NAME:-

SUBJECT:- Control System (Electrical)

INSTITUTE:- MADE EASY

18/03/2014

Control s/s

(1).— Control system (Nagrath & Gopal)

(2).— K. Ogata

Part-(1): INTRODUCTION TO CONTROL SYSTEM

(1) Consider a liquid level control s/s whose control objective is to keep the water level in the time at a height 'h'.

(2). Controller is an automatic device with error s/g $E(s)$ as I/p and controller o/p $P(s)$ affecting the dynamics of the plant to achieve the control objective.

Controller o/p, $P = f(e)$ where, e = error

∴ Controller o/p is a fn of error.

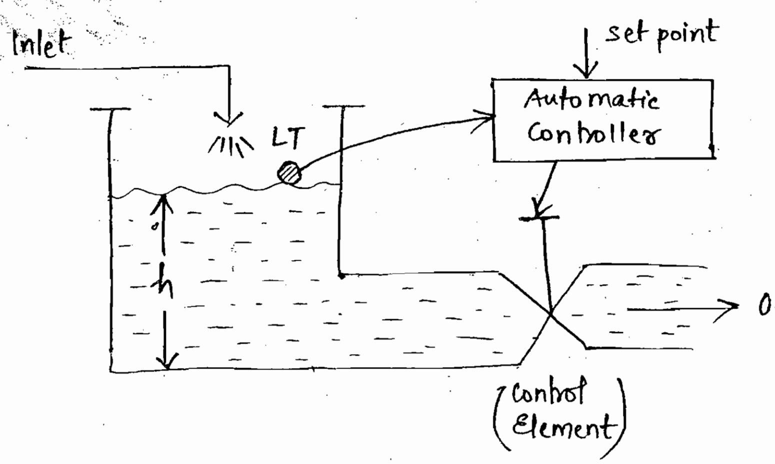
(3). The different modes of controller operation are proportional, proportional + integral and proportional + integral + derivative

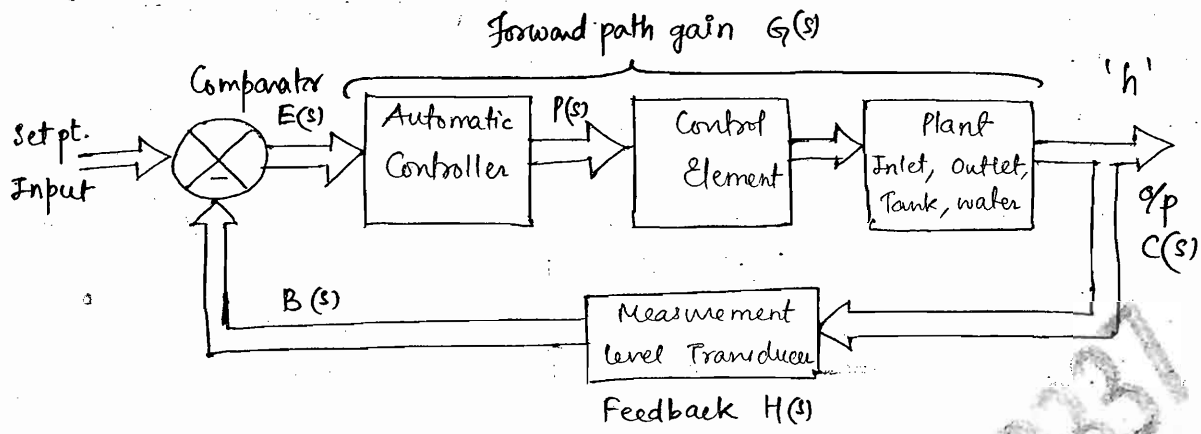
(4). There are 2 basic control loop configurations

(i). Closed loop OR feedback control s/s –

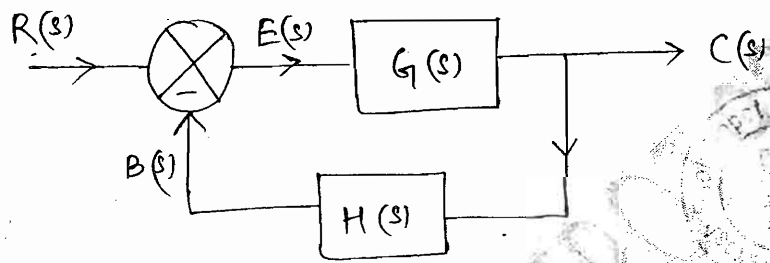
– In this configuration, the changes in the o/p are measured through feedback and compared with the I/p OR set point to achieve the control objective.

Feedback implies measurement (sensors or transducers).

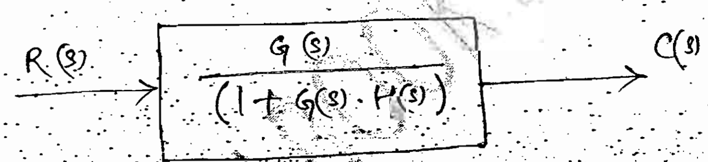




Control canonical form :



Equivalent mathematical form



$$E(s) = R(s) - B(s)$$

$$\frac{C(s)}{G(s)} = R(s) - C(s)\cdot H(s)$$

$$C(s) = R(s)\cdot G(s) - C(s)\cdot G(s)\cdot H(s)$$

$$C(s) = \frac{G(s)\cdot R(s)}{1+G(s)\cdot H(s)}$$

$$\therefore \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)\cdot H(s)}$$

(ii). Open loop Control s/s –

(1). They are conditional control s/s formulated under the condition that the s/s is not subjected to any type of disturbances.

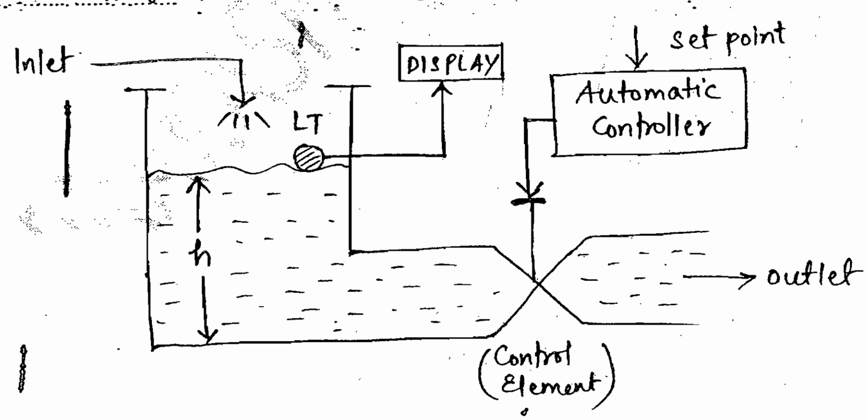
(2). In this configuration the feedback or measurement is not connected to the forward path or controller (open loop).

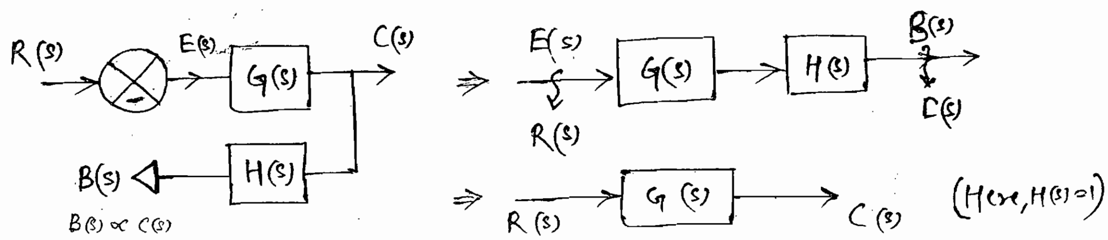
(3). Feedback in open loop s/s except for displaying the information about o/p has no major significance, this insignificantness of feedback is turned as elimination of feedback.

(4). Open loop s/s are more stable than closed loop s/s (without disturbances) because the delays associated with open loop s/s are less compared to close loop s/s.

(5). Performance analysis is not applicable to open loop s/s because they are not subjected to any disturbances and are stable s/s.

Eg. Traffic light s/s, Washing m/c, bread toaster etc.

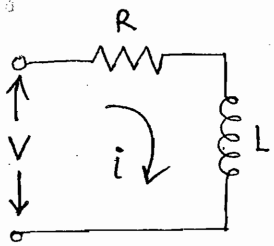




Concept of Transfer function:

It is a mathematical model representing a control s/s relating I/p and O/p in the form of ratio

i.e. $TF = \dfrac{O/p}{I/p}$



$v = iR + L\dfrac{di}{dt}$

(in time domain)

Applying Laplace Transform

$$V(s) = I(s)\cdot R + L\cdot s\cdot I(s)$$

$$V(s) = I(s)\,(R + sL)$$

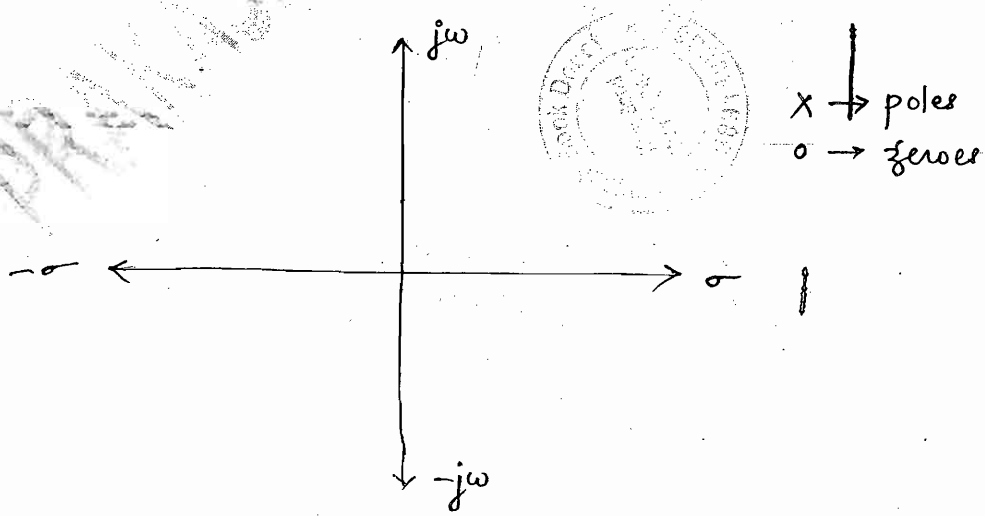
$$\frac{I(s)}{V(s)} = \frac{1}{R + sL}$$

$$\therefore \frac{I(s)}{V(s)} = \frac{1/L}{\left(R/L + s\right)}$$

$$TF = F(s) = \frac{N(s)}{D(s)} = \frac{C(s)}{R(s)} = \frac{K\,(s+Z_1)(s+Z_2)\cdots}{(s+P_1)\,(s+P_2)\cdots} \quad \text{——(1)}$$

Time Constant form: $(1+Ts)$

$$TF = F(s) = \frac{N(s)}{D(s)} = \frac{C(s)}{R(s)} = \frac{K\,(1+T_a s)(1+T_b s)}{(1+T_1 s)\,(1+T_2 s)} \quad \text{——(2)}$$



s-plane (OR space plane)

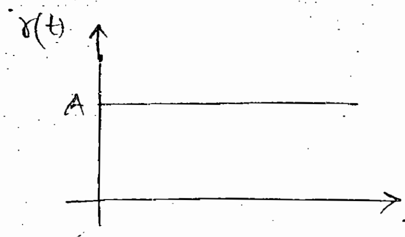
(1). From eqn (1), TF of LTI s/s may be defined as the ratio of Laplace transform of o/p to Laplace transform of I/p under the assumption that the s/s initial conditions are zero.

(2). Poles and zeroes are those critical frequencies which make the TF ∞ and zero respectively.

Singularity functions and TF :

The standard time domain test s/g are also known as singularity functions, define the TF of LTI s/s.

(1). **Step signal**



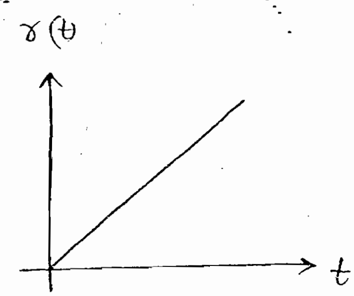
$r(t) = A \cdot u(t)$

$u(t) = \begin{cases} 1 & , t > 0 \\ 0 & , t < 0 \end{cases}$

At t=0, there is no analysis as initial conditions are ignored.

$R(s) = \frac{A}{s}$

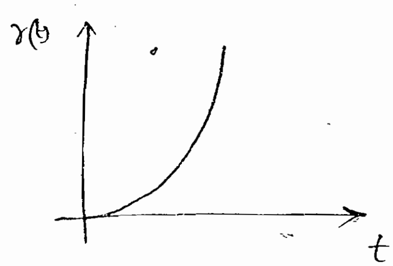
(2). **Ramp signal**



$r(t) = \begin{cases} A \cdot t & , t > 0 \\ 0 & , t < 0 \end{cases}$

$R(s) = A/s^2$

(3). **Parabolic signal**



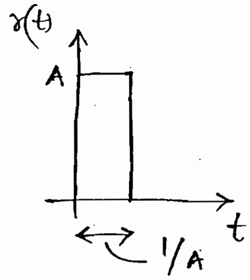
$r(t) = \begin{cases} A \cdot \frac{t^2}{2} & , t > 0 \\ 0 & , t < 0 \end{cases}$

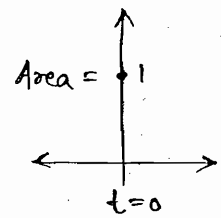
$R(s) = A/s^3$

(4). Impulse signal

$$r(t) = \begin{cases} 1 & , t = 0 \\ 0 & , t \neq 0 \end{cases}$$

$R(s) = 1$





Let $F(s) = \dfrac{C(s)}{R(s)} = TF$

$C(s) = F(s) \cdot R(s)$

Let $R(s) = $ Impulse s/g $= 1$

$\therefore C(s) = $ Impulse response

$= F(s) \cdot 1$

$= F(s) \Rightarrow TF$

$\therefore \mathcal{L}\begin{pmatrix}\text{Impulse} \\ \text{Response}\end{pmatrix} = TF$ (Weighting function)

Relations b/w Responses :

$\dfrac{d}{dt}\begin{pmatrix}\text{Parabolic} \\ \text{Response}\end{pmatrix} = $ Ramp Response

$\dfrac{d}{dt}\begin{pmatrix}\text{Ramp} \\ \text{Response}\end{pmatrix} = $ Step Response

$\dfrac{d}{dt}$ (Step Response) = Impulse Response

$\therefore \mathcal{L}$ (Impulse Response) $= TF$

Q. The impulse response of a s/s is $c(t) = -te^{-t} + 2e^{-t}$ $(t > 0)$. Its open loop TF will be

(a) $\dfrac{2s+1}{(s+1)^2}$ ~~(b)~~. $\dfrac{2s+1}{s^2}$ (c). $\dfrac{2s+1}{s+1}$ (d) $\dfrac{2s+1}{s}$

Sol. $\mathcal{L}\{c(t)\} = C(s) = \frac{-1}{(s+1)^2} + \frac{2}{(s+1)}$

$$= \frac{-1+2s+2}{(s+1)^2}$$

$$\therefore TF = \frac{2s+1}{(s+1)^2} \quad \text{(close loop TF)}$$

$$\frac{G(s)}{1+G(s)\cdot H(s)} = \frac{2s+1}{(s+1)^2}$$

Put $H(s)=1$, then

$$(s+1)^2 \cdot G(s) = (2s+1) + (2s+1)\,G(s)$$

$$G(s) = \frac{2s+1}{\{(s+1)^2-(2s+1)\}} = \frac{2s+1}{s^2} \quad \text{(open loop TF)}$$

Short cut method:

Open loop TF, $G(s) = \frac{\text{Num.}}{(\text{Den.} - \text{Num.})}$ (if $H(s)=1$)

$$G(s) = \frac{2s+1}{(s+1)^2-(2s+1)} = \frac{2s+1}{s^2}$$

Q What is open loop DC gain of unity -ve feedback s/s having closed loop TF $\frac{s+4}{s^2+7s+13}$?

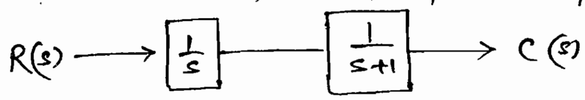
(a). 4/13 (b). 4/9 (c). 4 (d). 13

Sol. $G(s) = \frac{s+4}{(s^2+7s+13)-s-4} = \frac{s+4}{s^2+6s+9}$

$$\text{DC gain of TF} = \lim_{s\to 0} \frac{s^n\cdot(s+z_1)}{s^n(s+p_1)}$$

$$\text{DC gain of open loop TF} = \lim_{s\to 0}\frac{s+4}{s^2+6s+9} = \frac{4}{9}$$

Q. What is impulse response of the s/s shown in figure?

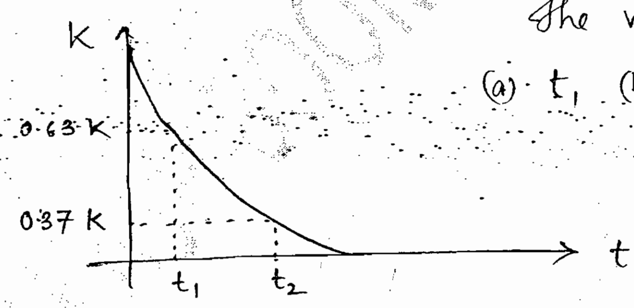


(a). $1-e^{-t}$ (b). $1+e^{-t}$ (c). e^{-t} (d). e^{t}

Sol: $TF = \frac{C(s)}{R(s)} = F(s) = \frac{1}{s(s+1)}$

$$\text{Impulse Response} = \mathcal{L}^{-1}(TF)$$
$$= \mathcal{L}^{-1}\left\{\frac{1}{s(s+1)}\right\}$$
$$= \mathcal{L}^{-1}\left(\frac{1}{s} - \frac{1}{s+1}\right)$$
$$\therefore C(t) = 1-e^{-t}$$

Q. The impulse response of a s/s having TF $\frac{C(s)}{R(s)} = \frac{K}{s+\alpha}$ as shown in figure.



The value of α will be

(a). t_1 (b) $\frac{1}{t_1}$ (c) t_2 (d) $\frac{1}{t_2}$

Sol. Impulse Response $= \mathcal{L}^{-1}\left(\frac{K}{s+\alpha}\right) = Ke^{-\alpha t}$

At $t = t_2$

$$Ke^{-\alpha t_2} = 0.37K$$
$$e^{-\alpha t_2} = 0.37$$
$$-\alpha t_2 = \ln(0.37)$$
$$-\alpha t_2 = -1$$
$$\therefore \alpha = 1/t_2$$

Q. A certain control s/s has I/p $r(t)$ and o/p $c(t)$. If the I/p is first passed through a block whose TF is e^{-s} and then applied to the s/s. The modified o/p will be

(a). $c(t)\cdot u(t-1)$ (b) $c(t-1)\,u(t)$ ~~(c)~~ $c(t-1)\,u(t-1)$ (d). None.

Sol. $C(s) = F(s)\cdot R(s)$

$$R(s) \longrightarrow \boxed{F(s)} \longrightarrow C(s)$$

(Response)

$C_M(s) = R(s)\cdot e^{-s}\cdot F(s)$

$$R(s) \longrightarrow \boxed{e^{-s}} \longrightarrow \boxed{F(s)} \longrightarrow C_M(s)$$

(Modified Response)

$C_M(s) = C(s)\cdot e^{-s}$

$$\mathcal{L}^{-1}\left(F(s)\cdot e^{-as}\right) = f(t-a)\cdot u(t-a)$$

$$\mathcal{L}^{-1}\{C(s)\cdot e^{-s}\} = c(t-1)\cdot u(t-1)$$

$$\therefore \mathcal{L}^{-1}\cdot C_M(s) = C_M(t) = c(t-1)\cdot u(t-1)$$

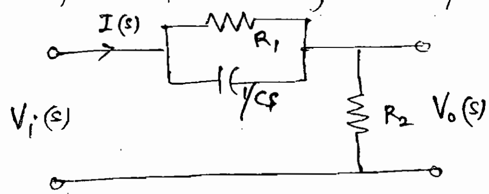
INTRODUCTION TO COMPENSATORS

Compensators in control s/s are used for improving the performance specifications i.e. transient and steady state response characteristics.

They are of 3 types:

(1). **Lead Compensator:**

It is used for improving the transient state or speed of response of the s/s.



$$V_i(s) = I(s)\left[\frac{R_1}{R_1 Cs+1} + R_2\right]$$

$$V_i(s) = I(s)\left(\frac{R_1+R_2+R_1R_2Cs}{R_1Cs+1}\right)$$

$$V_o(s) = I(s) \cdot R_2$$

$$\frac{V_o(s)}{V_i(s)} = \frac{R_2(R_1Cs+1)}{R_1+R_2+R_1R_2Cs}$$

Here, $T = R_1C$

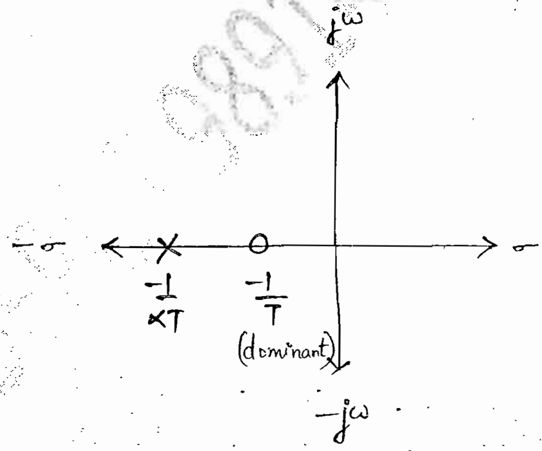
α = Attenuation constant $= \frac{R_2}{R_1+R_2}$ $(\alpha < 1)$

$$\frac{V_o(s)}{V_i(s)} = \frac{R_2(R_1Cs+1)}{R_1+R_2\left(1+\frac{R_1R_2Cs}{R_1+R_2}\right)}$$

$$= \frac{\alpha(1+Ts)}{(1+\alpha Ts)}$$

zero at $s = \frac{-1}{T}$

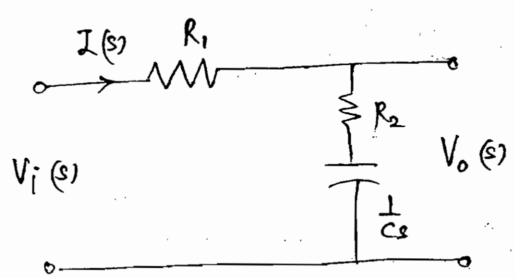
Pole at $s = \frac{-1}{\alpha T}$



<u>Note</u>: Adding a zero to a s/s TF in terms of compensators represents Lead compensator.

(2). <u>Lag Compensator</u>:

It is used for improving the steady state response characteristic of the s/s i.e. elimination of steady state error b/w o/p and I/p.



$$V_i(s) = I(s)\cdot\left(R_1+R_2+\frac{1}{Cs}\right)$$

$$= I(s)\left(\frac{R_1Cs+R_2Cs+1}{Cs}\right)$$

$$V_o(s) = I(s)\left(R_2+\frac{1}{Cs}\right)$$

$$= I(s) \cdot \left(\frac{R_2 C s + 1}{C s} \right)$$

$$\frac{V_o(s)}{V_i(s)} = \frac{R_2 C s + 1}{R_1 C s + R_2 C s + 1}$$

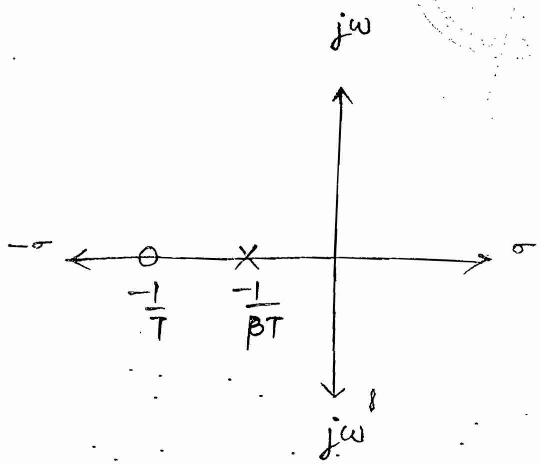
$$T = R_2 C \quad ; \quad \beta = \frac{1}{\alpha} = \frac{R_1 + R_2}{R_2} \quad (\beta > 1)$$

$$\frac{V_o(s)}{V_i(s)} = \frac{R_2 C s + 1}{R_2 C s \frac{(R_1 + R_2)}{R_2} + 1}$$

$$= \frac{1 + Ts}{1 + \beta T s}$$

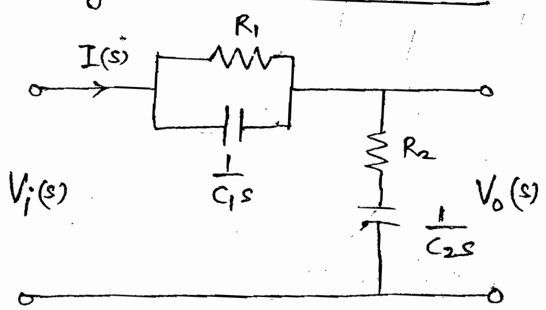
Zero at $s = -\frac{1}{T}$

Pole at $s = -\frac{1}{\beta T}$



Note: Adding a pole to a s/s T.F in terms of compensators represent Lag compensator.

(3). Lag - Lead Compensator :



It is used for improving both transient and steddy state response characteristics.
It exhibits the characteristics of both lead and lag in its frequency response

Lead

$$\alpha = \frac{R_2}{R_1+R_2}$$

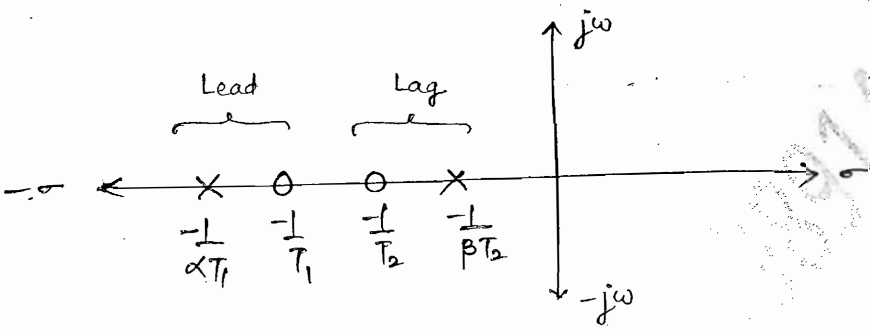
$$T_1 = R_1 C_1$$

Lag

$$\beta = \frac{R_1+R_2}{R_2}$$

$$T_2 = R_2 C_2$$

$$\frac{V_o(s)}{V_i(s)} = \frac{\alpha(1+T_1 s)(1+T_2 s)}{(1+\alpha T_1 s)(1+\beta T_2 s)}$$



Mechanical systems :

All mechanical systems are classified into 2 types:

(1) Mechanical translational system :

In translational s/s,

I/p = Force (F)

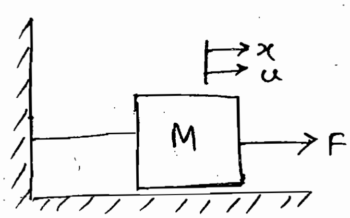
o/p = Linear displacement (x) OR Linear velocity (v)

The 3 ideal elements are:

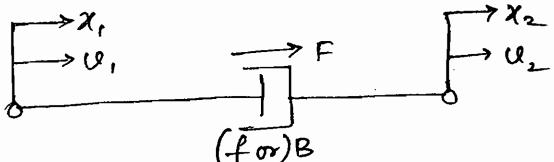
(1) Mass element

(a). $F = M \cdot \frac{dv}{dt}$

(b). $F = M \cdot \frac{d^2x}{dt^2}$



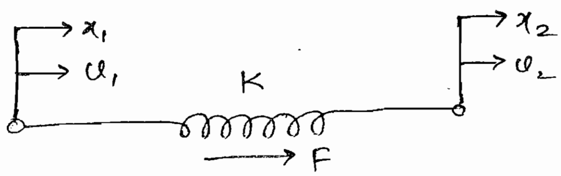
(2). Damper Element (Friction) :



(a). $F = f\cdot(v_1 - v_2) = f\cdot v$ $(v = v_1 - v_2)$

(b). $F = f\cdot\frac{d}{dt}(x_1 - x_2) = f\cdot\frac{dx}{dt}$ $(x = x_1 - x_2)$

(3). Spring Element (Stiffness) :



(a). $F = K\int(v_1 - v_2)\,dt = K\int v\,dt$ $(v = v_1 - v_2)$

(b) $F = K(x_1 - x_2) = Kx$ $(x = x_1 - x_2)$

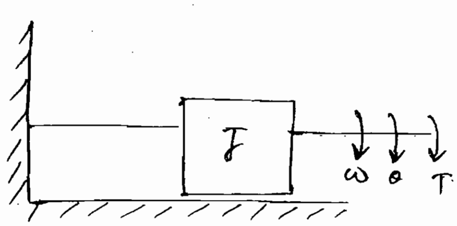
(2). Mechanical Rotational systems :

I/p = Torque (T)

O/p = Angular displacement (θ) or Angular velocity (ω)

the 3 ideal elements are :

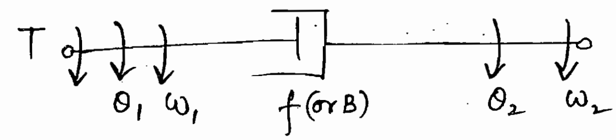
(1) Inertia Element



(a). $T = J\cdot\frac{d\omega}{dt}$

(b). $T = J\cdot\frac{d^2\theta}{dt^2}$

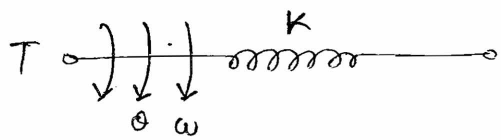
(2) Torsional damper element (friction):



(a). $T = f \cdot (\omega_1 - \omega_2) = f\omega \qquad (\omega = \omega_1 - \omega_2)$

(b). $T = f \cdot \frac{d}{dt}(\theta_1 - \theta_2) = f \cdot \frac{d}{dt}\theta \qquad (\theta = \theta_1 - \theta_2)$

(3). Spring element (stiffness):



(a). $T = K \int \omega \, dt$

(b) $T = K\theta$

Analogous systems:

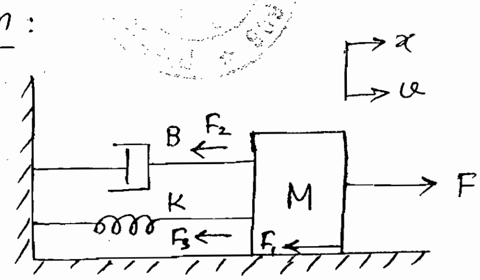
The electrical equivalents of mechanical elements are known as analogous systems.

Mechanical Translational system:

$F = F_1 + F_2 + F_3$

$F = M\frac{dv}{dt} + Bv + K\int v \, dt$

$F = M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx \quad \text{—— (1)}$

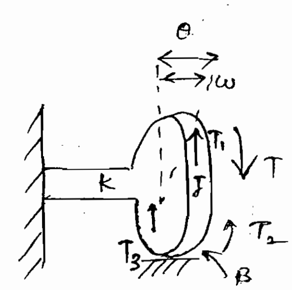


Mechanical Rotational s/s –

$T = T_1 + T_2 + T_3$

$T = J\frac{d\omega}{dt} + B\omega + K\int \omega dt$

$T = J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K\theta \quad \text{—— (2)}$



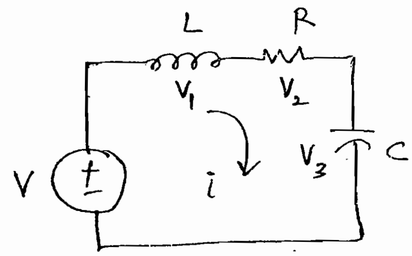
Electrical s/s :

$$V = V_1 + V_2 + V_3$$

$$V = L\frac{di}{dt} + Ri + \frac{1}{C}\int i\,dt$$

$$V = L\frac{d^2q}{dt^2} + R\cdot\frac{dq}{dt} + \frac{q}{C} \quad \text{——(3)}$$

$\because$ $i = dq/dt$ (q = charge)



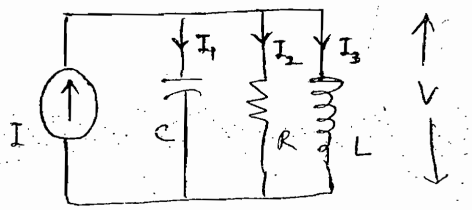
Electrical s/s :

$$I = I_1 + I_2 + I_3$$

$$I = C\frac{dV}{dt} + \frac{V}{R} + \frac{1}{L}\int V\,dt$$

$$I = C\frac{d^2\phi}{dt^2} + \frac{1}{R}\cdot\frac{d\phi}{dt} + \frac{\phi}{L} \quad \text{——(4)}$$

$\because$ $V = d\phi/dt$ (ϕ = flux)



Comparing eqns (1) $\rightarrow$ (4). We get,

(1). F – T – V Analogy

(2). F – T – I Analogy

F	T	V	I
M	J	L	C
B	B	R	1/R
K	K	1/C	1/L
v	ω	i	V
x	θ	q	ϕ

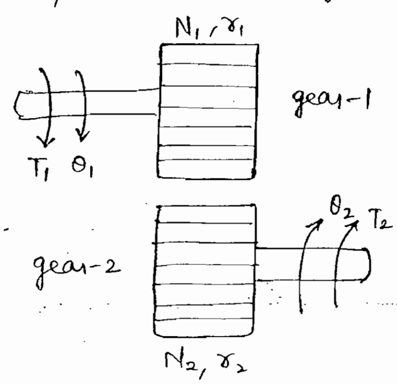
Note : Mass or inertia elements and spring elements are known as conservative elements because they are analogous to Inductors and capacitors. (energy storage elements).

GEARS :

(1). Gears are mechanical devices which are used as intermediate elements b/w electrical motor and load.

(2). They are used for stepping up and stepping down either torque or speed.

(3). They are analogous to electrical X^rs



N = No. of teeth on the circumference of gear wheel.

r = radius of gear wheel (m)

θ = Ang. displacement (radians)

T = Torque on gear wheel (N-m)

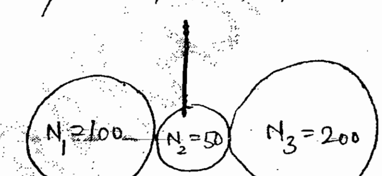
Dynamics of gear train:

$$\frac{N_1}{N_2} = \frac{T_1}{T_2} = \frac{r_1}{r_2} = \frac{\theta_2}{\theta_1} = \frac{\omega_2}{\omega_1} = \frac{\alpha_2}{\alpha_1}$$

for n-gear wheels connected in series

$$\frac{N_x}{N_y} = \frac{T_x}{T_y} = \frac{r_x}{r_y} = \frac{\theta_y}{\theta_x} = \frac{\omega_y}{\omega_x} = \frac{\alpha_y}{\alpha_x}$$

Eg.



(a) $T_1 = 10$ Nm find T_2 and T_3 = ?

(b) $\omega_1 = 20$ r/s (cw) find ω_2 & ω_3 = ?

(a). $\frac{N_1}{N_2} = \frac{T_1}{T_2} \Rightarrow \frac{100}{50} = \frac{10}{T_2} \Rightarrow T_2 = 5$ Nm

$\frac{N_1}{N_3} = \frac{T_1}{T_3} \Rightarrow \frac{100}{200} = \frac{10}{T_3} \Rightarrow T_3 = 20$ Nm

(b). $\frac{N_1}{N_2} = \frac{\omega_2}{\omega_1} \Rightarrow \frac{100}{50} = \frac{\omega_2}{20} \Rightarrow \omega_2 = 40$ r/s (CCW)

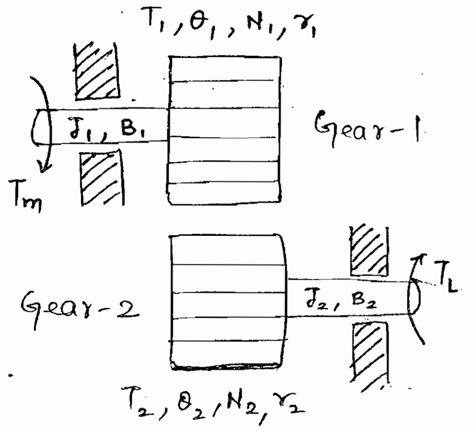
$\frac{N_1}{N_3} = \frac{\omega_3}{\omega_1} \Rightarrow \frac{100}{200} = \frac{\omega_3}{20} \Rightarrow \omega_3 = 10$ r/s (CW)

Observations:

(1). $N_1 > N_2$; $T \downarrow, \omega \uparrow$

(2). $N_1 = N_2$ (Buffer)

There is no change in speed (or) Torque.



$T_m \rightarrow$ Motor torque (Nm)

$T_1 \rightarrow$ Torque on gear-1 due to T_m (Nm)

$T_2 \rightarrow$ Torque on gear-2 due to T_1 (Nm)

$T_L \rightarrow$ Torque on load due to T_2 (Nm)

$J, B \rightarrow$ Inertia & friction of gear

$$T_M = J_1 \frac{d^2\theta_1}{dt^2} + B_1 \frac{d\theta_1}{dt} + T_1$$

$$T_2 = J_2 \frac{d^2\theta_2}{dt^2} + B_2 \frac{d\theta_2}{dt} + T_L$$

Now, $\frac{T_1}{T_2} = \frac{N_1}{N_2} \Rightarrow T_1 = \left(\frac{N_1}{N_2}\right) \cdot T_2$

$$T_1 = \left(\frac{N_1}{N_2}\right)\left(J_2 \frac{d^2\theta}{dt^2} + B_2 \frac{d\theta_2}{dt} + T_L\right)$$

$$T_m = J_1 \frac{d^2\theta_1}{dt^2} + B_1 \frac{d\theta_1}{dt} + \left(\frac{N_1}{N_2}\right) J_2 \frac{d^2\theta_2}{dt^2} + \left(\frac{N_1}{N_2}\right) \cdot \frac{d\theta_2}{dt} + \left(\frac{N_1}{N_2}\right) T_L$$

I. Equivalent Inertia and friction of motor side gear (gear-1)

$$\frac{N_1}{N_2} = \frac{\theta_2}{\theta_1} = \frac{\dot{\theta}_2}{\dot{\theta}_1} = \frac{\ddot{\theta}_2}{\ddot{\theta}_1} \Rightarrow \dot{\theta}_2 = \left(\frac{N_1}{N_2}\right) \cdot \dot{\theta}_1 \; ; \; \ddot{\theta}_2 = \left(\frac{N_1}{N_2}\right) \ddot{\theta}_1$$

$$T_m = J_1 \frac{d^2\theta_1}{dt^2} + B_1 \frac{d\theta_1}{dt} + \left(\frac{N_1}{N_2}\right)^2 J_2 \frac{d^2\theta_1}{dt^2} + \left(\frac{N_1}{N_2}\right)^2 B_2 \frac{d\theta_1}{dt} + \left(\frac{N_1}{N_2}\right) T_L$$

$$T_m = \left\{J_1 + \left(\frac{N_1}{N_2}\right)^2 J_2\right\} \frac{d^2\theta_1}{dt^2} + \left\{B_1 + \left(\frac{N_1}{N_2}\right)^2 \cdot B_2\right\} \frac{d\theta_1}{dt} + \left(\frac{N_1}{N_2}\right) \cdot T_L$$

$$\therefore J_{eq_1} = J_1 + \left(\frac{N_1}{N_2}\right)^2 J_2$$

$$\therefore B_{eq_1} = B_1 + \left(\frac{N_1}{N_2}\right)^2 B_2$$

II. Equivalent inertia and friction of load side gear (gear-2)

$$\frac{N_1}{N_2} = \frac{\theta_2}{\theta_1} = \frac{\dot{\theta}_2}{\dot{\theta}_1} = \frac{\ddot{\theta}_2}{\ddot{\theta}_1} \Rightarrow \dot{\theta}_1 = \left(\frac{N_2}{N_1}\right)\dot{\theta}_2 \quad ; \ddot{\theta}_1 = \left(\frac{N_2}{N_1}\right)\cdot\ddot{\theta}_1$$

$$T_m = \left(\frac{N_2}{N_1}\right) J_1 \frac{d^2\theta_2}{dt^2} + \left(\frac{N_2}{N_1}\right)\cdot B_1 \frac{d\theta_2}{dt} + \left(\frac{N_1}{N_2}\right)\cdot J_2 \frac{d^2\theta_2}{dt^2} + \left(\frac{N_1}{N_2}\right) B_2 \frac{d\theta_2}{dt} + \left(\frac{N_1}{N_2}\right) T_L$$

$$\left(\frac{N_2}{N_1}\right) T_m = \left(\frac{N_2}{N_1}\right)^2 J_1 \frac{d^2\theta_2}{dt^2} + \left(\frac{N_2}{N_1}\right)^2 B_1 \frac{d\theta_2}{dt} + J_2 \frac{d^2\theta_2}{dt^2} + B_2 \frac{d\theta_2}{dt} + T_L$$

$$\left(\frac{N_2}{N_1}\right) T_m = \left\{\left(\frac{N_2}{N_1}\right)^2 J_1 + J_2\right\} \frac{d^2\theta_2}{dt} + \left\{\left(\frac{N_2}{N_1}\right)^2 B_1 + B_2\right\} \frac{d\theta_2}{dt} + T_L$$

$$\therefore J_{eq_2}' = \left(\frac{N_2}{N_1}\right)^2 J_1 + J_2$$

$$\therefore B_{eq_2} = \left(\frac{N_2}{N_1}\right)^2 B_1 + B_2$$

Nodal method:

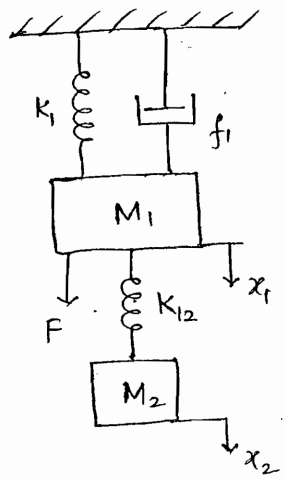
(1). No. of nodes is equal to no. of displacements.

(2). Take an additional node which is a reference node.

(3). connect the mass (or) inertia elements b/w the principle node and reference only.

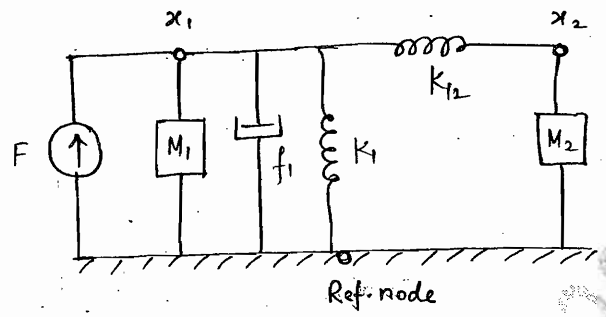
(4). connect spring and damping elements either b/w the principles node (or) b/w the principle node and reference depending on their position.

(5). Obtain the nodal diagram and write the describing differential eqns at each node.



Nodal Diagram

(Mech. Equivalent diagram)



At node x_1,

$$F = M_1 \frac{d^2x_1}{dt^2} + f_1 \frac{dx_1}{dt} + K_1 x_1 + K_{12}(x_1 - x_2)$$

At node x_2,

$$0 = M_2 \frac{d^2x_2}{dt^2} + K_{12}(x_2 - x_1)$$

Transfer Function : $X_1(s)/F(s)$

$$F(s) = (M_1 s^2 + f_1 s + K_1 + K_{12}) X_1(s) - K_{12} X_2(s)$$

$$0 = (M_2 s^2 + K_{12}) X_2(s) - K_{12} X_1(s)$$

$$F(s) = \left\{ (M_1 s^2 + f_1 s + K_1 + K_{12}) - \frac{K_{12}^2}{M_2 s^2 + K_{12}} \right\} X_1(s)$$

$$F(s) = \left\{ \frac{(M_1 s^2 + f_1 s + K_1 + K_{12})(M_2 s^2 + K_{12}) - K_{12}^2}{(M_2 s^2 + K_{12})} \right\} X_1(s)$$

$$\frac{X_1(s)}{F(s)} = \frac{(M_2 s^2 + K_{12})}{(M_1 s^2 + f_1 s + K_1 + K_{12})(M_2 s^2 + K_{12}) - K_{12}^2}$$

Observations:

1 mass element → order → 2

2 mass element → order → 4

3 mass element → order → 6

⋮ ⋮ ⋮ ⋮

n mass elements → order → 2n

F-V Analogy :

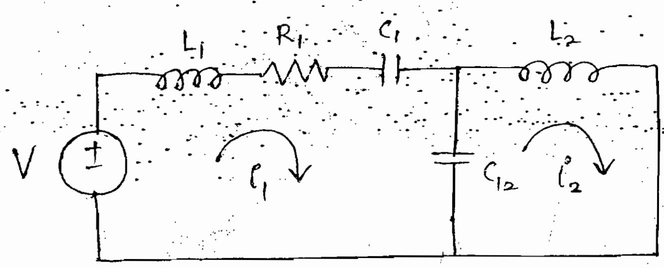
$$V = L_1 \frac{d^2 q_1}{dt^2} + R_1 \frac{dq_1}{dt} + \frac{q_1}{C_1} + \frac{q_1 - q_2}{C_{12}}$$

$$0 = L_2 \frac{d^2 q_2}{dt^2} + \frac{q_2 - q_1}{C_{12}}$$

$$i_1 = \frac{dq_1}{dt} \Rightarrow q_1 = \int i_1 dt \quad ; \quad i_2 = \frac{dq_2}{dt} \Rightarrow q_2 = \int i_2 dt$$

$$V = L_1 \frac{di_1}{dt} + R_1 i_1 + \frac{1}{C_1}\int i_1 dt + \frac{1}{C_{12}}\int (i_1 - i_2) dt$$

$$0 = L_2 \frac{di_2}{dt} + \frac{1}{C_{12}} \int (i_2 - i_1) dt$$



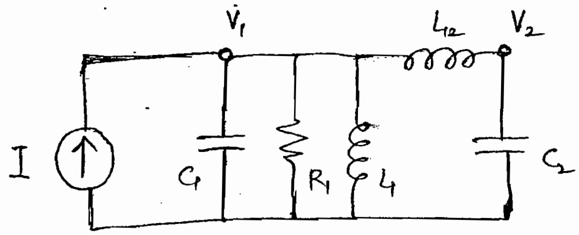
F-I Analogy :

$$I = C_1 \frac{d^2\phi_1}{dt^2} + \frac{1}{R_1} \cdot \frac{d\phi_1}{dt} + \frac{\phi_1}{L_1} + \frac{\phi_1 - \phi_2}{L_{12}}$$

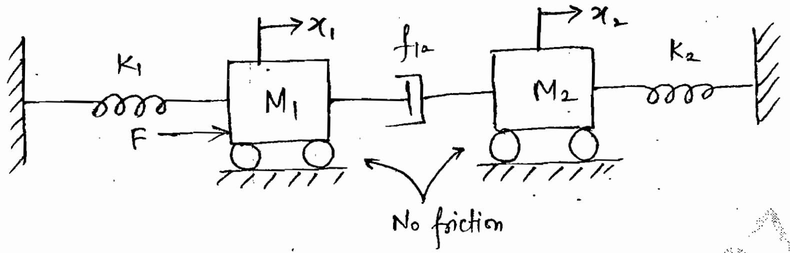
$$0 = C_2 \frac{d^2\phi_2}{dt^2} + \frac{\phi_2 - \phi_1}{L_{12}}$$

$$I = C \frac{dV_1}{dt} + \frac{V_1}{R} + \frac{1}{L_1}\int V_1 dt + \frac{1}{L_{12}} \int (V_1 - V_2) dt$$

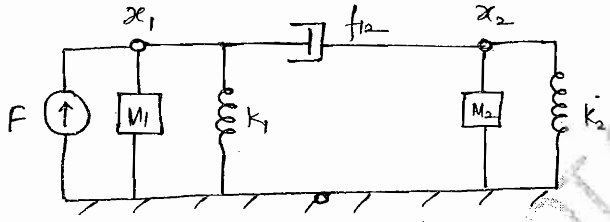
$$0 = C_2 \frac{dV_2}{dt} + \frac{1}{L_{12}} \int (V_2 - V_1) dt$$



Conv. Q2.



Nodal Diagram:



At Node x_1,

$$F = M_1 \frac{d^2 x_1}{dt^2} + f_{12} \frac{d}{dt}(x_1 - x_2) + K_1 x_1$$

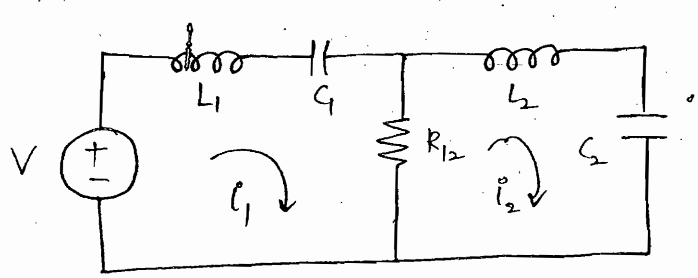
At node x_2,

$$0 = M_2 \frac{d^2 x_2}{dt^2} + f_{12} \frac{d}{dt}(x_2 - x_1) + K_2 x_2$$

F-V Analogy

$$V = L_1 \frac{di_1}{dt} + R_{12}(i_1 - i_2) + \frac{1}{C_1}\int i_1 \, dt$$

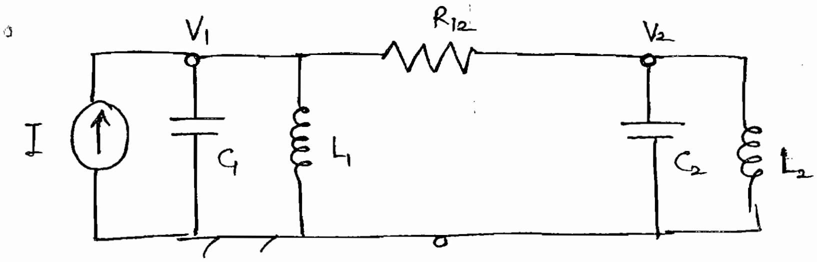
$$0 = L_2 \frac{di_2}{dt} + R_{12}(i_2 - i_1) + \frac{1}{C_2}\int i_2 \, dt$$



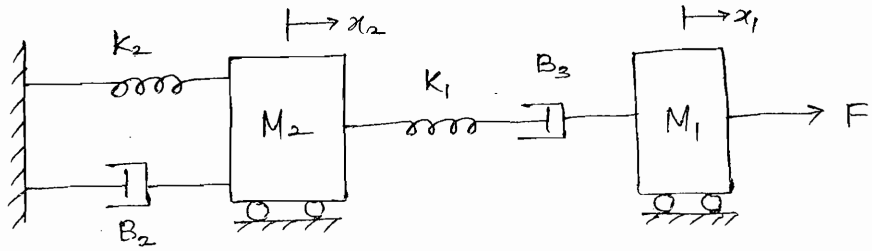
F-I Analogy:

$$I = C_1 \frac{dV_1}{dt} + \frac{V_1 - V_2}{R_{12}} + \frac{1}{L_1}\int V_1\, dt$$

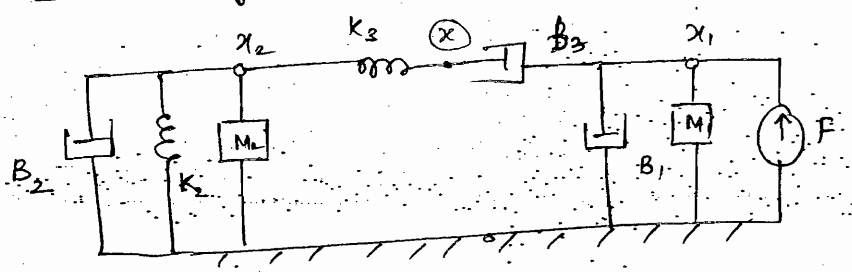
$$0 = C_2 \frac{dV_2}{dt} + \frac{V_2 - V_1}{R_{12}} + \frac{1}{L_2}\int V_2\, dt$$



Q.



Nodal Diagram:



At node x_1,

$$F = M_1 \frac{d^2x_1}{dt^2} + B_1 \frac{dx_1}{dt} + B_3 \cdot \frac{d}{dt}(x_1 - x)$$

At dummy node x,

$$0 = B_3 \frac{d}{dt}(x - x_1) + K_3(x - x_2)$$

At node x_2,

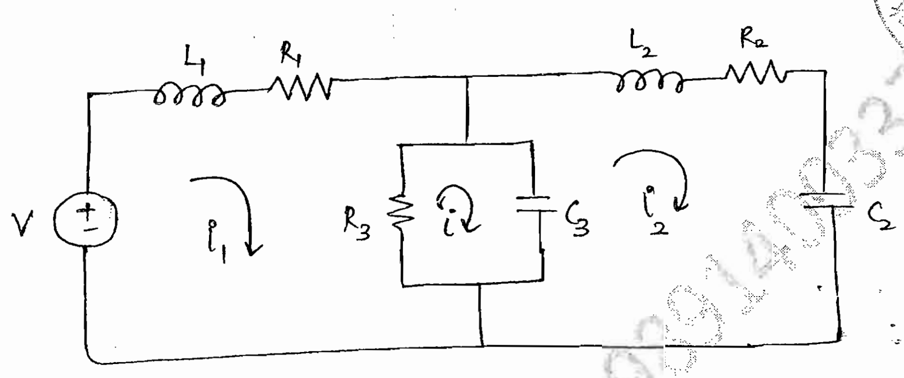
$$0 = M_2 \frac{d^2x_2}{dt^2} + B_2 \cdot \frac{dx_2}{dt} + K_2 x_2 + K_3(x_2 - x)$$

F-V analogy:

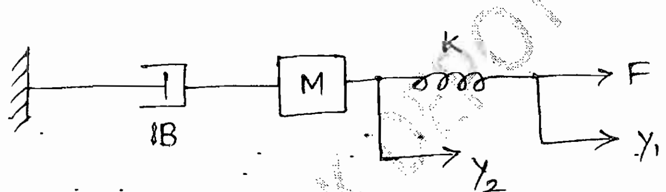
$$V = L_1 \frac{di_1}{dt} + R_1 i_1 + R_3 (i_1 - i)$$

$$0 = R_3 (i - i_1) + \frac{1}{C_3} \int (i - i_2) dt$$

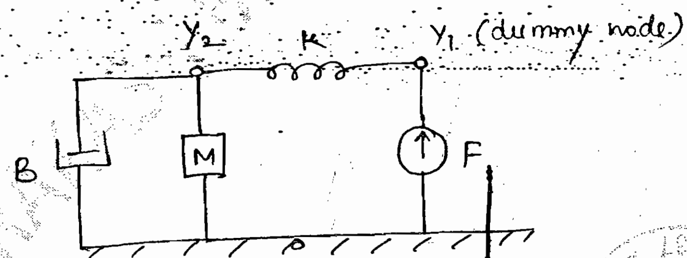
$$0 = L_2 \frac{di_2}{dt} + R_2 i_2 + \frac{1}{C_2} \int i_2 \, dt + \frac{1}{C_3} \int (i_2 - i) dt$$



Q5.



Nodal Diagram:



At dummy node Y_1,

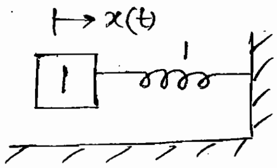
$$F = K(Y_1 - Y_2)$$

At node Y_2,

$$0 = M \frac{d^2 Y_2}{dt^2} + B \frac{dY_2}{dt} + K(Y_2 - Y_1)$$

$$\therefore K(Y_1 - Y_2) = M \frac{d^2 Y_2}{dt^2} + B \cdot \frac{dY_2}{dt}$$

Q4.



Given : $M = K = 1$

$$F = M\frac{d^2x}{dt^2} + Kx$$

$$F = \frac{d^2x}{dt^2} + x$$

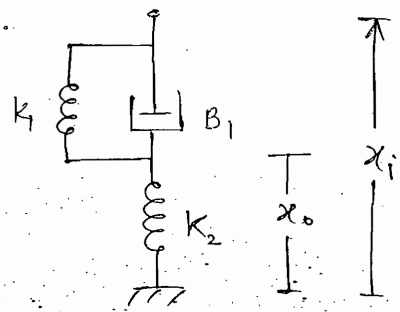
$$F(s) = (s^2+1)\,X(s)$$

Given : $F(s)$ = unit impulse $= 1$

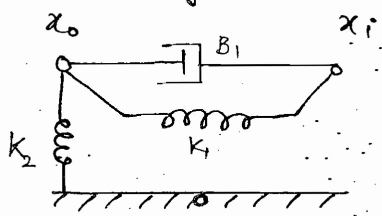
$$X(s) = \frac{1}{s^2+1}$$

$$\therefore\ x(t) = \sin t$$

Q3.



Nodal Diagram :



$x_0, x_i \rightarrow$ dummy nodes as there is no mass element.

At node x_0,

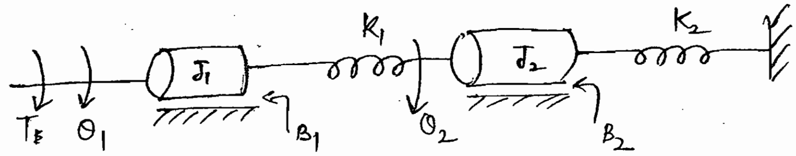
$$0 = K_2 x_0 + K_1(x_0 - x_i) + B_1\frac{d}{dt}(x_0 - x_i)$$

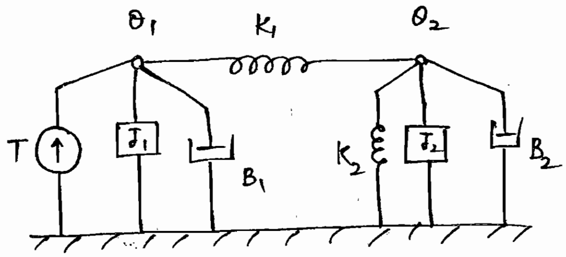
$$0 = K_2 X_0(s) + K_1 X_0(s) - K_1 X_i(s) + B_1 s\cdot X_0(s) - B_1 s X_i(s)$$

$$0 = X_0(s)\{K_2 + K_1 + sB_1\} - X_i(s)(K_1 + B_1 s)$$

$$\therefore\ \frac{X_0(s)}{X_i(s)} = \frac{K_1 + sB_1}{K_1 + K_2 + sB_1}$$

Q.





At node θ_1,

$$T = J_1 \frac{d^2\theta_1}{dt^2} + B_1 \frac{d\theta_1}{dt} + K(\theta_1 - \theta_2)$$

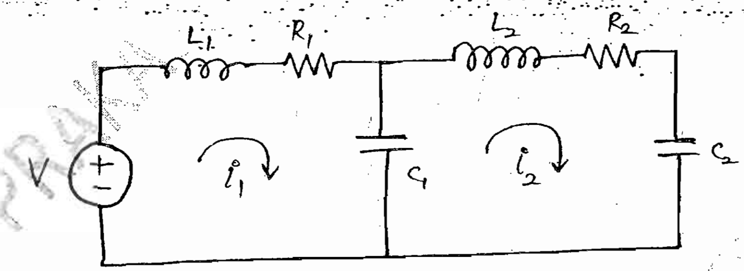
At node θ_2,

$$0 = J_2 \frac{d^2\theta_2}{dt^2} + B_2 \frac{d\theta_2}{dt} + K_2\theta_2 + K_1(\theta_2 - \theta_1)$$

T–V Analogy:

$$V = L_1 \frac{di_1}{dt} + R_1 i_1 + \frac{1}{C_1}\int (i_1 - i_2)\,dt$$

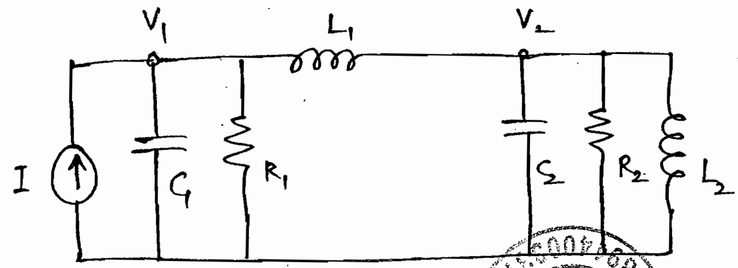
$$0 = L_2 \frac{di_2}{dt} + R_2 i_2 + \frac{1}{C_2}\int i_2\,dt + \frac{1}{C_1}\int (i_2 - i_1)\,dt$$



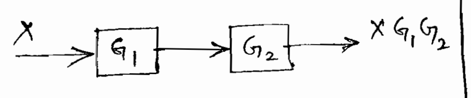
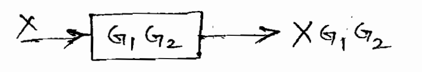
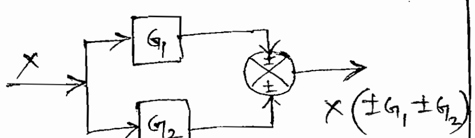
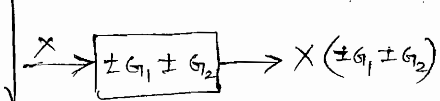
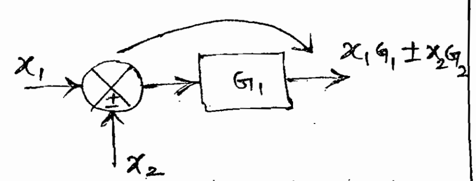
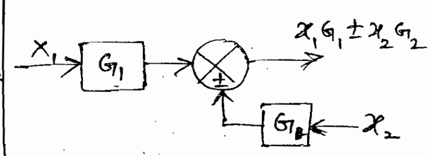
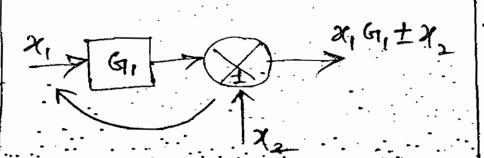
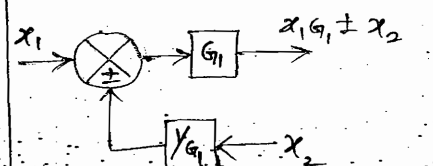
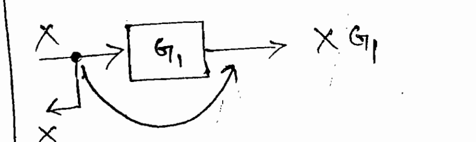
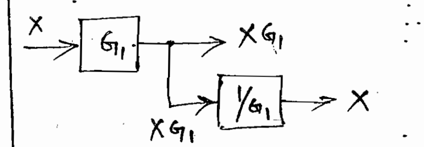
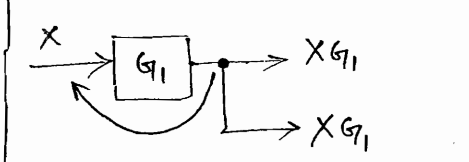
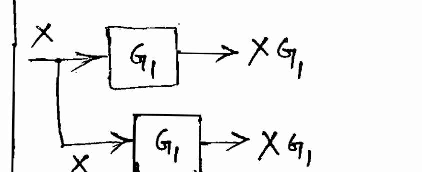
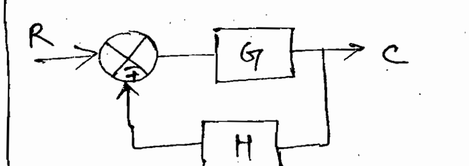
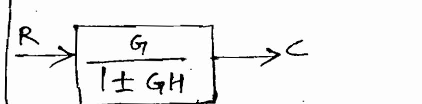
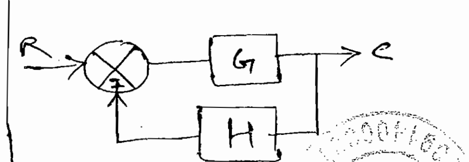
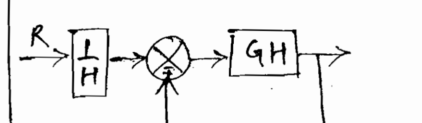
T–I Analogy:

$$I = C_1 \frac{dV_1}{dt} + \frac{V_1}{R_1} + \frac{1}{L_1}\int (V_1 - V_2)\,dt$$

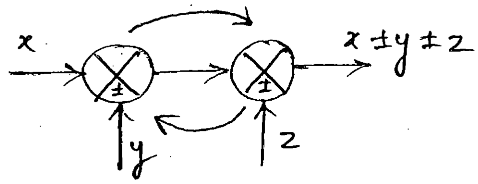
$$0 = C_2 \frac{dV_2}{dt} + \frac{V_2}{R_2} + \frac{1}{L_2}\int V_2\,dt + \frac{1}{L_1}\int (V_2 - V_1)\,dt$$

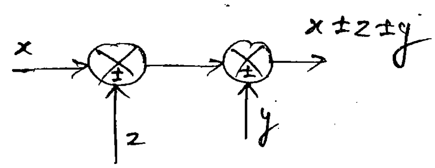


Block diagrams :

Rules	Original diagram	Equivalent diagram
(1). Combining blocks in series		
(2). Combining blocks in parallel		
(3). Shifting the summing element after the block.		
(4). Shifting the summing element before the block		
(5). Shifting the take-off point after the block.		
(6). Shifting the take-off point before the block.		
(7). TF of closed loop Control s/s.		
(8). Block diagram transformation		 $\frac{C}{R} = \frac{1}{H} \cdot \frac{GH}{1 \pm GH} = \frac{G}{1 \pm GH}$

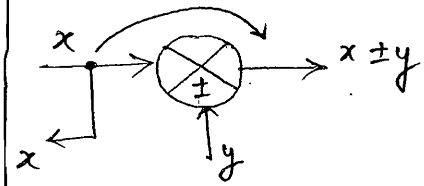
(9). Interchanging the summing elements

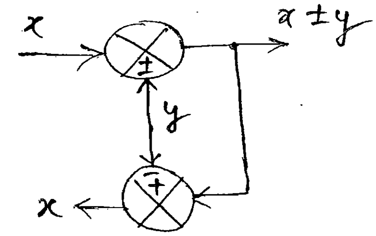




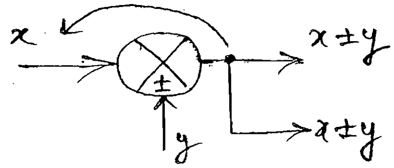
CRITICAL RULES : (10) & (11)

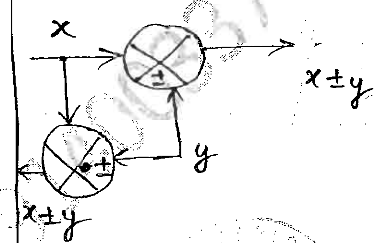
(10). Shifting the take off point after the suming element





(11). Shifting the take off point before the suming element.

