

SYALLABUS -

1. BASICS
2. STEADY STATE A.C. CIRCUITS (RESONANCE)
3. THEOREMS
4. TRANSIENT
5. TWO PORT
6. GRAPH THEORY
7. FILTERS
8. SYNTHESIS.

① Magnetic coupled circuit.

② Filters

③ Two port

④ Graph theory

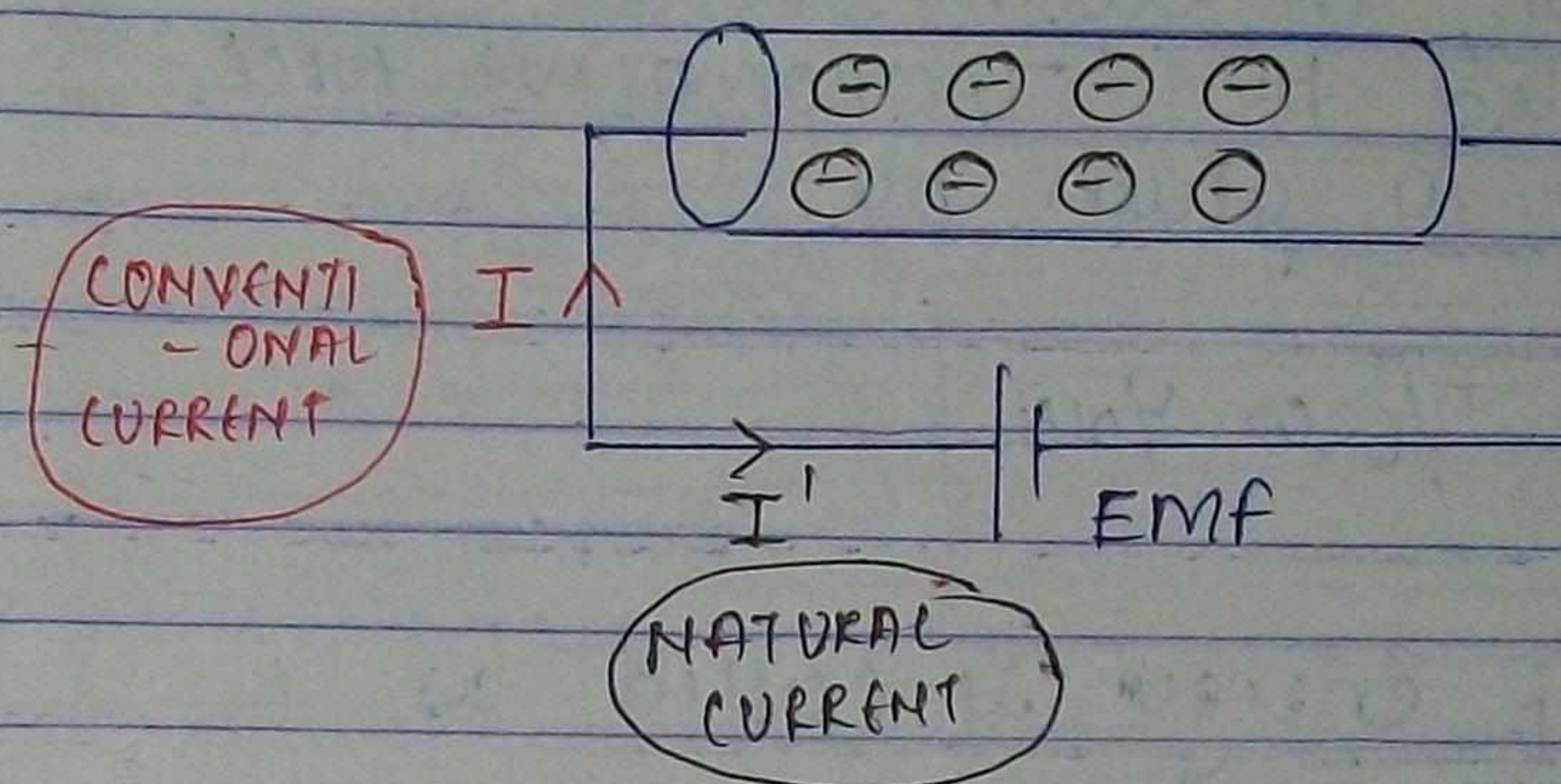
⑤ Laplace.

⑥ Synthesis.

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BASIC CONCEPTS

17/10/11



- The basic quantity in the electric circuit is charge. The charge of an e^- is given by

$$e^- = -1.602 \times 10^{-19} \text{ C}$$

- The flow of e^- s is called as current.

(OR)
The time rate of charge is also called as current

$$I = \frac{dQ}{dt} \quad \text{C/s or Amp.}$$

- By using conventional current KVL & KCL equations are developed.
- To move a e^- from one point to other point in particular direction external

force is required, in electric circuit external force is provided by ELECTROMOTIVE FORCE (EMF). And it is given by.

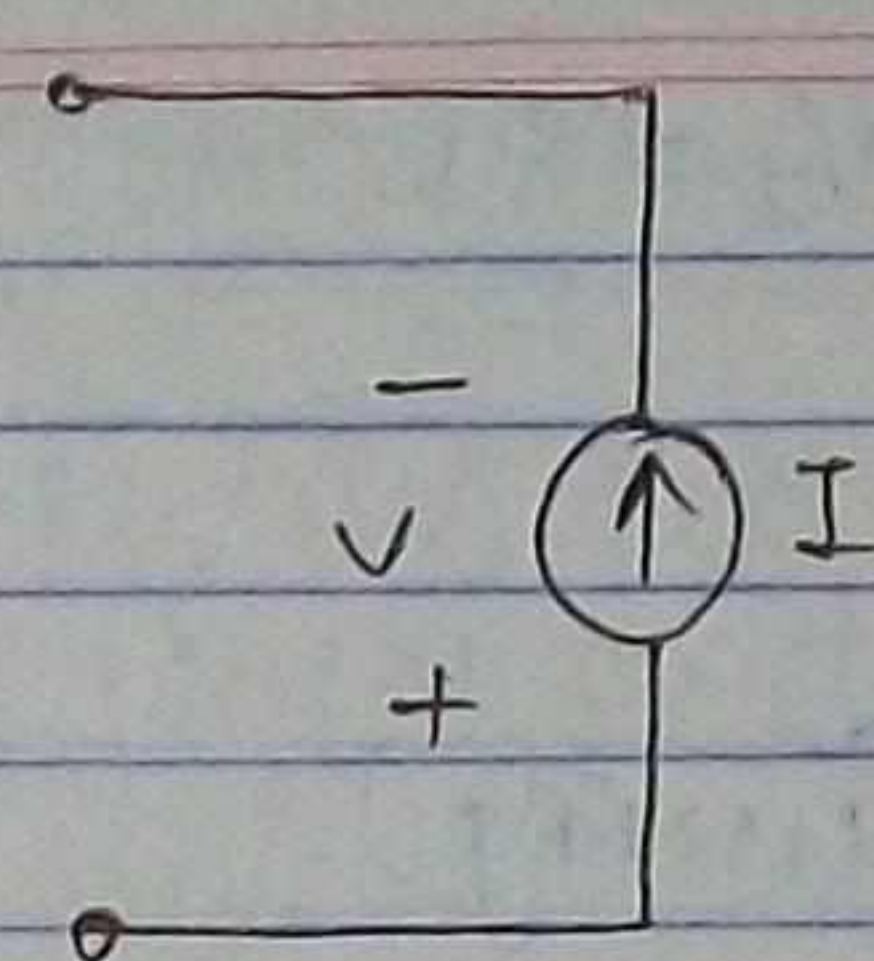
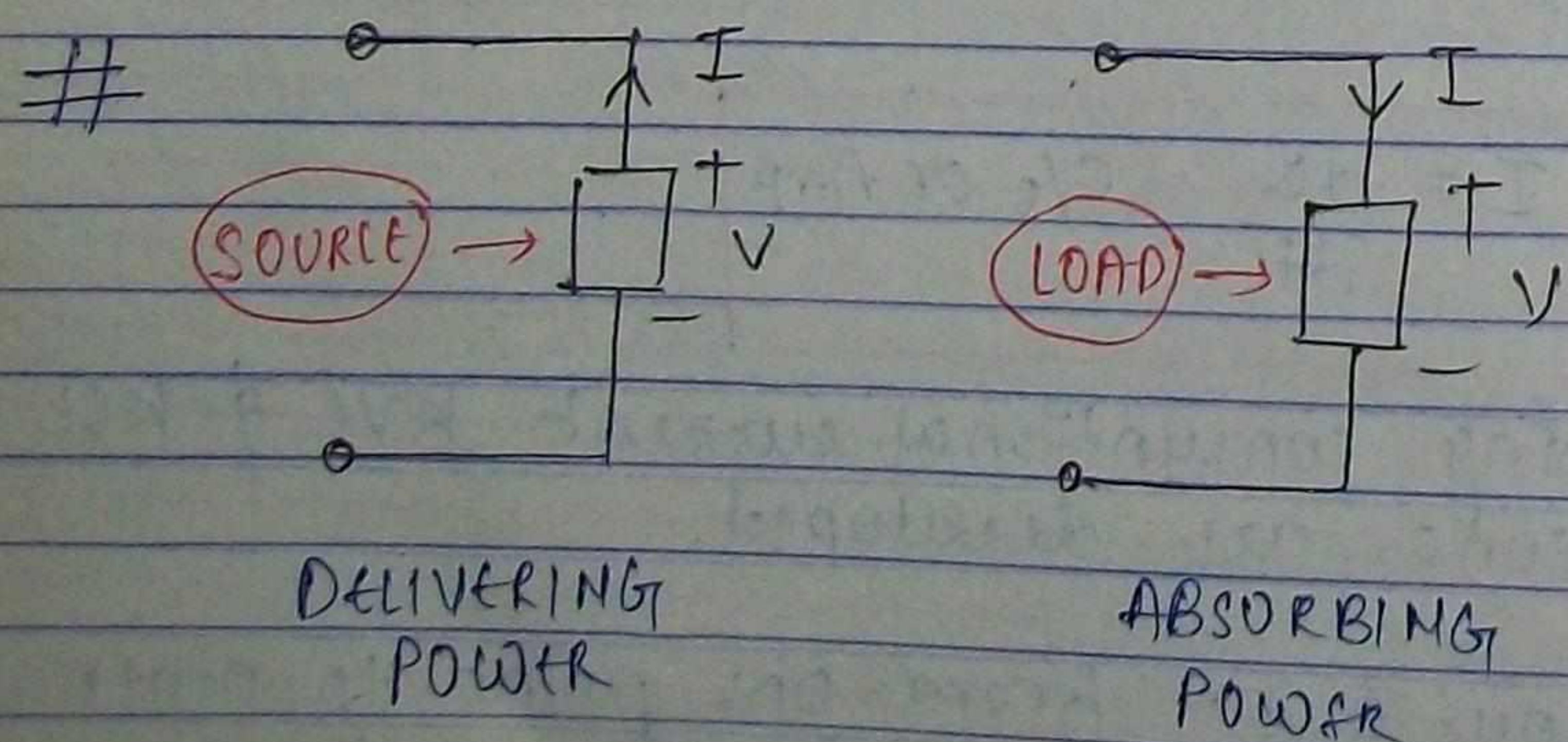
$$V = \frac{dW}{dQ} \text{ J/C or Volts.}$$

- The time rate of energy is called as power and given by:

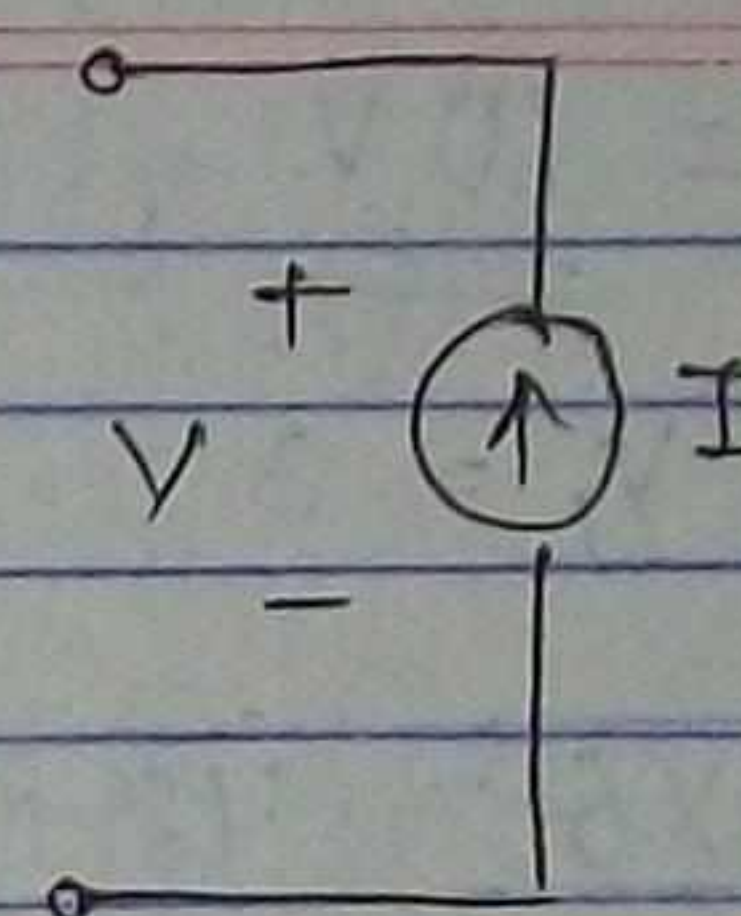
$$P = \frac{dW}{dt} \text{ J/s or watt}$$

$$P = \frac{dW}{dQ} \cdot \frac{dQ}{dt}$$

$$P = V \cdot I$$



ABSORBING
POWER

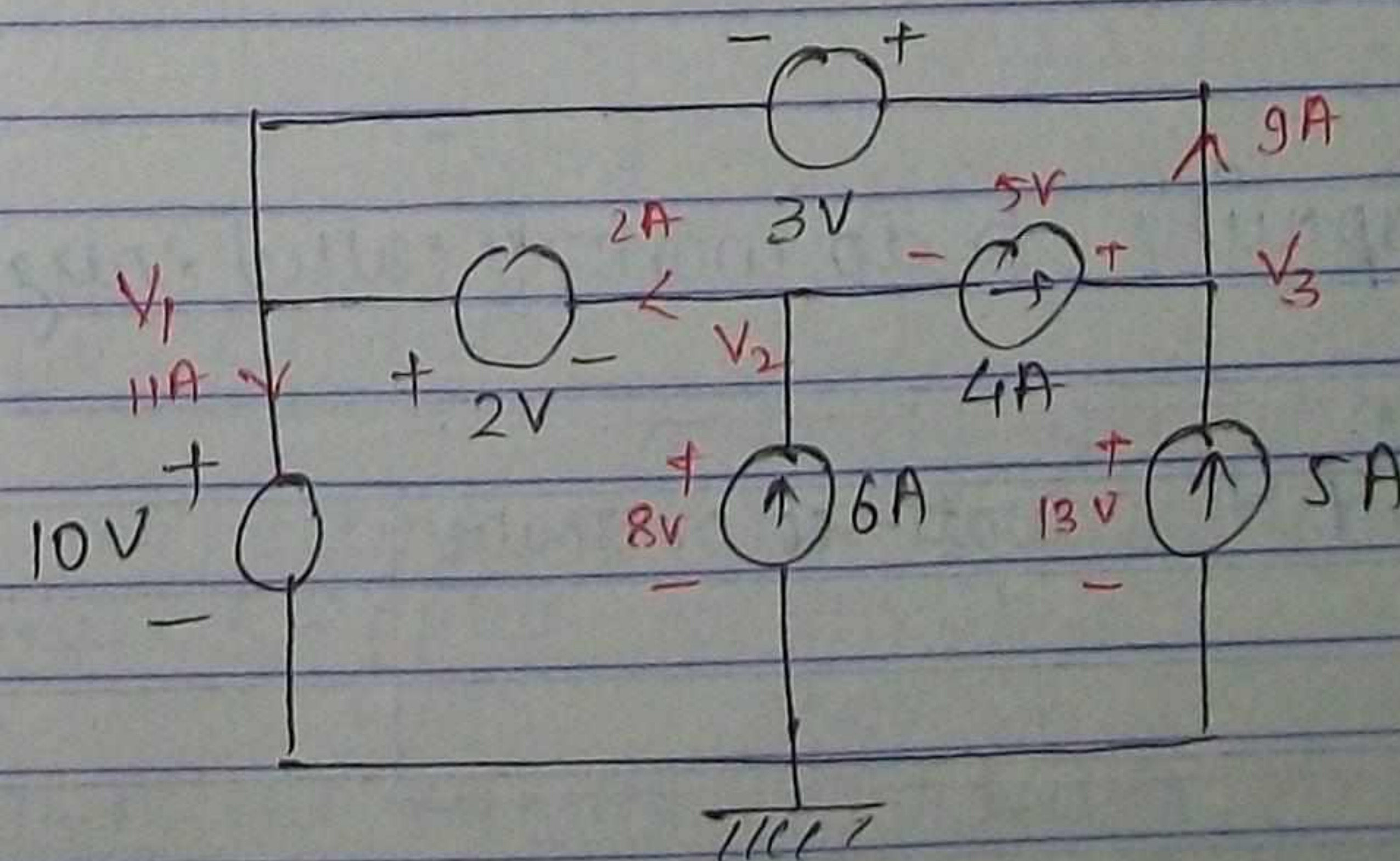


DELIVERING
POWER

NOTE -

- ① when current is entering at positive terminal element is absorbing power.
- ② when current is leaving from the terminal element is delivering power.

Ques find power of each element of circuit:



Soln: $V_1 = 10V$, $V_1 - V_2 = 2V$, $V_2 = 8V$

$V_3 - V_1 = 3$ $V_3 = 13V$

$P_{6A} = 8 \times 6 = 48W$ (delivering)

$P_{5A} = 13 \times 5 = 65W$ (delivering)

$P_{4A} = 4 \times 5 = 20W$ (delivering)

$P_{3V} = 3 \times 9 = 27W$ (absorbing)

$P_{2V} = 2 \times 2 = 4W$ (Delivering)

$P_{10V} = 10 \times 11 = 110W$ (absorbing)

$(P_T)_{\text{absorbing}} = (P_T)_{\text{delivering}}$ for all circuits

ENERGY -

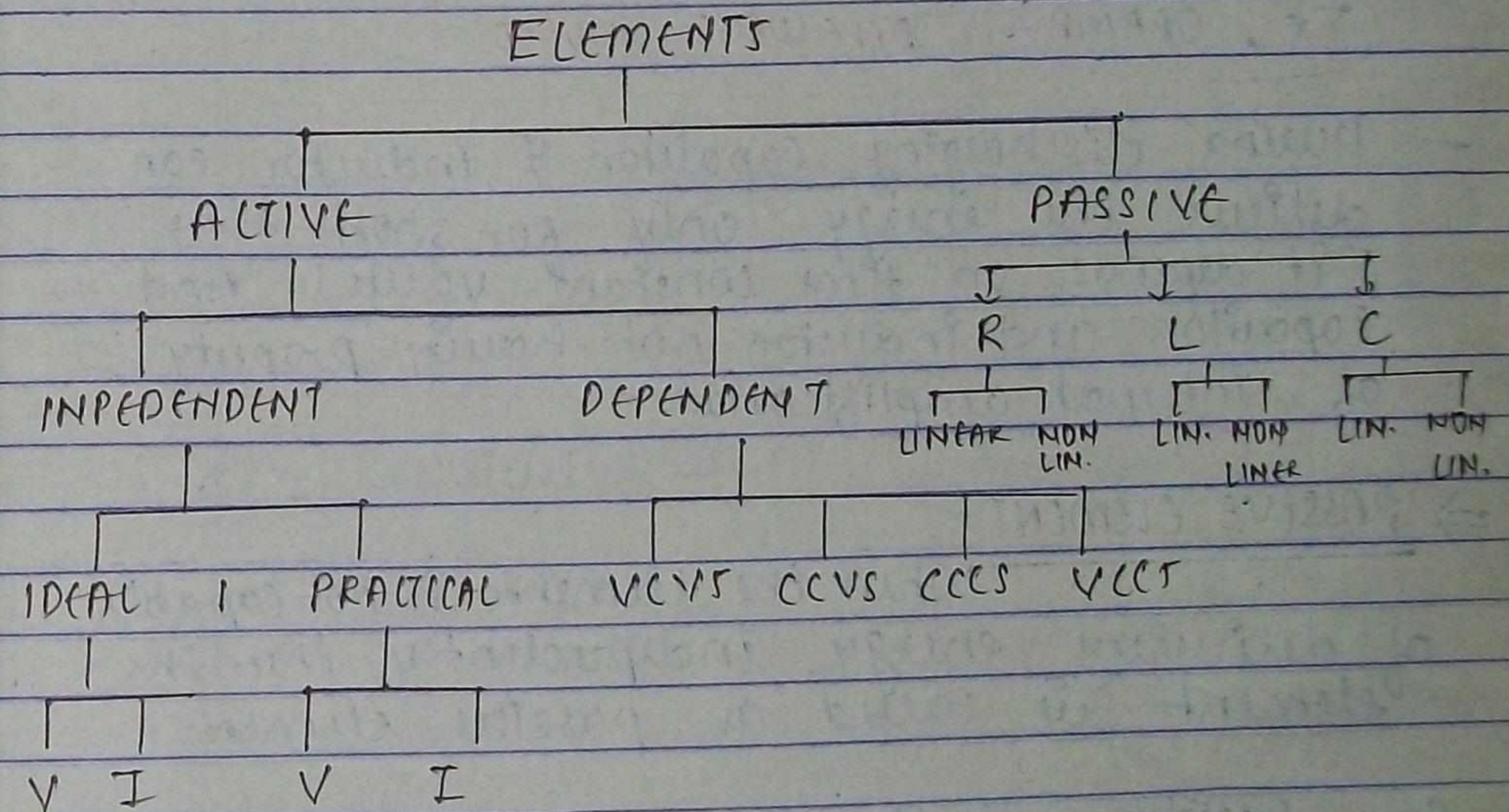
The capacity to do work is called energy

$$W = \int_0^t P dt \text{ watt-sec or Joules}$$

BIDIRECTIONAL - BILATERAL
UNI DIRECTIONAL - UNILATERAL

⇒ CLASSIFICATION OF ELEMENTS -

- ① Active and Passive.
- ② Linear and Non linear
- ③ uni-directional and Bidirectional
- ④ Time variant and invariant.
- ⑤ Lumped and distributed.



⇒ ACTIVE ELEMENTS -

when the element is capable of delivering energy independently for long time (approx. ∞ time)

(OR)
when the element is having property of

Internal amplification then the element is called as active elements.

Ex - Voltage source, current source, Transistor and OP-AMP.

$V, I \rightarrow$ INDEPENDENT SOURCE

$Tx, OPAMP \rightarrow$ DEPENDENT SOURCE

- During discharging capacitor & inductor can deliver the energy only for short time (it depends on time constant value). And capacitor and inductor not having property of internal amplification.

$\Rightarrow$ PASSIVE ELEMENT -

When the element is not capable of delivering energy independently then the element is called as passive element.

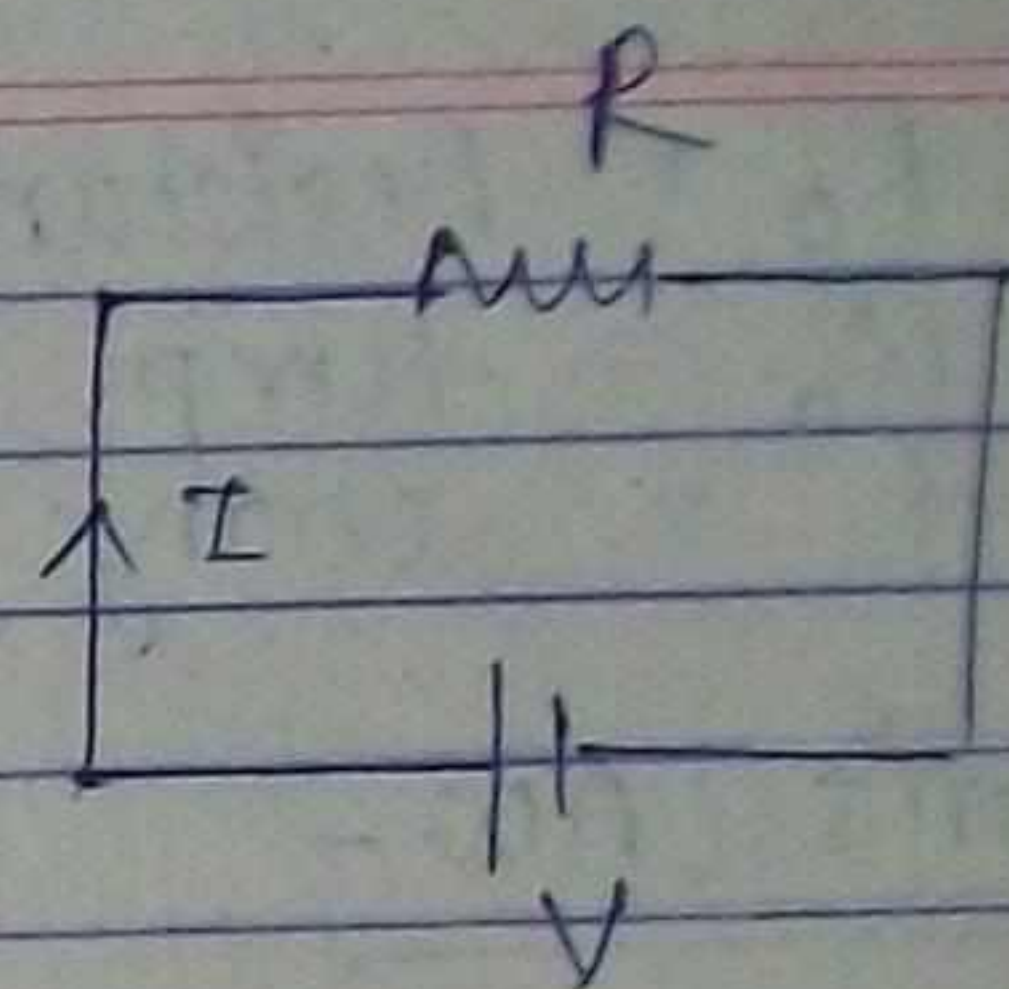
Ex - Resistor, bulb, transformer.

$$V_1 I_1 = V_2 I_2$$

I/P POWER equal
O/P POWER

$\Rightarrow$ RESISTOR -

- Resistance is a property of a resistor, it opposes flow of current (friction to flow of electrons). By doing so, it converts electric energy into heat energy.



$$P = I^2 R$$

$$W = I^2 R t$$

↓
HEAT

$$R = \frac{\rho L}{A}$$

ρ = Resistivity Ωm

- CONDUCTOR
- SEMICONDUCTOR
- INSULATOR

- When ρ of a conductor equal zero, then it is called as superconductor.

Ex - At 4.015 K mercury acts as superconductor.

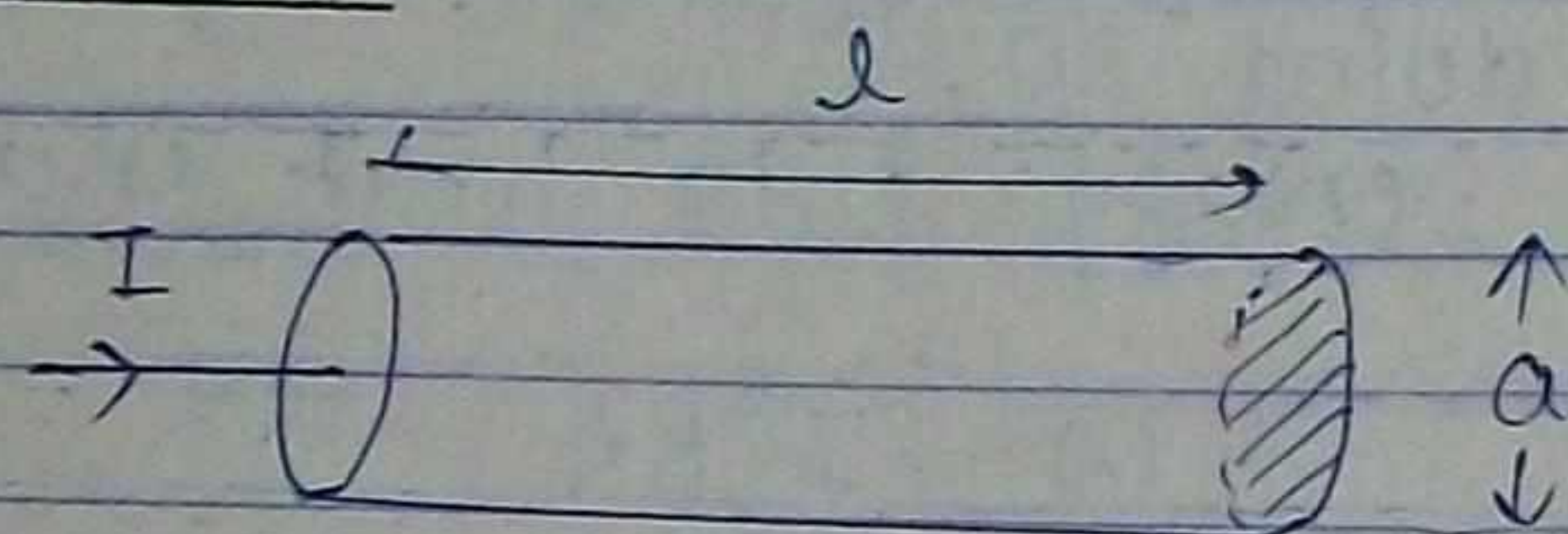
$$R_t = R_0 (1 + \alpha_0 t)$$

R_0 = Resistance of the material at 0°C .

α_0 = temp. coefficient.

t = change in temp.

Ohm's LAW -



- At constant temp. current density J is directly prop. to electric field intensity E

$$J \propto E$$

$$J = \sigma E$$

$$\frac{I}{a} = \frac{1}{\rho} \frac{V}{l}$$

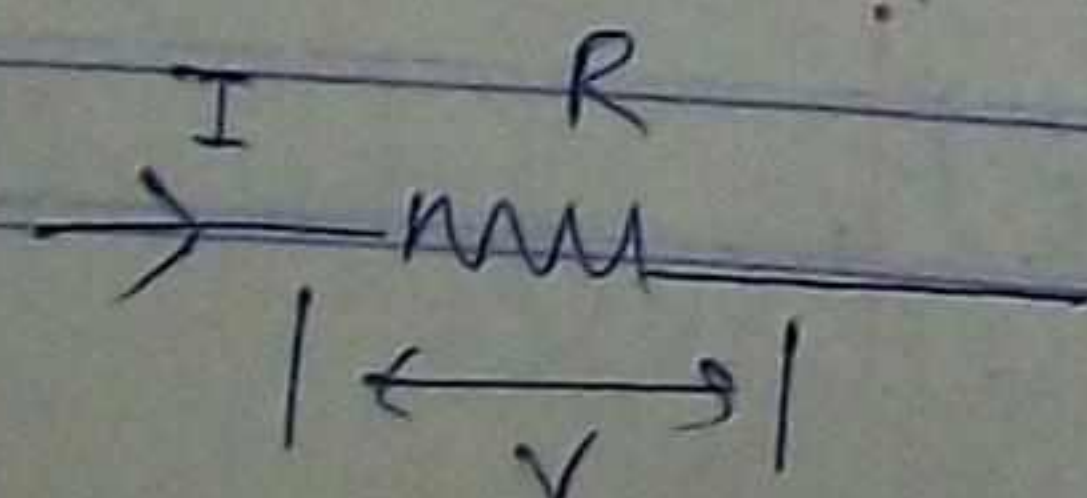
$$\frac{V}{I} = \frac{\rho l}{a} = R$$

$$J = \frac{I}{a}, E = \frac{V}{l}$$

$$\sigma = \frac{1}{\rho}, R = \frac{\rho l}{a}$$

rho ohm/m A

- At constant temp. potential difference across the element is directly prop. to current flow into element



$$V \propto I$$

$$V = RI$$

$$R = \frac{V}{I} = \text{CONSTANT}$$

- Ohm's law is valid when temp. and conductivity of the material are constant.

#

$$J = \sigma E$$

Ohm's LAW
1st form

$$V = RI$$

2nd form

$$G = \frac{1}{R} \text{ mho or Siemens}$$

$$I = GV$$

3rd form

$$V = R \frac{dQ}{dt}$$

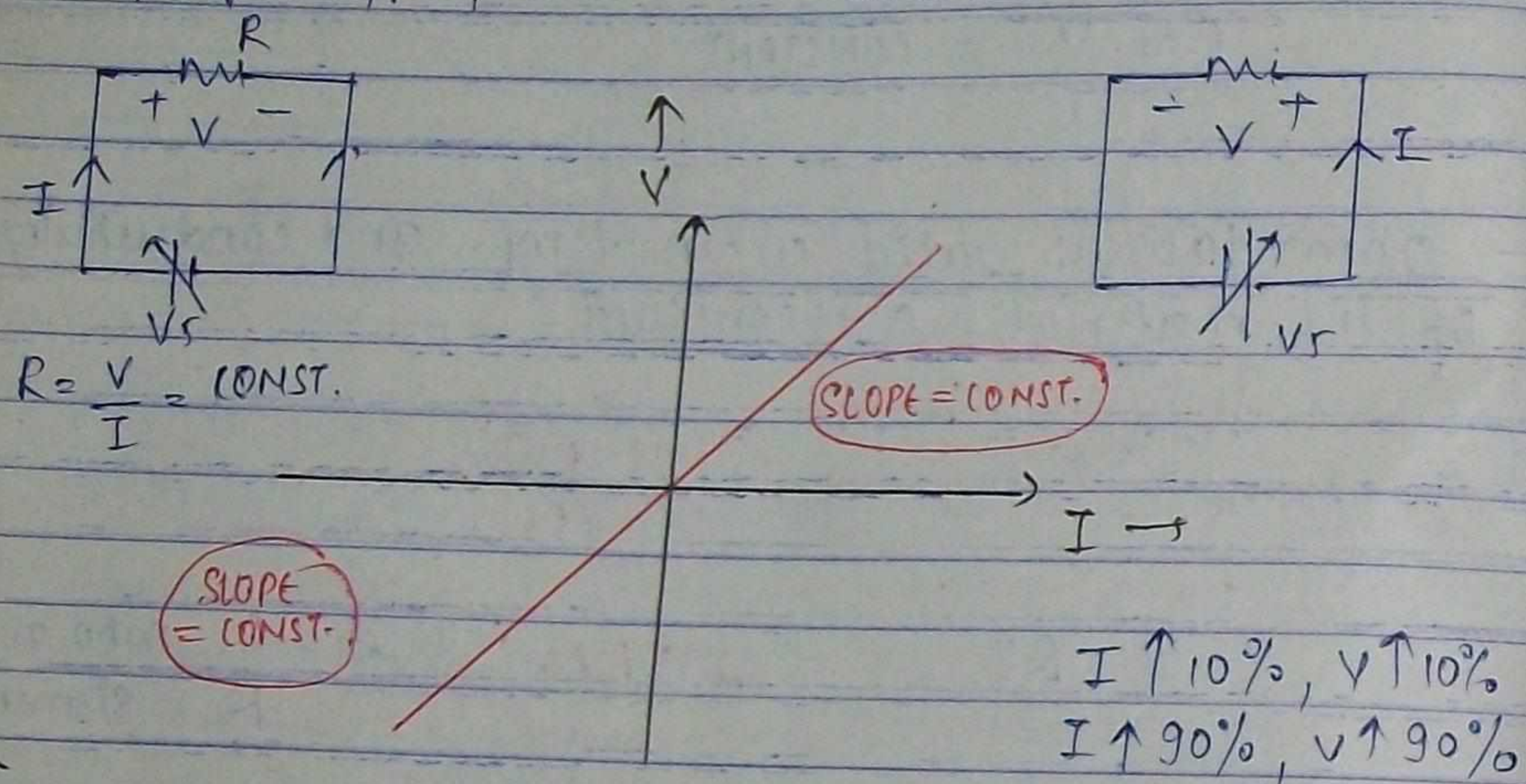
4th form

- When element properties and characteristics independent on the direction of current then the element is called a bidirectional element.

- When element obeys the Ohm's law then

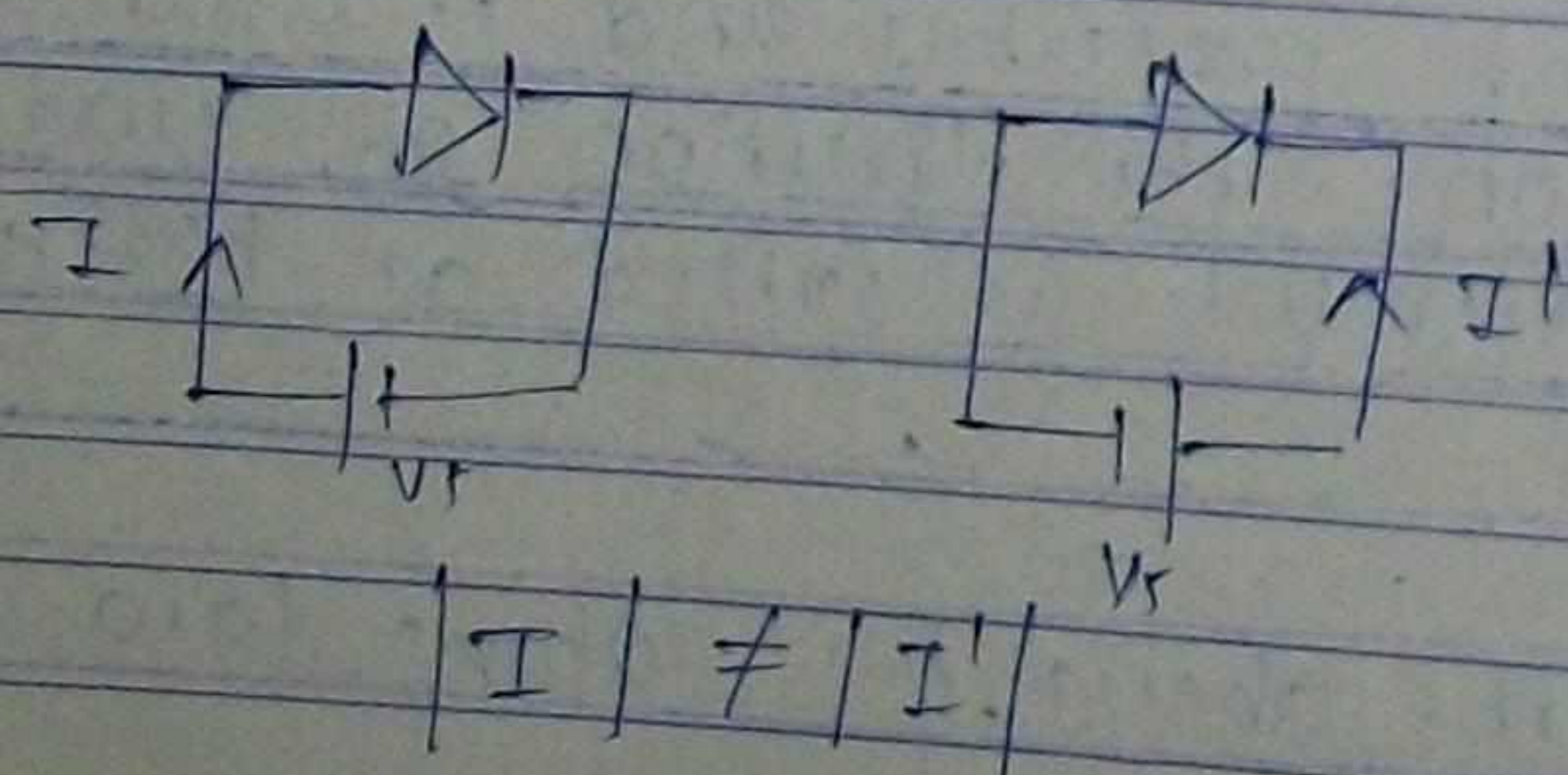
The element is called as linear resistor.

- Every linear element obeys the bidirectional property, but not vice versa.

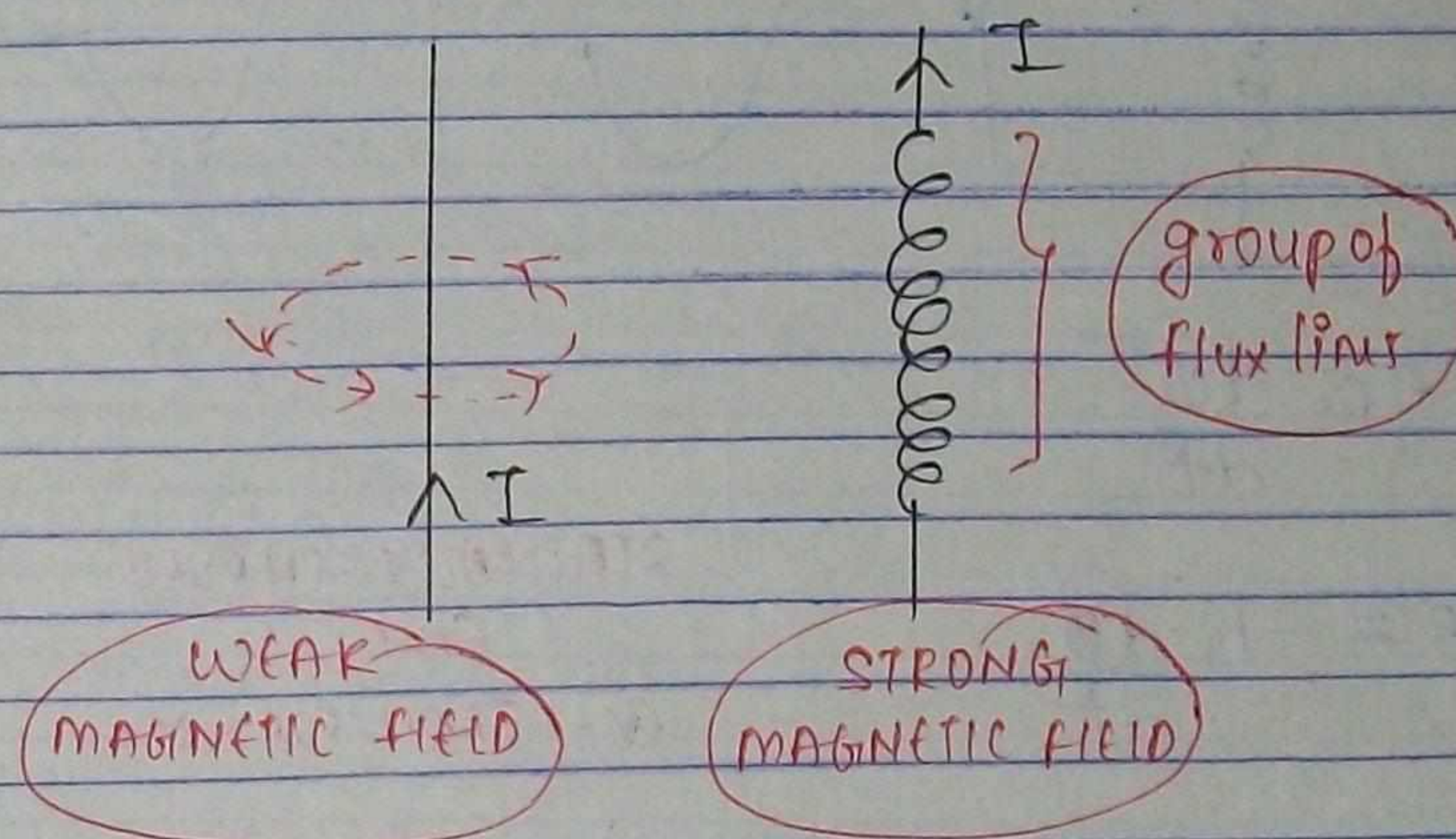


- When element properties and characteristics depend on direction of the current then the element is called as unidirectional element.

- When element does not obey the ohm's law then the element is called as non-linear resistor.



⇒ INDUCTOR -



FARADAY'S 1st LAW -

When conductor cuts magnetic lines of forces an emf induced into the conductor.

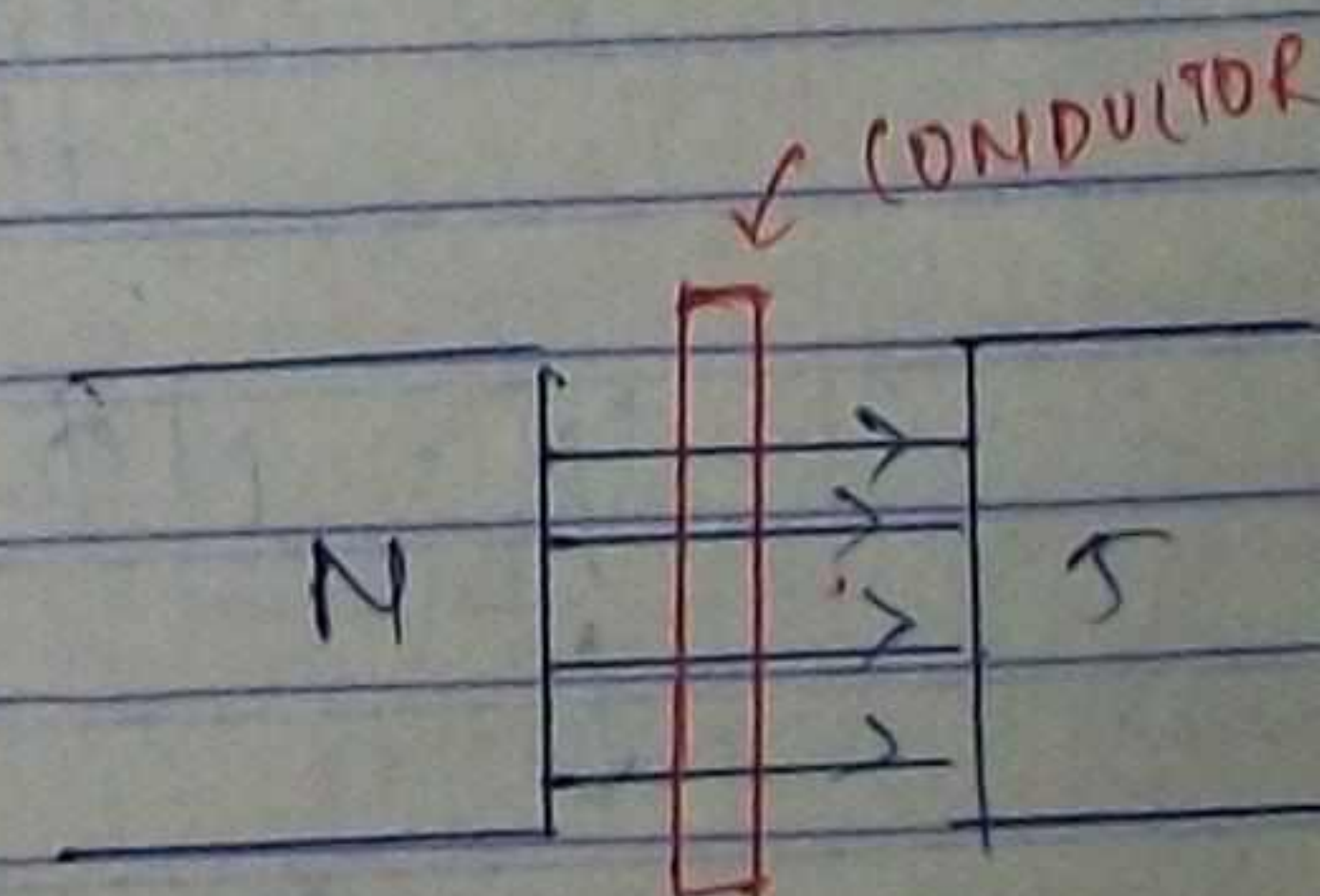
FARADAY'S 2nd LAW -

Emf induced in the conductor is directly prop. of rate of change of flux.

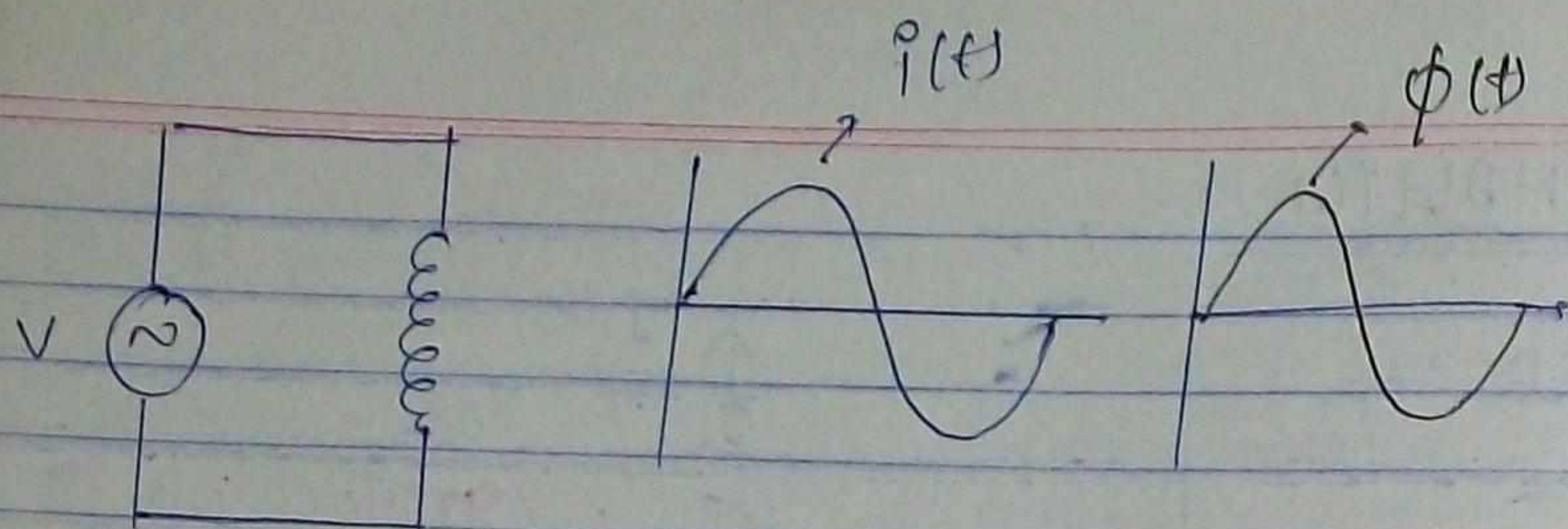
$$e \propto \frac{d\phi}{dt}$$

$$e = -N \frac{d\phi}{dt}$$

N = NO. of conductor.



DYNAMICALLY INDUCED EMF
EX - GENERATOR



$$e \propto \frac{d\phi}{dt}$$

$$e = -N \frac{d\phi}{dt}$$

STATICALLY INDUCED

EMF

Ex - TRANSFORMER

N = No. of turns.

- with respect to lenz's law induced voltage opposes its cause of existence.

V, i, ϕ, e



$$V = N \frac{d\phi}{dt}$$

$$\psi = N\phi$$



flux linkage

$$V = \frac{d\psi}{dt}$$

$$\phi \propto i$$

$$\psi \propto \phi$$

$$\psi \propto i$$

$$\psi = Li$$

$$V = L \frac{di}{dt}$$

$$L = \frac{V}{di/dt}$$

$$\psi = N\phi$$

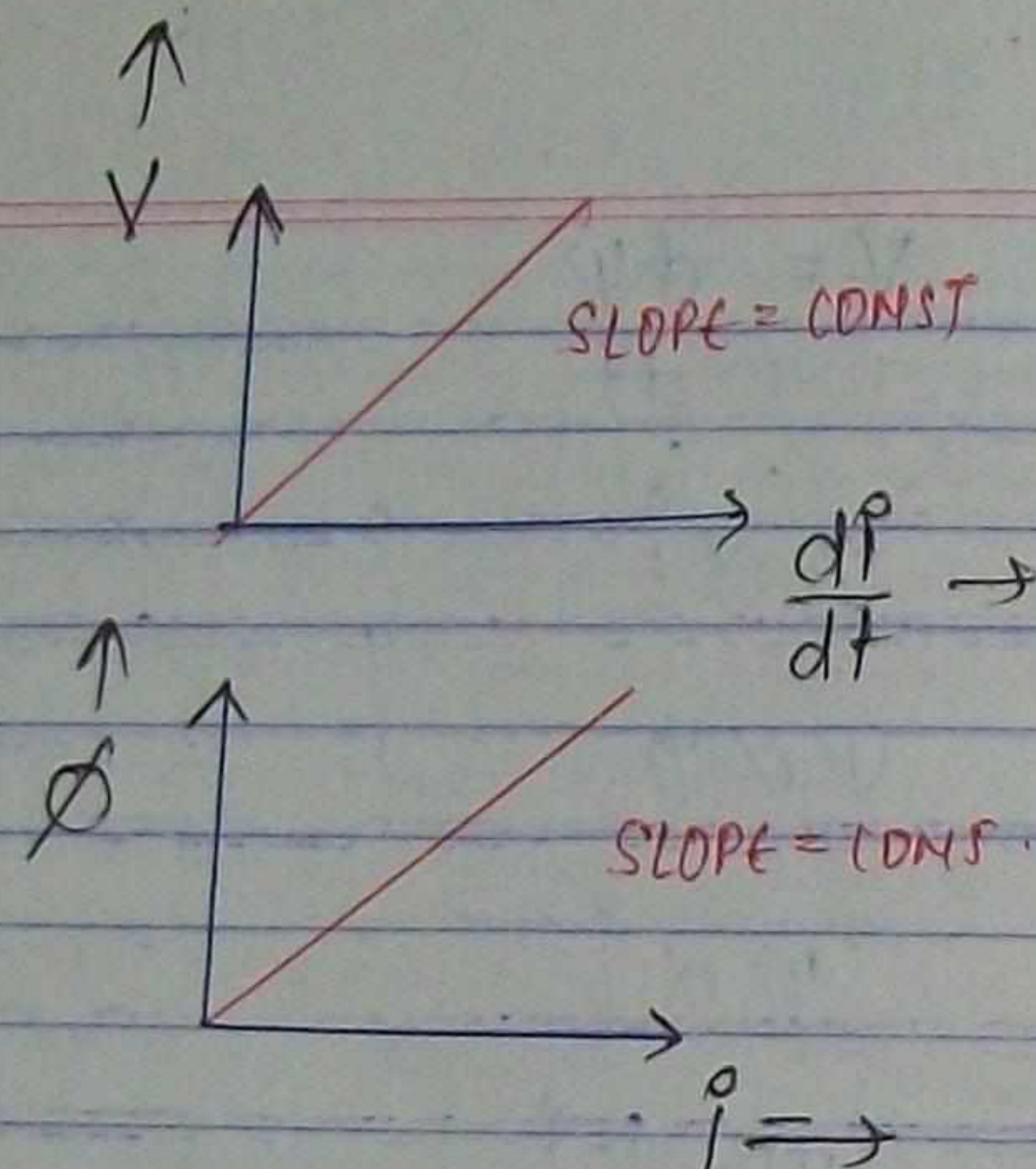
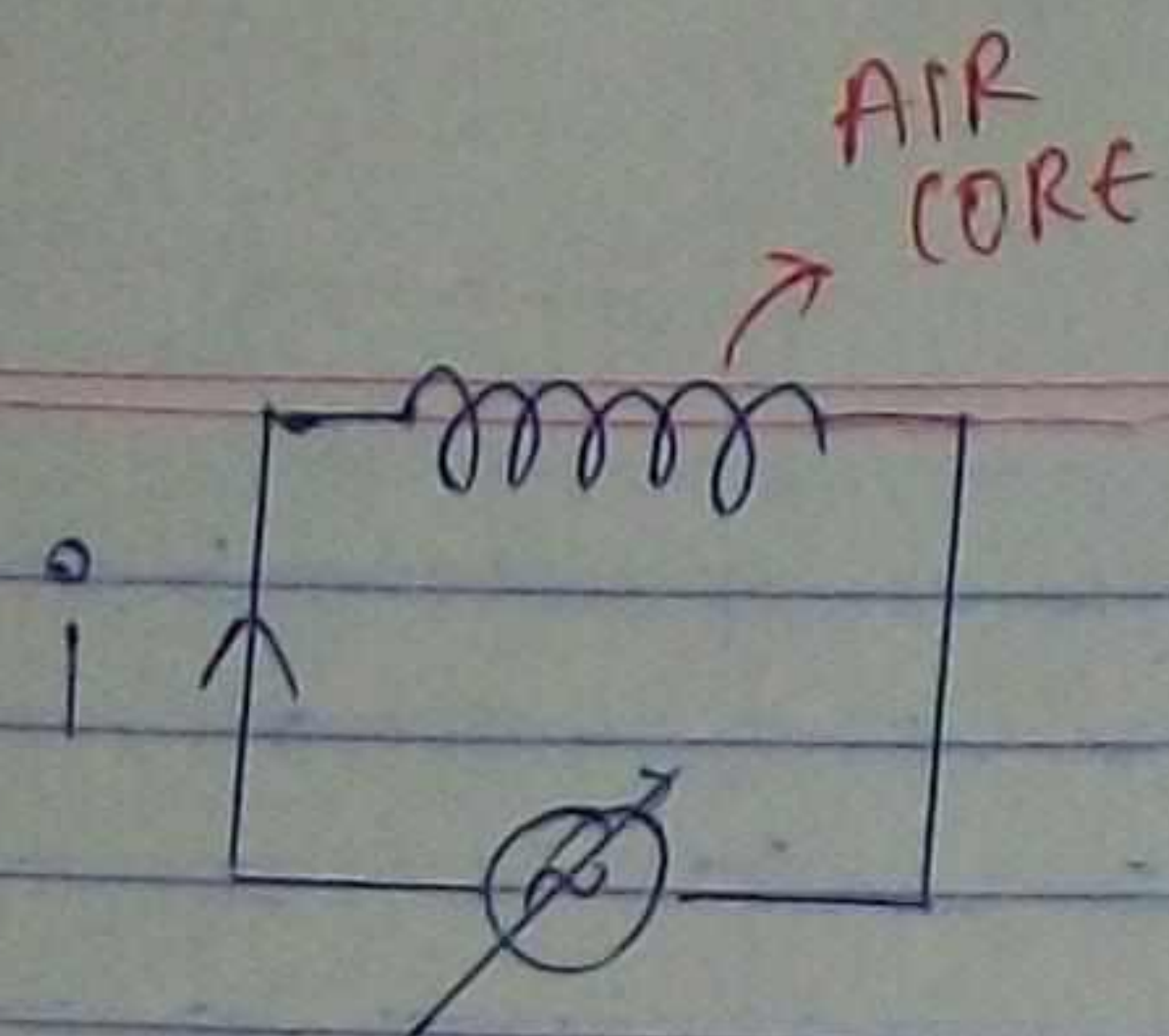
$$\psi = Li$$

$$L = \frac{N\phi}{i}$$

Henry.

- when inductance of the inductor is independent of the current magnitude then the inductor is called as linear inductor.

Ex - air core inductor



$i \uparrow 10\%, \phi \uparrow 10\%$
 $i \uparrow 90\%, \phi \uparrow 90\%$

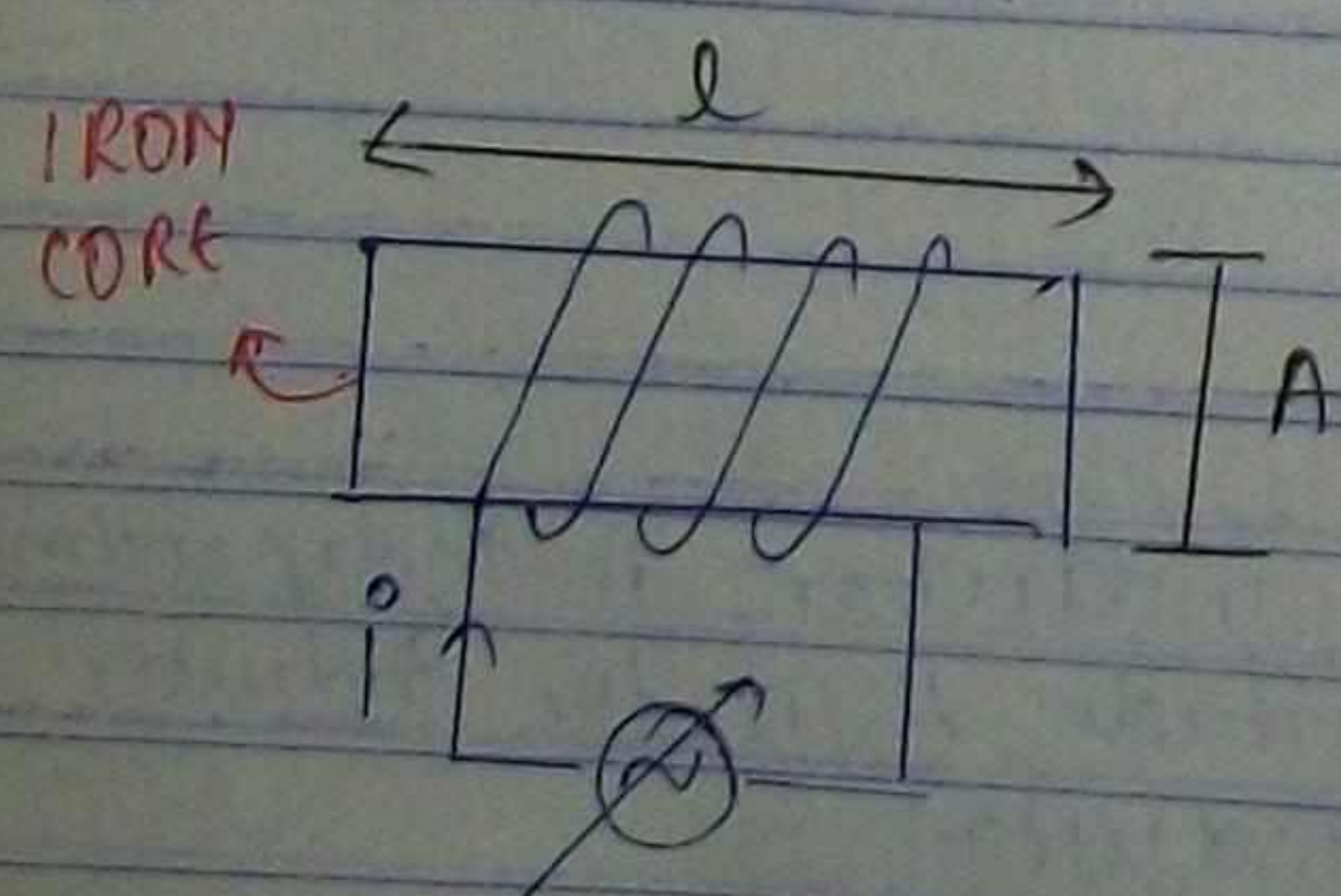
$$V = L \frac{di}{dt}$$

$$L = \frac{V}{di/dt}$$

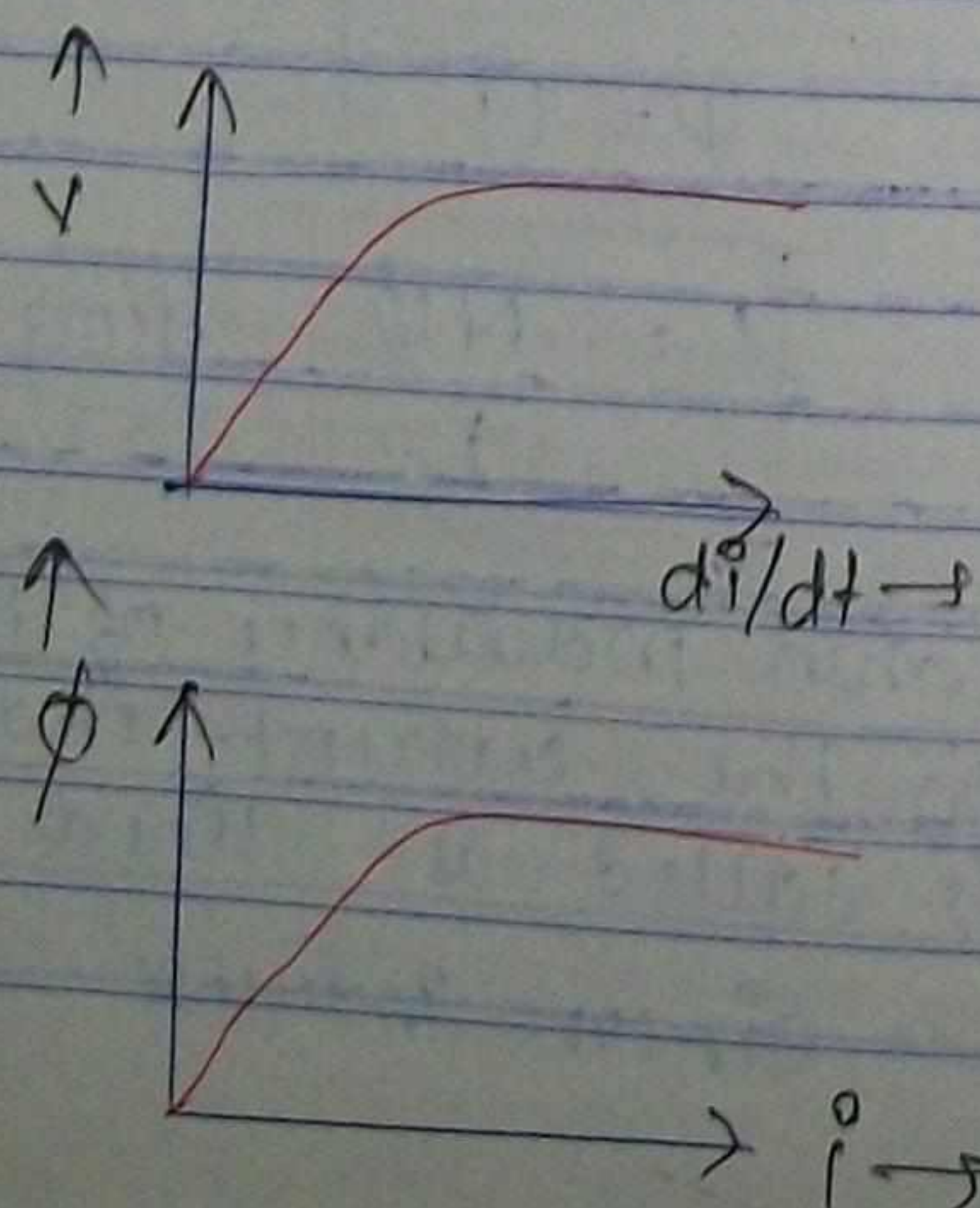
$$L = \frac{N\phi}{i} \text{ H.}$$

— When Inductance of the Inductor depends on current magnitude then the Inductor is called as Non linear Inductor.

Ex - Iron core Inductor.



$$L = \frac{N\phi}{i} \text{ H.}$$



$i \uparrow 10\%, \phi \uparrow 10\%$

$i \uparrow 60\%, \phi \uparrow 60\%$

$i \uparrow 90\%, \phi = \text{CONSTANT}$

SATURATION

#

ELECTRIC CIRCUIT

① $i = \frac{\text{EMF}}{R}$

② $i = \frac{\text{EMF}}{\int \frac{l}{a}}$

MAGNETIC CIRCUIT

① $\phi = \frac{\text{MMF}}{S}$

② $\phi = \frac{Ni}{\frac{l}{\mu_0 \mu_r}}$

$\phi = \frac{Ni}{\frac{l}{\mu_0 \mu_r}} \text{ --- ①}, \quad L = \frac{N\phi}{i} \text{ --- ②}$

from ① and ②: NR

$$L = \frac{N^2 \mu_0 \mu_r a}{l}$$

μ_0 = absolute permeability = $4\pi \times 10^{-7} \text{ H/m}$
 μ_r = relative permeability = 1 (air)
 l = length of core
 a = area of crosssection of core.

44

$$L = \frac{N^2}{\frac{l}{\mu_0 \mu_r}} = \frac{N^2 \mu_0 \mu_r}{l}$$

#

$$V = L \frac{di}{dt}$$

OHM'S LAW
5th form

$$i = \frac{1}{L} \int_{-\infty}^t V dt$$

OHM'S LAW
6th form

Power -

$$P = Vi$$

$$P = L \frac{di}{dt} \cdot i$$

ENERGY -

$$W = \int P dt$$

$$W = \int L \frac{di}{dt} i dt$$

$$W = \frac{1}{2} Li^2$$

- Power dissipation in ideal inductor is zero.
Since internal resistance equal to zero.

- Inductor stores energy in the form of magnetic fields (kinetic energy).

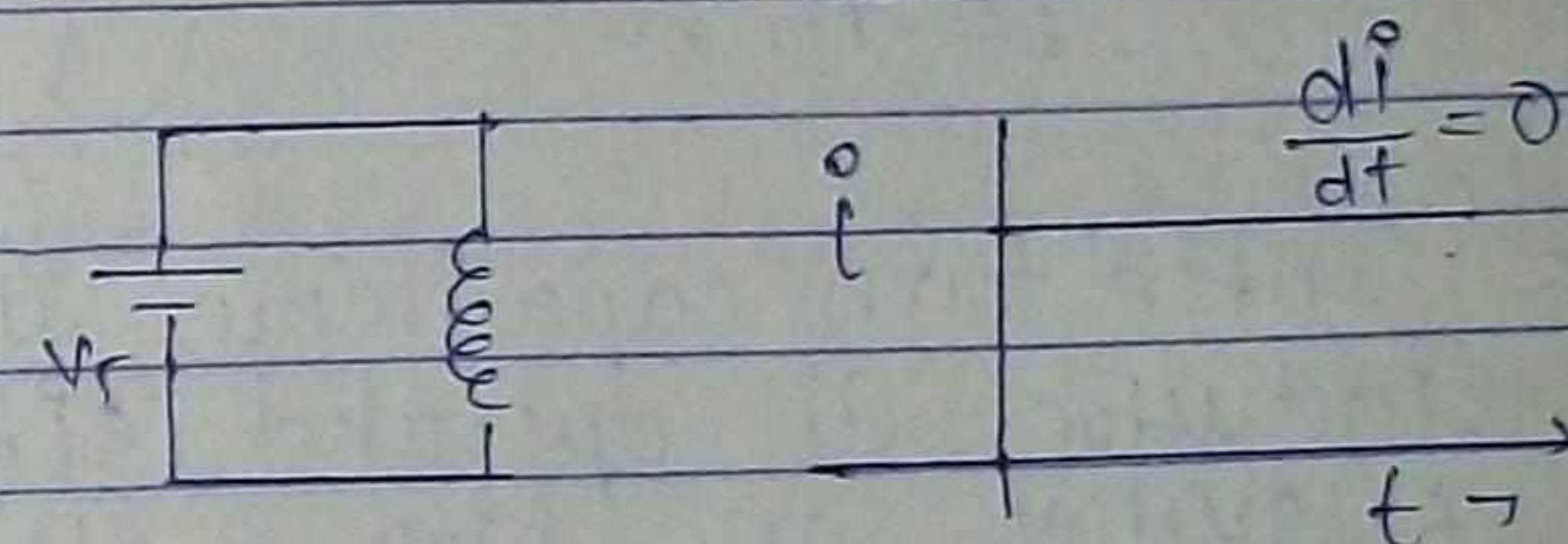
⇒ CONCLUSIONS -

- Under steady state conditions for DC source inductor acts as a short circuit.

$$V = L \frac{di}{dt}$$

$$\frac{di}{dt} = 0$$

$$V = 0$$



SHORT CIRCUIT

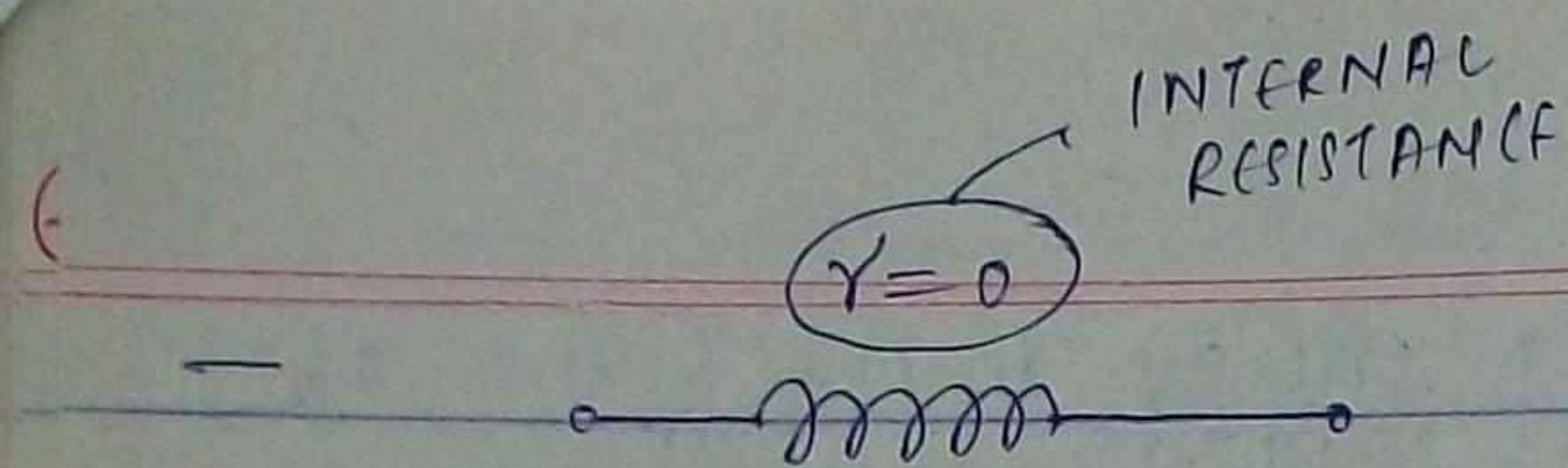
- Inductor does not allow sudden change of current, since for sudden change of current ∞ voltage is required.

$$V = L \frac{di}{dt}$$

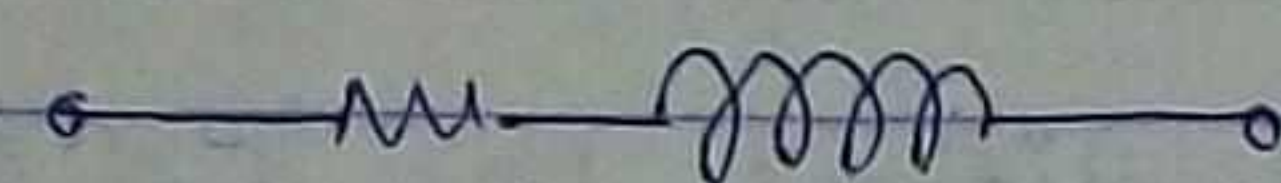
when $dt \rightarrow 0$ $V = \infty$

- Inductor does not allow sudden change of current, since practical inductor circuit have time constants.

$$T = \frac{L}{R}$$



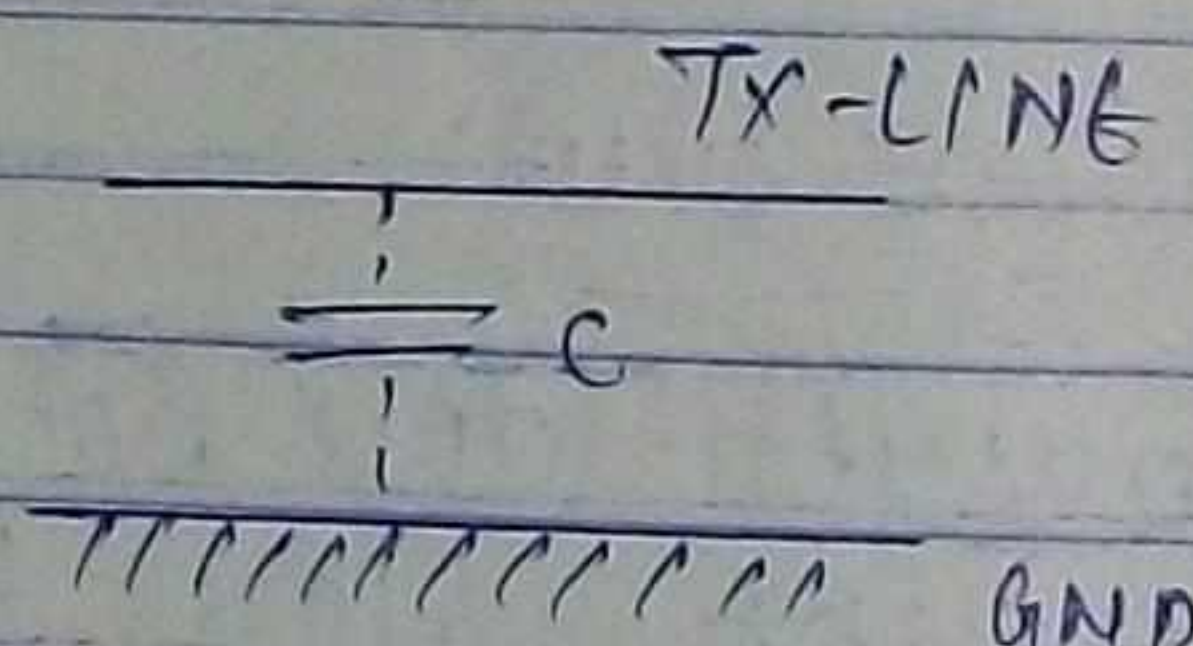
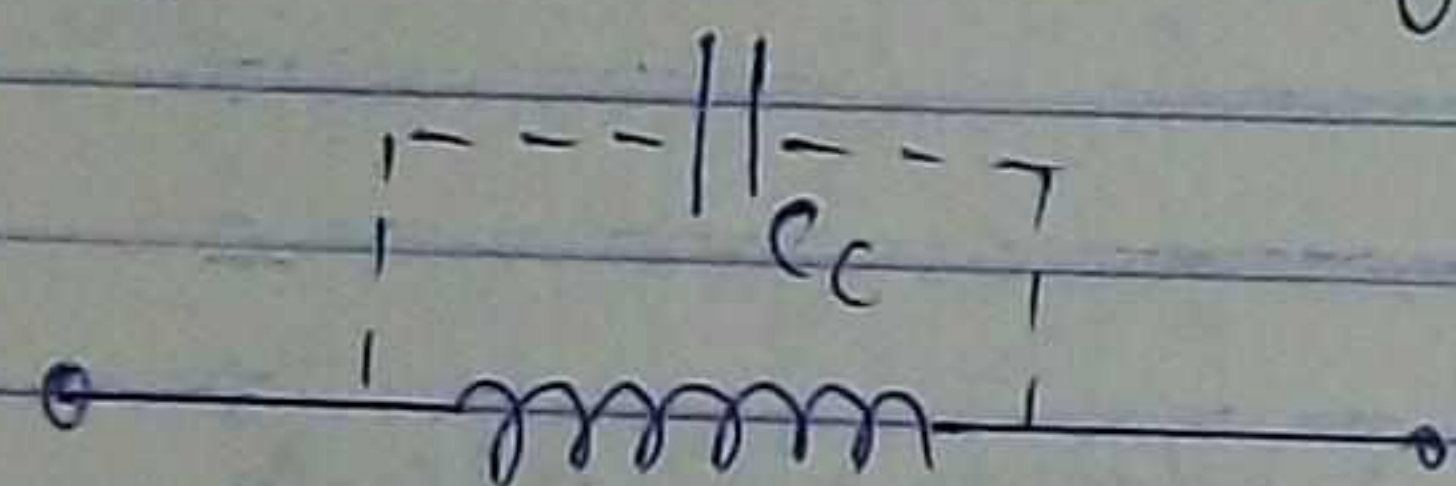
IDEAL
INDUCTOR



PRACTICAL
INDUCTOR

$$r = m\Omega$$

- Inter turn capacitance is present when inductor is operated either at high frequency or high voltage.

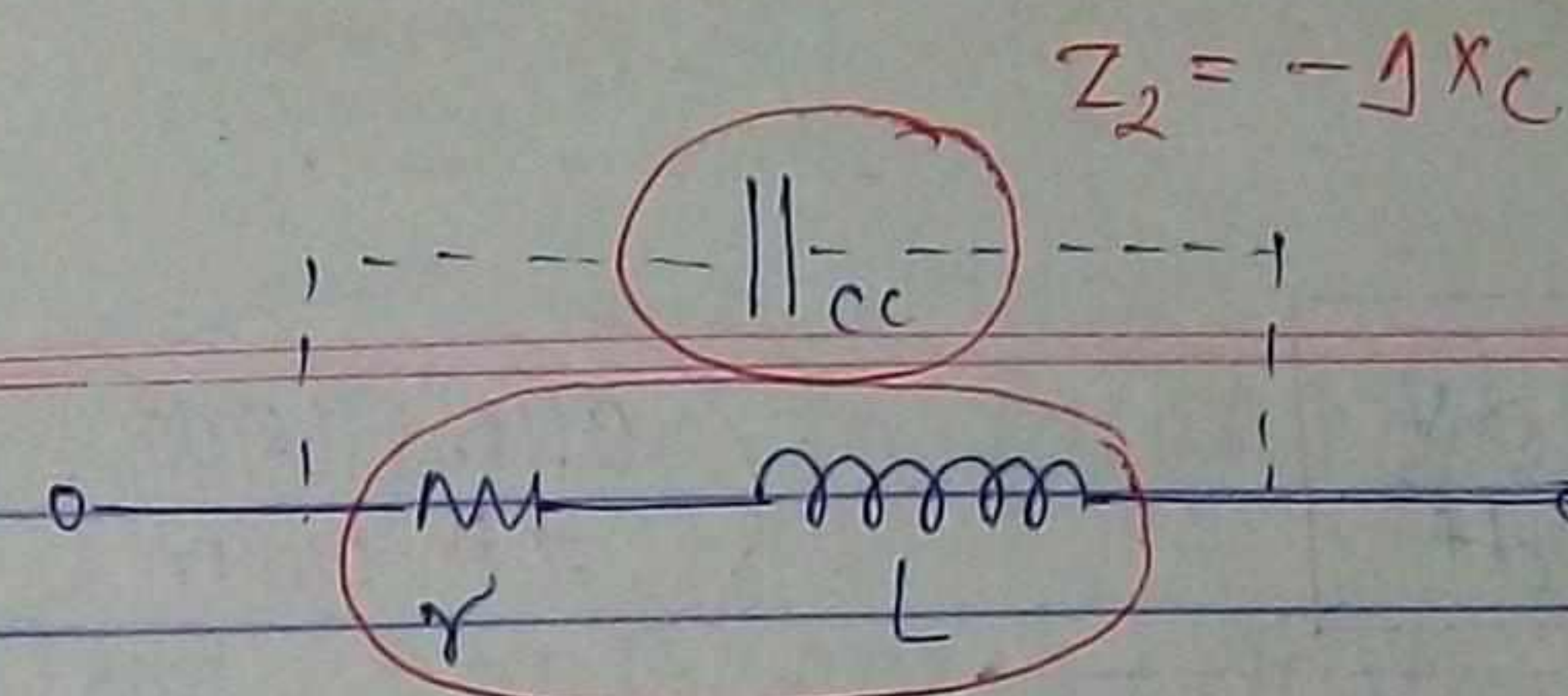


C_c = INTER TURN CAPACITANCE

(OR)
SELF CAPACITANCE

$$X_c = \frac{1}{2\pi f C}, \quad X_L = 2\pi f L$$

- when inductor is having self resistance and self capacitance then equivalent impedance of the inductor:



$$Z_{eq} = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

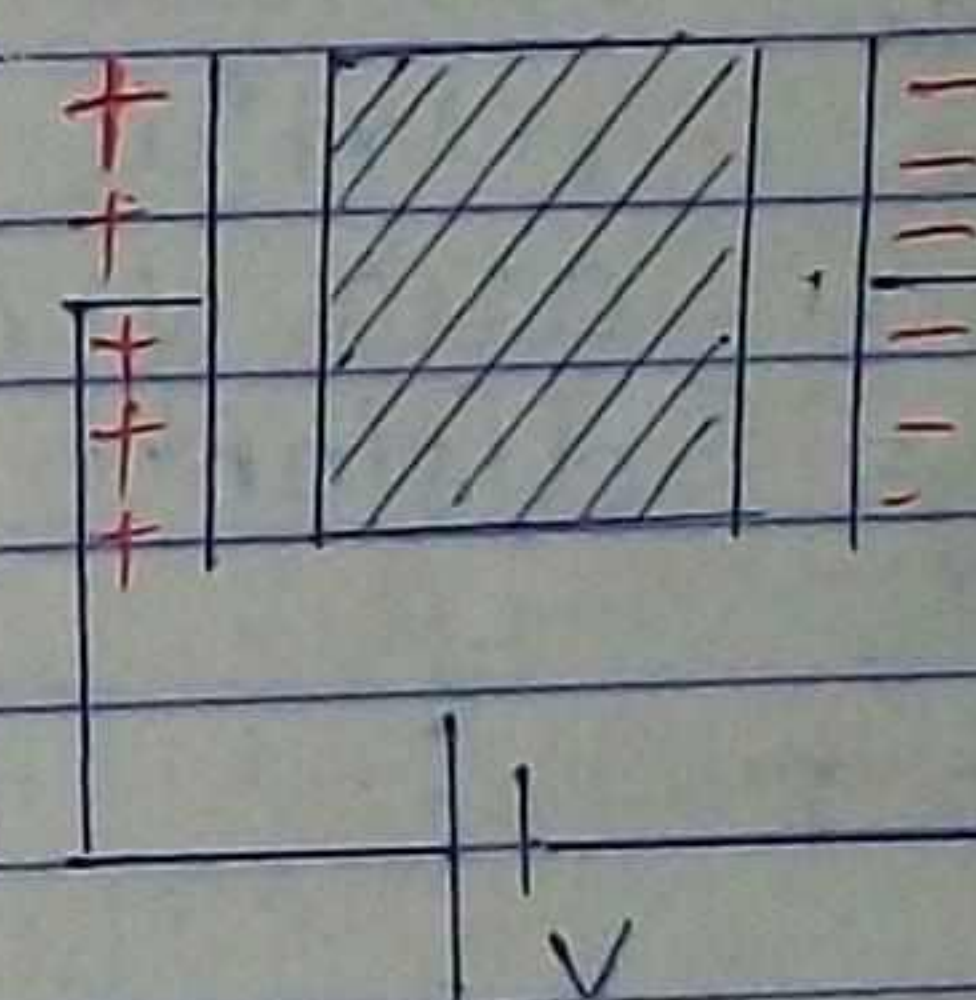
$$Z_{eq} = \frac{(-jX_c)(r + jX_L)}{r + jX_L - jX_c} = \frac{X_c X_L - X_c r j}{r + j(X_L - X_c)}$$

⇒ CAPACITOR -

$$Q \propto V$$

$$Q = CV$$

$$C = \frac{Q}{V} \quad \text{f or} \quad \frac{\text{Col}}{\text{Volts}}$$



$$Q = CV$$

$$\frac{dQ}{dt} = C \frac{dV}{dt}$$

$$i = C \frac{dV}{dt}$$

$$C = \frac{i}{dV/dt}$$

$$i = C \frac{dv}{dt}$$

OHM'S LAW
7th FORM

$$V = \frac{1}{C} \int_{-\infty}^t i dt$$

OHM'S LAW
8th FORM

$C = \frac{\epsilon A}{d}$

$$\epsilon = \epsilon_0 \cdot \epsilon_r$$

ϵ_0 = absolute permittivity = 8.85 pF/m

ϵ_r = relative permittivity = 1 (air)

A = area of each plate.

d = distance b/w plates.

Power - $P = V i$

$$P = V C \frac{dv}{dt}$$

ENERGY - $W = \int P dt$

$$W = \int V C dv$$

$$W = \frac{1}{2} C V^2$$

- Power dissipation in ideal capacitor is zero.

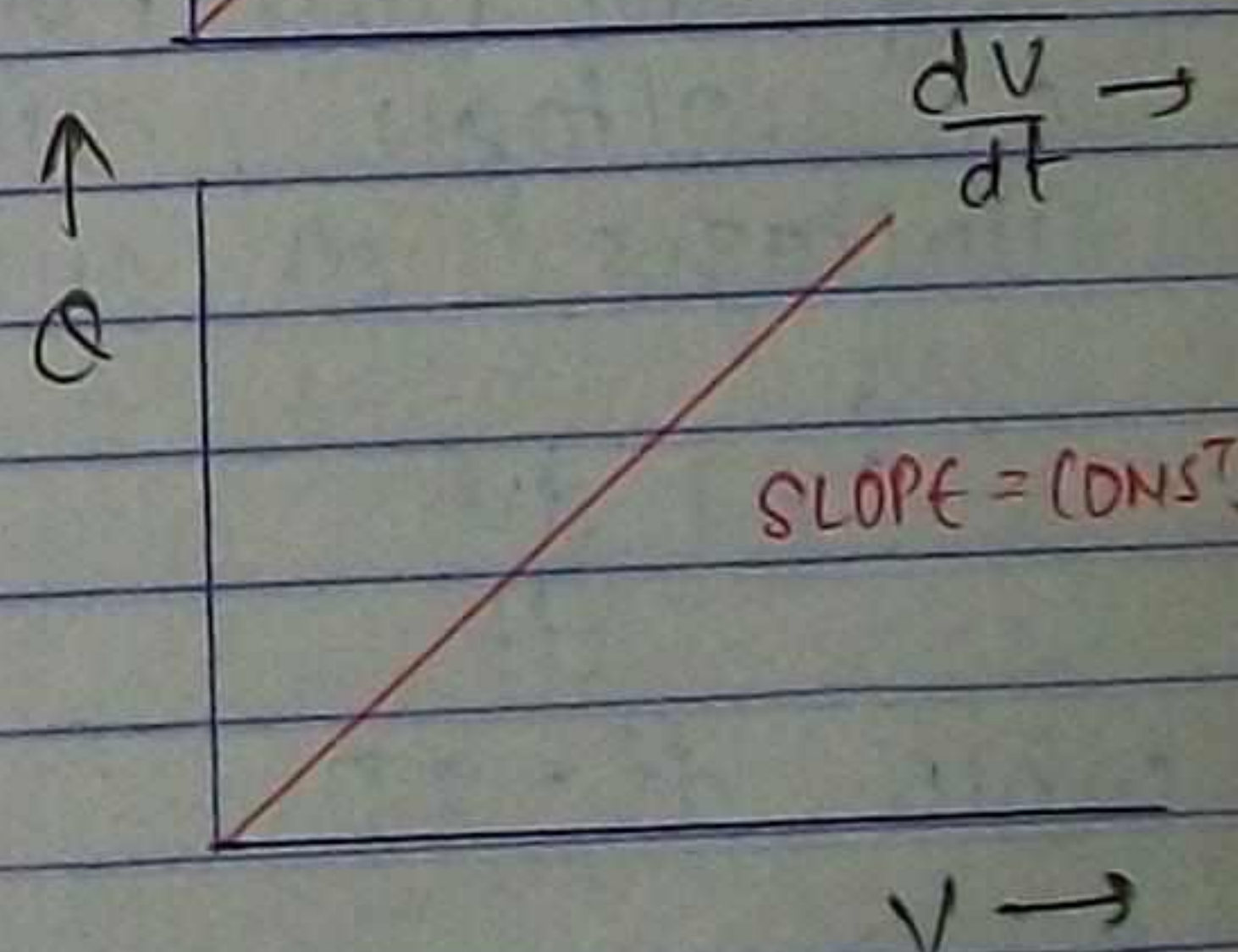
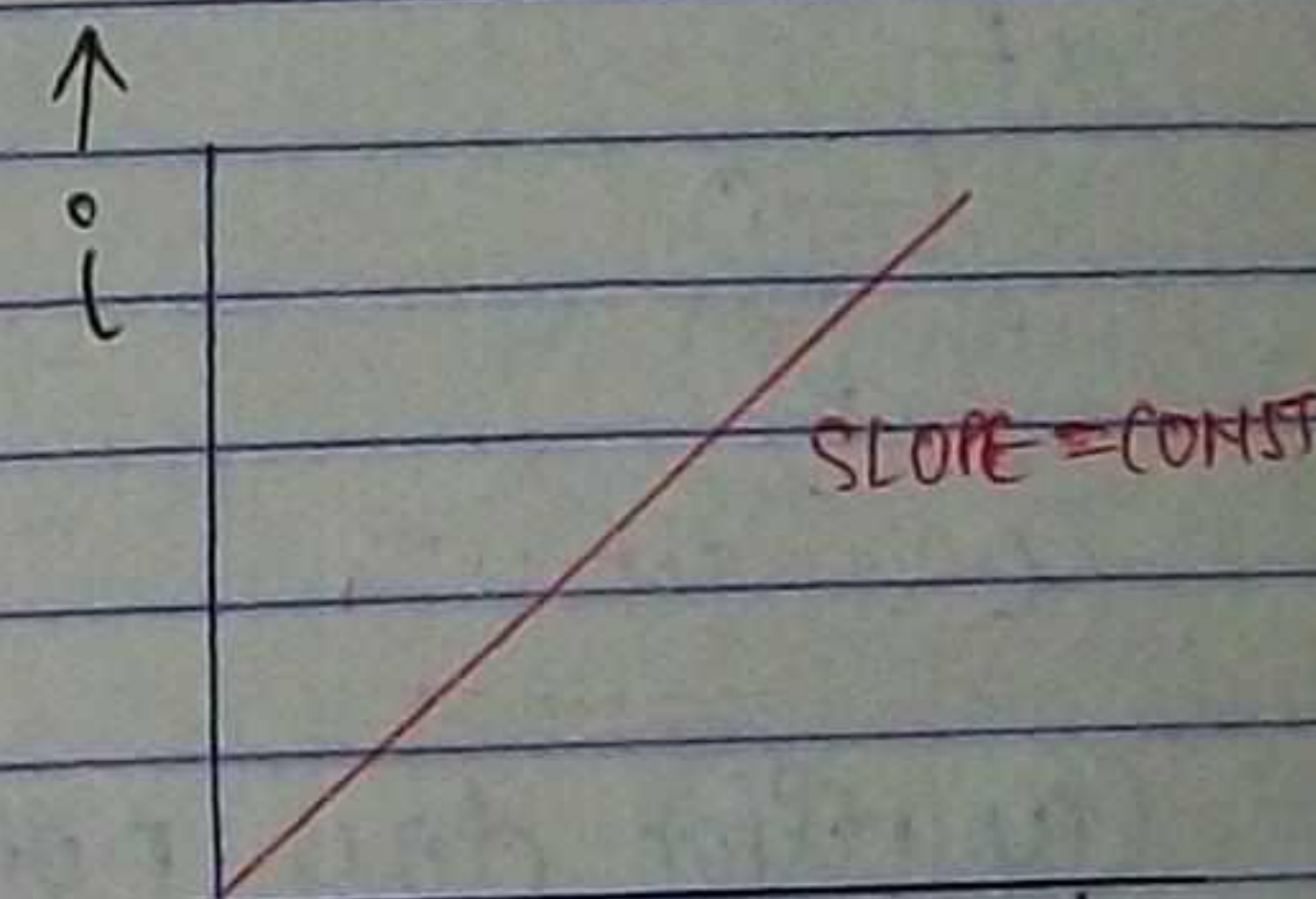
- Capacitor stores energy in the form of electric field (Potential energy).

- when capacitance of the capacitor are independent of the voltage magnitude then the capacitor is called as linear capacitor.

$$C = \frac{Q}{V} = \text{CONST.}$$

$V \uparrow 10\%$, $Q \uparrow 10\%$

$V \uparrow 90\%$, $Q \uparrow 90\%$



$$i = C \frac{dv}{dt}$$

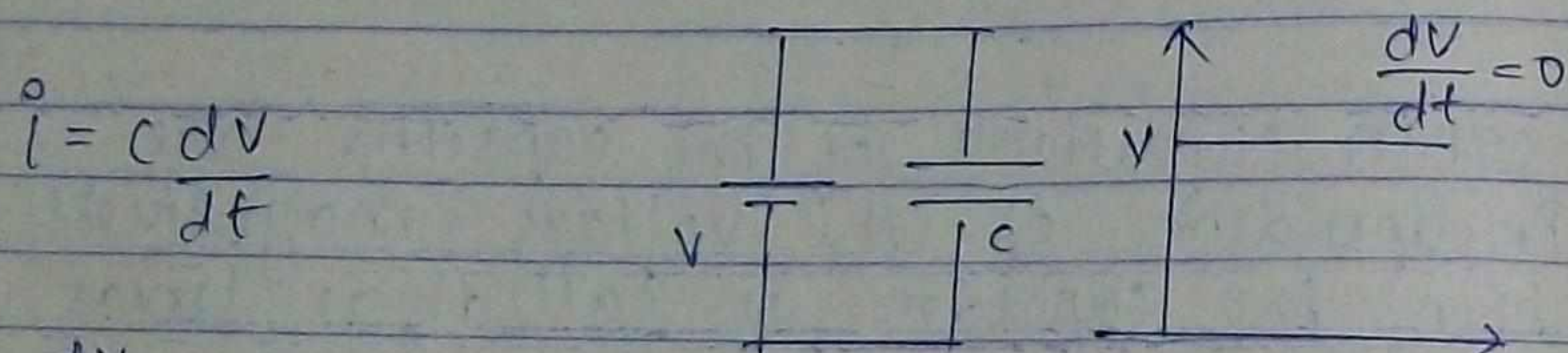
$$C = \frac{i}{dv/dt}$$

- when capacitance of a capacitor depends on voltage magnitude then the capacitor is called as non linear capacitor.
Ex- Varactor diode.

$$C = \frac{Q}{V} = \text{VARIABLE}$$

⇒ CONCLUSIONS -

- Under steady state condition for d.c supply capacitor acts as an open circuit.



