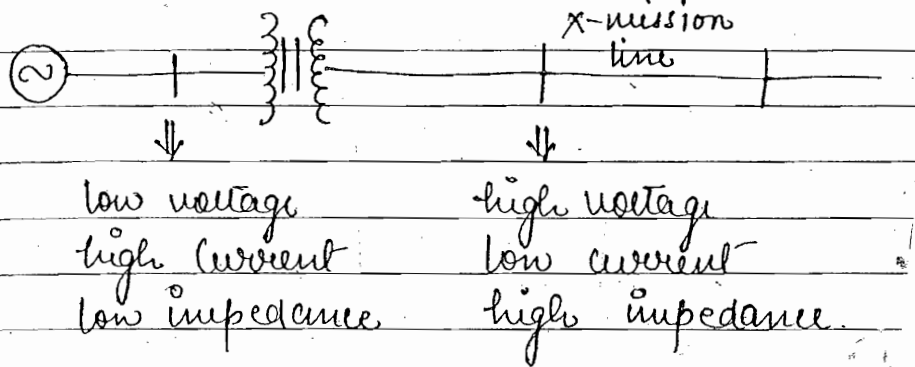


# Per Unit System

Date \_\_\_\_\_



→ The impedance will be same either H.V. & L.V. side by using per unit system.

→ X-men provide discontinuity as it has low voltage in one side and high voltage in second side. And two different voltage can't connect together.

*always give this answer*  
per unit → "unit less quantity"

(i) A per unit Method uses per unit values

(ii) A per unit value is a unit less quantity.

$$\text{p.u.} = \frac{\text{Actual value in some units}}{\text{Base or Reference value in the same units}}$$

$$\star \text{ p.u.} \times 100 = \% \text{ value}$$

ex:- actual voltage = 400 KV, base voltage = 100 KV

$$\text{Voltage in (p.u.)} = \frac{400}{100} = 4 \text{ p.u.}$$

$$1 \text{ p.u.} = 100 \text{ KV}$$

ex: -  $V_{actual} = 400 \text{ KV}$        $V_{base} = 400 \text{ KV}$

$$V_{p.u.} = \frac{400}{400} = 1 \text{ p.u.}$$

$$1 \text{ p.u.} = 400 \text{ KV}$$

ex: -  $V_{actual} = 400 \text{ KV}$

$$V_{base} = 0.0000001 \text{ p.u.}$$

$$V_{p.u.} = \frac{400}{0.1 \times 10^{-12}}$$

{ It's better to work with actual value }

### \* Advantages :-

- (i) It simplified power system calculation.
- (ii) It avoids the discontinuity problems posed by the presence of x-mers in power sys. network.

### • Selection of Base Values :-

- ↳ We have 4 quantity in pow sys. for which we select base values. → voltage, power, current, impedance.
- ↳ We only select base values for power and voltage. Base values for current and impedance we derived.
- ↳ For active and Reactive power KW and KVAR, only select KVA as base value. No, need to select KW<sub>base</sub> and KVAR<sub>base</sub> values.

## • Single Phase System :-

↳ Base power  $\begin{cases} \rightarrow KVA_b \\ \rightarrow MVA_b \end{cases}$  (may be in Kilo or Mega)

↳ Base voltage  $\rightarrow KV_b$  (always in Kilo volts).

↳ Base Current (in Amps)  $\begin{cases} \rightarrow \frac{KVA_b}{KV_b} \\ \rightarrow \frac{MVA_b}{KV_b} = \text{Kilo Amps.} \\ = \frac{MVA_b \times 1000}{KV_b} \end{cases}$

↳ Base Impedance (in  $\Omega$ )  $\begin{cases} \rightarrow \frac{(KV_b)^2}{MVA_b} \\ \rightarrow \frac{(KV_b)^2 \times 1000}{KVA_b} \end{cases}$

↳ voltage drop is always in phase

↳ We generally assume the star connect<sup>n</sup> fashion. Delta connect<sup>n</sup> produce circulating current, produce heating and harmonic that's why we don't use generally.

## • 3-phase System :-

↳ Base power  $\begin{cases} \rightarrow KVA_{b,3\phi} = 3 KVA_b \\ \rightarrow MVA_{b,3\phi} = 3 MVA_b \end{cases}$

↳ Base voltage  $\rightarrow K V_{b, line}$  (always line voltage)  
 $= \sqrt{3} K V_{b, phase}$

$$K V_b = \frac{K V_{b, line}}{\sqrt{3}}$$

$$\sqrt{3} K V_b I_b = K V A_{b 3\phi}$$

↳ Base Current  $I_b$  (in Amps)  $\rightarrow \frac{K V A_{b, 3\phi}}{\sqrt{3} \times K V_{b, line}}$   $\left\{ \begin{array}{l} \sqrt{3} V_L I_L = S_{3\phi} \\ I_L = \frac{S_{3\phi}}{\sqrt{3} V_L} \end{array} \right.$   
 $\rightarrow \frac{M V A_{b, 3\phi} \times 1000}{\sqrt{3} \times K V_{b, line}}$

↳ Base Impedance  $Z_b$  (in Ohms)  $\rightarrow \frac{(K V_{b, line})^2}{M V A_{b, 3\phi}}$  ( $Z_b$  in 3 $\phi$ )  
 $= \frac{(\sqrt{3} K V_b)^2}{3 \times M V A_b}$   
 $= \frac{(K V_b)^2}{M V A_b}$  (Impedance in per phase)  
 ( $Z_b$  in 1 $\phi$ )  
 $\rightarrow \frac{(K V_{b, line})^2 \times 1000}{K V A_{b, 3\phi}}$

$$Z_b \text{ in } 3\phi = Z_b \text{ in } 1\phi$$

General  $Z_b \text{ (1-}\phi\text{)} = \frac{(K V_b)^2}{M V A_b} = \frac{(K V_b)^2}{K V A_b} \times 1000$

$\frac{Z_b}{(3-\phi)}$  3  $\phi$   $K V_b \rightarrow$  line voltage  
 $K V A_b \rightarrow$  3 $\phi$  power

## • 3- $\phi$ System

↳ Single line diagram  $\rightarrow$  show balance condition

\_\_\_\_\_ R.

\_\_\_\_\_ Y

\_\_\_\_\_ B.

↳ Neutral of bus is called zero power bus bcoz it carry no current and power.

\_\_\_\_\_  $Z_n$   $\leftarrow$  N

↳ ZPB only valid for balance system.

$$I_n = I_R + I_Y + I_B = 0$$

$$V_n = I_n Z_n = 0$$

$$P_n = V_n I_n^* = 0$$

↳ Whenever we use p.u. values we are working with per phase value or 1- $\phi$  bases

↳ But final values are in actual form, Multiple p.u. with base values. (Base values should of 3- $\phi$  base values in 3 $\phi$  case)

• A single line diagram representation of a power system network indicated the original power system is a 3- $\phi$  and working under balance condition.

• The advantages of balance 3- $\phi$  sys. is, by working on 1- $\phi$  basis claim can be made for 3- $\phi$  analysis

• The neutral of balance 3- $\phi$  system is known as zero power bus. In unbalanced system

ZPB shifts to ground. bcoz potential of ground is zero.

- Whenever you change the base values, p.u. values will also get change.

$$Z_{pu(Old)} = \frac{Z_{actual}}{Z_{base(Old)}} = \frac{Z_{actual}}{(KV_b)_{old}^2 / MVA_{b,old}}$$

$$Z_{pu(New)} = \frac{Z_{actual}}{Z_{b(New)}} = \frac{Z_{actual}}{(KV_b)_{new}^2 / MVA_{b,new}}$$

$$Z_{pu(New)} = Z_{pu(Old)} \times \frac{Z_{actual}}{(KV_b)_{new}^2} \times \frac{MVA_{b,new} \times (KV_b)_{old}^2}{Z_{actual} \times MVA_{b,old}}$$

$$Z_{pu(New)} = Z_{pu(Old)} \times \frac{MVA_{b(New)} \times (KV_b)_{old}^2}{MVA_{b(Old)} \times (KV_b)_{new}^2}$$

→ How To Select the Base Values when X-mel is connected in Network: -

$S_b = 25 \text{ MVA}$      $V_b = 10 \text{ KV}$

②  $G_1$   
 10KV, 20MVA  $X_{G1} = j0.1 \Omega$   
 actual value

②  $G_2$   
 10KV, 15MVA  $X_{G2} = j0.2 \text{ p.u.}$   
 p.u. value  
 This p.u. value need to convert into Z new one.

②  $G_3$   
 10KV, 25MVA  $X_{G3} = 10\%$   
 $= 0.1 \text{ p.u. value}$

common base values are not same  
 need to change in 2 p.u. new value.

25MVA  
 $\left. \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} \right\} \parallel \left\{ \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} \right.$

$X_{T1} = j0.2 \text{ p.u.}$   
 10KV / 100KV

common base values are same No. need to change  $Z_{pu}$ .

Base value in L.V.  
side of x-mec ←  
25 MVA, 10 KV.

In G<sub>2</sub>

10 KV, 15 MVA are old values  
10 KV, 25 MVA are new values

$$Z_b = \frac{V_b^2}{S_b} = \frac{10^2}{25} = 4 \Omega$$

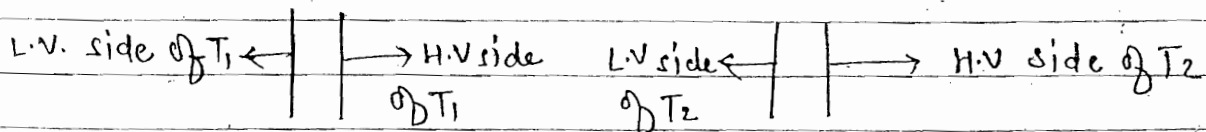
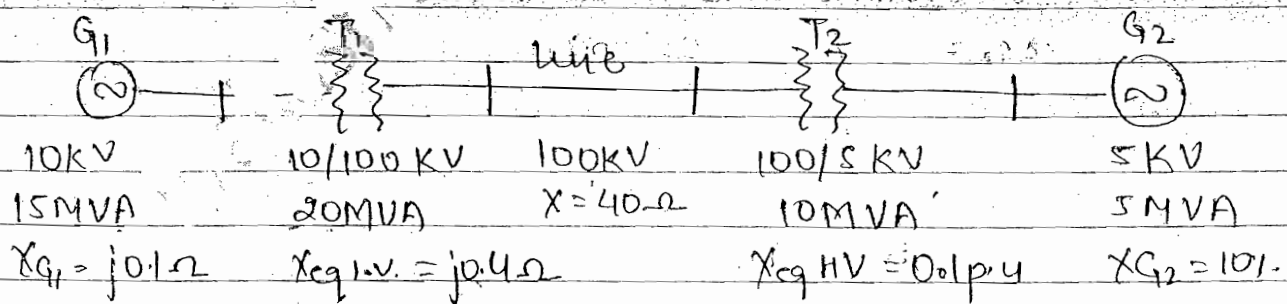
$$Z_{G1} = j \frac{0.1}{4} = j 0.025 \text{ p.u.}$$

$$Z_{G2} = j 0.2 \times \frac{25}{15} \times \left(\frac{10}{10}\right)^2 =$$

$$Z_{G3} = 0.1 \text{ p.u.}$$

- Transmission lines have always high voltage
- x-mec of x-mission line sides never connected to L.V.

• Selection of Base Values when the T/F is present in N/W



$$MVA_b = 15 \quad MVA_b = 15 \quad MVA_b = 15 \quad \therefore MVA_b = 15$$

$$KV_b = 10KV \quad KV_b = 100KV \quad KV_b = 100KV \quad KV_b = 5KV$$

$$Z_b = \frac{10^2}{15} \quad Z_b = \frac{100^2}{15} \quad Z_b = \frac{100^2}{15} \quad Z_b = \frac{5^2}{15}$$

$$= 6.66 \Omega$$

$$= 666.67 \Omega$$

$$= 666.67 \Omega$$

$$= 1.66 \Omega$$



→ In the network containing x-mers base values must be selected according to some rule:-

- (i) Two sets of base values must be selected for either side of x-mer
- (ii) A common base power is selected for either side of x-mer as well as for the entire N/w.
- (iii) Two different voltages are selected at the base voltages at for L.V. and H.V. side of x-mer in such a way their ratio must be equal to transformation ratio ( $K$ ) of original x-mer.

Solution  $G_1$  (on L.V. side of  $T_1$ )

$$X_{G1} = j0.1 \Omega$$

$$X_b = 6.66 \Omega$$

$$X_{pu} = \frac{0.1}{6.66} = 0.015 \text{ p.u.}$$

$G_2$  (on L.V. side of  $T_2$ )

$$X_{G2} = 10\% = 0.1 \text{ p.u.}$$

$$= ?$$

on 5 KV, 5 MVA.

on 5 KV, 15 MVA

new base values.

$$X_{G2} = 0.1 \times \frac{15}{5} \times \left(\frac{5}{5}\right)^2$$

$$= j0.3 \text{ p.u.}$$

⇒ on HV side impedance will ↑ so multiple with transformation ratio square  $K = 10$   $\frac{H.V}{L.V} = \frac{100}{10} = 10$

$$T_1 \quad X_{eq \text{ L.V}} = j0.4 \Omega$$

$$X_{eq \text{ H.V}} = K^2 \times j0.4$$

$$= 10^2 \times 0.4$$

$$= 40 \Omega$$

Impedance will be same  
in either H.V. and L.V. side

$$X_{T1} = \frac{j 0.4}{6.66 \Omega} = j 0.06 \text{ p.u.} \quad (\text{L.V.})$$

or

$$= \frac{j 40 \Omega}{666.67 \Omega} = j 0.06 \text{ p.u.} \quad (\text{H.V.})$$

$$T_2 \quad X_{T2} = j 0.1 \text{ p.u. on } 100 \text{ KV, } 10 \text{ MVA}$$

$$I = ? \quad \text{on } 100 \text{ KV, } 15 \text{ MVA}$$

$$X_{T2} = 0.1 \times \frac{15}{10} \times \left( \frac{100}{100} \right)^2$$

$$= j 0.15 \text{ p.u.}$$

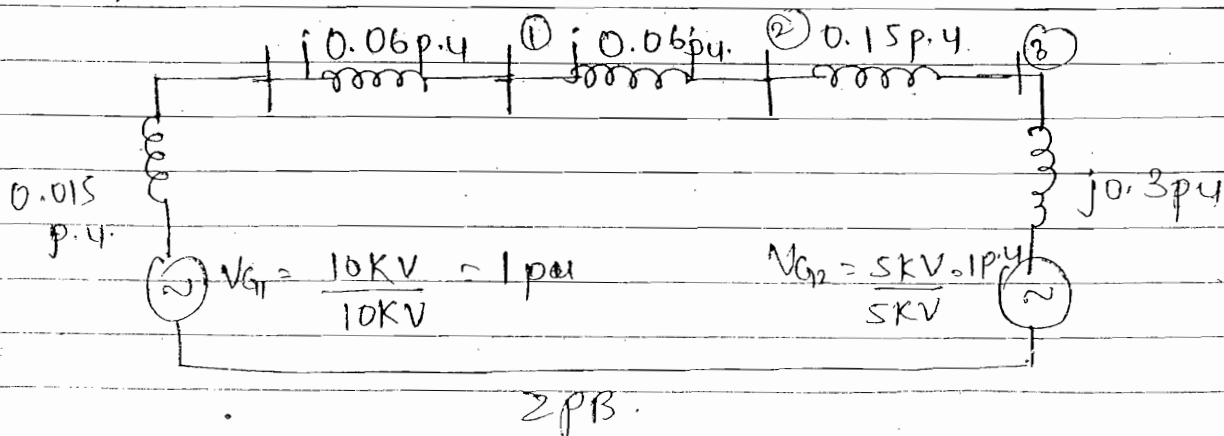
Line on H.V. side of  $T_1$  &  $T_2$

$$X_{line} = 40 \Omega$$

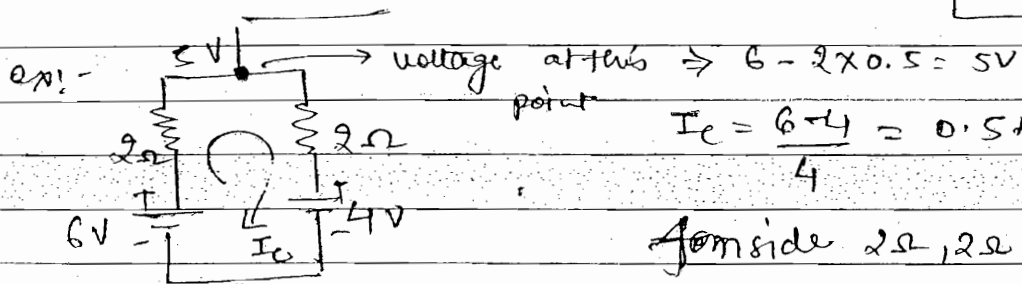
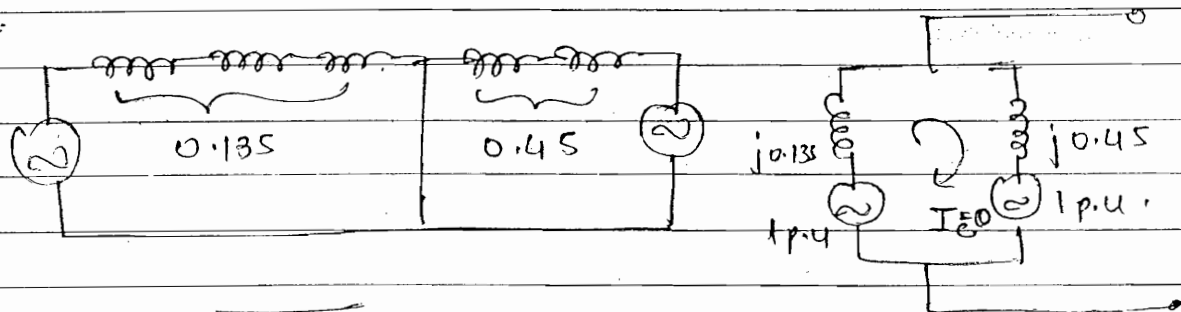
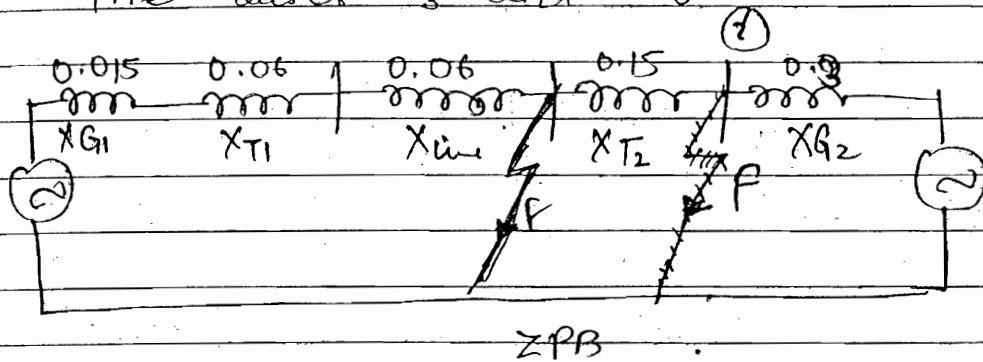
$$X_{line} = \frac{j 40 \Omega}{666.67 \Omega}$$

$$= j 0.06 \text{ p.u.} \quad \text{Ans}$$

• Per Unit Equivalent Diagram:-



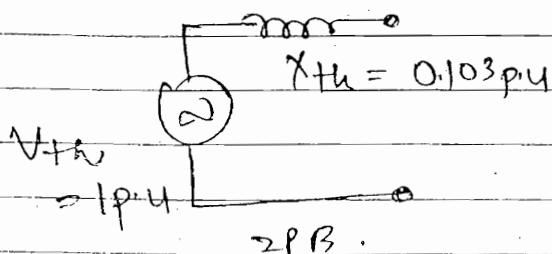
- If a fault occur at bus 3, reduce the network into their equivalent network across the buses 3 and 2PB



$$I_e = \frac{6-4}{4} = 0.5 \text{ A}$$

For side  $2\Omega, 2\Omega$  are in series

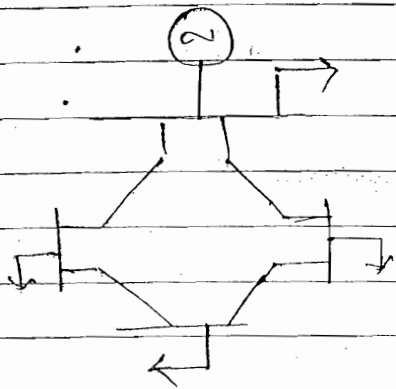
$$X_{th} = j0.135 \parallel j0.45 = j0.103$$



# Network Matrices

Date 8<sup>th</sup> June 2014

- ↳ leaving out (not connected the element to pow system) generator, the rest of the pow. sys. is purely passive
- ↳ for conducting various studies on p.s., we require properties of the N/w
- ↳ N/w Matrices provides properties of passive pow. sys. N/w.



↳ By removing generator from N/w whole p.s. N/w become passive.

fig. passive p.s. N/w.

→ N/w matrices can be found dependency upon the frame of reference.

frame of Reference

- Bus frame
- loop frame
- Branch frame

Based on Bus frame - the N/w eq<sup>n</sup> is -

$$I_{bus} = Y_{bus} V_{bus} \quad \text{or} \quad V_{bus} = Z_{bus} I_{bus}$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

↳ Network Matrices based on the bus frame of reference are popular and widely use over the others. Bcoz bcoz for the changes in the network matrices can be modified easily, can be formulated easily and they are programmable.

• Based on Bus frame of Reference.

| Bus admittance Matrix<br>( $Y_{BUS}$ )               |                                                | Bus Impedance Matrix<br>( $Z_{BUS}$ )                                                                                            |                                          |
|------------------------------------------------------|------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------|------------------------------------------|
| direct inspection<br>Method                          | Singular<br>Transform <sup>n</sup><br>Method   | Inverse<br>Method                                                                                                                | using $Z_{BUS}$<br>building<br>Algorithm |
| (used when element<br>not having mutual<br>coupling) | (used element having<br>mutual coupling)       | $Z_{BUS} = Y_{BUS}^{-1}$<br>(never used bcoz<br>size of $M \times M$ is very<br>large the no. of<br>buses are large<br>in $m$ .) |                                          |
| • easy and convenient                                | • uses graph theory<br>• $Y_{BUS} = A^T [y] A$ |                                                                                                                                  |                                          |

- for the  $n$  bus,  $Z_{BUS}$  already exist.
- Modification is easy compare to forming the  $Z_{BUS}$  bcoz modification required on only few row or column.

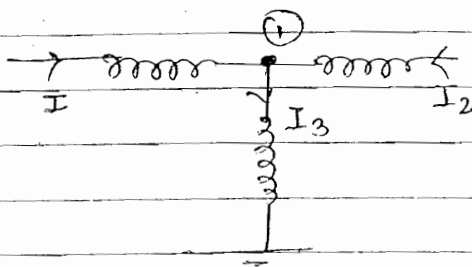
• Network Topologies: - OR Graph theory: -

- ↳ geometrical representation of n<sup>th</sup> the objects. Made for Robotics.
- ↳ few elements of passive and active element → circuit
- ↳ large no. of passive and active element → Network

→ Network analysis means finding current through and voltage across every branch of a network.

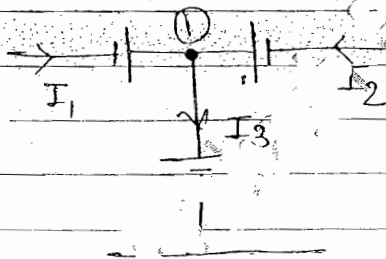
→ No Network analysis can be carried by using either loop analysis or nodal analysis.

→ The bases of loop analysis is KVL and for nodal analysis is KCL.



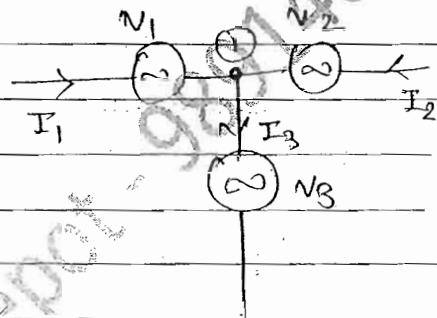
$$I_3 = I_1 + I_2$$

purely passive AC N/w.



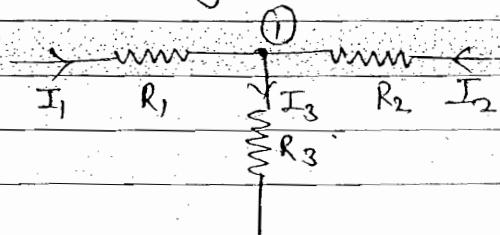
$$I_3 = I_1 + I_2$$

purely active dc N/w



$$I_3 = I_1 + I_2$$

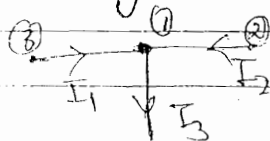
purely active ac N/w.



$$I_3 = I_1 + I_2$$

purely passive dc N/w.

- for the formation of network equation we don't require element we require structure of network.
- Different Network having same structure have same N/w equation.



$$I_3 = I_1 + I_2$$

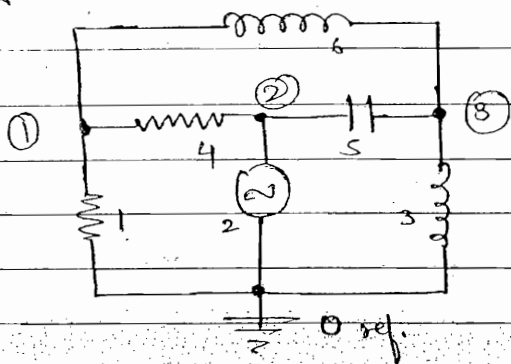
graph.

→ The KCL and KVL do not depend on the nature of the elements but depends upon the structure of network. (graph)

| elements \ Nodes | (1)           | (2)                | (3)           |
|------------------|---------------|--------------------|---------------|
| A (I)            | -1<br>coming. | 0<br>not connected | +1<br>leaving |

→ graph theory study for Network analysis not for N/w circuit.

★



No. of nodes = 4

No. of elements = 6

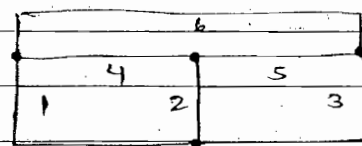


fig. Graph of the N/w.

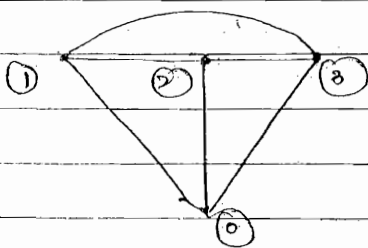


fig. Graph of the N/w shape is different

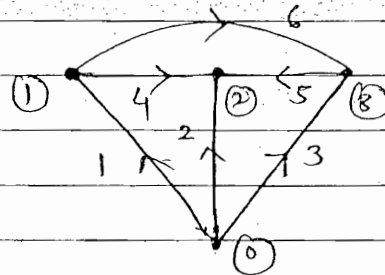


fig. oriented (well connected) graph (linear or planar graph)

→ If the lines connecting another graph will become non-linear or non-planar graph

→ If we have one node not connected to graph it is called not connected graph



- (2) sub graph.

↳ sub graph is also well connected graph but not having any close loop. called tree of the graph.

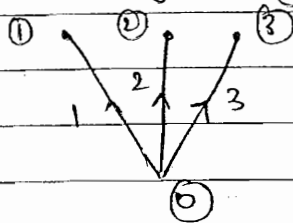
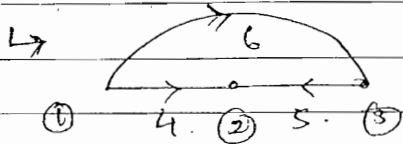
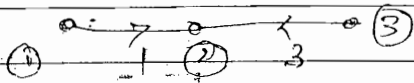


fig. Tree of the graph.

elements 1, 2, & 3 called twigs

$$t = n - 1$$

$n = \text{No. of Nodes.}$



$$\text{Tree} + \text{Co-tree} = \text{Main graph}$$

co-tree (not present in tree graph)

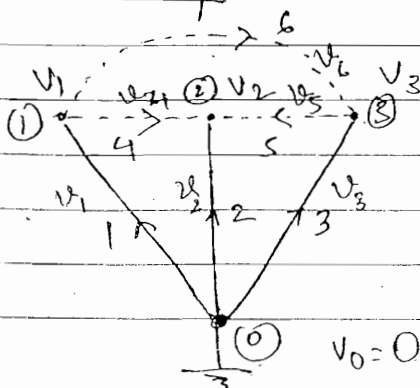
- 4, 5, 6 are links or chords

$$l = e - t = e - (n - 1)$$

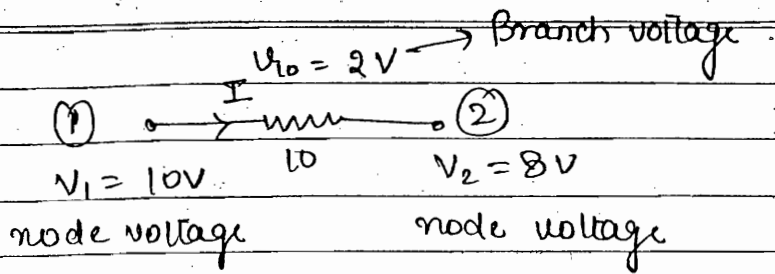
$$l = e - n + 1$$

NOTE (1) When twig element is added it creates a new node

(2) When link element is added it creates a new loop



$V_0, V_1, V_2, V_3 \rightarrow \text{Node voltages}$

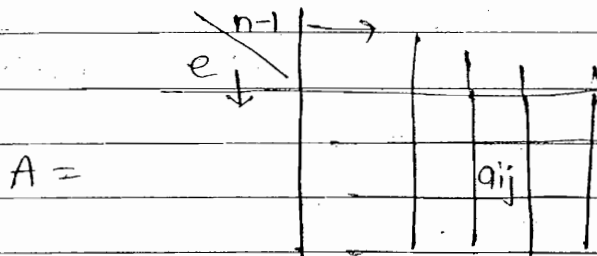


$$I = \frac{10-8}{2} = 1 \text{ amp.}$$

$$V_{10} = 2 \times 1 = 2V.$$

$$\begin{aligned} V_1 &= V_0 - V_1 = -V_1 \\ V_2 &= V_0 - V_2 = -V_2 \\ V_3 &= V_0 - V_3 = -V_3 \\ V_4 &= V_2 - V_1 \\ V_5 &= V_3 - V_2 \\ V_6 &= V_1 - V_3 \end{aligned} \quad \left. \begin{array}{l} \therefore V_0 = 0 \\ \text{Network equations} \end{array} \right\}$$

• Element - Node Incidence Matrix  $[A]$



$i = \text{element}$   
 $j = \text{node}$

$$e \times (n-1)$$

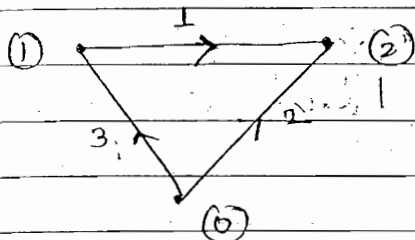
$a_{ij} = \begin{cases} +1 & \text{if } i^{\text{th}} \text{ element is incident to } j^{\text{th}} \text{ node and oriented away} \\ -1 & \text{if element is towards } j^{\text{th}} \text{ node} \\ 0 & \text{if is not incident to } j^{\text{th}} \text{ node} \end{cases}$

|       | $e \backslash n-1$ | (1) | (2) | (3) |
|-------|--------------------|-----|-----|-----|
| $A =$ | 1                  | -1  | 0   | 0   |
|       | 2                  | 0   | -1  | 0   |
|       | 3                  | 0   | 0   | -1  |
|       | 4                  | +1  | -1  | 0   |
|       | 5                  | 0   | -1  | +1  |
|       | 6                  | +1  | 0   | -1  |

Q

A =

|   | ①  | ②  |
|---|----|----|
| 1 | 1  | 4  |
| 2 | 0  | -1 |
| 3 | -1 | 0  |



By relation of node voltage matrix and A we get branch voltage Matrix

$$V = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix}$$

Branch voltage matrix

$$V = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

node voltage matrix

$$V = A[V]$$

## • $Y_{bus}$ Formation

⑨ Direct Inspection Method  
for a 3-Bus p.s N/w.

$$Y_{Bus} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix}$$

↳ Diagonal element  $\rightarrow Y_{ii}$

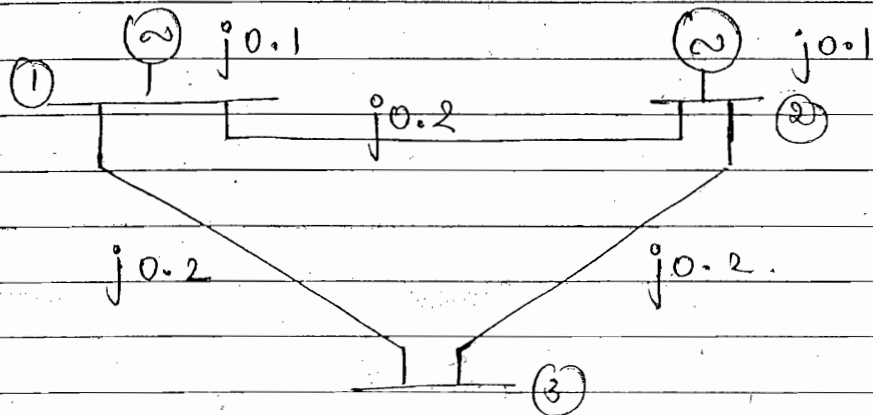
= Total admittance connected To  $i^{th}$  Bus  
for  $i=1, 2, \dots, n$

↳ Off-diagonal elements  $\rightarrow Y_{ik}$

= Negative value of series admittance connected b/w the buses ① & ②

NOTE:- If Buses (i) & (k) are not connected  
then  $Y_{ik} = 0$

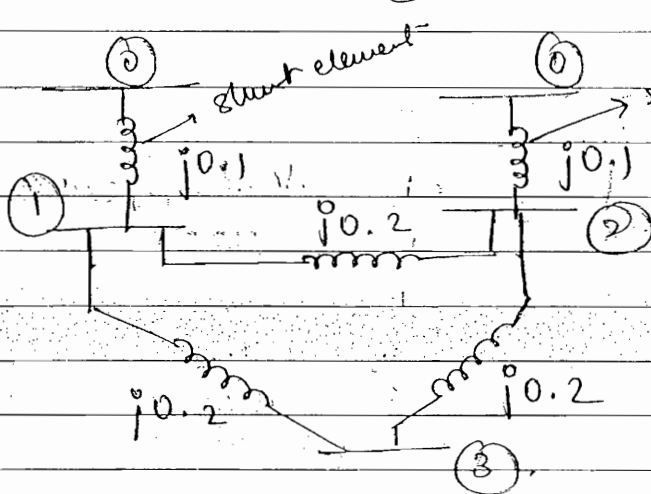
Problem page No. 48.



$$Y_{22} = ?$$

$$\text{Ans} = -j20$$

Solution



$$= \frac{1}{j0.1} = -j10$$

$$= \frac{1}{j0.2} = -j5$$

$$Y_{Bus} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix}$$

$$Y_{11} = -j10 - j5 - j5 = -j20$$

$$Y_{22} = -j10 - j5 - j5 = -j20$$

$$Y_{33} = -j5 - j5 = -j10$$

$$Y_{12} = Y_{21} = -(-j5) = j5$$

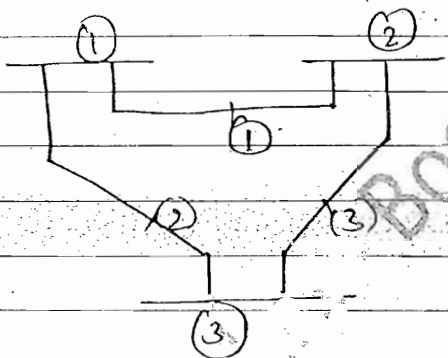
$$Y_{23} = Y_{32} = -(-j5) = j5$$

$$Y_{31} = Y_{13} = -(-j5) = j5$$

$$Y_{BUS} = \begin{bmatrix} -j20 & j5 & j5 \\ j5 & -j20 & j5 \\ j5 & j5 & -j10 \end{bmatrix}$$

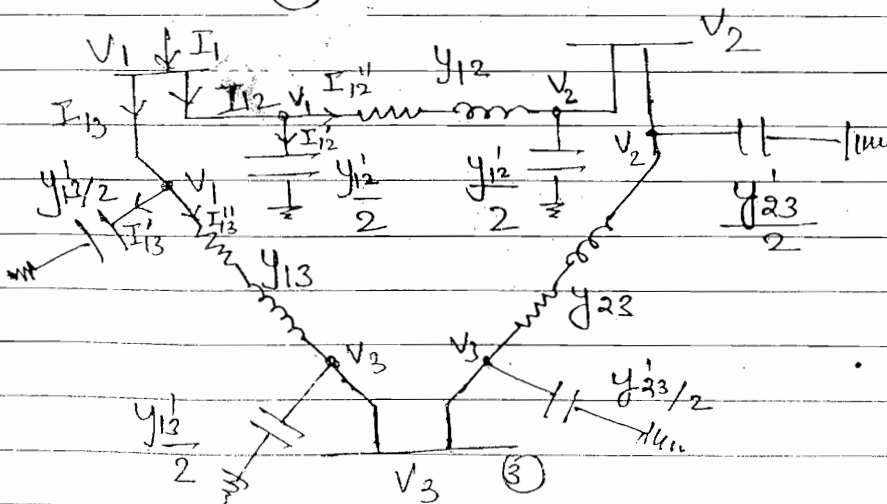
- shunt element present in diagonal element not in off-diagonal element
- If sum of all elements in each row of  $Y_{BUS}$  matrix is zero then the corresponding Bus is not having shunt elements. If it is not zero corresponding bus having shunt element

Problem: page no. 48 Q. 13 (c)



$Z_{ik}$  = series impedance of the line

$\frac{Y'_{ik}}{2}$  = half line charging admittance of the line



$Y_{ik}$  = Series admittance of the line

$$= \frac{1}{Z_{ik}} = \frac{R_{ik}}{R_{ik}^2 + X_{ik}^2} - j \frac{X_{ik}}{R_{ik}^2 + X_{ik}^2}$$

$$\text{ex: } Y_{12} = \frac{R_{12}}{R_{12}^2 + X_{12}^2} - j \frac{X_{12}}{R_{12}^2 + X_{12}^2}$$

$I_1$  = Current injected into 1<sup>st</sup> Bus

$$= I_{12} + I_{13}$$

$$= (I_{12}' + I_{12}'') + (I_{13}' + I_{13}'')$$

$$= \left( (V_1 - 0) \frac{Y_{12}'}{2} + (V_1 - V_2) Y_{12} \right) + \left( (V_1 - 0) \frac{Y_{13}'}{2} + (V_1 - V_3) Y_{13} \right)$$

$$= \left( \frac{Y_{12}'}{2} + \frac{Y_{13}'}{2} + Y_{12} + Y_{13} \right) V_1 + (-Y_{12}) V_2 + (-Y_{13}) V_3$$

$$\boxed{I_1 = Y_{11} V_1 + Y_{12} V_2 + Y_{13} V_3}$$

$Y_{11}$  = Total admittance connected to

$$= \frac{Y_{12}'}{2} + \frac{Y_{13}'}{2} + Y_{12} + Y_{13}$$

$Y_{12}$  = Negative Value of Series admittance connected to/w buses (1) & (2)

$$= -Y_{12}$$

$$Y_{13} = -Y_{13}$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 + Y_{23} V_3$$

$$Y_{21} = -Y_{12} = Y_{12}$$

$$Y_{23} = -Y_{23} = Y_{32}$$

$$Y_{22} = \frac{Y_{12}'}{2} + \frac{Y_{23}'}{2} + Y_{12} + Y_{23}$$

→ susceptance → means no real part.  $R=0$ .

$\{y_{12} \rightarrow \text{shunt susceptance}\}$

Problem 15 pag no. 48

$$Y = \begin{bmatrix} -13 & 10 & 5 \\ 10 & -18 & 10 \\ 5 & 10 & -13 \end{bmatrix} \quad y_{12} = ?$$

$$Y_{11} = \frac{y_{12}}{2} + \frac{y_{13}}{2} + y_{12} + y_{13}$$

$$-13 = y_{12}' - y_{12} - y_{13} = y_{12}' - 10 = 5 = y_{12}' - 15$$

$$\frac{y_{12}'}{2} = 15 - 13$$

$$\boxed{y_{12}' = 2} \quad \text{Ans. ✓}$$

Solution  $\left( \frac{y_{12}'}{2} + \frac{y_{13}}{2} + y_{12} + y_{13} \right) - y_{12} - y_{13} = 2$

$$\frac{y_{12}'}{2} + \frac{y_{13}}{2} = 2 \quad (1)$$

$$-y_{12}' + \frac{y_{13}}{2} + \frac{y_{12}'}{2} + y_{12} + y_{13} - y_{13} = 2$$

$$\frac{y_{13}}{2} + \frac{y_{12}'}{2} = 2 \quad (2)$$

$$-y_{13} - y_{12}' + \frac{y_{13}}{2} + \frac{y_{12}'}{2} + y_{13} + y_{12} = 2$$

$$\frac{y_{13}}{2} + \frac{y_{12}'}{2} = 2 \quad (3)$$

eqn  $\frac{y_{13}}{2} = 2 - \frac{y_{12}'}{2}$

eqn (8)

$$\frac{y_{13}}{2} + 2 - \frac{y_{12}}{2} = 2$$

$$\frac{y_{13}}{2} - \frac{y_{12}}{2} = 0 \quad \text{--- (4)}$$

eqn (1) &amp; (4)

$$\frac{y_{12}}{2} + \frac{y_{13}}{2} = 2$$

$$-\frac{y_{12}}{2} + \frac{y_{13}}{2} = 0$$

$$\frac{y_{13}}{2} = 2$$

$$y_{13} = 2$$

$$y_{12} = y_{13} = 2 \text{ A}$$

Problem 2. p.No. 46

Problem 4 p.No. 46.

$$\Rightarrow \frac{1}{j0.1} = -j10$$

$$\Rightarrow \frac{1}{j0.2} = -j5 \quad \Rightarrow \frac{1}{-j0.08} = j12.5$$

$$\Rightarrow \frac{1}{-j20} = +j0.05$$

$$\Rightarrow \frac{1}{j0.1} = -j10$$

$$Y_{22} = -j10 - j10 + j0.05$$

$$Y_{11} = -j10 - j5 = -j15$$

$$= -j19.95 \text{ A}$$

Ans. (b)

Problem 3.  $Y = Z^{-1}$ 

$$|Z| = 0.5$$

$$Y_{22} = \frac{0.9}{0.5} = 1.8 \text{ A}$$

(d)

### • Properties of $Y_{bus}$ :-

- (i) It is square Matrix.
- (ii) It is a symmetric Matrix:  $Y_{ik} = Y_{ki}$   $Y_{bus} = Y_{bus}^T$
- (iii) It is a sparsity Matrix  
(More no. of elements are zero). sparsity  $\approx 100\%$ .

Advantage of sparsity  $\rightarrow$   $\uparrow$  speed of C.P.U., less space of memory required.

$\rightarrow$  We prefer  $Y_{bus}$ ,  $Z_{bus}$  is totally opposite to  $Y_{bus}$ , less no. of elements.

$\rightarrow$   $Z_{bus}$  Matrix use in short-circuit NW. becoz it will give more info. about non-zero element.

- If a Matrix consist of more no. of elements as zero it is said to be sparsity Matrix.  
ex:- Null Matrix has 100% sparsity degree

The advantage of sparsity are

- (i) Reduce the memory requirement
- (ii)  $\uparrow$  C.P.U speed
- (iii) less execution time

NOTE:-  $\checkmark$  As the load flow studies have to be performed under on line, the sparsity techniques are quite useful.

$\checkmark$  As the  $Y_{bus}$  is sparsity Matrix its inverse,  $Z_{bus} = Y_{bus}^{-1}$ , is in full Matrix

$\checkmark$  With more no. of non-zero elements  $Z_{bus}$  provides properties of the NW more efficiently. Therefore short-ckt studies uses  $Z_{bus}$ .

Problem no. 9 Page No. 47 Am (B).

Problem no. 6.  $\rightarrow$  diagonal element  $\neq 0$ . (new)

$$Y_{Bus} = \begin{bmatrix} \text{diagonal elem} \\ = 100 \end{bmatrix}_{100 \times 100}$$

Total no. of elements in  
 $Y_{Bus} = 10,000$

Total no. of non-zero elements = 10% of 10,000  
 $= 1000$

$$Y_{Bus} = \begin{bmatrix} & & 450 \\ & Y_{12} \neq 0 & \\ Y_{21} & & \\ 450 & & \end{bmatrix}$$

diagonal element  $\neq 0$

- The no. of non-zero off-diagonal element present in either in upper triangle or lower triangle of  $Y_{Bus}$  gives no. of transmission lines

Q. 50% sparse.

Total no. of non-zero = 50% of 10,000  
 $= 5000$

diagonal elements  $> 5000 - 100$   
 $= 4990$

No. of elements in upper  $\Delta = \frac{4990}{2} = 2495$

- Upper  $\Delta$  and lower  $\Delta$  sum gives Total no. of transmission lines.

# • Singular Transformation Method :-

According to this Method.

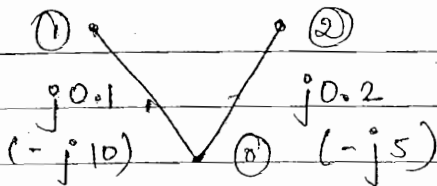
$$[Y_{Bus}] = [A^T] [Y] [A]$$

A: Element Node Incidence Matrix

$$[Y] = [Z]^{-1} = \text{primitive admittance Matrix}$$

= Z: primitive impedance Matrix.

ex: -



using Direct Inverse Method

$$Y_{Bus} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$

$$= \begin{bmatrix} -j10 & 0 \\ 0 & -j5 \end{bmatrix}$$

using Singular T-matrix Method

| e \ n | (1) | (2) |
|-------|-----|-----|
| 1     | -1  | 0   |
| 2     | 0   | -1  |

$$A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \quad A^T = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

Z = primitive Impedance Matrix

| e \ e | 1    | 2    |
|-------|------|------|
| 1     | j0.1 | 0    |
| 2     | 0    | j0.2 |

exe

$$Y = Z^{-1} = \begin{bmatrix} -j10 & 0 \\ 0 & -j5 \end{bmatrix}$$

According to Singular TX-M Method

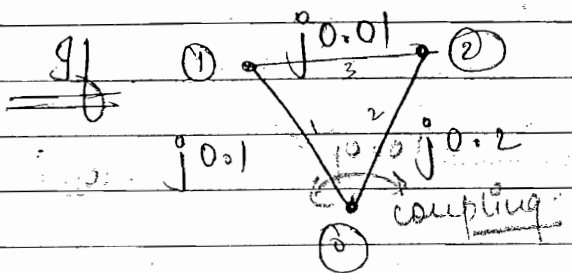
Date \_\_\_\_\_

Problem,

$$Y_{bus} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -j10 & 0 \\ 0 & -j5 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} j10 & 0 \\ 0 & j5 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -j10 & 0 \\ 0 & -j5 \end{bmatrix}$$



$$Y_{bus} = \begin{bmatrix} -j110 & j100 \\ j100 & -j105 \end{bmatrix}$$

$$A = \begin{array}{c|cc} e \backslash n-1 & (1) & (2) \\ \hline 1 & -1 & 0 \\ 2 & 0 & -1 \\ 3 & +1 & -1 \end{array}$$

$$A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ 0 & -1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & -1 \end{bmatrix}$$

$$Z = \begin{array}{c|cc} e \backslash e & (1) & (2) & \text{coupling} \\ \hline 1 & j110 & j100 & 0 \\ 2 & j100 & j105 & \\ 3 & & & \end{array}$$

$$Z = \begin{bmatrix} j0.1 & j0.01 \\ j0.01 & j0.2 \end{bmatrix}$$

$$|Z| = 0.0199 \quad Z^{-1} = \begin{bmatrix} -10j & +j.5 \\ +j.5 & -j5 \end{bmatrix}$$

$$Y_{bus} = A^T [y] A$$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} -10j & j5 \\ j5 & -j5 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 10j + 0 & -5j \\ -5j & +5j \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -10j & +5j \\ +5j & -5j \end{bmatrix}$$

Date 9/6/14

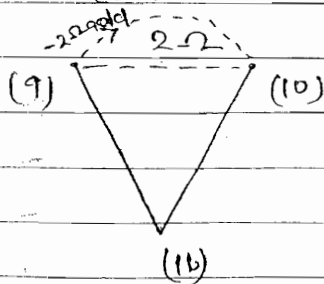
## • Zbus Building Algorithm:-

- Zbus formation by using inverse method has following disadvantages:-

- (i) Owing to the size of Ybus which is quite large finding the inverse of such a big Matrix is always difficult.
- (ii) In pow. sys. network changes regularly take place. The changes like - element addition, element deletion, and element impedance value change take places. Whenever such change take place, the network is modified. Everytime when the modification takes place we need to have a modified Zbus. Forming the Zbus from the beginning through Ybus is always difficult and time consuming.