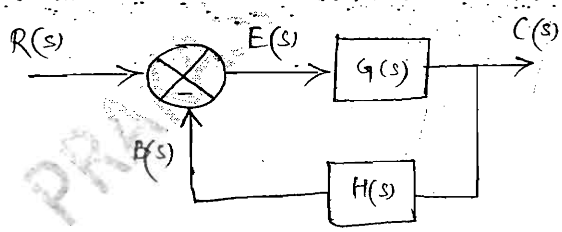


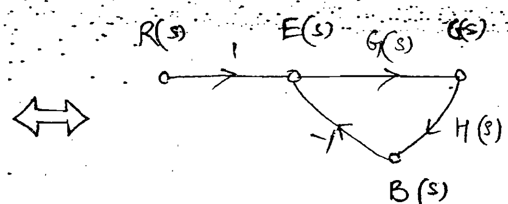


Signal flow Graphs: (SFG)

It is a graphical representation of control s/s in which nodes representing each of the s/s variables are connected by direct branches.

S.F.G for CLCS





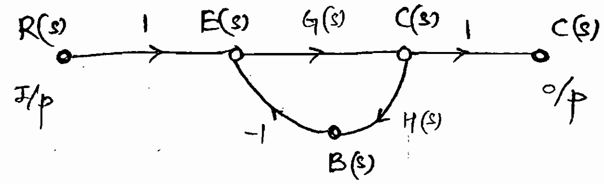
R(s), E(s), C(s), B(s) → s/s Variables (nodes)

Terminology of S.F.G's

(1). Node : It represents the s/s variable and is equal to the sum of all incoming s/g's at it.

(2). I/p node (or) source node : It is a node having only o/t going branches.

(3) O/p node (or) sink node : It is node having only incoming branches



Extend C(s) by gain of 1 becoz it is an actual o/p of s/s.

(4). Mixed (or) chain node : It is a node having both incoming and outgoing branches.

(5). Path : It is the traversal of the connected branches in the dirn of branch arrow such that no node is traversed more than once.

(6). Forward path : It is a path from I/p node to O/p node

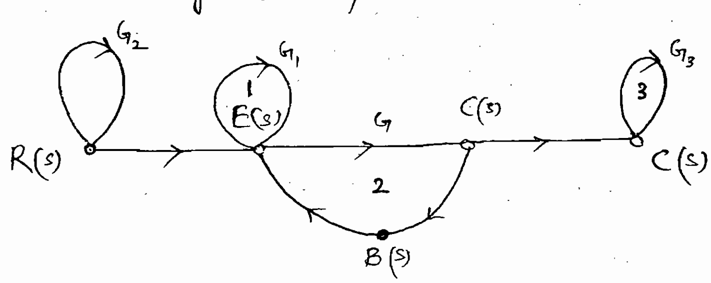
(7). Loop : It is a path which starts and ends at the same node.

* On a defined I/p, self loops are not valid.

Note: self loops on defined I/p nodes are not valid loops and should not be considered when writing the TF.

Loops or self loops on defined O/p nodes are valid loops

(8). Non-touching loops : 2 or more loops are said to be non-touching if they do not have a common node



Mason's gain formulae:

Overall gain (or) Transfer function $= \dfrac{\sum P_K \cdot \Delta_K}{\Delta} = \dfrac{P_1\Delta_1 + P_2\Delta_2 + \cdots + P_K\Delta_K}{\Delta}$

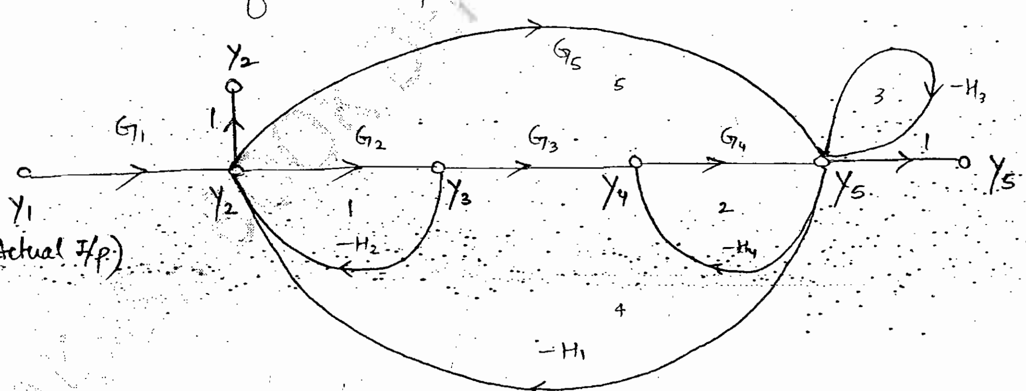
where, $P_K \rightarrow$ path gain of K^{th} forward path.

$\Delta = 1 - \left[\begin{array}{l}\text{sum of loop gain}\\ \text{of all individual loops}\end{array}\right] + \left[\begin{array}{l}\text{sum of gain products}\\ \text{of 2 non-touching loops}\end{array}\right]$

$- \left[\begin{array}{l}\text{sum of gain products of}\\ \text{3 non-touching loops}\end{array}\right] + \cdots$

$\Delta_K \rightarrow$ It is that value of "Δ" obtained by removing all the loops touching K^{th} forward path.

Eg.



Case:(1). $Y_5/Y_1 = ?$

(1). forward paths:

$P_1 = G_1 G_2 G_3 G_4$

$P_2 = G_1 G_5$

(2). To find Δ:

(1). Individual loops

$I_1 = -G_2 H_2$, $I_4 = -G_2 G_3 G_4 H_1$

$I_2 = -G_4 H_4$, $I_5 = -G_5 H_1$

$I_3 = -H_3$

(2). Two Non-touching loops

$L_1 = I_1 I_2 = G_2 H_2 G_4 H_4$

$L_2 = I_1 I_3 = G_2 H_2 H_3$

(3). To find Δ_1 and Δ_2 :

$$\Delta_1 = \Delta_2 = 1$$

$$\frac{Y_5}{Y_1} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$\boxed{\therefore \frac{Y_5}{Y_1} = \frac{G_1G_2G_3G_4 + G_1G_5}{\Delta}} \quad \text{—— (I)}$$

where,

$$\Delta = 1 - \left\{ \begin{matrix} -G_2H_2 - G_4H_4 - H_3 \\ -G_2G_3G_4H_1 - G_5H_1 \end{matrix} \right\} + \left\{ \begin{matrix} G_2H_2G_4H_4 \\ + G_2H_2H_3 \end{matrix} \right\}$$

Case : (2) : To find $Y_5/Y_2 = \dfrac{\left[\frac{Y_5}{Y_1}\right]}{\left[\frac{Y_2}{Y_1}\right]}$ ($\because$ Mason's gain formulae is applicable only b/w actual I/p and o/p node).

To find Y_2/Y_1 :

(1). Forward path

$$P_1 = G_1$$

(2). To find Δ :

'Δ' is independent of forward path.

(3). To find Δ_1 :

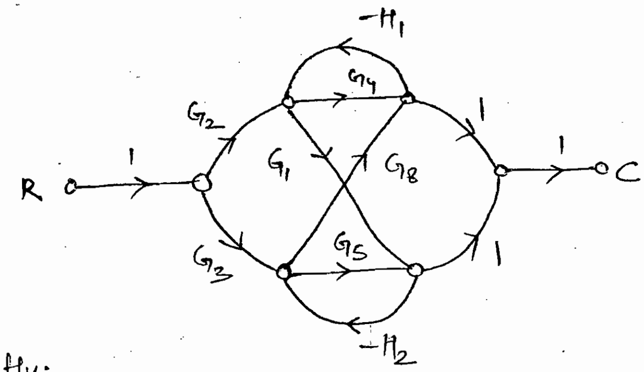
$$\Delta_1 = 1 - (-G_4H_4 - H_3)$$

$$= 1 + G_4H_4 + H_3$$

Now, $$\boxed{\frac{Y_2}{Y_1} = \frac{P_1\Delta_1}{\Delta} = \frac{G_1(1 + G_4H_4 + H_3)}{\Delta}} \quad \text{—— (II)}$$

$$\frac{(I)}{(II)} = \frac{Y_5}{Y_2} = \frac{G_1G_2G_3G_4 + G_1G_5}{G_1(1 + G_4H_4 + H_3)} = \frac{(G_2G_3G_4 + G_5)}{(1 + G_4H_4 + H_3)}$$

conv
Q4.

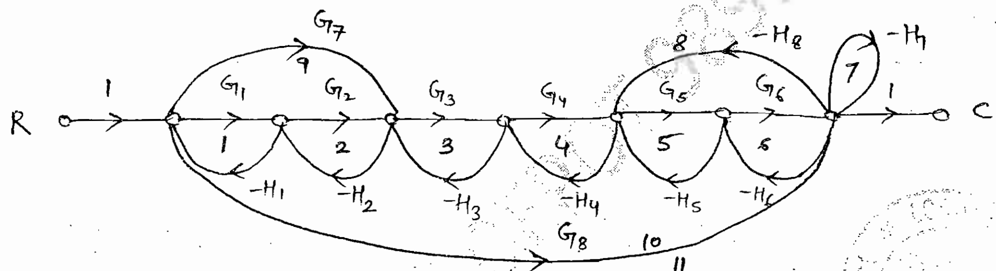


forward paths = 6
Individual loops = 3
2 Non-touching loops = 1

Directly:

$$\frac{C}{R} = \frac{G_2G_4(1+G_5H_2) + G_3G_5(1+G_4H_1) + G_2G_1(1) + G_3G_8(1) + G_2G_1(-H_2)G_8(1) + G_3G_8(-H_1)G_1(1)}{1-(-G_4H_1 - G_5H_2 + G_1G_8H_1H_2) + (G_4H_1G_5H_2)}$$

Q.



find :

(a). No. of forward paths

(b). No. of feedback paths

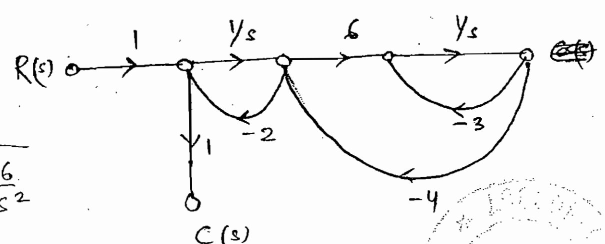
(a) 3, 8 (b). 3, 9 (c) 3, 10 (d) 3, 11 ✓

$L_9 = G_7(-H_2)(-H_1)$; $L_8 = G_5G_6(-H_8)$

$L_{10} = G_8(-H_8)(-H_4)(-H_3)(-H_2)(-H_1)$

$L_{11} = G_8(-H_6)(-H_5)(-H_4)(-H_3)(-H_2)(-H_1)$

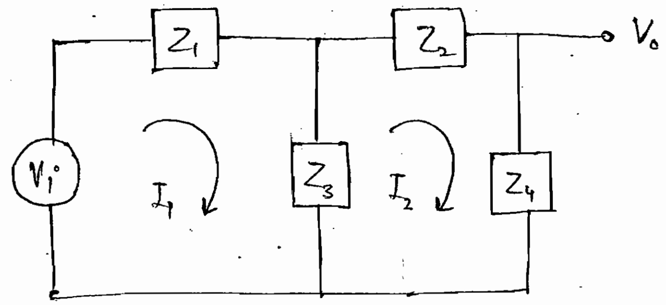
Q11.

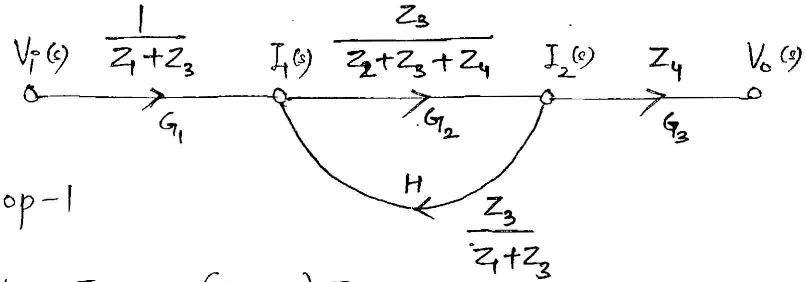


$$\frac{C(s)}{R(s)} = \frac{1\left(1-\left(\frac{-3}{s}-\frac{24}{s}\right)\right)}{1-\left(-\frac{2}{s}-\frac{3}{s}-\frac{24}{s}\right)+\frac{6}{s^2}}$$

$$= \frac{\frac{s+27}{s}}{\frac{s^2+29s+6}{s^2}} = \frac{s(s+27)}{s^2+29s+6}$$

Q12.





Loop-1

$$V_i = I_1 Z_1 + (I_1 - I_2) Z_3$$

$$V_i = (Z_1 + Z_3) I_1 - I_2 Z_3$$

$$I_1 = \frac{V_1}{(Z_1+Z_3)} + I_2 \cdot \frac{Z_3}{(Z_1+Z_3)}$$

Loop-2

$$0 = I_2 (Z_2 + Z_4) + (I_2 - I_1) Z_3$$

$$I_1 Z_3 = I_2 (Z_2 + Z_4 + Z_3)$$

$$I_2 = \frac{Z_3}{Z_2 + Z_3 + Z_4} \cdot I_1$$

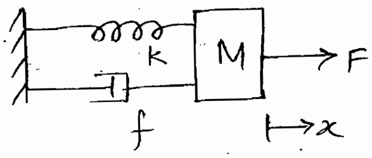
Now,

$$V_0 = I_2 \cdot Z_4$$

$$\therefore G_2 = Z_3 / (Z_2 + Z_3 + Z_4)$$

$$H = Z_3 / (Z_1 + Z_3)$$

Conv Q3.



$$F = M\frac{d^2x}{dt^2} + f\frac{dx}{dt} + Kx$$

$$F(s) = (Ms^2 + fs + K)\cdot X(s)$$

$$\frac{X(s)}{F(s)} = \frac{1}{(Ms^2+fs+K)}$$

$$\frac{X(s)}{F(s)} = \frac{1/M}{s^2 + \frac{f}{M}.s + \frac{K}{M}}$$

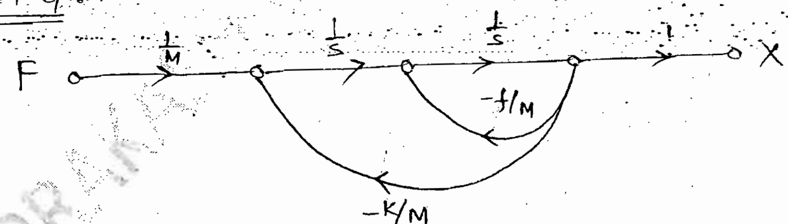
State Diagram :

$$\frac{X(s)}{F(s)} = \frac{1/M}{s^2\left(1 + \frac{f}{M}\cdot\frac{1}{s} + \frac{K}{Ms^2}\right)}$$

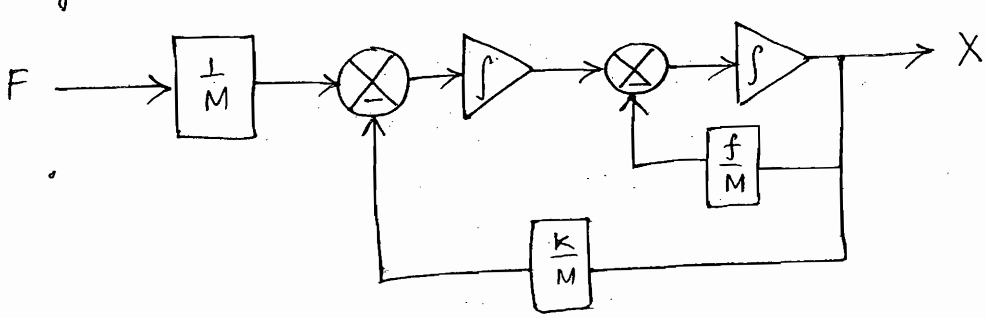
converting into Mason's gain formulae form.

$$\frac{X(s)}{F(s)} = \frac{\frac{1/M}{s^2}}{1 - \left(\frac{-f/M}{s} - \frac{K/M}{s^2}\right)}$$

SFG :



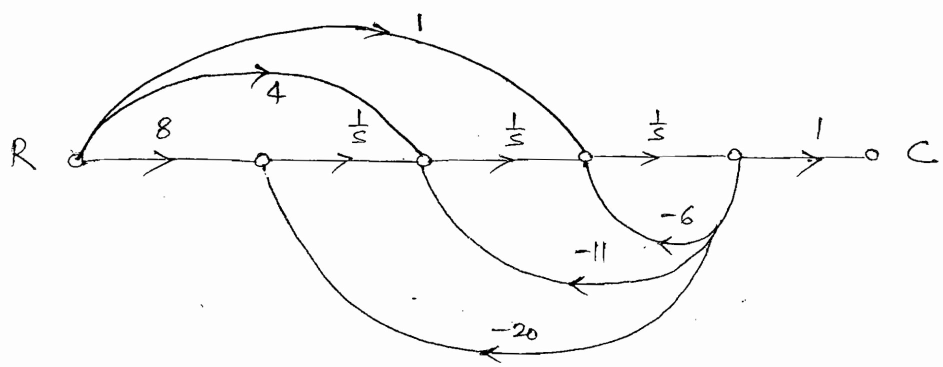
Integrated Based electronic ckt :



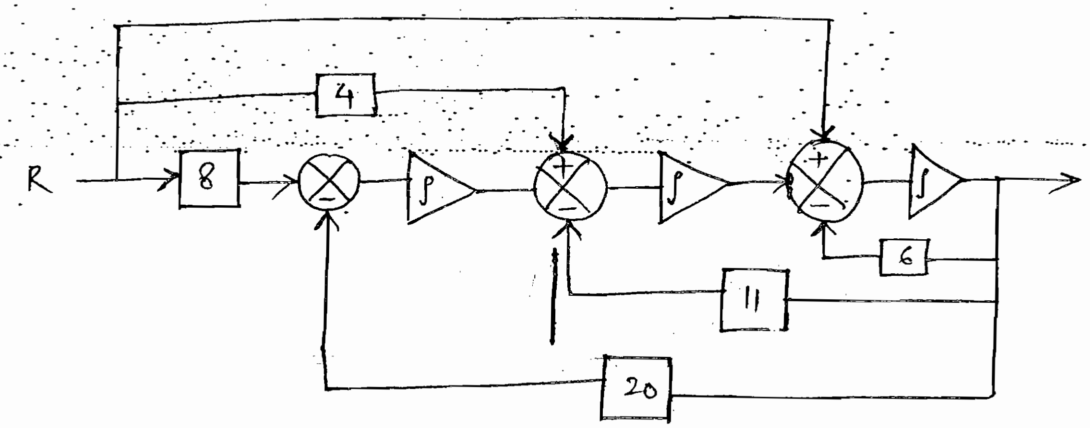
Eg. $$\frac{C(s)}{R(s)} = \frac{s^2+4s+8}{s^3+6s^2+11s+20}$$

$$= \frac{\frac{s^2}{s^3}+\frac{4s}{s^3}+\frac{8}{s^3}}{1+\frac{6s^2}{s^3}+\frac{11s}{s^3}+\frac{20}{s^3}}$$

$$= \frac{\frac{1}{s}+\frac{4}{s^2}+\frac{8}{s^3}}{1-\left(-\frac{6}{s}-\frac{11}{s^2}-\frac{20}{s^3}\right)}$$



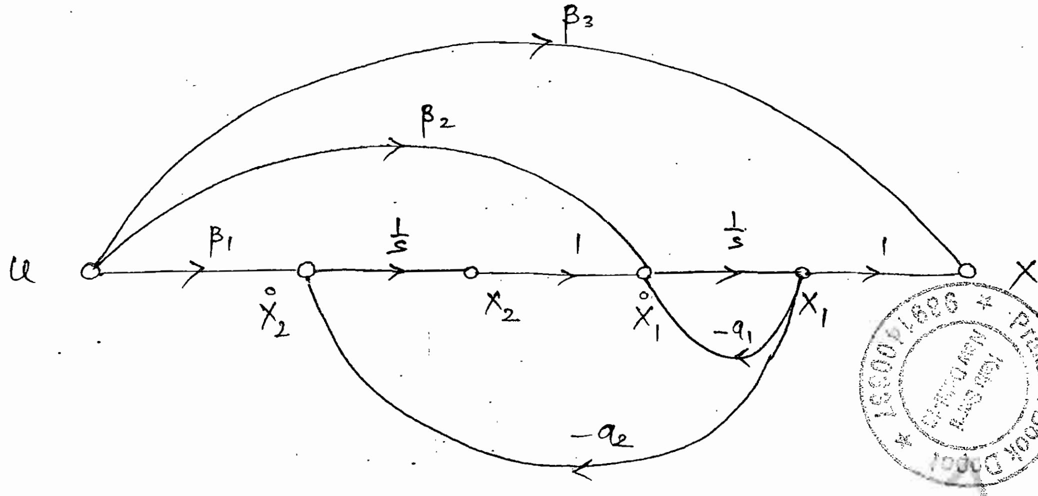
State Diagram:



Conv.

Q5. TF = $X(s)/U(s)$?

$$X = X_1 + \beta_3 u$$
$$\dot{X}_1 = -aX_1 + X_2 + \beta_2 u$$
$$\dot{X}_2 = -a_2 X_1 + \beta_1 u$$



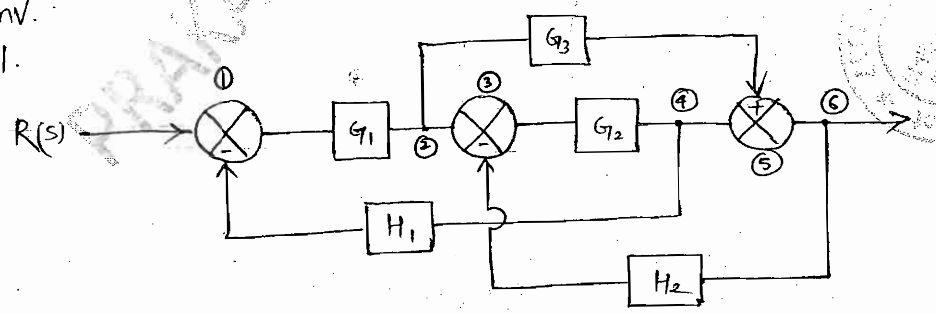
$$\frac{X(s)}{U(s)} = \frac{\frac{\beta_1}{s}(1) + \frac{\beta_2}{s}(1) + \beta_3\left\{1-\left(-\frac{a_1}{s} - \frac{a_2}{s^2}\right)\right\}}{1-\left(-\frac{a_1}{s} - \frac{a_2}{s^2}\right)}$$

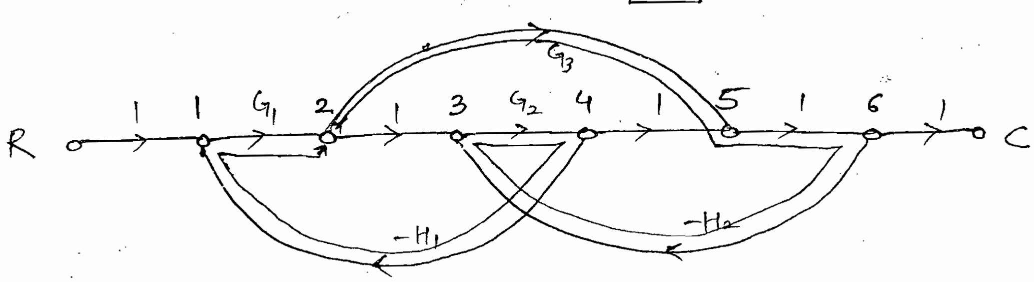
$$= \frac{\frac{\beta_1}{s} + \frac{\beta_2}{s} + \beta_3\left(1+\frac{a_1}{s}+\frac{a_2}{s^2}\right)}{\left(1+\frac{a_1}{s}+\frac{a_2}{s^2}\right)}$$

$$= \frac{s\beta_1 + s\beta_2 + \beta_3(s^2+a_1 s + a_2)}{(s^2+a_1 s+a_2)}$$

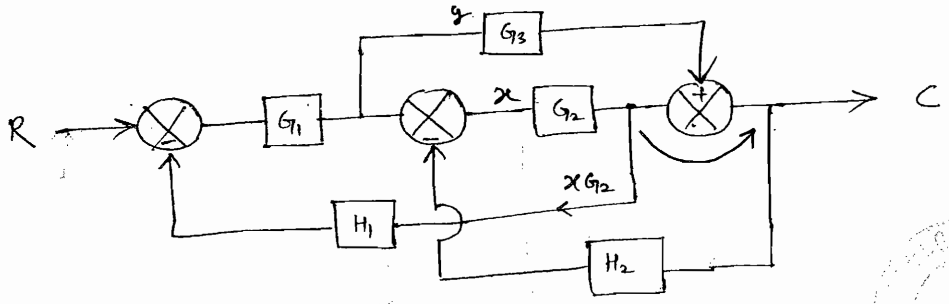
$$= \frac{s^2\beta_3 + s(\beta_1+\beta_2+a_1\beta_3) + \beta_3 a_2}{(s^2+a_1 s+a_2)}$$

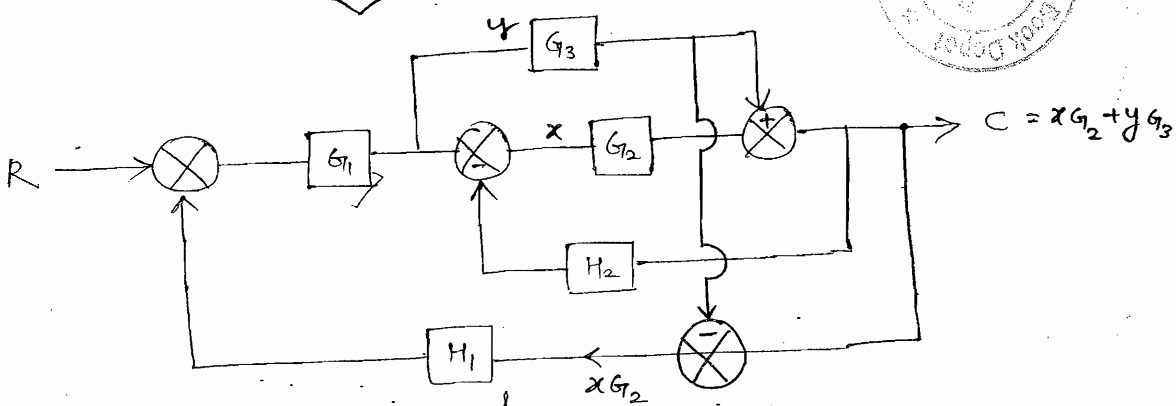
conv.
Q1.

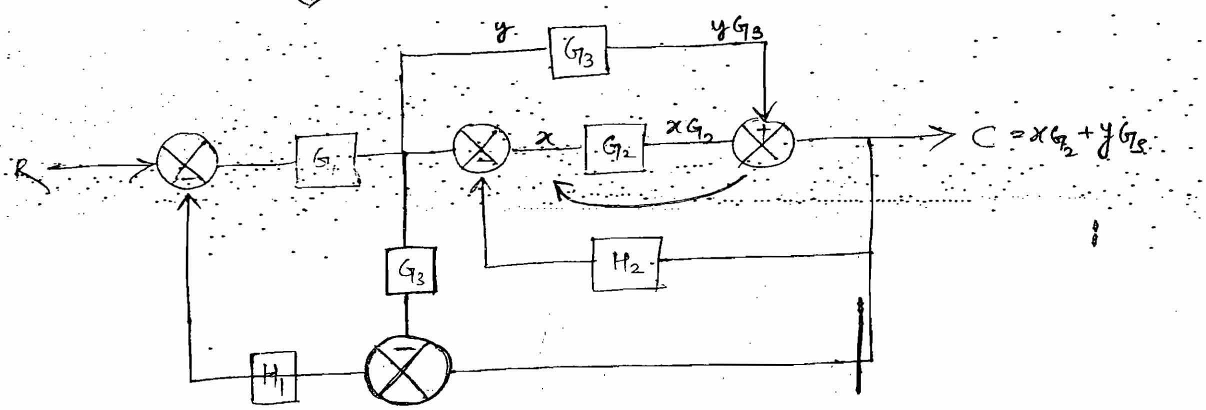


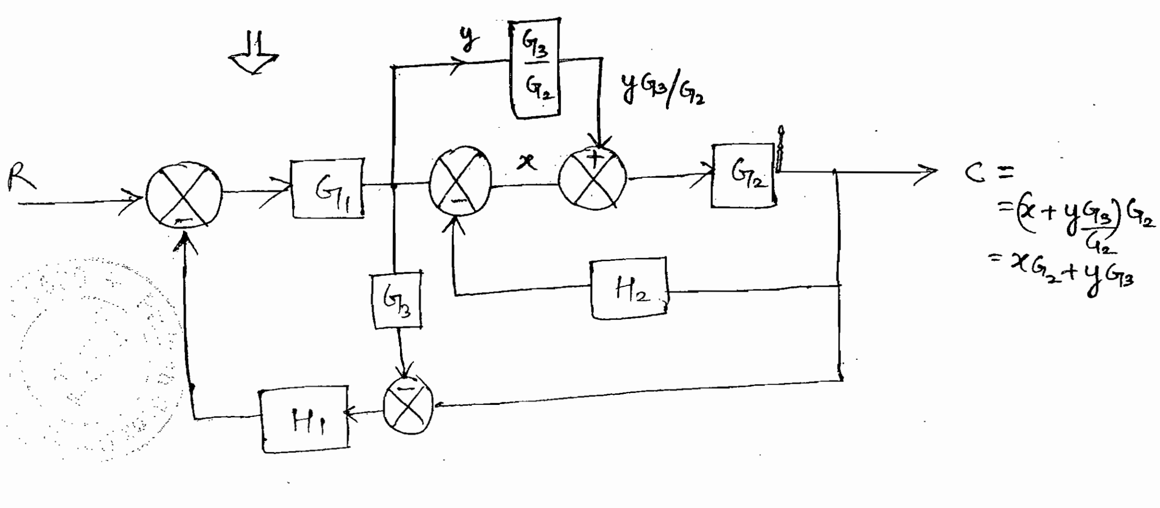


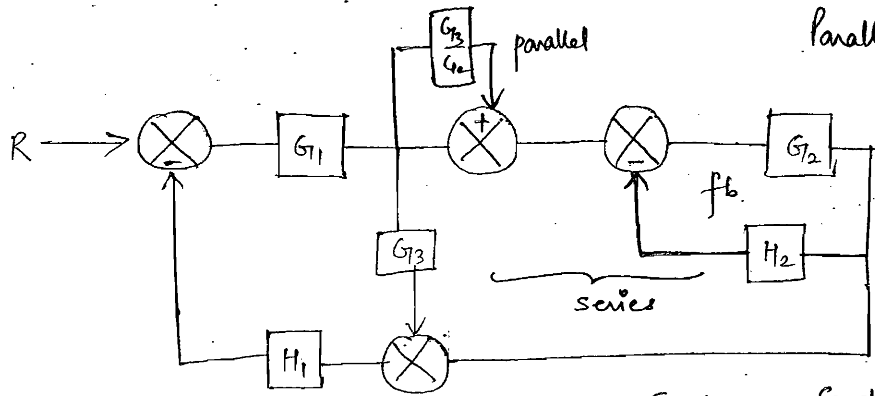
$$\frac{C(s)}{R(s)} = \frac{G_1G_2(1) + G_1G_3(1)}{1-\left(-G_1G_2H_1 - G_2H_2 + G_1G_2G_3H_1H_2\right)}$$









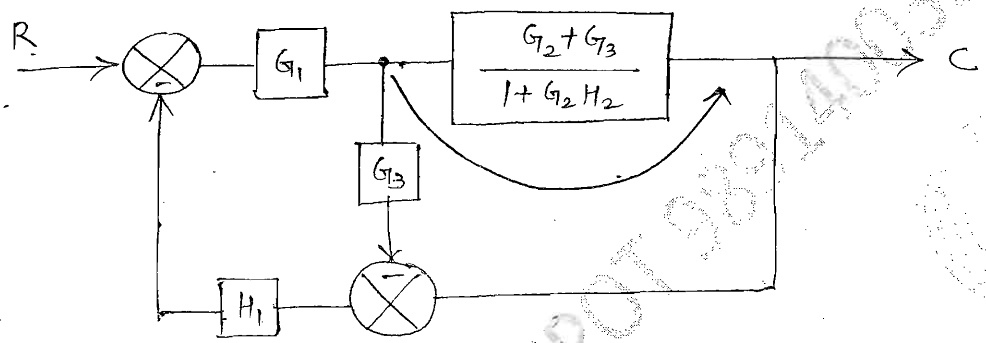


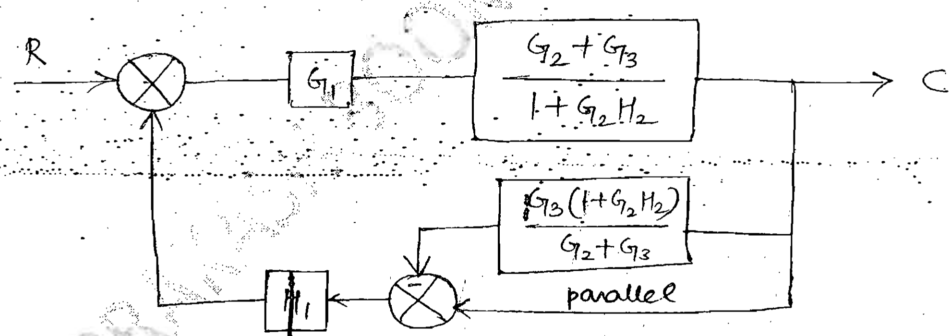
$$\text{Parallel} = 1 + \frac{G_3}{G_2} = \frac{G_2 + G_3}{G_2}$$

$$fb = \frac{G_2}{1 + G_2 H_2}$$

$$\text{Series} = \frac{G_2 + G_3}{1 + G_2 H_2}$$

$\Downarrow$

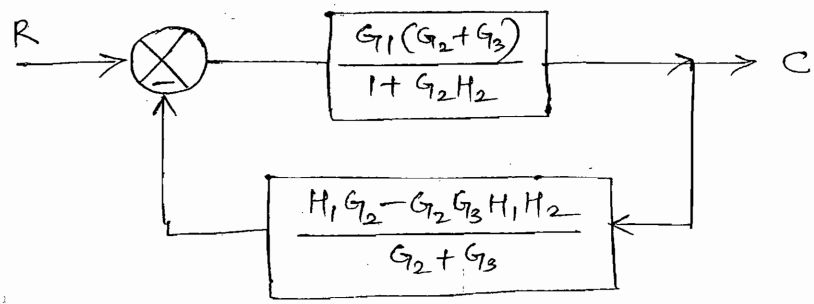




$$\text{Parallel} = 1 - \frac{G_3(1+G_2H_2)}{G_2+G_3} = \frac{G_2 + \cancel{G_3} - \cancel{G_3} - G_2G_3H_2}{G_2+G_3}$$

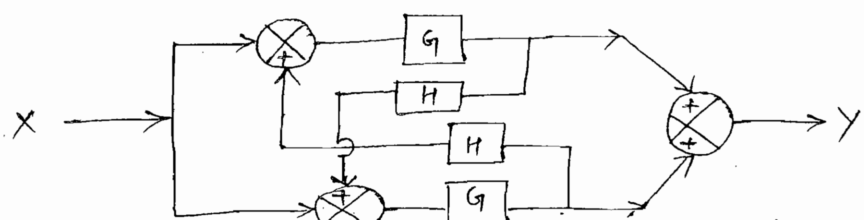
In series with H_1

$$= \frac{H_1G_2 - G_2G_3H_1H_2}{G_2+G_3}$$

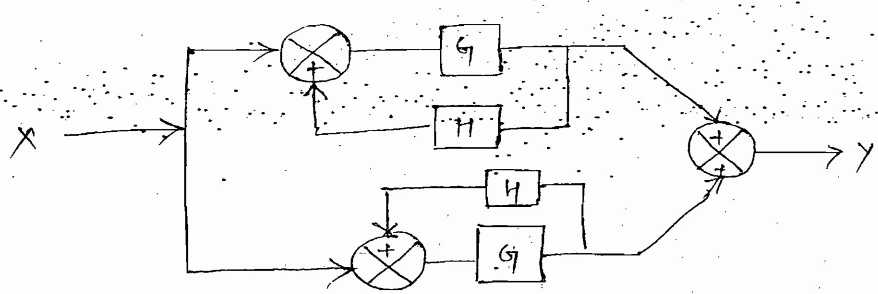


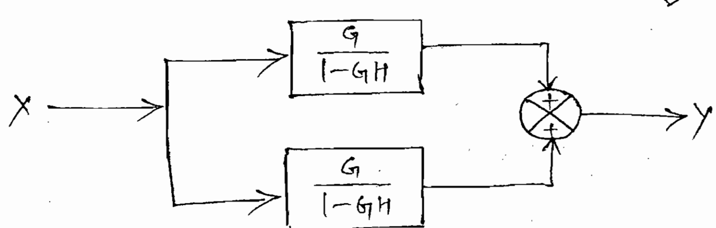
$$\frac{C}{R} = \frac{G_1G_2 + G_1G_3}{1 + G_1G_2H_1 + G_2H_2 - G_1G_2G_3H_1H_2}$$

Q. Y/X equals (a). $\frac{2G}{1-GH}$ (b). $\frac{G}{1-GH}$ (c) $\frac{2G}{1-2GH}$ (d) $\frac{G}{1-2GH}$



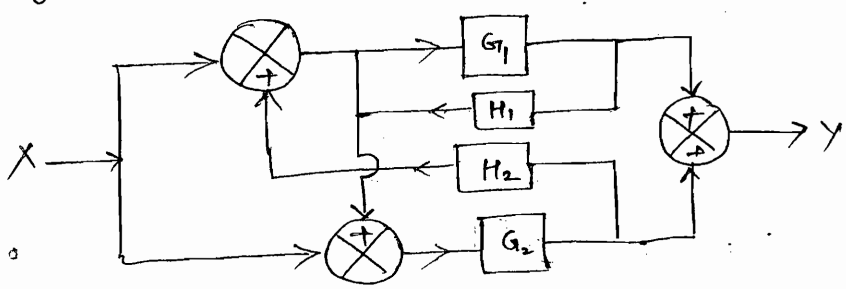
Sol.





$$\frac{Y}{X} = \frac{2G}{1-GH}$$

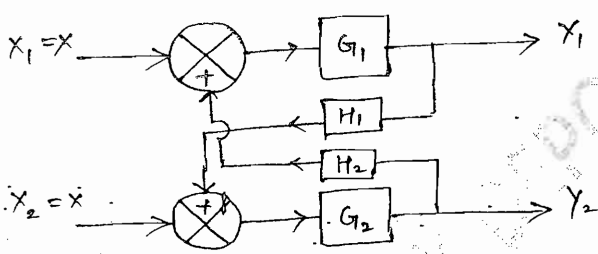
If it is like:



By S.F.G's,

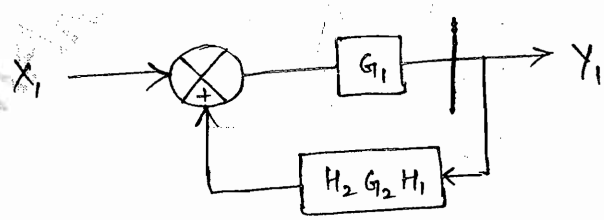
$$\frac{Y}{X} = \frac{G_1 + G_2 + G_1 H_1 G_2 + G_2 H_2 G_1}{1 - G_1 H_1 G_2 H_2}$$

Block Diagram:



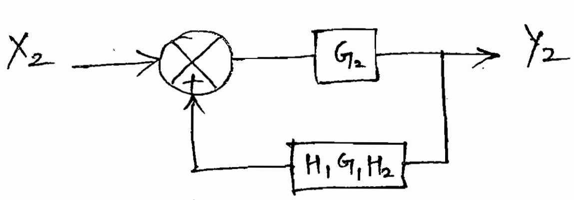
for MIMO s/s, use superposition,

Case (1): $\left.\frac{Y_1}{X_1}\right|_{Y_2 = X_2 = 0}$



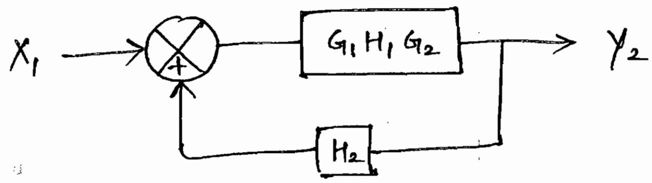
$$\frac{Y_1}{X_1} = \frac{G_1}{1 - G_1 H_1 G_2 H_2}$$

Case (2): $\left.\frac{Y_2}{X_2}\right|_{X_1 = Y_1 = 0}$



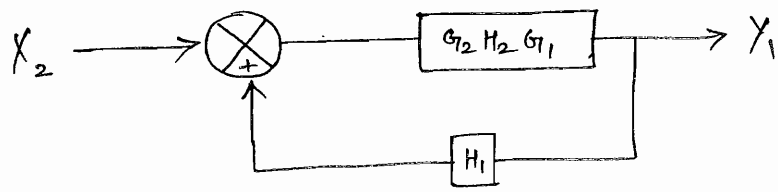
$$\frac{Y_2}{X_2} = \frac{G_2}{1 - G_1 H_1 G_2 H_2}$$

Case (3): $\left.\dfrac{Y_2}{X_1}\right|_{Y_1 = X_2 = 0}$



$$\frac{Y_2}{X_1} = \frac{G_1 H_1 G_2}{1 - G_1 H_1 G_2 H_2}$$

Case (4):



$$\frac{Y_1}{X_2} = \frac{G_2 H_2 G_1}{1 - G_1 H_1 G_2 H_2}$$

$$\frac{Y}{X} = \frac{G_1 + G_2 + G_1 H_1 G_2 + G_2 H_2 G_1}{1 - G_1 H_1 G_2 H_2}$$

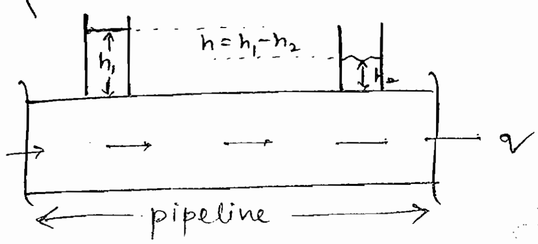
Tra

Transfer Functions of physical systems

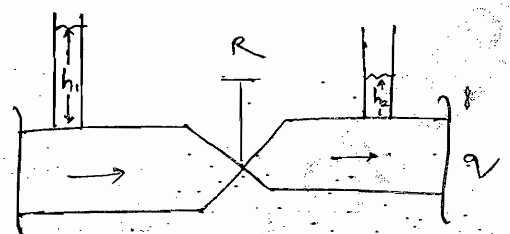
A physical s/s is said to be constituted of 5 elements:

(1). Resistance element
(2). Capacitance element
(3). Time constant element
(4). Oscillatory element
(5). Dead time element.

(1). Resistance Element :



R : Hydraulic Resistance

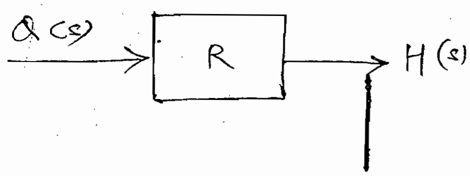


$$h \propto q$$

$$h = q \cdot R$$

$$H(s) = Q(s) \cdot R$$

$$\frac{H(s)}{Q(s)} = R$$

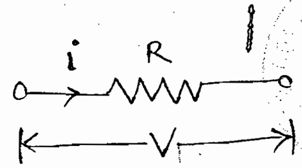


It is analogous to "Electrical Resistance".

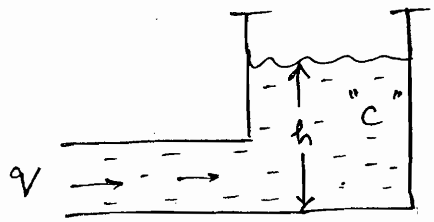
$$V = iR$$

$$V(s) = I(s) \cdot R$$

$$\frac{V(s)}{I(s)} = R$$



(2). Capacitance Element -



$C \rightarrow$ Hydraulic capacitance
($\Leftarrow$ Area/ volume of the tank)

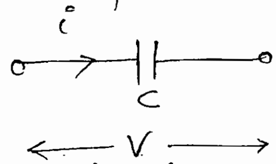
$$q \propto \frac{dh}{dt}$$

$$q = c\,\frac{dh}{dt}$$

$$Q(s) = Cs \cdot H(s)$$

$$\frac{H(s)}{Q(s)} = \frac{1}{Cs}$$

It is analogous to "Electrical capacitance".

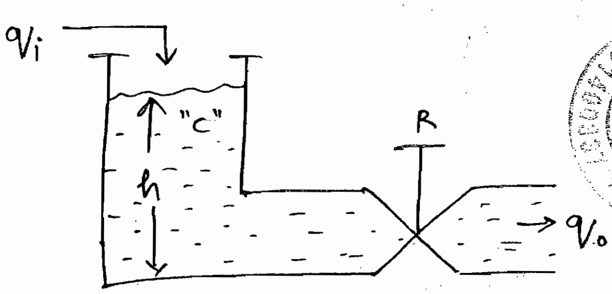


$$i = c\,\frac{dv}{dt}$$

$$I(s) = sC \cdot V(s)$$

$$\frac{V(s)}{I(s)} = \frac{1}{sC}$$

(3). Time constant Element



$$q_i - q_o \propto \frac{dh}{dt}$$

$$q_i - q_o = c\,\frac{dh}{dt}$$

$$q_i = c\,\frac{dh}{dt} + q_o$$

$$h = R q_o \Rightarrow q_o = \frac{h}{R}$$

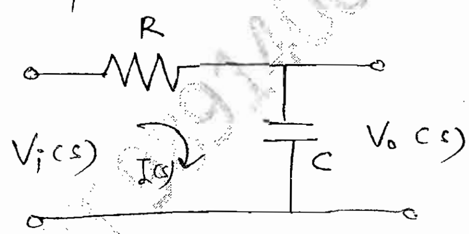
$$q_i = \frac{C\,dh}{dt} + \frac{h}{R}$$

$$R q_i = RC \frac{dh}{dt} + h$$

$$R\, Q_i(s) = (RCs + 1)\, H(s)$$

$$\frac{H(s)}{Q_i(s)} = \frac{R}{(RCs+1)}$$

It is analogous to RC N/w.



$$V_i(s) = I(s)\left(R + \frac{1}{Cs}\right)$$

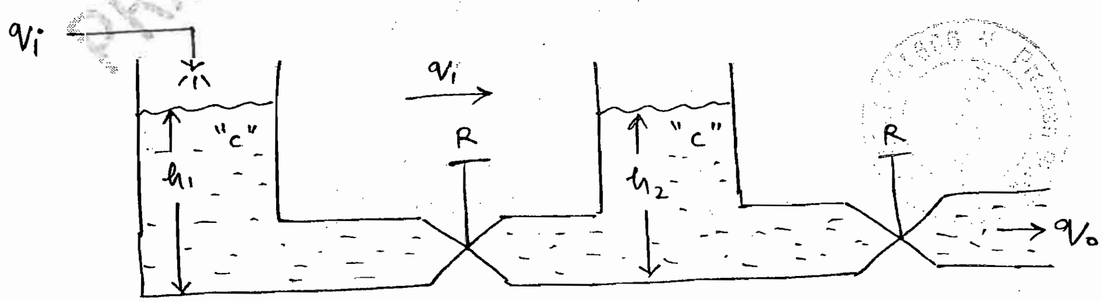
$$V_i(s) = I(s)\left(\frac{RCs+1}{Cs}\right)$$

$$V_o(s) = I(s) \cdot \frac{1}{Cs}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{RCs+1}$$

Interacting and Non-interacting systems:

2 or more time time constant elements are cascaded to study the loading effects b/w them.



$$q_i - q_1 = \frac{C\,dh_1}{dt}$$

$$Q_i(s) - Q_1(s) = sC \cdot H_1(s) \quad \text{—(1)}$$

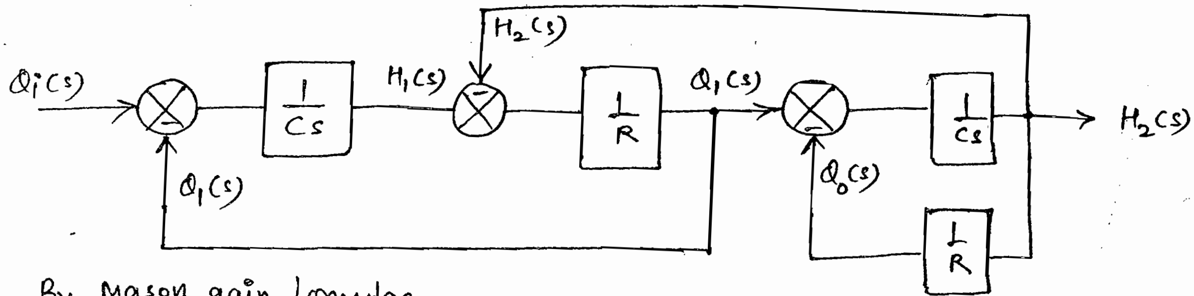
$$h_1 - h_2 = R q_1 \Rightarrow H_1(s) - H_2(s) = R\, Q_1(s) \quad \text{—(2)}.$$

$$q_1 - q_0 = \frac{c\,dh_2}{dt}$$

$$Q_1(s) - Q_0(s) = sC \cdot H_2(s) \quad \text{——— (3)}$$

$$h_2 = R\,q_0$$

$$H_2(s) = R\,Q_0(s) \quad \text{——— (4)}$$



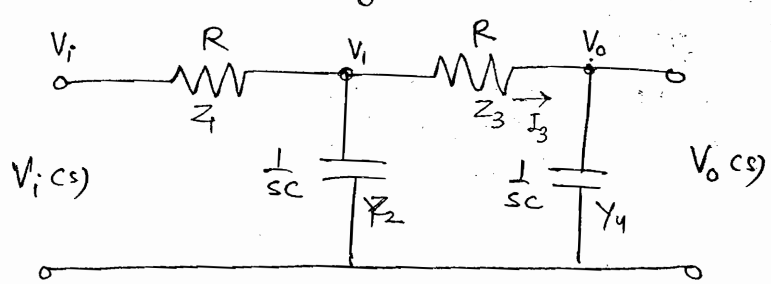
By Mason gain formulae,

$$\frac{H_2(s)}{Q_i(s)} = \frac{\frac{1}{RC^2s^2}}{1-\left[\frac{-1}{RCs} - \frac{1}{RCs} - \frac{1}{RCs}\right] + \frac{1}{R^2C^2s^2}}$$

$$\frac{H_2(s)}{Q_i(s)} = \frac{\frac{1}{RC^2s^2}}{\frac{R^2C^2s^2 + 3RCs + 1}{R^2C^2s^2}}$$

$$\therefore \frac{H_2(s)}{Q_i(s)} = \frac{R}{(R^2C^2s^2 + 3RCs + 1)}$$

Electrical Interacting s/s -



Current through Y_4, $I_3 = V_0 Y_4 = V_0 Cs$

to find V_1:

$$I_3 = \frac{V_1 - V_0}{Z_3} \Rightarrow \quad V_1 = I_3 Z_3 + V_0$$
$$V_1 = V_0 CsR + V_0$$
$$V_1 = V_0 (RCs+1)$$

Current through (Y_2), $I_2 = V_1 Y_2 = V_0 (1+RCs) Cs$
$$= V_0 (Cs + R^2 C^2 s^2)$$

Current through (Z_1), $I_1 = I_2 + I_3$
$$= V_0 (Cs + RC^2 s^2) + V_0 Cs$$
$$= V_0 (2Cs + RC^2 s^2)$$

To find V_i :

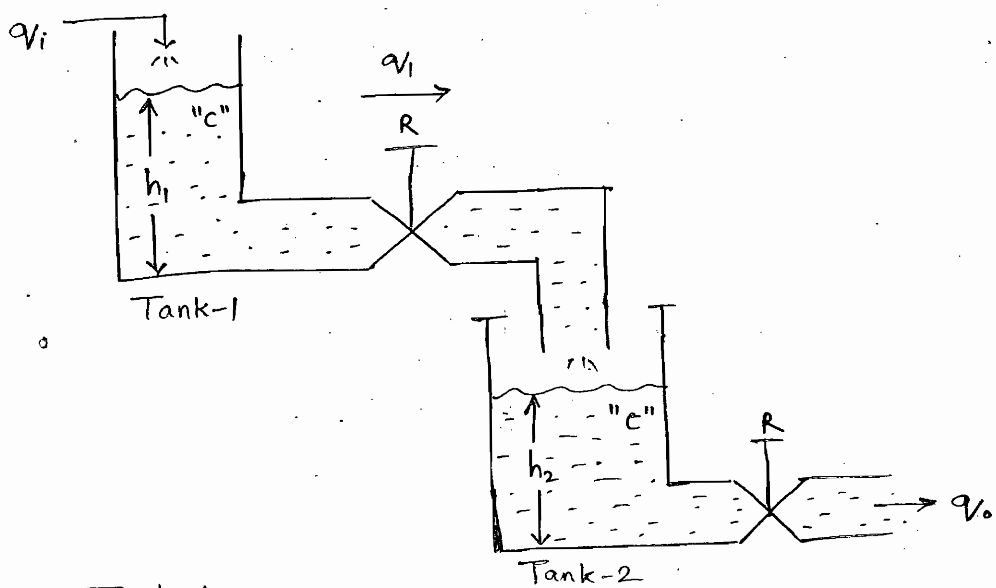
$$V_i = I_1 Z_1 + V_1$$
$$V_i = V_0 (2Cs + RC^2 s^2) \cdot R + V_0 (1+RCs)$$
$$= V_0 (2RCs + R^2 C^2 s^2 + 1 + RCs)$$
$$V_i = V_0 (R^2 C^2 s^2 + 3RCs + 1)$$

$$\therefore \frac{V_0(s)}{V_i(s)} = \frac{1}{(R^2 C^2 s^2 + 3RCs + 1)}$$

Note :

When 2 time constant elements cascaded interactively, the overall TF of such an arrangement is not the product of 2 individual TF due to loading effects.

Non-interacting systems :



Tank-1

$$q_i - q_1 = C\frac{dh_1}{dt}$$

$$q_i = C\frac{dh_1}{dt} + q_1$$

$$h_1 = Rq_1 \Rightarrow q_1 = \frac{h_1}{R}$$

$$q_i = C\frac{dh_1}{dt} + \frac{h_1}{R}$$

$$Rq_i = RC\frac{dh_1}{dt} + h_1$$

$$R\,Q_i(s) = (RCs+1)\,H_1(s) \quad \text{—— (1)}$$

Tank-2

$$q_1 - q_0 = C\frac{dh_2}{dt}$$

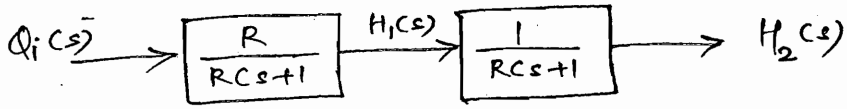
$$q_1 = C\frac{dh_2}{dt} + q_0$$

$$\frac{h_1}{R} = C\frac{dh_2}{dt} + \frac{h_2}{R} \qquad (\because h_2 = Rq_0)$$

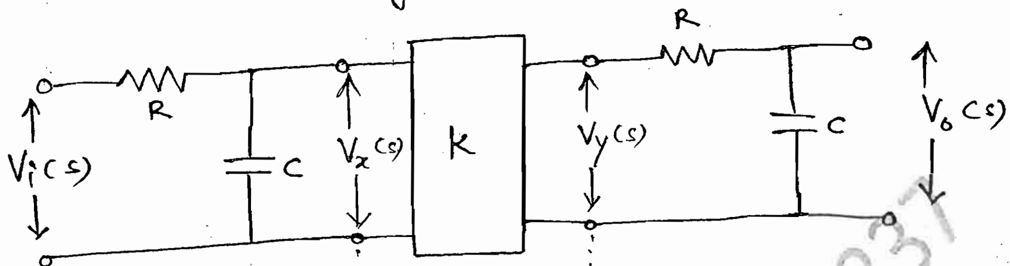
$$h_1 = RC\frac{dh_2}{dt} + h_2$$

$$H_1(s) = (RCs+1)\,H_2(s)$$

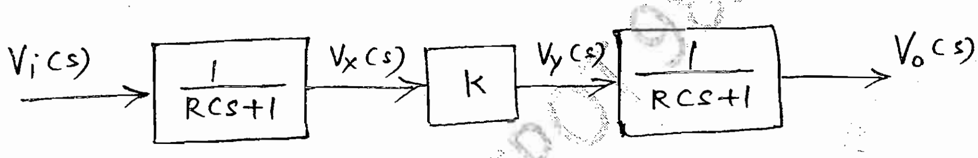
$$\therefore \frac{H_2(s)}{Q_i(s)} = \frac{R}{(RCs+1)^2}$$



Electrical Non-interacting s/s.



$$\frac{V_x(s)}{V_i(s)} = \frac{1}{(RCs+1)} \qquad K = \frac{V_y(s)}{V_x(s)} \qquad \frac{V_o(s)}{V_y(s)} = \frac{1}{(RCs+1)}$$



$$\therefore \quad \frac{V_o(s)}{V_i(s)} = \frac{K}{(RCs+1)^2}$$

Note :

When 2 time constant elements are cascaded non-interactively, the overall TF of such an arrangement is the product of 2 individual TF's. due to absence of loading effect.

Servomechanism

A servomechanism is an electromechanical system whose I/p is electrical voltage and o/p is mechanical position.

They are also known as inverse transducers.

Servo → I/p and o/p characteristics should be approximately linear.

They are used as basic control s/s components.

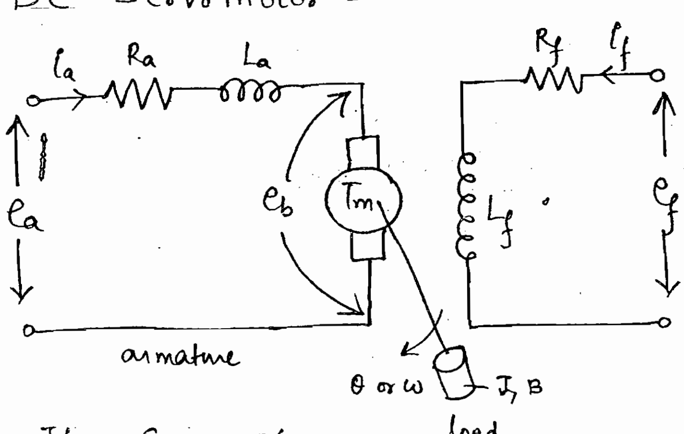
The term servo in control s/s implies low power and low frequency applications, also the I/p & o/p char. must be approximately linear.

Most common application of servomechanism is servo motors.

Servomotors

- AC servomotors
- DC servo motors
  - Armature Control
  - Field Control

~~Armature Controlled~~ DC Servomotor –



$I/p = e_a$; $o/p = \theta$ (or) ω

Armature controlled DC Servomotor:

Principle of Operation:

Air gap flux (ϕ) $\propto$ field current (i_f)

$$\phi = K_f i_f \quad \text{—— (1)}$$

$$T_m \propto i_a \phi$$

$$T_m \propto i_a K_f i_f$$

$$T_m = K_1 i_a K_f i_f$$

$$T_m = K_T i_a \quad \text{—— (2)}$$ where K_T = motor torque constant

Now,

Back emf (e_b) $\propto$ speed (ω)

$$e_b = K_b \omega \quad \text{—— (3)}$$

Analysis of Armature ckt:

$$e_a = i_a R_a + L_a \frac{di_a}{dt} + e_b$$

$$e_a - e_b = i_a R_a + L_a \frac{di_a}{dt} \quad \text{—— (4)}$$

At load,

$$T_m = J\frac{d\omega}{dt} + B\omega \quad \text{—— (5)}$$

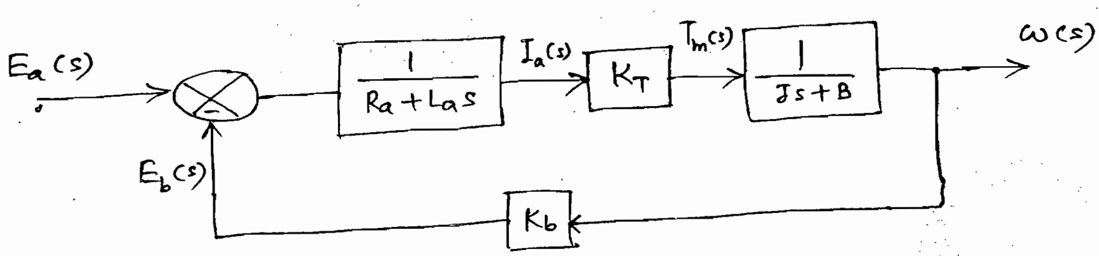
Transfer functions:

$$T_m(s) = K_T I_a(s) \quad \text{—— (6)}$$

$$E_b(s) = K_b \omega(s) \quad \text{—— (7)}$$

$$E_a(s) - E_b(s) = I_a(s)(R_a + L_a s) \quad \text{—— (8)}$$

$$T_m(s) = (Js+B)\,\omega(s) \quad \text{—— (9)}$$



Since, $L_a \approx 0$

$$\frac{\omega(s)}{E_a(s)} = \frac{\frac{K_T}{R_a(Js+B)}}{1+\frac{K_T\cdot K_b}{R_a(Js+B)}}$$

$$= \frac{K_T}{R_a(Js+B)+K_T K_b}$$

$$\frac{\omega(s)}{E_a(s)} = \frac{K_T/R_a}{(Js+B)+\underbrace{\frac{K_T K_b}{R_a}}_{f}}$$

$$= \frac{K_T/R_a}{Js+f}$$

Note: Armature controlled DC servomotor is a single time constant feedback control system.

Field controlled DC servomotors:

$I/p = e_f$; $O/p = \theta$ (or) ω.

Principle of operation:

Air gap flux (ϕ) $\propto$ field current (i_f)

$$\phi \propto i_f$$

$$\phi = K_f i_f \quad \text{—— (1)}$$

$$T_m \propto i_a \phi$$

$$T_m \propto K_f i_f i_a$$

$$T_m = K_1 K_f i_a i_f \qquad \text{(where } K_T = \text{motor torque constant)}$$

$$T_m = K_T i_f \quad \text{—— (2)}$$

Back emf (e_b) $\propto$ speed (ω)

$$e_b = K_b \omega \quad \text{——— (3)}$$

Analysis of field ckt:

$$e_f = i_f R_f + L_f \frac{di_f}{dt} \quad \text{——— (4)}$$

At load,

$$T_m = J\frac{d\omega}{dt} + B\omega \quad \text{——— (5)}$$

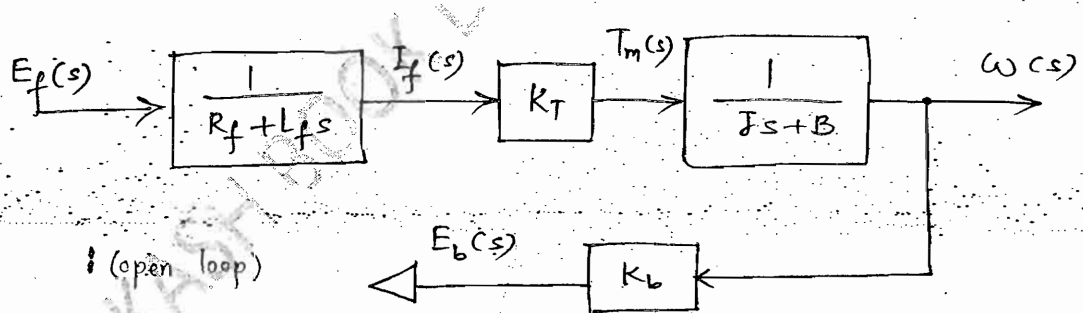
Transfer fn:

$$T_m(s) = K_T I_f(s) \quad \text{——— (6)}$$

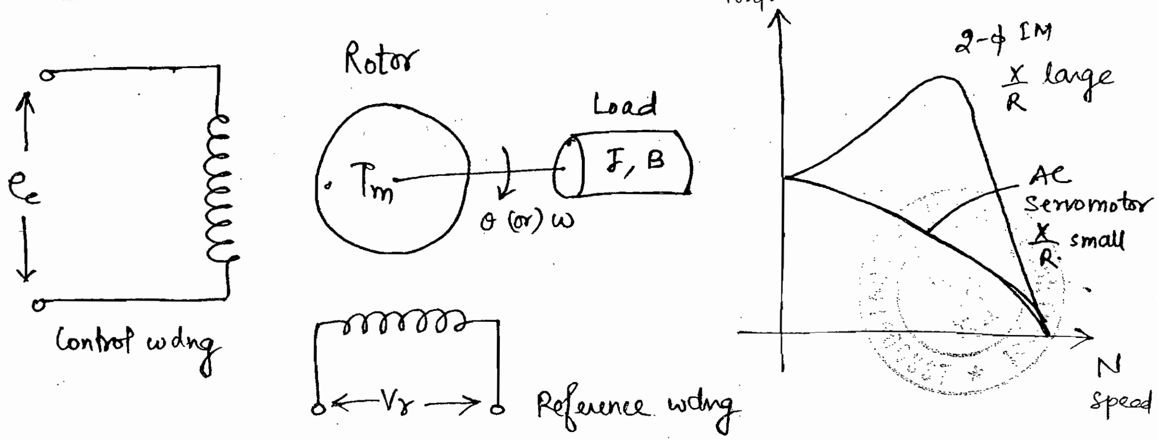
$$E_b(s) = K_b\, \omega(s) \quad \text{——— (7)}$$

$$E_f(s) = I_f(s)\,(R_f + L_f s) \quad \text{——— (8)}$$

$$T_m(s) = (Js+B)\cdot \omega(s) \quad \text{——— (9)}$$



AC Servomotor :



(1). It is constructionally similar to 2-ϕ IM.