OPEN CIRCUIT

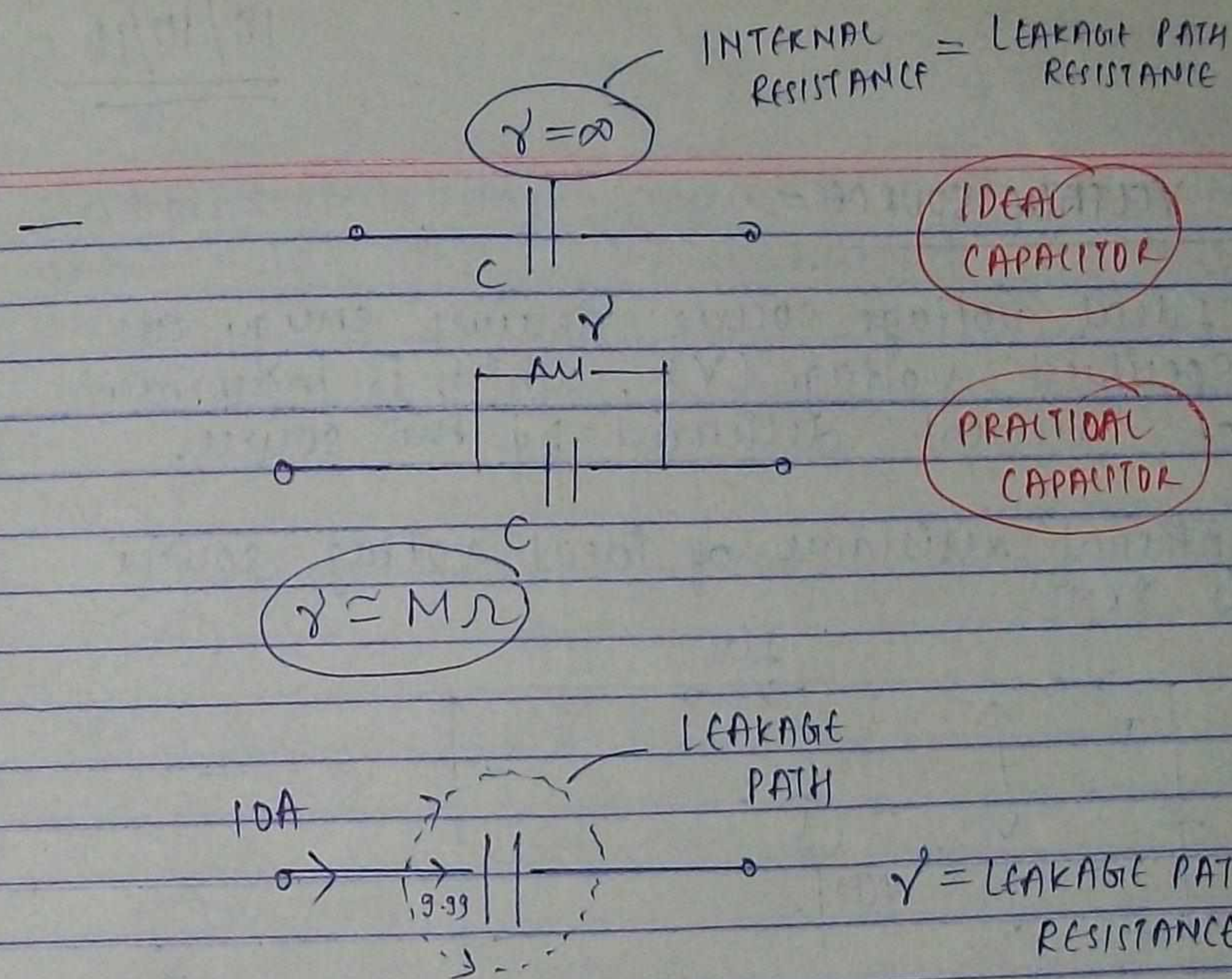
- Capacitor does not allow sudden change of voltage. Since for sudden change of voltage ∞ current is required.

$$i = C \frac{dv}{dt}$$

when $dt \rightarrow 0$ $i = \infty$.

- Capacitor does not allow sudden change of voltage since practical capacitor circuit have finite value of time constant

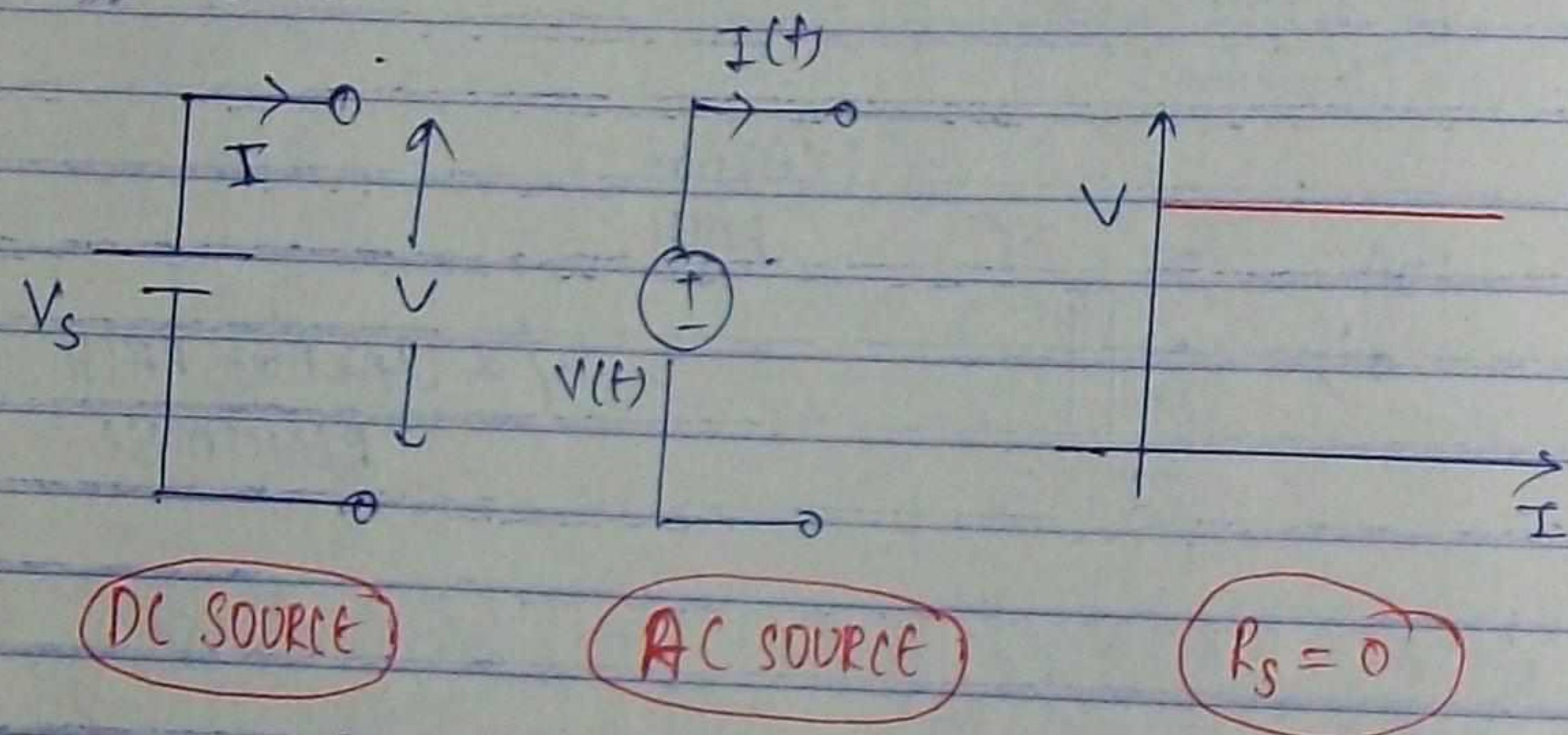
$$T = RC$$



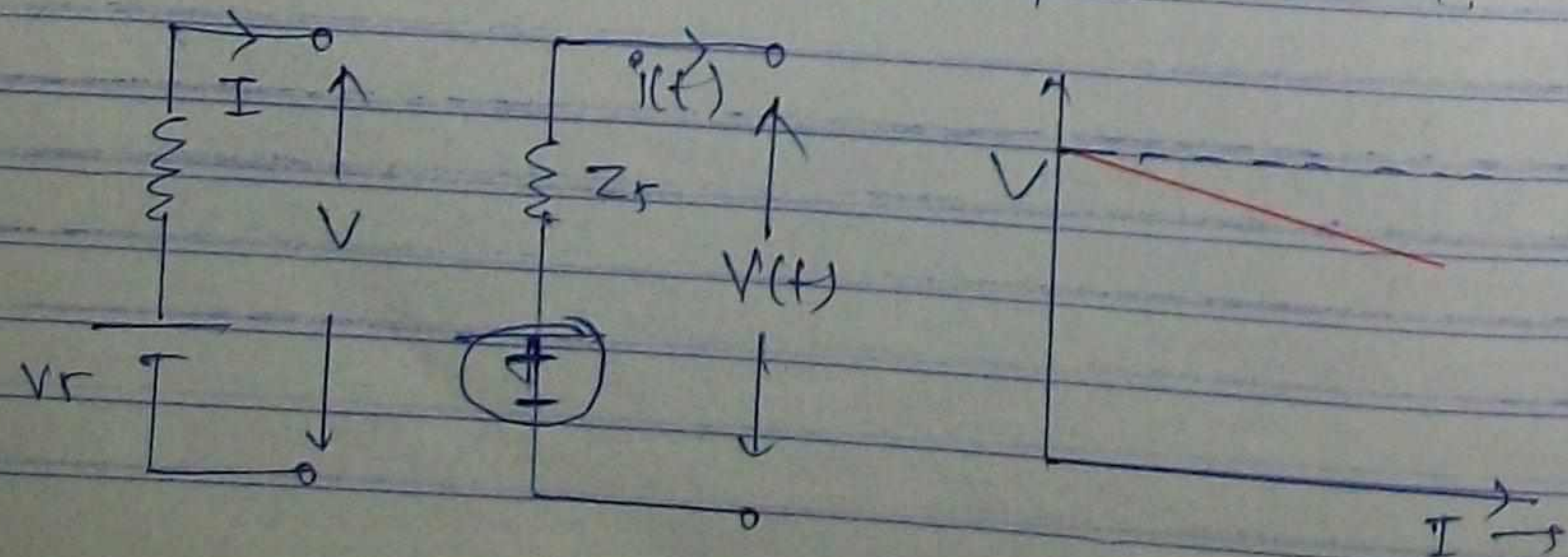
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⇒ VOLTAGE SOURCE -

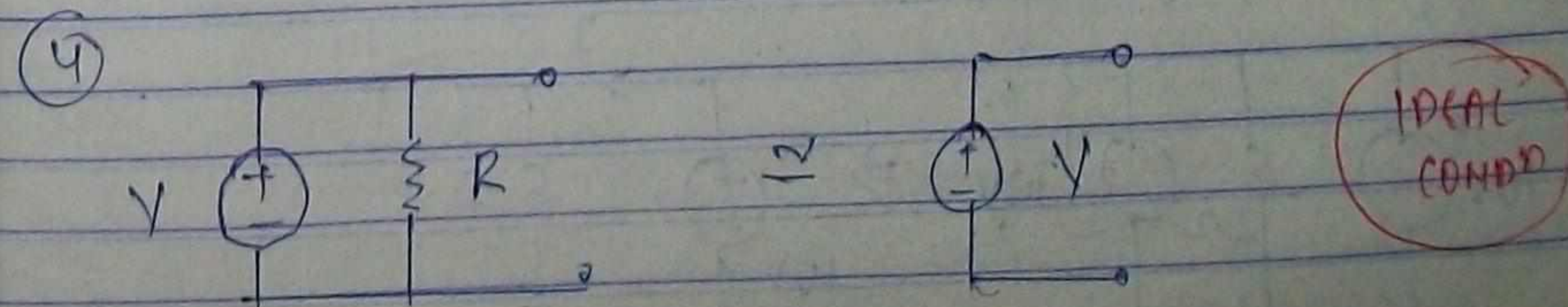
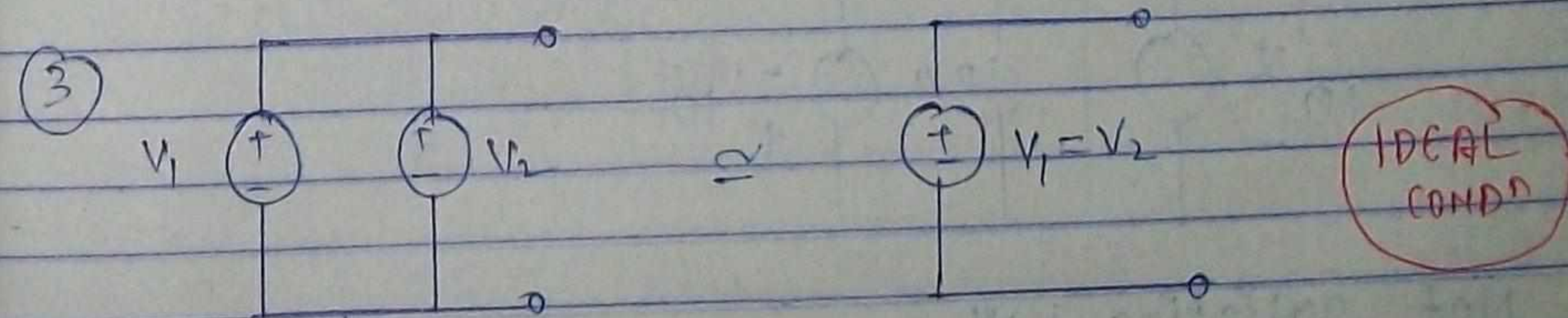
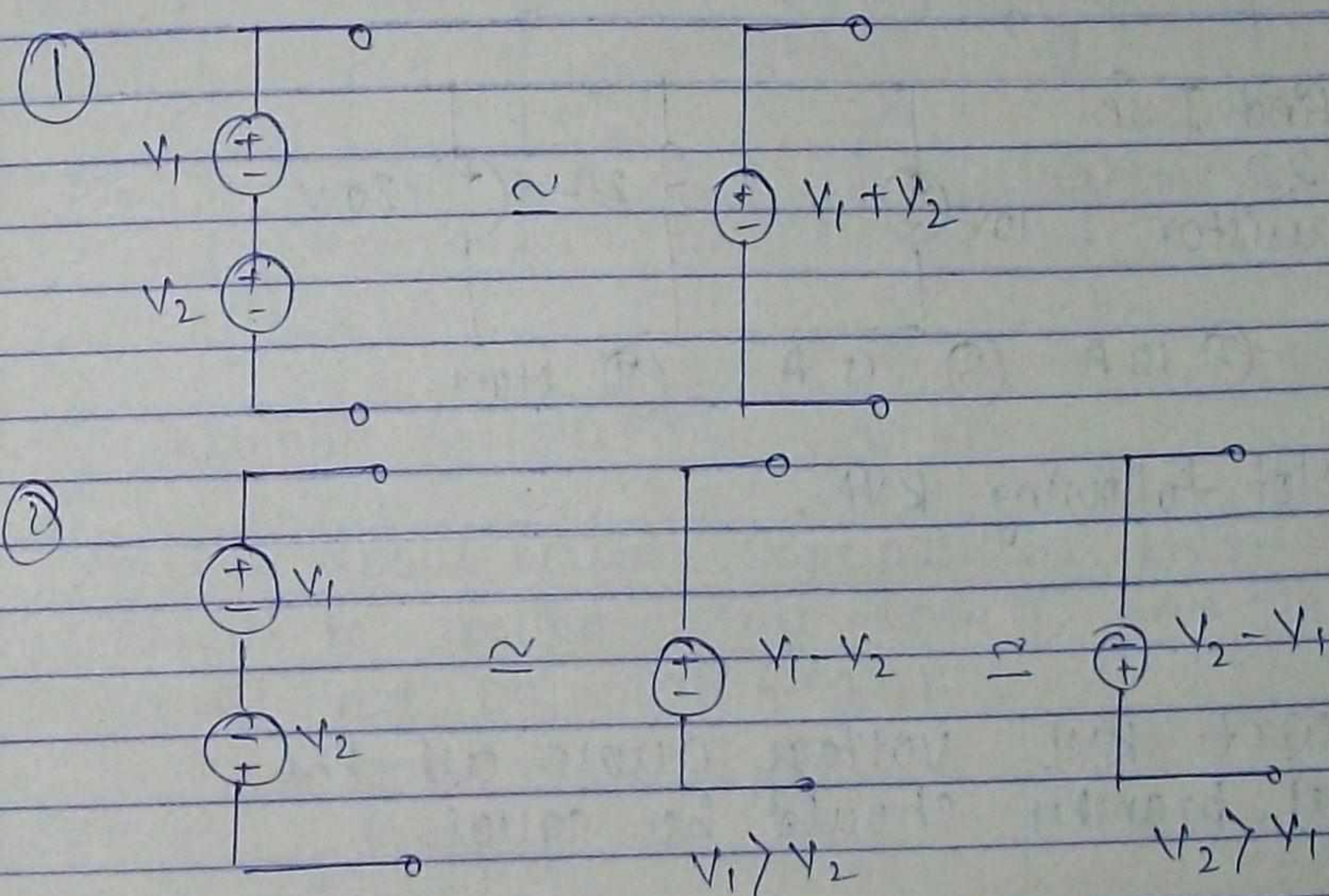
- Ideal voltage source delivers energy at specified voltage (V), which is independent of current delivered by the source.
- Internal resistance of ideal voltage source is zero.

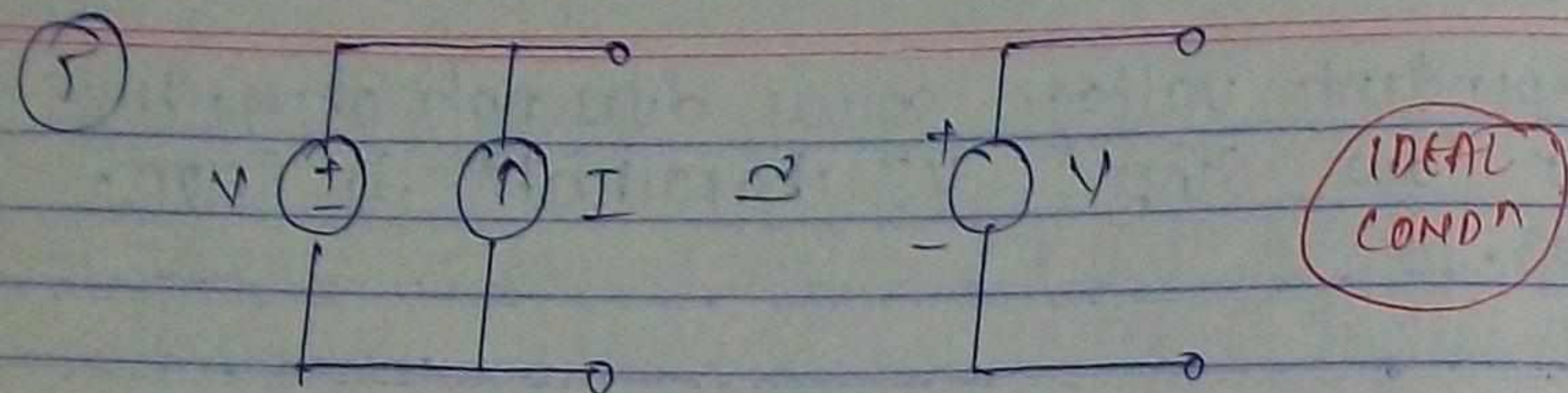


- Practical voltage source delivers energy at specified voltage (V) which depends on current delivered by the source.



- Independent voltage source does not obey Ohm's law. Since, VI characteristic is non-linear.





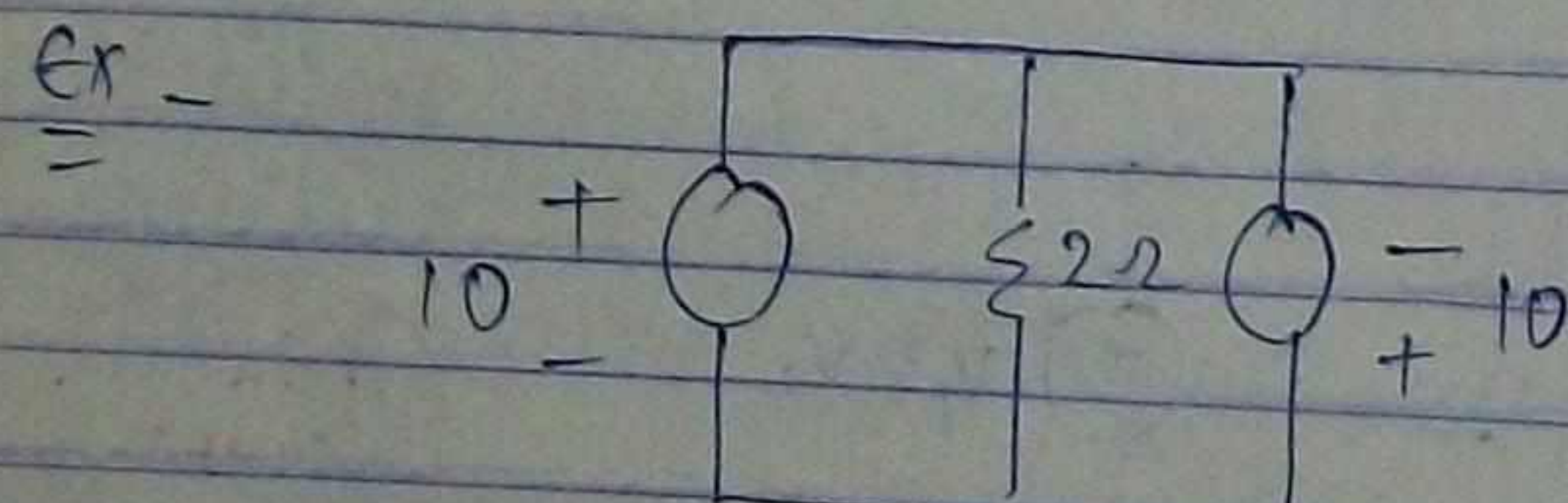
Ques. find I in 2Ω resistor.

- (1) 5A (2) 10A (3) 15A (4) None.

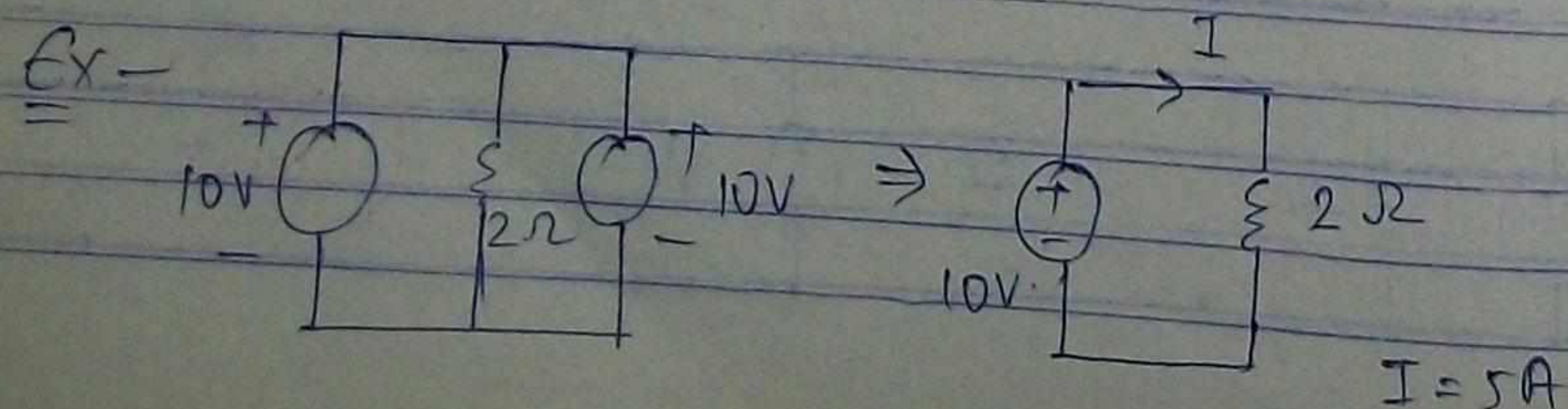
Solⁿ: Not following KVL.

NOTE -

Wrt KVL voltage across all the parallel branches should be equal.



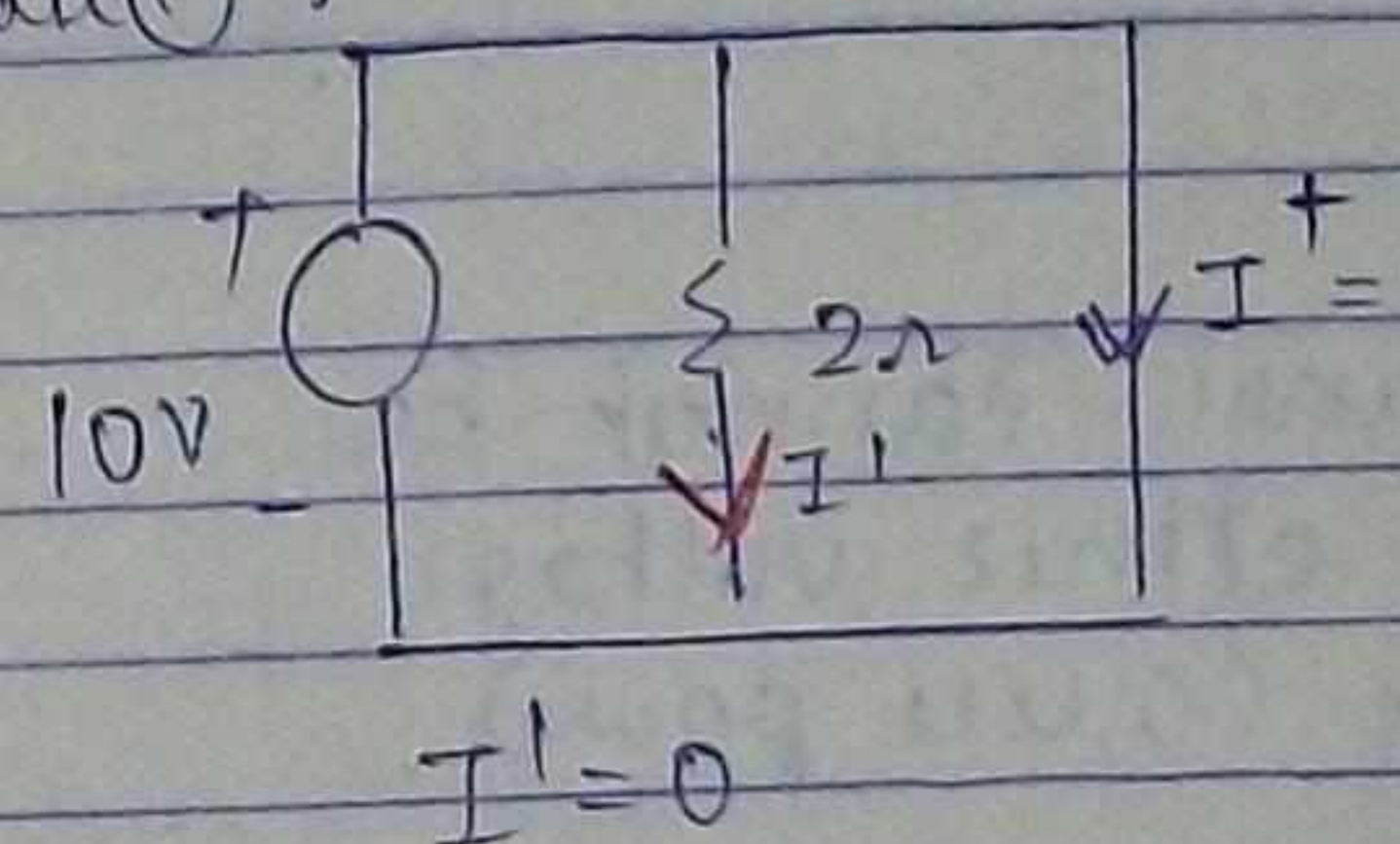
Not satisfying KVL.



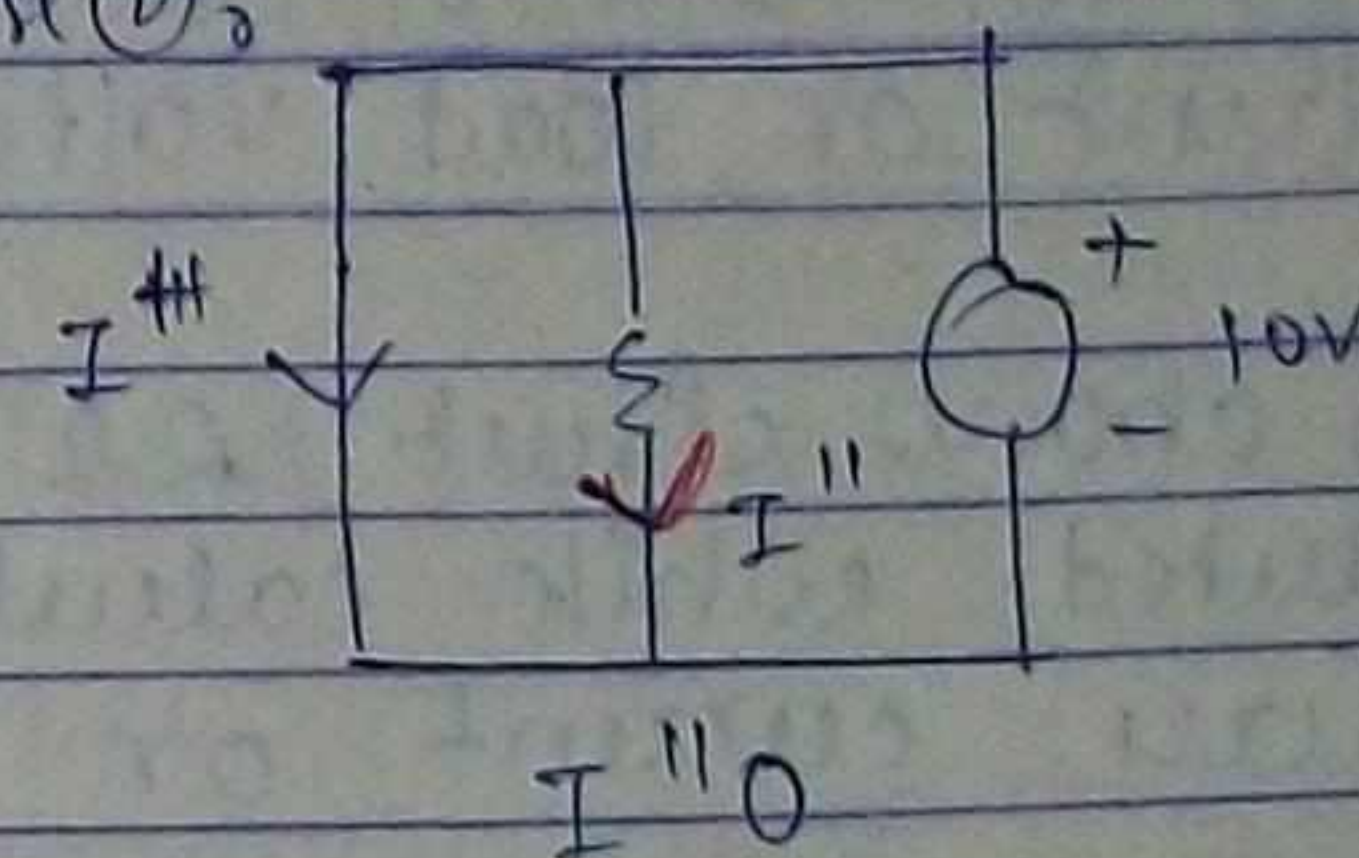
NOTE -

- By superposition theorem:

Case (1):



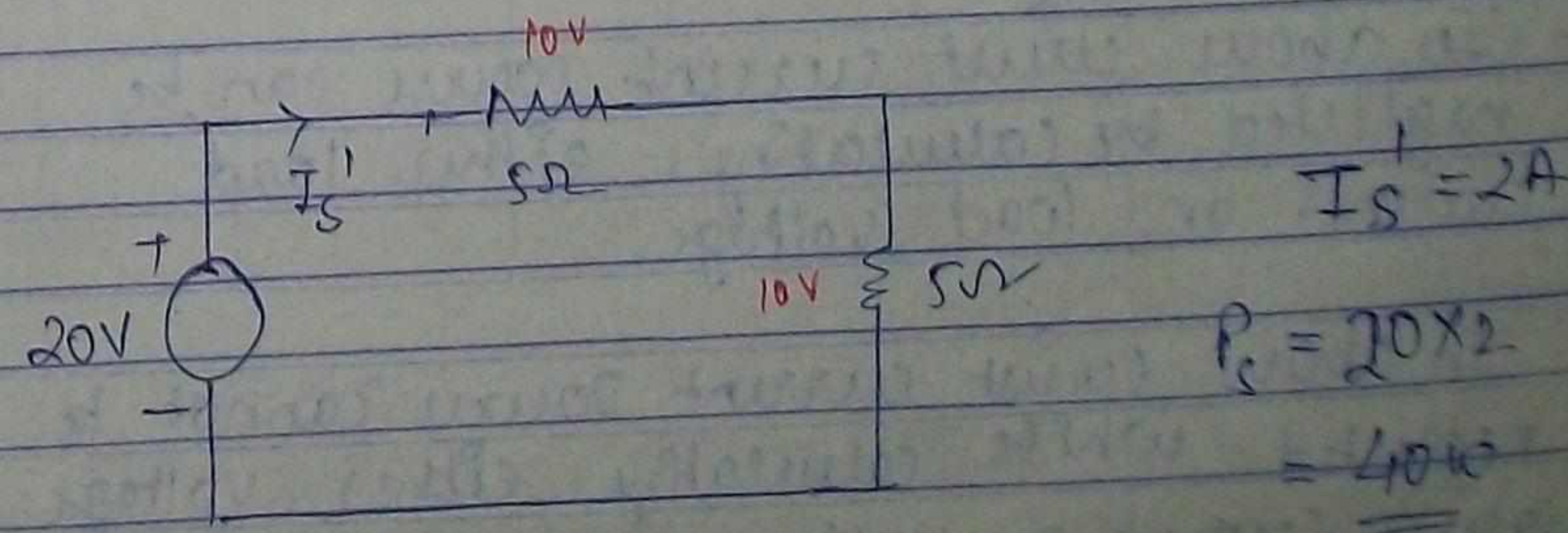
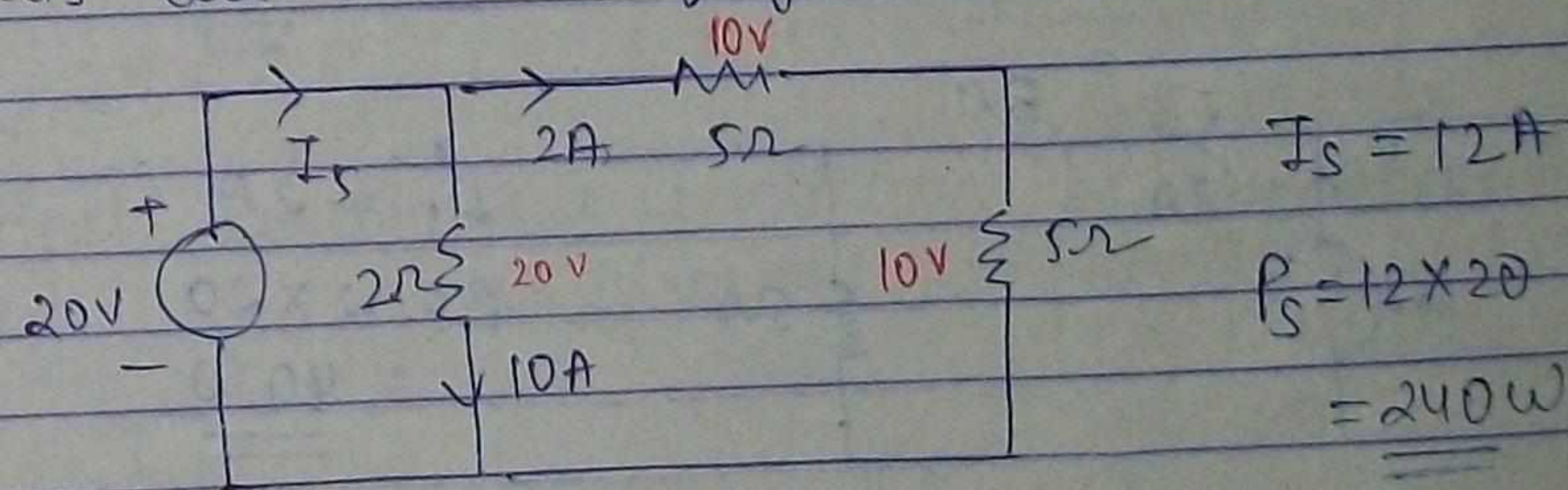
Case (2):



Cannot follow KVL

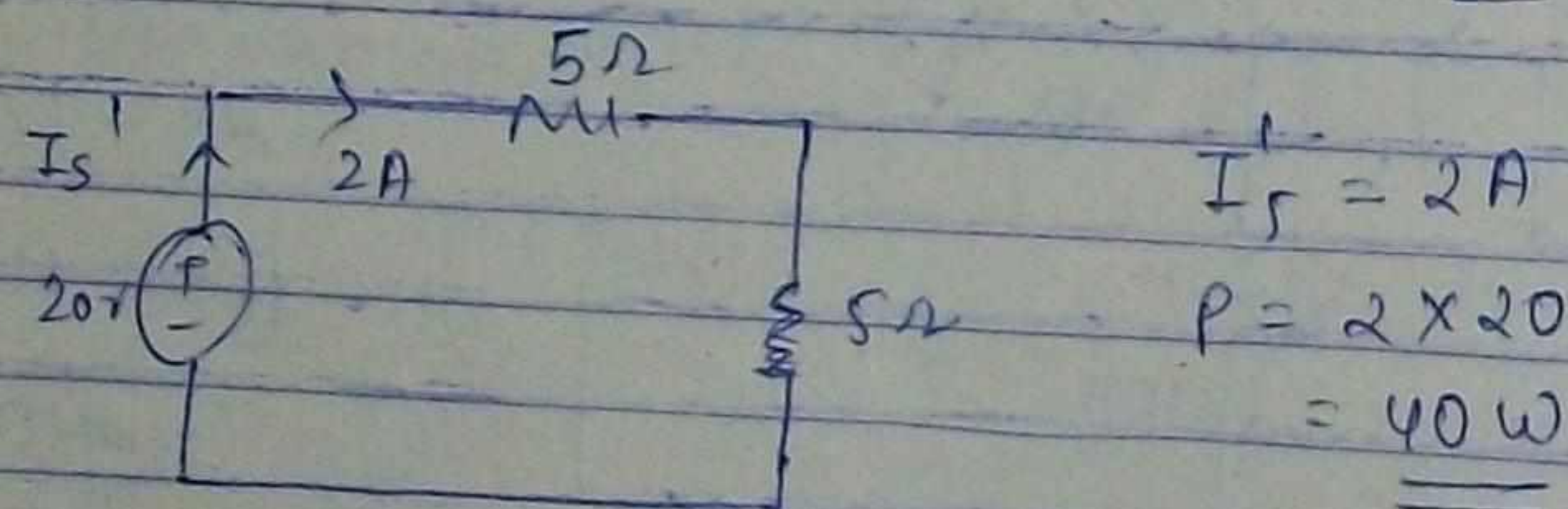
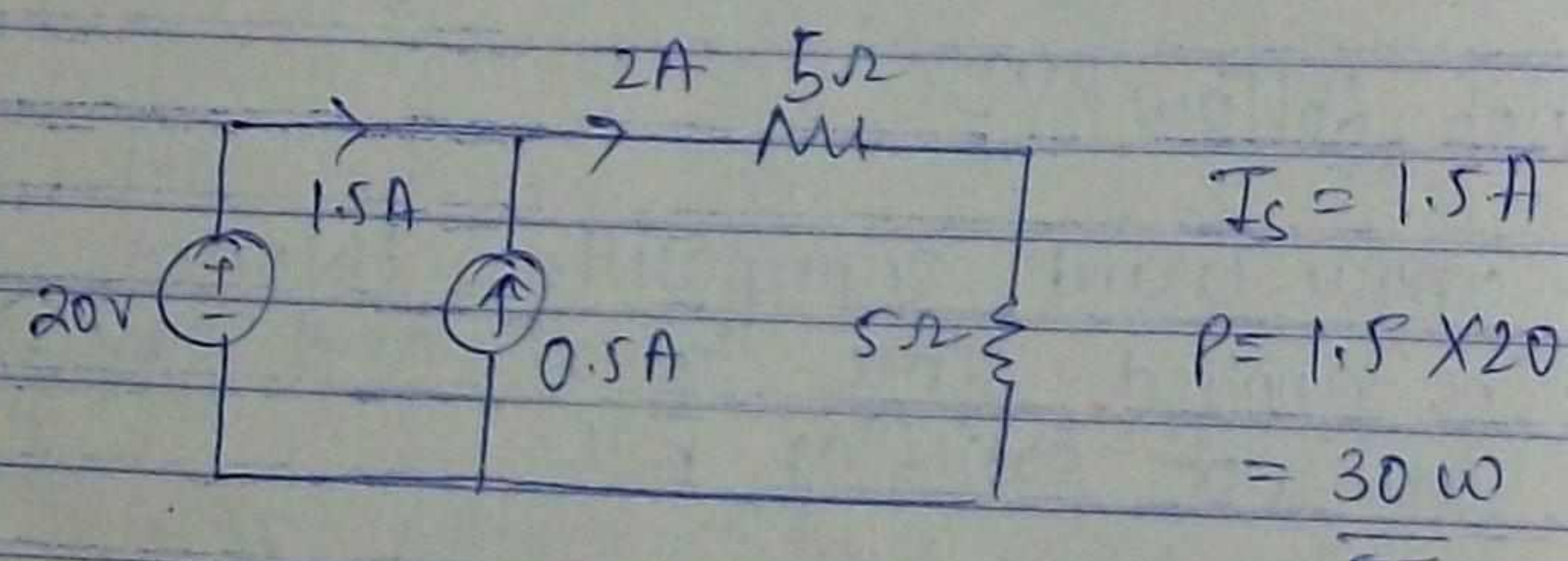
for the above circuit superposition theorem cannot be applied since Case (1) and Case (2) circuits are not satisfying KVL

Ex -



NOTE -

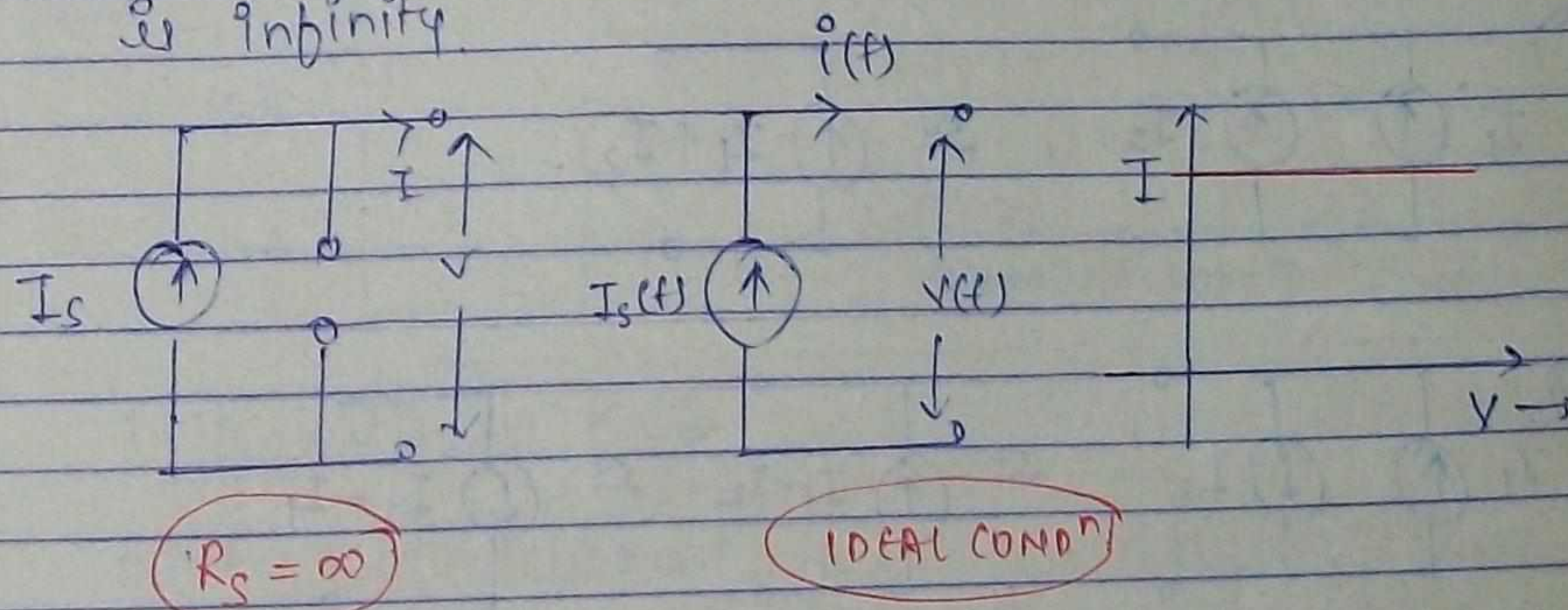
- In above circuit 2Ω resistance can be neglected while calculating either load current or load voltage.
- In above circuit 2Ω resistance cannot be neglected while calculating either voltage source current or voltage source power.



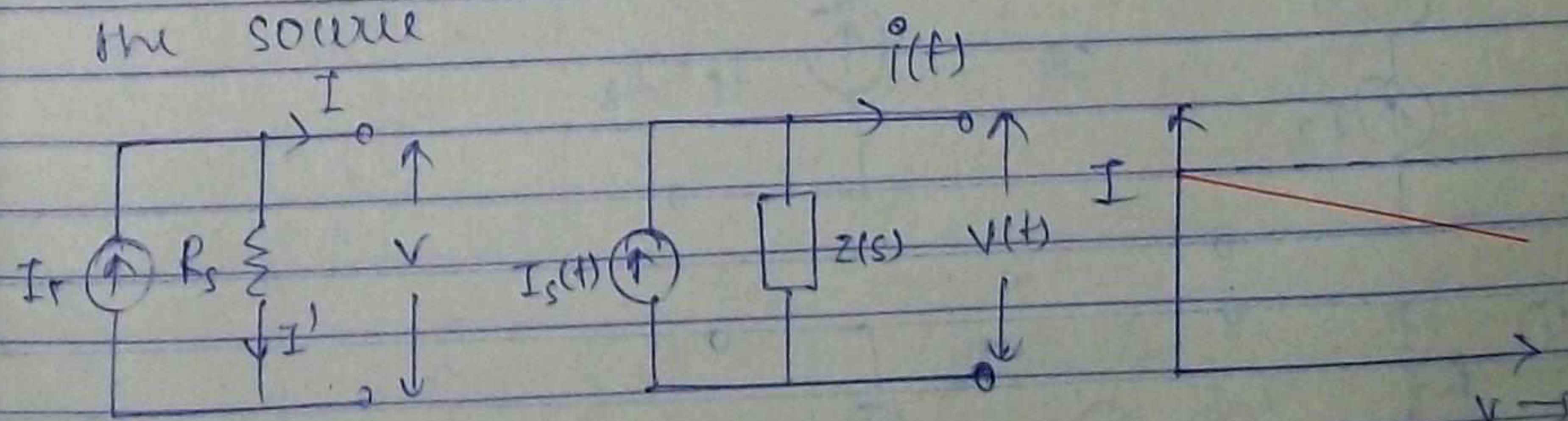
- In above circuit current source can be neglected by calculating either load current or load voltage.
- In above circuit current source cannot be neglected while calculating either voltage source current or voltage source power.

⇒ CURRENT SOURCE -

- Ideal current source delivers energy at specified current (I) which is independent of voltage across the source.
- Internal resistance of ideal current source is infinity.



- Practical current source delivers energy at specified current (I) which depends on voltage across the source.



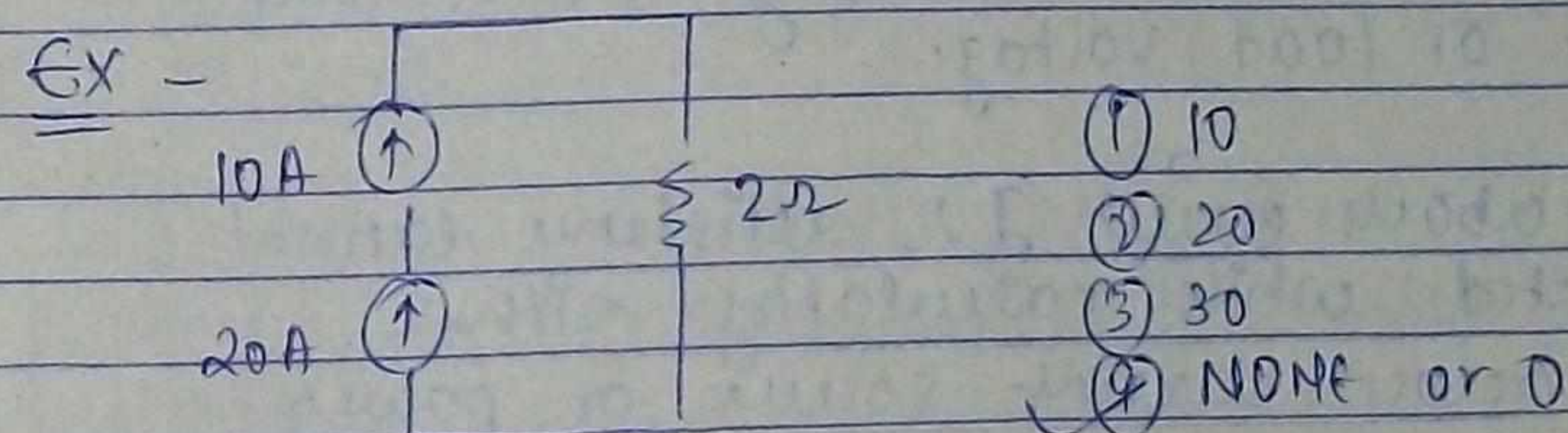
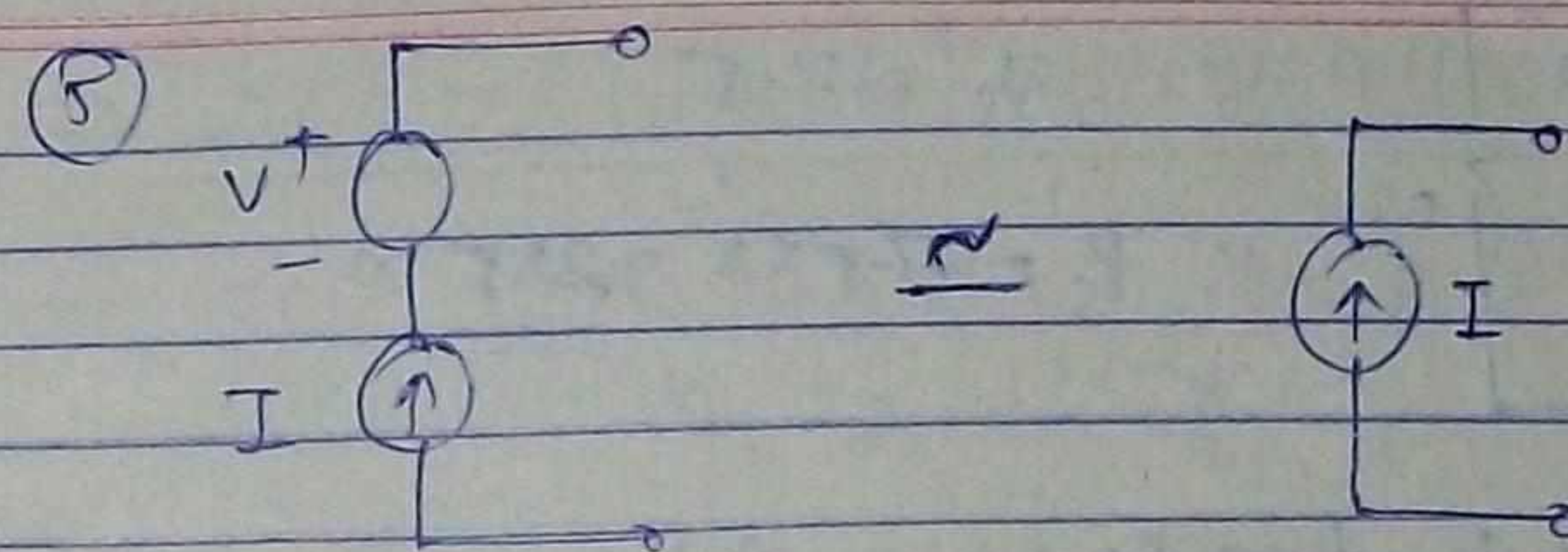
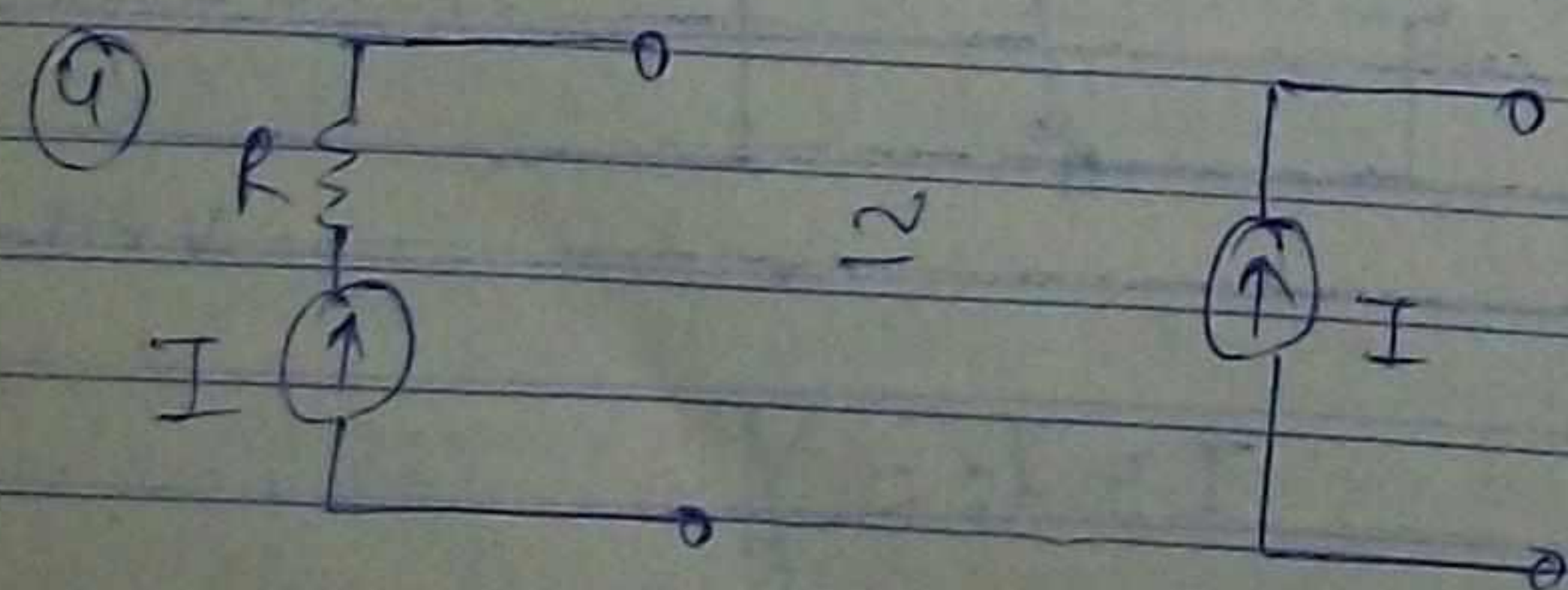
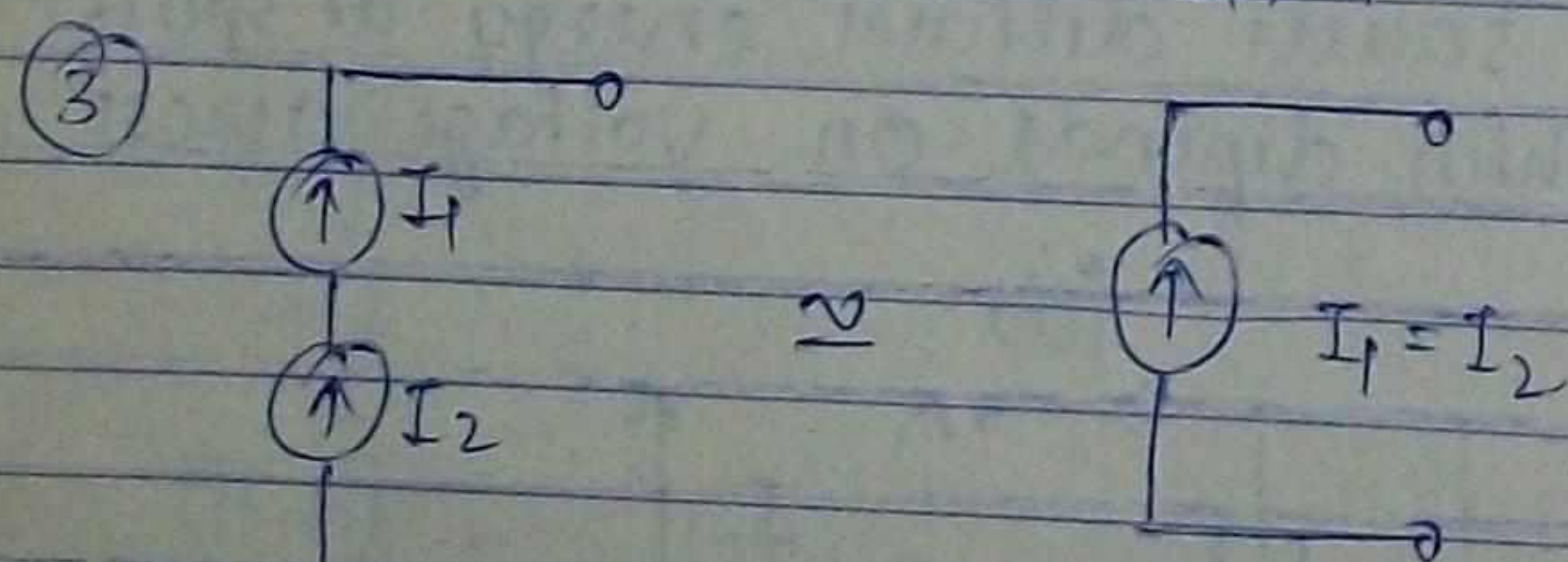
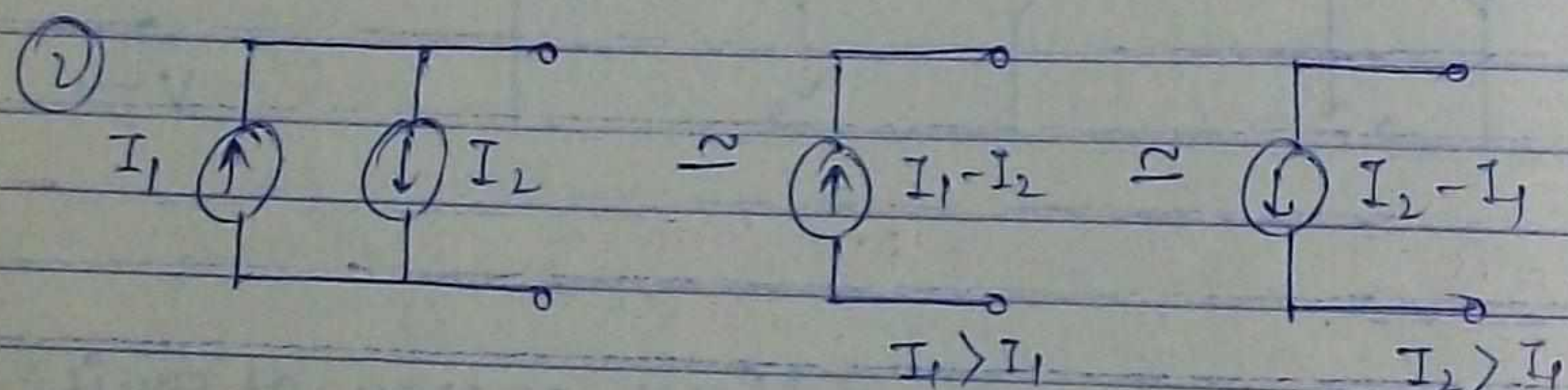
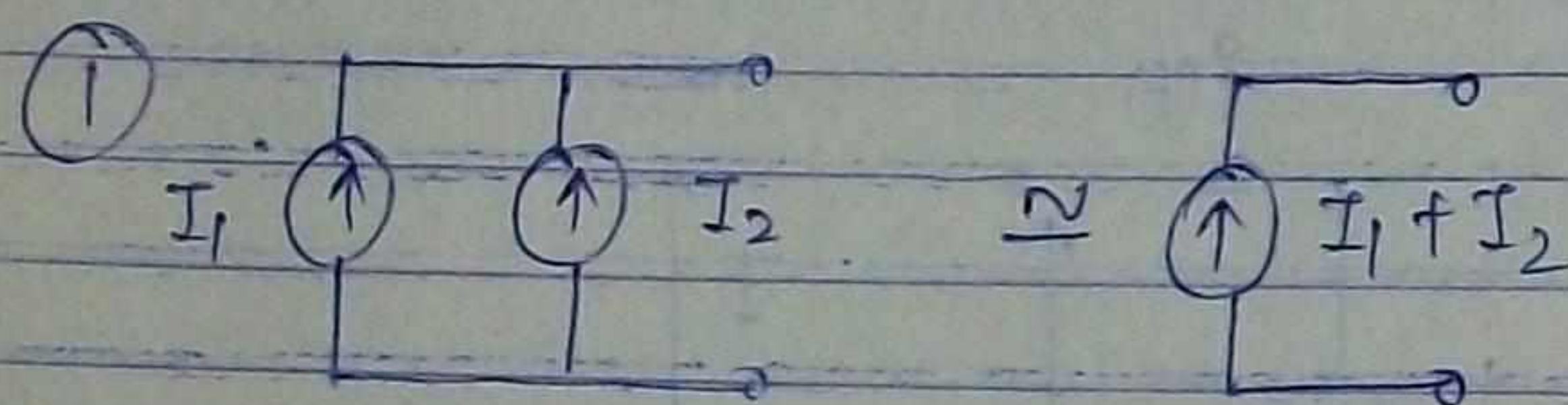
$$I_s = I + I'$$

$$I = I_s - I'$$

$$I = I_s - \frac{V}{R_s}$$

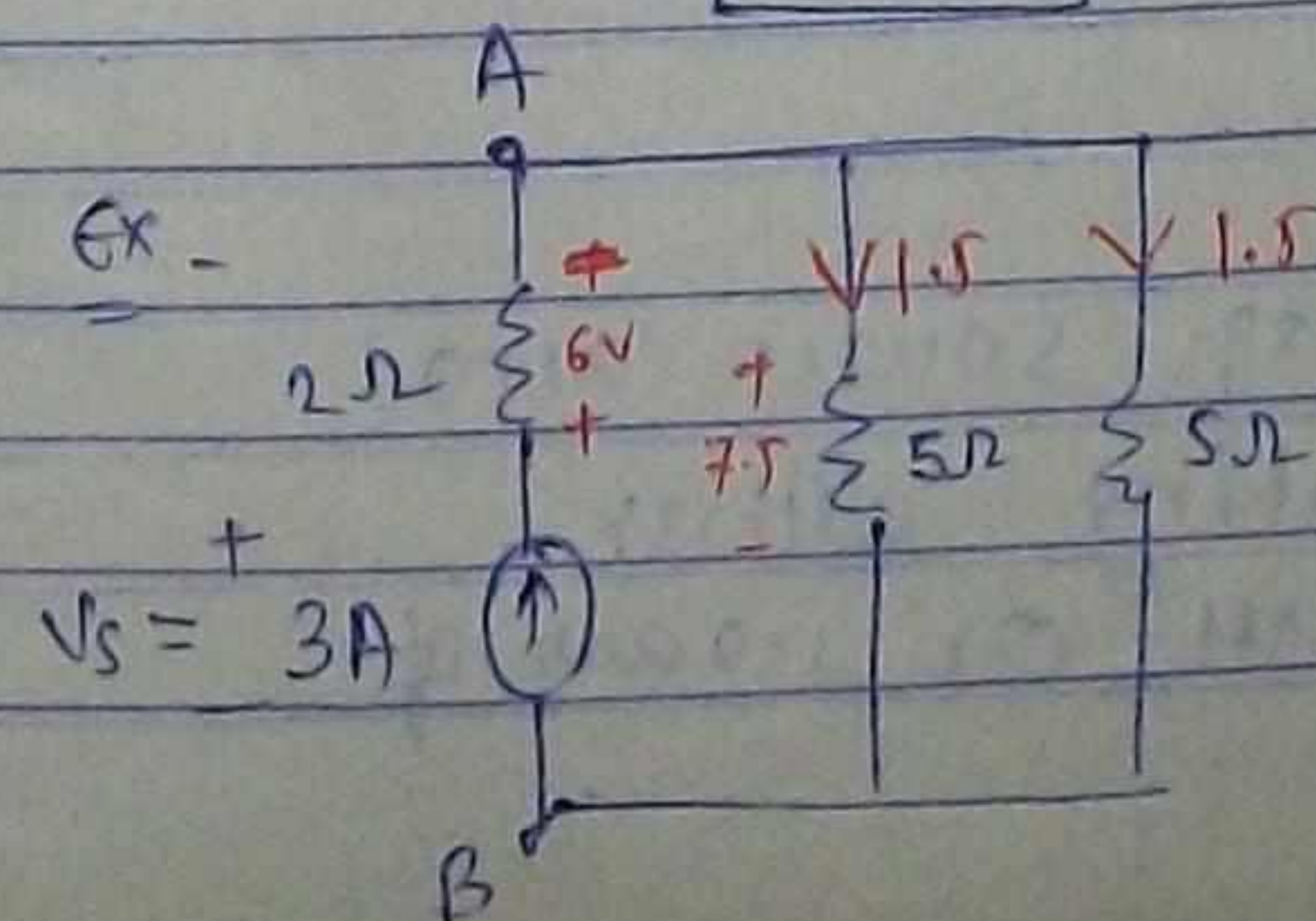
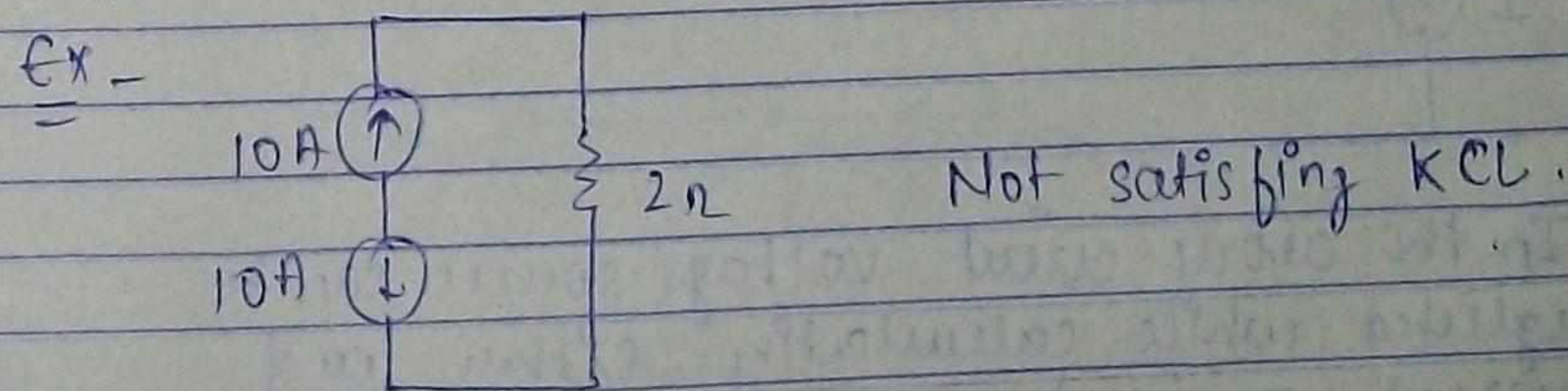
- Independent current source does not obey's the ohm's law. Since, $V-I$ characteristic is non-linear.

- In the real time system no independent current source exist.



Not satisfying KCL.

- For KCL current flow into all the series elements should be equal.

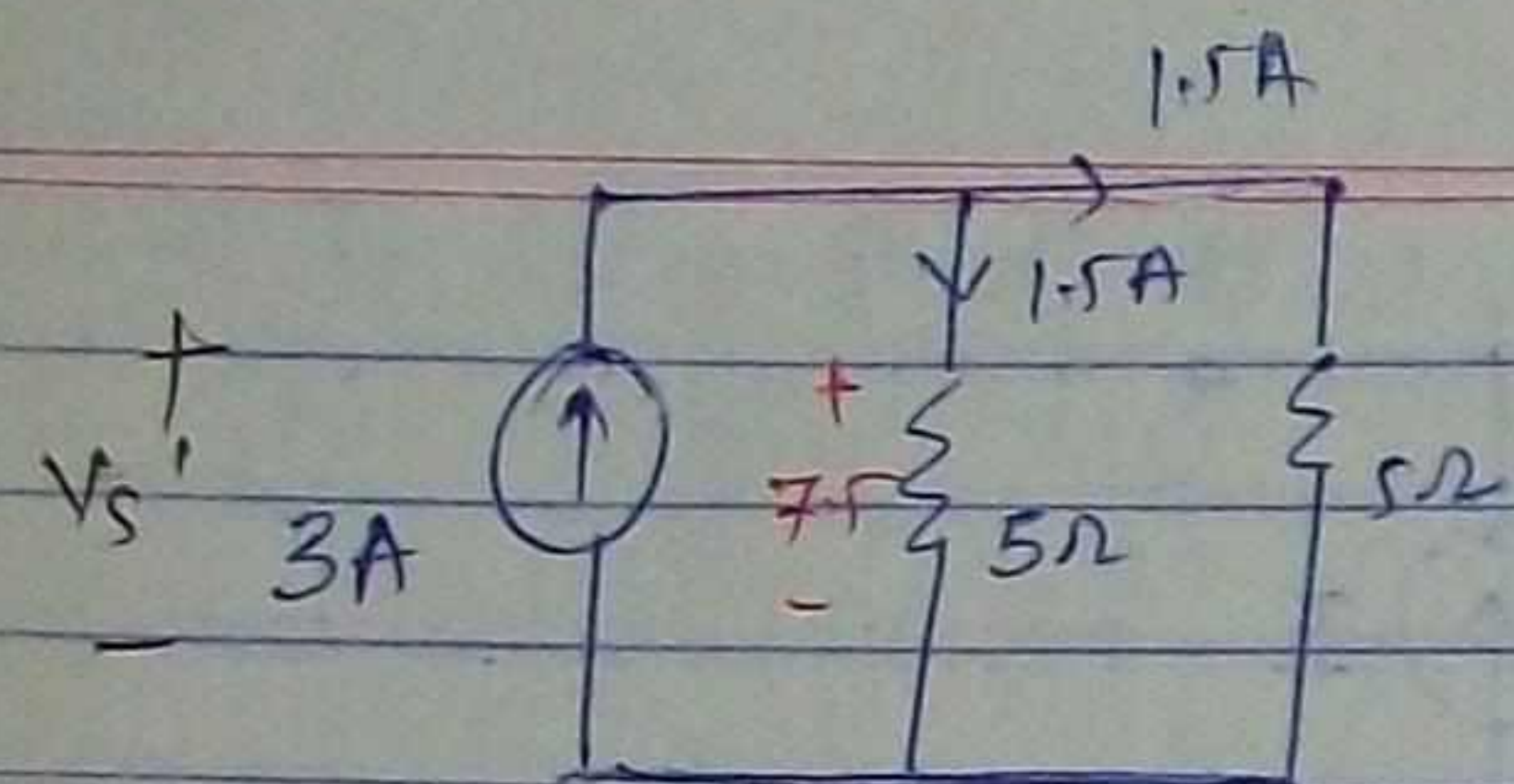


$$V_{AB} = V_s - 6 = 7.5$$

$$V_s = 13.5$$

$$P = 13.5 \times 3$$

$$= 40.5 \text{ W}$$



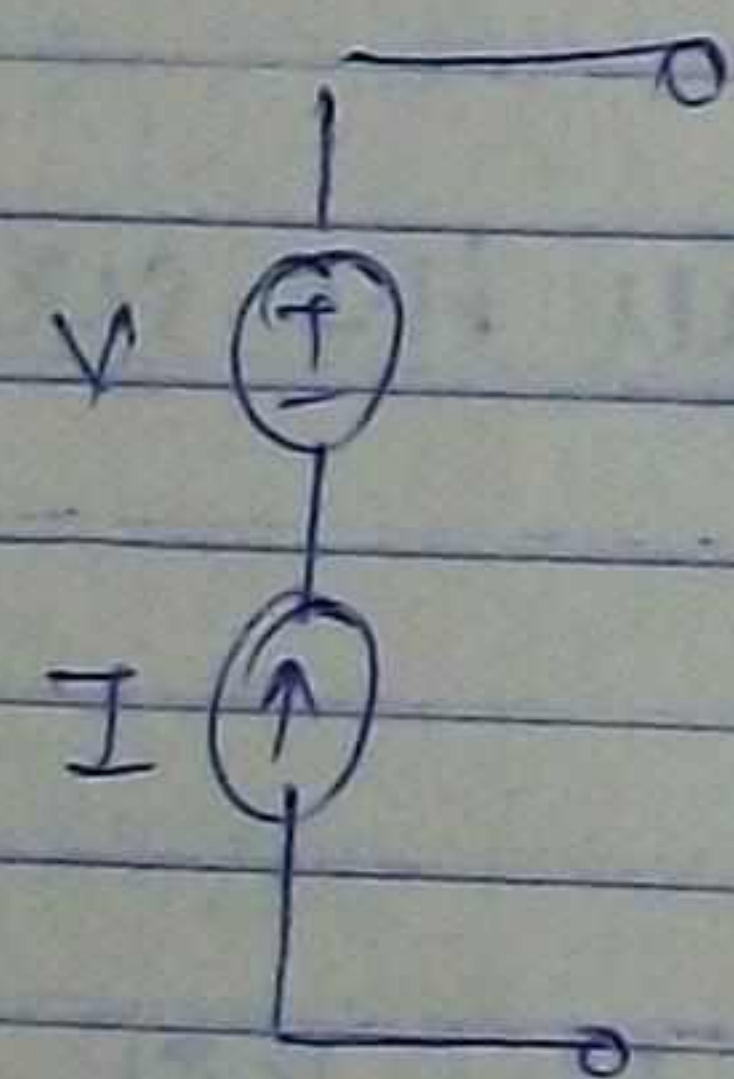
$$V_s' = 7.5$$

$$P_s' = 7.5 \times 3 = \underline{\underline{22.5 \text{ W}}}$$

NOTE -

- In above circuit two (2) resistances can be neglected while calculating either load current or load voltage.

- In the above circuit 2Ω resistance cannot be neglected while calculating either voltage across current source or power of the current source.



- In the above circuit voltage source can be neglected while calculating either load current or load voltage.

- In the above circuit voltage source cannot be neglected while calculating either voltage across current source or power of the current source.

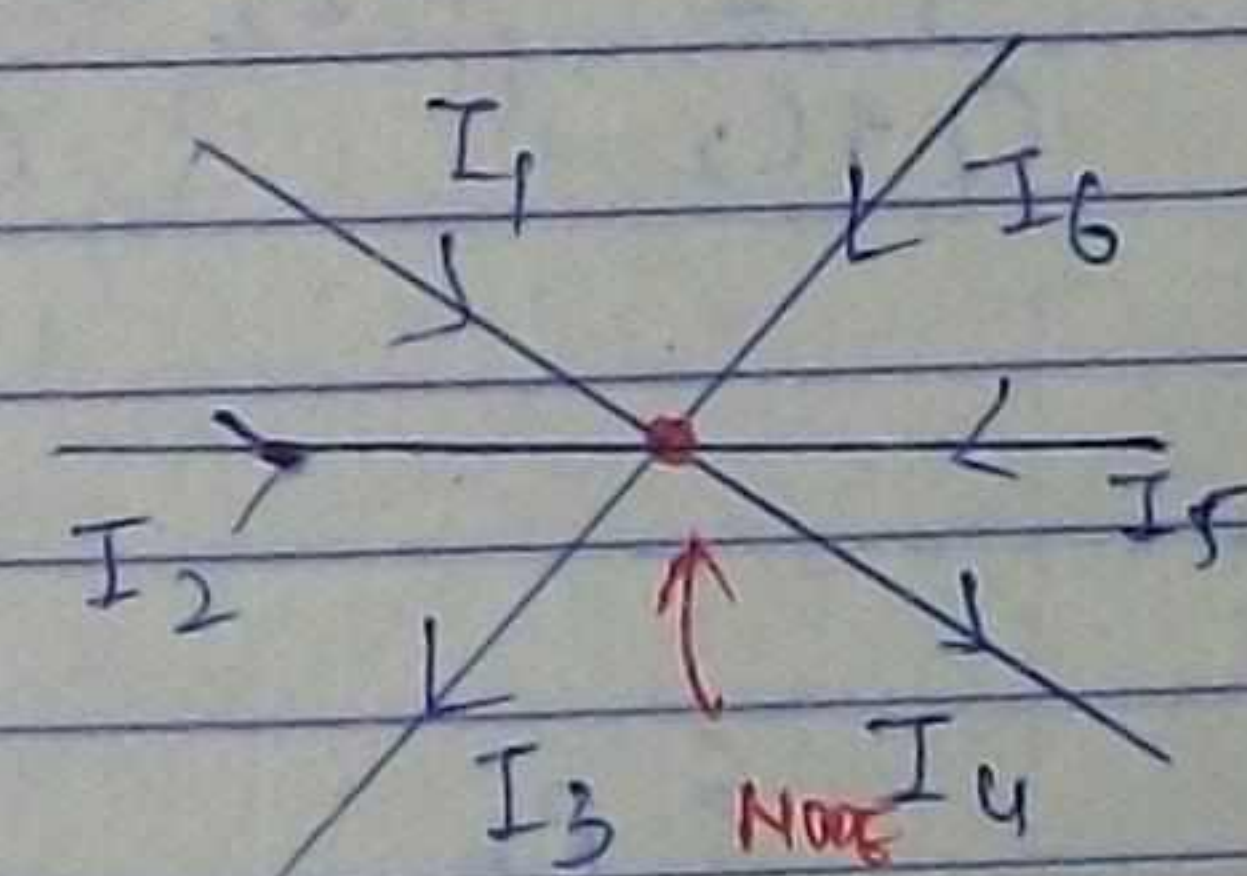
⇒ KCL - (LAW OF CONSERVATION OF CHARGE)

- KCL works based on the principle of law of conservation of charge.

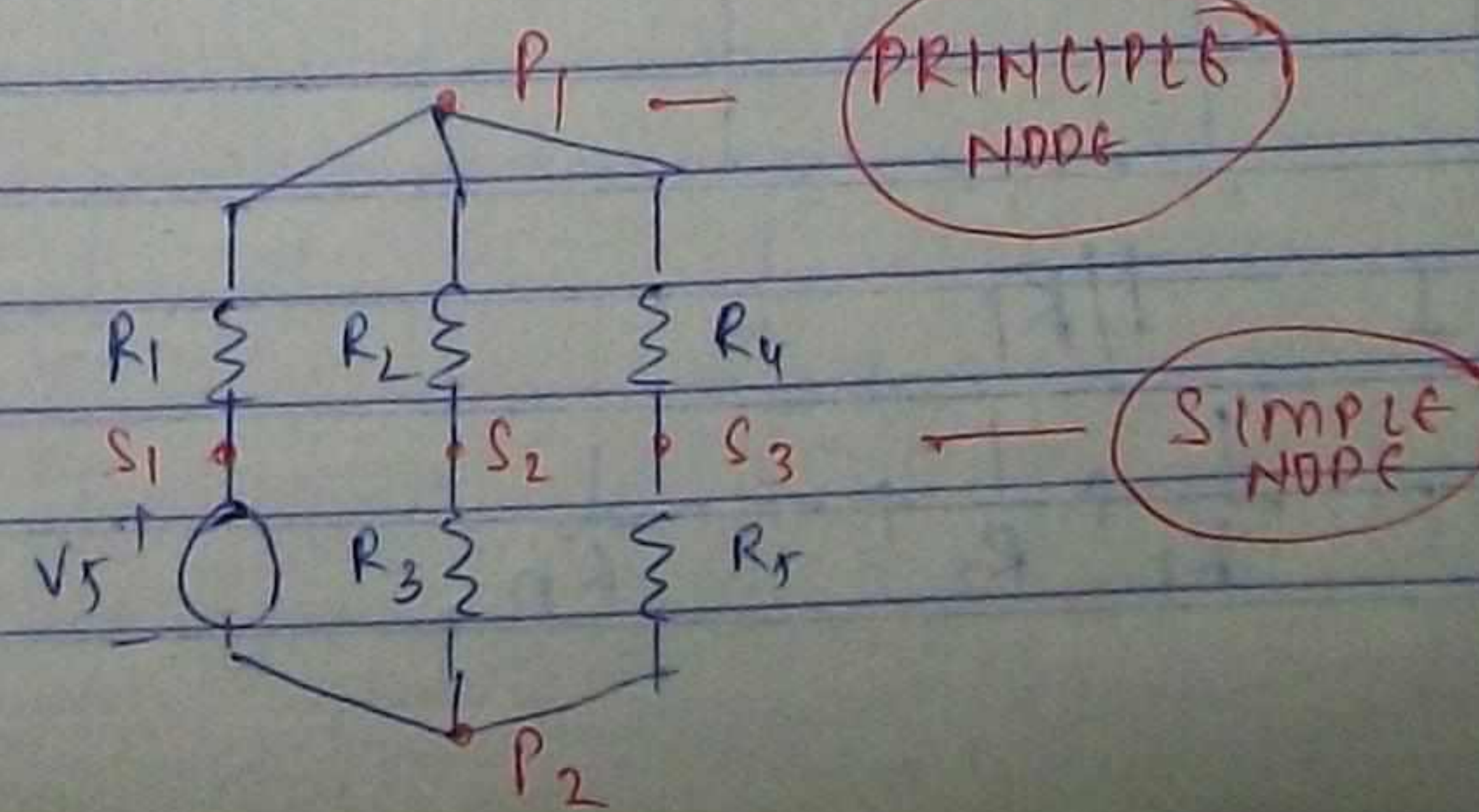
- KCL states that "Algebraic sum of the current meeting at a point is equal to zero."

- When two elements are connected together then the common point is called as simple node.

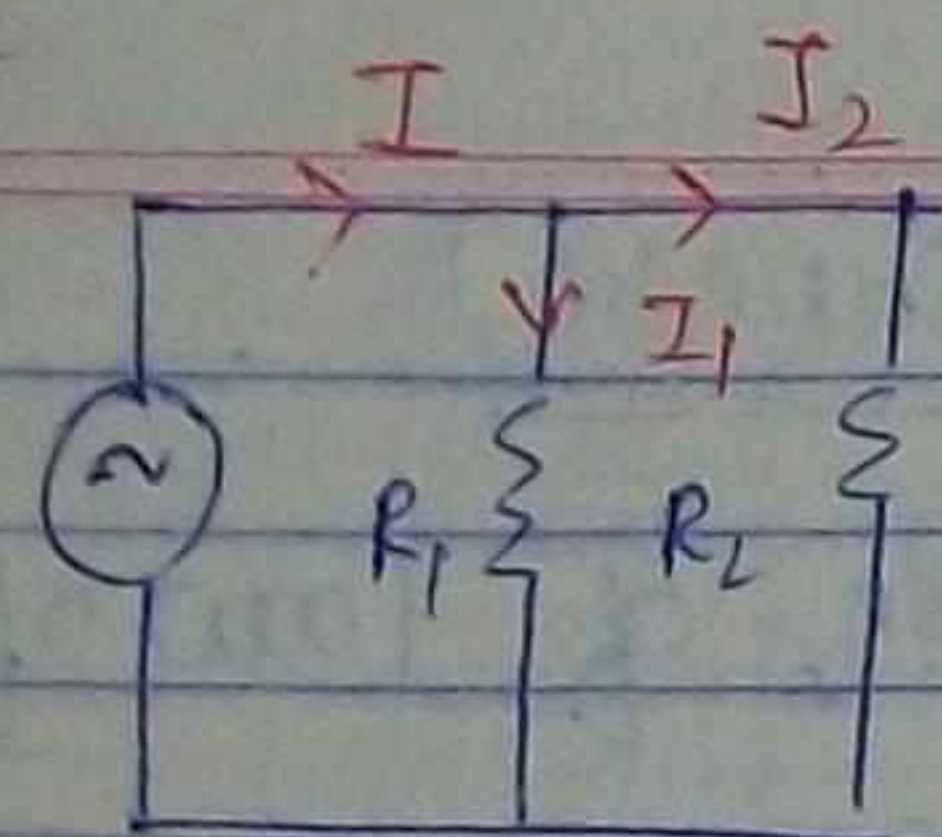
- When more than two elements are connected together then the common point is known as principle node.



$$I_1 + I_2 - I_3 - I_4 + I_5 - I_6 = 0$$

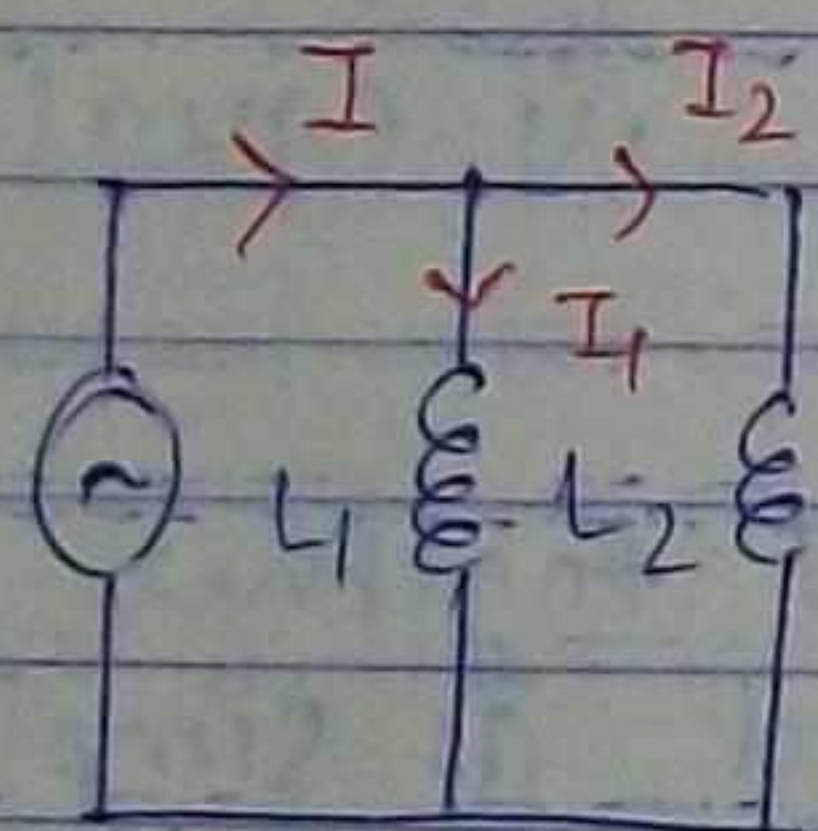


#



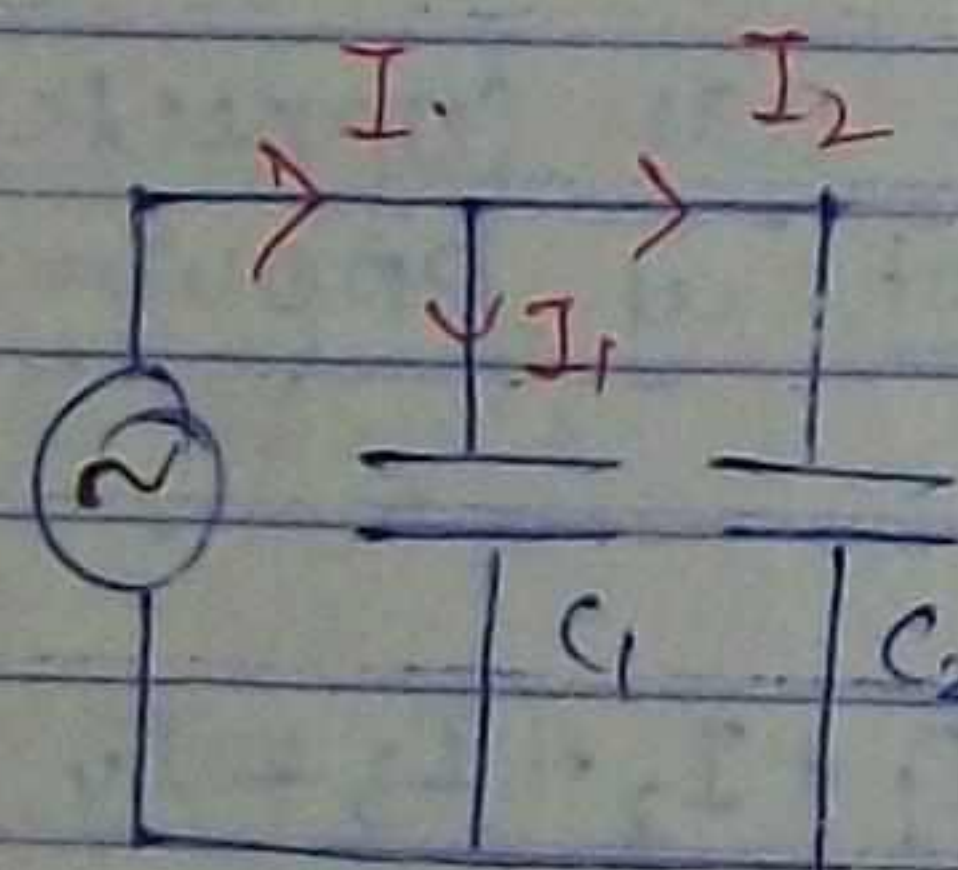
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$I_1 = \frac{R_2}{R_1 + R_2} I, \quad I_2 = \frac{R_1}{R_1 + R_2} I$$



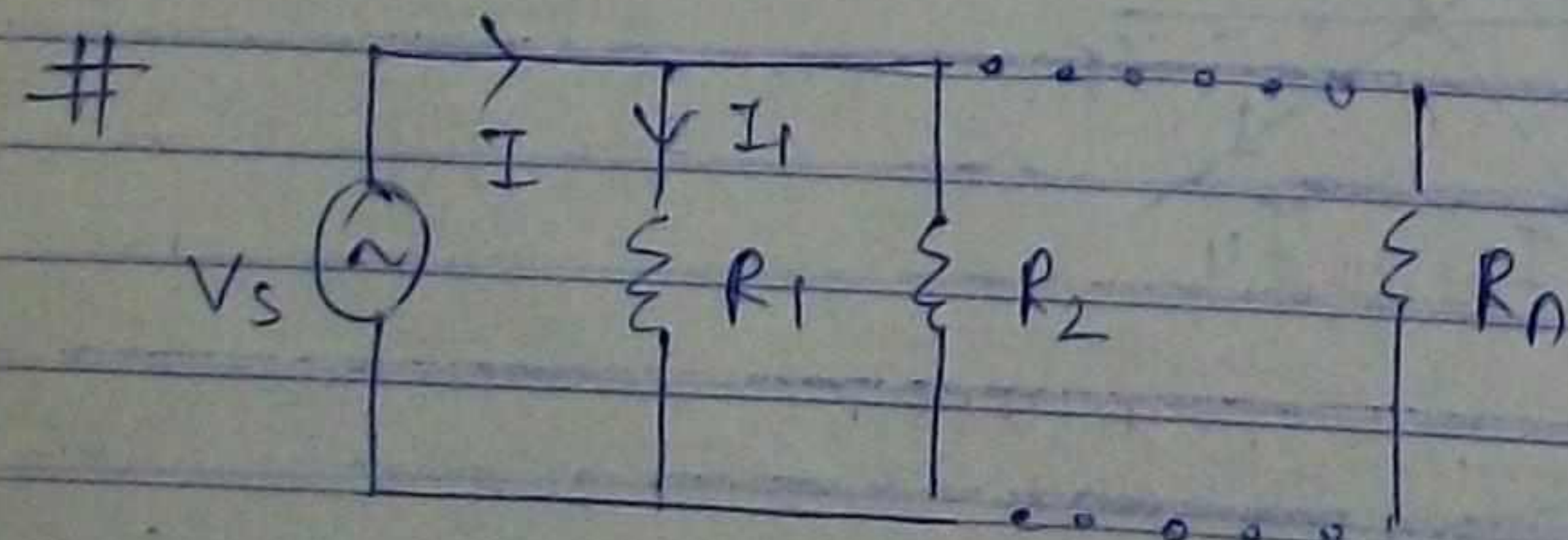
$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$$

$$I_1 = \frac{L_2}{L_1 + L_2} I, \quad I_2 = \frac{L_1}{L_1 + L_2} I$$



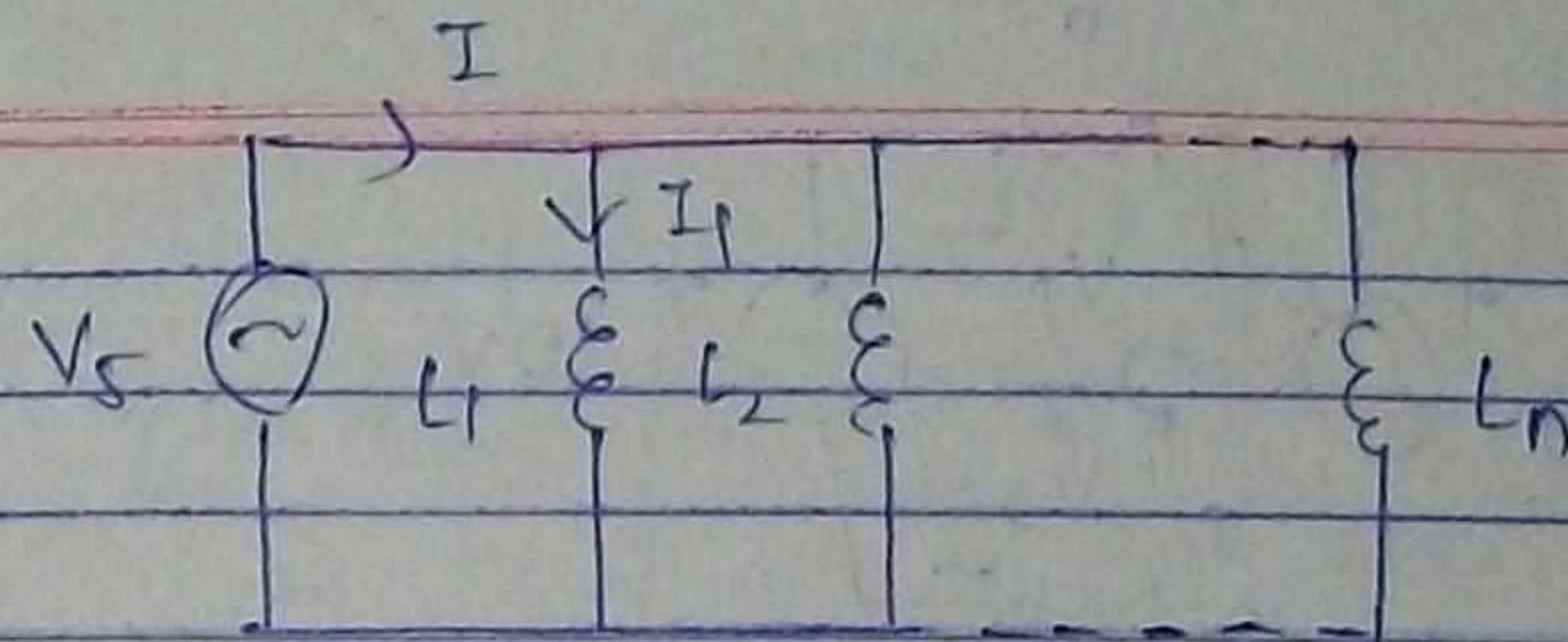
$$C_{eq} = C_1 + C_2$$

$$I_1 = \frac{C_2}{C_1 + C_2} I, \quad I_2 = \frac{C_1}{C_1 + C_2} I$$



$$I_1 = I = \frac{V_s}{R_1 + R_2 + \dots + R_n}$$

$$\frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}$$

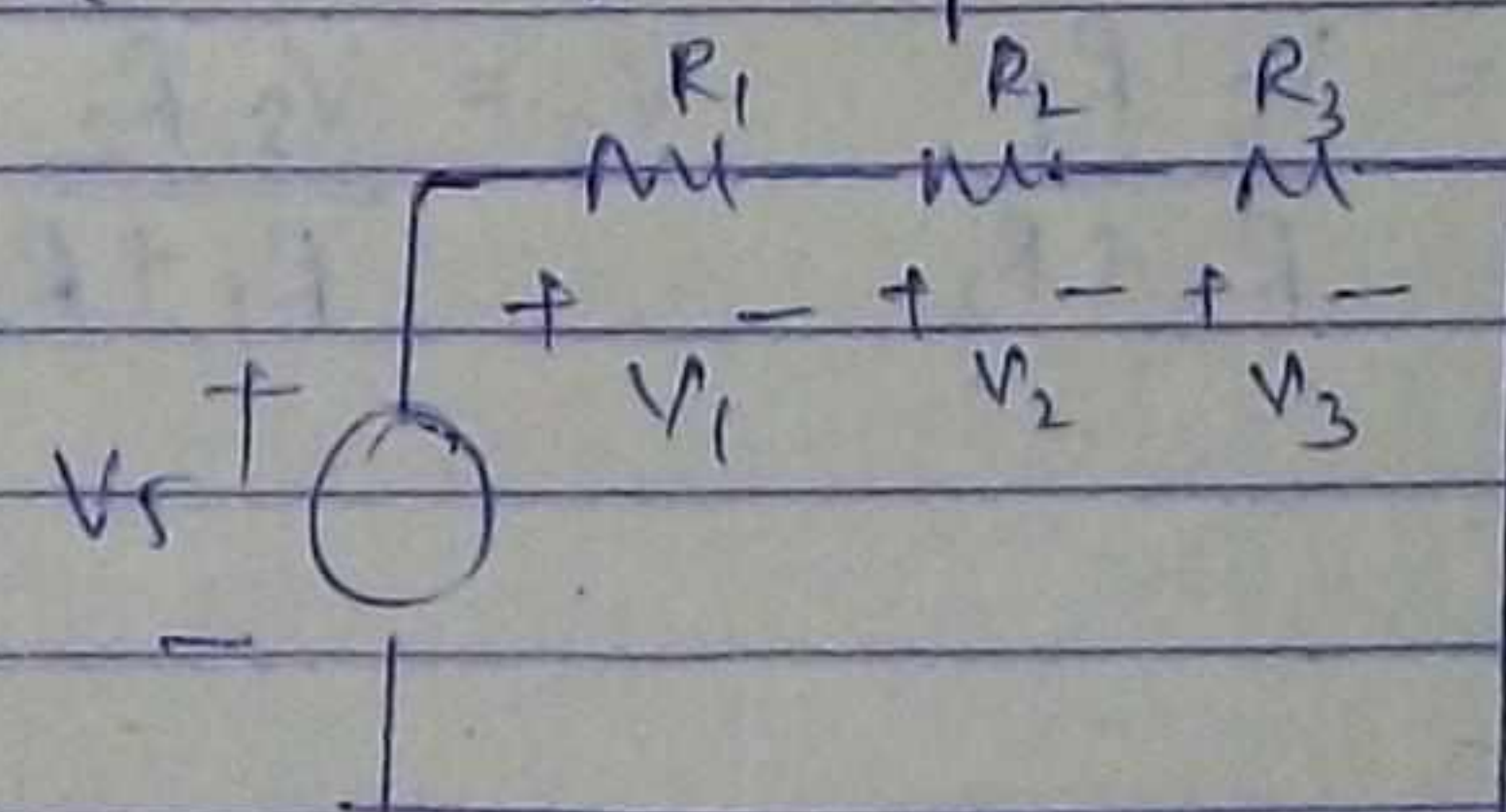


$$I_1 = I = \frac{V_s}{\frac{1}{L_1} + \frac{1}{L_2} + \dots + \frac{1}{L_n}}$$

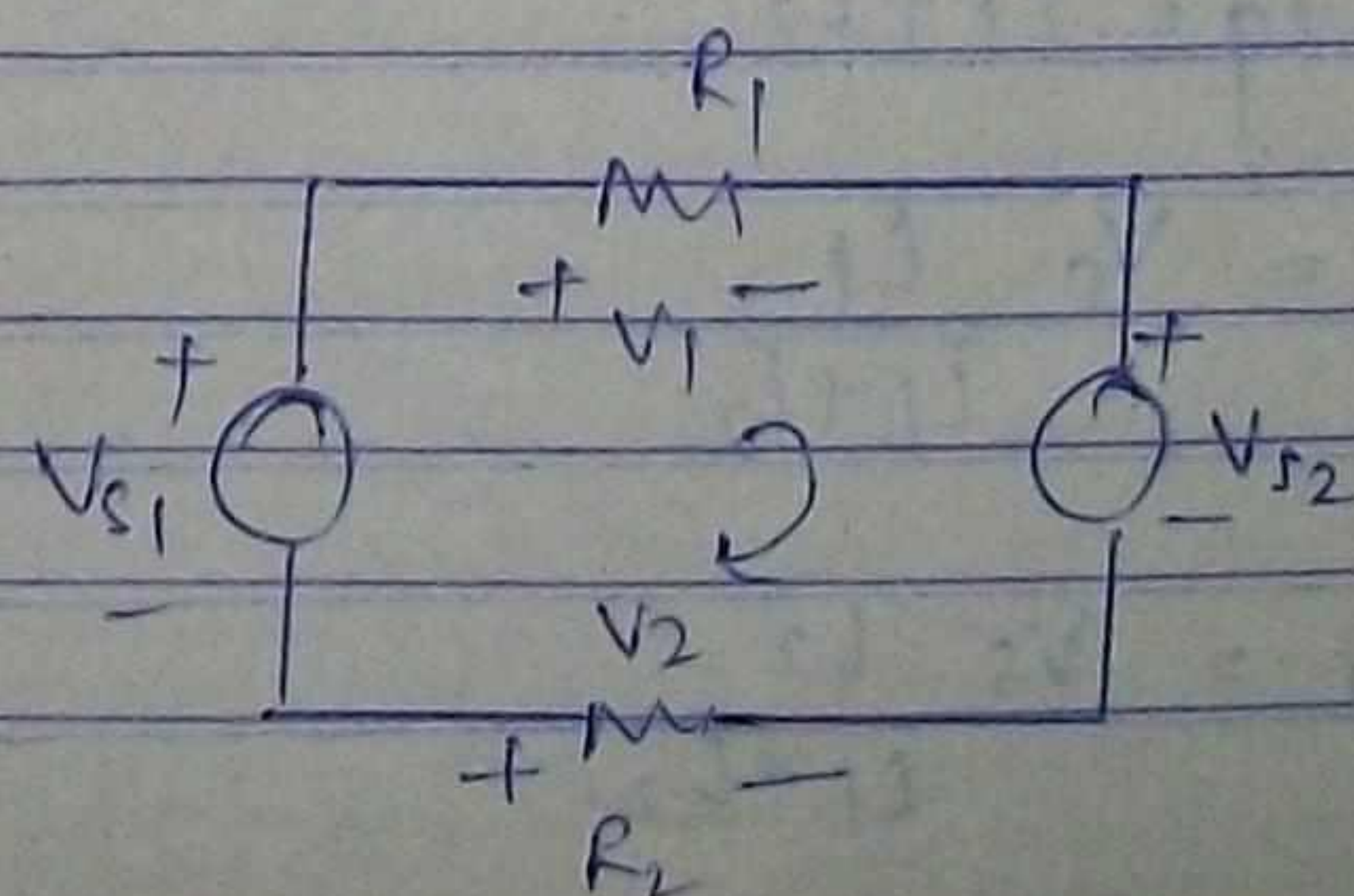
⇒ KVL - (LAW OF CONSERVATION OF ENERGY)

- KVL work based on the principle of law of conservation of energy.

