EMT :-

1. Static Electro-Magnetic Fields. → 40% IES (9/10 ques)
2. EM Waves
3. VI Waves — Transmission lines
4. Guided Waves

] 80% (Gate)

5. Radiated Waves — Antennas] 10%

Text Books :-

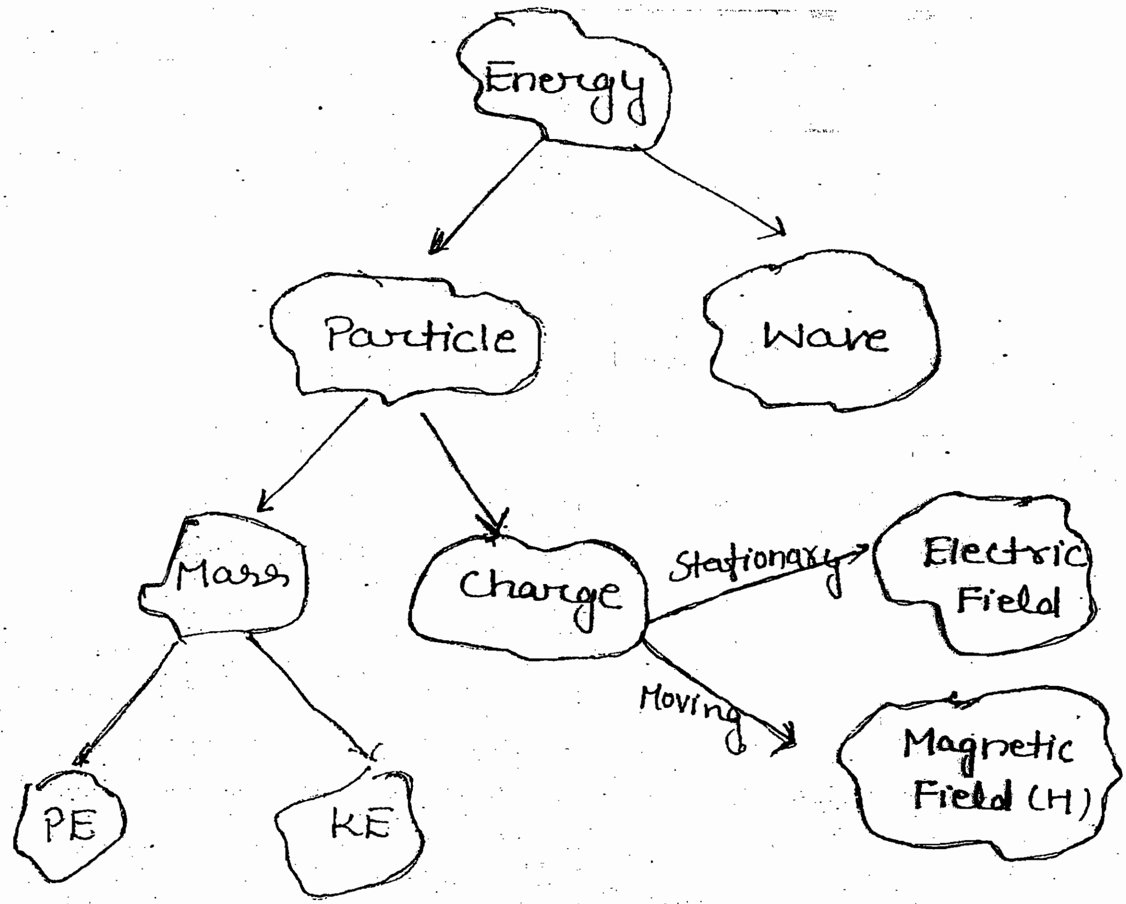
1. William Hayt (For theory of first chapter)
2. For vector calculus → N.O Sadiku
3. For problems → Schaum Series
4. Jorden → For EM Waves & Guided Waves
5. John D Ryder → For VI Waves
6. K.D Prasad & Balanis → Antenna
7. Nathan Ida → For whole EMT

→ VSRS22@GMAIL.COM

→ VSR Suresh

Lecture - 1

Static Electromagnetic Fields



Electric Field :-

It is a format of energy that is all around a charge & influences other charges near by

Magnetic Field (H) :-

It is a format of energy that is all around a moving charge & influences other moving charges nearby.

NOTE :-

The word influence in electric fields is a linear accelerating attractive or repulsive force on a charge resulting in straight line path. This is Coulomb's law and hence electric

... charge particles ...

$$\boxed{\vec{F} = QE = ma} \longrightarrow \text{Linear path}$$

energised

The word influence in magnetic field is only on moving charges such that forces perpendicular to velocity and the field

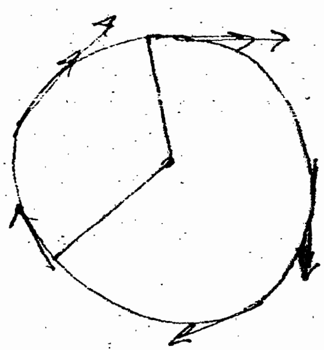
F $\perp$ displacement

So that workdone = 0

This is Lorentz law and particle acquire circular path

$$\boxed{\vec{F} = Q(v \times B) = \frac{mv^2}{r}} \longrightarrow \text{Circular path}$$

deflection



Summary :-

1) Charge is stationary then $\longrightarrow$ Electric Field (Static)

$\longrightarrow$ Time invariant

$\longrightarrow$ Space Varying

eg:- (i) accumulated charge

(ii) D.C voltage

Electric field is energy in the above discussion

2. Charge is moving / flowing. without accelerati or with constant velocity or linearly with time

$$Q = Kt$$

$$\frac{dQ}{dt} = K = I \quad (\text{DC current})$$

and DC current cause magnetic field whic is static in nature

→ Magnetic field is the energy in above discussion.

3. Charge is moving with acceleration which creates Electric field ~~energy~~ $E(t)$ ~~an~~ as well as magnetic field $H(t)$ which are power.

Basic Terms & Definitions :-

There is a measure of the field strength at any point in the field

(I) $\vec{E}$ → Electric field Intensity (V/m or N/C)

per unit length

(II) $\vec{H}$ → Magnetic field ~~flux~~ intensity ~~density~~ (Amp/m)

(III) $\vec{B}$ → Magnetic flux density [Weber/m² or Tesla]

(IV) $\vec{D}$ → Electric flux density (Coulombs/m²)

per unit area

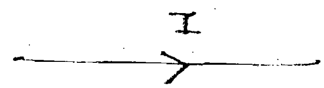
(V) ϵ → Permittivity of the medium

The ability to permit /allow/ hold electric field in that medium

(II) Current Carrying Wire :-

→ Scalar

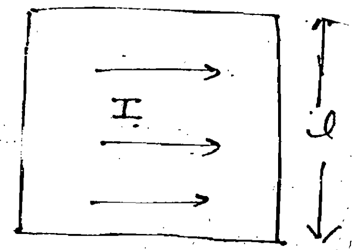
→ Amp



(III) Surface current :-

→ $\vec{K} = \frac{dI}{dl}$ = Amp/m

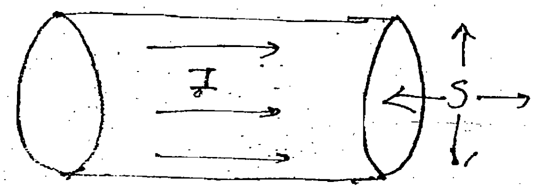
→ vector



(IV) Solid conductor (J) :-

→ vector

→ $J = \frac{dI}{ds}$ = Amp/m^2



Vector Calculus :-

It is a study of directional integrations & directional derivatives.

Directional Integration :-

It is the study of the total effects or cummulative effects of a phenomeno in a specific direction in a specific region

Directional derivative :-

It is the study of the instantaneous or rate of change analysis of a phenomenon in a specific direction in a specific region

eg:- Del-operator

$$\nabla \rightarrow \frac{\partial}{\partial x} a_x + \frac{\partial}{\partial y} a_y + \frac{\partial}{\partial z} a_z$$

→ It is used to study the rate of change of various space varying quantities in 3d-space

→ Del is called as vector spatial derivative operators

∇ (vector operator) → Scalar function f → ∇f → Gradient of Scalar → vector form

∇ (vector operator) → Vector function $\vec{A}$ → $\nabla \cdot \vec{A}$ → Divergence of vector (Scalar function)

∇ (vector operator) → Vector function $\vec{A}$ → $\nabla \times \vec{A}$ → curl of vector (Vector function)

∇ (vector operator) → ∇ → $\nabla \cdot \nabla$ → ∇^2 → Scalar Laplacian operator

NOTE :-

Vector Identities :-

$\nabla \times \nabla f = 0$

$\nabla \times (\nabla f) = 0$

curl (Grad. of scalar) = 0

$\nabla \cdot (\nabla \times \vec{A}) = 0$

Div. (curl of vector) = 0

$\nabla \times (\nabla \times A) = \nabla (\nabla \cdot A) - \nabla^2 A$

Divergence & Out flow:—

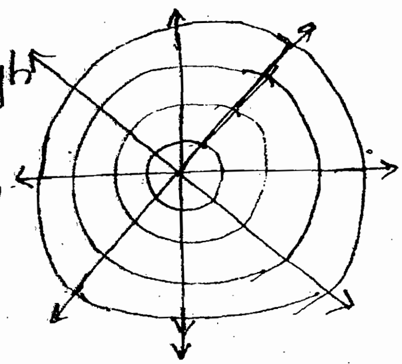
i) Consider a cause or source which has effects spread outward from the cause

eg:-(I) light from a bulb

(II) air velocity from a punctured tyre

For all such phenomenon the strength dec. as the area of expansion inc. such that total outflow is same

The total outflow through any enclosing surface is always the same (constant) and this constant depends on central cause



Total outflow = ↑Strength x Area ↓

= constant. $\propto$ cause

The strength for all such phenomenon can be expressed as the constant per unit area or cause/area i.e. called as density.

NOTE:—

i) If the cause is Q coulomb charge then the repulsive force or attractive force is called as electric flux and the strength is called as flux density (D) such that

$$\oint D \cdot ds = \Psi_{e\,total} \propto Q$$

If proportionality constant is 1 then this called Gauss Law in integral form

$$\Rightarrow \boxed{\oint D \cdot ds = Q}$$

NOTE 1:-

→ If the surface is not completely enclosing then the effects are the partial flux crossing i.e.

$$\int \vartheta \cdot ds = \psi_e \neq \psi_{e\,\text{total}}$$

This is not a Gauss law

→ Every Closed surface (vector) is identified by a finite (scalar) volume

eg:- $4\pi r^2 \xrightarrow{\text{sphere}} \frac{4}{3}\pi r^3$

$2\pi r h \xrightarrow{\text{cylinder}} \pi r^2 h$

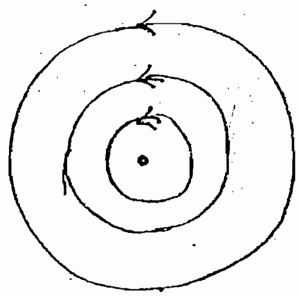
$6a^2 \xrightarrow{\text{cube}} a^3$

→ Closed surface having direction while finite volume doesn't having direction

Circulation and curl :-

Cause or Source →

Effects → around the cause



→ The total circulation in any closed length is always a constant and this constant depends on the central cause.

eg:- air velocity under the fan

Total circulation = strength ↓ × length ↑

= constant ∝ Cause

$$\text{Strength} = \frac{\text{constant}}{\text{length}} \text{ or } \frac{\text{cause}}{\text{length}}$$

$\longrightarrow$ Intensity

Cause or Source $\longrightarrow$ I ampere current

Effects $\longrightarrow$ Magnetic line field

Strength $\longrightarrow$ Magnetic field intensity ($\vec{H}$)

$$\boxed{\oint_{\text{closed}} H \cdot dl = I}$$

This is ampere's law in integral form

Note - 1 :-

Every closed line is identified with a finite area enclosed

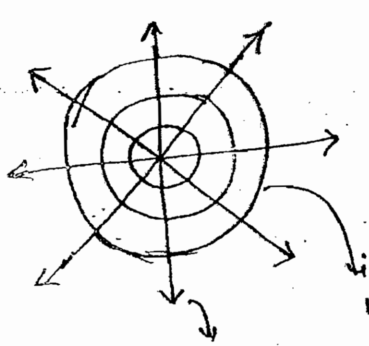
closed line $\rightarrow$ vector

area enclosed $\downarrow$ vector

$2\pi r \xrightarrow{\text{circle}} \pi r^2$

$4a \xrightarrow{\text{square}} a^2$

Summary 1 :-



$\psi_{e\,\text{total}} = Q$ i.e. outflow total = Strength × area

= Cause

$\hookrightarrow$ Electric flux has the units of coulomb's

ψ is constant

ψ's direction & direction of divergence

$\hookrightarrow$ closed surface element

Strength around the cause $\vec{D} = \frac{dQ}{ds} = \frac{C}{m^2}$

where ds = any closed surface element

Strength at the cause $= \frac{dQ}{dv} = \frac{d}{dl}\left(\frac{dQ}{ds}\right)$

$= \nabla \cdot D$ = Divergence of D

$= \frac{\text{outflow}}{\text{volume}} = \rho_v$

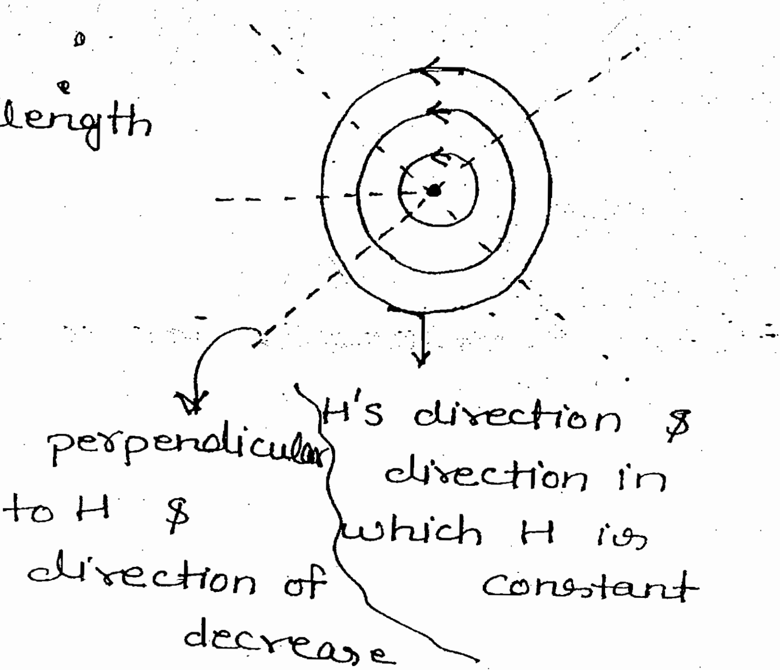
$\boxed{\nabla \cdot D = \rho_v}$ → Gauss law in point form

→ The (·) dot product signifies that D's change or derivative exists only when we move in direction of D. This is called as directional derivative

→ The (·) dot product indicates a surface (vector) derivative resulting in volume (scalar) derivative

Summary 2 :-

Total circulation = strength × length = current



Total Circulation = strength x length

= current

$$H = \frac{\text{Strength around the current}}{} = \frac{dI}{dl} = \frac{\text{Amp}}{\text{m}}$$

where dl = any close line element

$$\text{Strength at the current} = \frac{d}{dl}\left(\frac{dI}{dl}\right) = \nabla \times H$$

= curl of H

$$= \frac{\text{circulation}}{\text{Area}} = J$$

$\boxed{\nabla \times H = J}$ → This is Ampere's law in point form

- The (X) cross product signifies that H changes only when we move perpendicular to H
- The (X) cross product implies a vector (line) derivative resulting a vector (surface) derivative

Summary-3 :-

Col - Q | Cause | I - Amps
Source
Gauss Law ← Mathematical → Ampere's Law
Effects

Density - Col/m^2 → D | Strength | $\vec{H}$ - amp/m → Intensity

ϵ | μ

Colomb's Law ← Physical → Lorentz's Law

Force b/w Causes

Intensity (V/m) | $\vec{E}$ | $\vec{B}$ → Weber/m^2 → Density

Summary-4 :-

Divergence & stoke's theorem :-

→ $$\boxed{\oint D \cdot ds = Q = \int \rho_v dv = \int (\nabla \cdot D) dv}$$

This is gauss divergence theorem

→ $$\boxed{\oint H \cdot dl = I = \int J \cdot ds = \int (\nabla \times H) \cdot ds}$$

Stoke's theorem

Note :-

Wrong Notations :-

→ $\int H \cdot dl = \int (\nabla \times H) \cdot ds$ → $\int D \cdot ds = \int (\nabla \cdot D) dv$

→ $\oint H \cdot dl = \int (\nabla \cdot H) ds$ → $\oint D ds = \int (\nabla \times D) dv$

→ $\oint H \cdot dl = \oint (\nabla \times H) \cdot ds$

Note :-

Maxwell equations having two formats, 2 field, 2 (·) dot, 2 (x) cross, 2 line, 2 surface, 4 eqn

Summary 5 :-

Maxwell's Equation

Integral form		Point form
1. $\oint D \cdot ds = Q$	[Gauss Law]	1. $\nabla \cdot D = \rho_v$
2. $\oint E \cdot dl = 0$	[Irrotational Vector]	2. $\nabla \times E = 0$
3. $\oint B \cdot ds = 0$	[Solenoidal]	3. $\nabla \cdot B = 0$
4. $\oint H \cdot dl = I$	[Ampere's Law]	4. $\nabla \times H = J$

Note :-

→ $\nabla \times E = 0.$ (Always)

curl is generally applied with intensity vector

$$\nabla \times \left(\frac{D}{\epsilon}\right) = 0$$

If ϵ = constant i.e. medium is homogeneous & isotropic

$$\Rightarrow \frac{1}{\epsilon} \nabla \times D = 0 \quad \Rightarrow \quad \boxed{\nabla \times D = 0}$$

→ Acc. to stoke's theorem (For integral form)

$$\oint E \cdot dl = \int (\nabla \times E) \cdot ds = 0$$

Note :-

i) $\nabla \cdot B = 0$ (Always)

Divergence is generally applied with density vector

$$\nabla \cdot (\mu H) = 0$$

If μ is constant i.e. medium is homogeneous and isotropic

$$\mu (\nabla \cdot H) = 0$$

$$\nabla \cdot H = 0 \longrightarrow \text{Also correct}$$

For integral form, apply divergence theorem

$$\oint B \cdot ds = \int (\nabla \cdot B)\, dv = 0$$

(ii) $\oint E \cdot dl = 0 \longrightarrow$ KVL

(iii) $\oint B \cdot ds = 0 \longrightarrow$ KCL

Coordinate Systems

It is a way of adressing/locating points in a 3d-space from a known reference

Reference :-

3 infinite mutually orthogonal planes

$\rightarrow$ XY, YZ & ZX planes

Cartesian Coordinate System :-

eg:- 1. Uniform plane waves

2. Rectangular wave guides

3. Capacitor plates

$\rightarrow$ Parameters (X, Y, Z)

$\rightarrow$ Unit vector (a_x, a_y, a_z)

Reference :-

1 infinite line

axial   z-axis

Cylinderical coordinate system

eg:- (I) Line charges

(II) I carrying waves wires

(III) cylindrical waveguide

Parameter → ρ, ϕ, z

Unit vectors → a_ρ, a_ϕ, a_z

Reference :-

1 single point

point   origin

Spherical coordinate systems

eg:-(I) point charges

(II) antennas

Parameter → r, θ, ϕ

Unit vectors → a_r, a_θ, a_ϕ

Relation b/w coordinate systems :-

$x \rightarrow y \rightarrow z$

$\rho \rightarrow \phi \rightarrow z$

$r \rightarrow \theta \rightarrow \phi$

Note:-

All the three co-ordinates systems are unit

- orthogonal
- orthonormal
- right handed system

Orthogonal :-

The (.) dot product of any two unit vectors of the same coordinate system is always zero

→ $a_x \cdot a_y = a_y \cdot a_z = a_z \cdot a_x = 0$

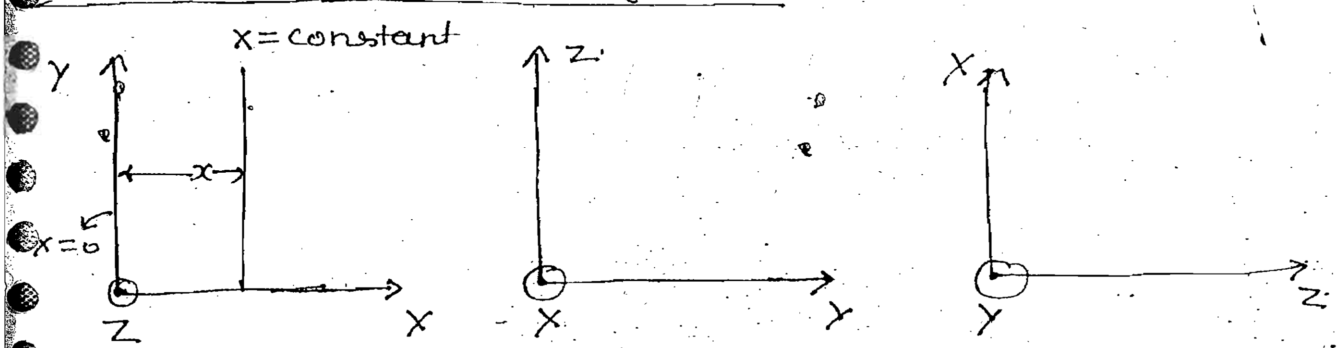
→ $a_x \cdot a_x = a_y \cdot a_y = a_z \cdot a_z = 1$

Orthonormal :-

The (x) cross product of any two unit vectors of the same co-ordinate system is always the third unit vector

$$a_x \times a_y = a_z$$
$$a_y \times a_z = a_x$$
$$a_z \times a_x = a_y$$

Cartesian Coordinate Systems:-



x → It is the shortest distance or perpendicular distance of the point from YZ plane

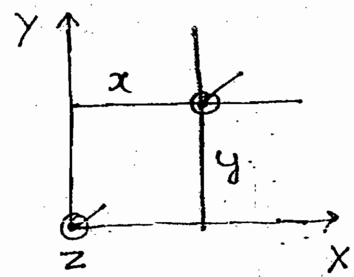
Note:-

The locus of all the points with x = constant is an infinite plane parallel to YZ plane

Range of X → $(-\infty, \infty)$

Note :-

The locus of all the points with X, Y = constant is an infinite line parallel to z-axis

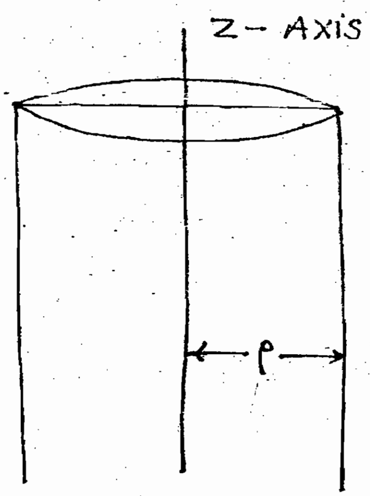


Cylindrical Coordinate System :-

ρ :→ It is the shortest distance / perpendicular distance / radial distance of the point from reference line

Note :-

The locus of all the points with ρ = constant is a concentric cylinder around a reference line

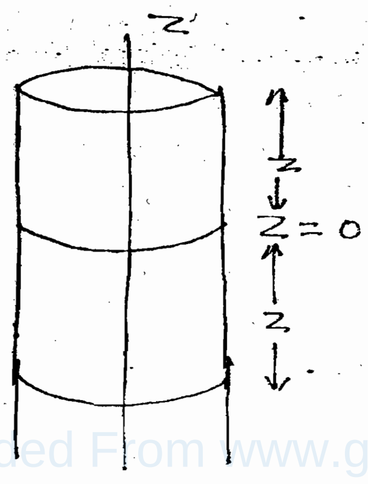


Range → $[0, \infty)$

Z :→

It is the height of the point along the reference line

Range :- $(-\infty, \infty)$

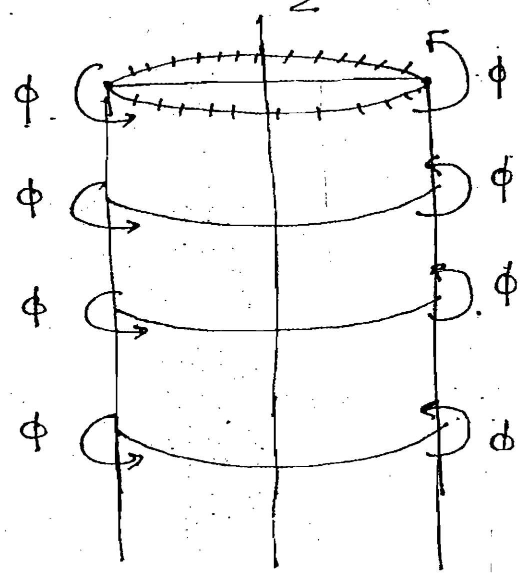


Note:-

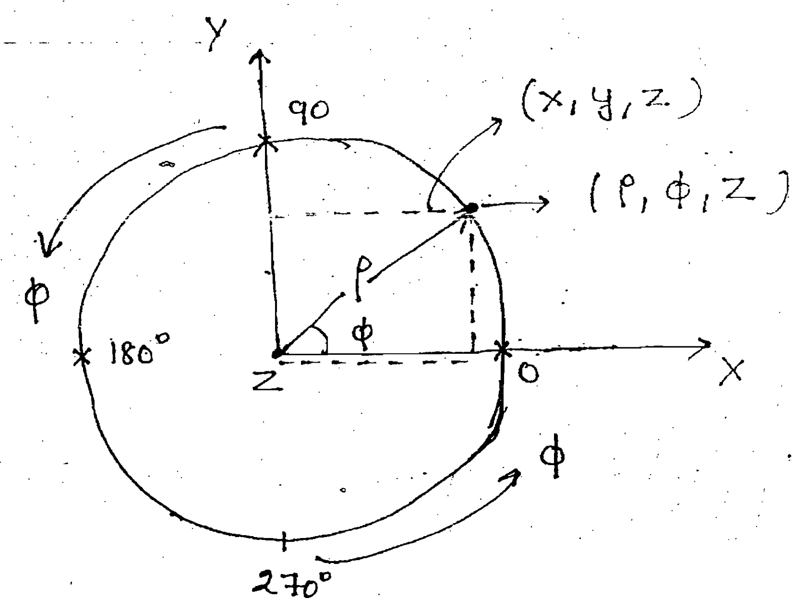
The locus of all the points with ρ, z = constant is a cocentric circle around the line.

ϕ :-

It is the orientation of the point around the reference line



Range :- $[0, 2\pi)$



Point Transformation :-

Cartesian $\longleftrightarrow$ cylindrical

$x = \rho \cos\phi$

$y = \rho \sin\phi$ $\qquad \rho = \sqrt{x^2 + y^2}$

$z = z$ $\qquad \phi = \tan^{-1}(y/x)$

Spherical Coordinate Systems !—

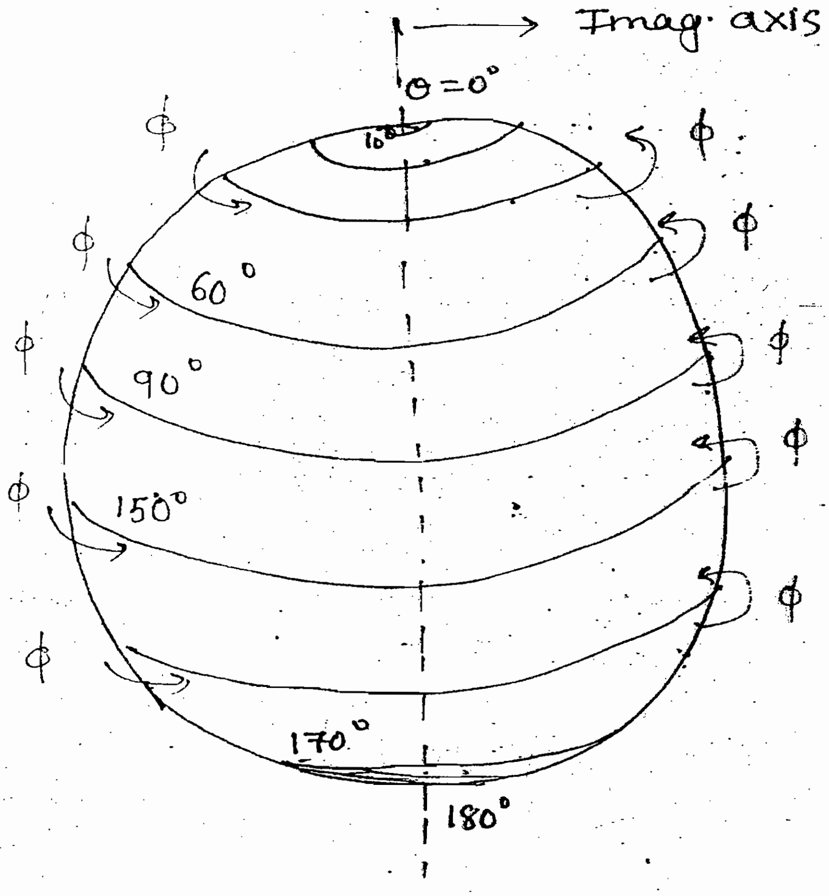
r :→

It is the radial distance or shortest distance of the point from reference point.

Note :-

The locus of the all the points with r is constant ~~as~~ is concentric sphere around the origin

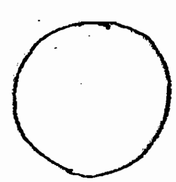
Range → $[0, \infty)$



θ :—

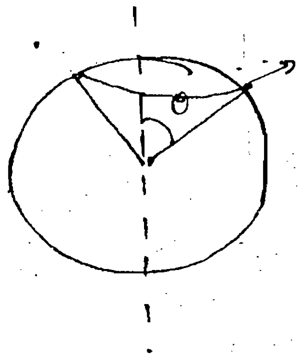
It is the angle of cone whose base is a circle on the r = constant sphere

Range → $[0, \pi]$



Identification of θ constant circle :-

Consider an imag. axis through the centre of the sphere and take a radial segment inclined by θ with imaginary axis. Rotate the radial segment which results in a cone whose base is a circle on the r = constant sphere



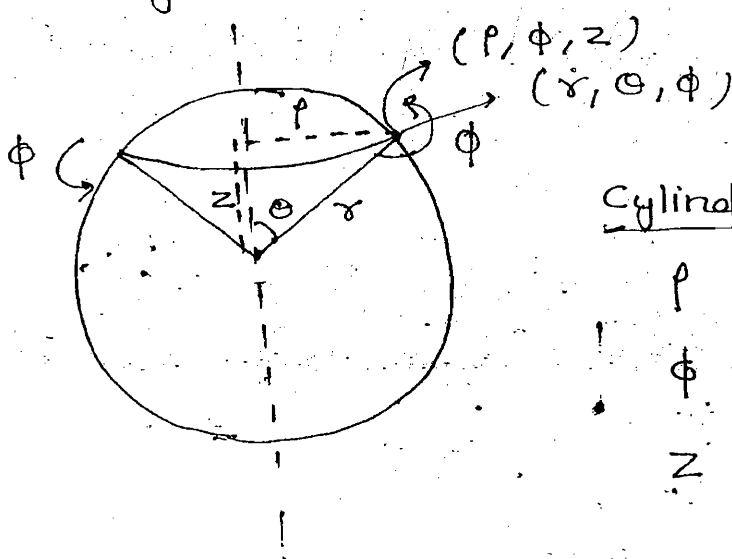
ϕ :-

It is the orientation angle around the imaginary axis

Range of $\phi \rightarrow [0, 2\pi]$

Point transformation :-

If z-axis of cylindrical coordinates coincide with imaginary axis of spherical coordinates then ϕ is same in both the coordinate system



Cylindrical ⟷ Spherical

$$\rho = r\sin\theta$$

$$\phi = \phi$$

$$z = r\cos\theta$$

Point Transformation :-

Cartesian $\longleftrightarrow$ Spherical

$$x = r\sin\theta\cos\phi$$

$$y = r\sin\theta\cdot\sin\phi$$

$$z = r\cos\theta \qquad r = \sqrt{x^2+y^2+z^2}$$

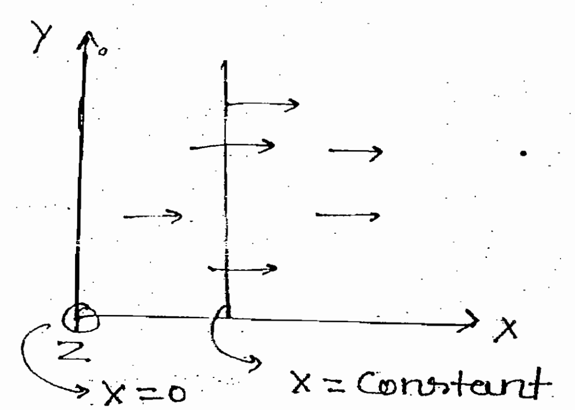
Unit Vectors & Orthogonality in coordinate system :-

Unit vector of Parameter :-

It has unit magnitude & a direction in which the parameter increases

1) Cartesian coordinate system :-

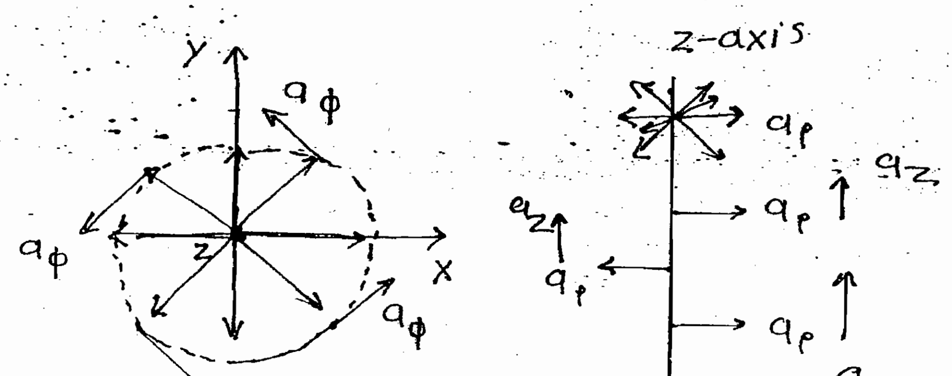
a_x :-



It is always normal to YZ plane or any plane which has X = constant

II) Cylindrical coordinate system :-

a_ρ :-



It is radially outward from the reference line

a_ϕ :-

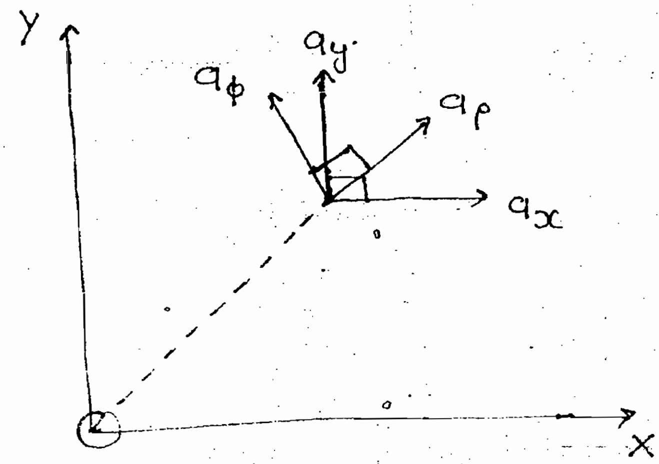
It is tangentially around the reference line

$$\boxed{a_\rho \perp a_\phi \perp a_z}$$

a_z :-

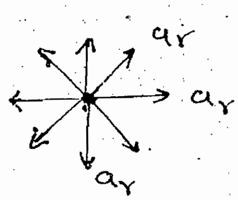
It is linearly along the reference line

Summary :-



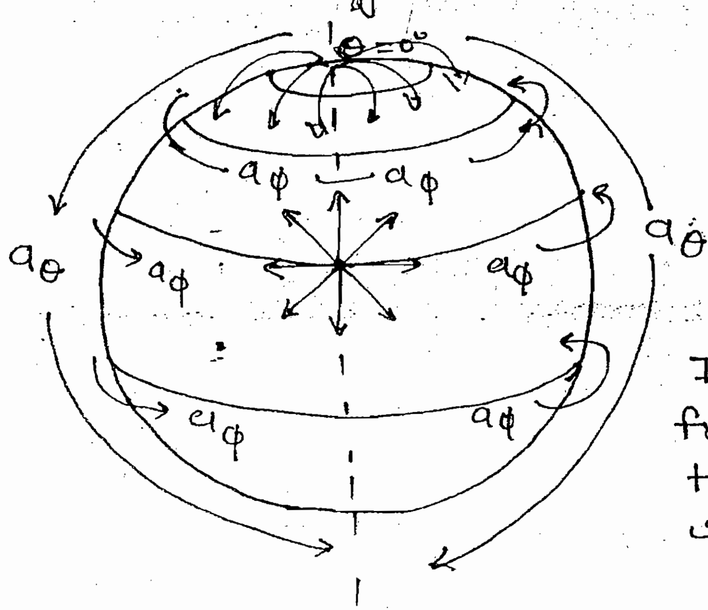
Spherical Coordinate System :-

a_r :-



It is radially outward from the origin

a_θ :-

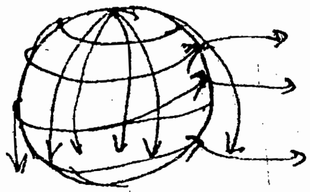


It is vertically from the top to the bottom of the sphere

It is horizontally around the imaginary axis.

ϕ = constant , $\theta \uparrow$

ϕ = constant ; $\theta \uparrow$



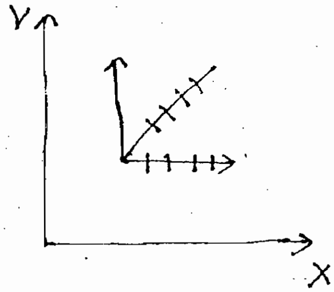
$$\boxed{a_r \perp a_\theta \perp a_\phi}$$

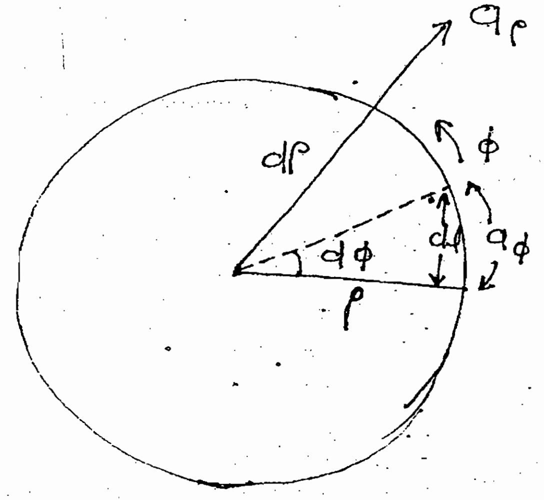
Lecture - 3

Line as a Vector :-

It has a magnitude equal to its length and a direction in which the length parameter increase

Cartesian :-

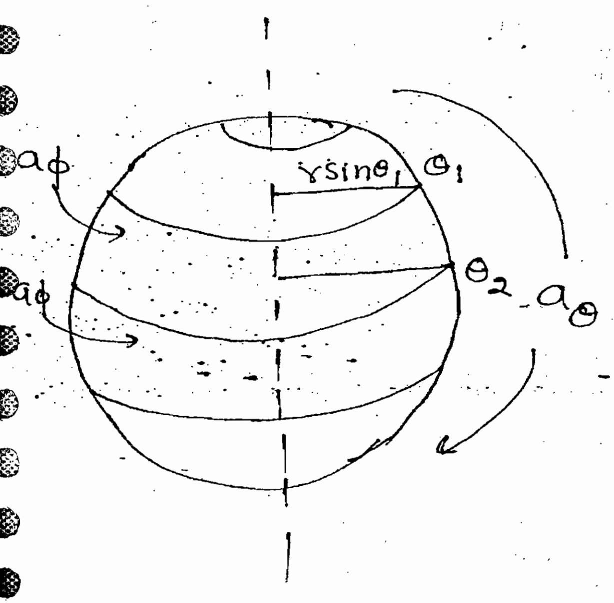


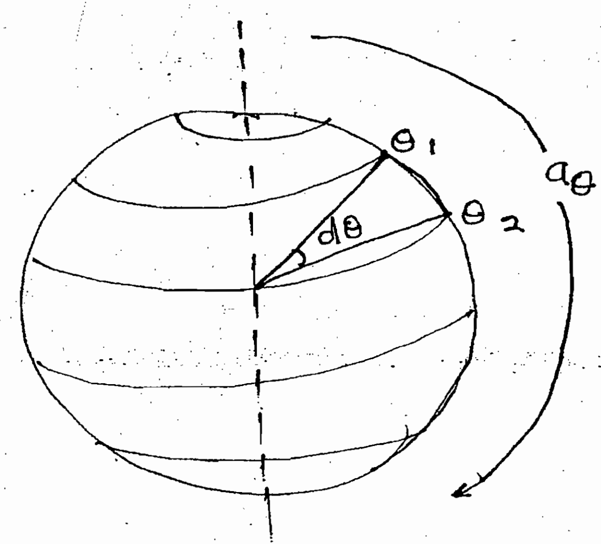


$$d\vec{l} = dx\, a_x + dy\, a_y + dz\, a_z$$

Spherical :-

$$d\vec{l} = dr\, a_r + r\, d\theta \cdot a_\theta + r\sin\theta \cdot d\phi\, a_\phi$$





In angular direction length is curvature

Note :-

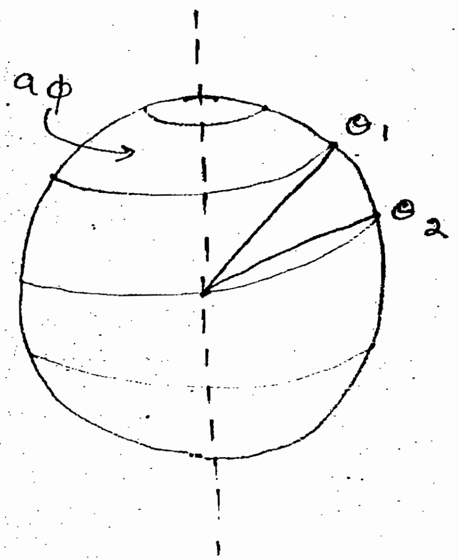
In angular directions length is a curvature not straight line or linear

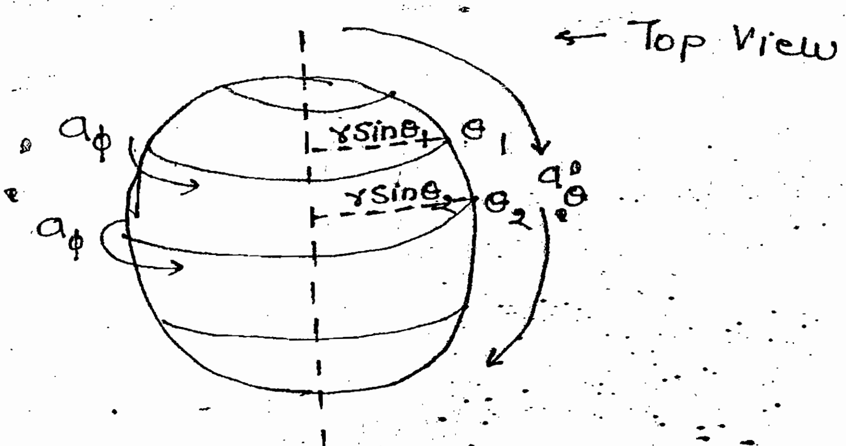
Curvature length = Radius x angle

Cartesian → $\vec{dl} = dx\, a_x + dy\, a_y + dz\, a_z$

Cylindrical → $\vec{dl} = d\rho\, a_\rho + \rho\, d\phi\, a_\phi + dz\, a_z$

Spherical → $\vec{dl} = dr\, a_r + r\, d\theta\, a_\theta + r\sin\theta\, d\phi\, a_\phi$





Note:-

In ϕ direction in spherical coordinating system curvature length is height on the sphere dependent i.e. it depends on the θ value of the circle.

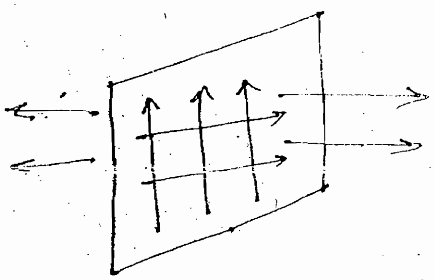
Summary :-

Parameters :-			Scaling Factors		
x	y	z	1	1	1
ρ	ϕ	z	1	ρ	1
r	θ	ϕ	1	r	$r\sin\theta$
u	v	w	h_1	h_2	h_3

$$\vec{dl} = h_1 du a_x + h_2 dv a_v + h_3 dw a_w$$

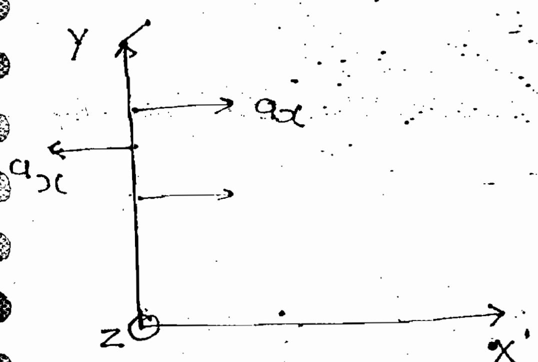
Surface as a vector :-

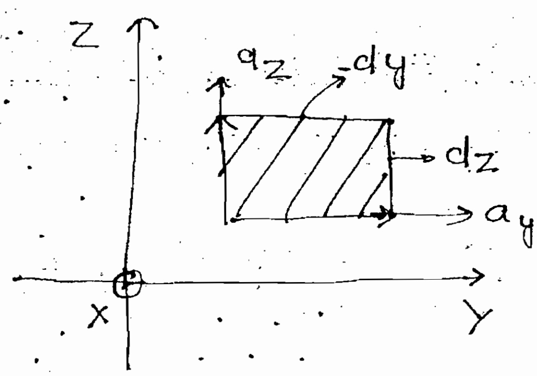
It has a magnitude equal to its area and a direction normal to the plane of surface



Direction is unique only when we take normal to the surface as there can be two tangential directions.

x = constant surface
$x = 0$ plane, YZ plane

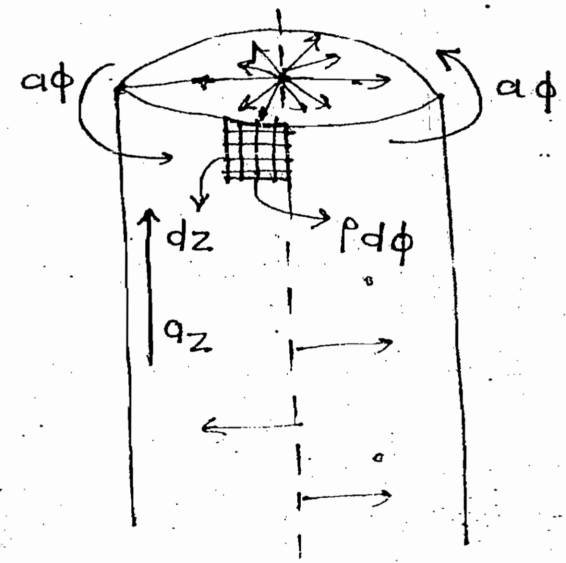


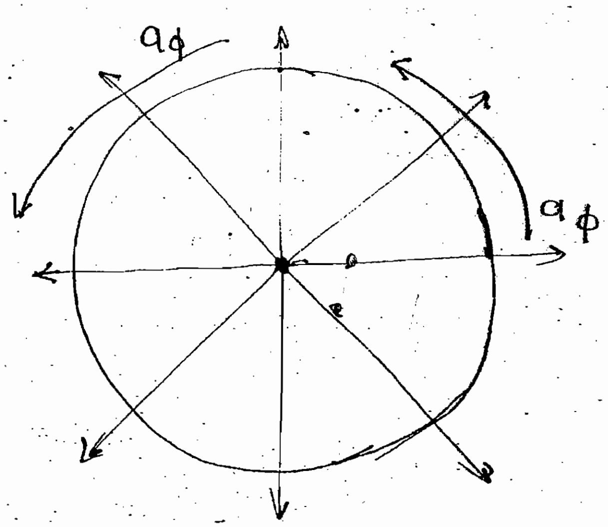


Cartesian :-

$$\boxed{\vec{ds} = \underbrace{dy\,dz}_{\text{mag.}}\,\underbrace{a_x}_{\text{direction}} + dz\,dx\,a_y + dx\,dy\,a_z}$$

Cylindrical coordinate system :-





Top view

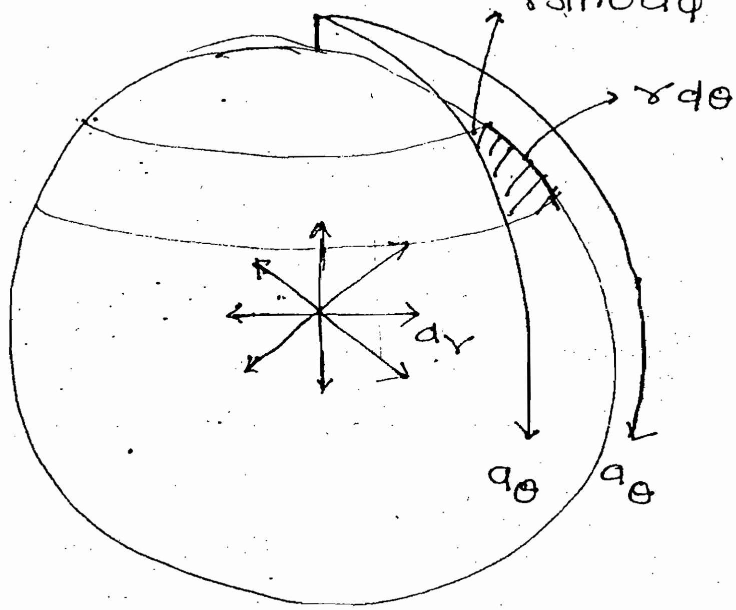
a_ρ = normal i.e. direction

a_ϕ , a_z = tangential → ∥ to surface

$$\vec{ds} = \rho\, d\phi\, dz\, a_\rho + dz\, d\rho\, a_\phi + \rho\, d\rho\, d\phi\, a_z$$

Spherical coordinate System :-

$r =$ constant, sphere



$a_r =$ normal
↓
direction

$a_\theta, a_\phi =$ tangential

$$\boxed{d\vec{s} = r^2 \sin\theta \, d\theta \, d\phi \, \underline{a_r} + r \sin\theta \, d\phi \, dr \, \underline{a_\theta} + r \, dr \, d\theta \, \underline{a_\phi}}$$

$$\boxed{ds = h_2 h_3 \, dv \, dw \, a_u + h_3 h_1 \, dw \, du \, a_v + h_1 h_2 \, du \, dv \, a_w}$$

Summary :-

$\left.\begin{array}{l}\text{1 parameter = Constant} \\ \text{2 parameter = variable}\end{array}\right] \rightarrow$ surface

Surface direction = Constant direction
= UNIQUE

$\left.\begin{array}{l}\text{2 Parameter = Constant} \\ \text{1 Parameter = Variable}\end{array}\right] \rightarrow$ line

Lines direction = Variable's direction
= UNIQUE

3 Parameters = variable] → Volume

3 Parameters = constant] → Point

Volume as a scalar :—

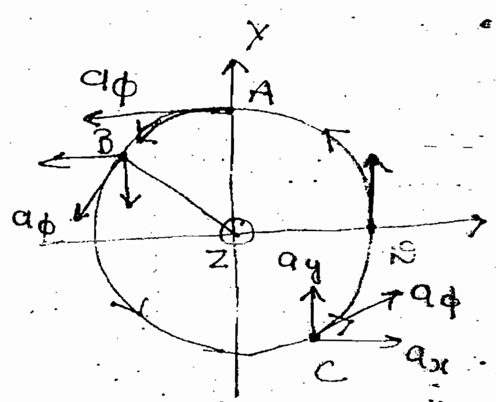
It has no unique direction & it is the scalar triple product of lengths in all three dimensions

$$dv = dx\,dy\,dz$$
$$= \rho\,d\rho\,d\phi\,dz$$
$$= r^2 \sin\theta\,dr\,d\theta\,d\phi$$
$$= h_1 h_2 h_3\,du\,dv\,dw$$

Workbook -1

2. At Point A

$a_\phi = -a_x$



divided by $\sqrt{2}$ to make magnitude = 1

At Point B

$$a_\phi = \frac{-a_x - a_y}{\sqrt{2}}$$

At Point C

$$a_\phi = \frac{a_x + a_y}{\sqrt{2}}$$

At point D

$$a_\phi = a_y$$

3. $\vec{D} = xy\,a_x + yz\,a_y + zx\,a_z \quad C/m^2$

$y = 2 \qquad 0 < x < 4$

$\qquad\qquad 0 < z < 2$

Since only 2 variable are present. Hence it is not a closed surface

$$ds = dx\,dz\,a_y$$

$$\int D \cdot ds = \Psi_e$$

$$\Rightarrow \Psi_e = \int_{x=0}^{4}\int_{z=0}^{2} yz\, dx\, dz \Big|_{y=2}$$

$$= 2\frac{z^2}{2}\Big|_0^2 \cdot x\Big|_0^4 = 16\, C \longrightarrow \text{flux}$$

4. $$\vec{D} = 5(\rho-3)^2 a_\rho$$

$\rho = 4$, $0 < \phi < \pi$

$-5 < z < 5$

$$ds = \rho\, d\phi\, dz\, a_\rho$$

$$\Psi_e = \int_{\phi=0}^{\pi}\int_{z=-5}^{5} 5(\rho-3)^2 a_\rho \cdot \rho\, d\phi\, dz\, a_\rho \Big|_{\rho=4}$$

$$= 20\, \phi\Big|_0^{\pi} z\Big|_{-5}^{5}$$

$$= 200\pi$$

Divergence and curl of vector :-

$$\vec{A} = A_u a_u + A_v a_v + A_w a_w$$

$$\nabla\cdot\vec{A} = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u}(h_2 h_3 A_u) + \frac{\partial}{\partial v}(h_3 h_1 A_v) + \frac{\partial}{\partial w}(h_1 h_2 A_w)\right]$$

Cartesian :-

$$\nabla\cdot A = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

Cylindrical :-

$$\nabla\cdot\vec{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho\cdot A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

Spherical :-

$$\nabla \cdot \vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, A_\theta)$$

$$+ \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

Workbook - 1

$$\vec{D} = \rho \cdot z \cos^2\phi\, a_z$$

$$\rho_v = ? \quad \text{at } (1, \pi/4, 3)$$

$$\nabla \cdot D = \rho_v$$

$$\nabla \cdot D = \frac{\partial(\rho z \cos^2\phi)}{\partial z}$$

$$= \rho\cos^2\phi = 1 \cdot \frac{1}{2} = 0.5\ C/m^3$$

5. $$\vec{D} = \rho \cdot z \cdot \cos^2\phi\, a_\rho$$

$$\nabla \cdot D = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho \cdot \rho z \cos^2\phi)$$

$$= \frac{1}{\rho} z\cos^2\phi \cdot 2\rho = 2 \cdot 3 \cdot \frac{1}{2} = 3\ C/m^3$$

7. $$\vec{F} = \rho a_\rho + \rho\sin^2\phi\, a_\phi - z a_z$$

$$\nabla \cdot \vec{F} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho \cdot \rho) + \frac{1}{\rho}\frac{\partial}{\partial\phi}(\rho \cdot \sin^2\phi) + \frac{\partial}{\partial z}(-z)$$

$$= 2 + 2\sin\phi\cos\phi - 1$$

$$= 1 + \sin 2\phi$$

Ans - D

Ques - GATE 2012

$$\vec{A} = k r^n a_r \qquad n = ?$$

If $\nabla \cdot A = 0$

Soln:- $\nabla \cdot \vec{A} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 k r^n)$

$$= \frac{k}{r^2} (n+2) r^{n+1} = 0$$

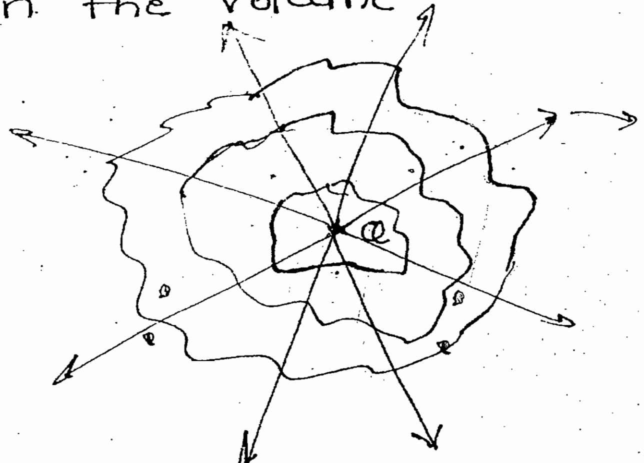
$$\Rightarrow n = -2 \quad \text{, Ans}$$

Curl of a vector $\vec{A}$:-

$$\nabla \times \vec{A} = \frac{1}{h_1 h_2 h_3} \begin{vmatrix} h_1 a_u & h_2 a_v & h_3 a_w \\ \frac{\partial}{\partial u} & \frac{\partial}{\partial v} & \frac{\partial}{\partial w} \\ h_1 A_v & h_2 A_v & h_3 A_w \end{vmatrix}$$

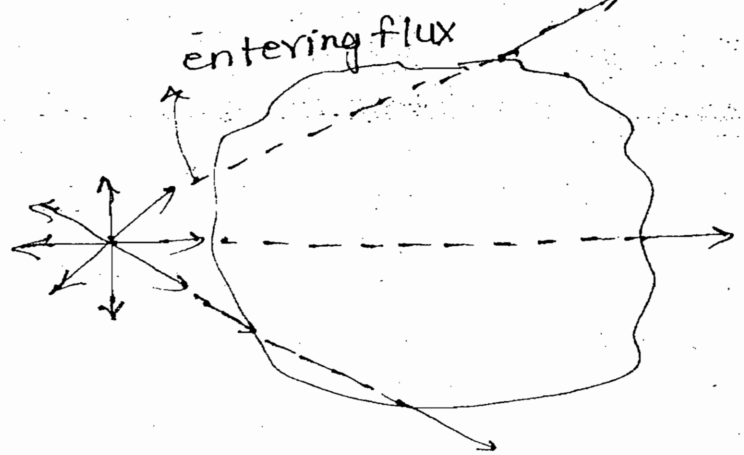
Static Electric Fields :-

Gauss Law :- The net (total) electric (effects) flux leaving any closed surface is always equal to the charge (cause) enclosed in the volume



Flux leaving

$$\oint D \cdot ds = Q$$



$$\oint D \cdot ds = 0$$

12

The word electric flux means the attractive or repulsive force on any +ve test charge placed in the electric field. Hence electric field or electric flux or lines of force physically represent the direction in which a test charge moves away when placed in the field.

