

SIGNAL AND SYSTEM

9/11/11

⇒ SIGNAL -

- Signal is a physical quantity that contains an information and varies with some independent variables such as time, distance, direction etc.
- Signals are described by mathematical funⁿ

⇒ 1-D SIGNAL -

- Signal is a function of one variable only.

⇒ MULTI DIMENSIONAL SIGNAL -

- Signal is a function of more than one variable.

Ex - TV. picture.

⇒ CHARACTERISTIC OF A SIGNAL -

(i) AMPLITUDE (A) -

It signifies the strength and units depends on nature of the signal.

for electrical signals - V, A or watts.

② PHASE (ϕ or θ) -

It signifies the direction and delay. units are degree or radian.

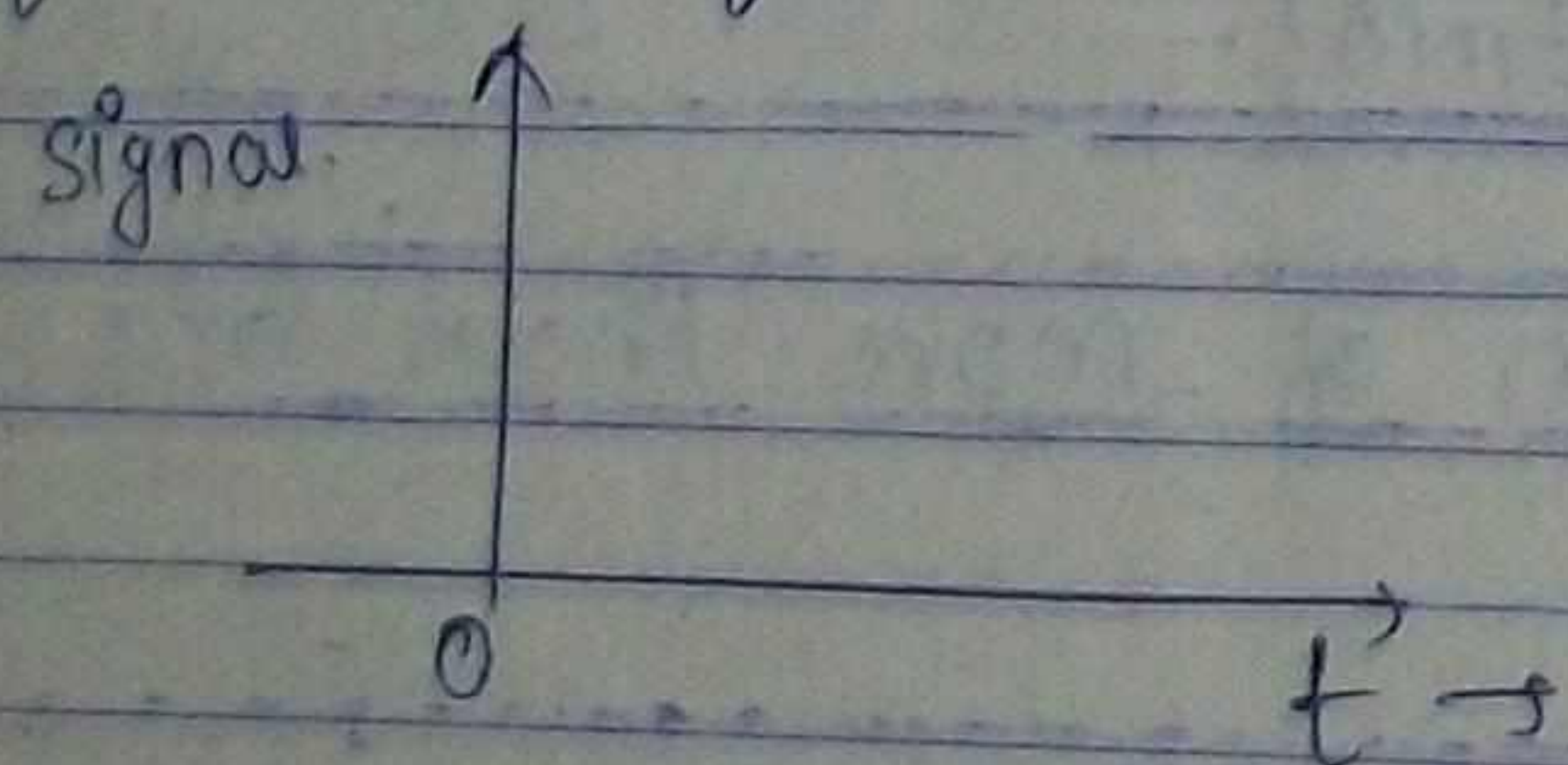
③ Time (t) -

It gives the instant of occurrence
units $\rightarrow$ seconds.

$\Rightarrow$ TYPES OF REPRESENTATION -

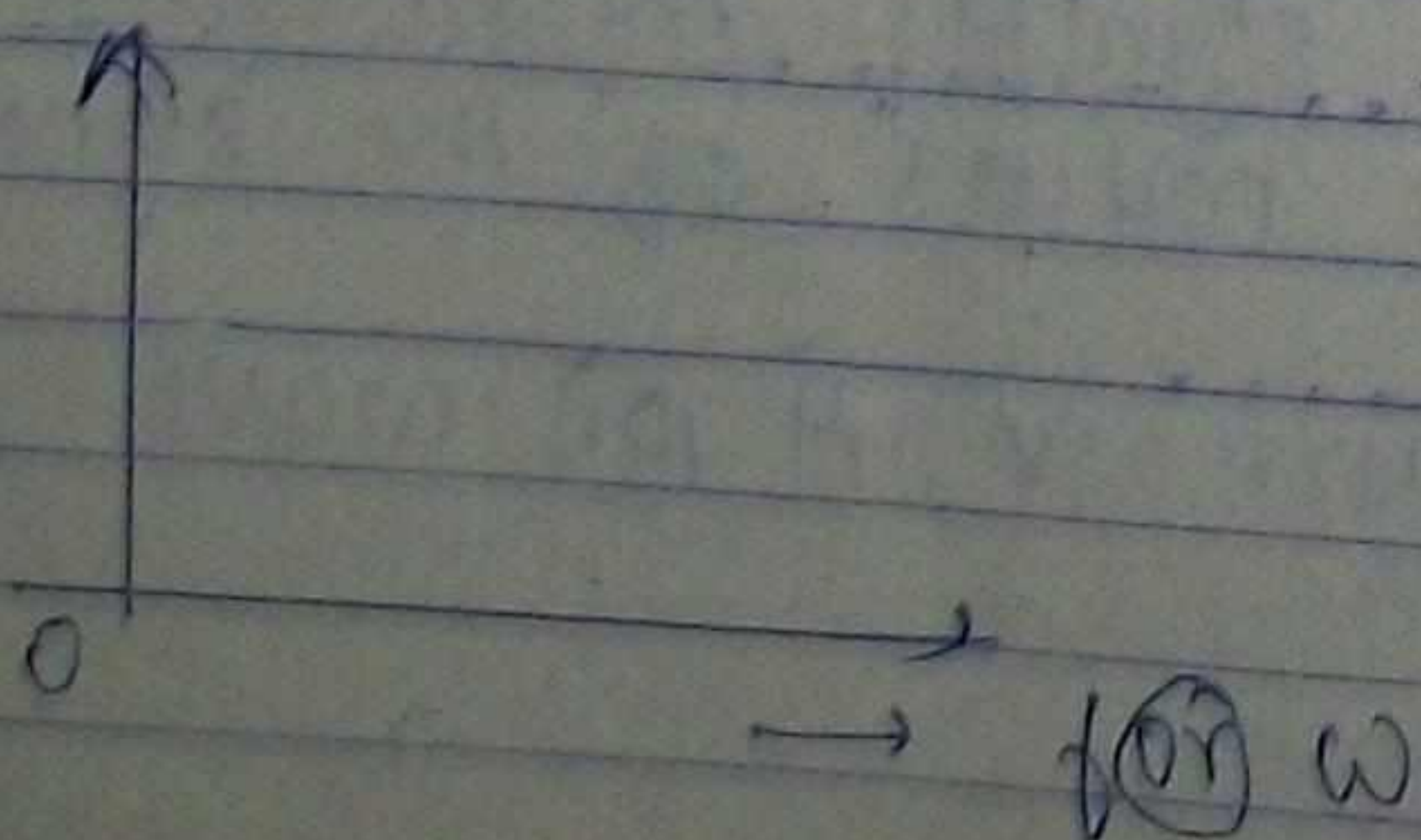
① TIME DOMAIN REPRESENTATION -

Representing the signal as a function of time.



② FREQ. DOMAIN REPRESENTATION -

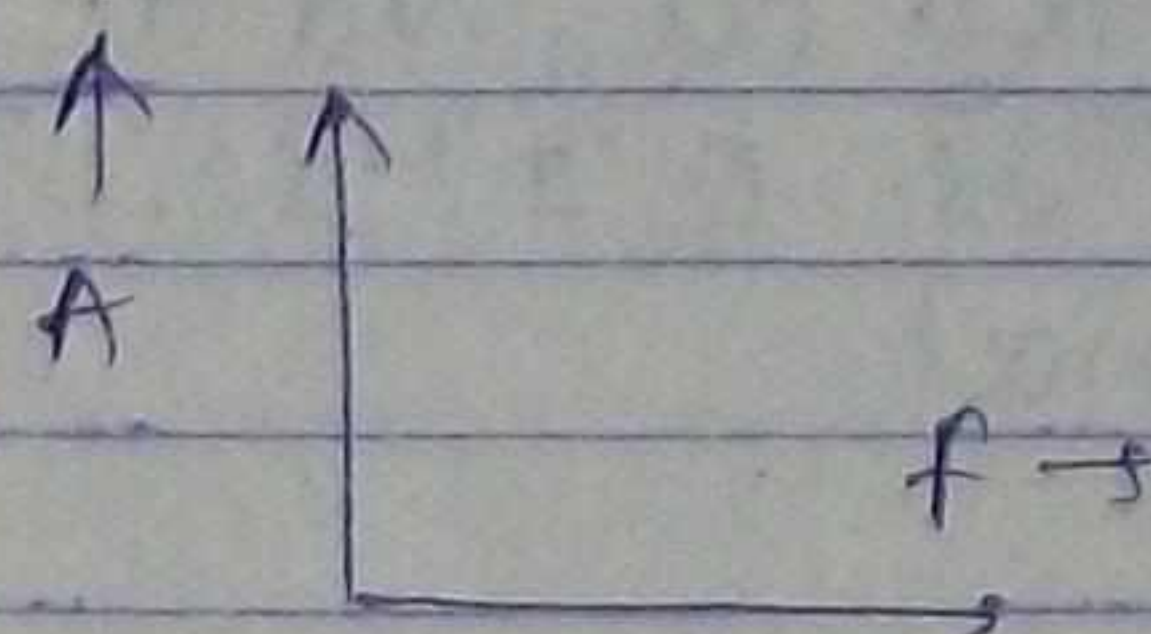
Representing the signal as the function of frequency.



SPECTRAL REPRESENTATION

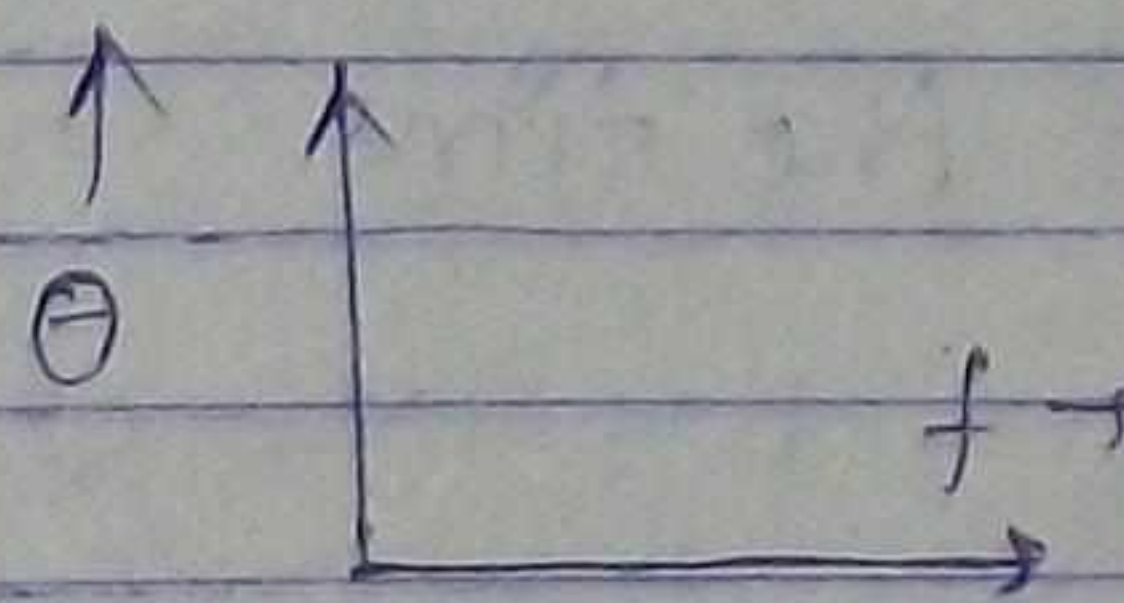
$\Rightarrow$ AMPLITUDE SPECTRUM -

plot B/w amplitude and freq.



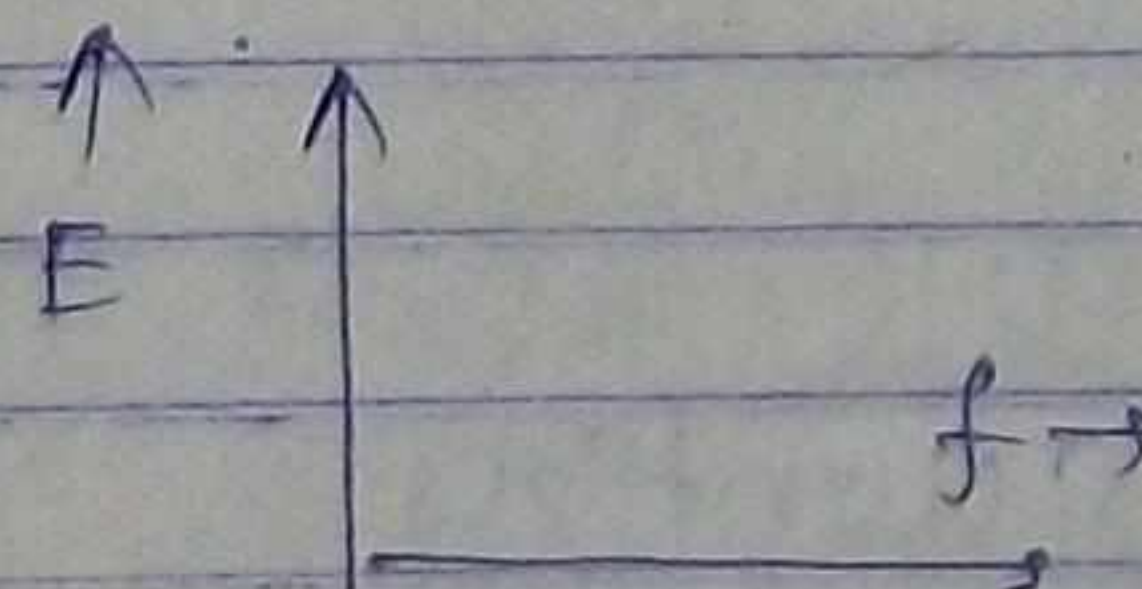
$\Rightarrow$ PHASE SPECTRUM -

plot B/w phase and freq.



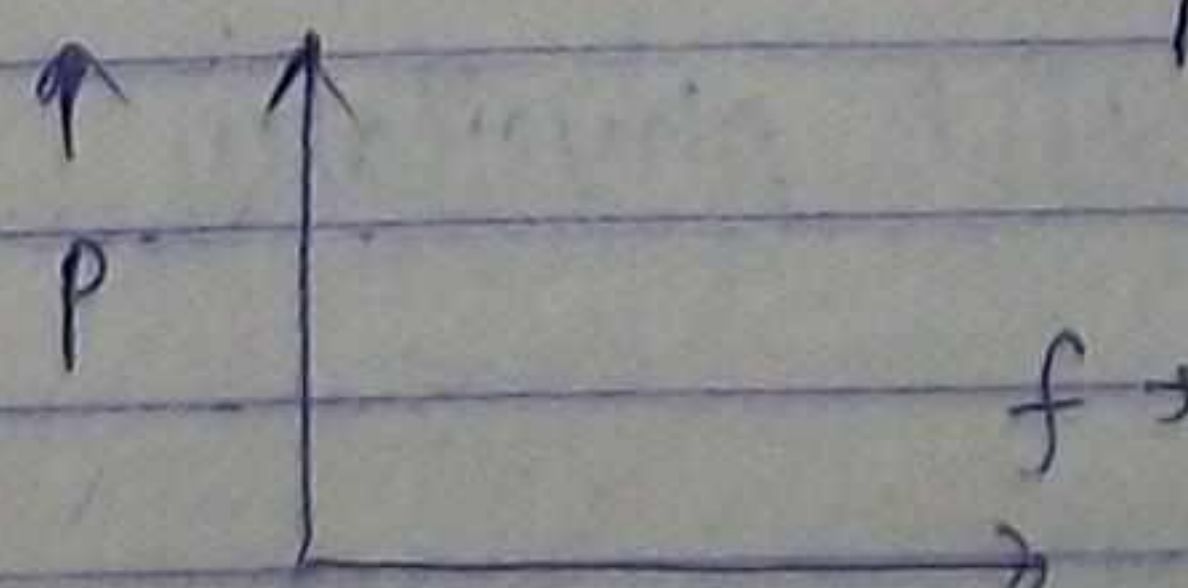
$\Rightarrow$ ENERGY SPECTRUM -

plot B/w energy and freq.



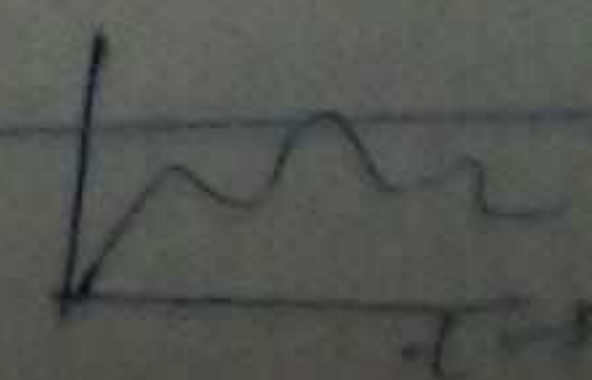
$\Rightarrow$ POWER SPECTRUM -

plot B/w power and freq.



$\Rightarrow$ CONTINUOUS TIME -

If the amplitudes are defined for the continuum values of time, then it is continuous time signal.



⇒ DISCRETE TIME -

If the amplitudes are defined at the integer values of time (i.e. $0, \pm 1, \pm 2, \dots$) then it is discrete time signal.

⇒ ANALOG TO DIGITAL CONVERSION -

① CACT -

Signal in which both the time and amplitude are continuous.

② CADT -

It is the signal after sampling and before quantizer.

③ DADT -

It is the signal after quantizer and before encoder.

④ DACT -

This signal does not exist physically.

⇒ REPRESENTATION OF SIGNAL -

① CONTINUOUS TIME -

$$x(t) = A \sin(\omega t + \phi)$$

$$= A \cos(\omega t + \phi)$$

$$= A e^{j(\omega t + \phi)}$$

A → amplitude

ω → analog angular freq.
(rad/sec)

$$\omega = 2\pi f \rightarrow \text{Analog freq. (Hz)}$$

$$\omega = \frac{2\pi}{T} \rightarrow \text{FUNDAMENTAL PERIOD (sec)}$$

② DISCRETE TIME -

$$x(n) = A \sin(\Omega n + \theta)$$

$$= A \cos(\Omega n + \theta)$$

$$= A e^{j(\Omega n + \theta)}$$

A → amplitude

Ω → digital angular freq.
(rad/sample)

(CYCLES/SAMPLE)

$$\Omega = 2\pi F \rightarrow \text{DIGITAL FREQ.}$$

$$\Omega = \frac{2\pi}{N} \rightarrow \text{TIME PERIOD OR NO. OF SAMPLE PER CYCLE (SAMPLES)}$$

1. SHIFTING
2. SCALING
3. INVERSION OR FOLDING

⇒ OPERATIONS ON SIGNALS -

- ① SHIFTING -
- ② SCALING
- ③ INVERSION

DC → average value

If is the centre of the signal wrt x-axis.

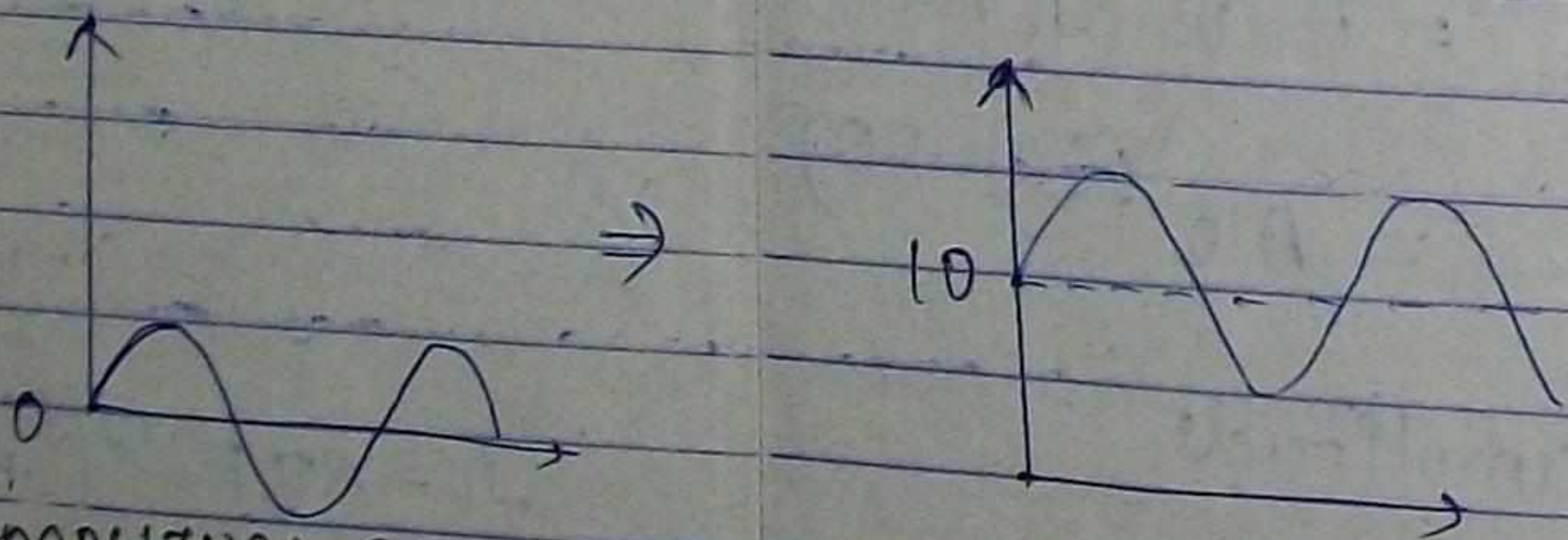
⇒ OPERATION OF AMPLITUDE -

- ① AMPLITUDE SHIFTING -

CLAMPING

- $c + x(t)$ is the signal $x(t)$ shifted in amplitude by c , where c is constant.

- for $c > 0$ upshift
for $c < 0$ downshift

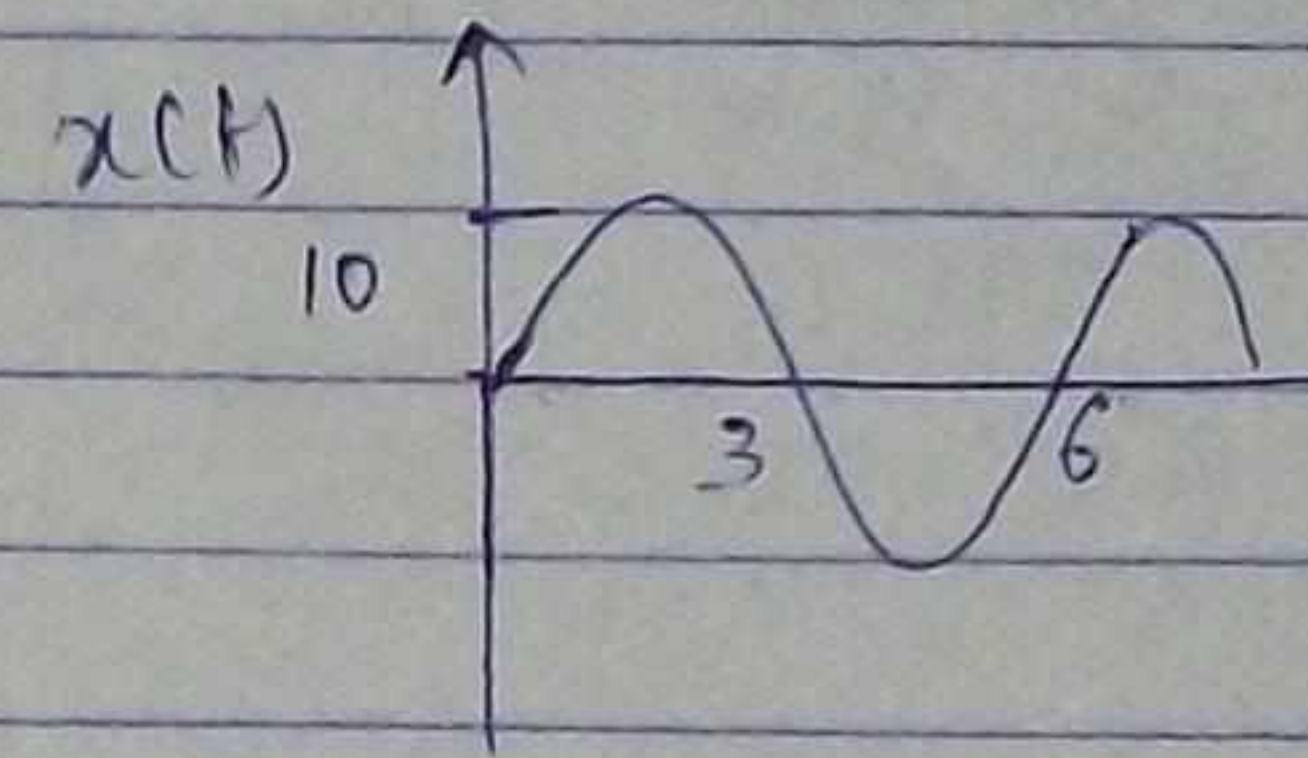


- ② AMPLITUDE SCALING -

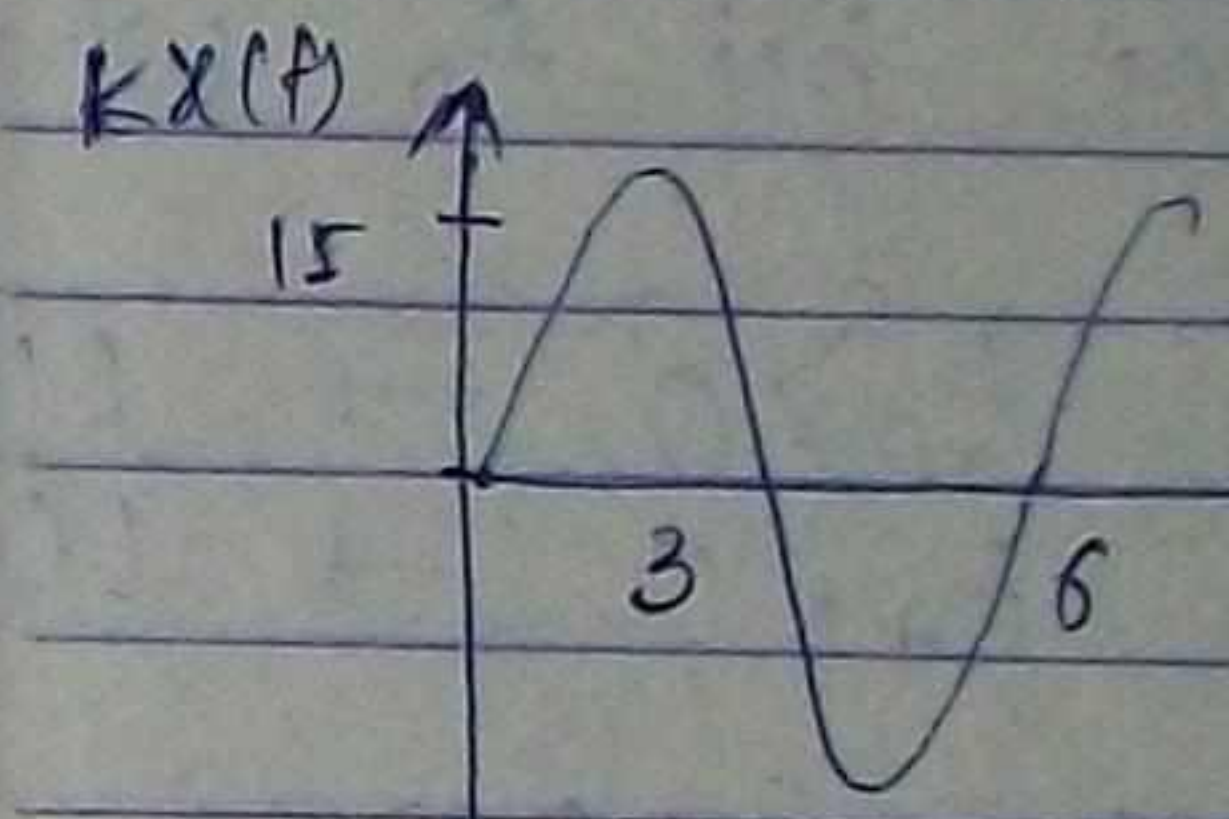
- $kx(t)$ is the signal $x(t)$ scaling in

amplitude by k where

$k \rightarrow$ constant.



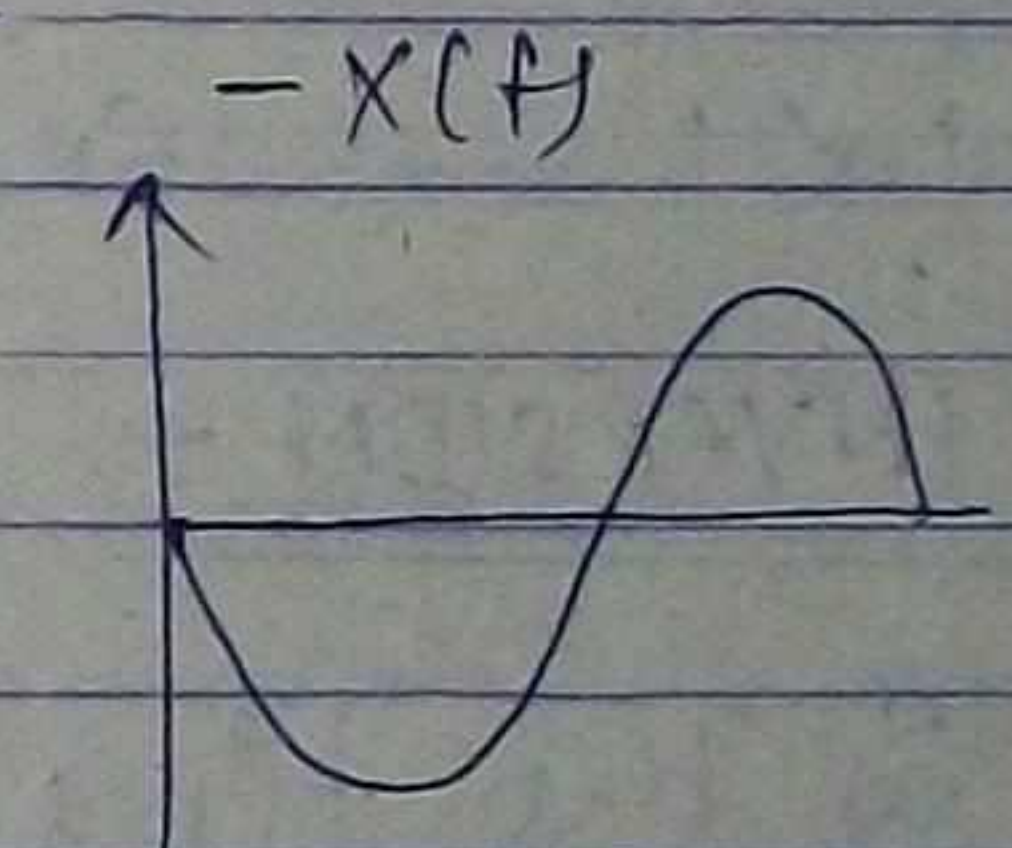
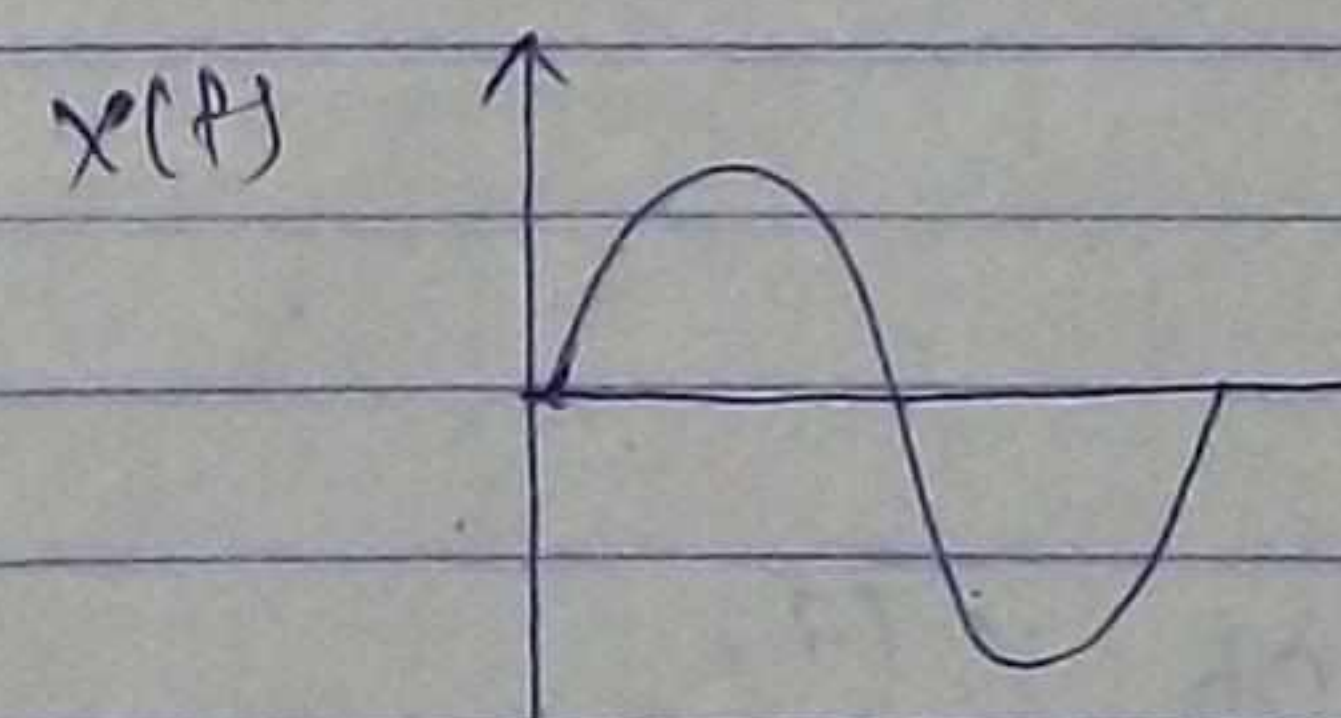
⇒



- for $|k| > 1 \rightarrow$ amplification
for $|k| < 1 \rightarrow$ attenuation.

- ③ AMPLITUDE INVERSION -

$[-x(t)]$ is the amplitude inversion of $x(t)$.
it is the reflection or folding about x-axis.



⇒ OPERATION ON TIME -

- ① TIME SHIFTING -

LEFT SHIFTING

ADVANCE

RIGHT SHIFTING

DELAY

— $x(t-t_0)$ is the signal $x(t)$ shifted in time by t_0 where t_0 is constant.

— for $t_0 > 0 \rightarrow$ RIGHT SHIFT
for $t_0 < 0 \rightarrow$ LEFT SHIFT

② TIME SCALING —

MULTIPLICATION
DIVISION

COMPRESSION

EXPANSION

— $x(at)$ is the signal $x(t)$ scaled in time by a , where $a \rightarrow$ constant.

— for $|a| > 1 \rightarrow$ compression
for $|a| < 1 \rightarrow$ expansion

③ TIME INVERSION —

$x(-t)$ is the time

— It is the folding

inversion of $x(t)$.

about y-axis.

Ques. for the signal sketch

① $x(t) + 3$

$x(t) - 3$

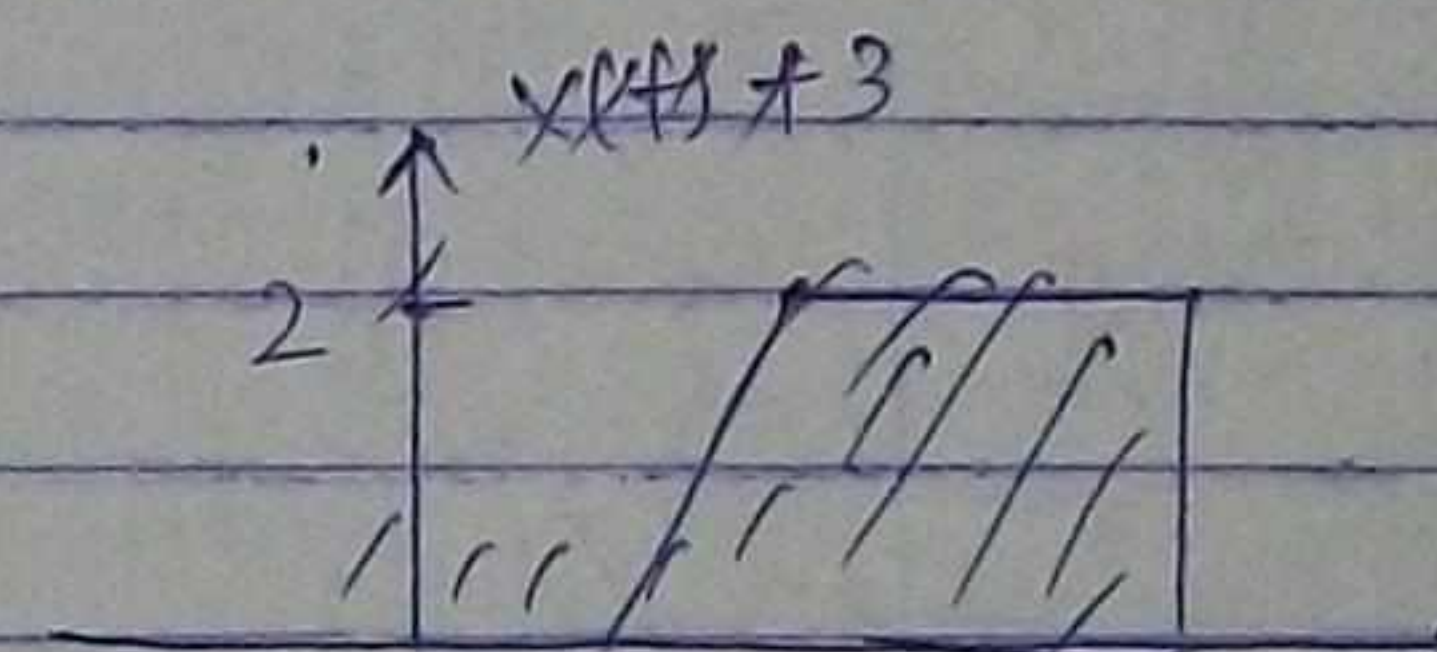
$-x(t)$

$3 - x(t)$

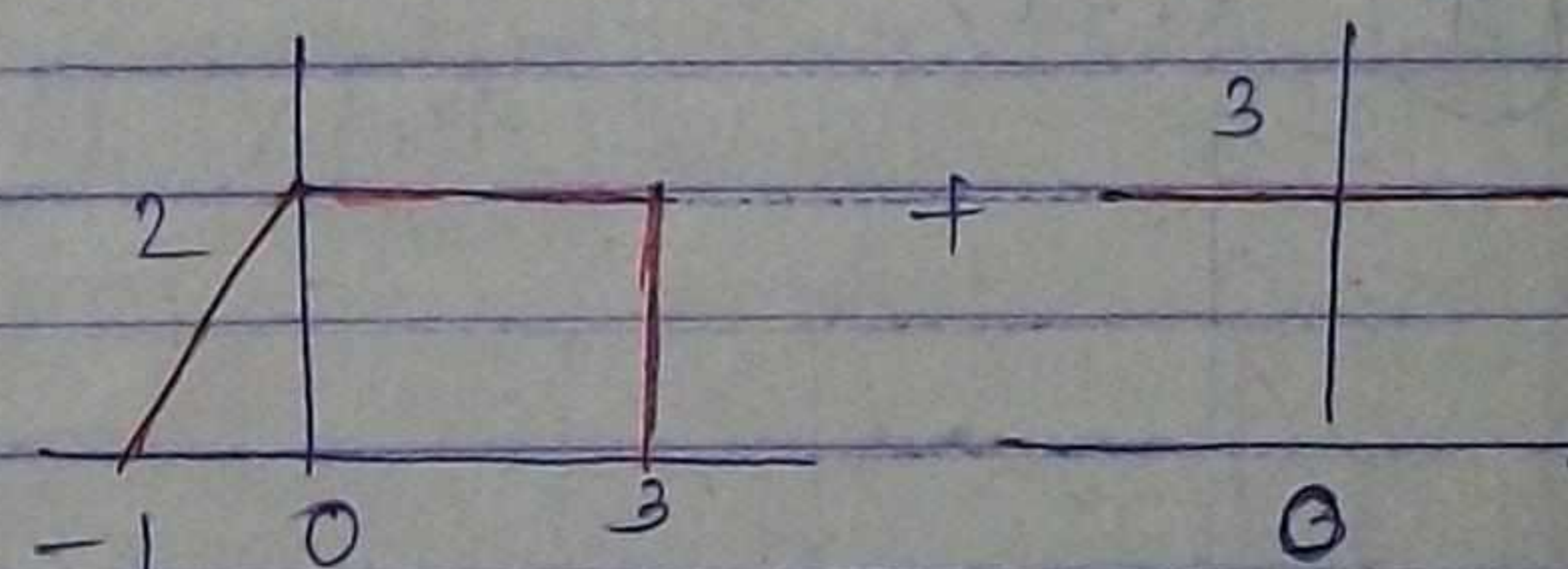
② $-3/5 x(t)$

$10 x(t)$

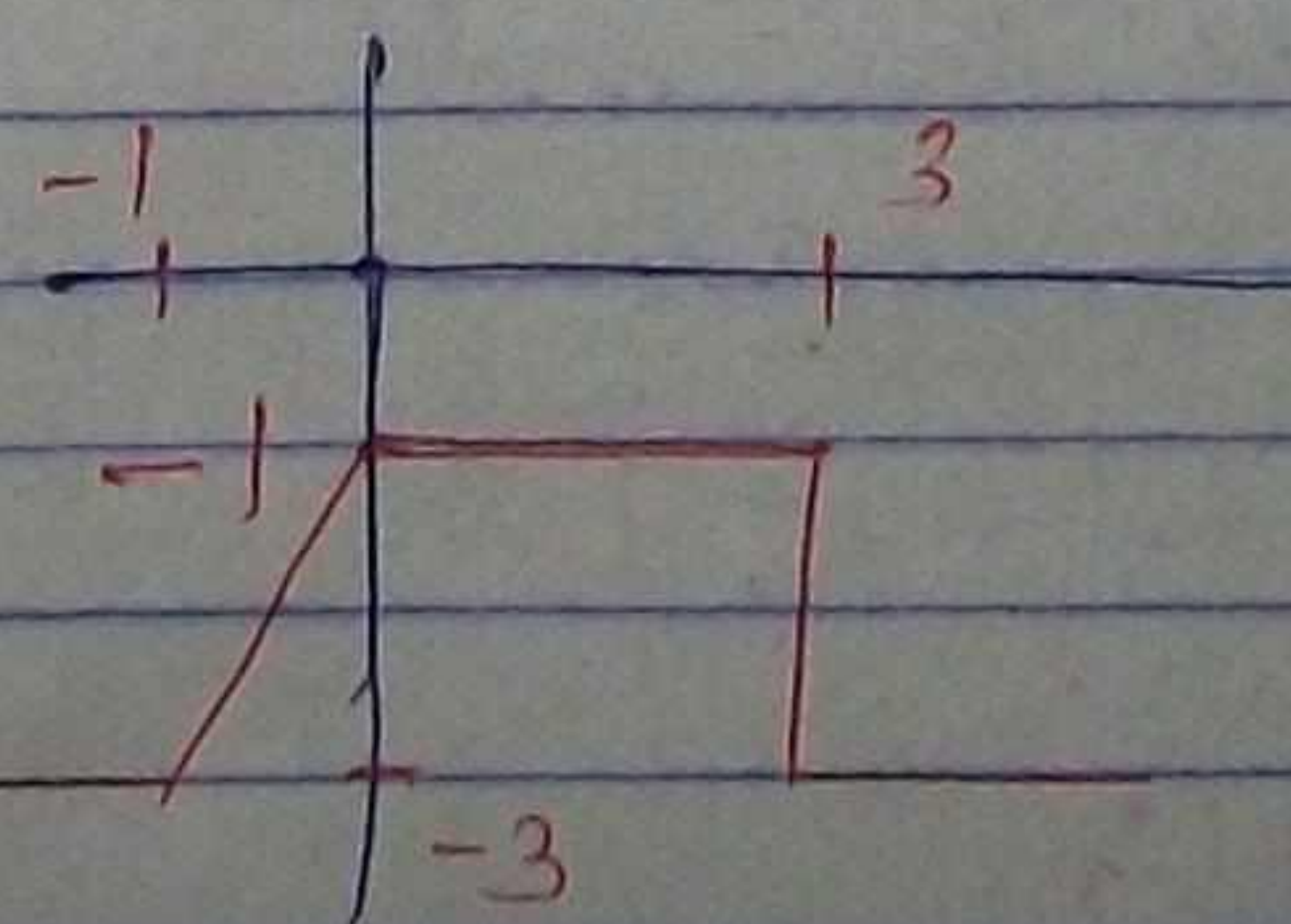
Solⁿ: ①



① $x(t) + 3$

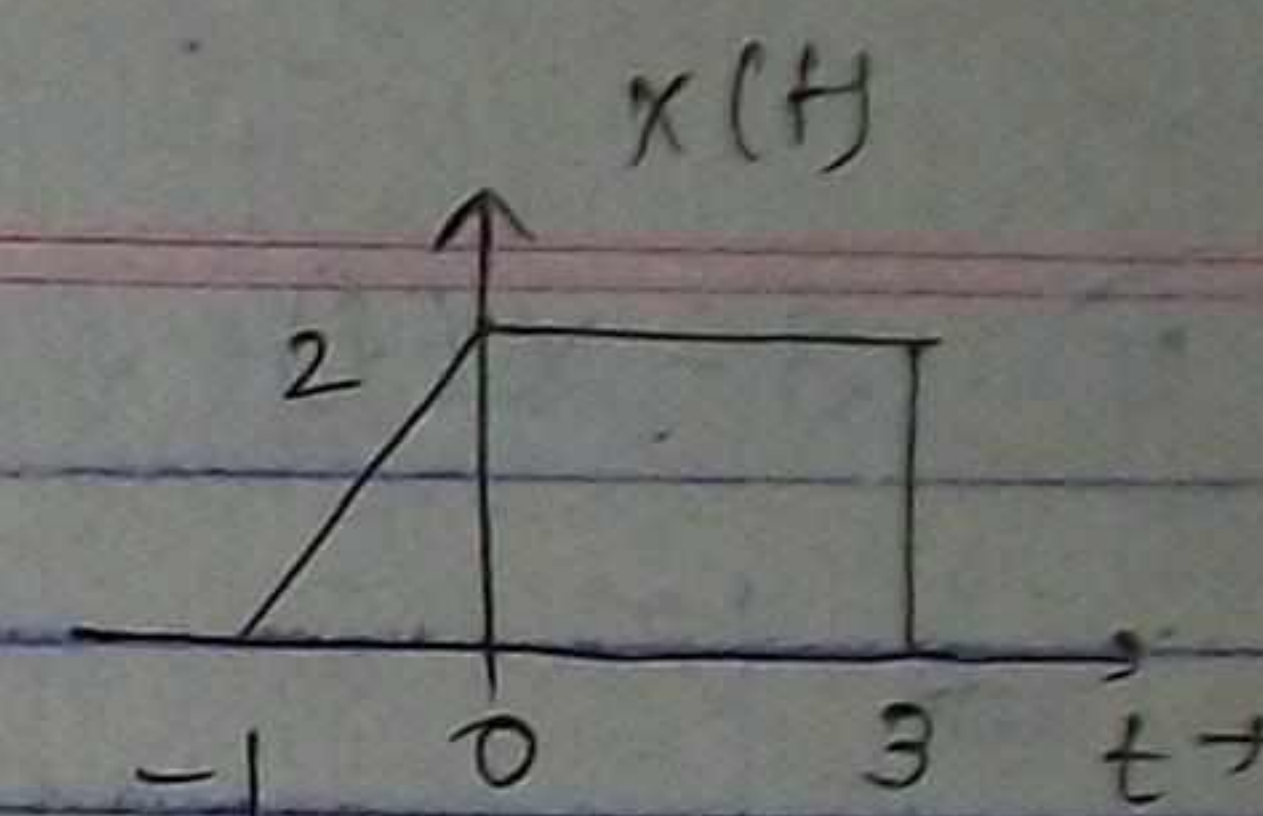
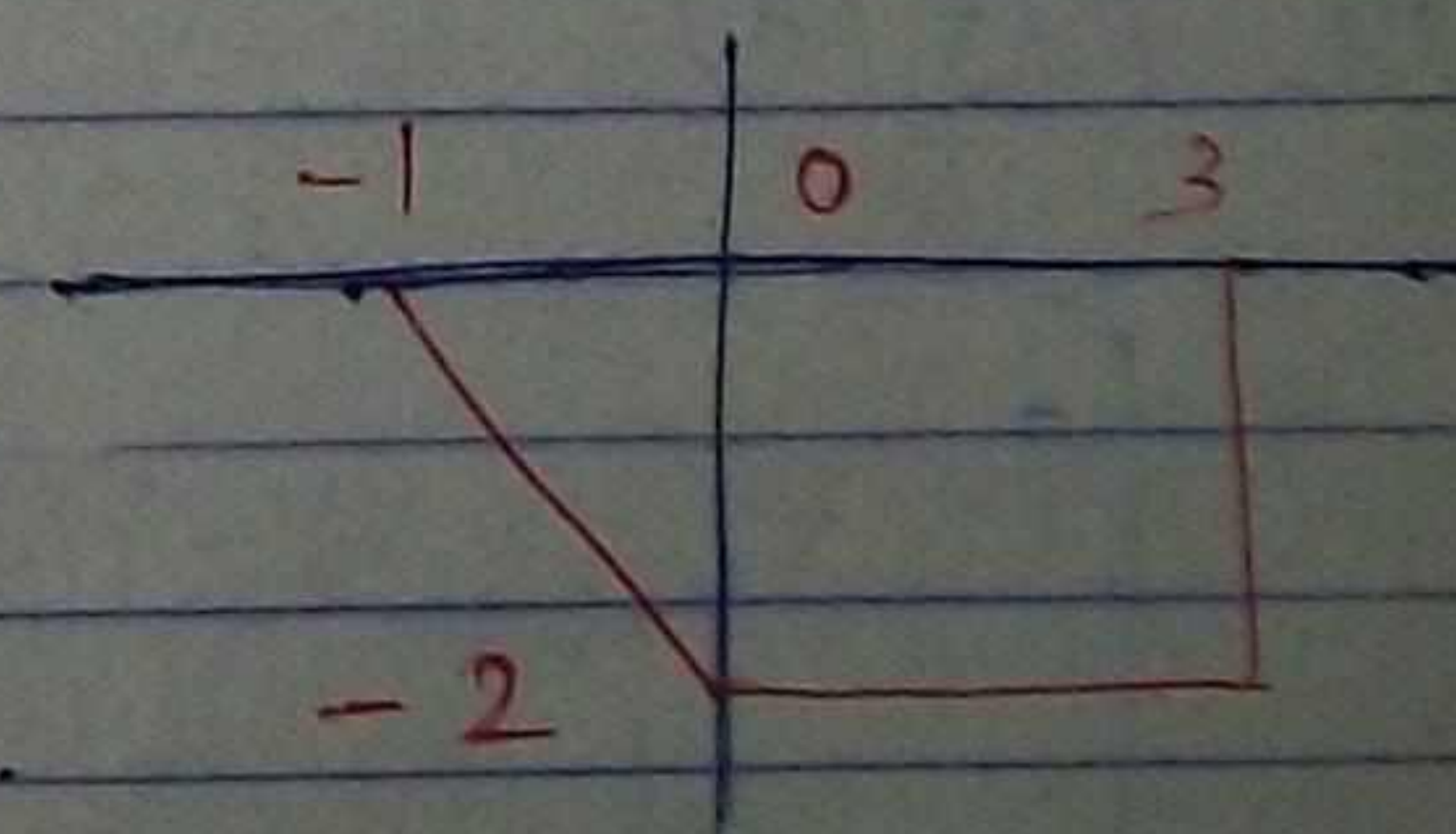


② $x(t) - 3$

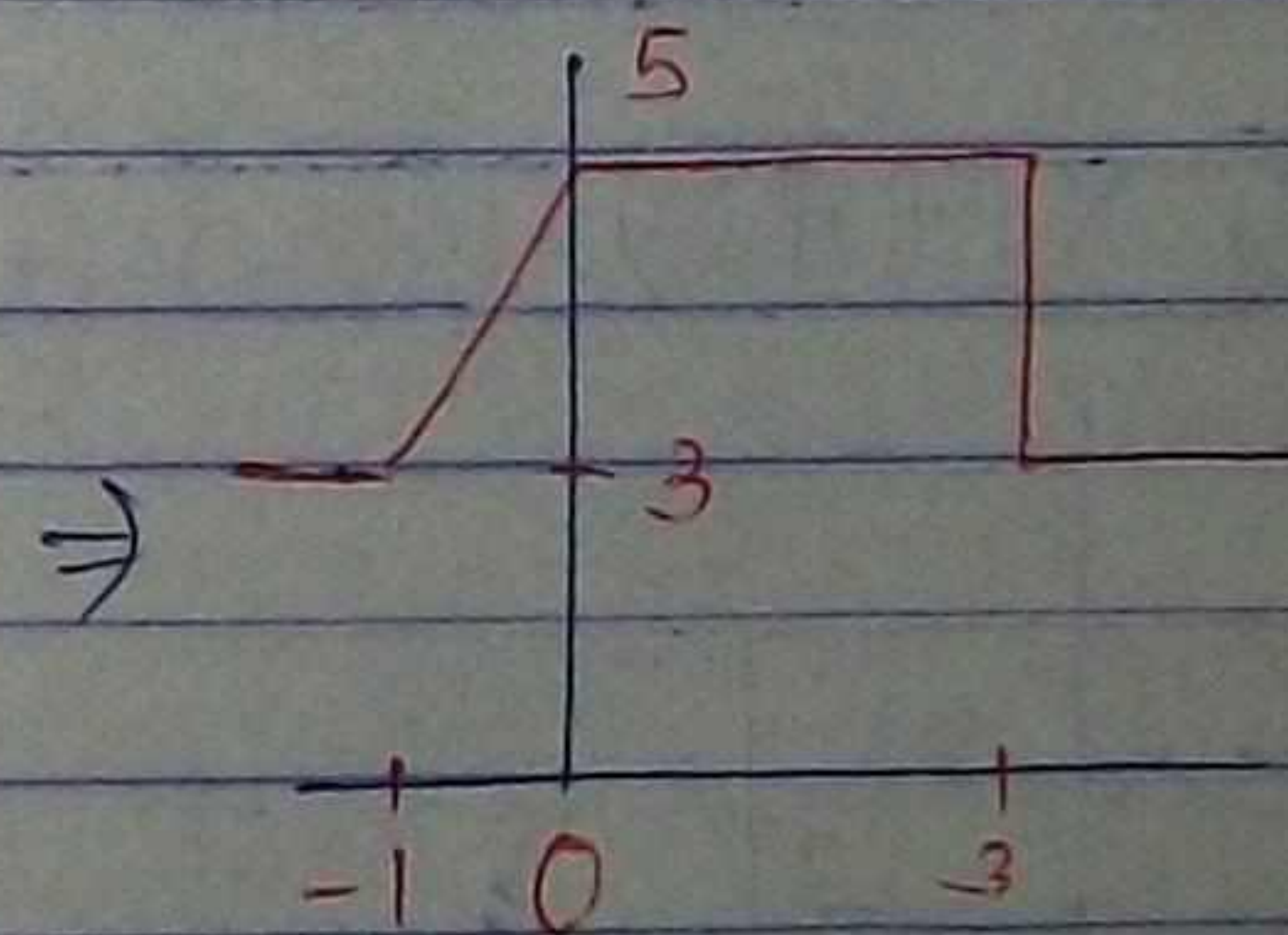


③

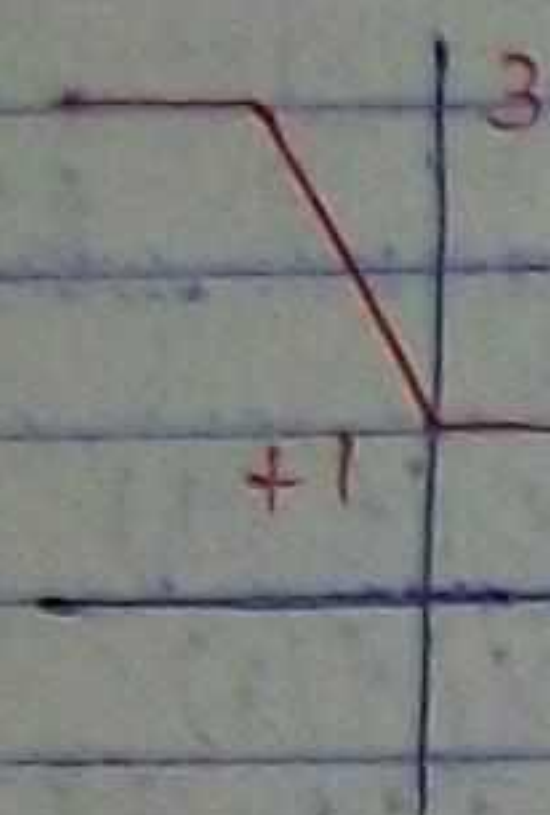
$-x(t)$



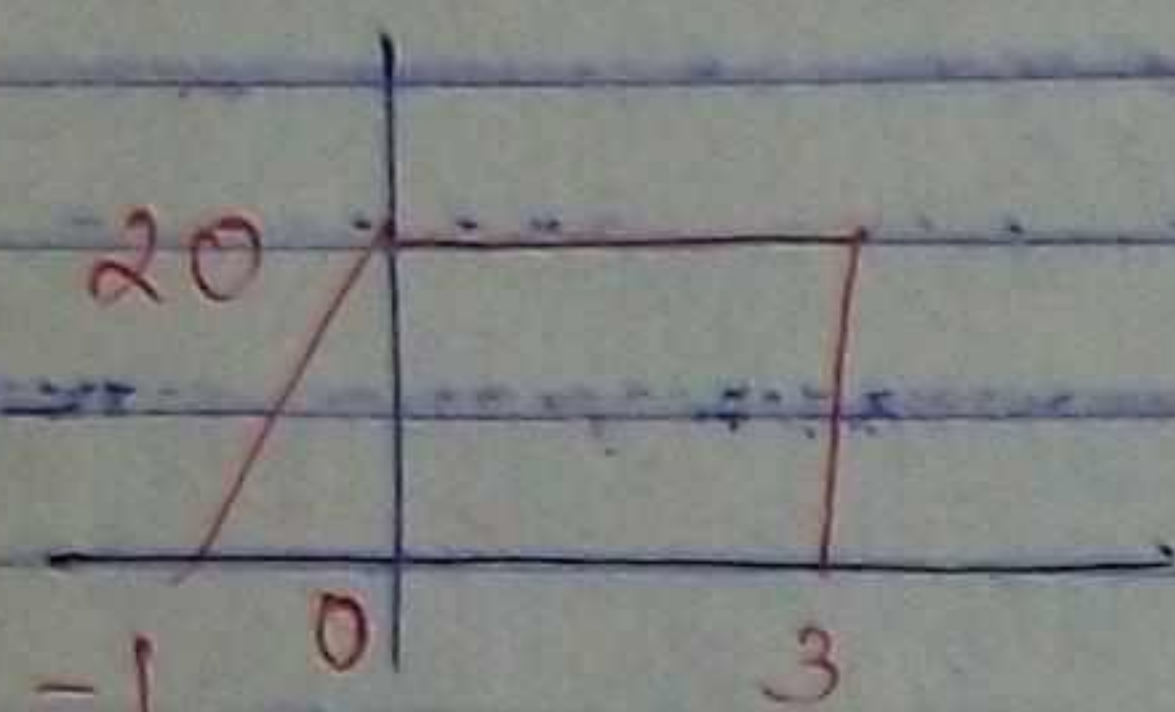
③ $x(t-2), x(t+1)$
 $x(2t), x(t/3)$
 $x(-2t), x(1-t)$
 $x(2t-3), x(-2t-3)$



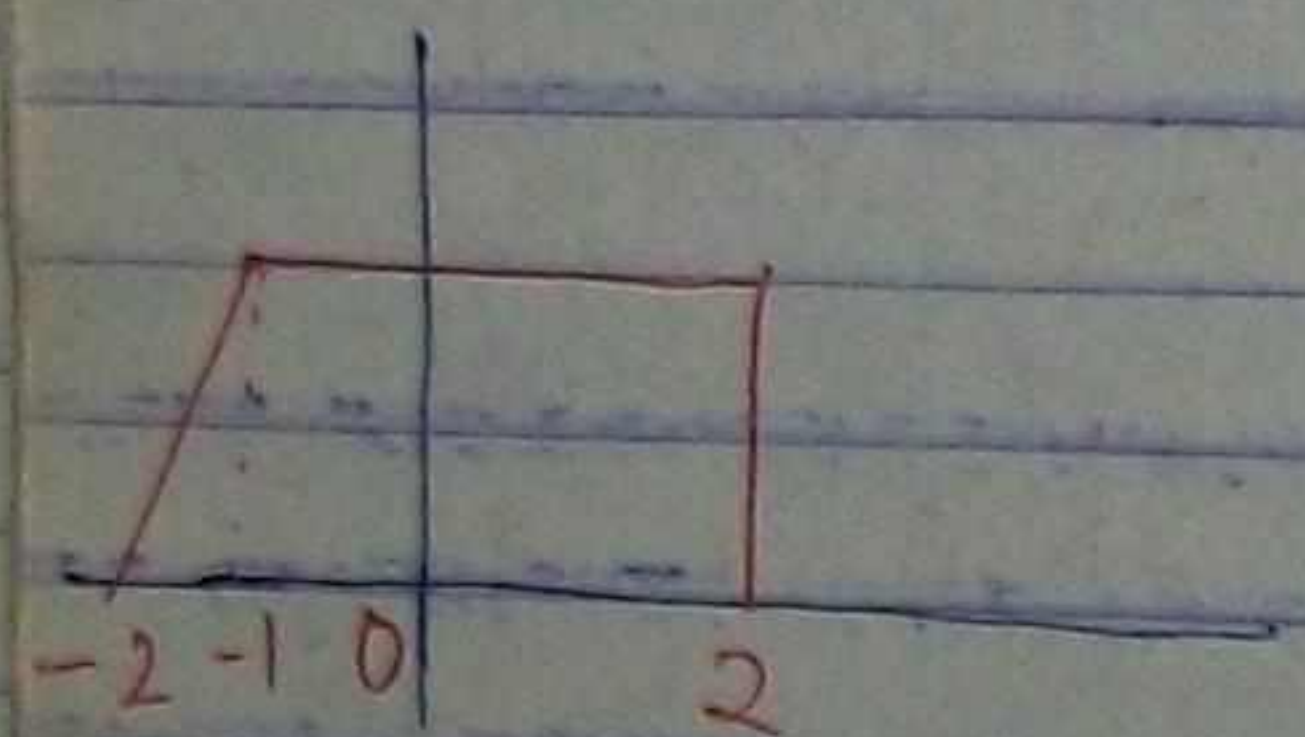
(4) $3 - x(t)$



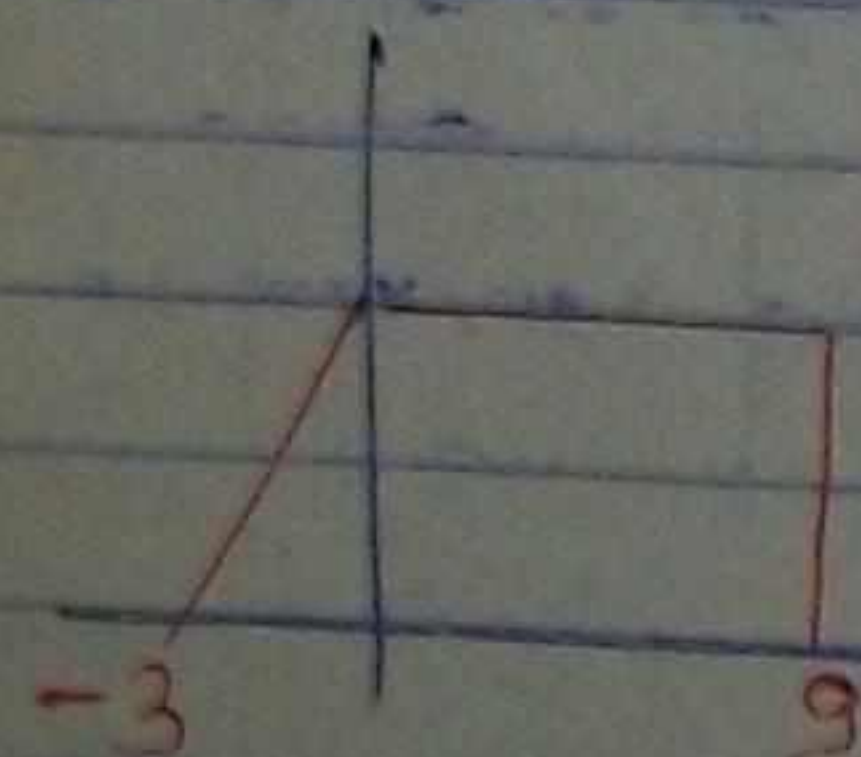
(6) $10 x(t)$



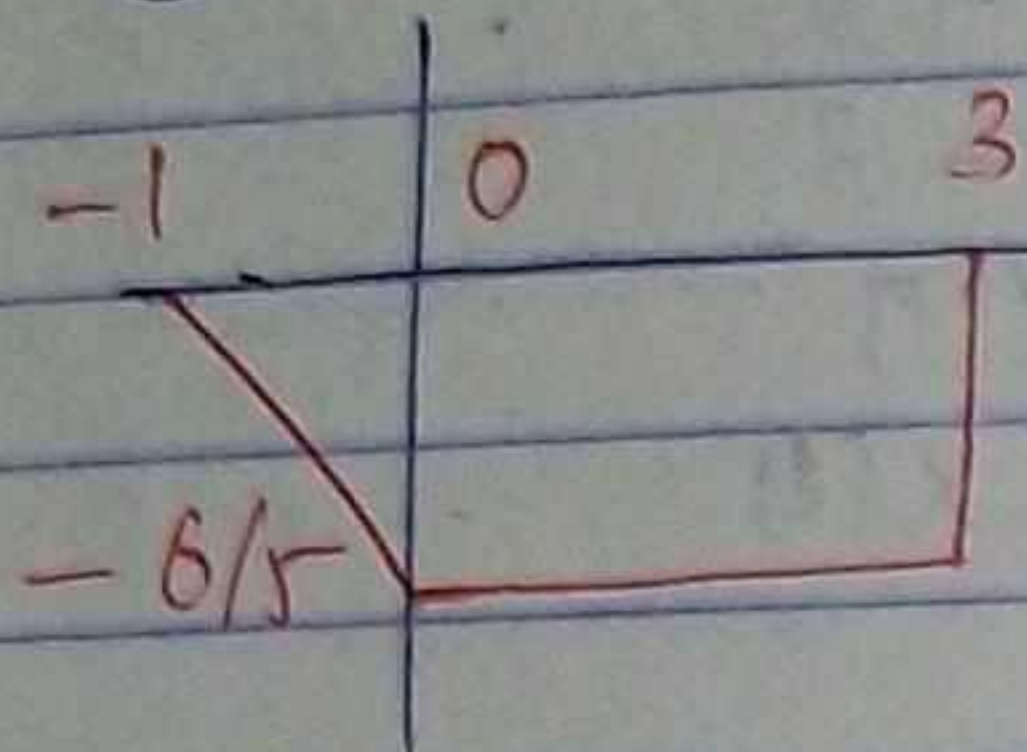
(8) $x(t+1)$



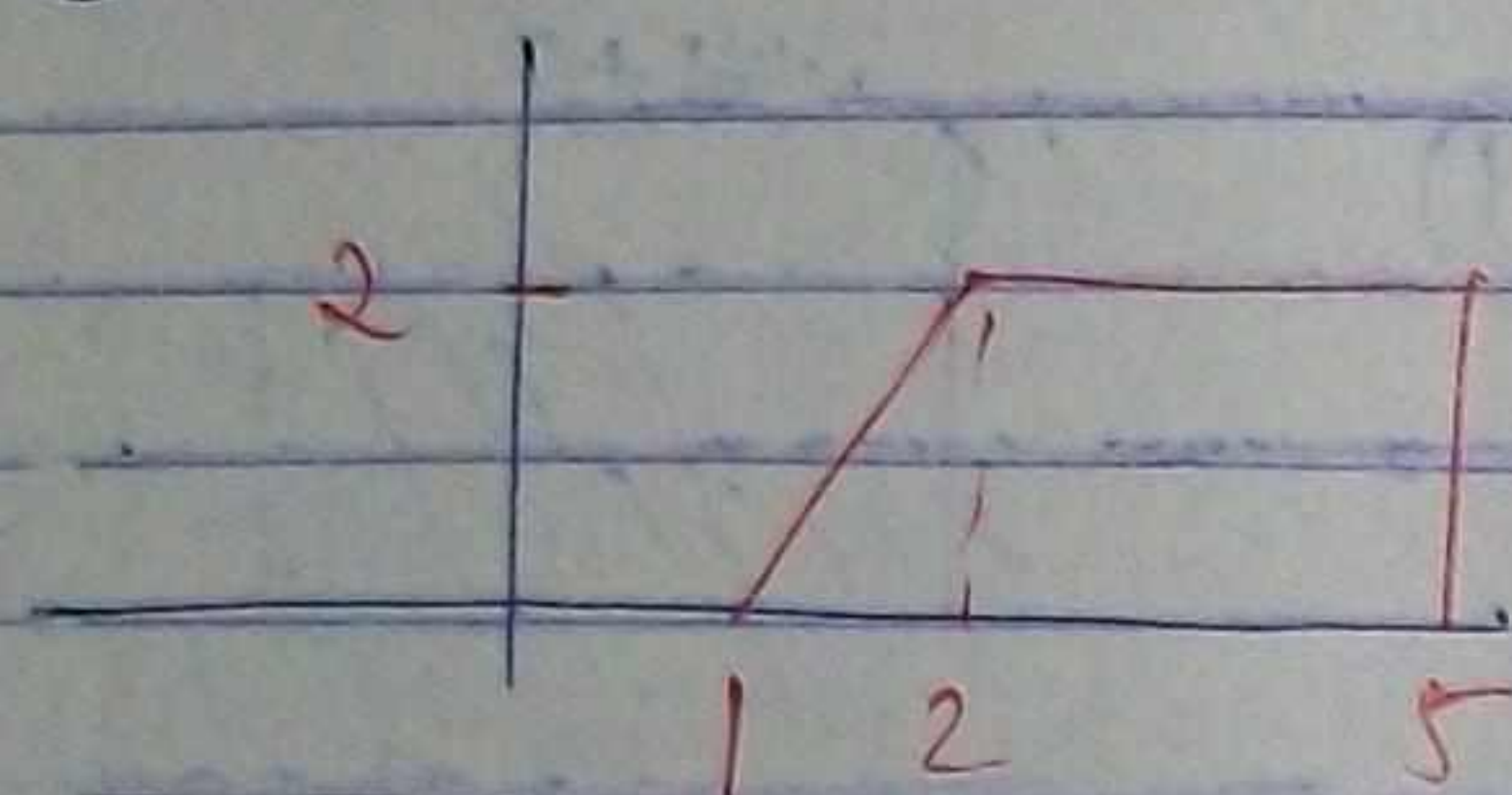
(10) $x(t/3)$



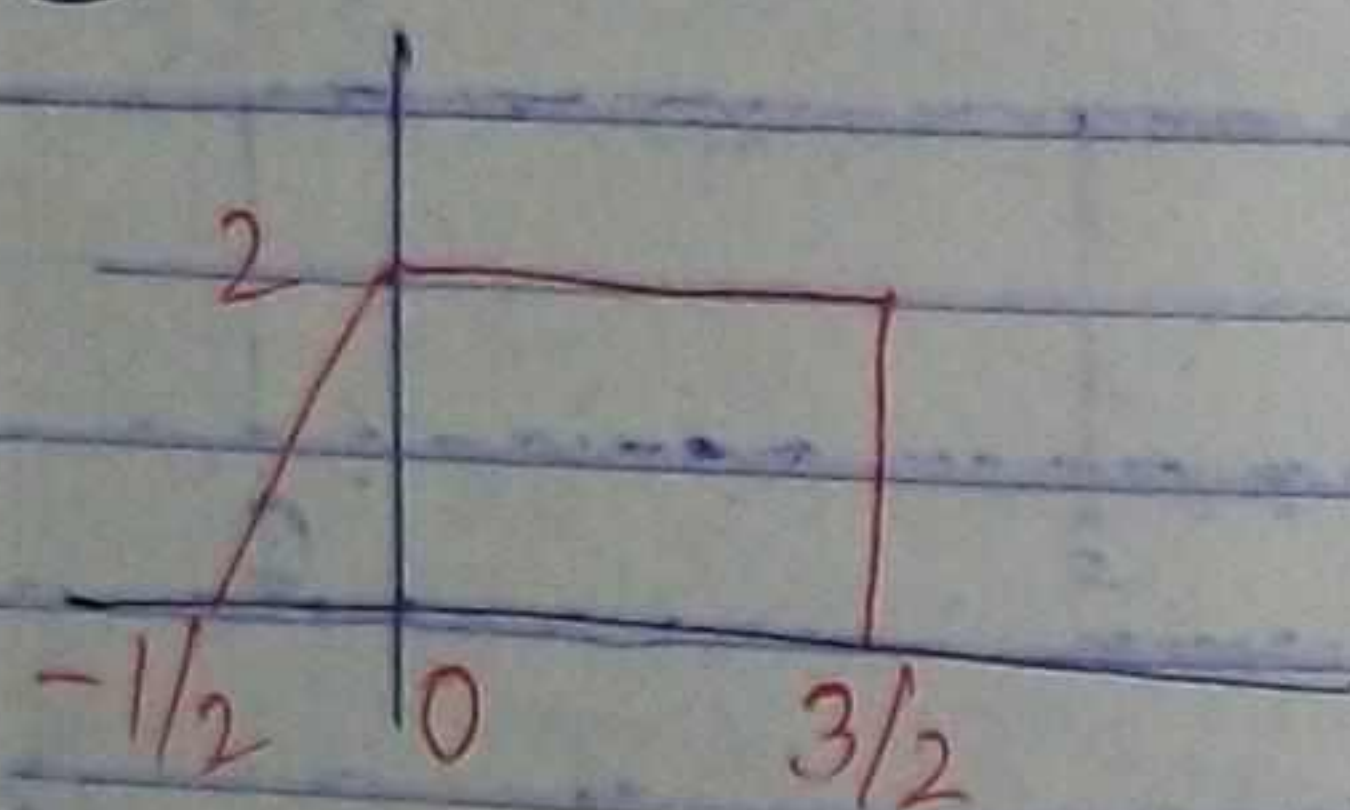
(5) $-3/5 x(t)$



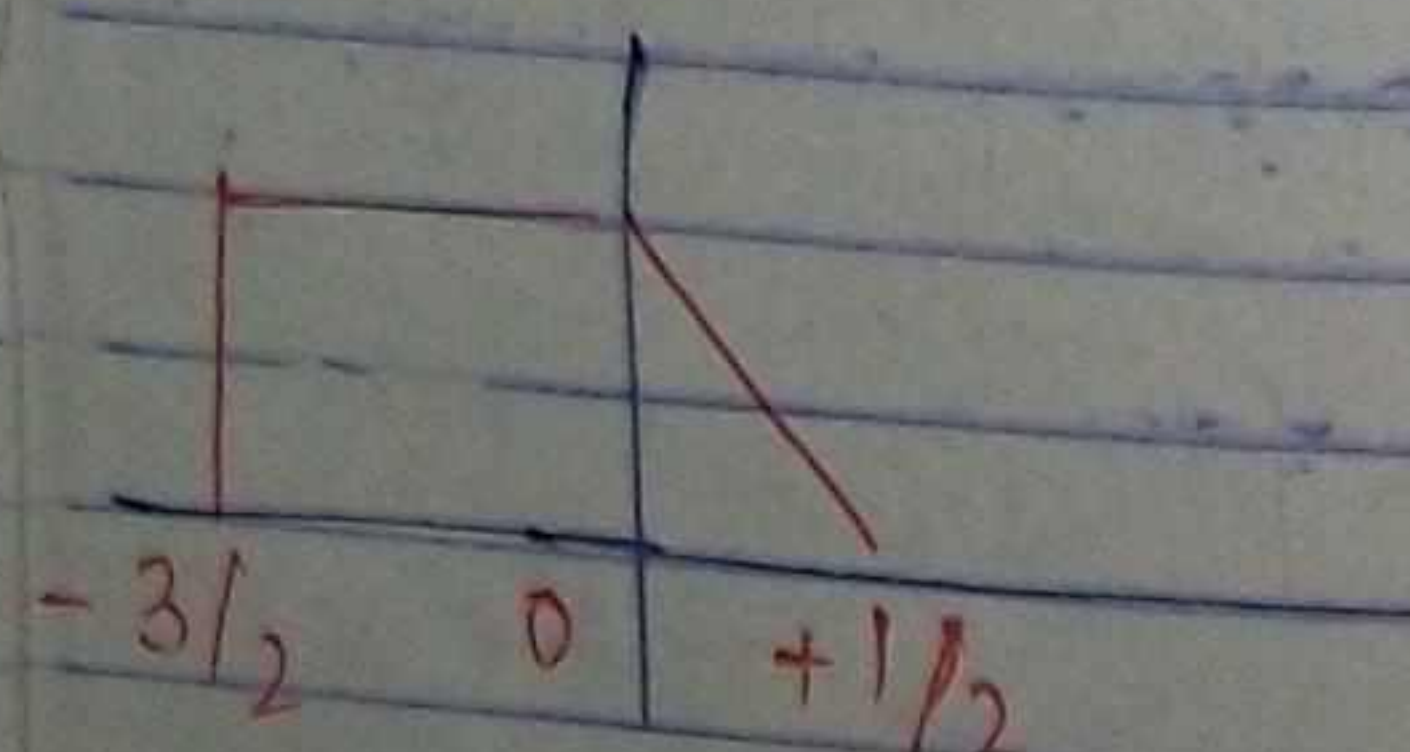
(7) $x(t-2)$



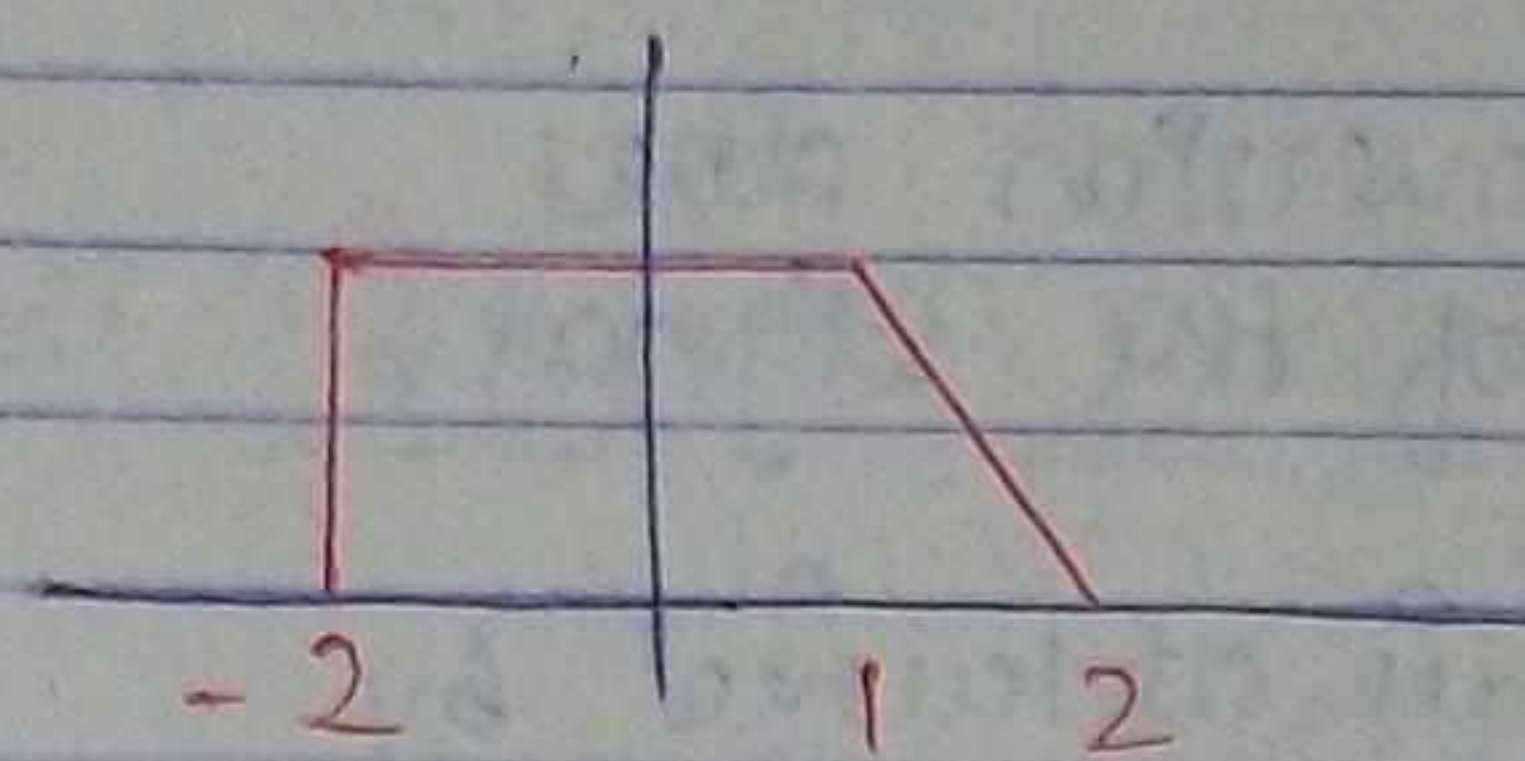
(9) $x(2t)$



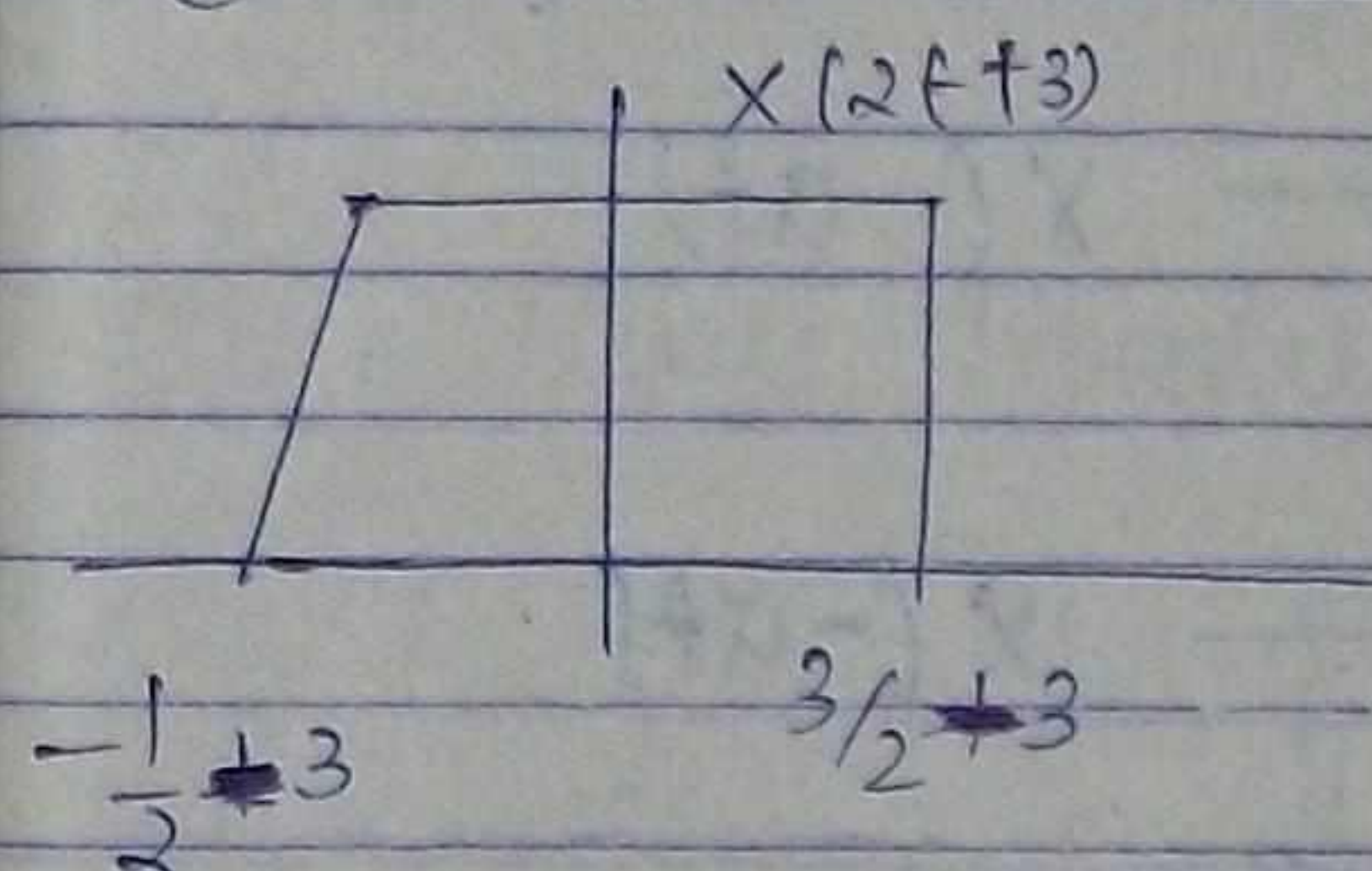
(11) $x(-2t)$



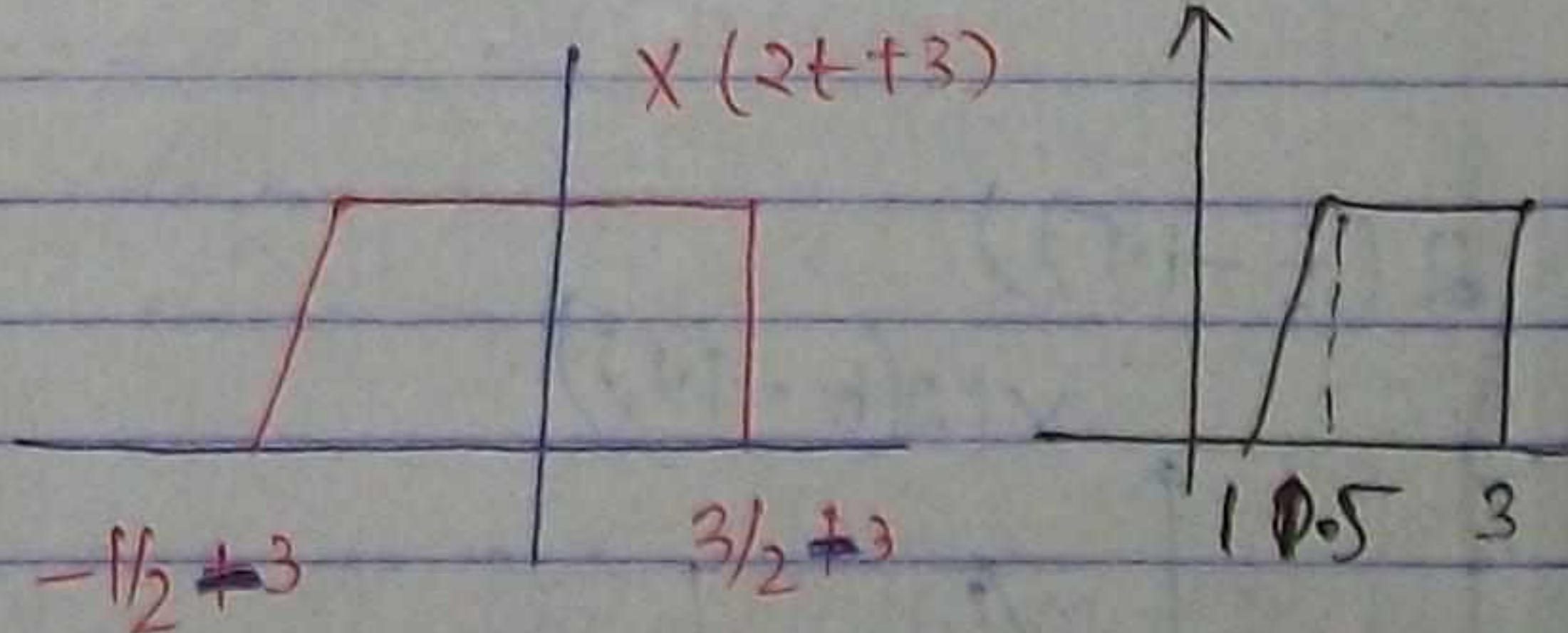
(12) $x(1-t)$



(14) $x(-2t-3)$



(15) $x(2t-3)$

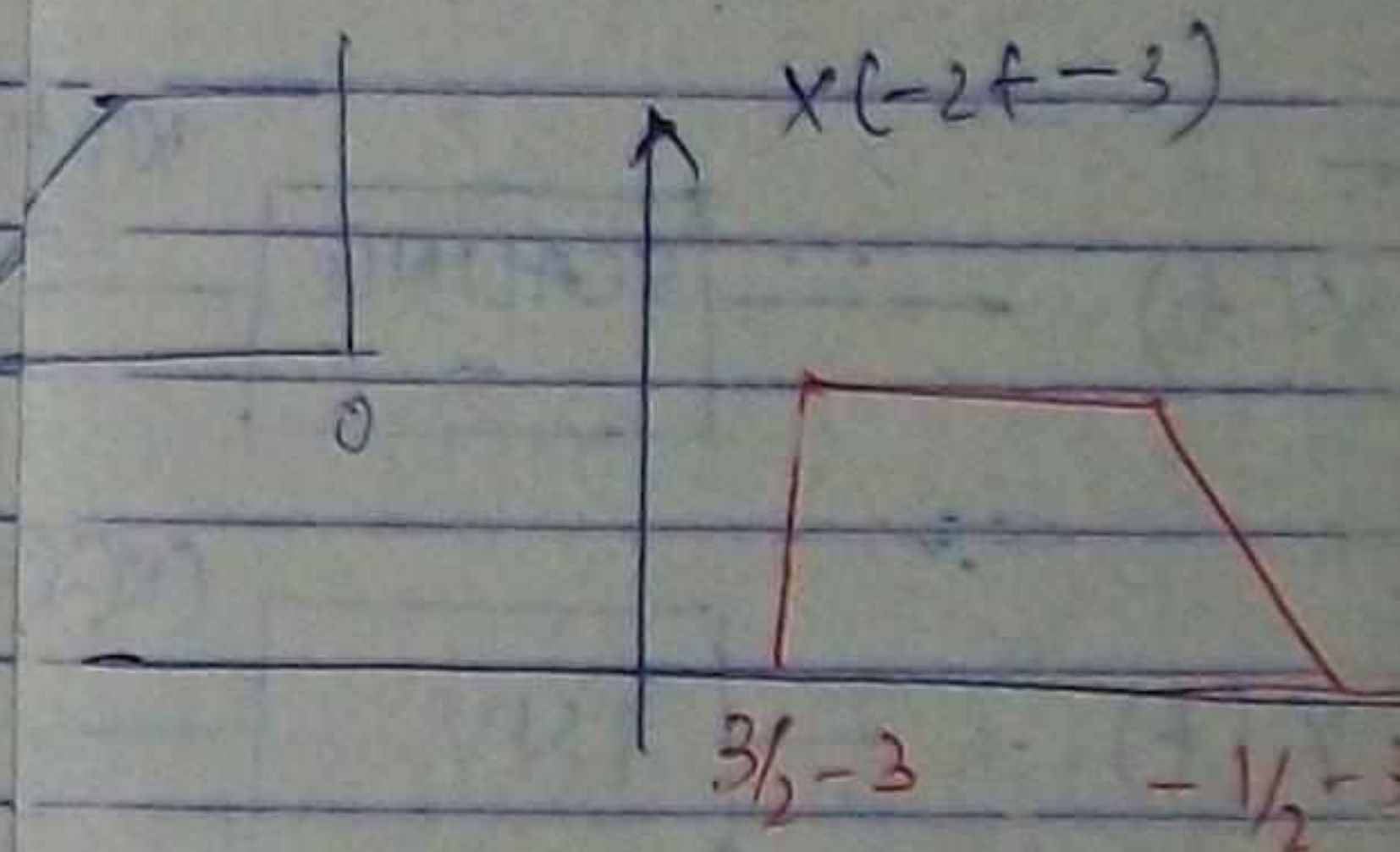
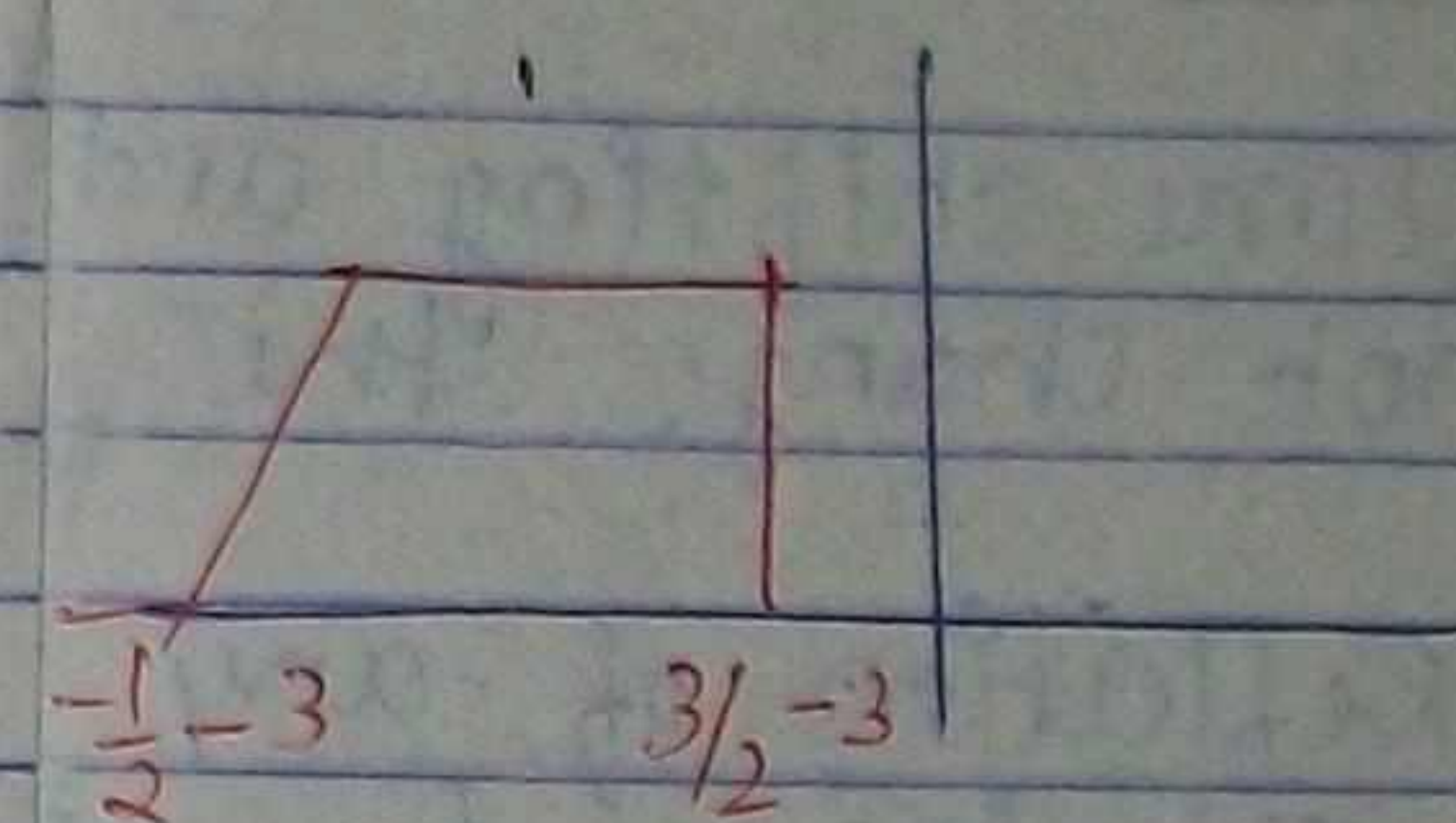


$x(at-b)$

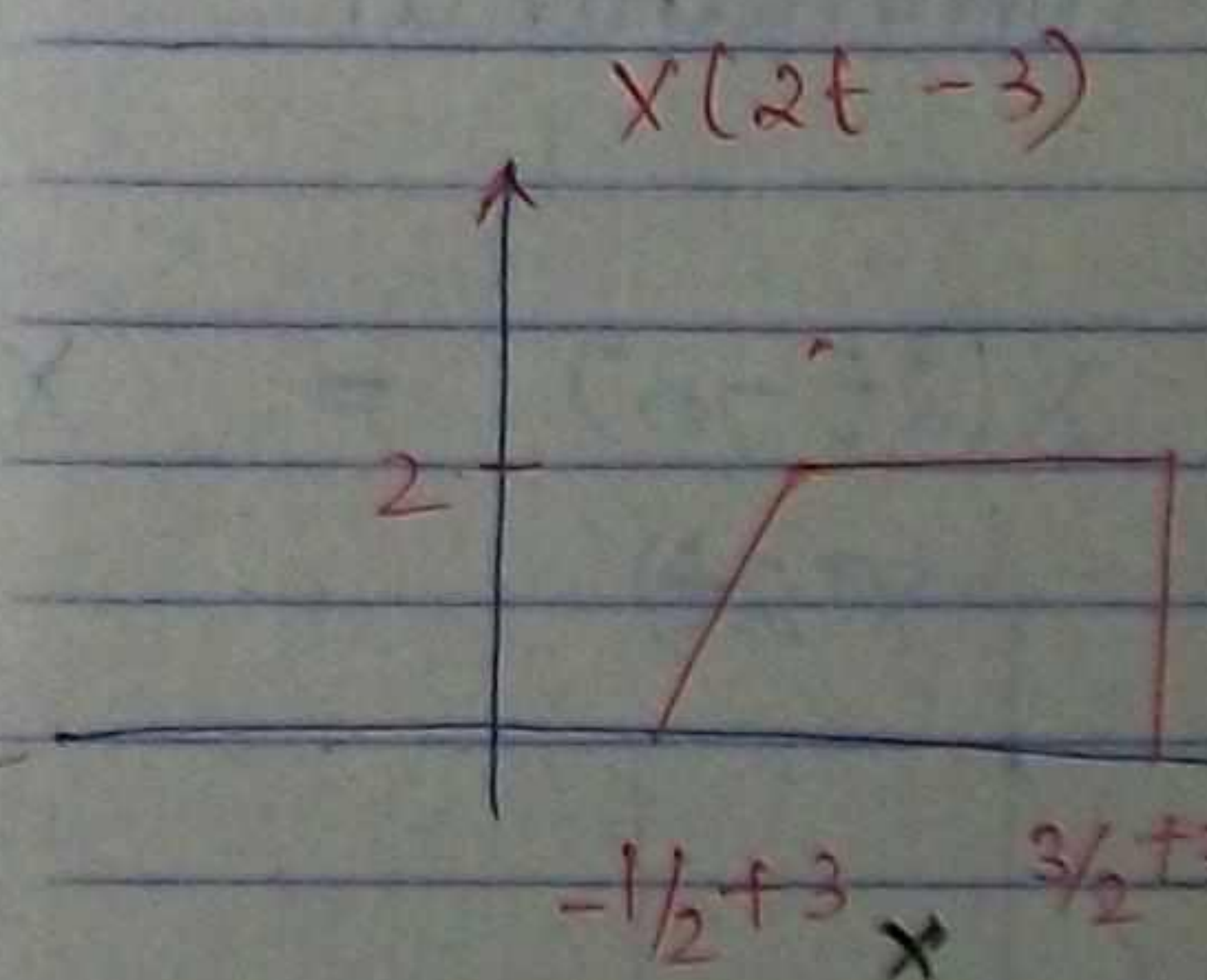
REFLECTION

Replacing
 $t \rightarrow -t$

(13) $x(2t+3)$



$x(2(t-3/2))$

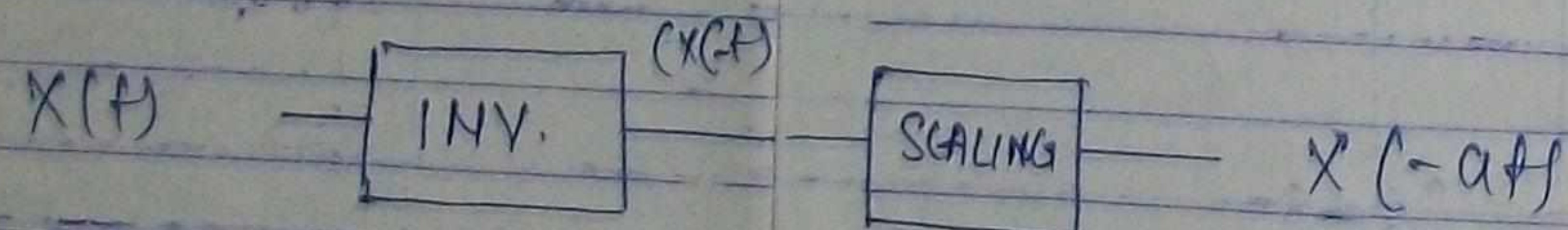
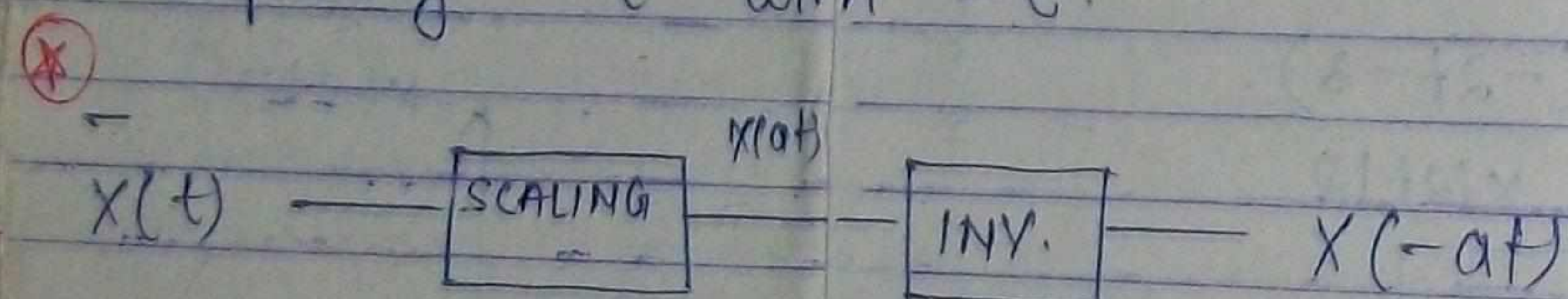


$x(-at-b)$

10/11/11

NOTE -

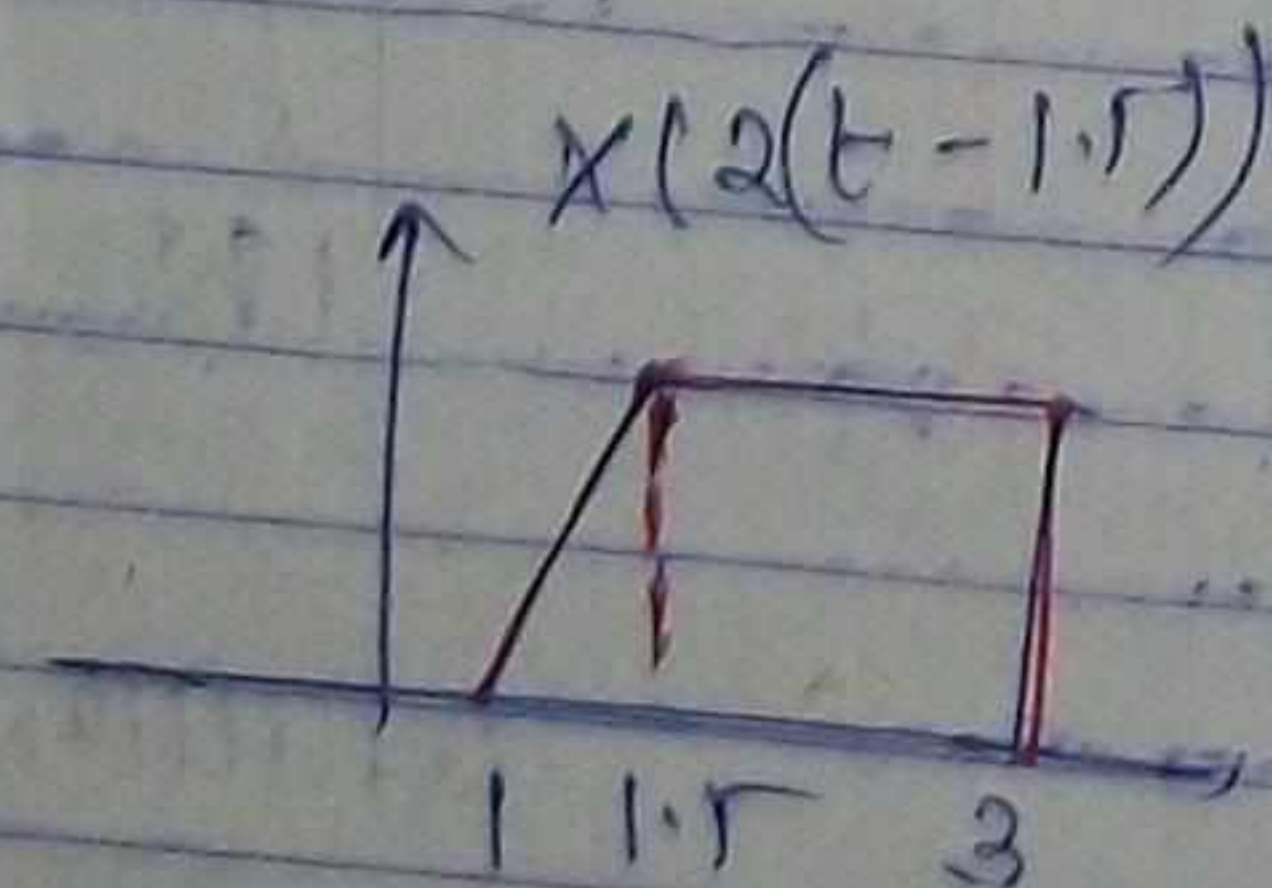
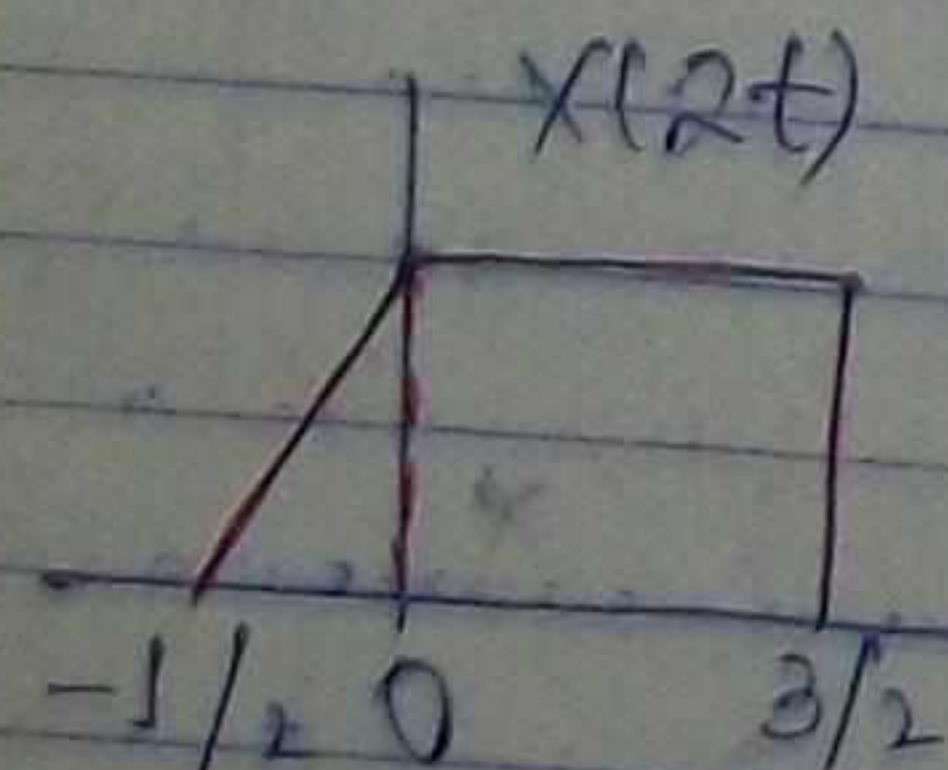
- Time shifting and time inversion does not change the width of the signal.
- Reflection of any signal is obtained by replacing t with $-t$.



Therefore Inversion commutative

and scaling are

$$X(2t-3) = X(2(t-1.5))$$



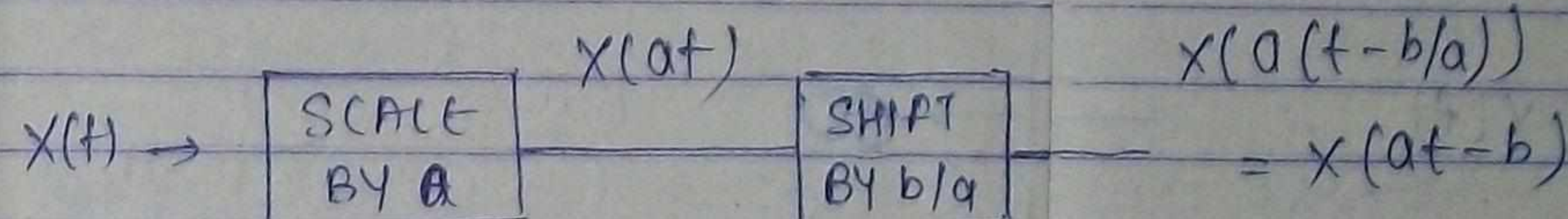
* (circled)

Always (circled)

SCALING

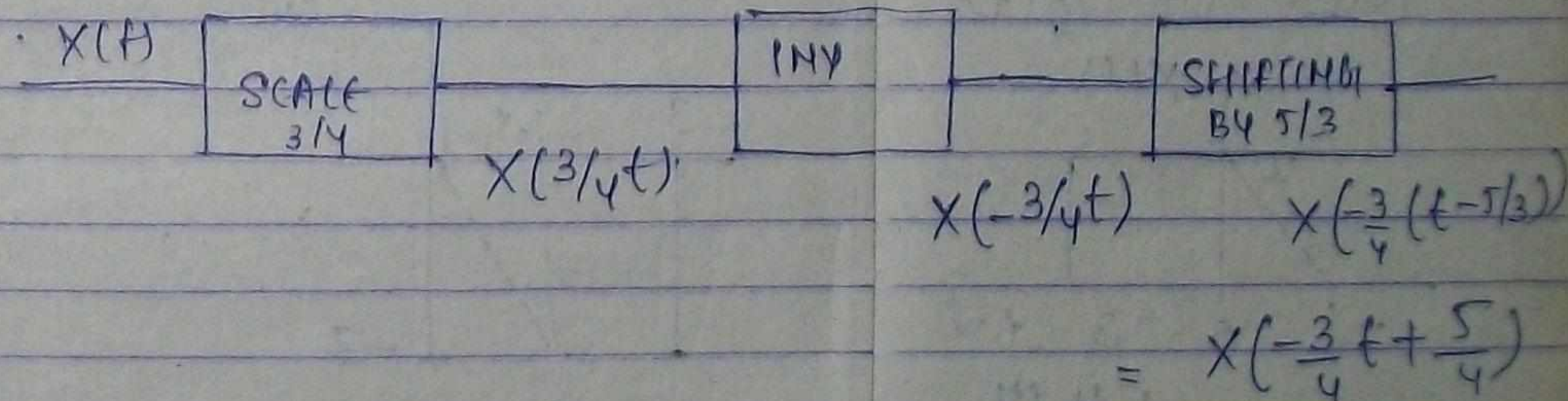
SHIFTING

⇒ PROCEDURE TO SKETCH $X(at-b)$ -



Ques. Write Procedure to sketch the signal $X[\frac{5-3t}{4}]$

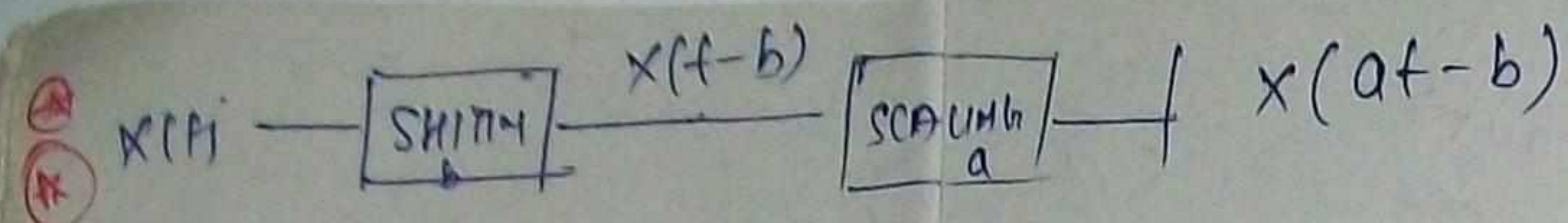
Solⁿ: $X(-\frac{3t}{4} + \frac{5}{4}) = X(-\frac{3}{4}(t - \frac{5}{3}))$



Ques. If the width of $X(t)$ is 7 units then find the width of following:

(1) $X(t-3) \rightarrow 7$

(2) $X(2-t) \rightarrow 7$



③ $x(3t) \rightarrow 7/3$

④ $x(-t/2) \rightarrow 14$

⑤ $1/3 x(t) \rightarrow 7$

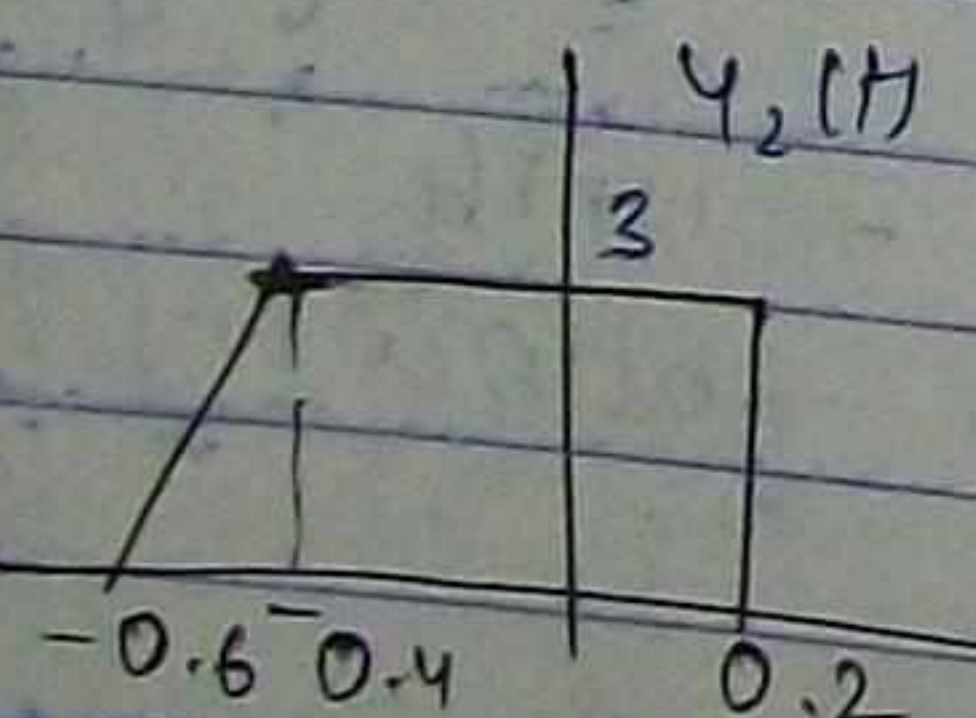
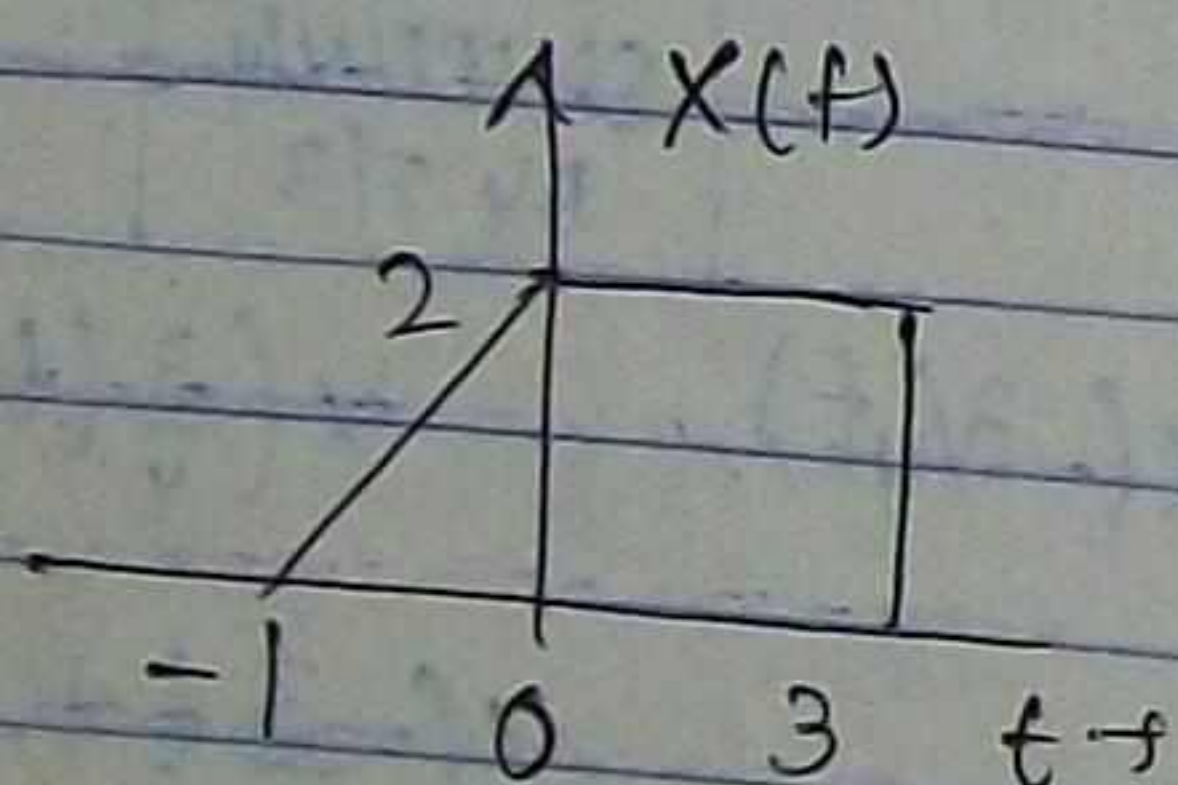
⑥ $2 + x(t) \rightarrow \infty$

NOTE

⑦ $x(-3t+5) \rightarrow 7/3$

⑧ $x\left(\frac{5t-4}{7}\right) \rightarrow 49/5$

Ques. Write the expn terms of $x(t)$

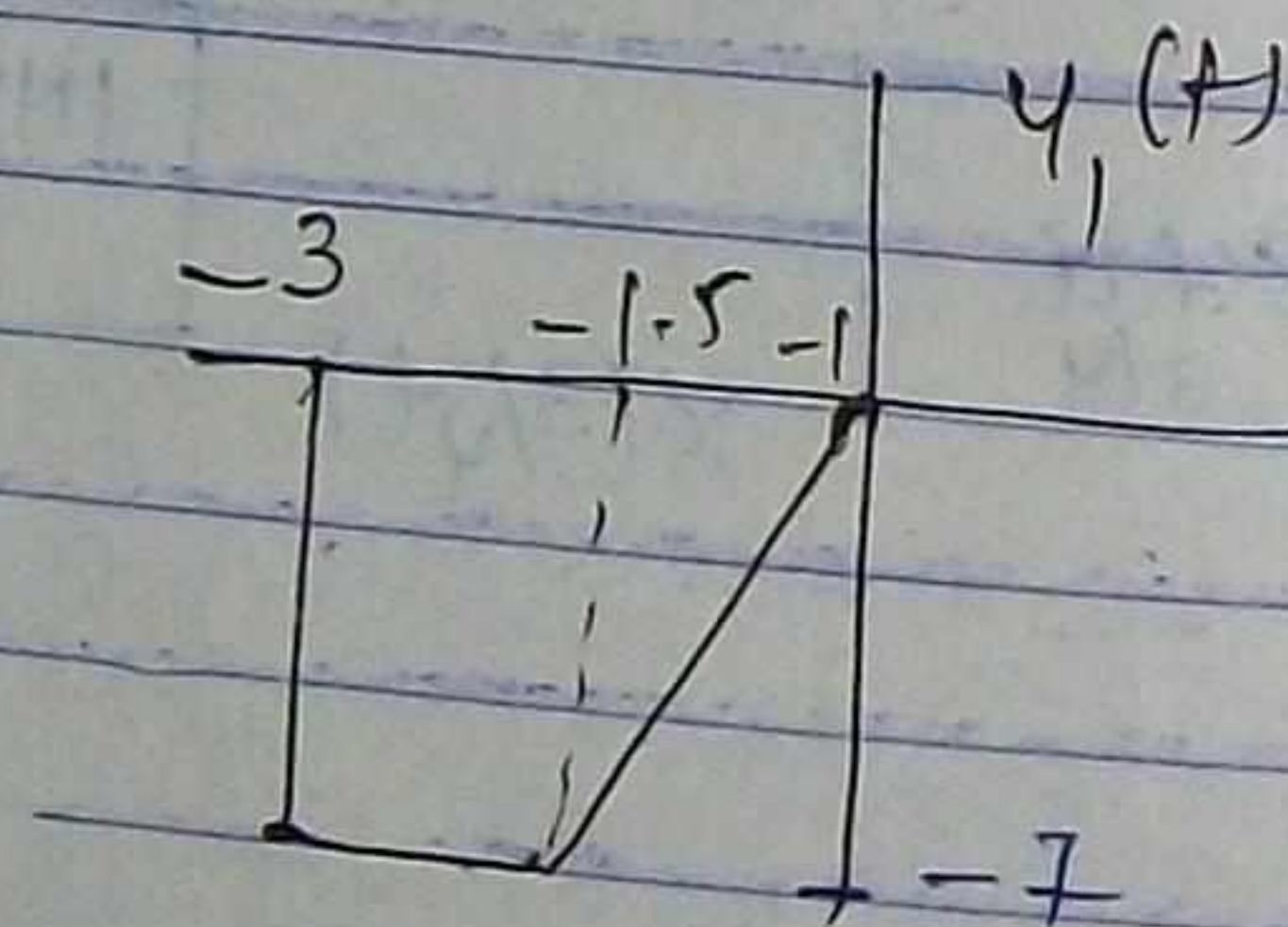


Soln:

$y_1(t) =$

$y_2(t) =$

of y_1 and y_2 in



$-7/2 x(-2t-3)$

$3/2 x(5t+2)$

Ques. Given that $x(t) = 0$

$= 3$

Write exp for

① $x(t-3)$

② $x(-t)$

③ $x(-2t+3)$

Soln: ① $x(t-3) = 0$

$= 3$

$= 3e^{-4(t-3)}$

$= 0$

② $x(-t) = 0$

$= 3$

$= 3e^{-4(-t)}$

$= 0$

③ $x(-2t+3) = 0$

$= 3e^{-4(-2t+3)}$

$= 3$

$= 0$

for $t < -2$

for $-2 < t < 0$

for $0 < t < 5$

for $t > 5$

$(t-3) < -2$

$(t < 1)$

$-2 < (t-3) < 0$

$(1 < t < 3)$

$0 < t-3 < 5$

$(3 < t < 8)$

$t > 5$

$(t < 8)$

$t > 2$

$2 > t > 0$

$0 > t > -5$

$t < -5$

$t > 5/2$

$5/2 > t > 3/2$

$3/2 > t > -1$

$t < -1$

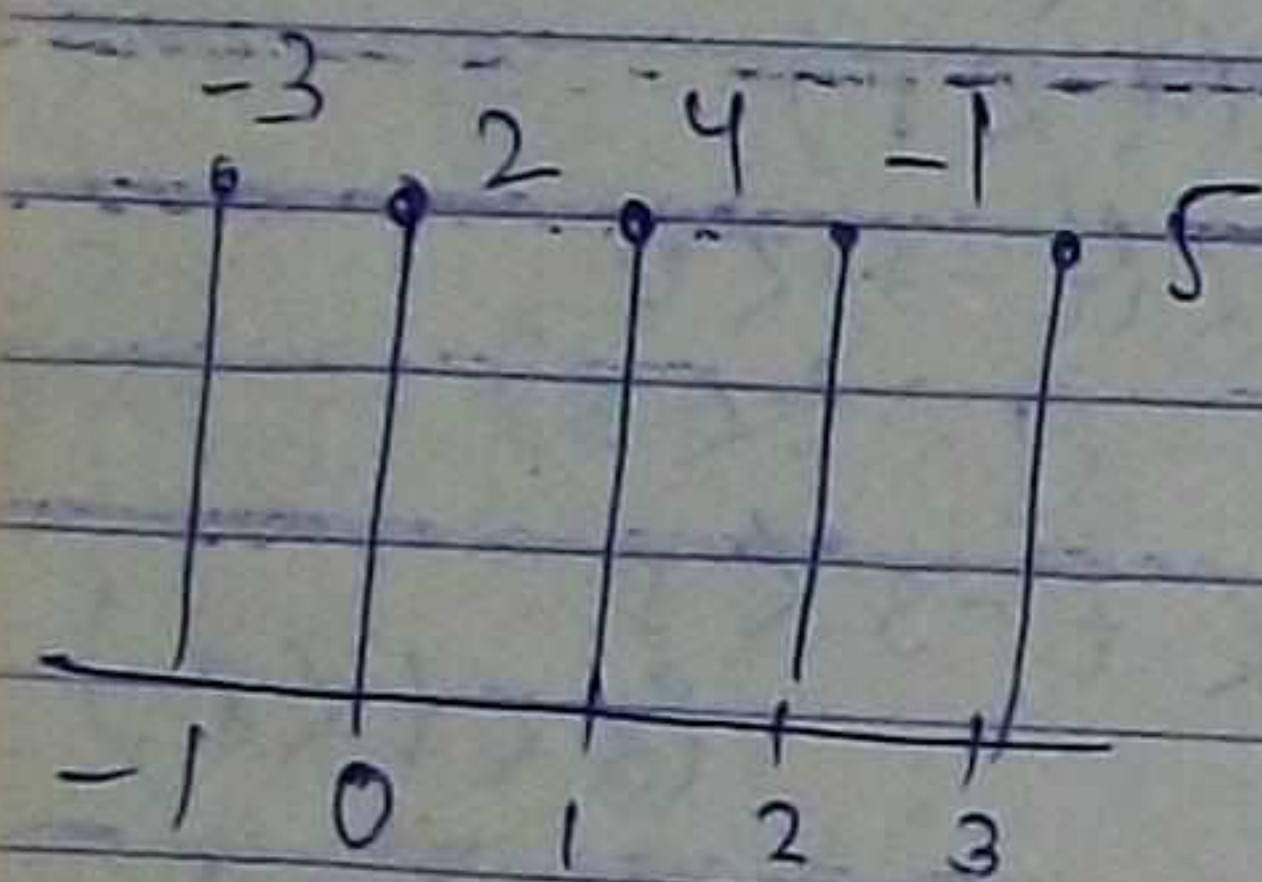
Ques. Given that

$$X(n) = \{-3, 2, 4, -1, 5\}$$

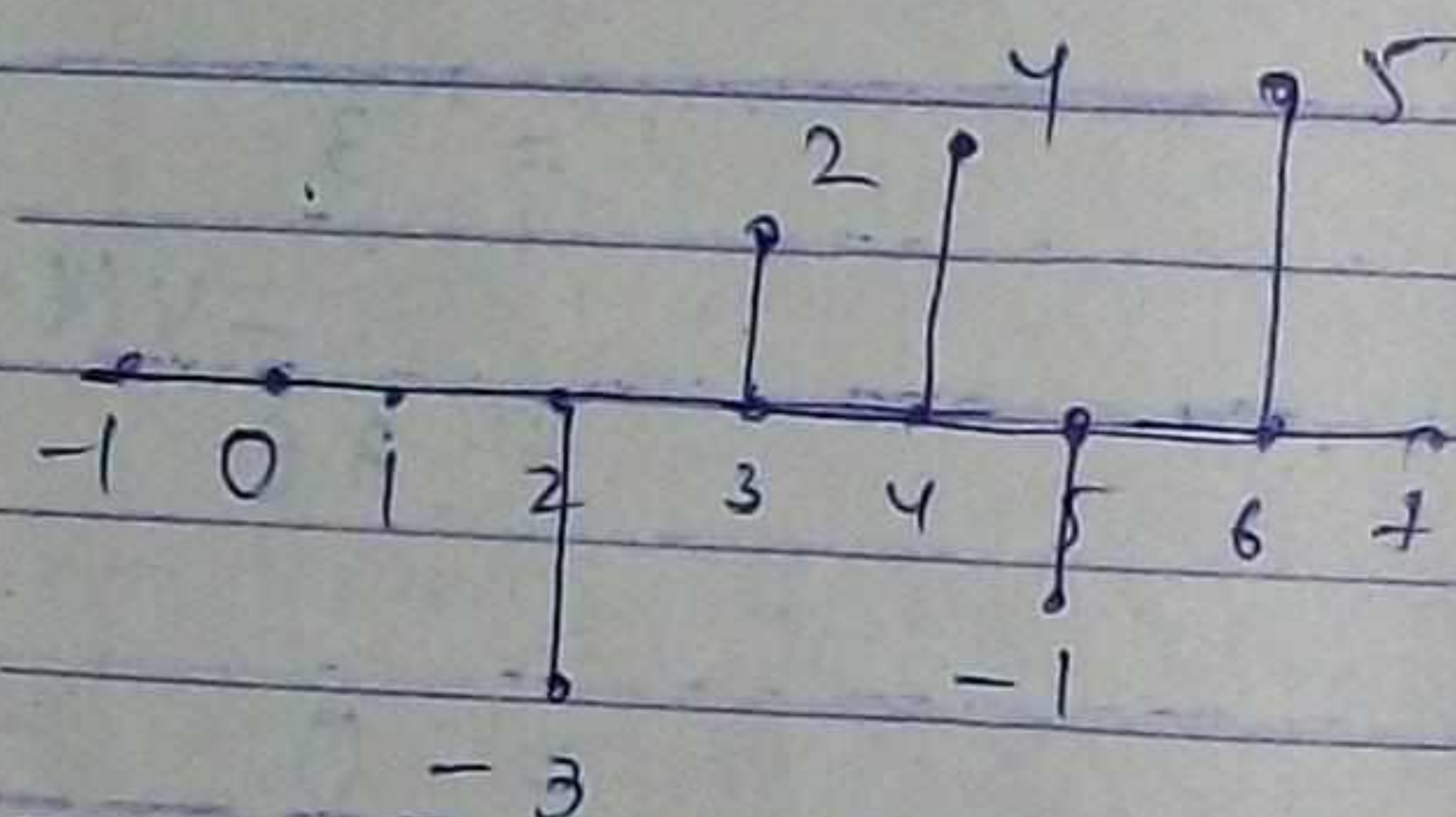
Sketch $X(n-3)$, $X(n+1)$, $X(-n)$,
 $X(1-n)$, $X(n/3)$, $X(2n)$

Solⁿs

① $X(n-3) = \{$



$\{0, 0, -3, 2, 4, -1, 5\}$



② $X(n+1) = \{$

$\{-3, 2, 4, -1, 5\}$

③ $X(-n) = \{$

$\{5, -1, 4, 2, -3\}$

④ $X(1-n) = \{$

$\{5, -1, 4, 2, -3\}$

⑤ $X(n/3) = \{$

$\{-3, 0, 0, 2, 0, 0, 4, 0, 0, -1, 0, 0, 5\}$

⑥ $X(2n) = \{$

$\{2, -1\}$

NOTE -

- Since Discrete-signal are defined at the integers value hence non integers values of n will be discarded.

- Expansion of D.T. signal is known as UP SAMPLING OR INTERPOLATION.

$$X\left(\frac{t}{n}\right) \rightarrow \text{INTERPOLATION}$$

- Compression is known as down sampling or decimation.

$$X(nt) \rightarrow \text{DECIMATION}$$

- Compression of D.T. signal results in loss of information.

Ques. Given that $X(n) = \{3, -1, 5, 6, -7, 8\}$

$X\left(\frac{n}{4}\right)$ using (a) 0-Interpolation
(b) step-Interpolation
(c) linear-Interpolation

Solⁿ: ① 0 - interpolation -

$$X(n/4) = \{ 3, 0, 0, -1, 0, 0, 5, 0, 0, 6, 0, 0, -7, 0, 0, 8 \}$$

② step - $1/P$ -

$$X(n/4) = \{ 3, 1, 1, -1, 1, 1, 5, 1, 1, 6, 1, 1, -7, 1, 1, 8 \}$$

③ linear IP -

$X(n/4) =$ first determine the difference.

b/w 3 and -1

$$\frac{-1-3}{4} = -1$$

$$\{ 2, 1, 0 \}$$

b/w 5 and -1

$$5 - \frac{(-1)}{4} = \frac{21}{4}$$

$$\{ 1\frac{1}{2}, 2, 7\frac{1}{2} \}$$

b/w 6 and 8

$$\frac{6-8}{4} = -\frac{1}{2}$$

$$\{ \frac{21}{4}, \frac{22}{4}, \frac{23}{4} \}$$

b/w -7 and 6

$$\frac{-7-6}{4} = -\frac{13}{4}$$

$$\{ \frac{11}{4}, -\frac{2}{4}, -\frac{15}{4} \}$$

b/w 8 and -7

$$\frac{8-(-7)}{4} = \frac{15}{4}$$

$$\{ -\frac{13}{4}, \frac{2}{4}, \frac{17}{4} \}$$

$$X(n/4) = \{ 3, 2, 1, 0, -1, 1\frac{1}{2}, 2, 7\frac{1}{2}, 5, -2, -1\frac{1}{2}, -7, -1\frac{3}{4}, -2, \frac{17}{4}, 8 \}$$

$$\frac{21}{4}, \frac{22}{4}, \frac{23}{4}$$

6

$$\{ -\frac{2}{4}, -\frac{15}{4}, -\frac{7}{4}, -\frac{13}{4}, \frac{2}{4}, \frac{17}{4}, 8 \}$$

(*)

✓ X(n)

INTERPOLATION BY a

DECIMATION BY a

X(n)

X(n)

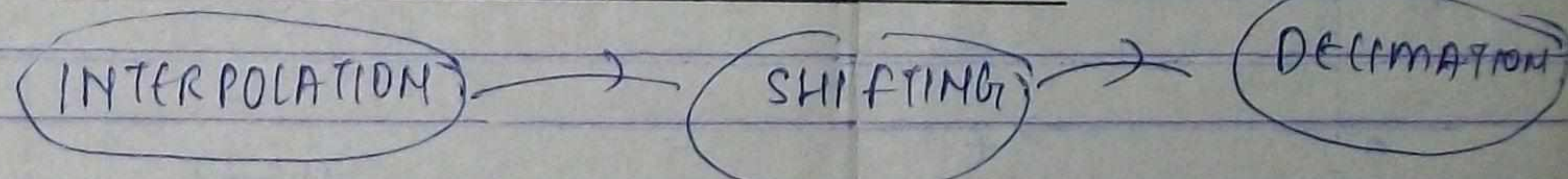
DECIMATION BY a

INTERPOLATION BY a

X'(n) ≠ X(n)

Therefore interpolation is the inverse of decimation but decimation is not the inverse of interpolation.