- KVL states that "Algebraic sum of the voltage in a closed loop is equal to zero."

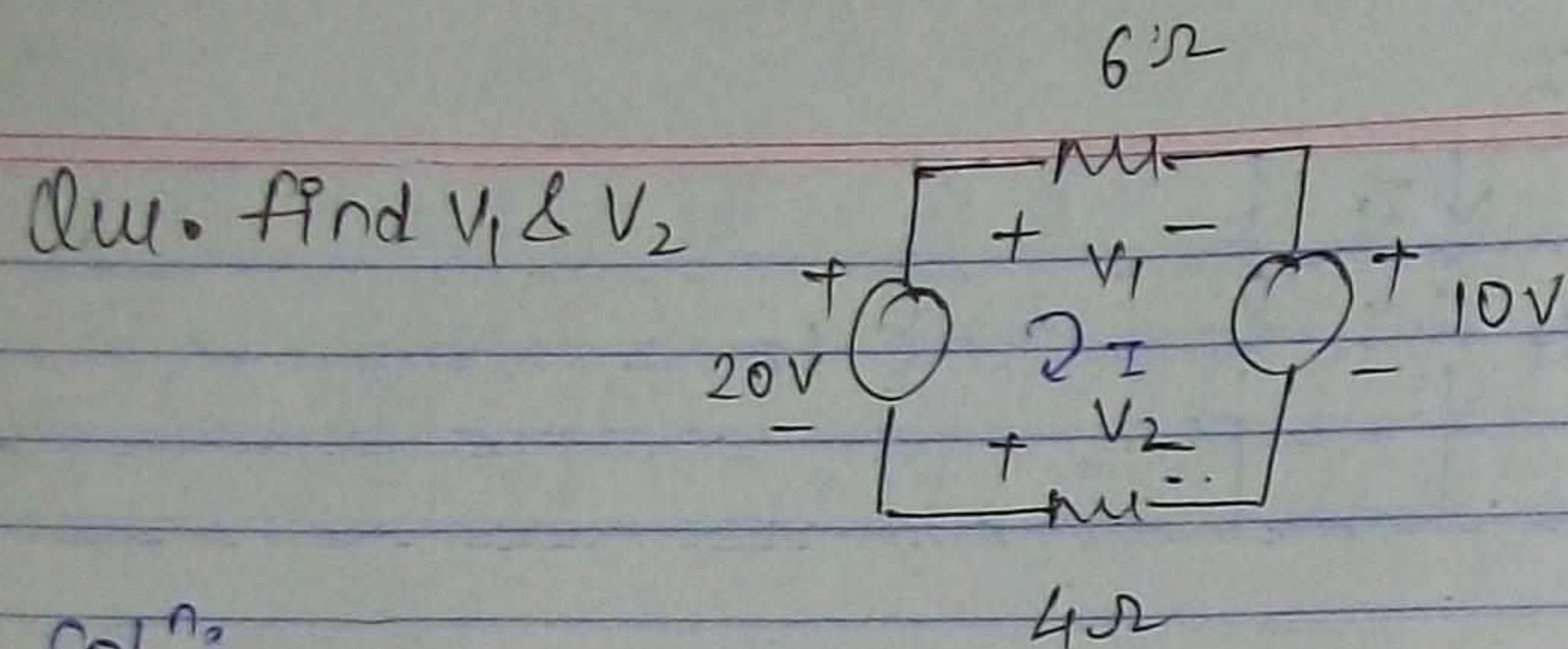


$$V_s = V_1 + V_2 + V_3$$



$$V_{s1} = V_1 + V_{s2} - V_2$$

$$V_{s1} + V_2 = V_1 + V_{s2}$$

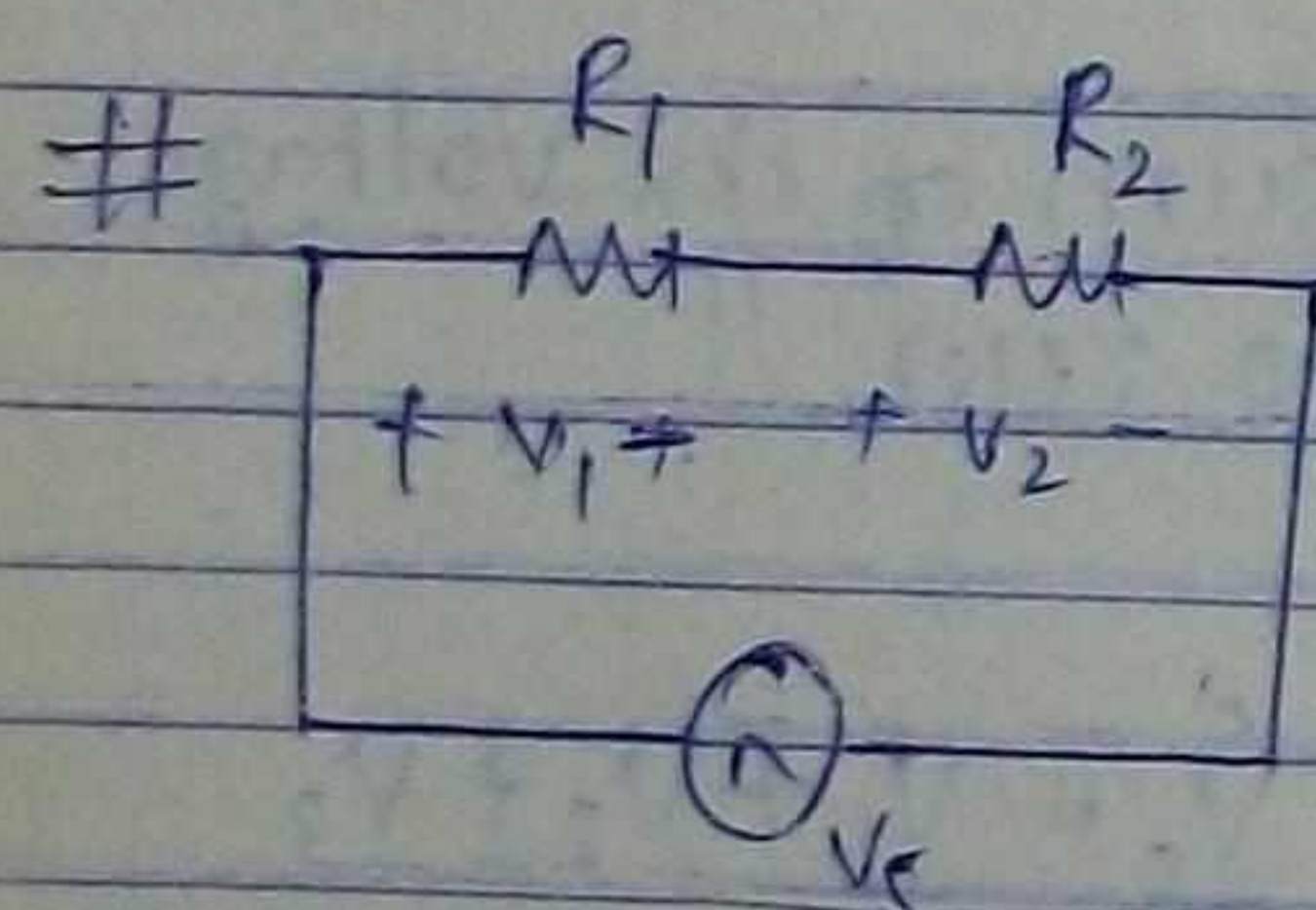


Solⁿ

$$20 = 6I + 10 + 4I$$

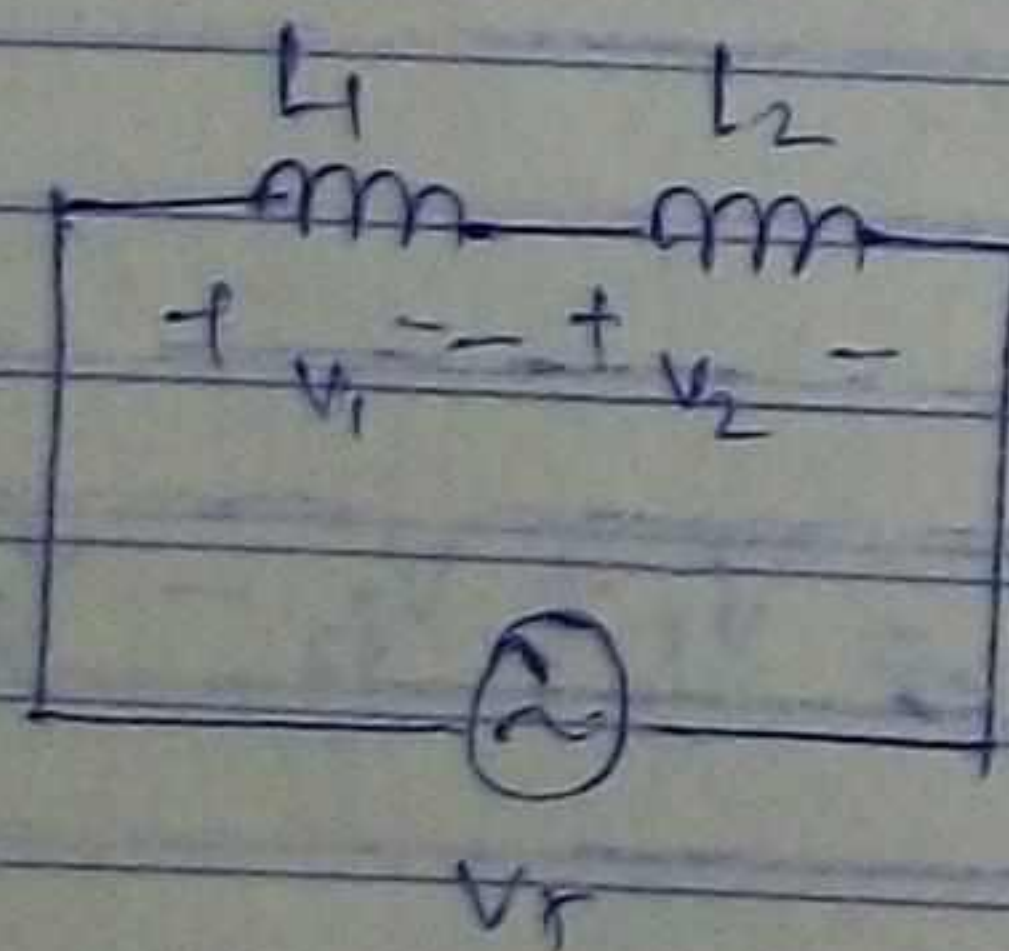
$$\frac{10}{(6+4)} = I = \underline{\underline{1A}}$$

$$V_1 = \underline{\underline{6V}} \quad V_2 = \underline{\underline{-4V}}$$



$$R_{eq} = R_1 + R_2$$

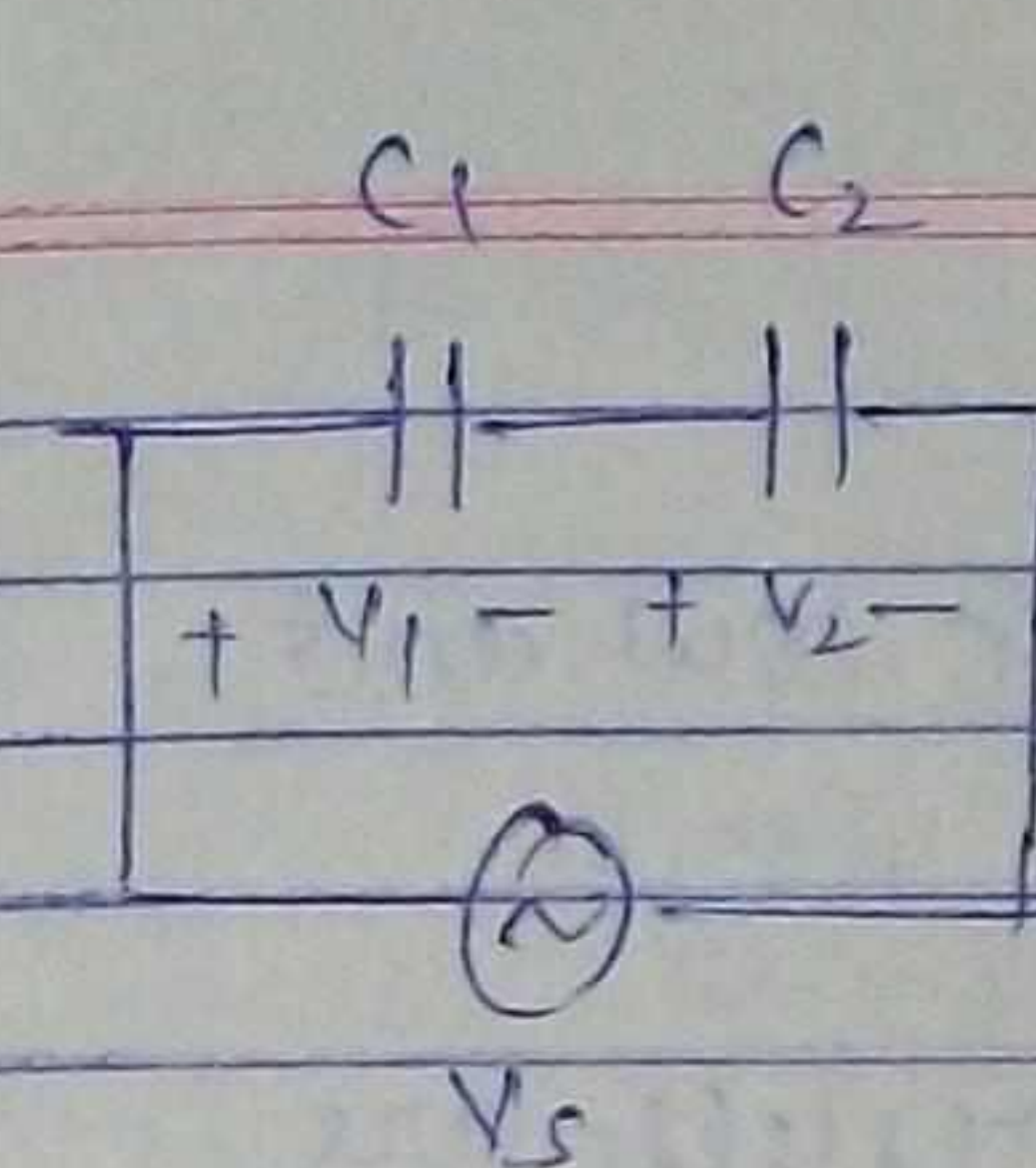
$$V_1 = \frac{V_s R_1}{R_1 + R_2} \quad V_2 = \frac{V_s R_2}{R_1 + R_2}$$



$$L_{eq} = L_1 + L_2$$

$$V_1 = V_s \frac{L_1}{L_1 + L_2}$$

$$V_2 = V_s \frac{L_2}{L_1 + L_2}$$

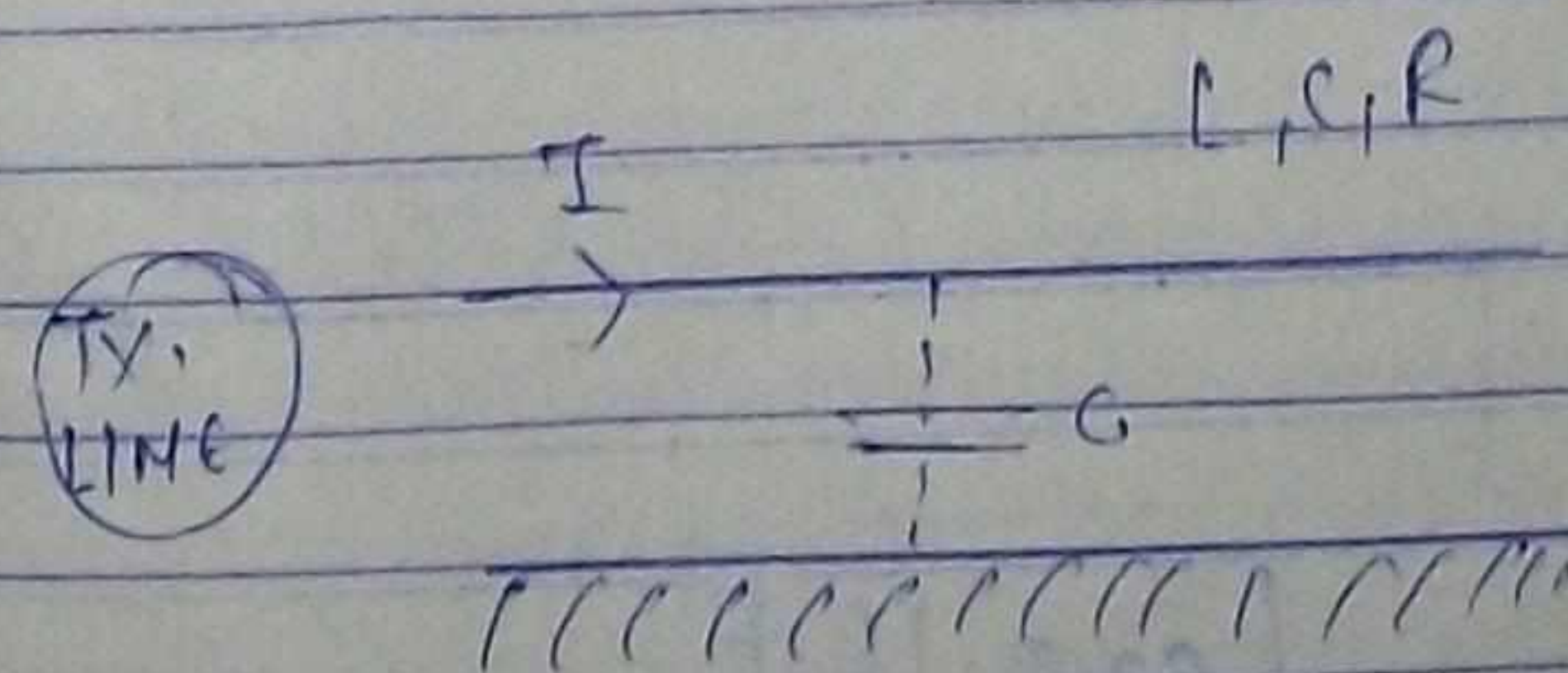
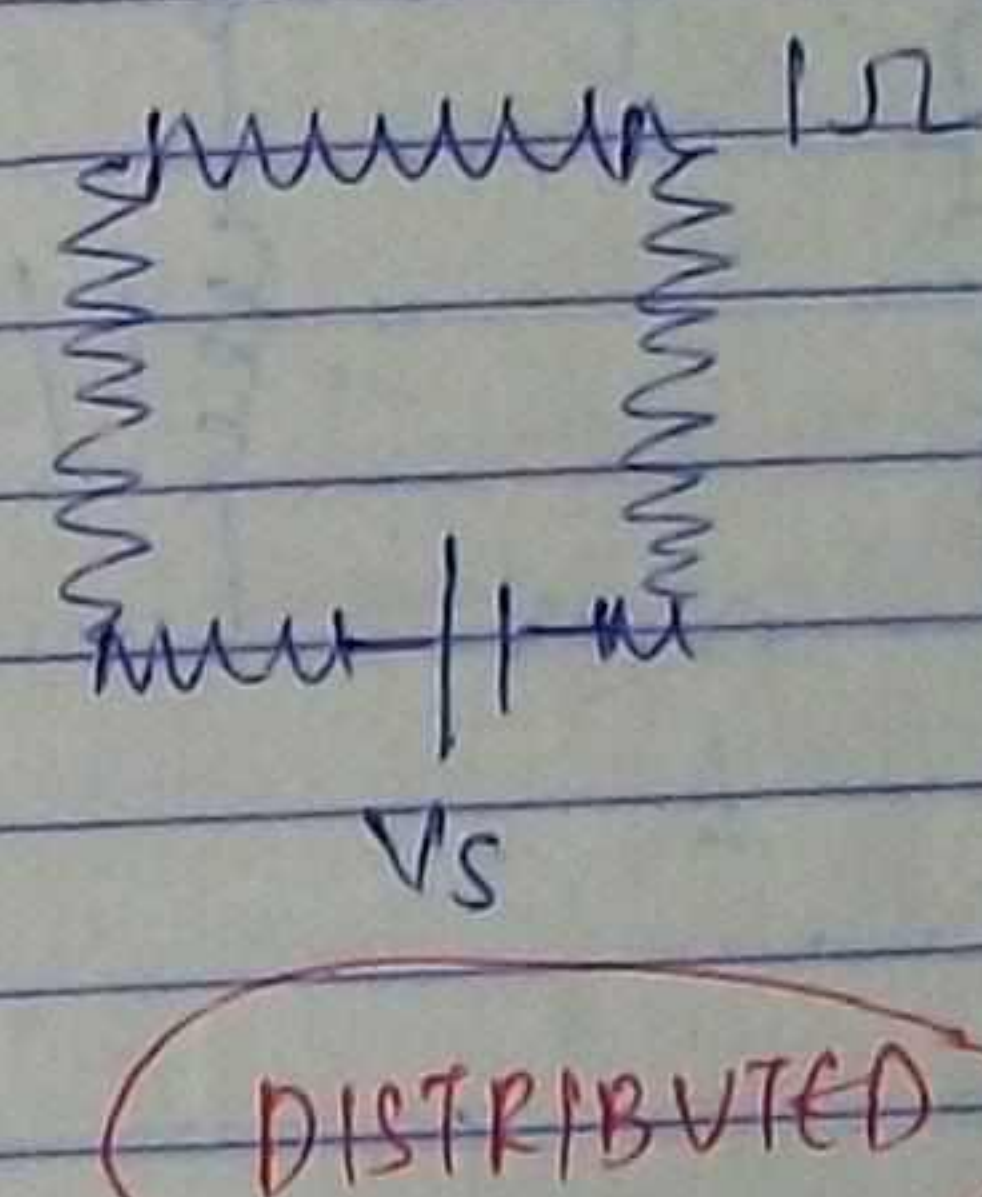
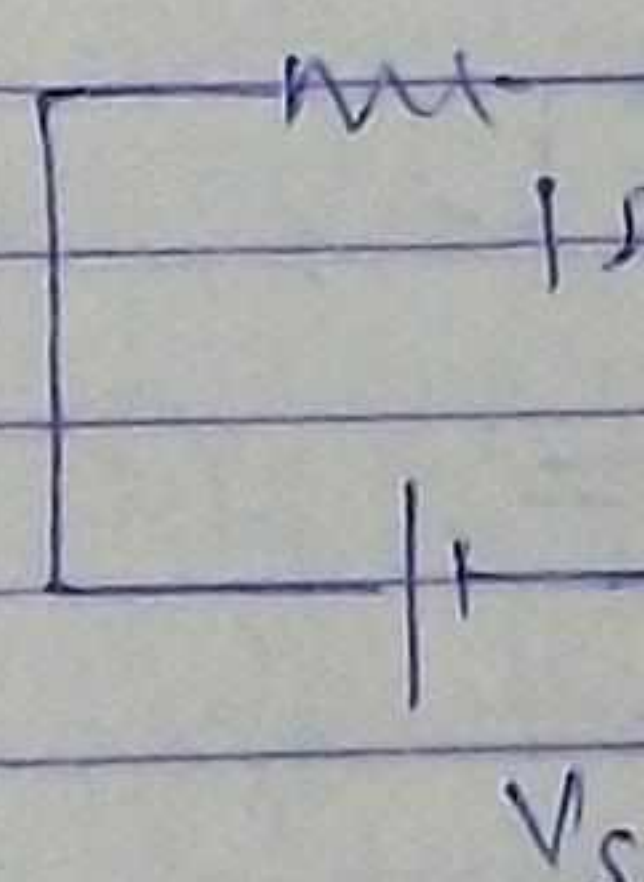
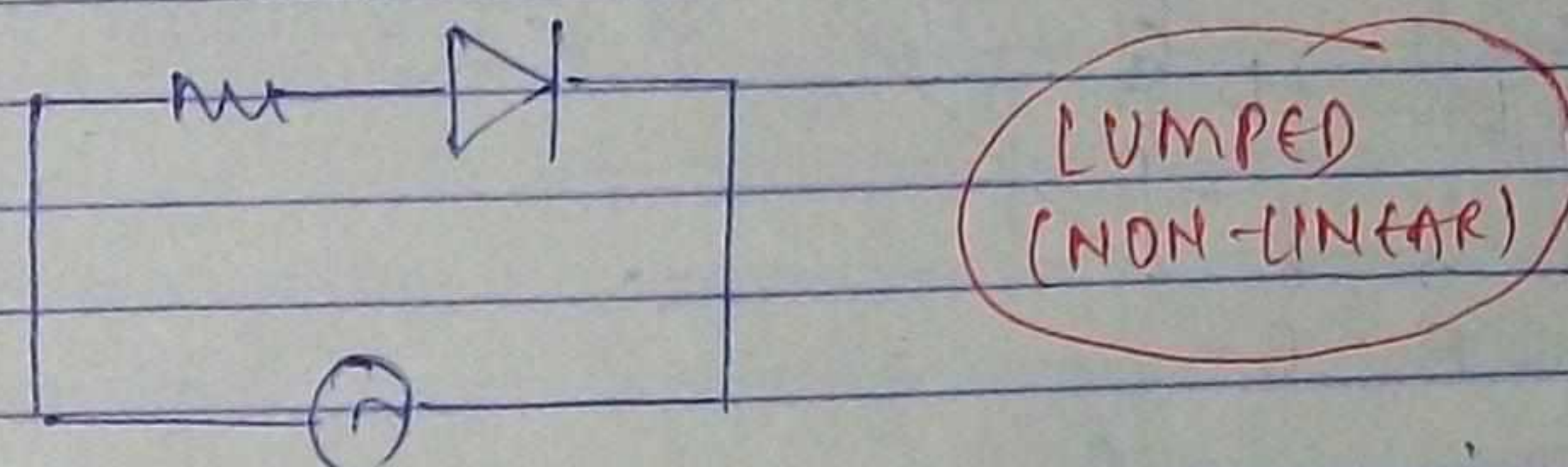
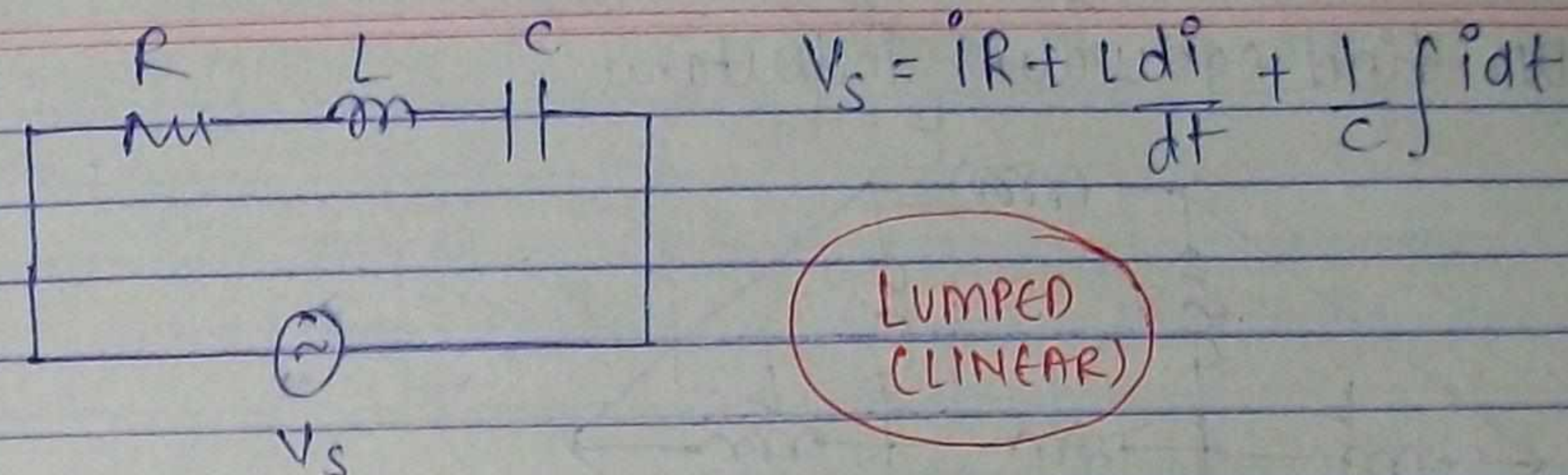


$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$V_1 = V_s \times \frac{C_2}{C_1 + C_2} \quad V_2 = V_s \times \frac{C_1}{C_1 + C_2}$$

⇒ CONCLUSIONS -

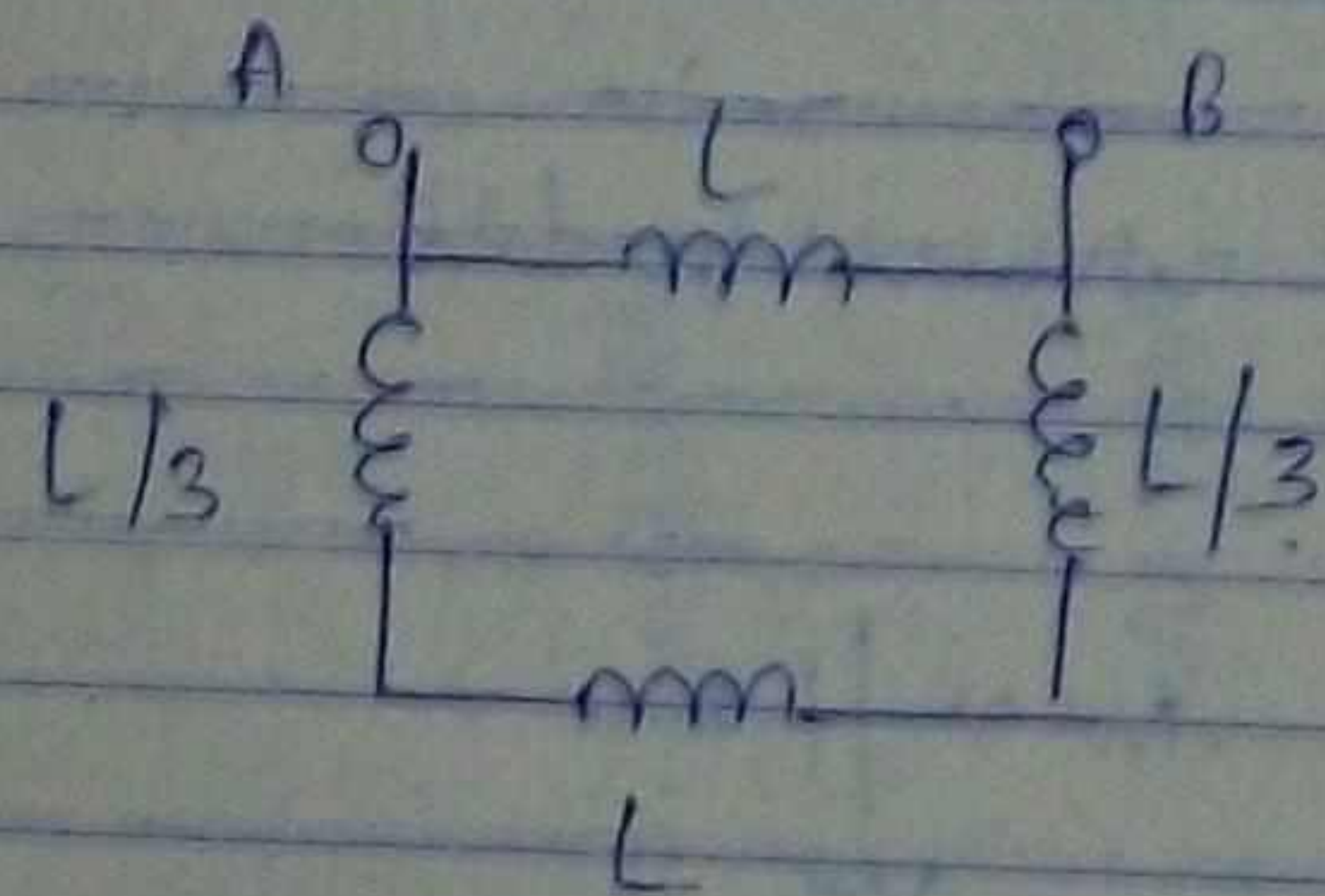
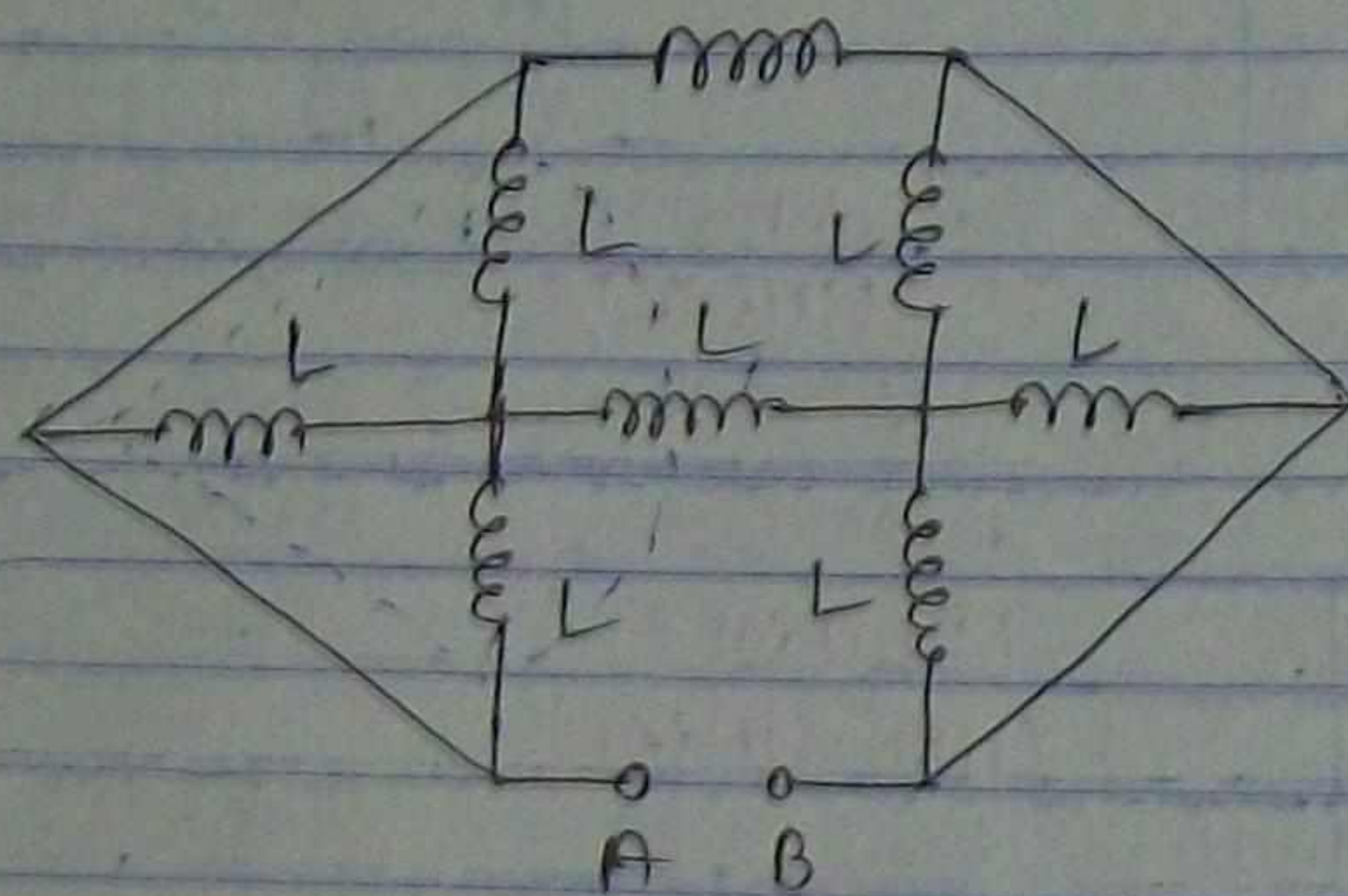
- field theory can be applied for low and high frequency application.
- In the field theory accurate results are obtained but developing mathematical eqn is complex.
- Network theory can be applied only for low frequency application.
- In the Network theory approximate results are obtained and developing mathematical equation is easy.
- KVL and KCL fails for the high frequency applications.
- KVL and KCL fails for distributed parameters since in the distributed parameter it is not possible to separate resistance, inductance, capacitance effect.
- Ohm's law can be applied for lumped (linear) and distributed parameter.
- KVL and KCL can be applied for lumped parameter (linear, non-linear, unidirectional, bidirectional, Time variant & invariant elements)



DISTRIBUTED

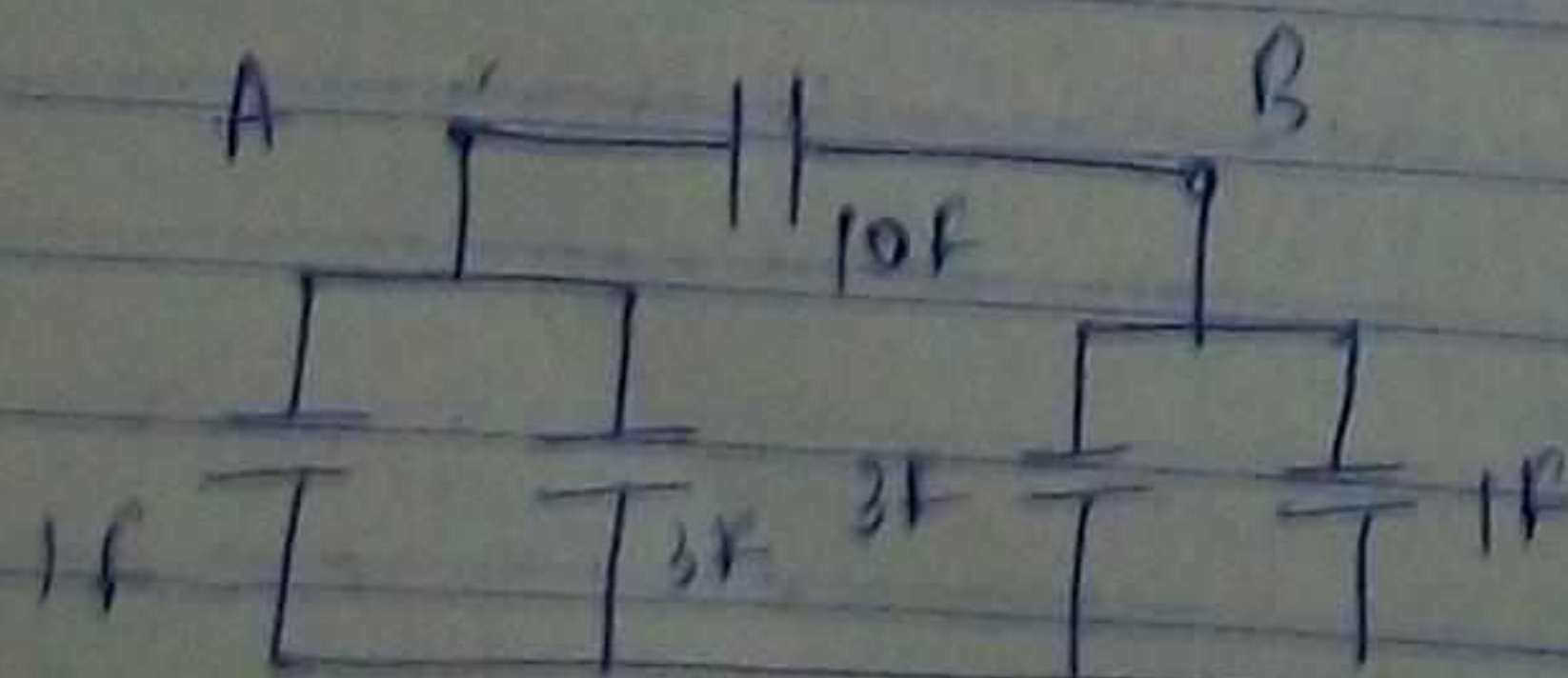
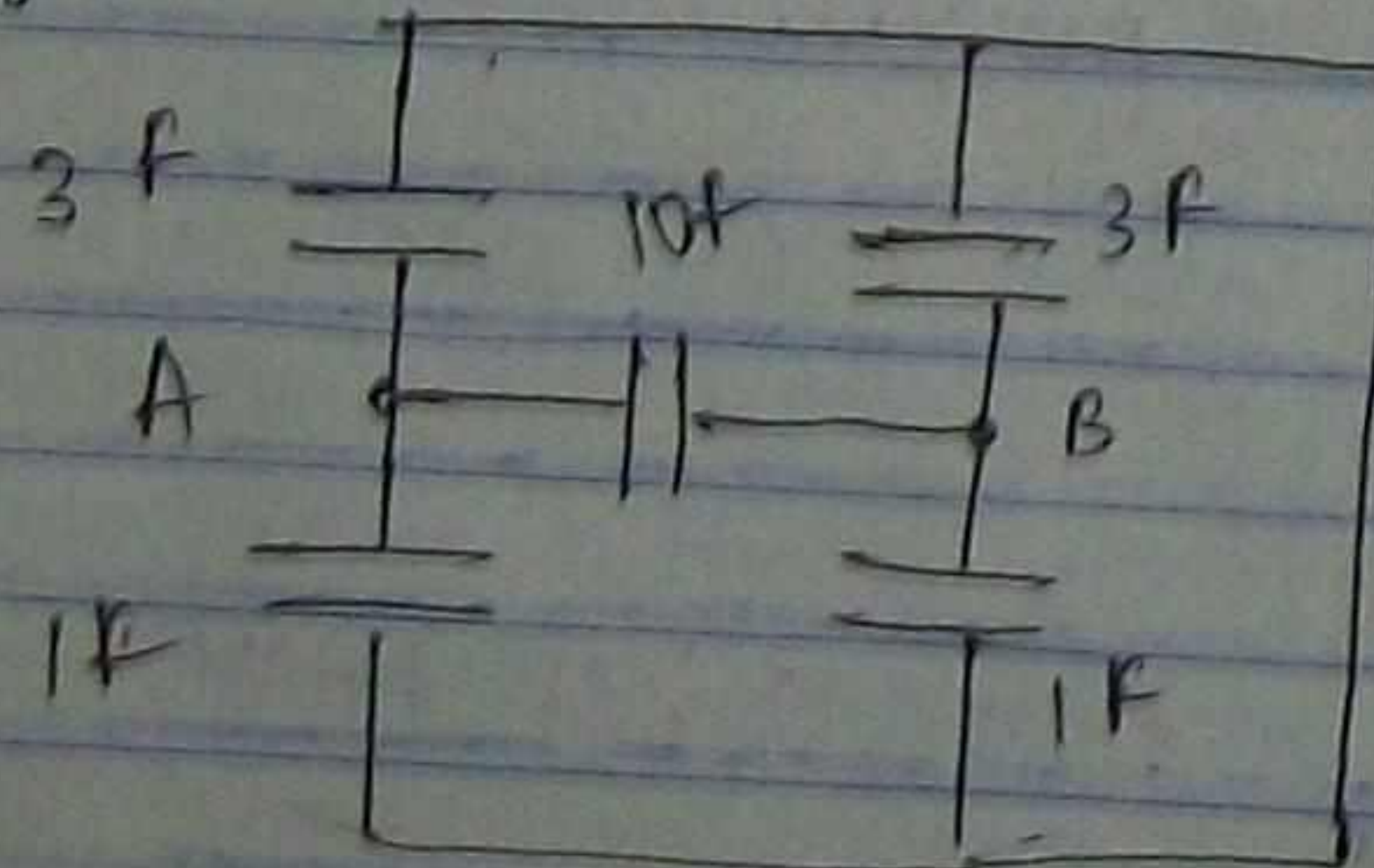
[$J = \sigma E$] OHM'S LAW

Ques. find equivalent inductance

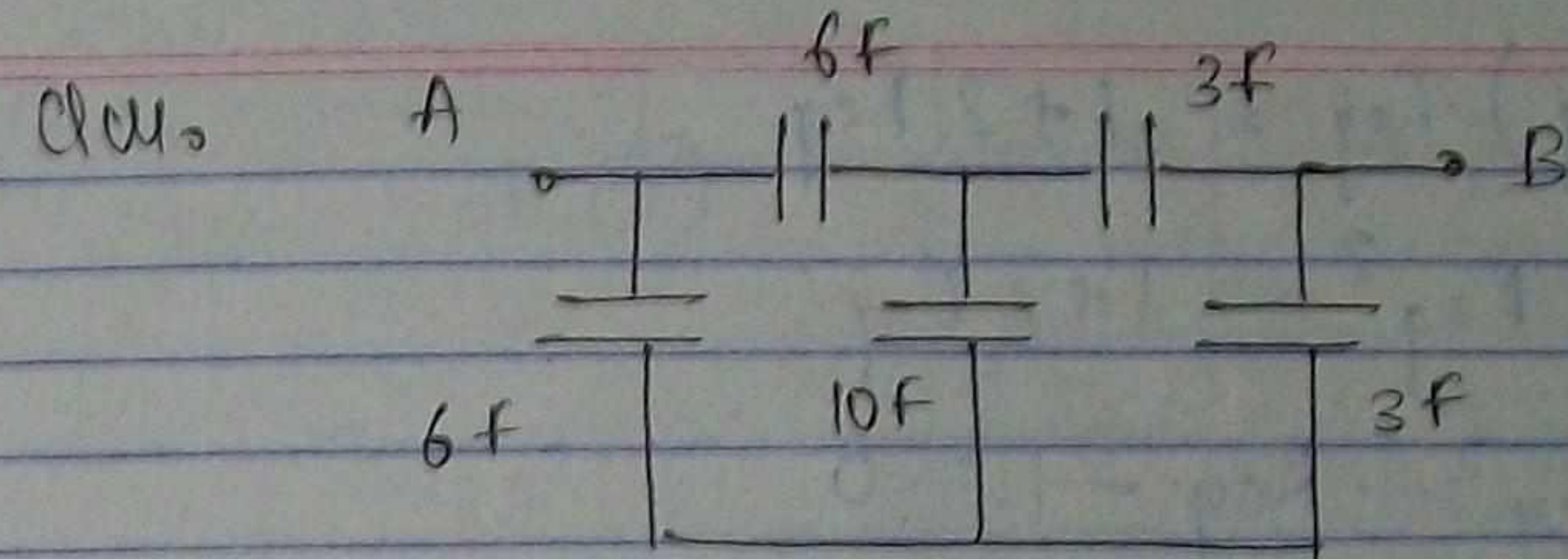


$$L_{eq} = \frac{5L}{8}$$

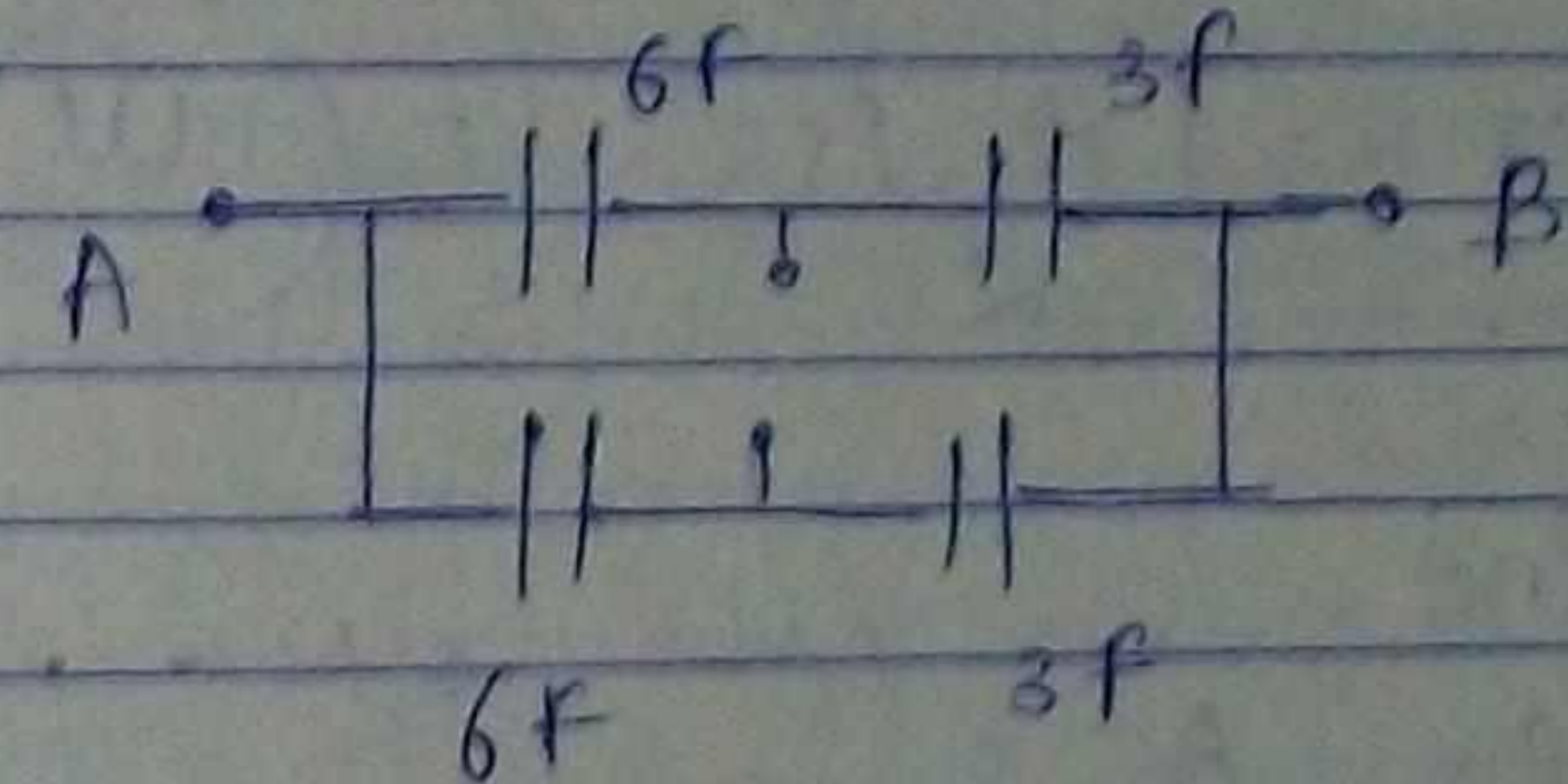
Ques.



$$L_{eq} = 12F$$

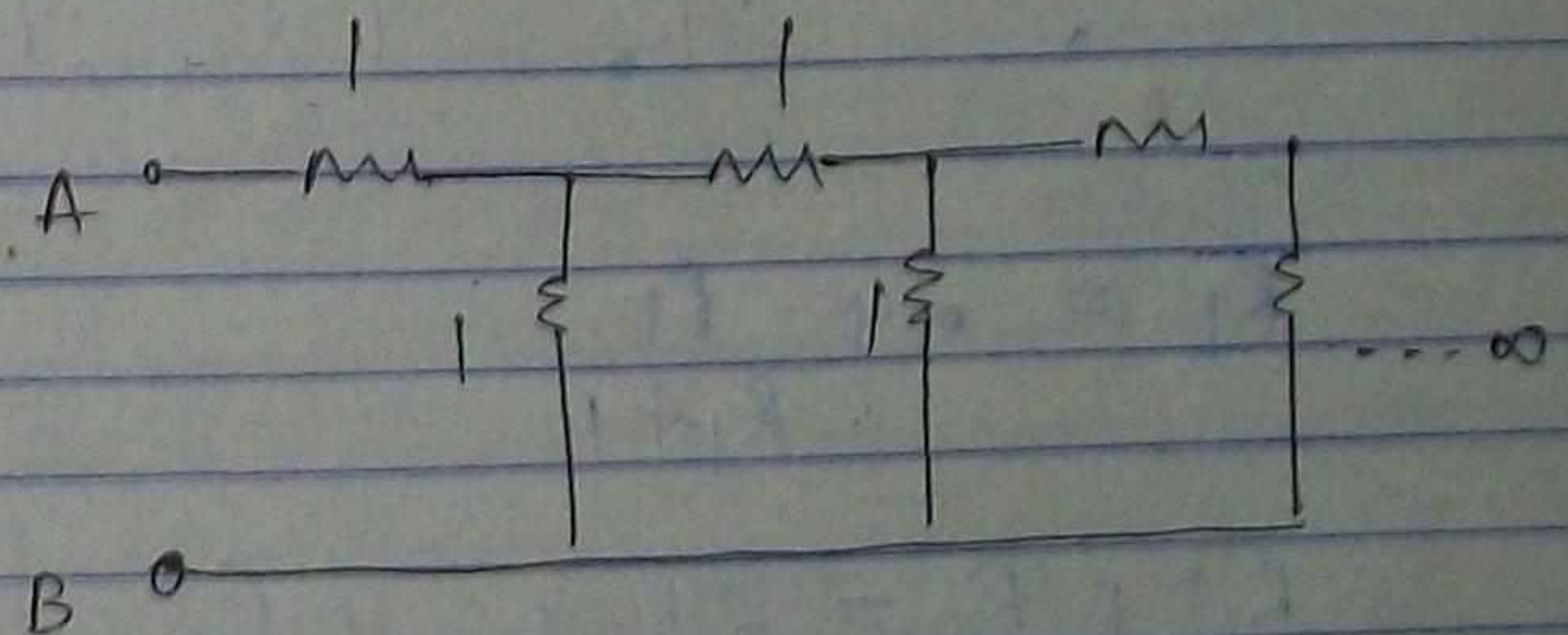


Solⁿ: Balanced bridge

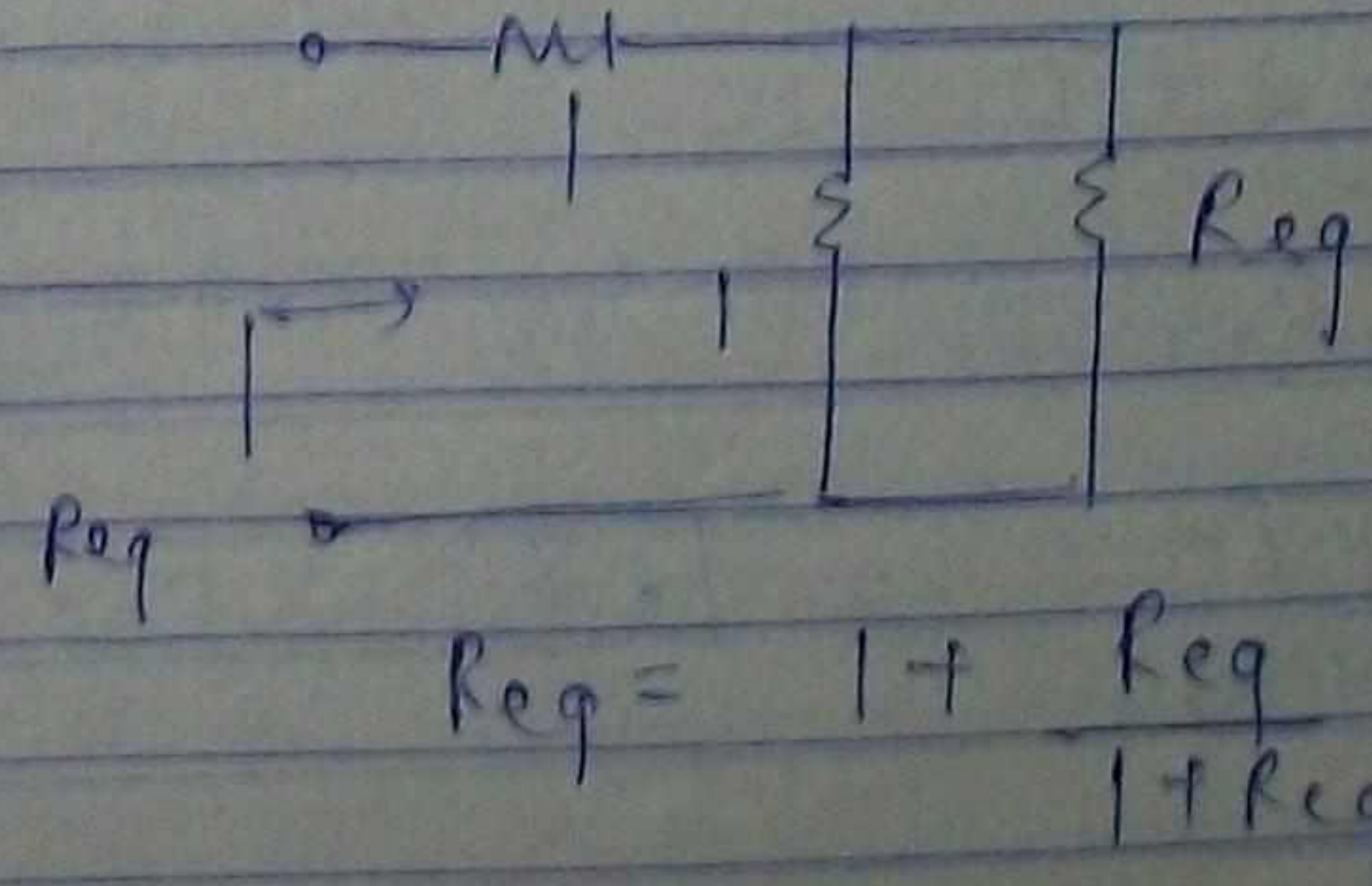


$$C_{eq} = 4F$$

Ques Req = ?



Solⁿ:

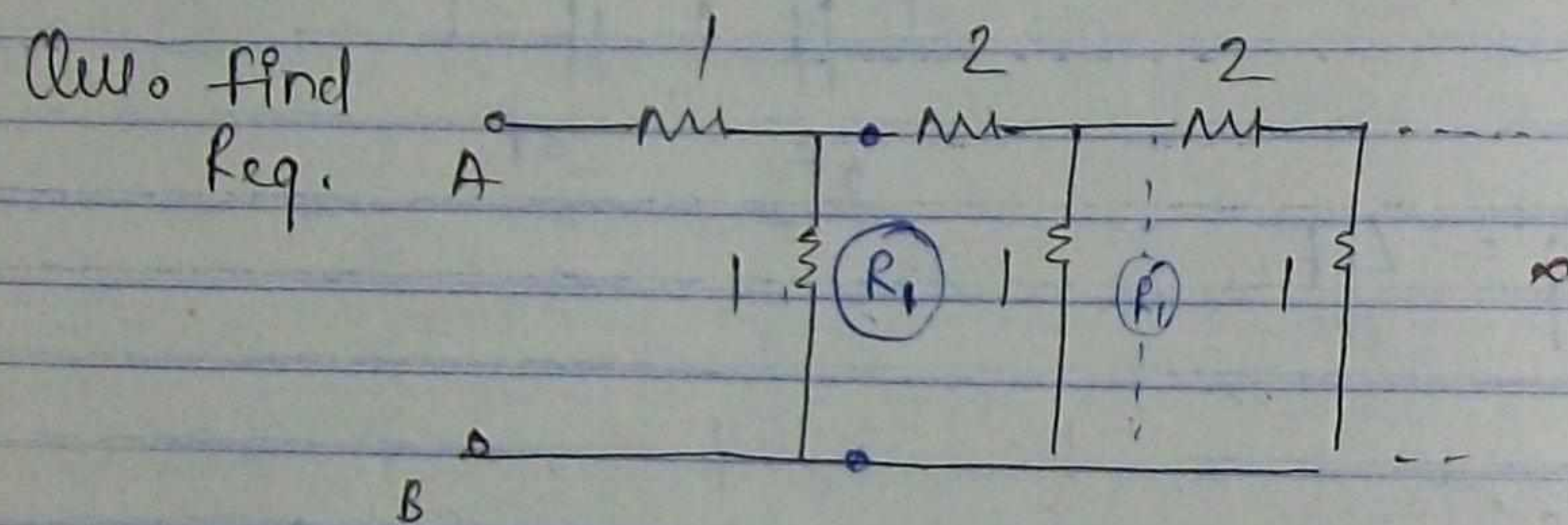


$$(1 + R_{eq}) R_{eq} = 1 + 2 R_{eq}$$

$$R_{eq} + R_{eq}^2 = 1 + 2 R_{eq}$$

$$R_{eq}^2 - R_{eq} - 1 = 0$$

$$R_{eq} = \frac{1 + \sqrt{5}}{2} = \text{+ve} \checkmark$$



$$R_1 = 2 + \frac{R_1}{R_1 + 1}$$

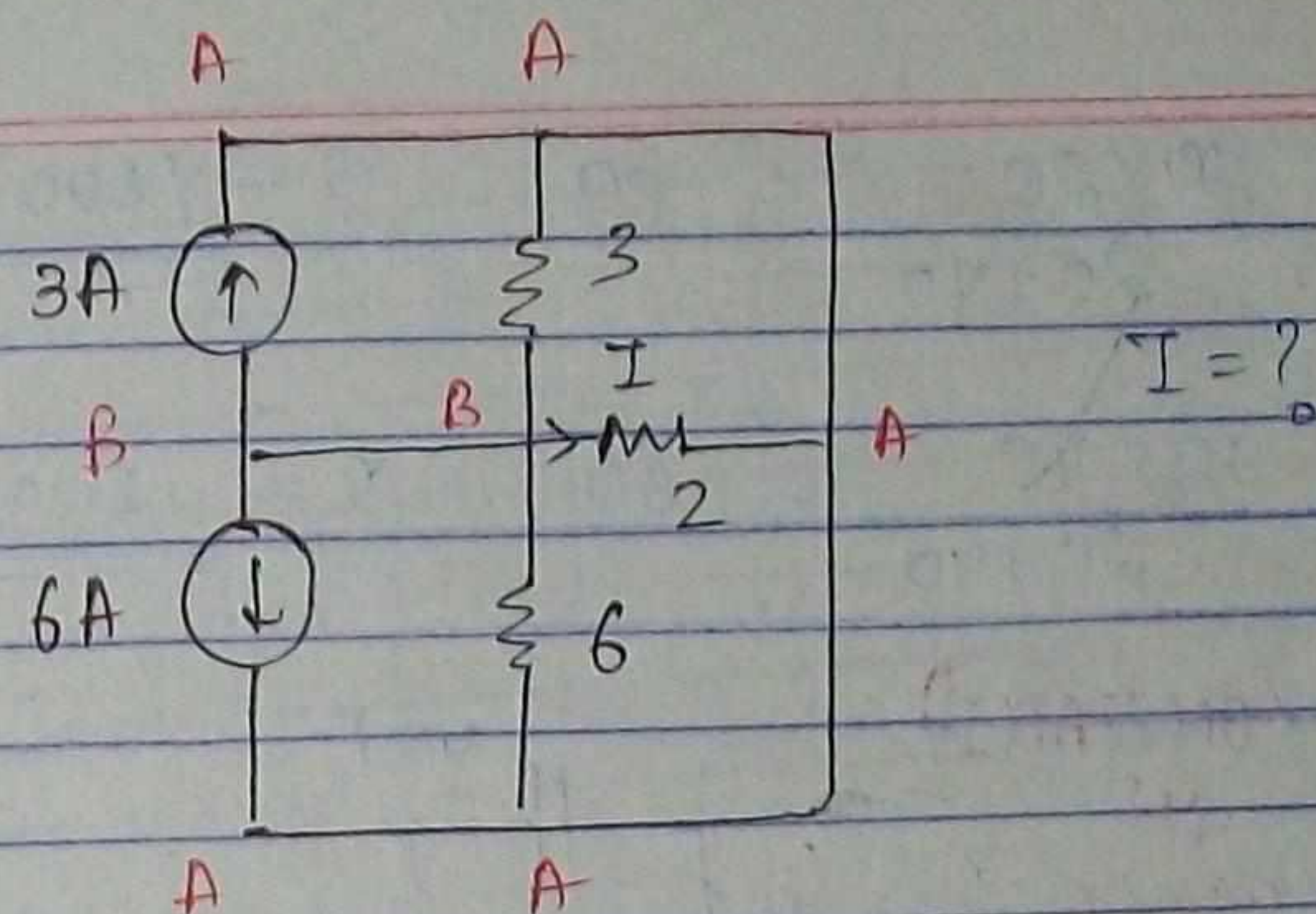
$$R_1^2 + R_1 = 2R_1 + 2 + R_1$$

$$R_1^2 - 2R_1 - 2 = 0$$

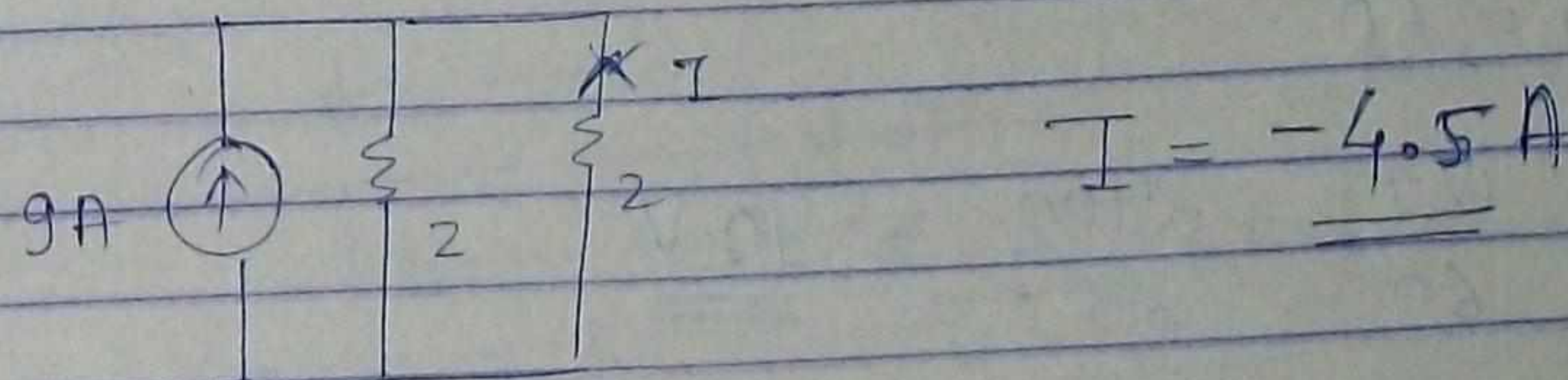
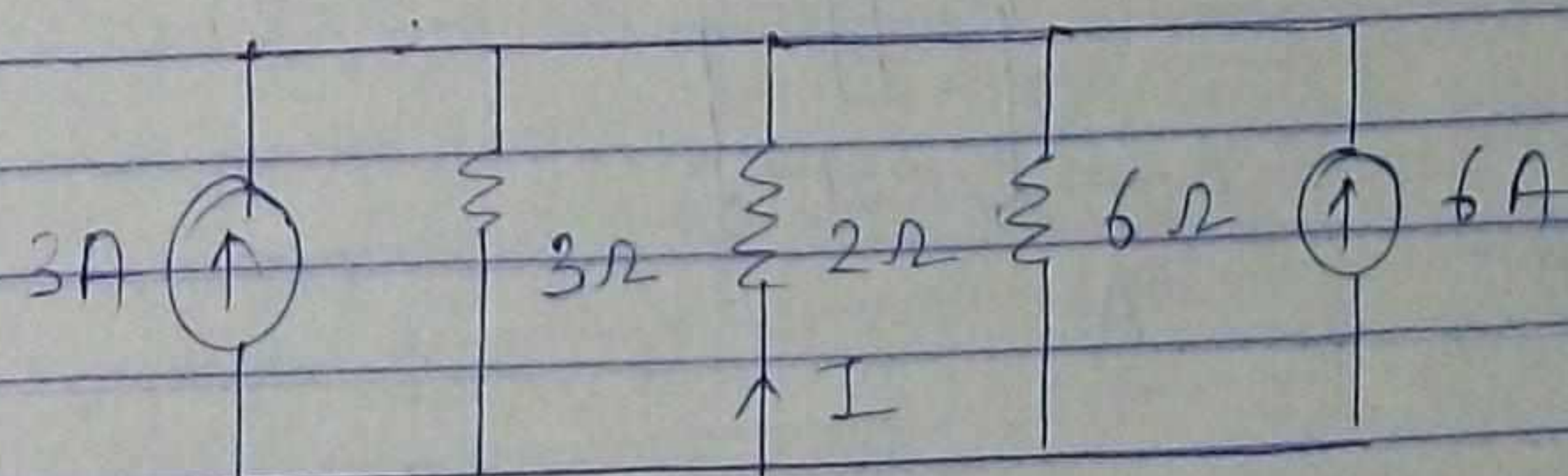
$$R_1 = 2.732$$

$$R_{eq} = \frac{1 + \frac{1 \times R_1}{1 + R_1}}{1 + R_1} = \underline{\underline{1.73}}$$

Ques.

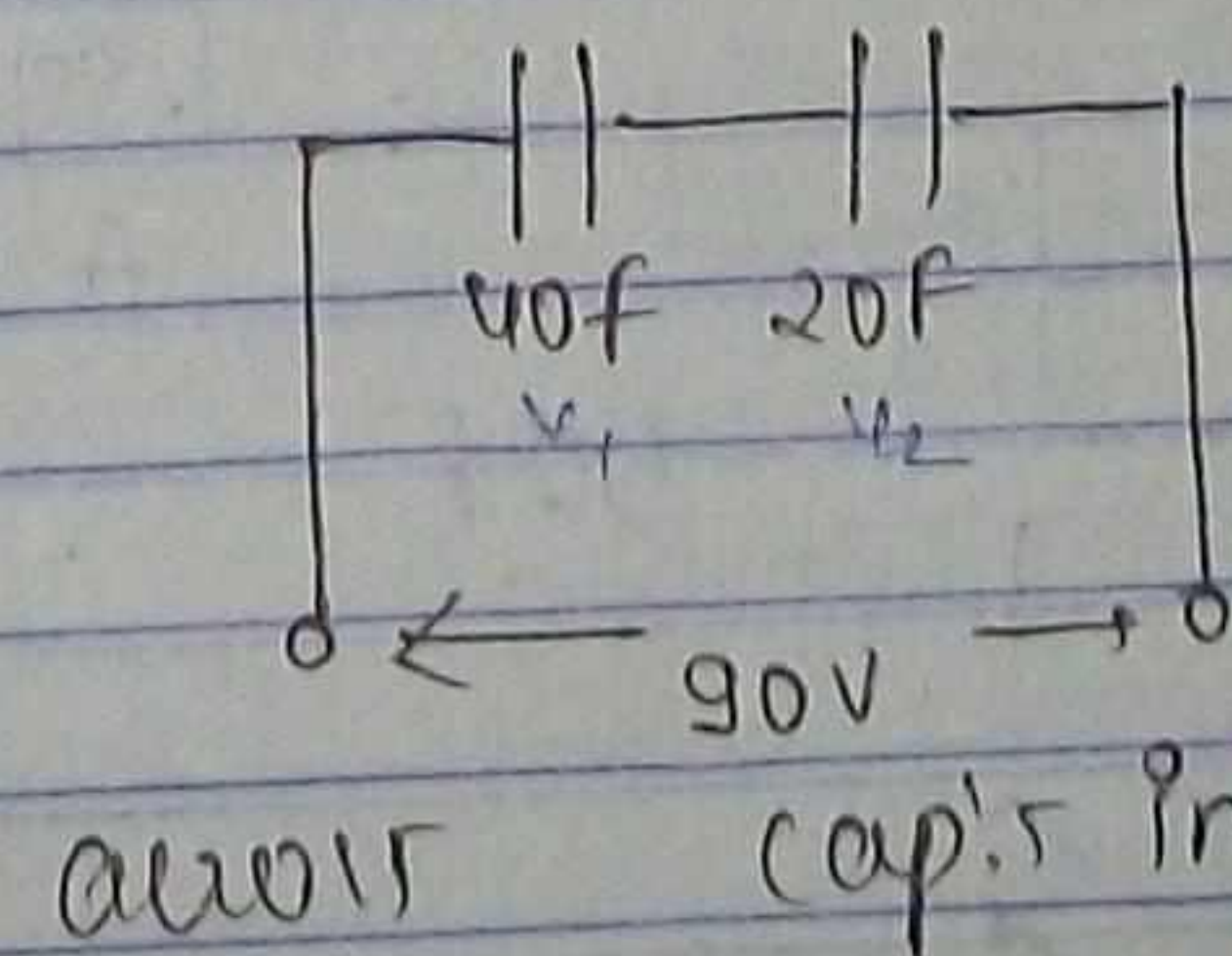


Solⁿ:



$$I = \underline{\underline{-4.5 A}}$$

Ques.



Solⁿ:

$$C_{eq} = \frac{40}{3}$$

$$Q = CV$$

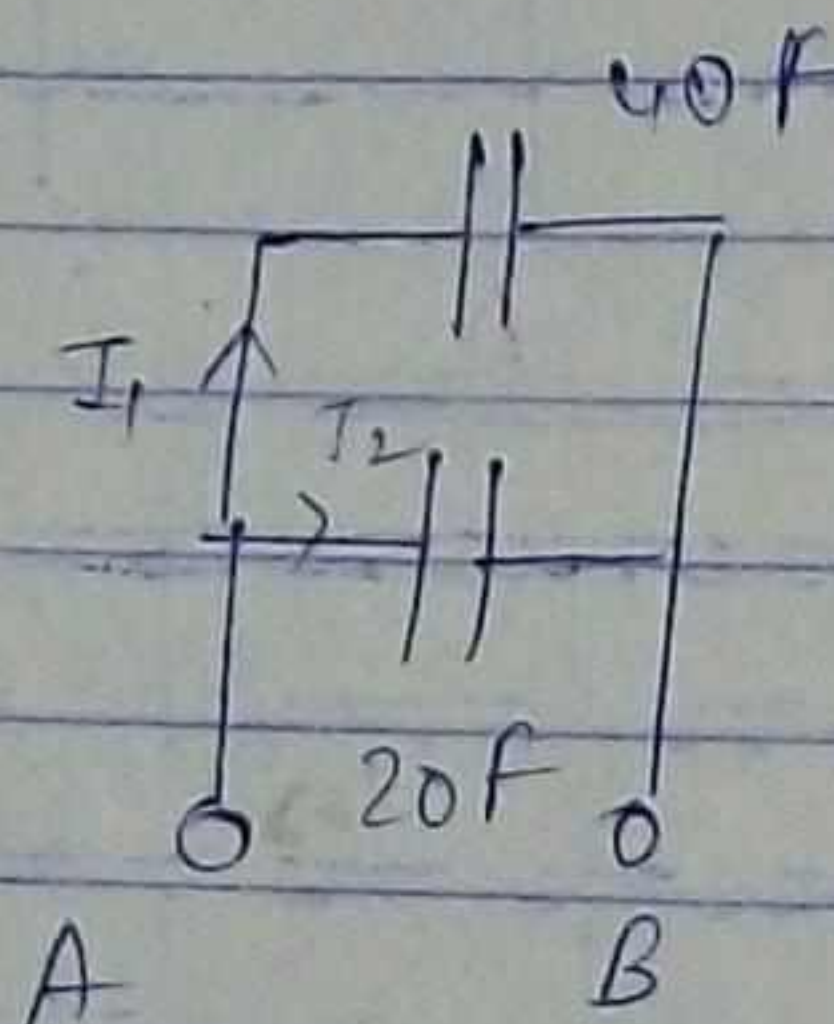
$$Q = \frac{40}{3} \times 90 = 1200 C$$

when two cap. are charged fully they are connected in parallel find voltage

$$V_1 = \frac{90 \times 20}{20 + 40} = 30 \quad Q = \underline{\underline{1200 C}}$$

$$V_2 = \frac{90 \times 40}{20 + 40} = 60 \quad Q = \underline{\underline{1200 C}}$$

Q = CONSTANT
IN
SERIES



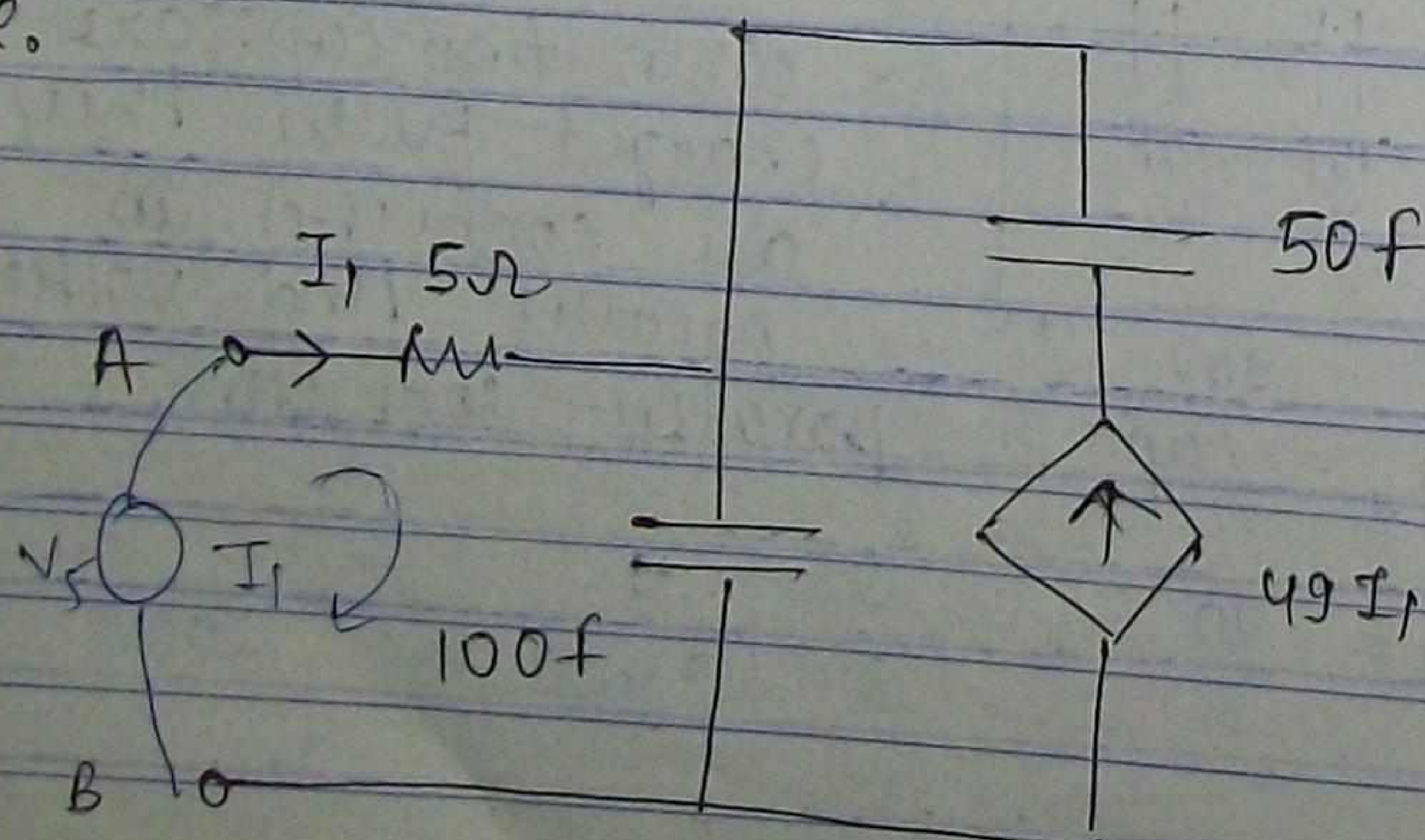
$$Q_T = 1200 + 1200 = 2400$$

$$C_{eq} = 60$$

$$V = \frac{Q_T}{C_{eq}} = \frac{2400}{60} = \underline{\underline{40 V}}$$

⊛

Que.



C_{eq} across
A & B

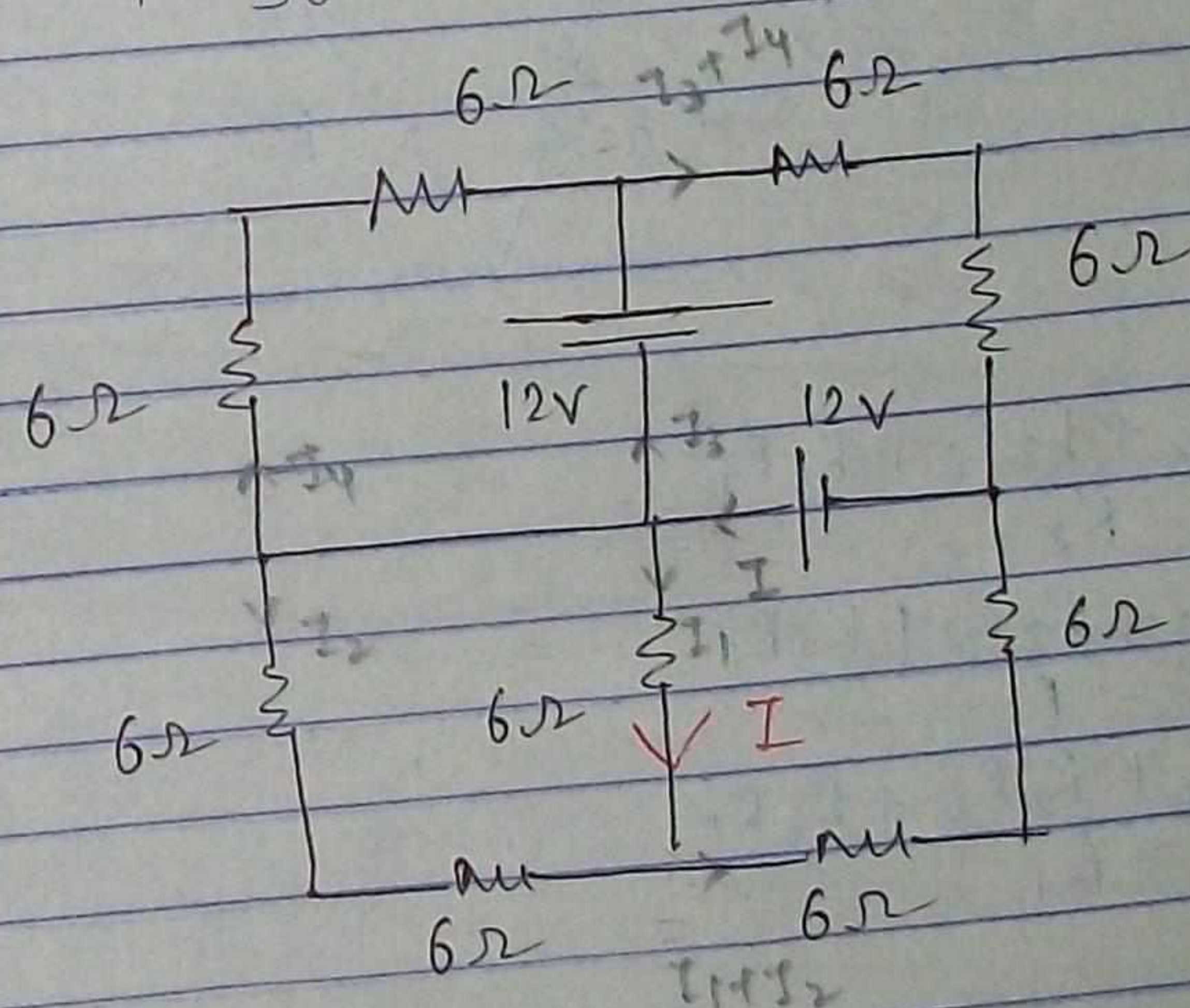
$$V_s = 5 I_1 + \frac{1}{100} \int 50 I_1 dt$$

$$V_s = 5 I_1 + \frac{50}{100} \int I_1 dt$$

$$V_s = R I_1 + \frac{1}{C_{eq}} \int I_1 dt$$

$$C_{eq} = \frac{100}{50} = \underline{\underline{2}}$$

Que.



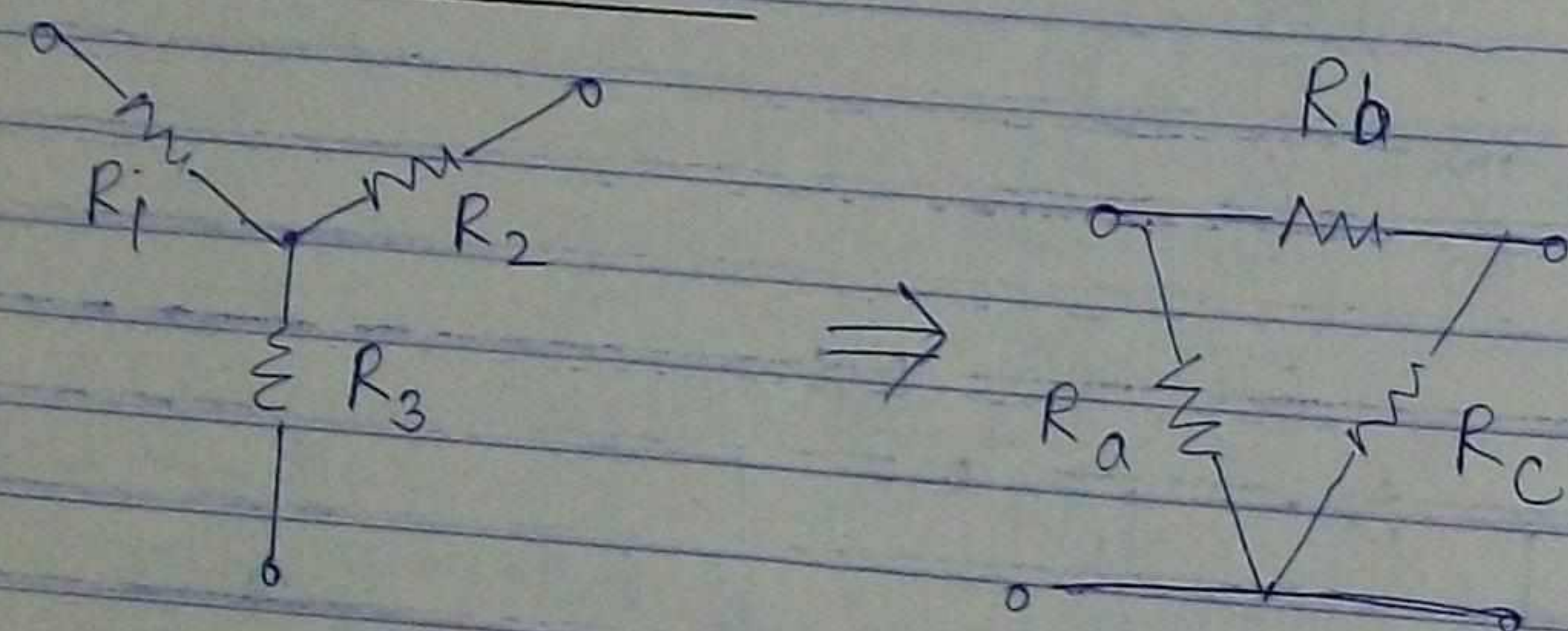
$I = ?$

$I = 0.5$

19/10/11

- when elements are connected neither in series nor in parallel, to reduce the network star-delta transformation is used.