→ When the surface is closed the total outflow or flux leaving is independent of density and area. i.e.

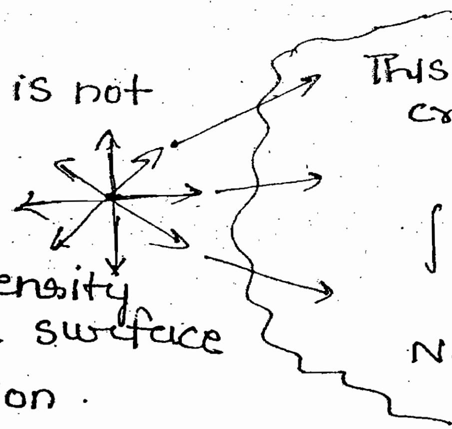
$$\oint D \cdot ds = Q$$

→ If the charge is outside the effects still exists but net effects is zero

$$\therefore \text{entering flux} = \text{leaving flux}$$

$$\oint D \cdot ds = 0$$

→ If the surface is not completely enclosing the flux crossing depends on the density of the flux and the surface area of consideration.



This is crossing flux

$$\int D \cdot ds = \psi_c$$

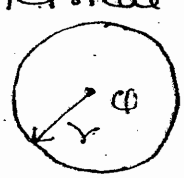
↓

Not Gauss Law

Application of Gauss Law :-

1) Electric field strength of a point charge :-

Consider a concentric symmetrical surface such that D is same everywhere



$$\oint D \cdot ds = Q$$

r = constant

a_r const-directed

$$\Rightarrow D \cdot S = Q$$

$$\Rightarrow D = \frac{Q}{S}$$

$$\Rightarrow \boxed{D = \frac{Q}{4\pi r^2} a_r} \quad C/m^2$$

→ Coulomb had a different measure of field strength i.e. the force measured & he related this with charge measure using ϵ. This is called as intensity

$$\vec{E} = \frac{\vec{F}}{Q_t} = \frac{\vec{D}}{\epsilon} = \frac{Q}{4\pi\epsilon r^2} a_r$$

↓

Intensity (N/C)

$$\Rightarrow \boxed{F = \frac{Q_1 Q_2}{4\pi\epsilon r^2}}$$

This is coulomb's law

Static Magnetic Fields :-

Biot - Savart's Law (Ampere's law for current elements) :-

It is derived from Ampere's law & consider a $\vec{dl}$ length I (DC current) carrying as a basic cause of H field

$I\vec{dl}$ — (Amp-m) is basic cause of H field

$$\vec{H} = \frac{I dl}{4\pi r^2} \times a_r$$

↓

Intensity (Amp/m).

→ $\vec{H}$ direction = I flow direction [cross] × Radial vector from current

→ Lorentz had a different measure of field strength i.e. density measure which is physically force per basic cause

$$\boxed{\vec{B} = \mu \vec{H} = \frac{\mu I dl \times a_r}{4\pi r^2} = \frac{\vec{F}}{\overrightarrow{Idl}} = \frac{\text{Newton}}{\text{Amp-m}}}$$

$$\vec{F} = Q\,(V \times B)$$

$$\Rightarrow dF = dQ\left(\frac{dl}{dt} \cdot B\cos\theta\right) = IdlB$$

$$\Rightarrow B = \frac{F}{Idl}$$

Note:–

→ $D \cdot E = \frac{\text{col}}{m^2} \times \frac{\text{Newton}}{\text{col}} = \frac{\text{Newton}}{m^2}$

= Electric pressure at that point

→ $B \cdot H = \frac{\text{Newton}}{\text{Amp-m}} \times \frac{\text{Amp}}{m} = \frac{\text{Newton}}{m^2}$

= Magnetic Pressure.

→ $\frac{\text{Newton} \times m}{m^2 \times m} = \frac{\text{Joules}}{m^3}$ = Energy density

→ $\frac{1}{2} D \cdot E = \frac{1}{2} \epsilon E^2$ = Energy density in electric field

→ $\frac{1}{2} B \cdot H = \frac{1}{2} \mu H^2$ = Energy density in H field

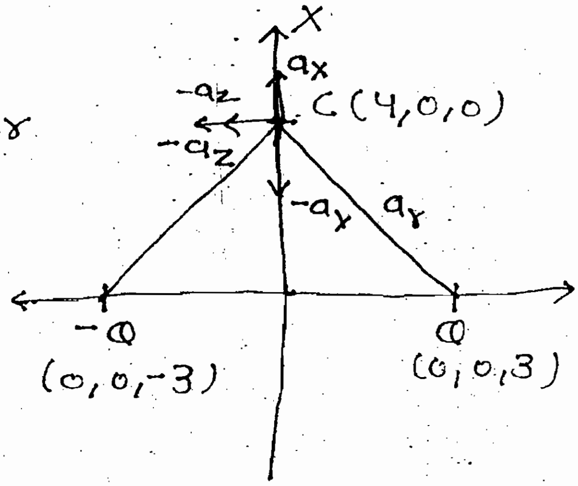
Workbook

$Q \rightarrow (0,0,3)$

$-Q \rightarrow (0,0,-3)$

E at C $(4,0,0)$

$$E = \frac{Q}{4\pi\epsilon r^2} a_r$$



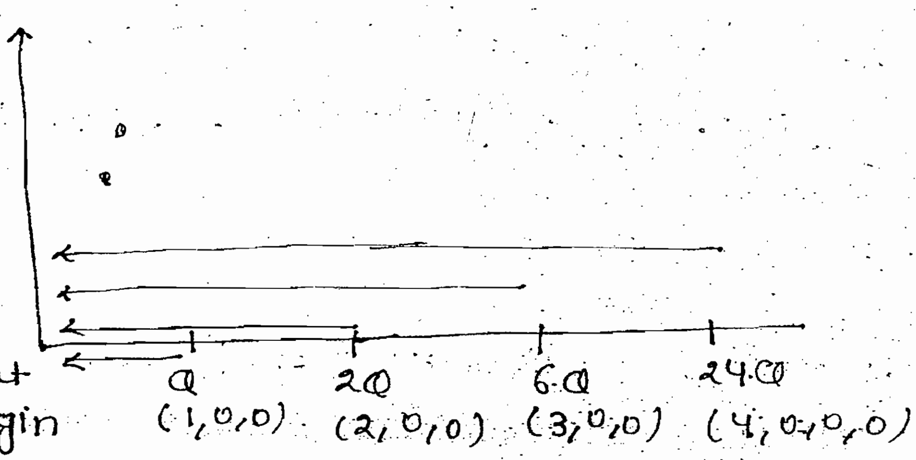
$E_1 = (a_x, -a_z)$
(Q)

$$= \frac{4a_x - 3a_z}{5}$$

$E_2 = (-a_x, -a_z)$
$(-Q)$

$$= \left(\frac{-4a_x - 3a_z}{5}\right)$$

$E_{total} = -a_z$
(direction)



$$E_T = E_1 + E_2 + E_3 + E_4$$

$$E = \frac{Q}{4\pi\epsilon r^2} a_r$$

$$a_r = -a_x$$

$$E_T = \left(\frac{Q}{4\pi\epsilon(1)^2} + \frac{2Q}{4\pi\epsilon(2)^2} + \frac{6Q}{4\pi\epsilon(3)^2} + \frac{24Q}{4\pi\epsilon(4)^2} \right) a_x$$

$$= \frac{Q}{4\pi\epsilon} \left(1 + \frac{1}{2} + \frac{2}{3} + \frac{3}{2} \right) (-a_x)$$

:- Net flux leaving $= \oint D \cdot ds = Q =$ charge enclosed

Sphere $\longrightarrow$ centre = origin

radius = 6m

$2C \longrightarrow (4,8,3) \longrightarrow d_1 = \sqrt{4^2+8^2+3^2} > 6$

$8C \longrightarrow (2,-1,-3) \longrightarrow d_2 = \sqrt{2^2+1^2+3^2} < 6$

$-12C \longrightarrow (-4,0,1) \longrightarrow d_3 = \sqrt{4^2+1^2} < 6$

Net flux leaving $= -4C$

:- Centre - $(2,-3,2)$

$d_1 = \sqrt{2^2+11^2+1^2} > 6 \longrightarrow$ out

$d_2 = \sqrt{0^2+2^2+5^2} < 6 \longrightarrow$ in

$d_3 = \sqrt{6^2+3^2+1^2} > 6 \longrightarrow$ out

Net flux leaving $= 8C$

:- Net flux leaving $= Q = \rho_L l$

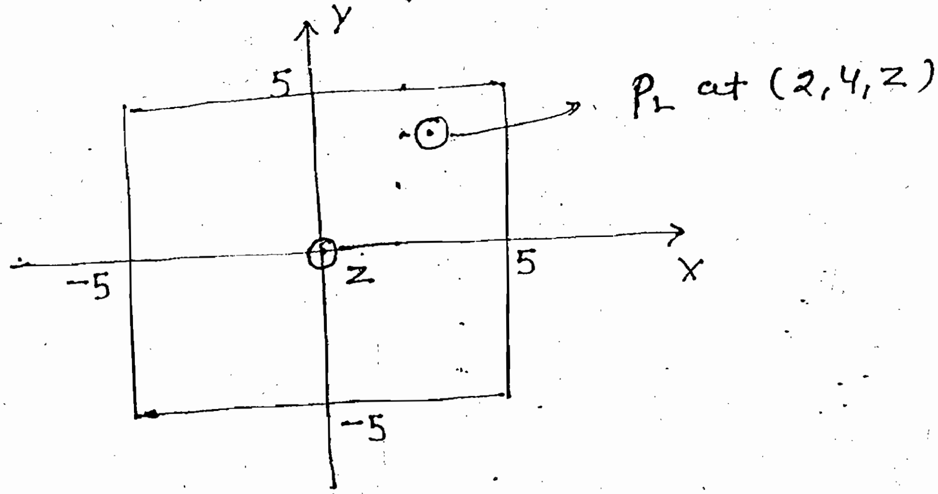
$\rho_L = 15\,nC/m$

at $x=2, y=4$ for all z

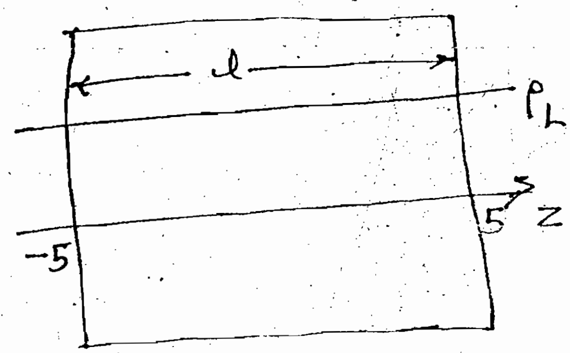
closed surface $= -5 < x < 5$

$-5 < y < 5$

$-5 < z < 5$

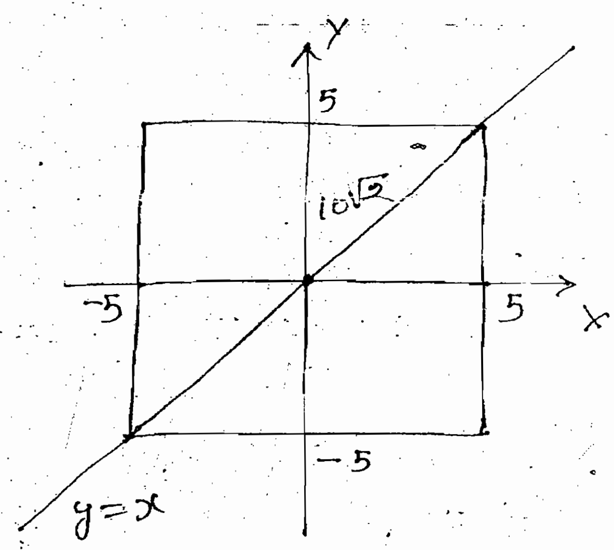


$$Q = 15\,\frac{nC}{m} \times 10$$
$$= 150\ nC \quad , \text{Ans}$$



13:-

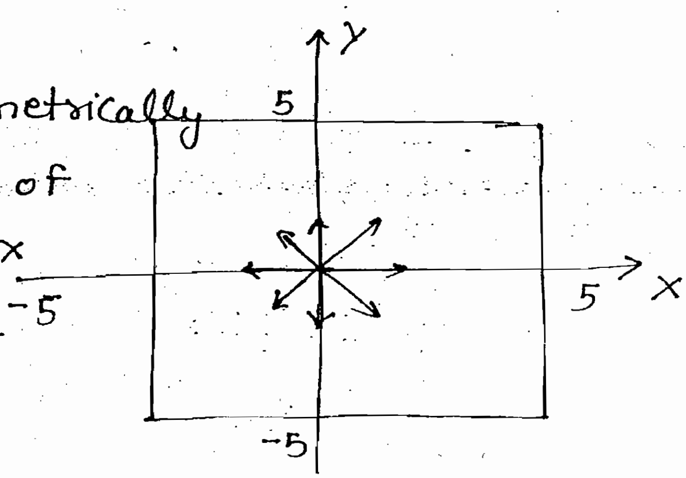
$$Q = 15 \times 10\sqrt{2}$$
$$= 150\sqrt{2}$$



14:-

$Q = 6\ nC$

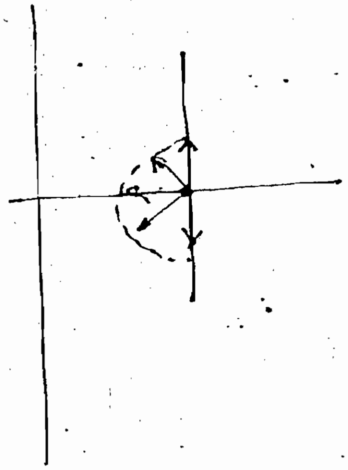
6nC → flux symmetrically crosses 6 sides of cube. Inc-flux cross per 1side



5.

z = 5m-plane

3nC, 14r2

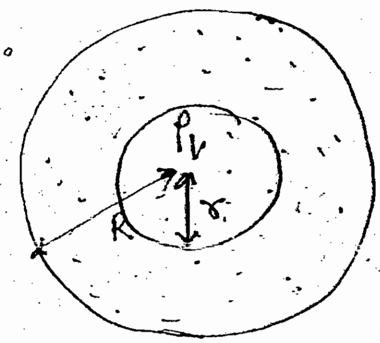


Note :- A point charge and a infinite sheet near by has half the flux crossing through

$$\oint_s D \cdot ds = Q$$

$\downarrow$

constant



$$D \cdot S = Q$$

$$\Rightarrow D = \frac{Q}{S}$$

$$= \frac{\rho_v \frac{4}{3}\pi r^3}{4\pi r^2} = \frac{\rho_v r}{3}$$

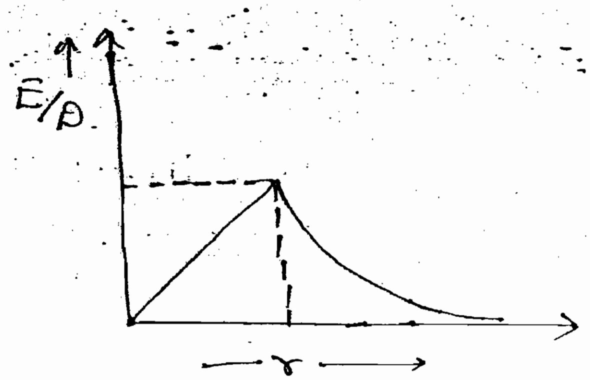
$$E = \frac{\rho_v r}{3\epsilon}$$

$$\boxed{E\left(\frac{R}{2}\right) = \frac{\rho_v R}{6\epsilon}}$$

$$D \cdot S = Q$$

$$\Rightarrow D = \frac{Q}{S}$$

$$= \frac{\rho_v \frac{4}{3}\pi R^3}{4\pi r^2}$$



$$D = \frac{\rho_V R^3}{3r^2}$$

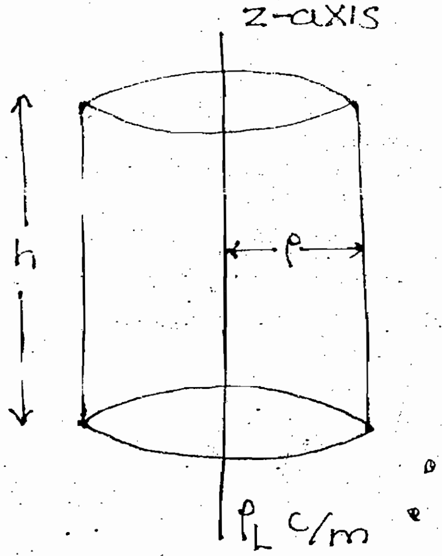
$$E = \frac{\rho_V R^3}{3\epsilon r^2}$$

$$\boxed{E(2R) = \frac{\rho_V R}{12\epsilon}}$$

Gauss Law Application-2 :-

Electric field strength of an infinite length ρ_L C/m line charge :-

Consider a cocentric symmetric Gaussian surface which is a cylinder around the charge



$$\oint \underbrace{D \cdot ds}_{\text{Constant}} = Q$$

ρ = Constant

a_ρ directed

$$D \cdot S = Q$$

$$\Rightarrow D = \frac{Q}{S}$$

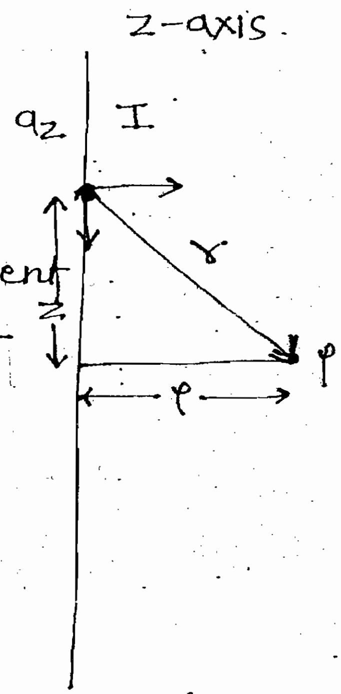
$$\Rightarrow D = \frac{\rho_L \cdot \cancel{h}}{2\pi\rho \cancel{h}}$$

$$\Rightarrow D = \frac{\rho_L}{2\pi\rho} a_\rho$$

$$\Rightarrow \boxed{E = \frac{\rho_L}{2\pi\epsilon\rho} a_\rho}$$

Magnetic field strength of an infinite length I carrying wire :-

$$H = \frac{I\,dl}{4\pi r^2} \times a_r$$



Consider a small dl length I current element at the z point height above the point

$$dl = dz\,a_z$$

$$r = \sqrt{\rho^2+z^2}$$

$$a_r = \frac{\vec{r}}{|r|}$$

$$= \frac{\rho a_\rho - z a_z}{\sqrt{\rho^2+z^2}}$$

$$dH = \frac{I\,dz\,a_z}{4\pi(\rho^2+z^2)} \times \left(\frac{\rho a_\rho - z a_z}{\sqrt{\rho^2+z^2}}\right)$$

$$H = \int_{z=-\infty}^{\infty} dH$$

$$\Rightarrow H = \frac{I\rho}{4\pi}\int_{z=-\infty}^{\infty} \frac{dz\,(a_z \times a_\rho)}{(\rho^2+z^2)^{3/2}}$$

Put $z = \rho\tan\theta$

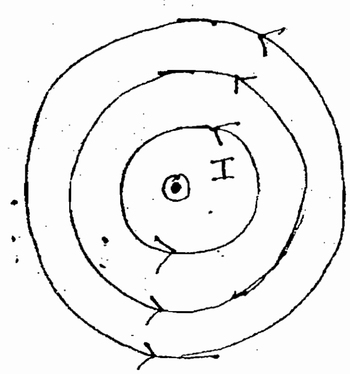
$$dz = \rho\sec^2\theta\,d\theta$$

$$(\rho^2+z^2)^{3/2} = \rho^3\sec^3\theta$$

$$H = \frac{I\rho}{4\pi}\int_{\theta=-\pi/2}^{\pi/2} \frac{\rho \sec^2\theta \, d\theta}{\rho^3 \sec^3\theta} = \frac{I}{4\pi\rho}\int_{-\pi/2}^{\pi/2} \cos\theta \cdot d\theta$$

$$\Rightarrow H = \frac{I}{2\pi\rho}(a_z \times a_\rho) = \frac{I}{2\pi\rho} a_\phi$$

Note :-



$$\oint H \cdot dl = I$$

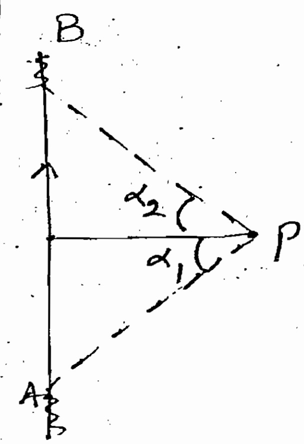
H · Length = current

$$H = \frac{I}{2\pi\rho} a_\phi$$

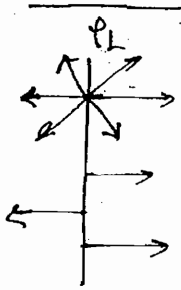
Extension :-

H - field of finite length I wire :-

$$\boxed{H = \frac{I}{4\pi\rho}(\sin\alpha_1 + \sin\alpha_2) a_\phi}$$



Summary :-



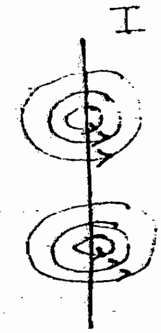
$\rho_L \rightarrow$ Line charge

$E \rightarrow$ Field

$$D = \frac{\rho_L}{2\pi\rho} a_\rho \qquad E = \frac{\rho_L}{2\pi\epsilon\rho} a_\rho$$

I carrying H-field

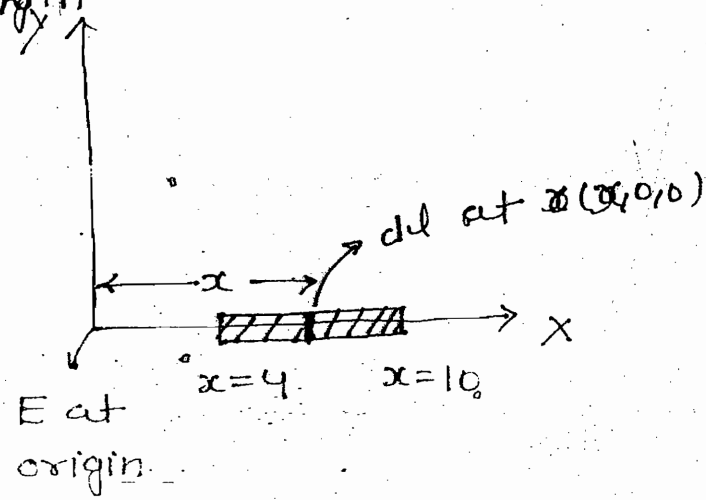
$$H = \frac{I}{2\pi\rho} a_\phi$$



$$B = \frac{\mu I}{2\pi\rho} a_\phi$$

7 :-

consider a dl length on the charge and apply the $\frac{Q}{4\pi\epsilon r^2} a_r$



Field calculation

$Q = \rho_L dx$

$r = x$

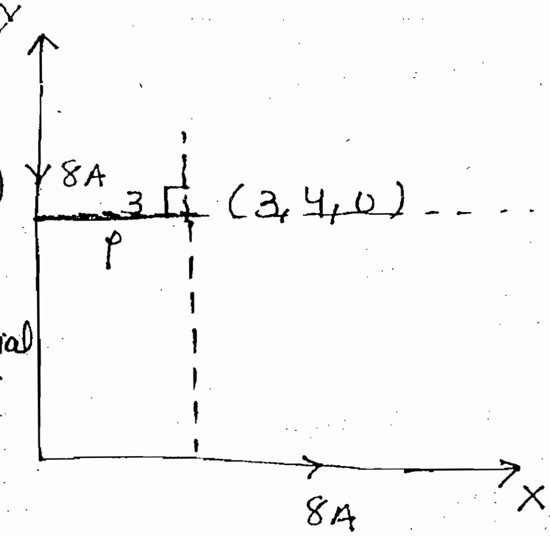
$a_r = -a_x$

$$dE = \frac{\rho_L dx}{4\pi\epsilon x^2}(-a_x)$$

$$E = \frac{\rho_L}{4\pi\epsilon}\int_{x=4}^{10} \frac{dx}{x^2}(-a_x)$$

$$= \frac{\rho_L}{4\pi\epsilon}\left(-\frac{1}{x}\right)_4^{10}(-a_x)$$

$$\underset{(Y\text{-axis})}{H} = \frac{8}{4\pi 3}\left(1 + \frac{4}{5}\right) \quad (-a_y \times a_x)$$



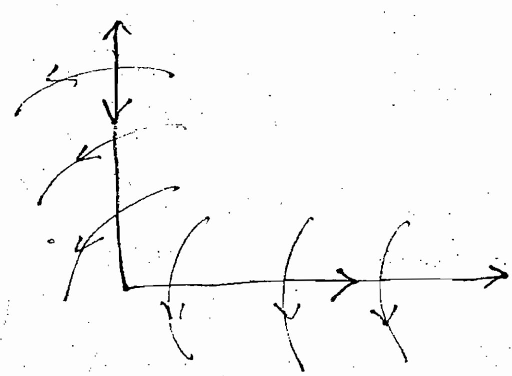
$$a_\phi = a_z \times a_\rho$$

$$\text{H-direction} = \underset{\text{direction}}{I} \times \underset{\text{dir.}}{\text{Radial}}$$

$$= \frac{8}{4\pi 3} \times \frac{9}{5}\,(a_z)$$

$$\underset{(x\text{-axis})}{H} = \frac{8}{4\pi \cdot 4}\left(\frac{3}{5} + 1\right)(a_x \times a_y)$$

$$= \frac{8}{16\pi} \times \frac{8}{5}\,(a_z)$$

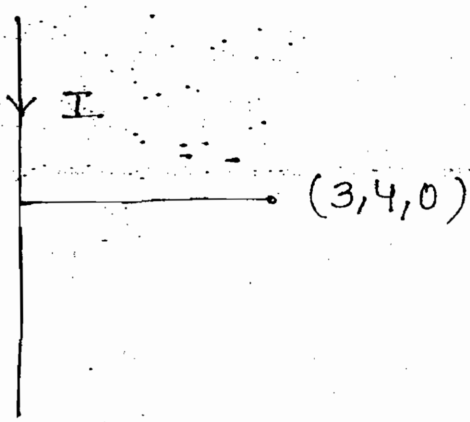


$$\underset{(\text{total})}{H} = \frac{2}{\pi}\,a_z$$

Ques:- Repeat the above problem if current is only entire y-axis

Soln:- $H = \dfrac{8}{2\pi \cdot 3}\,a_z$

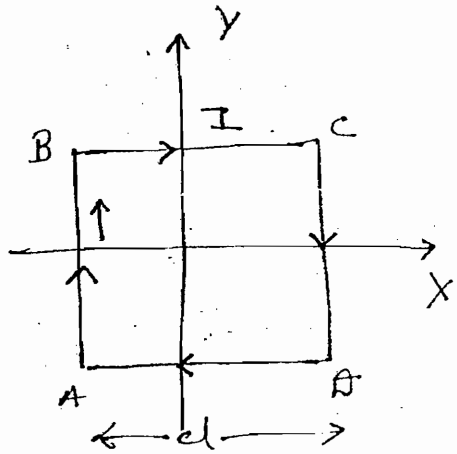
$= \dfrac{1.33}{\pi}\,a_z$



19

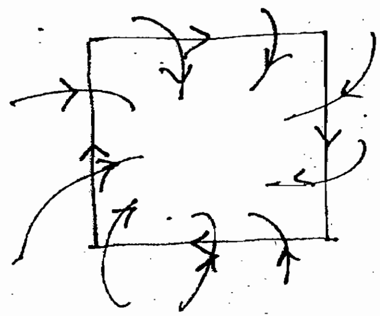
$$H_{AB} = a_y \times a_x = -a_z$$

$$H_{BC} = a_x \times -a_y = -a_z$$



Note:-

→ For any symmetric current distribution H field at geometric centre is always max



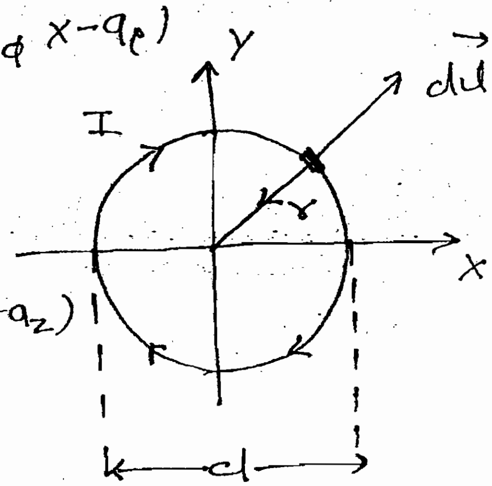
→ For any symmetric charge distribution E field at the geometric centre is always zero

$$H = \frac{I}{4\pi \cdot \frac{d}{2}} \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) \times 4 \, (-a_z)$$

$$= \frac{2\sqrt{2}\, I}{\pi d} (-a_z) = 0.9 \frac{I}{d} (-a_z)$$

20

$$dH = \frac{I \, \vec{dl}}{4\pi r^2} (-a_\phi \times -a_\rho)$$

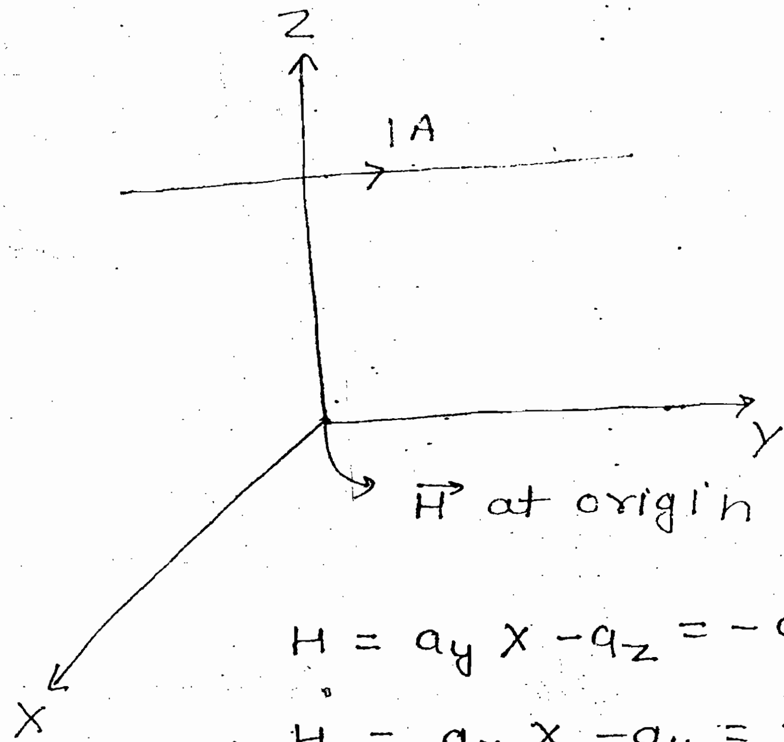


$$H = \int dH$$

$$= \frac{I \, 2\pi r}{4\pi r^2} (-a_z)$$

$$= \frac{I}{2r} = \frac{I}{d} (-a_z)$$

22 :-



$H = a_y \times -a_z = -a_x$

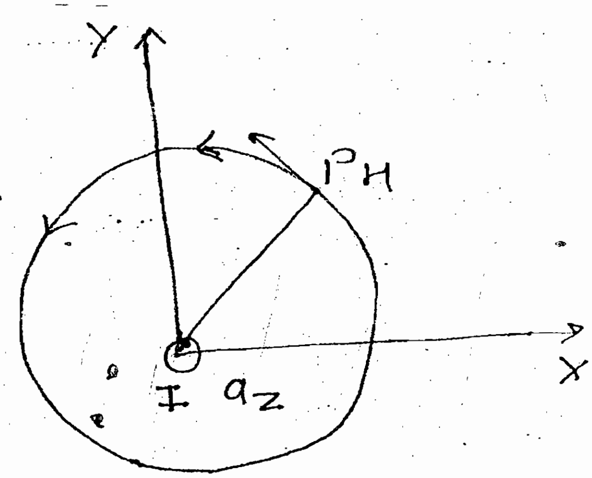
$H = a_x \times -a_y = -a_z$

Ans - d

23.

$H_\phi = a_z \times a_\rho$

Ans - (c)



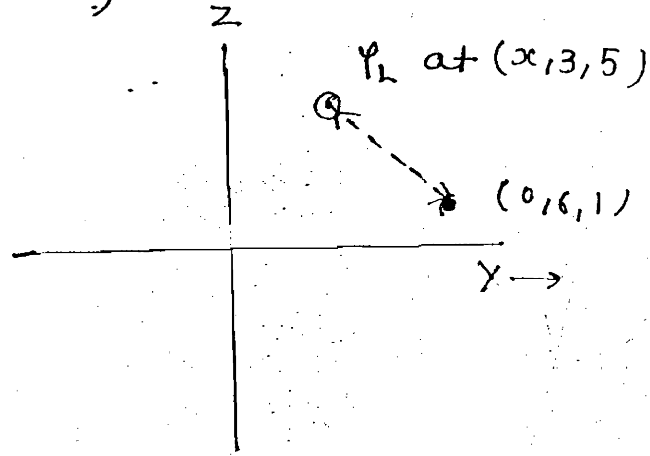
Lecture – 5

2.

ρ_L at $y=3$, $z=5$ for all x

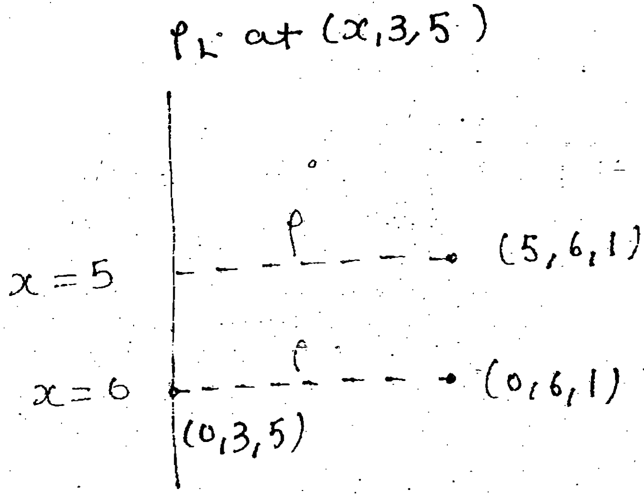
Given $\vec{E}$ at $(0,6,1)$

$\vec{E} = ?$ at $(5,6,1)$



$$E = \frac{\rho_L}{2\pi\epsilon\rho} a_\rho$$

$$E \propto \frac{1}{\rho}$$



For both point ρ is same

$$\rho = \sqrt{3^2+4^2} = 5$$

Note :–

In general z-axis $\longrightarrow$ $(0,0,z)$

Any point (x,y,z)

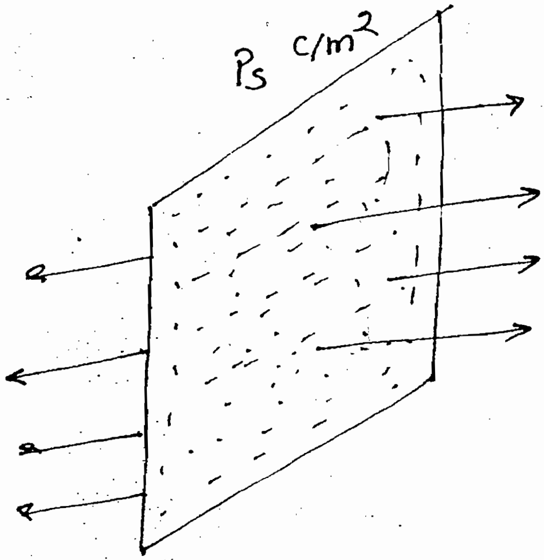
$$\rho = \sqrt{x^2+y^2}$$

Any line parallel to x-axis $\longrightarrow$ (x,a,b)

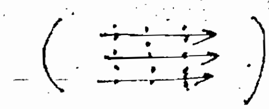
Any point $\longrightarrow$ (x,y,z)

$$\rho = \sqrt{(y-a)^2+(z-b)^2}$$

Electric field strength of an infinite sheet of ρ_s C/m² charge density :-



→ The electric field is completely normal to the sheet and the field lines are parallel to themselves



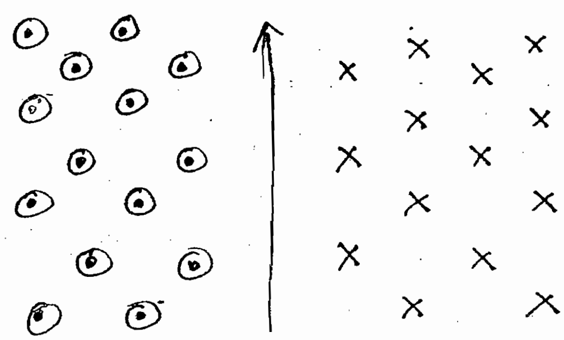
→ The flux density is same everywhere. The field is uniform

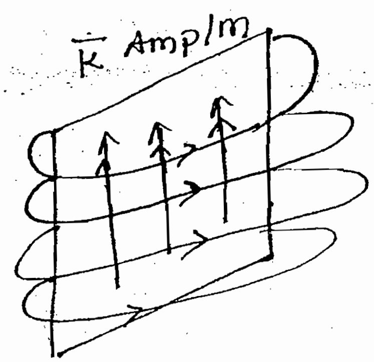
$$\rightarrow \quad D = \frac{\rho_s}{2} a_N$$

$$\rightarrow \quad E = \frac{D}{\epsilon} = \frac{\rho_s}{2\epsilon} a_N \Rightarrow \boxed{E = \frac{\rho_s}{2\epsilon} a_N}$$

**

Magnetic field strength of an infinite sheet of ρ_s C/m² charge density :-





→ The magnetic field is completely normal to the current and the field lines are parallel to themselves and to the sheet

→ The flux density is same everywhere i.e. field is uniform

→ $$H = \frac{\vec{K}}{2} \times a_N \quad \text{Amp/m}$$

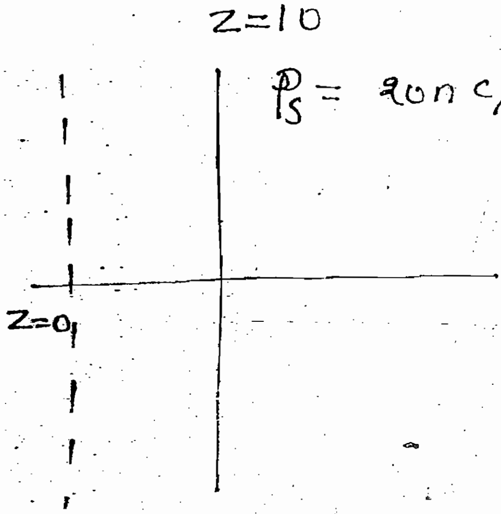
and $$\boxed{B = \frac{\mu \vec{K}}{2} \times a_N}$$

24. $a_N = \pm a_z$

At $z = 0$, $a_N = -a_z$

$$E = \frac{\rho_s}{2\epsilon} a_N$$

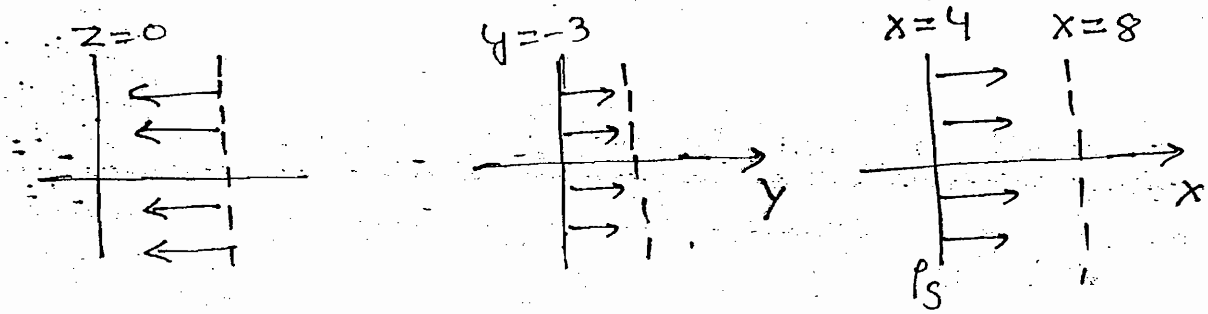
$$= \frac{20 \times 10^{-9}}{2 \times \frac{1}{36\pi \times 10^{9}}} (-a_z)$$



25. $x = 4 \quad \rho_{s_1} = 18\, nC/m^2 \longrightarrow E_1 = \pm a_x$

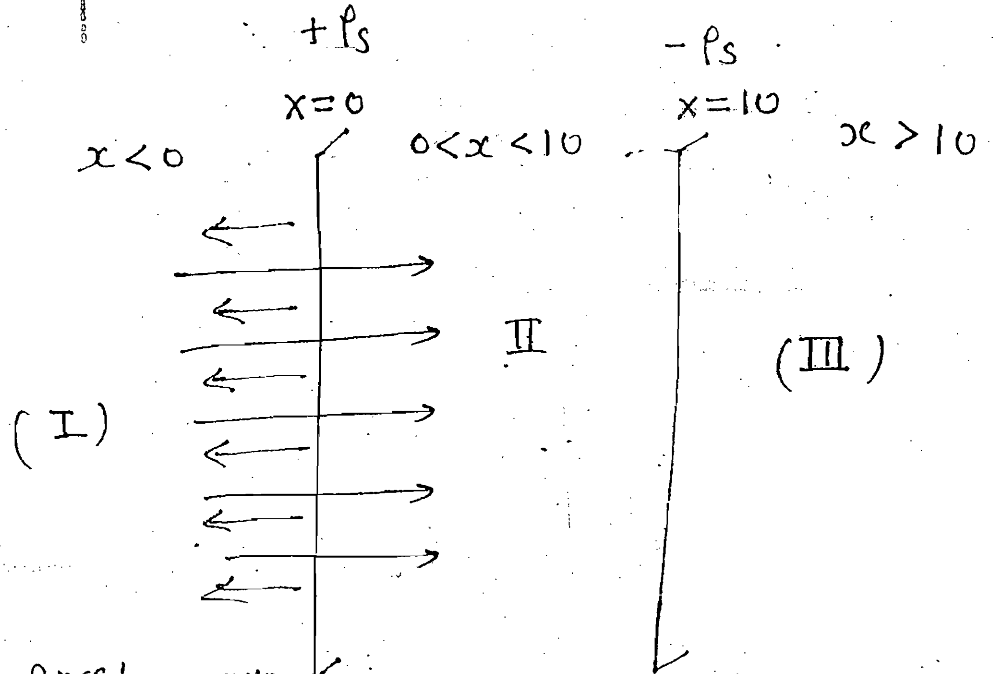
$y = -3 \quad \rho_{s_2} = 9\, nC/m^2 \longrightarrow E_2 = \pm a_y$

$z = 0 \quad \rho_{s_3} = -24\, nC/m^2 \longrightarrow E_3 = \pm a_z$



$$E_3 = \frac{24}{2\epsilon}(-a_z) \qquad E_2 = \frac{9}{2\epsilon} a_y \frac{nN}{C} \qquad E_1 = \frac{18}{2\epsilon} a_x \frac{nN}{C}$$

$$E_T = \frac{3}{2\epsilon}\left(6a_x + 3a_y - 8a_z\right) \frac{nN}{C}$$



<u>In first region</u>

$$E_1 = \frac{\rho_s}{2\epsilon}(-a_x)$$

$$E_2 = \frac{\rho_s}{2\epsilon}(a_x)$$

$$\vec{E}_{Net} = 0$$

<u>In third region</u>

$$\vec{E}_{Net} = 0$$

<u>In second region</u> :-

$$E_1 = \frac{\rho_s}{2\epsilon}a_x \qquad E_2 = \frac{\rho_s}{2\epsilon}a_x$$

$$E = \frac{\rho_s}{\epsilon}a_x \quad 0 < x < 10$$

$$= 0 \quad \text{elsewhere}$$

7. $\vec{K} = 30\, a_z$ mA/m $\longrightarrow$ $Y=0$ plane $\Rightarrow$ zx plane

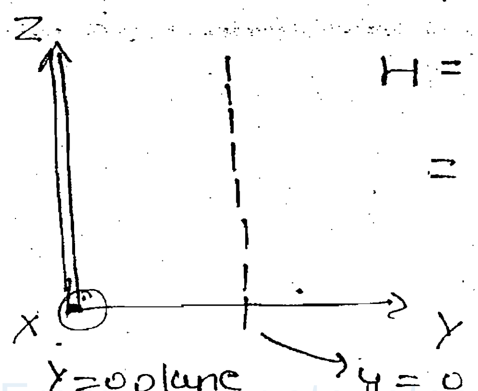
$H = \frac{\vec{K}}{2} \times a_N$

at $y=2$

$a_N = \pm a_y$

for $y=20$

$a_N = a_y$



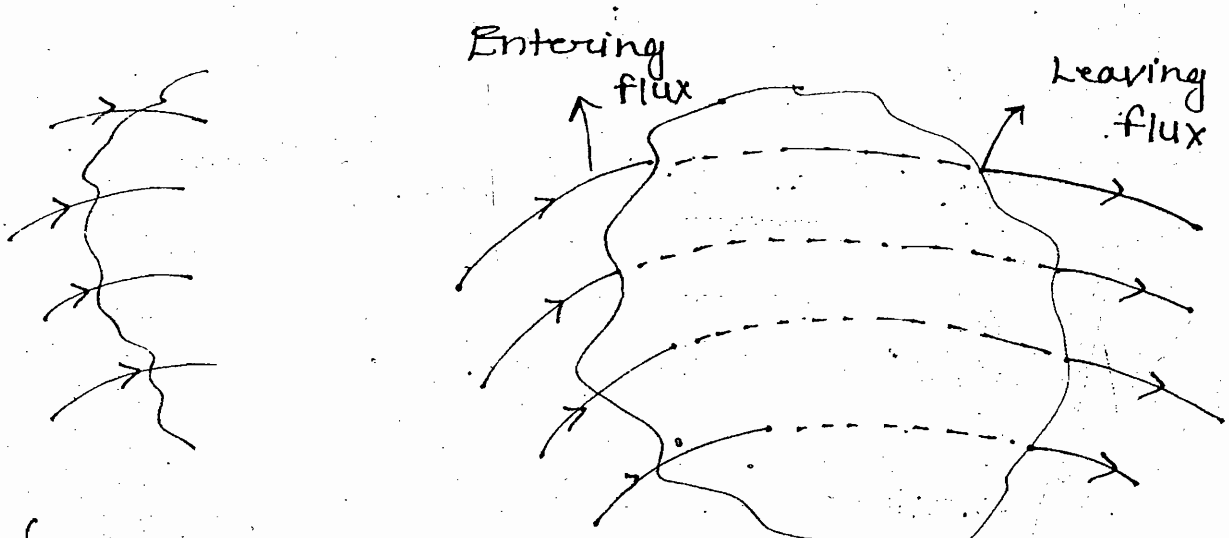
$H = \frac{30}{2} a_z \times a_y$

$= -15 a_x$ mA/m

Ans-(A)

Closed surface Integral of B — Maxwell's III Equation :-

→ Magnetic field always forms closed lines around the current and has no starting / ending point for flux lines i.e. No sources / sinks exists for B flux lines



$\int B \cdot ds = \psi_m$

This is not Maxwell's equation

$\oint B \cdot ds = 0$

This is Maxwell III equation

→ For any closed surface

Entering flux = Leaving flux

$$\oint B \cdot ds = 0$$

B field is solenoidal (No divergence)

$$\nabla \cdot B = 0$$

Note :-

$$\oint D \cdot ds = 0 \begin{cases} \rightarrow Q = 0 \\ \rightarrow +Q, -Q \end{cases} \Rightarrow \boxed{\oint B \cdot ds = 0}$$

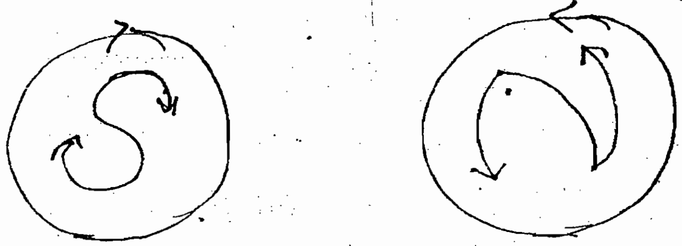
Always

→ By comparison, equals of charge in E fields donot exist in B fields i.e. every cause of B field is a dipole, i.e. Magnetic monopoles don't exist eg:- Bar Magnet , N/S - Dipole

best example:- I current carrying wire

→ I always flows only when both the polarities exist and are connected hence it is always treated as magnetic dipoles.

→ I always flows in closed circuits and every closed I wire is a magnetic dipole



Clockwise → South pole

Anticlockwise → North pole

→ Cause - current - closed

Entering current = Leaving current (Junction)

Effect - B field - closed around the cause

Entering flux = Leaving flux (closed surface)

$$\boxed{\oint B \cdot ds = 0}$$ → KCL in Magnetic Field

Potential, Gradient, closed line Integral in E field :-

Potential is a ~~square~~ scalar measure of E field strength in terms of energy at any point in the field

V (Volts) Potential at any point = $\dfrac{\text{Workdone by the charge to reach the point}}{\text{charge}}$

$$V = \frac{W}{Q} = \frac{\text{Joules}}{\text{coulomb}}$$

$\Rightarrow$ Work = Force · displacement

$$\Rightarrow dW = \vec{F}\cdot\vec{dl} = Q\vec{E}\cdot\vec{dl}$$

= Workdone in field / force direction

= Workdone on the charge

→ Work is done on the charge when it moves in the field direction and hence the charge acquires energy

$$\boxed{V = \frac{W}{Q} = -\int \vec{E}\cdot\vec{dl}}$$ → scalar potential function of space

$$V_{AB} = -\int_{\text{Ref } B}^{\text{Ref } A} \vec{E}\cdot\vec{dl} = \text{Potential diff b/w } A \,\&\, B$$

If ref. B has $V_B = 0$, then V_A is called as absolute potential at A.

<u>Potential function of a point charge</u> :—

$$V = -\int \vec{E}\cdot\vec{dl}$$

Spherical coord.

$$= -\int \frac{Q}{4\pi\epsilon r^2} a_r \, dr \, a_r = \frac{Q}{4\pi\epsilon_0 r}$$

$$\Rightarrow \boxed{V = \frac{Q}{4\pi\epsilon r}}$$

$\vec{E} \rightarrow$ Intensity $- \frac{1}{r^2}$ decrease

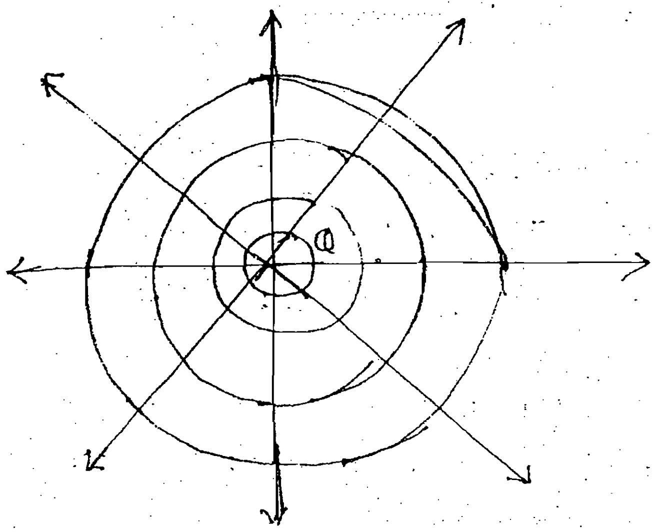
a_r directed vector

$V \longrightarrow$ Potential $- \frac{1}{r}$ decrease

– scalar –

If $r =$ constant then $V =$ constant.

$\longrightarrow$ cocentric sphere $\rightarrow$ surface



→ The family of ~~cocentri~~ equipotential surfaces represent the potential distribution and variation of potential in the region and similar to vector intensity line

Potential function of a line charge:–

$$V = -\int \vec{E} \cdot \vec{dl}$$

$$= -\int \frac{\rho_L}{2\pi\epsilon\rho} a_\rho \, d\rho \, a_\rho$$

$$\Rightarrow V = \frac{\rho_L}{2\pi\epsilon} \ln\left(\frac{1}{\rho}\right)$$

If $\rho =$ constant $\Rightarrow V =$ constant

Equipotential surfaces are cocentric cylinders

$\vec{E}$ – Intensity – $\frac{1}{\rho}$ decrease

a_ρ directed vector

$V \rightarrow$ Potential – logarithmic decrease

– scalar

Potential function of a sheet charge (Uniform) :-

$$V = -\int \vec{E} \cdot \vec{dl}$$

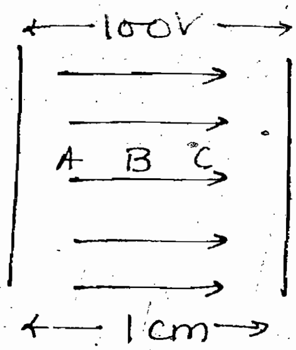
$$\Rightarrow V = -Ed$$

$$\Rightarrow E = -\frac{V}{d}$$

$\vec{E} \rightarrow$ Intensity – Uniform

V – Potential – linear decrease

eg :- Capacitor Plates

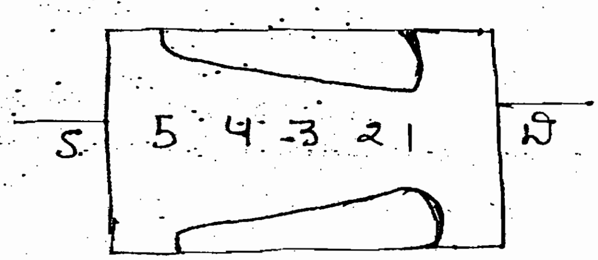


E_A or E_B or E_C

$$= \frac{100}{10} = 10\ \text{kV/m}$$

$$V_A > V_B > V_C$$

FET Channel



28.

$$V = -\int \vec{E}\cdot\vec{dl} = -\int_{(0,0,0)} (y a_x + x a_y)\cdot(dx\, a_x + dy\, a_y).$$

$$V = -\int_{x=0}^{4} y\,dx - \int_{y=0}^{1} x\,dy$$

Using the straight line path or equation

$$\frac{y-1}{1-0} = \frac{x-4}{4-0}$$

$$x = 4y$$

$$= -\int_0^4 \frac{x}{4}\,dx - \int_0^1 4y\,dy$$

$$= -\frac{1}{8}(16) - \frac{4}{2}\times 1$$

$$= -4V \text{ , Ans.}$$

Note :—

In line integral,

$\int f(x,y,z)\,dx$ We have to transform y & z in terms of x using the known relationship

i.e. path of integration

$$\int f(x,y,z)\,dx = \int g(x)\,dx$$

29.

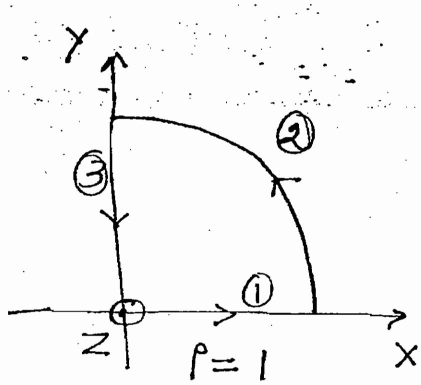
$$\vec{A} = 2\rho\cos\phi\, a_\rho$$

$$\oint A\cdot dl = 3 \rightarrow \int A\cdot dl$$

(1) $dl = d\rho\, a_\rho$

$\phi = 0$

$A = 2\rho\, a_\rho$



$$\int A\cdot dl = \int_{\rho=0}^{1} 2\rho\, a_\rho\, d\rho \cdot a_\rho$$

$$= \rho^2 \Big|_0^1 = 1$$

$2 \rightarrow$ $dl = \rho\, d\phi\, \underline{a_\phi}$ $\quad (\because a_\phi \cdot a_\rho = 0)$

$$\int A\cdot dl = 0$$

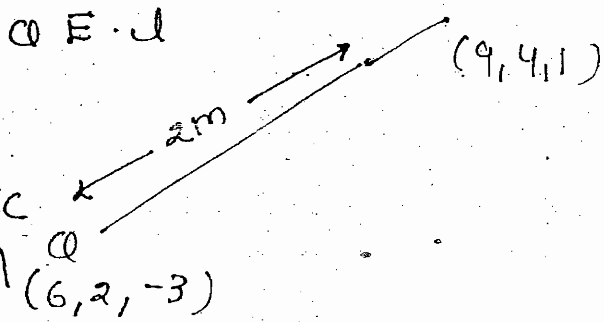
$3 \rightarrow$ $dl = d\rho\, a_\rho$

ρ from 1 to 0

$\phi = 90 \quad \Rightarrow \quad A = 0$

$30 \rightarrow$ $\vec{E} = 4a_x - 3a_y + 5a_z \longrightarrow$ Uniform

$$W = Q\int \vec{E}\cdot\vec{dl} = Q\,E\cdot l$$



Unit length vector $= \dfrac{3a_x + 2a_y + 4a_z}{\sqrt{3^2+2^2+4^2}} \times 2 \quad 5C$

$(\because$ 2m length vector$)$

$$W = 5\,(4a_x - 3a_y + 5a_z)\cdot \frac{(3a_x + 2a_y + 4a_z)\times 2}{\sqrt{29}}$$

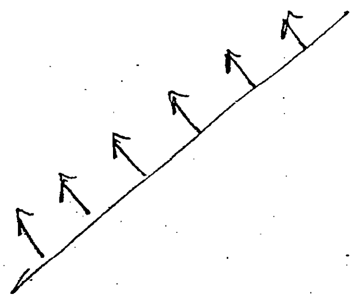
$$= \frac{260}{\sqrt{29}}\ J$$

Potential Gradient :-

Scalar surface equation (f) $\xrightarrow{\nabla f}$ Vector (Normal) to the surface (direction)

eg:- Linear surface

$$f = 4x + 7y - 15z = 18$$



$$\nabla f = 4a_x + 7a_y - 15a_z$$

eg:- Non-linear surface

$$f = 4xy - 15x^2yz$$

$$\nabla f = (4y - 30xyz)a_x + (4x - 15xz^2)a_y + 15x^2y\,a_z$$

$$V = -\int \vec{E} \cdot \vec{dl}$$

$$dV = -\vec{E}\, dl \cos\theta$$

$$\Rightarrow \boxed{\frac{dV}{dl} = -E\cos\theta}$$

Case-(1) :-

If $\theta = 90^\circ \Rightarrow V = \text{constant}$

→ The direction of E field is always normal to equi-potential surfaces.