⇒ PRECEDENCE RULE FOR D.T. SIGNAL -

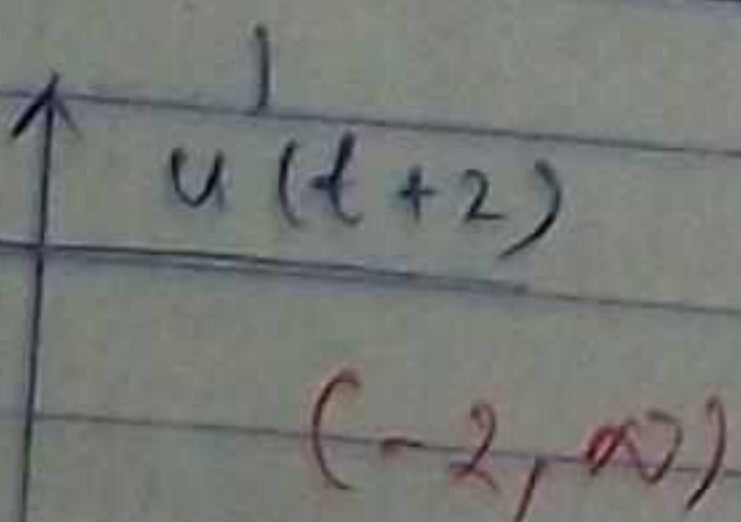
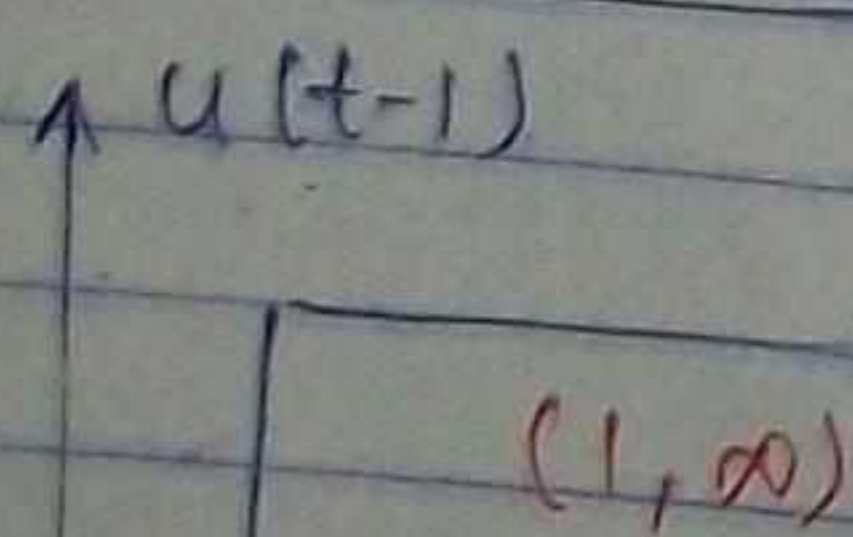
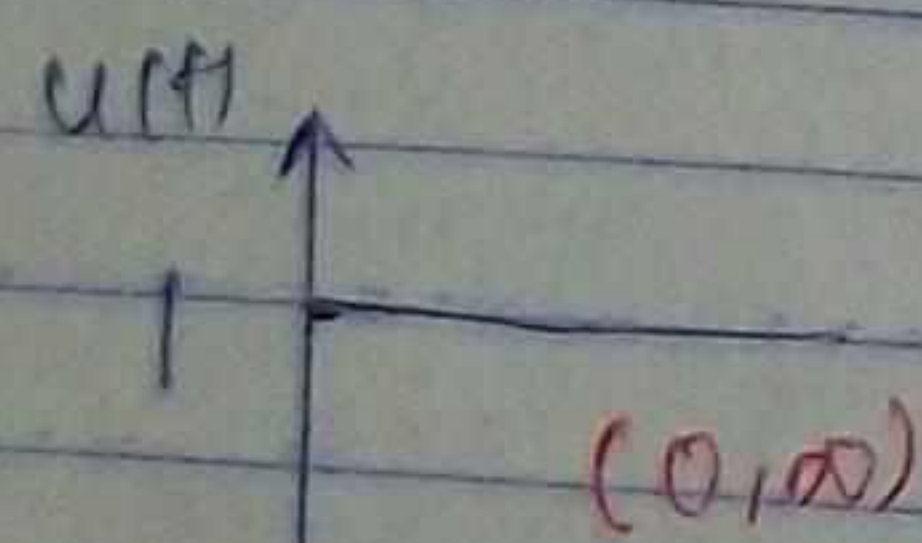
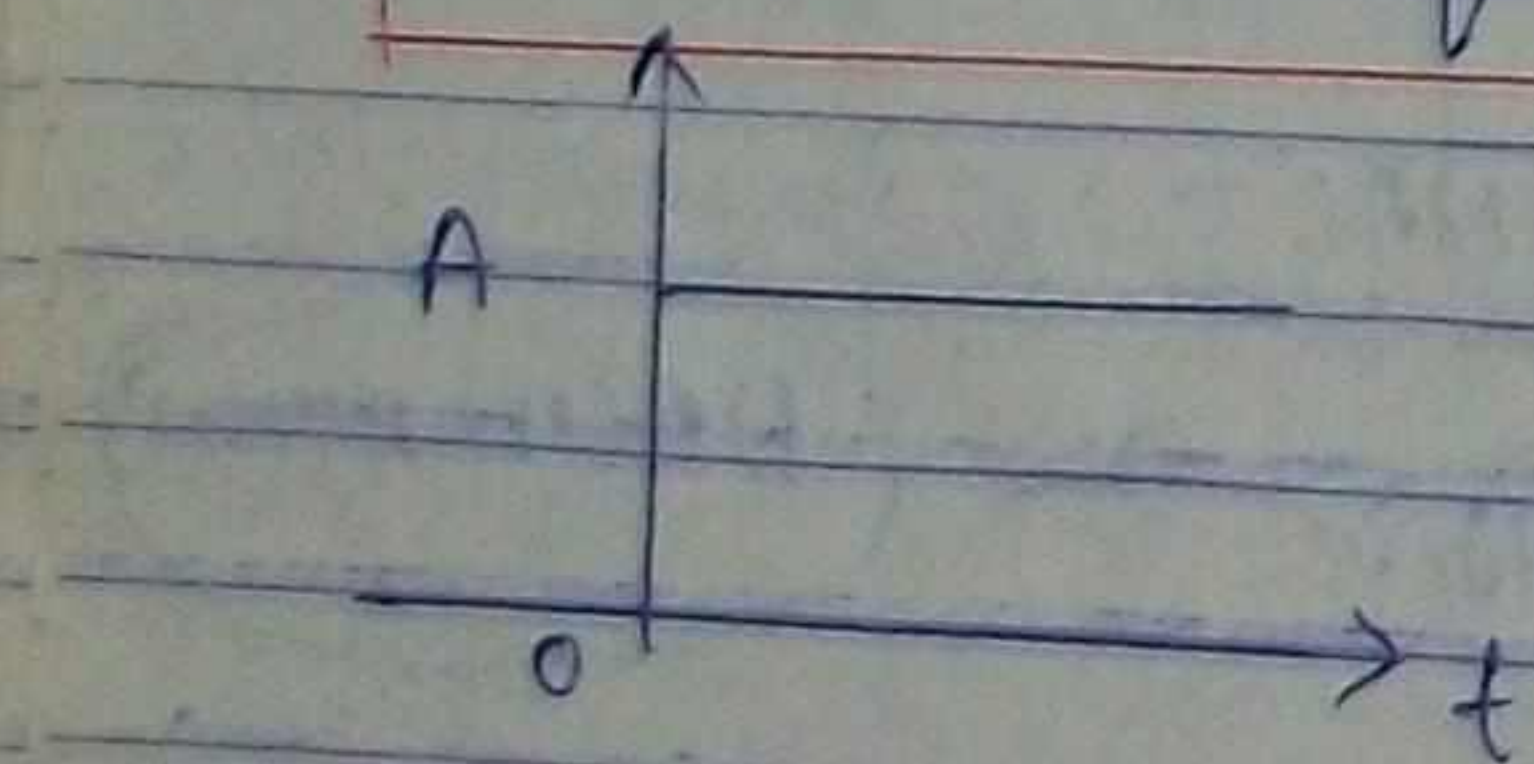


⇒ BASIC SIGNALS -

① STEP SIGNAL ($u(t)$) -

- It is used to represent sudden increment and sudden decrement and also to specify the limit of the signal.

$$\begin{aligned} A u(t) &= A & \text{for } t > 0 \\ &= 0 & \text{for } t < 0 \\ &= \text{undefined} & \text{for } t = 0 \end{aligned}$$



$u(-t)$

$(-\infty, 0)$

$u(-t-1)$

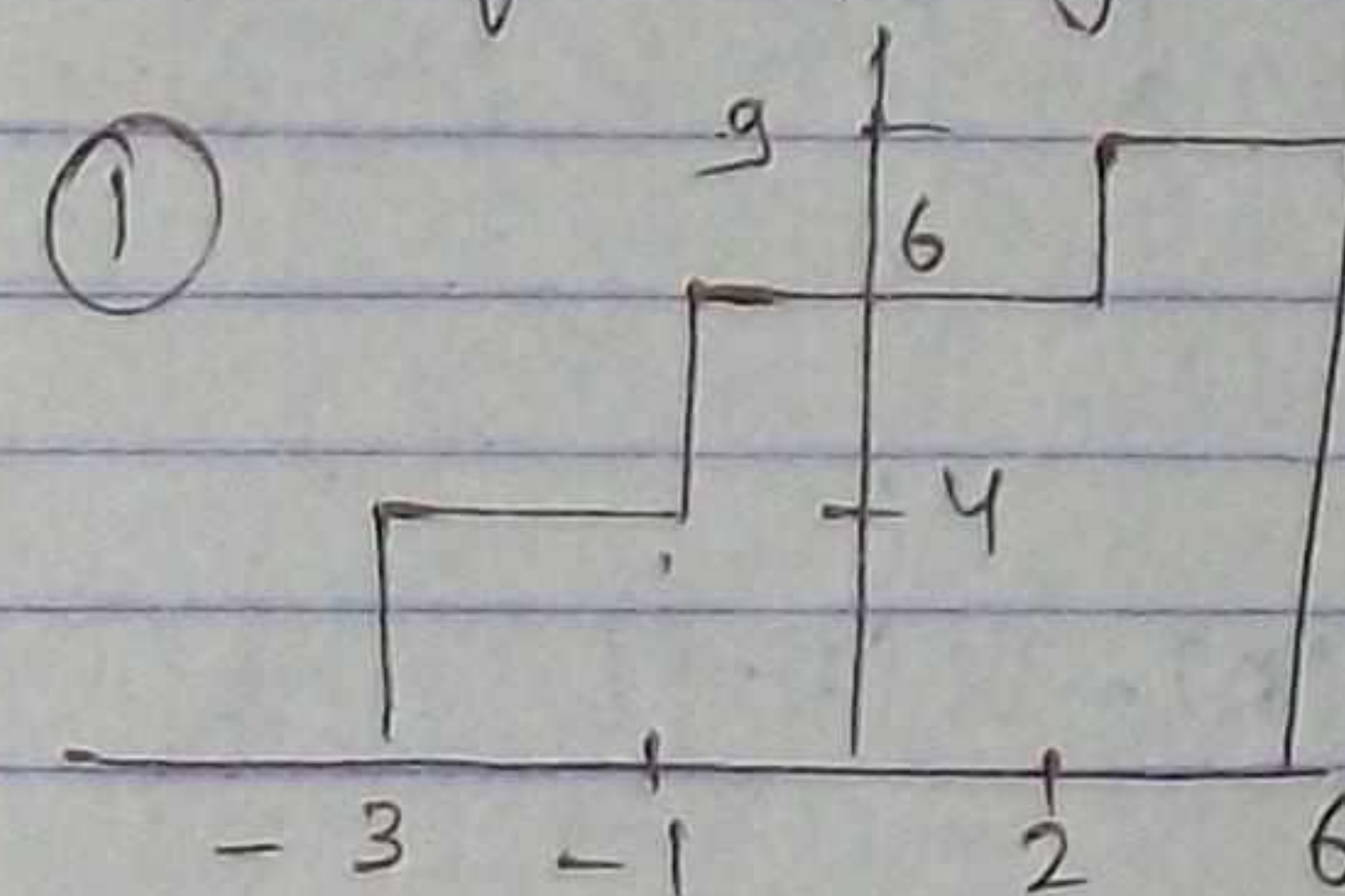
$(-\infty, -1)$

$u(-t+2)$

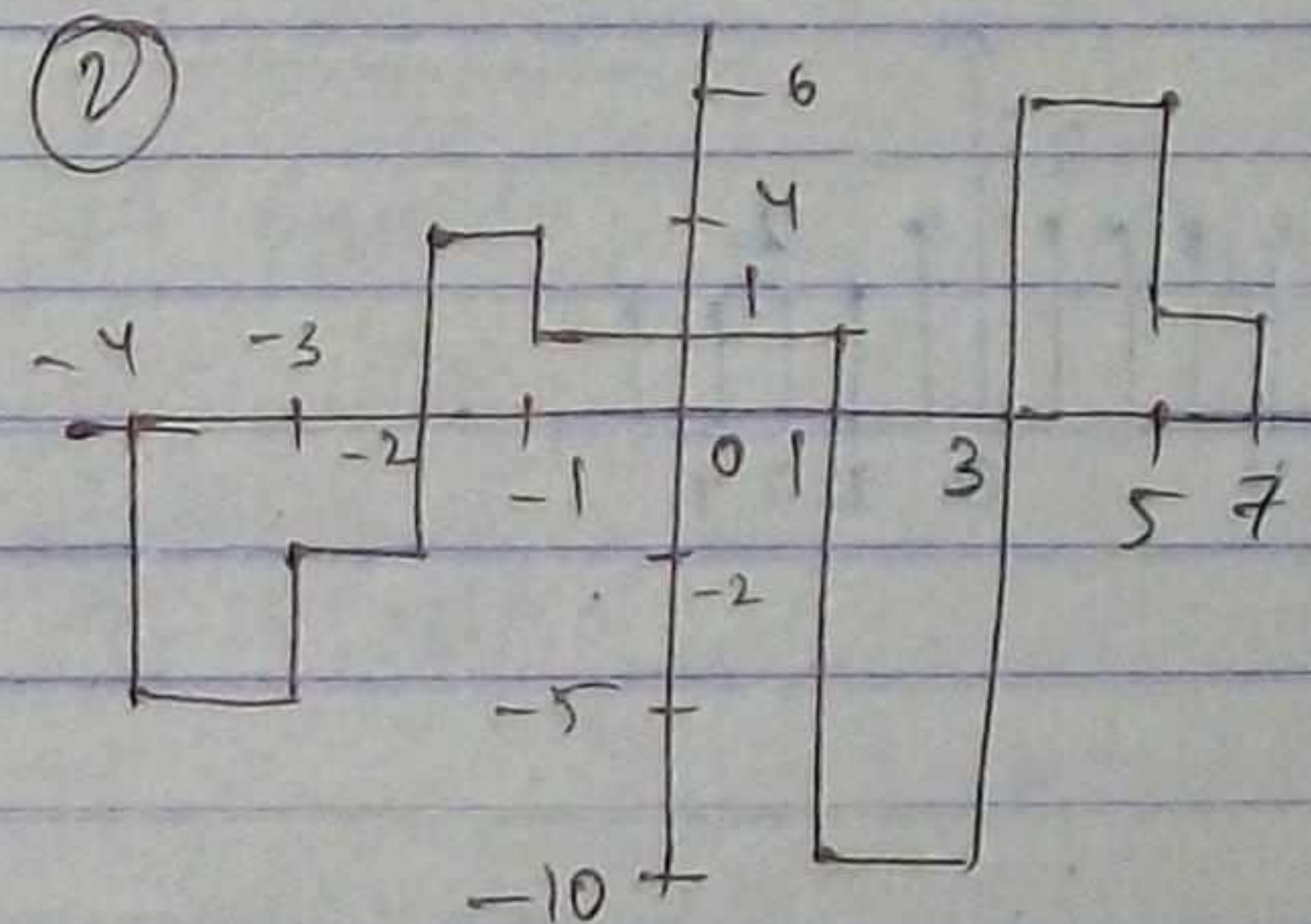
$(-\infty, 2)$

- The operations of $u(-t)$ are opposite to $u(t)$.

Ques. Write the expn for following in terms of step signals.

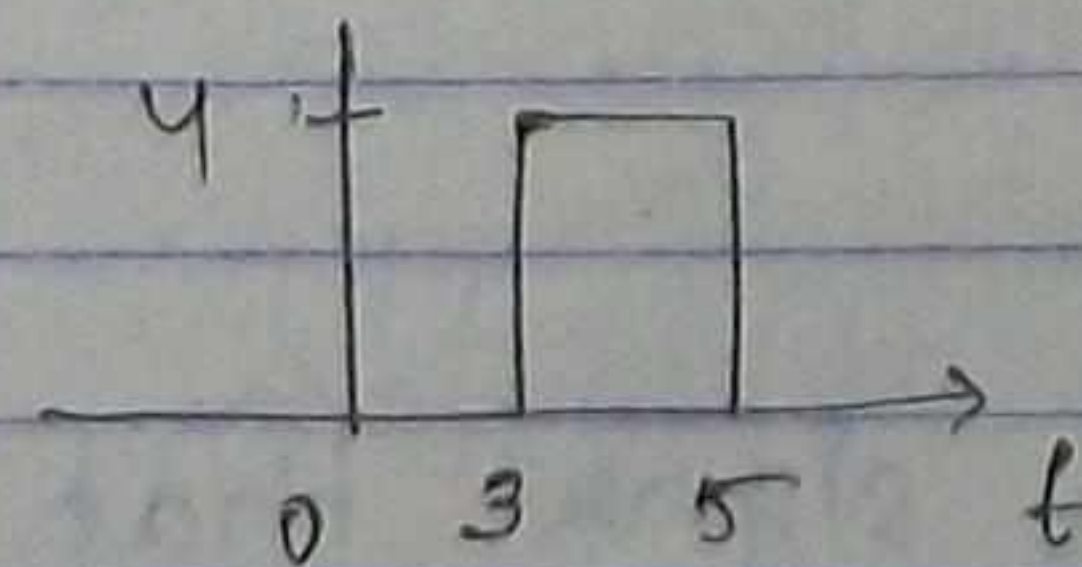


$$\begin{aligned} &4 u(t+3) + 2 u(t+1) \\ &+ 3 u(t-2) - 9 u(t-6) \end{aligned}$$



$$\begin{aligned} &-5 u(t+4) + 3 u(t+3) \\ &+ 6 u(t+2) - 3 u(t+1) \\ &- 11 u(t-1) + 16 u(t-3) \\ &- 5 u(t-5) - u(t-7) \end{aligned}$$

Ques. Write possible expn in terms of $u(t)$ for:



$$① \quad 4 u(t-3) - 4 u(t-5)$$

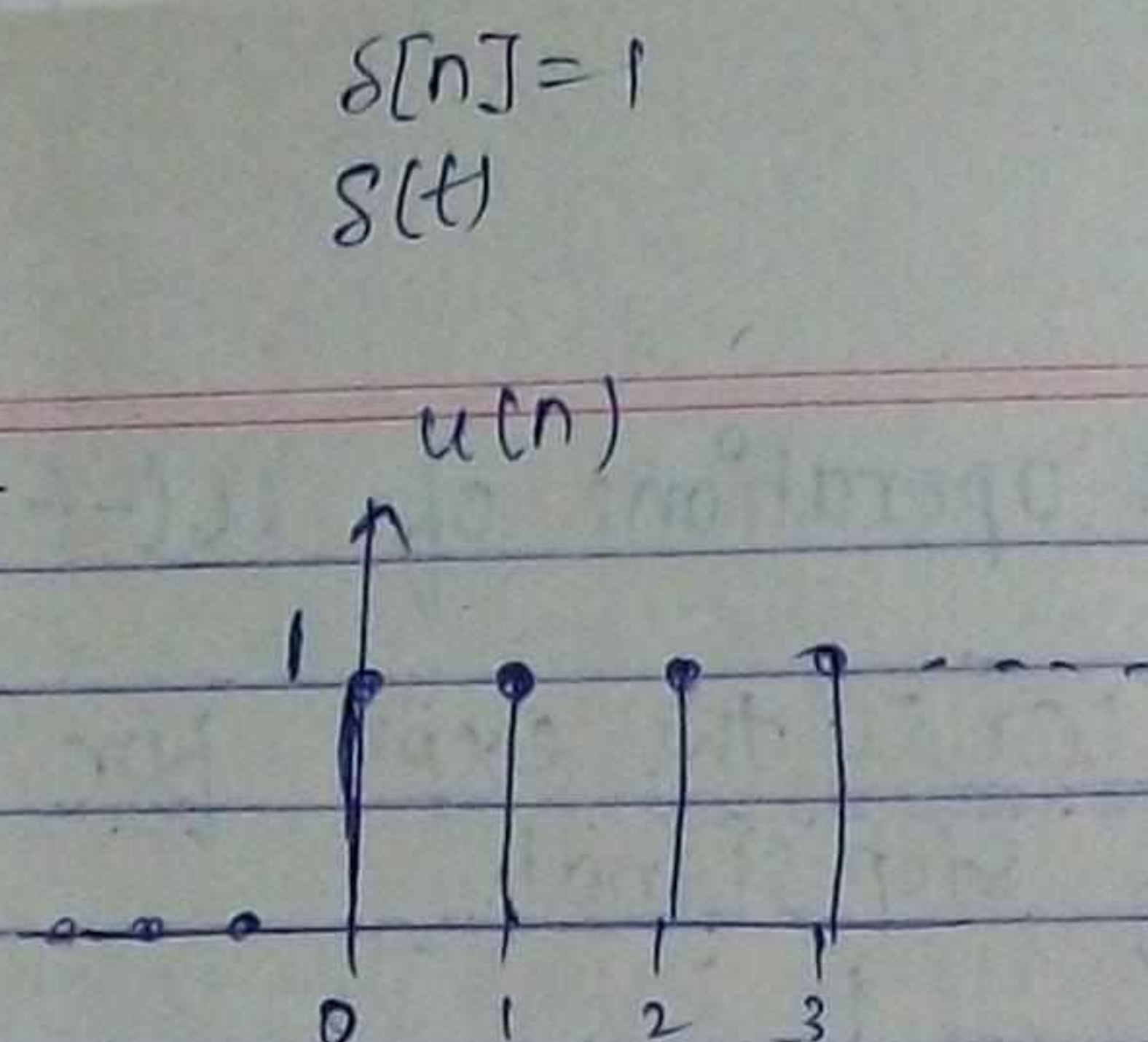
$$② \quad 4 u(-t+5) - 4 u(-t+3)$$

$$③ \quad 4 [u(t-3) \cdot u(-t+5)]$$

② DISCRETE TIME STEP -

$$u(n) = A, n \geq 0$$

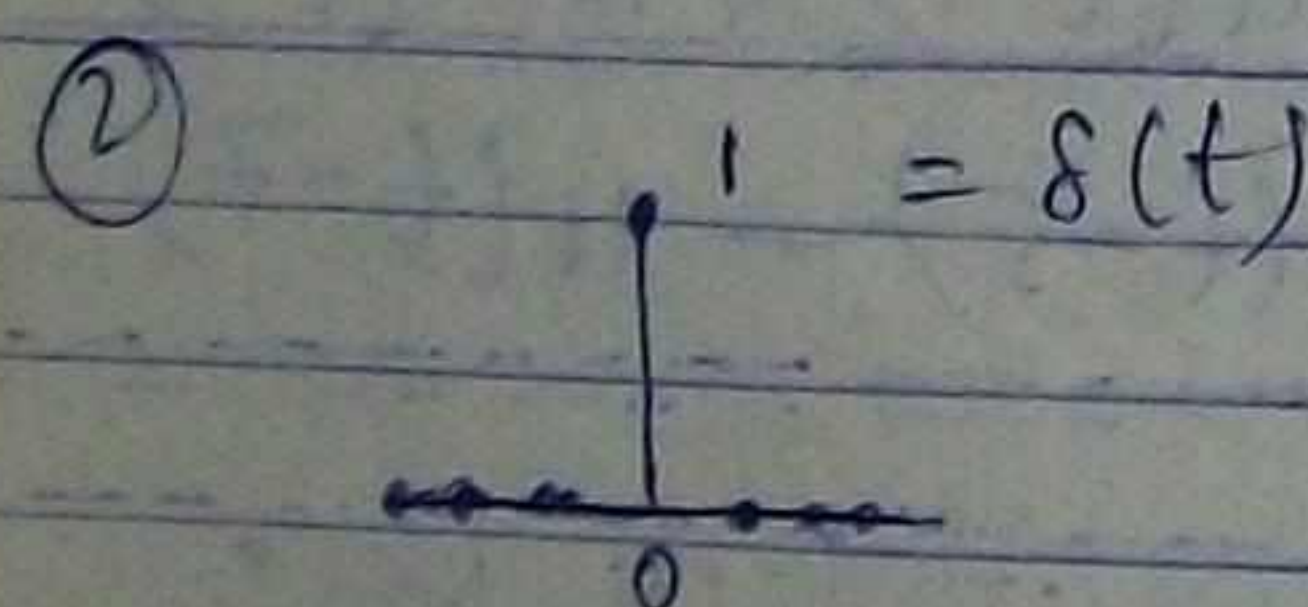
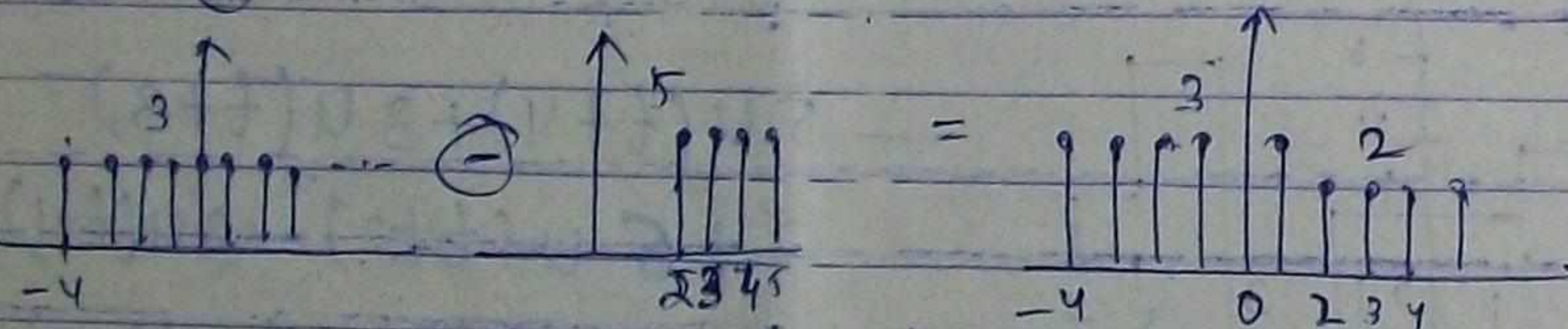
$$= 0, n < 0$$



Ques: Sketch the following:

① $3u(n+4) - 5u(n-2)$ ② $u(n) - u(n-1)$

Solⁿ: ①



③ IMPULSE SIGNAL $\delta(t)$ -

- It is used to represent the signal that occurs and vanishes in zero time such as, lightning, thunderbird.

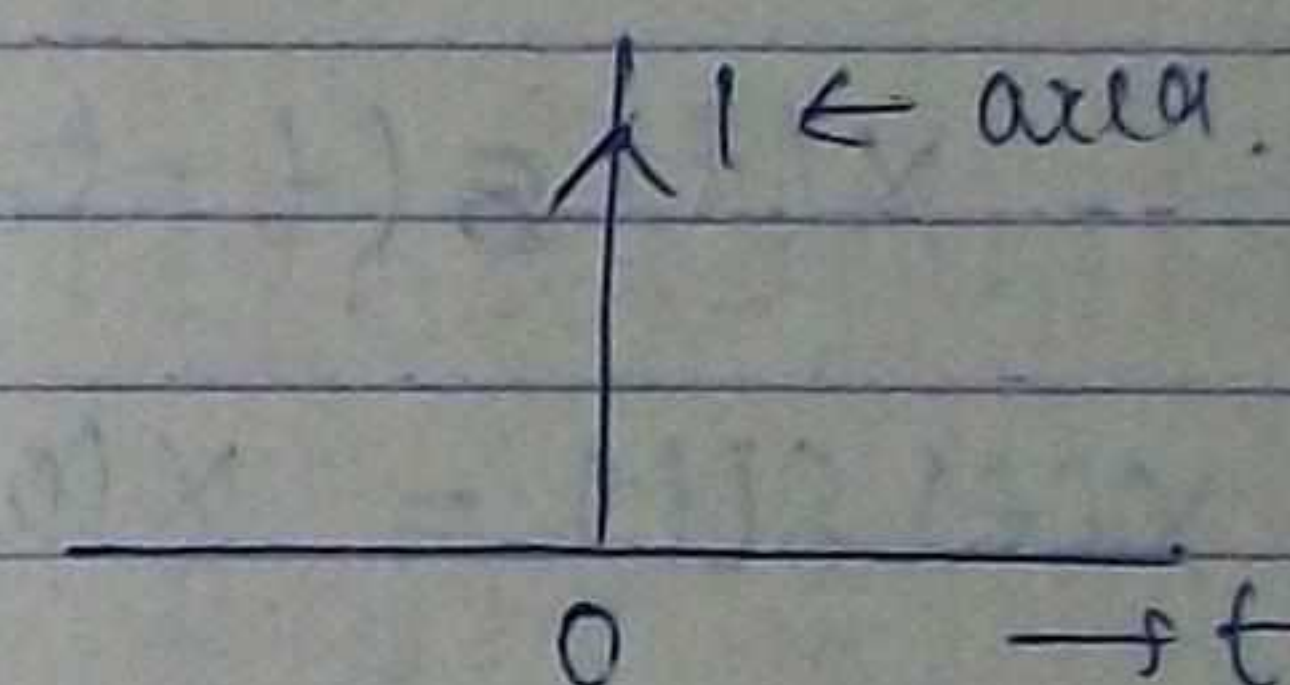
- Irrespective of the shape of the signal

The area under the signal is unity. Such that width is very small and height is very large then it is known as impulse funⁿ.

$$\delta(t) = 0 \quad \text{for } t \neq 0$$

$$= \infty \quad \text{for } t = 0$$

area = 1



PROPERTIES -

① Area: $\int_{-\infty}^{\infty} \delta(t-t_0) dt = 1$

② scaling: $\delta(a(t-t_0)) = \frac{1}{|a|} \delta(t-t_0)$

$$\delta(at) = \frac{1}{|a|} \delta(t)$$

③ $\delta(-(t-t_0)) = \delta(t-t_0)$

$$\delta(-t) = \delta(t)$$

④ Sampling property:

area = strength

- Multiplication of signal $x(t)$ with impulse produces impulse only. But the strength of the impulse is governed by the signal at the location impulse.

$$x(t) \delta(t - t_0) = x(t_0) \delta(t - t_0)$$

$$x(t) \delta(t) = x(0) \delta(t)$$

⑤ Shifting - (sampling of impulse)

$$\int_{-\infty}^{\infty} x(t) \delta(t - t_0) dt = x(t_0)$$

$$\int_{-\infty}^{\infty} x(t) \delta(t) dt = x(0)$$

$$\int_a^b x(t) \delta(t - t_0) dt = x(t_0) \quad \text{for } a < t_0 < b$$

$$= 0 \quad \forall t_0 < a < b$$

$$= 0 \quad \forall a < b < t_0$$

$$= \text{undefined} \quad \forall (a = b)$$

$$\delta(t - t_0) = \frac{d}{dt} u(t - t_0)$$

$$\delta(t) = \frac{d}{dt} u(t)$$

$$u(t) = \int_{-\infty}^t \delta(\tau) d\tau.$$

$$\text{Impulse Response} = \frac{d}{dt} \times (\text{Step Response})$$

Ques. Simplify.

$$\textcircled{1} \delta(\omega) \quad \textcircled{2} e^{-3t} \delta(3-t) \quad \textcircled{3} (1+3t+2t^2) \delta(2t-3)$$

$$\textcircled{4} t \delta(t) \quad \textcircled{5} \delta(-3t) \quad \textcircled{6} \frac{2+j3\omega}{1-j5\omega} \delta(\omega+1)$$

$$\textcircled{7} \cos(3\tau) \delta(\tau - \frac{\pi}{3}) \quad \textcircled{8} \frac{\sin k\omega}{\omega} \delta(\omega).$$

$$\text{Sol}^n: \textcircled{1} \delta(\omega) = \delta(2\pi f) = \frac{1}{2\pi} \delta(f)$$

$$\textcircled{2} e^{-3t} \delta(3-t) = e^{-3t} \delta(t-3) = e^{-9} \delta(t-3)$$

$$\textcircled{3} t \delta(t) = 0 \cdot \delta(t) = \underline{0}$$

$$\textcircled{4} \delta(-3t) = \delta(3t) = \frac{1}{3} \delta(t)$$

$$\begin{aligned} \textcircled{5} & \delta(2t-3) + 3t \delta(2t-3) + 2t^2 \delta(2t-3) \\ &= \frac{1}{2} [\delta(t-1.5) + 3t \delta(t-1.5) + 2t^2 \delta(t-1.5)] \\ &= \frac{1}{2} [1 + 3(1.5) + 2(1.5)^2] \delta(t-1.5) \end{aligned}$$

$$(5) \frac{2+3j\omega}{1-5j\omega} g(\omega+1)$$

$$= \frac{2+3(-1)j}{1-5(-1)j} g(\omega+1)$$

$$(6) \cos(3\tau) g(\tau-\pi/3)$$

$$= \cos(3 \cdot \pi/3) g(\tau-\pi/3) = -g(\tau-\pi/3)$$

$$(7) \frac{\sin k\omega}{\omega} g(\omega) = \frac{0}{0}$$

$$\frac{(\cos k\omega) \cdot k}{1} g(\omega) = k g(\omega)$$

Ques simplify:

$$(1) \int_{-\infty}^{\infty} e^{-3t} g(t-1.2) dt$$

$$= e^{-3 \times 1.2} = e^{-18/5}$$

$$(2) \int_0^{50} e^{-3t} g(t-1.2) dt$$

$$= e^{-18/5}$$

$$(3) \int_{-\infty}^0 e^{-3t} g(t-1.2) dt = 0$$

$$(4) \int_{-10}^4 e^{-3t} g(t-1.2) dt = e^{-3(1.2)} = e^{-3.6}$$

Ques. If the step response of LTI system is $e^{-3t} u(t)$ find the impulse response.

Soln:

$$g(t) = \frac{d}{dt} u(t) = \frac{d}{dt} (e^{-3t} u(t))$$

$$\text{Impulse resp.} = -3 e^{-3t} u(t) + e^{-3t} u'(t)$$

$$= -3 e^{-3t} u(t) + e^{-3t} g(t)$$

$$= -3 e^{-3t} u(t) + e^{-3(0)} g(t)$$

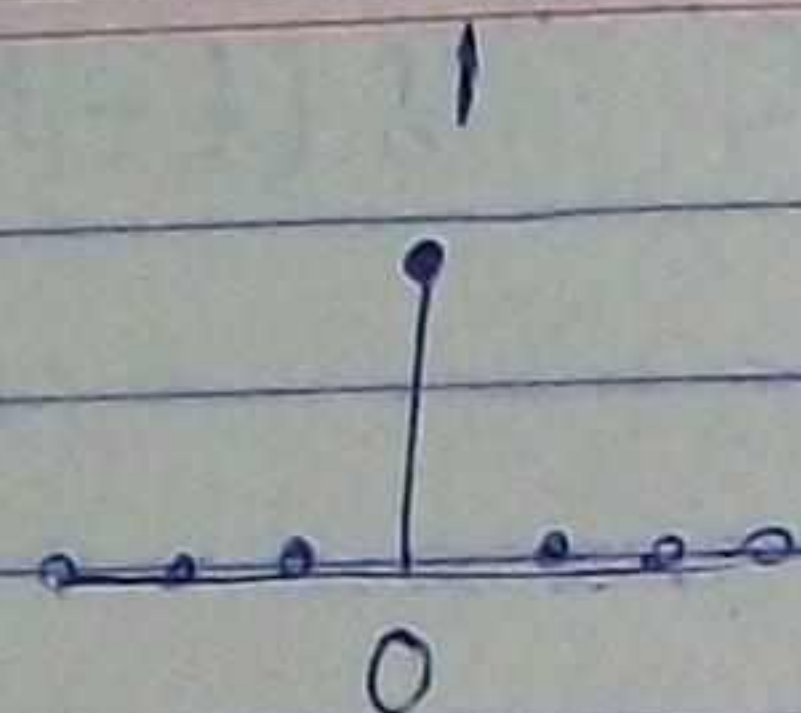
$$= g(t) - 3 e^{-3t} u(t)$$

(4) DISCRETE TIME IMPULSE

- unlike $g(t)$, $g(n)$ does not represent the area under the signal rather it represents the height.

$$\delta[n] = 0 \quad \text{for } n \neq 0$$

$$= 1 \quad \text{for } n = 0$$



PROPERTIES -

$$(1) \delta[an] = \delta[n]$$

$$(2) x[n] \delta[n-k] = x[k] \delta[n-k]$$

$$(3) \sum_{k=-\infty}^{\infty} x[k] \delta[n-k] = x[n]$$

$$(4) \delta[n] = u[n] - u[n-1]$$

IMPULSE RESPONSE = $\delta[n] - \delta[n-1]$

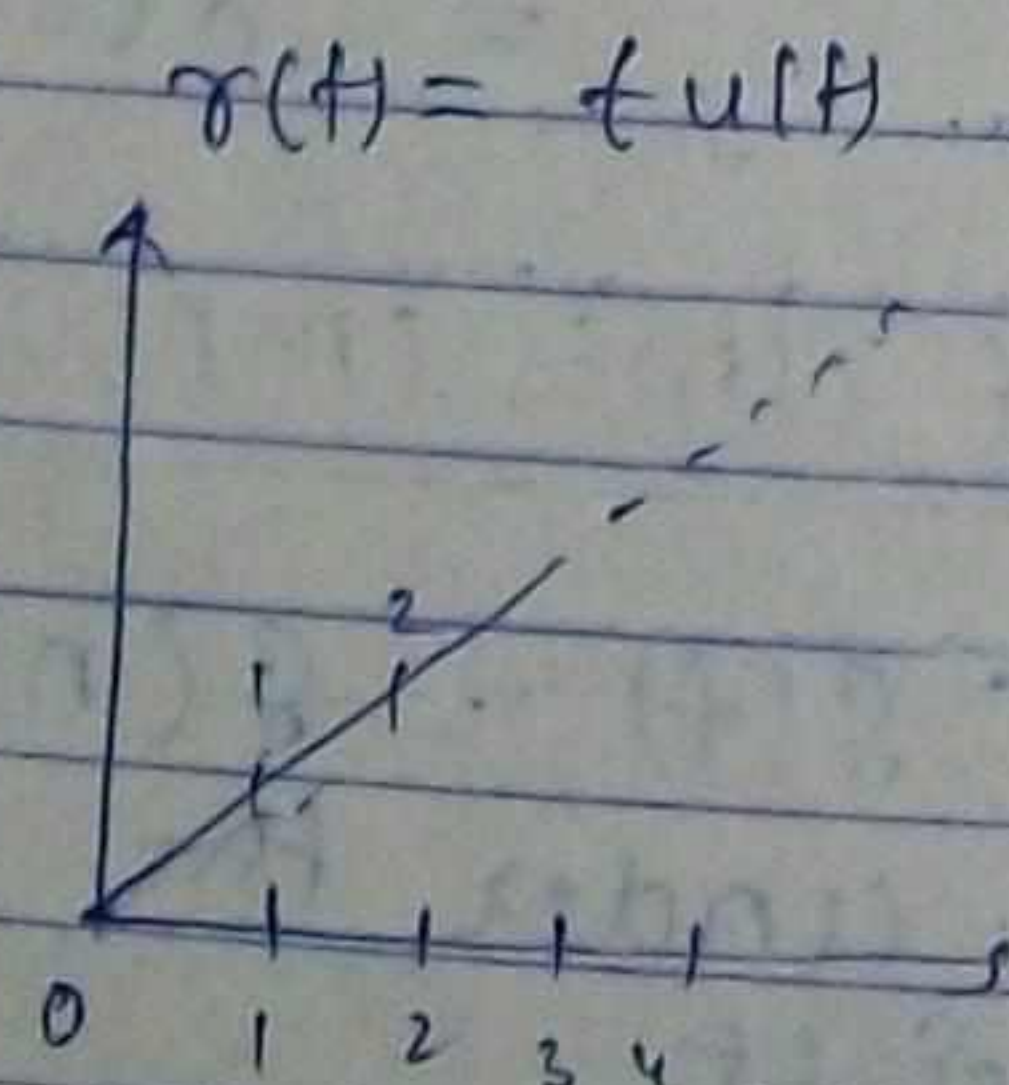
$\delta[n]$ = step response

(5) RAMP SIGNAL -

$$r(t) = t \quad \text{for } t \geq 0$$

$$= 0 \quad \text{for } t < 0$$

$$(6) r(t) = t u(t)$$



$$3r(t) = r(3t)$$

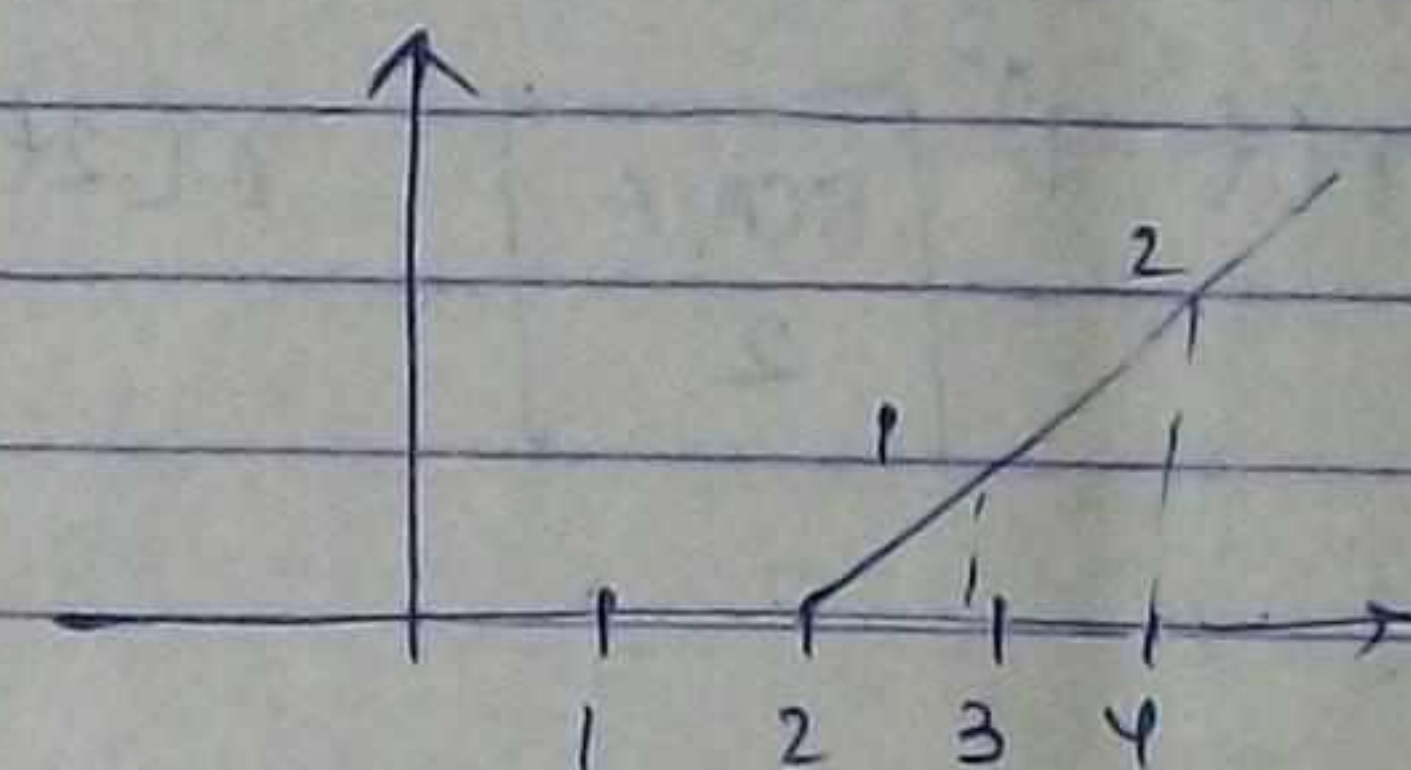
$$3t u(t) = 3t u(3t)$$

Q. Sketch the following

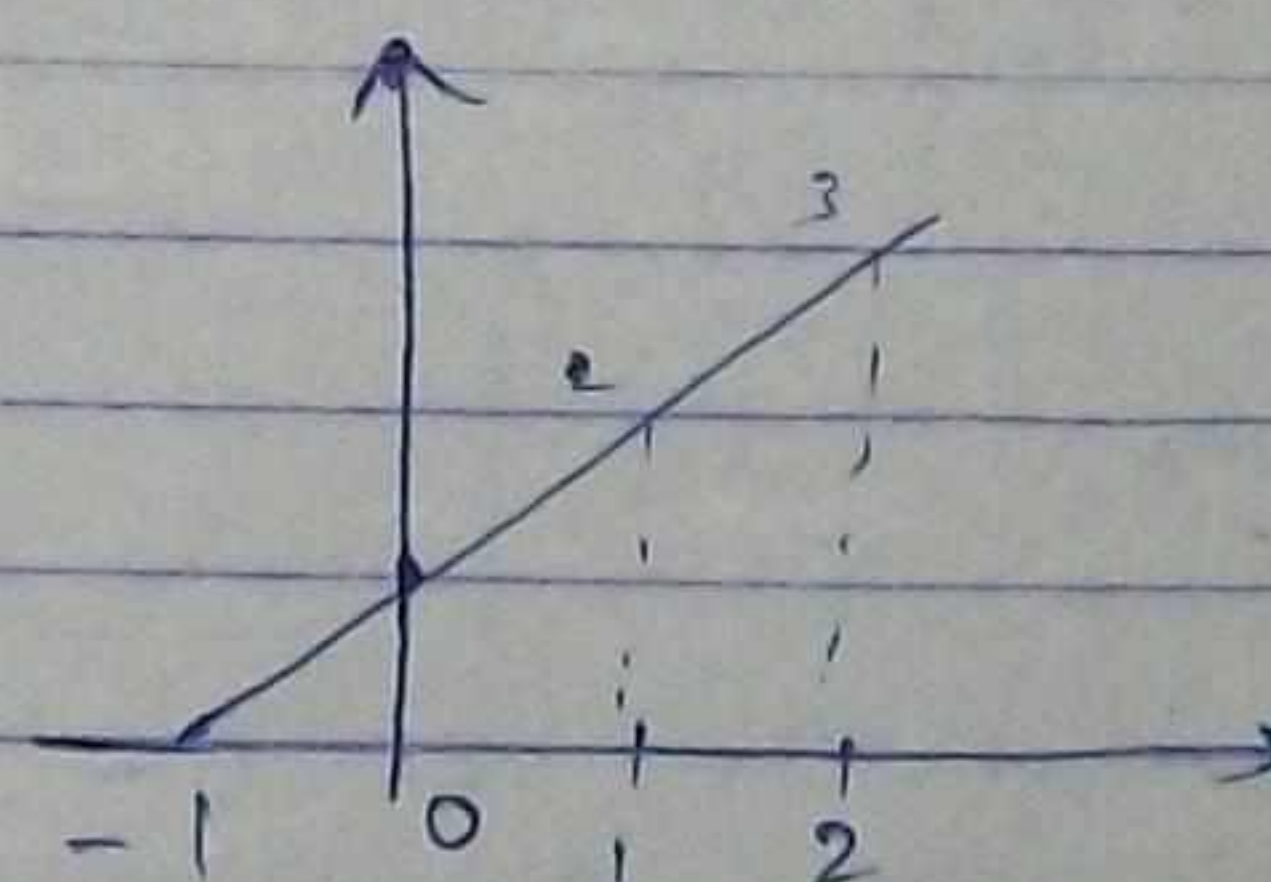
$$(1) r(t-2), r(t+1), r(2t), 3r(t), r(-t), -r(t), -r(-t), r(2t-3), r(-2t-3)$$

$$(2) r(t) + u(t), r(t) + 2u(t-3), r(t) - u(t), r(t) - 3u(t-2), r(t) - r(t-3), r(t) - 2r(t-3)$$

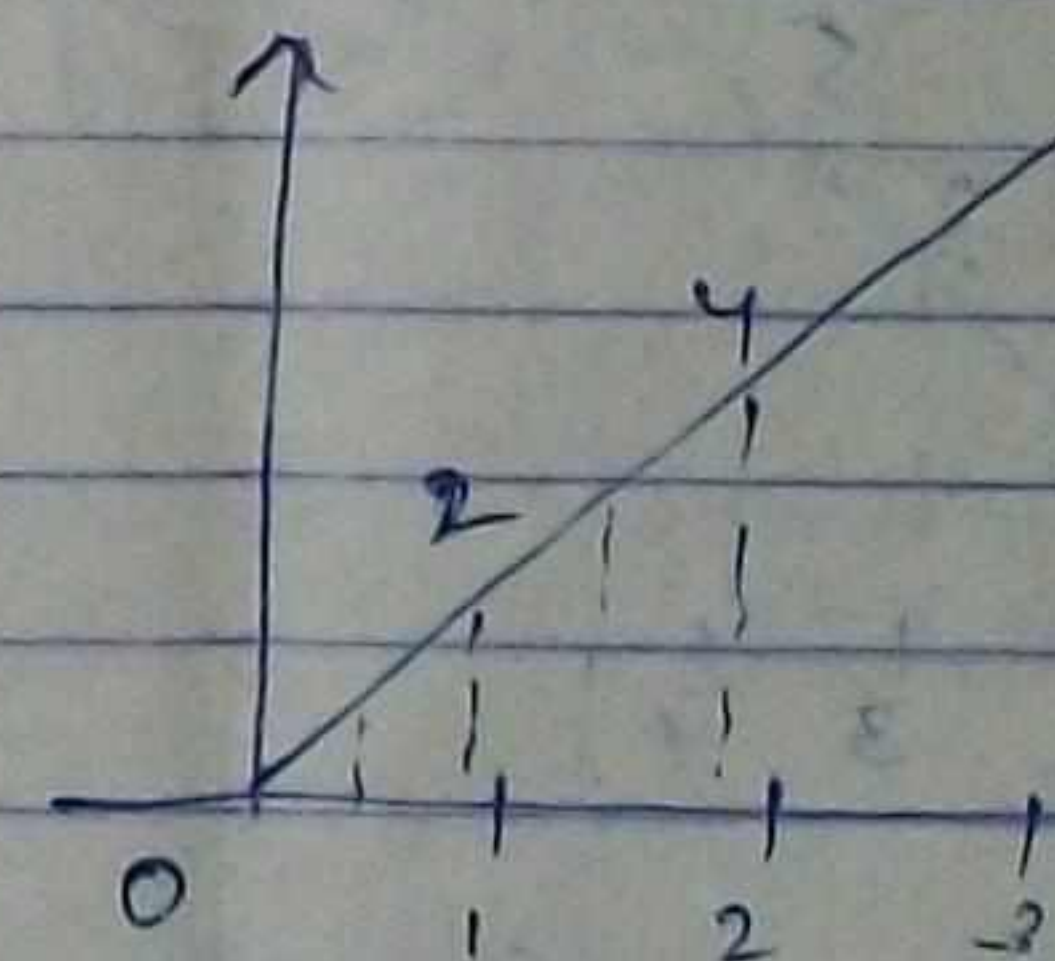
Soln: (1) $r(t-2) = (t-2) u(t-2)$



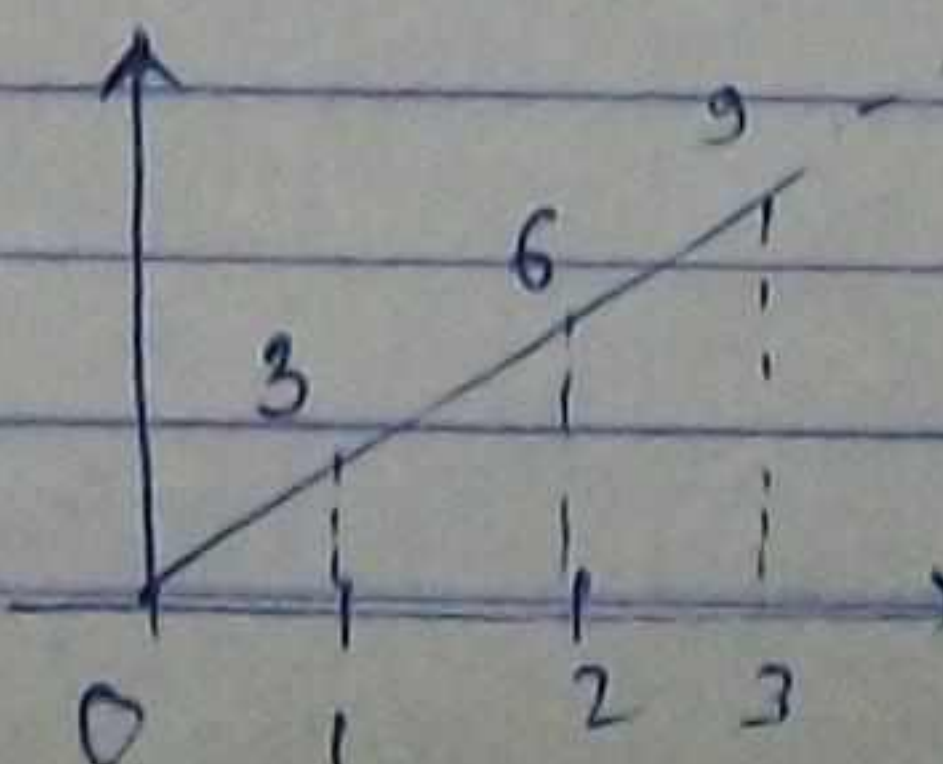
$$(2) r(t+1) = (t+1) u(t+1)$$



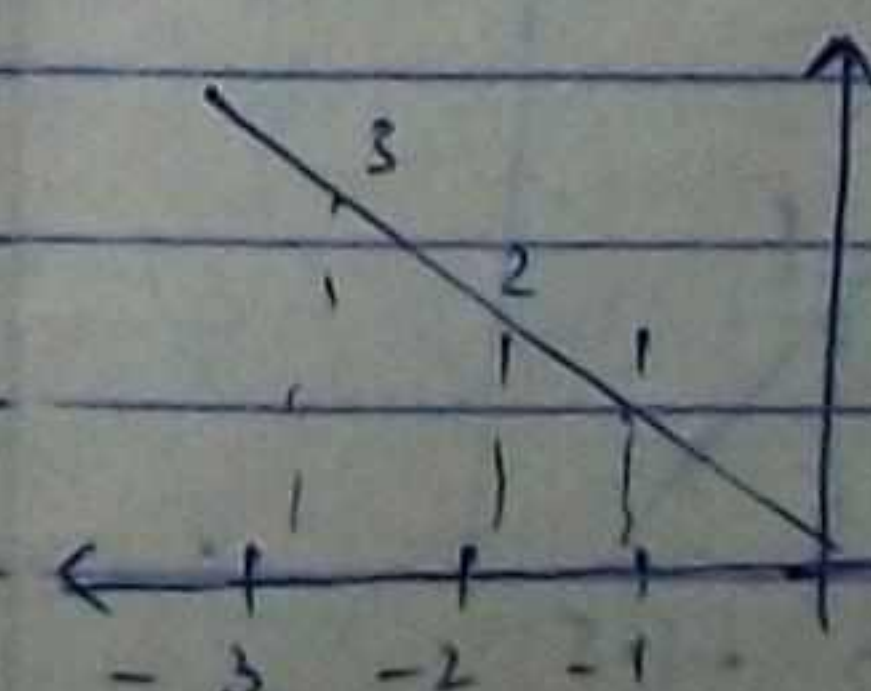
$$(3) r(2t) = 2t u(2t)$$



$$(4) 3r(t) = 3t u(t)$$

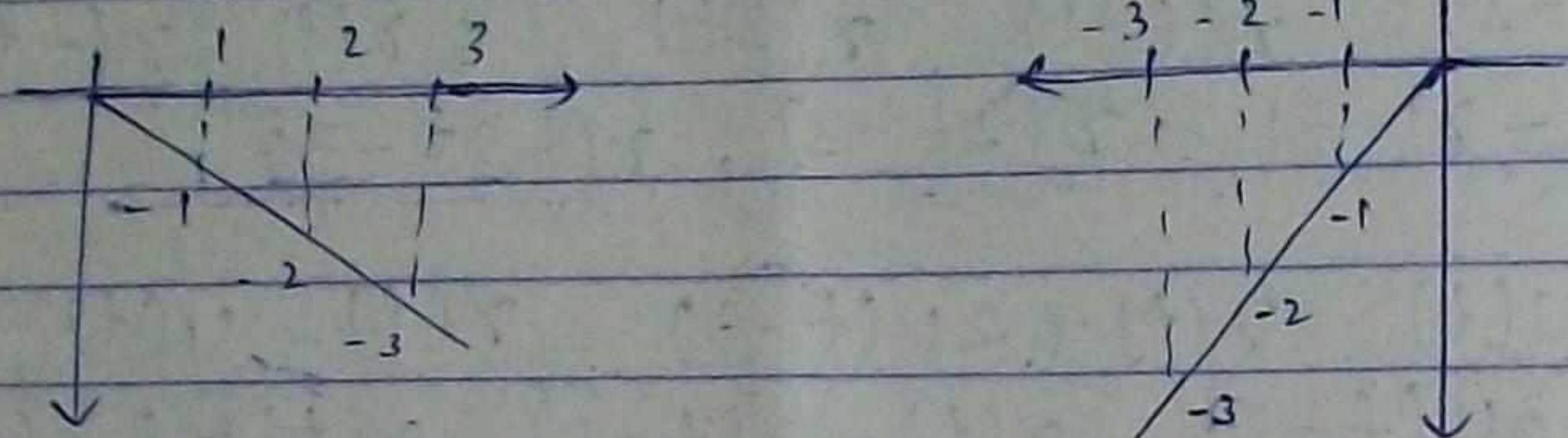


$$(5) r(-t) = -t u(-t)$$

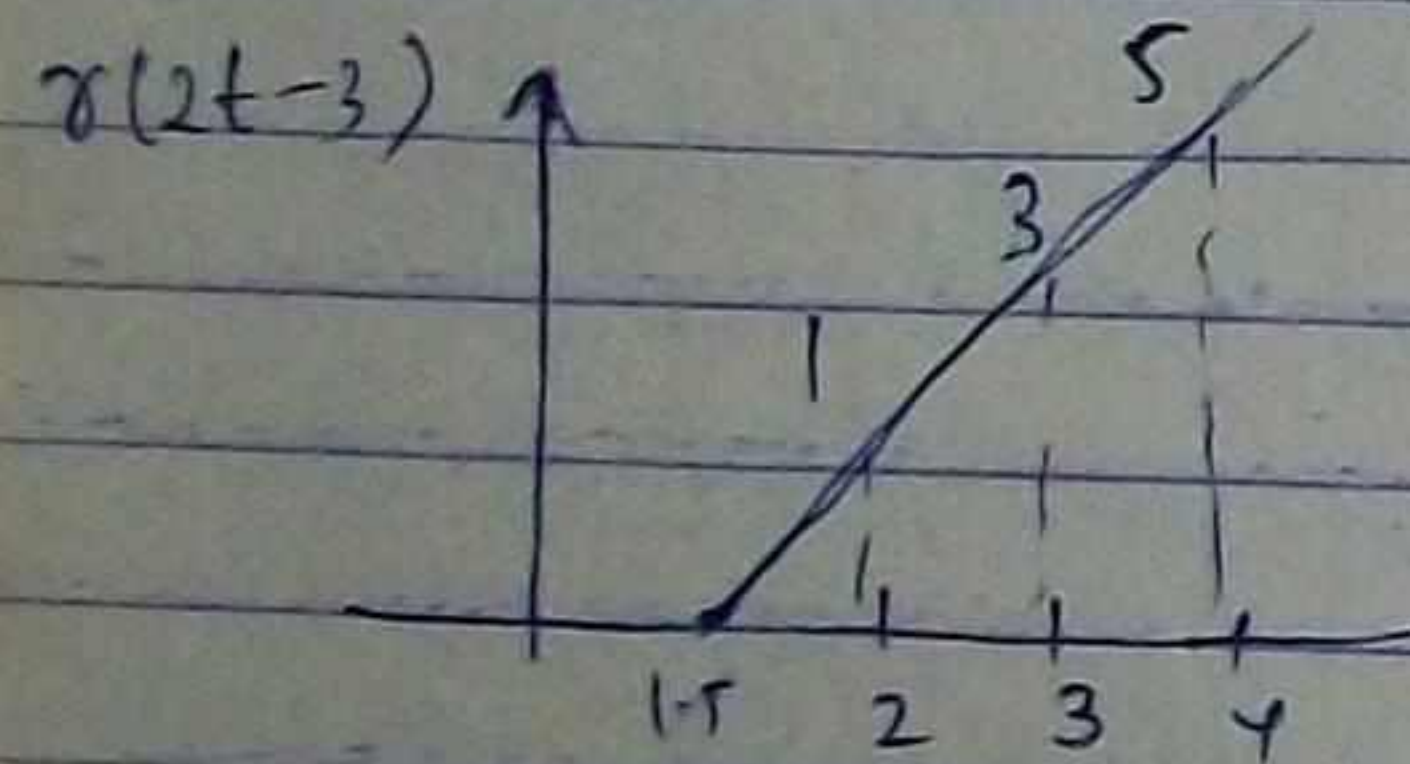
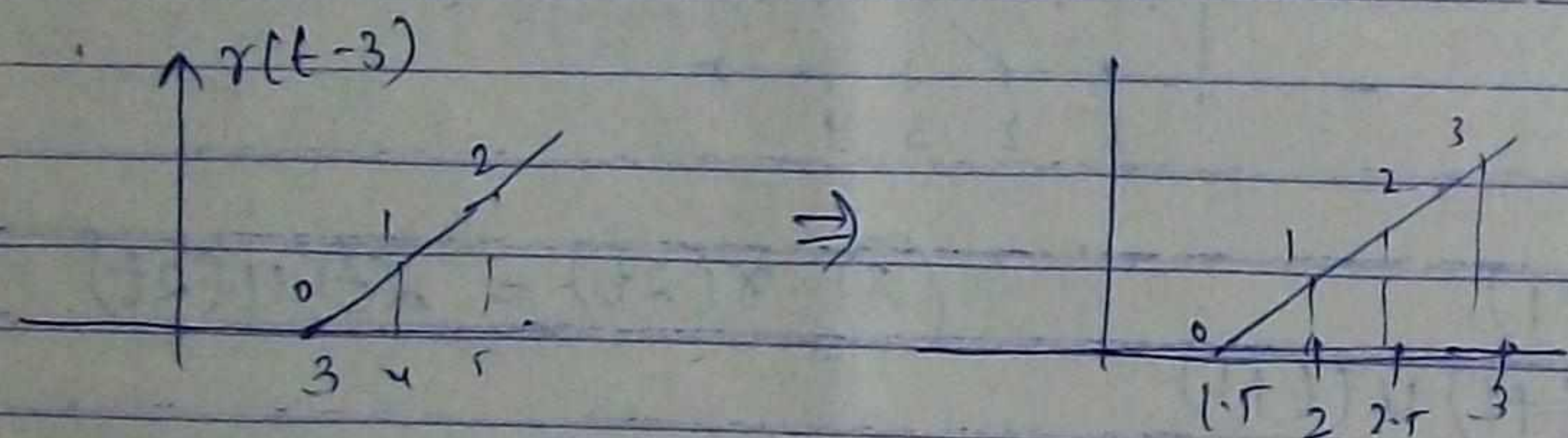
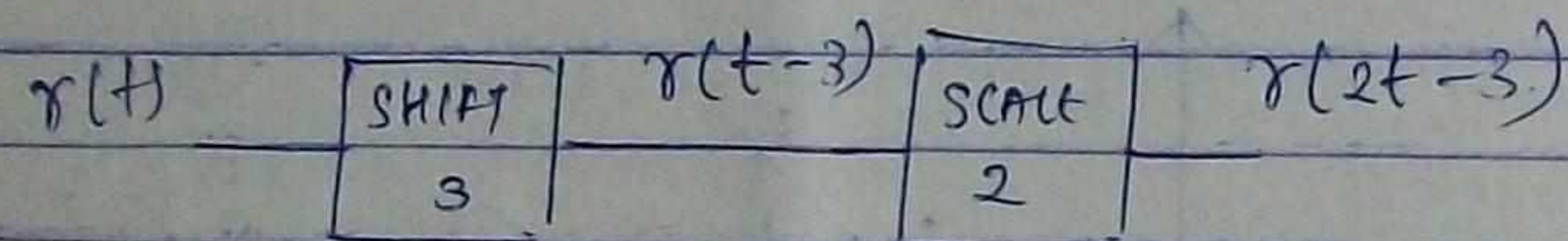


$$r(2t-1.5)$$

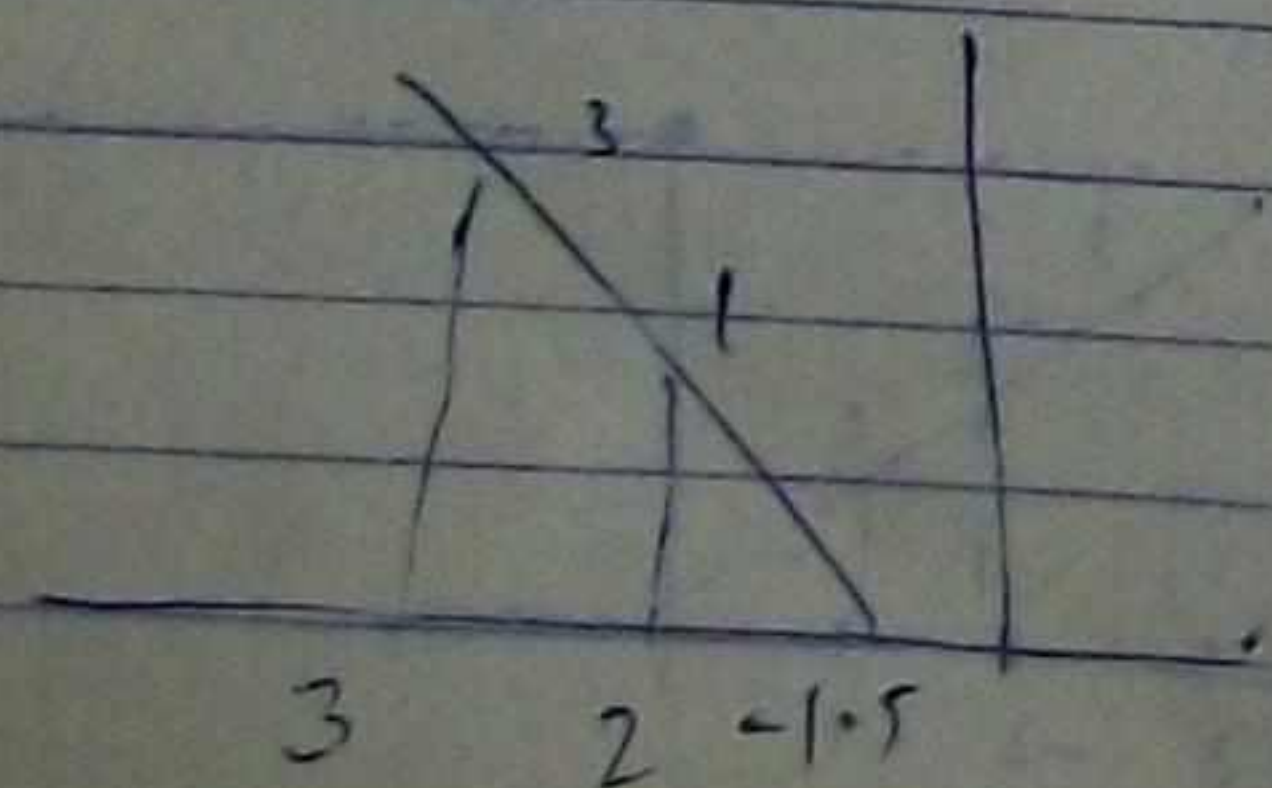
(6) $-r(t) = -t u(t)$ (7) $-r(-t) = -t u(-t)$



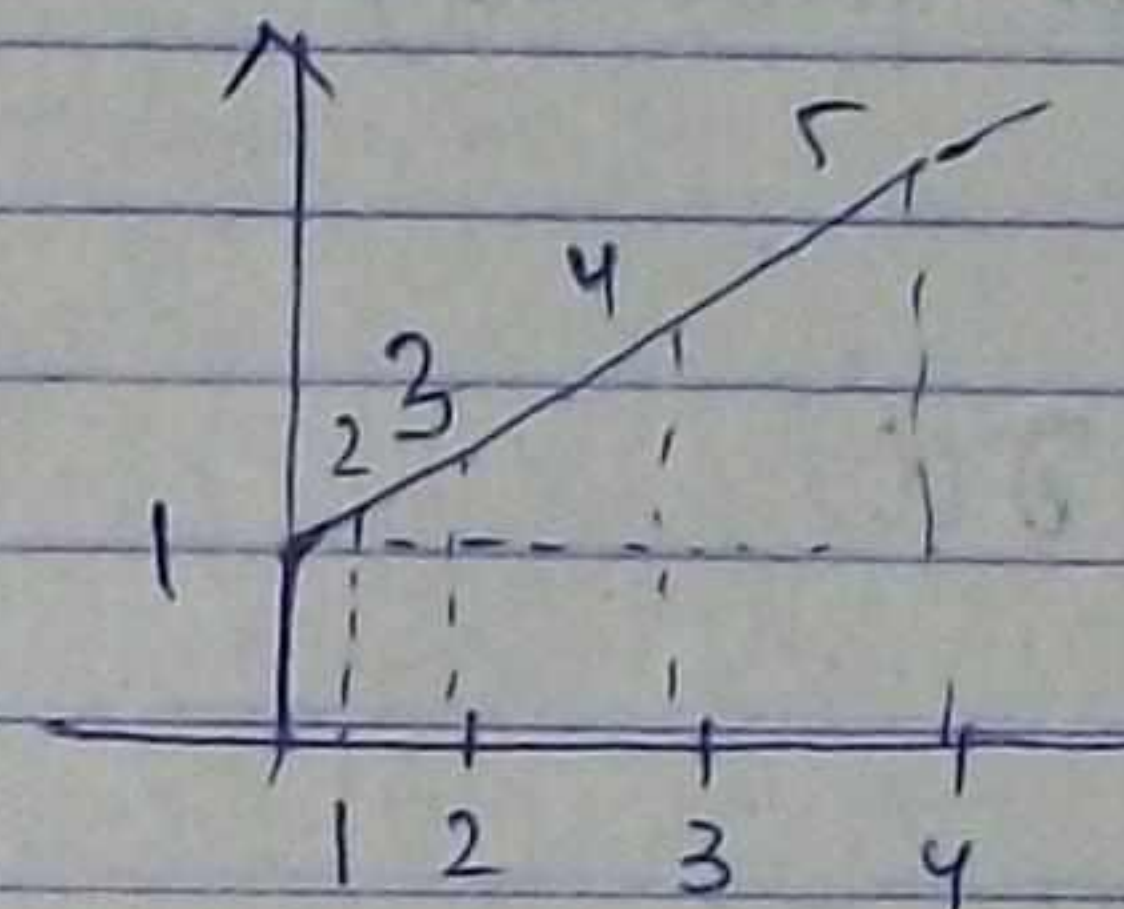
(8) $r(2t-3) = (2t-3) u(2t-3)$



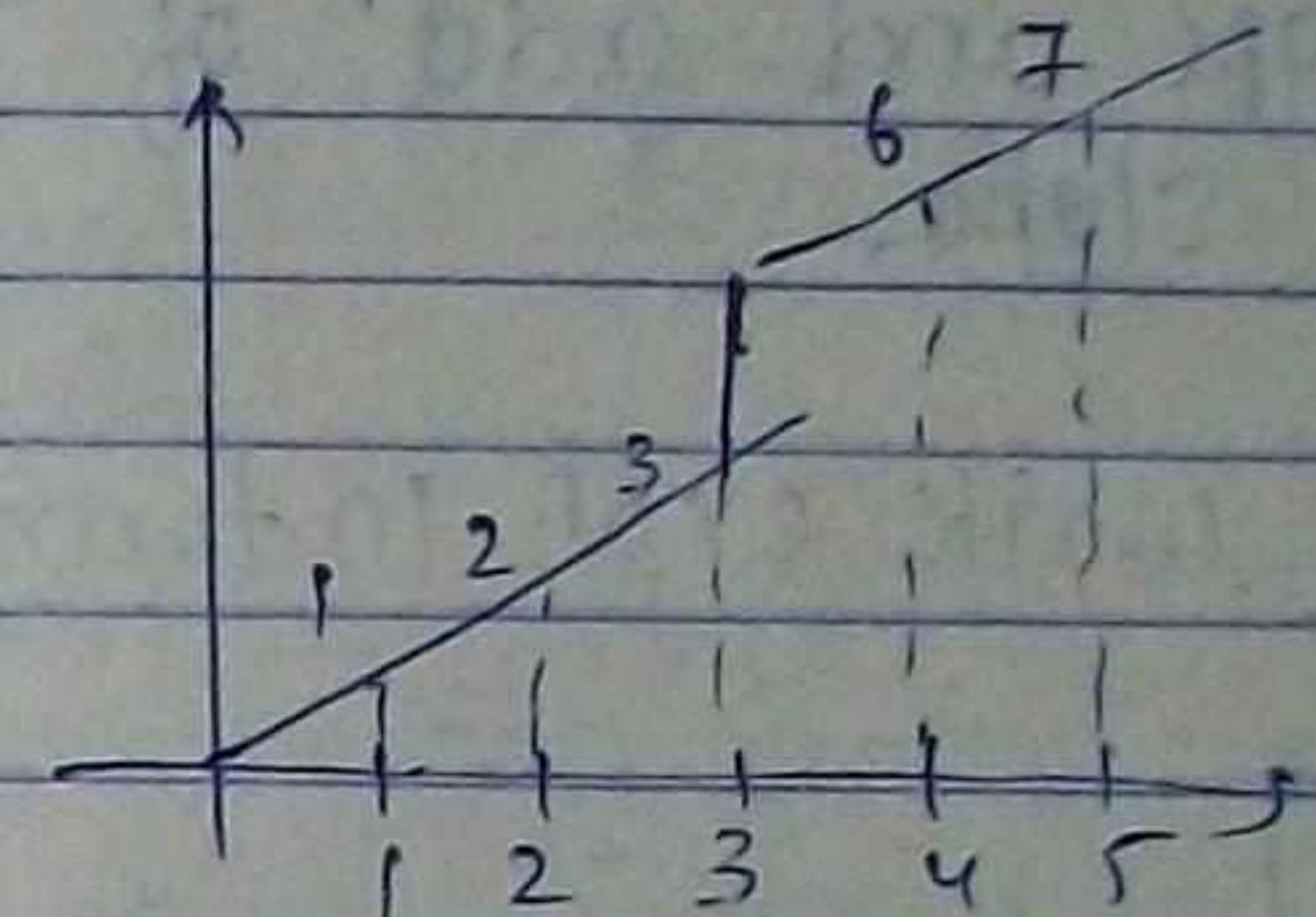
(9) $r(-2t-3) = -(2t+3) u(2t+3)$



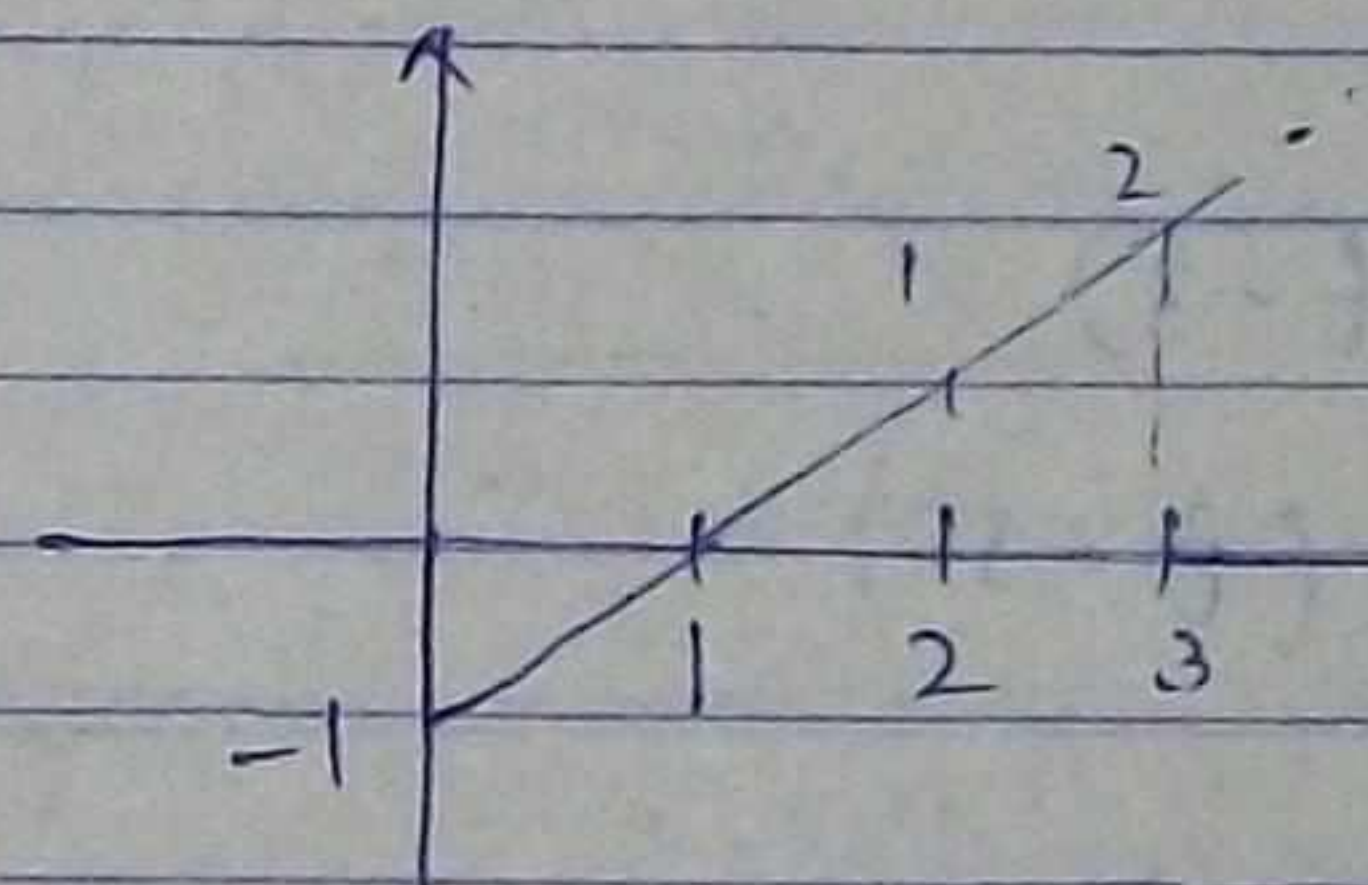
(10) $r(t) + u(t)$



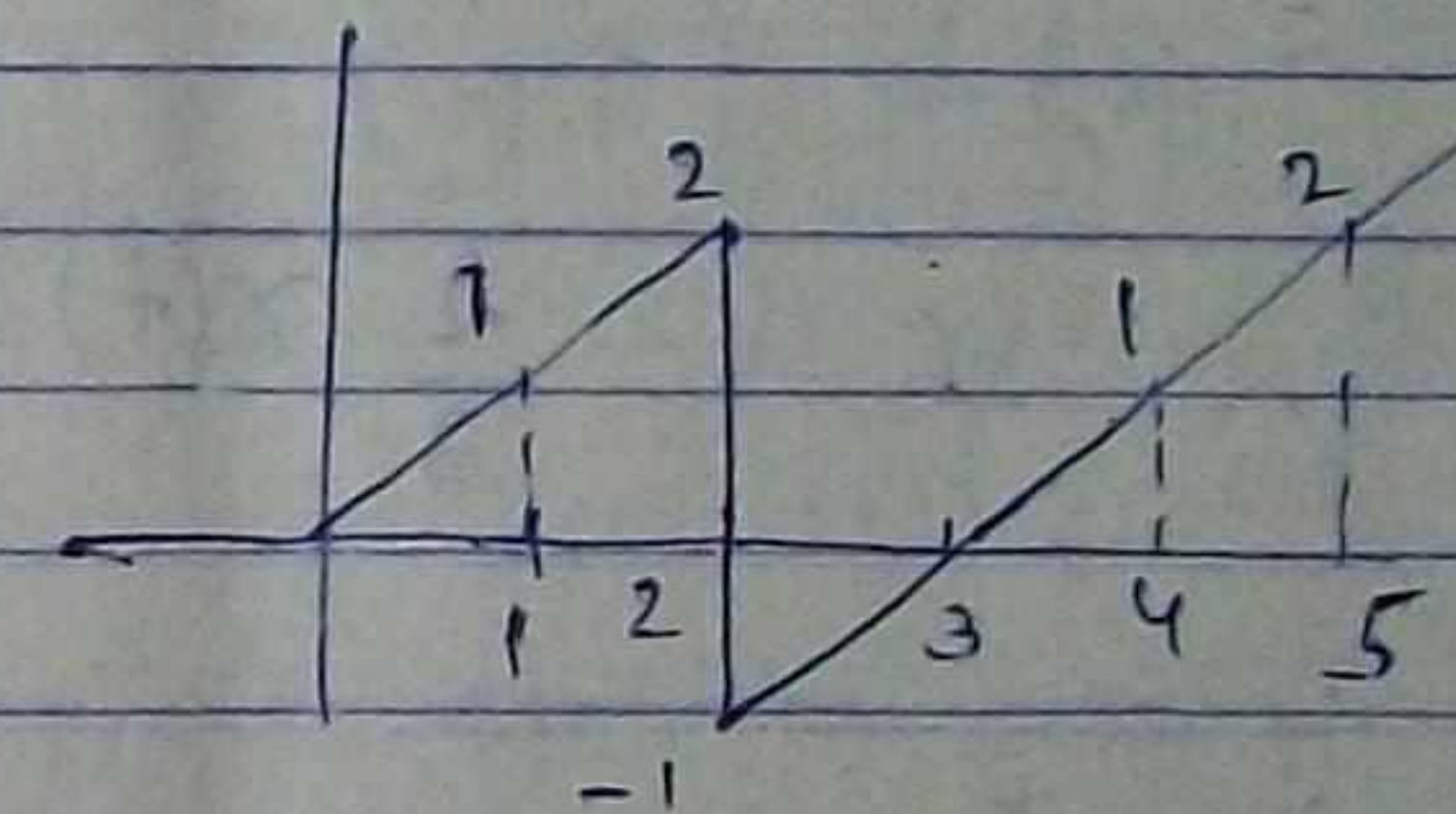
(11) $r(t) + 2u(t-3)$



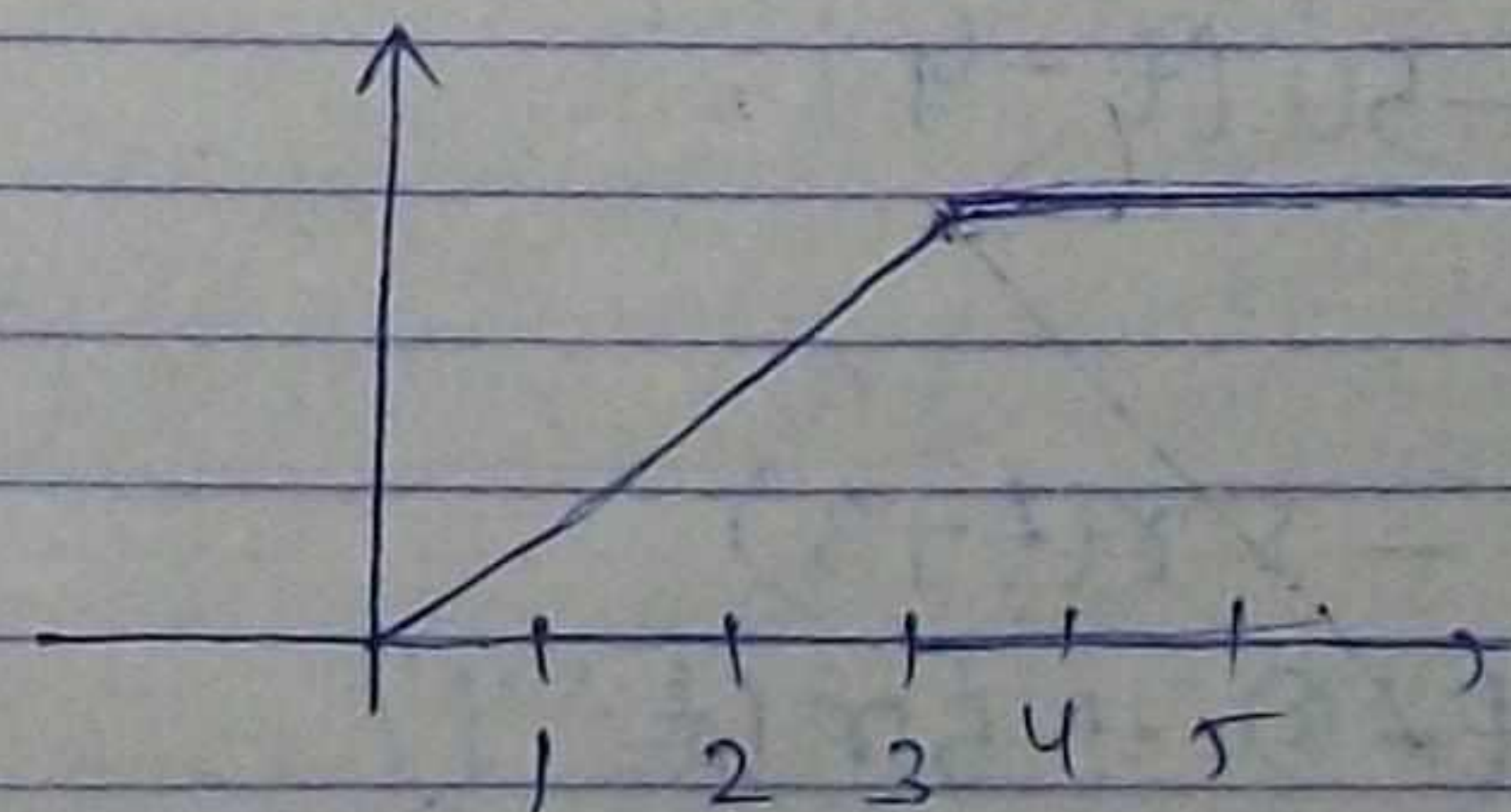
(12) $r(t) - u(t)$



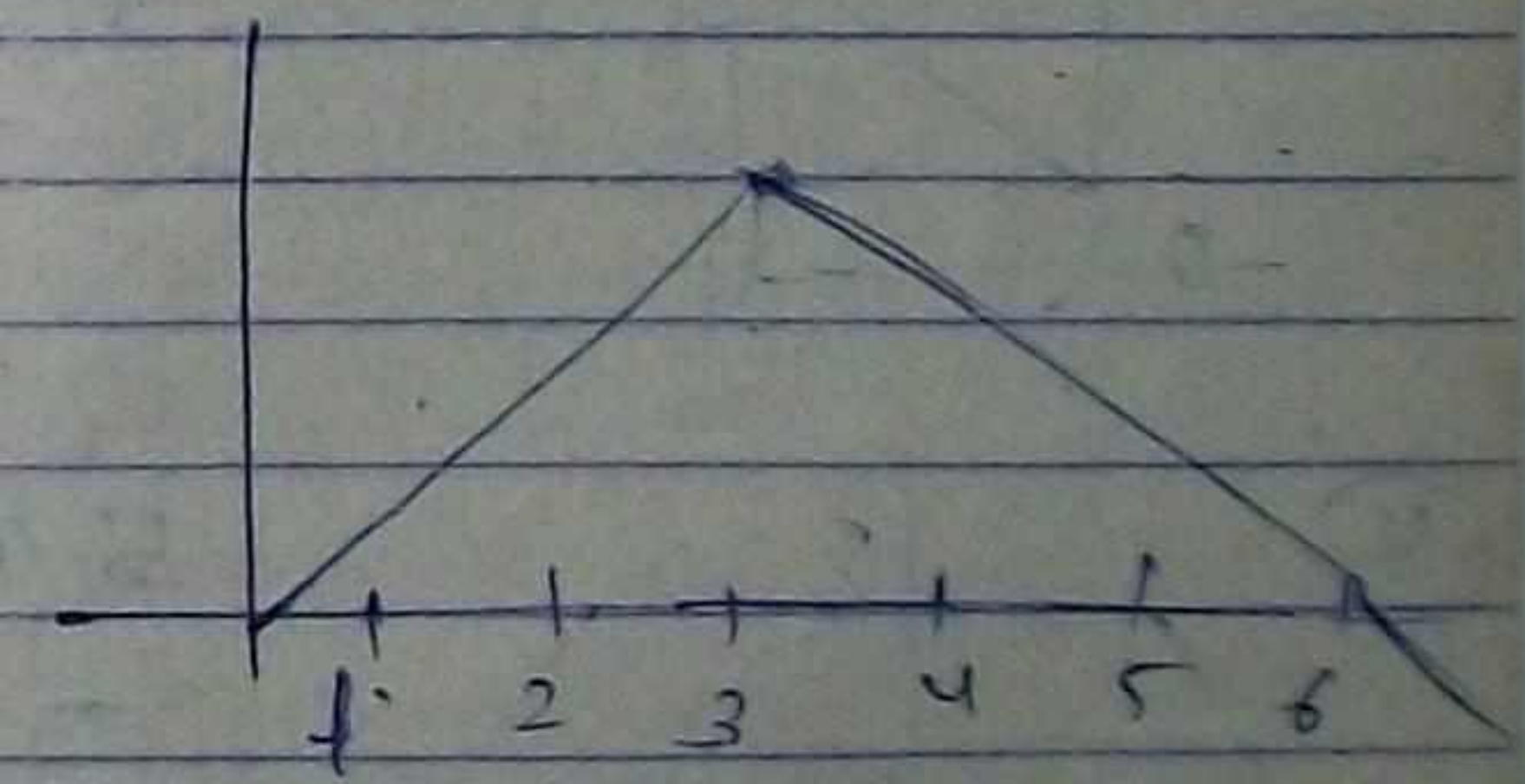
(13) $r(t) - 3u(t-2)$



(14) $r(t) - r(t-3)$



(15) $r(t) - 2r(t-3)$



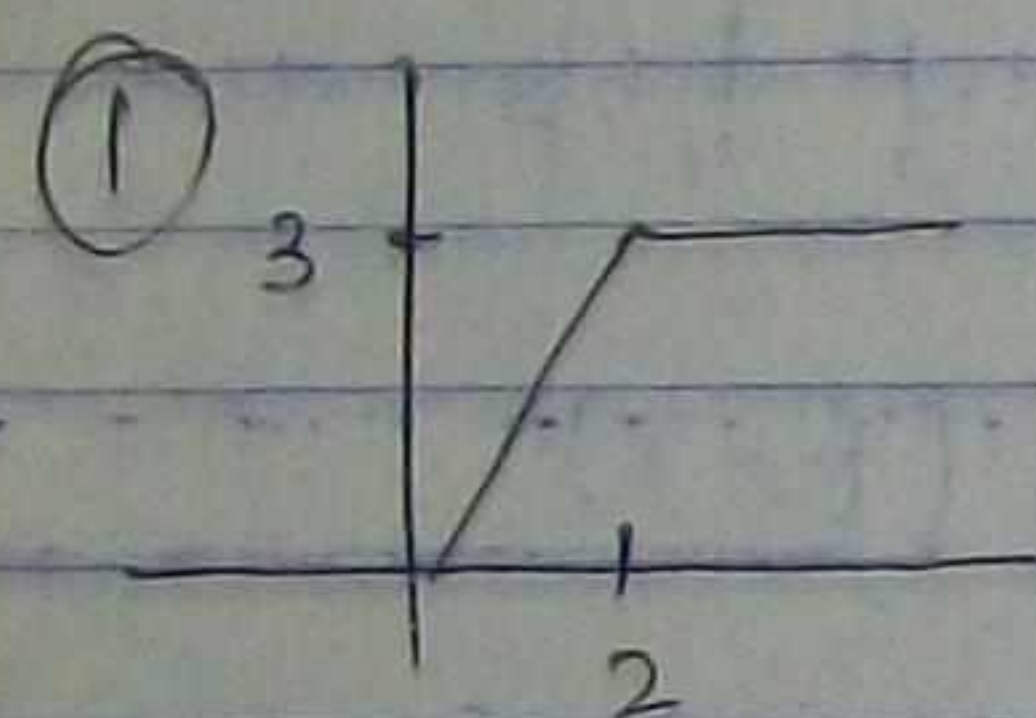
$$r(t) - 2r(t-3)$$

$$= r(t) - r(t-3) - r(t-3)$$

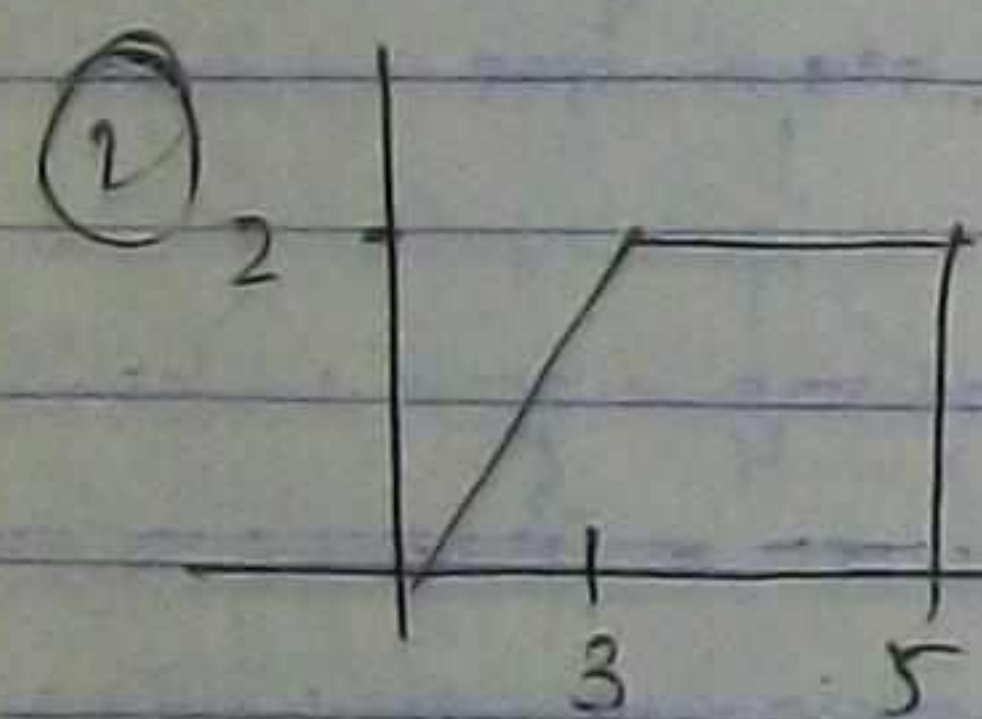
* NOTE -

- subtraction of the ramp signal decreases the slope and addⁿ of ramp signal increases the slope.

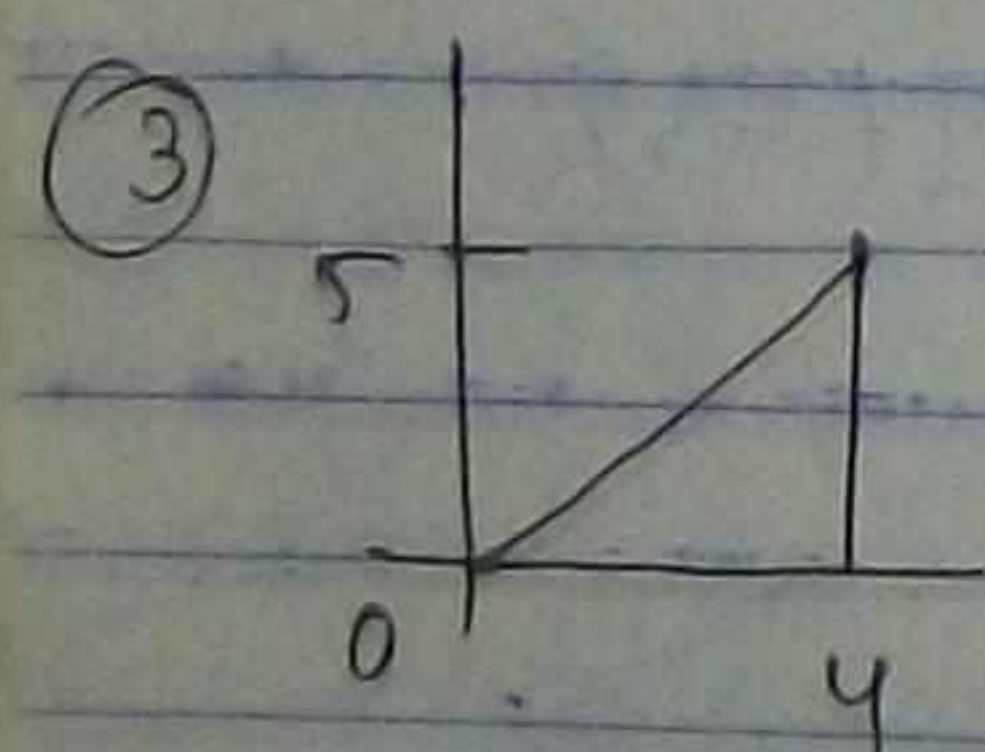
Ques. write exprⁿ in terms of $u(t)$ & $r(t)$.



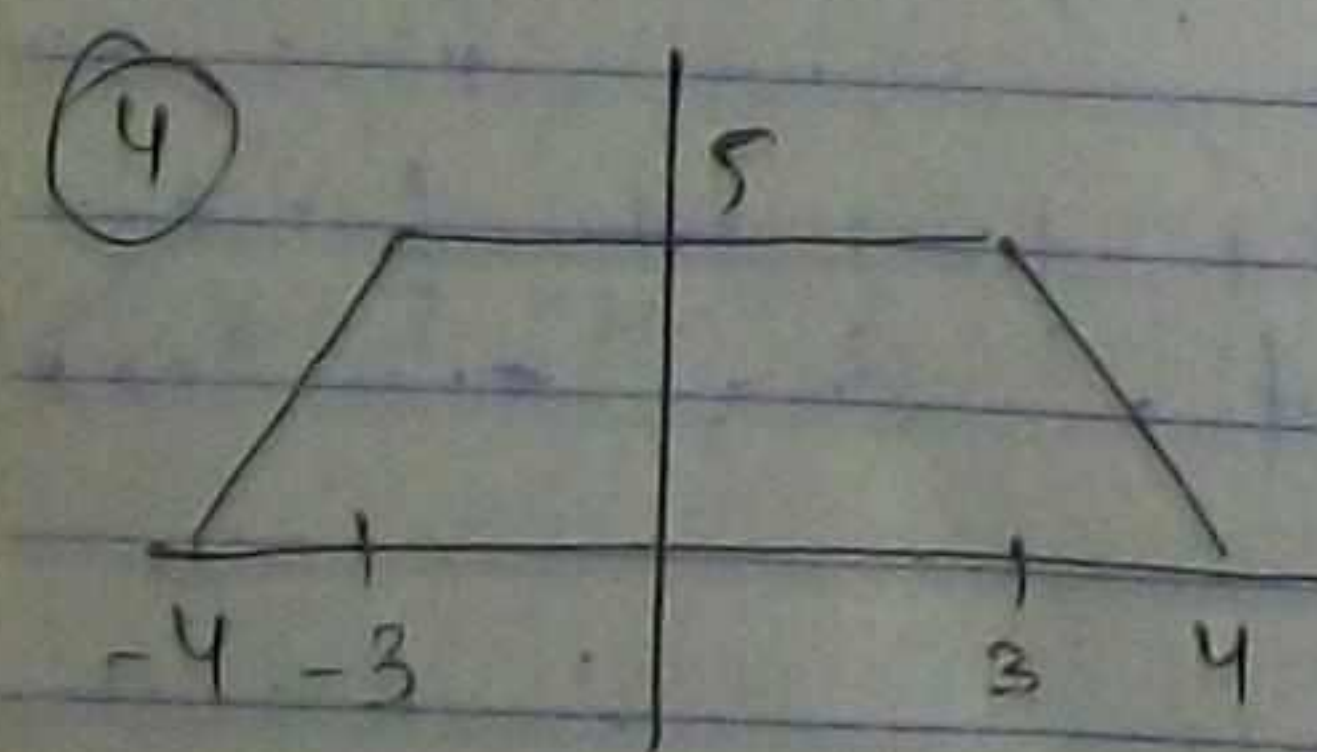
$$\frac{3}{2} r(t) - \frac{3}{2} r(t-2)$$



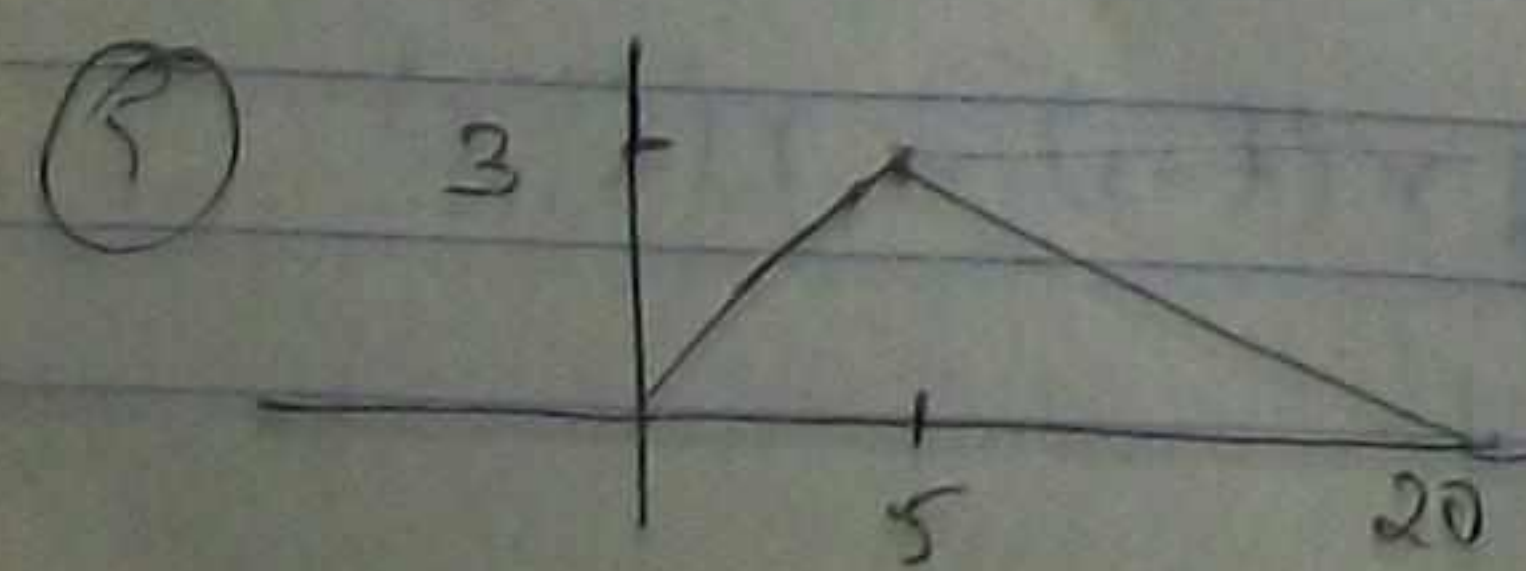
$$\frac{2}{3} r(t) - \frac{2}{3} r(t-3) - 2 u(t-5)$$



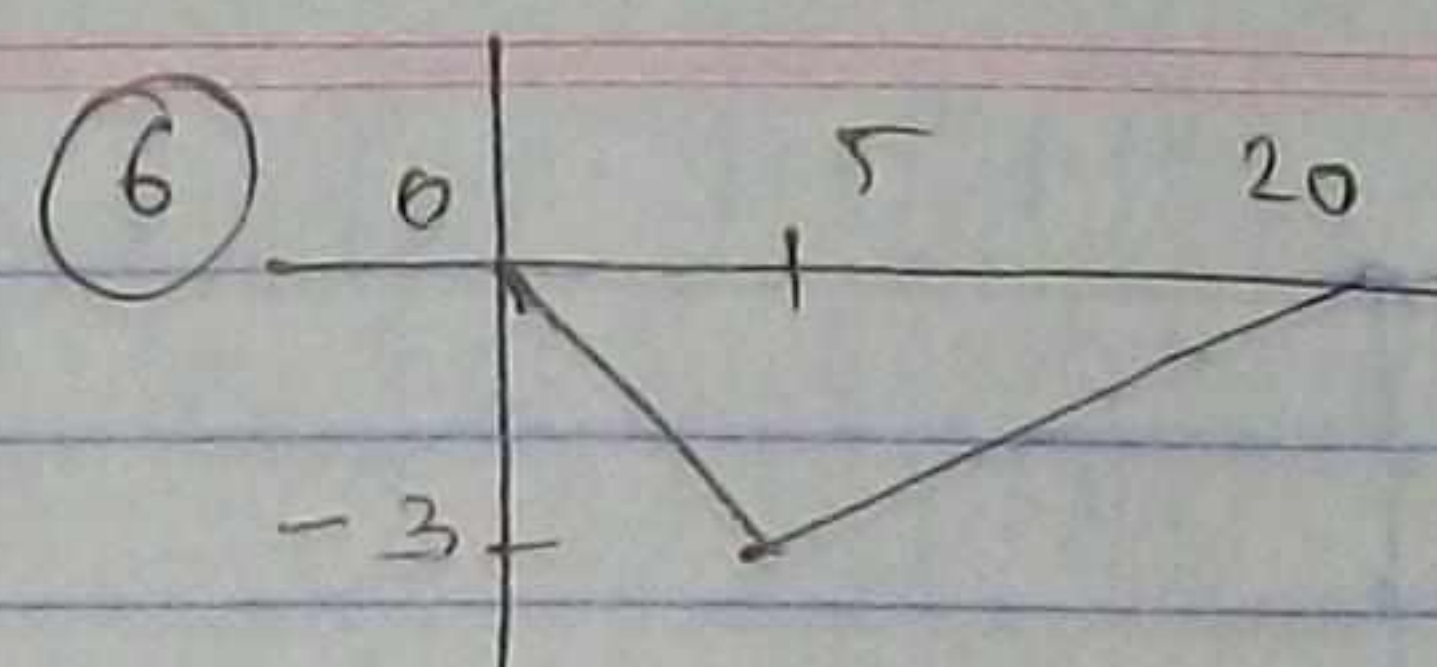
$$\frac{5}{4} r(t) - \frac{5}{4} r(t-4) - 5 u(t-4)$$



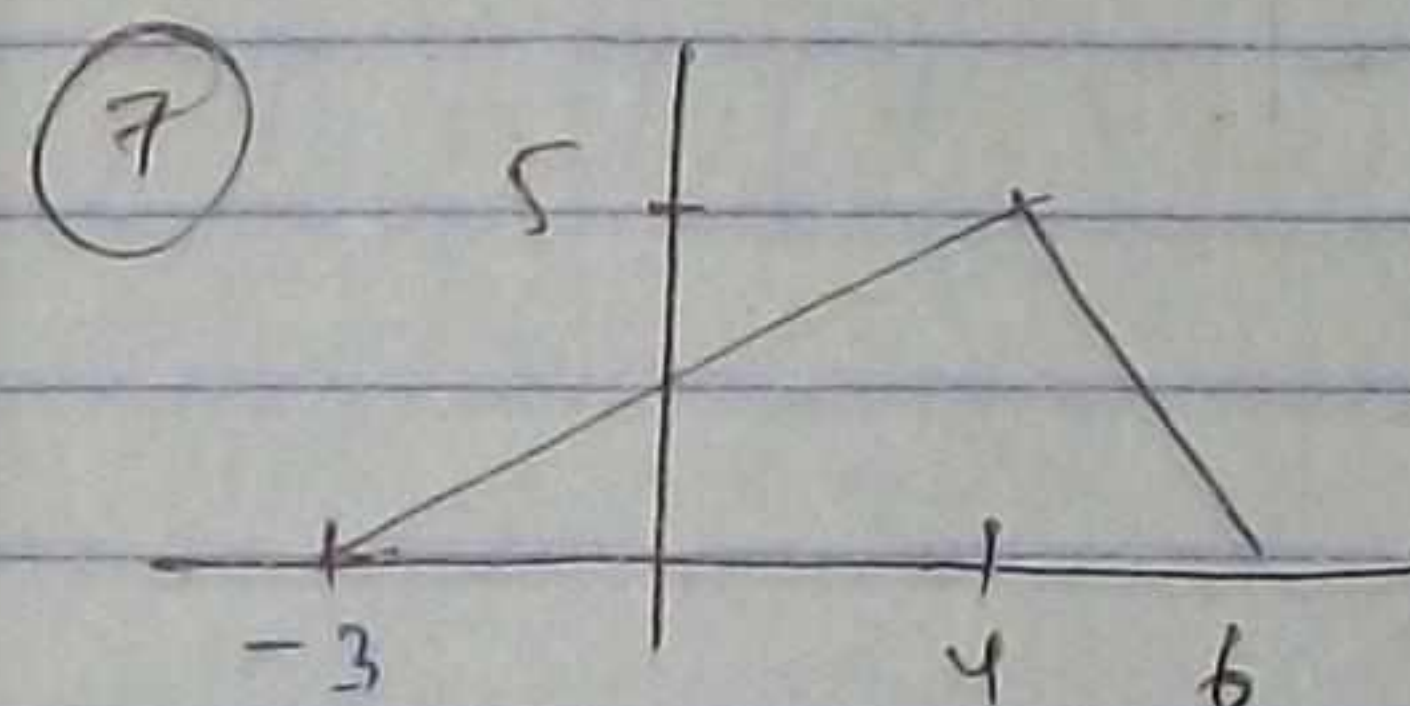
$$5 r(t+4) - 5 r(t+3) - 5 r(t-3) + 5 r(t-4)$$



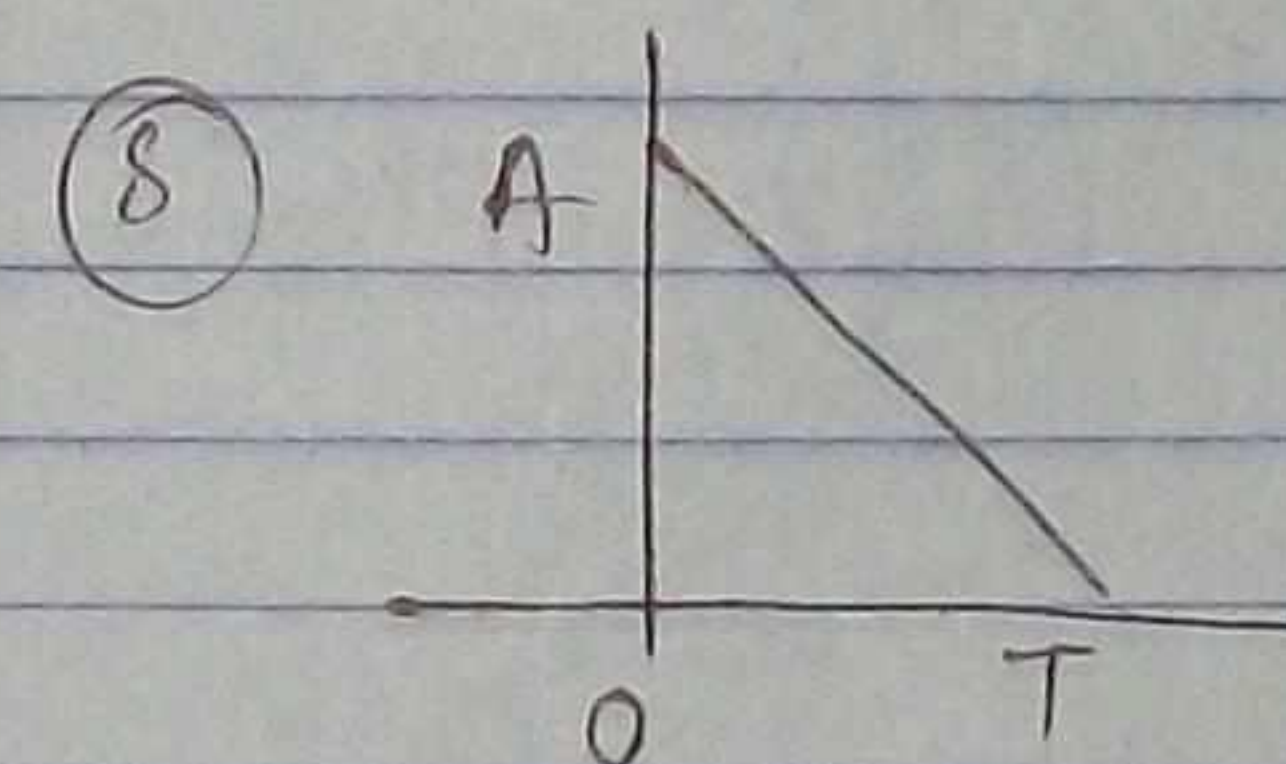
$$\frac{3}{5} r(t) - \frac{3}{5} r(t-5) - \frac{3}{15} r(t-5) + \frac{3}{15} r(t-20)$$



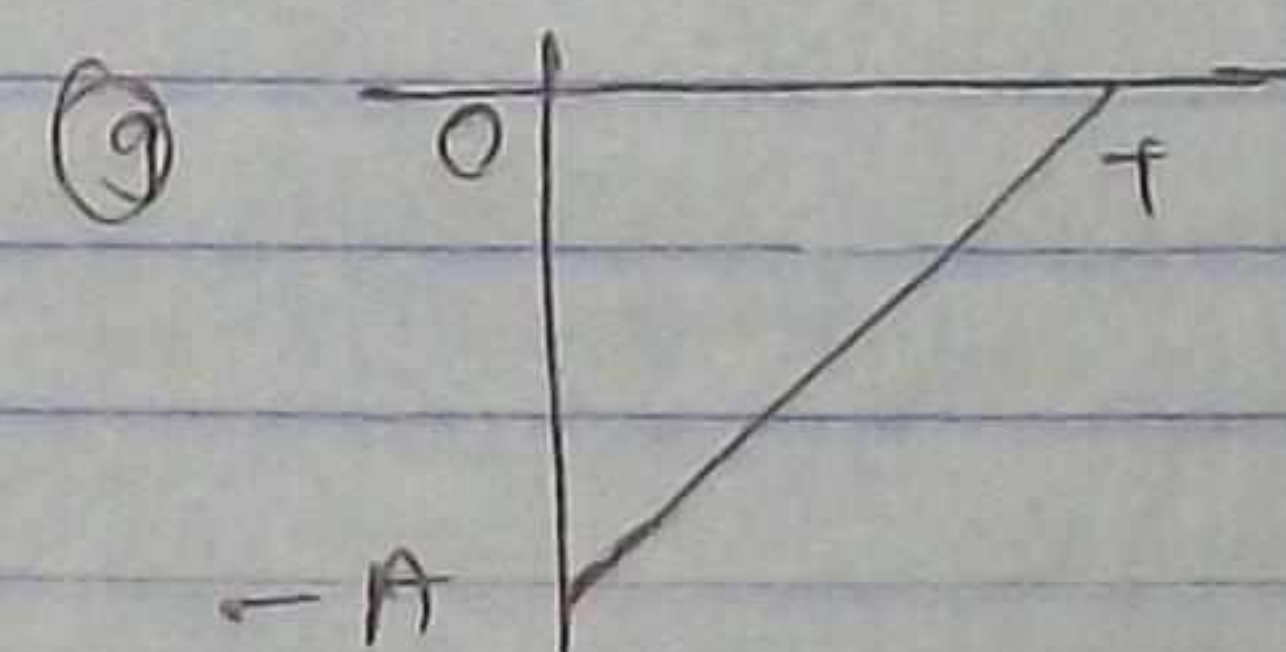
$$-\frac{3}{5} r(t) + \frac{3}{5} r(t-5) + \frac{3}{15} r(t-5) - \frac{3}{15} r(t-20)$$



$$\frac{5}{7} r(t+3) - \frac{5}{7} r(t-4) - \frac{5}{2} r(t-4) + \frac{5}{2} r(t-6)$$



$$-\frac{A}{T} r(t+T) u(-t+T) + A u(t) - \frac{A}{T} r(t) + \frac{A}{T} r(t-T)$$



$$+\frac{A}{T} r(t+T) - A u(t) + \frac{A}{T} r(t) - \frac{A}{T} r(t-T)$$

* (6)

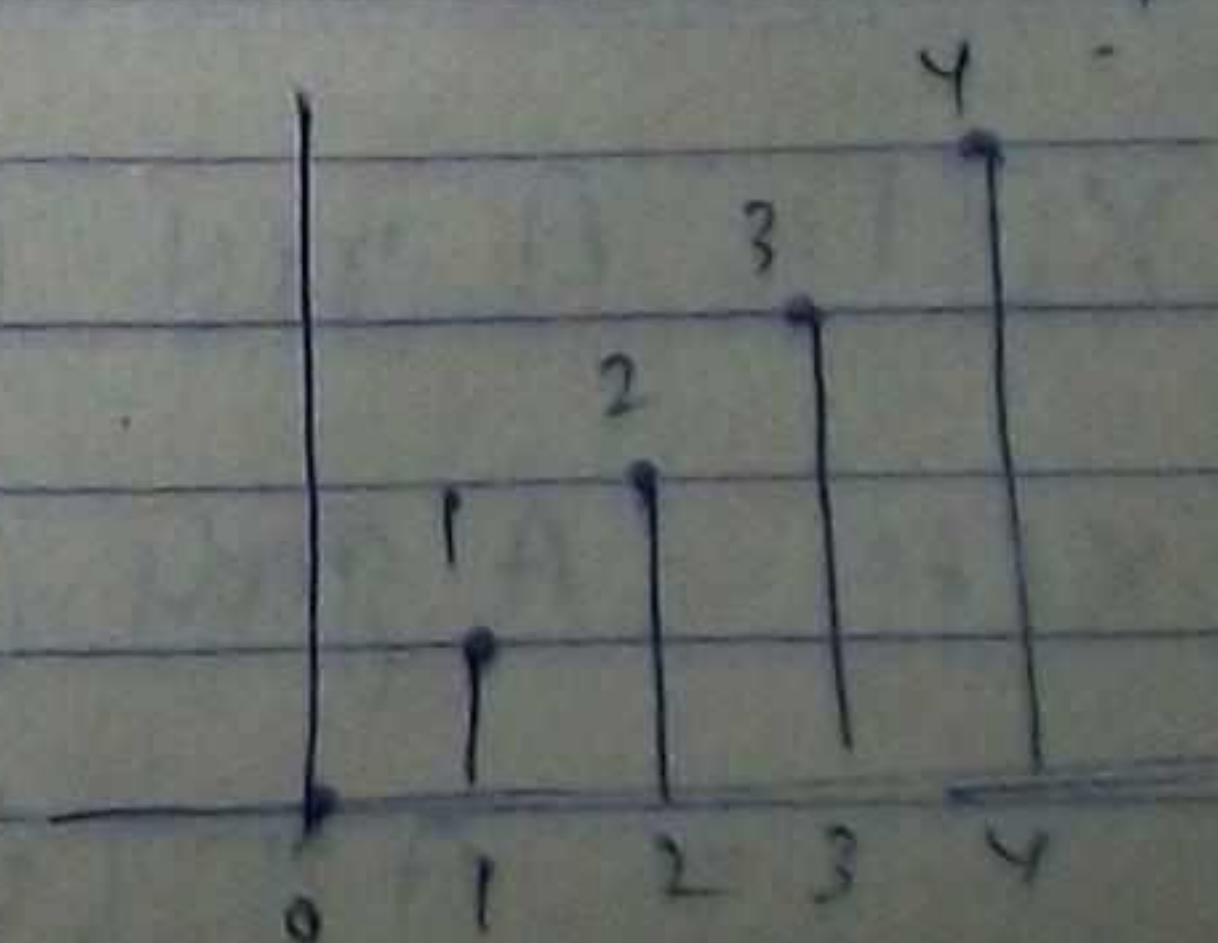
DISCRETE TIME RAMP -

$$r[n] = n \quad \text{for } n \geq 0$$

$$= 0 \quad \text{for } n < 0$$

(or)

$$r(n) = n u(n)$$

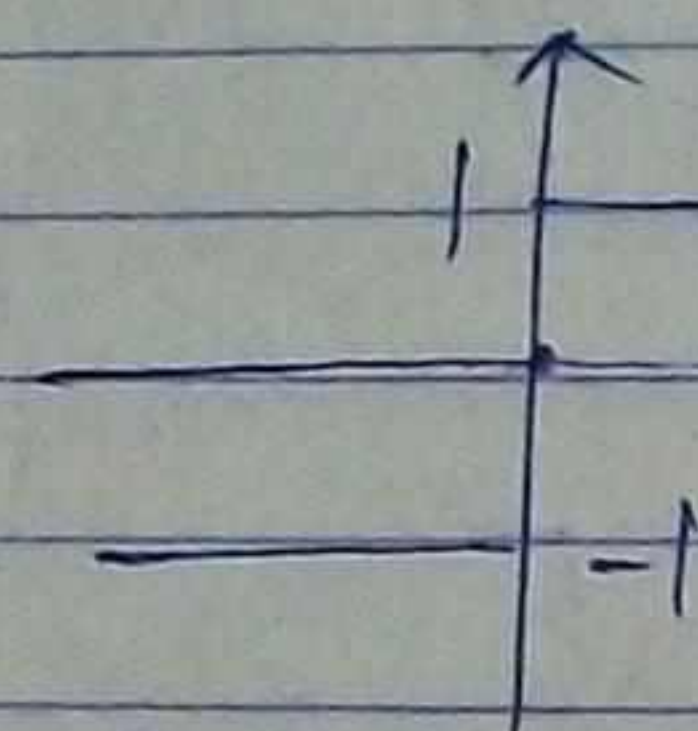


7) SIGNUM FUNCTION -

$$\text{Sgn}(t) = 1 \quad t > 0$$

$$= -1 \quad t < 0$$

$$= \text{undefined} \quad t = 0$$



or

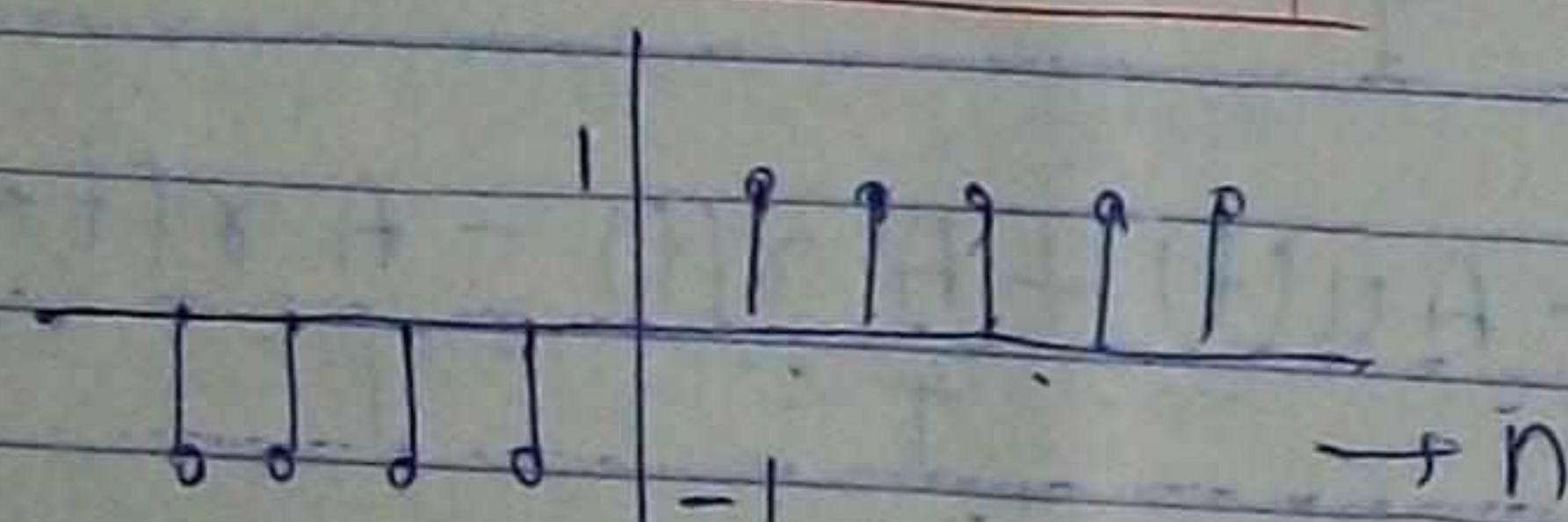
$$\text{Sgn}(t) = u(t) - u(-t)$$

8) SIGNUM FOR DISCRETE TIME -

$$\text{Sgn}(n) = 1 \quad n > 0$$

$$= -1 \quad n < 0$$

$$= 0 \quad n = 0$$



$$\text{Sgn}(n) = u[n] - u[-n]$$

9) RECTANGULAR FUNⁿ - (Gate funⁿ)

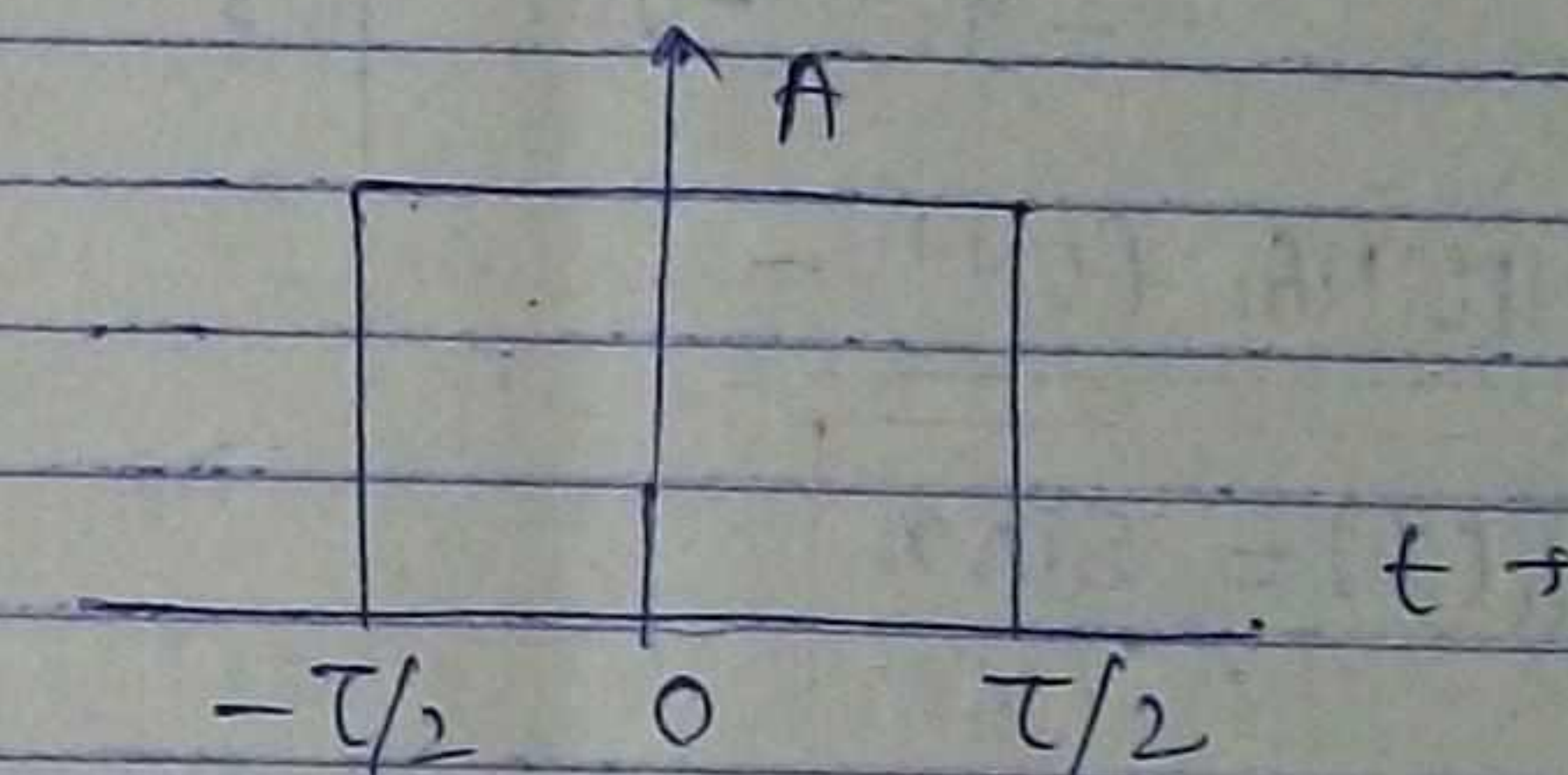
$$x(t) = A \text{ rect} \left(\frac{t-t_0}{\tau} \right)$$

$$x(t) = A \text{ gate} \left(\frac{t-t_0}{\tau} \right)$$

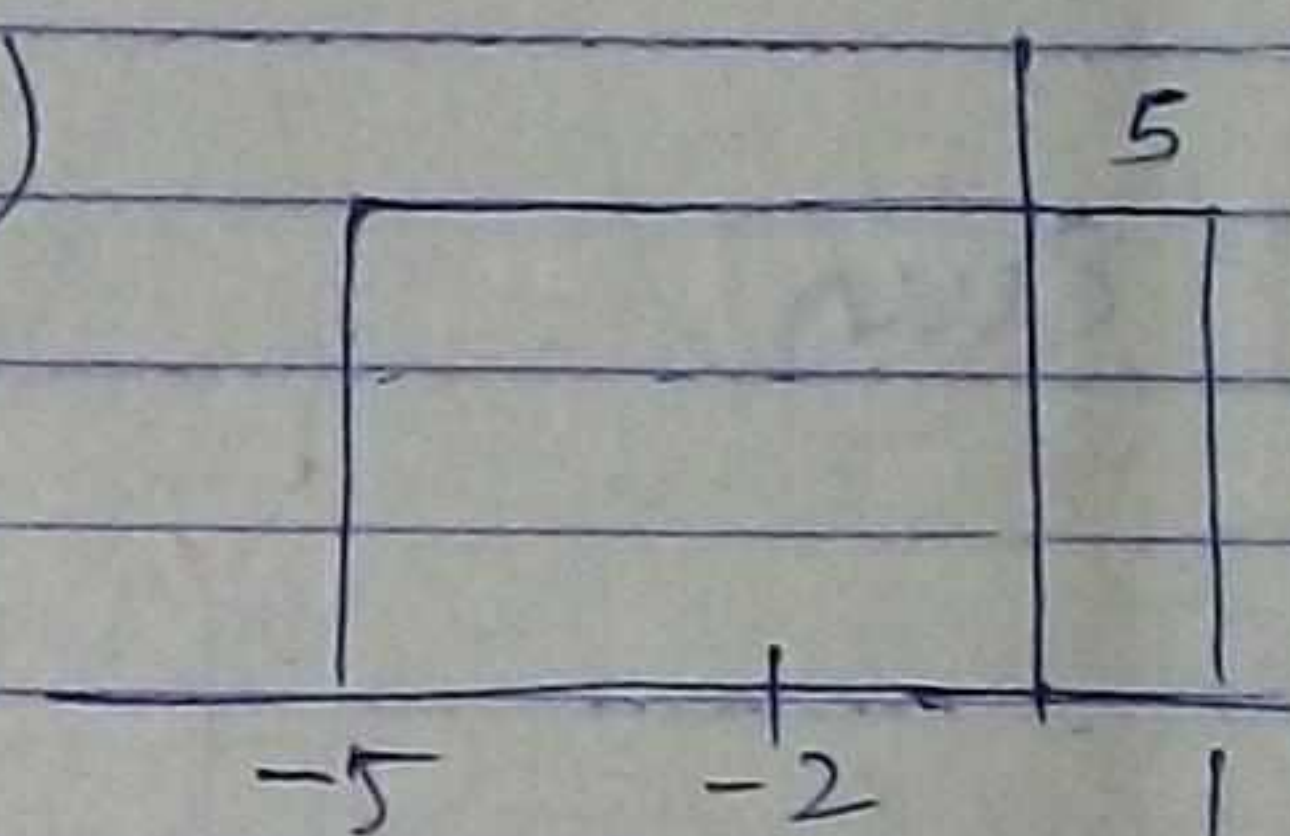
$$= A \pi \left(\frac{t-t_0}{\tau} \right)$$

A: Height
 t_0 : centre of pulse
 τ : width.