(2). out of 2 wdngs placed in quadrature, one of wdng is excited by constant voltage and is known as reference wdng.

(3). The torque developed by rotor depends on control wdng voltage.

(4). In servo applications, the rotor is built with high resistance so that its X/R ratio is small and the torque speed characteristics are approximately linearized.

(5). In TF modelling the torque is made proportional to control wdng voltage by assuming the resistance and inductance of control wdng to be negligible.

$$T_m \propto e_c$$

$$T_m = K_m e_c$$

$$K_m = \frac{T_o}{e_c} \quad \text{where, } T_o = \text{stall torque}$$

$$\therefore T_m(s) = K_m E_c(s) \quad \text{---(I)}$$

At load,

$$T_m = J\frac{d\omega}{dt} + B\omega$$

$$T_M(s) = (Js + B)\,\omega(s) \quad \text{---(II)}$$

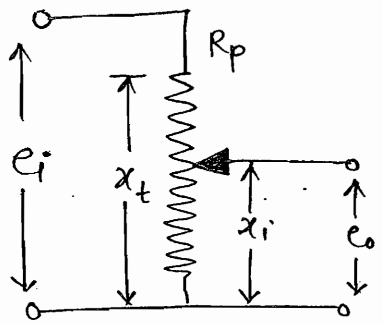
$$K_m E_c(s) = (Js+B)\,\omega(s)$$

$$\frac{\omega(s)}{E_c(s)} = \frac{K_m}{(Js+B)}$$

$$\frac{\omega(s)}{E_c(s)} = \frac{K_m}{Js+(B-M)}$$

where, M = slope of N-T characteristics.
(Correction factor)

Potentiometer



(1). It is a variable resistive displacement transducer used as error detector in control s/s applications.

(2). A pair of potentiometers act as error detectors

I/p = wiper displacement (x_i)

O/p = e_o

Total resistance of POT = R_p

Resistance/unit length = $\frac{R_p}{x_t}$

Resistance for wiper disp. of x_i units = $\frac{R_p \cdot x_i}{x_t}$

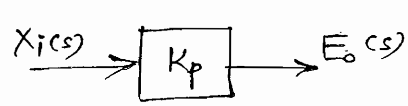
Applying Voltage Divider Rule:

$$e_o = \frac{\frac{R_p}{x_t} \cdot x_i}{R_p} \cdot e_i$$

$$e_o = \frac{x_i}{x_t} e_i$$

Let K_p = POT gain = $\frac{e_i}{x_t}$ (V/mm)

$$E_o(s) = K_p X_i(s)$$



Tachometers –

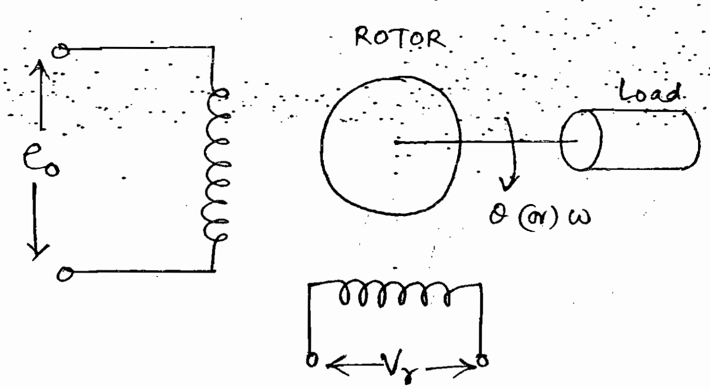
They are speed transducers used as feedback elements in control s/s applications.

They are of 2 types:

(1). DC Tachometer : This is a small DC generator whose I/p is mechanical speed and o/p is electrical voltage proportional to the speed.

(2). AC Tachometer : It is constructionally similar to AC servomotor. It is also called as drag cup generator because the rotor is of drag cup shape.

In AC tachometer as the rotor rotates the rate of change of flux induces an emf (e_0) which is directly proportional to speed.



(a). $e_0 \propto$ speed

$e_0 \propto d\theta/dt$

$e_0 = K_T \, d\theta/dt$

$E_0(s) = K_T \cdot s \cdot \theta(s)$

$$\theta(s) \longrightarrow \boxed{sK_T} \longrightarrow E_0(s)$$

(b). $e_0 \propto \omega$

$e_0 = K_T \, \omega$

$E_0(s) = K_T \, \omega(s)$

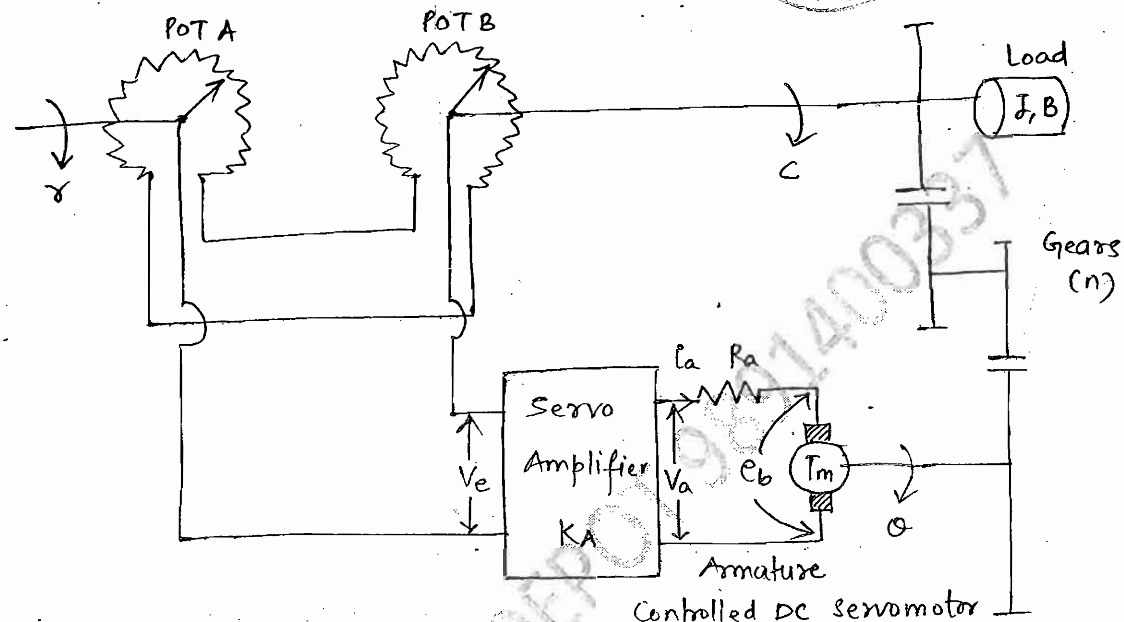
where, $K_T \rightarrow$ tachometer gain

$$\omega(s) \longrightarrow \boxed{K_T} \longrightarrow E_0(s)$$

Q. The TF of a tachometer $E(s)/\theta(s)$ is

(a). Ks (b). K/s (c). Ks^2 (d) K

Position Control system –



I/p = r ; O/p = C

Principle of operation:

(i). At potentiometer:

$$V_e \propto r - c$$

$$V_e = K_p (r - c)$$

$$V_e(s) = K_p (R(s) - C(s)) \quad \text{——— (1)}$$

(2). At Amplifier:

$$V_a \propto V_e$$

$$V_a = K_A V_e$$

$$V_a(s) = K_A \cdot V_e(s) \quad \text{——— (2)}$$

(iii). Analysis of Armature controlled DC servomotor:

$$T_m(s) = K_T I_a(s) \quad \text{——— (3)}$$

$e_b \propto d\theta/dt$

$$e_b = K_b \cdot \frac{d\theta}{dt}$$

$$E_b(s) = K_b \cdot s \cdot \theta(s) \quad \text{——} (4)$$

$$V_a - e_b = i_a R_a$$

$$V_a(s) - E_b(s) = I_a(s) \cdot R_a \quad \text{——} (5)$$

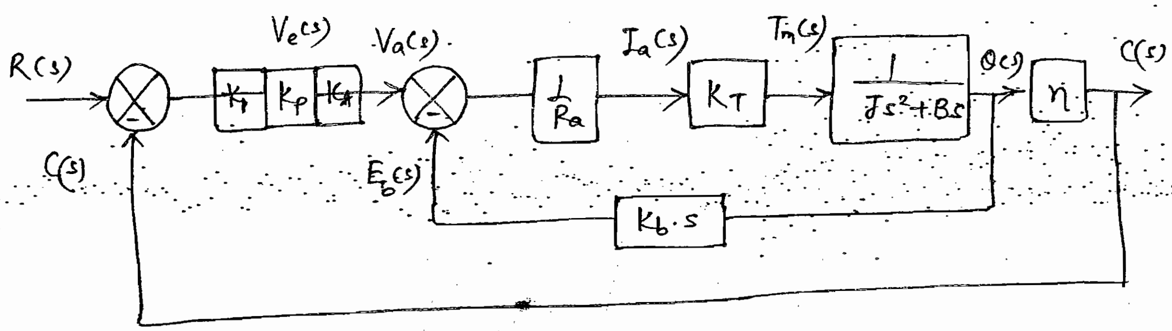
$$T_m = J\frac{d^2\theta}{dt^2} + B \cdot \frac{d\theta}{dt}$$

$$T_m(s) = (Js^2 + Bs)\ \theta(s) \quad \text{——} (6)$$

(iv). Gears:

$c \propto \theta$

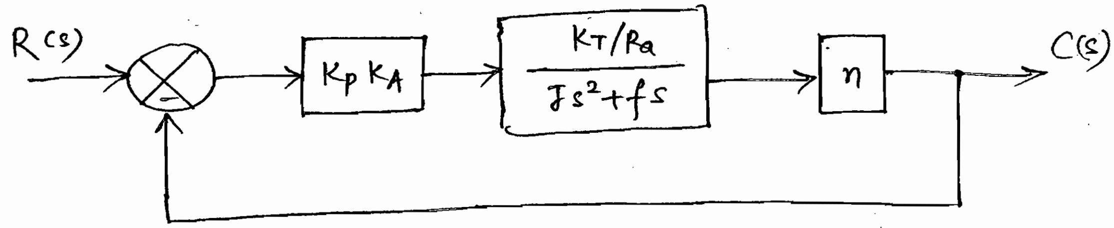
$$C(s) = n\ \theta(s) \quad \text{——} (7)$$



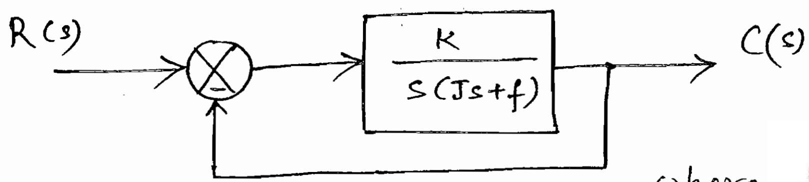
Inner loop:

$$\frac{\dfrac{K_T}{R_a(Js^2+Bs)}}{1 + \dfrac{K_T \cdot K_b \cdot s}{R_a(Js^2+Bs)}} = \frac{K_T}{R_a(Js^2+Bs) + K_T K_b s}$$

$$= \frac{K_T/R_a}{\underbrace{Js^2 + Bs + \frac{K_T K_b}{R_a} \cdot s}_{fs}} = \frac{K_T/R_a}{Js^2 + fs}$$

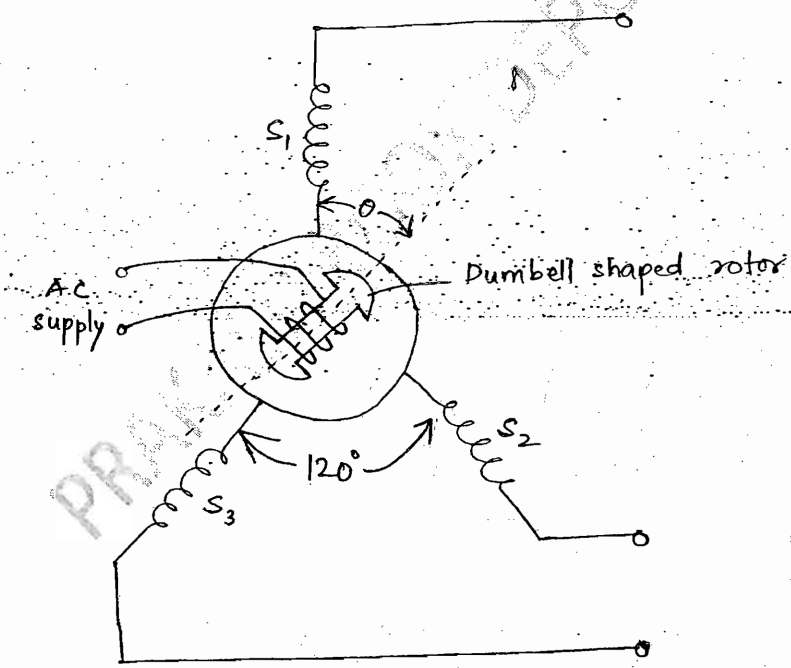


⇓



where, $K = \frac{K_A K_T K_p \eta}{R_a}$

Synchro –



(1). It is commercially known as SELSYN (or) AUTOSYN.

(2). It is an electromagnetic transducer which converts angular position of the rotor into proportional voltages.

(3). It is constructionally similar to 3-ϕ alternator. but operationally based on X^r action.

(4). When the dumbell shaped rotor is excited by an AC voltage, sinusoidally time varying fluxes are induced in the stator and rotor periphery depending on the angular position of the rotor, voltages of unequal magnitudes and phase difference of $120°$ are induced in all the 3 stator wdngs.

(5). When the rotor is inclined along any one of the stator axis completely, then max^m voltage will be induced across that wdng and almost zero voltages are induced in other 2 wdngs, this position of rotor is known as electrical zero position.

(6). A pair of synchros known synchro transmitter and synchro control X^r (Receiver) act as error detector.

(7). The rotor of synchro control X^r is circular in shape because it act as servomechanism.

22/03/2014

Part (2)

Time Domain Analysis

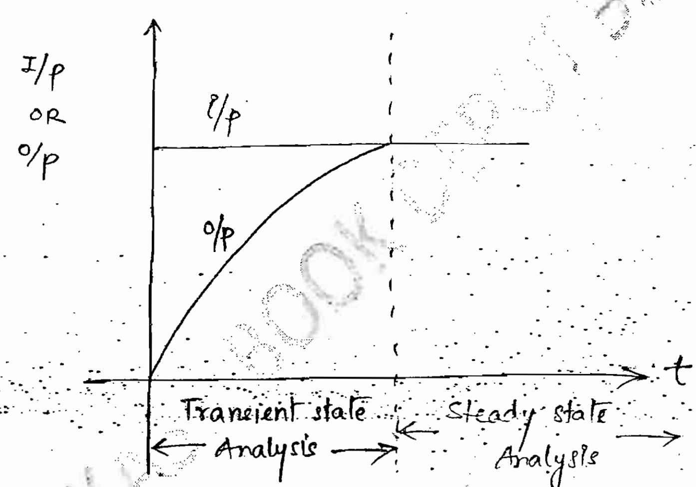
The time response analysis of a control s/s is divided into 2 parts :

(1). Transient state Analysis

It deals with the nature of response of a s/s when subjected to an I/p.

(2). Steady state Analysis

It deals with the estimation of magnitude of steady state error b/w O/p and I/p.



Standard Test signals :

(1). Sudden Inputs → Step signal — Bounded (constant)

(2). Velocity type Inputs → Ramp signal

(3). Acceleration type Inputs → Parabolic signal

Time domain Analysis — linear, Non-linear

(4). Sudden shocks → Impulse signal → stability

Note : for evaluating the transient state analysis, the step s/g is preferred becoz the magnitude of step s/g does not change with time.

Type & Order :

(1). Every TF representing a control s/s has some type and order.

(2). The steady state analysis depend upon the type of control s/s.

(3). The type of control s/s is obtained from open loop TF $G(s)H(s)$.

(4). The no. of open loop poles occuring at origin determines the type of control s/s.

Let $G(s)H(s) = \dfrac{K(1+T_a s)}{s^P (1+T_1 s)}$

$P = 0$; Type - 0 s/s

$P = 1$; Type - 1 s/s

⋮

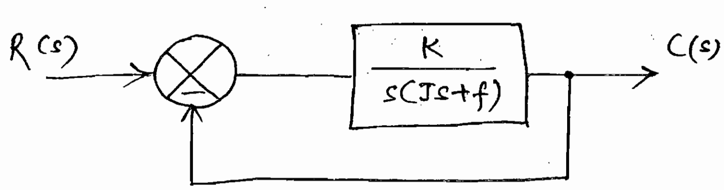
$P = n$; Type - n s/s

(5). The transient state analysis depends on order of the control s/s.

(6). The order of control s/s is obtained from close loop TF $G(s)/[1+G(s)H(s)]$

(7). The highest power of characteristic eqn $1+G(s)H(s)=0$ determines the order of control s/s.

Eg. Position control s/s.



$G(s) = \dfrac{K}{s^1(Js+f)}$ ⇒ Type - 1 s/s

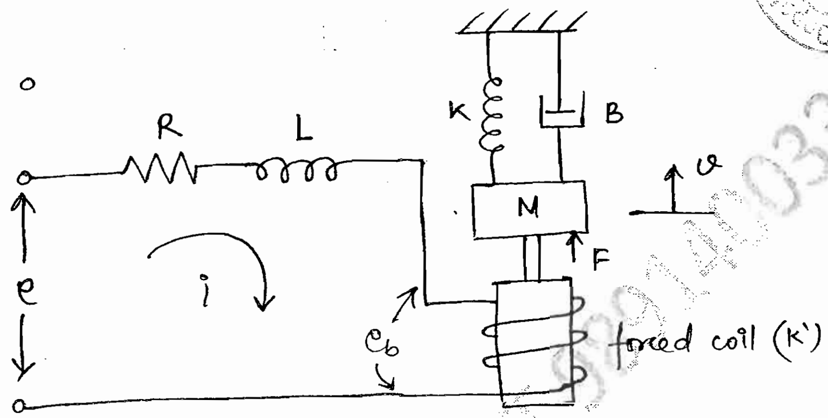
Characteristic Eqn:

$$1 + G(s) = 0$$

$$1 + \frac{K}{s(Js+f)} = 0$$

$$\therefore \; Js^2 + fs + K = 0 \quad \Rightarrow \text{Order} - 2.$$

Eg.



$$e = iR + L\frac{di}{dt} + e_b$$

$$e - e_b = iR + L\frac{di}{dt}$$

$$E(s) - E_b(s) = I(s)\,(R + Ls) \quad \text{———(1)}$$

e_b (Transducted Voltage) $\propto$ speed

$$e_b \propto v$$

$$e_b = K_T \cdot v \qquad K_T \rightarrow \text{Tachometer gain}$$

$$E_b(s) = K_T \cdot V(s) \quad \text{———(2)}$$

Force coil:

$$F \propto i$$

$$F = K' i$$

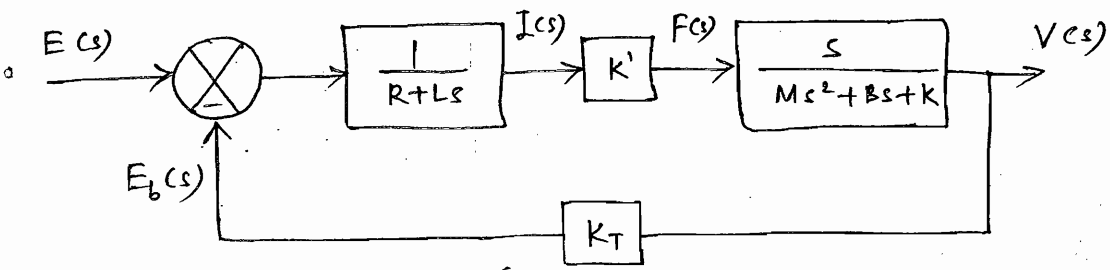
$$F(s) = K' I(s) \quad \text{———(3)}$$

At mech s/s:

$$F = M\frac{dv}{dt} + Bv + K\int v\,dt$$

$$F(s) = \left(Ms + B + \frac{K}{s}\right) V(s)$$

$$F(s) = \left(\frac{Ms^2 + Bs + K}{s}\right) \cdot V(s) \quad \text{—— (4)}$$



(Tachometer fb)

$$G(s)H(s) = \frac{K' K_T\, s}{(R+Ls)(Ms^2+Bs+K)} \quad ; \text{Type-0 s/s}$$

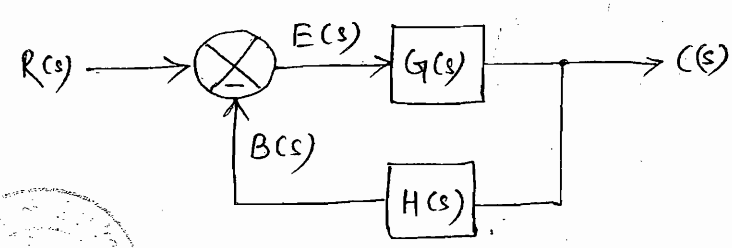
$$1 + G(s)H(s) = 0$$

$$(R + L(s))(Ms^2 + Bs + K) + K' K_T s = 0$$

$\Rightarrow$ Order - 3 s/s.

Steady State Response Analysis:

: To obtain an expression for error :



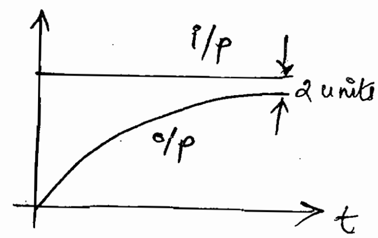
$$E(s) = R(s) - B(s)$$

$$E(s) = R(s) - H(s) \cdot C(s)$$

$$E(s) = R(s) - H(s) \cdot G(s) \cdot E(s)$$

$$E(s) = \frac{R(s)}{1 + G(s)(H(s))} \quad \text{..... error ratio}$$

$$\underset{t\to\infty}{Lt}\ e(t) = 2 \text{ units } (e_{ss})$$



$$e_{ss} = \underset{t\to\infty}{Lt}\ e(t)$$

Applying final value theorem,

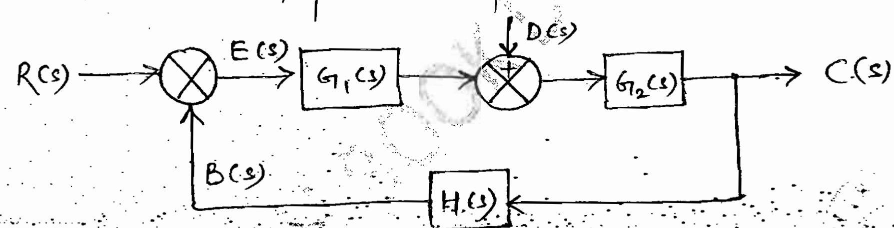
$$e_{ss} = \underset{s\to 0}{Lt}\ s \cdot E(s)$$

$$\boxed{e_{ss} = \underset{s\to 0}{Lt}\ \frac{s \cdot R(s)}{1 + G(s) \cdot H(s)}}$$

$$e_{ss} = \frac{\underset{s\to 0}{Lt}\ s \cdot R(s)}{1 + \underset{s\to 0}{Lt}\ G(s) \cdot H(s)}$$

Note: That's why we say e_{ss} depends on i/p and type of s/s.

To obtain expression for error with disturbances:



$$E(s) = R(s) - B(s)$$

$$E(s) = R(s) - C(s) \cdot H(s)$$

$$C(s) = [E(s)\ G_1(s) + D(s)]\ G_2(s)$$

$$E(s) = R(s) - E(s)\, G_1(s)\, G_2(s) H(s) - D(s)\, G_2(s)\, H(s)$$

$$E(s)\left[1 + G_1(s) G_2(s) H(s)\right] = R(s) - D(s)\, G_2(s)\, H(s)$$

$$E(s) = \frac{R(s)}{1 + G_1(s)\, G_2(s)\, H(s)} - \frac{D(s)\, G_2(s)\, H(s)}{1 + G_1(s) G_2(s) H(s)}$$

$$e_{ss} = \underset{s\to 0}{Lt}\ \frac{s \cdot R(s)}{1 + G_1(s) G_2(s) H(s)} - \underset{s\to 0}{Lt}\ \frac{D(s) \cdot G_2(s)\ H(s)}{1 + G_1(s) G_2(s) H(s)}$$

Q4. $e_{ss} = -\underset{s\to 0}{Lt} \dfrac{s \cdot D(s) \cdot G_2(s)}{1+G_1(s)\, G_2(s)}$

for $D(s) = 1/s$ (given)

$$e_{ss} = -\underset{s\to 0}{Lt} \frac{G_2(s)}{1+G_1(s) \cdot G_2(s)}$$

$$\downarrow |e_{ss}| = \frac{G_2}{1+G_1 G_2} = \frac{1}{\uparrow G_1 + \frac{1}{G_2}}$$

Steady state errors: for different types of I/p's.

(1). Step I/p: $R(s) = \dfrac{A}{s}$

$$e_{ss} = \underset{s\to 0}{Lt} \frac{s \cdot \frac{A}{s}}{1+G(s) \cdot H(s)}$$

$$= \frac{A}{1+\underset{s\to 0}{Lt}\, G(s) \cdot H(s)} = \frac{A}{1+K_p}$$

$K_p \rightarrow$ position error constant

$$K_p = \underset{s\to 0}{Lt}\, G(s)\, H(s)$$

(2). Ramp I/p : $R(s) = \dfrac{A}{s^2}$

$$e_{ss} = \underset{s\to 0}{Lt} \frac{s \cdot \frac{A}{s^2}}{1+G(s) \cdot H(s)}$$

$$= \underset{s\to 0}{Lt} \frac{A}{s+s\, G(s) H(s)}$$

$$= \frac{A}{\lim\limits_{s\to 0} s \cdot G(s) H(s)} = \frac{A}{K_v}$$

K_v : Velocity error constant

$$K_v = \lim_{s\to 0} s \cdot G(s) H(s)$$

(c) Parabolic I/p : $R(s) = A/s^3$

$$e_{ss} = \lim_{s\to 0} \frac{s \cdot \frac{A}{s^3}}{1 + G(s)H(s)}$$

$$= \lim_{s\to 0} \frac{A}{s^2 + s^2 G(s) H(s)}$$

$$= \frac{A}{\lim_{s\to 0} s^2 G(s) H(s)} = \frac{1}{K_a}$$

K_a : acceleration error constant

$$K_a = \lim_{s\to 0} s^2 G(s) H(s)$$

$\therefore$ K_p, K_v and K_a $\Rightarrow$ static error constants.

Steady state error for different types of s/s :

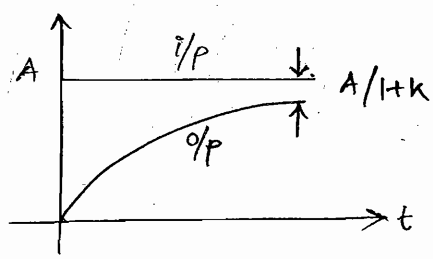
Type – 0 s/s :

$$G(s)H(s) = \frac{K(1+T_a s)}{(1+T_1 s)}$$

(1) Step I/p : $K_p = \lim_{s\to 0} G(s) H(s) = K$

$$e_{ss} = \frac{A}{1+K_p}$$

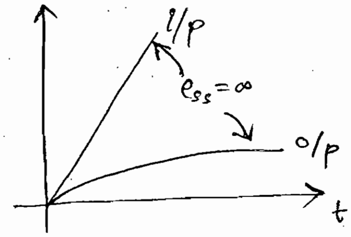
$$e_{ss} = \frac{A}{1+K}$$



(2) Ramp I/p : $K_v = \lim_{s\to 0} s\, G(s) H(s) = 0$

$$e_{ss} = \frac{A}{K_v}$$

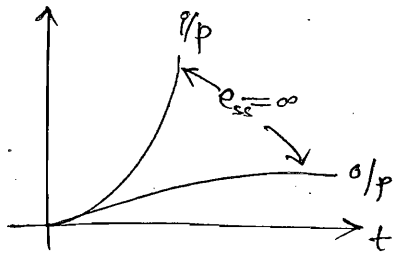
$$e_{ss} = \infty$$



(3). Parabolic I/p : $K_a = \lim_{s \to 0} s^2 \cdot G(s) \cdot H(s) = 0$

$e_{ss} = \dfrac{A}{K_a}$

$e_{ss} = \dfrac{A}{0} = \infty$



I/p s/g \ Type	Step I/p	Ramp I/p	Parabolic I/p
Type - 0	$\frac{A}{1+K}$ $K_p = K$	∞ $K_v = 0$	∞ $K_a = 0$
Type - 1	0 $K_p = \infty$	A/K $K_v = K$	∞ $K_a = 0$
Type - 2	0 $K_p = \infty$	0 $K_v = \infty$	A/K $K_a = K$

Observations:

(1). finite error, $\downarrow e_{ss} \propto \frac{1}{K\uparrow}$

(2). The max^m type no. for LTI s/s is 2. Beyond type - 2, the s/s exhibits non-linear characteristics more dominantly also the s/s tends to become unstable as the gain increases

Q3. $G(s) = \dfrac{10}{s^2(4+s)}$

$r(t) = 2 + 3t + 4t^2$

$R(s) = \dfrac{2}{s} + \dfrac{3}{s^2} + \dfrac{8}{s^3} = \dfrac{2s^2+3s+8}{s^3}$

$e_{ss} = \lim_{s \to 0} \dfrac{s \cdot \dfrac{2s^2+3s+8}{s^3}}{1 + \dfrac{10}{s^2(4+s)}} = \lim_{s \to 0} \dfrac{2s^2+3s+8}{s^2 + \dfrac{10}{(4+s)}} = \dfrac{8}{10/4}$

$= 3.2$ units

Short cut : $G(s) = \dfrac{10}{s^2(s+4)}$

Type – 2 s/s

$$R(s) = \frac{2}{s} + \frac{3}{s^2} + \frac{8}{s^3}$$

$$e_{ss} = 0 + 0 + \frac{A}{K}$$

$A = 8$

$$K = \lim_{s\to 0} s^2 \cdot \frac{10}{s^2 \cdot (4+s)} = \frac{10}{4}$$

$$e_{ss} = \frac{8}{10/4} = 3.2 \text{ units}$$

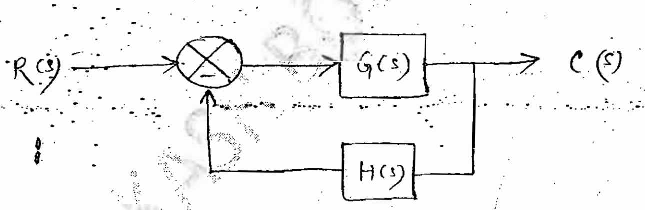
Q5. Type – 0 s/s $\xrightarrow[\text{(Integrator)}]{1/s}$ Type – 1

$$e_{ss} = \frac{A}{1+K} \qquad\qquad e_{ss} = \frac{1}{K}$$

$$0.2 = \frac{1}{1+K} \qquad\qquad = \frac{1}{4}$$

$\Rightarrow K = 4$ $\qquad\qquad = 0.25$ units

Q4.



$$r(t) = (1 - t^2)\,3 \cdot u(t) = \frac{3}{s} - \frac{6}{s^3}$$

$$e_{ss} = \lim_{s\to 0} \frac{s \cdot \left(\frac{3}{s} - \frac{6}{s^3}\right)}{1 + G(s)H(s)}$$

$$e_{ss} = \lim_{s\to 0} \frac{3}{1+G(s)H(s)} - \lim_{s\to 0} \frac{6}{s^2 + s^2 G(s)H(s)}$$

$$e_{ss} = \frac{3}{1 + \lim_{s\to 0} G(s)H(s)} - \frac{6}{\lim_{s\to 0} s^2 G(s)H(s)}$$

$$\therefore e_{ss} = \frac{3}{1+K_p} - \frac{6}{K_a}$$

short cut :

$$R(s) = \frac{3}{s} - \frac{6}{s^3}$$

$$e_{ss} = \frac{\quad}{1+K} - \frac{\quad}{K}$$

$$\therefore\ e_{ss} = \frac{3}{1+K_p} - \frac{6}{K_a}$$

Q2. $e_{ss} = 5\% = \frac{5}{100} = \frac{1}{20} = \frac{1}{K_v} \Rightarrow K_v = 20$

Ramp I/p $\rightarrow$ Type - 1 s/s.

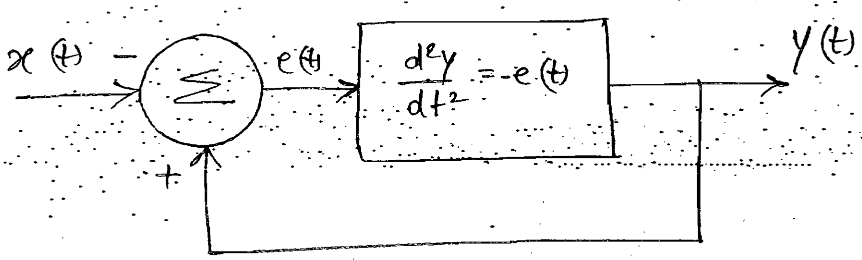
Q1. $e_{ss} = \frac{1}{1+K}$

$K = K_p$

$K_p = 10/50$

$$e_{ss} = \frac{1}{1+K_p} = \frac{1}{1+\frac{10}{50}} = \frac{50}{60} = 0.83 \text{ units.}$$

Q.



for $x(t) = t \cdot u(t)$ · find $e(t) = ?$

(a) sin t (b) cos t (c) − sint (d) − cost

Sol. $e(t) = -x(t) + y(t)$

$E(s) = -X(s) + Y(s)$

$$\frac{d^2y}{dt^2} = -e(t)$$

$s^2 Y(s) = -E(s)$

$$Y(s) = -\frac{E(s)}{s^2}$$

$$E(s) = -X(s) - \frac{E(s)}{s^2}$$

$$E(s)\left(1+\frac{1}{s^2}\right) = -X(s)$$

$$E(s) = \frac{-X(s)\cdot s^2}{(1+s^2)}$$

Not $x(t) = t\cdot u(t)$

$$X(s) = \frac{1}{s^2}.$$

$$\therefore E(s) = \frac{-\frac{1}{s^2}\cdot s^2}{(1+s^2)} = \frac{-1}{(s^2+1)}$$

$$\therefore e(t) = -\sin t$$

Error Series –

$$E(s) = \frac{R(s)}{1+G(s)H(s)}$$

Let $F(s) = \dfrac{1}{1+G(s)H(s)}$

$$E(s) = R(s)\cdot F(s)$$

$$\mathcal{L}^{-1}E(s) = \mathcal{L}^{-1}\{R(s)\cdot F(s)\}$$

$$e(t) = \int_0^\infty f(T)\, r(t-T)\, dT$$

Expanding $r(t-T)$ using Taylor series:

$$r(t-T) = r(t) - T\dot{r}(t) + \frac{T^2}{2!}\ddot{r}(t) - \frac{T^3}{3!}\dddot{r}(t) + \cdots$$

$$e(t) = r(t)\cdot\int_0^\infty f(T)\, dT - \dot{r}(t)\int_0^\infty T\cdot f(T)\, dT + \frac{\ddot{r}(t)}{2!}\int_0^\infty T^2 f(T)\, dT + \cdots$$

Defining "Dynamic error constants".

$$K_0 = \int_0^\infty f(T)\,dT \;;\; K_1 = -\int_0^\infty T f(T)\,dT \;;\; K_2 = \int_0^\infty T^2 f(T)\,dT$$

$$e(t) = K_0 r(t) + K_1 \dot{r}(t) + \frac{K_2}{2!}\ddot{r}(t) + \frac{K_3}{3!}\dddot{r}(t) + \cdots$$ ....error series

$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{t\to\infty} e(t)$$

To find dynamic error constants

$$\mathcal{L} f(t) = F(s) = \int_0^\infty f(T)\cdot e^{-sT}\,dT$$

$$\lim_{s\to 0} F(s) = \lim_{s\to 0}\int_0^\infty f(T)\cdot e^{-sT}\,dT = \int_0^\infty f(T)\,dT = K_0 \quad \text{——(1)}$$

Now, $$\frac{d}{ds}F(s) = \frac{d}{ds}\int_0^\infty f(T)\,e^{-sT}\,dT = -\int_0^\infty T f(T)\,e^{-sT}\,dT$$

$$\lim_{s\to 0}\frac{d}{ds}F(s) = \lim_{s\to 0} -\int_0^\infty T\cdot f(T)e^{-sT}\,dT = -\int_0^\infty f(T)\cdot dT = K_1 \quad \text{——(II)}$$

Hence, $$K_0 = \lim_{s\to 0} F(s)$$

$$K_1 = \lim_{s\to 0}\frac{d}{ds}(F(s))$$

$$K_2 = \lim_{s\to 0}\frac{d^2}{ds^2}F(s)$$

$\vdots \qquad \vdots \qquad \vdots$ where, $$F(s) = \frac{1}{1+G(s)H(s)}$$

Relationship b/w static and dynamic error constants:

Let $$G(s)H(s) = \frac{100}{s(s+2)}$$

I. Static Error Constants:

$$K_p = \lim_{s\to 0}\frac{100}{s(s+2)} = \infty$$

$$K_v = \lim_{s\to 0} s\cdot\frac{100}{s(s+2)} = 50$$

$$K_a = \lim_{s\to 0} s^2\cdot\frac{100}{s(s+2)} = 0$$

II. Dynamic Error constants :

$$F(s) = \frac{1}{1+G(s)\,F(s)} = \frac{1}{1+\frac{100}{s(s+2)}}$$

$$K_0 = \lim_{s\to 0} F(s) = \lim_{s\to 0}\frac{1}{1+\frac{100}{s(s+2)}} = \frac{1}{1+\infty} = 0$$

$$\therefore K_0 = \frac{1}{1+K_p}$$

$$K_1 = \lim_{s\to 0}\frac{d}{ds}F(s) \Rightarrow \frac{d}{ds}\cdot\frac{1}{1+\frac{100}{s(s+2)}} = \frac{d}{ds}\cdot\frac{s(s+2)}{(s^2+2s+100)}$$

$$= \frac{(s^2+2s+100)(2s+2) - s(s+2)(2s+2)}{(s^2+2s+100)^2}$$

$$K_1 = \lim_{s\to 0}\frac{(s^2+2s+100)(2s+2) - s(s+2)(2s+2)}{(s^2+2s+100)^2}$$

$$= \frac{(0+0+100)(0+2) - 0(2)(2)}{(0+0+100)^2} = \frac{1}{50}$$

$$\therefore K_1 = \frac{1}{K_v} \quad \text{and} \quad K_2 = \frac{1}{K_a}$$

Note: The dynamic error constants are inversely related to static error constants but they need not be always direct reciprocal values.

Transient state Analysis:

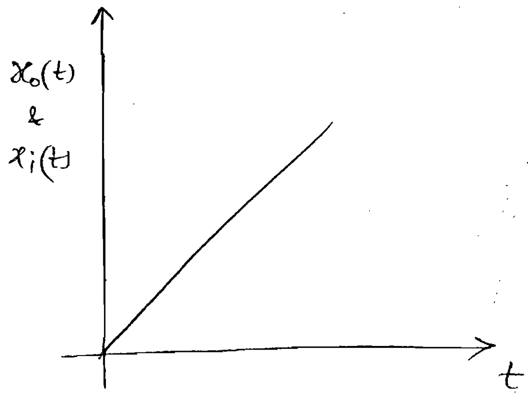
$$\frac{X_o(s)}{X_i(s)} = \frac{b_o}{a_n s^n + a_{n-1} s^{n-1} + \cdots a_1 s + a_o}$$

(1). Zero Order system:

$$\frac{X_o(s)}{X_i(s)} = \frac{b_o}{a_o}$$

Let $K = \text{gain} = b_o/a_o$

$$\frac{X_o(s)}{X_i(s)} = K$$



$$X_i(s) \longrightarrow \boxed{K} \longrightarrow X_o(s)$$

Eg. Sensor (or) Transducers

(2). I[st] order system:

$$\frac{X_o(s)}{X_i(s)} = \frac{b_o/}{a_1 s + a_o} = \frac{b_o/a_o}{\left(\frac{a_1}{a_o} s + 1\right)}$$

Let $K = b_o/a_o = \text{gain}$

$T = \text{time constant element} = \frac{a_1}{a_o}$

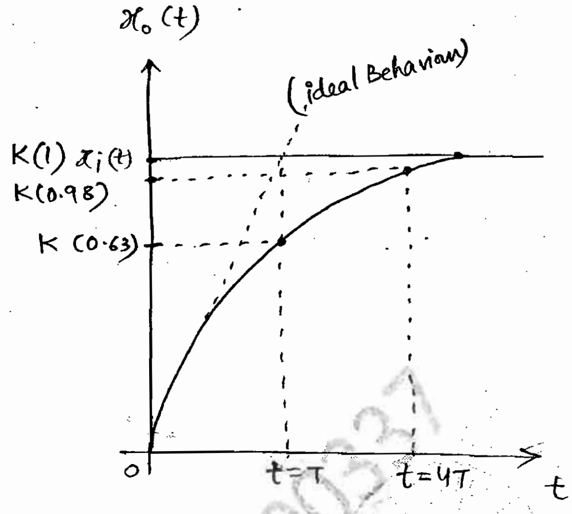
$$\frac{X_o(s)}{X_i(s)} = \frac{K}{1 + Ts}$$

Eg. RC N/ω

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{RCs + 1} \quad ; \quad T = RC$$

Transient Analysis:

Let $X_i(s) = \frac{1}{s}$ (unit step)



$$X_o(s) = \frac{K}{s(1+Ts)}$$

$$= K\left(\frac{1}{s} - \frac{T}{1+Ts}\right)$$

$$= K\left(\frac{1}{s} - \frac{1}{s+1/T}\right)$$

$$x_o(t) = K\left(1-e^{-t/T}\right)$$

At $t=0$,

$$x_o(t) = K(1-e^0) = K(1-1) = 0$$

At $t=T$,

$$x_o(t) = K(1-e^{-1}) = K(1-0.37) = K(0.63)$$

At $t=4T$,

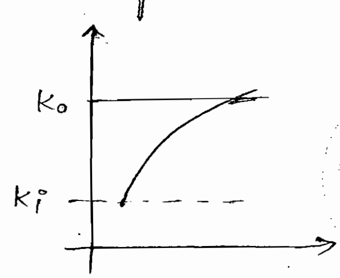
$$x_o(t) = K(1-e^{-4}) = K(1-0.02) = K(0.98)$$

(1). Time constant is defined as time taken by the response of the s/s to reach 63% of the final value.

(2). Thermal s/s, liquid level s/s (Hydraulic s/s), pneumatic s/s, thermometers (mercury based) and RC & RL N/w are examples of 1st order s/s.

If there is some initial condition, then modified response will be

$$x_o(t) = K_o + (K_i - K_o)e^{-t/T}$$



(3) Second Order s/s :

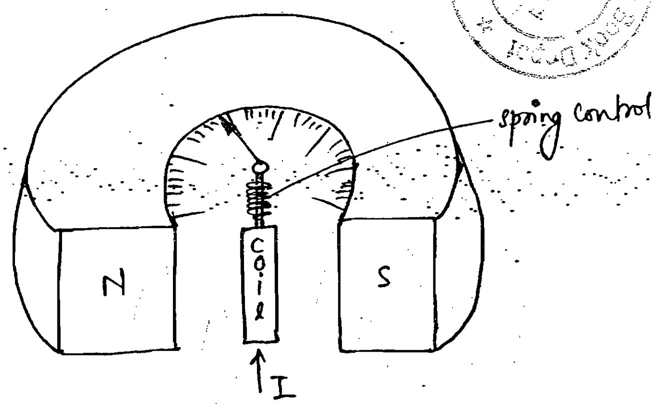
(1). The response of 2^{nd} order or higher order s/s exhibits continuous & sustained oscillations about the steady state value of the I/p with the frequency known as undamped natural frequency ω_n rad/sec

(2). These oscillations in the response are damped to the steady state value of I/p using appropriate damping measures.

(3). Damping is mathematically expressed as damping ratio ξ (or zeta).

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

Eg. PMMC



I/p = Deflecting torque (T_d)

O/p = Angular deflection of pointer (θ)

J = Inertia of moving s/s

B = Inherent friction

K = spring constant

$$T_d = J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K\theta$$

$$T_d(s) = \left(Js^2 + Bs + K\right)\theta(s)$$

$$\frac{\theta(s)}{T_d(s)} = \frac{1}{Js^2+Bs+K} = \frac{1/J}{s^2+\frac{B}{J}s+\frac{K}{J}}$$

Comparing.

$$s^2 + 2\xi\omega_n s + \omega_n^2 = s^2 + \frac{B}{J}s + \frac{K}{J}$$

$$\therefore \omega_n = \sqrt{K/J} \text{ rad/s} \quad \& \quad 2\xi\omega_n = \frac{B}{J}$$

$$\therefore \xi = \frac{B}{2\sqrt{KJ}} \qquad \xi = \frac{B\sqrt{J}}{2J\sqrt{K}}$$

$\xi \propto B$ (i.e. damping $\propto$ friction)

Effect of damping on nature of Response :

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$$s = \frac{+2\xi\omega_n \pm \sqrt{4\xi^2\omega_n^2 - 4\omega_n^2}}{2}$$

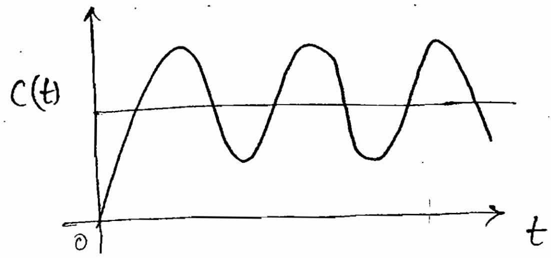
$$= -\xi\omega_n \pm \omega_n\sqrt{\xi^2-1}$$

$D = \xi^2 - 1 = 0 \Rightarrow \xi = 1$

$D = \xi^2 - 1 < 0 \Rightarrow \xi < 1$

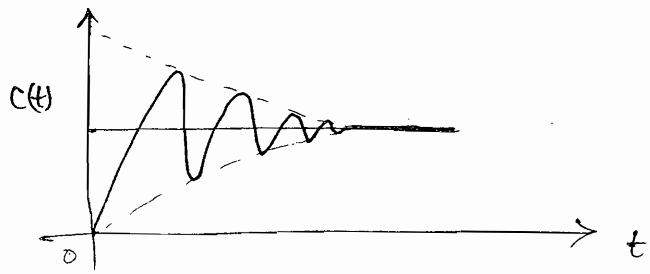
$D = \xi^2 - 1 > 0 \Rightarrow \xi > 1$

Case (1): Undamped case ($\xi = 0$)

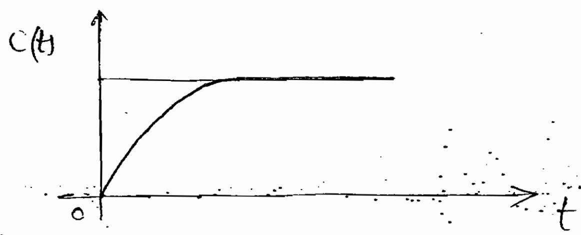


ω_n rad/s

Case (2): Underdamped case ($0 < \xi < 1$)

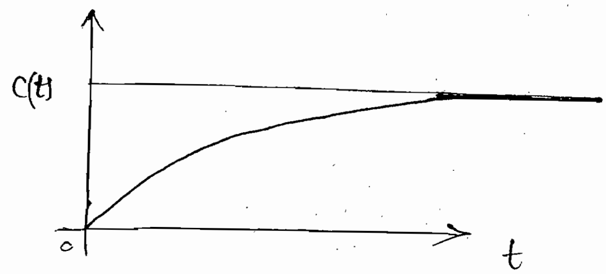


Case (3): Critically damped case ($\xi = 1$)



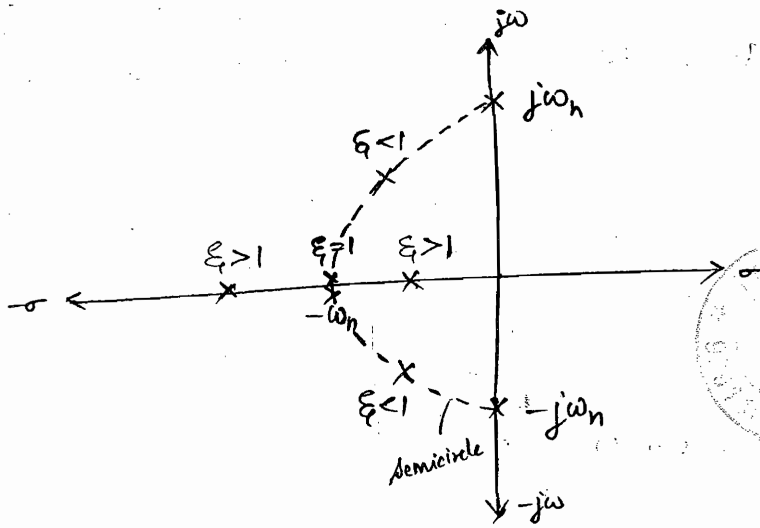
(practically not possible)
to construct non-interacting s/s
100% accuracy is not obtained

Case (4): Overdamped case ($\xi > 1$)



(slow & sluggish) response

(1). Most of the control s/s are designed $\xi < 1$ because the response can be analysed using more no. of performance specification.



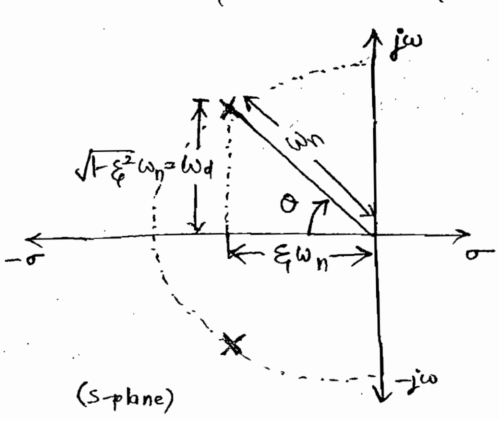
(2). The root locus of 2^{nd} order s/s is a semicircular path with the radius of ω_n and break away point at $-\omega_n$ on the -ve real axis where the damping ratio ξ is varying. $(0<\xi<1)$.

(3). The optimum values of damping ratio are b/ω in 0.3 to 0.7.

* for Robotics, $\xi \simeq 0.98$ is used.
* 100% accuracy is not achieved, so, of 2 non-interacting s/s's so $\xi<1$ or $\xi>1$. So s/s characteristics may fall in case (2) and case (4) respectively.

Characteristics of underdamped s/s.

$$s = -\xi\omega_n \pm \omega_n\sqrt{\xi^2-1}$$



(1). $\cos\theta = \frac{\xi\omega_n}{\omega_n}$

$\theta = \cos^{-1}\xi$

(2). Damping cofficient (or) Damping factor (or) Actual damping

$\alpha = \xi\omega_n$

(3). Time constant of underdamped response

$$T = \frac{1}{\alpha} = \frac{1}{\xi \omega_n}$$

(4). Damped Natural frequency (ω_d) :

$$\omega_d = \omega_n \sqrt{1-\xi^2} \quad \text{rad/s}$$

(5). $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

$$s = -\xi\omega_n \pm j\,\omega_d$$

$$\Rightarrow s^2 + 2\xi\omega_n s + \omega_n^2 = (s+\xi\omega_n)^2 + \omega_d^2$$

(6). Damping ratio $= \dfrac{\text{Actual damping}}{\text{Critical damping}} = \dfrac{\xi \cancel{\omega_n}}{\cancel{\omega_n}} = \xi$

Actual damping $= \xi\omega_n$

At $\xi = 1$, actual damping becomes critical damping

Critical damping $= \omega_n$

* $\cos\theta = \xi$
* $\sin\theta = \sqrt{1-\xi^2}$
* $\tan\theta = \sqrt{1-\xi^2} \,/\, \xi$

Transient analysis (Underdamped response) :

Let $R(s) = \frac{1}{s}$

$$C(s) = \frac{\omega_n^2}{s(s^2 + 2\xi\omega_n s + \omega_n^2)}$$

$$= \frac{1}{s} - \frac{(s + 2\xi\omega_n)}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$= \frac{1}{s} - \frac{(s+\xi\omega_n)}{(s+\xi\omega_n)^2 + \omega_d^2} - \frac{\xi\cancel{\omega_n}}{(s+\xi\omega_n)^2 + \omega_d^2} \times \frac{\omega_d}{\cancel{\omega_n}\sqrt{(1-\xi^2)}}$$

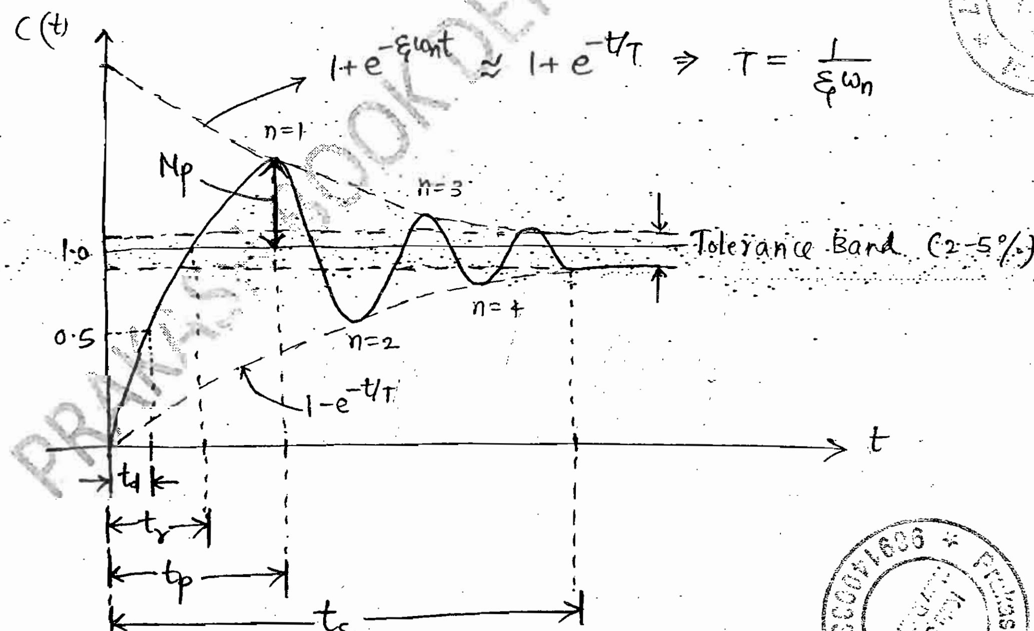
$$C(s) = \frac{1}{s} - \frac{s+\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2} - \frac{\xi}{\sqrt{1-\xi^2}} \cdot \frac{\omega_d}{(s+\xi\omega_n)^2+\omega_d^2}$$

$$c(t) = 1 - e^{-\xi\omega_n t}\cos\omega_d t - e^{-\xi\omega_n t} \cdot \frac{\xi}{\sqrt{1-\xi^2}} \cdot \sin\omega_d t$$

$$= 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \cdot \left\{ \underbrace{\sqrt{1-\xi^2}}_{B} \cdot \cos\omega_d t + \underbrace{\xi}_{A} \sin\omega_d t \right\}$$

$$A\sin\omega t + B\cos\omega t = \sqrt{A^2+B^2} \cdot \sin\left(\omega t + \tan^{-1} B/A\right)$$

$$\boxed{c(t) = \underbrace{1}_{\text{steady state response}} - \underbrace{\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \cdot \sin\left\{\omega_d t + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right\}}_{\text{transient state response}}}$$



(1). <u>Delay time (t_d)</u> :

$$t_d = \frac{1+0.7\xi}{\omega_n} \text{ sec.}$$

(2). Rise time (t_r) :

At $t=t_r$; $c(t)=1$

$$c(t)\Big|_{t=t_r} = 1-\frac{e^{-\xi\omega_n t_r}}{\sqrt{1-\xi^2}}\cdot\sin(\omega_d t_r+\theta)=1$$

$$\Rightarrow \frac{e^{-\xi\omega_n t_r}}{\sqrt{1-\xi^2}}\cdot\sin(\omega_d t_r+\theta)=0$$

Since, $\sin(\omega_d t_r+\theta)=0$

$$\omega_d t_r+\theta=\pi$$

$$\boxed{t_r=\frac{\pi-\theta}{\omega_d}}\ \text{sec} \qquad \text{where, } \theta=\tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}$$

(3). Peak time : (t_p) :

It is time taken by the response of the s/s to reach $\max^m$ value.

At $t=t_p$; $c(t)=\max^m$ value

$$\frac{d}{dt}\,c(t)=0 \Rightarrow \frac{d}{dt}\left[1-\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\cdot\sin(\omega_d t+\theta)\right]=0$$

$$0-\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\cos(\omega_d t+\theta)\cdot\omega_d+\sin(\omega_d t+\theta)\cdot\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\cdot(-\xi\omega_n)=0$$

$$\sin(\omega_d t+\theta)\cdot\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\cdot\xi\omega_n=\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\cdot\omega_d\cdot\cos(\omega_d t+\theta)$$

$$\frac{\sin(\omega_d t+\theta)}{\cos(\omega_d t+\theta)}=\frac{\omega_d}{\xi\omega_n}=\frac{\omega_n\sqrt{1-\xi^2}}{\omega_n\cdot\xi}=\frac{\sqrt{1-\xi^2}}{\xi}$$

$$\tan(\omega_d t+\theta)=\tan\theta$$

$\tan(n\pi + \theta) = \tan\theta$

$\therefore \omega_d t = n\pi$

Since, $t = t_p$

$\omega_d t_p = n\pi$

$\boxed{t_p = \frac{n\pi}{\omega_d}}$ sec. where $n \in$ odd for overshoots
$n \in$ even for undershoots

$\therefore t_p = \pi/\omega_d$ sec. for 1^{st} overshoot

(4) <u>settling time (t_s):</u>

for 2% tolerance band, $t_s = 4T = \frac{4}{\xi\omega_n}$ sec

for 5% tolerance band, $t_s = 3T = \frac{3}{\xi\omega_n}$ sec

(5). <u>Maximum peak overshoot (M_p):</u>

$$C(t)\Big|_{t=t_p=\pi/\omega_d} = 1 - \frac{e^{-\xi\omega_n \frac{\pi}{\omega_d}}}{\sqrt{1-\xi^2}} \sin\left(\omega_d \cdot \frac{\pi}{\omega_d} + \theta\right)$$

$$= 1 + e^{-\frac{\xi\omega_n\pi}{\omega_n(1-\xi^2)^{1/2}}}$$

$$M_p = c(t)\Big|_{t=t_p} - 1$$

$$M_p = e^{-\xi\pi/\sqrt{1-\xi^2}} \simeq e^{-\xi n\pi/\sqrt{1-\xi^2}}$$

(6). <u>No. of cycles:</u>

$\omega_d = 2\pi f_d$

$f_d = \frac{\omega_d}{2\pi}$ cycles/sec.

2% tolerance band $\Rightarrow t_s \times f_d = \frac{4 f_d}{\xi\omega_n}$ (cycles)

5% TB $\Rightarrow$ $t_s \cdot f_d = \frac{3 f_d}{\xi \omega_n}$ (cycles)

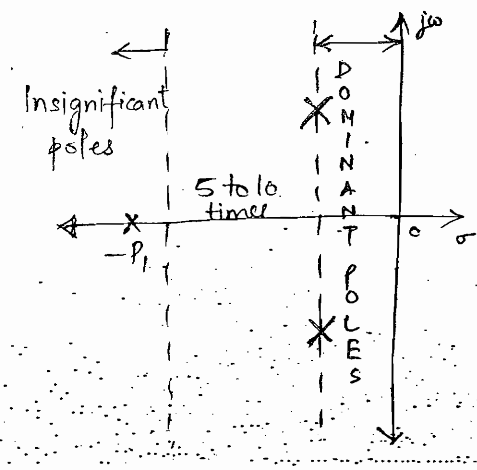
(7). Time period (or) Time interval of damped sinusoid:

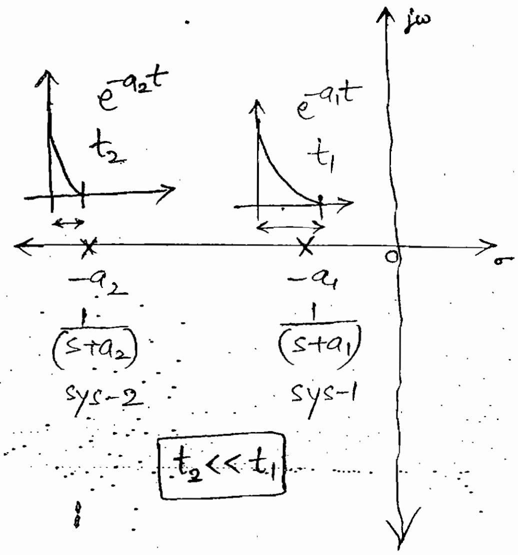
$$T = \frac{1}{f_d} \text{ sec}$$

Time Response analysis for Higher Order s/s:

Consider a 3rd order characteristic eqn, $s^3 + ps^2 + qs + K = 0$

factorize: $(s+P_1)(s^2 + q_1 s + K_1) = 0$





(1). The time response of higher order s/s is obtained by approximating to 2nd order s/s w.r.t dominant poles.

(2). The time domain specifications obtained for approximated lower order s/s are valid for the original s/s.

Q. The IInd order approx using dominate pole concept is

$$T(s) = \frac{5}{(s+5)(s^2+s+1)}$$

(a). $\frac{5}{s^2+s+1}$ ~~(b)~~ $\frac{1}{s^2+s+1}$ (c) $\frac{5}{(s+1)(s+5)}$ (d) $\frac{1}{(s+1)(s+5)}$

Sol. $T(s) = \frac{5}{(s+5)(s^2+s+1)}$

$$= \frac{5}{5\left(\frac{s}{5}+1\right)(s^2+s+1)} = \frac{1}{\left(\frac{s}{5}+1\right)(s^2+s+1)} \cong \frac{1}{(s^2+s+1)}$$

$\times$

Note: When approximating a higher order TF to a lower order TF, eliminate the insignificant poles after converting the TF in Time constant form.

conv Q5.