① STAR → DELTA -

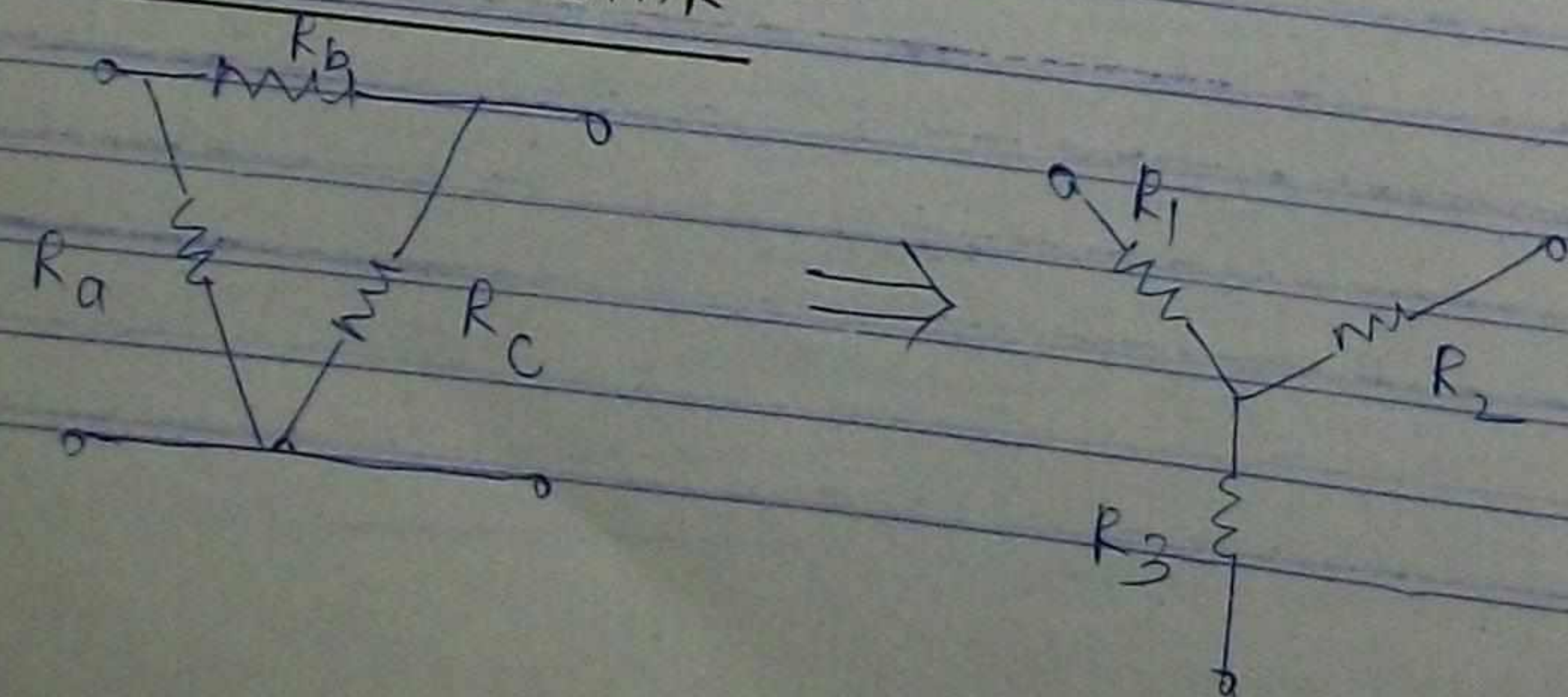


$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

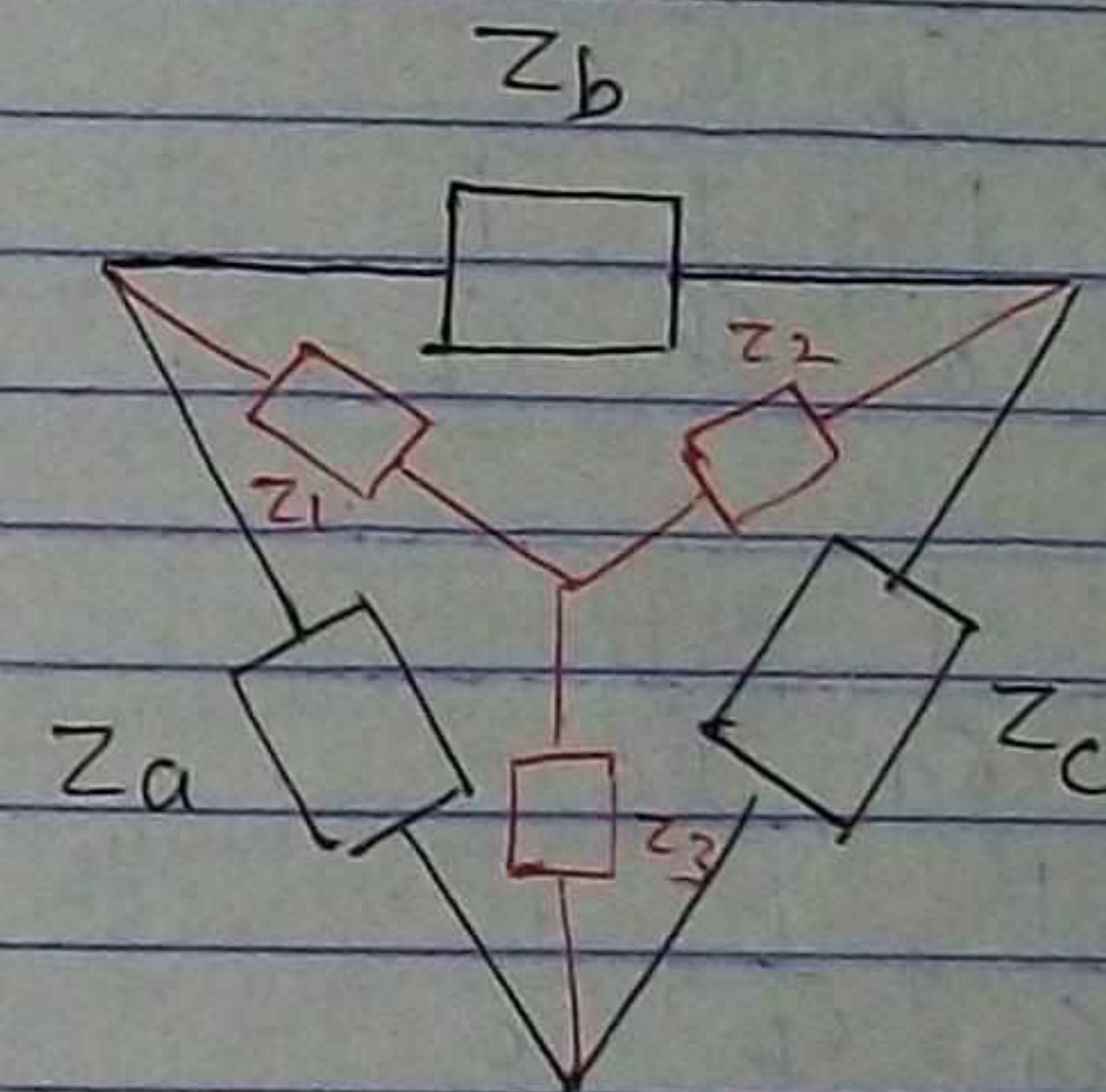
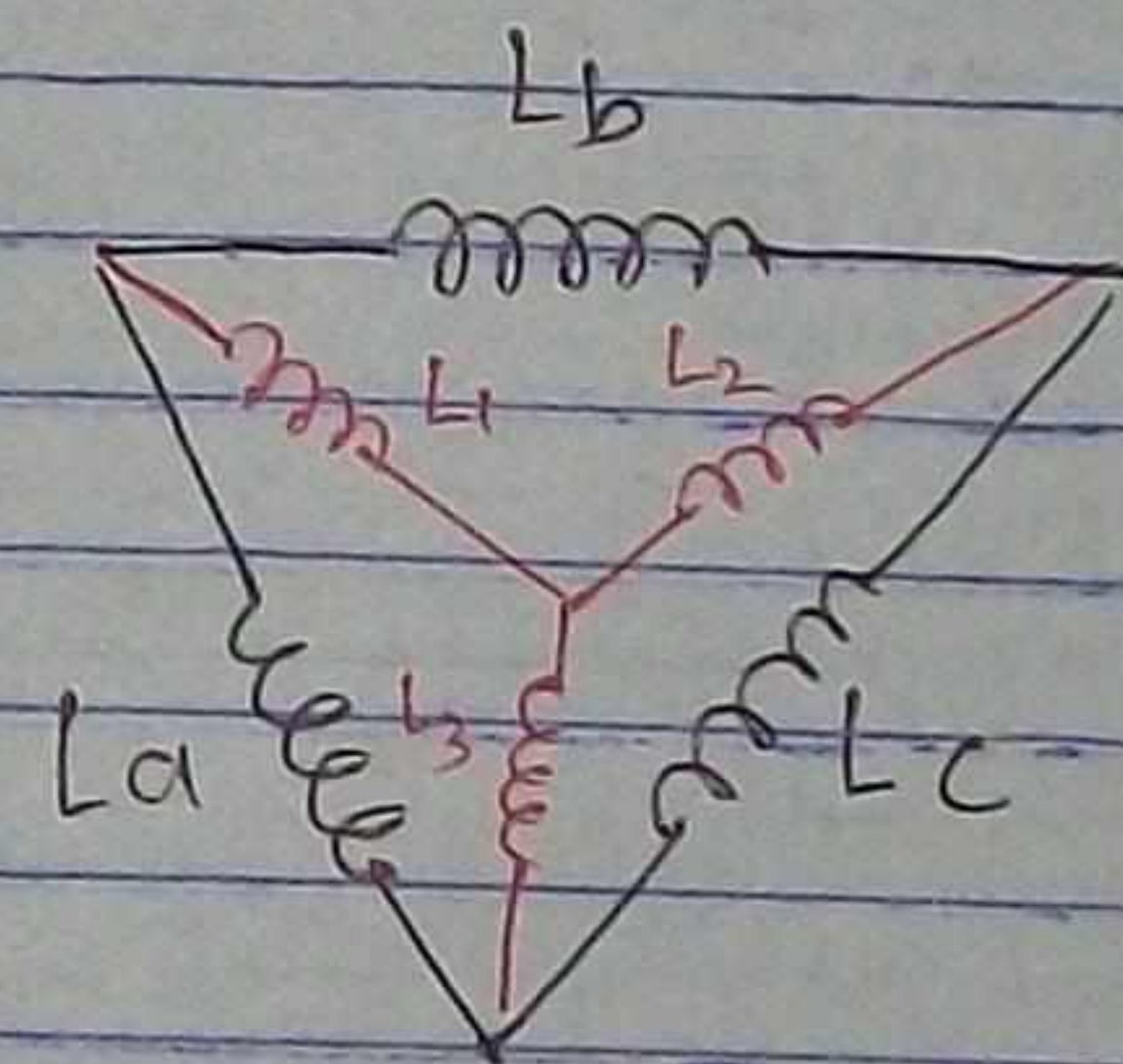
② DELTA → STAR -



$$R_1 = \frac{R_a R_b}{R_a + R_b + R_c}$$

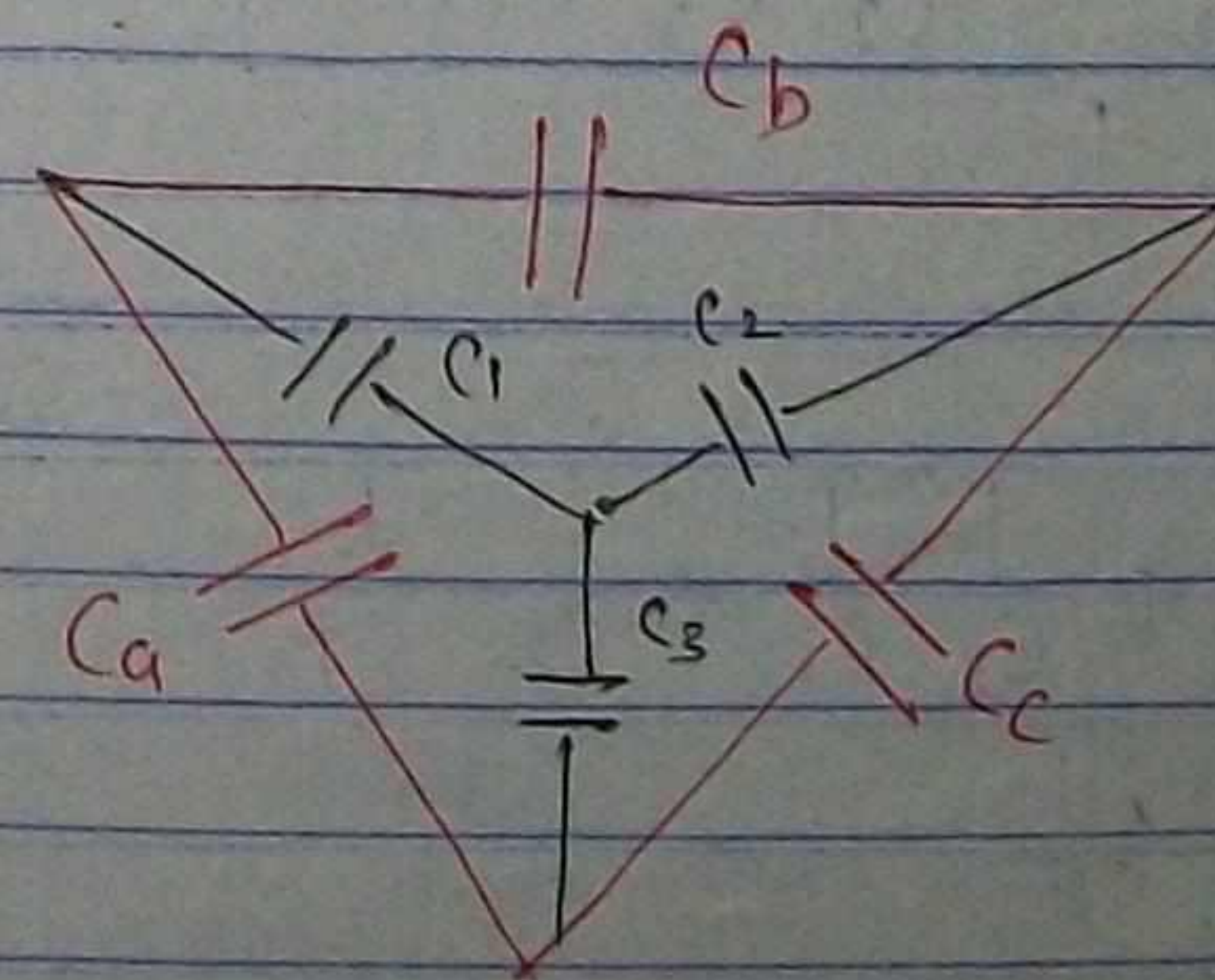
$$R_2 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_c R_a}{R_a + R_b + R_c}$$



- The procedure of transformation either from $\Delta \rightarrow \star$ or $\star \rightarrow \Delta$ for the resistor, inductor and impedance is same.

FOR CAPACITANCE



① DELTA → STAR -

$$\frac{1}{C_1} = \frac{\frac{1}{C_a} \times \frac{1}{C_b}}{\frac{1}{C_a} + \frac{1}{C_b} + \frac{1}{C_c}}$$

$$\frac{1}{C_2} = \frac{\frac{1}{C_b} \times \frac{1}{C_c}}{\frac{1}{C_a} + \frac{1}{C_b} + \frac{1}{C_c}}$$

$$\frac{1}{C_3} = \frac{\frac{1}{C_a} \times \frac{1}{C_c}}{\frac{1}{C_a} + \frac{1}{C_b} + \frac{1}{C_c}}$$

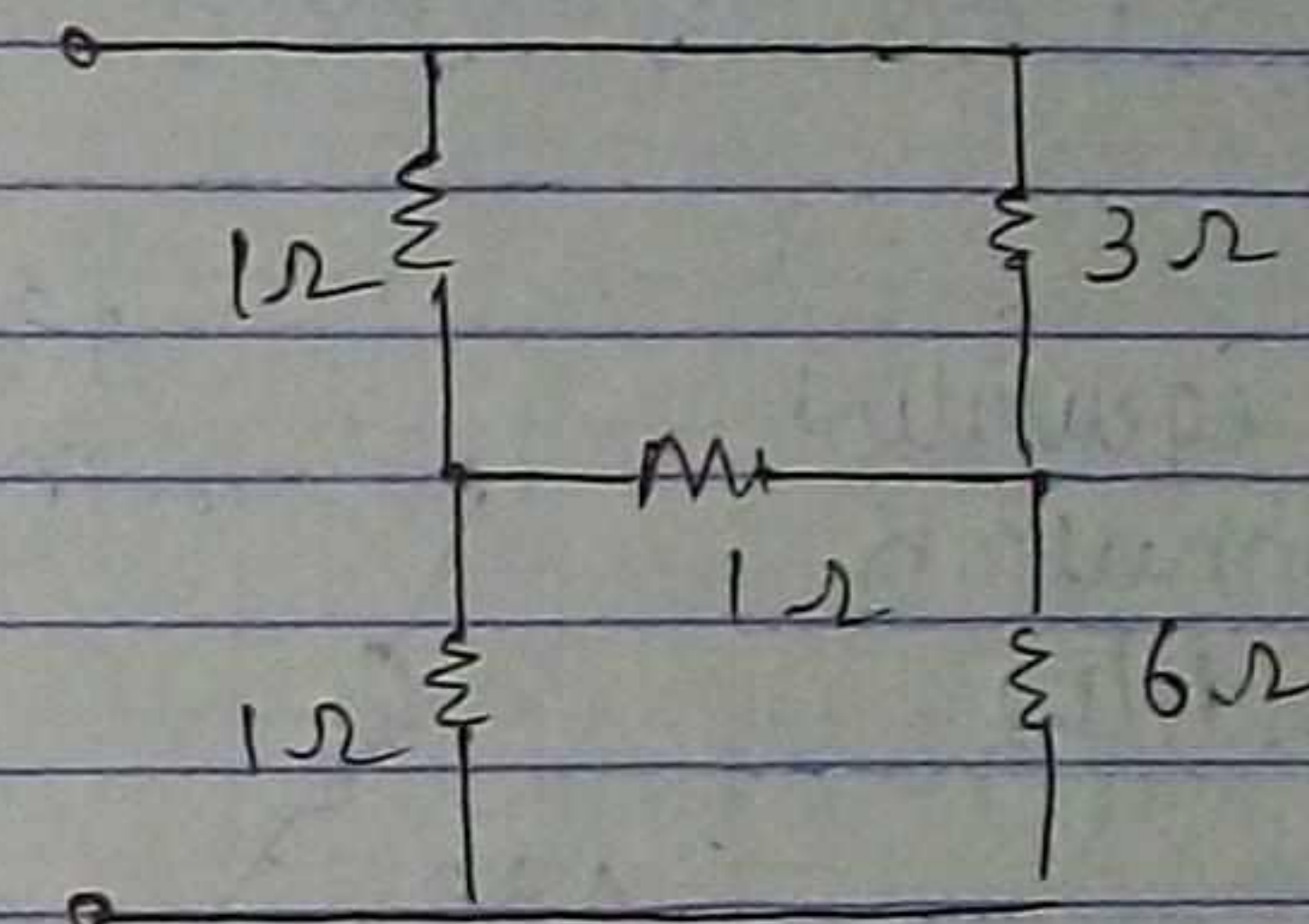
② STAR → DELTA -

$$\frac{1}{C_a} = \frac{\frac{1}{C_1} \cdot \frac{1}{C_2} + \frac{1}{C_2} \cdot \frac{1}{C_3} + \frac{1}{C_3} \cdot \frac{1}{C_1}}{\frac{1}{C_2}}$$

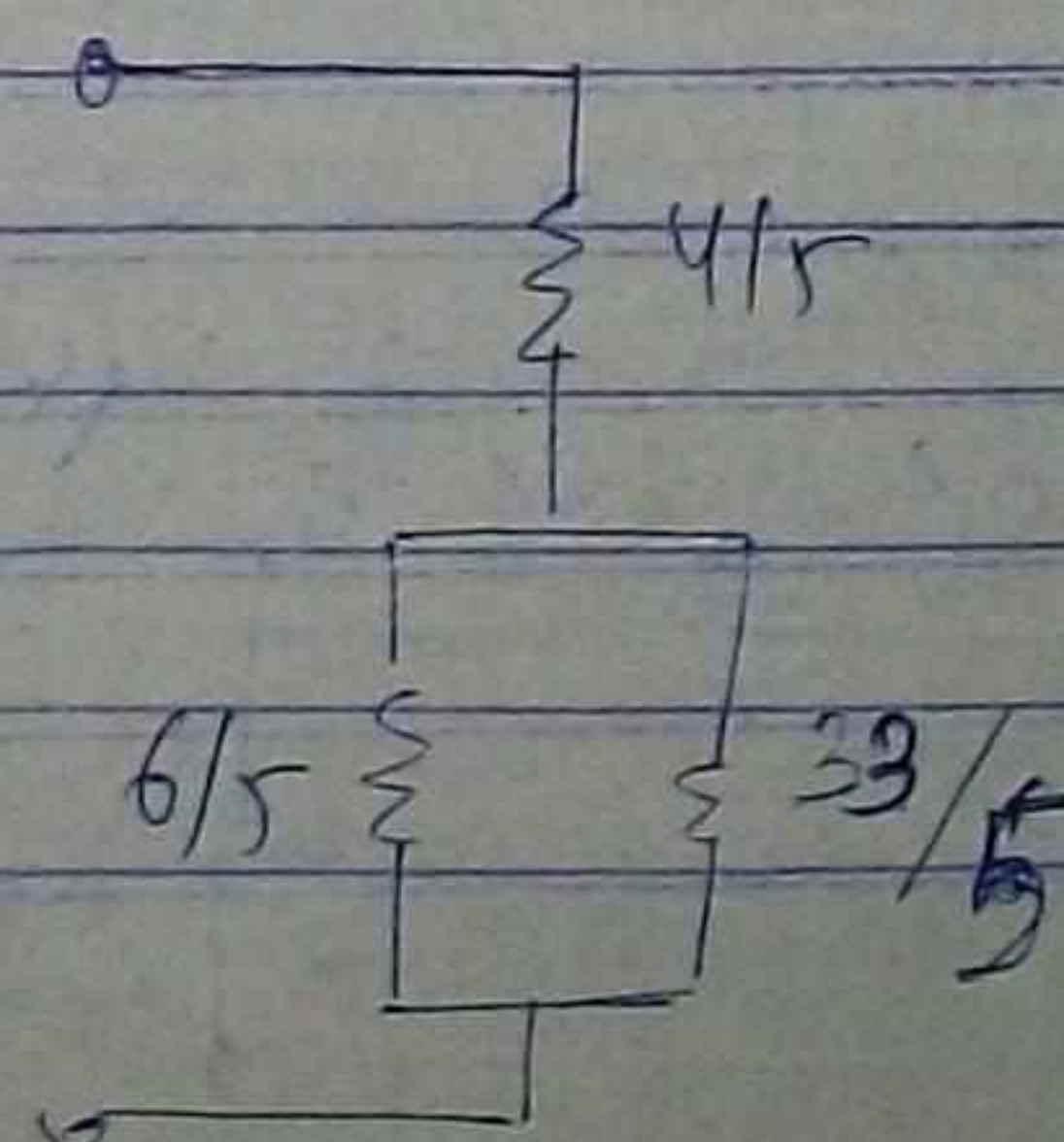
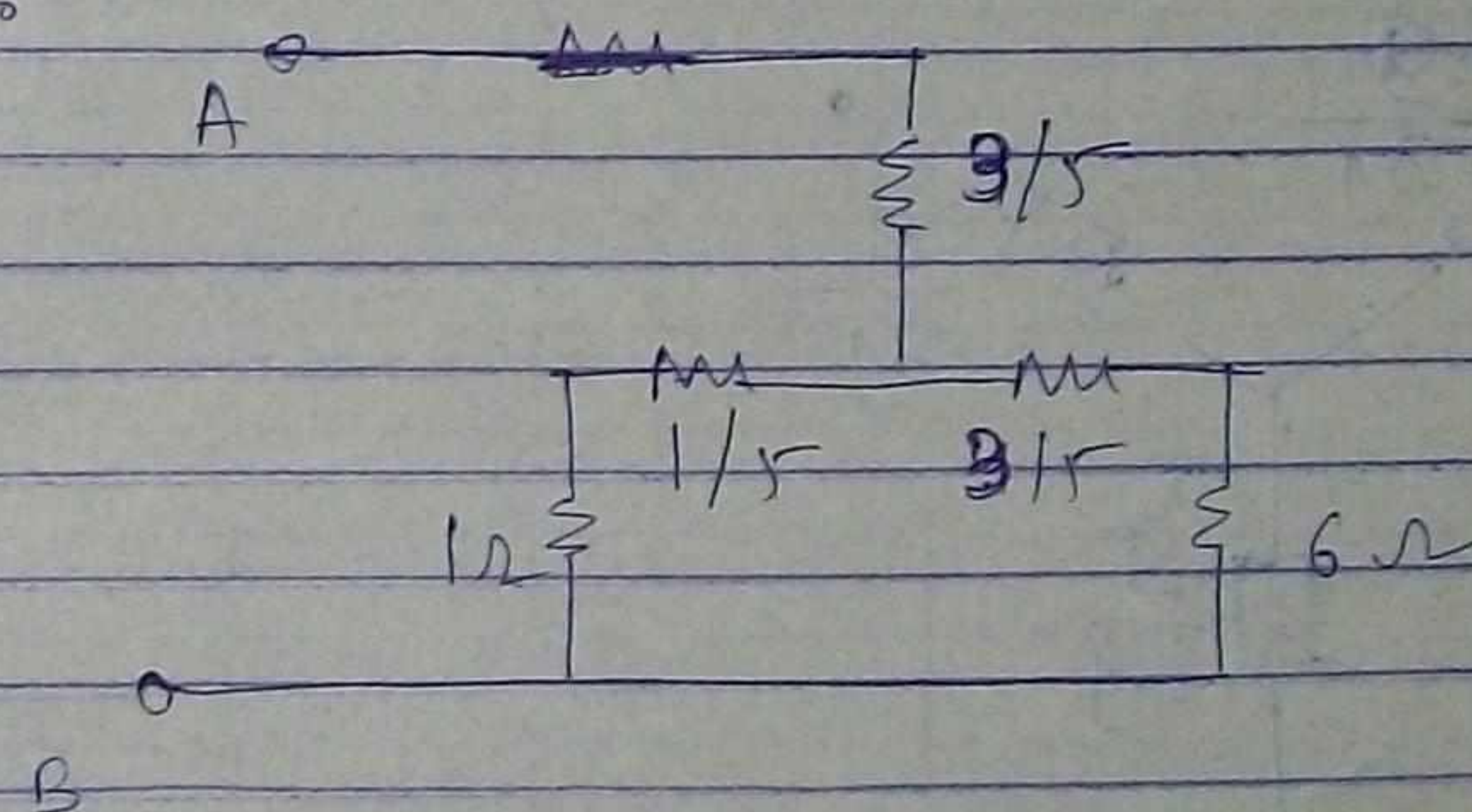
$$\frac{1}{C_b} = \frac{\frac{1}{C_1} \cdot \frac{1}{C_2} + \frac{1}{C_2} \cdot \frac{1}{C_3} + \frac{1}{C_3} \cdot \frac{1}{C_1}}{\frac{1}{C_3}}$$

$$\frac{1}{C_c} = \frac{\frac{1}{C_1} \cdot \frac{1}{C_2} + \frac{1}{C_2} \cdot \frac{1}{C_3} + \frac{1}{C_3} \cdot \frac{1}{C_1}}{\frac{1}{C_1}}$$

Ques. find Req.



Solⁿ:

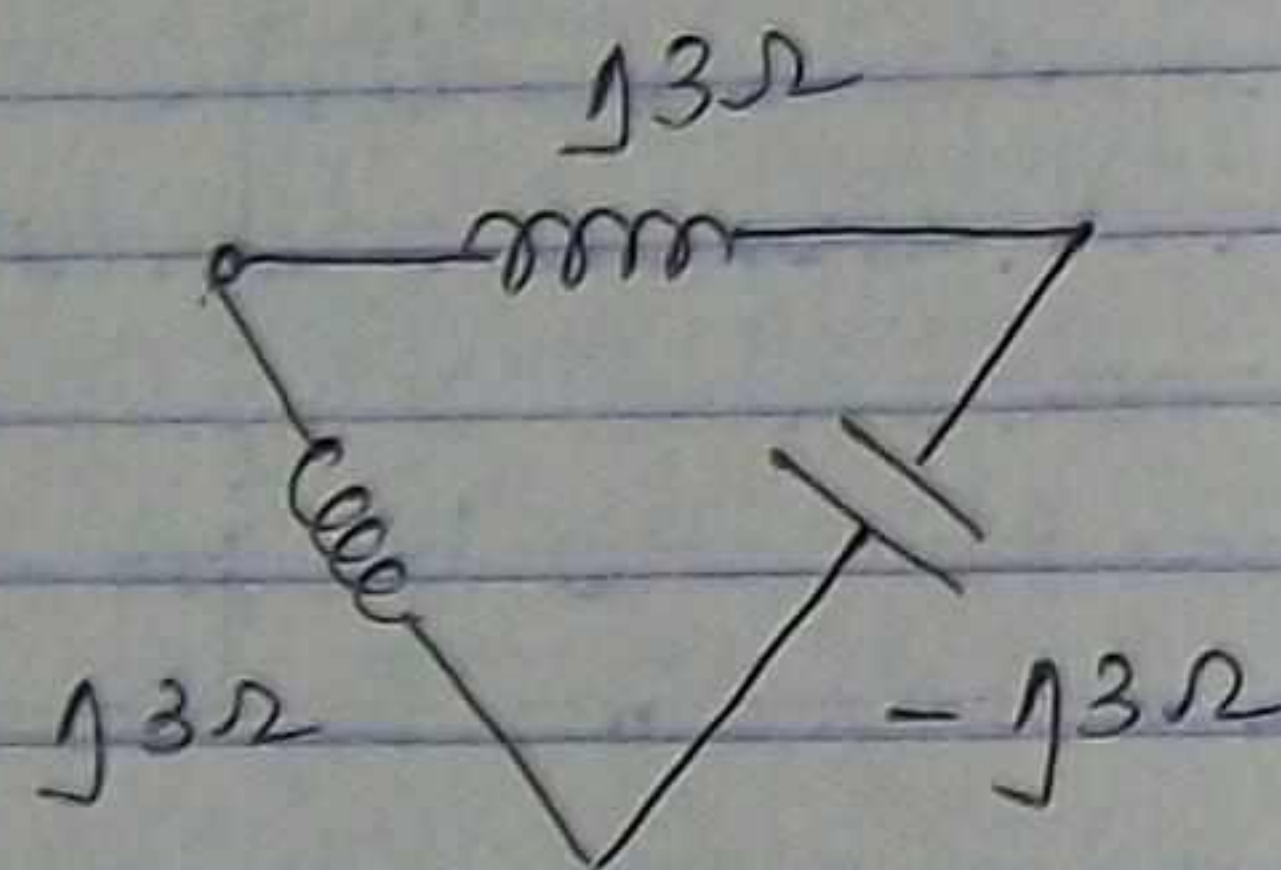


$$R_{eq} = \frac{21}{13} = 1.625$$

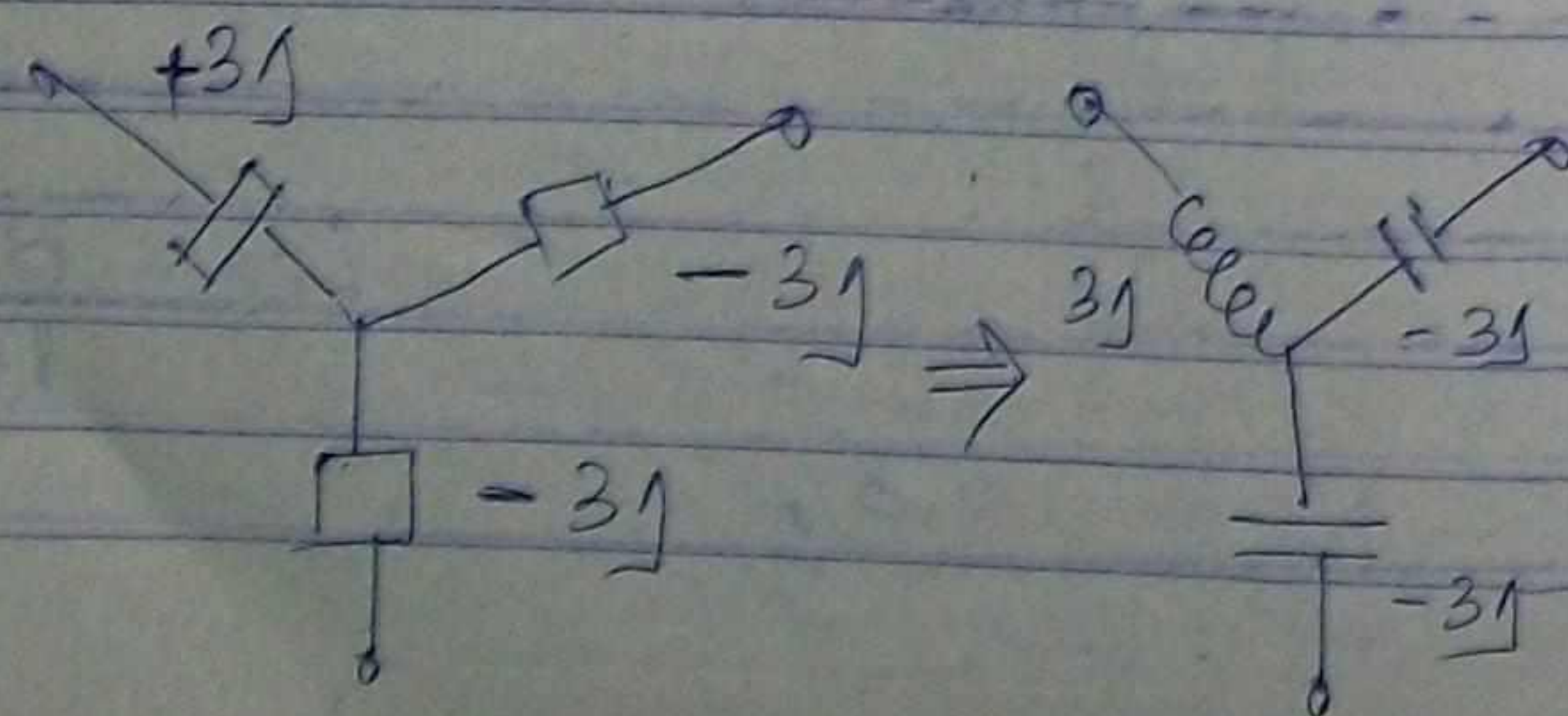
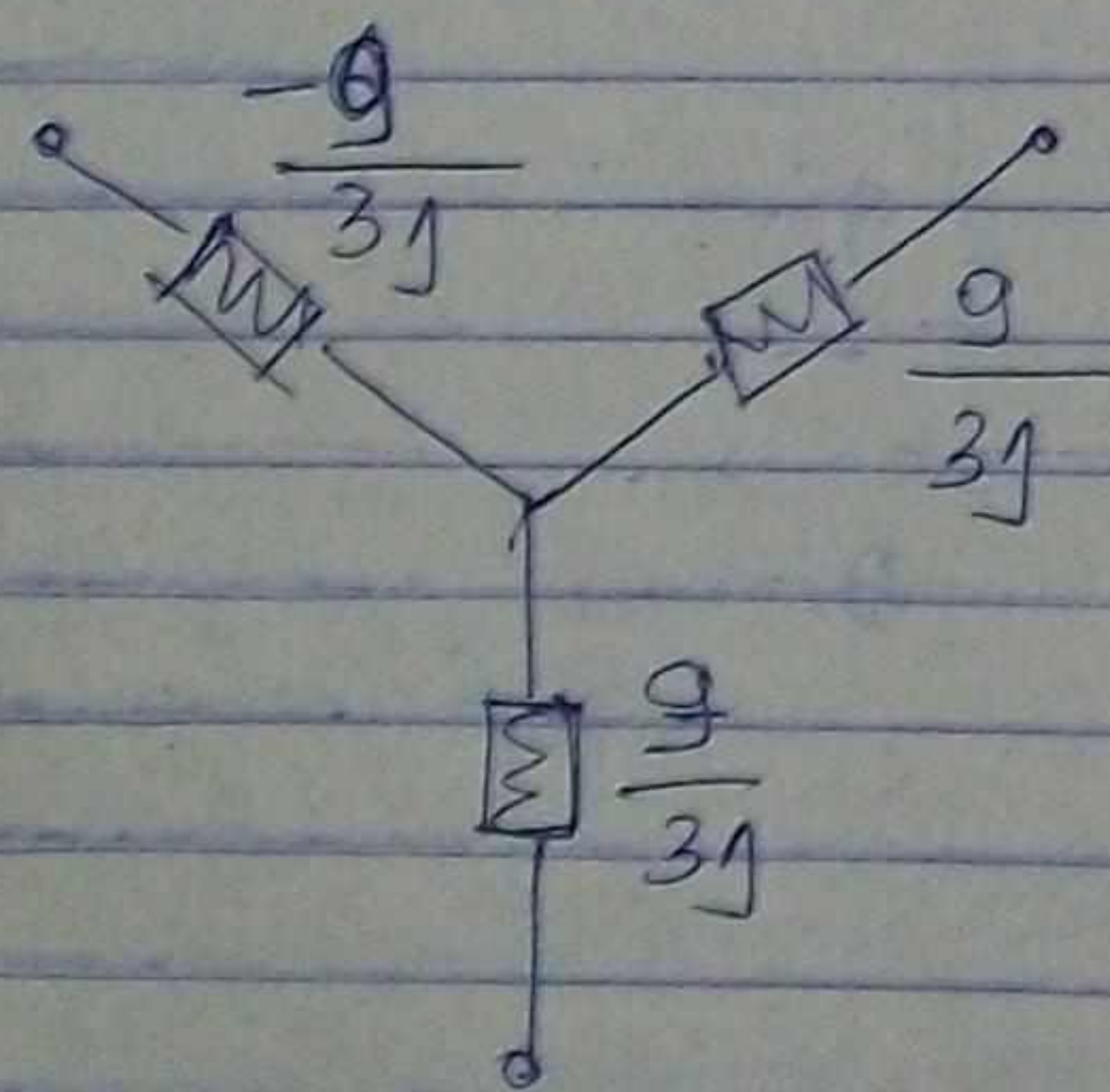
- When resistors of equal value are transformed to ($\star \rightarrow \Delta$) Resistance increased by 3 times.

- When capacitors of equal value transform from ($\star \rightarrow \Delta$) capacitance decrease by 3 times.

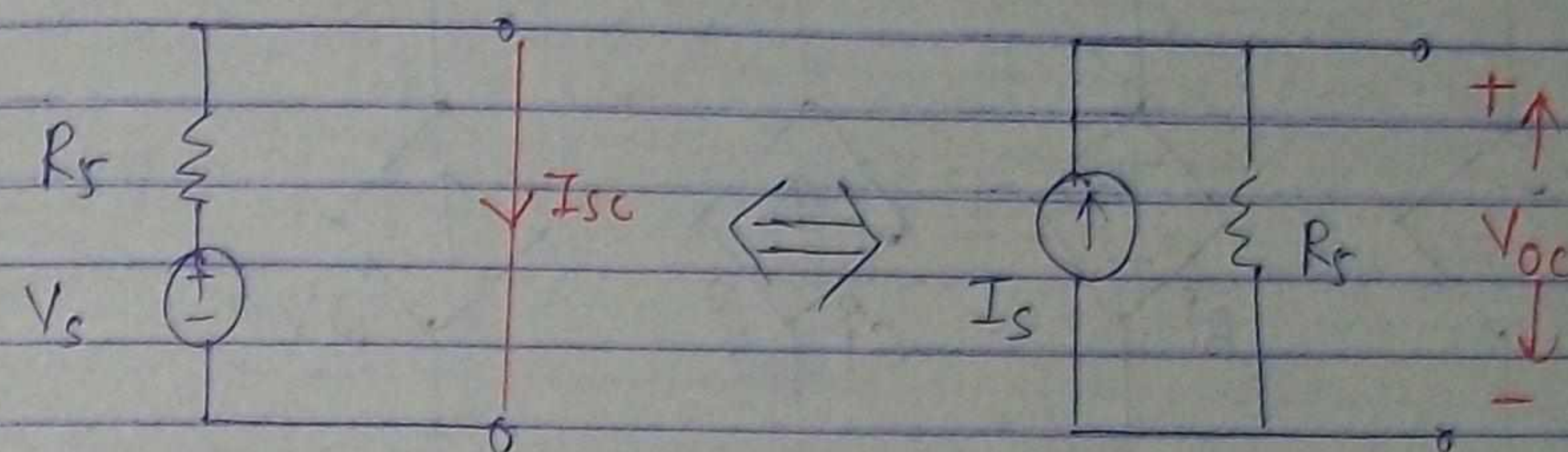
Qw. Obtain equivalent star connection for the N/w.



Solⁿ:



⇒ SOURCE TRANSFORMATION -

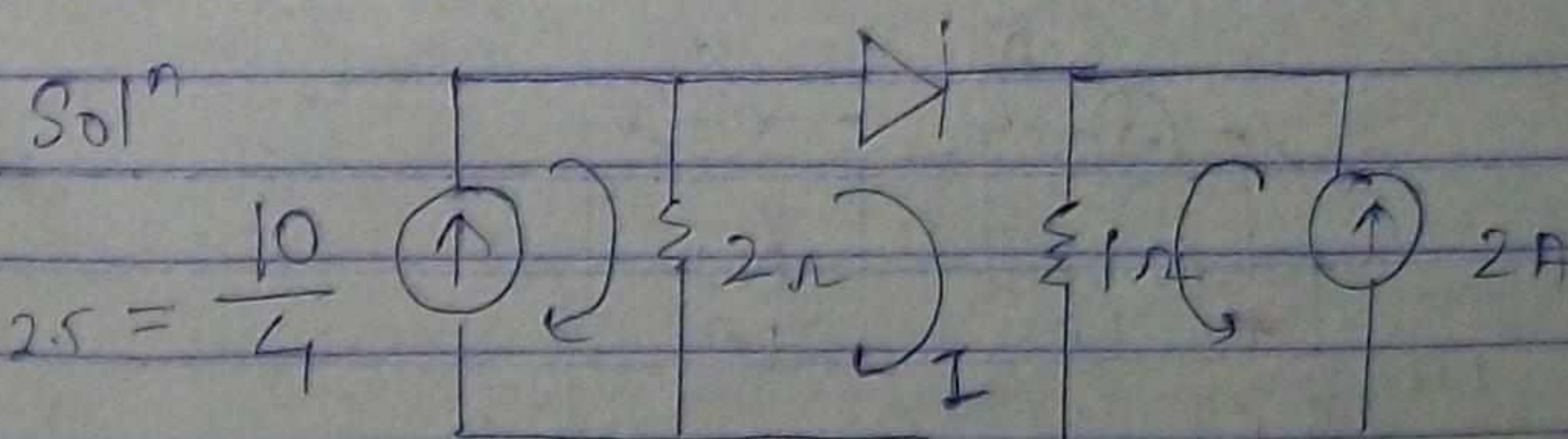
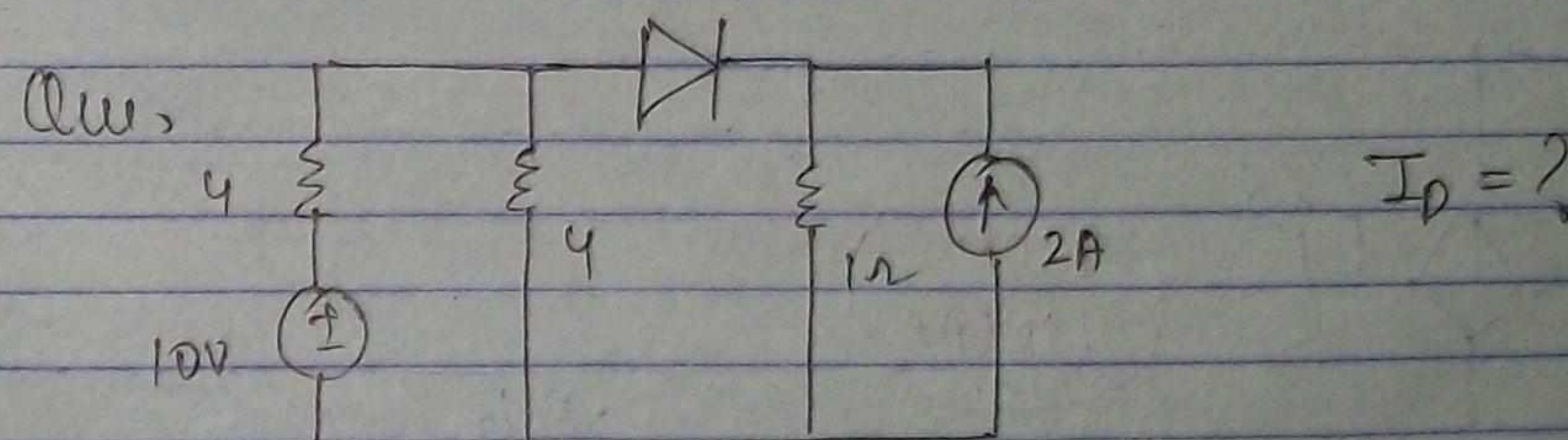


$$I_s = I_{sc} = \frac{V_s}{R_s}$$

$$R_s = R_s$$

$$V_s = V_{oc} = I_s R_s$$

$$R_s = R_s$$



$$1(I+2) + 2(I-2.5) = 0$$

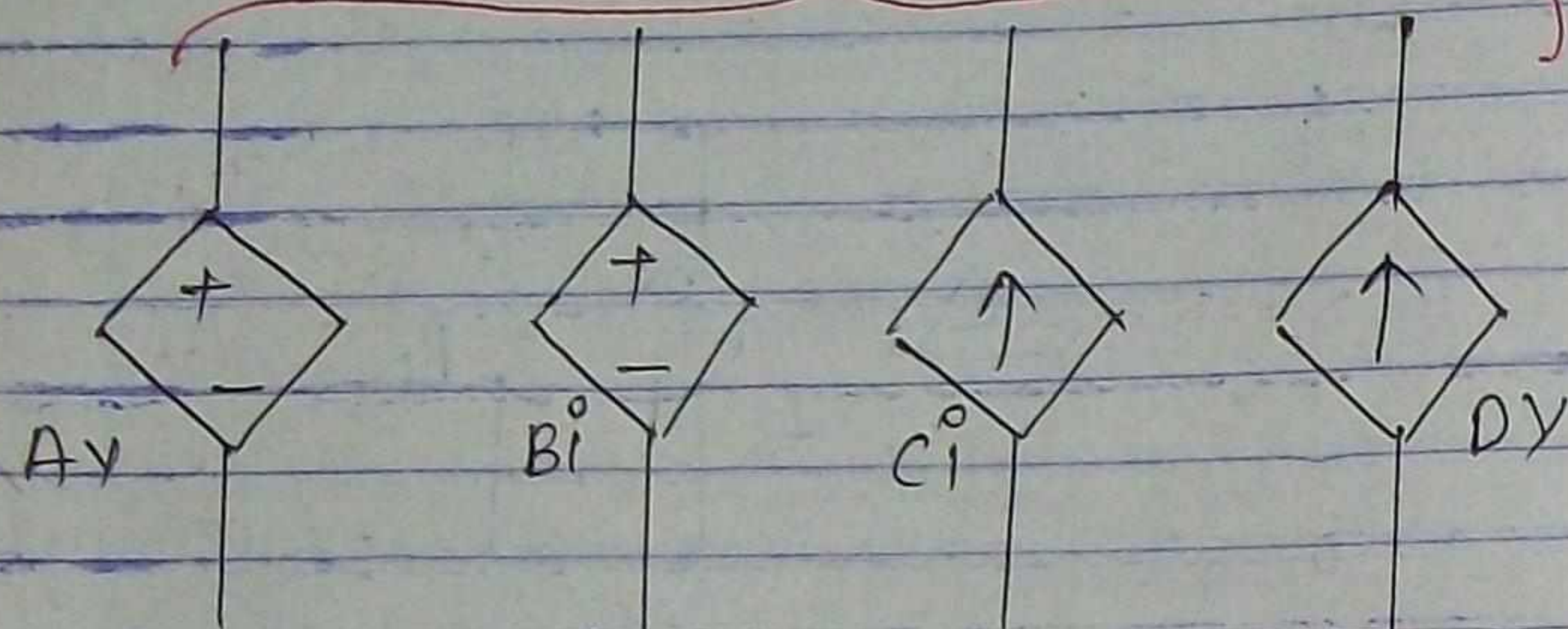
$$I+2 + 2I = 5$$

$$3I = 3$$

$$I = 1$$

LINEAR
DEPENDENT
SOURCE

⇒ DEPENDENT SOURCE -



VCVS

CCVS

CCCS

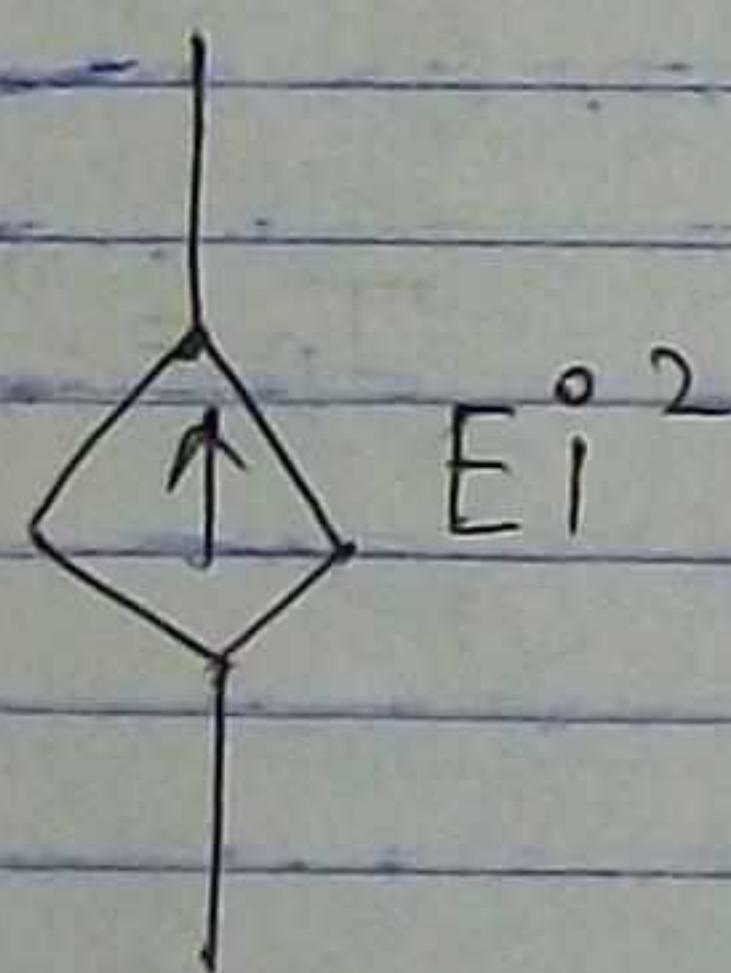
VCCS

A → NO
UNIT

B → OHM
(V/I)

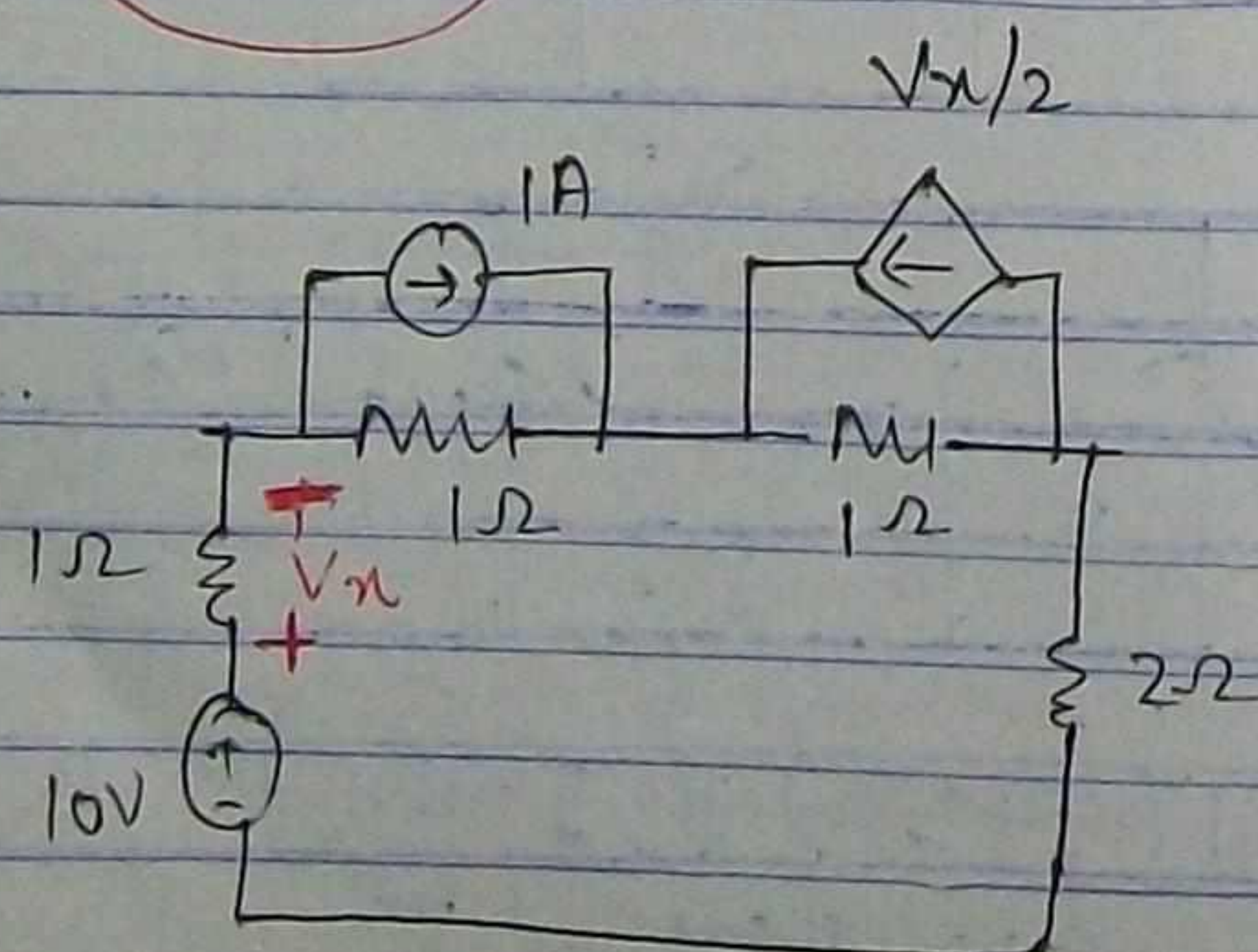
C → NO
UNIT

D → MHO
(I/V)



NON
LINEAR

Ques Find I in
2Ω.



$$V_x = 1I$$

$$10 = V_x + 1(I-1) + 1\left(I + \frac{V_x}{2}\right) + 2I$$

$$10 = I + I - 1 + I + \frac{I}{2} + 2I$$

$$19 = 5I + \frac{I}{2} = 3.5I$$

$$I = \frac{19}{3.5} = \frac{38}{7} \approx 5.43$$

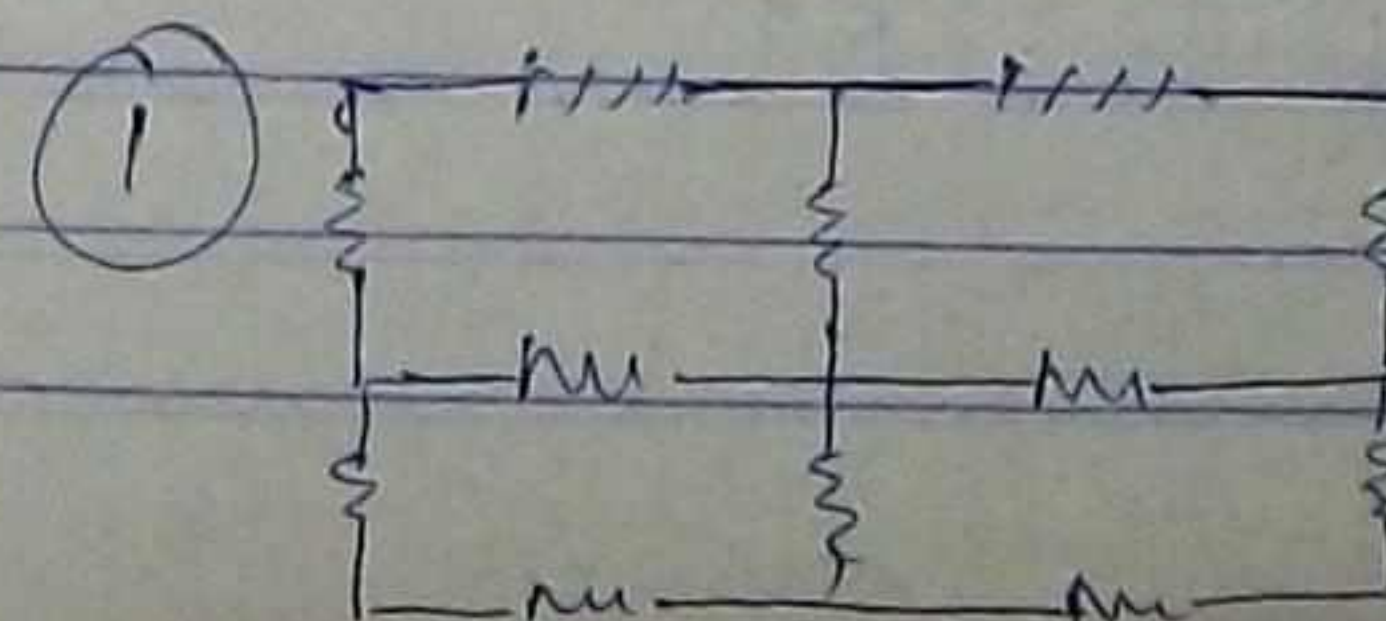
- while applying source transformation for dependent source wherever dependent source magnitude depends without disturbing that element transformation can be applied.

⇒ MESH ANALYSIS -

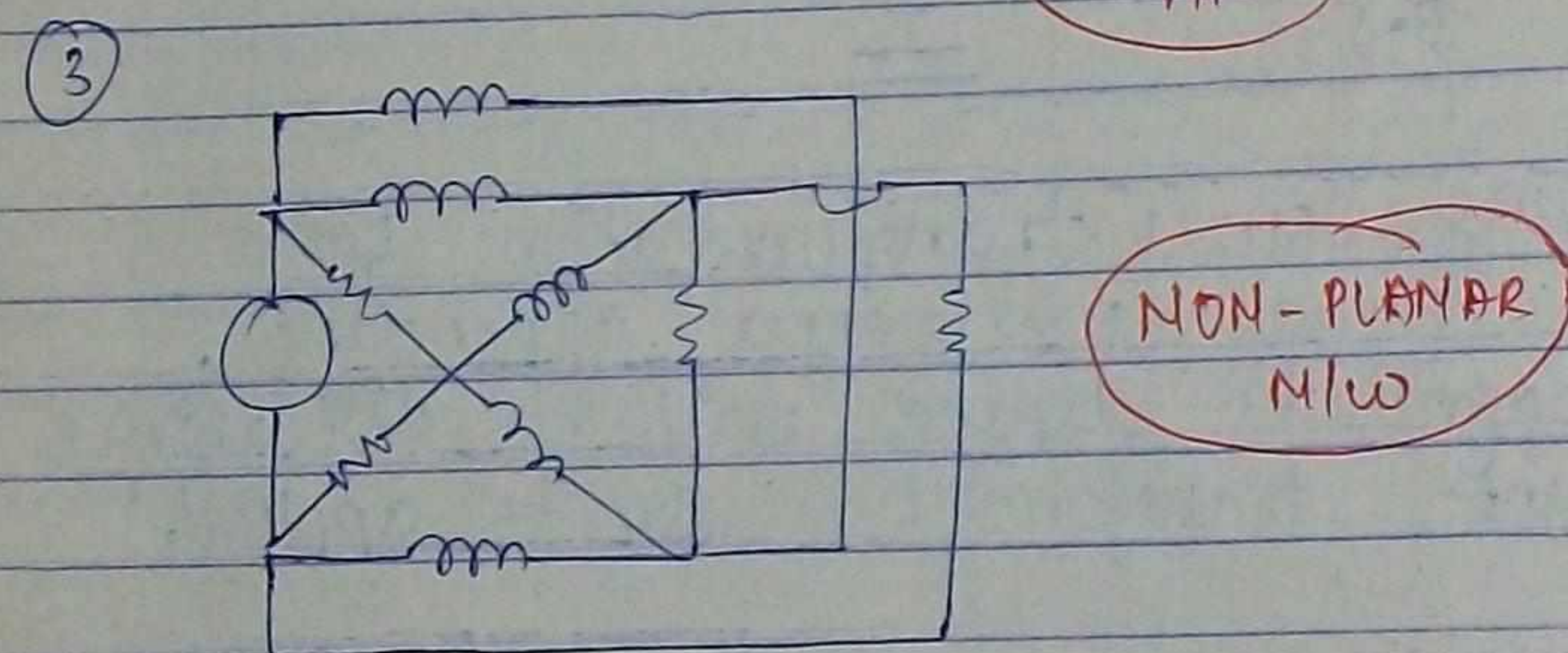
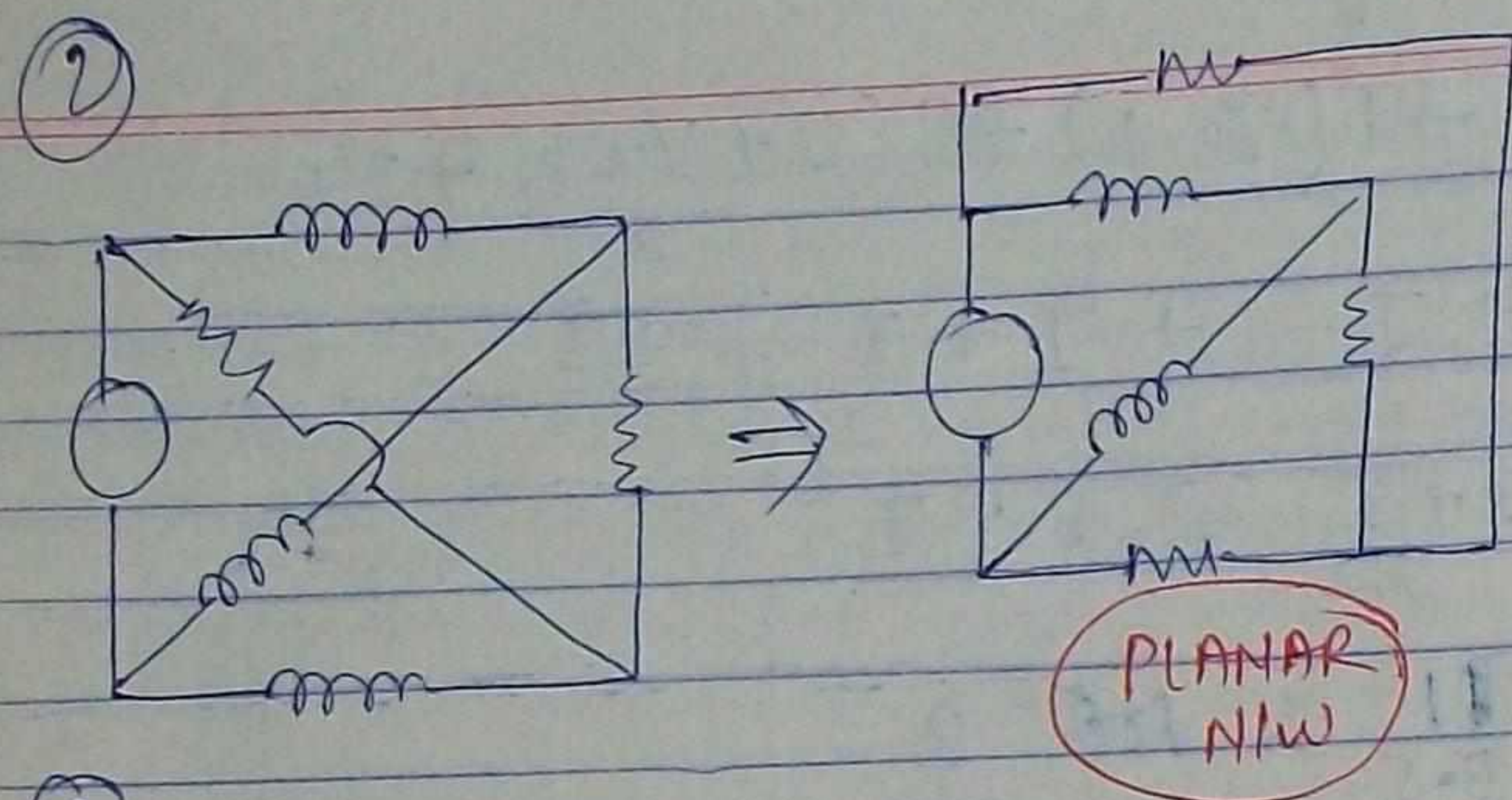
- Mesh is a loop, it does not consist of any inner loop.

- when a Network is drawn on plane without any crossover then the Network is called as planar N/w.

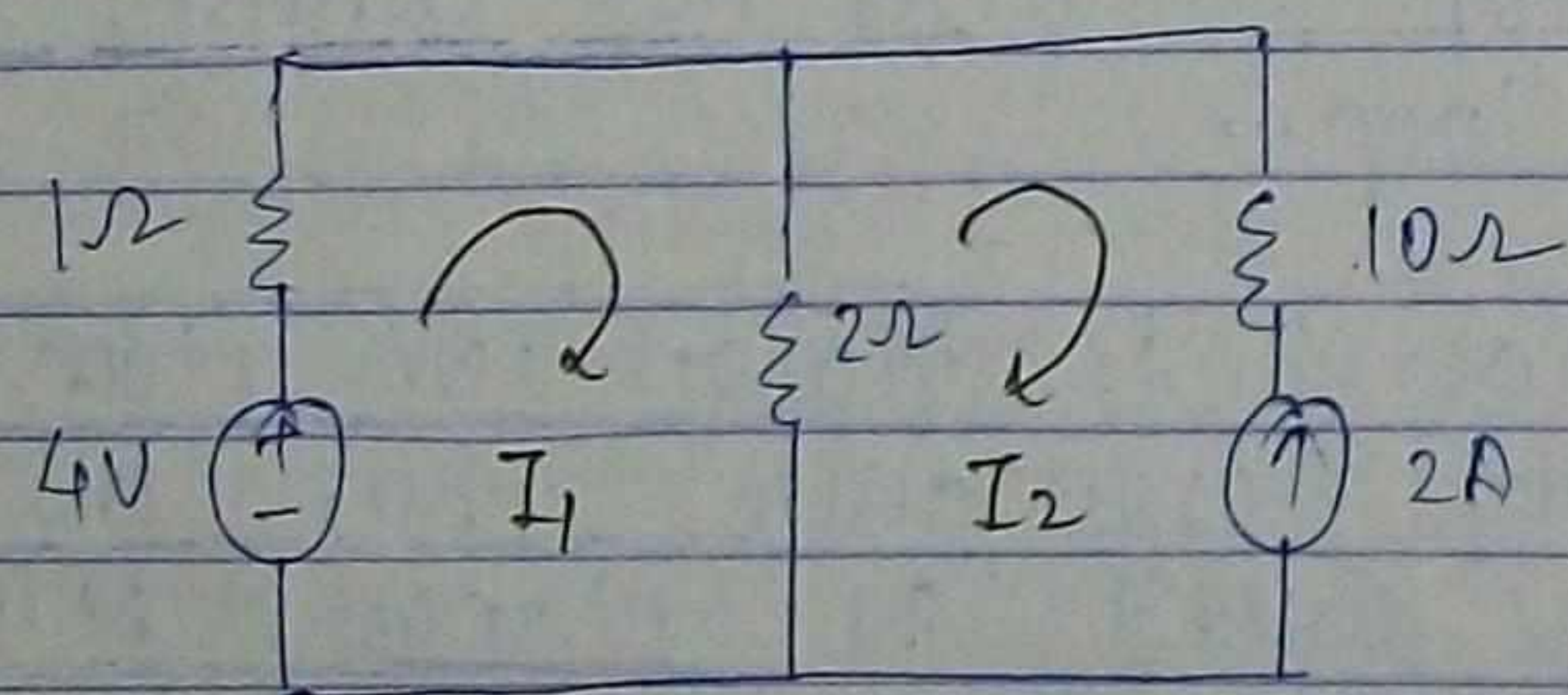
- Mesh analysis can be applied only for planar Network.



PLANAR
N/W



⇒ PROCEDURE OF MESH ANALYSIS -



① Identify total no. of meshes.

② assign the current direction for each mesh.

③ develop KVL eqⁿ for each mesh.

④ By solving KVL eqⁿ find loop currents.

$$-4 + 3I_1 - 2I_2 = 0 \quad \text{--- (1)}$$

$$I_2 = -2 \quad \text{--- (2)}$$

$$I_1 = 0$$

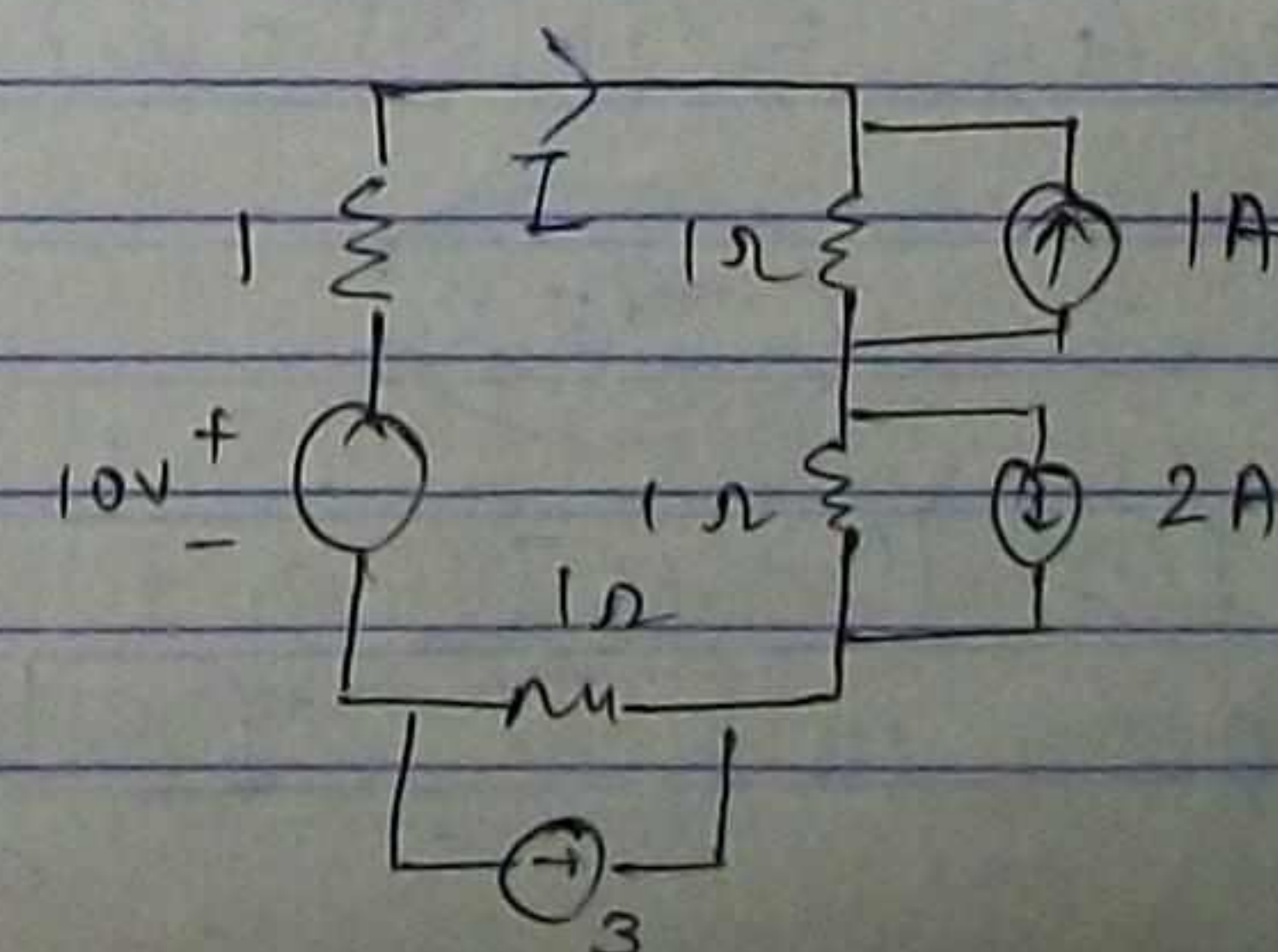
-	TOTAL NO. OF EQUATIONS	=	TOTAL NO. OF MESHES	= 2
-	TOTAL NO. OF EQ ⁿ	=	$b - (N - 1)$	$= 3 - (2 - 1) = 2$

b = NO. of branches (total)

N = Total No. of nodes.

- To find loop currents in the above circuit minimum one equation is required.

Ques. find I



$$10 = I + 1(I+1) + 1(I-2) + 1(I+3)$$

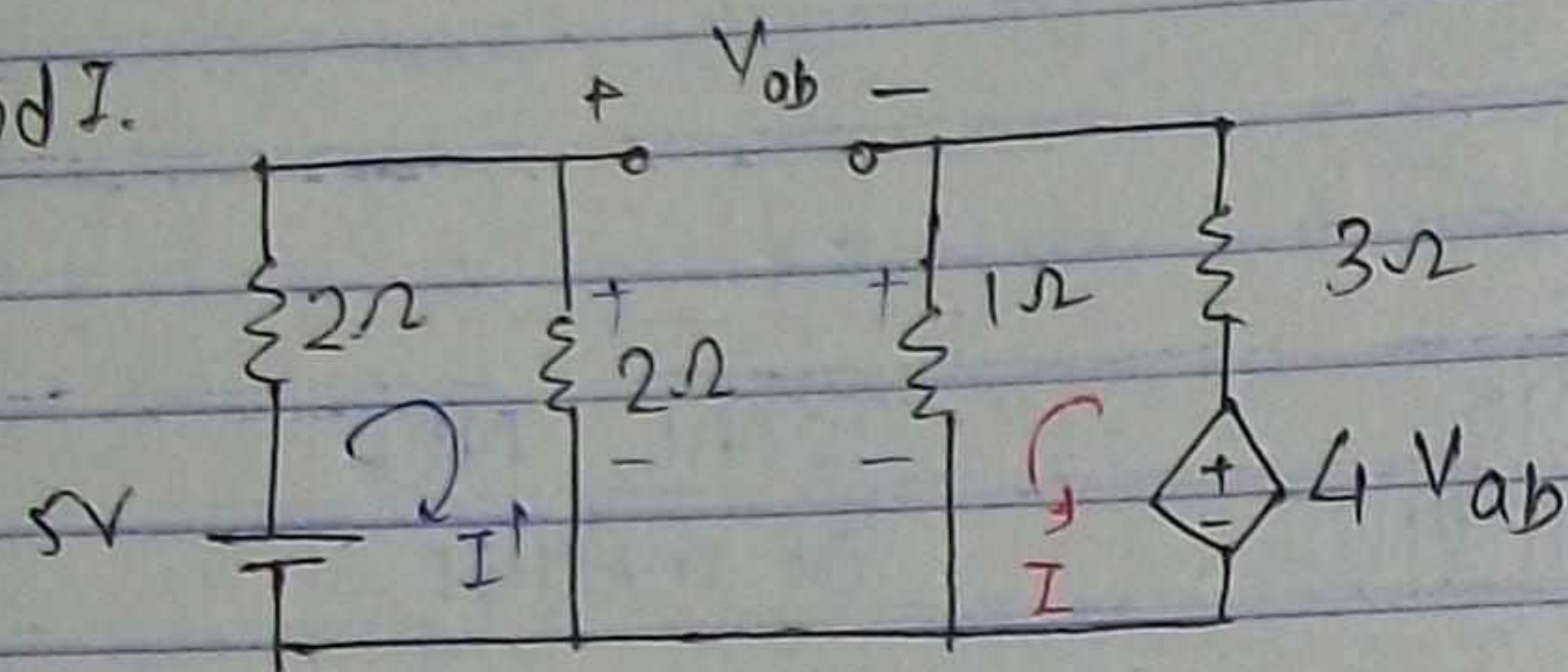
$$10 = I + I + 1 + I - 2 + I + 3$$

$$10 - 1 + 2 - 3 = 4I$$

$$8 = 4I$$

$$I = 2A$$

Ques. Find I .



$$5 = (2+2)I' \quad I' = 5/4$$

$$V_{ab} = 2I' - I$$

$$= 2.5 - I$$

$$4V_{ab} = 3I + I' = 4I$$

$$4(2.5 - I) = 4I = 4(2.5 - I)$$

$$40 = 40$$

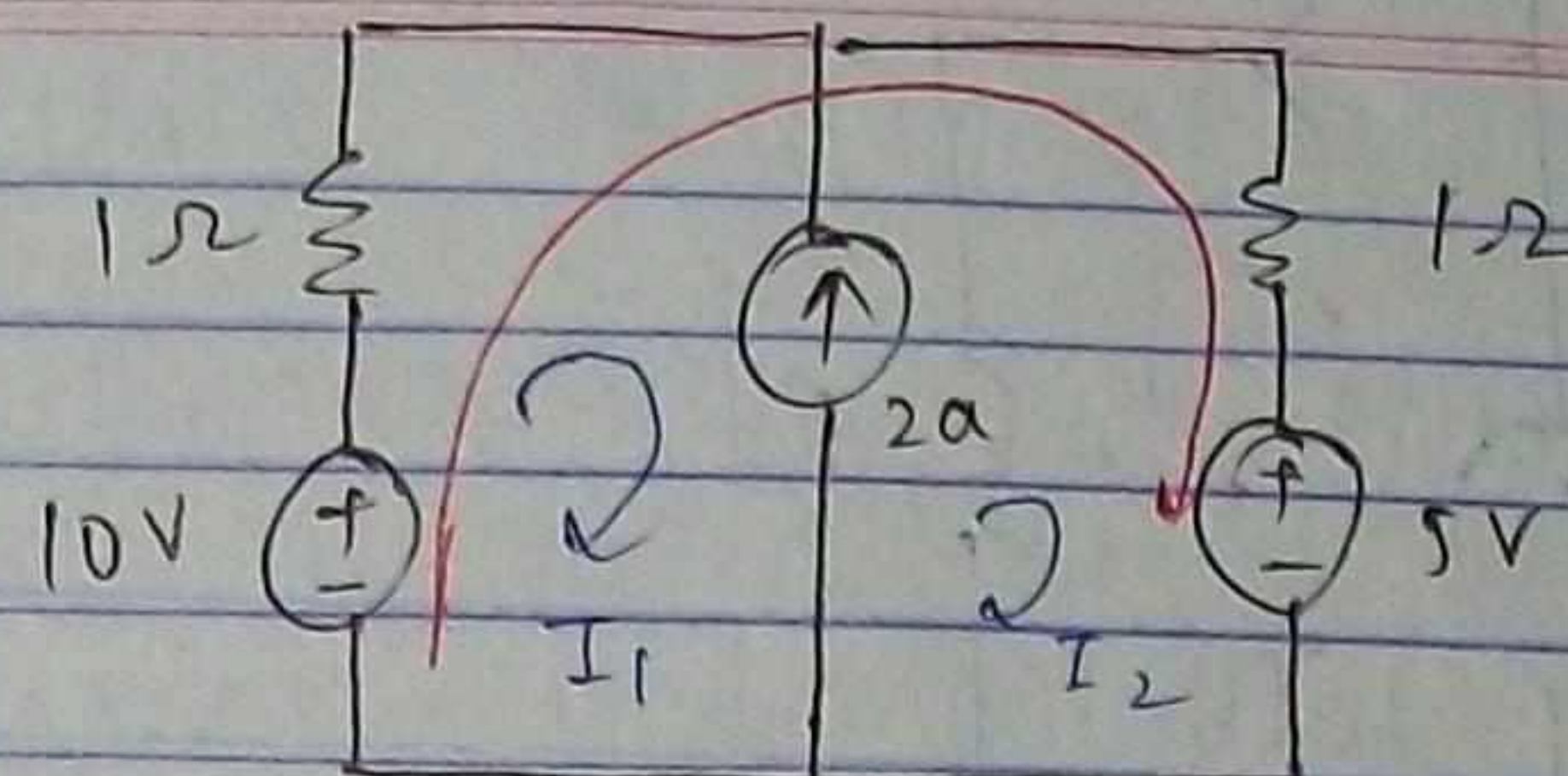
$$I = 0$$

$$4I = 10 - 4I$$

$$8I = 10$$

$$I = \frac{10}{8} = \frac{5}{4} = 1.2$$

Ques.



$$I_1 = ? \quad I_2 = ?$$

$$10 = I_1 + I_2 + 5$$

$$I_1 + I_2 = 5$$

$$-I_1 + I_2 = 2$$

$$I_2 = 7/2 = 3.5$$

$$I_1 = 3/2 = 1.5$$

⇒ SUPER MESH -

- When current source branch is common for two meshes it is possible to find solⁿ by using super mesh technique.

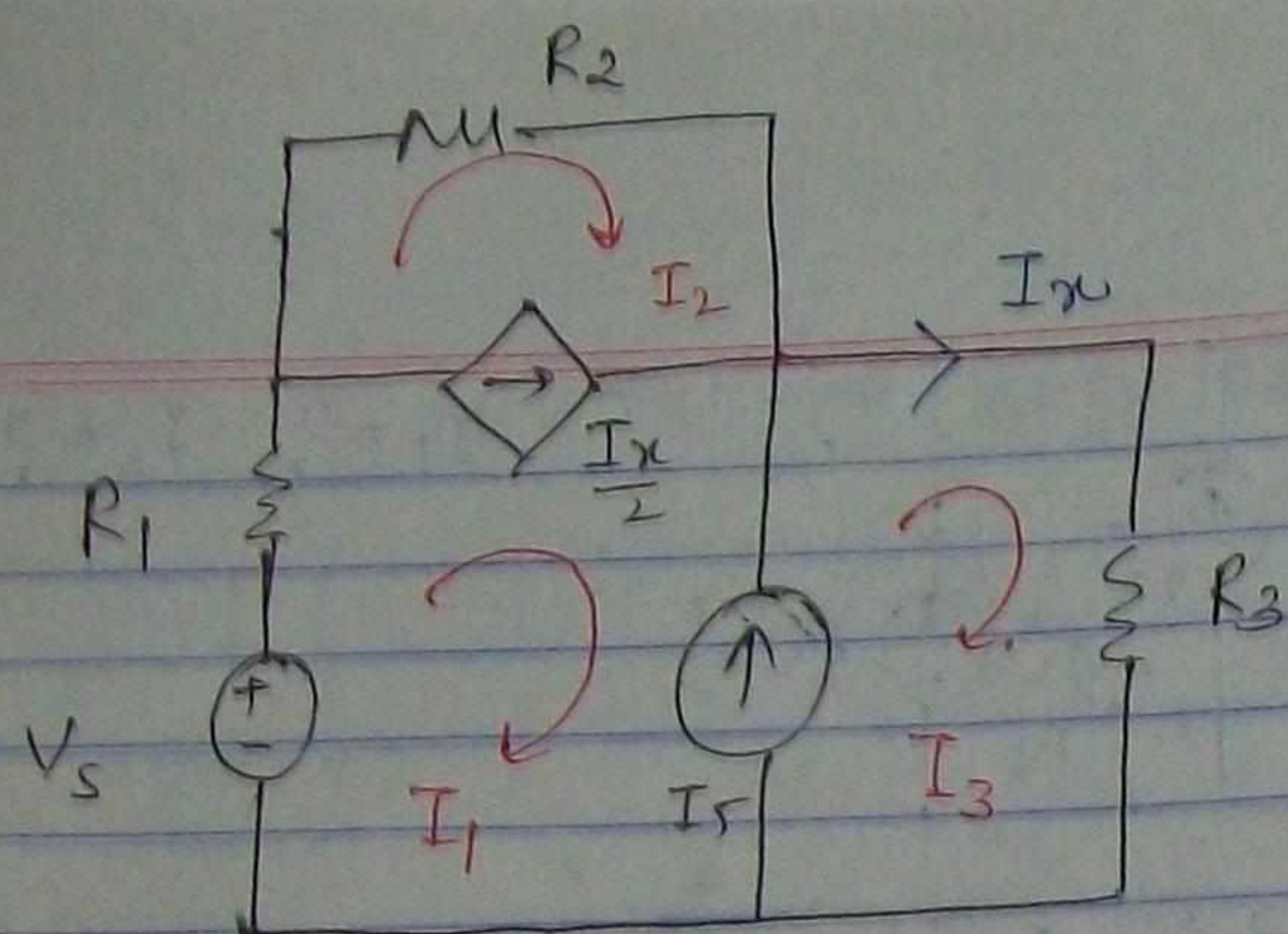
MESH ANALYSIS

→ KVL + OHM'S LAW

SUPER MESH ANALYSIS

→ KVL + KCL + OHM'S LAW

Ques. Write mesh eqn for the circuit shown.



Solⁿ:

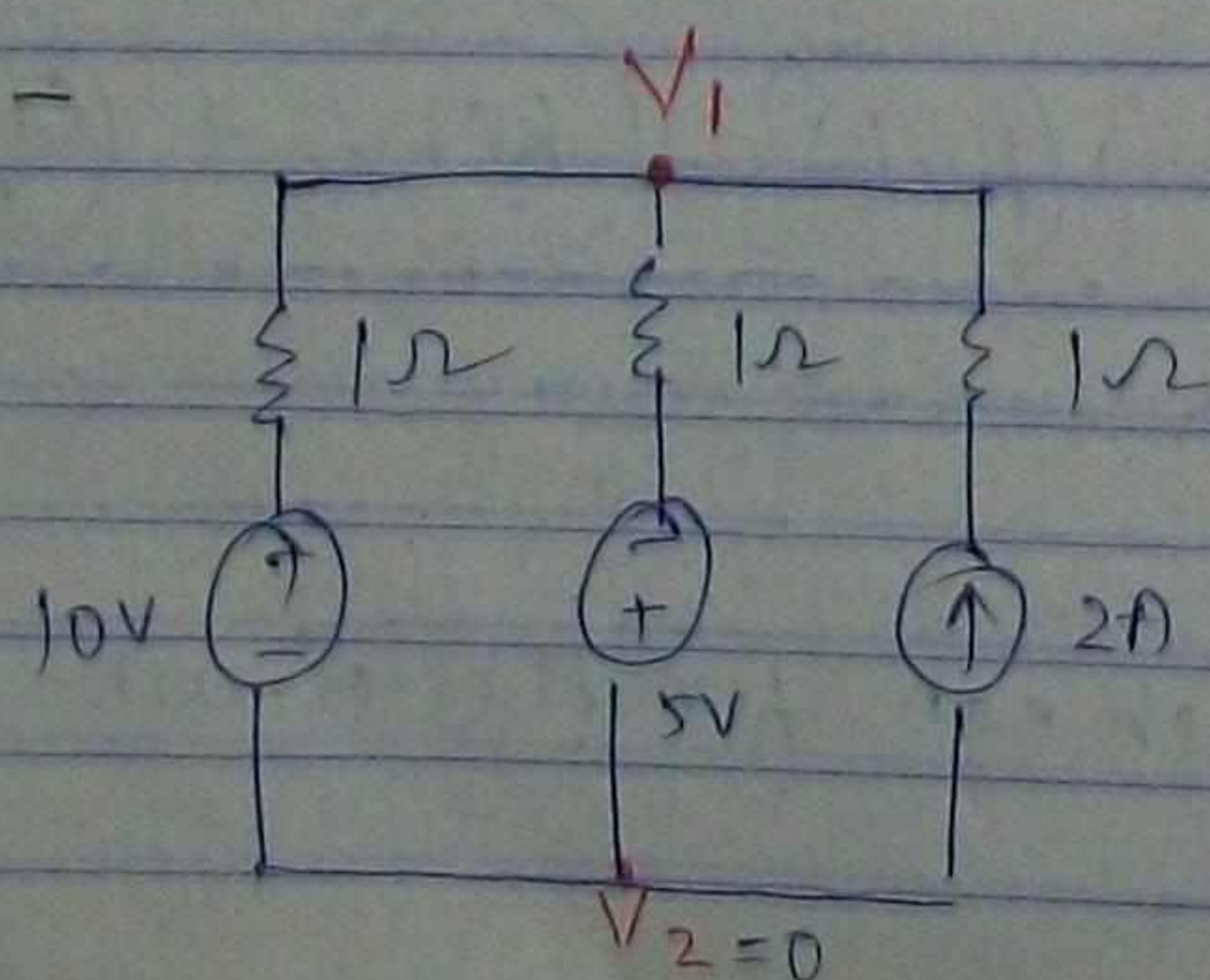
$$V_s = I_1 R_1 + I_2 R_2 + I_3 R_3 \quad \text{--- (KVL)}$$

$$\left. \begin{aligned} I_3 - I_1 &= I_s \\ I_1 - I_2 &= \frac{I_x}{2} \end{aligned} \right\} \text{--- (KCL)}$$

⇒ NODAL ANALYSIS -

- Nodal analysis can be applied for planar and non-planar networks.

PROCEDURE -



① Identify total no. of Nodes.

② Assign the voltage at each node, one of the nodes is taken as a reference node, and reference node potential should be equal to GND potential.

③ Develop KCL eqⁿ at each non reference node.

④ By solving KCL eqⁿ find node voltages.

$$\frac{V_1 - 10}{1} + \frac{V_1 + 5}{1} - 2 = 0$$

$$V_1 = 7/2 = \underline{\underline{3.5V}}$$

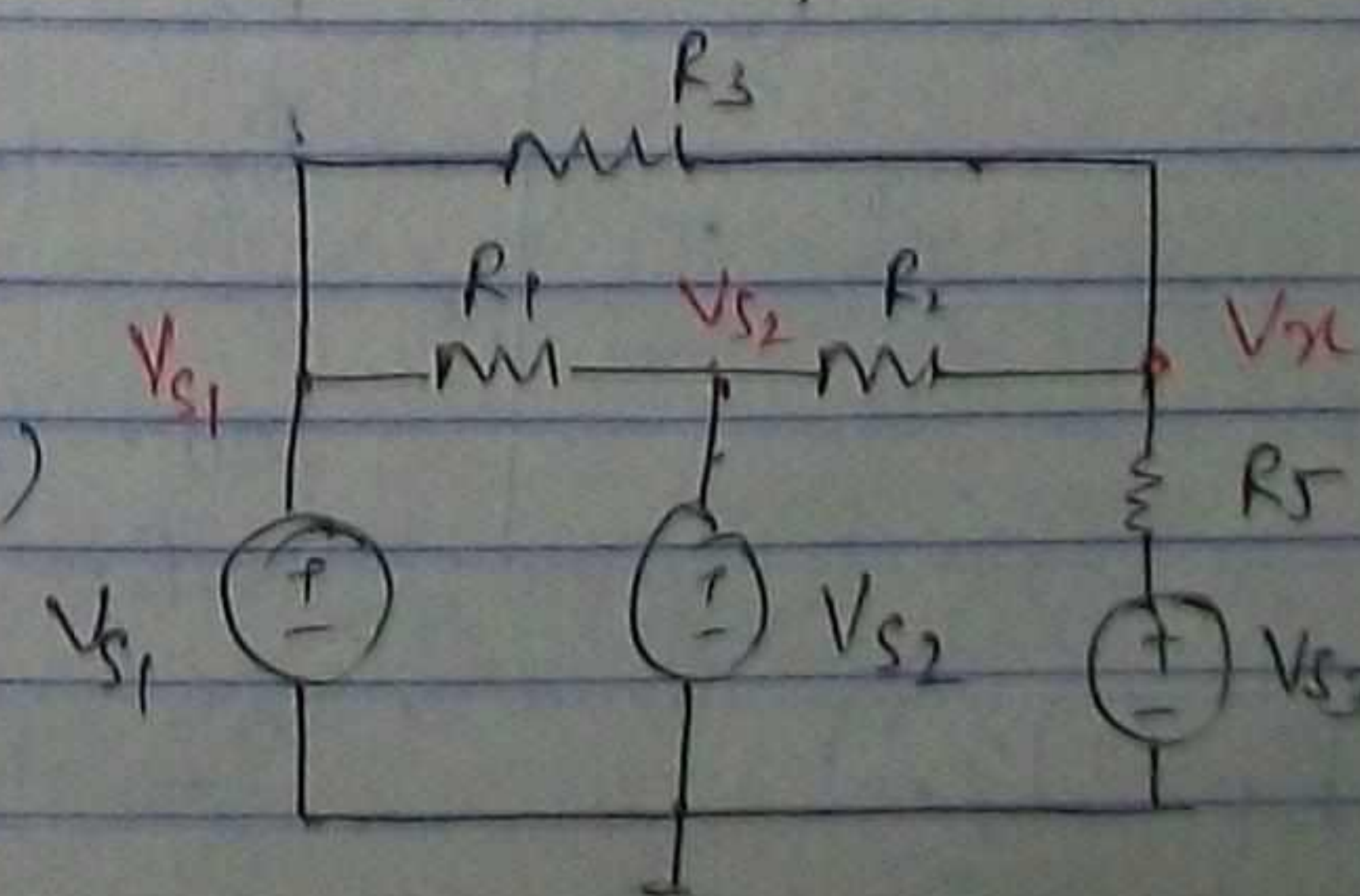
$$\boxed{\text{TOTAL NO. of eq}^n = N - 1}$$

$N = \text{No. of Nodes (total)}$

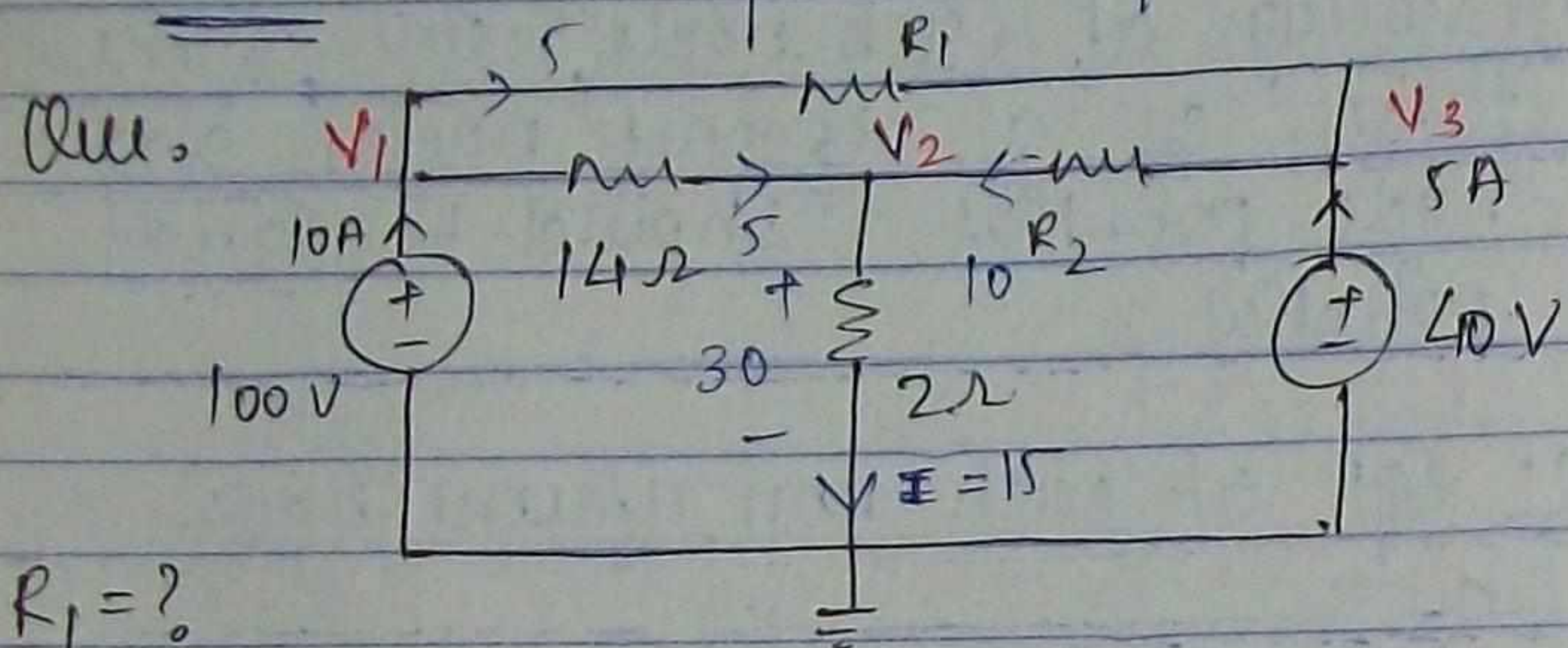
Ques

No. of eq = 3 (4-1)

min. NO. of eqⁿ = 1



— To find Node voltage in the above N/w min. one eqn is required.



$$R_1 = ?$$

$$R_2 = ?$$

Solⁿ:

$$\frac{V_2 - 100}{14} + \frac{V_2}{2} + \frac{V_2 - 40}{R_2} = 0$$

$$V_2 \left(\frac{1}{14} + \frac{1}{2} + \frac{1}{R_2} \right) - \frac{100}{14} + \frac{-40}{R_2} = 0$$

By KCL at Node 4 $I = 15A$

$$\therefore V_2 = 30V$$

$$\frac{100 - 30}{14} = I'$$

$$\frac{70}{14} = I' = 5$$

$$R_1 = 12\Omega$$

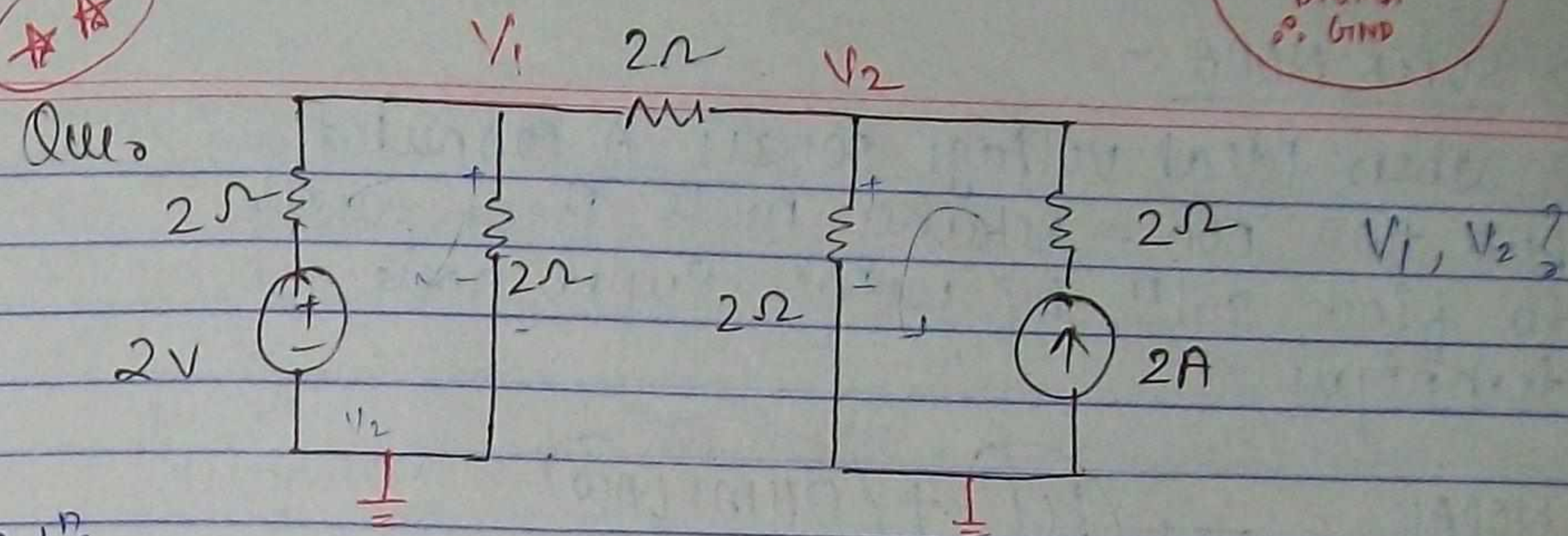
$$\frac{40 - 30}{R_2} = 10$$

$$R_2 = 1\Omega$$

$$\frac{100 - 40}{R_1} = 5$$

$$\frac{60}{5} = R_1$$

★ ★

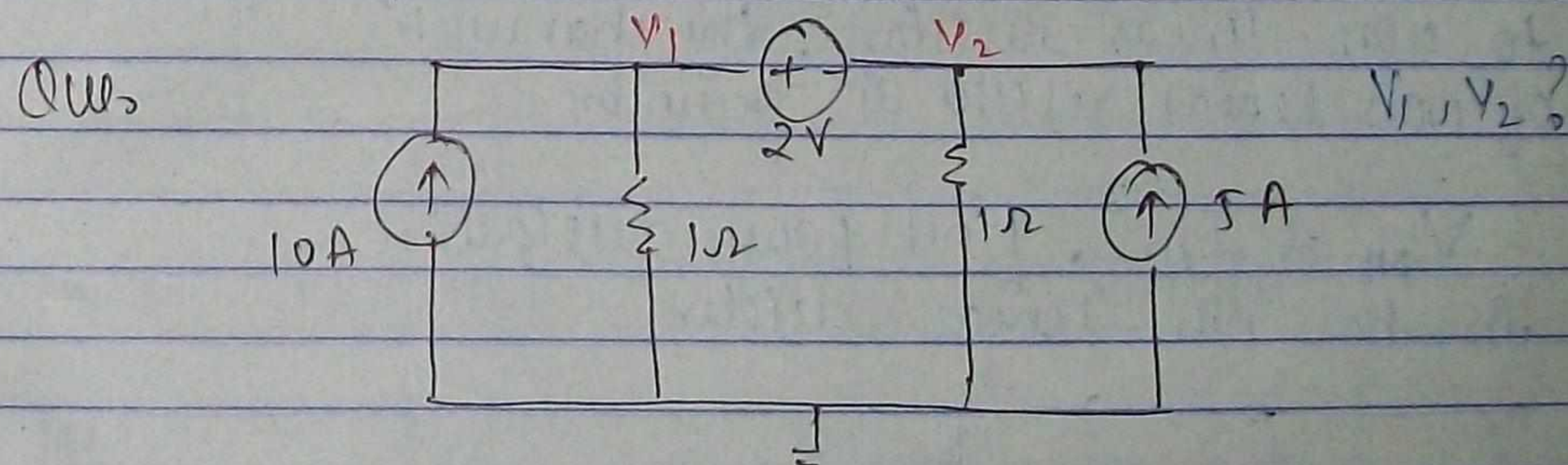


Solⁿ:

$$\frac{V_1 - 2}{2} + \frac{V_1}{2} + \frac{V_1 - V_2}{2} = 0 \quad \text{--- (1)}$$

$$\frac{V_2}{2} + \frac{V_2 - V_1}{2} = 2 \quad \text{--- (2)}$$

$$V_1 = 1.6V \quad V_2 = 2.8V$$



$$-10 + \frac{V_1}{1} + \frac{V_2}{1} - 5 = 0$$

$$V_1 + V_2 = 15 \quad \text{--- (KCL)} \quad V_1 = 17/2$$

$$V_1 - V_2 = 2 \quad \text{--- (KVL)} \quad V_2 = \frac{17}{2} - 2 = \frac{13}{2}$$

⇒ SUPER NODE -

- when ideal voltage source is connected b/w two non-reference node it is possible to find solⁿ by using super node technique.