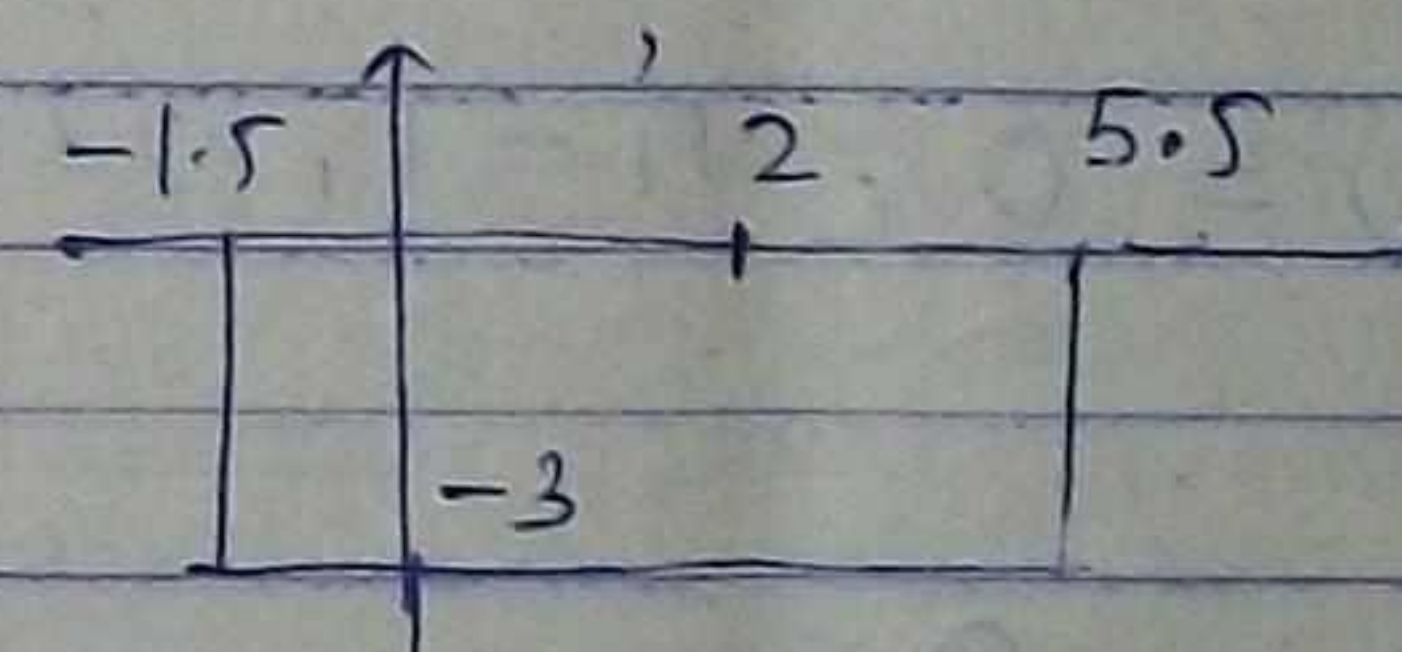
$$A \text{ rect} \left(\frac{t}{\tau} \right)$$



$$5 \text{ gate} \left(\frac{t+2}{6} \right)$$



$$-3 \pi \left(\frac{t-2}{7} \right)$$



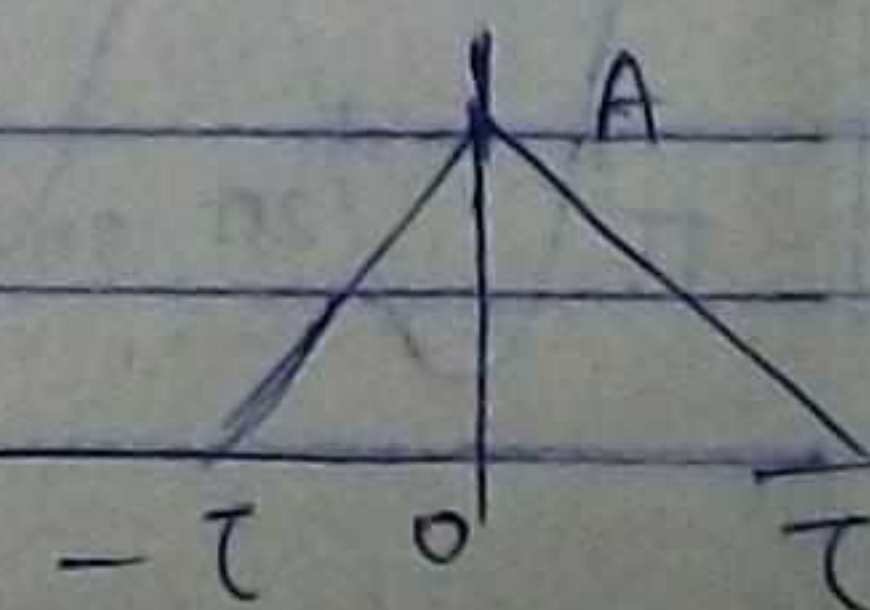
10) TRIANGULAR FUNⁿ -

$$x(t) = A \text{ Tri} \left(\frac{t-t_0}{\tau} \right)$$

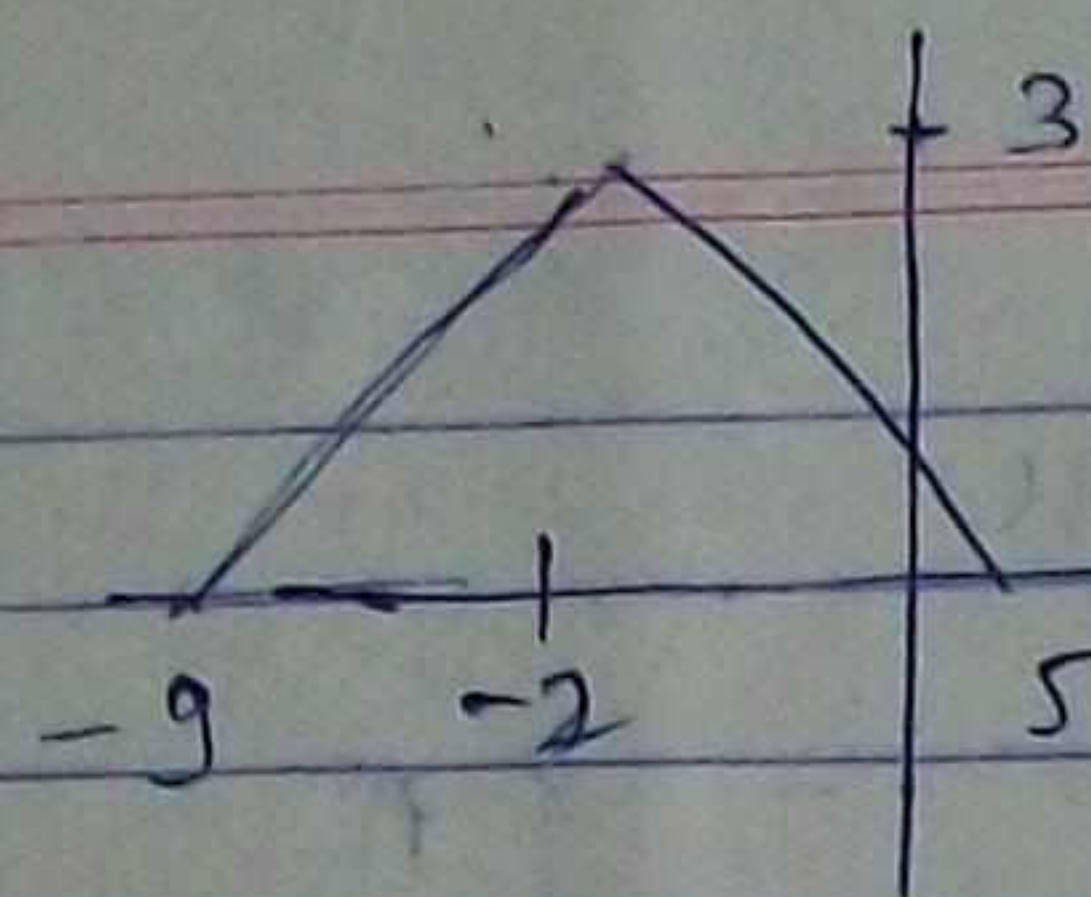
$$= A \Delta \left(\frac{t-t_0}{\tau} \right)$$

A: HEIGHT
 t_0 : CENTRE
 τ : BASE WIDTH / HALF WIDTH

$$A \text{ trig} \left(\frac{t}{\tau} \right)$$



$$3\Delta\left(\frac{t+2}{7}\right)$$



⑪ SAMPLING FUNⁿ -

$$S_a(x) = \frac{\sin x}{x}$$

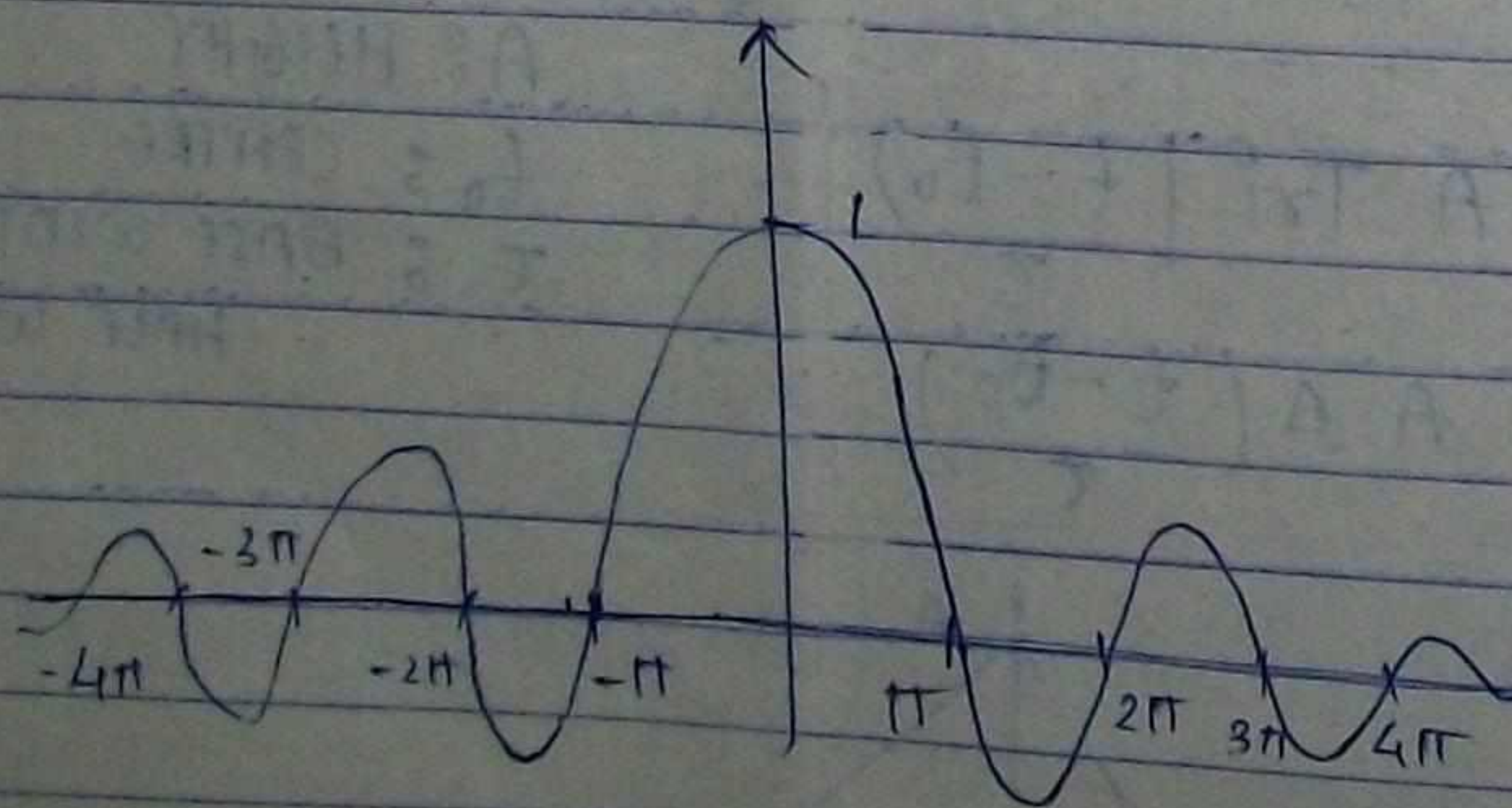
PROPERTIES -

① $S_a(-x) = S_a(x) = \text{Even}$

② $\lim_{x \rightarrow 0} S_a(x) = 1$

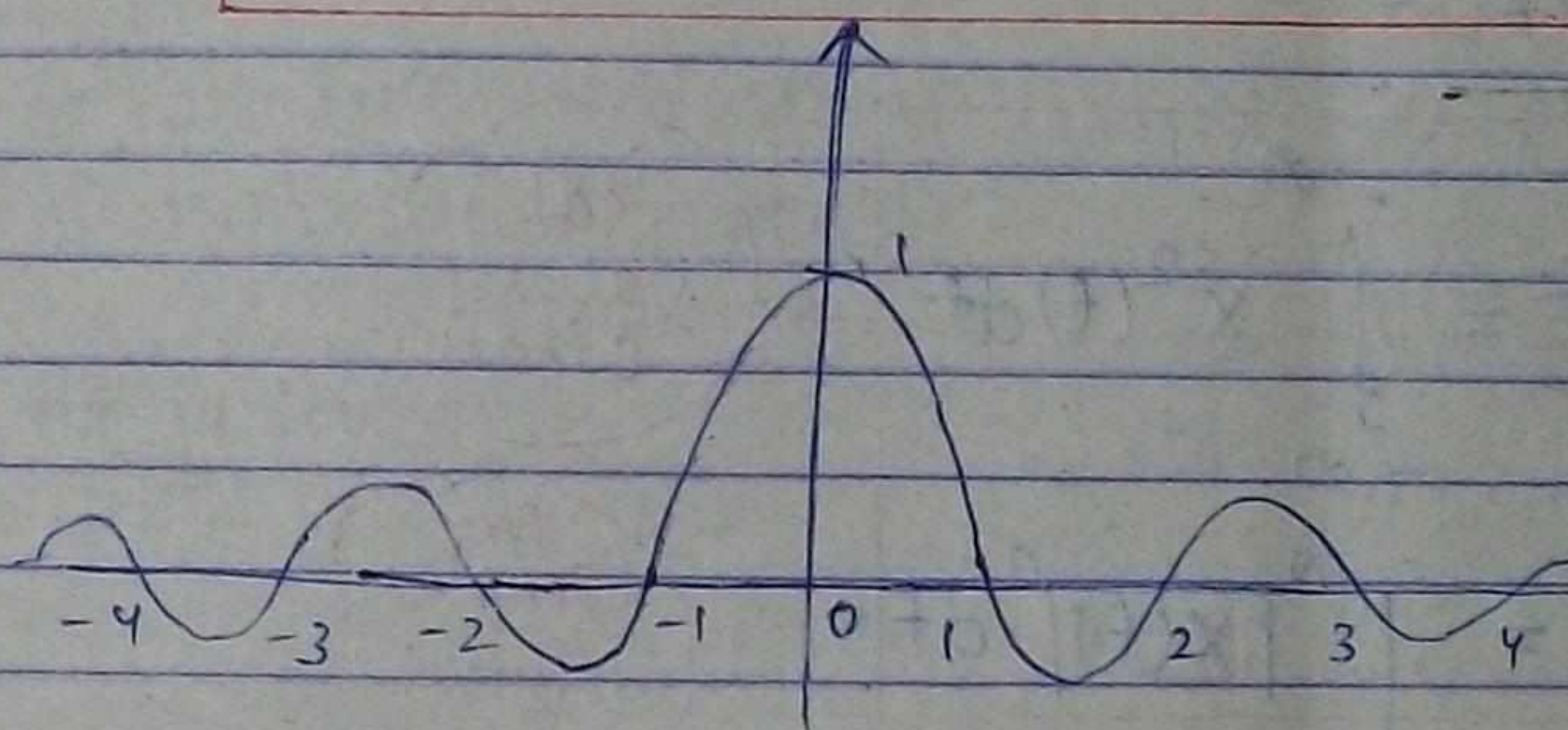
③ $\lim_{x \rightarrow n\pi} S_a(x) = 0, n = 1, 2, 3, \dots$

④ $\lim_{x \rightarrow \infty} S_a(x) = 0$



⑫ SINC FUNCTION -

$$\text{Sinc}(x) = \frac{\sin \pi x}{\pi x} = S_a(\pi x)$$



⇒ AREA OF Sgn FUNⁿ -

for CONTINUOUS
TIME

$$\text{Area} = \int_{-\infty}^{\infty} x(t) dt$$

V-sec
or
A-sec

for DISCRETE
TIME

$$\text{Area} = \sum_{n=-\infty}^{\infty} x(n)$$

V or A

⇒ ABSOLUTE VALUE -

for CONTINUOUS
TIME

$$\text{ABSOLUTE INTEGRAL} = \int_{-\infty}^{\infty} |x(t)| dt$$

V-sec
A-sec

unit of laplace - $\frac{V \cdot \text{sec}}{A \cdot \text{sec}}$

FOR DISCRETE
TIME

$$\text{ABSOLUTE SUMMATION} = \sum_{n=-\infty}^{\infty} |x(n)|$$

⇒ ENERGY OF SIGNAL -

FOR CONTINUOUS
TIME

$$E = \int_{-\infty}^{\infty} x^2(t) dt$$

Real
valued
signal

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

for
Complex
signal

unit → $V^2 \text{ sec}$
or $A^2 \text{ sec}$

FOR DISCRETE
TIME

$$E = \sum_{n=-\infty}^{\infty} x^2(n)$$

REAL
VALUED
SIGNAL

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

COMPLEX
SIGNAL

unit → V^2 or A^2

⇒ POWER -

$$P = \frac{1}{T} \int_T |x(t)|^2 dt$$

FOR PERIODIC
SIGNAL

$$P = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |x(t)|^2 dt$$

NON PERIODIC
SIGNAL

unit → A^2 or V^2 (normalised power $P=1$)

- In communications power represent the normalised power.

FOR DISCRETE
TIME

$$P = \frac{1}{N+1} \sum_{n=0}^N |x(n)|^2$$

FOR
PERIODIC
SIGNAL

$$P = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N |x(n)|^2$$

FOR
NON PERIODIC
SIGNAL

Queso find the energy of the following:

① $e^{-at} u(t)$ ② $e^{at} u(-t)$ ③ $(e^{-at} + e^{-bt}) u(t)$

④ A (d.c) ⑤ $A e^{j\omega_0 t}$ ⑥ $A \sin(\omega t + \theta)$

⑦ $A \cos(\omega t + \theta)$ ⑧ $u(t)$ ⑨ $r(t)$

Soln: ① $E = \int_0^{\infty} (e^{-at})^2 dt = \int_0^{\infty} e^{-2at} dt$
 $= -\frac{1}{2a} [e^{-2a(\infty)} - 1]$

for $a < 0$ $E = \frac{1}{2a} (e^{\infty} - 1) = \infty$
 $a > 0$ $E = \frac{1}{2a} (e^{-\infty} - 1) = -\frac{1}{2a}$

$x = \infty$ $|x| > 1$
 $= 0$ $|x| < 1$
 $= 1$ $x = 1$

$e^{3t} u(t) \rightarrow E = \infty$ $a < 0$ $a = -3$

$e^{-3t} u(t) \rightarrow E = 1/6$ $a > 0$ $a = 3$

(2) $E = \int_{-\infty}^0 (e^{at})^2 dt$
 $= \frac{1}{2a} [1 - e^{-2a\infty}]$

for $a < 0$ $E = \frac{1}{-2a} (1 - e^{\infty}) = \infty$

for $a > 0$ $E = \frac{1}{2a} [1 - e^{-\infty}] = \frac{1}{2a}$

$e^{3t} u(-t) \rightarrow E = 1/6$

$e^{-3t} u(-t) \rightarrow E = \infty$

(3) $(e^{-at} + e^{-bt}) u(t)$

$E = \int_0^{\infty} (e^{-at} + e^{-bt})^2 dt = \int_0^{\infty} (e^{-2at} + e^{-2bt} + 2e^{-(a+b)t}) dt$

$E = \frac{1}{2a} + \frac{1}{2b} + \frac{2}{a+b}$ $a, b > 0$

$e^{-3t} u(t) + e^{5t} u(t) \Rightarrow (e^{-3t} + e^{5t}) u(t) \rightarrow (E = \infty)$

$e^{-3t} u(t) + e^{-5t} u(t) \rightarrow \frac{1}{6} + \frac{1}{10} + \frac{2}{8} = E$

(4) $E = \int_{-\infty}^{\infty} A^2 dt = \infty$

(5) $E = \int_{-\infty}^{\infty} |A e^{j\omega t}|^2 dt = \int_{-\infty}^{\infty} A^2 dt = \infty$

(6) $E = \int_{-\infty}^{\infty} |A \sin(\omega t + \theta)|^2 dt$
 $= \int_{-\infty}^{\infty} \frac{A^2}{2} (1 - \cos(2\omega t + 2\theta)) dt$
 $= \frac{A^2}{2} \int_{-\infty}^{\infty} dt - \frac{A^2}{2} \int_{-\infty}^{\infty} \cos(2\omega t + 2\theta) dt$
 $= \infty$

11/11/11

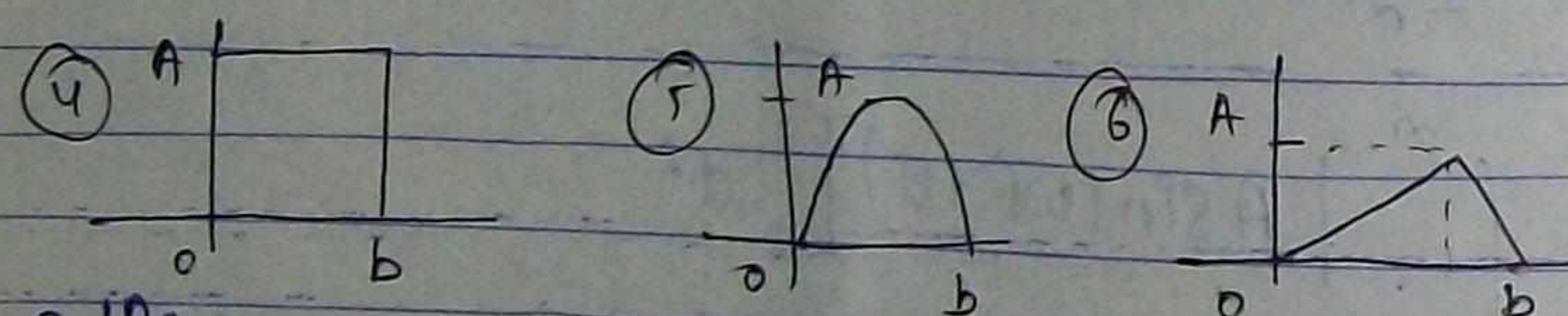
$$(7) E = \int_{-\infty}^{\infty} |A \cos(\omega t + \theta)|^2 dt = \underline{\underline{\infty}}$$

$$(8) E = \int_0^{\infty} 1^2 dt = \underline{\underline{\infty}}$$

$$(9) E = \int_0^{\infty} t^2 dt = \underline{\underline{\infty}}$$

Ques. find the K:

(1) $3e^{-5t}$ (2) -4 (3) $\sin(4t - 30^\circ)$



Soln:

(1) $3e^{-5t} \rightarrow E = \frac{9}{2(5)} = \frac{9}{10}$

(2) $E = \infty$ (dc)

(3) $\sin(4t - 30^\circ) = \infty$

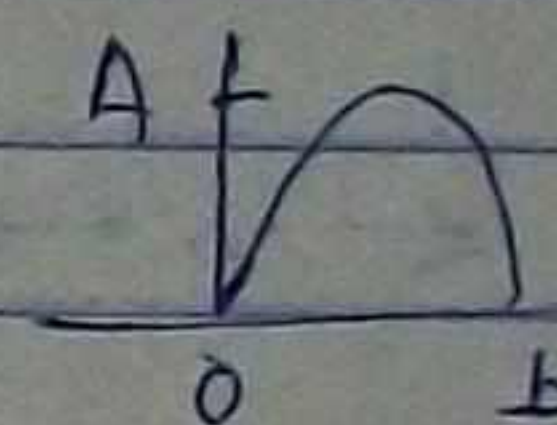
AA

(4) $x(t) = A$, for $0 < t < b$
 $= 0$, otherwise

$$E = \int_0^b A^2 dt = \underline{\underline{A^2 b}}$$

AA

(5) $x(t) = (T=2b) \quad \omega = \frac{2\pi}{T} = \frac{\pi}{b}$



$$x(t) = A \sin\left(\frac{\pi}{b} t\right) \quad 0 < t < b$$

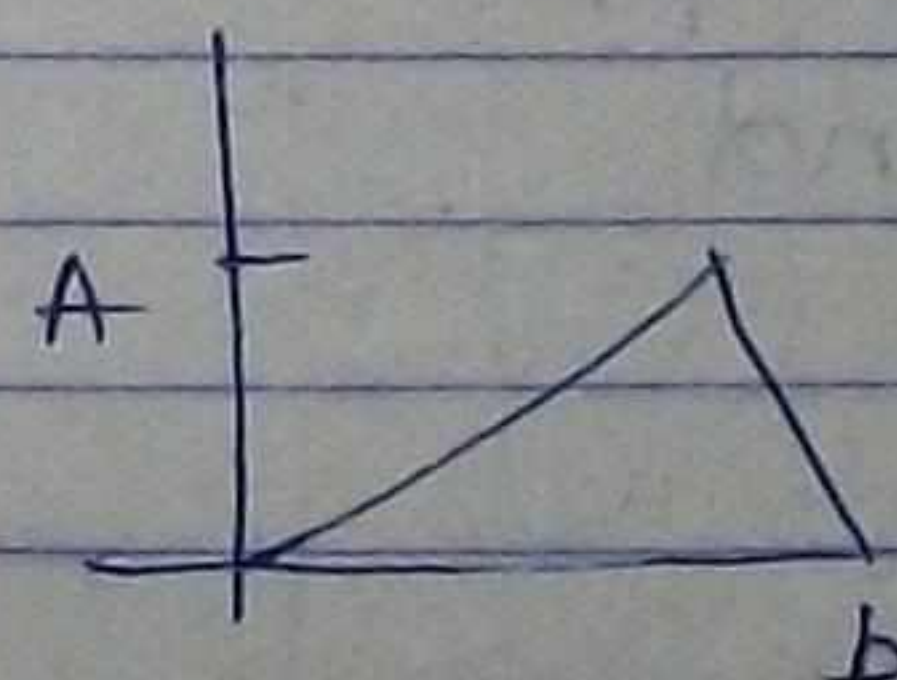
$$= 0 \quad \text{otherwise.}$$

$$E = \int_0^b A^2 \sin^2\left(\frac{\pi}{b} t\right) dt = \int_0^b \frac{A^2}{2} dt - \int_0^b \frac{A^2}{2} \cos\left(\frac{2\pi}{b} t\right) dt$$

$$E = \underline{\underline{\frac{A^2 b}{2}}}$$

AA

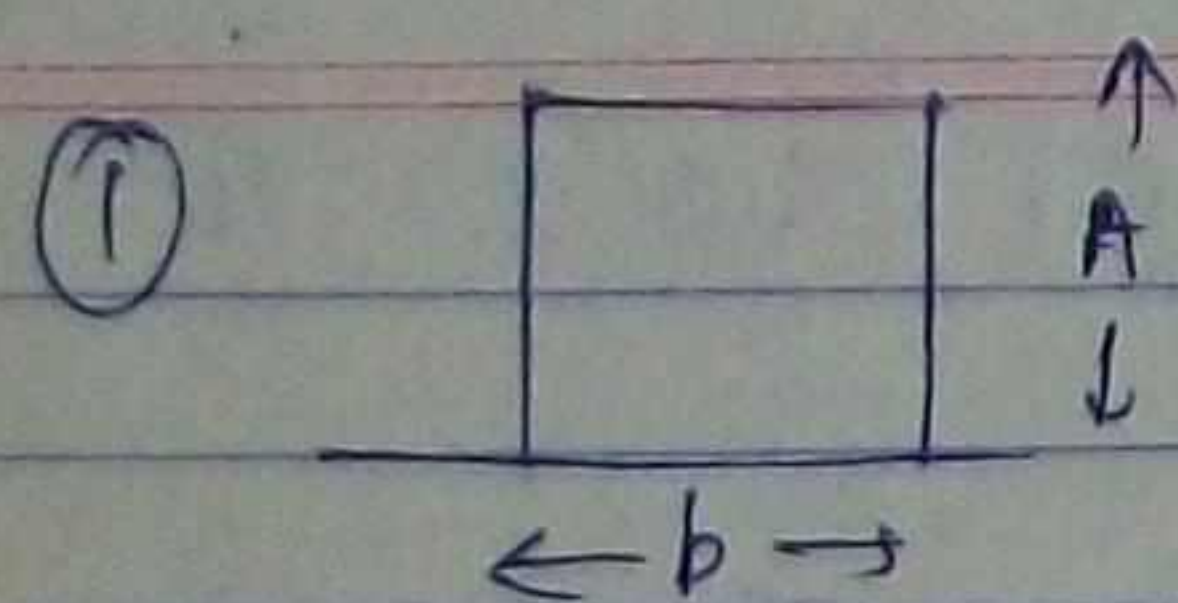
(6)



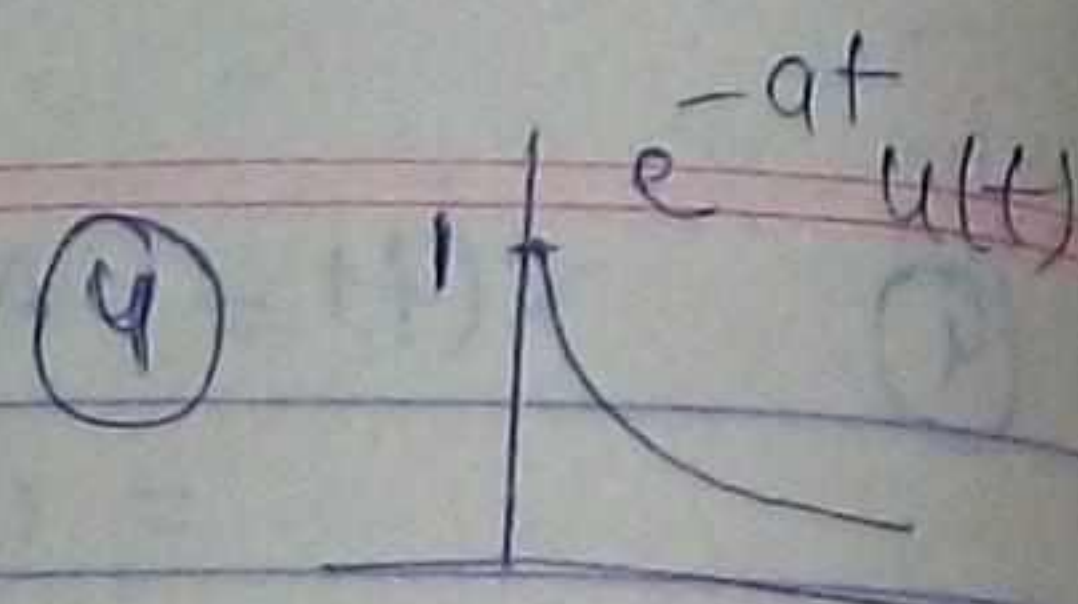
$$E = A^2 \cdot \frac{b}{3}$$

- In general

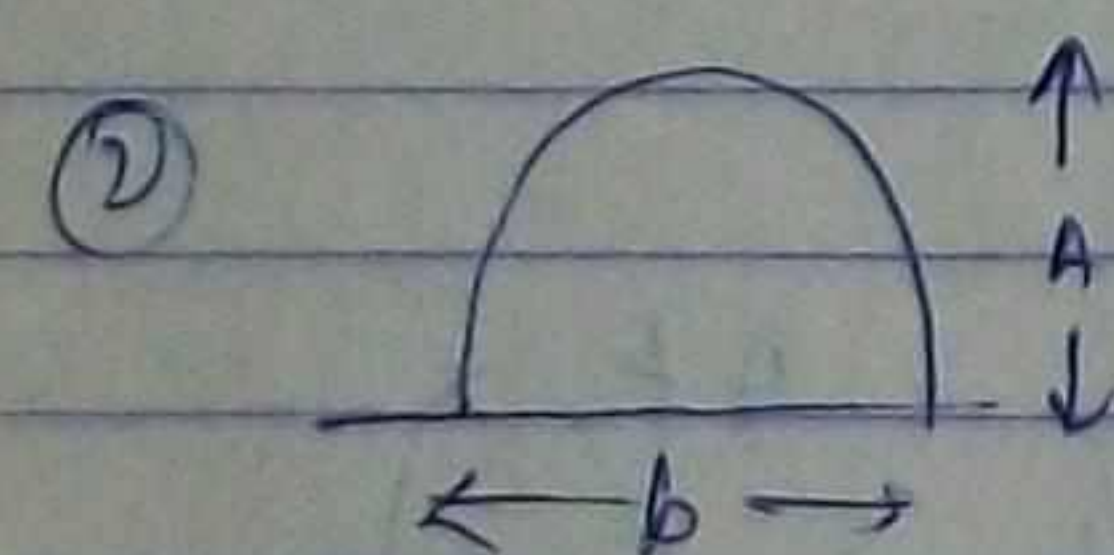
Energy of signal = (HEIGHT)² x WIDTH



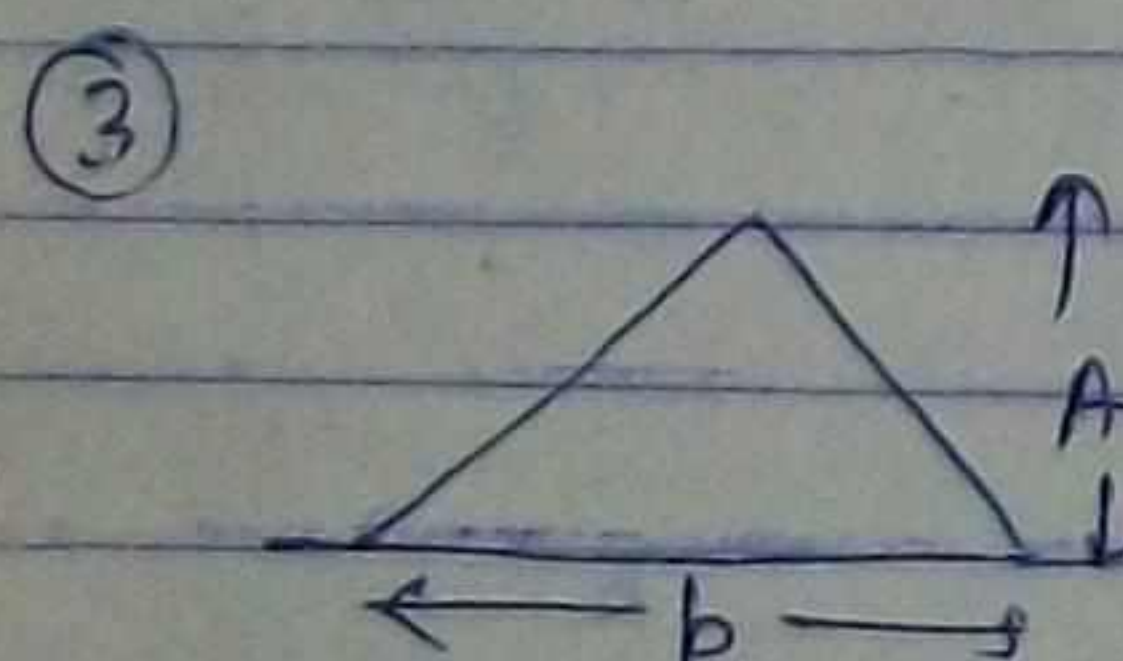
$$E = A^2 \cdot b$$



$$E = \frac{1}{2a}$$



$$E = A^2 \cdot \frac{b}{2}$$

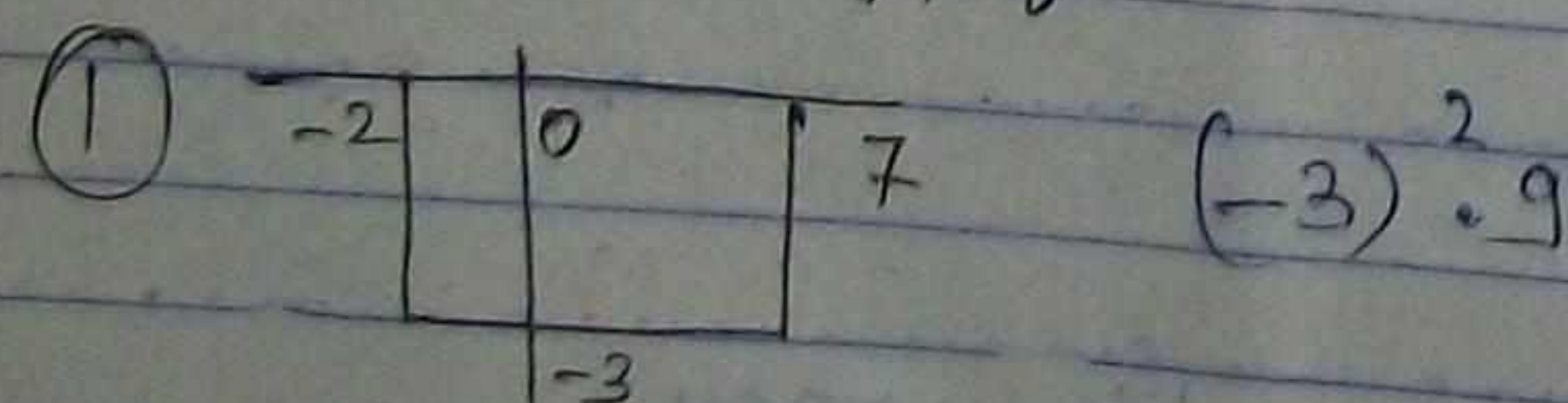


$$E = A^2 \cdot \frac{b}{3}$$

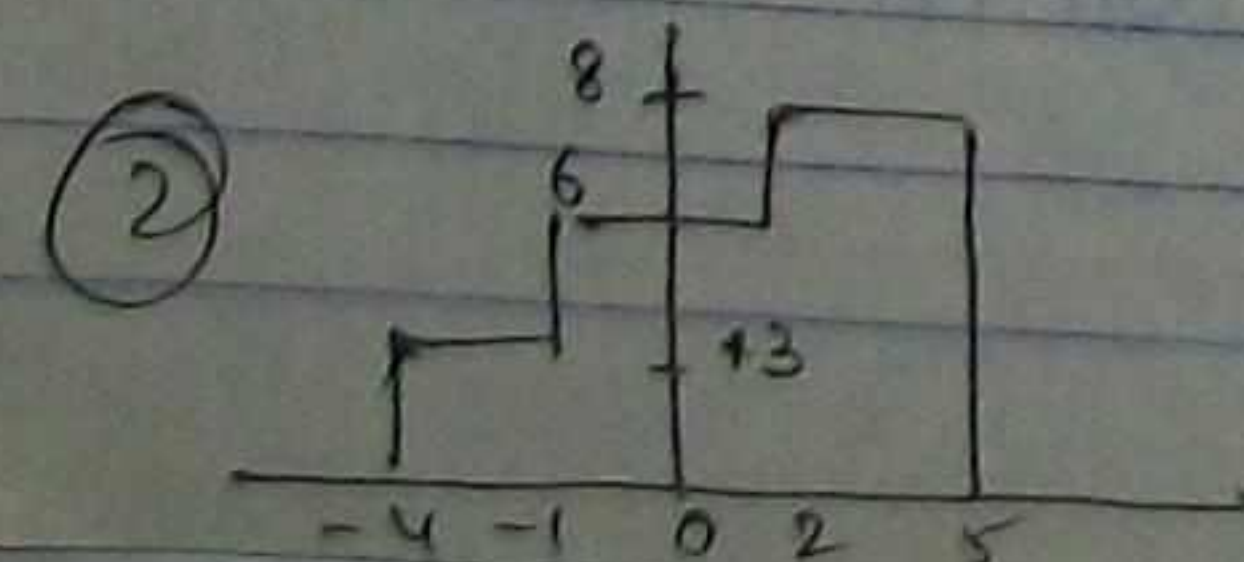
NOTE -

- Energy is always positive.
- Energy of the signal is directly prop. to the square of the amplitude and width of the signal.

Ques. Find Energy of

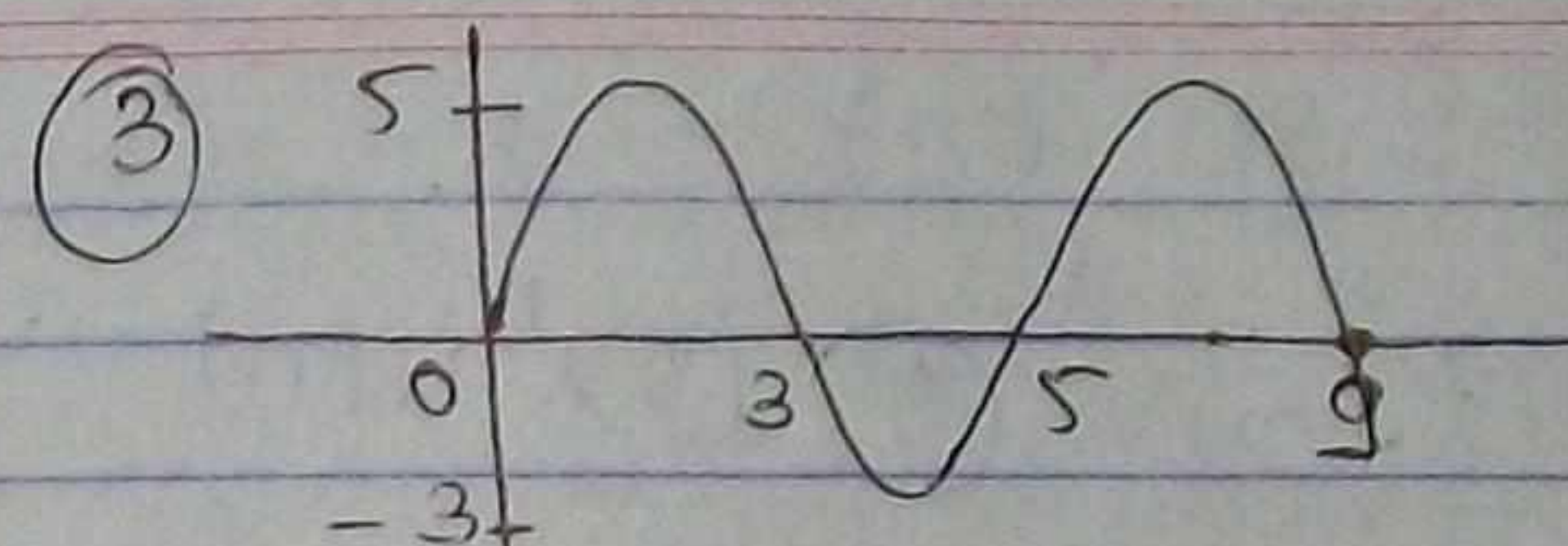


$$(-3)^2 \cdot 9$$

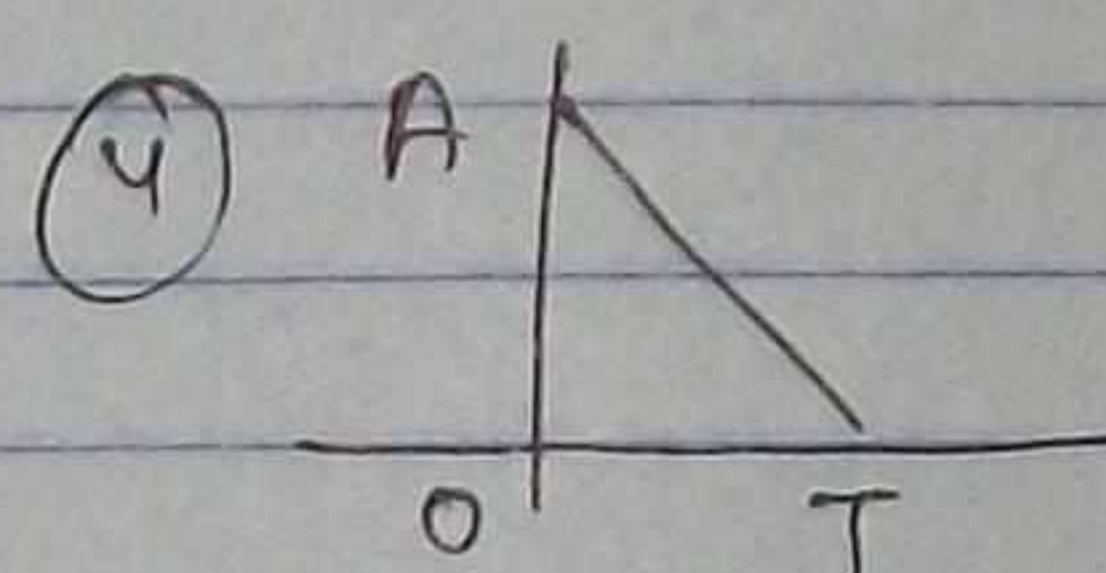


$$E = 3^2 \cdot 3 + 6^2 \cdot 3 + 8^2 \cdot 3$$

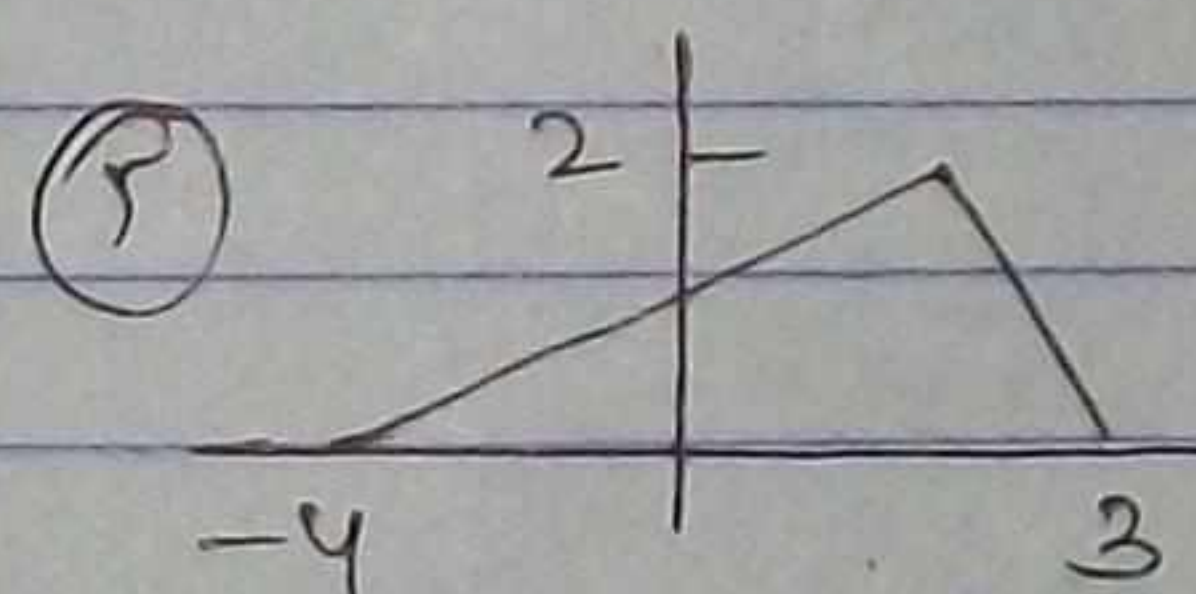
for periodic signal $E \rightarrow \infty$ DC SIGNAL $E \rightarrow \infty$



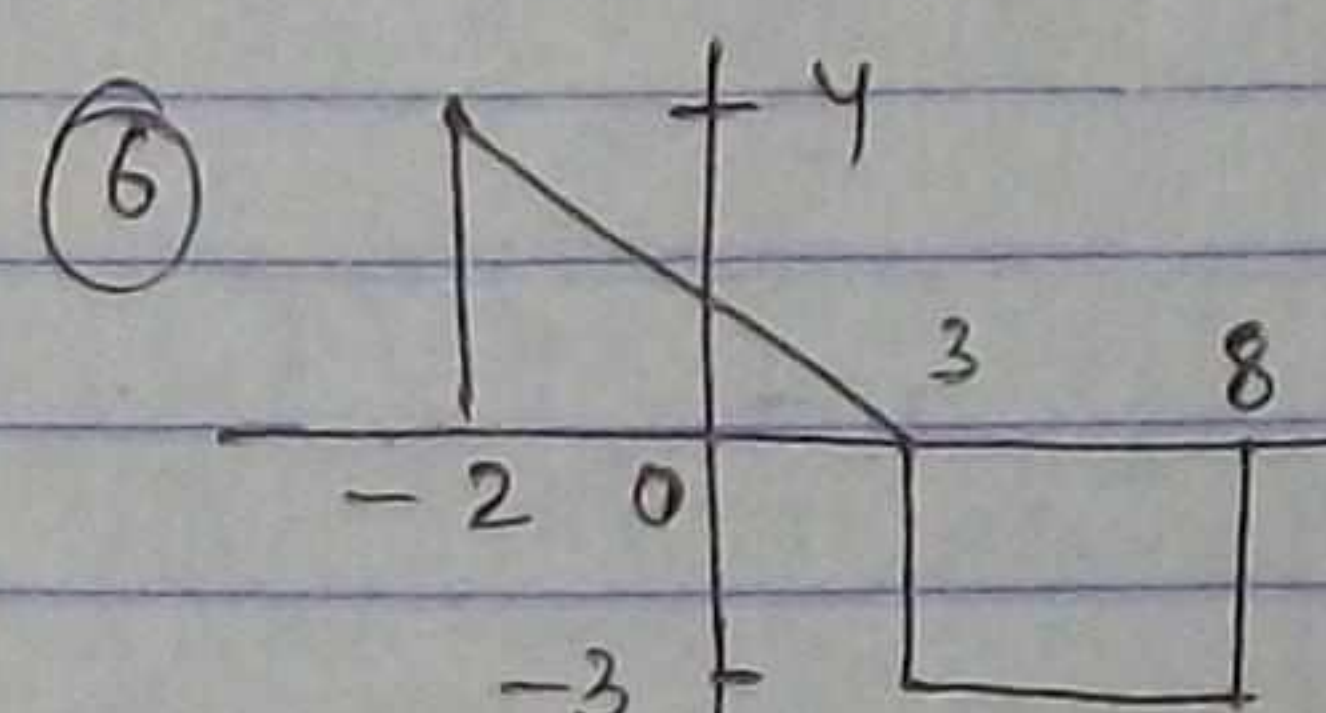
$$E = 5^2 \cdot \frac{3}{2} + 3^2 \cdot \frac{2}{2} + 5^2 \cdot \frac{4}{2}$$



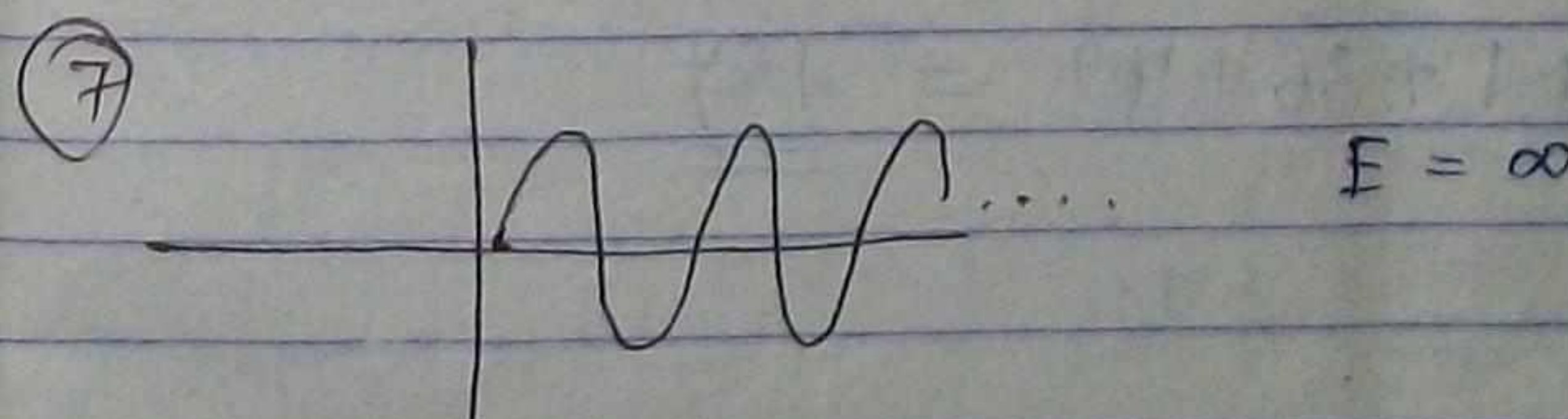
$$E = A^2 \cdot \frac{T}{3}$$



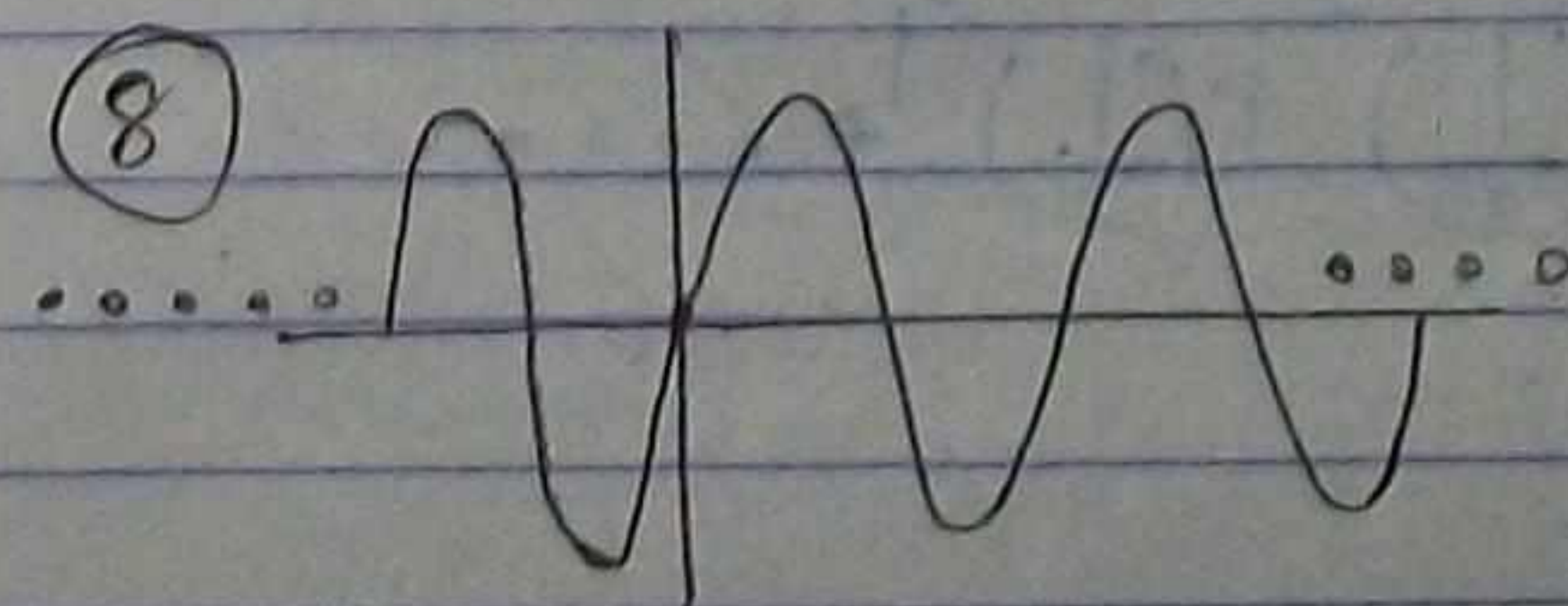
$$E = 2^2 \cdot \frac{7}{3}$$



$$E = 4^2 \cdot \frac{5}{3} + (-3)^2 \cdot 5$$



$$E = \infty$$



$$E = \infty$$

Ques. find Energy of:

① $x(n) = \{-3, 5, 2, 1, 6, 7\}$ ⑦ $(5)^n u(n)$

② $(\frac{1}{2})^n u(n)$ ⑤ $u(n)$

③ $a^n u(n)$ ⑥ $\text{sgn}(n)$

④ $[(\frac{1}{3})^n + (\frac{1}{5})^n] u(n)$

Soln: $E = \sum_{n=-\infty}^{\infty} |x(n)|^2$

① $E = \sum_{n=-2}^3 |x(n)|^2$

$= 9 + 25 + 4 + 1 + 36 + 49 = \underline{134}$

② $(\frac{1}{2})^n u(n)$

$E = \sum_{n=0}^{\infty} (\frac{1}{2})^{2n} = 1 + (\frac{1}{2})^2 + (\frac{1}{2})^4 + \dots$

$= 1 + \frac{1}{4} + (\frac{1}{4})^2 + \dots$

$\frac{a}{1-r} = \frac{1}{1-1/4} = \frac{4}{3}$

$\frac{a}{1-r}$

$r < 1$

③ $x = (5)^n u(n)$

$25 > 1$

$E = \sum_{n=0}^{\infty} (5)^{2n} = 1 + 25 + (25)^2 + \dots = \infty$

④ $a^n u(n)$

$E = \sum_{n=0}^{\infty} a^{2n} = 1 + a^2 + (a^2)^2 + \dots$

$E = \frac{1}{1-a^2} \quad |a^2| < 1$

$= \infty \quad |a^2| > 1$

⑤ $(\frac{1}{3})^n u(n) + (\frac{1}{5})^n u(n)$

$E \neq \frac{1}{1-(1/3)^2} + \frac{1}{1-(1/5)^2} \neq \frac{1}{1-1/9}$

$[(\frac{1}{3})^n + (\frac{1}{5})^n] u(n)$

$E = \sum_{n=0}^{\infty} \left[(\frac{1}{9})^n + (\frac{1}{25})^n + 2(\frac{1}{15})^n \right]$

$= \frac{1}{1-1/9} + \frac{1}{1-1/25} + \frac{2}{1-1/15} = \frac{15}{4}$

$$(6) u(n) =$$

$$E = \sum_{n=0}^{\infty} 1^2 = 1+1+1+\dots = \infty$$

$$(7) \text{sgn}$$

$$E = \sum_{n=0}^{\infty} 1^2 + \sum_{n=-\infty}^0 (-1)^2 = \infty$$

$$(8) x(n)$$

$$E = \sum_{n=0}^{\infty} n^2 = 1+2^2+3^2+\dots = \infty$$

$$(9) \left(-\frac{1}{2}\right)^n u(n)$$

$$E = \sum_{n=-\infty}^{\infty} \left(\frac{1}{4}\right)^n = 1 + \frac{1}{4} + \left(\frac{1}{4}\right)^2 + \dots$$

$$= \frac{1}{1-1/4} = \frac{4}{3}$$

$$(10) 5^n u(-n)$$

$$E = \sum_{n=-\infty}^0 25^n = 0 + \frac{1}{25} + \left(\frac{1}{25}\right)^2 + \dots = \frac{25}{24}$$

$$E = \sum_{n=0}^3 |x(n)|^2$$

$$(11) x(n) = \{10, (-2+2j), -2, (-2-2j)\}$$

$$E = 10^2 + |(-2+2j)|^2 + 2^2 + |(-2-2j)|^2$$

$$= 100 + 4 + 4 + 4 + 4 + 4 = 120$$

(12) Q. If the Energy of $x(t)$ is 8 unit then Energy of following:

$$(1) x(t-2) \rightarrow 8$$

$$(2) x(t+1) \rightarrow 8$$

$$(3) x(-t) \rightarrow 8$$

$$(4) x(3t) \rightarrow 8/3$$

$$(5) x(-t/0.2) \rightarrow 8 \times 0.2$$

$$(6) x(-3t+2) \rightarrow 8/3$$

$$(7) 1/5 x(0.2t) \rightarrow \frac{1}{25} \times \frac{8}{0.2}$$

$$(8) 3-x(t) \rightarrow \infty$$

$$(9) x\left(\frac{3-4t}{5}\right) \rightarrow \frac{8}{4/5}$$

AA

$$E \propto \text{WIDTH}$$

$$E \propto (\text{Amplitude})^2$$

Ques find power:

$$\begin{aligned} x(t) &= A \cos(\omega t + \theta) \\ &= A \sin(\omega t + \theta) \\ &= A e^{j(\omega t + \theta)} \end{aligned} \left\{ \begin{array}{l} \text{Periodic} \\ \text{with} \\ \text{period } T \end{array} \right.$$

$$= A(\text{dc})$$

$$(1) P = \frac{1}{T} \int_T |x(t)|^2 dt$$

$$P = \frac{1}{T} \int_T A^2 \cos^2(\omega t + \theta) dt$$

$$= \frac{1}{T} \int_T \frac{A^2}{2} dt + \underbrace{\frac{1}{T} \int_T \frac{A^2}{2} \cos(2\omega t + 2\theta) dt}_{=0}$$

$$= \frac{1}{T} \cdot \frac{A^2}{2} \cdot T$$

$$= \frac{A^2}{2}$$

for $A \sin \theta$
 $A \cos \theta$

$$P = A^2/2$$

(*)

$$(3) P = \frac{1}{T} \int_T |A e^{j(\omega t + \theta)}|^2 dt$$

$$= \frac{1}{T} \cdot A^2 \cdot T = \underline{\underline{A^2}}$$

(*)

$$(4) A \rightarrow \text{dc}$$

$$P = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |x(t)|^2 dt$$

$$= \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |A^2| dt$$

$$= \lim_{T \rightarrow \infty} \frac{2T A^2}{2T} = \underline{\underline{A^2}}$$

(*)

$$(5) u(t)$$

$$P = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T 1 dt = \frac{1}{2}$$

for $A u(t) \rightarrow A^2/2$

(*)

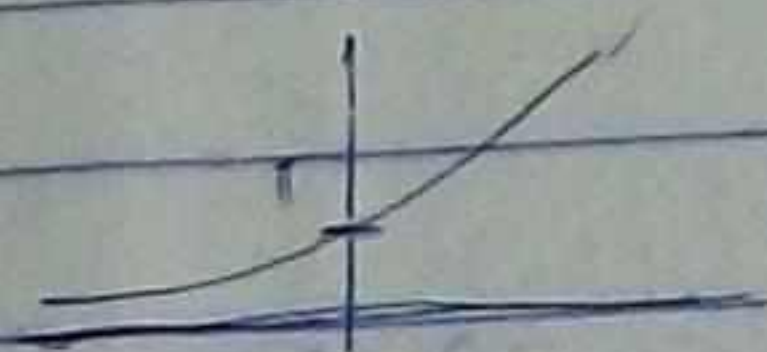
$$(6) x(t)$$

$$P = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T t dt$$

$$= \lim_{T \rightarrow \infty} \frac{1}{2T} \left(\frac{T^3}{3} \right) = \lim_{T \rightarrow \infty} \frac{T^2}{6} = \infty$$

④ $x(t) = e^{at} u(t)$ for $a > 0$

PROB



⑤ $x(t) = e^{at} u(t)$ for $a < 0$

Solⁿ:

$$P = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_0^{\infty} e^{2at} dt$$

$$= \lim_{T \rightarrow \infty} \frac{1}{4aT} [e^{2a(\infty)} - e^{(0)}]$$

= undefined for $a > 0$

= 0 for $a < 0$

NOTE -

- power is always +ve.
- power is independent of freq. and phase. It depends only on amplitude.
- ~~power is linear~~. Total power associated with signal is equal to sum of the power associated with the individual frequencies.

FOURIER
TRIGONOMETRIC
SERIES

ex ① $x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)$

$$P_{\text{TOTAL}} = a_0^2 + \sum_{n=1}^{\infty} \frac{a_n^2 + b_n^2}{2}$$

② $x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$

EXPONENTIAL
FOURIER SERIES

$$P_{\text{TOTAL}} = \sum_{n=-\infty}^{\infty} |C_n|^2$$

Ques. find the power of:

① $-4 \rightarrow P = 16$

② $(3-4j)e^{-j5t} \rightarrow P = 25$

③ $5 \cos(3t - 45^\circ) \rightarrow P = 5^2/2$

④ $3 \sin^2(\frac{\pi}{5}t) \rightarrow \frac{3}{2} - \frac{3}{2} \cos(\frac{2\pi}{5}t) = \left(\frac{3}{2}\right)^2 + \left(\frac{3}{2}\right)^2$

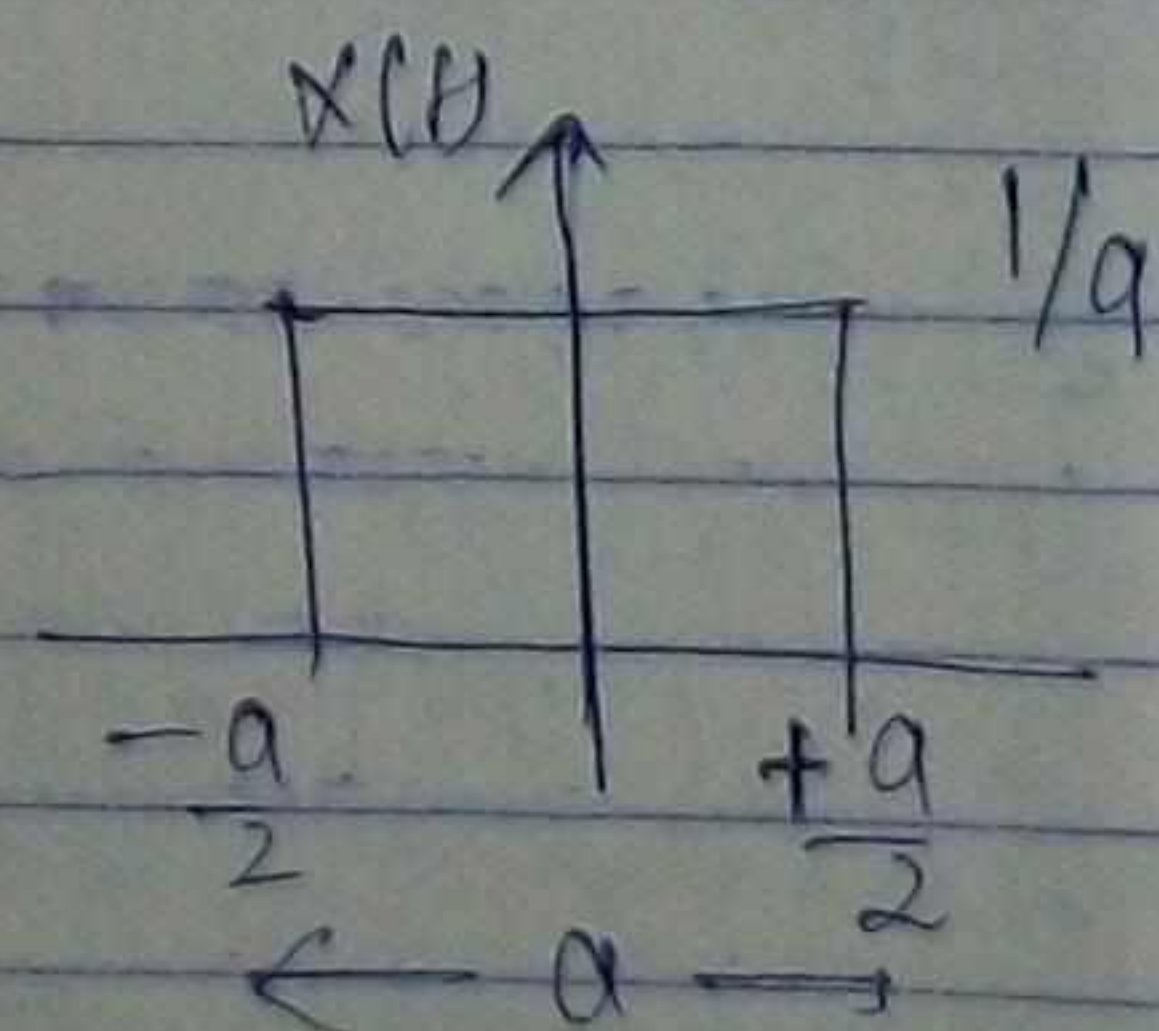
⑤ $\cos(3t) \sin(5t) \rightarrow \frac{1}{2} (\sin 2t) + \frac{1}{2} \sin 8t$
 $= \left(\frac{1}{2}\right)^2 \cdot \frac{1}{2} + \left(\frac{1}{2}\right)^2 \cdot \frac{1}{2} = 1/4$

⑥ $\left[\sin(\frac{\pi}{5}t) + \cos(3\pi t)\right]^2 \rightarrow \sin^2(\frac{\pi}{5}t) + \cos^2(3\pi t) + 2\sin(\frac{\pi}{5}t)\cos(3\pi t)$

$$\begin{aligned}
 &= \frac{1}{2} - \frac{1}{2} \cos(t) + \frac{1}{2} + \frac{1}{2} (\cos(t) + \sin(t) + \sin(t)) \\
 &\Rightarrow \left(\frac{1}{2}\right)^2 \frac{1}{8} + \left(\frac{1}{2}\right)^2 \frac{1}{2} + \left(\frac{1}{2}\right)^2 \frac{1}{8} + \left(\frac{1}{2}\right)^2 \frac{1}{2} + \left(\frac{1}{2}\right)^2 \frac{1}{2} + \left(\frac{1}{2}\right)^2 \frac{1}{2} \\
 &= 2\left(\frac{1}{2}\right)^2 = 4/4 = 1
 \end{aligned}$$

⇒ DERIVATIVE OF IMPULSE - $(\delta'(t))$

- If any funⁿ is singular then its derivative and integral can't be determine directly.
- Since impulse is singular funⁿ its derivative can't be determine directly.



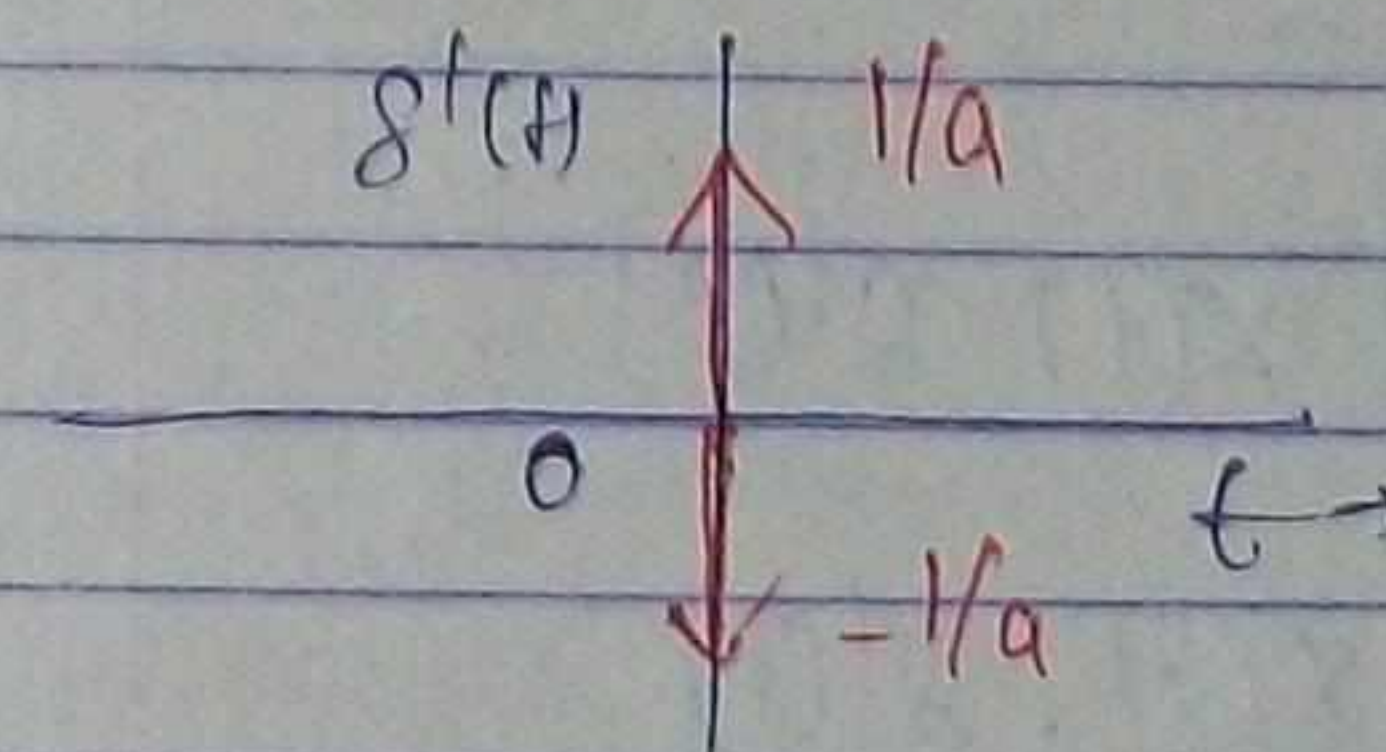
$$x(t) = \frac{1}{a} u\left(t + \frac{a}{2}\right) - \frac{1}{a} u\left(t - \frac{a}{2}\right)$$

$$\lim_{a \rightarrow 0} x(t) = \delta(t)$$

$$\delta'(t) = \lim_{a \rightarrow 0} x'(t)$$

$$= \lim_{a \rightarrow 0} \left[\frac{1}{a} u'(t + a/2) - \frac{1}{a} u'(t - a/2) \right]$$

$$\delta'(t) = \lim_{a \rightarrow 0} \left[\frac{1}{a} \delta(t + a/2) - \frac{1}{a} \delta(t - a/2) \right]$$



$\delta'(t) \rightarrow$ doublet

$\delta''(t) \rightarrow$ quadruplet

$\delta'''(t) \rightarrow$ octaplet.

$$\left. \begin{aligned} \delta'(t) &= 0 && \text{For } t \neq 0 \\ &= \text{undefined} && \text{for } t = 0 \end{aligned} \right\}$$

and

$$\int_{-\infty}^{\infty} \delta'(t) dt = 0$$

PROPERTIES OF DOUBLET -

$$\textcircled{1} \delta'(a(t - t_0)) = \frac{1}{a|a|} \delta'(t - t_0)$$

$$\textcircled{2} \delta'(-(t-t_0)) = -\delta'(t-t_0)$$

$$\delta'(-t) = -\delta'(t)$$

$$\textcircled{3} [x(t) \delta(t)]' = x'(t) \delta(t) + x(t) \delta'(t)$$

$$[x(t) \delta(t)]' = x'(t) \delta(t) + x(t) \delta'(t)$$

$$x(t) \delta'(t) = x'(t) \delta(t) + x(t) \delta'(t)$$

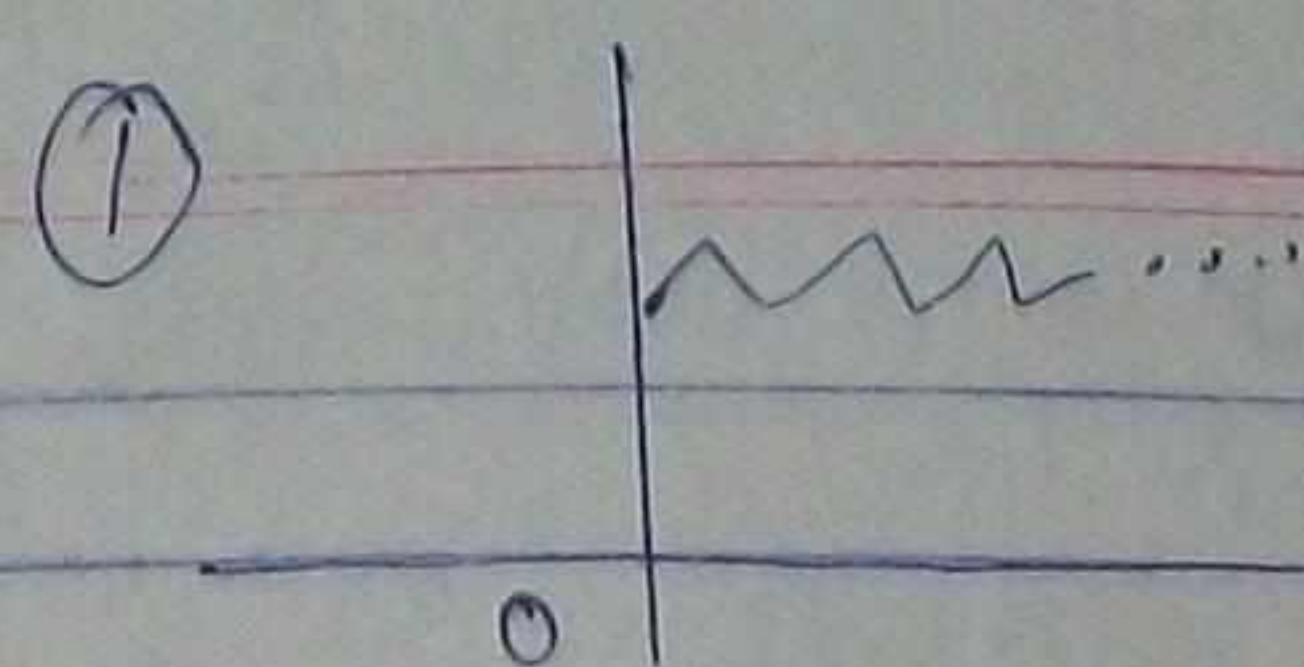
$$x(t) \delta'(t) = x'(t) \delta(t) - x'(0) \delta(t)$$

⇒ CLASSIFICATION OF SIGNAL -

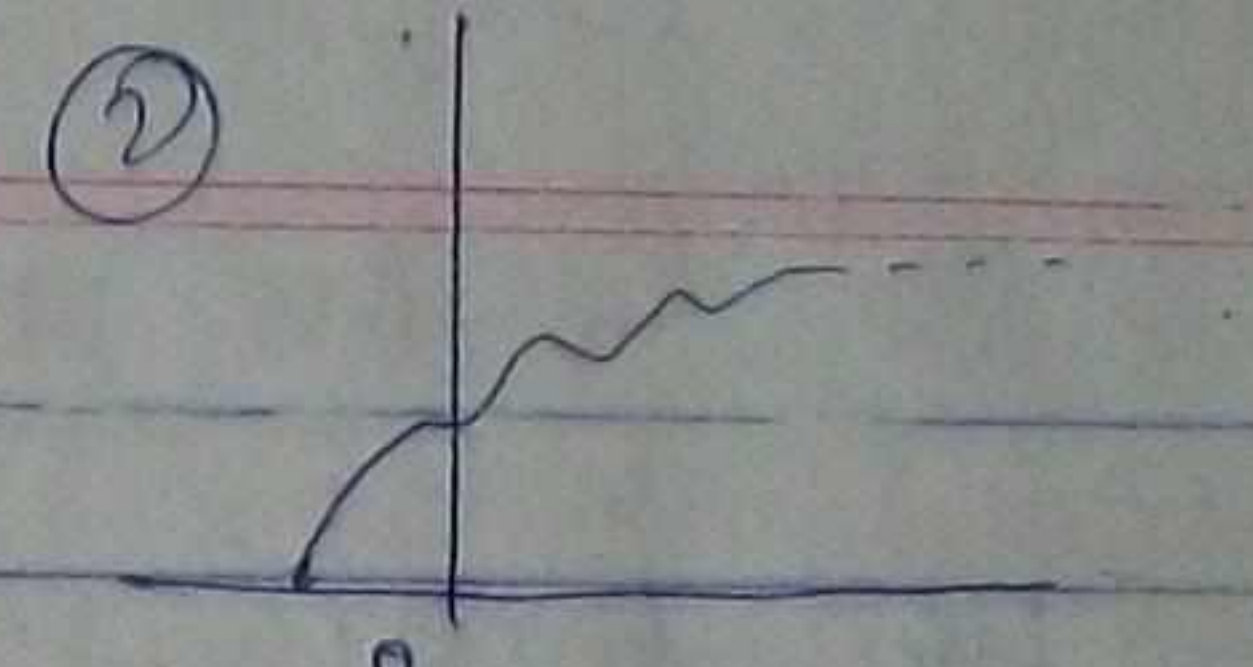
\textcircled{1} RIGHT SIDE & LEFTSIDE SIGNAL -

- Signal $x \rightarrow +\infty$ is Right sided
- Signal $x \rightarrow -\infty$ is left sided
- Signal $-\infty$ to $+\infty$ is everlasting signal
- Signal that has finite width is finite time signal.

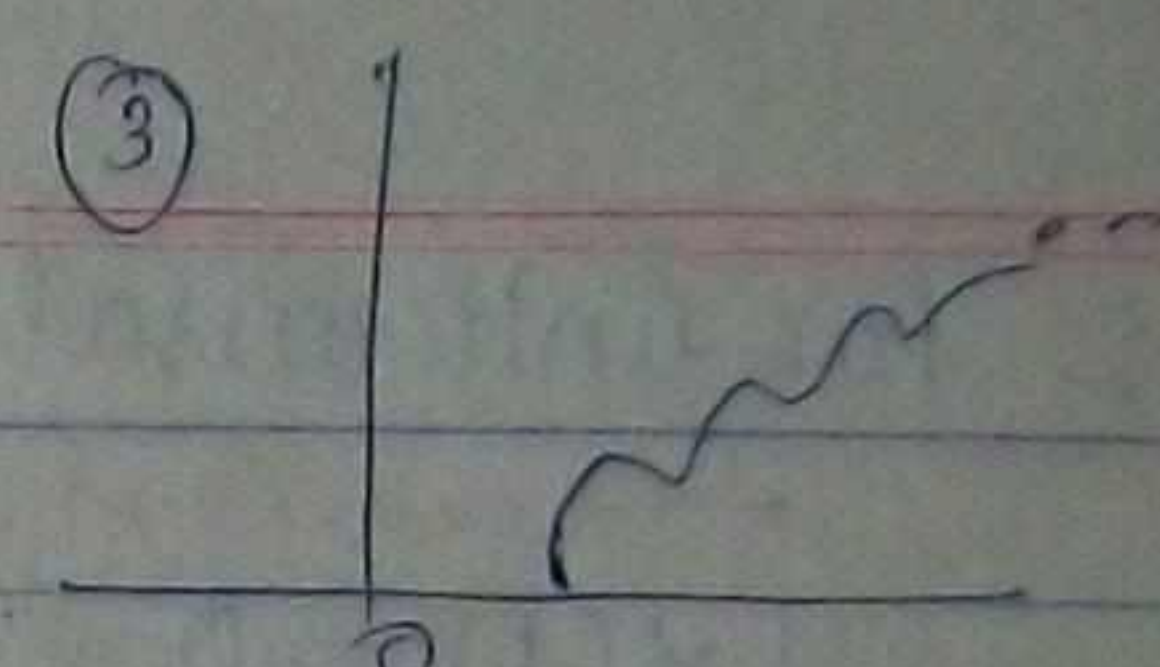
C : CAUSAL
AC : ANTI CAUSAL
NC : NON CAUSAL



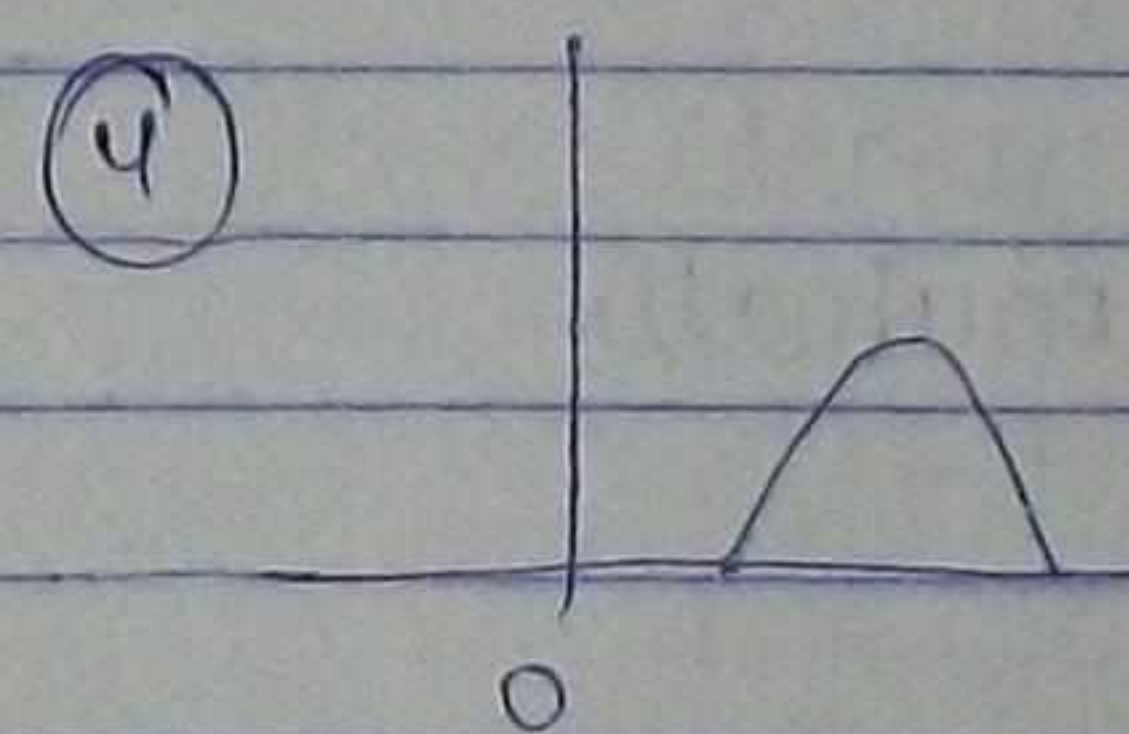
RIGHT SIDE (C)



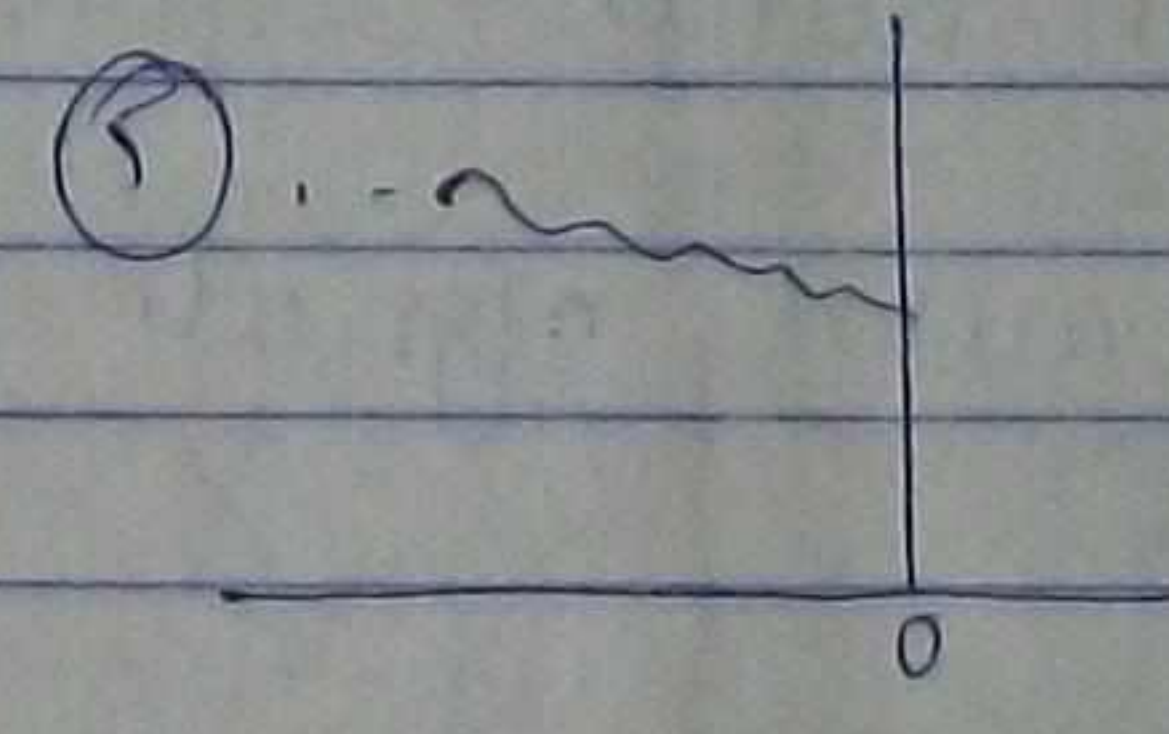
RIGHT SIDE (NC)



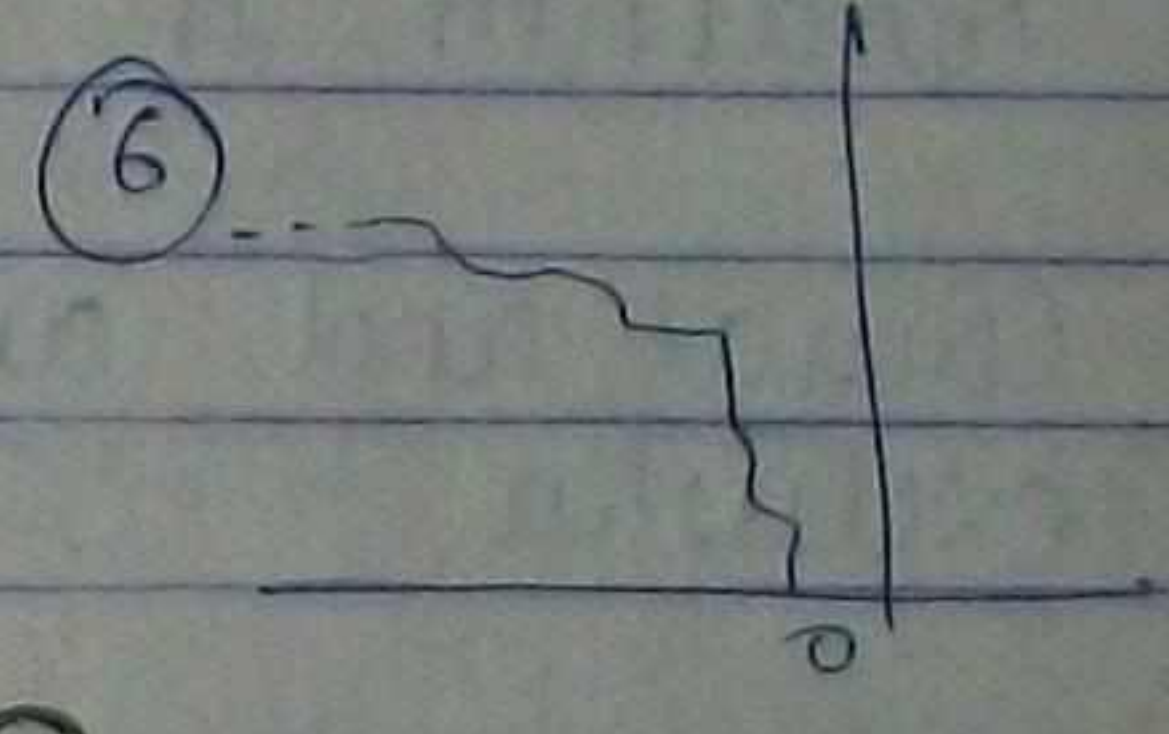
RIGHT SIDE (C)



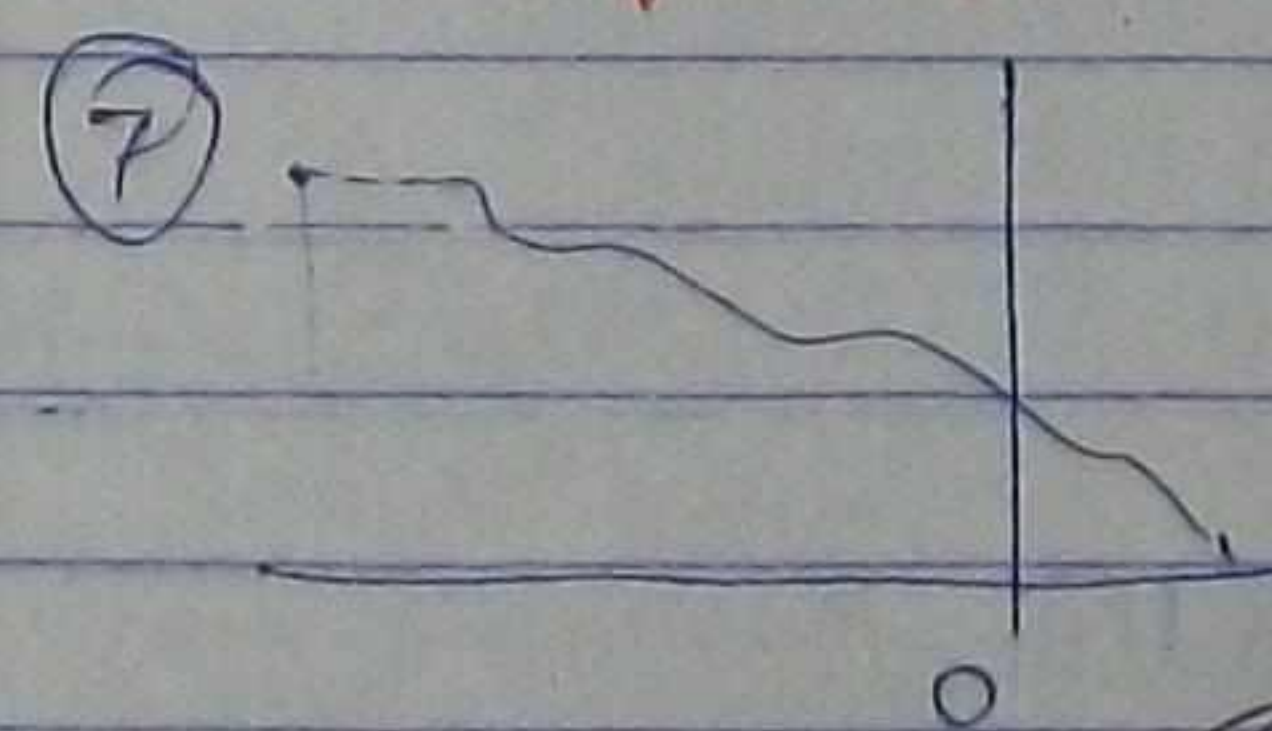
RIGHT SIDE FINITE (C)



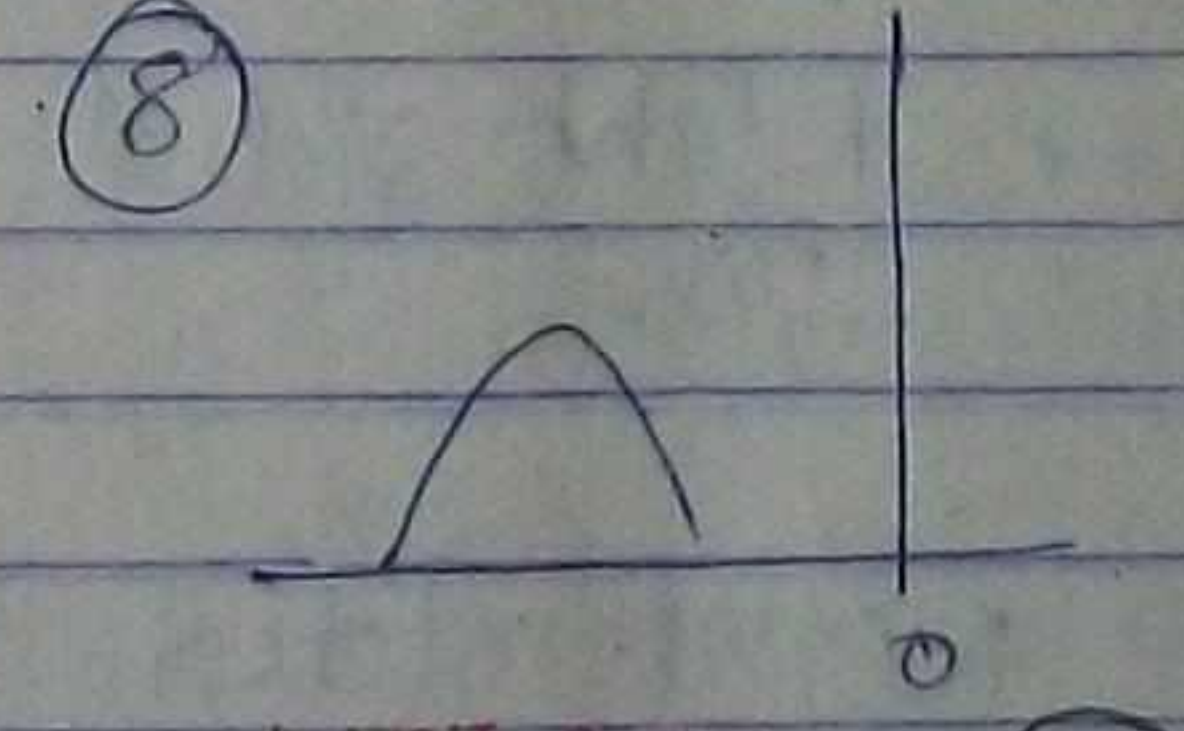
LEFT SIDE (AC)



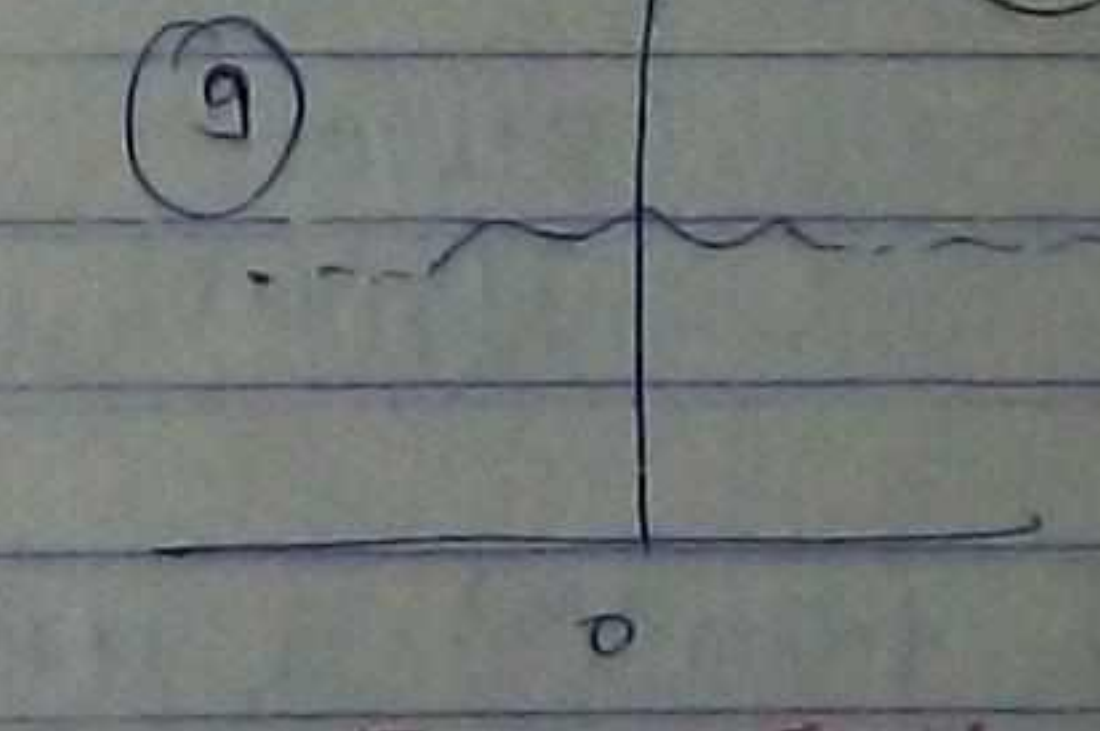
LEFT SIDE (AC)



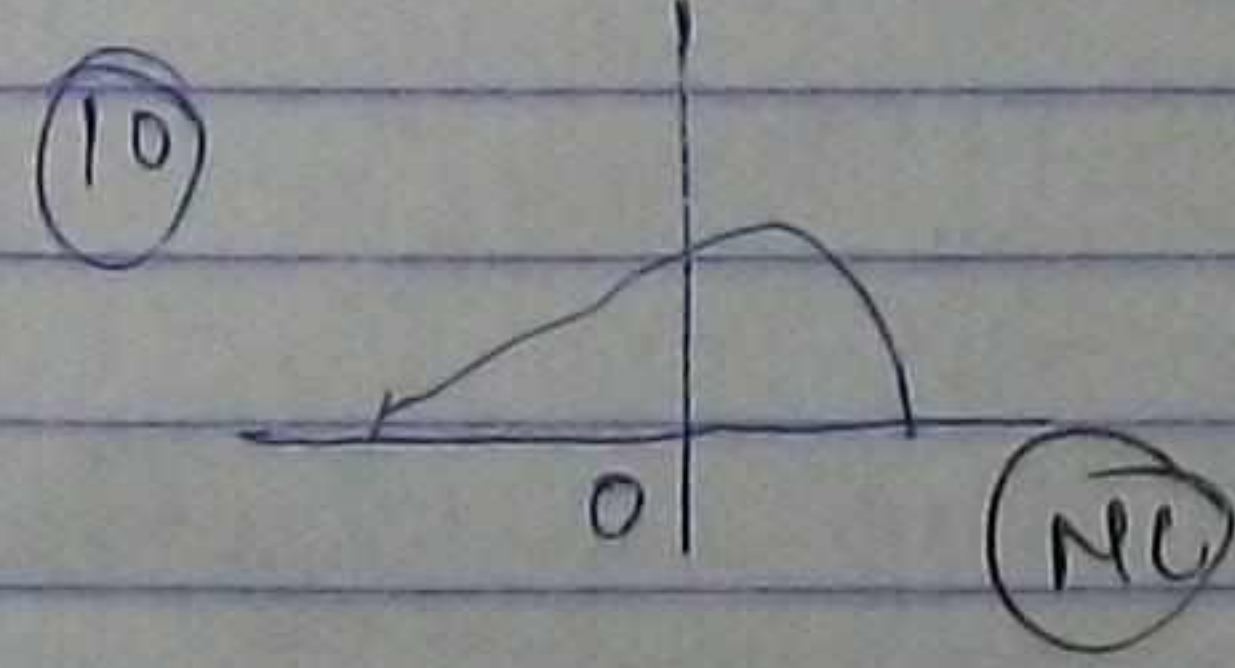
LEFT SIDE (NC)



LEFT SIDE FINITE (AC)



EVERLASTING (NC)



FINITE TIME TWO SIDED (NC)

\textcircled{2} CAUSAL, NON-CAUSAL & ANTI-CAUSAL -

- for any causal signal

$$x(t) = 0 \quad \text{for } t < 0$$

- for anticausal

$$x(t) = 0 \quad \text{for } t > 0$$

- Signal that does not satisfy the above condition is noncausal signal.

- Causal and anticausal signals are mutually exclusive.

- from ①, ③ and ④ every:

causal signal $\rightarrow$ Right sided sign

- from ② every Right sided signal need not to be a causal signal.

- for ⑤, ⑥ and ⑧ every:

anticausal signal $\rightarrow$ Left sided

- from ⑦ every left sided signal need not to be an anticausal signal.

- Some signal are there which are neither causal nor anticausal, they are known as non-causal.

③ EVEN AND ODD SIGNAL -

- for every even / symmetric signal

$$x(t) = x(-t) \quad \text{for all } t.$$

- for every odd / antisymmetric signal.

$$x(t) = -x(-t) \quad \text{for all } t$$

- A signal that does not satisfy the above condition is neither even nor odd.

- Even and odd signal are mutually exclusive.

- Any signal can be expressed as the sum of even and odd parts of it.

$$x(t) = x_e(t) + x_o(t)$$

- Even and odd parts are mutually exclusive.

$$x_e(t) = \frac{1}{2} [x(t) + x(-t)]$$

$$x_o(t) = \frac{1}{2} [x(t) - x(-t)]$$

- for signals

$$\text{even} + \text{even} = \text{even}$$

$$\text{odd} + \text{odd} = \text{odd}$$

$$\text{even} + \text{odd} = \text{No symmetry}$$

$$\text{Even} \cdot \text{Even} = \text{Even}$$

$$\text{odd} \cdot \text{odd} = \text{even}$$

$$\text{odd} \cdot \text{even} = \text{odd}$$

$$\int_T \text{Even} = 2 \int_{T/2} \text{Even}$$

$$\int_T \text{odd} = 0$$

Ques. Test the symmetry, hence determine the even and odd parts of folⁿ:

$$(1) \quad x(t) = 1 + 2t + 3t^2$$

$$x(-t) = 1 - 2t + 3t^2 \neq x(t) \\ \neq -x(-t)$$

neither even nor odd

means contain both parts $\begin{cases} e^{j\theta} = \cos\theta + j\sin\theta \\ e^{-j\theta} = \cos\theta - j\sin\theta \end{cases}$

$$x_e(t) = \frac{1}{2} [2 + 6t^2] = 1 + 3t^2$$

$$x_{\text{odd}}(t) = \frac{1}{2} [4t] = 2t$$

$$(2) \quad e^{-jt} = x(t) \quad (\text{NO symmetry})$$

$$x(-t) = e^{jt}$$

$$x_e(t) = \frac{1}{2} (e^{-jt} + e^{jt}) = \cos t$$

$$x_o(t) = \frac{1}{2} (e^{-jt} - e^{jt}) = -j \sin t$$

$$(3) \quad e^t = x(t), \quad x(-t) = e^{-t} \quad (\text{NO symmetry})$$

$$x_e(t) = \frac{1}{2} (e^t + e^{-t}) = \cosh t$$

$$x_o(t) = \frac{1}{2} (e^t - e^{-t}) = \sinh t$$

$$(4) \quad x(t) = \cos t \quad (\text{even fun}^n) \\ x(-t) = \cos t$$

$$(5) \quad x(t) = \cosh t \quad (\text{even fun}^n) \\ x(-t) = \cosh t$$

$$(6) \quad x(t) = \sin t = \frac{e^{jt} - e^{-jt}}{2j}$$

$$x(-t) = \sin(-t) = \frac{e^{-jt} - e^{jt}}{2j}$$

ODD
FUNⁿ

$$(7) \quad x(t) = \sinh t$$

$$x(-t) = -\sinh t$$

ODD
FUNⁿ

$$(8) \quad x(t) = u(t)$$

$$x(-t) = u(-t)$$

NO symmetry

$$x_e(t) = \frac{1}{2} [u(t) + u(-t)] = \frac{1}{2} \begin{array}{c} | \\ \hline \end{array}$$

$$x_o(t) = \frac{1}{2} [u(t) - u(-t)] = \frac{1}{2} \begin{array}{c} \hline | \\ \hline \end{array}$$

$$= \frac{1}{2} \operatorname{sgn}(t)$$

$$u(t) = \frac{1}{2} + \frac{1}{2} \operatorname{sgn}(t)$$

$$(9) \quad \operatorname{sgn}(t) = x(t)$$

$$x(-t) = -\operatorname{sgn}(-t)$$

ODD FUN

$$\operatorname{sgn}(t) = u(t) - u(-t) = 2u(t) - 1$$

$$(10) \quad e^{-3t} \cos(2t) = x(t)$$

$$x(-t) = e^{3t} \cos(2t)$$

No symmetry

$$x_e(t) = \cosh 3t \cdot \cos 2t$$

$$x_o(t) = -\sinh 3t \cdot \cos 2t$$

$$(11) \quad x(t) = e^{-3t} u(t)$$

$$x(-t) = e^{3t} u(-t)$$

No symmetry

$$x_e(t) = \frac{1}{2} [e^{-3t} u(t) + e^{3t} u(-t)] = \frac{1}{2} [e^{-3|t|}]$$

$$x_o(t) = \frac{1}{2} [e^{-3t} u(t) - e^{3t} u(-t)] =$$

$$(12) \quad x(t) = \sin t + \cos 3t + \sin 5t \cdot \cos 7t$$

$$x(-t) = -\sin t + \cos 3t - \sin 5t \cdot \cos 7t$$

$$x_e(t) = \cos 3t$$

$$x_o(t) = \sin t + \sin 5t \cos 7t$$

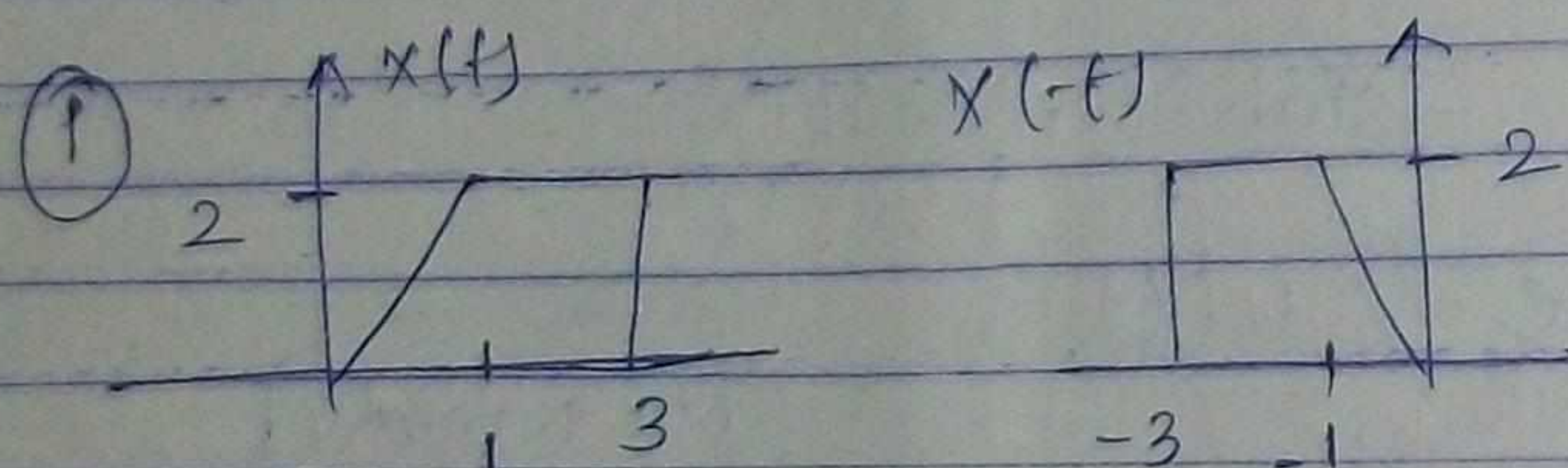
No
symmetry

$$(13) x(t) = [t + \sin t]^2 = t^2 + \sin^2 t + 2t \sin t$$

$$x(-t) = t^2 + \sin^2 t + 2t \sin t$$

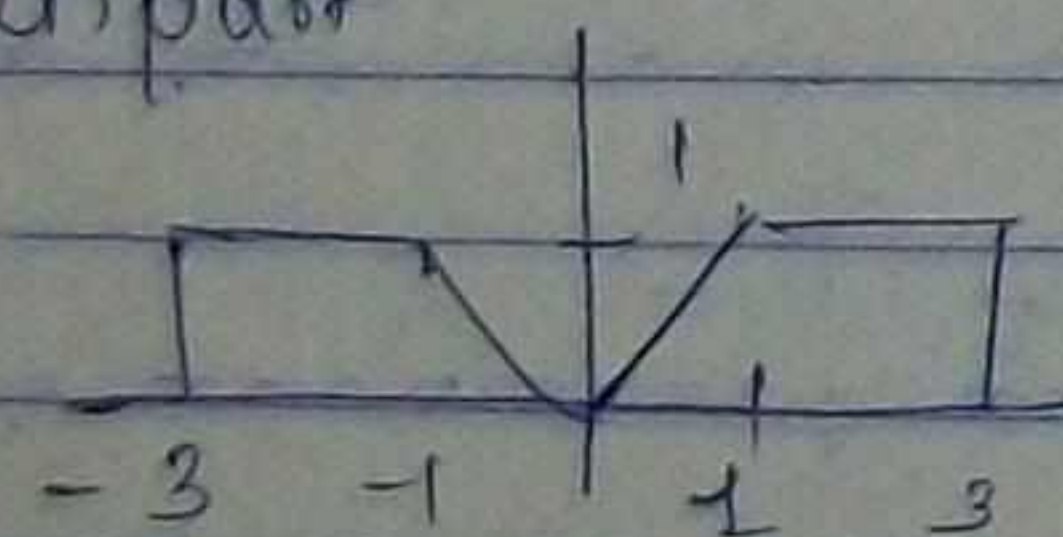
even funⁿ

Ques. Sketch even and odd parts of:

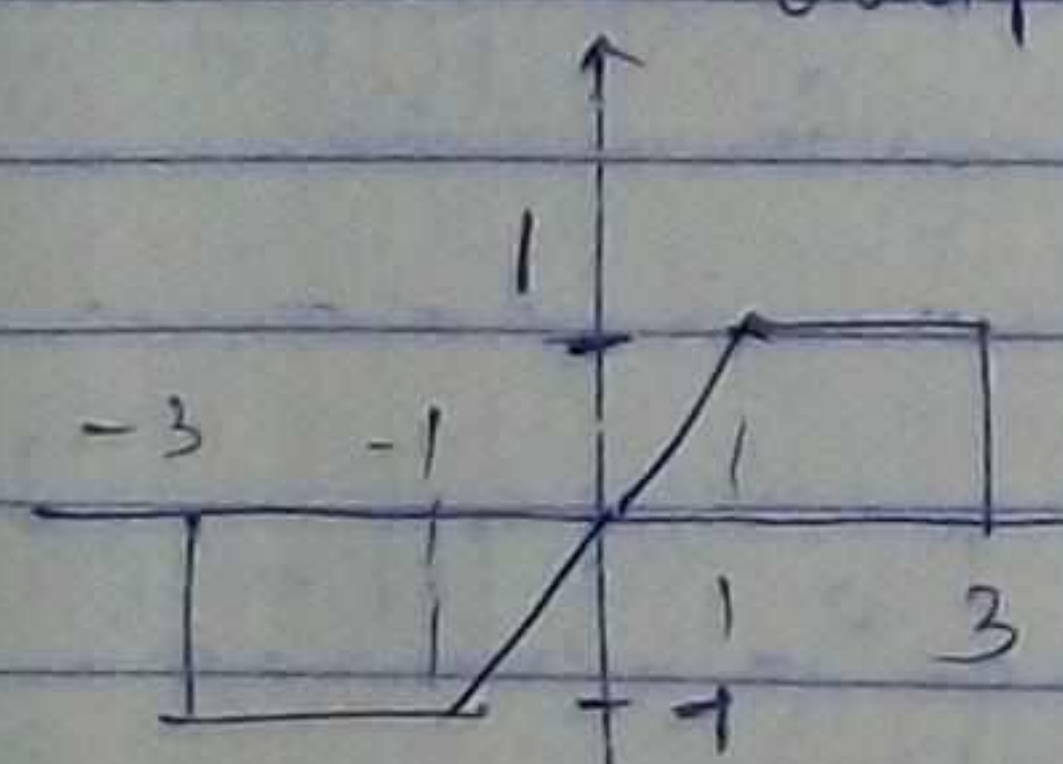


Neither even nor odd

Even part



Odd part



NOTE -

Symmetry condⁿ for real valued signals and imaginary valued signal remain same, whereas for the complex valued signal it is known as conjugate symmetry and conjugate antisymmetry.

- let $y(t)$ be the img. valued signal.

$y(t)$ is said to be conjugate symmetric if:

$$y(t) = y^*(-t)$$

- $y(t)$ is said to be conjugate antisymmetric if:

$$y(t) = -y^*(-t)$$

Ques. Test the symmetry of the following:

$$(1) x(t) = jt$$

ODD FUNⁿ

$$x(-t) = -jt = -x(t)$$

$$(2) x(t) = 3 - jt$$

$$x(-t) = 3 + jt$$

$$x^*(-t) = 3 - jt$$

conjugate even symmetry

Ques. Test symmetry and find even & odd parts:

$$(1) x(n) = \{-3, 2, 4, 5, 1\}$$

$$x(-n) = \{1, 5, 4, 2, -3\}$$

No symmetry

$$X_e(f) = \left\{ 1/2, 5/2, 1/2, 2, 1/2, 5/2, 1/2 \right\}$$

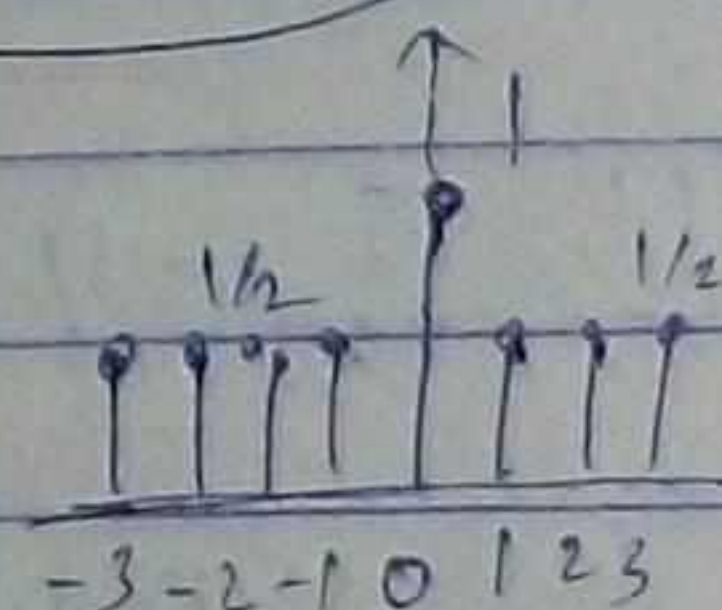
$$X_o(f) = \left\{ -1/2, -5/2, -7/2, 0, 7/2, 5/2, 1/2 \right\}$$

② $x(n) = u(n)$

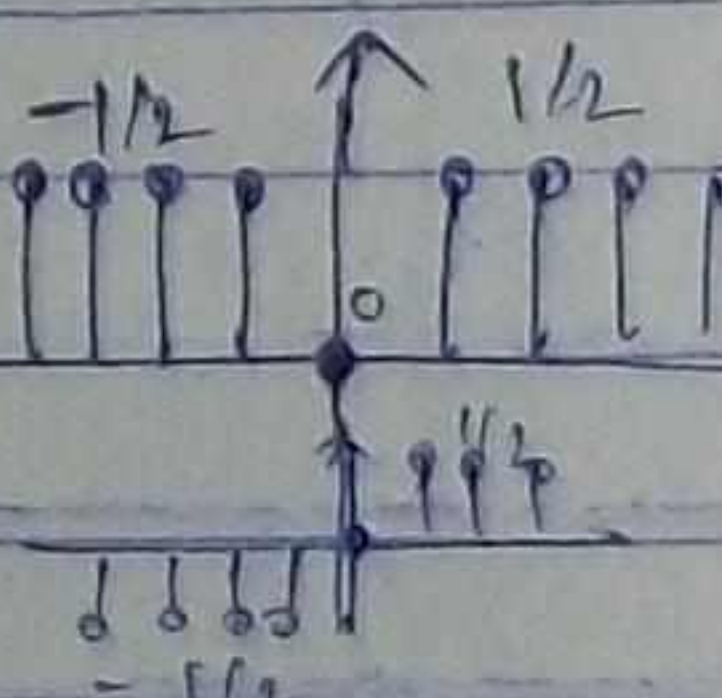
No symmetry

$$x(-n) = u(-n)$$

$$X_e(n) = \frac{1}{2} [u(n) + u(-n)]$$



$$X_o(n) = \frac{1}{2} [u(n) - u(-n)]$$



③ $x(n) = \text{sgn}(n)$

$$x(-n) = -\text{sgn}(-n)$$

ODD
FORM

④ $x(n) = (1/2)^n u(n)$

$$x(n) = (1/2)^n u(-n)$$

No symmetry

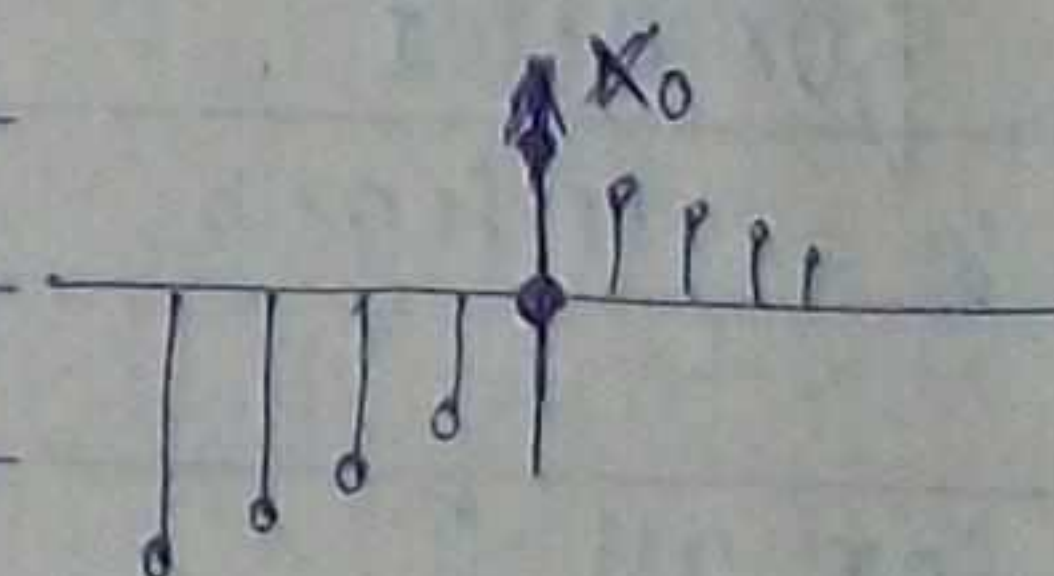
$$X_e(n) = \frac{1}{2} \left[\left(\frac{1}{2}\right)^n u(n) + \left(\frac{1}{2}\right)^n u(-n) \right]$$



$$X_e(n) = 1 \quad \text{for } n=0$$

$$= \left(\frac{1}{2}\right)^{|n|+1} \quad \text{for } n \neq 0$$

$$X_o(n) = \frac{1}{2} \left[\left(\frac{1}{2}\right)^n u(n) - \left(\frac{1}{2}\right)^{-n} u(-n) \right]$$



⑤ $x(n) = \{ 10, (-2+2j), -2, (-2-2j) \}$

$$x(-n) = \{ (-2-2j), -2, (-2+2j), 10 \}$$

$$x^*(-n) = \{ -2+2j, -2, -2-2j, 10 \}$$

$\neq x(n)$ neither conjugate symmetric
 $\neq -x(n)$ nor conjugate symmetric

$$X_e(n) = \frac{1}{2} [x(n) + x^*(-n)] = \{ (1+j), -1, (-1-j), 10, (-1+j), -1, (1-j) \}$$

$$X_o(n) = \frac{1}{2} [x(n) - x^*(-n)] =$$

$$= \{$$

④ PERIODIC AND NON PERIODIC SIGNAL -

(A) FOR CONTINUOUS TIME -

- Signal $x(t)$ is said to be periodic if:

$$x(t) = x(t + kT) \quad \text{for all } t, \\ k = \text{Integer}$$

or

$$x(t) = x(t + T) \quad \text{for all } t.$$

- The smallest value of T for which the above condⁿ is satisfied is known as fundamental period or periodicity.

- If there is no value of T to satisfy the above condⁿ, then signal is known as non periodic or Aperiodic signal. The fundamental period of non periodic signal is undefined.

- The condⁿ of periodicity is satisfied iff the signal repeats itself from $-\infty$ to ∞ . Otherwise it is non periodic signal. Therefore every periodic signal is everlasting signal but everlasting signal is not periodic.

PROCEDURE TO TEST PERIODICITY IF SIGNAL IS THE COMBⁿ OF TWO OR MORE SIGNAL $\{x_1(t) + x_2(t) + \dots\}$

METHOD ①: USING FUNDAMENTAL PERIODS -

① Find the fundamental period of each signal is $T_1, T_2, T_3, \dots$

② Find the ratio test is:

$$\frac{T_1}{T_2}, \frac{T_2}{T_3}, \frac{T_3}{T_4}, \dots$$

③ If all the ratios are rational then it is periodic. If one of the ratio is irrational then non periodic.

④ If periodic fundamental period is given by $T = \frac{\text{LCM of Numerator of } (T_1, T_2, T_3, \dots)}{\text{HCF of Denominator of } (T_1, T_2, T_3, \dots)}$



METHOD ②: USING FUNDAMENTAL FREQUENCY -

① Find the fundamental freq. of each signal is $\omega_1, \omega_2, \omega_3, \dots$

② Find the ratio test is:

$$\frac{\omega_1}{\omega_2}, \frac{\omega_2}{\omega_3}, \frac{\omega_3}{\omega_4}, \dots$$

③ find all If all the ratios are rational then it is periodic and if any one is irrational then it is nonperiodic.

④ If periodic fundamental freq is given by:

$$f = \frac{\text{HCF of Num of } (\omega_1, \omega_2, \omega_3, \dots)}{\text{LCM of Deno. of } (\omega_1, \omega_2, \omega_3, \dots)}$$

Ques Test the periodicity of:

① $x(t) = 4 \cos 3t \cdot u(t)$
nonperiodic (not everlasting signal) because

② $-10 \sin(5\pi t - 30^\circ)$ $\omega = \frac{2\pi}{T} = 5\pi$

$$\omega = \frac{2\pi}{T} = 5\pi \quad T = 2/5$$

$$x(t+T) = -10 \sin(5\pi(t+2/5) - 30^\circ)$$

$$= -10 \sin((5\pi t - 30^\circ) - 2\pi)$$

$$= -10 \sin(5\pi t - 30^\circ)$$

Periodic

$$T = 2/5$$

$$f = 5/2 \text{ Hz}$$

$$\omega = 5\pi \text{ Hz}$$

③ $x(t) = 3 \cos^2(0.5\pi t)$

- addⁿ or subⁿ of DC does not change the periodicity

$$x(t) = \frac{3}{2} + \frac{3}{2} \cos(\pi t)$$

$$\omega = \pi \quad T = 2 \text{ sec}$$

$$x(t+2) = \frac{3}{2} + \frac{3}{2} \cos(\pi(t+2))$$

$$= \frac{3}{2} + \frac{3}{2} \cos(\pi t + 2\pi)$$

$$= \frac{3}{2} + \frac{3}{2} \cos(\pi t)$$

Periodic

④ $x(t) = 10 e^{j(5/7 t + 30^\circ)}$

$$\omega = 5/7 \quad T = 14\pi/5$$

$$x(t + \frac{14\pi}{5}) = 10 e^{j(5/7(t + \frac{14\pi}{5}) + 30^\circ)}$$

$$= 10 e^{j(5t + 30^\circ)} \cdot e^{j2\pi}$$

$$= 10 e^{j(5t + 30^\circ)}$$

PERIODIC

$$⑤ \quad x(t) = \sin(\sqrt{3}t)$$

$$\omega = \sqrt{3} \quad T = 2\pi/\sqrt{3}$$

$$x(t + 2\pi/\sqrt{3}) = \sin(\sqrt{3}t + 2\pi) \\ = \sin(\sqrt{3}t)$$

PERIODIC

$$⑥ \quad 1 + 2t + 3t^2$$

NON PERIODIC

no fundamental period.

* The fundamental period of a C.T. signal can be rational or irrational.

Ques. Test the periodicity, hence determine the fundamental T and f.

$$① \quad x(t) = \cos(5t) \cdot \sin(7t)$$

$$= \frac{1}{2} [\sin(12t) + \sin(2t)]$$

$$\omega_1 = 12, \quad \omega_2 = 2$$

$$\omega = 2, 12$$

$$\frac{\omega_1}{\omega_2} = \frac{2}{12} = \frac{1}{6} \rightarrow \text{Rational} \rightarrow \text{periodic}$$

$$\omega = \frac{\text{HCF}(2, 12)}{\text{LCM}(1, 12)} = 2 \text{ rad/sec}$$

$$T = \frac{2\pi}{2} = \pi \text{ sec} \quad f = \frac{1}{\pi} \text{ Hz}$$

— The first term represent the 1st harmonic and the 2nd term represent the 6th harmonic of the periodic signal with fundamental freq $\omega = 2 \text{ rad/sec}$.

$$② \quad x(t) = \cos\left(\frac{\pi}{3}t\right) - \sin(10t - 30^\circ)$$

$$\omega_1 = \frac{\pi}{3} \quad \omega_2 = 10$$

Aperiodic

$$\frac{\omega_1}{\omega_2} = \frac{\pi}{30} = \text{Irrational}$$

$$③ \quad x(t) = 5 \cos(5\pi t) - 3 \sin\left(\frac{\pi}{0.3}t - 45^\circ\right) + \sin\left(\frac{5\pi}{3}t\right)$$

$$\omega_1 = 5\pi \quad \omega_2 = \pi/0.3 \quad \omega_3 = 5\pi/3$$

$$\frac{\omega_1}{\omega_2} = \frac{5\pi}{\pi/0.3} = \frac{15}{10}$$

$$\frac{\omega_2}{\omega_3} = \frac{\pi/0.3}{5\pi/3} = \frac{50}{9}$$

$$\omega = \frac{\text{HCF}(15, 10, 3\pi)}{\text{LCM}(1, 0.3, 5)} = \frac{\pi}{15}$$

$$\omega = \pi/15 \quad T = 30 \text{ sec}$$

(4) $\sin(50t) + \cos\left(\frac{3}{10}t\right) - \sin\left(\frac{3}{10}t\right) - \cos\left(\frac{7}{3}t\right)$

$$\frac{\omega_1}{\omega_2} = \frac{50 \times 10}{3} = \frac{500}{3}$$

$$\frac{\omega_2}{\omega_3} = \frac{3 \times 10}{10 \times 3} \quad \frac{\omega_3}{\omega_4} = \frac{3 \times 3}{10 \times 7}$$

$$\omega = \frac{\text{HCF}(50, 3, 3, 7)}{\text{LCM}(1, 10, 10, 3)} = \frac{1}{30} \text{ rad/s}$$

$$T = 60\pi$$

(B) FOR DISCRETE SIGNAL -

- A signal $x(n)$ is said to be periodic if:

$$x(n) = x(n + mN) \quad \text{for all } n, \quad m = \text{integer}$$

(or)

$$x(n) = x(n + N) \quad \text{for all } n$$

- The smallest value of 'N' for which the above condition is satisfied is known as the fundamental period or periodicity.

- If there is no value to satisfy the above condition then it is known as non periodic or Aperiodic signal. The fundamental period of non periodic signal is undefined.

- for periodicity let

$$x(n) = A \cos(\omega n + \theta) \quad \text{--- (1)}$$

$$x(n+N) = A \cos(\omega(n+N) + \theta)$$

$$= A \cos(\omega n + \theta + \omega N) \quad \text{--- (2)}$$

for periodicity eq (2) must reduce to eq (1), it is possible iff and only iff

$$\omega N = 2\pi k$$

where $k = \text{integer}$.

$$N = \left(\frac{2\pi}{\omega}\right) k, \quad k = \text{integer}$$

- Since N is integer and the ratio $(2\pi/\omega)$ must be rational. then only it is periodic otherwise it is non periodic.

12/11/11

Ques. Test the periodicity of the foll^g:

$$(1) x(n) = 10 \cos(3\pi n) u(n)$$

Aperiodic

$$(2) x(n) = -4 \sin^2(0.2\pi n - 30^\circ)$$

$$= -2 + 2 \cos(0.4\pi n - 60^\circ)$$

$$\Omega = 0.4\pi$$

$$N = \frac{2\pi}{\Omega} = \frac{2\pi}{0.4\pi} = \frac{1}{0.2} = 5 \quad \text{Rational Periodic}$$

$$N=5$$

$$(3) 4e^{-j(100n)} = x(n)$$

$$\Omega = 100$$

Non Periodic

$$N = \frac{2\pi}{100} = \frac{\pi}{50} \quad \text{Irrational}$$

$$(4) x(n) = 15 \cos\left(\frac{30}{7}n\right)$$

$$\Omega = \frac{30}{7}$$

$$N = \frac{2\pi \times 7}{30} = \frac{14}{30}\pi$$

Aperiodic

Irrational

A

$$(5) x(n) = 7 \cos\left(\frac{\pi}{3}n - 30^\circ\right) - 5 \sin\left(\frac{2\pi}{10}n\right)$$

PERIODIC

$$\Omega_1 = \frac{\pi}{3} \quad \Omega_2 = \frac{2\pi}{10}$$

$$N_1 = 6$$

$$N_2 = 10$$

$$N = \frac{\text{HCF}(\pi, 2\pi)}{\text{LCM}(3, 10)} = \frac{\pi}{30}$$

$$N = \frac{2\pi \times 30}{\pi} = 60$$

- In discrete time combination of periodic signal is always periodic.

Since both signals are periodic the combⁿ is also periodic.

Fundamental period $N = \text{LCM}(N_1, N_2)$

$$= \text{LCM}(6, 10) = 30$$

A

$$(6) x(n) = 4 \sin(7\pi n) - 13 \cos(6n)$$

$$N_1 = \frac{2\pi}{7\pi} = \frac{2}{7}K$$

$$N_2 = \frac{2\pi}{6}$$

Non periodic

$$N_1 = 2 \quad \text{for } K=7$$

Since second is non-periodic hence the combⁿ is also non periodic.

⑤ POWER AND ENERGY SIGNAL -

- for every energy signal energy is finite and power is zero, i.e.

$$0 < E < \infty, P = 0$$

- for every power signal power is finite and the energy is infinite i.e.

$$0 < P < \infty, E = \infty$$

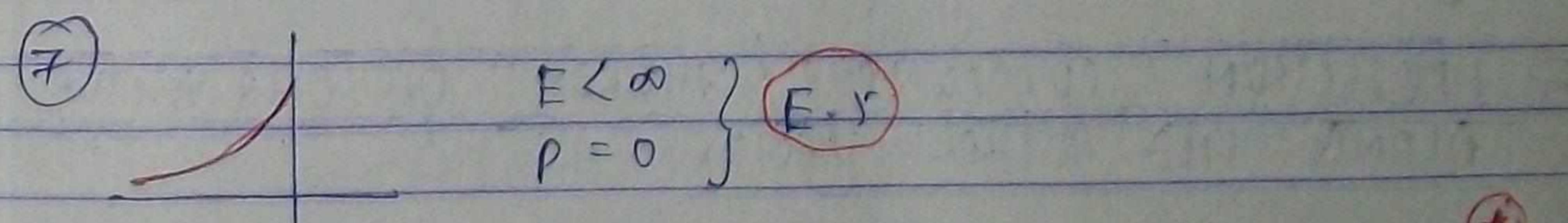
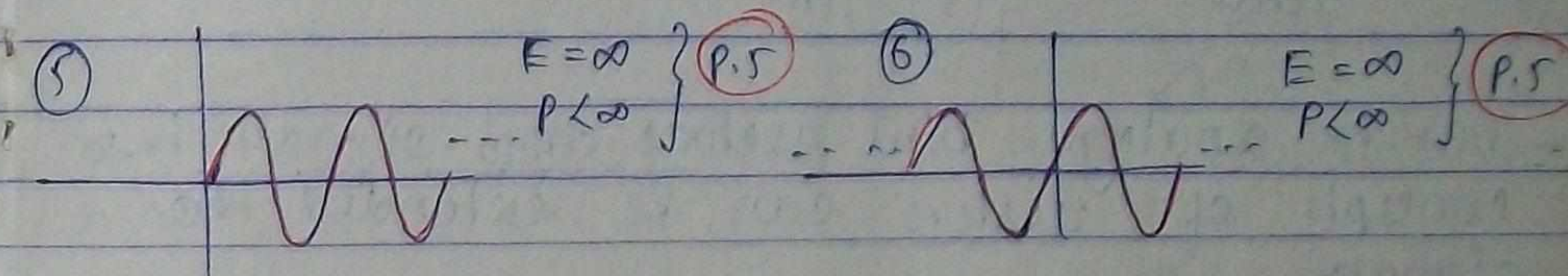
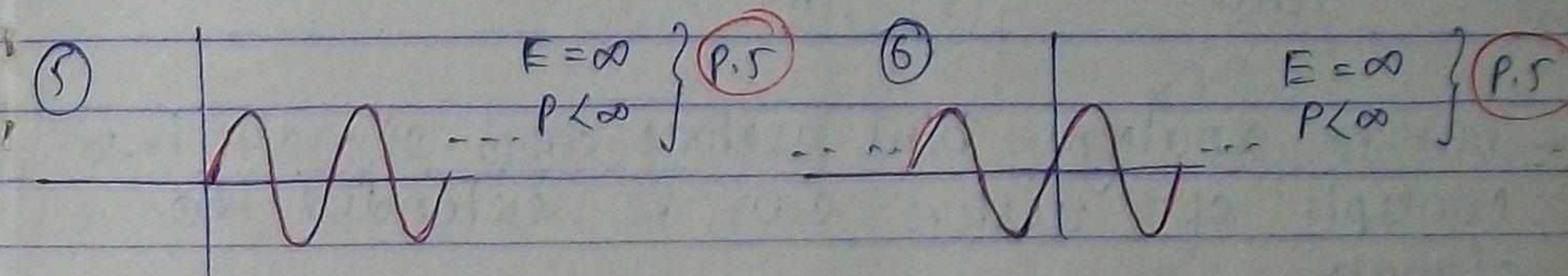
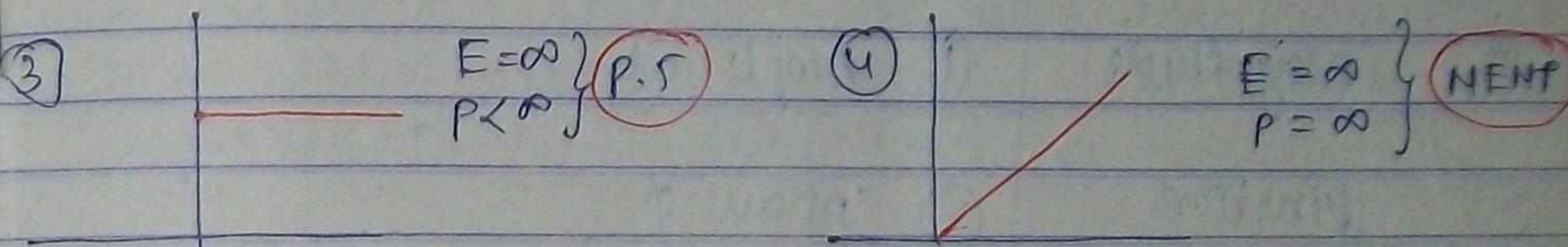
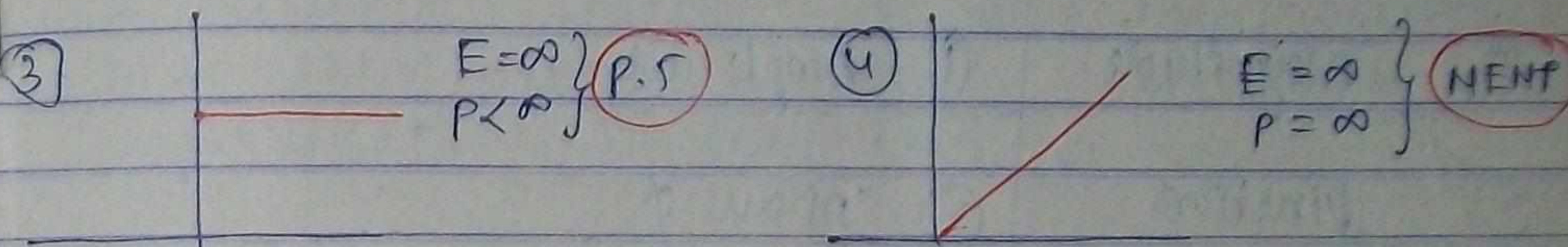
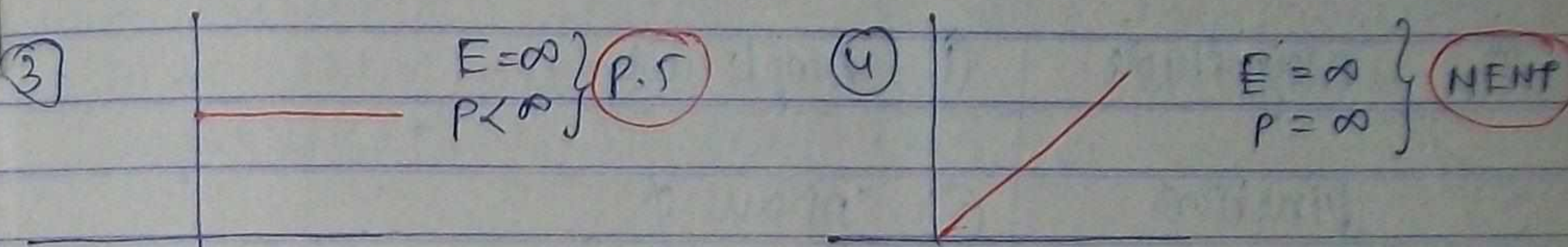
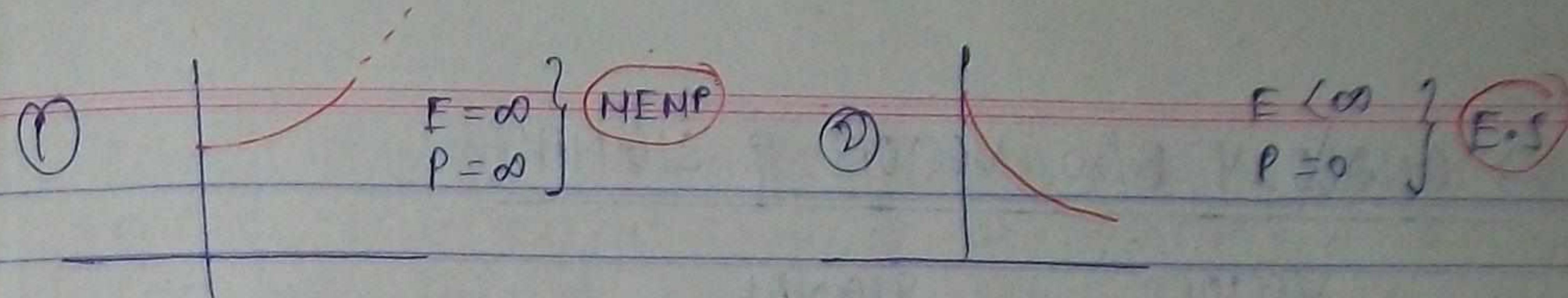
- The signal for which both power and energy tends to infinite is neither energy signal nor power signal.

$$P = \infty, E = \infty \quad (\text{ramp})$$

- Power signal and energy signals are mutually exclusive.

- for some signals energy is finite and for some other signals power is finite but no signal possesses the finite power and finite energy.

- for every energy signal its amplitude tends to zero as $|t| \rightarrow \infty$



- from ⑥ every periodic signal is power signal

- from ⑤ every power signal need not be periodic.

- from ② and ⑦ every energy signal is non-periodic.