(1). $15\% < M_p < 30\%$

$$e^{-\pi\xi/\sqrt{1-\xi^2}} = 0.15 \Rightarrow \xi = 0.55$$

$$\theta = \cos^{-1}\xi = \cos^{-1}(0.55) = 57^\circ$$

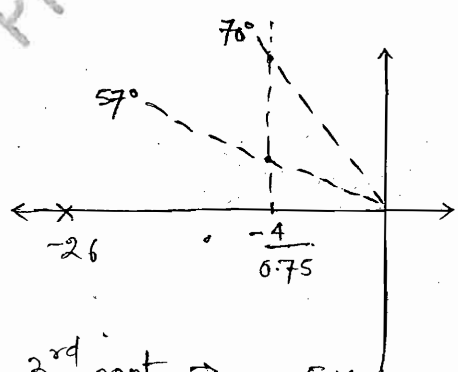
(2). $e^{-\pi\xi/\sqrt{1-\xi^2}} = 0.3 \Rightarrow \xi = 0.3$

$$\theta = \cos^{-1}(0.3) = 70^\circ$$

$t_s < 0.75$ sec

$t_s = 0.75$ sec.

$$\frac{4}{\xi\omega_n} = 0.7 \Rightarrow \xi\omega_n = \frac{4}{0.75}$$



(b). 3^{rd} root $\Rightarrow -5 \times \frac{4}{0.75} = -26.$

(c). $\% M_p = 30\%$

$$\xi = 0.35$$

$$t_s = 0.75$$

$$\frac{4}{\xi\omega_n} = 0.75$$

$$\frac{4}{0.35 \times \omega_n} = 0.75 \Rightarrow \omega_n = 15 \text{ rad/s}$$

$$s^2 + 2\xi\omega_n s + \omega_n^2$$

$$\Rightarrow s^2 + 2(0.35)(15)s + 15^2$$

$$\Rightarrow s^2 + 10.5s + 225$$

Now, char. eqn is

$$(s+26)(s^2 + 10.5s + 225) = 0$$

$$\underbrace{s^3 + 36.5s^2 + 498s} + 5850 = 0$$

$$1 + \frac{5850}{s^3 + 36.5s^2 + 498s} = 0$$

$$1 + G(s) = 0$$

$$G(s) = \frac{5850}{s(s^2 + 36.5s + 498)}$$

Q3. $G(s) = \dfrac{K(s+2)}{s^3 + \alpha s^2 + 4s + 1}$

$$1 + G(s) = 0$$

$$s^3 + \alpha s^2 + 4s + 1 + K(s+2) = 0$$

$$s^3 + \alpha s^2 + s(4+K) + 2K + 1 = 0 \quad \text{--- (1)}$$

Let $(s+a)(s^2 + bs + c) = 0$

and, $b = 2\xi\omega_n$, $c = \omega_n^2 = 9$

$$= 2(0.2)(3)$$

$$= 1.2$$

$$\therefore (s+a)(s^2 + 1.2s + 9) = 0$$

$s^3 + s^2(1.2+a) + s(9+1.2a) + 9a = 0$ —— (11)

$\alpha = 1.2 + a$

$4 + K = 9 + 1.2a$

$2K + 1 = 9a$

$2(9 + 1.2a - 4) + 1 = 9a$

$18 + 2.4a - 8 + 1 = 9a$

$11 = 6.6a \Rightarrow a = 2.8$

and $K = 9 + 1.2(2.8) - 4 \Rightarrow K = 7$

Q. The characteristic eqn of a control s/s is

$$s(s^2 + 6s + 13) + K = 0$$

The value of 'k' such that the char. eqn has a pair of complex conjugate roots whose real part is -1 will be

(a) 10 (b) 20 (c) 25 (d) 30

Sol. $s^3 + 6s^2 + 13s + K = 0$ —— (1)

$(s+a)(s^2 + bs + c) = 0$

$$s = \frac{-b \pm \sqrt{b^2 - 4c}}{2}$$

Real part $= \frac{-b}{2} = -1 \Rightarrow b = 2$

$(s+a)(s^2 + 2s + c) = 0$

$s^3 + 2s^2 + cs + as^2 + 2as + ac = 0$

$s^3 + s^2(2+a) + s(c+2a) + ac = 0$ —— (11)

$2 + a = 6 \Rightarrow a = 4$

$c + 2a = 13 \Rightarrow c = 5$

$ac = K \Rightarrow K = 20$

conv. Q2. $G(s) = \frac{K}{s(sT+1)}$

$1 + G(s) = 0$

$s(sT+1) + K = 0$

$Ts^2 + s + K = 0$

$s^2 + \frac{1}{T}\cdot s + \frac{K}{T} = 0$

$\omega_n = \sqrt{K/T}$ rad/s

$2\xi\sqrt{K/T} = \frac{1}{T}$

$\xi = \frac{1}{2\sqrt{KT}} \propto \frac{1}{\sqrt{K}}$

Case (1) : $\% M_p = 40\%$

$M_p = 0.4$

$e^{-\pi\xi/\sqrt{1-\xi^2}} = 0.4$

$\xi = \xi_1 = 0.28$

Let $K = K_1$

Case (2) : $\% M_p = 60\%$

$M_p = 0.6$

$e^{-\pi\xi/\sqrt{1-\xi^2}} = 0.6$

$\xi = \xi_2 = 0.16$

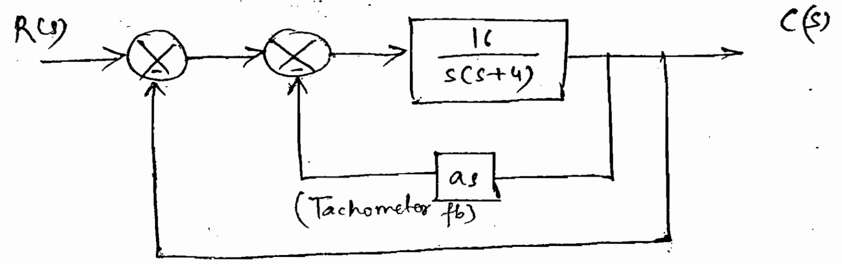
Let $K = K_2$

Now, $\frac{\xi_1}{\xi_2} = \sqrt{\frac{K_2}{K_1}}$

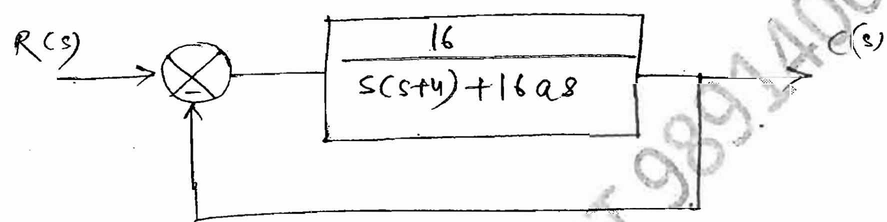
$\frac{K_2}{K_1} = \left(\frac{\xi_1}{\xi_2}\right)^2 = \left(\frac{0.28}{0.16}\right)^2 = 3$

$\therefore K_2 = 3K_1$

Conv.
Q1.



$$\frac{\frac{16}{s(s+4)}}{1+\frac{16as}{s(s+4)}} = \frac{16}{s(s+4)+16as}$$



(a). $\% M_p = 1.5\%$

$M_p = 0.015$

$e^{-\pi\xi/\sqrt{1-\xi^2}} = 0.015 \Rightarrow \xi = 0.8$

$1 + G(s) = 0$

$$1 + \frac{16}{s(s+4)+16as} = 0$$

$$s^2 + s(4+16a) + 16 = 0$$

$\omega_n = 4$ rad/s

$2\xi\omega_n = 4 + 16a$

$2(0.8)\times 4 = 4 + 16a$

$6.4 - 4 = 16a \Rightarrow a = 0.15$

(b). $e_{ss}\big|_{\text{unit ramp i/p}}$

(i). Without 'a' : $a = 0$

$$G(s) = \frac{16}{s(s+4)}$$

$$e_{ss} = \lim_{s\to 0} \frac{s \cdot \frac{1}{s^2}}{1 + \frac{16}{s(s+4)}} = \lim_{s\to 0} \frac{1}{s + \frac{16}{s+4}} = 0.25 \text{ units}$$

(ii). with 'a' : $a = 0.15$

$$G(s) = \frac{16}{s(s+4) + 16 \times 0.15 s}$$

$$= \frac{16}{s(s+6.4)}$$

$$e_{ss} = \lim_{s\to 0} \frac{s \cdot \frac{1}{s^2}}{1 + \frac{16}{s(s+6.4)}} = \lim_{s\to 0} \frac{1}{s + \frac{16}{(s+6.4)}} = 0.4 \text{ units}$$

(c). $G(s) = \frac{K}{s(s+4) + K \times 0.15 s}$

{Let s/s gain $= K$

$\because e_{ss} \propto \frac{1}{K}$ }

$$e_{ss} = \lim_{s\to 0} \frac{s \cdot \frac{1}{s^2}}{1 + \frac{K}{s(s+4) + K \times 0.15 s}}$$

$$0.25 = \frac{4 + 0.15K}{K}$$

$$0.25K = 4 + 0.15K$$

$$\Rightarrow K = 40$$

Checking : $1 + \frac{40}{s(s+4) + 40 \times 0.15 s}$

$$s^2 + 10s + 40 = 0$$

$$\Rightarrow \omega_n = \sqrt{40} = 6.32 \text{ rad/sec}$$

$$\Rightarrow \xi = \frac{10}{2 \times 6.32} = 0.8 \text{ (same)}$$

Effect of Tachometer fb:

Case (1): without Tachometer fb $(a=0)$:

$$G(s) = \frac{16}{s(s+4)}$$

$$1 + \frac{16}{s(s+4)} = 0 \Rightarrow s^2+4s+16 = 0$$

$$\omega_n = 4 \text{ rad/s}$$

$$\xi = \frac{4}{2\times 4} = 0.5 \text{ rad/s}$$

$$T = \frac{1}{0.5\times 4} = 0.5 \text{ sec}$$

$$\boxed{\begin{aligned} \omega_n &= 4 \text{ rad/s} \\ \xi &= 0.5 \\ T &= 0.5 \text{ sec} \end{aligned}}$$

Case (2): with Tachometer fb $a = 0.15$

$$G(s) = \frac{16}{s(s+4) + 16\times 0.15s}$$

$$= \frac{16}{s(s+6.4)}$$

$$1 + \frac{16}{s(s+6.4)} \Rightarrow s^2 + 6.4s + 16 = 0$$

$$\omega_n = 4 \text{ rad/s}$$

$$\xi = \frac{6.4}{2\times 4} = 0.8$$

$$T = \frac{1}{0.8\times 4} = 0.3 \text{ sec}$$

$$\boxed{\begin{aligned} \omega_n &= 4 \text{ rad/s} \\ \xi &= 0.8 \\ T &= 0.3 \text{ sec} \\ T\downarrow &;\ \xi\uparrow \end{aligned}}$$

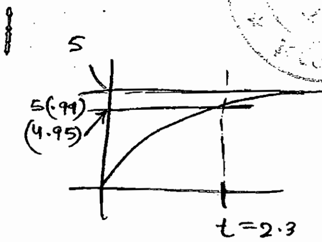
Q17. $H(s) = \dfrac{Y(s)}{X(s)} = \dfrac{1}{s+2}$

$x(t) = 10\, u(t) \Rightarrow X(s) = 10/s$

$Y(s) = \dfrac{10}{s(s+2)} = 5\left(\dfrac{1}{s} - \dfrac{1}{s+2}\right)$

$y(t) = 5(1-e^{-2t})\cdot u(t) = \dfrac{99}{100}\times 5 = 4.95$

$\Rightarrow \ln(e^{-2t}) = \ln(0.01) \Rightarrow t = 2.3$ sec.



Q13. $G(s) = \dfrac{10K}{s(1+0.1s)}$

(a). $e_{ss} < 2^\circ$

$1\text{ rps} \longrightarrow 2\pi \text{ rad/s}$

$\frac{1}{2}\text{ rps} \longrightarrow \pi \text{ rad/s}$

$180^\circ \longrightarrow \pi$

$0.2^\circ \longrightarrow ? \quad \left\{\dfrac{0.2}{180} \times \pi \text{ rad.}\right\}$

$$e_{ss} = \lim_{s\to 0} \frac{\cancel{s}\cdot \frac{\pi}{s^{\cancel{2}}}}{1 + \frac{10K}{s(1+0.1s)}} = \frac{0.2\cancel{\pi}}{180}$$

$$\frac{\pi}{10K} = \frac{0.2\pi}{180} \Rightarrow K = 90$$

(b). $1 + \dfrac{10(90)}{s(1+0.1s)} = 0$

$0.1s^2 + s + 900 = 0$

$s^2 + 10s + 9000 = 0$

$\therefore \omega_n = \sqrt{9000} = 95 \text{ rad/s}$

$\therefore \xi = \dfrac{10}{2 \times 95} \Rightarrow \xi = 0.05$

Q11. $\dfrac{d^2y}{dt^2} + \dfrac{3dy}{dt} + 2y = x(t)$

Short cut :
At $t=0$,
$y(t) = 0$.
(a) ✓

$(s^2+3s+2)\, Y(s) = X(s)$

$$\frac{Y(s)}{X(s)} = \frac{1}{s^2+3s+2}$$

$$Y(s) = \frac{1}{(s+1)(s+2)} \times \frac{2}{s}$$

$$Y(s) = \frac{2}{s(s+1)(s+2)} = \frac{1}{s} - \frac{2}{s+1} + \frac{1}{s+2}$$

$$y(t) = \left(1 - 2e^{-t} + e^{-2t}\right) u(t)$$

Q12. $$4\frac{d^2c(t)}{dt^2} + 8\frac{dc(t)}{dt} + 16\,c(t) = 16\,u(t)$$

$$(4s^2 + 8s + 16)\,C(s) = 16\,U(s)$$

$$\frac{C(s)}{U(s)} = \frac{16}{4s^2+8s+16} = \frac{4}{s^2+2s+4}$$

$$\omega_n = 2 \text{ rad/s}$$

$$\xi = \frac{2}{2\times 2} = 0.5$$

Q9. Unit step Response : $1 - e^{-5t} - 5t\,e^{-5t} \quad (t>0)$

$\frac{d}{dt}$(Step response) = Impulse response

$$\frac{d}{dt}\left(1 - e^{-5t} - 5t\,e^{-5t}\right) = 0 + 5e^{-5t} - 5(-5t\,e^{-5t} + e^{-5t})$$

$$= 25t\,e^{-5t}$$

$$\mathcal{L}(\text{Impulse Response}) = T.F = \frac{25}{(s+5)^2} = \frac{25}{s^2+10s+25}$$

$$\therefore \omega_n = 5 \text{ rad/s}$$

$$\therefore \xi = \frac{10}{2\times 5} = 1$$

Q8. $$\frac{H(s+c)}{(s+a)(s+b)}$$

(1). $u(t) \longrightarrow 2 + De^{-t} + Ee^{-3t}$

(2). $e^{2t}\,u(t) \longrightarrow F e^{-t} + G e^{-3t}$

(1). $$\frac{H(s+c)}{s(s+a)(s+b)} = \frac{K_1}{s} + \frac{K_2}{s+a} + \frac{K_3}{s+b}$$

$$= 2 + De^{-t} + Ee^{-3t}$$

$a = 1$
$b = 3$

$$\lim_{s\to 0} \frac{\cancel{s} H(s+c)}{\cancel{s}(s+a)(s+b)} = \lim_{t\to\infty} (2 + De^{-t} + Ee^{-3t})$$

$$\frac{Hc}{ab} = 2$$

$$Hc = 2ab = 6$$

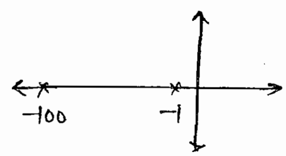
(2). $$\frac{H\cancel{(s+c)}}{\cancel{(s+2)}(s+a)(s+b)} \Leftrightarrow Fe^{-t} + Ge^{-3t}$$

$\therefore c = 2$

and $H = 3$

Q7: $$G(s) = TF = \frac{100}{(s+1)(s+100)}$$

$$= \frac{1}{(1+s)(1+s/100)}$$



$T_1 = 1$ sec. and $T_2 = \frac{1}{100} = 0.01$ sec

$\therefore$ Time constant $= T_1 = 1$ sec

2% of T.B. $\Rightarrow t_s = 4T_1 = 4(1) = 4$ sec

For overdamped s/s, dominant pole (time constant is considered).

Q6. $$1 + \frac{K}{s(s+1)} = 0$$

$\boxed{\xi \propto \frac{1}{\sqrt{K}}}$ general relation.

$$s^2 + s + K = 0$$

$$\omega_n = \sqrt{K} \text{ rad/s}$$

$$\xi = \frac{1}{2\sqrt{K}}$$

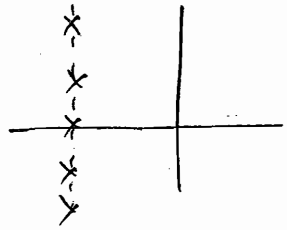
When $K = \infty$, $\xi = 0$

Q. The s/s is originally critically damped. If the gain is doubled, the it will become:

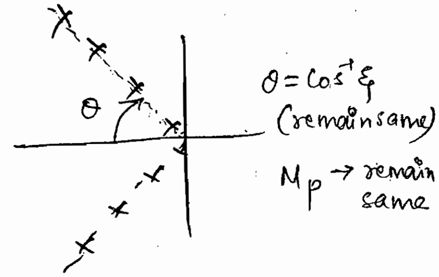
(a) Overdamped ✓(b) Underdamped (c) Undamped (d) Critically damped.

Q. Which of the following set of underdamped s/s will exhibit same overshoot.?

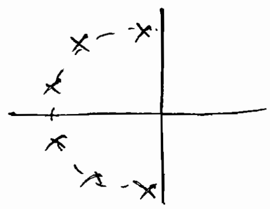
(a).



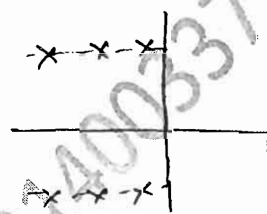
(b)



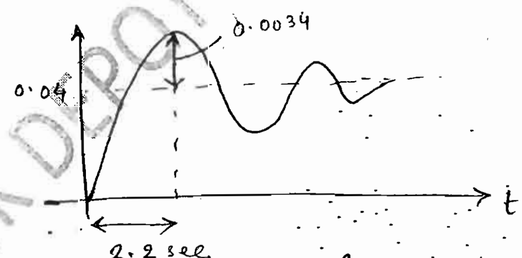
(c)



(d).



Q. The response of II order s/s is shown below. find ξ and ω_n ?



Sol. $M_p = 0.0034$

$e^{-\pi\xi/\sqrt{1-\xi^2}}$ ~~= 0.0034~~

is applicable only when i/p is unity. equivalent value has to be found if i/p is not unity.

$0.04 \longrightarrow 0.0034$

$1 \longrightarrow ?$

$\frac{1}{0.04} \times 0.0034 = 0.085$

$e^{-\xi\pi/\sqrt{1-\xi^2}} = 0.085 \Rightarrow \xi = 0.6$

And, $t_p = \frac{\pi}{\omega_d}$

$2.2 = \frac{\pi}{\omega_n (1-0.6^2)^{1/2}} \Rightarrow \omega_n = 1.8$ rad/s

Q10. $TF = \frac{16}{s^2+4s+16}$; $t_p = \frac{3\pi}{\omega_d} = \frac{3\pi}{4\sqrt{1-0.5^2}}$

$\omega_n = 4$ r/s , $\xi = 0.5$ $= \frac{1.5\pi}{\sqrt{3}}$ sec

25/03/14

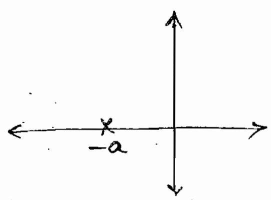
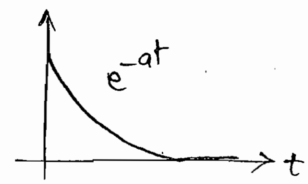
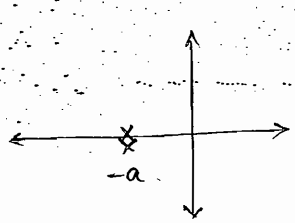
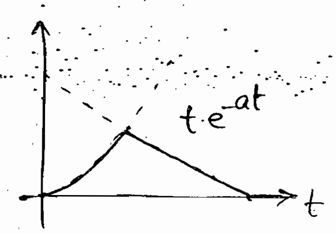
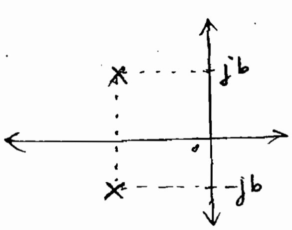
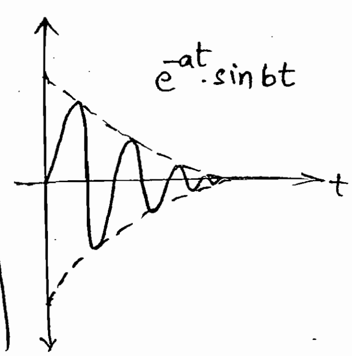
Part-III Stability

(1). The stability of LTI s/s may be defined as when the s/s is subjected to bounded i/p, the o/p should be bounded.

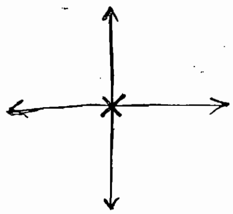
(2). BIBO implies the impulse response of the s/s should tend to zero as time $t \to \infty$.

(3). Stability of the s/s depends on roots of the char. eqn $[1 + G(s)H(s) = 0]$ i.e. closed loop poles.

Impulse Response and stability :

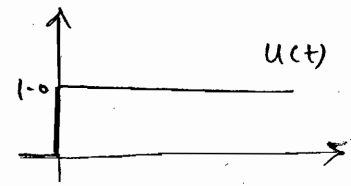
C.L Pole locations	Stability criteria	Impulse response
(1).  $F(s) = \frac{1}{s+a}$	Absolutely stable	
(2).  $F(s) = \frac{1}{(s+a)^2}$	Absolutely stable	
(3).  $F(s) = \frac{1}{(s+a)^2 + b^2}$	Absolutely stable	

(4).

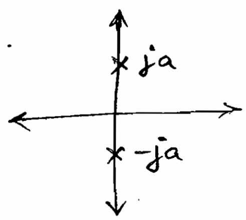


$F(s) = \frac{1}{s}$

Marginally (or) Critically stable

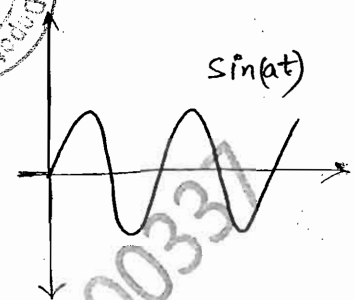


(5).

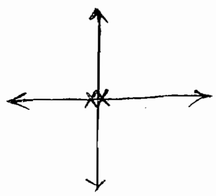


$F(s) = \frac{1}{s^2+a^2}$

Marginally (or) Critically stable

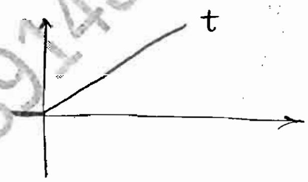


(6).

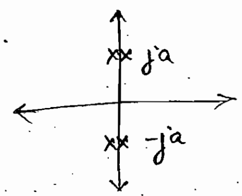


$F(s) = 1/s^2$

Unstable

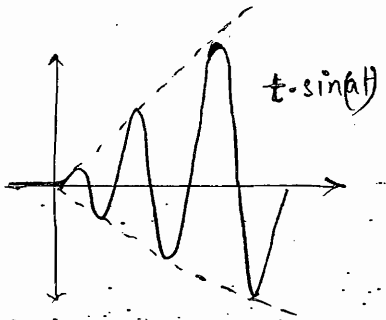


(7).

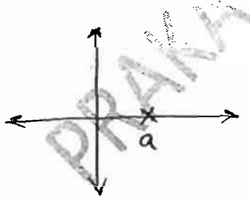


$F(s) = \frac{1}{(s^2+a^2)^2}$

Unstable

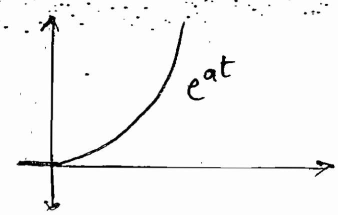


(8).

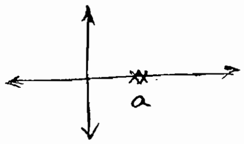


$F(s) = \frac{1}{(s-a)}$

Unstable

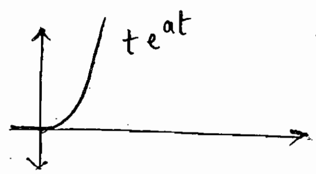


(9).

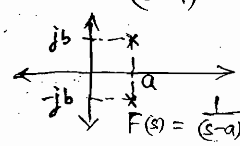


$F(s) = \frac{1}{(s-a)^2}$

Unstable

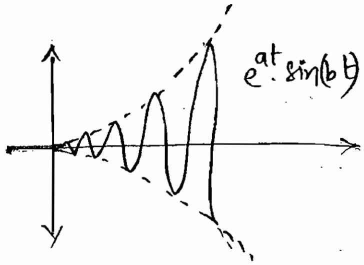


(10).



$F(s) = \frac{1}{(s-a)^2+b^2}$

Unstable



Q2. $P(s) = s^4 + 8s^3 + 15s^2 + 16s + 5 = 0$

Routh Hurwitz criteria:

Routh Array:

s^4	1	18	5
s^3	8	16	0
s^2	$b_1 = 16$	$b_2 = 5$	0
s^1	$c_1 = 13.5$	0	0
s^0	$d_1 = 5$	0	0

$$b_1 = \frac{8\times 18 - 1\times 16}{\boxed{8}} = 16 \quad ; \quad b_2 = \frac{8\times 5 - 1\times 0}{\boxed{8}} = 5$$

$$c_1 = \frac{16\times 16 - 8\times 5}{\boxed{16}} = 13.5$$

$$d_1 = \frac{13.5\times 5 - 16\times 0}{\boxed{13.5}} = 5$$

* sign of 1^{st} column only depends on 1^{st} column elements. That's why we take 1^{st} column only.

Difficulty - (i):

Q3.

s^5	1	2	3
s^4	1	2	15
s^3	ϵ	-12	0
s^2	$\frac{2\epsilon+12}{\epsilon}$ $(+\infty)$	15	0
s^1	$\frac{-24\epsilon - 144 - 15\epsilon^2}{2\epsilon+12}$ (-12)	0	0
s^0	15	0	0

To check for sign changes:

(i). $\lim_{\epsilon\to 0} \frac{2\epsilon+12}{\epsilon} = 2 + \frac{12}{\epsilon} = +\infty$

(ii). $\lim_{\epsilon\to 0} \frac{-24\epsilon - 144 - 15\epsilon^2}{2\epsilon+12} = -12$

2 sign changes:

$+\infty \rightarrow -12$

$-12 \rightarrow +15$

$\therefore$ 2 CLPoles in RHS of s-plane

Difficulty - (1) When the 1^{st} element of any row is zero while rest of the row has atleast one non-zero term, then in such cases, substitude a small +ve no. ϵ in place of zero and evaluate the rest of the Routh array. Check for sign changes in the 1^{st} column by taking limit $\epsilon \to 0$

Q4.

s^6	1	8	20	16
s^5	2	12	16	0
s^4	2	12	16	0
s^3	8	24	0	0
s^2	6	16	0	0
s^1	2.6	0	0	0
s^0	16	0	0	0

Construct an auxilliary eqn (A) :

$$A(s) = 2s^4 + 12s^2 + 16$$

$$\frac{dA(s)}{ds} = \underline{8}s^3 + \underline{24}s + 0$$

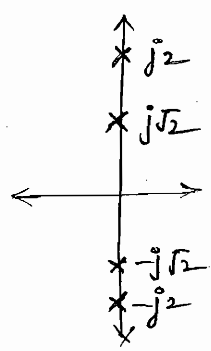
The roots of $A(s)$ = Poles lying on $j\omega$ axis

$$= \frac{-12 \pm \sqrt{144 - 8\times 16}}{4}$$

$$= -2, -4$$

$\therefore (s^2+2)(s^2+4) = 0$

$s = \pm j\sqrt{2}$ or $s = \pm j2$



$\therefore$ S/s is marginally stable

Roots (on $j\omega$ axis) = 4

Roots (LHS) = 2

Q5.

s^5	2	4	2
s^4	1	2	1
s^3	$\cancel{0}$ 4	$\cancel{0}$ 4	$\cancel{0}$ 0
s^2	1	1	0
s^1	$\cancel{0}$ 2	$\cancel{0}$ 0	$\cancel{0}$ 0
s^0	1	0	0

$A_1(s) = s^4 + 2s^2 + 1$

$\frac{d}{ds}(A_1(s)) = 4s^3 + 4s$

$A_2(s) = s^2 + 1$

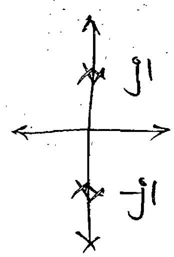
$\frac{d}{ds}(A_2(s)) = 2s$

The roots of $A_1(s) = \frac{-2 \pm \sqrt{4-4}}{2} = -1, -1$

$(s^2+1)(s^2+1) = 0 \Rightarrow s = \pm j1, \pm j1$

Roots ($j\omega$ axis) $= 4$

Roots (LHS) $= 1$



(Unstable)

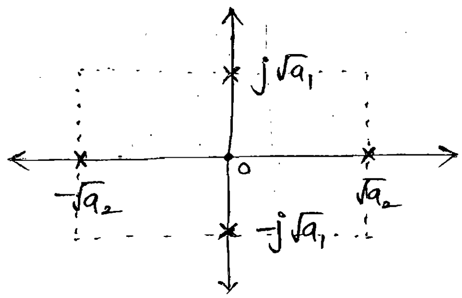
<u>Difficulty (2)</u> : When one complete row of routh array becomes zero, then in such cases, construct an auxilliary eqn $A(s)$, differentiate it to get new cofficients to complete the rest of the routh array.

Check for the multiplicity of auxilliary eqn roots on $j\omega$ axis to comment on stability.

Note: The roots of auxilliary eqn is always be symmetrical about origin

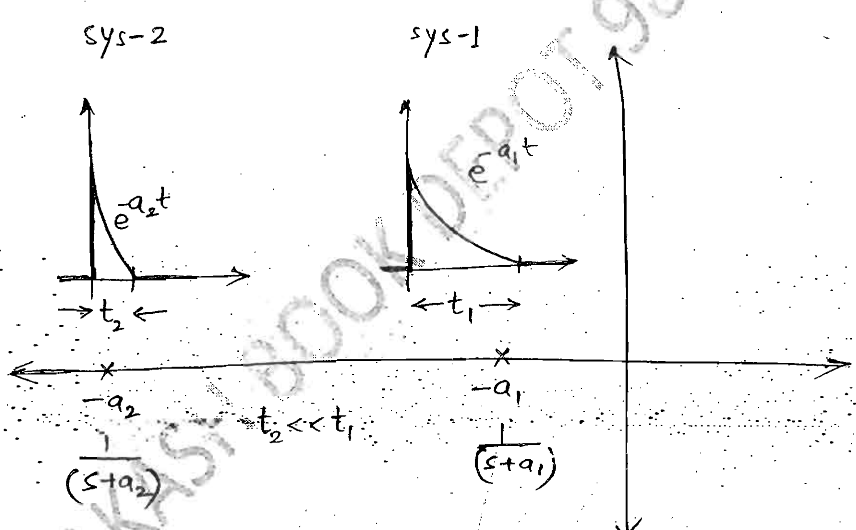
Suppose : $A(s) = (s^2+a_1)(s^2-a_2) = 0$

$$s = \pm j\sqrt{a_1} \;,\; \pm\sqrt{a_2}$$



All roots are symmetrical about origin.

Relative stability analysis using Routh Array :

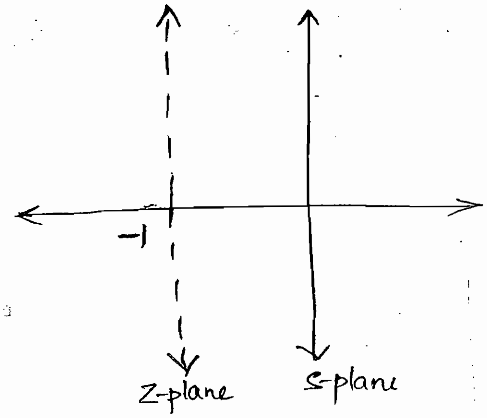


(1). Both s/s -1 and s/s -2 are said to be absolutely stable.

(2). s/s -2 is said to be relatively more stable than s/s -1 becoz $t_2 << t_1$.

Let $P(s) = s^3 + 7s^2 + 25s + 39 = 0$

Check whether the roots are lying more -vely w.r.to -1 ?



$$s+1 = z$$
$$s = z-1$$
$$P(z) = (z-1)^3 + 7(z-1)^2 + 25(z-1) + 39 = 0$$
$$= z^3 + 4z^2 + 14z + 20 = 0$$

z^3	1	14
z^2	4	20
z^1	9	0
z^0	20	0

Difficulty (3):

Relative stability analysis using Routh array is not feasible for higher order polynomials becoz it involves shifting of origin of s-plane more -vely

Q6. $s^5 + 15s^4 + 85s^2 + 274s + 120 = 0$

Sol. Put $s+1=0 \Rightarrow s=-1$

$$(-1)^5 + 15(-1)^4 + 85(-1)^2 + 274(-1) + 120 = 0$$

Hence, $s=-1$ is lying on given polynomial, remaing 4 roots are lying on LHS of $s+1=0$

→ If on putting $s=-1$ in polynomial, $P(s)$. we get $P(s)\big|_{s=-1} = +ve$, it implies that root is lying on LHS of $s+1=0$

→ If $P(s)\big|_{s=-1} = -ve$, it implies that root is lying on RHS of $s+1=0$.

Conditionally stable s/s -

A s/s is said to be conditionally stable, if its stability depends on more than one parameters.

Q7. $s^4 + 2s^3 + 3s^2 + 2s + K = 0$

s^4	1	3	K
s^3	2	2	0
s^2	2	K	0
s^1	$\frac{4-2K}{2}$	0	0
s^0	K	0	0

(1). $\frac{4-2K}{2} > 0$

$4-2K > 0$

$2K < 4$

$K < 2$

(2). $K > 0$

$\therefore 0 < K < 2$

Now, $K_{mar} = 2$

s^1 row $= 0$

$A(s) = 2s^2 + K_{mar}$

$= 2s^2 + 2 = 0$

$s^2 + 1 = 0 \Rightarrow s = \pm j1$

$j\omega_{mar} = \pm j\omega = \pm j1$

$\therefore \omega = 1$ rad/s.

Q9. $1 + \frac{K(s-2)^2}{(s+2)^2} = 0$

$(s+2)^2 + K(s-2)^2 = 0$

$s^2 + 4s + 4 + Ks^2 + 4K - 4Ks = 0$

$s^2(1+K) + s(4-4K) + 4K + 4 = 0$

s^2	$1+K$	$4K+4$
s^1	$4-4K$	0
s^0	$4+4K$	0

(1). $1+K > 0 \Rightarrow K > -1$

(2). $4-4K > 0$

$4K < 4 \Rightarrow K < 1$

But $K \geq 0$ (given).

$\therefore 0 \leq K < 1$

Q8. $1 + \dfrac{K}{(s^2+2s+2)(s+2)}$

$$(s^2+2s+2)(s+2)+K=0$$

$$s^3+2s^2+2s+2s^2+4s+4+K=0$$

$$s^3+s^2(4)+s(6)+4+K=0$$

s^3	1	6
s^2	4	4+K
s^1	$\frac{24-(4+K)}{4}$	0
s^0	4+K	0

(1). $\dfrac{24-(K+4)}{4} > 0$

$K < 20$

(2). $K+4 > 0$

$K > -4$

$\therefore \quad -4 < K < 20$

Now, $K_{max} = 20$

$$A(s) = 4s^2+(4+20)=0$$

$$4s^2+24=0$$

$$s^2+6=0$$

$$s=\pm j\sqrt{6}$$

$$\therefore \omega = \sqrt{6} \text{ rad/s.}$$

Short cut method: (3^{rd} order s/s):

$$s^3+4s^2+6s+4+K=0$$

(1). Product of external cofficients < Product of internal Coff. ⇒ stable

(2). Product of ext. coff. = Product of int. coff. ⇒ Marginaly ~~Unstal~~ stable

(3). Product of ext. coff. > Product of int. coff. ⇒ Unstable

$K+4=24 \Rightarrow K_{max}=20$

$A(s) = 4s^2+(4+K)=0$

$s=\pm j\sqrt{6} \Rightarrow \omega=\sqrt{6}$ rad/s.

Q10. $G(s) = \dfrac{K(s+1)}{s^3+as^2+2s+1}$

$1 + G(s) = 0$

$s^3 + as^2 + 2s + 1 + K(s+1) = 0$

$s^3 + as^2 + s(2+K) + K + 1 = 0$

$K+1 = a(K+2)$

$a = \dfrac{K+1}{K+2}$

$A(s) = as^2 + (K+1) = 0$

$as^2 = -(K+1)$

$s^2 = -\dfrac{(K+1)}{a}$

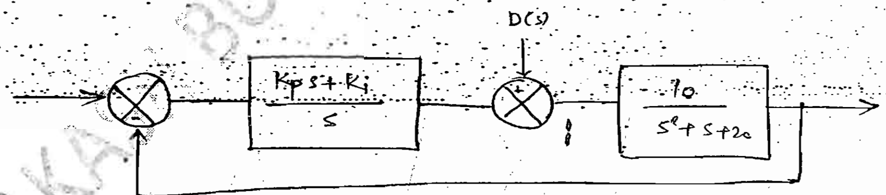
$s^2 = -(K+2)$

$s = \pm j\sqrt{K+2} = \pm j\omega = j2$

$\therefore \sqrt{K+2} = 2 \Rightarrow K = 2$

and $a = \dfrac{2+1}{4} = 0.75$

conv Q1.



$1 + \dfrac{10(K_p s + K_i)}{s(s^2+s+20)}$

$s^3 + s^2 + 20s + 10K_p s + 10K_i = 0$

$s^3 + s^2 + s(20+10K_p) + 10K_i = 0$

s^3	1	$20+10K_p$
s^2	1	$10K_i$
s^1	$20+10K_p-10K_i$	0
s^0	$10K_i$	0

(1). $10K_i > 0$
$K_i > 0$

(2). $20 + 10K_p - 10K_i > 0$
$K_p > K_i - 2$

conn Q2.

$$1 + \frac{K(s+2)^2}{s(s^2+1)(s+4)} = 0$$

$$s(s^2+1)(s+4) + K(s+2)^2 = 0$$

$$s^4 + 4s^3 + s^2(1+K) + s(4+4K) + 4K = 0$$

s^4	1	$1+K$	$4K$
s^3	4	$4+4K$	0
s^2	$\cancel{0}\,\epsilon$	$4K$	0
s^1	$\frac{(4+4K)\epsilon - 16K}{\epsilon}$	0	0
s^0	$4K$	0	0

Condition : (1) :

(1). If $\lim_{\epsilon \to 0}$ is applied to the 1st column elements, the s/s is unstable for all value of $K > 0$. (i.e. 2 sign changes)

Condition (2) : for stability :

(1) . $4K > 0 \Rightarrow K > 0$

(2). $$\frac{(4+4K)\epsilon - 16K}{\epsilon} > 0$$

$$4 + 4K - \frac{16K}{\epsilon} > 0$$

$$\frac{16K}{\epsilon} < 4 + 4K$$

$$\epsilon > \frac{16K}{4K+4}$$

If ϵ is depending on some parameter (like K).
ϵ is +ve no., then s/s will be stable in this condition.
Hence, s/s is conditionally stable.

Q3. $G(s) = \dfrac{K(s+\alpha)}{s(s+2)(s+4)^2}$

(a).

$$1 + \frac{K(s+\alpha)}{s(s+2)(s+4)^2} = 0$$

$$s(s+2)(s+4)^2 + K(s+\alpha) = 0$$

$$s^4 + 10s^3 + 32s^2 + (K+32)s + K\alpha = 0$$

s^4	1	32	$K\alpha$
s^3	10	$K+32$	0
s^2	$\dfrac{288-K}{10}$	$K\alpha$	0
s^1	$\dfrac{\dfrac{(288-K)(K+32)}{10} - 10K\alpha}{\dfrac{288-K}{10}}$	0	0
s^0	$K\alpha$	0	0

(1) $\dfrac{288-K}{10} > 0 \Rightarrow K < 288$

(2) $\dfrac{\dfrac{(288-K)(K+32)}{10} - 10K\alpha}{\left(\dfrac{288-K}{10}\right)} > 0$

$$(288-K)(K+32) - 100K\alpha > 0$$

$$K\alpha < \frac{(288-K)(K+32)}{100}$$

$$\therefore\ 0 < K\alpha < \frac{(288-K)(K+32)}{100}$$

(b). Let $K = 200$

$$K\alpha = \frac{(288-200)(200+32)}{100} = 1$$

$$G(s) = \frac{200(s+1)}{s(s+2)(s+4)^2}$$

$$e_{ss} = \lim_{s\to 0} \cdot \frac{s \cdot \frac{1}{s^2}}{1 + \frac{200(s+1)}{s(s+2)(s+4)^2}}$$

$$= \frac{16 \times 2}{200} = 0.16 \quad \text{i.e. } 16\% \quad \in (15, 20)\%.$$

Hence, values choosen are correct. i.e. for $K = 200, \alpha = 1$.

Root Locus -

(1). The root locus is defined as locus of C.L. poles obtained when s/s gain K is varied from 0 to ∞.

(2). Root locus plots determine the relative stability of a s/s w.r.to variations in s/s gain K.

Angle and Magnitude conditions:

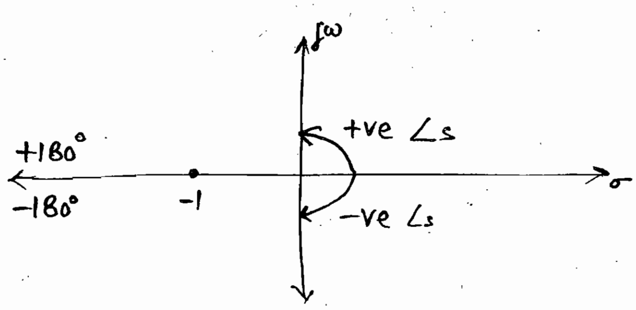
(1). The angle condition is used for checking whether any point lying on root locus or not and also the validity of RL shape for C.L. poles.

$$1 + G(s)H(s) = 0$$

$$G(s)H(s) = -1 + j0$$

$$\angle G(s)H(s) = 180^\circ - \tan^{-1}\left(\frac{0}{1}\right) = 180^\circ \approx \pm 180^\circ$$

$$\therefore \angle G(s)H(s) = \pm(2q+1)180^\circ$$



(s-plane)

(2). The $\angle$ condition may be stated as for a point to lie on RL, the $\angle$ evaluated at that point must be an odd multiple of $\pm 180°$.

(3). The magnitude condition is used for finding the value of s/s gain k at any point on RL.

$$G(s)H(s) = -1 + j0$$

$$|G(s)H(s)| = |-1 + j0|$$

$$|G(s)H(s)| = \sqrt{(-1)^2 + 0^2}$$

$$\boxed{\therefore |G(s)H(s)| = 1}$$

Q4. $s_1 = -3 + j4$

$s_2 = -3 - j2$

$$G(s)H(s) = \frac{K}{(s+1)^4}$$

$$G(s)H(s)\Big|_{s_1 = -3+j4} = \frac{K}{(-3+j4+1)^4} = \frac{K+j0}{(-2+j4)^4} = M_1 \angle -466°$$

$$G(s).H(s)\Big|_{s_2 = -3-j2} = \frac{K}{(-3-j2+1)^4} = \frac{K+j0}{(-2-j2)^4} = M_2 \angle +540°$$

As $540°$ is odd multiple of $180°$

$\therefore s_2$ lies on RL and s_1 does not lie on RL.

Q5. $s = -1 + j1$

$$G(s) = \frac{K}{s(s^2+7s+12)}$$

$$G(s)\Big|_{s=-1+j1} = \frac{K}{(-1+j1)[(-1+j1)^2 + 7(-1+j1) + 12]} = \frac{K+j0}{(j-1)(5+5j)}$$

$$= \frac{K}{\sqrt{2}\cdot 5\sqrt{2}} \angle -180°$$

$$|G(s)| \angle G(s) = \frac{K}{10} \angle -180^\circ$$

$$\therefore |G(s)| = \frac{K}{10}$$

$$1 = K/10 \Rightarrow K = 10$$

Construction Rules of RL :

Rule (1) : The root locus is symmetrical about Re-axis.
$[G(s) H(s) = -1]$.

Rule (2) : Let P = No. of open loop poles
Z = No. of open loop zeroes

Since, $P > Z$

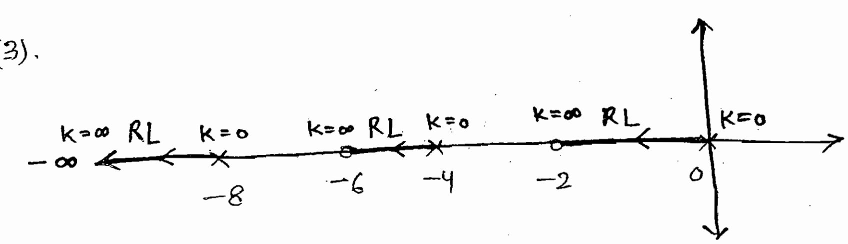
Then, No. of Branches of RL $= P$
No. of Branches terminating at zeroes $= Z$
No. of branches terminating at $\infty = P - Z$

Rule (3) : A point on Re-axis is said to be on RL if to the right side of this point, the sum of open loop poles and zeroes is odd.

Eg. $G(s) = \dfrac{K(s+2)(s+6)}{s(s+4)(s+8)}$

(2). $P = 3$
$Z = 2$
$P - Z = 1$

(3).



$$\frac{G(s)}{1+G(s)} = \frac{K(s+2)(s+6)}{s(s+4)(s+8) + K(s+2)(s+6)}$$

C.L. poles $\Rightarrow$ char. eqn.

$$s(s+4)(s+8) + K(s+2)(s+6) = 0$$

when $K = 0$,

$$s(s+4)(s+8) = 0$$

$$\Rightarrow s = 0, -4, -8.$$

$$s(s+4)(s+8) = -K(s+2)(s+6)$$

$$(s+2)(s+6) = \frac{s(s+4)(s+8)}{-K}$$

when $K = \infty$,

$$(s+2)(s+6) = 0$$

$$\Rightarrow s = -2, -6$$

Rule (4): Angle of Asymptotes

P–Z Branches terminating at ∞ will go along certain straight lines known as asymptotes of RL. Therefore no. of asymptotes is equal to P–Z.

$$\theta = \frac{(2q+1)180^\circ}{P-Z}, \quad q = 0, 1, 2, 3, \ldots$$

Eg. $P-Z = 2$

$$\theta_1 = \frac{[2(0)+1]180^\circ}{2} = 90^\circ$$

$$\theta_2 = \frac{[2(1)+1]180^\circ}{2} = 270^\circ$$

Q6. $s(s+4)(s^2+2s+\cancel{5}) + K(s+1) = 0$

$$1 + \frac{K(s+1)}{s(s+4)(s^2+3s\cancel{+8})} = 0$$

$$\therefore G(s)H(s) = \frac{K(s+1)}{s(s+4)(s^2+3s\cancel{+8})} = \frac{K(s+1)}{s^2(s+4)(s+3)}$$

$P = 4$; $Z = 1$, $P - Z = 3$

$$\theta_1 = \frac{[2(0)+1]\,180^\circ}{3} = 60^\circ$$

$$\theta_2 = \frac{[2(1)+1]\,180^\circ}{3} = 180^\circ$$

$$\theta_3 = \frac{[2(2)+1]\,180^\circ}{3} = 300^\circ$$

$$\angle \text{ b/w 2 Asymptotes} = \frac{2\pi}{(P-Z)}$$

Rule (5): centroid

It is the intersection point of asymptotes on the Re-axis

It may or may not be a part of RL

$$\text{Centroid} = \frac{\sum \text{Real part of open loop poles} - \sum \text{Real part of open loop zeroes}}{P-Z}$$

Q7. $s^3 + 5s^2 + (K+6)s + K = 0$

$s^3 + 5s^2 + 6s + K(s+1) = 0$

$$1 + \frac{K(s+1)}{(s^3+5s^2+6s)} = 0$$

$$\therefore G(s)H(s) = \frac{K(s+1)}{s(s^2+5s+6)}$$

$$= \frac{K(s+1)}{s(s+3)(s+2)}$$

Zeroes at $s = -1 + j0$

$\underline{-1}$

Poles at $s = 0 + j0$

$= -2 + j0$

$= -3 + j0$

$\underline{-5}$

$$\text{Centroid} = \frac{-5-(-1)}{2} = -2 \Rightarrow (-2, 0).$$

Rule (6): Break away points

They are those points where multiple roots of char. eqn occur

Procedure:

(1). Contruct $1+G(s)H(s)=0$

(2). Write 'K' in terms of 's'.

(3). Find $dK/ds=0$.

(4). The roots of $dK/ds=0$ will give B.A points

(5). To test valid B.A point substitute in step(2).
If $K=+ve \Rightarrow$ valid B.A points.

General predictions about B.A. points:

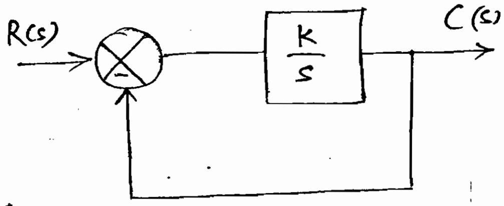
(1). The branches of RL either approach or leave the B.A points at an ∠ of $\pm 180°/n$ where n = no. of branches approaching or leaving the B.A. point.

(2). ~~Whenever~~ The complex conjugate part for the branches of RL approaching or leaving the B.A point is a circle.

(3). Whenever there are 2 adjacently placed poles on the Re-axis ~~at~~ with the section of Re-axis b/w them as a part of RL, there exist a B.A. point b/w the adjacently placed poles.

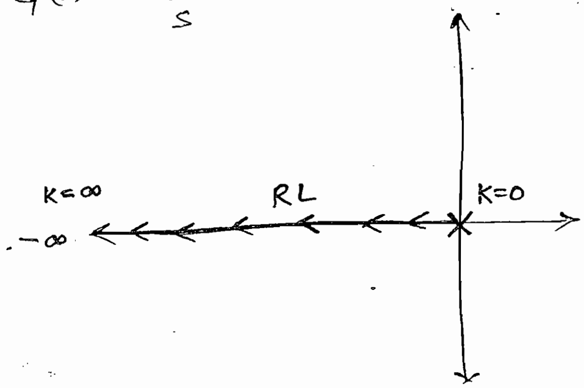
(4). Whenever there is a zero on Re-axis and to the left side of that zero, if there are no poles and zeroes on Re-axis with the entire section of Re-axis to the LHS of that zero as a part of RL, then there exist a B.A point to the LHS of that zero.

First Order s/s -

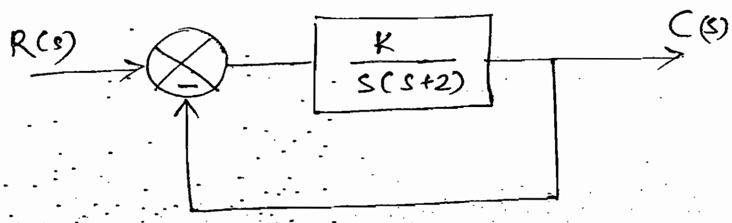


$$\frac{C(s)}{R(s)} = \frac{K}{s+K}$$

$$G(s) = \frac{K}{s}$$



Second Order s/s -

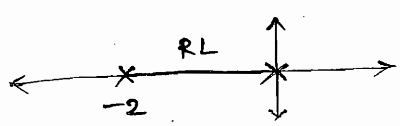


$$\frac{C(s)}{R(s)} = \frac{K}{s^2+2s+K}$$

$$G(s) = \frac{K}{s(s+2)}$$

(2). $P = 2$; $Z = 0$; $P - Z = 2$

(3).



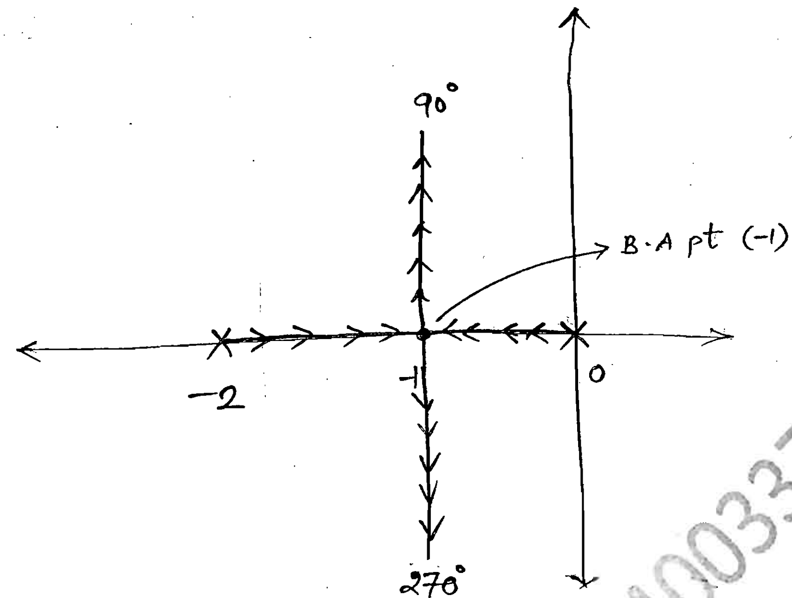
(4). $\theta_1 = 90^\circ$; $\theta_2 = 270^\circ$

(5). Centroid $= \dfrac{0 + (-2) - 0}{2} = -1$ i.e. $(-1, 0)$

(6). B.A points : $s^2 + 2s + K = 0$

$$K = -s^2 - 2s$$

$$\frac{dK}{ds} = -2s - 2 = 0 \Rightarrow s = -1$$



3rd Order s/s -

(Effect of adding poles to a TF) :

$$G(s) = \frac{K}{s(s+2)(s+4)}$$

(2) · $P = 3$; $Z = 0$; $P - Z = 3$

(3)

(4) · $\theta_1 = 60°$, $\theta_2 = 180°$, $\theta_3 = 300°$

(5) · centroid $= \dfrac{0 + (-2) + (-4) - 0}{3} = -2$

(6) · B-A points

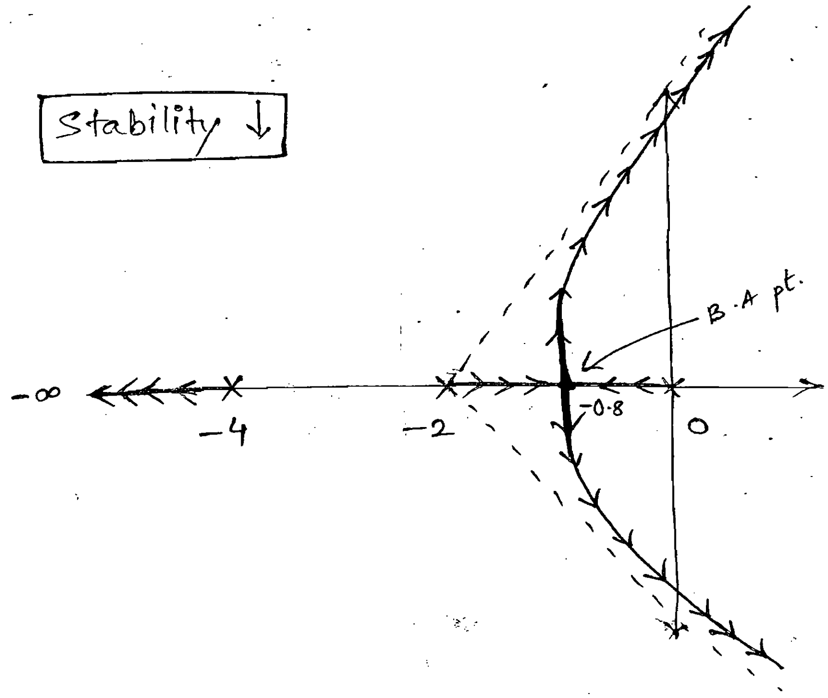
$$s^3 + 6s^2 + 8s + K = 0$$

$$K = -s^3 - 6s^2 - 8s$$

$$\frac{dK}{ds} = 3s^2 + 12s + 8 = 0$$

$$\therefore \quad s = -0.8, \ -3.15$$

* Stability of higher s/s ↓ as we ↑ the the order of s/s.

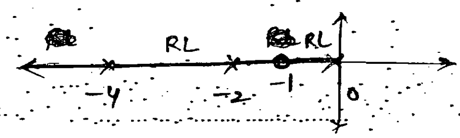


Effect of adding zeroes to a TF :

$$G(s) = \frac{K(s+1)}{s(s+2)(s+4)}$$

(2). $P=3$; $Z=1$; $P-Z=2$

(3).

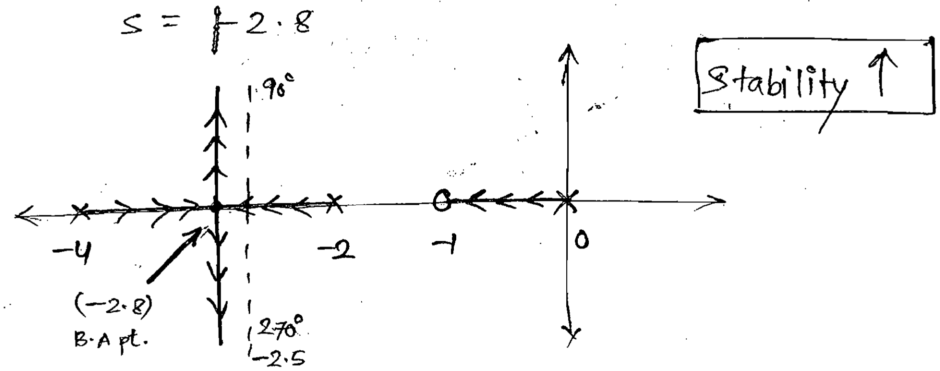


(4). $\theta_1 = 90°$, $\theta_2 = 270°$

(5). $\dfrac{0+(-2)+(-4)-(-1)}{2} = -2.5$

(6). B-A points

$s = -2.8$



* It's impossible to have for any s/s

$Z > P$

Becoz it is absolutely 100% stable and 100% accurate that is not possible practically.

Hence, at most, $Z = P$

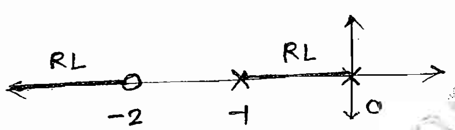
otherwise for most of the s/s's, $Z < P$.

Prediction-(4):

$$G(s) = \frac{K(s+2)}{s(s+1)}$$

(2). $P = 2$; $Z = 1$; $P - Z = 1$

(3).



(6). B·A points:

$$s(s+1) + K(s+2) = 0$$

$$K = \frac{-s^2 - s}{(s+2)}$$

$$\frac{dK}{ds} = \frac{(s+2)(-2s-1) - (-s^2-s)(1)}{(s+2)^2} = 0$$

$$-2s^2 - s - 4s - 2 + s^2 + s = 0$$

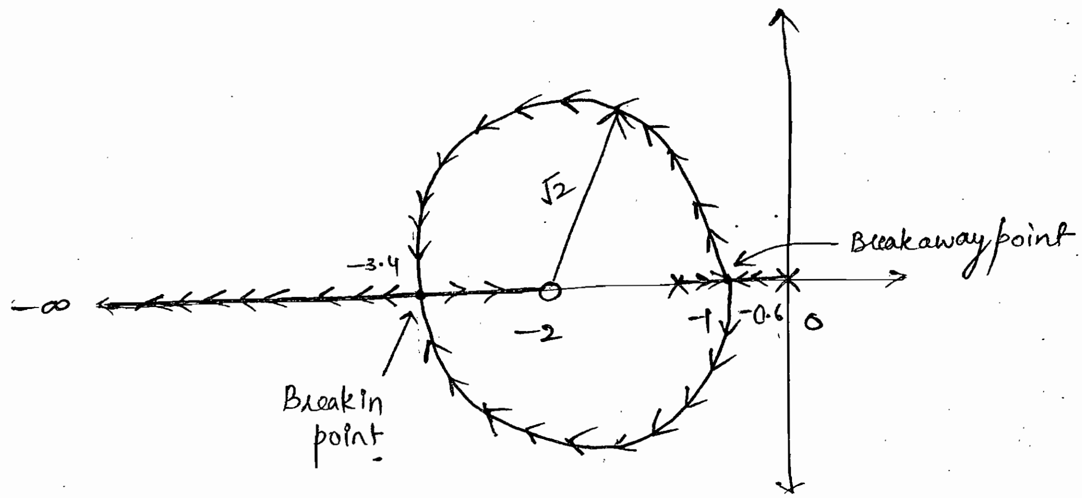
$$-s^2 - 4s - 2 = 0$$

$$s^2 + 4s + 2 = 0$$

$$s = \frac{-4 \pm \sqrt{16-8}}{2}$$

$s = -2 \pm \sqrt{2}$ (Centre ± Radius)

$s = -0.6, -3.4$



$$G(s) = \frac{K(s+b)}{s(s+a)}$$

Let $s = x + jy$

$$G(s) = \frac{K(x+jy+b)}{(x+jy)(x+jy+a)}$$

$$= \frac{K[(x+b)+jy]}{x^2+jxy+jxy-y^2+ax+jay}$$

$$= \frac{K[(x+b)+jy]}{(x^2-y^2+ax)+j(2xy+ay)}$$

$$\tan^{-1}\left(\frac{y}{x+b}\right) - \tan^{-1}\left(\frac{2xy+ay}{x^2+ax-y^2}\right) = 180^\circ$$

$$\frac{y}{x+b} - \frac{ay+2xy}{x^2-y^2+ax} = 0$$

$$x^2-y^2+ax - (x+b)(a+2x) = 0$$

$$x^2-y^2+\cancel{ax}-\cancel{ax}-ab-2x^2-2bx = 0$$

$$-x^2-y^2-ab-2bx = 0$$

$$x^2+y^2+2bx = -ab$$

$$x^2+y^2+2bx+b^2 = -ab+b^2$$

$$(x+b)^2+y^2 = b(b-a)$$

Centre $\rightarrow (-b,0)$

Radius $\rightarrow \sqrt{b(b-a)}$

Eg. $a=1$, $b=2$ $\therefore$ centre $(-2,0)$

Radius $= \sqrt{2(2-1)} = \sqrt{2}$ units

$$\boxed{\text{B.A points} = [\text{Centre} \pm \text{Radius}]}$$

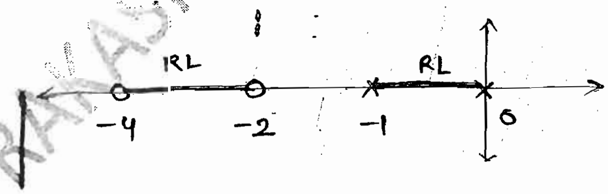
Prediction (5):

Whenever there are 2 adjacently placed zeroes on Re-axis with the section of Re-axis b/w them as Root locus then there exist a B.A point b/w the adjacently placed zeroes.

$$G(s) = \frac{K(s+2)(s+4)}{s(s+1)}$$

(2). $P=2$; $Z=2$; $P-Z=0$

(3).



(6). B. A points :

$$s(s+1)+K(s^2+6s+8)=0$$

$$K = \frac{-s^2-s}{s^2+6s+8}$$

$$\frac{dk}{ds}=0 \Rightarrow \frac{(s^2+6s+8)(-2s-1)+(s^2+s)(2s+6)}{(s^2+6s+8)} = 0$$

$$5s^2+16s+8=0$$

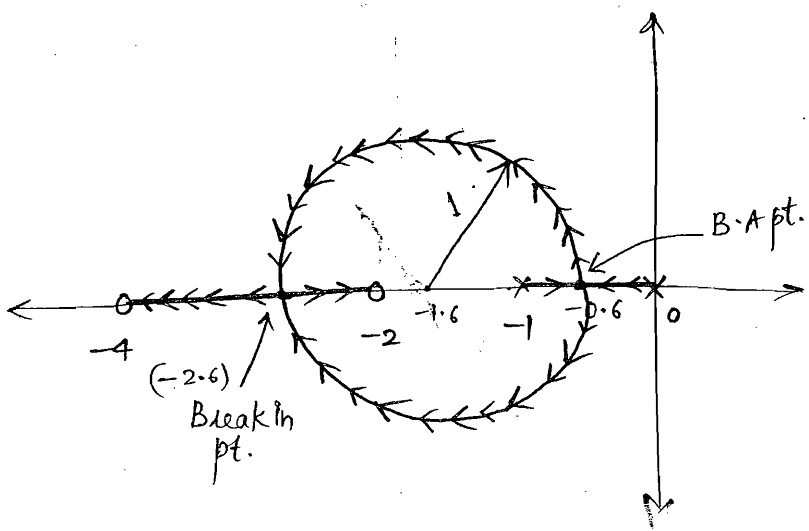
$$s = \frac{-16 \pm \sqrt{256-160}}{10} = -1.6 \pm 1$$

$\Rightarrow s = -0.6, -2.6$

Centre, ± radius

Centre → -1.6

Radius → 1



Rule (7):

Intersection of RL with imaginary axis, the roots of auxilliary eqn at $K = K_{max}$ from Routh array gives the intersection of RL with imaginary axis.

$$G(s) = \frac{K}{s(s+2)(s+4)} \Rightarrow s^3 + 6s^2 + 8s + K = 0$$

(7).

s^3	1	8
s^2	6	K
s^1	$\frac{48-K}{6}$	0
s^0	K	0

(1). $48 - K > 0$

$K < 48$

(2). $K > 0$

$K_{max} = 48$

$$A(s) = 6s^2 + 48 = 0$$

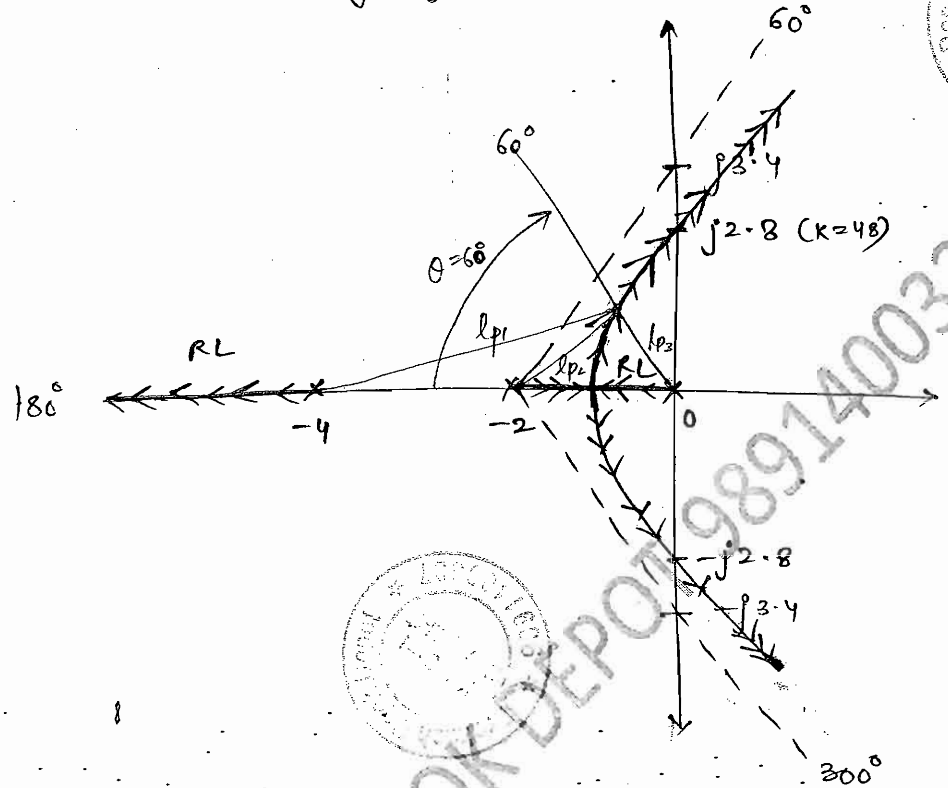
$$s^2 + 8 = 0$$

$$s = \pm j\sqrt{8} = \pm j2.8$$

Intersection of asymptotes with $j\omega$ axis-

$\tan\theta = y/x = \frac{y}{2} = \tan 60^\circ = \sqrt{3} \Rightarrow y = 2\sqrt{3}$

$\therefore y = \pm j3.4$



<u>Short cut</u> : $G(s) = \frac{K}{s(s+a)(s+b)}$

Intersection of RL with $j\omega$ axis $= \pm j\sqrt{ab}$

Q. find 'K' at $\xi = 0.5$ from RL ?

$\theta = \cos^{-1}\xi = \cos^{-1}(0.5) = 60^\circ$

$K = \frac{\text{Product of vector lengths of poles}}{\text{Product of vector lengths of zeroes}} = \frac{l_{p_1} \times l_{p_2} \times l_{p_3}}{1}$

Graphical method to find K.