✓ Zbus building algorithm is the procedure for updating existing Zbus for the network change take place. The time consuming and calculation involved is less in updating Zbus rather than forming from beginning.

### • Element deletion Case:-



→ The  $2\Omega$  resistance across the buses (9) & (10) has to be deleted.

procedure:- add  $-2\Omega$  across the buses (9) & (10)

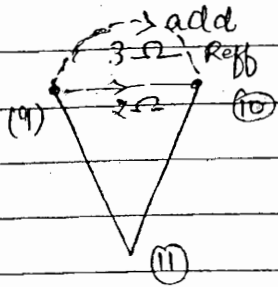
$$R_{eq} = \frac{2 \times (-2)}{2 - 2} = \infty$$

(open ct)

change

Date \_\_\_\_\_

- Element Impedance value takes place. Case:-



→ Impedance value of element changes from  $2\Omega$  to  $3\Omega$

Procedure:- add a resistance equal to  $R_{eff}$  ( $R_{effective}$ )

$$R_{eff} \parallel 2\Omega = 3\Omega$$

$$\frac{2 \cdot R_{eff}}{2 + R_{eff}} = 3\Omega$$

$$\Rightarrow 2R_{eff} = 6 + 3R_{eff}$$

$$\Rightarrow R_{eff} = -6\Omega$$

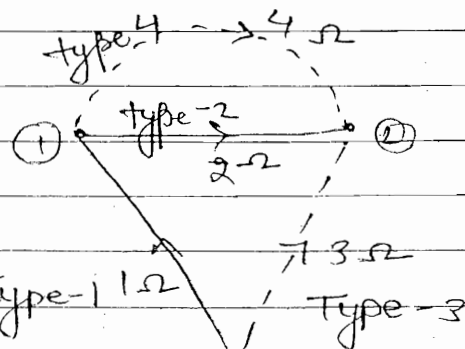
→ parallel addition means adding a link. series addition have difficulty of calculation.

type 1 Modification - twig is added b/w reference and new node

type 2 Modification - twig is added b/w old bus other than ref and new bus

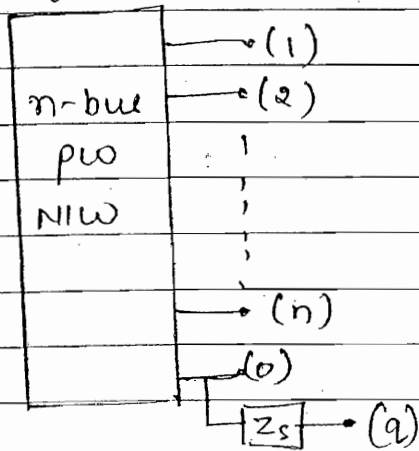
type 3 Modification - link is

added b/w ref and bus node



type 4 Modification:- line is added b/w two bus other than ref.

# • Type - 1 Modification.



$$Z_{bus, old} = \begin{bmatrix} Z_{11} & Z_{12} & \dots & Z_{1n} \\ Z_{21} & Z_{22} & \dots & Z_{2n} \\ \vdots & \vdots & & \vdots \\ Z_{n1} & Z_{n2} & \dots & Z_{nn} \end{bmatrix}_{n \times n}$$

① An element with reference impedance  $Z_s$  is added b/w the reference bus (0) and a new bus (q).

$$Z_{bus, new} = \begin{bmatrix} Z_{bus, old} & \begin{matrix} Z_{1q} \\ Z_{2q} \\ \vdots \\ Z_{nq} \end{matrix} \\ \begin{matrix} Z_{q1} & Z_{q2} & \dots & Z_{qn} \end{matrix} & Z_{qq} \end{bmatrix}_{(n+1) \times (n+1)}$$

↖  $q^{th}$  column

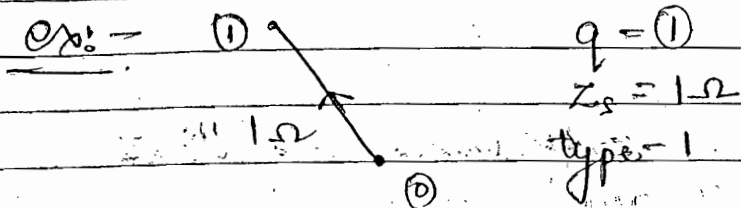
$q^{th}$  row.

procedure :- (1) Set all the off diagonal elements of  $q^{th}$  row /  $q^{th}$  column to zero.

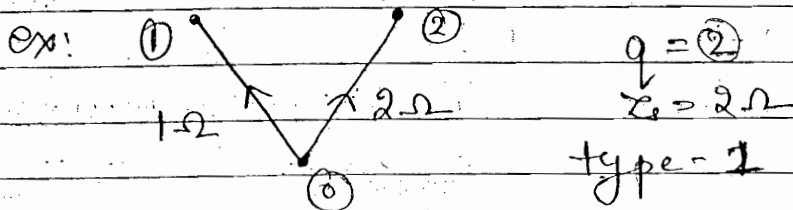
$$Z_{iq} = Z_{qi} = 0 \quad i = 1, 2, \dots, n \quad i \neq q$$

(2) Set the diagonal element  $Z_{qq} = Z_s$

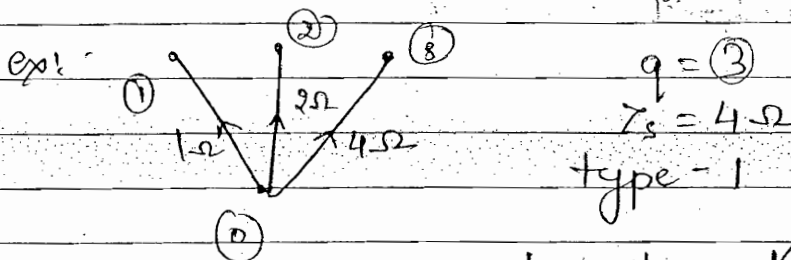
$$Z_{bus, new} = \begin{bmatrix} Z_{bus, old} & \begin{matrix} 0 \\ 0 \\ \vdots \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 & \dots & 0 \end{matrix} & Z_s \end{bmatrix}_{(n+1) \times (n+1)}$$



$$Z_{BUS} = \begin{matrix} & ① \\ ① & 1\Omega \end{matrix} \quad Z_{qq} = Z_s = 1\Omega$$



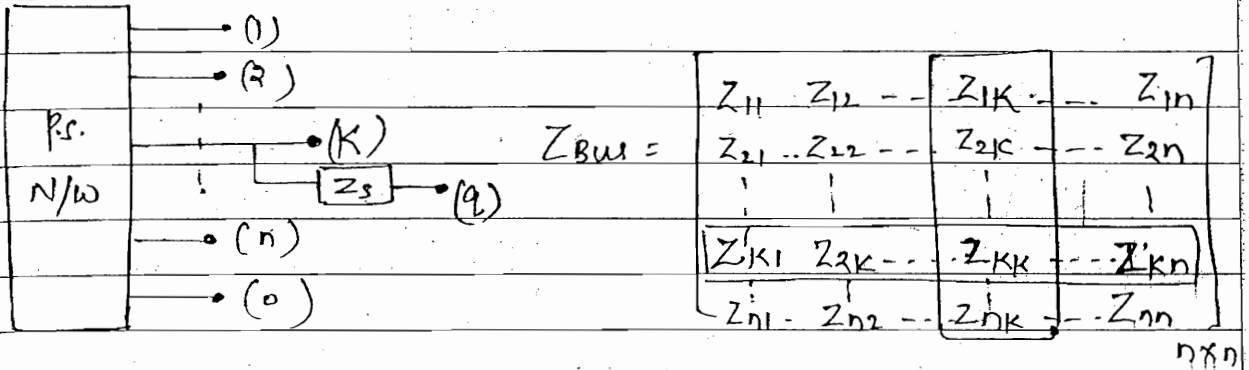
$$Z_{BUS} = \begin{matrix} & ① & ② \\ ① & 1\Omega & 0 \\ ② & 0 & 2\Omega \end{matrix}_{2 \times 2} \quad Z_{qq} = Z_{22} = 2\Omega$$



$$Z_{BUS} = \begin{matrix} & ① & ② & ③ \\ ① & 1 & 0 & 0 \\ ② & 0 & 2 & 0 \\ ③ & 0 & 0 & 4 \end{matrix}_{3 \times 3} \quad \begin{matrix} \nearrow \\ Z_{BUS, old} \end{matrix}$$

## • Type 2 Modification

- An element with self impedance  $Z_s$  is added b/w the already existing bus (K) and a new bus (q)



$$Z_{BUS, new} = \begin{bmatrix} Z_{11} & Z_{12} & \dots & Z_{1K} & \dots & Z_{1n} \\ Z_{21} & Z_{22} & \dots & Z_{2K} & \dots & Z_{2n} \\ \vdots & \vdots & & \vdots & & \vdots \\ Z_{K1} & Z_{K2} & \dots & Z_{KK} & \dots & Z_{Kn} \\ \vdots & \vdots & & \vdots & & \vdots \\ Z_{n1} & Z_{n2} & \dots & Z_{nK} & \dots & Z_{nn} \\ Z_{q1} & Z_{q2} & \dots & Z_{qK} & \dots & Z_{qn} \\ Z_{q1} & Z_{q2} & \dots & Z_{qK} & \dots & Z_{qn} \end{bmatrix} \quad (n+1) \times (n+1)$$

Procedure (1) Copy  $K^{th}$  row and paste it as  $q^{th}$  row

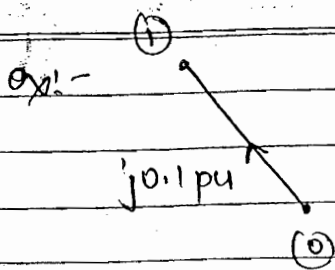
(2) Copy  $K^{th}$  column and paste it as  $q^{th}$  column

(3)  $Z_{qq} = Z_{KK} + Z_s$

$$Z_{BUS, new} = \begin{bmatrix} Z_{11} & Z_{12} & \dots & Z_{1K} & \dots & Z_{1n} & Z_{1K} \\ Z_{21} & Z_{22} & \dots & Z_{2K} & \dots & Z_{2n} & Z_{2K} \\ \vdots & \vdots & & \vdots & & \vdots & \vdots \\ Z_{K1} & Z_{K2} & \dots & Z_{KK} & \dots & Z_{Kn} & Z_{KK} \\ \vdots & \vdots & & \vdots & & \vdots & \vdots \\ Z_{n1} & Z_{n2} & \dots & Z_{nK} & \dots & Z_{nn} & Z_{nK} \\ Z_{K1} & Z_{K2} & \dots & Z_{KK} & \dots & Z_{Kn} & Z_{KK} + Z_s \end{bmatrix} \quad (n+1) \times (n+1)$$

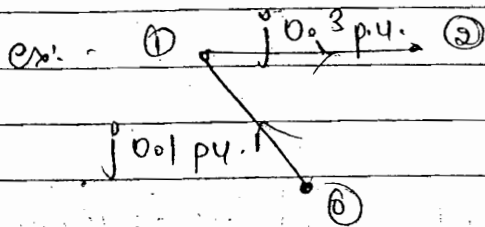
K → Old Bus  
q → new Bus.

Date \_\_\_\_\_



q = 1  
 $Z_s = j0.1 \text{ pu}$   
 Type - 1

$Z_{bus} = \begin{bmatrix} (1) & \\ & (2) \end{bmatrix} \begin{bmatrix} j0.1 \\ j0.1 \end{bmatrix}$



K = 1

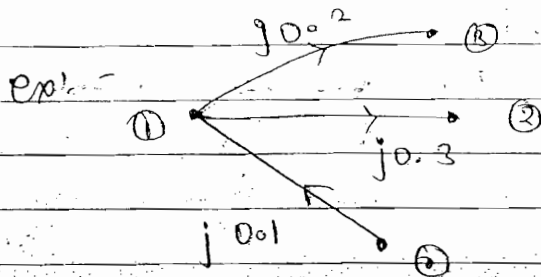
q = 2

$Z_s = j0.3$

type-2

$Z_{bus} = \begin{bmatrix} (1) & (2) \\ (1) & j0.1 & j0.1 \\ (2) & j0.1 & j0.4 \end{bmatrix}$

$Z_{22} = Z_{11} + Z_s$   
 $= j0.1 + j0.3$   
 $= j0.4$



K = 1

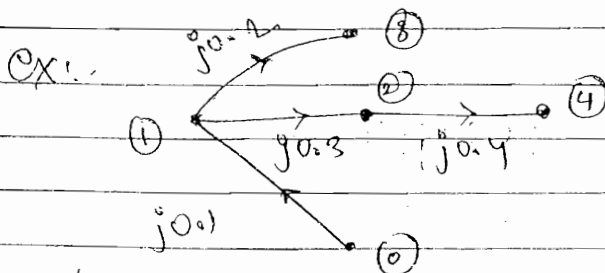
q = 3

$Z_s = j0.2$

type

$Z_{bus} = \begin{bmatrix} (1) & (2) & (3) \\ (1) & j0.1 & j0.1 & j0.1 \\ (2) & j0.1 & j0.4 & j0.1 \\ (3) & j0.1 & j0.1 & j0.3 \end{bmatrix}$

$Z_{33} = Z_{11} + Z_s$   
 $= j0.1 + j0.2$   
 $= j0.3$



K = 2

q = 4

$Z_s = j0.4$

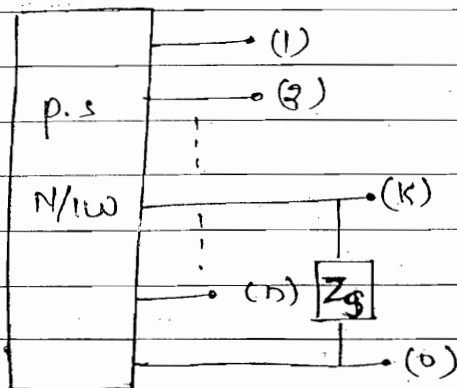
type-2

|             | ①        | ②      | ③      | ④      |
|-------------|----------|--------|--------|--------|
| $Z_{BUS} =$ | ① $j0.1$ | $j0.1$ | $j0.1$ | $j0.1$ |
|             | ② $j0.1$ | $j0.4$ | $j0.1$ | $j0.4$ |
|             | ③ $j0.1$ | $j0.1$ | $j0.3$ | $j0.1$ |
|             | ④ $j0.1$ | $j0.4$ | $j0.1$ | $j0.8$ |

$$\begin{aligned}
 Z_{44} &= Z_{22} + Z_s \\
 &= j0.4 + j0.4 \\
 &= j0.8
 \end{aligned}$$

### • Type 3 Modification:-

- An element with self impedance  $Z_s$  is added b/w the reference bus (0) and old bus (K)



$$Z_{BUS, old} = \begin{bmatrix} Z_{11} & Z_{12} & \dots & Z_{1K} & \dots & Z_{1n} \\ Z_{21} & Z_{22} & \dots & Z_{2K} & \dots & Z_{2n} \\ \vdots & \vdots & & \vdots & & \vdots \\ Z_{K1} & Z_{K2} & \dots & Z_{KK} & \dots & Z_{Kn} \\ \vdots & \vdots & & \vdots & & \vdots \\ Z_{n1} & Z_{n2} & \dots & Z_{nK} & \dots & Z_{nn} \end{bmatrix}_{n \times n}$$

$$Z_{BUS, new} = Z_{BUS, old} - \frac{1}{Z_{KK} + Z_s} \begin{bmatrix} Z_{1K} \\ Z_{2K} \\ \vdots \\ Z_{KK} \\ \vdots \\ Z_{nK} \end{bmatrix}_{n \times 1} \begin{bmatrix} Z_{K1} & Z_{K2} & \dots & Z_{KK} & \dots & Z_{Kn} \end{bmatrix}_{1 \times n}$$

K<sup>th</sup> column      K<sup>th</sup> Row

$$Z_{ij, new} = Z_{ij, old} - \frac{1}{Z_{KK} + Z_s} \times Z_{iK} \times Z_{Kj}$$

Ex:-  $i=3$  ;  $j=2$  ;  $K=4$

$$Z_{32, new} = Z_{32, old} - \frac{1}{Z_{44} + Z_s} \times Z_{34, old} \times Z_{42, old}$$

Q. Page No. 65. Q. 30

(B) Ar

$$Z_{22\text{new}} = Z_{22\text{old}} - \frac{1}{Z_{22} + Z_s} \times Z_{21} \times Z_{22}$$

$$= j0.3408 - \frac{1}{j0.3408 + j0.2} \times j0.2586 \times j0.3408$$

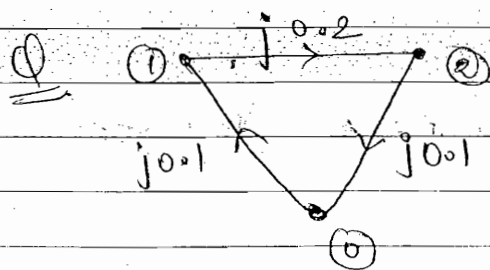
$$= j0.3408 - \frac{1}{j0.3408 + j0.2} \times j0.3408 \times j0.3408$$

$$= j0.3408 - (j1.84 \times 0.116) = j0.1260 \Omega$$

$$Z_{23\text{new}} = Z_{23\text{new}} - \frac{1}{Z_{22} + Z_s} \times Z_{22} \times Z_{23}$$

$$= j0.2586 - \frac{1}{j0.3408 + j0.2} \times j0.3408 \times j0.2586 = j0.0956 \Omega$$

Ar



type 1

$$q = 1$$

$$Z_s = j0.1$$

$$Z_{\text{bus}} = \textcircled{1} \begin{bmatrix} j0.1 \end{bmatrix}$$

type 2  $q = 2$ 

$$Z_s = j0.2$$

|   | ①      | ②      |
|---|--------|--------|
| ① | $j0.1$ | $j0.1$ |
| ② | $j0.1$ | $j0.3$ |

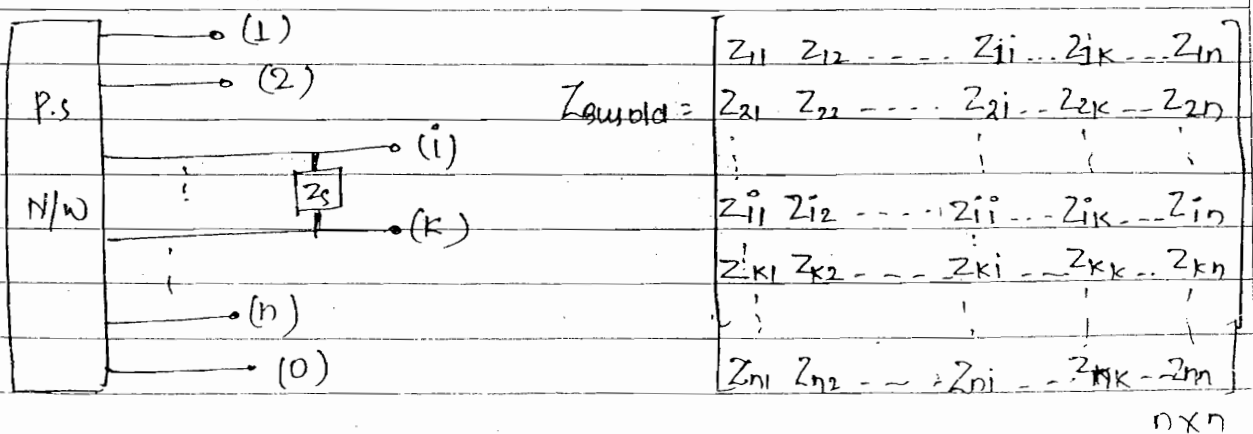
type 3

link is connected  
btw. bus ② and ③.

$$\begin{aligned}
 Z_{bus\text{new}} &= \begin{bmatrix} j0.1 & j0.1 \\ j0.1 & j0.3 \end{bmatrix} - \frac{1}{j0.3+j0.1} \times \begin{bmatrix} j0.1 \\ j0.3 \end{bmatrix} \begin{bmatrix} j0.1 & j0.3 \end{bmatrix} \\
 &= \begin{bmatrix} j0.1 & j0.1 \\ j0.1 & j0.3 \end{bmatrix} - \frac{1}{j0.3+j0.1} \times (-) \begin{bmatrix} 0.01 & 0.03 \\ 0.03 & 0.09 \end{bmatrix} \\
 &= \begin{bmatrix} j0.1 & j0.1 \\ j0.1 & j0.3 \end{bmatrix} + \frac{1}{j0.4} \begin{bmatrix} 0.01 & 0.03 \\ 0.03 & 0.09 \end{bmatrix} \\
 &= \begin{bmatrix} j0.1 & j0.1 \\ j0.1 & j0.3 \end{bmatrix} - j2.5 \begin{bmatrix} 0.01 & 0.03 \\ 0.03 & 0.09 \end{bmatrix} \\
 &= \begin{bmatrix} j0.1 & j0.1 \\ j0.1 & j0.3 \end{bmatrix} - \begin{bmatrix} j0.025 & j0.075 \\ j0.075 & j0.225 \end{bmatrix} \\
 &= \begin{bmatrix} j0.075 & j0.025 \\ j0.025 & j0.075 \end{bmatrix} \quad \text{Ans}
 \end{aligned}$$

### • Type - 4 Modification.

- An element with self impedance  $Z_s$  is added b/w two old buses (i) & (k)



from  $i^{th}$  column subtract  $k^{th}$  column and

Date

$$Z_{bus, new} = Z_{bus, old} - \frac{1}{Z_s + Z_{kk} + Z_{ii} - 2Z_{ik}} \times$$

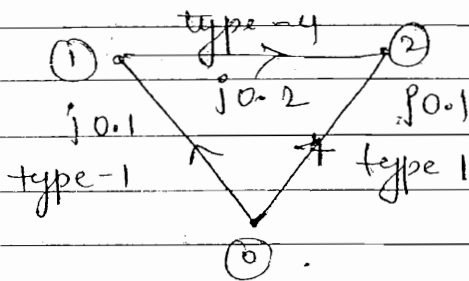
$$\begin{bmatrix} Z_{ii} - Z_{ik} \\ Z_{2i} - Z_{2k} \\ \vdots \\ Z_{ni} - Z_{nk} \end{bmatrix} \times \begin{bmatrix} Z_{ii} - Z_{ki} & Z_{i2} - Z_{2k} & \dots & Z_{in} - Z_{kn} \end{bmatrix} \times \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix}$$

write as column

$n \times 1$

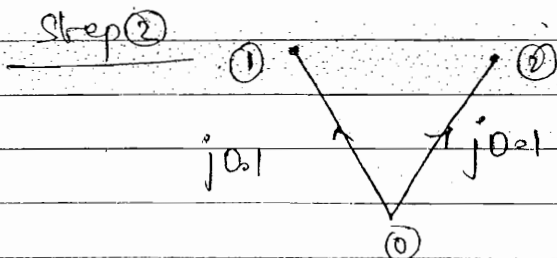
from  $i^{th}$  Row subtract  $k^{th}$  row and write as row.

ex<sup>o</sup>:- Obtain  $Z_{bus}$ .



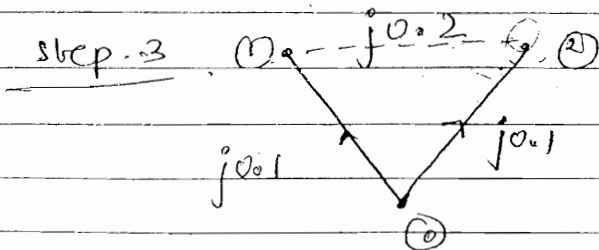
Step 1

$$Z_{bus} = \begin{bmatrix} 1 & 0 \\ j0.1 & 0 \end{bmatrix}$$



$$Z_{bus} = \begin{bmatrix} 1 & 0 \\ j0.1 & 0 \\ 0 & j0.1 \end{bmatrix}$$

$g = 0.2$   
 $Z_s = j0.1$



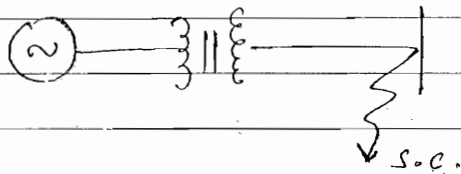
$i = 1$      $j = 2$      $k = 2$   
 $Z = j0.2$     Type-4

$$Z_{bus} = \begin{bmatrix} j0.1 & 0 \\ 0 & j0.1 \end{bmatrix} - \frac{1}{j0.2 + j0.1 + j0.1 + 2 \times 0} \begin{bmatrix} j0.1 \\ -j0.1 \end{bmatrix} \begin{bmatrix} j0.1 & -j0.1 \end{bmatrix}$$

$$\begin{aligned}
 Z_{BUS} &= \begin{bmatrix} j0.1 & 0 \\ 0 & j0.1 \end{bmatrix} + \frac{1}{j0.4} \begin{bmatrix} j0.01 & -j0.01 \\ -j0.01 & +j0.01 \end{bmatrix} \\
 &= \begin{bmatrix} j0.1 & 0 \\ 0 & j0.1 \end{bmatrix} + j2.5 \begin{bmatrix} j0.01 & -j0.01 \\ -j0.01 & +j0.01 \end{bmatrix} \\
 &= \begin{bmatrix} j0.075 & j0.025 \\ j0.025 & j0.075 \end{bmatrix}
 \end{aligned}$$

### • Short-Circuit Analysis or Fault Analysis

↳ Short circuit → low impedance path, abnormal current flow.



$I_{sc} \rightarrow$  s.c. current

$V I_{sc} =$  short ckt KVA or fault level.

↓  
It will give the breaking capacity of C.B.

- The purpose of conducting of short circuit or fault analysis is to determine breaking capacity of C.B.

✓ The short circuit studies are performed well before than any other studies

$$R \ll X$$

$$Z_{total} \downarrow \quad I_{sc} \uparrow$$

→ S.C. current is limited by generator,  $X_{\text{mer}}$ ,  $X_{\text{mission}}$  reactances, so flowing current is lagging one.

→ Due to S.C. current voltage become low bcoz of high demand of reactive power.

"Short Circuit" A short circuit is a low voltage, high current, highly lagging, low power factor phenomenon.

↳ If fault current is highly lagging and fault demands lagging reactive power.

→ The main cause for the stability <sup>problem</sup> is short circuit. During S.C. active power = 0. The entire ~~active~~ <sup>mechanical</sup> power

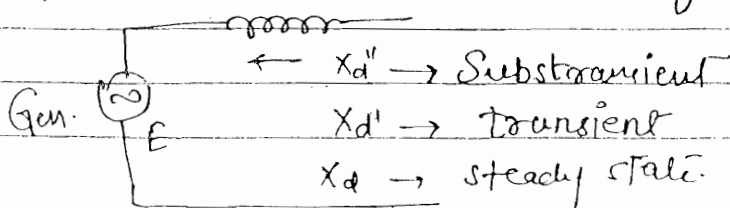
mechanical I/p stores as kinetic energy and this leads to problem of stability.

→ When fault occurs generator starts delivering reactive power and stops active power.

### Assumptions:-

(1) Generator is represented as a voltage source behind the reactance.

(2) Saliency (non-uniform air gap length of salient pole rotor alternator) of alternator is neglected.



above 100mA  $\rightarrow$  we get shock

150mA - 200mA  $\rightarrow$  paralysis  
200mA - 250mA  $\rightarrow$  Burn  
250mA - 300mA  $\rightarrow$  Stop (heart)

Date \_\_\_\_\_

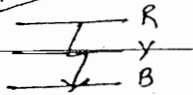
(3) X-men is represented as a series reactance element if the p.u. method is used.

(4) The resistance & capacitance effects are neglected.

### Fault Analysis

Symmetrical /  
Balanced F.A.

3  $\phi$  fault  $\downarrow$



Current will remain  
0 whether ground  
is involved or not.

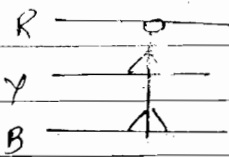
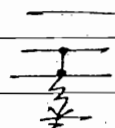
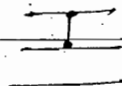
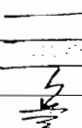
$$I_R + I_Y + I_B = 0$$

Unsymmetrical /  
unbalanced F.A.

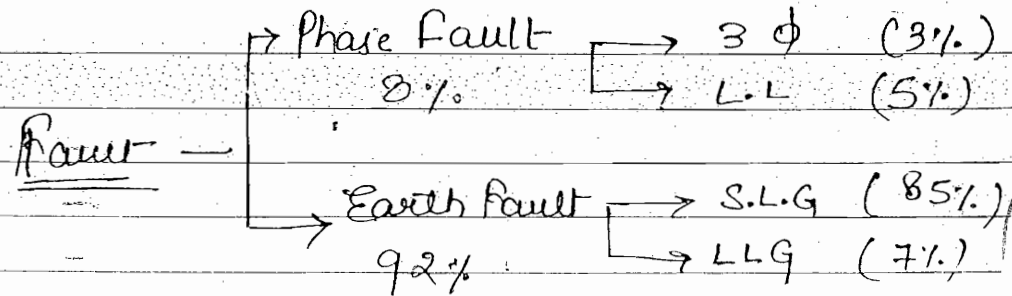
SLG

L-L

L-L-G



Due to zero current it will damage the body.  
If ground is involved, nothing will happen as  
it balance condition



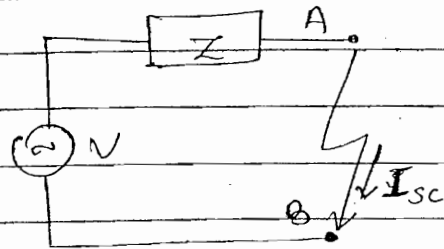
✓ Most Severe fault  $\rightarrow$  3  $\phi$   
✓ least Severe fault  $\rightarrow$  L-G fault

rated value = base value

# Symmetrical Fault Analysis

Date: 10<sup>th</sup> June 2014

↳ During fault, due to de-magnetising effect generator stop delivering active power so  $N=0$



$V_b \leftarrow V$  = Rated voltage of the equipment  
 $I_b \leftarrow I$  = Rated current of —

$$Z_{Base} = \frac{V}{I} \quad \text{--- (1)}$$

$Z$  = Internal Impedance of —

↳ fault impedance is not included in  $3\phi$ .

$$I_{sc} = \frac{V}{Z} \quad \text{--- (2)}$$

$$\begin{aligned} Z_{pu} &= \frac{Z}{Z_{Base}} \\ &= \frac{Z}{V/I} \\ &= \frac{I}{V/Z} \end{aligned}$$

$$\therefore \boxed{Z_{pu} = \frac{I}{I_{sc}}} \quad \text{--- (4)}$$

NOTE: - The ratio of Rated Current to the short circuit current gives p.u. impedance of the equipment

$$I_{sc} = \frac{I}{Z_{pu}} \quad I_{sc} \text{ (in p.u.)} = \frac{I_{pu}}{Z_{pu}}$$

∴  $I$  (in p.u.) is always equal to 1.

$$\boxed{I_{sc} \text{ (in p.u.)} = \frac{1}{Z_{pu}}} \quad \text{--- (4)}$$

NOTE: For the power sys. network the S.C. current across the faulted terminal

can be obtain by taking reciprocal of thevenin's equivalent p.u. impedance

$$I_{sc}(\text{in p.u.}) = \frac{1}{Z_{th}(\text{in p.u.})}$$

$\therefore Z_{th}$  = Thevenin's equivalent impedance.

$$Z_{pu.} = \frac{I}{I_{sc}} \quad \% Z = \frac{I}{I_{sc}} \times 100$$

$$\Rightarrow \boxed{I_{sc} = \frac{I \times 100}{\% Z}}$$

Multiply both sides with rated voltage 'V'

$$V I_{sc} = V I \times \frac{100}{\% Z}$$

Short circuit KVA we get. (rated KVA is taken as base KVA)

$$\boxed{\text{Short circuit KVA} = \frac{\text{Rated or Base KVA} \times 100}{\% Z}} \quad (5)$$

$$I_{sc} = I \times \frac{1}{Z_{pu.}} \Rightarrow V I_{sc} = V I \frac{1}{Z(\text{p.u.})} \Rightarrow \frac{V I_{sc}}{V I} = \frac{1}{Z(\text{p.u.})}$$

$$\begin{aligned} \text{Short circuit power (in p.u.)} &= \frac{1}{Z(\text{p.u.})} \\ &= \frac{I_{sc}}{I} = \frac{I_{sc}}{1} \\ &= I_{sc} \end{aligned}$$

• Bez the formula for S.C. current and S.C. power in pu. we say  $\left(\frac{1}{Z_{pu.}}\right)$ , Therefore if the S.C

$$\text{S.c power p.u} = I_{sc} \text{ in p.u} = \frac{1}{Z_{th} \text{ (in p.u.)}}$$

Date \_\_\_\_\_

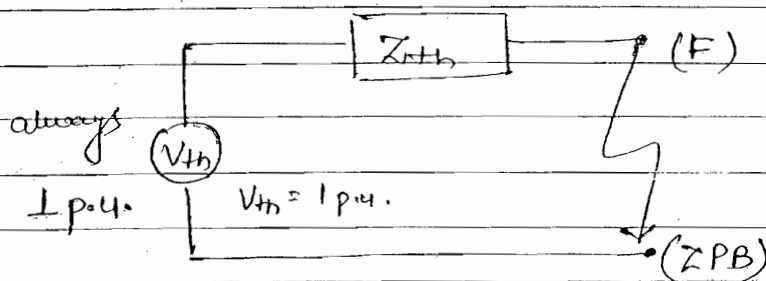
Current is  $X \text{ p.u.}$ . Then <sup>S.c</sup> power is also  $X \text{ p.u.}$

• Procedure for Symmetrical fault Analysis:-

Step 1 :- By choosing appropriate base values, convert the given single line diagram p.u. N/W into p.u. equivalent reactance diagram.

Step 2 :- Identify faulted Bus terminals (F) & (ZPB)

Step 3 :- Across the faulted bus terminals reduce the network into Thevenin's equivalent ckt

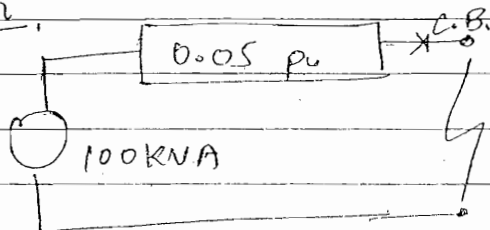


$$I_{sc} \text{ (in p.u.)} = \frac{1}{Z_{th} \text{ (in p.u.)}} = \text{Short ckt power (in p.u.)}$$

$$\left[ \text{Short ckt KVA} = \frac{\text{Short ckt power (in p.u.)} \times \text{Common base KVA}}{\text{KVA}} \right]$$

Problem :- A 100 KVA equipment has 5% impedance  
Its S.B. KVA = \_\_\_\_\_

Solution



$$\begin{aligned} I_{sc} \text{ in p.u.} &= \frac{1}{Z \text{ in p.u.}} \\ &= \frac{1}{0.05} \\ &= 20 \text{ p.u.} \end{aligned}$$

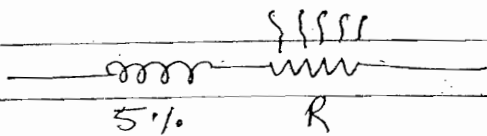
$$S.C. \text{ KVA in p.u.} = I_{sc} \text{ in p.u.} = 20 \text{ p.u.}$$

$$\begin{aligned} S.C. \text{ KVA} &= S.C. \text{ KVA in p.u.} \times \text{common base value} \\ &= 20 \times 100 \\ &= 2000 \text{ KVA} \end{aligned}$$

→ CB should have 2000 KVA breaking capacity.

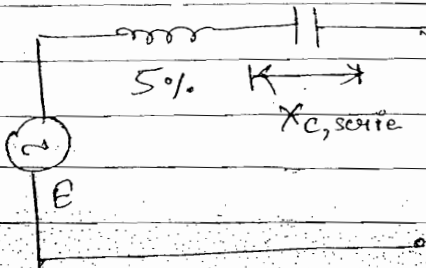
• To limit  $I_{sc}$  :-

(a) Resistance is not considered



disadv :- Continuous power losses

(b) Series capacitor is not considered



Disadv :- under normal condition

$$I \quad X_c \quad V = I X_c$$

• under for S.C. condition

$$I_{sc} \quad X_c$$

During S.C., voltage drop is

$$V_{sc} = I_{sc} X_c$$

high due to  $I_{sc}$ , which is several times larger than rated current  $I$

$$I_{sc} > I$$

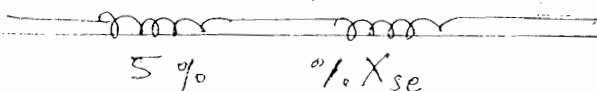
$$V_{sc} > V$$

So, we have to provide heavy insulation which is very costly and expensive for its prevention.

Review ques

• During S.C. heavy voltage drop across capacitor damage the insulation.

(c) Series Reactors are used to limit  $I_{sc}$  :-



linear  $\rightarrow$  obey Ohm's law + super position.

Date \_\_\_\_\_

advantage: - (1) No power loss problem.

(2) No insulation breakdown problems.

✓ Inductor is linear until the saturation.

$$L = \frac{N\Phi}{I}$$

$\{ I \uparrow, \Phi \uparrow \rightarrow L$  remain constant

$\{ I \downarrow, \Phi \downarrow \rightarrow - " - " - " - " -$

It there is saturat<sup>n</sup> problem  $L \downarrow$ ,  $I \uparrow$  its repeat after some time  $L$  down.

✓ Resistor is linear if temp. remain constant.

✓ Capacitor is linear until the breakdown insulation take place.

$C \uparrow$  (charge storage),  $V \uparrow$  (voltage) after some time breakdown take place and diffusion will happen, no capacitor.

• To avoid the problem of saturat<sup>n</sup> as in the case of series reactor, they are design with no core or air core.

ex. - A 100 KVA equip. has 5% impedance. To limit the S.C KVA to 250, the % of reactance of series reactor required.

Ans  $I_{sc} \text{ in p.u.} = \frac{1}{Z_{in p.u.}} = \frac{1}{0.05} = 20 = \text{S.C. power in p.u.}$

$$\text{Short/cut power} = 20 \times 100 = 2000 \text{ KVA.}$$

$$I_{sc} \text{ in pu} = \frac{1}{0.05 + X_e}$$

$$250 = \frac{1}{0.05 \times \% X_e} \times 100$$

Date \_\_\_\_\_

$$X_{se} + 0.05 = \frac{100}{250} = 0.4$$

$$X_{se} = 0.4 - 0.05 = 0.35$$

$$\% X_{se} = 35\% \quad \text{Ans}$$

OR

$$S_{ckt} \text{ KVA} = \frac{1}{8} \times 2000$$

$$I_{sc} = \frac{1}{Z_{in \text{ p.u.}}}$$

$$I_{sc}' = \frac{1}{8} \times I_{sc}$$

$$Z_{pu}' = 8 \times Z_{pu}$$

$$= 8 \times 5 = 40\%$$

$$\% X_e = 40 - 5 = 35\%$$

Page NO. 61

Solution 2  $X_{dnew} = 0.4 \times \frac{100}{75} \times \left(\frac{10}{11}\right)^2$

$$= 0.44 \text{ p.u.}$$

Solution 3  $X_{p.u.new} = 2 \times \frac{100}{500} \times (1)^2 = 0.4$

$$M_{p.u.new} = 20 \times \frac{100}{500} \times (1)^2 = 4$$

Ref. Stability Chapter :-

$$\text{Inertia Constant } M = \frac{GH}{180f} \quad \boxed{M \propto H}$$

$$H_{eb} = \frac{(GH)_{rated}}{G_{base}} \quad G: \text{Rated MVA}$$

$$M_{eb} = H_{eb} = \frac{G_{rated} \times \text{MVA}_{rated}}{G_{base}} = \frac{500 \times 20}{100} = 100$$

Solution 4  $X_{G1} = j0.25 \text{ pu}$  on 250 MVA, 15KV.  
 $= ?$  on 100 MVA, 15KV

$$X_{G1} = 0.25 \times \frac{100}{250} \times 1^2$$

$$= j0.1 \text{ pu.}$$

$$X_{G2} = j0.1 \text{ pu. on 100 MVA, 15KV}$$

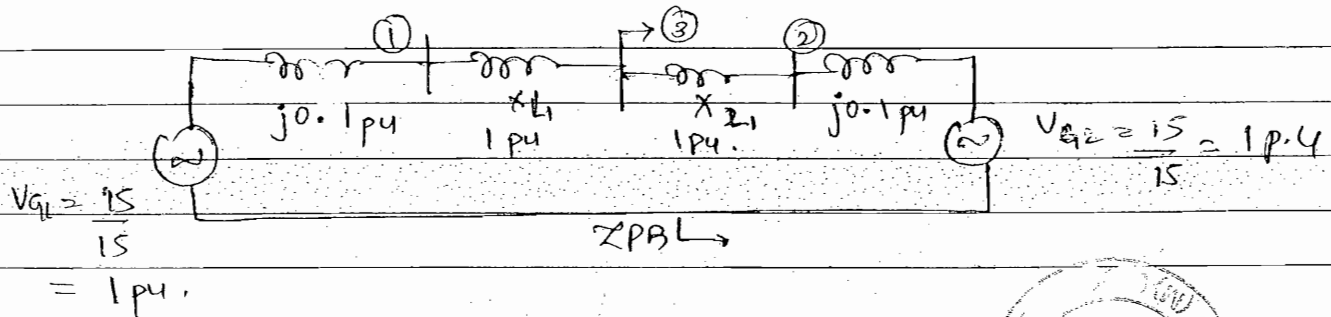
$$= ) \text{ on 100 MVA, 15KV.}$$

$$= j0.1 \text{ same.}$$

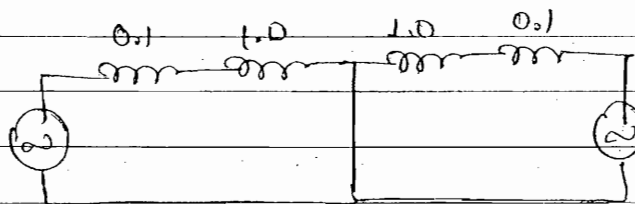
$$Z_{\text{base}} = \frac{V^2}{\text{MVA}} = \frac{15^2}{100} = 2.25 \Omega$$

$$X_{L1} = X_{L2} = j0.225 \Omega / \text{km} \times 10 \text{ km} = j2.25 \Omega$$

$$= j1 \text{ pu.}$$



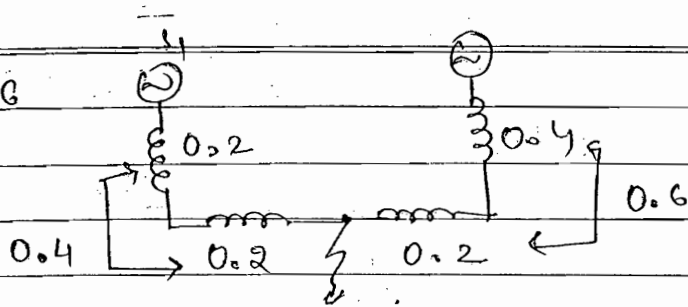
Solution 5.



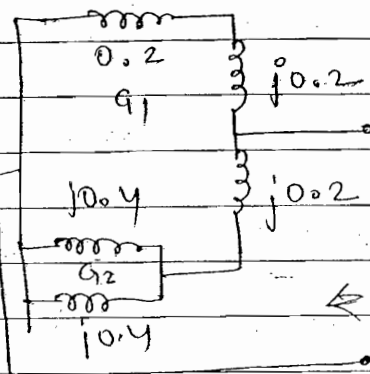
$$R_{th} = \frac{1.0 \times 1.0}{1.0 + 1.0} = 0.55$$

$$I_{sc} \text{ pu} = \frac{1}{2 \text{ pu.}} = 1.818 \text{ pu.} \approx \text{S.C. MVA pu.}$$

$$\text{S.C. power} = 1.818 \times 100 = 181.8 \text{ kVA}$$

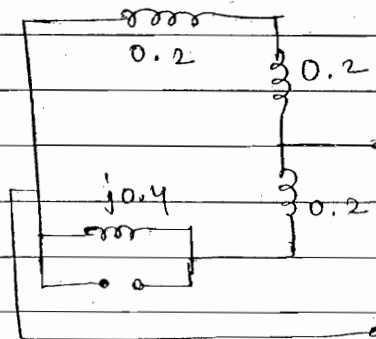
Solution 6

$$Z_{th} = 0.4 \parallel 0.6 = \frac{0.4 \times 0.6}{0.4 + 0.6} = 0.24$$



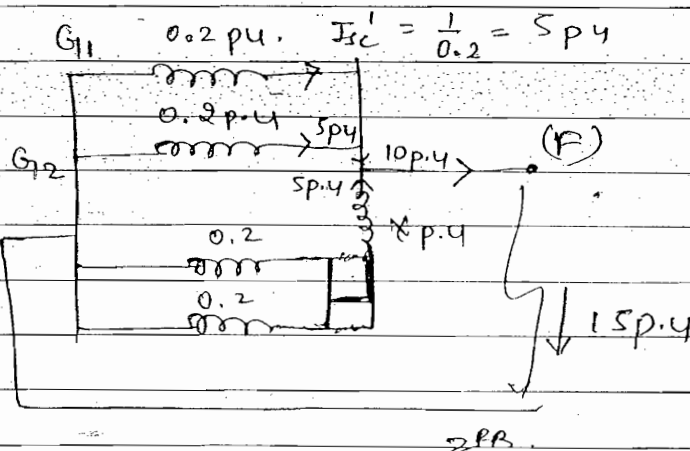
$$\leftarrow X' = 0.2 \text{ pu.}$$

2 PB



$$\leftarrow X = 0.24 \text{ pu.}$$

$$\% \text{ change} = \frac{0.24 - 0.2}{0.2} \times 100 = 20\%$$

Solution 7

$$0.1 + j0.1$$

$$n = 0.1 \text{ pu.}$$

2 PB

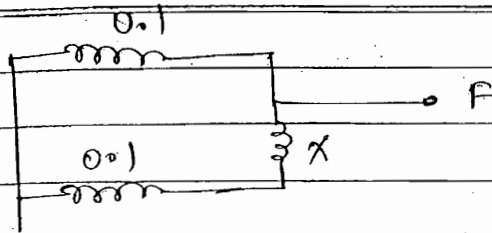
Method

$$\text{Base MVA} = 100$$

$$\text{SKT MVA} = 1500$$

$$\text{MVA}_{pu} = \frac{1500}{100} = 15 \text{ pu} = I_{sc} \text{ pu.}$$

$$\frac{1}{Z_{th}} = I_{sc} \quad Z_{th} = \frac{1}{15} = 0.067 \text{ pu.}$$



$$0.067 = 0.1 \parallel (0.1 + x)$$

$$0.067 = \frac{0.1(0.1 + x)}{0.1 + 0.1 + x}$$

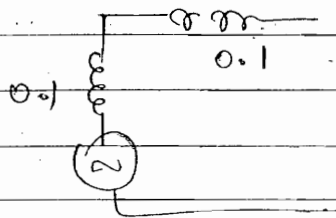
$$= \frac{0.1(0.1 + x)}{0.2 + x}$$

$$\Rightarrow 0.0134 * 0.067 * x = 0.01 + 0.1x$$

$$\& 0.0134 - 0.01 = x(0.1 - 0.067)$$

$$x = 0.1 \text{ p.u.}$$

Solution 9



$$Z_{th} = 0.2$$

$$I_{sc} = 5 = \text{S.C. MVA p.u.}$$

$$\text{Sc. MVA} = 5 \times 100$$

$$= 500 \text{ MVA A.M.}$$

## • Unbalanced / Unsymmetrical Fault Analysis

$$\begin{bmatrix} I_R \\ I_Y \\ I_B \end{bmatrix} = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \end{bmatrix} + \begin{bmatrix} \cdot \\ \cdot \\ \cdot \end{bmatrix} + \begin{bmatrix} \cdot \\ \cdot \\ \cdot \end{bmatrix}$$

↓

unbalance  
condition

3 set of balance condition

$$I_R = I_{R1} + I_{R2} + I_{R0}$$

$$I_Y = I_{Y1} + I_{Y2} + I_{Y0}$$

$$I_B = I_{B1} + I_{B2} + I_{B0}$$

balance vector

unbalance  
vector

↓  
+ve

↓  
-ve

↓  
zero

seq. comp

seq. comp

seq. comp

→ -ve and zero sequence are purely passive element having no voltage source.

### • Symmetrical Component -

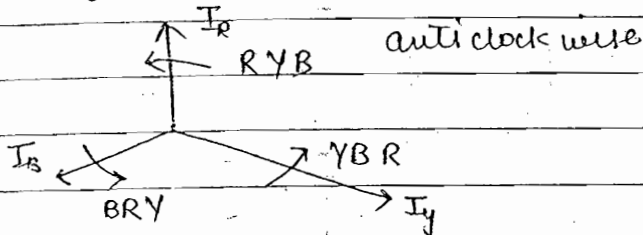
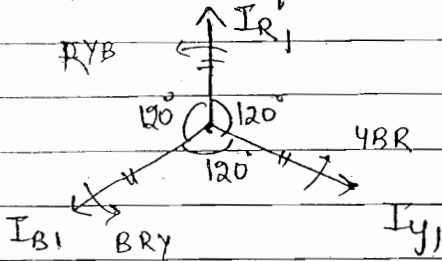


fig. original unbalance vector.