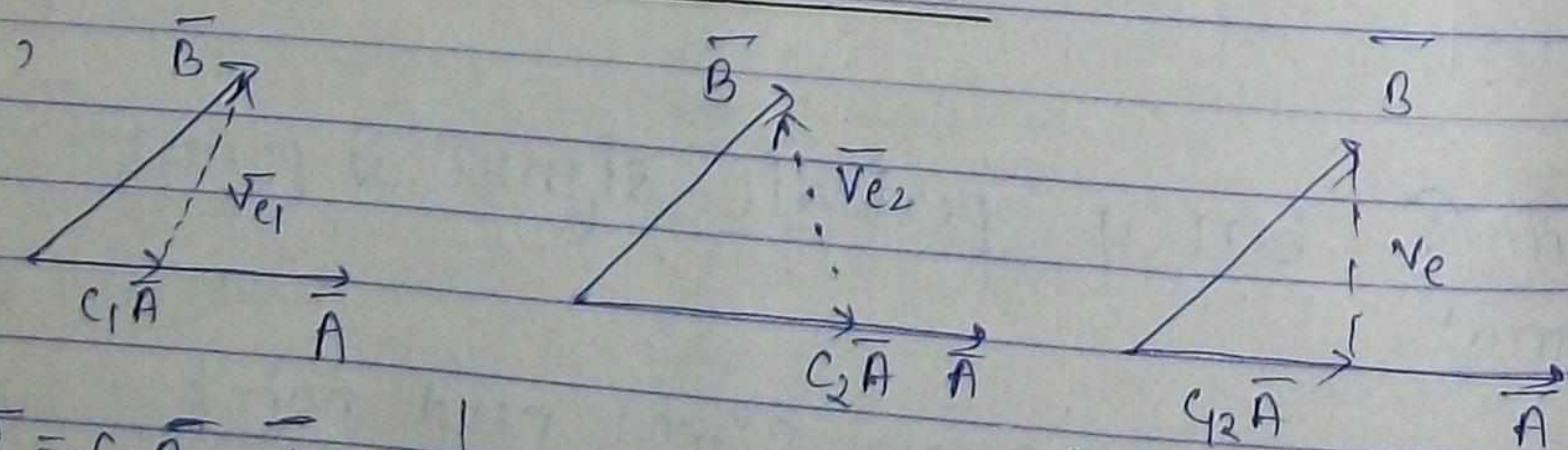
- from ①, ③, ④ every non-periodic signal need not be energy signal.

⇒ ANALOGY B/W VECTORS & SIGNAL -

VECTORS	SIGNAL
① Magnitude	① Amplitude
② Direction	② phase ϕ
③ Time	③ Time

- Due to analogy b/w vectors and signals the concepts of vectors can be extended to signals.

⇒ PROJECTION / COMPONENT / APPROXⁿ OF ONE VECTOR ALONG THE OTHER VECTOR -

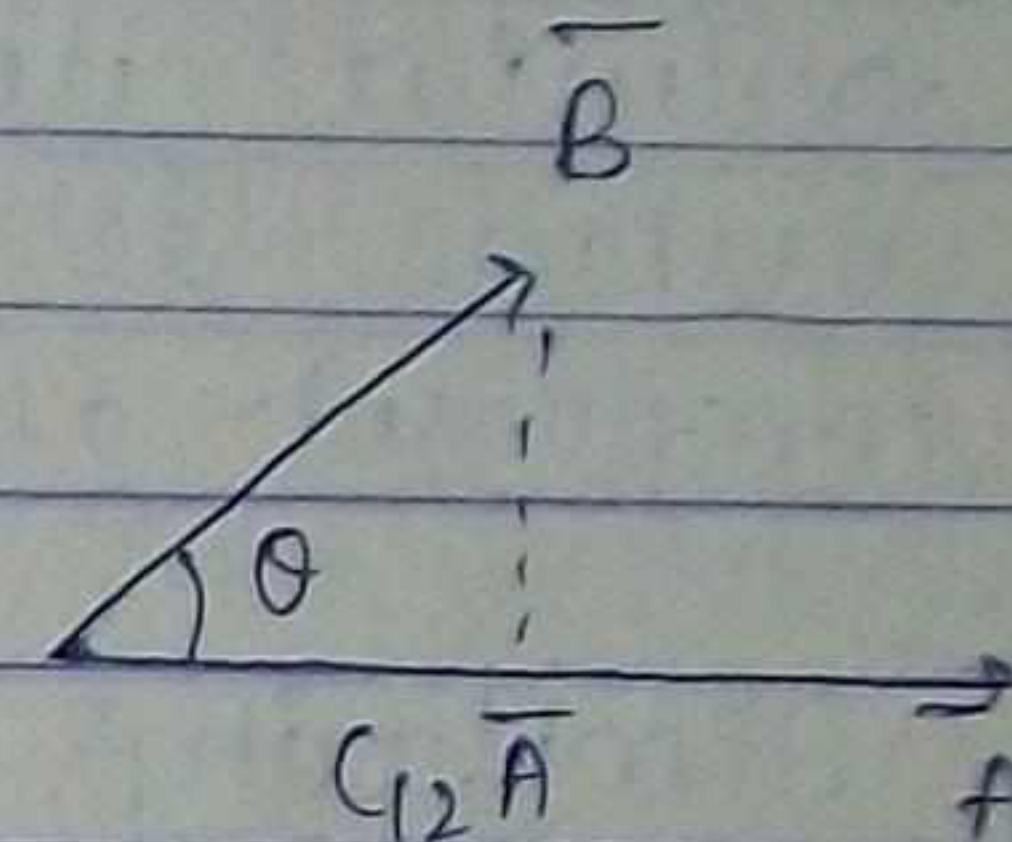


$$\begin{array}{l|l|l}
 \vec{B} = C_1 \vec{A} + \vec{V}_{e1} & C_2 \vec{A} + \vec{V}_{e2} = \vec{B} & C_{12} \vec{A} + \vec{V}_e = \vec{B} \\
 \vec{V}_{e1} = \vec{B} - C_1 \vec{A} & \vec{V}_{e2} = \vec{B} - C_2 \vec{A} & \vec{V}_e = \vec{B} - C_{12} \vec{A}
 \end{array}$$

min
error

- while approximating one vector along the other more the error should be minimized. Error can be minimized by projecting the vector in $\perp^r$ manner. Therefore to find the projection of one vector along the other vector the third representation is considered.

⇒ PROJECTION OF $\vec{B}$ ALONG $\vec{A}$ -



$$\text{Projection of } \vec{B} \text{ along } \vec{A} = |\vec{B}| \cos \theta$$

$$C_{12} |\vec{A}| = |\vec{B}| \cos \theta$$

$$C_{12} |\vec{A}|^2 = |\vec{A}| |\vec{B}| \cos \theta$$

$$C_{12} = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}|^2}$$

- (1)

Similarly A along B

$$C_{21} = \frac{\vec{A} \cdot \vec{B}}{|\vec{B}|^2}$$

$$\text{If } \vec{A} = \vec{B}, \quad C_{12} = 1$$

$$\text{If } \theta = 90^\circ, \quad C_{12} = 0.$$

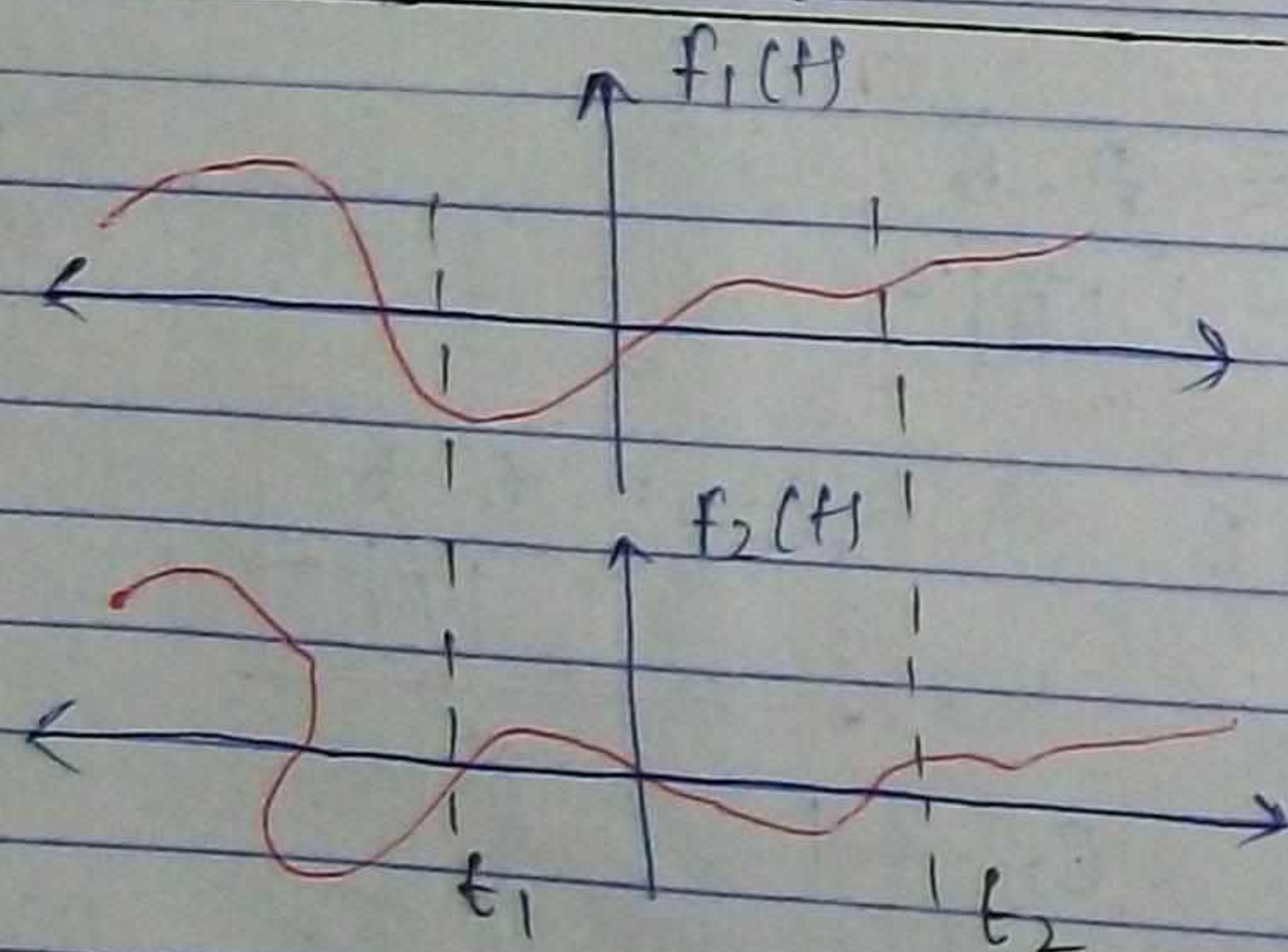
- $C_{12} = 1$ implies that $\vec{A}$ and $\vec{B}$ are identical vectors

- $C_{12} = 0$ implies $\bar{A}$ and $\bar{B}$ are mutually orthogonal vectors. Therefore C_{12} represents the degree of similarity b/w $\bar{A}$ and $\bar{B}$.

- If $C_{12} = 0$ then $\bar{A}$ and $\bar{B}$ are said to be mutually orthogonal vectors (or) mutually exclusive vectors. (or) mutually independent vectors. i.e. one vector does not contain the component of the other vector.

- Similarity b/w $\bar{A}$ and $\bar{B}$ can completely be determined from the numerator of eq (1) i.e. Dot product.

⇒ APPROXIMATION / PROJECTION OF FUNCTION $f_1(t)$ ALONG THE FUNⁿ $f_2(t)$ OVER INTERVAL $[t_1, t_2]$



- from the analogy b/w vectors and signal approxⁿ / projection of $f_1(t)$ along $f_2(t)$ can be written as:

(*)

$$C_{12} = \frac{\int_{t_1}^{t_2} f_1(t) f_2(t) dt}{\int_{t_1}^{t_2} f_2^2(t) dt} \quad (2)$$

- If $C_{12} = 1$, $f_1(t)$ and $f_2(t)$ are ~~orth~~ identical functions over interval $[t_1, t_2]$.

- If $C_{12} = 0$, $f_1(t)$ and $f_2(t)$ are orthogonal funⁿ over interval $[t_1, t_2]$.

- If two functions are orthogonal they are said to be mutually exclusive funⁿ (or) mutually independent funⁿ. i.e. one funⁿ does not contain the component of the other funⁿ. Therefore similarity b/w two function can completely be determined from the numerator integral of eq (2).

(*)

- If $\int_{t_1}^{t_2} f_1(t) \cdot f_2(t) dt = 0$, $f_1(t)$ and $f_2(t)$ are orthogonal funⁿ.

(*)

- If $\int_{t_1}^{t_2} f_1(t) \cdot f_2(t) dt = 1$, $f_1(t)$ and $f_2(t)$ are orthonormal funⁿ.

Ques. Prove that the follⁿ. funⁿ are orthogonal over the interval (t_0, t_0+T) , where $T = \frac{2\pi}{\omega_0}$, for the integer values of m & n .

① $\cos(m\omega_0 t) \cdot \cos(n\omega_0 t)$

② $\sin(m\omega_0 t) \cdot \sin(n\omega_0 t)$

③ $\cos(m\omega_0 t) \cdot \sin(n\omega_0 t)$

Solⁿ: ① $\int_{t_0}^{t_0+T} \cos(m\omega_0 t) \cdot \cos(n\omega_0 t) dt$

$$= \frac{1}{2} \int_{t_0}^{t_0+T} \cos(m-n)\omega_0 t dt + \frac{1}{2} \int_{t_0}^{t_0+T} \cos(m+n)\omega_0 t dt$$

$= 0$ for $(m \neq n)$

i.e. $\cos(0\omega_0 t), \cos(\omega_0 t), \cos(2\omega_0 t) \dots$ } orthogonal over (t_0, t_0+T)

② $\int_{t_0}^{t_0+T} \sin(m\omega_0 t) \sin(n\omega_0 t) dt$

$$= \frac{1}{2} \int_{t_0}^{t_0+T} \cos(m-n)\omega_0 t dt + \frac{1}{2} \int_{t_0}^{t_0+T} \cos(m+n)\omega_0 t dt$$

$= 0$, for $m \neq n$

i.e. $\sin(0\omega_0 t), \sin(\omega_0 t), \sin(2\omega_0 t) \dots$ } orthogonal over (t_0, t_0+T)

③ $\int_{t_0}^{t_0+T} \cos(m\omega_0 t) \cdot \sin(n\omega_0 t) dt$

$$= \frac{1}{2} \int_{t_0}^{t_0+T} \sin(m-n)\omega_0 t dt + \frac{1}{2} \int_{t_0}^{t_0+T} \sin(m+n)\omega_0 t dt$$

$= 0$ for all m and n
($\sin$ and $\cos$ are always orthogonal)

— Since infinite sine and infinite cosine are mutually exclusive, any physical quantity can be expressed as the sum of infinite sine and cosines, which is known as Fourier series representation.

FOURIER SERIES

⇒ CONDITION FOR THE EXISTENCE OF FOURIER SERIES
(DIRICHLET'S CONDⁿ)

- Let $x(t)$ be periodic with the fundamental period T .

① Signal should be absolutely integrable over one period.

$$\int_T |x(t)| dt < \infty$$

② Signal should have finite maxima & minima over one period and each of them must also be finite.

③ Signal must have finite discontinuity over one period and each of them must also be finite.

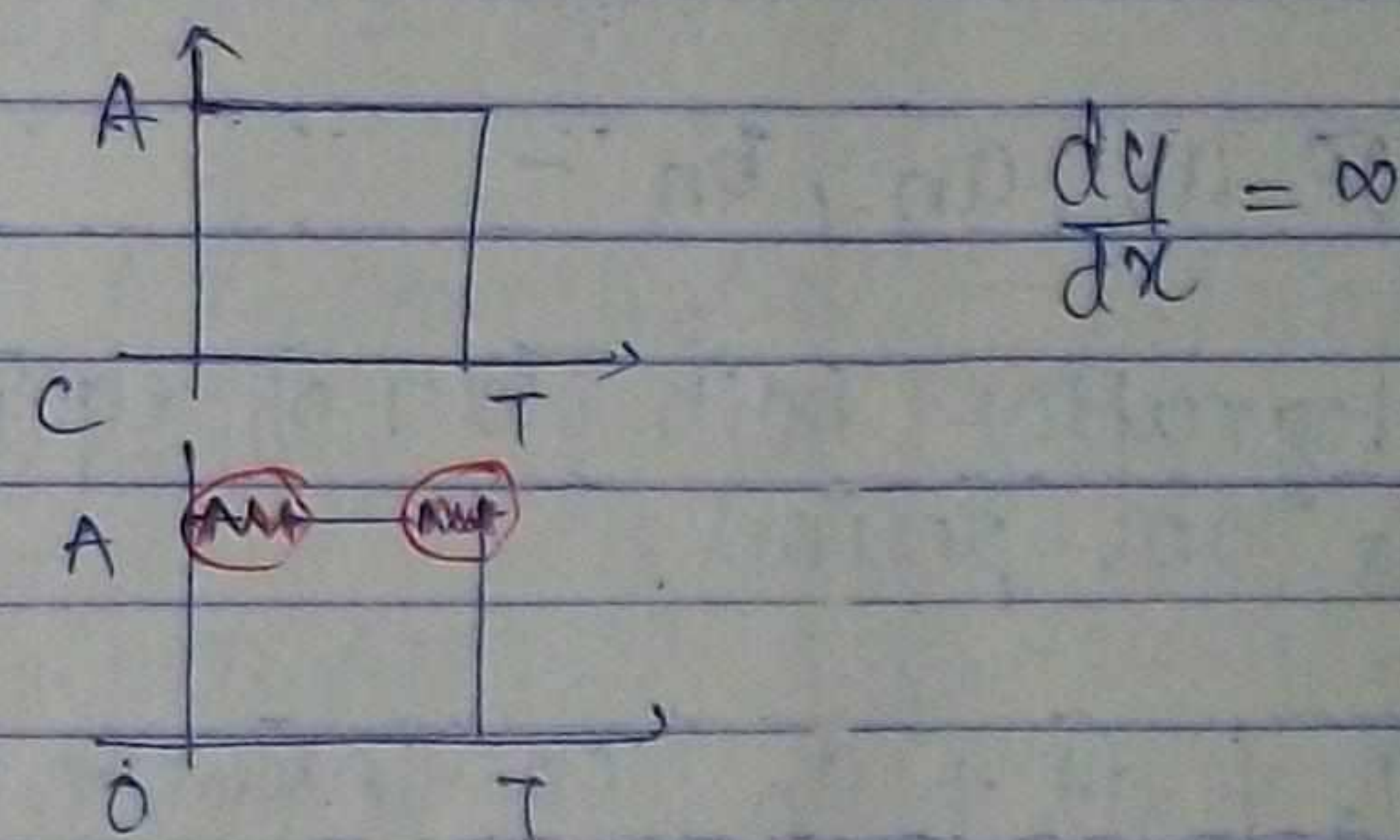
- first condⁿ is weak condⁿ and ② and ③ are strong condⁿ.

- ① is necessary condⁿ.
② and ③ are sufficient condⁿ.

- If the amplitude of the signal increases or decreases suddenly, it cannot be

represented completely in Fourier series, if it is represented in Fourier series it produces oscillation at the sudden change. These oscillations are known as GIBB'S PHENOMENON.

- Gibbs phenomenon is due to the slow convergence of Fourier series.



⇒ REPRESENTATION OF FOURIER SERIES -

$$x(t) = \sum \left(\begin{array}{l} a_0 \cos \omega_0 t, a_1 \cos \omega_0 t, a_2 \cos 2\omega_0 t, \dots \\ b_0 \sin \omega_0 t, b_1 \sin \omega_0 t, b_2 \sin 2\omega_0 t, \dots \end{array} \right)$$

$$x(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)]$$

⇒ FORMS OF FOURIER SERIES -

- ① TRIGONOMETRIC form
- ② POLAR form
- ③ EXPONENTIAL form

(A) TRIGONOMETRIC FOURIER SERIES -

$$x(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)] \quad \text{--- (1)}$$

a_0, a_n, b_n are T.F.S coefficients and $\omega_0 = \frac{2\pi}{T}$

EVALUATION OF a_0, a_n, b_n -

① a_0 : Integrating both sides of eq (1) wrt T over one period.

$$\int_T x(t) dt = a_0 \int_T dt + a_n \int_T \cos(n\omega_0 t) dt + b_n \int_T \sin(n\omega_0 t) dt$$

$$a_0 = \frac{1}{T} \int_T x(t) dt$$

- a_0 is the dc component, or average value of the $x(t)$.

② a_n : multiplying both sides of eq (1) with $\cos(m\omega_0 t)$ and integrating wrt T over one period,

$$\int_T x(t) \cos(m\omega_0 t) dt = a_0 \int_T \cos(m\omega_0 t) dt +$$

$$a_n \int_T \cos(m\omega_0 t) \cos(n\omega_0 t) dt + b_n \int_T \cos(m\omega_0 t) \sin(n\omega_0 t) dt \quad \text{--- (2)}$$

$$I = 0 \quad \text{--- (3)}$$

$$II = \frac{1}{2} \int_T \cos(m-n)\omega_0 t dt + \frac{1}{2} \int_T \cos(m+n)\omega_0 t dt$$

$$= 0, m \neq n \\ = T/2, m = n \quad \text{--- (4)}$$

$$III = \frac{1}{2} \int_T \sin(m-n)\omega_0 t dt + \frac{1}{2} \int_T \sin(m+n)\omega_0 t dt$$

$$= 0, \text{ for all } m \text{ and } n \quad \text{--- (5)}$$

Substituting (3), (4), (5) in eq (2):

$$a_n = \frac{2}{T} \int_T x(t) \cos(n\omega_0 t) dt$$

③ b_n : multiply both sides of eq (1) with $\sin(m\omega_0 t)$ and integrating wrt T over one period,

$$\int_T x(t) \sin(m\omega_0 t) dt = a_0 \underbrace{\int_T \sin(m\omega_0 t) dt}_{\text{IV}} + \underbrace{a_n \int_T \sin(m\omega_0 t) \cos(n\omega_0 t) dt}_{\text{V}} + \underbrace{b_n \int_T \sin(m\omega_0 t) \sin(n\omega_0 t) dt}_{\text{VI}} \quad (6)$$

$$\text{IV} = 0 \quad (7)$$

$$\begin{aligned} \text{VI} &= \frac{1}{2} \int_T \cos(m-n)\omega_0 t dt - \frac{1}{2} \int_T \cos(m+n)\omega_0 t dt \\ &= \begin{cases} 0 & m \neq n \\ T/2 & m = n \end{cases} \quad (9) \end{aligned}$$

$$\begin{aligned} \text{V} &= \frac{1}{2} \int_T \sin(m-n)\omega_0 t dt + \frac{1}{2} \int_T \sin(m+n)\omega_0 t dt \\ &= 0 \quad (8) \end{aligned}$$

Substituting (7), (8), (9) in (6):

$$b_n = \frac{2}{T} \int_T x(t) \sin(n\omega_0 t) dt$$

SYMMETRY CONDⁿ IN T.F.S

① EVEN SYMMETRY -

for every even funⁿ $b_n = 0$
Therefore TFS of even funⁿ contain DC (a_0) and cosine terms (a_n).

$$X_e(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t)$$

② ODD SYMMETRY -

for every odd funⁿ $a_0 = 0$ $a_n = 0$
Therefore TFS of odd funⁿ contain sine terms only.

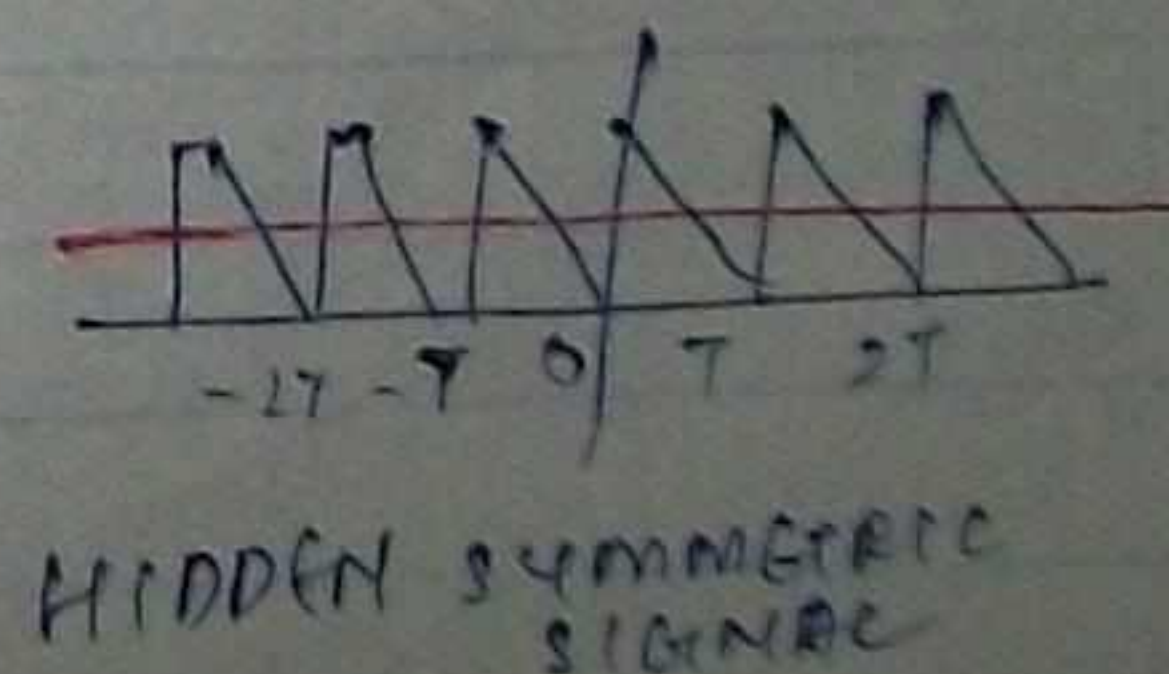
$$X_o(t) = \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)$$

③ HIDDEN SYMMETRY -

(obstacle)
A symmetry that is obscured by DC is known as hidden symmetry. It can be obtained by adding or subtracting the DC.

- TFS of hidden symmetric signal contain DC and sine terms.

$$X_H(t) = a_0 + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)$$



(4) HALF WAVE SYMMETRY - If the alternate half cycles are symmetric then it is known as half wave symmetry.

for every half wave symmetric signal

$$x(t) = \pm x(t \pm \frac{T}{2})$$

$$\omega_0, 3\omega_0, 5\omega_0, \dots$$

- TFS of half wave symmetric signal contains odd harmonics only i.e. $\omega_0, 3\omega_0, \dots$

if $x(t) = +x(t \pm T/2)$ HWE

$= -x(t \pm T/2)$ HWO

- TFS of half wave even symmetric signal contains odd harmonics of cosine terms.

- TFS of half wave odd symmetric signal contains odd harmonic of sine terms.

(B) POLAR FORM / ALTERNATE FORM / COMPACT FORM - form T.F.S -

$$x(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)] \quad (10)$$

$$x(t) = S_0 + \sum_{n=1}^{\infty} S_n \cos(n\omega_0 t + \phi_n) \quad (11)$$

Eqn (11) is known as polar form (or) compact form which is used to represent the spectrum of the signal $x(t)$.

Equating (10) and (11):

$$a_0 + \sum_{n=1}^{\infty} (a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)) = S_0 + \sum_{n=1}^{\infty} [S_n \cos \phi_n \cos(n\omega_0 t) - S_n \sin \phi_n \sin(n\omega_0 t)]$$

By comparing:

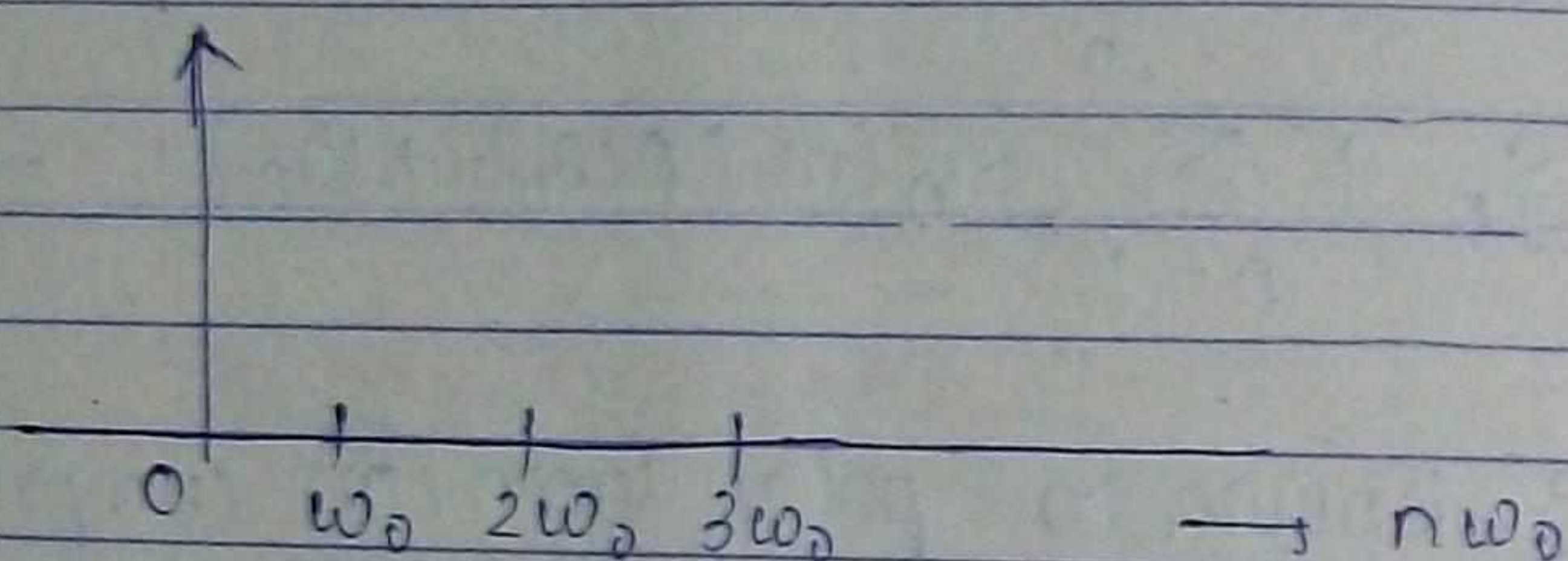
$$S_0 = a_0$$

$$\cos \phi_n = \frac{a_n}{S_n}, \quad \sin \phi_n = -\frac{b_n}{S_n}$$

$$\phi_n = -\tan^{-1}\left(\frac{b_n}{a_n}\right)$$

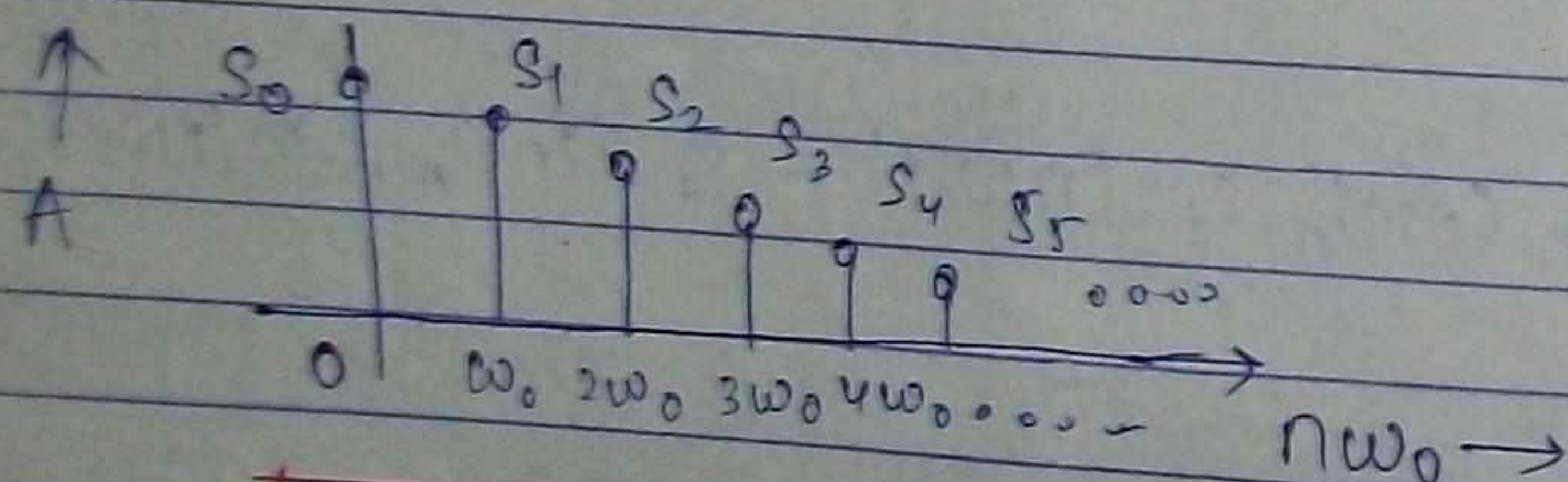
SPECTRA OF T.F.S -

$$x(t) = S_0 + S_1 \cos(\omega_0 t + \phi_1) + S_2 \cos(2\omega_0 t + \phi_2) + S_3 \cos(3\omega_0 t + \phi_3) + \dots$$



- Therefore the spectra of T.F.S is one sided line spectra of one sided discrete spectra.

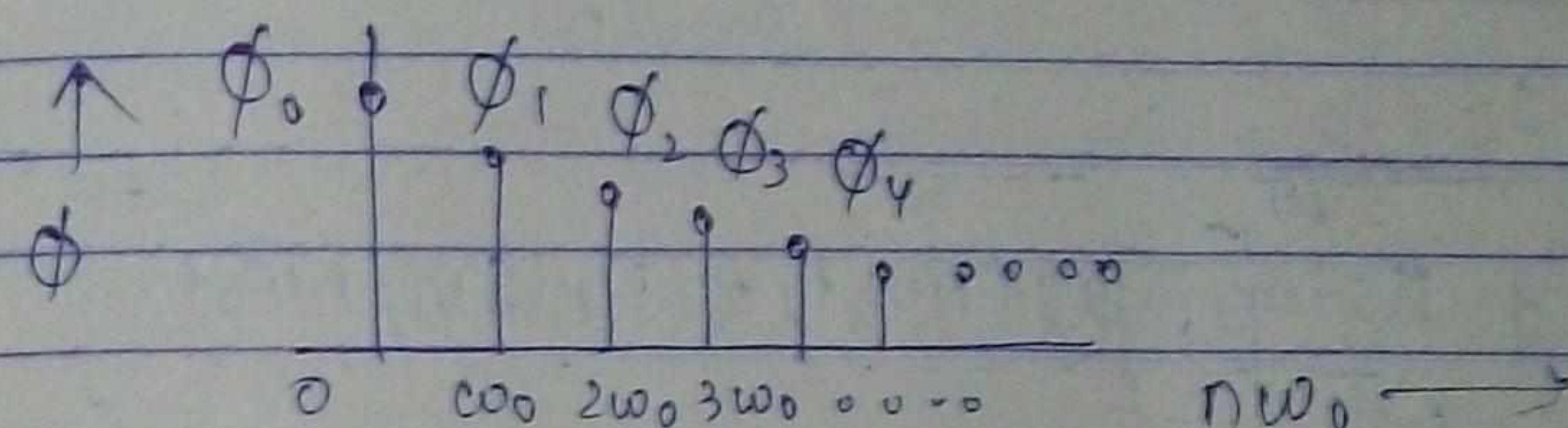
MAGNITUDE / AMP. SPECTRUM -



$$S_0 = a_0, \quad n=0$$

$$S_n = \sqrt{a_n^2 + b_n^2}, \quad \text{for } n \neq 0$$

PHASE SPECTRUM -



- phase spectrum is also one sided line spectrum
(or) one sided discrete spectrum.

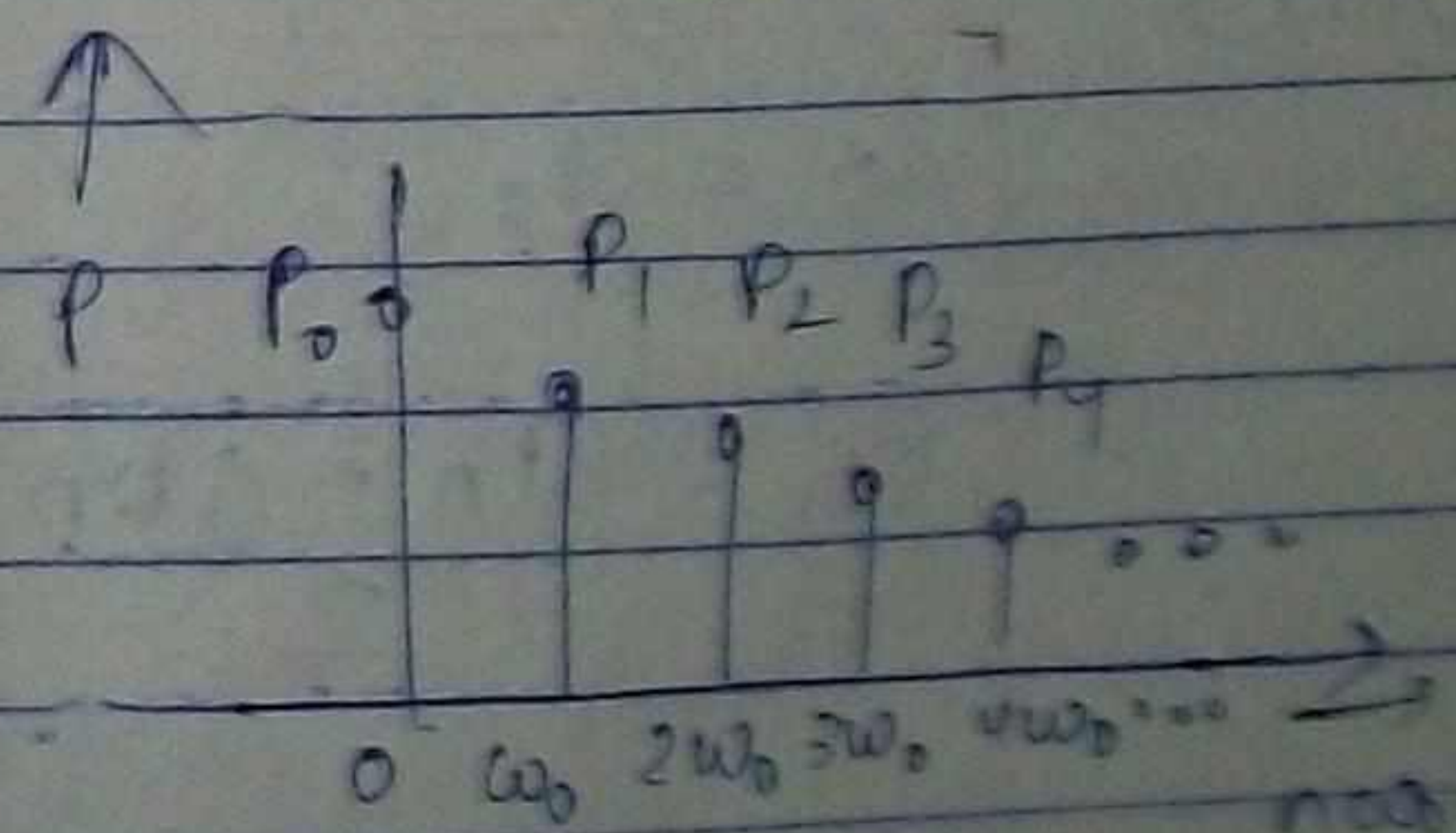
$$\phi_0 = 0, \quad \text{+ve d.c.} \\ \phi_0 = \pi, \quad \text{-ve d.c.}$$

$$\phi_n = -\tan^{-1}\left(\frac{b_n}{a_n}\right), \quad \text{for } n \neq 0.$$

POWER SPECTRUM -

$$x(t) = S_0 + \sum_{n=1}^{\infty} S_n \cos(n\omega_0 t + \phi_n)$$

$$P_{\text{total}} = P_0 + \sum_{n=1}^{\infty} P_n$$



$$P_0 = S_0^2 \quad \text{for } n=0$$

$$P_n = \frac{S_n^2}{2} \quad \text{for } n \neq 0$$

$$= \frac{a_n^2 + b_n^2}{2} \quad \text{for } n \neq 0$$

(c) EXPONENTIAL FOURIER SERIES -

- for EFS:

$$x(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)]$$

$$= a_0 + \sum_{n=1}^{\infty} a_n \left(\frac{e^{jn\omega_0 t} + e^{-jn\omega_0 t}}{2} \right) + b_n \left(\frac{e^{jn\omega_0 t} - e^{-jn\omega_0 t}}{2j} \right)$$

$$x(t) = a_0 + \sum_{n=1}^{\infty} \left(\frac{a_n - j b_n}{2} \right) e^{jn\omega_0 t} + \left(\frac{a_n + j b_n}{2} \right) e^{-jn\omega_0 t}$$

$$x(t) = c_0 + \sum_{n=1}^{\infty} c_n e^{jn\omega_0 t} + c_{-n} e^{-jn\omega_0 t}$$

where,

$$c_0 = a_0$$

$$c_n = \frac{a_n - j b_n}{2}$$

$$c_{-n} = \frac{a_n + j b_n}{2}$$

RR

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

(12)

Eq (12) is known as exponential Fourier series.

where, c_n are EFS coefficients or complex coefficients.

$$\text{where, } c_0 = \frac{1}{T} \int_T x(t) dt \quad \text{for } n=0$$

$$c_n = \frac{1}{T} \int_T x(t) e^{-jn\omega_0 t} dt, \quad \text{for } n \neq 0$$

⇒ RELATION B/W TFS and EFS COEFFICIENTS -

$$a_0 = c_0$$

$$a_n = c_n + c_{-n}$$

$$b_n = j(c_n - c_{-n})$$

⇒ SYMMETRY CONDⁿ IN EFS -

① EVEN SYMMETRY -

for every even $\sin^n$ $b_n = 0$

$$c_0 = a_0$$

$$c_n = \frac{a_n}{2} = c_{-n}$$

Therefore if $x(t)$ is real then its EFS coefficient are real and equal.

② ODD SYMMETRY -

for every odd $\sin^n$ $a_0 = 0$, $a_n = 0$

$$c_0 = 0$$

$$c_n = -j \frac{b_n}{2}$$

$$c_n = -c_{-n}$$

$$c_{-n} = j \frac{b_n}{2}$$

If $x(t)$ is odd then EFS coefficient are imaginary and odd.

SPECTRUM OF EFS -

$$\begin{aligned} x(t) &= \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t} \\ &= c_0 + c_1 e^{j\omega_0 t} + c_2 e^{j2\omega_0 t} + \dots \\ &\quad + c_{-1} e^{-j\omega_0 t} + c_{-2} e^{-j2\omega_0 t} + \dots \end{aligned}$$

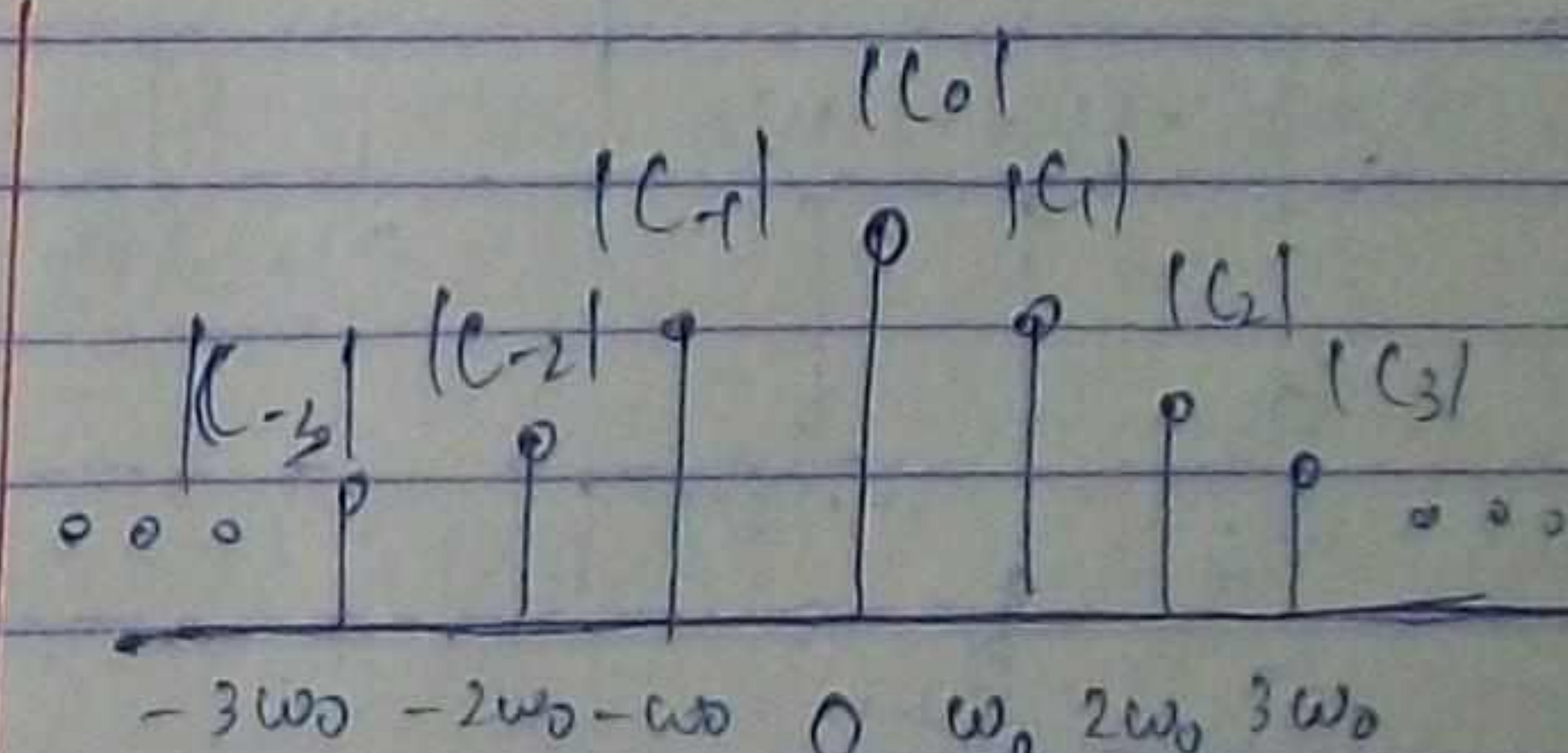
- Spectrum of EFS is two sided line spectra or two sided discrete spectra which is designed at $0, \pm\omega_0, \pm2\omega_0, \dots$

MAGNITUDE SPECTRUM -

$$|c_0| = |a_0|$$

$$|c_n| = \frac{1}{2} (a_n^2 + b_n^2)^{1/2}$$

$$|c_n| = |c_{-n}|$$



- Therefore magnitude spectrum is an even function of ω , i.e. symmetric about the vertical axis passing through $\omega = 0$.

$$\begin{aligned} (\text{MAG})_{\text{EFS}} &= (\text{MAG})_{\text{EFS}} \quad \forall n=0 \\ &= \frac{(\text{MAG})_{\text{EFS}}}{2} \quad \forall n \neq 0 \end{aligned}$$

PHASE SPECTRUM -

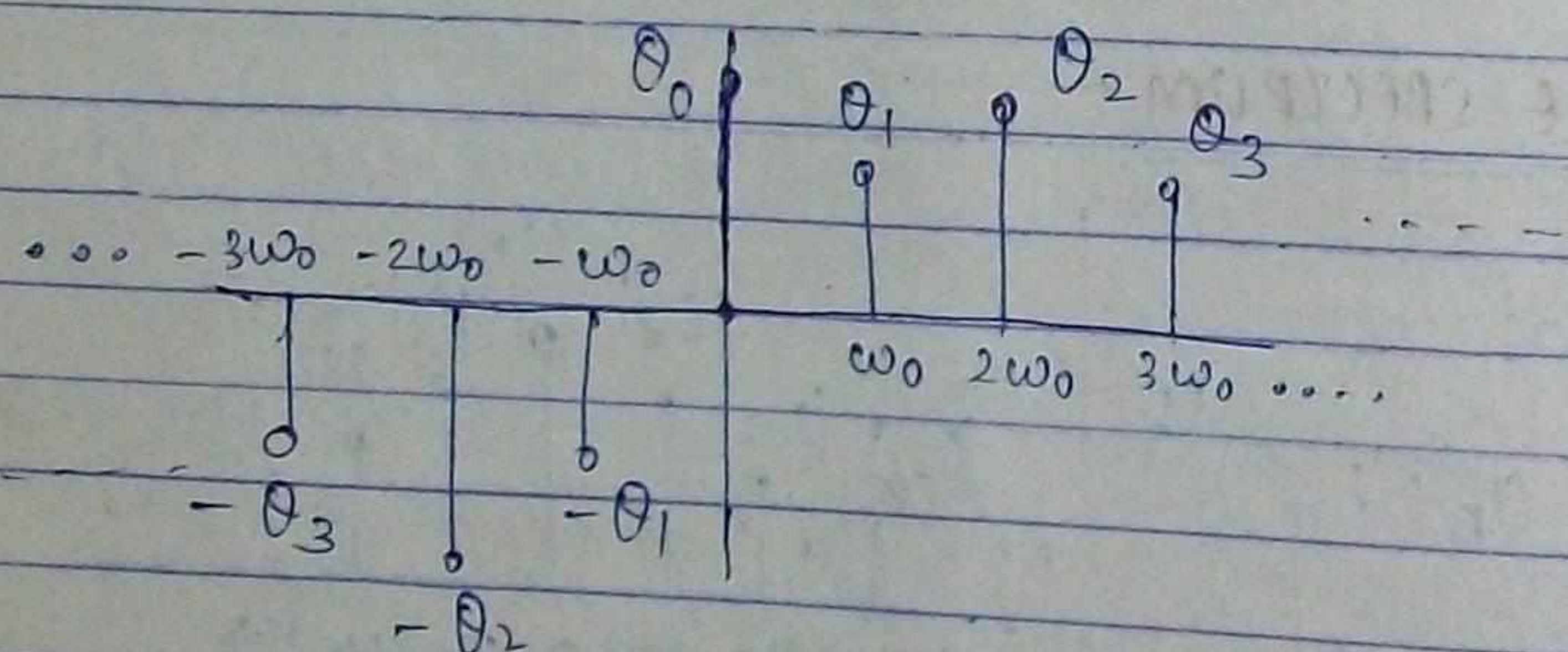
$$\theta = 0^\circ \quad \text{for +ve dc}$$

$$\theta = \pi \quad \text{for -ve dc.}$$

$$\theta_n = -\tan^{-1}\left(\frac{b_n}{a_n}\right), \quad \text{for } n > 0$$

$$\theta_{-n} = \tan^{-1}\left(\frac{b_n}{a_n}\right) \quad \text{for } n < 0$$

$$\theta_n = -\theta_{-n}$$



- Therefore phase spectrum is odd funⁿ of ω
i.e. antisymmetric about the vertical axis passing through the origin.

$$(\text{Ph})_{\text{EFS}} = (\text{Ph})_{\text{TFS}}, \quad \text{for } n \geq 0$$

POWER SPECTRA -

$$P_{\text{TOTAL}} = \sum_{n=-\infty}^{\infty} |C_n|^2$$

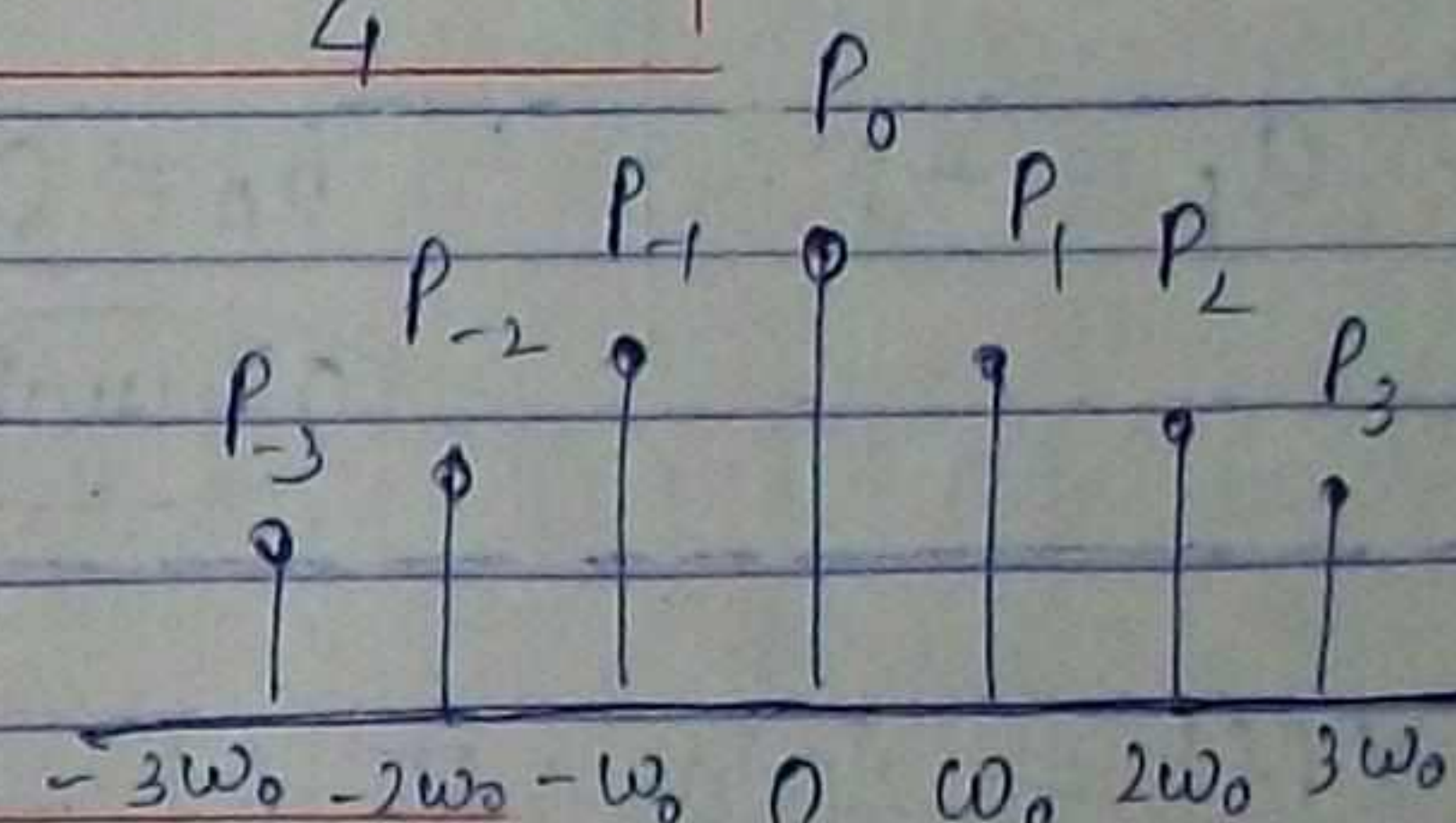
$$P_n = P_{-n}$$

$$P_0 = |C_0|^2$$

$$P_n = |C_n|^2 = \frac{a_n^2 + b_n^2}{4}$$

$$P_{-n} = |C_{-n}|^2 = \frac{a_n^2 + b_n^2}{4}$$

- Therefore power spectra is even function of ω



$$P_{\text{EFS}} = (P)_{\text{TFS}}, \quad \text{for } n = 0$$

$$= \frac{(P)_{\text{TFS}}}{2}, \quad \text{for } n \neq 0.$$

Ques. find the TFS coefficient for the following:

① $x(t) = 10 \sin^2\left(\frac{3\pi}{7}t\right)$

(*) first check the periodicity and everlasting nature.

$$x(t) = 5 - 5 \cos\left(\frac{6\pi}{7}t\right)$$

$$\omega_0 = 6\pi/7 \text{ r/s}$$

Comparing with,

$$x_1(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{6\pi}{7}nt\right) + b_n \sin\left(\frac{6\pi}{7}nt\right)$$

$$a_0 = 5 \quad | \quad a_n = 0$$

$$a_1 = -5 \quad | \quad b_n = 0$$

Otherwise

$$(2) x(t) = 4 \cos\left(\frac{\pi}{10}t\right) - 5 \sin(10t - 30^\circ)$$

$$\omega_1 = \frac{\pi}{10}, 10$$

Non periodic

Since it is Aperiodic Fourier series is not applicable

$$(3) x(t) = 15 \cos\left(\frac{3\pi}{7}t - 30^\circ\right) - 10 \cos(4\pi t) + 15 \sin(0.7\pi t) - 25 \sin(15\pi t)$$

Periodic

$$\omega = \frac{14}{3}, \frac{1}{2}, \frac{20}{7}, \frac{2}{15}$$

$$\omega = \frac{3\pi}{7}, \frac{4\pi}{10}, \frac{7\pi}{10}, \frac{15\pi}{10}$$

$$\frac{\omega_1}{\omega_2} = \frac{3}{28}, \quad \frac{\omega_2}{\omega_3} = \frac{40}{7}, \quad \frac{\omega_3}{\omega_4} = \frac{7}{150}$$

$$\omega_0 = \frac{\text{GCD}(3\pi, 4\pi, 7\pi, 15\pi)}{\text{LCM}(7, 1, 10, 1)} = \frac{\pi}{70}$$

TPS :

$$a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{\pi}{70}nt\right) + b_n \sin\left(\frac{\pi}{70}nt\right) =$$

$$7.5\sqrt{3} \cos\left(\frac{3\pi}{7}t\right) - 7.5 \sin\left(\frac{3\pi}{7}t\right) - 10 \cos(4\pi t) +$$

$$15 \sin(0.7\pi t) - 25 \sin(15\pi t)$$

Comparing

$$a_0 = 0$$

$$a_{30} = 7.5\sqrt{3}$$

$$b_{30} = -7.5$$

$$a_{280} = -10$$

$$b_{49} = 15$$

$$b_{1050} = -25$$

Otherwise

$$a_n = 0$$

$$b_n = 0$$

$$\frac{n\pi}{70} = \frac{3\pi}{7} \quad n=30$$

$$\frac{n\pi}{70} = 4\pi \quad n=280$$

$$\frac{n\pi}{70} = 0.7\pi \quad n=49$$

$$\frac{n\pi}{70} = 15\pi \quad n=1050$$

$$(4) \quad x(t) = 4 \sin(15t) - 14 \sin\left(\frac{3}{5}t + 45^\circ\right) - 20 \cos(0.3t)$$

$$\omega = 15, \frac{3}{5}, 0.3 \quad \Rightarrow \quad \omega_0 = \frac{3}{10}$$

Periodic

TFS:

$$a_0 + \sum_{n=1}^{\infty} \left[a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t) \right]$$

$$= 4 \sin(15t) - \frac{14}{\sqrt{2}} \sin\left(\frac{3}{5}t\right) - \frac{14}{\sqrt{2}} \cos\left(\frac{3}{5}t\right) - 20 \cos(0.3t)$$

$$a_0 = 0$$

$$b_{50} = 4$$

$$b_2 = -14/\sqrt{2}$$

$$a_2 = -14/\sqrt{2}$$

$$a_1 = -20$$

$$a_n = 0$$

$$b_n = 0$$

otherwise,

$$\frac{3 \cdot n}{10} = 15$$

$$\frac{3}{10} n = \frac{3}{5}$$

$$\frac{2}{3} \cdot \frac{10}{5} = \frac{2}{3} \cdot 2 = \frac{4}{3}$$

Q. find the EFS coefficient of the following:

$$(1) \quad x(t) = 10 \sin(5t - 30^\circ)$$

$$\omega = 5$$

$$C_0 = 0$$

$$C_n = 1 + j \frac{10}{2} \angle + 87^\circ$$

$$x_1(t) = -j5 \left(\frac{e^{j(5t-30^\circ)}}{e} - \frac{e^{-j(5t-30^\circ)}}{e} \right)$$

$$\sum_{n=-\infty}^{\infty} C_n e^{j5nt} = -j5 e^{j30^\circ} e^{j5t} + j5 e^{-j30^\circ} e^{-j5t}$$

$$\text{comparing, } C_1 = -j5 e^{-j30^\circ}$$

$$C_{-1} = j5 e^{j30^\circ}$$

$$C_0 = 0$$

$$C_n = 0 = C_{-n}, \text{ otherwise.}$$

$$(2) \quad x(t) = 15 \cos\left(\frac{\pi}{10}t\right) - 10 \sin(3t)$$

NON PERIODIC

Hence EFS is not applicable.

$$(3) X_3(t) = -4 \sin(14t) + 10 \cos(5/7t - 30^\circ) - 15 \cos(0.3t - 45^\circ)$$

$$\omega = 14, 5/7, 3/10 \quad \omega_0 = 1/70$$

$$\sum_{n=-\infty}^{\infty} c_n e^{j \frac{n}{70} t} = -4 \sin(14t) + 10 \cos(5/7t - 30^\circ) - 15 \cos(0.3t - 45^\circ)$$

$$= j2 e^{j14t} - j2 e^{-j14t} + 5 e^{j5/7t} e^{-j30^\circ} + 5 e^{-j5/7t} e^{j30^\circ}$$

$$+ 7.5 e^{j0.3t} e^{-j45^\circ} + 7.5 e^{-j0.3t} e^{j45^\circ}$$

$$7.5 e^{j0.3t} e^{-j45^\circ} + 7.5 e^{-j0.3t} e^{j45^\circ}$$

Comparing:

$$c_0 = 0$$

$$c_{21} = -7.5 e^{+j45^\circ}$$

$$c_{980} = j2$$

$$c_{-21} = -7.5 e^{+j45^\circ}$$

$$c_{-980} = -j2$$

$$c_{50} = 5 e^{-j30^\circ}$$

$$c_{-50} = 5 e^{j30^\circ}$$

$$\frac{n}{70} = 14$$

$$n = 980$$

$$\frac{5}{7} = \frac{n}{70}$$

$$n = 10$$

$$\frac{n}{70} = \frac{3}{10}$$

$$n = 21$$

Ques. TFS coeff. of periodic signal is given by:

$$a_0 = -10$$

$$b_7 = 15$$

$$a_{20} = -0.3$$

$$a_{25} = \frac{3}{5}$$

$$b_{25} = -4$$

find the corresponding EFS coefficient.

$$\text{Sol}^n \quad a_0 = c_0 \quad c_n = \frac{a_n - b_n}{2} \quad c_{-n} = \frac{a_n + b_n}{2}$$

$$c_0 = -10$$

$$c_{20} = \frac{a_{20} - j b_{20}}{2} = -0.015$$

$$c_{-20} = -0.15$$

$$c_7 = \frac{a_7 - b_7}{2} = -j \frac{15}{2} = -j 7.5$$

$$c_{-7} = j 7.5$$

$$c_{25} = \frac{a_{25} - b_{25}}{2} = \frac{3/5 - j(-4)}{2} = \frac{3 + j20}{10}$$

$$c_{-25} = \frac{3 - j20}{10}$$

Ques. find the spectra of the following: (TFS) USING

$$(1) X_1(t) = 10 \sin^2(50t - 30^\circ)$$

$$\phi_n = \text{always} +ve$$

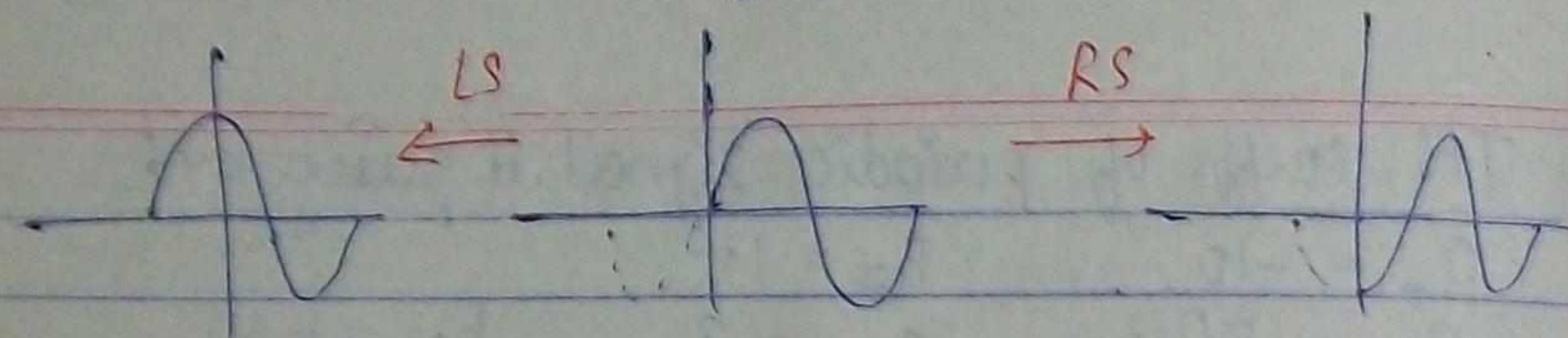
$$= \cos \theta$$

$$= \sin(\theta + 90^\circ)$$

$$\sin \theta$$

$$= -\cos \theta$$

$$= \sin(\theta - 90^\circ)$$



$$= -\sin \theta$$

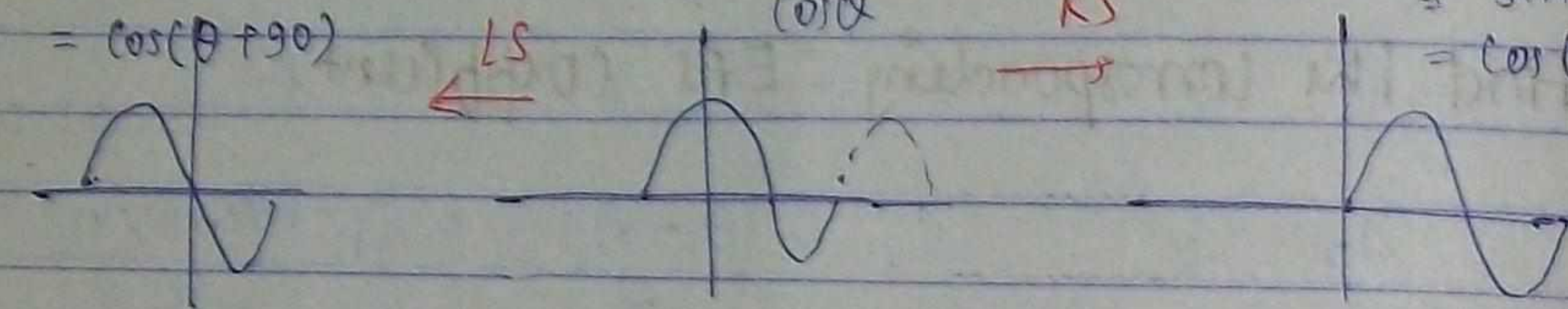
$$= \cos(\theta + 90^\circ)$$

$$\cos \theta$$

$$RS$$

$$= \sin \theta$$

$$= \cos(\theta - 90^\circ)$$



$$(1) X(t) = 10 \sin^2(100t - 30^\circ)$$

$$X(t) = 5 - 5 \cos(100t - 60^\circ)$$

$$\omega_0 = 100 \text{ rad/s}$$

$$S_0 + \sum_{n=1}^{\infty} S_n \cos(100nt + \phi_n) =$$

$$= 5 - 5 \cos(100t - 60^\circ)$$

$$= 5 + 5 \cos(100t - 60 + 180^\circ)$$

$$\text{Comparing, } S_0 = 5$$

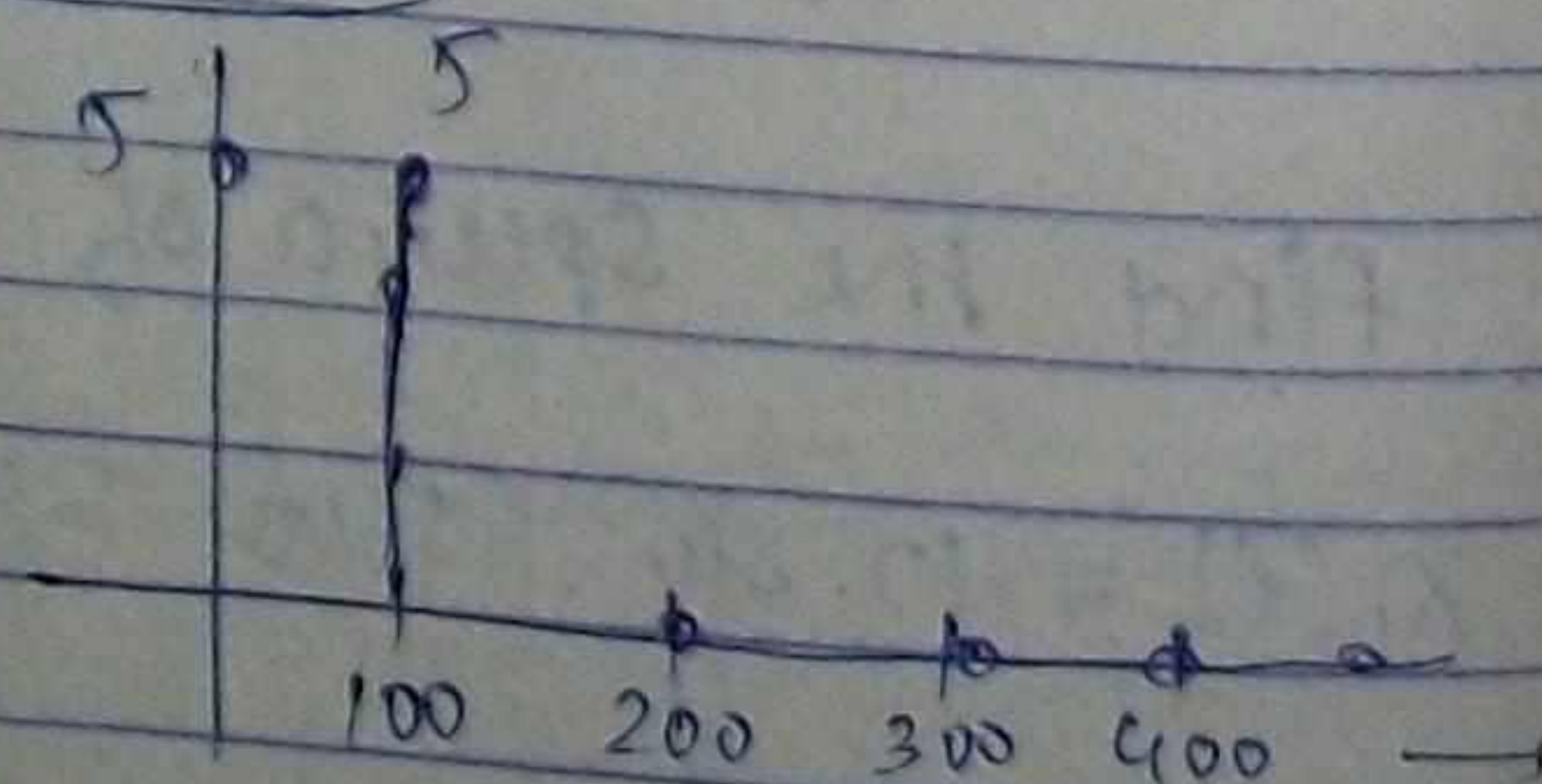
$$S_1 = 5$$

$$\phi_1 = 120^\circ$$

$$S_n = 0, \phi_n = 0$$

otherwise.

MAG. SPECTRUM

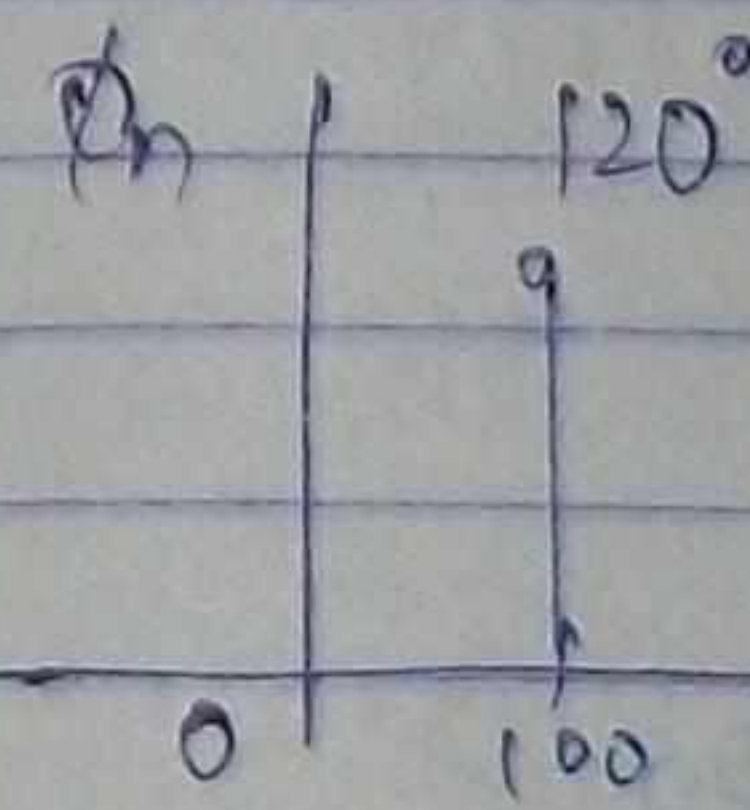


13/11/11

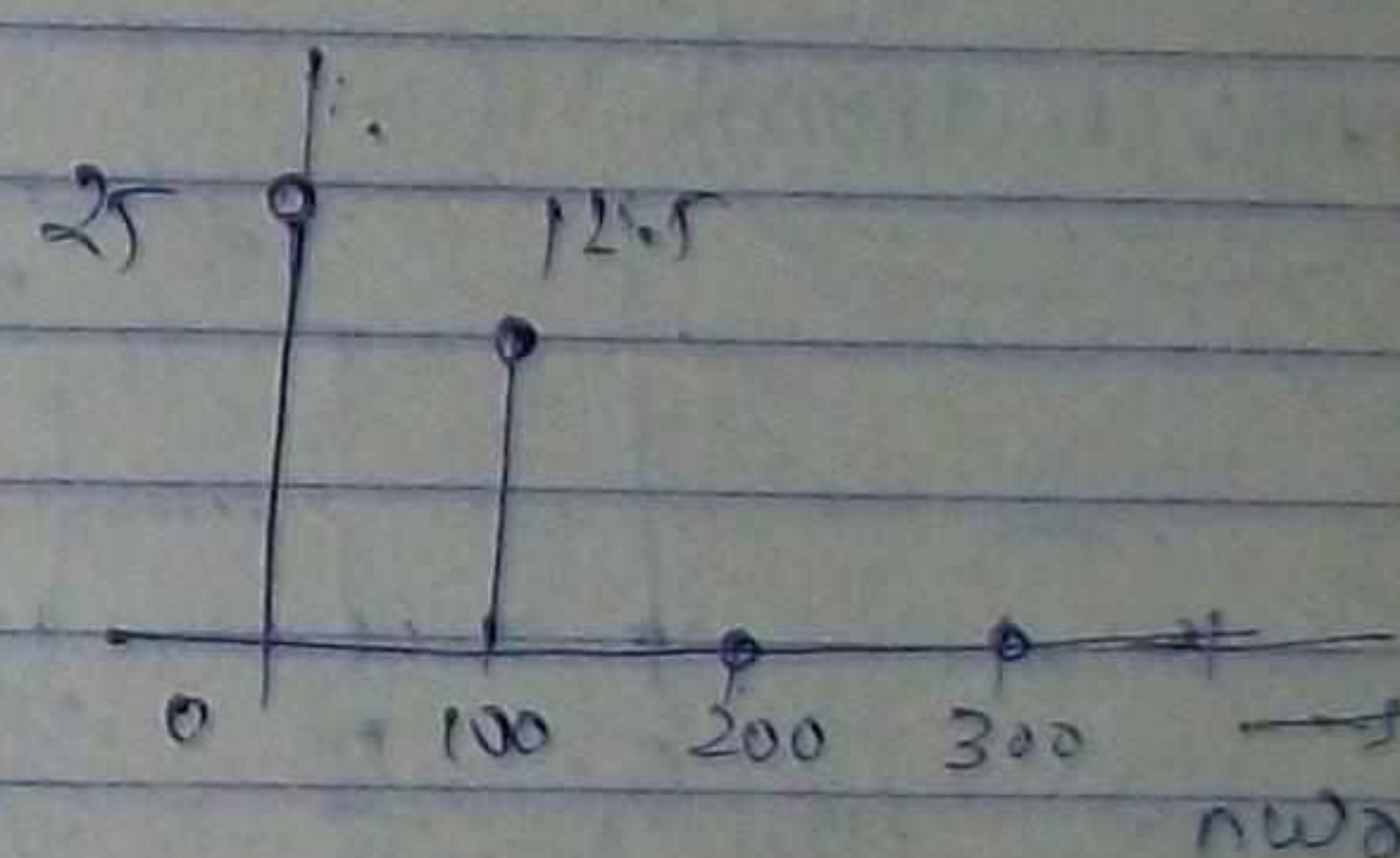
$$P = 100^2$$

$$P_n = \frac{S_n^2}{2}$$

PHASE SPECTRUM



POWER SPECTRUM



$$(2) X(t) = 20 \cos\left(\frac{\pi}{3}t - 45^\circ\right) - 10 \cos(5\pi t) - 15 \sin\left(\frac{3\pi}{7}t\right)$$

$$\omega = \frac{\pi}{3}, 5\pi, \frac{3\pi}{7}$$

$$\omega_0 = \pi/21$$

$$S_0 + \sum_{n=1}^{\infty} S_n \cos\left(\frac{\pi}{21}nt + \phi_n\right) = 20 \cos\left(\frac{\pi}{3}t - 45^\circ\right) + 10 \cos(5\pi t + \pi) + 15 \cos\left(\frac{3\pi}{7}t + 90^\circ\right)$$

Comparing:

$$S_0 = 0$$

$$S_7 = 20, \phi_7 = -45^\circ$$

$$S_{105} = 10, \phi_{105} = \pi$$

$$S_9 = 15, \phi_9 = 90^\circ$$

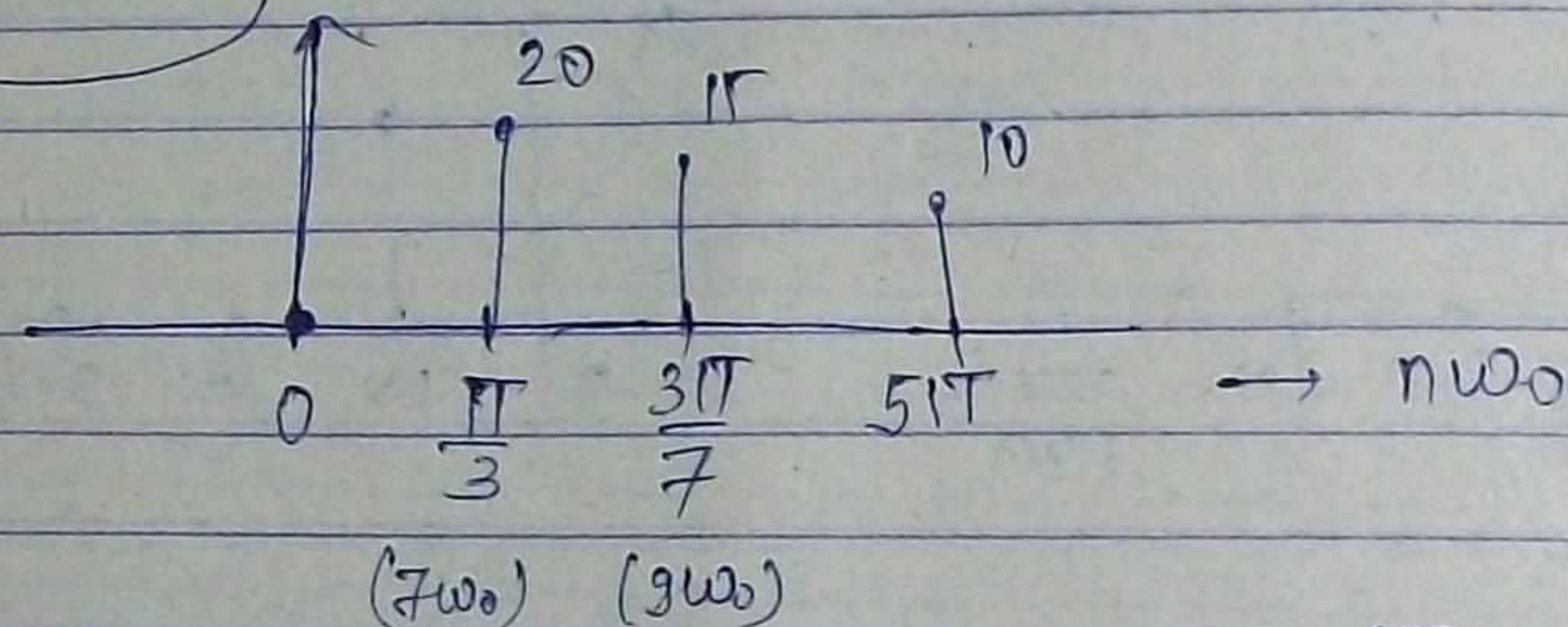
$$\frac{n\pi}{21} = \frac{\pi}{3} \quad n=7$$

$$\frac{n\pi}{21} = 5\pi \quad n=105$$

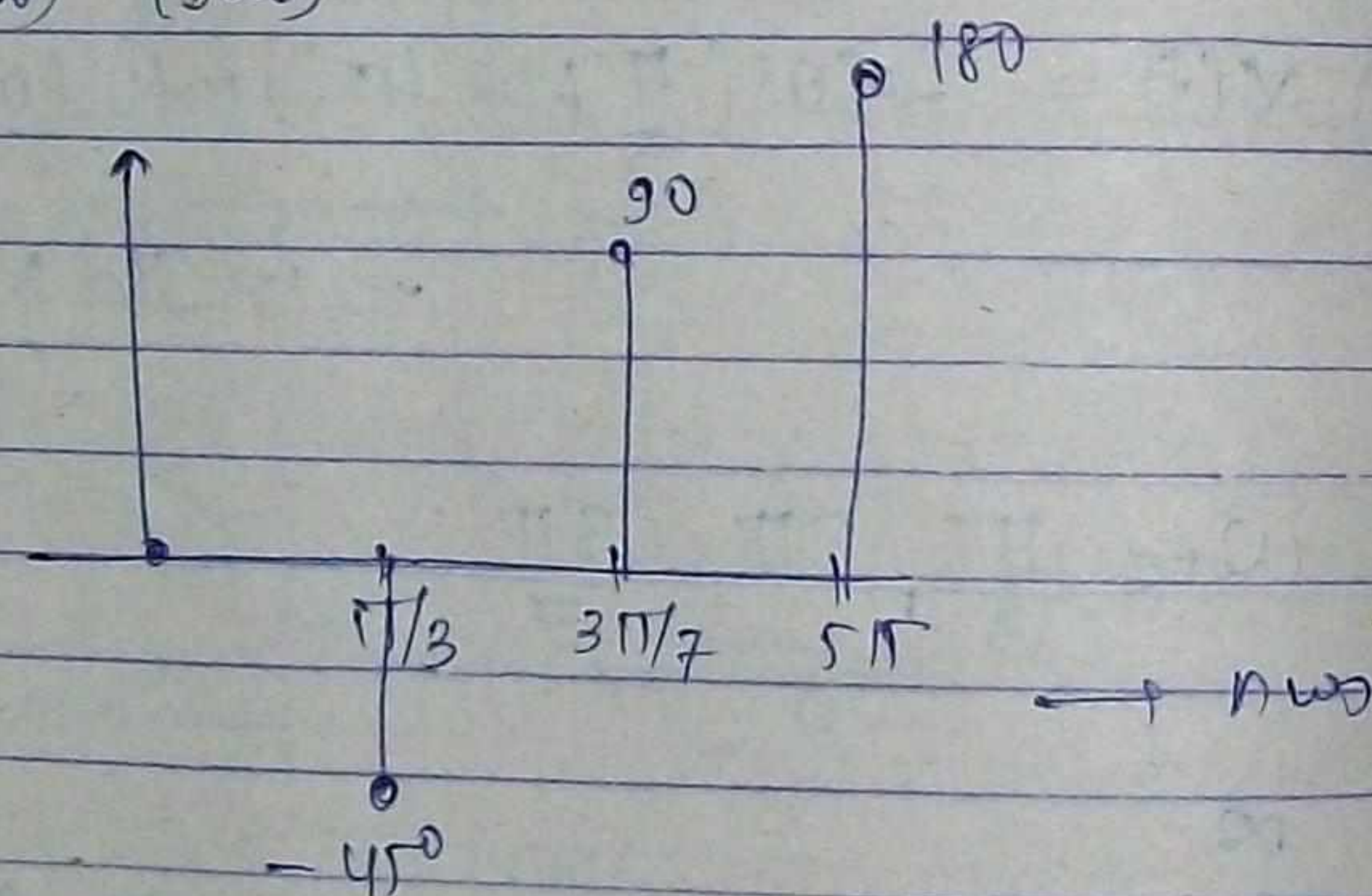
$$\frac{n\pi}{21} = \frac{3\pi}{7} \quad n=9$$

$$P_0 = 0 \quad P_7 = 20^2/2 \quad P_{105} = 10^2/2 \quad P_9 = 15^2/2$$

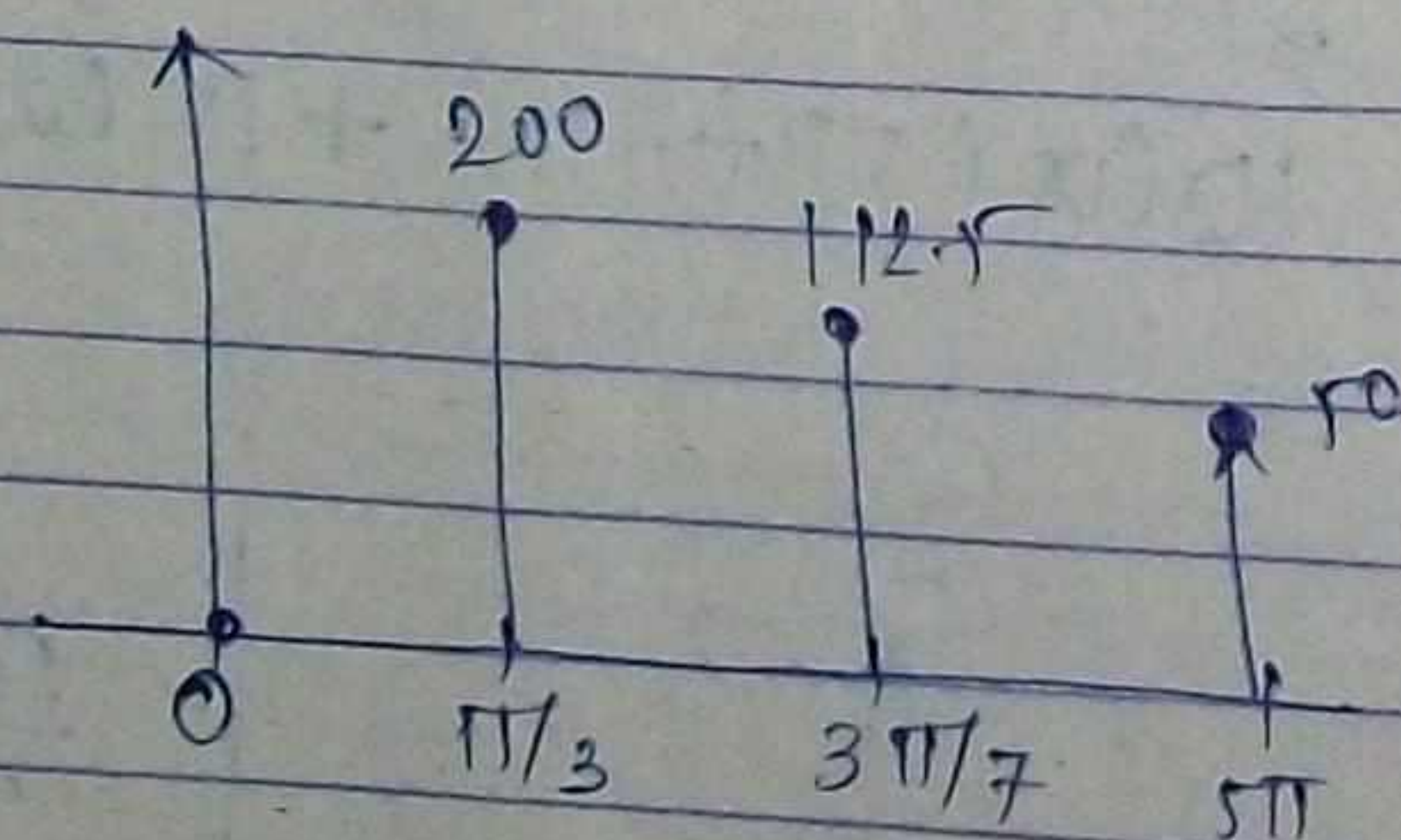
MAG. SPECTRUM



PHASE SPECTRUM



POWER SPECTRUM



Ques. find the exponential coefficient of the following.

$$(1) \quad X(t) = -3 \sin^2(0.7\pi t - 40^\circ)$$

$$X(t) = -\frac{3}{2} + \frac{3}{2} \cos(1.4\pi t - 80^\circ)$$

$$X(t) = -1.5 + 1.5 \cos(1.4\pi t - 80^\circ)$$

$$\omega_0 = 1.4\pi \quad \text{Now, EFS}$$

$$\sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t} = -1.5 + 0.75 e^{j1.4\pi t - j80^\circ} + 0.75 e^{-j1.4\pi t + j80^\circ}$$

Comparing:

$$C_0 + C_1 e^{j1.4\pi t} + C_2 e^{j2 \times 1.4\pi t} + \dots = -1.5 + 0.75 e^{j1.4\pi t - j80^\circ} + 0.75 e^{-j1.4\pi t + j80^\circ}$$

$$C_0 = -1.5$$

$$C_1 = 0.75 e^{-j80^\circ}, \quad C_{-1} = 0.75 e^{j80^\circ}$$

$$(2) \quad X(t) = -4 \cos(10t - 30^\circ) + 15 \cos\left(\frac{5t}{7}\right) - 13 \sin(0.3t)$$

$$\omega = 10, \frac{5}{7}, \frac{3}{10} \quad \omega_0 = 1/70$$

Now, EFS

$$\sum_{n=-\infty}^{\infty} c_n e^{j \frac{n}{70} t} = -2 e^{j 10 t} \cdot e^{-j 30^\circ} - 2 e^{-j 10 t} \cdot e^{j 30^\circ} + 7.5 e^{j 5 t} + 7.5 e^{-j 5 t} + 6.5 j$$

Comparing, $e^{j 10.3 t} - 6.5 j e^{-j 10.3 t}$

$$c_{700} = -2 e^{-j 30^\circ} \quad \frac{n}{70} = 10$$

$$c_{-700} = -2 e^{j 30^\circ} \quad n = 700$$

$$c_{50} = 7.5 \quad \frac{n}{70} = \frac{5}{7} \quad n = 50$$

$$c_{-50} = 7.5 \quad \frac{n}{70} = \frac{0.5}{10} \quad n = 21$$

$$c_{21} = 6.5 j$$

$$c_{-21} = -6.5 j$$

Ques. Sketch the spectra of follⁿ, in EFS.

(i) $x(t) = -5 \cos^2(0.7t - 60^\circ)$ $\omega_0 = 1.4$

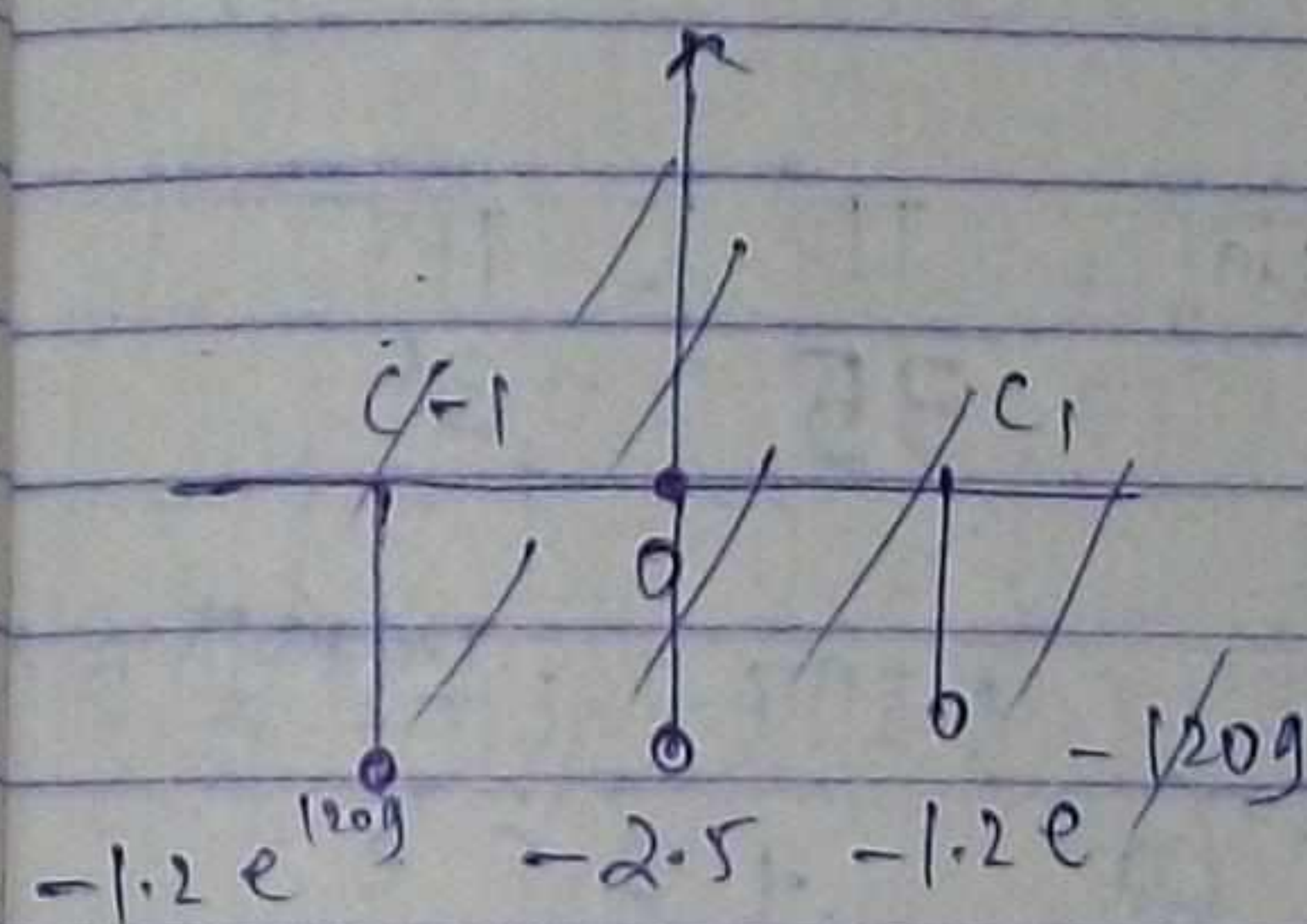
$$x(t) = -2.5 \cos(1.4t - 120^\circ) - 2.5$$

$$\sum_{n=-\infty}^{\infty} c_n e^{j 1.4 t} = -1.2 e^{j 1.4 t - 120^\circ} - 1.2 e^{-j 1.4 t + 120^\circ} - 2.5$$

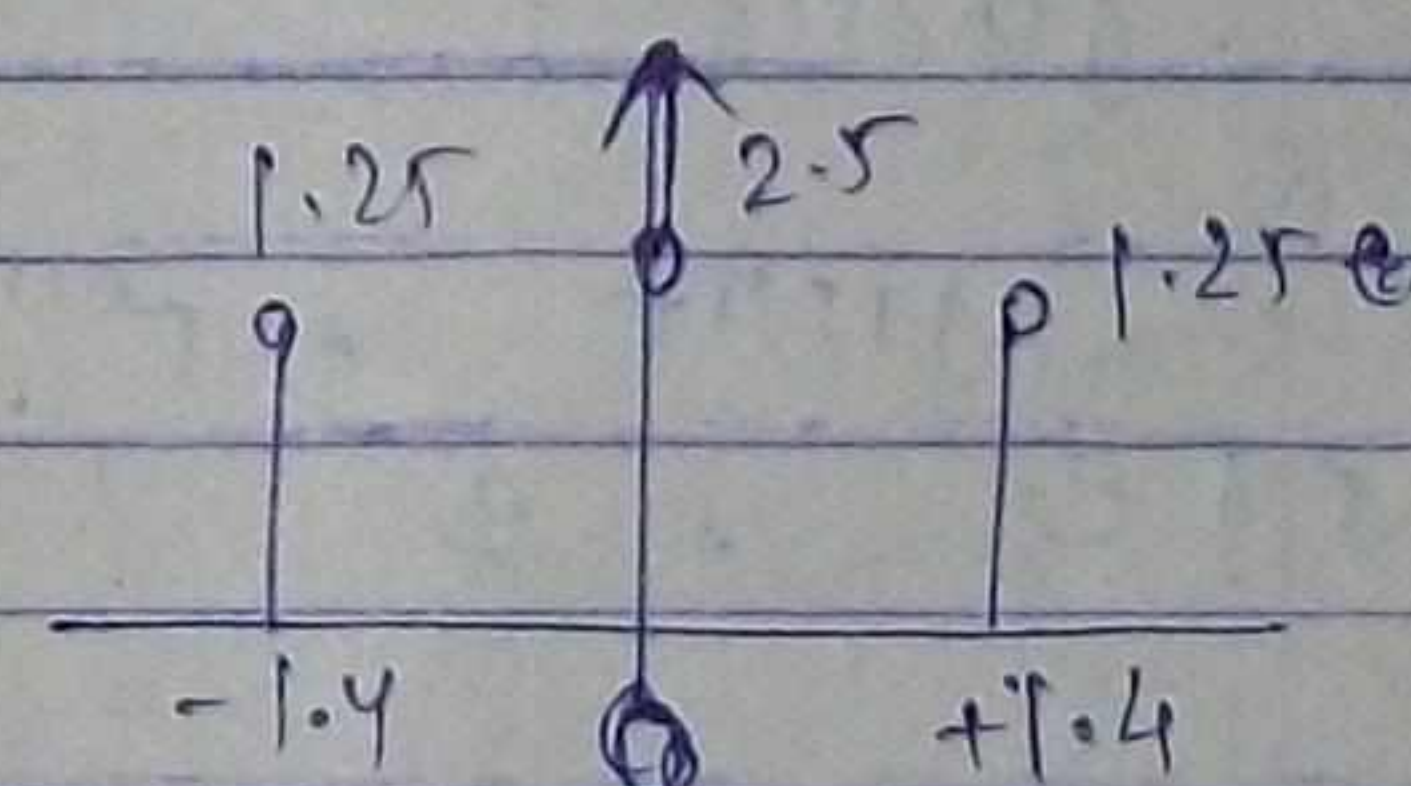
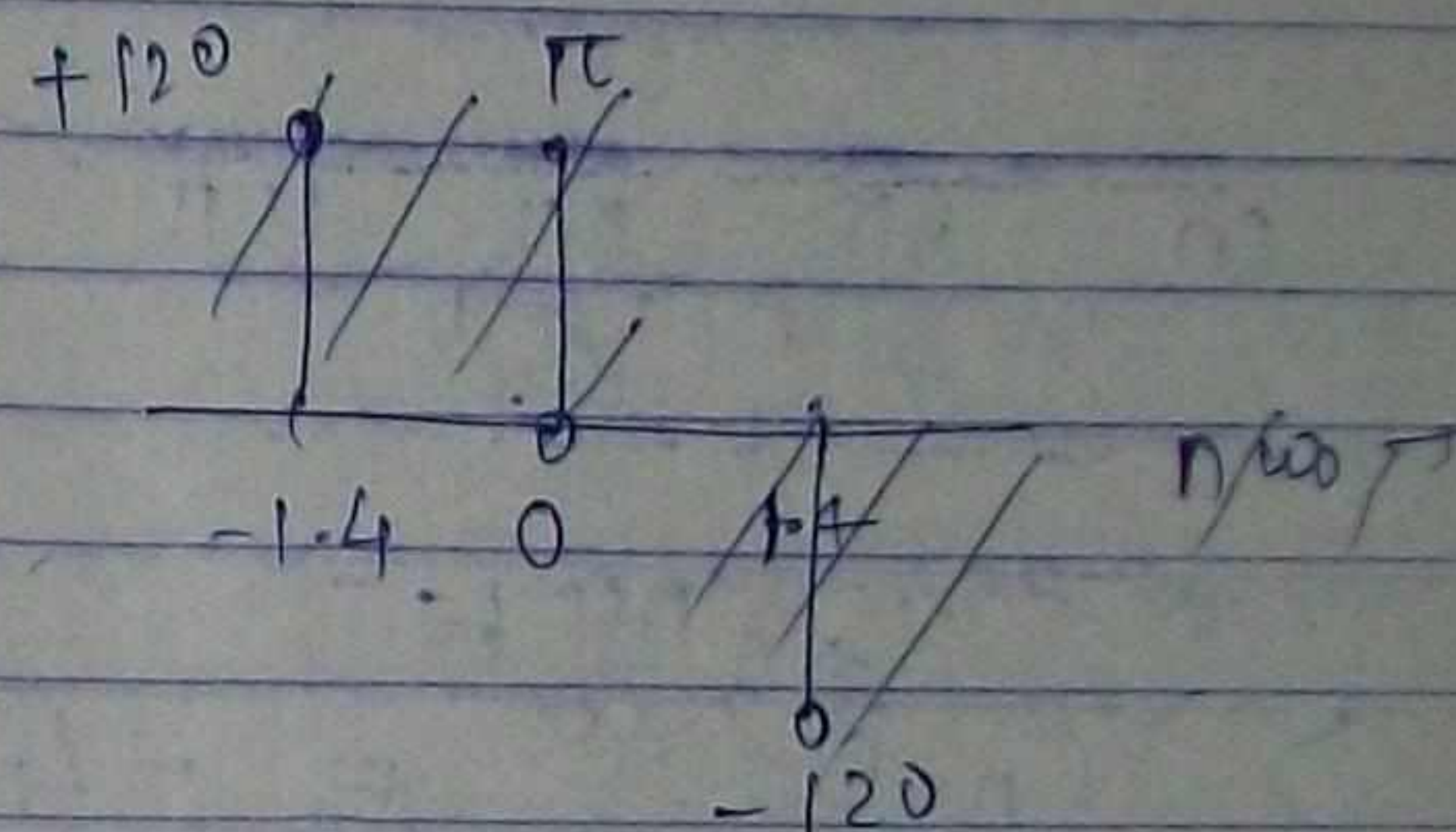
Periodic

$$c_0 = -2.5 \quad c_1 = -1.2 e^{-j 120^\circ} \quad c_{-1} = -1.2 e^{j 120^\circ}$$

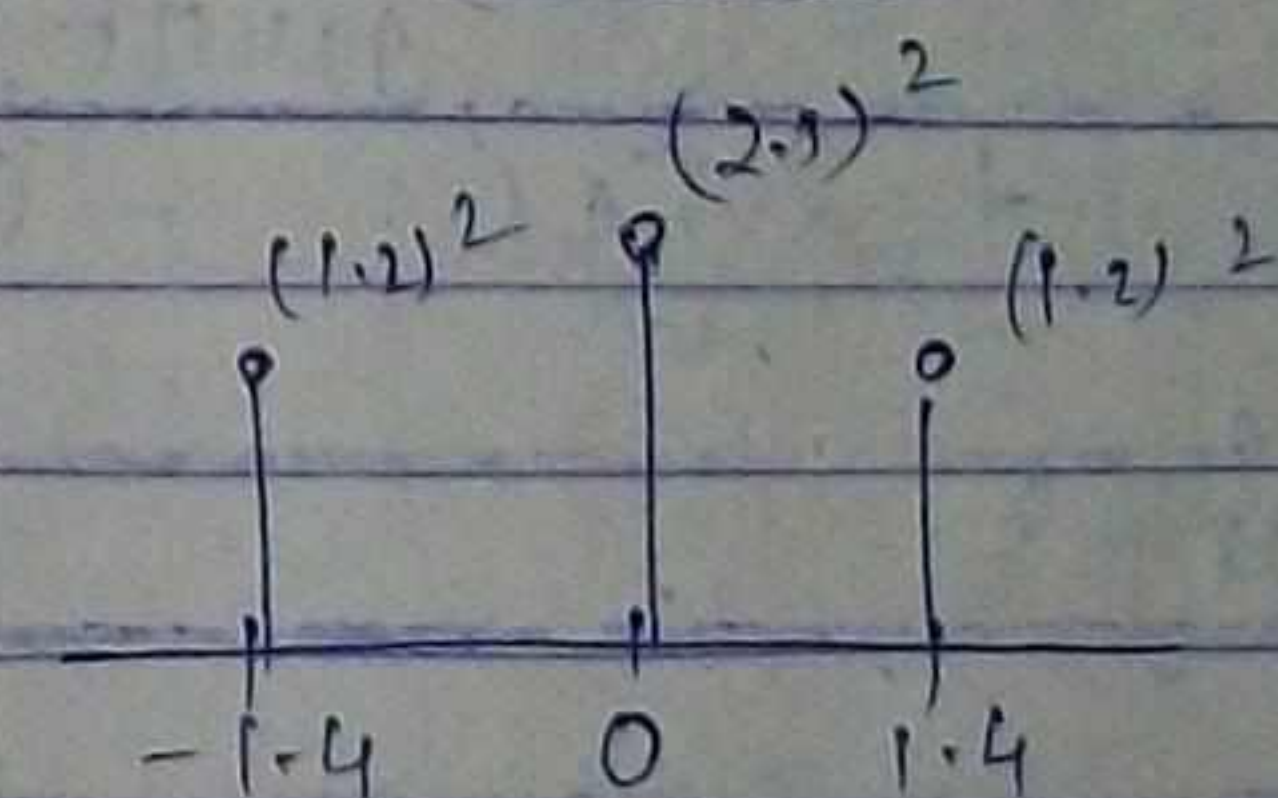
MAGN. SPECTRUM



PHASE SPECTRUM



POWER SPECTRUM



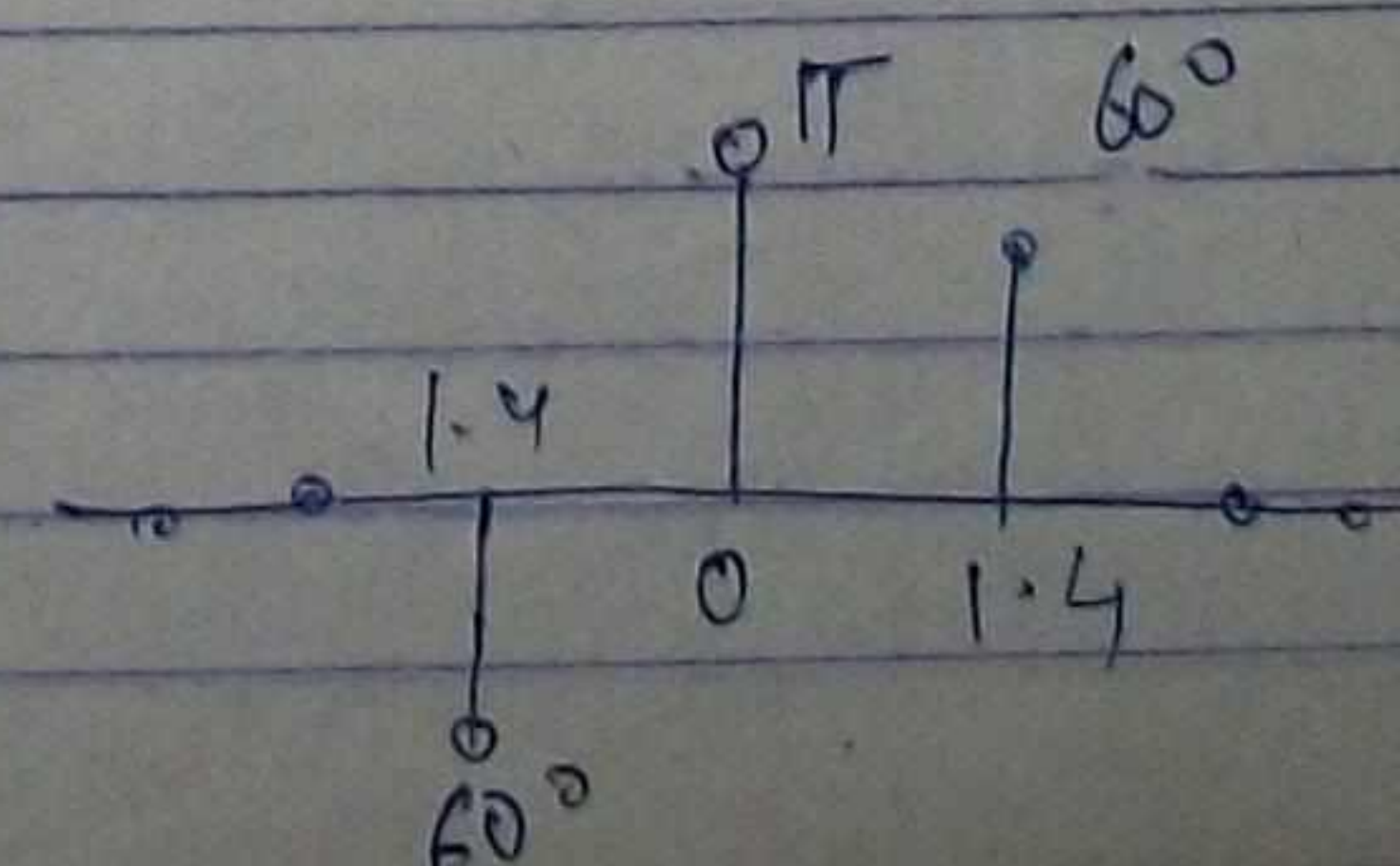
PHASE SPECTRUM

$$\theta_0 = \pi$$

$$a_n = c_n + c_{-n}$$

$$b_n = j(c_n - c_{-n})$$

$$\theta_1 = \tan^{-1} \left(\frac{b_1}{a_1} \right) = \tan^{-1} \left(\frac{0}{-1.25} \right) - 120^\circ = 60^\circ$$



$$(2) \quad x(t) = -4 + 5 \cos\left(\frac{3\pi}{7}t\right) - \cos(0.2\pi t) - 5 \sin(15\pi t - 45^\circ)$$

$$\omega = \frac{3\pi}{7}, \frac{2\pi}{10}, \frac{15\pi}{7}$$

$$\omega_0 = \frac{\pi}{35} = \frac{\pi}{35}$$

$$\sum_{n=-\infty}^{\infty} c_n e^{j \frac{n\pi}{35} t} = -4 + 2.5 e^{j \frac{3\pi}{7} t} + 2.5 e^{-j \frac{3\pi}{7} t} - 0.5 e^{j 0.2\pi t} - 0.5 e^{-j 0.2\pi t} + 2.5 j e^{j 15\pi t - j 45^\circ} + 2.5 j e^{-j 15\pi t + j 45^\circ}$$

Comparing:

$$c_0 = -4$$

$$c_{15} = 2.5, c_{-15} = 2.5$$

$$c_7 = -0.5, c_{-7} = -0.5$$

$$c_{525} = 2.5 j e^{-j 45^\circ}$$

$$c_{-525} = -2.5 j e^{j 45^\circ}$$

$$\frac{3\pi}{7} = \frac{n\pi}{35}$$

$$n = 15$$

$$\frac{n\pi}{35} = \frac{2\pi}{10}$$

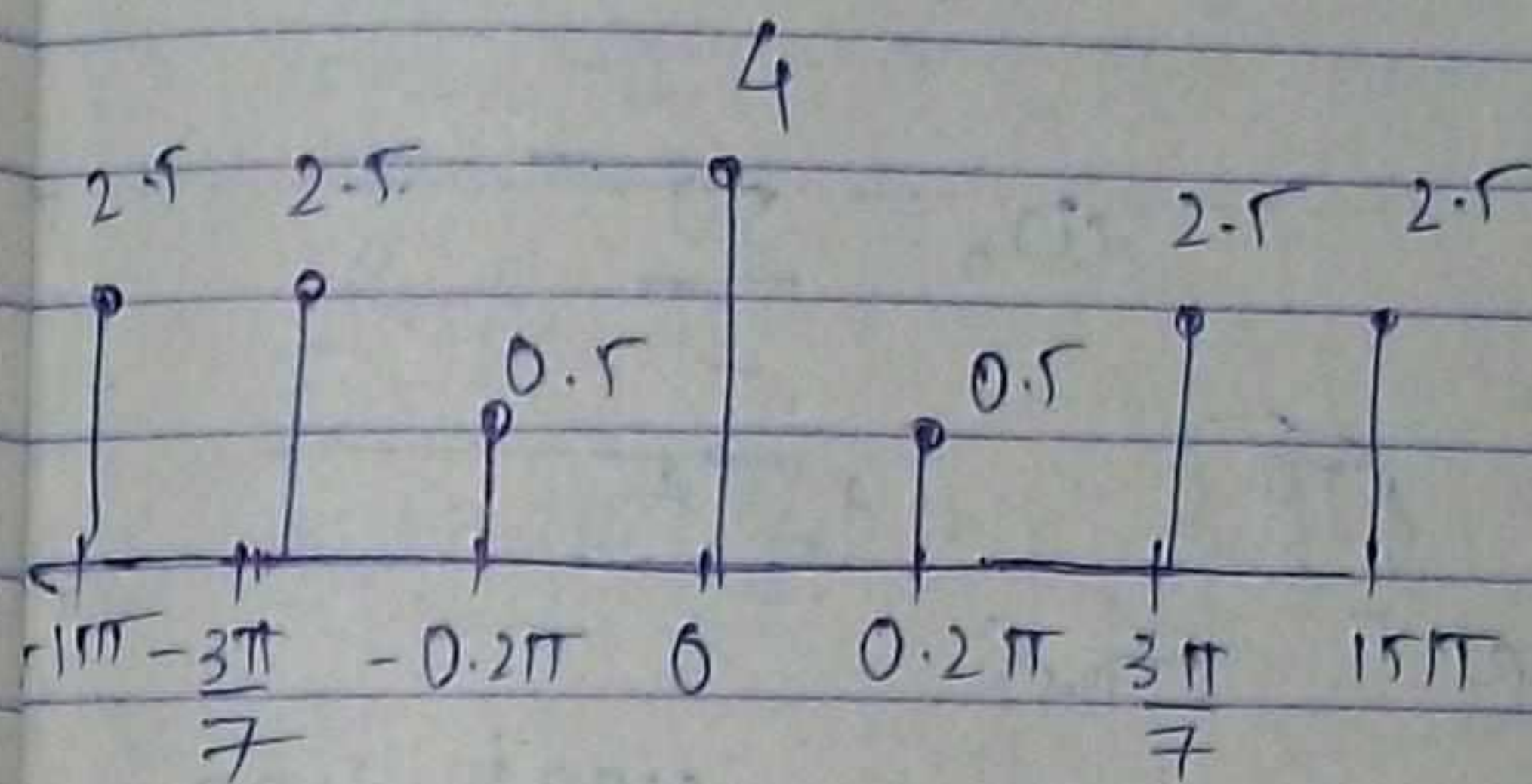
$$\frac{n\pi}{35} = 15\pi$$

$$n = 15 \times 35$$

$$\frac{35}{15} = 17.5$$

$$17.5 \times 2.5 = 43.75$$

Magnitude spectrum



Phase spectrum

$$\phi_0 = \pi$$

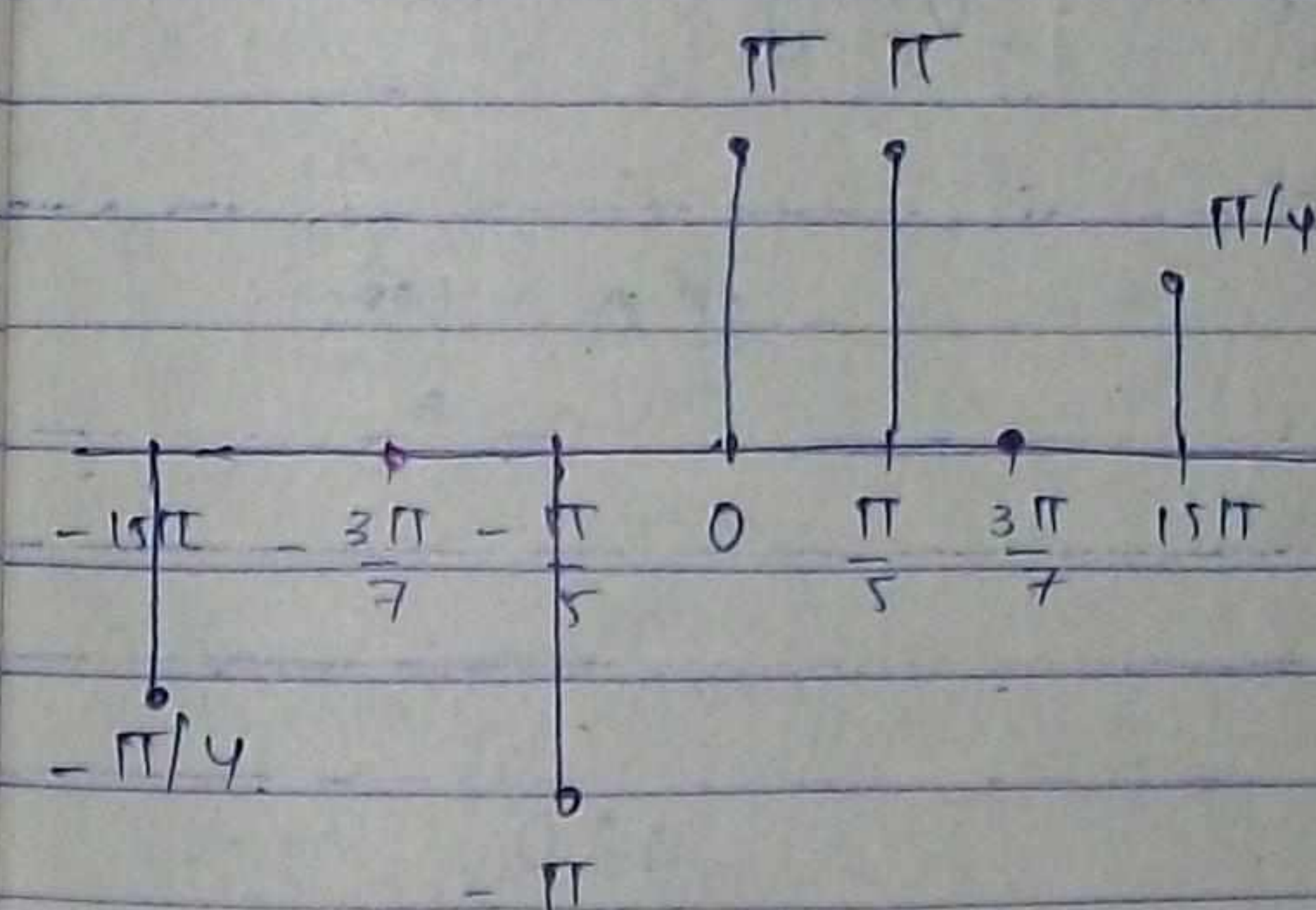
$$\theta_{15} = \tan^{-1}\left(\frac{b_n}{a_n}\right) - \frac{3\pi}{7} = \tan^{-1}(0) - \frac{3\pi}{7}$$

$$= 0$$

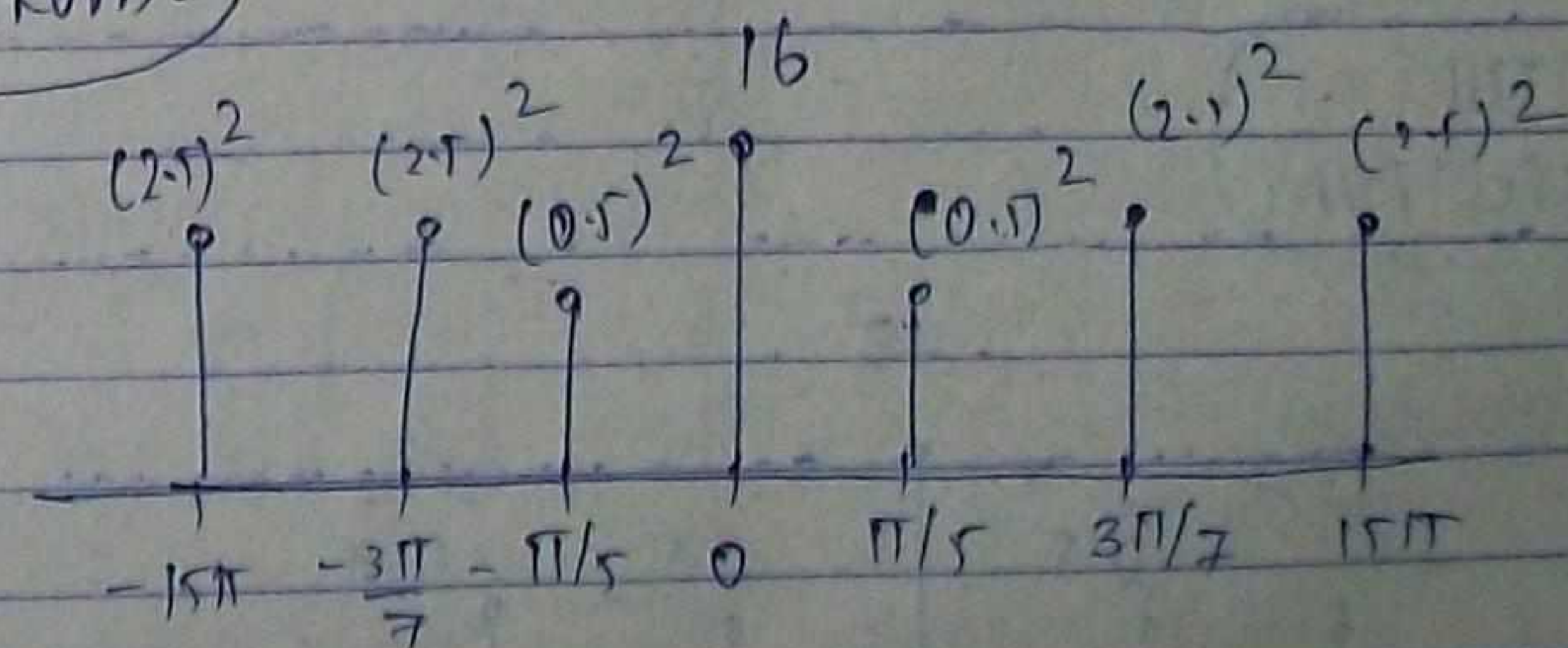
$$\theta_{-15} = 0$$

$$\theta_7 = \pi$$

$$c_{525} = \tan^{-1}\left(\frac{2.5}{0}\right) - 45^\circ = 90^\circ - 45^\circ = 45^\circ$$



Power spectrum



$$P_0 = |c_0|^2$$

$$P_n = |c_n|^2$$

$$(3) \quad x(t) = 13 + \cos\left(\frac{50}{3}t\right) - \sin(100t - 30^\circ)$$

$$x(t) =$$

$$\omega = \frac{50}{3}, 100$$

$$\omega_0 = \frac{50}{3}$$

$$\sum_{n=-\infty}^{\infty} c_n e^{j \frac{50n}{3} t}$$

$$= 13 + e^{j \frac{50}{3} t} - e^{-j \frac{50}{3} t}$$

$$+ 0.5 e^{j 100t - j 30} - 0.5 e^{-j 100t + j 30}$$

$$c_0 = 13$$

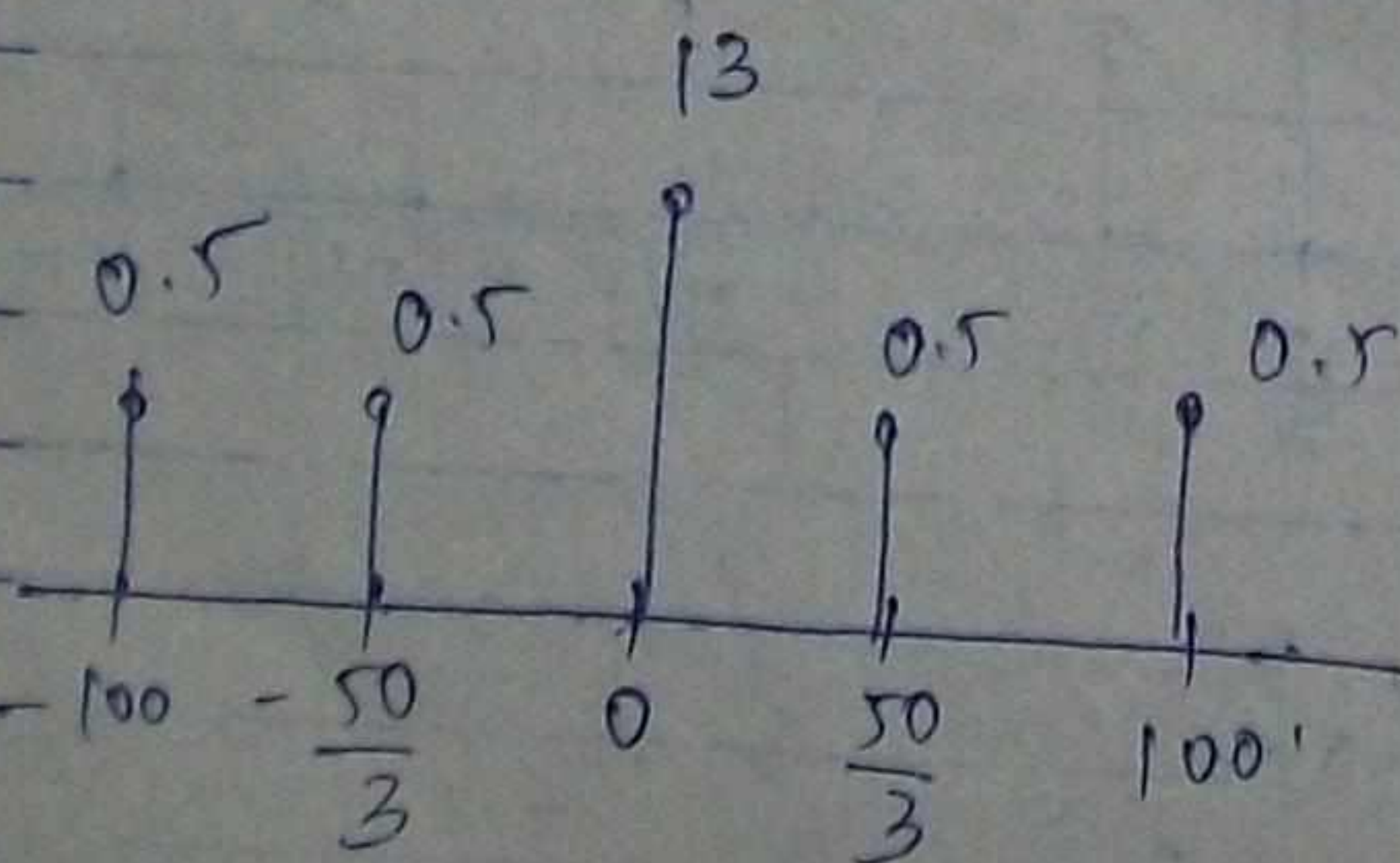
$$c_1 = 0.5$$

$$c_{-1} = 0.5$$

$$c_6 = 0.5 e^{-j 30}$$

$$c_{-6} = -0.5 e^{j 30}$$

MAGNITUDE SPECTRUM

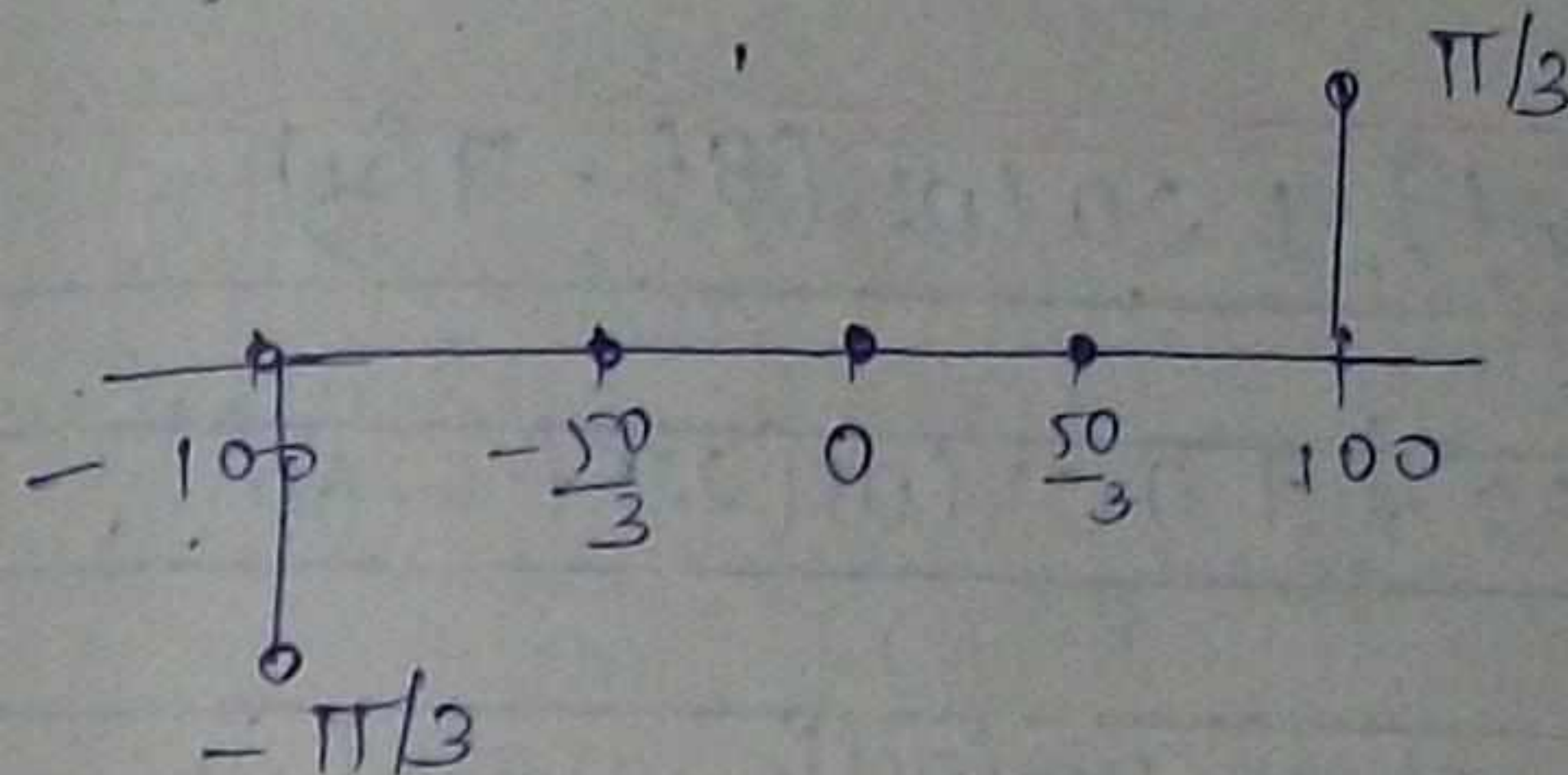


PHASE SPECTRUM

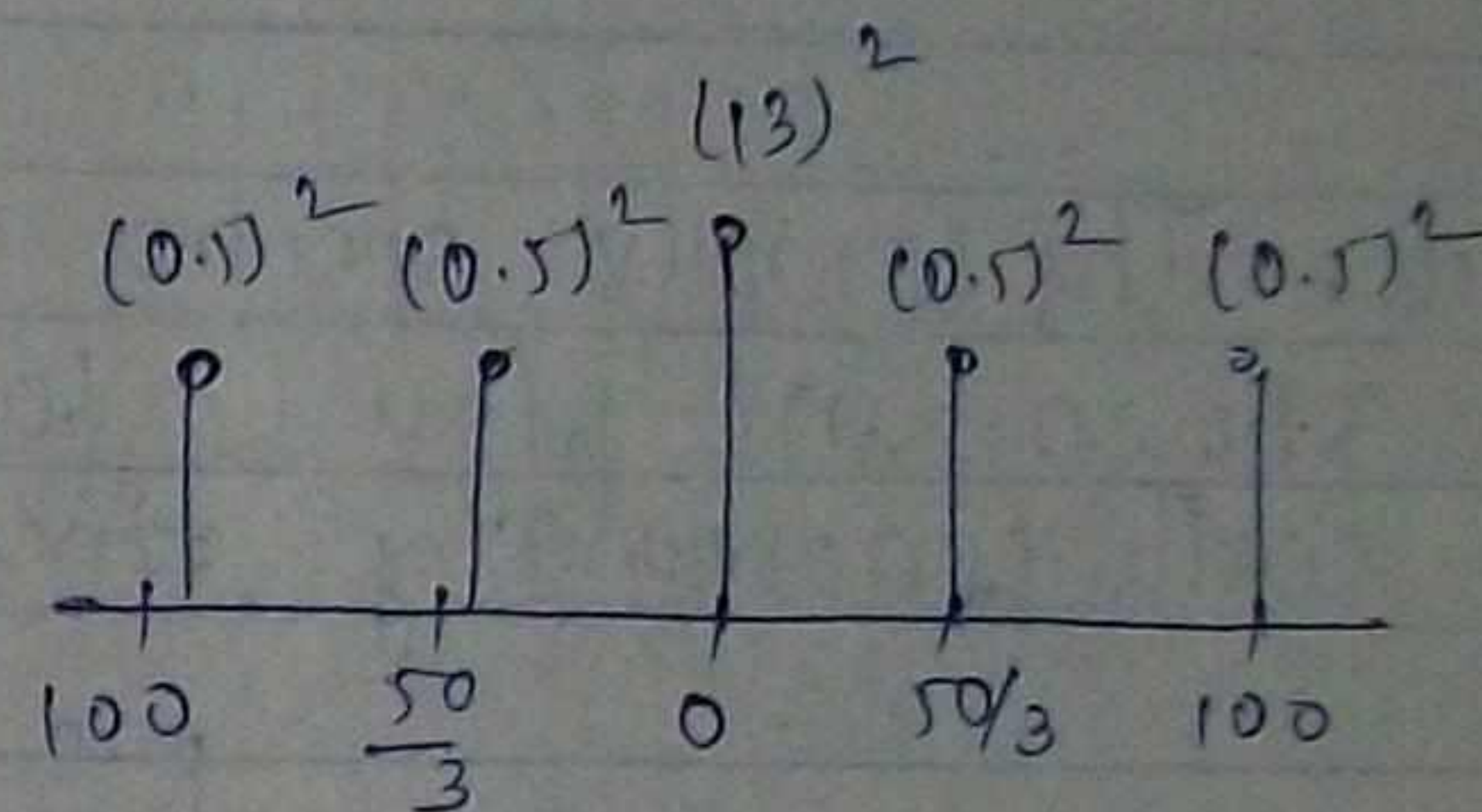
$$\theta_0 = 0$$

$$\theta_1 = 0$$

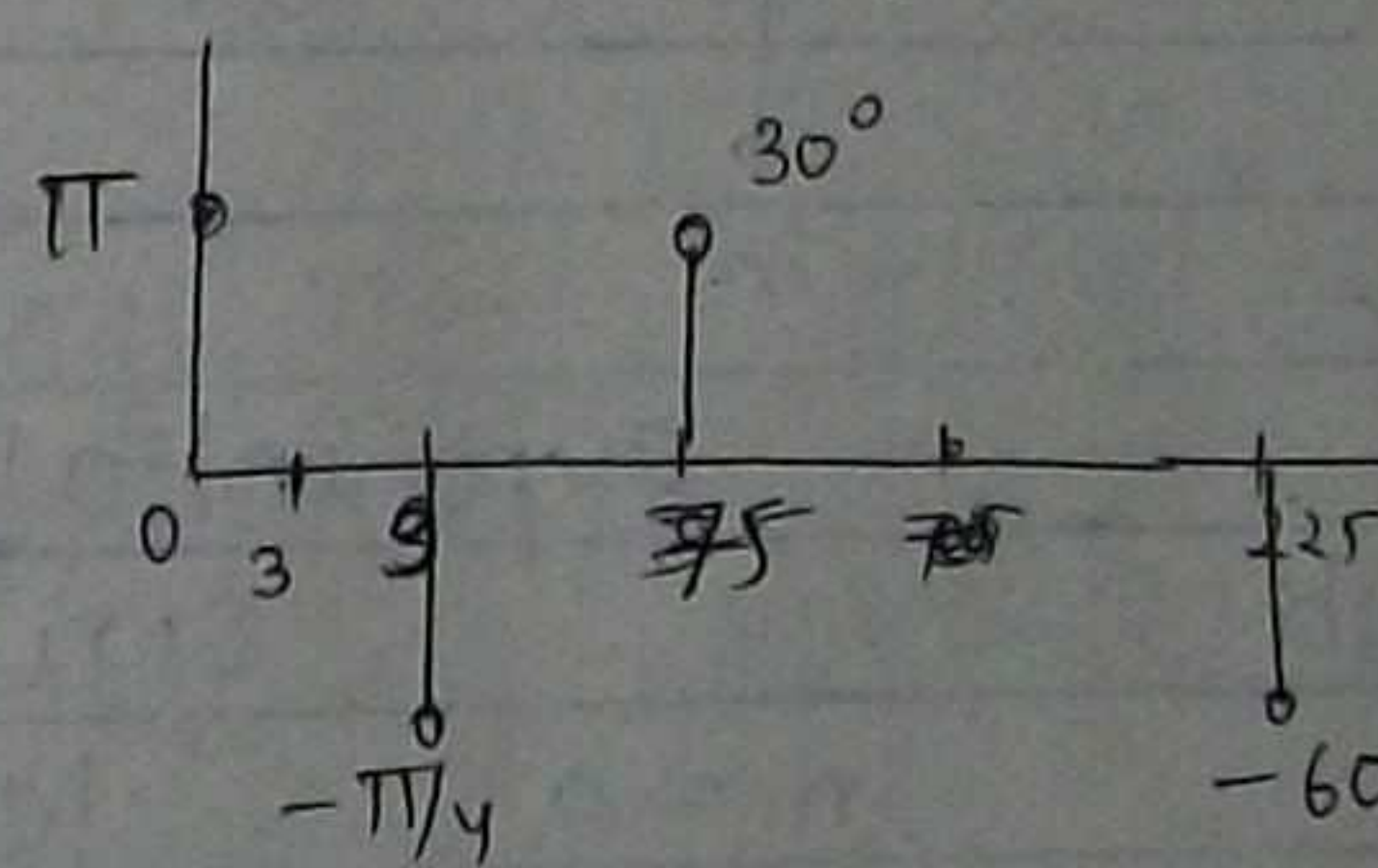
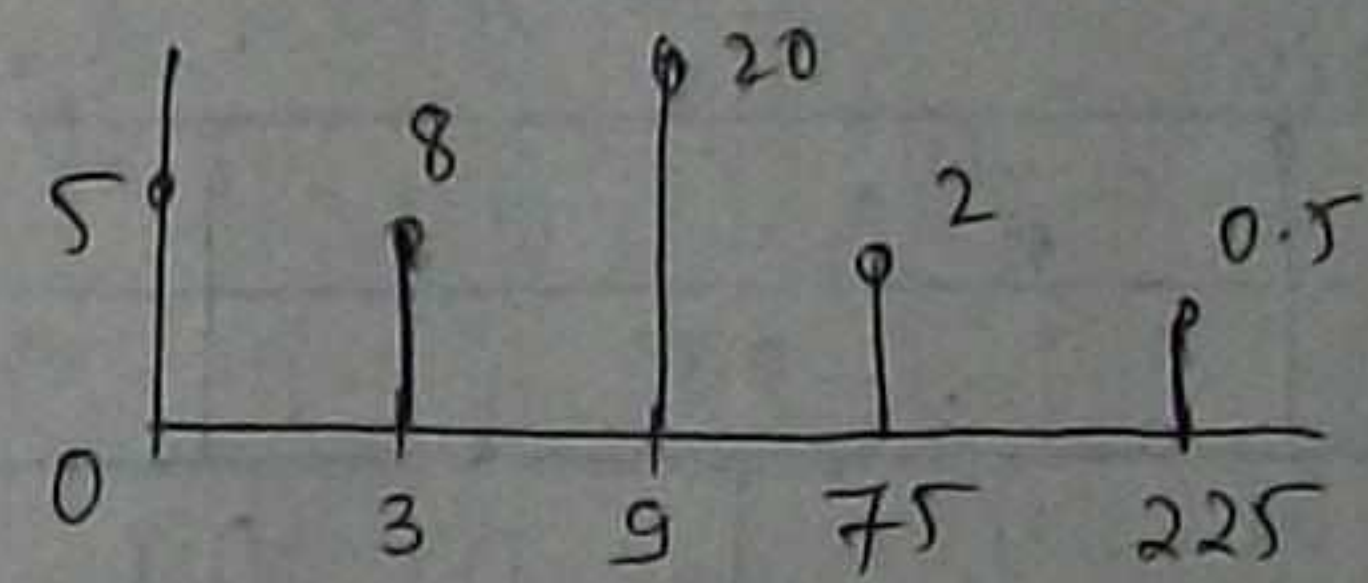
$$\theta_6 = \tan^{-1}\left(\frac{b_n}{a_n}\right) - 30 = \tan^{-1}\left(\frac{0.5}{0}\right) - 30 = 90 - 30 = 60^\circ$$



POWER SPECTRUM



Ques. The spectra of a periodic signal $x(t)$ with $\omega_0 = 1.5 \text{ rad/s}$ is as shown write the corresponding time signal.