Rule (8) : Angle of departure and arrival

(1). The angle of departure is obtained when complex poles terminate at ∞.

(2). The angle of arrival is obtained when poles terminate at complex zeroes.

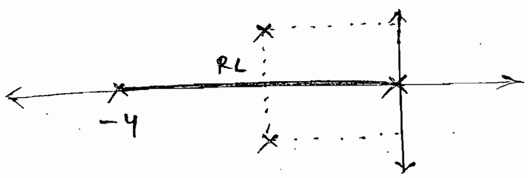
$$\phi_D = 180^\circ + \phi \qquad \text{where } \phi = \sum \phi_z - \sum \phi_P$$

$$\phi_A = 180^\circ - \phi$$

Q8. $G(s)H(s) = \dfrac{K}{s(s+4)(s^2+4s+20)}$

(2). $P = 4$; $Z = 0$; $P - Z = 4$

(3).



(4). ~~con~~ $\theta_1 = 45^\circ$, $\theta_2 = 135^\circ$, $\theta_3 = 225^\circ$, $\theta_4 = 315^\circ$

(5). centroid $= \dfrac{0 + (-2) + (-2) + (-4) - 0}{4} = -2$

Short cut method:

Avg. value of Re poles $= \dfrac{0 + (-4)}{2} = -2$

If, Avg value of Re poles = Real part of complex poles

there will be 3 B.A points

If, Avg value of Re poles $\neq$ Real part of complex poles

there will be only one B.A point.

Nature of B.A points.

Absolute value of avg. value of Re poles $\Rightarrow$ $2 \times x = 20$

$x = 10$

(1). $x \geq 5 \Rightarrow$ There will be one real & 2 complex B.A points.

(2). $x < 5 \Rightarrow$ There will be 3 Real B.A points.

Q8. Avg. value of Real poles $= \frac{0+(-4)}{2} = -2$

Real part of complex zero

$$= -b/2$$
$$= -\frac{4}{2} = -2$$

$\therefore$ There will be 3 B.A points.

Q9. Avg value of Real poles $= \frac{0+(-4)}{2} = -2$

Real part of complex zero $= -b/2 = -2$

There will be 3 B.A points.

Nature of B.A points:

absolute value $= |-2| = 2$

$2 \times x = 5 \Rightarrow x = 2.5 < 5$ (3 Real B.A points)

(6). B.A points:

$$K = -s^4 - 8s^3 - 36s^2 - 80s$$

$$\frac{dK}{ds} = 4s^3 + 24s^2 + 72s + 80$$

$$s = -2, \ -2 \pm j2.45$$

<u>Note</u>: Use angle condition to check the validity of complex B.A points.

$$G(s)\Big|_{s=-2+j2.45} = \frac{K+j0}{(-2+j2.45)(2+j2.45)(0+j6.45)(0-j1.55)}$$

$$= -180^\circ \quad \therefore \text{Valid.}$$

(7). $s^4 + 8s^3 + 36s^2 + 80s + K = 0$

s^4	1	36	K
s^3	8	80	0
s^2	26	K	0
s^1	$\frac{2080-8K}{26}$	0	0
s^0	K	0	0

(1). $2080 - 8K > 0$
$8K < 2080$
$K < 260$
$\therefore K_{max} = 260$

(2). $K > 0$

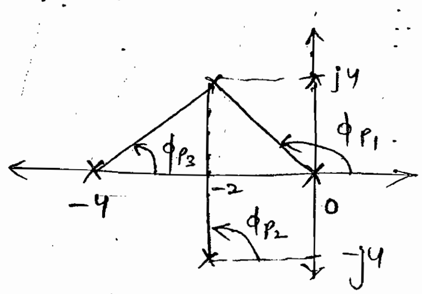
$$A(s) = 26s^2 + K_{max} = 0$$
$$26s^2 + 260 = 0$$
$$s^2 + 10 = 0 \Rightarrow s = \pm j\,3.16$$

Intersection of Asymptotes with $j\omega$ axis:

$y = \tan 45^\circ \times 2 = 2 = \pm j2$

(8). Angle of departure:

$\phi_{P_1} = 180^\circ - \tan^{-1}\left(\frac{4}{2}\right)$
$= 180^\circ - \tan^{-1}(2)$
$= 180^\circ - 63.4^\circ$
$= 116.6^\circ$



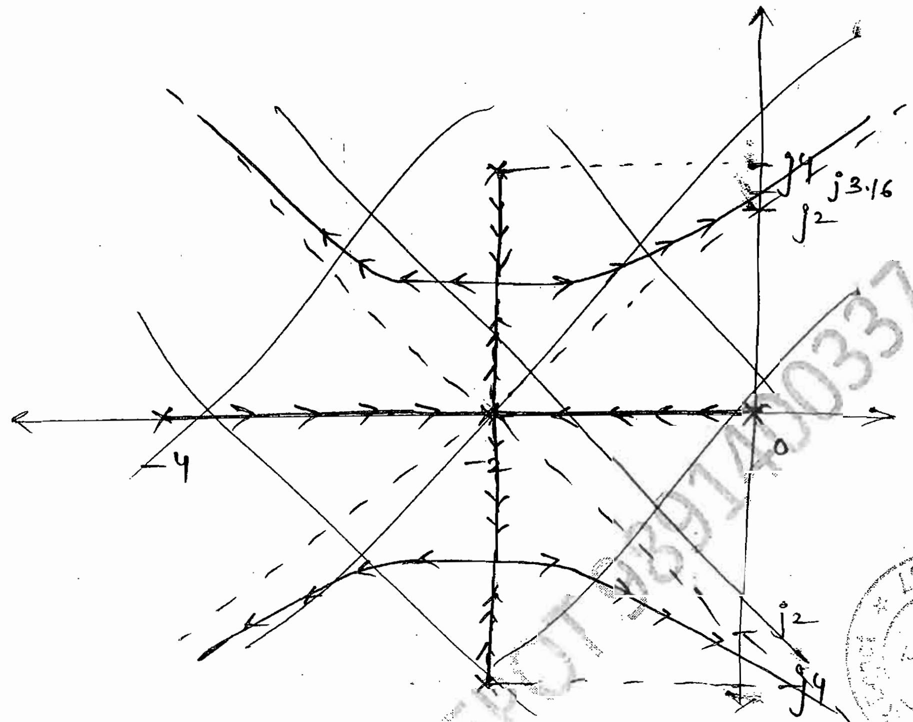
$\phi_{P_2} = 90^\circ$

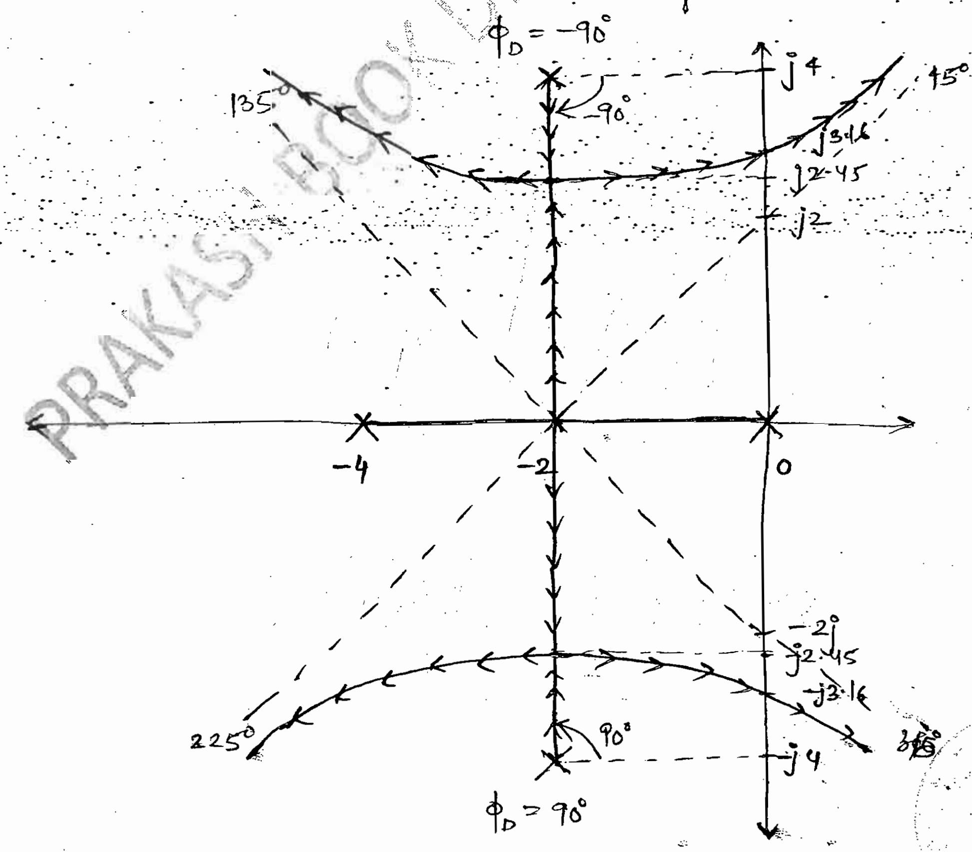
$\phi_{P_3} = \tan^{-1}\left(\frac{4}{2}\right) = \tan^{-1}(2) = 63.4^\circ$

$\phi_z = 0^\circ$

$\phi = \Sigma\phi_z - \phi_P$
$= 0^\circ - (116.6^\circ + 63.4^\circ + 90^\circ)$
$= -270^\circ$

$$\phi_D = 180^\circ + \phi$$
$$= 180^\circ - 270^\circ$$
$$= -90^\circ$$

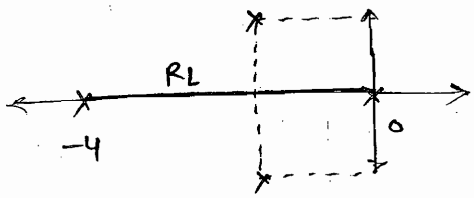




Q9: $G(s) = \dfrac{K}{s(s+4)(s^2+4s+5)}$

(2). $P = 4$, $Z = 0$; $P - Z = 4$

(3).



(4). $\theta_1 = 45^\circ$, $\theta_2 = 135^\circ$, $\theta_3 = 225^\circ$, $\theta_4 = 300^\circ$.

(5). centroid $= \dfrac{0-2-2-4-0}{4} = -2$

(6). B.A points.

$s^4 + 8s^3 + 21s^2 + 20s + K = 0$

$K = -s^4 - 8s^3 - 21s^2 - 20s$

$\dfrac{dK}{ds} = 4s^3 + 24s^2 + 42s + 20 = 0$

$s = -2, -0.78, -3.2$

(7).

s^4	1	21	K
s^3	8	20	0
s^2	18.5	K	0
s^1	$\dfrac{370-8K}{18.5}$	0	0
s^0	K	0	0

$370 - 8K > 0$

$8K < 370$

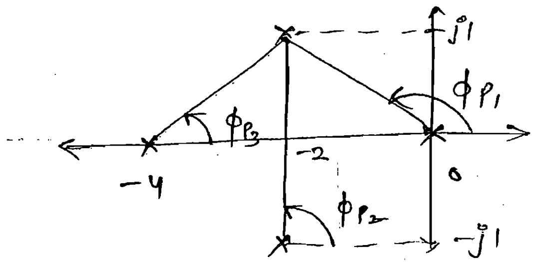
$K < 46.25 \Rightarrow K_{max} = 46.25$

$A(s) = 18.5s^2 + 46.25 = 0$

$\Rightarrow s = \pm j1.58$

$y = \tan 45^\circ \times 2 = 2 \pm j2$

(8). Angle of departure:



$\phi_{P_1} = 180^\circ - \tan^{-1}\left(\frac{1}{2}\right) = 153.5^\circ$

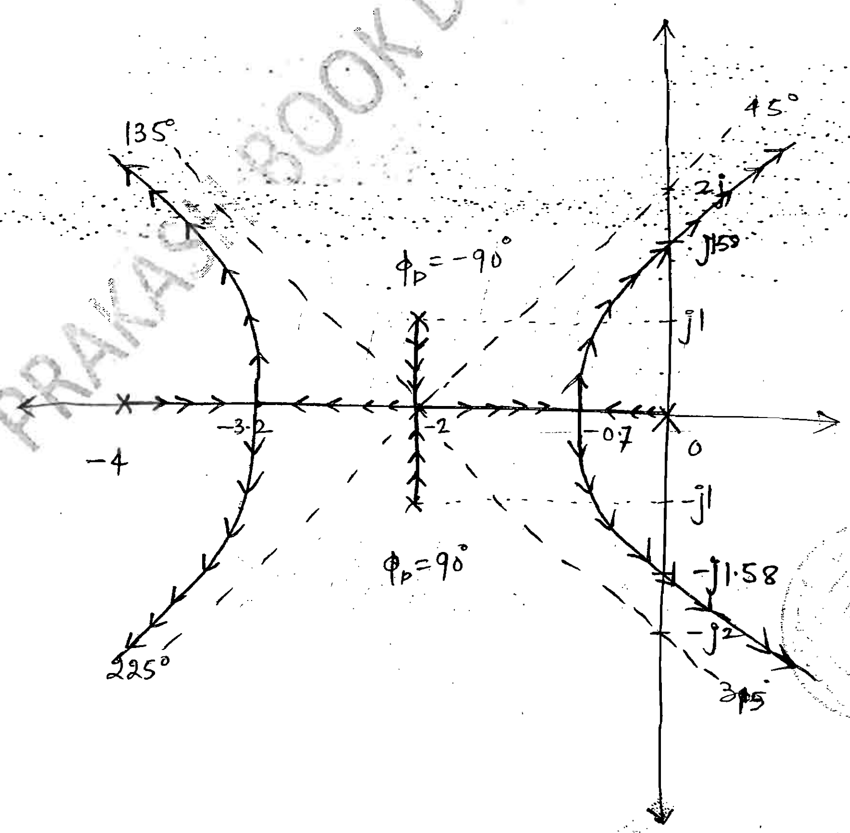
$\phi_{P_2} = 90^\circ$

$\phi_{P_3} = \tan^{-1}(1/2) = 26.5^\circ$

$\phi = 0^\circ - (153.5^\circ + 90^\circ + 26.5^\circ)$

$= -270^\circ$

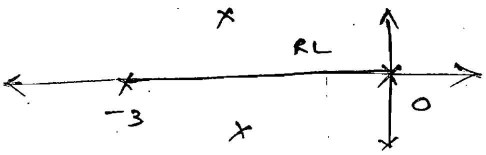
$\phi_D = 180^\circ + \phi = 180^\circ - 270^\circ = -90^\circ$



CONV Q1. $G(s) = \dfrac{K}{s(s+3)(s^2+3s+4.5)}$

(2). $P = 4$, $Z = 0$, $P - Z = 4$

(3).



(4). $\theta_1 = 45°$, $\theta_2 = 135°$, $\theta_3 = 225°$, $\theta_4 = 300°$.

(5). Centroid $= \dfrac{0 + (-3) + (-1.5) + (-1.5)}{4} = -1.5$

(6). B.A pts:

$$s^4 + 6s^3 + 13.5s^2 + 13.5s + K = 0$$

$$K = -s^4 - 6s^3 - 13.5s^2 - 13.5s$$

$$dK/ds = 4s^3 + 18s^2 + 27s + 13.5 = 0$$

$$\therefore s = -1.5, -1.5, -1.5.$$

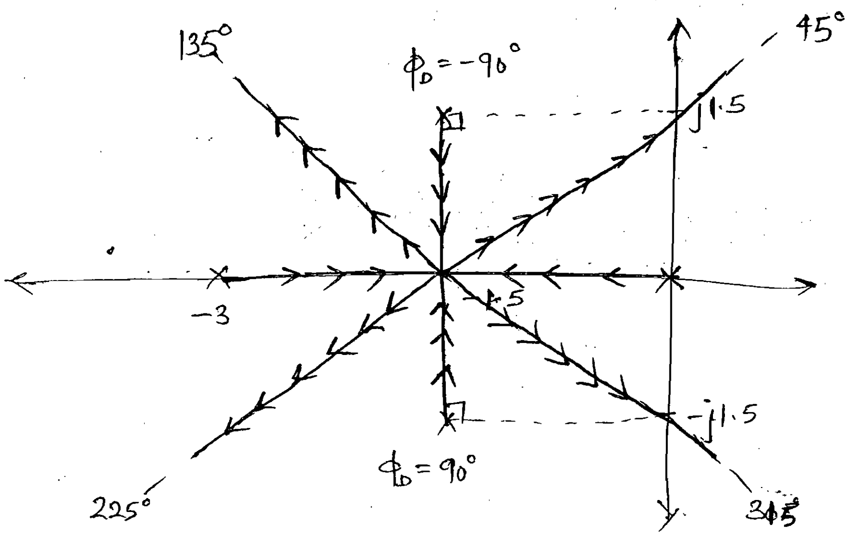
(7). Intersection of RL with $j\omega$ axis:

$$s = \pm j1.5$$

Intersection of asymptotes with $j\omega$ axis:

$$y = \pm j1.5$$

(8). Angle of departure: $\phi_D = \mp 90°$



CONV Q2.

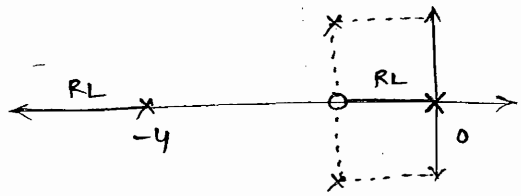
$$\underbrace{s(s+4)(s^2+2s+2)} + K(s+1) = 0$$

$$1 + \frac{K(s+1)}{s(s+4)(s^2+2s+2)} = 0$$

$$G(s)H(s) = \frac{K(s+1)}{s(s+4)(s^2+2s+2)}$$

(2). $P = 4$, $Z = 1$; $P - Z = 3$

(3).



(4). $\theta_1 = 60^\circ$, $\theta_2 = 180^\circ$; $\theta_3 = 300^\circ$

(5). Centroid $= \dfrac{0 + (-4) + (-1) + (-1) - (-1)}{3} = -1.6$

(6). B.A pts. $\rightarrow$ NIL

(7).

s^4	1	10	K
s^3	6	$K+8$	0
s^2	$\frac{52-K}{6}$	K	0
s^1	$\dfrac{\frac{(52-K)(K+8)}{6} - 6K}{\left(\frac{52-K}{6}\right)}$	0	0
s^0	K	0	0

(1). $\dfrac{(52-K)(K+8)}{6} - 6K = 0$

$(52-K)(K+8) - 36K = 0$

$K^2 - 8K - 416 = 0 \Rightarrow K_{mar} = 24.78,\ -16.78\,(\times)$

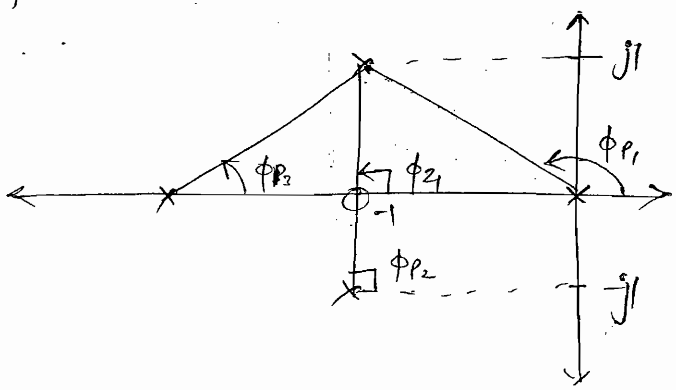
$A(s) = \left(\dfrac{52-K}{6}\right)s^2 + K = 0$

$\left(\dfrac{52-24.78}{6}\right)s^2 + 24.78 = 0 \Rightarrow s = \pm j\,2.38$

Intersection with Asymptotes:

$y = \tan 60^\circ \times 1.6 = \sqrt{3} \times 1.6 = 2.77 = \pm j2.77$

(8). Angle of departure:



$\phi_{P_1} = 180^\circ - \tan^{-1}(1) = 135^\circ$

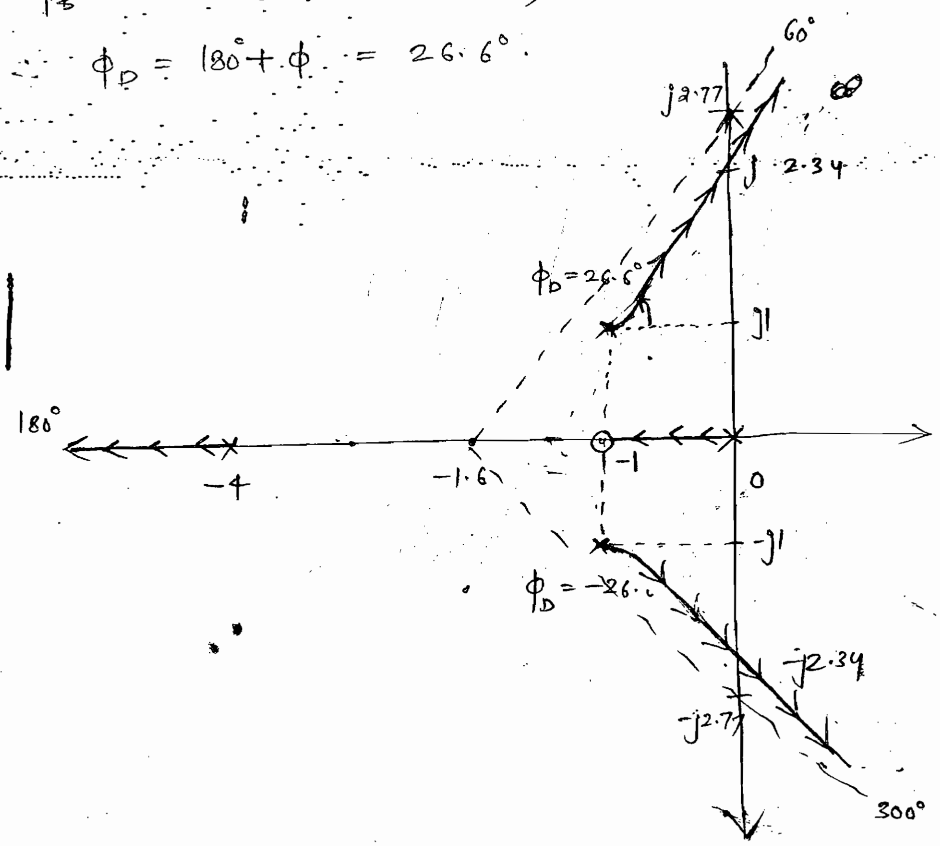
$\phi_{P_2} = 90^\circ$

$\phi_{P_3} = \tan^{-1}(1/3) = 18.4^\circ$

$\phi_{Z_1} = 90^\circ$

$\phi = 90^\circ - (90^\circ + 18.4^\circ + 135^\circ) = -153.4^\circ$

$\therefore \phi_D = 180^\circ + \phi = 26.6^\circ.$



Effects of feedback

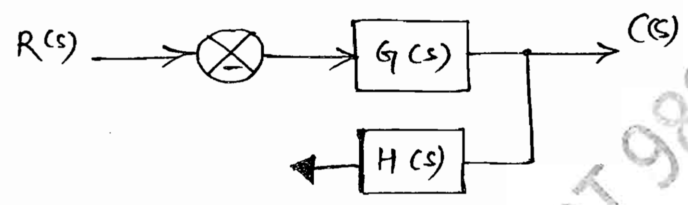
Sensitivity Analysis

Let α = A variable that changes its value

β = A parameter that changes the value of 'α'.

$$S_{\beta}^{\alpha} = \frac{\%\text{ change in } \alpha}{\%\text{ change in } \beta} = \frac{\partial\alpha/\alpha}{\partial\beta/\beta} = \frac{\beta}{\alpha}\cdot\frac{\partial\alpha}{\partial\beta}$$

(1). Open Loop s/s :



Let $M(s)$ = O.L.C.S

$M(s) = G(s)\,H(s)$

Let α = OLCS = $M(s)$

$\beta = G(s)$

$$S_{G(s)}^{M(s)} = \frac{G(s)}{M(s)}\cdot\frac{\partial M(s)}{\partial G(s)}$$

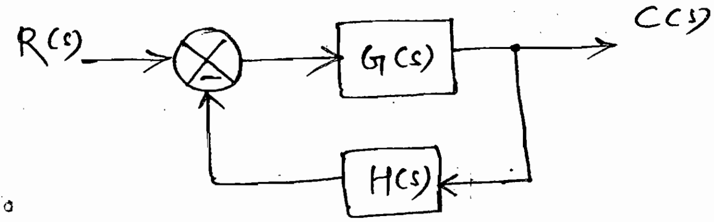
$$M(s) = G(s)\,H(s)$$

$$\frac{M(s)}{G(s)} = \frac{1}{H(s)}$$

$$\frac{\partial M(s)}{\partial G(s)}\left[G(s)\,H(s)\right] = H(s)$$

$$\therefore\ S_{G(s)}^{M(s)} = \frac{1}{H(s)}\cdot H(s) = 1$$

(2). Close Loop C.S :



$$M(s) = \text{C.L.C.S}$$
$$= \frac{G(s)}{1+G(s)H(s)}$$

$\alpha = M(s)$; $\beta = G(s)$

$$S^{M(s)}_{G(s)} = \frac{G(s)}{M(s)} \cdot \frac{\partial M(s)}{\partial G(s)}$$

$$M(s) = \frac{G(s)}{1+G(s)H(s)}$$

$$\frac{G(s)}{M(s)} = 1+G(s)H(s)$$

$$\frac{\partial M(s)}{\partial G(s)} = \frac{\partial}{\partial G(s)} \left\{ \frac{G(s)}{1+G(s)H(s)} \right\}$$

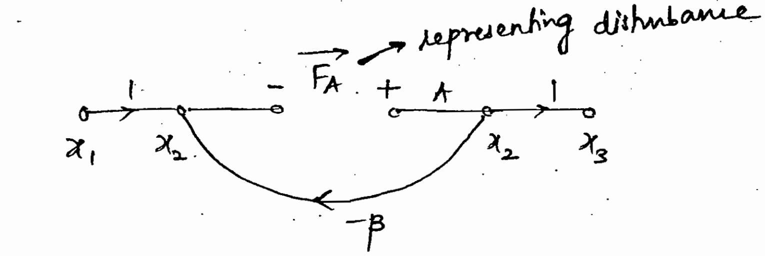
$$= \frac{1+G(s)H(s) - G(s)H(s)}{[1+G(s)H(s)]^2} = \frac{1}{[1+G(s)H(s)]^2}$$

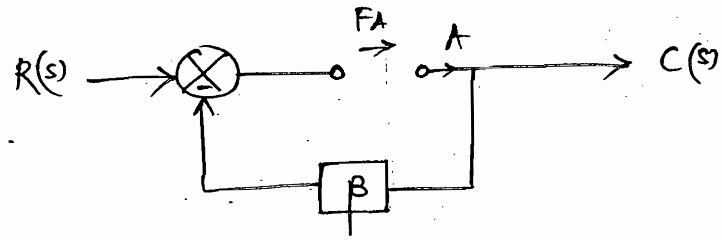
$$S^{M(s)}_{G(s)} = [1+G(s)H(s)] \cdot \frac{1}{[1+G(s) \cdot H(s)]^2}$$

$$\therefore S^{M(s)}_{G(s)} = \frac{1}{[1+G(s)H(s)]}$$

$[1+G(s)H(s)]$ = Noise Reduction factor (or) Return Difference

Q1.





Return difference $= 1 + G(s) \cdot H(s)$

$= 1 + A \cdot \beta$

Sensitivity of C.L. s/s w.r.to $H(s)$:

$\alpha = M(s)$; $\beta = H(s)$

$$S^{M(s)}_{H(s)} = \frac{H(s)}{M(s)} \cdot \frac{\partial M(s)}{\partial H(s)}$$

$$M(s) = \frac{G(s)}{1 + G(s) H(s)}$$

$$\frac{M(s)}{H(s)} = \frac{G(s)}{[1 + G(s) \cdot H(s)] H(s)} \quad \text{---(1)}$$

$$\frac{H(s)}{M(s)} = \frac{H(s) \cdot [1 + G(s) H(s)]}{G(s)}$$

$$\frac{\partial M(s)}{\partial H(s)} = \frac{\partial}{\partial H(s)} \left\{ \frac{G(s)}{1 + G(s) H(s)} \right\}$$

$$= \frac{(1 + G(s) H(s)) \cdot 0 - G(s) \cdot G(s)}{[1 + G(s) \cdot H(s)]^2}$$

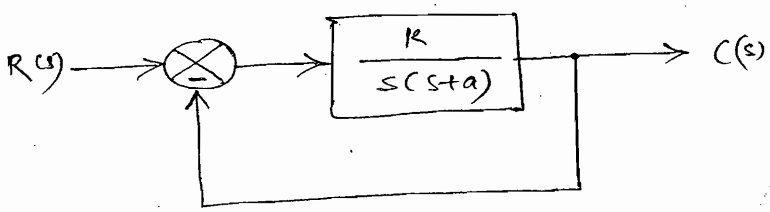
$$= \frac{-G(s)^2}{[1 + G(s) H(s)]^2}$$

$$S_{H(s)}^{M(s)} = \frac{H(s)\cdot[1+G(s)H(s)]}{G(s)} \times \frac{-G(s)^2}{[1+G(s)H(s)]^2}$$

$$S_{H(s)}^{M(s)} = \frac{-G(s)H(s)}{1+G(s)H(s)}$$

$$\therefore \left|S_{H(s)}^{M(s)}\right| = \frac{G(s)H(s)}{1+G(s)H(s)}$$

Q3.



$$G(s) = \frac{K}{s(s+a)}$$

$$e_{ss} = \lim_{s\to 0} \frac{s\cdot\frac{1}{s^2}}{1+\frac{K}{s(s+a)}} = \frac{a}{K}$$

(I) $$S_K^{e_{ss}} = \frac{K}{e_{ss}}\cdot\frac{\partial e_{ss}}{\partial K}$$

$$e_{ss} = \frac{a}{K}$$

$$\frac{e_{ss}}{K} = \frac{a}{K^2}$$

$$\frac{K}{e_{ss}} = \frac{K^2}{a}$$

$$\frac{\partial e_{ss}}{\partial K} = \frac{\partial}{\partial K}\left(\frac{a}{K}\right) = -\frac{a}{K^2}$$

$$S_K^{e_{ss}} = \frac{K^2}{a}\cdot\frac{-a}{K^2} = -1$$

$$\therefore \left|S_K^{e_{ss}}\right| = 1$$ ie. unity

(ii). $S_a^{e_{ss}} = \frac{a}{e_{ss}} \cdot \frac{\partial e_{ss}}{\partial a}$

$$e_{ss} = a/K$$

$$\frac{a}{e_{ss}} = K$$

$$\frac{\partial e_{ss}}{\partial a} = \frac{1}{\partial a}\left(\frac{a}{K}\right) = \frac{1}{K}$$

$$\therefore S_a^{e_{ss}} = K \cdot \frac{1}{K} = 1 \quad \text{i.e. unity}$$

<u>Note</u>: The sensitivity of steady state error w.r.to K and a is same.

Q7. fig-1 $T(s) = \frac{\frac{25K}{s(s+5)}}{1 + \frac{25K}{s(s+5)}} = \frac{25K}{s(s+5)+25K}$

$$S_K^{T(s)} = \frac{K}{T(s)} \frac{\partial T(s)}{\partial K}$$

$$\frac{K}{T(s)} = \frac{s(s+5)+25K}{25}$$

$$\frac{\partial T(s)}{\partial K} = \frac{\partial}{\partial K}\left\{\frac{25K}{s(s+5)+25K}\right\}$$

$$= \frac{[s(s+5)+25K]25 - 25K \cdot 25}{[s(s+5)+25K]^2}$$

$$= \frac{25s(s+5)}{[s(s+5)+25K]^2}$$

$$S_K^{T(s)} = \frac{\cancel{[s(s+5)+25K]}}{\cancel{25}} \times \frac{\cancel{25}\, s(s+5)}{[s(s+5)+25K]^{\cancel{2}}}$$

$$= \frac{s(s+5)}{s(s+5)+25K}$$

$$S_{K=1}^{T(s)} = \frac{s(s+5)}{s^2+5s+25}$$

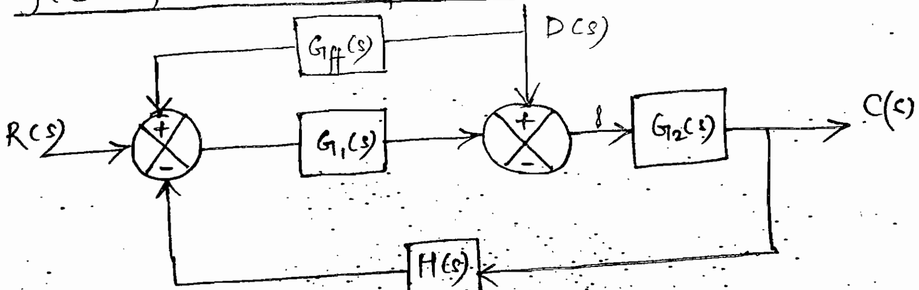
$$S_{K=1}^{T(j\omega)} = \frac{j\omega(j\omega+5)}{(25-\omega^2+j5\omega)}$$

$$\left|S_{K}^{T(j\omega)}\right| = \frac{\omega\sqrt{\omega^2+5^2}}{\sqrt{(25-\omega^2)^2+(5\omega)^2}}$$

$$\left|S_{K=1}^{T(j5)}\right| = \frac{5\sqrt{50}}{\sqrt{25^2}}$$

$$= \frac{5 \cdot 5\sqrt{2}}{25} = \sqrt{2} = 1.414$$

<u>feed forward compensation</u> –



$$\left.\frac{C(s)}{R(s)}\right|_{D(s)=0} = \frac{G_1(s)\,G_2(s)}{1+G_1(s)\,G_2(s)\,H(s)}$$

$$\left.\frac{C(s)}{D(s)}\right|_{R(s)=0} = \frac{G_2(s) + G_{ff}(s)\,G_1(s)\,G_2(s)}{1+G_1(s)\,G_2(s)\,H(s)}$$

$$C(s) = \frac{R(s)\cdot[G_1(s)\,G_2(s)] + D(s)\,[G_2(s) + G_{ff}(s)\,G_1(s)\,G_2(s)]}{1+G_1(s)\,G_2(s)\,H(s)}$$

* The condition for feed forward controller to eliminate the effect of disturbances in the s/s is $G_{ff}(s) = \frac{-1}{G_1(s)}$.

Q6. $G_1(s) = \dfrac{(s+a)\,K(s+c)}{(s+b)\,(s+d)}$

~~for D(s) = 0~~ $G_c(s) = \dfrac{(s+b)(s+d)}{K(s+a)(s+c)}$

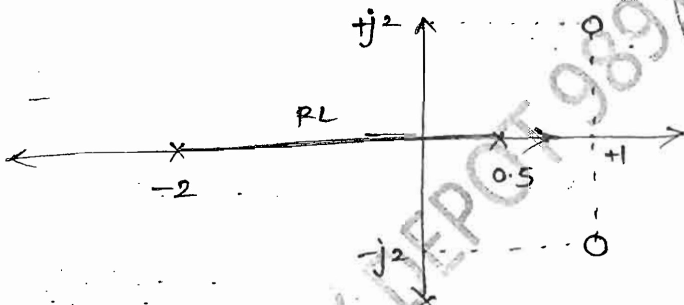
Root locus :

conv.

Q3. $G(s) = \dfrac{K(s^2-2s+5)}{(s+2)\,(s-0.5)}$

(2). $P = 2$, $Z = 2$; $P - Z = 0$

(3).



(6). B.A point :

$$(s+2)(s-0.5) + K(s^2-2s+5) = 0$$

$$K = \frac{-s^2-1.5s+1}{s^2-2s+5}$$

$$\frac{dk}{ds} = 3.5s^2 - 12s - 5.5 = 0$$

$$s = -0.4 \checkmark, \; 3.8$$

(7). Intersection of ~~Asyt~~ RL with $j\omega$ axis.

$$s^2(1+K) + s(1.5-2K) + 5K-1 = 0$$

s^2	$1+K$	$5K-1$
s^1	$1.5-2K$	0
s^0	$5K-1$	

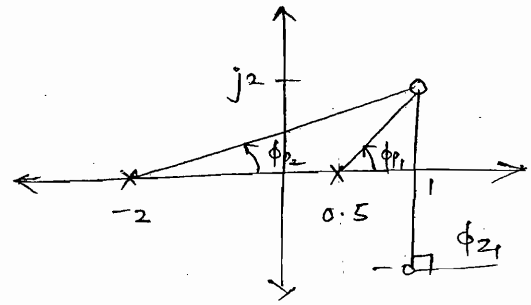
$K_{mar} \Rightarrow 1.5 = 2K$

$K_{mar} = 0.75$

$A(s) = (1+K)s^2 + (5K-1) = 0$

$s = \pm j1.25$

(8). Angle of arrival :



$\phi_{z_1} = 90°$

$\phi_{p_1} = \tan^{-1}\left(\frac{2}{0.5}\right) = 76°$

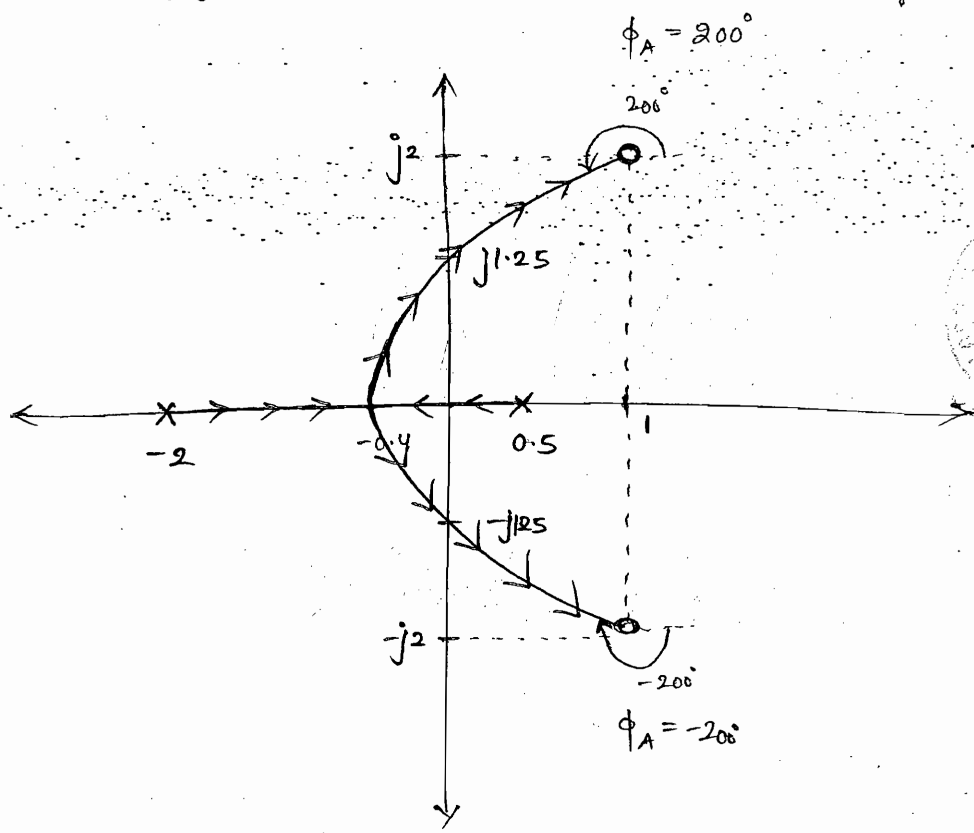
$\phi_{p_2} = \tan^{-1}\left(\frac{2}{3}\right) = 34°$

$\phi = 90° - (76° + 34°) = -20°$

$\phi_A = 180° - \phi$

$= 180° - (-20°)$

$= 200°$



Q1. $$1+\frac{K(s+a)}{s^2(s+b)}=0$$

$$s^3+bs^2+Ks+aK=0$$

s^3	1	K
s^2	b	aK
s^1	$\frac{bK-aK}{b}$	0
s^0	aK	0

(1). $bK-aK>0$

$bK>aK$

$K(b-a)>0$

$K>0$

(2). $aK>0$

$K>0$

$\therefore K_{max}=0$

$$A(s)=bs^2+aK=0$$

$$s^2=0 \Rightarrow s=0$$

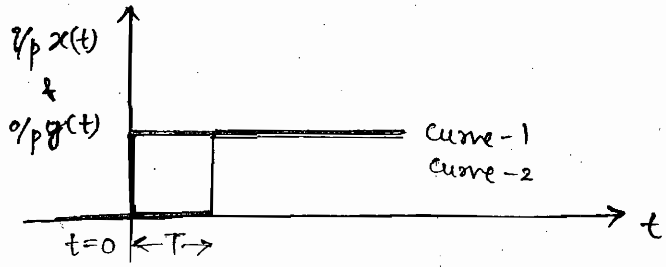
(No intersection of RL with $j\omega$ axis)

s/s is stable for $K>0$. → should be in LHS.

Q10. $0<K<1$ → Poles are real and unequal (overdamped)

$K>5$ → Again poles are real and unequal (overdamped).

Analysis of s/s having dead time (or) transportation lag :



For curve-1,

$$o/p\ y(t) = i/p\ x(t)$$

For curve-2,

$$o/p\ y(t) = x(t-T)$$

Applying LT. We get,

$$Y(s) = X(s) \cdot e^{-Ts}$$

$$\frac{Y(s)}{X(s)} = e^{-Ts}$$

(I) Time Domain Approximation:

T.D. Analysis, RH, RL plots

$$y(t) = x(t-T) = x(t) - T\dot{x}(t) - \frac{T^2}{2!}\ddot{x}(t) + \cdots$$

$$y(t) = x(t) - T\dot{x}(t)$$

$$Y(s) = X(s) - Ts\,X(s)$$

$$\frac{Y(s)}{X(s)} = 1 - Ts$$

$$\therefore\ e^{-Ts} \simeq 1 - Ts$$

Eg. $G(s) = \frac{Ke^{-s}}{s(s+3)} = \frac{K(1-s)}{s(s+3)}$

(1). Dead time is one of the forms of non-linearity and is approximated as zero in RHS of s-plane.

(2). TF having poles or zeroes in RHS of s-plane are called as Non-min^m phase functions.

Non- min^m phase fn's:

$$\angle F(s)\Big|_{\omega=\infty} \neq -(P-Z)\cdot 90^\circ \qquad \{P=2, Z=1, P-Z=1\}$$

Eg. $$G(s) = \frac{K(1-s)}{s(s+3)}$$

$$G(j\omega) = \frac{K(1-j\omega)}{j\omega(j\omega+3)}$$

$$= \frac{(K+j0)(1-j\omega)}{(0+j\omega)(j\omega+3)}$$

$$\angle G(j\omega) = \frac{\angle 0^\circ - \angle \tan^{-1}\omega}{\angle 90^\circ + \angle \tan^{-1}(\omega/3)}$$

$$\angle G(j\omega) = -90^\circ - \tan^{-1}\omega - \tan^{-1}(\omega/3)$$

$$\angle G(j\omega)\Big|_{\omega=\infty} = -90^\circ - 90^\circ - 90^\circ = -270^\circ$$

$$\neq -(P-Z)\,90^\circ.$$

(3). LTI TF should be min^m phase fn's i.e. their poles and zeroes must lie in the LHS of s-plane only.

Min^m phase fn's:

Eg. $$G(s) = \frac{K(1+s)}{s(s+3)}$$

$$\angle G(j\omega) = -90^\circ + \tan^{-1}\omega - \tan^{-1}(\omega/3)$$

$$\angle G(j\omega)\Big|_{\omega=\infty} = -90^\circ + 90^\circ - 90^\circ = -90^\circ$$

$$= -(P-Z)\cdot 90^\circ$$

(4).

$$G(s) = \frac{Ke^{-s}}{s(s+3)} \simeq \frac{K(1-s)}{s(s+3)} = \frac{-K(s-1)}{s(s+3)}$$

since "s" (i.e. Time constant) cannot be -ve, (1-s) factor should be expressed as -(s-1) in time domain methods.

Complimentary RL or Inverse RL :
(CRL) (IRL)

$$G(s)H(s) = 1$$

(1). Angle condition :

$$\angle G(s)H(s) = 0^\circ = \pm(2q)180^\circ$$

(2). Magnitude condition :

$$|G(s)H(s)| = 1$$

Construction Rules of CRL :

Rule (1): The CRL is symmetrical about Re-axis.
$[G(s)H(s) = 1]$.

Rule (2): same as RL.

Rule (3): A point on Re-axis is said to be on CRL if ~~to the right~~ the right side of this point, the sum of open loop poles and zeroes is even.

Rule (4): Angle of Asymptotes:

$$\theta = \frac{(2q)180^\circ}{P-Z} \quad \text{where, } q = 0, 1, 2, \dots$$

Rule (5): Centroid

same as RL.

Rule (6): Break away points

same as RL.

Rule (7): Intersection of CRL with $j\omega$ axis.

same as RL

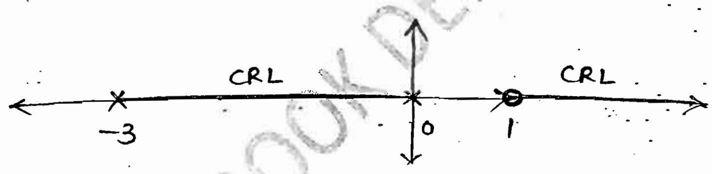
Rule (8): Angle of departure and arrival:

$$\phi_D = 0^\circ + \phi \qquad \text{where } \phi = \Sigma\phi_z - \Sigma\phi_P$$
$$\phi_A = 0^\circ - \phi$$

Q. $G(s) = \dfrac{-K(s-1)}{s(s+3)}$

(2). $P = 2$; $Z = 1$; $P - Z = 1$

(3).



(4) B.A points:

$$1 + \left[\frac{-K(s-1)}{s(s-3)}\right] = 0$$

$$s(s+3) + K(1-s) = 0$$

$$K = \frac{-s^2 - 3s}{(1-s)}$$

$$\frac{dK}{ds} = \frac{(1-s)(-2s-3) + (s^2+3s)(-1)}{(1-s)^2} = 0$$

$$-2s - 3 + 2s^2 + 3s - s^2 = 0$$

$$s^2 - 2s - 3 = 0$$

$$s = \frac{2 \pm \sqrt{4+12}}{2} = 1 \pm 2 \quad \text{(Centre ± radius)}$$

$$\therefore s \Rightarrow -1, 3.$$

Q.(7). $s(s+3)+K(1-s)=0$

$s^2+s(3-K)+K=0$

s^2	1	K
s^1	3-K	0
s^0	K	

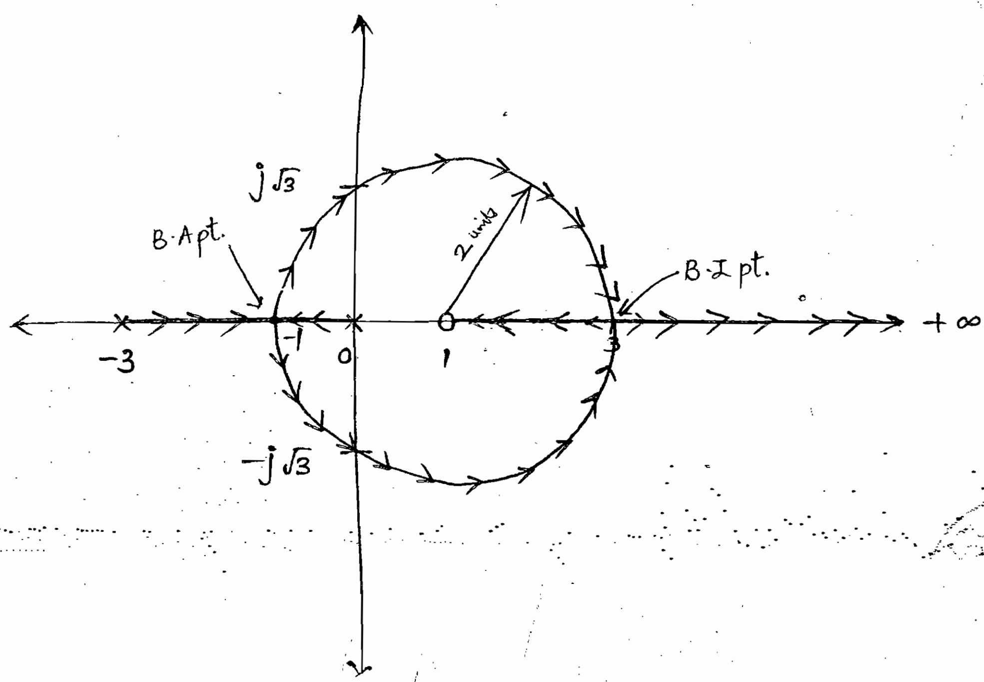
$3-K>0$

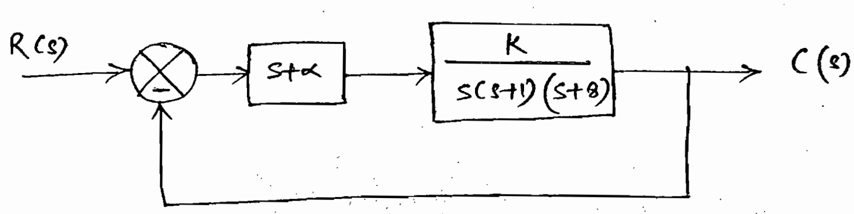
$K<3$

$K_{mar}=3$

$A(s)=s^2(1)+K \Rightarrow s^2+3=0$

$\Rightarrow s=\pm j\sqrt{3}$





$$G(s)=\frac{K(s+\alpha)}{s(s+1)(s+8)}$$

* we cannot draw its RL, becoz. location of zero is unknown. So we draw root contours.

Root Contours :

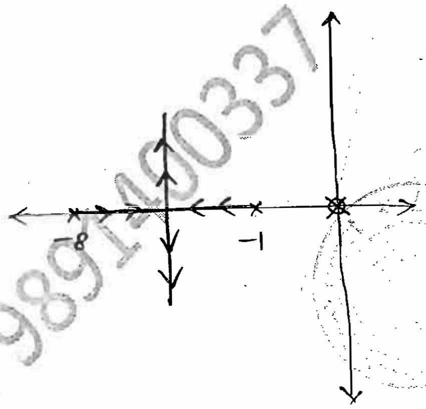
Roots contours are multiple RL diagrams obtained by varying multiple parameters in a TF drawn on same s-plane

$$G(s) = \frac{K(s+\alpha)}{s(s+1)(s+8)}$$

Root Contours :

Case (1) : $\alpha = 0$

$$G(s) = \frac{K\cdot \cancel{s}}{\cancel{s}(s+1)(s+8)}$$



Case (2) : $1 + \frac{K(s+\alpha)}{s(s+1)(s+8)}$

$$s(s+1)(s+8) + K(s+\alpha) = 0$$

$$\underbrace{s(s+1)(s+8) + Ks} + K\alpha = 0$$

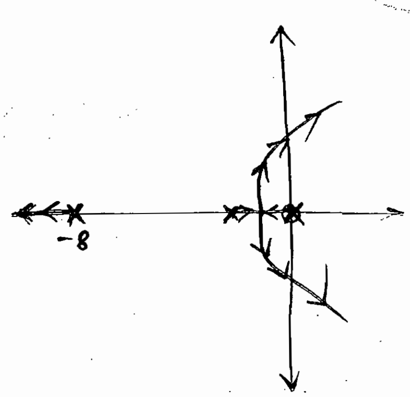
$$1 + \frac{K\alpha}{s(s+1)(s+8)+Ks} = 0$$

$$\therefore G(s)H(s) = \frac{K\alpha}{s(s+1)(s+8)+Ks}$$

Put $K=1$ (min^m value if not given)

$$G(s)H(s) = \frac{\alpha}{s(s+1)(s+8)+s}$$

$$= \frac{\alpha}{s(s^2+9s+9)}$$



Q. find B.A pts. for K=10. ?

$$G(s)H(s) = \frac{10\alpha}{s(s+1)(s+8)+10s}$$

Let $10\alpha = K'$

$$G(s)H(s) = \frac{K'}{s(s+1)(s+8)+10s}$$

$$1 + \frac{K'}{s(s^2+9s+18)} = 0$$

$$s^3 + 9s^2 + 18s + K' = 0$$

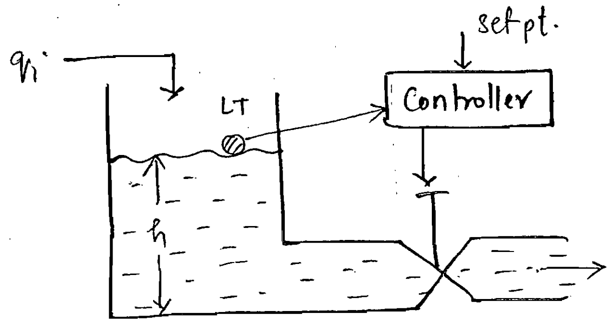
$$K' = -s^3 - 9s^2 - 18s$$

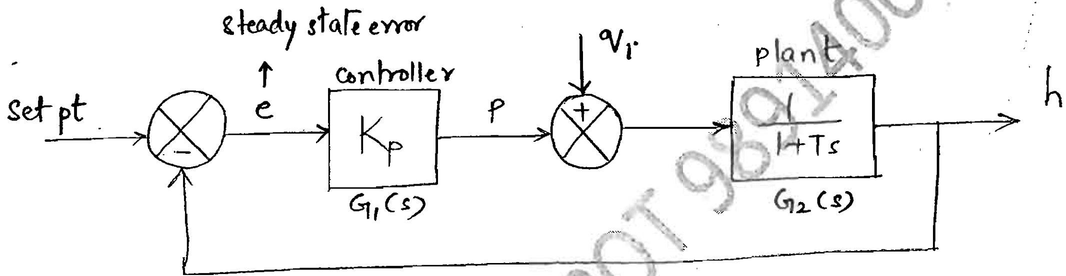
$$\frac{dK}{ds} = 3s^2 + 18s + 18 = 0$$

$$\Rightarrow s = -1.26, -4.73$$

Industrial Controllers

(1). Proportional mode :





$$|e_{ss}| = \lim_{s \to 0} \frac{s \cdot q_i(s) \cdot G_2(s)}{1 + G_1(s) \cdot G_2(s)}$$

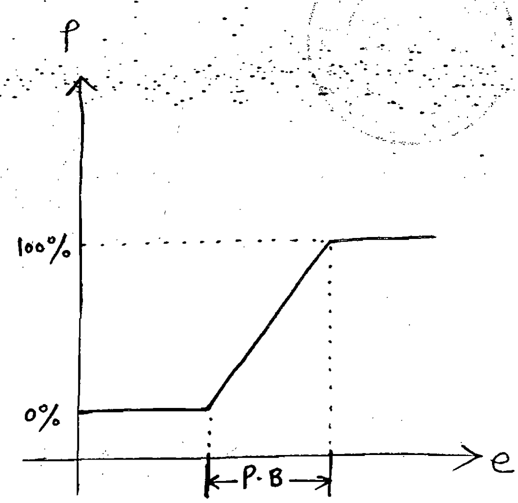
$P \propto e$

$P = K_p e$

$P(s) = K_p E(s)$

$K_p \rightarrow$ proportional gain

$$E(s) \longrightarrow \boxed{K_p} \longrightarrow P(s)$$



Proportional Band,

$$PB = \frac{100}{K_p}$$

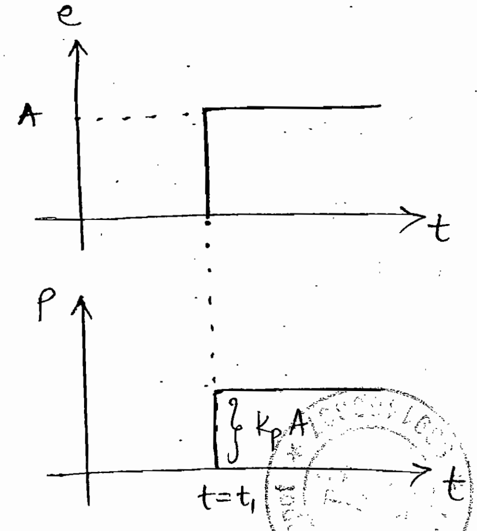
$p = K_p e$

$p = K_p A \qquad (e = A)$

$$|e_{ss}| = \lim_{s\to 0} \frac{s \cdot \frac{A}{s} \cdot \frac{1}{1+Ts}}{1 + \frac{K_p}{1+Ts}}$$

$$\boxed{|e_{ss}| = \frac{A}{1+K_p}} \quad \text{(offset)}$$

$$\text{offset} \propto \frac{1}{K_p}$$



(1). It is a natural extension of ON/OFF controller (NL).

(2). The band of errors where every value of error has a unique value of controller o/p is known as propotional band.

(3). The disadvantages of a protional controller is it exhibits a permanent residual error known as offset at its operating point.

Q20. $p = K_p e$

$100 = K_p \cdot 1 \Rightarrow K_p = 100$

$PB = \frac{100}{K_p} = \frac{100}{100} = 1$

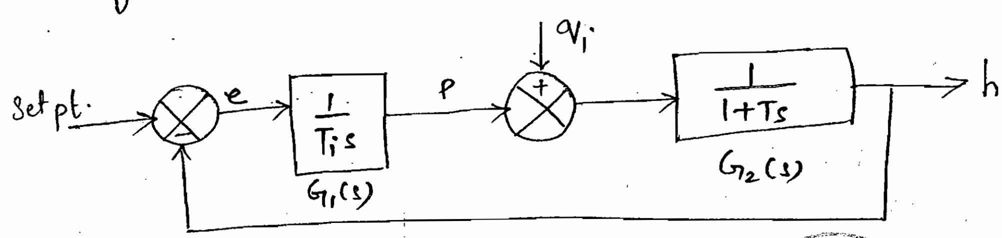
$\% PB = 1 \times 100 = 100\%$

$100\%\ PB \longrightarrow 100V$

$20\%\ PB \longrightarrow ?$

$\frac{20}{100} \times 100 = 20V \quad \Rightarrow \quad \boxed{1V,\ 20V}$ Ans.

(2). Integral controller:



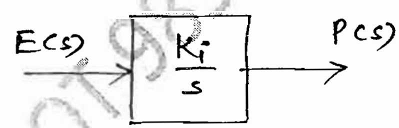
$$\frac{dP}{dt} \propto e$$

$$\frac{dP}{dt} = K_i e$$

$K_i \rightarrow$ Integral scaling

$$P = K_i \int e\, dt$$

$$P(s) = \frac{K_i}{s} \cdot E(s)$$

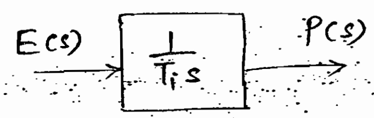


Defining " Reset time".

$$T_i = 1/K_i$$

$$P = \frac{1}{T_i} \int e\, dt$$

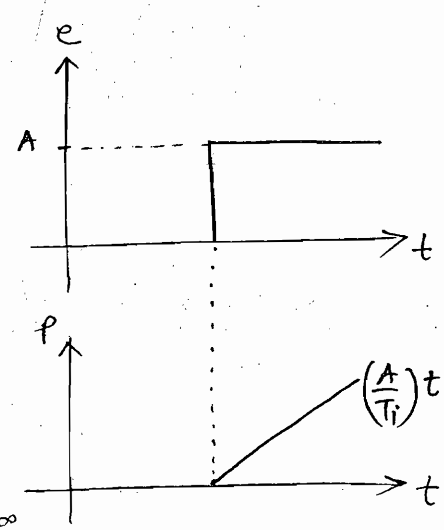
$$P(s) = \frac{1}{T_i s} \cdot E(s)$$



$$P = \frac{1}{T_i} \int e\, dt$$

$$= \frac{1}{T_i} \int A\, dt \quad (\because e = A)$$

$$= \frac{A}{T_i} t$$



Let $e = \sin\omega t$

$$\Rightarrow P = \frac{1}{\omega T_i} \cdot \sin(\omega t - \pi/2)$$

$$|e_{ss}| = \lim_{s \to 0} \frac{s \cdot \frac{A}{s} \cdot \frac{1}{1+Ts}}{1 + \frac{1}{T_i s(1+Ts)}} = \frac{A}{1+\infty}$$

$$\boxed{|e_{ss}| = 0}$$

(1). The disadvantage of integral controller is its response to errors is sluggish. However, it is capable of eliminating the steady state error completely in the s/s.

(3). __Derivative (or) Rate mode :__

$P \propto de/dt$

$P = K_d \, de/dt$

$K_d \rightarrow$ rate constant

$P(s) = K_d \cdot s \cdot E(s)$

$$E(s) \longrightarrow \boxed{K_d s} \longrightarrow P(s)$$

Defining "Rate Time".

$T_d = K_d$

$P = T_d \cdot \frac{de}{dt}$

$\therefore P(s) = T_d \cdot s \cdot E(s)$

$$E(s) \longrightarrow \boxed{sT_d} \longrightarrow P(s)$$

$P = T_d \cdot \frac{d}{dt}(A) \quad (e = A)$

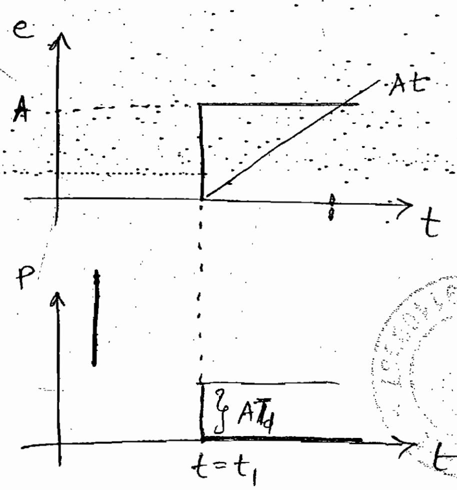
$P = 0$

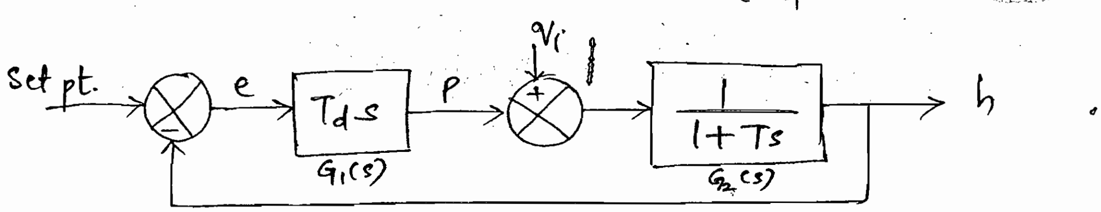
$P = T_d \cdot \frac{d}{dt}(At) \quad (e = At)$

$P = AT_d$

Let $e = \sin\omega t$

$\Rightarrow P = \omega T_d \sin(\omega t + \pi/2)$





$$|e_{ss}| = \lim_{s\to 0} \frac{s \cdot \frac{A}{s^2} \cdot \frac{1}{1+Ts}}{1 + \frac{T_d s}{1+Ts}} = \lim_{s\to 0} \frac{\frac{A}{1+Ts}}{\frac{s + s^2 T_d}{1+Ts}} = \frac{A}{0} = \infty$$

$$\boxed{|e_{ss}| = \infty}$$

(1). The disadvantage of derivative controller is it cannot respond to sudden errors.

(2). It is also known as anticipatory controller because it sends a control s/g in anticipation of error.

- Becoz of these disadvantages, we never use them alone.