Case-(II):-

$$\text{If } \theta = 0°/180° \Rightarrow \left.\frac{dV}{dl}\right|_{max} = |E|$$

→ The magnitude of E intensity is always the maximum rate of change of potential per unit length ~~If~~

Case-(III):-

$$\text{If } \theta = 0° \Rightarrow |E| = -\left.\frac{dV}{dl}\right|_{max}$$

→ The direction of E intensity is always the direction in which potential decreases by maximum

Note:-

The operation is physically called as gradient

Hence $E = -\nabla V$ = potential gradient is field intensity.

→ Gradient signifies the maximum rate of change of scalar along with the direction of change.

eg:- Given $V = 4(x^2-y^2)$ for all z

Find the equation of equipotential surface passing through (3,1,9)

Soln:- V at (3,1,9) $= 4(9-1) = 32V$

All x & y when V = 32V is the equipotential surface

$$\Rightarrow x^2 - y^2 = 8$$

Summary:-

Equation of equipotential surface OR Voltage function (V) $\longrightarrow \nabla V \rightarrow$ Vector Intensity function (E) which is normal to equi-potential surfaces

Mathematically, ∇V.

$$\boxed{\nabla V = \frac{1}{h_1}\frac{\partial V}{\partial u} a_u + \frac{1}{h_2}\frac{\partial V}{\partial v} a_v + \frac{1}{h_3}\frac{\partial V}{\partial w} a_w}$$

eg:- If $V = \frac{4\cos\theta}{r^2}$

Find $\vec{E}$ at $(2, \pi/4, \pi)$

Soln:- $E = -\nabla V$

$$\nabla V = \frac{1}{1}\frac{\partial}{\partial r}\left(\frac{4\cos\theta}{r^2}\right) a_r + \frac{1}{r}\frac{\partial}{\partial \theta}\left(\frac{4\cos\theta}{r^2}\right) a_\theta$$

$$= 4\cos\theta\left(\frac{-2}{r^3}\right) a_r + \frac{4}{r^3}(-\sin\theta) a_\theta$$

$$E = \frac{4}{r^3}\left(2\cos\theta\, a_r + \sin\theta\, a_\theta\right)\Big|_{(2, \pi/4, \pi)}$$

$$= \frac{1}{2}\left(\sqrt{2}\, a_r + \frac{1}{\sqrt{2}} a_\theta\right)$$

32.

(a) $V = 0$ $\quad \nabla V = 6xy\, a_x + (3x^2 - z) a_y - y a_z$

$\neq 0$ at $(1, 0, -1)$

(b) $x^2 y = 1$ in XY plane

$V = 3(1) - y(0)$

$V = 3 \longrightarrow$ equipotential surface

(c) At $(2, -1, 4) \Rightarrow V = -8$

$V = 3(4)(-1) - (-1)4 = -8$

(d) Normal vector at P $= \frac{\nabla V}{|\nabla V|}\Big|_{(2,-1,4)} = \frac{-12a_x + 8a_y - a_z}{\sqrt{12^2 + 8^2 + 1}}$

$= (-0.83\hat{x} + 0.55\hat{y} + 0.07\hat{z})$

Closed line integral of electric field - Maxwell's II Eqn :-

$$\oint E \cdot dl = 0$$

→ Potential difference exists b/w 2 different points but b/w the same point potential difference is zero

→ Potential is unique at a point at a time

→ Workdone in moving a charge in any closed path is always zero

→ In any closed path energy acquired = energy lost

i.e Energy acquired in field direction = energy lost opposite to field direction.

→ Electric field is a conservative field & never forms closed loops.

i.e. Irrotational vector ($\nabla \times E = 0$)

→ Workdone in moving a charge b/w two points is independent of path of consideration.

→ This is KVL in electric fields.

Potential, Vector Potential and Ampere's Law in H field :-

→ Potential is called as MMF or Magneto Motive force and is a scalar measure of magnetic field strength at any point in the field.

$$V_m = MMF = \int \vec{H} \cdot \vec{dl} = Amp$$

$$H = \nabla V_m$$

$$\boxed{\oint H \cdot dl = I}$$

This is Ampere's law in Integral form

$$\boxed{\nabla \times H = J}$$

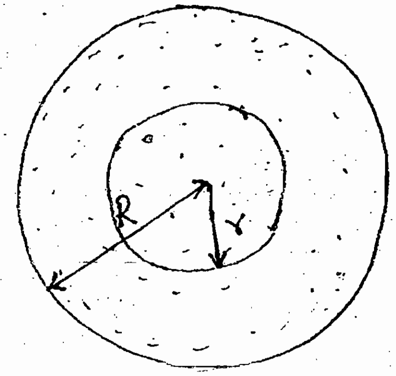
This is Ampere's law in point form

Statement :-

The ~~net circulation~~ effects of magnetic field in any closed line is always equal to the current crossing the surface enclosed.

Circulation = Strength × length
= current

Consider a closed ~~ring~~ line symmetrical & cocentric with the I flow direction such that H = constant everywhere



$$\oint \underset{\downarrow \atop \text{constant}}{H} \cdot dl = I$$

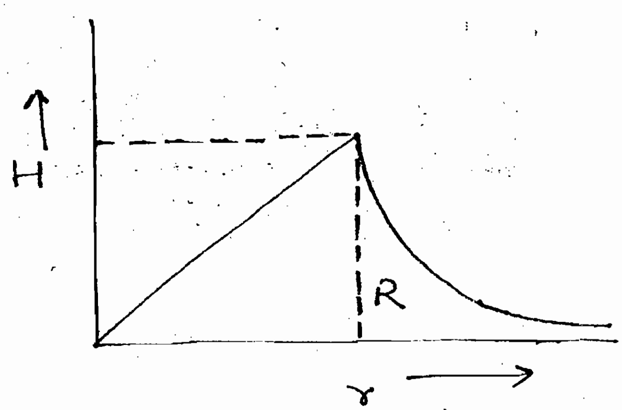
H · Length = current crossing

$r < R$

$$H = \frac{\frac{I}{\pi R^2} \cdot \pi r^2}{2\pi r} = \frac{Ir}{2\pi R^2}$$

$r > R$

$$H = \frac{I}{2\pi r}$$



Note :-

Point → $1/r^2$

Line → $1/\rho$

Sheet → Uniform

Solid → r

Vector Potential :-

Limitation of Scalar Potential (MMF) :-

→ The definition of potential in electric field is consistent with its nature i.e. irrotational nature

$$E = -\nabla V$$

$$\nabla \times E = 0$$

$$\nabla \times (-\nabla V) = 0$$

Curl (Grad of scalar) = 0

→ A similar definition in magnetic field for MMF is not consistence with Ampere's law

$$H = \nabla V_m$$

$$\nabla \times H = J$$

$$\nabla \times (\nabla V_m) = 0 = J$$

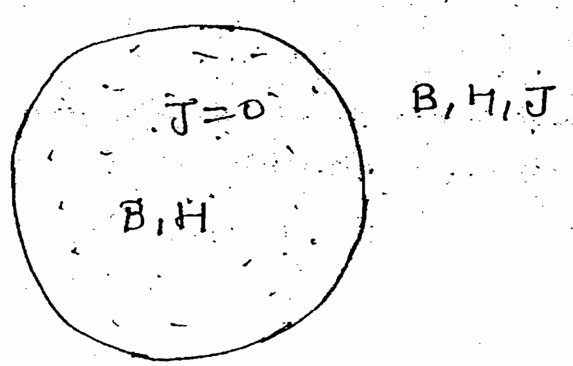
Hence MMF is defined and exists in only those regions where $J = 0$

i.e. outside current flowing regions

i.e. free space conditions and outside conductor

eg:- (i) MMF b/w windings of solenoids

(ii) MMF in air gaps in machines



Vector Potential $\vec{A}$:-

→ The basic definition of vector potential is in accordance to nature of magnetic field i.e. solenoidal nature. Hence if curl of A = B then A is called as vector potential

$$B = \nabla \times \vec{A}$$

$$\nabla \cdot B = 0$$

$$\nabla \cdot (\nabla \times \vec{A}) = 0$$

Div. (curl of vector) = 0

→ The term vector potential has a units weber/m which physically signifies the work for current element

$$\vec{A} = \text{Weber/m} = \frac{\text{Joules}}{\text{Amp-m}}$$

$$= \frac{W}{I\,d\vec{l}}$$

and as $I\,d\vec{l}$ is a vector, $\vec{A}$ is a vector quantity.

Note:-

$$\frac{\text{Weber}}{\text{Second}} = \text{Volts}$$

$$\frac{\text{Weber}}{\text{second}} = \frac{\text{Joules}}{\text{Coulomb}}$$

$$\frac{\text{Weber}}{\text{m}} = \frac{\text{Joules}}{\text{Amp-m}}$$

$$V = \frac{W}{Q} = \frac{Q}{4\pi\epsilon r}$$

$$\boxed{\vec{A} = \frac{W}{I\vec{dl}} = \frac{\mu I \vec{dl}}{4\pi r}}$$

A's direction is along I flow direction & is a solenoidal vector.

Summary 1:–

→ Q – coulomb – point charge – Basic cause – E field
(scalar)

→ $I\vec{dl}$ – Amp-m – current element – H field
(vector)

→ Q – $\rho_L\, dl$ – $\rho_s\, ds$ – $\rho_v\, dv$

→ $I\vec{dl}$ – $\vec{K}\, ds$ – $\vec{J}\, dv$

→ $\vec{E}$ – Intensity – Strength – Force – $\frac{\vec{F}}{Q}$ – $\frac{1}{\epsilon}$ dependent

→ $\vec{B}$ – Density – Strength – Force – $\frac{\vec{F}}{I\vec{dl}}$ – μ dependent

→ $\vec{D}$ – Density – strength – $\frac{dQ}{ds} = C/m^2$

→ $\vec{H}$ – Intensity – strength – $\frac{dI}{dl}$ = Amp/m

→ V – Potential – strength – Work = W/Q → Scalar
(Volts)

→ $\vec{A}$ – Potential – strength – Work = $\frac{W}{Idl}$ → Vector.
(Weber/m)

Summary :-

Points → Line → Surface ; Line → Surface → Volume

Scalar function $\xrightarrow[\text{Gradient}]{\nabla}$ Vector f^n Intensity (per m)

eg:- V volts $\longrightarrow$ ∇V $\longrightarrow$ E volts/m

Vector function Intensity (per m) $\xrightarrow[\text{curl}]{\nabla \times}$ Vector f^n Density (per m^2)

eg:- Amp/m $\vec{H}$ $\longrightarrow$ $\nabla \times H$ $\longrightarrow$ J Amp/m^2

Web/m $\vec{A}$ $\longrightarrow$ $\nabla \times \vec{A}$ $\longrightarrow$ $\vec{B}$ Web/m^2

Vector function Density (per m^2) $\xrightarrow{\nabla \cdot}$ Scalar f^n Volume (per m^3)

eg:- $\vec{D}$ (C/m^2) $\longrightarrow$ $\nabla \cdot \vec{D}$ $\longrightarrow$ ρ_v C/m^3

33 $\oint A \cdot dl = \frac{\text{Weber}}{m} \times m = \text{Webers}$ ↓ flux

OR

$$\oint \vec{A} \cdot \vec{dl} = \int (\nabla \times A) \cdot ds = \int B \cdot ds = \psi_m$$

34.- Stoke's theorem

$\vec{V} = \nabla \times \vec{A}$

C → closed line

S_c → surface

B

$$\oint A \cdot dl = \int (\nabla \times A) \cdot ds = \int \vec{V} \cdot \vec{ds}$$

Laplace / Poisson's Equations :-

→ They are second order differential equations relating volume charge and potential developed inside / outside the charge

$$\nabla \cdot D = \rho_v$$

$$\nabla \cdot [\epsilon(-\nabla V)] = \rho_v$$

If ϵ = constant then

$$\boxed{\nabla^2 V = -\frac{\rho_v}{\epsilon}} \longrightarrow \text{Poisson's Equation}$$

For $\rho_v = 0$, i.e. outside the charge, charge free region

$$\boxed{\nabla^2 V = 0} \longrightarrow \text{Laplace Equation.}$$

Note :-

Inspite of being II order differential equation they always have a unique solution i.e. voltage is a single valued function of space

This is called as ~~uni~~ Uniqueness theorem

Mathematically

$$\nabla^2 V = \nabla \cdot (\nabla V)$$

$$\nabla \cdot (\nabla V) = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u}\left(\frac{h_2 h_3}{h_1}\frac{\partial V}{\partial u}\right) + \frac{\partial}{\partial v}\left(\frac{h_3 h_1}{h_2} \cdot \frac{\partial V}{\partial v}\right) + \frac{\partial}{\partial w}\left(\frac{h_1 h_2}{h_3}\frac{\partial V}{\partial w}\right)\right]$$

Extension :—

$\vec{A}$ Vs $\vec{J}$ in Magnetic Fields

$$\nabla \times H = \vec{J}$$

$$\Rightarrow \nabla \times \left(\frac{B}{\mu}\right) = J$$

$$\Rightarrow \nabla \times (\nabla \times A) = \mu J$$

$$\Rightarrow \nabla(\nabla \cdot A) - \nabla^2 A = \mu J$$

$$\Rightarrow \boxed{\nabla^2 A = -\mu J}$$

For current free region

$$\boxed{\nabla^2 A = 0}$$

35

$$V = -\frac{6r^5}{\epsilon_0}$$

1. $\nabla^2 V = -\rho_v/\epsilon$ 2. $Q = \int \rho_v dv$

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 \frac{\partial}{\partial r}\left(-\frac{6r^5}{\epsilon}\right)\right] = -\frac{\rho_v}{\epsilon}$$

$$\Rightarrow \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \cdot 6 \cdot 5 \cdot r^4\right) = \rho_v$$

$$\frac{30}{r^2} 6 \cdot r^5 = \rho_v$$

$$\Rightarrow \rho_v = 180 r^3$$

$$Q = \int_{r=0}^{1}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} 180 r^3 \cdot r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$= 180 \frac{r^6}{6}\Big|_0^1 (-\cos\theta)_0^{\pi} (\phi)_0^{2\pi} = 120\pi C \text{, Ans.}$$

36. $\rho_v = -10^{-8}(1+10\rho)$

V on $\rho = 5cm$

Given $\rho = 2cm$ V & $E = 0$

$$\nabla^2 V = -\frac{\rho_v}{\epsilon}$$

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \cdot \frac{\partial V}{\partial \rho}\right) = \frac{10^{-8}(1+10\rho)}{\frac{1}{36\pi \times 10^9}}$$

$$= 360\pi(1+10\rho)$$

$$\rho \frac{\partial V}{\partial \rho} = 360\pi \int (\rho + 10\rho^2)\, d\rho$$

$$\rho \frac{\partial V}{\partial \rho} = 360\pi \left(\frac{\rho^2}{2} + \frac{10\rho^3}{3}\right)$$

$$\Rightarrow V = 360\pi \int \left(\frac{\rho}{2} + \frac{10\rho^2}{3}\right) d\rho$$

$$= 360\pi \left(\frac{\rho^2}{4} + \frac{10\rho^3}{9}\right)_{2\times 10^{-3}}^{5\times 10^{-2}}$$

$$\Rightarrow \boxed{V = 0.5V}$$

37. $$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = -\frac{\rho_v}{\epsilon}$$

$$= 20 \cdot 3 \cdot 2 \cdot x + 10 \cdot 4 \cdot 3\, y^2 \Big|_{(2,0)} = -\frac{\rho_v}{\epsilon}$$

$$\Rightarrow \boxed{\rho_v = -240\epsilon_0}$$

38. ϕ — 1 dimension function — satisfying — laplace

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} = 0$$

$$\frac{\partial\phi}{\partial x} = c_1 = \text{Rate of change is constant}$$

$$\frac{\phi_2 - \phi_1}{d} = \frac{\phi_3 - \phi_2}{2d}$$

$$\phi = c_1 x + c_2 \longrightarrow \text{Linear equation}$$

Ans — B

39. $V = \sinh x \cdot \cos ky \, e^{pz}$

$$\nabla^2 V = 0 \qquad \frac{e^{x} - e^{-x}}{2} = \sinh x$$

$$\nabla^2 V = 1V - k^2 V + p^2 V = 0$$

$$\Rightarrow \boxed{k = \sqrt{1+p^2}}$$

Boundary Conditions for Electric fields

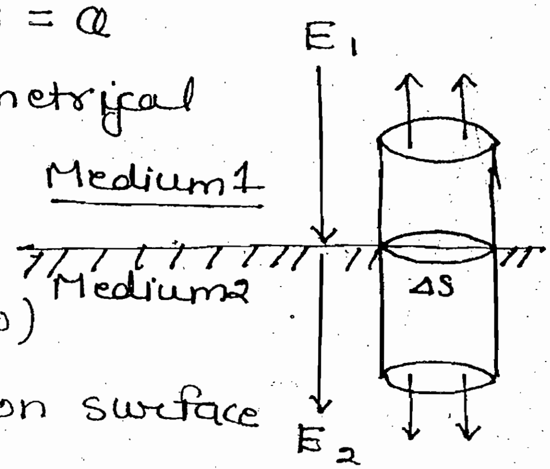
→ If a field is spread out into two different medium and known the field in one region. The field in the adjacent region can be calculated under two conditions

Case – (I) :–

Electric field is normal to the boundary.

Using Gauss law $\oint D \cdot ds = Q$

consider a surface symmetrical in both the medium



$$\boxed{D_2 \cdot \Delta S - D_1 \Delta S = 0}$$

$(\because Q = 0)$

If charge is present on surface then

$$\boxed{D_2 \Delta S - D_1 \Delta S = \rho_S \Delta S}$$

$$D_{n_1} = D_{n_2}$$

$$D_{n_2} - D_{n_1} = \rho_S$$

Statement :–

The normal components of electric flux density is same on either side but otherwise discontinous when a surface charge density exists on the boundary

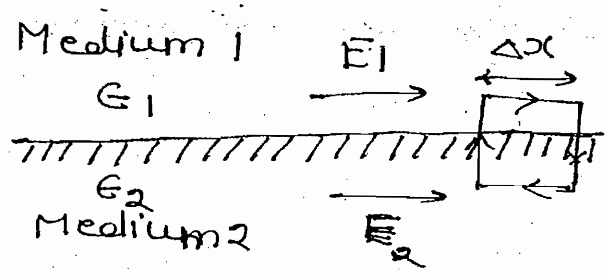
Case – (II) :–

Electric field is ~~parallel~~ tangential to boundary :–

Using $\oint E \cdot dl = 0$

$$E_1 \Delta x - E_2 \Delta x = 0$$

$$\Rightarrow \boxed{E_1 = E_2}$$



$$E_{t_1} = E_{t_2}$$

The tangential components of electric field intensity is always continous

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$x = 0$ Plane

$YZ \to$ Plane

$x < 0$ $x > 0$

$\epsilon_{R_1} = 1.5$ $\epsilon_{R_2} = 2.5$

$E_{n_1} = 2a_x$

$E_{t_1} = -3a_y + a_z$

$E_1 = 2a_x - 3a_y + a_z$

$$E_{t_1} = E_{t_2} = -3a_y + a_z$$

$$D_{n_1} = 1.5 \times 2a_x \times \epsilon_0 \quad (D_{n_1} = \epsilon_0 \epsilon_R \times E_n)$$

$$\cancel{= 3a_x} \quad \cancel{= D_{n_2}}$$

$$= 3\epsilon_0 a_x \longrightarrow D_{n_2} = 3\epsilon_0 a_x$$

$$E_{n_2} = \frac{3\epsilon_0 a_x}{2.5\epsilon_0} = 1.2 a_x$$

$$D_1 = \epsilon_0 (3a_x - 4.5a_y + 1.5a_z)$$

$$D_2 = \epsilon_0 (3a_x - 7.5a_y + 2.5a_z)$$

$$\boxed{E_2 = 1.2a_x - 3a_y + a_z}$$ Ans

Note :-

→ The value of vector dec. then its projection dec. i.e moves away from normal and moves towards the surface

→ The projection of normal component is dec. in E_2 when compare to E_1. Hence the field is shifting away from the normal. It can be verified with inc. tangential component when Comparing D_1 & D_2

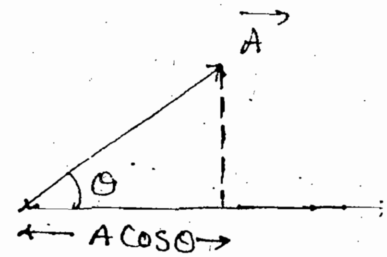
Sur

A's scalar projection on B

$$= A\cos\theta$$

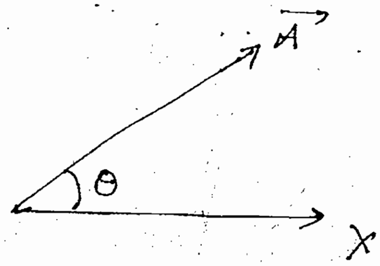
$$= \frac{\cancel{A}\ A\cdot B}{\cancel{A}\ B}$$

$$= \frac{A\cdot B}{|B|}$$



A's projection in x

$$= \frac{A\cdot a_x}{|a_x|}$$



A's vector projection on B

$$= (A\cos\theta)\cdot \hat{B}$$

$$= \frac{(A\cdot B)}{|B|}\hat{B}$$

$$= \frac{(A\cdot B)}{|B|^2}\vec{B}$$

of

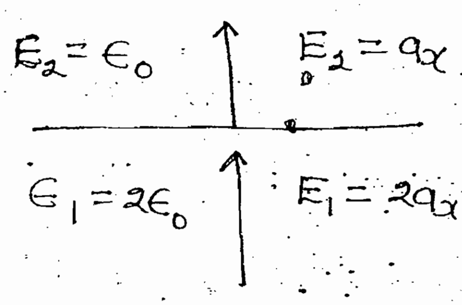
13.

41.

$$D_2 - D_1 = \rho_s$$

$$1\cdot\epsilon_0 - 2\cdot 2\epsilon_0 = \rho_s$$

$$\boxed{\rho_s = -3\epsilon_0}$$



42.

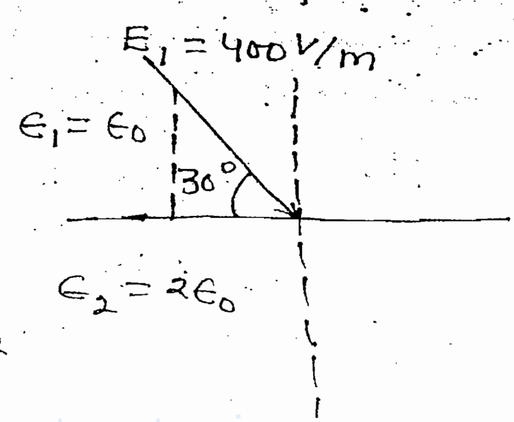
$$E_{t_1} = E_{t_2}$$

$$E_1\cos 30 = E_{t_2}$$

$$\Rightarrow E_{t_2} = 200\sqrt{3}$$

$$\epsilon_0 E_1 \sin 30^\circ = 2\epsilon_0 E_{n_2}$$

$$\Rightarrow E_{n_2} = 10$$



Ohm's law and Continuity Equation:

$$\int (\nabla \cdot J)\, dv = \oint J \cdot ds = I = \frac{dQ}{dt} = \frac{d}{dt}\int \rho_v\, dv = \int \frac{\partial \rho_v}{dt}\, dv$$

i.e. $$I = \frac{dQ}{dt} = \frac{d}{dt}\int \rho_v\, dv = \int \frac{\partial \rho_v}{dt}\, dv$$

$$\Rightarrow \quad I = \oint J \cdot ds = \int (\nabla \cdot J)\, dv$$

Divergence theorem

$$\boxed{\nabla \cdot J = \frac{\partial \rho_v}{\partial t}}$$

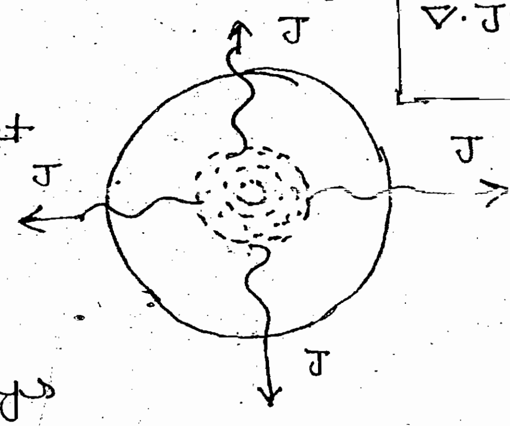
→ current has outflow and divergence at a rate depending on decrease of volume charges with time

$$\boxed{\nabla \cdot J = -\frac{\partial \rho_v}{\partial t}}$$

→ Practically

Entering current = Leaving current

charge can neither be created nor be destroyed



$$\left.\begin{array}{l} \oint J \cdot ds = 0 \\ \nabla \cdot J = 0 \end{array}\right] \longrightarrow \text{Always}$$

→ For a uni-directional current flow

$$\frac{\partial J}{\partial x} = \frac{\partial \rho_v}{\partial t}$$

$$\partial J = \partial \rho_v \frac{\partial x}{\partial t}$$

Integrating on both sides

$$J = \rho_v V_d$$

where $V_d = \dfrac{\partial x}{\partial t}$ = drift velocity

V_f = free velocity $= \sqrt{\frac{2qV}{m}}$

= vacuum tubes, CRO

= few $10^6 - 10^7$ m/s

$$\boxed{V_f \propto \sqrt{E}}$$

where $V_d = \frac{\partial x}{\partial t}$ = drift velocity

= charge in a material (bulk)

= few cm/sec

$V_d \propto E \Rightarrow \boxed{V_d = \mu E}$ → mobility of the particle

$J = \rho_v v_d$

$J = \rho_v \mu E$

$\Rightarrow \boxed{J = \sigma E}$ → Ohm's law in point form

where σ = conductivity (mho/m) $= \rho_v \mu$

= ability to allow current

= available free carriers × mobility of carriers

Case-(1) :-

$\sigma = \infty$ → very good conductor

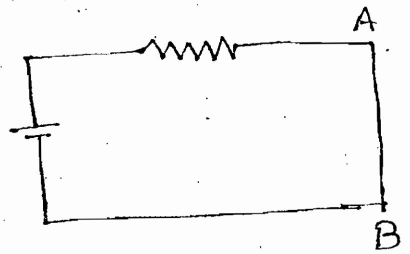
$\frac{J}{\sigma} = E = 0$

→ Electric field cannot exist in a very good conductor

$\rightarrow \quad V = \int 0 \cdot dl = \text{constant}$;

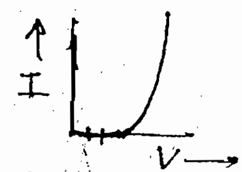
conductor is always a equi-potential region

→ Potential difference or voltage cannot exist in a very good conductor only current can flow but voltage difference ~~good~~ conductor cannot exist



$V_{AB} = 0$ but current flows

$V \uparrow \quad V_{AB} = 0 \quad I_{AB} \uparrow$



eg:- (I) diode current after cuting voltage

(II) Accumulation — does not exist — only flow exists

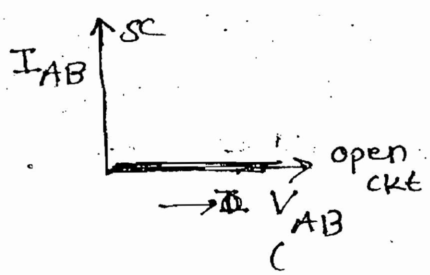
Case - (II) :-

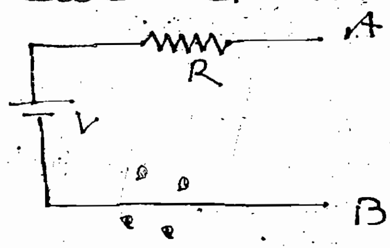
$\sigma = 0$ very good dielectric

$J = 0$

→ current cannot exist in a very good dielect

→ Voltage or potential difference or Electric field can exist but flow current exist





→ Accumulation can exist but flow can't exist.

$$\nabla \cdot J = -\frac{\partial \rho_v}{\partial t}$$

$$J = \sigma E$$

$$\nabla \cdot (\sigma E) = -\frac{\partial \rho_v}{\partial t}$$

$$\Rightarrow \quad \sigma (\nabla \cdot E) = -\frac{\partial \rho_v}{\partial t}$$

· The ρ_v is time solution is

$$\boxed{\rho_v(t) = \rho_{v_0} \cdot e^{-\frac{t\sigma}{\epsilon}}}$$

Note:—

Every charge density exponentially spread on the medium at a rate depending on $\frac{\epsilon}{\sigma}$ that called the relaxation time

$$\frac{\epsilon}{\sigma} = \text{Relaxation time} = \frac{\text{Farad}}{m \times \frac{\text{mho}}{m}}$$

$$= \text{ohms} \times \text{farad}$$

$$= \text{second}$$

Boundary conditions at conductor surfaces:—

(I) $E_{t_1} = E_{t_2}$ (General)

As $\sigma = \infty$ along the conductor

$\frac{J}{\sigma} = E = 0$ along the surface

$$\Rightarrow \boxed{E_{tang.} = 0}$$

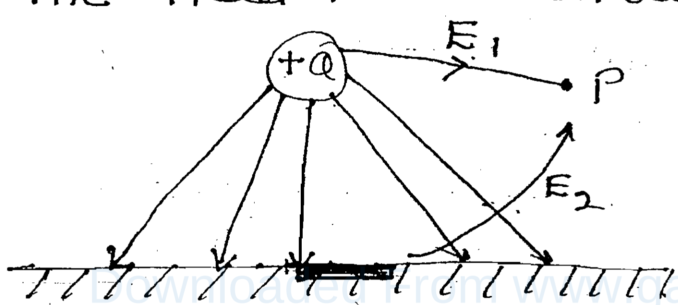
(II) $D_{n_2} - D_{n_1} = \rho_s$ (General)

Electric field can be normal to a conductor which depends on ρ_s on the surface

$$\boxed{D_{normal} = \rho_s}$$

Conductors, Induction, Method of Images:—

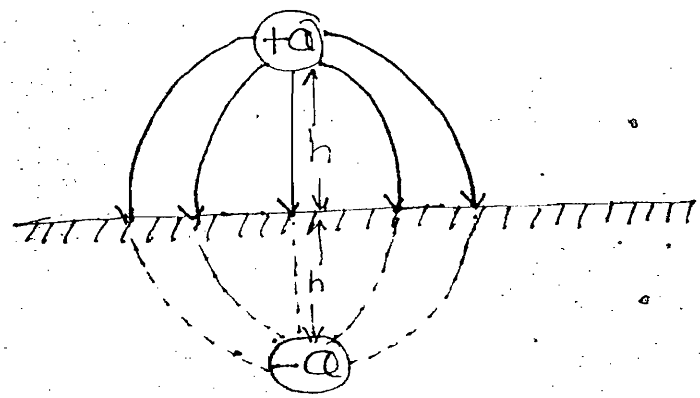
→ When a charge is placed near a conductor surface the field is as shown below



→ The field has tangential component which displaces the free carrier. Hence they are periodically accumulated all ~~the other~~ along the conductor surface. This is called as induced charge

→ The resultant field at any point is the vector sum of field due to actual charge and induced charge

→ This field is such that the tangential component are removed and has only normal components as shown below.



→ The field appears to be a dipole field with negative charge called as image charge below the conductor surface

Summary :—

Every charge and its induction effects are represented by a image charge below the conductor obeying all rules of light and optics

44.

$$D_1 = 2(a_x - \sqrt{3}a_z)\ C/m^2 = \rho_s$$

$$\rho_s = |D_n| = 2\sqrt{1+3} = 4$$

45.

$$E_n = 2\ V/m$$

$$D_n = 80\epsilon_0 E_n = \rho_s$$

$$\Rightarrow \rho_s = 80 \times 8.8 \times 10^{-12} \times 2$$

$$= 1.41 \times 10^{-9}\ C/m^2$$

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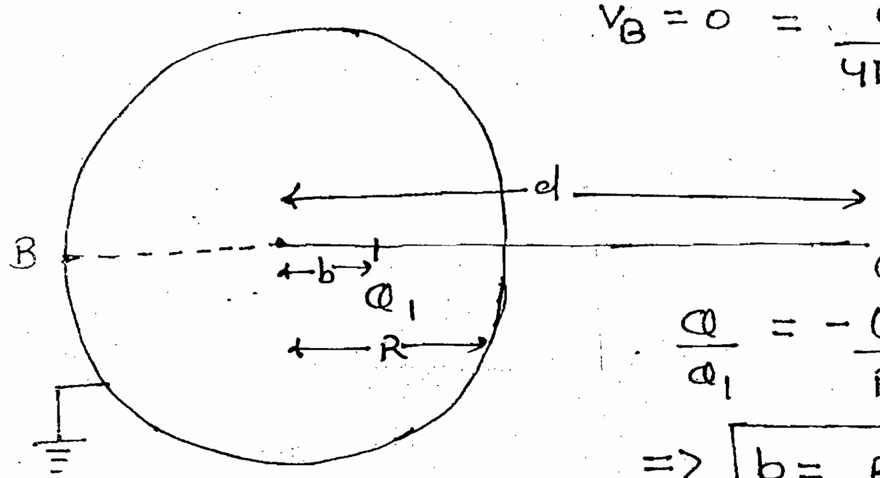
6.

$$V_A = 0 = \frac{Q}{4\pi\epsilon(d-R)} + \frac{Q_1}{4\pi\epsilon(R-b)}$$

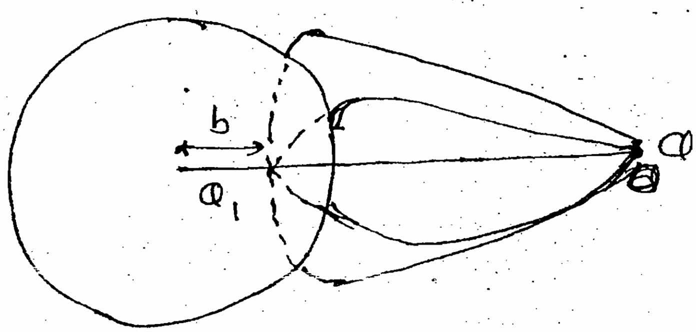
$$V_B = 0 = \frac{Q}{4\pi\epsilon(d+R)} + \frac{Q_1}{4\pi\epsilon(R+b)}$$



$$\frac{Q}{Q_1} = -\frac{(d-R)}{R-b} = -\frac{(d+R)}{(R+b)}$$

$$\Rightarrow \boxed{b = \frac{R^2}{d}} \quad \& \quad \boxed{Q_1 = -\frac{R}{d}}$$

Note:-



7. 4μC , Ans

Energy Density in Electric fields $\left(\frac{dW_E}{dV} = \frac{1}{2}\epsilon E^2\right)$:-

Consider $Q_1, Q_2, Q_3 ------ Q_n$ point charges assemble in a region

Total energy of the Electric field = Total Energy expended in assembling the charges

$W_1 = 0$ $\qquad V_{21}$ = Potential at 2nd due to 1st charge

$W_2 = -Q_2 V_{21}$

$W_3 = -Q_3 V_{31} - Q_3 V_{32}$

$W_4 = -Q_4 V_{41} - Q_4 V_{43}$

⋮

$W_n = -Q_n V_{n1} - Q_n V_{n2} ---- Q_n V_{n\,n-1}$

$$Q_2 V_{21} = Q_2 \cdot \frac{Q_1}{4\pi\epsilon r_{21}} = Q_1 V_{12}$$

Substitute subscripts can be interchange without change in meaning of value

$$W_1 = 0$$

$$W_2 = -Q_1 V_{12}$$

$$W_3 = -Q_1 V_{13} - Q_2 V_{23}$$

$$W_4 = -Q_1 V_{14} - Q_2 V_{24} - Q_3 V_{34}$$

$$\vdots$$

$$W_n = -Q_1 V_{1n} - Q_2 V_{2n} \cdots - Q_{n-1} V_{n-1 \cdot n}$$

$$W_E = W_1 + W_2 + W_3 + \cdots W_n$$

total energy

$$2W_E = -Q_1 V_1 - Q_2 V_2 - Q_3 V_3 \cdots\cdots Q_n V_n$$

$$\Rightarrow \boxed{W_E = -\frac{1}{2} \sum_{i=1}^{n} Q_i V_i}$$

For a continous charge distribution

$$W_E = -\frac{1}{2} \int \rho_v V dv = -\frac{1}{2} \int (\nabla \cdot D) V dv = \int \frac{1}{2} D(-\nabla V) dv$$

$$\Rightarrow W_E = \int \frac{1}{2} (D \cdot E) dv$$

$$\frac{dW_E}{dv} = \text{Energy density}$$

$= \text{Strength of energy at any point}$

$$= \frac{1}{2} (D \cdot E) = \frac{1}{2} \epsilon E^2$$

Extension :-

$$\frac{dW_H}{dv} = \text{Magnetic Energy density}$$

$$= \frac{1}{2} B \cdot H = \frac{1}{2} \mu H^2$$

Capacitors and Inductors :-

→ Capacitance is the ability to confine Electric field in a finitely small region:

$$C = \text{Farad} = \frac{\oint D \cdot S}{\int E \cdot dl} = \frac{\epsilon \oint E \cdot ds}{\int E \cdot dl} = \frac{Q}{V}$$

→ It is the ratio with charge utilized to the potential developed by the charge.

Specific Geometries :-

→ Parallel Plates
→ cocentric cylinders
→ Cocentric sphere

Inductance :-

Inductance is the ability to confine H field in a finitely small region

$$L = \text{Henry} = \frac{\int B \cdot ds}{\oint H \cdot dl} = \frac{\mu \int H \cdot ds}{\oint H \cdot dl} = \frac{\Psi_m}{I}$$

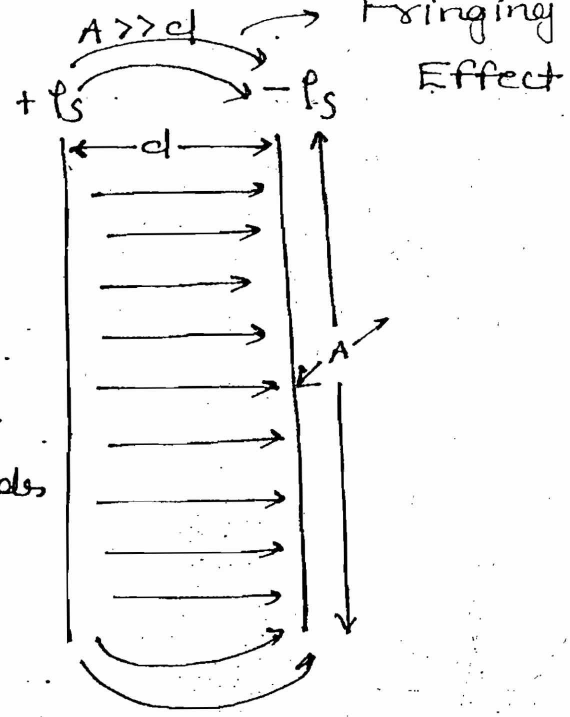
→ It is the ratio of the flux developed to the current utilized by the flux

Specific Geometries :-

→ Solenoids
→ cocentric cylinder
→ Torroids.

Parallel Plate Capacitors :-

Parallel Plate Capacitors :-



$$E = \frac{\rho_s}{\epsilon}$$

$$C = \frac{Q}{V} = \frac{\rho_s A}{\frac{\rho_s}{\epsilon} d} = \frac{\epsilon A}{d}$$

Capacitance is independent of Q or V and always depends on Area and distance or physical dimensions

$$W_E = \frac{1}{2}\epsilon E^2 (Ad)$$

$$= \frac{1}{2}\frac{\epsilon A}{d}(Ed)^2$$

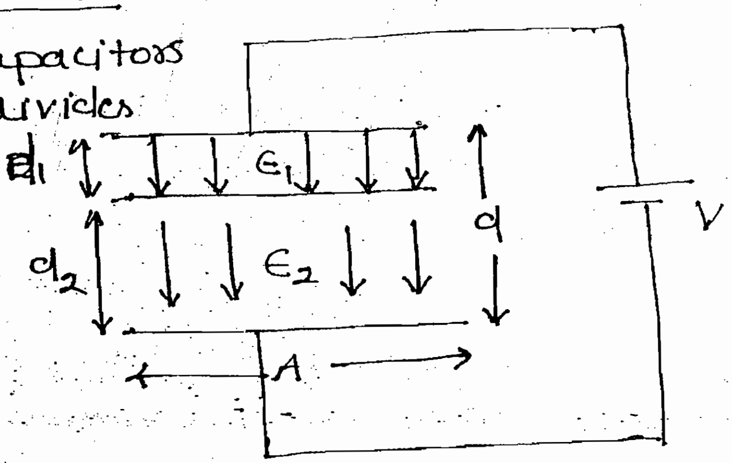
$$= \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV$$

Multiple Dielectrics in Capacitors :-

Case-(1) :-

Equal areas of cross-section unequal width of the dielectrics :-

They are two capacitors in series as voltage divides



$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

$$= \frac{\frac{\epsilon_1 A}{d_1}\cdot\frac{\epsilon_2 A}{d_2}}{\frac{\epsilon_1 A}{d_1} + \frac{\epsilon_2 A}{d_2}}$$

$$D_1 = D_2 \Rightarrow \epsilon_1 E_1 = \epsilon_2 E_2 \Rightarrow \frac{\epsilon_1 V_1}{d_1} = \frac{\epsilon_2 V_2}{d_2}$$

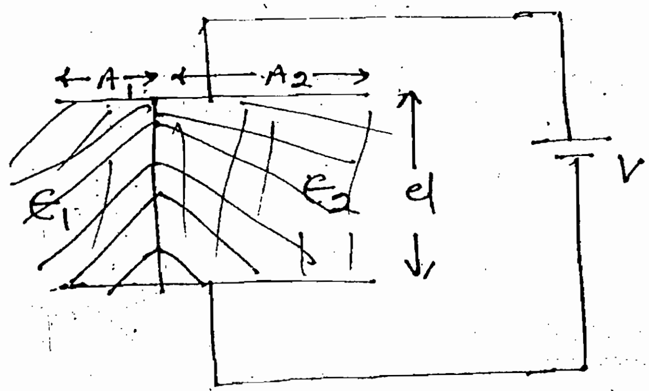
$$\Rightarrow \boxed{\frac{V_1}{V_2} = \left(\frac{\epsilon_2}{\epsilon_1}\right)\left(\frac{d_1}{d_2}\right)}$$

Case-(II.)

Unequal areas of cross-section and equal width of the dielectrics :-

$$C_{eq} = C_1 + C_2$$

$$= \frac{\epsilon_1 A_1 + \epsilon_2 A_2}{d}$$



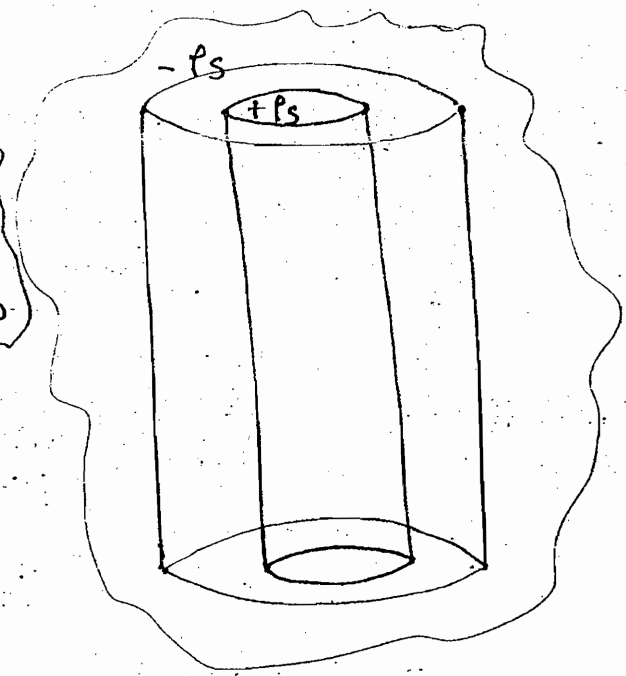
Cocentric Cylinder Capacitance :-

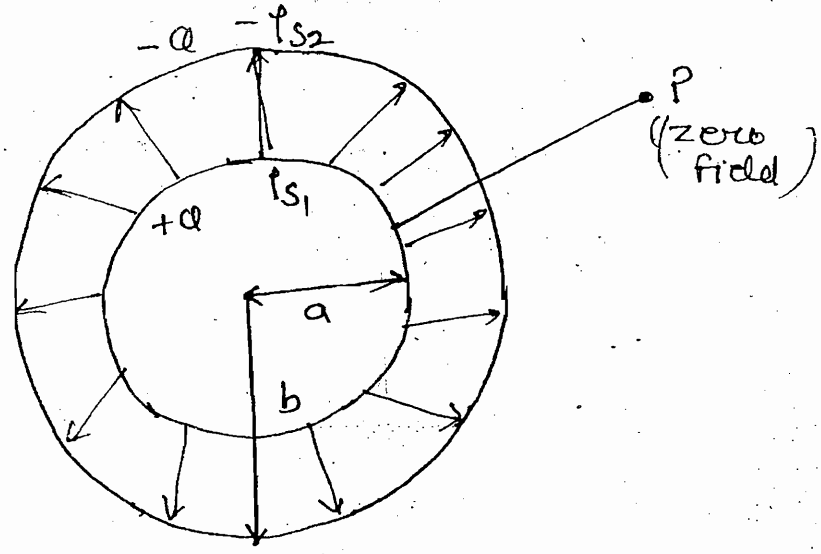
Two cocentric cylinders with equal and opposite charge density may not have confined flux but two equal and opposite charges on concentric cylinders has flux confined only b/w the cylinders

$$Q_{net} = Q_{out} + Q_{in}$$

$$= -\rho_s 2\pi bh + \rho_s 2\pi ah$$

$$\neq 0$$





$$D \propto \rho_S \propto \frac{1}{\rho}$$

→ The field of a sheet of cylindrical charge obeys same geometry as a line charge

$$C = \frac{Q}{V} = \frac{\rho_L h}{\frac{\rho_L}{2\pi\epsilon} \ln\left(\frac{b}{a}\right)}$$

$$\Rightarrow \boxed{C = \frac{2\pi\epsilon h}{\ln(b/a)}}$$

Extension:-

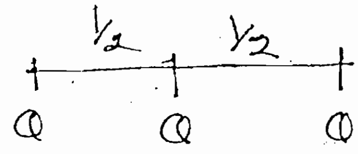
Cocentric sphere capacitance

$$C = \frac{Q}{V}$$

$$\boxed{C = \frac{Q}{\frac{Q}{4\pi\epsilon}\left(\frac{1}{a} - \frac{1}{b}\right)} = \frac{4\pi\epsilon}{\left(\frac{1}{a} - \frac{1}{b}\right)}}$$

30

$$W_{E1} = 0 + Q \cdot \frac{Q}{4\pi\epsilon 1/2} + Q \cdot \frac{Q}{4\pi\epsilon \cdot 1/2} + Q \cdot \frac{Q}{4\pi\epsilon \cdot 1} = \frac{5Q^2}{4\pi\epsilon}$$



$$\boxed{W_{E_2} = \frac{5Q^2}{8\pi\epsilon} = \frac{W_{E1}}{2}}$$

Note:—

$$W_E \propto QV \propto \frac{1}{r}$$

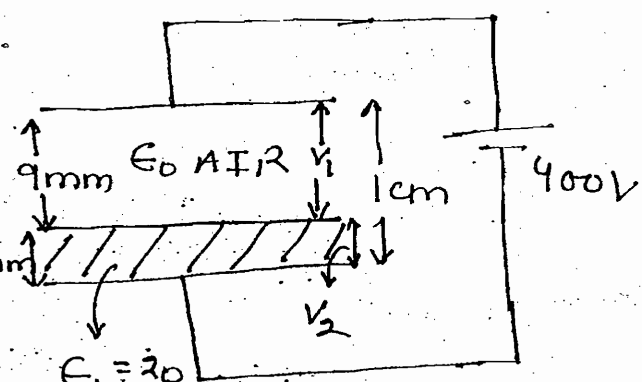
31 $$\frac{1}{2}(J \cdot A) = \frac{\text{Amp}}{m^2} \times \frac{\text{Joules}}{\text{Amp-m}} = \frac{\text{Joules}}{m^3}$$

OR

$$\Rightarrow \frac{1}{2}(J \cdot A) = \frac{1}{2}((\nabla \times H) \cdot A) = \frac{1}{2}(H(\nabla \times A))$$

$$= \frac{1}{2} B \cdot H$$

32. $$E_{AIR} = \frac{V_{AIR}}{9\,mm}$$



$$V_1 + V_2 = 400 \text{ —(I)}$$

$$\frac{V_1}{V_2} = \left(\frac{\epsilon_2}{\epsilon_1}\right)\left(\frac{d_1}{d_2}\right) \text{—(II)}$$

$$\frac{V_1}{V_2} = \frac{20\epsilon_0}{\epsilon_0} \times \frac{9\,mm}{1\,mm} = 180$$

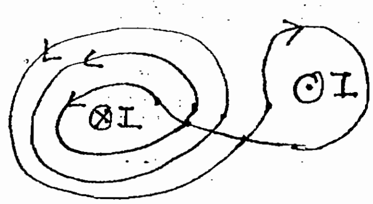
$$V_1 = 180 V_2$$

Ans → $V_1 = 44\ kV/m$

34. Circulation $= \oint H \cdot dl = I$

$= I + I + I - (-I$

$= 4I$, Ans.



Time Varying Fields and Maxwell's Equation

Static Fields - Maxwell's Equations :-

Integral Form	Point form
(i) $\oint D \cdot ds = Q$	$\nabla \cdot D = \rho_V$
(ii) $\oint E \cdot dl = 0$	$\nabla \times E = 0$
(iii) $\oint B \cdot ds = 0$	$\nabla \cdot B = 0$
(iv) $\oint H \cdot dl = I$	$\nabla \times H = J$

Note :-

- The surface ~~and~~ Integral and Divergence expressions are consistently the same in static / time varying fields
- The line integral and curl expression are modified in time varying fields to explain AC voltages and currents

Open Integral's (Not Maxwell Equations) :-

1. $\int D \cdot ds = \psi_e = $ coulombs
2. $\int E \cdot dl = V = $ EMF $=$ volts
3. $\int B \cdot ds = \psi_m = $ Webers
4. $\int H \cdot dl = V_m = $ MMF $=$ Amps

time

Maxwell 2nd Equation and Faraday's Law :-

Faraday's Law Statement :-

EMF or Voltage is induced even in a closed conductor when the magnetic flux crossing the surface changes with time.

i.e. Rate of change of magnetic flux is equal to induced EMF

$$\oint E \cdot dl = V = -\frac{d\psi_m}{dt} = -\frac{d}{dt}\int B \cdot ds$$

due to Lenz's Law

$$= \int -\frac{\partial B}{\partial t} \cdot ds$$

Lenz's Law :—

The induced EMF ~~allows~~ always opposes the basic changing flux (Cause)

$$\oint E \cdot dl = \int -\frac{\partial B}{\partial t} \cdot ds$$

$$\oint E \cdot dl = V = -\frac{d\psi_m}{dt} = -\frac{d}{dt}\int B \cdot ds = \int -\frac{\partial B}{\partial t} \cdot ds$$

Apply stoke's theorem $= \int \nabla \times E \cdot ds$

$$\boxed{\nabla \times E = -\frac{\partial B}{\partial t}}$$

Modified Maxwell's Second Equation

Note:—

Potential is unique at a time at a point in a space but changes with time and Hence the modification

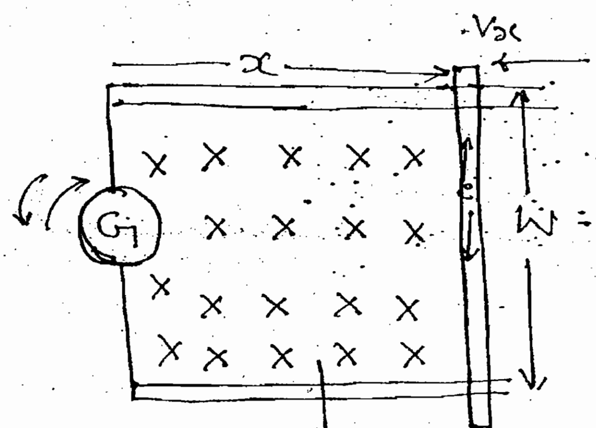
Sliding Rail Experiment :—

$$V = -\frac{d\psi_m}{dt}$$

$$\Rightarrow V = -\frac{d}{dt}(B \cdot A)$$

$$\Rightarrow \boxed{V = -B \cdot W \frac{dx}{dt}}$$

$$\Rightarrow \boxed{V = -B \cdot W \cdot V_x}$$



Static Uniform B field

Note:-

→ When a conducting rod is moving in a magnetic field a force is exist on the electron obeying Lorentz Law. This displaces the electrons towards one side. This accumulation is called as induced voltage

→ As the rod moves to and fro the voltage polarity changes. This is called as AC voltage.

→ The already displaced $\bar{e}$ opposes another further coming $\bar{e}$'s. This is called as Lenz's law

$$F_y = \cancel{q}(V_x \times B_z) = \cancel{q}E$$

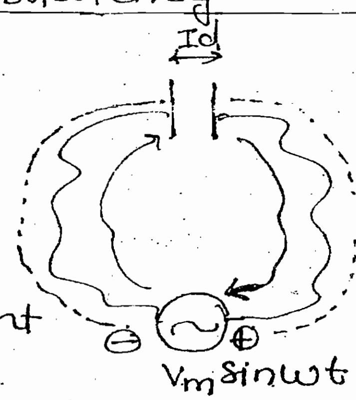
$$\Rightarrow \frac{dx}{dt} B \cdot \sin\theta = \frac{V}{W} \quad \cancel{q}C$$

$$\Rightarrow V = \frac{d}{dt}(xBW) = \frac{d}{dt}(BA) \quad (\theta = 90^\circ)$$

$$\Rightarrow V = \frac{d\psi_m}{dt}$$

Maxwell's IV Equation and Inconsistency of Ampere's Law:-

→ When a capacitor is connected with AC harmonic voltage there is a current flowing in the wire and plates obeying ohm's law like any linear element



$$I = C\frac{dV}{dt}$$

$$I = C \cdot W \cdot \underbrace{V_m \sin(\omega t}_{} \underbrace{+ 90^\circ}_{\text{Phase}})$$

$e^{j90} = j$ → Phase

$$I = j\omega C\, V_m \sin \omega t$$

$$V = \left(\frac{1}{j\omega C}\right) I$$

→ There is no current flowing b/w plates as their is dielectric Hence the circuit is not closed But Ampere's law state that current flows only closed circuits. Hence Maxwell's modified Ampere's law as

$$\oint H \cdot dl = I_c + I_d$$

$$\nabla \times H = J_c + J_d$$

Using equation of continuity

$$\nabla \cdot J_d = \frac{\partial \rho_v}{\partial t} = \frac{\partial (\nabla \cdot D)}{\partial t} = \nabla \cdot \frac{\partial D}{\partial t} \Rightarrow$$

$$\Rightarrow J_d = \frac{\partial D}{\partial t} \quad \& \quad I_d = \int \frac{\partial D}{\partial t} \cdot ds$$

Maxwell IV Equation is

$$\oint H \cdot dl = I_c + \int \frac{\partial D}{\partial t} \cdot ds$$

$$\Rightarrow \nabla \times H = J_c + \frac{\partial D}{\partial t}$$

where I_c = Conduction current
OR
= Moving $\bar{e}$'s

→ When an AC voltage applied to the capacitor plate the plates are alternatively charge and discharge. This continues takes place as the polarity changes. Hence this self establish continuity

→ As ρ_s on the plates changes with time, D b/w the plates changes with time. Hence $\frac{\partial D}{\partial t}$ is

$$\frac{C/m^2}{\text{second}} = Amp/m^2 = J_d$$

This is also a format of current

This is called as Displacement current density.

Summary 1:—

$$\oint E \cdot dl = \int -\frac{\partial B}{\partial t} \cdot ds$$

$$\oint H \cdot dl = \int \frac{\partial D}{\partial t} \cdot ds + J_c$$

→ A time varying magnetic flux is a cause of Voltage

→ A time varying electric flux is a format of current

$$\boxed{\frac{\text{Weber}}{\text{second}} = \text{Volts}} \qquad \boxed{\frac{\text{Coulomb}}{\text{sec}} = \text{Amp}}$$

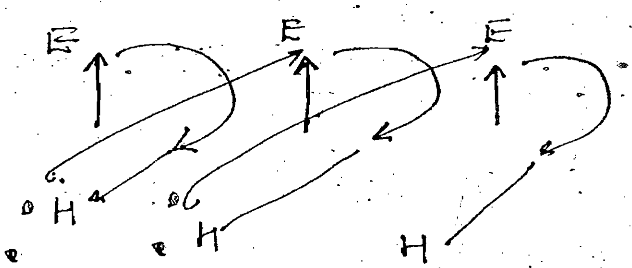
$$\begin{tikzcd}
\text{Cause: Voltage} \arrow[r] & \text{Effect: Electric field} \arrow[d, "\text{time}"] \\
\text{H} \cdot \text{Field (Effect)} \arrow[u, "\text{time}"] & \text{current (Cause)} \arrow[l]
\end{tikzcd}$$

Summary 2:-

Space varying Electric field ← $\nabla \times E = -\frac{\partial B}{\partial t}$ → time varying B field

$$\nabla \times H = \frac{\partial D}{\partial t} + J_c$$

→ A time varying E field produces a space varying orthogonal H field and vice-versa



$$\text{H field} \underset{\text{space}}{\overset{\text{time}}{\rightleftarrows}} \text{E} \cdot \text{Field}$$

→ Accumulation leads to flow and flow leads to accumulation sustaining each other.