Trigonometric series in polar form from graph

Soln:

Harmonics

$$1.5n = 3 \Rightarrow n = 2$$

$$1.5n = 9 \Rightarrow n = 6$$

$$1.5n = 75 \Rightarrow n = 50$$

$$1.5n = 225 \Rightarrow n = 150$$

$$S_0 = 5$$

$$S_2 = 8$$

$$S_6 = 20$$

$$S_{50} = 2$$

$$S_{225} = 0.5$$

$$\phi_0 = \pi$$

$$\phi_2 = 0$$

$$\phi_6 = -\pi/4$$

$$\phi_{50} = 30^\circ$$

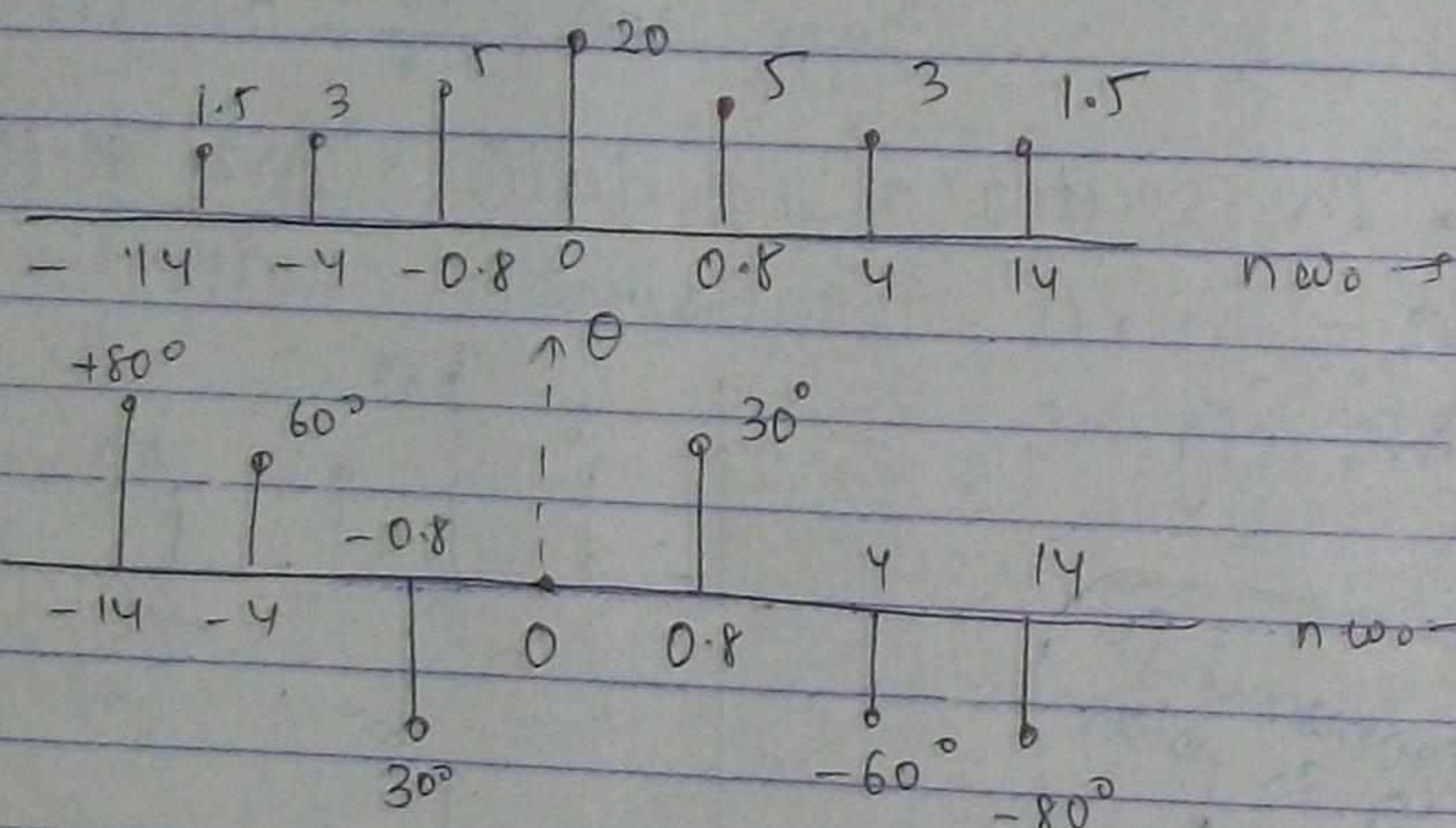
$$\phi_{225} = -60^\circ$$

Polar form

$$x(t) = S_0 + \sum_{n=1}^{\infty} S_n^{(\omega)} (1.5nt + \phi_n)$$

$$x_1(t) = -5 + 8 \cos(13t) + 20 \cos(9t - \pi/4) + 2 \cos(75t + 30^\circ) + 0.5 \cos(225t - 60^\circ)$$

Ques. The complex spectra of a periodic signal is shown in figure for the $\omega_0 = 0.2 \text{ rad/s}$ write corresponding time signal.



Solⁿ: from graph $\rightarrow$ Exponential Fourier Series

$0.2n = 0.8$	$n = 0$	$ C_0 = 20$	$\phi = 0$
$0.2n = 4$	$n = 4$	$ C_4 = 5$	$\theta_4 = 30^\circ$
$0.2n = 20$	$n = 20$	$ C_{20} = 3$	$\theta_{20} = -60^\circ$
$0.2n = 14$	$n = 70$	$ C_{70} = 1.5$	$\theta_{70} = -80^\circ$

$$\text{EFS: } x(t) = \sum_{n=-\infty}^{\infty} C_n e^{j0.2nt}$$

$$\begin{aligned} \theta_{-4} &= -30^\circ \\ \theta_{-20} &= 60^\circ \\ \theta_{-70} &= +80^\circ \end{aligned}$$

$$C_0 + C_1 e^{j0.2t} + \dots = \sum_{n=-\infty}^{\infty} C_n e^{j0.2nt} = x(t)$$

$$C_n = |C_n| e^{j\theta_n}, \quad C_{-n} = |C_{-n}| e^{j\theta_{-n}}$$

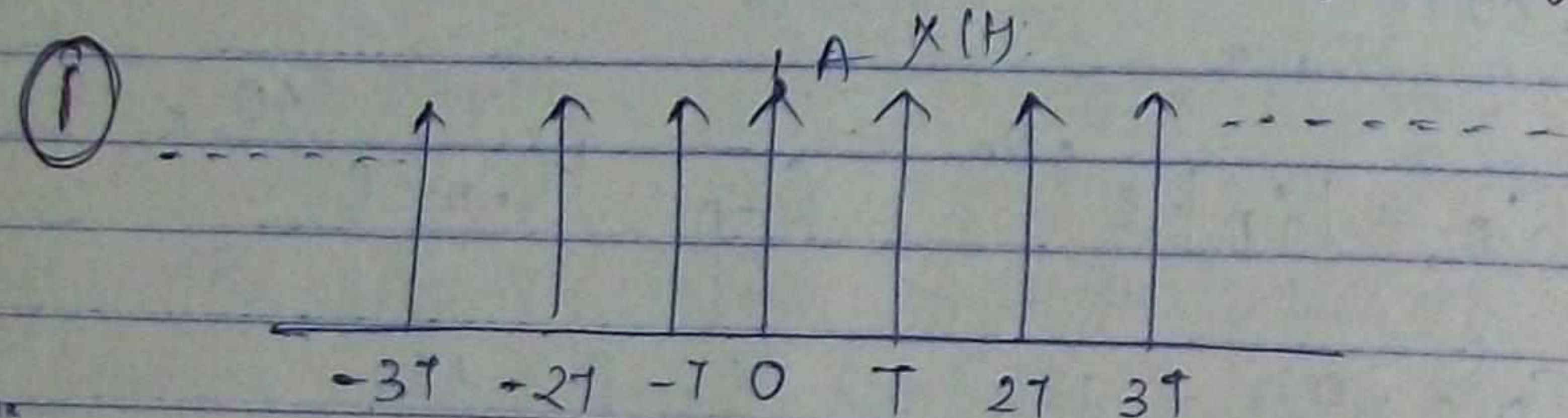
$$\begin{cases} C_0 = 20 \\ C_4 = 5 e^{j30^\circ} \\ C_{20} = 3 e^{-j60^\circ} \\ C_{70} = 1.5 e^{-j80^\circ} \end{cases} \quad \begin{cases} C_{-4} = 5 e^{-j30^\circ} \\ C_{-20} = 3 e^{j60^\circ} \\ C_{-70} = 1.5 e^{j80^\circ} \end{cases}$$

$$\begin{aligned} x(t) &= 20 + 5 e^{j0.8t} e^{j30^\circ} + 5 e^{-j0.8t} e^{-j30^\circ} \\ &+ 3 e^{j4t} e^{-j60^\circ} + 3 e^{-j4t} e^{j60^\circ} + 1.5 e^{j14t} e^{-j80^\circ} + 1.5 e^{-j14t} e^{j80^\circ} \end{aligned}$$

$$\begin{aligned} x(t) &= 20 + 5 \left[e^{j(0.8t+30^\circ)} + e^{-j(0.8t-30^\circ)} \right] \\ &+ 3 \left[e^{j(4t-60^\circ)} + e^{-j(4t-60^\circ)} \right] \\ &+ 1.5 \left[e^{j(14t-80^\circ)} + e^{-j(14t-80^\circ)} \right] \end{aligned}$$

$$x(t) = 20 + 10 \cos(0.8t + 30^\circ) + 6 \cos(4t - 60^\circ) + 3 \cos(14t - 80^\circ)$$

Ques. find the EFS Representation of the follⁿ:



Solⁿ:

- $x(t)$ represent the periodic train of impulses with periodicity T and the strength of each impulse is A .

$$T_0 = T \quad \omega_0 = 2\pi/T$$

$$x(t) = A \delta(t) \quad , \quad t = 0$$

$$= 0 \quad , \quad 0 < t < T$$

EFS:

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{j \frac{2\pi}{T} n t} \quad - \textcircled{1}$$

$$C_0 = \frac{1}{T} \int_T x(t) dt \quad , \quad n=0$$

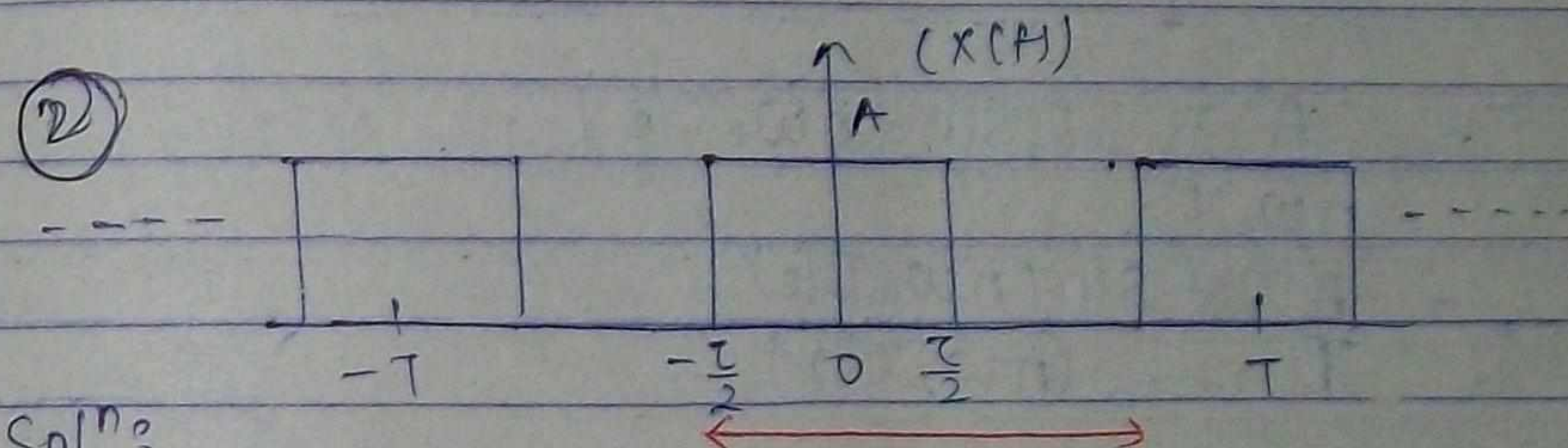
$$C_n = \frac{1}{T} \int_T x(t) e^{-jn\omega_0 t} dt \quad , \quad n \neq 0$$

$$C_0 = \frac{1}{T} \int_0^T A \delta(t) dt = \frac{A}{T} \quad - \textcircled{2}$$

$$C_n = \frac{1}{T} \int_0^T A \delta(t) e^{jn\omega_0 t} dt = \frac{A}{T} \quad - \textcircled{3}$$

from ①, ②, ③:

$$x(t) = \frac{A}{T} \sum_{n=-\infty}^{\infty} e^{j \frac{2\pi}{T} n t}$$



Solⁿ:

- $x(t)$ represent periodic train of pulses with periodicity T , height A and width τ

$$x(t) = A \quad , \quad -\tau/2 < t < \tau/2$$

$$= 0 \quad , \quad -\tau/2 < t < (T - \tau/2)$$

EFS:

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{j \frac{2\pi}{T} n t} \quad - \textcircled{1}$$

$$C_0 = \frac{1}{T} \int_T x(t) dt$$

$$C_0 = \frac{1}{T} \int_{-\tau/2}^{\tau/2} A dt + \int_{\tau/2}^{T-\tau/2} 0 dt$$

$$C_0 = \frac{A\tau}{T}$$

$$C_n = \frac{1}{T} \int_{-T/2}^{T/2} A e^{-jn\omega_0 t} dt = \frac{A}{T} \left[\frac{e^{-jn\omega_0 t}}{-jn\omega_0} \right]_{-T/2}^{T/2}$$

$$= \frac{-A}{jn\omega_0 T} \left[e^{-jn\omega_0 T/2} - e^{jn\omega_0 T/2} \right]$$

$$= \frac{A}{jn\omega_0 T} 2 \sin(n\omega_0 T/2)$$

$$= \frac{AT}{T} \frac{\sin(n\omega_0 T/2)}{(n\omega_0 T/2)}$$

$$= \frac{AT}{T} \text{Sa}(n\omega_0 T/2)$$

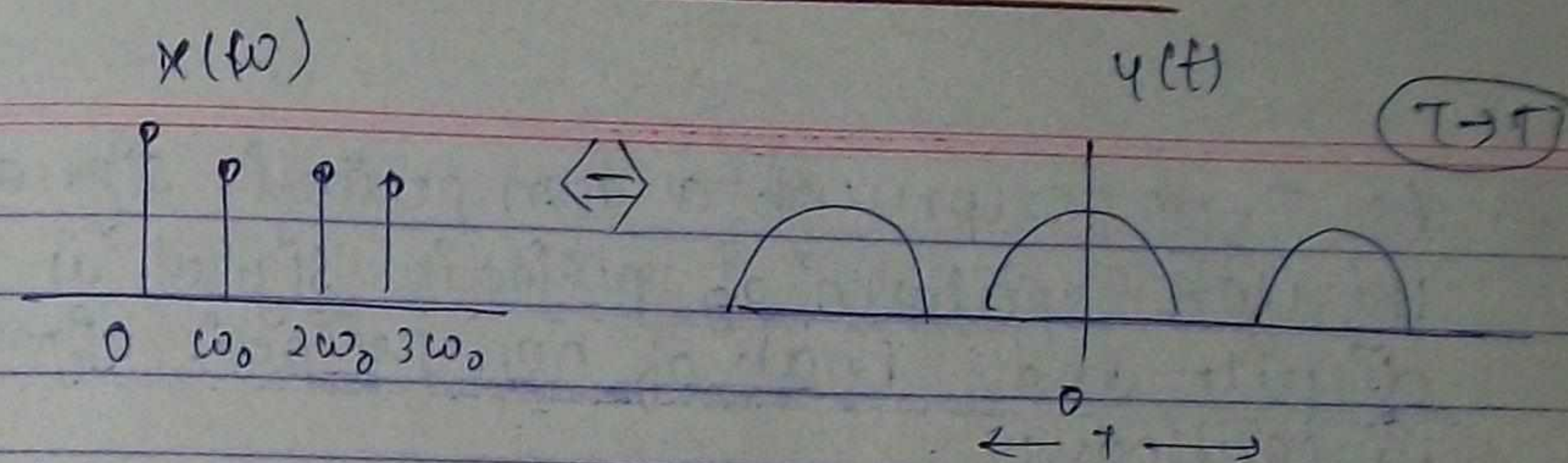
$$= \frac{AT}{T} \text{sinc}\left(n \frac{\tau}{T}\right)$$

$$\omega_0 = \frac{2\pi}{T}$$

$$\therefore \text{EFS} = \frac{AT}{T} \sum_{n=-\infty}^{\infty} \text{Sa}(n\omega_0 T/2) e^{j \frac{2\pi n}{T} t}$$

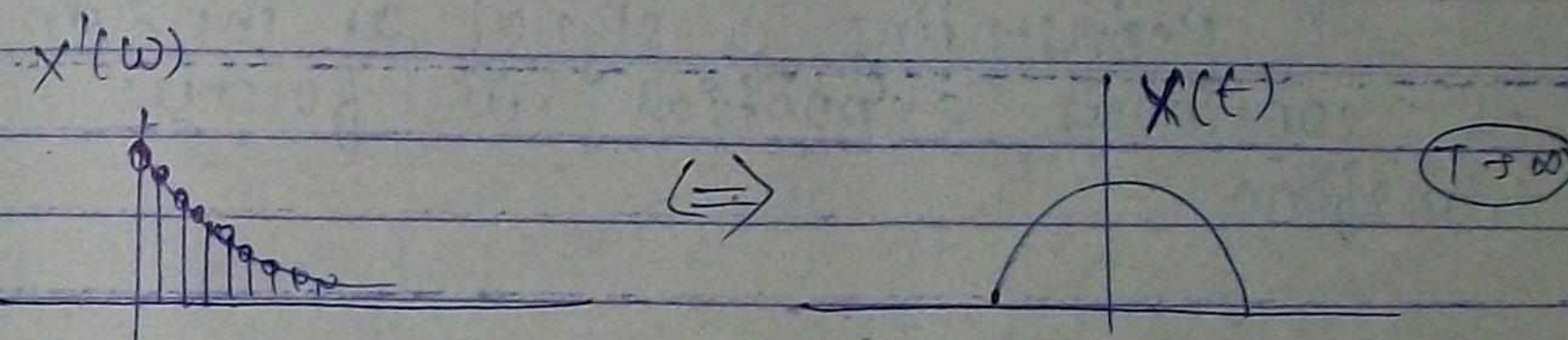
$$= \frac{AT}{T} \sum_{n=-\infty}^{\infty} \text{sinc}\left(\frac{n\tau}{T}\right) e^{j \frac{2\pi n}{T} t}$$

FOURIER TRANSFORM



$$\text{as } \omega_0 = \frac{2\pi}{T}$$

$T \uparrow, \omega_0 \downarrow$ and $T \rightarrow \infty, \omega_0 = 0$



$$X(t) = \lim_{T \rightarrow \infty} y(t)$$

$$\text{N.P.} = \lim_{T \rightarrow \infty} \text{Periodic}$$

$$f.T = \lim_{T \rightarrow \infty} f.S.$$

$$\text{CONT.} = \lim_{T \rightarrow \infty} \text{DISCRETE}$$

The spectrum of periodic signal is discrete as $T \rightarrow \infty$ increase, ω_0 decreases hence spectrum become more dense, as $T \rightarrow \infty$ $\omega_0 \rightarrow 0$ and spectrum becomes continuous

→ As $T \rightarrow \infty$ represent a non periodic signal therefore spectrum of periodic signal is discrete and that of non periodic signal is continuous.

★ Representing the signal as the discrete sum of exponential is Fourier series.
Representing a signal as the sum of continuous exponential is Fourier transform.

⇒ CONDITION FOR THE EXISTENCE OF F.T. — (DIRICHLET'S CONDITIONS)

① Signal should be absolutely integrable from $-\infty$ to ∞ .

$$\int_{-\infty}^{\infty} |x(t)| dt < \infty$$

② Signal must have finite no. of maxima and minima, from $-\infty$ to ∞ , and each of them must also be finite.

③ Signal must have finite discontinuities from $-\infty$ to ∞ , and each of them must also be finite.

① → necessary condⁿ
② & ③ → necessary & sufficient condⁿ.

⇒ FOURIER TRANSFORM —

Fourier transform of $x(t)$ is denoted by:

$$F\{x(t)\} = X(\omega) = X(j\omega)$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

V-sec
or
A-sec

Inverse Fourier transform of $x(t)$ is

$$F^{-1}\{X(\omega)\} = x(t)$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$$

V or A

$$x(t) \xleftrightarrow{F.T.} X(\omega)$$

$$\# F\{x(t)\} = X(f) = X(jf) = X(2\pi f) = X(j2\pi f)$$

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt$$

IFT: $X(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi ft} df$

F.T.
 $X(t) \leftrightarrow X(f)$

⇒ FOURIER TRANSFORM OF STANDARD SIGNALS -

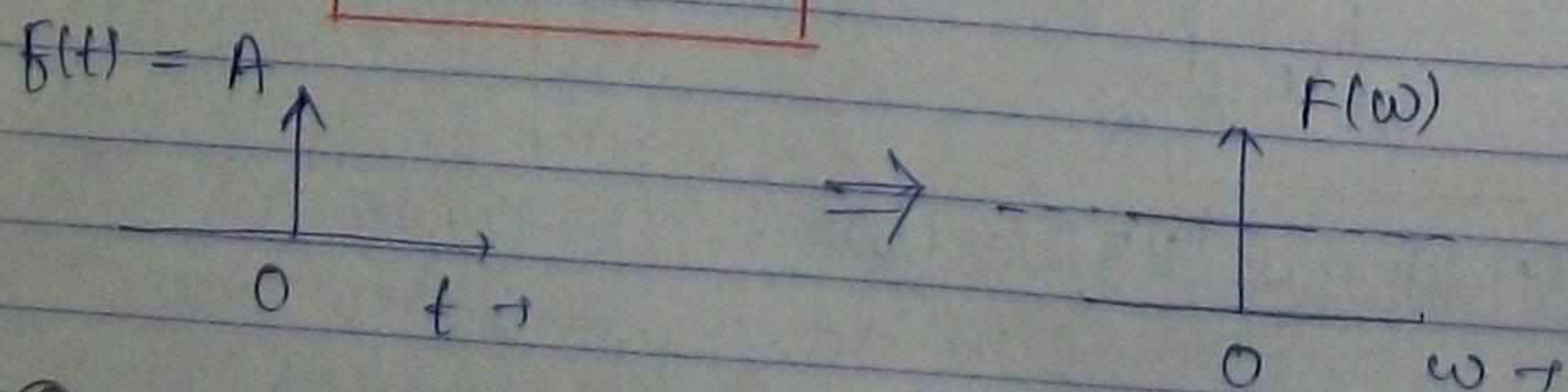
① IMPULSE FUNCTION -

$$X(t) = A \delta(t)$$

$$X(\omega) = \int_{-\infty}^{\infty} A \delta(t) e^{-j\omega t} dt$$

$$X(\omega) = A \cdot e^{-j\omega(0)}$$

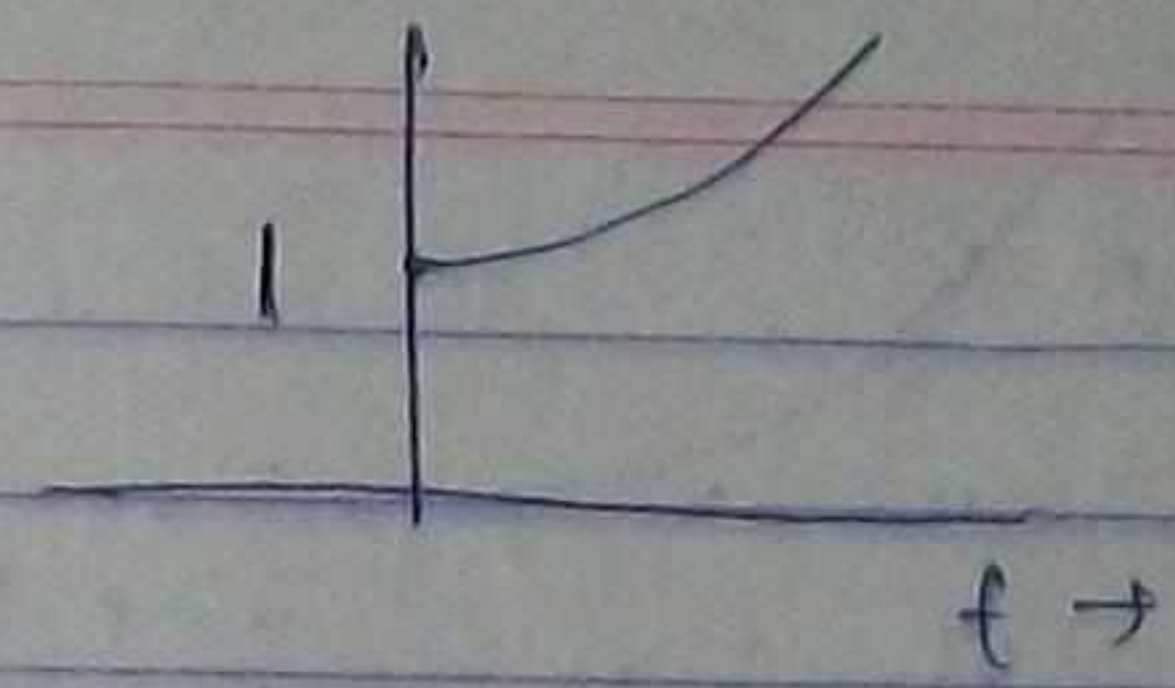
$$X(\omega) = A$$



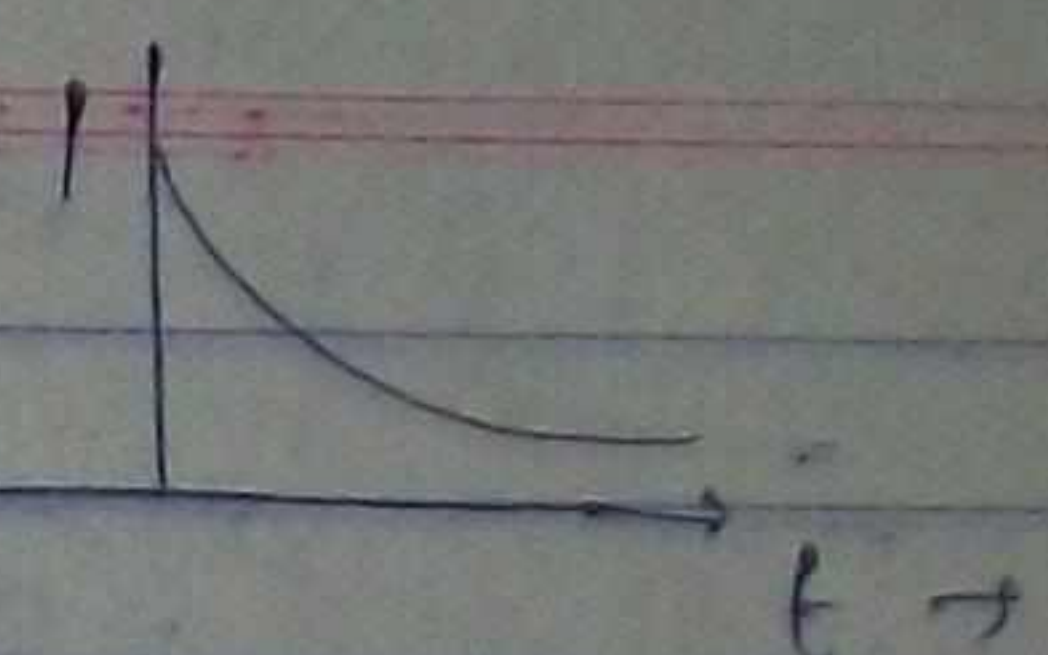
② EXPONENTIAL FUNCTION -

(A) $x(t) = e^{-at} u(t)$

$(a < 0)$



$(a > 0)$



- for $a < 0$ the signal is not absolutely integrable hence its f.T. does not exist for $a < 0$, it is defined for $a > 0$ only.

$$\begin{aligned} X(\omega) &= \int_{-\infty}^{\infty} e^{-at} \cdot e^{-j\omega t} dt \\ &= \int_{0}^{\infty} e^{-(a+j\omega)t} dt \\ &= \frac{-1}{(a+j\omega)} \left[e^{-(a+j\omega)\infty} - e^{-0} \right] \end{aligned}$$

$$X(\omega) = \frac{1}{a+j\omega} \quad a > 0$$

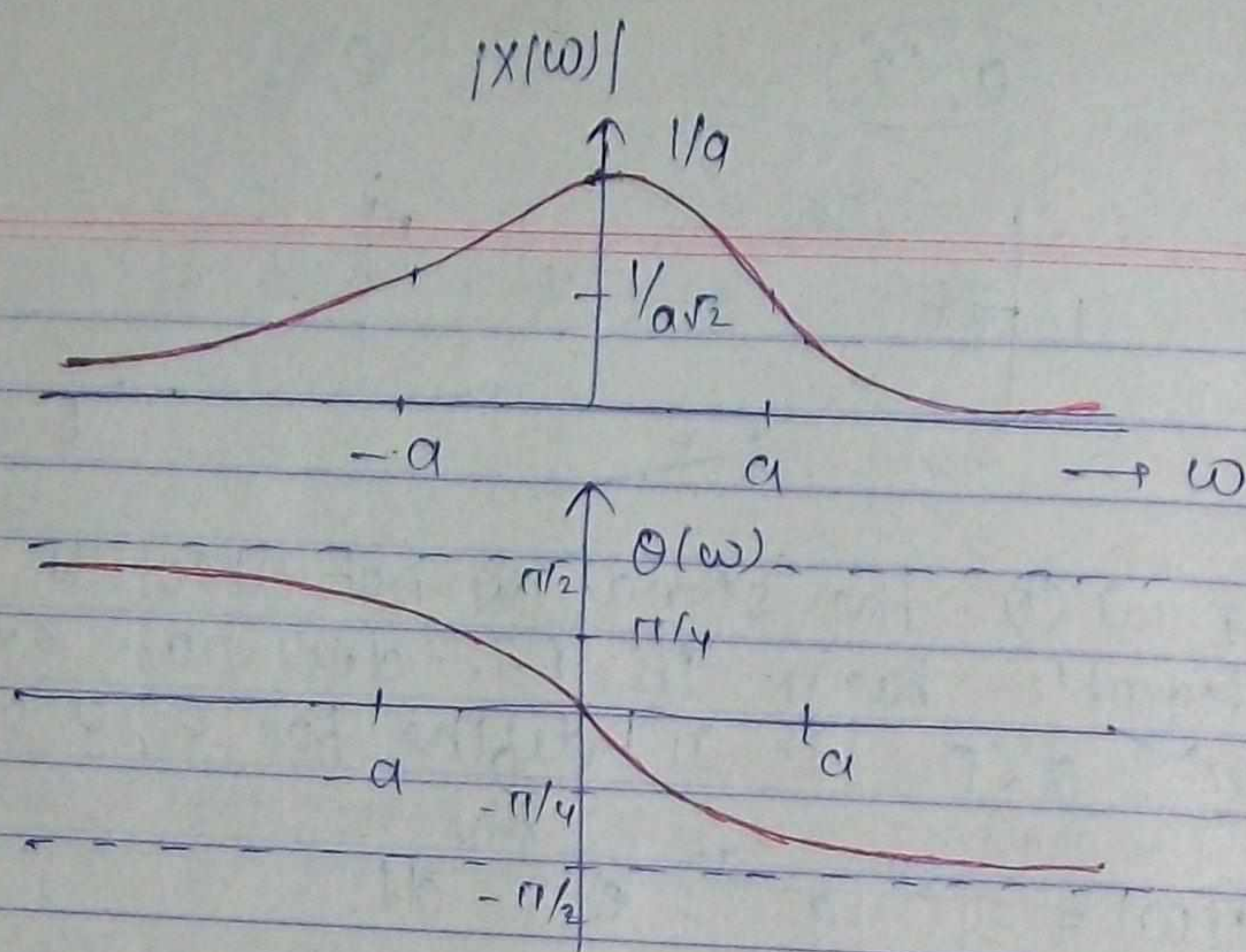
$$F\{e^{-at} u(t)\} \rightarrow \frac{1}{a+j\omega} \quad a > 0$$

$$F^{-1}\left\{\frac{1}{a+j\omega}\right\} \rightarrow e^{-at} u(t)$$

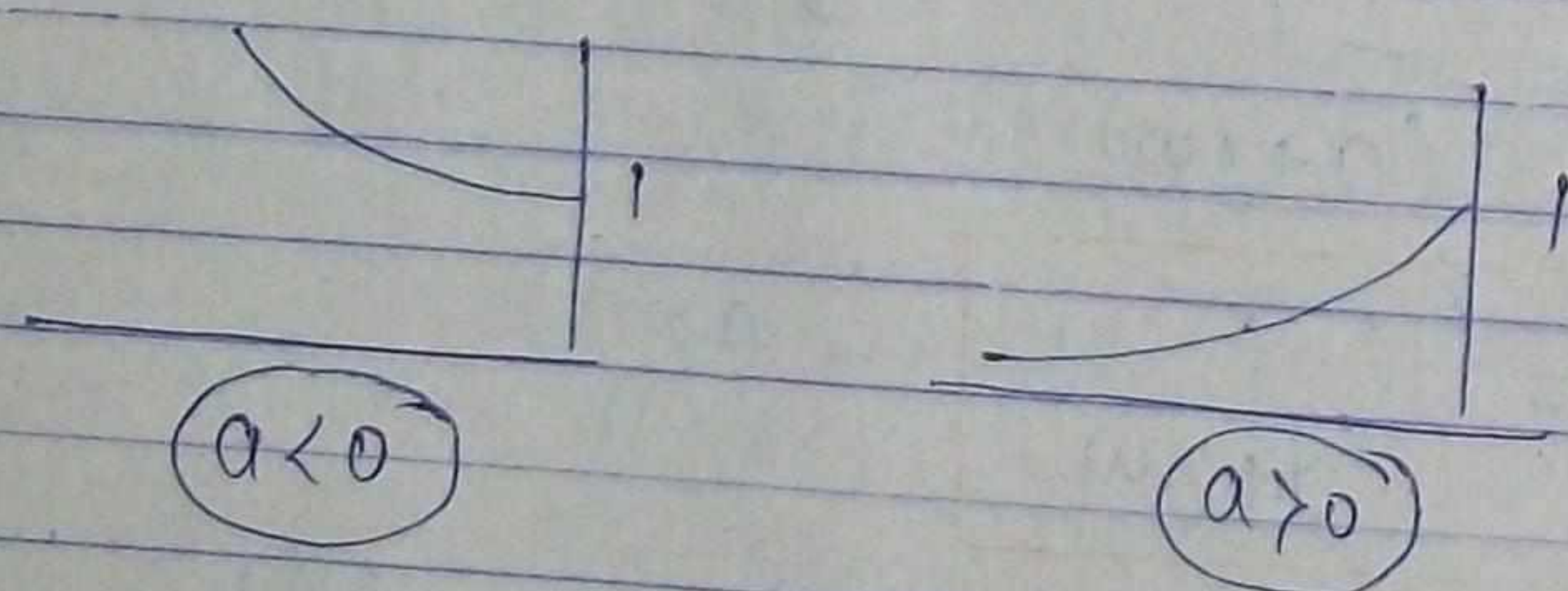
Now $|X(\omega)| = \frac{1}{(a^2 + \omega^2)^{1/2}}$

$$\theta(\omega) = -\tan^{-1}(\omega/a)$$

ω	$ X(\omega) $	$\theta(\omega)$
0	$1/a$	0
a	$1/a\sqrt{2}$	$-\pi/4$
∞	0	$-\pi/2$



(B) $x(t) = e^{at} u(-t)$



for $a > 0$

$$X(\omega) = \int_{-\infty}^0 e^{at} \cdot e^{-j\omega t} dt$$

$$= \int_{-\infty}^0 e^{+(a-j\omega)t} dt$$

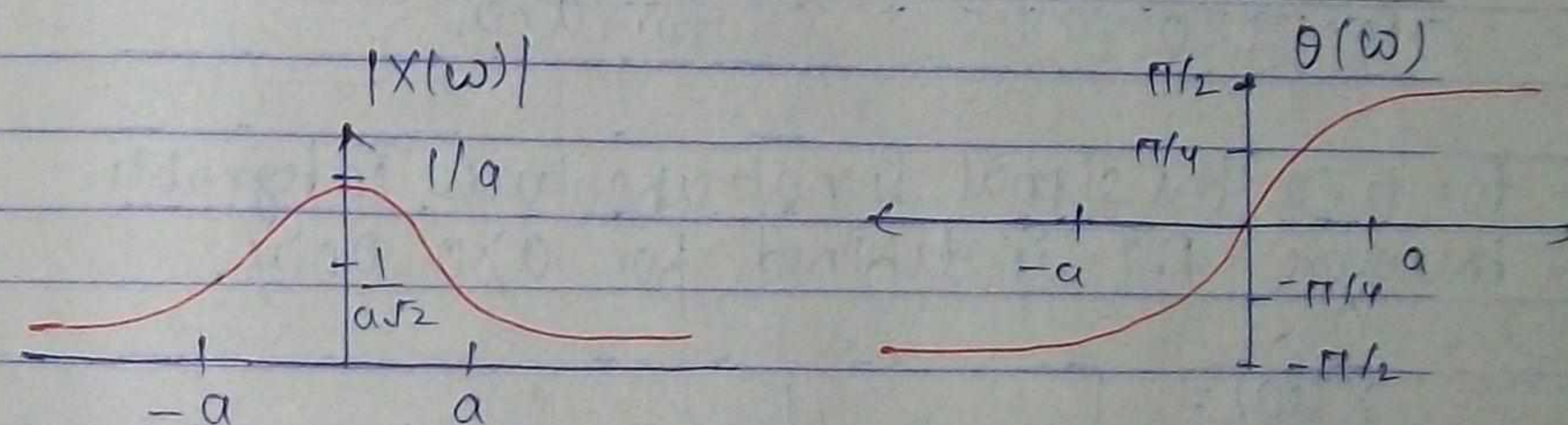
$X(\omega) = \frac{1}{a-j\omega}$

for $a > 0$

$$F\{e^{at} u(-t)\} = \frac{1}{a-j\omega} \quad a > 0$$

$$F^{-1}\left\{\frac{1}{a-j\omega}\right\} = e^{at} u(-t)$$

$$X(\omega) = \frac{1}{(a^2 + \omega^2)^{1/2}} \quad \theta(\omega) = \tan^{-1}\left(\frac{\omega}{a}\right)$$



Ex (1) $e^{-3t} u(t) \rightarrow \frac{1}{3+j\omega}$

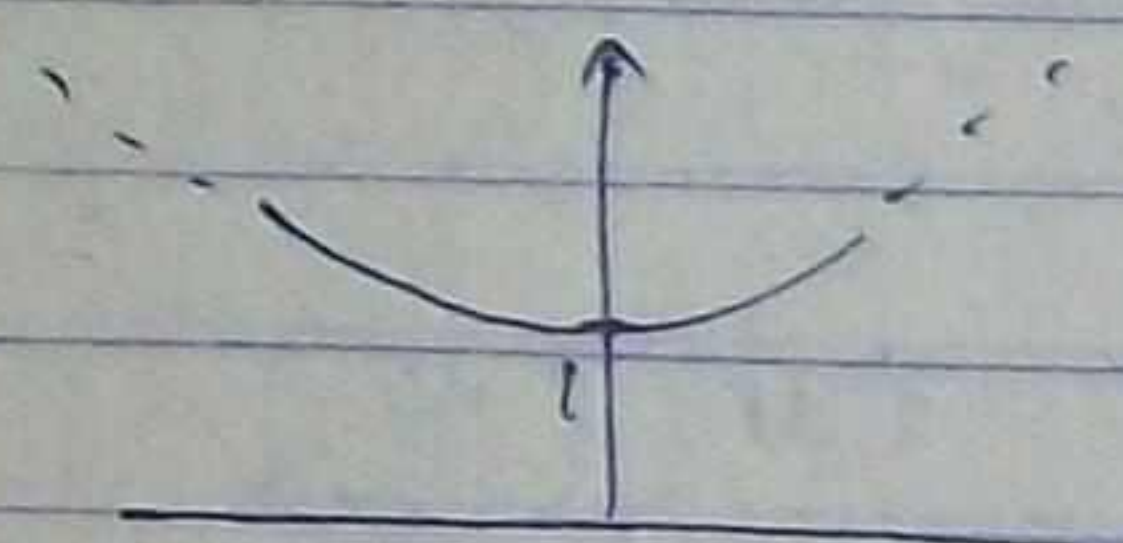
(2) $e^{2t} u(t)$, $a = -2 < 0$
do not exist

(3) $e^{2t} u(-t) \rightarrow \frac{1}{2-j\omega}$

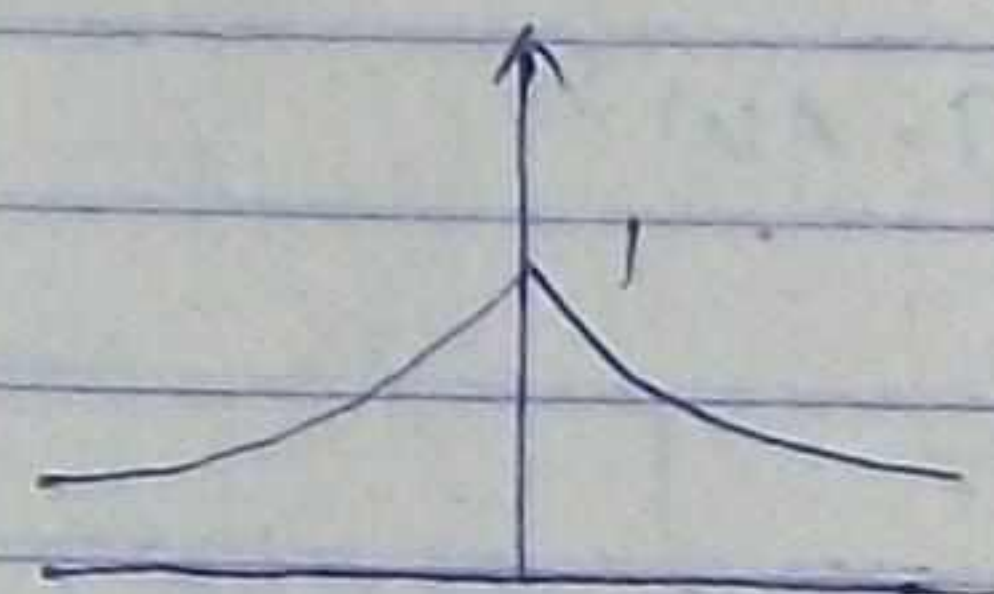
(4) $e^{-3t} u(-t)$, $a = -3 < 0$
do not exist

(c) $x(t) = e^{-a|t|}$

$$x(t) = e^{-at} u(t) + e^{at} u(-t)$$



for $a < 0$



for $a > 0$.

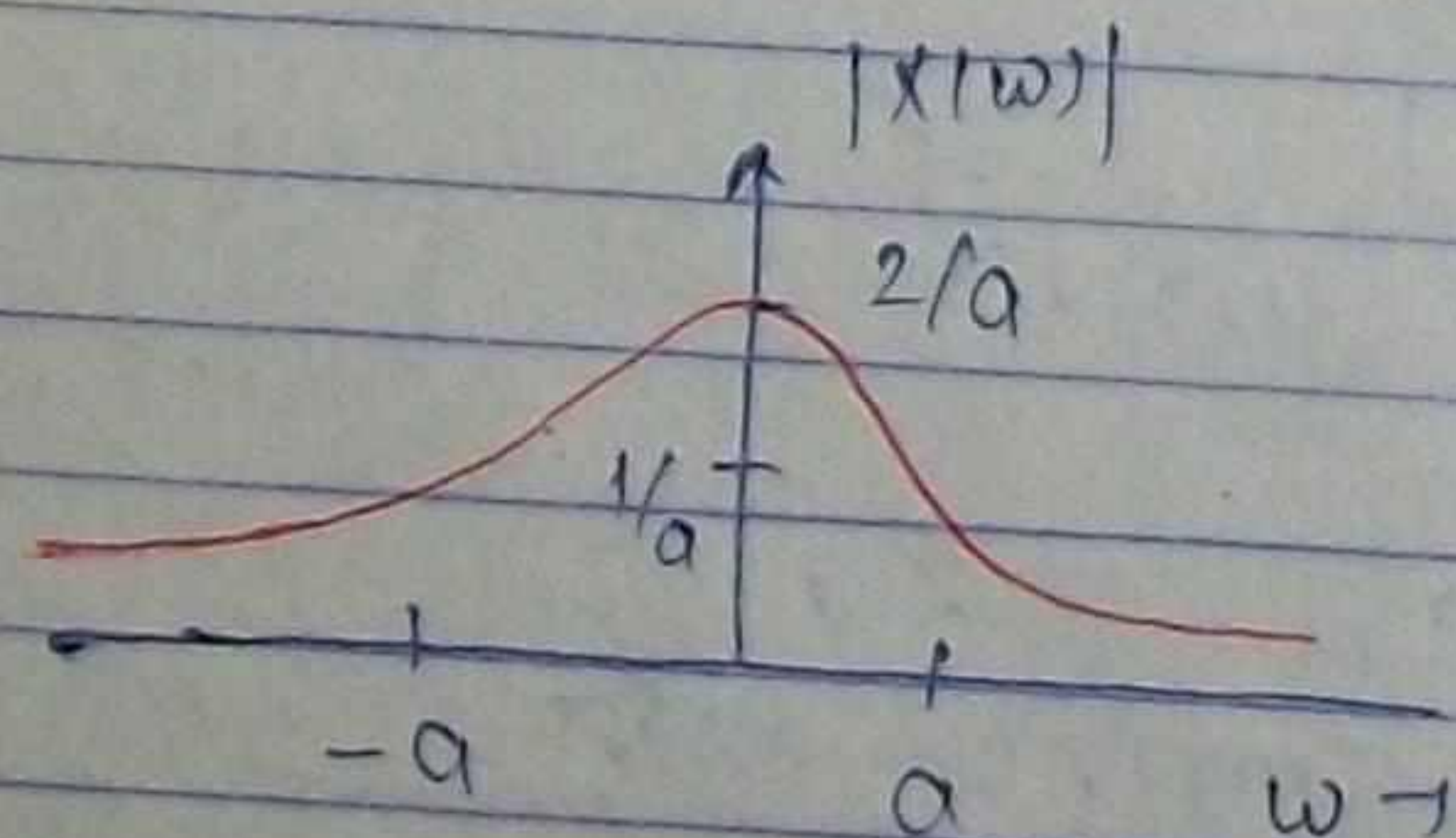
- for $a < 0$ the signal is not absolutely integrable therefore f.T is defined for $a > 0$ only.

$$X(\omega) = \frac{1}{a + j\omega} + \frac{1}{a - j\omega}$$

$$X(\omega) = \frac{2a}{a^2 + \omega^2}$$

$$|X(\omega)| = \frac{2a}{a^2 + \omega^2}$$

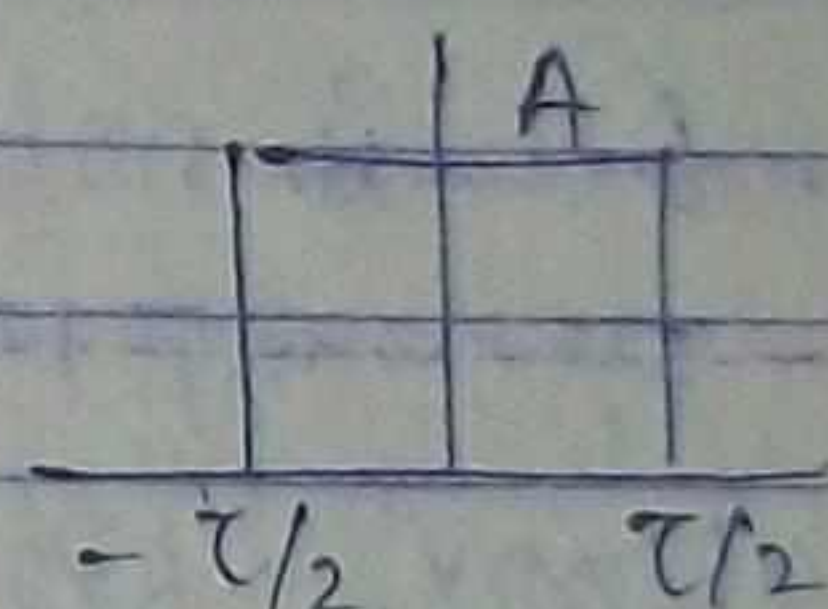
$$\angle X(\omega) = 0^\circ$$



ω	$ X(\omega) $	$\angle X(\omega)$
0	$2/a$	0
a	$1/a$	0
∞	0	0

(3) GATE FUNCTION -

$$x(t) = A \text{ rect}(t/\tau)$$



$$X(\omega) = \int_{-\tau/2}^{\tau/2} A e^{-j\omega t} dt$$

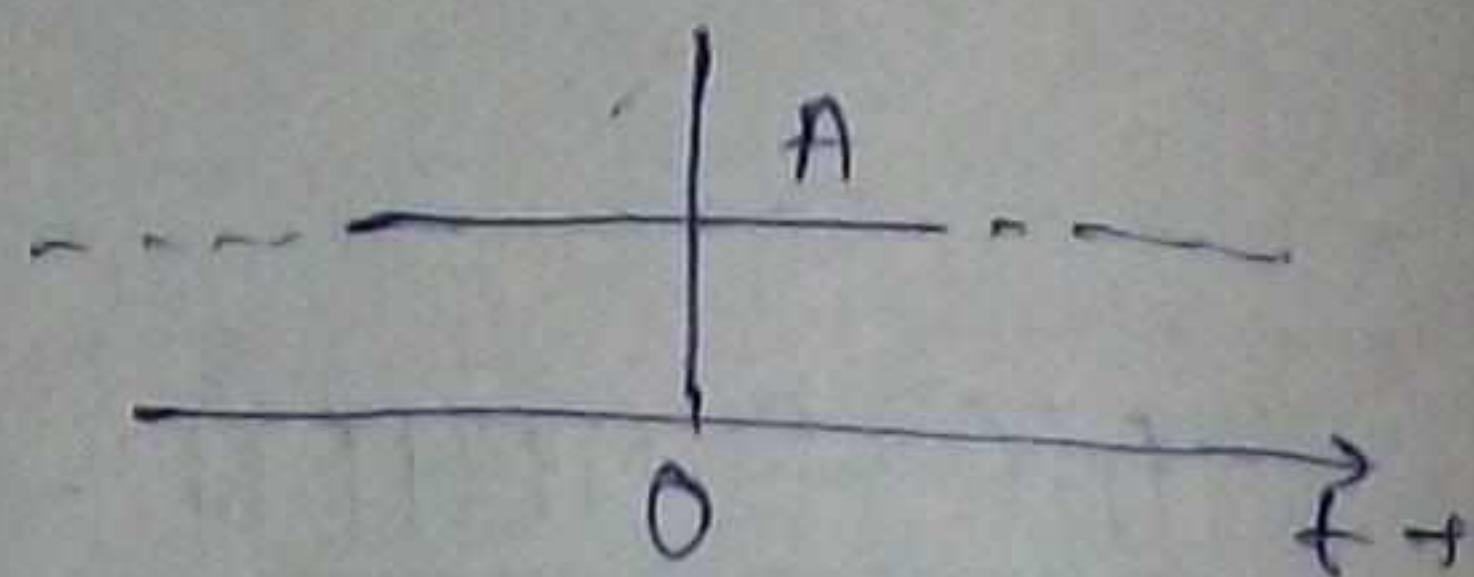
$$X(\omega) = \frac{A}{\omega} \left[\frac{e^{j\omega\tau/2} - e^{-j\omega\tau/2}}{j} \right]$$

$$X(\omega) = \frac{A}{\omega} 2 \sin(\omega\tau/2) = A \frac{\sin(\omega\tau/2)}{\omega/2}$$

$$\begin{aligned} X(\omega) &= A\tau \text{sinc}(\omega\tau/2) \\ &= A\tau \text{sinc}(\tau/T) \\ &= A\tau \text{sinc}(\pi\tau/T) \end{aligned}$$

④ CONSTANT FUNCTION -

$$x(t) = A$$



- DC is not absolute integrable hence its F.T. can't be determined directly, it can approx. from a gate pulse. by considering the gate pulse of height A width τ & centre '0'. and by assuming $\tau \rightarrow \infty$.

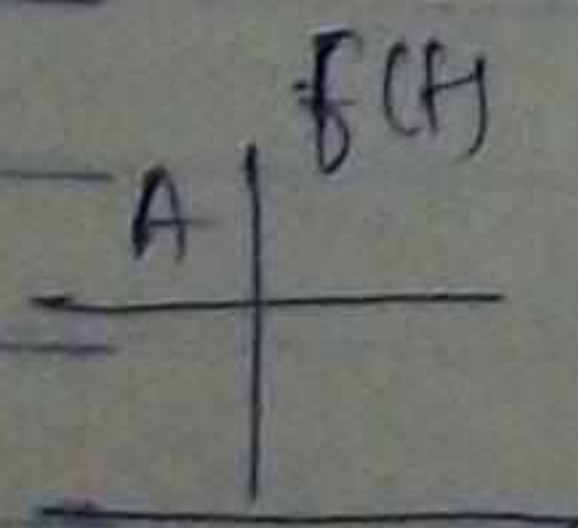
$$\begin{aligned} F\{A\} &= \lim_{\tau \rightarrow \infty} F\{A \text{ rect}(t/\tau)\} \\ &= \lim_{\tau \rightarrow \infty} A\tau \text{Sa}(\omega\tau/2) \end{aligned}$$

$$\lim_{k \rightarrow \infty} \frac{k}{\pi} \text{Sa}(kp) = \delta(p) \quad p \rightarrow \text{domain}$$

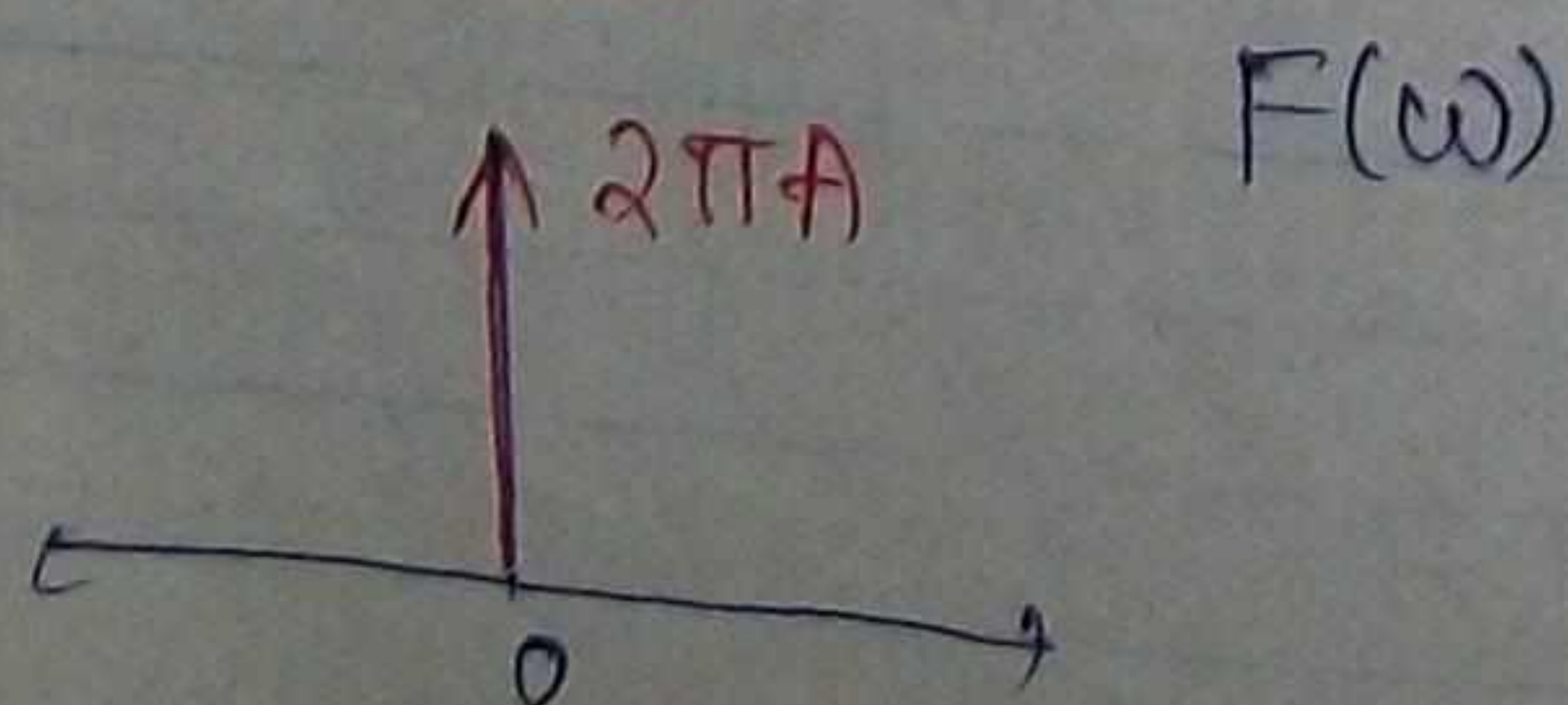
$$\lim_{k \rightarrow \infty} \frac{k}{\pi} \text{Sa}(k(p-p_0)) = \delta(p-p_0)$$

$$F\{A\} = 2A\pi \lim_{\tau \rightarrow \infty} \frac{(\tau/2)}{\pi} \text{Sa}(\omega\tau/2)$$

$$F\{A\} = 2A\pi \delta(\omega)$$

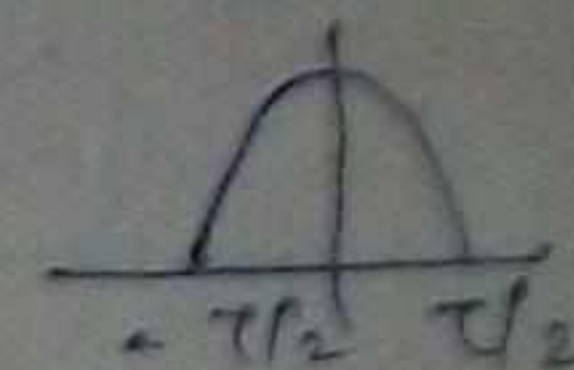
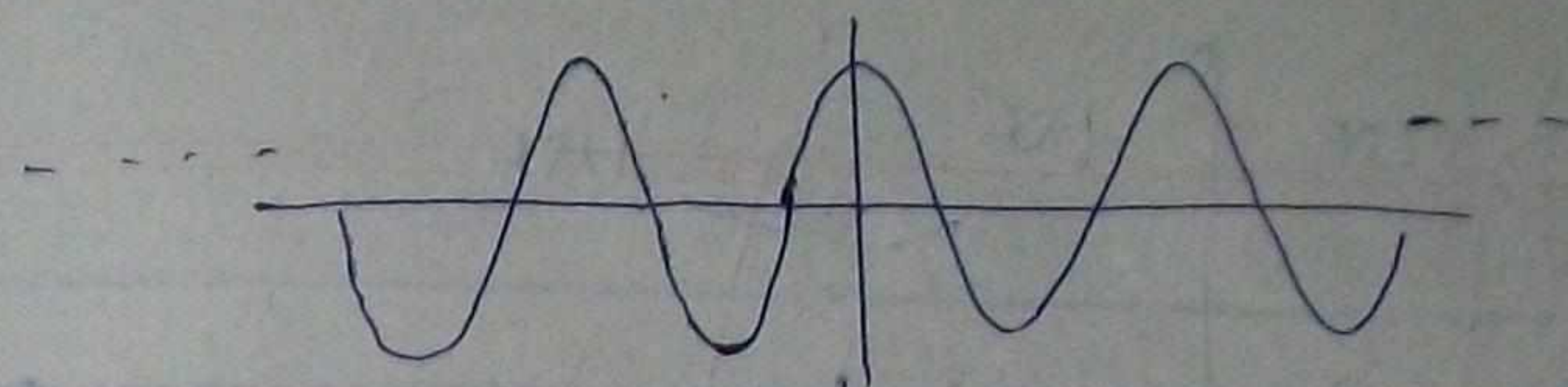


$\Leftrightarrow$



⑤ EVERLASTING COSINE -

$$x(t) = \cos \omega_0 t$$

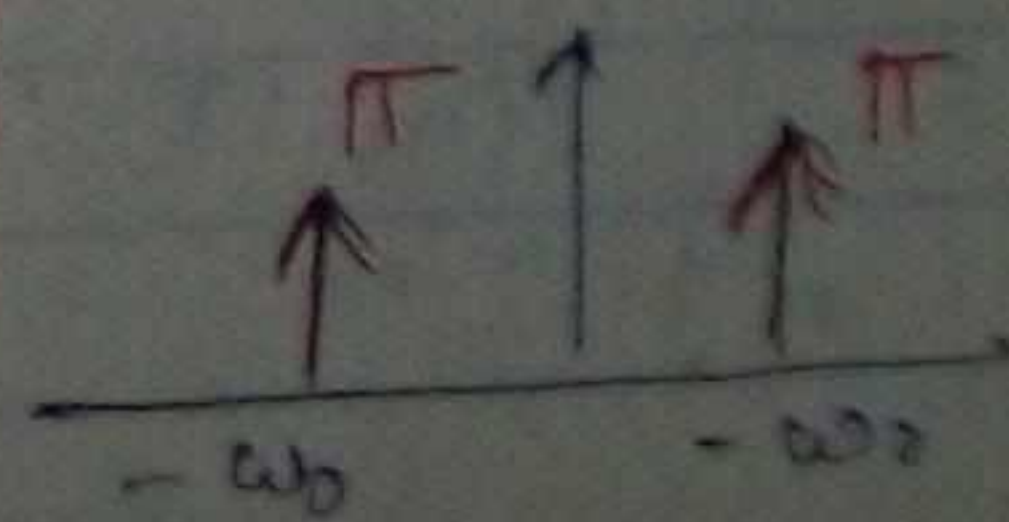


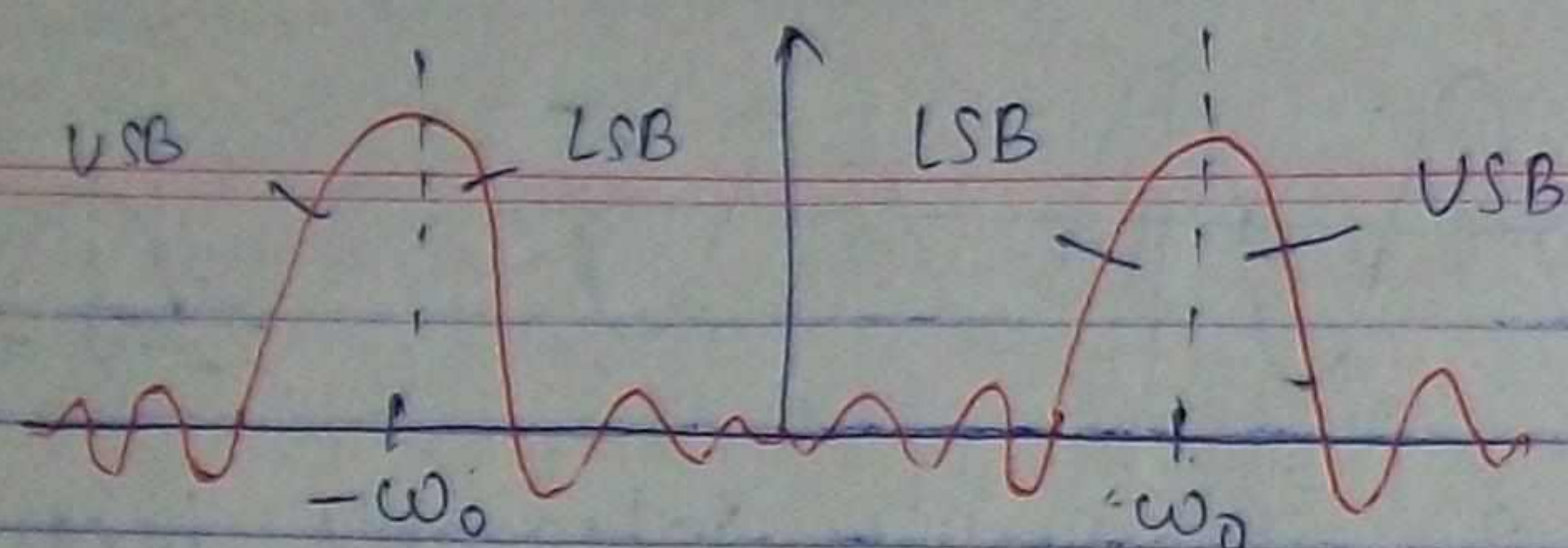
- $\cos \omega_0 t$ is not absolute integrable hence its Fourier transform cannot be determined directly it can be approximated by considering, a cosine funⁿ from $-\tau/2$ to $\tau/2$ and by letting $\tau \rightarrow \infty$

$$\begin{aligned} F\{\cos \omega_0 t\} &= \lim_{\tau \rightarrow \infty} \int_{-\tau/2}^{\tau/2} \cos(\omega_0 t) e^{-j\omega t} dt \\ &= \lim_{\tau \rightarrow \infty} \int_{-\tau/2}^{\tau/2} \left(\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2} \right) e^{-j\omega t} dt \\ &= \frac{1}{2} \lim_{\tau \rightarrow \infty} \int_{-\tau/2}^{\tau/2} e^{-j(\omega - \omega_0)t} dt + \frac{1}{2} \lim_{\tau \rightarrow \infty} \int_{-\tau/2}^{\tau/2} e^{-j(\omega + \omega_0)t} dt \\ &= \frac{1}{2} \lim_{\tau \rightarrow \infty} \tau \text{Sa}((\omega - \omega_0)\tau/2) + \frac{1}{2} \lim_{\tau \rightarrow \infty} \tau \text{Sa}((\omega + \omega_0)\tau/2) \\ &\quad \text{< from f.T of rect. pulse >} \end{aligned}$$

$$X(\omega) = \frac{1}{2} 2\pi \delta(\omega - \omega_0) + \frac{1}{2} 2\pi \delta(\omega + \omega_0) \quad \text{< from f.T of DC >}$$

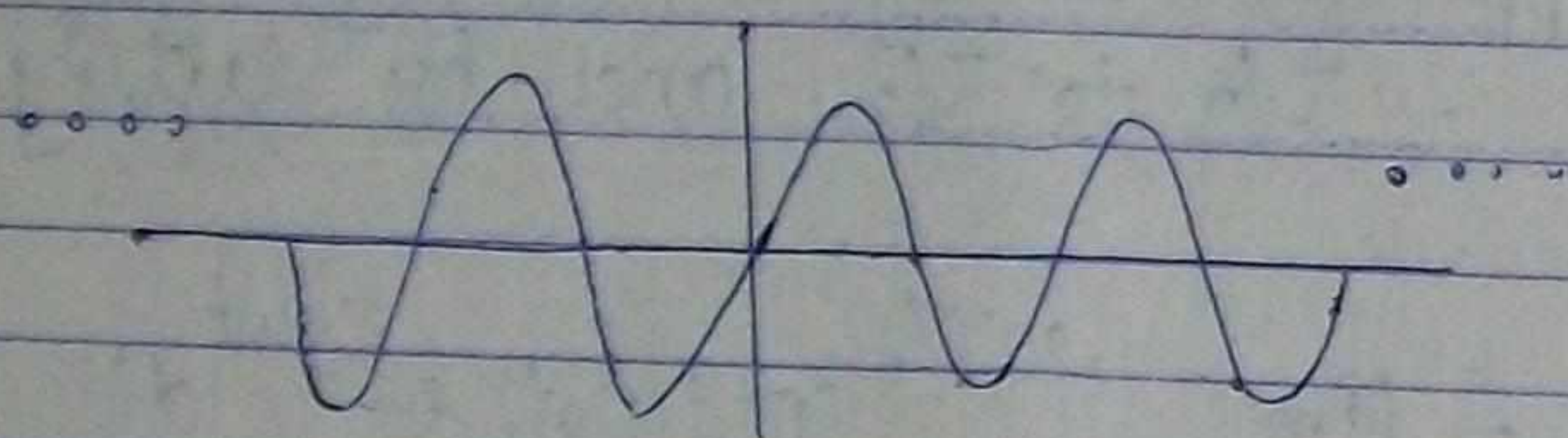
$$X(\omega) = \pi \delta(\omega - \omega_0) + \pi \delta(\omega + \omega_0)$$





⑥ EVERLASTING SINE FUNCTION -

$$x(t) = \sin \omega_0 t$$



— $\sin \omega_0 t$ is not absolutely integrable hence its F.T. can be determined directly.

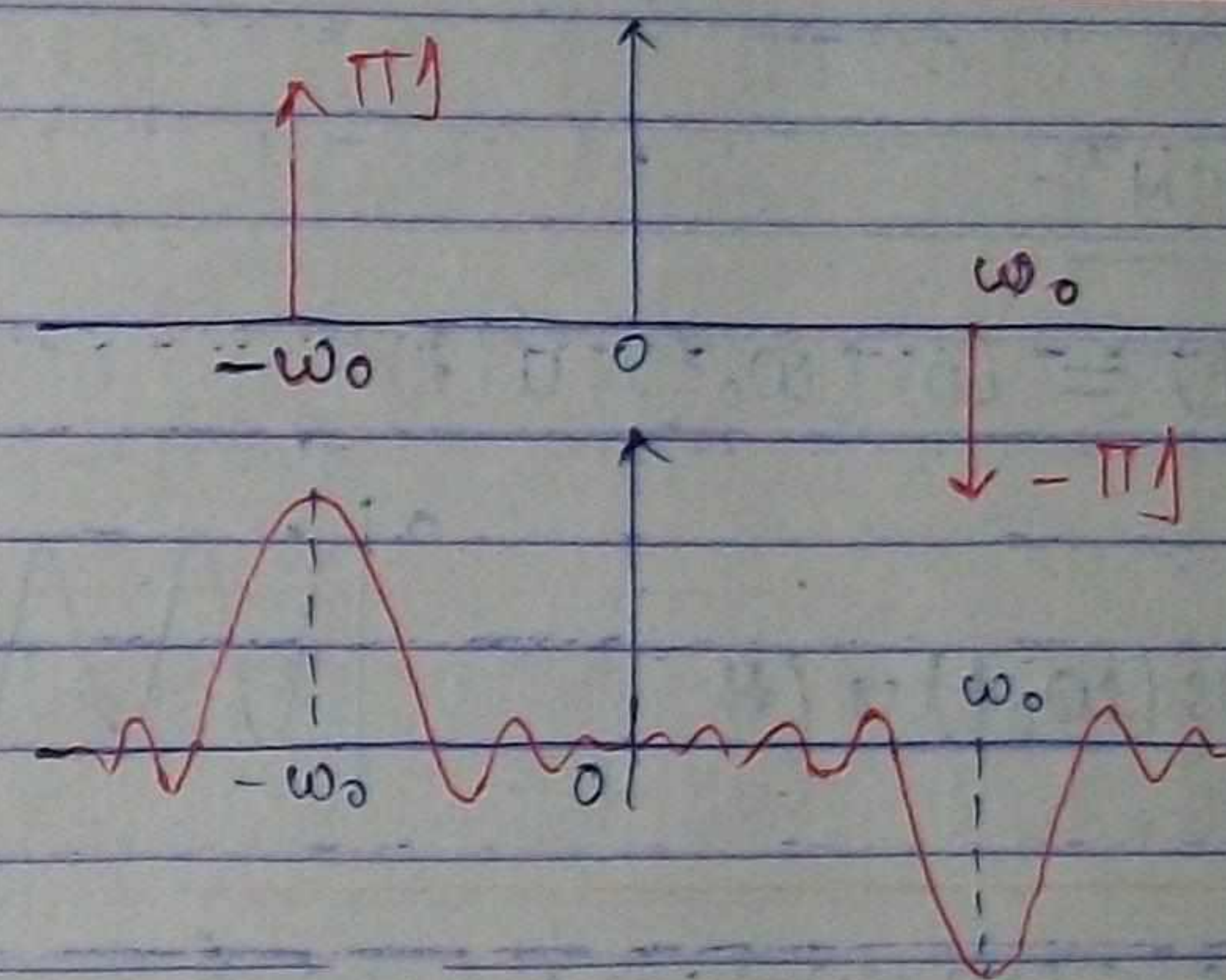
$$\begin{aligned} F\{\sin \omega_0 t\} &= \lim_{\tau \rightarrow \infty} \int_{-\tau/2}^{\tau/2} \sin(\omega_0 t) e^{-j\omega t} dt \\ &= \lim_{\tau \rightarrow \infty} \int_{-\tau/2}^{\tau/2} \left(\frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{j} \right) e^{-j\omega t} dt \end{aligned}$$

$$= \frac{1}{j} \lim_{\tau \rightarrow \infty} \int_{-\tau/2}^{\tau/2} e^{-(j\omega - j\omega_0)t} dt - \frac{1}{j} \lim_{\tau \rightarrow \infty} \int_{-\tau/2}^{\tau/2} e^{-(j\omega + j\omega_0)t} dt$$

$$= \frac{1}{j} \lim_{\tau \rightarrow \infty} \tau \text{Sa}((\omega - \omega_0)\tau/2) - \frac{1}{j} \lim_{\tau \rightarrow \infty} \tau \text{Sa}((\omega + \omega_0)\tau/2)$$

$$= \frac{1}{2j} 2\pi \delta(\omega - \omega_0) - \frac{1}{2j} 2\pi \delta(\omega + \omega_0)$$

$$X(\omega) = j\pi \delta(\omega + \omega_0) - j\pi \delta(\omega - \omega_0)$$



⑦ EVERLASTING EXPONENTIAL -

$$x(t) = e^{j\omega_0 t}$$

$$x(t) = \cos \omega_0 t + j \sin \omega_0 t$$

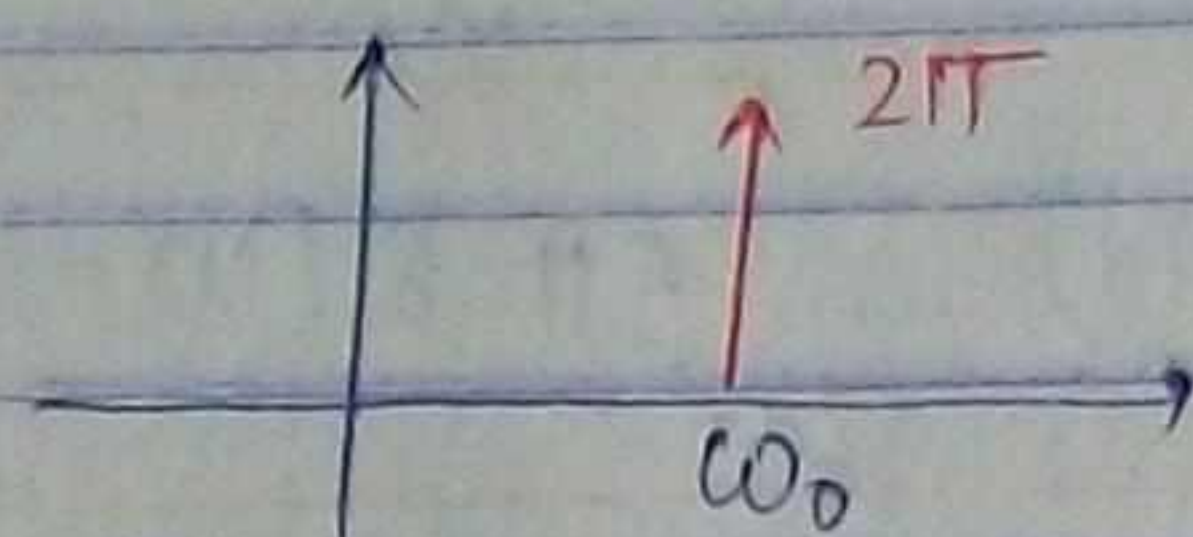
$$X(\omega) = F\{\cos \omega_0 t + j \sin \omega_0 t\}$$

$$X(\omega) = \pi \delta(\omega + \omega_0) + \pi \delta(\omega - \omega_0) +$$

$$j [j\pi \delta(\omega + \omega_0) - j\pi \delta(\omega - \omega_0)]$$

$$= 2\pi \delta(\omega - \omega_0)$$

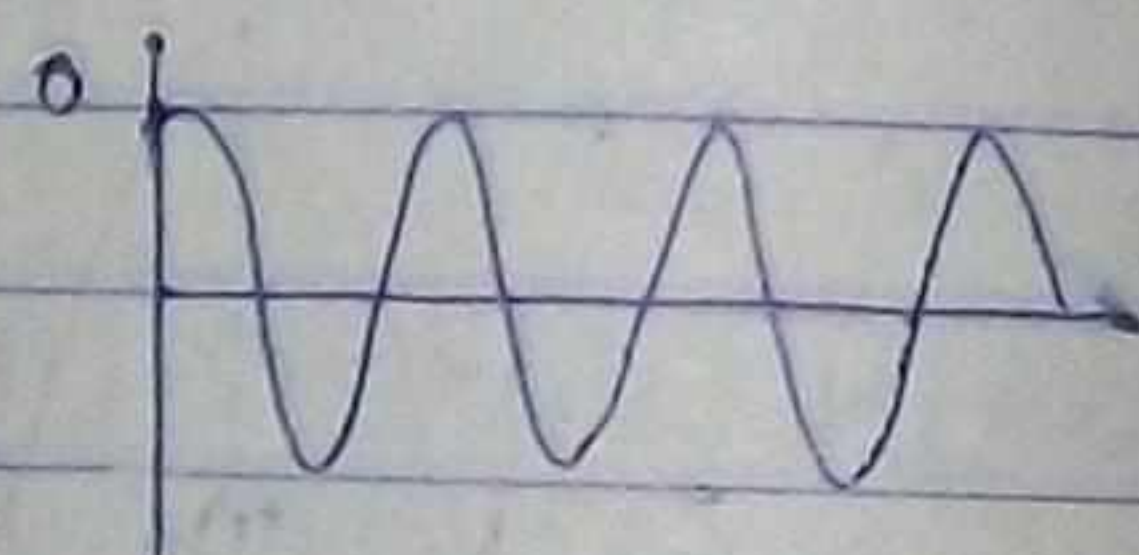
$$X(\omega) = 2\pi\delta(\omega - \omega_0)$$



⑧ COSINE FUNⁿ -

$$x(t) = \cos(\omega_0 t) u(t)$$

(A) $x(t) = \cos(\omega_0 t) u(t)$



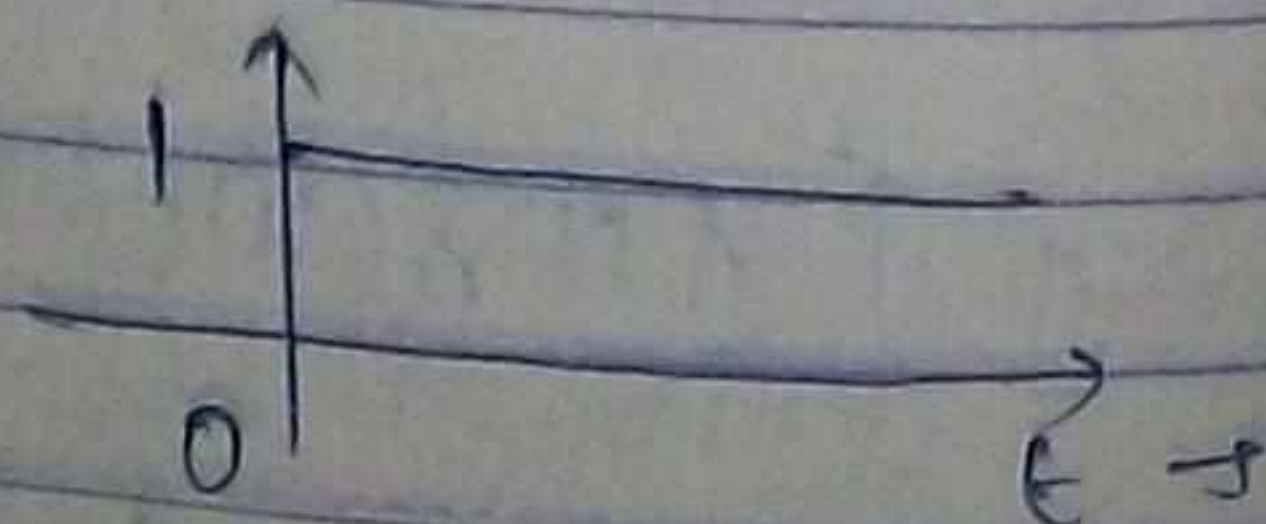
$$X(\omega) = \frac{\pi}{2} \{ \delta(\omega + \omega_0) + \delta(\omega - \omega_0) \} + \frac{j\omega}{\omega_0^2 - \omega^2}$$

(B) $x(t) = \sin(\omega_0 t) u(t)$

$$X(\omega) = j\frac{\pi}{2} \{ \delta(\omega + \omega_0) - \delta(\omega - \omega_0) \} + \frac{\omega_0}{\omega_0^2 - \omega^2}$$

⑨ UNIT STEP FUNCTION -

$$x(t) = u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$



- Step signal is not absolute integrable hence its f.t can't be determined directly. It can be approx. from $\cos \omega_0 t u(t)$ by letting $\omega_0 \rightarrow 0$.

$$F\{u(t)\} = \lim_{\omega_0 \rightarrow 0} F\{\cos(\omega_0 t) u(t)\}$$

$$= \lim_{\omega_0 \rightarrow 0} \frac{\pi}{2} \{ \delta(\omega + \omega_0) + \delta(\omega - \omega_0) \} + \frac{j\omega}{\omega_0^2 - \omega^2}$$

$$X(\omega) = \pi\delta(\omega) + \frac{1}{j\omega}$$

Similarly

$$x(t) = u(-t)$$

$$F\{u(-t)\} = \pi\delta(-\omega) + \frac{1}{j(-\omega)}$$

$$F\{u(-t)\} = \pi\delta(\omega) - \frac{1}{j\omega}$$

⑩ SIGNUM FUNCTION -

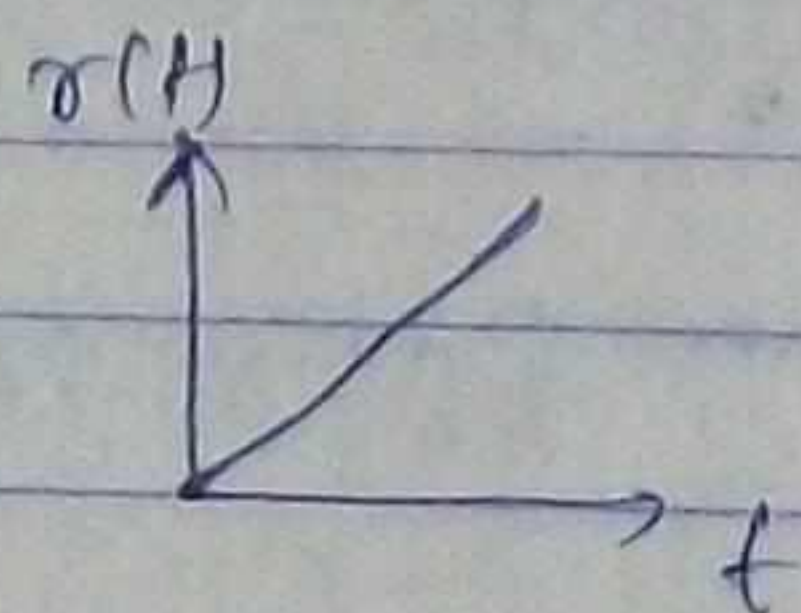
$$x(t) = \text{sgn}(t) = u(t) - u(-t)$$

$$X(\omega) = F\{u(t)\} - F\{u(-t)\}$$

$$X(\omega) = \frac{2}{j\omega}$$

(II) RAMP SIGNAL -

$$x(t) = r(t)$$



- Ramp signal is not absolute integrable hence its Fourier transform cannot be determined as there is no signal to approximate the ramp signal. Hence its F.T. does not exist.

* If the signal is neither power nor energy then its F.T. does not exist.

- for energy signal Fourier transform can be determined directly and for power signal F.T. can be determined by indirect method.

⇒ FOURIER TRANSFORM TABLE -

SIGNAL	F.T. IN ' ω '	F.T. IN ' f '
(1) $A\delta(t)$	A	A
(2) A	$2\pi A\delta(\omega)$	$A\delta(f)$
(3) $u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$	$\frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$
(4) $u(-t)$	$\pi\delta(\omega) - \frac{1}{j\omega}$	$\frac{1}{2}\delta(f) - \frac{1}{j2\pi f}$
(5) $\text{sgn}(t)$	$\frac{2}{j\omega}$	$\frac{1}{j\pi f}$
(6) $\cos(\omega_0 t)$	$\pi \left[\delta(\omega + \omega_0) + \delta(\omega - \omega_0) \right]$	$\frac{1}{2} \left[\delta(f + f_0) + \delta(f - f_0) \right]$
(7) $\sin(\omega_0 t)$	$j\pi \left[\delta(\omega + \omega_0) - \delta(\omega - \omega_0) \right]$	$\frac{j}{2} \left[\delta(f + f_0) - \delta(f - f_0) \right]$
(8) $\cos\omega_0 t \cdot u(t)$	$\frac{\pi}{2} \left[\delta(\omega + \omega_0) + \delta(\omega - \omega_0) \right] + \frac{j\omega}{\omega_0^2 - \omega^2}$	$\frac{1}{4} \left[\delta(f + f_0) + \delta(f - f_0) \right] + \frac{jf}{2\pi(f_0^2 - f^2)}$

(9) $\sin \omega_0 t u(t)$	$\frac{1}{2} \left[\frac{1}{j(\omega + \omega_0)} - \frac{1}{j(\omega - \omega_0)} \right]$	$\frac{1}{4} \left[\frac{1}{s(b + b_0)} - \frac{1}{s(b - b_0)} \right]$
	$+ \frac{\omega_0}{\omega_0^2 - \omega^2}$	$+ \frac{b_0}{2\pi(b_0^2 - b^2)}$
(10) $e^{j\omega_0 t}$	$2\pi \delta(\omega - \omega_0)$	$2\pi \delta(b - b_0)$
(11) $e^{-at} u(t)$	$\frac{1}{a + j\omega}$	$\frac{1}{a + j2\pi f}$ $a > 0$
(12) $e^{at} u(-t)$	$\frac{1}{a - j\omega}$	$\frac{1}{a - j2\pi f}$ $a > 0$
(13) $e^{-a t }$	$\frac{2a}{a^2 + \omega^2}$	$\frac{2a}{a^2 + (2\pi f)^2}$ $a > 0$
(14) $\text{Arect}\left(\frac{t}{\tau}\right) \rightarrow \text{AT Sa}\left(\omega \frac{\tau}{2}\right)$		
(15) $x(t) \rightarrow$ Does not exist.		

FOURIER TRANSFORM OF PERIODIC SIGNAL -

- Periodic signals are not absolutely integrable hence its Fourier transform can't be determined directly. But the EFS representation of the periodic signal is absolutely integrable hence the F.T. of

periodic signal is approximated from the exponential Fourier series.

- Let signal $x(t)$ be periodic is fundamental period T

$$\omega_0 = \frac{2\pi}{T}$$

$$F\{x(t)\} = F\left\{\sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}\right\}$$

$$= \sum_{n=-\infty}^{\infty} C_n F\{e^{jn\omega_0 t}\}$$

$$X(\omega) = 2\pi \sum_{n=-\infty}^{\infty} C_n \delta(\omega - n\omega_0)$$

$$X(\omega) = 2\pi \left[C_0 \delta(\omega) + C_1 \delta(\omega - \omega_0) + \dots \right] + \left[C_{-1} \delta(\omega + \omega_0) + C_{-2} \delta(\omega + 2\omega_0) + \dots \right]$$

- Therefore Fourier transform of a periodic signal contain train of impulses that are repeating for every ω_0 from $-\infty$ to ∞ and strength of each impulse is same as 2π times the corresponding coeff. in EFS representation of its.

⇒ PROPERTIES OF FOURIER TRANSFORM -

$$x(t) \xleftrightarrow{\text{F.T.}} X(\omega) = |X(\omega)| e^{j\theta(\omega)}$$

$$y(t) \xleftrightarrow{\text{F.T.}} Y(\omega)$$

$$h(t) \xleftrightarrow{\text{F.T.}} H(\omega)$$

a, b, t_0 and ω_0 : constant.

① LINEARITY -

- Fourier transform is linear operation. i.e. F.T. obeys principle of superposition and homogeneity.

$$a x(t) + b y(t) \longleftrightarrow a X(\omega) + b Y(\omega)$$

② TIME SHIFTING -

$$x(t - t_0) \longleftrightarrow e^{-j\omega t_0} X(\omega)$$

$$= |X(\omega)| e^{j\theta(\omega)} \cdot e^{-j\omega t_0}$$

$$= |X(\omega)| e^{j(\theta(\omega) - \omega t_0)}$$

$$x(t + t_0) \longleftrightarrow e^{j\omega t_0} X(\omega)$$

EFFECT : Shifting a signal in time domain causes phase shift in the freq. (i.e. multiplication by complex exponential) mean, time shifting does not change the magnitude spectrum where as phase spectrum will be shifted by the amount ωt_0

③ FREQUENCY SHIFTING - MODULATION

$$X(\omega - \omega_0) \longleftrightarrow e^{j\omega_0 t} x(t)$$

$$X(\omega + \omega_0) \longleftrightarrow e^{-j\omega_0 t} x(t)$$

- Therefore shifting in freq. causes phase shift in the time domain and vice versa.

④ TIME SCALING -

$$x(at) \longleftrightarrow \frac{1}{|a|} X(\omega/a)$$

Same in frequency domain

COMPRESSION IN T

EXPANSION IN ω

EFFECT : Compression in time domain causes expansion in the freq. domain by the same factor and vice versa.

⑤ TIME INVERSION -

$$x(-t) \longleftrightarrow x(-\omega)$$

⑥ CONVOLUTION IN TIME -

$$x(t) * h(t) \longleftrightarrow x(\omega) \cdot H(\omega)$$

- Convolution in time causes multiplication of their transforms in the frequency.

⑦ CONVOLUTION IN FREQUENCY -

$$X(\omega) * H(\omega) \longleftrightarrow 2\pi x(t) h(t)$$

⑧ DIFFⁿ IN TIME -

$$\frac{dx(t)}{dt} \longleftrightarrow j\omega x(\omega)$$

$$\frac{d^n x(t)}{dt^n} \longleftrightarrow (j\omega)^n x(\omega)$$

- Time differentiation eliminates the DC and enhances the a.c.

⑨ DIFFⁿ IN FREQUENCY -

$$\frac{d x(\omega)}{d\omega} \longleftrightarrow -jt x(t)$$

⑩ INTEGRⁿ IN TIME -

$$\int_{-\infty}^t x(t) dt \longleftrightarrow \frac{1}{j\omega} x(\omega) + \pi \delta(\omega) x(\omega)$$

DUE TO D.C.

$$= \frac{1}{j\omega} x(\omega) + \pi x(0) \delta(\omega)$$

⑪ INTEGRⁿ IN FREQUENCY -

$$\int_0^\omega x(\omega) d\omega \longleftrightarrow \frac{1}{-jt} x(t)$$

even funⁿ
freq. spectrum

⑫ DUALITY / SYMMETRY -

$$x(t) \longleftrightarrow x(\omega)$$

$$2\pi x(-\omega) \longleftrightarrow X(t)$$

⇒ F.T. OF REAL FUNCTION -

- If $x(t)$ is real then its F.T is conjugate symmetric i.e.

$$x(\omega) = x^*(-\omega)$$

eg:

$$e^{-2t} u(t) \longleftrightarrow \frac{1}{2+j\omega} = x(\omega)$$

$$x(-\omega) = \frac{1}{2-j\omega}$$

$$x^*(-\omega) = \frac{1}{2+j\omega}$$

⇒ F.T. OF REAL & EVEN FUNCTION -

- If $x(t)$ is real and even then its F.T. is real and even. i.e.

$$x(\omega) = x(-\omega)$$

eg: $x(t) = e^{-3|t|}$

$$x(\omega) = \frac{6}{9+\omega^2}$$

$$x(-\omega) = \frac{6}{9+\omega^2} = x(\omega)$$

⇒ F.T OF REAL & ODD FUNCTION -

- If $x(t)$ is real and odd then its F.T is imaginary and odd. i.e.

$$x(\omega) = -x(-\omega)$$

eg: $x(t) = \text{sgn}(t)$

$$x(\omega) = \frac{2}{j\omega}$$

$$x(-\omega) = \frac{-2}{j\omega} = -x(\omega)$$

⇒ F.T. OF IMAG. FUNCTION -

- If $x(t)$ is imag. then its F.T is real function.
- If $x(t)$ is imag and odd its F.T is Real and odd funn.

$R \rightarrow \text{Complex}$

$R \& E \rightarrow R \& E$

$R \& O \rightarrow I \& O$

⇒ PARSEVAL'S ENERGY THEOREM -

- It is used to find the energy of the signal.

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} |x(\omega)|^2 d\omega$$

⇒ AREA UNDER $x(t)$ -

$$\text{Area of } x(t) = \int_{-\infty}^{\infty} x(t) dt$$

$$= x(\omega) \Big|_{\omega=0}$$

$$= x(0)$$

$$x(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$x(0) = \int_{-\infty}^{\infty} x(t) dt$$

⇒ AREA UNDER $x(\omega)$ -

$$\text{Area of } x(\omega) = 2\pi x(0)$$

$$= \int_{-\infty}^{\infty} x(\omega) d\omega$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(\omega) e^{j\omega t} d\omega$$

$$x(0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(\omega) d\omega$$

Ques. find f.T. of the following:

① $-0.5 \rightarrow -\pi \delta(\omega)$

② $\delta(-3t) \rightarrow \frac{1}{3} \delta(\omega/3) = 1/3$

③ $\delta(3-2t) \rightarrow \frac{1}{2} \delta(\omega/2) \cdot e^{-\frac{3j\omega}{2}}$

$$\delta(2(t-1.5)) = \frac{e^{-3/2 j\omega}}{2}$$

④ $e^{-2t} \delta(4-t) \rightarrow \delta(\omega) \cdot e^{-j4\omega}$

$$= e^{-2(4)} \delta(t-4) = e^{-8} \delta(t-4)$$

$$\rightarrow e^{-8} \cdot e^{-j4\omega}$$

⑤ $e^{3t} u(t) \quad a < 0 \quad a = -3$

Does not exist.

⑥ $e^{-2t} u(-t) \quad a = -2 < 0$

does not exist

⑦ $e^{-3t} u(t) \rightarrow \frac{1}{3+j\omega}$

$$\textcircled{8} \quad e^{-3t u(t)} = e^{-3t} \text{ for } t > 0$$

$$= e^0 \text{ for } t < 0$$

$$e^{-3t u(t)} = e^{-3t} u(t) + u(-t)$$

$$= \frac{1}{3+j\omega} + \pi \delta(\omega) - \frac{1}{j\omega}$$

$$\textcircled{9} \quad e^{-3(t-2)} u(t)$$

$$= e^{-3t} \cdot e^6 u(t) \leftrightarrow e^6 \cdot \frac{1}{3+j\omega}$$

$$\textcircled{10} \quad e^{-3t} u(t-2)$$

$$= e^{-3(t-2)} \cdot e^{-6} u(t-2)$$

$$= e^{-6} \cdot \frac{1}{3+j\omega} \cdot e^{-2j\omega} = \frac{e^{-2(3+j\omega)}}{(3+j\omega)}$$

$$\textcircled{11} \quad \cos(3t) \rightarrow \pi \delta(\omega+3) + \pi \delta(\omega-3)$$

$$\textcircled{12} \quad [\cos(3t)] u(t)$$

$$\leftrightarrow \frac{\pi}{2} [\delta(\omega+3) - \delta(\omega-3)] + \frac{j\omega}{9-\omega^2}$$

$$\textcircled{13} \quad [\cos 3(t-2)] u(t)$$

$$= \cos(3t-6) u(t)$$

$$= \cos 6 \cdot \cos 3t u(t) + \sin 6 \cdot \sin 3t u(t)$$

$$\leftrightarrow \cos 6 \cdot \left[\frac{\pi}{2} (\delta(\omega+3) + \delta(\omega-3)) + \frac{j\omega}{9-\omega^2} \right]$$

$$+ \sin 6 \cdot \left[\frac{\pi}{2} (\delta(\omega+3) - \delta(\omega-3)) + \frac{3}{9^2-\omega^2} \right]$$

$$\textcircled{14} \quad [\cos 3(t-2)] u(t-2)$$

$$\leftrightarrow \left[\frac{\pi}{2} (\delta(\omega+3) - \delta(\omega-3)) + \frac{j\omega}{9-\omega^2} \right] e^{-2j\omega}$$

$$\textcircled{15} \quad \cos 3t u(t-2)$$

$$= \cos 3(t-2+2) u(t-2)$$

$$= \cos 6 \cdot \cos 3(t-2) u(t-2) - \sin 6 \cdot \sin 3(t-2) u(t-2)$$

$$\leftrightarrow \cos 6 \cdot e^{-j2\omega} \left[\frac{\pi}{2} \{ \delta(\omega+3) + \delta(\omega-3) \} + \frac{j\omega}{9-\omega^2} \right]$$

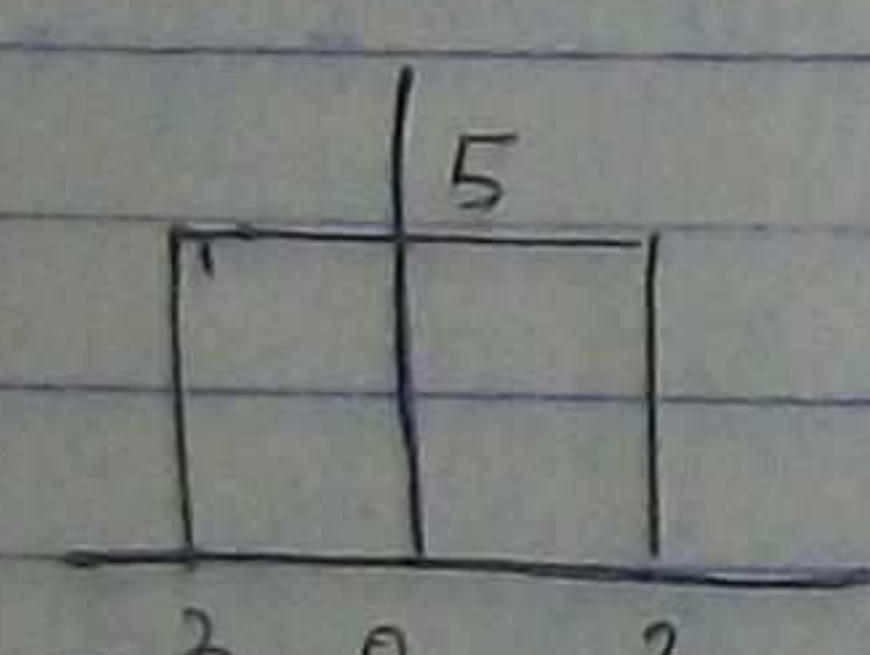
$$- \sin 6 \cdot e^{-j2\omega} \left[\frac{\pi}{2} (\delta(\omega+3) - \delta(\omega-3)) + \frac{j}{9-\omega^2} \right]$$

(16) $e^{j3t} \leftrightarrow 2\pi \delta(\omega-3)$

(17) $5 \text{rect}(t/7) \rightarrow (5 \times 7) \text{Sa}(\omega 7/2)$

~~(18)~~

Que. find the fourier transform of:

(1)  $5 \text{rect}(t/6) \leftrightarrow (5 \times 6) \text{Sa}(\omega 6/2)$

Method (1):

$$X(t) = 5u(t+3) - 5u(t-3)$$

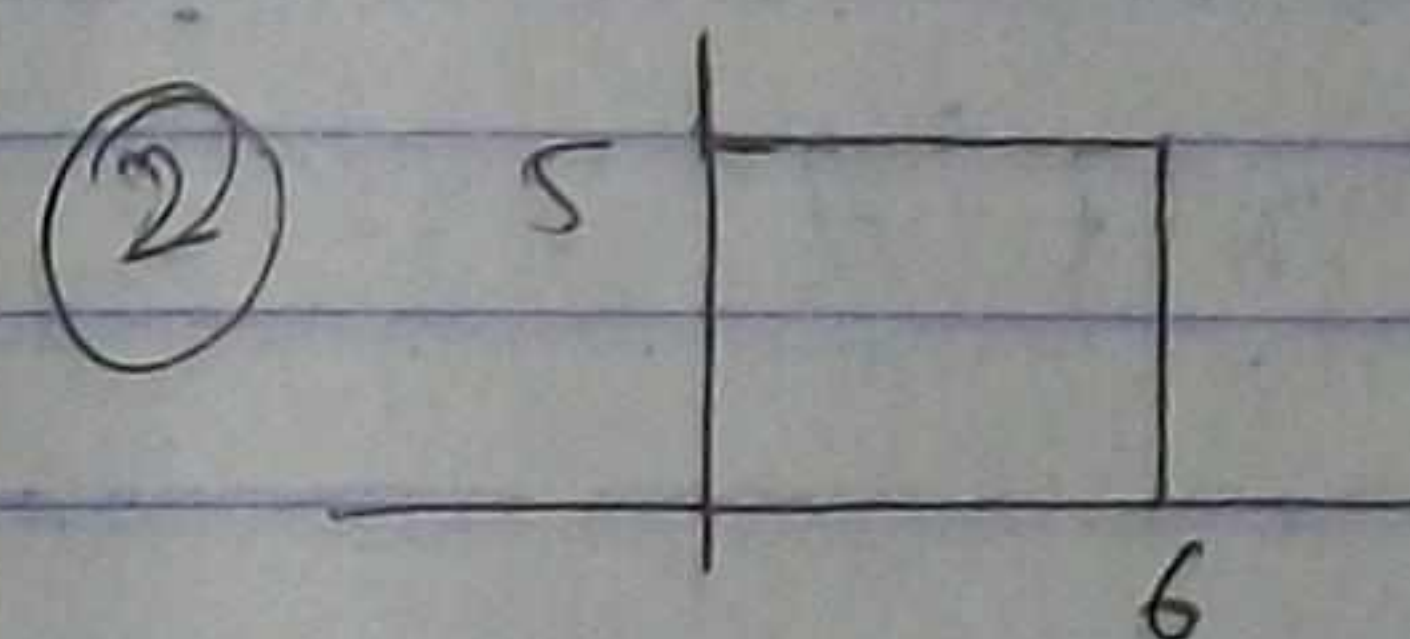
$$X(\omega) = 5 \left[\pi \delta(\omega) + \frac{1}{j\omega} \right] (e^{j3\omega} - e^{-j3\omega})$$

$$= 10 \left[\pi \delta(\omega) + \frac{1}{j\omega} \right] \sin(3\omega)$$

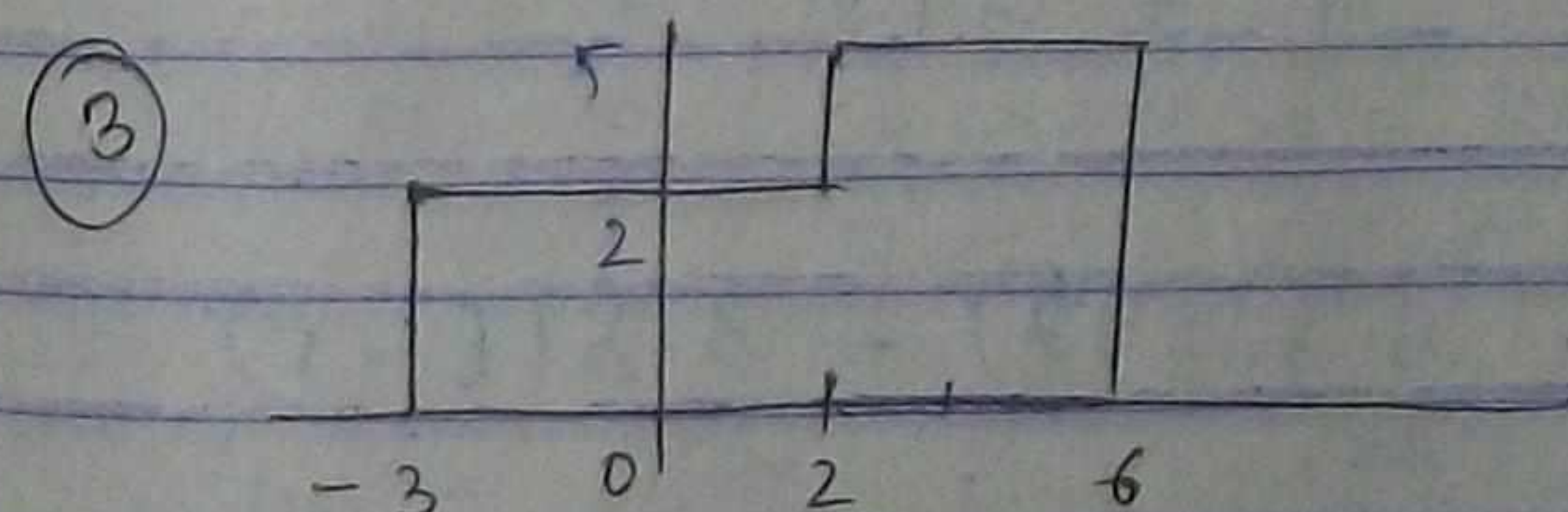
$$= 10 \sin(3\omega) \pi \delta(\omega) + \frac{10 \sin(3\omega)}{j\omega}$$

$$= \frac{10 \sin(3\omega)}{\omega}$$

$$= 30 \text{Sa}(3\omega)$$



$$X(\omega) = e^{-j3\omega} 30 \text{Sa}(3\omega)$$



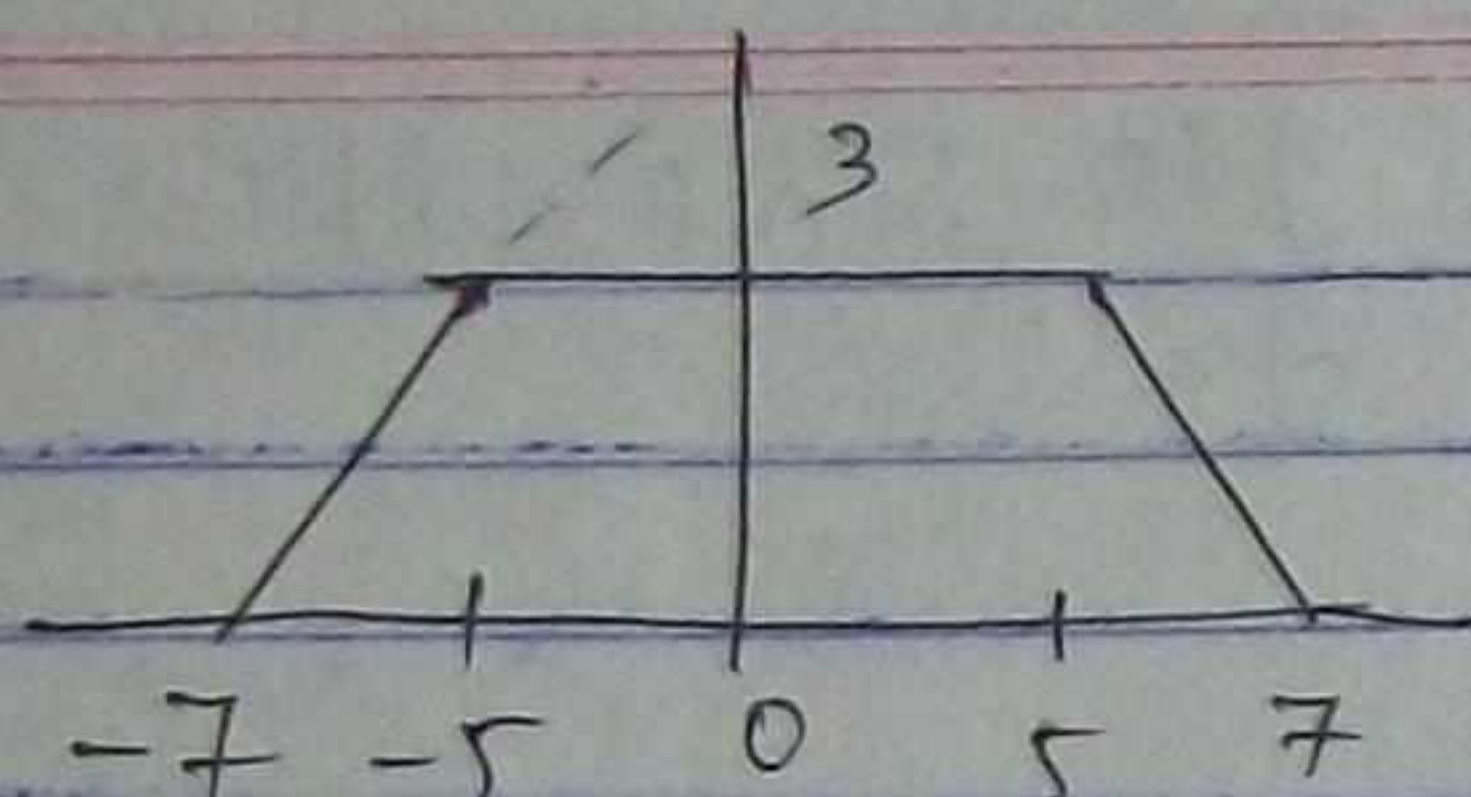
$$X(t) = 2u(t+3) + 3u(t-2) - 5u(t-6)$$

$$X(t) = \left(\pi \delta(\omega) + \frac{1}{j\omega} \right) \left[2e^{j\omega 3} + 3e^{-j\omega 2} + (-5)e^{-j\omega 6} \right]$$

Property used:
Time shifting
Linearity

TIME DIFFⁿ
TIME SHIFTING
LINEARITY

④



$$x(t) = \frac{3}{2} r(t+7) - \frac{3}{2} r(t+5) - \frac{3}{2} r(t-5) + \frac{3}{2} r(t-7)$$

$$x'(t) = \frac{3}{2} u(t+7) - \frac{3}{2} u(t+5) - \frac{3}{2} u(t-5) + \frac{3}{2} u(t-7)$$

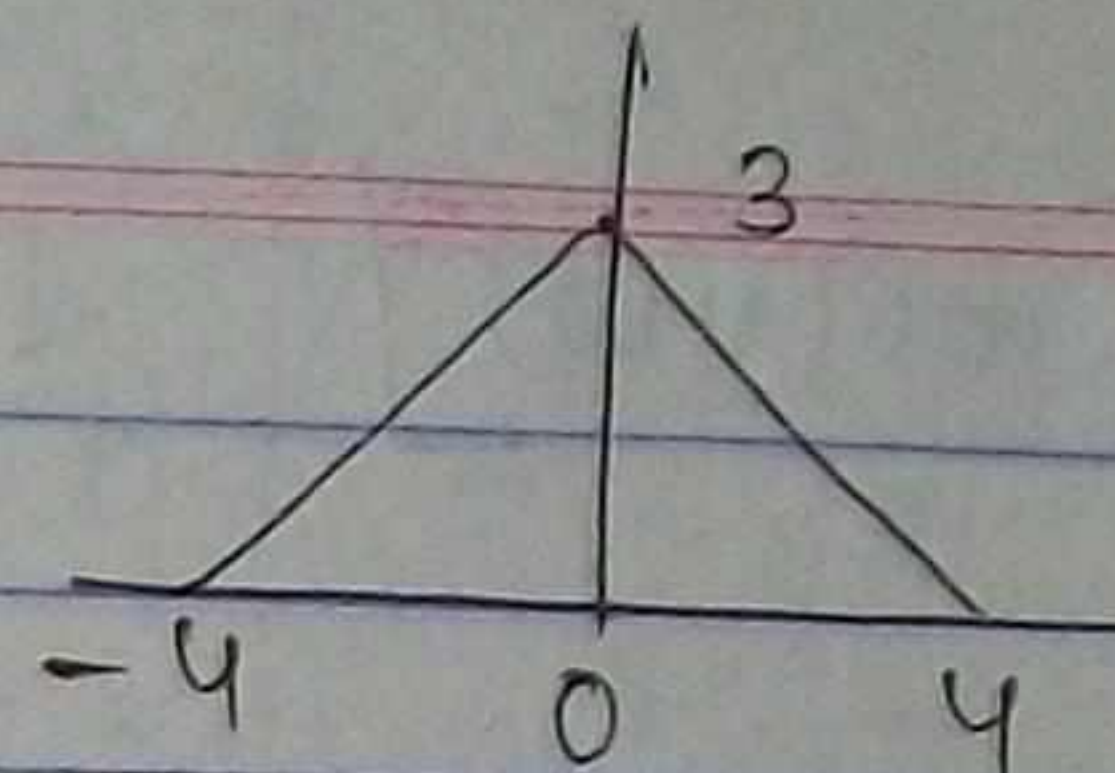
$$x''(t) = \frac{3}{2} \delta(t+7) - \frac{3}{2} \delta(t+5) - \frac{3}{2} \delta(t-5) + \frac{3}{2} \delta(t-7)$$

$$x(\omega) (j\omega)^2 = \frac{3}{2} [e^{j7\omega} - e^{j5\omega} - e^{-j5\omega} + e^{-j7\omega}]$$

$$x(\omega) = \frac{3}{2(j\omega)^2} [e^{j7\omega} - e^{j5\omega} - e^{-j5\omega} + e^{-j7\omega}]$$

$$= \frac{3}{(j\omega)^2} [\cos 7\omega - \cos 5\omega]$$

⑤



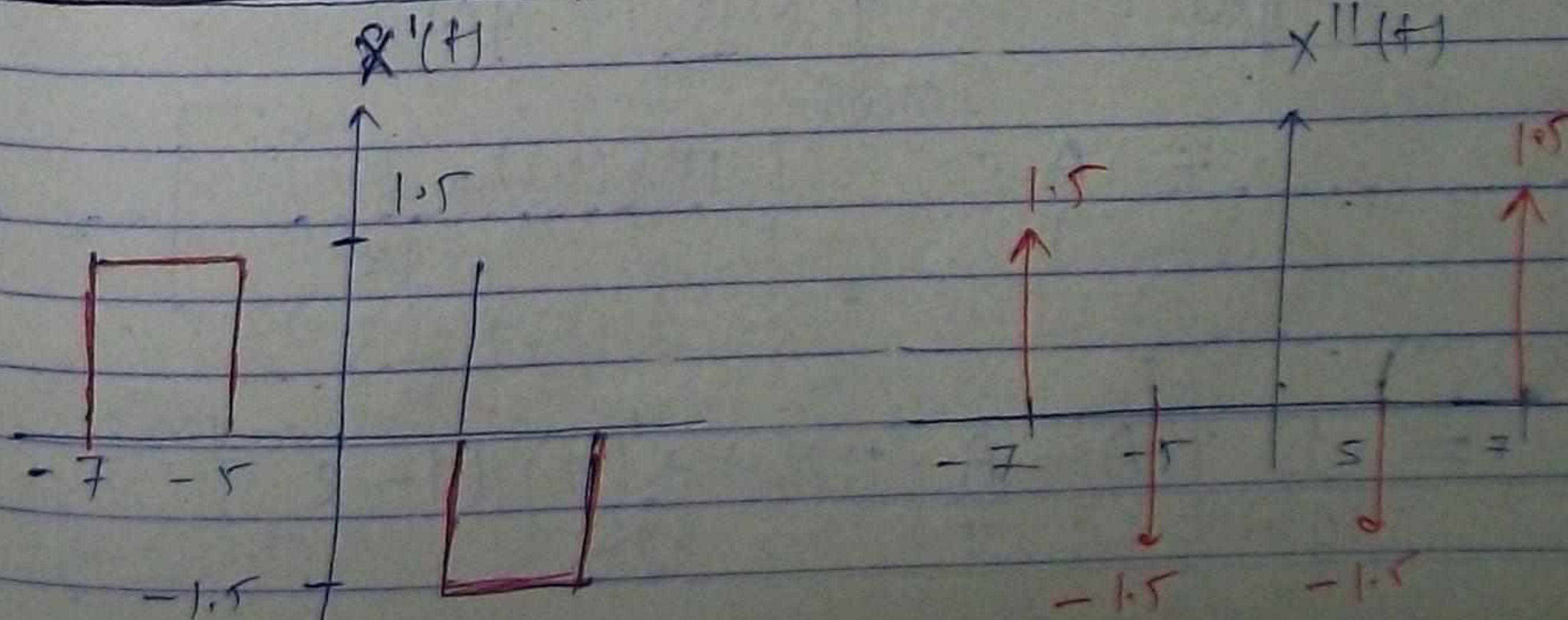
$$x(t) = \frac{3}{4} r(t+4) - \frac{3}{2} r(t) + \frac{3}{4} r(t-4)$$

$$x'(t) = \frac{3}{4} u(t+4) - \frac{3}{2} u(t) + \frac{3}{4} u(t-4)$$

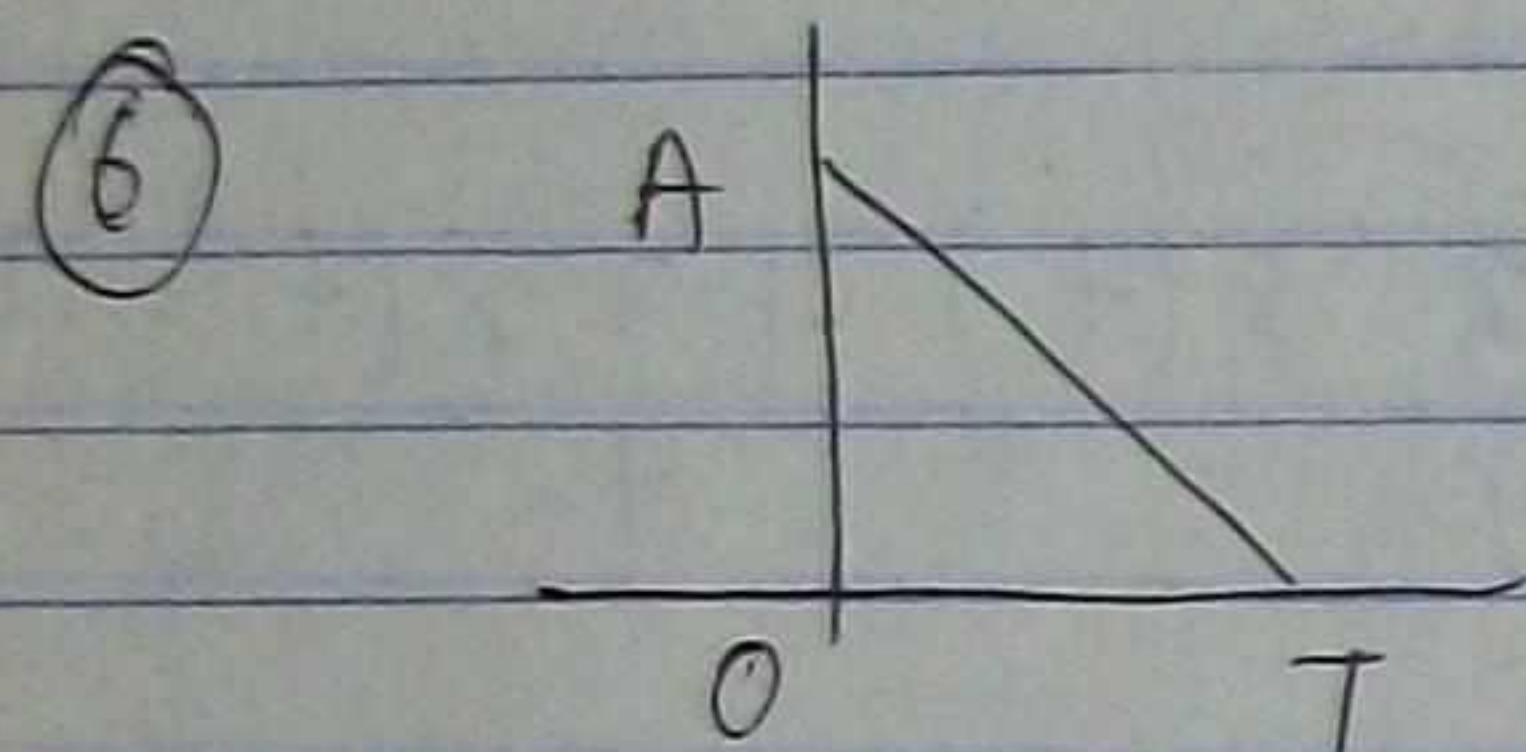
$$x''(t) = \frac{3}{4} \delta(t+4) - \frac{3}{2} \delta(t) + \frac{3}{4} \delta(t-4)$$

$$x(\omega) (j\omega)^2 = \frac{3}{4} e^{j4\omega} - \frac{3}{2} + \frac{3}{4} e^{-j4\omega}$$

$$x(\omega) = \frac{1}{(j\omega)^2} \cdot \left[\frac{3}{2} (\cos 4\omega - 1) \right]$$



$$X(\omega) = \frac{3}{2\omega^2} [1 - \cos(4\omega)]$$



$$x(t) = \underbrace{-A \tau(t)}_{\text{wrong}} + \frac{A}{T} \tau(t-T)$$

or

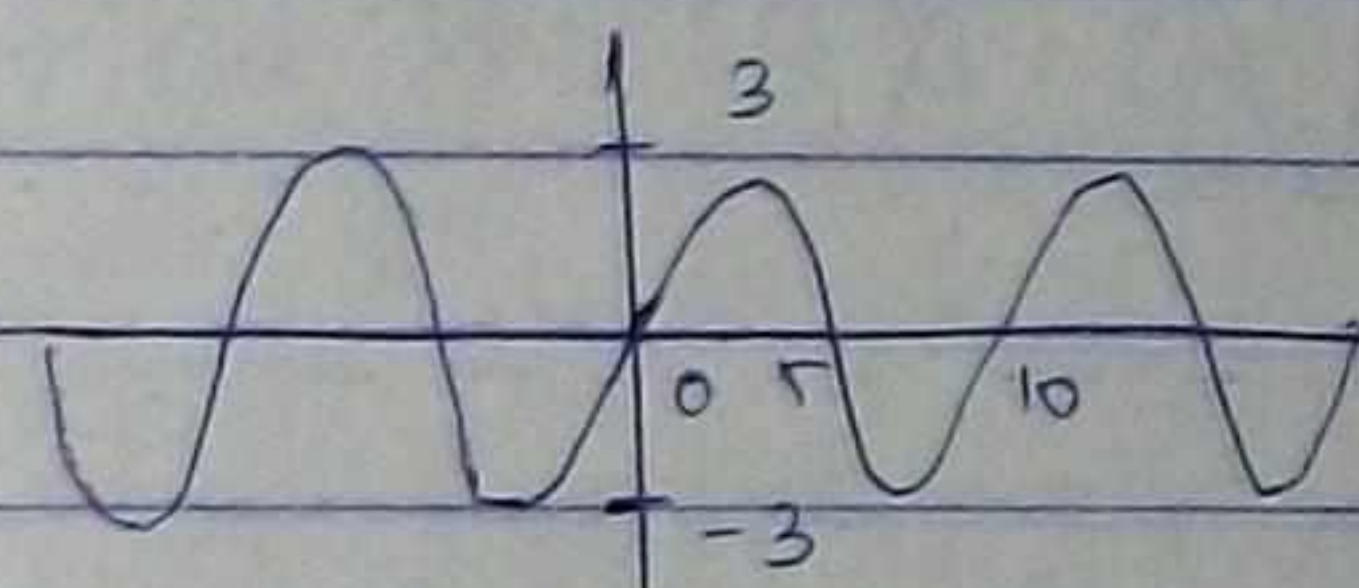
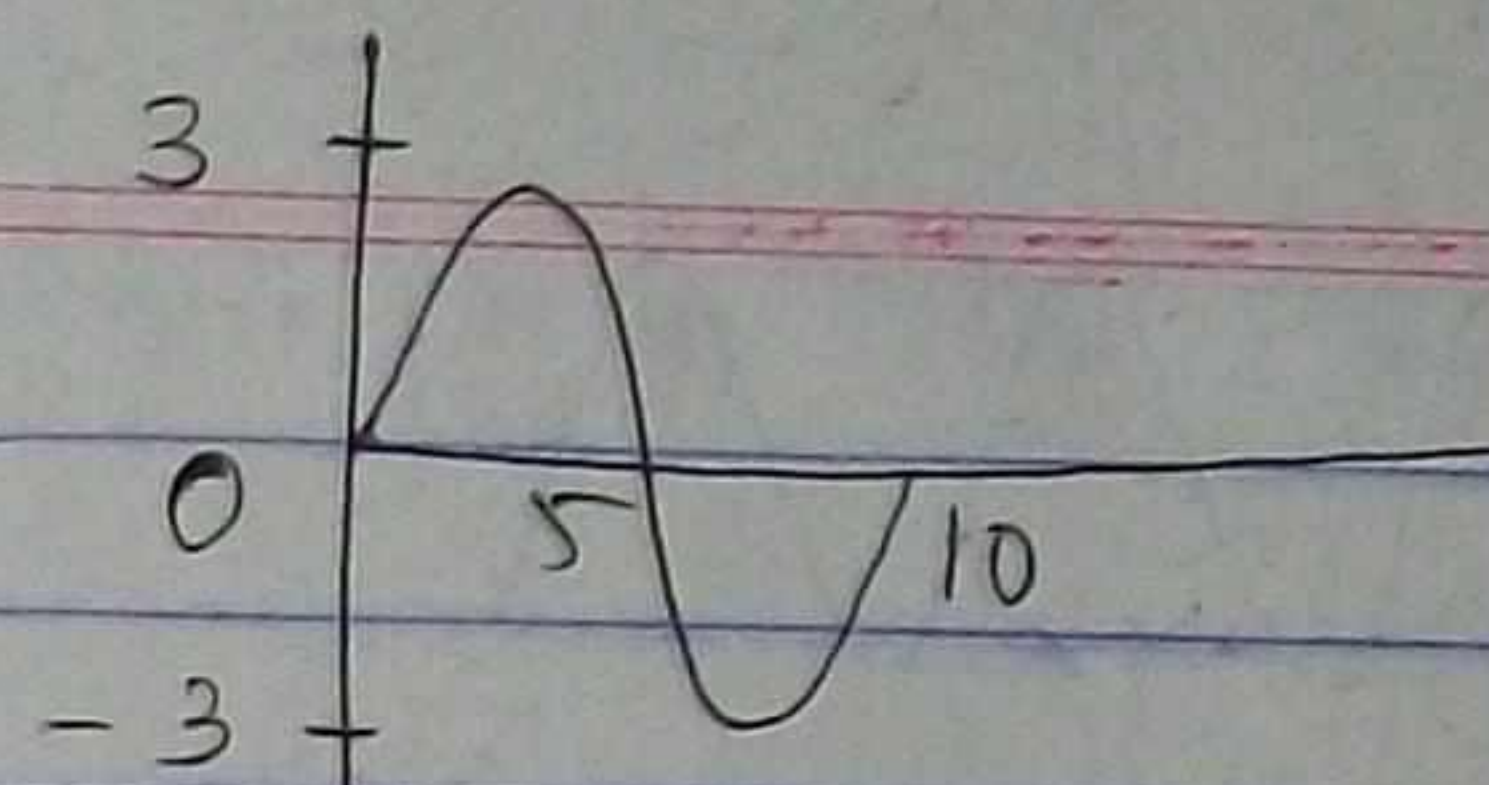
$$x(t) = A u(t) - \frac{A}{T} \tau(t) + \frac{A}{T} \tau(t-T)$$

$$x'(t) = A \delta(t) - \frac{A}{T} u(t) + \frac{A}{T} u(t-T)$$

$$X(\omega) = \frac{1}{j\omega} \left[A - \frac{A}{T} \left(\pi \delta(\omega) + \frac{1}{j\omega} \right) + \frac{A}{T} e^{-j\omega T} \left(\pi \delta(\omega) + \frac{1}{j\omega} \right) \right]$$

$$X(\omega) = \frac{1}{j\omega} \left[A - \frac{A}{T} \left(\pi \delta(\omega) + \frac{1}{j\omega} \right) (1 - e^{-j\omega T}) \right]$$

⑦



$$T = 10$$

$$\omega = \frac{2\pi}{T} = \frac{\pi}{5}$$

$$y_1(t) =$$

$$y_2(t) =$$

method ①

$$X(t) = y_1(t) \cdot y_2(t)$$

$$= 3 \sin\left(\frac{\pi t}{5}\right) [u(t) - u(t-10)]$$

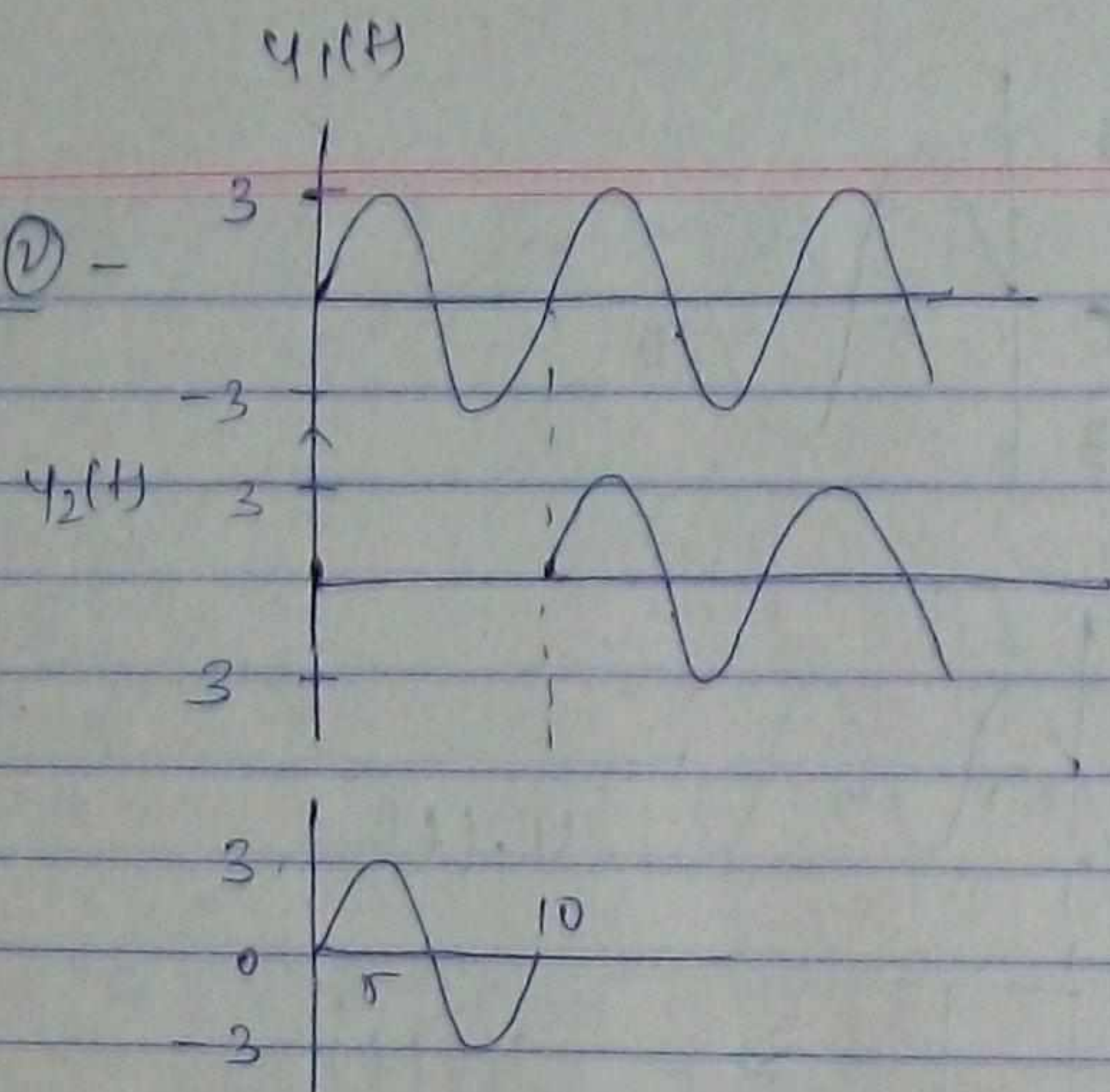
$$X(t) = 3 \sin\left(\frac{\pi t}{5}\right) u(t) - 3 \sin\left(\frac{\pi t}{5}\right) u(t-10)$$

$$= 3 \sin\left(\frac{\pi t}{5}\right) u(t) - 3 \sin\left(\frac{\pi (t-10)}{5} + 2\pi\right) u(t-10)$$

$$= 3 \sin\left(\frac{\pi t}{5}\right) u(t) - 3 \sin\left(\frac{\pi (t-10)}{5}\right) u(t-10)$$

$$= 3(1 - e^{-j10\omega}) \left[\frac{j\pi}{2} \left\{ \delta\left(\omega + \frac{\pi}{5}\right) - \delta\left(\omega - \frac{\pi}{5}\right) \right\} + \frac{\pi/5}{(j\pi/5)^2 - \omega^2} \right]$$