Composite Controller modes :

P + I mode :

$$P = K_p e + \frac{K_p}{T_i}\int e\,dt.$$

$$P(s) = K_p\left(1 + \frac{1}{T_i s}\right) \cdot E(s)$$

$$E(s) \longrightarrow \boxed{K_p\left(1+\frac{1}{T_i s}\right)} \longrightarrow P(s)$$

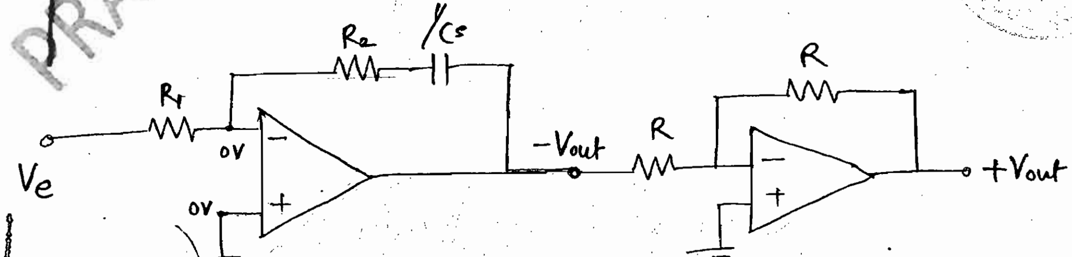
Effect on transient state :

Let $e = \sin\omega t$.

$$P = K_p \sin\omega t + \frac{K_p}{T_i}\int \sin\omega t\, dt$$

$$P = K_p \sin\omega t - \frac{K_p}{\omega T_i}\cos\omega t$$

$$P = \sqrt{K_p^2 + \left(\frac{K_p}{\omega T_i}\right)^2} \cdot \sin\left\{\omega t - \tan^{-1}\left(\frac{1}{\omega T_i}\right)\right\}$$



$$\frac{V_e}{R_1} = \frac{-V_{out}}{\frac{R_2 Cs + 1}{Cs}} \Rightarrow \frac{V_e}{R_1} = \frac{-V_{out} \cdot Cs}{R_2 Cs + 1} \Rightarrow -V_{out} = \frac{V_e (R_2 Cs + 1)}{R_1 Cs}$$

$$\Rightarrow -V_{out} = \frac{V_e R_2 Cs}{R_1 Cs} + \frac{V_e}{R_1 Cs} \Rightarrow V_{out} = V_e\left(\frac{R_2}{R_1}\right) + \left(\frac{R_2}{R_1}\right)\frac{1}{R_2 C}\int V_e\, dt$$

$$\boxed{K_p = \frac{R_2}{R_1}} \qquad \boxed{T_i = R_2 C}$$

Effects:

(1). It is capable of improving the steady-state response characteristics of the s/s only. i.e. elimination of s.s. error. b/w o/p and i/p.

(2). The integral controller eliminates the offset of propotional controller.

(3). For sinusoidal i/p, the phase of the controller o/p lags by $\tan^{-1}\left(\frac{1}{\omega T_i}\right)$. Hence, it is similar to phase lag compensator.

(4). In terms of filtering property it is a low pass filter.

(5). It increases rise time. t_r.

(6). It reduces BW.

(7). It reduces stability of the s/s.

(8). It increases t_p and reduces M_p

(9). The P+I controllers increases type and order of the s/s by one.

* It is also valid for only I i.e. (P+I specifications = only I)

P+D mode:

$$P = K_p e + K_p T_d \frac{de}{dt}$$

$$P(s) = K_p(1+T_d s) \cdot E(s)$$

$$E(s) \longrightarrow \boxed{K_p(1+T_d s)} \longrightarrow P(s)$$

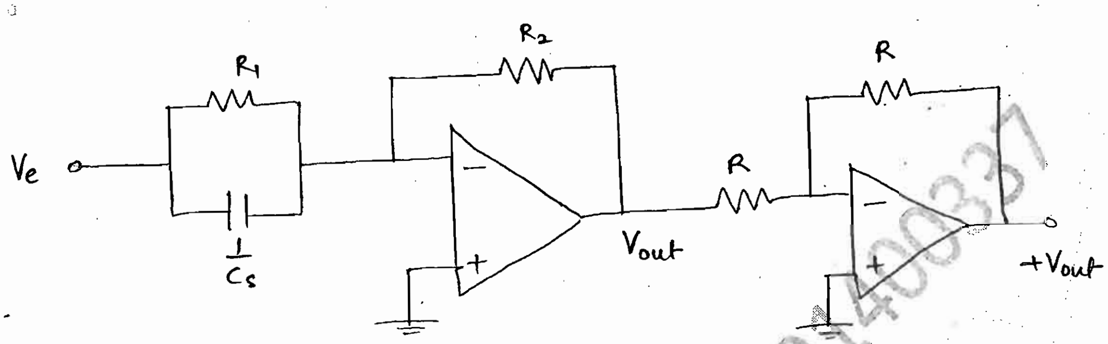
Effect on transient state:

Let $e = \sin\omega t$

$$P = K_p \sin\omega t + K_p T_d \frac{d}{dt}(\sin\omega t)$$

$$P = K_p \sin\omega t + \omega K_p T_d \cos\omega t$$

$$P = \sqrt{K_p^2 + (\omega K_p T_d)^2} \cdot \sin\left\{\omega t + \tan^{-1}(\omega T_d)\right\}$$



$$\frac{V_e}{\dfrac{R_1}{R_1Cs+1}} = \frac{-V_{out}}{R_2}$$

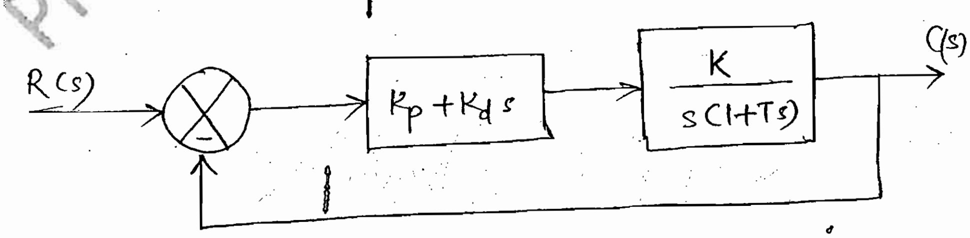
$$-V_{out} = \frac{V_e (R_1Cs+1) R_2}{R_1}$$

$$-V_{out} = \frac{R_2}{R_1} \cdot V_e + \frac{R_2}{R_1} R_1 C s\, V_e$$

$$+V_{out} = \left(\frac{R_2}{R_1}\right) V_e + \left(\frac{R_2}{R_1}\right) R_1 C \frac{dV_e}{dt} = \left(\frac{R_2}{R_1}\right) \left[V_e + R_1 C \frac{dV_e}{dt}\right]$$

$$\boxed{K_p = R_2/R_1} \qquad \boxed{T_d = R_1 C}$$

Conv. Q4.



I. Without P+D controller

$$G(s) = \frac{K}{s(1+Ts)}$$

Type = 1 & order = 2

With P+D controller

$$G(s) = \frac{K(K_p + K_d s)}{s(1+Ts)}$$

Type = 1 & Order = 2

II. <u>With P controller only</u>

$$G(s) = \frac{K \cdot K_p}{s(1+Ts)}$$

$$1 + \frac{K \cdot K_p}{s(1+Ts)} = 0$$

$$Ts^2 + s + KK_p = 0$$

$$s^2 + \frac{1}{T} \cdot s + \frac{KK_p}{T} = 0$$

$$\omega_n = \sqrt{\frac{KK_p}{T}} \; r/s$$

$$2\xi\omega_n = \frac{1}{T}$$

$$\xi = \frac{1}{2\sqrt{KK_pT}}$$

<u>With P+D controller</u>

$$1 + \frac{K(K_p + K_d s)}{s(1+Ts)} = 0$$

$$Ts^2 + s(1+KK_d) + KK_p = 0$$

$$s^2 + \frac{s(1+KK_d)}{T} + \frac{KK_p}{T} = 0$$

$$\omega_n = \sqrt{\frac{KK_p}{T}} \; r/s$$

$$2\xi\omega_n = \frac{1+KK_d}{T}$$

$$\xi = \frac{1+KK_d}{2\sqrt{KK_pT}}$$

$$e_{ss} = \lim_{s\to 0} \frac{s \cdot \frac{1}{s^2}}{1 + \frac{K(K_p + K_d s)}{s(1+Ts)}}$$

$$e_{ss} = \frac{1}{KK_p} \text{ (offset)}$$

<u>Effects:</u>

(1). It is used for improving the transient state response of the s/s only.

(2). For sinusoidal i/p the phase of the controller o/p leads by $\tan^{-1}(\omega T_d)$. Hence, it is similar to phase lead compensator.

(3). In terms of filtering property it is High Pass filter.

(4). It reduces t_r.

(5). It increases BW.

(6). It amplifies noise and hence reduces S/N ratio (noise comes only at high frequency operation).

(7). It increases stability of the s/s.

(8). It increases ξ and hence reduces M_p.

P+I+D mode :

(1). It is used for improving both transient and steady state response of the s/s.

(2). It is similar to lag-lead compensator.

(3). It reduces t_r

(4). It increases BW.

(5). It reduces S/N ratio.

(6). It increases stability of the s/s.

(7). It reduces M_p as the ξ increases.

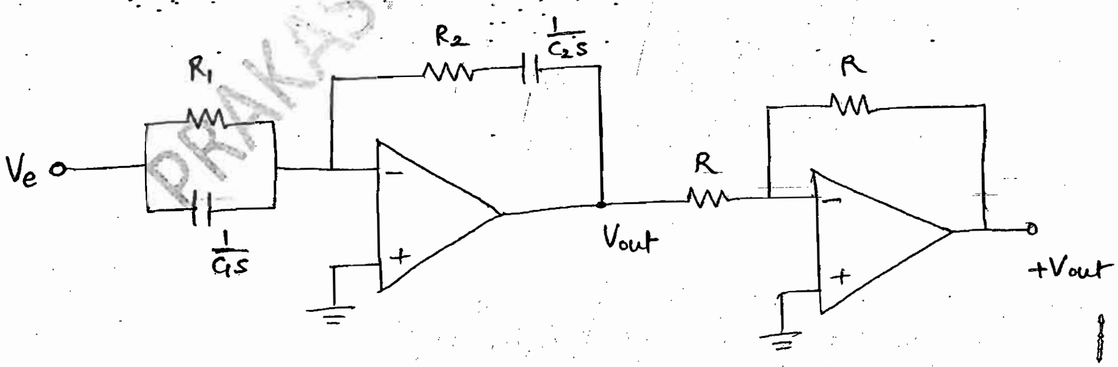
(8). It eliminates steady state error b/w o/p and i/p.

(9). It increases type and order of the s/s by one.

$$P = K_p e + \frac{K_p}{T_i}\int e\,dt + K_p T_d \frac{de}{dt}$$

$$P(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right) E(s)$$

$$E(s) \longrightarrow \boxed{1 + \frac{1}{T_i s} + T_d s} \longrightarrow P(s)$$



$$\frac{V_e}{\dfrac{R_1}{R_1C_1s+1}} = \frac{-V_{out}}{\dfrac{R_2C_2s+1}{C_2s}}$$

$$-V_{out} = V_e \cdot \frac{(R_1C_1s+1)(R_2C_2s+1)}{R_1C_2s}$$

$$-V_{out} = \frac{V_e \cancel{R_1} C_1 R_2 C_2 s^{\cancel{2}}}{\cancel{R_1} C_2 \cancel{s}} + \frac{V_e}{R_1 C_2 s} + V_e\, s\left(\frac{R_1C_1 + R_2C_2}{R_1 C_2 s}\right)$$

$$-V_{out} = V_e\left(\frac{R_1C_1}{R_1C_2} + \frac{R_2}{R_1}\cdot\frac{C_2}{C_2}\right) + \frac{V_e}{R_1C_2 s} + R_2 C_1 s V_e$$

$$+V_{out} = \left(\frac{R_2}{R_1}\right) V_e + \left(\frac{R_2}{R_1}\right)\cdot\frac{1}{R_2C_2}\int V_e\,dt + \left(\frac{R_2}{R_1}\right) R_1 C_1 \frac{dV_e}{dt}$$

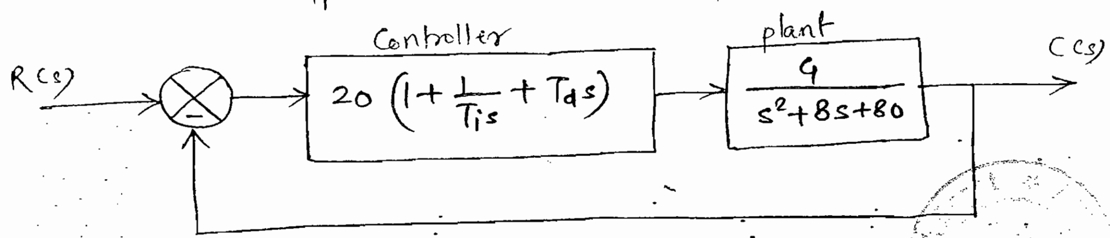
$$\boxed{K_p = \frac{R_2}{R_1}} \qquad \boxed{T_i = R_2 C_2} \qquad \boxed{T_d = R_1 C_1}$$

(10) - In terms of filtering property, it is a Band Reject filter.

* Tuning of I and D controllers is possible only in composite mode.

conv. Q1.

$$\theta_c(s) = 20\left(1 + \frac{1}{T_i s} + T_d s\right) E(s)$$



(a). $G(s) = \dfrac{20(1+T_d s)\cdot 4}{(s^2+8s+80)}$

$$1 + G(s) = 0$$

$$1 + \frac{80(1+T_d s)}{(s^2+8s+80)} = 0$$

$$s^2 + 8s + 80 + 80(1+T_d s) = 0$$

$$s^2 + (8 + 80T_d)s + 160 = 0$$

$$\omega_n = \sqrt{160} = 12.64 \text{ r/s}$$

$$2\xi\omega_n = 8 + 80T_d$$

$$\xi = \frac{8+80T_d}{2\times 12.64} = 1 \text{ (given)} \Rightarrow T_d = 0.2 \text{ sec}$$

(b). $G(s) = \dfrac{(20T_i s + 20 + 4T_i s^2)4}{T_i s(s^2+8s+80)}$

$$1 + \frac{80T_i s + 80 + 16T_i s^2}{T_i s(s^2+8s+80)} = 0$$

$$T_i s^3 + 8T_i s^2 + 80T_i s + 80T_i s + 80 + 16T_i s^2 = 0$$

$$T_i s^3 + 24T_i s^2 + 160T_i s + 80 = 0$$

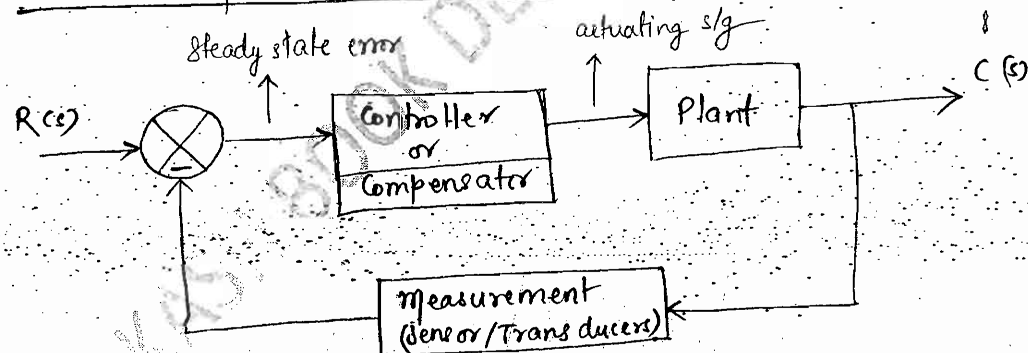
$$s^3 + 24s^2 + 160s + \frac{80}{T_i} = 0$$

For stability,

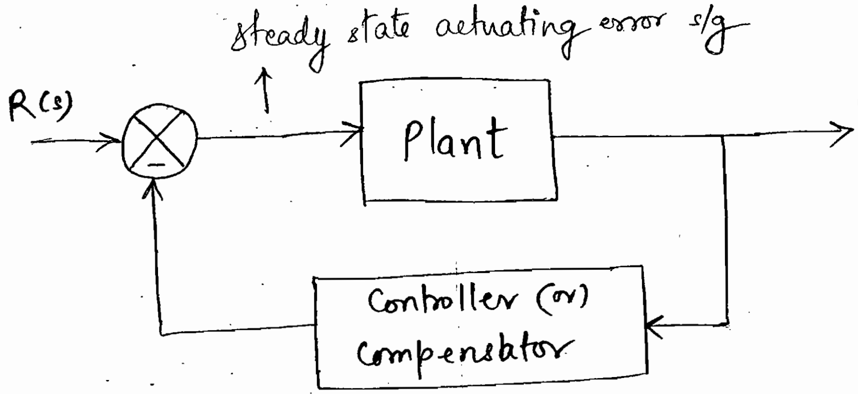
$$24 \times 160 \geq \frac{80}{T_i}$$

$$T_i \geq \frac{80}{24 \times 160} \Rightarrow T_i = 0.02 \text{ sec}$$

Error compensation and o/p compensation :

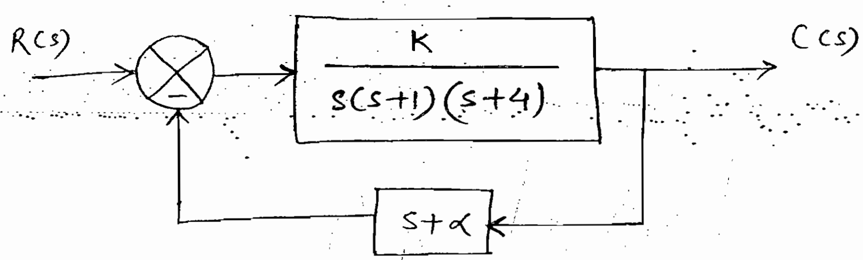


A controller or compensator placed in the forward path compensates for steady state error and sends an actuating s/g affecting the dynamics of the plant to achieve the control objective. In error compensation for finding the type of the s/s fb should be either unity or should be specified as measuring elements i.e. $H(s)$.



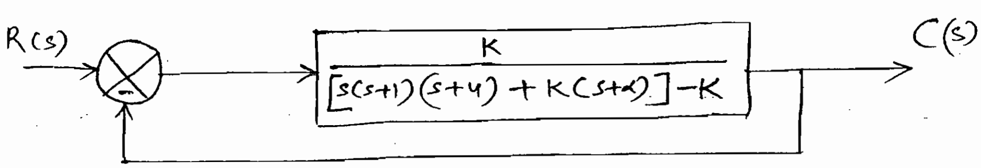
A controller (or) compensator placed in the fb path compensates for changes in o/p and steady state actuating error affects the dynamics of the plant to achieve the control objective. In o/p compensation to find the type of the s/s and ss error convert the control s/s into unity fb s/s (ufb).

Eg. Find e_{ss} for unit step i/p ?



$$TF = \frac{\frac{K}{s(s+1)(s+4)}}{1+\frac{K(s+\alpha)}{s(s+1)(s+4)}} = \frac{K}{s(s+1)(s+4)+K(s+\alpha)}$$

$$G(s) = \frac{K}{[s(s+1)(s+4)+K(s+\alpha)]-K}$$



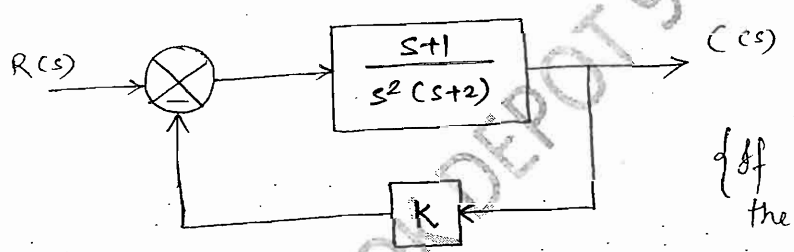
Type – 0 s/s :

$$e_{ss} = \lim_{s \to 0} \frac{s \cdot \frac{1}{s}}{1 + \frac{K}{[s(s+1)(s+4) + Ks + K\alpha] - K}}$$

$$= \frac{1}{1 + \frac{K}{K\alpha - K}}$$

$$= \frac{K\alpha - K}{K\alpha - K + K} = \frac{K(\alpha - 1)}{K\alpha} = \frac{\alpha - 1}{\alpha}$$

Q. Find e_{ss} for unit step i/p ?



{ If $H(s) = K$ then only. $G(s)\,H(s) \Rightarrow$ }

Here it can proportional controller also.

$$TF = \frac{\frac{s+1}{s^2(s+2)}}{1 + \frac{K(s+1)}{s^2(s+2)}}$$

$$= \frac{s+1}{s^2(s+2) + Ks + K}$$

$$G(s) = \frac{s+1}{[s^2(s+2) + Ks + K] - (s+1)} \quad \text{— Type – 0 s/s}$$

$$e_{ss} = \lim_{s \to 0} \frac{\cancel{s} \cdot \frac{1}{\cancel{s}}}{1 + \frac{s+1}{[s^2(s+2) + Ks + K] - (s+1)}} = \frac{1}{1 + \frac{1}{K-1}}$$

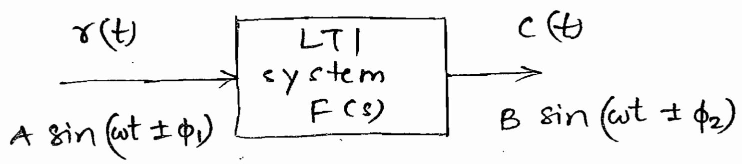
$$= \frac{K-1}{K-1+1} = \frac{K-1}{K}$$

Part – IV

Frequency Domain Analysis

(1). When any s/s is subjected to sinusoidal i/p, the o/p is also sinusoidal having different magnitude and phase angle but same i/p frequency ω rad/s.

(2). Frequency response analysis implies varying ω from 0 to ∞ and observing corresponding variations in magnitude and phase angle of the response.



$$B = A \cdot |F(s)|$$
$$\phi_2 = \pm\phi_1 + \angle F(s)$$

Let $\dfrac{C(s)}{R(s)} = F(s)$

Put $s = j\omega$, $F(j\omega) \rightarrow$ Sinusoidal TF $\cong$ Sinusoidal Response

$$F(j\omega) = |F(j\omega)| \cdot \angle F(j\omega)$$

Frequency response plots:

(1). **Polar plots**:

Absolute values of $|F(j\omega)|$ & $\angle F(j\omega)$ (degrees) Vs "ω"

(2). **Bode plots**:

decibel (db) values of $|F(j\omega)|$
$20\log|F(j\omega)|$ & $\angle F(j\omega)$ (degrees) Vs "$\log\omega$"

Q1. $X(t) \longrightarrow \boxed{\frac{1}{s+1}} \longrightarrow Y(t)$ for $X(t) = \sin t$. find $y(t) = ?$

Sol.

$$f(s) = \frac{Y(s)}{X(s)} = \frac{1}{s+1}$$

$$f(j\omega) = \frac{1}{j\omega+1} = \frac{1+j0}{1+j\omega} = \frac{\sqrt{1^2+0^2}}{\sqrt{1^2+\omega^2}} \cdot \angle -\tan^{-1}\omega = \frac{1}{\sqrt{1+\omega^2}} \cdot \angle -\tan^{-1}\omega$$

given: $X(t) = \sin \omega t = \sin t \Rightarrow \omega = 1$ rad/s

$$\therefore f(j\omega) = \frac{1}{\sqrt{2}} \angle -45^\circ$$

$$y(t) = \frac{1}{\sqrt{2}} \cdot \sin(t - 45^\circ)$$

Q2. $$G(s) = \frac{(s^2+9)(s+2)}{(s+3)(s+4)(s+5)}$$

$$G(j\omega) = \frac{(-\omega^2+9)(j\omega+2)}{(j\omega+3)(j\omega+4)(j\omega+5)} = 0$$

$$-\omega^2+9 = 0 \Rightarrow \omega = 3 \text{ rad/s.}$$

Frequency response analysis of IInd order s/s

$$F(s) = \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$= \frac{1}{\left(\frac{s}{\omega_n}\right)^2 + \frac{2\xi\omega_n s}{\omega_n^2} + 1}$$

$$F(j\omega) = \frac{1}{\left(\frac{j\omega}{\omega_n}\right)^2 + \frac{2\xi j\omega}{\omega_n} + 1}$$

$$f(j\omega) = \frac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2 + j\,2\xi\left(\frac{\omega}{\omega_n}\right)} \quad \text{— sinusoidal TF}$$

$$|F(j\omega)| = \frac{1}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

Its dB value is

$$-20\log\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}$$

<u>Asymptotic approximations:</u>

$$-20\log\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2}$$

<u>Case: (1)</u> for low frequency, $1 >>> \left(\frac{\omega}{\omega_n}\right)^2$

$$-20\log\sqrt{1} = 0\text{ dB}$$

<u>Case: (2)</u> for High frequency, $\left(\frac{\omega}{\omega_n}\right)^2 >>> 1$

$$-20\log\sqrt{\left[\left(\frac{\omega}{\omega_n}\right)^2\right]^2} = -40\log\left(\frac{\omega}{\omega_n}\right) \quad \text{—(1)}$$

$$= -40\log\omega + 40\log\omega_n$$

$$y = m\,x + c$$

$$\text{slope } (m) = -40\text{ dB/dec.}$$

<u>Corner frequency</u>

$$0 = -40\log(\omega/\omega_n)$$

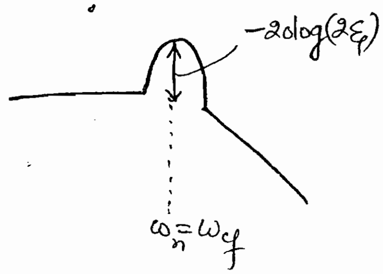
$$\log(\omega/\omega_n) = 0$$

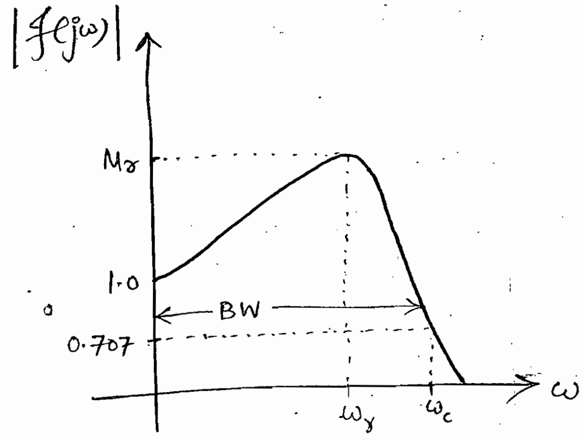
$$\therefore \omega = \omega_{cf} = \omega_n \text{ r/s}$$

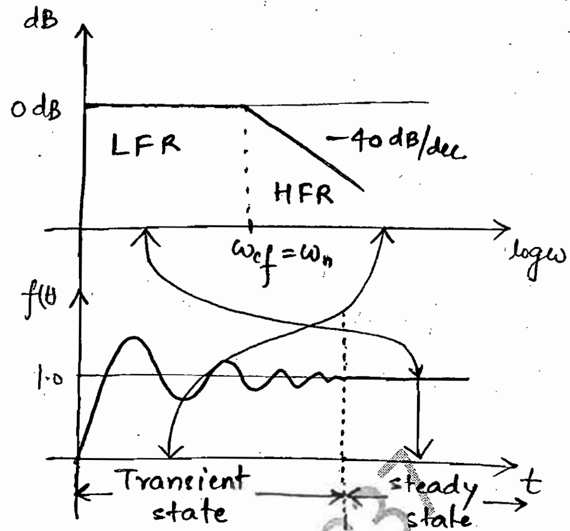
<u>Error at ω_{cf}</u>

At $\omega = \omega_{cf} = \omega_n$

$$-20\log\sqrt{(1-1)^2+(2\xi)^2} = -20\log(2\xi)$$







frequency domain specifications :

(1). Resonant frequency (ω_r) :

It is defined as the frequency at which the magnitude has max^m value.

$$|f(j\omega)| = \frac{1}{\sqrt{(1-u^2)^2+(2\xi u)^2}} \qquad \text{where, } u = \frac{\omega}{\omega_n}$$

At $\omega = \omega_r$, $u_r = \frac{\omega_r}{\omega_n}$

$$\frac{d}{du_r}\left[(1-u_r^2)^2 + (2\xi u_r)^2\right]^{-1/2} = 0$$

$$-\frac{1}{2}\left[(1-u_r^2)^2+(2\xi u_r)^2\right]^{-3/2} \cdot \frac{d}{du_r}\left[(1-u_r^2)^2+(2\xi u_r)^2\right] = 0$$

$$2(1-u_r^2)(-2u_r) + 4\xi^2 \cdot (2u_r) = 0$$

$$-4u_r + 4u_r^3 + 8\xi^2 u_r = 0$$

$$-1 + u_r^2 + 2\xi^2 = 0$$

$$u_r^2 = 1-2\xi^2$$

$$u_r = \sqrt{1-2\xi^2}$$

$$\therefore \quad \omega_r = \omega_n\sqrt{1-2\xi^2} \quad r/s$$

For "ω_r" to be real and +ve,

$$2\xi^2 < 1 \Rightarrow \boxed{\xi < \frac{1}{\sqrt{2}}}$$

For "ω_d" to be real and +ve,

$$\omega_d = \omega_n\sqrt{1-\xi^2} \Rightarrow \boxed{\xi < 1}$$

ω_r and ω_d are corelated.

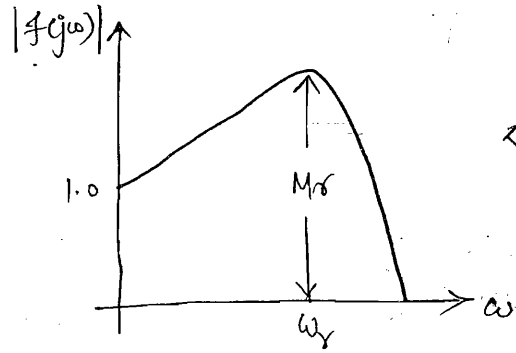
(2). <u>Resonant peak (or) Peak magnitude (M_r):</u>

It is the max^m value of magnitude occuring at resonant frequency ω_r.

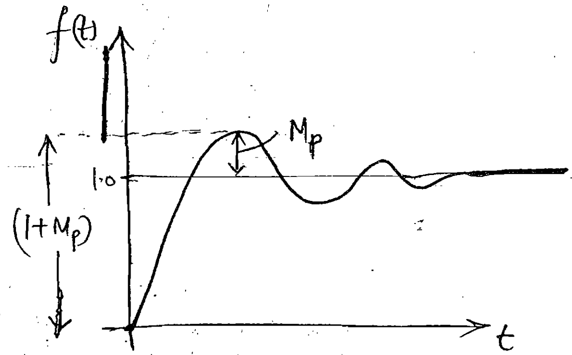
$$|f(j\omega)|_{\omega=\omega_r} = M_r$$

$$M_r = \frac{1}{\sqrt{\left[1-\left(\frac{\cancel{\omega_n}\sqrt{1-2\xi^2}}{\cancel{\omega_n}}\right)^2\right]^2 + \left(\frac{2\xi\cancel{\omega_n}\sqrt{1-2\xi^2}}{\cancel{\omega_n}}\right)^2}}$$

$$= \frac{1}{2\xi\sqrt{1-\xi^2}}$$



$\Leftrightarrow$



$\xi < \frac{1}{\sqrt{2}}$; $M_r > 1$

$\xi = \frac{1}{\sqrt{2}}$; $M_r = 1$

$\xi > \frac{1}{\sqrt{2}}$; No M_r

Q10.

$$M(j\omega) = \frac{100}{100-\omega^2 + 10\sqrt{2}\,j\omega}$$

$$= \frac{1}{1-\left(\frac{\omega}{10}\right)^2 + j\left(\frac{10\sqrt{2}\,\omega}{100}\right)}$$

$$= \frac{1}{1-\left(\frac{\omega}{10}\right)^2 + j\left(\frac{\sqrt{2}\,\omega}{10}\right)} = \frac{1}{1-\left(\frac{\omega}{\omega_n}\right)^2 + j\,2\xi\frac{\omega}{\omega_n}}$$

$\therefore \omega_n = 10$ rad/s & $\frac{2\xi}{\omega_n} = \frac{\sqrt{2}}{10} \Rightarrow \xi = \frac{1}{\sqrt{2}}$

$\Rightarrow M_r = 1$ (peak)

(3). Band width :

It is the range of frequencies over which the magnitude has a value of $\frac{1}{\sqrt{2}}$. It indicates the speed of response of the s/s.

Wider BW $\Rightarrow$ faster Response

$\boxed{BW \propto \frac{1}{t_r}}$ where $t_r \rightarrow$ rise time

(4). Cut-off frequency :

It is defined as the frequency at which the magnitude has the value of $1/\sqrt{2}$. It indicates the ability of the s/s to distinguish s/g from noise.

$$|F(j\omega)| = \frac{1}{\sqrt{(1-u^2)^2 + (2\xi u)^2}} \quad \text{where } u = \frac{\omega}{\omega_n}$$

At $\omega = \omega_c$ = cut off frequency

$$u = u_c = \frac{\omega_c}{\omega_n}$$

$$|F(j\omega)| = \frac{1}{\sqrt{(1-u_c^2)^2 + (2\xi u_c)^2}} = \frac{1}{\sqrt{2}}$$

$$(1-u_c)^2 + (2\xi u_c)^2 = 2$$

$$1+u_c^4 - 2u_c^2 + 4\xi^2 u_c^2 - 2 = 0$$

$$u_c^4 + u_c^2(4\xi^2-2) - 1 = 0$$

$$u_c^2 = \frac{-(4\xi^2-2) \pm \sqrt{(4\xi^2-2)^2 - 4(-1)}}{2}$$

$$u_c^2 = 1-2\xi^2 \pm \sqrt{4\xi^4 - 4\xi^2 + 2}$$

$$u_c = \sqrt{1-2\xi^2 + \sqrt{2-4\xi^2+4\xi^4}}$$

$$\omega_c \text{ (or) } BW = \omega_n \sqrt{1-2\xi^2 + \sqrt{2-4\xi^2+4\xi^4}} \quad \text{rad/s}$$

Linear approximation,

$$\omega_c \text{ (or) } BW = \omega_n(-1.19\xi + 1.85)$$

Frequency domain analysis of dead time (or) transportation lag

$$\frac{Y(s)}{X(s)} = F(s) = e^{-Ts}$$

Put $s = j\omega$

$$F(j\omega) = e^{-j\omega T} = \cos\omega T - j\sin\omega T$$

$$|e^{-j\omega T}| = \sqrt{\cos^2\omega T + \sin^2\omega T} = 1$$

$$\angle e^{-j\omega T} = \tan^{-1}\left(\frac{-\sin\omega T}{\cos\omega T}\right) = -\omega T \text{ (radians)}$$

$$= -\frac{\omega T}{\pi} \times 180^\circ \text{ (degrees)}$$

$$= -57.3\,\omega T \text{ degrees}$$

$$\boxed{e^{-j\omega T} = 1 \angle -57.3\,\omega T \text{ (degrees)}}$$

Stability from frequency response plots :

$$1 + G(s)H(s) = 0$$

$$G(s)H(s) = -1$$

Put $s = j\omega \Rightarrow G(j\omega)H(j\omega) = -1 + j0$ (critical point)

Stability criteria :

(1). Gain crossover frequency (ω_{gc}) :

$$\left|G(j\omega)H(j\omega)\right|_{\omega=\omega_{gc}} = 1 \text{ or } 0\,dB$$

(2). Phase crossover frequency (ω_{pc}) :

$$\angle G(j\omega)H(j\omega)\Big|_{\omega=\omega_{pc}} = -180^\circ$$ (min^m phase s/s)

(3). Gain Margin (GM): It is the "allowable gain".

$$\left|G(j\omega)H(j\omega)\right|_{\omega=\omega_{pc}} = x \; ; \; GM = \frac{1}{x}$$

$$GM(dB) = 20\log\left(\frac{1}{x}\right)$$

(4). Phase Margin (PM) : It is the "allowable phase lag".

$$\angle G(j\omega)H(j\omega)\Big|_{\omega=\omega_{gc}} = \phi \; ; \; PM = 180^\circ + \phi$$

Stable	$\Rightarrow$ GM & PM = +ve	$\Rightarrow$	$\omega_{gc} < \omega_{pc}$
Unstable	$\Rightarrow$ GM & PM = −ve	$\Rightarrow$	$\omega_{gc} > \omega_{pc}$
Marginally stable	$\Rightarrow$ GM = PM = 0	$\Rightarrow$	$\omega_{gc} = \omega_{pc}$

GM & PM for IInd order s/s :

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$G(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2-\omega_n^2} = \frac{\omega_n^2}{s^2+2\xi\omega_n s}$$

$$G(s) = \frac{\omega_n^2}{s(s+2\xi\omega_n)}$$

$$G(j\omega) = \frac{\omega_n^2}{j\omega(j\omega+2\xi\omega_n)} = \frac{\omega_n^2+j0}{(0+j\omega)(j\omega+2\xi\omega_n)}$$

$$|G(j\omega)| = \frac{\omega_n^2}{\omega\sqrt{\omega^2+4\xi^2\omega_n^2}}$$

$$\angle G(j\omega) = \frac{0^\circ}{(90^\circ)\cdot\tan^{-1}\left(\frac{\omega}{2\xi\omega_n}\right)} = -90^\circ-\tan^{-1}\left(\frac{\omega}{2\xi\omega_n}\right)$$

$$G(j\omega) = \frac{\omega_n^2}{-\omega^2+2\xi\omega_n j\omega} = \frac{\omega_n^2+j0}{-\omega^2+j\,2\xi\omega\omega_n}$$

$$\angle G(j\omega) = \frac{0^\circ}{180^\circ-\tan^{-1}\left(\frac{2\xi\omega_n}{\omega}\right)} = -180^\circ+\tan^{-1}\left(\frac{2\xi\omega_n}{\omega}\right)$$

At $\omega=\omega_{pc}=\infty$ rad/s $\Rightarrow \angle G(j\omega) = -180^\circ$

$$|G(j\omega)|_{\omega=\omega_{pc}=\infty} = x = 0$$

$$GM = \frac{1}{x} = \frac{1}{0} = \infty$$

$$\therefore \boxed{\begin{array}{l}\omega_{pc} = \infty \text{ rad/s} \\ GM = \infty\end{array}}$$

At $\omega = \omega_{gc}$

$$\frac{\omega_n^2}{\omega\sqrt{\omega^2+4\xi^2\omega_n^2}} = 1$$

$$\omega_n^4 = \omega^2\left(\omega^2+4\xi^2\omega_n^2\right)$$

$$\omega^4 + \omega^2 4\xi^2\omega_n^2 - \omega_n^4 = 0$$

$$\omega^2 = \frac{-4\xi^2\omega_n^2 \pm \sqrt{16\xi^4\omega_n^4+4\omega^4}}{2}$$

$$\omega^2 = -2\xi^2\omega_n^2 \pm \omega_n^2\sqrt{4\xi^4+1}$$

$$\omega^2 = -2\xi^2\omega_n^2 + \omega_n^2\sqrt{4\xi^4+1}$$

$$\boxed{\omega = \omega_{gc} = \omega_n\sqrt{-2\xi^2+\sqrt{4\xi^4+1}}}\ \text{rad/s}$$

$$\angle G(j\omega)\Big|_{\omega=\omega_{gc}} = \phi = -90^\circ - \tan^{-1}\left(\frac{\cancel{\omega_n}\cdot\sqrt{-2\xi^2+\sqrt{4\xi^4+1}}}{2\xi\cancel{\omega_n}}\right)$$

$$PM = 180^\circ + \phi$$

$$\boxed{PM = 90^\circ - \tan^{-1}\left(\frac{\sqrt{-2\xi^2+\sqrt{4\xi^4+1}}}{2\xi}\right)}$$

$$\angle G(j\omega)\Big|_{\omega=\omega_{gc}} = \phi = -180^\circ + \tan^{-1}\left[\frac{2\xi\cancel{\omega_n}}{\cancel{\omega_n}\sqrt{-2\xi^2+\sqrt{4\xi^4+1}}}\right]$$

$$PM = 180^\circ + \phi$$

$$\boxed{PM = \tan^{-1}\left[\frac{2\xi}{\sqrt{-2\xi^2+\sqrt{4\xi^4+1}}}\right]}$$

Approximation : $\boxed{PM = 100\,\xi}$

Q3. $\% M_p = 50\% \Rightarrow M_p = 0.5$

$$e^{-\pi\xi/\sqrt{1-\xi^2}} = 0.5$$

$$\Rightarrow \xi = 0.215$$

$$T = \frac{1}{f_d} = 0.2 \text{ sec.} \Rightarrow f_d = 5 \text{ Hz}$$

$$\omega_d = 2\pi f_d = 2\pi(5) = 10\pi = 31.41 \text{ rad/s}$$

$$\omega_n\sqrt{1-\xi^2} = 31.41 \text{ rad/s}$$

$$\omega_n\sqrt{1-(0.215)^2} = 31.41 \text{ rad/s}$$

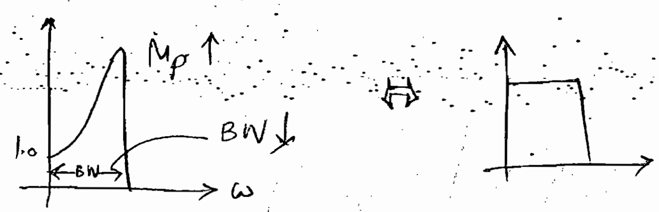
$$\Rightarrow \omega_n = 32.16 \text{ rad/sec}$$

$$\omega_r = \omega_n\sqrt{1-2\xi^2}$$

$$= 32.16\sqrt{1-2(0.215)^2}$$

$$\therefore \omega_r = 30.63 \text{ rad/s}$$

Q4. Sharp cut off characteristics $\Rightarrow$ less BW



Q5. $$1 + \frac{100}{s(s+10)} = 0$$

$$s^2 + 10s + 100 = 0 \Rightarrow \omega_n = 10 \text{ rad/s}$$

$$2\xi\omega_n = 10 \Rightarrow \xi = 0.5$$

$$\omega_r = \omega_n\sqrt{1-2\xi^2} = 10\sqrt{1-2(0.5)^2} = 7.07 \text{ rad/s}$$

$$BW = \omega_n\left[-1.19\xi + 1.85\right] = 10(-1.19 \times 0.5 + 1.85)$$

$$= 12.7 \text{ rad/s}$$

Q6. $G(s) = \dfrac{2\sqrt{3}}{s(s+1)} \Rightarrow G(j\omega) = \dfrac{2\sqrt{3}}{j\omega(j\omega+1)}$

$$G(j\omega) = \frac{2\sqrt{3}}{\omega\sqrt{1+\omega^2}} \angle -90^\circ - \tan^{-1}\omega$$

At $\omega = \omega_{gc}$,

$$\frac{2\sqrt{3}}{\omega\sqrt{1+\omega^2}} = 1 \Rightarrow \omega^2(\omega^2+1) = 12$$

$$\omega^4 + \omega^2 - 12 = 0$$

$$\omega^2 = \frac{-1 \pm \sqrt{1+48}}{2}$$

$$\omega^2 = -0.5 \pm 3.5$$

$$\omega^2 = -4, 3$$

$$\omega = \omega_{gc} = \sqrt{3} \text{ rad/s.}$$

$$\angle G(j\omega)\Big|_{\omega=\omega_{gc}=\sqrt{3} \text{ r/s}} = \phi$$

$$\phi = -90^\circ - \tan^{-1}\sqrt{3} = -90^\circ - 60^\circ = -150^\circ$$

$$PM = 180^\circ + \phi = 180^\circ - 150^\circ = 30^\circ$$

Short cut method: $1 + \dfrac{2\sqrt{3}}{s(s+1)} = 0$

$$s^2 + s + 3.4 = 0 \Rightarrow \omega_n = \sqrt{3.4} = 1.86 \text{ rad/s}$$

$$2\xi \times 1.86 = 1 \Rightarrow \xi = 0.27$$

$$PM = 100\xi = 100(.27) \approx 27^\circ = 30^\circ.$$

Q7. $G(s) = \dfrac{as+1}{s^2}$ $\qquad \dfrac{1}{\sqrt{\sqrt{2}}} = 0.~~84~~ \approx 0.9$

$$1 + \frac{as+1}{s^2} = 0 \Rightarrow s^2 + as + 1 = 0$$

$$\omega_n = 1 \text{ rad/s}$$

$$\xi = \frac{a}{2}$$

given: $PM = 45^\circ$, $PM = 100\xi \Rightarrow \xi = 0.45$ $\boxed{\therefore a = 0.9}$

Q9. $G(s) = \dfrac{3e^{-2s}}{s(s+2)}$

$$G(j\omega) = \frac{3\,e^{-2j\omega}}{j\omega(j\omega+2)}$$

$$|G(j\omega)| = \frac{3(1)}{\omega\sqrt{\omega^2+4}}$$

$$\angle G(j\omega) = \frac{(-57.3\times 2\omega)}{(90^\circ)\ \tan^{-1}(\omega/2)} = -90^\circ - 114.6\,\omega - \tan^{-1}\left(\frac{\omega}{2}\right)$$

(degrees)

At $\omega = \omega_{gc}$,

$$\frac{3}{\omega\sqrt{\omega^2+4}} = 1$$

$$9 = \omega^2(\omega^2+4)$$

$$\omega^4 + 4\omega^2 - 9 = 0$$

$$\Rightarrow \quad \omega^2 = -5.6,\ 1.6$$

$$\omega^2 = 1.6 \quad \Rightarrow \quad \omega = \omega_{gc} = \sqrt{1.6} = 1.26 \text{ rad/s}$$

$$\angle G(j\omega)\Big|_{\omega=\omega_{gc}} = \phi = -90^\circ - 114.6(1.26) - \tan^{-1}\left(\frac{1.26}{2}\right)$$

$$= -267.5^\circ$$

$$PM = 180^\circ + \phi$$

$$= 180^\circ - 267.5^\circ$$

$$= -87.5^\circ$$

for ω_{pc}, Hit and trial method,

$$\angle G(j\omega) = -90^\circ - 114.6\omega - \tan^{-1}\left(\frac{\omega}{2}\right)$$

Put $\omega = 0.63$ rad/s

$$\angle G(j\omega) = -90^\circ - 114.6(0.63) - \tan^{-1}(0.63/2) = -179.88^\circ$$

$$\approx -180^\circ.$$

$\therefore\ \omega = \omega_{pc} = 0.63$ rad/s

Q8. $G(s) = \dfrac{1}{s(s^2+s+1)}$

$$G(j\omega) = \frac{1}{j\omega(1-\omega^2+j\omega)}$$

$$|G(j\omega)| = \frac{1}{\omega\sqrt{(1-\omega^2)^2+\omega^2}} \quad ; \quad \angle G(j\omega) = -90^\circ - \tan^{-1}\left(\frac{\omega}{1-\omega^2}\right)$$

for ω_{pc},

$$\angle G(j\omega) = -90^\circ - \tan^{-1}\left(\frac{\omega}{1-\omega^2}\right) = -180^\circ$$

$$\tan^{-1}\left(\frac{\omega}{1-\omega^2}\right) = 90^\circ \Rightarrow \omega^2 - 1 = 0$$

$$\Rightarrow \omega_{pc} = 1 \text{ rad/s}$$

$$|G(j\omega)|_{\omega=\omega_{pc}} = x = \frac{1}{1\sqrt{(1-1)^2+1^2}} = 1$$

$$GM = \frac{1}{x} = 1 \Rightarrow GM(dB) = 20\log(1) = 0 \text{ dB}$$

Q12. $G(s) = \dfrac{s+5}{s^2+4s+9}$

$$G(j\omega) = \frac{j\omega+5}{9-\omega^2+j4\omega}$$

$$\angle G(j\omega)\Big|_{\omega<3} = \tan^{-1}\left(\frac{\omega}{5}\right) - \tan^{-1}\left(\frac{4\omega}{9-\omega^2}\right)$$

$$\angle G(j\omega)\Big|_{\omega>3} = \tan^{-1}\left(\frac{\omega}{5}\right) - 180^\circ + \tan^{-1}\left(\frac{4\omega}{9-\omega^2}\right)$$

$$\angle G(j\omega)\Big|_{\omega=0} = 0^\circ - 0^\circ = 0^\circ$$

$$\angle G(j\omega)\Big|_{\omega=\infty} = 90^\circ - 180^\circ + 0^\circ = -90^\circ$$

$\therefore \angle G(j\omega)$ varies from $0^\circ \to -90^\circ$

Q20. $G(s) = \dfrac{e^{-Ts}}{s(s+1)}$; $G(j\omega) = \dfrac{e^{-j\omega T}}{j\omega(j\omega+1)}$

$$\angle G(j\omega) = -\omega T - \frac{\pi}{2} - \tan^{-1}\omega \quad \text{rad.}$$

$$\angle G(j\omega)\Big|_{\omega=\omega_1} = 0$$

$$-\frac{\pi}{2} - \omega_1 T - \tan^{-1}\omega_1 = 0$$

$$-\tan^{-1}\omega_1 = \frac{\pi}{2} + \omega_1 T$$

$$-\omega_1 = \tan\left(\frac{\pi}{2} + \omega_1 T\right)$$

$$-\omega_1 = -\cot\omega_1 T$$

$$\therefore \omega_1 = \cos\omega_1 T$$

Conv. Q2. $1 + \dfrac{K}{s(s+a)} = 0 \Rightarrow s^2 + as + K = 0$

$K = \omega_n^2$, $a = 2\xi\omega_n$

Given: $M_r = 1.04$

$$\frac{1}{2\xi\sqrt{1-\xi^2}} = 1.04 \Rightarrow \xi = 0.6,\ 0.8$$

$\therefore \xi = 0.6$ $(\because \xi < 0.707)$

$\omega_r = 11.55$ r/s

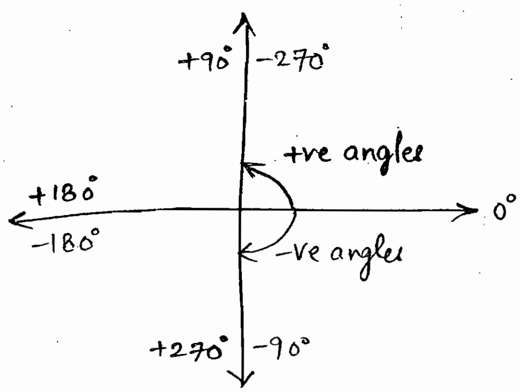
$$\omega_n\sqrt{1-2\xi^2} = 11.55 \Rightarrow \omega_n = 22.2 \text{ rad/s}$$

$\therefore$ $K = (22.2)^2 = 492.84$

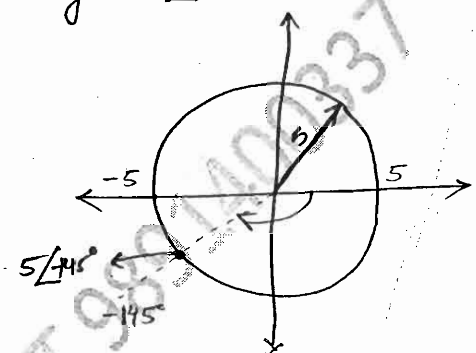
$a = 2(0.6)(22.2) = 26.64$

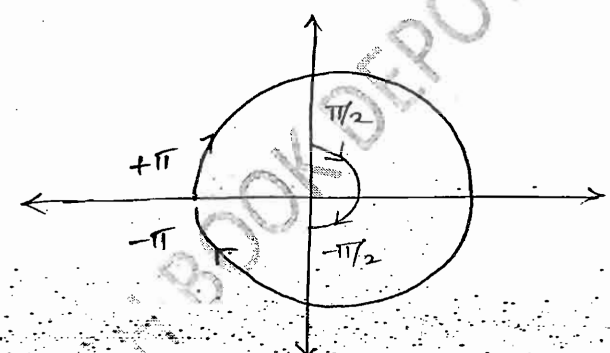
Polar Plots :

It is a plot of absolute values of magnitude and phase angle in degrees of open loop TF $[G(j\omega)H(j\omega)]$ Vs ω drawn on polar coordinates.



Eg. $5\angle -145°$





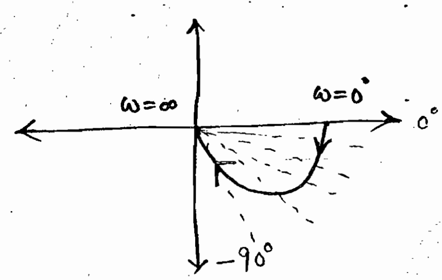
$$G(s) = \frac{1}{s+1} = \frac{1}{1+s}$$

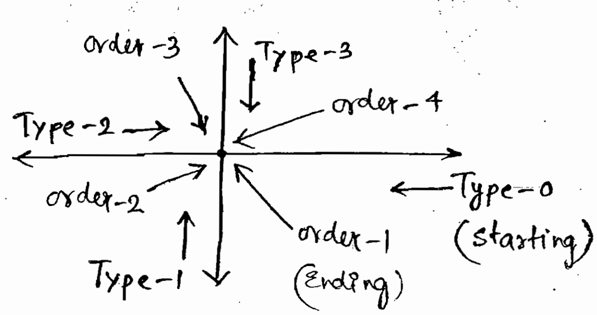
$$G(j\omega) = \frac{1}{1+j\omega}$$

$$|G(j\omega)| = \frac{1}{\sqrt{1+\omega^2}}$$

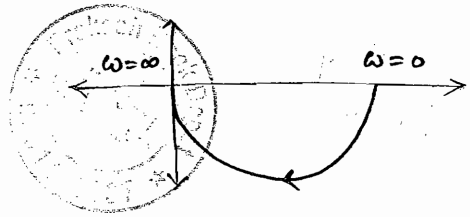
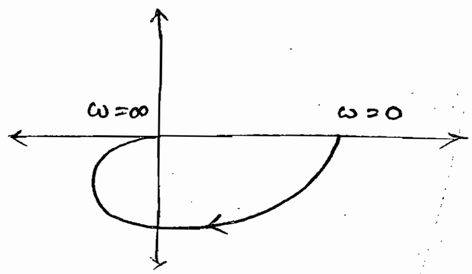
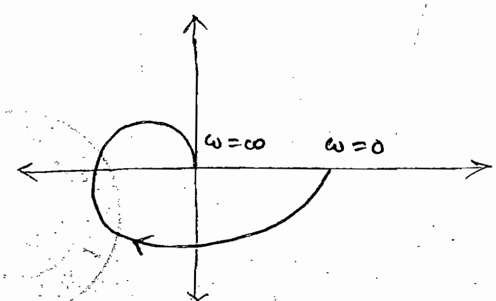
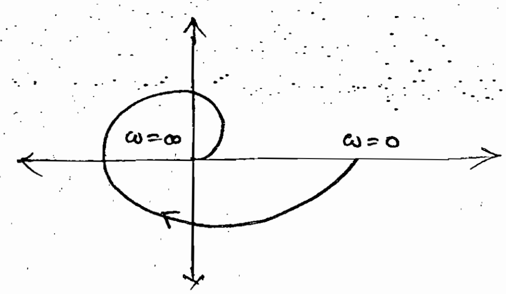
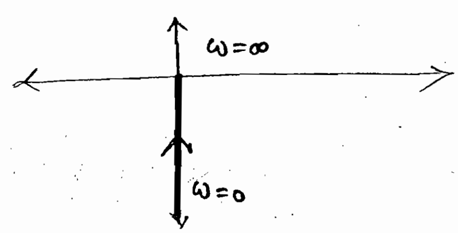
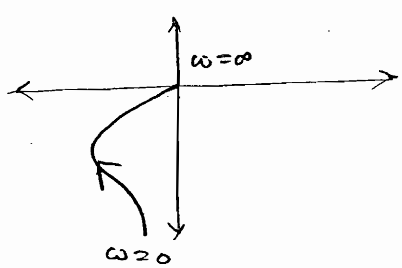
$$\angle G(j\omega) = -\tan^{-1}\omega$$

ω	0		∞
$\lvert G(j\omega)\rvert$	1		0
$\angle G(j\omega)$	0°		−90°



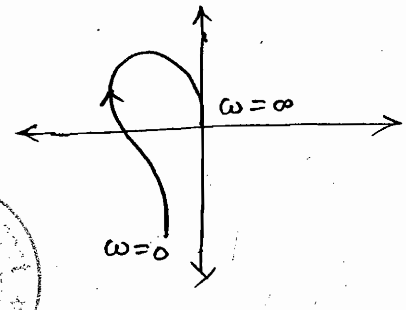


General shapes of polar plots:

Type / order	Polar plot
(1). **Type-0 / order-1** $G(s) = \frac{1}{(1+Ts)}$	
(2). **Type-0 / order-2** $G(s) = \frac{1}{(1+T_1 s)(1+T_2 s)}$	
(3). **Type-0 / order-3** $G(s) = \frac{1}{(1+T_1 s)(1+T_2 s)(1+T_3 s)}$	
(4). **Type-0 / order-4** $G(s) = \frac{1}{(1+T_1 s)(1+T_2 s)(1+T_3 s)(1+T_4 s)}$	
(5). **Type-1 / order-1** $G(s) = \frac{1}{s} = \frac{1}{j\omega} = \frac{1}{\omega}\angle -90^\circ$	
(6). **Type-1 / order-2** $G(s) = \frac{1}{s(1+Ts)}$	

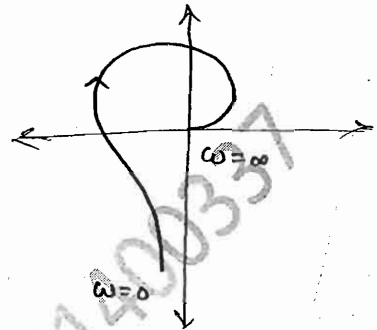
(7) · Type -1 / order-3

$$G(s) = \frac{1}{s(1+T_1 s)(1+T_2 s)}$$



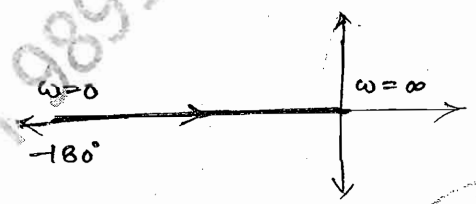
(8). Type-1 / order -4

$$G(s) = \frac{1}{s(1+T_1 s)(1+T_2 s)(1+T_3 s)}$$



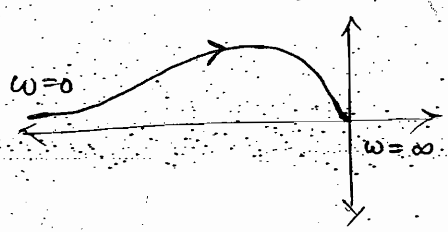
(9). Type -2 / order -2

$$G(s) = \frac{1}{s^2} = \frac{1}{(j\omega)^2} = \frac{1}{\omega^2} \angle -180^\circ$$



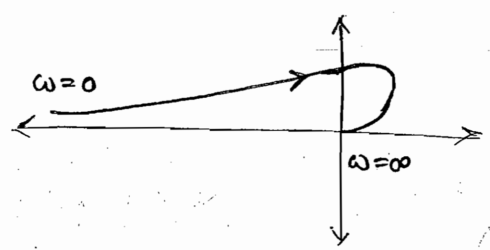
(10). Type -2 / order - 3

$$G(s) = \frac{1}{s^2(1+Ts)}$$



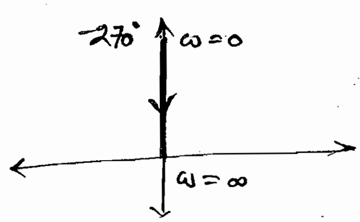
(11). Type - 2 / order - 4

$$G(s) = \frac{1}{s^2(1+T_1 s)(1+T_2 s)}$$



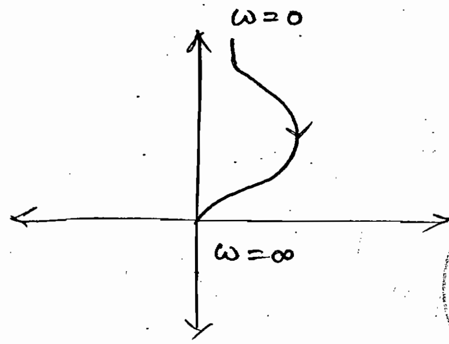
(12). Type- 3 / order - 3

$$G(s) = \frac{1}{s^3} = \frac{1}{(j\omega)^3} = \frac{1}{\omega^3} \angle -270^\circ$$



(13). Type-4 / order-4

$$G(s) = \frac{1}{s^3(1+Ts)}$$



Effect on adding zeroes on shape of polar plots:

for minimum or non-minimum phase functions when zeroes are present in the TF, it is necessary to check the intersection of polar plot with -ve real axis b/w ω=0 and ω=∞ points.

conv Q4. $G(s) = \frac{s+2}{(s+1)(s-1)} = \frac{-2(1+0.5s)}{(1+s)(1-s)}$

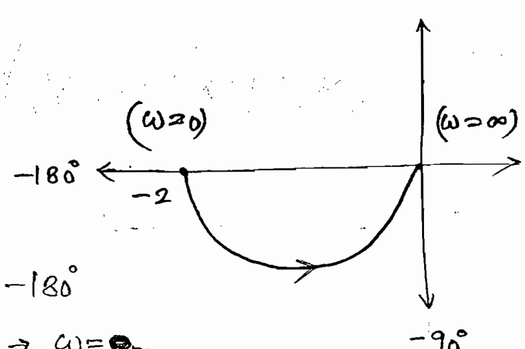
$$G(j\omega) = \frac{-2(1+0.5j\omega)}{(1+j\omega)(1-j\omega)}$$

<u>Note:</u> -ve gain $(-K+j0)$ contributes $-180°$ for all "ω".

$$|G(j\omega)| = \frac{2\sqrt{1+(0.5\omega)^2}}{(1+\omega^2)}$$

$$\angle G(j\omega) = \frac{(-180°)(\tan^{-1}(0.5\omega))}{(\tan^{-1}\omega)(-\tan^{-1}\omega)} = -180° + \tan^{-1}(0.5\omega)$$

ω	0		∞
$\lvert G(j\omega)\rvert$	2		0
$\angle G(j\omega)$	-180°		-90°



<u>Checking:</u> $-180° + \tan^{-1}(0.5\omega) = -180°$

$\tan^{-1}(0.5\omega) = 0 \Rightarrow \omega = 0$.
(already we have)

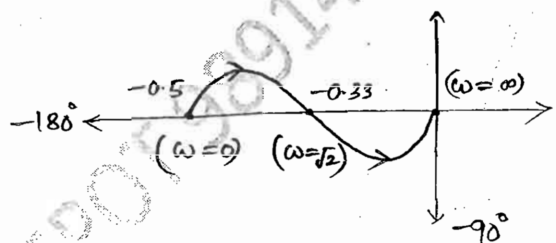
Now, $G(s) = \dfrac{s+2}{(s+1)(s-4)} = \dfrac{-0.5\,(1+0.5s)}{(1+s)(1-0.25s)}$

$$G(j\omega) = \frac{-0.5\,(1+0.5j\omega)}{(1+j\omega)(1-0.25j\omega)}$$

$$|G(j\omega)| = \frac{0.5\sqrt{1+(0.5\omega)^2}}{\sqrt{1+\omega^2}\,\sqrt{1+(0.25\omega)^2}}$$

$$\angle G(j\omega) = \frac{(-180^\circ)(\tan^{-1}0.5\omega)}{(\tan^{-1}\omega)(-\tan^{-1}0.25\omega)} = -180^\circ - \tan^{-1}\omega + \tan^{-1}0.5\omega + \tan^{-1}0.25\omega$$

ω	0	$\sqrt{2}$	∞
$\lvert G(j\omega)\rvert$	0.5	0.33	0
$\angle G(j\omega)$	-180°	-180°	90°



Checking: $-180^\circ - \tan^{-1}\omega + \tan^{-1}0.5\omega + \tan^{-1}0.25\omega = -180^\circ$

$$\tan^{-1}\omega = \tan^{-1}0.5\omega + \tan^{-1}0.25\omega$$

$$\omega = \frac{0.5\omega + 0.25\omega}{1 - 0.125\omega^2}$$

$$1 - 0.125\omega^2 = 0.75 \Rightarrow \omega = \omega_{pc} = \sqrt{2}\ \text{rad/s}$$

$$|G(j\omega)|_{\omega=\omega_{pc}=\sqrt{2}\ r/s} = x = \frac{0.5\sqrt{1+(0.5\times\sqrt{2})^2}}{\sqrt{1+(\sqrt{2})^2}\,\sqrt{1+(0.25\times\sqrt{2})^2}} = 0.33$$

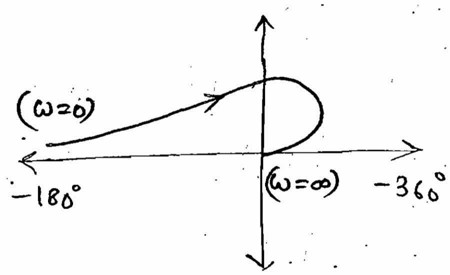
Note: For type-2 and type-3 min^m phase fns when zeroes are added before the location of poles the polar plot intersects the −ve real axis as many time as there are zeroes.

eg. $G(s) = \dfrac{1}{s^2(1+s)(1+2s)}$

$\angle G(j\omega) = -180^\circ - \tan^{-1}\omega - \tan^{-1}2\omega$

$\omega = 0$; $|G(j\omega)| = \infty$; $\angle G(j\omega) = -180^\circ$

$\omega = \infty$; $|G(j\omega)| = 0$; $\angle G(j\omega) = -360^\circ$



Now,

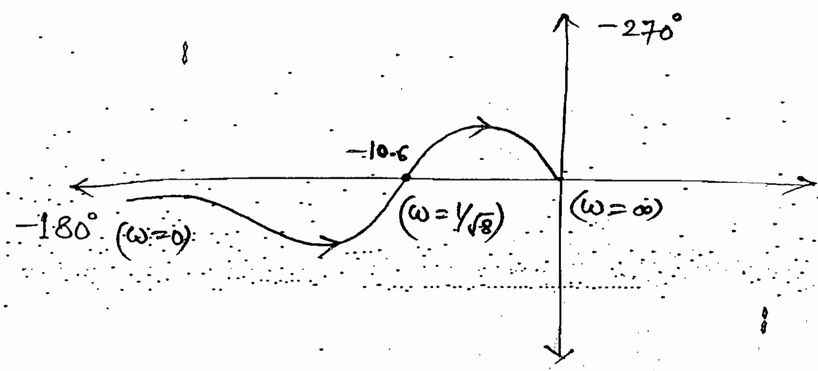
$G(s) = \dfrac{(1+4s)}{s^2(1+s)(1+2s)}$ $\{(1+4s)$ is added$\}$ – adding zero

$s = 0, -1, -0.5$ (poles)
$s = -0.25$ (zero)

$\angle G(j\omega) = -180^\circ - \tan^{-1}\omega - \tan^{-1}2\omega + \tan^{-1}4\omega$

$\omega = 0$; $|G(j\omega)| = \infty$; $\angle G(j\omega) = -180^\circ$

$\omega = \infty$; $|G(j\omega)| = 0$; $\angle G(j\omega) = -270^\circ$



checking: $-180^\circ - \tan^{-1}\omega - \tan^{-1}2\omega + \tan^{-1}4\omega = -180^\circ$

$\tan^{-1}4\omega = \tan^{-1}\omega + \tan^{-1}2\omega$

$4\omega = \dfrac{\omega + 2\omega}{1 - 2\omega^2}$

$4 - 8\omega^2 = 3 \Rightarrow \omega = \omega_{pc} = \dfrac{1}{\sqrt{8}}$ rad/s

$$|G(j\omega)|_{\omega=\omega_{pc}=1/\sqrt{8}} = x = \frac{\sqrt{1+(4/\sqrt{8})^2}}{\left(\frac{1}{\sqrt{8}}\right)^2 \sqrt{1+\left(\frac{1}{\sqrt{8}}\right)^2}\sqrt{1+\left(\frac{2}{\sqrt{8}}\right)^2}}$$

$= 10.6$

Stability from polar plots :

(1). Stable :

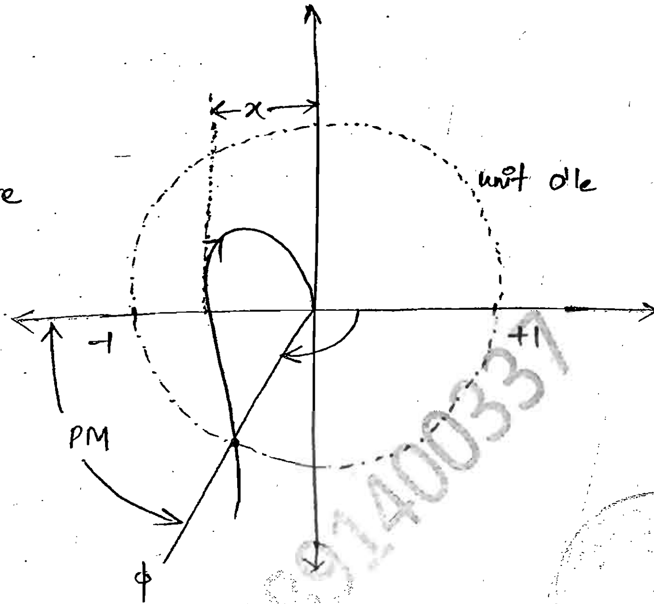
$GM = \frac{1}{x}$ $\quad (\because x < 1)$

$GM(dB) = 20\log\left(\frac{1}{x}\right) = +ve$

$PM = \phi - (-180^\circ)$

$= \phi + 180^\circ$

$= +ve$



(2). Marginally stable :

$x = 1$

$GM = \frac{1}{x} = 1$

$GM(dB) = 20\log x$

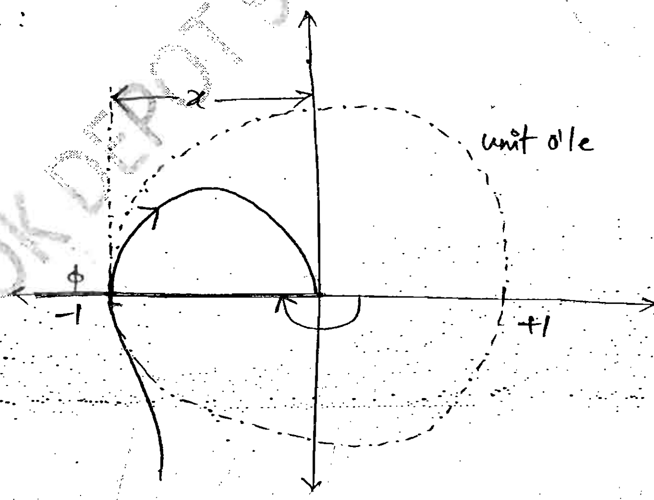
$= 20\log(1)$

$= 0 \text{ dB}$

$PM = 180^\circ + \phi$

$= 180^\circ - 180^\circ$

$= 0^\circ$



(3). Un-stable :

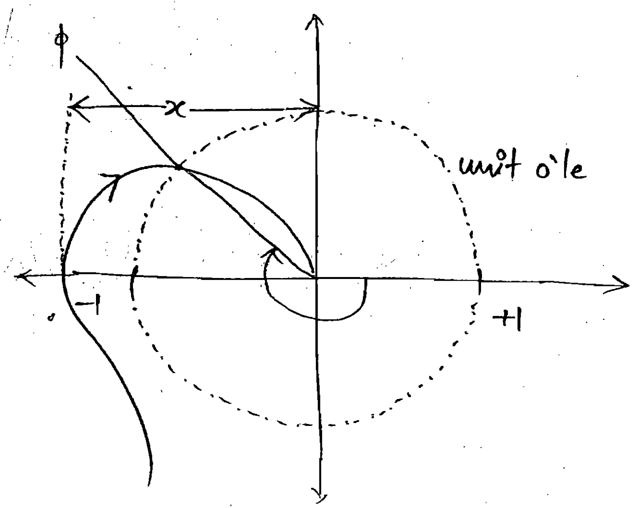
$x > 1$

$GM = \frac{1}{x}$

$GM(dB) = 20\log\left(\frac{1}{x}\right) = -ve$

$PM = 180^\circ + \phi$

$= -ve$



Concept of Enclosement and Encirclement :

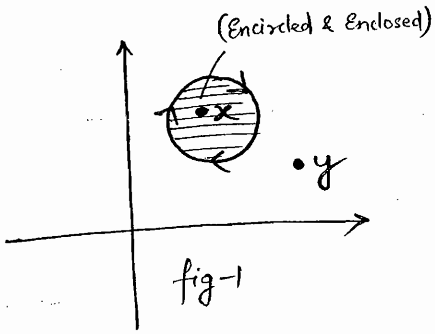


fig-1

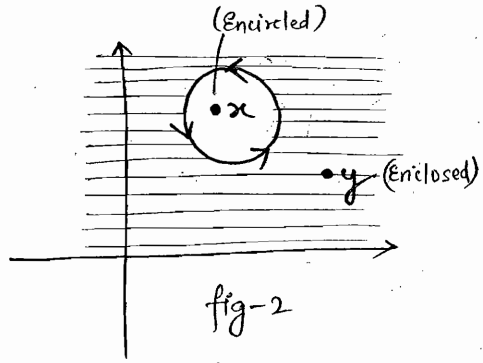


fig-2

(1). A point is said to be enclosed by a contour (ie shape) if it lies to the right side of the dirn of the contour.