Summary 3:-

$$\nabla \times E = -\mu \frac{\partial H}{\partial t}$$

$$\nabla \times H = \epsilon \frac{\partial E}{\partial t} + \sigma E$$

→ μ = Henry/m , ϵ = Farad/m , σ = mho/m →

For E to transform to H and vice-versa the material and its permitting abilities are also important.

i.e. Material consists decides the E/H dynamics in the medium

→ $\vec{E}$ – Volts/m , $\vec{H}$ – amps/m , ∇ – perm

Note:–

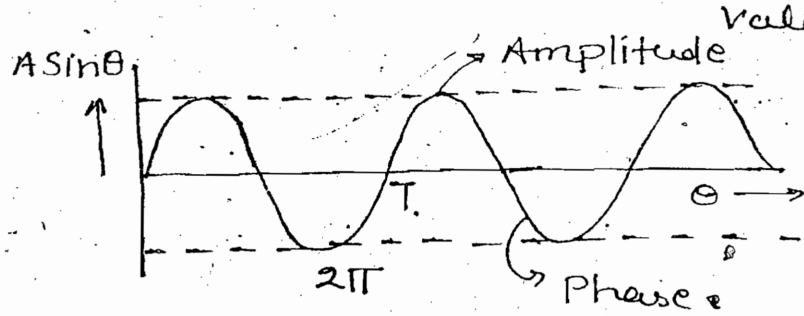
→ For sustain oscillation or ever existing phenomenon are produc when the time derivative of E and H is always back the same function and such a function called as Harmonic function.

Harmonic Functions:–

They are 2 dimension quantities and having 3 formats

→ $A\sin\theta$
→ $A\cos\theta$
→ $Ae^{j\theta}$

Amplitude → Domain 1 → Peak value
Phase → Domain 2 → Instanteous value



Property 1:–

The phase should be a linear function of the variable

(i) $\theta \propto t$ t→time Harmonic

$\Rightarrow \theta = \omega t$ ω = Phase shift constant per unit time

$= \frac{2\pi}{T}$ rad/second

(ii) $\theta \propto z$ → Space Harmonic

$\Rightarrow \theta = \beta z$ β = Phase shift constant per unit length $= \frac{2\pi}{\lambda}$ rad/m

Property 2 :—

The derivative of every harmonic has to be back the same function shifted orthogonally by 90°

$$A\sin\omega t \xrightarrow{\text{I-derivative}} \omega A\sin(\omega t+90^\circ) \xrightarrow[\text{der.}]{\text{II}} \omega^2 A\sin(\omega t+180^\circ)$$

$$\xrightarrow{\text{III derivative}} \omega^3 A\sin(\omega t+270)$$

$$Ae^{j\omega t} \xrightarrow[\text{der}]{\text{I}} \omega A e^{j(\omega t+90)} \xrightarrow[\text{der.}]{\text{II}} \omega^2 A e^{j(\omega t+180)}$$

$$\xrightarrow{\text{III-der.}} \omega^3 A e^{j(\omega t+270)}$$

$(\because\ j^3 = -j = e^{j270})$

$(-1 = e^{j180})$

$(\because\ j = e^{j90})$

→ All harmonics obey the basic property the second order derivative is back the same function

→ They satisfied the differential equation

$$D^2 - M^2 = 0$$

$$D^2 + M^2 = 0$$

eg:- E/H field equation in free space

Electro magnetic wave equation in free space

$$\nabla \times E = -\mu \frac{\partial H}{\partial t}$$

$$\nabla \times H = \epsilon \frac{\partial E}{\partial t}$$

Taking ∇ on both sides

$$\nabla \times (\nabla \times H) = \epsilon \frac{\partial}{\partial t}(\nabla \times E)$$

$$-\nabla^2 H = \epsilon \frac{\partial}{\partial t}\left(-\mu \frac{\partial H}{\partial t}\right)$$

Space Harmonic

Time Harmonic

$$\boxed{\begin{aligned} \nabla^2 H &= \epsilon\mu \frac{\partial^2 H}{\partial t^2} \\ \nabla^2 E &= \epsilon\mu \frac{\partial^2 E}{\partial t^2} \end{aligned}}$$

These are called as E/H wave equations in free space

eg-(2) :- V/I Equations in LC circuits

$$I = C\frac{dV}{dt}$$

$$V = -L\frac{dI}{dt} = -L\frac{d}{dt}\left(C\frac{dV}{dt}\right)$$

$$\Rightarrow \quad V = -LC\frac{d^2V}{dt^2}$$

$$\Rightarrow \quad \boxed{\frac{d^2V}{dt^2} = -\frac{1}{LC}V}$$

and $\boxed{\dfrac{d^2I}{dt^2} = -\dfrac{1}{LC}I}$

By comparison

$$\boxed{\omega = \frac{1}{\sqrt{LC}}}$$

Types of Exponential Functions :-

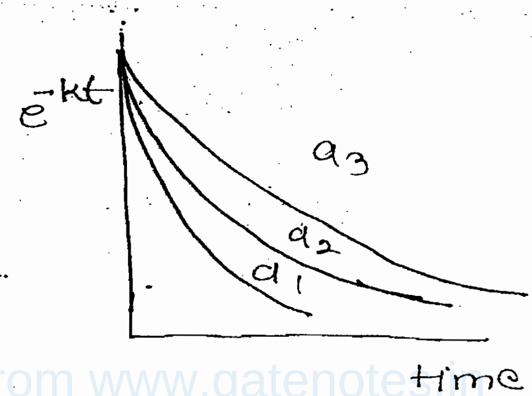
$\rightarrow$ e^{-Kt} vs t

(1) Case - (1) :-

$K = a =$ +ve real no.

$a_1 > a_2 > a_3$

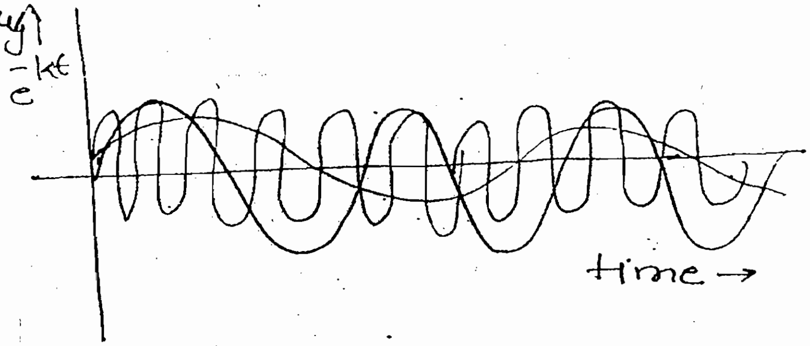
$\rightarrow$ Real Exponential.



II) Case – (II) :–

$k = j\omega$ = purely imaginary no

ω = Phase Shift Constant

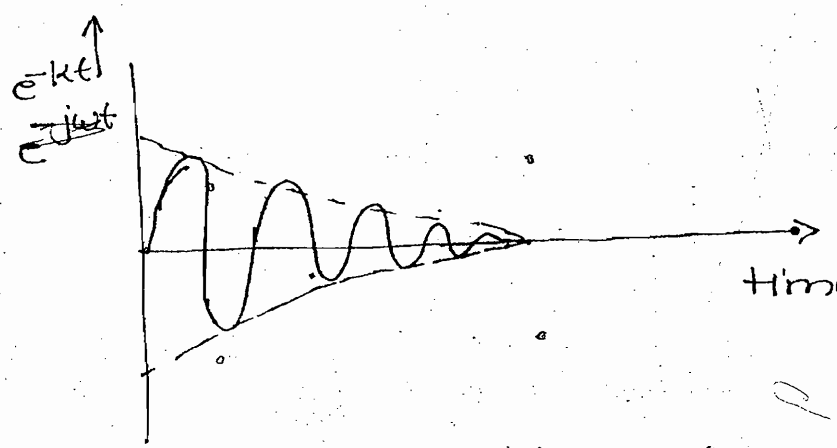


III) Case – (III) :–

$k = a + j\omega$

$(e^{-at})\ (e^{-j\omega t})$

$A \cdot e^{j\theta}$



Note :–

→ j stands for an orthogonal shift from domain 1 to a second independent domain 2

eg :– (I) east and north displacements

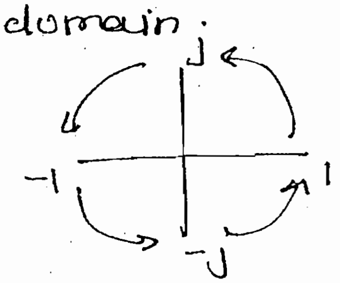
(II) Resistance and reactance of a impedance

(III) Amplitude and phase of a harmonic

→ A scaling of j in domain 1 is a shift of 90° in domain 2

eg :– (I) $j\omega t\, e^{-j\omega t} \longrightarrow \omega e^{j(\omega t + 90^\circ)}$

(II) S-domain :



$1 \times j = j = 1 \angle 90^\circ$

$j \times j = -1 = 1 \angle 180^\circ$

$-1 \times j = -j = 1 \angle 270^\circ$

$-j \times j = 1 \angle 360^\circ$

12

EM Propogation in General Materials :- (σ, ϵ, μ)

Using Maxwell's Equation

$$\nabla \times H = \sigma E + \epsilon \frac{\partial E}{\partial t}$$

$$\nabla \times E = -\mu \frac{\partial H}{\partial t}$$

→ Every EM wave needs a time harmonic source at one end

$$\left.\begin{array}{l} E_s = E_0 e^{j\omega t} \\ H_s = H_0 e^{j\omega t} \end{array}\right\} \omega = \text{frequency in rad/s}$$

Range of ω :-

- f/ω → Hz/kHz → AF Wave
- → MHz → Radio frequency Wave
- → GHz/10^{12} Hz → Microwave
- → $10^{15} - 10^{18}$ Hz → Light wave
- → $10^{20} - 10^{22}$ Hz → X-rays / Gamma rays
- → 10^{25} Hz → Cosmic Rays

→ EM Wave Spectrum

$$\nabla \times H = \sigma E + \epsilon \frac{\partial E}{\partial t} = \sigma \underline{E_0 e^{j\omega t}} + \epsilon j\omega \underline{E_0 e^{j\omega t}}$$

$$\cancel{\nabla \times E = -\mu \frac{\partial H}{\partial t}} \qquad = (\sigma + j\omega\epsilon) E$$

$$\Rightarrow \nabla \times H = (\sigma + j\omega\epsilon) E \quad \text{—(1)}$$

$$\nabla \times E = -j\omega\mu H \longrightarrow \text{(II)}$$

$$\nabla \times E = \mu \frac{\partial H}{\partial t}$$

$$= -\mu j \omega H_0 e^{j\omega t} = -j\omega\mu H$$

$$\nabla \times E = -j\omega\mu H \quad \text{--- (II)}$$

Eq-(I) & (II) are called as Maxwell Equations for time Harmonic source

Take H from (II) & put in (I)

$$H = \frac{\nabla \times E}{-j\omega H}$$

$$\Rightarrow \quad \nabla \times \left(\frac{\nabla \times E}{-j\omega\mu} \right) = (\sigma + j\omega\epsilon) E$$

$$\Rightarrow \quad \nabla \underbrace{(\nabla \cdot E)}_{0} - \nabla^2 E = -j\omega\mu(\sigma + j\omega\epsilon) E$$

For a charge free region

$$\nabla^2 E = j\omega\mu(\sigma + j\omega\epsilon) E \quad \text{--- (III)}$$

Similarly

$$\underbrace{\nabla^2}_{\text{Space}} H = j\omega\mu(\sigma + j\omega\epsilon) \underbrace{H}_{\text{time}} \quad \text{--- (IV)}$$

These are called as Helmholtz's Propogation Equations.

Note :-

→ If the source is the time harmonic then the effect is space harmonic propogation in the medium.

Lecture - 9

$$\nabla^2 E = j\omega\mu(\sigma + j\omega\epsilon)E \quad \text{— (III)}$$

$$\nabla^2 H = j\omega\mu(\sigma + j\omega\epsilon)H \quad \text{— (IV)}$$

Consider ∇ in one ~~direction~~ dimension of z only

E is in x direction, H is in y-direction

Consider $\sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = \gamma$

$$\frac{\partial^2 E_x}{\partial z^2} = \gamma^2 E_x \quad \text{— (III)}$$

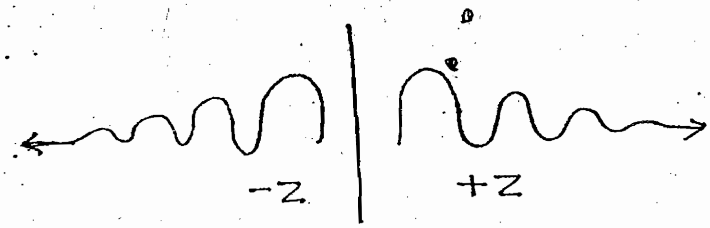
$$\frac{\partial^2 H_y}{\partial z^2} = \gamma^2 H_y \quad \text{— (IV)}$$

The E(z) and H(z) solution is.

$$E(z)_x = (c_1 e^{-\gamma z} + c_2 e^{\gamma z})\, a_z \quad \text{— (V)}$$

$$H(z)_y = (\underbrace{c_3 e^{-\gamma z}}_{+z} + \underbrace{c_4 e^{\gamma z}}_{-z})\, a_y \quad \text{— (VI)}$$

The solution physically represent two waves ravelling on either sides from a planar source and both decaying. This is called as uniform plane wave



Considering only the +z solution and c_1 & c_3 being the initial source values. The final E/H wave solution is

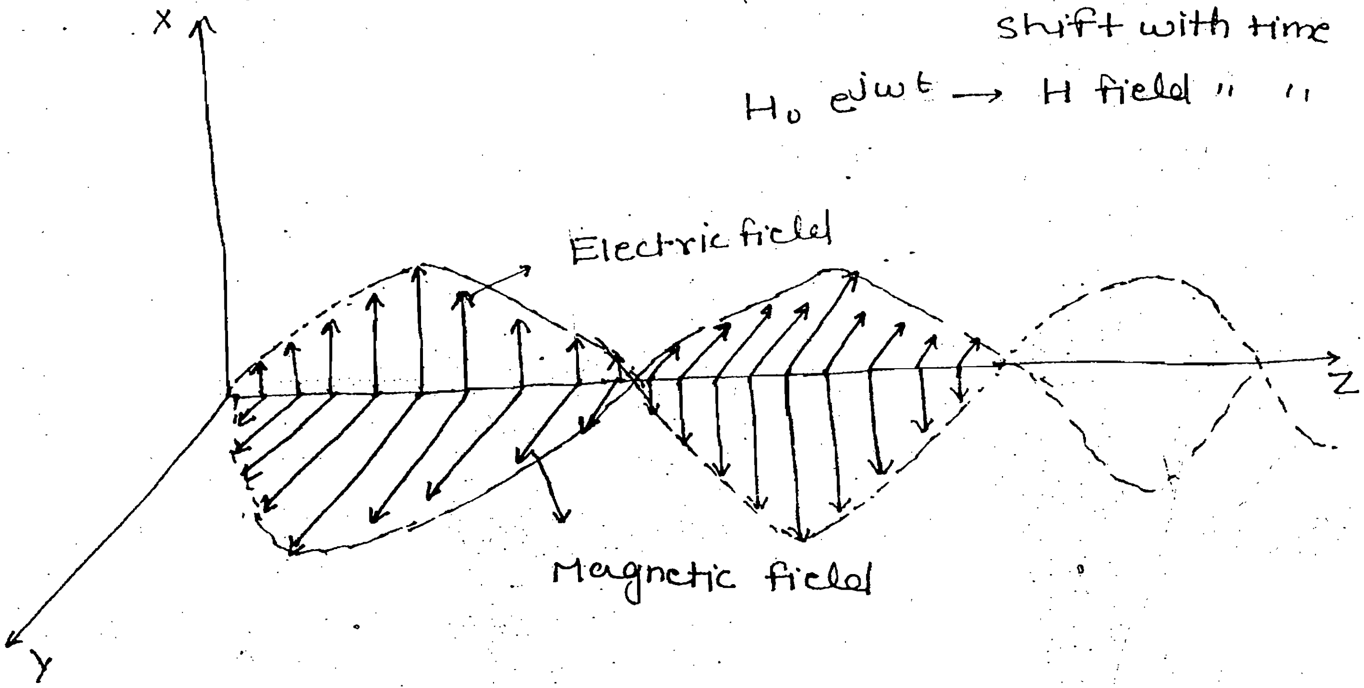
$$E(z,t)_x = (E_0 e^{j\omega t} \cdot e^{-\gamma z})\, a_x$$

$$H(z,t)_y = (H_0 e^{j\omega t} \cdot e^{-\gamma z})\, a_y$$

The solution is a product solution of time and space harmonics.

$E_0 e^{j\omega t} \rightarrow$ electric field shift with time

$H_0 e^{j\omega t} \rightarrow$ H field " "



→ Electric and H field perpendicular and shifts simultaneously with time

→ Cross product result in transverse and dot product result in longitudinal.

Propogation constant (γ) :-

γ is the constant of the exponential with z that decides the course of propagation i.e. γ is called as propagation constant.

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = \alpha + j\beta$$

$$E(z,t) = \left(E_0 e^{-\alpha z} \cdot e^{j\omega t} e^{-j\beta z}\right) a_x$$

$$\boxed{\begin{aligned} E(z,t)_x &= (E_0 e^{-\alpha z})\, e^{j(\omega t - \beta z)}\, a_x \\ H(z,t)_y &= (H_0 e^{-\alpha z})\, e^{j(\omega t - \beta z)}\, a_y \end{aligned}}$$

field direction

Amplitude

Phase

Note-1:-

Every EM wave is harmonic whose amplitude exponentially decays at α rate

α = attenuation constant

Note-2:-

Every EM wave is a harmonic whose phase independently but linearly changes with time and space.

Note-3:-

The harmonic E & H have direction and obeying the basic transverse nature with propogation

$$\boxed{\underset{(\text{Dir})}{E} \times \underset{(\text{Dir})}{H} = \underset{(\text{Dir.})}{\text{Propogation constant}}}$$

Intrinsic Wave Impedance (η):-

$$E \rightleftharpoons H$$

$$\eta = \frac{E}{H} = \frac{\text{Volts/m}}{\text{Amp/m}} = \text{ohm}$$

When E is transformed to H & vice-versa the rate of transformation is called as intrinsic impedance of the medium

$$\nabla \times E = -j\omega\mu H$$

$$\frac{\partial}{\partial z} a_z \times E a_x = -j\omega\mu H a_y$$

$$\frac{\partial E}{\partial z} = -j\omega\mu H$$

As $E = E_0 e^{-\gamma z}$

$$\Rightarrow \quad -\gamma E = -j\omega\mu H$$

$$\Rightarrow \quad \frac{E}{H} = \frac{j\omega\mu}{\sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}}$$

$$\Rightarrow \boxed{\frac{E}{H} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \text{complex} = R + jX$$

The transformation has a resistance which is a loss (α) in transformation and a reactance which leads to phase shift (β)

$$\gamma = \alpha + j\beta$$

$$\eta = R + jX$$

Workbook-2

1) $V = -\dfrac{d\psi_m}{dt} = -\dfrac{d}{dt}\left(\dfrac{\lambda}{3}t^3\right) = -\dfrac{\lambda}{3}\cdot 3t^2 \Big|_{t=3s} = 9V$

$$\Rightarrow \boxed{\lambda = -1}$$

(2) $V = -\dfrac{d\psi_m}{dt} = -\dfrac{d}{dt}(BA) = -\pi r^2 \dfrac{dB}{dt}$

$= -\pi \times (0.1)^2\, 10\, [-\sin(120\pi t)\cdot 120\pi)$

$= 118 \sin(120\pi t)$ Ans

3) B $\rightarrow$ Linearly decrease with time $B(t) = -kt$

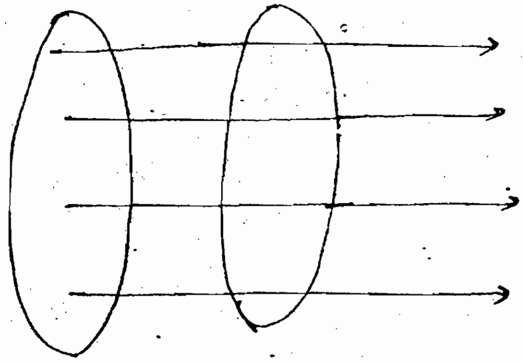
$\rightarrow$ Uniform decrease with space

$\therefore V = -\dfrac{d\psi_m}{dt} = -\dfrac{d}{dt}(BA) = -A(-k) = \text{Constant}$

$= \text{DC voltage}$

→ Upto open circuit E flows and emf is induced

→ Induced emf converts into current in coil 1

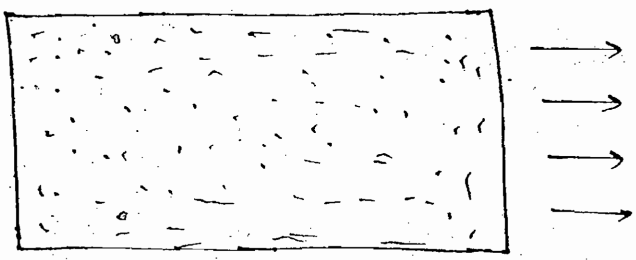


Ans- D

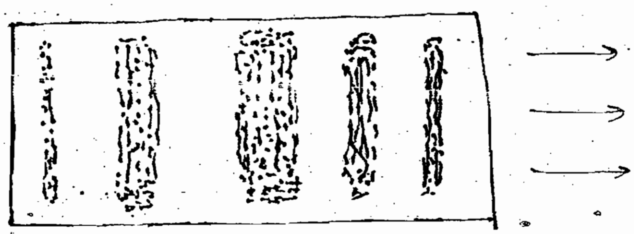
$V = 0$ $\quad$ $V \neq 0$

$I \neq 0$ $\quad$ $I = 0$

Note:-



DC current

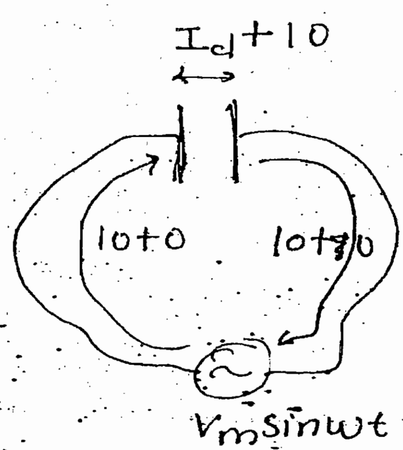


AC current

4.

$I_c = I_d$

$\oint H \cdot dl = I_c + I_d \neq 2I_d$ or $2I_c$

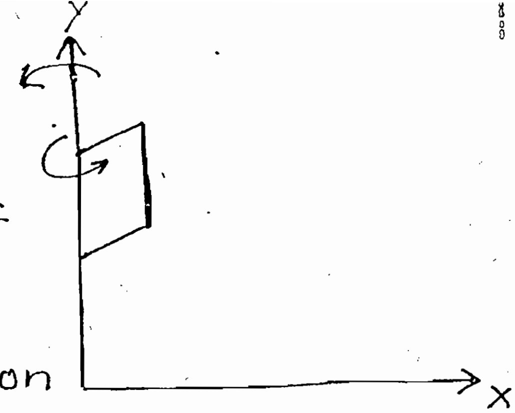


$J_d = \epsilon \dfrac{\partial E}{\partial t}$

$I_d = \epsilon A \cdot j\omega E = j\omega \dfrac{\epsilon A}{d} V$

Ans-(a)

5. $B(t)x$



Uniform and Harmonic in time

$\downarrow$

Transformer Action

$\downarrow$

AC Voltages and harmonic B(t) being involved

$$V = -\frac{d\psi_m}{dt} = -\frac{d}{dt}(BA) = \underbrace{-B\frac{dA}{dt}}_{\text{Rotation}} - A\frac{dB}{dt}$$

$B(t) = \cos(\omega t) \longrightarrow$ AC voltage

<u>Ans - (c)</u>

6. Radiation $\rightarrow$ EM waves $\rightarrow$ generate due to cross product

7. $\gamma = \alpha + j\beta$

Amplitude $= |E|$

$|E| = E_0 e^{-\alpha z} = \frac{1}{e} E_0$ when $z = 20m$

$\Rightarrow \alpha z = 1 \Rightarrow \alpha = \frac{1}{20}$

$\theta \propto z \Rightarrow \theta = \beta z$

$\Rightarrow \beta = \frac{\pi/6}{20m} = \frac{\pi}{120}$

$$\boxed{\gamma = \frac{1}{20} + j\frac{\pi}{120}}$$

$|E| \longrightarrow 20\%$ of initial value

$$|E| \longrightarrow 20 \longrightarrow \overset{E_0}{100} \longrightarrow z = 5m$$

$$|E| \longrightarrow 40 \longrightarrow 100 \longrightarrow z = ?$$

It is not a linear function

$$|E| = E_0 e^{-\alpha z}$$

$$\Rightarrow 20 = 100 e^{-\alpha 5} \qquad \Rightarrow e^{5\alpha} = 5$$

$$\Rightarrow 5\alpha = \ln 5$$

$$\Rightarrow \alpha = \frac{\ln 5}{5}$$

$$40 = 100 e^{-\alpha z}$$

$$\Rightarrow e^{\alpha z} = \frac{100}{40} = 2.5$$

$$\Rightarrow \alpha z = \ln 2.5$$

$$\Rightarrow z = \frac{5 \ln 2.5}{\ln 5} = 2.8, \text{ Ans.}$$

Note:-

Change is depending on function value

Case-(1) :-

$$|E| = E_0 e^{-\alpha z}$$

EM Wave Propogation in free space

$\frac{2}{\alpha}$

$\frac{3}{\alpha}$

$$E_0 = 100$$

$$\downarrow z = \frac{1}{\alpha}$$

$$\frac{100}{e} = 37$$

$$\downarrow z = \frac{1}{\alpha}$$

$$\frac{100}{e^2} = 13$$

$$\downarrow z = \frac{1}{\alpha}$$

$$\frac{100}{e^3} = 4$$

Case - (i) :—

EM Wave Propogation in free space $\begin{bmatrix} \sigma = 0, \epsilon = \epsilon_0 \\ \mu = \mu_0 \end{bmatrix}$

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = j\omega\sqrt{\mu_0\epsilon_0}$$ = purely imaginary

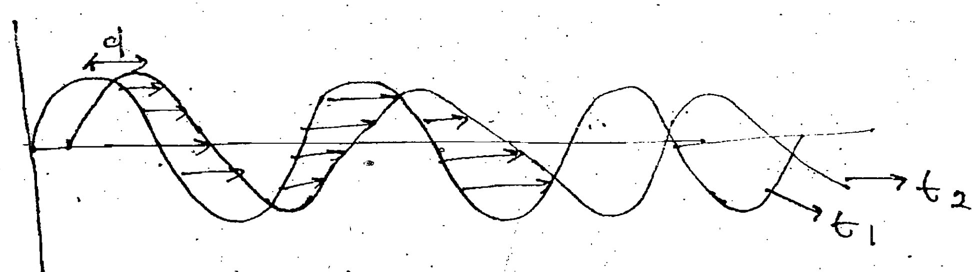
$\Rightarrow$ $\alpha = 0$ No attenuation of EM wave in free space

$\beta = \omega\sqrt{\mu_0\epsilon_0}$ = only phase shift and propogation constant exists

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \sqrt{\frac{\mu_0}{\epsilon_0}} = \text{real}$$

$$= \sqrt{4\pi \times 10^{-7} \times 36\pi \times 10^{9}} = 120\pi = 377$$

Phase Velocity (V_p) :—



$t_2 - t_1$

The distance travelled by any in-phase point per unit time is called as phase velocity.

$$V_p = \frac{d}{t}$$

$$= \frac{\lambda}{T} = \lambda f = \frac{2\pi\lambda}{2\pi T}$$

$$\Rightarrow V_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu_0\epsilon_0}} = \frac{1}{\sqrt{\mu_0\epsilon_0}}$$

$$\Rightarrow V_p = 3 \times 10^8 \text{ m/s}$$

case-(II) :-

EM Wave propogation in Ideal dielectrics

→ Ideal dielectrics = Perfect / lossless dielectrics

→ $\sigma = 0$, $\epsilon = \epsilon_0 \epsilon_R$, $\mu = \mu_0$

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = j\omega\sqrt{\mu_0\epsilon_0\epsilon_R}$$

= purely imaginary

=> $\alpha = 0$ i.e. No attenuation of EM wave in ideal dielectrics

$\beta = \omega\sqrt{\mu_0\epsilon_0\epsilon_r}$ = only phase shift and property exist

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \sqrt{\frac{\mu_0}{\epsilon_0\epsilon_R}} = \text{real}$$

$$= \sqrt{\frac{4\pi \times 10^{-7} \times 36\pi \times 10^9}{\epsilon_R}} = \frac{120\pi}{\sqrt{\epsilon_R}} = \frac{377}{\sqrt{\epsilon_R}}$$

Phase Velocity (V_p) :-

$$V_p = \frac{\lambda}{T} = \lambda f = \frac{2\pi\lambda}{2\pi T}$$

$$\Rightarrow V_p = \frac{\omega}{\beta_0} = \frac{\omega}{\omega\sqrt{\mu_0\epsilon_0}} = \frac{1}{\sqrt{\mu_0\epsilon_0\epsilon_r}}$$

$$\Rightarrow \boxed{V_p = \frac{3\times10^8}{\sqrt{\epsilon_r}}} \text{ m/s}$$

Q. $E(x,t) = 25\sin(\omega t + 4x)\, a_y$

By comparison, propogation direction → $-a_x$

$\beta = 4 = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{\pi}{2}$

$f = \frac{3\times10^8}{\frac{\pi}{2}} = \frac{600}{\pi}$ MHz.

Method-1 :-

$$\nabla \times E = -j\omega\mu H = -\mu \frac{\partial H}{\partial t}$$

$$H = -\frac{1}{\mu} \int (\nabla \times E)\, dt$$

$$\nabla \times E = \begin{vmatrix} a_x & a_y & a_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & E_y & 0 \end{vmatrix}$$

Method 2 :-

$$E(x,t) = 25 \sin(\omega t + 4x)\, a_y$$

Step-1 :- $\dfrac{|E|}{|H|} = |\eta|$

Step-2 :- when η is real E & H have the same harmonic and phase

Step-3 :- E dir $\times$ H dir = Prop. dir. (Basic Transverse Nature)

$$H(x,t) = \left(\frac{25}{120\pi}\right) \sin(\omega t + 4x)\,(-a_x \times a_y)$$

$$= \left(\frac{-25}{120\pi}\right) \sin(\omega t + 4x)\, a_z$$

10. $E(y,t) = 25 \sin(10^8 t - y)\, a_z$

const

free space

lossless dielectrics

$\omega = 10^8$

$\beta = 1$

$V_p = \dfrac{\omega}{\beta} = 10^8 \neq 3\times10^8$

or $V_p = \dfrac{3\times10^8}{\sqrt{\epsilon_R}} = 10^8 \Rightarrow \epsilon_R = 9$

Propogation direction $\longrightarrow a_y$

$$f = \frac{10^8}{2\pi}$$

$$\lambda = \frac{\cancel{1}}{\cancel{2\pi}} 2\pi$$

$$H(y,t) = \left(\frac{25}{\frac{120\pi}{\sqrt{9}}}\right) \sin(10^8 t - y)(a_y \times a_z)$$

$$= \left(\frac{5}{8\pi}\right) \sin(10^8 t - y)\, a_x$$

1. $H = (0.5\, e^{-0.1x}) \cos(10^6 t - 2x)\, a_z$

$\omega = 10^6$ rad/second

$\beta = 2 = \frac{2\pi}{\lambda} \Rightarrow \lambda = 3.14$

Propogation $\longrightarrow +\hat{a}_x$

$\rightarrow$ ~~Propogation~~ Polarization means E field direction.

Note :-

Polarization direction is always E field orientation and direction

2. Free Space

$$E = 50 \sin(10^7 t + \beta z)\, a_y$$

$$\rightarrow \quad \beta = \frac{10^7}{3 \times 10^8} = \frac{1}{30} = \frac{2\pi}{\lambda}$$

$$\Rightarrow \lambda = 60\pi = 188\text{m}$$

$$\rightarrow \quad K = \beta = \frac{1}{30} = 0.033$$

$\rightarrow$ Free space, no attenuation

13. $H = 0.5\, e^{-0.1x} \sin(10^6 t - 2x)\, a_z$

$$V_p = \frac{10^6}{2} = 5\times 10^5$$

$$\gamma = 0.1 + j2$$

direction $\longrightarrow$ $+a_x$

14. $\sigma = 0 \quad \epsilon_R = 81$

$\omega = 6\pi \times 10^8 \;, \quad \beta = ?$

$$V = \frac{3\times 10^8}{\sqrt{81}} = \frac{6\pi \times 10^8}{\beta}$$

$$\Rightarrow \beta = 18\pi$$

15. $\sigma = 0 \quad \mu_R = 2 \quad \epsilon_R = 8$

$$\eta = \sqrt{\frac{\mu_0 \mu_R}{\epsilon_0 \epsilon_R}} = 120\pi \sqrt{\frac{2}{8}} \simeq 188\,\Omega$$

16. $H = 0.1 \cos(\omega t - \beta z)\, a_x$

$$E = (0.1 \times 377) \cos(\omega t - \beta z)\,(a_x \times a_z)$$

$$\simeq -37.7 \cos(\omega t - \beta z)\, a_y$$

Ans - D

17. $$\beta = \omega \sqrt{\mu_0 \epsilon_0 \epsilon_R} = \frac{2\pi}{\lambda}$$

$$\lambda \propto \frac{1}{\sqrt{\epsilon}}$$

$$\frac{\lambda_0}{\lambda} = \sqrt{\frac{\epsilon_0 \epsilon_R}{\epsilon_0}} = \frac{2}{1} \Rightarrow \epsilon_R = 4$$

18. $E = 20\, e^{-j5z} a_x - 25\, e^{-j5z} a_y$

$$H = \left(\frac{20}{120\pi}\right) e^{-j5z} a_y - 25\, e^{-j5z}(-a_x)$$

$$= \left(\frac{25}{120\pi}\right) e^{-j5z} a_x + \left(\frac{20}{120\pi}\right) e^{-j5z} a_y$$

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$$\beta = \frac{\theta}{z} = \frac{2\pi}{\lambda}$$

3mm

90

$$= \frac{\pi/2}{3mm} = \frac{2\pi}{\lambda}$$

$$\Rightarrow \lambda = 12mm$$

$$\lambda f = 12 \times 10^{-3} \times 10 \times 10^{9} = \frac{3\times10^{8}}{\sqrt{\epsilon_R}}$$

$$\Rightarrow \epsilon_R = 6.25$$

20

$$E = A \cos\left(\omega t - \frac{\omega}{c} z\right) a_y$$

$$\frac{\omega}{c} = \beta$$

$$H = \left[\frac{A}{\sqrt{\mu/\epsilon}}\right] \cos\left(\omega t - \frac{\omega}{c} z\right)(-a_x)$$

$$= -A\sqrt{\frac{\epsilon}{\mu}} \cos\left(\omega t - \frac{\omega}{c} z\right) a_x$$

Ans-D

21

$$\nabla^2 E + K^2 E = 0$$

$$\nabla^2 E = \gamma^2 E \quad D^2 m^2 = 0$$

$$e^{-\gamma z} + c^{\gamma z} \leftarrow \nabla^2 E - \gamma^2 E = 0$$

if $\gamma = j\beta$

$$e^{-j\beta z} + e^{j\beta z} \leftarrow \nabla^2 E + \beta^2 E = 0$$

$(+z)$ $\qquad (D^2 + m^2 = 0)$

Ans-C

22.

$$E(x, z, t)_y = 25 \sin(\omega t - 3x + 4z)\, a_y$$

Unit Prop. direction $= \dfrac{3a_x - 4a_z}{5}$

$$\beta = \sqrt{\beta_x^2 + \beta_z^2} = 5 = \frac{2\pi}{\lambda}$$

Note :-

$$\nabla^2 E = \mu\epsilon \frac{\partial^2 E}{\partial t^2}$$

$$\frac{\partial^2 E}{\partial z^2} - \mu\epsilon \frac{\partial^2 E}{\partial t^2} = 0$$

$$E(z,t) = E_0 e^{j\omega t} \cdot e^{-j\beta z}$$

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial z^2} - \mu\epsilon \frac{\partial^2 E}{\partial t^2} = 0$$

$$E(x,z,t) = E_0 e^{j\omega t} \cdot e^{-j\beta_x \cdot x} \cdot e^{-j\beta_z \cdot z}$$

It is a product solution of time, x & z harmonics.

$$E(x,z,t) = 25 \sin(\omega t - 3x + 4z)\, a_y$$

$$H(x,z,t) = \left(\frac{25}{120\pi}\right) \sin(\omega t - 3x + 4z)$$

$$\frac{(3a_x - 4a_z)}{5} \times a_y$$

$$= \left(\frac{25}{120\pi}\right) \sin(\omega t - 3x + 4z) \left(\frac{3a_z + 4a_x}{5}\right)$$

$E(x,z,t)_y$

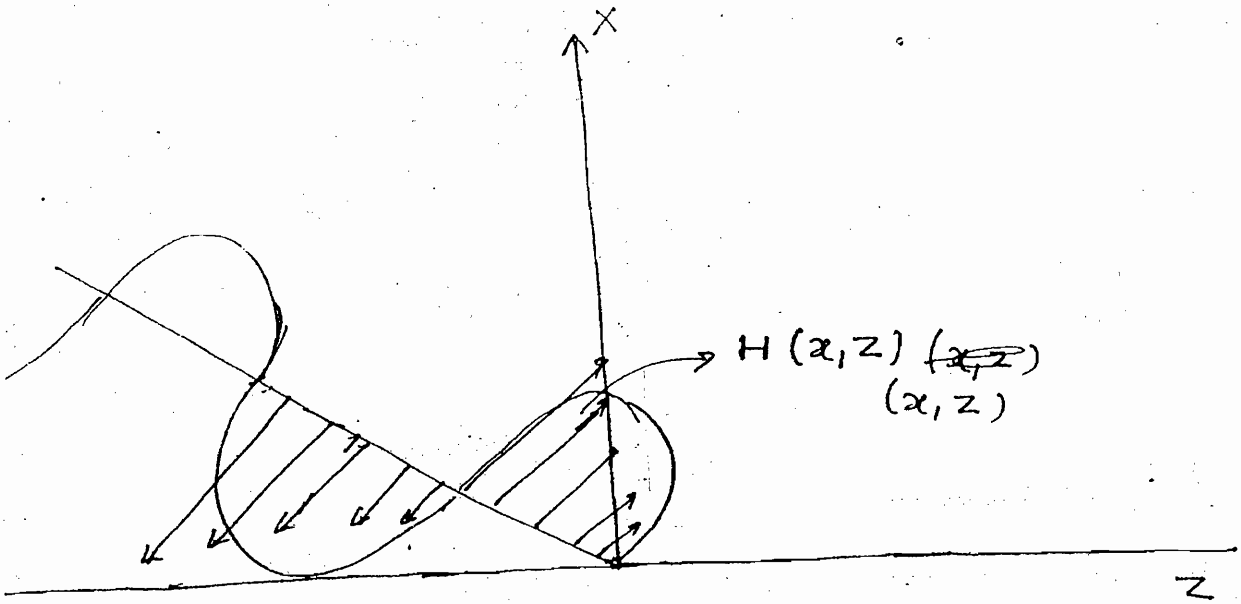
$H(x,z,t)\,(x,z)$

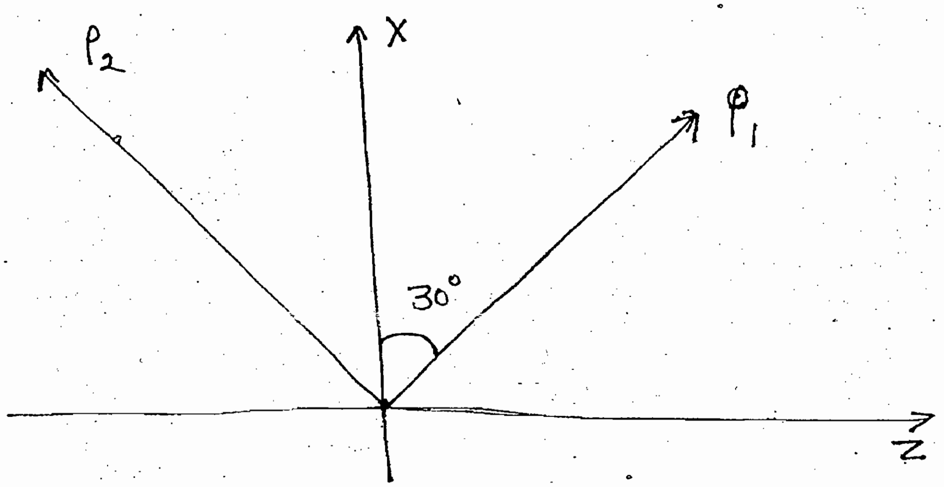
Prop. direction $\rightarrow 3a_x - 4a_z$

H direction $\rightarrow 4a_x + 3a_z$

E direction $\rightarrow a_y$ $\quad$ ($\because$ dot product $= 0$)

$\boxed{E \perp H \perp P}$ $\rightarrow$ Basic Transverse Nature





90° to Y-axis ⇒ ZX Plane

30° to the X-axis

$$E(x,z,t) = E_o e^{j\omega t} \cdot e^{-j\beta_x \cdot x} \cdot e^{\pm j\beta_z \cdot z}$$

$$\tan 30 = \frac{\beta_z}{\beta_x} = \frac{1}{\sqrt{3}}$$

$$\beta_x = \sqrt{3}\,\beta_z$$

$$\beta = \frac{2\pi}{\lambda} = \sqrt{\beta_x^2 + \beta_z^2} = \sqrt{(\sqrt{3}\,\beta_z)^2 + \beta_z^2}$$

$$\Rightarrow \beta_z = \frac{\pi}{\lambda} \qquad \beta_x = \frac{\sqrt{3}\,\pi}{\lambda}$$

Ans – (a)

24.

$$\beta = 280\pi = \frac{2\pi}{\lambda}$$

$$\lambda = \frac{1}{140}$$

$$f = 14 \times 10^9$$

$$\lambda f = 10^8 = \frac{3\times 10^8}{\sqrt{\epsilon_R}} \Rightarrow \epsilon_R = 9$$

$$\frac{E_p}{3} = \frac{120\pi}{\sqrt{9}} \Rightarrow E_p = 120\pi \text{ , Ans}$$

Q4

Lecture - 10

Note:—

R, L, G, C $\longrightarrow$ Primary constant — $\frac{V}{I}$ Waves
|
Transmission line

$\mu, \sigma, \epsilon \longrightarrow$ Material constant — $E/_H$ Waves
|
Materials

$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$	$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$
$z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$	$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$

Note:— (1)

$$\mu = \frac{|\gamma\eta|}{\omega}$$

$$\sigma = \text{Real}\left[\gamma/\eta\right]$$

$$\epsilon = \frac{\text{Imag}\left[\gamma/\eta\right]}{\omega}$$

Note 2:—

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \text{complex} = \frac{E_x}{H_y} = |\eta|\angle\theta$$

$$= \left|\frac{E_x}{H_y}\right|\angle\theta$$

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→ In general material E_x and H_y are orthogonal in directions but have a phase shift or time delay which is η's phase

Example :- $E \rightarrow \sin(\omega t - \beta z) a_x$

$H \rightarrow \sin(\omega t - \beta z - \theta) a_y$

Note 3 :-

$$\gamma = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

$$\alpha^2 - \beta^2 + j2\alpha\beta = j\omega\mu\sigma - \omega^2\mu\epsilon$$

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right)}$$

$$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right)}$$

Loss Tangent or Dissipation Factor :-

$\frac{\sigma}{\omega\epsilon}$ decides the propogation aspects α or β of the EM Waves

If $\frac{\sigma}{\omega\epsilon} >> 1 \Rightarrow$ very good conductor

α is very large $\Rightarrow$ EM Wave propogation is very tough

→ EM waves cannot easily propogate in good conductors.

→ The propogation aspects depends on $\frac{\sigma}{\omega\epsilon}$ and hence on frequency - ω also

eg:- $\frac{\sigma}{\omega\epsilon} >> 1$ For humanbody upto light frequency

$\frac{\sigma}{\omega\epsilon} << 1$ For human body beyond X-ray frequen eg:- $\frac{\sigma}{\omega\epsilon} >> 1$ For earth upto MHz only

$\frac{\sigma}{\omega\epsilon} << 1$ for earth beyond microwaves

Summary:-

The term $\frac{\sigma}{\omega\epsilon} = \left|\frac{J_c}{J_d}\right|$ = loss tangent and hence decides the behaviour of the material

$J_c >> J_d \Rightarrow$ good conductor

$J_d >> J_c \Rightarrow$ good dielectric

$$\frac{J_c}{J_d} = \frac{\sigma E}{\epsilon \frac{\partial E}{\partial t}} = \frac{\sigma E}{j\omega\epsilon E} = \frac{\sigma}{\omega\epsilon}$$

Case-(III):-

EM Wave Propogation in very good conductors ($\sigma >> \omega\epsilon$)

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} \simeq \sqrt{j\omega\mu\sigma}$$

$$= \sqrt{\omega\mu\sigma \angle 90} = \sqrt{\omega\mu\sigma}\ \angle \pm 45$$

$$= \sqrt{\frac{\omega\mu\sigma}{2}} + j\sqrt{\frac{\omega\mu\sigma}{2}}$$

$$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} \simeq \sqrt{\frac{j\omega\mu}{\sigma}} \simeq \sqrt{\frac{\omega\mu}{\sigma}\angle 90^\circ}$$

$$= \sqrt{\frac{\omega\mu}{\sigma}}\ \angle 45 = \sqrt{\frac{\omega\mu}{2\sigma}} + j\sqrt{\frac{\omega\mu}{2\sigma}}$$

$$R = X = \sqrt{\frac{\omega\mu}{2\sigma}}$$

Note (1) :–

→ η's real part (R) = Cause of attenuation (α)

η's Imaginary part (X) = Cause of phase shift (β)

η's phase = 45° means E_x & H_y are delayed in time by 45°

Skin Depth or Depth of penetration :–

$$|E| = E_0 e^{-\alpha z}$$

$$E_0 = 100$$

$$100 \xrightarrow[z=\frac{1}{\alpha}]{1/e} 37 \xrightarrow[z=\frac{2}{\alpha}]{1/e^2} 13 \xrightarrow[3/\alpha]{1/e^3} 4$$

→ The first $1/\alpha$ distance travelled by the wave is where majority of the ~~wave~~ amplitude is attenuated. This distance is called as skin depth. Hence skin depth is the distance travelled by the wave where the wave leakage to $1/e$ times (37%) of the initial value.

$$\delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\sigma}} = \sqrt{\frac{2\sigma}{\omega\mu}} \cdot \frac{1}{\sigma} = \frac{1}{\sigma R_s}$$

where R_s = η's real part = skin resistance

25. $V_p = \frac{\omega}{\beta} = \frac{\omega}{\alpha} = \omega\delta = 2\pi \times 2\times10^5 \times 4\times10^{-6}$

$= 1.6\pi \ m/s \cong 5 \ m/s$

6. $\omega' = 4\omega \qquad \delta = \sqrt{\frac{2}{\omega\mu\sigma}} \Rightarrow \delta' = \frac{\delta}{2}$

$V_p' = \omega'\delta' = 4\omega\frac{\delta}{2} = 2V_p = 10 \ m/s$

Note :-

$\frac{\sigma}{\omega\epsilon} = 0$ Free space $\longrightarrow$ $3\times10^8 \ m/s$

Ideal dielectric $\longrightarrow$ $\frac{3\times10^8}{\sqrt{\epsilon_R}}$

$\frac{\sigma}{\omega\epsilon}\uparrow$ General Material $\longrightarrow$ $10^5 - 10^6 \ m/s$

$\frac{\sigma}{\omega\epsilon} >> 1$ Good conductor $\longrightarrow$ few m/s

$\frac{1}{\sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1\right)}}$

$$V_p = \frac{\cancel{\omega}}{\cancel{\omega}\sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1\right)}}$$

27. $\frac{\sigma}{\omega\epsilon} = 1.73$

η's phase $= \sqrt{\frac{j\omega\mu}{\sigma+j\omega\epsilon}} = \sqrt{\frac{\omega\mu \angle 90}{K \angle \tan^{-1}\left(\frac{\omega\epsilon}{\sigma}\right)}}$

$= \sqrt{K' \angle 90 - \tan^{-1}\left(\frac{\omega\epsilon}{\sigma}\right)}$

η's phase $= \frac{90 - \tan^{-1}\left(\frac{\omega\epsilon}{\sigma}\right)}{2} = \frac{\tan^{-1}(\sigma/\omega\epsilon)}{2} = \frac{\tan^{-1}\sqrt{3}}{2} = \frac{60}{2} = 30^\circ$

Note:-

η's phase = E_x to H_y Phase

$\frac{\sigma}{\omega\epsilon} = 0$ Free space $\rightarrow$ Ideal dielectric $\rightarrow$ $\theta = 0$

$\frac{\sigma}{\omega\epsilon} \uparrow$ General Material $\rightarrow$ $20^\circ - 30^\circ$

$\frac{\sigma}{\omega\epsilon} >> 1$ Good conductor $\rightarrow$ 45°

$\frac{\tan^{-1}\left(\frac{\sigma}{\omega\epsilon}\right)}{2}$

28.

$J_c = J_d$

$\frac{\sigma}{\omega\epsilon} = 1$

$\Rightarrow \frac{\sigma}{2\pi f\epsilon} = 1$

$f = \frac{10^{-2}}{2\pi \times 4 \times \frac{1}{36\pi \times 10^9}} = 45\text{ MHz}$

29.

$\frac{I_c}{I_d} = \frac{J_c}{J_d} = \frac{\sigma}{\omega\epsilon}$

$\Rightarrow I_d = \frac{I_c\,\omega\epsilon}{\sigma} = \frac{1 \times 2\pi \times 50 \times 8.8 \times 10^{-12}}{58}$

Ans - (B)

30.

$\eta = \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{\omega\mu}{\sigma}}\,\angle 45^\circ$

$= R + jX$ Ans - D

31. $\eta = 0.02 \angle 45^\circ$ Ans-(a)

32. lossy dielectric, $\frac{\sigma}{\omega\epsilon} << 1$

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = \sqrt{j\omega\mu \cdot j\omega\epsilon\left(1 + \frac{\sigma}{j\omega\epsilon}\right)}$$

$$= j\omega\sqrt{\mu\epsilon}\left(1 + \frac{\sigma}{j2\omega\epsilon}\right) = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}} + j\omega\sqrt{\mu\epsilon}$$

Power, Power density and Poynting Vector :-

→ Energy that can be transformed into other formats is said to be a power format

$$\frac{\text{Joules}}{\text{second}} = \text{Watts}$$

Using Maxwells IV Equation

$$\nabla \times H = \sigma E + \epsilon \frac{\partial E}{\partial t}$$

Take $\cdot E$ on both sides

$$(\nabla \times H) \cdot E = \sigma E^2 + \epsilon \frac{\partial E}{\partial t} \cdot E$$

$$\Rightarrow -\nabla \cdot (E \times H) + H \cdot (\nabla \times E) = \sigma E^2 + \epsilon \frac{\partial E}{\partial t} \cdot E$$

$$\Rightarrow -\nabla \cdot (E \times H) - \sigma E^2 = \epsilon \frac{\partial E}{\partial t} \cdot E + \mu \frac{\partial H}{\partial t} \cdot H$$

$$= \frac{\partial}{\partial t}\left(\frac{1}{2}\epsilon E^2 + \frac{1}{2}\mu H\right)^2$$

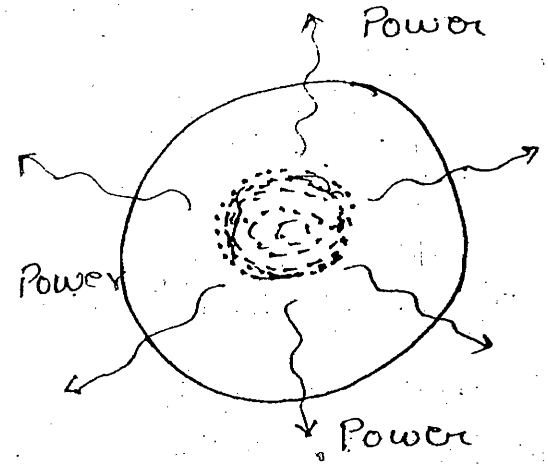
Take $\int dv$ on both sides

$$\int \nabla \cdot (E \times H)\, dv + \int \sigma E^2 dv = -\frac{\partial}{\partial t}\int \left(\frac{1}{2}\epsilon E^2 + \frac{1}{2}\mu H^2\right) dv$$

Using Divergence theorem

$$\oint (E \times H)\, ds + \int \sigma E^2 dV = \text{Rate of decrease of E/H}$$

Energy inside a volume



The LHS is a EM wave power crossing the surface

Hence $E \times H$ = Power density of EM Wave
= Strength of power of EM Wave
= Poynting Vector of the EM Wave
= It has the direction of propogation

$$E(z,t)_x \times H(z,t)_y = P(z,t)_z$$

= Power density at a time at a point in space

= Instantaneous Poynting Vector

$P(z)_{avg.}$ = Time Average Poynting Vector

$$= E_{RMS} \cdot H_{RMS} = \frac{E_0 e^{-\alpha z}}{\sqrt{2}} \cdot \frac{H_0 e^{-\alpha z}}{\sqrt{2}}$$

$$P(z)_{avg.} = \frac{1}{2} E_0 H_0 e^{-2\alpha z} = \frac{1}{2} \frac{E_0^2}{\eta} e^{-2\alpha z} = \frac{1}{2} \eta H_0^2 e^{-2\alpha z}$$

Note 1 :-

The average power decays at 2α rate exponentially in the medium

→ This power lost in the wave is acquired by the medium as ohmic power

$$J \cdot E \quad \text{or} \quad \sigma E^2 \quad \text{Watts}/m^3$$

Note 2 :-

Alternatively

$$P(z)_{avg.} = \frac{1}{2}(E \times H^*)$$

* conjugate operation means average power is independent of harmonic and its phase and depends only on amplitude.