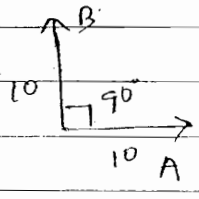
These unbalance vector will be resolved into balance +ve, -ve, zero sequence components.

→ +ve Sequence Components: - These balanced component vectors



have the same sequence of original vector.

### • operator 'a'

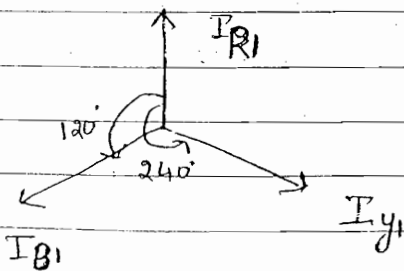


$$A = 10 \angle 0^\circ$$

$$B = 10 \angle 90^\circ = jA$$

$$a = 1 \angle 120^\circ \text{ shift } 120^\circ \text{ anticlockwise}$$

$$a^2 = 1 \angle 240^\circ \text{ shift } 240^\circ$$



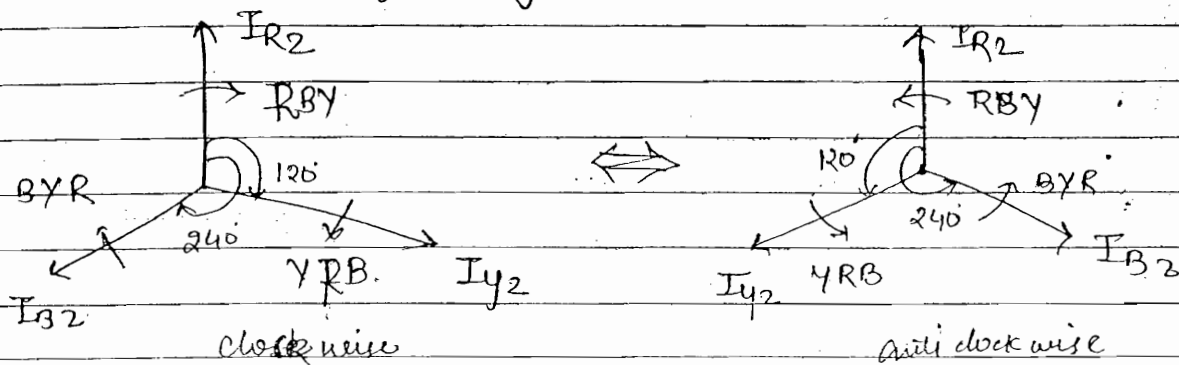
$$I_{R1} = I_{R1} \angle 0^\circ \text{ ref.}$$

$$I_{Y1} = I_{R1} \angle 240^\circ = a^2 I_{R1}$$

$$I_{B1} = I_{R1} \angle 120^\circ = a I_{R1}$$

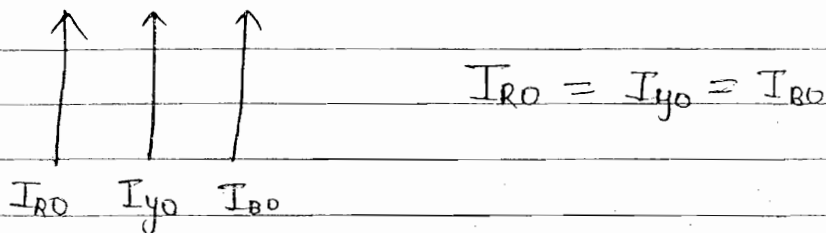
✓ Operator a Transform Y phase and B phase components into R phase and avoids 3- $\phi$  quantities.

→ Sequence Component - These balanced vector component have the sequence opposite to that of original vectors.



$$\begin{aligned} I_{R2} &= I_{R2} \angle 0^\circ \text{ reference} \\ I_{Y2} &= a I_{R2} = I_{R2} \angle 120^\circ \\ I_{B2} &= a^2 I_{R2} = I_{R2} \angle 240^\circ \end{aligned}$$

→ Zero Sequence Component - These components vector have no sequence.



$$I_R = I_{R1} + I_{R2} + I_{R0} \quad \text{--- (1)}$$

$$\begin{aligned} I_Y &= I_{Y1} + I_{Y2} + I_{Y0} \\ &= a^2 I_{R1} + a I_{R2} + I_{R0} \quad \text{--- (2)} \end{aligned}$$

$$\begin{aligned} I_B &= I_{B1} + I_{B2} + I_{B0} \\ &= a I_{R1} + a^2 I_{R2} + I_{R0} \quad \text{--- (3)} \end{aligned}$$

$$\begin{bmatrix} I_R \\ I_Y \\ I_B \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{R0} \\ I_{R1} \\ I_{R2} \end{bmatrix} \quad -(4)$$

In concise form:-

$$[I]_{R \times B} = [A] [I]_{012} \quad -(5)$$

where  $A$  = operator Matrix

NOTE:- operator  $A$  Matrix is also known as transform matrix. Transforms unbalanced conditions into balanced conditions and vice-versa.

### • Relations of operator ' $a$ '

$$(1) \quad a = 1 \angle 120^\circ = -0.5 + j0.866$$

$$(2) \quad a^2 = 1 \angle 240^\circ = -0.5 - j0.866$$

$$(3) \quad a^3 = 1 \angle 360^\circ = 1$$

$$(4) \quad a^4 = 1 \angle 480^\circ = a^3 \cdot a = a$$

$$(5) \quad a^5 = a^3 \cdot a^2 = a^2$$

$$(6) \quad \boxed{1 + a + a^2 = 0}$$

$$(7) \quad 1 - a^2 = 1 - (-0.5 - j0.866)$$

$$= 1.5 + j0.866$$

$$= \frac{3}{2} + j \frac{\sqrt{3}}{2}$$

$$= \sqrt{3} \left[ \frac{\sqrt{3}}{2} + j \frac{1}{2} \right]$$

$$= \sqrt{3} \angle +30^\circ$$

$$(8) \quad 1 - a = \sqrt{3} \angle -30^\circ$$

$$(9) \quad [I]_{R \times B} = [A] [I]_{012}$$

$$(10) \quad [I]_{012} = [A]^{-1} [I]_{R \times B}$$

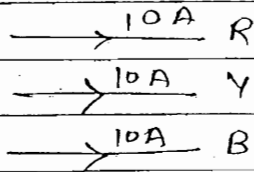
$$A^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$$

Date 11<sup>th</sup> June 14

Observation 1 : (1) If a 3 $\phi$  system is balanced, only the sequence components are present.

ex: - In a balanced 3 $\phi$  sys. the current in each phase is 10A. The phase sequence is RYB. Find sym. components.

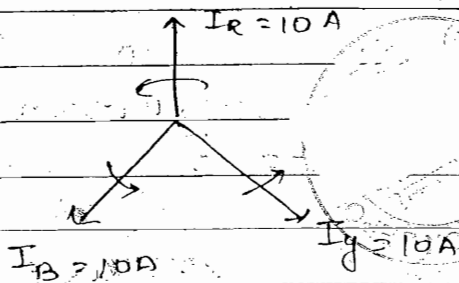
Solve



$$I_R = 10 \angle 0^\circ \text{ refer.}$$

$$I_Y = 10 \angle 240^\circ = a^2 I_0$$

$$I_B = 10 \angle 120^\circ = a I_0$$



$$\begin{bmatrix} I_{R0} \\ I_{R1} \\ I_{R2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_R \\ I_Y \\ I_B \end{bmatrix}$$

$$\begin{bmatrix} I_{R0} \\ I_{R1} \\ I_{R2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} 10 \angle 0^\circ \\ a^2 10 \\ a 10 \end{bmatrix}$$

$$I_{R0} = \frac{1}{3} [10 + a^2 10 + a 10] = \frac{10}{3} [1 + a + a^2] = 0$$

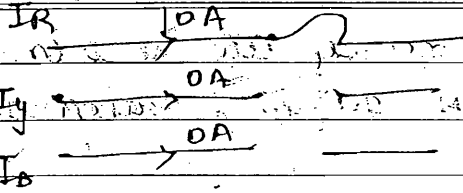
$$I_{R1} = \frac{1}{3} [10 + a^3 10 + a^3 10] = \frac{30}{3} = 10A$$

$$I_{R2} = \frac{1}{3} [10 + a^4 10 + a^2 10] = \frac{10}{3} [1 + a + a^2] = 0$$

Observation 2 : If a 3 $\phi$  system is unbalanced all the three sequence component may be present.

ex: - In a balanced 3 $\phi$  sys. the current in each phase is 10A. phase sequence is RYB. Now, the fuses in Y & B ph are Blown off, Find sym. component.

Solution



$$\begin{bmatrix} I_{R0} \\ I_{R1} \\ I_{R2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}$$

$$I_{R0} = \frac{1}{3} [10 + 0 + 0] = \frac{10}{3} \text{ A}$$

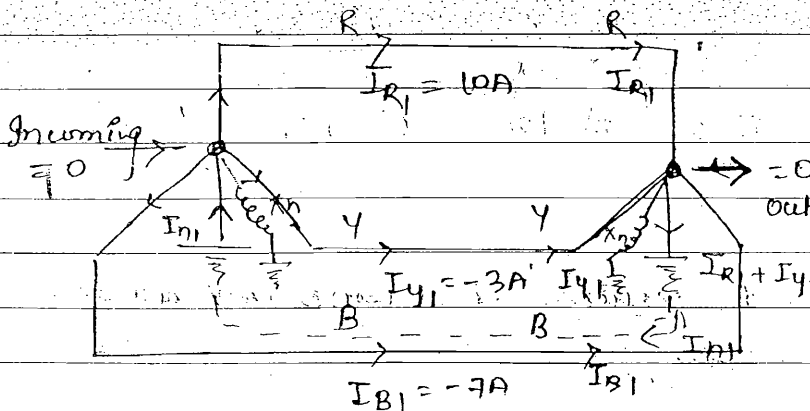
$$I_{R1} = \frac{1}{3} [10 + 0 + 0] = \frac{10}{3} \text{ A}$$

$$I_{R2} = \frac{1}{3} [10 + 0 + 0] = \frac{10}{3} \text{ A}$$

Observation 3: For the flow of +ve, -ve seq. currents, return path through ground is not compulsory  
 (but current leaving = 0)

OR. The condition of neutral (ungrounded), solidly grounded or reactance grounded) has got no effect for the flow of +ve/-ve seq. components.

$$\therefore I_{R1} + I_{y1} + I_{B1} = 0 \quad \text{also} \quad I_{R2} + I_{y2} + I_{B2} = 0$$



• Neutral point is ungrounded.

• Incoming current = 0

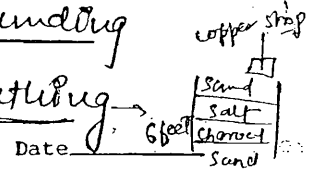
• -ve current means flowing in opp direction

• Solidly rotors are grounded.

$$I_{n1} = I_{R1} + I_{y1} + I_{B1} = 0$$

• If  $X_n$  effect the current than consider  $X_n$  but  $I_{n1} = 0$   
 so voltage drop is also = 0 (not necessary)

- Connecting neutral point to ground called grounding
- Connecting equipment body to ground called earthing



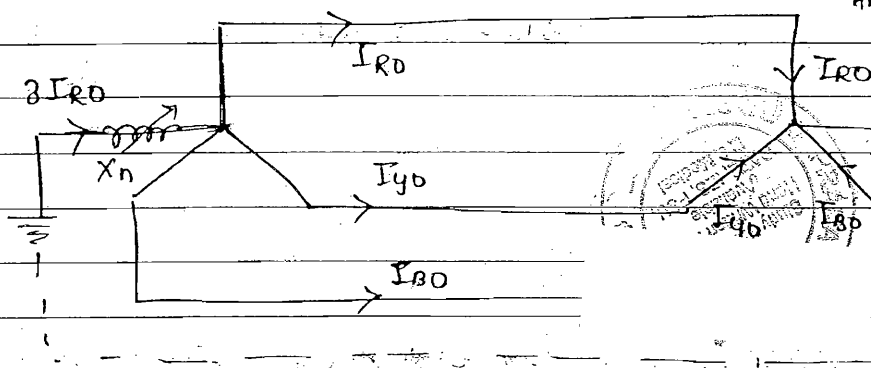
→ NO return path → means any one or two path out of 3- $\phi$  path will act as return path.

Observation 4:- For the flow of zero sequence currents, return path through ground is compulsory.

OR The condition of neutral has got effect for the flow of zero sequence currents.

$$I_{R0} + I_{Y0} + I_{B0} = 3 I_{R0}$$

→  $X_n$  is used to limit the earth fault (not phase fault)



At the fault occur it goes through ground by using  $X_n$  we can limit it

• If not grounded incoming current to supply is 0 and current leaving to load is  $3 I_{R0}$ , not satisfy the KCL, "NO current will flow"

• The neutral ground reactance has to be considered in zero seq. ckt.

→ Ground having 0  $\Omega$  and act as perfect conductor

→ If all the loads are 3- $\phi$  → Then 3 $\phi$ , 3 wire

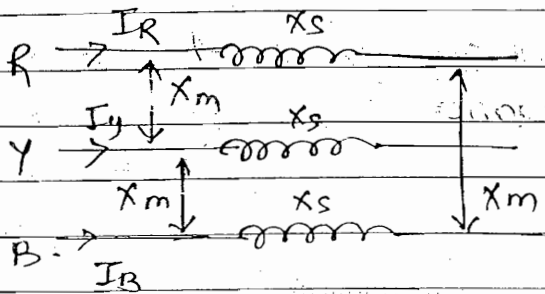
If load is 1- $\phi$

→ Then 3 $\phi$ , 4 wire.

→ In transmission lines NO return path, bcoz they are feeder not distributor.

Observation - 5: The original 3- $\phi$  sys (R Y B) is a mutually jointed N/w. where as, the sequence N/w (+ve, -ve & zero) are mutually disjointed N/w's.

→ Consider X-connection line



$X_s \rightarrow$  self Reactance  
 $X_m \rightarrow$  mutual Reactance.

→ Sys is mutually coupled.  
 → current in coil 1 induced voltage in coil 2.

$$\begin{bmatrix} V_R \\ V_Y \\ V_B \end{bmatrix} = \begin{bmatrix} X_s & X_m & X_m \\ X_m & X_s & X_m \\ X_m & X_m & X_s \end{bmatrix} \begin{bmatrix} I_R \\ I_Y \\ I_B \end{bmatrix}$$

→ If the off-diagonals  $\neq 0$  then they are mutually coupled.  
 → In condensed form:-

$$\begin{aligned} [V]_{RYB} &= [X]_{RYB} [I]_{RYB} \\ [A] [V]_{012} &= [X]_{RYB} [A] [I]_{012} \\ [V]_{012} &= \underbrace{[A]^{-1} [X]_{RYB} [A]}_{\text{"sequence impedance Matrix"}} [I]_{012} \end{aligned}$$

$$[X]_{012} = [A]^{-1} [X]_{RYB} [A]$$

$$[X]_{012} = \begin{bmatrix} X_s + 2X_m & 0 & 0 \\ 0 & X_s - X_m & 0 \\ 0 & 0 & X_s - X_m \end{bmatrix}$$

$$X_0 = X_s + 2X_m$$

$$X_1 = X_2 = X_s - X_m$$

2 of zero seq. N/w.

2 of the f -ve seq. N/w.

NOTE:- In the seq. Impedance Matrix shown above, the off-diagonal elements are zero. This means three seq. N/w are mutually disjoint.

- The values of seq. Impedances are  $X_1 = X_2 = X_s - X_m$  and  $X_0 = X_s + 2X_m$ .

Problem 10

$$E_a = 10 \angle 0^\circ \text{ V}$$

$$E_b = 10 \angle 90^\circ \text{ V}$$

$$E_c = 10 \angle 120^\circ \text{ V}$$

$$E_a = 10 \angle 0^\circ \text{ V}$$

$$I_a = \frac{E_a}{j^2} = \frac{10 \angle 0^\circ}{j^2} = 5 \angle -90^\circ$$

$$E_b =$$

$$j^2$$

$$j^2$$

$$I_b = \frac{E_b}{j^2} = \frac{10 \angle 90^\circ}{j^2} = 10 \angle 120^\circ$$

$$I_c = \frac{E_c}{j^2} = \frac{10 \angle 120^\circ}{j^2}$$

$$= \frac{1}{j^2} = \frac{1}{-1} = -1$$

$$= 3.3 \angle -180^\circ$$

$$I_c = 2.5 \angle 30^\circ$$

$$I_{q1} = \frac{1}{3} [I_a + a I_b + a^2 I_c]$$

$$= \frac{1}{3} [5 \angle -90^\circ + 3.3 \angle -180^\circ + 120^\circ + 2.5 \angle 30^\circ + 240^\circ]$$

$$= \frac{1}{3} [5 \angle -90^\circ + 3.3 \angle -60^\circ + 2.5 \angle 270^\circ]$$

$$= 3.49 \angle -80.9^\circ$$

$$= 3.5 \angle -81^\circ$$

Solution 11  $X_1 = X_d - X_m = 15$

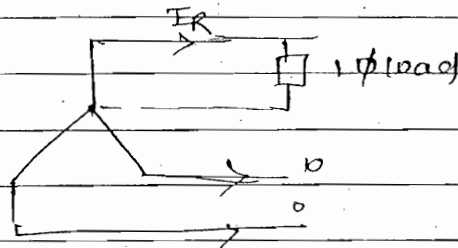
$X_0 = X_s + 2X_m = 48$

$+ 3X_m = 33$

$X_m = 11$

$X_s = 15 + X_m = 15 + 11 = 26 \Omega$

Solution 20



$$\begin{bmatrix} I_{R0} \\ I_{R1} \\ I_{R2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_R \\ 0 \\ 0 \end{bmatrix} \Rightarrow I_{R1} = I_{R2} = I_{R0} = \frac{I_R}{3}$$

• Sequence Impedances:-

✓ For Generator  $X_{G1} \approx X_{G2}$  (nearly equal to)

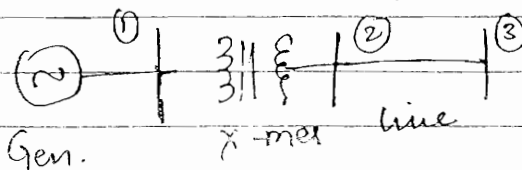
• strictly speaking  $X_{G2}$  is slightly less than  $X_{G1}$

$X_{G0} \ll X_{G1}$  # (Gen. is kept at starting. no ground impedance).

✓ For static devices like x-mes & x-line sequence is not important

$X_0 \gg X_1$   $X_1 = X_2$  # (ground impedance is added to x-line)

• Sequence Network:-



convert into 3 seq. N/w.

## • Sequence Network:

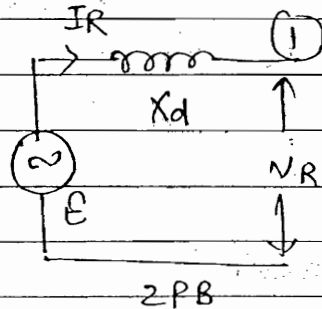
• Generator Representation :- In the original N/w

- ① used in symm. fault analysis, generator is
- ② represented as a constant voltage source  $E$  behind the reactance  $X_d$ .

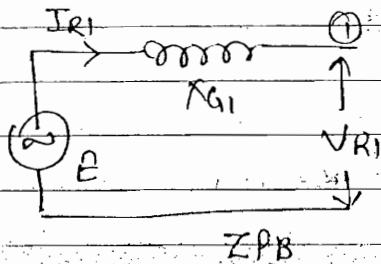
### Original N/w

$I_R \rightarrow$  phase current

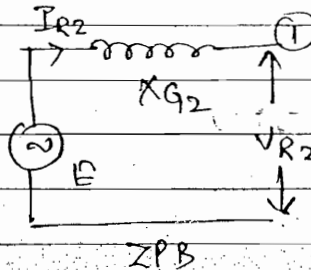
$V_R \rightarrow$  phase voltage terminal.



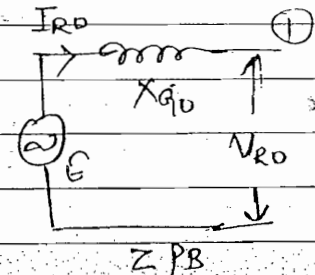
### +ve Seq. N/w



### -ve Seq. N/w



### Zero Seq. N/w

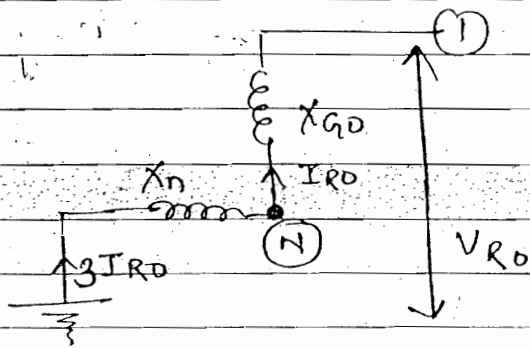
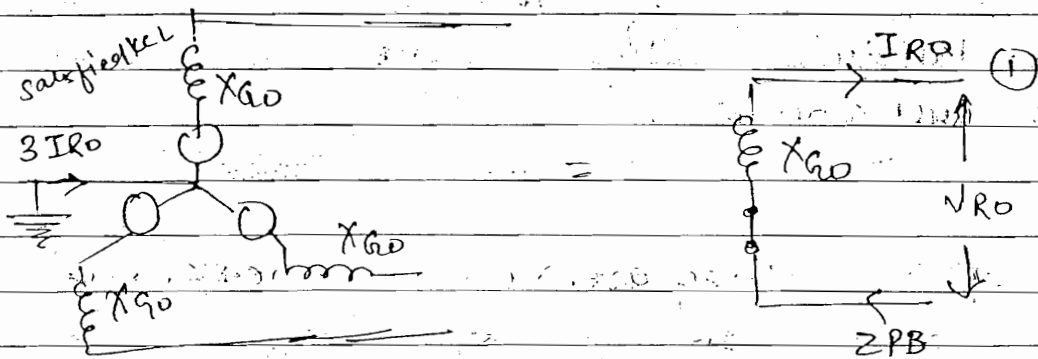
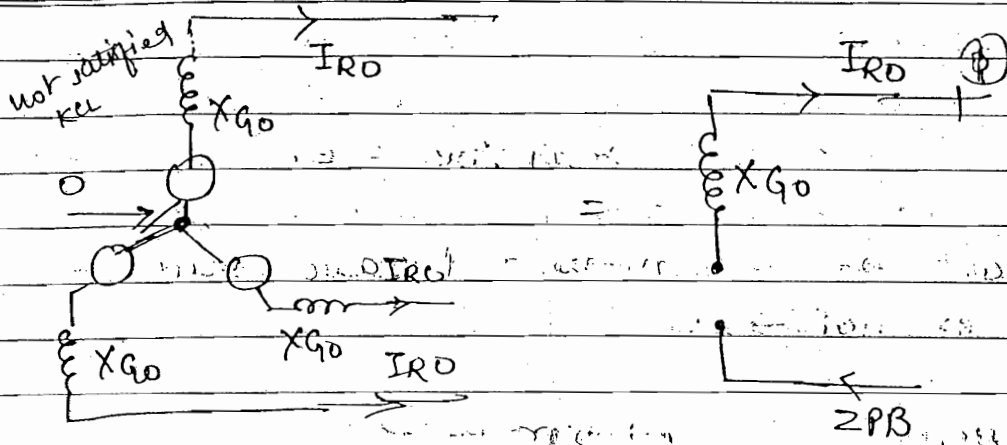


### Interview

- Ques. Why generator are  $\gamma$  connected not  $\Delta$  connected?
- $\rightarrow$  In  $\Delta$  connect<sup>n</sup> it produce circulating current, heating harmonics.

Condition to identify:- (1) Whether the voltage source is <sup>generator</sup> producing power or not. If it is then represent  $E = E$ , If not then  $E = 0$ .

(2) Find the neutral condition, whether neutral is affecting the seq. N/w or not. If it effect modify it if not, leave the db as it is.



$$V_{R0} = 3I_{R0} X_n + I_{R0} X_{G0}$$

$$= I_{R0} (3X_n + X_{G0})$$

$$V_{R0} = I_{R0} (X_{G0} + 3X_n)$$

### Summary :-

- (1) Only +ve seq. network contain the voltage source.
- (2) -ve and zero seq. network don't contain voltage source
- (3) All voltage sources must be represented by 1p.u.
- (4) The condition of neutral as ~~not~~ has no effect in representing generator both in +ve & -ve seq. n/w.

(1) In the seq. N/w  $\rightarrow$  seq. of the and original phase is RYB. rotor is rotating in anticlockwise direction, so generator is produce <sup>the seq. voltage</sup> ~~power~~  $\therefore E_{\text{gen}} = E$ .

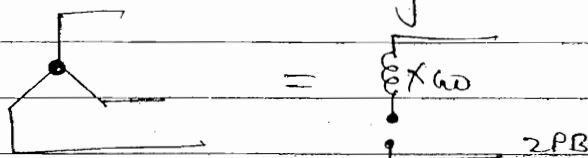
(2) In -ve seq. N/w  $\rightarrow$  seq of -ve and original phase is opposite, original  $\rightarrow$  RYB anticlock and -ve seq.  $\rightarrow$  RBY clock

is not possible, rotor will rotate in both direct at same time. for producing -ve seq. voltage rotor should rotate in clock wise. which is not possible so gen will not produce -ve seq. voltage  $\therefore E = 0$

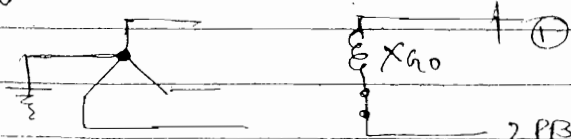
(3) In zero seq. N/w  $\rightarrow$  seq are in same direct<sup>n</sup> at the same time this is possible only when rotor is stationary. When rotor is stationary gen will not produce <sup>zero seq. voltage</sup> ~~power~~ so no zero seq. voltage  $\therefore E = 0$

(5) The condition of neutral has got effect in representing generator in zero sequence Network

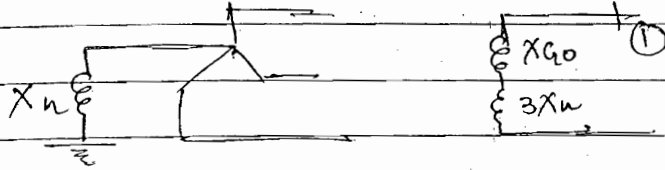
(6) If the neutral is ungrounded shown  $\rightarrow$  o.c.



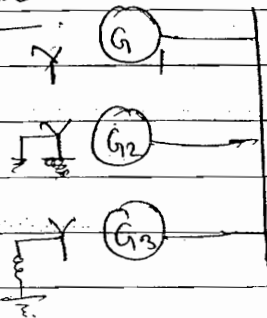
(7) If the neutral is ~~solidly~~ <sup>solidly</sup> grounded shown  $\rightarrow$  s.c



(8) If the neutral is reactance grounded then  $3X_n$  should be added to  $X_{G0}$  to get the total zero sequence imp reactance of generator.



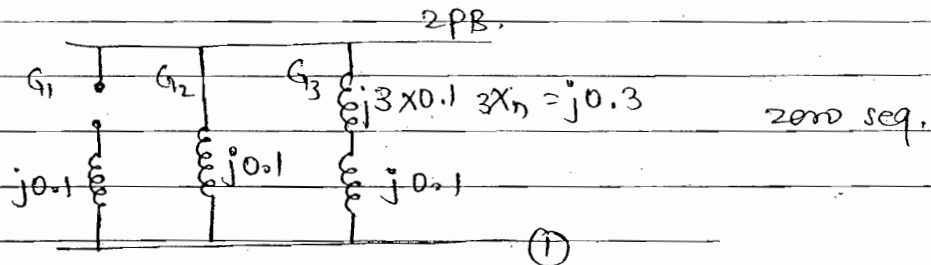
Problem



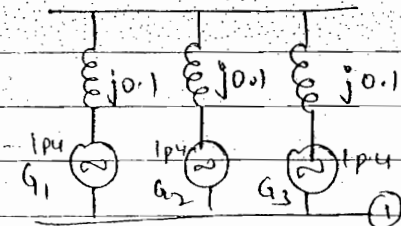
$$X_{G1} = X_n = j0.1$$

Show the seq. N/w.

Solution

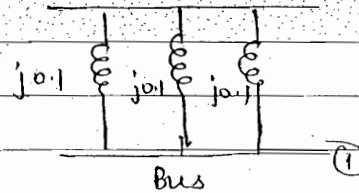


2PB.



pos seq. N/w.

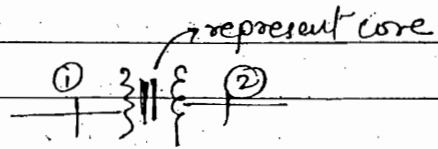
2PB.



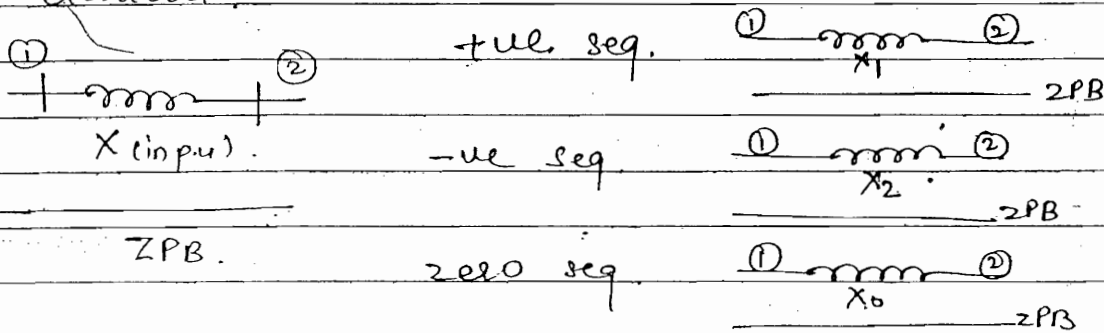
neg seq. N/w.

# Transformer Representation:-

→ X-mtr connected b/w bus 1 & 2

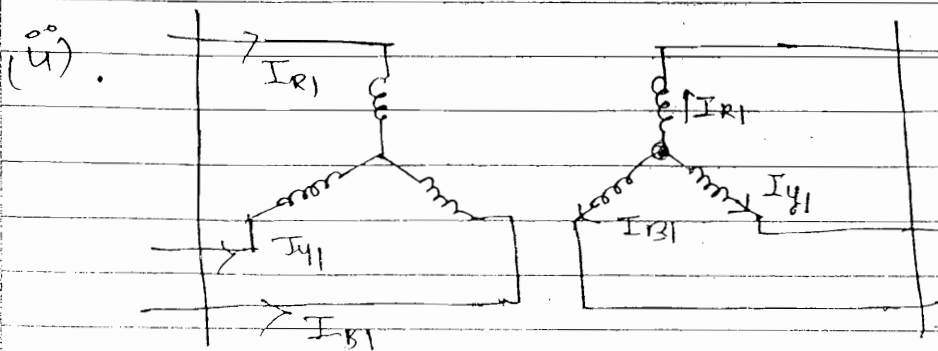
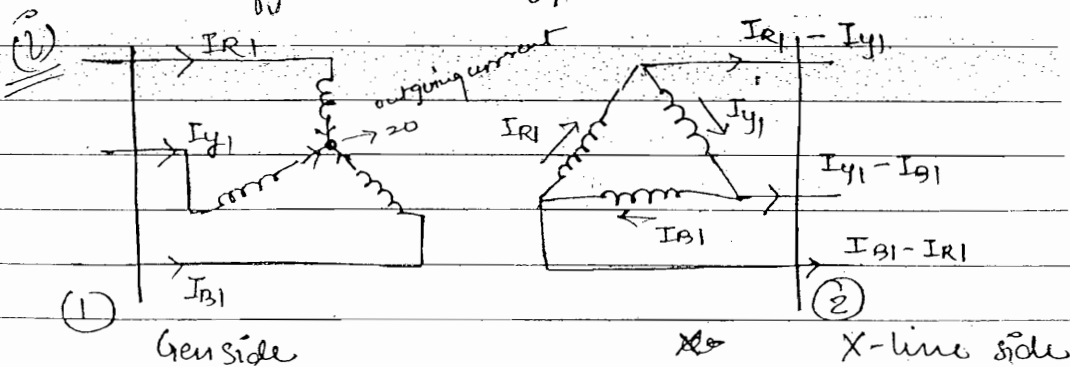


In the original N/w used in Symmetrical fault analysis, X-mtr is represented as a series reactance element.



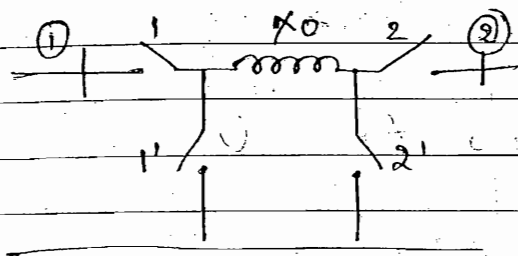
Conditions to verify:- primary & secondary wdg is Y or Δ there is an effect or not.

② If Y connect<sup>n</sup> neutral will effect the wdg. connecti<sup>n</sup> or not. If it effect we modify, otherwise leave it as.



Date 12<sup>th</sup> June 2014

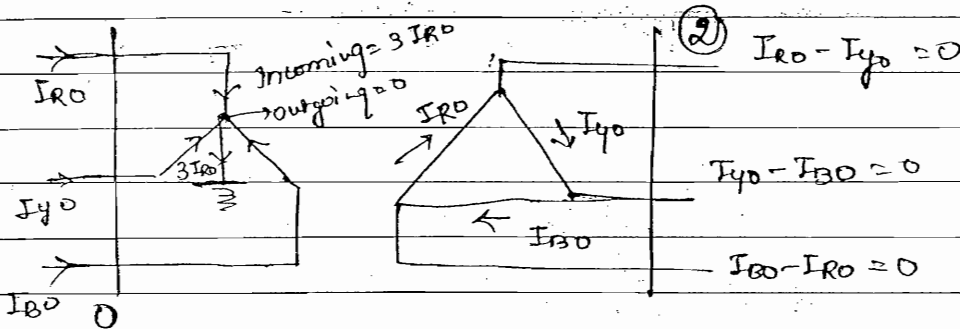
- Switch diagram (use to represent X-mtr zero seq.)  
NB.



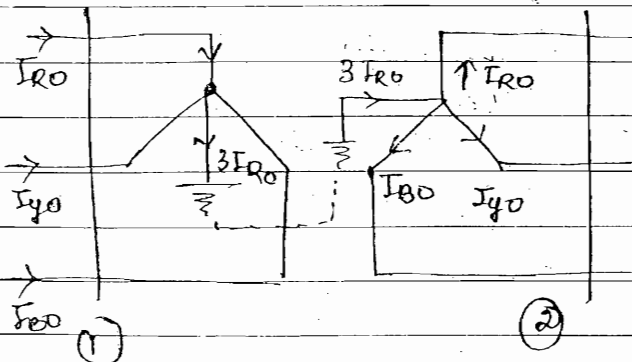
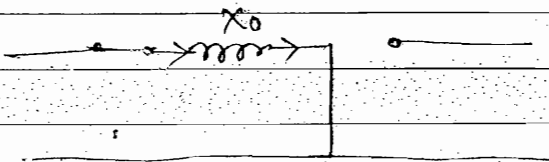
- 1, 1' → primary switches.
- 2, 2' → secondary switches.
- 1, 2 → Series switches.
- 1', 2' → shunt switches.

ZPB.

- A series switches when X-mtr wdg is Y connected and neutral grounded.
- A shunt switches close when the X-mtr wdg is  $\Delta$  conn.



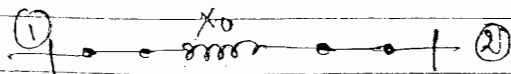
- switch 1, 2' closed.



- switch 1, 2 closed.

→ If the neutral ground imped reactance present then add

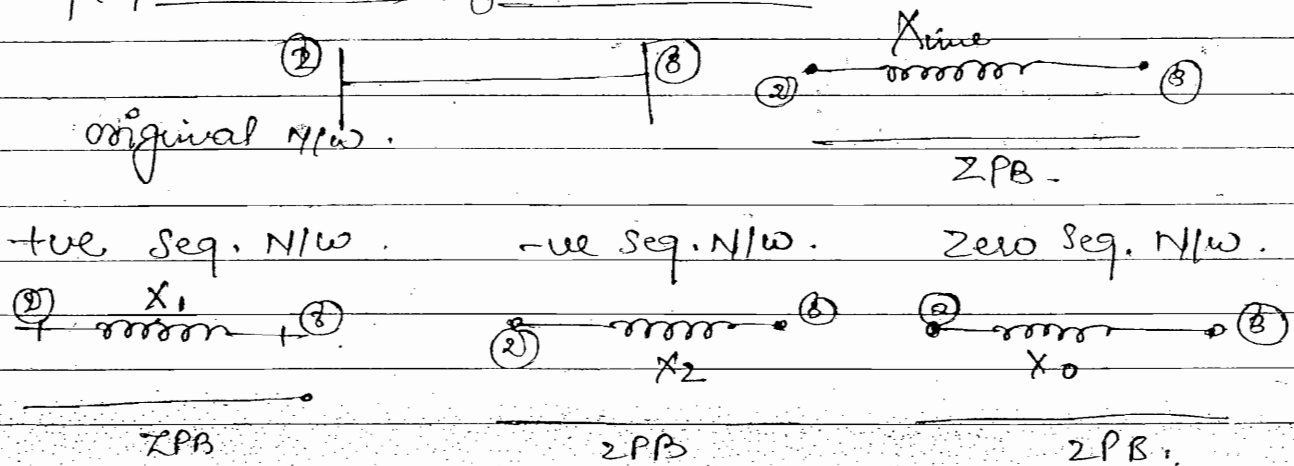
$$X_0 + 3X_{n1} - 13X_{n2} -$$



## • Summary

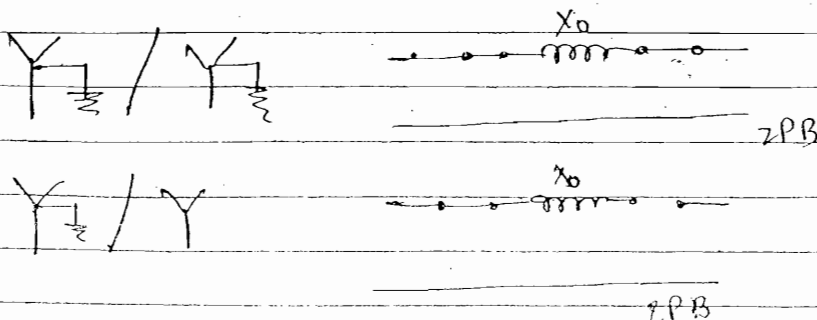
- (1) The type of wdg and the condition of neutral have got no effect in representing X-line both<sup>in</sup> +ve and -ve seq. N/w.
- (2) The type of wdg and the condition of neutral has got effect in representing X-line in zero seq. N/w. We use the switch diagram for this purpose.

## • Representation of TX-lines :-



↳ In all the 3-seq. N/w. X-line is represented as a series reactance element without verifying any condition as such. No condition exist for verification.

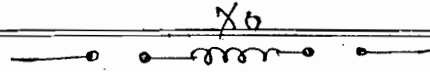
## Problem 4 Page NO. 66



driving point  $\rightarrow$  diagonal point

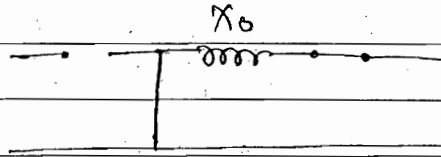
Date \_\_\_\_\_

Y/Y



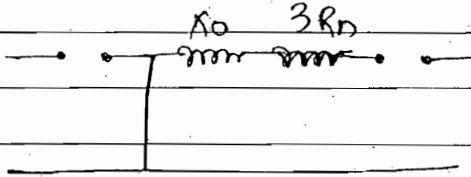
ZPB

$\Delta/Y$



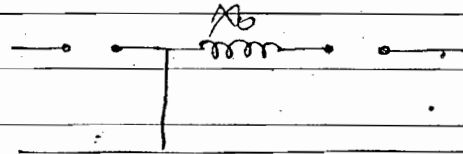
ZPB

$\Delta/Y_{\frac{1}{2}R}$



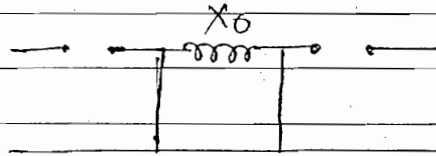
ZPB

$\Delta/Y$



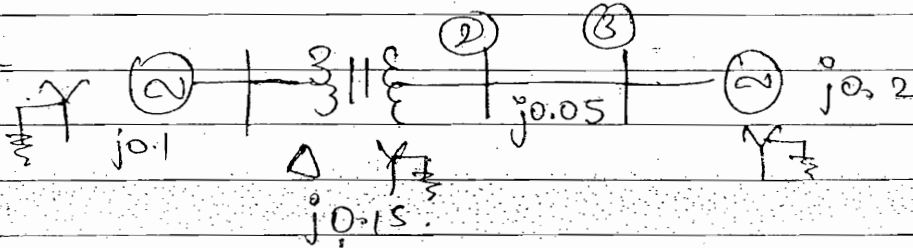
ZPB

$\Delta/\Delta$



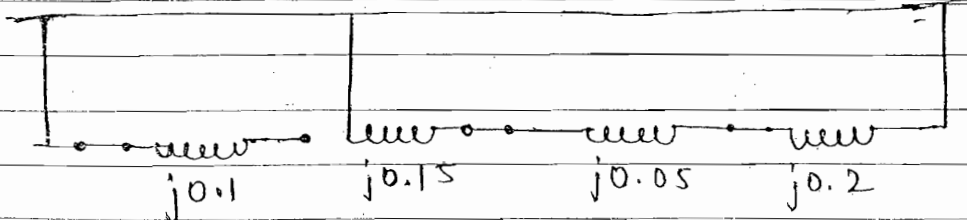
ZPB

Problem



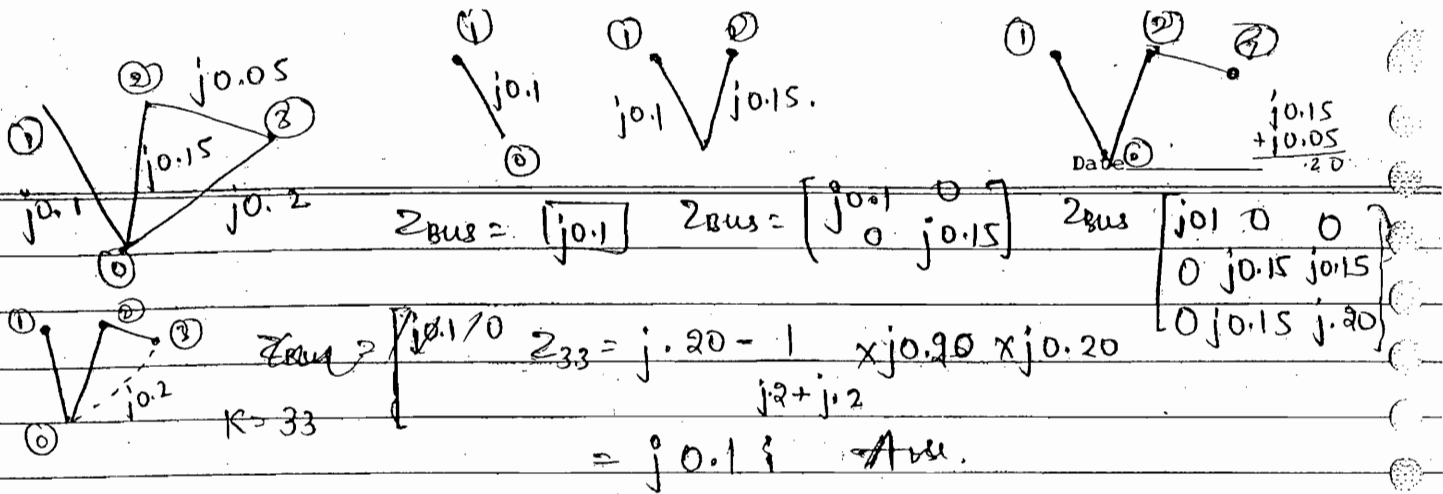
- $\rightarrow$  p.u. zero seq. reactances are given in fig.
- $\rightarrow$  find zero seq. driving point reactance at the bus (3).

ZPB

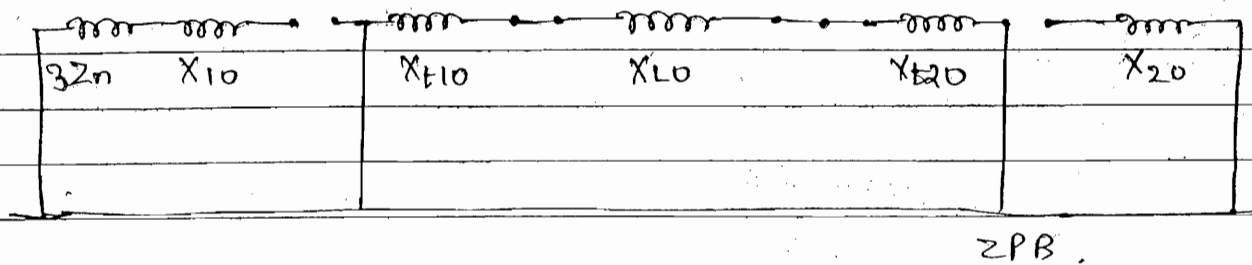


Zero seq.  
N/w

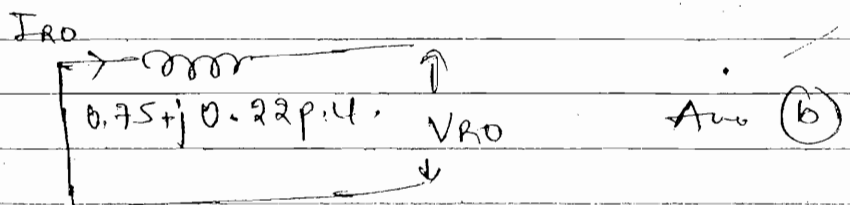
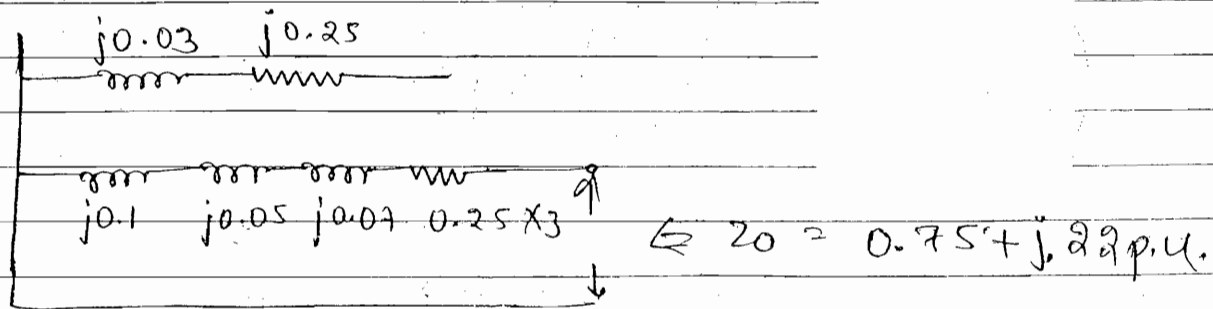
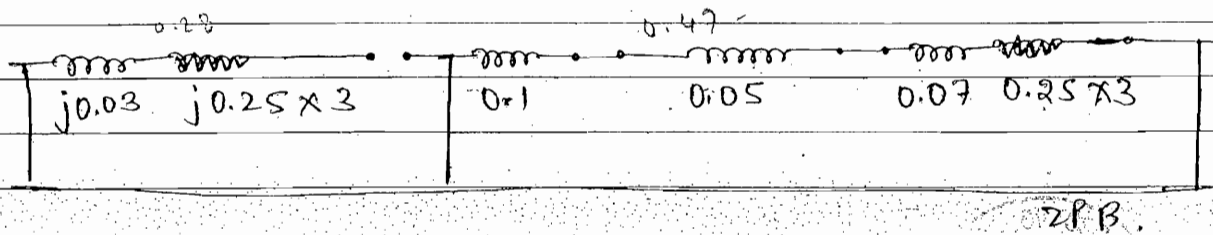
diagonal element called driving point  
off-diagonal element called transferred admittance



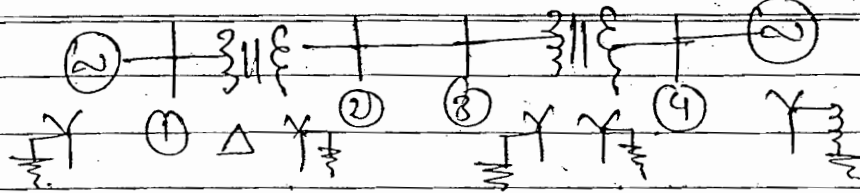
Problem 5.



Problem 14, Page No. 63.