NODAL ANALYSIS → KCL + OHM'S LAW

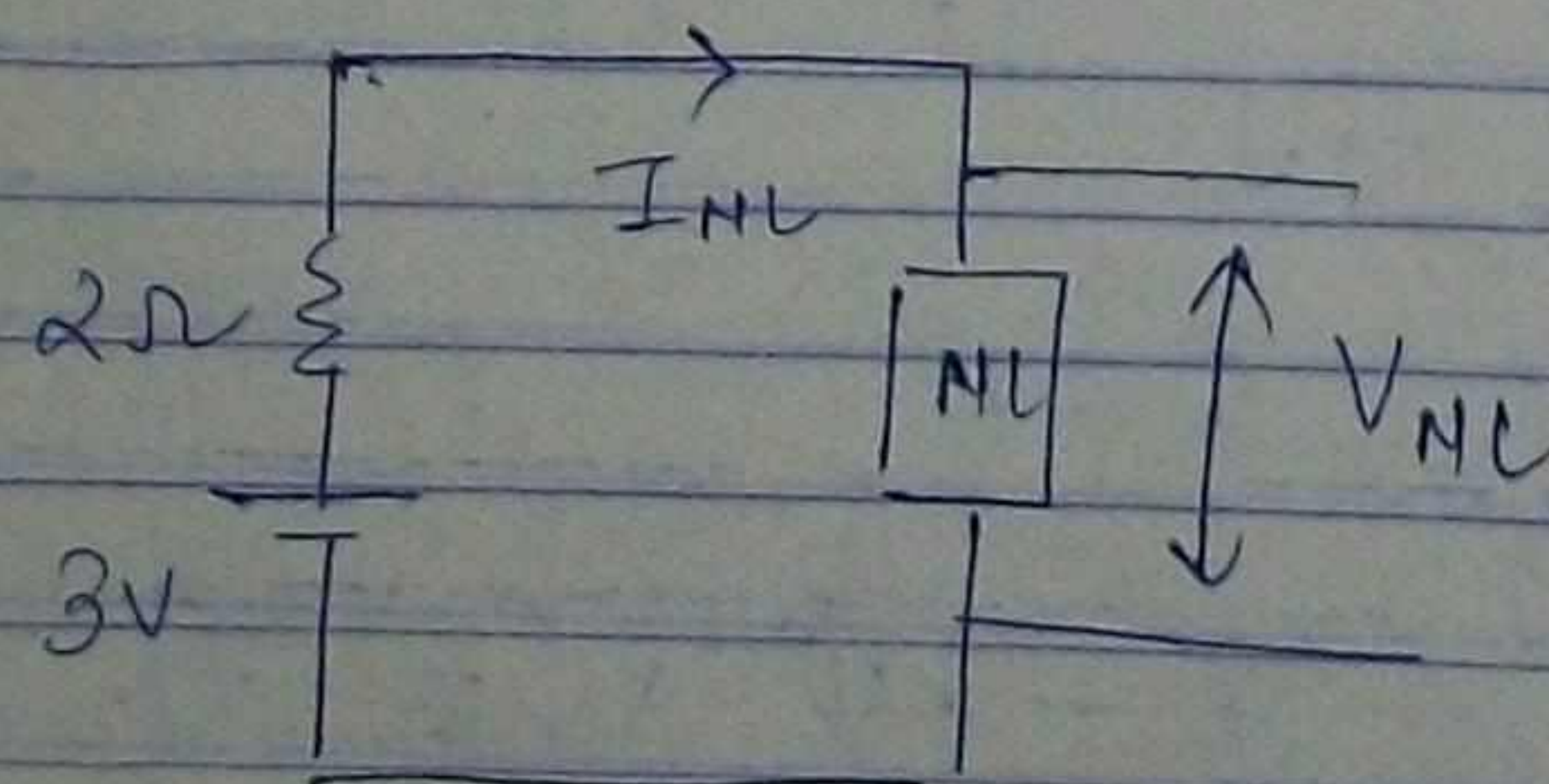
SUPER NODE TECH. → KCL + KVL + OHM'S LAW

Ques

A practical DC voltage source of 3V with internal resistance of 2Ω is connected to non-linear resistor, the characteristic of non linear resistor is given by

$V_{NL} = I_{NL}^2$. find power dissipation in the non-linear resistor.

Solⁿ:



$$2I_{NL} + V_{NL} = 3$$

$$2I_{NL} + I_{NL}^2 = 3$$

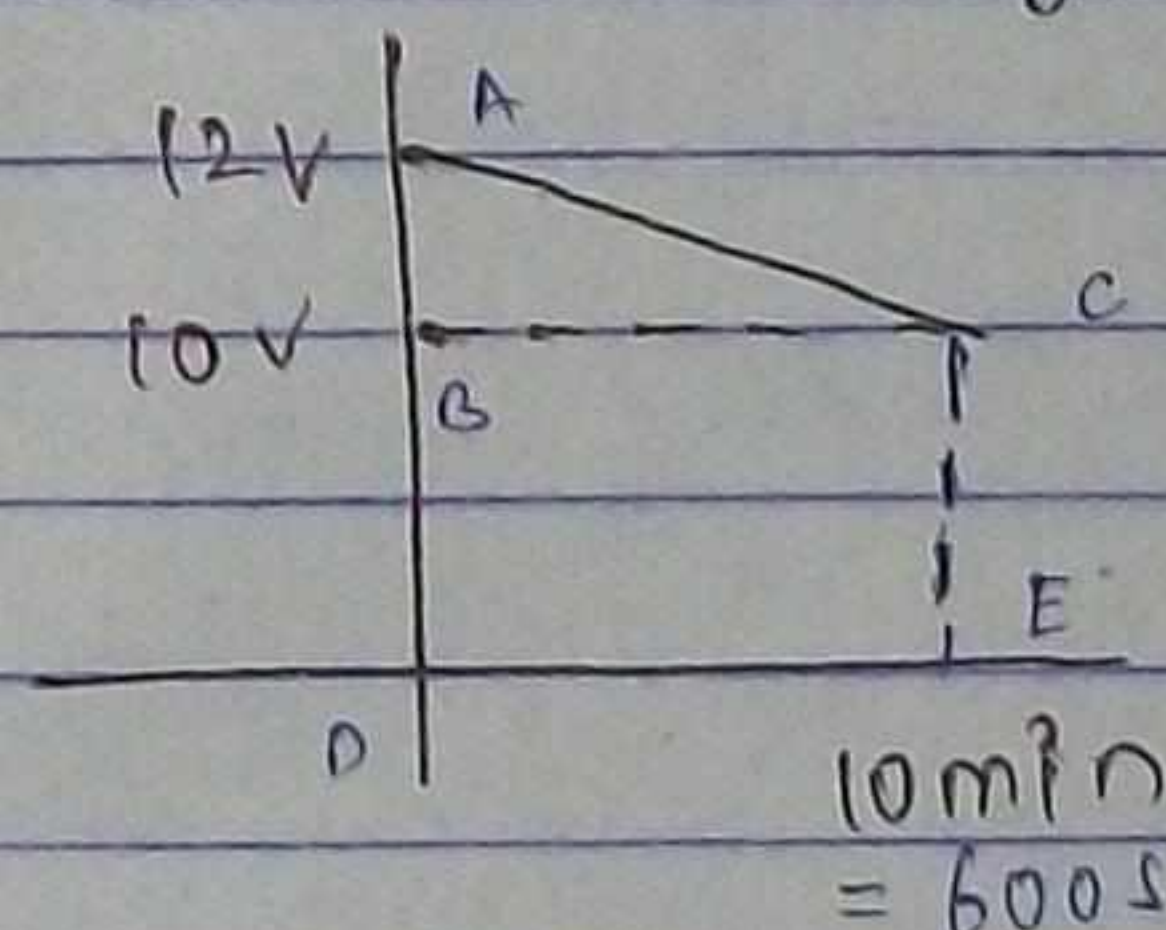
$$I_{NL}^2 + 2I_{NL} - 3 = 0$$

$$I_{NL} = \frac{-2 \pm \sqrt{4 + 12}}{2} = \frac{-2 \pm 4}{2}$$

$$I_{NL} = 1$$

$$\begin{aligned} \text{Power dissipated} &= V_{NL} \times I_{NL} \\ &= I_{NL}^3 = 1 \text{ watt} \end{aligned}$$

Ques. A fully charged mobile phone is good for 10 min. talk time. During the talk time battery delivers a constant current of 2A the voltage characteristic of battery is as shown



find energy of the battery during talk time.

Solⁿ: ① method: (area under curve)

$$T_{\text{area}} = \Delta ABC + \text{BCDE}$$

$$= \frac{1}{2} \times 600 \times 2 + 10 \times 600 = 6600$$

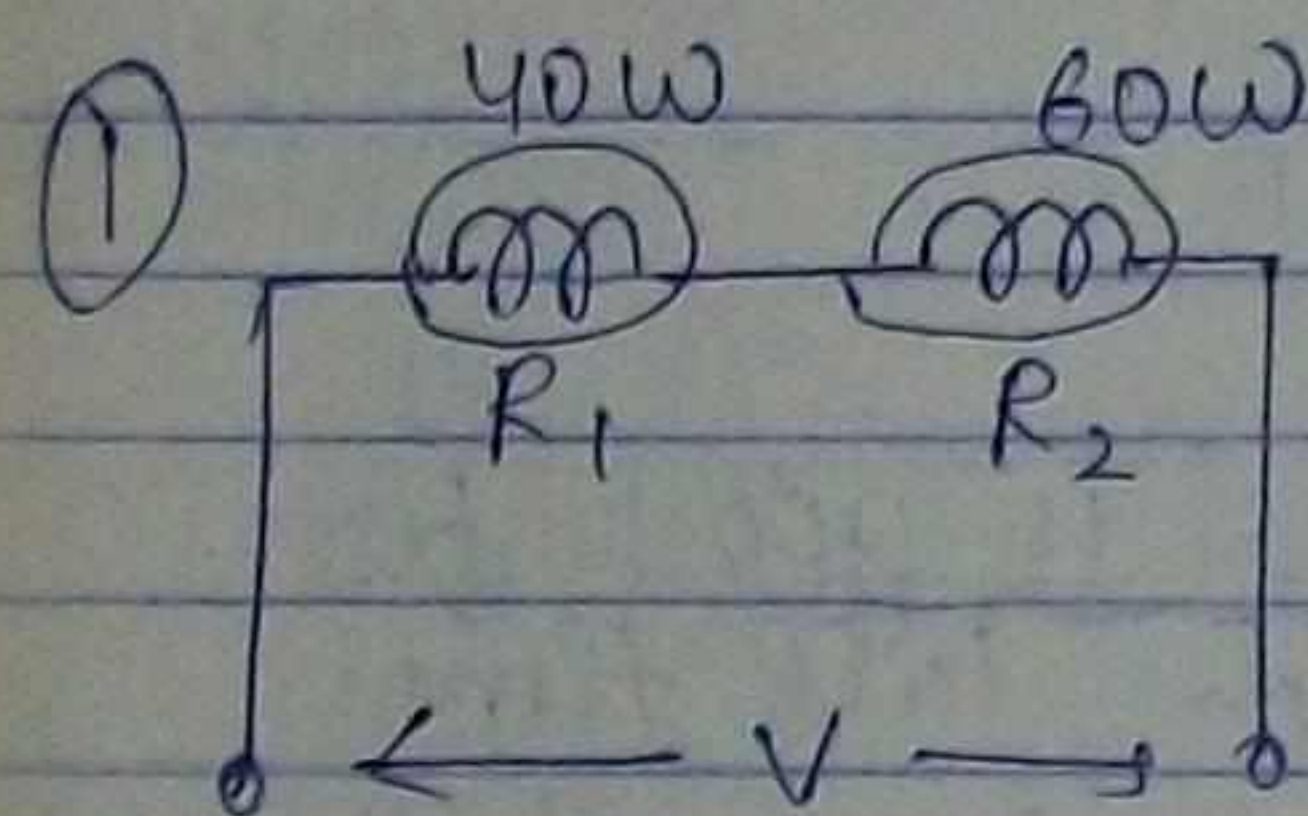
$$\therefore Vt = 6600$$

$$\begin{aligned} \text{Energy } W &= Vt \times I = 6600 \times 2 \\ &= 13.2 \text{ KJ} \end{aligned}$$

⇒ RATING -

$V \rightarrow 230V$
 $f \rightarrow 50Hz$
 $P \rightarrow \text{variable}$

Bulb Rating



$$P = \frac{V^2}{R}$$

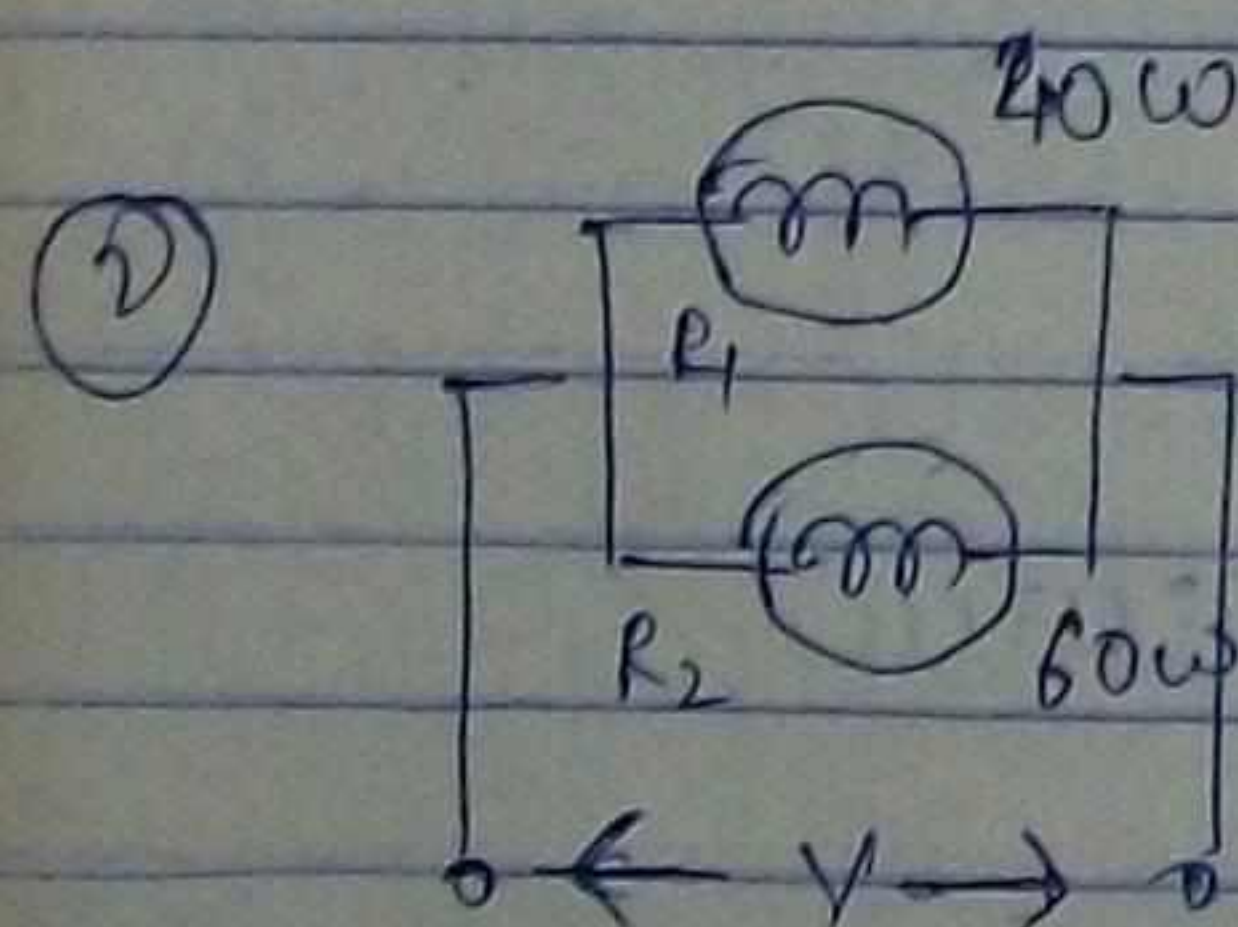
$$P \propto 1/R$$

$$R_1 > R_2$$

$$P_1 > P_2$$

$$P_1 = i^2 R_1$$

$$P_2 = i^2 R_2$$



$$R_1 > R_2$$

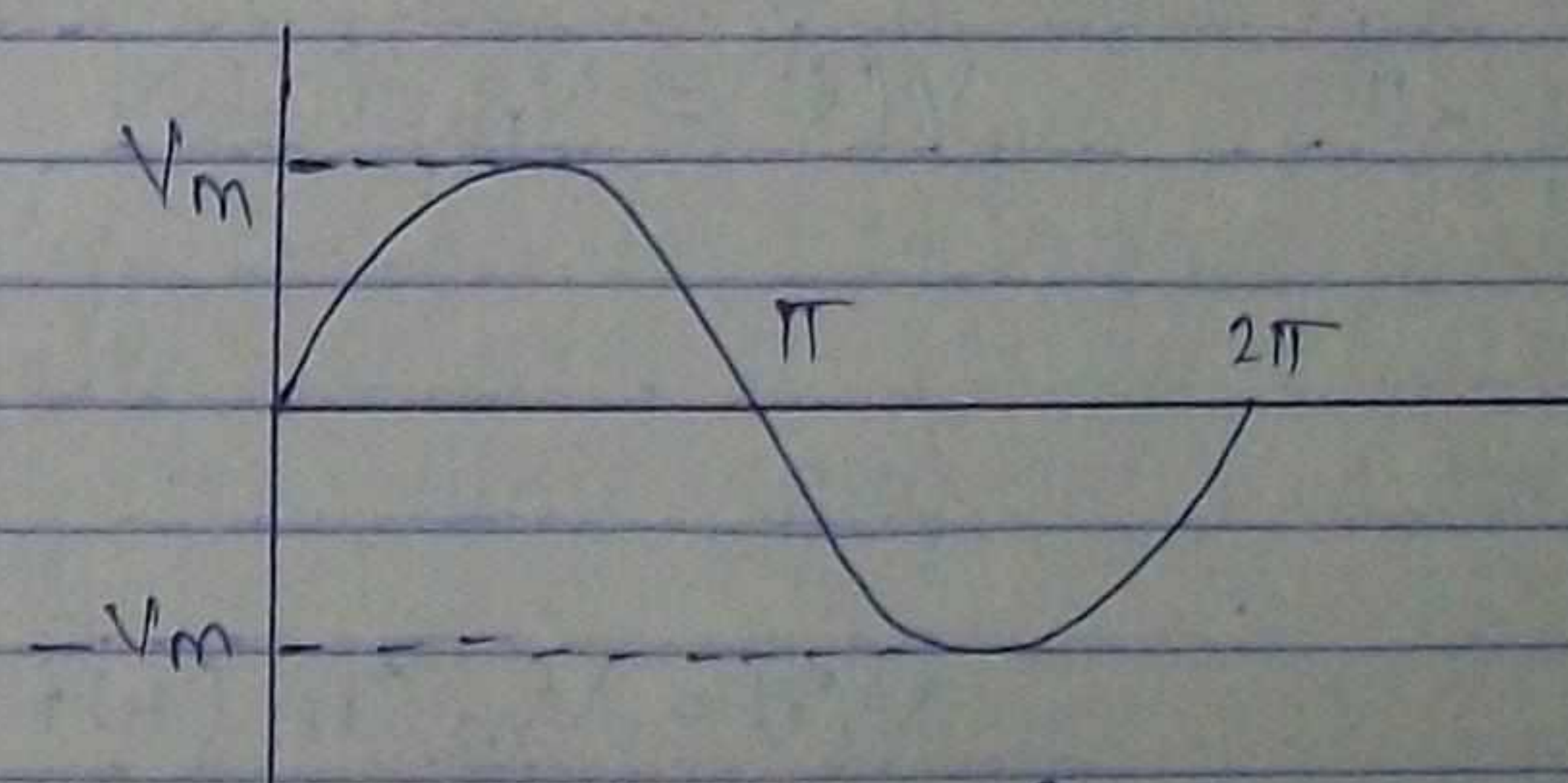
$$P_1 < P_2$$

$$P_1 = \frac{V^2}{R_1}, \quad P_2 = \frac{V^2}{R_2}$$

STEADY STATE AC CIRCUITS

⇒ ADVANTAGE OF SINE WAVE -

- It is easy to handle mathematically. (either differential or integral of the sine function can be rewrite in terms of sine function)
- The natural phenomenon like motion of the simple pendulum and response of undamped system exhibits sinusoidal character.
- Any periodic waveform can be expressed in terms of sine function by using fourier analysis
- It is easy to generate in lab.

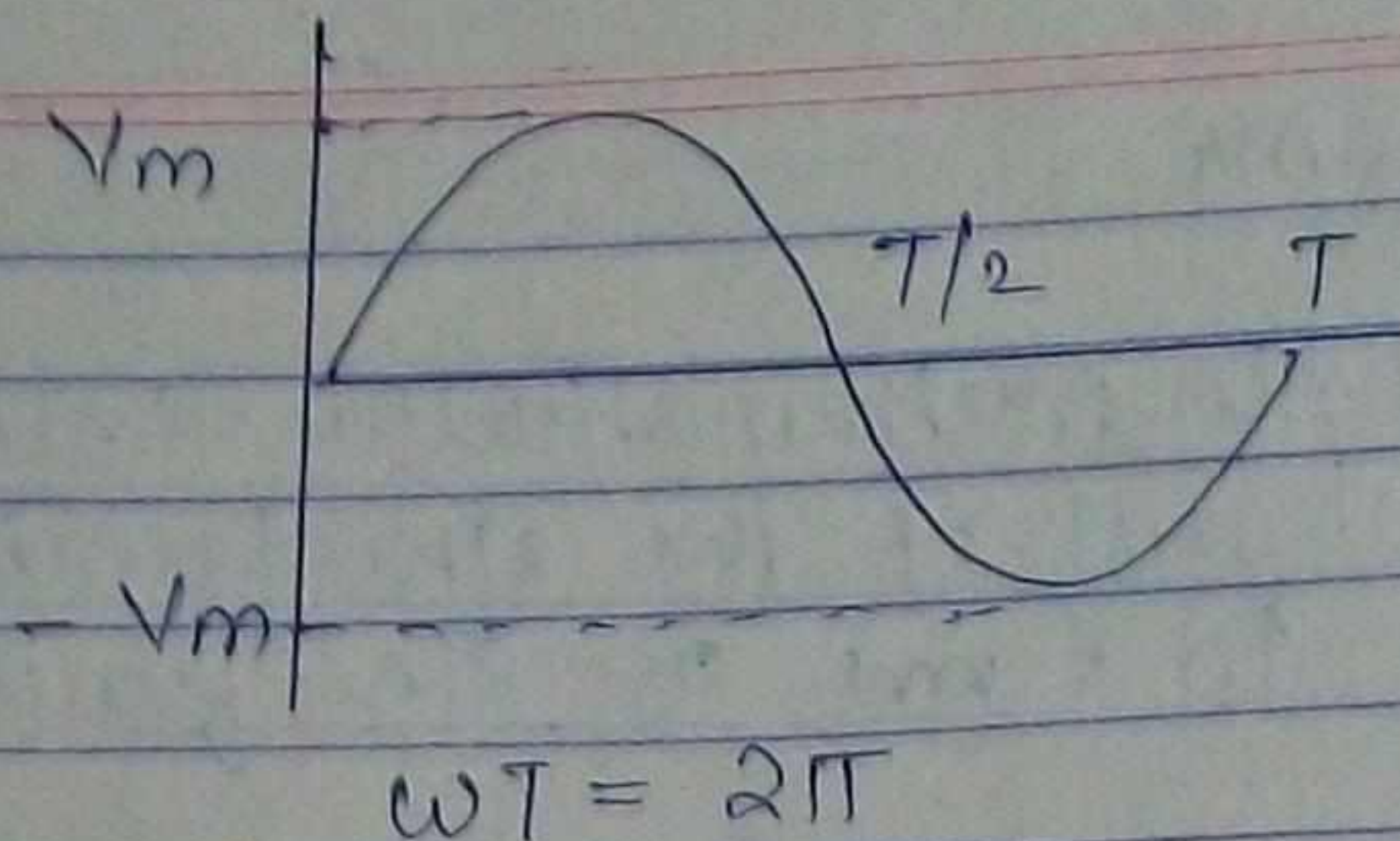


$$V(t) = V_m \sin \omega t$$

$V_m = \text{peak (or) max. value}$

$\omega = \text{angular freq (rad/sec)}$

$\omega t = \text{argument (radian)}$



$$T = \frac{2\pi}{\omega} \text{ sec}$$

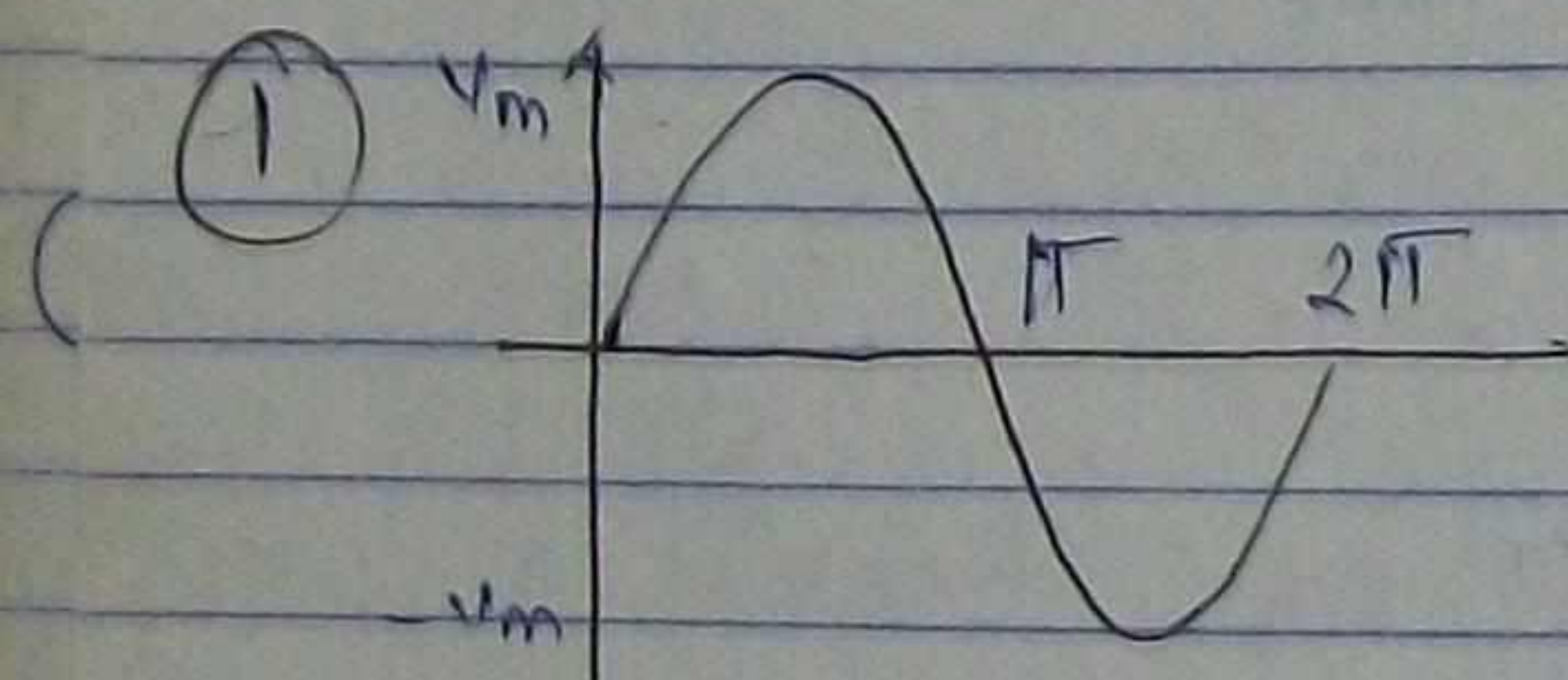
TIME PERIOD

$$f = \frac{1}{T}$$

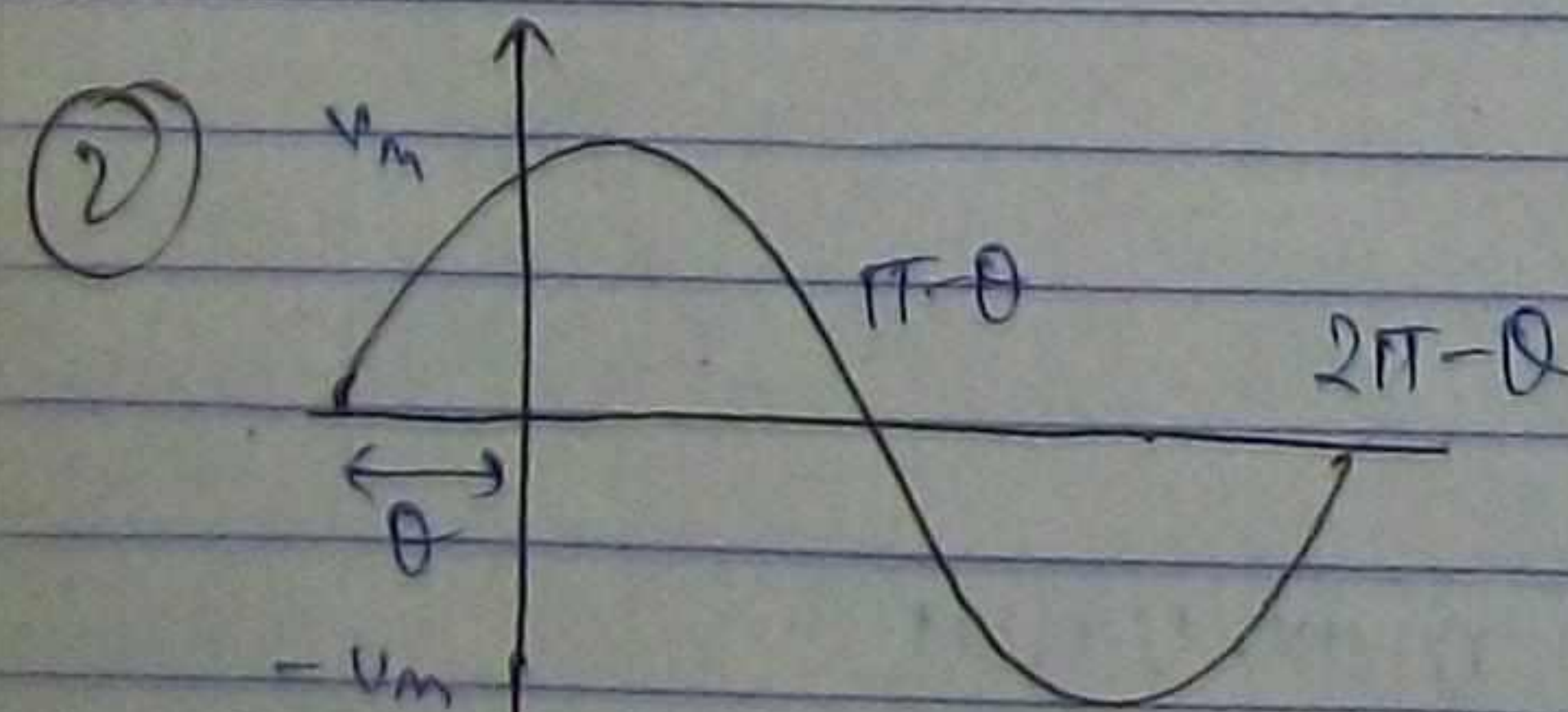
$$f = \frac{\omega}{2\pi} \text{ Hz or cycle/sec}$$

FREQUENCY

⇒ MATHEMATICAL REPRESENTATION-

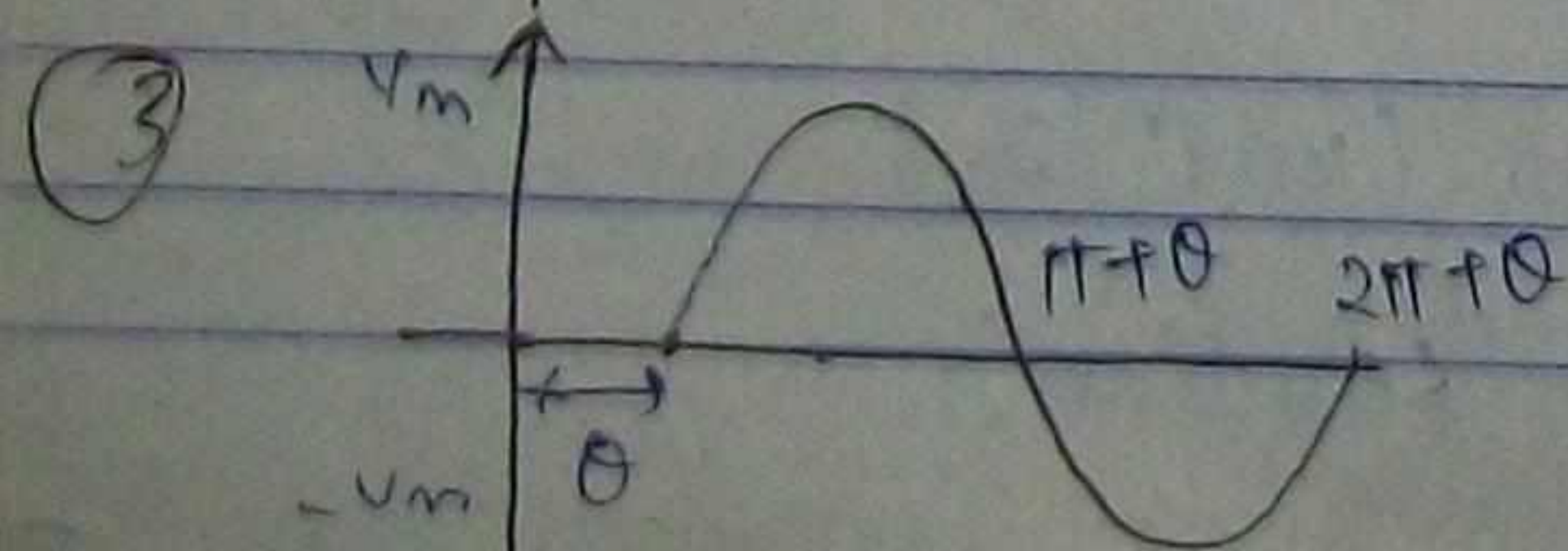


$$V(t) = V_m \sin \omega t$$



$$V(t) = V_m \sin(\omega t + \theta)$$

LEADING



$$V(t) = V_m \sin(\omega t - \theta)$$

LAGGING

- w.r.t to 1st waveform 2nd waveform is leading by an angle θ .

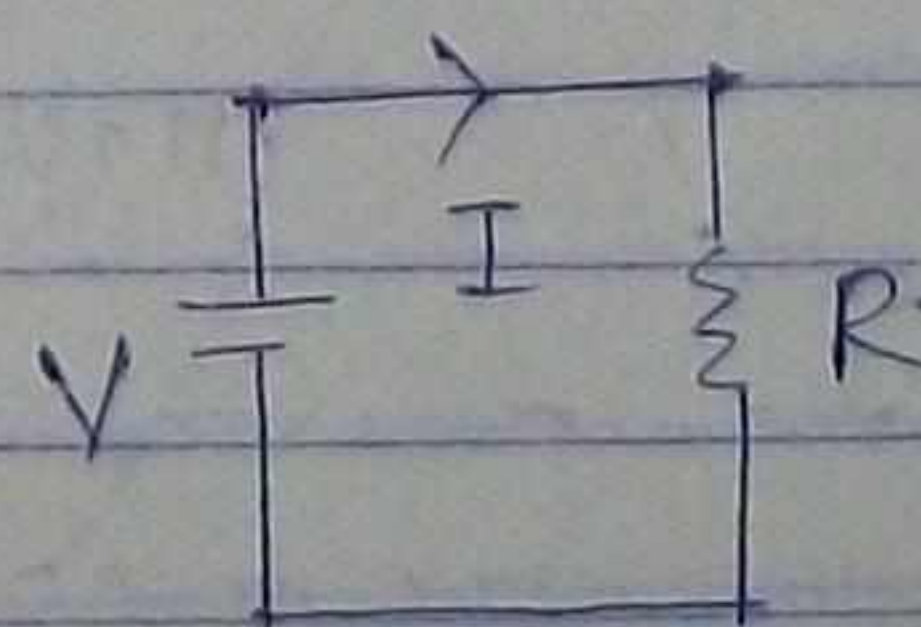
- w.r.t to 1st waveform 3rd waveform is lagging by an angle θ .

- w.r.t to 2nd waveform 3rd waveform is lagging by an angle 2θ .

⇒ RMS VALUE -

- RMS value is defined based on heating effect of the waveform.

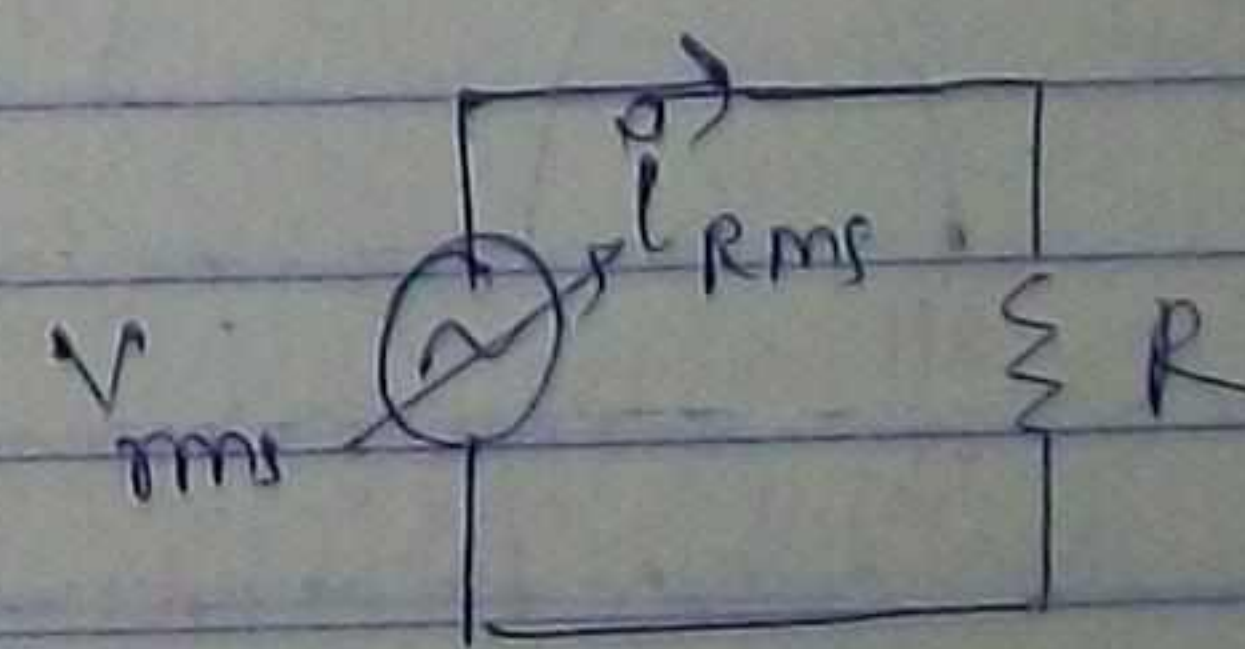
- The voltage at which heat dissipation in AC circuit is equal to heat dissipation in DC circuit is called as V_{rms} , provided both AC and DC circuit have equal value of resistance and operated for same time.



$$P = I^2 R$$

$$W = I^2 R t$$

↓
HEAT DC.



$$P = i^2 R$$

$$W = i^2 R t$$

↓
HEAT AC

$$W_{DC} = W_{AC}$$

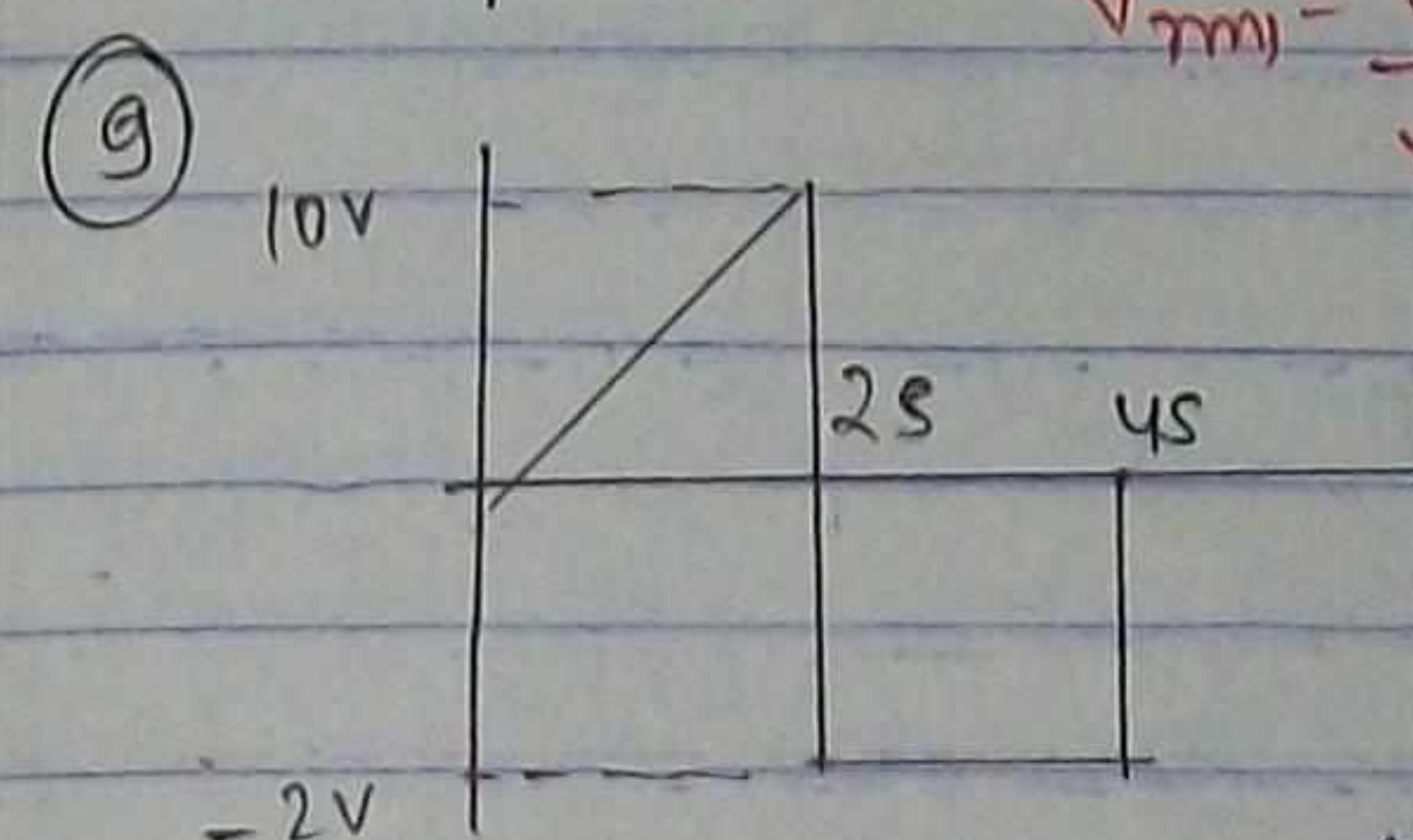
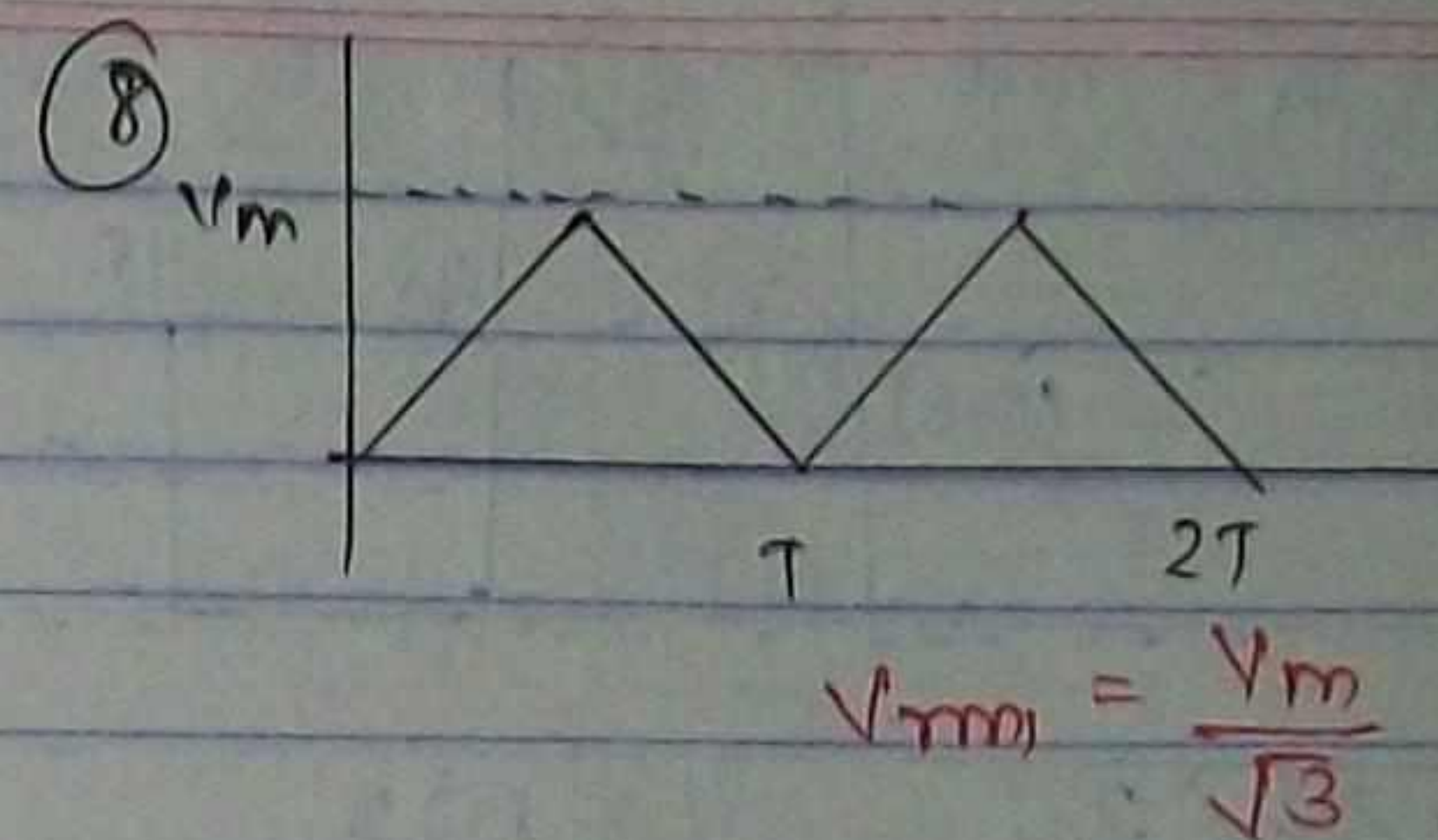
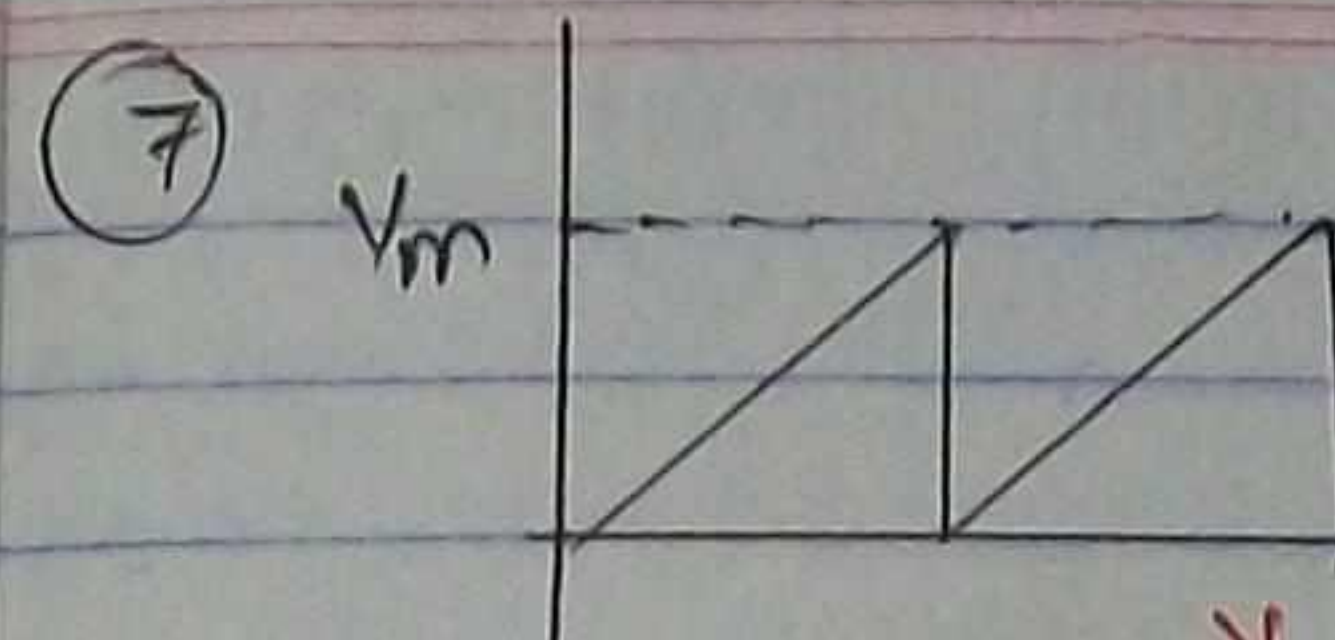
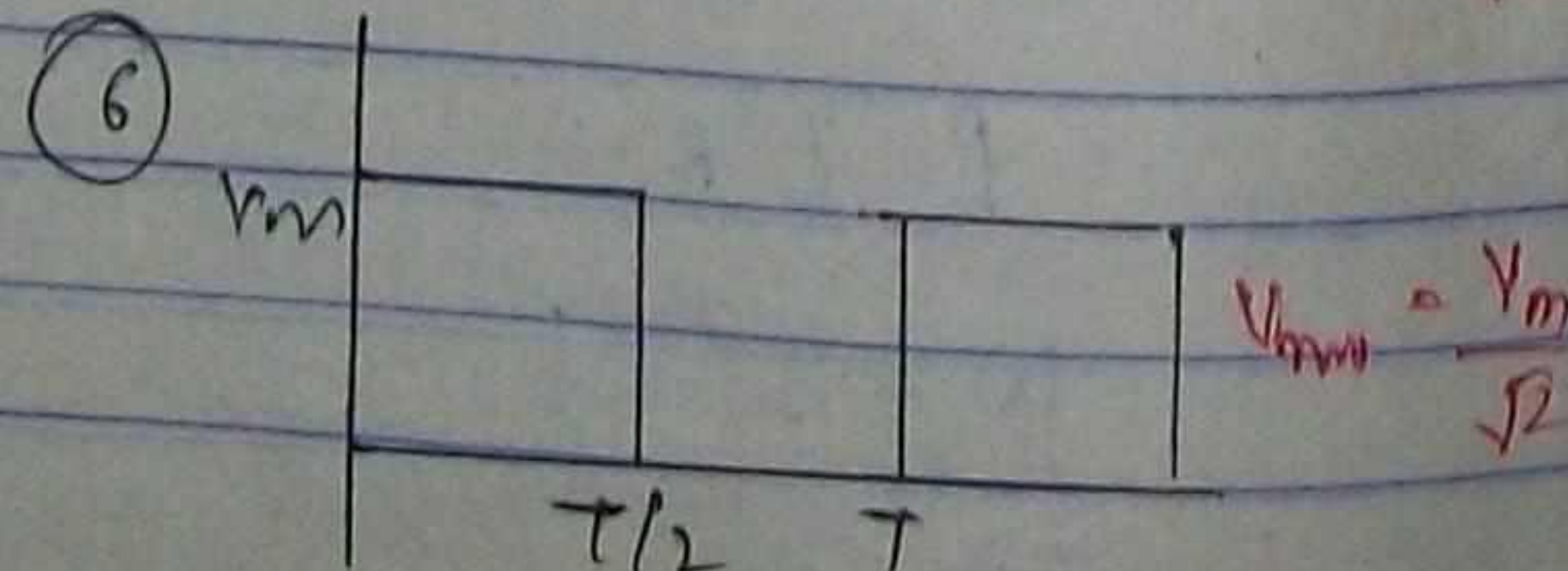
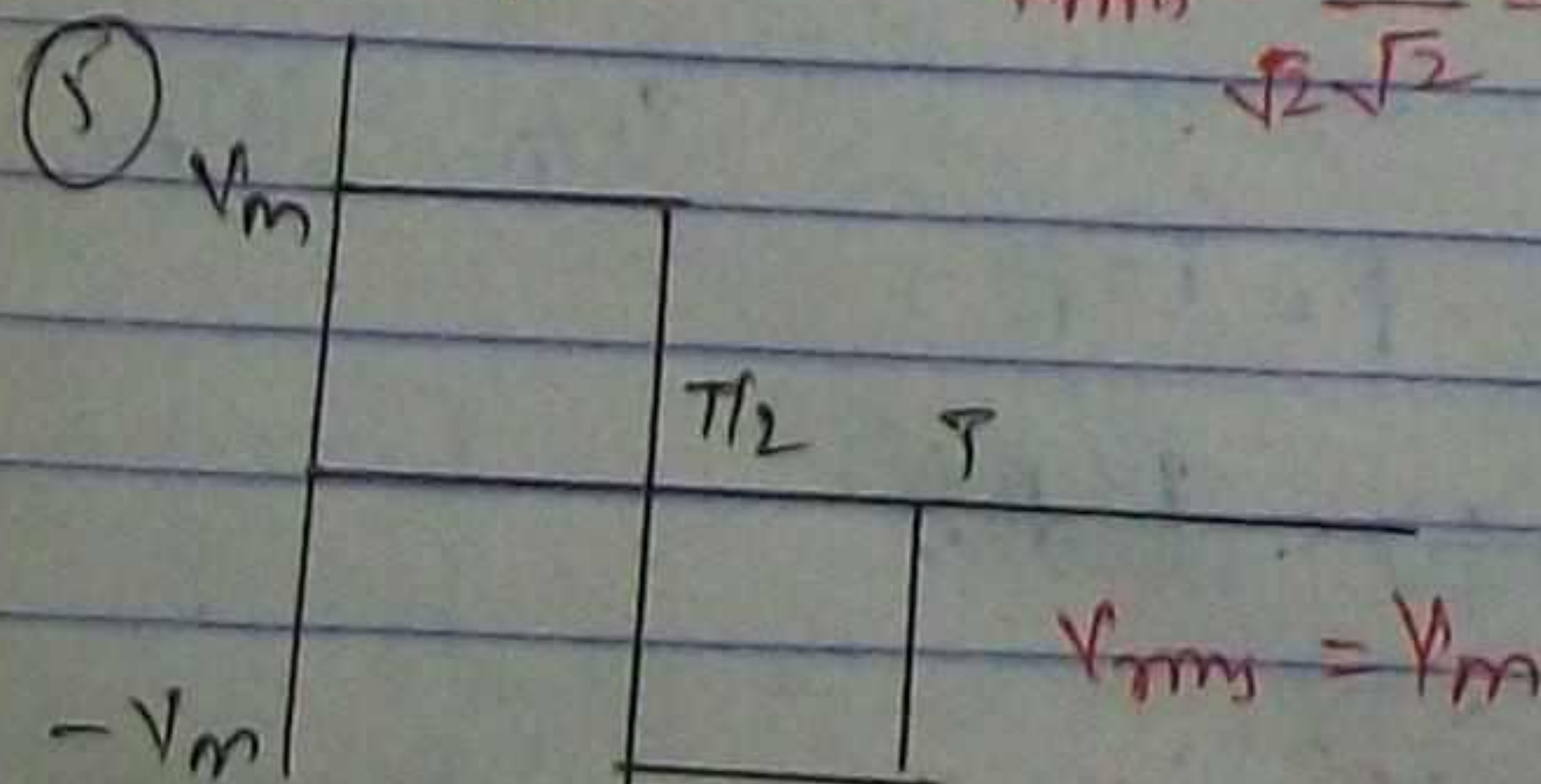
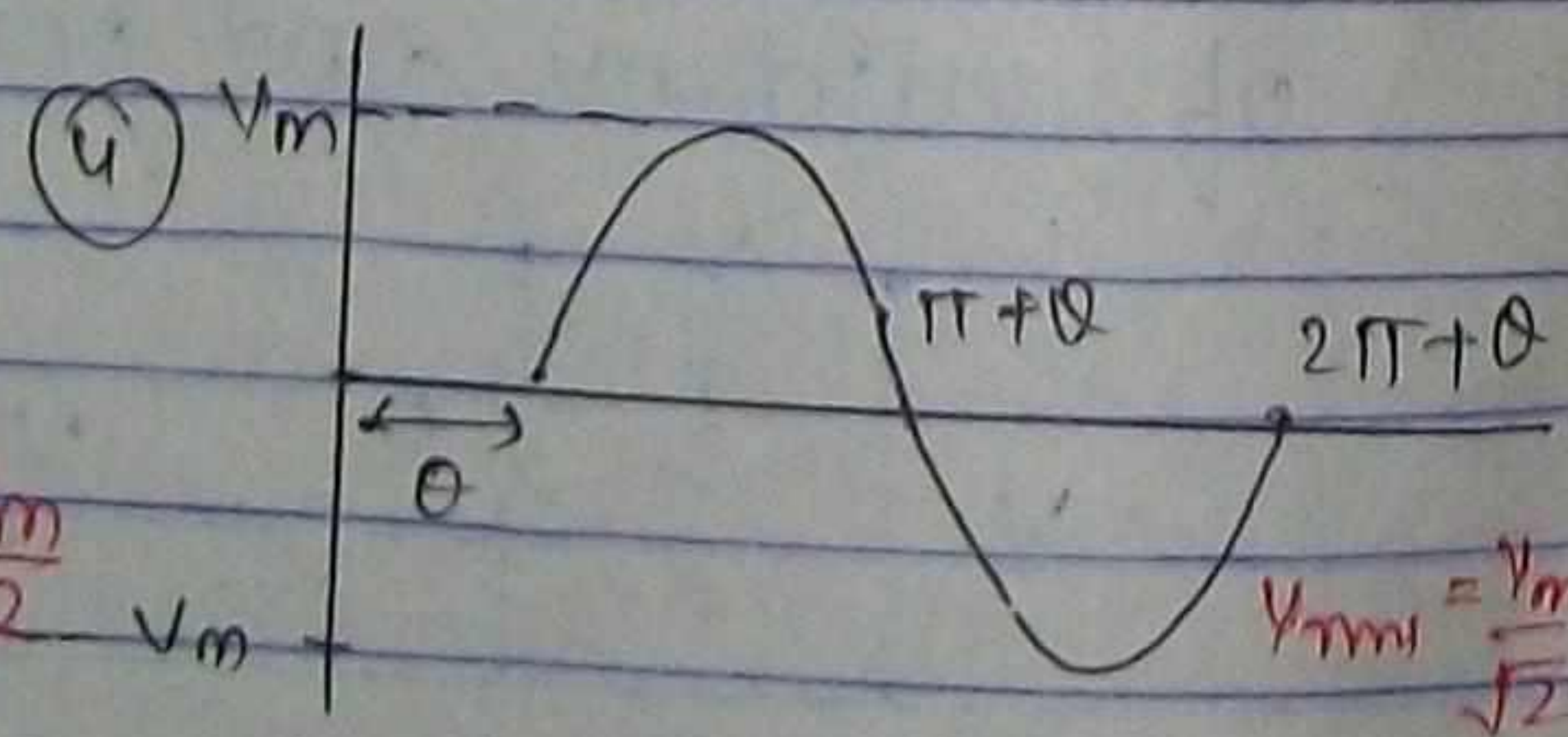
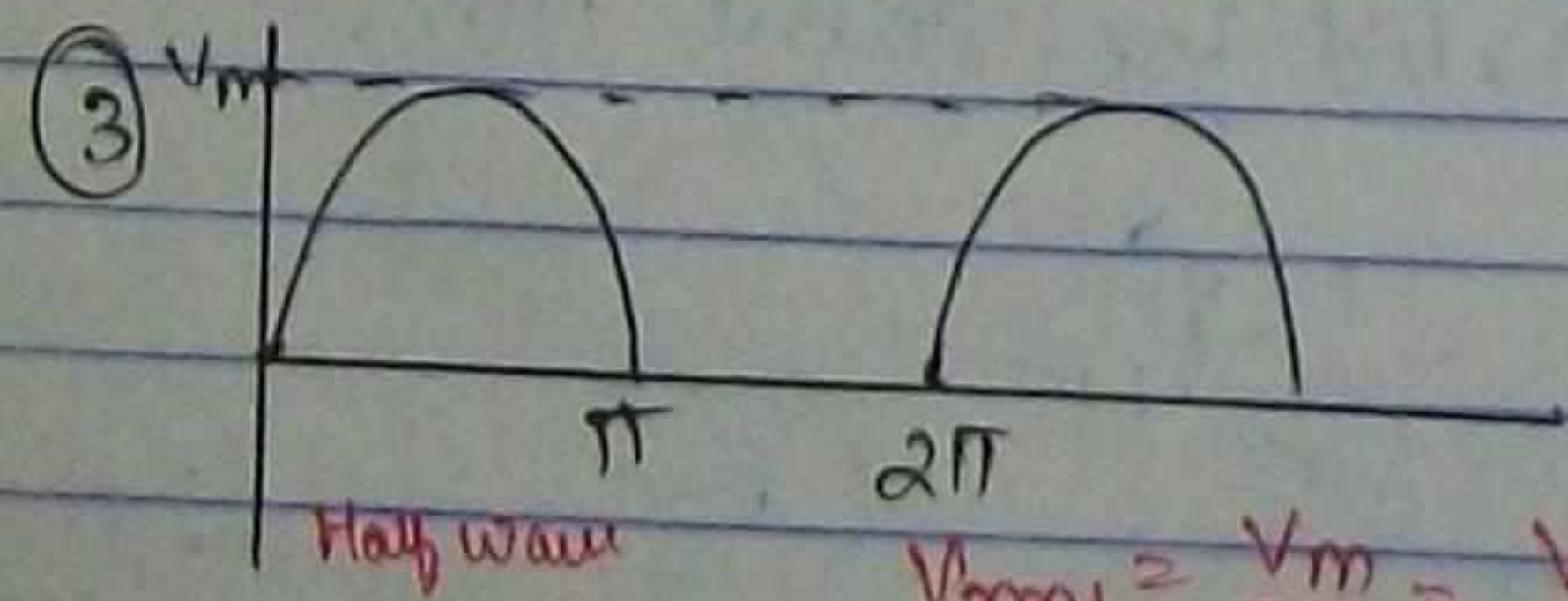
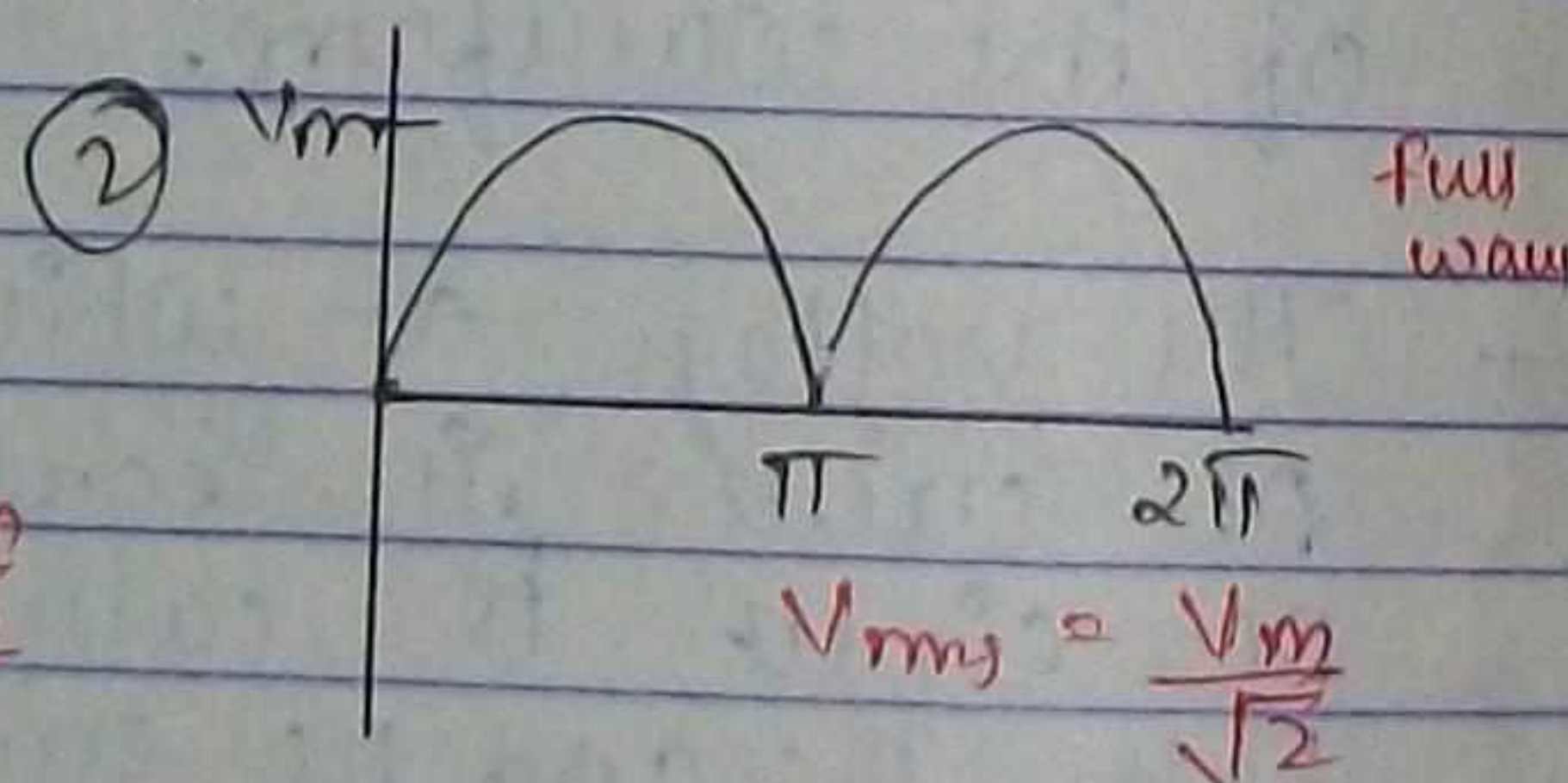
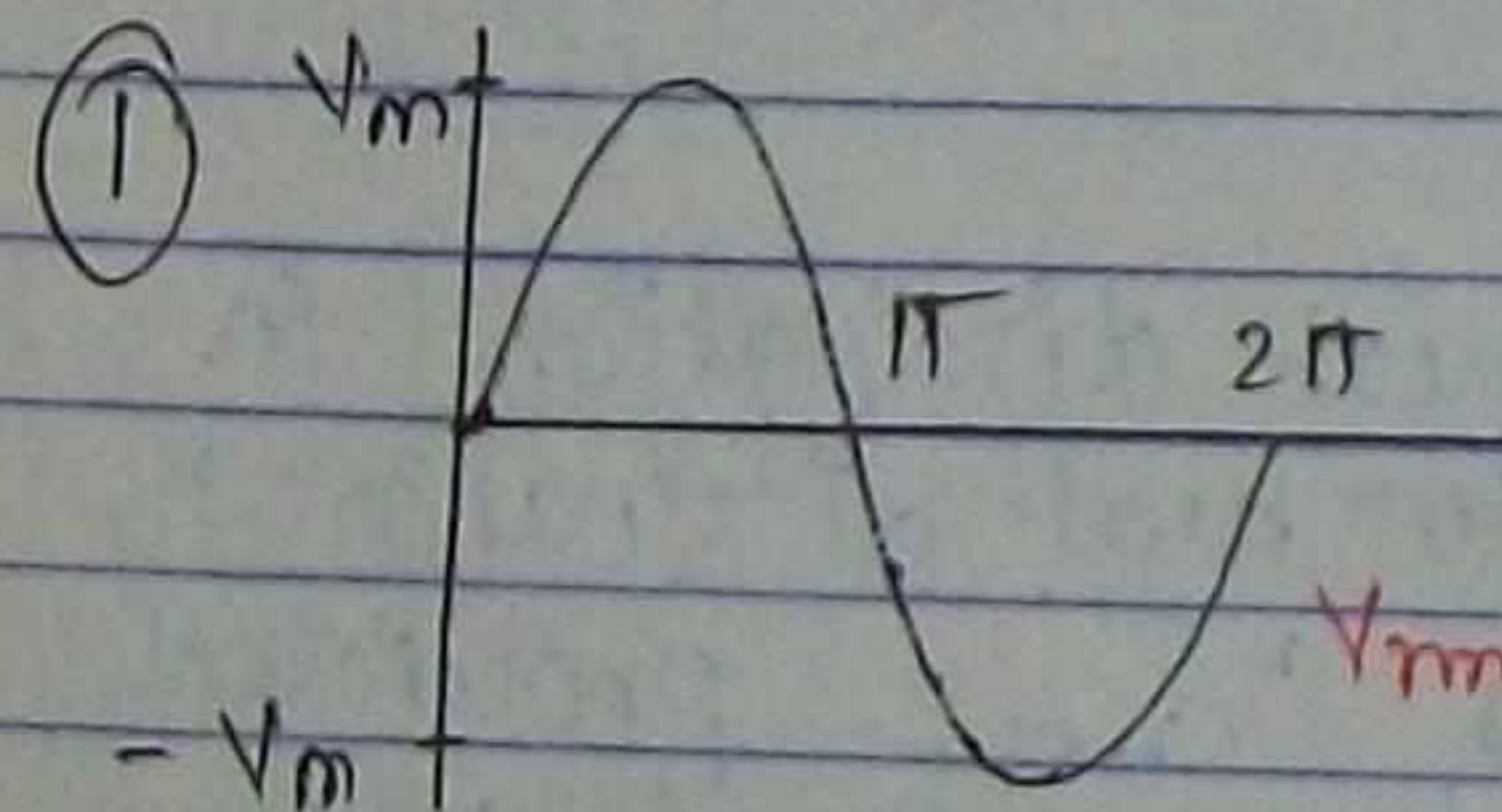
20/10/11

⇒ GENERAL EXPRESSION FOR RMS VALUE -

$$V_{RMS} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} v^2 d\omega t}$$

$$V_{RMS} = \sqrt{\frac{1}{T} \int_0^T v^2 dt}$$

Q. find the rms value of the waveforms:



Soln ① $V_{RMS} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} (V_m \sin \omega t)^2 d\omega t}$

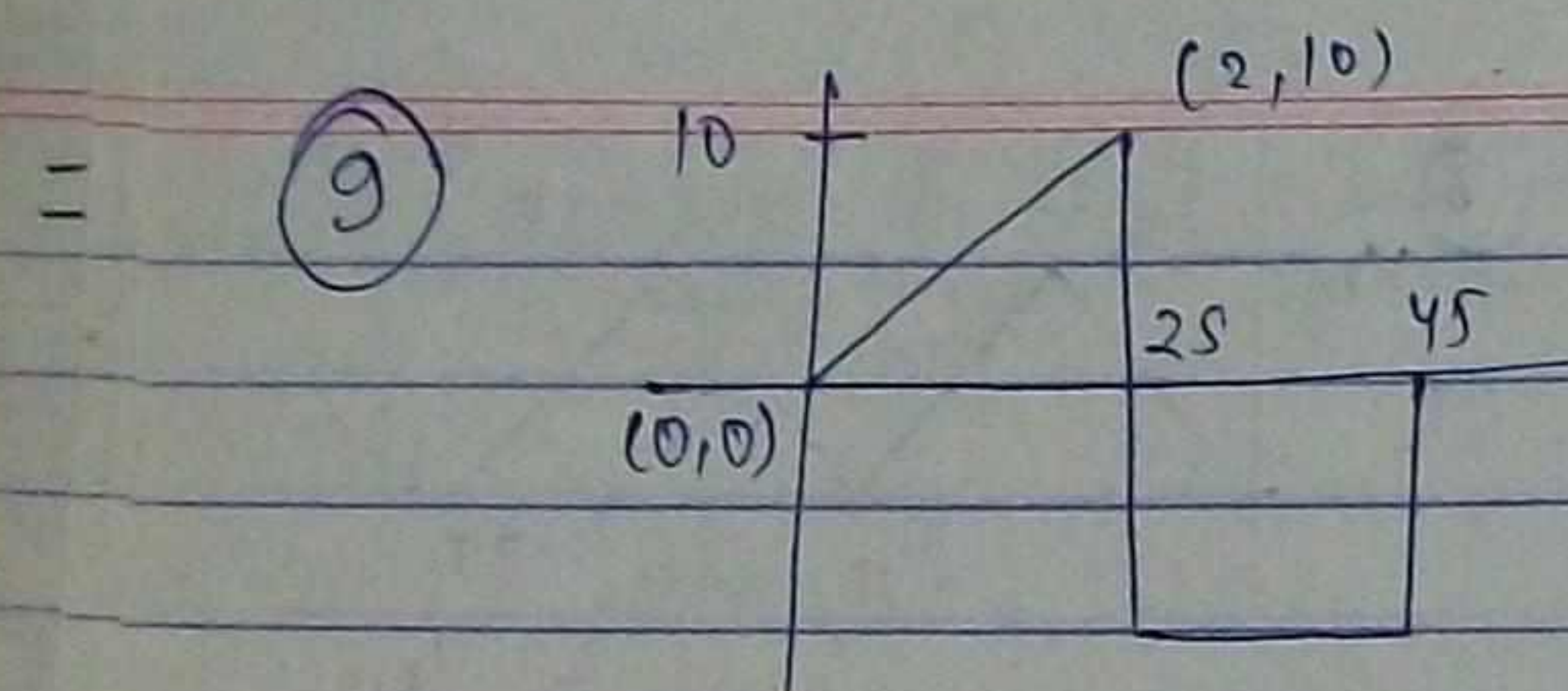
$$V_{RMS} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} \left(\frac{1 - \cos 2\omega t}{2} \right) d\omega t}$$

$$V_{RMS} = \frac{V_m}{\sqrt{2}}$$

⑥ $V_{RMS} = \sqrt{\frac{1}{T} \int_0^{T/2} v_m^2 dt + \int_{T/2}^T 0 dt}$

$$= \sqrt{\frac{V_m^2}{T} \cdot \frac{T}{2}}$$

$$= \frac{V_m}{\sqrt{2}}$$



$$0 < t < 2 \quad y = mx$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10 - 0}{2 - 0} = 5$$

$$V = 5t$$

$$V = \sqrt{\frac{1}{4} \left[\int_0^2 (5t)^2 dt + \int_2^4 (-2)^2 dt \right]}$$

$$V = \sqrt{\frac{1}{4} \left[25 \left(\frac{t^3}{3} \right)_0^2 + 4 \left(t \right)_2^4 \right]}$$

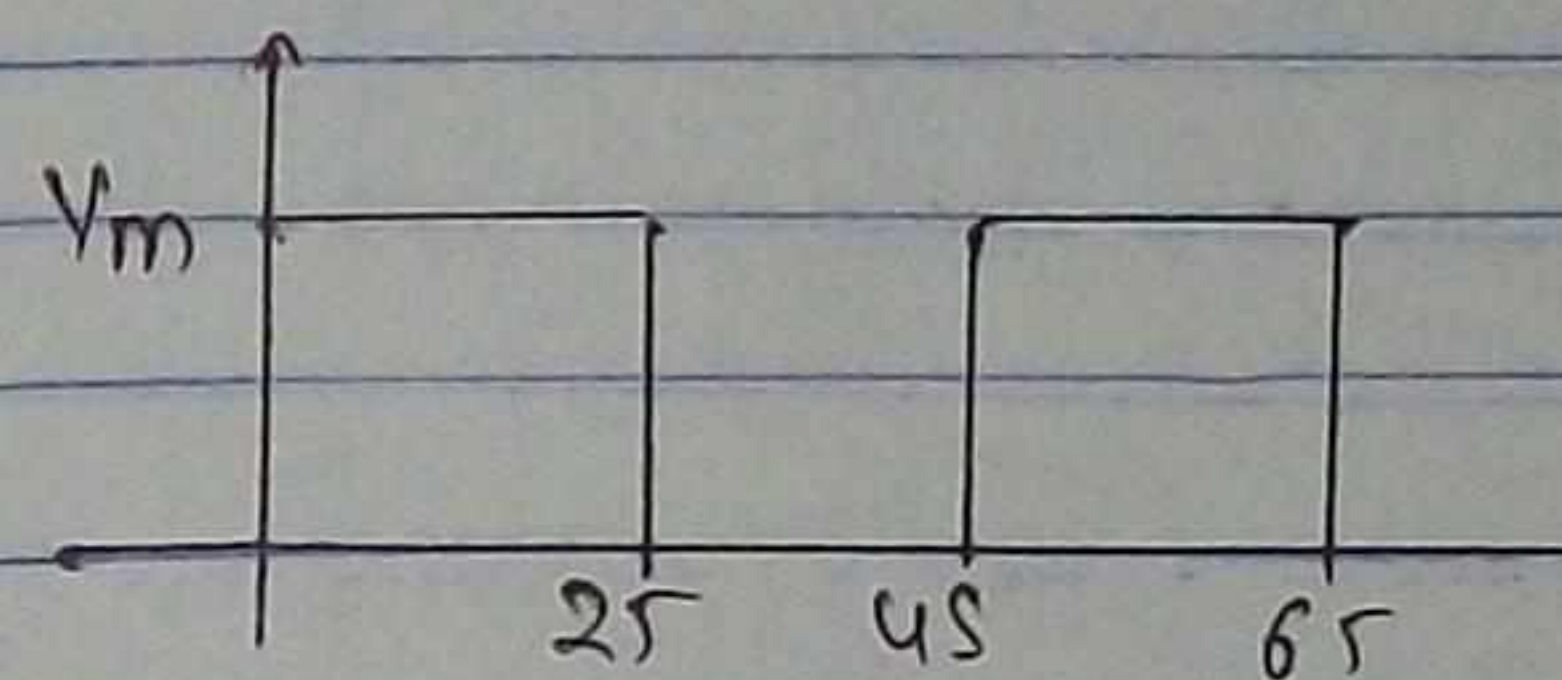
$$V = \sqrt{\frac{1}{4} \left[\frac{25 \times 8}{3} + 8 \right]}$$

$$V = \sqrt{2 \left(\frac{25}{3} + 1 \right)}$$

$$V = \sqrt{\frac{56}{3}} = \sqrt{18.6} = 4.3$$

active power $\rightarrow P_{avg}$

Ques. find power dissipation in resistor for given waveform.