> In free space

$$P_{avg} = \frac{1}{2} E_0 H_0 = \frac{1}{2}\frac{E_0^2}{\eta} = \frac{1}{2}\eta H_0^2$$

Workbook :-

3. $P_{avg.} = \frac{1}{2}\eta H_0^2 \, (-a_y)$

$H_0 = 0.1$

$= \frac{1}{2} \cdot 120\pi \cdot (0.1)^2 (-a_y) \longrightarrow \underline{B}$

34. $W = \int P_{avg.} \, ds$

$= \int \frac{1}{2}\frac{E_0^2}{\eta} a_x \cdot ds \, a_x$ $\quad x = \text{constant}$

$= \frac{1}{2} \cdot \frac{60 \cdot 60}{120\pi} \pi \times 4^2 = 240W$

35 C

36 (1) $H = 50\sin(\omega t - \beta z)a_x + 150\sin(\omega t - \beta z)a_y$

$P_{avg} = \frac{1}{2}\eta H_0^2 \, (a_z)$

$H = 50\sin(\omega t - \beta z)\left(\frac{a_x + 3a_y}{\sqrt{10}}\right)\sqrt{10}$

$$= (50\sqrt{10}) \sin(\omega t - \beta z) \left(\frac{a_x + 3a_y}{\sqrt{10}}\right)$$

$$P_{avg} = \frac{1}{2} \cdot 120\pi \cdot (50\sqrt{10})^2 a_z$$

$$H_0 = 50\sqrt{10}$$

(ii) $H = 50 \sin(\omega t - \beta z) a_x + 150 \cos(\omega t - \beta z) a_x$

$$= 50 \sin(\omega t - \beta z) a_x (1 + 3j)$$

$$= 50 (1 + 3j) \sin(\omega t - \beta z) a_x$$

Note:—

Orthogonal field components in time ~~and~~ or space have same effect

(iii) $H = 50 \sin(\omega t - \beta z) a_x + 150 \cos(\omega t - \beta z) a_y$

$$H = 50 \sin(\omega t - \beta z)(a_x + 3j a_y)$$

$$P_{avg} = \frac{1}{2}(E \times H^*)$$

$$E = (50\eta) \sin(\omega t - \beta z)(-a_y) + (150\eta) \cos(\omega t - \beta z)(a_x)$$

$$= (50\eta) \sin(\omega t - \beta z)(3j a_x - a_y)$$

$$P_{avg} = \frac{1}{2} \cdot 50 \cdot 120\pi (3j a_x - a_y) \times 50 (a_x - 3j a_y)$$

$$= \frac{1}{2} \cdot 120\pi \cdot 50^2 (a_z + 9a_z)$$

$$= \frac{1}{2} \cdot 120\pi (50\sqrt{10})^2 a_z$$

37.

$$E = (a_x + j a_y)\, e^{jkz - j\omega t}$$

$$H = \frac{k}{\omega\mu} (a_y + j a_x)\, e^{jkz - j\omega t}$$

$$P_{avg.} = \frac{1}{2} (a_x + j a_y) \times \frac{k}{\omega\mu} (a_y - j a_x)$$

$$= \frac{k}{2\omega\mu} (a_z - a_z) = 0 \rightarrow \text{Null vector}$$

<u>Conventional</u> :-

$$E = 100\, (j a_x + 2 a_y - j a_z)\, e^{j\omega t}$$

$$H = (-a_x + j a_y + a_z)\, e^{j\omega t}$$

$$P_{avg.} = \frac{1}{2} (E \times H^*) = \frac{1}{2}\, 100\, (j a_x + 2 a_y - 2 a_z) \times (-a_x - j a_y + a_z)$$

$$= 50 \begin{vmatrix} a_x & a_y & a_z \\ j & 2 & -j \\ -1 & -j & 1 \end{vmatrix}$$

$$= 50\, (3 a_x + 3 a_z)$$

$$|P_{avg.}| = 150\sqrt{2}$$

$$P_{avg.} \text{ direction} = (a_x + a_z)$$

$$\text{Unit } P_{avg.} \text{ direction} = \frac{a_x + a_z}{\sqrt{2}}$$

Wave Polarization :-

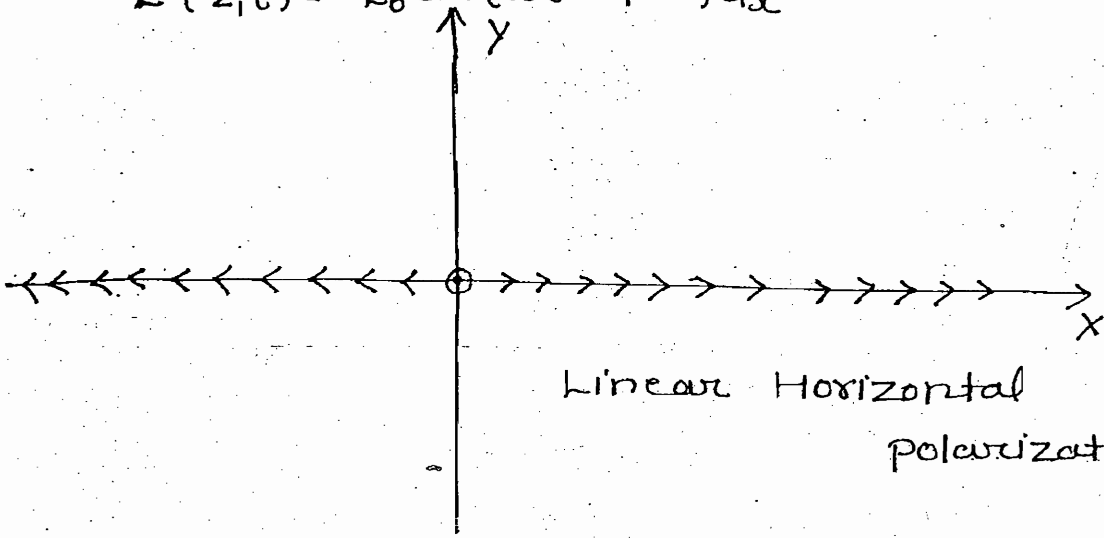
It is the electric field orientation & the possible planar components of the E field satisfying the basic transverse nature

EM Wave is propogating along z-axis in the following cases :-

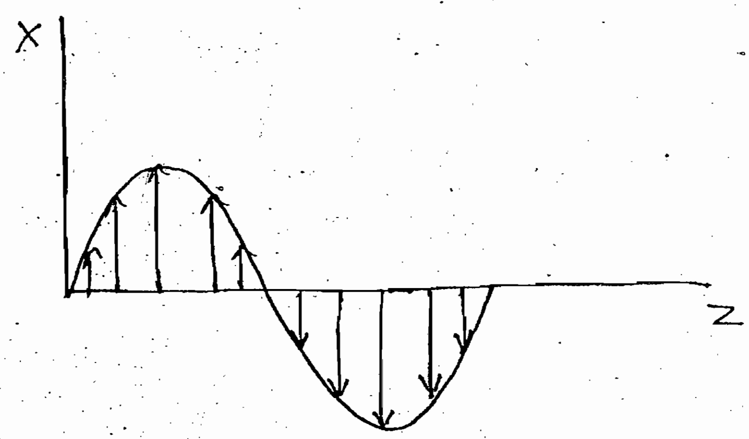
Case (1) :-

E field is oriented in x-direction

$$E(z,t) = E_0 \sin(\omega t - \beta z)\, a_x$$



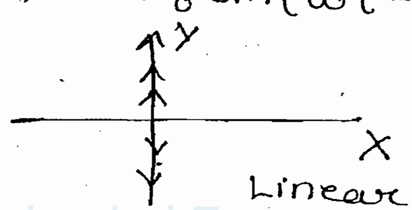
Linear Horizontal polarization



Case -(II) :-

E. field is oriented in y-direction

$$E(z,t) = E_0 \sin(\omega t - \beta z)\, a_y$$

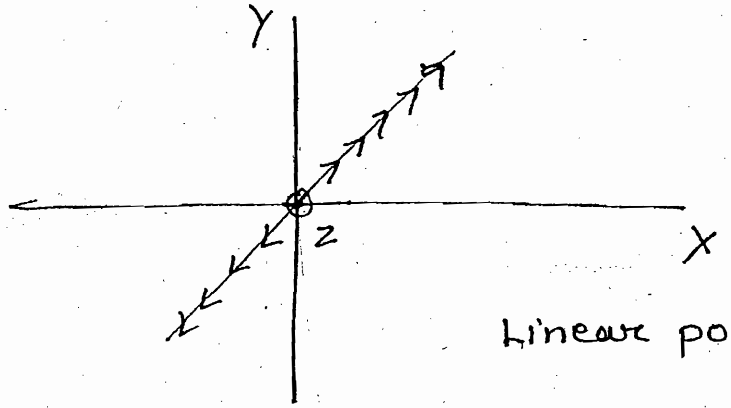


Linear vertical polarization

Case-(III) :-

$\bar{E}$ field is oriented in x & y directions

$$E(z,t) = E_1 \sin(\omega t - \beta z) a_x + E_2 \sin(\omega t - \beta z) a_y$$



Linear polarization

Note :-

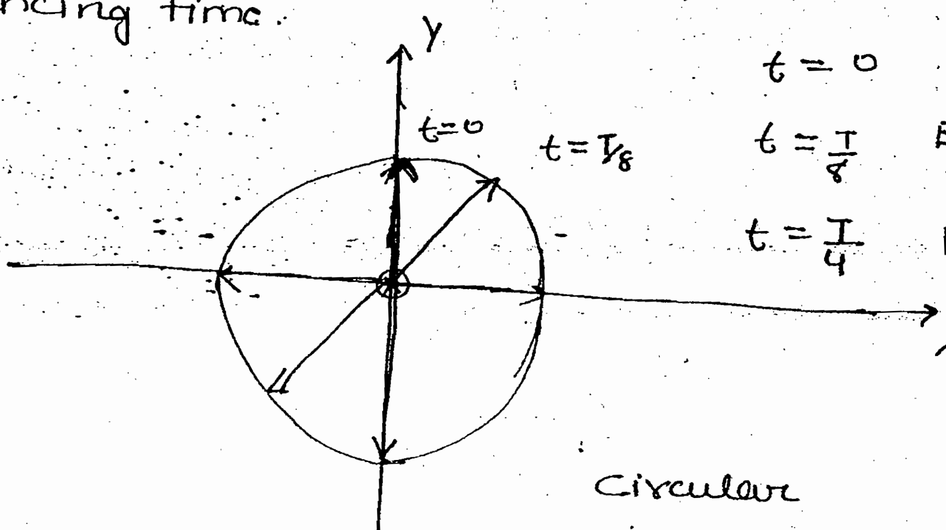
If the EM waves has single E field components or two planar components both inphase the wave is said to be linearly polarized

Case-(IV) :-

E field is oriented in x & y directions

$$E(z,t) = E_0 \sin(\omega t - \beta z) a_x + E_0 \cos(\omega t - \beta z) a_y$$

→ Polarization is identified by studying the trace of the E field on the $z=0$ plane for various advancing time.



$t = 0 \quad E_x = 0, \; E_y = E_0$

$t = \frac{T}{8} \quad E_x = \frac{E_0}{\sqrt{2}}, \; E_y = \frac{E_0}{\sqrt{2}}$

$t = \frac{T}{4} \quad E_x = E_0, \; E_y = 0$

Circular Polarization.

Summary:-

If the EM Waves has two planar components of E field both out of phase at 90° and equal amplitude, the wave is circularly polarised.

Note:-

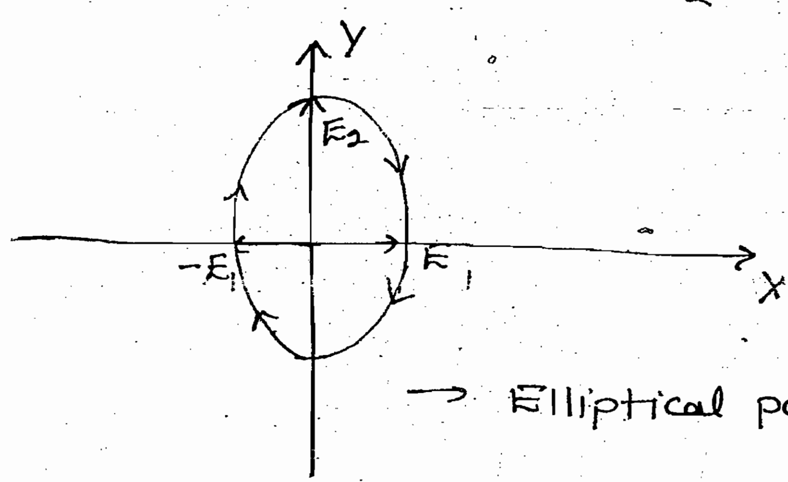
Sense of rotation left or right, if the left hand thumb points towards propagation direction and the closed finger along advancing time, the wave is left circularly polarised

eg:- out of paper - propogation clockwise time advancement - left circular

Case-(V):-

E field is oriented in x & y directions

$$E(z,t) = E_1 \sin(\omega t - \beta z)\, a_x + E_2 \cos(\omega t - \beta z)\, a_y$$

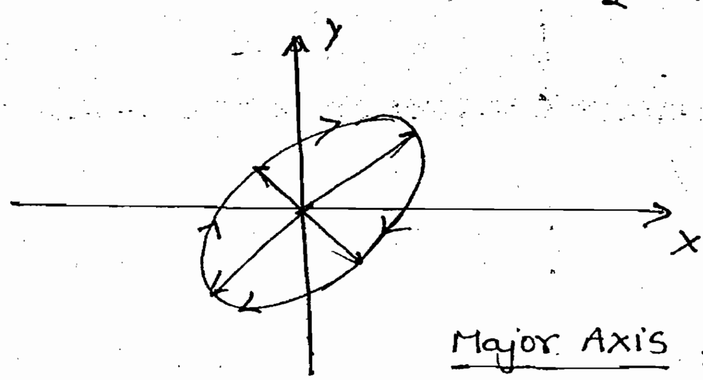


→ Elliptical polarization

Case-(VI):-

E field is oriented in x & y direction

$$E(z,t) = E_1 \sin(\omega t - \beta z)\, a_x + E_2 \sin(\omega t - \beta z + \theta)\, a_y$$



$$\frac{\text{Major Axis}}{\text{Minor Axis}} = AR = [1, \infty)$$

Circle Linear

Workbook :-

3 (I) $\vec{E} = 25\sin(\omega t + 4x)(a_y + 6a_z)$

$\rightarrow$ Linear

II) $\vec{E} = 25\sin(\omega t + 4x)a_y + 25\cos(\omega t + 4x)a_z$

$\rightarrow$ Circular

III) $\vec{E} = 25\sin(\omega t + 4x + 60°)a_y + 25\cos(\omega t + 4x + 60°)a_z$

$\rightarrow$ Circular

(IV) $\vec{E} = 25\sin(\omega t + 4x)a_y + (25\sqrt{2})\sin(\omega t + 4x + 45°)a_z$

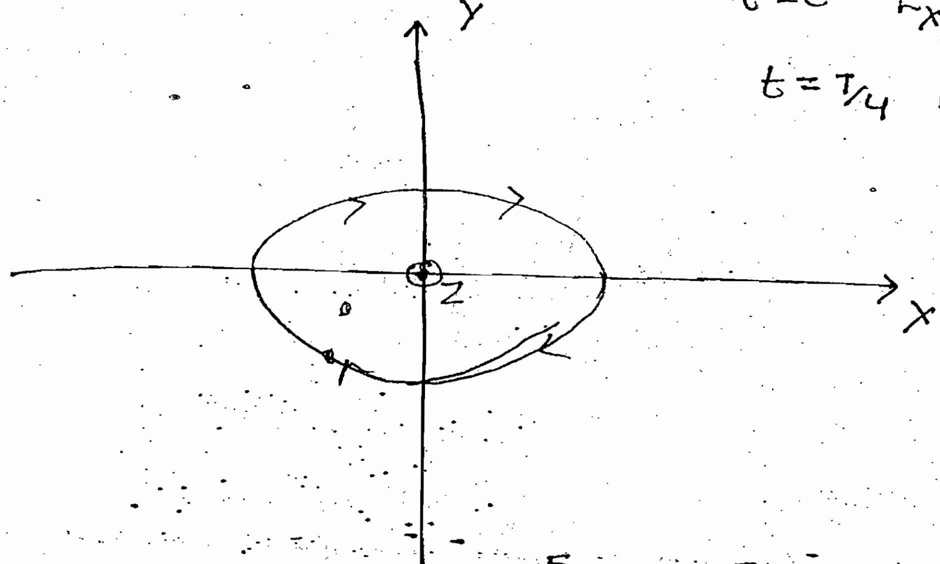
$\rightarrow$ Elliptical

V) Elliptical

VI) Circular

9. $\theta = 60° \rightarrow$ elliptical

10. $\theta = 90°$



$t = 0 \quad E_x = E_1 \quad E_y = 0$

$t = T/4 \quad E_x = 0 \quad E_y = E_2$

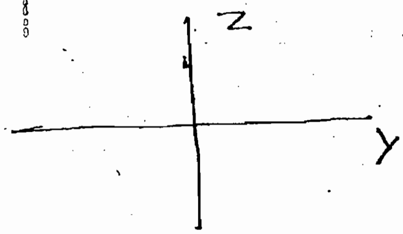
Summary :-

$$\left.\begin{array}{l} E_x \rightarrow E_1\cos\omega t \\ E_y \rightarrow E_2\sin\omega t \end{array}\right\} e^{-jkz}$$

E_x	E_y	Prop.	Polarization
cos	-sin	z	Left
cos	sin	z	Right
cos	-sin	-z	Right
-sin	cos	z	Right

$(a_y + j a_z)\, e^{j\beta x}$

a_y	a_z
$\downarrow$	$\downarrow$
sin	cos
$\downarrow$	$\downarrow$
cos	−sin

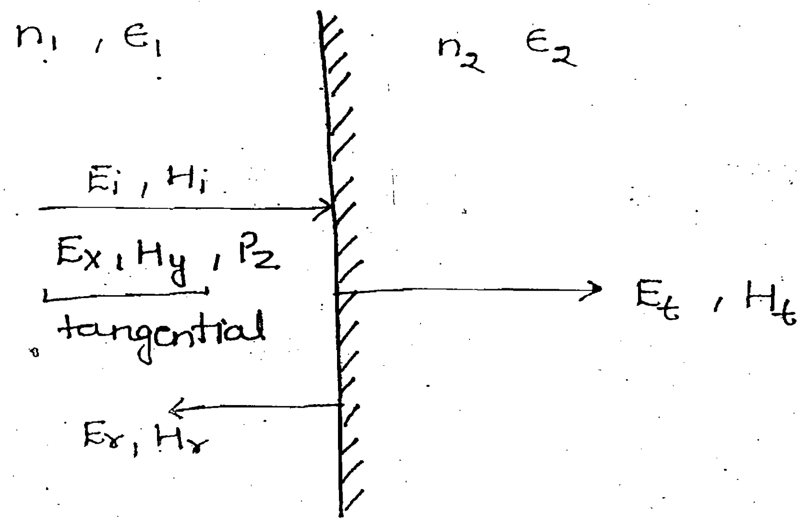


Lecture-11

Reflections / Transmissions of EM Waves.

(I) Normal Incidence (II) Dielectric - Dielectric

$z<0$ $z=0$ (XY Plane)



(I) $E_t > E_i$ or $E_t < E_i$ $\longrightarrow$ It depends on ϵ_1 & ϵ_2

(II) $H_t > H_i$ if $E_t < E_i$

$H_t < H_i$ if $E_t > E_i$

(III) $P_t < P_i$ $\longrightarrow$ Always

E_x	H_y	Prop. z	
a_x	a_y	a_z	$\rightarrow$ Incidence
a_x	$-a_y$	$-a_z$	$\rightarrow$ Reflection
$-a_x$	a_y	$-a_z$	

$\rightarrow$ When reflections occur either E or H negates along with propogation direction but not both.

$$E_i = \eta_1 H_i$$

$$E_t = \eta_2 H_t$$

$$E_r = -\eta_1 H_r$$

$\rightarrow$ For normal incidence the E/H fields are tangential.

$$E_{t_1} = E_{t_2} \longrightarrow E_i + E_r = E_t$$

$$H_{t_1} = H_{t_2}$$

$$E_i + E_r = E_t$$

$$1 + \frac{E_r}{E_i} = \frac{E_t}{E_i}$$

$$\Rightarrow \boxed{1 + \Gamma = \tau} \quad \text{——} \; (1)$$

where $\Gamma = \frac{E_r}{E_i}$

= Reflection coefficient for the E fields

$\tau = \frac{E_t}{E_i}$ = Transmission coefficient for the E fields

$$H_i + H_r = H_t$$

$$\frac{E_i}{\eta_1} - \frac{E_r}{\eta_1} = \frac{H_t}{\eta_2}$$

$$E_i - E_r = \frac{\eta_1}{\eta_2} E_t$$

$$\boxed{\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}}$$

Note :–

$$\Gamma = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2} = -\Gamma_E$$

$$\Gamma_P = \Gamma_E \cdot \Gamma_H = -\Gamma_E^2 = -\Gamma_H^2$$

$$\boxed{1 + \Gamma = \tau}$$ Always - Either E or H or Power

(IV) If $\eta_1 = \sqrt{\frac{\mu_0}{\epsilon_1}}$ & $\eta_2 = \sqrt{\frac{\mu_0}{\epsilon_2}}$

$$\Gamma_E = \frac{\sqrt{\epsilon_1} - \sqrt{\epsilon_2}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}}$$

If $\epsilon_1 > \epsilon_2 \Rightarrow \Gamma_E = +ve \quad \tau_E > 1$

$$E_t > E_i$$

$$\Rightarrow \Gamma_H = -ve, \quad \tau_H < 1$$

$$H_t < H_i$$

$\Gamma_P = -ve, \quad \tau_P < 1 \longrightarrow$ Always

Workbook :-

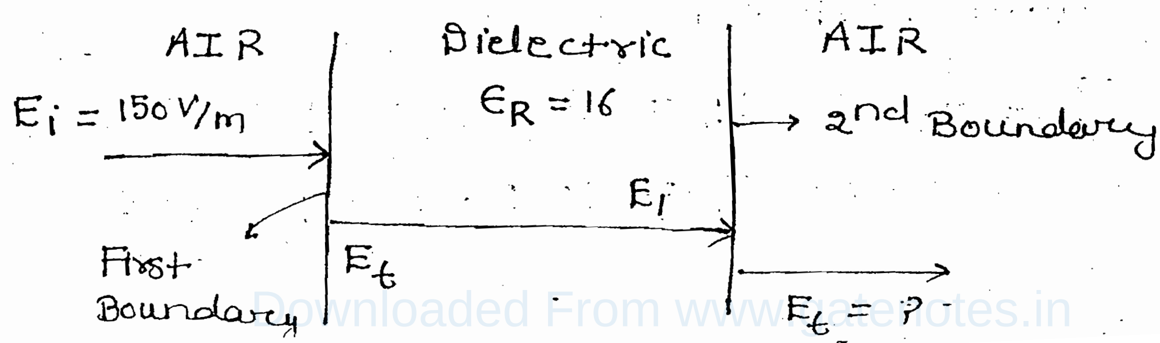
12. $\frac{\text{Air}}{\epsilon_0}$ - $\frac{\text{Dielectric}}{4\epsilon_0}$ $\quad \tau_P = ?$

$$\Gamma_E = \frac{\sqrt{\epsilon_0} - \sqrt{4\epsilon_0}}{\sqrt{\epsilon_0} + \sqrt{4\epsilon_0}} = \frac{1-2}{1+2} = -\frac{1}{3}$$

$$\Gamma_H = +\frac{1}{3} \qquad \Gamma_P = -\frac{1}{9}$$

$$\Rightarrow \tau_P = 1 - \frac{1}{9} = \frac{8}{9}$$

13.



At the 1st boundary

$$\Gamma_E = \frac{1-\sqrt{16}}{1+\sqrt{16}} = \frac{-3}{5}$$

$$\tau_E = 1-\frac{3}{5} = \frac{2}{5} = \frac{E_t}{E_i} \Rightarrow E_t = \frac{2}{5} \times 150 \text{ V/m}$$

$$= 60 \text{ V/m}$$

E_i at the 2nd Boundary = 60 V/m

$$\Gamma_E = \frac{\sqrt{16}-\sqrt{1}}{\sqrt{16}+\sqrt{1}} = \frac{3}{5}$$

$$\tau_E = 1+\frac{3}{5} = \frac{8}{5} = \frac{E_t}{E_i}$$

$$E_t = \frac{8}{5} \times 60 = 96 \text{ V/m}$$

14. $$\Gamma_E = \frac{1-\sqrt{4}}{1+\sqrt{4}} = \cancel{\frac{-1}{3}}\ -\frac{1}{3}$$

$$\tau_E = 1-\frac{1}{3} = \frac{2}{3} = \frac{E_t}{E_i}$$

$$E_t = \frac{2}{3} E_0 \cos(\omega t - 2\beta z)\, a_y$$

$$(\beta' = \omega\sqrt{\mu_0 \epsilon_0 \epsilon_R} = 2\beta)$$

Note:-

$\alpha, \beta, V, \eta, V_p, \lambda \rightarrow$ Propogation aspects & material aspects and change as σ, ϵ, μ changes

$\omega \rightarrow$ source aspect / time aspect and is same for every material.

15. $\Gamma_E = -\frac{1}{3}$ $\qquad H_i = \cos(10^8 t - \beta z)\, a_y$

$$E_i = 120\pi \cdot \cos(10^8 t - \beta z)\, a_x$$

$$\frac{10^8}{\beta} = 3 \times 10^8 \quad \Rightarrow \quad \beta = \frac{1}{3}$$

$$E_r = \left(-\frac{1}{3} \cdot 120\pi\right) \cos\left(10^8 t + \frac{z}{3}\right) a_x$$

16.

$$\frac{E_t}{E_r} = \frac{E_t}{E_i \frac{E_r}{E_i}} = \frac{\tau}{\Gamma} = \frac{1+\Gamma}{\Gamma} = -2$$

$$\Gamma = -\frac{1}{3} = \frac{1-\sqrt{\epsilon_2/\epsilon_1}}{1+\sqrt{\epsilon_2/\epsilon_1}} \Rightarrow \frac{\epsilon_2}{\epsilon_1} = 4$$

48. EM Wave reflections at conductors :-

$$\Gamma_E = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \qquad \eta_1 = 120\pi = 377$$

$$\eta_2 = \sqrt{\frac{j\omega\mu}{\sigma}} = 0$$

→ EM wave power is completely reflected at conductors

$$\Gamma_E = \frac{E_r}{E_i} = 1$$

$$E_i + E_r = E_t = 0$$

$$\tau_E = 0$$

$$\Gamma_H = \frac{H_r}{H_i} = 1$$

Ans-(c)

$$H_i + H_r = H_t = H_{max}$$

$$\tau_H = 2$$

47.

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \sqrt{\frac{\mu}{\epsilon}} \quad (\text{if } \sigma = 0) \text{ real}$$

$$= \sqrt{\frac{\mu^*}{\epsilon^*}} \quad (\text{if } \sigma \neq 0) \text{ complex}$$

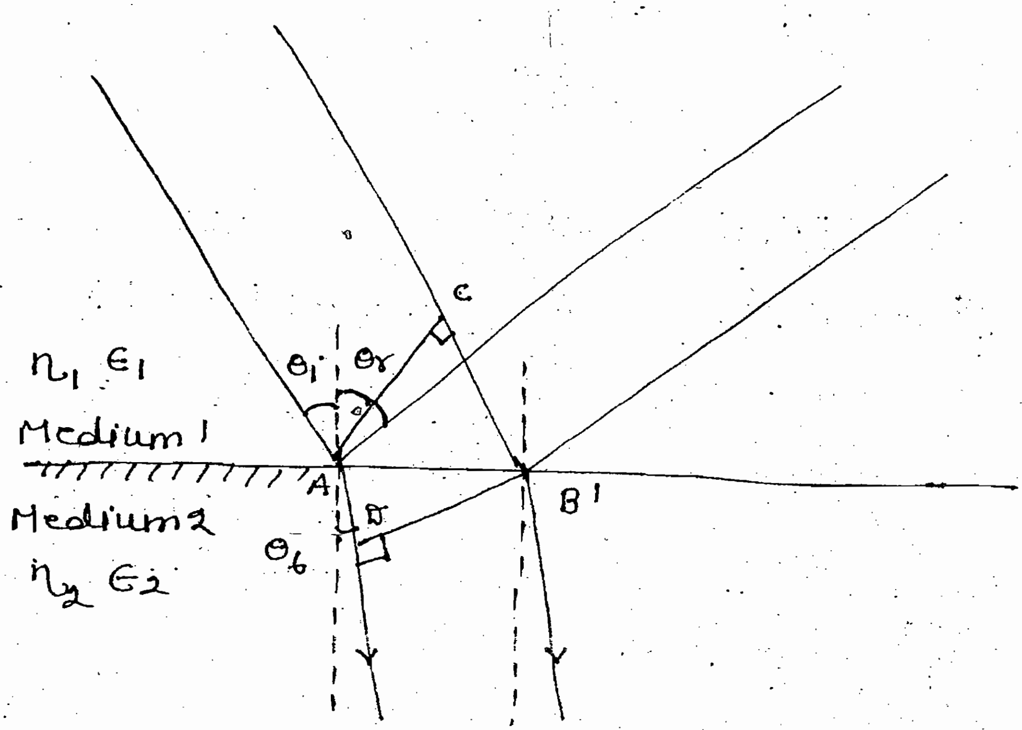
As $\mu^* = \epsilon^*$

$$\eta = \sqrt{\frac{\mu^*}{\epsilon^*}} = 1$$

$$\Gamma = \frac{1-377}{1+377} \Rightarrow \Gamma \cong -1$$

Ans - (c)

Oblique (Inclined) Incidence :-



→ A/C are inphase points in medium 1

→ CB is the extra distance travelled in medium 1

→ B/D are inphase points in medium 2

→ AD is the extra distance travelled in medium 2

$$\frac{CB}{AD} = \frac{V_1}{V_2} = \text{Refracting Index of med. 2 w.r.t med 1}$$

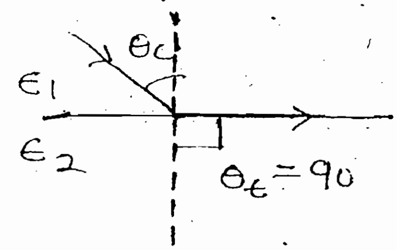
$$\frac{AB\cos(90-\theta_i)}{AB\cos(90-\theta_t)} = \frac{\frac{1}{\sqrt{\mu_0\epsilon_1}}}{\sqrt{\mu_0\epsilon_2}}$$

$$\boxed{\frac{\sin\theta_i}{\sin\theta_t} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}} \longrightarrow \text{Refracting Index}$$

Snell's Ist law → $\boxed{\theta_i = \theta_r}$

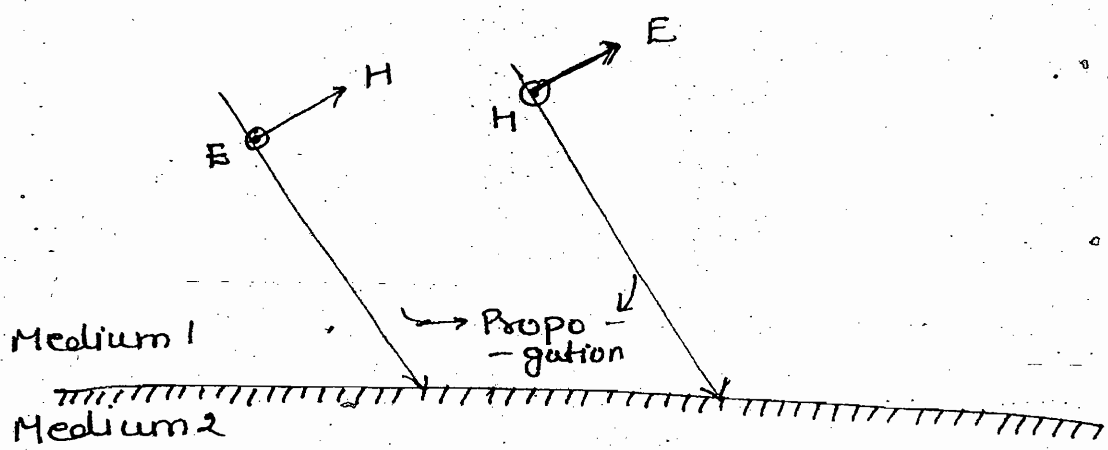
If $\theta_t = 90^\circ$ then $\theta_i = \theta_c$ = critical angle

For all $\theta_i > \theta_c$ complete reflections and zero transmission takes place



$$\boxed{\sin\theta_c = \sqrt{\frac{\epsilon_2}{\epsilon_1}}}$$

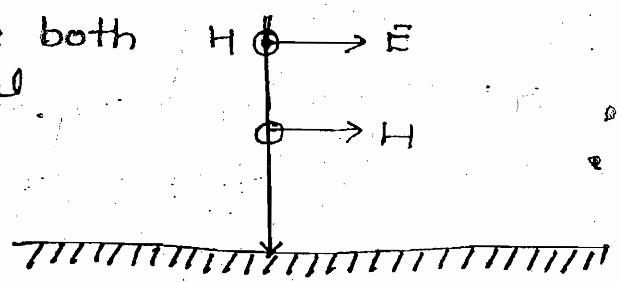
S & P Polarized Waves :-



S-Polarized Horizontal Polarization

P-Polarized vertical polarization.

In normal incidence both fields are Horizontal



Applying the Boundary conditions,

$$E_{t_1} = E_{t_2}$$

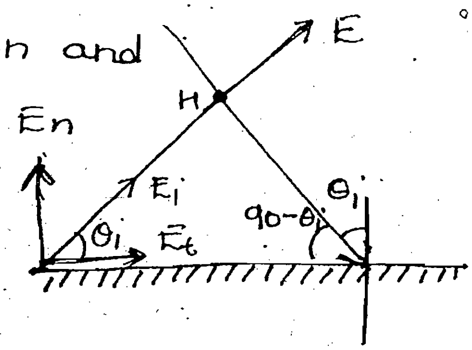
For s-polarized waves

$$E_i + E_r = E_t$$

$$\boxed{1 + \Gamma_s = \tau_s} \quad \text{——— (1s)}$$

Note:–

The angle b/w propogation and normals is the same angle b/w fields and surface



For p-polarised waves

$$E_i \cos\theta_i + E_r \cos\theta_r = E_t \cos\theta_t$$

$$E_i + E_r = E_t \frac{\cos\theta_t}{\cos\theta_i}$$

$$\boxed{1 + \Gamma = \tau \frac{\cos\theta_t}{\cos\theta_i}} \quad \text{— 1p}$$

similarly using $H_{t_1} = H_{t_2}$

$$\Gamma_s = \frac{\eta_2 \sec\theta_t - \eta_1 \sec\theta_i}{\eta_2 \sec\theta_t + \eta_1 \sec\theta_i}$$

$$\Gamma_p = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

Note:–

$\Gamma_s \neq \Gamma_p$ in oblique incidence

but when $\theta_i = \theta_r = \theta_t = 0$ then

$$\Gamma_s = \Gamma_p = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

As $\eta_1 = \sqrt{\frac{\mu_0}{\epsilon_1}}$ $\quad \eta_2 = \sqrt{\frac{\mu_0}{\epsilon_2}}$

$$\Gamma_s = \frac{\sqrt{\epsilon_1}\cos\theta_i - \sqrt{\epsilon_2}\cos\theta_t}{\sqrt{\epsilon_1}\cos\theta_i + \sqrt{\epsilon_2}\cos\theta_t}$$

$$\frac{\sin\theta_i}{\sin\theta_t} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

$$\sin\theta_t = \sqrt{\frac{\epsilon_1}{\epsilon_2}}\sin\theta_i$$

$$\cos\theta_t = \sqrt{1-\frac{\epsilon_1}{\epsilon_2}\sin^2\theta_i}$$

$$\Gamma_s = \frac{\cos\theta_i - \sqrt{\frac{\epsilon_2}{\epsilon_1}-\sin^2\theta_i}}{\cos\theta_i + \sqrt{\frac{\epsilon_2}{\epsilon_1}-\sin^2\theta_i}}$$

$$\Gamma_p = \frac{\frac{\epsilon_2}{\epsilon_1}\cos\theta_i - \sqrt{\frac{\epsilon_2}{\epsilon_1}-\sin^2\theta_i}}{\frac{\epsilon_2}{\epsilon_1}\cos\theta_i + \sqrt{\frac{\epsilon_2}{\epsilon_1}-\sin^2\theta_i}}$$

If $\Gamma_p = 0$ i.e. zero reflections & complete transmission

$$\left(\frac{\epsilon_2}{\epsilon_1}\right)^2(1-\sin^2\theta_i) = \frac{\epsilon_2}{\epsilon_1} - \sin^2\theta_i$$

$$\sin^2\theta_i\left(1-\left(\frac{\epsilon_2}{\epsilon_1}\right)^2\right) = \frac{\epsilon_2}{\epsilon_1} - \left(\frac{\epsilon_2}{\epsilon_1}\right)^2$$

$$\sin^2\theta_i = \frac{\frac{\epsilon_2}{\epsilon_1}\left(1-\frac{\epsilon_2}{\epsilon_1}\right)}{\left(1+\frac{\epsilon_2}{\epsilon_1}\right)\left(1-\frac{\epsilon_2}{\epsilon_1}\right)}$$

$$\sin\theta_i = \sqrt{\frac{\epsilon_2}{\epsilon_1+\epsilon_2}}$$

$$\tan\theta_i = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

This $\theta_i = \theta_B$ is called as Brewster angle for p-polarised wave

If $\Gamma_s = 0$

$$\cos^2\theta_i = \frac{\epsilon_2}{\epsilon_1} - \sin^2\theta_i$$

$$\Rightarrow \epsilon_2 = \epsilon_1$$

Brewster angle and zero reflections cannot occur for s-polarized waves.

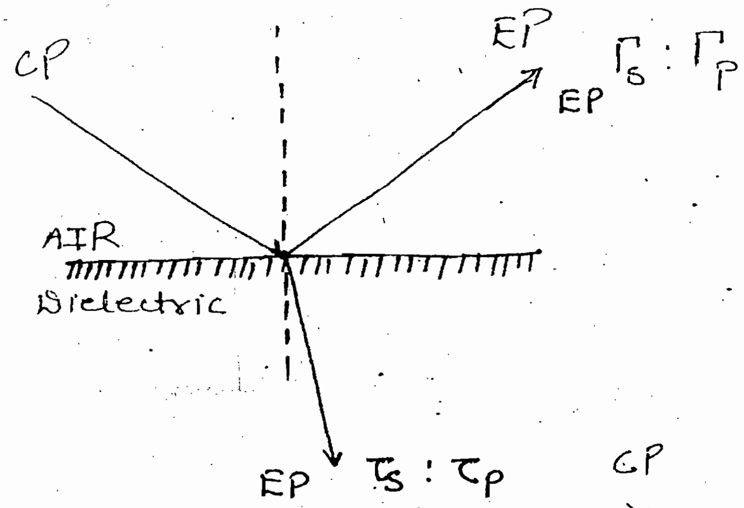
Summary :-

zero transmission, complete reflections θ_c = critical angle	zero reflections, complete transmission θ_B = Brewster angle
$\sin\theta_c = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$,	2. $\tan\theta_B = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$
$\epsilon_1 > \epsilon_2$ is a required condition	3. No such medium restriction
No such polarization restriction	4. Exists only for p-polarized waves
For all $\theta_i > \theta_c$ → the same effect	5. At $\theta_i = \theta_B$ only this occurs

Workbook :-

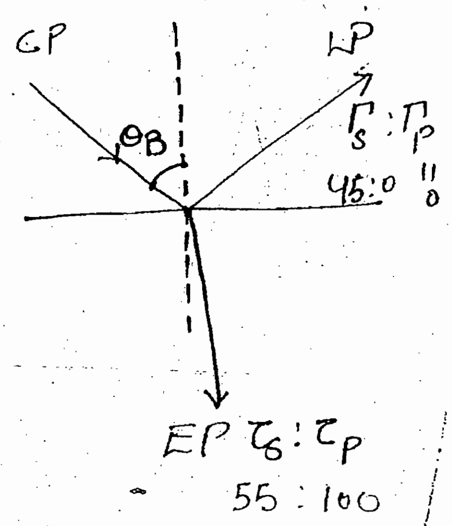
19.

RCP → S + P



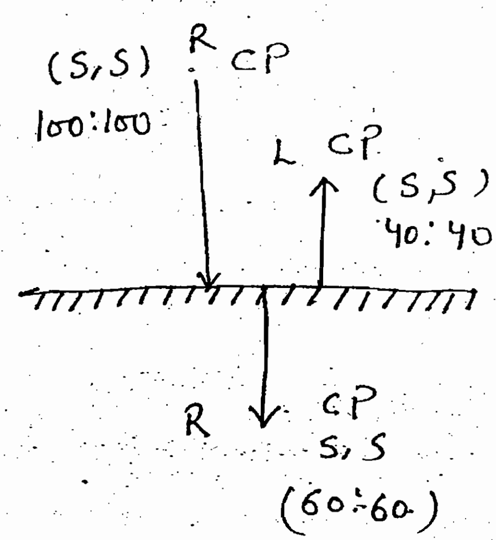
$\tan\theta_B = \tan 60^\circ = \sqrt{\frac{\epsilon_R \epsilon_0}{\epsilon_0}}$

$\Rightarrow \epsilon_R = 3$

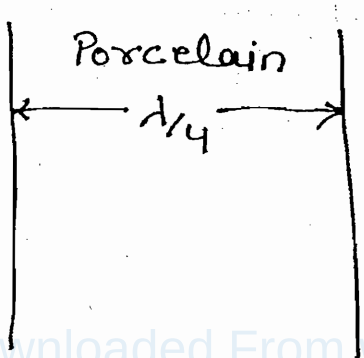


50 Linearly Polarised

51. b



52.



$\text{thickness} = \frac{\lambda}{4} = \left(\frac{3 \times 10^8}{\sqrt{\epsilon_R} \times 10 \times 10^9 \times 4} \right)$

20

53. μ_R [$J_b \rightarrow$ Bound current Density
$\rightarrow$ Due to spinning / revolving electrons constituting a magnetic dipole.

σ [$J_c \rightarrow$ conduction current - moving electron

ϵ [$J_d \rightarrow$ displacement current $\rightarrow$ changing Electric flux

41. $$E = 3\sin(\omega t - \beta z)a_x + 6\sin(\omega t - \beta z + 75^\circ)a_y$$

$$E_0 = \sqrt{3^2+6^2} = 3\sqrt{5}$$

$$P_{avg} = \frac{1}{2}\frac{E_0^2}{\eta}a_z$$

$$= \frac{1}{2}\frac{(3\sqrt{5})^2}{120\pi} \cong 58\text{ mW}$$

OR

$$E = 3\sin(\omega t - \beta z)(a_x + 2e^{j75}a_y)$$

$$H = \left(\frac{3}{120\pi}\right)\sin(\omega t - \beta z)(a_y - 2e^{j75}a_x)$$

$$P_{avg} = \frac{1}{2}\cdot 3(a_x + 2e^{j75}a_y)\times\left(\frac{3}{120\pi}(a_y - 2e^{-j75}a_x)\right)$$

$$= \frac{1}{2}\frac{3\cdot 3}{120\pi}(a_z + 4a_z)$$

$$= \frac{1}{2}\frac{(3\sqrt{5})^2}{120\pi}a_z$$

TRANSMISSION LINES

Energy Propogation

MHz/GHz

V/I Waves | E/H Waves

→ $\sigma = \infty$ | → $\sigma = 0$

→ conductors | → Free space

→ Wired or point to point | → Broadcast wireless

→ Extremely low frequency | → Extremely high frequency

$f < kHz$ | $f > 10^{12}\ Hz$

V_S, I_S → ω source → R, L, G, C Primary constant → Transmission line

↓

$V(x)$
$I(x)$

↓

Propogation

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

E_s, H_s $\longrightarrow$ ω source $\longrightarrow$ μ, σ, ϵ Material constant $\longrightarrow$ Medium $\longrightarrow$ $E(z)$, $H(z)$

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

Primary constants :-

Resistance :-

→ It is the resistance of the conducting material all along the length of the line.

→ $R = \rho \frac{l}{A}$ $\qquad R \propto l$.

It is distributed all along the length

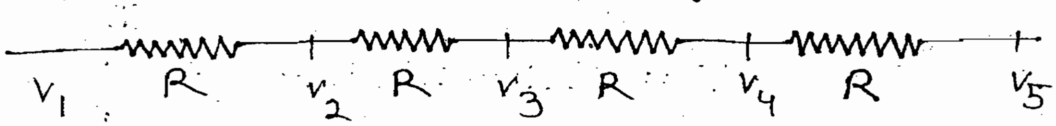
$R = \frac{R}{l} = \text{ohm/m}$

↓

Primary constant

→ It is the resistance at any instance on the line

→ It appears in series along the line

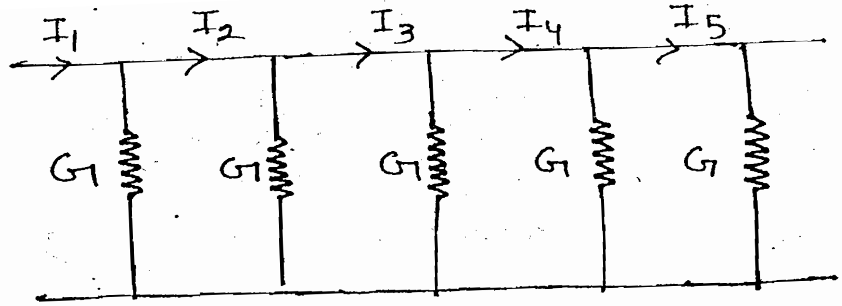


→ It causes a voltage decay all along the line

Conductance :-

→ It is the conductance of the insulating material b/w the lines

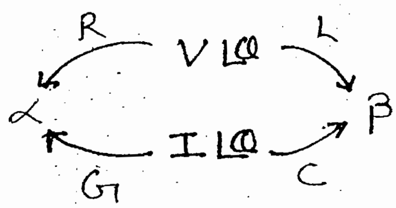
→ It appears in shunt b/w the lines

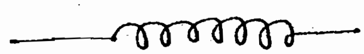


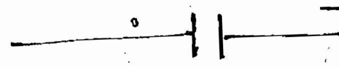
→ It causes current decay

→ It is distributed all along the length

$$G = \frac{G}{l} = \frac{mho}{m}$$

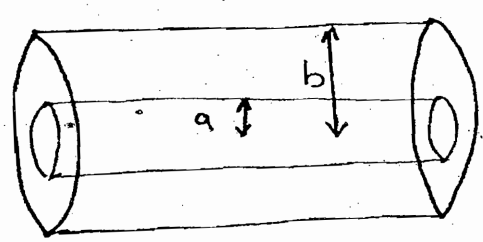


 → flow

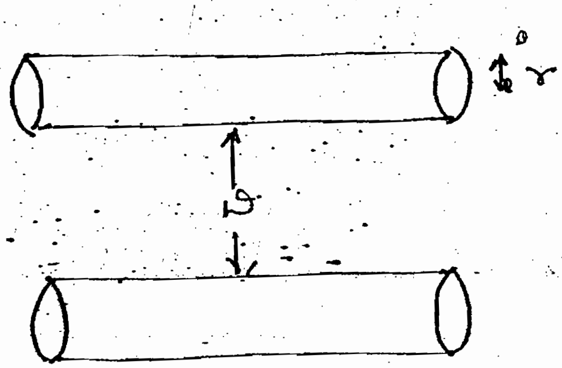
 → accumulati or

<u>Capacitance</u> :-

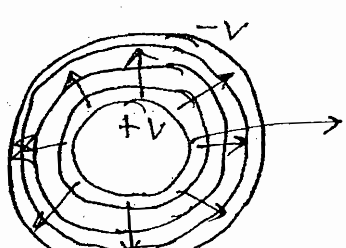
→ It is due to the voltages on the line there is a surrounded E field and hence capacitance



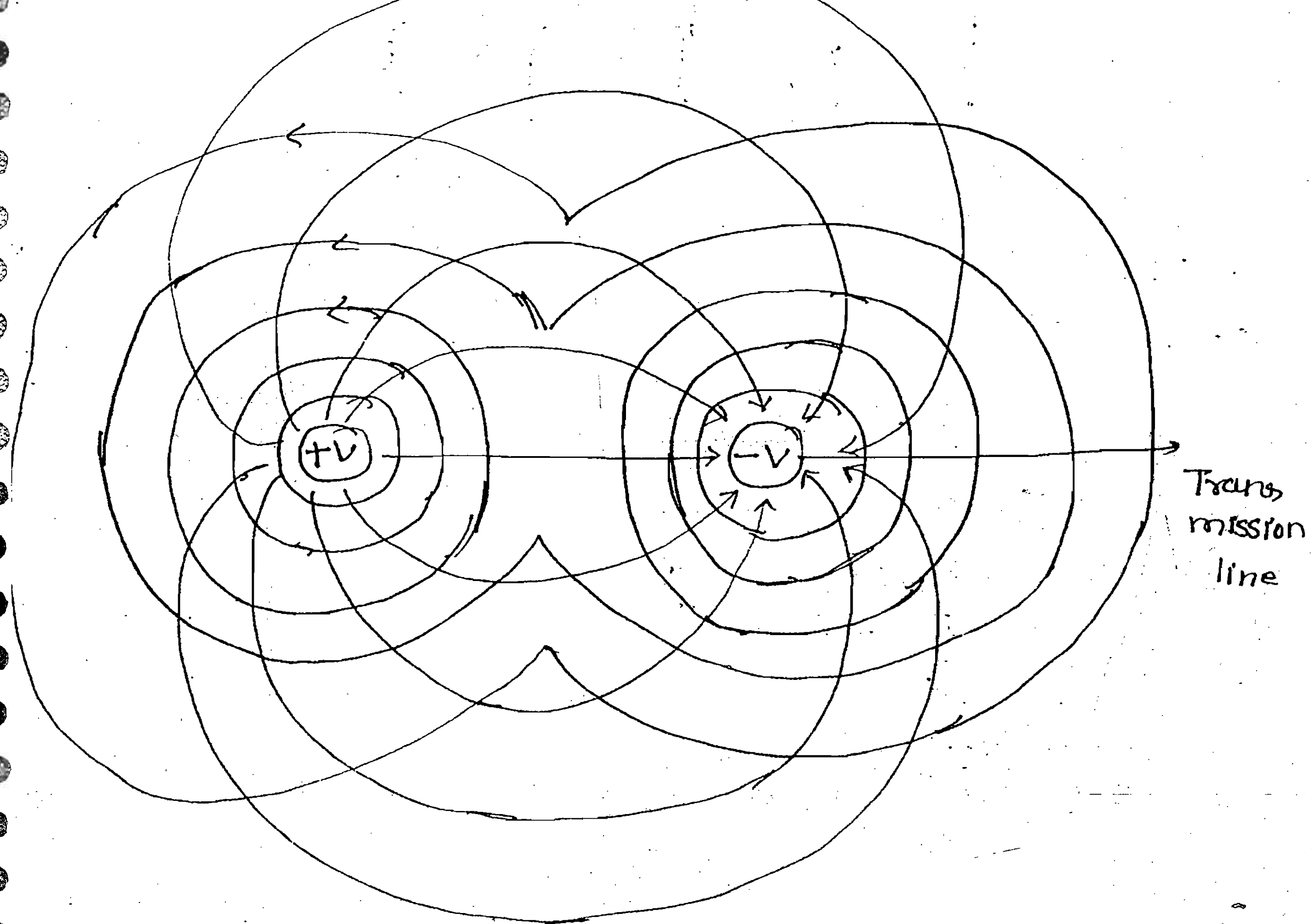
→ Co-axial cable



→ Parallel Wire.



→ Transmission line.



$$C = \frac{2\pi\epsilon l}{\ln(b/a)} \longrightarrow \text{co-axial cable}$$

$$C = \frac{\pi\epsilon l}{\ln(D/r)} \longrightarrow \text{Parallel wire}$$

→ $C \propto l$ → capacitance is distributed

→ $c = \frac{C}{l} = \frac{\text{Farads}}{m}$

→ It appears in shunt b/w the cables

Inductance :-

→ It is due to the currents on the line there is a surrounded H-field and hence inductance

→ $L = \frac{\mu l \ln(b/a)}{2\pi}$ → co-axial cable

$L = \frac{\mu l}{\pi} \cdot \ln(D/r)$ → parallel wire

→ L is distributed i.e. $L \propto l$

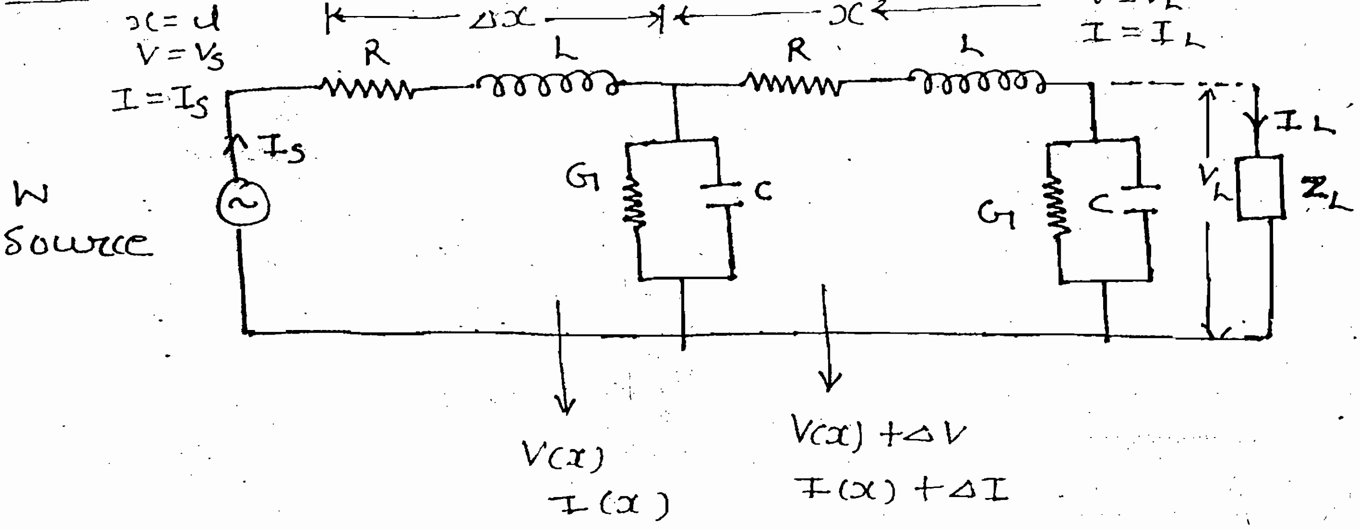
$$\boxed{L = \frac{L}{l} \ \frac{\text{Henry}}{m}}$$

Note :-

$$\boxed{LC = \underset{L}{\text{Distributed}} \times \underset{C}{\text{Distributed}} = \mu\epsilon}$$

For any transmission line and for any geometry.

V/I Equations on the line :-



Z_{series} of Δx length $= (R + j\omega L)\cdot \Delta x$

Y_{shunt} of Δx length $= (G + j\omega C)\cdot \Delta x$

$$\Delta V = I\,(R + j\omega L)\cdot \Delta x$$

$$\frac{\Delta V}{\Delta x} = \frac{dV}{dx} = (R + j\omega L)\,I \quad \text{— (I)}$$

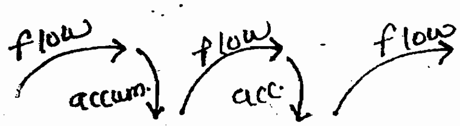
$$\frac{\Delta I}{\Delta x} = \frac{dI}{dx} = (G + j\omega C)\,V \quad \text{— (II)}$$

Take I from (I) & put in (II)

$$\frac{d}{dx}\left[\left(\frac{1}{R + j\omega L}\right)\frac{dV}{dx}\right] = (G + j\omega C)\,V$$

$$\frac{d^2V}{dx^2} = (R + j\omega L)(G + j\omega C) \quad \text{— (III)}$$

$$\frac{d^2I}{dx^2} = (R + j\omega L)(G + j\omega C) \quad \text{— (IV)}$$



$\longrightarrow$ Meaning of Harmonic

Note :-

If the source is time harmonic at one end the effect is space harmonic all along the length of the line.

Let $\sqrt{(R+j\omega L)(G+j\omega C)} = \gamma$ (per m)
↓
Unit

Eq-(III) & (IV) can be written as

$$\frac{d^2V}{dx^2} - \gamma^2 V = 0$$

$$\frac{d^2I}{dx^2} - \gamma^2 I = 0$$

The V/I solution on the line is

$$V(x) = c_1 e^{-\gamma x} + c_2 e^{\gamma x} \quad \text{—— (V)}$$

$$I(x) = c_3 e^{-\gamma x} + c_4 e^{\gamma x} \quad \text{—— (VI)}$$

Applying initial conditions, $(x=0, V=V_L, I=I_L)$

$$V_L = c_1 + c_2 \quad \text{—— (VII)}$$

$$I_L = c_3 + c_4 \quad \text{—— (VIII)}$$

Using eq-(I) in eq-(V)

$$\frac{dV}{dx} = -\gamma c_1 e^{-\gamma x} + \gamma c_2 e^{\gamma x} = (R+j\omega L) I$$

Put $x=0$

$$c_2 - c_1 = \frac{(R+j\omega L) I_L}{\sqrt{(R+j\omega L)(G+j\omega C)}}$$

$$= \sqrt{\frac{R+j\omega L}{G+j\omega C}}\, I_L = I_L Z_0$$

where $Z_0 = \sqrt{\dfrac{R+j\omega L}{G+j\omega C}}$

$$I_L Z_0 = c_2 - c_1 \quad \text{—— (IX)}$$

Similarly $\dfrac{V_L}{Z_0} = c_4 - c_3$ —— (X)

$$\boxed{c_1 = \frac{V_L - I_L Z_0}{2} \qquad c_2 = \frac{V_L + I_L Z_0}{2}}$$

$$\boxed{C_3 = \frac{I_L - V_L/Z_0}{2}, \quad C_4 = \frac{I_L + V_L/Z_0}{2}}$$

$$V(x) = \frac{V_L}{2}\left[\left(1 - \frac{Z_0}{Z_L}\right)e^{-\gamma x} + \left(1 + \frac{Z_0}{Z_L}\right)e^{\gamma x}\right]$$

$$V(x) = \frac{V_L(Z_L + Z_0)}{2Z_L}\left[e^{\gamma x} + \left(\frac{Z_L - Z_0}{Z_L + Z_0}\right)e^{-\gamma x}\right]$$

$$V(x) = V_0\left(e^{\gamma x} + \left(\frac{Z_L - Z_0}{Z_L + Z_0}\right)e^{-\gamma x}\right)$$

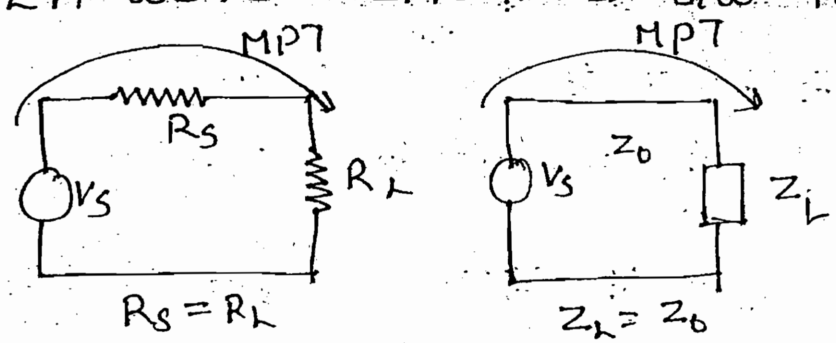
$$I(x) = \frac{I_L(Z_L + Z_0)}{2Z_0}\left[e^{\gamma x} + \left(\frac{Z_0 - Z_L}{Z_0 + Z_L}\right)e^{-\gamma x}\right]$$

$e^{\gamma x} \longrightarrow -x$, source-load forward wave

$e^{-\gamma x} \longrightarrow +x$, load-source, reflected wave

→ Every transmission line has two waveforms i.e. the forward and reflected and the reflected wave does not exist in one single condition i.e. $Z_L = Z_0$. This is called as matched condition and this is a condition where complete source power is absorbed by the load.