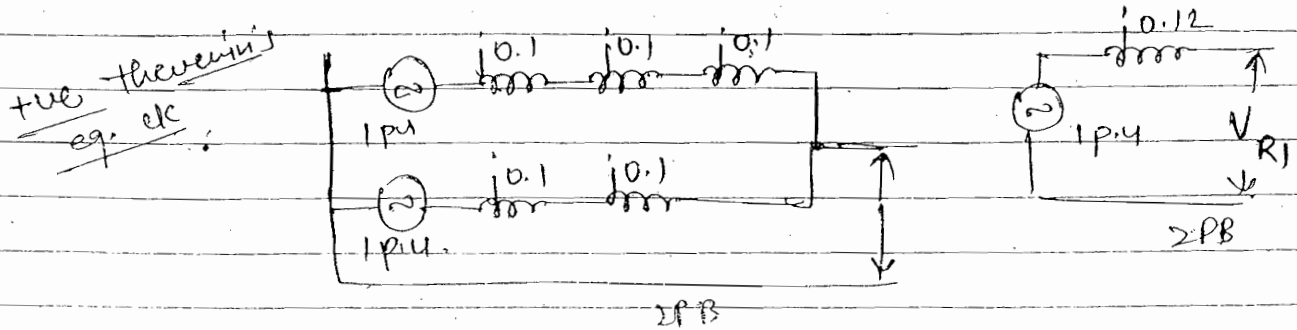
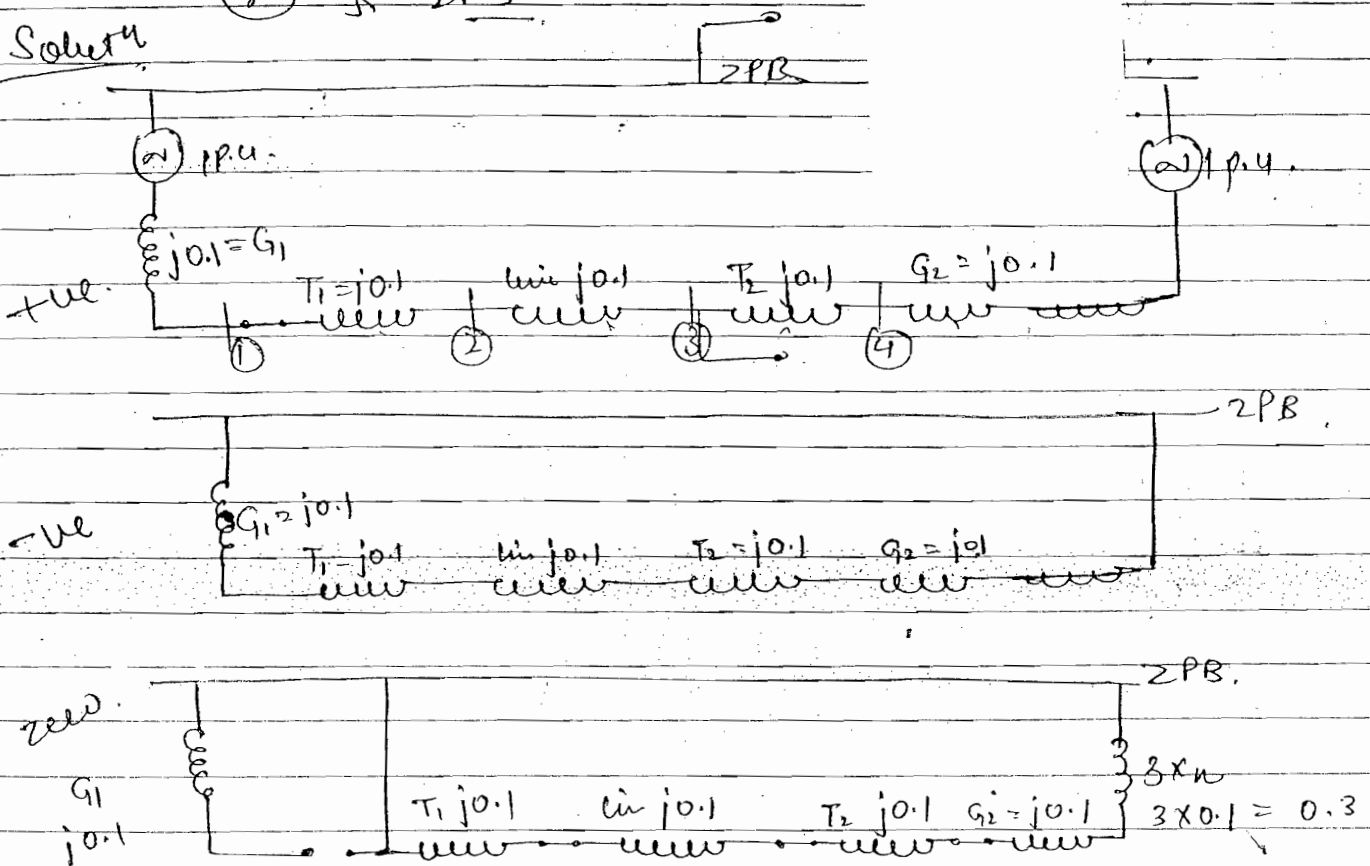
Problem



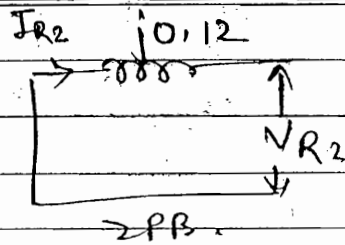
In the above Nlw. all the equipment has.  $X_1 = X_2 = X_0 = X_n = j0.1 \text{ pu}$ .

- (a) Draw the 3 seq. Nlw.
- (b) obtain thevenin's equivalent ckt across bus (3) & 2PB

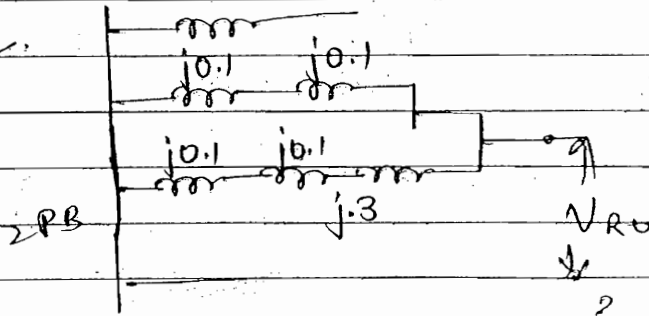
Solution



-ve terminal in  
circuit

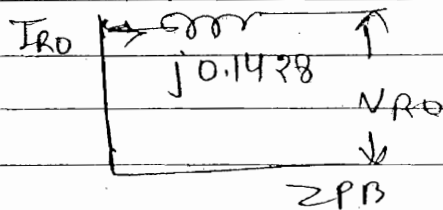


zero terminal  
circuit

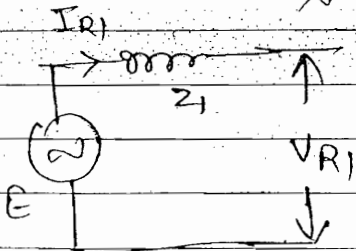


$$Z_0 = 0.2 \parallel 0.5$$

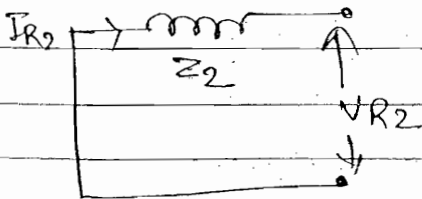
$$= 0.1428 \text{ p.u.}$$



### Formulas For Seq. Voltages -

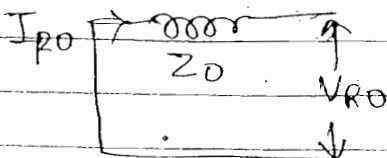


$$V_{R1} = E - I_{R1} Z_1$$



$$V_{R2} = E - I_{R2} Z_2$$

$$V_{R2} = -I_{R2} Z_2$$

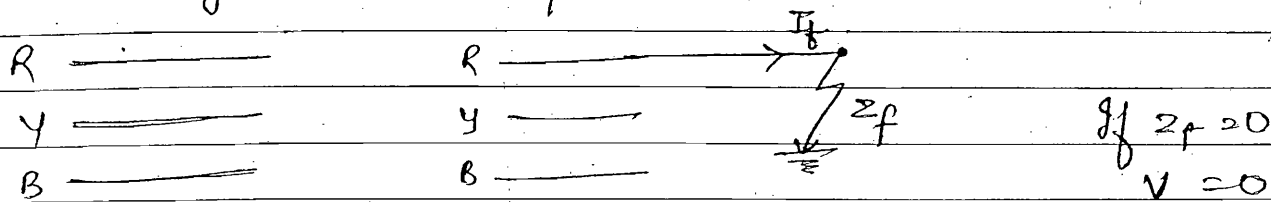


$$V_{R0} = E - I_{R0} Z_0$$

$$V_{R0} = -I_{R0} Z_0$$

# Unsymmetrical Fault Analysis <sup>Date</sup> \_\_\_\_\_

- Single line to Ground fault (SLG fault)



before fault

during fault

$$I_R + I_Y + I_B = 0$$

$$I_R = I_Y = I_B = 0$$

$$I_f = I_R$$

$$V_f = V_R = I_f Z_f$$

$$= I_R Z_f$$

Relation b/w Seq. Currents.

$$\begin{bmatrix} I_{R0} \\ I_{R1} \\ I_{R2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_f \\ 0 \\ 0 \end{bmatrix}$$

$$I_{R0} = \frac{1}{3} I_f = I_{R1} = I_{R2}$$

$$I_{R0} = I_{R1} = I_{R2} = \frac{I_f}{3} \quad (1)$$

Magnitude of Seq. Currents.

$$V_f = V_R = I_f Z_f = I_R Z_f = 3 I_{R1} Z_f$$

$$V_{R1} + V_{R2} + V_{R0} = 3 I_{R1} Z_f$$

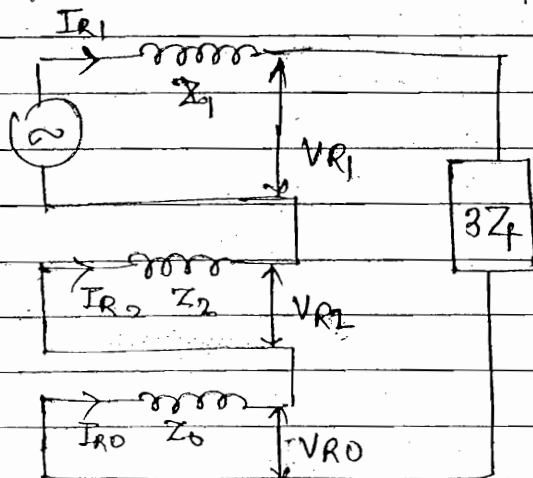
$$E - I_{R1} Z_1 - I_{R2} Z_2 - I_{R0} Z_0 = 3 I_{R1} Z_f \quad \therefore I_{R1} = I_{R0} = I_{R2}$$

$$E - I_{R1} (Z_1 + Z_0 + Z_2 + 3Z_f) = 0$$

$$I_{R1} = I_{R2} = I_{R0} = \frac{E}{Z_1 + Z_0 + Z_2 + 3Z_f}$$

$$I_{f, LG} = 3I_{R1} = 3I_{R2} = 3I_{R0} = \frac{3E}{Z_1 + Z_2 + Z_0 + 3Z_f}$$

- Sequence N/w of SLG fault.



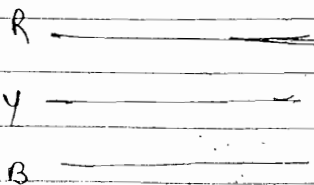
- Summary: (1) for SLG fault all the seq. N/w are connect in series.

$$(2) \quad I_{R1} = I_{R2} = I_{R0} = \frac{E}{Z_1 + Z_2 + Z_0 + 3Z_f}$$

$$(3) \quad I_{f, LG} = 3I_{R1} = 3I_{R2} = 3I_{R0} = \frac{3E}{Z_1 + Z_2 + Z_0 + 3Z_f}$$

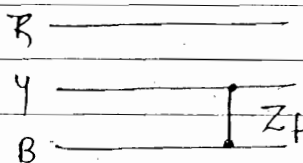
- Double line Fault:-

Before fault

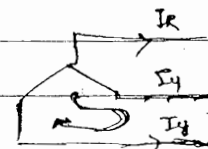


$$I_R = I_Y = I_B = 0$$

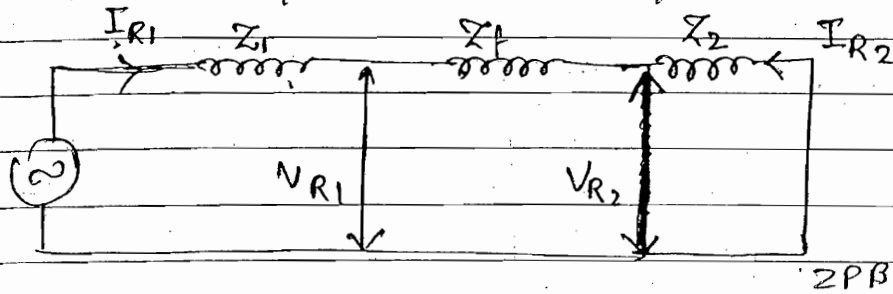
During fault



$$I_{f, LL} = I_Y = -I_B$$



Thavenin equivalent +ve Seq. Network.



$$\begin{bmatrix} I_{R0} \\ I_{R1} \\ I_{R2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_R \\ I_f \\ I_B \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} 0 \\ +I_f \\ -I_f \end{bmatrix}$$

$$I_{R0} = \frac{1}{3} [0 + aI_f - a^2I_f] = 0$$

$$I_{R1} = \frac{1}{3} [0 + aI_f - a^2I_f]$$

$$= \frac{1}{3} [a(1-a)I_f]$$

$$= \frac{\sqrt{3} \angle 90^\circ I_f}{3}$$

$$= \frac{I_f \angle 90^\circ}{\sqrt{3}}$$

$$I_{R2} = \frac{1}{3} [0 + a^2I_f - aI_f] \Rightarrow (a^2 - a)I_f$$

$$= -\frac{I_f \angle 90^\circ}{\sqrt{3}}$$

$$I_{R1} = -I_{R2}$$

$$I_{R0} = 0$$

Relat<sup>n</sup> b/w Seq. Current.

$$I_{R1} = -I_{R2} \quad \& \quad I_{R0} = 0 \quad \text{--- (1)}$$

Magnitude of Seq. Current.

$$I_{R1} = -I_{R2} = \frac{E}{Z_1 + Z_2 + Z_f}$$

$$I_{fLL} = I_y$$

$$I_{fLL} = I_y = I_{R0} + a^2 I_{R1} + a I_{R2} \\ = (a^2 - a) I_{R1}$$

$$\left\{ \begin{aligned} a^2 - a &= -0.5 - j0.866 - (-0.5 + j0.866) \\ &= -j1.732 = -j\sqrt{3} \end{aligned} \right\}$$

$$|I_{fLL}| = \sqrt{3} I_{R1} = \sqrt{3} I_{R2} = \frac{\sqrt{3} E}{Z_1 + Z_2 + Z_f}$$

• Summary :-

(1) For double line fault +ve & -ve seq. N/w are connected in series opposition.

$$(2) \quad I_{R1} = -I_{R2} = \frac{E}{Z_1 + Z_2}$$

(3) The magnitude of double line fault <sup>current</sup> will be

$$I_{fLL} = \sqrt{3} I_{R1} = \sqrt{3} I_{R2} = \frac{\sqrt{3} E}{Z_1 + Z_2}$$



$$3 \times 0.05 \left\{ \begin{array}{l} \text{---} \\ j0.04 \quad j0.5 \end{array} \right.$$

Date \_\_\_\_\_

$$V_n = 3.37 \angle -90^\circ \times 0.05 \angle 90^\circ$$

$$= 0.1685 \text{ p.u.}$$

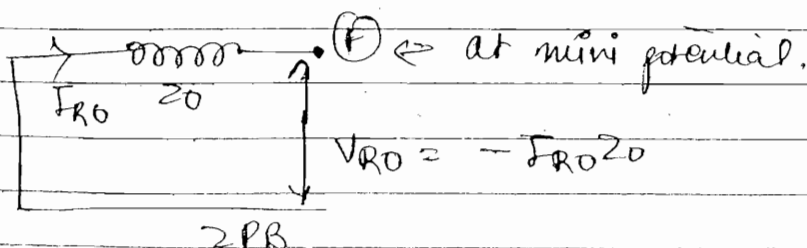
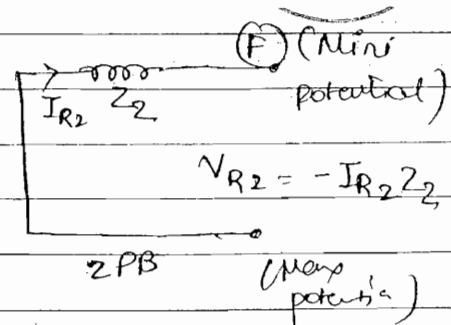
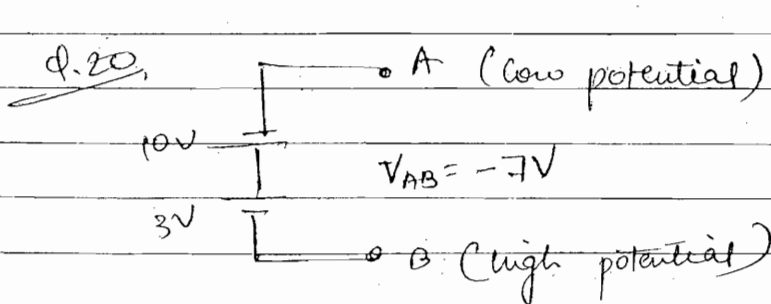
$$V_{ph, \text{Base}} = \frac{6.6 \times 1000}{\sqrt{3}} = 3810 \text{ V}$$

$$V_n = 0.1685 \times 3810$$

$$= 642.07 \text{ V}$$

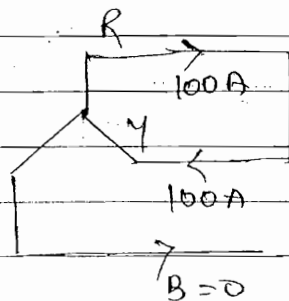
- Voltage b/w high and low voltage  $\rightarrow$  ground to any phase use "phase voltage"
- Voltage b/w high and high voltage  $\rightarrow$  b/w lines. use "line voltages"

$$Q.17 \quad \frac{3 \times 1}{0.15 + 0.15 + 0.05} = I_{f, Lg} = 8.553$$



Problem.. 28

Q Determine the magnitude of seq. current in a 3 $\phi$ , 3 wire  
 3/w when fault occur b/w R, Y phase  
 fault current  $\rightarrow 100\text{A}$

Solution

$$\begin{bmatrix} I_{R0} \\ I_{R1} \\ I_{R2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} 100 \\ -100 \\ 0 \end{bmatrix}$$

$$I_{R0} = \frac{1}{3} [100 - 100 + 0] = 0$$

$$I_{R1} = \frac{1}{3} [100 - 100 \cdot a] = \frac{100(1-a)}{3} = \frac{100 \times \sqrt{3} \angle -30^\circ}{3}$$

$$= 57.73 \angle -30^\circ$$

$$= 57.73 (\cos(-30) + j \sin(-30))$$

$$= 28.86 \text{ A } 50 - j 28.86 \text{ A}$$

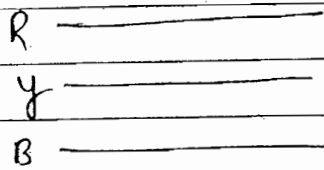
$$I_{R2} = \frac{1}{3} [100 - a^2 \cdot 100] = \frac{100}{3} [1 - a^2] = \frac{100 \cdot \sqrt{3} \angle 30^\circ}{3}$$

$$= 57.73 \angle 30^\circ$$

$$= 50 + j 28.86 \text{ A}$$

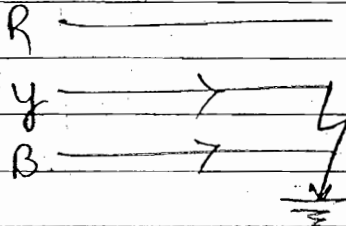
- Double line to Ground. (LLG fault).

Before fault



$$I_R = I_B = I_Y = 0$$

During fault



$$I_R = 0$$

$$I_{fug} = I_Y + I_B$$

Thevenin's eq. ~~and~~ Seq. N/w.

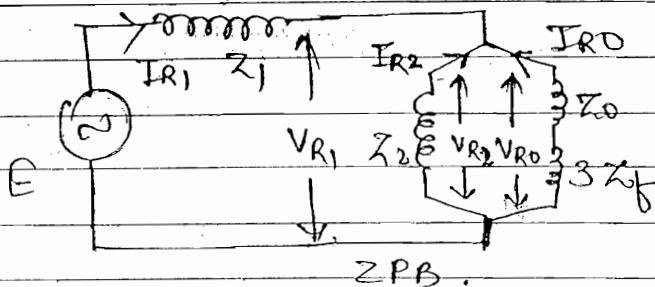


Fig. for LLG fault.

Relation B/w Seq. Current.

$$I_{R1} = -[I_{R2} + I_{R0}]$$

$$V_{R1} = V_{R2} = V_{R0}$$

$$I_{R1} = \frac{E}{Z_1 + (Z_2 \parallel Z_0 + 3Z_f)}$$

$$I_{R2} = -I_{R1} \times \frac{Z_0 + 3Z_f}{Z_2 + Z_0 + 3Z_f}$$

$$I_{R0} = -I_{R1} \times \frac{Z_2}{Z_2 + Z_0 + 3Z_f}$$

- Calculate  $I_R = I_{R1} + I_{R2} + I_{R0}$

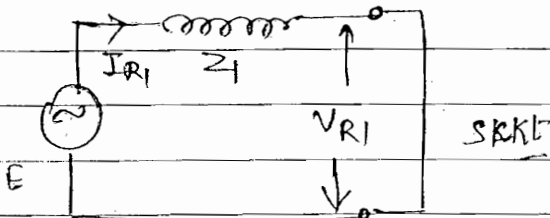
$$I_y = I_{R0} + a^2 I_{R1} + a I_{R2}$$

$$I_B = I_{R0} + a I_{R1} + a^2 I_{R2}$$

$$I_{f,3\phi} = I_y + I_B$$

### • 3- $\phi$ Fault

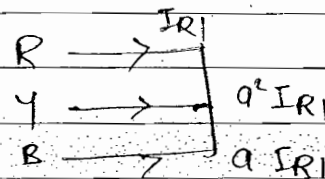
→ +ve and zero N/W is not present. Work only with +ve



• All the sequence voltage, including the sequence voltage are zeros.

$$V_{R1} = V_{R2} = V_{R0} = 0$$

$$I_{f,3\phi} = I_{R1} = \frac{E}{Z_1}$$



Problem 92 50 MVA, 11 KV.

$$I_{f,3\phi} = -j5 \text{ pu}$$

$$I_{fLL} = -j4 \text{ p.u.}$$

$$Z_1 = \frac{E}{I_{f,3\phi}} = \frac{1}{-j5 \text{ pu}} = 0.2 \text{ p.u.}$$

$$I_{fLL} = \frac{\sqrt{3} \times 1}{Z_1 + Z_2}$$

$$Z_1 + Z_2 = \frac{0.31}{-j4} = 0.95$$

$$Z_2 = 0.023 \text{ } 0.05$$

For Impedance always deal with phase values

Date \_\_\_\_\_

Problem 18

220 kV

3 $\phi$  fault = 4000 MVA

SLG fault = 5000 MVA

$$I_{f, 3\phi} = \frac{4000 \text{ MVA}}{\sqrt{3} \times 220 \text{ kV}} = 10.49 \text{ kA}$$

$$I_{f, 3\phi} = \frac{220 \times \sqrt{3}}{Z_1}$$

$$Z_1 = \frac{220 \times \sqrt{3}}{10.49} = 12.1 \Omega$$

$$I_{f, LG} = \frac{5000}{\sqrt{3} \times 220} = 13.12 \text{ kA}$$

$$I_{f, LG} = \frac{220 \sqrt{3}}{Z_1} = 3E$$

$$Z_0 + Z_2 + Z_1 = \frac{220 \sqrt{3}}{13.12 \text{ kA}}$$

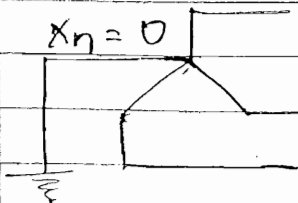
$$= 29.04 \Omega$$

$$\text{Assume } Z_1 = Z_2 = 12.1$$

$$\therefore Z_0 = 29.04 - 2 \times 12.1 = 4.84 \Omega$$

• Comparison of 3 $\phi$  & SLG Fault:-

(a) Solidly Grounded Alternator  $\rightarrow$



$$I_{f, 3\phi} = \frac{E}{X_{G1}}$$

$$I_{f, LG} = \frac{3E}{X_{G1} + X_{G2} + X_{G0} + 3X_f}$$

$$= \frac{3E}{\sqrt{3} \times 0}$$

0

Special Case

Date \_\_\_\_\_

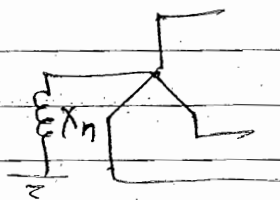
$$= \frac{3E}{X_{G1} + (\approx X_{G1}) + (\ll X_{G1})}$$

$$\approx \frac{3E}{2X_{G1}} \approx 1.5 \frac{XE}{X_{G1}}$$

LQ fault is more severe than the 3 $\phi$  fault.

$$\approx 1.5 \times I_{f, 3\phi}$$

(b) Generator Neutral Grounded with Reactance  $X_n$ .



$$I_{f, 3\phi} = \frac{E}{X_{G1}}$$

$$I_{f, LQ} = \frac{3E}{X_{G1} + X_{G2} + X_{G0} + 3X_n}$$

$$\text{If } X_n = \frac{1}{3} [X_{G1} - X_{G0}] \text{ say}$$

$$I_{f, LQ} = \frac{3E}{X_{G1} + X_{G2} + X_{G0} + 3 \times \frac{1}{3} [X_{G1} - X_{G0}]}$$

$$= \frac{3E}{2X_{G1} + X_{G2}} = \frac{3E}{3X_{G1}} \quad \therefore X_{G1} = X_{G2}$$

$$I_{f, 3\phi} = \frac{E}{X_{G1}}$$

$$\text{If } X_n \text{ is -}$$

|                                               |                            |
|-----------------------------------------------|----------------------------|
| $\rightarrow < \frac{1}{3} [X_{G1} - X_{G0}]$ | $I_{f, LQ} > I_{f, 3\phi}$ |
| $\rightarrow = \frac{1}{3} [X_{G1} - X_{G0}]$ | $I_{f, LQ} = I_{f, 3\phi}$ |
| $\rightarrow > \frac{1}{3} [X_{G1} - X_{G0}]$ | $I_{f, 3\phi} > I_{f, LQ}$ |

(c) X-mtr and X-line.  $\div$

$$I_{f, 3\phi} = \frac{E}{X_1}$$

$$I_{f, LQ} = \frac{3E}{X_1 + (\approx X_1) + (\gg X_1)}$$

$$\approx \frac{3E}{>3X_1} < \frac{E}{X_1}$$

$$I_{f, LG} < I_{f, 3\phi}$$

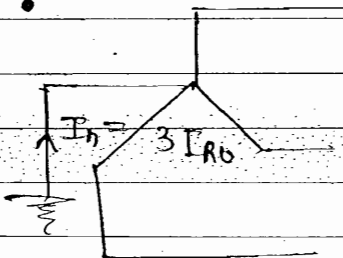
Problem 16  $X_n = \frac{1}{3} [X_{G1} - X_{G0}]$

$$X_n = \frac{1}{3} [0.1 - 0.05] = 0.0166 \text{ p.u.}$$

Problem 24  $I_{R1} = -(I_{R2} + I_{R0})$  LLG fault.

Problem 25  $I_{Y0} = I_{B0} = 0$  B coz. no return path. so.  
 $V_{Y0} = V_{B0} = 0$  also.

phase voltage are indeterminate coz no data for calculation but in line voltage zero seq. component will cancel each other.

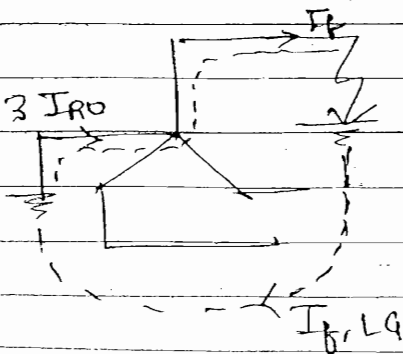


$$I_n = I_R + I_Y + I_B$$

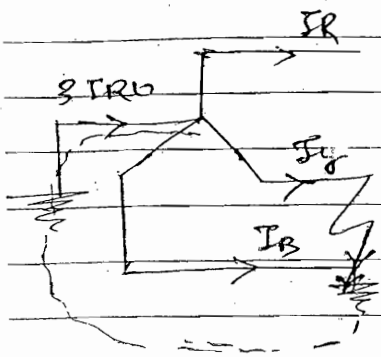
$$= (I_{R0} + I_{R1} + I_{R2}) + (I_{R0} + a^2 I_{R1} + a I_{R2}) + (I_{R0} + a I_{R1} + a^2 I_{R2})$$

$$I_n = 3I_{R0}$$

• LG fault



$$I_f = 3I_{R1} = 3I_{R2} = 3I_{R0}$$

• LLG fault

$$I_R = 0 \Rightarrow I_{R1} + I_{R2} + I_{R0} = 0 \quad (1)$$

$$I_{f,LLG} = I_Y + I_B$$

$$= (I_{R0} + a^2 I_{R1} + a I_{R2}) + (I_{R0} + a I_{R1} + a^2 I_{R2})$$

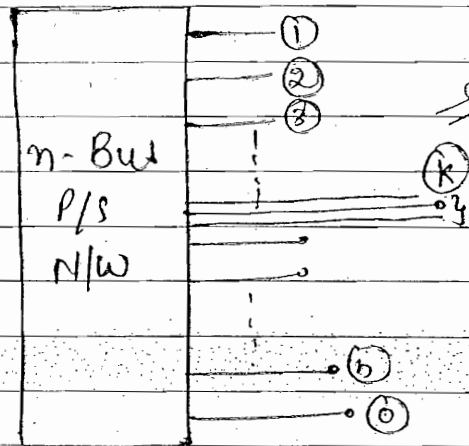
$$= 2 I_{R0} + (a + a^2) I_{R1} + (a + a^2) I_{R2}$$

$$= 2 I_{R0} - I_{R1} - I_{R2}$$

$$= 2 I_{R0} - (I_{R1} + I_{R2})$$

$$= 2 I_{R0} - (-I_{R0})$$

$$\boxed{I_{f,LLG} = 3 I_{R0}}$$

• Fault Analysis using  $Z_{bus}$  :-

containing { 3-phase voltages at faulty Bus  
3 phase current at faulty Bus  
3 phase voltages at healthy Bus

- fault at Bus k.
- healthy Bus is 3.

Step 1:- Depending upon type of fault, calculate the three sequence current at bus (k).

1. fault always occur at bus k.
2. Before fault current at all the buses are equal to zero.
3. The three phase currents at Bus (k)

Step 2:-

$$\left. \begin{array}{l} I_R \\ I_Y \\ I_B \end{array} \right\} (k)$$

$$\begin{bmatrix} I_R \\ I_Y \\ I_B \end{bmatrix}^{(K)} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{R0}^K \\ I_{R1}^K \\ I_{R2}^K \end{bmatrix}$$

Sequence voltage:

$$\begin{bmatrix} V_{R1}^{(1)} \\ V_{R1}^{(2)} \\ \vdots \\ V_{R1}^{(i)} \\ \vdots \\ V_{R1}^{(K)} \\ \vdots \\ V_{R1}^{(n)} \end{bmatrix} = \begin{bmatrix} E \\ E \\ \vdots \\ E \\ \vdots \\ E \\ \vdots \\ E \end{bmatrix} - \begin{bmatrix} Z_{11} & Z_{12} & \dots & Z_{1i} & \dots & Z_{1K} & \dots & Z_{1n} \\ Z_{21} & Z_{22} & \dots & Z_{2i} & \dots & Z_{2K} & \dots & Z_{2n} \\ \vdots & \vdots & & \vdots & & \vdots & & \vdots \\ Z_{i1} & Z_{i2} & \dots & Z_{ii} & \dots & Z_{iK} & \dots & Z_{in} \\ \vdots & \vdots & & \vdots & & \vdots & & \vdots \\ Z_{K1} & Z_{K2} & \dots & Z_{Ki} & \dots & Z_{KK} & \dots & Z_{Kn} \\ \vdots & \vdots & & \vdots & & \vdots & & \vdots \\ Z_{n1} & Z_{n2} & \dots & Z_{ni} & \dots & Z_{nK} & \dots & Z_{nn} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ \vdots \\ I_{R1}^{(K)} \\ \vdots \\ 0 \end{bmatrix}$$

(1)  
n x n

Step 2 .. Sequence voltages at faulted Bus K.

$$V_{R1}^{(K)} = E - Z_{KK}^{(1)} I_{R1}^{(K)}$$

$$V_{R2}^{(K)} = 0 - Z_{KK}^{(2)} I_{R2}^{(K)}$$

$$V_{R0}^{(K)} = -Z_{KK}^{(0)} I_{R0}^{(K)}$$

3  $\phi$  voltages at Bus(K)

$$\begin{bmatrix} V_R \\ V_Y \\ V_B \end{bmatrix}^{(K)} = [A] \begin{bmatrix} V_{R0}^K \\ V_{R1}^K \\ V_{R2}^K \end{bmatrix}$$

Step 3 :- Sequence voltage at bus(i),  $i \neq K$ 

$$V_{R1}^{(i)} = E - Z_{iK}^{(1)} I_{R1}^{(K)}$$

$$V_{R2}^{(i)} = -I_{R2}^{(K)} Z_{iK}^{(2)}$$

$$V_{R0}^{(i)} = -I_{R0}^{(K)} Z_{iK}^{(0)}$$

3  $\phi$  voltages at Bus (i),

$$\begin{bmatrix} V_R \\ V_Y \\ V_B \end{bmatrix}^{(i)} = [A] \begin{bmatrix} V_{R0}^{(i)} \\ V_{R1}^{(i)} \\ V_{R2}^{(i)} \end{bmatrix}$$

• 3  $\phi$  fault

A 3  $\phi$  fault occurs at Bus (K) (Not -ve and zero seq. current).

$$I_{R1}^{(K)} = I_{f3\phi}^{(K)}$$

$$I_{R1}^{(K)} = \frac{E}{Z_{KK}^{(1)}}$$

$$\begin{cases} V_{R1}^{(K)} = E - Z_{KK}^{(1)} I_{R1}^{(K)} \\ 0 = E - Z_{KK}^{(1)} I_{R1}^{(K)} \\ I_{R1}^{(K)} = \frac{E}{Z_{KK}^{(1)}} \end{cases}$$

$$I_{R2}^{(K)} = I_{R0}^{(K)} = 0$$

$\rightarrow$  3  $\phi$  currents at Bus (K)

$$\begin{array}{l} R \rightarrow I_{R1}^{(K)} \\ Y \rightarrow a^2 I_{R1}^{(K)} \\ B \rightarrow a I_{R1}^{(K)} \end{array}$$

$\rightarrow$  3  $\phi$  voltages at Bus K :- all the sequence voltages are zero for 3- $\phi$  fault

$$V_{R1}^{(K)} = V_{R2}^{(K)} = V_{R0}^{(K)} = 0$$

$$\Rightarrow V_R = V_Y = V_B = 0$$

$\rightarrow$  3  $\phi$  voltages at Bus (i)

$$V_{R1}^{(i)} = E - I_{R1}^{(K)} Z_{ik}^{(1)}$$

$$V_{R2}^{(i)} = V_{R0}^{(i)} = 0$$

$$\begin{array}{l} R \rightarrow V_{R1}^{(i)} \\ Y \rightarrow a^2 V_{R1}^{(i)} \\ B \rightarrow a V_{R1}^{(i)} \end{array} \quad \textcircled{1}$$

$$i = 1, 2, \dots, n; K \neq i$$

• SLG Fault :- A LG fault occurs at Bus (K)

$$I_{R1}^{(K)} = I_{R2}^{(K)} = I_{R0}^{(K)} = \frac{E}{Z_{KK}^{(1)} + Z_{KK}^{(2)} + Z_{KK}^{(0)}}$$

$$I_{f1g}^{(K)} = \frac{3E}{Z_{KK}^{(1)} + Z_{KK}^{(2)} + Z_{KK}^{(0)}}$$

Calculate sequence voltage at Bus (i) & (K). Calculate the 3- $\phi$  voltage at bus (i) & (K)

$$\begin{bmatrix} I_R \\ I_Y \\ I_B \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{R1}^{(K)} \\ I_{R1}^{(K)} \\ I_{R1}^{(K)} \end{bmatrix}$$

$$I_R = 3 I_{R1}^{(K)}$$

$$I_Y = I_{R1}^{(K)} + I_{R1}^{(K)} a^2 + I_{R1}^{(K)} a$$

$$= I_{R1}^{(K)} (1 + a^2 + a)$$

$$= 0$$

$$I_B = I_{R1}^{(K)} + I_{R1}^{(K)} a + I_{R1}^{(K)} a^2$$

$$= 0$$

$$V_{R1}^K = E - I_{R1}^K Z_{KK}^{(1)}$$

$$V_{R2}^K = - I_{R2}^K Z_{KK}^{(2)}$$

$$V_{R0}^K = - I_{R0}^K Z_{KK}^{(0)}$$

$$\begin{bmatrix} V_R^K \\ V_Y^K \\ V_B^K \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{R0}^K \\ V_{R1}^K \\ V_{R2}^K \end{bmatrix}$$

$$V_{R1}^i = E - I_{R1}^K Z_{ik}^{(1)}$$

$$V_{R2}^i = - I_{R2}^K Z_{ik}^{(2)}$$

$$V_{R0}^i = - I_{R0}^K Z_{ik}^{(0)}$$

$$\begin{bmatrix} V_R^i \\ V_Y^i \\ V_B^i \end{bmatrix} = [A] \begin{bmatrix} V_{R0}^i \\ V_{R1}^i \\ V_{R2}^i \end{bmatrix}$$

Problem 28

$$I_{R1}^{(2)} = I_{f, 3\phi}^{(2)} = \frac{1}{Z_{22}^{(1)} + j0.24} = \frac{1}{4.18 \angle -90^\circ \text{ p.u.}}$$

$$V_{R1}^{(1)} = 1 \angle 0^\circ - Z_{12}^{(1)} I_{R1}^{(2)} = 1 \angle 0^\circ - 0.08 \angle 90^\circ \times 4.18 \angle -90^\circ = 0.667 \text{ p.u.}$$

$$V_{R1}^{(3)} = 1 \angle 0^\circ - Z_{32}^{(1)} I_{R1}^{(2)} = 1 \angle 0^\circ - 0.16 \angle 90^\circ \times 4.18 \angle -90^\circ = 0.33 \text{ p.u.}$$

• LL Fault : A LL fault occurs at Bus R

$$I_{R1}^K = -I_{R2}^K \quad I_{R0}^K = 0$$

$$I_{R1}^K = \frac{E}{Z_{KK}^{(1)} + Z_{KK}^{(2)}} \quad V_{R0}^K = 0$$

$$I_{f, LL} = \frac{\sqrt{3} E}{Z_{RK}^{(1)} + Z_{RK}^{(2)}}$$

$$\begin{bmatrix} I_R \\ I_Y \\ I_B \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ I_Y \\ -I_Y \end{bmatrix} = [A] \begin{bmatrix} 0 \\ I_{R1}^K \\ -I_{R1}^K \end{bmatrix}$$

$$V_{R1}^K = E - I_{R1}^K Z_{1K}^{(1)}$$

$$V_{R2}^K = -I_{R2}^K Z_{2K}^{(2)}$$

$$\begin{bmatrix} V_R \\ V_Y \\ V_B \end{bmatrix}^K = [A] \begin{bmatrix} 0 \\ V_{R1}^K \\ V_{R2}^K \end{bmatrix}$$

$$V_{R1}^i = E - I_{R1}^K Z_{1K}^{(1)}$$

$$V_{R2}^i = -I_{R2}^K Z_{2K}^{(2)}$$

$$\begin{bmatrix} V_R \\ V_Y \\ V_B \end{bmatrix}^i = [A] \begin{bmatrix} 0 \\ V_{R1}^i \\ V_{R2}^i \end{bmatrix}$$

# LOAD FLOW / POWER FLOW STUDIES

Date: \_\_\_\_\_

• study well for interview and for conventional quest<sup>n</sup>.

• Study have three stages:-

- (1) N/w Modelling (electrical equivalent N/w)
- (2) Mathematical Modelling (equat<sup>n</sup>, calculat<sup>n</sup>)
- (3) Solution stages

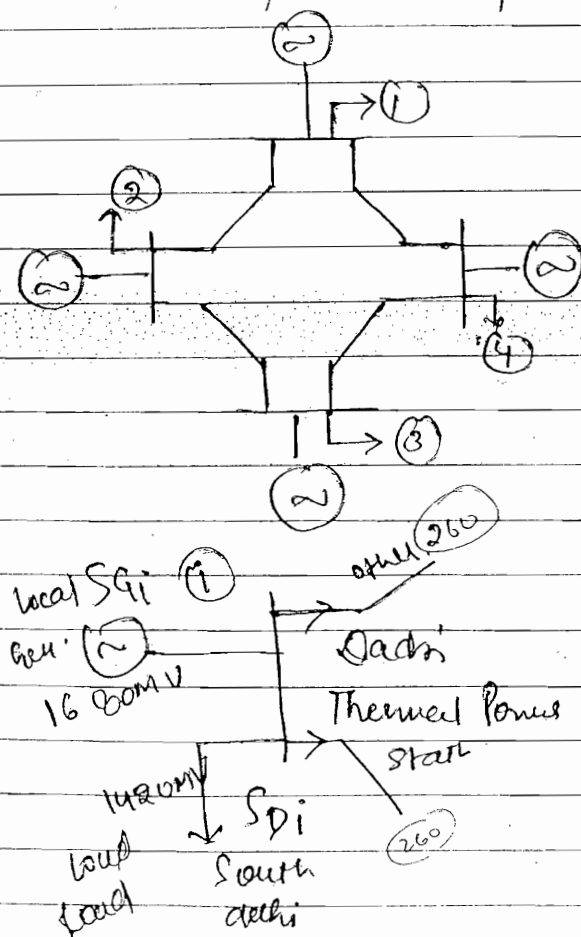
{ For steady state study  $\rightarrow$  algebraic equat<sup>n</sup> used  
 { For dynamic study  $\rightarrow$  differential equat<sup>n</sup> used

• load flow study is Steady State Study:

$\rightarrow$  Information from LF studies.

In the stage - 1 at each Bus we obtain 4 Bus quantities.

- (i) Injected active power  $P_i$
- (ii) Injected reactive power  $Q_i$
- (iii) Magnitude of Bus voltage  $|V_i|$
- (iv) load / Torque angle  $\delta_i$



$$S_{Gi} = P_{Gi} + j Q_{Gi}$$

= Complex power generated at  $i^{th}$  Bus.

$$S_{Di} = \text{Complex power demand at } i^{th} \text{ Bus}$$

$$= P_{Di} + j Q_{Di}$$

active power demand      reactive power demand

active and reactive power are net power that can be supply to other x-line.

$$S_i = S_{Gi} - S_{Di}$$

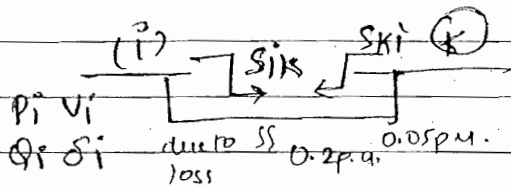
If  $S_i > 0$  +ve exporting Bus.  
 $S_i < 0$  -ve importing Bus. (as we as)

$\delta_i$  = phase difference of  $i^{\text{th}}$  Bus w.r.t Ref Bus.

$$\delta_2 = \angle V_2 - \angle V_{\text{ref}}$$

$$\delta_{10} = \angle V_{10} - \angle V_{\text{ref}}$$

→ we determine line flows / losses.



$S_{ik}$  = complex power flowing from  $i^{\text{th}}$  Bus to  $k^{\text{th}}$  Bus.

$$S_{ik} = -1 + j0.9$$

$$S_{ki} = \frac{1.2 + j0.85}{0.2 + j0.05}$$

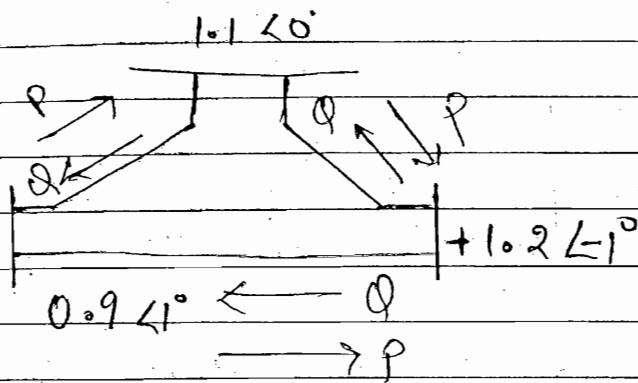
→ we get info about how much power is flowing  
 → we get info about how much power losses taking place

NOTE (i) Active power always flow from leading angle Bus to lagging angle Bus.

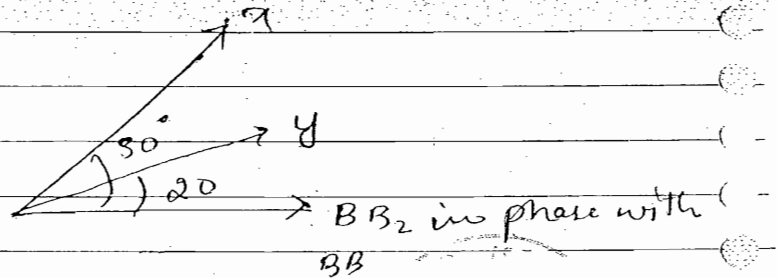
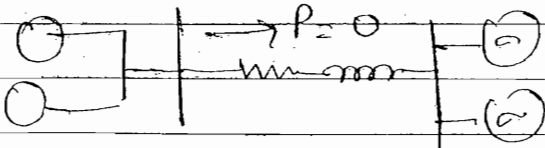
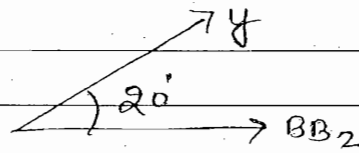
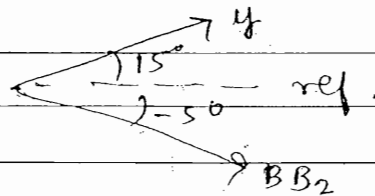
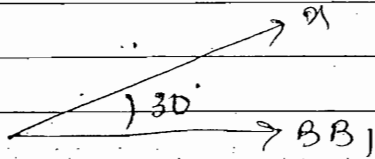
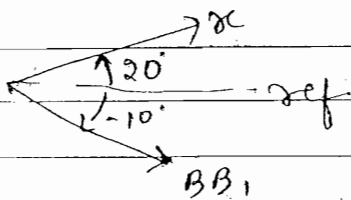
(ii) Reactive power always flow from high potential Bus to low potential Bus.

(iii) No angle difference  $\Rightarrow$  No active power flow

(iv) If no potential difference No reactive power flow.



Problem 10 QNO. 47.

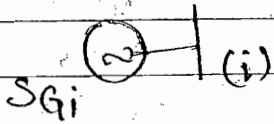


x leads y By  $10^\circ$ .