Soln:

$$P_{avg} = \frac{V_{rms}^2}{R} = \frac{(V_m/\sqrt{2})^2}{R} = \frac{V_m^2}{2R}$$

$$P_{avg} = \frac{P_{peak}}{2}$$

$$(1) P_{avg} = \frac{P_{peak}}{\sqrt{2}}$$

$$(2) P_{av} = \frac{P_{peak}}{2}$$

$$(3) P_{av} = \frac{P_{peak}}{\sqrt{3}}$$

#

$$P_{rms} = P_{peak} = P_{avg} = \frac{V_m^2}{R}$$

#

$$\frac{P_{d.c.}}{P_{a.c.}} = \frac{I_{avg}^2 R}{I_{rms}^2 R}$$

Ques. find RMS value of:

$$V(t) = 3 + \sin t + \sin 3t$$

Soln:

NOTE

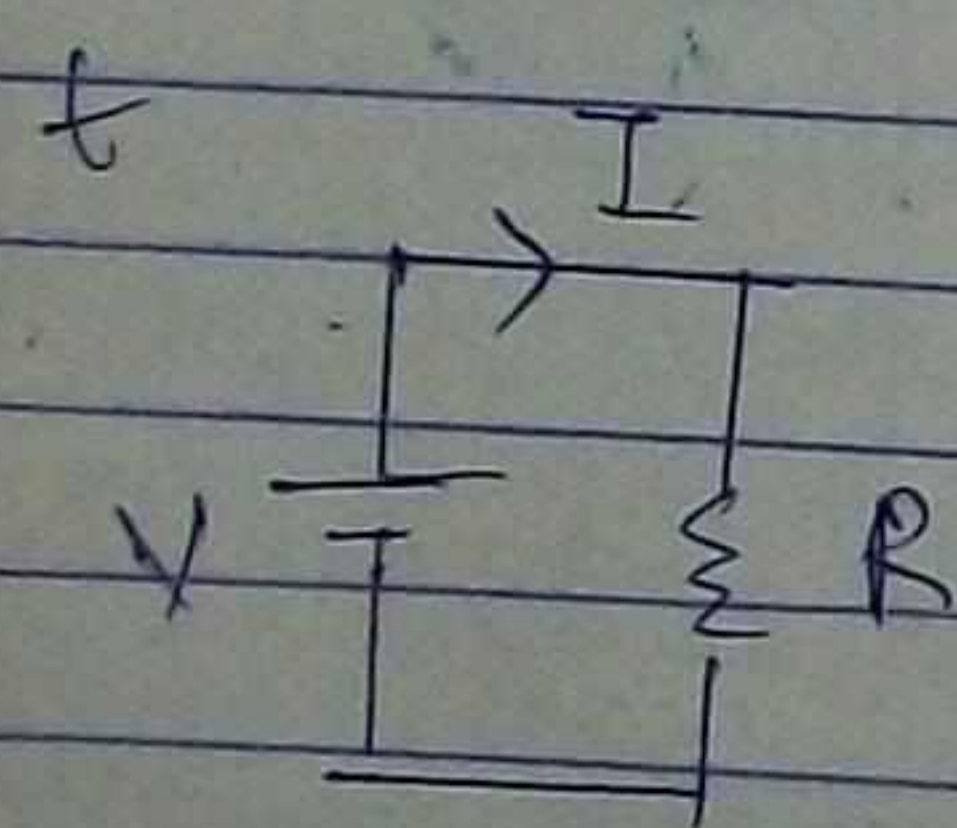
$$V_{rms} = \sqrt{V_{rms1}^2 + V_{rms2}^2 + V_{rms3}^2 + \dots}$$

$$V_{rms} = \sqrt{3^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2}$$

$$= \sqrt{10}$$

⇒ AVERAGE VALUE -

- Average value is defined based on charge transfer in the circuit.
- The voltage at which charge transfer in a.c. circuit is equal to charge transfer in d.c. circuit is called V_{avg} . provided both a.c. and d.c. circuit have equal value of resistance and operated for same time.



$$I = \frac{V}{R}$$

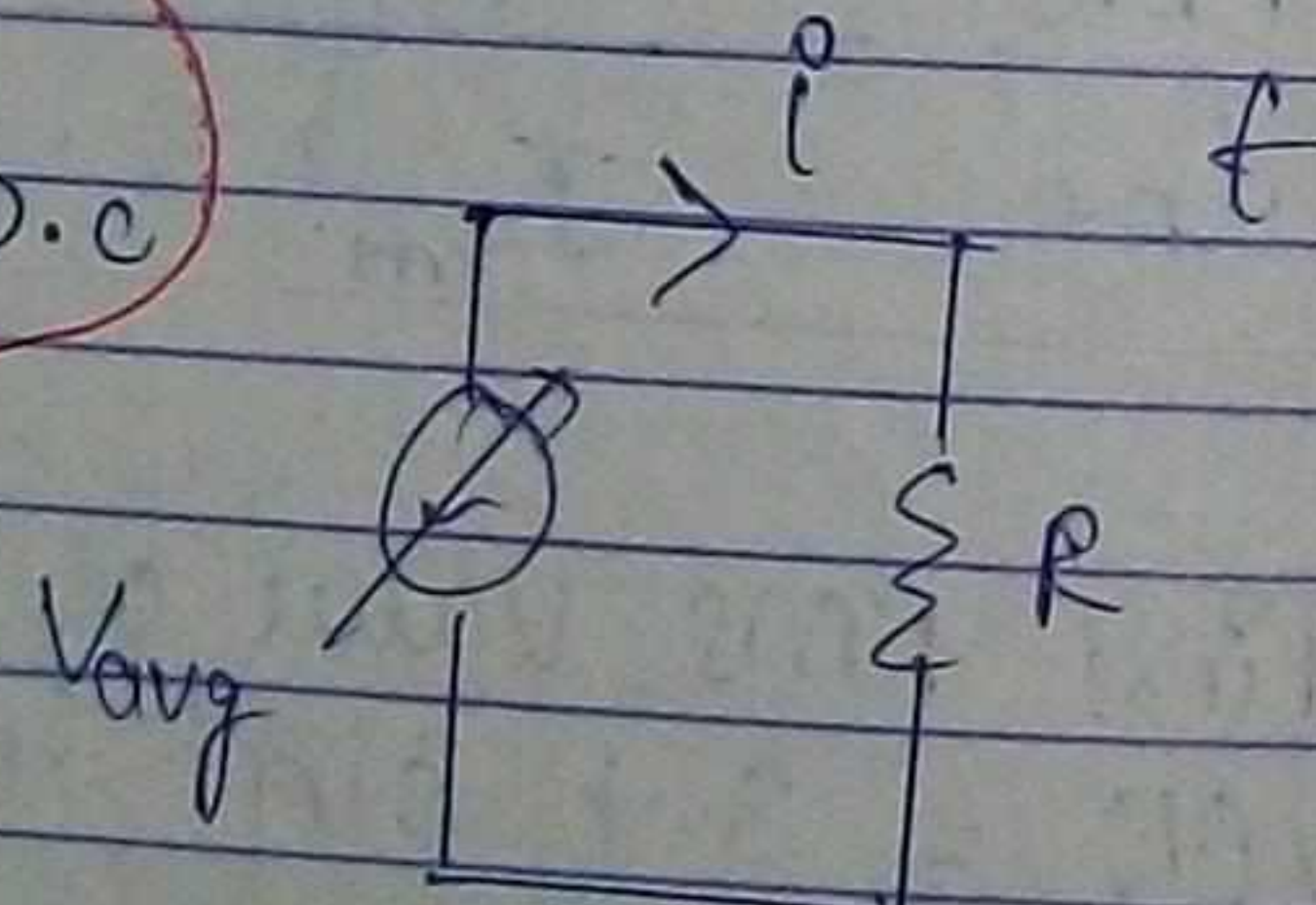
$$I = Q$$

$$t$$

$$Q = It$$

↓
D.C.

$$Q_{A.C.} = Q_{D.C.}$$



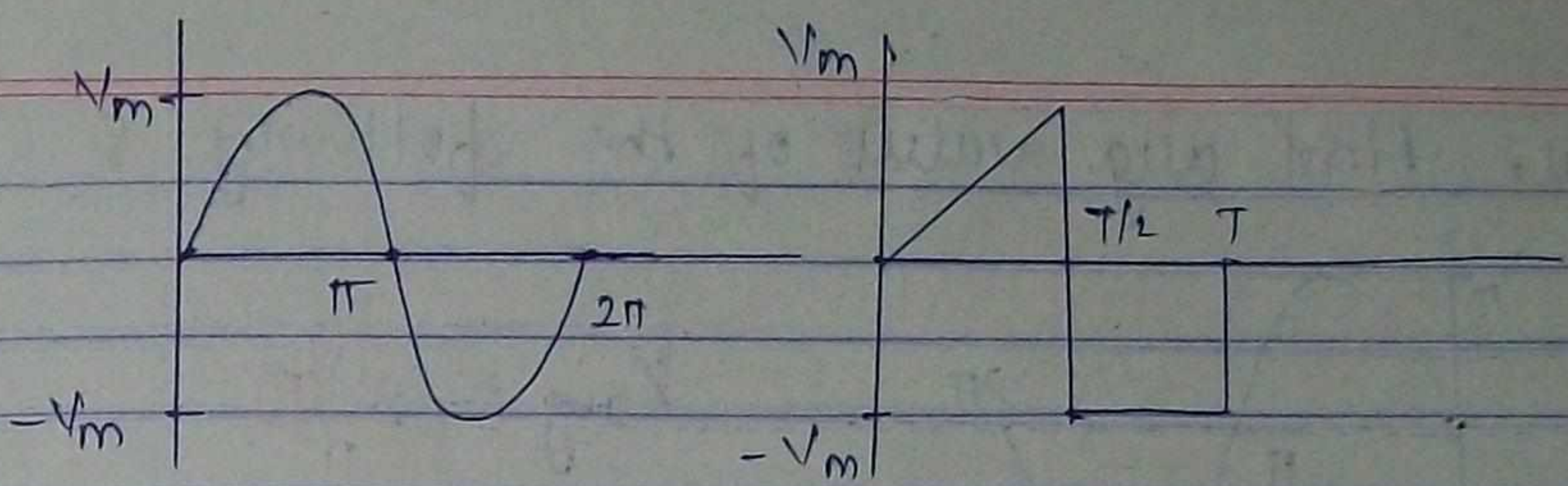
$$i = V/R$$

$$i = Q/t$$

$$Q = it$$

↓
A.C.

$$\text{form factor} = \frac{V_{rms}}{V_{avg}}$$



SYMMETRICAL

UNSYMMETRICAL

- Average value of complete cycle of symmetrical wave equal to zero.
- for analysis while finding average value of symmetrical wave (either voltage or current waveform) only half cycle is considered
- while finding average value of unsymmetrical wave time taken to complete full cycle is considered.

$$V_{avg} = \frac{1}{\pi} \int_0^{\pi} V \, d\omega t$$

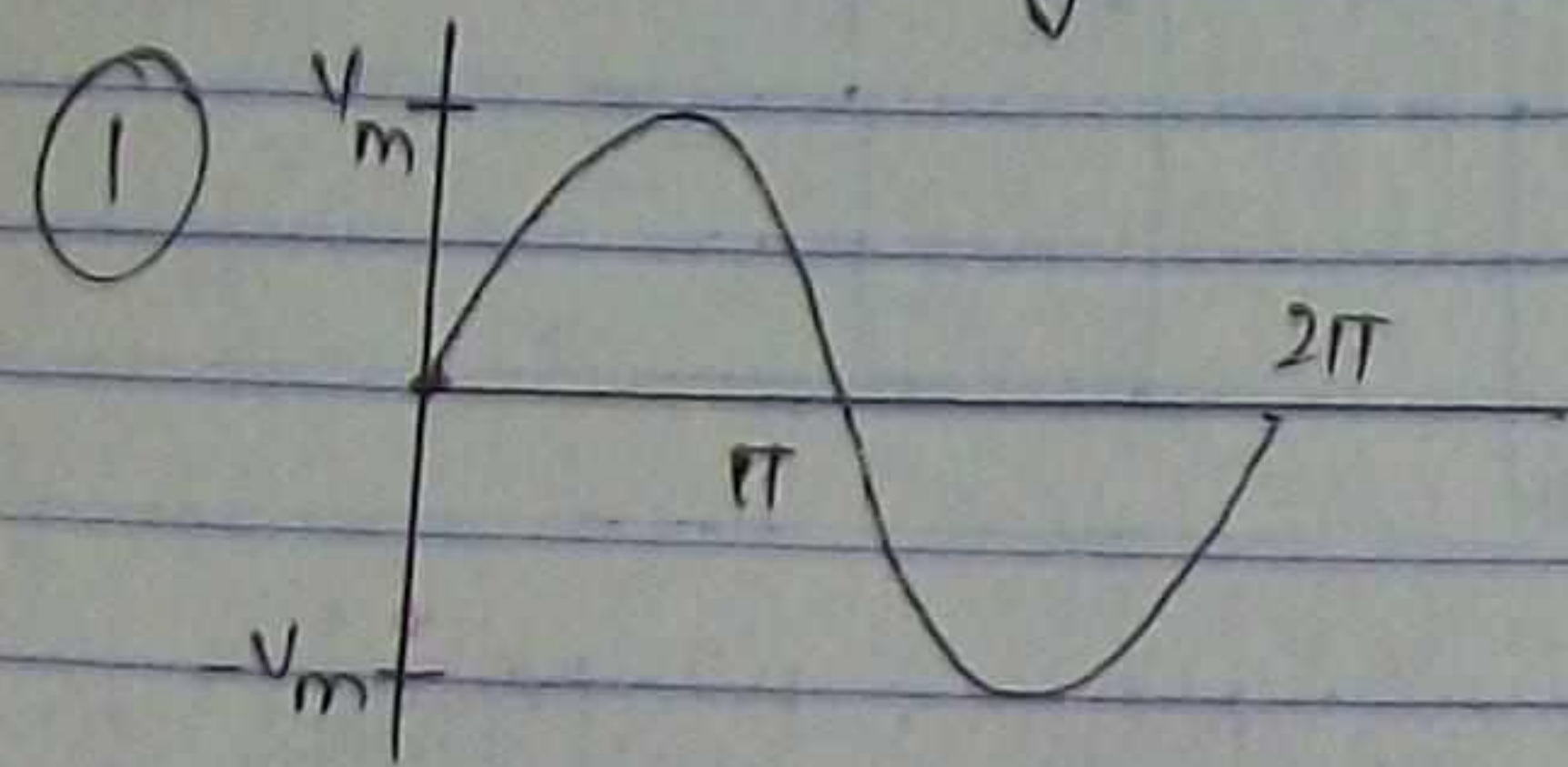
FOR
SYMMETRICAL

$$V_{avg} = \frac{1}{T} \left[\int_0^{T/2} v \, dt + \int_{T/2}^T v \, dt \right]$$

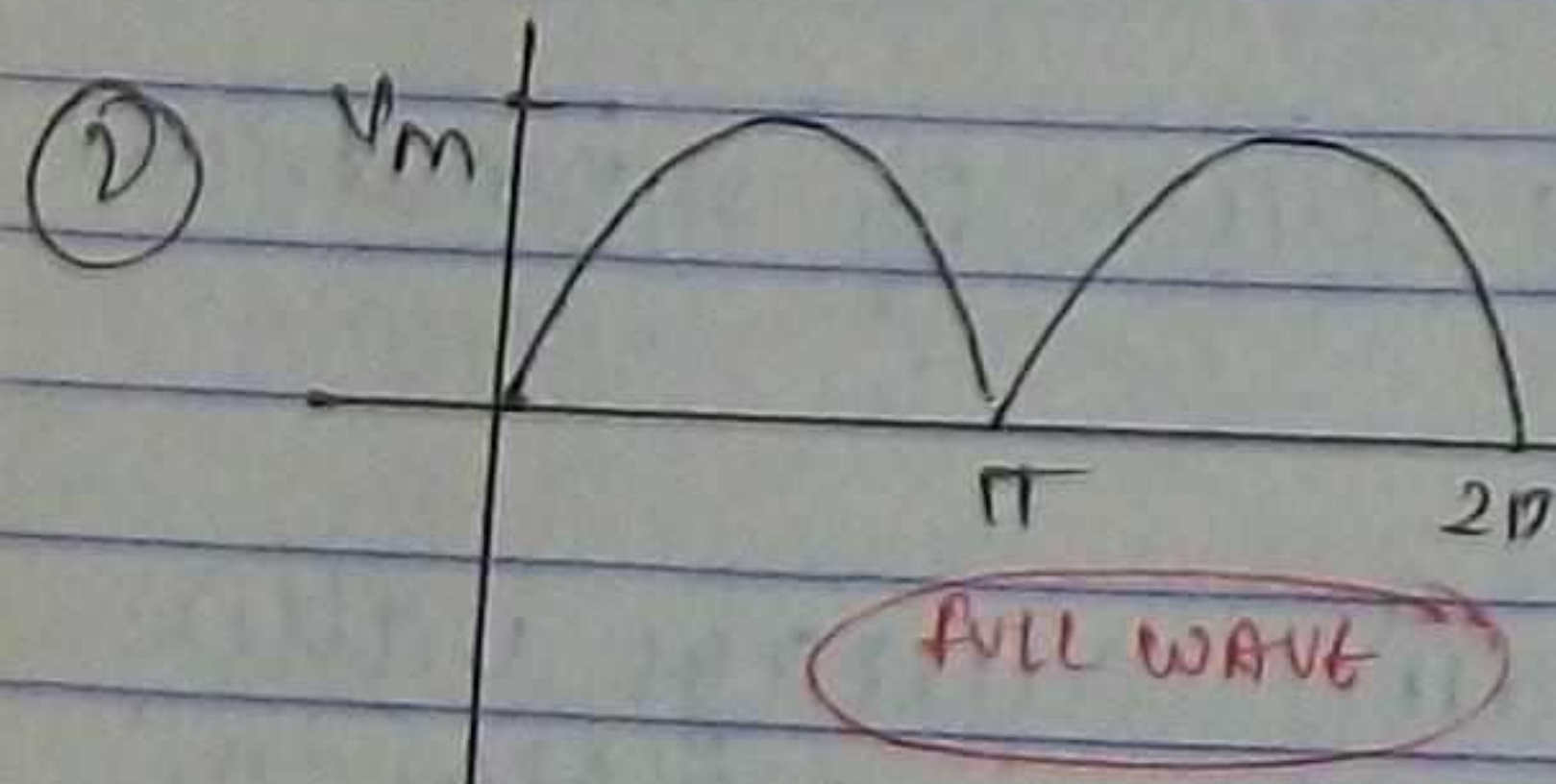
FOR
UNSYMMETRICAL

★★

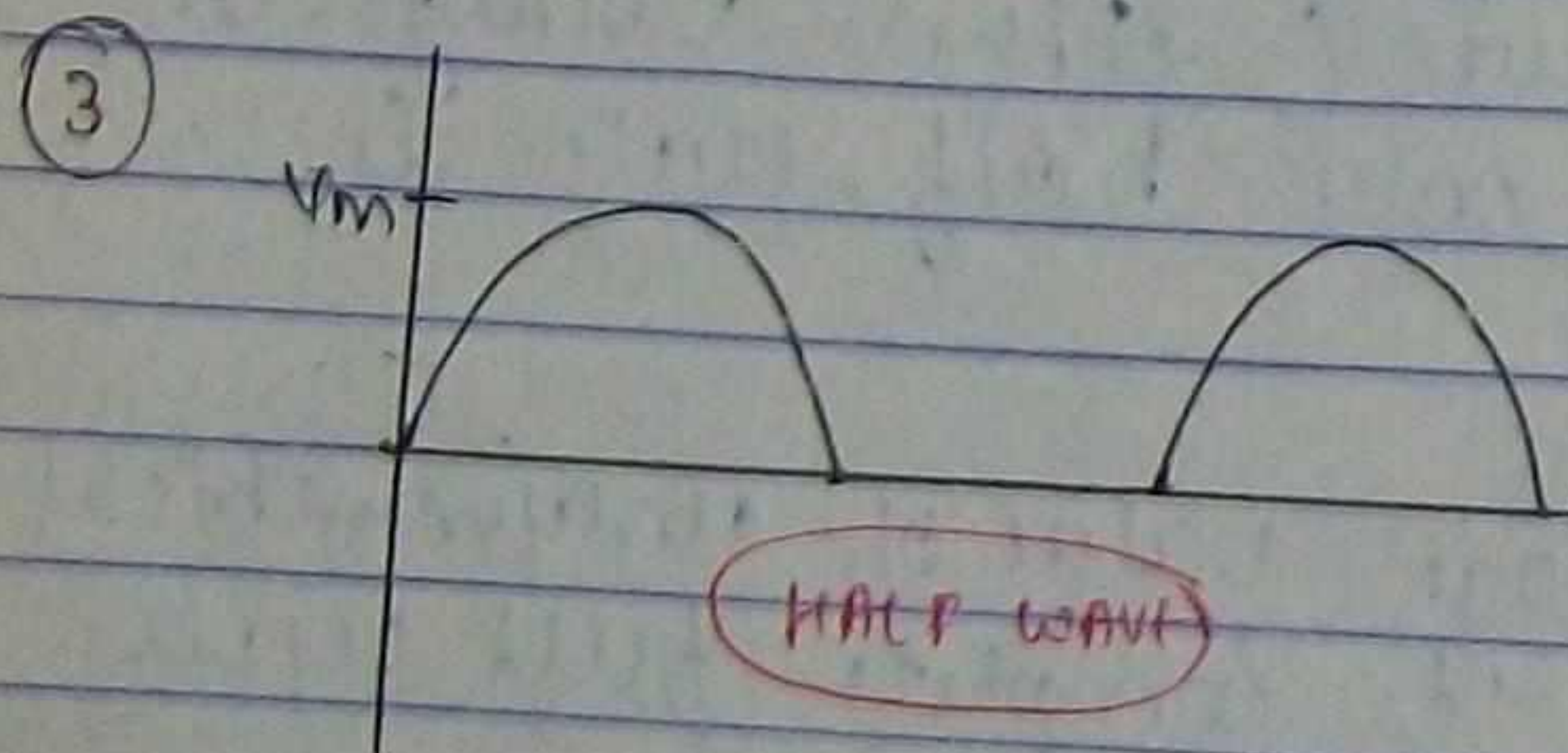
Ques. find avg. value of the following. :



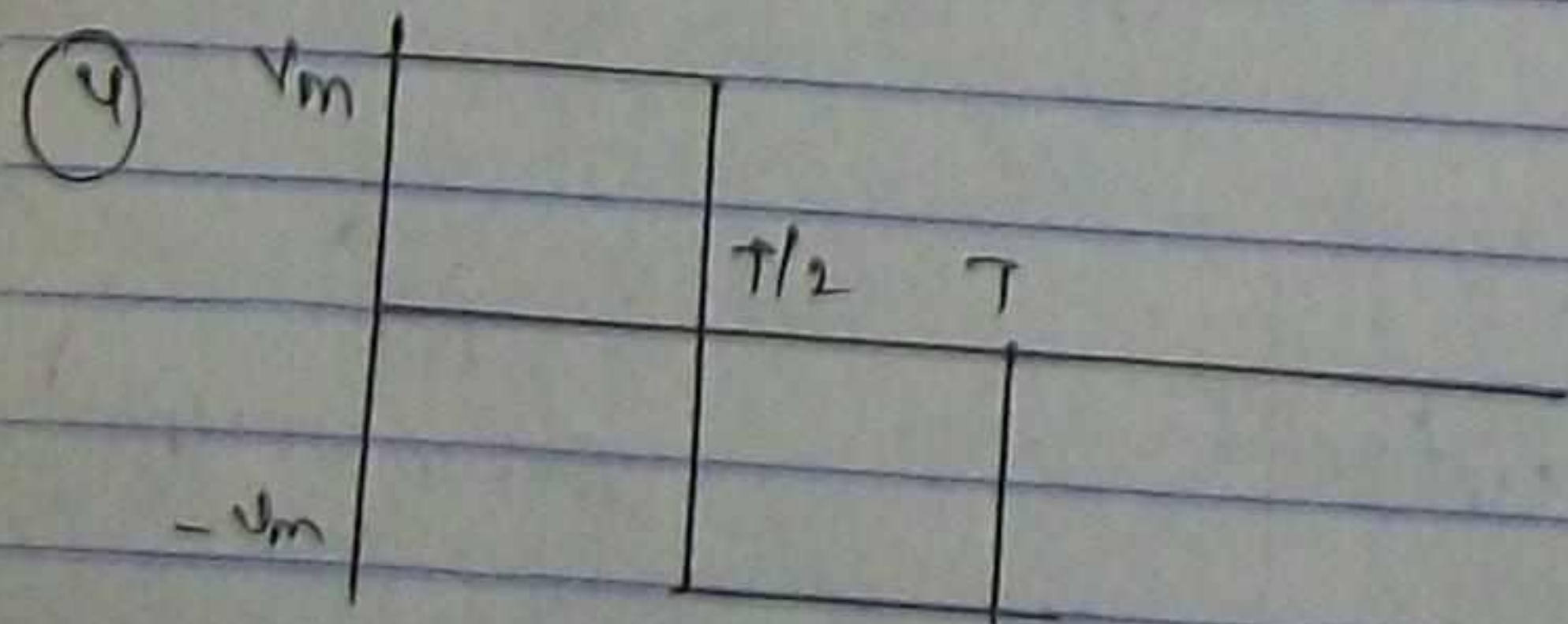
$$V_{avg} = \frac{2V_m}{\pi}$$



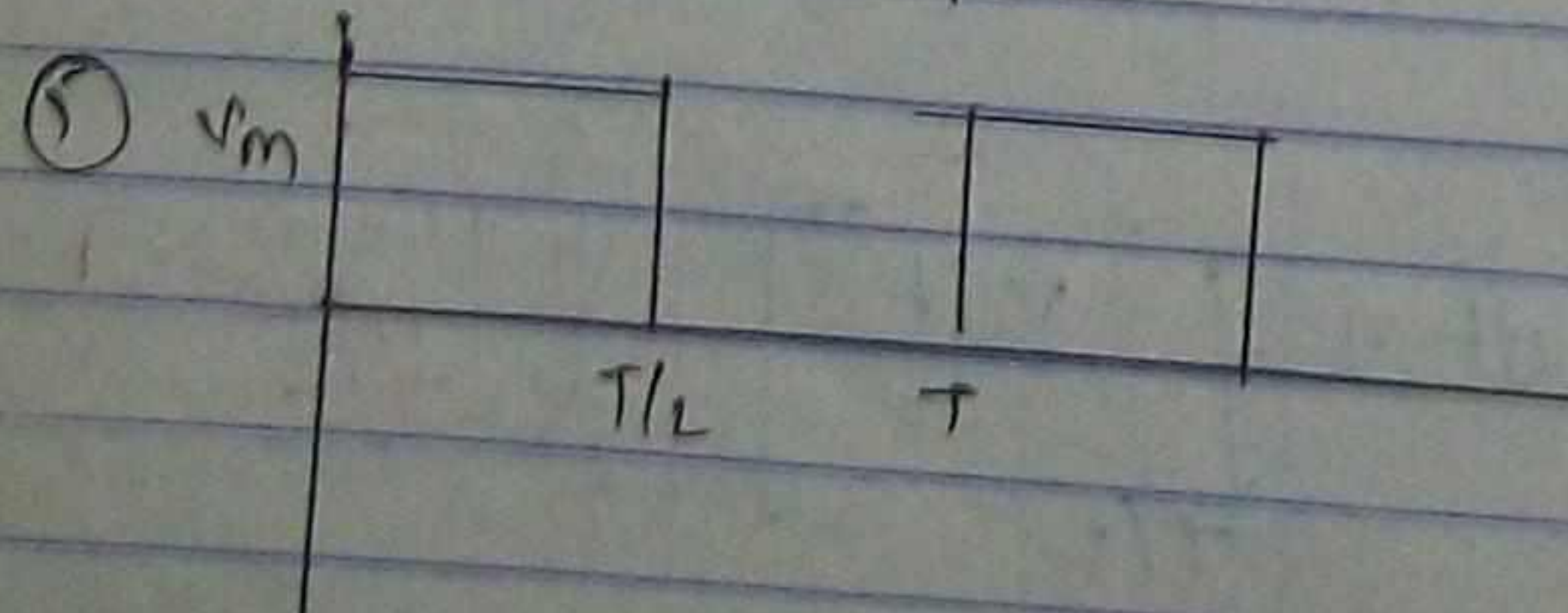
$$V_{avg} = \frac{2V_m}{\pi}$$



$$V_{avg} = \frac{V_m}{\pi}$$

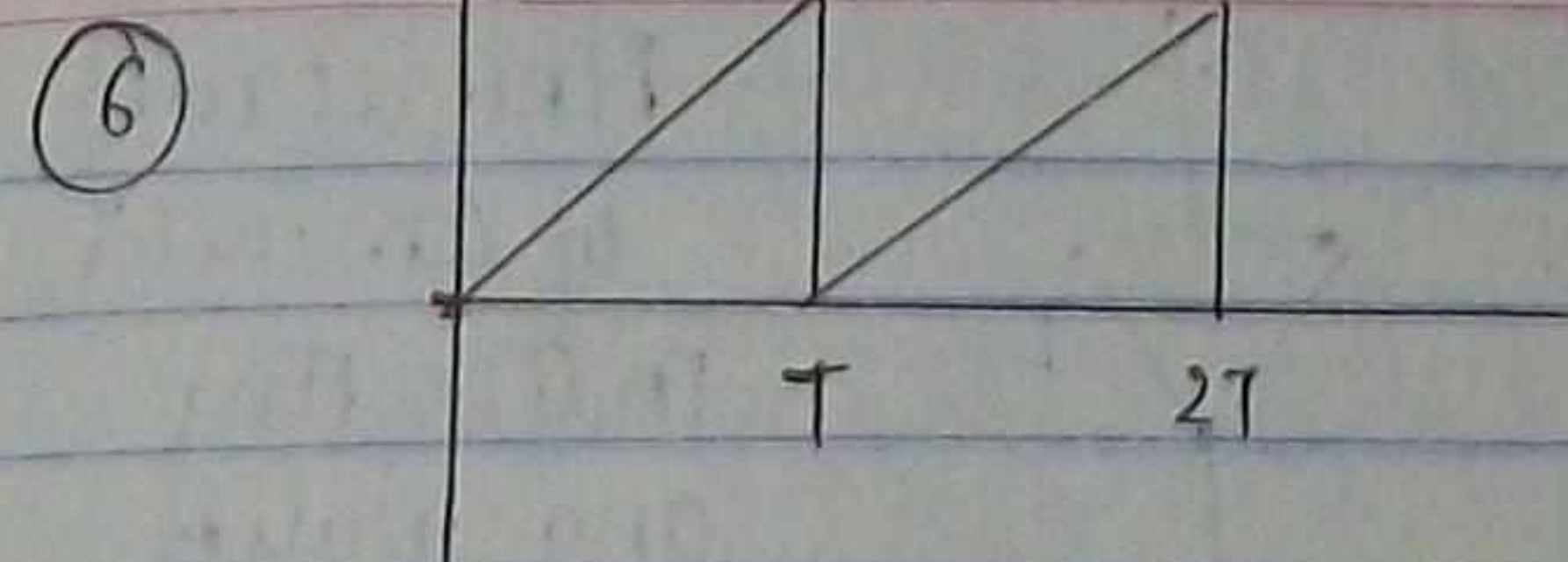


$$V_{rms} = V_{avg} = V_m$$

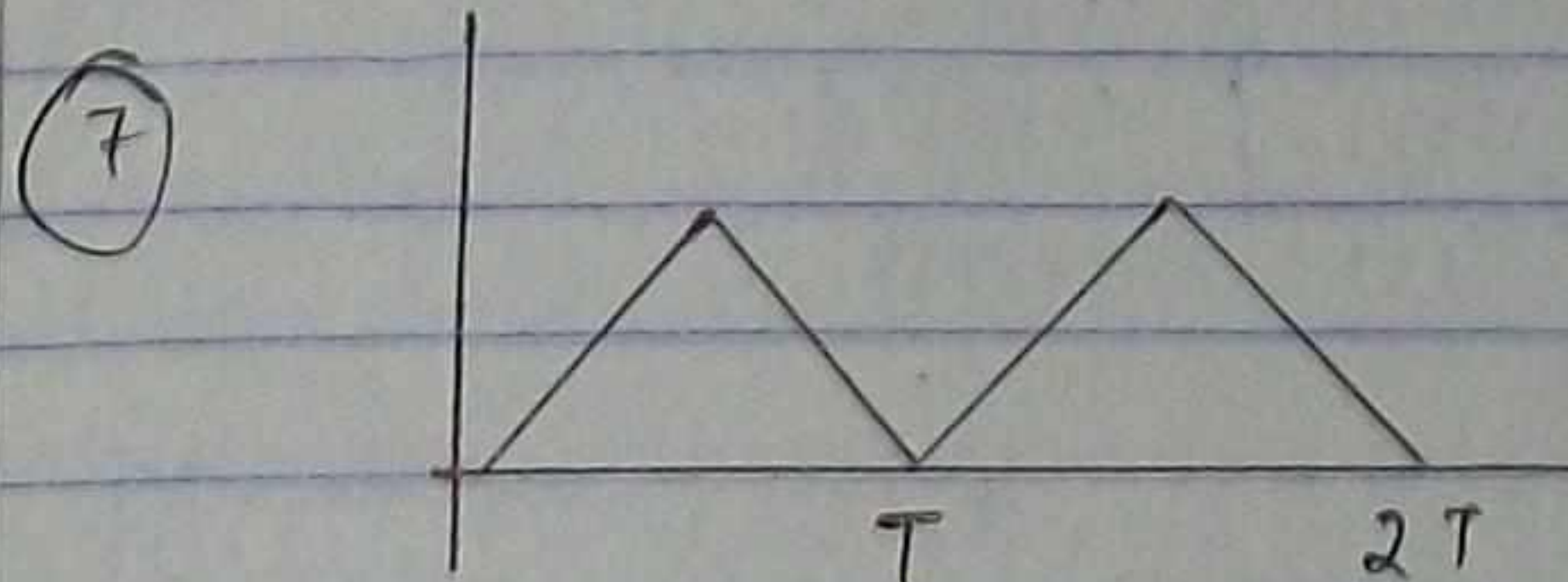


$$V_{avg} = \frac{V_m}{2}$$

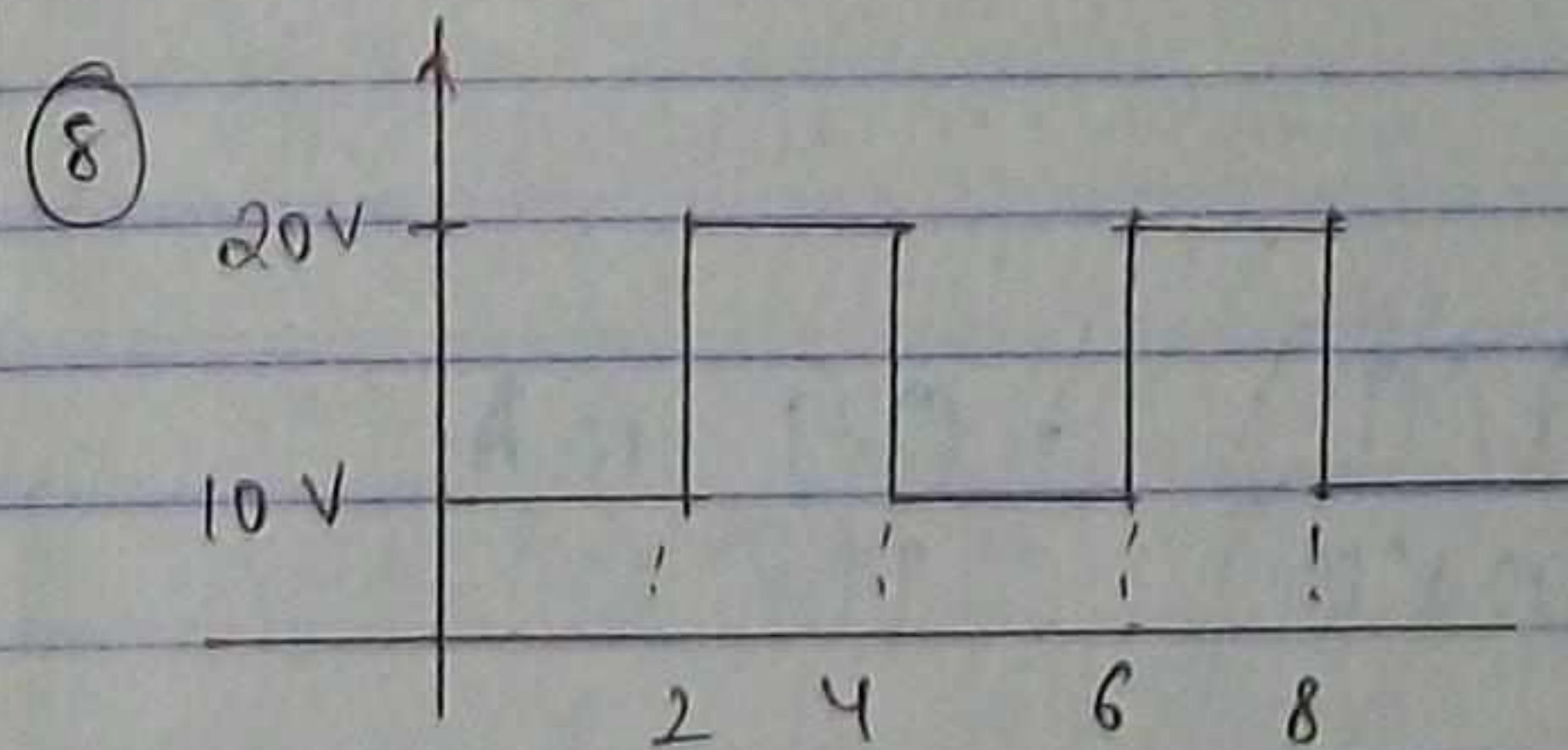
$$\frac{1}{T/2} \int_0^{T/2} v_m dt = \frac{v_m}{T/2} \left(t \right)_{0}^{T/2} = \frac{v_m}{T/2} \left(\frac{T}{2} - 0 \right) = v_m$$



$$V_{avg} = \frac{V_m}{2}$$



$$V_{avg} = \frac{V_m}{2}$$



Solⁿ: ① $V_{avg} = \frac{1}{T} \int_0^T v_m \sin \omega t dt = \frac{2V_m}{\pi}$

②
$$V_{avg} = \frac{1}{4} \left[\int_0^2 10 dt + \int_2^4 20 dt \right]$$

$$= \frac{1}{4} [10 \times 2 + 20 \times 2]$$

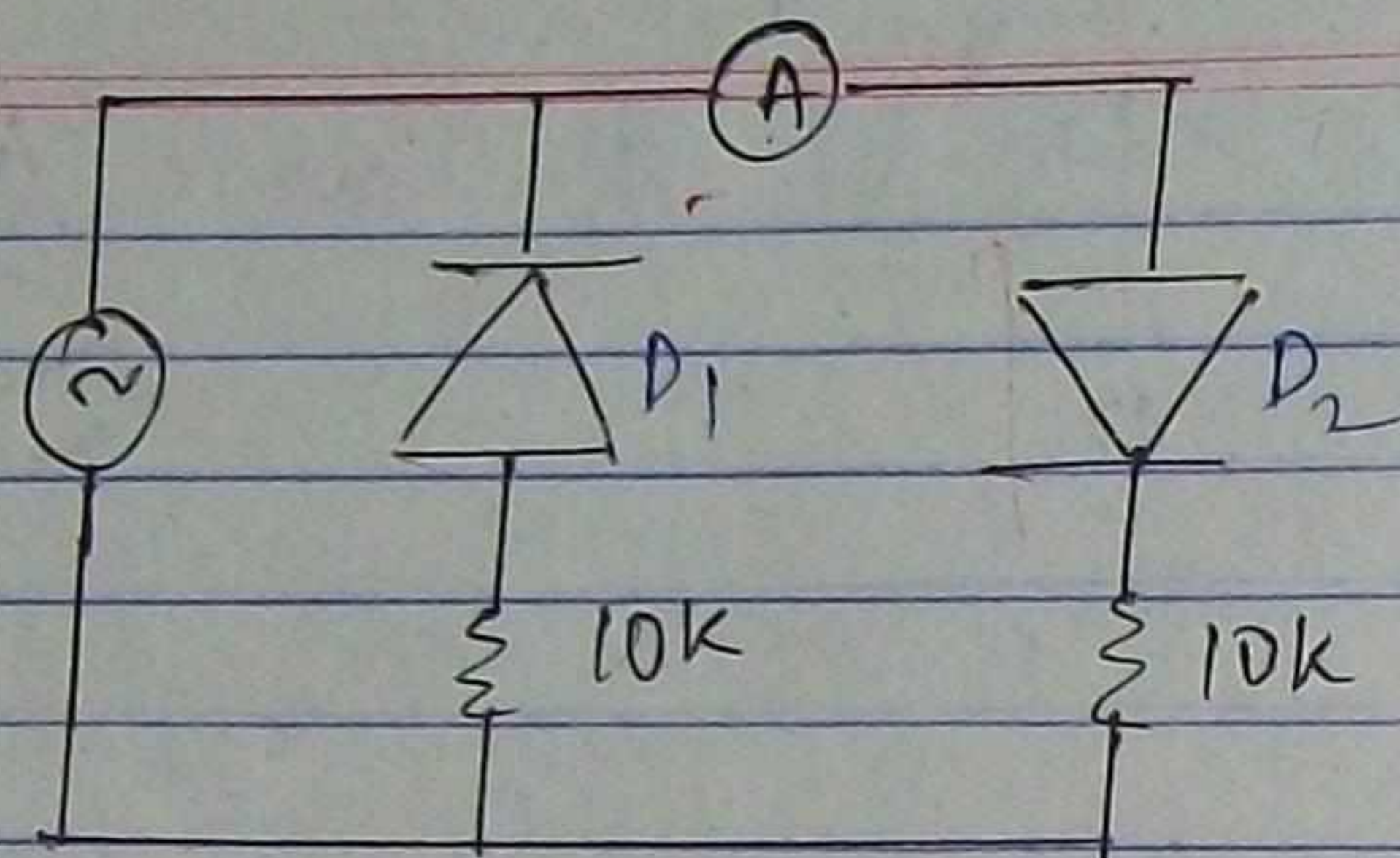
$$= \frac{60}{4} = \frac{28}{2} = 12.5$$

$$= 15$$

★★

Que.

$V(t) = 4 \sin \omega t$



find reading of ammeter indicating avg value

Solⁿ:

Half wave rectifier

$$V_{av} = \frac{V_m}{\pi} = \frac{4}{\pi}$$

$$I_{av} = \frac{V_{av}}{10 \times 10^3} = \frac{4/\pi}{10 \times 10^3} = 0.4 \text{ mA}$$

⇒ FORM FACTOR -

form factor is the ratio of RMS value of waveform to average value of waveform.

$$\text{form factor} = \frac{V_{rms}}{V_{avg}}$$

⇒ PEAK FACTOR -

Peak factor is the ratio of max. value of waveform to RMS value of waveform

$$\text{PEAK FACTOR} = \frac{V_m}{V_{rms}}$$

To justify about shape of the waveform form factor and peak factor concepts are introduced.

for sine wave:

$$\text{form factor} = \frac{V_{rms}}{V_{avg}} = \frac{V_m/\sqrt{2}}{2V_m/\pi} = 1.11$$

$$\text{Peak factor} = \frac{V_m}{V_{rms}} = \frac{V_m}{V_m/\sqrt{2}} = \sqrt{2}$$

Power system.

11 kV

33 kV

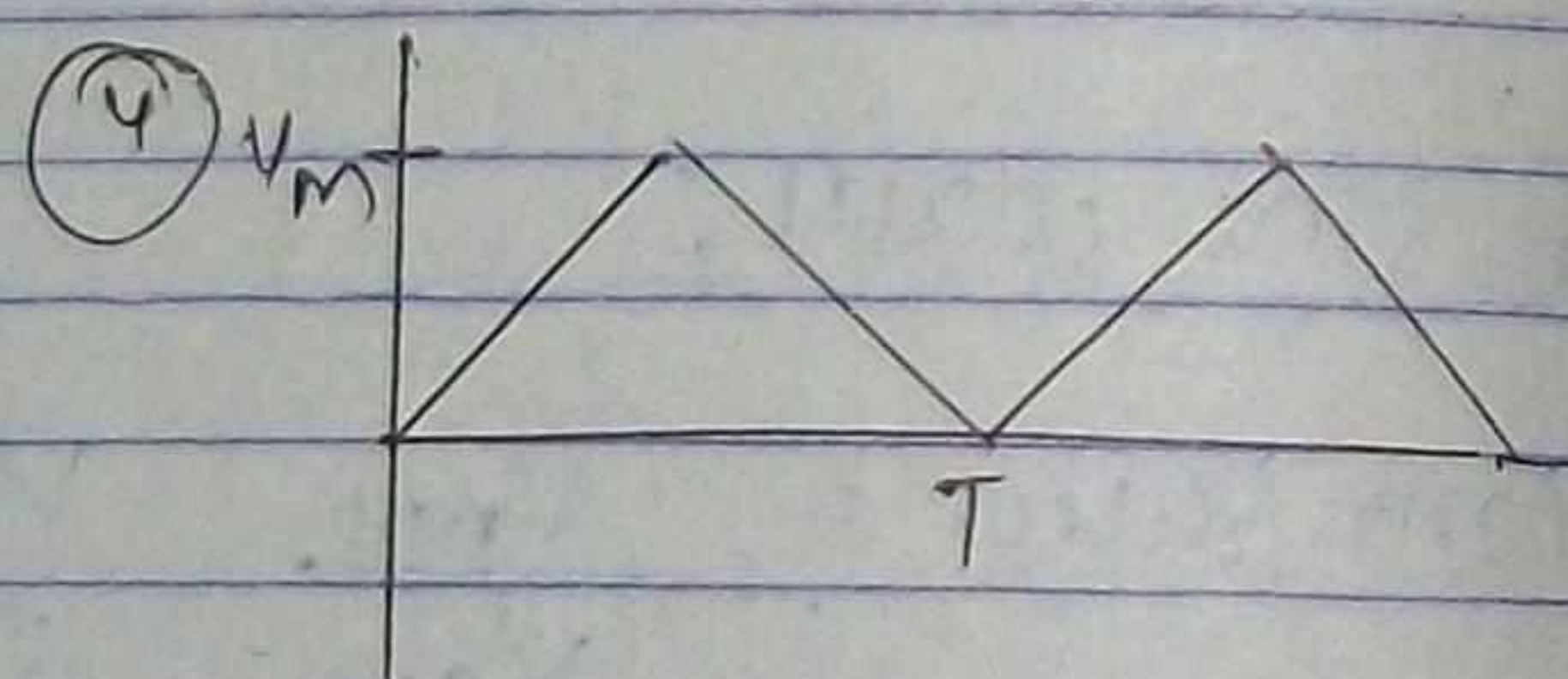
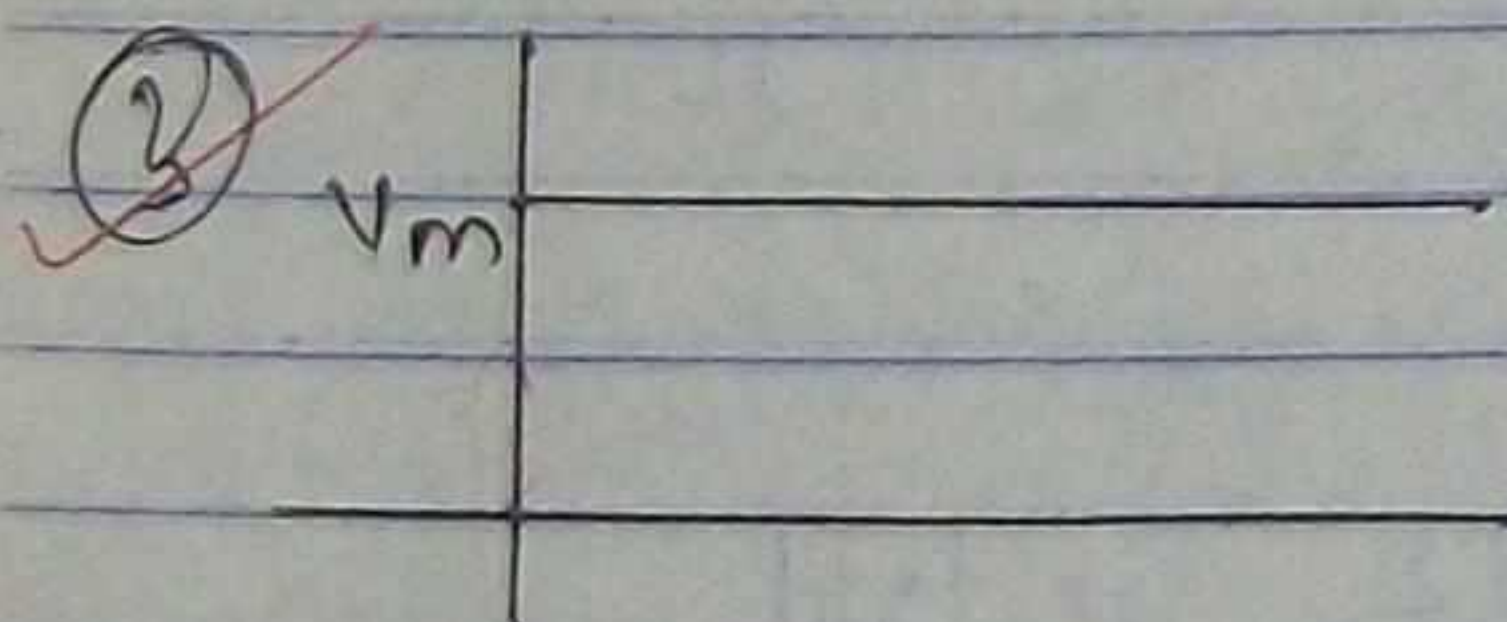
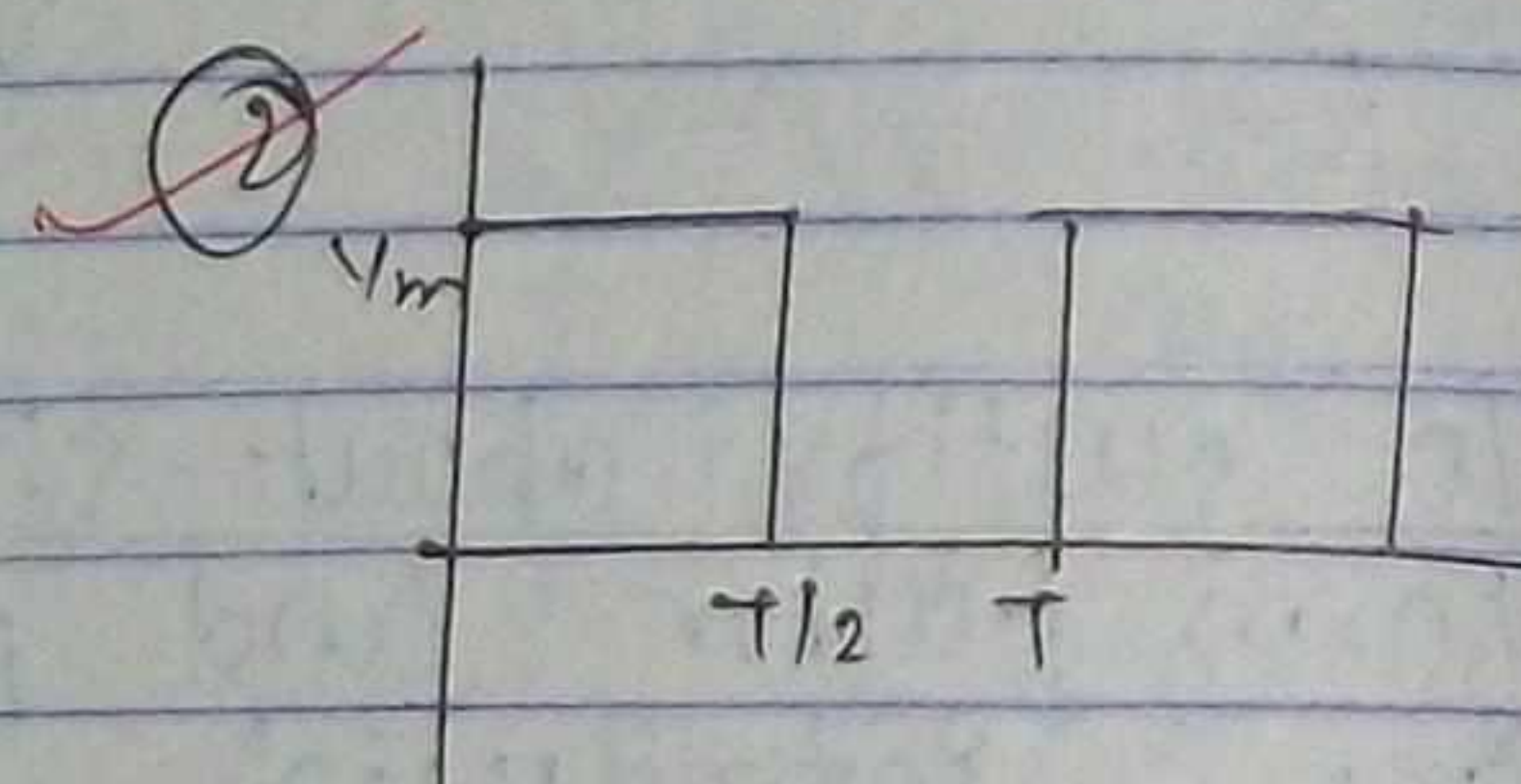
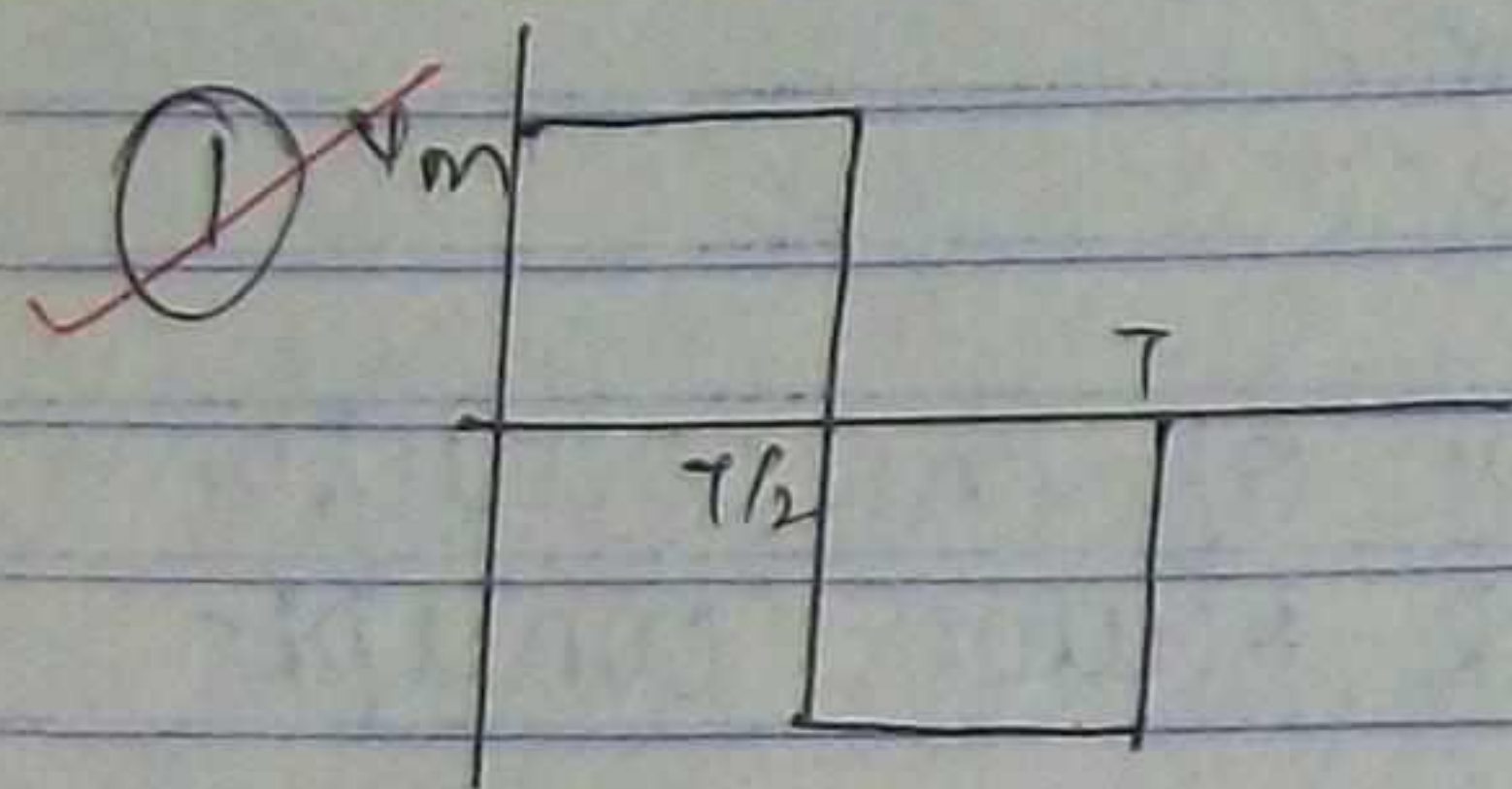
66 kV

132 kV

220 kV

form factor

Ques. which of the wave have form factor = Peak factor



Solⁿ: ① $V_{avg} = V_{rms} = V_m$

form factor = 1, Peak factor = 1

② $V_{rms} = V_m / \sqrt{2}$ $V_{avg} = V_m / 2$

form factor = $\sqrt{2}$ Peak factor = $\sqrt{2}$

③ $V_{avg} = V_{rms} = V_m = V$

form factor = Peak factor = 1

④ $V_{rms} = V_m / \sqrt{3}$ $V_{avg} = V_m / 2$

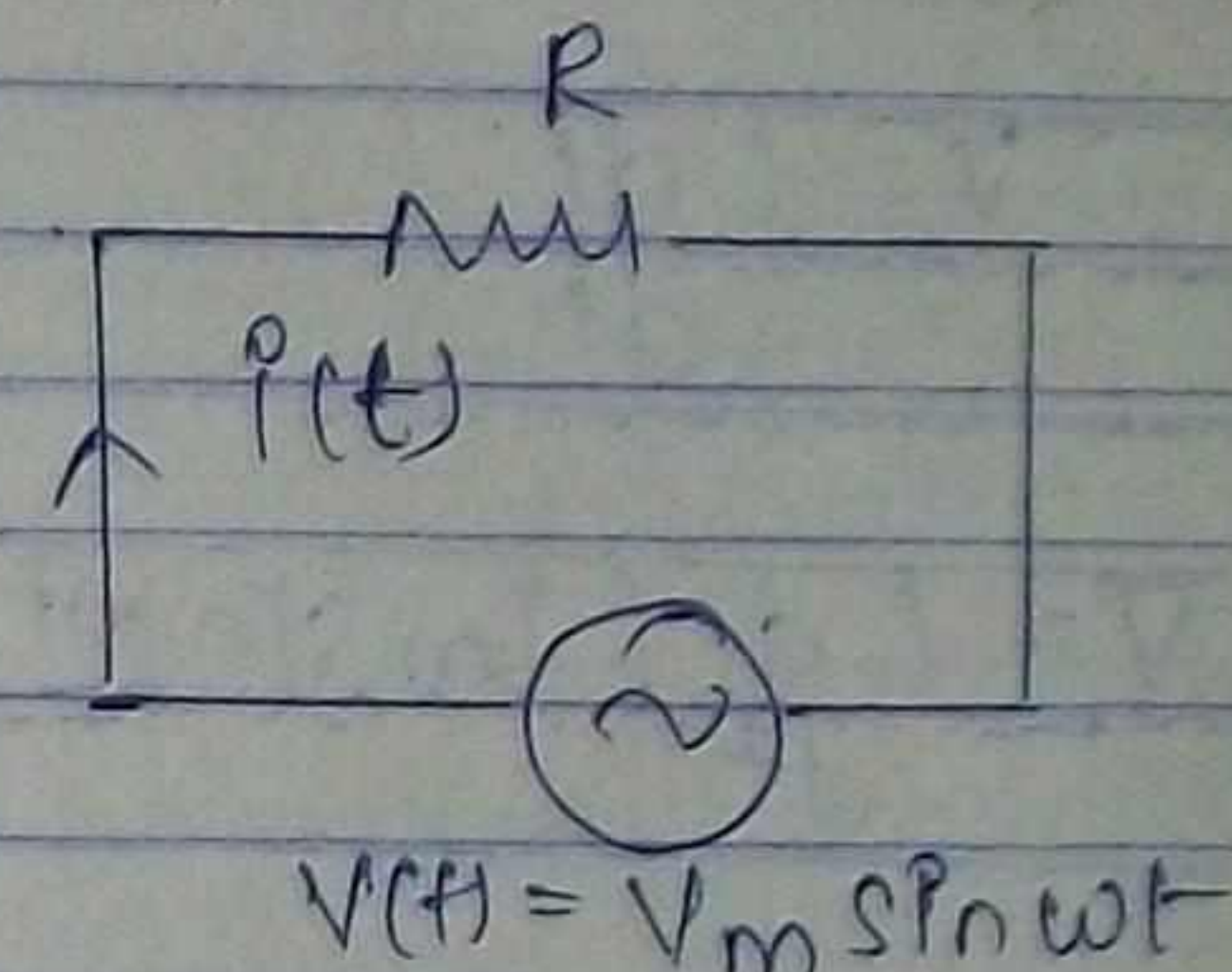
form factor = $\frac{2}{\sqrt{3}}$ Peak factor = $\sqrt{3}$

⇒ A.C. SOURCE ACROSS RESISTOR -

$$i(t) = \frac{V(t)}{R}$$

$$i(t) = \frac{V_m \sin \omega t}{R}$$

$$i(t) = I_m \sin \omega t$$

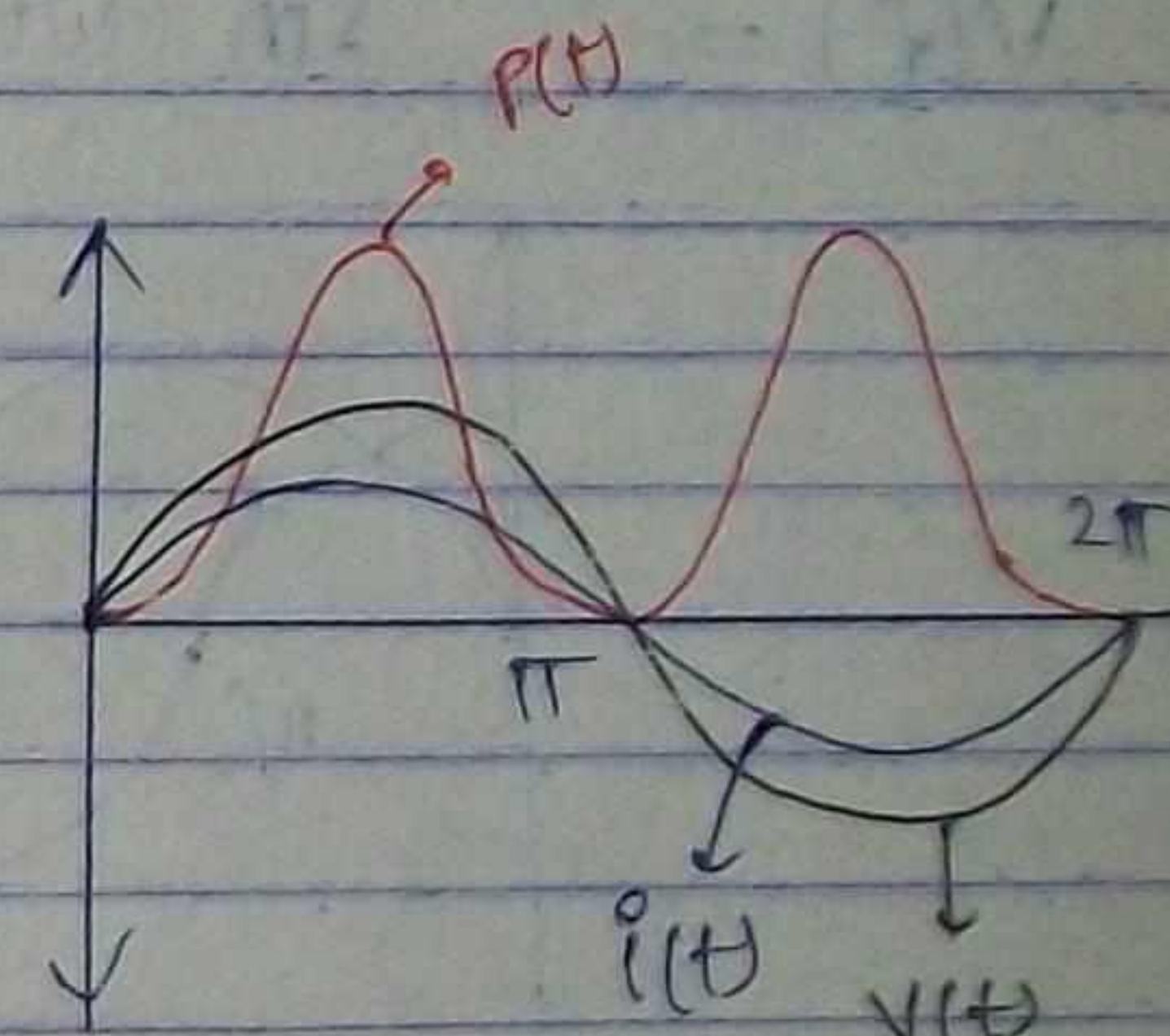


INSTANTANEOUS POWER -

$$P(t) = V(t) i(t)$$

$$P(t) = V_m \sin \omega t \cdot I_m \sin \omega t$$

$$P(t) = \frac{V_m I_m}{2} (1 - \cos 2\omega t)$$



AVERAGE POWER -

$$P_{avg} = \frac{1}{2\pi} \int_0^{2\pi} P(t) d\omega t$$

$$P_{avg} = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}$$

$$P_{avg} = V_{rms} \cdot I_{rms}$$

$$f = 50 \text{ Hz}$$

freq of V or I

$$f_p = 2f$$

$$f_p = 100 \text{ Hz or C/s}$$

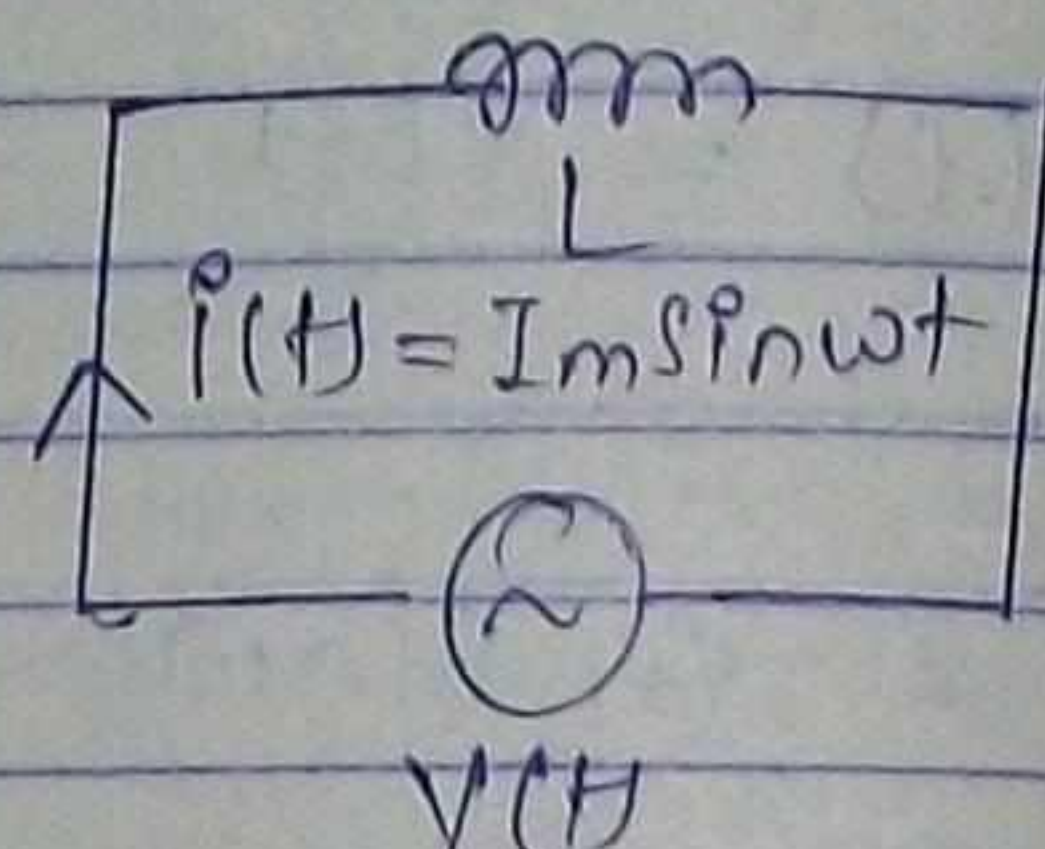
⇒ A-C. SOURCE ACROSS INDUCTOR -

$$V = L \frac{di}{dt}$$

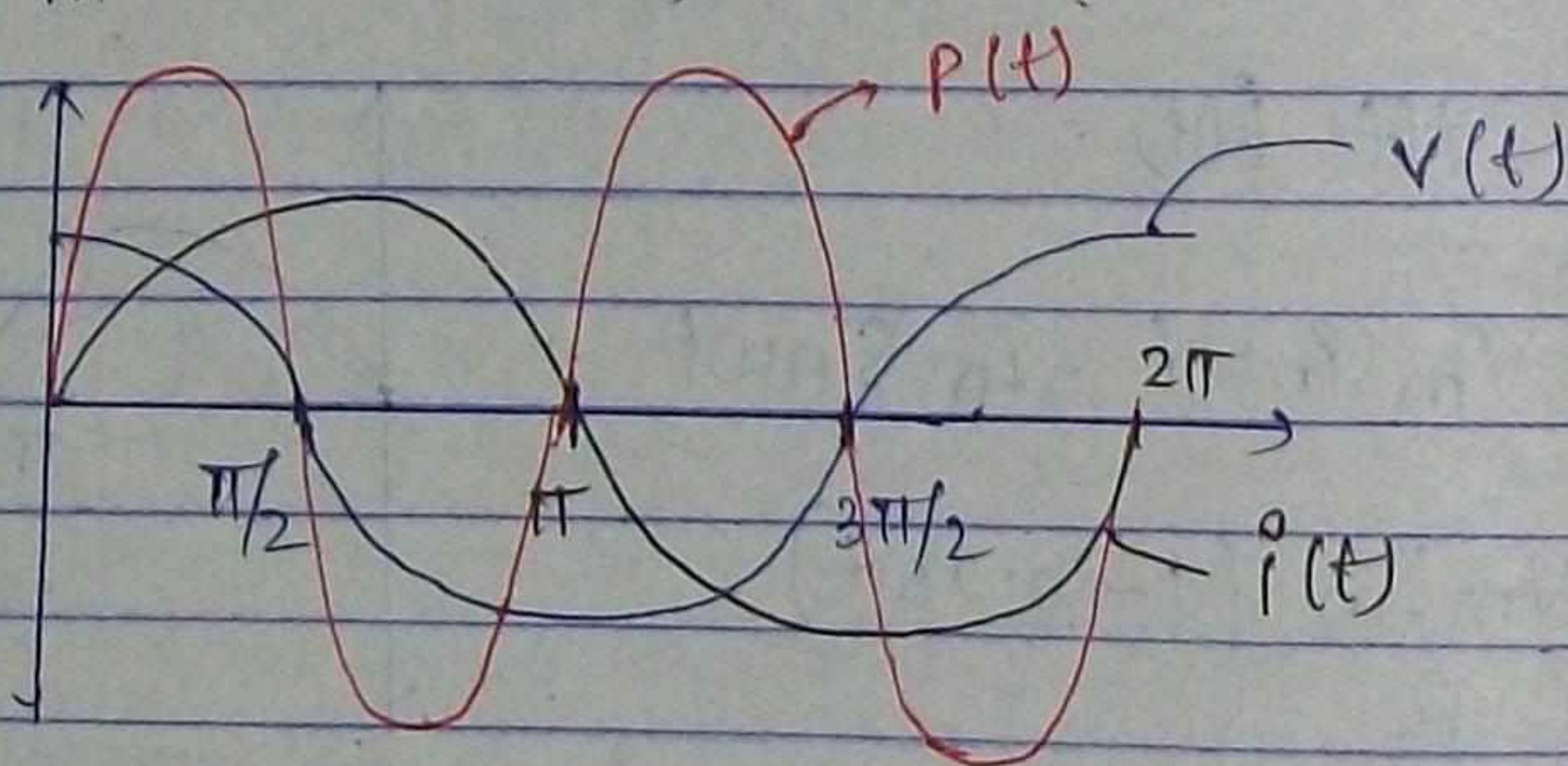
$$V = L \frac{d(I_m \sin \omega t)}{dt}$$

$$V = \omega L I_m \cos \omega t$$

$$V(t) = V_m \sin(\omega t + 90^\circ)$$



$$[X_L = \omega L \text{ (INDUCTIVE REACTANCE)}]$$



INSTANTANEOUS POWER -

$$P(t) = V(t) i(t)$$

$$P(t) = V_m \cos \omega t \cdot I_m \sin \omega t$$

$$P(t) = \frac{V_m I_m \sin 2\omega t}{2}$$

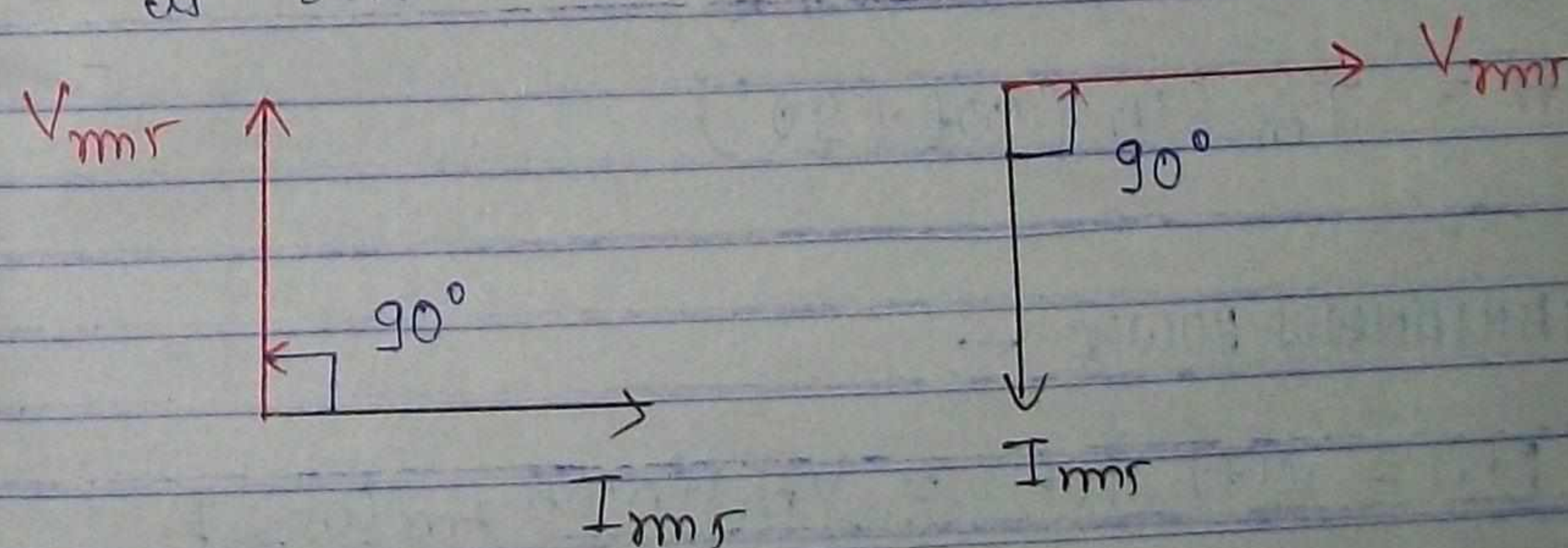
Average Power -

$$P_{avg} = \frac{1}{2\pi} \int_0^{2\pi} P(t) d\omega t = 0$$

$$f = 50 \text{ Hz (supply freq.)}$$

$$f_p = 2f = 100 \text{ Hz (power freq.)}$$

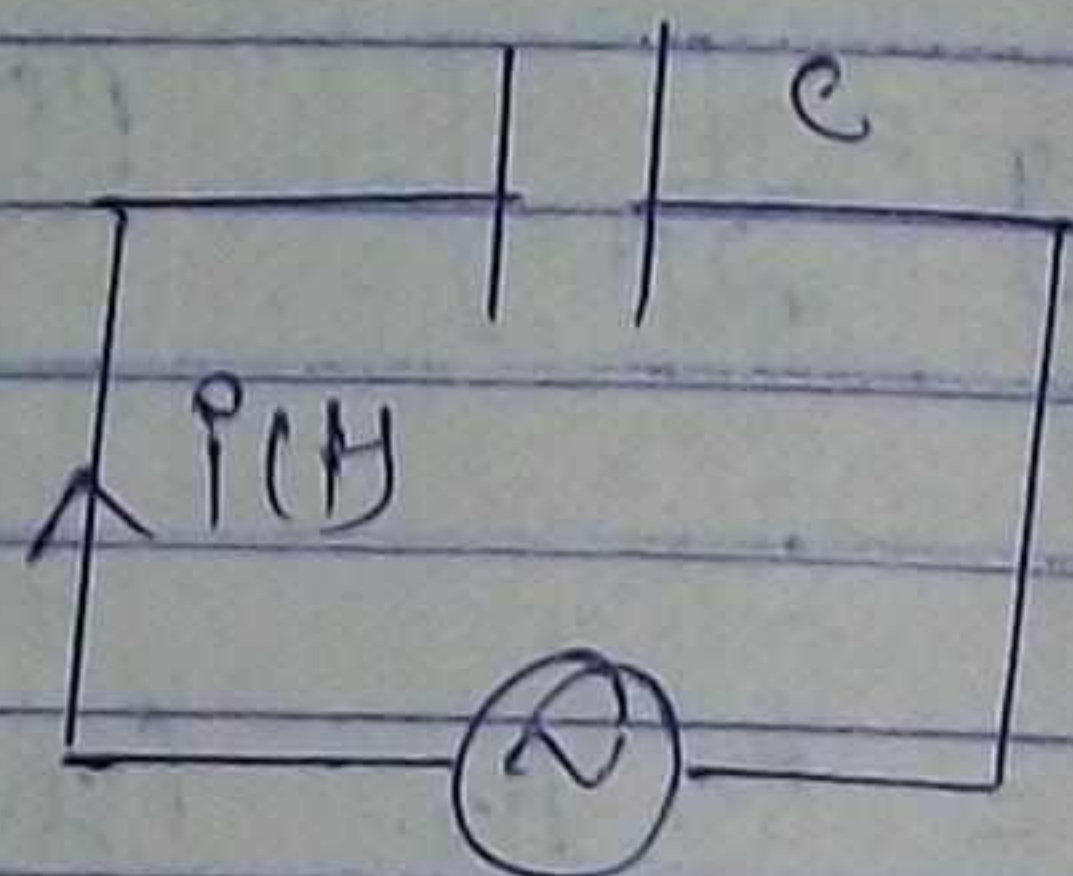
- In the positive half cycle of the power inductor takes energy from the source and in the negative half cycle of power inductor delivers energy to the source. Thereby net power taken by the source is zero.



⇒ AC SOURCE ACROSS CAPACITOR -

$$i = C \frac{dv}{dt}$$

$$i(t) = C \frac{d}{dt} (V_m \sin \omega t)$$



$$V(t) = V_m \sin \omega t$$

$$i(t) = \omega C V_m \cos \omega t$$

$$i(t) = \frac{V_m}{1/\omega C} \sin(\omega t + 90^\circ)$$

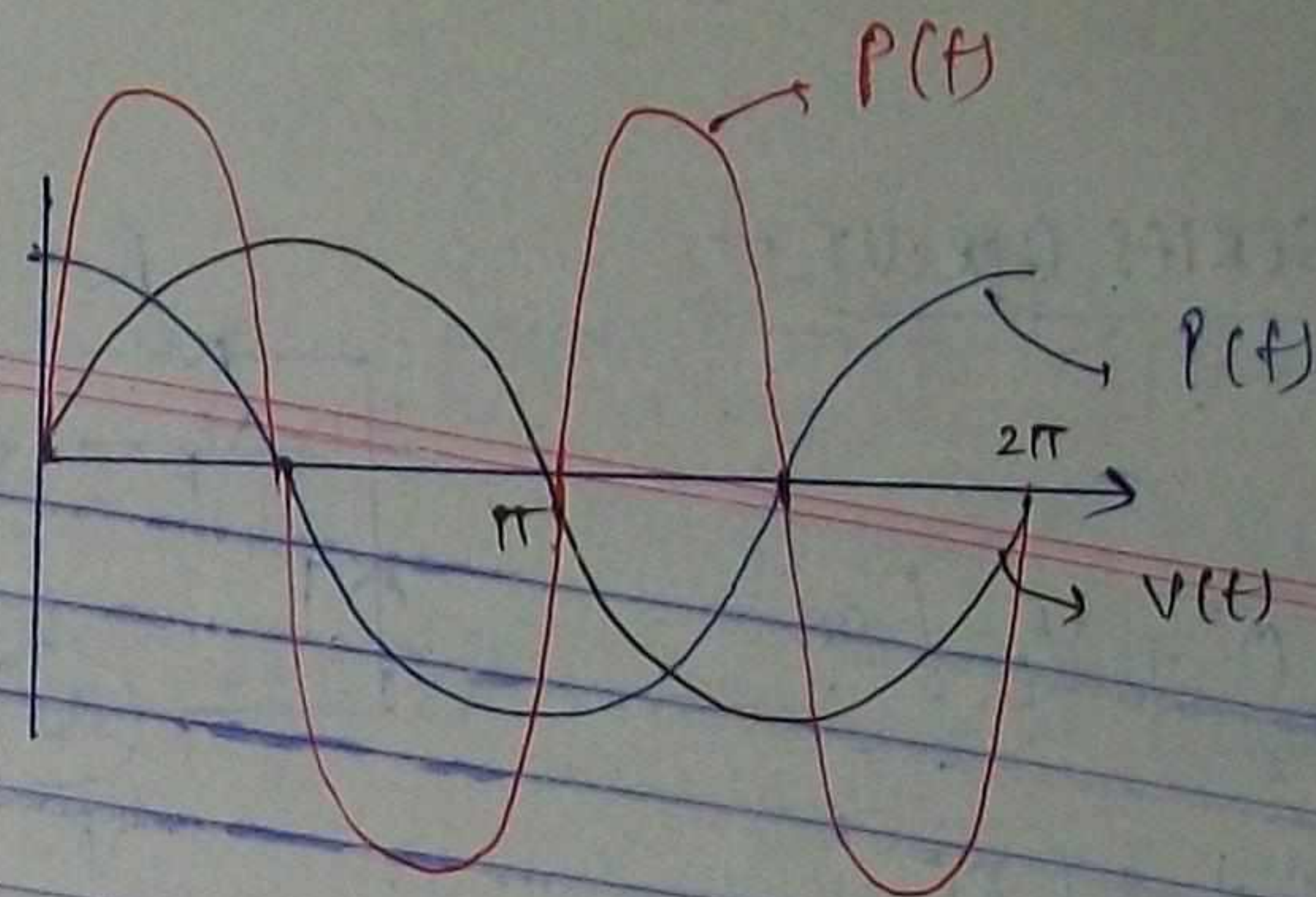
$$\left(X_C = \frac{1}{\omega C} \right) \text{ (CAPACITANCE IMPEDANCE)}$$

$$i(t) = I_m \sin(\omega t + 90^\circ)$$

INSTANTANEOUS POWER -

$$P(t) = V(t) i(t) = V_m \sin \omega t I_m \cos \omega t$$

$$P(t) = \frac{V_m I_m}{2} \sin 2\omega t$$

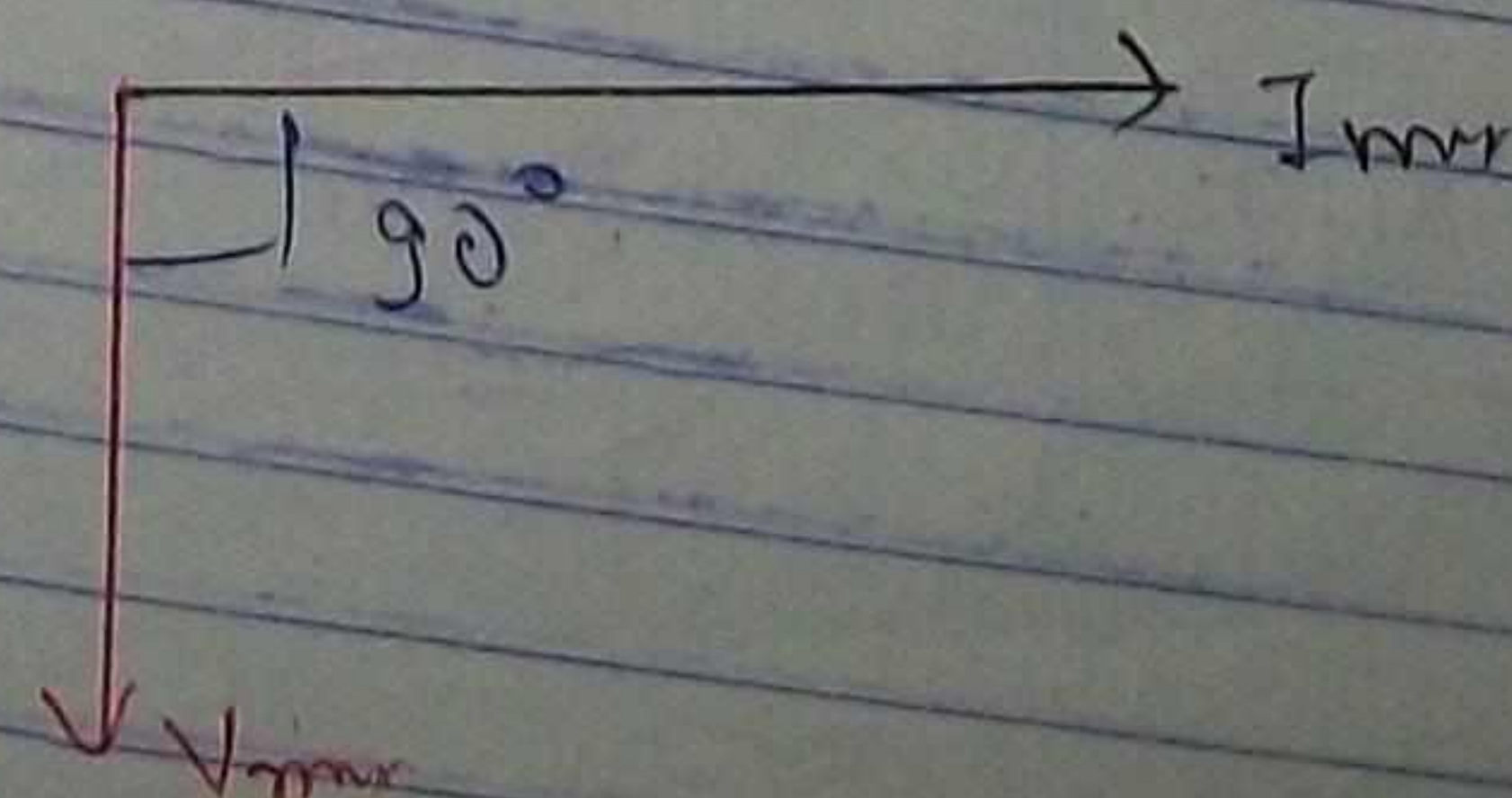
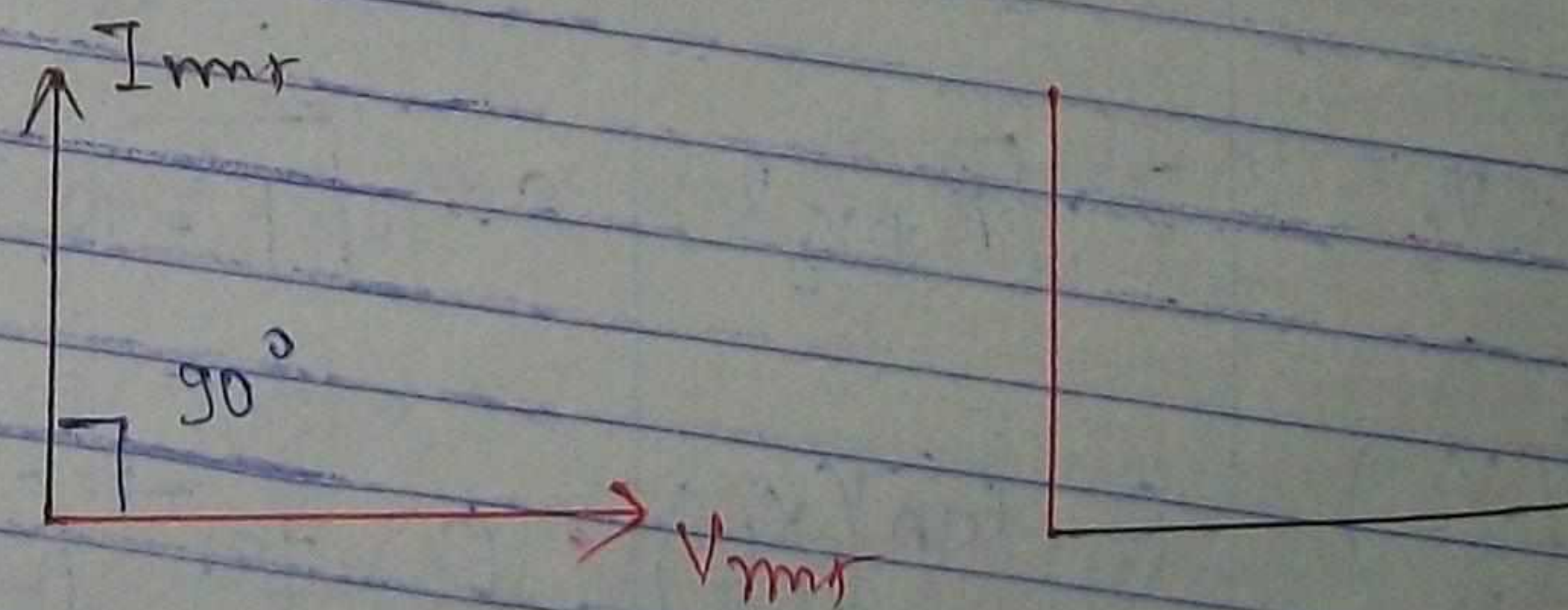


AVERAGE POWER -

$$P_{avg} = \frac{1}{2\pi} \int_0^{2\pi} P(t) d\omega t = 0$$

$$f = 50 \text{ Hz (supply freq)}$$

$$f_p = 2 \times 50 = 100 \text{ Hz (power frequency)}$$



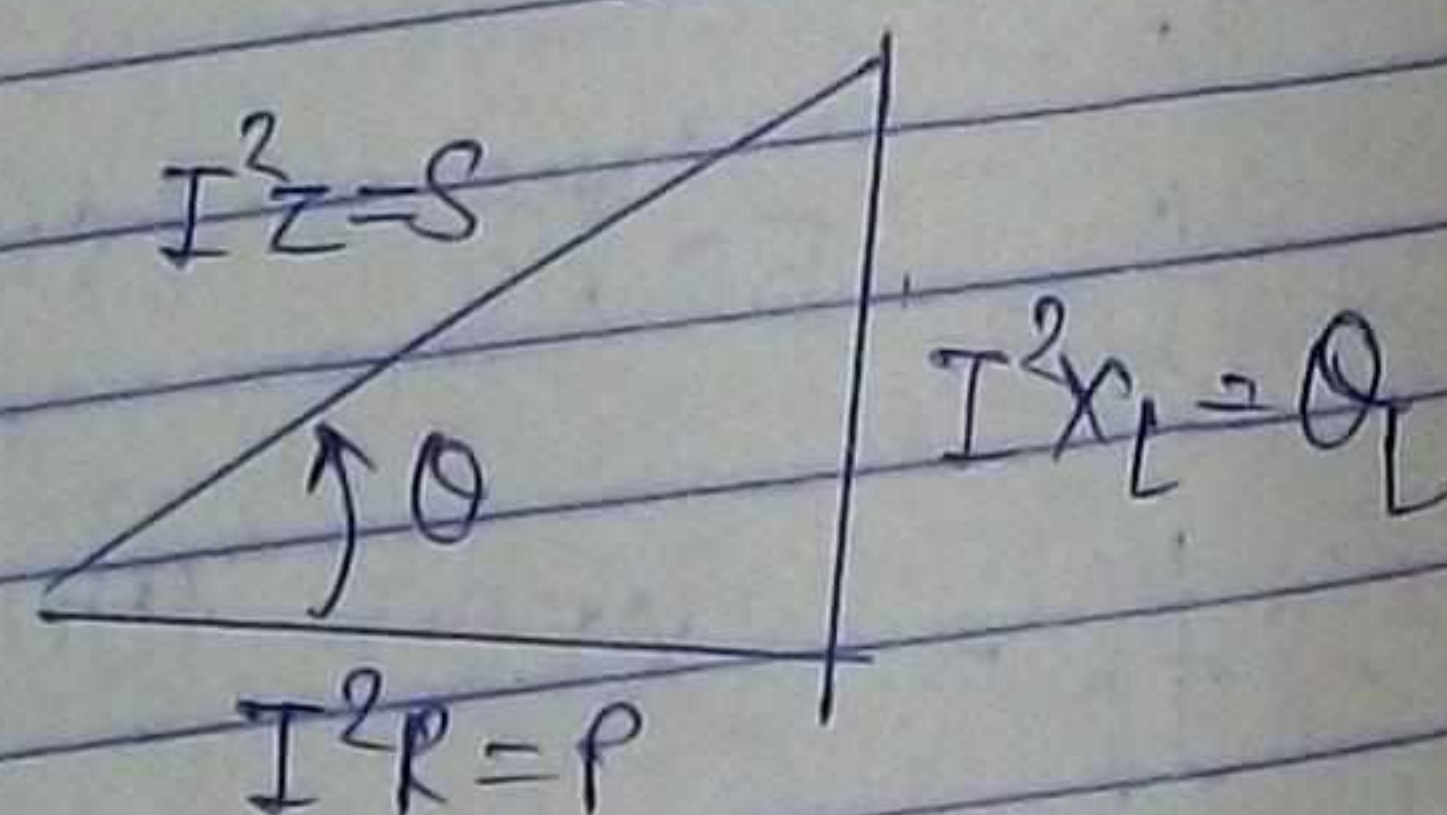
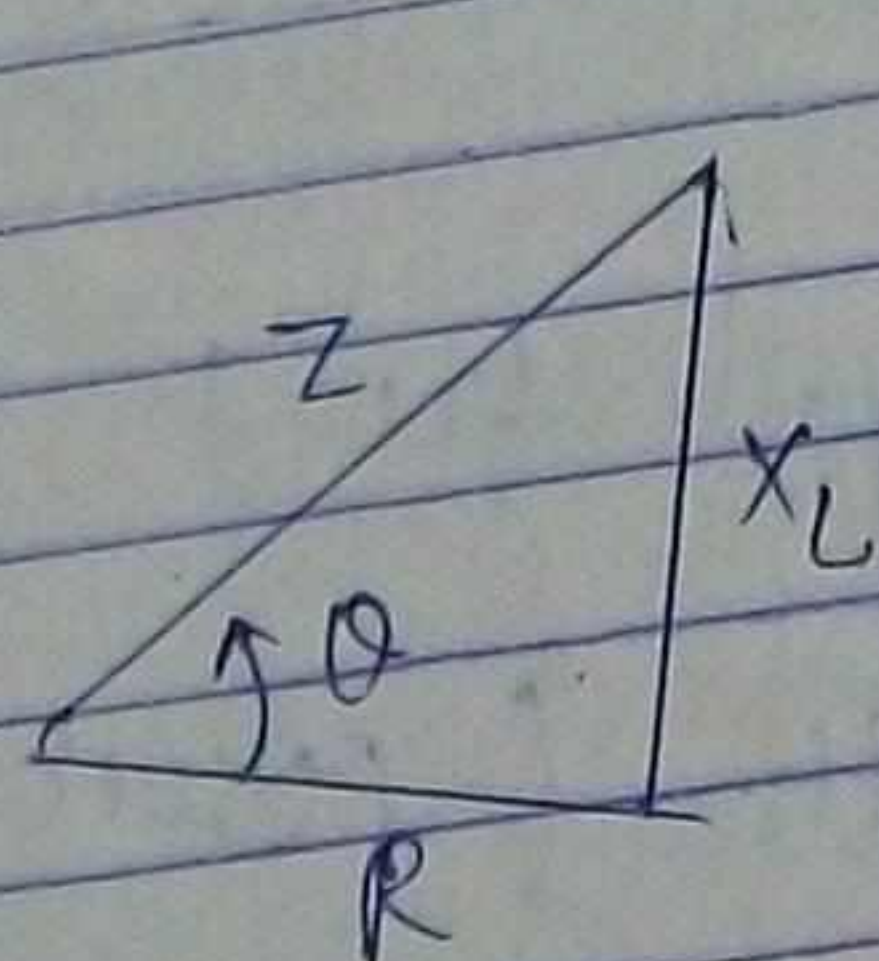
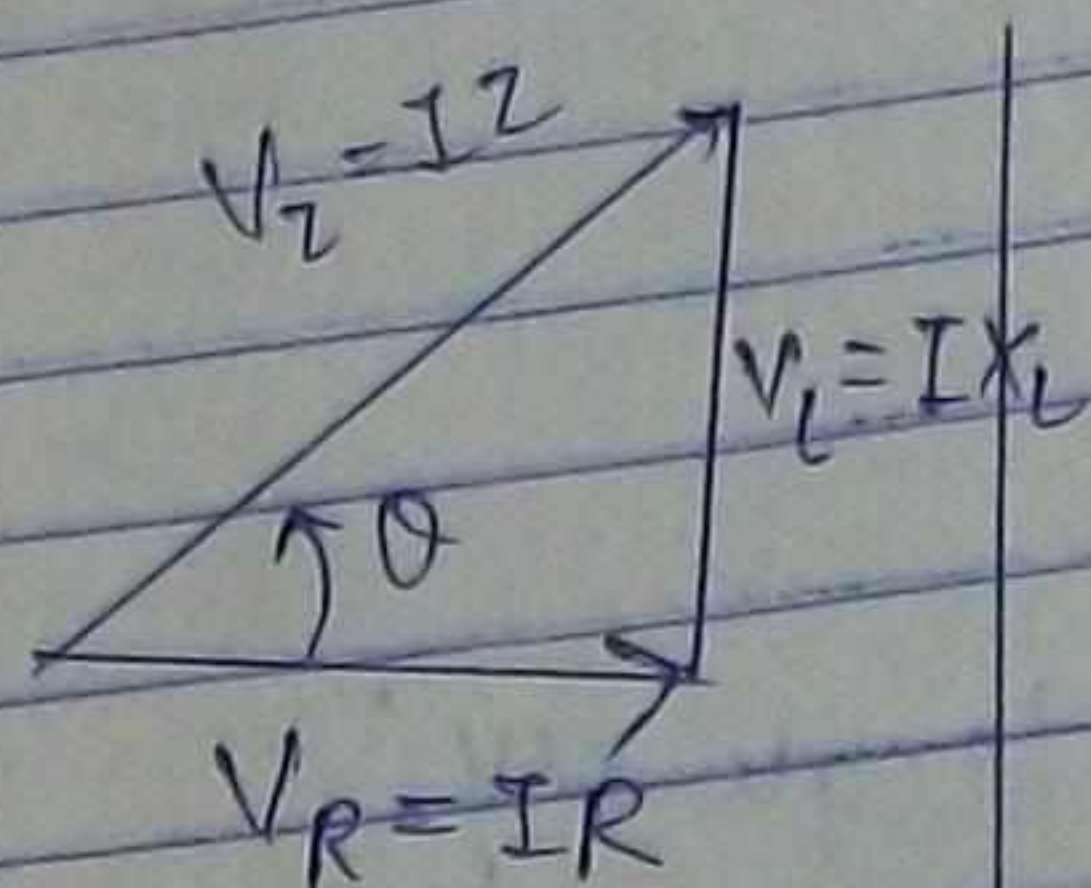
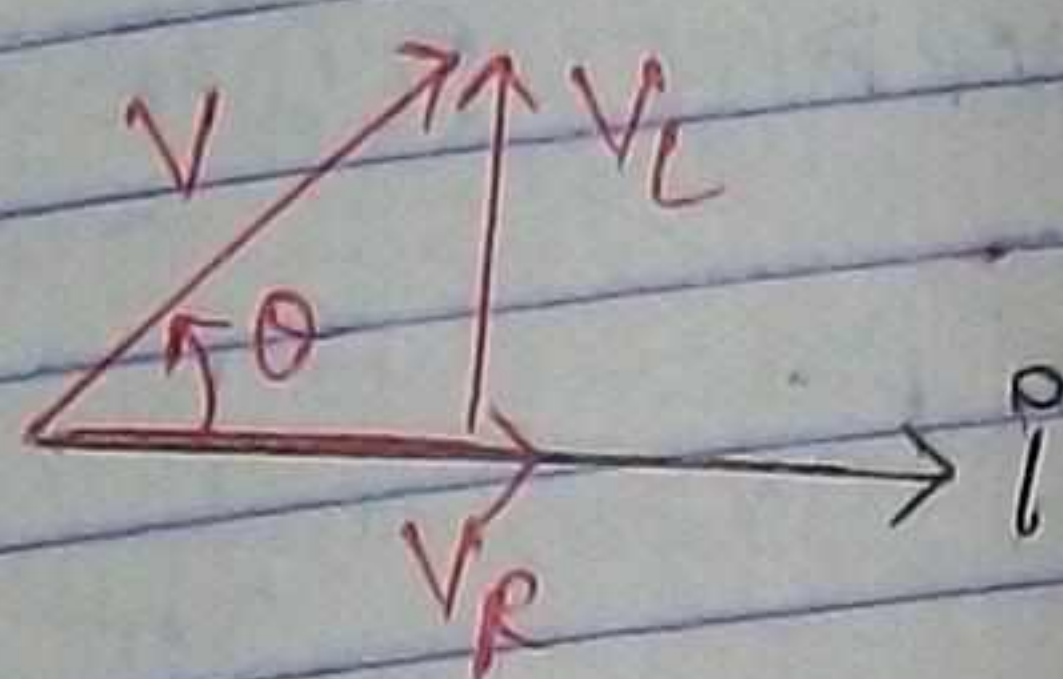
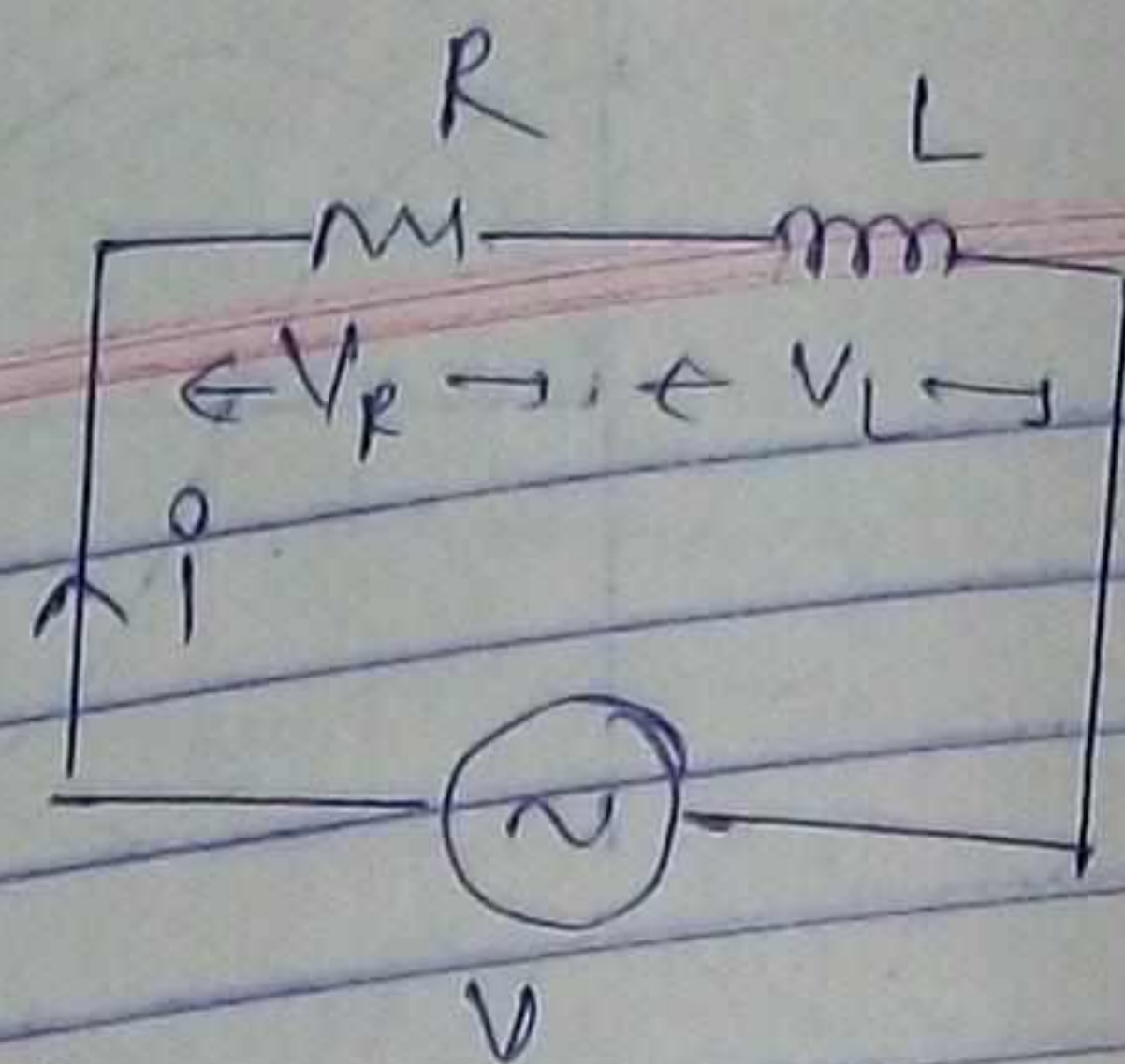
⇒ RL SERIES CIRCUIT -

By KVL

$$V = V_R \angle 0^\circ + V_L \angle 90^\circ$$

$$IZ = IR + jIX_L$$

$$Z = R + jX_L$$



$$V = \sqrt{V_R^2 + V_L^2}$$

$$Z = \sqrt{R^2 + X_L^2}$$

$$S = \sqrt{P^2 + Q_L^2}$$

$$\theta = \tan^{-1} \left(\frac{V_L}{V_R} \right)$$

$$\theta = \tan^{-1} \left(\frac{X_L}{R} \right)$$

$$\theta = \tan^{-1} \left(\frac{Q_L}{P} \right)$$

VOLTAGE
Δ

IMPEDANCE
Δ

POWER
Δ

where,

$P \rightarrow$ active power (or) true power

(or) Real power (or) average power

(or) effective power. [watts]

$Q_L \rightarrow$ inductive reactive power [VAR]

(voltampere-reactive)

$S \rightarrow$ apparent power (or) complex power

[VA] volt-ampere

INSTANTANEOUS POWER -

$$P(t) = V(t) I(t)$$

$$P(t) = V_m \sin(\omega t + \theta) \cdot I_m \sin \omega t$$

AVERAGE POWER -

$$P_{avg} = \frac{1}{2\pi} \int_0^{2\pi} P(t) d\omega t$$

$$P_{avg} = \frac{V_m I_m}{2} \cos \theta = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos \theta = V_{rms} \cdot I_{rms} \cos \theta$$

$$\text{Power factor, } \cos \theta = \frac{V_R}{V} = \frac{R}{Z} = \frac{P}{S}$$

LAGGING

① — Power factor angle indicates position of current phasor w.r.t voltage phasor.

⊕ — while defining power factor for any combination of the current voltage phasor is taken as a reference, since

① In the real time system only independent voltage source exist. (source voltage is constant).

② In the real time system loads are connected in parallel (load voltage is constant).

⊕ — Power factor indicates contribution of resistive load in the total load of the system.