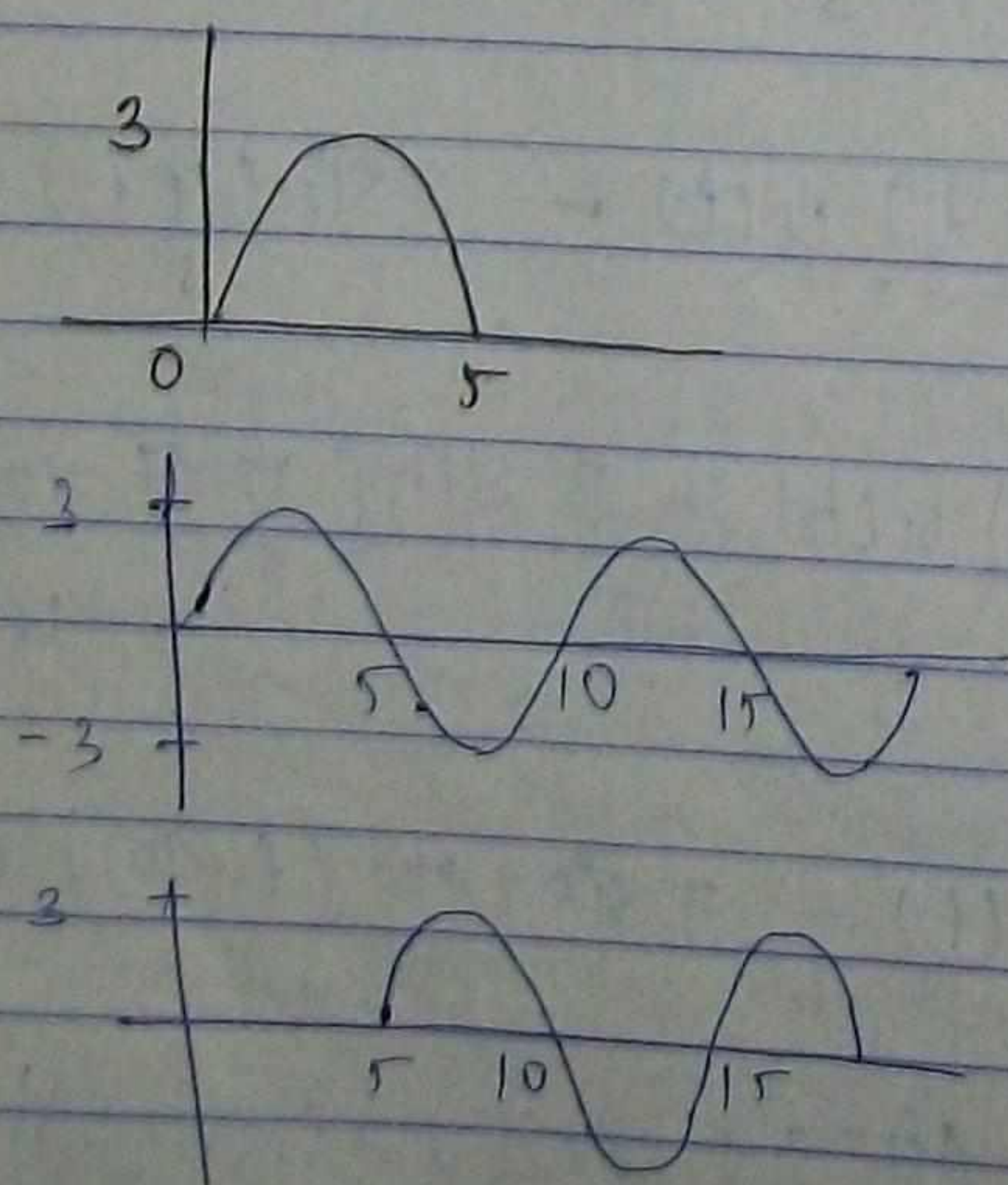
METHOD ② -



$$x(t) = y_1(t) - y_2(t)$$

$$= 3 \sin\left(\frac{\pi}{5} t\right) u(t) - 3 \sin\left(\frac{\pi}{5} (t-10)\right) u(t-10)$$

⑧



$$\omega = \frac{2\pi}{T} = \frac{\pi}{5}$$

SHIFTING → SCALING
① ②

14/11/11

$$x(t) = 3 \sin\left(\frac{2\pi}{5} t\right) u(t) + 3 \sin\left(\frac{2\pi}{5} (t-5)\right) u(t-5)$$

Ques find the F.T. of the following:

① $x(t) \leftrightarrow \frac{3}{2+j\omega}$ (given)

② $x(t-2) \leftrightarrow \frac{3}{2+j\omega} e^{-2j\omega}$

③ $x(-t) \leftrightarrow \frac{3}{2-j\omega}$

④ $e^{-j5t} x(t) \leftrightarrow \frac{3}{2+j(\omega+5)} + j\frac{3}{2+j(\omega/2)}$

⑤ $x(-2t-3) \leftrightarrow \frac{+1}{2} e^{+j3\omega} \frac{3}{2+j(\omega/2)}$

⑥ $k \frac{d}{dt} [x(t) * x(t/1)] \frac{1}{k}$

$$= e^{+3/2 j\omega} \cdot \frac{1}{2} \frac{3}{2-j(\omega/2)}$$

$$= \frac{3}{4-j\omega} e^{j3/2 \omega}$$

$$\textcircled{6} \quad e^{j\omega t} \frac{d}{dt} [x(t) * x(t-1)]$$

$$\longleftrightarrow j(\omega-1) \left(\frac{3}{2+j(\omega-1)} \right)^2 e^{j(\omega-1)t}$$

Ques. find the F.T. of following:

$$\textcircled{1} \quad \frac{1}{jt}$$

* If the f.T. of any signal is not present in the F.T. table then use the duality property by substituting $t = \omega$, $\omega = -t$

$$x(t) \longleftrightarrow X(\omega)$$

$$X(t) \longleftrightarrow 2\pi x(-\omega)$$

$$x(t) = \text{sgn}(t) \longleftrightarrow X(\omega) = \frac{2}{j\omega}$$

$$X(t) = \frac{2}{j\omega} \longleftrightarrow 2\pi X(-\omega)$$

$$= 2\pi \text{sgn}(-\omega)$$

$$= -2\pi \text{sgn}(\omega)$$

$$\frac{1}{j\omega} \longleftrightarrow -\pi \text{sgn}(\omega)$$

$$\textcircled{2} \quad \frac{1}{3+jt}$$

$$e^{-3t} u(t) \longleftrightarrow \frac{1}{3+j\omega}$$

$$\frac{1}{3+jt} \longleftrightarrow 2\pi e^{-3(-\omega)} u(-\omega)$$

$$= 2\pi e^{3\omega} u(\omega)$$

$$\textcircled{3} \quad \frac{1}{2-jt}$$

$$e^{2t} u(-t) \longleftrightarrow \frac{1}{2-j\omega}$$

$$\frac{1}{2-jt} \longleftrightarrow e^{2(-\omega)} u(-(-\omega)) \times 2\pi$$

$$= 2\pi e^{-2\omega} u(\omega)$$

*

$$\textcircled{4} \quad \frac{1}{5+t^2}$$

$$e^{-\sqrt{5}|t|} \longleftrightarrow \frac{2\sqrt{5}}{5+\omega^2}$$

$$\frac{1}{2\sqrt{5}} e^{-\sqrt{5}|t|} \longleftrightarrow \frac{1}{5+\omega^2}$$

$$\frac{1}{5+t^2} \leftrightarrow 2\pi \times \frac{1}{2\sqrt{5}} e^{-\sqrt{5}|\omega|}$$

$$= \frac{\pi}{\sqrt{5}} e^{-\sqrt{5}|\omega|}$$

(*)

(5) $13 \text{Sa}(7t)$

$$\frac{13}{14} \text{rect}\left(\frac{t}{14}\right) \leftrightarrow 13 \text{Sa}(7\omega)$$

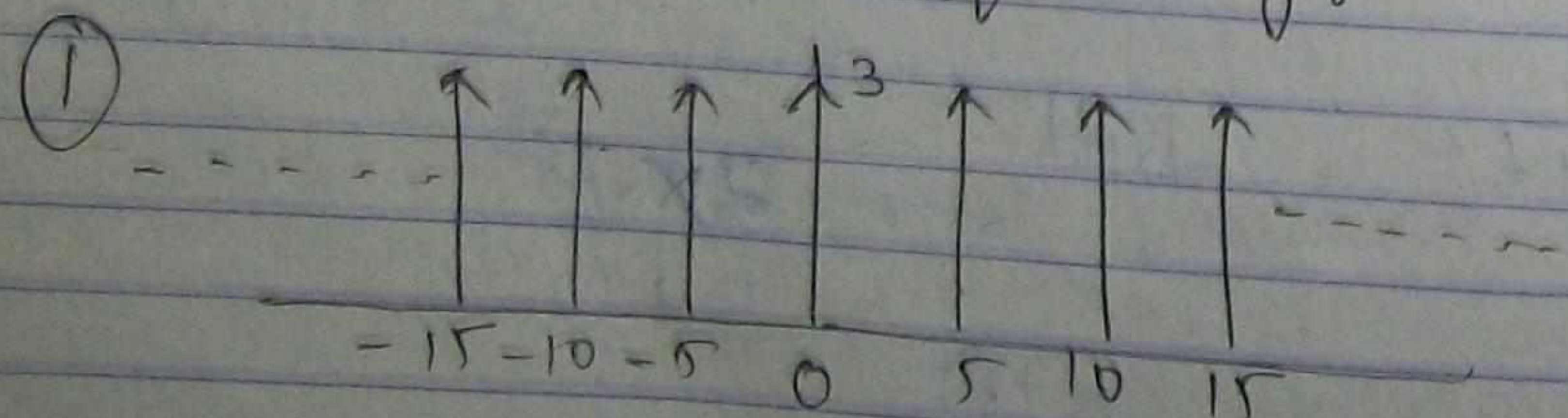
$$13 \text{Sa}(7t) \leftrightarrow 2\pi \times \frac{13}{14} \text{rect}\left(\frac{-\omega}{14}\right)$$

$$= \frac{13\pi}{7} \text{rect}\left(\frac{-\omega}{14}\right)$$

$$= \frac{13\pi}{7} \text{rect}\left(\frac{\omega}{14}\right)$$

(*)

Ques. Find the F.T. of the following:



Solⁿ: $T=5$, $\omega_0 = 2\pi/5$

$$F\{P.S\} = 2\pi \sum_{n=-\infty}^{\infty} C_n \delta(\omega - n\omega_0)$$

$x(t)$ is the periodic train of impulse with periodicity $T=5$ Strength '3' and centre '0' for impulse train:

from exponential F.S:

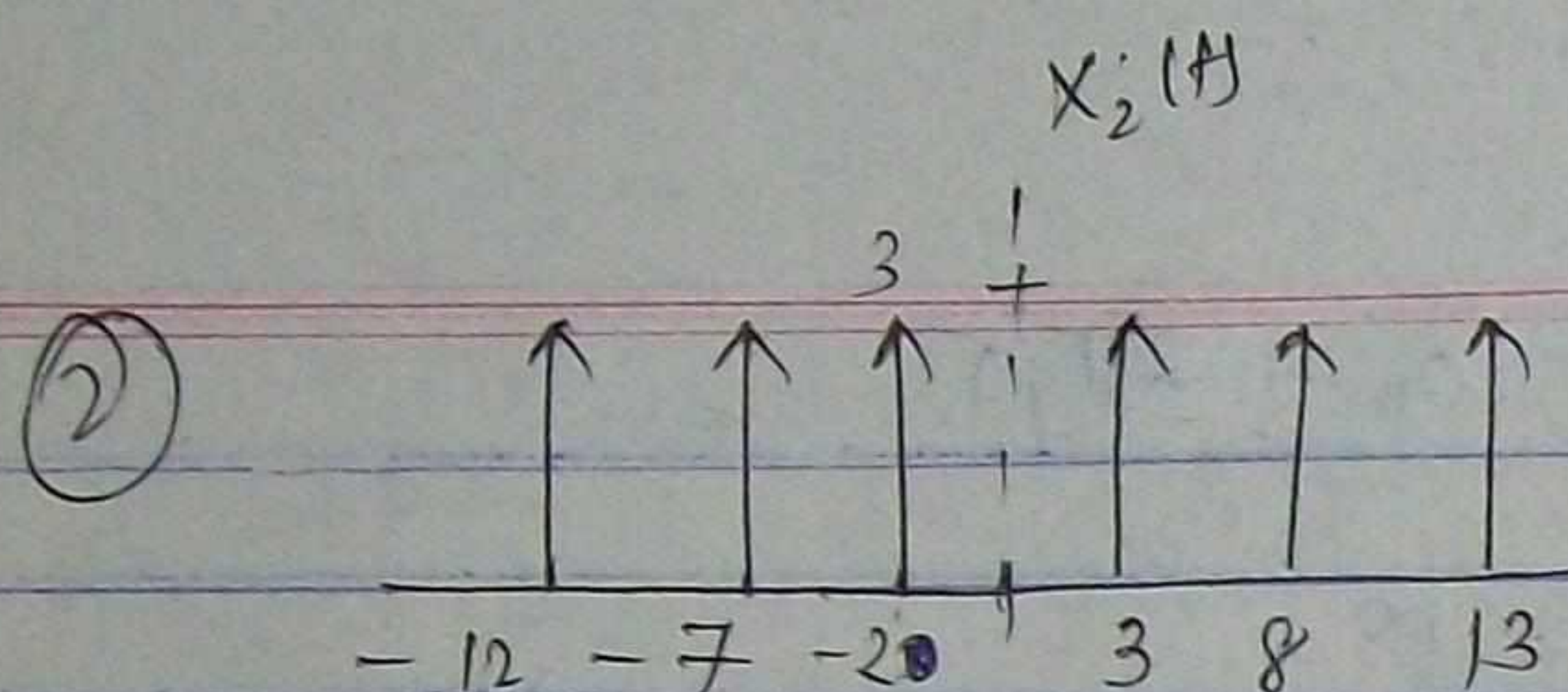
$$C_n = \frac{A}{T} \quad \text{< from EAs >}$$

$$C_n = \frac{3}{5}$$

$$F\{P.S\} = 2\pi \sum_{n=-\infty}^{\infty} \frac{3}{5} \delta\left(\omega - n\frac{2\pi}{5}\right)$$

$$= 2\pi \times \frac{3}{5} \sum_{n=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi n}{5}\right)$$

$$X_1(\omega) = \frac{6\pi}{5} \sum_{n=-\infty}^{\infty} \delta(\omega - 0.4\pi n)$$

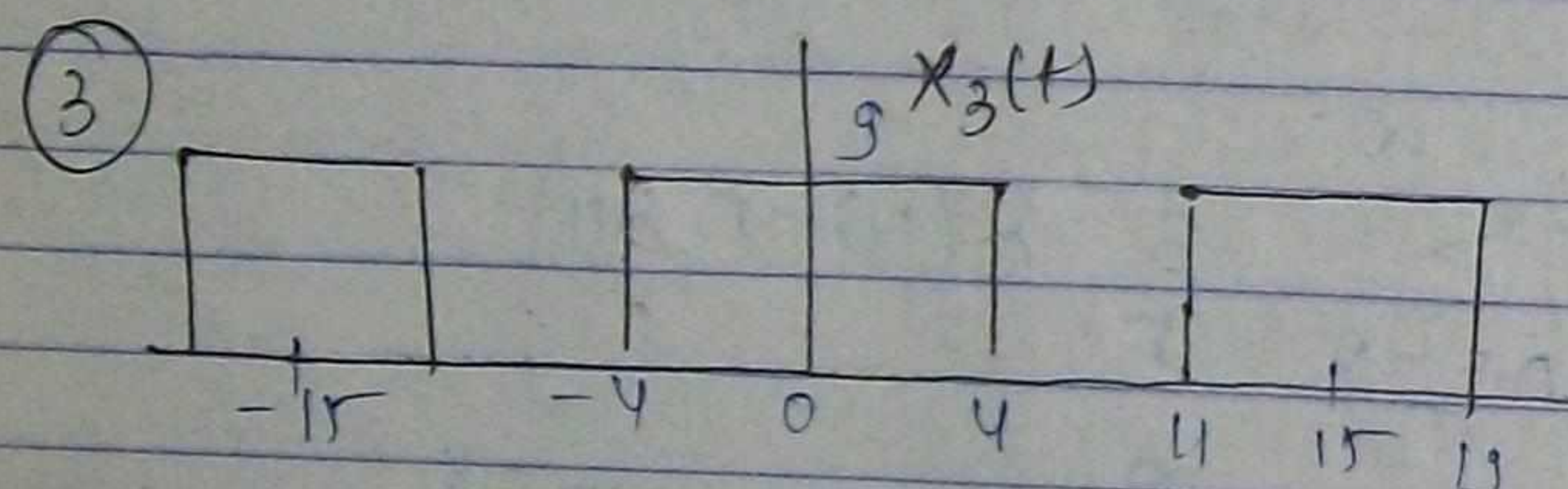


Time Shifting

$x_2(t)$ is the train of impulse shifted to right side by three unit or shifted to the left side by 2 unit i.e.

$$X(\omega) = e^{-j3\omega} \left(\frac{6\pi}{5} \sum_{n=-\infty}^{\infty} \delta(\omega - 0.4\pi n) \right)$$

$$(or) = e^{j2\omega} \left(\frac{6\pi}{5} \sum_{n=-\infty}^{\infty} \delta(\omega - 0.4\pi n) \right)$$



$x_3(t)$ is the periodic train of pulse with centre '0', $A = 9$, $\tau = 8$, $T = 15$

$$\omega_0 = 2\pi/15$$

for periodic train of pulse C_n is:

$$C_n = \frac{1}{T} \int_{-T/2}^{T/2} x(t) dt$$

$$C_n = \frac{1}{T} \int_{-4}^4 9 \times \text{rect}(t/8) dt$$

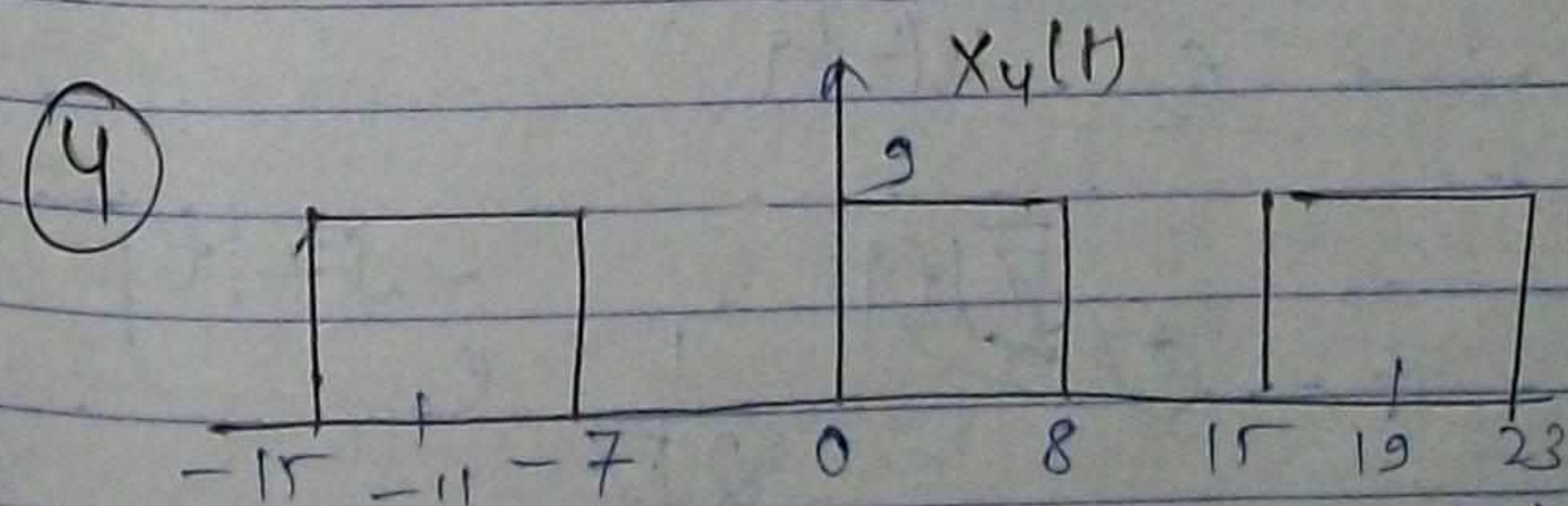
$$C_n = \frac{A\tau}{T} \text{Sa}(\omega T/2)$$

$$C_n = \frac{9 \times 8}{15} \text{Sa}(\omega \times \frac{2\pi}{15} \times \frac{8}{2})$$

$$C_n = \frac{A\tau}{T} \text{Sa}(n\omega_0 \tau/2)$$

$$= \frac{9 \times 8}{15} \text{Sa}(n \times \frac{2\pi}{15} \times \frac{8}{2})$$

$$X(\omega) = 2\pi \sum_{n=-\infty}^{\infty} \frac{72}{15} \frac{\text{Sa}(\frac{8\pi n}{15})}{\delta(\omega - \frac{2\pi n}{15})}$$



$x_4(t)$ is same as $x_3(t)$ shifted to right by 3 unit or shifted to left by 11 unit

$$X_4(\omega) = e^{-j4\omega} \left[2\pi \sum_{n=-\infty}^{\infty} \frac{72}{15} \frac{\text{Sa}(\frac{8\pi n}{15})}{\delta(\omega - \frac{2\pi n}{15})} \right]$$

$$(08) X(\omega) = e^{+j11\omega} \left[\sum_{n=-\infty}^{\infty} \frac{72}{15} \text{sinc}\left(\frac{8\pi n}{15}\right) \delta\left(\omega - \frac{2\pi n}{15}\right) \right]$$

Ques. Find the inverse FT of the following:

$$(1) -4 \longleftrightarrow -4\delta(t)$$

$$(2) f(\omega) \longleftrightarrow 1/2\pi$$

$$(3) \delta(-3\omega) \longleftrightarrow 1/6\pi$$

$$(4) \delta(3-2\omega) \longleftrightarrow \frac{1}{4\pi} e^{+3/2 j t}$$

$$(5) \frac{1}{0.3+j\omega} \longleftrightarrow e^{-0.3t} u(t)$$

$$(6) \frac{1}{\sqrt{2}-j\omega} \longleftrightarrow e^{\sqrt{2}t} u(-t)$$

$$(7) \frac{1}{7+\omega^2} \longleftrightarrow \frac{1}{2\sqrt{7}} e^{-\sqrt{7}|t|}$$

$$(8) \frac{1}{2+j5\omega} \longleftrightarrow \frac{1}{5} e^{-2/5 t} u(t)$$

$$(9) \frac{1}{3+j(\omega-5)} \longleftrightarrow e^{j5t} e^{-3t} u(t)$$

$$(10) \frac{1}{5-j(\omega-3)} \longleftrightarrow e^{j3t} e^{5t} u(-t)$$

$$(11) \frac{j5\omega+14}{(\omega)^2+j5\omega+6} = \frac{j5\omega+14}{(j\omega+3)(j\omega+2)}$$

$$X(\omega) = \frac{4}{(2+j\omega)} + \frac{1}{(3+j\omega)}$$

$$X(t) = 4 e^{-2t} u(t) + e^{-3t} u(t)$$

$$(12) X(\omega) = \frac{\omega^2+9-j}{\omega^2+9} = 1 + \frac{-j}{\omega^2+9}$$

$$X(\omega) = 1 + \frac{-j}{\omega^2+9}$$

$$X(t) = \delta(t) - \frac{3}{2} e^{-3|t|}$$

$$(13) \quad X(\omega) = \frac{-2\omega^2 + j15\omega + 14}{(j\omega)^2 + j5\omega + 6}$$

$$X(\omega) = -2 + \frac{j5\omega + 2}{(j\omega)^2 + j5\omega + 6}$$

$$= -2 + \frac{13}{(j\omega + 3)} + \frac{-8}{(j\omega + 2)}$$

$$x(t) = 2\delta(t) + 13e^{-3t}u(t) - 8e^{-2t}u(t)$$

Ques. find the IFT of following

$$X(\omega) \xleftrightarrow{f-t} e^{j5t} e^{-3t} \cos(2t) u(t)$$

$$(1) \quad X(\omega - 5) \leftrightarrow e^{j5t} e^{-3t} \cos(2t) u(t)$$

$$(2) \quad X(-\omega) \leftrightarrow e^{+3t} \cos(2t) u(-t)$$

$$(3) \quad e^{j0.3\omega} X(\omega) \leftrightarrow e^{-3(t+0.3)} \cos(2(t+0.3)) u(t+0.3)$$

$$(4) \quad X(2\omega - 3) \leftrightarrow \frac{e^{+j1.5t}}{2} e^{-3t/2} \cos(t) u(t/2)$$

Ques. find the IFT of following

$$(1) \quad u(\omega)$$

$$u(t) \leftrightarrow \pi\delta(\omega) + \frac{1}{j\omega}$$

$$u(\omega) \leftrightarrow \frac{1}{2\pi} \left[\delta(-t) + \frac{1}{j(-t)} \right]$$

$$= \frac{1}{2\pi} \left[\pi\delta(t) - \frac{1}{jt} \right] = \frac{2/\pi^2 \delta(t)}{jt} + \frac{1}{2\pi jt}$$

** If the I.F.T of the signal is not present in the F-T table then use duality property by substituting $(\omega = -t)$.*

DUALITY PROPERTY IN f -

$$x(t) \leftrightarrow X(f)$$

$$X(t) \leftrightarrow x(-f)$$

$$= \frac{2/\pi^2 \delta(f)}{jt} + \frac{1}{2\pi jt}$$

$$(2) u(t-\omega) \leftrightarrow \frac{1}{2\pi} \left[\pi \delta(-t) + \frac{1}{jt} \right]$$

$$= \frac{1}{2\pi} \left[\delta(t) \pi + \frac{1}{jt} \right]$$

$$(3) e^{-2\omega} u(\omega) \leftrightarrow \frac{1}{2\pi} \left[\pi \delta(t-2) + \frac{1}{j(t-2)} \right]$$

$$(4) e^{3\omega} u(t-\omega) \leftrightarrow \frac{1}{2\pi} \left[\pi \delta(t+3) + \frac{1}{j(t+3)} \right]$$

$$(5) \text{sgn}(\omega) \quad \text{or} \quad \frac{1}{2\pi} \left[\frac{1}{3+jt} \right]$$

$$x(t) = \text{sgn}(t) \leftrightarrow \frac{2}{j\omega}$$

$$\frac{2}{jt} \leftrightarrow 2\pi \text{sgn}(-\omega)$$

$$\text{sgn}(\omega) = -\frac{1}{\pi jt}$$

$$(6) e^{-5|\omega|}$$

$$e^{-5|t|} \leftrightarrow \frac{10}{25 + \omega^2}$$

$$\frac{10}{25 + t^2} \leftrightarrow e^{-5|\omega|} \times 2\pi$$

$$e^{-5|\omega|} \leftrightarrow \frac{5}{\pi(25 + t^2)}$$

$$(7) \text{rect}(13\omega)$$

$$\text{rect}(T/\tau) \leftrightarrow A\tau \text{sa}(\omega T/2)$$

$$\text{rect}(\tau) \leftrightarrow \tau \text{sa}(\omega \tau/2)$$

$$\text{rect}(T/1/13) \leftrightarrow \frac{1}{13} \text{sa}(\omega/26)$$

$$\text{rect}(13T) \leftrightarrow \frac{1}{13} \text{sa}(\omega/26)$$

$$\frac{1}{13} \text{sa}(t/26) \leftrightarrow 2\pi \text{rect}(13\omega)$$

$$\text{rect}(13\omega) \leftrightarrow \frac{1}{26\pi} \text{sa}(t/26)$$

LT1

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⇒ CONVOLUTION -

CONVOLUTION $\begin{cases} \text{CT} \rightarrow \text{linear conv.} \\ \text{DT} \rightarrow \begin{cases} \text{linear conv.} \\ \text{circular conv.} \end{cases} \end{cases}$

Ex -

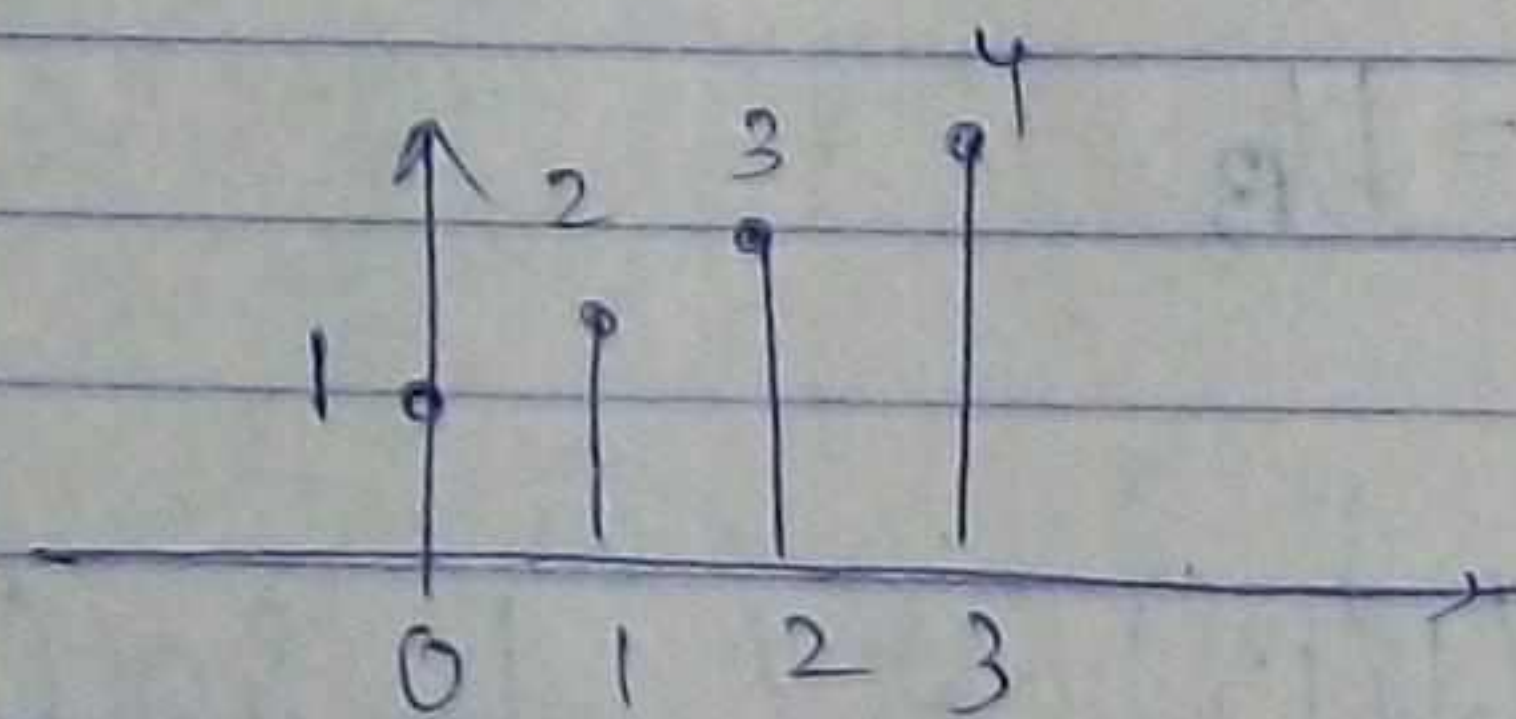
$$x(n) = \{1, 2, 3, 4\}$$

$$x(0) = 1$$

$$x(1) = 2$$

$$x(2) = 3$$

$$x(3) = 4$$



$$x(n) = 1\delta(n) + 2\delta(n-1) + 3\delta(n-2) + 4\delta(n-3)$$

$$= x(0)\delta(n) + x(1)\delta(n-1) + x(2)\delta(n-2) + x(3)\delta(n-3)$$

$$= \sum_{k=0}^3 x(k)\delta(n-k)$$

Generalised,

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\delta(n-k)$$

$$x(n) = x(n) * \delta(n)$$

$$y(n) = x(n) * h(n)$$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

Similarly
for
continuous
time

$$y(t) = x(t) * h(t)$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

- Therefore representing of one signal in terms of other signal is known as convolution of those two signals.

- O/P of any system is the convolution of the I/P signal and system function.

$$x(t) \rightarrow [h(t)] \rightarrow y(t)$$

$$y(t) = x(t) * h(t)$$

⇒ CONTINUOUS - TIME CONVOLUTION -

$$\text{let } y(t) = x(t) * h(t)$$

PROPERTIES -

① COMMUTATIVE -

$$x(t) * h(t) = h(t) * x(t)$$

$$\int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau = \int_{-\infty}^{\infty} h(\tau) x(t-\tau) d\tau$$

② ASSOCIATIVE -

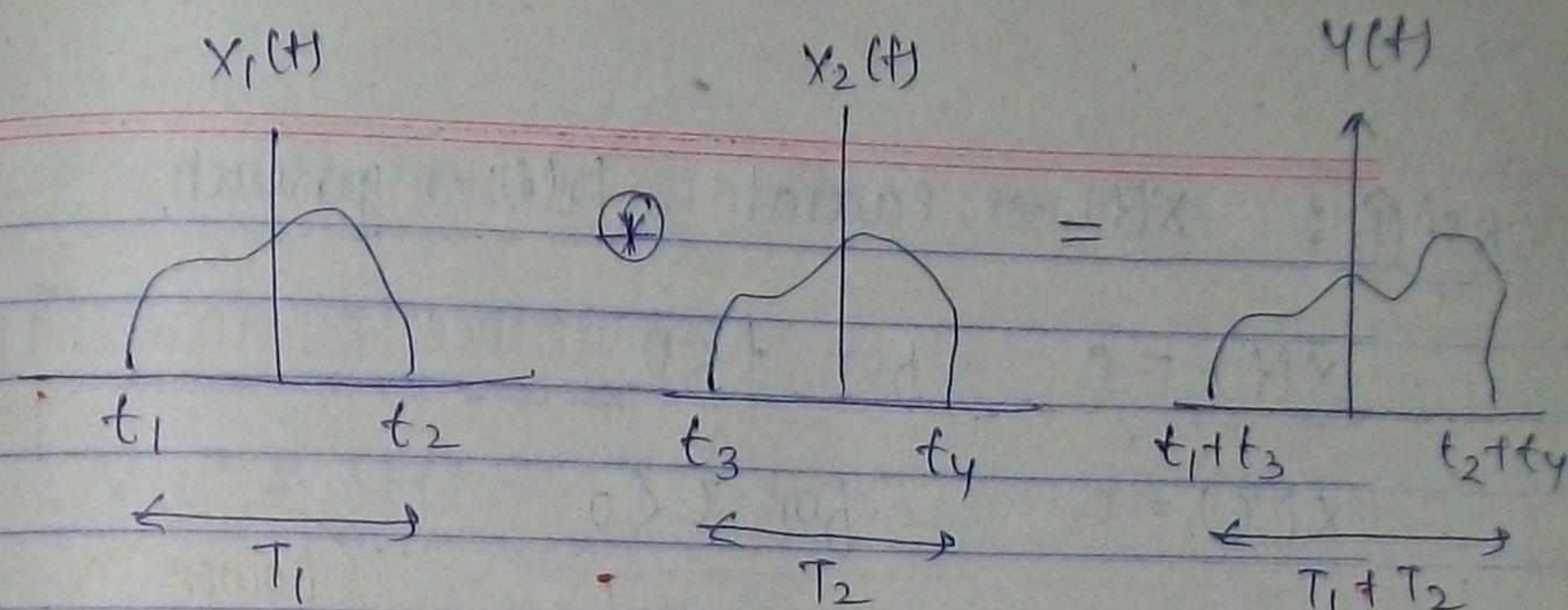
$$x(t) * [h_1(t) * h_2(t)] = [x(t) * h_1(t)] * h_2(t)$$

③ DISTRIBUTIVE -

$$x(t) * [h_1(t) + h_2(t)] = [x(t) * h_1(t)] + [x(t) * h_2(t)]$$

④ WIDTH PROPERTY -

— width of the convolved signal added up.



$$\text{If } y(t) = x(t) * h(t)$$

$$y(t-T_1-T_2) = x(t-T_1) * h(t-T_2)$$

⑤ DIFFERENTIATION -

$$\text{If } y(t) = x(t) * h(t)$$

$$\frac{d}{dt} x(t) * h(t) = \frac{d}{dt} y(t)$$

$$x(t) * \frac{d}{dt} h(t) = \frac{d}{dt} y(t)$$

⑥ LIMITS OF CONVOLUTION INTEGRALS -

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$$

for general $x(t)$ and $h(t)$.

CASE ①: $x(t) \rightarrow$ causal $h(t) \rightarrow$ general.

$$x(t) = 0 \quad \text{for } t < 0$$

$$x(\tau) = 0 \quad \text{for } \tau < 0$$

$$\int_0^{\infty} x(\tau) h(t-\tau) d\tau$$

LOWER
LIMIT
CHANGE

CASE ②: $x(t) \rightarrow$ general $h(t) \rightarrow$ causal

$$h(t) = 0 \quad \text{for } t < 0$$

$$h(t-\tau) = 0 \quad (t-\tau) < 0$$

$$h(t-\tau) = 0 \quad \text{for } t < \tau$$

$$\int_{-\infty}^t x(\tau) h(t-\tau) d\tau$$

UPPER
LIMIT
CHANGE

CASE ③: $x(t) \rightarrow$ causal $h(t) \rightarrow$ causal

$$\int_0^t x(\tau) h(t-\tau) d\tau$$

BOTH
LIMIT
CHANGE

$\Rightarrow$ CONVOLUTION OF STANDARD SIGNALS -

① IMPULSE FUNCTION -

$$x(t) * \delta(t) = \int_{-\infty}^{\infty} x(\tau) \delta(t-\tau) d\tau = x(t)$$

Therefore convolⁿ of any signal with impulse result in the same signal.

$$\checkmark x(t) * \delta(t-T) = x(t-T)$$

$$\checkmark \delta(t) * \delta(t) = \delta(t)$$

$$u(t) * \delta(t) = u(t)$$

$$\gamma(t) * \delta(t) = \gamma(t)$$

$$e^{-at} x(t) * \delta(t) = e^{-at} x(t)$$

$$\text{ex - ① } \delta(t-1) * \delta(3-2t)$$

$$\delta(t-1) * \delta(-2(t-1.5)) \Rightarrow \delta(t-1) * \delta(2(t-1.5))$$

$$\Rightarrow \delta(t-1) * \frac{1}{2} \delta(t-1.5) \Rightarrow \frac{1}{2} \delta(t-1-1.5)$$

$$\textcircled{2} \quad u(t+1) * \delta(3-t)$$

$$u(t+1) * \delta(t-3)$$

$$u(t+1-3) \Rightarrow u(t-2)$$

$\textcircled{3} \quad \delta(-t+2) * \delta(t-5)$	$u(-t+2) * \delta(t-5)$
$\delta(t-2) * \delta(t-5)$	$u(-t+2-5)$
$\delta(t-7)$	$u(-t-3)$

$$\textcircled{4} \quad e^{-3t} u(t-2) * \delta(1.5-t)$$

$$e^{-3t} u(t-2) * \delta(t-1.5)$$

$$e^{-3(t-1.5)} u(t-2-1.5)$$

② STEP FUNCTION -

$$u(t) * u(t) = \int_{-\infty}^{\infty} u(t) u(t-\tau) d\tau$$

$$= \int_0^t 1 \cdot 1 d\tau$$

$$= \frac{t}{1} \quad (t > 0)$$

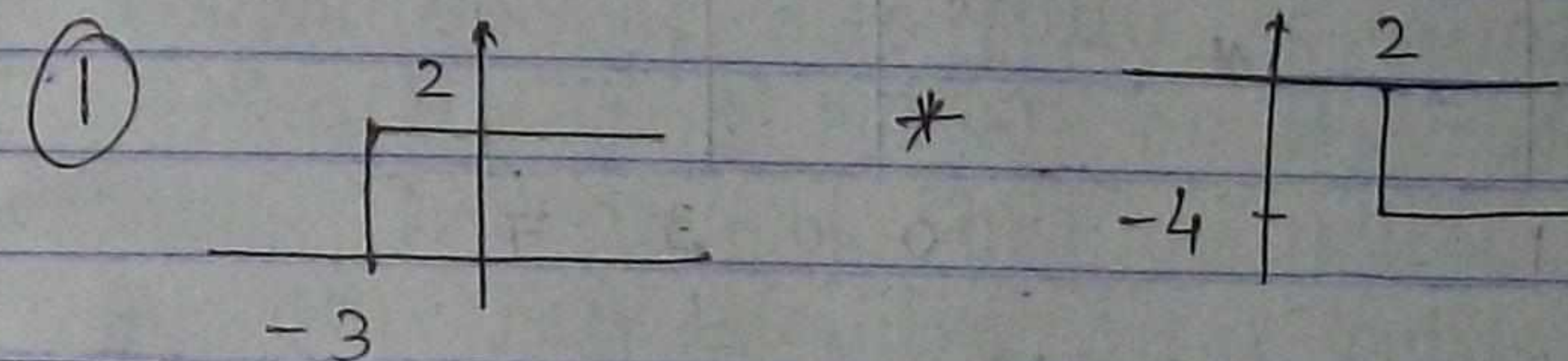
$$= r(t)$$

RAMP

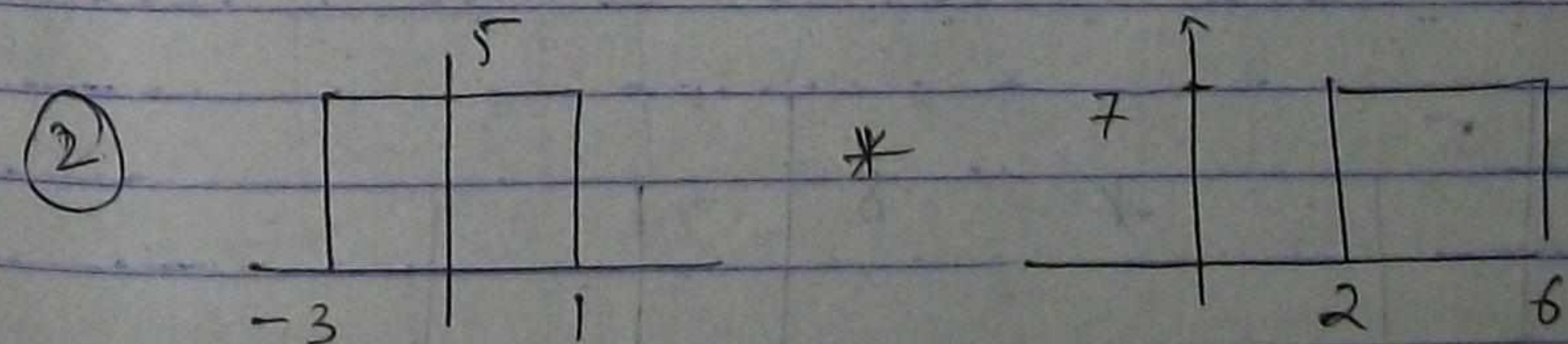
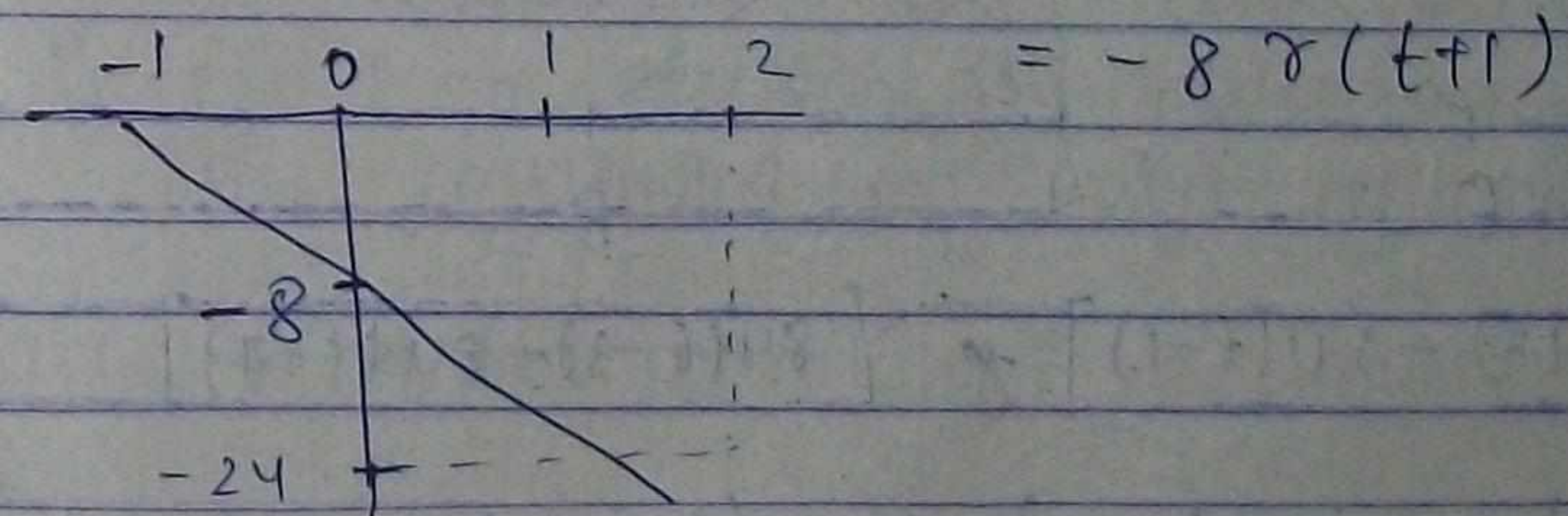
$$u(t) * u(t) = r(t)$$

$$u(t-T_1) * u(t-T_2) = r(t-T_1-T_2)$$

Ques Evaluate.



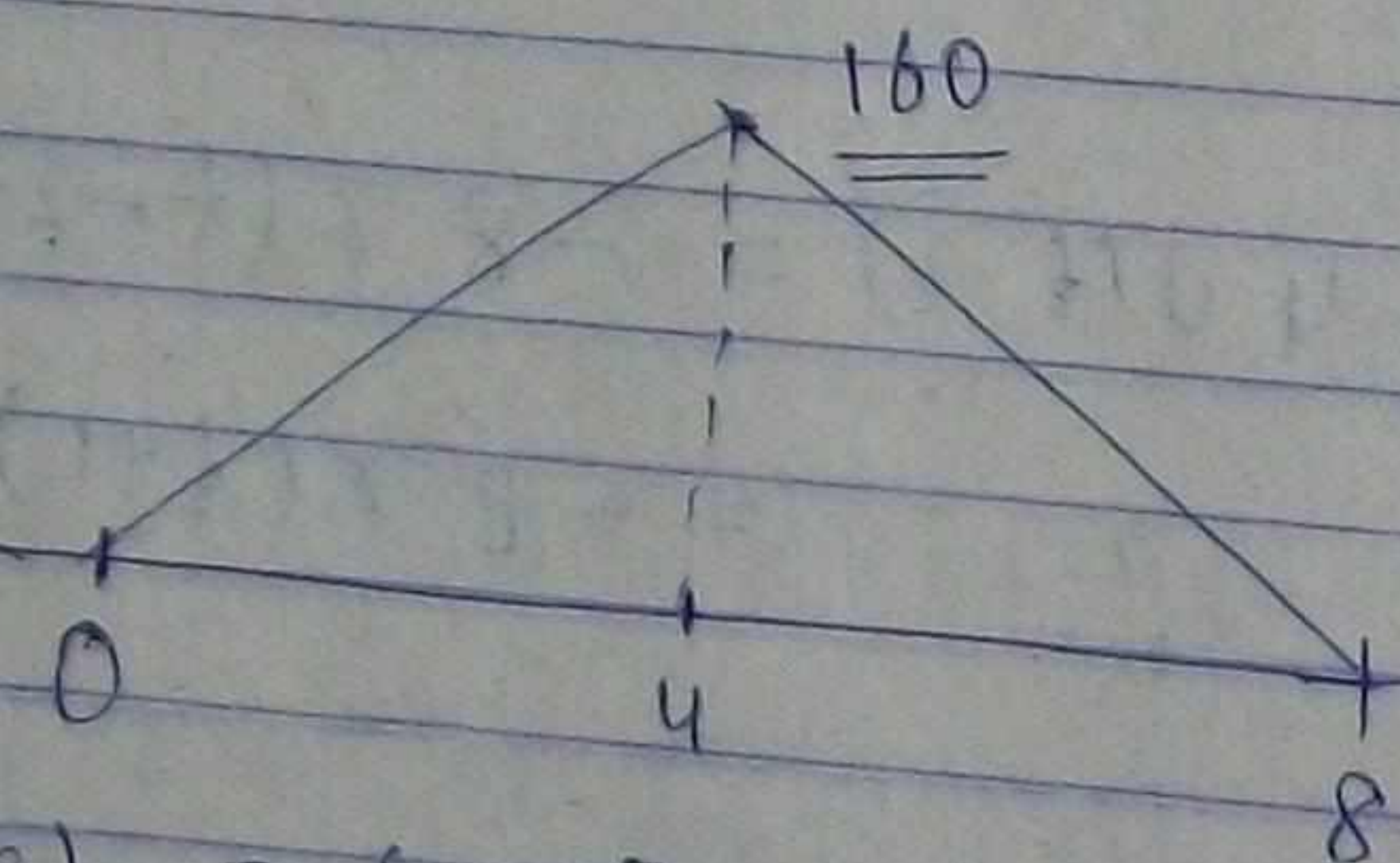
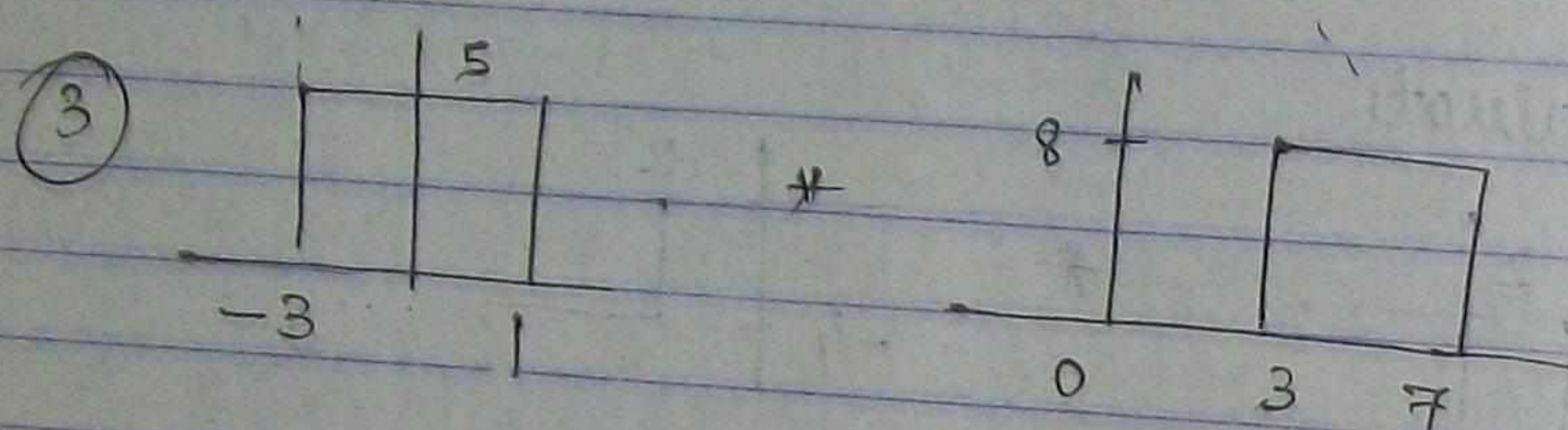
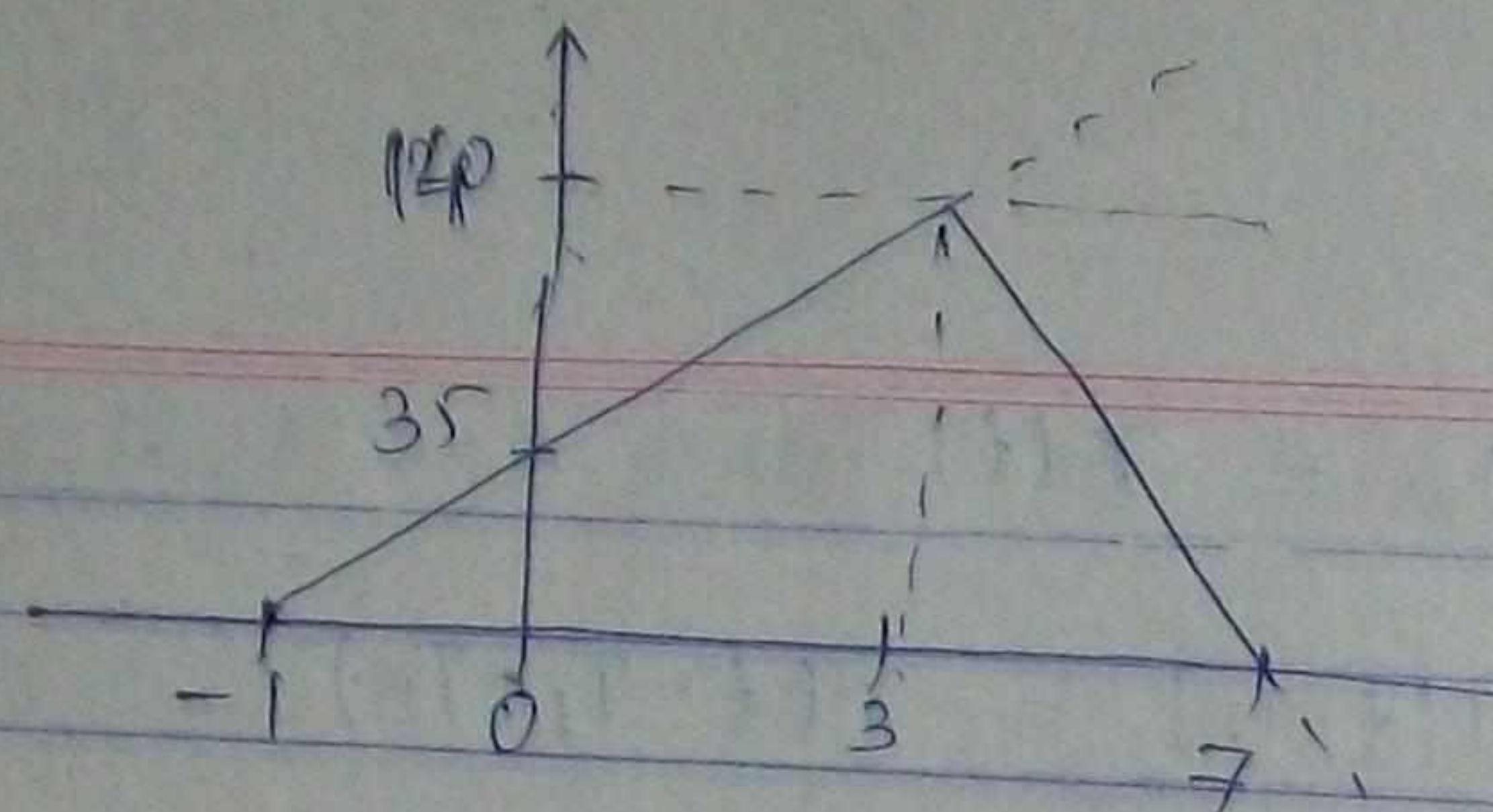
$$2u(t+3) * -4u(t-2) = -8r(t-2+3)$$



$$[5u(t+3) - 5u(t-1)] * [7u(t-2) - 7u(t-6)]$$

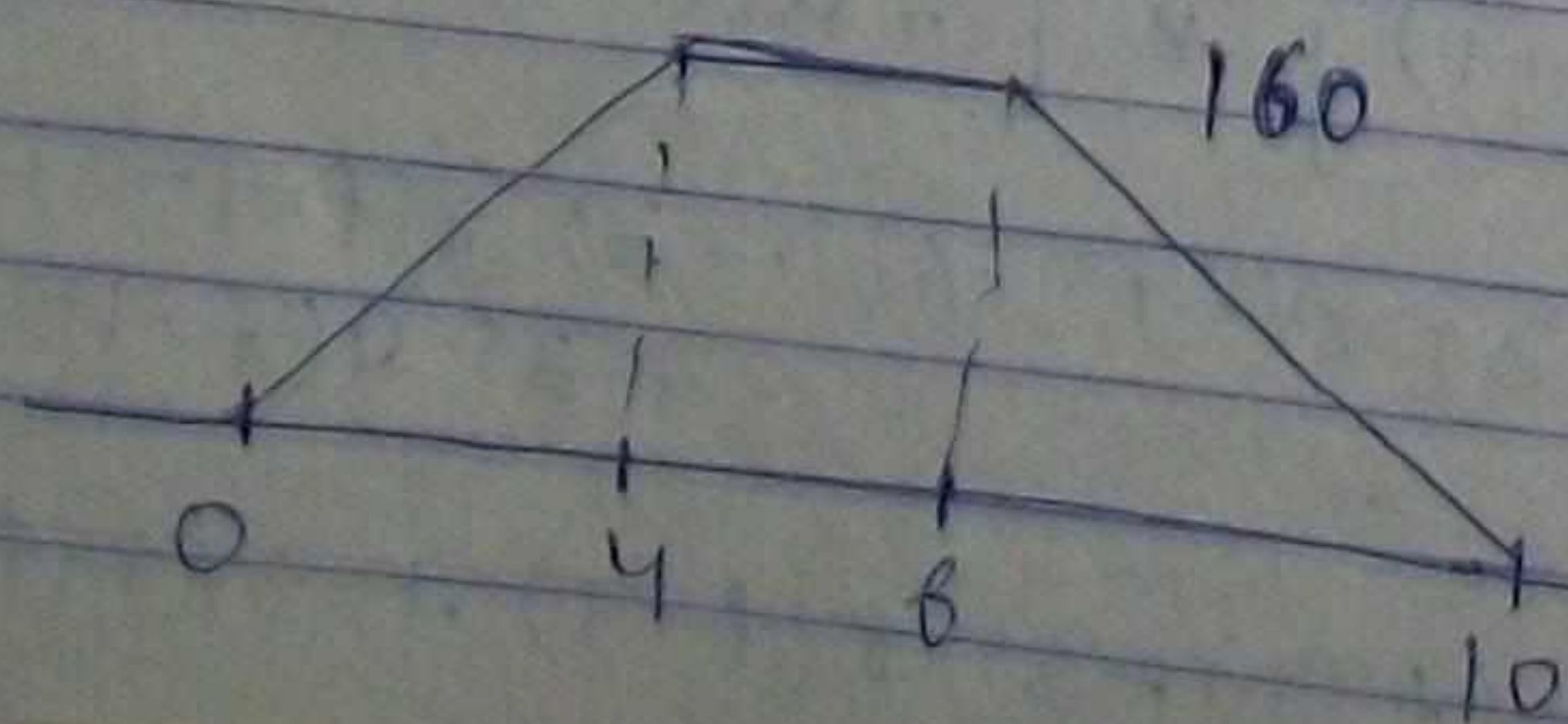
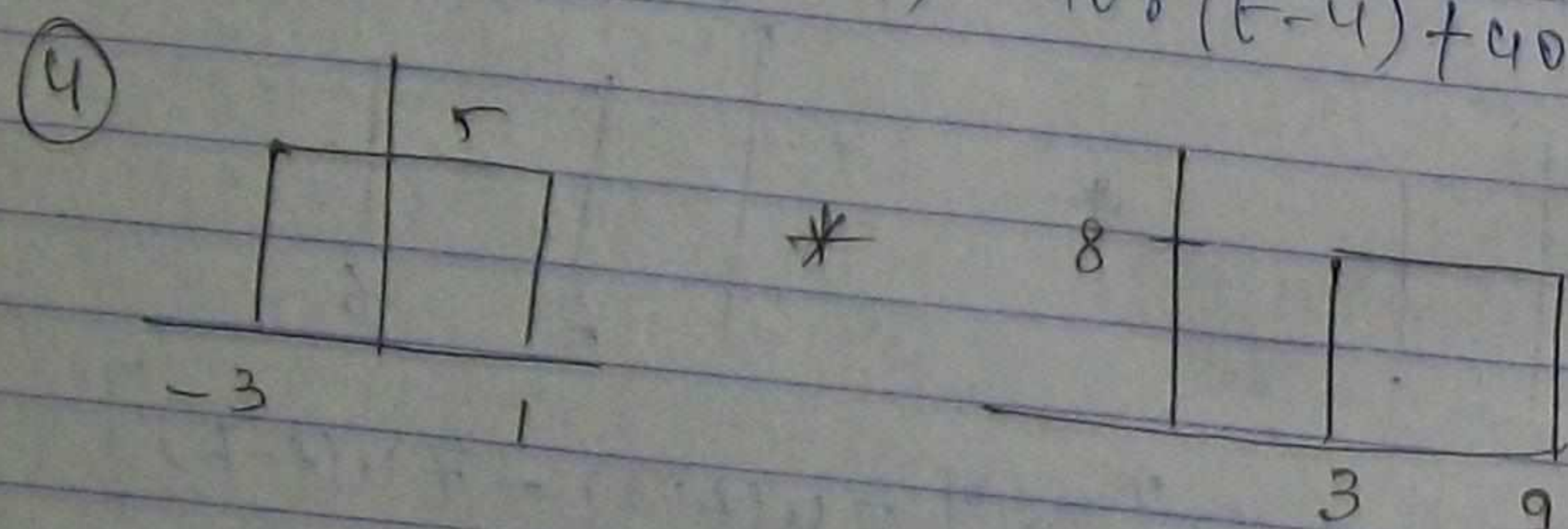
$$35r(t+3-2) - 35r(t+3-6) - 35r(t-1-2) + 35r(t-1-6)$$

$$35r(t+1) - 35r(t-3) - 35r(t-3) + 35r(t-7)$$



$$[5u(t+3) - 5u(t-1)] * [8u(t-3) - 8u(t-7)]$$

$$= 40r(t) - 40r(t-4) - 40r(t-4) + 40r(t-8)$$



$(-3, 1) (3, 9)$
 $0, 6, 4, 10$
 $(0, 4, 6, 10)$

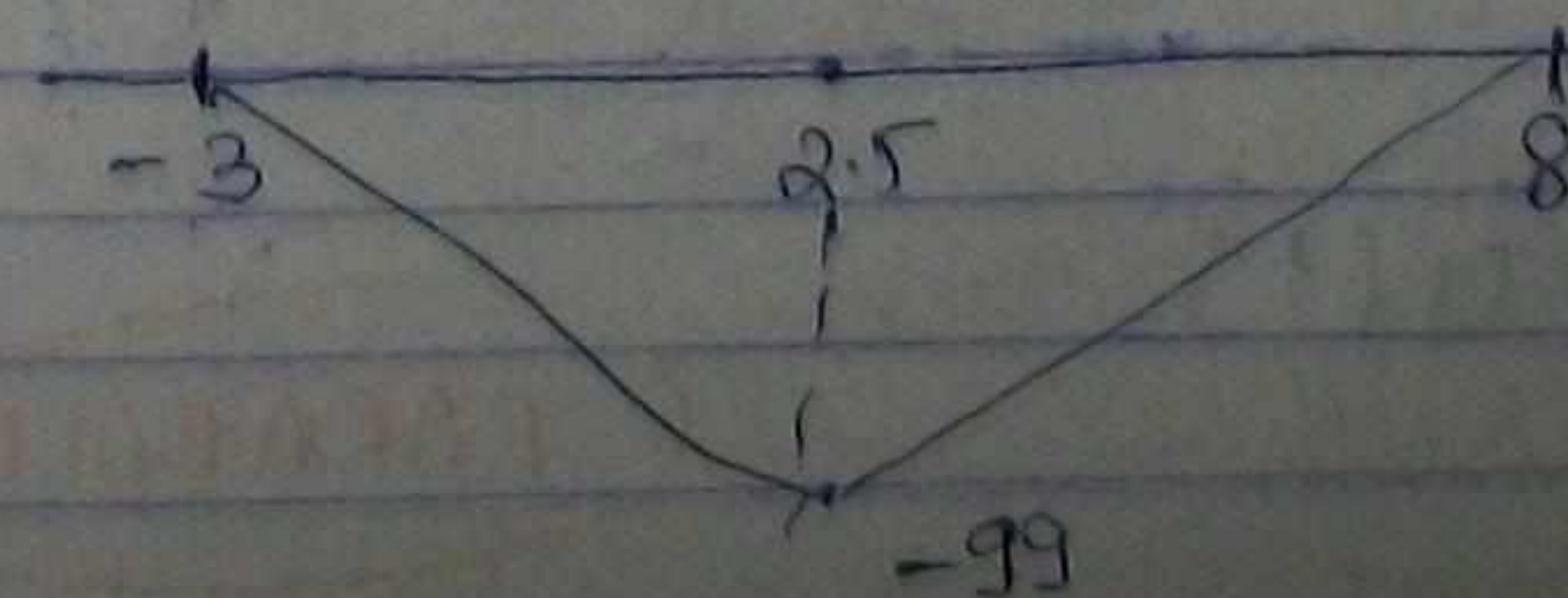
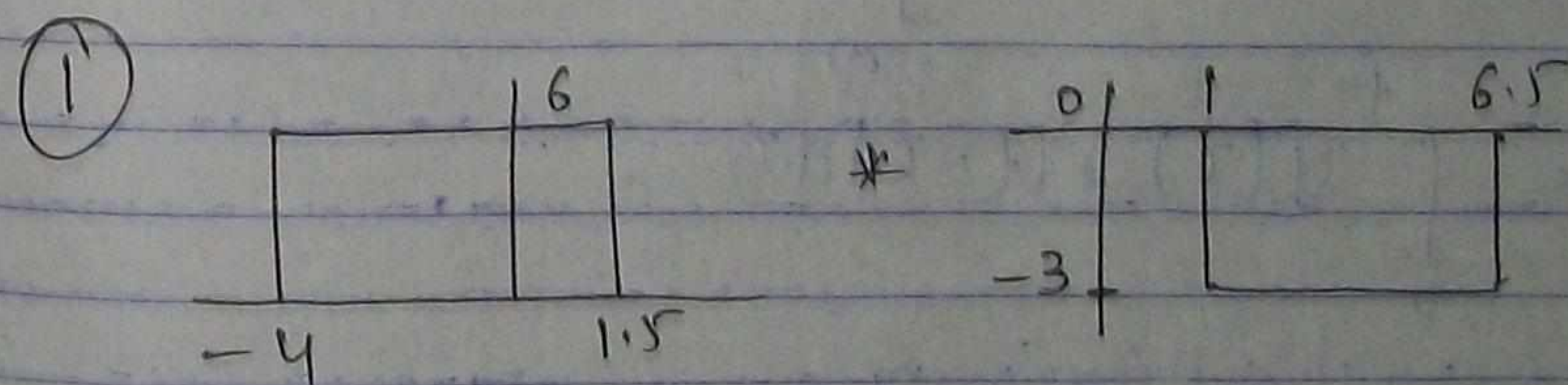
$$[5u(t+3) - 5u(t-1)] * [8u(t-3) - 8u(t-9)]$$

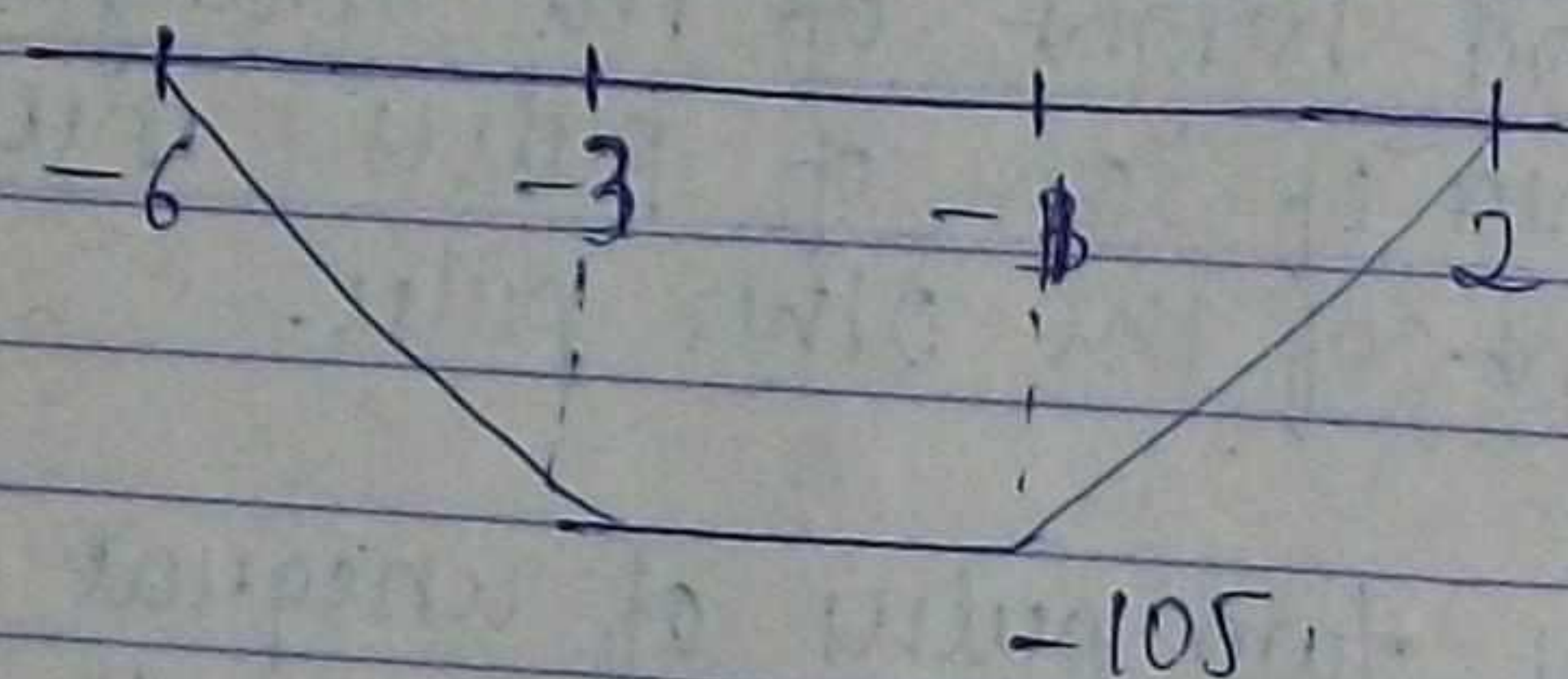
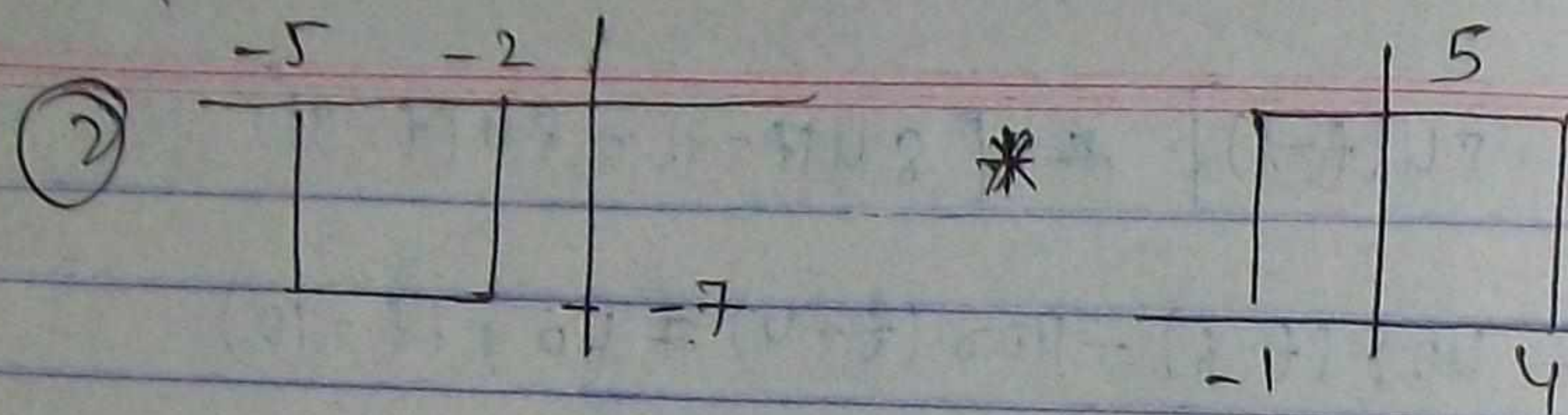
$$= 40r(t) - 40r(t-6) - 40r(t-4) + 40r(t-10)$$

⑤ Convolution of two pulses of equal width is a triangle and height of the triangle is same as the area of one of pulses multiplied by the height of the other pulse.

⑥ Convolution of two pulses of unequal width is a trapezoidal and height of the trapezoid is equal to area of the pulse of smaller width multiplied by height of the larger pulse.

Ques. find the convoln of:





③ STEP & RAMP IMPULSE FUNCTION -

$$u(t) * r(t) = \int_{-\infty}^{\infty} u(\tau) r(t-\tau) d\tau$$

$$= \int_0^t u(\tau) r(t-\tau) d\tau$$

$$= \int_0^t (t-\tau) d\tau$$

$$= t^2 - \frac{t^2}{2} = \frac{t^2}{2} \quad t > 0$$

$$= \frac{t^2}{2} u(t)$$

PARABOLA

In general.

$$u(t-T_1) * r(t-T_2) = \frac{(t-T_1-T_2)^2}{2} u(t-T_1-T_2)$$

$$= \frac{(t-T_1-T_2)^2}{2}, \quad t > (T_1+T_2)$$

④ EXPONENTIAL & STEP -

$$u(t) * e^{-at} u(t)$$

$$= \int_{-\infty}^{\infty} u(\tau) \cdot e^{-a(t-\tau)} u(t-\tau) d\tau$$

$$= \int_0^t 1 \cdot e^{-a(t-\tau)} d\tau$$

$$= e^{-at} \int_0^t e^{a\tau} d\tau$$

$$= \frac{e^{-at}}{a} (e^{at} - 1)$$

$$= \frac{1}{a} (1 - e^{-at}), \quad t > 0$$

$$= \frac{1}{a} (1 - e^{-at}) u(t)$$

Valid
for
both
 $a > 0$ &
 $a < 0$

Ques find the convolⁿ of followⁿ:

$$(1) u(t) * e^{-3t} u(t)$$

$$= \frac{1}{3} (1 - e^{-3t}) u(t)$$

$$(2) u(t) * e^{3t} u(t)$$

$$= \frac{1}{-3} (1 - e^{3t}) u(t)$$

$$(3) u(t-1) * e^{-3t} u(t)$$

$$= \frac{1}{3} (1 - e^{-3(t-1)}) u(t-1)$$

$$(4) u(t-2) * e^{-3t} u(t+1)$$

$$= \frac{1}{3} (1 - e^{-3(t-2+1)}) u(t-2+1)$$

$$= \frac{1}{3} (1 - e^{+3(t-1)}) u(t-1)$$

(5) RAMP WITH RAMP FUNCTION

$$r(t) * r(t)$$

$$= \int_{-\infty}^{\infty} r(\tau) \cdot r(t-\tau) d\tau$$

$$= \int_0^t \tau \cdot (t-\tau) d\tau$$

$$= \int_0^t (\tau t - \tau^2) d\tau$$

$$= \frac{t \cdot t^2}{2} - \frac{t^3}{3} = \frac{t^3}{2} - \frac{t^3}{3}$$

$$= \frac{t^3}{6}, \quad t > 0$$

$$t u(t) * t u(t) = \frac{t^3}{6}$$

General,

$$t^m u(t) * t^n u(t) = \frac{t^{m+n+1}}{(m+n+1)!}, \quad t > 0$$

⑥ EXPONENTIAL FUNCTION —

$$e^{-at} u(t) * e^{-bt} u(t) \quad \text{for } a \neq b$$

$$= \int_{-\infty}^{\infty} e^{-a\tau} u(\tau) \cdot e^{-b(t-\tau)} u(t-\tau) d\tau$$

$$= \int_0^t e^{-a\tau} \cdot e^{-b(t-\tau)} d\tau$$

$$= e^{-bt} \int_0^t e^{-(a+b)\tau} d\tau$$

$$= \frac{e^{-bt}}{-(a-b)} \left[\frac{-(a-b)t}{e} - 1 \right]$$

$$= \frac{e^{-bt}}{-(a-b)} \left[e^{-(a-b)t} - 1 \right]$$

$$= \frac{1}{b-a} (e^{-at} - e^{-bt}) u(t)$$

$$e^{-at} u(t) * e^{-bt} u(t) \quad \text{for } a=b$$

$$= \int_{-\infty}^{\infty} e^{-a\tau} u(\tau) \cdot e^{-a(t-\tau)} u(t-\tau) d\tau$$

$$= \int_0^t e^{-a\tau} \cdot e^{-a(t-\tau)} d\tau$$

$$= \int_0^t e^{-at} \cdot 1 d\tau$$

$$= e^{-at} \cdot t u(t)$$

$$e^{-at} u(t) * e^{-bt} u(t)$$

$$= \frac{1}{b-a} (e^{-at} - e^{-bt}) u(t) \quad , \quad \text{for } a \neq b$$

$$= t e^{-at} u(t) \quad , \quad \text{for } a=b$$

Que Evaluate the following:

(1) $e^{-3t} u(t) * e^{-5t} u(t)$

$$= \frac{1}{2} (e^{-3t} - e^{-5t}) u(t)$$

(2) $e^{-3t} u(t) * e^{3t} u(t)$

$$= \frac{1}{-6} (e^{-3t} - e^{+3t}) u(t)$$

(3) $e^{-3t} u(t) * e^{-3t} u(t)$

$$= t \cdot e^{-3t} u(t)$$

(4) $e^{5t} u(t) * e^{3t} u(t)$

$$= \frac{1}{+2} (e^{5t} - e^{3t}) u(t)$$

(5) $e^{5t} u(t) * e^{5t} u(t)$

$$= t e^{5t} u(t)$$

- Convolution of any signal with impulse is known as impulse response

- Convolution of the signal with step function is known as step response.

Que. find the step response of the LTI system with $h(t) = e^{-5t} u(t)$.

Solⁿ: $y(t) = h(t) * u(t)$

$$= e^{-5t} u(t) * u(t)$$

$$= \frac{1}{5} (1 - e^{-5t}) u(t)$$

Que. Compare

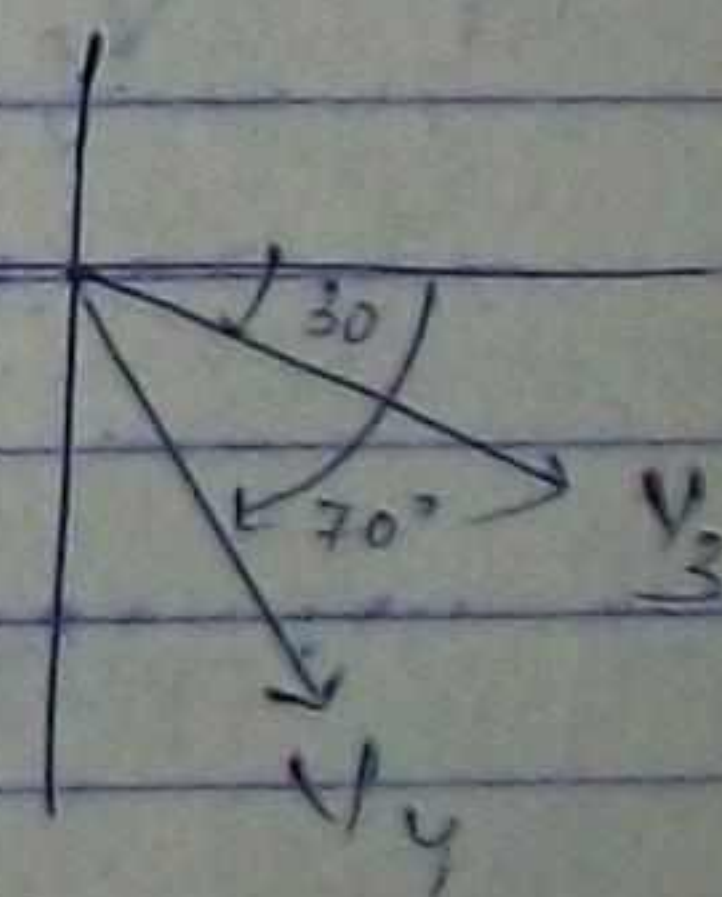
$$V_3 = 10 \cos(5t - 30^\circ)$$

$$V_4 = 20 \cos(3t - 70^\circ)$$

Solⁿ:

$$V_3 = 10 \angle -30^\circ \quad \omega = 5$$

$$V_4 = 20 \angle -70^\circ \quad \omega = 3$$



leading
lagging
cannot be
compare

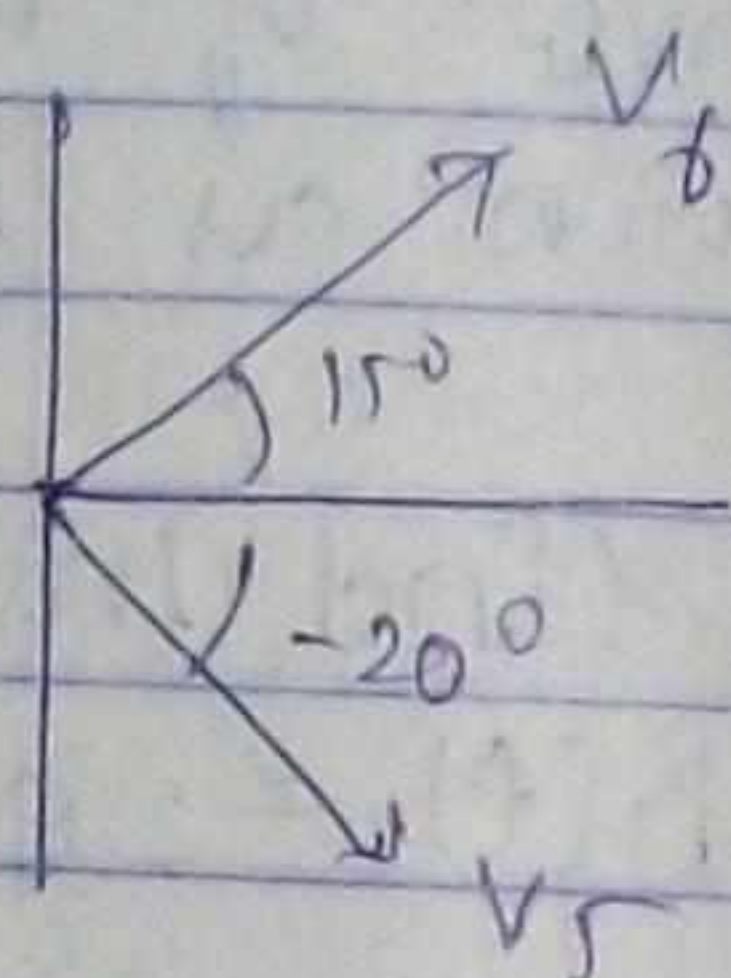
as freq are different
cannot be compare

Ques. Compare $V_5 = 10 \cos(3t - 20^\circ)$
 $V_6 = -5 \sin(3t - 75^\circ)$

$$V_5 = 10 \angle -20^\circ \quad \omega = 3$$

$$V_6 = 5 \angle 15^\circ \quad \omega = 3$$

$$= 5 \angle 15^\circ \quad \omega = 3$$



Cannot compare

both angles must be either clockwise or anticlockwise

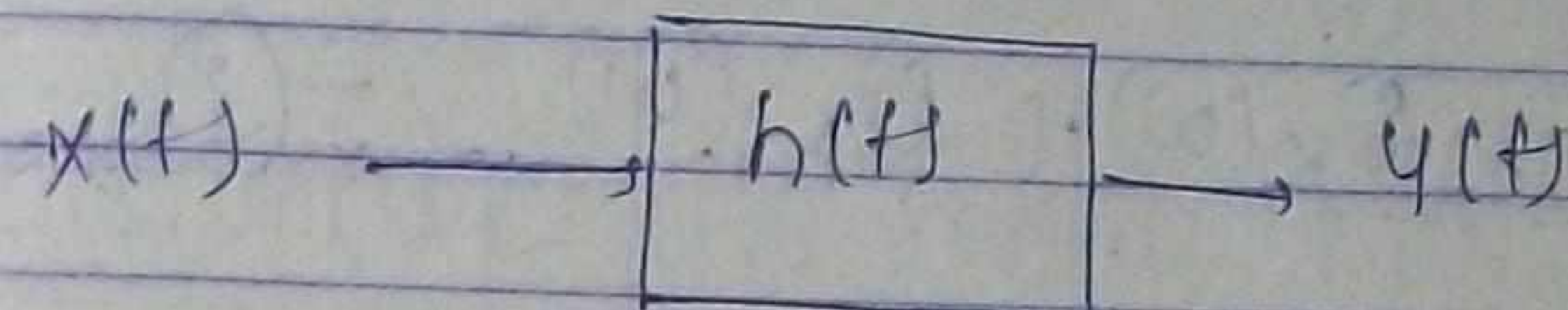
$$V_5 = 10 \angle -20^\circ = 10 \angle 340^\circ = 10 \angle -20^\circ$$

$$V_6 = 5 \angle 15^\circ = 5 \angle 15^\circ = 5 \angle -345^\circ$$

Therefore V_5 is leading V_6 by 325°

or V_6 is lagging V_5 by 325°

SIGNAL TRANSMISSION THROUGH LINEAR SYSTEMS -



$$x(t) \xleftrightarrow{FT} |X(\omega)| e^{j\theta_x(\omega)}$$

$$y(t) \xleftrightarrow{FT} |Y(\omega)| e^{j\theta_y(\omega)}$$

$$h(t) \xleftrightarrow{FT} |H(\omega)| e^{j\theta_h(\omega)}$$

$|H(\omega)|$: magnitude spectrum

$\theta_h(\omega)$: phase spectrum

- for every system linear system,

$$y(t) = x(t) * h(t)$$

Taking FT on each side

$$Y(\omega) = X(\omega) H(\omega) \quad - (1)$$

$$|Y| e^{j\theta_y(\omega)} = |X(\omega)| e^{j\theta_x(\omega)} \cdot |H(\omega)| e^{j\theta_h(\omega)}$$

By comparing:

$$|Y(\omega)| = |X(\omega)| \cdot |H(\omega)| \quad - (2)$$

$$\theta_Y(\omega) = \theta_X(\omega) + \theta_H(\omega) \quad - (3)$$

Squaring both side of eq (2)

$$|Y(\omega)|^2 = |X(\omega)|^2 \cdot |H(\omega)|^2$$

$$\Psi_o(\omega) = \Psi_i(\omega) \cdot \Psi_h(\omega) \quad - (4)$$

where,

$$\Psi_o(\omega) = |Y(\omega)|^2 \quad - (5)$$

$$\Psi_i(\omega) = |X(\omega)|^2 \quad - (6)$$

$$\Psi_h(\omega) = |H(\omega)|^2 \quad - (7)$$

SPECTRAL DENSITY
 $\Psi(\omega)$

- from (2) magnitude response of o/p is equal to the product of magnitude response of i/p and transfer function.
- from (3) phase response of o/p is equal to the addition of phase responses of i/p and transfer function.
- from (4) the spectral density funⁿ of the o/p is the product of spectral density funⁿ of the i/p and T.f.

- from (1) every system modifies the i/p signal in accordance with the system characteristics.

⇒ DISTORTION LESS TRANSMISSION -

- A transmission is said to be distortion less if the o/p is the exact replica of the i/p signal i.e.

$$x(t) \longrightarrow \boxed{H(t)} \longrightarrow y(t)$$

$$y(t) = x(t)$$

$$Y(\omega) = X(\omega) \quad - (8)$$

Comparing (8) and (1)

$$H(\omega) = 1$$

I.F.T

$$h(t) = \delta(t) \quad - (10)$$

- from (10) to have a distortion less transmission $h(t)$ must be characterised by the impulse function. (since impulse include all freq.) the transmission become distortionless.
- for every practical system there exist a finite delay b/w the o/p and i/p and some attenuation.

Therefore for a practical system to be distortionless, O/P must be of the form:

$$y(t) = K x(t - t_0), \quad |K| < 1$$

F.T.

$$y(\omega) = K e^{-j\omega t_0} x(\omega) \quad \text{--- (11)}$$

Comparing (11) and (1):

$$H(\omega) = K e^{-j\omega t_0} \quad \text{--- (12)}$$

Eq (12) represent the transfer function of the practical distortionless system.

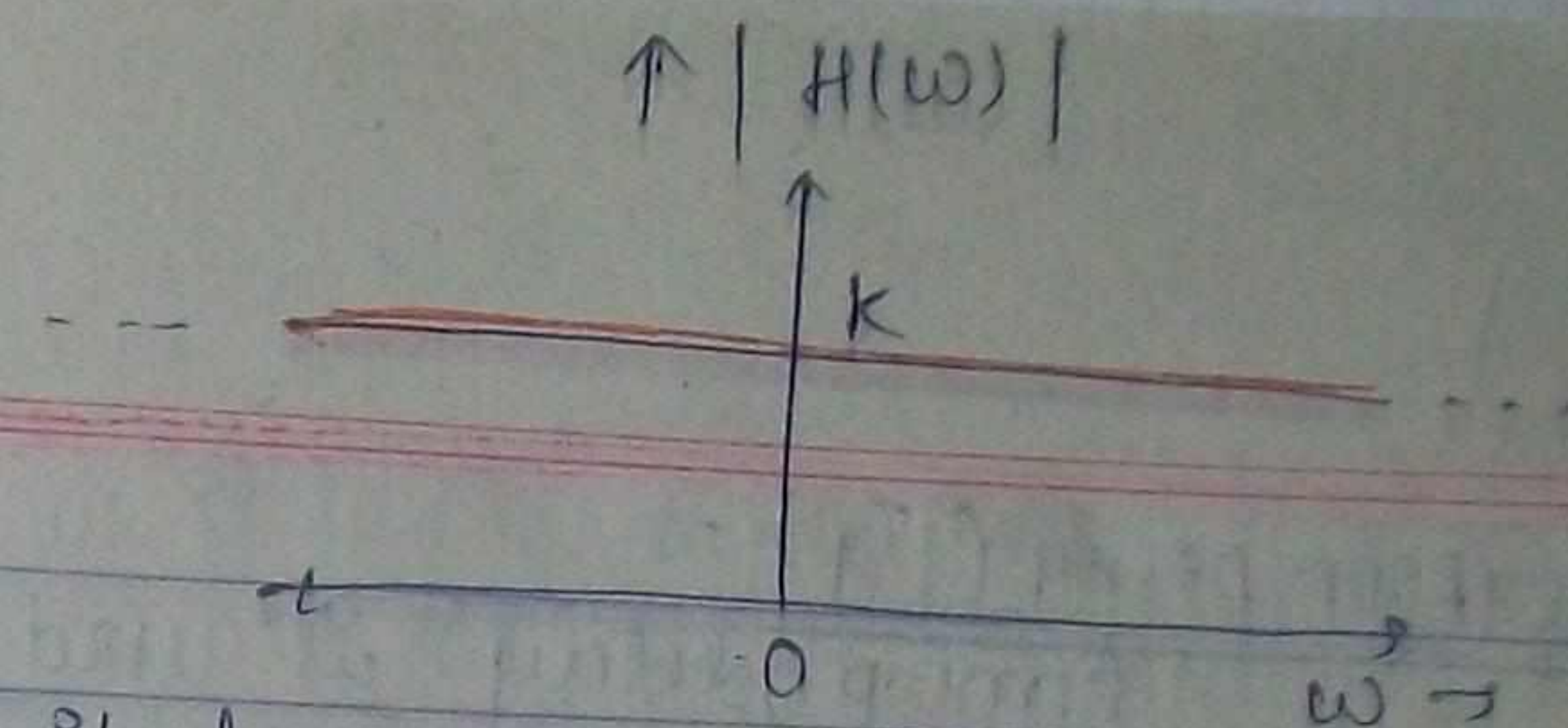
$$|H(\omega)| e^{j\theta_h(\omega)} = K e^{-j\omega t_0}$$

$$|H(\omega)| = K \quad \text{--- (13)}$$

$$\theta_h(\omega) = -\omega t_0 \quad \text{--- (14)}$$

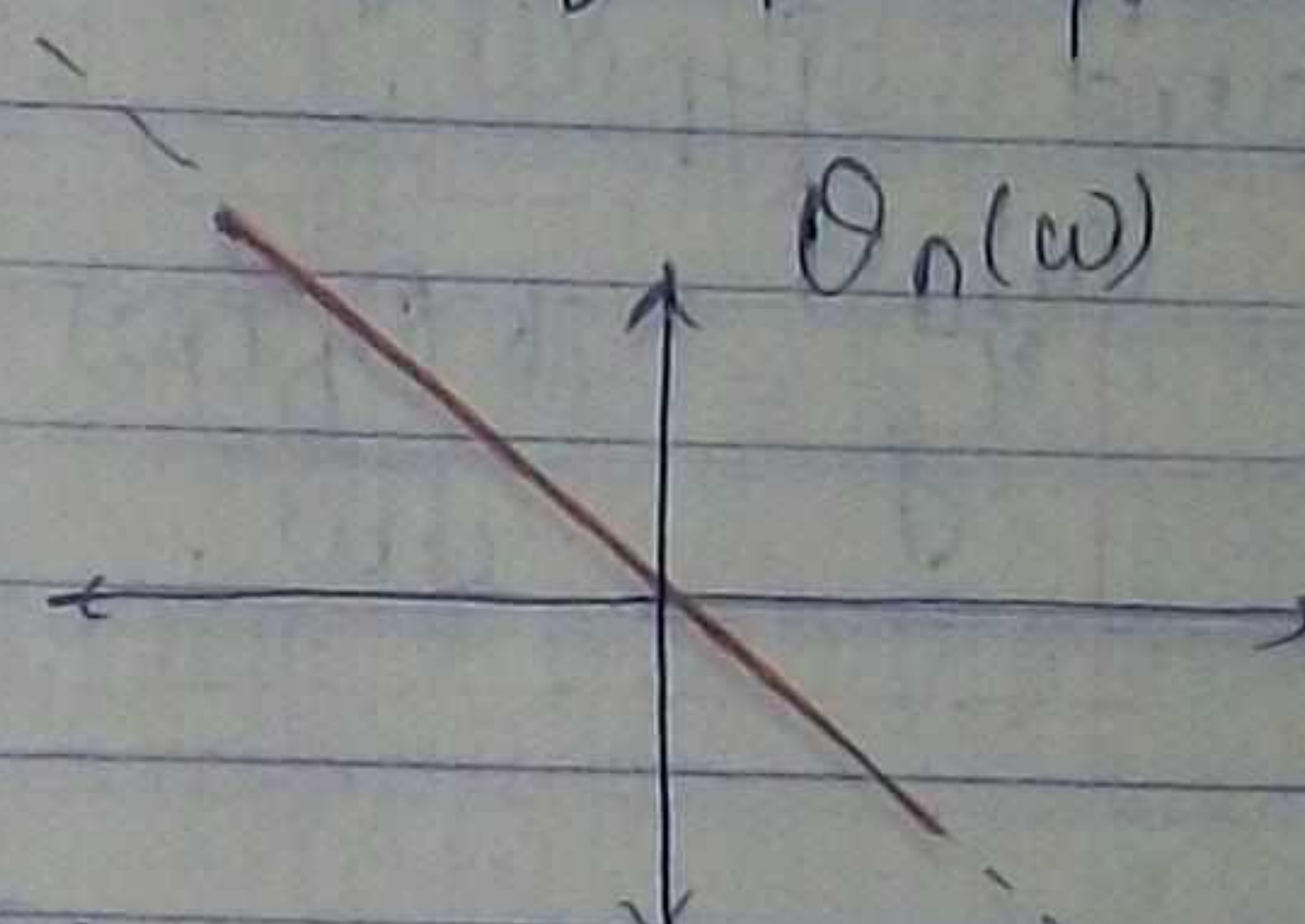
Eq (13) represent magnitude response and
Eq (14) represent phase response of distortionless system (practical)

Magnitude response



Therefore magnitude response of distortionless system is independent of frequency.

Phase Response



Phase response of distortionless system varies linearly with ω .

--- from (14) $t_0 = -\frac{\theta_h(\omega)}{\omega}$

$$\tau_p = \frac{\theta_h(\omega)}{\omega} \quad \text{--- (15)}$$

$$\tau_g = \frac{d\theta_h(\omega)}{d\omega} \quad \text{--- (16)}$$

PHASE DELAY (τ_p) —

phase delay is used to find the delay associated with a single freq. component.

$$\tau_p = \frac{\theta_h(\omega)}{\omega}$$

GROUP DELAY (τ_g) -

Group delay is used to find the delay associated with a group of frequencies, such as envelope of modulated signals.

$$\tau_g = \frac{d\theta_h(\omega)}{d\omega}$$

Since,

$$\theta_h(\omega) = \tan^{-1} \frac{\text{Im}[H(\omega)]}{\text{Re}[H(\omega)]}$$

TRANSFER FUNCTION -

- T.F. is the ratio of transform of O/P to the transform of I/P under zero initial conditions.

- T.F. is applicable for Linear time invariant system only.

$$H(\omega) = \frac{Y(\omega)}{X(\omega)}$$

16/11/11

Ques. find the phase delay and group delay of an LTI system characterised by the impulse response $-e^{-3t}u(t)$.

Solⁿ:

$$Y(t) = X(t) * \delta(t)$$

$$H(t) = -e^{-3t}u(t)$$

$$H(\omega) = \frac{-1}{3+j\omega}$$

$$\begin{aligned}\theta(\omega) &= \tan^{-1}\left(\frac{0}{-1}\right) - \tan^{-1}\left(\frac{\omega}{3}\right) \\ &= \pi - \tan^{-1}\left(\frac{\omega}{3}\right)\end{aligned}$$

$$\tau_p = -\frac{\theta(\omega)}{\omega} = -\frac{\pi - \tan^{-1}(\omega/3)}{\omega}$$

$$\tau_g = \frac{d\theta(\omega)}{d\omega} = \frac{3}{9+\omega^2}$$

Ques. The response of a LTI system is given by $e^{-2t}u(t)$ for the excitation $e^{3t}u(-t)$. Find the phase delay & τ_g .

Solⁿ:

$$\frac{1}{2+j\omega} = H(\omega) \cdot \frac{1}{3-j\omega}$$

$$\frac{1}{s} \quad \frac{1}{s^2+1} \quad \frac{1}{1+s^2} \quad \tan^{-1}(s)$$

$$H(\omega) = \frac{3-j\omega}{2+j\omega}$$

$$\begin{aligned} \theta(\omega) &= \tan^{-1}\left(-\frac{\omega}{3}\right) - \tan^{-1}\left(\frac{\omega}{2}\right) \\ &= -\tan^{-1}\left(\frac{\omega}{3}\right) - \tan^{-1}\left(\frac{\omega}{2}\right) \end{aligned}$$

$$\tau_p = \frac{\tan^{-1}(\omega/3) + \tan^{-1}(\omega/2)}{\omega}$$

$$\begin{aligned} \tau_g &= -\frac{1}{1+(\omega/3)^2} \cdot \frac{1}{3} - \frac{1}{1+(\omega/2)^2} \cdot \frac{1}{2} \\ &= \frac{3}{9+\omega^2} - \frac{2}{4+\omega^2} \end{aligned}$$

$\Rightarrow$ phase delay & group delay are constant.

Ques. P-20 Q-25

Soln:

$$x(t) = 2 \cos(2t) \quad h(t) = e^{-2t} \text{ u}(t)$$

$$y(t) = x(t) * h(t)$$

$$Y(\omega) = |Y(\omega)| e^{j\theta_Y(\omega)}$$

$$x(t) = 2 \cos(2t) \quad \omega = 2$$

$$H(\omega) = \frac{1}{2+j\omega} \quad |H(\omega)| = \frac{1}{(4+\omega^2)^{1/2}}$$

$$\theta_h(\omega) = -\tan^{-1}(\omega/2)$$

$$\begin{aligned} |Y(\omega)| &= |H(\omega)| |X(\omega)| & \theta_Y(\omega) &= \theta_X(\omega) + \theta_h(\omega) \\ &= 2 \cdot \frac{1}{(4+\omega^2)^{1/2}} & &= 0^\circ - \tan^{-1}(\omega/2) \\ & & &= -\tan^{-1}\left(\frac{2}{2}\right) = -\frac{\pi}{4} \end{aligned}$$

$$\text{at } \omega = 2$$

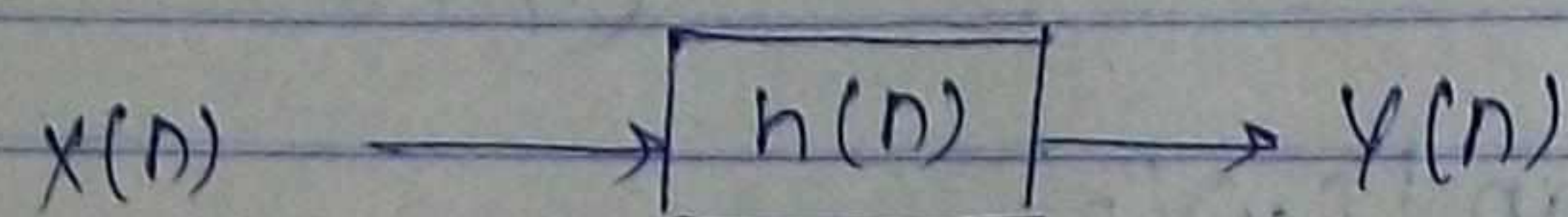
$$= 1/\sqrt{2}$$

$$y(t) = \frac{1}{\sqrt{2}} \cos\left(2t - \frac{\pi}{4}\right)$$

$$= 2^{-0.5} \cos(2t - 0.25\pi)$$

⇒ DISCRETE TIME CONVOLUTION -

(A) LINEAR CONVOLUTION -



$$y(n) = x(n) * h(n)$$

PROPERTIES -

(1) COMMUTATIVE -

$$x(n) * h(n) = h(n) * x(n)$$

$$\sum_{k=-\infty}^{\infty} x(k) h(n-k) = \sum_{k=-\infty}^{\infty} h(k) x(n-k)$$

(2) ASSOCIATIVE -

$$x(n) * [h_1(n) * h_2(n)] = [x(n) * h_1(n)] * h_2(n)$$

(3) DISTRIBUTIVE -

$$x(n) * [h_1(n) + h_2(n)] = [x(n) * h_1(n)] + [x(n) * h_2(n)]$$

(4) WIDTH PROPERTY -

$$x(n) * h(n) = y(n)$$

then,

$$x(n-N_1) * h(n-N_2) = y(n-N_1-N_2)$$

L : Length of $x(n)$

m : Length of $h(n)$

N : Length of $y(n)$

$$N = L + m - 1$$

(5) LIMITS OF CONVOLUTION -

$$x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k) h(n-k) \quad \text{for general } x(n) \text{ and } h(n)$$

$$= \sum_{k=0}^{\infty} x(k) h(n-k), \quad \begin{matrix} x(n) \rightarrow \text{causal} \\ h(n) \rightarrow \text{general} \end{matrix}$$

$$= \sum_{k=-\infty}^n x(k) h(n-k), \quad \begin{matrix} x(n) \rightarrow \text{general} \\ h(n) \rightarrow \text{causal} \end{matrix}$$

$$= \sum_{k=0}^n x(k) h(n-k), \quad \begin{matrix} x(n) \rightarrow \text{causal} \\ h(n) \rightarrow \text{causal} \end{matrix}$$

⇒ CONVOLUTION OF STANDARD SIGNAL -

① IMPULSE FUNCTION -

$$x(n) * \delta(n)$$

$$= \sum_{k=-\infty}^{\infty} x(k) \delta(n-k)$$

$$= x(n)$$

$$x(n) * \delta(n-N) = x(n-N)$$

$$\delta(n-1) * \delta(n+5) = \delta(n-1+5) = \delta(n+4)$$

$$u(n-1) * \delta(n-3) = u(n-4)$$

$$(3)^n u(n-2) * \delta(n-2) = 3^{n-2} u(n-4)$$

② STEP FUNCTION -

$$u(n) * u(n)$$

$$= \sum_{k=-\infty}^{\infty} u(k) u(n-k)$$

$$= \sum_{k=0}^n 1 \cdot 1 = 1+1+1 \dots (n+1) \text{ times}$$

$$= (n+1) \quad n \geq 0$$

$$= (n+1) u(n)$$

③ $u(n)$ AND $r(n)$

$$u(n) * r(n)$$

$$= \sum_{k=-\infty}^{\infty} u(k) r(n-k)$$

$$= \sum_{k=0}^n (n-k) = \sum_{k=0}^n n - \sum_{k=0}^n k$$

$$= \frac{n(n+1)}{2} - \frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{2} u(n)$$

④ $u(n) * a^n u(n)$

$$u(n) * a^n u(n) = \sum_{k=-\infty}^{\infty} u(k) a^{n-k} u(n-k)$$

$$= \sum_{k=0}^n a^{n-k} = a^n \sum_{k=0}^n a^{-k}$$

* CONVOLUTION

$$= a^n \sum_{k=0}^n \left(\frac{1}{a}\right)^k$$

$$\sum_{k=m}^n (r)^k = \frac{r^{n+1} - r^m}{r-1}$$

$$= a^n \frac{(1/a)^{n+1} - (1/a)^0}{\frac{1}{a} - 1}$$

$$= \left[\frac{1 - a^{n+1}}{1 - a} \right] u(n)$$

$$\Rightarrow 2^n u(n) * u(n) = \frac{2^{n+1} - 1}{2 - 1} = 2^{n+1} - 1$$

$$= (2^{n+1} - 1) u(n)$$

$$\begin{aligned} (1/3)^n u(n) * u(n) &= \left(\frac{(1/3)^{n+1} - 1}{1/3 - 1} \right) u(n) \\ &= -\frac{3}{2} \left((1/3)^{n+1} - 1 \right) u(n) \end{aligned}$$

$$\frac{2^n}{r-1}$$

(5) $x(n) * x(n) =$

$$x(n) * x(n) = \sum_{k=-\infty}^{\infty} x(k) x(n-k)$$

$$= \sum_{k=0}^n k(n-k)$$

$$= \sum_{k=0}^n (nk - k^2) = n \sum_{k=0}^n k - \sum_{k=0}^n k^2$$

$$= \left[\frac{n \cdot n(n+1)}{2} - \frac{n(n+1)(2n+1)}{6} \right] u(n)$$

(6) $a^n u(n) * b^n u(n)$ for $a \neq b$

$$a^n u(n) * b^n u(n) = \sum_{k=-\infty}^{\infty} a^k u(k) \cdot b^{n-k} u(n-k)$$

$$= \sum_{k=0}^n a^k \cdot b^n \cdot b^{-k} = b^n \sum_{k=0}^n a^k \cdot b^{-k}$$

$$= b^n \sum_{k=0}^n \left(\frac{a}{b}\right)^k = b^n \frac{(a/b)^{n+1} - (a/b)^0}{a/b - 1}$$

$$= b^n \cdot \frac{(a/b)^{n+1} - 1}{a/b - 1} = b^{n+1} \frac{(a/b)^{n+1} - 1}{a - b}$$

$$= \left(\frac{a^{n+1} - b^{n+1}}{a - b} \right) u(n)$$

⑦ $a^n u(n) * b^n u(n)$ for $a = b$

$$= \sum_{k=-\infty}^{\infty} a^k u(k) \cdot b^{n-k} u(n-k)$$

$$= b^n \sum_{k=0}^n \left(\frac{a}{b} \right)^k$$

$$= (n+1) a^n u(n) \quad \text{for } a = b$$

Ques. Convolution -

① $2^n u(n) * 3^n u(n)$

$$= \frac{2^{n+1} - 3^{n+1}}{2 - 3} = 3^{n+1} - 2^{n+1}$$

② $2^n u(n) * (-2)^n u(n)$

$$= \frac{2^{n+1} - (-2)^{n+1}}{2 + 2}$$

③ $2^n u(n) * 2^n u(n)$

$$= (n+1) 2^n u(n)$$

④ $\left(\frac{1}{3}\right)^n u(n) * \left(\frac{3}{5}\right)^n u(n)$

$$= \frac{\left(\frac{1}{3}\right)^{n+1} - \left(\frac{3}{5}\right)^{n+1}}{\frac{1}{3} - \frac{3}{5}}$$

⇒ CONVOLUTION OF FINITE TIME SEQUENCES -

Let $x(n)$ and $h(n)$ are finite time sequences.

$$y(n) = x(n) * h(n)$$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k) h(n-k), \quad \text{for all } n$$

— (1)

PROCEDURE -

① Find the limits of $y(n)$

$$\text{Lower limit of } y(n) = \text{Lower limit of } x(n) + \text{Lower limit of } h(n)$$

$$\text{Upper limit of } y(n) = \text{Upper limit of } x(n) + \text{Upper limit of } h(n)$$

② Find the limits of k :
limit of k are same as limit of n
in $x(n)$

③ Evaluate $y(n)$ using eq (1) for all the value of n by substituting the limits of k .

Ques. Find the linear convolⁿ of $x(n)$ and $h(n)$.

$$x(n) = \{3, -5, 2, 4\} \quad h(n) = \{5, 2, 4\}$$

Solⁿ: **METHOD (1)**:

$$y(n) = x(n) * h(n)$$

① limit of $x(n) = (-1, 2)$
limit of $h(n) = (-1, 1)$
limit of $y(n) = (-2, 3)$

② limit of $k = (-1, 2)$

$$y(n) = \sum_{k=-1}^2 x(k) h(n-k)$$

for $\{n = -2, -1, 0, 1, 2, 3\}$

for $n = -2$

$$\begin{array}{r} 425 \\ 3524 \\ 6-25 \quad -15 \end{array}$$

$$y(-2) = \sum_{k=-1}^2 x(k) h(-2-k)$$

$$= x(-1)h(-3) + x(0)h(-2) + x(1)h(-1) + x(2)h(0)$$

$$= 3 \cdot 5 + (-5)(0) + 2(0) + 4(0)$$

$$= 15$$

for $n = -1$

$$y(-1) = \sum_{k=-1}^2 x(k) h(-1-k)$$

$$y(0) = 12$$

$$y(1) = 4$$

$$y(2) = 16$$

$$y(3) = 16$$

$$y(n) = \{15, -19, 12, 4, 16, 16\}$$

METHOD (2) -

$h(n)$	5	2	4	$y(0)$ diagonal
$x(n)$				
3	15	6	12	
-5	25	-10	-20	
2	10	4	8	
4	20	8	16	

$$\begin{array}{cccccccc} -4 & 5 & 6 & 1 & 2 & 0 & 2 & -1 \\ -2 & -1 & 1 & 2 & 8 & & & \\ & 5 & 4 & 5 & -3 & -2 & & \\ & -1 & 0 & 1 & 2 & 3 & & \end{array}$$

Ques. $y(n) = x(n) * h(n)$

$$x(n) = \{-4, 5, 6, 1, 2, 0, 2, -1\}$$

$$h(n) = \{-3, -2, 1, 4, 5\} \quad y(-1) = ?$$

$$y(n) = \sum_{k=-2}^4 x(k) h(n-k) \quad \text{for } n = -5, -4, -3, \dots, 5$$

$$y(-1) = \sum_{k=-2}^4 x(k) h(-1-k)$$

$$= -2$$

⇒ CIRCULAR CONVOLUTION -

- Unlike the linear convolution to find the circular convolⁿ sequences must have the same length.
- Both the sequence must start from $n=0$
- The length of the convolved signal is same as the length of the signal to be convolved.
- In circular convolⁿ samples must be shifted in the circular manner.

$$\begin{array}{cccccc} -4 & -3 & 6 & 10 & -26 \\ -2 & -1 & 1 & 2 & 8 & \\ & 5 & 4 & 5 & -3 & -2 \\ & -1 & 0 & 1 & 2 & 3 \end{array}$$

4 point

Ques. find the circular convolⁿ of $x(n)$ and $h(n)$.

$$x(n) = \{2, -3, 4, 5\} \quad h(n) = \{5, 4, 1, 2\}$$

solⁿ:

$\begin{array}{c} -3 \\ 2 \end{array}$	$\begin{array}{c} 1 \\ 4 \end{array}$	$=$	$\begin{array}{c} -3+8+25+8 \\ 38 \end{array}$
$\begin{array}{c} -3 \\ 2 \end{array}$	$\begin{array}{c} 2 \\ 5 \end{array}$	$=$	$\begin{array}{c} -3+16+25+4 \\ 42 \end{array}$
$\begin{array}{c} -3 \\ 2 \end{array}$	$\begin{array}{c} 1 \\ 5 \end{array}$	$=$	$\begin{array}{c} -12+4+10+10 \\ 12 \end{array}$
$\begin{array}{c} -3 \\ 2 \end{array}$	$\begin{array}{c} 2 \\ 5 \end{array}$	$=$	$\begin{array}{c} -6+4+20+10 \\ 28 \end{array}$
$\begin{array}{c} -3 \\ 2 \end{array}$	$\begin{array}{c} 1 \\ 5 \end{array}$	$=$	$\begin{array}{c} -15+16+5+4 \\ 10 \end{array}$
$\begin{array}{c} -3 \\ 2 \end{array}$	$\begin{array}{c} 2 \\ 5 \end{array}$	$=$	$\begin{array}{c} -6+20+20+2 \\ 36 \end{array}$

$= \{38, 42, 12, 28, 10, 36\}$ (wrong)

METHOD -

$$x(n) * h(n) = \begin{bmatrix} -3 & 2 & 5 & 4 \\ 4 & -3 & 2 & 5 \\ 5 & 4 & -3 & 2 \\ 2 & 5 & 4 & -3 \end{bmatrix} \begin{bmatrix} 2 \\ -3 \\ 4 \\ 5 \end{bmatrix}$$

$$= \{42, 28, 6, 20\}$$

find five point circular convolⁿ of

$$\textcircled{2} \quad h(n) = \{1, 2, 3, 4\} \quad y(n) =$$

$$x(n) = \{2, 3, 5\}$$

Solⁿ: five point

$$x(n) = \{2, 3, 5, 0, 0\}$$

$$h(n) = \{1, 2, 3, 4, 0\}$$

$$h(-n) = \{0, 4, 3, 2, 1\}$$

$$x(n) \cdot h(-n) = \{17, 27, 27, 22, 7\}$$

⇒ LINEAR CONVOLUTION USING CIRCULAR CONVOLUTION.

Let lengths

Let L be length of $x(n)$
 M be length of $h(n)$
 N be length of $y(n)$

$$N = L + M - 1$$

- ① Increase the length of $x(n)$ and $h(n)$ to N
- ② Find the N point circular convolⁿ of $x(n)$ and $h(n)$ the result is equal to linear convolution

Ques. $x(n) = \{2, 3, 5\}$ find linear convolⁿ of $x(n) * h(n)$
 $h(n) = \{3, 4\}$

$$x(n) * h(n) = \{6, 17, 27, 20, 0\}$$

$$L = 3, \quad M = 2$$

$$N = 3 + 2 - 1 = 4$$

Circular convolⁿ

2	0	5	3
3	2	0	5
5	3	2	0
0	5	3	2

3
4
0
0

$$= \{6, 17, 27, 20\}$$

Ques compute linear convolⁿ using circular convolⁿ.

Solⁿ: $x(n) = \{-1, 3, 4\}$

$$h(n) = \{1, 2, 3, 4\}$$

$$x(n) * h(n) = \{-1, 1, 7, 13, 24, 18\}$$

⇒ CLASSIFICATION OF SYSTEM

$x(n), y(n)$: PRESENT VALUE

$x(n-k), y(n-k)$: for $k < 0$, future
for $k > 0$, past.

① STATIC & DYNAMIC SYSTEM - (MEMORYLESS AND MEMORY)

- If the present o/p at any instant depends only on present value then the system is known as memoryless or static, otherwise it is known as dynamic system.

Ex - ① $3x(n) - 2x(n-1) \leftrightarrow$ Dynamic

② $\frac{1}{3x(n) - 2x(n-1)} \leftrightarrow$ Dynamic

③ $x(-n) \leftrightarrow$ Dynamic

④ $x^2(n) \leftrightarrow$ Static

⑤ $x(n^2) \leftrightarrow$ Dynamic

⑥ $x(2n) \leftrightarrow$ Dynamic

⑦ $x(2n-k) \leftrightarrow$ Dynamic ($k \neq n$)

⑧ $n x(n) \leftrightarrow$ static

⑨ $\log x(n) \leftrightarrow$ static

⑩ $\cos(x(n)) \leftrightarrow$ static

⑪ $a^{x(n)} \leftrightarrow$ static.

$y(n) = x(-n)$

$\left. \begin{array}{l} n = -2 \quad y(-2) = x(2) \\ n = 0 \quad y(0) = x(0) \\ n = 2 \quad y(2) = x(-2) \end{array} \right\}$ Dynamic

② CAUSAL & NON CAUSAL SYSTEM - (NON ANTICIPATORY & NON-CAUSAL ⇒ ANTICIPATORY)

- If the present o/p, at any instant is independent of does not depend on future value then the system is known as causal system.

Otherwise it is known

- as non-causal system.

Ex ① $3x(n) - 2x(n-1) \leftrightarrow$ Causal

② $\frac{1}{3x(n) - 2x(n-1)} \leftrightarrow$ non causal

③ $x(-n) \leftrightarrow$ non causal

④ $x^2(n) \leftrightarrow$ causal

⑤ $x(n^2) \leftrightarrow$ non causal

⑥ $x(2n) \leftrightarrow$ non causal

⑦ $x(2n-k) \leftrightarrow$ causal $(k > n)$

⑧ $n \times(n) \rightarrow$ causal

⑨ $\log x(n) \rightarrow$ causal

⑩ $\cos(x(n)) \rightarrow$ causal

⑪ $a^{x(n)} \rightarrow$ causal

* Every static system is causal, but every causal system need not be static.

③ TIME VARIANT & INVARIANT SYSTEM -

- If the relationship b/w I/P and O/P, for any instant is independent of time then the system is known as time invariant. otherwise it is time variant.

$y(n-k)$: shifted O/P < every 'n' will be replaced by 'n-k' >

$y(n,k)$: It is the O/P for shifted I/P. (delay). $x(n-k)$

If $y(n-k) = y(n,k) \rightarrow$ T.I.V

$y(n-k) \neq y(n,k) \rightarrow$ T.V.

Ex -

① $y(n) = x(-n)$

$y(n-k) = x(-(n-k))$ - ①

$y(n,k) = x(-n-k)$ - ②

① $\neq$ ② $\rightarrow$ T.V

② $3x(n) - 2x(n-1) \leftrightarrow$ I.V

③ $\frac{1}{3x(n) - 2x(n-1)} \leftrightarrow$ I.V

④ $x(-n) \leftrightarrow$ V

⑤ $x^2(n) \leftrightarrow$ I.V

$$(6) \quad x(n^2) \longleftrightarrow \text{Variant}$$

$$(7) \quad x(2n) \longleftrightarrow \text{Variant}$$

$$(8) \quad x(2n-k) \longleftrightarrow \text{Variant}$$

$$(9) \quad n x(n) \longleftrightarrow \text{Variant}$$

$$(10) \quad \log x(n) \longleftrightarrow \text{I.V}$$

$$(11) \quad \cos(x(n)) \longleftrightarrow \text{I.V}$$

$$(12) \quad a^{x(n)} \longleftrightarrow \text{I.V}$$

(4) LINEAR AND NON LINEAR SYSTEM -

- A system is said to be linear if it obeys the principle of superposition and homogeneity. Otherwise it is non-linear system.
for $x(n)$

$$x_1(n) \longleftrightarrow y_1(n)$$

$$x_2(n) \longleftrightarrow y_2(n)$$

$$x_1(n) + x_2(n) \longleftrightarrow y_1(n) + y_2(n)$$

LINEAR
SYSTEM

$$\text{Ex - } (1) \quad y(n) = 3x(n) - 2x(n-1) \rightarrow \text{linear}$$

$$(2) \quad y(n) = \frac{1}{3x(n) - 2x(n-1)} \rightarrow \text{Non linear}$$

$$\frac{1}{3x_1(n) - 2x_1(n-1)} \longleftrightarrow y_1(n)$$

$$\frac{1}{3x_2(n) - 2x_2(n-1)} \longleftrightarrow y_2(n)$$

$$= \frac{1}{3(x_1(n) + x_2(n)) - 2[x_1(n-1) + x_2(n-1)]}$$

$$= \frac{1}{[3x_1(n) - 2x_1(n-1)] + [3x_2(n) - 2x_2(n-1)]}$$

$$\neq y_1(n) + y_2(n)$$

$$(3) \quad x(-n) \rightarrow \text{linear}$$

$$(4) \quad x^2(n) \rightarrow \text{non linear}$$

$$(5) \quad x(n^2) \rightarrow \text{linear}$$

$$(6) \quad x(2n) \rightarrow \text{linear}$$

$$(7) \quad x(2n-k) \rightarrow \text{linear}$$

(8) $n x(n) \rightarrow$ linear.

(9) $\log x(n) \rightarrow$ non linear.

(10) $\cos(x(n)) \rightarrow$ non linear.

(11) $a^{x(n)} \rightarrow$ non linear.

(5) STABLE AND UNSTABLE SYSTEM -

- A system is said to be stable if it produces the bounded o/p for the bounded i/p. Otherwise it is unstable.
- i.e. the signal should be bounded in amplitude.

$$y(n) = x(n) * h(n)$$

$$= \sum_{k=-\infty}^{\infty} x(k) h(n-k)$$

$$= \sum_{k=-\infty}^{\infty} h(k) x(n-k)$$

To ensure the stability $|y(n)| < \infty$.

$$\sum_{k=-\infty}^{\infty} |h(k)| |x(n-k)| < \infty$$

$$\left| \sum_{k=-\infty}^{\infty} x(k) h(n-k) \right| < \infty$$
$$\left| \sum_{k=-\infty}^{\infty} h(k) x(n-k) \right| < \infty \quad \text{--- (1)}$$

Since $|x(n)|$ is finite $|x(n-k)|$ is also finite.

Therefore to satisfy eq (1)

$$\sum_{k=-\infty}^{\infty} |h(k)| < \infty$$

Therefore any S.D.T. system is stable if and only if the system function $|H(n)|$ is absolutely summable otherwise it is unstable.

$$\sum_{k=-\infty}^{\infty} |h(n)| < \infty$$

- for continuous time stable system, system function $h(t)$ must be absolutely integrable. Otherwise it is unstable.

$$\int_{-\infty}^{\infty} |h(t)| dt < \infty$$

- Stability is determined from the system funⁿ $|h(n)|$

- $h(n)$ can be formed from $y(n)$ by substituting $x(n) = \delta(n)$ sinu.

$$y(n) = x(n) * h(n)$$

$$y(n) = \delta(n) * h(n)$$

$$y(n) = h(n)$$

Ex- (1) $y(n) = \frac{1}{3x(n) - 2x(n-1)}$

$$h(n) = \frac{1}{3\delta(n) - 2\delta(n-1)}$$

$$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=-\infty}^{\infty} \left| \frac{1}{3\delta(n) - 2\delta(n-1)} \right|$$

$$= \left| \frac{1}{3\delta(-\infty) - 2\delta(-\infty-1)} \right| + \dots$$

$$+ \dots + \left| \frac{1}{3\delta(0) - 2\delta(-1)} \right| + \dots$$

17/11/11

$$= \left| \infty + \dots + \right| = \infty \text{ (unstable)}$$

(2) $3x(n) - 2x(n-1)$

(3) $x(-n)$

(4) $x^2(n)$

(5) $a^{x(n)} \Rightarrow a^{\delta(n)} \Rightarrow \text{unstable.}$

(6) INVERTIBLE AND NON-INVERTIBLE SYSTEM -

- If it is possible to determine unique I/P from the O/P then it is known as invertible system otherwise it is non-invertible system.

- Therefore in invertible system the relation b/w the O/P and I/P is 1 to 1 i.e. unique, if it is many to one the system is non-invertible.

- Integral system and differential system without limits and initial condⁿ is non-invertible.

Similarly Laplace transform and z-transform with ROC is non-invertible.

- ④ characteristic of every linear system pass through the origin. Since I/P-O/P relationship is given from 0 to ∞ they represent the characteristic of causal system

LAPLACE TRANSFORM

$$ROC = \sigma > \sigma_c$$

- Signal that are not converged by fourier transform are represented in the freq. using laplace transform.
- L.T. is also known as 'COMPLEX FOURIER TRANSFORM'

$$L\{x(t)\} = X(s)$$

LT.:

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt$$

$$= \int_0^{\infty} x(t) e^{-st} dt$$

V-sec
A-sec

BILATERAL
L.T.

unilateral
L.T.

where, $s = \sigma + j\omega$

ILT:

$$L^{-1}\{X(s)\} = x(t) = \frac{1}{j2\pi} \int_{\sigma - j\infty}^{\sigma + j\infty} X(s) e^{st} ds$$

V or A

$$x(t) \xleftrightarrow{L.T.} X(s)$$

⇒ CONDITION FOR THE EXISTENCE OF L.T. -

$$|X(s)| < \infty$$

$$\left| \int_{-\infty}^{\infty} x(t) e^{-st} dt \right| < \infty$$

low pass



$$\int_{-\infty}^{\infty} |x(t)| |e^{-\sigma t}| |e^{-j\omega t}| dt < \infty \quad (1)$$

where,

$$|e^{-j\omega t}| = 1$$

The finite value of eq (1) depends on the product.

$$|x(t)| |e^{-\sigma t}| < \infty$$

Therefore the finite value of Laplace transform depends on σ i.e. $\text{Real}\{s\}$.

⇒ RELATION B/W F.T AND L.T -

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt$$

$$= \int_{-\infty}^{\infty} x(t) e^{-\sigma t} e^{-j\omega t} dt$$

$$L\{x(t)\} = F\{x(t) e^{-\sigma t}\}$$

for $\sigma = 0$

$$L\{x(t)\} = F\{x(t)\}, \quad X(\omega) = X(s) \Big|_{s=j\omega}$$

(*)

Therefore f.T is the special case of L.T in which $\sigma = 0$ i.e. it is possible to obtain the Fourier transform from L.T. and vice versa by substituting $s = j\omega$ if and only if the corresponding time signal is absolutely integrable, otherwise it is not possible.

⇒ LAPLACE TRANSFORM OF STANDARD SIGNALS -

(1) $x(t) = A \delta(t)$

$$X(s) = \int_{-\infty}^{\infty} A \delta(t) e^{-st} dt = A e^{-s(0)}$$

$$x(t) \longleftrightarrow A, \text{ for all } s$$

(2) $x(t) = e^{-at} u(t) \quad \forall (0, \infty) \text{ R.S.S}$

$$X(s) = \int_{-\infty}^{\infty} e^{-at} e^{-st} dt$$

$$= \int_0^{\infty} e^{-(s+a)t} dt = \frac{-1}{(s+a)} \left[e^{-(s+a)t} \right]_0^{\infty}$$

$$X(s) = \frac{-1}{s+a} \left[e^{-(s+a)t} \Big|_{t=0}^{\infty} - 1 \right] \quad (2)$$

(Case 1): $(s+a) < 0$, $(s+a) \rightarrow -(s+a)$

$s < -a$, $(\sigma + j\omega) < -a$, $\sigma < -a$

$\text{Re}\{s\} < -a$

from (2) $X(s) = \frac{-1}{(s+a)} [e^{\infty} - 1] = \infty$

(Case 2): $(s+a) > 0$, $s > -a$

from (2)

$X(s) = \frac{-1}{(s+a)} [e^{-\infty} - 1]$

$= \frac{-1}{s+a} [0 - 1] = \frac{1}{s+a}$

$L\{e^{-at} u(t)\} = \frac{1}{s+a}, s > -a$

$L^{-1}\left\{\frac{1}{s+a}\right\} = e^{-at} u(t), s > -a$

(3) $x(t) = -e^{-at} u(-t), (-\infty, 0)$ LSS

$X(s) = -\int_{-\infty}^0 e^{-at} e^{-st} dt$

$= -\int_{-\infty}^0 e^{-(s+a)t} dt$

$= \frac{1}{s+a} [e^{-(s+a)t}]_{-\infty}^0$

$= \frac{1}{s+a} [1 - e^{(s+a)\infty}] = \textcircled{3}$

(Case 1): $(s+a) > 0$, $s > -a$

from (3)

$X(s) = \frac{1}{(s+a)} [1 - e^{\infty}] = -\infty$

(Case 2): $(s+a) < 0$, $s < -a$

$X(s) = \frac{1}{(s+a)} [1 - e^{-\infty}] = \frac{1}{(s+a)}$

$L\{-e^{-at} u(-t)\} = \frac{1}{s+a}, s < -a$

$L^{-1}\left\{\frac{1}{s+a}\right\} = -e^{-at} u(-t), s < -a$

$$L^{-1} \left\{ \frac{1}{s+a} \right\} = e^{-at} u(t), \quad s > -a$$

$$= -e^{-at} u(-t), \quad s < -a$$

$$(4) x(t) = e^{-a|t|}$$

$$= e^{-at} u(t) + e^{at} u(-t)$$

$$X(s) = \frac{1}{s+a} - \frac{1}{s-a}, \quad -a < s < a$$

for $a > 0$ only.