(2). A point is said to be encircled if the contour is a closed path.

(3). In fig-2, point y is said to be enclosed whereas point x is said to be encircled in anti clockwise dirn.

(4). In polar plots if the critical point $(-1+j0)$ is not enclosed then the s/s is said to be stable.

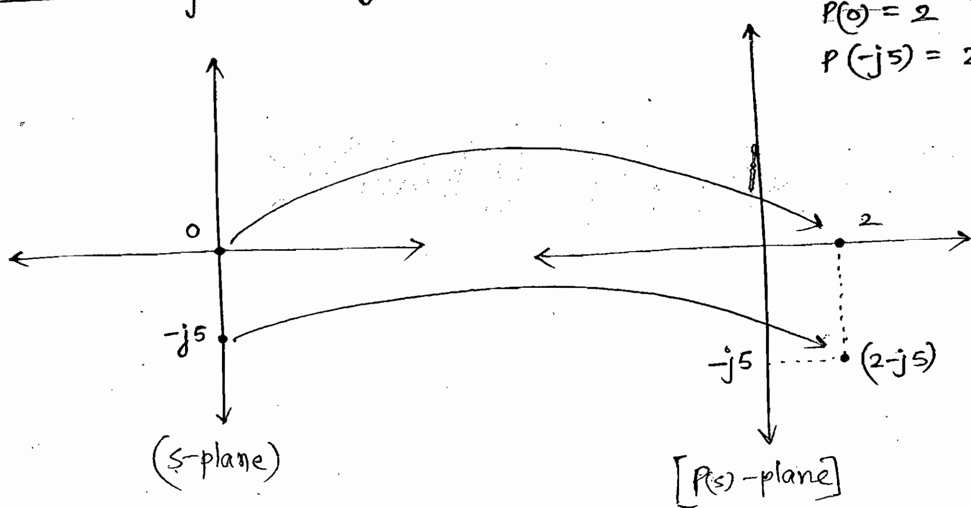
Theory of Nyquist plots :

I. Principle of Mapping :

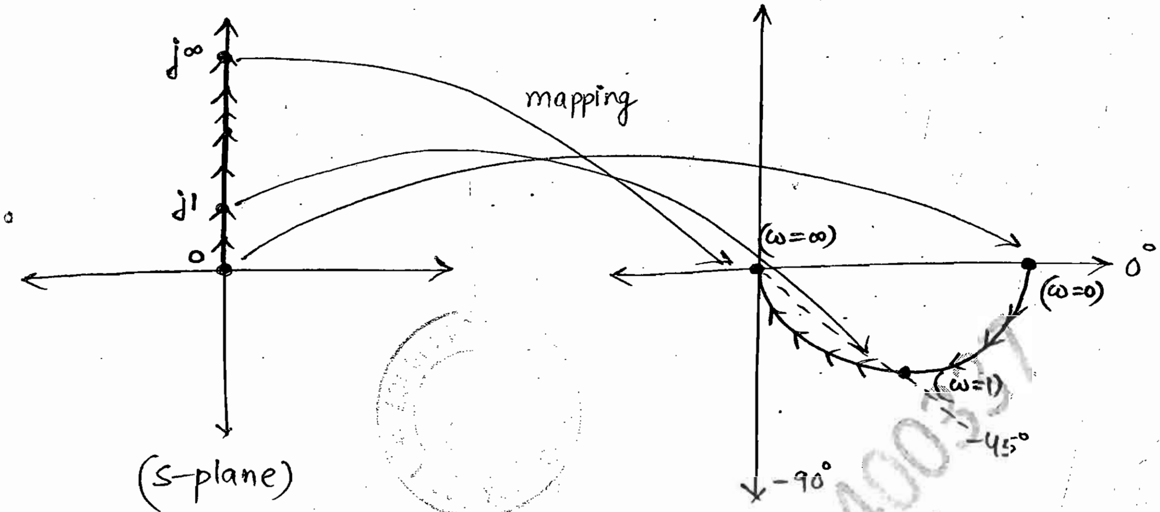
$P(s) = s+2$

$P(0) = 2$

$P(-j5) = 2-j5$



Eg. Polar plots.

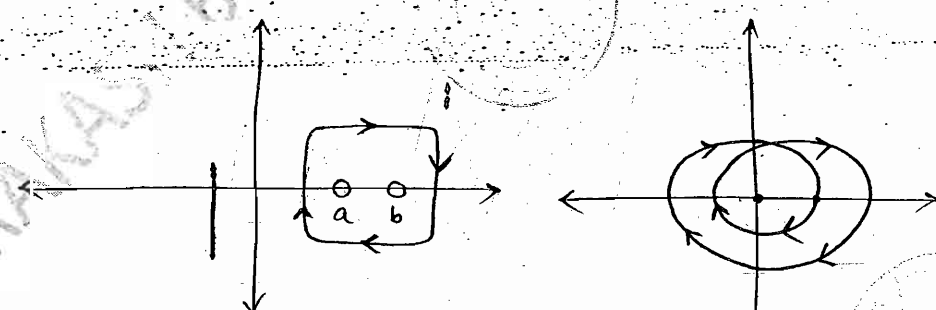


$$P(s) = G(s) = \frac{1}{s+1} = \frac{1}{\sqrt{1+\omega^2}} \angle -\tan^{-1}\omega$$

The mapping theorem states that any point of s-plane will get mapped on corresponding P(s)-plane where P(s) is any function of s.

II. Principle of Argument:

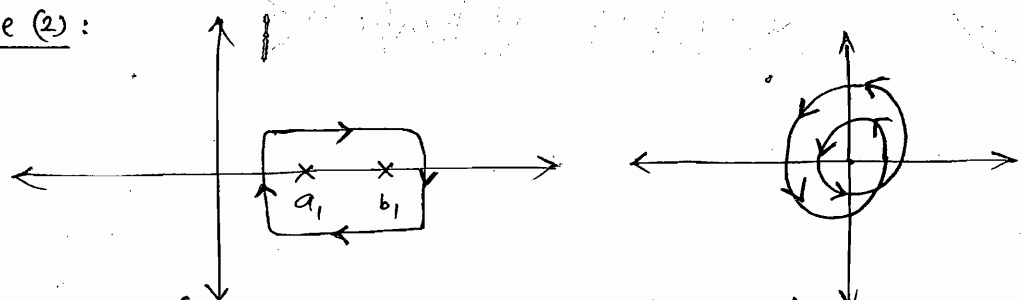
Case (1):



(s-plane)

$P(s) = (s-a)(s-b)$

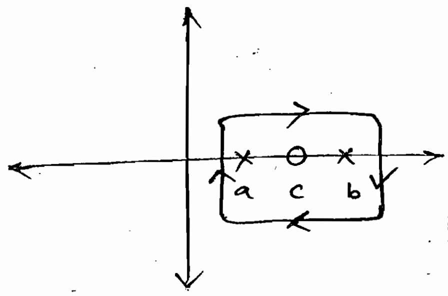
Case (2):



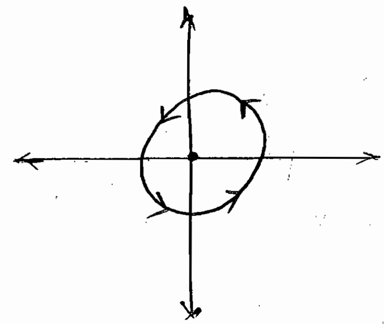
(s-plane)

$P(s) = \dfrac{1}{(s-a_1)(s-b_1)}$

Case -(3):



(s-plane)



P(s) -plane

$$P(s) = \frac{(s-c)}{(s-a)(s-b)}$$

$N = P - Z$

$N \rightarrow$ No. of encirclements

= +ve (anticlockwise)

= −ve (clockwise)

$P \rightarrow$ No. of poles in RHS of s-plane

$Z \rightarrow$ No. of zeroes in RHS of s-plane

(1). The principle of argument may be stated as "if the s-plane closed contour encloses P poles and Z zeroes $(P > Z)$ in RHS of s-plane, then the origin of P(s)-plane is encircled $(P-Z)$ times in anticlockwise dir^n".

<u>Nyquist stability criteria</u> :

$$G(s)H(s) = \frac{K(s \pm z_1)}{s(s \pm P_1)}$$

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K(s \pm z_1)}{s(s \pm P_1)} = 0$$

$$\frac{s(s\pm P_1)+K(s\mp Z_1)}{s(s\pm P_1)}=0 \quad \text{--- (1)}$$

$s(s\pm P_1)+K(s\mp Z_1)$ → CL poles

$s(s\pm P_1)$ → O.L poles

Applying $N=P-Z$ to eqn (1).

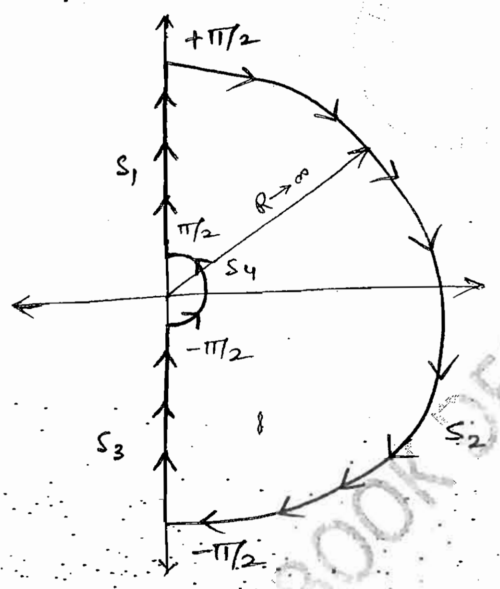
P = No. of OL poles in RHS of s-plane

Z = No. of CL poles in RHS of s-plane

(1). $Z=0$

(2). $N=P$

Nyquist path



To map s_1 :

Polar plot

To map s_2 :

Put $s=\lim_{R\to\infty} R\cdot e^{j\theta}$.

$\theta=[+\pi/2 \to -\pi/2]$

To map s_3 :

Inverse polar plot

To map s_4 :

Put $s=\lim_{r\to 0} r e^{j\theta}$

$\theta=[-\pi/2 \to \pi/2]$

Q. without constructing Nyquist plot find the no. of encirclement about $-1+j0$?

$$G(s)=\frac{10}{(s+2)(s+5)}$$

Sol.

$$1+G(s)=0 \Rightarrow 1+\frac{10}{(s+2)(s+5)}=0 \Rightarrow s^2+7s+20=0.$$

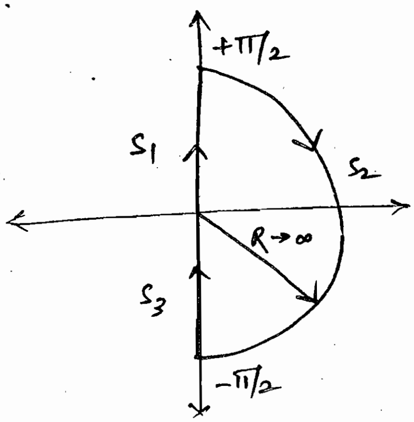
s^2	1	20
s^1	7	0
s^0	20	

$\Rightarrow P=0$
$\Rightarrow Z=0$

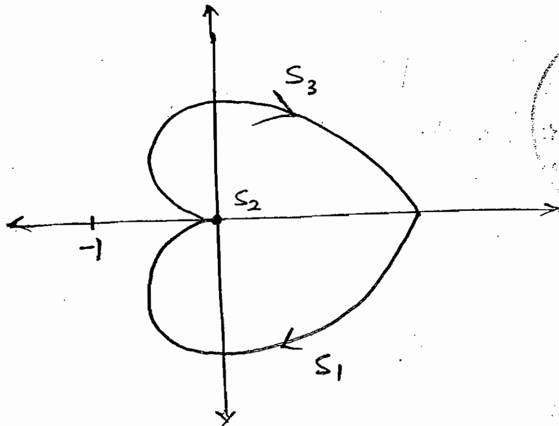
$N=P-Z$
$=0-0$
$=0$.

Q. Plot Nyquist of $G(s) = \dfrac{10}{(s+2)(s+5)}$.

Sol:



(s-plane)



$G(s) = \dfrac{10}{(s+2)(s+5)}$

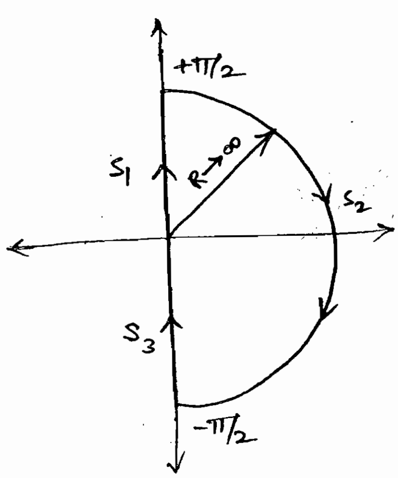
To map s_2: $G(s) \approx \dfrac{10}{s^2}$

$$= \lim_{R\to\infty} \frac{10}{(Re^{j\theta})^2}$$

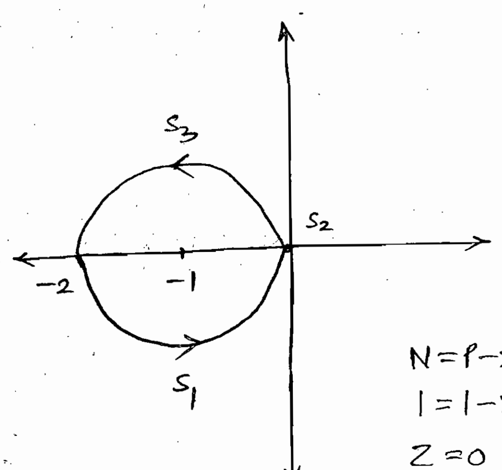
$$= \lim_{R\to 0} \frac{10}{R^2} \cdot e^{-2j\theta} = 0 \cdot e^{-2j\theta}$$

$N = 0$, $P = 0$ } $N = P - Z \Rightarrow Z = 0$ (STABLE s/s)

Q. $G(s) = \dfrac{s+2}{(s+1)(s-1)}$



(s-plane)



$G(s) = \dfrac{(s+2)}{(s+1)(s-1)}$

$N = P - Z$

$1 = 1 - Z$

$Z = 0$

(STABLE s/s)

CHECKING:

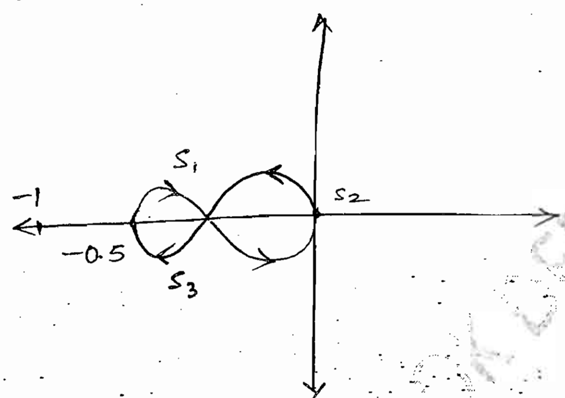
$$1 + \frac{s+2}{(s+1)(s-1)} = 0$$

$$s^2 - 1 + s + 2 = 0$$

$$s^2 + s + 1 = 0$$

$$\begin{array}{c|cc} s^2 & 1 & 1 \\ s^1 & 1 & 0 \\ s^0 & 1 & 0 \end{array} \Rightarrow \left.\begin{array}{l} Z = 0 \\ P = 1 \end{array}\right\} \begin{array}{l} N = P - Z \\ \quad = 1 - 0 \\ \quad = 1 \end{array}$$

Q. $G(s) = \dfrac{s+2}{(s+1)(s-4)}$



$N = P - Z$

$0 = 1 - Z$

$Z = 1$ (UNSTABLE s/s)

CHECKING:

$$1 + \frac{(s+2)}{(s+1)(s-4)} = 0$$

$$s^2 - 3s - 4 + s + 2 = 0$$

$$s^2 - 2s - 2 = 0$$

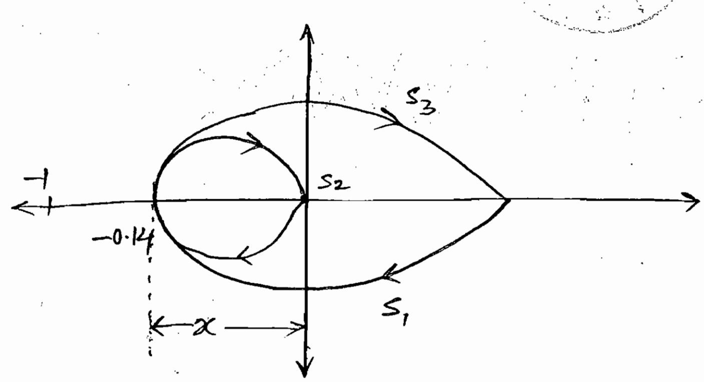
$$\begin{array}{c|cc} s^2 & 1 & -2 \\ s^1 & -2 & 0 \\ s^0 & -2 & 0 \end{array} \Rightarrow Z = 1$$

$N = P - Z$

$N = 1 - 1$

$N = 0$.

Q. $G(s) = \dfrac{100}{(s+2)(s+4)(s+8)}$



$N = P - Z = 0$

$0 - Z = 0$

$Z = 0$

(STABLE s/s)

Short cut method:

$$\frac{100}{(-\omega^2+6j\omega+8)(j\omega+8)}$$

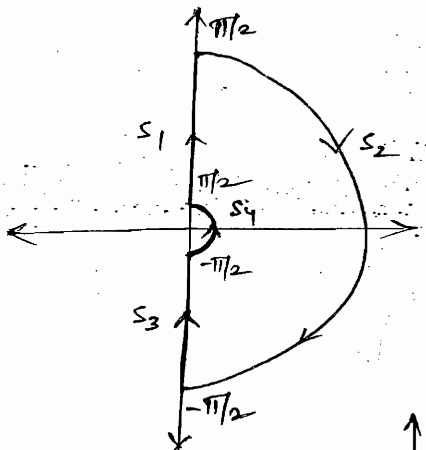
$$\frac{100}{-j\omega^3-8\omega^2-6\omega^2+48j\omega+8j\omega+64}$$

$$\frac{100}{64-14\omega^2+j(56\omega-\omega^3)}$$

$$56\omega-\omega^3=0 \Rightarrow \omega^2=56 \Rightarrow \omega=\omega_{pc}=\sqrt{56}$$
$$=7.4 \text{ rad/s}$$

$$x=\frac{100}{64-14(56)}=-0.14$$

Q. $G(s)=\dfrac{5}{s(1+2s)}$



To Map s_4:

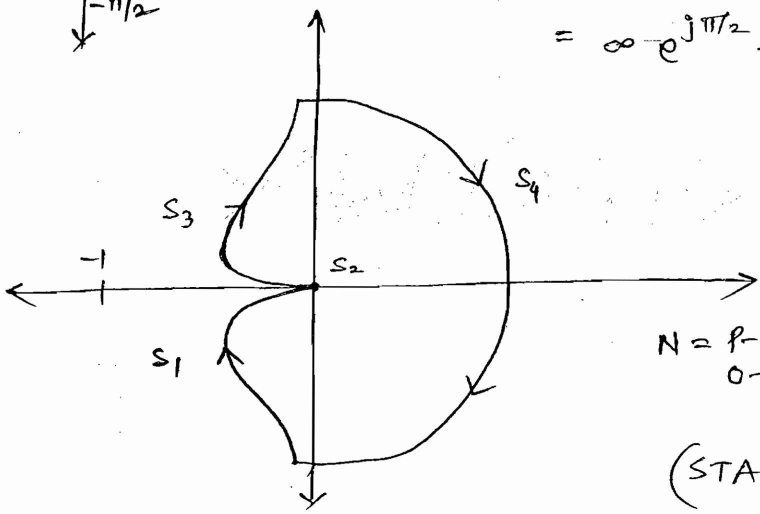
$$G(s)=\frac{5}{s}$$

$$=\lim_{r\to 0}\frac{5}{re^{j\theta}}$$

$$=\infty e^{-j\theta}$$

$$\theta \to \left(\frac{-\pi}{2}\to\frac{\pi}{2}\right)$$

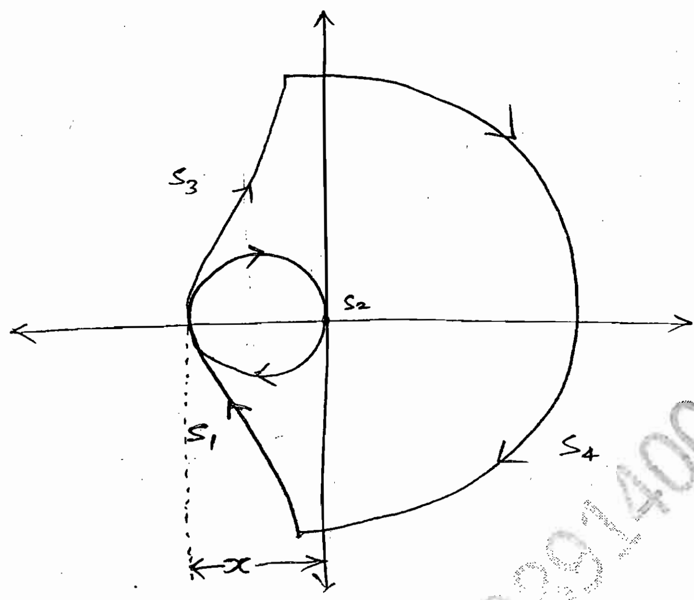
$$=\infty e^{j\pi/2}\to \infty e^{-j\pi/2}$$



$N=P-Z=0$
$0-Z=0$
$Z=0$

(STABLE s/s)

Q. $G(s) = \dfrac{K}{s(s+1)(s+2)}$



To map s_4 :

$$G(s) = \frac{K}{s(1)(2)} = \frac{0.5K}{s} = \lim_{\delta \to 0} \frac{0.5K}{re^{j\theta}} = \infty e^{-j\theta}$$

$$\theta = -\pi/2 \to \pi/2 \Rightarrow \infty e^{j\pi/2} \to \infty e^{-j\pi/2}$$

$$\frac{K}{j\omega(-\omega^2+3j\omega+2)} = \frac{K}{-3\omega^2+j(2\omega-\omega^3)}$$

$$2\omega-\omega^3=0 \Rightarrow \omega = \omega_{pc} = \sqrt{2} \text{ r/s}$$

$$x = \frac{-K}{3\omega^2} = \frac{-K}{3\times2} = -K/6$$

$$x = \frac{K}{6} \Rightarrow GM = \frac{1}{x} = \frac{6}{K} \qquad \boxed{GM \propto \frac{1}{K}}$$

If $\quad \dfrac{-K}{6} = 1 \Rightarrow K_{max} = 6$

Case (1) : $K > 6$ (Unstable)

$N = -2 = 0-2 \Rightarrow Z=2$

Case (2) $K < 6$ (Stable)

$N = 0 = 0-2 \Rightarrow Z=0$

Q19. $G(s) = \dfrac{K(s+3)(s+5)}{(s-2)(s-4)}$

$-1.33K = -1$

$K_{max} = 1/1.33$ (0.75)

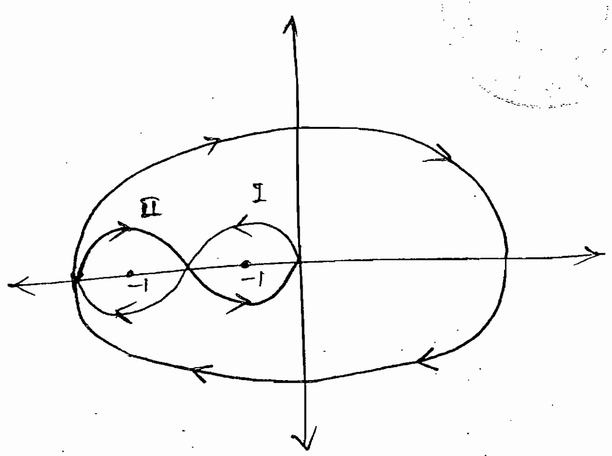
Case -(1): $K < 1/1.33$; $N = P - Z = 0$

$2 - Z = 0 \Rightarrow Z = 2$

Case -(2): $K > 1/1.33$; $N = P - Z = 2$

$2 - Z = 0 \Rightarrow Z = 0$ (Stable)

Q18.



Given : $P = 0$

(I). $N = P - Z$

$0 = 0 - Z$

$Z = 0$

(STABLE)

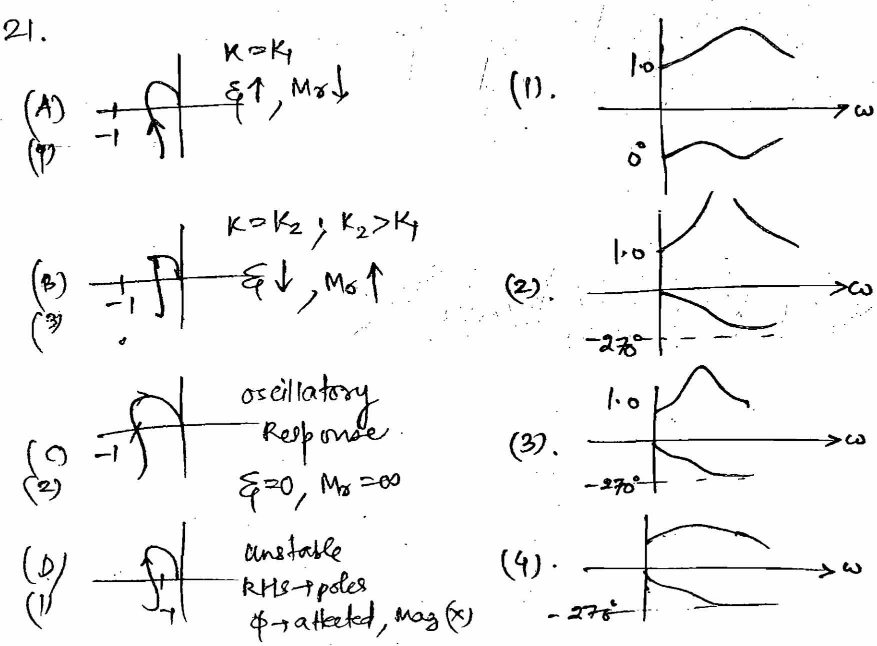
(II). $N = P - Z$

$-2 = 0 - Z$

$Z = 2$

(UNSTABLE)

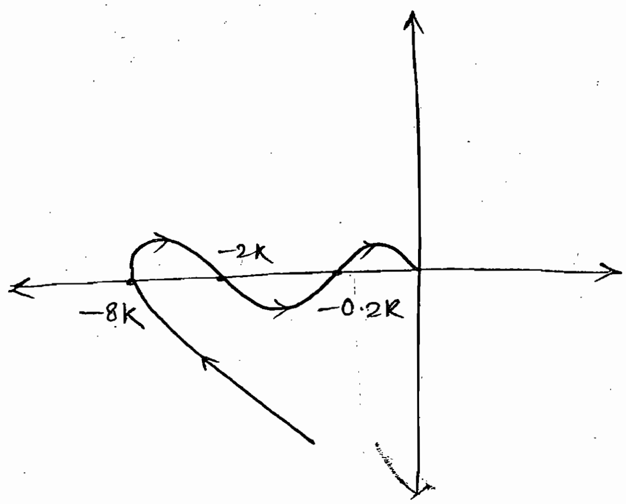
Q21.



$\xi \propto \dfrac{1}{\sqrt{K}}$

$K\uparrow, \xi\downarrow, M_r\uparrow$

Q22.



(1). $-0.2K = -1 \Rightarrow K_{max} = 5$

$K < 5 \longrightarrow$ stable

(2). $-2K = -1 \Rightarrow K_{max} = 1/2$

$K > 1/2 \longrightarrow$ Stable

$\frac{1}{2} < K < 5$

&

(3). $-8K = -1 \Rightarrow K_{max} = 1/8$

$K < 1/8 \longrightarrow$ stable

$K < \frac{1}{8}$

Q15. $G(s) = \dfrac{1}{s(1+T_1 s)(1+T_2 s)}$

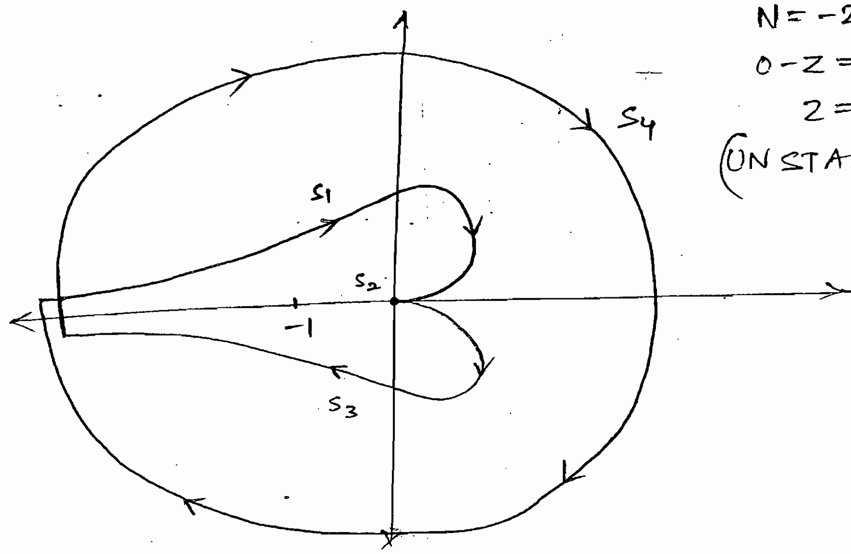
$$= \frac{1}{j\omega(1+j\omega T_1)(1+j\omega T_2)} = \frac{1}{j\omega\left[1+j\omega(T_1+T_2)-\omega^2 T_1 T_2\right]}$$

$$= \frac{1}{-\omega^2(T_1+T_2)+j\left(\omega-\omega^3 T_1 T_2\right)}$$

$$\omega - \omega^3 T_1 T_2 = 0 \Rightarrow \omega = \omega_{pc} = \frac{1}{\sqrt{T_1 T_2}}$$

$$x = \frac{-1}{\frac{1}{T_1 T_2}\cdot(T_1+T_2)} = \frac{-T_1 T_2}{(T_1+T_2)} \Rightarrow GM = \frac{1}{x} = \frac{T_1+T_2}{T_1 T_2}$$

Q. $G(s) = \dfrac{1}{s^2(1+s)(1+2s)}$



$N = -2$

$0 - Z = -2$

$Z = 2$

(UNSTABLE)

To Map s_4: $G(s) = 1/s^2$

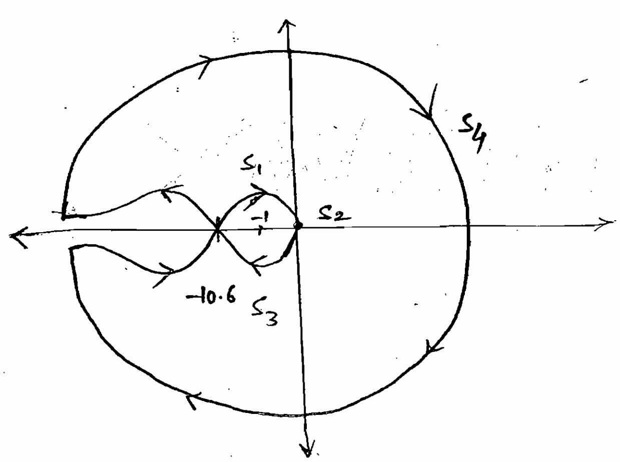
$$= \lim_{r\to 0} \frac{1}{(re^{j\theta})^2}$$

$$= \lim_{r\to 0} \frac{1}{r^2} \cdot e^{-2j\theta}$$

$$= \infty e^{-2j\theta} \quad (\theta \to -\pi/2 \to \pi/2)$$

$$= \infty e^{j\pi} \to \infty e^{-j\pi}$$

Q. $G(s) = \dfrac{1+4s}{s^2(1+s)(1+2s)}$



Q. $G(s) = \dfrac{(s+1)}{s^2(s-2)}$

To Map S_1:

$$G(s) = \frac{(1+s)}{-2s^2(1-0.5s)} = \frac{-0.5(1+s)}{s^2(1-0.5s)}$$

$$G(j\omega) = \frac{-0.5(1+j\omega)}{(j\omega)^2(1-0.5j\omega)}$$

$$|G(j\omega)| = \frac{0.5\sqrt{1+\omega^2}}{\omega^2\sqrt{1+(0.5\omega)^2}}$$

$$\angle G(j\omega) = \frac{(-180^\circ)(\tan^{-1}\omega)}{(180^\circ)(-\tan^{-1}0.5\omega)}$$

$$= -360^\circ + \tan^{-1}\omega + \tan^{-1}0.5\omega$$

ω	0	∞
$\lvert G(j\omega)\rvert$	∞	0
$\angle G(j\omega)$	-360°	-180°

To map S_4: $(-1 = e^{j\pi})^*$

$$G(s) = \frac{1}{s^2(-2)}$$

$$= \frac{0.5}{s^2[-1]}$$

$$= \lim_{r\to 0} \frac{0.5}{(re^{j\theta})^2}(e^{j\pi})$$

$$= \frac{0.5}{\lim_{r\to 0} r^2 \cdot e^{j2\theta} \cdot e^{j\pi}}$$

$$= \infty \cdot e^{-j(2\theta+\pi)}$$

$$\theta \to -\pi/2 \to \pi/2$$

$$\Rightarrow \infty e^{-j0} \to \infty e^{-j2\pi}$$

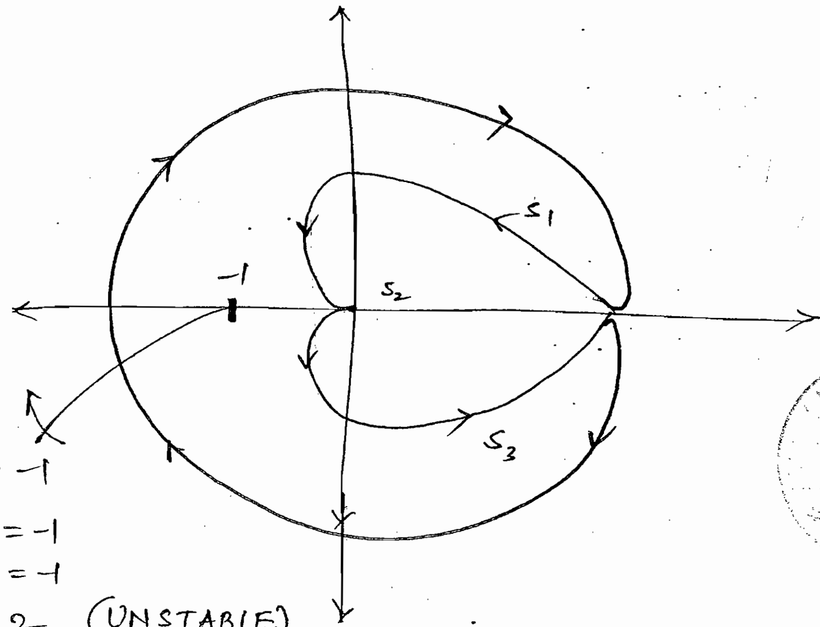
$$1 + \frac{s+1}{s^2(s-2)} = 0$$

$$s^3 - 2s^2 + s + 1 = 0$$

s^3	1	1
s^2	-2	1
s^1	1.5	0
s^0	1	0

$\Rightarrow Z = 2$

$$N = P - Z$$
$$= 1 - 2$$
$$= -1$$



$N = -1$

$P - Z = -1$

$1 - Z = -1$

$Z = 2$ (UNSTABLE)

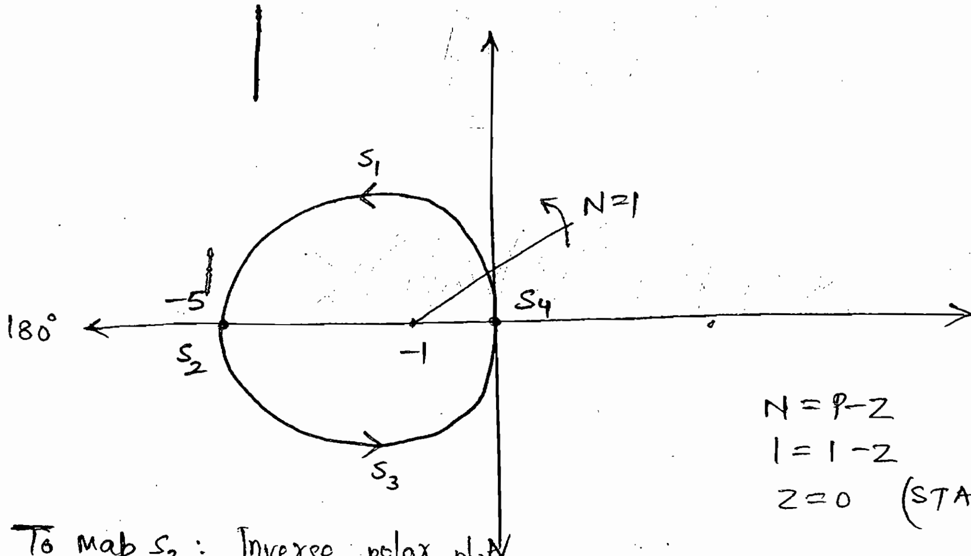
Conv Q4. $\quad G(s) = \dfrac{s}{(1-0.2s)}$

To Map s_1: Polar plot

$$G(j\omega) = \frac{j\omega}{(1-0.2j\omega)}$$

ω	0	∞
M	0	5
ϕ	90°	180°

$$|G(j\omega)| = \frac{\omega}{\sqrt{1+(0.2\omega)^2}} \quad ; \quad \angle G(j\omega) = 90^\circ + \tan^{-1}(0.2\omega)$$



$N = P - Z$

$1 = 1 - Z$

$Z = 0$ (STABLE)

To map s_3: Inverse polar plot

To Map s_2 :

$$G(s) = \frac{s}{-0.2s} = -5$$

To Map s_4 :

$$G(s) = s$$
$$= \lim_{r \to 0} re^{j\theta} = 0 \cdot e^{j\theta}.$$

Q. $G(s) = \frac{K(1+s)^2}{s^3}$

To Map s_1 : Polar plot

$$G(j\omega) = \frac{K(1+j\omega)^2}{(j\omega)^3}$$

ω	0	∞
$\lvert G(j\omega)\rvert$	∞	0
$\angle G(j\omega)$	-270°	-90°

$$|G(j\omega)| = \frac{K}{\omega^3}(1+\omega^2)$$

$$\angle G(j\omega) = -270^\circ + 2\tan^{-1}\omega$$

To Map s_4 :

$$G(s) = \frac{K}{s^3} = \frac{K}{\lim_{r \to 0} (re^{j\theta})^3} = \lim_{r \to 0} \frac{K}{r^3} e^{-3j\theta}$$

$$= \infty e^{-j3\theta} \quad \left[\theta \Rightarrow \frac{\pi}{2} \to -\frac{\pi}{2}\right]$$

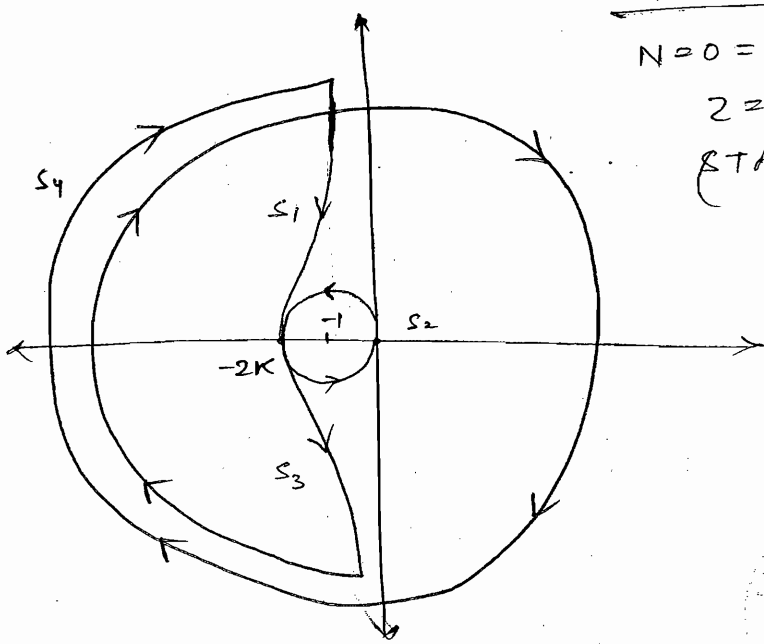
$$\Rightarrow \infty e^{j3\pi/2} \to \infty e^{-j3\pi/2}$$

To find x :

$$-270^\circ + 2\tan^{-1}\omega = -180^\circ$$
$$2\tan^{-1}\omega = 90^\circ$$
$$\tan^{-1}\omega = 45^\circ \Rightarrow \omega = \omega_{pc} = 1 \text{ rad/s}$$

$$|G(j\omega)|_{\omega=\omega_{pc}=1 \text{ r/s}} = x = \frac{K(1+1)}{1} = 2K$$

$$-2K = -1 \Rightarrow K_{max} = 0.5$$



$K > 0.5$

$N = 0 = 0 - Z$

$Z = 0$

(STABLE)

Q. $G(s) = \dfrac{10e^{-s}}{s(1+5s)}$

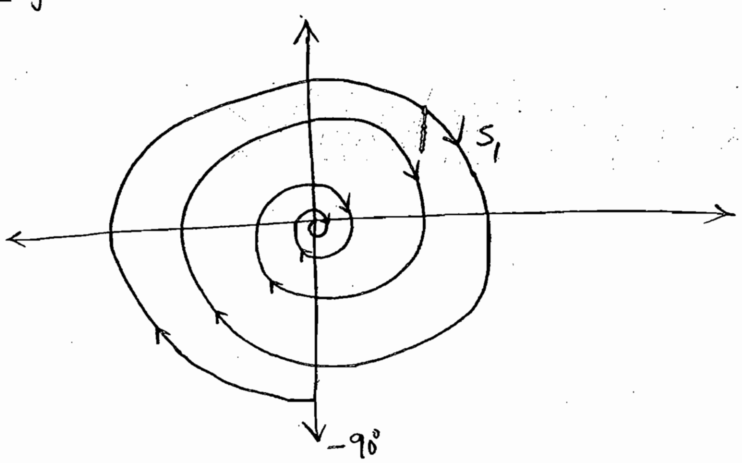
To map s_1: Polar plot

$$G(j\omega) = \frac{10 e^{-j\omega}}{j\omega(1+5j\omega)}$$

ω	0	∞
$\lvert G(j\omega)\rvert$	∞	0
$\angle G(j\omega)$	-90°	$-\infty^\circ$

$$|G(j\omega)| = \frac{10 \times 1}{\omega\sqrt{1+(5\omega)^2}}$$

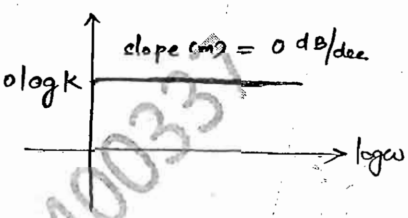
$$\angle G(j\omega) = -90^\circ - 57.3\omega - \tan^{-1} 5\omega$$



* Nyquist is not suitable for finding stability of s/s with dead time. Hence, GM and PM are used for finding stability of such s/s.

Bode Plots :

(1). System gain ($\pm K$) :



$F(j\omega) = \pm K + j0$

$|F(j\omega)| = \sqrt{(\pm K)^2 + 0^2} = K$

Its dB value is $20\log|F(j\omega)| = 20\log K$

(2). Integral & derivative factors : (Poles and zeroes at origin) $s^{\pm n}$

$F(s) = (s)^{\pm n}$

$F(j\omega) = (j\omega)^{\pm n} = (0 + j\omega)^{\pm n}$

$|F(j\omega)| = \left[\sqrt{0^2 + \omega^2}\right]^{\pm n}$

$= (\omega)^{\pm n}$

Its dB value is

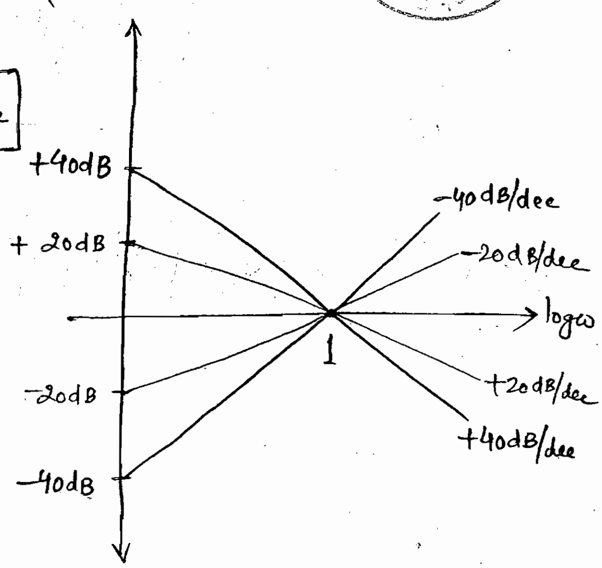
$20\log(\omega)^{\pm n} = \underbrace{\pm 20n}_{m}\underbrace{\log\omega}_{x}$ —(1)

$\boxed{\text{slope (m)} = \pm 20n \text{ dB/dec}}$

$\pm 20n\log\omega = 0$

$\log\omega = 0$

$\Rightarrow \omega = 1$ rad/s



(3). <u>First order factors</u> $(1 \pm Ts)^{\pm 1}$

$$F(j\omega) = (1 \pm j\omega T)^{\pm 1}$$

$$|F(j\omega)| = \left[\sqrt{1+(\omega T)^2}\right]^{\pm 1}$$

Its dB value is

$$20 \log \left[\sqrt{1+(\omega T)^2}\right]^{\pm 1} = \pm 20 \log \sqrt{1+(\omega T)^2} \quad \text{——— (1)}$$

<u>Asymptotic Approximation:</u>

<u>Case (1)</u>: Low frequency $[1 >>> (\omega T)^2]$

$$\pm 20 \log \sqrt{T} = 0 \text{ dB}$$

<u>Case (2)</u>: High frequency $[(\omega T)^2 >>> 1]$

$$\pm 20 \log \sqrt{(\omega T)^2} = \pm 20 \log \omega T \quad \text{——— (2)}$$

$$= \pm 20 \log \omega \pm 20 \log T$$

$$[m x \pm c]$$

$$\boxed{\text{Slope } (m) = \pm 20 \text{ dB/dec}}$$

<u>Corner frequency (ω_{cf})</u>:

$$0 = \pm 20 \log(\omega T)$$

$$\log(\omega T) = 0$$

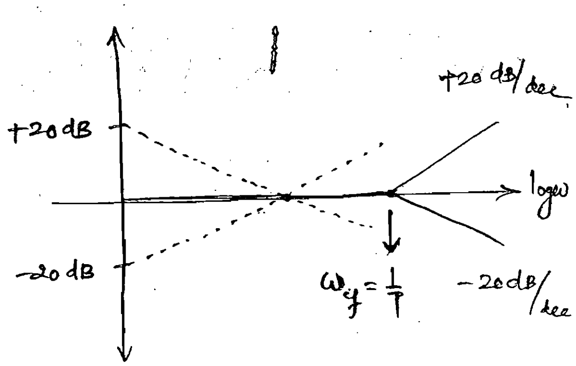
$$\omega T = \log^{-1}(0) = 1$$

$$\therefore \omega = \omega_{cf} = 1/T \text{ rad/s}$$

Eg. $(s \pm 2)^{\pm 1} = \left(1 \pm \frac{s}{2}\right)^{\pm 1}$

$T = 1/2$

$\Rightarrow \omega_{cf} = \frac{1}{T} = 2$ rad/s



Error at ω_{cf}:

At $\omega = \omega_{cf} = 1/T$

$$\pm 20\log\sqrt{1+\left(\frac{1}{T}\cdot T\right)^2} = \pm 20\log\sqrt{2} = \pm 3\,dB$$

(4). Quadratic factors:

$$\left(s^2+2\xi\omega_n s+\omega_n^2\right)^{\pm 1}$$

$$\left[\frac{s^2}{\omega_n^2}+\frac{2\xi s}{\omega_n}+1\right]^{\pm 1}$$

Put $s=j\omega$

$$\left[\frac{j^2\omega^2}{\omega_n^2}+\frac{2\xi j\omega}{\omega_n}+1\right]^{\pm 1}$$

$$\left[1-\left(\frac{\omega}{\omega_n}\right)^2+j2\xi\left(\frac{\omega}{\omega_n}\right)\right]^{\pm 1}$$

$$\pm 20\log\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2+\left(2\xi\frac{\omega}{\omega_n}\right)^2}$$

0 dB (LFR) $\qquad$ $\pm 40\log(\omega/\omega_n)$ (HFR)

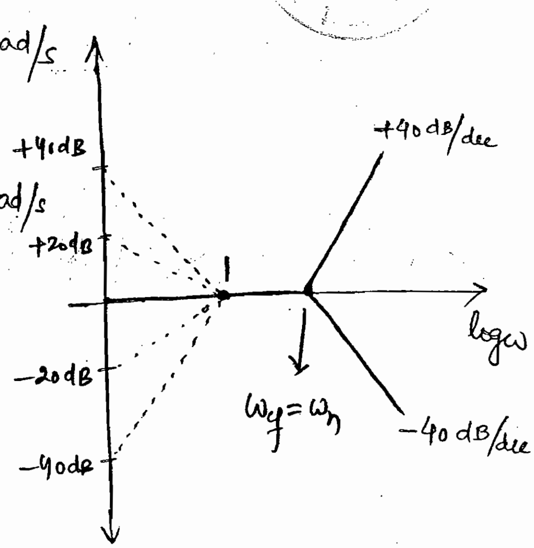
Slope (m) = ± 40 dB/dec

Corner frequency, $\omega_{cf} = \omega_n$ rad/s

Eg. $(s^2+4s+16)^{\pm 1}$

$\omega_n^2 = 16 \Rightarrow \omega_{cf} = \omega_n = 4$ rad/s

Error at ω_{cf}: $\pm 20\log(2\xi)$

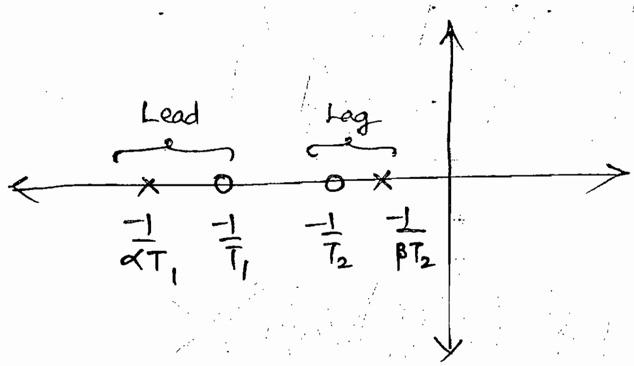


Factor	Corner frequency	Magnitude	slope
(1). s/s Gain (K)	—	$20 \log K$	0 dB/dec
(2). $(j\omega)^{\pm n}$	—	$\pm 20 n \log \omega$	$\pm 20 \times n$ dB/dec
(3). $(1 \pm j\omega T)^{\pm 1}$	$\frac{1}{T}$	$\pm 20 \log(\omega T)$	± 20 dB/dec
(4). $\left[1-\left(\frac{\omega}{\omega_n}\right)^2 + j2\xi\frac{\omega}{\omega_n}\right]^{\pm 1}$	ω_n	$\pm 40 \log\left(\frac{\omega}{\omega_n}\right)$	± 40 dB/dec

Bode plot for Lag Lead Compensator:

$$F(s) = \frac{\alpha (1+T_1 s)(1+T_2 s)}{(1+\alpha T_1 s)(1+\beta T_2 s)}$$

$$F(j\omega) = \frac{\alpha (1+j\omega T_1)(1+j\omega T_2)}{(1+j\omega\alpha T_1)(1+j\omega\beta T_2)}$$

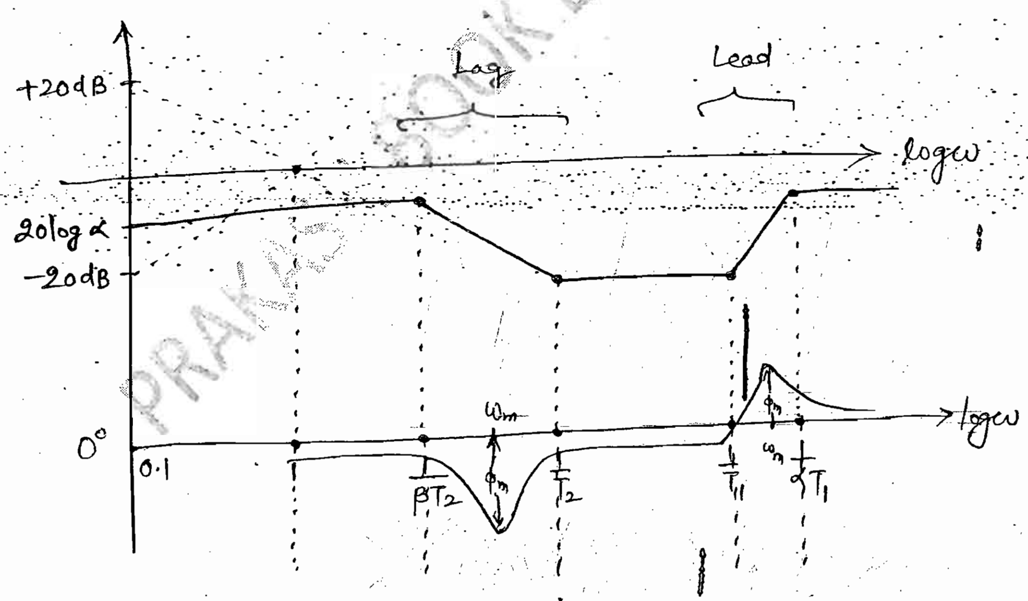


$$\angle F(j\omega) = \tan^{-1}(\omega T_1) - \tan^{-1}(\omega\alpha T_1) + \tan^{-1}(\omega T_2) - \tan^{-1}(\omega\beta T_2)$$

ω				
$\angle F(j\omega)$				

Magnitude table:

Factor	Cf	Magnitude
(1). $K = \alpha$	–	$20 \log \alpha$
(2). $(j\omega)^{\pm n}$	–	Nil
(3). $\frac{1}{(1+j\omega\beta T_2)}$	$\frac{1}{\beta T_2}$	-20 dB/dec
(4). $1+j\omega T_2$	$\frac{1}{T_2}$	$+20$ dB/dec
(5). $1+j\omega T_1$	$\frac{1}{T_1}$	$+20$ dB/dec
(6). $\frac{1}{(1+j\omega\alpha T_1)}$	$\frac{1}{\alpha T_1}$	-20 dB/dec



$$\omega_m = \sqrt{\frac{1}{\beta T_2} \cdot \frac{1}{T_2}} = \frac{1}{T_2\sqrt{\beta}}$$

$$\omega_m = \sqrt{\frac{1}{T_1} \cdot \frac{1}{\alpha T_1}} = \frac{1}{T_1\sqrt{\alpha}}$$

Characteristics of phase lead compensator :

(1). The phase lead compensator shifts the gain crossover ω_{gc} to higher values where the desired phase margin is acceptable hence, it is effective when the slope of the uncompensated s/s near the ω_{gc} is low

(2). The max^m phase lead occurs at the geometric mean of the 2 corner frequencies.

$$\angle F(j\omega) = \phi = \tan^{-1}(\omega T_1) - \tan^{-1}(\omega \alpha T_1)$$

$$\tan\phi = \tan\left[\tan^{-1}(\omega T_1) - \tan^{-1}(\omega\alpha T_1)\right]$$

$$\tan\phi = \frac{\omega T_1 - \omega\alpha T_1}{1+(\omega T_1)^2\alpha}$$

$$\tan\phi = \frac{\omega T_1(1-\alpha)}{1+(\omega T_1)^2\alpha}$$

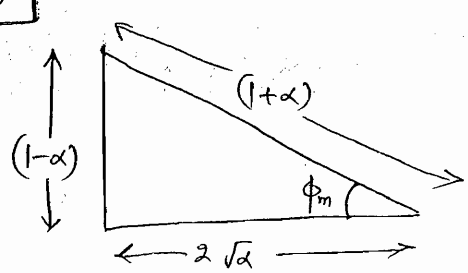
At $\omega = \omega_m = \frac{1}{T\sqrt{\alpha}}$; $\phi = \phi_m$

$$\tan\phi_m = \frac{\frac{1}{T_1\sqrt{\alpha}}\cdot T_1(1-\alpha)}{1+\left(\frac{1}{T_1\sqrt{\alpha}}\cdot T_1\right)^2\alpha} = \frac{1-\alpha}{2\sqrt{\alpha}}$$

$$\therefore \boxed{\phi_m = \tan^{-1}\left(\frac{1-\alpha}{2\sqrt{\alpha}}\right)}$$

$$\sin\phi_m = \frac{1-\alpha}{1+\alpha}$$

$$\boxed{\phi_m = \sin^{-1}\left(\frac{1-\alpha}{1+\alpha}\right)}$$



$$\sin\phi_m (1+\alpha) = 1-\alpha$$

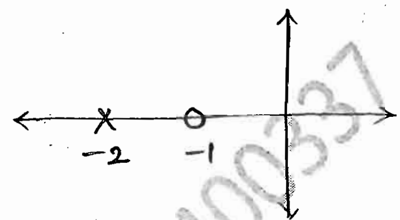
$$\alpha \sin\phi_m + \sin\phi_m = 1-\alpha$$

$$\alpha(1+\sin\phi_m) = 1-\sin\phi_m$$

$$\alpha = \frac{1-\sin\phi_m}{1+\sin\phi_m}$$

(3). Polar Plot :

$$F(s) = \frac{s+1}{s+2}$$

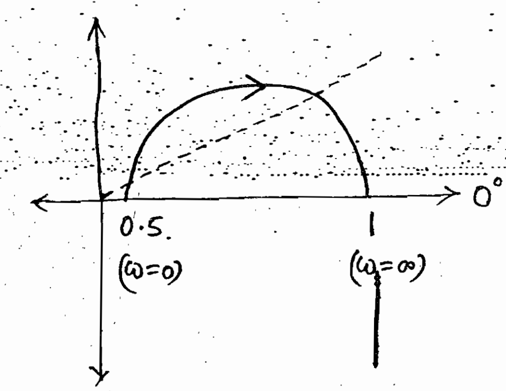


$$F(s) = \frac{0.5(1+s)}{(1+0.5s)} \Rightarrow \frac{0.5(1+j\omega)}{(1+0.5j\omega)}$$

$$|F(j\omega)| = \frac{0.5\cdot\sqrt{1+\omega^2}}{\sqrt{1+(0.5\omega)^2}}$$

$$\angle F(j\omega) = \tan^{-1}\omega - \tan^{-1}(0.5\omega)$$

ω	0	1	∞
$\lvert F(j\omega)\rvert$	0.5		1
$\angle F(j\omega)$	0°	18°	0°



Characteristics of phase lag compensator:

(1) The phase lag compensator shifts the gain crossover frequency (ω_{gc}) to lower value where the desired PM is acceptable hence, it is effective when the slope of uncompensated s/s near the ω_{gc} is high.