⊕ — In the real time system always high power factor is desirable since

(Power factor = 1 ideally
= 0.8 practically)

① losses in the system are reduced

② voltage drop is reduced.

$$P = I^2 R = VI \cos \theta$$

$$Q_L = I^2 X_L = VI \sin \theta$$

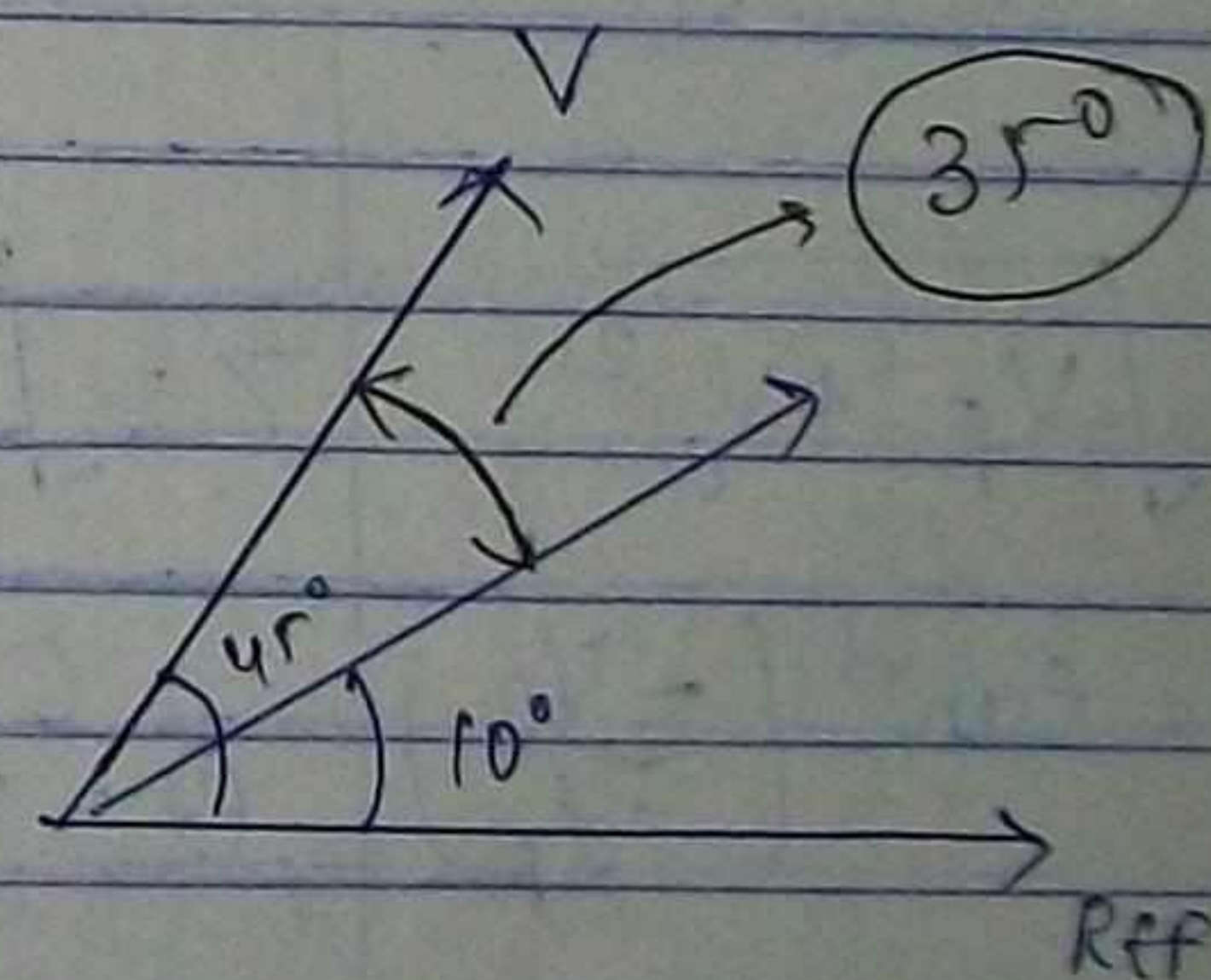
$$S = I^2 Z = VI^*$$

Ex — $V = 10 \angle 45^\circ$, $I = 5 \angle 10^\circ$

$$S = VI^*$$

$$S = 10 \angle 45^\circ \cdot 5 \angle -10^\circ$$

$$S = 50 \angle 35^\circ$$



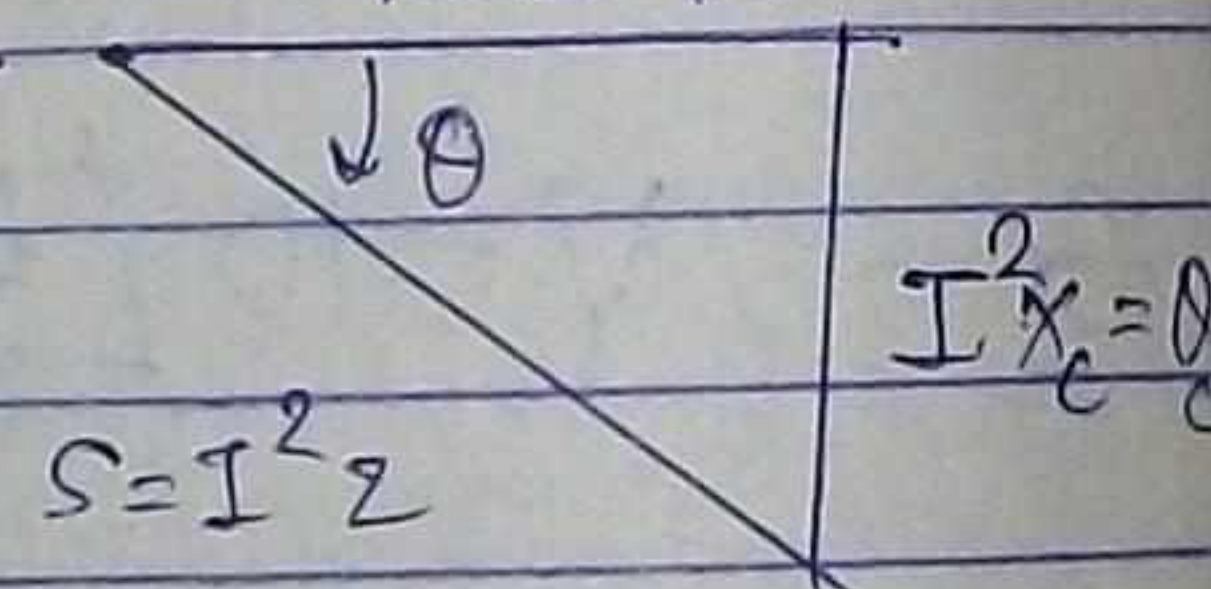
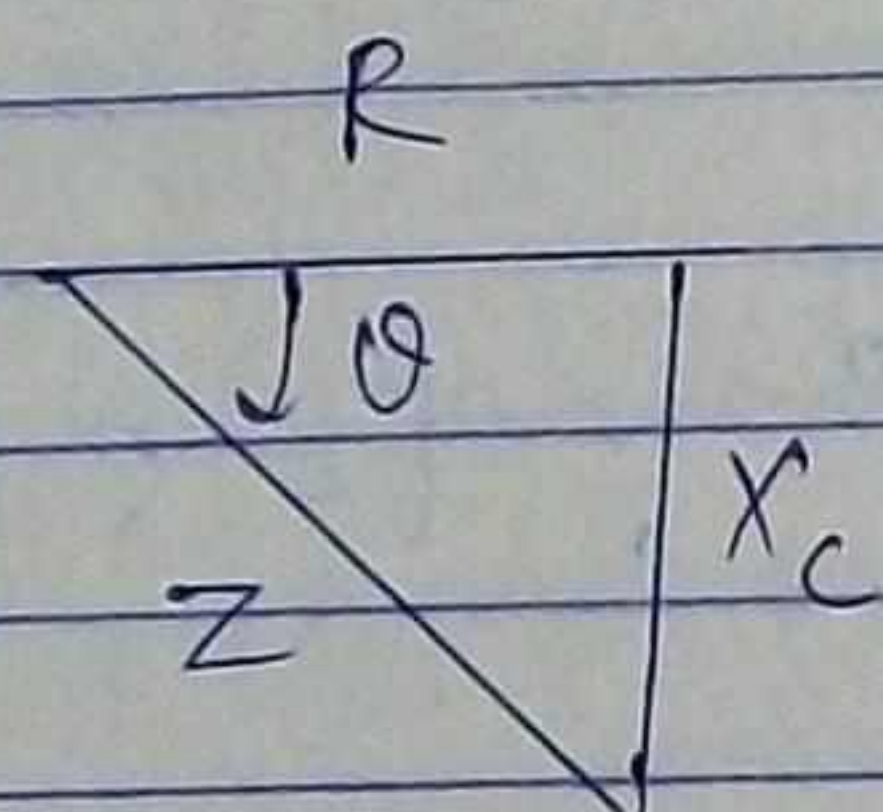
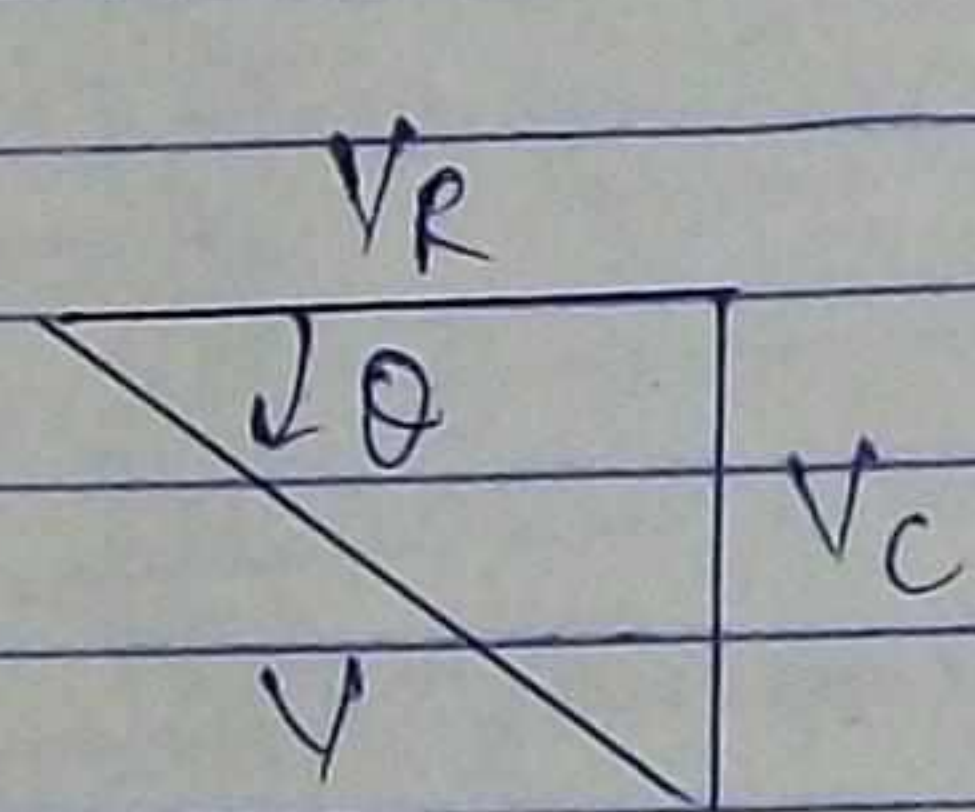
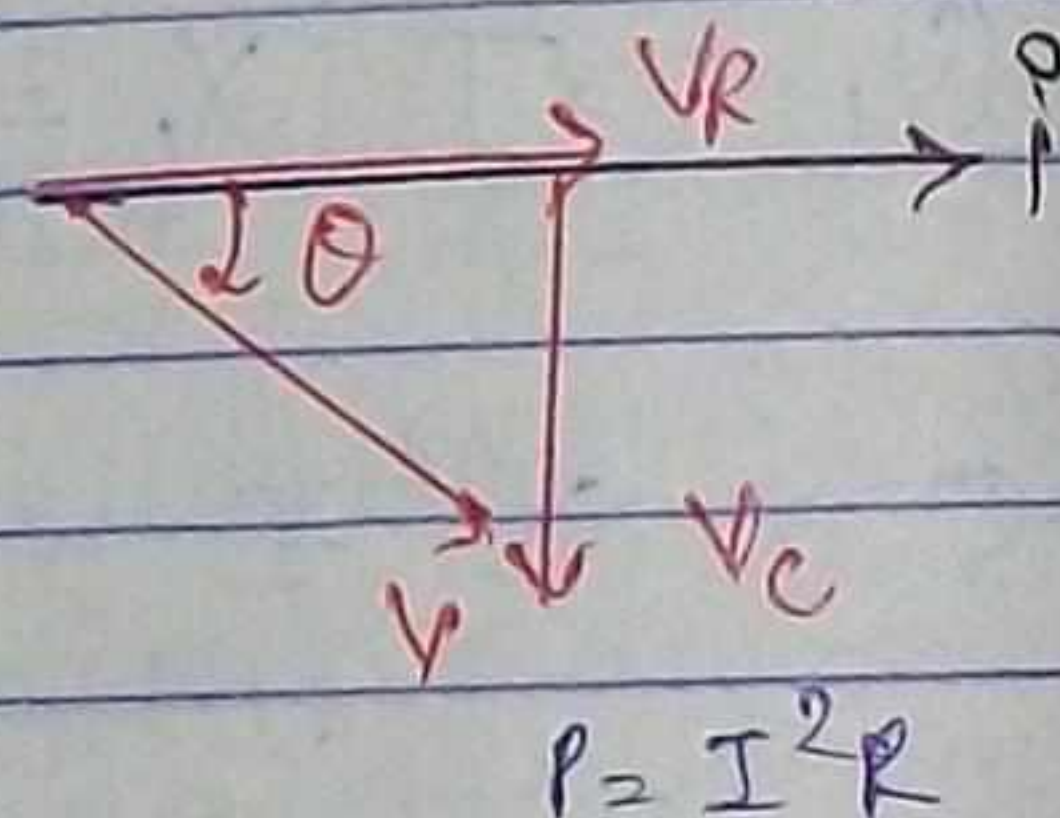
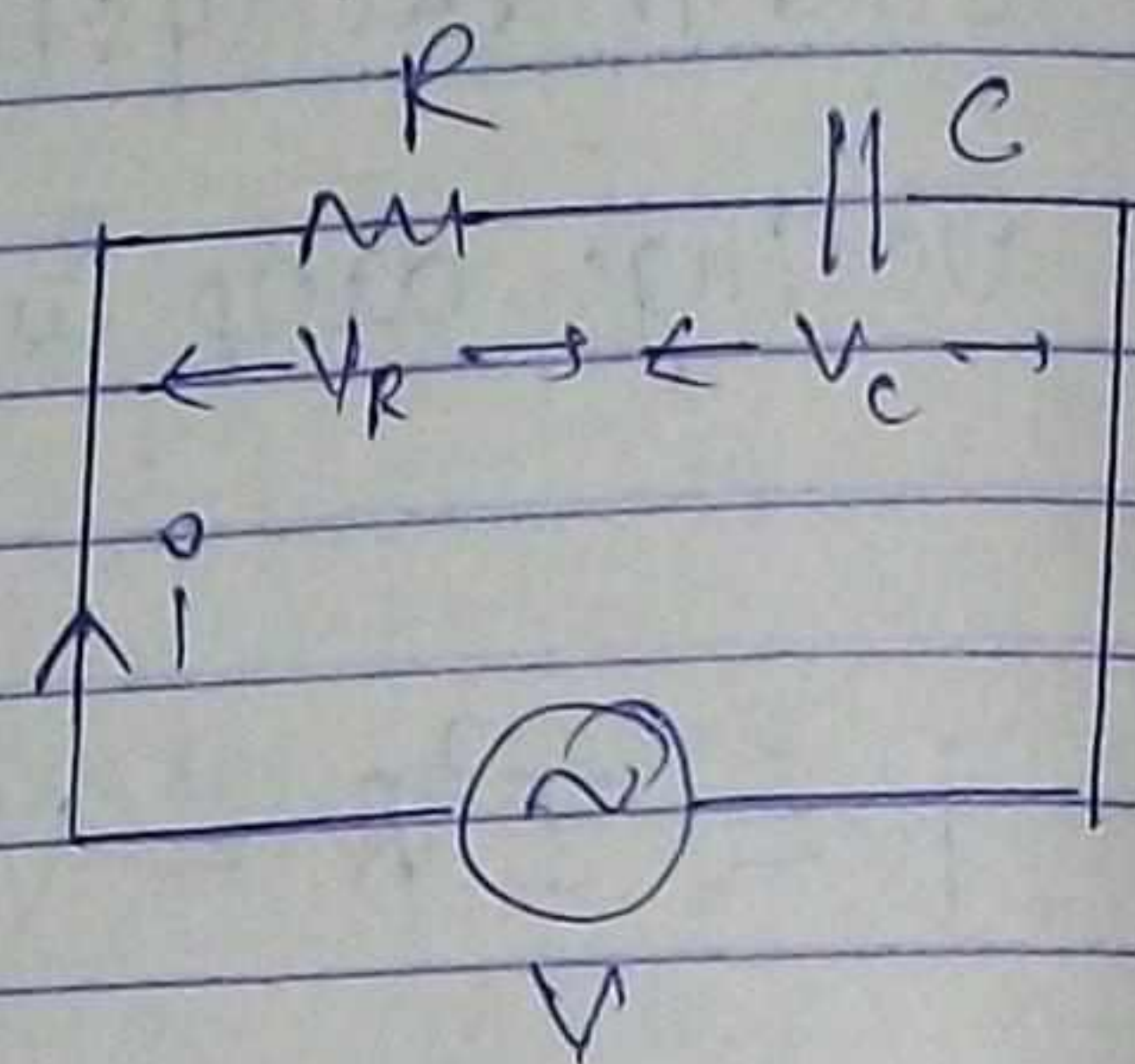
⇒ RC SERIES CIRCUIT -

By KVL,

$$V = V_R \angle 0 + V_C \angle -90$$

$$IZ = IR + [-jIX_C]$$

$$Z = R - jX_C$$



$$V = \sqrt{V_R^2 + V_C^2}$$

$$Z = \sqrt{R^2 + X_C^2}$$

$$S = \sqrt{P^2 + Q_C^2}$$

$$\theta = \tan^{-1} \left(\frac{-V_C}{V_R} \right)$$

$$\theta = \tan^{-1} \left(\frac{-X_C}{R} \right)$$

$$\theta = \tan^{-1} \left(\frac{-Q_C}{P} \right)$$

VOLTAGE



IMPEDANCE



POWER



$$\text{POWER FACTOR} = \cos \theta = \frac{V_R}{V} = \frac{R}{Z} = \frac{P}{S}$$

LEADING

$$V(t) = V_m \sin(\omega t + \theta)$$

$$V_m \angle \theta$$

→ vector representation

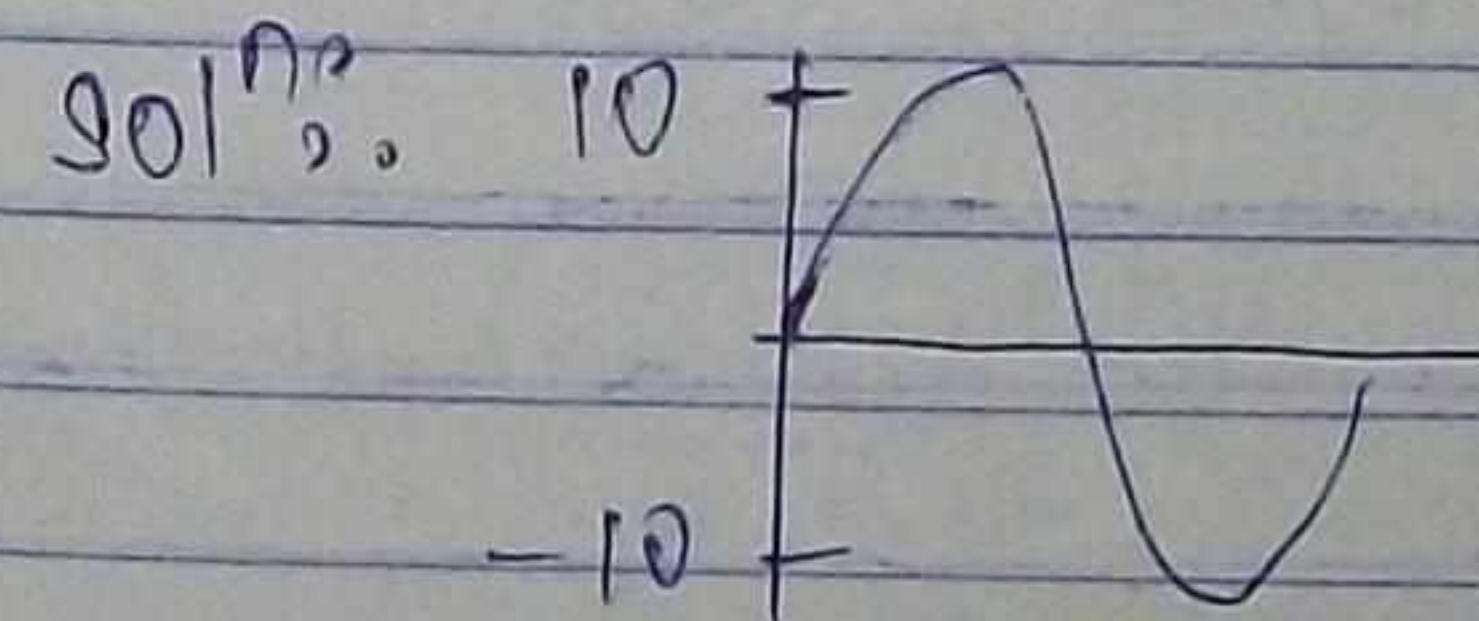
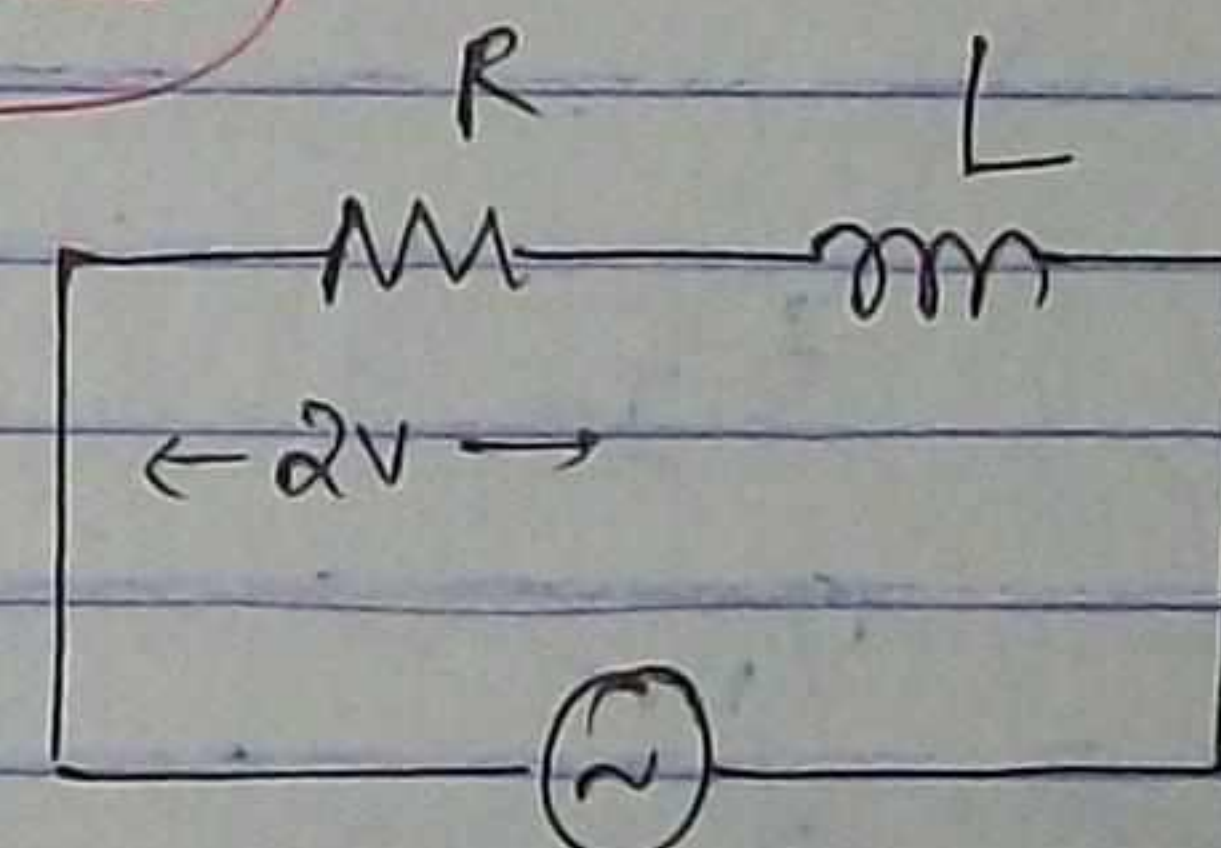
$$\frac{V_m}{\sqrt{2}} \angle \theta$$

→ phasor representation.

$$V = 10 \angle 30^\circ$$

rms value

Ques. find V_L



$$V = \sqrt{V_R^2 + V_L^2}$$

$$V_m = 10, \quad V_{rms} = \frac{10}{\sqrt{2}}$$

$$\frac{10}{\sqrt{2}} = \sqrt{2^2 + V_L^2}$$

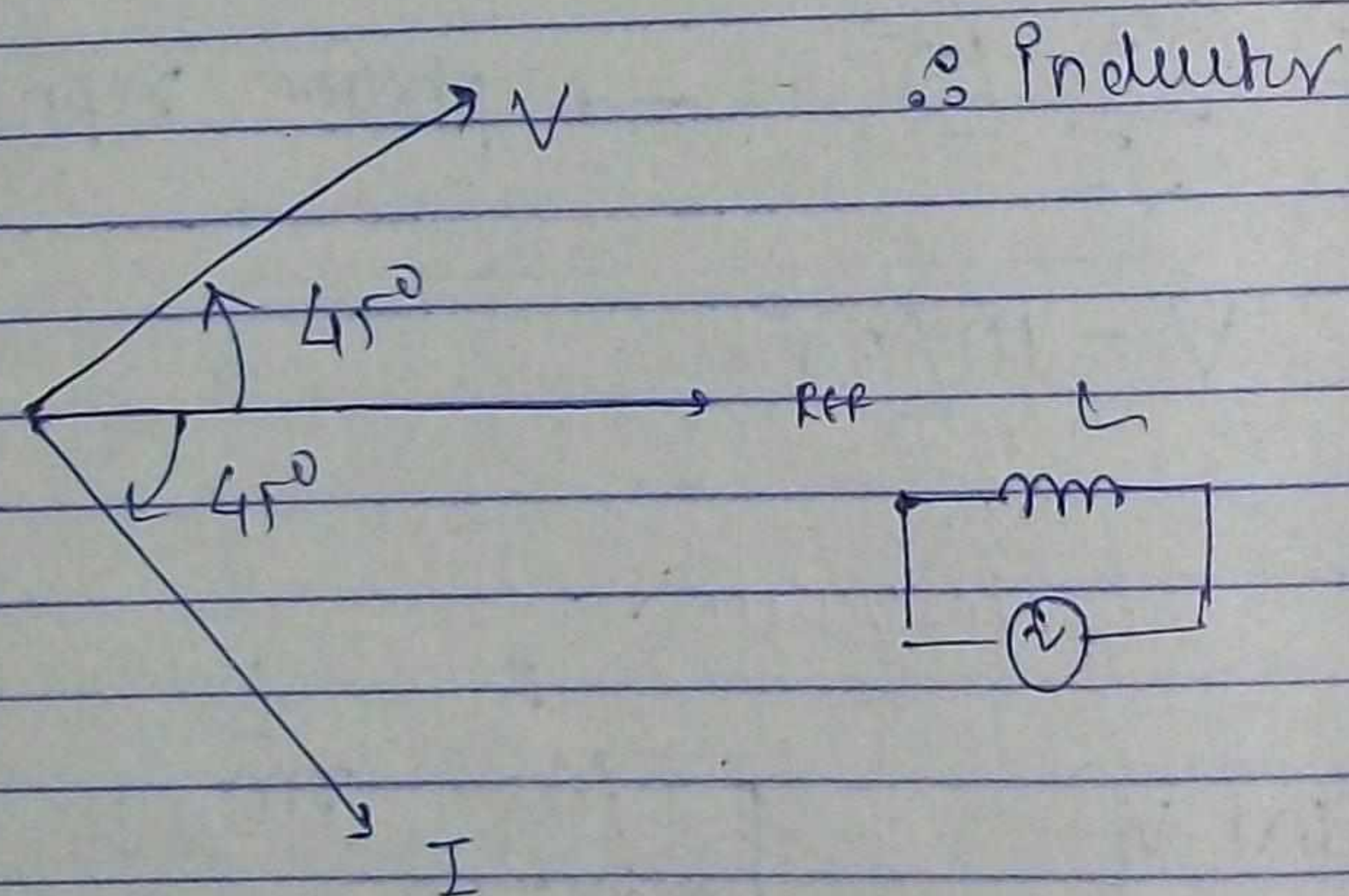
$$V_L = \sqrt{46}$$

Ques. find circuit element for given VI eqⁿs

$$V(t) = 9 \sin(t + 45^\circ)$$

$$I(t) = 3 \sin(t - 45^\circ)$$

Solⁿ:



$$X_L = \frac{V}{I} = \frac{V_{rms}}{I_{rms}} = \frac{9/\sqrt{2}}{3/\sqrt{2}} = 3$$

$$X_L = 3$$

$$\omega L = 3$$

$$L = 3 \text{ H}$$

$$\omega = 1$$

Ques. find circuit element for given VI eqⁿs

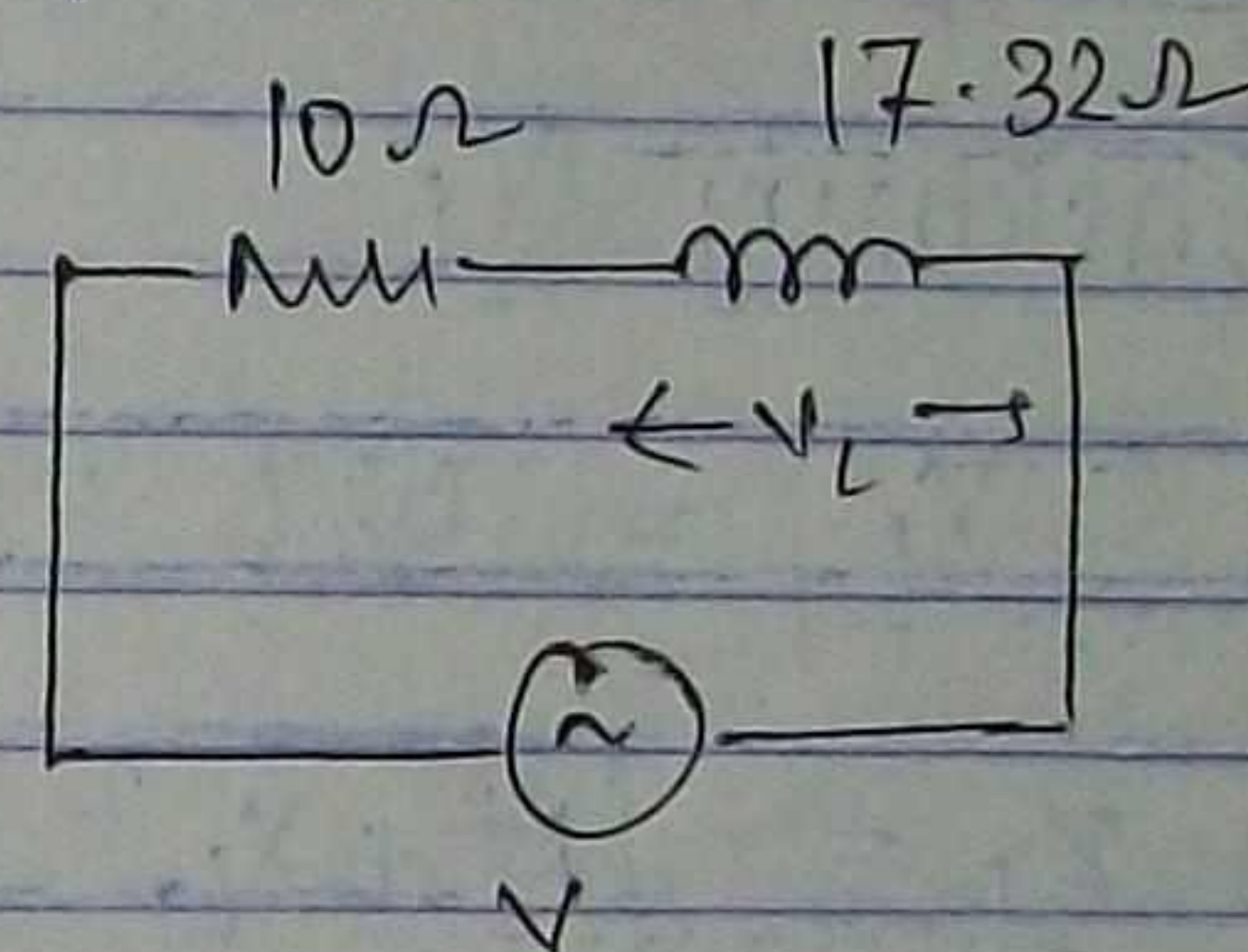
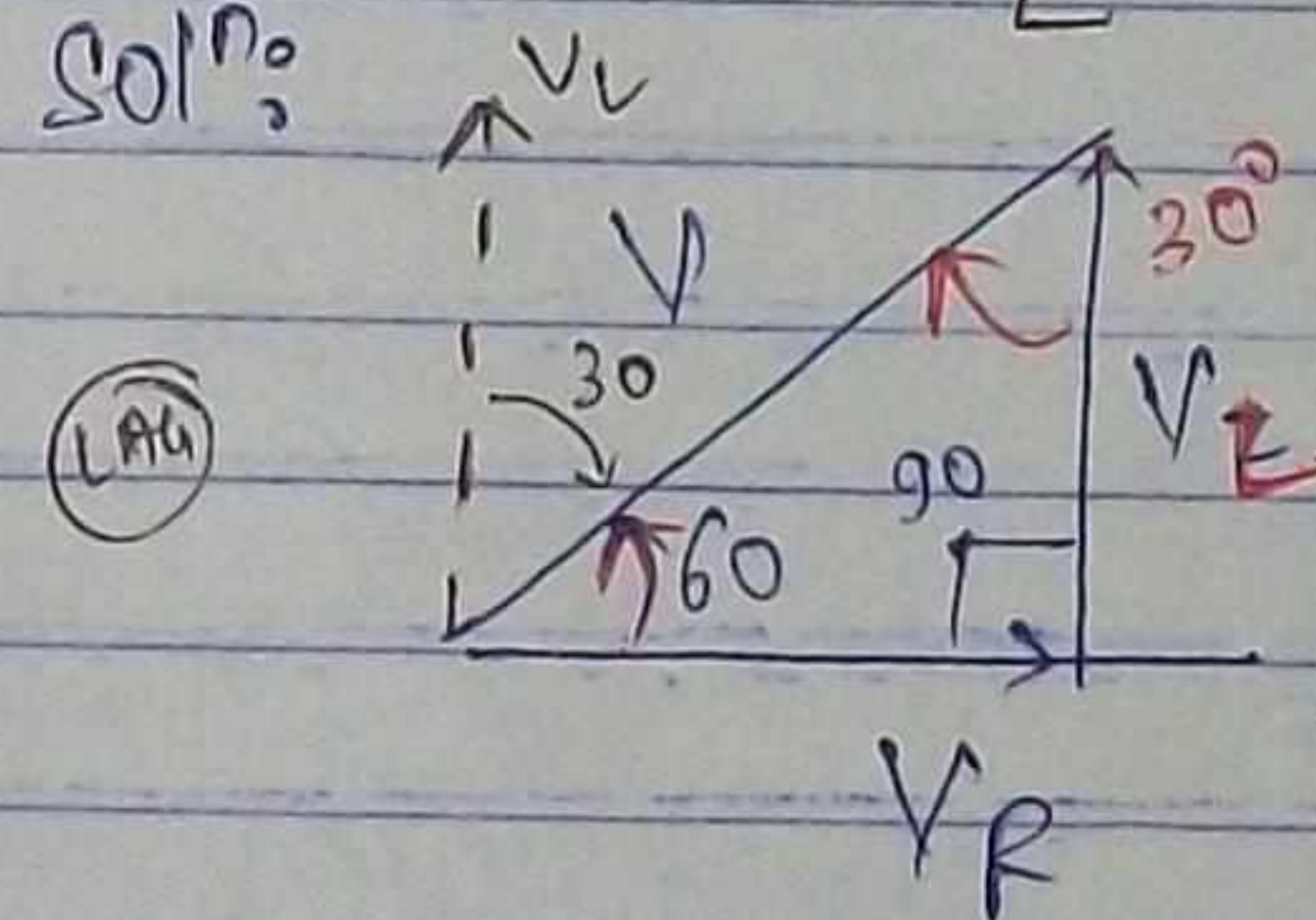
$$V = 9 \sin(t + 30^\circ)$$

$$I = 3 \sin(2t + 60^\circ)$$

— By using above eqⁿ it is not possible to design the circuit since freq. of voltage and current are unequal.

Ques. find angle made by source voltage wrt. V_L

Solⁿ:



$$\cos \theta = \frac{V_R}{V}$$

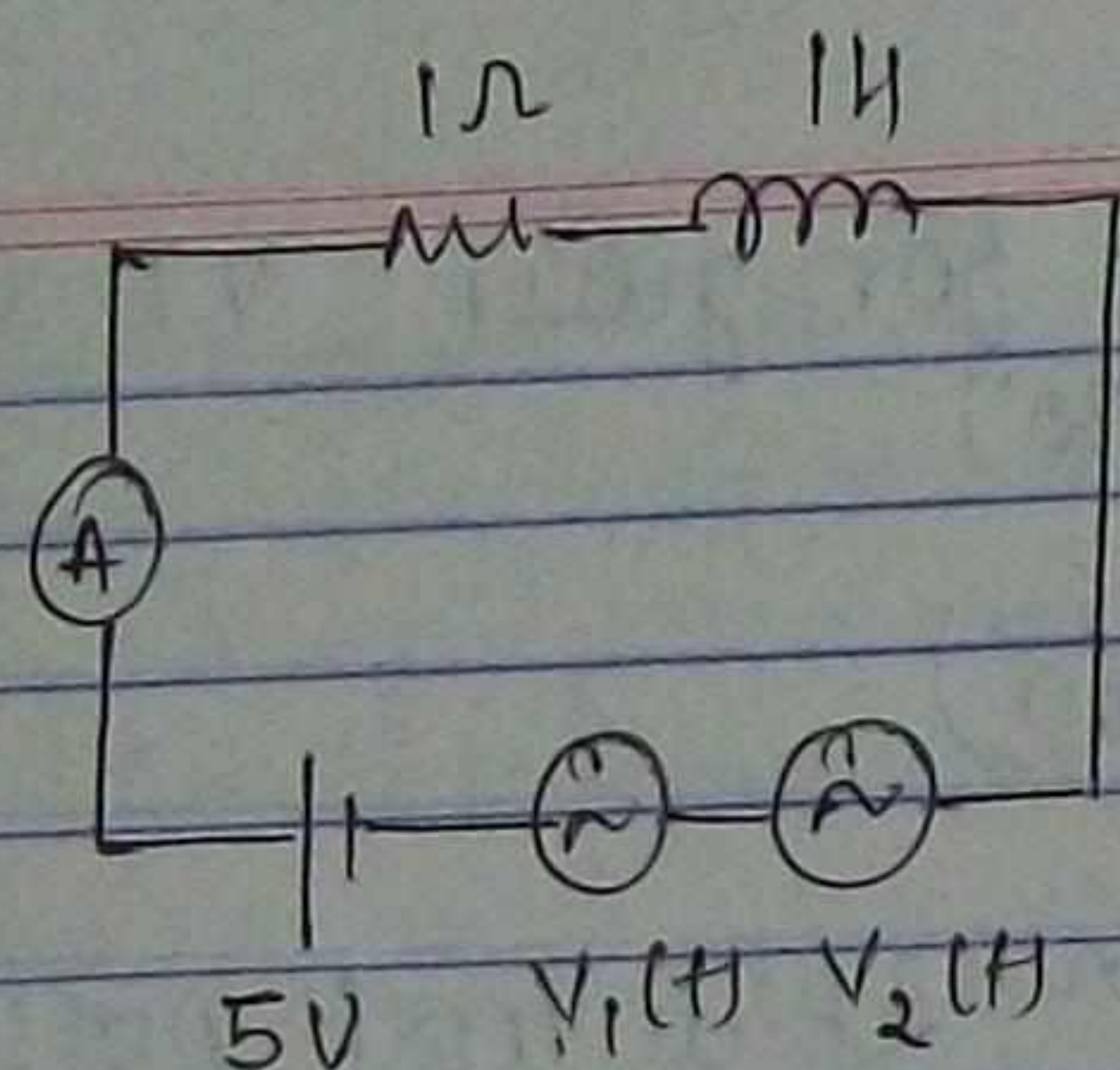
$$\theta = \tan^{-1} \left(\frac{X_L}{R} \right)$$

$$\theta = \tan^{-1} \left(\frac{17.32}{10} \right) = 60^\circ$$

Ans. -30° ✓

★★

Ques.



Find ammeter reading and power factor?

$$V_1(t) = 10 \sin t$$

$$V_2(t) = 10\sqrt{5} \sin 2t$$

Soln:

By superposition:

(1) Considering V_1

$$X_{L1} = \omega_1 L = 1$$

$$Z_1 = \sqrt{R^2 + X_{L1}^2} = \sqrt{2}$$

$$I_1 = \frac{V_1}{Z_1} = \frac{10/\sqrt{2}}{\sqrt{2}} = 5$$

(2) Considering V_2

$$X_{L2} = \omega_2 L = 2$$

$$Z_2 = \sqrt{1^2 + 2^2} = \sqrt{5}$$

$$I_2 = \frac{V_2}{Z_2} = \frac{10\sqrt{5}/\sqrt{2}}{\sqrt{5}} = 10/\sqrt{2}$$

(3) Considering DC source

$$X_{L3} = \omega_3 L = 0$$

$L \rightarrow$ Short circuit

$$I_3 = \frac{V_3}{R} = \frac{5}{1} = 5$$

Net current $I = \sqrt{5^2 + \left(\frac{10}{\sqrt{2}}\right)^2 + 5^2}$

frequency are unequal

$$I = \sqrt{I_1^2 + I_2^2 + I_3^2}$$

$$I = 10 \text{ A}$$

Cannot use sine freq. are diff.

power factor

$$\cos \theta = \frac{R}{Z} = \frac{P}{S} = \frac{V_R}{V}$$

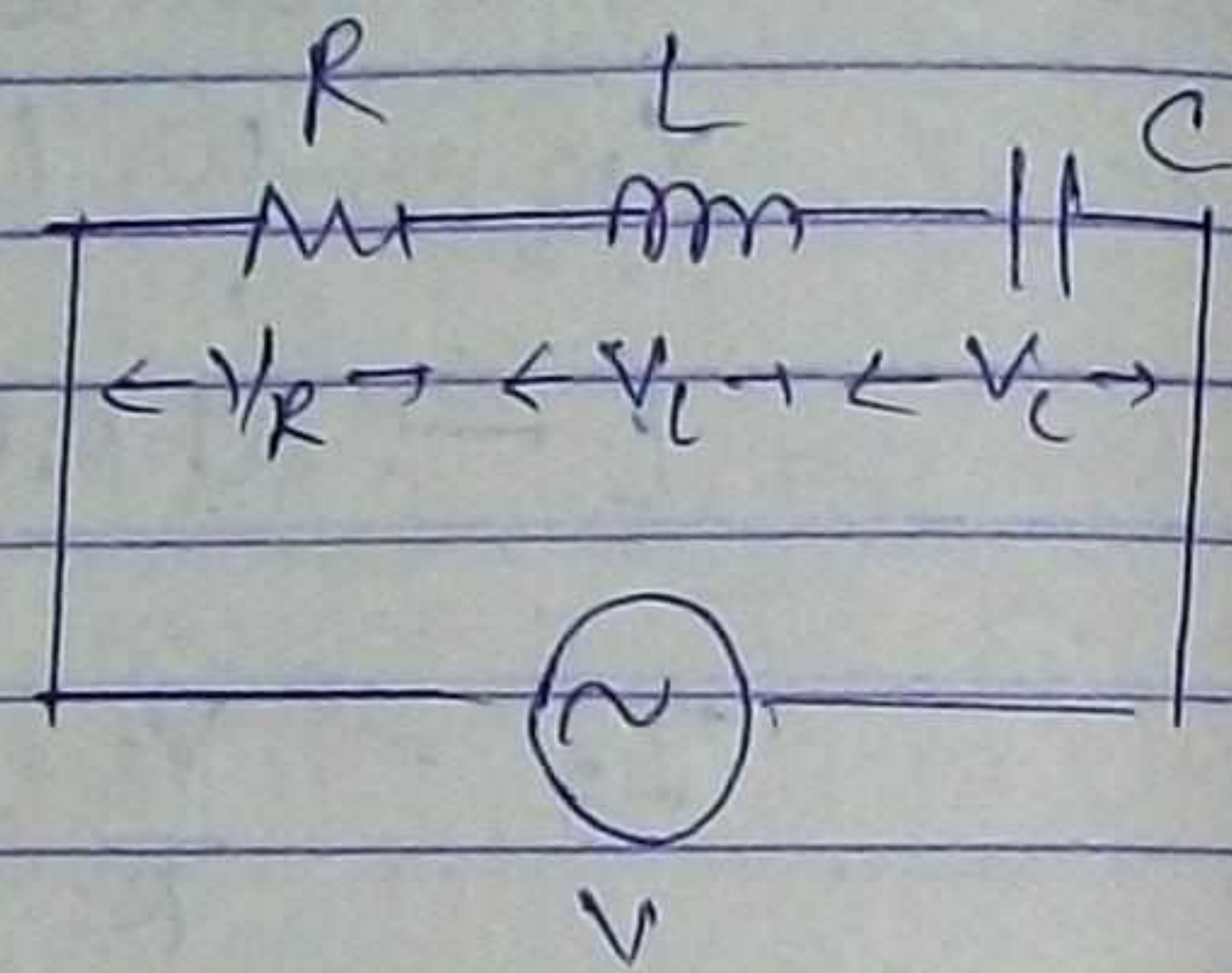
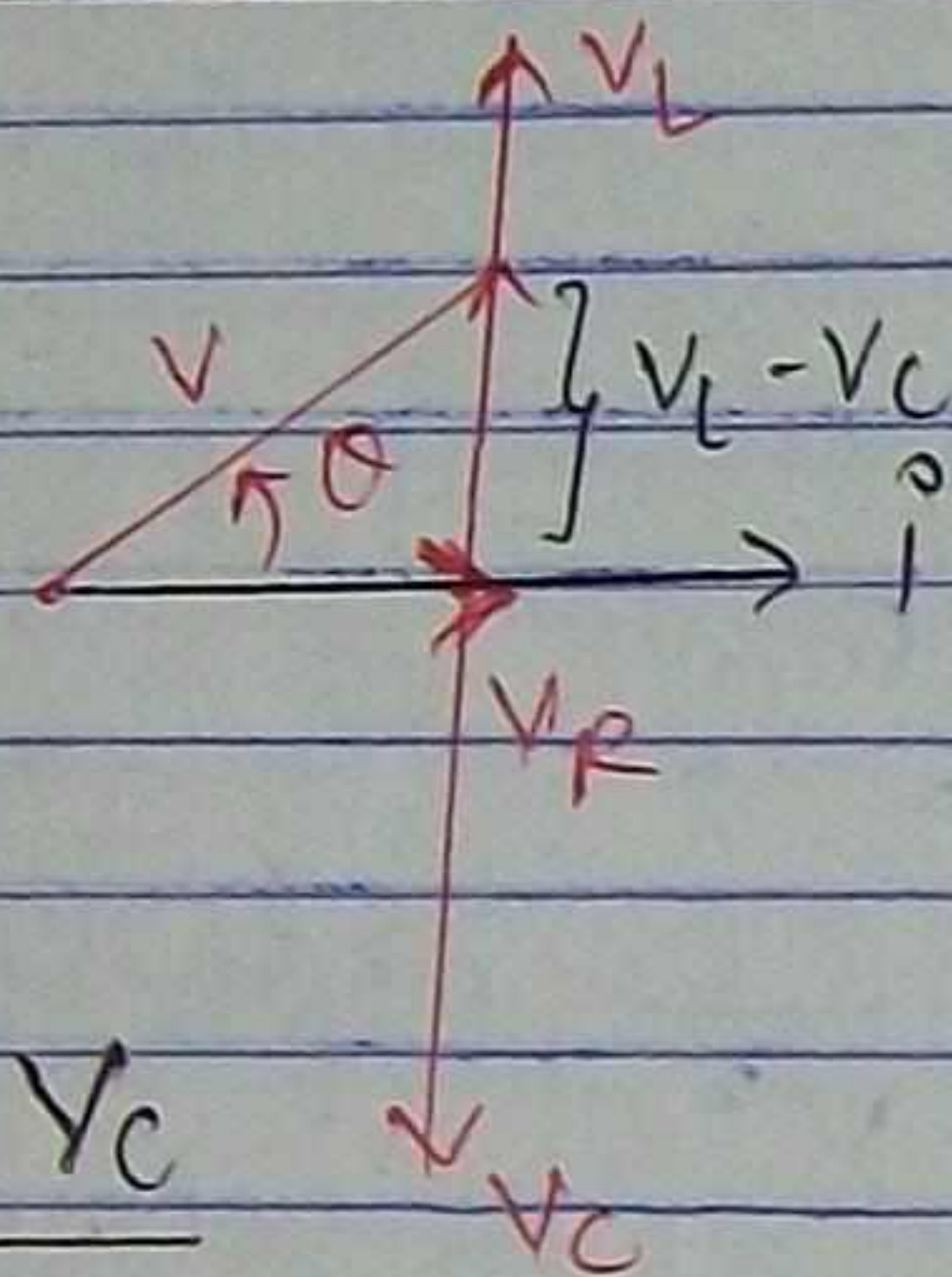
$$\cos \theta = \frac{P}{S} = \frac{i^2 R}{V i}$$

$$\cos \theta = \frac{i R}{V} = \frac{10 \times 1}{10}$$

$$V_{eff} = \sqrt{\left(\frac{10}{\sqrt{2}}\right)^2 + \left(\frac{10\sqrt{5}}{\sqrt{2}}\right)^2 + 5^2} =$$

21/10/11

⇒ RLC SERIES CIRCUIT —



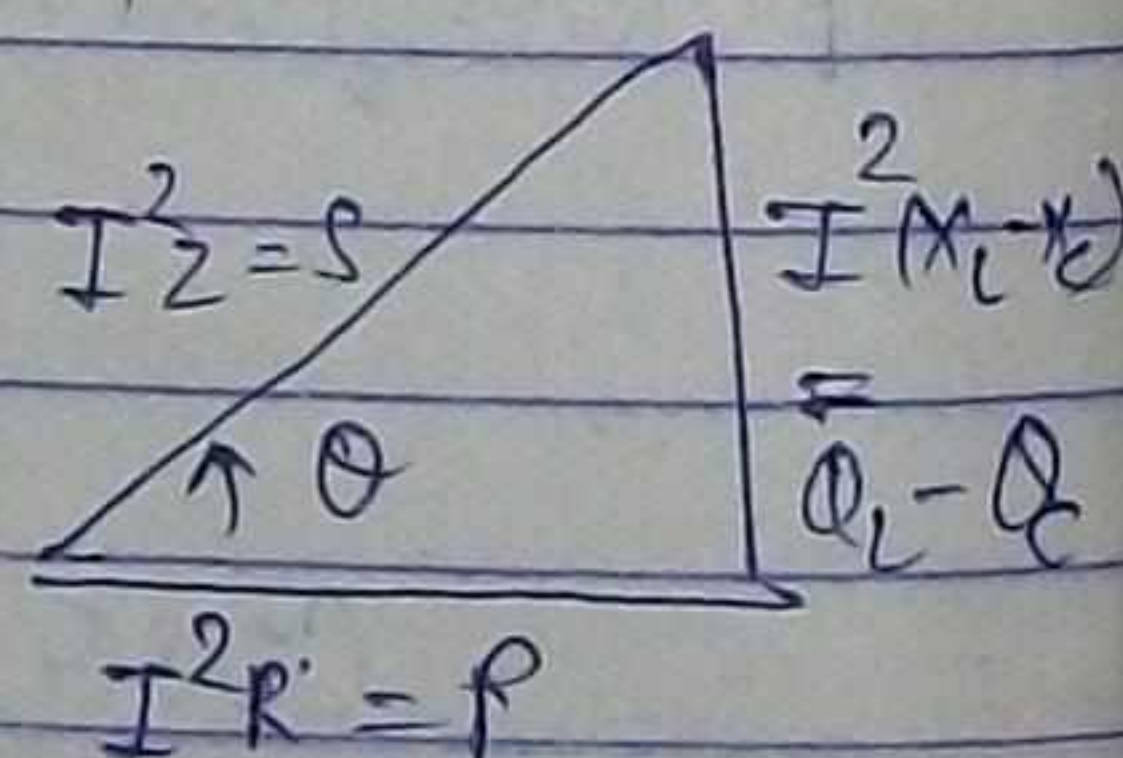
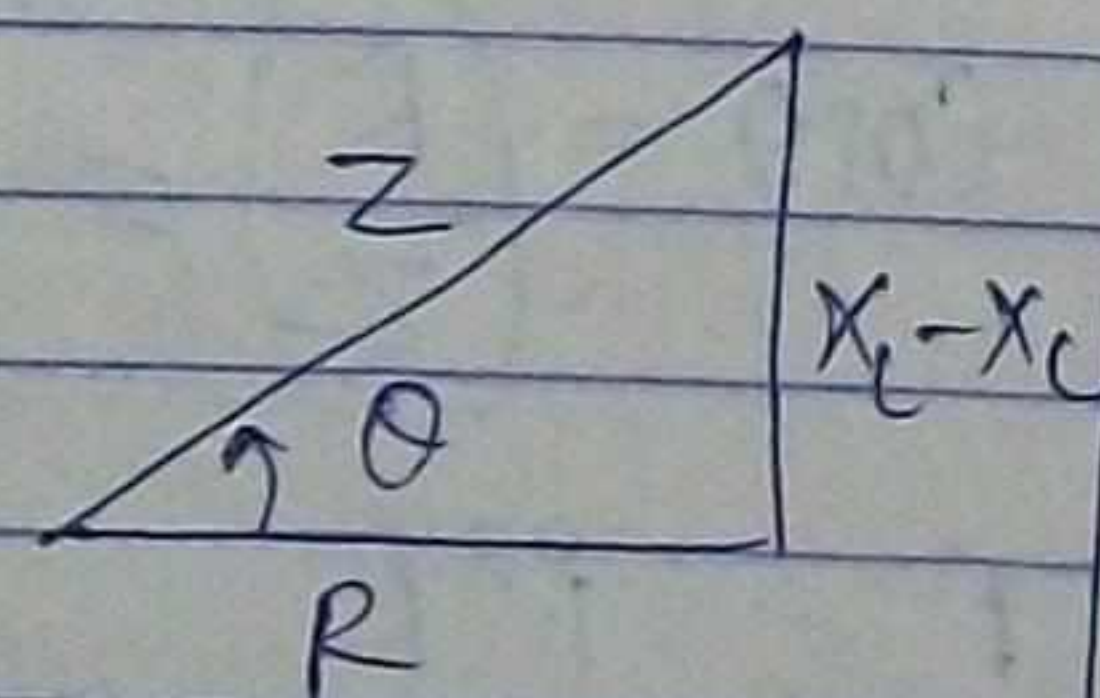
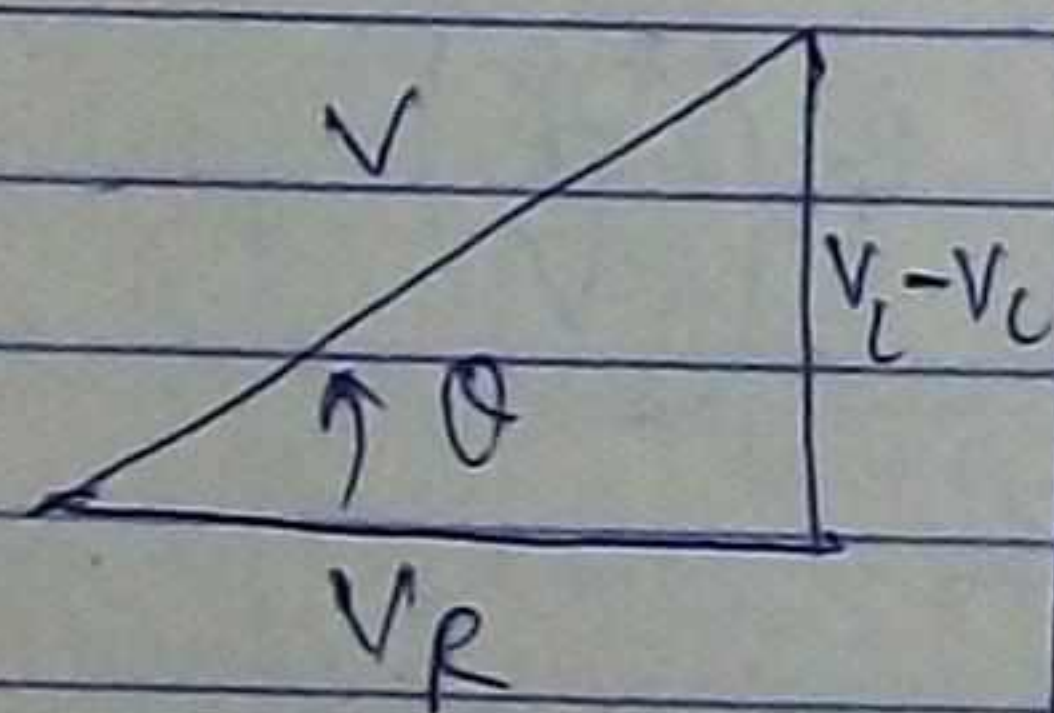
(i) $X_L > X_C$

By KVL,

$$V = V_R \angle 0^\circ + V_L \angle 90^\circ + V_C \angle -90^\circ$$

$$IZ = IR + jIX_L - jIX_C$$

$$Z = R + j(X_L - X_C)$$



$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

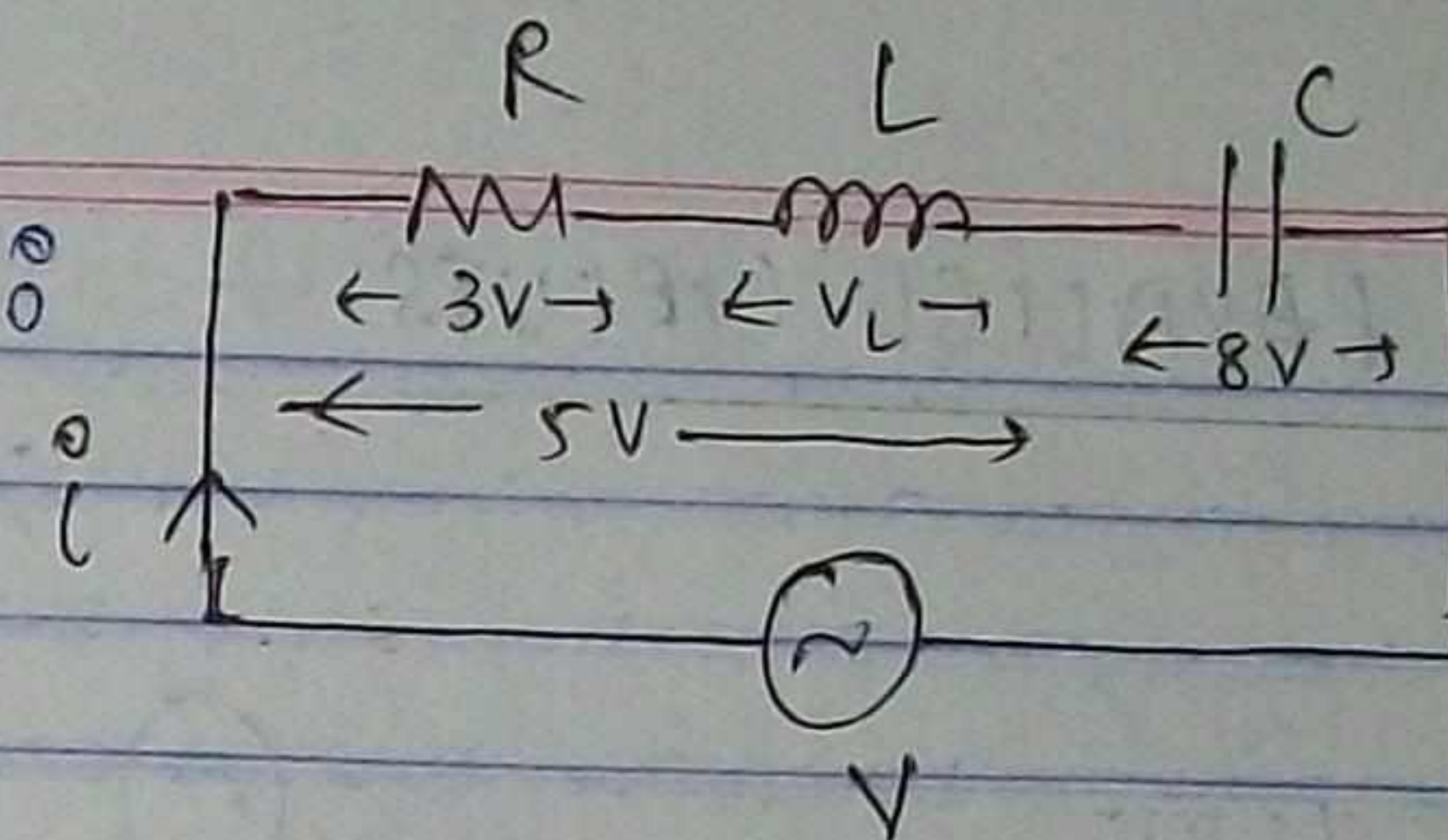
$$S = \sqrt{P^2 + (Q_L - Q_C)^2}$$

$$\theta = \tan^{-1} \left(\frac{V_L - V_C}{V_R} \right)$$

$$\theta = \tan^{-1} \left(\frac{X_L - X_C}{R} \right)$$

$$\theta = \tan^{-1} \left(\frac{Q_L - Q_C}{P} \right)$$

Ques. find V_L & V_C :



Solⁿ :

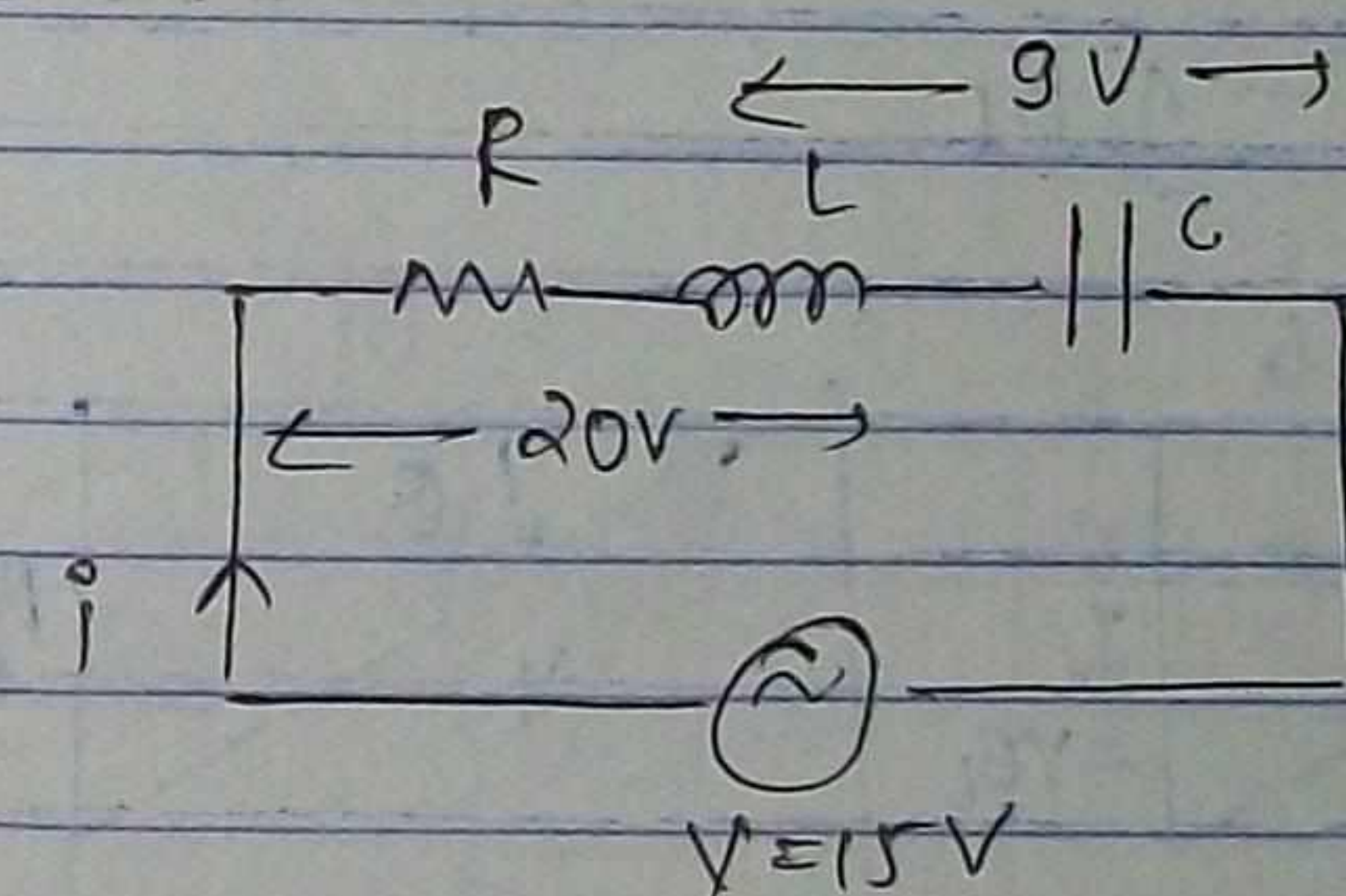
$$5 = \sqrt{3^2 + V_L^2} \quad , \quad V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$V_L = 4$$

$$= \sqrt{3^2 + (4 - 8)^2}$$

$$V = 5$$

Ques. find V_C :



Solⁿ :

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$15^2 = V_R^2 + 9^2$$

$$V_R = 12$$

$$20^2 = V_R^2 + (V_L)^2$$

$$400 - 144 = V_L^2$$

$$V_L = 16$$

$$V_L - V_C = 9$$

$$V_C = 16 - 9 = 7$$

$$V_C = 7V$$

⇒ R-L PARALLEL CIRCUITS -

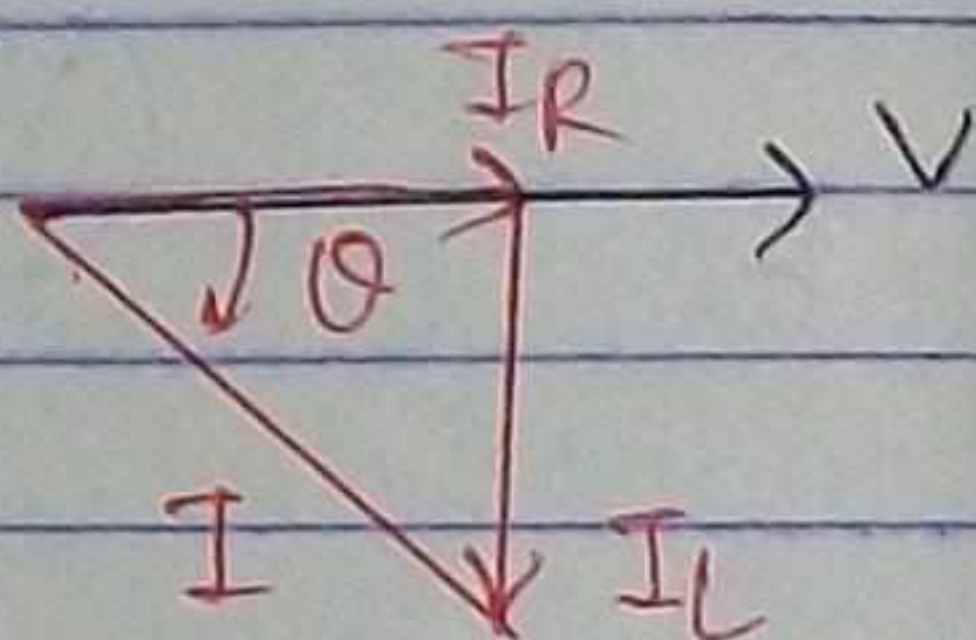
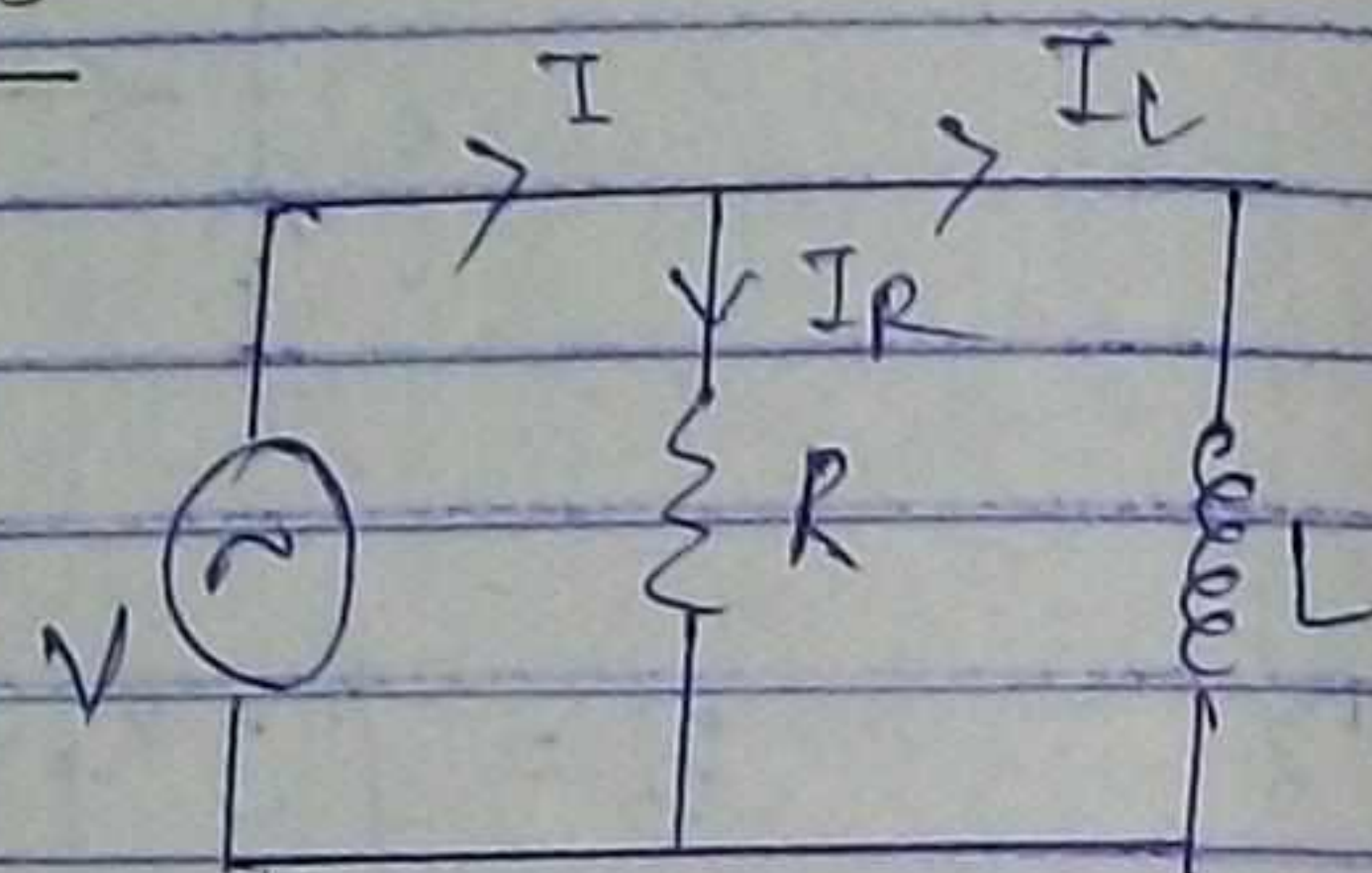
By KCL -

$$I = I_R + I_L \angle -90^\circ$$

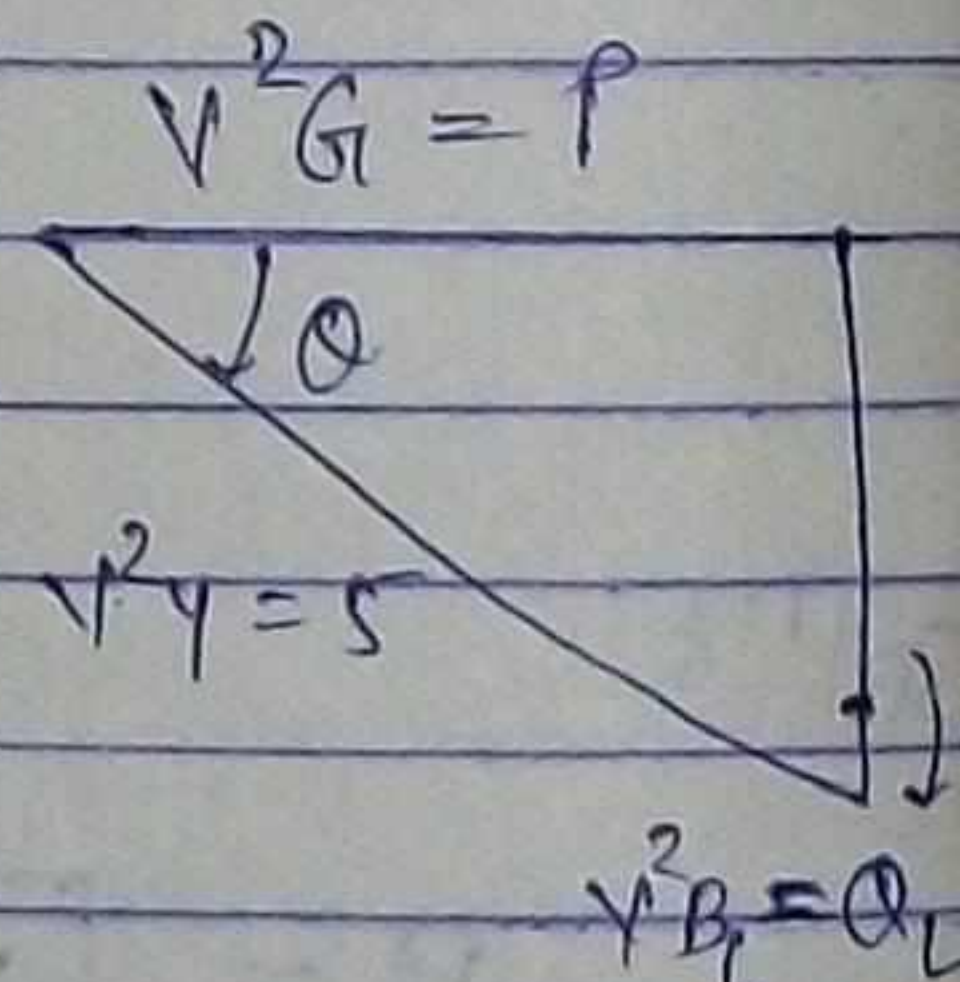
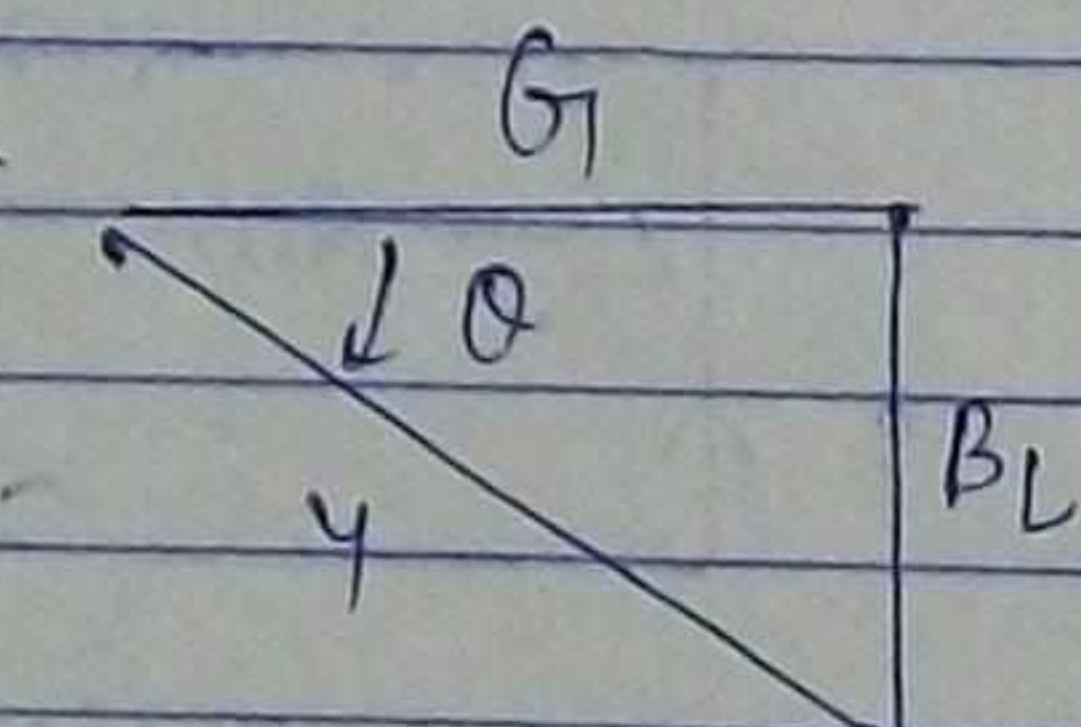
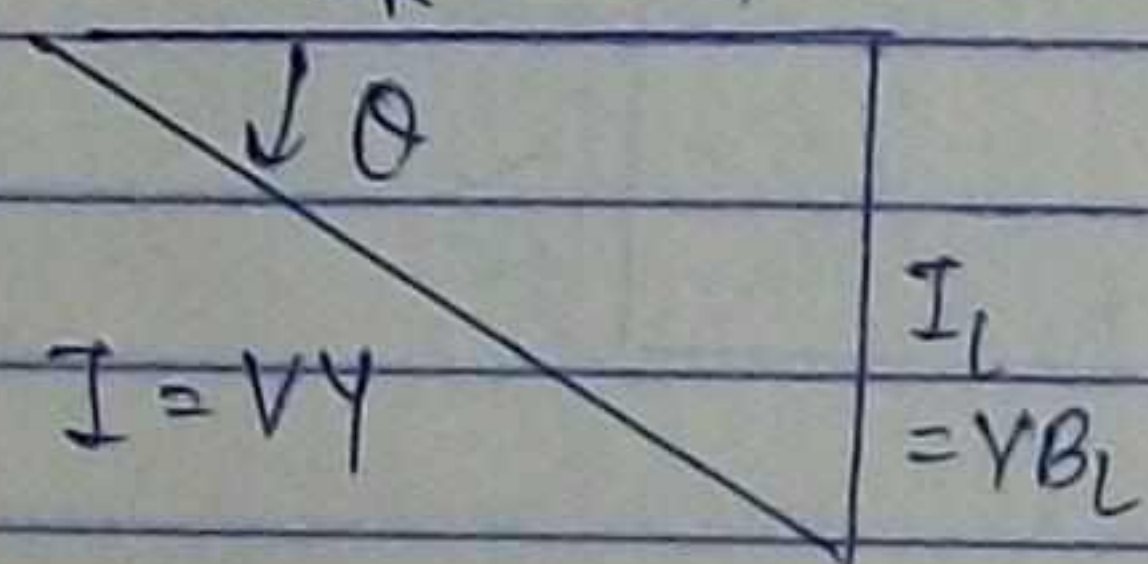
$$\frac{V}{Z} = \frac{V}{R} - j \frac{V}{X_L}$$

$$VY = VG - jVB_L$$

$$Y = G - jB_L$$



$$I_R = VG$$



$$I = \sqrt{I_R^2 + I_L^2}$$

$$Y = \sqrt{G^2 + B_L^2}$$

$$S = \sqrt{P^2 + Q_L^2}$$

$$\theta = \tan^{-1} \left(\frac{-I_L}{I_R} \right)$$

$$\theta = \tan^{-1} \left(\frac{-B_L}{G} \right)$$

$$\theta = \tan^{-1} \left(\frac{-Q_L}{P} \right)$$

POWER FACTOR,

$$P.F. = \cos \theta = \frac{I_R}{I} = \frac{G}{Y} = \frac{P}{S}$$

(LAGGING)

⇒ R-C PARALLEL CIRCUITS -

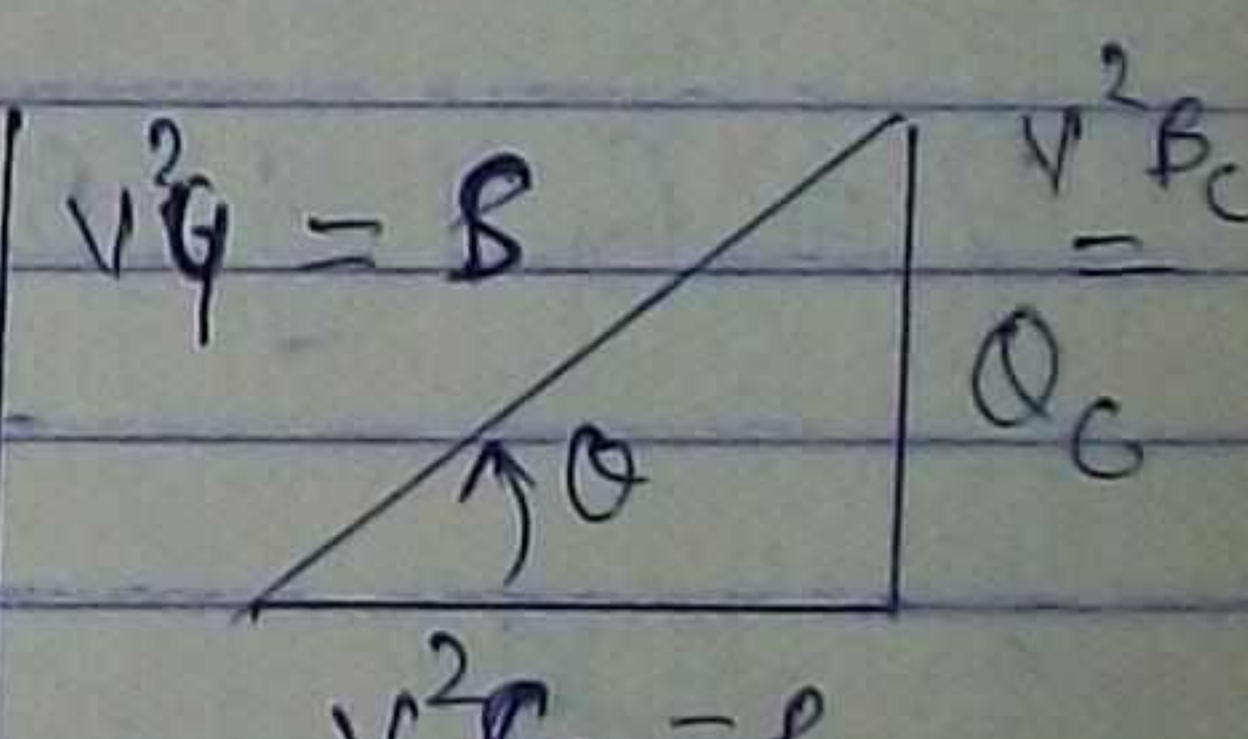
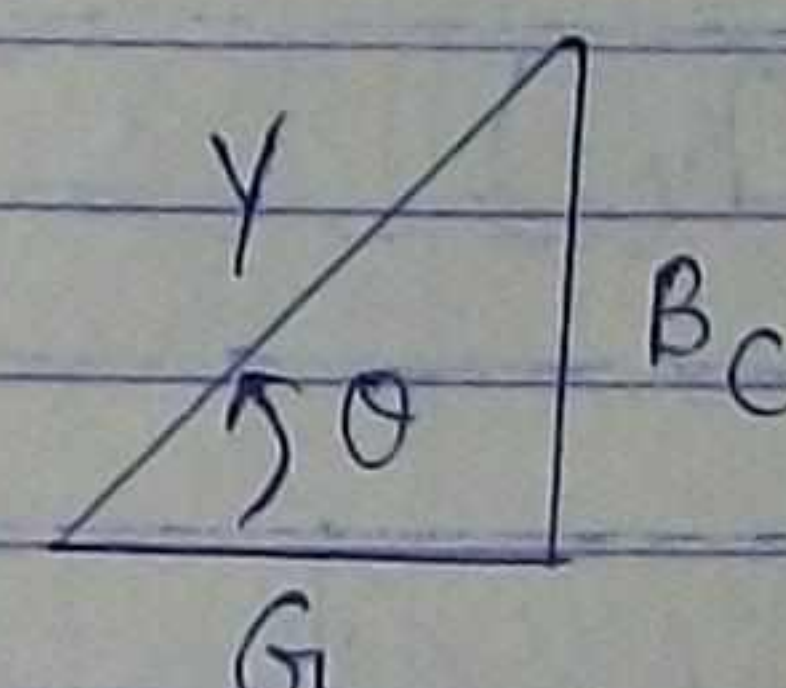
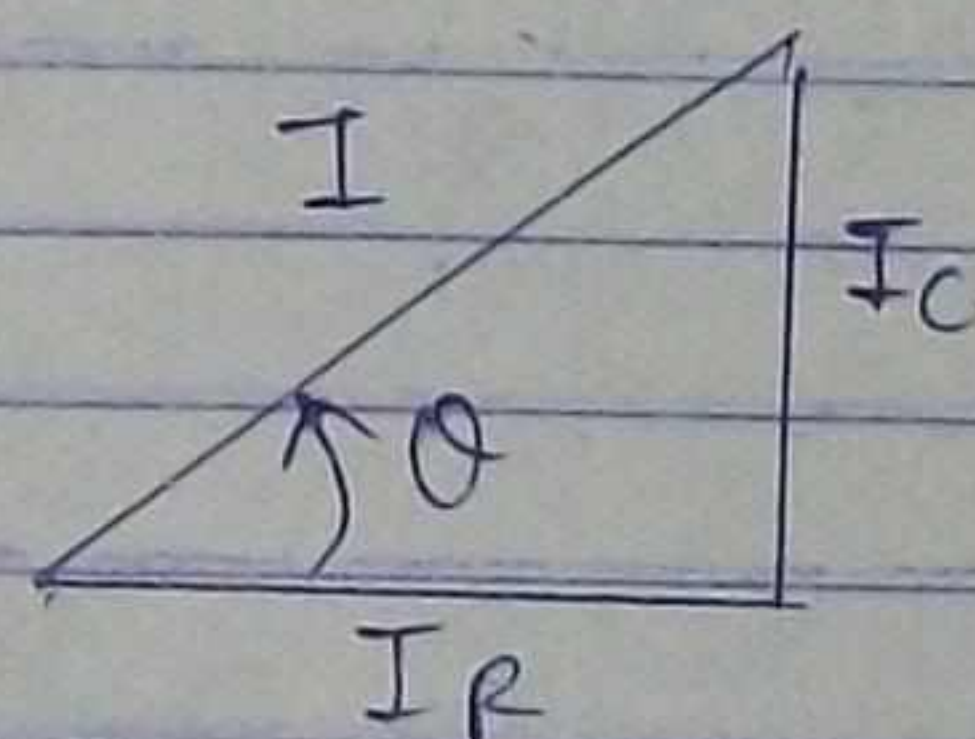
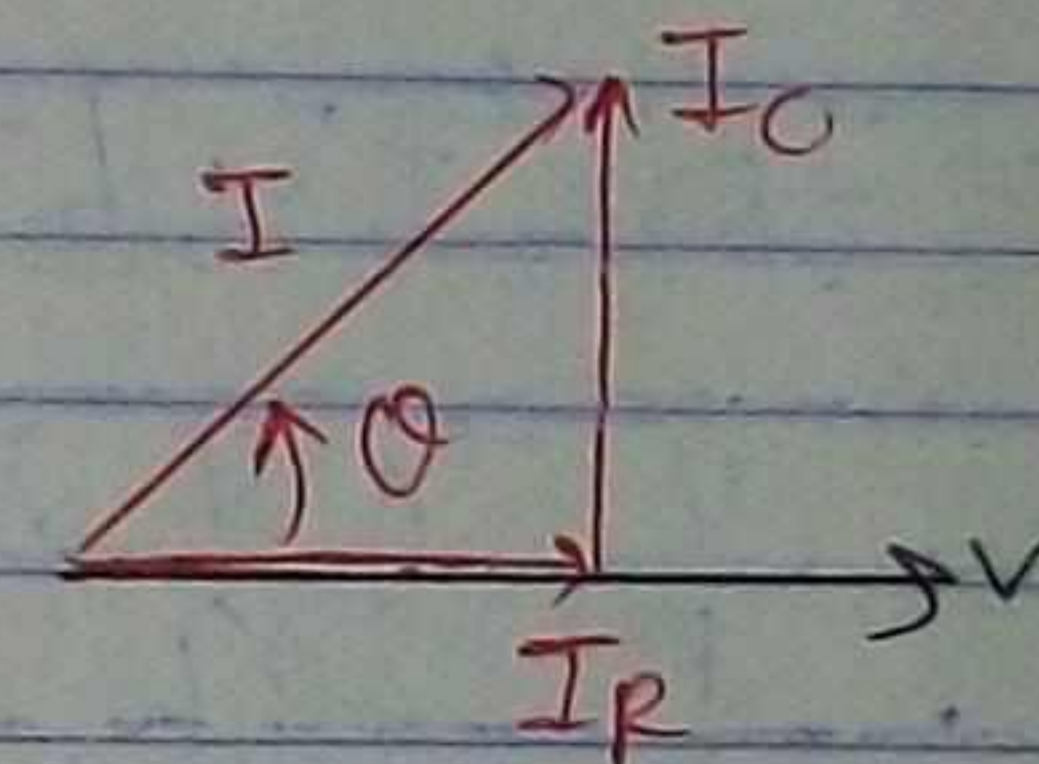
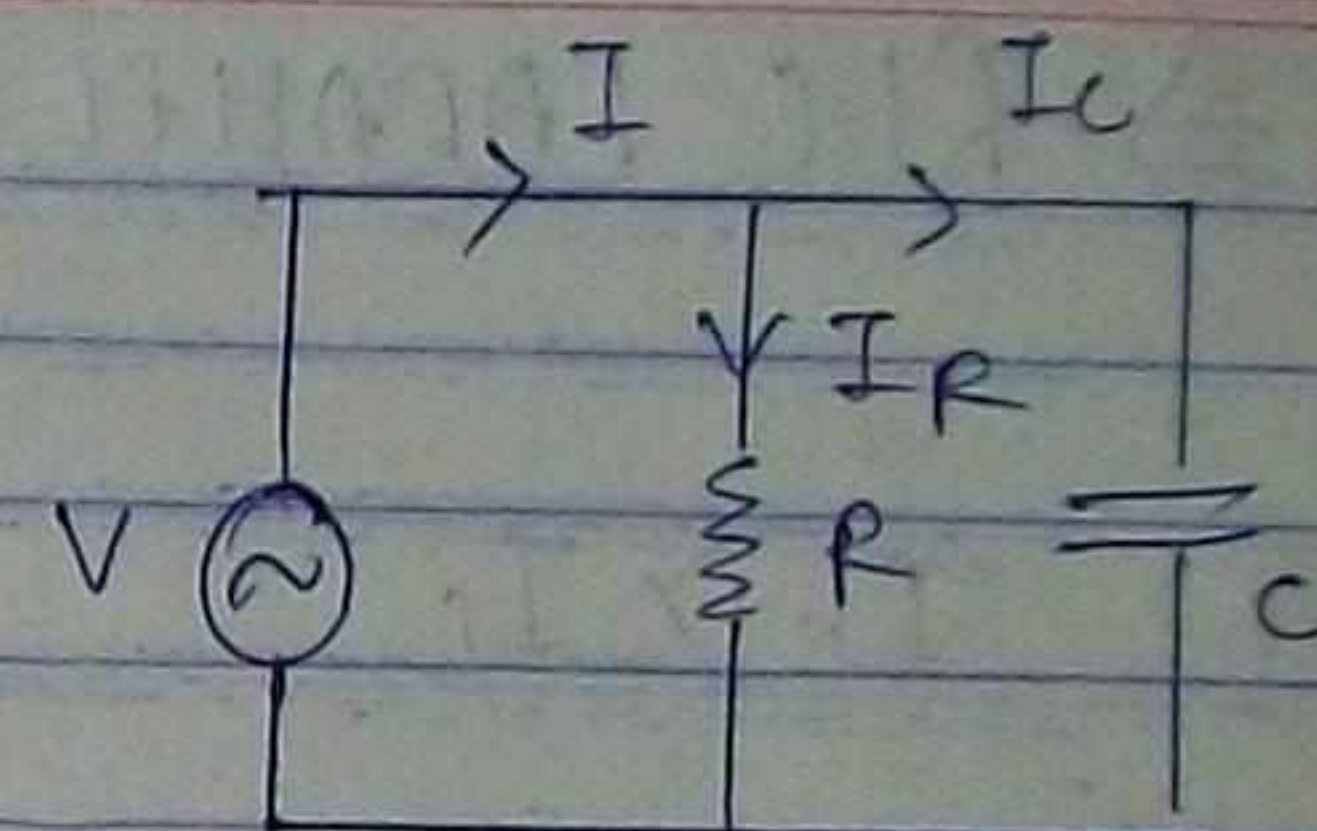
By KCL -

$$I = I_R \angle 0 + I_C \angle 90^\circ$$

$$\frac{V}{Z} = \frac{V}{R} + j \frac{V}{X_C}$$

$$VY = VG + jVB_C$$

$$Y = G + jB_C$$



$$I = \sqrt{I_R^2 + I_C^2}$$

$$Y = \sqrt{G^2 + B_C^2}$$

$$S = \sqrt{P^2 + Q_C^2}$$

$$\theta = \tan^{-1} \left(\frac{I_C}{I_R} \right)$$

$$\theta = \tan^{-1} \left(\frac{B_C}{G} \right)$$

$$\theta = \tan^{-1} \left(\frac{Q_C}{P} \right)$$

POWER FACTOR,

$$P.F. = \cos \theta = \frac{I_R}{I} = \frac{G}{Y} = \frac{P}{S}$$

(LEADING)