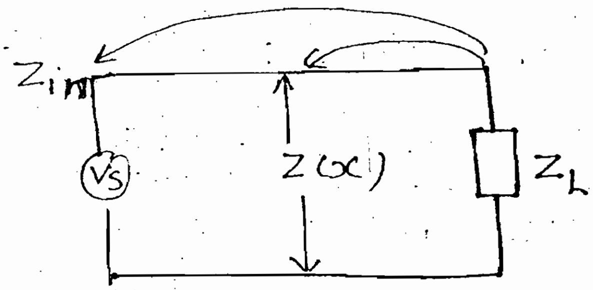
→ This is similar to max power transfer in N/w or EM wave reflections b/w materials



Note:-

- Z_0 is a unique impedance of the line with which if the line is terminated the reflections on the line and hence called as characteristic impedance

Impedance on line (z) and i/p impedance (Z_{in}):-



Note:-

Due to the loaded one end and primary constan n the line, impedance is different on different point of line such that

$$Z(x) = \frac{V(x)}{I(x)} \qquad Z_{in} = Z(l)$$

$$V(x) = \frac{V_L}{Z_L}\left(\frac{(Z_L+Z_0)}{2}e^{\gamma x} + \frac{(Z_L-Z_0)}{2}e^{-\gamma x}\right)$$

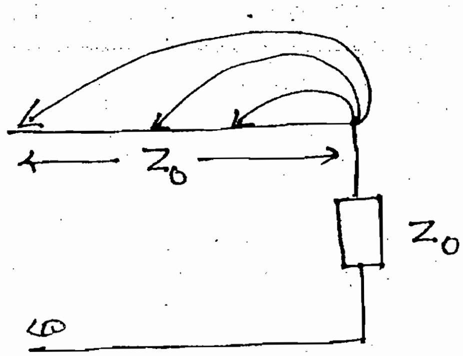
$$V(x) = V_L \cosh(\gamma x) + I_L Z_0 \sinh(\gamma x)$$

Similarly

$$I(x) = I_L \cosh(\gamma x) + \frac{V_L}{Z_0}\sinh(\gamma x)$$

$$Z(x) = Z_0\left[\frac{Z_L \cosh(\gamma x) + Z_0 \sinh(\gamma x)}{Z_0 \cosh(\gamma x) + Z_L \sinh(\gamma x)}\right]$$

If $Z_L = Z_0$ then $Z(x) = Z_0 = Z_L = Z_{in}$



Matched line

→ For a matched line the impedance anywhere on the line is also same Z_0

Note:-

Z_0 is that unique impedance with which if the line is terminated impedance on the line is also the same

Short circuit and open circuit lines:-

→ If $Z_L = 0$ ⇒ SC line.

$$Z_{in} = Z_{SC} = Z_0 \tanh(\gamma x)$$

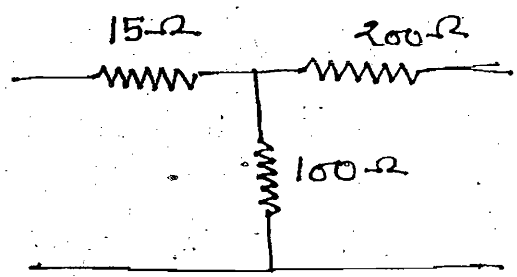
→ If $Z_L = \infty$ ⇒ O.C line

$$Z_{in} = Z_{OC} = Z_0 \coth(\gamma x)$$

$$\Rightarrow \boxed{Z_0 = \sqrt{Z_{SC} \cdot Z_{OC}}}$$

It is geometric mean of S.C and O.C i/p impedances.

eg:-

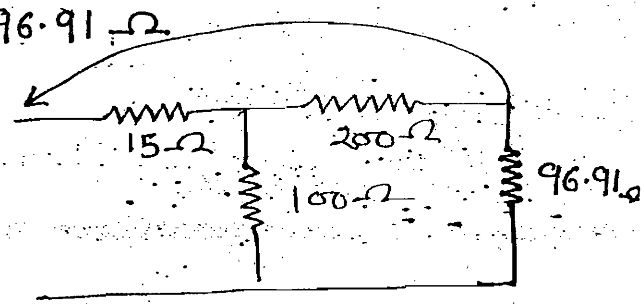


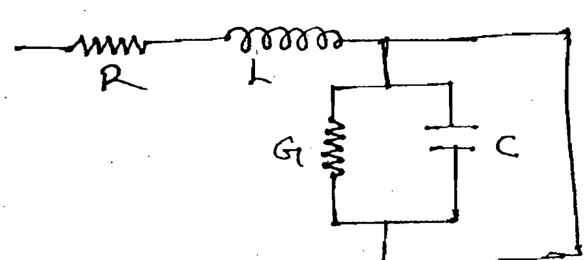
$$Z_{OC} = 115\Omega$$

$$Z_{SC} = 15 + \frac{100 \cdot 200}{100 + 200}$$

$$= 81.67\Omega$$

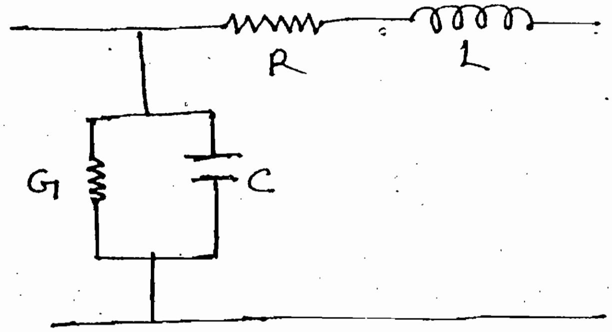
$$Z_0 = \sqrt{81.67 \times 115} = 96.91\Omega$$

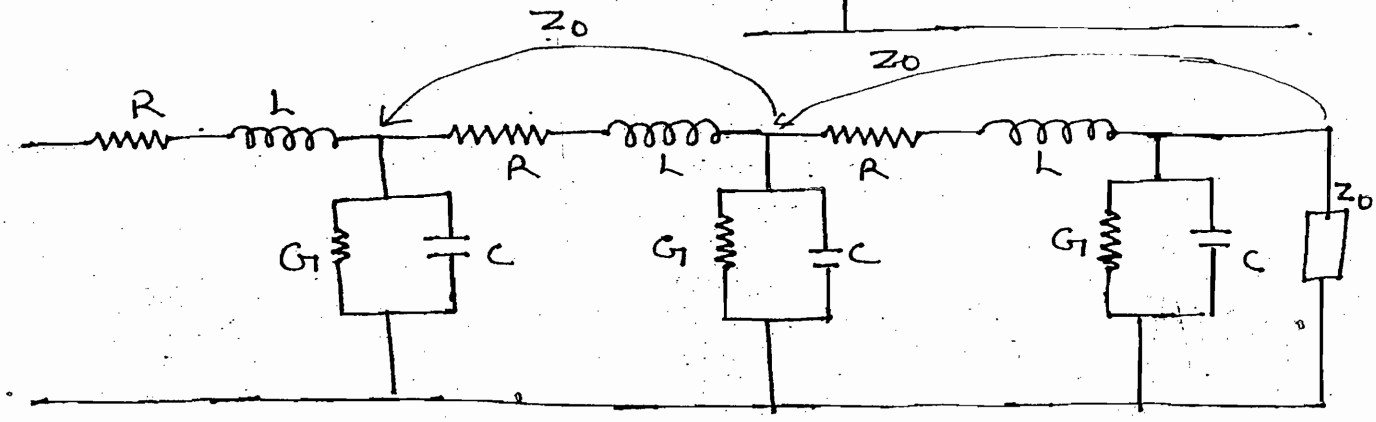




$$Z_{SC} = R + j\omega L$$

$$Z_{oc} = \frac{1}{G + j\omega C}$$





Note:—

- Any N/w either discrete or continous when terminated with its characteristics impedance the same impedance appears on other end
- For a transmission line whose every RLGC branch having Z_0 characteristics impedance when terminated with Z_0 at one end the loading effect appears to be the same anywhere on the line.
- Lossless line and distortionless line,

$$\gamma = \sqrt{R}$$

Lossless line and distortionless line:—

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \text{propogation constant}$$

$$= \alpha + j\beta$$

If $\alpha = 0$ everywhere on the line, the line is said to be lossless

(I) $R = G = 0$

(II) $\left.\begin{array}{l}\omega L >> R \\ \omega C >> G\end{array}\right\}$ High frequency lines

$$\gamma = j\omega\sqrt{LC} \quad \text{(purely imaginary)}$$

$$= j\beta$$

$$\beta = \omega\sqrt{LC}$$

$$V_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu_0 \epsilon_0 \epsilon_R}} = \frac{3\times10^8}{\sqrt{\epsilon_R}}$$

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{L}{C}} = \text{real}$$

$$= \sqrt{\frac{\mu_0 \ln(b/a)}{2\pi} \frac{\ln(b/a)}{2\pi\epsilon}}$$

$$= \sqrt{\frac{\mu_0}{\epsilon_0 \epsilon_R}} \cdot \frac{\ln(b/a)}{2\pi}$$

$$\boxed{Z_0 = \frac{60 \ln(b/a)}{\sqrt{\epsilon_R}}} \longrightarrow \text{Co-Axial cable}$$

→ Z_0 is a design aspect of the line and depends on the cable dimensions but not on R, L, G, C or operational frequency of the line.

$$\boxed{Z_0 = \frac{120 \ln(b/a)}{\sqrt{\epsilon_R}}} \longrightarrow \text{Parallel wire.}$$

Distortionless line:-

If the phase is linear the wave is said to distortionless i.e. $\beta \propto \omega$ → For distortionless line

→ Equal Rise time = Fall time is characteristic of distortionless line

Series arm time constant = Shunt arm time constant

$$\frac{L}{R} = \frac{C}{G} \Rightarrow \boxed{LG = RC}$$

$$\gamma = \sqrt{R\left(1+\frac{j\omega L}{R}\right) G\left(1+\frac{j\omega C}{G}\right)}$$

$$= \sqrt{RG}\left(1+\frac{j\omega L}{R}\right) = \text{complex}$$

$$Z_0 = \sqrt{\frac{R(1+j\omega L/R)}{G(1+j\omega C/G)}} = \sqrt{\frac{R}{G}} = \text{Real}$$

Note:–

Every lossless line $\Rightarrow$ Distortionless $\not\Rightarrow$ lossless

$R = G = 0 \longrightarrow LG = RC$

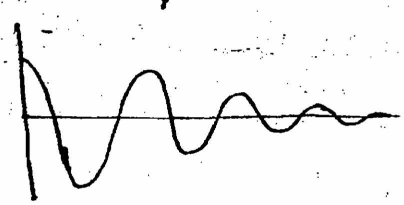
$\beta = \omega\sqrt{LC} \longrightarrow \beta \propto \omega$

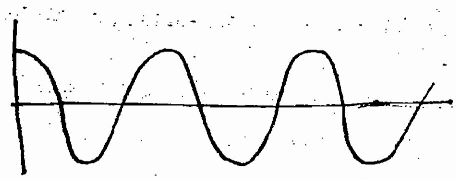
Workbook:–

1) $$Z_{in} = Z_0\left[\frac{Z_L \cosh(j\beta l) + Z_0 \sinh(j\beta l)}{Z_0 \cosh(j\beta l) + Z_L \sinh(j\beta l)}\right]$$

$$\cosh\theta = \frac{e^{\theta}+e^{-\theta}}{2} \qquad \cosh(j\theta) = \frac{e^{j\theta}+e^{-j\theta}}{2}$$

$\downarrow$ $\qquad\qquad\qquad\qquad$ $\downarrow = \cos\theta$





$\sinh(j\theta) = j\sin\theta$

$$Z_{in} = Z_0 \left[\frac{Z_L \cos\beta l + j Z_0 \sin\beta l}{Z_0 \cos\beta l + j Z_L \sin\beta l} \right]$$

(I) $l = \frac{\lambda}{8}$ $\quad \beta l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{8} = \frac{\pi}{4}$

$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0}{Z_0 + jZ_L} \right]$$

(II) $l = \lambda/4$ $\quad \beta l = \frac{\pi}{2}$ $\quad Z_{in} = \frac{Z_0^2}{Z_L}$

(III) $l = \frac{\lambda}{2}$ $\quad \beta l = \pi$ $\quad Z_{in} = Z_L$

$Z_L = j50$ $\quad Z_0 = 50\Omega$

$$Z_{in} = 50 \left[\frac{j50 + j50}{50 + j50} \right] = \infty \rightarrow O/C$$

$$\underset{(\lambda/4)}{Z_{in}} = \frac{50 \times 50}{j\,50} = -j\,50\,\Omega$$

$$\underset{(\lambda/2)}{Z_{in}} = Z_L = j50\,\Omega$$

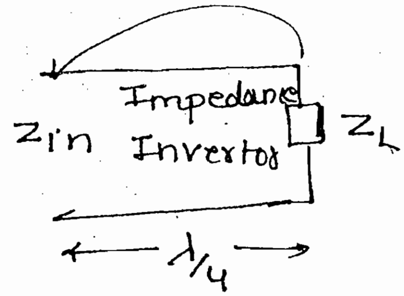
Note 1:-

→ βl is a critical aspect of the line that decides the nature and performance of the line and it is called as electrical paths line

→ length/λ relationship is a critical aspect.

Note 2:-

$l = \lambda/4$ $\quad Z_{in} = \frac{Z_0^2}{Z_L}$



It's one end impedance has opposite nature to other end impedance hence called as impedance invertor

Note 3:-

$$l = \lambda/2 \qquad Z_{in} = Z_L$$

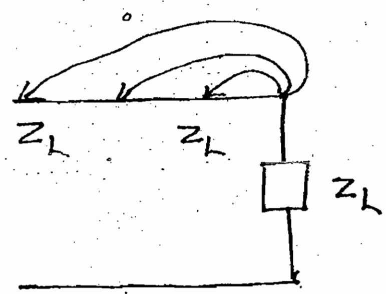
It is called as impedance reflector

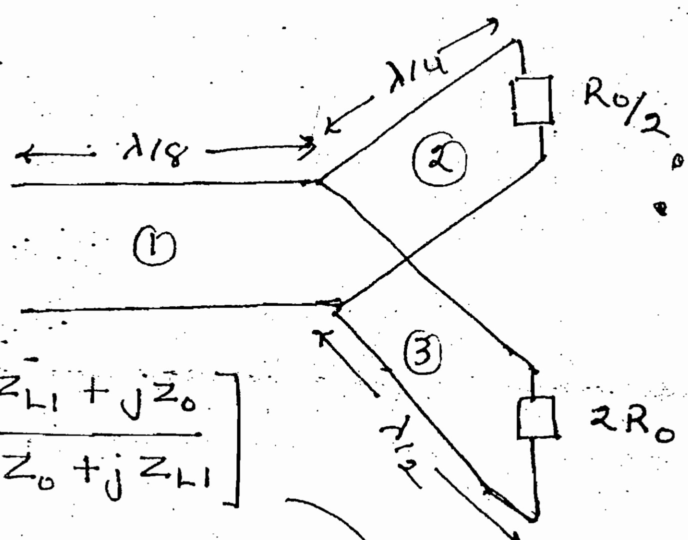
Note 4:-

If $l \to 0$ or $\lambda \to \infty$ where $\beta l \to 0$

$$Z(x) = Z_L$$

For electrically short line the wire is a simple connecting wire or a short ckt.





$$Z_{in2}\,(\lambda/4) = \frac{R_0^2}{R_0/2} = 2R_0$$

$$Z_{in3}\,(\lambda/2) = 2R_0$$

$$Z_{in1}\,(\lambda/8) = Z_0\left[\frac{Z_{L1} + jZ_0}{Z_0 + jZ_{L1}}\right]$$

$$Z_{L1} = Z_{in2} \,||\, Z_{in3}$$

$$= R_0\left[\frac{R_0 + jR_0}{R_0 + jR_0}\right]$$

$$= R_0$$

When loads ... interchanged

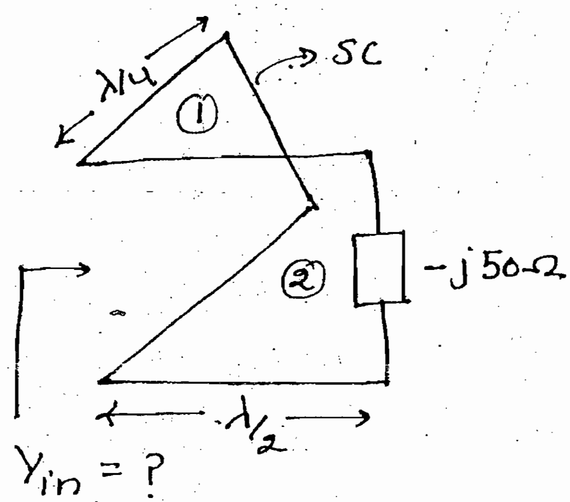
$$\underset{(\lambda/4)}{Z_{in2}} = \frac{R_0^2}{2R_0} = \frac{R_0}{2}$$

$$\underset{(\lambda/2)}{Z_{in3}} = \frac{R_0}{2}$$

$$\underset{(\lambda/8)}{Z_{in1}} = Z_0\left[\frac{Z_{L1} + jZ_0}{Z_0 + jZ_{L1}}\right] = R_0\left[\frac{R_0/4 + jR_0}{R_0 + j\frac{R_0}{4}}\right]$$

$$= R_0\left[\frac{1+j4}{4+j}\right]$$

$$Y_{in} = Y_{in1} + Y_{in2}$$



$$\underset{(\lambda/4)}{Y_{in1}} = \frac{Z_L}{Z_0^2} = 0$$

$$\underset{(\lambda/2)}{Y_{in2}} = \frac{1}{Z_L} = \frac{1}{-j50}$$

$$= j0.02$$

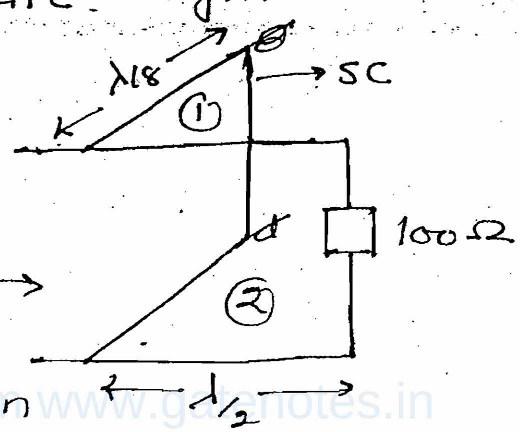
$$Z_{in} = (Z_{in1} \parallel Z_{in2})$$

$$= (\infty \parallel -j50) = -j50$$

$$Y_{in} = j0.02 \mho$$

Stub ⟹ SC line of finite length

$$Y_{in} = Y_{in1} + Y_{in2}$$



$$\underset{(\lambda/8)}{Z_{in1}} = Z_0\left[\frac{0 + jZ_0}{Z_0 + j0}\right]$$

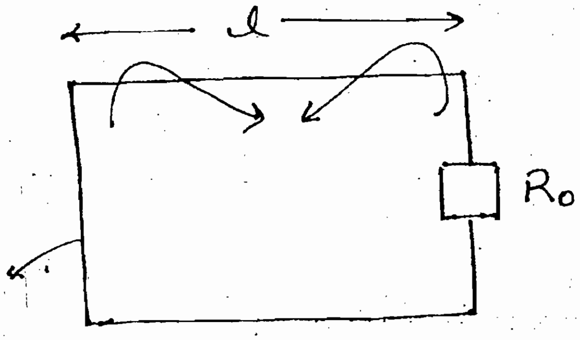
$$= jZ_0 = j50\,\Omega$$

$$Y_{in1} = \frac{1}{j50} = -j0.02$$

$$Y_{in2} = \frac{1}{Z_L} = \frac{1}{100} = 0.01$$

$$Y_{in} = Y_{in1} + Y_{in2} = 0.01 - j0.02 \ \mho$$

8.

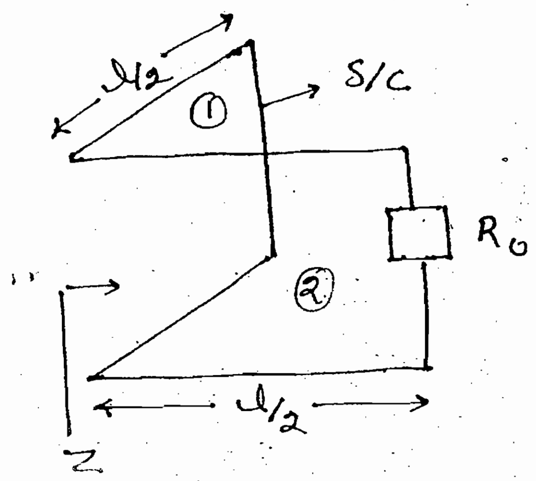


$$l = \lambda/4$$

$$Z_{in1} = R_0 \left[\frac{0 + jR_0}{R_0 + j0}\right]_{SC}$$

$$= jR_0$$

$$Z_{in2} = R_0$$



$$Z = \frac{jR_0 \cdot R_0}{jR_0 + R_0}$$

$$= \frac{jR_0}{j+1}$$

$l = \lambda \quad Z = R_0$

$l = \lambda/8 \quad Z = SC$

9. $V_p = 2\times10^8 \quad f = 10\,MHz \quad \lambda = 20m$

$l = 10m \quad l = \lambda/2$

$$Z_{in} = Z_L = 30 - j40\,\Omega$$

10. $f' = 5\,MHz \quad \lambda' = 40m \quad l = \frac{\lambda'}{4}$

$$Z_{in} = \frac{Z_0^2}{Z_L} = \frac{50\times50}{30-j40} = 30 + j40$$

11. $Z_{in} = j60 = Z_{sc} = Z_0 \tanh(j\beta l)$

$$= jZ_0 \tan\beta l$$

$$\beta l = \frac{\pi}{4}$$

$$= jZ_0$$

$Z_0 = 60\,\Omega$

2. $f' = 2f$

$12\,MHz \longrightarrow 24\,MHz$

$$\lambda' = \lambda/2$$

$$\beta' l = 2\beta l = \pi/2$$

$$\therefore Z_{in} = Z_{sc} = jZ_0 \tan \pi/2 = \infty$$

3. $f = 12\,MHz$

$$Z_{in} = j60 = j\omega L \Rightarrow L = \frac{60}{2\pi \times 12 \times 10^6}$$

$$= \frac{2.5}{\pi}\,\mu H$$

Summary :—

$$Z_{sc} = jZ_0 \tan \beta l$$

$$\left.\begin{array}{l} 0 < \beta l < \frac{\pi}{2} \\ 0 < l < \lambda/4 \end{array}\right\} Z_{in} \text{ is inductive}$$

$$\left.\begin{array}{l} \frac{\pi}{2} < \beta l < \pi \\ \lambda/4 < l < \lambda/2 \end{array}\right\} Z_{in} \text{ is capacitive}$$

$$Z_{oc} = -jZ_0/\tan \beta l$$

Z_{in} inductive $\longrightarrow$ Z_{in} is capacitive

Z_{in} is capacitive $\longrightarrow$ Z_{in} is inductive.

→ one end of a line is S.C or O.C then the other end is purely reactive ..

14. (a) $Z_{in} = -jZ_0$ For s/c line $\lambda/8$ length ✗

$\to jZ_0$

✓(b) $Z_{in} = \pm j\infty$ for SC line $\lambda/4$ length

$$Z_{sc} = jZ_0 \tan\beta l$$

$$Z_{in} = \frac{Z_0^2}{Z_L}$$

✗(c) $Z_{in} = -jZ_0 \longrightarrow$ o/c line $\rightarrow \lambda/2$ length

$\downarrow$

$$Z_{in} = Z_L$$

✓(d) $Z_{in} = Z_0$

15. $$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{LC}{C^2}} = \frac{\sqrt{\mu_0 \epsilon_0 \epsilon_R}}{C} = \frac{\sqrt{\epsilon_R}}{V_p \cdot C}$$

16. (a) $\alpha =$ even , $\beta =$ odd ✗

(b) $\alpha =$ constant $\beta = k\omega$ ✓

(c) ✗ $\frac{L}{C} =$ constant $= \frac{R}{G}$

(d) Matched line ($Z_L = Z_0$) ✗

17. $\beta l = \omega\sqrt{LC}\, l = 108°$

Reflection Coefficient :—

$$V(x) = \underbrace{V_0 e^{j\beta x}}_{V_f} + \underbrace{V_0 \left(\frac{Z_L - Z_0}{Z_L + Z_0}\right) e^{-j\beta x}}_{V_r}$$

$\frac{V_r}{V_f}$ = Reflection coefficient for the voltages anywhere on the line $= \left(\frac{Z_L - Z_0}{Z_L + Z_0}\right) e^{-j2\beta x}$

$= \Gamma(x)$

$$\boxed{\Gamma(x) = \frac{Z(x) - Z_0}{Z(x) + Z_0}}$$

At $x = 0$ Γ at load $= \frac{Z_L - Z_0}{Z_L + Z_0}$ = complex

$= |\Gamma| \angle\theta = \left|\frac{V_r}{V_f}\right| \angle\theta$

$|\Gamma| \rightarrow$ It is the ratio of reflected voltage to the forward voltage amplitude

$\angle\theta \rightarrow$ It is the time delay or phase difference b/w the reflected and forward voltage.

Γ is the measure of mismatch b/w the expected impedance and the actual impedance Z_L at the load.

$\Gamma(x)$ is the measure of mismatch b/w the expected impedance Z_0 and the actual impedance $Z(x)$ anywhere on the line.

Note :-

(I) $$\Gamma_I = \frac{Z_0 - Z_L}{Z_0 + Z_L} = -\Gamma_V$$

If V_f and I_f they are inphase

$$V_0 e^{j\beta x} + V_0 \left(\frac{Z_L - Z_0}{Z_L + Z_0}\right) e^{-j\beta x}$$

$$I_0 e^{j\beta x} - I_0 \left(\frac{Z_L - Z_0}{Z_L + Z_0}\right) e^{-j\beta x}$$

then V_r and I_r are out of phase by 180^0

(II) $$0 \leq |\Gamma| \leq 1$$

Matched line

Complete Reflection (Mismatch)

Lecture - 13

Four Cases of complete mismatch on a line ($|\Gamma| = 1$) :-

Case -(1) :-

$Z_L = jR_0$ = Inductive load

$Z_0 = R_0$ = lossless line

$$\therefore |\Gamma| = 1$$

$V_f \rightarrow \sin$

$V_r \rightarrow \cos$

$$\Gamma = \frac{jR_0 - R_0}{jR_0 + R_0} = \frac{j-1}{j+1}$$

$$= \frac{\sqrt{2} \angle 135}{\sqrt{2} \angle 45^\circ}$$

$$= 1 \angle 90^\circ = j$$

Case-(II) :-

$$Z_L = -jR_0 = \text{capacitive - Load}$$

$$Z_0 = R_0 = \text{lossless line}$$

$$\Gamma = \frac{-jR_0 - R_0}{-jR_0 + R_0} = \frac{-j-1}{-j+1}$$

$$= \frac{\sqrt{2}\,\angle 45^\circ}{\sqrt{2}\,\angle 135} = -j$$

Note :-

Real Power dissipation in any resistive load is always zero.

Case-(III) :-

$$Z_L = 0 \Rightarrow \text{S.C load}$$

$$Z_0 = R_0 = \text{lossless line}$$

$$\Gamma = \frac{0 - Z_0}{0 + Z_0} = -1 = 1\,\angle 180^\circ$$

$V_f \rightarrow \sin$

$V_r = -\sin$

$$V_f + V_r = 0$$

Voltage at S.C is always zero

Case-(IV) :-

$$Z_L = \infty \Rightarrow \text{O.C load}$$

$$Z_0 = R_0 = \text{lossless line}$$

$$\Gamma = \frac{1 - Z_0/Z_L}{1 + Z_0/Z_L} = 1\,\angle 0$$

$$V_f + V_r = V_{max}$$

Voltage at O.C is always maximum

$$|\Gamma| = 1$$

$V_f \rightarrow \sin$

$V_r = \sin$

Standing Wave and SWR :-

$$\beta x - (-\beta x + \theta) = 2n\pi$$

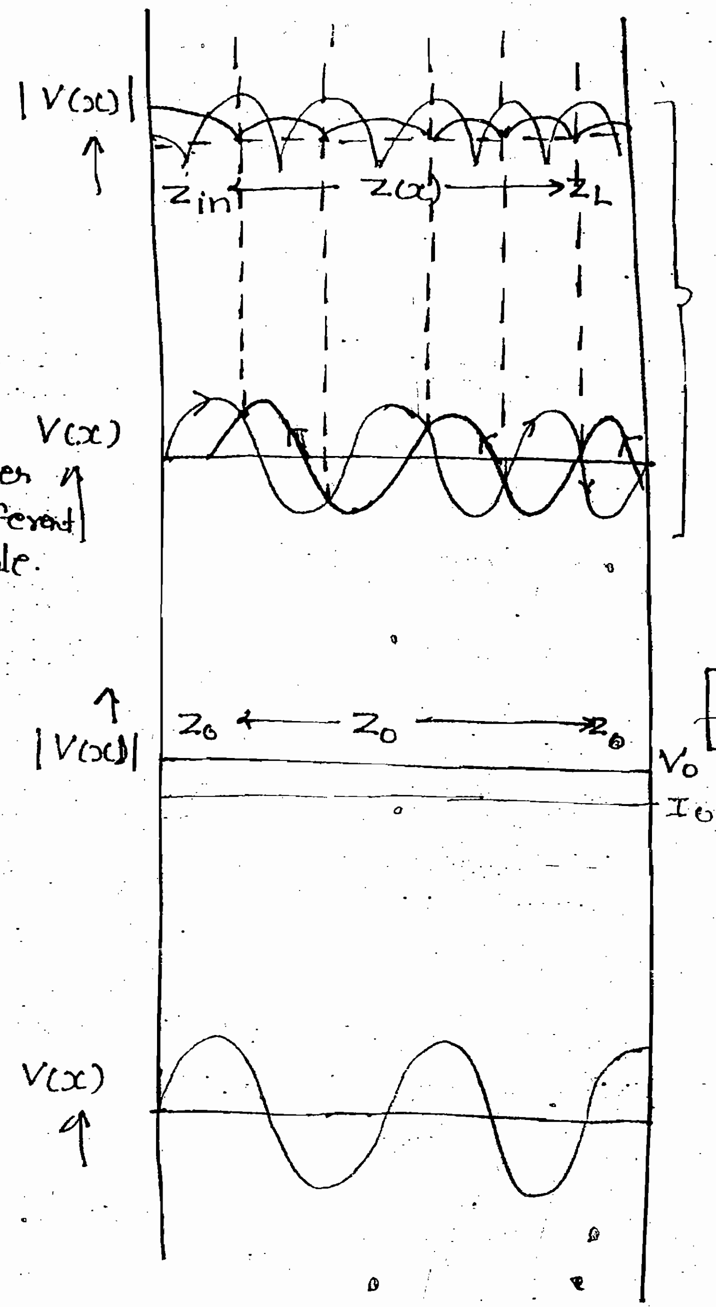
$n = 0, 1, 2 \ldots \quad |V(x)|$

$$\boxed{2\beta x_{max} = 2n\pi + \theta}$$

Voltage Maxima
→ current minima
→ Impedance maxima

Mismatched line

These two waves have different amplitude.



$$\boxed{2\beta x_{min} = (2n+1)\pi + \theta}$$

Voltage minimas
→ current maximas
→ Impedance minima.

Matched line.

$$V(x) = V_0 e^{j\beta x} + V_0 |\Gamma| e^{j\theta} e^{-j\beta x}$$
$$= V_0 e^{j\beta x} + V_0 |\Gamma| e^{j(-\beta x + \theta)}$$

The magnitude plot of a single harmonic matched line as a constant value straight line graph. The magnitude plot of two harmonics mismatched line as shown below :-

$$|V(x)| = V_{max} = V_0 + V_0 |\Gamma| \quad \text{when they are inphase}$$

$$\beta x - (-\beta x + \theta) = 2n\pi$$

$|V(x)|$

$n = 0, 1, 2$

$$\boxed{2\beta x_{max} = 2n\pi + \theta}$$

Voltage Maxima Positions.

$|V(x)| = V_{min} = V_0 - V_0|\Gamma|$ when they are out of phase

$$\boxed{2\beta x_{min} = (2n+1)\pi + \theta}$$

$\longrightarrow$ Voltage Minima

→ The magnitude plot of two harmonics travelling in opposite direction having periodic interference results in addition and cancelation leading to standing wave formation

Note 1 :-

The current also has identical standing wave pattern with the voltage maxima coincide with current minima

Note 2 :-

As the impedance is different at different point of line voltage and current distribution is also extremely spread on the line such that

$$Z_{max} = \frac{V_{max}}{I_{min}}$$

$$Z_{min} = \frac{V_{min}}{I_{max}}$$

Note 3 :-

$$\left.\begin{array}{l} \beta x - \frac{2\pi}{\lambda}x - \text{Periodic} - 2\pi \\ x \rightarrow \text{Periodic} - \lambda \end{array}\right\} \text{Harmonic}$$

$$\left.\begin{array}{l} 2\beta x - \frac{2\pi}{\lambda/2}x \rightarrow \text{Periodic} - 2\pi \\ x \longrightarrow \text{Periodic} - \lambda/2 \end{array}\right\} \text{Standing wave}$$

Maxima to Maxima distance on standing wave is $\lambda/2$

Standing Wave Ratio (SWR):-

It can be VSWR or CSWR

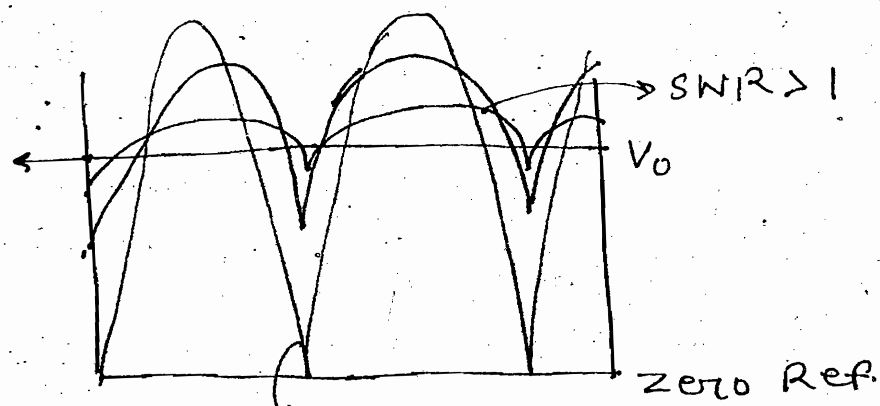
$$= \frac{V_{max}}{V_{min}} \text{ or } \frac{I_{max}}{I_{min}}$$

$$\boxed{SWR = \frac{V_0 + V_0|\Gamma|}{V_0 - V_0|\Gamma|} = \frac{1+|\Gamma|}{1-|\Gamma|}}$$

$$0 \leq |\Gamma| \leq 1$$

$$\boxed{1 \leq SWR < \infty}$$

Matched line — Complete Mismatch



Summary:-

$Z_0 \downarrow\uparrow$, $V_0 \uparrow\downarrow$, $I_0 \uparrow\downarrow$ } SWR

SWR is a measure of mismatch on the line

$$Z_{max} = \frac{V_{max}}{I_{min}} = \frac{V_0 + V_0|\Gamma|}{I_0 - I_0|\Gamma|}$$

$$Z_{max} = Z_0 \cdot SWR$$

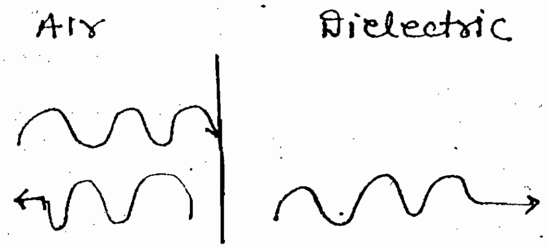
$$Z_{min} = Z_0/SWR$$

Workbook :-

8.

$$ESWR = 5 = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$|\Gamma| = \frac{5-1}{5+1} = \frac{2}{3}$$



0 or 180°

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{2}{3}\angle\theta = \pm\frac{2}{3}$$

$\eta_1 \rightarrow 120\pi$ (real)

$\eta_2 \rightarrow \dfrac{120\pi}{\sqrt{\epsilon_R}}$ (real)

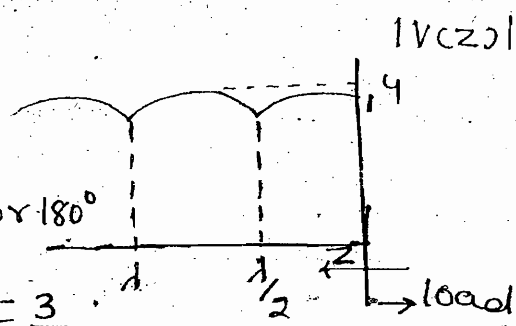
As $\eta_2 < \eta_1$

$$\Gamma = -\frac{2}{3} = \frac{\eta_2 - 120\pi}{\eta_2 + 120\pi} \Rightarrow \eta_2 = 24\pi$$

9.

$$VSWR = 4$$

$$|\Gamma| = \frac{4-1}{4+1} = \frac{3}{5}$$



0° or 180°

$$\Gamma = \frac{Z_L - 50}{Z_L + 50} = \frac{3}{5}\angle\theta = \pm\frac{3}{5}$$

Γ → real, Z_L → real, 50 → real

Vmin is exactly at the load, Zmin is at the load

$$Z_L < Z_0$$

$\Rightarrow$ Γ is negative

$$\Gamma = -\frac{3}{5} = \frac{Z_L - 50}{Z_L + 50} \Rightarrow Z_L = 12.5\,\Omega$$

→ 1st voltage maxima occurs at $\lambda/4$

$$2 \cdot \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = 2n\pi + \theta \quad (n=0) \Rightarrow \boxed{\theta = \pi}$$

Note:-

→ If Z_L, Z_0 are real

$Z_L > Z_0$

Γ is real & positive

Γ's phase is 0

$2\beta x_{max} = 2n\pi + \theta \Rightarrow x_{max} = 0$

First voltage maxima exactly occurs

eg:- OC line where $\Gamma = +1$

→ If Z_L, Z_0 are real

$Z_L < Z_0$

Γ is real and negative

First voltage minima exactly occurs

eg:- SC line where $\Gamma = -1$

→ $$VSWR = \frac{1 + \left|\frac{Z_L - Z_0}{Z_L + Z_0}\right|}{1 - \left|\frac{Z_L - Z_0}{Z_L + Z_0}\right|}$$

If Z_L, Z_0 are real & $Z_L > Z_0$

$$VSWR = \frac{1 + \frac{Z_L - Z_0}{Z_L + Z_0}}{1 - \frac{Z_L - Z_0}{Z_L + Z_0}}$$

$$\boxed{VSWR = \frac{Z_L}{Z_0}}$$

If Z_L, Z_0 are real

$Z_L < Z_0$ $\boxed{VSWR = ?}$

20 S 24. $\Gamma(\lambda/8) = ?$

$|\Gamma|$ Γ at load $= 0.6 \angle 60^\circ$

Γ $$\Gamma(x) = \frac{Z(x) - Z_0}{Z(x) + Z_0} = \frac{Z_L - Z_0}{Z_L + Z_0} e^{-}$$

$$\Gamma(\lambda/8) = \Gamma(0)\, e^{-j2\beta \cdot \frac{\lambda}{8}}$$

$$= 0.6\, e^{j60}\, e^{-j2 \cdot \frac{2\pi}{\lambda} \cdot \frac{\lambda}{8}}$$

21. 1st ← 17

7cm

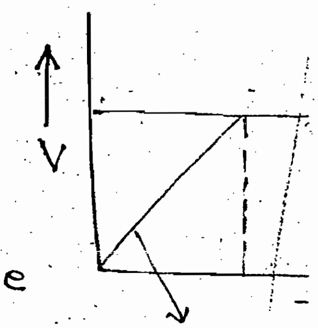
n=0

$$= 0.6\, e^{j60}\, e^{-j90}$$

$$= 0.6 \angle -30^\circ$$

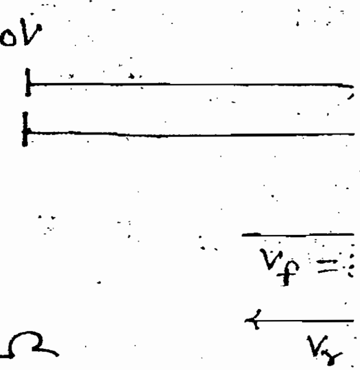
2 25. 30 V D.C Battery

D.C Transients



During the transient state even DC has time variations and all AC fundamentals are applicable

$V_f + V_r =$ Adding



$$\Gamma = \frac{10}{30} = \frac{Z_L - 50}{Z_L + 50}$$

$$\Rightarrow Z_L = 100\,\Omega$$

12.

$$I_L \text{ (Steady state)} = \frac{30}{Z_L}\,\Omega$$

n=0

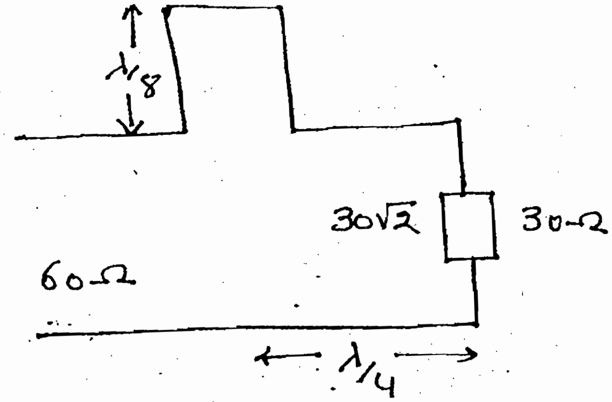
2.2

$$= \frac{30}{100}$$

$$= 0.3 \text{ Amp.}$$

26.

$$VSWR = \frac{1+|\Gamma|}{1-|\Gamma|}$$



$$= \frac{\sqrt{17}+1}{\sqrt{17}-1}$$

$$= 1.63$$

$$Z_{L_1} = Z_{in2} + Z_{in3}$$

$$= 60 + j30\,\Omega$$

$$Z_{in2} = j30\,\Omega$$

$$Z_{in3} = \frac{(30\sqrt{2})^2}{30} = 60\,\Omega$$

$$\Gamma = \frac{Z_{L_1} - jZ_0}{Z_{L_1} + jZ_0} = \frac{60 + j30 - 60}{60 + j80} = \frac{j}{j+4}$$

$$|\Gamma| = \frac{1}{\sqrt{17}}$$

27.

$$V_f = 25 \sin(bx - 75^\circ)$$

$$\Gamma = 0.6 \angle -30^\circ$$

$$V_r = ?$$

$$\frac{|V_r|}{|V_f|} = 0.6 \qquad |V_r| = 15$$

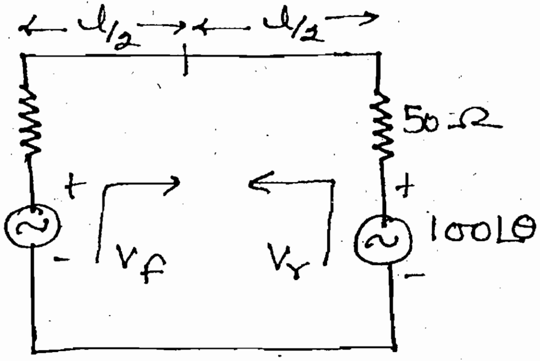
$$V_r \text{ phase} = V_f \text{ phase} + \Gamma_{phase}$$

$$= -75^\circ - 30 = -105^\circ$$

$$V_r = 15 \sin(-bx - 105^\circ)$$

28. V_f & V_r are inphase and add up at centre

I_f & I_r cancel at the centre



$$|I_f| = |I_r|$$

$$\Rightarrow I_{min} = 0$$

$$Z_{max} = \frac{V_{max}}{I_{min}} = \infty$$

29. $$VSWR = \frac{Z_L}{Z_0} = \frac{75}{50} = 1.5$$

Smith Chart (Circle Diagrams):-

→ It is used to calculate Γ and VSWR for a known Z_L & Z_0.

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$= \frac{Z_L/Z_0 - 1}{Z_L/Z_0 + 1}$$

→ It is used normalised or relative ratio of Z_L/Z_0 for its calculation

→ It is a rectangular graph or polar plot of Γ_r Vs Γ_i having two families on circles

→ constant R circles

→ constant X circles.

where $R = \text{Real}\left[Z_L/Z_0\right]$

$X = \text{Imag}\left[Z_L/Z_0\right]$

$$\Gamma_r + j\Gamma_i = \frac{R+jX-1}{R+jX+1} = \frac{(R-1)+jX}{(R+1)+jX}$$

$$\left(\Gamma_r - \frac{R}{R+1}\right)^2 + \Gamma_i^2 = \left(\frac{1}{R+1}\right)^2$$ constant R circles Equation

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{X}\right)^2 = \left(\frac{1}{X}\right)^2$$ constant X circles Equation.

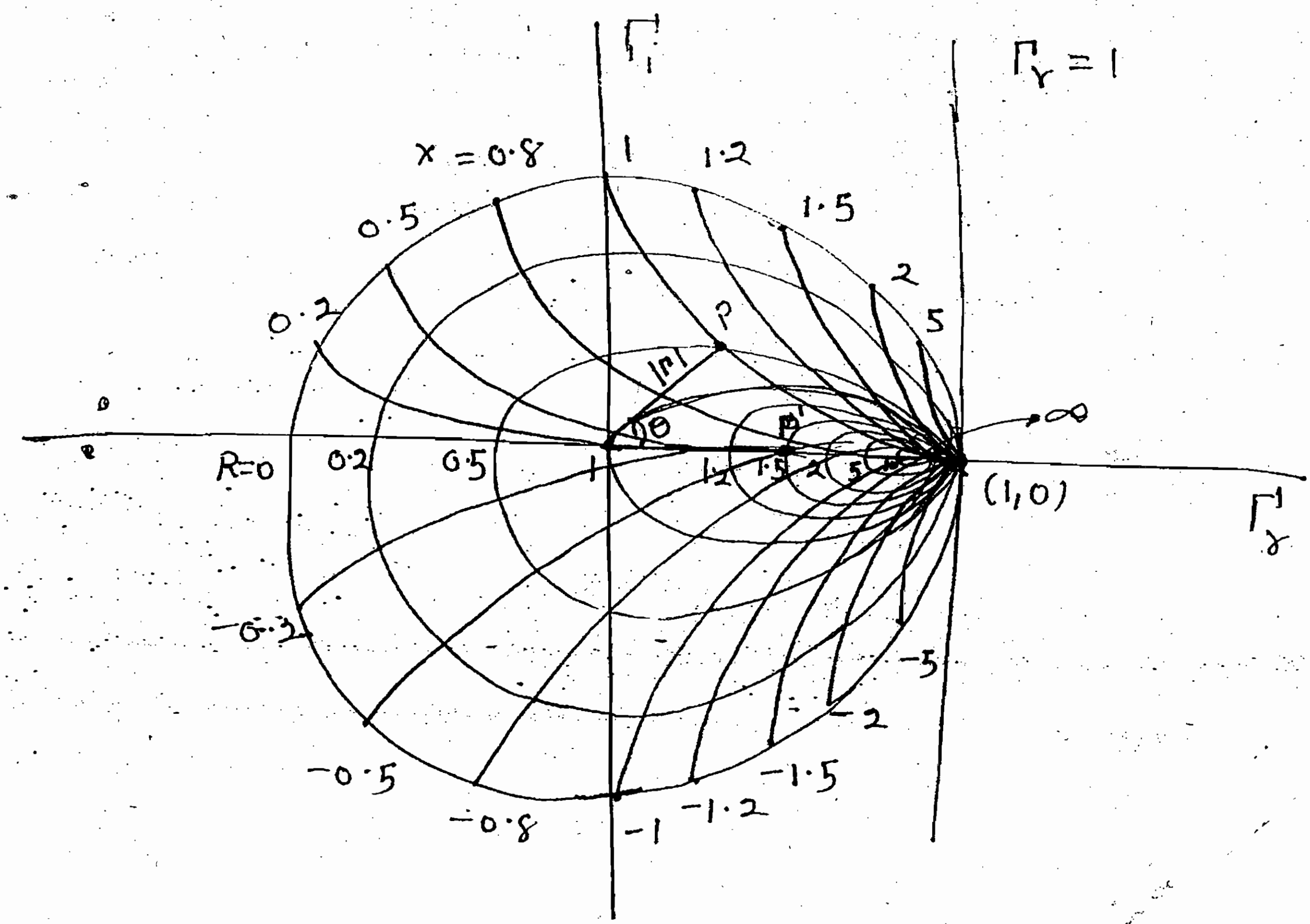
Properties of constant R circles:-

(a) Centres − $\left(\frac{R}{R+1}, 0\right)$

(b) Radius − $\left(\frac{1}{R+1}\right)$

(c) They all pass through (1,0)

(d) Range $(0, \infty)$



1. The R=0 circle is the biggest possible R circle and it is pheriphary or boundary of Smith chart.
2. All circles R = 0 to 1 extend into the four quadrants the circles with R>1 confined in the 1st and IV quadrants
3. All the R-circles are concurrent with their centres on a straight line, and have a common tangent $\Gamma_r = 1$ axis

Properties of constant X-circles:-

a) Centres - $(1, \frac{1}{X})$ (b) Radius - $(1/X)$

c) They all pass through (1,0)

d) Range $(-\infty, \infty)$

1. X = 0 circle is the biggest possible X circle and it is Γ_r axis
2. All circles with X = 0 to 1 extend into the first two quadrant but circles with X > 1 confined in the 1st quadrant
3. All the X-circles are concurrent and have a common tangent with Γ_r axis

Note :-

1. The centres of R circles lie on the common tangent of X-circle and Vice-versa. Hence R and X constitute a pair of orthogonal families of circles ~~Be~~
2. A horizontal movement on the X=0 circle ~~has~~ as R value increasing and a clockwise movement on the R=0 circle as X increasing inductively (+ly)

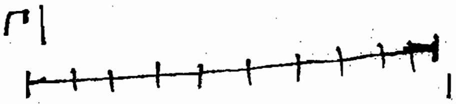
Calculation of Γ :-

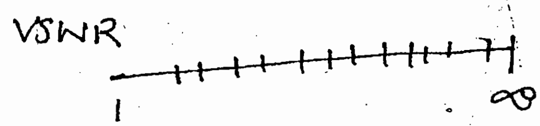
For a known Z_L and Z_0 calculate $\frac{Z_L}{Z_0} = R + jX$ and locate the corresponding point of intersection of R and X circle

→ Join the point P to origin the OP length $= |\Gamma|$

→ OP segments inclination with Γ_r axis is Γ's phase

→ End





Calculation of VSWR :-

→ For a known $|\Gamma|$ identify a point P' on the the $|\Gamma|$ real axis such that OP' length $= |\Gamma|$. The R circle value at P' is VSWR

$$R + jX \Big|_{X=0} = \frac{Z_L}{Z_0} = VSWR$$

Γ_r axis

Identify following points on the Smith's Chart :-

(1) $Z_L = jR_0$, $Z_0 = R_0$

$$\frac{Z_L}{Z_0} = \underset{\downarrow}{0} + j\underset{\downarrow}{1}$$

$R = 0 \qquad X = 1$

$$\Gamma = j$$

(11) $Z_L = -jR_0$

$Z_0 = R_0$ bottom

(III) $Z_L = 0$

$Z_0 = R_0$

$$\frac{Z_L}{Z_0} = \underbrace{0}_{R} + j\underbrace{0}_{X}$$

$\Gamma = -1$ Extremely left

(IV) $Z_L = \infty$

$Z_0 = R_0$

Extremely Right

(V) $Z_L = R_0$

$Z_0 = R_0$

$\longrightarrow$ centre $\Gamma = 0$

Lecture -14

Impedance Matching Technique:-

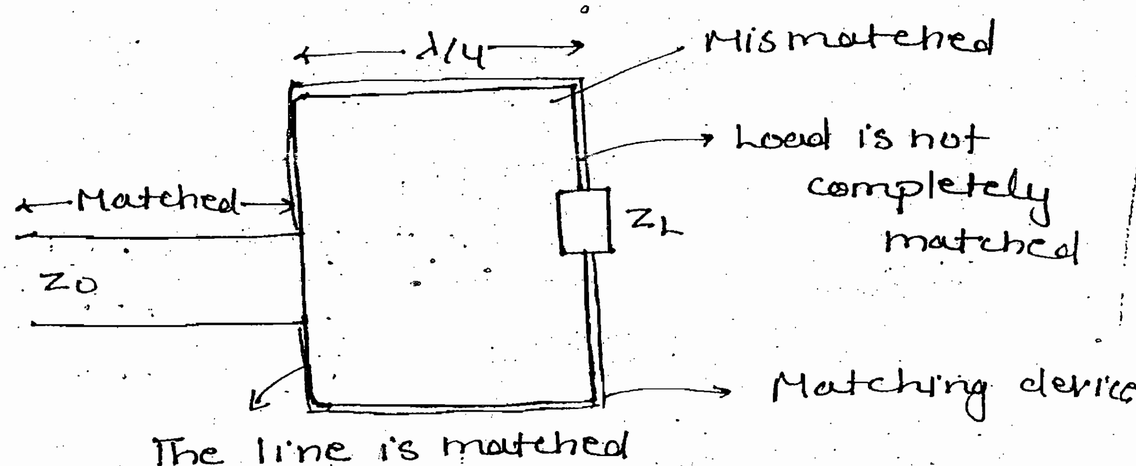
- Active (Bias)
  - CB Amp
  - CC Amp
- Passive
  - Series
    - $\lambda/4$ → Quaterwave transformer
  - Shunt → Stub matching

$$Z_0 = \frac{60 \ln(b/a)}{\sqrt{\epsilon_r}}$$
$= \text{few } 10\,\Omega \text{ to } 100\,\Omega$

& $Z_L = \text{few } \Omega - k\Omega$

($Z_L \neq Z_0$ Most often)

Series ($\lambda/4$) Quaterwave transformer:-



The termination of Z_0 line $\doteq$ Z_{in} of $\lambda/4$ matching device

$$= \frac{Z_0'^2}{Z_L} = Z_0$$

Design needs → $\boxed{Z_0' = \sqrt{Z_0 Z_L}}$

eg:- $50\,\Omega$ → line $100\,\Omega$ → load

$$\Gamma_{(load)} = \frac{100-50}{100+50} = \frac{1}{3}$$

Using a $\lambda/4$ transformer

$$Z_0' = \sqrt{50 \times 100} \simeq 70\,\Omega$$

$$\Gamma_{(at\ load)} = \frac{100-70}{100+70} < \frac{1}{3}$$

Disadvantage:-

i) The technique is suited for a narrow range of frequencies as the length of the transformer is frequency dependent.

ii). $Z_0' = \sqrt{Z_0 \cdot Z_L}$ → The technique is suited only for lossless line and resistive load.

Shunt - Stub matching:-

A stub is S.C or O.C line of a pre-calculated length, placed at a precalculated position such that the line is matched from the stub to the source

Design of a stub:-

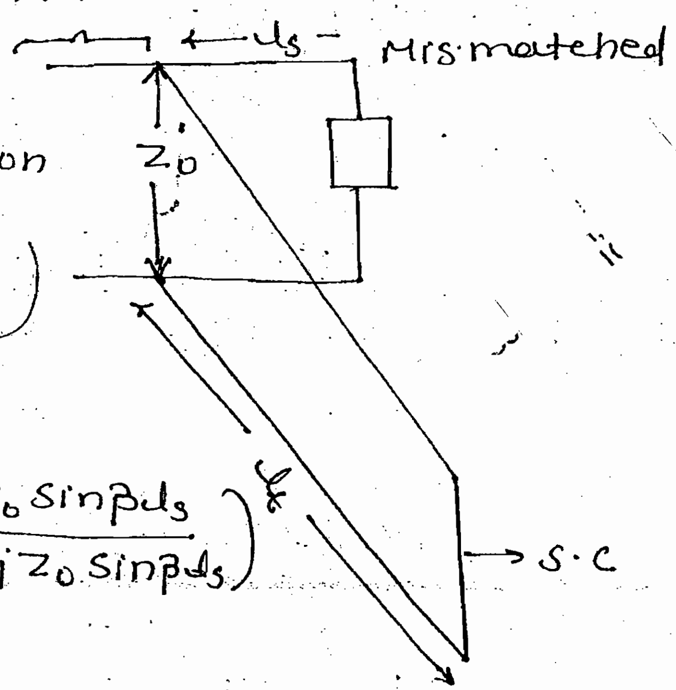
Step-(1):-

Identify a length l_s on the line from the load where the impedance has a real part Z_0

$$Z(l_s) = Z_0 \pm jX$$

l_s = stub position

$$l_s = \frac{\lambda}{2\pi} \tan^{-1}\left(\sqrt{\frac{Z_L}{Z_0}}\right)$$



$$Z(l_s) = Z_0\left(\frac{Z_L \cos\beta l_s + jZ_0 \sin\beta l_s}{Z_0 \cos\beta l_s + jZ_0 \sin\beta l_s}\right) \text{ \&}$$

$$Z(l_s) = \text{Real}\left[\frac{Z_L \cos\beta l_s + jZ_0 \sin\beta l_s}{Z_0 \cos\beta l_s + jZ_0 \sin\beta l_s}\right] = 1$$

Step-(II):-

At this identified position place an equal and opposite reactance in shunt such that cancel to the existing reactance i.e.

$$Z(l_s) = Z_0 \pm jX (\mp jX)$$

Stub

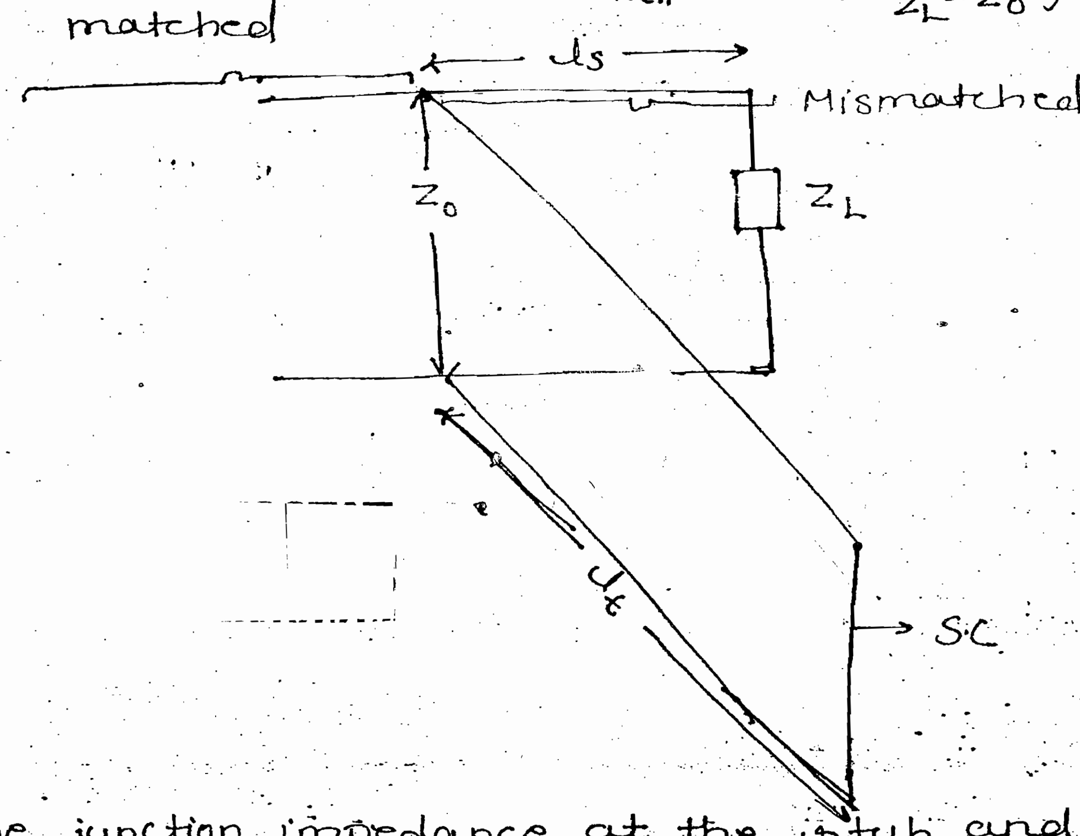
$$Y(l_s) = Y_0 \pm jB (\mp jB)$$

Step-(III):-

This reactance is realised from a SC or O.C stub of a finite length l_t such that

$$Z_{SC} = jZ_0 \tan \beta l_t = -\text{Imag}[Z(l_s)]$$

$$l_t = \text{stub length} = \frac{\lambda}{2\pi} \tan^{-1}\left(\frac{\sqrt{Z_L Z_0}}{Z_L - Z_0}\right)$$



The junction impedance at the stub and the line purely real as Z_0 and hence matched from the stub to the source

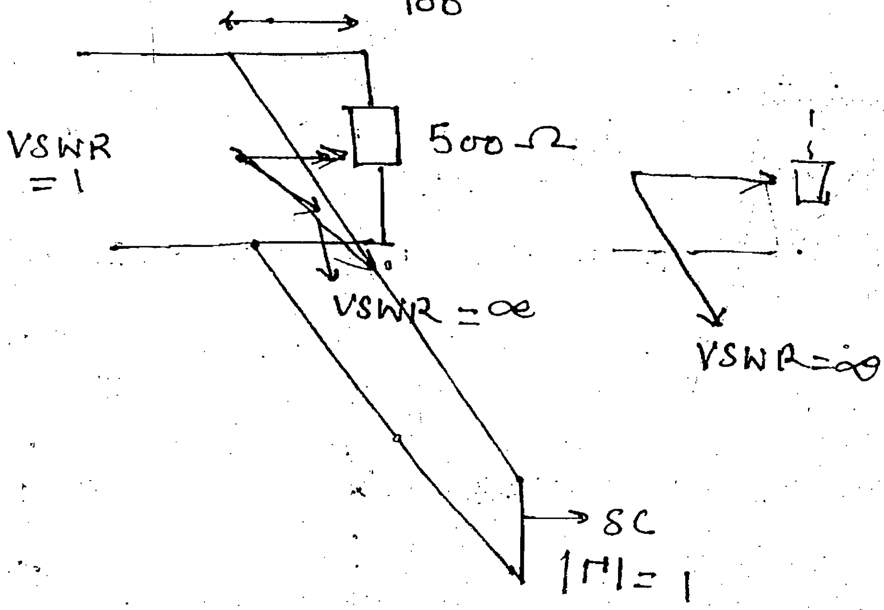
As the frequency on the line changes the stub length and stub position both have to adjusted which is mechanically inconvenient however in SC stub it is easy to using movable shorts

NOTE:-

For wide range of frequencies we use double stub matching where l_{s_1} and l_{s_2} are two stub position fixed and l_{t_1} and l_{t_2} are varied.