⇒ LAPLACE TRANSFORM TABLE —

$$(1) A \delta(t) \longleftrightarrow A \quad \text{+ a.u.s}$$

$$(2) e^{-at} u(t) \longleftrightarrow \frac{1}{s+a}, \quad s > -a$$

$$(3) e^{at} u(t) \longleftrightarrow \frac{1}{s-a}, \quad s > a$$

$s < -a$

$$(4) e^{jat} u(t) \longleftrightarrow \frac{1}{s-j a}, \quad s > 0$$

$$(5) e^{-jat} u(t) \longleftrightarrow \frac{1}{s+j a}, \quad s > 0$$

$$(6) u(t) \longleftrightarrow \frac{1}{s}, \quad s > 0$$

$$(7) \cosh at u(t) \longleftrightarrow \frac{s}{s^2 - a^2}, \quad s > a$$

$$(8) \sinh at u(t) \longleftrightarrow \frac{a}{s^2 - a^2}, \quad s > a$$

$$(9) \cos at u(t) \longleftrightarrow \frac{s}{s^2 + a^2}, \quad s > 0$$

$$(10) \sin at u(t) \longleftrightarrow \frac{a}{s^2 + a^2}, \quad s > 0$$

$$(11) -e^{-at} u(-t) \longleftrightarrow \frac{1}{s+a}, \quad s < -a$$

$$(12) -u(-t) \longleftrightarrow \frac{1}{s}, \quad s < 0$$

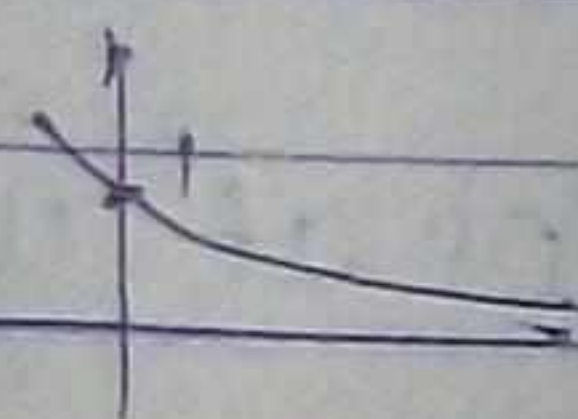
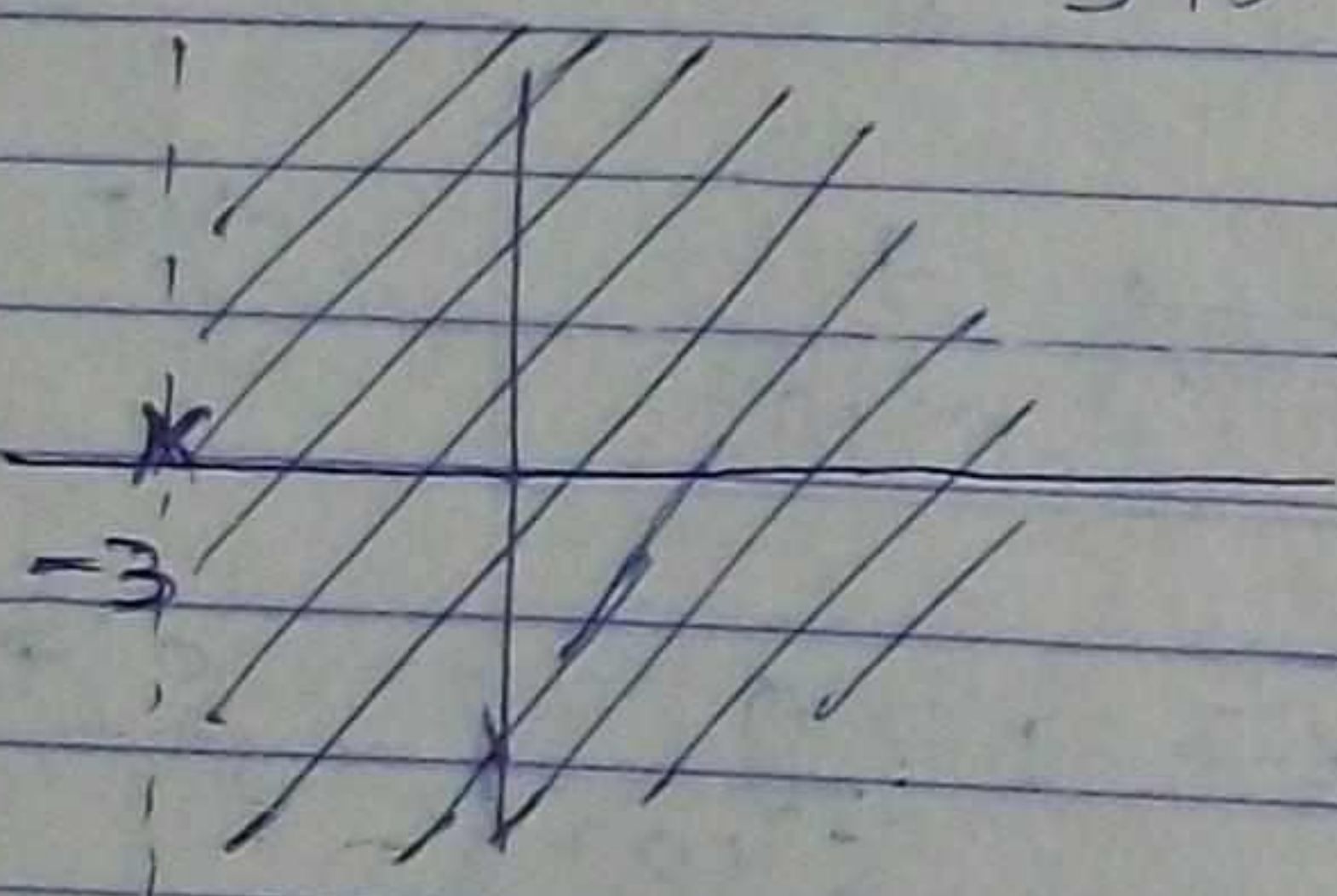
$$(13) x(t) = t u(t) \longleftrightarrow \frac{1}{s^2}, \quad s > 0$$

$$t^n u(t) = \frac{n!}{s^{n+1}}, \quad s > 0$$

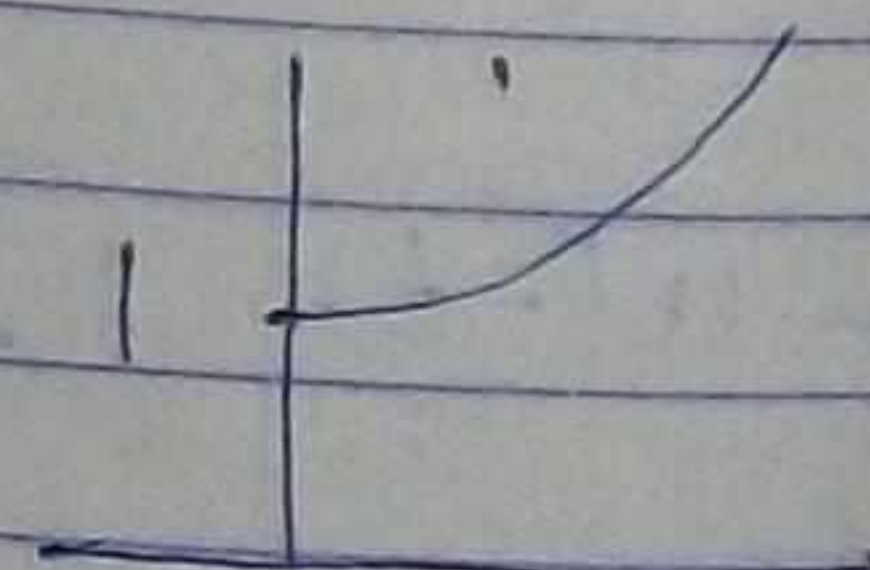
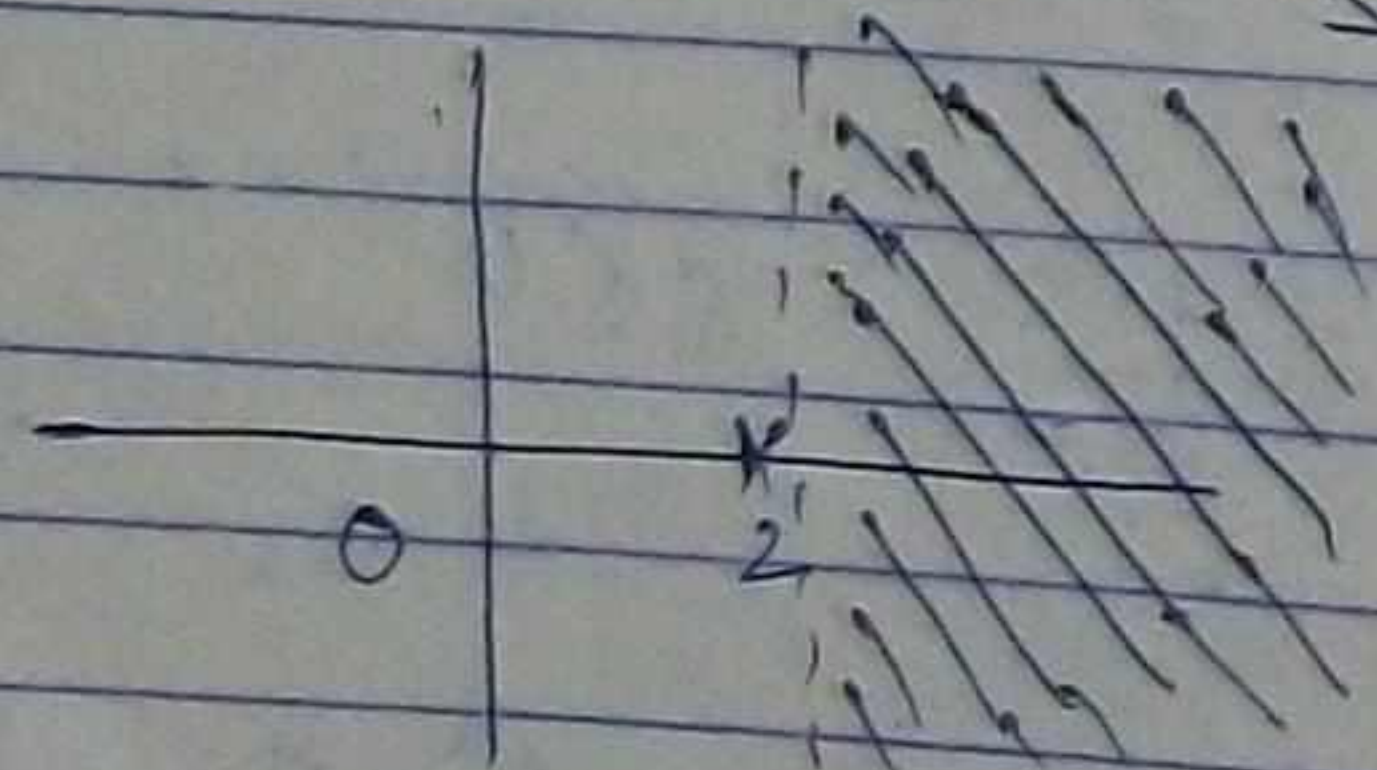
$$(14) \quad t e^{-at} u(t) \leftrightarrow \frac{1}{(s+a)^2}, \quad s > -a$$

$$(15) \quad t^n e^{-at} u(t) \leftrightarrow \frac{n!}{(s+a)^{n+1}}, \quad s > -a$$

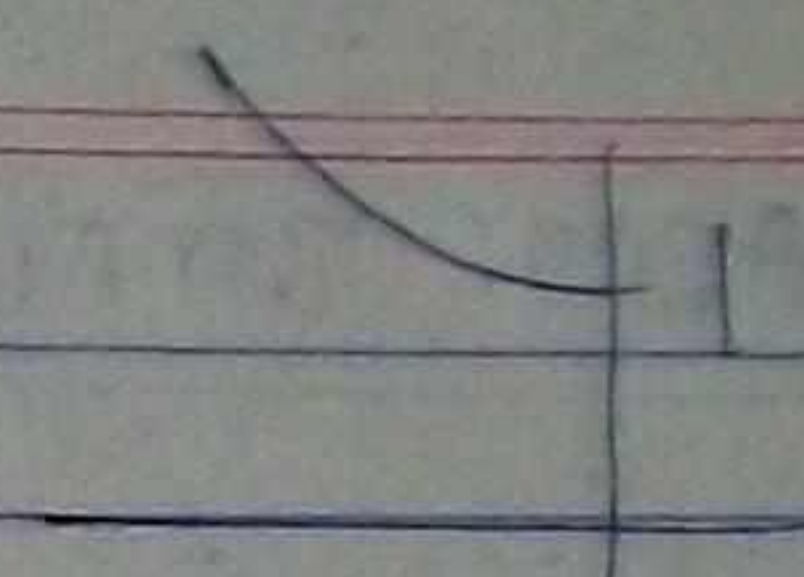
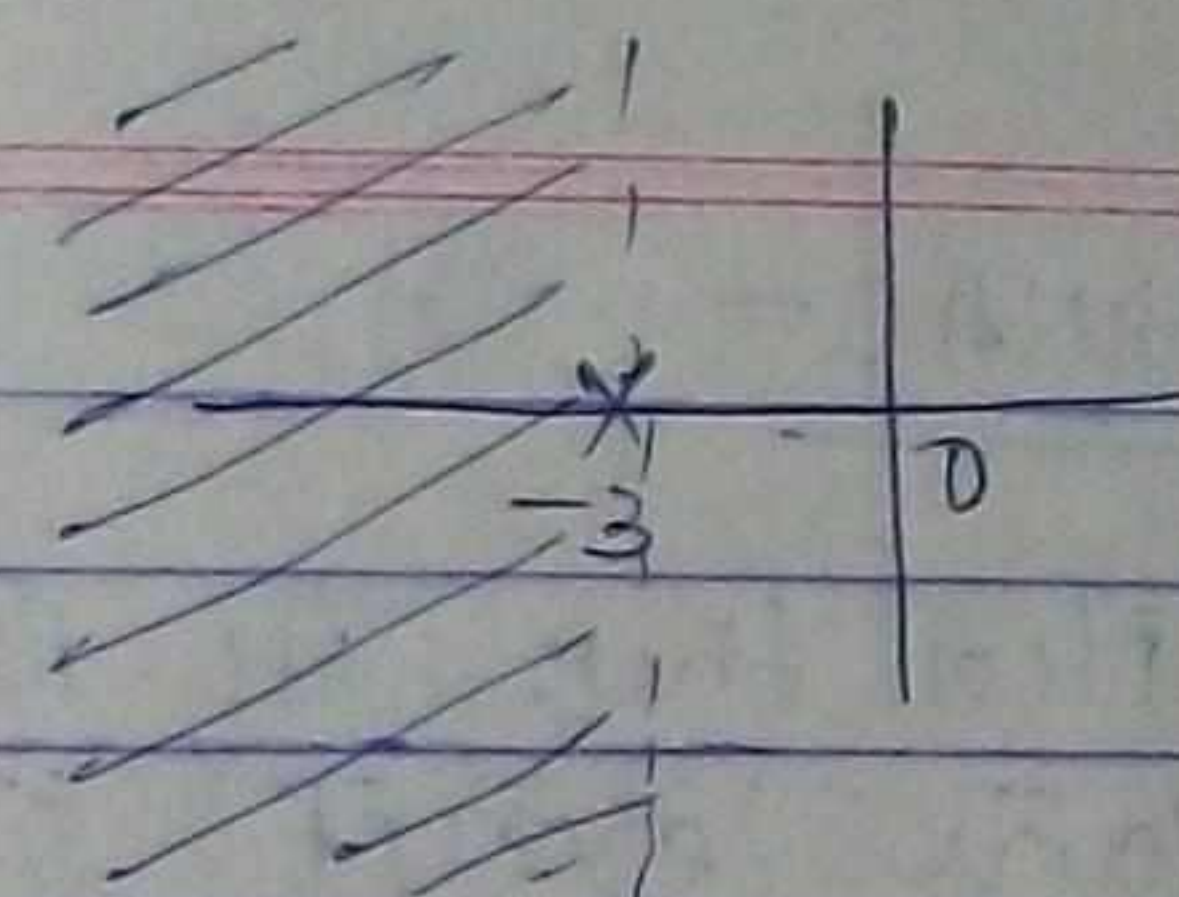
$$\text{Ex } (1) \quad e^{-3t} u(t) \leftrightarrow \frac{1}{s+3}, \quad s > -3 \quad \text{ROC}$$



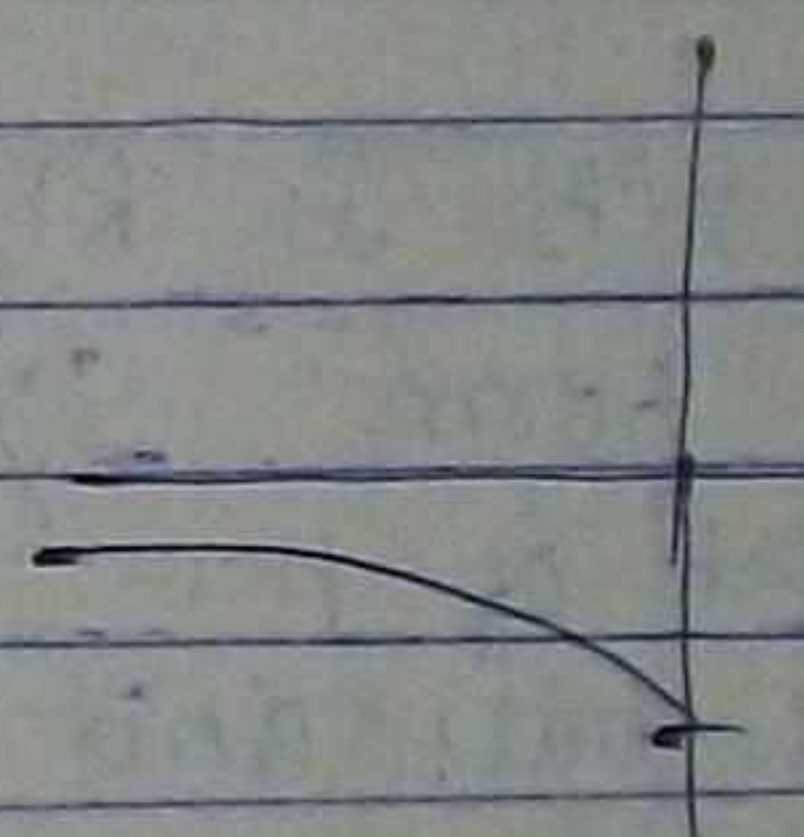
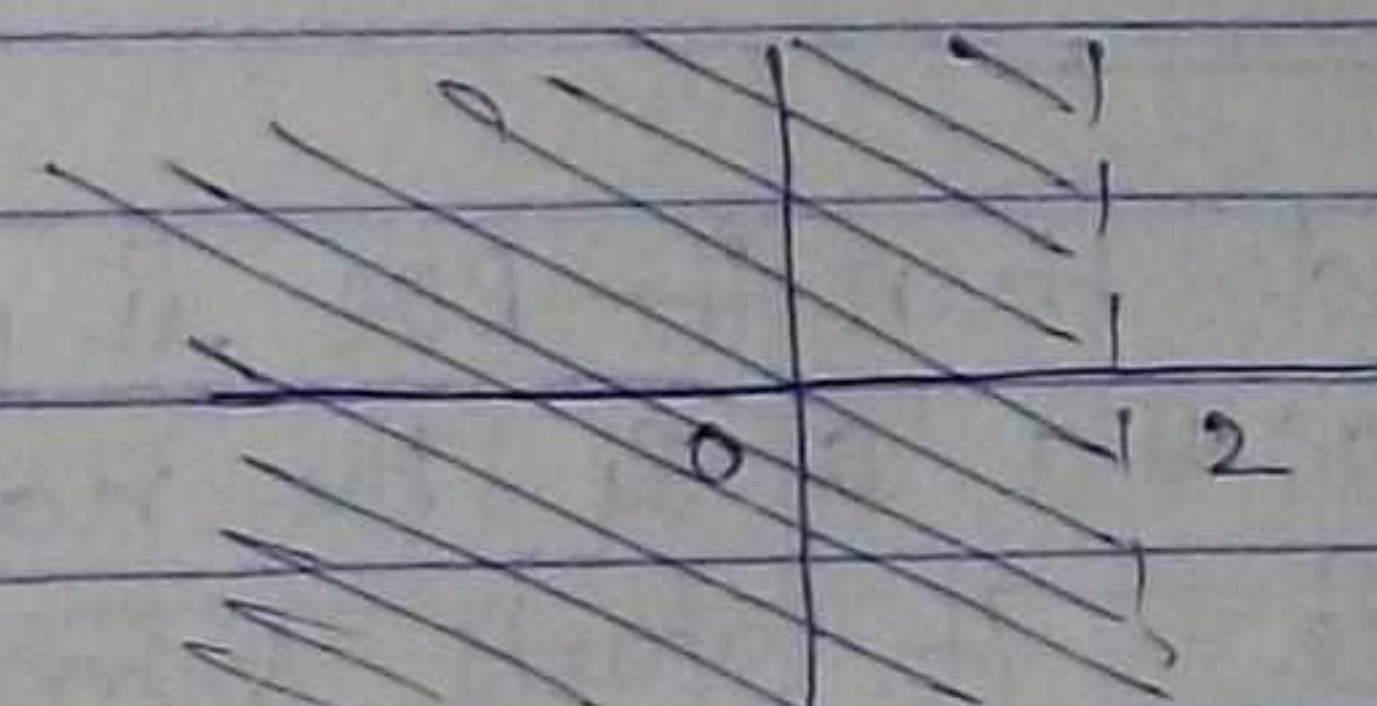
$$(2) \quad e^{2t} u(t) \leftrightarrow \frac{1}{s-2}, \quad s > 2 \quad \text{ROC}$$



$$(3) \quad e^{-3t} u(-t) \leftrightarrow -\frac{1}{s+3}, \quad s < -3 \quad \text{ROC}$$



$$(4) \quad -e^{2t} u(-t) \leftrightarrow \frac{1}{s-2}, \quad s < 2 \quad \text{ROC}$$



$\Rightarrow$ REGION OF CONVERGENCE OF L.T.

- It is the range of value of ' σ ' or ' s ' for which the L.T. is finite.

- ROC of L.T. is a strip parallel to $j\omega$ axis.

$\Rightarrow$ PROPERTIES OF ROC

(1) ABOUT POLES

ROC of L.T. does not include any poles but bounded by the poles.
R: Because at poles L.T. becomes ∞ .

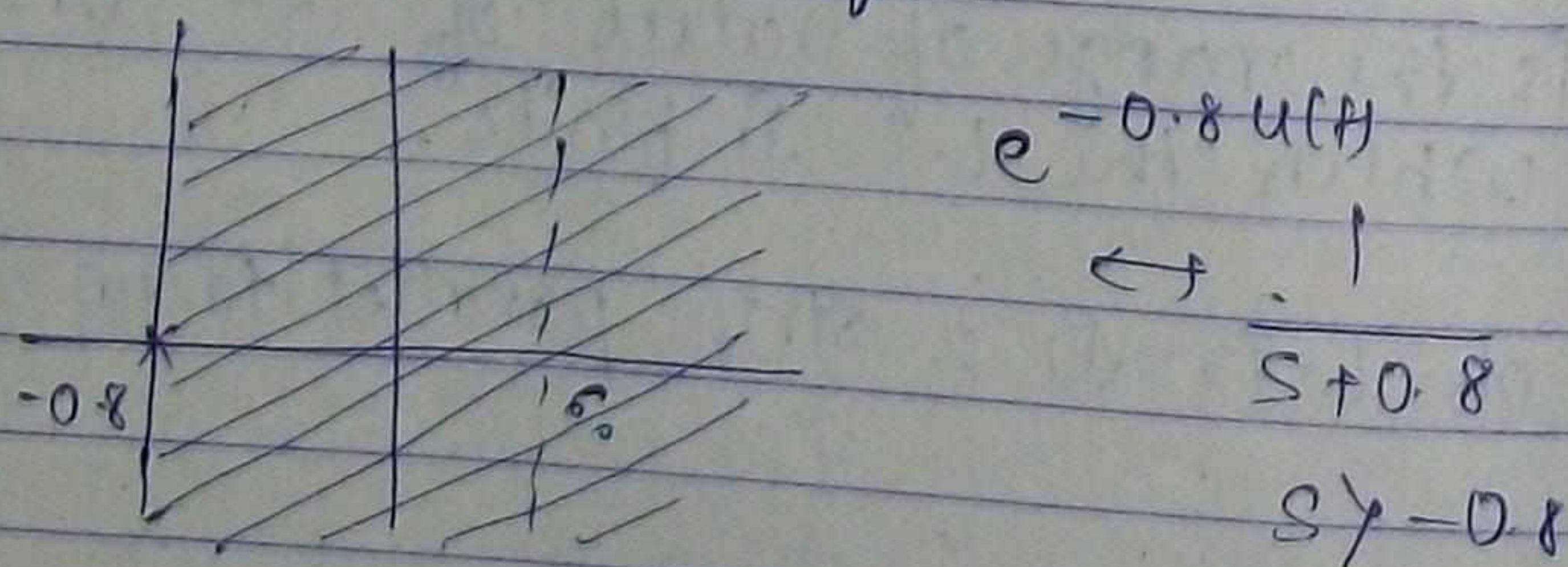
② ABOUT TIME LIMITED SIGNAL -

If $x(t)$ is time limited then its ROC will be entire s -plane except $s=0$ and $s=\pm\infty$.

③ ABOUT RIGHT SIDE SIGNALS -

(A) If $x(t)$ is Right sided then its ROC is of the form $\sigma > a$, where 'a' is the real part of the pole. i.e. it starts from the pole and extends to ∞ .

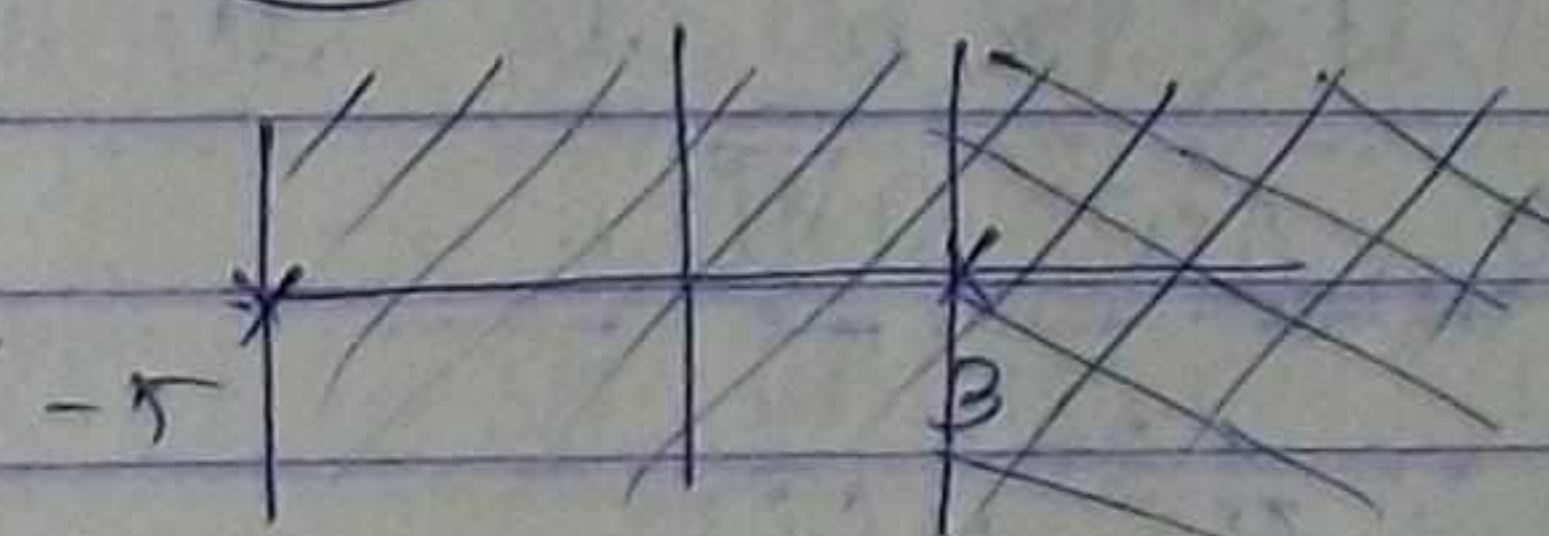
If a line $\sigma = \sigma_0$ is in the ROC then all values of σ for which $\sigma > \sigma_0$ will also be in the ROC of it.



(B) If $x(t)$ is the combn of Right sided signal then its ROC will be Right side to the right most pole.

Ex- $e^{-5t}u(t) + e^{3t}u(t) \leftrightarrow \frac{1}{s+5} + \frac{1}{s-3}$

Common ROC $\sigma > 3$

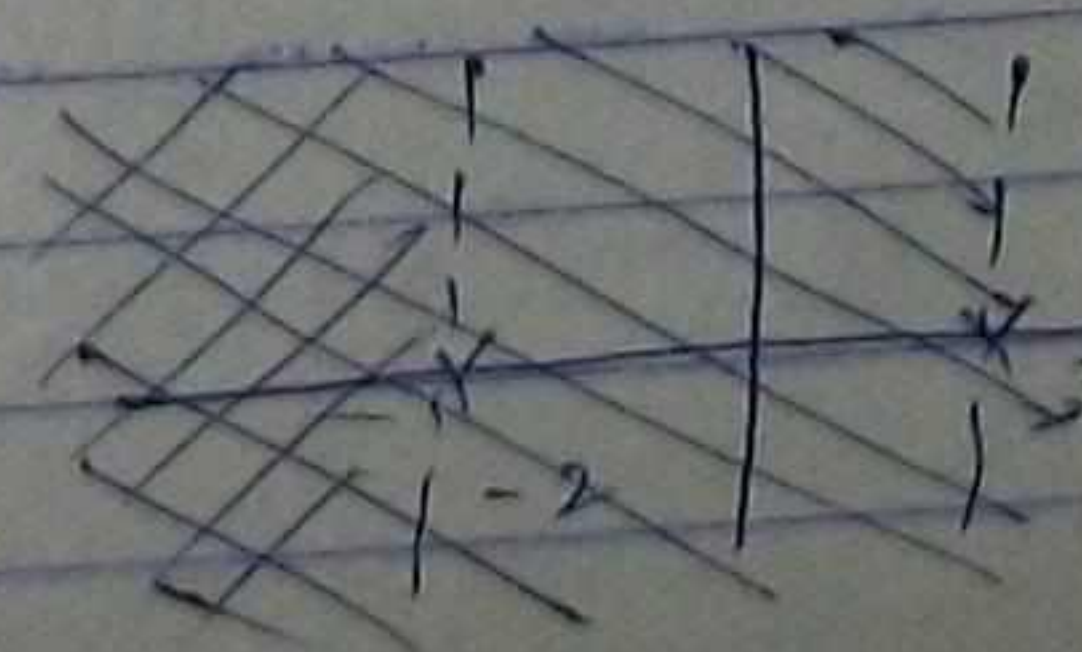


④ ABOUT LEFT SIDED SIGNALS -

(A) Let $x(t)$ is L.S.S then its ROC is of the form $\sigma < b$ where b is the real part of the pole. i.e. it starts from the pole and it extends to $-\infty$ and If a line $\sigma = \sigma_0$ is in the ROC then all values of σ for which $\sigma < \sigma_0$ will also be in the ROC of it.

(B) If $x(t)$ is the combination of left side signal then its ROC will be Left side to the left most pole.

Ex- $e^{-2t}u(-t) + e^{3t}u(-t) = -\frac{1}{s+2} - \frac{1}{s+3} \quad \sigma < -2$

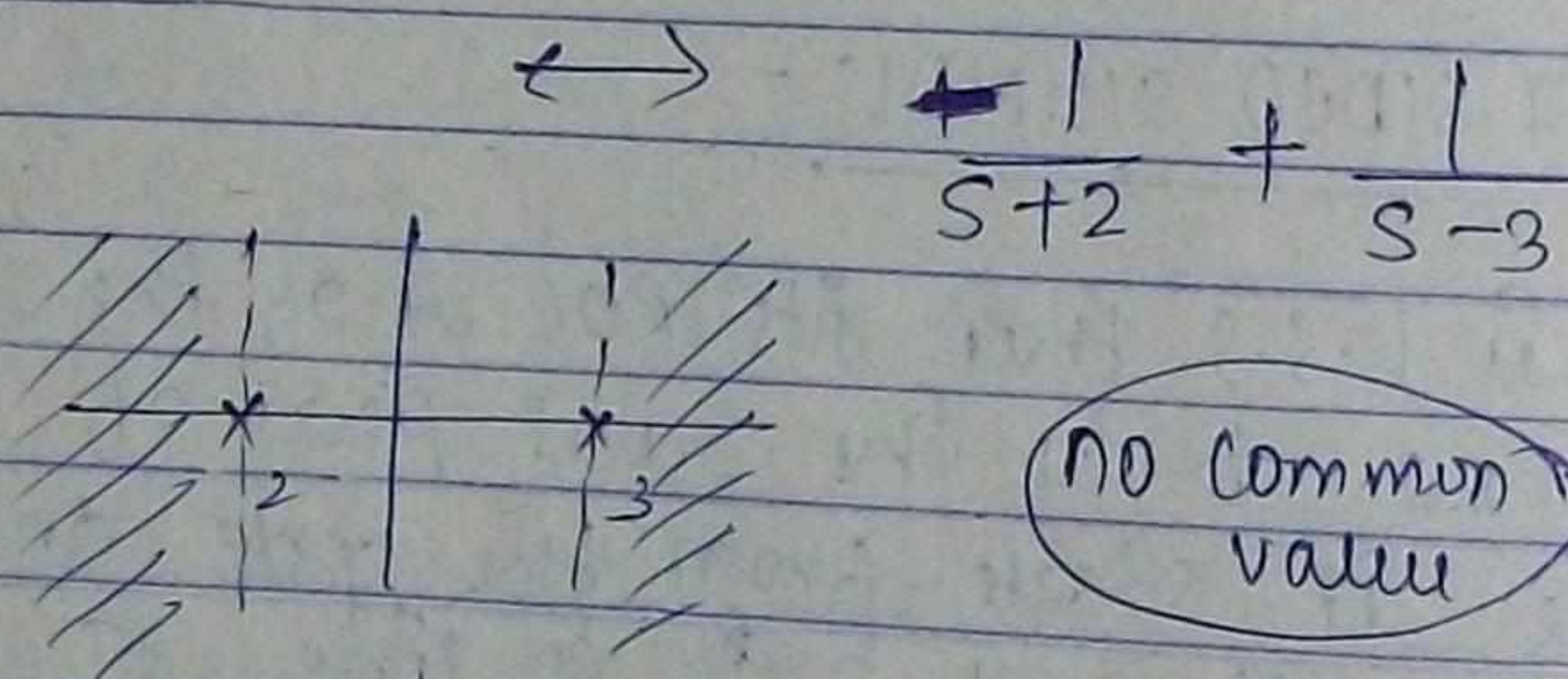


⑤ ABOUT TWO SIDED SIGNAL -

If $x(t)$ is two sided signal then its ROC is of the form

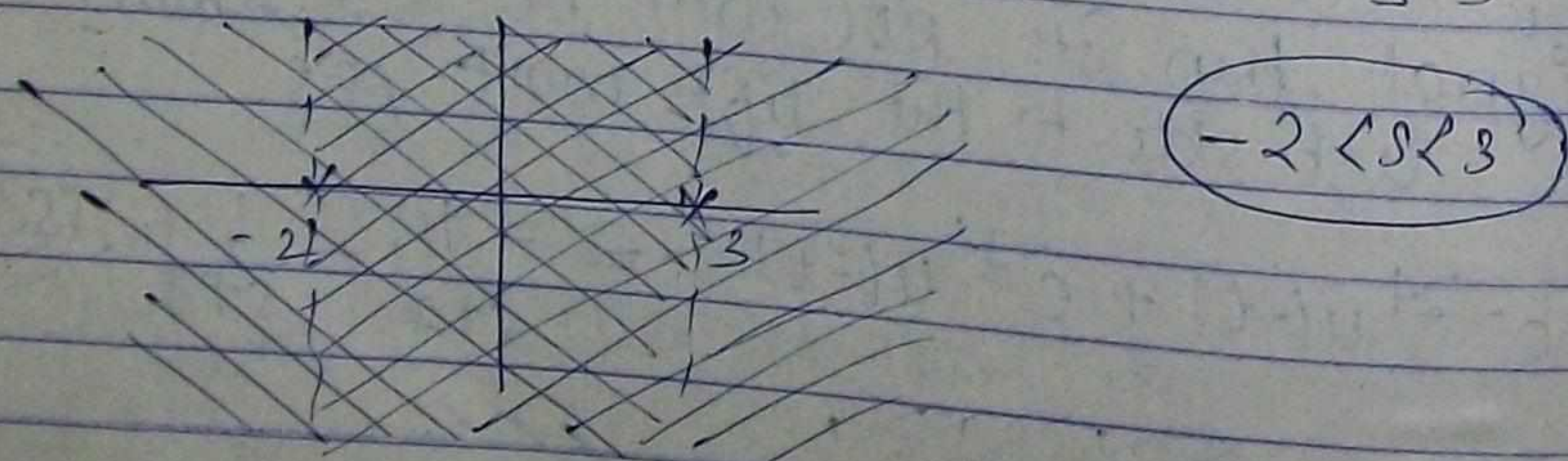
$a < s < b$ i.e. it is a strip parallel to jw axis.

$$f_x = e^{-2t} u(-t) + e^{3t} u(t)$$



as there is no common ROC its L.T. does not exist

$$f_x = e^{-2t} u(-t) + e^{3t} u(t) \longleftrightarrow \frac{1}{s+2} - \frac{1}{s-3}$$



⑥ ABOUT STABILITY -

The given system is stable if and only if its ROC includes imaginary axis. Otherwise it is unstable.

⇒ PROPERTIES OF LAPLACE TRANSFORM -

$$x(t) \longleftrightarrow X(s), \text{ ROC: } R_1$$

$$y(t) \longleftrightarrow Y(s), \text{ ROC: } R_2$$

a, b, t_0 & s_0 are constants.

① LINEARITY -

L.T. is linear operation.

$$a x(t) + b y(t) \longleftrightarrow a X(s) + b Y(s); R_1 \cap R_2$$

② TIME SHIFTING -

$$x(t - t_0) \longleftrightarrow e^{-s t_0} X(s); R_1 \mid (R_1 \pm R_0)$$

③ SHIFTING IN S-DOMAIN -

$$X(s - s_0) \longleftrightarrow e^{s_0 t} x(t); R_1 \pm R_0$$

④ SCALING -

$$x(at) \leftrightarrow \frac{1}{|a|} x(s/a) ; s/a$$

⑤ INVERSION -

$$x(-t) \leftrightarrow x(-s) ; -s$$

⑥ CONVOLUTION -

$$x(t) * y(t) \leftrightarrow x(s) \cdot y(s) ; R_1 \cap R_2$$

⑦ DIFFⁿ IN TIME -

$$\frac{d}{dt} x(t) \leftrightarrow sX(s) - x(0) ; R_1$$

$$\frac{d^2}{dt^2} x(t) \leftrightarrow s^2 X(s) - s x(0) - x'(0) ; R_1$$

⑧ INTEGRATION IN TIME -

$$\int_0^t x(t) dt \leftrightarrow \frac{1}{s} X(s) ; R_1$$

$$\int_{-\infty}^t x(t) dt \leftrightarrow \frac{1}{s} X(s) + \int_{-\infty}^0 x(t) dt ; R_1$$

Initial condition

⑨ DIFFⁿ IN S -

$$\frac{d}{ds} x(s) \leftrightarrow -t x(t) ; R_1$$

⑩ INTEGRATION IN S -

$$\int_0^s x(s) ds \leftrightarrow -\frac{1}{t} x(t) ; R_1$$

⑪ INITIAL VALUE THEOREM -

$$\lim_{t \rightarrow 0} x(t) = x(0) = \lim_{s \rightarrow \infty} sX(s)$$

⑫ FINAL VALUE THEOREM -

It is applicable for STABLE system only. (all poles in L.H.S)

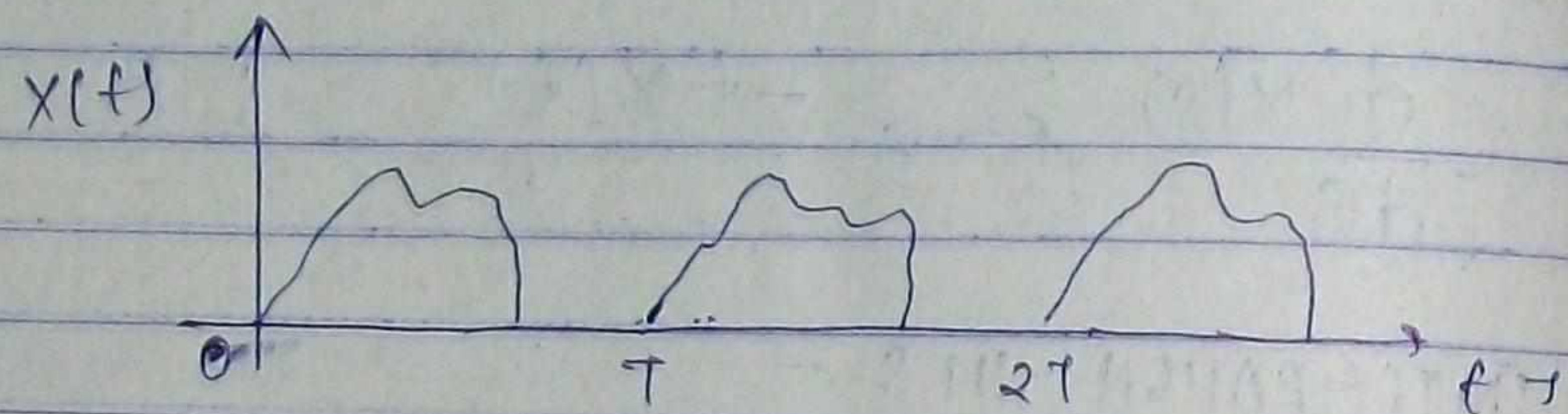
$$\lim_{t \rightarrow \infty} x(t) = x(\infty) = \lim_{s \rightarrow 0} sX(s)$$

⇒ LAPLACE TRANSFORM OF CAUSAL PERIODIC FUNCTION -

- L.T. of periodic signal does not exist.

- A signal that repeats from 0 to ∞ for every 'T' value is known as causal periodic function.

Let $x(t)$ be causal periodic



A causal periodic signal can be expressed as the linear combⁿ of the first cycle and delayed versions of the first cycle.

Let $x_1(t)$ be the first cycle of $x(t)$.

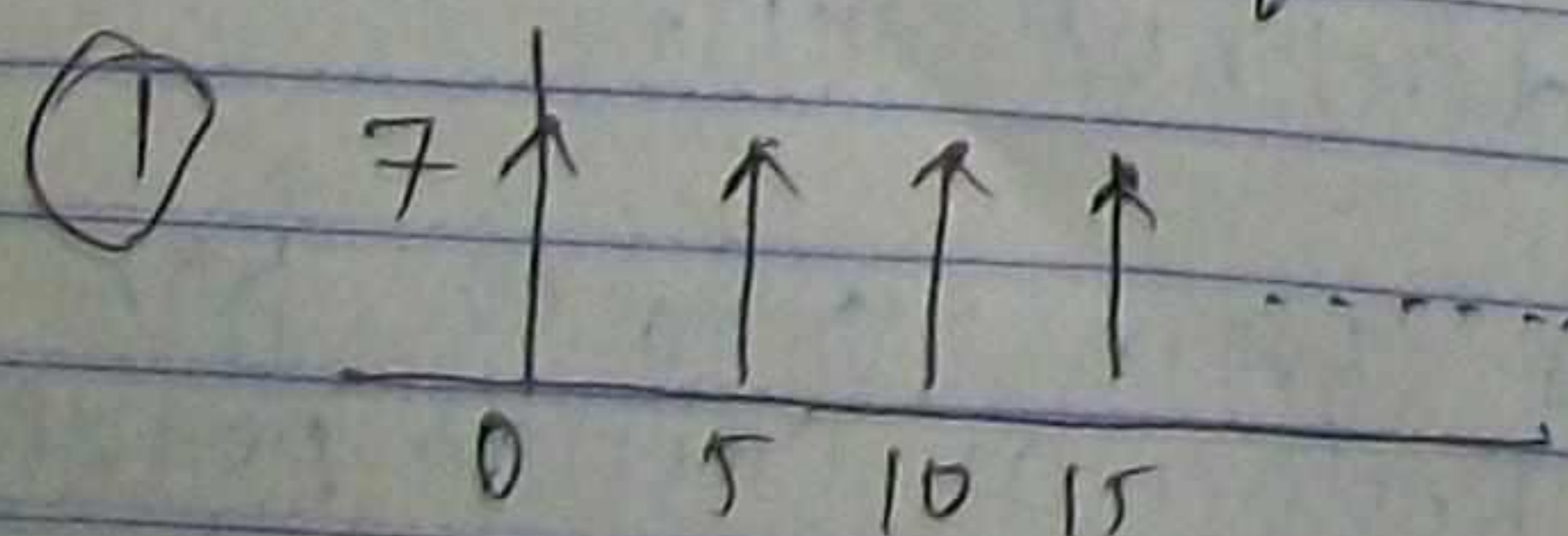
$$x(t) = x_1(t) + x_1(t-T) + x_1(t-2T) + \dots$$

$$X(s) = X_1(s) [1 + e^{-sT} + e^{-2sT} + \dots]$$

$$X(s) = X_1(s) \cdot \frac{1}{1 - e^{-sT}}, \quad |e^{-sT}| < 1$$

$$X(s) = \frac{X_1(s)}{1 - e^{-sT}}, \quad |e^{-sT}| < 1$$

Q. Find the L.T of following:

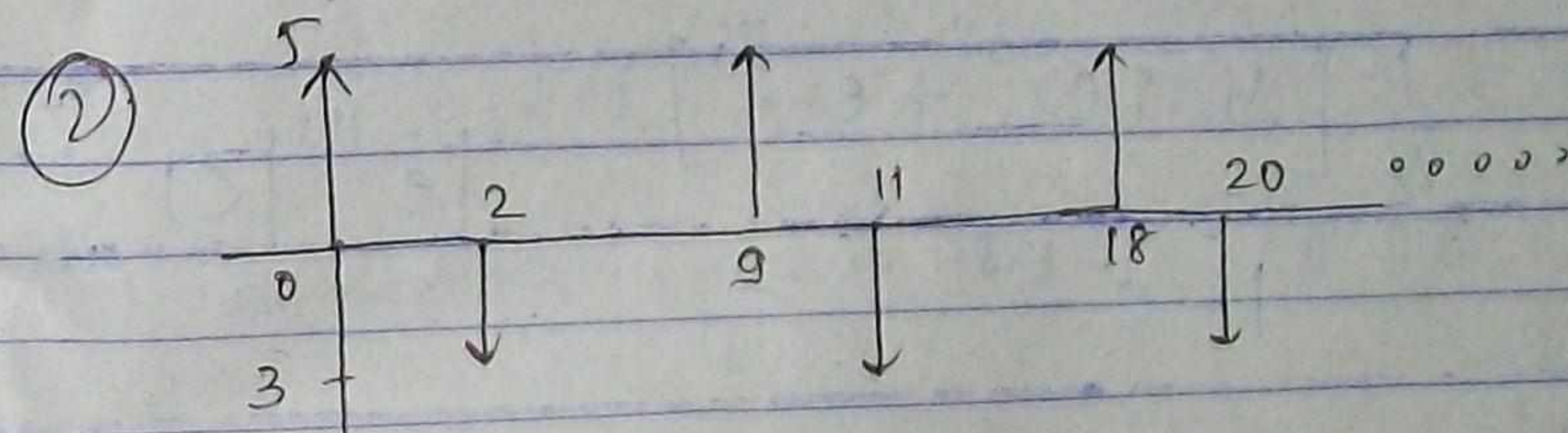


$$T = 5$$

Solⁿ: $x(t) = 7\delta(t) + 7\delta(t-5) + 7\delta(t-10) + \dots$

$$x(t) = 7\delta(t) [1 + e^{-s5} + e^{-10s} + e^{-15s} + \dots]$$

$$X(s) = 7\delta(t) \cdot \frac{1}{1 - e^{-5s}} = \frac{7}{1 - e^{-5s}}, \quad |e^{-5s}| < 1$$



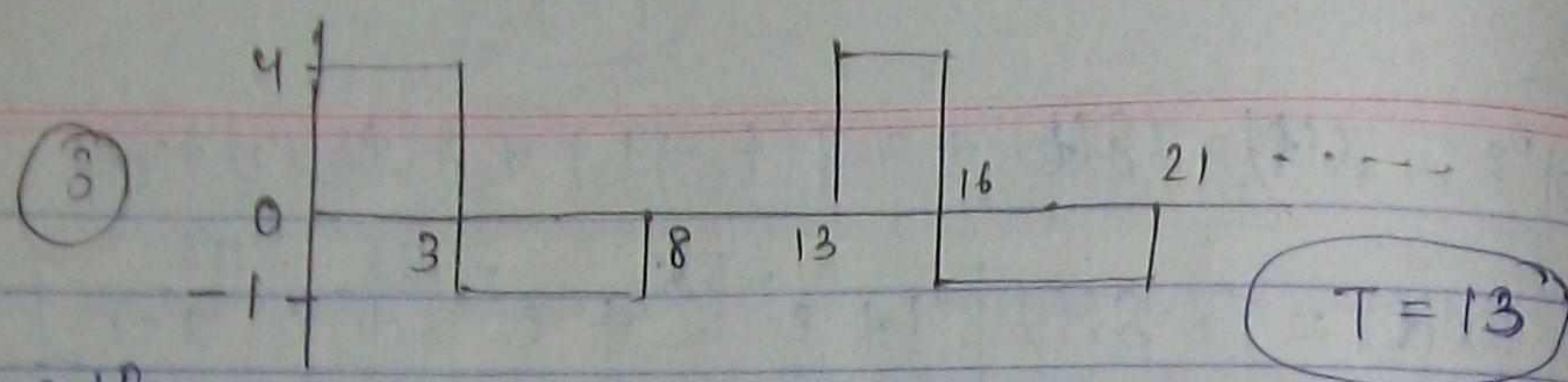
$$x(t) = 5\delta(t) [1 + e^{-9s} + e^{-18s} + \dots]$$

$$- 3\delta(t-2) [1 + e^{-9s} + e^{-18s} + \dots]$$

$$= [5\delta(t) - 3\delta(t-2)] [1 + e^{-9s} + e^{-18s} + \dots]$$

$$= [5\delta(t) - 3\delta(t-2)] [1 + e^{-9s} + e^{-18s} + \dots]$$

$$= \frac{5 - 3e^{-2s}}{1 - e^{-9s}}, \quad |e^{-9s}| < 1$$

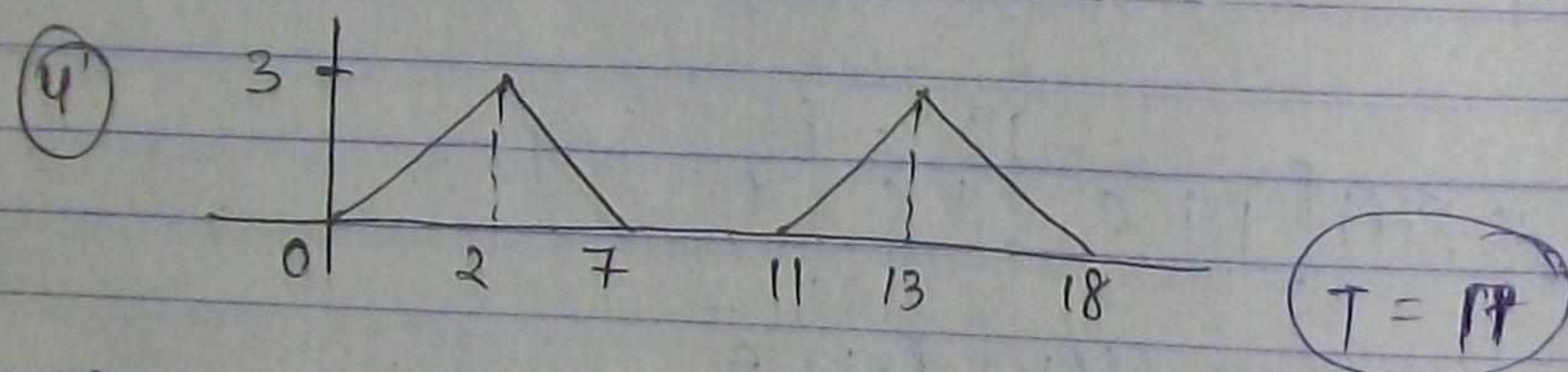


Solⁿ:

$$x_1(t) = 4u(t) - 5u(t-3) + u(t+8)$$

$$X(s) = \frac{1}{s} [4 - 5e^{-3s} + e^{-8s}]$$

$$X(s) = \frac{1}{s} [4 - 5e^{-3s} + e^{-8s}] \quad |e^{-13s}| < 1$$



Solⁿ:

$$x_1(t) = \frac{3}{2}r(t) - \frac{3}{2}r(t-2) + \frac{3}{5}r(t+7) - \frac{3}{5}r(t-2)$$

$x_1(s)$

$$x_1(t) = \frac{3}{2}r(t) - \frac{21}{10}r(t-2) + \frac{3}{5}r(t+7)$$

$$X_1(s) = \frac{1}{25} \left(\frac{3}{2} - \frac{21}{10}e^{-2s} + \frac{3}{5}e^{7s} \right)$$

$$X(s) = \frac{X_1(s)}{1 - e^{-11s}} \quad |e^{-11s}| < 1$$

Ques find L.T. of the follⁿ:

① $\delta(-t) \leftrightarrow 1, \forall s$

② $\delta(-0.3t) \leftrightarrow 1/0.3, \forall s$

③ $e^{-3t}\delta(2-t) \leftrightarrow e^{-3(2)}\delta(2-t) \leftrightarrow e^{-6}\delta(t-2) = e^{-6}e^{-2s}$

④ $\delta(-2t+3) \leftrightarrow e^{-1.5s} \times 1/2. \quad (-\infty < s < \infty)$

⑤ $e^{-3t}u(t) \leftrightarrow \frac{1}{(s+3)}, s > -3$

⑥ $e^{-3t}u(-t) \leftrightarrow \frac{-1}{s+3}, s < -3$

⑦ $\delta(t+3) \rightarrow e^{3s}, s < \infty$

⑧ $\delta(t-1) + \delta(t+2) \rightarrow e^{-s} + e^{2s}, -\infty < s < \infty$

⑨ $e^{-3t}u(t) \rightarrow e^{-3t}u(t) + u(-t)$

$$= \frac{1}{s+3} - \frac{1}{s} \quad -3 < s < 0$$

$$(10) e^{-3|t|} \longleftrightarrow \frac{1}{s+3} - \frac{1}{s-3}, \quad -3 < s < 3$$

$$(11) \frac{-3|t-1|}{3} \longleftrightarrow e^{-s} \left[\frac{1}{s+3} - \frac{1}{s-3} \right], \quad -3 < s < 3$$

$$(12) e^{-3(t-1)} u(t) \longleftrightarrow e^3 e^{-3t} u(t) \longleftrightarrow e^3 \frac{1}{s+3}, \quad s > -3$$

$$(13) e^{-3(t-1)} u(t-1) \longleftrightarrow e^{-s} \cdot \frac{1}{s+3}, \quad s > -3$$

$$(14) e^{-3(t+1)} u(t+1) \longleftrightarrow e^s \frac{1}{s+3}, \quad -3 < s < \infty$$

* Shifting Right side signal to right side and left side signal to the left side does not change the ROC, but R-side signal to L-side and L-side signal to R-side changes the ROC.

$$(15) e^{-3t} u(t-1) \longleftrightarrow e^{-3} e^{-3(t-1)} u(t-1) \longleftrightarrow e^{-3} \cdot e^{-s} \cdot \frac{1}{s+3}, \quad s > -3$$

$$(16) \cos 3(t-2) \cdot u(t-2) = \cos 6 \cdot \cos 3(t-2) u(t-2) - \sin 6 \cdot \sin 3(t-2) u(t-2) = \cos 6 \cdot e^{-2s} \frac{s}{s^2+9} - \sin 6 e^{-2s} \frac{3}{s^2+9}, \quad s > 0$$

$$(16) \cos(3t) u(t-2) \longleftrightarrow \cos 6 \cdot \cos(3t) u(t) + \sin 6 \cdot \sin(3t) u(t) \longleftrightarrow \cos 6 \cdot \frac{s}{s^2+9} + \sin 6 \cdot \frac{3}{s^2+9}, \quad s > 0$$

$$(17) t^2 u(t-1) \longleftrightarrow (t-1+1)^2 u(t-1) \longleftrightarrow [(t-1)^2 + 1 + 2(t-1)] u(t-1) \rightarrow (t-1)^2 u(t-1) + u(t-1) + 2(t-1) u(t-1) \rightarrow e^{-s} \cdot \frac{2}{s^3} + e^{-s} \cdot \frac{1}{s} + 2 \cdot e^{-s} \cdot \frac{1}{s^2}, \quad s > 0$$

$$= \frac{1}{s^2} \left[s e^{-s} + 2 e^{-s} + \frac{2 e^{-s}}{s} \right], \quad (s > 0)$$

Ques. find the Laplace of the bell's

$$x(t) \xrightarrow{L.T.} \frac{5}{s+3}, \quad s > -3$$

$$(18) x(t-2) \longleftrightarrow e^{-2s} \frac{5}{s+3}, \quad s > -3$$

$$\frac{u}{v} = \frac{v \frac{du}{ds} + u \frac{dv}{ds}}{v^2}$$

$$(2) \quad x(-t) \longleftrightarrow \frac{s}{3-s} \quad (-s > -3) \quad (s < 3)$$

$$(3) \quad x(-0.5t) \longleftrightarrow \frac{1}{0.5} \cdot \frac{s}{\left(\frac{s}{-0.5}\right) + 3} \quad \frac{s}{-0.5} > -3 \quad (s < 1.5)$$

$$(4) \quad e^{-2t} x(t) \longrightarrow \frac{s}{(s+2)+3} \quad (s+2) > -3 \quad (s > -5)$$

$$(5) \quad x(2t-3) \longleftrightarrow e^{-1.5s} \cdot \frac{1}{2} \cdot \frac{s}{(s/2)+3} \quad \frac{s}{2} > -3 \quad (s > -6)$$

$$(6) \quad t x(t) \longleftrightarrow -\frac{d}{ds} \frac{s}{s+3} = -\frac{s+1}{(s+3)^2} = \frac{s}{(s+3)^2} \quad (s > -3)$$

$$(7) \quad x(t+2) \longrightarrow e^{2s} x(s), \quad (-3 < s < \infty)$$

Ques. find the Laplace transform of

18/11/11

Ques. find the I.L.T of the following:

$$(1) \quad x(s) = \frac{3}{s+2} \quad \text{for} \quad (a) \quad s > -2 \quad (b) \quad s < -2.$$

Soln: (1) $s > -2$ (2) $s < -2$

$$x(t) = \frac{-2t}{3e} u(t) \quad x(t) = -3e^{-2t} u(-t)$$

$$(2) \quad x(s) = \frac{s}{s+3} \quad \text{for} \quad (a) \quad s > -3 \quad (b) \quad s < -3$$

Soln: $x(s) = 1 - \frac{3}{s+3}$

$$x(t) = \delta(t) - 3e^{-3t} u(t) \quad s > -3$$

$$x(t) = \delta(t) + 3e^{-3t} u(-t) \quad s < -3$$

$$(3) \quad x(s) = \frac{s^2}{s^2+9}$$

$$x(t) = \frac{s^2+9}{s^2+9} - \frac{9}{s^2+9}$$

$$x(t) = \delta(t) - 9 \sin 3t u(t)$$

$$(4) \quad X(s) = \frac{5s - 8}{s^2 + 5s + 6}$$

$$a) \quad s > -2$$

$$b) \quad s < -3$$

$$c) \quad -3 < s < -2$$

$$\text{soln: } X(s) = \frac{A}{(s+3)} + \frac{B}{(s+2)}$$

$$X(s) = \frac{+23}{(s+3)} + \frac{-18}{(s+2)}$$

$$X(t) = 23e^{-3t} u(t) - 18e^{-2t} u(t) \quad (s > -2)$$

$$X(t) = -23e^{-3t} u(-t) + 18e^{-2t} u(-t) \quad (s < -3)$$

$$X(t) = 23e^{-3t} u(t) + 18e^{-2t} u(-t) \quad (-3 < s < -2)$$

$$(5) \quad X(s) = \frac{s^2 + 15s + 13}{s^2 + 5s + 6}$$

$$a) \quad s > -2$$

$$b) \quad s < -3$$

$$c) \quad -3 < s < -2$$

$$= \frac{s^2 + 5s + 6}{(s^2 + 5s + 6)} + \frac{10s + 7}{(s^2 + 5s + 6)}$$

$$= 1 + \frac{10s + 7}{(s+3)(s+2)}$$

$$= 1 + \frac{+23}{(s+3)} + \frac{-13}{(s+2)}$$

$$x(t) = \delta(t) + 23e^{-3t} u(t) + 13e^{-2t} u(-t) \quad (s > -2)$$

$$X(t) = \delta(t) + 23e^{-3t} u(t) + 13e^{-2t} u(-t) \quad (s < -3)$$

$$X(t) = \delta(t) + 23e^{-3t} u(t) + 13e^{-2t} u(-t) \quad (-3 < s < -2)$$

Ques. Given that $e^{-2t} \cos(3t) u(t) \xleftrightarrow{\text{I.T.}} X(s)$
then find, I.T. of

$$(1) \quad X(s+2) \leftrightarrow e^{-2t} e^{-2t} \cos(3t) u(t)$$

$$(2) \quad e^{-3s} X(-s) \leftrightarrow e^{+2(t+3)} \cos(3(t+3)) u(t+3)$$

$$e^{-2(t+3)} \cos(3(-t+3)) u(-t+3)$$

$$(3) \quad X(-s) \leftrightarrow e^{+2t} \cos(3t) \cdot u(-t)$$

$$(4) \quad X(-0.3s) \leftrightarrow \frac{1}{0.3} e^{2t/0.3} \cos\left(\frac{3 \cdot t}{0.3}\right) u\left(\frac{-t}{0.3}\right)$$

$$(5) \quad X(2s-3) \leftrightarrow e^{-1.5t} e^{-2t} \cos(3t) u(t) \leftrightarrow \frac{1}{2} e^{-1.5t} e^{-\frac{2t}{2}} \cos\left(\frac{3t}{2}\right) u\left(\frac{t}{2}\right)$$

$$\frac{d}{dt} x(t) = -s x(t)$$

$$(5) X(s)(s-2) \leftrightarrow sX(s) - 2X(s)$$

$$\leftrightarrow + \frac{d}{dt} e^{-2t} \cos 3t u(t) - 2 e^{-2t} \cos 3t u(t)$$

$$\leftrightarrow -2 e^{-2t} \cos 3t u(t) + (-3 \sin 3t) e^{-2t} u(t)$$

$$+ \underbrace{e^{-2t} \cos(3t) \delta(t)}_{\delta(t)} - 2 e^{-2t} \cos 3t u(t)$$

$$\leftrightarrow -4 e^{-2t} \cos(3t) u(t) - 3 e^{-2t} \sin 3t u(t) + \delta(t)$$

Z-TRANSFORM

ROC =
circle with
centre origin

- It is a counterpart of Laplace transform which is used to represent the discrete time signals in the frequency domain.

$$\mathcal{Z}\{x(n)\} = X(z)$$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

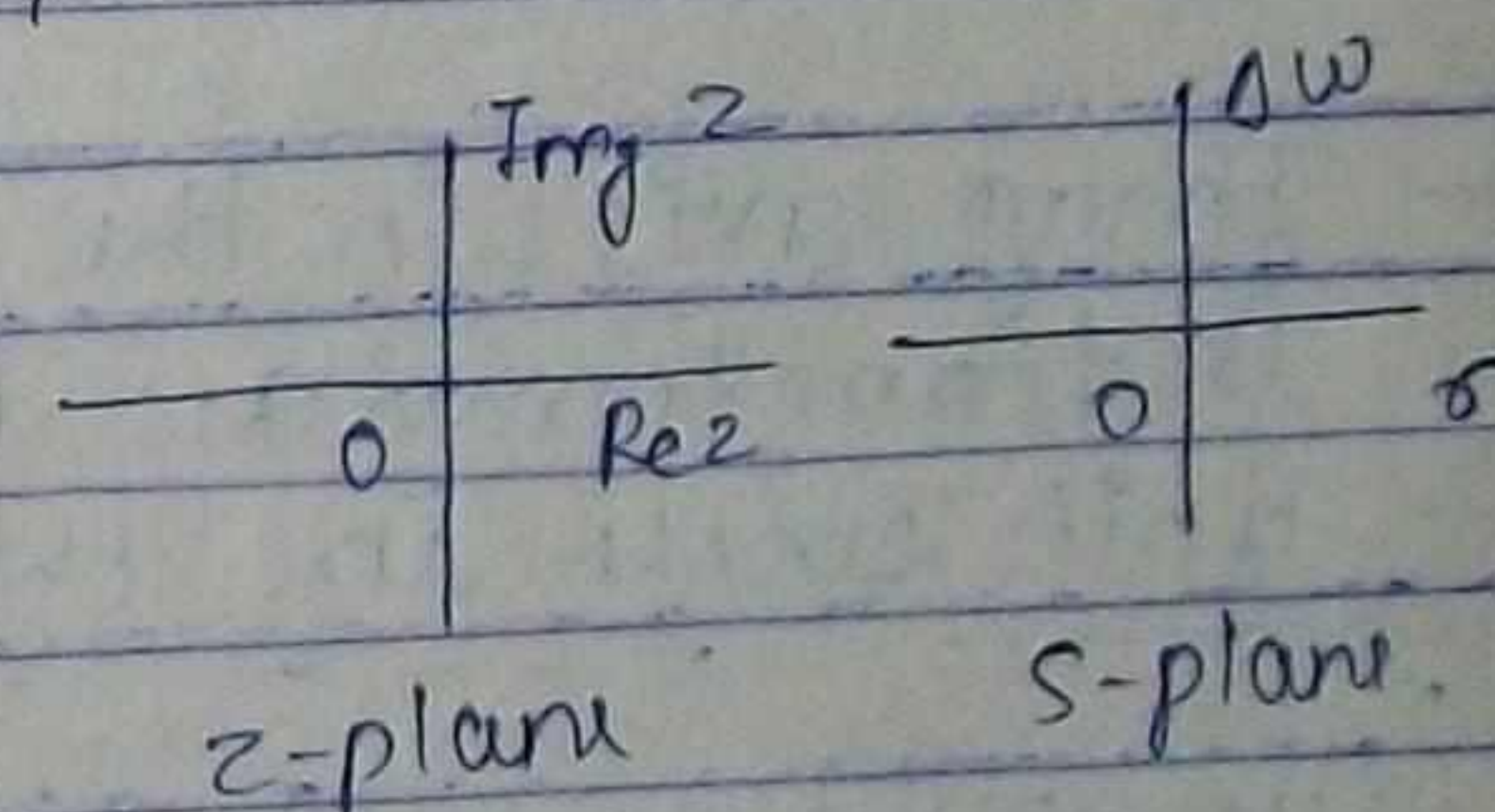
BILATERAL
Z.T.

$$= \sum_{n=0}^{\infty} x(n) z^{-n}$$

UNILATERAL
Z.T.

$\Rightarrow$ RELⁿ B/w ANALOG PLANE & DIGITAL PLANE -

s-plane $\rightarrow$ analog
z-plane $\rightarrow$ digital



- Locus of z is a circle of radius $|z|$
since

$$z = |z| e^{jkz} \quad \text{--- (1)}$$

$$z = e^s$$

$$|z| e^{j\omega} = e^{\sigma} e^{j\omega}$$

$$|z| = e^{\sigma} \quad - (2)$$

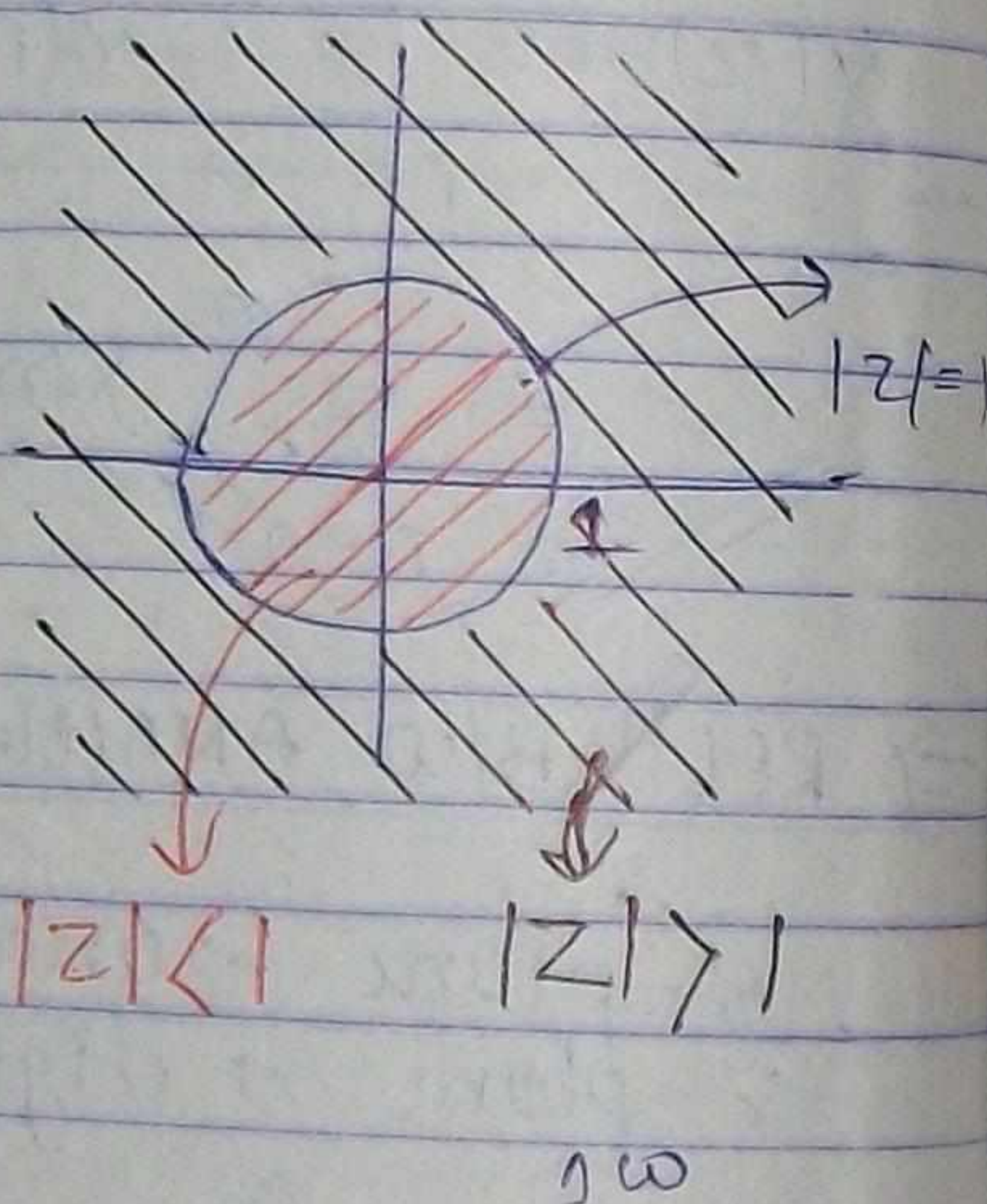
$$\Omega = \omega \quad - (3)$$

- Eqn (2) relates analog poles to digital poles
- Eqn (3) relates analog freq to digital freq.

$$\text{for } \sigma = 0 \Rightarrow |z| = 1$$

$$\sigma < 0 \Rightarrow |z| < 1$$

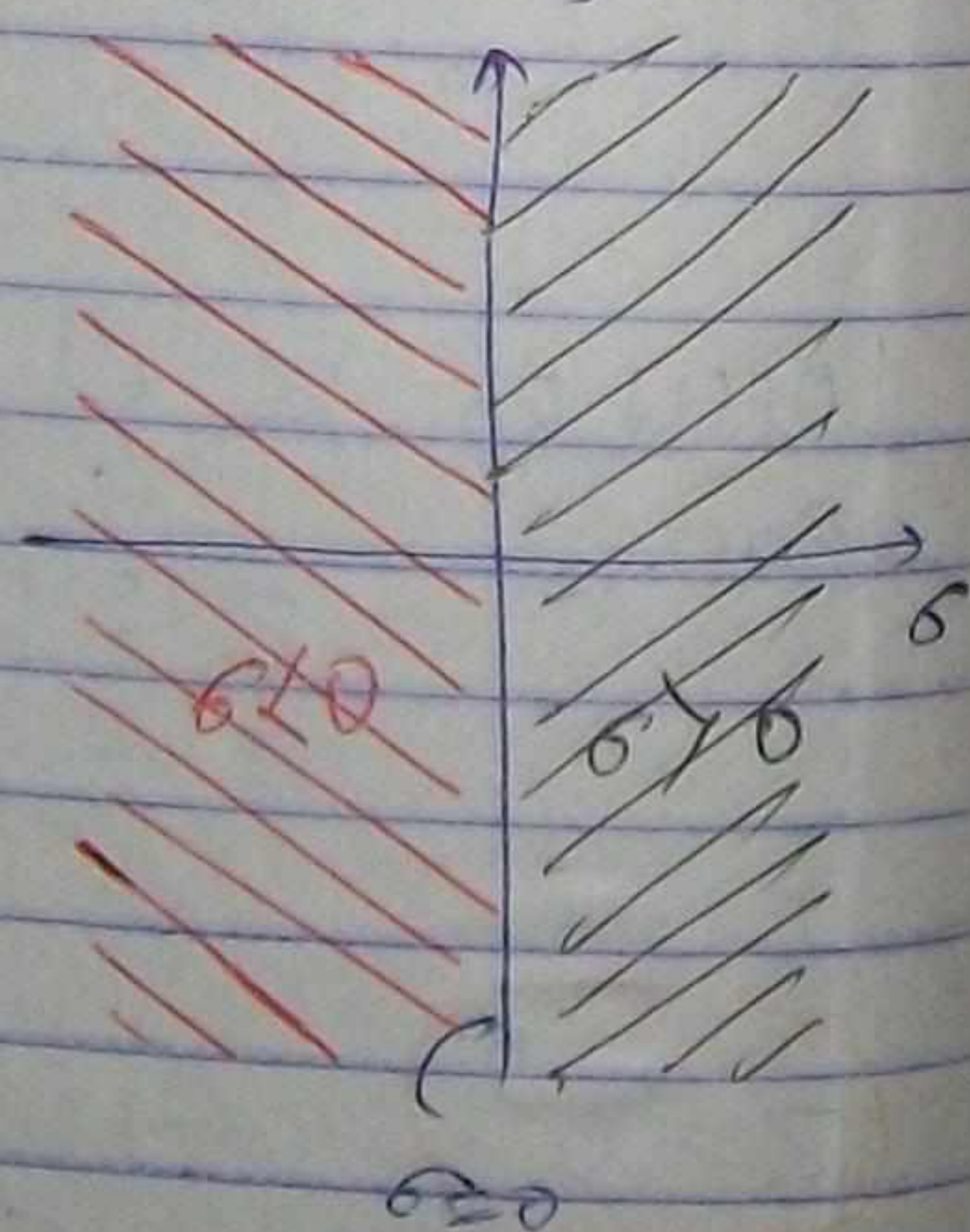
$$\sigma > 0 \Rightarrow |z| > 1$$



- Imag. axis in the s-plane is transformed as the unit circle in the z-plane

- LHS of s-plane is transformed as the interior of the unit circle in z-plane

- RHS of s-plane is transformed as the exterior of the unit circle.



- A causal digital system is stable iff all the poles are in the interior of the unit circle.

If one of the pole is on the perimeter of the unit circle then it is marginally stable.

If there are repeated pole on the perimeter of the unit circle then it is unstable.

If a pole lies in the exterior of the unit circle then it is unstable.

- Stability of anticausal system is opposite to that of the causal system.

- ⊛ In general a D.T. system is stable if it ROC includes unit circle otherwise it is unstable.

- Stability of DT system can completely be determined from the unit circle.

⇒ Z-TRANSFORM OF STANDARD PAIR -

$$(1) x(n) = A \delta(n)$$

$$= A \sum_{n=-\infty}^{\infty} \delta(n) \cdot z^{-n} = A z^{-0} = A$$

$$(2) \quad x(n) = \{1, 2, 3, 4\}$$

Solⁿ: limit $\rightarrow (0, 3)$

here called finite time right side signal

$$X(z) = \sum_{n=0}^3 x(n) z^{-n}$$

$$= x(0) z^{-0} + x(1) z^{-1} + x(2) z^{-2} + x(3) z^{-3}$$

$$= 1 + 2z^{-1} + 3z^{-2} + 4z^{-3}$$

$\forall z$ except zero

$$(or) \quad |z| > 0$$

$$(3) \quad x(n) = \{4, 3, 2, 1\}$$

$$X(z) = \sum_{n=-3}^0 x(n) z^{-n}$$

$$= x(-3) z^3 + x(-2) z^2 + x(-1) z^1 + x(0) z^0$$

$$= 4z^3 + 3z^2 + 2z^1 + 1$$

$$-\infty < z < \infty$$

$$|z| < \infty$$

finite time two sided signal

$$(4) \quad x(n) = \{4, 3, 2, 1, 2, 3, 4\}$$

$$X(z) = \sum_{n=-3}^3 x(n) z^{-n}$$

$$= 4z^3 + 3z^2 + 2z^1 + 1 + 2z^{-1} + 3z^{-2} + 4z^{-3}$$

$$0 < |z| < \infty$$

$$(5) \quad x(n) = a^n u(n) \rightarrow (0, \infty) \quad \text{ITRS}$$

$$X(z) = \sum_{n=0}^{\infty} a^n z^{-n}$$

$$= 1 + az^{-1} + (az^{-1})^2 + (az^{-1})^3 + \dots$$

$$= \frac{1}{1 - (az^{-1})}, \quad |az^{-1}| < 1$$

$$= \frac{z}{z-a}, \quad |z| > |a| \quad |z| > a$$

$$(6) \quad x(n) = -a^n u(-n) \rightarrow (-\infty, 0) \quad \text{ITLS}$$

$$X(z) = \sum_{n=-\infty}^0 a^n z^{-n}$$

$$= - \left[1 + a^{-1}z + (a^{-1}z)^2 + (a^{-1}z)^3 + \dots \right]$$

$$= - \left[\frac{1}{1 - a^{-1}z} \right], \quad |a^{-1}z| < 1$$

$$= \frac{a}{z-a}, \quad |z| < a$$

(7) $x(n) = -a^n u(-n-1) \rightarrow (-\infty, -1)$ ITCS

$$Y(z) = - \sum_{n=-\infty}^{-1} a^n z^{-n}$$

$$= - \left[a^{-1}z + (a^{-1}z)^2 + (a^{-1}z)^3 + \dots \right]$$

$$= -a^{-1}z \left[1 + a^{-1}z + (a^{-1}z)^2 + \dots \right]$$

$$= -a^{-1}z \frac{1}{1 - a^{-1}z}, \quad |a^{-1}z| < 1$$

$$= \frac{z}{z-a}, \quad |z| < a$$

$$\frac{z}{z-a} \begin{cases} |z| > a \rightarrow u(n) \\ |z| < a \rightarrow -u(-n-1) \end{cases}$$

$\Rightarrow$ Z-TRANSFORM TABLE -

(1) $A \delta(n) \longleftrightarrow A$

(2) $a^n u(n) \longleftrightarrow \frac{z}{z-a}, \quad |z| > a$

(3) $u(n) \longleftrightarrow \frac{z}{z-1}, \quad |z| > 1$

(4) $e^{j\omega_0 n} u(n) \longleftrightarrow \frac{z}{z - e^{j\omega_0}}, \quad |z| > 1$

(5) $e^{-j\omega_0 n} u(n) \longleftrightarrow \frac{z}{z - e^{-j\omega_0}}, \quad |z| > 1$

(6) $\cos(\omega_0 n) u(n) \longleftrightarrow \frac{z^2 - z \cos \omega_0}{z^2 - 2z \cos \omega_0 + 1}, \quad |z| > 1$

(7) $\sin(\omega_0 n) u(n) \longleftrightarrow \frac{z \cos \omega_0}{z^2 - 2z \cos \omega_0 + 1}, \quad |z| > 1$

(8) $-a^n u(-n-1) \longleftrightarrow \frac{a}{z-a}, \quad |z| < a$

(9) $-u(-n-1) \longleftrightarrow \frac{1}{z-1}, \quad |z| < 1$

$$(10) -a^n u(n-1) \leftrightarrow \frac{z}{z-a}, \quad |z| < a$$

$$(11) -u(n-1) \leftrightarrow \frac{z}{z-1}, \quad |z| < 1$$

$$\begin{aligned} (*) \quad a^{|n|} &= a^n u(n) + a^{-n} u(-n-1) \\ &= \frac{z}{z-a} + \frac{z}{z-1/a} \quad \left(a < |z| < \frac{1}{a} \right) \end{aligned}$$

valid for
 $|a| < 1$

⇒ ROC OF Z-TRANSFORM -

It is the range of value of z for which the z -transform is finite.

- ROC of z -transform is a circle in the z -plane centered about $z=0$ or origin.

⇒ PROPERTY OF ROC -

(1) ABOUT POLES -

ROC of z -transform does not include any poles but bounded by the poles because at poles z -transform becomes ∞ .

(2) ABOUT TIME LIMITED SIGNAL -

If $x(n)$ is time limited then its ROC will be entire z -plane except $z=0$ and $|z|=\infty$.

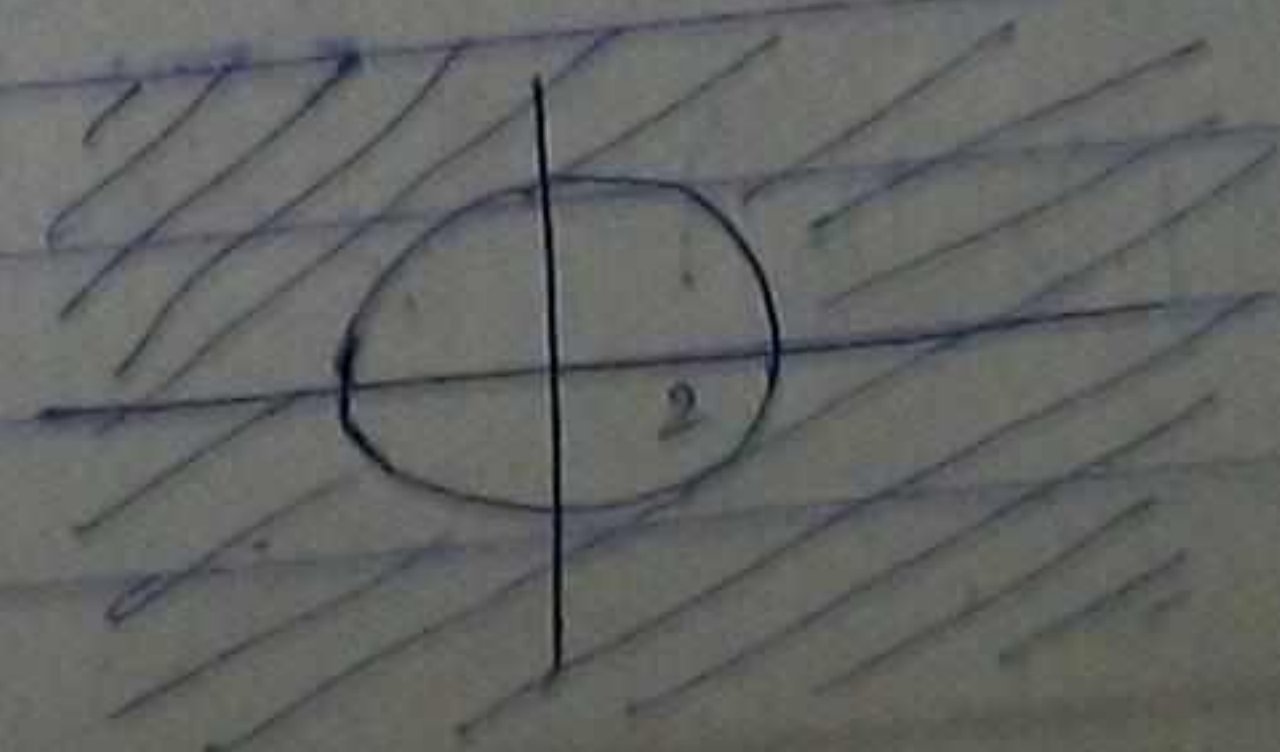
Ex - problem (1), (2), (3), (4)

(3) ABOUT RIGHT SIDED SIGNAL -

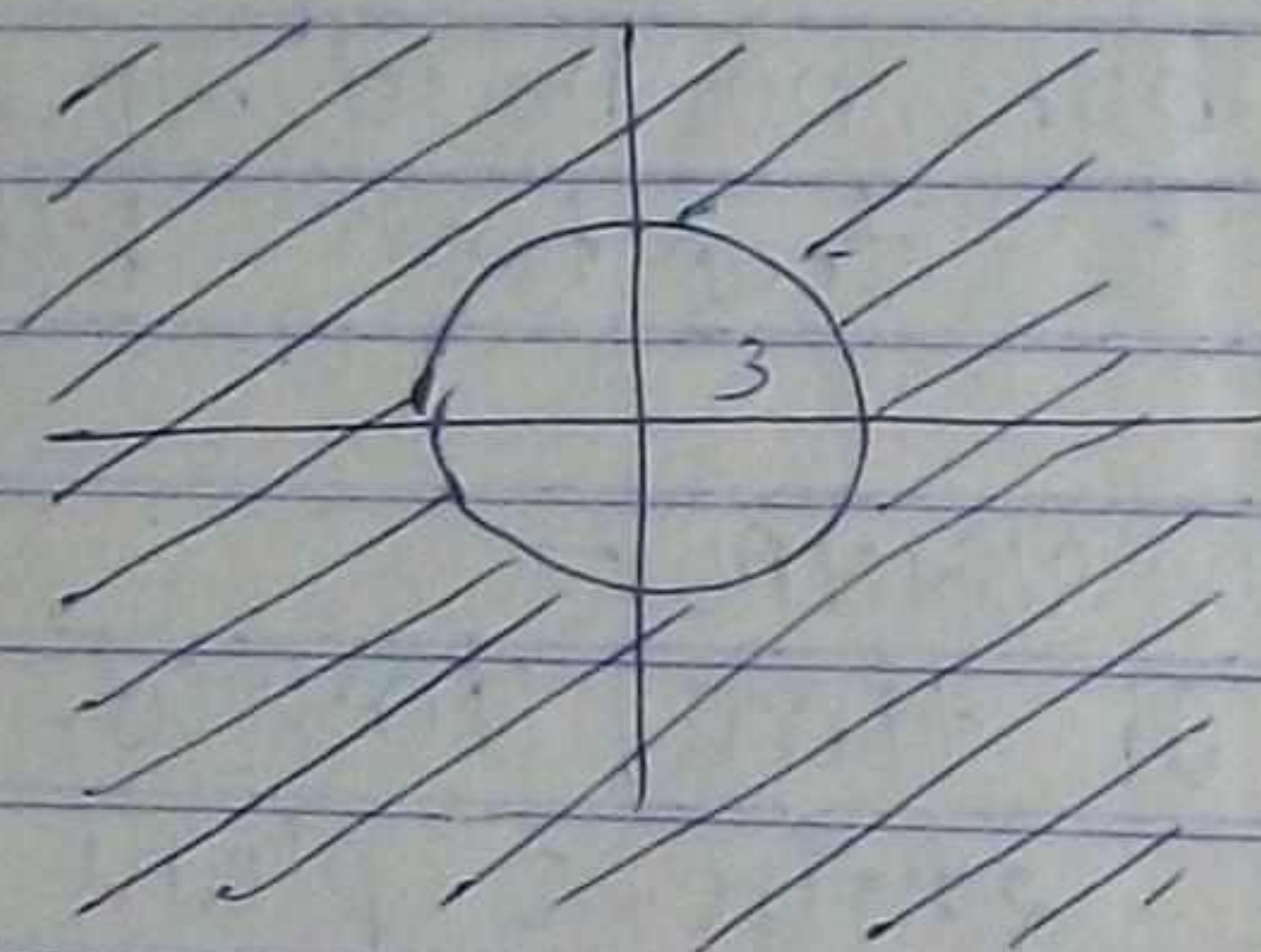
(A) If $x(n)$ is RSS then its ROC is of the form

ROC $\rightarrow |z| > a$ where a is the magnitude of the pole, i.e. it is outside the circle of radius ' a '. If a circle $|z| = z_0$ is in the ROC then all values of z for which $|z| > z_0$ will also be in the ROC of it.

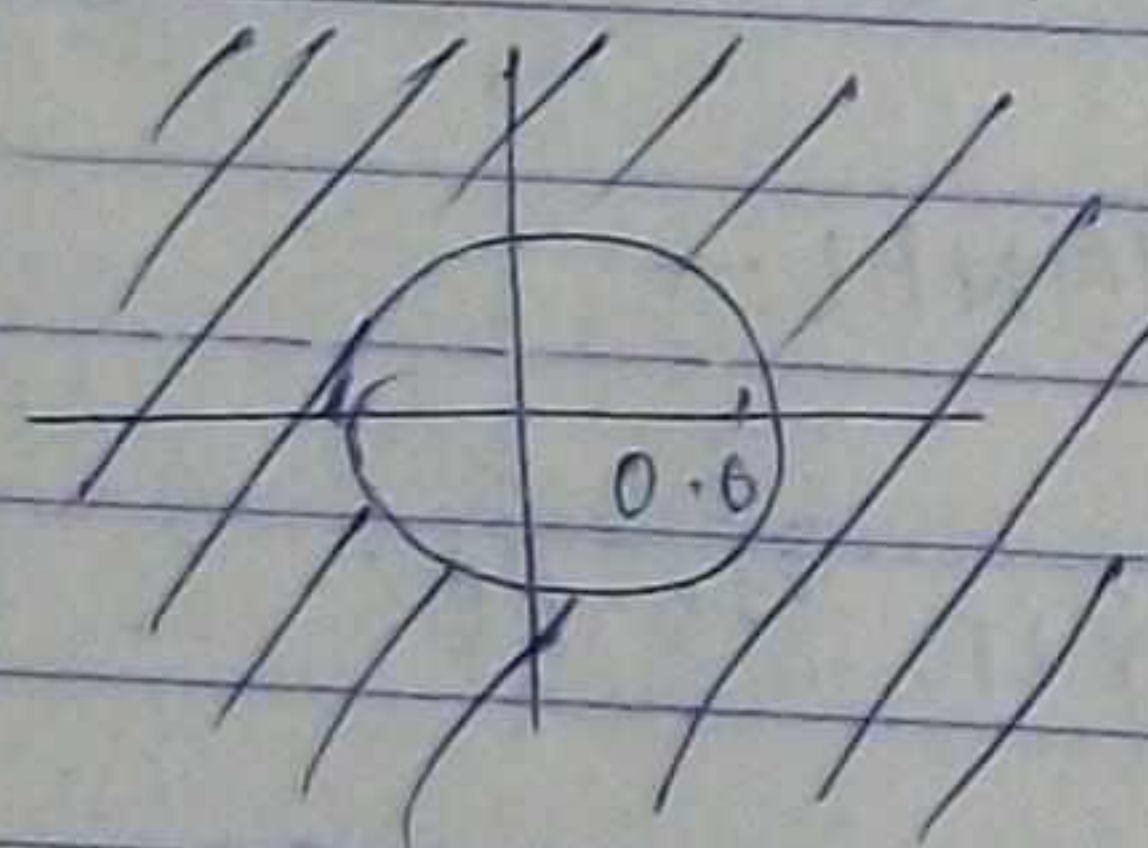
$$\text{Ex (1)} \quad z^n u(n) \leftrightarrow \frac{z}{z-2}, \quad |z| > 2$$



② $(-3)^n u(n) \leftrightarrow \frac{z}{z+3}, |z| > 3$



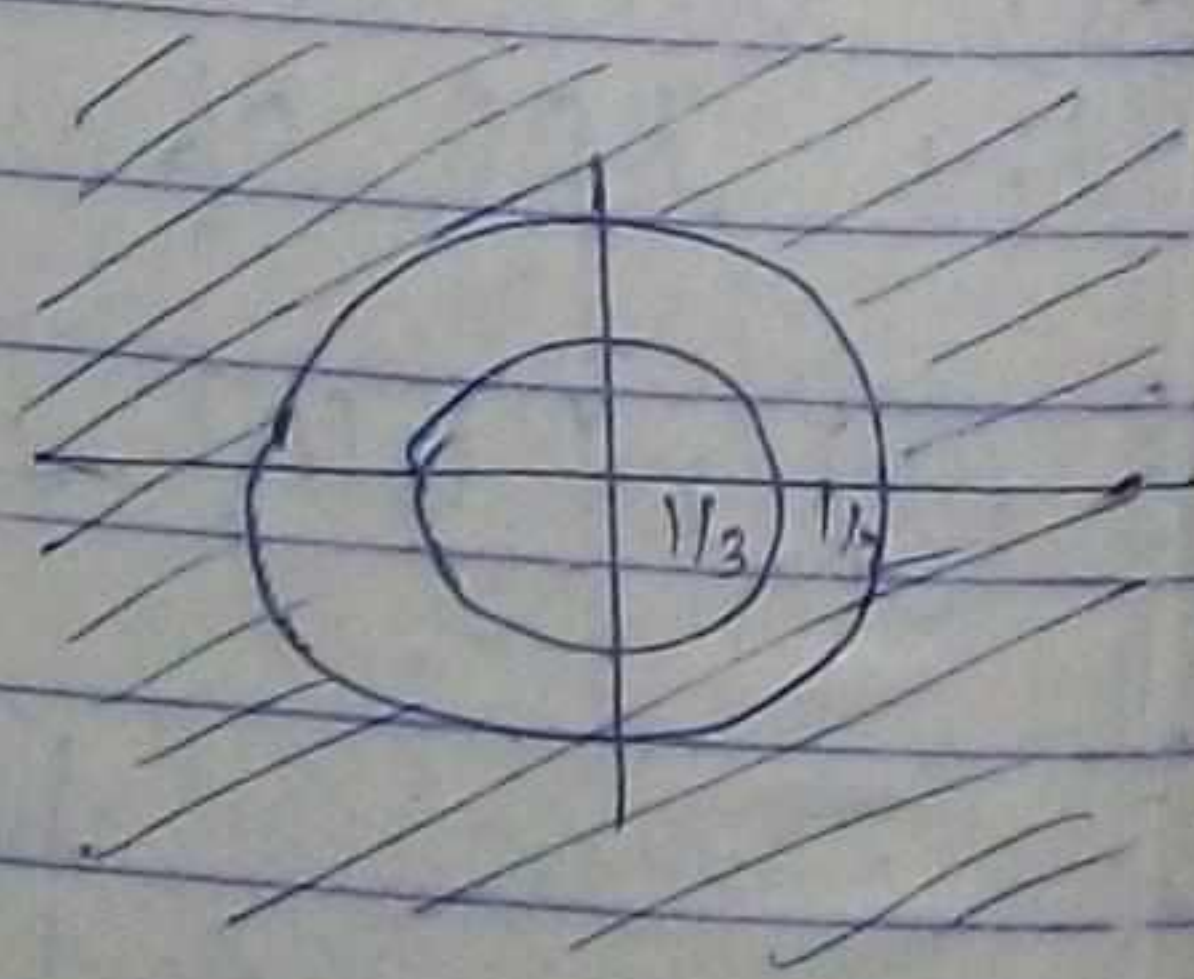
③ $(-\frac{3}{5})^n u(n) \leftrightarrow \frac{z}{z+3/5}, |z| > 3/5$



(B) If $x(n)$ is the combⁿ of Right side signal then its ROC will be $|z| > z_{max}$ i.e. outside the outermost circle.

Ex - $(\frac{1}{2})^n u(n) - (\frac{1}{3})^n u(n)$

$\frac{z}{z-1/2} - \frac{z}{z-1/3}$

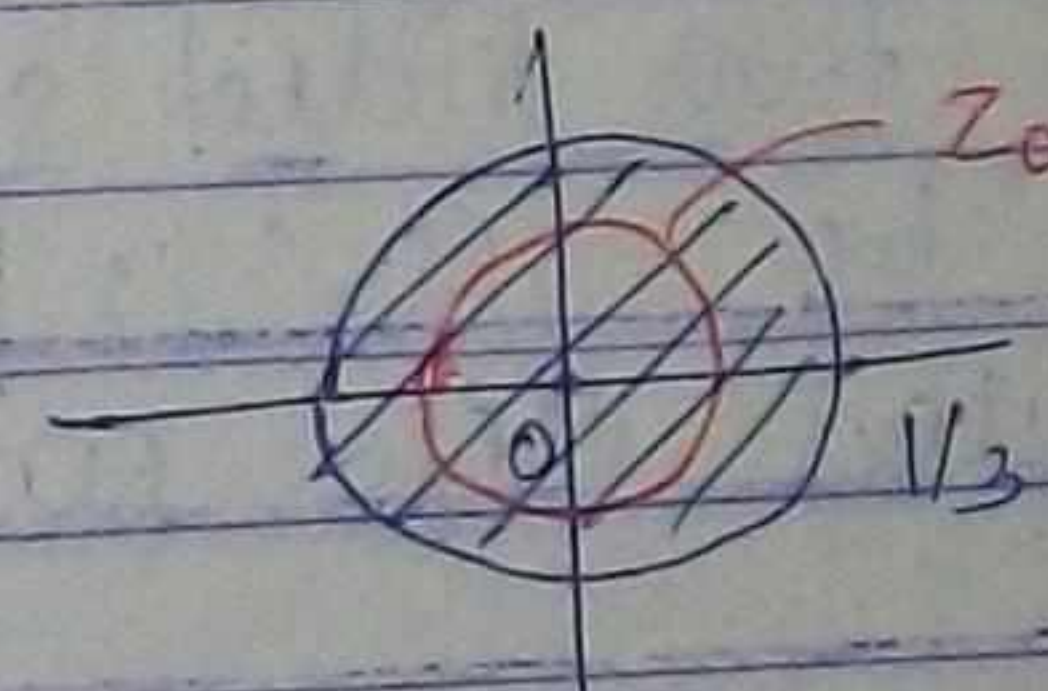


Common ROC $|z| > 1/2$ $|z| > 1/3$

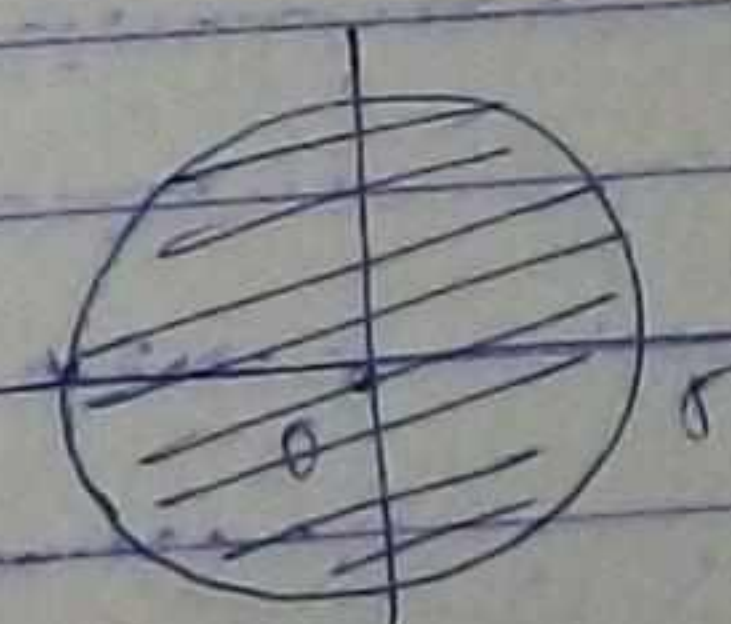
④ ABOUT LEFT SIDE SIGNAL -

(A) If $x(n)$ is LSS then its ROC is of the form $|z| < b$ where b is the magnitude of the pole i.e. it is the interior of the circle of radius 'b'. and if a circle $|z| = z_0$ is in the ROC then all values of z for which $|z| < z_0$ will also be in the ROC of it.

Ex - ① $(-\frac{1}{3})^n u(-n) \leftrightarrow -\frac{(-1/3)}{z+1/3}, |z| < 1/3$



② $(5)^n u(-n-1) \leftrightarrow \frac{z}{z-5}, |z| < 5$



(B) If $x(n)$ is the combination of left side signal then its ROC will be $|z| < z_{min}$ i.e. interior of inner most circle.

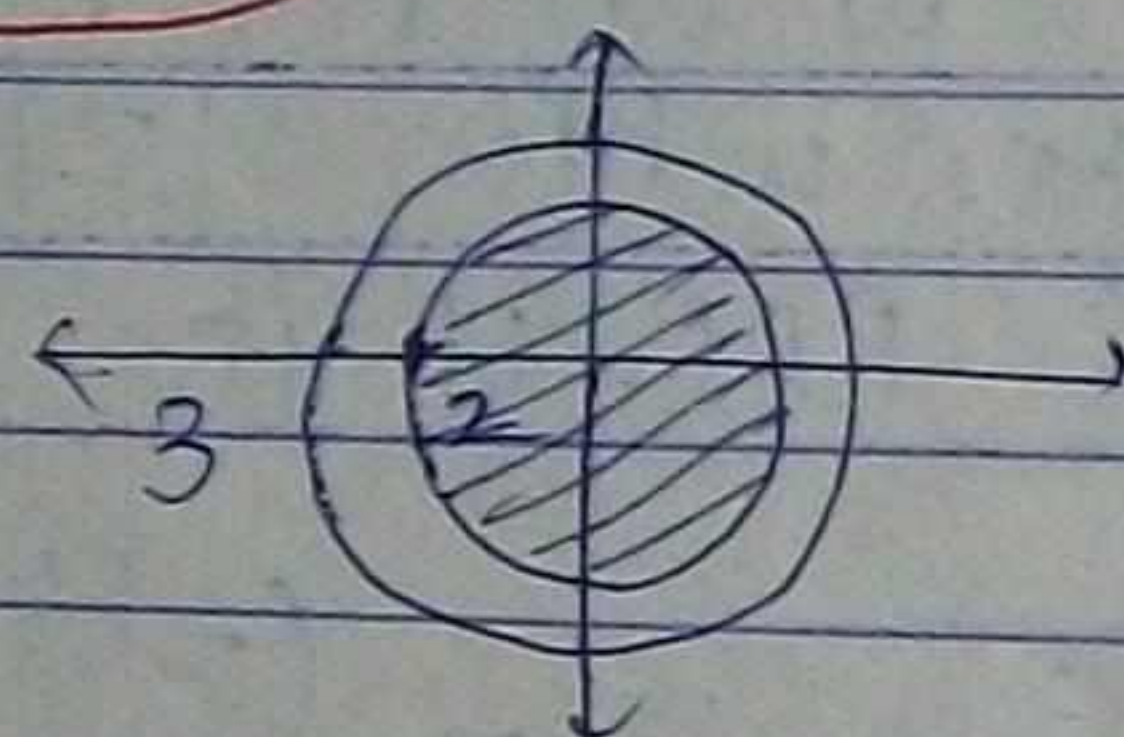
Ex $(-2)^n u(n-1) + 3^n u(-n)$

$\leftrightarrow \frac{-Z}{Z+2} - \frac{3}{Z-3}$

Common
ROC

$|Z| < 2$

$|Z| < 3$



⑤ ABOUT TWO SIDED SIGNALS -

If $x(n]$ is two sided signal then its ROC will be $a < |Z| < b$ i.e. it is the circular disc centered about $z=0$.

Ex - $2^n u(n) + 3^n u(-n-1)$

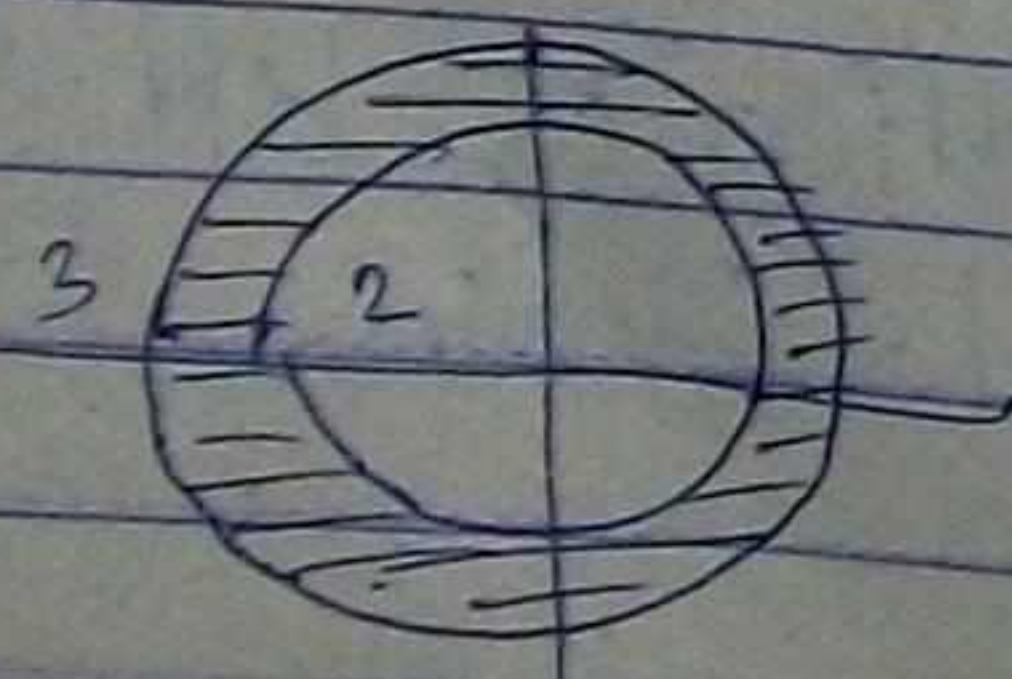
$\leftrightarrow \frac{Z}{Z-2} - \frac{Z}{Z-3}$

$|Z| > 2$

$|Z| < 3$

Common

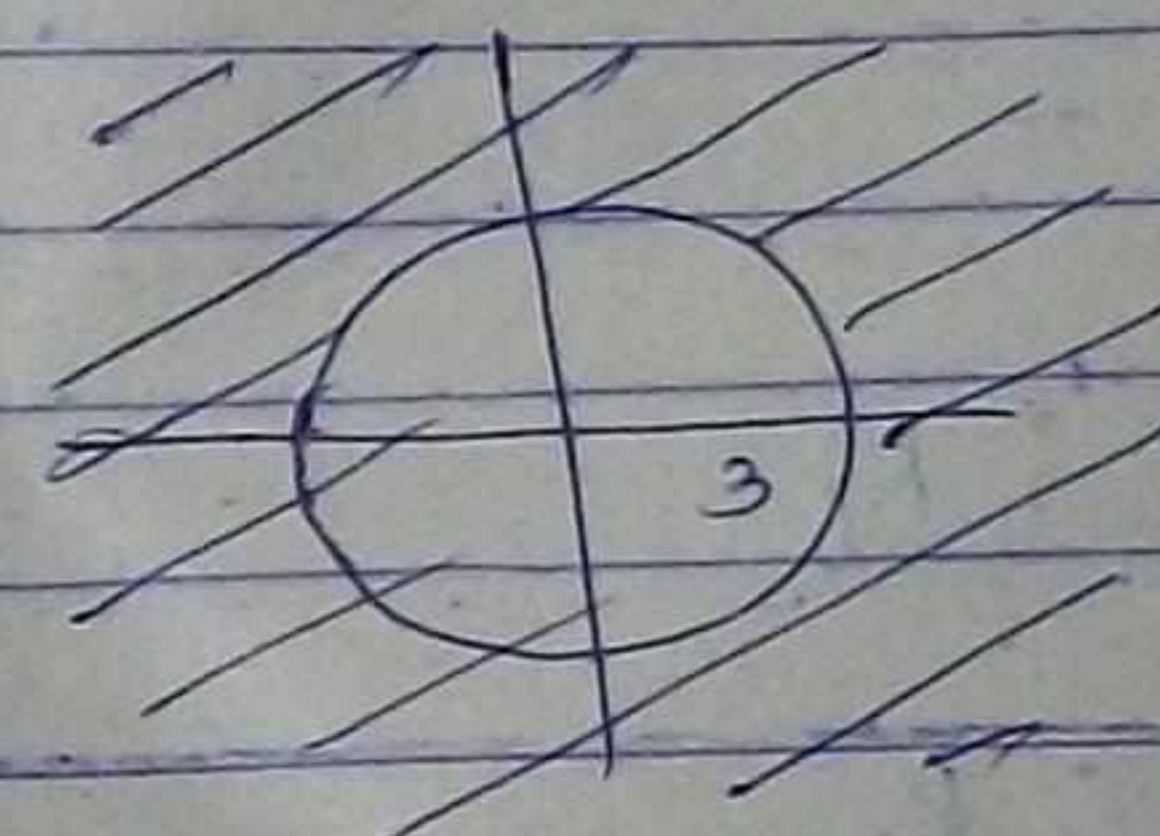
$2 < |Z| < 3$



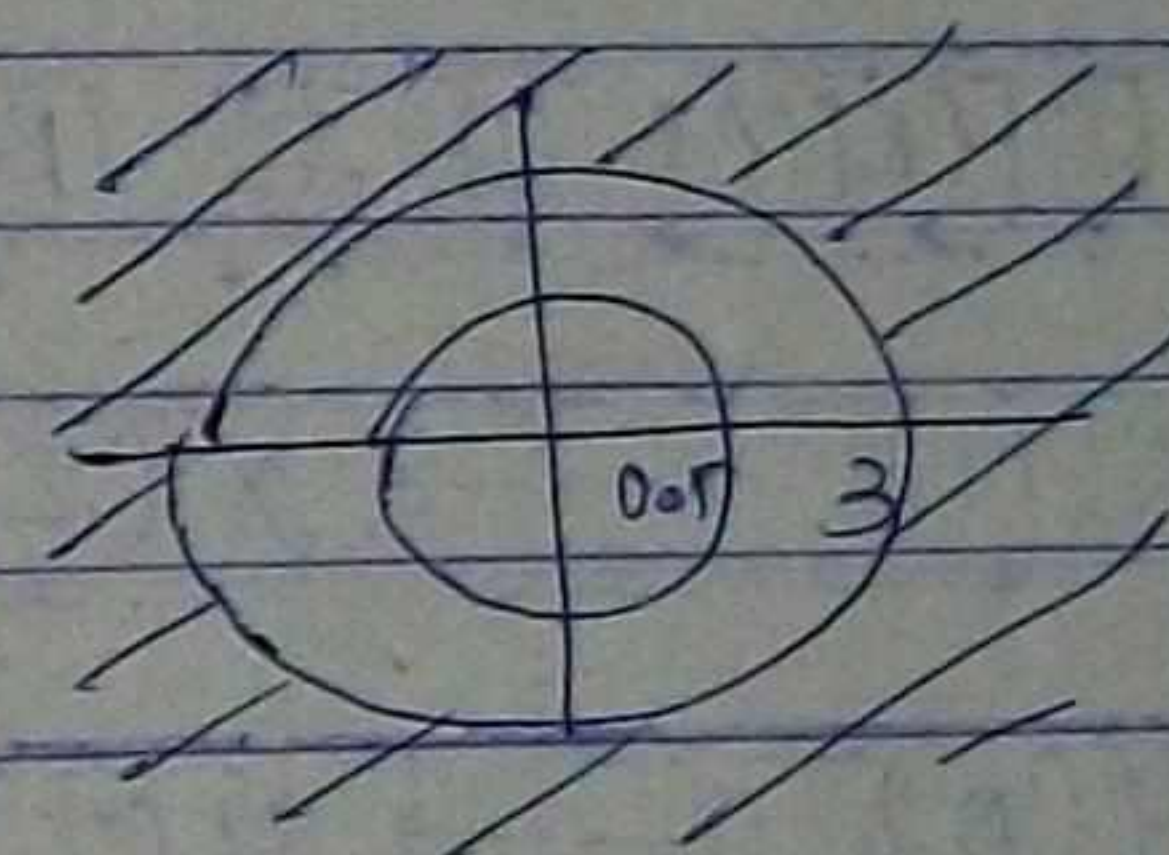
⑥ ABOUT STABILITY -

The given circle is stable if and only if its ROC include unit circle otherwise it is unstable.

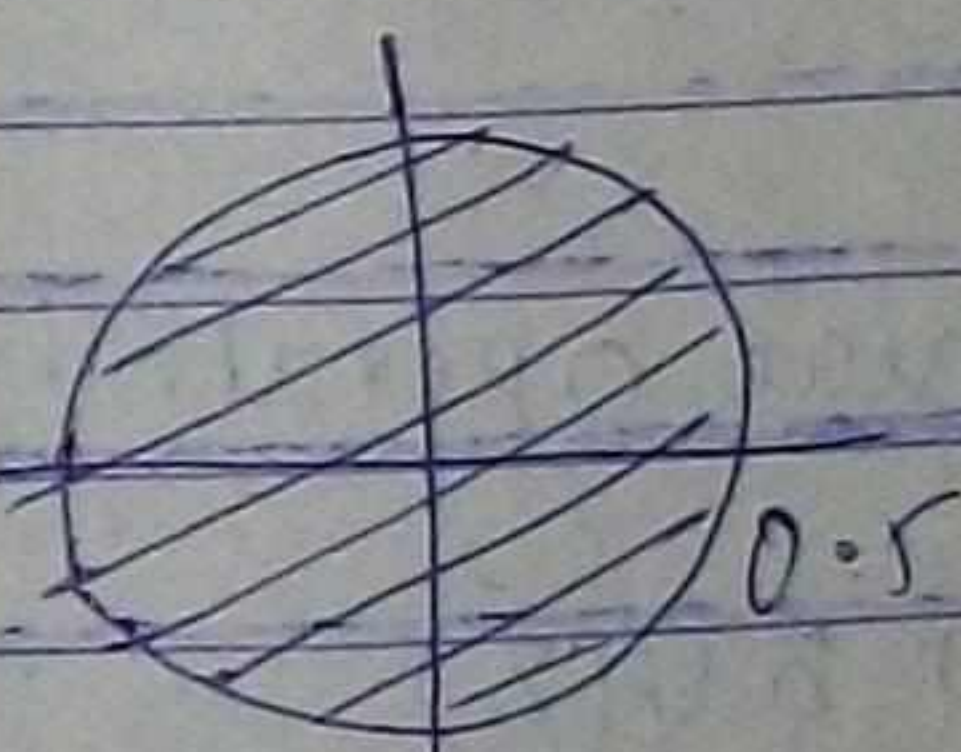
Ex



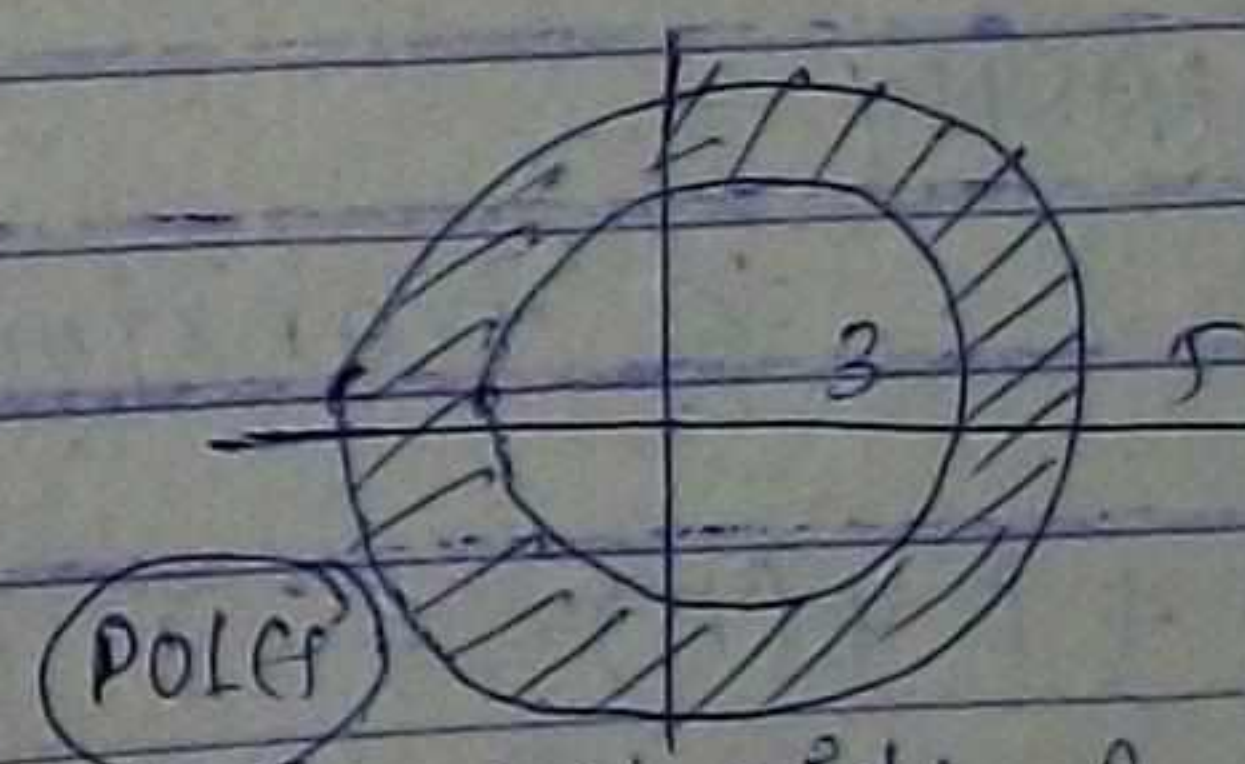
Right sided
unstable



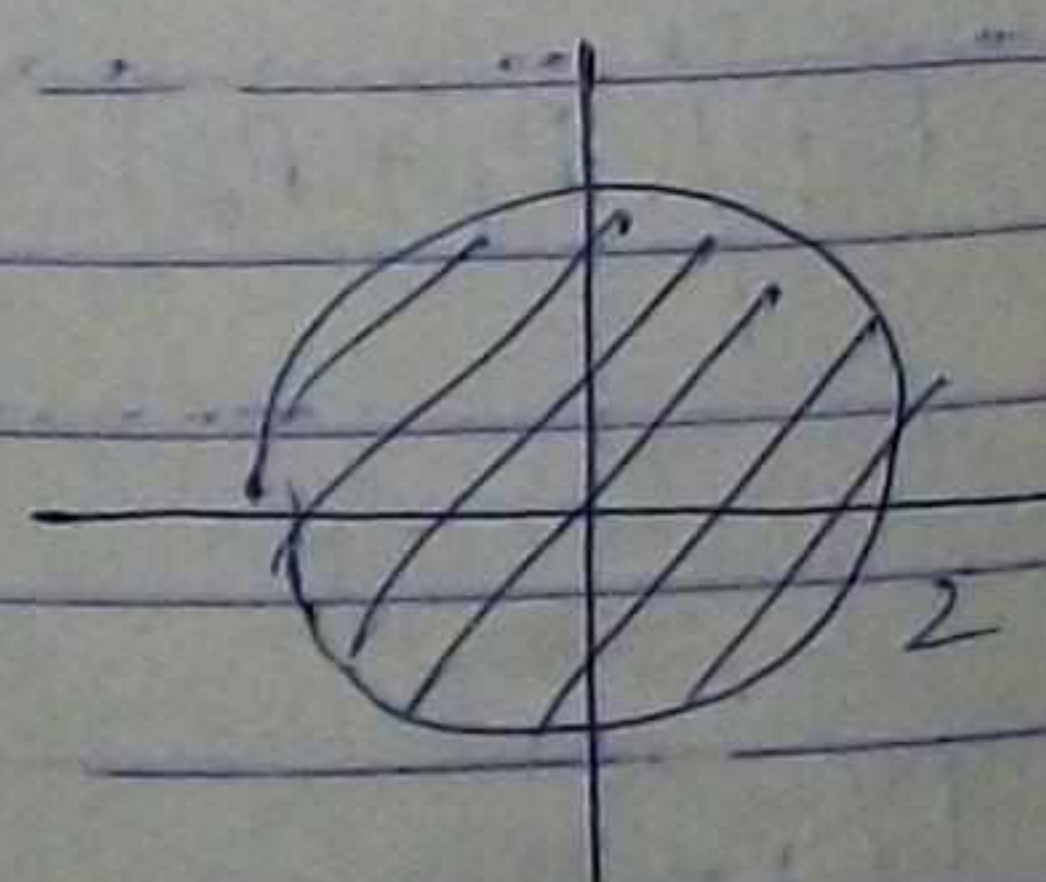
Two right sided
unstable signal.



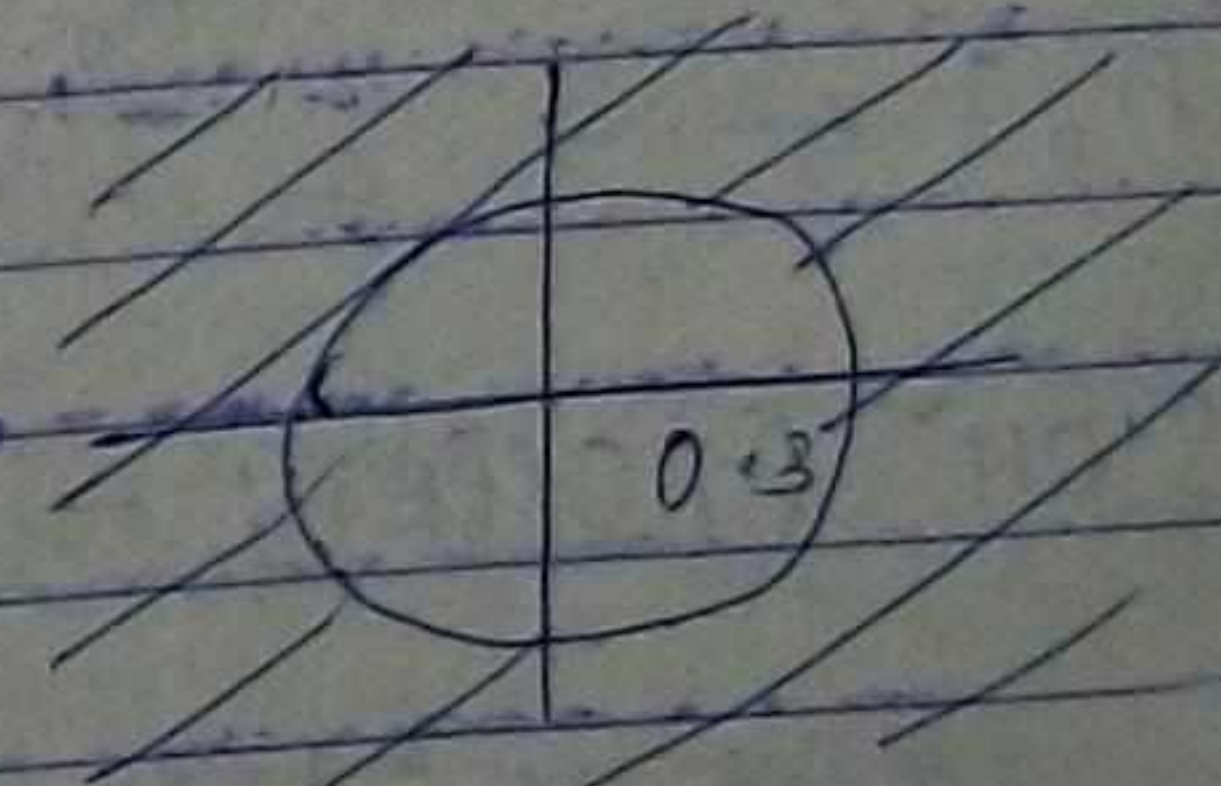
Left sided
unstable



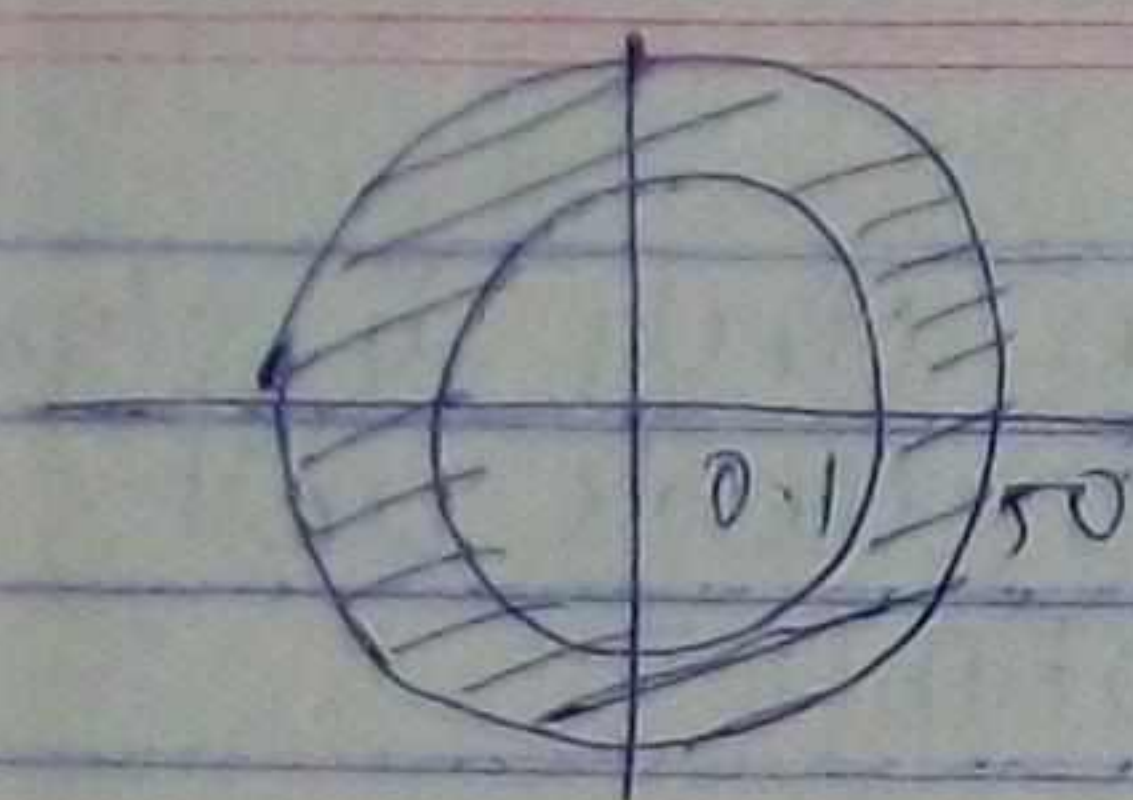
POLE
3 - left side } comb
5 - right side }
unstable system



Left sided
stable



Right sided
stable.



0.1 - Right sided } (Combⁿ)
 50 - Left sided }
 Stable

⇒ PROPERTIES OF Z-TRANSFORM -

$$x(n) \xleftrightarrow{ZT} X(z), \text{ ROC: } R_1$$

$$y(n) \xleftrightarrow{\quad} Y(z), \text{ ROC: } R_2$$

a, b, n_0 and z_0 are constant

① LINEARITY -

Z-transform is linear operation.

$$ax(n) + by(n) \xleftrightarrow{\quad} aX(z) + bY(z), \text{ } R_1 \cap R_2$$

② TIME SHIFTING -

$$x(n - n_0) \xleftrightarrow{\quad} z^{-n_0} X(z), \text{ } R_1 / (R_1 + R_0)$$

③ INVERSION PROPERTY -

$$x(-n) \xleftrightarrow{\quad} X\left(\frac{1}{z}\right), \text{ } 1/|z|$$

④ SCALING -

$$a^n x(n) \xleftrightarrow{\quad} X\left(\frac{z}{a}\right), \text{ } \frac{|z|}{a}$$

⑤ CONVOLUTION -

$$x(n) * y(n) \xleftrightarrow{\quad} X(z)Y(z), \text{ } R_1 \cap R_2$$

⑥ DIFF. IN TIME -

$$n x(n) \xleftrightarrow{\quad} -z \frac{d}{dz} X(z), \text{ } R_1$$

⑦ INITIAL VALUE THEOREM -

$$\lim_{n \rightarrow 0} x(n) = x(0) = x(0^+), \\ = \lim_{z \rightarrow \infty} x(z).$$

⑧ FINAL VALUE THEOREM -

for final value theorem
 poles must be interior of the unit circle

$$\lim_{n \rightarrow \infty} x(n) = x(\infty) = \lim_{z \rightarrow 1} (1 - z^{-1}) X(z)$$

Ques. find the Z-transform of :

$$(1) (-2)^n u(n) \longleftrightarrow \frac{z}{z+2}, |z| > 2$$

$$(2) \left(\frac{1}{2}\right)^n u(n) \longleftrightarrow \frac{z}{z-1/2}, |z| > 1/2$$

$$(3) \left(\frac{-1}{2}\right)^n u(n) \longleftrightarrow \frac{z}{z+1/2}, |z| > 1/2$$

$$(4) \left(\frac{1}{2}\right)^n u(n) \longleftrightarrow \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^n + \sum_{n=-\infty}^{-1} 1 z^n$$

$$\begin{aligned} \left(\frac{1}{2}\right)^n u(n) &= \left(\frac{1}{2}\right)^n u(n) + u(-n+1) \\ &= \frac{z}{z-1/2} - \frac{1}{z+1}, \quad \frac{1}{2} < |z| < 1 \end{aligned}$$

$$(5) \left(\frac{1}{2}\right)^{n+1} u(n) \longleftrightarrow \left(\frac{1}{2}\right)^{-1} \left(\frac{1}{2}\right)^n u(n)$$

$$\Rightarrow 2 \cdot \frac{z}{z-1/2}, |z| > 1/2$$

$$\Rightarrow \frac{2z}{z-1/2}, |z| > 1/2$$

$$(6) \left(\frac{1}{2}\right)^{n-1} u(n-1) \longleftrightarrow z^{-1} \cdot \frac{z}{z-1/2}, |z| > 1/2$$

$$(7) \left(\frac{1}{2}\right)^n u(n-1) \longleftrightarrow \left(\frac{1}{2}\right) \cdot \frac{z}{z-1/2} \cdot z^{-1}, |z| > 1/2$$

$$(8) \left(\frac{1}{2}\right)^{|n|} \longleftrightarrow \left(\frac{1}{2}\right)^n u(n) + \left(\frac{1}{2}\right)^{-n} u(-n)$$

$$\longleftrightarrow \frac{z}{z-1/2} - \frac{z}{z-2}, \quad \frac{1}{2} < |z| < 2$$

$$(9) \left(\frac{1}{2}\right)^{|n-1|} \longleftrightarrow z^{-1} \left[\frac{z}{z-1/2} - \frac{z}{z-2} \right], \quad \frac{1}{2} < |z| < 2$$

$$(10) \left(\frac{1}{2}\right)^n u(-n) \longleftrightarrow \frac{-1/2}{z-1/2}, |z| < 1/2$$

$$(11) \left(\frac{1}{2}\right)^n u(-n-1) \longleftrightarrow \frac{-z}{z-1/2}, |z| < 1/2$$

Ques. Given that $x(n) \longleftrightarrow \frac{z}{z+3}, |z| > 3$

$$(1) x(n+1) \longleftrightarrow z \cdot \frac{z}{z+3}, \quad \infty > |z| > 3$$

(uv)

$$(2) \quad x(n-1) \leftrightarrow z^{-1} \cdot \frac{z}{z+3}, \quad |z| > 3$$

$$(3) \quad x(-n) \leftrightarrow \frac{1/z}{1/z+3}, \quad |1/z| > 3$$

$$(4) \quad 5^n x(n) \leftrightarrow \frac{z/5}{z/5+3}, \quad |z/5| > 3$$

$$(5) \quad n x(n) \leftrightarrow -\frac{d}{dz} \cdot \frac{z}{z+3}, \quad |z| > 3$$

$$= - \left[\frac{1}{(z+3)} + \frac{z \cdot 1}{(z+3)^2} \right]$$

$$= \frac{-3z}{(z+3)^2}$$

⇒ INVERSE Z-TRANSFORM -

(1) LONG DIVISION METHOD (OR) POWER SERIES METHOD -

(2) PARTIAL FRACTION EXPⁿ

(3) RESIDUE / INVERSE INTEGRAL METHOD -

(1) POWER SERIES -

This method is used to find the few coefficient of given signal.

- This method involves in dividing the numerator with the denominator, and division is performed according to the (ROC).

- If the ROC indicates the right side signal then performed the division so as to obtain the -ve powers of z . (i.e. z^{-1})

- If the ROC indicates the left side signal then performed the division so as to obtain the +ve power of z . (z)

- Positive power of z can only be obtained by writing the numerator & denomⁿ. polynomial in ascending order of z .

Ques. find the first three coefficients of the following signals.

$$(1) \quad x(z) = \frac{1}{1-z^{-1}}$$

$$(a) \quad |z| > 1$$

$$(b) \quad |z| < 1$$

19/11/19

① $|z| < 3 \rightarrow$ left side $\rightarrow$ +ve power

$$6 + 5z + z^2 \over 13 + 15z + 2z^2 \left(\frac{13}{6} + \frac{25z}{36} - \frac{13}{216} z^2 \right)$$

$$13 + \frac{65z}{6} + \frac{13z^2}{6}$$

$$\frac{25}{6} z - \frac{1}{6} z^2$$

$$\frac{25}{6} z + \frac{125}{36} z^2 + \frac{25}{36} z^2$$

$$\frac{13}{216} + \frac{13}{36} z^2 - \frac{25}{36} z^3$$

② PARTIAL FRACTION EXPANSION -

This is suitable for the funⁿ that contains the simple poles and order of the funⁿ less than three.

Q. find the IZT using partial fraction method of the following:

① $x(z) = \frac{z}{z+5}$ for

(a) $|z| > 5$ (b) $|z| < 5$

Solⁿ: (A) $|z| > 5$

$$\frac{z}{z+5} \rightarrow (-5)^n u(n)$$

(B) $|z| < 5$

$$\frac{z}{z+5} \rightarrow -(-5)^n u(-n-1)$$

② $x(z) = \frac{5z^2 - 8z}{z^2 + 5z + 6}$ for

(a) $|z| > 3$ (b) $|z| < 2$ (c) $2 < |z| < 3$

Solⁿ: $x(z) = \frac{5z^2 - 8z}{(z+3)(z+2)} = \left[\frac{+23}{(z+3)} + \frac{-18}{(z+2)} \right] z$

(B) $|z| < 2$

$$x(z) = \frac{23z}{(z+3)} - \frac{18z}{(z+2)}$$

$$x(z) = \frac{23z}{(z+3)} - \frac{18z}{(z+2)}$$

① $|z| > 3$

$$x(z) = 23(-3)^n u(n) - 18(-2)^n u(n)$$

② $|z| < 2$

$$x(z) = -23(-3)^n u(-n-1) + 18(-2)^n u(-n-1)$$

$$s(n-1)$$

$$(3) \quad 2 < |z| < 3$$

$$X(z) = -23(-3)^n u(-n-1) - 18(-2)^n u(n)$$

$$(3) \quad X(z) = \frac{2z^3 - 15z^2 + 14z}{z^2 + 5z + 6} \quad \text{for}$$

$$(a) \quad |z| > 3 \quad (b) \quad |z| < 2 \quad (c) \quad 2 < |z| < 3$$

$$\text{Soln: } X(z) = z \left[\frac{2z^2 - 15z + 14}{(z+3)(z+2)} \right] \quad \text{Num = den}$$

$$= z \left[\frac{\text{improper fun}^n}{(z+3)(z+2)} \right]$$

$$= z \left[\frac{2z^2 - 15z + 14}{z^2 + 5z + 6} \right]$$

$$= z \left[2 + \frac{-25z + 2}{(z+2)(z+3)} \right]$$

$$= z \left[2 + \frac{-77}{(z+3)} + \frac{52}{(z+2)} \right]$$

$$= 2z - \frac{77z}{(z+3)} + \frac{52z}{(z+2)}$$

$$X(n) = 2\delta(n+1) + 52(-2)^n u(n) - 77(-3)^n u(n) \quad 3 < |z| < \infty$$

$$X(n) = 2\delta(n+1) + 52(-2)^n - 77(-3)^n$$

$$(1) \quad |z| > 3$$

$$X(n) = 2\delta(n) + 52(-2)^n u(n) - 77(-3)^n u(n)$$

$$(2) \quad |z| < 2 \quad |z| < \infty$$

$$X(n) = 2\delta(n+1) - 52(-2)^n u(-n-1) + 77(-3)^n u(-n-1)$$

$$(3) \quad 2 < |z| < 3$$

$$X(n) = 2\delta(n+1) + 52(-2)^n u(n) + 77(-3)^n u(-n-1)$$

(3) INVERSE INTEGRAL METHOD -

- Residues are evaluated at the poles.

- Residue at pole $z=a$ of first order is given by

$$R_{z=a} = \lim_{z \rightarrow a} \frac{d}{dz} \left[X(z)(z-a)z^{n-1} \right] \bigg|_{z=a}$$

1st order

- Residue at pole $z=a$ of m^{th} order is given by

$$\text{Res}_{z=a} = \frac{1}{(m-1)!} \frac{d^{m-1}}{dz^{m-1}} \left[X(z)(z-a)^m z^{n-1} \right] \bigg|_{z=a}$$

m^{th} order

The time signal is equal to sum of residues.

$$x(n) = \sum \text{Residue}$$

- This method is applicable for proper function only.

Ques. Find IZT using residue method.

$$(1) \quad x(z) = \frac{z}{z+5}$$

$$x(n) = R_{z=-5} \text{ (1st order)}$$

$$= \frac{z}{z+5} \cdot (z+5) z^{n-1} \Big|_{z=-5}$$

$$= (-5)^n$$

$$(2) \quad x(z) = \frac{5z^2 + 15z}{(z+1)(z-2)}$$

$$\begin{aligned} \text{Soln: } x(n) &= z \left(\frac{5z + 15}{(z+1)(z-2)} \right) \\ &= z \left(\frac{-10/3}{(z+1)} + \frac{25/3}{(z-2)} \right) \end{aligned}$$

$$x(n) = R_{z=-1} + R_{z=2}$$

$$= \frac{z(-10/3)}{(z+1)} \cdot (z+1) z^{n-1} \Big|_{z=-1}$$

$$+ \frac{25/3 z}{(z-2)} \cdot (z-2) z^{n-1} \Big|_{z=2}$$

$$= -\frac{10}{3} (-1)^n + \frac{25}{3} (2)^n$$

$$(3) \quad x(z) = \frac{5z^2 - 8z}{(z+1)(z-2)^2}$$

$$x(n) = R_{z=-1} + R_{z=2}$$

$$= \frac{5z^2 - 8z}{(z+1)(z-2)^2} \cdot (z+1) z^{n-1} \Big|_{z=-1}$$

$$+ \frac{1}{2-1} \frac{d}{dz} \left(\frac{5z^2 - 8z}{(z-2)^2} \cdot (z-2)^2 \cdot z^{n-1} \right) \Big|_{z=2}$$

$$= -\frac{13}{9} (-1)^n + \left(\frac{d}{dz} \frac{(5z-8)z^n}{(z-2)} \right) \Big|_{z=2}$$

REALIZATION

$$\frac{5}{3} - \frac{2}{5} = \frac{13}{9}$$

$$= -\frac{13}{9}(-1)^n + \left[\frac{z^n 5 + 5z - 8nz}{(z+1)} + z^n \frac{(5z-8)(-1)}{(z+1)^2} \right] \Big|_{z=-2}$$

$$= -\frac{13}{9}(-1)^n + \left[\frac{2^n 5}{3} + \frac{2 \cdot n 2^{n-1}}{3} + \frac{2^n \cdot (-2)}{9} \right]$$

$$= -\frac{13}{9}(-1)^n + \left[2^n \left(\frac{13}{9} \right) + \frac{2}{3} n \cdot 2^{n-1} \right]$$

$$= -\frac{13}{9}(-1)^n + \frac{13}{9} 2^n + \frac{n \cdot (2)}{3} 2^{n-1}$$

⇒ REALIZATION -

- Pictorial representation of the T.F. is known as realization.