## • Network Modelling

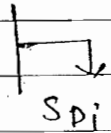
(1) In load flow studies

(i) Generators are represented as complex power source.



$$S_{Gi} = P_{Gi} + jQ_{Gi}$$

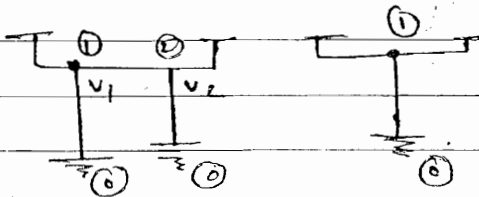
(ii) Similar to generators loads are also represented as complex power demands.



$$S_{Di} = P_{Di} + jQ_{Di}$$

(iii)  $\pi$ -line is represented as a  $\pi$  NW with the series admittance ( $y_{ik}$ ) and half line charging admittance

$\therefore$  T NW is better than  $\pi$  NW. In  $\pi$  NW we have to solve more no. of eqn and % of regulat<sup>n</sup> is high compare to T NW. But  $\pi$  NW is mathematical convenience. T NW is more efficient.



$$\% \text{ Regulat}^n \quad N_{\pi, \pi} < V_{\pi, \pi}$$

$\therefore$  long shoot is not use they are different eq<sup>n</sup>

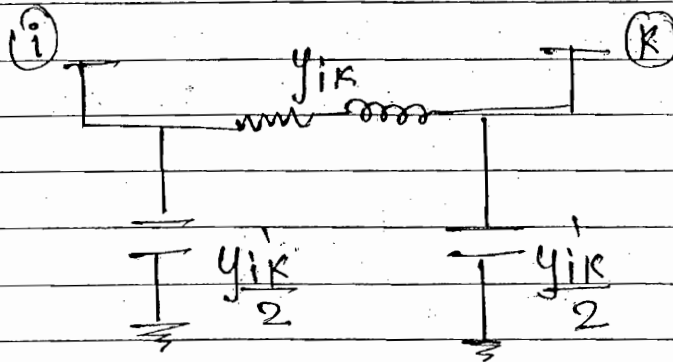
$$y_{ik}^1 = j \frac{\omega C_{ik}}{2} \quad \text{half line charge}$$

$$Z_{ik} = \text{series impedances of the line} = R_{ik} + jX_{ik}$$

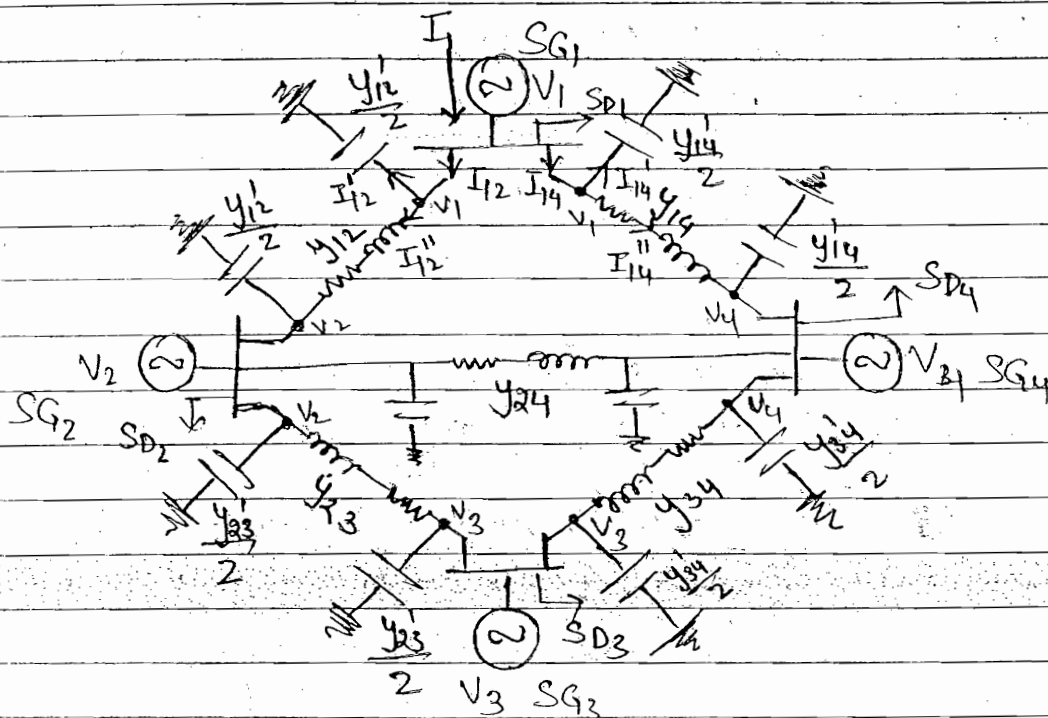
$$y_{ik}^0 = \frac{1}{Z_{ik}} = \frac{R_{ik}}{R_{ik}^2 + j^2 X_{ik}^2} - j \frac{X_{ik}}{R_{ik}^2 + X_{ik}^2}$$

$$= G_{ik} - jB_{ik}$$

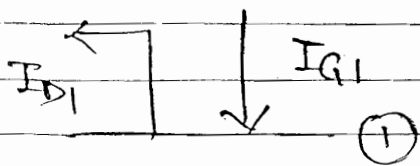
$y_{ik} \rightarrow$  It is not a electrical parameter It is a mathematical parameter.



We are using  
 $\pi$  NW box  
of mathematical  
component.



Mathematical Modelling.

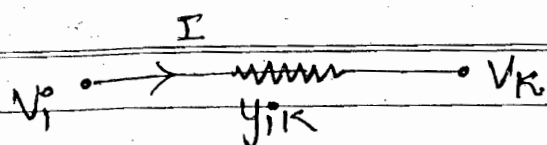


Injected current into 1<sup>st</sup> Bus -

$$I_1 = I_{G1} - I_{D1}$$

$$I_1 = I_{12} + I_{14}$$

$$I_1 = (I_{12}' + I_{12}'') + (I_{14}' + I_{14}'')$$



Date: \_\_\_\_\_

$$I = (V_i - V_k) y_{ik}$$

$$I_1 = \left( (V_1 - 0) \frac{y_{12}}{2} + (V_1 - V_2) y_{12} \right) + \left( (V_1 - 0) \frac{y_{14}}{2} + (V_1 - V_4) y_{14} \right)$$

$$= V_1 \left( \frac{y_{12}}{2} + \frac{y_{14}}{2} + y_{12} + y_{14} \right) + (-V_2 y_{12}) + (-V_4 y_{14}) + (0)V_3$$

$$\bullet I_1 = Y_{11} V_1 + Y_{12} V_2 + Y_{14} V_4 + Y_{13} V_3$$

$Y_{11}$  = Total admittance connected to 1<sup>st</sup> Bus

$$= \frac{y_{12}}{2} + \frac{y_{14}}{2} + y_{12} + y_{14}$$

$Y_{12}$  = Negative value of Series admittance connected b/w the buses (1) & (2).

$Y_{13} = 0$  (No direct connect<sup>n</sup> b/w (1) & (3))

$$Y_{14} = -Y_{41}$$

$$\bullet I_2 = Y_{21} V_1 + Y_{22} V_2 + Y_{23} V_3 + Y_{24} V_4$$

$$Y_{12} = Y_{21} = -Y_{12}$$

$$Y_{22} = \frac{y_{12}}{2} + \frac{y_{23}}{2} + \frac{y_{24}}{2} + y_{12} + y_{23} + y_{24}$$

$$Y_{23} = Y_{32} = -Y_{23}$$

$$Y_{24} = Y_{42} = -Y_{24}$$

General Expression :-

$$I_i = Y_{i1} V_1 + Y_{i2} V_2 + Y_{i3} V_3 + \dots + Y_{ii} V_i + \dots + Y_{in} V_n$$

$i = 1, 2, \dots, n-1, n$

$$= \sum_{k=1}^n Y_{ik} V_k \quad \text{--- (2)}$$

• In Matrix form:-

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

$$[I_{Bus}]_{n \times 1} = [Y_{Bus}]_{n \times n} [V_{Bus}]_{n \times 1}$$

- $Y_{Bus}$  is preferred in load flow studies bcoz it is sparsity matrix.
- The advantages of sparsity techniques are reduce memory requirement and  $\uparrow$  C.P.U speed.
- Sparsity techniques are useful for online applicat<sup>n</sup>.
- Since  $Y_{Bus}$  is a sparsity Matrix its inverse  $Z_{Bus}$  is full Matrix.
- $Z_{Bus}$  is dominated by non-zero elements.
- With more no. of non-zero elements  $Z_{Bus}$  matrix provides properties of the network more properly and used is used in s.c studies.
- The sum of all the element in each row of  $Y_{Bus}$  matrix is zero then corresponding row of  $Y_{Bus}$  not having shunt element. If it is non-zero the corresponding Bus has having shunt element.
- The non-zero off diagonal elements either is upper or lower triangle gives the no. of x-lines in pow. sys. N/w.
- If the degree of sparsity  $\downarrow$  no. of x-line  $\uparrow$  and vice-versa.

- ✓ for inductive load  $P + jQ$
- ✓ for capacitive load  $P - jQ$

Date \_\_\_\_\_



Complex power injected into  $i^{\text{th}}$  Bus

$$S_i = P_i + jQ_i = V_i I_i^*$$

• when  $V_i I_i^*$  conjugate

for the inductive load  $S = P + jQ$

→  $V = V \angle 0^\circ$  Ref.  $I = I \angle -\theta$  lags  $V$ .

$$S = VI$$

$$= V \angle 0^\circ I \angle -\theta$$

$$= VI \angle -\theta$$

$$= VI (\cos(-\theta) + j \sin(-\theta))$$

$$= VI \cos \theta - j VI \sin \theta$$

$$= P - jQ$$

If we take

$$S = VI^*$$

$$= VI \cos \theta + j VI \sin \theta$$

$$= P + jQ$$

$$S_i = V_i \left[ \sum_{k=1}^n Y_{ik} V_k \right]^*$$

$$S_i^* = V_i^* \sum_{k=1}^n Y_{ik}^* V_k$$

$$= |V_i| \angle -\delta_i \sum_{k=1}^n Y_{ik} V_k$$

$$\therefore S_i^* = P_i + j(-Q_i)$$

$$\begin{cases} P_i = \{ \text{Real term of } S_i^* \} \\ Q_i = - \{ \text{Imag. term of } S_i^* \} = -(-Q) = +Q \\ Q_i = - \text{Imag } \{ S_i^* \} \end{cases}$$

$$S_i^* = V_i^* I_i = P_i - jQ_i$$

$$= V_i^* \sum_{k=1}^n Y_{ik} V_k$$

\* The eqn of  $P_i$  and  $Q_i$  are static load flow  
equation  
 Date \_\_\_\_\_

Let  $V_i = |V_i| \angle \delta_i$   
 $V_i^* = |V_i| \angle -\delta_i$

$$Y_{ik} = G_{ik} - jB_{ik} = |Y_{ik}| \angle -\gamma_{ik}$$

$$V_k = |V_k| \angle \delta_k$$

$$\begin{cases} |Y_{ik}| = \sqrt{G_{ik}^2 + B_{ik}^2} \\ \gamma_{ik} = \tan^{-1} \left( \frac{B_{ik}}{G_{ik}} \right) \end{cases}$$

By substituting the terms,

$$S_i^* = P_i - jQ_i$$

$$= V_i^* I_i$$

$$= |V_i| \angle -\delta_i \sum_{k=1}^n |Y_{ik}| \angle -\gamma_{ik} \cdot |V_k| \angle \delta_k$$

$$= \sum_{k=1}^n |V_i| |Y_{ik}| |V_k| \angle -(\delta_i + \gamma_{ik} - \delta_k)$$

$$\checkmark P_i = \text{Real} \{ S_i^* \}$$

$$= \sum_{k=1}^n |V_i| |V_k| |Y_{ik}| \cos(\delta_i - \delta_k + \gamma_{ik})$$

$$\checkmark Q_i = -\text{Imag} \{ S_i^* \}$$

$$= \sum_{k=1}^n |V_i| |V_k| |Y_{ik}| \sin(\delta_i - \delta_k + \gamma_{ik})$$

$$V_i^* I_i = P_i - jQ_i = S_i^*$$

$$I_i = \frac{P_i - jQ_i}{V_i^*}$$

$$I_i = \sum_{k=1}^n V_k Y_{ik}$$

$$\sum_{k=1}^n V_k Y_{ik} = \frac{P_i - jQ_i}{V_i^*}$$

$$Y_{ii} V_i + \sum_{\substack{k=1 \\ k \neq i}}^n Y_{ik} V_k = \frac{P_i - jQ_i}{V_i^*}$$

$$\left| V_i = \frac{1}{Y_{ii}} \left[ \frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{k=1 \\ k \neq i}}^n Y_{ik} V_k \right] \right| = |V_i| \angle \delta_i$$

polar form

•  $P_i, Q_i$  are simultaneous eq<sup>n</sup>

↳ The load flow equations are non-linear & simultaneous algebraic equation

Objective question: Gauss Seidal → use to linear or non-linear simultaneous eq<sup>n</sup>

• GAUSS SEIDEL METHOD

↳ No. of unknown, no. of eq<sup>n</sup>

$$y_1 = f_1(x_1, x_2) = x_1^2 - 2 \log x_1 x_2 - x_2^2 \quad \text{--- (1)}$$

$$y_2 = f_2(x_1, x_2) = x_1 - 2x_1^2 + \log x_1 x_2 + x_2^2 \quad \text{--- (2)}$$

above eq<sup>n</sup> are non-linear simultaneous ~~equat<sup>n</sup>~~ <sup>equat<sup>n</sup></sup>

$$x_1 \quad f_1(x_1, x_2) = 0$$

$$x_2 \quad f_2(x_1, x_2) = 0$$

$$f_1(x_1, x_2) = 0 = x_1^2 - 2 \log x_1 x_2 - x_2^2$$

$$\Rightarrow x_1 = f_3(x_1, x_2) = \sqrt{2 \log x_1 x_2 + x_2^2} \quad \text{--- (3)}$$

$$f_2(x_1, x_2) = 0 = -2x_1^2 + \log x_1 x_2 + x_2^2$$

$$\Rightarrow x_2 = \sqrt{2x_1^2 - \log x_1 x_2} = f_4(x_1, x_2) \quad \text{--- (4)}$$

$x_1^0 = x_2^0 = 5 \rightarrow$  guess values.

1<sup>st</sup> iteration  $x_1^{(1)} = f_3(x_1^0, x_2^0)$

$$= \sqrt{2 \log x_1^0 x_2^0 + x_2^0}$$

$$x_2^{(1)} = f_4(x_1^{(1)}, x_2^0)$$

$$= \sqrt{2 x_1^{(1)} - \log x_1^{(1)} x_2^0}$$

Check for convergence.

$$|x_i^{(n)} - x_i^{(n-1)}| \leq \epsilon \quad i=1,2$$

2nd iteration

$$x_1^{(2)} = f_3(x_1^{(1)}, x_2^{(1)})$$

$$x_2^{(2)} = f_4(x_1^{(2)}, x_2^{(1)})$$

NOTE:-

The calculat<sup>n</sup> will reduce from  $4n$  to  $2n$   
Unknown variables will  $\rightarrow$  \_\_\_\_\_

C.P.U. time of execut<sup>n</sup> will reduce  
Solut<sup>n</sup> will reduce.

## • Classification of Buses:-

$\rightarrow$  85% Buses are Load / PQ Bus.

specified quantity  $\rightarrow P_i, Q_i$   
unspecified quantity  $\rightarrow V_i, \delta_i$

It is purely Importing Bus.

$\rightarrow$  15% Buses are Gen / PV / Voltage controlled.

$\rightarrow$  gen. Must be connected

By default generator connected bus is called PV Bus  
 ————— load ————— PQ Bus

Date \_\_\_\_\_

for PV Buses. Spec quantity  $\rightarrow P_i, |V_i|$   
 Unspec quantity  $\rightarrow Q_i, \delta_i$

$\rightarrow$  Every P-V bus is generator bus but vice-versa is not true.

• Slack Bus :- The bus having largest no of gen called slack.

$\rightarrow$  Every load is PQ bus but vice versa is not true bco some time PQ bus act as exporting bus.

$\rightarrow$  Reference bus will generate the power which is required from losses.

$\rightarrow$  When the  $Q$  is within the limit, we can PV bus. If value of  $Q$  exceed  $Q_{max}$  that mean  $V \downarrow$ , we have to  $\uparrow$  the exciter. if value of  $Q$  reduced below  $Q_{min}$  that mean  $V \uparrow$ , demagnetisation should happen.

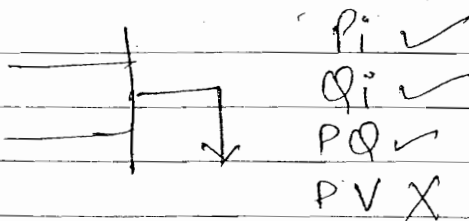
$\rightarrow$  solution possible  $\rightarrow$  convergence

$\rightarrow$  solution not possible  $\rightarrow$  divergence.

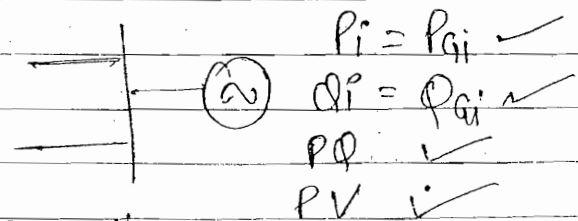
$\rightarrow$  without slack bus load flow is not possible.

$\rightarrow$  slack bus and PQ is compulsory.

$\rightarrow$  Main role of slack bus  $\rightarrow$  To meet the local load demand and meet the difference of gen. and demand goes to losses.



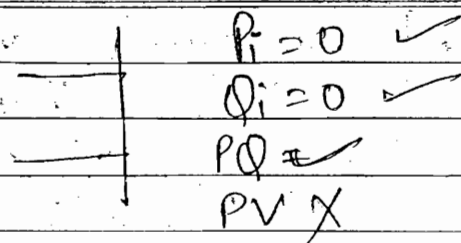
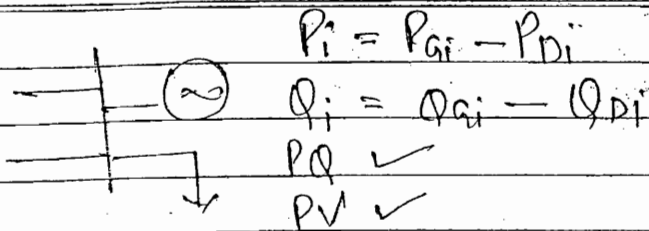
importing



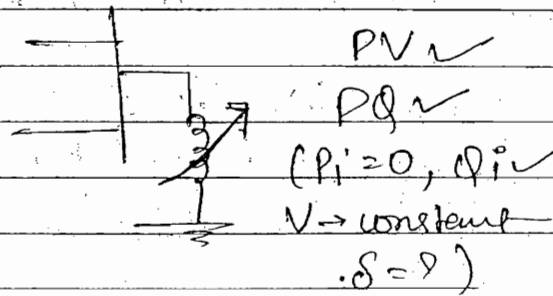
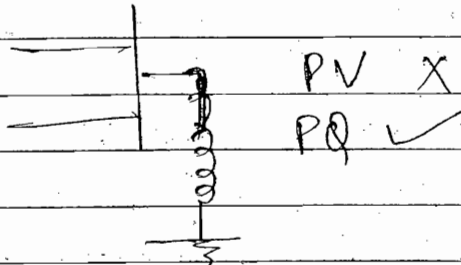
exporting

gen is present so

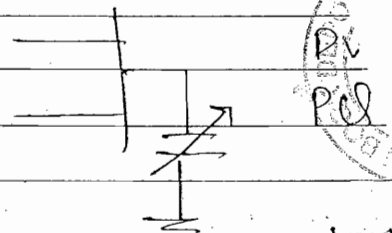
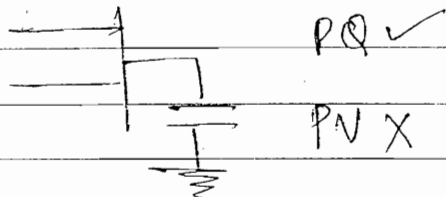
PV ✓



Both ex. in



Reactor is ~~not~~ constant don't  
compensate any leading power.



if not controlling lagging so PV ✗

control lagging maintain voltage constant

Case 1 → One slack bus and remaining are PQ Bus

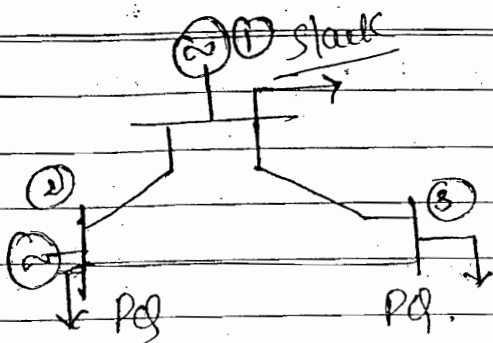
Case 2 → One slack Bus, some are PV Bus and  
all remaining are PQ Bus.

→ For coding convenient Slack Bus is given no. 1. then  
PV buses, then PQ Buses.

GS Method :-

Case-1 :- PV Buses are absent -

Stage 2 :- To determine Bus the Bus quantities.



using line data from  $[Y_{bus}]_{3 \times 3}$

$$y_{ik} = \frac{R_{ik}}{R_{ik}^2 + X_{ik}^2} - j \frac{X_{ik}}{R_{ik}^2 + X_{ik}^2}$$

line data

| line no. | from Bus to Bus | $R_{ik}$ | $X_{ik}$ | $y_{ik}/2$ |
|----------|-----------------|----------|----------|------------|
| L1       | (1) — (2) (K)   | $R_{12}$ | $X_{12}$ | $y_{12}/2$ |
| L2       | (2) — (3)       | $R_{23}$ | $X_{23}$ | $y_{23}/2$ |
| L3       | (3) — (1)       | $R_{31}$ | $X_{31}$ | $y_{31}/2$ |

→ using Direct Inspection Method Form  $[Y_{bus}]_{3 \times 3}$

$$Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix}$$

• Bus data given:

Slack  $V_1, \delta_1 = 0$   
 PQ, PQ  $P_2, Q_2, P_3, Q_3$   
 (2) (3)

• Bus data to find

$P_1, Q_1, V_2, \delta_2, V_3, \delta_3$

$$P_i = \sum_{k=1}^n |V_i| |V_k| |Y_{ik}| \cos(\delta_i - \delta_k + \gamma_{ik})$$

$$Q_i = \sum_{k=1}^n |V_i| |V_k| |Y_{ik}| \sin(\delta_i - \delta_k + \gamma_{ik})$$

$$V_i = \frac{1}{Y_{ii}} \left[ \frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{k=1 \\ k \neq i}}^n Y_{ik} V_k \right]$$

- Never calculate of power of slack bus at beginning bcoz value of voltages of all buses are unknown.

- Guess values are, called  $\rightarrow$  flat start

$$V_2^0, V_3^0 = 1 \text{ p.u.} \quad \delta_2^0, \delta_3^0 = 0 \text{ rad}$$

$$V_2^{(1)} = \frac{1}{Y_{22}} \left[ \frac{P_2 - jQ_2}{V_2^{(0)} \angle -\delta_2} - Y_{21} V_1^{(0)} - Y_{23} V_3^{(0)} \right]$$

$$= |V_2|^{(1)} \angle \delta_2^{(1)} \quad \text{polar coordinates.}$$

$$V_3^{(1)} = \frac{1}{Y_{33}} \left[ \frac{P_3 - jQ_3}{V_3^{(0)} \angle -\delta_3} - Y_{31} V_1^{(0)} - Y_{32} V_2^{(1)} \right]$$

$$= |V_3|^{(1)} \angle \delta_3^{(1)}$$

Check for convergence. -

$r = \text{iteration no.}$

$$|V_i^r - V_i^{r-1}| \leq \epsilon, \quad |\delta_i^r - \delta_i^{r-1}| \leq \epsilon$$

$i = 2, 3$

$\rightarrow$  Get the convergence is occur at the end of  $r^{\text{th}}$  iteration

$$V_2^{(r)} = \frac{1}{Y_{22}} \left[ \frac{P_2 - jQ_2}{V_2^{r-1} \angle -\delta_2^{r-1}} - Y_{21} V_1 - Y_{23} V_3^{r-1} \right]$$

$$= |V_2|^r \angle \delta_2^r$$

$$V_3^{(r)} = \frac{1}{Y_{33}} \left[ \frac{P_3 - jQ_3}{V_3^{r-1} \angle -\delta_3^{r-1}} - Y_{31} V_1 - Y_{32} V_2^r \right]$$

$$= |V_3|^r \angle \delta_3^r$$

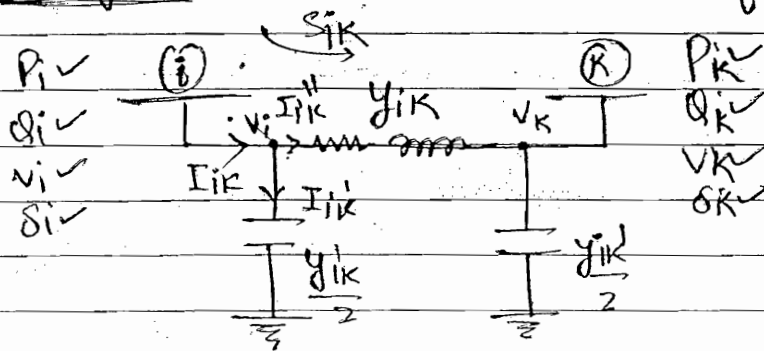
$r = \text{current iteration}$   
 $r-1 = \text{previous iteration}$

Now, as a last step in stage-1 Calculate slack Bus power  $P_1$  &  $Q_1$ ,

$$P_1 = |V_1| |V_2| |Y_{12}| \cos(\gamma_{12}) + |V_1| |V_2|^2 |Y_{12}| \cos(\delta_1 - \delta_2 + \gamma_{12}) \\ + |V_1| |V_3|^2 |Y_{13}| \cos(\delta_1 - \delta_3 + \gamma_{13})$$

Similarly calculate  $Q_1$

• Stage 2 :- (To determine line flows / losses)



$$I_{ik} = I_{ik}' + I_{ik}'' \\ = \frac{V_i}{y_{ik}} \frac{y_{ik}'}{2} + (V_i - V_k) y_{ik}$$

$$S_{ik} = P_{ik} + j Q_{ik} = V_i I_{ik}^*$$

$$= V_i \left[ \frac{y_{ik}'}{2} V_i^* + (V_i^* - V_k^*) y_{ik} \right]$$

$$S_{ki} = V_k \left[ V_k^* \frac{y_{ik}'}{2} + (V_k^* - V_i^*) y_{ki} \right]$$

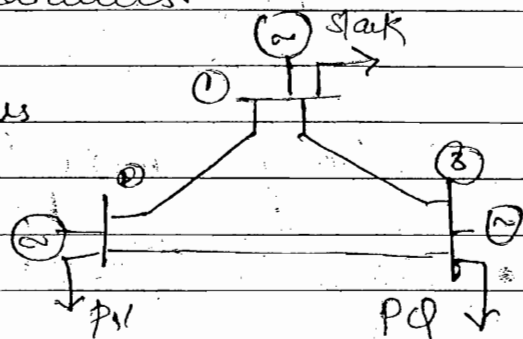
$$\text{losses in line} = S_{ik} + S_{ki}$$

Case 2 PV Buses are present.

Stage 1 :- To determine Bus Quantities.

→ Using the line data form  $Y_{bus}$

• flat start



$$V_3^0 = 1 \text{ p.u.}$$

$$\delta_2^0 = \delta_3^0 = 0 \text{ rad.}$$

Given data :-

$$V_1, \delta_1, P_2, V_2, P_3, Q_3, \\ Q_{2, \min}, Q_{2, \max}$$

→ Calculate  $Q_2^0$

$$Q_2^{(0)} = |V_2| |V_1| |Y_{21}|$$

$$\sin(\delta_2^0 - \delta_1 + \alpha_{21}) +$$

To find  $P_1, Q_1, Q_2, \delta_2, \\ V_3, \delta_3$

$$|V_2| |V_2| |Y_{22}| \sin \delta_{22} + |V_2| |V_3| |Y_{23}| \sin(\delta_2^0 - \delta_3 + \alpha_{23})$$

→ Check  $Q_2^0$  for limits

$$Q_{2, \min} \leq Q_2^0 \leq Q_{2, \max}$$

$$V_2^{(1)} = \frac{1}{Y_{22}} \left[ \frac{P_2 - jQ_2}{|V_2^0| \angle -\delta_2} - Y_{21} V_1 - Y_{23} V_3^0 \right]$$

$$= |V_2^{(1)}| \angle \delta_2^{(1)}$$

Omit  $|V_2|^{(1)}$  but retain  $\delta_2^{(1)}$  and set

$$V_2^{(1)} = |V_2| \angle \delta_2^{(1)}$$

$$V_3^{(0)} = \frac{1}{Y_{33}} \left[ \frac{P_3 - jQ_3}{|V_3^{(0)}| \angle \delta_3^{(0)}} - Y_{31} V_1 - Y_{32} V_2^{(1)} \right]$$

$$= |V_3^{(1)}| \angle \delta_3^{(1)}$$

→ Check for Convergence

$$|V_3^r - V_3^{r-1}| \leq \epsilon, \quad |\delta_i^r - \delta_i^{r-1}| \leq G \quad i=2,3$$

→ If convergence occurs, then calculate  $P_1, Q_1$  and find values of  $Q_2$

→ Then go to stage - 2 calculation.

→ If convergence doesn't occur go for Next iteration.

→ After completing 'r' no. of iteration the reactive power limitation violates at Bus (2)

→ Convert PV Bus into PQ Bus and start iteration from the beginning.

→ When PV bus is treated as PQ buses.

PV

PQ

$P_2$

$P_2$

, Same values is used.

$Q_2 = ?$

$Q_2$

$$\begin{cases} = Q_{2, \min} \\ = Q_{2, \max} \end{cases}$$

If  $Q_2^r > Q_{2, \max}$  Then  $Q_2 = Q_{2, \max}$

$Q_2^r < Q_{2, \min}$  Then set  $Q_2 = Q_{2, \min}$

• If  $Q$  crosses its limit then change to PQ Bus.



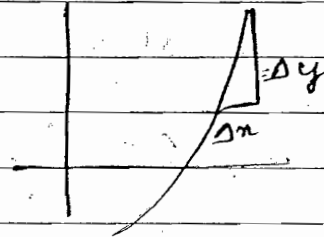
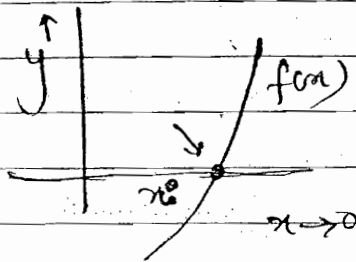
Date 15/6/2014

# • Newton-Rapson Method (differential)

(a) Single Valued Method:-

ex:-  $y = f(x) = x^2 - \log x + 1$  Nonlinear single value eqn  
 $y = f(x) = 0$

$y = f(x^0) = 0$   $x = x^0$  Guess value



$$\frac{dy}{dx} = \frac{\Delta y}{\Delta x}$$

$$\Delta y = \left( \frac{dy}{dx} \right) \Delta x$$

slope.

$x^{(1)} = \text{New Root}$   
 $= x^0 + \Delta x^0$

$f(x^{(1)}) = f(x^0 + \Delta x^0) \neq 0$

$y^{(1)} = y^0 + \Delta y^0 = f(x^0) + \left( \frac{dy}{dx} \right)^0 \Delta x^0 = 0$

$\Delta x^0 = - \frac{f(x^0)}{\left( \frac{dy}{dx} \right)^0}$

$x^{(1)} = x^0 + \Delta x^0$

→ check for convergence

$\Delta x^{(1)} = - \frac{f(x^{(1)})}{\left( \frac{dy}{dx} \right)^{(1)}}$

next  $x^{(2)} = x^{(1)} + \Delta x^{(1)}$

(b) Multi valued Function

$y_1 = f_1(x_1, x_2) = x_1^2 - \log x_1 x_2 + x_2^2 = 0$   
 $y_2 = f_2(x_1, x_2) = -2x_1^2 + \log x_1 x_2 + x_2^2 = 0$

$x_1 = x_1^0, x_2 = x_2^0$  Guess values

$y_1^0 = f_1(x_1^0, x_2^0) \neq 0$   $y_2^0 = f_2(x_1^0, x_2^0)$

$x_1^{(1)} = x_1^0 + \Delta x_1^0$

$x_2^{(1)} = x_2^0 + \Delta x_2^0$

Taylor's Series

Date \_\_\_\_\_

$$y_1^{(1)} = y_1^0 + \Delta y_1^0 = f_1(x_1^0, x_2^0) = f_1(x_1^0 + \Delta x_1^0, \Delta x_2^0 + x_2^0) = 0$$

$$\Rightarrow f_1(x_1^0 + x_2^0) + \left(\frac{\partial f_1}{\partial x_1}\right)^0 \Delta x_1^0 + \left(\frac{\partial f_1}{\partial x_2}\right)^0 \Delta x_2^0 = 0 \quad \text{--- (1)}$$

Similarly

$$\Rightarrow f_2(x_1^0, x_2^0) + \left(\frac{\partial f_2}{\partial x_1}\right)^0 \Delta x_1^0 + \left(\frac{\partial f_2}{\partial x_2}\right)^0 \Delta x_2^0 = 0 \quad \text{--- (2)}$$

Equat<sup>n</sup> (1) & (2) in Matrix Form.

$$\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}^0 \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix} = - \begin{bmatrix} f_1(x_1, x_2) \\ f_2(x_1, x_2) \end{bmatrix}^0$$

↑ Jacobian Matrix      ↑ Increment Matrix

In condensed form.

$$[\Delta x]^0 = - [J^0]^{-1} [f^0]$$

update

$$x_1^{(1)} = x_1^0 + \Delta x_1^0$$

$$x_2^{(1)} = x_2^0 + \Delta x_2^0$$

N-R Method for load flow :-  
n values

$$P_i = f_1(\delta, |V|) \quad \text{n values}$$

$$Q_i = f_2(\delta, |V|)$$

$$\Delta P_i = \left(\frac{\partial P_i}{\partial \delta_1}\right) \Delta \delta_1 + \left(\frac{\partial P_i}{\partial \delta_2}\right) \Delta \delta_2 + \dots + \left(\frac{\partial P_i}{\partial \delta_n}\right) \Delta \delta_n +$$

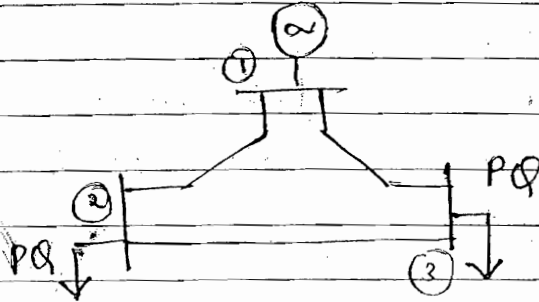
$$\left(\frac{\partial P_i}{\partial V_1}\right) \Delta V_1 + \left(\frac{\partial P_i}{\partial V_2}\right) \Delta V_2 + \dots + \left(\frac{\partial P_i}{\partial V_n}\right) \Delta V_n$$

$$\Delta Q_i = \left( \frac{\partial Q_i}{\partial \delta_1} \right) \Delta \delta_1 + \left( \frac{\partial Q_i}{\partial \delta_2} \right) \Delta \delta_2 + \dots + \left( \frac{\partial Q_i}{\partial \delta_n} \right) \Delta \delta_n +$$

$$\left( \frac{\partial Q_i}{\partial V_1} \right) \Delta V_1 + \left( \frac{\partial Q_i}{\partial V_2} \right) \Delta V_2 + \dots + \left( \frac{\partial Q_i}{\partial V_n} \right) \Delta V_n$$

for  $i = 2, 3, \dots, n$  if 1

- first bus is slack Bus voltage and angle remain same in all iteration so it will not change



|       |                                          |                                          |                                     |                                     |                   |                  |
|-------|------------------------------------------|------------------------------------------|-------------------------------------|-------------------------------------|-------------------|------------------|
| $P_2$ | $\frac{\partial P_2}{\partial \delta_2}$ | $\frac{\partial P_2}{\partial \delta_3}$ | $\frac{\partial P_2}{\partial V_2}$ | $\frac{\partial P_2}{\partial V_3}$ | $\Delta \delta_2$ | 1 element remove |
| $P_3$ | $\frac{\partial P_3}{\partial \delta_2}$ | $\frac{\partial P_3}{\partial \delta_3}$ | $\frac{\partial P_3}{\partial V_2}$ | $\frac{\partial P_3}{\partial V_3}$ | $\Delta \delta_3$ |                  |
| $Q_2$ | $\frac{\partial Q_2}{\partial \delta_2}$ | $\frac{\partial Q_2}{\partial \delta_3}$ | $\frac{\partial Q_2}{\partial V_2}$ | $\frac{\partial Q_2}{\partial V_3}$ | $\Delta V_2$      | 1 element remove |
| $Q_3$ | $\frac{\partial Q_3}{\partial \delta_2}$ | $\frac{\partial Q_3}{\partial \delta_3}$ | $\frac{\partial Q_3}{\partial V_2}$ | $\frac{\partial Q_3}{\partial V_3}$ | $\Delta V_3$      |                  |

(2n-2) x 1

Power Mismatch Matrix
Jacobian Matrix
Increment Matrix

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & N \\ J & L \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix}$$

Submatrix of Jacobian J  
(2n-2) x (2n-2)

✓ Always start with 22, not with 12<sup>2,1</sup> as 1 → slack Bus.

$$H = \frac{\partial P}{\partial \delta}; \quad H_{22} = \frac{\partial P_2}{\partial \delta_2}$$

$$H_{32} = \frac{\partial P_3}{\partial \delta_2}$$

$$N = \frac{\partial P}{\partial |V|} ; N_{22} = \frac{\partial P_2}{\partial |V_2|}$$

$$\bar{J} = \frac{\partial Q}{\partial \delta} ; \bar{J}_{22} = \frac{\partial Q_2}{\partial \delta_2}$$

$$L = \frac{\partial Q}{\partial |V|} ; L_{22} = \frac{\partial Q_2}{\partial |V_2|}$$

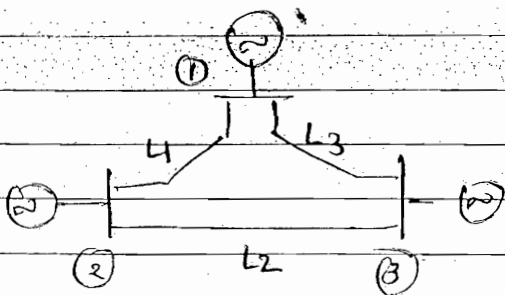
✓ When  $V$  and  $\delta$  are perfect value,  $P$  values of calculated equal to specified value. That means power mismatch is zero.

$$m_{spec} \leftarrow P_2^o = P_2 \rightarrow r_{spec}$$

$$\Delta P_2 = P_2^o - P_2$$

Case 1 :- PV are absent

stage 1 :- Find Bus quantities.



→ Using line data form  $Y_{bus}$ .

Not in position to calc.

$$\Delta P_1 = P_1^o - P_1$$

$$\Delta Q_1 = Q_1^o - Q_1$$

→ NOT specified in Slack Bus

Bus data :-

Given →  $V_1, \delta_1, P_2, Q_2, P_3, Q_3$

find →  $P_1, Q_1, V_2, \delta_2, V_3, \delta_3$

flat Start :-  $V_2^0 = V_3^0 = 1 \text{ p.u.}$   
 $\delta_2^0 = \delta_3^0 = 0 \text{ rad.}$

- Calculate P & Q values from P.D. Bus

$$P_2^0 = |V_2|^0 |V_1| |Y_{21}| \cos(\delta_2^0 - \delta_1 + \gamma_{21}) + |V_2|^0 |V_2| |Y_{22}| \cos(\delta_2^0) + |V_2|^0 |V_3| |Y_{23}| \cos(\delta_2^0 - \delta_3 + \gamma_{23})$$

$\neq 1/2^0$

Similarly calculate  $P_3^0, Q_2^0, Q_3^0$

- Calculate power mismatch.

$$\Delta P_2^0 = P_2^0 - P_2$$

Similarly find  $\Delta P_3^0, \Delta Q_2^0, \Delta Q_3^0$

- Fill up  $[J]^0$  element.

$$\left( \frac{\partial P_2}{\partial \delta_2} \right)^0 = H_{22}^0 = -|V_2|^0 |V_1| |Y_{21}| \sin(\delta_2^0 - \delta_1 + \gamma_{21}) + 0 - |V_2|^0 |V_3| |Y_{23}| \sin(\delta_2^0 - \delta_3 + \gamma_{23})$$

$$\left( \frac{\partial P_2}{\partial \delta_3} \right)^0 = H_{23}^0 = +|V_2|^0 |V_1| |Y_{21}| (\sin(\delta_2^0 - \delta_2^0 + \gamma_{23}))$$

$$\left( \frac{\partial P_2}{\partial V_2} \right)^0 = |V_1| |Y_{21}| \cos(\delta_2^0 - \delta_1 + \gamma_{21}) + 2 |V_2|^0 |Y_{22}| \cos(\delta_2^0) + |V_3| |Y_{23}| \cos(\delta_2^0 - \delta_3 + \gamma_{23})$$

$$\left( \frac{\partial P_2}{\partial V_3} \right)^0 = |V_2|^0 |Y_{23}| \cos(\delta_2^0 - \delta_3 + \gamma_{23})$$

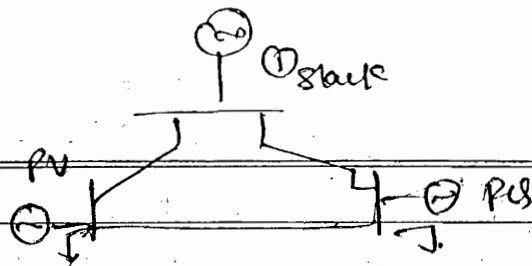
→ Find incremental matrix by multiplying inverse of jacobian and power mismatch matrix

→ updates the values.  $V_2, \delta_2, V_3, \delta_3$

→ checks for convergence.

Case 2

stage 1



→ using line data form  $Y_{bus}$ .

Bus data Given: -  $V_1, \delta_1, P_2, V_2, Q_{max}, Q_{min}, P_3, Q_3$

To find: -  $P_1, Q_1, \delta_2, Q_2, \delta_3, V_3$

flat start: -  $V_3^0 = 1 \text{ p.u.}$   $\delta_2^0 = \delta_3^0 = 0 \text{ rad.}$

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_2 \\ \Delta Q_3 \end{bmatrix}_{(2n-2-2) \times 1} = \begin{bmatrix} \frac{\partial P_2}{\partial V_2} & \frac{\partial P_3}{\partial V_2} \\ \frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial \delta_3} & \frac{\partial Q_2}{\partial V_2} & \frac{\partial Q_2}{\partial V_3} \\ \frac{\partial Q_3}{\partial \delta_2} & \frac{\partial Q_3}{\partial \delta_3} & \frac{\partial Q_3}{\partial V_2} & \frac{\partial Q_3}{\partial V_3} \end{bmatrix}_{(2n-2-n) \times (2n-2-n)} \begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \\ \Delta V_2 \\ \Delta V_3 \end{bmatrix}_{(2n-2) \times 1}$$

remove  $\Delta Q_2$   $\downarrow$  remove  $\Delta V_2$  is guess, constant no need.

remove  $\downarrow$  No. of PV buses

Stage 2 1) Calculate  $Q_2^0$  using guess values.

2) Check for limits

3) If not OK! - Convert PV bus into PQ buses

4) OK! -

4) Calculate P & Q values for PQ buses and only P values for PV buses

5) calculate power mismatches

6) calculate elements reduced order [J]

7) calculate increments using

$$\begin{bmatrix} \Delta \delta \\ \Delta V \end{bmatrix} = [J]^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}$$

8) update  $\delta$  &  $|V|$  values

9) check for convergences

- (10) If convergence occurs cal. P<sub>2</sub> & go to stage-2  
 otherwise Repeat the process

Problem 27.  $n = 300$   
 $2n = 600$

Out of 20 buses 1 is  
 slack bus remaining are  
 taken as PV bus

$$3 \times 600 - 1 = 599$$

taken as PV bus

$$2n - 2 - n \xrightarrow{25+19}$$

$$3 \times 2 \times 600 - 2 - 42$$

bus of slack bus of PV bus.

Parameter for Comparison

G-S Method

NR Method

(i) Complexity of Iteration

Easy

Complex

(ii) Time for each iteration

less

More

(iii) Accuracy

less

Accuracy  
 highest

(iv) Type of convergence

linear

quadratic

(v) Convergence Rate

Slow

fast

(vi) Total no. of Iteration work  
 To  $\uparrow$  of pow dys.

Increases

fixed

(vii) Guarantee for convergence

Not guaranteed

guaranteed

(viii) Dependency of slack bus selection

highly depended

less depended

(ix) Acceleration

external

internal

(x) Suitability

for small p.s.  
 + offline  
 studies

for large  
 size  
 p-s.

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & N \\ J & L \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix}$$

$$H = \frac{\partial P}{\partial \delta} \begin{matrix} \nearrow 20 \\ \searrow 40 \\ \quad 100 \end{matrix}$$

$$N = \frac{\partial P}{\partial |V|} \begin{matrix} \nearrow 0.01 \\ \searrow -0.01 \\ \quad 0.1 \end{matrix}$$

$$J = \frac{\partial Q}{\partial \delta} \begin{matrix} \nearrow 0.1 \\ \searrow 0.01 \end{matrix}$$

$$L = \frac{\partial Q}{\partial |V|} \begin{matrix} \nearrow 10 \\ \searrow 30 \\ \quad 40 \end{matrix}$$

$\therefore P$  is strongly depend on  $\delta$  and less to  $V$   
 $\therefore Q$  is strongly depend on  $V$  and less to  $\delta$

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & 0 \\ 0 & L \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix} \text{ Decoupled LF (DCFL) Method}$$

- (1) Total No. of calculation are reduced.  
 (2) free memory space increases.

$$\Delta P = H \Delta \delta$$

$$\Rightarrow \Delta \delta = \Delta P H^{-1}$$

$$\Delta Q = L \Delta |V|$$

$$\Delta |V| = \Delta Q L^{-1}$$

Increments are obtained through inverse of a small size matrix

Assumptions : (Fast Decoupled Method)  
FDLP

- (1) Neglect  $R$  of the system.

$$Y_{bus} = j[B] \quad \text{Integer Matrix}$$

- (2)  $\delta_i - \delta_k \ll 0 \text{ rad}$

$$H_{ii} = L_{ii}$$

$$H_{ik} = -L_{ik}$$

NOTE: (1) The advantage of this method is 1 Bus is a integer matrix require less memory space to store

(2) The size of Jacobian is reduced by 25%

(3) Total no. of calculation are reduced.

(4) The memory required to store Jacobian matrix is reduced.

## • ECONOMIC POWER DISPATCH

→ Economic power dispatch problem deals with allocation of load amongst the units in service in such a way that the total cost of generation is minimum.

It is an optimization problem

$$G = G_1 + G_2 + \dots + G_n = \sum_{i=1}^n C_i$$

Objective function.  $\Rightarrow$  "Minimize  $(G)$ "  
Total cost of generation.

Constraints  $\begin{cases} \rightarrow \text{Equality} & (\text{direct effect on obj. func}^n) \\ \rightarrow \text{Inequality} & (\text{indirect effect - u - }) \end{cases}$

• Equality constraints

(1)  $P_D = \sum_{i=1}^n P_i$  ... without losses

$P_D = \sum_{i=1}^n P_i + P_{\text{loss}}$  with losses

(2) LF problem should converge

• Inequality constraints

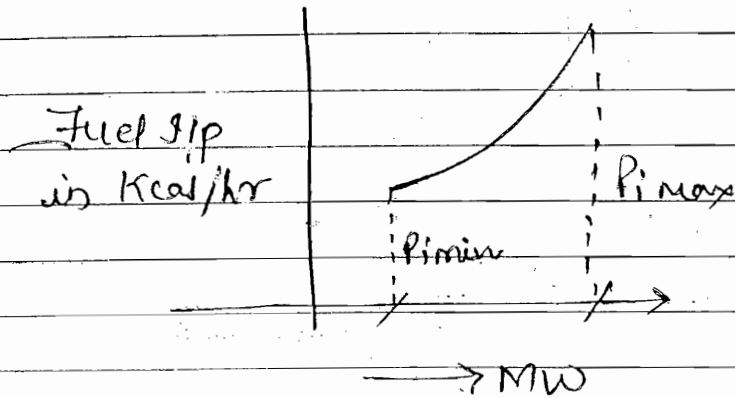
(1) Generator power limits

$P_{\min} \leq P_i \leq P_{\max}$

(2) LF Problem should converge

• Characteristic of Thermal Units :-

• Heat Curve



$$F_i(P_i) = \alpha_i P_i^2 + \beta_i P_i + c_i \quad \text{Kcal/hr}$$

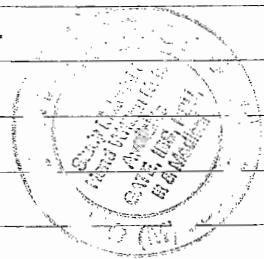
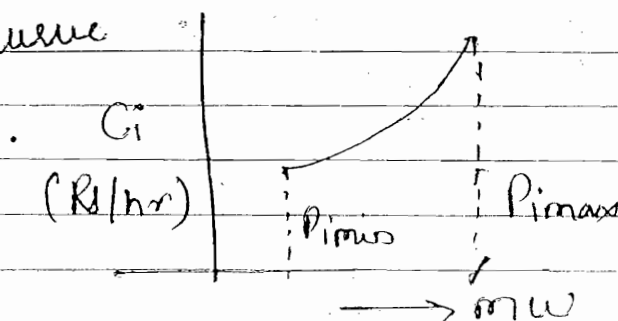
→ slope of this curve is known as "Heat Rate"

$$\frac{dF_i}{dP_i} = \alpha_i' P_i + \beta_i' \quad \text{Kcal/MWh}$$

→ efficient Thermal unit shall have less Heat Rate

$$\begin{array}{lcl} F_i \rightarrow \text{Kcal/hr} & & \\ \frac{F_i}{CV} \rightarrow \frac{\text{Kcal/hr}}{\text{Kcal/kg}} \rightarrow \text{kg/hr} & \rightarrow & \text{Rs/hr} \\ \downarrow & & \downarrow \\ \text{calorific value of fuel} & & \end{array}$$

• Cost Curve



$$C_i(P_i) = a_i P_i^2 + b_i P_i + c_i \quad \text{Rs/hr.}$$

$\therefore a_i, b_i, c_i = \text{cost coefficient.}$

→ Slopes of this curve

$$\frac{dC_i}{dP_i} = IC_i = a_i P_i + b_i \quad \text{Rs/mwh}$$

derivative  
ES. coefficient  
Incremental cost of Generation

• Economic Dispatch: (1) by Neglecting losses  
also called (Lagrange's Method)

Objective function  $GT = C_1 + C_2 + \dots + C_n$

$$= \sum_{i=1}^n C_i$$

Constraint:  $\sum_{i=1}^n P_i = P_D$

Define Lagrange's function: -

$$L(P_i) = \sum_{i=1}^n C_i - \lambda \left( \sum_{i=1}^n P_i - P_D \right)$$

$\downarrow$                        $\downarrow$                        $\downarrow$   
 Rs/hr                      Rs/mwh                      mw

$$\frac{dL(P_i)}{d(P_i)} = \frac{dC_i}{dP_i} - \lambda(1-0) = 0$$

$$0 = \frac{dC_i}{dP_i} - \lambda \quad i=1, 2, \dots, n$$

$$\boxed{\frac{dC_i}{dP_i} = \lambda}$$

$$\frac{dC_1}{dP_1} = \frac{dC_2}{dP_2} = \frac{dC_3}{dP_3} = \dots = \frac{dC_n}{dP_n} = \lambda$$

$$i=1, 2, \dots, n$$

$$\frac{dC_1}{dP_1} = \frac{dC_2}{dP_2} = \dots = \frac{dC_n}{dP_n} = \lambda$$

∴

Co-ordinate equation load is optimally allocated if all the units have same Incremental Cost production (ICP) and equal to  $\lambda$ .