WORKBOOK:-

$VSWR = \frac{500}{100} = 5$



1. $VSWR = 3$

$$|\Gamma|_V = \frac{3-1}{3+1} = \frac{1}{2}$$

$$\Gamma_P = -\frac{1}{2} \cdot \frac{1}{2} = -\frac{1}{4} = 25\%$$

32. $$Z_0' = \sqrt{225 \times 250} = 240\,\Omega$$

33. $Z_L = 75 - j40$

$Z_0 = 75$

Note:-

Stub can never be placed at node

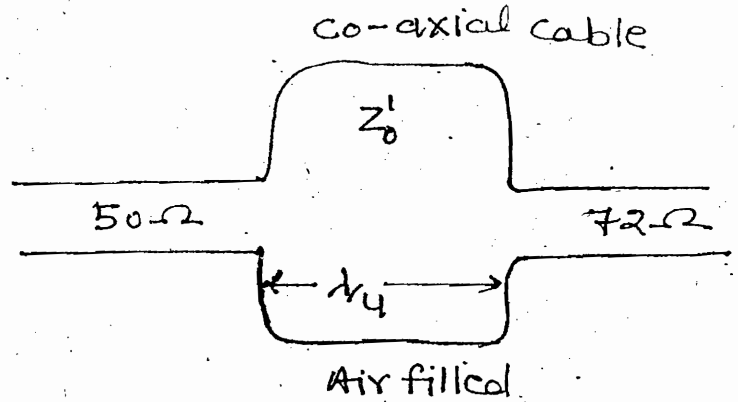
4. OC → stubs → More difficult to realize

SC → stubs

Ans-(a)

5. → D

36.



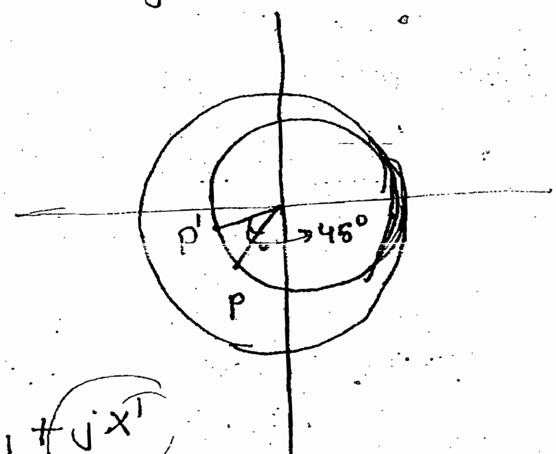
$$Z_0' = \sqrt{50 \cdot 72}$$
$$= 60\,\Omega$$
$$= \frac{60 \ln(b/a)}{\sqrt{\epsilon_R}}$$

$\ln(b/a) = 1 \Rightarrow \frac{b}{a} = e = 2.71 \Rightarrow b = 27\text{ mm}$

37. $0.5 - j0.3 = \frac{Z_L}{Z_0}$ = Normalized value

Actual value $= (0.5 - j0.3) \times 50$
$= 25 - j15$

38. Clocking on
R = constant



Z at P = $R + jX_1$

Z' at P' = $R + jX_2$

with $X_2 > X_1$

Z' at P' = $R + jX_1 + jX'$ → inductive

Ans – (a)

39. (i) $\frac{50 + j50}{50} = 1 + j1$ ($1 \to R$, $1 \to X$)

(ii) $\frac{10 + j10}{50} = 0.2 + j0.2$ ($0.2 \to R$, $0.2 \to X$)

(iii) $\frac{25 - j50}{50} = 0.5 - j1$ ($0.5 \to R$, $1 \to X$)

40 (i) one revolution for 360° – length – λ/2
↓
All impedances repeat for λ/2

(II) Clockwise movement $X \uparrow$

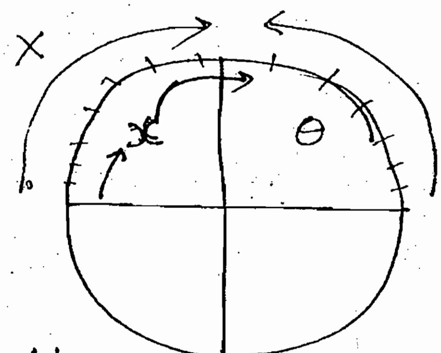
towards generator $x \uparrow$

$$Z(x) = Z_0 \left[\frac{Z_L \cos\beta x + j Z_0 \sin\beta x}{Z_0 \cos\beta x + j Z_L \sin\beta x} \right]$$

$$= j Z_0 \tan\beta x$$

$x \uparrow \quad X \uparrow$

(III) $\Gamma(x) = \Gamma(0) e^{-j2\beta x}$



Three scales
$\rightarrow X$
$\rightarrow \theta$
$\rightarrow x$

Three scales on the Boundary

(I) X – Reactance of the load

(II) x – Length on the line

(III) Γ's phase

All the three are related scales

$\rightarrow$ one is sufficient

(IV) Impedance (Wrong statement)

Conventional :–

$$V(x) = \frac{V_L (Z_L + Z_0)}{2 Z_L} \left[e^{\gamma x} + \frac{Z_L - Z_0}{Z_L + Z_0} e^{-\gamma x} \right]$$

$$= V_L \cosh \gamma x$$

If $\gamma = j\beta \qquad V(x) = V_L \cos\beta x$ — (1)

OR

$$V(x) = V_L \cosh\gamma x + I_L Z_0 \sinh\gamma x$$

$$V(x) = V_L \cosh\gamma x$$

$$I(x) = I_L \cosh(\gamma x) + \frac{V_L}{Z_0} \sinh(\gamma x)$$

$$I(x) = \frac{V_L}{Z_0} \sinh(\gamma x')$$ [illegible]

If $\gamma = j\beta$ $\quad I(x) = \frac{jV_L}{Z_0} \sin\beta x$

$V_L = 12V$

$Z_0 = 60\,\Omega$

$x = \lambda/4$

$l = \lambda/2$

$\longrightarrow$ if $\gamma = j\beta$ $\quad V(x) = V_L \cos\beta x$

$$Z_L = 100 - \frac{j}{\omega C} \qquad = 100 - j180\,\Omega$$

$$= 100 - \frac{j}{2\times\pi\times10^8 \times \frac{1}{36\pi\times10^9}}$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \qquad \text{VSWR}$$

$$Z_{max} = Z_0 (SWR)$$

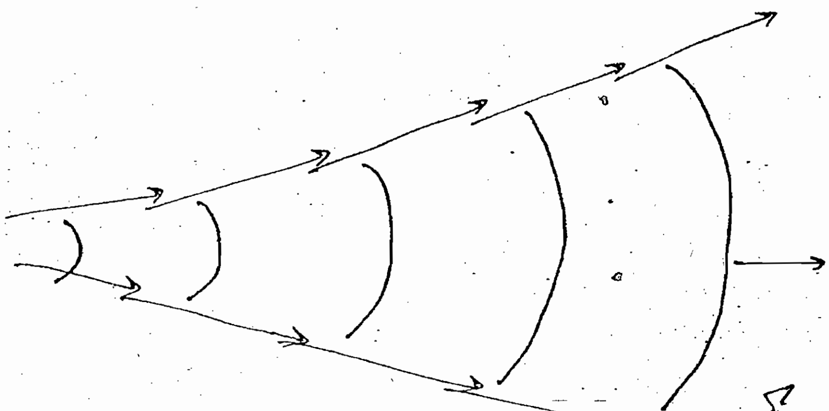
$$Z_{min} = Z_0 / SWR$$

Wave Guides :-

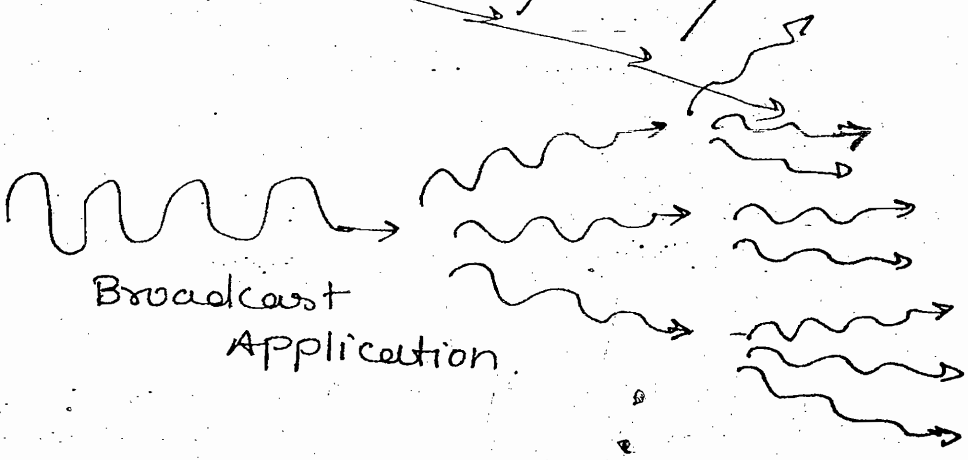
Uniform Plane Wave $\longrightarrow$ $\bar{E}(z,t)_x$, $\bar{H}(z,t)_y$

Wavefront

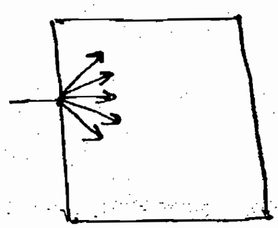
Dispersive Beams $\longrightarrow$ $\bar{E}(x,y,z,t)_{(x,y,z)}$
$H(x,y,z,t)_{(x,y,z)}$



Spherical Wavefront



Broadcast Application



Scattering
Diffraction
Diffusion

→ All practical EM waves are dispersive in nature and hence they obey Huygen's wave principle that every ray is a source of secondary emission. This is the cause of diffusion, diffraction and scattering properties of EM Waves

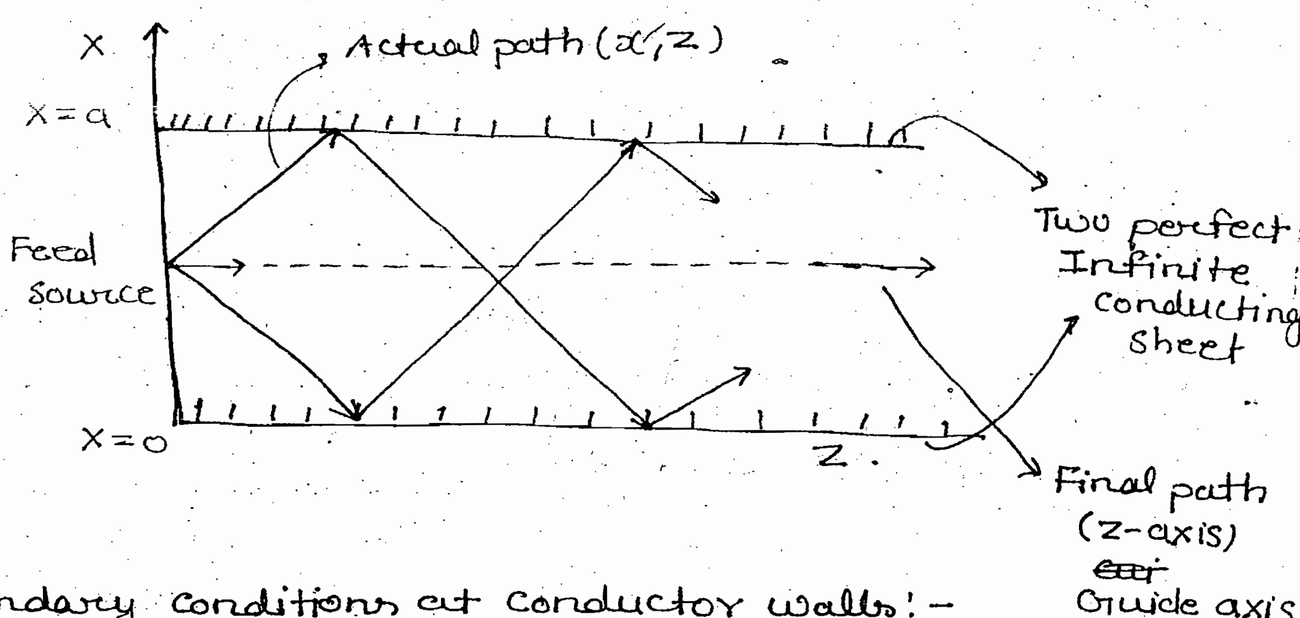
→ This is an advantage in broadcast application but a serious limitation in point to point communication. Hence waveguide are used to restrict the wave with a specific bound

$E(x,z,t)$, $H(x,z,t)$ } one dimension restriction in x, one dimension propogation in z → Parallel plane waveguide

eg:- earth and ionosphere guiding

$E(x,y,z,t)$, $H(x,y,z,t)$ } 2 dimension restriction in x, 1 dimension propogation in z → Rectangular Waveguide

Parallel Plane Waveguides:-



Boundary conditions at conductor walls:-

$E_{tang} = 0$ at $x=0$, $x=a$

$E(x)_{tang} = 0$ at $x=0$, $x=a$

$E(x)_y$ & $E(x)_z = 0$ at Guide walls

Propogation along the Guide Axis:-

Using $\nabla^2 E = \gamma^2 E$ Helmoltz's Equation

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial z^2} = -\omega^2 \mu \epsilon E$$

Let $E(z)$ be any natural harmonic $e^{-\bar{\gamma} z}$ as the z side is un-restricted

where $\bar{\gamma} = \gamma_z =$ Propogation consist in guide axis.

$$\frac{\partial^2 E}{\partial x^2} + \bar{\gamma}^2 E = -\omega^2 \mu \epsilon E$$

$$\frac{\partial^2 E}{\partial x^2} = -(\gamma^2 + \omega^2 \mu \epsilon) E$$

where $\bar{\gamma}^2 + \omega^2 \mu \epsilon = \gamma_x^2$

where $\gamma_x =$ Propogation constant in x side or restricted side

$$\frac{\partial^2 E}{\partial x^2} = -\gamma_x^2 E$$

The $E(x)$ solution is also harmonic

$$E(x) = c_1 \sin(\gamma_x x) + c_2 \cos(\gamma_x x)$$

The restricted side propogation has to be trignometric harmonic only

Applying the Boundary conditions

at $x = 0$

$$E(0)_{tang} = 0 + c_2 = 0$$

$$\Rightarrow c_2 = 0$$

Note:- The tangential E field harmonic has to be a 'sin' only is the restricted side

at $x = a$

$$E(a)_{tang} = c_1 \sin(\gamma_x a) = 0$$

$$\boxed{\gamma_x = \frac{m\pi}{a}} \qquad m = 0, 1, 2, 3 \ldots$$

Note :-

The restricted side propogation constant can ake only discrete solutions but not any continous value

Finally $\bar{\gamma} = \sqrt{\left(\frac{m\pi}{a}\right)^2 - \omega^2\mu\epsilon}$

Concept 1 :-

(f_c) cut off frequency of the guide

If $\left(\frac{m\pi}{a}\right)^2 > \omega^2\mu\epsilon$

then $\bar{\gamma} = \bar{\alpha} + j0$ No propogation along the guide axis

If $\omega^2\mu\epsilon > \left(\frac{m\pi}{a}\right)^2$

$$\omega > \frac{m\pi}{a\sqrt{\mu\epsilon}}$$

then $\bar{\gamma} = 0 + j\bar{\beta}$

The wave travels along the guide axis without attenuation.

$$\cancel{\omega > \frac{m\pi c}{a}}$$

Note :-

Every waveguide has a minimum cut-off frequency below which there cannot be propogation i.e. there is a max. wavelength above which there cannot be propogation and this wavelength is comparable to the guide dimensions

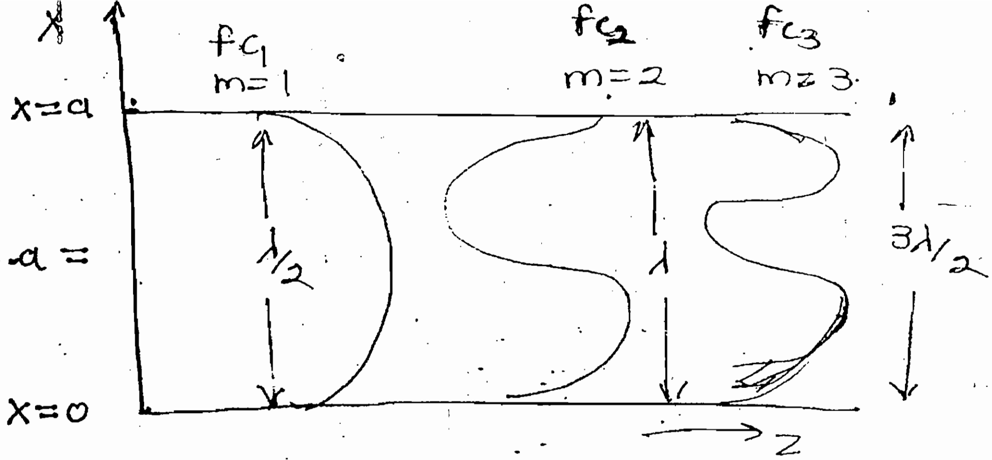
Hence $\omega_c > \frac{m\pi c}{a}$, $f_c > \frac{mc}{2a}$, $\lambda_c > \frac{2a}{m}$

$$\Rightarrow \omega_c = \frac{m\pi c}{a}$$

$$f_c = \frac{mc}{2a}$$

$$\lambda_c = \frac{2a}{m}$$

9μ



At exact cut-off frequency where $\bar{\gamma} = 0$, the wave resonats b/w guide walls and oscillates b/w the walls for 'm' such frequencies.

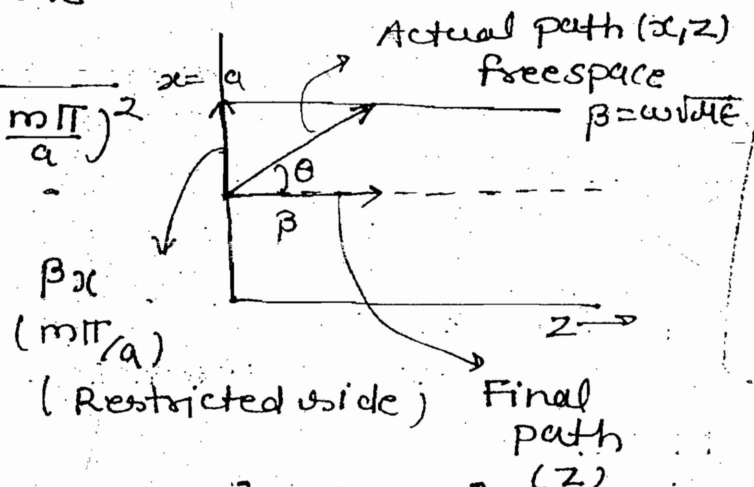
Concept 2 :-

Wave Angle or Tilt Angle :-

$$\bar{\beta} = \sqrt{\omega^2\mu\epsilon - \left(\frac{m\pi}{a}\right)^2}$$

$$\bar{\gamma} = \sqrt{\left(\frac{m\pi}{a}\right)^2 - \omega^2\mu\epsilon}$$

$$= j\sqrt{\omega^2\mu\epsilon - \left(\frac{m\pi}{a}\right)^2}$$



$$\beta^2 = \beta_x^2 + \bar{\beta}^2$$

$$\bar{\beta} = \sqrt{\omega^2\mu\epsilon - \left(\frac{m\pi}{a}\right)^2}$$

Phase velocity along the guide axis, $\bar{V}_p = \frac{\omega}{\bar{\beta}} = \frac{\omega}{\sqrt{\omega^2\mu\epsilon - \left(\frac{m\pi}{a}\right)^2}}$

$$\bar{V}_p = \frac{1}{\sqrt{\mu\epsilon - \left(\frac{m\pi}{a\omega}\right)^2}}$$

$$= \frac{1/\sqrt{\mu\epsilon}}{\sqrt{1 - \left(\frac{m\pi}{a\omega\sqrt{\mu\epsilon}}\right)^2}}$$

$$\boxed{\bar{V}_p = \frac{c}{\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}}}$$

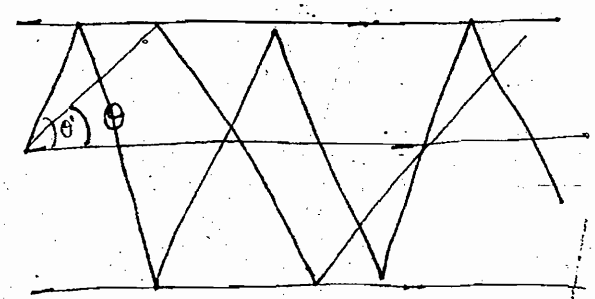
$$\bar{\beta} = \beta\cos\theta$$

$$\frac{2\pi}{\bar{\lambda}} = \frac{2\pi}{\lambda}\cos\theta$$

$$\Rightarrow \bar{\lambda} = \frac{\lambda}{\cos\theta}$$

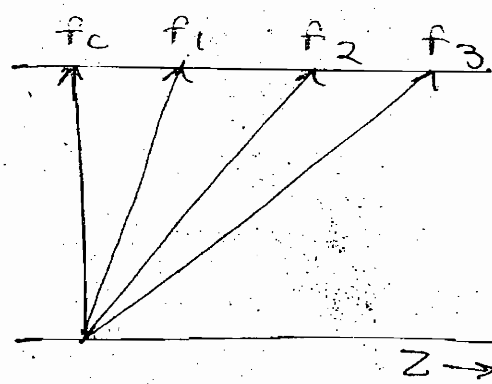
$$\Rightarrow \bar{\lambda} f = \frac{\lambda f}{\cos\theta}$$

$$\boxed{\bar{V}_p = \frac{c}{\cos\theta}}$$



By comparison

$$\boxed{\sin\theta = \frac{f_c}{f}}$$



Note:-
Every frequency has a unique tilt angle and unique velocity inside the guide so that two different frequencies never overlap to each other

$f_3 > f_2 > f_1 > f_c$

Concept 3:-

Group Velocity ($\bar{V}_g$):-

$$\bar{V}_p = \frac{c}{\cos\theta} \Rightarrow \bar{V}_p > c \rightarrow \text{Always}$$

In linear conditions, $\beta \propto \omega$

velocity is phase ~~vel~~ velocity $V_p = \frac{\omega}{\beta}$

eg!- lossless EM Waves in free space
lossless VI Waves in transmission line

$$\beta = \omega\sqrt{\mu\epsilon}$$

$$\beta = \omega\sqrt{LC}$$

But in dispersive condition β is not linear with ω

eg:- $V_g = \frac{d\omega}{d\bar{\beta}}$ = Group velocity

$$\bar{\beta} = \left[\sqrt{\omega^2\mu\epsilon - \left(\frac{m\pi}{a}\right)^2}\right] \rightarrow \text{dispersive conditions}$$

β is not linear with ω

$$\frac{d\bar{\beta}}{d\omega} = \frac{1}{\cancel{2}\sqrt{\omega^2\mu\epsilon - \left(\frac{m\pi}{a}\right)^2}} \cdot \cancel{2}\omega\mu\epsilon$$

$$= \frac{\mu\epsilon}{\sqrt{\mu\epsilon - \left(\frac{m\pi}{a\omega}\right)^2}}$$

$$= \frac{\sqrt{\mu\epsilon}}{\sqrt{1 - \left(\frac{m\pi}{a\omega\sqrt{\mu\epsilon}}\right)^2}}$$

$$= \frac{1}{c\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}}$$

$$\bar{V_g} = c\cos\theta$$

$$\bar{V_g} < c \longrightarrow \text{Always}$$

$$\boxed{\bar{V_g} \cdot \bar{V_p} = c^2} \longrightarrow \text{Always}$$

Concept 4:-

Modes of operation

Concept 4 :-

Modes of Operation :-

Note :-

Mode stands for physical connection of field which can be axial or longitudinal feed

→ The no. of field connection decides the integer m of that mode and this m decides the cut-off freq of the mode

→ The mode can be identified from field such that m stands for no. of half cycles b/w the guide walls ~~and~~ or no. of maxima b/w the guide walls.

Lecture - 15

Transverse Electric (TE), TM and TEM Modes:—

$$E(x,z,t)_{(x,y,z)}$$
$$H(x,z,t)_{(x,y,z)}$$

Using Maxwells equation

$$\nabla \times E = -j\omega\mu H \qquad \nabla \times H = j\omega\epsilon E$$

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -j\omega\mu H_x$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z$$

$$\begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = \nabla \times E$$

→ All y derivatives are zero

→ All z derivatives are back the same function with $\bar{\gamma}$ scaling

$$\nabla \times E = -j\omega\mu H \qquad \nabla \times H = j\omega\epsilon E$$

$$\bar{\gamma} E_y = -j\omega\mu H_x \qquad \bar{\gamma} H_y = j\omega\epsilon E_x$$

$$-\bar{\gamma} E_x - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y \qquad -\bar{\gamma} H_x - \frac{\partial H_z}{\partial x} = j\omega\epsilon E_y$$

$$\frac{\partial E_y}{\partial x} = -j\omega\mu H_z \qquad \frac{\partial H_y}{\partial x} = j\omega\epsilon E_z$$

$$H_x = \frac{-\bar{\gamma} E_y}{j\omega\mu}$$

$$-\bar{\gamma}\left(\frac{-\bar{\gamma} E_y}{j\omega\mu}\right) - \frac{\partial H_z}{\partial x} = j\omega\epsilon E_y$$

$$\bar{\gamma}^2 E_y - j\omega\mu \frac{\partial H_z}{\partial x} = -\omega^2 \mu\epsilon E_y$$

$$(\bar{\gamma}^2 + \omega^2\mu\epsilon) E_y = j\omega\mu \frac{\partial H_z}{\partial x}$$

(I) $$\frac{\partial H_z}{\partial x} = \frac{\gamma_x^2}{j\omega\mu} E_y$$

(II) $$\frac{\partial H_z}{\partial x} = \frac{-\gamma_x^2}{\bar{\gamma}} H_x$$

(III) $$\frac{\partial E_z}{\partial x} = \frac{-\gamma_x^2}{\bar{\gamma}} H_y$$

(IV) $$\frac{\partial E_z}{\partial x} = \frac{-\gamma_x^2}{\bar{\gamma}} E_x$$

Note :-

The axially directed field components determine the complete standing of the wave.

If $E_z = 0$ → physical connection

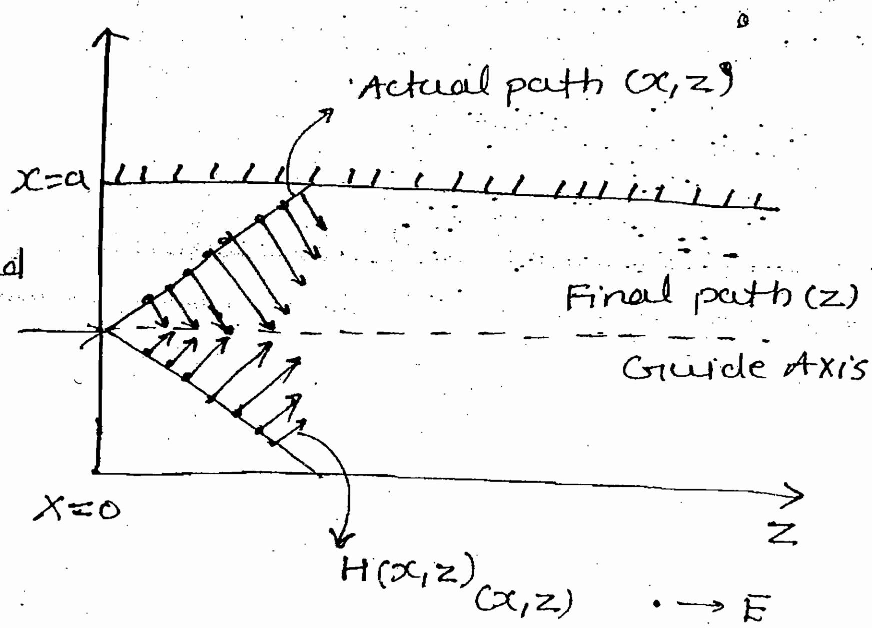
then $E_x = H_y = 0$ The waves becomes

$E(x, z, t)_y$

$H(\cancel{x}, \cancel{z}, t)_{(x,z)}$

$E \perp$ Guide Axis

The wave is called as TE wave



If $H_z = 0$. then $H_x = E_y = 0$

the wave becomes $H(x, z, t)_y$

$E(x, z, t)_{x,z}$

~~E ⊥~~ $\boxed{H \perp \text{Guide Axis}}$

The wave is called as TM Waves

$\boxed{E \perp H \perp \text{Propogation}}$

TE Wave solutions in Parallel - Plane Wave guides :-

The waves has $(E_x = E_z = H_y = 0)$ H_y

$$E(x, z, t)_y = E_{y0} \cdot \sin\left(\frac{m\pi}{a} x\right) e^{-\bar{\gamma} z} e^{j\omega t} a_y$$

$$E(x, z, t)_x = H_{x0} \cdot \sin\left(\frac{m\pi}{a} x\right) e^{-\bar{\gamma} z} e^{j\omega t} a_x$$

$$H(x, z, t)_z = H_{z_0} \cos\left(\frac{m\pi}{a} x\right) e^{-\bar{\gamma} z} e^{j\omega t} a_z$$

It is a product (AND) solution of time, x & z

Harmonics

$E(t)/H(t) \longrightarrow e^{j\omega t}$ Source harmonic

$E(z)/H(z) \longrightarrow e^{-\bar{\gamma} z}$ Natural Harmonic

$E(x)/H(x) \longrightarrow$ Trignometric Harmonic

$E(x)_{tang} \longrightarrow$ 6 Sin' Harmonic

$E(x)_y$ or $E(x)_z$

$$\frac{\partial H_z}{\partial x} = (\ \ ) H_x$$
$$= (\ \ ) E_y$$

If $m = 0$ the TE wave does not exist and hence $m = 1$ is the least value required for propogation

TM Wave Solutions in Parallel Plane Waveguides :-

$(\cancel{\#}\ H_z = H_x = E_y = 0)$

$$H(x,z,t)_y = H_{y_0} \cos\left(\frac{m\pi}{a}x\right) e^{-\bar{\gamma}z} e^{j\omega t} a_y$$

$$E(x,z,t)_x = E_{x0} \cos\left(\frac{m\pi}{a}x\right) e^{-\bar{\gamma}z} e^{j\omega t} a_x$$

$$E(x,z,t)_z = E_{z0} \sin\left(\frac{m\pi}{a}x\right) e^{-\bar{\gamma}z} e^{j\omega t} a_z$$

If $m = 0$, $E_z = 0$

The wave becomes $E(z,t)_x$
$H(z,t)_y$

This wave is called as ~~TEM waves~~ TEM Waves with $E_z = H_z = 0$

Properties of TEM Waves :-

I) It has $m = 0$ $\bar{\gamma} = \gamma = j\omega\sqrt{\mu\epsilon}$

- i.e. Propogates only along guide axis i.e. $\theta = 0$ → Always

II) It has $f_c = 0$ i.e. No cut off frequency

III) It has $m = 0$ i.e. No multi-mode connection mechanism

Summary :-

$\bar{E}(x,t)$, $H(x,t)$ → Wave at f_c, $\theta = 90^\circ$, $\bar{\gamma} = 0$

(f_{c_1}, f_{c_2} - - - - f_{c_m} exists)

$E(z,t)_x$, $H(z,t)_y$ → TEM wave, $\theta = 0^\circ$ $\bar{\gamma} = \gamma = j\omega\sqrt{\mu\epsilon}$

$m = 0$, $f_c = 0$

3. $E(x,z,t)$, $H(x,z,t)$ → TE → E_y, H_x, H_z → θ, f_c, mode, $\bar{v}_p$, $\bar{v}_g$

$E(x,z,t)$, $H(x,z,t)$ → TM → H_y, E_x, E_z → θ, f_c, mode, $\bar{v}_p$, $\bar{v}_g$

Waveguides (Workbook) :-

1) $f = 22\ GHz$ $\quad \sin\theta = \frac{f_c}{f}$ $\quad f_c = \frac{mc}{2a}$

$$f_{c_1} = \frac{1.38 \times 10^8}{2.25 \times 10^{-2}} = 6\ GHz$$

1st Mode – $6\ GHz \longrightarrow \infty$

1st Mode $\quad \sin\theta = \frac{6}{22}$

IInd Mode $12\ GHz \longrightarrow \infty$

III Mode $\quad \sin\theta = \frac{18}{22}$

IIIrd Mode $18\ GHz \longrightarrow \infty$

II) $f = 40\ GHz$

$$f_c = 36 \times 10^9 = \frac{3.3 \times 10^8}{2 \times a} \Rightarrow a = 1.25\ cm$$

III) $\sin\theta = \frac{36}{40} = \frac{f_{c_3}}{f_3} = \frac{f_{c_7}}{f_7} = \frac{84}{f_7}$

3rd Mode $\longrightarrow 36$

7th Mode $\longrightarrow 84$

$f_7 = 93.3\ GHz$

IV) $f = 2\ GHz$

$$f_{c_1} = \frac{1.3 \times 10^8}{\sqrt{9} \cdot 2 \times 3 \times 10^{-2}} = \frac{5}{3} = 1.67\ GHz$$

1st $\rightarrow 1.67 \rightarrow \infty$

2nd $\rightarrow 3.34 \rightarrow \infty$

$$\sin\theta = \frac{5/3}{2} = \frac{5}{6}$$

Note:-

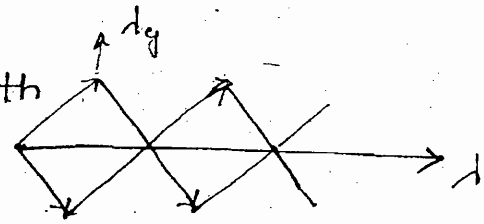
1. $\bar{r}, \bar{\beta}, \bar{\lambda}, \bar{V_p}, \bar{V_g}$, TRANSVERSE $\longrightarrow$ w.r.t guide axis

$$\eta_{TE} = \frac{E_{trans}}{H_{trans}} = \frac{E_y}{H_x} = \frac{E_{total}}{H_{total}\cos\theta} = \frac{120\pi}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

9n:

$$\eta_{TM} = \frac{E_x}{H_y} = \frac{E_{total}\cos\theta}{H_{total}} = 120\pi\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

3. $\bar{\lambda} = \frac{\lambda}{\cos\theta} = \lambda_g$ = Guide Wavelength



$$\lambda_g = \frac{\lambda}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{\lambda}{\sqrt{1-\left(\frac{\lambda}{\lambda_c}\right)^2}}$$

$$1-\left(\frac{\lambda}{\lambda_c}\right)^2 = \left(\frac{\lambda}{\lambda_g}\right)^2$$

$$\boxed{\frac{1}{\lambda^2} = \frac{1}{\lambda_c^2} + \frac{1}{\lambda_g^2}}$$

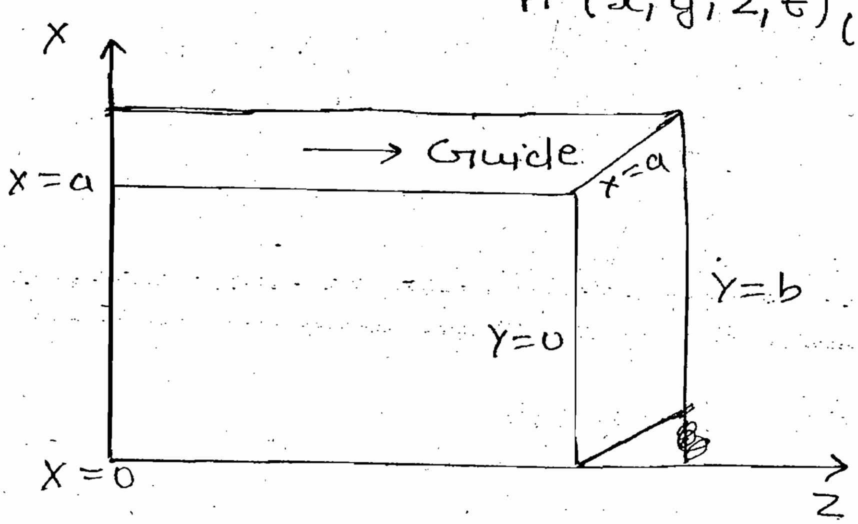
$\frac{c}{f}$ free space wavelength

$\frac{2a}{m}$ cut-off wavelength

Rectangular Waveguides:-

The waves are $E(x, y, z, t)\,(x, y, z)$

$H(x, y, z, t)\,(x, y, z)$



→ It is a single conductor hollow structure used to confine EM wave in 2 dimension using 4 walls

$x = 0 \quad x = a$

$y = 0 \quad y = b$

It has $\gamma_x = \frac{m\pi}{a}$

$\gamma_y = \frac{n\pi}{b}$

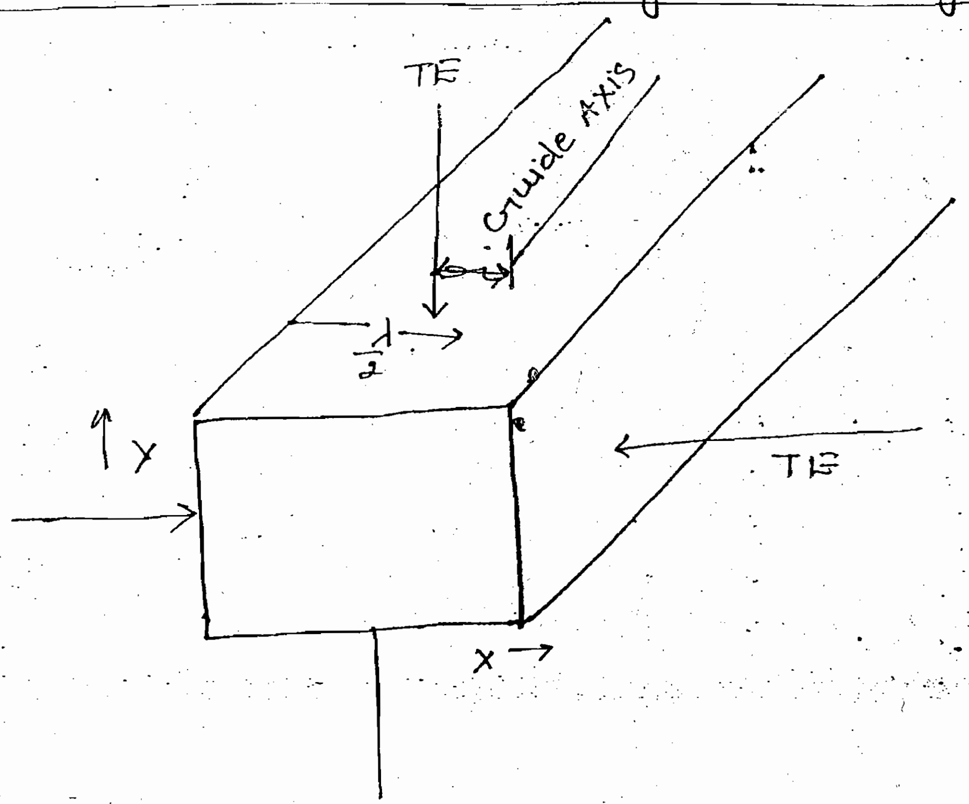
Applying $\nabla^2 E = -\omega^2 \mu \epsilon E$

$$\bar{\gamma} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2 \mu \epsilon}$$

$$\omega_c = \text{cut-off frequency} = \left(\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}\right) c$$

$$f_c = \left(\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}\right) \frac{c}{2}$$

Modes and feeds in Rectangular Waveguides:-



E_y or E_x

but $E_z = 0$

A horizontal or vertical field always gives a suitable E field but not along the guide axis

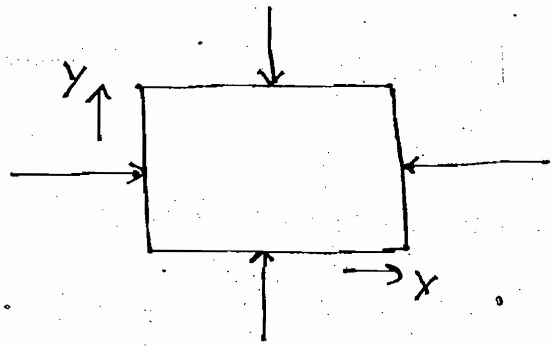
The no. of out of phase feed connections decides the integers m and n.

The modes are always designated as TE_{mn}

H_x & H_y but $H_z = 0$

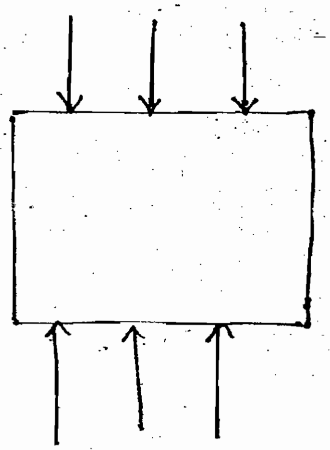
A lateral or axial feed always gives a suitable H field but not along the guide axis

(I)



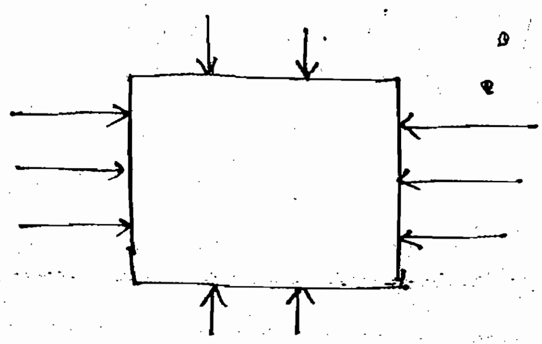
TE_{11}

(II)



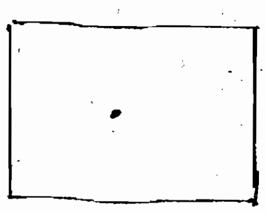
TE_{03}

(III)



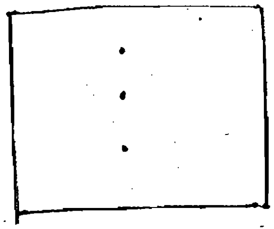
~~TE_{32}~~ TE_{23}

(IV)



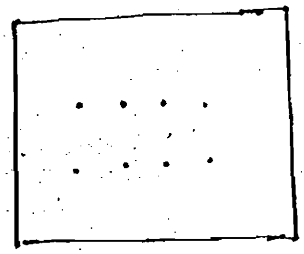
TM_{11}

(v)



TM_{13}

(vi)



TM_{42}

Note:-

TM_{M0} or TM_{0N} modes are even sent and donot exist in rectangular waveguides.

Such a connection mechanism doesn't exist

(i) $$f_c = \left(\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}\right)\frac{c}{2}$$

For both TE & TM are same so TE_{74}, TM_{74} are same cutoff called degenerate mode

Two different modes having the same f_c but different connection mechanism) are said to be degenerate modes.

(iii) For $m=1$, $n=0$
or $m=0$, $n=1$
i.e. TE_{10} & TE_{01} mode have the least f_c

f_c (TE_{10}) $= \frac{c}{2a}$

f_c (TE_{01}) $= \frac{c}{2b}$

If $a > b$ TE_{10} is dominent mode

$a < b$ TE_{01} is dominent mode

→ Broadside dimensions decides the dominent mode

→ Narrowside dimensions decides the maximum operable voltage and power handling ability

TM Wave Solutions in Rectangular Waveguide ($H_z = 0$) :-

The wave is $E(x, y, z, t)_{(x,y,z)}$
$H(x, y, z, t)_{(x,y)}$

$$E(x,y,z,t)_z = E_{z0} \cdot \sin\left(\frac{m\pi}{a} \cdot x\right) \sin\left(\frac{n\pi}{b} y\right) e^{-\gamma z} e^{j\omega t} a_z$$

$$E(x,y,z,t)_x = E_{x0}\left(\frac{m\pi}{a}\right) \cos\left(\frac{m\pi}{a} x\right) \sin\left(\frac{n\pi}{b} y\right) e^{-\gamma z} e^{j\omega t} a_x$$

$$E(x,y,z,t)_y = E_{y0}\left(\frac{n\pi}{b}\right) \sin\left(\frac{m\pi}{a} x\right) \cos\left(\frac{n\pi}{b} y\right) e^{-\gamma z} e^{j\omega t} a_y$$

$$H(x,y,z,t)_x = H_{x0}\left(\frac{n\pi}{b}\right) \sin\left(\frac{m\pi}{a} x\right) \cos\left(\frac{n\pi}{b} y\right) e^{-\gamma z} e^{j\omega t} a_x$$

$$H(x,y,z,t)_y = H_{y0}\left(\frac{n\pi}{a}\right) \cos\left(\frac{m\pi}{a} x\right) \sin\left(\frac{n\pi}{b} y\right) e^{-\gamma z} e^{j\omega t} a_y$$

It is a product solution of x, y, z, t harmonics

$E(t)/H(t) \longrightarrow e^{j\omega t} \longrightarrow$ Source harmonic

$E(z)/H(z) \longrightarrow e^{-\gamma z} \longrightarrow$ Natural Harmonic

$H/E(x \text{ or } y) \longrightarrow$ Trignometric harmonic

$\left.\begin{array}{l} E(x)_y \text{ or } E(x)_z \\ E(y)_x \text{ or } E(y)_z \end{array}\right\}$ Tangential harmonic $\}$ 'sin' Harmonics

If $m=0$ & $n \neq 0$ or $m \neq 0$ & $n = 0$

TM waves donot exist i.e. they are evanescent modes

TE Wave Solutions in Rectangular Waveguide ($E_z=0$):-

The wave is $E(x,y,z,t)_{(x,y)}$

$H(x,y,z,t)_{(x,y,z)}$

$$H(x,y,z,t)_z = H_{z0} \cdot \cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right) e^{-\bar{\gamma}z} e^{j\omega t} a_z$$

$$E(x,y,z,t)_x = E_{x0}\left(\frac{n\pi}{b}\right)\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right) e^{-\bar{\gamma}z} e^{j\omega t} a_x$$

$$E(x,y,z,t)_y = E_{y0}\left(\frac{m\pi}{a}\right)\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right) e^{-\bar{\gamma}z} e^{j\omega t} a_y$$

$$H(x,y,z,t)_x = H_{x0}\left(\frac{m\pi}{a}\right)\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right) e^{-\bar{\gamma}z} e^{j\omega t} a_x$$

$$H(x,y,z,t)_y = H_{y0}\left(\frac{n\pi}{b}\right)\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right) e^{-\bar{\gamma}z} e^{j\omega t} a_y$$

If $m\neq 0$ & $n=0$

The TE_{m0} wave exists $E(x,z,t)_y$

$H(x,z,t)_{(x,z)}$

If $m=0$ & $n\neq 0$

The TE_{0n} wave exists $E(y,z,t)_x$

$H(y,z,t)_{(y,z)}$

If $E_z = H_z = 0$ the wave cannot exist

or $m=n=0$

i.e. TEM waves donot exist in rectangular waveguide.

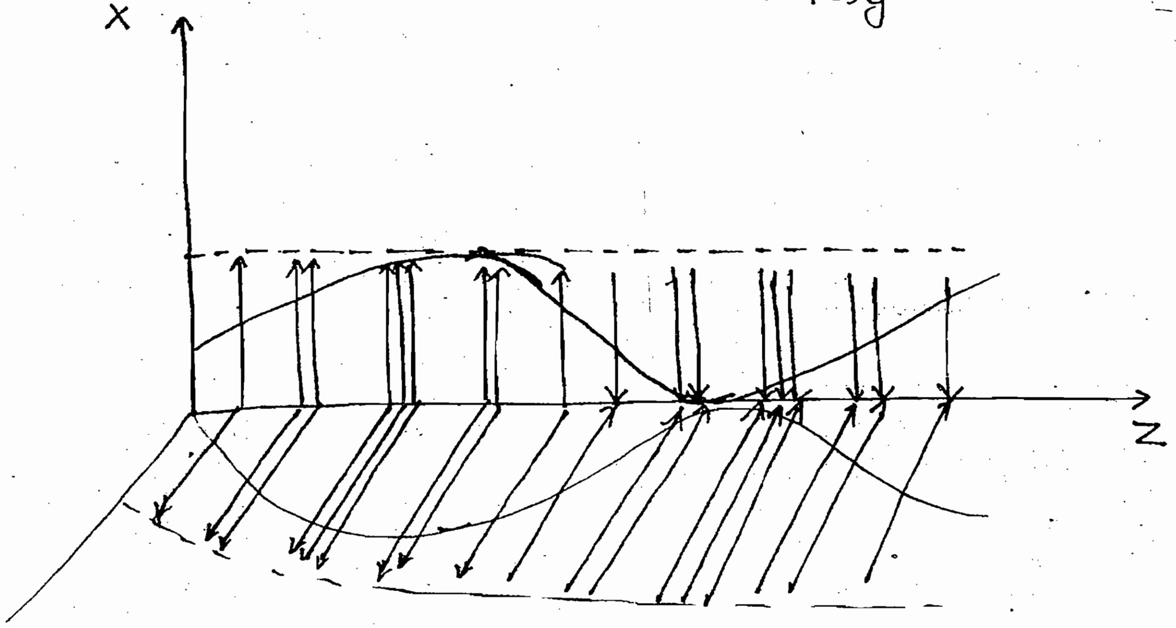
In general TEM waves donot exist in all single conductor guides

TEM needs two distinct conductor for its existence.

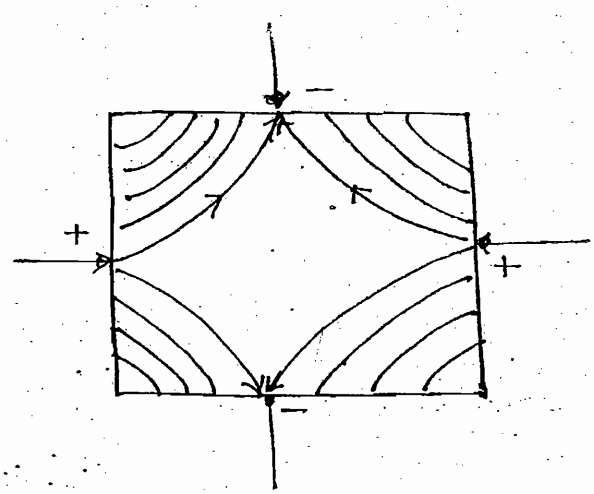
Field line Representations of guided waves:-

(i) Uniform plane wave — $E(z,t)_x$

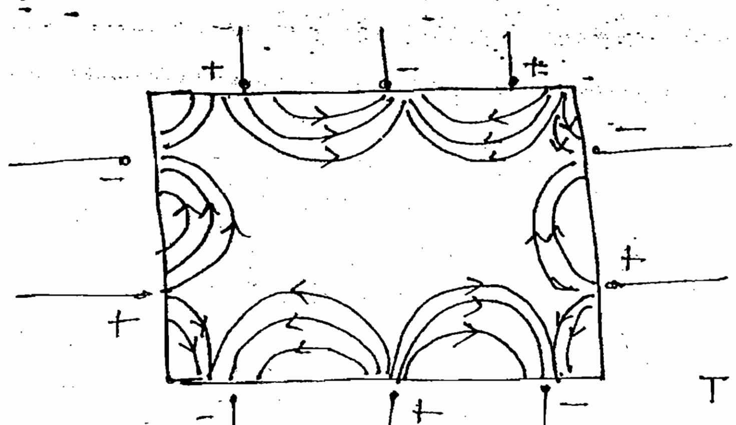
$H(z,t)_y$



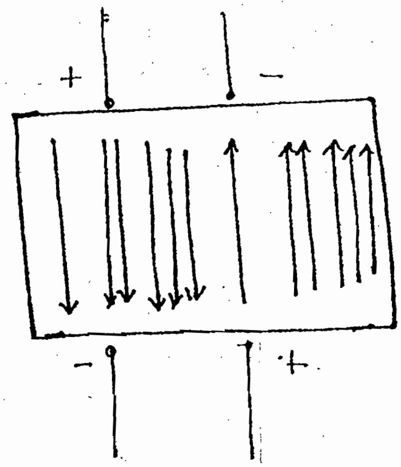
TE Waves:- (E_x, E_y)



TE_{11}

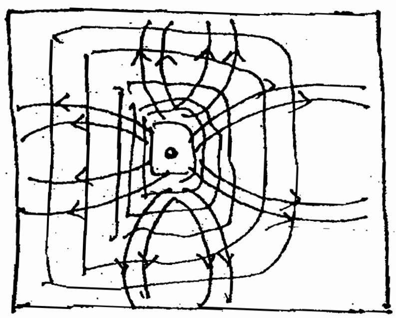


TE_{32}



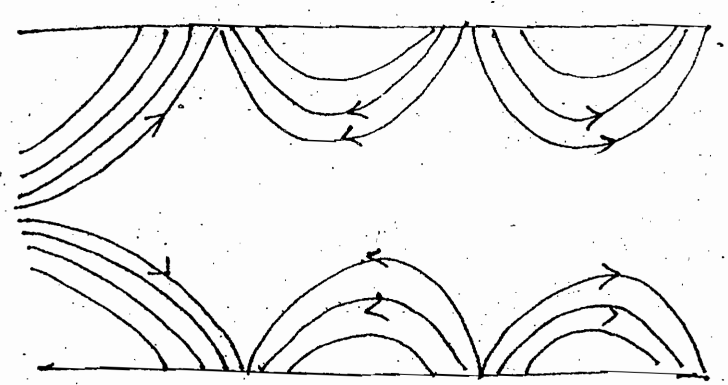
TE_{20}

TM Waves (E_x, E_y, E_z) :-



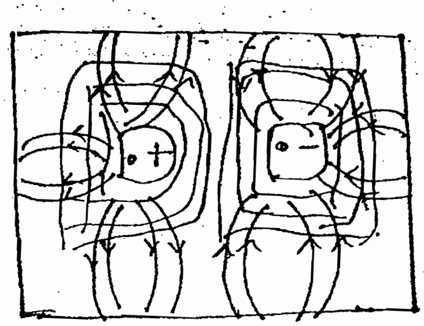
~~TE~~

TM_{11}



$\longrightarrow z$

$\underline{TM_{21}}$:-



TM_{21}

10K

With a guide axis out of board and both E & H fields traced on the board the wave is called as TEM waves else TE or TM

eg:- Co-axial cable and all other low frequency Transmission line. So TEM waves is low frequency energy transfer format

TM/TE $\rightarrow$ High frequency transmission waveguide

Workbook!-

1. $\frac{1}{\lambda^2} = \frac{1}{\lambda_c^2} + \frac{1}{\lambda_g^2}$

$\frac{3\times10^8}{2.5\times10^9} = 12cm$

TE_{10} $\rightarrow$ $2a = 20cm$

$\lambda_g =$

6. $f_c = \frac{1\times3\times10^8}{\sqrt{4}\times2\times3\times10^{-2}}$

$= 2.5GH$

7. $v_p > c$

3. $f_c = 10\times10^9 = \frac{1\times3\times10^8}{2\times a}$

$a = 1.5cm$

$a \cdot b > 2$

$a > b$

9. $\eta_{TE} = \frac{120\pi}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$

$= \frac{377}{\sqrt{1-\left(\frac{90}{30}\right)^2}}$

$= 400\Omega$

$f_c = \frac{c}{2a} = 10GHz$

10. $E(x,z,t)_y$

$\rightarrow$ Not depends on y

$\Rightarrow n=0$

TE_{m0}

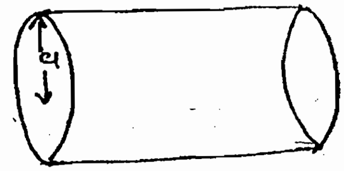
$\sin\left(\frac{2\pi}{a}x\right)$

$\sin(\omega t - \beta z)$

TE_{20}

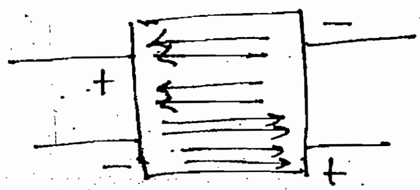
11 TEM exists → single conductor

(B)



12. No cut off freq → TEM exists   Ans - A

13. TE_{02}



14. $f_c = \dfrac{3\times10^8}{2\times2\times10^{-2}} = \dfrac{3\times10^8}{2\times\sqrt{\epsilon_R}\times1\times10^{-2}} \Rightarrow \epsilon_R = 4$

15 Note:— Parallel waveguide

TEM — $(0-\infty)$

TE_1/TM_1 — $(f_{c_1}-\infty)$

TE_2/TM_2 — $(f_{c_2}-\infty)$

15. ① m = 1 n = 0
② m = 0 n = 1
③ m = 1 n = 1

$$f_c = \left(\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}\right)\frac{c}{2}$$

4 × 3 cm

① → TE_{10} $f_{c_1} = \dfrac{c}{2a} = 3.75\ \text{GHz}$ $(3.75\to\infty)$ GHz

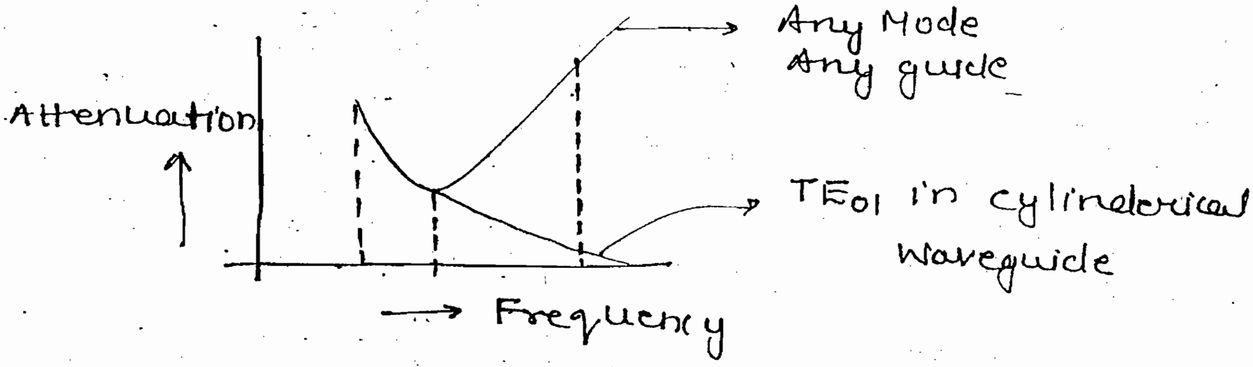
TE_{01} $f_{c_2} = \dfrac{c}{2b} = 5\ \text{GHz}$ $(5\ \text{GHz}\to\infty$

$\dfrac{TE_{11}}{TM_{11}}$ $f_{c_3} =$ $6.25\ \text{GHz}$ $(6.25\ \text{GHz}\to\infty)$

From (3.75 − 5) GHz there is strictly single mode operation.