(2) The max^m phase lag occurs at the geometric mean of the 2 corner frequencies.

$$\angle F(j\omega) = \phi = \tan^{-1}(\omega T_2) - \tan^{-1}(\omega \beta T_2)$$

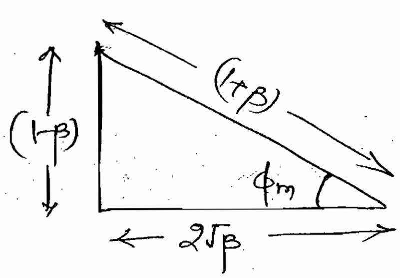
$$\tan\phi = \tan\left[\tan^{-1}(\omega T_2) - \tan^{-1}(\omega \beta T_2)\right]$$

$$\tan\phi = \frac{\omega T_2 - \omega\beta T_2}{1+(\omega T_2)^2\beta} = \frac{\omega T_2(1-\beta)}{1+(\omega T_2)^2\beta}$$

$$\text{At } \omega = \omega_m = \frac{1}{T_2\sqrt{\beta}} \quad ; \phi = \phi_m$$

$$\tan\phi_m = \frac{\frac{1}{T_2\sqrt{\beta}} T_2(1-\beta)}{1+\left(\frac{T_2}{T_2\sqrt{\beta}}\right)^2 \beta} = \frac{1-\beta}{2\sqrt{\beta}}$$

$$\therefore \phi_m = \tan^{-1}\left(\frac{1-\beta}{2\sqrt{\beta}}\right)$$



$$\sin\phi_m = \frac{1-\beta}{1+\beta}$$

$$\phi_m = \sin^{-1}\frac{(1-\beta)}{(1+\beta)}$$

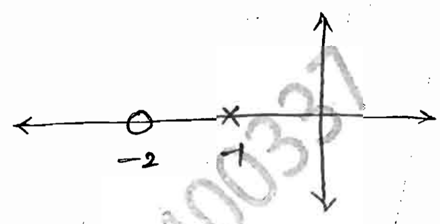
$$\sin\phi_m (1+\beta) = 1-\beta$$

$$\beta \sin\phi_m + \beta = 1 - \sin\phi_m$$

$$\beta = \frac{1-\sin\phi_m}{1+\sin\phi_m}$$

(3). <u>Polar plot :</u>

$$F(s) = \frac{s+2}{s+1}$$



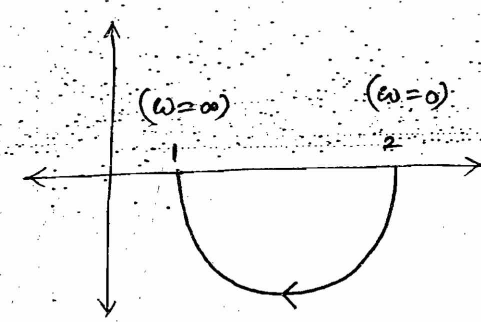
$$F(s) = \frac{2(1+0.5s)}{(1+s)}$$

$$F(j\omega) = \frac{2(1+0.5j\omega)}{(1+j\omega)}$$

$$|F(j\omega)| = \frac{2\sqrt{1+(0.5\omega)^2}}{\sqrt{1+\omega^2}}$$

$$\angle F(j\omega) = \tan^{-1}(0.5\omega) - \tan^{-1}\omega$$

ω	0		∞
$\lvert F(j\omega)\rvert$	2		1
$\angle F(j\omega)$	0°	-18°	0°



Q1. LEAD

$$\frac{s+a}{s+b} = \frac{s+1}{s+2} \quad (b>a)$$

LAG

$$\frac{s+p}{s+q} = \frac{s+2}{s+1} \quad (p>q)$$

Q2. $G_c(s) = \dfrac{5(1+0.3s)}{(1+0.1s)} = \dfrac{1+Ts}{1+KTs}$

$T = 0.3$; $KT = 0.1$

$K = \dfrac{0.1}{0.3} = \dfrac{1}{3}$ $\quad \therefore \alpha = \dfrac{1}{3}$

LEAD COMPENSATOR

$$\phi_m = \sin^{-1}\left(\frac{1-1/3}{1+1/3}\right) = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$$

Q3. $\alpha = \frac{R_2}{R_1+R_2} \Rightarrow \phi_m = \sin^{-1}\left[\frac{1-\frac{R_2}{R_1+R_2}}{1+\frac{R_2}{R_1+R_2}}\right] = \sin^{-1}\left(\frac{R_1}{R_1+2R_2}\right)$

Q4. $\frac{1+aTs}{1+Ts} = \frac{1+T_1 s}{1+\alpha T_1 s}$

$T_1 = aT$; $\alpha T_1 = T$

$\alpha \cdot aT = T \Rightarrow \alpha = \frac{1}{a}$

$$\phi_m = \sin^{-1}\left[\frac{1-1/a}{1+1/a}\right] = \sin^{-1}\left(\frac{a-1}{a+1}\right)$$

Q6. $\frac{K(1+0.3s)}{(1+0.17s)} = \frac{1+Ts}{1+\alpha Ts}$

$T = 0.3$; $\alpha T = 0.17$

$\alpha = \frac{0.17}{0.3} = 0.56$

$T = R_1 C = 0.3 = R_1 \times 10^{-6} \Rightarrow R_1 = 300\ k\Omega$

$\alpha = \frac{R_2}{R_1+R_2} = 0.56 \Rightarrow R_2 = 400\ k\Omega$

Q10. $\frac{1+2s}{1+0.2s} = \frac{1+Ts}{1+KTs}$

$T = 2$; $KT = 0.2$

$K = \frac{0.2}{2} = 0.1$

Q11. (b). $\frac{s+9.9}{s+3} \rightarrow$ LAG (d) . $\frac{s+6}{s} = 1+\frac{6}{s} \rightarrow$ PI $\left(K_p + \frac{K_i}{s}\right)$

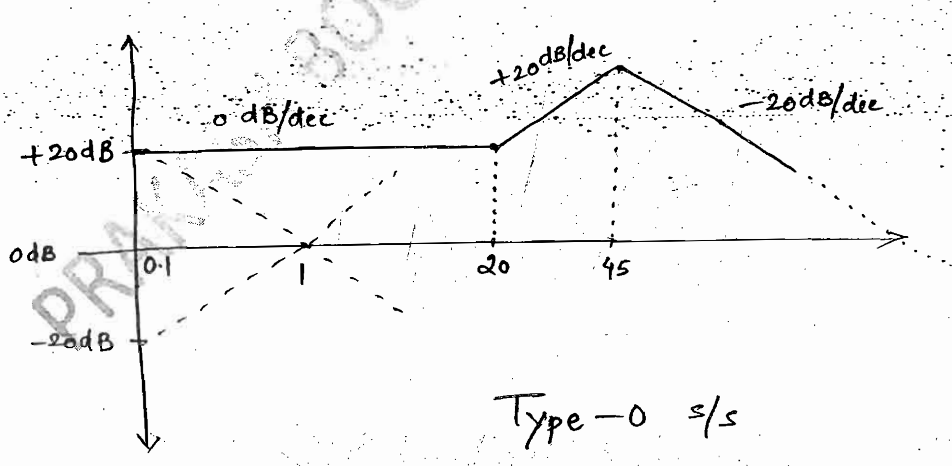
(c). Cannot be ans. (b). Becoz ξ & ω_n are transient state char.

Q. $G(s) = \dfrac{10^3\,(s+20)}{(s^2+210s+2000)}$

$$= \frac{10^3 \times 20\,(1+s/20)}{2000\left(1+\frac{210s}{2000}+\frac{s^2}{2000}\right)}$$

$$= \frac{10\,(1+j\omega/20)}{\left[1-\frac{\omega^2}{2000}+j\frac{2\omega}{200}\right]}$$

Factor	cf	Magnitude	
(1). $K=10$	–	$20\log(10) = 20\,dB$	
(2). $(j\omega)^{\pm n}$	–	Nil	
(3). $(1+j\omega/20)$	20 +20	$+20\log(\omega/20)$	$(+20\,dB/dec)$
(4). $\dfrac{1}{1-\frac{\omega^2}{2000}+j\frac{21\omega}{200}}$	45 −20	$-40\log\left(\frac{\omega}{45}\right)$	$(-40\,dB/dec)$



Type – 0 s/s

$K_v = K_a = 0$

$20\log K_p = 20$

$\log K_p = 1$

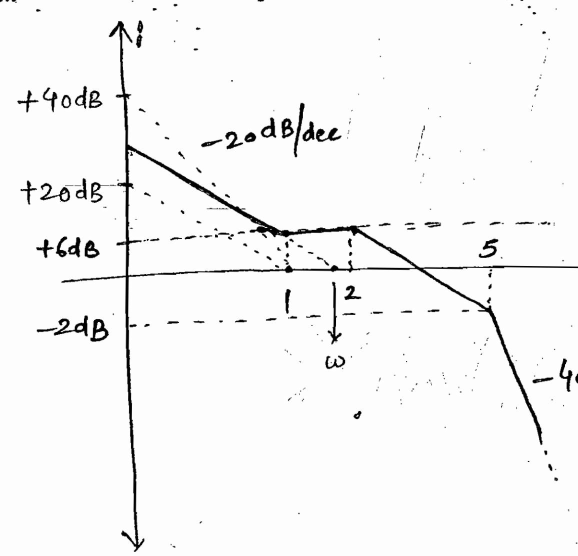
$\therefore K_p = 10$

Q. $G(s) = \dfrac{20(s+1)}{s(s+2)(s+5)} = \dfrac{20}{s(s+5)}\left[\dfrac{(s+1)}{(s+2)}\right]$ LEAD COMP.

$= \dfrac{2(1+s)}{s(1+0.5s)(1+0.2s)}$

$= \dfrac{2(1+j\omega)}{j\omega(1+0.5j\omega)(1+0.2j\omega)}$

Factor	cf	Magnitude
(1) $K=2$	–	$20\log 2 = 6\,dB$
(2). $1/j\omega$	–	$-20\log\omega$ (−20 dB/dec)
(3). $1+j\omega$	1	$+20\log\omega$ (+20 dB/dec)
(4). $\dfrac{1}{1+0.5j\omega}$	2	$-20\log 0.5\omega$ (−20 dB/dec)
(5). $\dfrac{1}{1+0.2j\omega}$	5	$-20\log 0.2\omega$ (−20 dB/dec)



At $\omega=5$.

$6 + (-20\log\omega) + (20\log\omega)$

$- 20\log 0.5\omega$

$= 6 - 20\log 0.5\omega \Rightarrow$

$= 6 - 20\log(0.5\times5) = -2dB$

$y = mx + c$

$= -20\log\omega + 6$

At $\omega = 0.1$,

$-20\log(0.1) + 6 = 20+6 = 26dB$

Type-1 s/s

$K_p = \infty,\ K_a = 0$

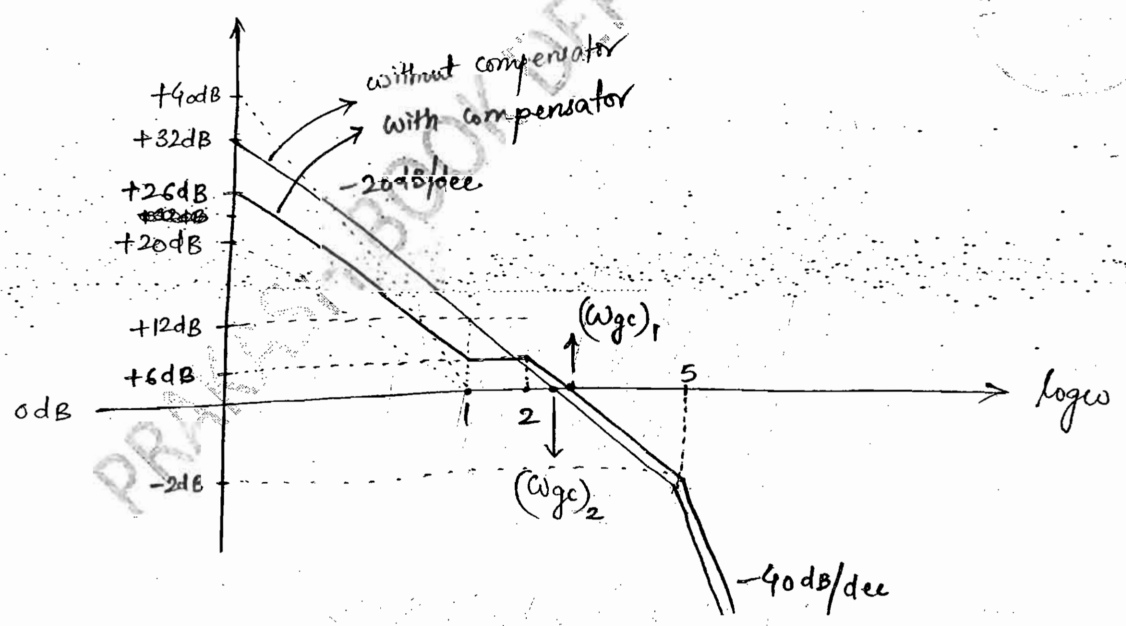
$+20\log K - 20\log\omega = 0 \Rightarrow K = K_v = \omega$

$$G(s) = \frac{20}{s(s+5)} \quad \text{w/o LEAD COMPENSATOR.}$$

$$G(s) = \frac{4}{s(1+0.2s)}$$

$$G(j\omega) = \frac{4}{j\omega(1+0.2j\omega)}$$

Factors	cf	Magnitude
(1). $K = 4$	–	$20\log 4 = 12\,dB$
(2). $\frac{1}{j\omega}$	–	$-20\log\omega$ ($-20\,dB/dec$)
(3). $\frac{1}{(1+0.2j\omega)}$	5	$-20\log\omega$ ($-20\,dB/dec$)



$$(\omega_{gc})_1 > (\omega_{gc})_2$$

Hence, LEAD COMPENSATOR SHIFTS ω_{gc} to the right side or high frequency side.

conv.
Q.1
$$G(s) = \frac{3(s+1)(s+700)}{s^2(s^2+18s+400)}$$

$$= \frac{3\times 700\,(1+s)\,(1+s/700)}{s^2 \cdot 400\,(s^2/400 + 18s/400 + 1)}$$

$$= \frac{5.2\,(1+j\omega)\left(1+\frac{j\omega}{700}\right)}{(j\omega)^2\left[1-\frac{\omega^2}{400}+j\,\frac{18 j\omega}{400}\right]}$$

$$\angle G(j\omega)\Big|_{\omega<20} = \frac{0^\circ\cdot(\tan^{-1}\omega)\,(\tan^{-1}\omega/700)}{(180^\circ)\left\{\tan^{-1}\left(\frac{18\omega}{400-\omega^2}\right)\right\}}$$

$$\angle G(j\omega)\Big|_{\omega>20} = \frac{(0^\circ)\cdot(\tan^{-1}\omega)\,(\tan^{-1}(\omega/700))}{(180^\circ)\left\{180^\circ-\tan^{-1}\left(\frac{18\omega}{400-\omega^2}\right)\right\}}$$

Magnitude table

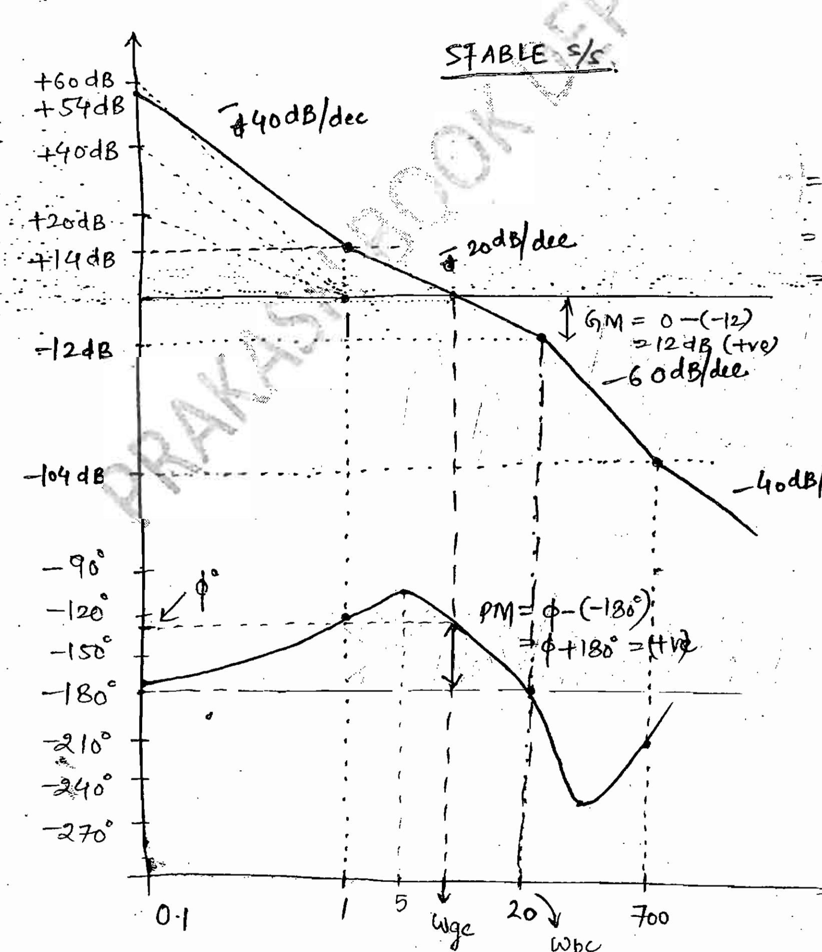
factor	cf	Magnitude
(1). $K=5.2$	—	$20\log(5.2) = 14$ dB
(2). $\frac{1}{(j\omega)^2}$		$-40\log\omega$ (-40 dB/dec)
(3). $(1+j\omega)$	1	$+20\log\omega$
(4) $\frac{1}{1-\frac{\omega^2}{400}+j\frac{18\omega}{400}}$	20	$-40\log(\omega/20)$ -40 dB/dec.
(5). $1+j\omega/700$	700	$+20\log\omega/700$ ($+20$ dB/dec)

Phas Angle Table

ω	$-180^\circ-\tan^{-1}\omega+\tan^{-1}\omega/700$	$-\tan^{-1}\frac{18\omega}{400-\omega^2}$ / $-180^\circ+\left[-\tan^{-1}\frac{18}{400-\omega^2}\right]$	ϕ_R
0.1	-180°	—	
10			
5			
20			
100			

Phase angle table

ω	-180°	$+\tan^{-1}\omega$	$+\tan^{-1}\frac{\omega}{700}$	$-\tan^{-1}\left(\frac{\omega}{400-\omega^2}\right)$ $-180^\circ + \left[-\tan^{-1}\left(\frac{18\omega}{400-\omega^2}\right)\right]$	ϕ_R
0.1	-180°	5.7°	0.008°	-0.25°	-174°
1	-180°	45°	0.08°	-0.25°	-137°
5	-180°	78.6°	0.4°	-13.5°	-114°
20	-180°	87°	1.6°	-90°	-181°
100	-180°	89.4°	8°	$-180^\circ+10.6^\circ=-169.4^\circ$	-251°
700	-180°	89.9°	45°	$-180^\circ+1.5^\circ=-178.5^\circ$	-223°



At $\omega = 20$,

$= 14 + (-40\log\omega) + 20\log\omega$

$= 14 - 40\log 20 + 20\log 20$

$= -12\,dB$

At $\omega = 700$,

$= 14 - 40\log\omega + 20\log\omega - 40\log \omega/20$

$= 14 - 40\log 700 + 20\log 700 - 40\log 700/20$

$= -104.6\,dB$

Type-2 s/s
$K_p = K_v = \infty$
$20\log K = -20\log\omega^2$
$K = K_a = \omega^2$

To find the range of "K" for stability using Bode plots :

Q13. $$G(s) = \frac{K(s+10)(s+20)}{s^3(s+100)(s+200)}$$

$$= \frac{\frac{K}{100}\left(1+\frac{s}{10}\right)\left(1+\frac{s}{20}\right)}{s^3\left(1+\frac{s}{100}\right)\left(1+\frac{s}{200}\right)}$$

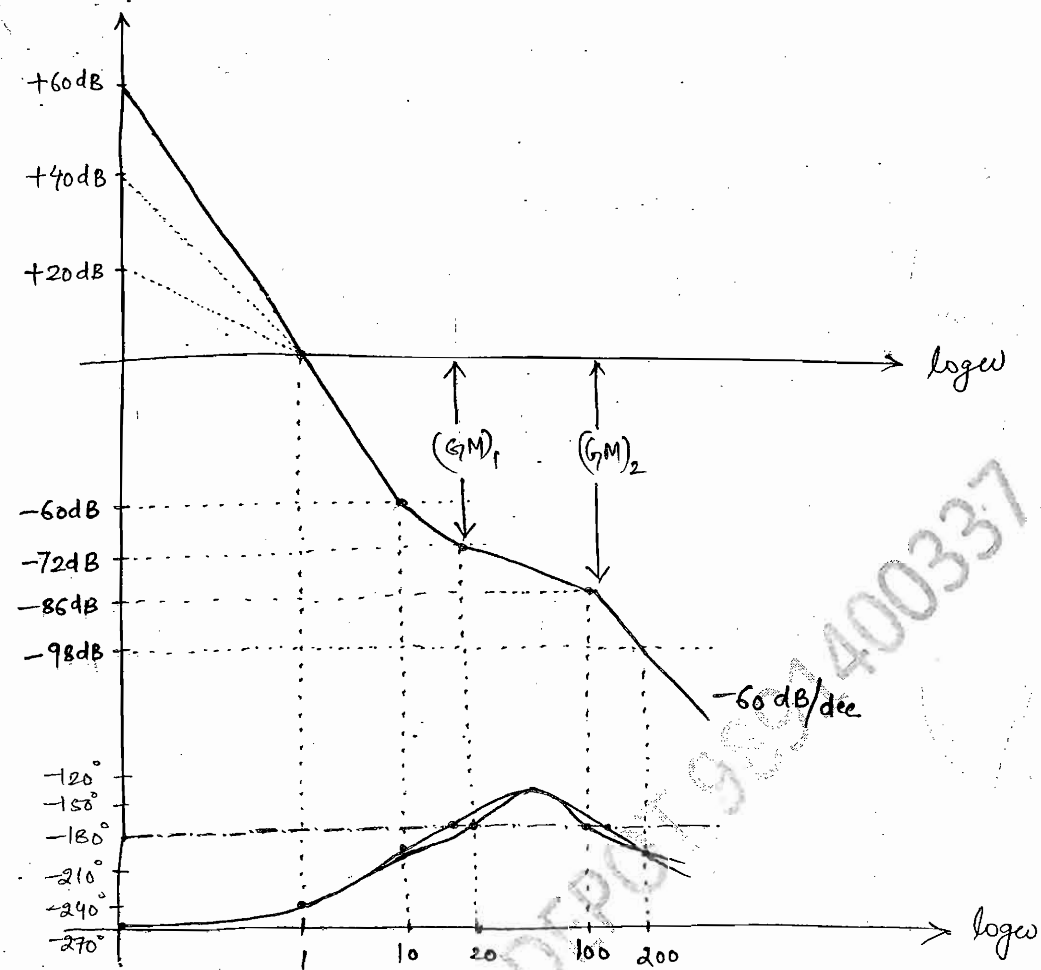
$$G(j\omega) = \frac{K'\left(1+\frac{j\omega}{10}\right)\left(1+\frac{j\omega}{20}\right)}{(j\omega)^3\left(1+\frac{j\omega}{100}\right)\left(1+\frac{j\omega}{200}\right)}$$

$$\angle G(j\omega) = \frac{\left[\tan^{-1}(\omega/10)\right]\cdot\left[\tan^{-1}(\omega/20)\right]}{(270^\circ)\left[\tan^{-1}(\omega/100)\right]\cdot\left[\tan^{-1}(\omega/200)\right]}$$

$$\phi_R = -270^\circ + \tan^{-1}\left(\frac{\omega}{10}\right) + \tan^{-1}\left(\frac{\omega}{20}\right) - \tan^{-1}\left(\frac{\omega}{100}\right) - \tan^{-1}\left(\frac{\omega}{200}\right)$$

Magnitude table | Phase angle Table

factor	C.f	Magnitude	ω	ϕ_R
(1). $K' = K/100$	—	$20\log K'$	0.1	-269°
(2). $\frac{1}{(j\omega)^3}$	—	$-60\log\omega$	1	-262°
(3). $(1+j\omega/10)$	10	$20\log(\omega/10)$	10	-207°
(4). $(1+j\omega/20)$	20	$20\log(\omega/20)$	20	-178°
(5). $\frac{1}{1+j\omega/100}$	100	$-20\log(\omega/100)$	50	-163°
			100	-178°
(6). $\frac{1}{1+j\omega/200}$	200	$-20\log(\omega/200)$	200	-207°



At $\omega = 10$,

$-60 \log\omega \Rightarrow -60 \log 10 = -60\,dB$

At $\omega = 20$,

$-60 \log\omega + 20\log(\omega/10) = -60\log 20 + 20\log 2 = -72\,dB$

At $\omega = 100$,

$-60\log\omega + 20\log(\omega/10) + 20\log(\omega/20)$

$-60\log 100 + 20\log 10 + 20\log 5 = -86\,dB$

At $\omega = 200$,

$-60\log 200 + 20\log 20 + 20\log 10 = -96\,dB$

$(GM)_1 = 0 - (-72) = 72\,dB$

$20\log K' = 72 \Rightarrow K' = 3981$

$\therefore K = 100K' = 398100$

$(GM)_2 = 0 - (-86) = 86\,dB$

$20\log K' = 86 \Rightarrow K' = 19952$

$K = 100K' = 1995200$

$\boxed{398100 < K < 1995200}$

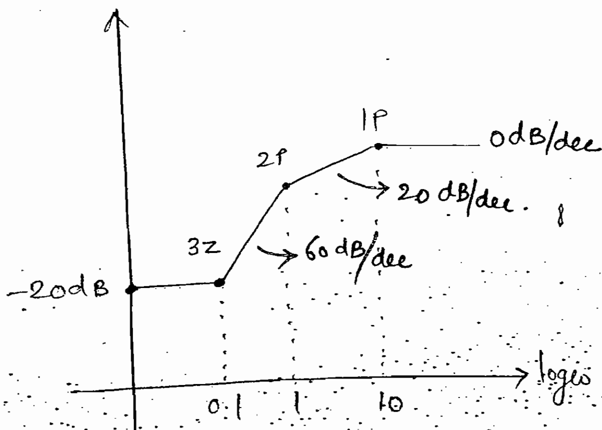
Inverse Bode Plot

(1). observe the starting slope, this will give the information of poles and zeroes at the origin.

(2). write the eqn $y = mx + c$, for the starting slope and put $c = 20\log K$.

(3). At every corner frequency, observe the change in slope. This will give the information of 1st order or quadratic factor.

(1).



$$G(s) = \frac{K(1+10s)^3}{(1+s)^2(1+0.1s)}$$

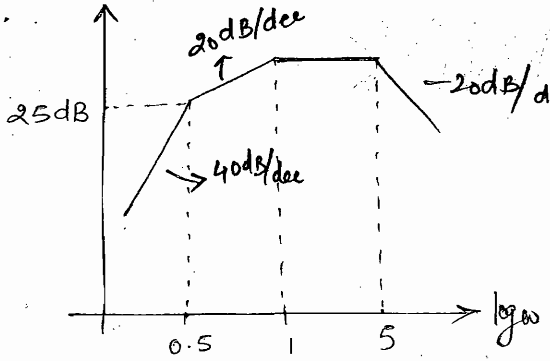
$$= \frac{0.1(1+10s)^3}{(1+s)^2(1+0.1s)}$$

At $\omega = 0.1$, $y = mx + c$

$$-20 = 0 \times \log(0.1) + c \Rightarrow c = -20$$

$$20\log K = c = -20 \Rightarrow K = 0.1$$

(2).



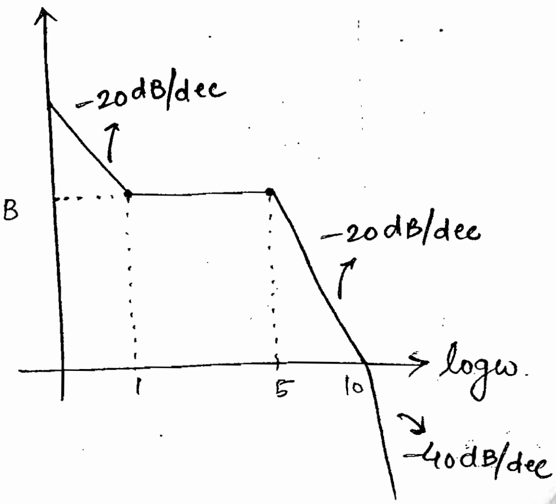
$$G(s) = \frac{70\,s^2}{(1+2s)(1+s)\left(1+\frac{s}{5}\right)}$$

At $\omega = 0.5$, $y = mx + c$

$$25 = 40\log(0.5) + c \Rightarrow c = 37$$

$$20\log K = 37 \Rightarrow K = 70$$

(3).



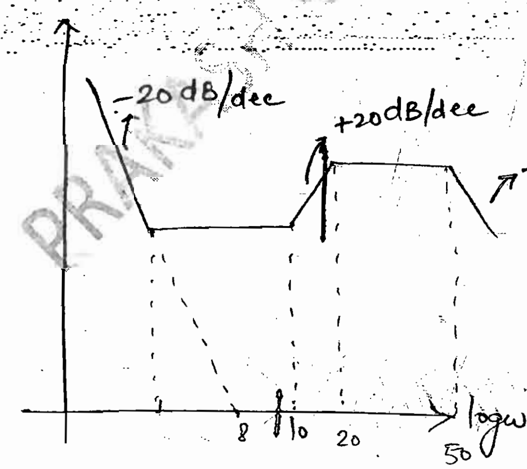
$$G(s) = \frac{10(1+s)}{s(1+0.2s)(1+0.1s)}$$

At $\omega = 1$, $y = mx + c$

$$y = 20 = -20\log(1) + c \Rightarrow c = 20$$

$$20\log K = 20 \Rightarrow K = 10$$

(4).



$$G(s) = \frac{8(1+s)(1+\frac{s}{10})}{s(1+\frac{s}{20})(1+\frac{s}{50})}$$

At $\omega = 8$, $y = mx + c$

$$0 = -20\log 8 + c \Rightarrow c = 18$$

$$20\log K = 18 \Rightarrow K = 8$$

$K_v = K = 8$ (ω)

Decade Scale:

$\omega_2 = 10\omega_1$

dB value magnitude $= \pm 20 \times n \log \omega$

Slope (m) $= \pm 20 \times n \log 10$

$= \pm 20 \times n$ dB/dec

Octave Scale:

$\omega_2 = 2\omega_1$

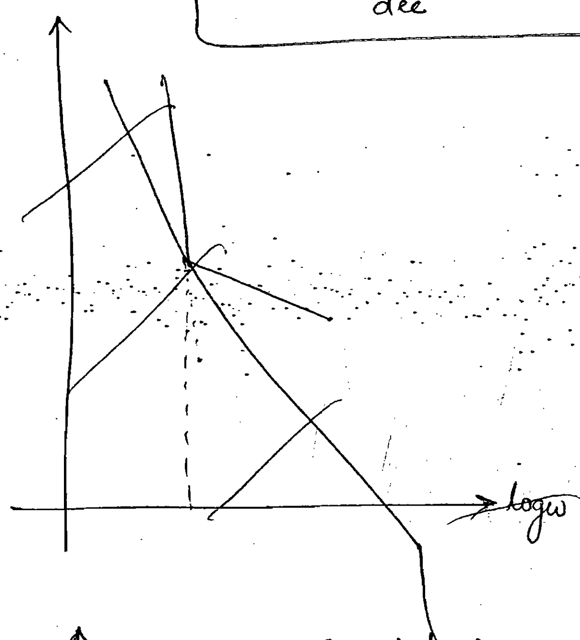
dB value magnitude $= \pm 20 \times n \log \omega$

$= \pm 20 \times n \log 2$

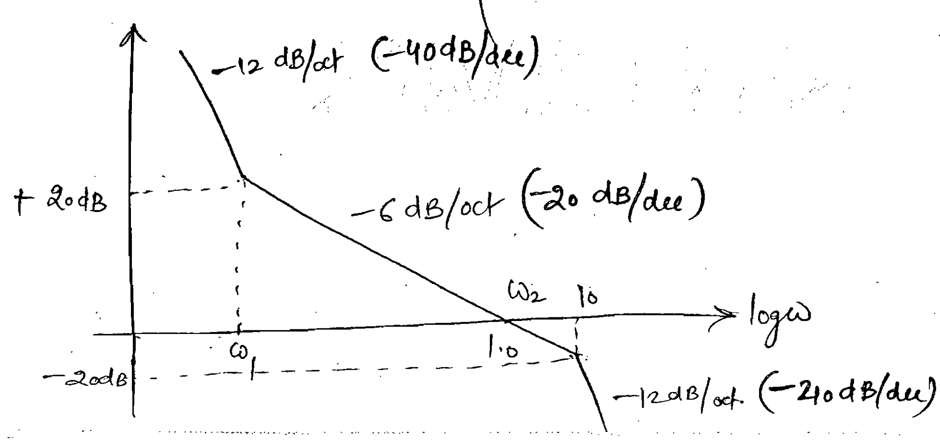
$= \pm 6 \times n$ dB/octave

$$\boxed{\pm 20 \times n \frac{dB}{dec} \cong \pm 6 \times n \frac{dB}{oct.}}$$

Q.



Q.



Second line (known parameters):

At $\omega = 1$, $y = mx + c$

$0 = -20\log(1) + c$

$c = 0$

At $\omega = \omega_1$, $y = mx + c$

$20 = -20\log\omega_1 + 0 \Rightarrow \omega_1 = 0.1\ r/s$

At $\omega = \omega_2$, $y = mx + c$

$-20 = -20\log\omega_2 + 0 \Rightarrow \omega_2 = \log^{-1}(1) = 10\ r/s$

First line: To find K?

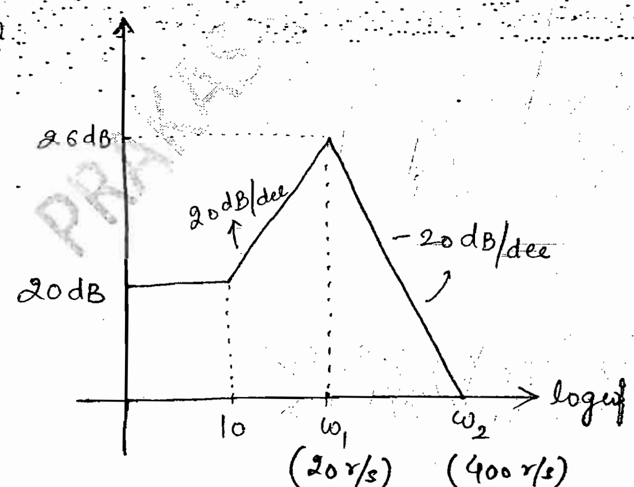
At $\omega = \omega_1 = 0.1$, $y = mx + c$

$20 = -40\log(0.1) + c \Rightarrow c = -20$

$20\log K = -20 \Rightarrow K = 0.1$

$$G(s) = \frac{0.1\ (1 + 10s)}{s^2\ (1 + 0.1s)}$$

Q



Second line:

At $\omega = 10$, $y = mx + c$

$20 = 20\log 10 + c$

$\Rightarrow c = 0$

At $\omega = \omega_1$

$26 = 20\log\omega_1$

$\omega_1 = 20\ rad/s$

Third line:

At $\omega = \omega_1 = 20$, $26 = -20\log(20) + c \Rightarrow c = 52$

At $\omega = \omega_2$, $0 = -20\log\omega_2 + 52 \Rightarrow \omega_2 = 400\ rad/s$

To find K : First line :

At $\omega = 10$, $y = mx + c$

$$20 = 0 \times \log(10) + c \Rightarrow c = 20$$

$$20 \log K = c = 20 \Rightarrow K = 10$$

$$G(s) = \frac{10\left(1+\frac{s}{10}\right)}{\left(1+\frac{s}{20}\right)}$$

M and N circles :

(Nichol charts)

(1). The Nichol's chart consist of magnitudes and phase angles of a closed loop s/s represented as a family of circles known as M & N circles.

(2). The Nichol's chart gives information about close loop frequency response of the s/s.

(3). M circles :

Let the CLTF $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$

Let $G(s) = x + jy$

$$\frac{C(s)}{R(s)} = \frac{x+jy}{x+1+jy}$$

The Magnitude [M]

$$M = \frac{\sqrt{x^2+y^2}}{\sqrt{(1+x)^2+y^2}} \Rightarrow M^2 = \frac{x^2+y^2}{(1+x)^2+y^2}$$

$$M^2\left[(1+x)^2+y^2\right] = x^2+y^2$$

$$M^2x^2 - x^2 + M^2y^2 - y^2 + 2xM^2 + M^2 = 0$$

$$x^2(M^2-1) + y^2(M^2-1) + 2xM^2 + M^2 = 0 \quad \text{--- (1)}$$

In eqn (1), if $M=1$, $\Rightarrow$ $2x+1=0$

It represents a straight line passing through $\left(\frac{-1}{2}, 0\right)$.

if $M \neq 1$ $\Rightarrow$

It represents a family of circles.

$$x^2+y^2+2x\frac{M^2}{M^2-1} + \frac{M^2}{M^2-1} = 0$$

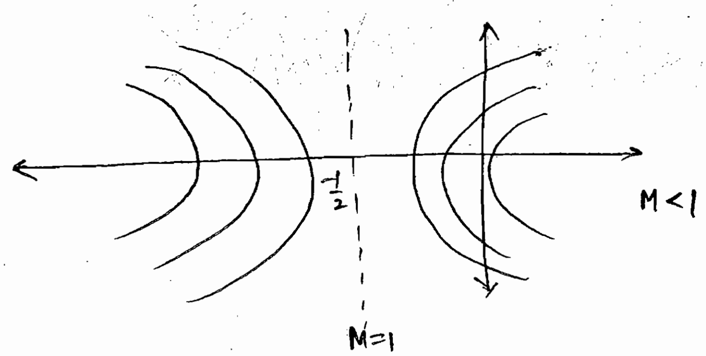
$$x^2+y^2+2x\frac{M^2}{M^2-1} + \underbrace{\frac{M^2}{M^2-1} + \frac{M^2}{(M^2-1)^2}}= \frac{M^2}{(M^2-1)^2}$$

$$x^2+y^2+2x\frac{M^2}{M^2-1} + \frac{M^4}{(M^2-1)^2} = \frac{M^2}{(M^2-1)^2}$$

$$\left[x+\frac{M^2}{M^2-1}\right]^2 + y^2 = \frac{M^2}{(M^2-1)^2}$$

Centre $= \left(-\frac{M^2}{M^2-1}, 0\right)$

Radius $= \frac{M}{M^2-1}$



(4). N circles :

Let α = phase angle of CL s/s

$N = \tan\alpha$ represents family of circles.

$$\alpha = \tan^{-1}\frac{Y}{X} - \tan^{-1}\frac{Y}{1+X}$$

$$N = \tan\alpha = \tan\left[\tan^{-1}\frac{Y}{X} - \tan^{-1}\frac{Y}{1+X}\right]$$

$$\frac{\frac{Y}{X} - \frac{Y}{1+X}}{1 + \frac{Y^2}{X(1+X)}} \Rightarrow X^2 + X + Y^2 - \frac{Y}{N} = 0$$

Adding $\frac{1}{4} + \left(\frac{1}{2N}\right)^2$ on both sides,

$$X^2 + X + \frac{1}{4} + Y^2 - \frac{Y}{N} + \left(\frac{1}{2N}\right)^2 = \frac{1}{4} + \left(\frac{1}{2N}\right)^2$$

$$\left[X + \frac{1}{2}\right]^2 + \left[Y - \frac{1}{2N}\right]^2 = \frac{1}{4} + \left(\frac{1}{2N}\right)^2$$

Centre $\rightarrow \left(\frac{-1}{2}, \frac{1}{2N}\right)$

Radius $= \sqrt{\frac{1}{4} + \left(\frac{1}{2N}\right)^2}$

All N circles intersect the Real axis b/w -1 and origin.

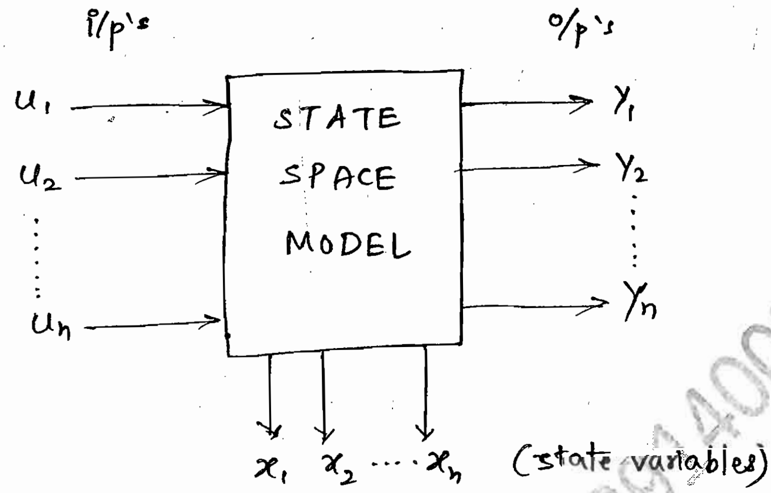
Q24. $X^2 + 2.25X + Y^2 + 1.125 = 0$

$$X^2 + 2X\frac{M^2}{M^2-1} + Y^2 + \frac{M^2}{M^2-1} = 0$$

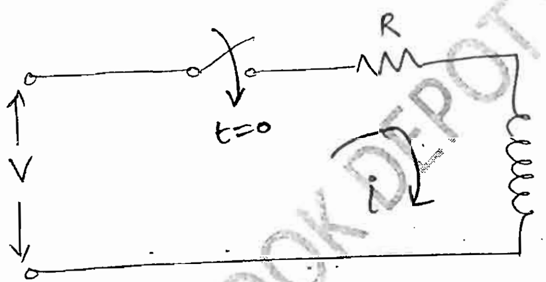
$$\frac{M^2}{M^2-1} = 1.125 \Rightarrow M = 3$$

Part - V

STATE SPACE ANALYSIS



Eg.



$$V = iR + L\frac{di}{dt}$$

$$\frac{di}{dt} = -\frac{R}{L}i + \frac{V}{L} \quad A/s$$

[STATE eqn of N/w]

~~I. STATE EQN~~: At $t = 0^-$

$$i(0^-) = i_L(0^-) = 0 \text{ Amps}$$

At $t = 0^+$,

$$i_L(0^+) = \frac{1}{L}\int_{t=0}^{0^+} V\,dt = 0\,A$$

I. STATE EQN:

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} A \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} B \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}$$

II OUTPUT EQN:

$$y(t) = C x(t) + D u(t)$$

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} C \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} D \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}$$

Type -(1): To obtain state model from differential Eqn:

$$\frac{Y(s)}{U(s)} = \frac{10}{s^3+4s^2+6s+8}$$

$$s^3 Y(s) + 4s^2 Y(s) + 6(s) Y(s) + 8 Y(s) = 10 U(s)$$

$$\frac{d^3y}{dt^3} + 4\frac{d^2}{dt^2}y + 6\cdot\frac{dy}{dt} + 8y = 10u$$

To define state variables:

Let $y = x_1$

$$\frac{dy}{dt} = \dot{x}_1 = x_2, \quad \frac{d^2y}{dt^2} = \dot{x}_2 = x_3, \quad \frac{d^3y}{dt^3} = \dot{x}_3$$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = 10u - 4x_3 - 6x_2 - 8x_1$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -8 & -6 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + 0$$

It is BUSH / COMPANION FORM.

Q7. $\frac{Y(s)}{U(s)} = \frac{1}{s^4+5s^3+8s^2+6s+3}$ ← Reverse order with reverse sign.

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\-3&-6&-8&-5\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} + \begin{bmatrix}0\\0\\0\\1\end{bmatrix}u$$

→ numerator

$$y = \begin{bmatrix}1&0&0&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}$$

Q8. $\frac{Y(s)}{U(s)} = \frac{2s+1}{s^2+7s+9}$

$s^2Y(s) + 7s\cdot Y(s) + 9Y(s) = 2s\,U(s) + U(s)$

$\frac{d^2y}{dt^2} + 7\frac{dy}{dt} + 9\cancel{\frac{dy}{dt}}y = 2\cdot\frac{du}{dt} + u$

Phase Variable form:

$\frac{Y(s)}{U(s)} = \frac{1}{(s^2+7s+9)}(2s+1)$ ← Reverse order

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0&1\\-9&-7\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} + \begin{bmatrix}0\\1\end{bmatrix}u$$

$$y = \begin{bmatrix}1&2\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}$$

Q. $\frac{Y(s)}{U(s)} = \frac{10(s^2+4s+8)}{s^3+6s^2+12s+10}$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -10 & -12 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} u$$

$$y = \begin{bmatrix} 8 & 4 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

<u>Type-(2)</u> To obtain TF from state model:

$$\dot{X}(t) = A X(t) + B u(t)$$

$$y(t) = C X(t) + D u(t)$$

Applying Laplace transform.

$$s X(s) - X(0) = A X(s) + B u(s)$$

$$y(s) = C X(s) + D u(s)$$

for TF, $X(0) = 0$

$$s X(s) = A X(s) + B u(s)$$

$$X(s)(sI - A) = B u(s)$$

$$X(s) = (sI-A)^{-1} B u(s)$$

$$y(s) = \{C \cdot (sI-A)^{-1} \cdot B + D\} u(s)$$

$$\boxed{\frac{y(s)}{u(s)} = C \cdot (sI-A)^{-1} B + D}$$ Transfer matrix

Q9. $\dot{x} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$

$$y = \begin{bmatrix} 1 \\ 1 \end{bmatrix}^T x$$

$$(sI-A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} s+1 & 0 \\ 0 & s+2 \end{bmatrix}$$

$$|sI-A| = (s+1)(s+2)$$

$$(sI-A)^{-1} = \begin{bmatrix} \frac{1}{s+1} & 0 \\ 0 & \frac{1}{s+2} \end{bmatrix}$$

$$(sI-A)^{-1}B = \begin{bmatrix} \frac{1}{s+1} & 0 \\ 0 & \frac{1}{s+2} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{s+1} \\ 0 \end{bmatrix}$$

$$C \cdot (sI-A)^{-1}B = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{s+1} \\ 0 \end{bmatrix} = \frac{1}{s+1}$$

Q2. $\dot{x}(t) = -2x(t) + 2u(t)$

$y(t) = 0.5x(t)$

$sX(s) = -2X(s) + 2U(s)$

$(s+2)X(s) = 2U(s)$

$X(s) = \frac{2}{s+2} U(s)$

$Y(s) = 0.5X(s)$

$Y(s) = \frac{2}{s+2} \times 0.5 U(s)$

$$\frac{Y(s)}{U(s)} = \frac{1}{s+2}$$

Type-3 : Stability from state model

$$\frac{Y(s)}{U(s)} = C\,(sI-A)^{-1}B + D$$

$$= C\cdot\frac{\operatorname{adj}(sI-A)}{|sI-A|}B + D$$

$$= C\cdot\frac{\operatorname{adj}(sI-A)B + D\,|sI-A|}{|sI-A|}$$

for Zeroes,

$$C\,\operatorname{adj}(sI-A)\cdot B + |sI-A|\cdot D = 0$$

for Poles, $1+G(s)H(s)=0 \Rightarrow |sI-A|=0$

Eigen values of s/s matrix [A] = CL poles

conv.

Q2. $\dot{x} = \begin{bmatrix} 0 & 1 \\ -20 & -9 \end{bmatrix}x + \begin{bmatrix} 0 \\ 1 \end{bmatrix}r$

$C = \begin{bmatrix} -17 & -5 \end{bmatrix}x + [1]\,r$

$$(sI-A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -20 & -9 \end{bmatrix} = \begin{bmatrix} s & -1 \\ 20 & s+9 \end{bmatrix}$$

$$\operatorname{Adj}(sI-A) = \begin{bmatrix} s+9 & -20 \\ 1 & s \end{bmatrix} = \begin{bmatrix} s+9 & 1 \\ -20 & s \end{bmatrix}$$

$$\operatorname{Adj}(sI-A)\cdot B = \begin{bmatrix} s+9 & 1 \\ -20 & s \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ s \end{bmatrix}$$

$$C\cdot\operatorname{Adj}(sI-A)\cdot B = \begin{bmatrix} -17 & -5 \end{bmatrix}\begin{bmatrix} 1 \\ s \end{bmatrix} = -17-5s$$

$$|sI-A| = s(s+9)+20$$

$$|sI-A|\cdot D = [s^2+9s+20]\,[1]$$

Zeroes $\Rightarrow$ $-17-5s+s^2+9s+20=0$

$$s^2+4s+3=0 \Rightarrow s=-1,-3$$

Poles $\Rightarrow$ $|sI-A|=0$

$$s^2+9s+20=0 \Rightarrow s=-4,-5$$

(s/s is stable)

Q6.

$$\dot{X} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 1 & 2 \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$u = [-0.5 \quad -3 \quad -5]\,X + V$$

$$\dot{X} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 1 & 2 \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}_{3\times1} [-0.5 \quad -3 \quad -5]_{1\times3}\, X + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} V$$

$$\dot{X} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 1 & 2 \end{bmatrix} X + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -0.5 & -3 & -5 \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} V$$

$$\dot{X} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} V$$

$$(sI-A) = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix} = \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 0 & 2 & s+3 \end{bmatrix}$$

$s(s(s+3)+2] + 1[0] + 0 = 0$

$s(s^2 + 3s + 2) = 0$

$s = 0, -1, -2.$

Type-4 : Controllability and observability

Controllability $\rightarrow$ To control the state variables.

observability $\rightarrow$ To measure the state variables.

Kalman's test :

$$Q_c = \begin{bmatrix} B & AB & A^2B & \dots & A^{n-1}B \end{bmatrix} = |Q_c| \neq 0$$

$$Q_0 = \begin{bmatrix} C^T & A^TC^T & (A^T)^2C^T & \dots & (A^T)^{n-1}C^T \end{bmatrix} = |Q_0| \neq 0$$

Q1. $\dot{X} = \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$

$y = \begin{bmatrix} 1 & 1 \end{bmatrix} x + 0$

$Q_c = [B \quad AB]$

$AB = \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$

$Q_c = \begin{bmatrix} 0 & 1 \\ 1 & -3 \end{bmatrix} = -1$

$\therefore$ Controllable

$Q_0 = [C^T \quad A^TC^T]$

$A^TC^T = \begin{bmatrix} 0 & 2 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$

$Q_0 = \begin{bmatrix} 1 & 2 \\ 1 & -2 \end{bmatrix} = -4$

$\therefore$ Observable

Q10. $\dot{X} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$; $y = [b \quad 0]\, x$

$Q_0 = [C^T \quad A^TC^T]$

$A^TC^T = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} b \\ 0 \end{bmatrix} = \begin{bmatrix} b \\ 2b \end{bmatrix}$

$Q_0 = \begin{bmatrix} b & b \\ 0 & 2b \end{bmatrix} = 2b^2 \neq 0$

$\therefore$ Observable for all value of b but not for $(b=0)$.

Type – 5 Solution of state Eqn

$$\dot{X}(t) = A X(t) + B u(t)$$

(a). Free Response : $[u(t) = 0]$.

$$\dot{X}(t) = A X(t) \Rightarrow X(t) = Ae^{At}$$

$$sX(s) - X(0) = AX(s)$$

$$(sI - A) X(s) = X(0)$$

$$X(s) = (sI-A)^{-1} \cdot X(0)$$

Resolvant matrix $\Rightarrow \phi(s) = (sI-A)^{-1}$

$$\boxed{X(t) = \left[\mathcal{L}^{-1} (sI-A)^{-1}\right] \cdot X(0)}$$

and $x(t) = e^{At} K$

$$\boxed{e^{At} = \phi(t) = \mathcal{L}^{-1} (sI-A)^{-1}}$$

It is state transition matrix $\phi(t)$.

(b). Forced Response :

$$sX(s) - X(0) = AX(s) + BU(s)$$

$$sX(s) - AX(s) = X(0) + BU(s)$$

$$(sI - A) X(s) = X(0) + BU(s)$$

$$X(s) = (sI-A)^{-1} X(0) + (sI-A)^{-1} \cdot B U(s)$$

$$\boxed{X(t) = \left\{\mathcal{L}^{-1} (sI-A)^{-1}\right\} X(0) + \mathcal{L}^{-1}\left[(sI-A)^{-1} B U(s)\right]}$$

Q3. $\dot{X}(t) = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} X$; $X(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$$(sI - A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} s-1 & 0 \\ -1 & s-1 \end{bmatrix}$$

$$\text{Adj}(sI-A) = \begin{bmatrix} s-1 & 0 \\ 0 & s-2 \end{bmatrix}$$

$$|sI-A| = (s-1)^2$$

$$(sI-A)^{-1} = \begin{bmatrix} \frac{1}{s-1} & 0 \\ \frac{1}{(s-1)^2} & \frac{1}{s-1} \end{bmatrix}$$

State transition matrix. $\phi(t)$.

$$\phi(t) = \begin{bmatrix} e^t & 0 \\ te^t & e^t \end{bmatrix}$$

Property of STM:

At $t=0$, $e^{A(0)} = I$,

$$x(t) = \begin{bmatrix} e^t & 0 \\ te^t & e^t \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} e^t \\ t\cdot e^t \end{bmatrix}$$

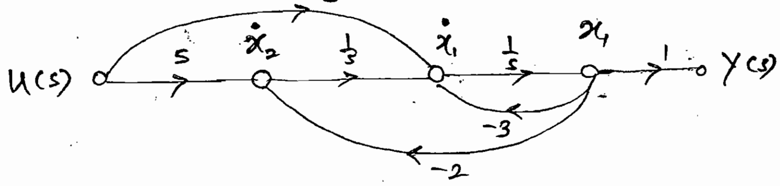
Type-6 STATE DIAGRAM

$$\frac{Y(s)}{U(s)} = \frac{3s+5}{s^2+3s+2}$$

(1). Phase Variable form or Observable canonical form.

$$TF = \frac{\frac{3}{s} + \frac{5}{s^2}}{1 - \left[-\frac{3}{s} - \frac{2}{s^2}\right]}$$

(Integrator in ~~series~~ cascade)



$$\dot{x}_1 = -3x_1 + x_2 + 3u$$

$$\dot{x}_2 = -2x_1 + 5u$$

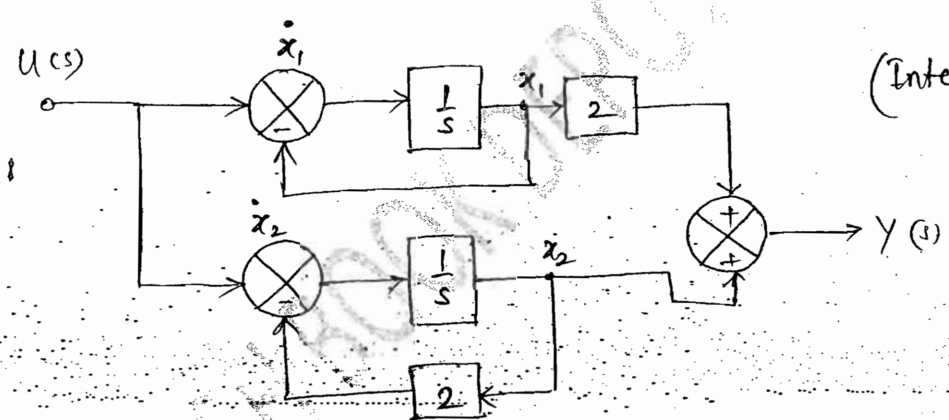
$$y = x_1$$

$$\begin{bmatrix}\dot{x}_1 \\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-3 & 1 \\ -2 & 0\end{bmatrix}\begin{bmatrix}x_1 \\ x_2\end{bmatrix} + \begin{bmatrix}3 \\ 5\end{bmatrix}u$$

$$y = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1 \\ x_2\end{bmatrix}$$

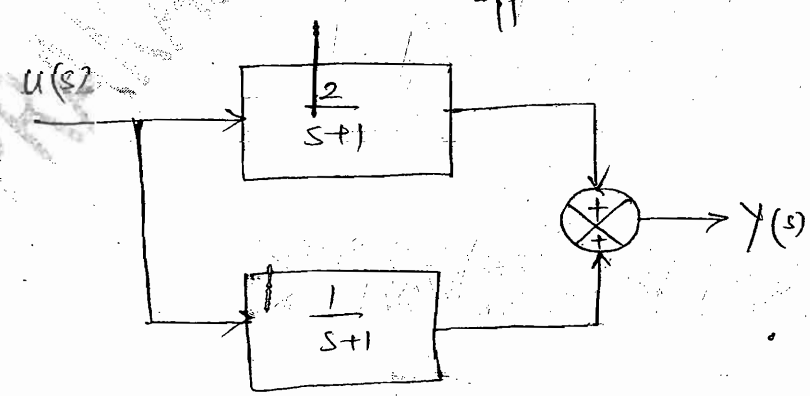
(2). Canonical Variable form (or) Controllable Canonical form

$$Y(s) = U(s)\cdot\left(\frac{2}{s+1}\right) + U(s)\frac{1}{(s+2)}$$



(Integrators in //)

$\Uparrow$

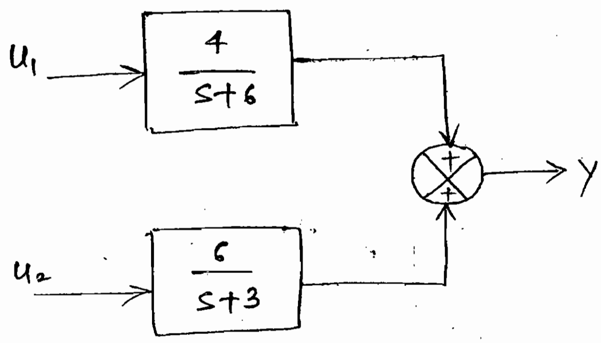


Residues (C)

$$\begin{bmatrix}\dot{x}_1 \\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-1 & 0 \\ 0 & -2\end{bmatrix}\begin{bmatrix}x_1 \\ x_2\end{bmatrix} + \begin{bmatrix}1 \\ 1\end{bmatrix}u \quad ; \quad y = \begin{bmatrix}2 & 1\end{bmatrix}\begin{bmatrix}x_1 \\ x_2\end{bmatrix}$$

(A) Eigenvalues (B) Unit matrix

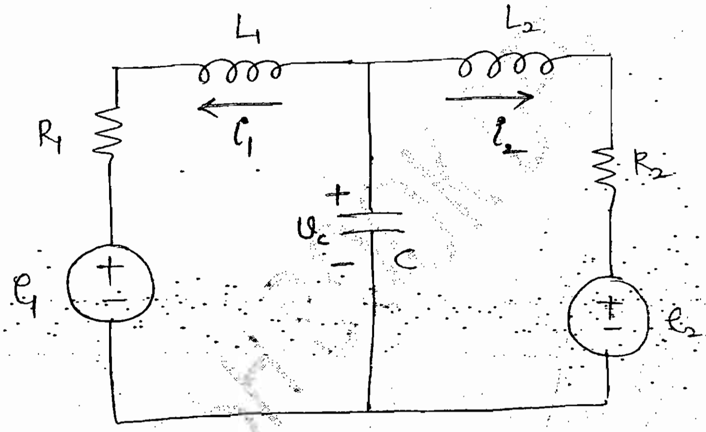
Eg.



$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -6 & 0 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$y = \begin{bmatrix} 4 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

conv.
Q1



Sol. Loop 1:

$$L_1 \frac{di_1}{dt} + i_1 R_1 + e_1 - v_c = 0$$

$$\frac{di_1}{dt} = -\frac{R_1}{L_1} i_1 + \frac{v_c}{L_1} - \frac{e_1}{L_1} \quad \text{—— (1)}$$

Loop 2:

$$L_2 \frac{di_2}{dt} + i_2 R_2 + e_2 - V_c = 0$$

$$\frac{di_2}{dt} = -\frac{R_2}{L_2} i_2 + \frac{v_c}{L_2} - \frac{e_2}{L_2} \quad \text{—— (2)}$$

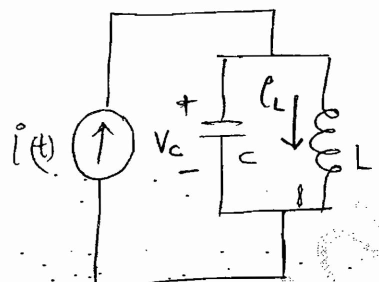
KCL at node V_c:

$$i_1 + i_2 + C\frac{dV_c}{dt} = 0$$

$$\frac{dV_c}{dt} = \frac{-i_1}{C} - \frac{i_2}{C} \quad \text{——— (3)}$$

$$\begin{bmatrix} \frac{di_1}{dt} \\ \frac{di_2}{dt} \\ \frac{dV_c}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R_1}{L_1} & 0 & \frac{1}{L_1} \\ 0 & -\frac{R_2}{L_2} & \frac{1}{L_2} \\ -\frac{1}{C} & -\frac{1}{C} & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ V_c \end{bmatrix} + \begin{bmatrix} -\frac{1}{L_1} & 0 \\ 0 & -\frac{1}{L_2} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$

Eg.



$$i(t) = i_L + i_c$$

$$i(t) = i_L + C\frac{dV_c}{dt}$$

$$\frac{dV_c}{dt} = \frac{-i_L}{C} + \frac{i(t)}{C} \quad \text{——— (1)}$$

$$L\frac{di_L}{dt} - V_c = 0$$

$$\Rightarrow \frac{di_L}{dt} = \frac{V_c}{L} \quad \text{——— (2)}$$

$$\begin{bmatrix} \frac{dV_c}{dt} \\ \frac{di_L}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{C} \\ \frac{1}{L} & 0 \end{bmatrix} \begin{bmatrix} V_c \\ i_L \end{bmatrix} + \begin{bmatrix} \frac{1}{C} \\ 0 \end{bmatrix} i(t)$$

STATE MODEL FOR ARMATURE CONTROLLED AND FIELD CONTROLLED DC SERVO MOTOR :

I. Armature controlled DC servomotor

$$e_a - e_b = i_a R_a + L_a \frac{di_a}{dt}$$

$$e_b = K_b \omega$$

$$T_m = K_T i_a$$

$$T_m = J \cdot \frac{d\omega}{dt} + B\omega$$

$$e_a - K_b \omega = i_a R_a + L_a \frac{di_a}{dt}$$

$$\frac{di_a}{dt} = -\frac{R_a}{L_a} i_a \mp \frac{K_b}{L_a}\omega + \frac{e_a}{L_a} \quad \text{——(1)}$$

$$K_T i_a = J \frac{d\omega}{dt} + B\omega$$

$$\frac{d\omega}{dt} = -\frac{B}{J}\omega + \frac{K_T}{J} \cdot i_a$$

$$\begin{bmatrix} \frac{di_a}{dt} \\ \frac{d\omega}{dt} \end{bmatrix} = \begin{bmatrix} -R_a/L_a & -K_b/L_a \\ K_T/J & -B/J \end{bmatrix} \begin{bmatrix} i_a \\ \omega \end{bmatrix} + \begin{bmatrix} +1/L_a \\ 0 \end{bmatrix} e_a$$

II. field Controlled servomotor :

$$e_f = i_f R_f + L_f \cdot \frac{di_f}{dt}$$

$$T_m = K_T \cdot i_f$$

$$T_m = J \frac{d\omega}{dt} + B\omega$$

$$K_T i_f = J \frac{d\omega}{dt} + B\omega$$

$$\frac{di_f}{dt} = -\frac{R_f}{L_f} \cdot i_f + \frac{e_f}{L_f} \quad \text{——(1)}$$

$$\frac{d\omega}{dt} = \frac{K_T}{J} i_f - \frac{B}{J}\omega \qquad \text{—— (2)}$$

$$\begin{bmatrix} \frac{di_f}{dt} \\ \frac{d\omega}{dt} \end{bmatrix} = \begin{bmatrix} -R_f/L_f & 0 \\ K_T/J & -B/J \end{bmatrix} \begin{bmatrix} i_f \\ \omega \end{bmatrix} + \begin{bmatrix} 1/L_f \\ 0 \end{bmatrix} e_f$$