Ex:- For a 2 unit system the IC values are:-

unit 1:  $\frac{dC_1}{dP_1} = 0.2P_1 + 25 \text{ Rs/mwh}$

unit 2:  $\frac{dC_2}{dP_2} = 0.1P_2 + 40 \text{ Rs/mwh}$

Solution

$$P_{\min} = 20 \text{ MW} \quad P_{\max} = 200 \text{ MW}$$

for the entire load range, give the schedule

Solution:- load can vary from 20 MW to 400 MW

Step 1 at the min load  $P_D = 20 \text{ MW}$

$$IC_1 = 29 \text{ Rs/mwh}$$

$$P_1 = 20$$

$$IC_2 = 40 \text{ Rs/mwh}$$

$$P_2 = 0$$

| $P_D$ | $P_1$ | $P_2$ | $\lambda$ | For what values of $P_1$   |
|-------|-------|-------|-----------|----------------------------|
| 20 MW | 20    | —     | —         | $IC_1 = 42 \text{ Rs/mwh}$ |
| 30    | 30    | —     | —         | $42 = 0.2P_1 + 25$         |
| 105   | 85    | 20    | 42 Rs/mwh | $P_1 = 85 \text{ MW}$      |
| 125   |       |       |           |                            |



Economic solution begins.

$$P_D = 125 \text{ MW}$$

$$P_1 + P_2 = 125 \text{ MW} \quad \text{--- (1)}$$

$$IC_1 = IC_2 \quad \text{--- (2)}$$

$$0.2P_1 - 0.1P_2 = 15 \quad \text{--- (2)}$$

$$P_1 + P_2 = 125 \times 0.1$$

$$0.2P_1 - 0.1P_2 = 15$$

$$P_1 = 91.66 \text{ MW}$$

$$P_2 = 33.33 \text{ MW}$$

| $P_D$  | $P_1$  | $P_2$  | $\lambda$    |                             |
|--------|--------|--------|--------------|-----------------------------|
| 20 MW  | 20 MW  | —      | —            |                             |
| 30 MW  | 20 MW  | —      | —            |                             |
| 105 MW | 85     | 20     | 42 Rs/MWh    | ← Economic soln begins.     |
| 125 MW | 91.66  | 33.33  | 43.33 Rs/MWh |                             |
| 200    | 116.67 | 83.33  | 48.31        |                             |
| 300    | 150    | 150    | 55           |                             |
| 325    | 158.3  | 166.67 | 56           |                             |
| 375    | 175    | 200 MW | 60           | ← Economic soln terminates. |

→ for what value of  $P_D$  ED soln terminates: —

at  $P_1 = 200 \text{ MW}$  ;  $IC_1 = \text{Rs } 65/\text{MW}$

at  $P_2 = 200 \text{ MW}$  ;  $IC_2 = \text{Rs } 60/\text{MW}$

$P_2 \rightarrow$  reaching 200 MW quickly

at  $IC_1 = 60 \Rightarrow P_1 = ?$   $P_1 = 175 \text{ MW}$

|     |     |     |   |
|-----|-----|-----|---|
| 380 | 180 | 200 | — |
| 390 | 190 | 200 | — |
| 400 | 200 | 200 | — |

→ When compare to equal sharing, calculate net saving in Rupees/hour when compare to optimum allocation.

$$P_D = 250 \text{ MW}$$

Allocation: 250 MW

|               | $P_1$  | $P_2$   |                         |
|---------------|--------|---------|-------------------------|
| Equal sharing | 125    | 125     |                         |
| optimal       | 133.33 | 116.66  | → $P_D   P_1   P_2   A$ |
|               | ↓ more | ↓ less  | 90                      |
|               | loss   | Profit. |                         |

$$C_1 = \int_{125}^{133.33} (0.2 P_1 + 25) dP_1$$

$$= \left[ \frac{0.2 P_1^2}{2} + 25 P_1 \right]_{125}^{133.33}$$

$$P_1 + P_2 = 90$$

$$0.2 P_1 - 0.1 P_2 = 15$$

$$P_1 = 133.33$$

$$P_2 = 116.66$$

$$= \text{Rs } 423.42 / \text{hour (loss)}$$

$$C_2 = \int_{125}^{116.66} (0.1 P_2 + 40) dP_2$$

$$= \left[ \frac{0.1 P_2^2}{2} + 40 P_2 \right]_{125}^{116.66}$$

$$= \text{Rs } -434.37 / \text{hr (Profit)}$$

$$\text{Net profit} = \text{Rs } 10.94 / \text{hr}$$

$$\text{Annual profit} = 10.94 \times 8760$$

$$= \text{Rs } 9583.44 \text{ Ans}$$

Problem (P8V2)  $I_{c1} = 0.2P_1 + 20$   $P_D = 700 \text{ mW}$   
 $I_{c2} = 0.3P_2 + 30$   $P_1 = ?$   
 $I_{c3} = 0.4P_3 + 25$   $P_2 = ?$   
 $P_3 = ?$

$$P_1 + P_2 + P_3 = 700 \quad \text{--- (1)}$$

for optimal allocation

$$I_{c2} = I_{c1}$$

$$0.3P_2 + 30 = 0.2P_1 + 20$$

$$P_2 = \frac{(0.2P_1 + 20 - 30)}{0.3}$$

$$P_2 = 0.67P_1 - 33.33 \quad \text{--- (2)}$$

$$I_{c3} = I_{c1}$$

$$0.4P_3 + 25 = 0.2P_1 + 20$$

$$P_3 = \frac{0.2P_1 + 20 - 25}{0.4}$$

$$P_3 = 0.5P_1 - 12.5 \quad \text{--- (3)}$$

Put (2) (3) in (1)

$$P_1 + 0.67P_1 - 33.33 + 0.5P_1 - 12.5 = 700$$

$$2.17P_1 - 45.83 = 700$$

$$P_1 = 343.70 \text{ mW}$$

$$P_2 = 196.15 \text{ mW}$$

$$P_3 = 159.61 \text{ mW}$$

Prob No. 48

Solve 16

$$I_{c1} = 25 + 0.2P_{G1}$$

$$I_{c2} = 32 + 0.02P_{G2}$$

$$P_1 + P_2 = 250 \quad \text{--- (2)}$$

$$b_1 + 2P_1 = B$$

$$b_1 + 2cP_1 = b + 4cP_2$$

Date: 2

$$P_1 = \frac{4.4P_2}{2.4}$$

$$0.2P_1 - 0.2P_2 = 7 \quad \text{--- (1)}$$

eqn (1) & (2)

$$P_1 = 142.5 \text{ MW}$$

$$P_2 = 107.5 \text{ MW}$$

$$P_1 + P_2 = 300$$

$$2P_1 + P_2 = 300$$

Soln 17

$$\frac{dC_1}{dP_1} = 0.11P_1 + 1$$

$$\frac{dC_2}{dP_2} = 0.06P_2 + 3$$

$$P_1 + P_2 = 250 \quad \text{--- (1)}$$

$$0.11P_1 - 0.06P_2 = 2 \quad \text{--- (2)}$$

eqn (1) & (2)

$$P_1 = 100 \text{ MW}$$

$$P_2 = 150 \text{ MW}$$

Soln 18

$$I_{C1} = 20 + 0.3P_1$$

$$I_{C2} = 30 + 0.4P_2$$

700 - 300 <sup>fixed</sup>

400

$$20 + 0.3P_1 = 30 + 0.4P_2$$

$$0.3P_1 - 0.4P_2 = 10$$

$$P_1 + P_2 = 400$$

$$\therefore I_{C2} = 30 \text{ Rs/MWh}$$

(Minimum cost)

$$P_2 = 300 \text{ MW (max)}$$

$$P_1 = 242.86 \text{ MW}, P_2 = 157.14 \text{ MW}$$

Soln 19

$$P_1 + P_2 = 700$$

Generator A  $\rightarrow$  450 MW  $\rightarrow$  Rs 600 / MWh

Generator B  $\rightarrow$  400 MW  $\rightarrow$  Rs 800 / MWh

Minimum cost  $\rightarrow$  450

$$P_2 = 700 - 450 = 250$$

(C)

$\lambda \rightarrow$  convert constraint into objective function

Date \_\_\_\_\_

Soln<sup>n</sup> 20.  $\frac{dF_1}{dP_1} = b + 2cP_1$   $\frac{dP_2}{dP_2} = b + 4cP_2$

$$\Rightarrow b + 2cP_1 = b + 4cP_2$$
$$P_1 = 2P_2$$

$$P_1 + P_2 = 300 \quad \Rightarrow \quad 2P_2 + P_2 = 300 \quad P_2 = 300/3$$
$$P_1 = 2 \times P_2 = 200 \quad P_2 = 100$$

- Economic Dispatch Problem Consideration of  
x-mission losses

Objective function  $G = \sum_{i=1}^n C_i$  "Min  $G$ "

Constraint  $\sum_{i=1}^n P_i = P_D + P_{loss}$

Define Lagrange's function

$$L(P_i) = \sum_{i=1}^n C_i - \lambda \left[ \sum_{i=1}^n P_i - P_D - P_{loss} \right]$$

To allocate load optimally

$$\frac{dL(P_i)}{dP_i} = 0$$

$$= \frac{dC_i}{dP_i} - \lambda \left[ 1 - 0 - \frac{\partial P_{loss}}{\partial P_i} \right]$$

multivariable  
function  
that's why  
partial diff.

$i = 1, 2, \dots, n$

$$0 = \frac{dC_i}{dP_i} - \lambda + \lambda \frac{\partial P_{loss}}{\partial P_i}$$

$$\frac{dC_i}{dP_i} = \lambda \left[ 1 - \frac{\partial P_{loss}}{\partial P_i} \right]$$

$$\left[ \begin{array}{c} 1 \\ 1 - \frac{\partial P_{\text{loss}}}{\partial P_i} \end{array} \right] \left[ \begin{array}{c} \frac{dC_i}{dP_i} \end{array} \right] = \lambda \quad \text{for } i=1, 2, \dots, n$$

Define penalty factor of  $i^{\text{th}}$  M/c as.

$$L_i = \frac{1}{1 - \partial P_{\text{loss}} / \partial P_i} \quad \text{for } i=1, 2, \dots, n$$

The co-ordinates of equation are.

$$L_1 \frac{dC_1}{dP_1} = L_2 \frac{dC_2}{dP_2} = \dots = L_n \frac{dC_n}{dP_n} = \lambda$$

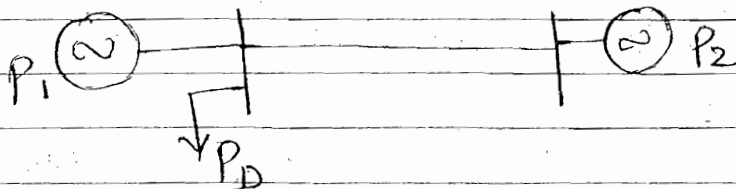
• Loss Equation In Terms of B-coefficient:-  
(1) for a 2-plant system:-

$$P_{\text{loss}} = B_{11} P_1^2 + 2B_{12} P_1 P_2 + B_{22} P_2^2$$

(2) for a 3-plant system:-

$$P_{\text{loss}} = B_{11} P_1^2 + 2B_{12} P_1 P_2 + B_{22} P_2^2 + B_{33} P_3^2 + 2B_{13} P_1 P_3 + 2B_{23} P_2 P_3$$

Case 1:- load connected near plant-1



loss equation is independent to  $B_{11}$  and  $B_{12}$

$$P_{\text{loss}} = B_{22} P_2^2 \quad B_{11} = B_{12} = B_{21} = 0$$

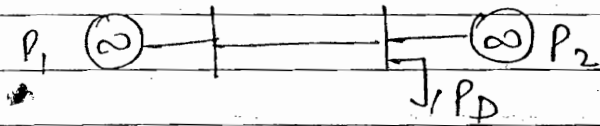
$$\frac{\partial P_{loss}}{\partial P_1} = 0$$

$$L_1 = \frac{1}{1-0} = 1$$

$$\frac{\partial P_{loss}}{\partial P_2} = 2B_{22}P_2$$

$$L_2 = \frac{1}{1-2B_{22}P_2} > 1$$

Case 2 load is connected near plant-2



$$P_{loss} = B_{11}P_1^2$$

$$B_{22} = B_{12} = B_{21} = 0$$

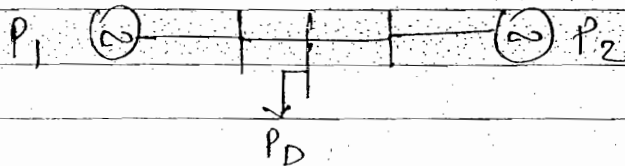
$$\frac{\partial P_{loss}}{\partial P_1} = 2B_{11}P_1$$

$$L_1 = \frac{1}{1-2B_{11}P_1} > 1$$

$$\frac{\partial P_{loss}}{\partial P_2} = 0$$

$$L_2 = \frac{1}{1-0} = 1$$

Case 3 load is connected b/w the plant 1 & 2.



$$P_{loss} = B_{11}P_1^2 + 2B_{12}P_1P_2 + B_{22}P_2^2$$

$$\frac{\partial P_{loss}}{\partial P_1} = 2P_1B_{11} + 2B_{12}P_2$$

$$L_1 = \frac{1}{1-(2P_1B_{11}+2B_{12}P_2)} > 1$$

$$\frac{\partial P_{loss}}{\partial P_2} = 2B_{12}P_1 + 2B_{22}P_2$$

$$L_2 = \frac{1}{1-(2B_{12}P_1+2B_{22}P_2)} > 1$$

Solution 21

$P_1 = 50 \text{ mW}$

$P_2 = 40 \text{ mW}$

$B_{11} = 0.001$

$B_{22} = 0.0025$

$B_{12} = +0.0005$

$$P_{\text{loss}} = P_1^2 B_{11} + P_2^2 B_{22} + 2 P_1 P_2 B_{12}$$

$$P_{\text{loss}} = 40.5$$

Solution 22

$\lambda = (0.012 P + 8) \quad \lambda = 25$

$$\frac{dP_L}{dP} = 0.2$$

$$\frac{dC_1}{dP} = 0.012P + 8$$

$$L \frac{dC}{dP} = \lambda$$

$$\& \frac{1}{1 - \frac{dR}{dL}} \times \frac{dC}{dP} = \lambda$$

$$\& \frac{1}{1 - 0.2} \times (0.012P + 8) = 25$$

$$\& 0.012P + 8 = 25 - 5$$

$$\& P = \frac{20 - 8}{0.012} = 1000 \text{ mW}$$

Solution 23

$B_{11} = 10^{-3} \text{ mW}^{-2}$

$P_1 = 100 \text{ mW}$

$P_2 = 125 \text{ mW}$

$$\frac{\partial P_{\text{loss}}}{\partial P_1} = 100 \times 10^{-3} \times 2$$

$$= 0.2$$

$$L_1 = \frac{1}{1 - 0.2} = 1.25$$

$$\frac{\partial P_{\text{loss}}}{\partial P_2} = 0$$

$$L_2 = \frac{1}{1 - 0} = 1$$

1.25 and 1  $\Delta P$

Sol<sup>n</sup> 24

$$P_1 = 100 \text{ MW} \quad P_2 = 100 \text{ MW} \quad P_3 = 100 \text{ MW}$$

$$P_1 + P_2 + P_3 = 120 \text{ MW}$$

(a) Plant 1 gener<sup>n</sup> is Maximum. Close to load.

Sol<sup>n</sup> 25

$$P_D = 40 \text{ MW}$$

$$I_{C1} = 10000 \text{ Rs/MWh} \quad I_{C2} = 12500 \text{ Rs/MWh}$$

$$P_{loss} = 0.5 P_1^2 \text{ (p.u.)}$$

$$100 \text{ MVA} = \text{Base}$$

$$L_1 \frac{dC_1}{dP_1} = L_2 \frac{dC_2}{dP_2} = \lambda$$

$$\therefore P_D \text{ is near to 2-plant.} \quad L_2 = 1$$

$$1 - 0.5 P_1^2 \neq 10000$$

$$1 \times 12500 = L_1 \times 10,000$$

$$L_1 = 1.25$$

$$1 - 0.5 P_1^2$$

$$\frac{1}{1 - \frac{\partial P_{loss}}{\partial P_1}} = 1.25$$

$$1 - 0.5 P_1^2 = \frac{1}{1.25} = 0.8 \quad \frac{\partial P_{loss}}{\partial P_1} = 0.2$$

$$P_1^2 = \frac{0.8 - 1}{-0.5} = 0.4$$

$P_{loss}$

$$\frac{\partial P_{loss}}{\partial P_1} = 0.5 \times 2 \times P_1 = 0.2$$

$$P_1 = 0.2 \text{ p.u.} \times 100$$

$$P_1 = 20 \text{ MW}$$

$$P_{loss} = 0.5 \times 0.2^2 = 0.02 \times 100 = 2 \text{ MW}$$

$$P_{load} = 40 \text{ MW}$$

$$P_1 + P_2 - P_D - P_{loss} = 0$$

$$20 + P_2 = 42$$

$$P_2 = 22 \text{ MW}$$

# P.S. STABILITY

Date 16/6/14

P & D<sup>2</sup>Lns      Hydro D↑ L↓ More inertia  
Thermal D↓ L↑ less inertia

- Power sys. stability depend as it is the ability of alternators working in parallel maintain synchronism after the disturbance.
- Gain of synchronism refers to power stability and loss of synchronism refers to instability.
- Stability problem always begin when the balance between mechanical i/p and electrical pow. o/p is upset. The tendency of rotor of alternator is now either to accelerate or decelerate.
- The power system problem of power sys. stability is further intensified due to large variation in inertia of alternators.
- If all the machines have same inertia they accelerate together and decelerate together and will not give stability problem. Such machines are known as coherent group of M/C.
- In practice coherent group of M/C never exist.
- Extraction of mechanical work produce by water jet will be more proper if

the hydro unit design with high inertia. These alternator have larger diameter, small axial length and have vertical configuration.

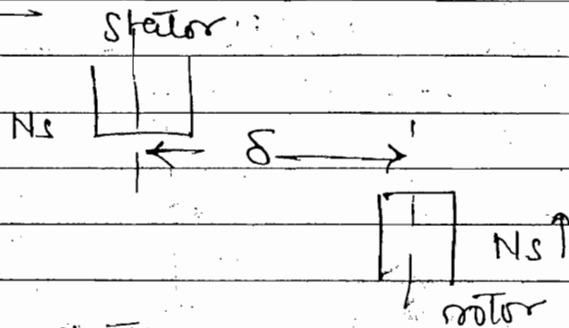
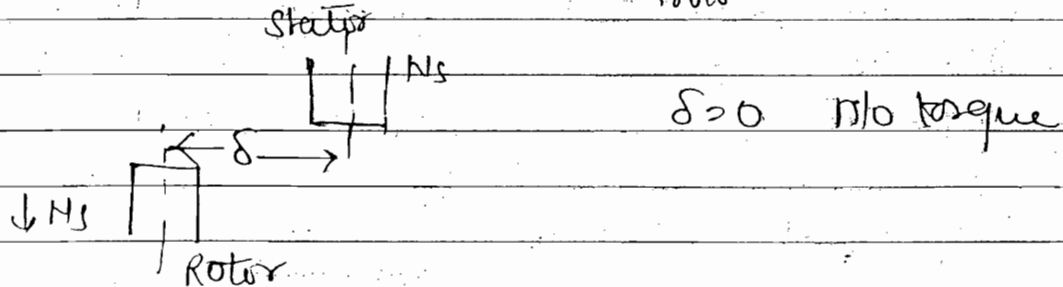
- Extraction of mechanical work from the steam will be more proper if thermal units are designed with low inertia. This machine have small diameter and longer axial length and have horizontal configuration.

- Forms of stability.
  - small signal Analysis
  - large signal / Transient stability Analysis

### Small Signal Analysis

### Transient stability Analysis

- |                                                                 |                                                                                                                                                                                     |
|-----------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| ① Amplitude of disturbance is small and occur for long duration | Amplitude of disturbance is large and occur suddenly.                                                                                                                               |
| ② System Non-linearities such as the inertia are neglected      | Non-linearities play vital role in the form of stability                                                                                                                            |
| ③ Ex:- Gradual increase or decrease in I/p or O/p etc.          | Ex:- Short circuit on X-line, Sudden loss of load, lightning etc.                                                                                                                   |
| ④ Mathematical Model: "linear-ized diff eq <sup>n</sup> "       | Mathematical Model: "Non linear 2 <sup>nd</sup> order diff eq <sup>n</sup> coupled with Non linear simultaneous algebraic. eq <sup>n</sup> " (Lof. eq <sup>n</sup> )<br>(Load Flow) |
| ⑤ Duration of study: - from few minutes to several hours        | ⑤ This phenomenon is very fast. Duration (sec Max)                                                                                                                                  |

Introduction partAlternatorMotorAlternator

$$P_{e0} = \frac{E V}{X} \sin \delta$$

$$\boxed{\delta \uparrow \quad P_{e0} \uparrow}$$

→ Mechanical I/p  $\uparrow$ ,  $\omega$  o/p, not suddenly  $\uparrow$  due to rotor inertia, (this extra energy store in rotor as kinetic energy) so  $\delta$  will not  $\uparrow$  suddenly, M/c accelerate,  $\delta \uparrow$ ,  $P_e \uparrow$

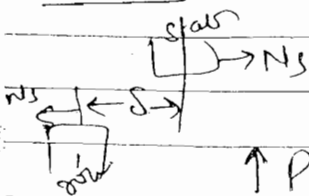
$$P_a = P_m - P_e$$

$$\text{when } P_m = P_e \quad P_a = 0$$

M/c comes back to  $N_s$  speed but distance does not change. But the electrical o/p is constant

Motor

load  $\uparrow$ , speed  $\downarrow$  ( $N_s \downarrow$ )  $\rightarrow \delta \uparrow, P_e \uparrow$



retardation power

$$\downarrow P_r = P_m - P_e \uparrow$$

$$\uparrow P_a = P_m - P_e \downarrow$$

$$\text{when } P_m = P_e$$

(this  $\uparrow$  due to  $\uparrow$  in  $\delta$ )

No accelerating power system becomes stable

Date 17/6/2014

- As long as mechanical I/p and electrical o/p exist in balance condition stability remain maintain.

### Small Signal analysis

### Transient stability analysis

⑤ Duration of study:- from few minutes to several hours.

This phenomenon is very fast Duration: 1 sec (Max)

### ⑥ Small Signal analysis

Steady state stability

Dynamic stability.

Actions of Control system (CS) components exciter/Governor are not included in (SSSB) steady state stability analysis and included in dynamic stability analysis.

Due to high time constants, the CS components are too slow to respond to this type of stability problem.

- Stability limit:- Max. amount of power that can flow into pt of disturbance is known as stability limit.

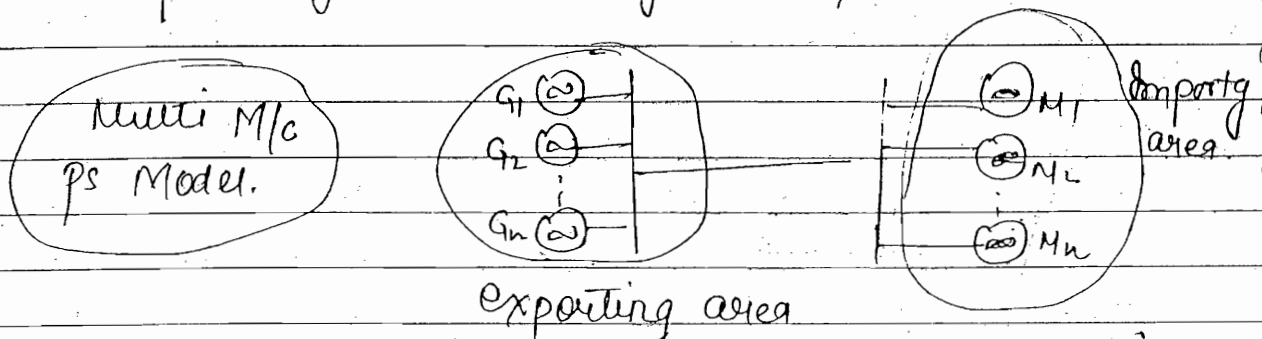
→ Steady state stability limit is always more than transient stability limit

→ Transient stability limit can be improve and can

be increased upto steady state limit beyond it is not possible.

## • Power System Model for Stability Studies:

→ Pow. Sys. Model called multi M/c ps. model bcoz we have so many M/c, generator, I.M. syn., DCM etc.



- All the generator shown in 1 area called exporting area
- All the Motor are shown in single (1) place called importing area.
- All generator or motor represent as equivalent generator or equivalent motor

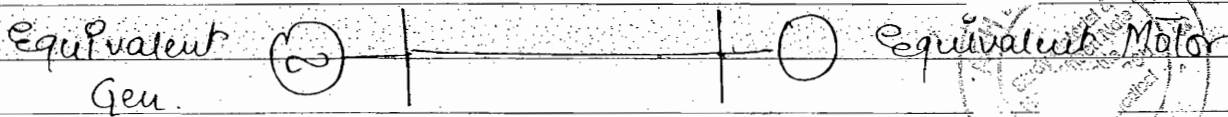
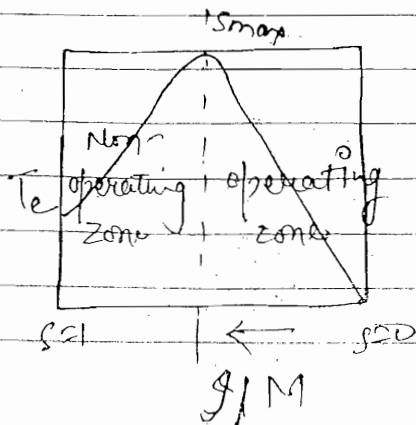


Fig. 2 M/c System.

→ Complex pow sys, we can't study, so two area is reduce to 2 M/c System.

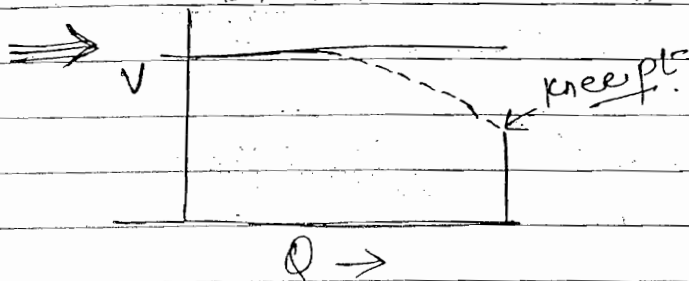


→ When  $s=0$ , stator rotor aligned in same direction, As the mechanical load applied,  $s \uparrow$ ,  $T \uparrow$ , upto  $s = s_{max}$  after  $s_{max}$ ,  $s \uparrow$  but  $T \downarrow$

and motor speed ↓, and after some time rotor blocked this causes instability.

→ Also called angle stability bcoz  $\delta$  is change due to change in load and <sup>due to</sup> difference b/w  $\text{M/P mechanical}$  and  $\text{electrical O/P}$ .

→ This is due to generator behaviour



→ Voltage will decrease due to load bcoz load gives lagging effect.

→ upto knee pt voltage stability maintain up above which system get voltage instability problem due to ↑ in reactive power demand. It is due to load behaviour

→ Power stability problem can also be known as angle stability problem

→ Power system stability problem is appearing in pow. sys due to dynamical behaviour of generator

→ Voltage stability problem occurs in power system due to load behaviour

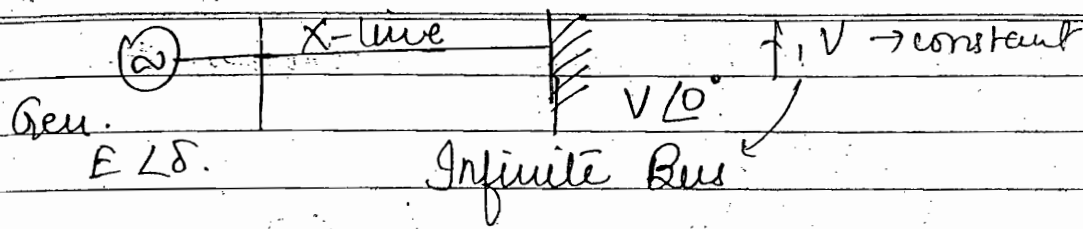


Fig. Single M/C connected to Infinite Bus sys.  
(SMIB)

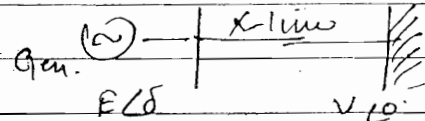
→ 2 M/C sys convert into above diagram include  $X\text{-line}$ .

⇒ We consider single M/C Infinite Bus sys in our pow. sys.

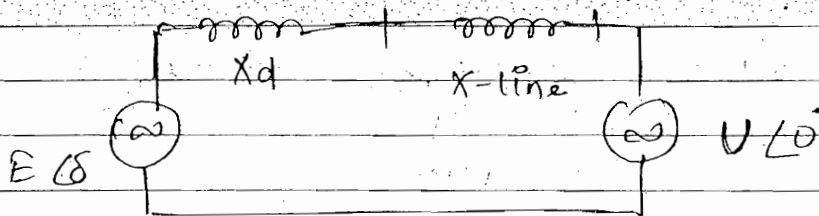
→ By taking any amount of active or reactive power voltage remain constant.

### • Power Angle Curve & Transfer Reactance

Let us consider a SMIB sys.



The equivalent ckt is :-



Let  $X = \text{Transfer Reactance} = X_d + X\text{-line}$

$$I = \frac{E \angle \delta - V \angle 0}{jX}$$

$$= \frac{E \angle \delta - 90}{X} - \frac{V \angle -90}{X}$$

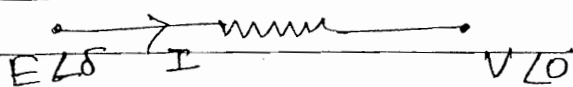
$$I^* = \frac{E}{X} \angle 90 - \delta - \frac{V}{X} \angle 90 = \frac{E}{X} \angle 90 - \delta - j \frac{V}{X}$$

Power Supplied by Generator to Infinite Bus is: -

$$S_e = VI^*$$

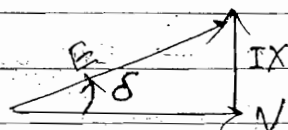
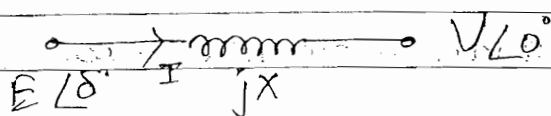
$$P_e = \text{Real} \{ S_e \} = \frac{EV}{X} \sin \delta$$

$$P_e = \frac{EV}{X} \sin \delta$$



$$\delta = 0^\circ$$

$\therefore$  No power will flow.



$\Rightarrow$  for the x-fer of active power b/w sending to receiving end reactance is compulsory.

$\Rightarrow$  Resistance can't create any angle difference as  $R$  is in same phase with  $I$ .

$\Rightarrow$  Angle difference is necessary for flow of power. power will always flow from non-zero  $\delta$  to zero  $\delta$ .

$\Rightarrow$  That's why  $X$  is called transfer reactance bcoz active power flow.

### • P- $\delta$ CURVES

$$P_e = \frac{EV}{X} \sin \delta$$

Compare with standard sine curve

$$P_e = P_{\max} \sin \delta$$

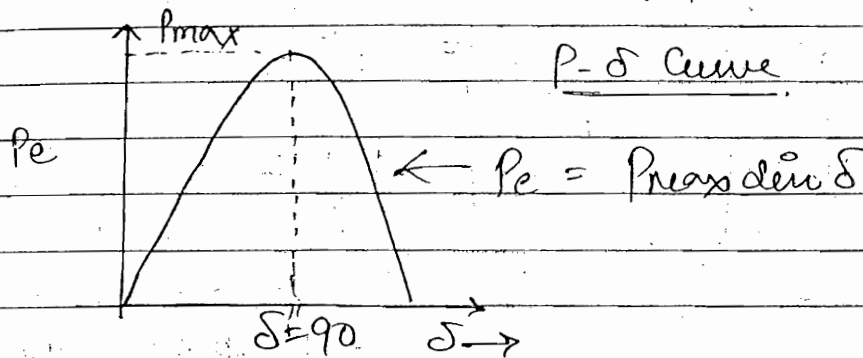
X-line should be reactive.

→ above steady state limit we can't x-fel power (if we generat)

Date \_\_\_\_\_

$$P_{max} = \frac{EV}{X}$$

→ steady state power limit.



(limit of x-line)

→ If the capacitor and inductor both are dominating parameters → resonance problem occur, sub syn. resonance problem.

### ⊙ Swing Equation:-

Prbl Bcr

⇒ The condition of Max Power of a X-line w/w is

$$X = \sqrt{3} R$$

→ To determine the dynamic of Rotor (acc or decel) we developed swing equat.

→ When the balance b/w the mechanical J/p and electrical pow O/p upsets, then the tendency of rotor is to either accelerate and/or decelerate.

$$T_a = T_m - T_e = \frac{1}{2} I \omega^2$$

$$T_q = I \frac{d^2 \delta}{dt^2}$$

$T_a$ : Accelerating torque

$$P_q = I \omega \frac{d^2 \delta}{dt^2}$$

$T_m$ : Mech J/p torque

$T_e$ : Elect O/p. Torque

$$= M \frac{d^2 \delta}{dt^2}$$

acceleration  $\propto$

$$\alpha = \text{acceleration} = \frac{d^2\theta}{dt^2}$$

Date \_\_\_\_\_

$I$ : Moment of Inertia

$$M = I\omega = \text{Inertia}$$

$\omega$ : angular speed

Constant.

$$T_{q.w} = T_{m.w} - T_{e.w} = \frac{1}{2} I \omega^2$$

$$P_a = P_m - P_e = \frac{1}{2} I \omega^2 = \frac{1}{2} M \omega$$

During transient period ' $\omega$ ' change regularly.

Therefore we can obtain multiple swing eq<sup>n</sup>. To avoid this, the value of  $M$  is obtained through another inertia constant ' $H$ '

→ { During transient period  $\omega, M$  change regularly }  
 so we have so many swing eq<sup>n</sup> which we  
 have to solve it is difficult to choose so we  
 obtain ' $H$ '

→ The value of  $M$  is calculated using another Inertia constant ' $H$ '.

$$H \text{ (Inertia constant)} = \frac{\text{Stored K.E in MJ}}{\text{Rating (G) of the M/C in MW}}$$

→  $H \rightarrow$  Independent to rating, size, weight of M/C

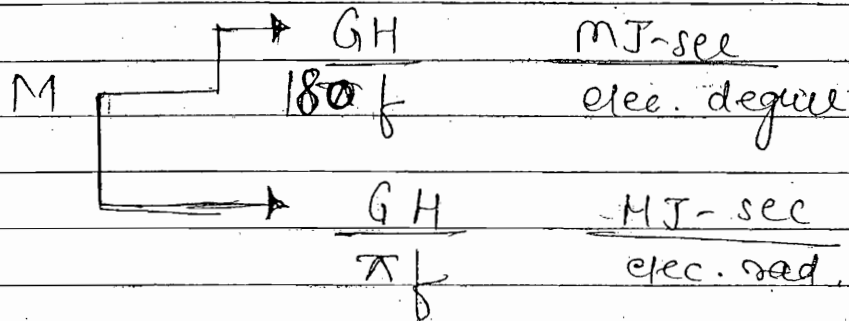
$$\text{Stored K.E. In MJ} = GH$$

But from the loss of M/C

$$\text{Stored K.E.} = \frac{1}{2} I \omega^2 = \frac{1}{2} M \omega$$

$$= \frac{1}{2} m \cdot \omega^2 = M \omega$$

$$\therefore M\pi f = GH$$



Swing equation:-

$$P_a = P_m - P_e = P_{max} \sin \delta$$

$$= P_m - P_{max} \sin \delta$$

$$P_a = M \frac{d^2 \delta}{dt^2}$$

### • Multi-Machine P/S:-

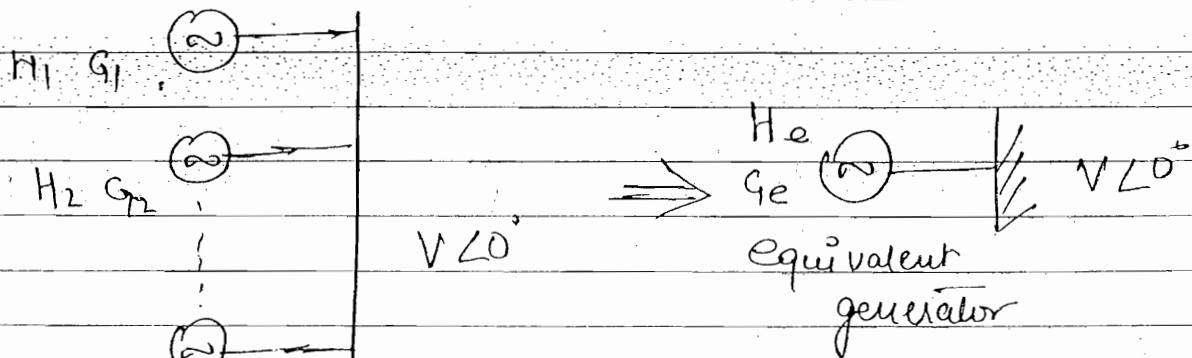


Fig:- SMIB Model.

PSV

completing objective ques

→ dynamic behaviour of  $G_e$  equal to  $G_1, G_2, \dots, G_n$  when K.E store in  $G_e$  is equal sum of all gen K.E.

Stored RoEs

$$G_e H_e = G_1 H_1 + G_2 H_2 + \dots + G_n H_n$$

unknown

$$H_e = \frac{G_1 H_1}{G_e} + \frac{G_2 H_2}{G_e} + \dots + \frac{G_n H_n}{G_e}$$

So calculate ~~the~~ at  $G_{Base}$ 

$$G_{Base} H_{eb} = G_1 H_1 + G_2 H_2 + \dots + G_n H_n$$

$$H_{eb} = \frac{G_1 H_1}{G_{Base}} + \frac{G_2 H_2}{G_{Base}} + \dots + \frac{G_n H_n}{G_{Base}}$$

Soln<sup>n</sup> 8.  $E_1 = 2.0 p.u.$   $E_2 = 1.3 p.u.$   $p = 0.5$

$$X_1 = 1.1 p.u.$$

$$X_2 = 1.2 p.u.$$

$$X_T = 0.5 p.u.$$

$$0.5 = \frac{2.0 \times 1.3}{(1.1 + 1.2 + 0.5)} \sin \delta$$

$$\sin \delta = 0.53$$

$$\delta = 32.58^\circ$$

Sol 25

$$G_1 = 500 \text{ MVA} \quad G_2 = 200 \text{ MVA}$$

$$H_1 = 5 \text{ MJ/MVA} \quad H_2 = 5$$

$$G_{Base} = 100 \text{ MVA}$$

$$H_{eb} = \frac{500 \times 5}{100} + \frac{200 \times 5}{100}$$

$$= 25 + 10$$

$$= 35 \text{ MJ/MVA}$$

Sol<sup>n</sup> (2) (d)

$$\text{Sol}^n (3) \quad G = 250 \quad H = 6 \quad G_{\text{Base}} = 100 \text{ mVA}$$

$$H_e = \frac{250 \times 6}{100} = 15 \text{ mJ/mVA}$$

$$\text{Sol}^n (5) \quad \text{stored energy} = H \times G = 8 \times 50 = 400 \text{ mJ}$$

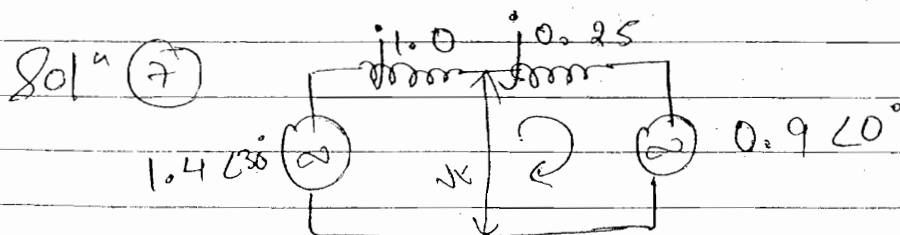
$$\text{Sol}^n (4) \quad P_g = 75 - 50 = 25 \text{ mW}$$

$$\alpha = \frac{d^2 \delta}{dt^2} = \frac{P_g}{m} = \frac{25}{180} \quad m = \frac{G H}{180 \text{ ft}}$$

$$\alpha = \frac{25 \times 180 \times 50}{100 \times 10} = 225$$

$$\text{Sol}^n (6) \quad \rho = \frac{E V}{P_2}$$

$$P_{\text{max}} = \frac{E V}{X} = \frac{1.0 \times 1.6}{(0.6 + 0.2)} = 2 \text{ p.u.}$$



$$I = \frac{1.4 \angle 30^\circ - 0.9 \angle 0^\circ}{j1 + j0.25} = \frac{0.5 \angle 70^\circ}{j1.25} = 1.4 \angle 30^\circ$$

$$= +0.56 \angle 0.96 \quad 1.212 + 0.91$$

$$= 0.61 \angle 24^\circ$$

$$V_t = 1.4 \angle 30^\circ - 0.61 \angle 24^\circ \times 1 \angle 90^\circ$$

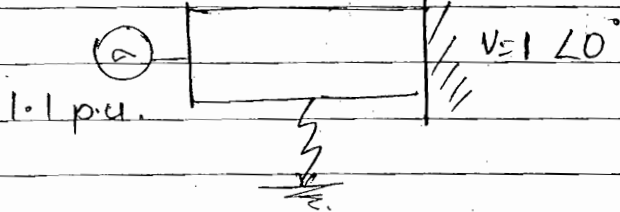
$$= 0.973 \quad (b) \text{ AL}$$

Sol<sup>n</sup> 8

$$\sin \delta = \frac{2.0 \times 1.3}{(1.1 + 1.2 + 0.5) \times 0.5}$$

$$\sin \delta = 0.53$$

$$\delta = 32.58^\circ$$

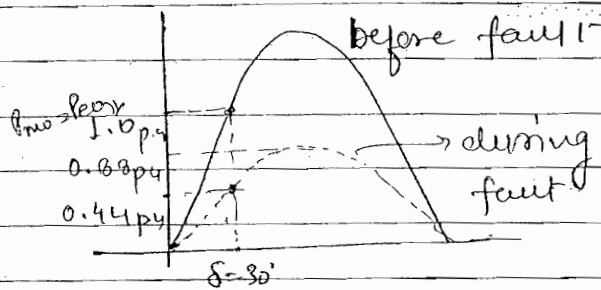
Sol<sup>n</sup> 9 (a)during  $x = 0.8 \text{ p.u.}$ Sol<sup>n</sup> 22

$$f = 50^\circ$$

$$G_1 = 10.0 \text{ mVA}$$

$$H_1 = 5$$

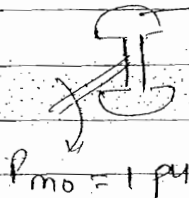
$$P_{eo} = P_{mo} = 1 \text{ p.u.} \quad \delta_o = 30^\circ$$



$$\text{Transfer reactance during fault} \\ X = \frac{1}{0.8} = 1.25 \text{ p.u.}$$

$$P_{max} = \frac{1.1 \times 1}{1.25} = 0.88 \text{ p.u.}$$

$$P_{eo} = 0.88 \times \sin 30^\circ \\ = 0.44 \text{ p.u.}$$



$$P_{mo} = 1 \text{ p.u.}$$

∴ Accelerating power

$$P_a = 1 - 0.44 = 0.56 \text{ p.u.} \quad (c)$$

(23)

$$P_a = 0.56 \text{ p.u.} = X \text{ p.u.} = \frac{100 \times \text{p.u.}}{\text{Base mVA} = 100 \text{ mVA}}$$

$$m = \frac{100 \times 8}{180 \times 8} = 0.056$$

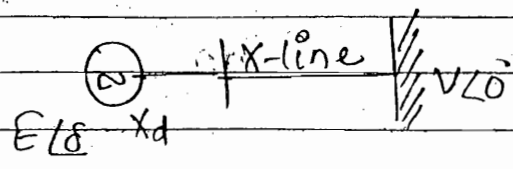
actual = Base  $\times$  p.u.

$$\alpha = \frac{P_a}{m} = \frac{0.56}{0.056} = \frac{100 \times}{0.056} = 1800 \times$$

$$25 \quad H_{eb} = \frac{5 \times 500}{100} + \frac{5 \times 200}{100} = 3.5 \text{ Ans}$$

## Steady state Stability:-

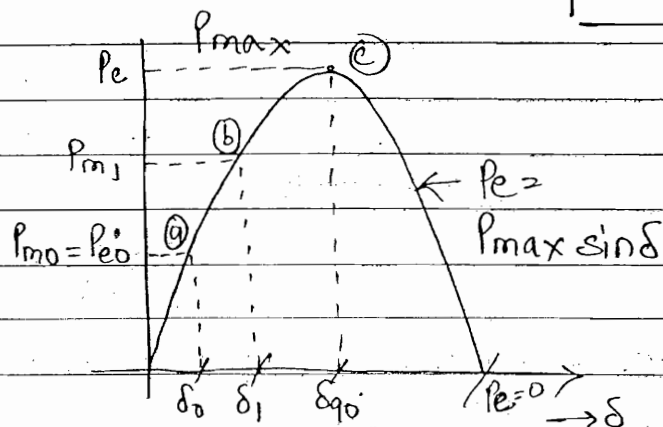
Consider a SMIB System



Transfer Reactance  $X = X_d + X_{line}$

$$P_{max} = \frac{E \cdot V}{X}$$

$$P_e = P_{max} \sin \delta \rightarrow \text{o/p eqn}$$



Initially system is operating at equilibrium at pt (a) with  $P_{m0} = P_{e0}$  at  $\delta_0$ .

Now the mech input raises from  $P_{m0}$  to  $P_{m1}$

→  $P_e$  cannot change instantly due to inertia.

→ excess mech I/p stores as K.E, motor starts acceleration  
 $\delta \uparrow, P_e \uparrow \quad P_a = P_m - P_e \downarrow$

→ upon reaching  $\delta = \delta_1$  at pt (b)  $P_a = 0$  &  $P_{m1} = P_{e1}$

→ System come to back equilibrium

→ Sys. is stable.