- 4 - types:

- ① Direct - form - I
- ② Direct - II / Canonic
- ③ Parallel realization
- ④ Cascade realization

① DIRECT FORM - I -

Ques. Realize the follⁿ. funⁿ in direct form - I and II.

$$\textcircled{1} \quad 8y(n) - 3y(n-1) + 5y(n-2) + 7y(n-3) = \frac{5}{3}x(n) - 3x(n-1) + 4x(n-2)$$

$$\text{Solⁿ: } y(n) = \frac{3}{8}y(n-1) + \frac{5}{8}y(n-2) + \frac{7}{8}y(n-3) = \frac{5}{24}x(n) - \frac{3}{8}x(n-1) + \frac{1}{2}x(n-2)$$

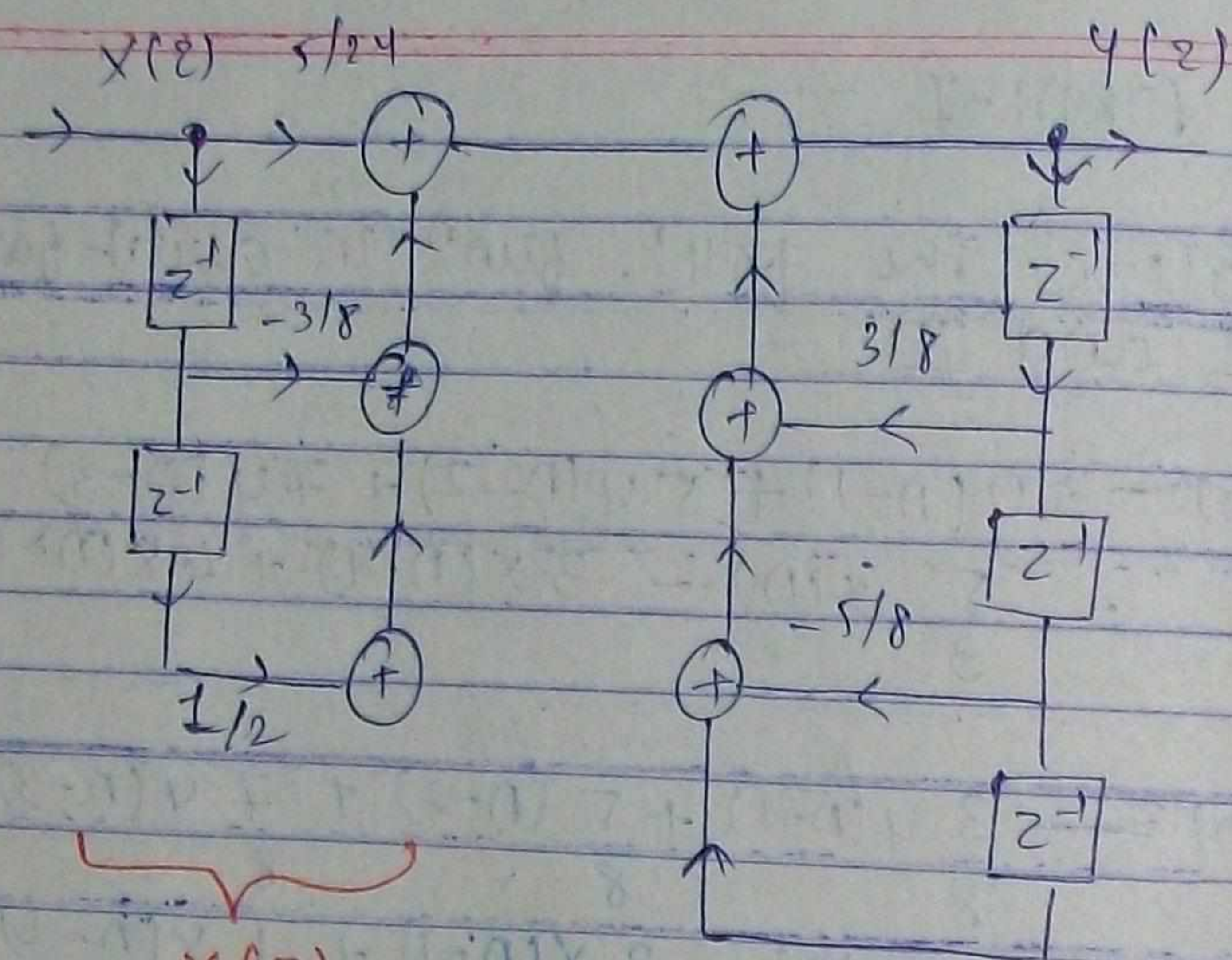
Z-transform on both side

$$Y(z) - \frac{3}{8}z^{-1}Y(z) + \frac{5}{8}z^{-2}Y(z) + \frac{7}{8}z^{-3}Y(z)$$

$$= \frac{5}{24}X(z) - \frac{3}{8}X(z) \cdot z^{-1} + \frac{1}{2}X(z)z^{-2}$$

$$Y(z) = \frac{5}{24}X(z) - \frac{3}{8}z^{-1}X(z) + \frac{1}{2}z^{-2}X(z) + \frac{3}{8}z^{-1}Y(z) - \frac{5}{8}z^{-2}Y(z) - \frac{7}{8}z^{-3}Y(z)$$

DIRECT FORM - I



$X(z)$
term

$Y(z)$
term

DIRECT FORM - II

$$\frac{Y(z)}{X(z)} = \frac{\left(\frac{5}{24} - \frac{3}{8}z^{-1} + \frac{1}{2}z^{-2}\right)}{\left(1 - \frac{3}{8}z^{-1} + \frac{5}{8}z^{-2} + \frac{7}{8}z^{-3}\right)}$$

$$\frac{W(z)}{X(z)} = \frac{Y(z)}{W(z)} = \frac{1}{\left(1 - \frac{3}{8}z^{-1} + \frac{5}{8}z^{-2} + \frac{7}{8}z^{-3}\right)} \cdot \left(\frac{5}{24} - \frac{3}{8}z^{-1} + \frac{1}{2}z^{-2}\right)$$

$$\frac{W(z)}{X(z)} = \frac{1}{\text{Denom}^n} \quad (1) \quad \frac{Y(z)}{W(z)} = \frac{\text{num}^n}{1} \quad (2)$$

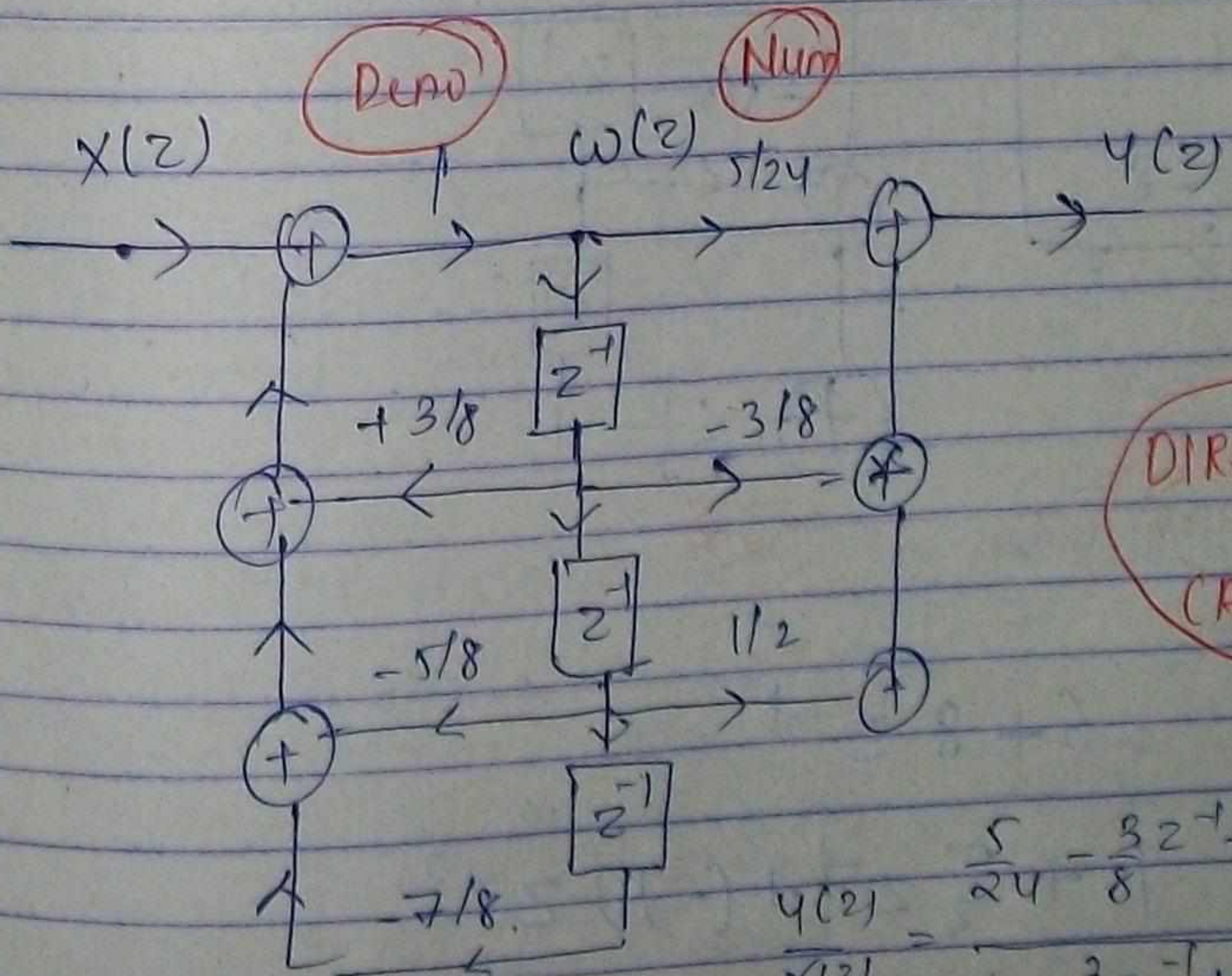
Posⁿ is must. (Standard form)

From (1), $\frac{W(z)}{X(z)} = \frac{1}{\left(1 - \frac{3}{8}z^{-1} + \frac{5}{8}z^{-2} + \frac{7}{8}z^{-3}\right)}$

$$W(z) = X(z) + \frac{3}{8}z^{-1}W(z) - \frac{5}{8}z^{-2}W(z) - \frac{7}{8}z^{-3}W(z) \quad (3)$$

From (2), $\frac{Y(z)}{W(z)} = \left(\frac{5}{24} - \frac{3}{8}z^{-1} + \frac{1}{2}z^{-2}\right)$

$$Y(z) = \frac{5}{24}W(z) - \frac{3}{8}z^{-1}W(z) + \frac{1}{2}z^{-2}W(z) \quad (4)$$



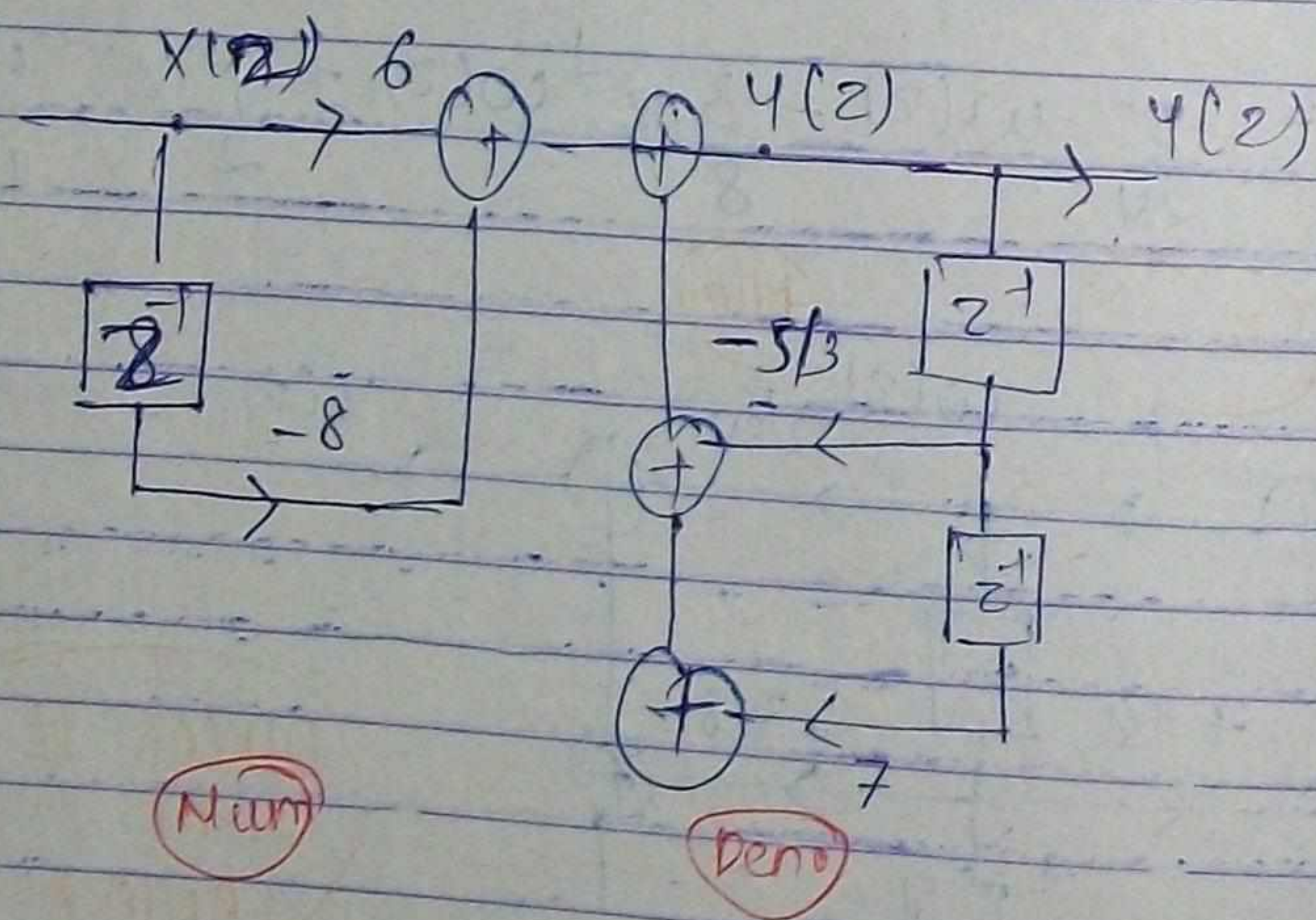
DIRECT - II
or
CANONICAL

$$\frac{Y(z)}{X(z)} = \frac{\frac{5}{24} - \frac{3}{8}z^{-1} + \frac{1}{2}z^{-2}}{1 - \frac{3}{8}z^{-1} + \frac{5}{8}z^{-2} + \frac{7}{8}z^{-3}}$$

- ★ In direct form - II no. of unit delays is equal to the order of the transfer function where as in direct form - I no. of delays is equal to no. of delay in I/P + no. of delays in O/P.

Ques. Sketch the direct form - I realisation of the following LTI system.

$$(1) \quad y(n) + \frac{5}{3}y(n-1) - 7y(n-2) = 6x(n) - 8x(n-1)$$

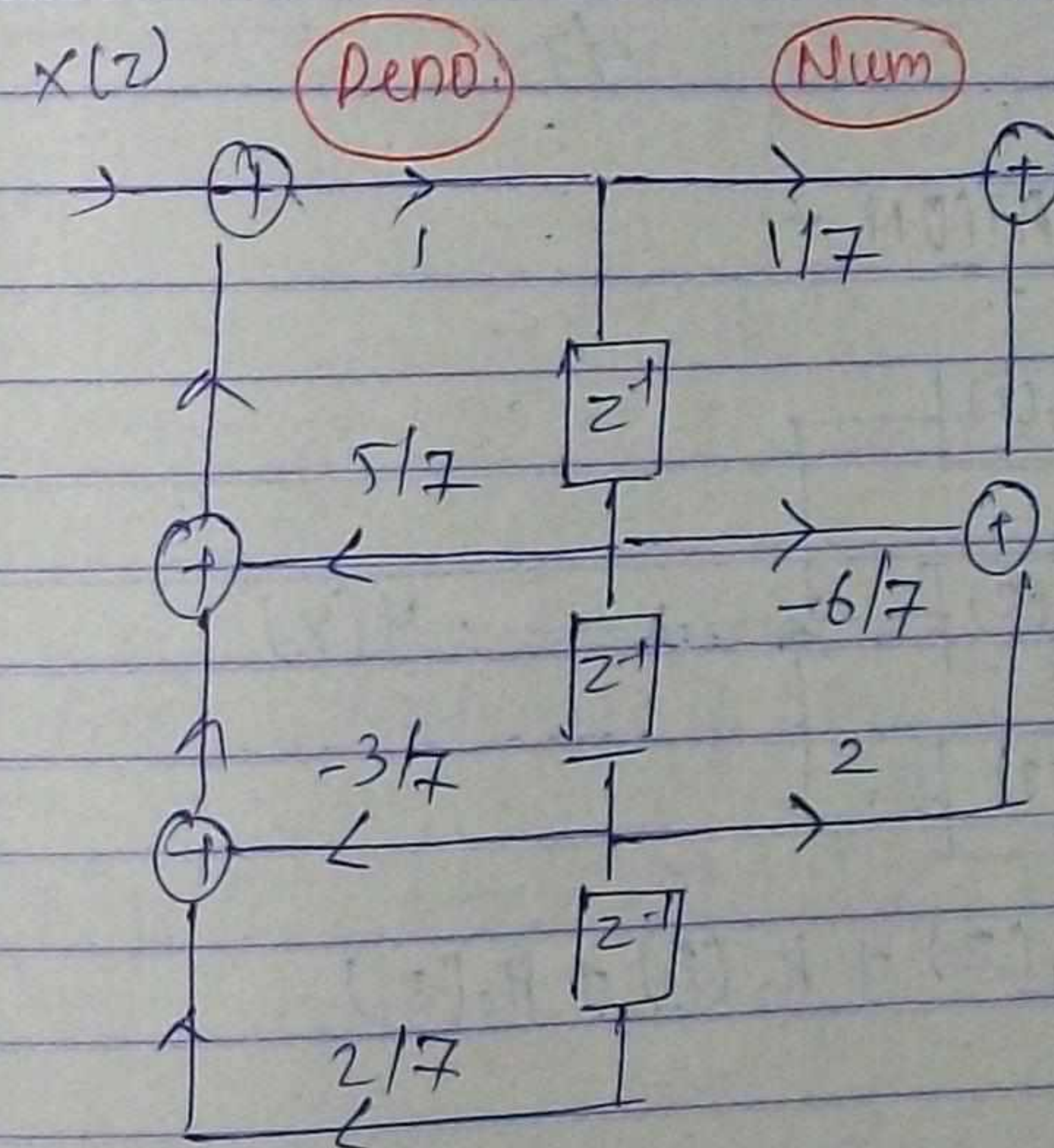


$$T.F = \frac{6 + 8z^{-1}}{1 + \frac{5}{3}z^{-1} + (-7)z^{-2}}$$

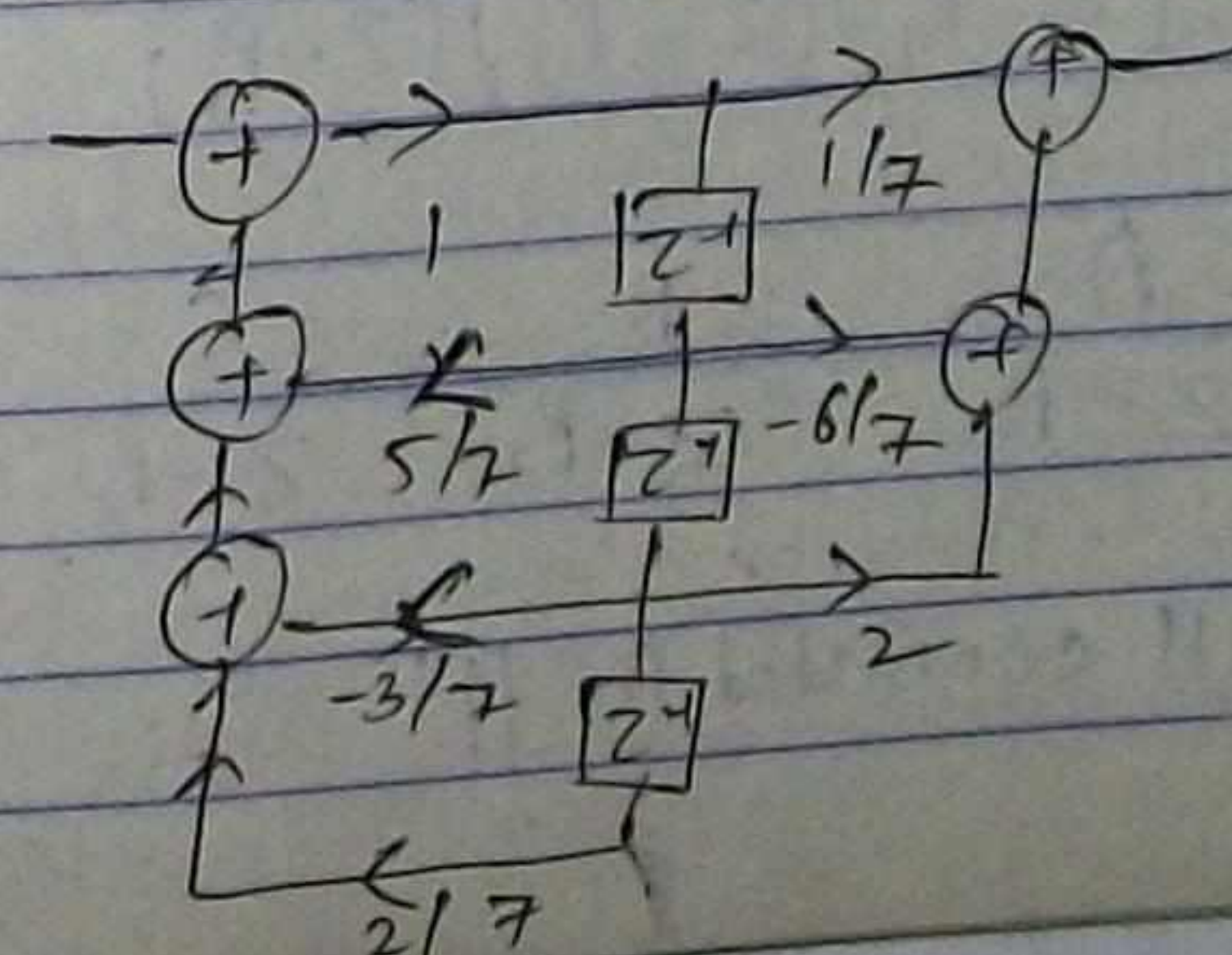
Realisation using Direct form - II.

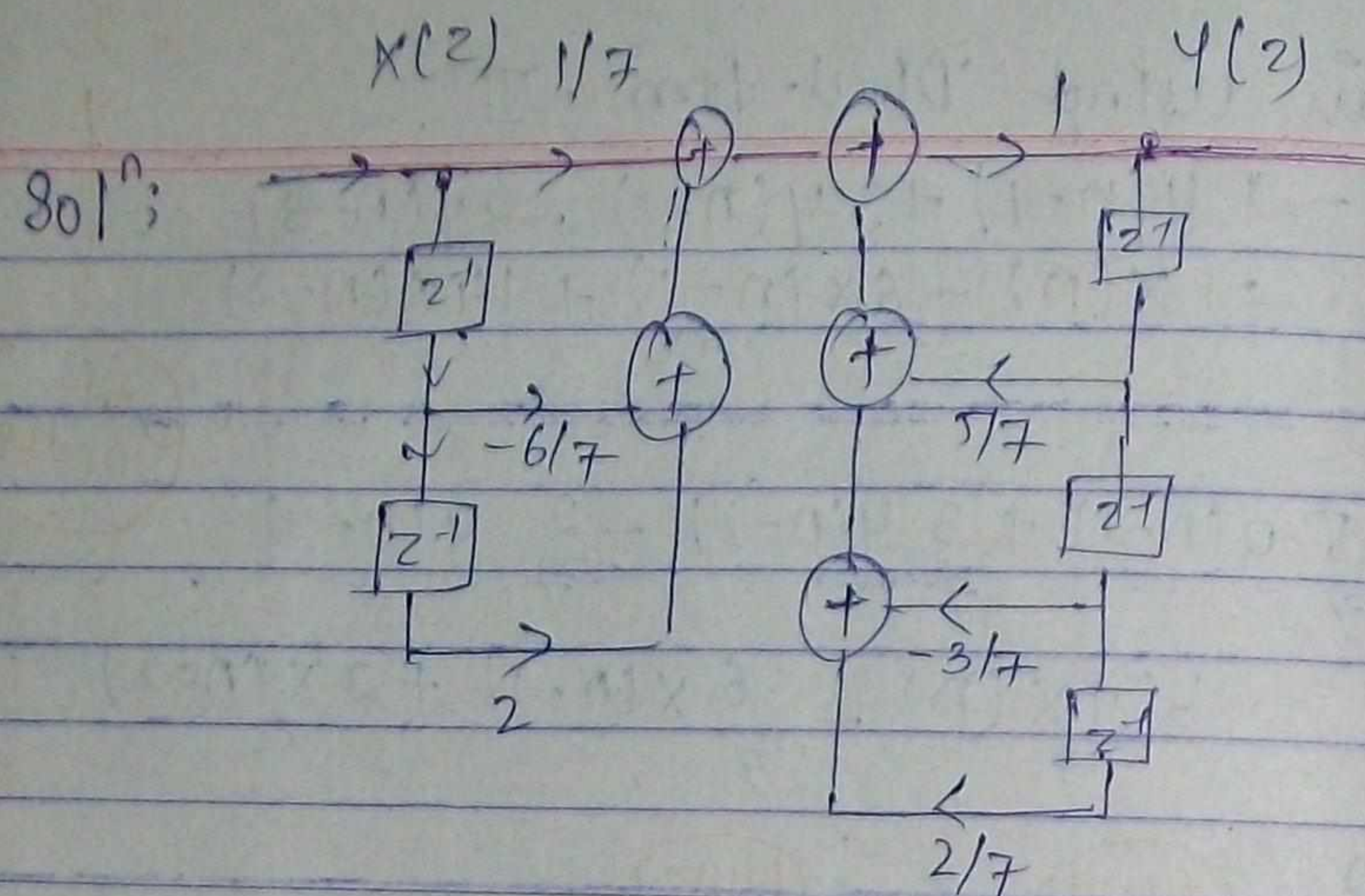
$$(2) \quad 7y(n) - 5y(n-1) + 3y(n-2) - 2y(n-3) = x(n) - 6x(n-1) + 14x(n-2)$$

$$y(n) = \frac{5}{7}y(n-1) + \frac{3}{7}y(n-2) - \frac{2}{7}y(n-3) + \frac{1}{7}x(n) - \frac{6}{7}x(n-1) + 2x(n-2)$$

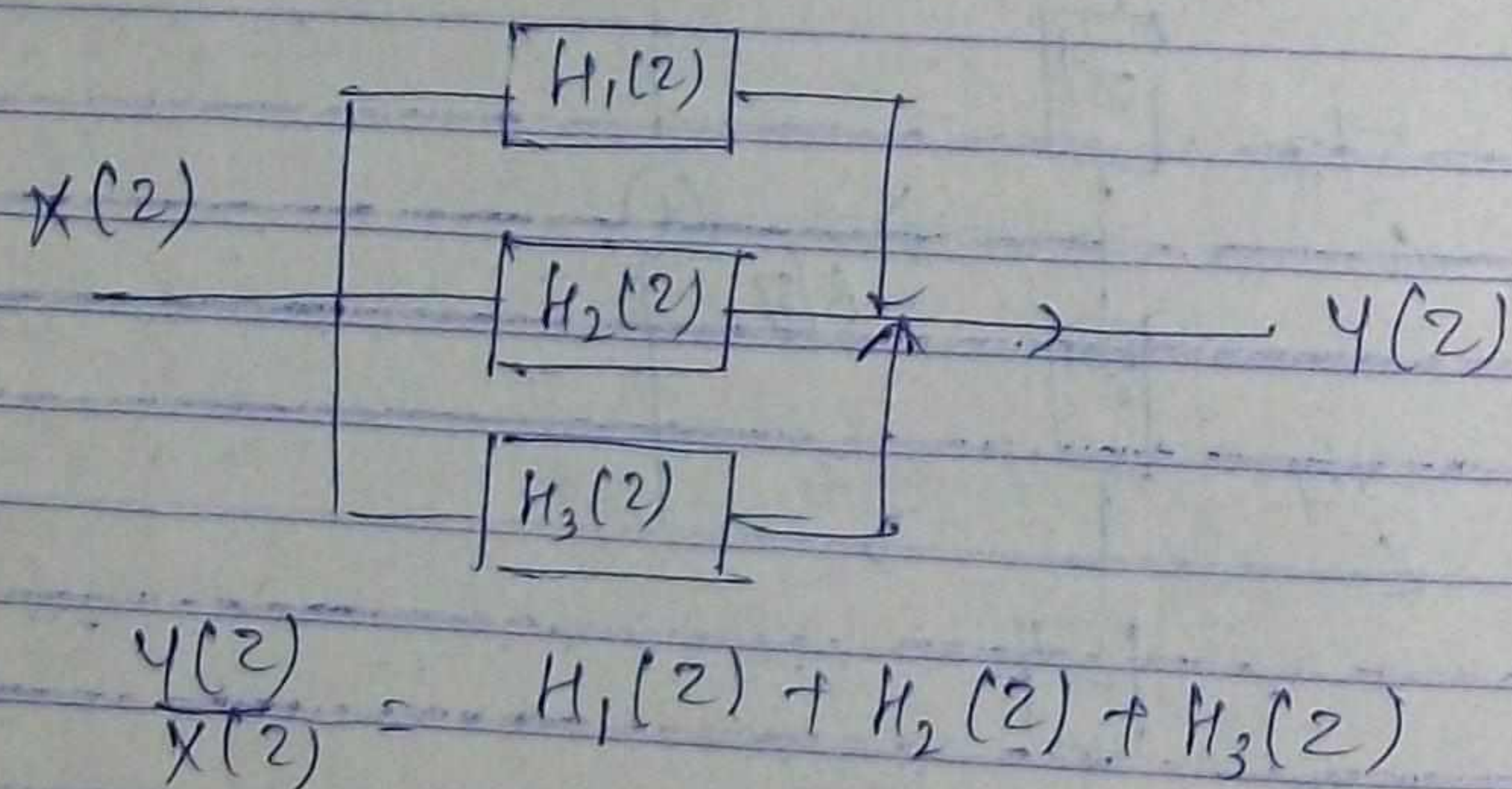


Ques. Sketch the direct form I for the following Direct form - II





③ PARALLEL REALISATION —

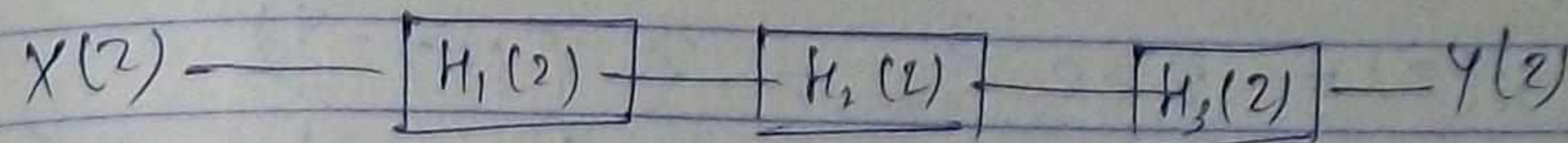


$$H(z) = \frac{N_r}{D_r} = \frac{N_r}{(z-p_1)(z-p_2)(z-p_3)}$$

$$= \frac{A}{z-p_1} + \frac{B}{z-p_2} + \frac{C}{z-p_3}$$

all connected in ||

④ CASCADE REALISATION —



$$\frac{Y(z)}{X(z)} = H_1(z) \cdot H_2(z) \cdot H_3(z)$$

$$H(z) = \frac{N_r}{D_r} = \frac{(z-z_1)(z-z_2)}{(z-p_1)(z-p_2)(z-p_3)}$$

$$= \frac{(z-z_1)}{(z-p_1)} \times \frac{(z-z_2)}{(z-p_2)} \times \frac{1}{(z-p_3)}$$

all in cascade,

Ques. find z transform of $s \cdot \frac{az}{(z-a)^2} = \frac{az^{-1}}{(1-az^{-1})^2}$ $|z| > a$

$$\textcircled{1} na^n u(n) \leftrightarrow -z \frac{d}{dz} \left(\frac{z}{z-a} \right)$$

$$\textcircled{2} -a^n u(-n) \leftrightarrow -z \frac{d}{dz} \left(\frac{a}{z-a} \right) \quad |z| < a$$

$$\textcircled{3} -na^n u(-n-1) \leftrightarrow -z \frac{d}{dz} \left(\frac{z}{z-a} \right) \quad |z| < a$$

$$= \frac{az}{(z-a)^2} = \frac{az^{-1}}{(1-az^{-1})^2}$$

Q-11 P-26.

Solⁿ:

$$X(z) = \frac{z + z^{-3}}{z + z^{-1}} = \frac{z (1 + z^{-4})}{z (1 + z^{-2})}$$

$$X(z) = \frac{1 + z^{-4}}{1 + z^{-2}}$$

$$\frac{1 + z^{-2}}{1 + z^{-2}} \left[\frac{1 + z^{-2} + 2z^{-4}}{-2z^{-6}} \right]$$

$$-z^2 + z^{-4}$$

$$-z^{-2} - z^{-4}$$

$$2z^{-4}$$

$$2z^{-4} + 2z^{-6}$$

$$-2z^{-6}$$

$$X(z) = 1 - z^{-2} + 2z^{-4} - 2z^{-6}$$

$$x(n) = \{1, 0, -1, 0, 2, 0, -2, \dots\}$$

Ques 1. P-27

$$R(z) = \frac{1 - e^{-\alpha T}}{(z-1)(z - e^{-\alpha T})} = \frac{A}{(z-1)} + \frac{B}{(z - e^{-\alpha T})}$$

$$R(z) = \frac{1}{z-1} - \frac{1}{(z - e^{-\alpha T})}$$

$$R(z) = u(n-1) - e^{-\alpha T(n-1)} u(n-1)$$

$$\frac{z}{z - e^{-\alpha T}} \leftrightarrow e^{-\alpha T n}$$

$$z^{-1} \cdot \frac{z}{z - e^{-\alpha T}} = \frac{1}{z - e^{-\alpha T}} \leftrightarrow e^{-\alpha T(n-1)}$$

Ques 2. P-27

$$x(n) = \{1, 2, -1, 1\}$$

$$X(z) = \sum_{n=-1}^2 x(n) z^{-n}$$

$$= x(-1)z + x(0)z^0 + x(1)z^{-1} + x(2)z^{-2}$$

$$= z + z - z^{-1} + z^{-2}, \quad 0 < |z| < \infty$$

DTFT is obtain from z-transform by substituting $z = e^{j\omega}$.

$$X(e^{j\omega}) = e^{j\omega} + 2 - e^{-j\omega} + e^{-j2\omega}$$

Ques 3. P-27

$$x_1(n) = \left(\frac{1}{4}\right)^n u(n-1) = \left(\frac{1}{4}\right) \left(\frac{1}{4}\right)^{n-1} u(n-1)$$

$$= \frac{1}{4} \left[\frac{z}{z-1/4} \right]$$

$$= \frac{1/4}{z-1/4}$$

$$x_2(n) = \left[1 + \left(\frac{1}{2}\right)^n \right] u(n)$$

$$= \left[\frac{z}{z-1} + \frac{z}{z-1/2} \right]$$

$$Y(z) = \frac{1}{4} \left[\frac{z}{z-1/4} \right] * \left[\frac{z}{z-1} + \frac{z}{z-1/2} \right]$$

$$Y(z) = \frac{1}{4} \frac{z (2z-1.5)}{(z-1)(z-1/2)(z-1/4)}$$

$$Y(z) = \frac{z}{4} \left[\frac{4/3}{z-1} + \frac{2}{z-1/2} + \frac{8/3}{z-1/4} \right]$$

$$y(n) = \left[\frac{1}{3} (1)^n + \frac{1}{2} \left(\frac{1}{2}\right)^n - \frac{1}{3} \left(\frac{1}{4}\right)^n \right] u(n-1)$$

Ques 4. P-27

Solⁿ: $H(z) = \frac{3-4z^{-1}}{1-3.5z^{-1}+1.5z^{-2}}$

$$H(z) = z \left[\frac{3z-4}{z^2-3.5z+1.5} \right] = z \left[\frac{3z-4}{(z-3)(z-0.5)} \right]$$

$$H(z) = \frac{2z}{z-3} + \frac{z}{z-0.5}$$

$$h(n) = 2(3)^n + (0.5)^n$$

hence (i) Stable system

$$h(n) = -2(3)^n u(-n-1) + (0.5)^n u(n)$$

$$ROC: 0.5 < |z| < 3$$

(ii) for causal system.

$$h(n) = [2(3)^n + (0.5)^n]$$

$$ROC: |z| > 3$$

DTFT

- DTFT is used to transform discrete time signal $x(n)$ into frequency domain.

- It transform DT. signal $x(n)$ as the continuous funⁿ of ω .

$$\text{DTFT of } x(n) = X(e^{j\omega})$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} \quad - (1)$$

- The periodicity of DT signal cannot exceed 2π units i.e. every DT periodic signal can uniquely be represented in freq. domain over $(0 \text{ to } 2\pi)$ or $(-\pi \text{ to } +\pi)$

$$\text{IDTFT of } X(e^{j\omega}) = x(n)$$

$$x(n) = \frac{1}{2\pi} \int_0^{2\pi} X(e^{j\omega}) e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega \quad - (2)$$

(*)

$$\text{From (2), } 2\pi X(0) = \int_0^{2\pi} X(e^{j\omega}) d\omega \quad - (3)$$

Eq (3) represent the area under DTFT

Ques: Given $x(n) = \{-3, -5, 6, 3\}$ find the area under DTFT.

$$\text{Sol}^n: \text{Area under DTFT} = 2\pi X(0) \\ = 2\pi \{-5\} = -10\pi$$

(*) for the existence of DTFT, It require absolute integration, where as Z-transform does not require absolute summation

(*) DTFT is a special case of Z.T. in which $z = e^{j\omega}$ but Z-transform is not a special case of DTFT. Therefore it is possible to obtain the DTFT from the Z-transform by substituting $z = e^{j\omega}$ if the corresponding time signal is absolute summable.

$$\begin{array}{ccc} \text{F.T} & \longrightarrow & \text{DTFT} \\ \uparrow & & \uparrow \\ \text{L.T} & \longrightarrow & \text{DFT (Z-TRANSFORM)} \end{array}$$

$$s = j\omega$$

$$z = e^{j\omega}$$

when signal is not absolute $\int$ then DTFT is obtain directly from FT.

$\Rightarrow$ DTFT TABLE -

① $A \delta(n) \leftrightarrow A$

② $u(n) \leftrightarrow \pi \delta(\omega) + \frac{1}{j\omega}$

③ $u(-n) \leftrightarrow \pi \delta(\omega) - \frac{1}{j\omega}$

④ $\cos(\omega_0 n) \leftrightarrow \pi \{ \delta(\omega + \omega_0) + \delta(\omega - \omega_0) \}$

⑤ $\sin(\omega_0 n) \leftrightarrow \pi j \{ \delta(\omega + \omega_0) - \delta(\omega - \omega_0) \}$

⑥ $a^n u(n) \leftrightarrow \frac{1}{1 - a e^{-j\omega}}, \text{ for } |a| < 1$
 $= \frac{e^{j\omega}}{e^{j\omega} - a}, \text{ for } |a| < 1$

⑦ $-a^n u(-n) \leftrightarrow \frac{a}{e^{j\omega} - a}, \text{ for } |a| > 1$

⑧ $-a^n u(-n-1) \leftrightarrow \frac{e^{j\omega}}{e^{j\omega} - a}, |a| > 1$

Ques. find the DTFT of the following:

① $x(n) = \{ 2, -5, 2, 3 \}$

Solⁿ: $x(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$

$= \sum_{n=-1}^2 x(n) e^{-j\omega n}$

$= x(-1) e^{j\omega} + x(0) + x(1) e^{-j\omega} + x(2) e^{-j2\omega}$

$= 2e^{j\omega} - 5 - 2e^{-j\omega} + 3e^{-j2\omega}$

② $x(n) = (2^n) u(n)$

$|a| = 2 > 1$ does not exist

③ $(-2)^n u(-n) \leftrightarrow \frac{-(-2)}{e^{j\omega} - 2}$
 $|2| > 1$

④ $(-\frac{3}{5})^n u(n) \leftrightarrow \frac{e^{j\omega}}{e^{j\omega} + 3/5}, |3/5| < 1$

⑤ $(\frac{1}{5})^n u(n) - (\frac{3}{5})^n u(n) \leftrightarrow \frac{e^{j\omega}}{e^{j\omega} - 0.2} - \frac{e^{j\omega}}{e^{j\omega} - 0.6}$

Ques. DFT of $x(n)$ is given by

$$x(n) \longleftrightarrow \frac{3}{1 - 5e^{-j\omega}}$$

① $x(n-1) \longleftrightarrow e^{-j\omega} \cdot \frac{3}{1 - 5e^{-j\omega}}$

② $x(n+2) \longleftrightarrow e^{2j\omega} \cdot \frac{3}{1 - 5e^{-j\omega}}$

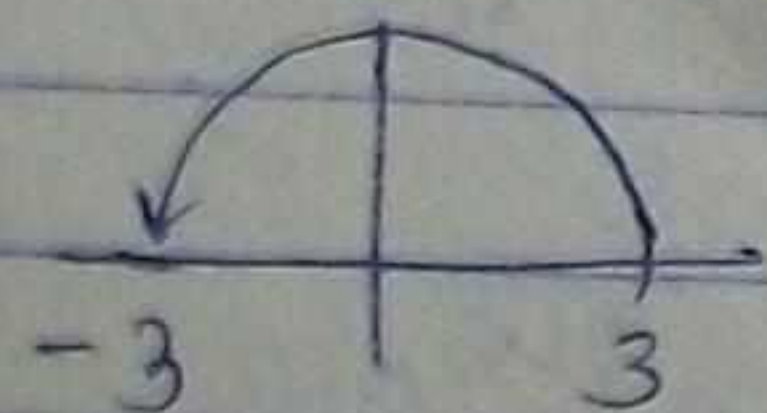
③ $x(-n) \longleftrightarrow x(e^{-j\omega}) = \frac{3}{1 - 5e^{j\omega}}$

④ $4^n x(n) \longleftrightarrow x\left(\frac{e^{j\omega}}{4}\right) = \frac{3}{1 - 5e^{\frac{j\omega}{4}}}$

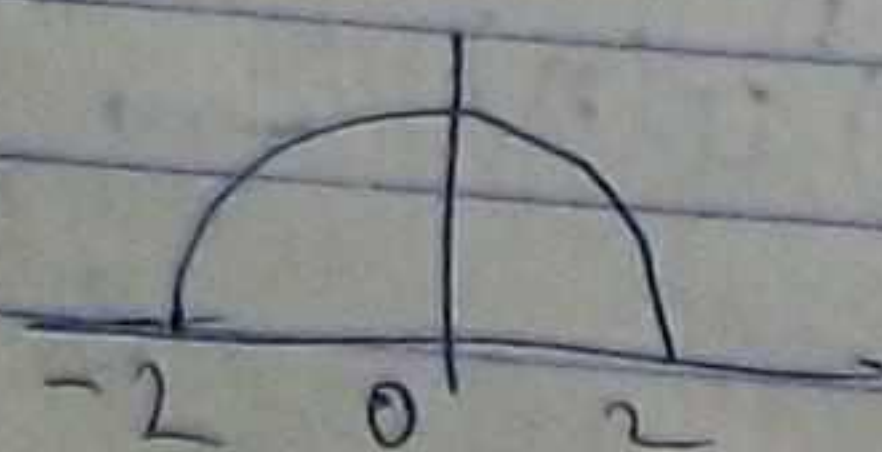
⇒ TWIDDLE FACTOR (w) -

$$w = e^{-j2\pi}$$

$$3e^{j\pi} = -3$$

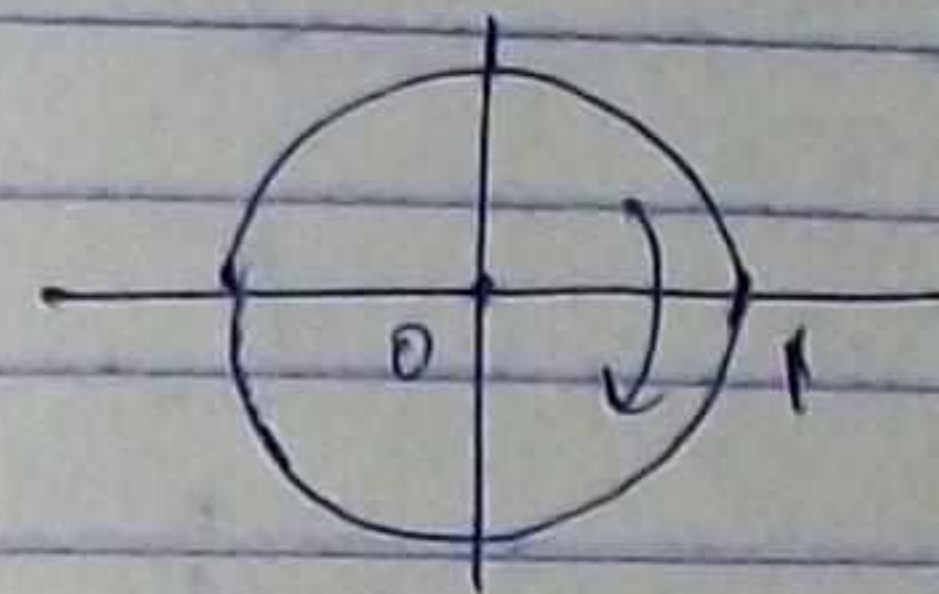


$$-2e^{-j\pi} = 2$$



ANTI CLK = $+w$
CLK = $-w$

Twiddle factor represent a unit circle that rotates in Clockwise by 2π .



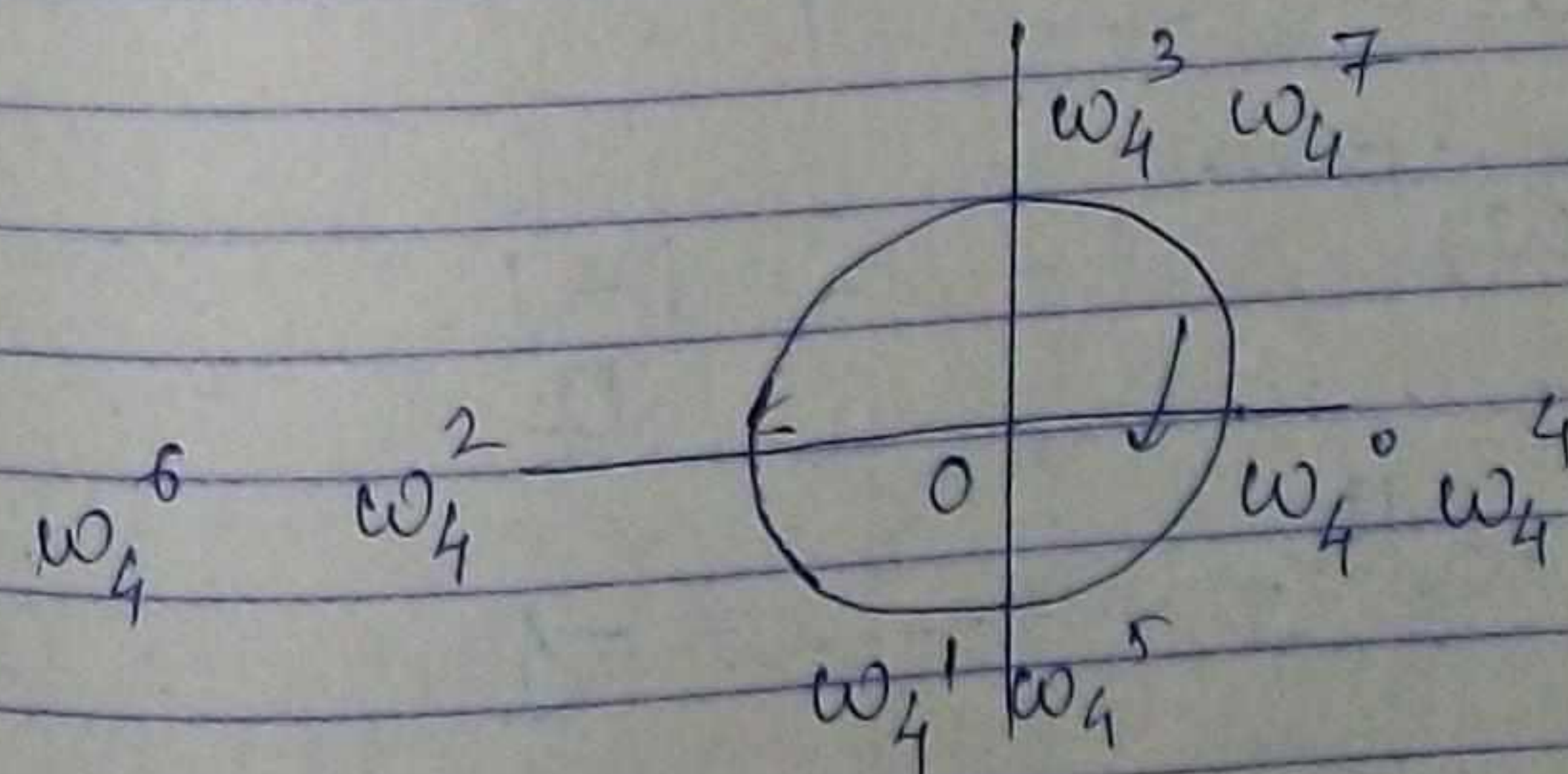
$$w_N = e^{-j\frac{2\pi}{N}}$$

$$w_N^{nk} = e^{-j\frac{2\pi}{N} \cdot nk}$$

⇒ EVALUATION OF TWIDDLE FACTOR -

for $N=4$

for $N=4$ divide the unit circle into 4 equal parts w.r.t x-axis



$$w_4^5 = e^{-j2\pi(5/4)} = \cos\left(\frac{5\pi}{2}\right) - j\sin\left(\frac{5\pi}{2}\right)$$

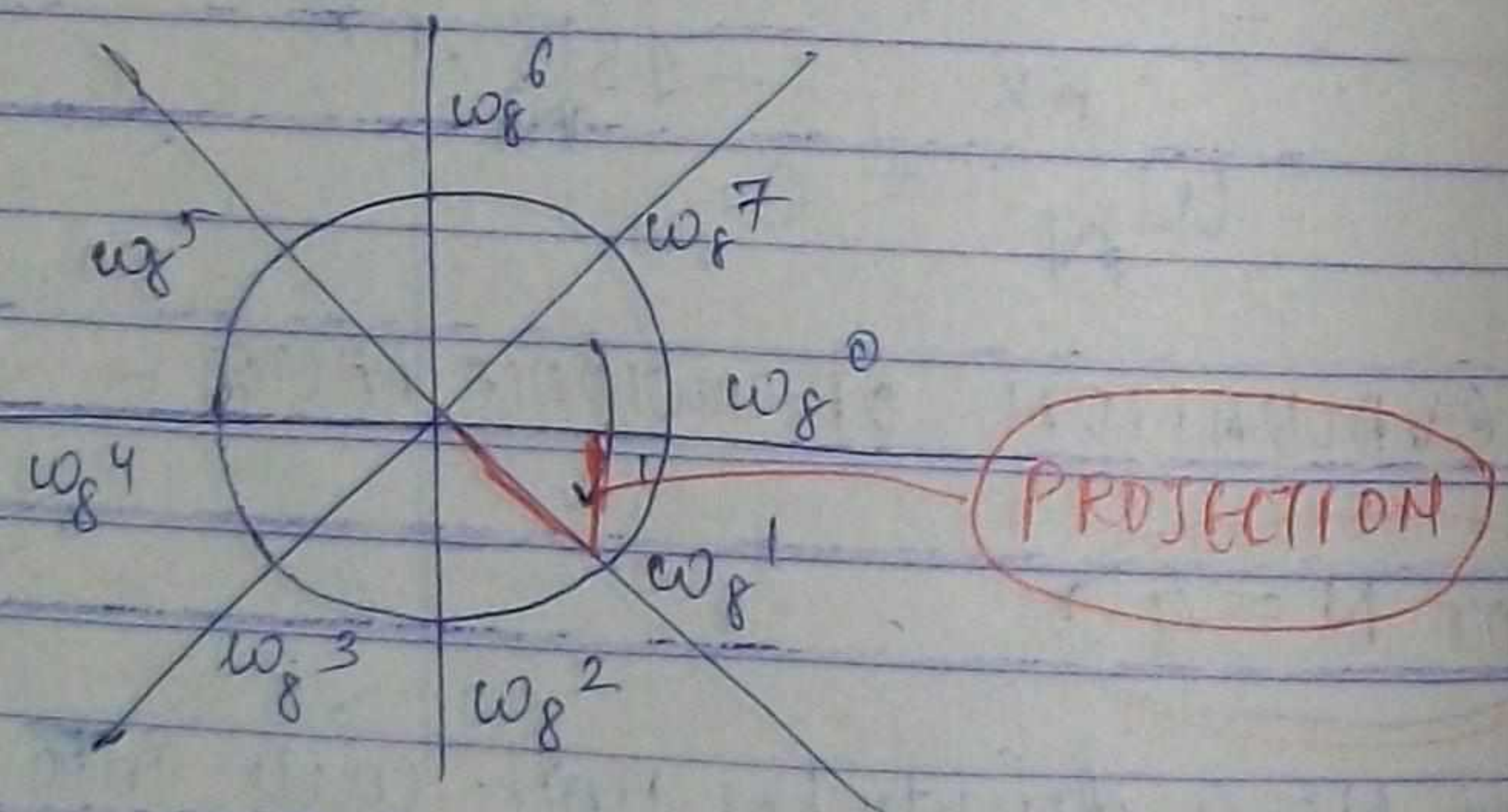
$$\omega_4^0 = \omega_4^4 = \omega_4^8 = \dots = 1$$

$$\omega_4^1 = \omega_4^5 = \omega_4^9 = \dots = -j$$

$$\omega_4^2 = \omega_4^6 = \omega_4^{10} = \dots = -1$$

$$\omega_4^3 = \omega_4^7 = \omega_4^{11} = \dots = j$$

FOR $N=8$



$$\omega_8^0 = \omega_8^8 = \omega_8^{16} = \dots = 1$$

$$\omega_8^1 = \omega_8^9 = \omega_8^{17} = \dots = \frac{1-j}{\sqrt{2}}$$

$$\omega_8^2 = \omega_8^{10} = \omega_8^{18} = \dots = -1$$

$$\omega_8^3 = \omega_8^{11} = \dots = -\frac{(1+j)}{\sqrt{2}}$$

$$\omega_8^4 = \omega_8^{12} = \dots = -1$$

$$\omega_8^5 = \omega_8^{13} = \dots = -\frac{(1-j)}{\sqrt{2}}$$

$$\omega_8^6 = \omega_8^{14} = \dots = -1$$

$$\omega_8^7 = \omega_8^{15} = \dots = \frac{1+j}{\sqrt{2}}$$

$\Rightarrow$ TWIDDLE FACTOR MATRIX -

$$[W]_{N \times N} = \begin{bmatrix} \omega_N^0 & \omega_N^0 & \omega_N^0 & \omega_N^0 & \dots & \omega_N^0 \\ \omega_N^0 & \omega_N^1 & \omega_N^2 & \omega_N^3 & \dots & \omega_N^{N-1} \\ \omega_N^0 & \omega_N^2 & \omega_N^4 & \omega_N^6 & \dots & \omega_N^{2(N-1)} \\ \omega_N^0 & \omega_N^3 & \omega_N^6 & \omega_N^9 & \dots & \omega_N^{3(N-1)} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \omega_N^0 & \omega_N^{N-1} & \omega_N^{2(N-1)} & \omega_N^{3(N-1)} & \dots & \omega_N^{(N-1)^2} \end{bmatrix}$$

$$[W]_{4 \times 4} = \begin{bmatrix} \omega_4^0 & \omega_4^0 & \omega_4^0 & \omega_4^0 \\ \omega_4^0 & \omega_4^1 & \omega_4^2 & \omega_4^3 \\ \omega_4^0 & \omega_4^2 & \omega_4^4 & \omega_4^6 \\ \omega_4^0 & \omega_4^3 & \omega_4^6 & \omega_4^9 \end{bmatrix}$$

⇒ DISCRETE FOURIER TRANSFORM ←

- If DFT both the time and freq. are contin. discrete.

N-point

DFT of $x(n) = X(k)$

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi nk}$$

for $k = 0, 1, 2, \dots, (N-1)$

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}nk}$$

for $k = 0, 1, 2, \dots, (N-1)$

$$X(k) = \sum_{n=0}^{N-1} x(n) \omega_N^{nk}$$

for $k = 0, 1, 2, \dots, (N-1)$

IDFT of $X(k) = x(n)$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j2\pi nk}, \text{ for } n = 0, 1, 2, \dots, (N-1)$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j\frac{2\pi}{N}nk}, n = 0, 1, 2, \dots, (N-1)$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} X(k) \cdot \omega_N^{-nk}, n = 0, 1, 2, \dots, (N-1)$$

$$x(n) \xleftrightarrow{\text{DFT}} X(k)$$

FOR 4-POINT DFT -

$$X(k) = \sum_{n=0}^3 x(n) \omega_4^{nk}, \text{ for } k = 0, 1, 2, 3$$

$$X(0) = \sum_{n=0}^3 x(n) \omega_4^{n \cdot 0} = x(0) \omega_4^0 + x(1) \omega_4^0 + x(2) \omega_4^0 + x(3) \omega_4^0$$

$$X(1) = \sum_{n=0}^3 x(n) \omega_4^{n \cdot 1} = x(0) \omega_4^1 + x(1) \omega_4^2 + x(2) \omega_4^3 + x(3) \omega_4^0$$

$$X(2) = \sum_{n=0}^3 X(n) \omega_4^{n \cdot 2} = X(0) \omega_4^0 + X(1) \omega_4^2 + X(2) \omega_4^4 + X(3) \omega_4^6$$

$$X(3) = \sum_{n=0}^3 X(n) \omega_4^{n \cdot 3} = X(0) \omega_4^0 + X(1) \omega_4^3 + X(2) \omega_4^6 + X(3) \omega_4^9$$

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} \omega_4^0 & \omega_4^0 & \omega_4^0 & \omega_4^0 \\ \omega_4^0 & \omega_4^1 & \omega_4^2 & \omega_4^3 \\ \omega_4^0 & \omega_4^2 & \omega_4^4 & \omega_4^6 \\ \omega_4^0 & \omega_4^3 & \omega_4^6 & \omega_4^9 \end{bmatrix} \begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix}$$

$$[X(k)] = [W]_{4 \times 4} [X(n)]$$

General
Expr

$$[X(k)] = [W]_{N \times N} [X(n)]$$

* N-point DFT requires N^2 multiplications and $[N \times (N-1)]$ addⁿ

The sequence to be followed to compute

Multiply $\rightarrow$ add $\rightarrow$ store

4 POINT IDFT

$$X(n) = \frac{1}{N} \sum_{k=0}^3 X(k) \cdot \omega_4^{-nk}$$

$$[X(n)] = \frac{1}{4} [W]_{4 \times 4}^* [X(k)]$$

For N-point -

$$[X(n)] = \frac{1}{N} [W]_{N \times N}^* [X(n)]$$

Ques Find the 4-point DFT of the sequence.

$$X(n) = \{1, 2, 3, 4\}$$

Solⁿ:

$$X(k) = \sum_{n=0}^3 X(n) e^{-j \frac{2\pi}{4} nk}$$

$$X(0) = \sum_{n=0}^3 X(n) e^{-j0n} = X(0) \omega_4^0 + X(1) \omega_4^0 + X(2) \omega_4^0 + X(3) \omega_4^0$$

$$[X(k)] = [W]_{4 \times 4} [X(n)]$$

$$\tan^{-1}\left(\frac{2}{-2}\right) = \pi - \tan^{-1}\left(\frac{2}{2}\right)$$

$$X(k) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

$$X(k) = \{10, (-2+2j), -2, -2-2j\}$$

$$|X(k)| = \{10, 2\sqrt{2}, 2, 2\sqrt{2}\}$$

$$\angle X(k) = \{0, 135^\circ, 180, 225\}$$

Ques. find the IDFT of $X(k) = \{10, (-2+2j), -2, (-2-j2)\}$

Soln:

$$X(n) = \frac{1}{4} [W]_{4 \times 4}^* [X(k)]$$

$$= \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

$$= \{1, 2, 3, 4\}$$

$$10 - 2 + 2j - 2 - 2 - 2j$$

(*)

Ques. Given that $X(n) = \{1, 2, 3, 4\}$
 $h(n) = \{2, 3, 4, 5\}$
 find the convolution using DFT.

Soln: $Y(n) = X(n) * h(n)$

DFT on both sides,

$$Y(k) = X(k) \cdot H(k)$$

$$X(k) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

$$H(k) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 14 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

$$X(k) H(k) = \{140, -j8, 4, 8j\}$$

$$Y(n) = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 140 \\ -j8 \\ 4 \\ 8j \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 36 \\ 38 \\ 36 \\ 30 \end{bmatrix}$$

Ques.

find the circular convolⁿ of $x(n)$ and $h(n)$

$$x(n) = \{1, 2, 3, 4\}$$

$$y(n) = \{2, 3, 4, 5\}$$

Solⁿ:

$$x(n) * h(n) = \begin{bmatrix} 1 & 4 & 3 & 2 \\ 2 & 1 & 4 & 3 \\ 3 & 2 & 1 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}$$

$$= \{36, 24, 36, 30\}$$

Ques.

Convolⁿ using DFT is equal to circular convolⁿ. where as convolⁿ using Laplace transform, F.T. and Z.T. is same as linear convolⁿ.

FAST FOURIER TRANSFORM

20/11/11

fast Fourier transform is another method to compute DFT of the signal in which the signal is decomposed the sum of sequences of smaller length.

NO. of arithmetic operations in FFT is given by $\frac{N}{2} \log_2 N$

⇒ TYPES OF FFT -

① DECIMATION IN TIME - FFT -

② DECIMATION IN FREQ - FFT.

⇒ DIT - FFT -

N-point DFT is given by

$$X(k) = \sum_{n=0}^{N-1} x(n) \omega_N^{nk}, \text{ for } k=0, 1, 2, \dots, (N-1)$$

for, $N=4$

$$X(k) = \sum_{n=0}^3 x(n) \omega_4^{nk}, \quad k=0, 1, 2, 3$$

Separating even and odd values of n

$$= \sum_{n=0}^1 x(2n) \omega_4^{2nk} + \sum_{n=0}^1 x(2n+1) \omega_4^{(2n+1)k}$$

$k=0, 1, 2, 3$

$$\omega_4^{2nk} = e^{-j\frac{2\pi}{4} \cdot 2nk} = e^{-j\frac{2\pi nk}{2}} = \omega_2^{nk}$$

$$= \sum_{n=0}^1 g_1(n) \omega_2^{nk} + \sum_{n=0}^1 g_2(n) \cdot \omega_4^{2nk} \cdot \omega_4^k$$

$k=0,1,2,3$

where

$$g_1(n) = x(2n) \quad - (1)$$

$$g_2(n) = x(2n+1) \quad - (2)$$

$$X(k) = \sum_{n=0}^1 g_1(n) \omega_2^{nk} + \omega_4^k \sum_{n=0}^1 g_2(n) \omega_2^{nk}$$

$k=0,1,2,3$

$$X(k) = G_1(k) + \omega_4^k G_2(k), \quad k=0,1,2,3 \quad - (3)$$

where

$$G_1(k) = \sum_{n=0}^1 g_1(n) \omega_2^{nk} \quad - (4)$$

$$G_2(k) = \sum_{n=0}^1 g_2(n) \omega_2^{nk} \quad - (5)$$

from (1)

$$g_1(0) = x(0)$$

$$g_1(1) = x(2)$$

from (2)

$$g_2(0) = x(1)$$

$$g_2(1) = x(3)$$

from (3),

$$X(0) = G_1(0) + \omega_4^0 G_2(0)$$

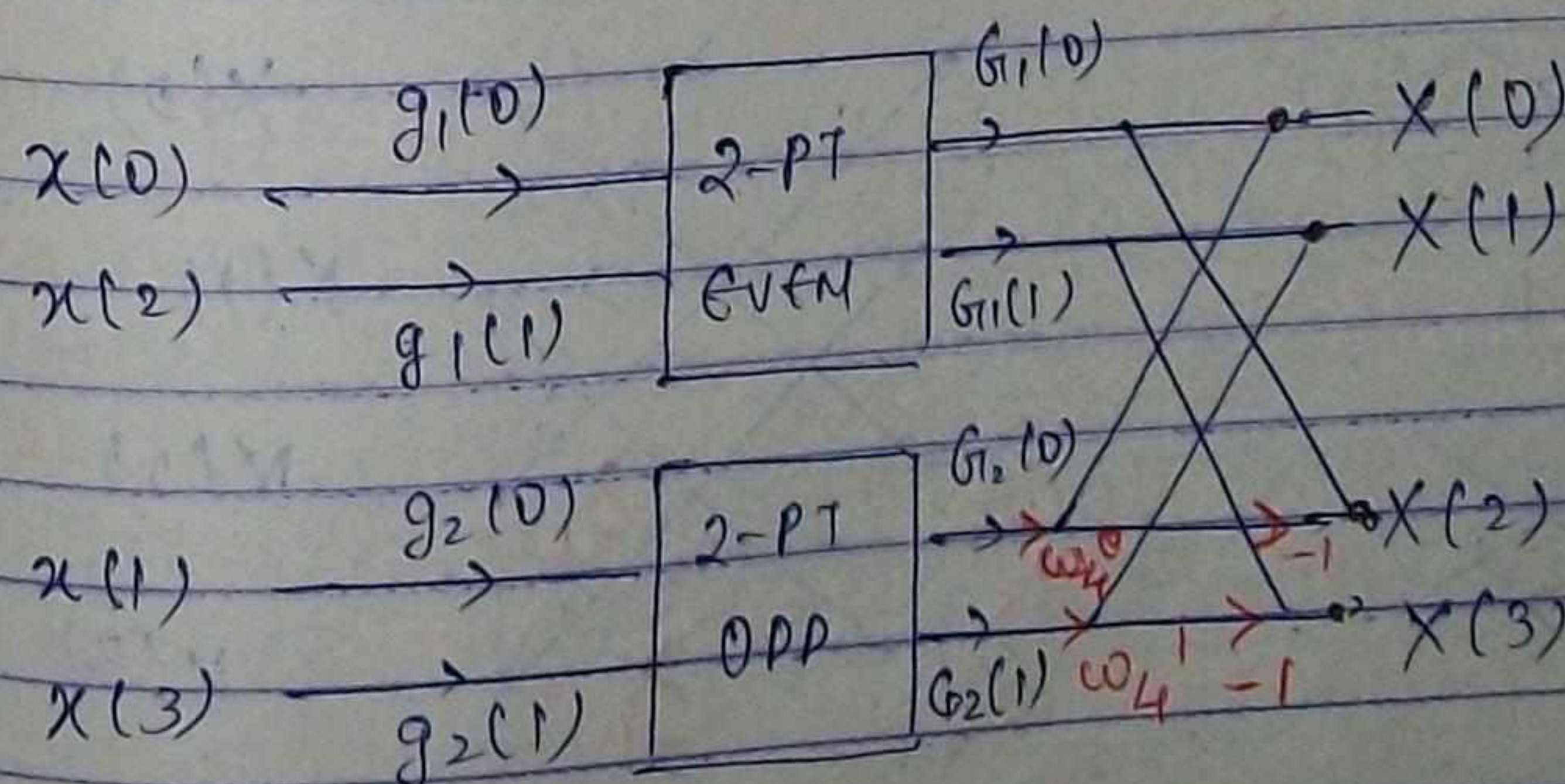
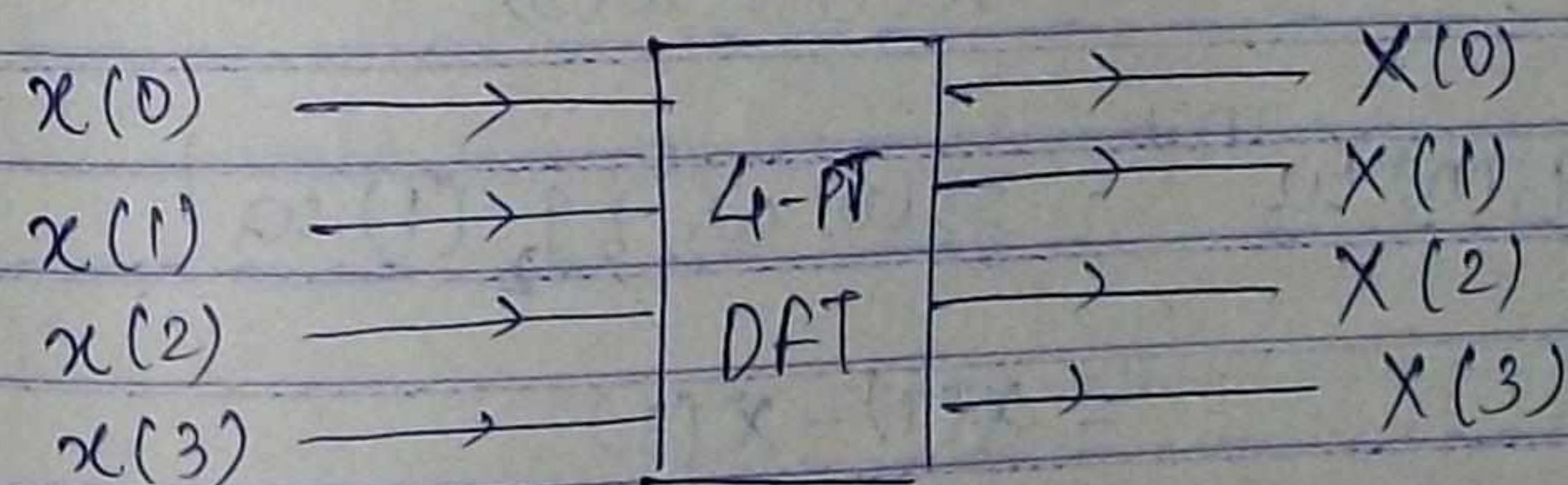
$$X(1) = G_1(1) + \omega_4^1 G_2(1)$$

$$X(2) = G_1(2) + \omega_4^2 G_2(2)$$

$$X(3) = G_1(3) + \omega_4^3 G_2(3)$$

$$X(2) = G_1(0) + \omega_4^0 G_2(0)$$

$$X(3) = G_1(1) + \omega_4^1 G_2(1)$$



Since,

from (4)

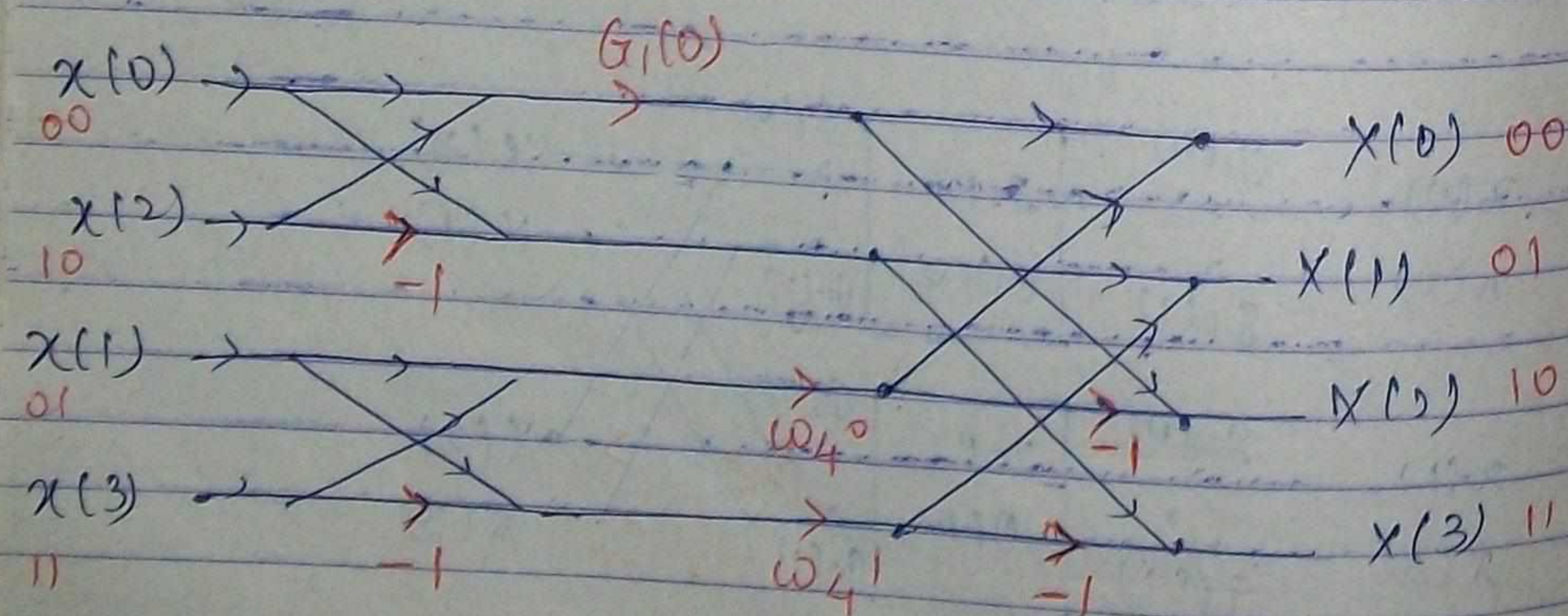
$$G_1(0) = \sum_{n=0}^1 g_1(n) \omega_2^{n \cdot 0} = g_1(0) \omega_2^0 + g_1(1) \omega_2^0 = x(0) + x(2)$$

$$G_1(1) = \sum_{n=0}^1 g_1(n) \omega_2^{n \cdot 1} = g_1(0) \omega_2^0 + g_1(1) \omega_2^1 = x(0) - x(2)$$

from (5)

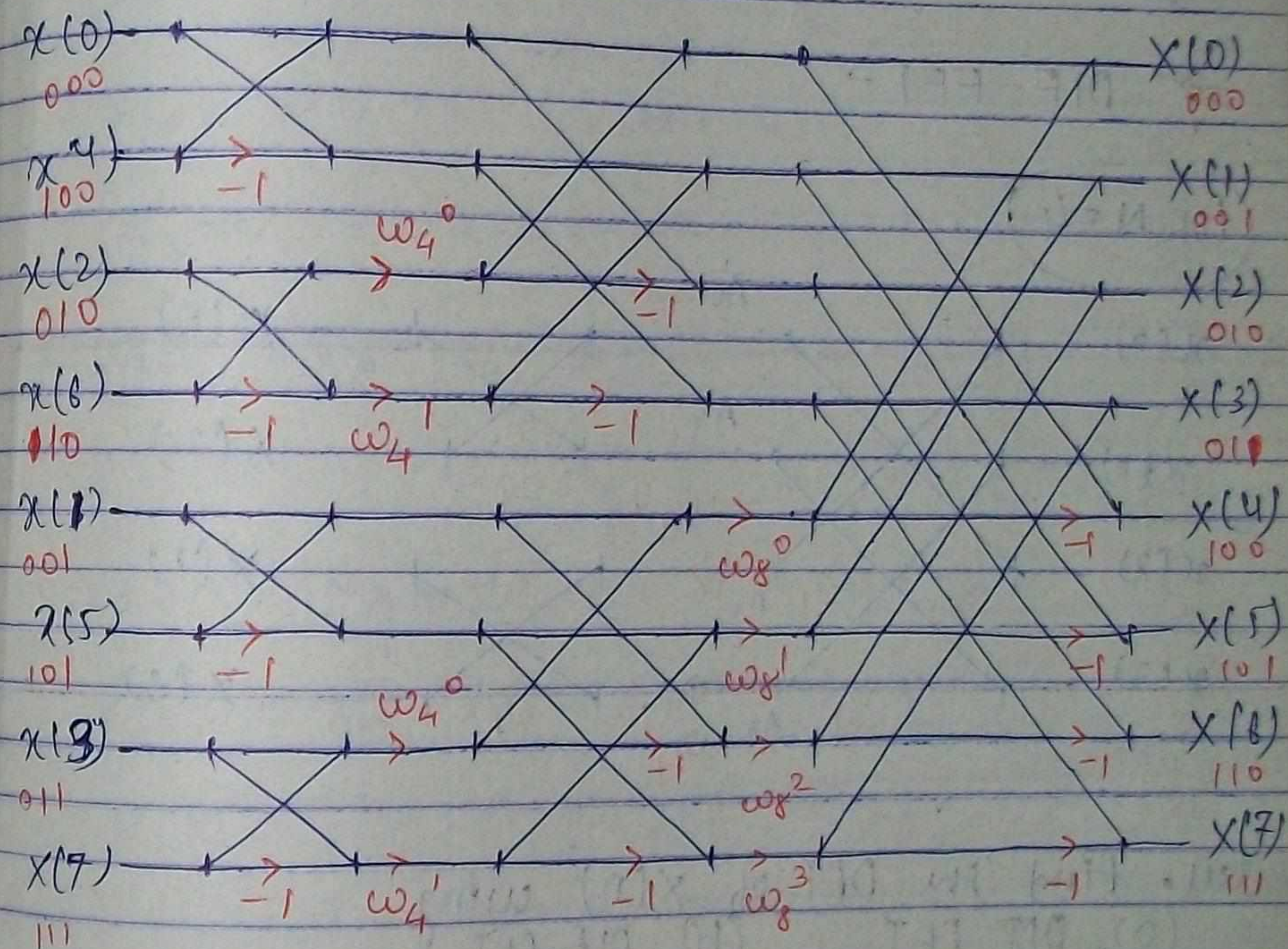
$$G_2(0) = \sum_{n=0}^1 g_2(n) \omega_2^{n \cdot 0} = g_2(0) \omega_2^0 + g_2(1) \omega_2^0 = x(1) + x(3)$$

$$G_2(1) = \sum_{n=0}^1 g_2(n) \omega_2^{n \cdot 1} = g_2(0) \omega_2^0 + g_2(1) \omega_2^1 = x(1) - x(3)$$



8-POINT DIT-FFT

SIGNAL FLOW GRAPH



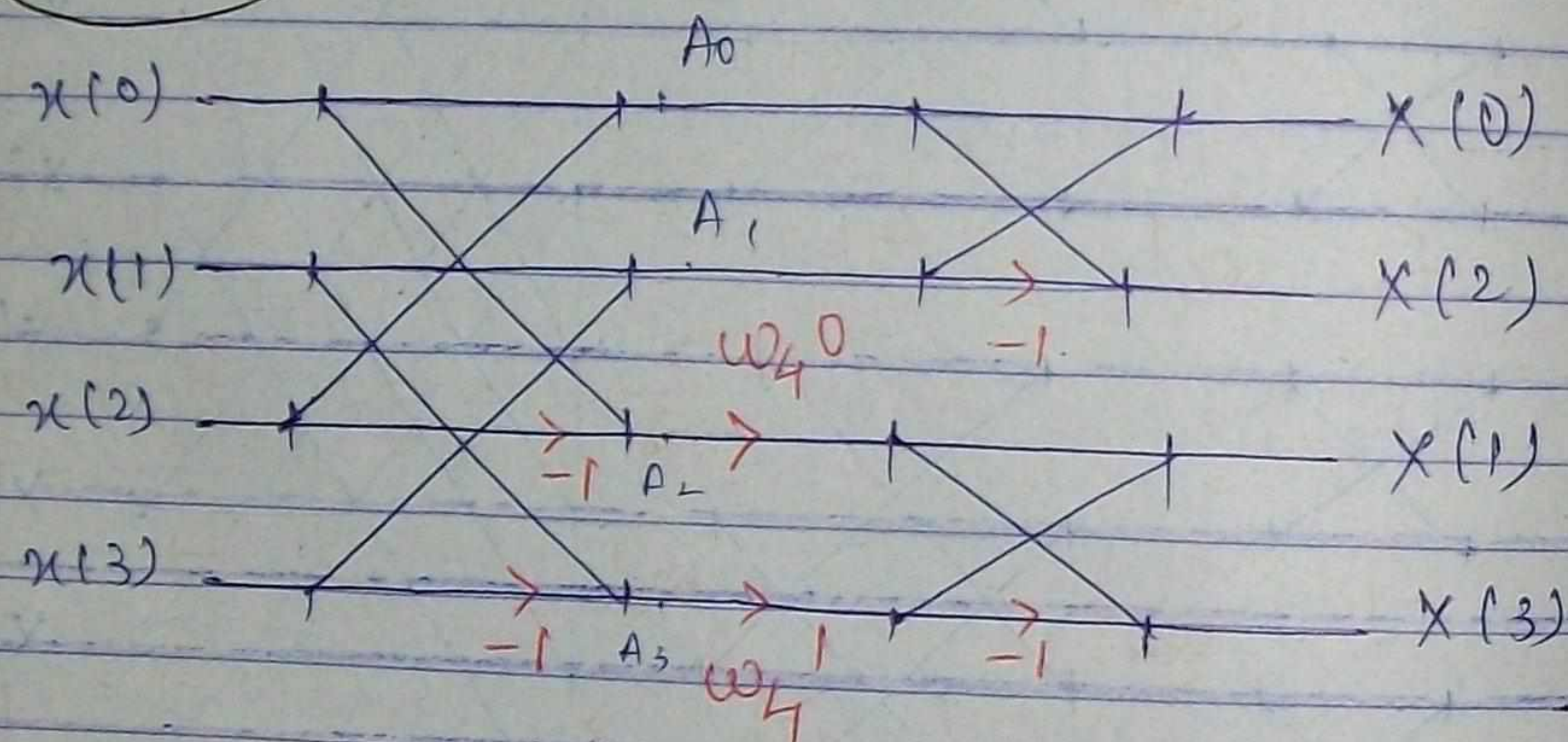
- In DIT FFT output are in serial order where as inputs are in the bit reversal order.

- In DIF FFT I/P are in serial order whereas O/P are in bit reversal order.

- No. of arithmetic operation in DIT are same as that of DIF-FFT.

⇒ DIF-FFT -

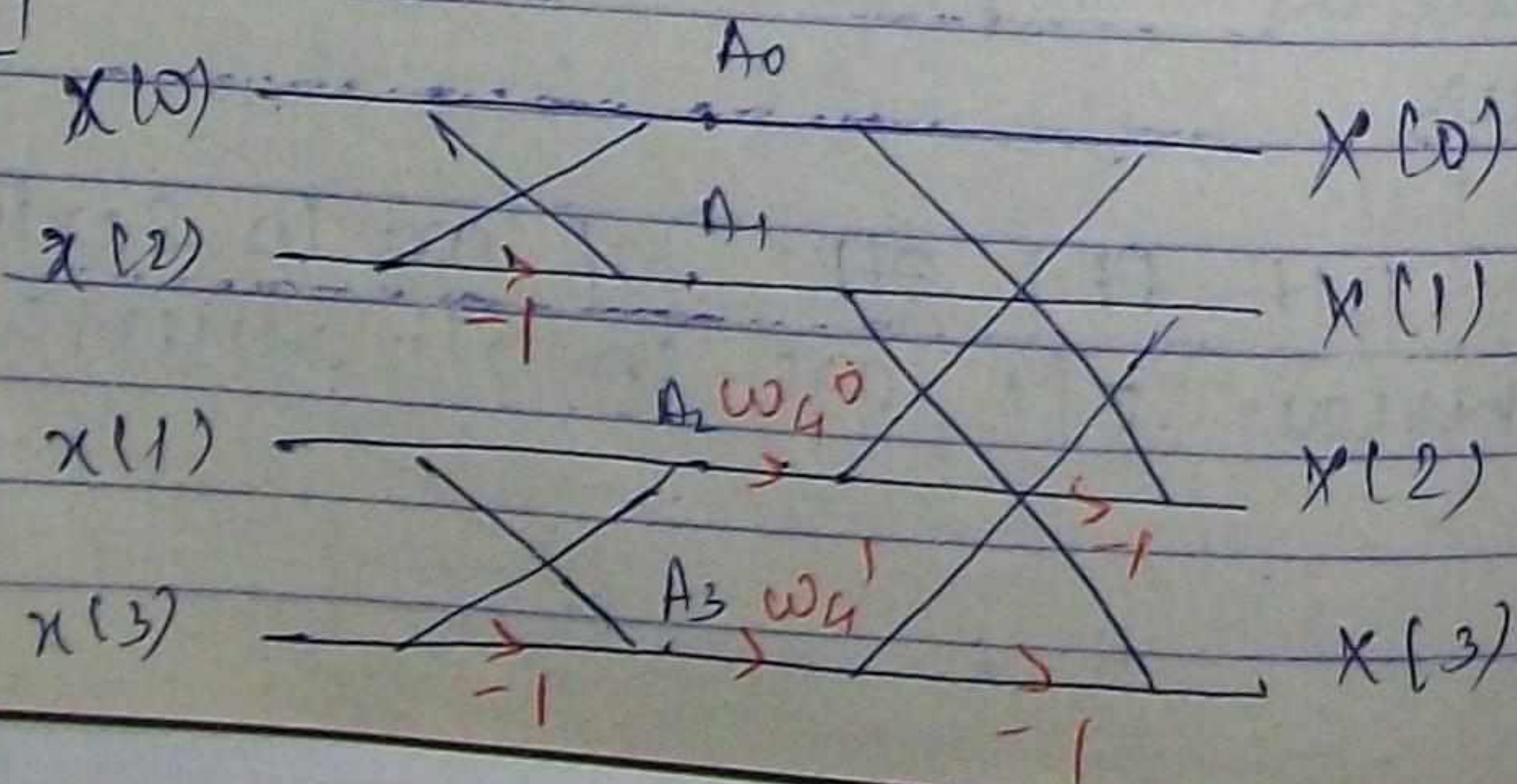
for $N=4$



Ques. Find the DFT of $x(n)$ using
(a) DIT-FFT (b) DIF-FFT

$$x(n) = \{1, 2, 3, 4\}$$

Solⁿ: (a) DIT-FFT



$$A_0 = x(0) + x(2) = 1 + 3 = 4$$

$$A_1 = x(0) - x(2) = 1 - 3 = -2$$

$$A_2 = x(1) + x(3) = 2 + 4 = 6$$

$$A_3 = x(1) - x(3) = 2 - 4 = -2$$

$$X(0) = A_0 + w_4^0 \cdot A_2 = 4 + 6 = 10$$

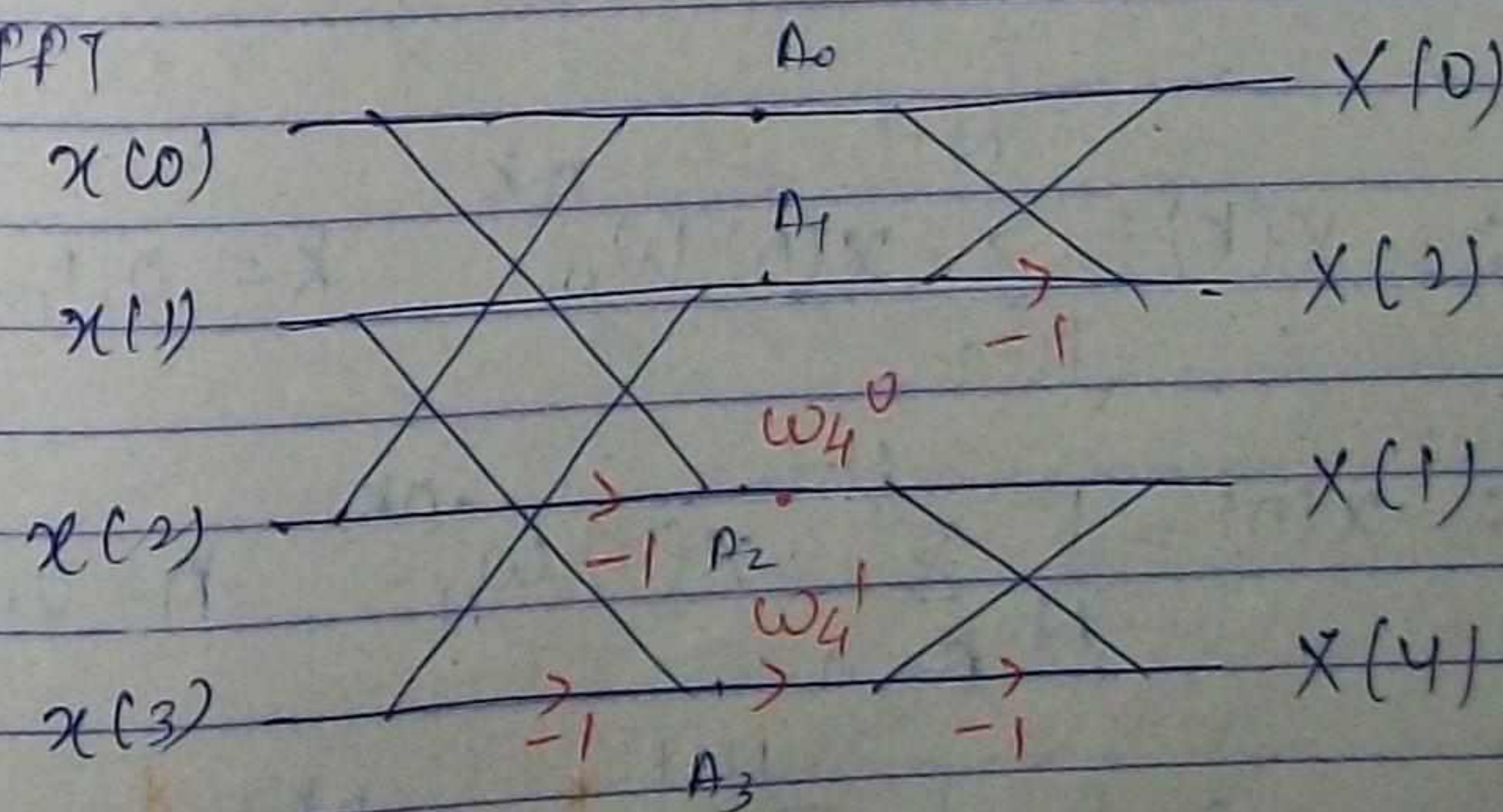
$$X(1) = A_1 + w_4^1 A_3 = -2 - j(-2) = -2 + j2$$

$$X(2) = A_0 + w_4^0 A_2 = 4 - 6 = -2$$

$$X(3) = A_1 - w_4^1 A_3 = -2 - 2j$$

$$X(k) = \{10, -2 + j2, -2, -2 - 2j\}$$

(b) DIF-FFT



$$A_0 = x(0) + x(2) = 3 + 1 = 4$$

$$A_1 = x(1) + x(3) = 2 + 4 = 6$$

$$A_2 = x(0) - x(2) = 3 - 1 = 2$$

$$A_3 = x(1) - x(3) = 2 - 4 = -2$$

$$x(0) = A_0 + A_1 = 10$$

$$x(2) = A_0 - A_1 = -2$$

$$x(1) = \omega_4^0 A_2 + \omega_4^1 A_3 = -2 - j(-2)$$

$$x(3) = \omega_4^0 A_2 - \omega_4^1 A_3 = -2 + j(-2)$$

$$x(k) = \{10, -2 + j2, -2, -2 - j2\}$$

$$\text{DFT: } X(k) = \sum_{n=0}^{N-1} x(n) \omega_N^{nk}, \quad k = 0, 1, \dots, (N-1)$$

$$\text{IDFT: } x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) \omega_N^{-nk}, \quad n = 0, 1, \dots, (N-1)$$

$$= \frac{1}{N} \left[\sum_{k=0}^{N-1} X(k) \omega_N^{nk} \right]^*$$

$$\text{IDFT} = \frac{1}{N} [\text{DFT O/P}]^*$$

⇒ PROCEDURE TO COMPUTE IDFT USING SIGNAL FLOW GRAPH OF FFT

STEP ①: Determine $X^*(k)$ i.e. conjugate of $X(k)$

STEP ②: Find the O/P for the I/P $X^*(k)$ using signal flow graph

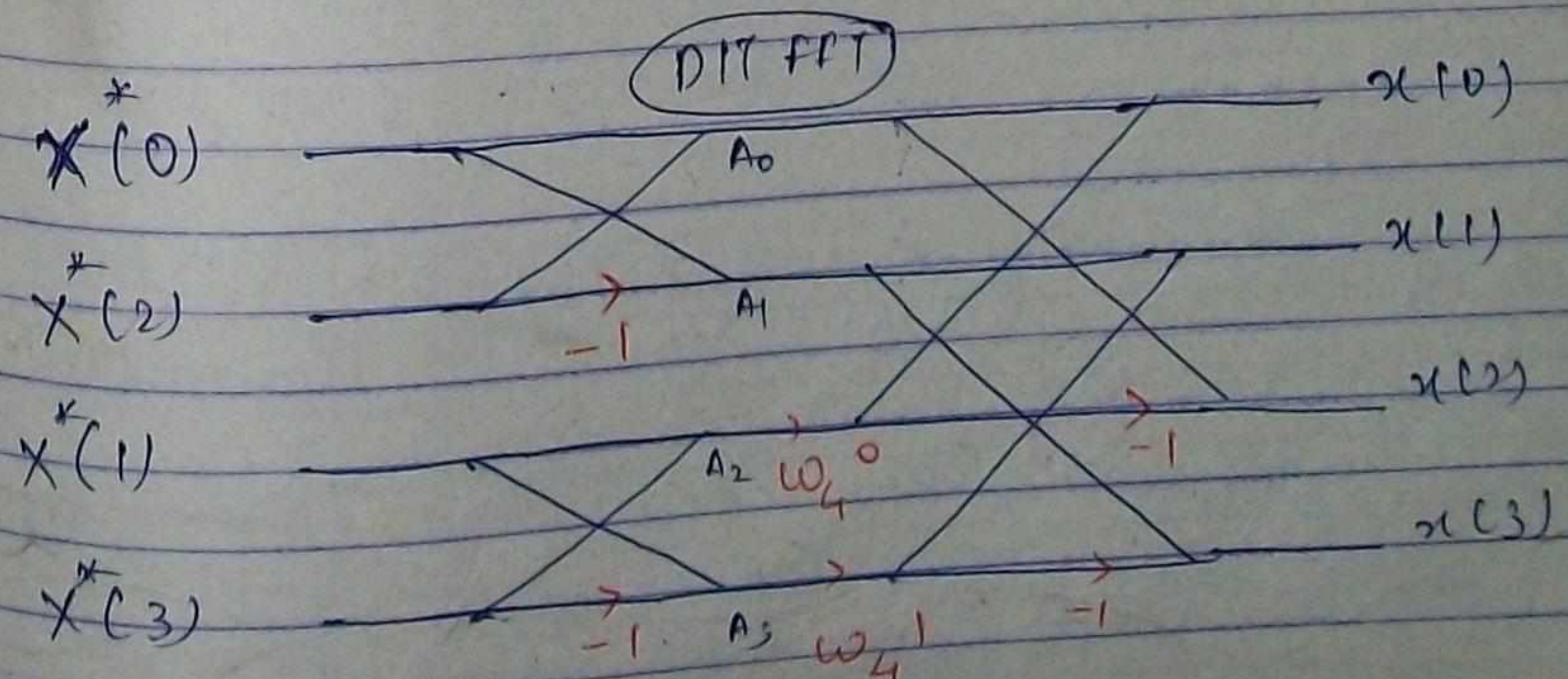
STEP ③: Take the conjugate of the sequence obtained in step ②.

STEP ④: Divide the sequence obtained in step ③ by N .

Ques. Find the IDFT using DIT-FFT

$$X(k) = \{10, -2 + j2, -2, -2 - j2\}$$

$$\text{Sol}^n: X^*(k) = \{10, -2 - j2, -2, -2 + j2\}$$



$$A_0 = X^*(0) + X^*(2) = 10 + (-2) = 8$$

$$A_1 = X^*(0) - X^*(2) = 10 - (-2) = 12$$

$$A_2 = X^*(1) + X^*(3) = -2 + 2 - 2 + 2 = -4$$

$$A_3 = X^*(1) - X^*(3) = -2 + 2 + 2 - 2 = -4$$

$$x(0) = A_0 + \omega_4^0 A_2 = 8 + 4 = 12$$

$$x(1) = A_1 + \omega_4^1 A_2 = 12 - 1(-4) = 16$$

$$x(2) = A_0 - \omega_4^0 A_2 = 8 - 4 = 4$$

$$x(3) = A_1 - \omega_4^1 A_2 = 12 - (-1)(-4) = 8$$

$$x(n) = \{4, 8, 12, 16\}$$

$$x(n) = \{1, 2, 3, 4\}$$

SAMPLING

⇒ SAMPLING THEOREM -

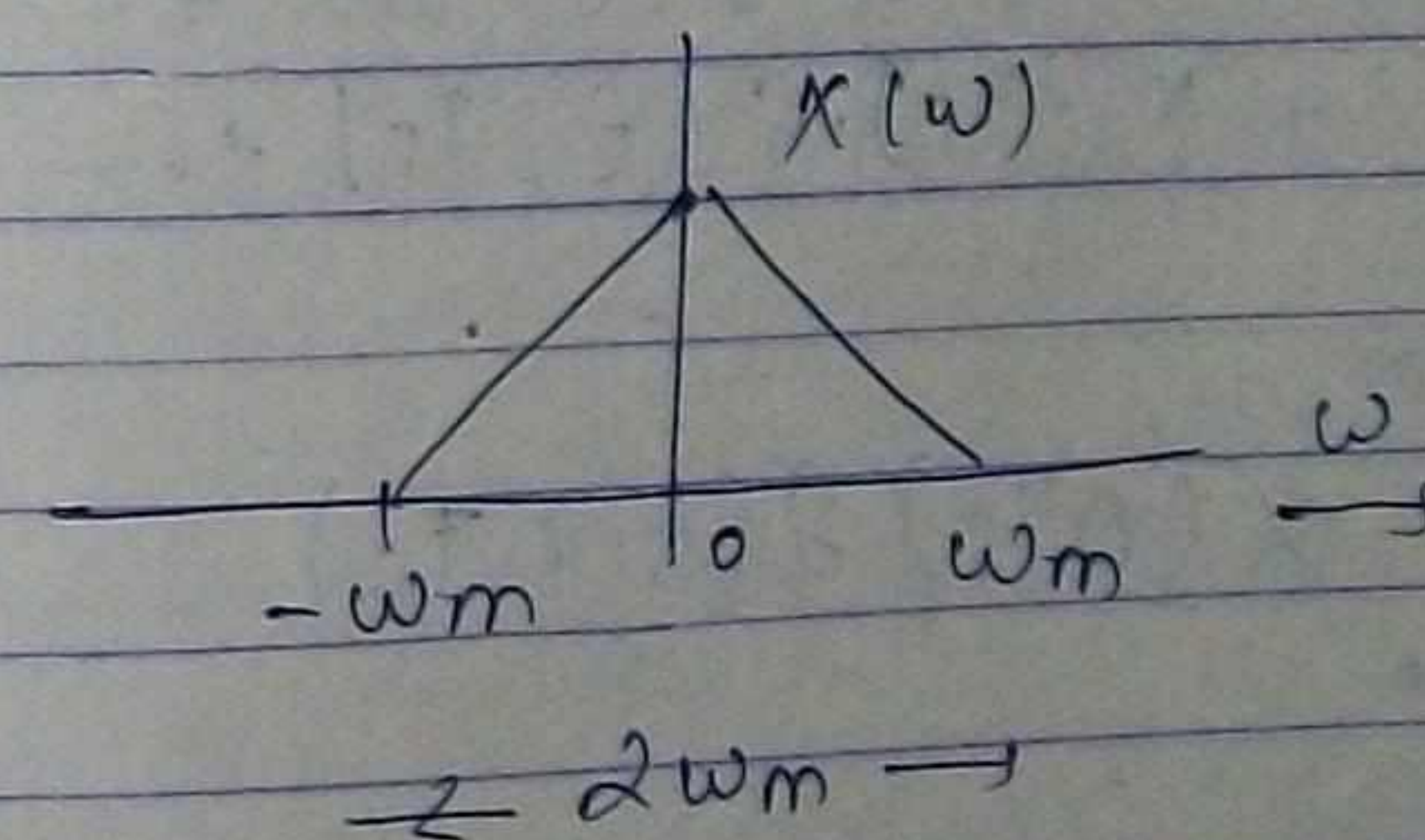
It states that 'Any bandlimited signal can uniquely be represented by collecting its sample at the regular intervals if and only if sampling freq. (f_s) is greater than or equal to twice the max. freq. (f_m) present in the signal.

$$\omega_s \geq 2\omega_m$$

$$f_s \geq 2f_m$$

Signal $x(t)$ is said to be band limited if

$$X(\omega) = 0, \text{ for } |\omega| > \omega_m$$



Bandlimited signal occupies finite range of frequencies.

Let $x(t)$: be the signal to be sampled
 $p(t)$: is the periodic train of impulse repeating for every T_s

$$P(t) = \sum_{n=-\infty}^{\infty} A \delta(t - nT_s) \quad (1)$$

$S(t)$: Is the sampled o/p.

$$S(t) = X(t) \cdot P(t) \quad (2)$$

$$= X(t) \cdot \sum_{n=-\infty}^{\infty} A \delta(t - nT_s)$$

$$= AX(t) \cdot [\delta(t) + \delta(t - T_s) + \delta(t - 2T_s) + \dots + \delta(t + T_s) + \delta(t + 2T_s) + \dots]$$

$$S(t) = X(t) \cdot \delta(t) + X(T_s) \delta(t - T_s) + \dots + X(-T_s) \delta(t + T_s) + \dots$$

$$S(t) = \sum_{n=-\infty}^{\infty} X(nT_s) \delta(t - nT_s) \quad (3)$$

Taking the Fourier transform of eq (2)

$$S(\omega) = \frac{1}{2\pi} X(\omega) * P(\omega)$$

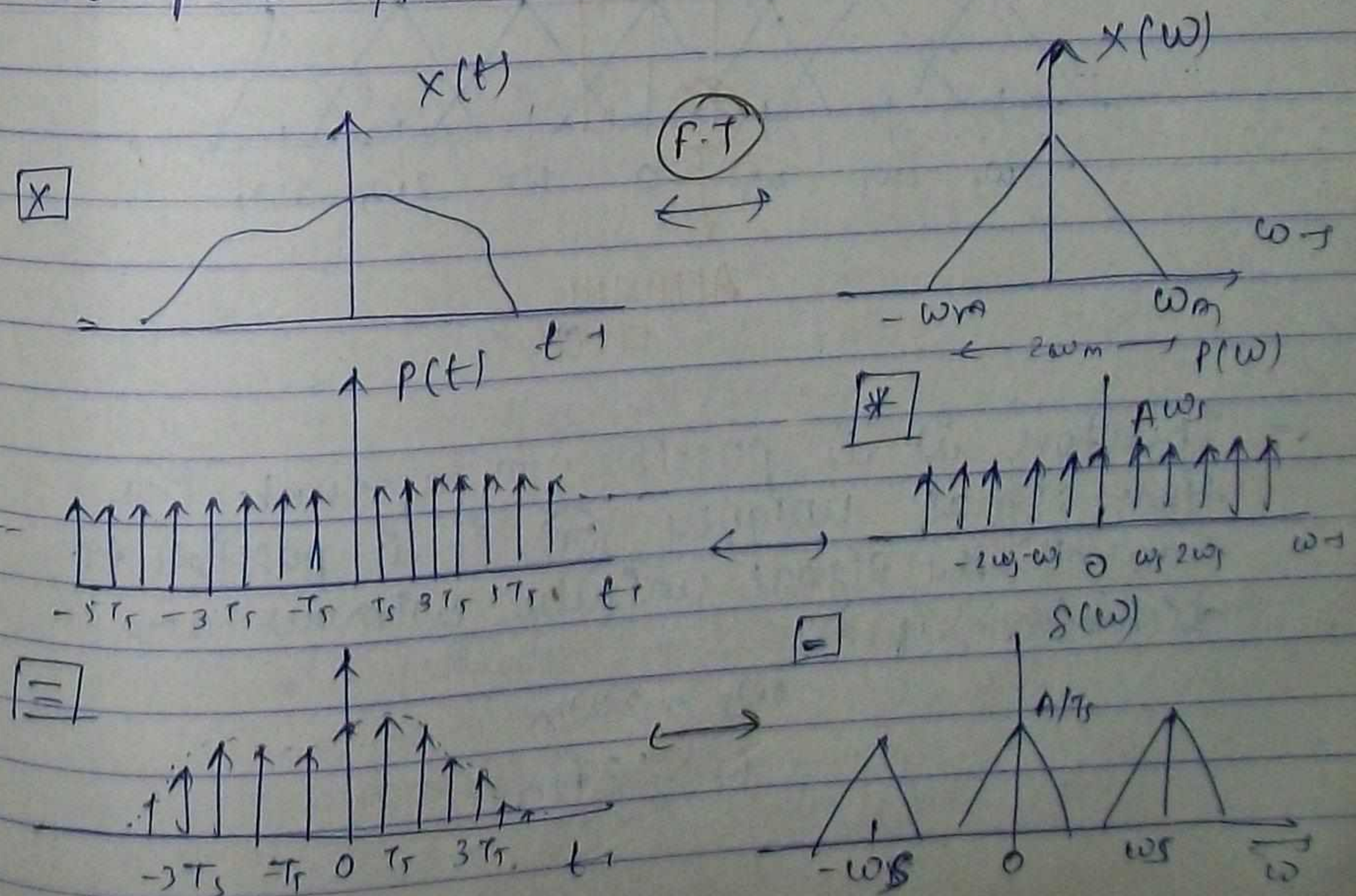
$$= \frac{1}{2\pi} X(\omega) * A \omega_s \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_s)$$

$$= \frac{A \omega_s}{2\pi} \left[X(\omega) * \delta(\omega) + \delta(\omega - \omega_s) + \dots + \delta(\omega + \omega_s) + \dots \right]$$

$$= \frac{A}{T_s} \left[X(\omega) + X(\omega - \omega_s) + \dots + X(\omega + \omega_s) + \dots \right]$$

$$S(\omega) = \frac{A}{T_s} \sum_{n=-\infty}^{\infty} X(\omega - n\omega_s) \quad (4)$$

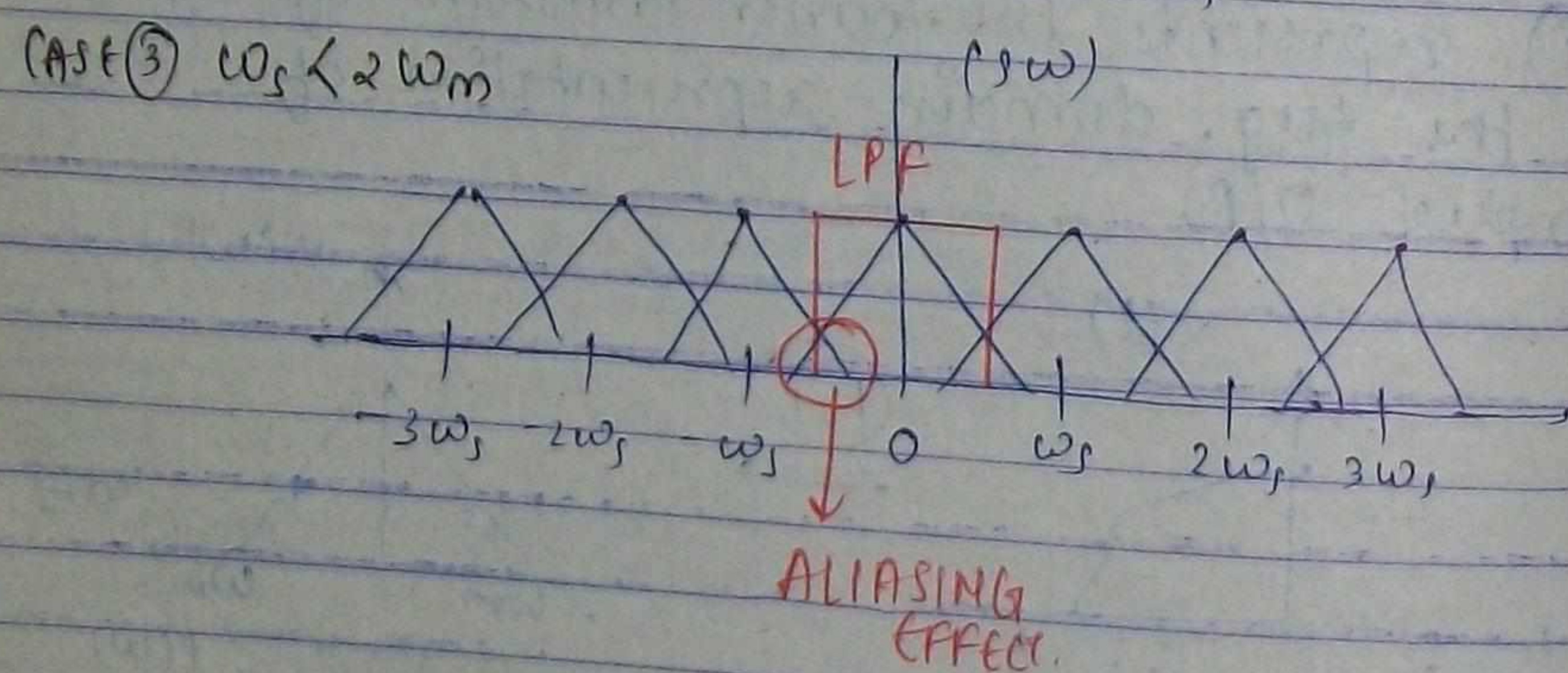
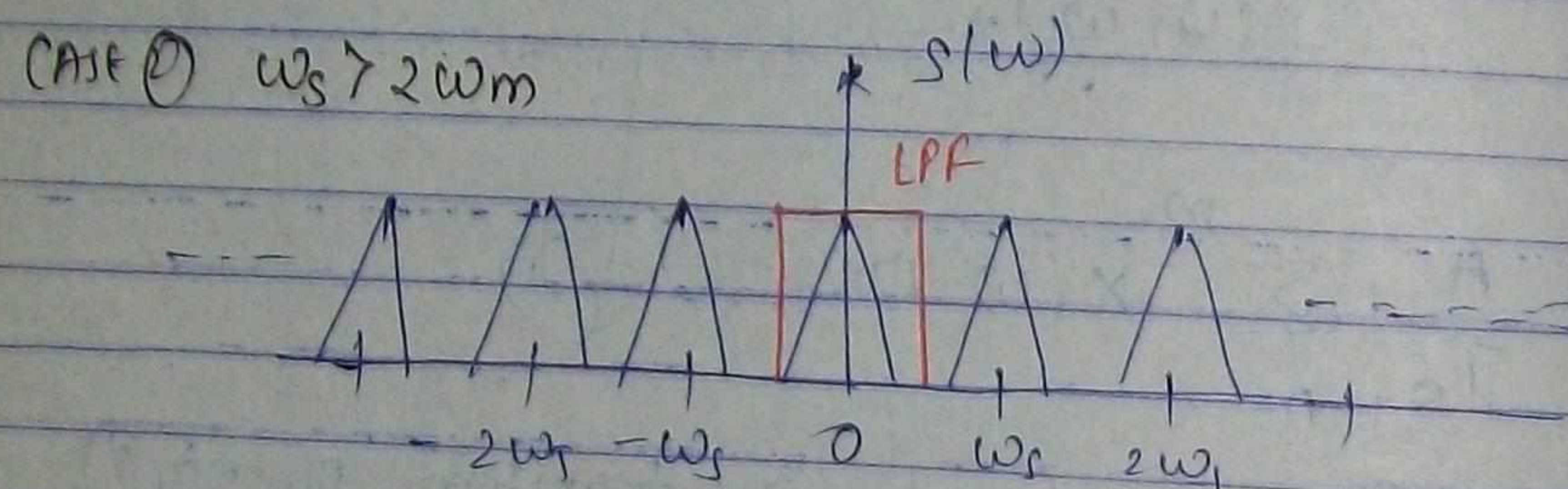
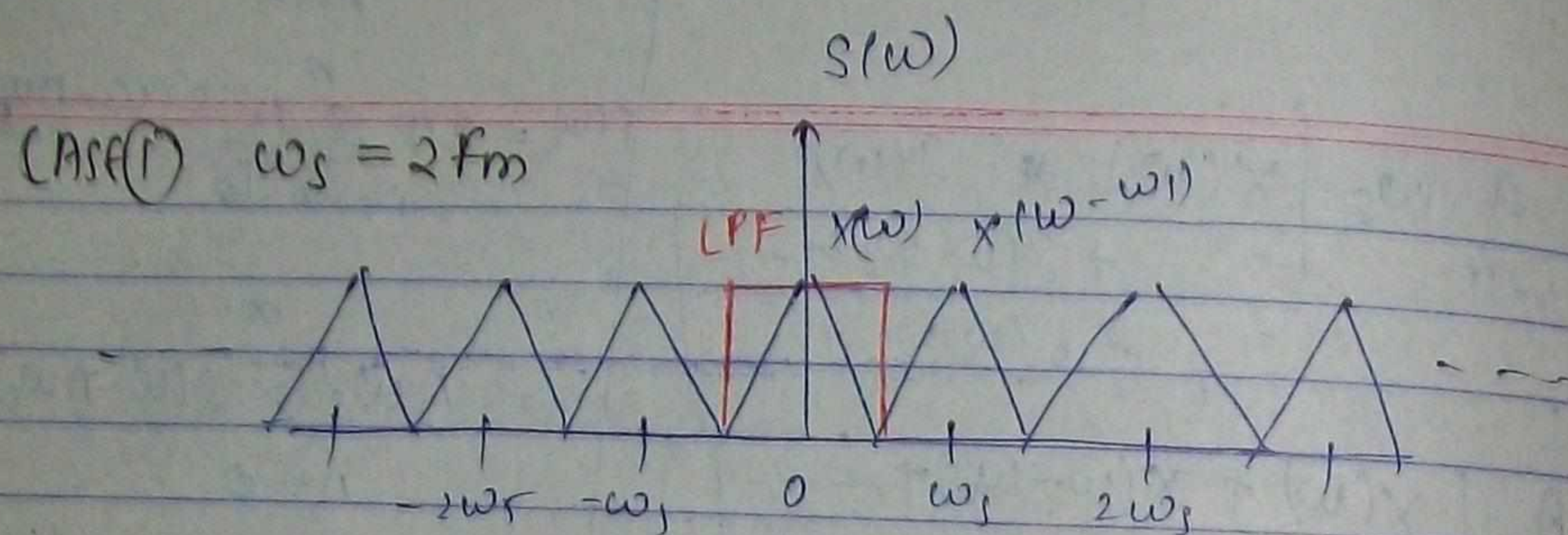
Eq (4) represents the Fourier transform of eq (3) i.e. the freq. domain representation of the sampled o/p.



$$F\{P(t)\} = 2\pi \sum_{n=-\infty}^{\infty} A \delta(\omega - n\omega_s)$$

$$P(\omega) = 2\pi \sum_{n=-\infty}^{\infty} \frac{A}{T_s} \delta(\omega - n\omega_s)$$

$$= A \omega_s \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_s)$$



Therefore it is possible to represent the time signal uniquely or it is possible to transmit the signal uniquely without any distortion if

$$\omega_s \geq 2\omega_m$$

$$f_s \geq 2f_m$$

if the min. freq. is to communicate any signal through a channel must be $2\omega_m$

$$\omega_s = 2\omega_m$$

Nyquist freq.

or
Minimum channel bandwidth

Ques find the min. sampling freq. required to transmit for the following signal.

(1) $10 \sin(50t) \cos(300t)$

Solⁿ: $5 \sin(350t) + 5 \sin(250t)$

$$\omega_m = 350$$

$$f_s = 2\omega_m$$

$$= 2 \times 350$$

$$= 700 \text{ rad/sec.}$$

(2) $-15 \sin(\pi/3t - 30^\circ) + 10 \cos(20\pi t - 30^\circ)$

$$\omega_m = 20\pi$$

$$= 2 \times 20 \times \pi = 40\pi \text{ rad/sec}$$

Ques Find Power of P.S.

$$x(t) = A \cos(\omega_1 t + \phi_1) + B \sin(\omega_2 t + \phi_2)$$

Solⁿ:

$$P = \frac{1}{T} \int_T x^2(t) dt$$

$$= \frac{1}{T} \int_T A^2 \cos^2(\omega_1 t + \phi_1) + B^2 \sin^2(\omega_2 t + \phi_2) + 2AB \cos(\omega_1 t + \phi_1) \sin(\omega_2 t + \phi_2) dt$$

$$= \frac{1}{T} \left[\frac{A^2}{2} \int_T dt + \frac{A^2}{2} \int_T \cos(2\omega_1 t + 2\phi_1) dt + \right.$$

$$\left. \frac{B^2}{2} \int_T dt - \frac{B^2}{2} \int_T \cos(2\omega_2 t + 2\phi_2) dt + \right.$$

$$\left. + AB \int_T \sin[(\omega_1 + \omega_2)t + (\phi_1 + \phi_2)] dt + \right.$$

$$\left. AB \int_T \sin[(\omega_1 - \omega_2)t - (\phi_1 - \phi_2)] dt \right]$$

$$= \frac{1}{T} \left[\frac{A^2}{2} T + \frac{B^2}{2} T \right] = \frac{A^2}{2} + \frac{B^2}{2}$$

$$P_T = P_1 + P_2$$

$$V_T = \sqrt{V_1^2 + V_2^2}$$

$$\frac{12.4}{2} = \frac{6^2}{2} = 31$$

Therefore total power of any signal is equal to sum of the power associated with individual frequencies.

$$P_T = P_1 + P_2 + \dots$$

Total RMS current is given by:

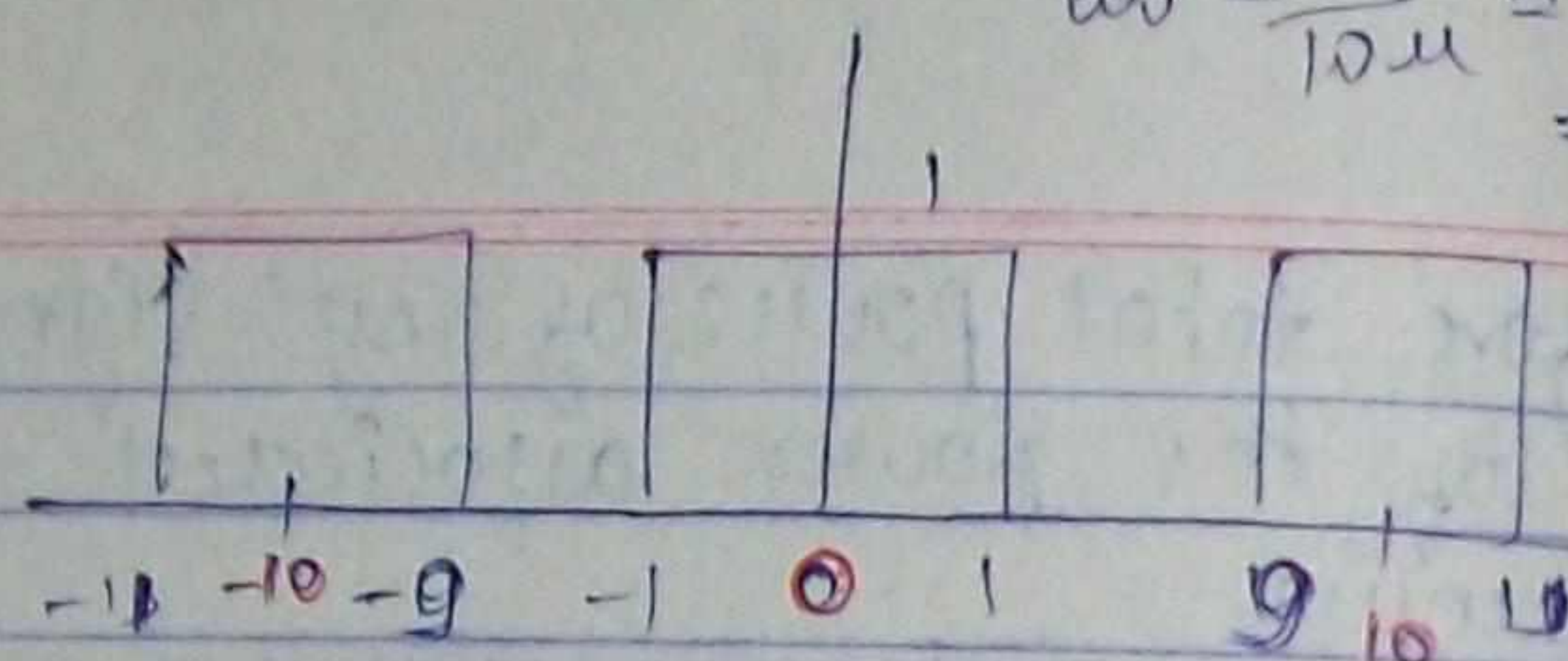
$$I_{rms} = \sqrt{I_{1r}^2 + I_{2r}^2 + I_{3r}^2 + \dots}$$

Total RMS voltage is given by:

$$V_{rms} = \sqrt{V_{1r}^2 + V_{2r}^2 + V_{3r}^2 + \dots}$$

*) Fourier series of periodic train of impulse is impulse train only and strength is equal to $\frac{A}{T}$

Ques 1 P-15



$$T = 10 \mu s$$

$$\omega_0 = \frac{2\pi}{T} = 0.2\pi \times 10^5$$

$$= 2\pi \times 10^4 \text{ rad/s}$$

Solⁿ:

$$a_0 = \frac{1}{10} \int_{-5}^{5} dt = \frac{2}{10} = 1/5$$

$$a_n = \frac{2}{10} \int_{-5}^{5} \cos(n\omega_0 t) dt$$

$$= \frac{2}{5} \int_{-5}^{5} \cos(n\omega_0 t) dt$$

$$= \left(\frac{\sin(n\omega_0 t)}{n\omega_0} \right)_{-5}^{5} \cdot \frac{2}{5}$$

$$= \frac{2}{(5 \times 10^{-6})(n \times 2\pi \times 10^4)} \sin(n \cdot 2\pi \times 10^4 \times 5 \times 10^{-6})$$

$$= \frac{2}{n\pi} \sin(0.2\pi n)$$

$$x(t) = \frac{1}{5} + \sum_{n=1}^{\infty} \frac{2}{n\pi} \sin(0.2\pi n) \cos(2\pi \times 10^4 n t)$$

Ques 2 P-15

$$f(t) = \sum_{n=-\infty}^{\infty} \frac{3}{4 + (n\pi)^2} e^{jn\pi t}$$

Solⁿ:

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t} \quad \text{--- (1)}$$

Efs is given by eq (1). Comparing (1) with given eqⁿ

$$c_n = \frac{3}{4 + (n\pi)^2} \quad \text{--- (2)} \quad \omega_0 = \pi$$

$$T = \frac{2\pi}{\omega_0} = \underline{\underline{2 \text{ sec}}}$$

$$\text{--- (3)} \quad = A \cos(3\pi t)$$

$$= A_3 \cos(3\omega_0 t) \quad \text{--- (3)}$$

Comparing the freq. eq (3) with ω_0 it is $3\omega_0$ is

A $\cos 3\pi t$ represent the 3rd harmonic of the fundamental freq $\omega_0 = \pi$ therefor $A = A_3$

from the rule b/w the Tfs and Efs coefficient

$$a_n = c_{n+G-3}$$

$$a_3 = c_{3+G-3} = \frac{3}{4 + (3\pi)^2} + \frac{3}{4 + (-3\pi)^2}$$

$$= \frac{6}{4 + (3\pi)^2}$$

⇒ GRAPHICAL EVALUATION OF THE CONVOLUTION -

$$y(t) = x(t) * h(t)$$

$$= \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau) d\tau$$

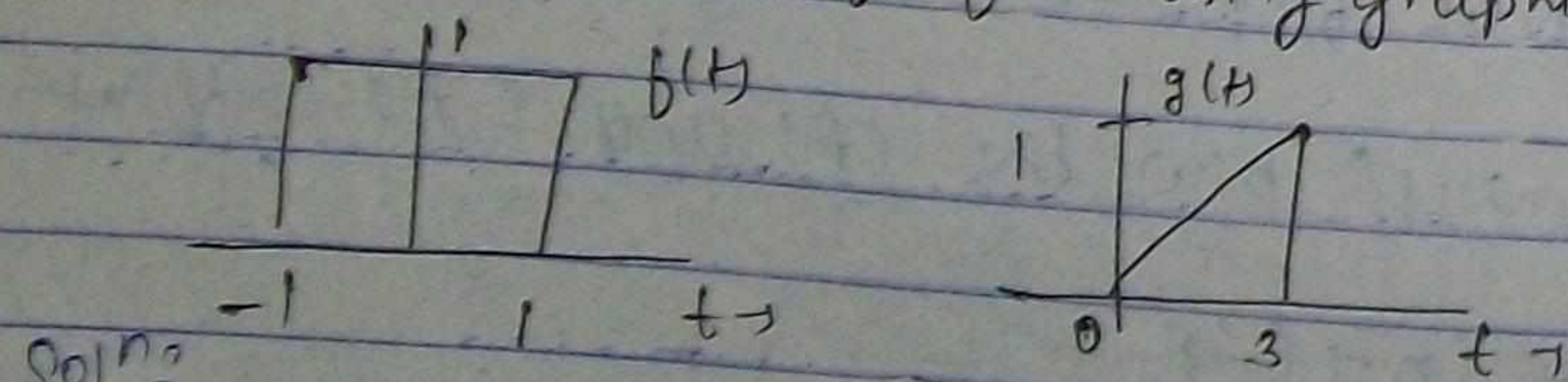
$$= \int_{-\infty}^{\infty} x(\tau) \cdot h(-\tau+t) d\tau$$

⇒ PROCEDURE -

① sketch $x(\tau)$ from $x(t)$ by changing t with τ .

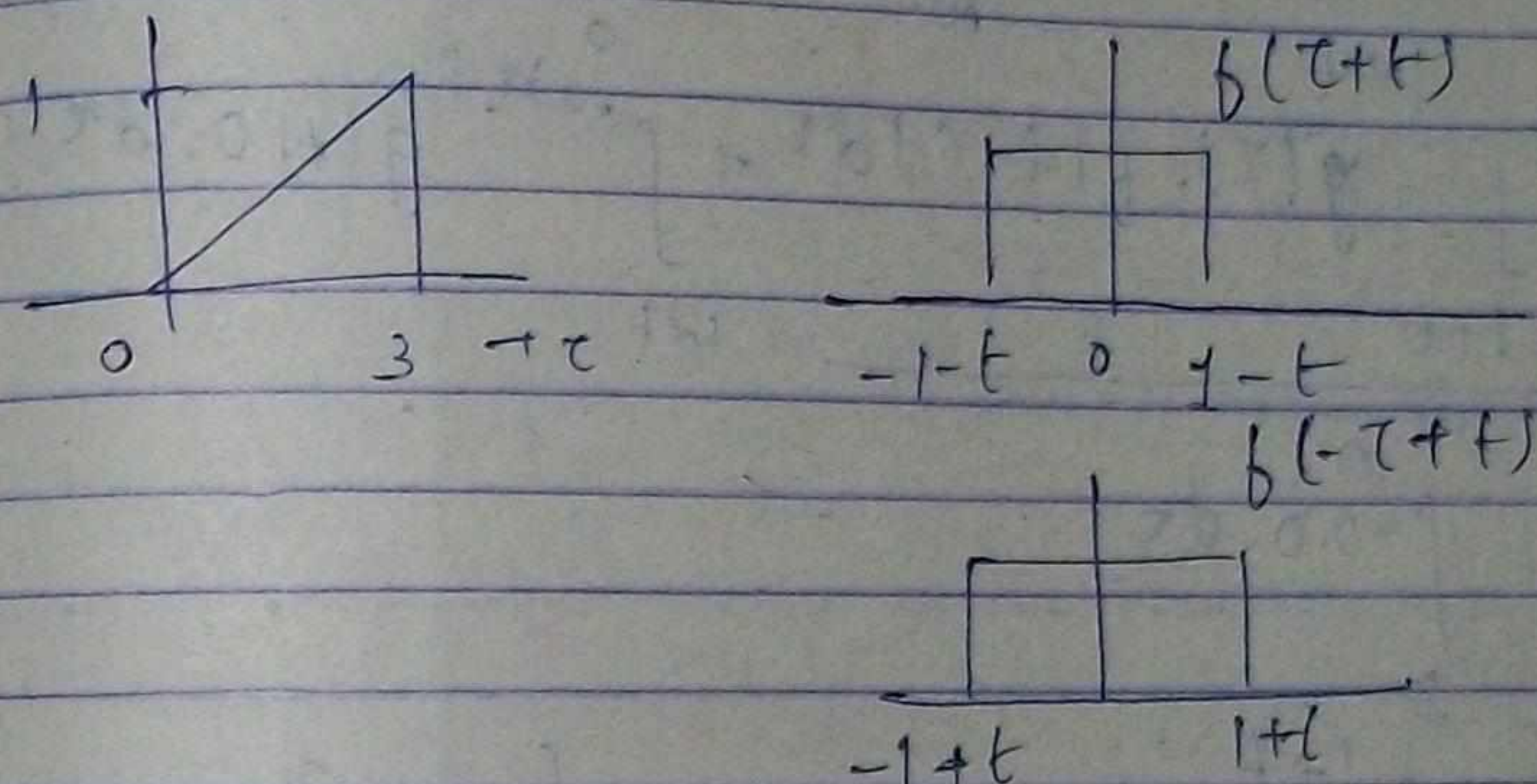
② sketch $h(-\tau+t)$ from $h(t)$ by replacing t with $-\tau+t$. then shift $h(-\tau+t)$ from $-\infty$ to ∞ for various values of t then evaluate the area under the product signal $x(\tau)h(t-\tau)$ using the convolution integral

Ques: find the convolⁿ of follⁿ using graphical method,

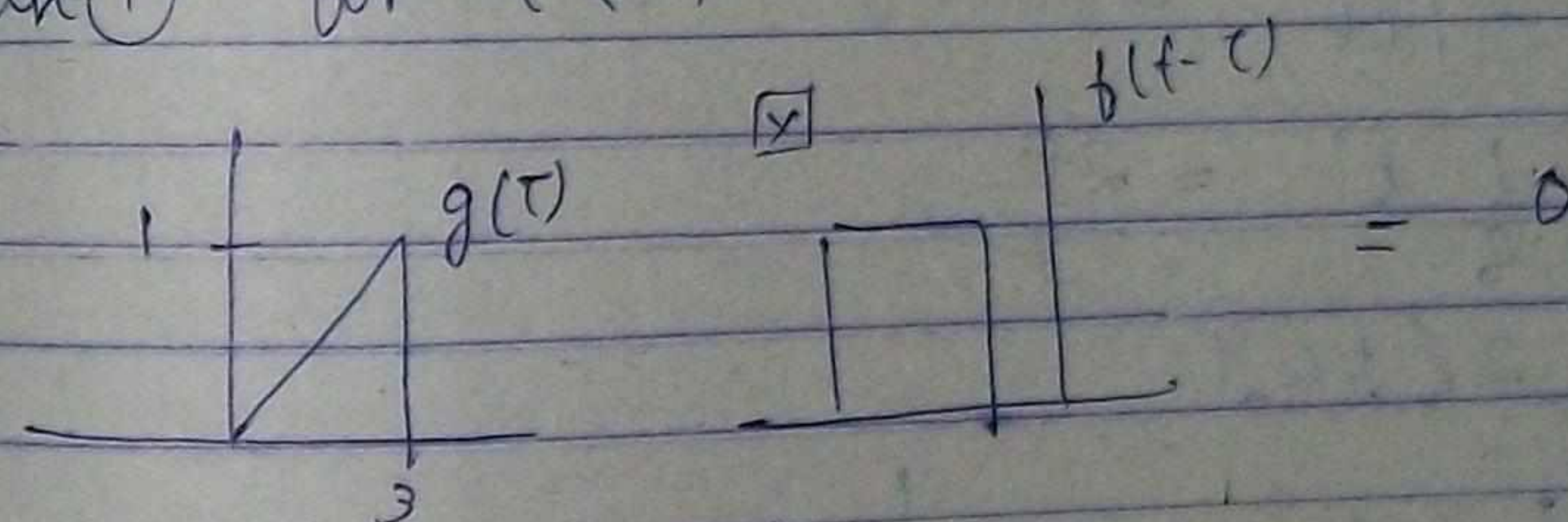


$$y(t) = g(t) * f(t) = \int_{-\infty}^{\infty} g(\tau) f(t-\tau) d\tau$$

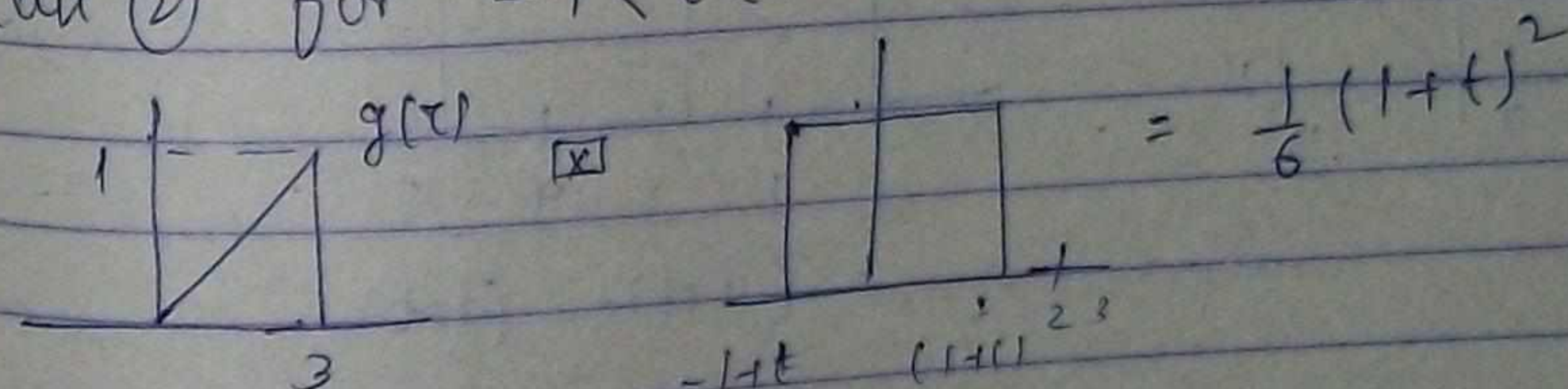
$$f(t) * h(t) = (-1, 1), (0, 3) \Rightarrow (-1, 1, 2, 4)$$



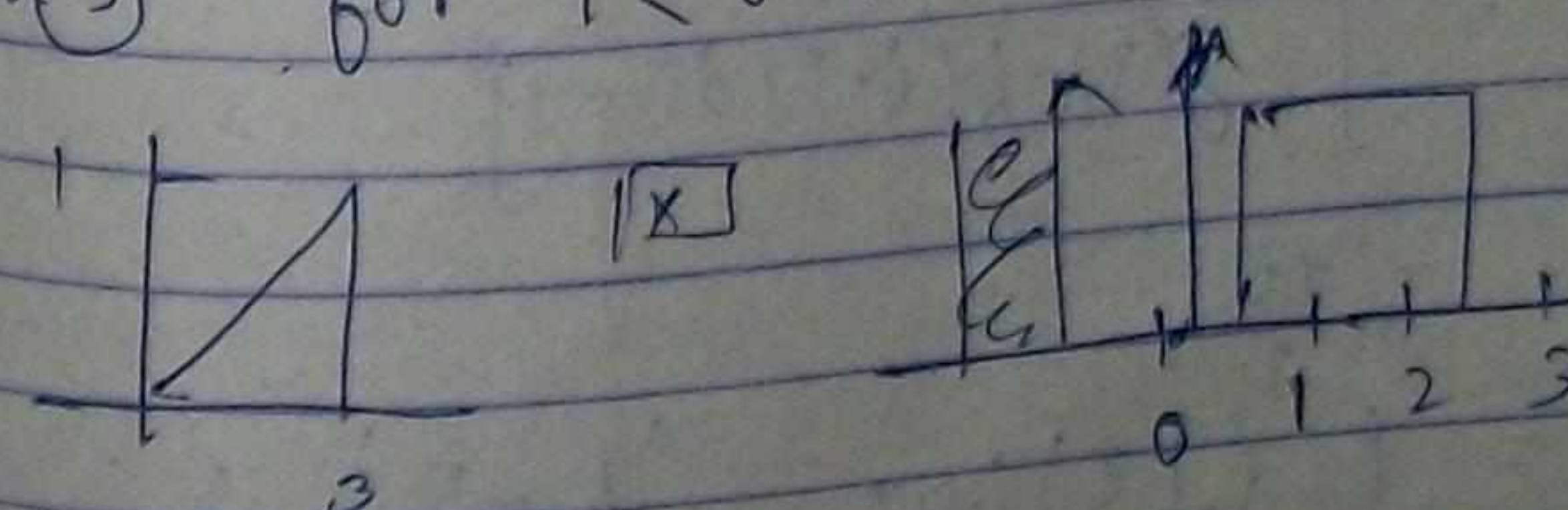
Case ① for $t < -1$



Case ② for $-1 < t < 1$



Case ③ for $1 < t < 2$



$$\int_{-\infty}^{\infty} g(\tau) \cdot f(t-\tau) d\tau = \int_{-\infty}^0 0 \cdot 0 d\tau + \int_0^{-1+t} g(\tau) \cdot 0 d\tau$$

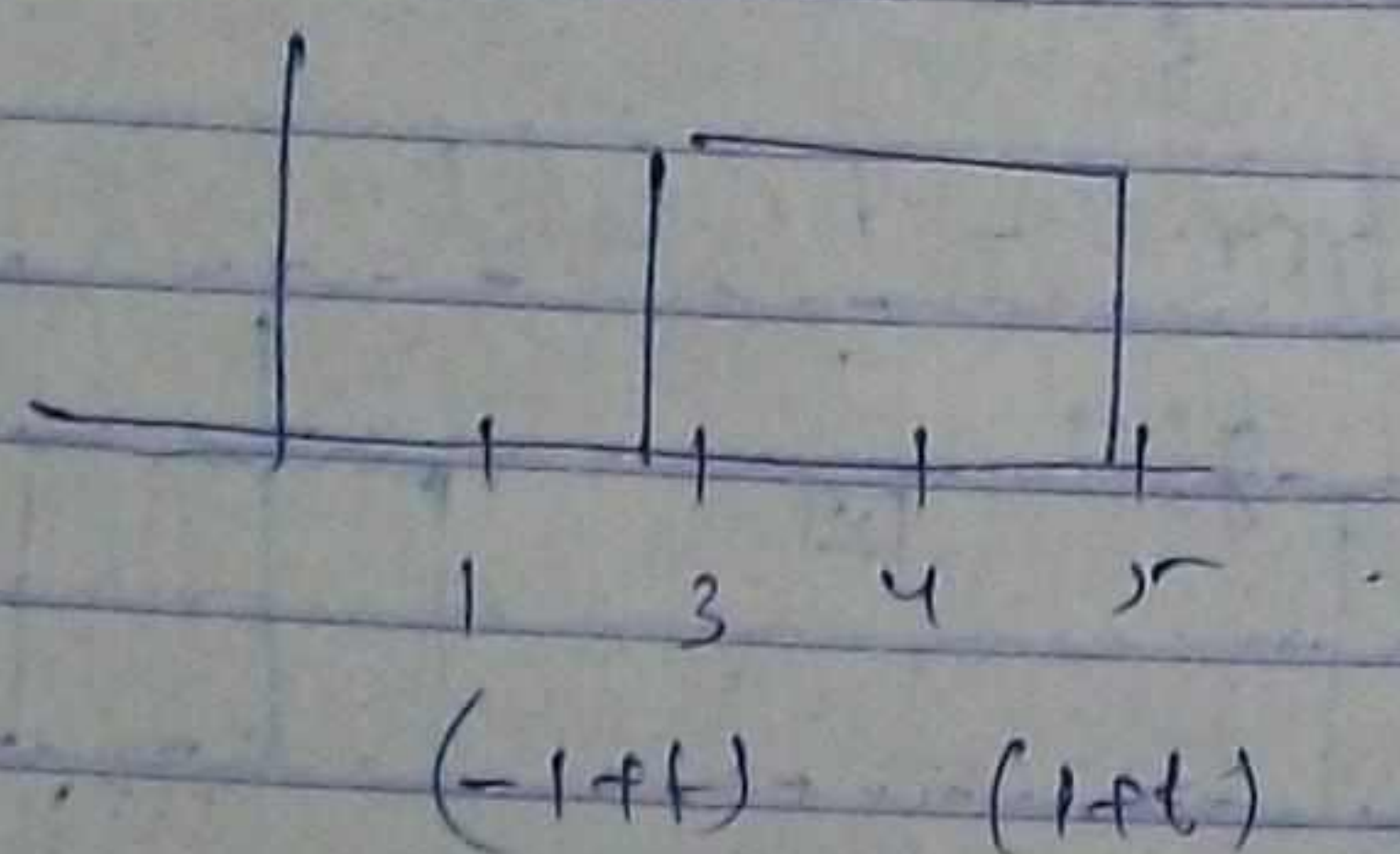
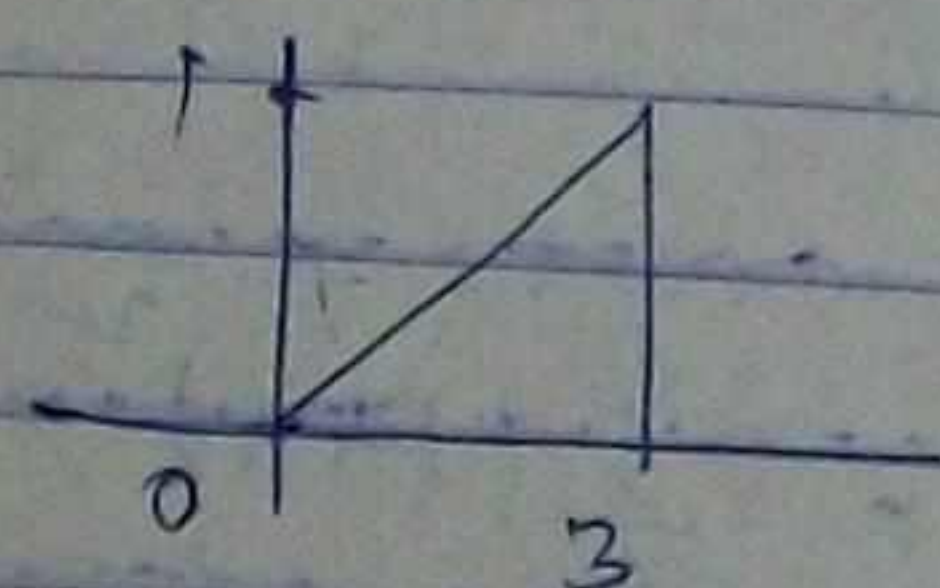
$$+ \int_{-1+t}^{1+t} g(\tau) \cdot f(t-\tau) d\tau + \int_{1+t}^3 g(1) \cdot 0 d\tau$$

$$+ \int_3^{\infty} 0 \cdot 0 d\tau$$

$$= \int_{-1+t}^{1+t} \frac{1}{3} \tau \cdot 1 d\tau = \frac{1}{6} \left[(1+t)^2 - (-1+t)^2 \right]$$

$$= \frac{1}{6} \cdot 4t = \frac{2t}{3}$$

Case (ii): $2 \leq t \leq 4$.



$$\int_{-\infty}^{\infty} g(\tau) \cdot f(t-\tau) d\tau = \int_{-1+t}^3 g(\tau) \cdot f(t-\tau) d\tau + \int_3^{\infty} g(\tau) \cdot f(t-\tau) d\tau$$

$$= \int_{-1+t}^3 \left(\frac{1}{3} \tau \right) \cdot 1 d\tau = \left(\frac{1}{6} \tau^2 \right) \Big|_{-1+t}^3$$

$$= \frac{1}{6} (-t^2 + 2t + 8)$$

Ans: $y(t) = 0$, $t < -1$

$$= \frac{1}{6} (t+1)^2, -1 \leq t \leq 1$$

$$= \frac{2}{3} t, 1 \leq t \leq 2$$

$$= -\frac{1}{6} (t^2 - 2t - 8), 2 \leq t \leq 4$$

$$= 0 \text{ for } t > 4$$