Note:-

Practically Graph :-



With practical non-ideal conducting walls, higher freq. are not preferred in lower modes

16. $\rightarrow$ D

17. $\rightarrow$ TE_{10} only

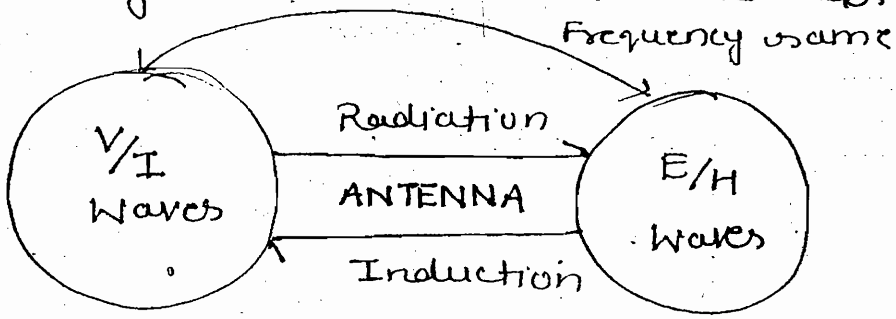
$$\frac{c}{2a} < \frac{c}{\lambda} < \frac{c}{2b}$$

$$\underset{TE_{10}}{f_c} < f < \underset{TE_{01}}{f_c}$$

$$2a > \lambda > 2b$$

ANTENNAS :-

- → Hertzian Dipole / Halfwave Dipole
- → Basic Terms and Definitions → 50%
- → Antenna Arrays
- → FRIIS - Free Space Propogation
- → Study / classification of Antennas



Hertzian Dipole as a Basic Radiating Element :-

It is a dl length $I(t) = I_m \sin\omega t$ (oscillatory or Harmonic) current element with $dl \ll \lambda$

(I) $$A(t) = \frac{\mu I(t) dl}{4\pi r}$$

(II) $$B(t) = \mu H(t) = \nabla \times \bar{A}$$

(III) $$\nabla \times H = \epsilon \frac{\partial E}{\partial t}$$ (IV) $$E(t) = \frac{1}{\epsilon}\int (\nabla \times H)\, dt$$

Every harmonic current produces a time / space harmonic E & H around it. This EM wave generation is called as radiation

Expression for Radiated fields of Hertzian Dipole :-

Radi-ated wave

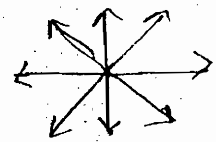
$$\begin{cases} E(r,\theta,\phi,t)_\theta = \left(\dfrac{I_m dl \cdot \sin\theta \cdot \omega}{4\pi\epsilon c^2 r}\right) \sin\omega t \cdot e^{-j\beta r} a_\theta \\ H(r,\theta,\phi,t)_\phi = \left(\dfrac{I_m dl \sin\theta\, \omega}{4\pi c r}\right) \sin\omega t \cdot e^{-j\beta r} a_\phi \end{cases}$$

$W \rightarrow E(z,t)_x = E_0 \, e^{j\omega t} \, e^{-\gamma z} \, a_x$

Properties of Radiated Waves :-

i) They travel radially outward and hence their amplitudes decrease as $\frac{1}{r}$ due to their power density decreases as $\frac{1}{r^2}$

→ Amplitude dec. but does not attenuate.



$$E_\theta \times H_\phi = P_r$$

$$\frac{E_\theta}{H_\phi} = \frac{1}{\epsilon c} = \frac{1}{\epsilon \sqrt{\frac{1}{\mu \epsilon}}} = \sqrt{\frac{\mu}{\epsilon}} = 120\pi \; \Omega$$

(II) The amplitude of the radiated wave is not omni-directional and depends on θ and ϕ i.e. Radiation is directive in nature.

(III) The amplitude of the radiated wave always depends on $\left(\frac{dl}{\lambda}\right)$ ratio

i.e. frequency decides radiated power

Total Power Radiated from a Hertzian Dipole :-

$$W_r = \oint P_{avg} \cdot ds$$

$$= \oint_{\text{Sphere}} \frac{1}{2} \frac{E_0^2}{\eta} a_r \cdot ds \cdot a_r$$

$$W_r = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \frac{1}{2} \left(\frac{I_m \, dl \sin\theta \, \omega}{4\pi \epsilon c^2 r} \right)^2 \frac{1}{120\pi} r^2 \sin\theta \, d\theta \, d\phi$$

$\left(\omega = \frac{2\pi c}{\lambda} \right)$

$$W_r = I_{rms}^2 \cdot 80\pi^2 \left(\frac{dl}{\lambda}\right)^2$$

The expression is similar to a resistor dissipating power as heat i.e. every antenna radiates power in the form of EM wave

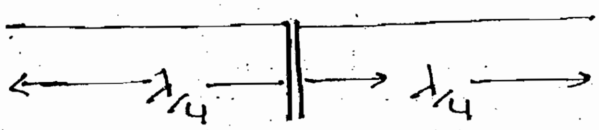
R_r = Radiation resistance of Hertzian Dipole

$$= 80\pi^2 \left(\frac{dl}{\lambda}\right)^2$$

→ R_r is a measure of radiated power for a given input current i.e. it should be as possible for a practical antenna

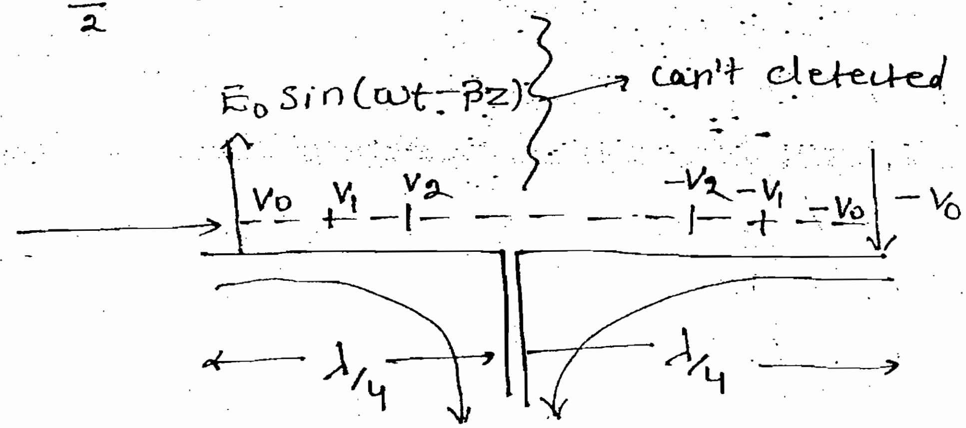
Halfwave dipole as a fundamental Antenna :-

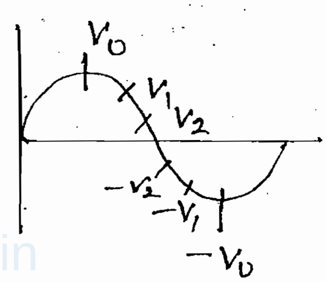
→ A centre feed $\lambda/_2$ - dipole is a transmission opened out by $\lambda/_4$ on either sides



→ The length of antenna is strictly depends on the frequency of operation

FM	GSM
$f = 100\,MHz$	$f = 1800\,MHz$
$\lambda = 3m$	λ = few cm
$\frac{\lambda}{2} = 1.5m$	





Note :-

When an EM waves having right frequency and right polarization travels along the axis of the antenna, it induced V_0 voltage at one end and progressively decrease voltage such that it is $-V_0$ at the other end. Hence the word half wave dipole

→ These voltages drive a current towards centre which is maximum in the line such that

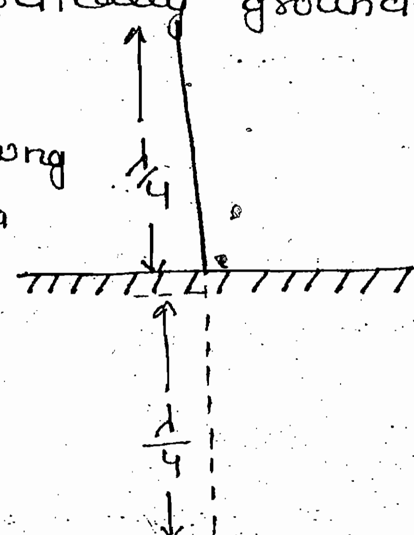
Assymptotic V/I distribution
$$\left.\begin{aligned} |V(z)| &= V_0 \sin \beta z \\ |I(z)| &= I_0 \cos \beta z \end{aligned}\right] \rightarrow z = -\frac{\lambda}{4} \text{ to } \frac{\lambda}{4}$$

* V_0 = −ve at other end bec. of phase diff = 180°

Quaterwave Monopole as a Practical Antenna :-

→ It is a low frequency vertically grounded single wire of $\lambda/4$ length

→ It is a base feed and along with its image works like a harmonic dipole.



Summary :-

A halfway dipole is an array of Hertzian dipoles with dl from $-\lambda/4$ to $\lambda/4$ and

$I_m = |I(z)| = I_0 \cos \beta z$
Assymptotic

Radiation Expressions for Halfwave Dipole :-

$$E(r,\theta,\phi,t)_\theta = \left(\frac{60 I_0}{r} \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right) \cdot \sin\omega t \cdot e^{-j\beta r} a_\theta$$

$$H(r,\theta,\phi,t)_\phi = \left(\frac{I_0}{2\pi r} \frac{\cos\left(\pi/2\cos\theta\right)}{\sin\theta}\right) \sin\omega t \cdot e^{-j\beta r} a_\phi$$

Total power radiated from Halfwave Dipole

$$W_r = \int \frac{1}{2} \frac{E_0^2}{\eta} \cdot d_r$$

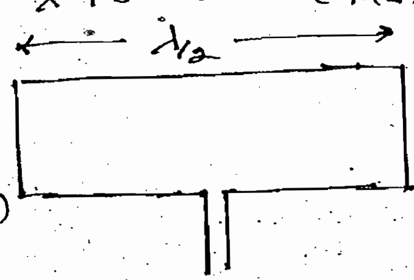
$$= \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \frac{1}{2} \left(\frac{60 I_0}{r} \cdot \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right)^2 \cdot \frac{1}{120\pi} r^2 \sin\theta d\theta d\phi$$

$$= I_{rms}^2 (73)$$

R_r for a Halfwave Dipole $= 73\,\Omega$

R_r for Quaterwave monopole $= 36.5\,\Omega$

R_r for a folded dipole $= 2^2 \times 73\,\Omega = 292\,\Omega$



R_r for n-Dipoles $= n^2 \times 73$

$(n^2 \times 273)$

Basic Terms and Definitions :-

1) Isotropic Antenna :-

It radiates power in all directions uniformly. Its E field pattern is independent of θ and ϕ

eg:- Broadcast antenna

(ii) Radiation Power Density :-

It is the strength of the radiated EM wave in any direction at any distance from the antenna

$$\frac{\text{Power}}{\text{Area}} \text{ or } \text{Watts}/m^2 = \frac{dW_r}{ds}$$

= Poynting vector of EM wave

$$= \frac{1}{2} \frac{E_0^2}{\eta} (r, \theta, \phi)$$

$$= V(r, \theta, \phi)$$

imp

iii) Radiation Power Intensity :-

It is the strength of the radiated EM wave in any direction from the antenna

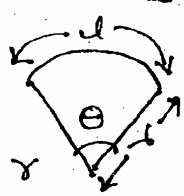
$$\frac{\text{Power}}{\text{direction}} = \underline{\text{Power/solid angle}}$$

$$= \text{with steradian} = \frac{dW_r}{d\Omega}$$

$l = \theta r$

If $\theta = 1$ then $l = r$

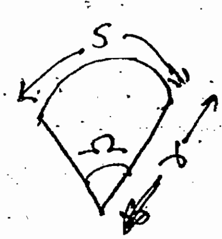
If $\theta = 6.28$ radian then $C = 6.28\, r$



$S = \Omega r^2$

$\Omega = 1$ Steradian

$S = r^2$



$\Omega = 12.56$ steradian

Total surface area $= 12.56\, r^2$

Any small incremental area, $ds = r^2 d\Omega = r^2 \sin\theta . d\theta\, d\phi$

$d\Omega = \sin\theta\, d\theta\, d\phi$

$$\frac{dW_r}{d\Omega} = \frac{dW_r}{ds} \cdot \frac{ds}{d\Omega}$$

$$= V(r, \theta, \phi)\, r^2 = \Psi(\theta, \phi)$$

eg:- Isotropic Antenna

$$\frac{dW_r}{ds} = \frac{W_r}{4\pi r^2} = V_{avg}.$$

$$\frac{dW_r}{d\Omega} = \frac{W_r}{4\pi} = \Psi_{avg}.$$

IV) Gain of an Antenna:-

$G_D \longrightarrow$ Directive gain

$G_P \longrightarrow$ Power gain

$D \longrightarrow$ Directivity

(a) Directive Gain (G_D):-

The radiation intensity of the antenna in a given direction to the radiation intensity of isotropic ~~ro~~ antenna

$$G_D = \frac{\Psi(\theta, \phi)}{\Psi_{avg}} = \frac{4\pi \Psi(\theta, \phi)}{W_r} = \frac{4\pi \Psi(\theta, \phi)}{\int \Psi(\theta, \phi)\, d\Omega}$$

$G_D > 1$ or $G_D < 1 \longrightarrow$ depends on direction

(b) Power Gain:-

$$G_P = \frac{4\pi \cdot \Psi(\theta, \phi)}{W_N}$$

W_r = Total o/p power

W_N = Total i/p power

$$G_p = \frac{4\pi \psi(\theta,\phi)}{W_r} \cdot \frac{W_r}{W_N}$$

$$= G_D \times \text{Efficiency of Radiation}$$

$$\text{Efficiency} = \frac{W_r}{W_r + W_l} = \frac{R_r}{R_r + R_l}$$

$$\text{Directivity} = G_D\Big|_{max}$$

$\boxed{D \geq 1}$ → Always

$D = 1$ for isotropic antenna

Radiation Pattern of an Antenna :-

It is a polar plot of radiation intensity indicating various directions around the antenna where radiation is finitely strong

HPBW : Half Power Beam width

BWFN : Beam Width Full Nulls

eg :- $|E| = \frac{k\sin\theta}{r}$

$\psi = \psi_0 \sin^2\theta$

$\theta = 0° / 180°$

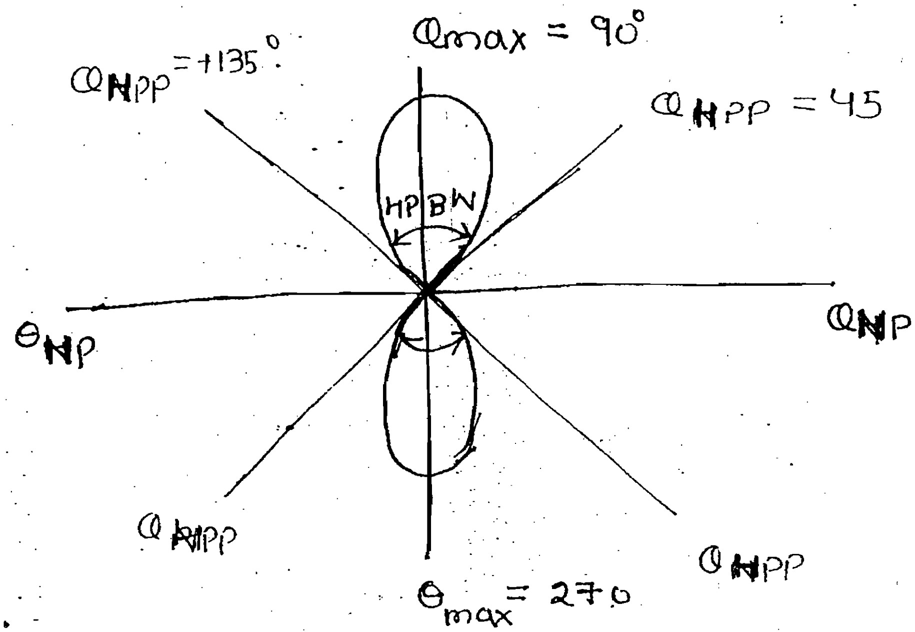
$|E| = 0$ → Null points

→ θ_{NP}

$\theta = 90^\circ/270^\circ$ $\qquad$ $\theta = 45^\circ/135^\circ/225^\circ/315^\circ$

$|E| = E_{max}$ $\qquad$ $|E| = \frac{E_{max}}{\sqrt{2}}$

$\rightarrow \theta_{max}$



θ_{HPBW} = HPP is next HPP in the maxima (Half power pt.)

$= 135^\circ - 45^\circ = 90^\circ$

The antenna considered has a ϕ independent pattern and hence a circular top view of the beam (for all ϕ)

In general

$$\theta_{HPBW} \times \phi_{HPBW} = \Omega_A \text{ Beam solid angle}$$

$$= \text{steradian} = \text{radian}^2$$

In circular

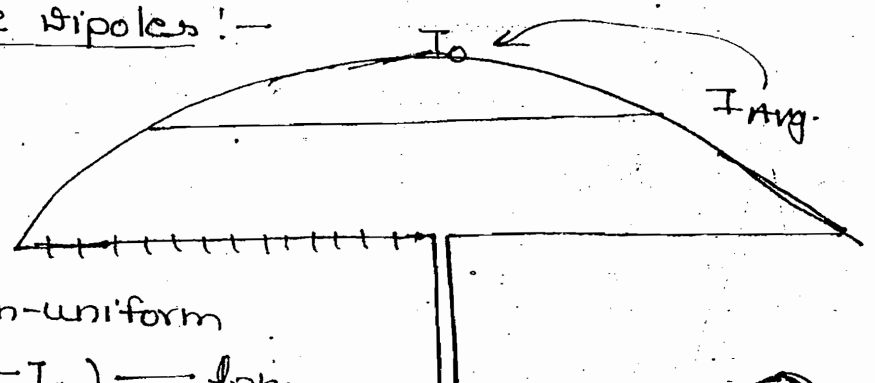
$$(\theta_{HPBW})^2 = \Omega_A$$

$$D \propto \frac{1}{\Omega_A}$$

$$\Rightarrow \boxed{D = \frac{4\pi}{\Omega_A}}$$

(VI) Effective length of an antenna :-

(a) For Halfwave dipoles :-



Non-uniform

W_r —— $(0 - I_0)$ —— l_{phy}

W_r —— I_{avg} —— l_{eff}

Note :-

→ An antenna radiates W_r power over its physical length and non-uniform currents everywhere. Effective length is a length required to radiate same power assuming uniform currents everywhere

$$I_{avg.} = \frac{1}{\pi}\int_0^{\pi} I_0 \sin t \, dt = \frac{2I_0}{\pi}$$

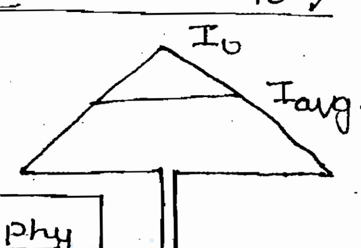
$$\boxed{l_{eff} = \frac{2 l_{phy.}}{\pi}}$$

(b) For electrically short dipoles ($l_{phy.} < \frac{\lambda}{10}$) :-

$|I(z)| \propto z$

Linear current

$$I_{avg.} = \frac{I_0}{2}$$



$$\boxed{l_{eff} = \frac{l_{phy}}{2}}$$

c) <u>For Hertzian Dipole</u> $\left(l_{phy} < \frac{\lambda}{25}\right)$:—

$$l_{phy} = l_{eff.} = dl$$

$$I_{avg} = I_m \longrightarrow \text{Uniform currents}$$

For Hertzian Dipole $R_r = 80\pi^2 \left(\frac{dl}{\lambda}\right)^2$

For any Dipole, $R_r = 80\pi^2 \left(\frac{l_{eff}}{\lambda}\right)^2$

For Electrically short Dipoles $R_r = 80\pi^2 \left(\frac{l}{2\lambda}\right)^2$

$$= 20\pi^2 \left(\frac{l}{\lambda}\right)^2$$

" " Monopoles $R_r = 10\pi^2 \left(\frac{l}{\lambda}\right)^2$

Monopoles over conducting earth $R_r = 10\pi^2 \left(\frac{2h}{\lambda}\right)^2$

$$= 40\pi^2 \left(\frac{h}{\lambda}\right)^2$$

For halfwave dipole $= 80\pi^2 \left(\frac{2l}{\pi\lambda}\right)^2$

$$= 320\pi^2 \left(\frac{l}{\lambda}\right)^2 = 80\,\Omega$$

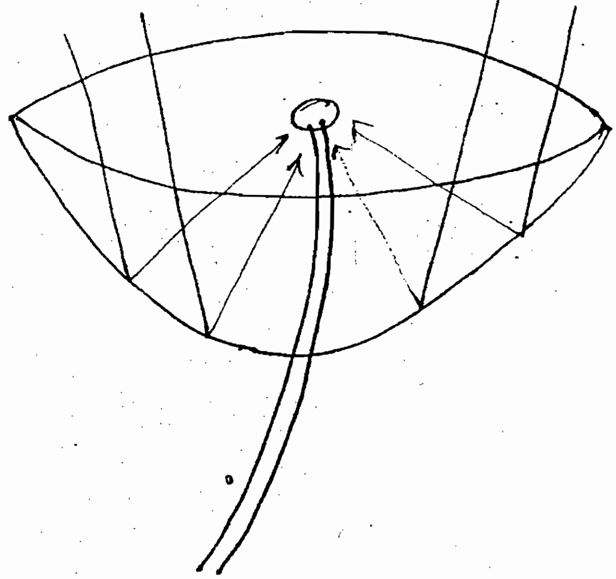
VII) <u>Capture Area or Effecting Area (A_c)</u> :—

In microwave antenna l = few cm

r = few 1000 km

A_c = capture Area = $\dfrac{\text{Power induced}}{\text{Poynting Vector}}$

$$\boxed{A_c = \frac{\lambda^2}{4\pi} D}$$



Workbook :-

$$D \rightarrow G_D\big|_{max} \longrightarrow \Psi(\theta, \phi) \longrightarrow U(r, \theta, \phi) \longrightarrow E(r, \theta, \phi)$$

Hertzian Dipole, $|E| = \dfrac{k\sin\theta}{r}$

$$U(r, \theta, \phi) = \frac{1}{2}\,\frac{k^2\sin^2\theta}{r^2 \cdot \eta}$$

$$\Psi(\theta, \phi) = \frac{1}{2}\,\frac{k^2\sin^2\theta}{\eta}$$

$$G_D = 4\pi^2 \cdot \frac{1}{2}\,\frac{k^2\sin^2\theta}{\eta}$$

$$\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} \frac{1}{2}\,\frac{k^2\sin^2\theta}{\eta}\sin\theta\, d\theta \cdot d\phi \qquad = \underline{2\sin^2\theta}$$

$$\int_{\theta=0}^{\pi} \sin^3\theta\, d\theta = \frac{4}{3}$$

$\Leftarrow$

$$= \frac{3}{2}\sin^2\theta$$

$D = GD|_{max} = 1.5$

For Halfwave Dipole, $D = 1.63$

$4 - \frac{\lambda}{2}$ Dipoles

$$W_r = 4 I_{rms}^2 \cdot 73 = 4\left(\frac{0.5}{\sqrt{2}}\right)^2 73 = 36.5 \text{ Watts}$$

3. $D = \frac{4\pi}{\Omega_A}$

$10 \log D = 44$ dB

$\Rightarrow D = \frac{4\pi}{(\theta_{HPBW})^2}$

$\Rightarrow D = 10^{4.4}$

$10^{4.4} = \frac{4 \times 3.14}{(\theta_{HPBW})^2} \times (57)^2$

4. Efficiency $= \frac{R_r}{R_r + R_d}$

R_r for $\lambda/8$ Dipole

$R_r \ \lambda/2$ — 73Ω

$R_r \ \lambda/4$ — 36.5Ω

$R_r \ \lambda/8$ — 18.25Ω

Efficiency $= \frac{18.25}{19.75} = 89\%$

5. $\lambda \rightarrow 492$ m

$d \rightarrow 124$ m

$\frac{\lambda}{4} = 123$ m

$R_r = 36.5\Omega$

6. $10 \log 4 = 6$dB

$G = 4$

$W_{IN} = 1$mW

lossless

$W_r = W_{IN}$

$\longrightarrow$ Ans- (B)

7. (B)

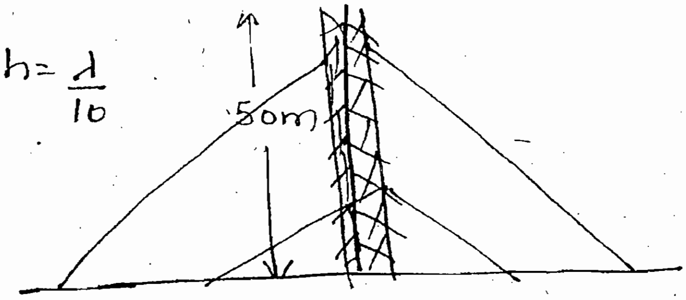
110

$h = 50 m$

$f = 60 kHz$

$\lambda = \frac{3 \times 10^8}{6 \times 10^5} = 500 m$

$h = \frac{\lambda}{10}$



$R_r = 40\pi^2 \left(\frac{50}{500}\right)^2$

$\cong 4\Omega = \frac{2\pi^2}{5}$

125 — $\lambda/4$ — 36.5 Ω

50 m — $\frac{\lambda}{10}$ — 4 Ω

$W_r = I_{rms}^2 R_r$

9. $\Psi_{avg} = \frac{100}{4\pi} = \frac{25}{\pi} = 7.96$

$V_{avg} = \frac{100}{4\pi \times (10 \times 10^3)^2} = 7.96 \times 10^{-8}$

$= 0.08 \mu W$

10. $\Psi \propto W_r \propto R_r \propto \frac{1}{\lambda^2} \times f^2$

strongly

$f' = f/2$

$\Psi' = \Psi/4$

11. $D = ?$ $\quad D = ?\ dB$

$D = \frac{4\pi \Psi(\theta, \phi)_{max}}{W_r}$

$= \frac{4\pi \cdot 150}{0.9\ (W_{IN})} = \frac{4\pi \cdot 150}{36\pi} = 16.67 = D$

$10 \log 16.67 = 11.76\ dB$

$D = 11.76\ dB$

12. 1 A — 1m — using

$\hookrightarrow I_{rms}$

$f = 10\,MHz$

$\lambda = \frac{3 \times 10^8}{10 \times 10^6} \times 30m$

$l = \frac{\lambda}{30}$

$W_r = I_{rms}^2 R_r$

$R_r = 80\pi^2 \left(\frac{1}{30}\right)^2 = 0.88\Omega$

$W_r = 0.88$

$\left(\frac{dl}{\lambda}\right) \begin{cases} \lambda/2 \rightarrow 73\Omega \\ \frac{\lambda}{4} \rightarrow 36.5\Omega \\ \frac{\lambda}{10} \rightarrow 4\Omega \\ \frac{\lambda}{30} \rightarrow 0.88\Omega \end{cases}$

13. $\theta_{HPBW} = 90^0$

14. A

15. Uniform currents

$l = 5m$

$\lambda = 100m$

$R_r = 80\pi^2 \left(\frac{dl}{\lambda}\right)^2$

$= 80\pi^2 \left(\frac{\lambda}{20\lambda}\right)^2 \cong 2\Omega$

16. A

18. $l = 1.5m$

$f = 100\,MHz$

$\lambda = 3m$

$l = \frac{\lambda}{2}$

$l = 15m$

$f = 10\,MHz$

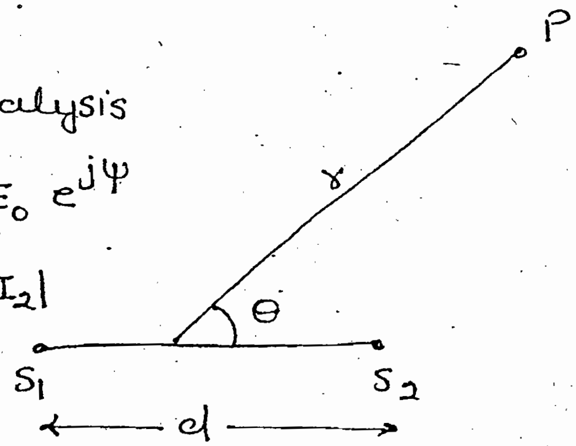
$\lambda = 30m$

$l = \frac{\lambda}{2}$

Halfwave Dipole

ANTENNA ARRAYS :-

2-Element Isotropic Array :-



$r >> d$ Far-zone analysis

$E_r = E_1 + E_2 = E_0 + kE_0 e^{j\psi}$

where $k = 1$ if $|I_1| = |I_2|$

ψ = Phase diff. b/w the two fields

= current having phase difference (α) + path difference giving phase

Path difference = $d\cos\theta$

phase diff. $= \frac{2\pi}{\lambda} d\cos\theta = \beta d\cos\theta$

$$\boxed{\psi = \alpha + \beta d\cos\theta}$$

$E_T = E_0(1 + \cos\psi + j\sin\psi)$

$|E_T| = |E_0|\sqrt{(1+\cos\psi)^2 + \sin^2\psi}$

$= |E_0|\sqrt{2 + 2\cos\psi}$

$|E_T| = 2E_0 \cos(\psi/2)$

Case-(1) :-

$d = \frac{\lambda}{2}$, $\alpha = 0$

$\psi = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} \cos\theta$

$|E_T| = 2E_0 \cos(\frac{\pi}{2}\cos\theta)$

If $\theta = 90°/270° \longrightarrow \theta_{max}$

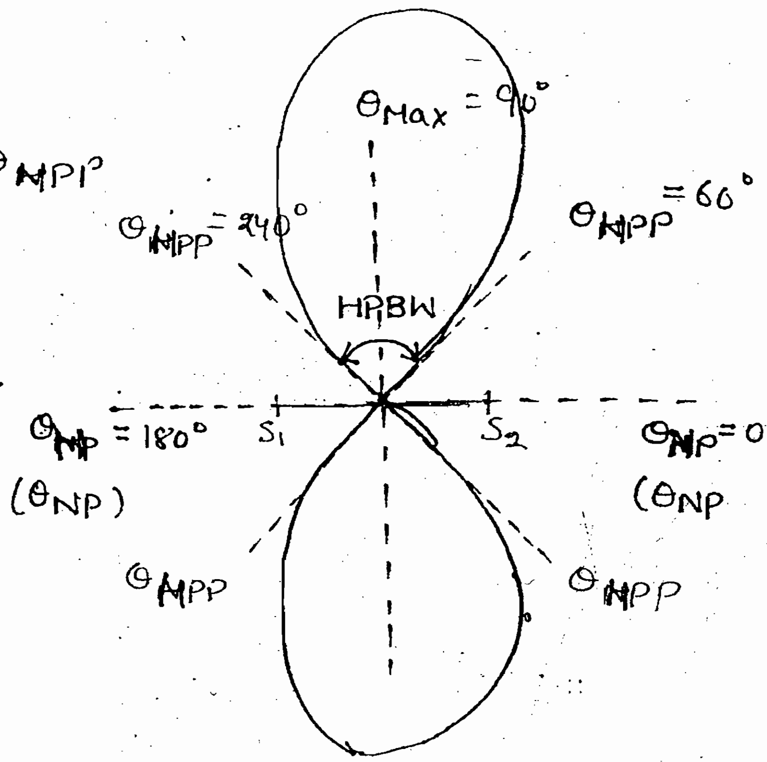
$|E_T| = 2E_0 = E_{max}$

If $\theta = 0^\circ / 180^\circ \longrightarrow \theta_{NP}$

$|E_T| = 0$

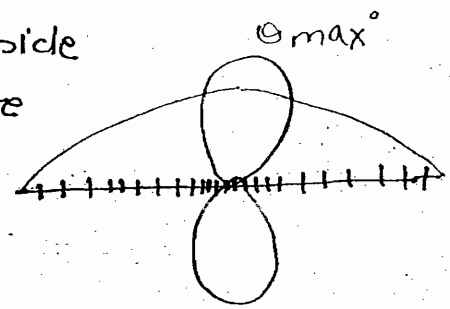
If $\theta = 60^\circ/120^\circ/240^\circ/300^\circ \longrightarrow \theta_{HPP}$

$|E_T| = \dfrac{E_{max}}{\sqrt{2}}$



Note :—

The currents on either side inphase and max. at the centre and halfwave dipole is also broad side pattern



Case-(ii) :-

$d = \dfrac{\lambda}{2}, \quad \alpha = \pi$

$\psi = \pi + \dfrac{2\pi}{\lambda} \cdot \dfrac{\lambda}{2} \cos\theta$

$|E_T| = 2E_0 \sin\left(\dfrac{\pi}{2}\cos\theta\right)$

If $\theta = 90^\circ/270^\circ \longrightarrow \theta_{NP}$

$|E_T| = 0$

If $\theta = 0^\circ/180^\circ \longrightarrow \theta_{max}$

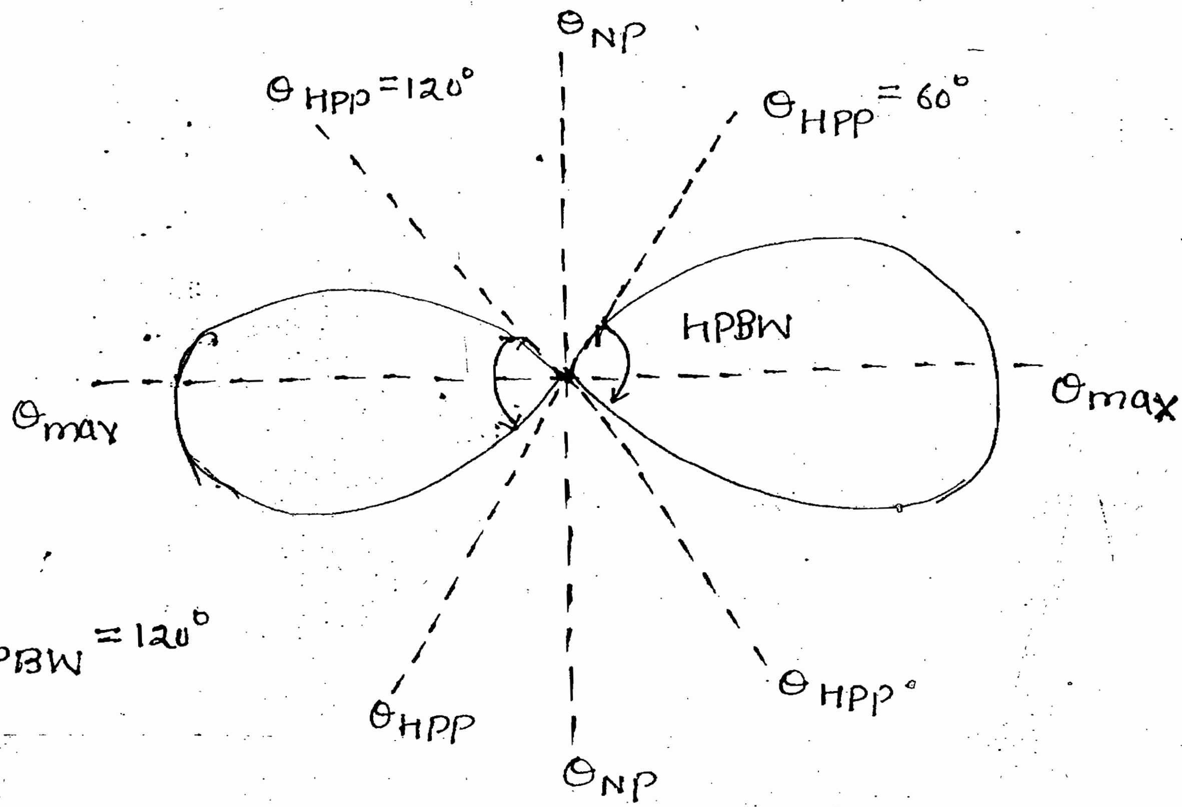
$|E_T| = E_{max} = 2E_0$

If $\theta = 60^\circ/120^\circ/240^\circ/300^\circ \longrightarrow \theta_{HPP}$

$|E_T| = \dfrac{E_{max}}{\sqrt{2}}$

This is called as End-fire Array whose

$\theta_{max} = 0°/180°$



$\theta_{HPBW} = 120°$

<u>Case-(III):-</u>

$d = \lambda$, $\alpha = 0°$

$E_T = 2E_0 \cos(\pi \cos\theta)$

If $\theta = 0°/90°/180°/270°$

$\longrightarrow \theta_{max}$

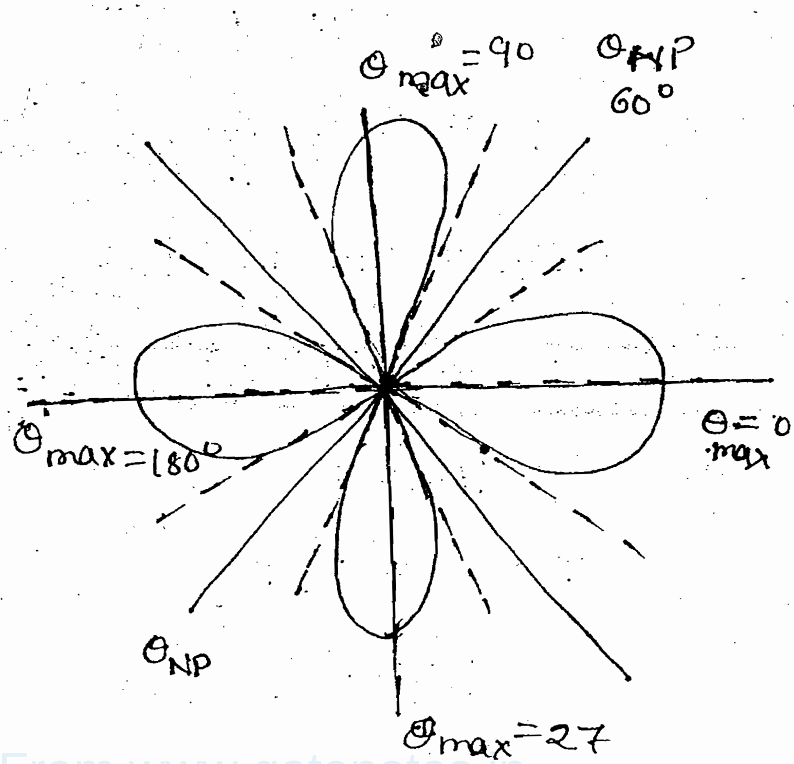
If $\theta = 0°/90°/180°/270°$

$\longrightarrow \theta_{max}$

If $\theta = 60°/120°/240°/300°$

$\longrightarrow \theta_{NP}$

$\cos\theta = \pm\frac{1}{4}$ or $\pm\frac{3}{4}$



In general θ_{max} can be designed towards a specific direction

With $\psi \to 0$

$$\alpha + \beta d \cos\theta_{max} = 0$$

$$\boxed{\cos\theta_{max} = \frac{-\alpha}{\beta d}}$$

For broadside array,

$$\theta_{max} = 90^\circ / 270^\circ$$

$$\Rightarrow \boxed{\alpha = 0}$$

Inphase currents

For end-fire array

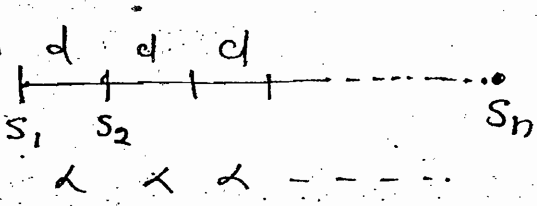
$$\theta_{max} = 0^\circ / 180^\circ \quad \Rightarrow \quad \boxed{\alpha = \pm \beta d}$$

~~θ_{max} -~~ ~~$\alpha = \pm$~~

Extension :-

For n elements, uniform, linear isotropic sources

$$E_T = E_0 (1 + e^{j\psi} + e^{j2\psi} + \dots e^{j(n-1)\psi}$$



G.P with $a = 1$ $r = e^{j\psi}$

$$E_T = \frac{E_0 (1 - e^{jn\phi})}{(1 - e^{j\psi})}$$

$$\boxed{|E_T| = \frac{E_0 \sin(n \cdot \psi/2)}{\sin(\psi/2)}}$$

If $\psi \to 0$

$$|E_T| = E_0 \frac{n(\psi/2)}{\psi/2} = nE_0$$

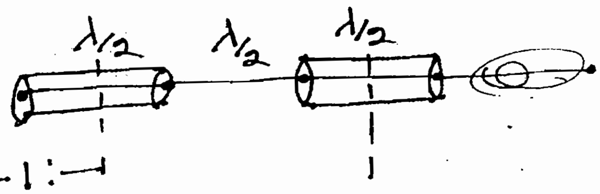
$$= E_{max}$$

If $n = 2$

$$|E_T| = 2E_0 \cos(\psi/2)$$

~~Workbook~~

9. Theorem of Multiplication of Patterns (Symm. Arrays) 2



Step-1:-

Identify the two points that symm. makes the group and Identify Eu unit pattern.

UNIT:-

$$d = \frac{\lambda}{2}, \quad \alpha = 0$$

2-Element Array

$$E_u = \cos\left(\frac{\pi}{2}\cos\theta\right)$$

Step 2:-

Identify the group formed from the two points and its pattern

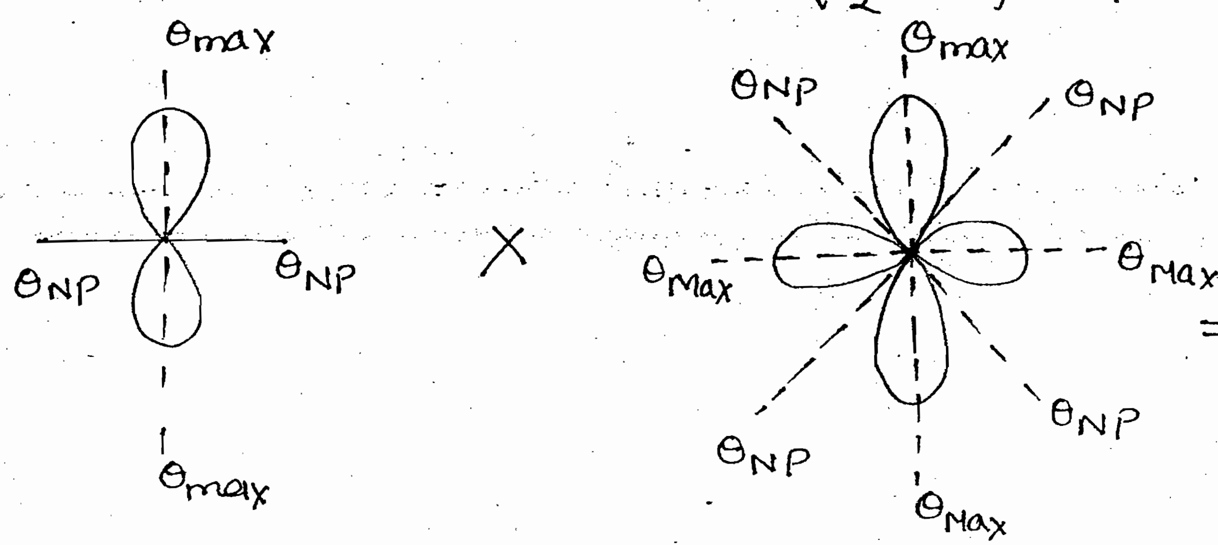
GROUP:- $d = \lambda \quad \alpha = 0$

$$E_g = \cos(\pi \cos\theta)$$

Step 3:-

~~Resultant~~ Resultant Pattern = Unit Pattern x Group Pattern

$$= \cos\left(\frac{\pi}{2}\cos\theta\right)\cos(\pi\cos\theta)$$



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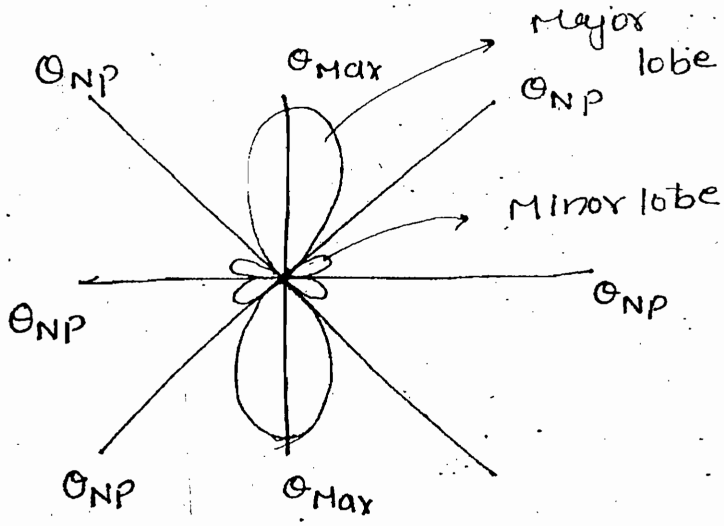


Workbook !-

19. C

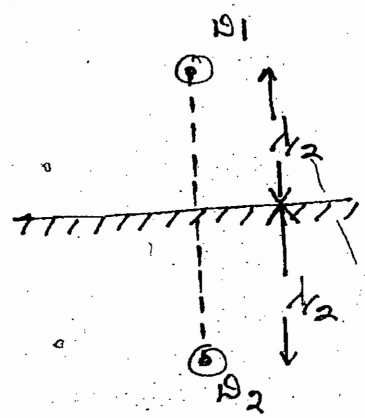
20 Unit - Dipole Antenna

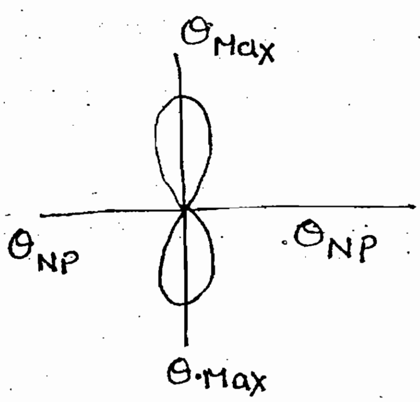
$$E_u = \cos\left(\frac{\pi}{2}\cos\theta\right)$$

GROUP → $d = \lambda$

$\alpha = \pi$ - 2 Dipoles

$$E_g = \sin(\pi\cos\theta)$$





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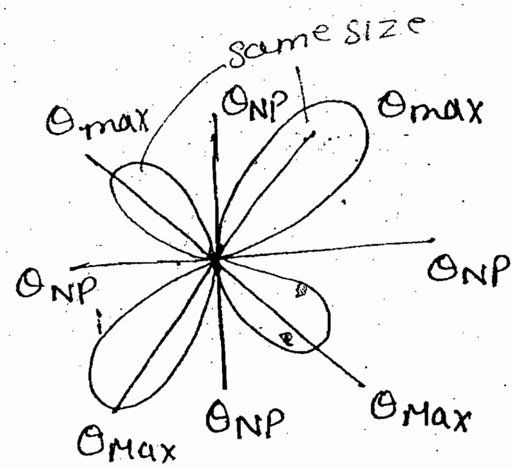

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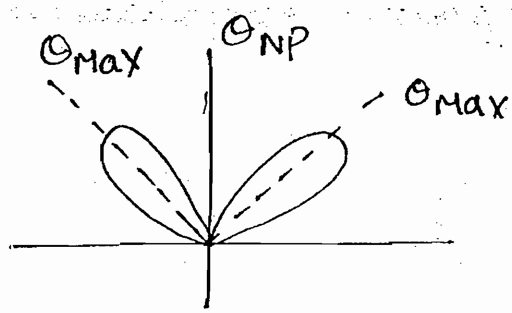

Ans - B

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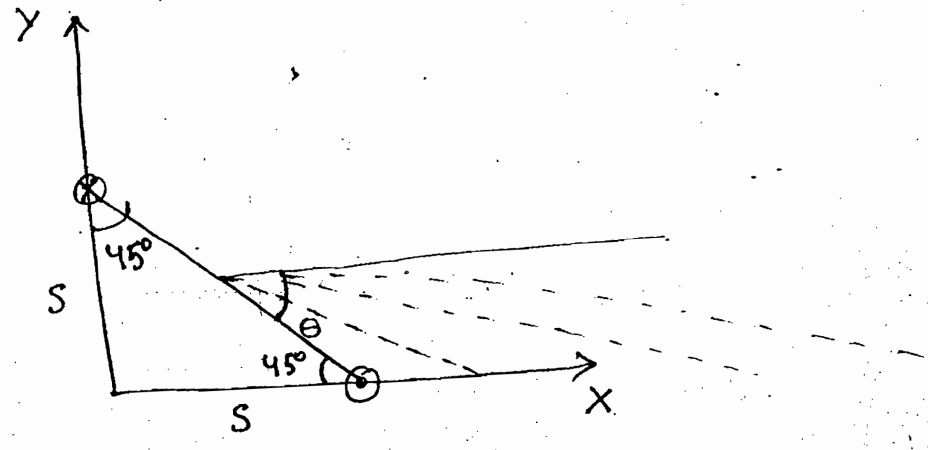
$\theta = \frac{\pi}{2}$ Plane $\longrightarrow$ $z = r\cos\theta = 0$ $\longrightarrow$ XY Plane



$d = \sqrt{2}\,S$, $\alpha = \pi$

$\theta = 45°$

$$\frac{E_T}{E_0} = 2\cos\left(\frac{\alpha + \beta d\cos\theta}{2}\right)$$

$$= 2\cos\left(\pi + \frac{2\pi}{\lambda}\cdot\frac{d}{2}\cdot\frac{1}{\sqrt{2}}\right)$$

$$= 2\sin\left(\frac{\pi S}{\lambda}\right)$$

22. $\theta_{max} = 60°$ off end-fire

$$\alpha + \beta d\cos 60° = 0$$

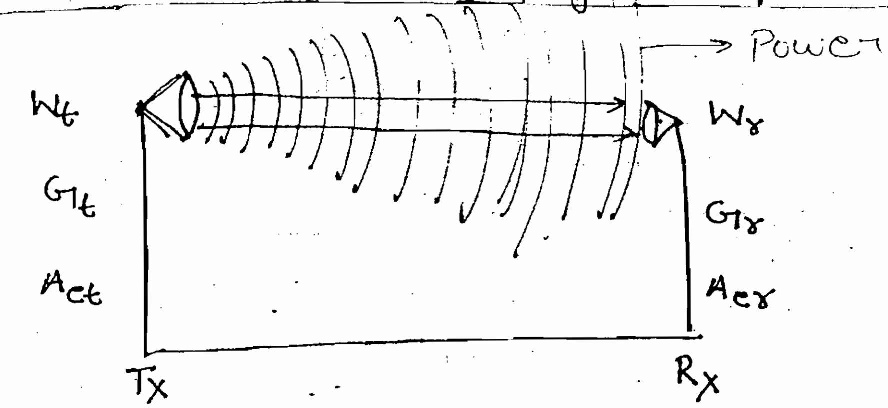
$$\alpha = -\frac{2\pi}{\lambda}\cdot\frac{\lambda}{4}\cdot\frac{1}{2} = -\frac{\pi}{4}$$

23. Unambigous Direction finding

— 4 lobe pattern

$d = \lambda$ $\alpha = 0°$

FRIIS – free Space Propogation Equation:–



$$\text{Power density at Rx} = \frac{W_t}{4\pi d^2} \cdot G_t$$

$$W_r \text{ at Rx} = \frac{W_t \cdot G_t \cdot A_{er}}{4\pi d^2}$$

$$A_{er} = \frac{\lambda^2 G_r}{4\pi}$$

$$W_r = \frac{W_t \cdot G_t \cdot G_r}{\left(\frac{4\pi d}{\lambda}\right)^2}$$

$$\left(\frac{4\pi d}{\lambda}\right)^2 = L_s = \text{Loss due to spatial dispersion.}$$

$$W_r(dBW) = W_t(dBW) + G_t(dB) + G_r(dB) - L_s(dB)$$

$$\frac{W_t G_t}{4\pi d^2} = \frac{1}{2}\frac{E_0^2}{\eta} = \frac{E_{rms}^2}{120\pi}$$

$$\boxed{E_{rms} = \frac{\sqrt{30\, W_t \cdot G_t}}{d}}$$

Workbook :—

17. $W_r(dBW) = 10\,dBW + 10\,dB + G_r(dB) - 100\,dB$

Tx – circularly polarized

Rx – Linearly polarised.

$\frac{1}{2}$ – Power received

due to polarization mismatch

$G_r = \frac{1}{2}$ $\quad 10\log\frac{1}{2} = -3\,dB$

$G = 2 = 3\,dB$

⟶ doubled

24. $W_r = \frac{W_t \cdot G_t \cdot A_{er}}{4\pi d^2} = 0.8\,W$

25. $W_r = \frac{W_t \cdot (G)^2}{\left(\frac{4\pi d}{\lambda}\right)^2}$ $\quad d = 30\,km$

$\lambda = \frac{3\times10^8}{1\times10^9} = 0.3\,m$

$W_r = -30\,dBm$ for $W_t = 1\,W$

$\left(\frac{W_r}{W_t} = -30\,dBm\right)$

$\left(\frac{W_r}{W_t} = -30\,dB\right)\times10^{-3}$

$\left(10\log\frac{W_r}{W_t} = -30\right)\times10^{-3}$

$\left(\frac{W_r}{W_t} = 10^{-3}\right)\cdot10^{-3}$ $\quad$ Ans – D

$\frac{W_r}{W_t} = 10^{-6}$

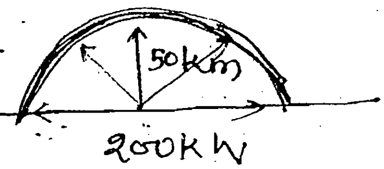
26

$$E_{rms} = \frac{\sqrt{30 \cdot 1000 \cdot 1.63}}{10 \times 10^3}$$

27.

$$P_{avg} = \frac{W_t}{2\pi d^2} a_r$$

$$= \frac{200 \times 10^3}{2\pi \times (50 \times 10^3)^2} \cdot a_r$$



$\longrightarrow \vartheta$

28.

E_{rms} — 5kms —— d

E'_{rms} — $\frac{E_{rms}}{\sqrt{2}}$ —— $d' = ?$

Further more ?

to detect 3dB decrease

3dB in power — $\frac{1}{2}$ — power

— $\frac{1}{\sqrt{2}}$ — field

$$E \propto \frac{1}{d}$$

$$E' = \frac{E}{\sqrt{2}}$$

$$d' = \sqrt{2}d = \sqrt{2} \times 5 \text{ kms} = 7 \text{ km}$$

Further more = 2 km

29. $E \propto \frac{1}{d}$

Conventional :-

$l = 10cm$ $\qquad f = 60MHz$

$\lambda = \frac{3 \times 10^8}{60 \times 10^6} \cong 5m$ $\qquad l = \frac{\lambda}{50} \rightarrow$ Hertzian dipole

$R_r = 80\pi^2 \left(\frac{dl}{\lambda}\right)^2$

$W_r = I_{rms}^2 R_r$ — (1)

$I_{rms} = \frac{10mA}{\sqrt{2}}$

$\text{Efficiency} = \frac{R_r}{R_r + 0.1}$

$E_{rms} = \frac{\overset{\text{milli}}{1mV}}{\underset{\text{meter}}{m}} = \frac{\sqrt{30 \cdot W_t \cdot G_t}}{d}$

$\Rightarrow 10^{-3} = \frac{\sqrt{30 \cdot W_t \cdot 1.5}}{d}$

At $\frac{\lambda}{50}$ lengths both monopole and dipole have same effect

$\rightarrow$ $f = 1500MHz$ $\qquad l = 10cm$

$\lambda = 20cm$ $\qquad l = \frac{\lambda}{2} \rightarrow$ halfwave dipole

$R_r = 73$

$E_{rms} = \frac{\sqrt{30 \cdot W_t \cdot 1.63}}{d}$

$\downarrow$

$= ?$