→ at pt (c)  $\delta = 90^\circ$ ,  $P_{m1} \uparrow \quad P_a = P_m - P_e \uparrow, \delta \uparrow$

$$\delta \uparrow \quad P_e \downarrow \quad P_a = P_m - P_e \uparrow \quad \delta \uparrow$$

After pt (c)  $P_e = \downarrow$  that mean imbalance → sys loose stable

$\delta = 0 \text{ to } 90^\circ \rightarrow$  stable

$\delta = 90^\circ \rightarrow$  critical stable

$\delta = \text{above } 90^\circ \rightarrow$  unstable

$$\frac{dP_e}{d\delta} = P_{\max} \cos \delta = \text{Synchronising power coefficient} - P_a$$

when  $\delta: 0 \text{ to } 90^\circ$

$$\frac{dP_e}{d\delta} > 0$$

system is very stable

at  $\delta: 90^\circ$

$$\frac{dP_e}{d\delta} = 0$$

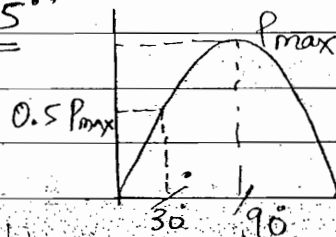
critically stable

for  $\delta > 90^\circ$

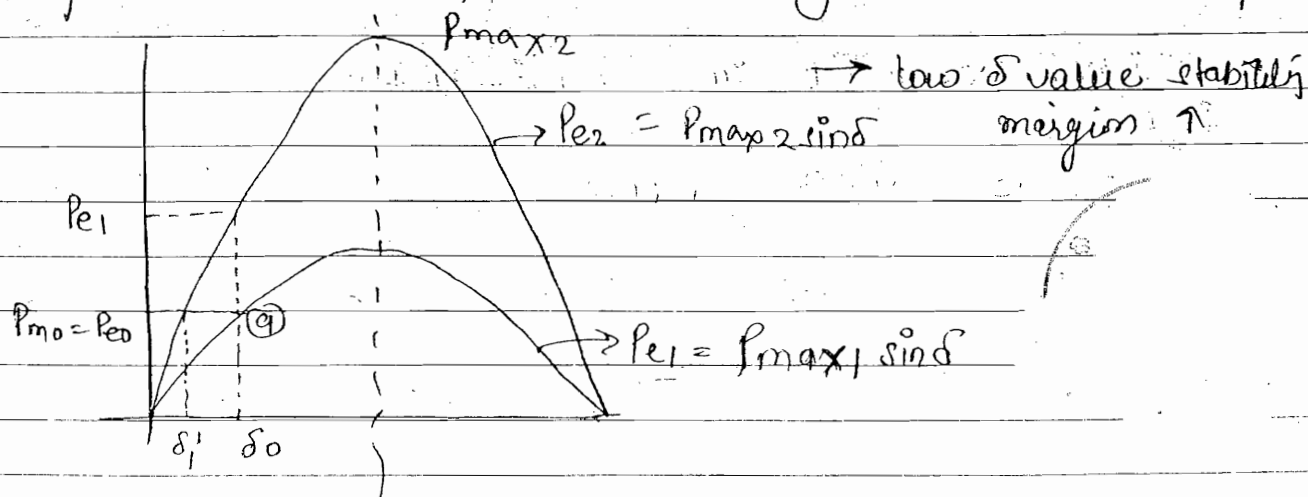
$$\frac{dP_e}{d\delta} < 0$$

System is unstable

- For the BETTER SYSTEM STABILITY  $\delta$  VALUE SHOULD IS MAINTAINED AROUND "30-45°"



- Methods to improve steady state stability:-



→ At the same value of  $\delta$ , more power can xfer

- By improving  $P_{max}$ 
  - at the same value of  $\delta$   $P_e$  can be more.
  - for the same power  $P_p$ ,  $\delta$  is lower value.

$$P_{max} = \frac{EV}{X}$$

operate the sys.  
at high voltage

Reduce  $X$ .

- 1) Use parallel lines
- 2) Use Bundle conductors

$$D_s \uparrow L = 2 \times 10^{-7} m \ln \frac{D_m}{D_s}$$

- 3) Use series capacitor

⇒ EHV → Bundle conductor to ~~reduce~~ improve stability

⇒ UHV → Hollow conductor to reduce wrong loss.

→ practical use shunt capacitor or shunt reactor

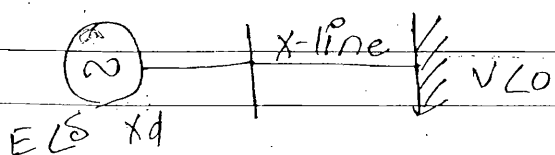
improve Pof ~~Reduce~~ ~~improve~~ ~~effect~~

series reactor → limit soc. current

series capacitor → steady state stability

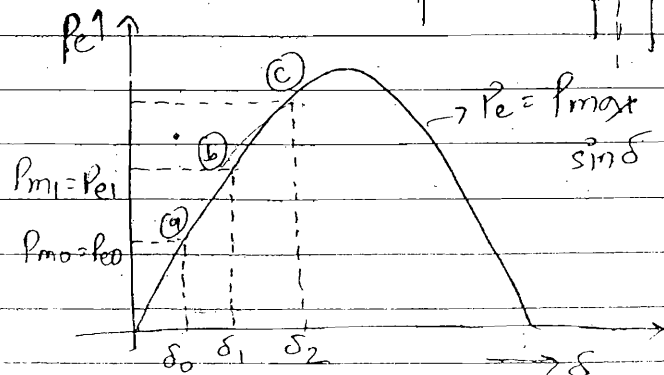
## • TRANSIENT STABILITY

Consider SMIB System



$$X = X_d + X_{line}$$

$$P_{max} = \frac{E \cdot V}{X}$$

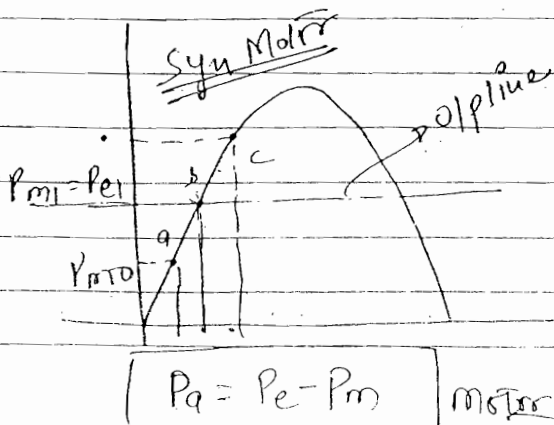


| Operating point | $P_a = P_m - P_e$ | $\delta$ value | Remarks        |
|-----------------|-------------------|----------------|----------------|
| a               | $P_a = 0$         | $= \delta_0$   | equilibrium pt |
| a               | $P_a > 0$         | $= \delta_0$   |                |

| Operating pt | $P_a = P_m - P_e$     | $\delta$ value      | $N$ value | Remarks                                 |
|--------------|-----------------------|---------------------|-----------|-----------------------------------------|
| a            | $P_a = 0$             | $= \delta_0$        | $= N_s$   | equilibrium pt                          |
| a            | $P_a > 0(\text{max})$ | $= \delta_0$        | $= N_s$   | Mech. I/p suddenly raised               |
| a-b          | $P_a > 0$             | $\delta \uparrow$   | $> N_s$   | $\delta \uparrow$ due to acceleration   |
| b            | $P_a = 0$             | $= \delta_1$        | $> N_s$   | $P_a = 0$                               |
| b-c          | $P_a < 0$             | $\delta \uparrow$   | $> N_s$   | $\delta \uparrow$ due to inertia        |
| c            | $P_a < 0(\text{max})$ | $= \delta_2$        | $= N_s$   | Inertial forces = 0                     |
| c-b          | $P_a < 0$             | $\delta \downarrow$ | $< N_s$   | $\delta \downarrow$ due to deceleration |
| b            | $P_a \geq 0$          | $= \delta_1$        | $< N_s$   | decelerating power = 0                  |
| b-a          | $P_a > 0$             | $\delta \downarrow$ | $< N_s$   | $\delta \downarrow$ due to inertia      |
| a            | $P_a > 0(\text{max})$ | $= \delta_0$        | $= N_s$   | Inertial forces = 0                     |

(Cycle Repeats)

- from b to a rotor store acc. pow. and speed will ↑
- Inertial forces present, until rotor come to originally position
- Rotor swing due to damping forces. (sustained oscillation)
- $P_a < 0(\text{max})$  Maximum de-accelerating. beoz inertial forces = 0
- b-c store de-accelerating energy
- pt b → last pt of de-acceleration  $< N_s$
- System is system with sustained oscillation.
- pt b → last pt of acceleration at  $> N_s$



- above b  $P_m$  is less  $P_e$  is more  
Rotor is at acceleration mode
- Below b  $P_m$  is more  $P_e$  is less  
Rotor is at deceleration mode
- Rotor acc. above b due to inertia
- Rotor deacc. below b due to inertia
- Rotor acc. above a due to acc.
- Rotor deacc. below c due to deacc.

→ Max. pt of acceleration → b (a to b)

→ Max. pt of de-acceleration → b (c to b)

## • Transient Stability Evaluation

- Equal Area Criterion:-

from Swing eq<sup>n</sup>  $P_a = M \frac{d^2\delta}{dt^2}$

$$\frac{d^2\delta}{dt^2} = \frac{P_a}{M} \quad (\text{Multiply } 2 \frac{d\delta}{dt} \text{ both sides})$$

$$2 \cdot \frac{d\delta}{dt} \frac{d^2\delta}{dt^2} = \frac{2 \cdot P_a}{M} \cdot \frac{d\delta}{dt}$$

$$\frac{d}{dt} \left( \frac{d\delta}{dt} \right)^2 = \frac{2}{M} \cdot P_a \cdot \frac{d\delta}{dt}$$

$$\left( \frac{d\delta}{dt} \right)^2 = \int \frac{2}{M} P_a \cdot d\delta$$

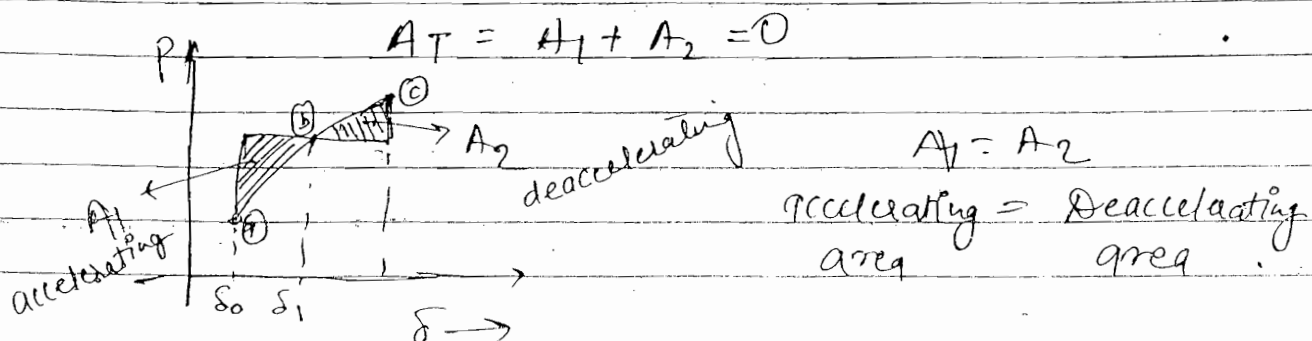
$$\frac{d\delta}{dt} = \sqrt{\int \frac{2}{M} P_a \cdot d\delta}$$

For system stability  $d\delta/dt = 0$ ,  $2/M \neq 0$

$$\therefore \int P_a d\delta = 0$$

→ This means total area under P- $\delta$  curve should be zero.

→ Total area can be zero, if total has two equal areas with opposite sign.



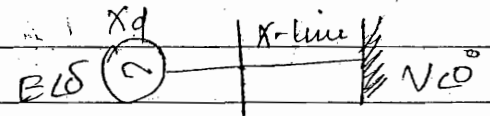
Date 18/6/12

- The concept of EAC is a graphical Method
- Can be applied to SMIB only not for multi-M/C pers.
- It gives absolute stability of the system.

## • Application of Equal Area Criterion (EAC):-

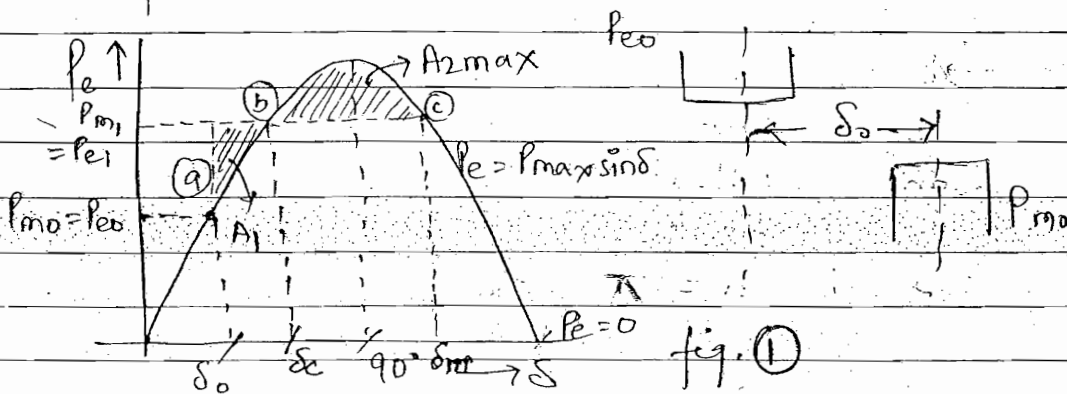
(1) Limiting Case:-

Consider SMIB system:-



$$X = X_d + X_{line} \quad P_{max} = \frac{EV}{X} \quad P_e = P_{max} \sin \delta$$

→ Initially the M/C is operating at its equilibrium at point (a) with  $P_{e0} = P_{m0}$  at  $\delta = \delta_0$



→ From the present value of  $P_{m0}$ , upto what value the mechanical input can be raised without losing stability?

upto pt c all Area equal to decel area. above pt c  $P_{m1}$  is large than  $P_{e1}$ ,  $P_{m1} \uparrow$ ,  $P_{e1} \downarrow$ ,  $\delta \uparrow$ ,  $P_{a1} \uparrow$ , unstable condition.

•  $\delta_c \rightarrow$  critical angle

→ If the CB isolate the ideal feeders when  $\delta$  value reaches  $\delta_c$ , the system is critical  $A_1 = A_2 \max$

→ The time taken for the rotor to reach upto  $\delta$  critical is known critical clearing time.

→ If CB is fast,  $A_1$  is smaller than  $A_2$ , it will operate before  $\delta_c$ , sys, more stable

### • Value of $\delta_c$

For the system to be stable  $A_1 = A_2 \max$

Area of accelerating  $A_1 = \text{area } a b c d = \text{Rectangle Area.}$

$$= P_{m0} (\delta_c - \delta_0) \quad \text{--- (1)}$$

Area of deceleration  $A_2 = \text{area DEF}$

$$= \int_{\delta_c}^{\delta_m} (P_{\max} \sin \delta - P_{m0}) d\delta$$

$$= P_{\max} [-\cos \delta]_{\delta_c}^{\delta_m} - P_{m0} [\delta]_{\delta_c}^{\delta_m}$$

$$= P_{\max} [\cos \delta_c - \cos \delta_m] - P_{m0} (\delta_m - \delta_c)$$

$$= P_{\max} \cos \delta_c - P_{\max} \cos \delta_m - P_{m0} \delta_m + P_{m0} \delta_c$$

$$\Rightarrow A_1 = A_2$$

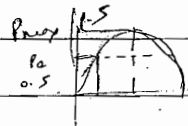
$$P_{m0} \delta_c - P_{m0} \delta_0 = P_{\max} \cos \delta_c - P_{\max} \cos \delta_m + P_{m0} \delta_c - P_{m0} \delta_m$$

$$0 = P_{\max} \cos \delta_c - P_{\max} \cos \delta_m - P_{m0} \delta_m + P_{m0} \delta_0$$

$$\cos \delta_c = \frac{P_{\max} \cos \delta_m - P_{m0} (\delta_m - \delta_0)}{P_{\max}} \quad \rightarrow (2)$$

$$\delta_c = \cos^{-1} \left[ \frac{P_{max} \cos \delta_m - P_{mo} (\delta_m - \delta_0)}{P_{max}} \right]$$

Problem 1 Page no. 56.



$$E = 1.5 \text{ pu}, V = 1 \text{ pu}, X_d = 1 \text{ pu}, X\text{-line} = 0.5 \text{ pu}.$$

$$P_{max} = \frac{E \cdot V}{X} = \frac{1.5 \times 1}{1 + 0.5} = 1.5$$

$$P_{mo} = 0.5 = 1.5 \sin \delta$$

$$\sin \delta = 0.33$$

$$\delta_0 = 19.26^\circ$$

$$\delta_m = \pi - \delta_0$$

$$= 3.14 - 19.26$$

$$=$$

• put in eq<sup>n</sup> (1)  $\delta_m = \pi - \delta_0$

$$P_{max} \cos \delta_c = P_{mo} (\pi - 2\delta_0) - P_{max} \cos \delta_0$$

$$\cos \delta_c = \frac{P_{mo} (\pi - 2\delta_0) - \cos \delta_0}{P_{max}}$$

$$P_{mo} = P_{eo} = P_{max} \sin \delta_0$$

$$\Rightarrow \frac{P_{mo}}{P_{max}} = \sin \delta_0$$

$$\delta_c = \cos^{-1} \{ (\pi - 2\delta_0) \sin \delta_0 - \cos \delta_0 \}$$

Solution  $P_{mo} = P_{eo} = 0.5 \text{ p.u.}$

$$P_{max} = \frac{1.5 \times 1}{0.5 + 1} = 1 \text{ p.u.}$$

$$\therefore 0.5 = 1 \times \sin \delta_0$$

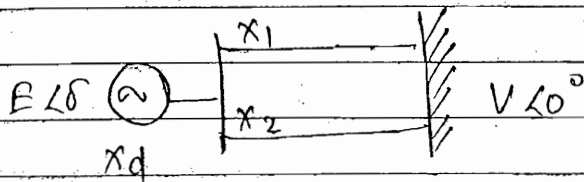
$$\delta_0 = 30^\circ$$

$$> 0.523 \text{ rad.}$$

$$\delta_c = \cos^{-1} \{ (3.14 - 2 \times 0.523) \sin 30^\circ - \cos 30^\circ \}$$

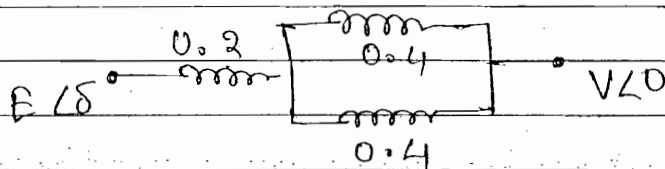
$$\delta_c = 79.6^\circ \quad \textcircled{D} \text{ Ans}$$

### (3) Fault Subsequent clearance By C.B.



(9) Before fault  $E = 1.2 \text{ pu}$   $V = 1 \text{ pu}$   $X_d = j0.2 \text{ pu}$   
 $X_1 = X_2 = j0.4 \text{ pu}$   $P_{m0} = P_{e0} = 1.5 \text{ pu}$

Transfer Reactance Before fault ( $X_b$ )



$$X_b = 0.2 + (0.4 \parallel 0.4)$$

$$= j0.4 \text{ pu.}$$

$$P_{mb} = \frac{EV}{X_b} = \frac{1.2 \times 1}{j0.4} = 3 \text{ pu.}$$

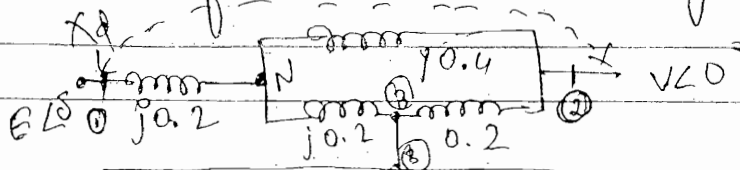
$$P_{eb} = P_{mb} \sin \delta = 3 \sin \delta$$

$$1.5 = 3 \sin \delta_0$$

$$\boxed{\delta_0 = 30^\circ}$$

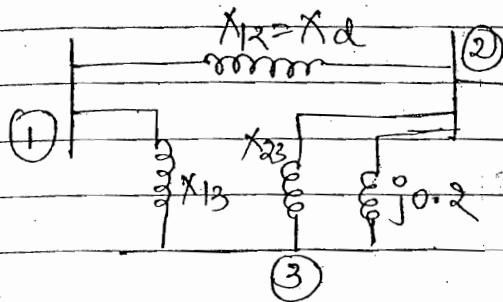
(6) Condition during fault  $\rightarrow$  3  $\phi$  fault occur at the middle pt. of line - 2

Transfer Reactance During fault ( $X_d$ )



$$P_{R2} =$$

Convert Y- reactance  $1n(j0.2)$ ,  $2n(j0.4)$  and  $3n(j0.2)$  into delta



$$X_{12} = \frac{X_{1n}X_{2n} + X_{1n} + X_{2n}}{X_{3n}}$$

$$= \frac{j0.2 \times j0.4 + j0.2 + j0.4}{j0.2}$$

$$= j1 \text{ p.u.}$$

$$X_{23} = \frac{X_{2n}X_{3n} + X_{2n} + X_{3n}}{X_{1n}}$$

$$= \frac{j0.4 \times j0.2 + j0.4 + j0.2}{j0.2}$$

$$= j1 \text{ p.u.}$$

$$P_{ma} = \frac{1.2 \times 1}{1} = 1.2 \text{ p.u.}$$

$$P_{eq} = 1.2 \sin \delta$$

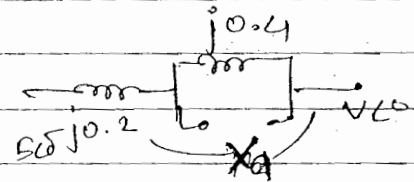
(c) After the fault (Post fault condition):

Two CBs connected at the 2-ends of line 2 isolates faulty feeder.

transfer reactance after the fault

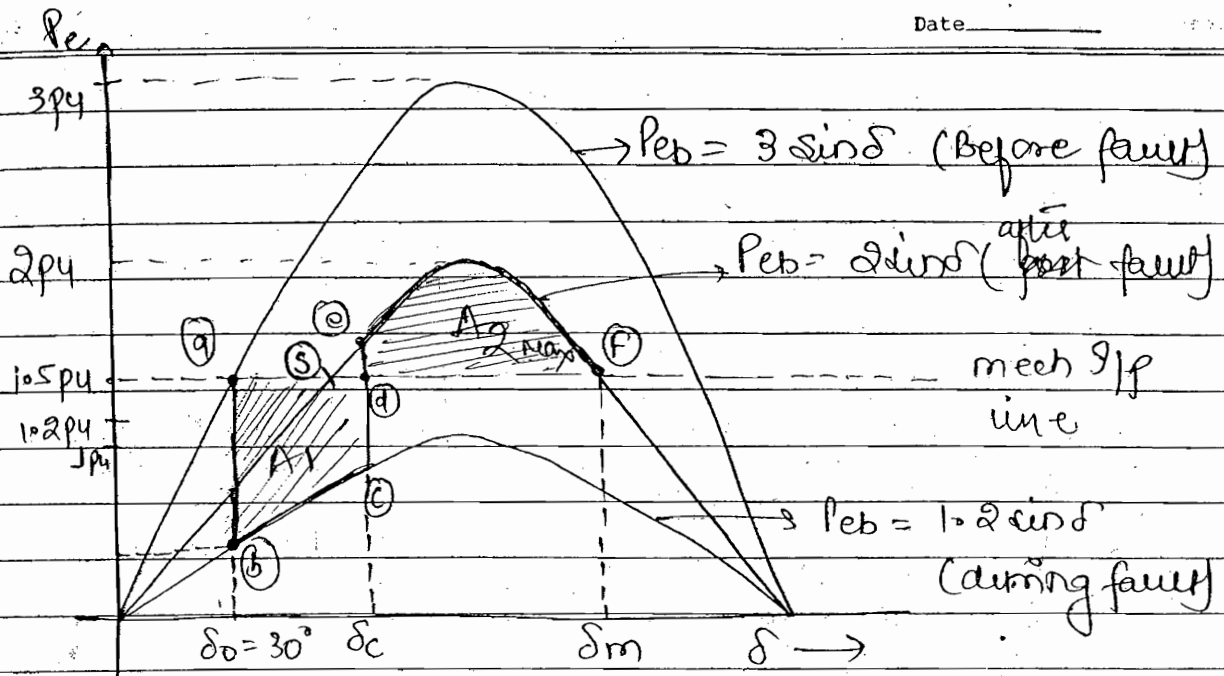
$$X_9 = j0.2 + j0.4$$

$$= j0.6$$



$$P_{ma} = \frac{1.2 \times 1}{0.6} = 2 \text{ p.u.}$$

$$P_{eq} = 2 \sin \delta$$



→ pt a is operating pt. When fault occur a pt shifted to pt (b), mechanical  $9/p >$  elec.  $9/p$ . Rotor will accelerate and reaches to pt c. At pt c LB will operate that is  $\delta_c$  (critical angle). pt c shift to pt (e). At pt (e) elec  $9/p >$  mech.  $9/p$ , rotor decelerate but due to inertia it accelerate and reaches to pt f ( $\delta = \delta_m$ ) where  $P_{mech} = P_{elec}$ . Rotor will swing in after fault come and finally come to pt e in stable condition.

→ CB operate at  $\delta_c$ ,  $A_1 = A_2 \max$ , sys. is critical stable.

→ for the system to be stable  $A_1 = A_2 \max$

$$A_1 = \int_{\delta_0}^{\delta_c} (P_{m0} - P_{md} \sin \delta) d\delta$$

$$A_1 = \left[ P_{m0} [\delta]_{\delta_0}^{\delta_c} - P_{md} [-\cos \delta]_{\delta_0}^{\delta_c} \right]$$

$$= P_{m0} (\delta_c - \delta_0) - P_{md} (\cos \delta_0 - \cos \delta_c)$$

$$= P_{m0} (\delta_c - \delta_0) + P_{md} \cos \delta_c - P_{md} \cos \delta_0$$

— (1)

$$A_2 = \int_{\delta_c}^{\delta_m} [P_{ma} \sin \delta - P_{mo}] d\delta$$

$$= [P_{ma} (-\cos \delta)_{\delta_c}^{\delta_m} - P_{mo} (\delta)_{\delta_c}^{\delta_m}]$$

$$= P_{ma} (\cos \delta_c - \cos \delta_m) - P_{mo} (\delta_m - \delta_c) \quad \text{--- (2)}$$

equating eqn (1) & (2).

$$\Rightarrow P_{mo} (\delta_c - \delta_o) + P_{md} \cos \delta_c - P_{md} \cos \delta_o = P_{ma} (\cos \delta_c - \cos \delta_m) - P_{mo} (\delta_m - \delta_c)$$

$$\Rightarrow P_{mo} \delta_c - \delta_o P_{mo} + P_{md} \cos \delta_c - P_{md} \cos \delta_o = P_{ma} \cos \delta_c - P_{ma} \cos \delta_m - P_{mo} \delta_m + P_{mo} \delta_c$$

$$\Rightarrow \cos \delta_c (P_{md} - P_{ma}) = -P_{mo} (\delta_m - \delta_o) - P_{ma} \cos \delta_m + P_{md} \cos \delta_o$$

$$\boxed{\cos \delta_c = \frac{P_{mo} (\delta_m - \delta_o) + P_{ma} \cos \delta_m - P_{md} \cos \delta_o}{P_{ma} - P_{md}}}$$

$\delta_m$  value :-  $P_e$  at pt (a) & (f) are same.

$$P_{mb} \sin \delta_o = P_{ma} \sin \delta_m$$

$$\sin \delta_m = \frac{P_{mb}}{P_{ma}} \sin \delta_o$$

$$\boxed{\delta_m = \pi - \sin^{-1} \left( \frac{P_{mb} \sin \delta_o}{P_{ma}} \right)}$$

$$d = \frac{180}{\pi} \times 2.19$$

Date \_\_\_\_\_

$$\delta_m = \pi - \sin^{-1} \left( \frac{3}{2} \times 0.5 \right)$$

$$\delta_m = 2.29 \text{ rad.}$$

$$\cos \delta_c = \frac{1.5 \times (2.29 \times -0.5) + 2 \times \cos(131.2) - 1.2 \times \cos(80)}{(2 \times -1.2)}$$

$$= \frac{2.685 + (-1.0317) - (-1.03)}{0.6}$$

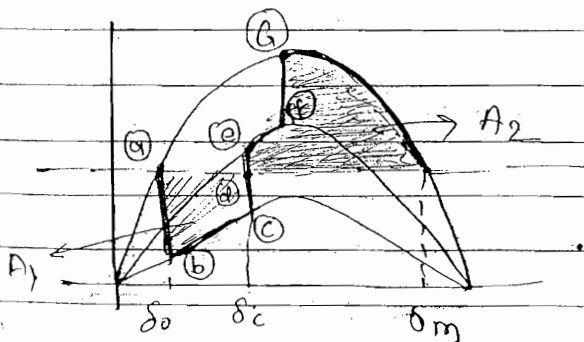
$$\delta_c = 68.65^\circ$$

### • Methods To Improve tr. Stability: -

(1) Use high Speed CB: - High speed C.B. Narrows down accelerating area, thereby the decelerating area available increases. This further increase transient stability margin.

(2) Use auto Reclosing CB: -

Sequence: O: Open C: close.  
 relay: O → 3 sec → C → O → 60 sec → C → O → 3 φ fault → C → O →  
 C.B. after the C.B. close fault remain present assume 3 φ fault occurs again close  
 go and clear the line



extra - de accelerating area

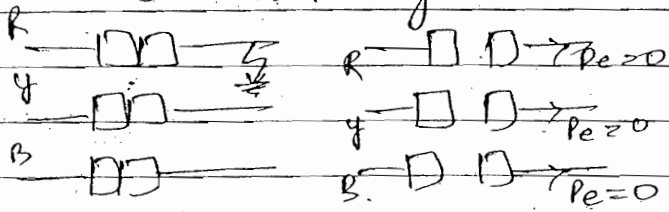
(3) Use fast field excitation system:- When rotor is accelerating, excitation system increase  $E'$  &  $V$  values.

$$P_e' = \frac{E' V \sin \delta}{X} \quad P_e' \uparrow \quad P_a \downarrow$$

any value of  $\delta$   $E' \uparrow$ , more power o/p.,  $P_a = P_m - P_e$ ,  $P_a \downarrow$

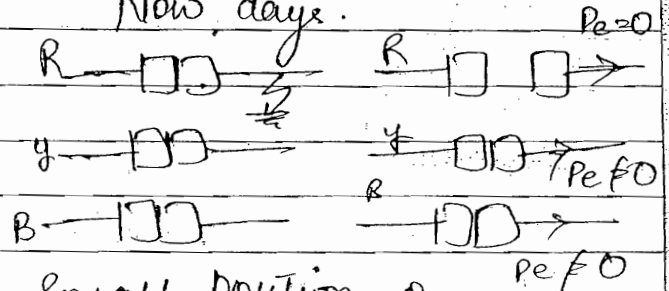
(4) Use single pole switches:- 85% are SLG fault.

In older days.



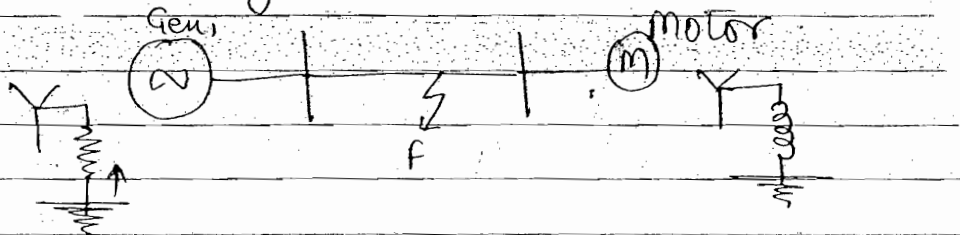
Entire Mechanical I/p becomes into R.E.

Now, days.



Small portion of mechanical I/p converts into R.E.

(5) Use high speed Governors:-



• Gen. is grounded with resistance. Fault is due to S.C. During S.C.  $P_e = 0$  and R.E. store in rotor due to which rotor accelerate heavily. For controlling this gen is grounded with resistance. fault current through resistance produce heat and decelerate the rotor.

• Motor In motor case  $P_e = 0$ , Rotor is decelerating if it is grounded with resistance,

it produce heat and further deaccelerate, deaccelerate + deaccelerate, further deaccelerate, to avoid this Motor is grounded with reactor.

Generator  $\rightarrow$  Resistor grounded  
Motor  $\rightarrow$  Reactor grounded.

$\rightarrow$  Governor will automatic close the valve when motor deaccelerate it open the valve, when motor accelerate it close the valve.

### Problem 16 Rpm 58

\* As long as the o/p is equal to mech. 91 p.u. doesn't loose stability

$$P_{mo} = 1 \text{ p.u.} = P_e'$$

$$P_e' = \frac{1 \times 1}{0.1 + X} \sin 130^\circ = 1$$

$$\Rightarrow 0.1 + X = \frac{1}{\sin 130^\circ}$$

$$\Rightarrow X = 0.46 - 0.1 = 0.66$$

$$X = 0.67 \text{ Ans}$$

Sol<sup>n</sup> 17  $E = 1 \text{ p.u.}$   $X_{\text{max}} = 0.12 \text{ p.u.}$   $V = 1.0$

$$P_{\text{max}} = 6.25 \text{ p.u.}$$

$$P_{\text{max}} = \frac{1 \times 1}{X} \sin 90^\circ$$

$$6.25 = \frac{1}{0.12 + 0.5X} \quad X = 0.08 \text{ p.u.}$$

$$P_{\text{max}}^H = \frac{1 \times 1}{0.12 + 0.5 \times 0.08} = 5 \text{ p.u. Ans}$$

$d \rightarrow \text{rad} \quad 1/180^\circ$

$\text{Rad} \rightarrow \text{deg} \quad \frac{180}{\pi}$

Date \_\_\_\_\_

Sol<sup>n</sup> 24  $P_{mo} = P_{eo} = 1 \text{ pu.}$   $P_{max} = 2.0$   $\delta_{max} = 110^\circ$

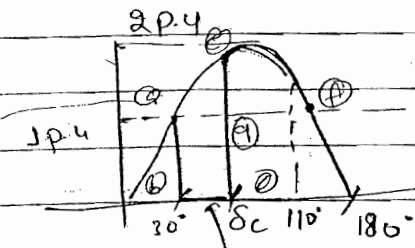
~~$\delta_c = \delta_{max}$~~   $=$

$P_{max} \sin \delta = P_{eo}$

$\sin \delta = \frac{1}{2} = 0.5$

$\Rightarrow \delta = 30^\circ$

$30^\circ \rightarrow 0.523 \text{ rad}$



$A_1 = 1(\delta_c - 0.523)$

$A_2 = \int_{\delta_c}^{110^\circ} (2 \sin \delta - 1) d\delta$

$1.919 \text{ rad}$

$= 2(-\cos \delta)_{\delta_c}^{110^\circ} - (110^\circ - \delta_c)$

$= 2(\cos \delta_c - \cos 110^\circ) - (1.919 - \delta_c)$

$A_1 = A_2$

$\delta_c - 0.523 = 2 \cos \delta_c + 0.684 - 1.919 + \delta_c$

$2 \cos \delta_c = 0.684 - 1.919 + 0.523$

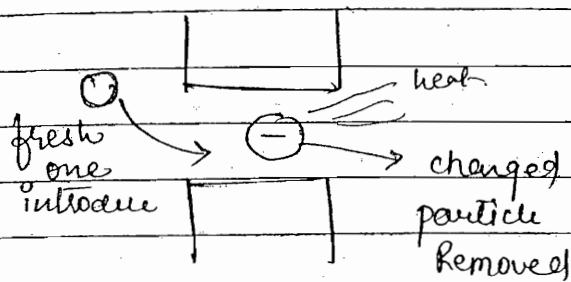
$\delta_c = 69.14^\circ$

Critical clearing Time:

$t_{cr} = \left( \frac{2H(\delta_{cr} - \delta_0)}{\pi f P_m} \right)^{1/2}$

long distance dielectric stress reduces and arc should be interrupted. However in circuit Breaker arc is maintain because of thermionic emission

## • Arc Interruption

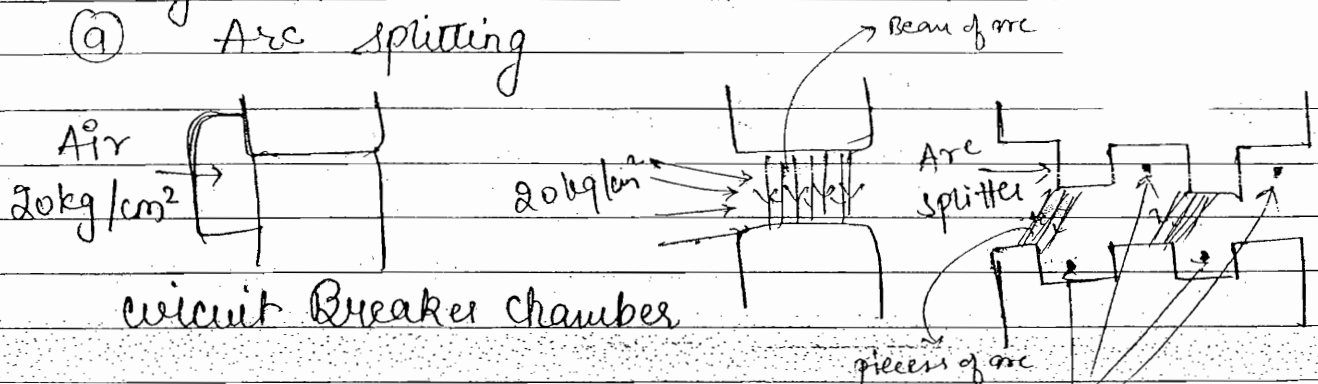


→ High Resistance Method  
used with DC CB.

→ Low Resistance / Natural Current Zero Method  
used with AC CB.

### (1) High Resistance Method :-

#### (a) Arc splitting



→ arc follow low resistance path, some points (-) are not present. Beam of arc is cut into pieces of arc. Magnitude of arc get reduce. And overall dielectric strength  $\uparrow$ .

Drawback:- Arc is concentrated on splitter, metal will get melt, vaporized and blast like bomb.

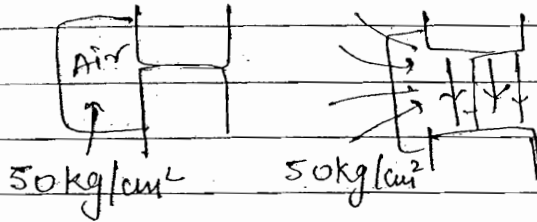
→ At time  $t$ , high resistance value is recovery so called high resistance.

(b) Arc lengthening. Arc length  $\uparrow$ , Arc Resistance  $\uparrow$ ,  
Arc Current Heat  $\downarrow$ .

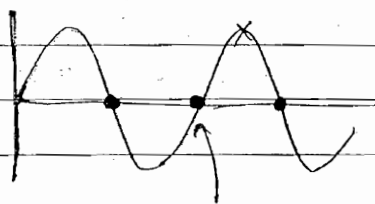
Drawback:  $\rightarrow$  It takes more time

$\rightarrow$  Arc length requires more.

(c) Arc Forced Blast: - Remove the heat very quickly.  
Drawback: mechanical force is high



(2) Low Resistance: - Natural Current Zero Method: -



$$\text{Natural } I = \frac{Q}{t}, \quad Q = 0$$

$$I = 0$$

$$\text{At peak pt (x)} \quad Q = Q_{\max}$$

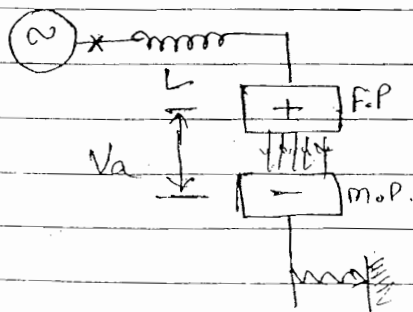
Natural Current Zero

$\rightarrow$  Charges coming from cathode keep on removing continuously and heat is also keep on removing. (Arc interrupted in first cycle)

$\rightarrow$  In ac C.B. Arc interruption takes place once the arc passes its natural current zero.

• Arc Restriking: -

Consider a p.c. with negligible resistance.



L: Inductance of the sys. upto the fault location.

$\rightarrow$  fault near generator L is low

$\rightarrow$  fault occur far L is high

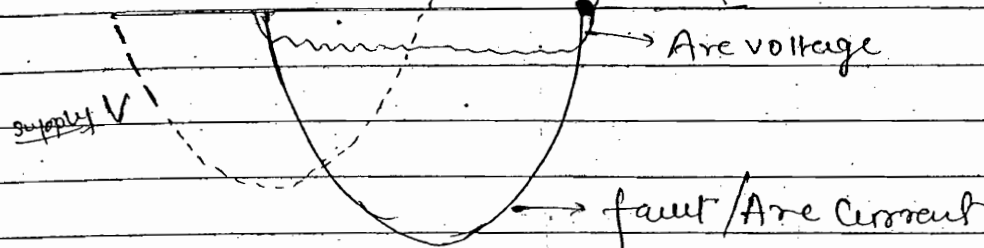
Arc voltage in phase with arc current.

dash-dash disappears

Transient Restriking voltage

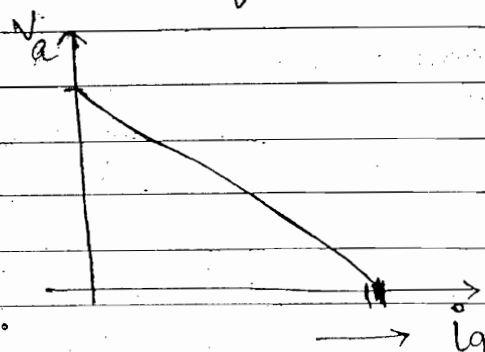
ARV

Date \_\_\_\_\_



$$V_a = \text{Arc Voltage} = I_a R_a$$

- (1) Arc has unity power factor
- (2) Arc flows through a pure resistance path.

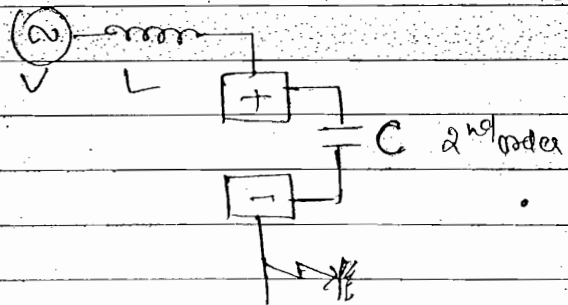


$\Rightarrow$  gap  $\uparrow$ ,  $R \uparrow$ ,  $I \downarrow$ ,  $V \uparrow$   
 $\Rightarrow$  gap  $\downarrow$ ,  $R \downarrow$ ,  $I \uparrow$ ,  $V \downarrow$

$\Rightarrow$  Arc has -ve Resistance characteristics.

Arc characteristics

• Time constant of 2nd order is oscillating.



• Arc disappears, O.C. condition capacitor will appear.

• Sys is oscillating due to presence of inductor and capacitor. capacitor we got due to O.C. condition so we get oscillatory waveform.

• In AC CB arc shall be interrupted at its natural current zero. At this time we have O.C. condition across the breaker contacts. At the instant when the arc is interrupted the

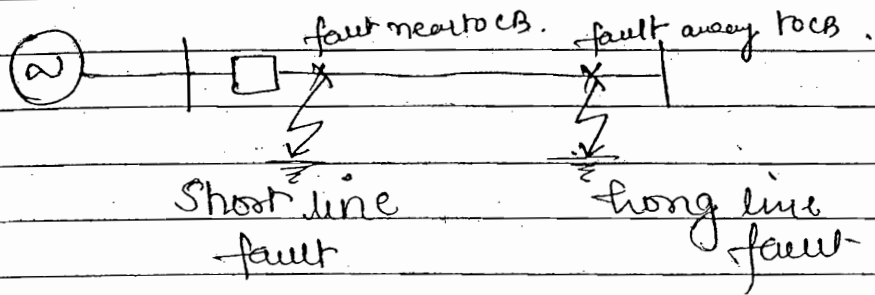
contacts are not cooled some heat remains in the gap. To addition this capacitor is appearing across the breaker point. Inductance and capacitance present in the circuit make the circuit oscillatory. At the instant when arc is interrupted instead of steady state value of voltage an oscillatory voltage appears across the breaker contacts. This voltage is known as transient Restriking voltage.

- The transient voltage that appears across the CB contact at the instant of arc interruption.

$$TRV = V_h = V_m \left( 1 - \cos \frac{t}{\sqrt{LC}} \right) \text{ KV}$$

→ For the particular value of LC, restriking voltage rises slowly. It will take more time to reach peak value. It have sufficient time. During this time air is blowing continuously, charge and heat removing. No condition help for restriking. Arc will be interrupted.

→ In CoB, arc may or may not restrike that depends on property of circuit. The property of circuit depends upon parameter of circuit. The property of circuit is such a way that restriking voltage rise slowly. This means that we have given with ample of time. During this time the heat prevail in the gap can be remove left charges can be remove etc. Restriking voltage soon after reaching the maximum value as no conditions are help the restriking voltage final arc interruption takes place.



- For a short :-  $L \downarrow C \downarrow t \uparrow \cos t \downarrow 1 - \cos t \uparrow V_r \uparrow$   
time fault  $\sqrt{LC}$   $\sqrt{LC}$   $\sqrt{LC}$
- For long line :-  $L \downarrow C \uparrow t \downarrow \cos t \uparrow 1 - \cos t \downarrow V_r \downarrow$   
fault  $\sqrt{LC}$   $\sqrt{LC}$   $\sqrt{LC}$

→ Short line fault > dangerous than long line fault.

→ In view of restriking voltage a short line fault are more dangerous than long line fault

- Rate of Restriking Rise of Restriking Voltage (RRRV) :-

$$RRRV = \frac{dV_m}{dt} = \frac{V_m}{\sqrt{LC}} \sin \frac{t}{\sqrt{LC}} \quad \text{KV}/\mu\text{sec}$$

- Max RRRV =  $\frac{V_m}{\sqrt{LC}}$  KV/ $\mu\text{sec}$ .

- Natural frequency of oscillation of RRRV

$$f_n = \frac{1}{2\pi\sqrt{LC}}$$

- Average RRRV =  $\frac{\text{Max value of } V_r}{\text{Time taken to reach the value}}$

$$\text{at } t = \pi\sqrt{LC}$$

$$V_r = V_m \left( 1 - \cos \frac{\pi\sqrt{LC}}{\sqrt{LC}} \right) = V_m (1 - \cos \pi) = 2V_m$$

$$\text{Avg. RRRV} = \frac{2V_m}{\pi\sqrt{LC}} \text{ kV/\mu sec.}$$

NOTE : - (1) For restriking free operation of CB- RRRV should be less.

(2) The rate at which heat is dissipated is less than compare to RRRV, then arc will restrike again, otherwise it interrupts

• Active Recovery Voltage : - The instantaneous voltage that appear across the breaker contact at the final instant of final arc interruption. The ~~same~~ value of this voltage is known as ARV.

• Current Chopping Phenomenon :-

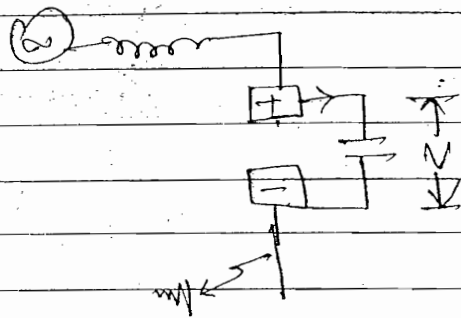
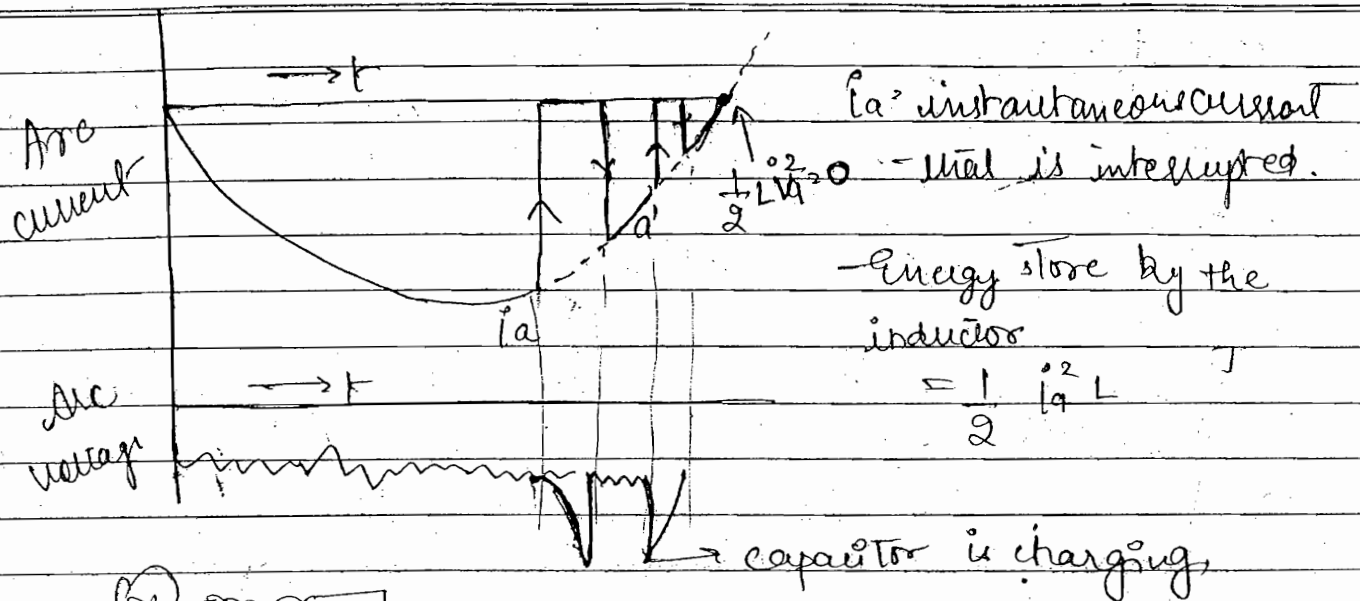
(1) While designing the strength of de-ionizing force most severe fault like 3 $\phi$  fault is considered.

(2) Consider the situation where the minimum fault condition are present in the system. The strength of de-ionizing force is so high and fault condition is so least, the de-ionizing force now will not wait until natural current zero. It has the sufficient strength to interrupt the arc before the natural current zero.

Def : - Interruption of Arc Before Natural Current Zero.

✓ dielectric stress and not condition sufficient for restriking again,  
when c.b voltage  $\uparrow$  bcoz of capacitor charging

Date \_\_\_\_\_



$$\frac{1}{2} C V^2 = \frac{1}{2} L i_a'^2$$

$$V = i_a' \sqrt{\frac{L}{C}} \rightarrow \text{prospective voltage}$$

→ capacitor is charging bcoz inductor gives its energy electromagnetic to electrostatic energy.

Q. What reason arc is interruption and restrike again?  
Ans. interruption:- high dielectric strength.

- Arc is interrupted well before natural current 0 bcoz strength of de-ionizing force is so high and severity of fault current is less.

arc is restriking bcoz of two reasons:-

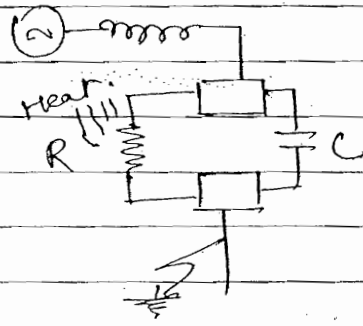
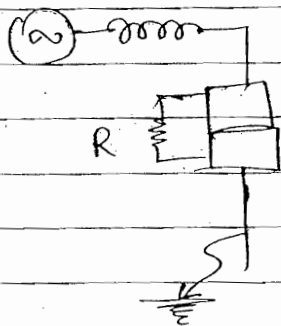
- (1) The prevailing heat in the gap.
- (2) The voltage across C.B. contact is rapidly  $\uparrow$  as the capacitor across the C.B. contact is charging due to electromagnetic energy present in the inductor.

Disadvantage - Transient over voltages known as switching over-voltage due appear in p.s.

- For insulation requirement for EHV lines is design based on switching over voltage

→ To avoid current chopping we apply resistance switching

### Resistance Switching



When  $R \rightarrow$  absent  
100% electromagnetic  
convert into  
electrostatic energy

→ Value of  $R$  so selected so that electromagnetic energy (majority) convert into heat and less to electrostatic energy. Chance of restriking voltage reduce and reduce chopping phenomenon reduce.

$$R = 0.5 \sqrt{\frac{L}{C}}$$

Sol<sup>n</sup> 19  $I_{base} = \frac{110}{\sqrt{3} \times 11} = 5.77 \text{ A.}$

$I_{sc} = \frac{1}{Z_{th}} = \frac{1}{0.19} = 5.26 \text{ p.u.}$

$I_{actual} = 5.77 \times 5.26 = 30.39 \text{ KA Ans}$

Sol<sup>n</sup> 20  $V_{r, \text{Max}} = 2 \times V_m \xrightarrow{\text{peak}}$   
 $= 2 \times \sqrt{2} \times \frac{17.32}{\sqrt{3}} \xrightarrow{\text{phase voltage}}$   
 $= 28.28 \text{ kV.}$

Sol<sup>n</sup> 21  $f = \frac{1}{2\pi \sqrt{LC}} = \frac{1}{2\pi \sqrt{15 \times 10^{-3} \times 2 \times 10^{-6}}}$   
 $= 29 \text{ kHz.}$

Sol<sup>n</sup> 22  $V = I_a \sqrt{\frac{L}{C}} = 10 \times \sqrt{\frac{1}{1 \times 10^{-6}}} = 100 \text{ kV.}$

Sol<sup>n</sup> 23 - (a)

Sol<sup>n</sup> 24 -  $R = 0.5 \sqrt{\frac{L}{C}} = 0.5 \times \sqrt{\frac{25 \times 10^{-3}}{25 \times 10^{-6}}} = 500 \Omega \text{ Ans}$

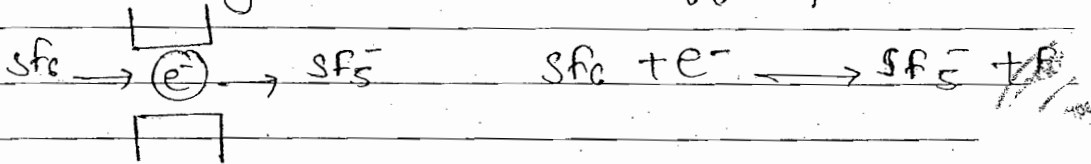
- ① ABCB's  $\rightarrow$  Air Blast CB
- ② SF<sub>6</sub> CB's  $\rightarrow$  Sulphur hexa fluoride CB
- ③ OC B's  $\rightarrow$  Oil CB
- ④ MO CB's  $\rightarrow$  Minimum oil CB
- ⑤ BO CB's  $\rightarrow$  Bulk Oil CB.
- ⑥ VCB's  $\rightarrow$  Vacuum

### • ABCB's

- CC phenomenon is most severe in case of ABCB's
- Resistance switching is mainly employed for ABCB's
- die pressure in ABCB's is maintain around  $30 \text{ kg/cm}^2$
- ABCB are most suitable for high speed and repeated operations
- ABCB's are most suitable for EHV and UHV applications 400 KV range.

### • SF<sub>6</sub> CB

- SF<sub>6</sub> CB's are more popular and widely used.
- SF<sub>6</sub> gas 3-5 times better than air bcz of its electronegative property.
- Electronegative means - affinity for electron



- SF<sub>6</sub> breakers are useful for all voltage application from 11 KV to 220 KV
- For interrupting low inductive and low capacitance currents without resistance switching the CB employed is SF<sub>6</sub> CB
- SF<sub>6</sub> gas has excellent thermal conductivity properties bcz of its low molecular weight

### • OCB's

- X-mec oil has two application.
  - It act as dielectric medium  $\rightarrow 50 \text{ KV/cm}$
  - It act as a cooling medium.
- At 658 K temp. X-mec oil decomposes and produces various gases. Out of these gases

In 1st cycle more heat produced as duration of arc current is more.

Date \_\_\_\_\_

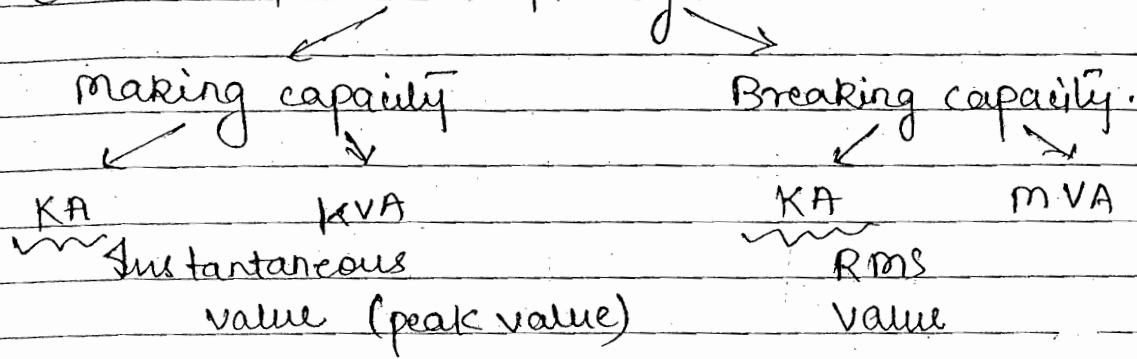
70% of gases is hydrogen which is a coolant medium.

- OCB has least current chopping
- OCB useful for all voltage applications from 11KV to 132KV
- Volume of oil required in max MOCB is only 300L. This is because oil in MOCB is used for arc interruption only whereas in BOCB it is used for insulating live parts.
- In OCB the strength of de-ionising force shall adjust in accordance to severity of fault & current. Hence these breakers has least tendency of current chopping.
- The chances for arc interruption is the subsequent natural current zero increases with OCB's but decreases in other types
- The problems with OCB's are:-
  - (1) Carbonisation of live parts
  - (2) Fire hazards.

### VCB's

- The vacuum pressure in VCB's is maintain around  $10^{-8}$  to  $10^{-6}$  Torr
- $1 \text{ Torr} = 1 \text{ mm of Hg pressure}$
- The principle of arc interruption in VCB's is condensation of arc products like Cu vapours.
- Maintenance is least in case of VCB's
- Suitable for remote and rural applications.
- For interruption high current - at low voltages (1KV range) VCB's are preferred.

## • Circuit Breaker Rating.

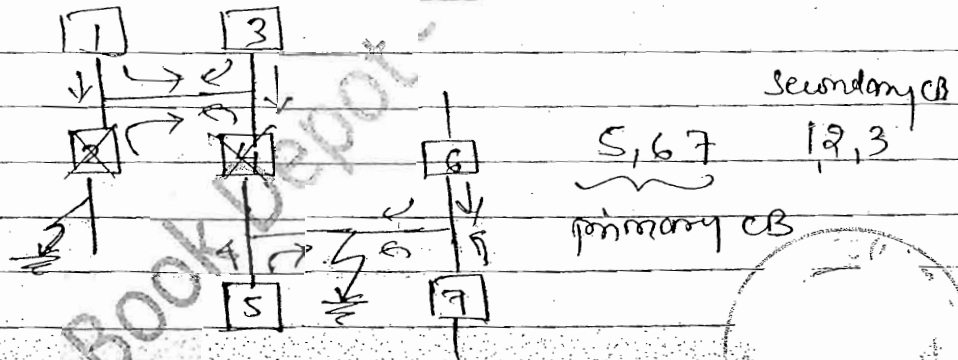


$$\text{Making capacity} = 2.55 \times \text{Breaking capacity}$$

- Making capacity at subtransient
- Breaking capacity at transient

Sol<sup>n</sup>. 17

Peq. No. 70



5, 6, 7, 3, 1, 2

2, 1, 3, 5, 6, 7

## • OVERHEAD LINE INSULATORS

- Application →
- To give necessary electrical clearance for power conductor
  - To give necessary mechanical support for power conductor.

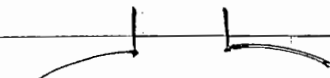
Material

- Toughened glass.
- porcelain

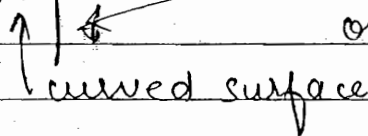
Types → Pin Insulators - upto 33 kV (Distribution System)

(If puncture take place whole unit to be replaced)

above 33 kV size ↑ unceram - mica



To increase creepage / flash over distances.



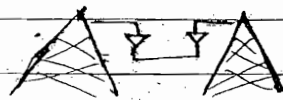
→ String insulators

| KV          | 66  | 132  | 220   | 400   |
|-------------|-----|------|-------|-------|
| No. of disc | 4-5 | 9-10 | 13-14 | 22-23 |

String

Suspension

strain



vertical down  
Tangent Tower

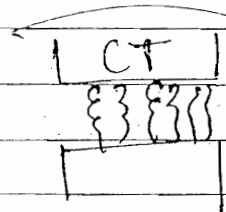


conductor

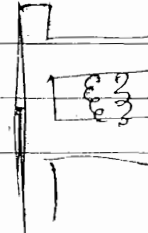
inclination more than 2°

Penetration Tower

→ Post Insulators : used to erect substation equipment.

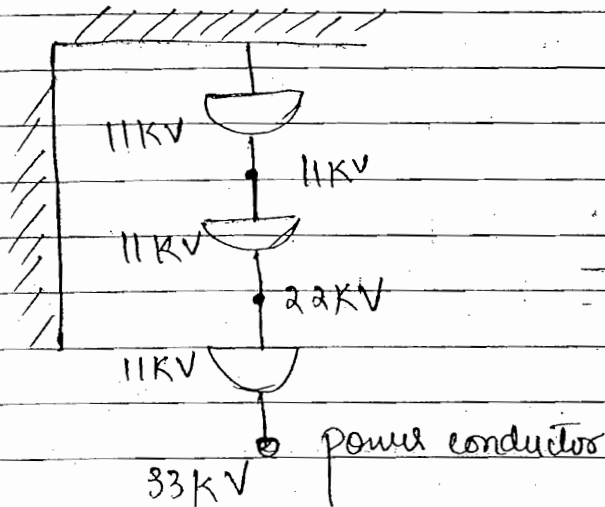


Shackle Insulator → The purpose is same as strain insulators, but are used in distribution



shackle

# Voltage Distribution Across String Insulators :-

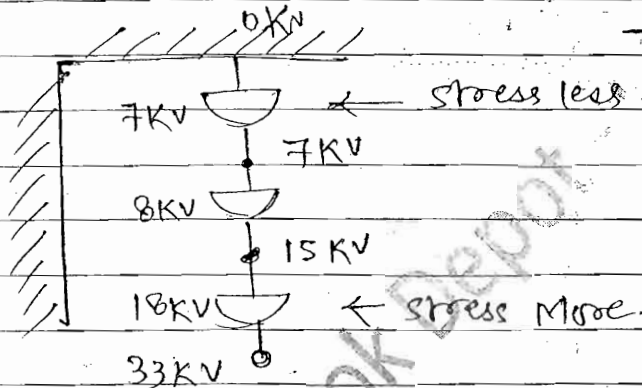


Required case :-

uniform distribution

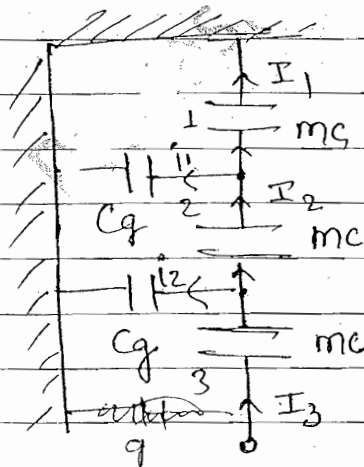
→ Dielectric is uniformly distributed

Practical



→ chances of failure is more.

mc → mutual Capacity = Insulator Capacitance.



→ ground capacitor exist due to pot diff b/w junc and ground

Cg → ground capacitor

→ capacitor exist b/w ground and junction

•  $I_3 > I_2 > I_1$

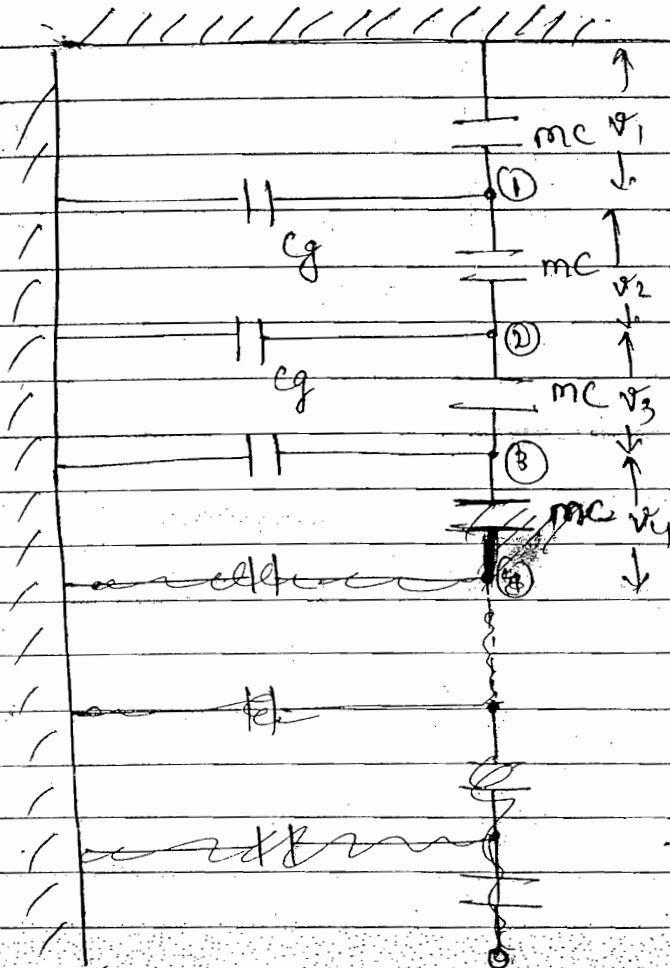
→ At every metallic junction we are loosing current

• In string insulator voltage distribution is not uniform bcoz of ground capacitor.

$m \Rightarrow \text{If } \frac{C_g}{mC} \text{ it is less than 1, take inverse } (m > 1) \text{ always}$

Date \_\_\_\_\_

→ 3 Insulator, 2 junction, 2 ground capacitor.



$V_i = \text{Voltage upto } i^{\text{th}} \text{ junction}$

$V_1 = V, V_2 = V_1 + V_2$

$V_3 = V_1 + V_2 + V_3$

$K = \text{No. of disc} = 4$

$n = \text{No. of pin junctions}$

$= K - 1$

$= 4 - 1 = 3$

$C_g = \text{Ground capacitance}$

$= 1 \mu F$

$m = \text{Ratio of two capacitance}$

$mC = \text{mutual capacitance}$

$= 10 \mu F$

$(mC > C_g)$

her  $m = \frac{mC}{C_g} = \frac{10}{1} > 1$

Max. voltage drop across the unit = 26 kV  
( $V_4$  is given)

$$V_{i+1} = V_i + \frac{V_i}{m}$$

for  $i = 1, 2, \dots, K$

$V = 1$

$$V_2 = V_1 + \frac{V_1}{10}$$

$V_1 = V$

$$= V_1 + \frac{V_1}{10}$$

$$= 1.1 V_1$$

$i = 2$ 

$$V_3 = V_2 + \frac{V_2}{m}$$

$$V_2 = V_1 + U_2$$

$$= 1.1V_1 + \frac{V_1 + U_2}{10}$$

$$= 1.1V_1 + \frac{V_1 + 1.1V_1}{10}$$

$$= 1.1V_1 + 0.21V_1$$

$$= 1.31V_1$$

 $i = 3$ 

$$V_4 = V_3 + \frac{V_3}{m}$$

$$= 1.31V_1 + \frac{V_2 + V_3 + U_1}{10}$$

$$= 1.31V_1 + 0.341V_1$$

$$V_4 = 1.651V_1$$

Given max voltage  $V_4 = 24 \text{ kV}$ .

$$V_1 = \frac{24}{1.651} = 14.54$$

$$V_2 = V_1 \times 1.1 = 14.54 \times 1.1 = 15.99$$

$$V_3 = V_2 \times 3.41 = 14.54 \times 3.41 = 49.58$$

• String Efficiency:-

$$= \frac{\text{voltage across}}{\text{No. of discs} \times \text{voltage across the unit near pow. conductor}} \times 100$$

Problem :  $K=3$  ;  $m=5$   $\therefore \eta = ?$

Soln : No. of disc. no. joints  $= K-1 = 2$ .

$$i=1 \quad u_2 = u_1 + \frac{V_1}{5} = 1.2u_1$$

$$i=2 \quad u_3 = u_2 + \frac{V_2}{5} = 1.1u_1 + \frac{u_1 + 1.1u_1}{5}$$

$$= 1.64u_1$$

$$V = u_1 + 1.2u_1 + 1.64u_1$$

$$= 3.84u_1$$

$$\% \eta = \frac{3.84u_1}{3 \times 1.64u_1} \times 100$$

$$= 78\%$$

• Methods of Improving String  $\eta$  :-

$$u_{i+1} = u_i + \frac{V_i}{m}$$

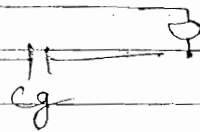
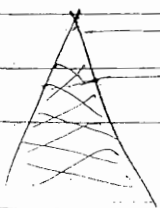
$$u_{i+1} \approx u_i \quad \text{if } \frac{V_i}{m} \approx 0$$

$$\% \eta_{V_{i+1}} = 100\%$$

$$m = \frac{mc}{c_g} \quad \text{if } \frac{V_i}{m} = 0 \text{ if } m = \infty$$

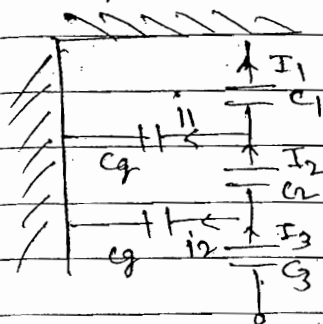
$$mc \uparrow \quad c_g \downarrow$$

Method 1 (1) Use larger cross arm



$c_g$  cannot be zero  
for longer cross  
arm  $\rightarrow$  mechanically  
not good

### (g) Capacitance grading:



$$I_1 < I_2 < I_3$$

$$I_3 X_{C3} \approx I_2 X_{C2} \approx I_1 X_{C1}$$

$$X_{C3} < X_{C2} < X_{C1}$$

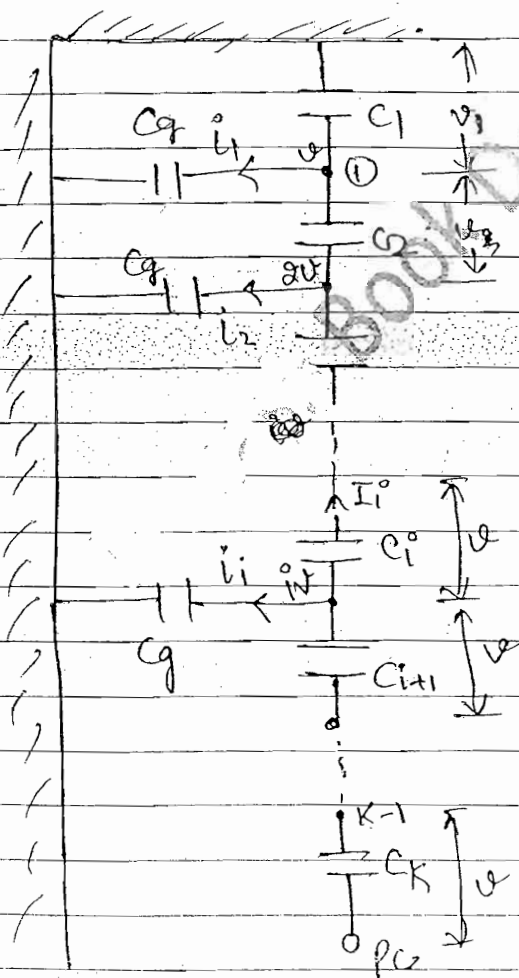
$$C_3 > C_2 > C_1$$

Effective  
quest

→ high current through  $X_{C3}$  and low current through high capacitance  $X_{C1}$ .

⇒ high capacitance is placed near power conductor.

⇒ low capacitance is placed near the arm.



$$\eta \% \approx 100$$

apply KCL at junction (i)

$$I_{i+1} = I_i + I_i$$

$$V_{C_{i+1}} = V_{C_i} + (I_i - 0)X_{C_i}$$

$$C_{i+1} = C_i + I_i \cdot C_i$$

for  $i = 1, 2, \dots, K-1$

Disadvantage:

- we have to maintain large no. of spars.
- no. of capacitor with diff. rating

ex. -  $K = 4$   $C_g = 1 \mu F$   $C_2 = 10 \mu F$

$$C_{i+1} = C_i + i \cdot C_g \quad i = 1, 2, 3 \dots$$

$i=2$   $C_3 = C_2 + 2 \times C_g$

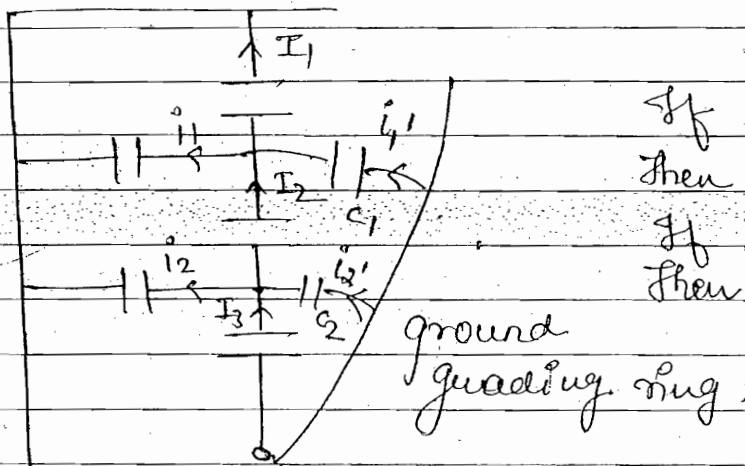
$$= 10 + 2 \times 1 = 12 \mu F$$

$i=3$   $C_4 = C_3 + 3 \times C_g$   
 $= 12 + 3 \times 1 = 15 \mu F$

$i=1$   $C_2 = C_1 + 1 \times C_g$   
 $C_1 = C_2 - C_g = 10 - 1 = 9 \mu F$

### (3) Static Shielding Method: -

Idea of this method is to compensate the loss of current at every pin junction.



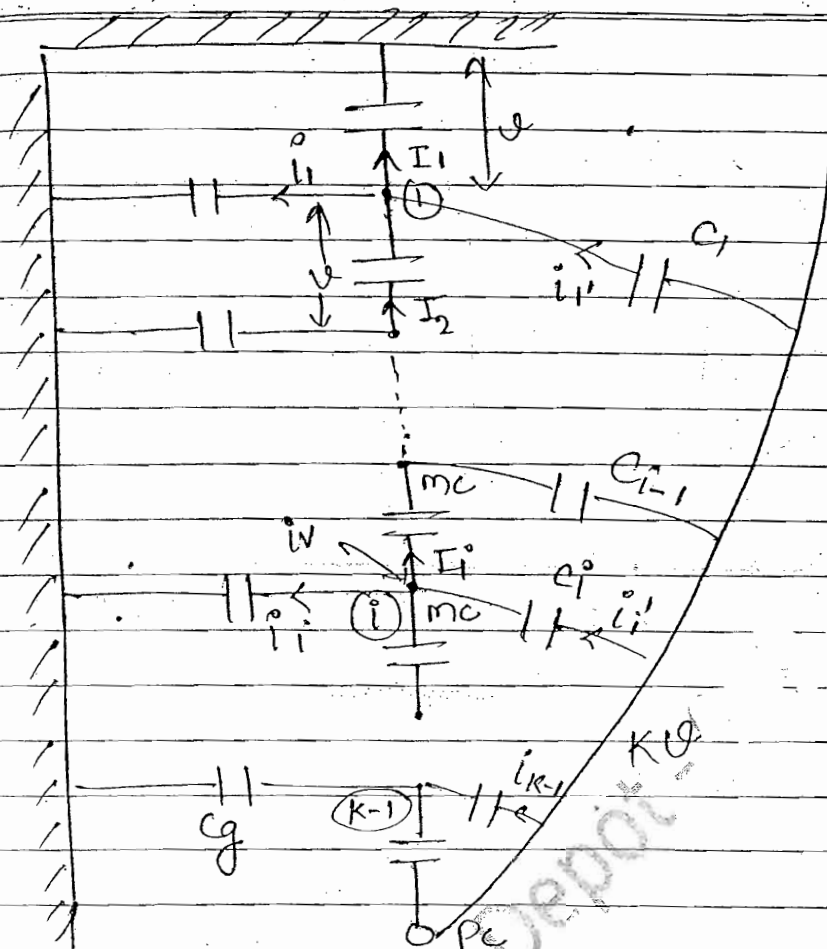
If  $i_{11} = i_1$

Then  $I_2 = I_1$

If  $i_{21} = i_2$

Then  $I_3 = I_2$

→ Same current is following.



$$\text{If } i_i' = i_i$$

$$I_{i+1} = I_i$$

$$\eta \% = 100\%$$

$$\omega C_i (KV - i_v) = (i_v - 0) \omega C_g$$

$$C_i = \left( \frac{i}{K-i} \right) \times C_g \quad \text{for } i = 1, 2, 3, \dots, K-1$$

ex:- Design guard ring.  $K=3$ ,  $C_g = 1 \mu F$

$$C_i = \left( \frac{i}{K-i} \right) \times C_g$$

$$i=1 \quad C_1 = \left( \frac{1}{3-1} \right) \times 1 = 0.5 \mu F$$

$$i=2 \quad C_2 = \left( \frac{2}{3-2} \right) \times 1 = 2 \mu F$$

Page no. 45

$$\text{Sol}^n 46. \quad \eta\gamma. = \frac{100\%}{4 \times 33.33} \times 100 = 75\% \text{ (d)}$$

$$\text{Sol}^n 48. \quad m = 5 \quad m = \frac{c}{0.2c} = 5.$$

$$u_2 = u_1 + \frac{u_1}{5}$$

$$= 1.2u_1$$

$$u_3 = u_2 + \frac{u_1 + u_2}{5} = 1.2u_1 + \frac{u_1 + 1.2u_1}{5}$$

=

## • Under Ground Cables :-

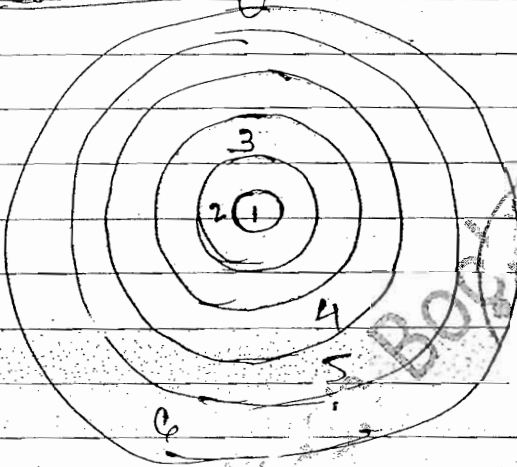
adv

- It protects beauty of city
- Distribution of power in bad weather cond<sup>n</sup> possible
- chances of failure is less

Disadv

- Cost ↑
- locating fault position is difficult.
- Tainting cables is difficult.
- Power transfer for long distances is not possible

Construction:



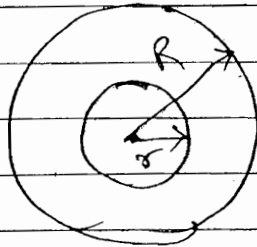
- (1) Al core → eddy current loss
- (2) Insulation → proper mech. strength
- (3) Sheathing → head  
→ Al  
→ To avoid entry of moisture  
→ Al is proper → low weight  
→ low cost.  
→ Being metal it partly protects from mech. damage.

- (4) Bedding → jute → To avoid frict<sup>n</sup> b/w sheathing & steel armoring
- (5) Steel armoring → steel tape. (To protect cable from mech. damage)
- (6) Serving → jute. (same as bedding only place is different)

## • Cable Parameters :-

- (1) Insulation Resistance :- Resistance offered for the leakage currents is called Insulation Resistance.

Insulation } Sheath.  
 Core } → leakage Currents



$R - r = \text{Insulation thickness}$

$$IR = \frac{\rho}{2\pi l} \ln \frac{R}{r} \quad \text{K}\Omega$$

Insulation Resistance

$l \uparrow \quad IR \downarrow$

Prob NO 43

Sol 47

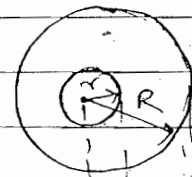
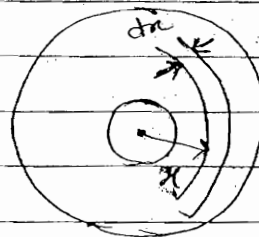
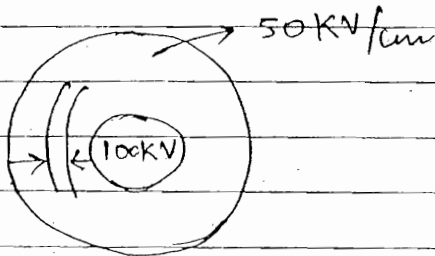
$l \downarrow - \frac{1}{2}$

$R \uparrow 2 \text{ times.}$

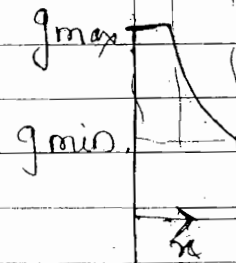
$$\begin{aligned} 20 &= \frac{20}{2} \\ &= 10 \end{aligned}$$

$$\begin{aligned} R \uparrow &= 8 \text{ M}\Omega \\ &= 2 \times 8 \text{ M}\Omega \\ &= 16 \text{ M}\Omega \quad \text{Ans} \end{aligned}$$

(2) Dielectric Stress (g)



$$g_{\max} = \frac{V}{r \ln \left( \frac{R}{r} \right)} \quad \text{KV/cm}$$



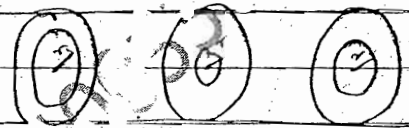
at  $r = r$  ;  $g_{\max} = \frac{V}{r \ln \left( \frac{R}{r} \right)} \quad \text{KV/cm}$

at  $x=R$  ;  $g_{\min} = \frac{V}{R \ln \frac{R}{r}} \text{ KV/cm}$

→ Internal layer has more dielectric stress., after some time cable will puncture bcoz of internal stress.

• Most Economical Size of the Cable.

$$g_{\max} = \frac{V}{r \ln(R/r)}$$



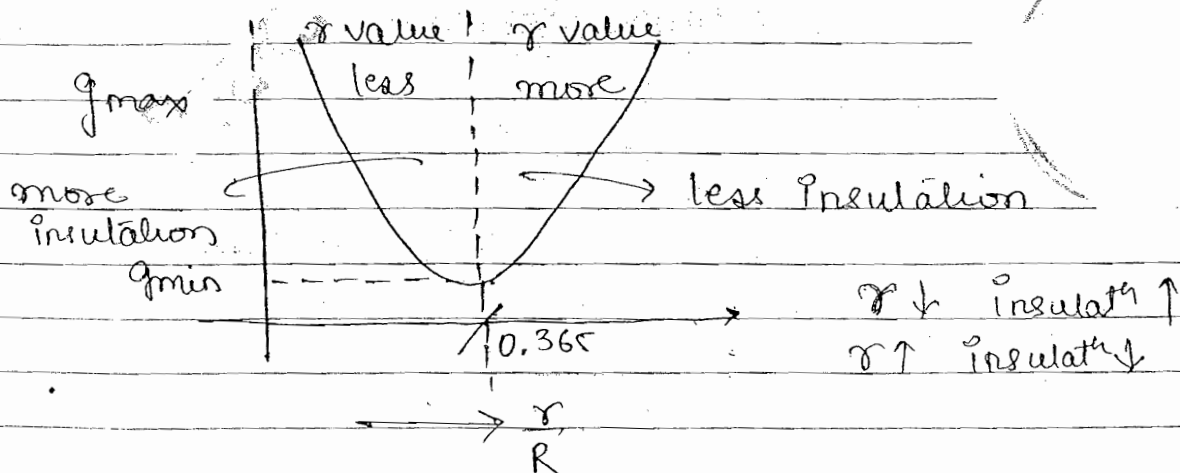
• What is the value of 'r' at which  $g_{\max}$  value is minimum?

$$\frac{dg_{\max}}{dr} = 0$$

Condition for minimum value is  $\frac{R}{r} = 2.718 = e$

→ at this value  $g_{\max}$  and  $g_{\min}$  <sup>diff.</sup> reduce, stress will ↓ so uniform stress.

$$\frac{r}{R} = 0.368$$



• providing more insulation is better than less insulation

- For stable and safely operation of cable ( $r/R$ ) should be less than 0.368 and ( $R/r$ ) should be more than 2.718

→ Find the most economical size of 1- $\phi$  100 KV cable provided with a dielectric material having dielectric strength 50 KV/cm.

Soln

$$g_{\max} = 50 \text{ KV/cm.}$$

$$V = 100 \text{ KV}$$

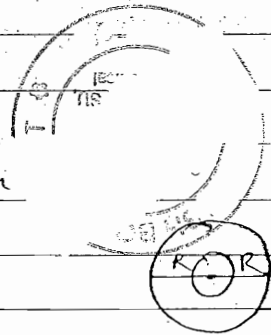
for most economical size,  $\frac{R}{r} = e$   $R = e r$ .

$$50 = \frac{100}{r \times \ln(R/r)}$$

$$r = \frac{100}{50} = 2 \text{ cm}$$

$$R = 2 e \text{ cm.}$$

$$\begin{aligned} \text{Overall size of cable} &= 2R \\ &= 4e = 4 \times 2.718 \\ &= 10.84 \text{ cm} // \end{aligned}$$



- Grading of cable :- By employing some method an attempt is made to reduce the difference b/w  $g_{\max}$  and  $g_{\min}$ . With the difference b/w  $g_{\max}$  and  $g_{\min}$  reduce, the stress is made uniform and the life of cable  $\uparrow$  improves.

- (1) Capacitance grading Method
- (2) Intersheaths grading

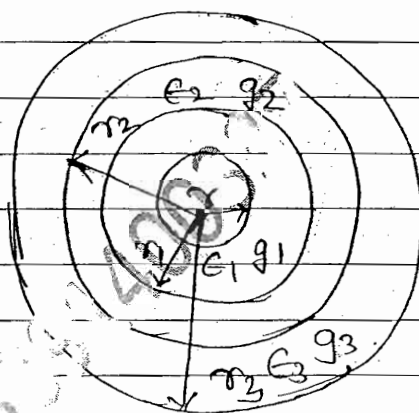
Capacitance gradingIntersheath grading

→ only one sheath is used

→ More no. of sheath used.

→ different dielectric materials are used

→ same dielectric material is used.

Materials with different  $\epsilon$  values but same  $g$  values.Materials with different  $\epsilon$  &  $g$  values.for  $r_1 < r_2 < R$   
 $\epsilon_1 > \epsilon_2 > \epsilon_3$ for  $r_1 < r_2 < R$   
 $\epsilon_1 g_1 > \epsilon_2 g_2 > \epsilon_3 g_3$ 

max

min

product

Q1 3 dielectric materials with same dielectric strength but different  $\epsilon$  values, 2, 3, 4 are suggested for capacitance grading. The placement of dielectric material w.r.t. core is —? Ans. 4, 3, 2,

Q2 The above dielectric material are having dielectric strength of 80 kV/cm, 50 kV/cm, 25 kV/cm respectively. The placement of dielectric material w.r.t. core is —? 2, 3, 4. Ans.

Solution

$$\epsilon_1 = 2$$

$$\epsilon_2 = 3$$

$$\epsilon_3 = 4$$

$$\epsilon_1 g_1 = 2 \times 80$$

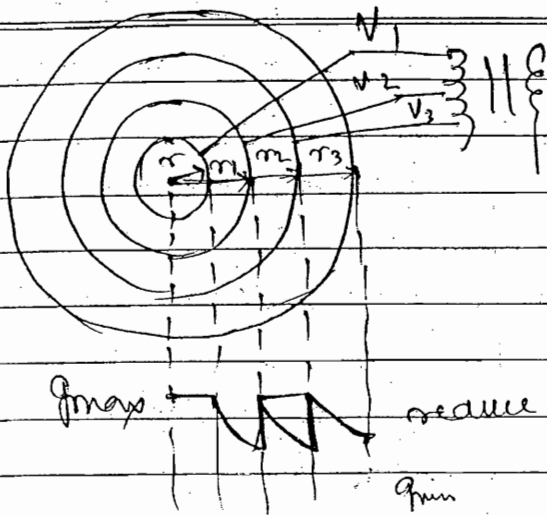
$$= 160$$

$$\epsilon_2 g_2 = 3 \times 50$$

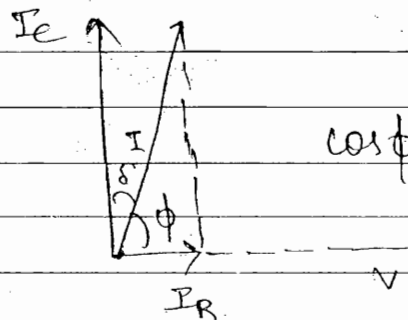
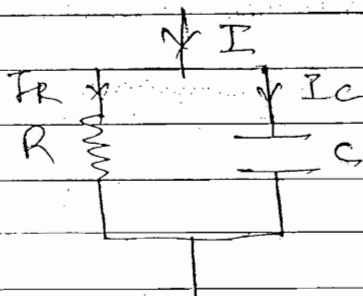
$$= 150$$

$$\epsilon_3 g_3 = 4 \times 25$$

$$= 100$$



Once the cable open, weld again is very difficult.



$$\phi = 90^\circ - \delta$$

$$\cos \phi = \cos (90^\circ - \delta)$$

$$= \sin \delta$$

→ p.f cable is  $\sin \delta$ , where  $\delta$  is known as dielectric loss angle

• Dielectric loss  $= V_{ph} I_R$

$$= V_{ph}^2 \omega c \tan \delta$$

If  $\delta$  is small  $\tan \delta = \delta$

$$\boxed{\text{Dielec. loss} = V_{ph}^2 \omega c \delta}$$

$$\left\{ \begin{array}{l} \frac{I_R}{I_C} = \tan \delta \\ I_R = I_C \tan \delta \\ = \frac{V_{ph} \tan \delta}{Y \omega c} \end{array} \right.$$

→ dangerous loss → dielectric loss.

Losses :-

- (1)  $I^2 R$  loss in core
- (2) dielectric loss in dielectric
- (3) due to eddy current sheath losses in sheath

→ available cables are 132 kV → called solid cable

→ above ~~132~~ 132 kV called → pressure cable  
132

→ popular available cable → XLPE

(cross line liquid polymer ethylene).

## • DISTRIBUTION

AC → Bulk power generation is possible

→ long distance x-line is possible

$\leftarrow d \rightarrow$   
 $\leftarrow \sqrt{2}V \rightarrow$   
stress.

$\leftarrow d \rightarrow$  DC → No stability problem, no capacitance, no surge.  
 $\leftarrow V \rightarrow$  min. losses etc. (stress value is low)

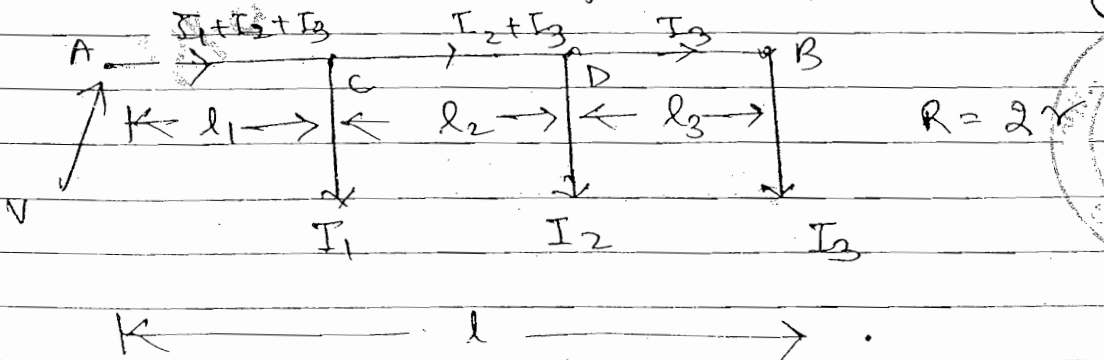
preferred → low cost of x-mission

generation → x-mission → distribution  
(a.c.) (d.c.) (a.c.).

→ HVDC problem of Reactive power supply.

### • Voltage drop Calculation:-

(a) DC distribution fed at one end only:-



r → resistance of go and return pair conductor

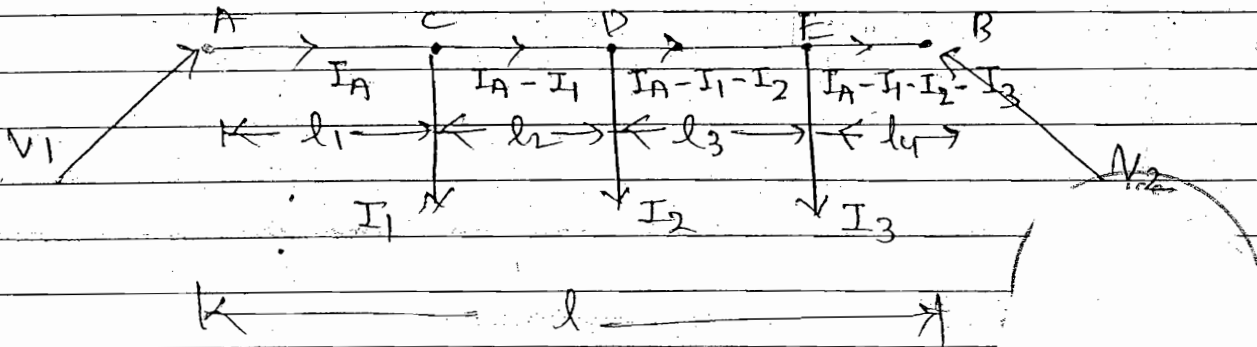
r → go

R → go and return

$$\text{Total voltage drop} = I_1 l_1 R + I_2 (l_1 + l_2) R + I_3 (l_1 + l_2 + l_3) R$$

$\rightarrow$  resistance of a conductor per unit length.

(b) DC distributor fed from both end.



$$\text{Total voltage drop} = V_1 - V_2$$

$$= I_A l_1 R + (I_A - I_1) l_2 R + (I_A - I_1 - I_2) l_3 R + (I_A - I_1 - I_2 - I_3) l_4 R$$

$\rightarrow$  calculate  $I_A$

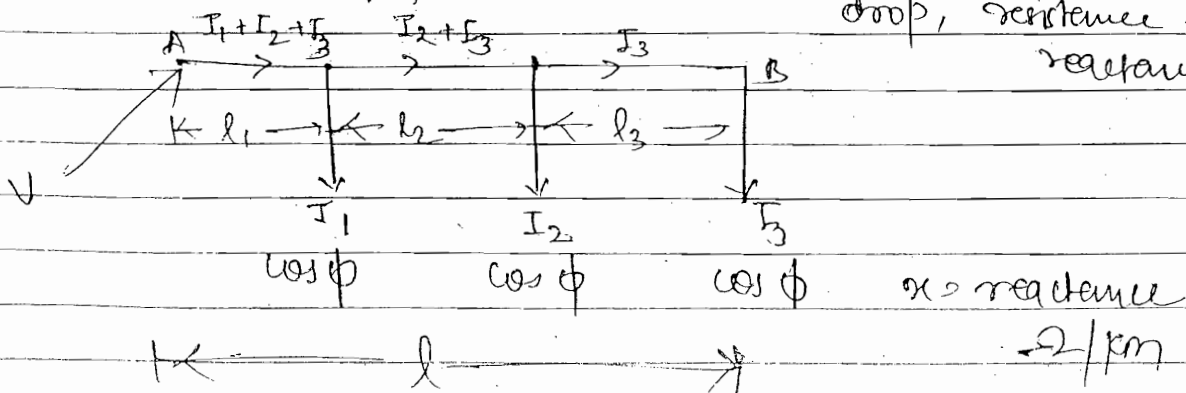
$\rightarrow$  calculate section areas

$\rightarrow$  calculate minimum potential point.  $\rightarrow$

• AC distributor

(c) Same p.f. load.

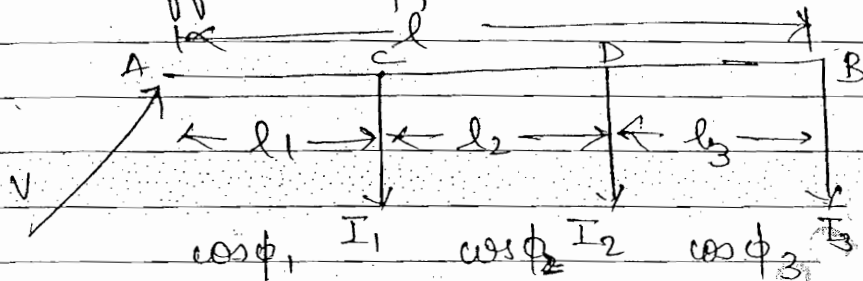
In it we have two loop, resistance + reactance.



Total voltage drop  $\rightarrow$

$$= I_1 l_1 (r \cos \phi + x \sin \phi) + I_2 (l_1 + l_2) (r \cos \phi + x \sin \phi) \\ + I_3 (l_1 + l_2 + l_3) (r \cos \phi + x \sin \phi)$$

(d) Different p.f.

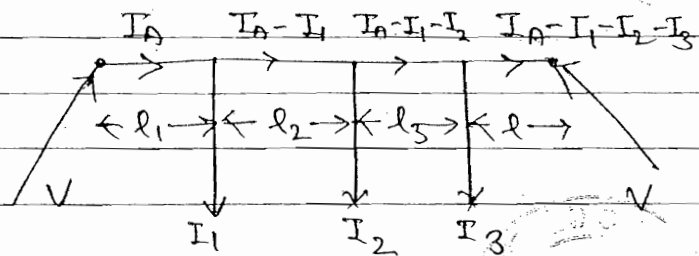
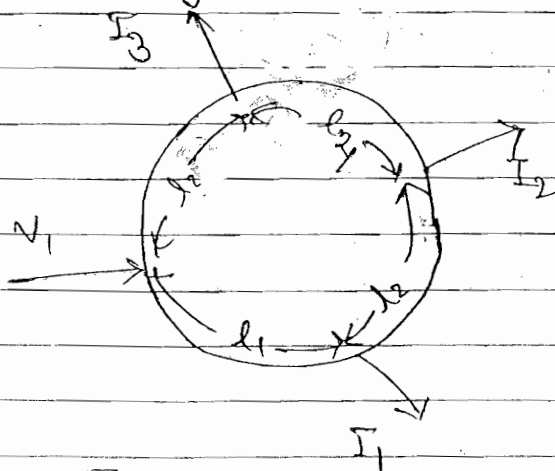


Total voltage drop  $\rightarrow$

$$= I_1 l_1 (r \cos \phi_1 + x \sin \phi_1) + I_2 (l_1 + l_2) (r \cos \phi_2 + x \sin \phi_2) \\ + I_3 (l_1 + l_2 + l_3) (r \cos \phi_3 + x \sin \phi_3)$$

• Ring Distribution:-

voltage difference is zero



Total voltage drop = 0

$$= I_A r l_1 + (I_A - I_1) r l_2 + (I_A - I_1 - I_2) r l_3 + (I_A - I_1 - I_2 - I_3) r l_4 = 0$$

95.RS-