⇒ RLC PARALLEL CIRCUITS -

① $I_C > I_L$

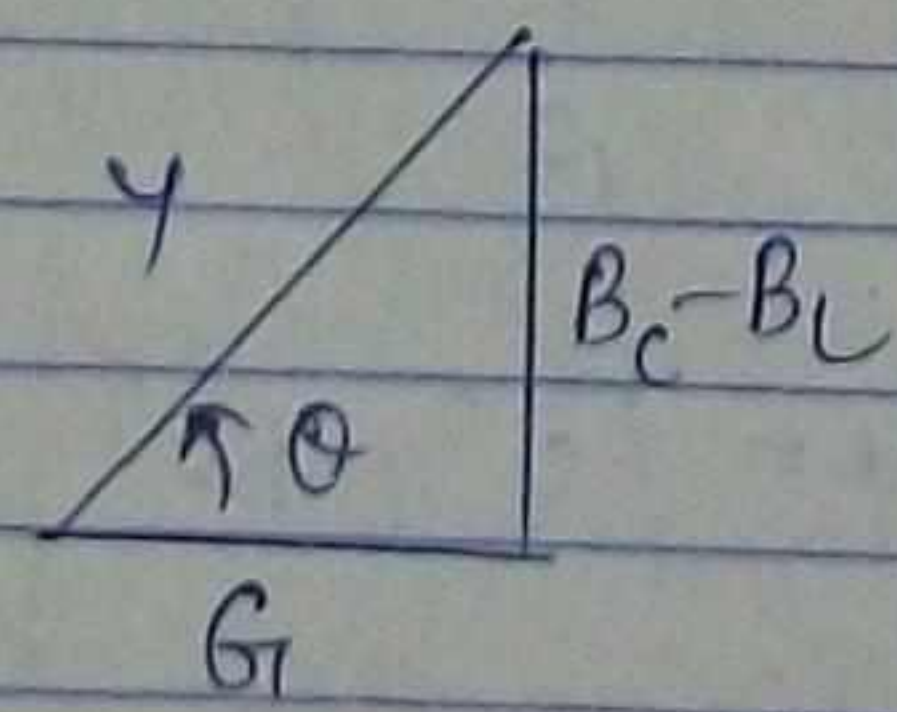
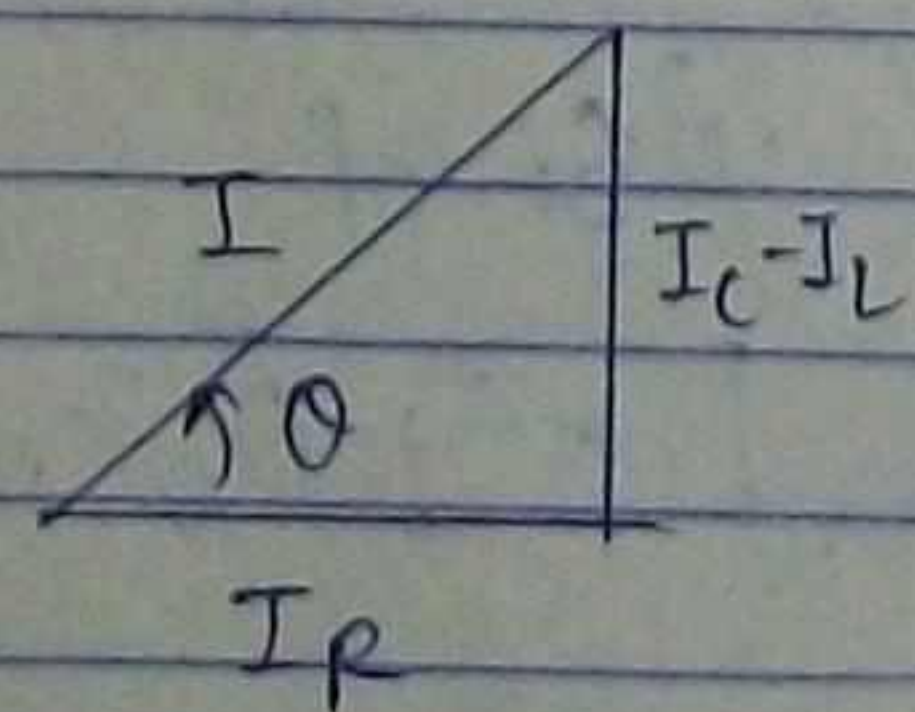
By KCL,

$$I = I_R \angle 0^\circ + I_L \angle -90^\circ + I_C \angle 90^\circ$$

$$\frac{V}{Z} = \frac{V}{R} - j \frac{V}{X_L} + j \frac{V}{X_C}$$

$$VY = VG + j(B_C - B_L)V$$

$$Y = G + j(B_C - B_L)$$

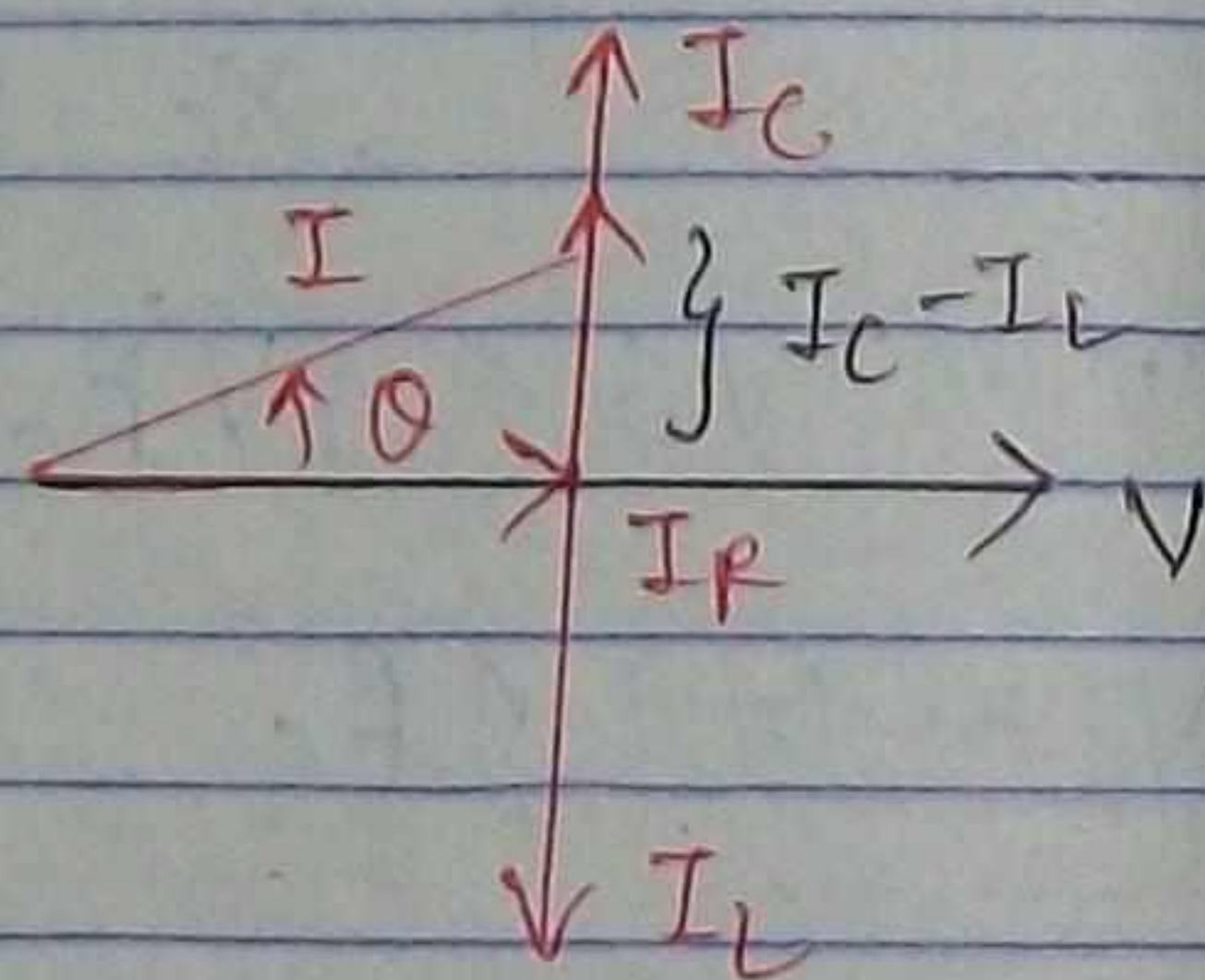
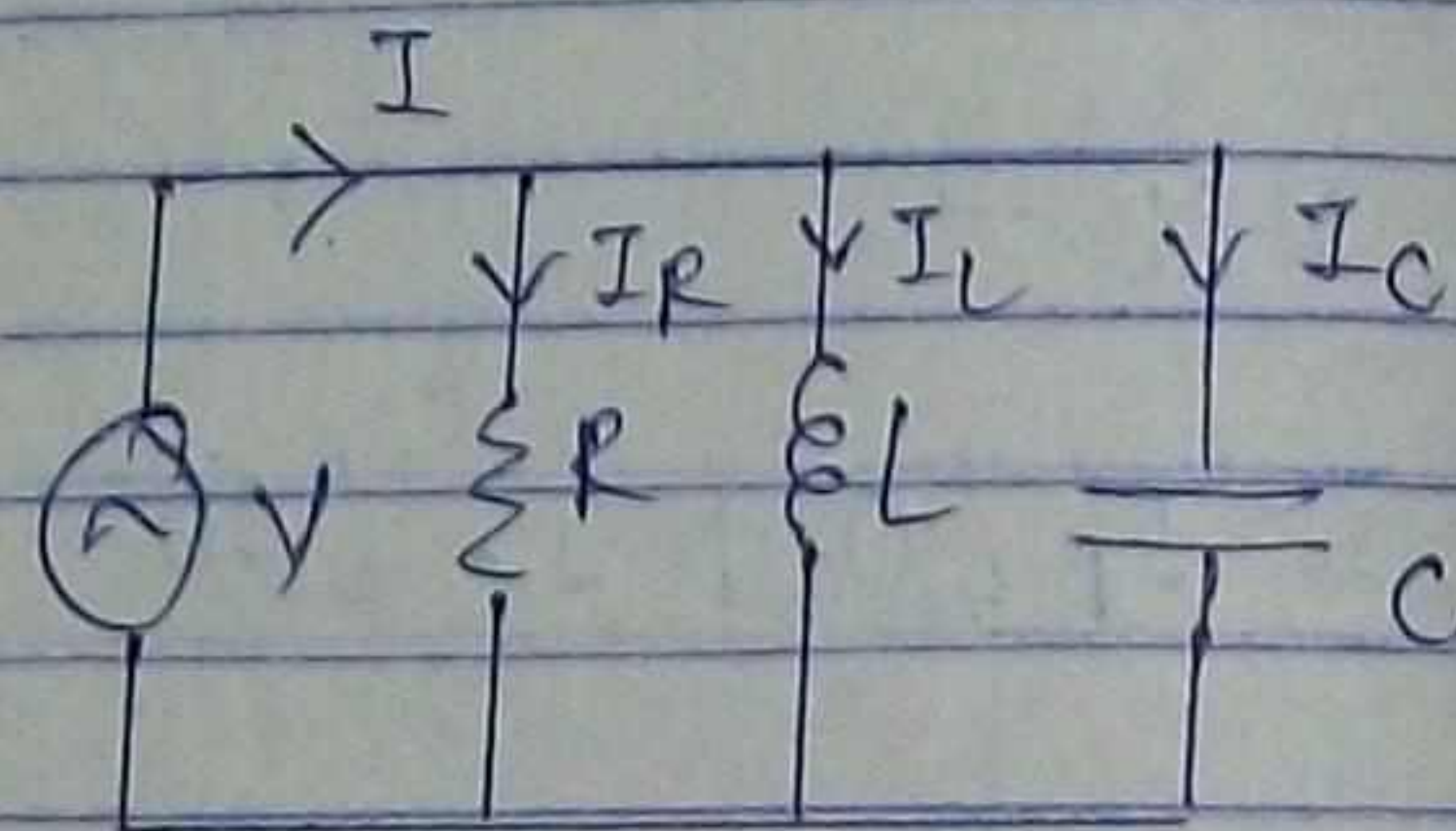


$$I = \sqrt{I_R^2 + (I_C - I_L)^2}$$

$$Y = \sqrt{G^2 + (B_C - B_L)^2}$$

$$\theta = \tan^{-1} \left(\frac{I_C - I_L}{I_R} \right)$$

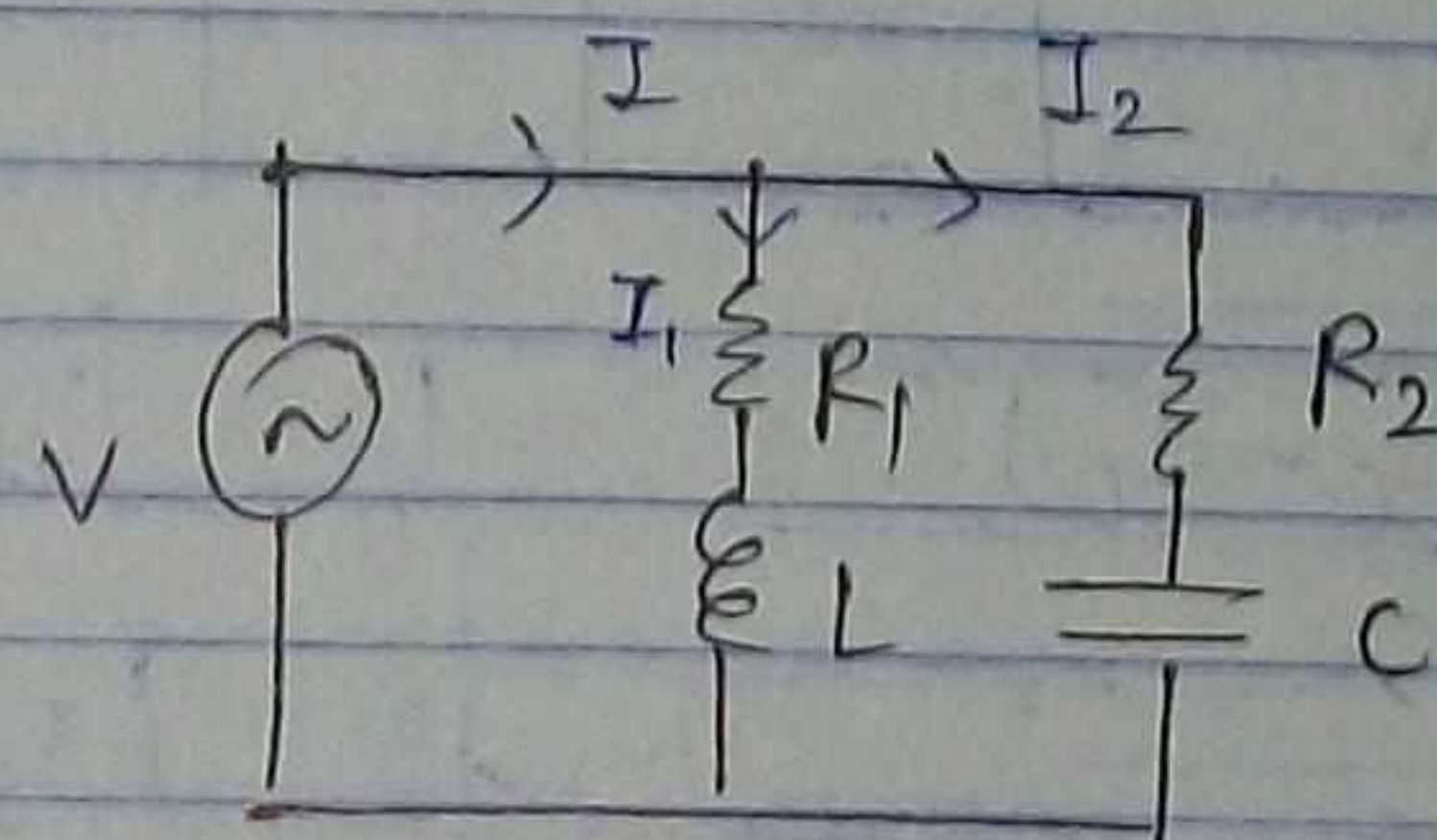
$$\theta = \tan^{-1} \left(\frac{B_C - B_L}{G} \right)$$



POWER
FACTOR

$$\cos \theta = \frac{I_R}{I}$$

#



$$I_1 = \frac{V}{R_1 + jX_L}$$

$$I_2 = \frac{V}{R_2 + jX_C}$$

$$\frac{V}{Z_1} = V \left[\frac{R_1}{R_1^2 + X_L^2} - j \frac{X_L}{R_1^2 + X_L^2} \right]$$

$$\frac{V}{Z_2} = V \left[\frac{R_2}{R_2^2 + X_C^2} + j \frac{X_C}{R_2^2 + X_C^2} \right]$$

$$Y_1 = G_1 - jB_L$$

$$Y_2 = G_2 + jB_C$$

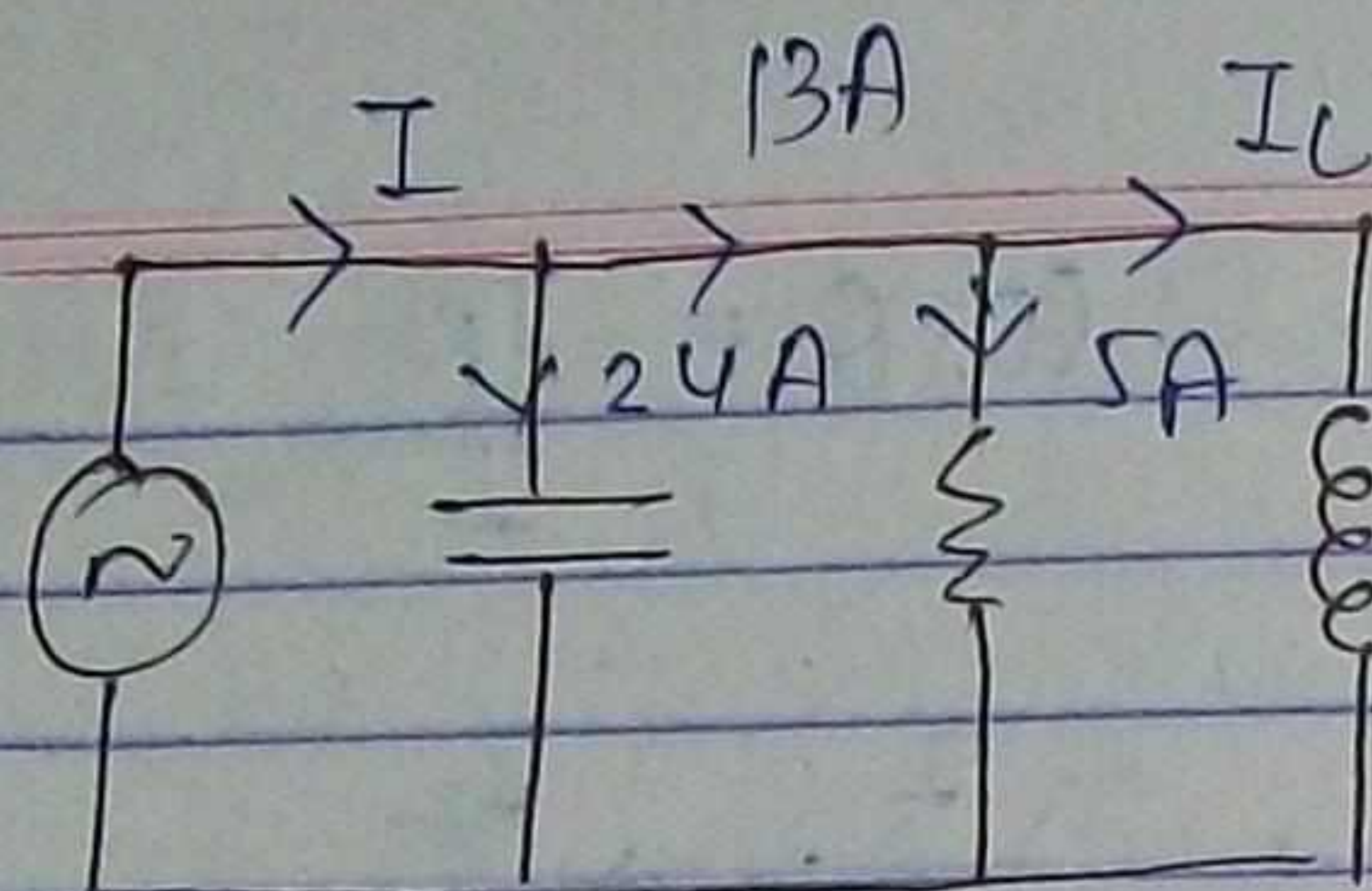
$$I = I_1 + I_2$$

$$VY_{eq} = VY_1 + VY_2$$

$$Y_{eq} = Y_1 + Y_2$$

$$Y_{eq} = (G_1 + G_2) + j(B_C - B_L)$$

Ques. Find I_L & I



Solⁿ:

$$I = \sqrt{I_R^2 + (I_C - I_L)^2}$$

$$13 = \sqrt{I_R^2 + I_L^2}$$

$$I = \sqrt{5^2 + (24 - I_L)^2}$$

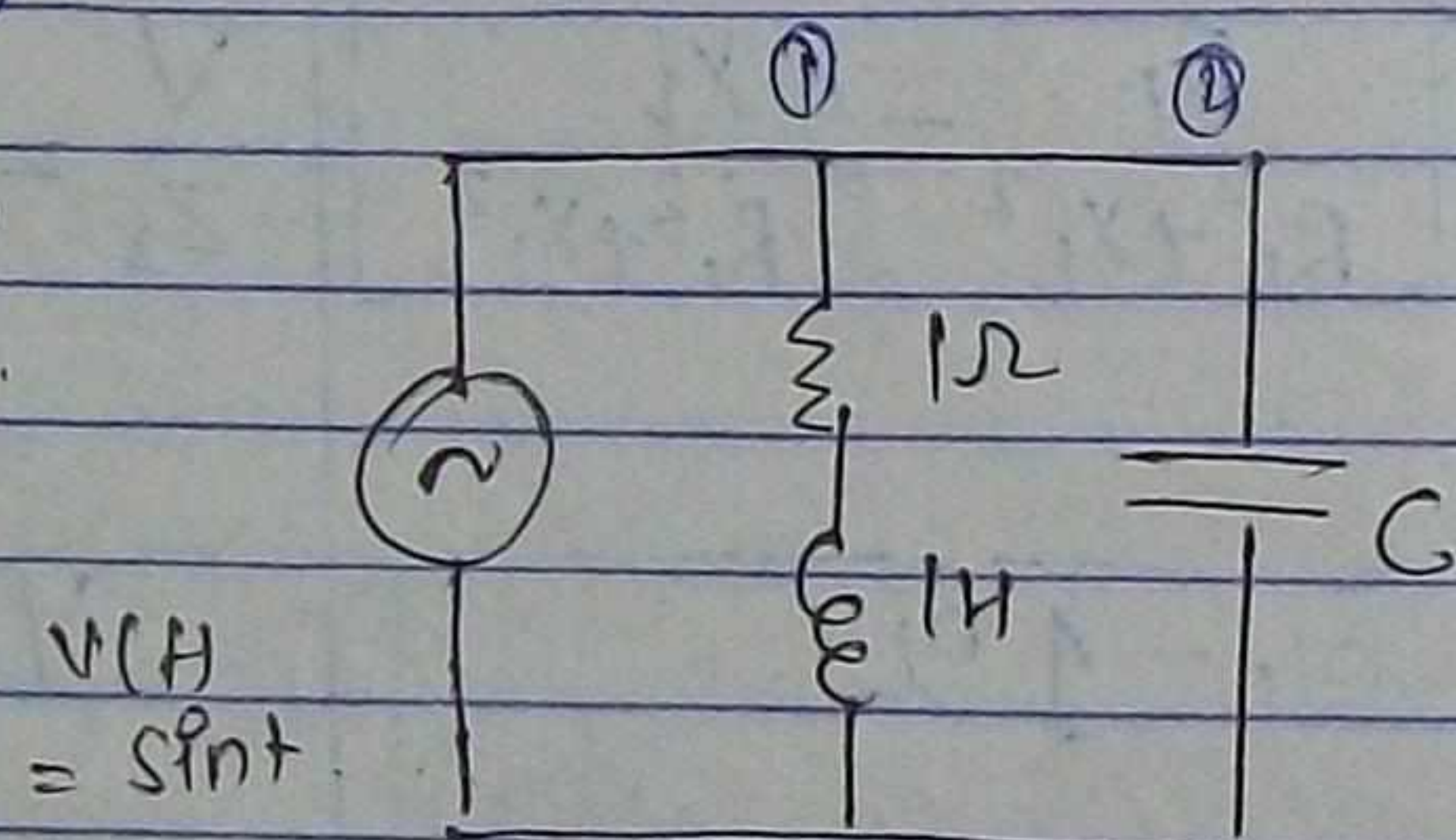
$$13 = \sqrt{5^2 + I_L^2}$$

$$I_L = 12$$

$$I = \sqrt{25 + 144} =$$

$$I = 13$$

Ques. Find C when
PF = 0.8 lag.



Solⁿ: for Branch (1):

$$X_L = \omega L = 1$$

$$G_1 = \frac{R_1}{R_1^2 + X_L^2} = \frac{1}{2}$$

$$B_L = \frac{X_L}{R_1^2 + X_L^2} = \frac{1}{2}$$

$$Y_1 = G_1 - jB_L = \frac{1}{2} - j\frac{1}{2}$$

for Branch (2):

$$Y_2 = jB_C = j\frac{1}{X_C}$$

$$Y_2 = j\omega C = jC$$

$$Y_{eq} = Y_1 + Y_2$$

$$= \frac{1}{2} - j\frac{1}{2} + jC$$

$$= \frac{1}{2} + j(C - \frac{1}{2})$$

$$PF = \cos \theta = \frac{G}{Y_{eq}} = 0.8$$

$$0.8 = \frac{1/2}{\sqrt{(1/2)^2 + (C - 1/2)^2}}$$

$$(0.8)^2 = \frac{(1/2)^2}{(1/2)^2 + (C - 1/2)^2}$$

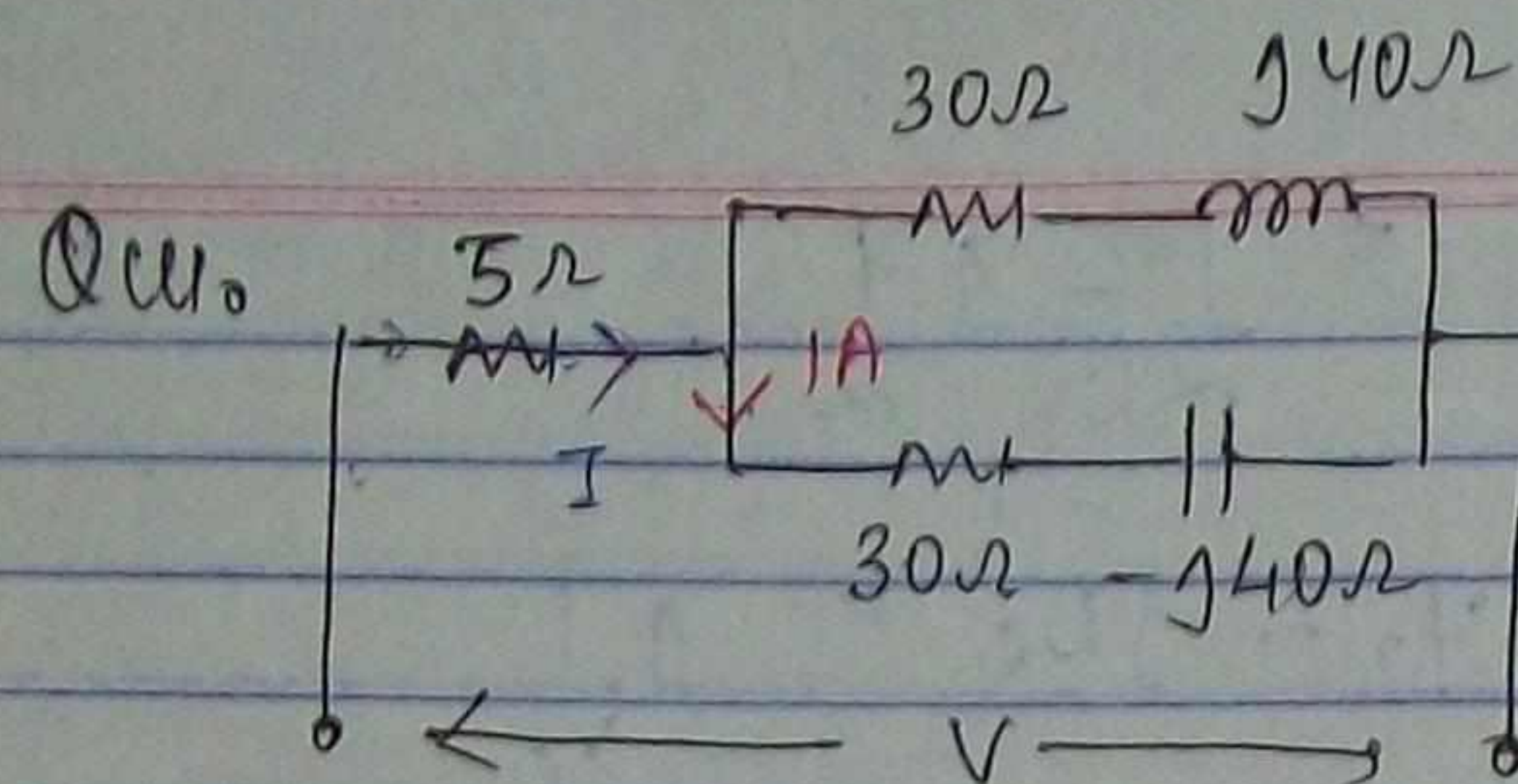
$$(0.8)^2 = \frac{(1/2)^2}{(1/2)^2 + (C - 1/2)^2}$$

$$C = \frac{7}{8}, \frac{1}{8}$$

$$\therefore C = 1/8$$

SINCE
LAGGING
 $B_L > B_C$

$$\frac{1}{2} > C$$



Find V.

Ans. 56V

Solⁿ: $I' = \frac{30 + j40}{30 + j40 + 30 - j40} \times I = 1A$

$I = 1.2 \angle -53.1^\circ$

$V = I R_{eq}$

$V = 1.2 \angle -53.1^\circ \left[5 + \frac{(30 + j40)(30 - j40)}{60} \right]$

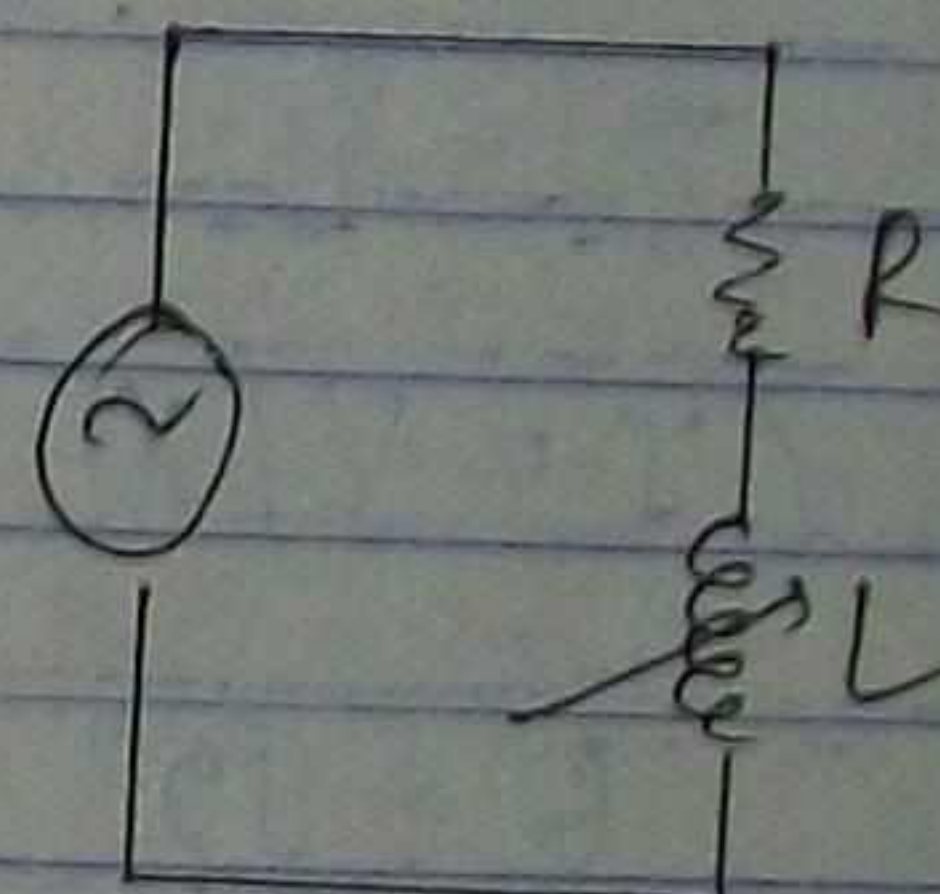
$V = 1.2 \times 46.66$

$V = 56 \angle -53.1^\circ$

LOCUS DIAGRAM -

- Locus diagrams are useful for analysis and designing of the circuits. ex - filters.
- The path traced by terminals of the current vectors by varying anyone of the circuit elements ~~or~~ by varying frequency is called as current Locus.

Ques. Draw the current locus for the circuit shown.



Solⁿ:

$X_L = 2\pi fL$

$X_L = 0$

$X_L \uparrow$

$X_L = \infty$

$Z = R$

$Z \uparrow$

$Z = \infty$

$\theta = 0$

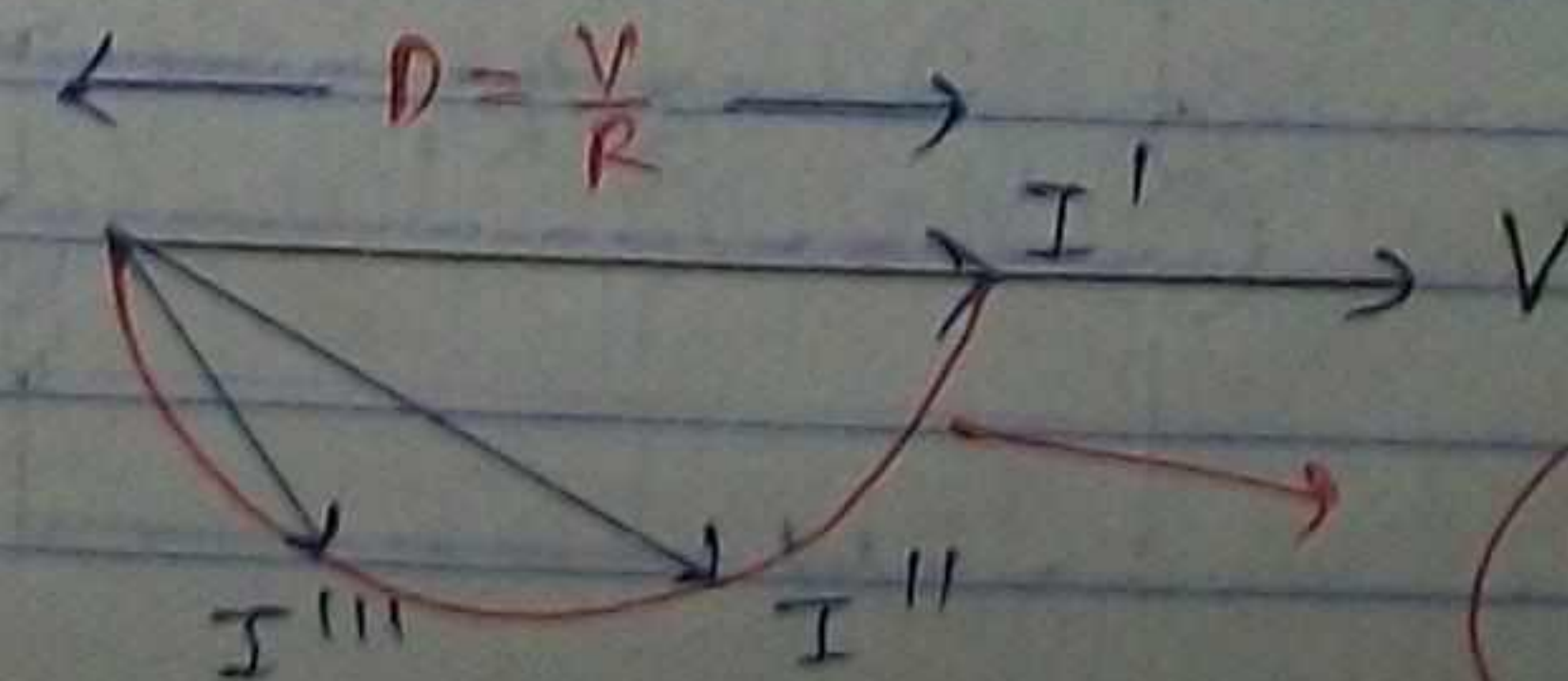
$I \downarrow$

$I = 0$

$I = V/R$

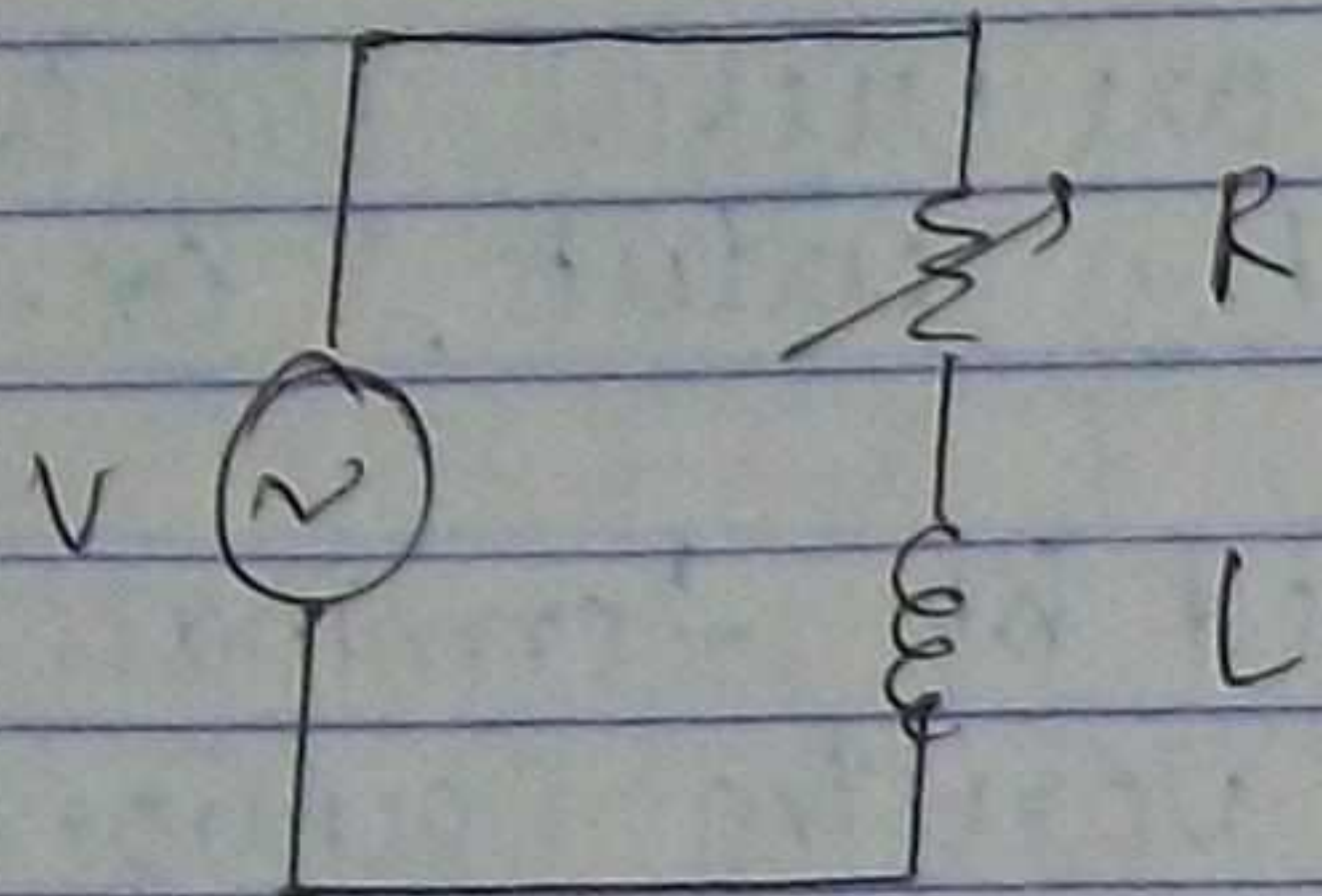
$\theta = \tan^{-1} \left(\frac{X_L}{R} \right) \uparrow$

DIAMETER



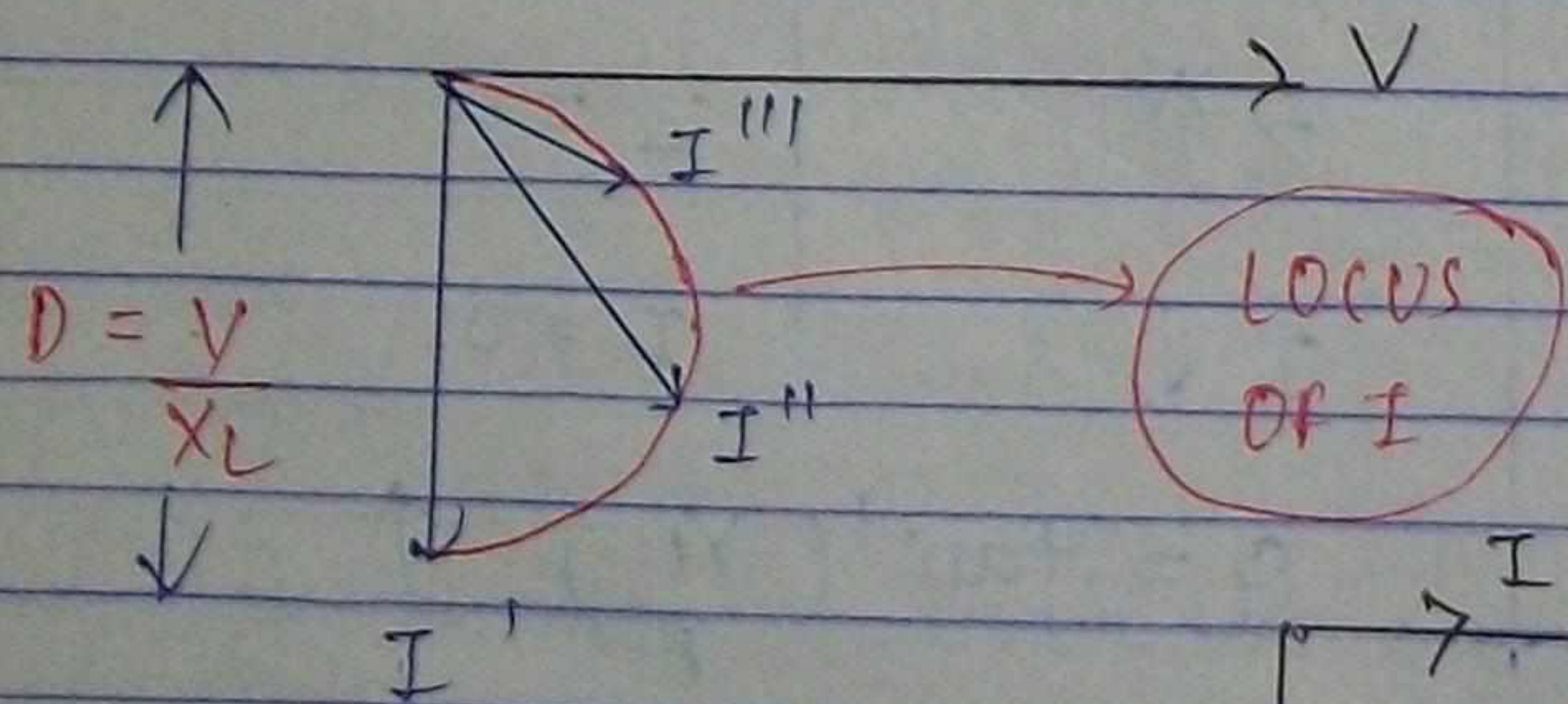
LOCUS OF I

Ques. Draw the current locus for the circuit shown:

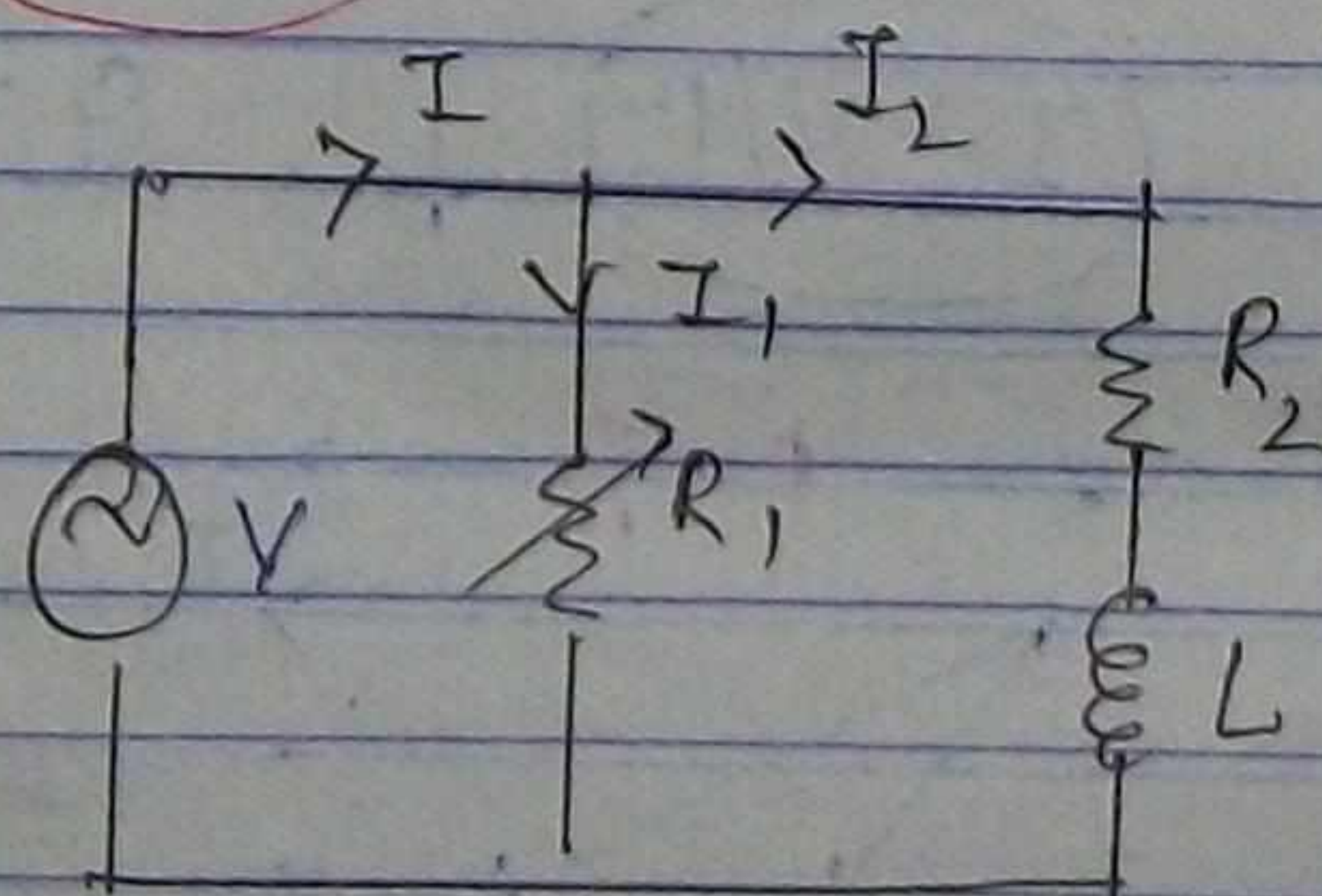


Solⁿ:

$R = 0$	$R \uparrow$	$R = \infty$
$Z = X_L$	$Z \uparrow$	$Z = \infty$
$I = V/X_L$	$I \downarrow$	$I = 0$
$\theta = 90^\circ$	$\theta = \tan^{-1}(X_L/R) \downarrow$	



Ques. Draw the current locus of I_1 and I of circuit



Solⁿ: let $R_1 = 0.1 \Omega$

$$I_1 = \frac{V}{R_1}$$

$$\theta_1 = 0$$

$$I = I_1 + I_2$$

$R_1 \uparrow$

$I_1 \downarrow$

$$\theta_1 = 0$$

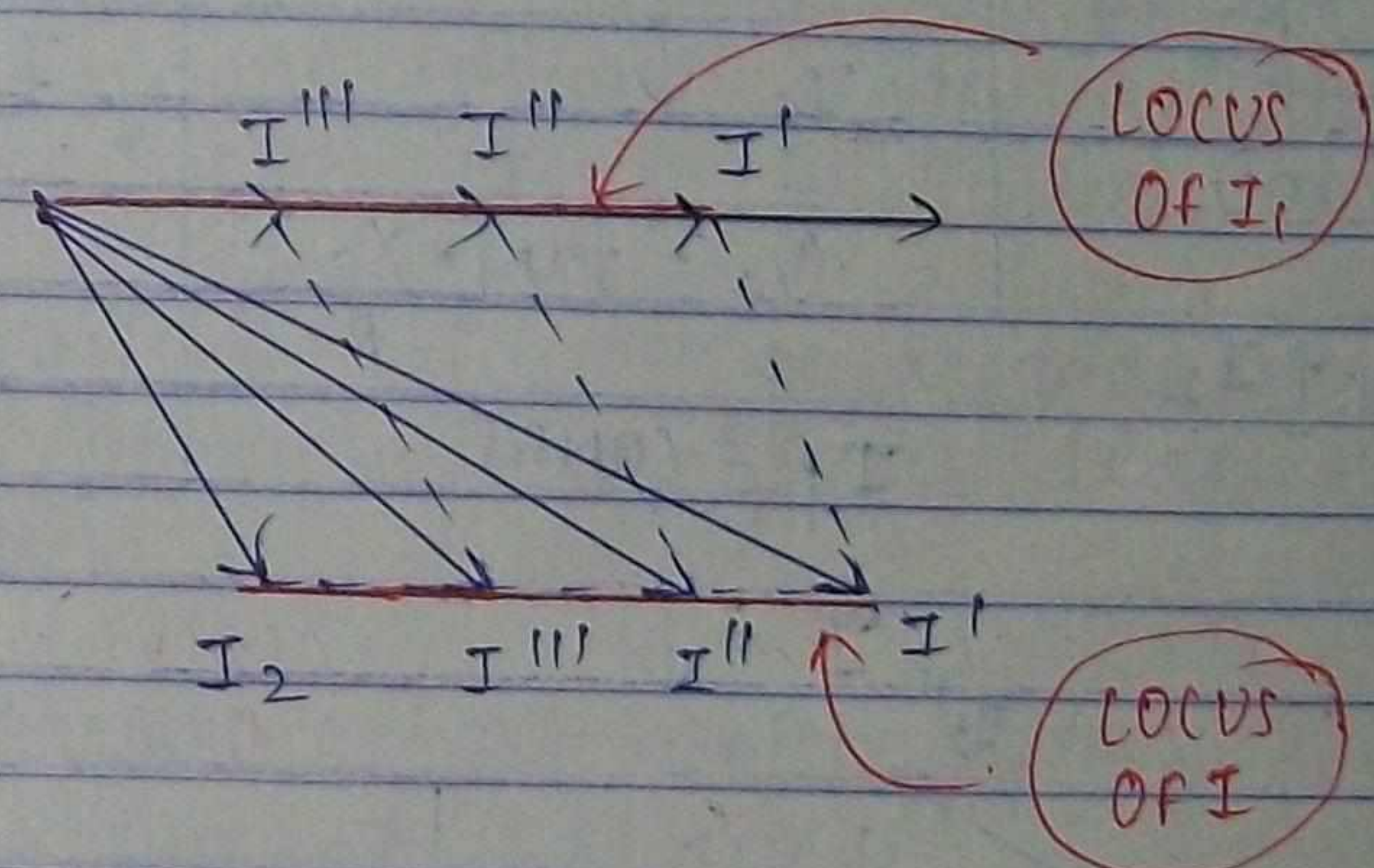
$$I_2 = \text{CONST}$$

$I \downarrow$

$R_1 \approx \infty$

$$I_1 = 0$$

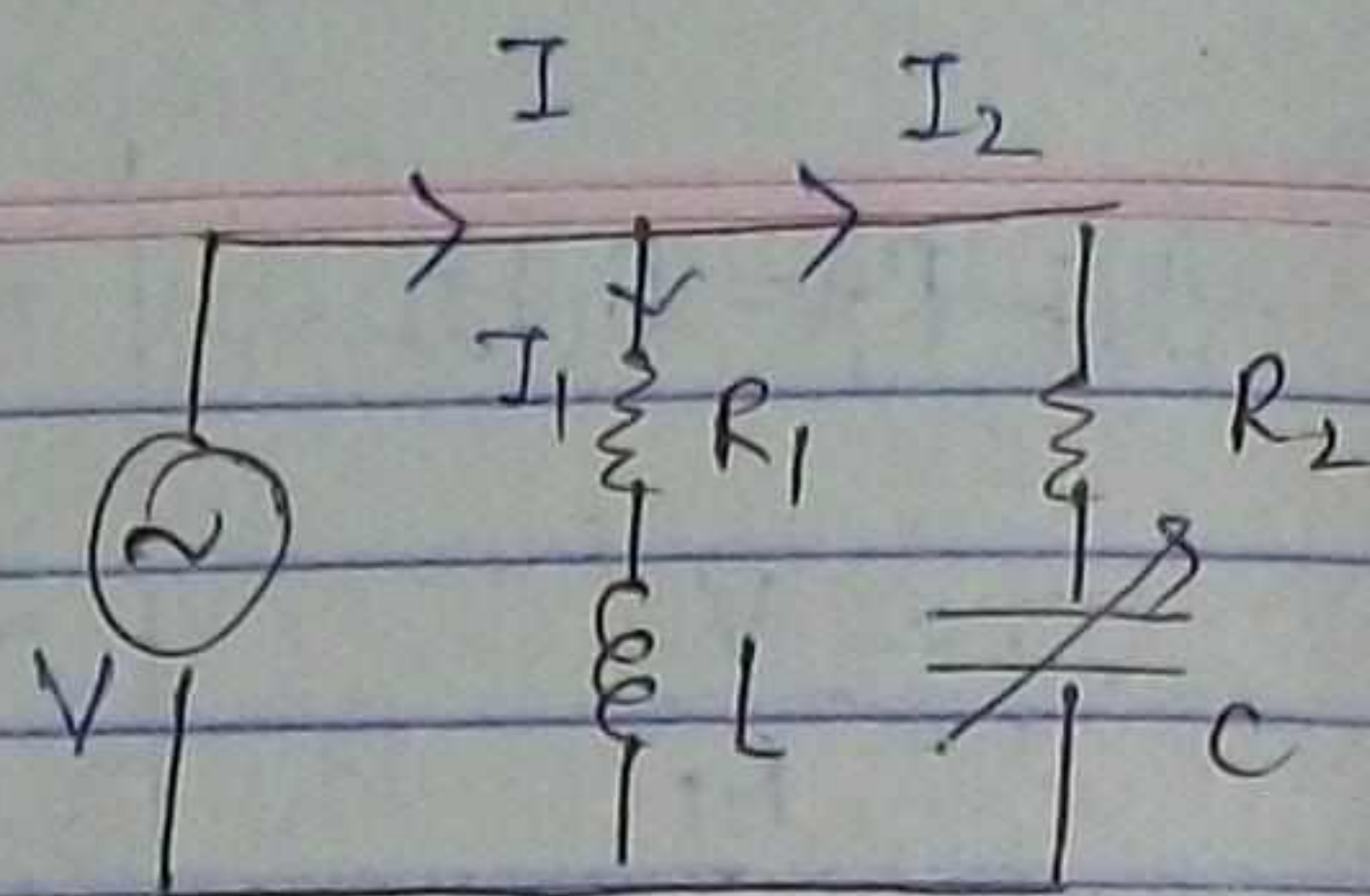
$$I = I_2$$



NOTE -

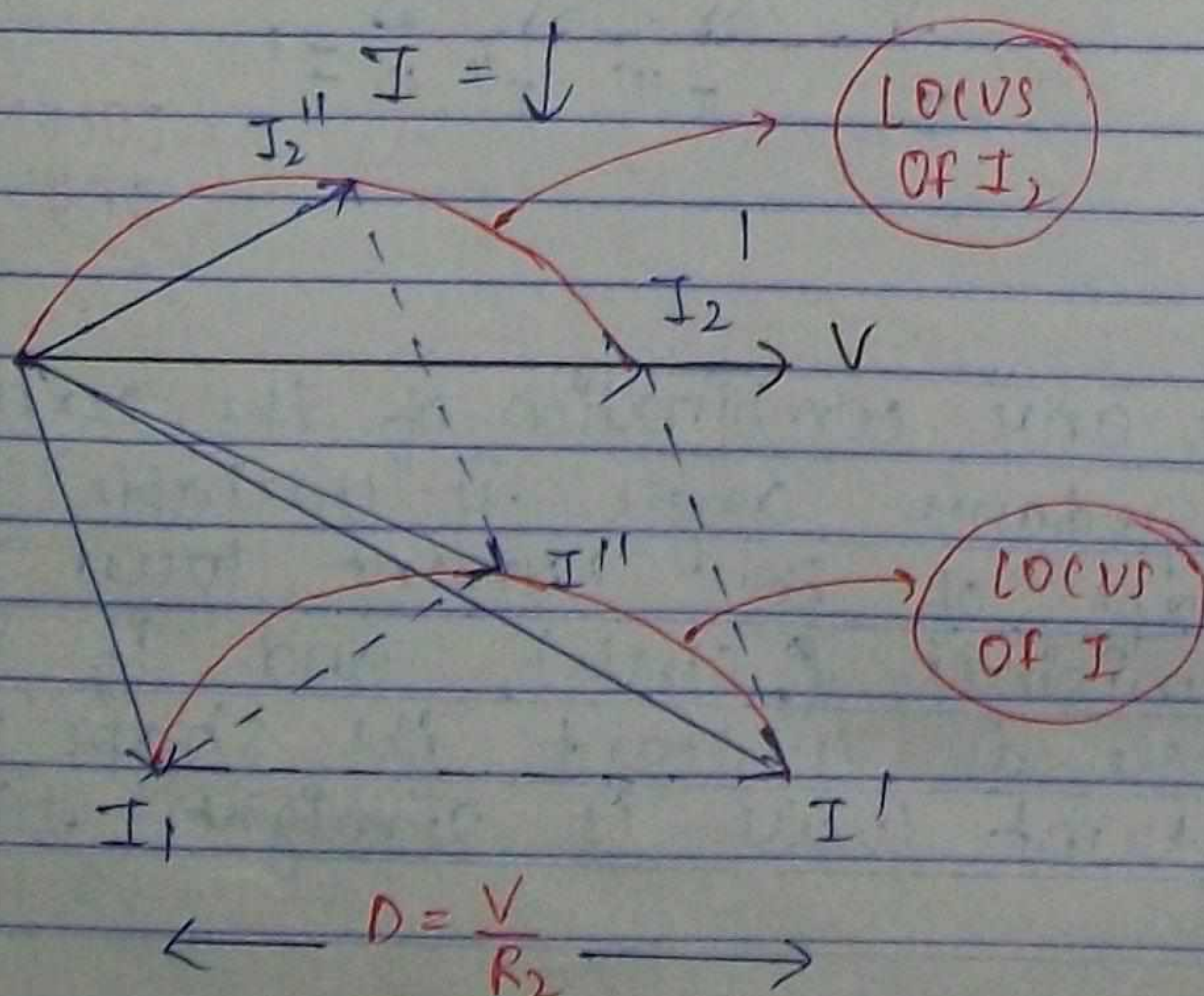
— for any combination of the circuit if impedance angle is variable the shape of the current locus is semicircle (circle). and if impedance angle is constant the shape of the current locus is straight line.

Ques. Draw the current locus of I_2 and I of circuit:



Solⁿ: $X_C = 1/2\pi fC$

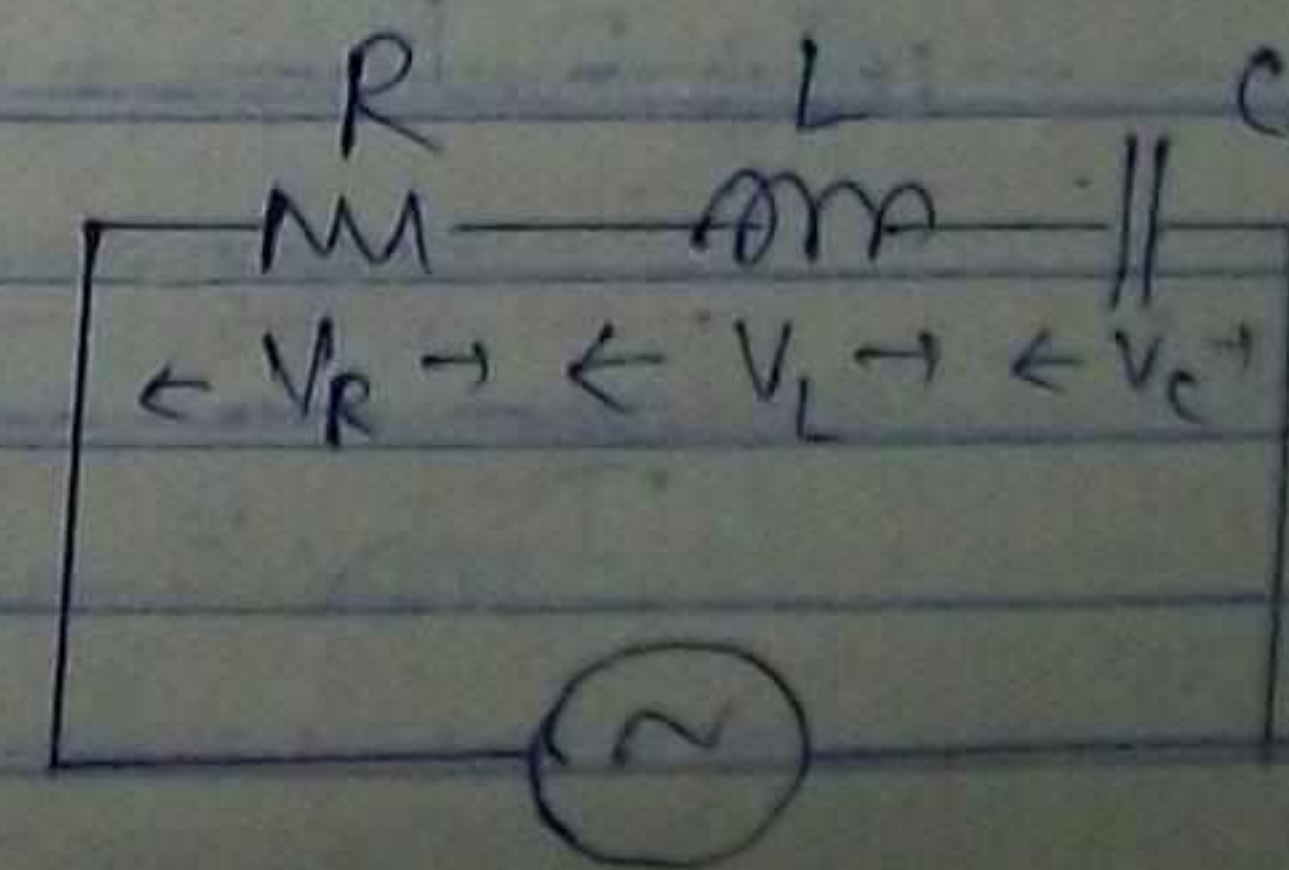
$C \gg 1$	$X_C \approx 0$	$X_C \uparrow$	$X_C \approx \infty$
$I_2 = \frac{V}{R_2}$		$Z_2 \uparrow$	$I_2 = 0$
$\phi_2 = 0$		$I_2 \downarrow$	$I = I_1$
$I = I_1 + I_2$		$\phi_2 = \tan^{-1} \left(\frac{-X_C}{R} \right)$	
		$I_1 = \text{CONST}$	



⇒ RESONANCE -

- For occurrence of resonance in any circuit (system) two energies are required, in electric circuit inductor is having energy in the form of magnetic field and capacitor is having energy in the form of electric field.
- When above two energies are present in a circuit at one instant wide variations are present in the circuit is called as RESONANCE.
- The circuit is said to be resonance when source voltage and source current are in phase.
- The frequency at which $X_L = X_C$ is called as resonant frequency.
- The f resonance frequency indicates the rate at which energy transformation is done b/w inductor and capacitor.

① SERIES RESONANCE -



By KVL,

$$V = V_R + V_L \angle 90^\circ + V_C \angle -90^\circ$$

at

$$V_L = V_C$$

$$IX_L = IX_C$$

$$\omega L = \frac{1}{\omega C}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/s}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$$

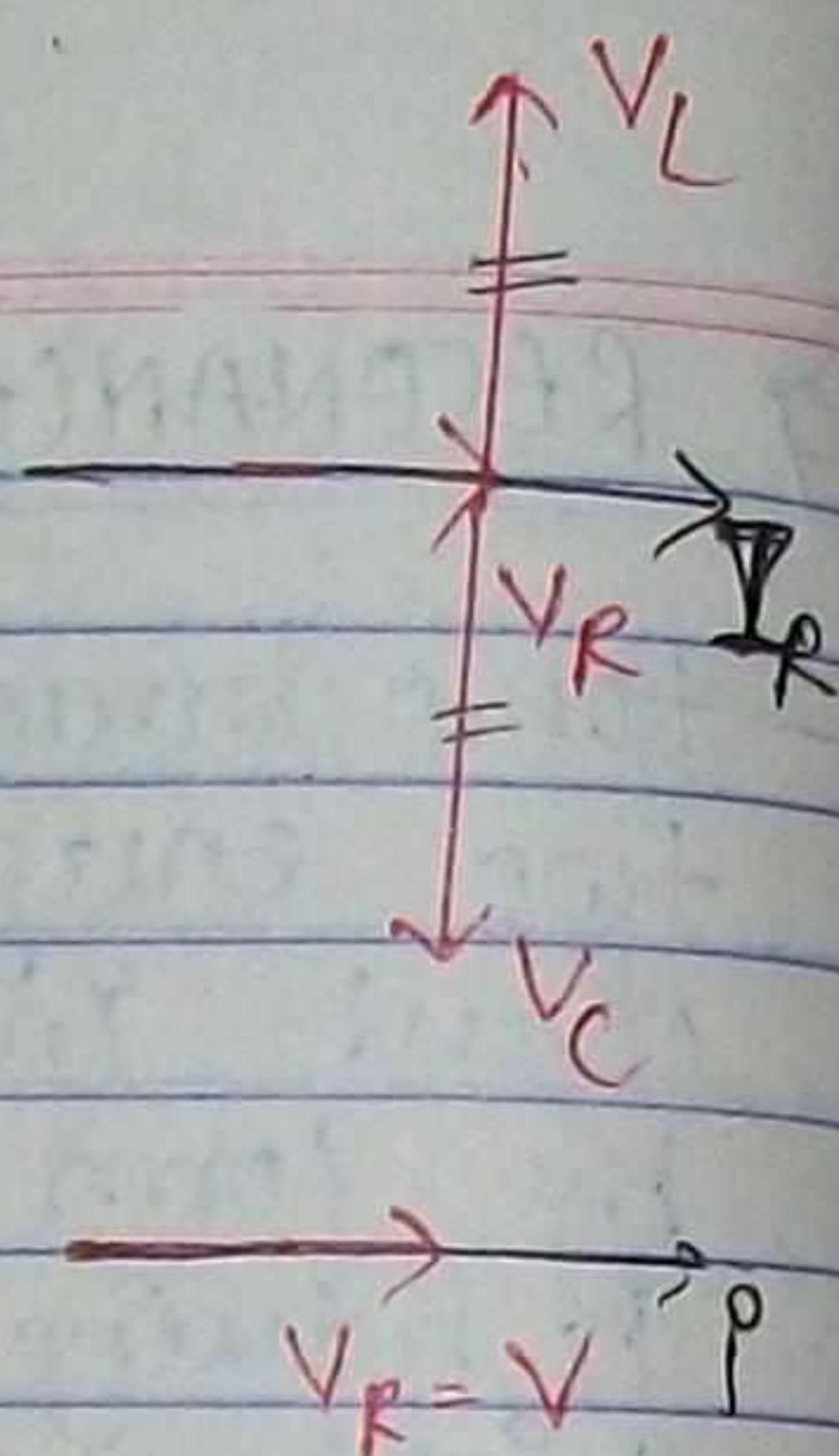
① IMPEDANCE -

$$Z = R + j(X_L - X_C)$$

$$Z_{\min} = R$$

② CURRENT -

$$I_{\max} = \frac{V}{Z_{\min}}$$



③ Power factor -

$$\cos \theta = 1$$

④

$$V_R = V$$

⑤ Net reactive voltage is equal to zero.

⑥ voltage across inductor or voltage across capacitor greater than source voltage. this phenomenon is called as VOLTAGE MAGNIFICATION.

APPLICATIONS -

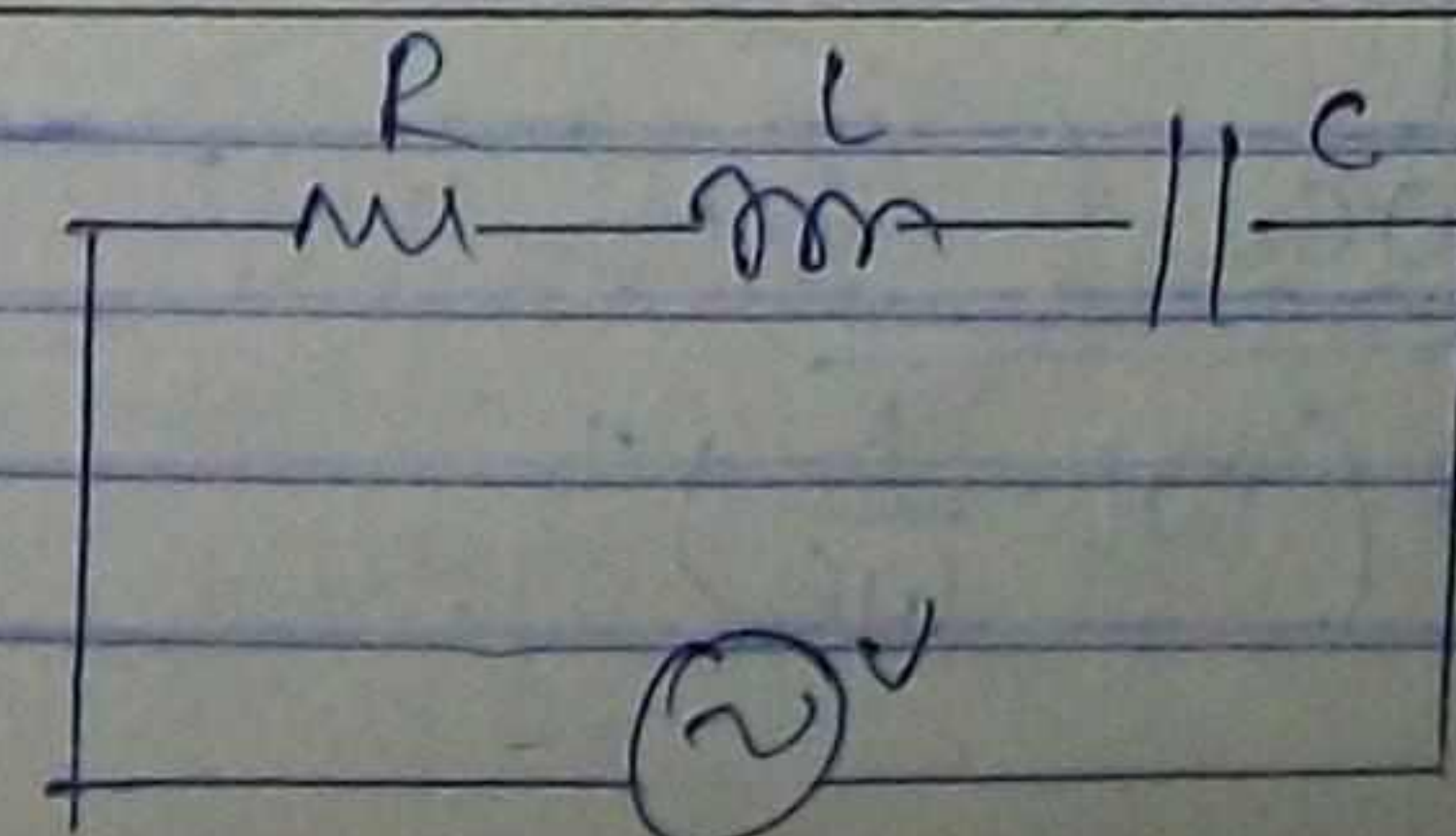
1. Oscillators

2. filters (band pass & band elimination filter)

3. Tuning circuits.

4. Induction heating.

⇒ VARIATION OF VOLTAGE ACROSS CAPACITOR & VOLTAGE ACROSS INDUCTOR WRT FREQ -



FOR
CAPACITOR

$$V_C = I X_C$$

$f = 0$	$f \uparrow$	$f \uparrow \uparrow$
$X_L = 2\pi fL$	$X_L \uparrow$ (100 Ω)	$X_L \uparrow \uparrow$
$X_C = \frac{1}{2\pi fC}$	$X_C \downarrow$ (1000 Ω)	$X_C \downarrow \downarrow$ complex
$X_L = 0$ $X_C = \infty$	$X = X_L - X_C \downarrow \downarrow$ (900 Ω)	$Z \uparrow \uparrow$
$X = X_L - X_C = \infty$	$Z \downarrow \downarrow$	$I \downarrow \downarrow$
$Z = \infty$	$I \uparrow \uparrow$	$V_C \downarrow$
$I = 0$	$V_C \uparrow$	
$V_C = V$		

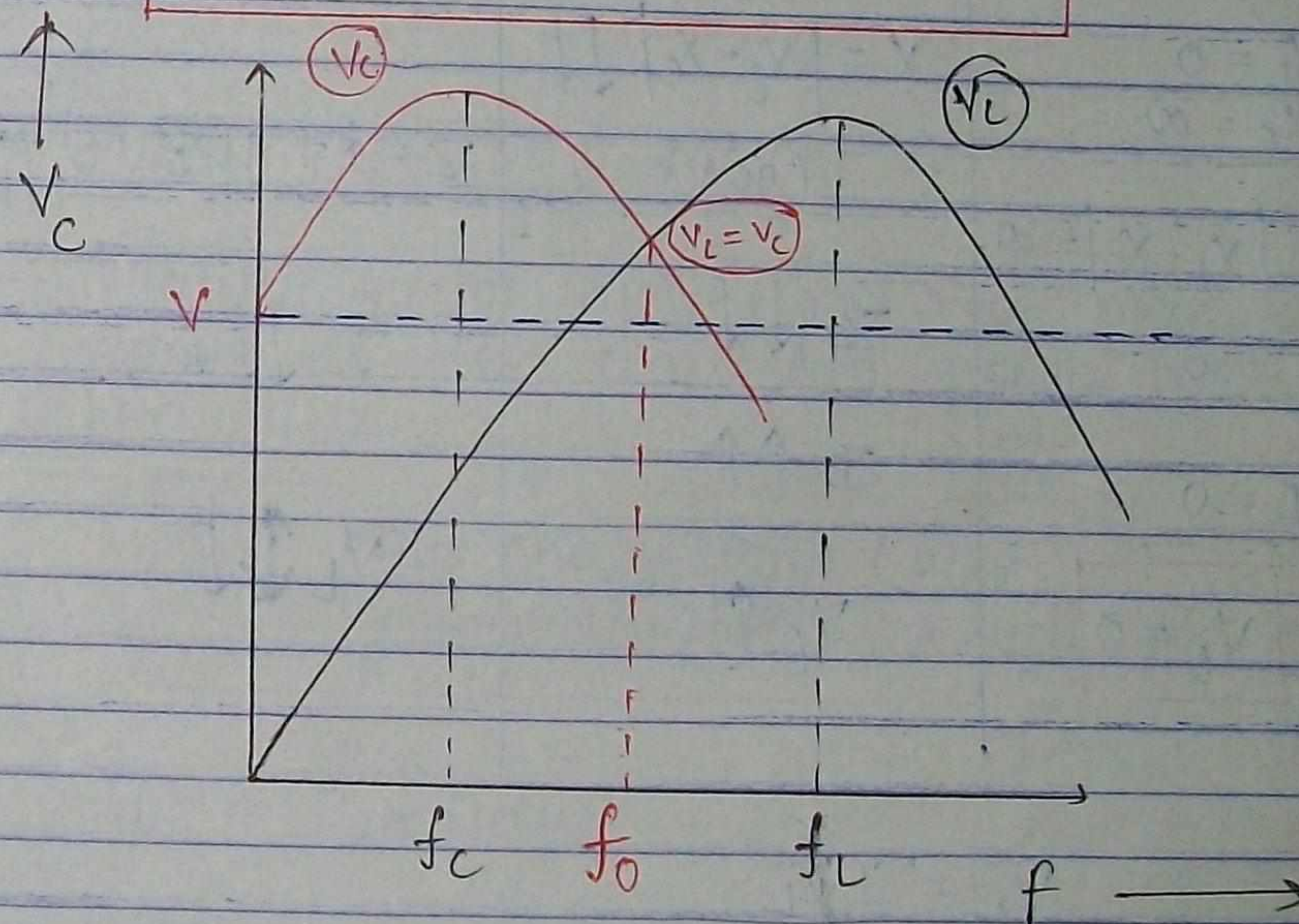
$$V_C = I X_C$$

$$V_C = \frac{V X_C}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$V_C = \frac{V \cdot \frac{1}{\omega C}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \quad \text{--- (1)}$$

differentiate V_C wrt ω and equating to zero
we get:

$$f_c = \frac{1}{2\pi \sqrt{LC}} \sqrt{1 - \left(\frac{R^2 C}{2L}\right)}$$



FOR
INDUCTOR

$$V_L = I X_L$$

$f=0$	$f \uparrow$	$f \uparrow \uparrow$
$X_L = 2\pi fL$ $X_C = \frac{1}{2\pi fC}$	$X_L \uparrow$ (low) $X_C \downarrow$ (high)	$X_L \uparrow \uparrow$ $X_C \downarrow \downarrow$ (very low)
$X_L = 0$ $X_C = \infty$	$X = X_L - X_C \downarrow \downarrow$ (low)	$Z = R + j(X_L - X_C) \uparrow \uparrow \uparrow$
$X = X_L - X_C = \infty$		
$Z = \infty$	$Z \downarrow \downarrow$	$I \downarrow \downarrow \downarrow$
$I = 0$	$I \uparrow \uparrow \uparrow$	
$V_L = 0$	$V_L \uparrow$	$V_L \uparrow \downarrow \downarrow$

$$V_L = I X_L$$

$$V_L = \frac{V X_L}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$V_L = \frac{V \omega L}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} \quad - (2)$$

diff. eq (2) wrt ω and equating to 0.

$$f_L = \frac{1}{2\pi \sqrt{LC}} \sqrt{\frac{1}{1 - \left(\frac{R^2 C}{2L}\right)}}$$

$\Rightarrow$ QUALITY FACTOR (Q-FACTOR) -

- Quality factor is the ratio of max. energy stored in the circuit to power dissipation per cycle.

$$Q = \frac{\text{Max. energy stored (wg)}}{\text{power dissipation per cycle.}}$$

$$i(t) = I_m \sin \omega t$$

$$V_C(t) = \frac{1}{C} \int i(t) dt$$

$$= \frac{1}{C} \int I_m \sin \omega t dt$$

$$= -\frac{1}{\omega C} I_m \cos \omega t$$

Max. energy stored $W_g = \frac{1}{2} L i^2 + \frac{1}{2} C V_c^2$

$$W_g = \frac{1}{2} L (I_m \sin \omega t)^2 + \frac{1}{2} C \left(-\frac{1}{\omega C} I_m \cos \omega t \right)^2$$

$$W_g = \frac{1}{2} L I_m^2 \sin^2 \omega t + \frac{1}{2} C \frac{I_m^2}{\omega^2 C^2} \cos^2 \omega t$$

$$\therefore \omega^2 = \frac{1}{LC}$$

$$\therefore L = \frac{1}{\omega^2 C}$$

$$W_g = \frac{1}{2} L I_m^2 \sin^2 \omega t + \frac{1}{2} L I_m^2 \cos^2 \omega t$$

$$W_g = \frac{1}{2} L I_m^2$$

$$W_g = \frac{1}{2} C V_{cm}^2$$

$$Q = \frac{1/2 L I_m^2}{\left(\frac{I_m}{\sqrt{2}} \right)^2 R \cdot \frac{1}{\omega}}$$

$$Q = \frac{\omega L}{R}$$

$$\left(\omega = \frac{1}{\sqrt{LC}} \right)$$

$$Q = \frac{V_L}{V} = \frac{V_C}{V}$$

$$Q = \frac{1}{R} \sqrt{\frac{L}{C}}$$

$$Q = \frac{\omega L}{R} = \frac{I X_L}{I R} = \frac{V_L}{V_R} = \frac{V_L}{V} \quad (V_R = V)$$

At resonance $V_L = V_C$

$$Q = \frac{V_L}{V} = \frac{V_C}{V}$$

$$Q = \frac{X_L}{R} = \frac{I^2 X_L}{I^2 R} = \frac{Q_L}{P}, \quad Q = \frac{Q_C}{P}$$

At resonance $X_L = X_C$

$$Q = \frac{X_L}{R} = \frac{X_C}{R} = \frac{\omega L}{R} = \frac{1}{\omega C R}$$

$$Q > 1$$

$$X_L > R$$

$$X_C > R$$

$$Q \propto \frac{1}{\text{Power loss } (P = I^2 R)}$$

CONDITION FOR RESONANCE

22/10/11

⇒ BANDWIDTH -

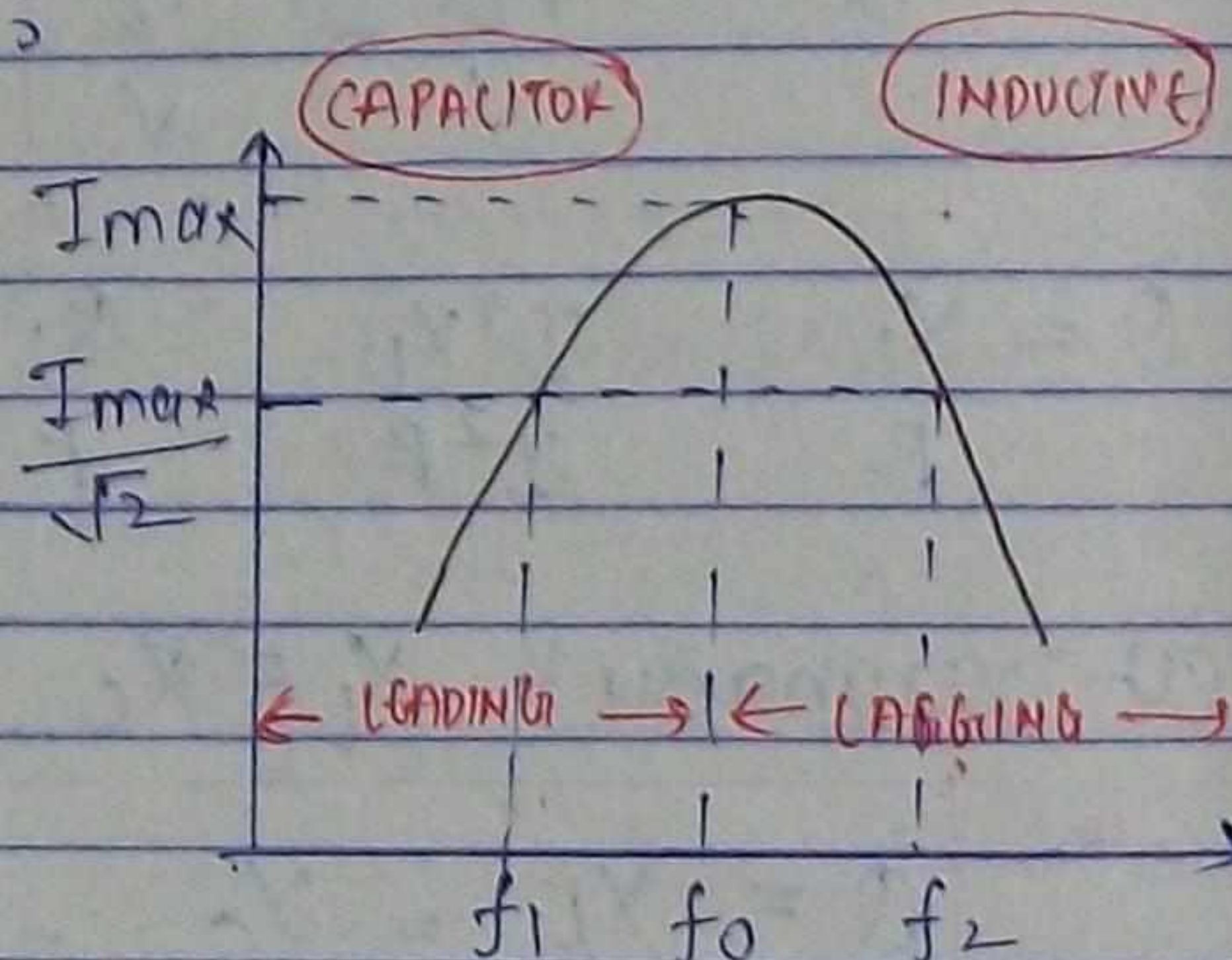
- when the curve is drawn between current and freq. then the curve is called as resonance curve.

- Bandwidth is the range of frequencies on either of the resonant frequency where the current falls from max. value to 70.7% of the max. value, and is given by:

$$BW = f_2 - f_1$$

f_2 = upper cutoff freq.

f_1 = lower cutoff freq.



① $f_0 \rightarrow I_m$, $Z = R$

$f_0 \rightarrow \frac{I_m}{\sqrt{2}}$, $Z = \sqrt{2} R$

② $f_0 \rightarrow \cos \theta = 1$

$f_1, f_2 \rightarrow Z = R \pm jX$

$$Z = \sqrt{R^2 + X^2} = \sqrt{2} R$$

on solving:

$$X = R$$

CAPACITIVE (f_1) -

$$Z = R - jX \text{ and } X = R$$

impedance angle = $\tan^{-1}(-\frac{X}{R})$
= -45°

$$I = \frac{V \angle 0}{Z \angle -45^\circ} = \frac{V \angle +45^\circ}{Z}$$

P.f angle = $+45^\circ$

$$\cos(45^\circ) = \frac{1}{\sqrt{2}}$$

LEADING

INDUCTIVE (f_2) -

$$Z = R + jX \text{ and } X = R$$

Impedance angle = $\tan^{-1}(\frac{X}{R}) = 45^\circ$

$$X = X_L - X_C$$

$$X_L = 2\pi f L$$

$$X_C = \frac{1}{2\pi f C}$$

$$f_1 \rightarrow X_C > X_L$$

$$X \rightarrow (-ve)$$

$$f_2 \rightarrow X_C < X_L$$

$$X \rightarrow (+ve)$$

P.f. angle = -45°

$$\cos(-45) = \frac{1}{\sqrt{2}}$$

LAGGING

(3) $f_1 \rightarrow X_C > X_L, X=R$

$$\frac{1}{\omega_1 C} - \omega_1 L = R \quad \text{--- (1)}$$

$f_2 \rightarrow X_L > X_C, X=R$

$$\omega_2 L - \frac{1}{\omega_2 C} = R \quad \text{--- (2)}$$

from (1) and (2) : (1) = (2) -

$$\omega_1 \omega_2 = \frac{1}{LC} \quad \text{--- (3)}$$

$$\omega_0^2 = \frac{1}{LC} \quad \text{--- (4)}$$

from (3) and (4) : (3) = (4) -

$$\omega_0^2 = \omega_1 \omega_2$$

★★

$$\omega_0 = \sqrt{\omega_1 \omega_2}$$

$$f_0 = \sqrt{f_1 f_2}$$

Adding eq (1) and (2) :

$$\frac{1}{C} \left[\frac{1}{\omega_1} - \frac{1}{\omega_2} \right] + L[\omega_2 - \omega_1] = 2R$$

$$\frac{1}{C} \left[\frac{\omega_2 - \omega_1}{\omega_1 \omega_2} \right] + L[\omega_2 - \omega_1] = 2R$$

Since, $\omega_1 \omega_2 = \frac{1}{LC} \quad L = \frac{1}{\omega_1 \omega_2 C}$

$$L[\omega_2 - \omega_1] + L[\omega_2 - \omega_1] = 2R$$

★★

$$B.W = \omega_2 - \omega_1 = \frac{R}{L} \text{ rad/sec}$$

$$BW = f_2 - f_1 = \frac{R}{2\pi L} \text{ Hz}$$

(4)

$$Q = \frac{\omega_0 L}{R} = \frac{\omega_0}{R/L}$$

QUALITY
FACTOR

Since $\omega_2 - \omega_1 = R/L$

★★

$$Q = \frac{\omega_0}{\omega_2 - \omega_1} = \frac{f_0}{f_2 - f_1}$$

⑤ $f_0 \rightarrow I_m$, $P = I_m^2 R$

$f_1, f_2 \rightarrow \frac{I_m}{\sqrt{2}}$, $P' = \left(\frac{I_m}{\sqrt{2}}\right)^2 R = \frac{I_m^2 R}{2} = \frac{P}{2}$

HALF POWER
FREQUENCY

OR

3dB
FREQUENCY

⇒ DAMPING RATIO AND QUALITY FACTOR -

By KVL,

$$V = iR + L \frac{di}{dt} + \frac{1}{C} \int i dt$$

$$V' = R \frac{di}{dt} + L \frac{d^2 i}{dt^2} + \frac{i}{C}$$

$$\frac{d^2 i}{dt^2} + \frac{R}{L} \frac{di}{dt} + \frac{i}{LC} = \frac{V'}{L}$$

Putting $(d/dt = D)$

$$\left(D^2 + \frac{R}{L} D + \frac{1}{LC}\right) i = 0$$

$$Q = \frac{\omega_0}{\omega_2 - \omega_1}$$

Comparing
with

$$s^2 + 2\delta\omega_n s + \omega_n^2 = 0$$

$$\omega_n = \frac{1}{\sqrt{LC}}$$

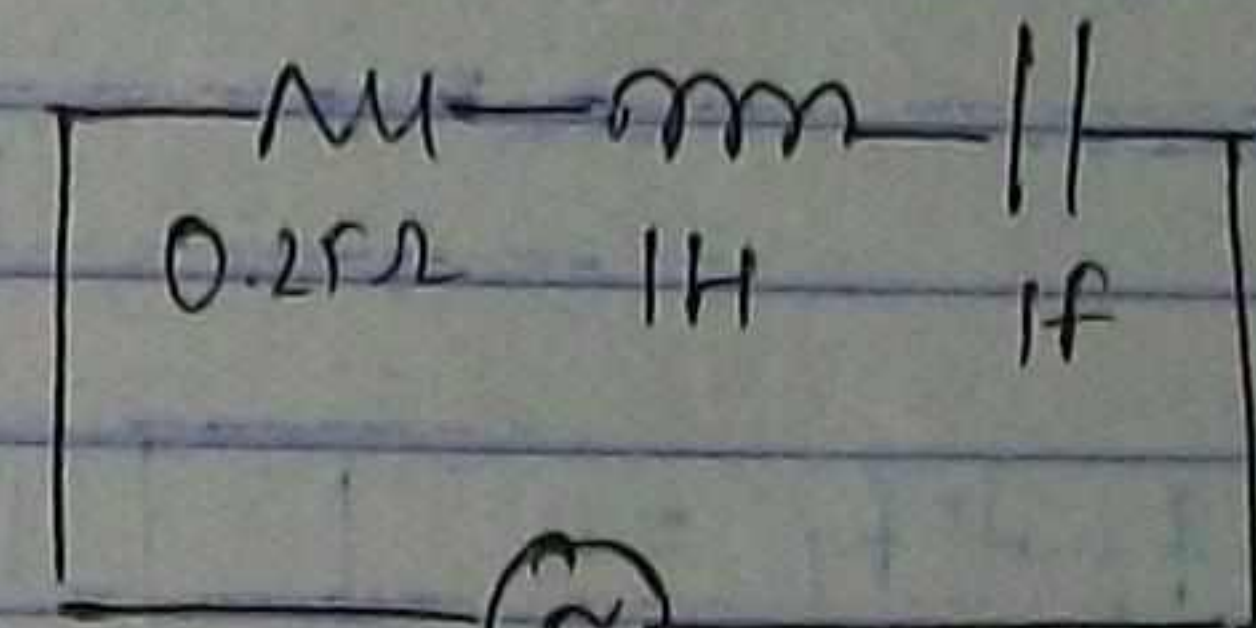
$$\delta = \frac{R}{2} \sqrt{\frac{C}{L}}$$

$$\delta = \frac{1}{2Q}$$

$$Q = \frac{1}{R} \sqrt{\frac{L}{C}}$$

Ques. find

f_0 , Q , BW, δ , f_1, f_2 ,
 I at f_0 .



$$V(t) = 10 \sin \omega t$$

Soln:

① $f_0 = \frac{1}{2\pi}$

② $Q = \frac{1}{R} \sqrt{\frac{L}{C}} = \frac{1}{0.25} \sqrt{\frac{1}{1}} = 4$

③ $BW = f_2 - f_1 = \frac{f_0}{Q} = \frac{1}{8\pi}$

④ $\delta = \frac{1}{2Q} = \frac{1}{8}$

$$(5) \quad f_1 f_2 = f_0^2 = \left(\frac{1}{2\pi} \right)^2$$

$$f_2 - f_1 = \frac{1}{8\pi}$$

$$(f_2 + f_1)^2 - (f_2 - f_1)^2 = 4f_1 f_2$$

$$(f_2 + f_1)^2 = 4 \cdot \frac{1}{4\pi^2} + \frac{1}{16\pi^2} \quad f_2 = 0.183$$

$$= \frac{17}{16\pi^2} \quad f_1 = 0.144$$

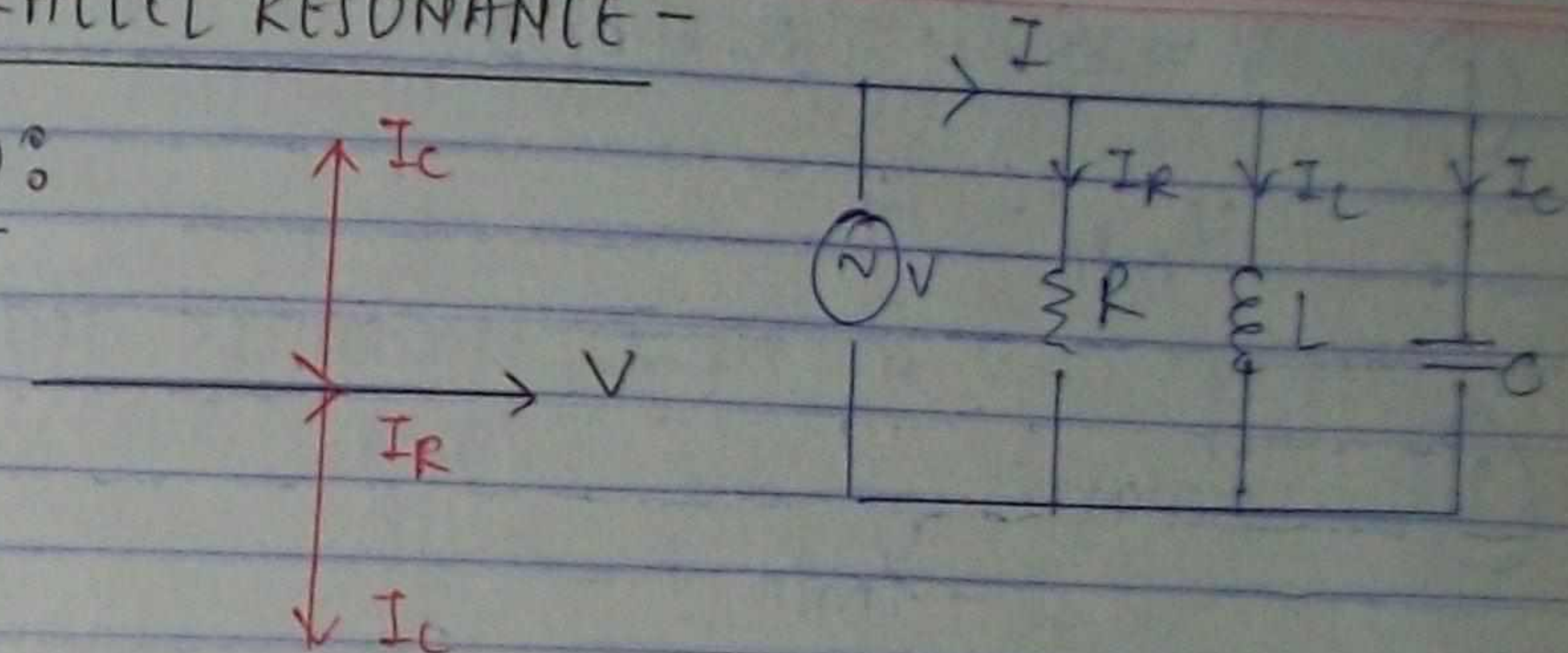
$$f_2 + f_1 = \frac{1}{\pi} \sqrt{\frac{17}{16}} = 0.328$$

$$(6) \quad I = \frac{V}{Z} = \frac{V}{R} = \frac{10/\sqrt{2}}{0.25}$$

$$I = \frac{40}{\sqrt{2}} \text{ A}$$

$\Rightarrow$ PARALLEL RESONANCE -

CASE (1):



$$I = I_R + I_L \angle -90^\circ + I_C \angle 90^\circ$$

$$I_L = I_C$$

$$\frac{V}{X_L} = \frac{V}{X_C}$$

$$B_L = B_C$$

$$\frac{1}{\omega L} = \omega C$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/sec}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$$

$$(1) \quad Y = G + j(B_C - B_L)$$

$$Y_{\min} = G$$

$$(2) \quad Z_{\max} = \frac{1}{Y_{\min}}$$

$$(3) \quad I_{\min} = \frac{V}{Z_{\max}}$$

$$(4) \quad \cos \theta = 1$$

$$(5) \quad I_R = I$$

Active component of current = I_R

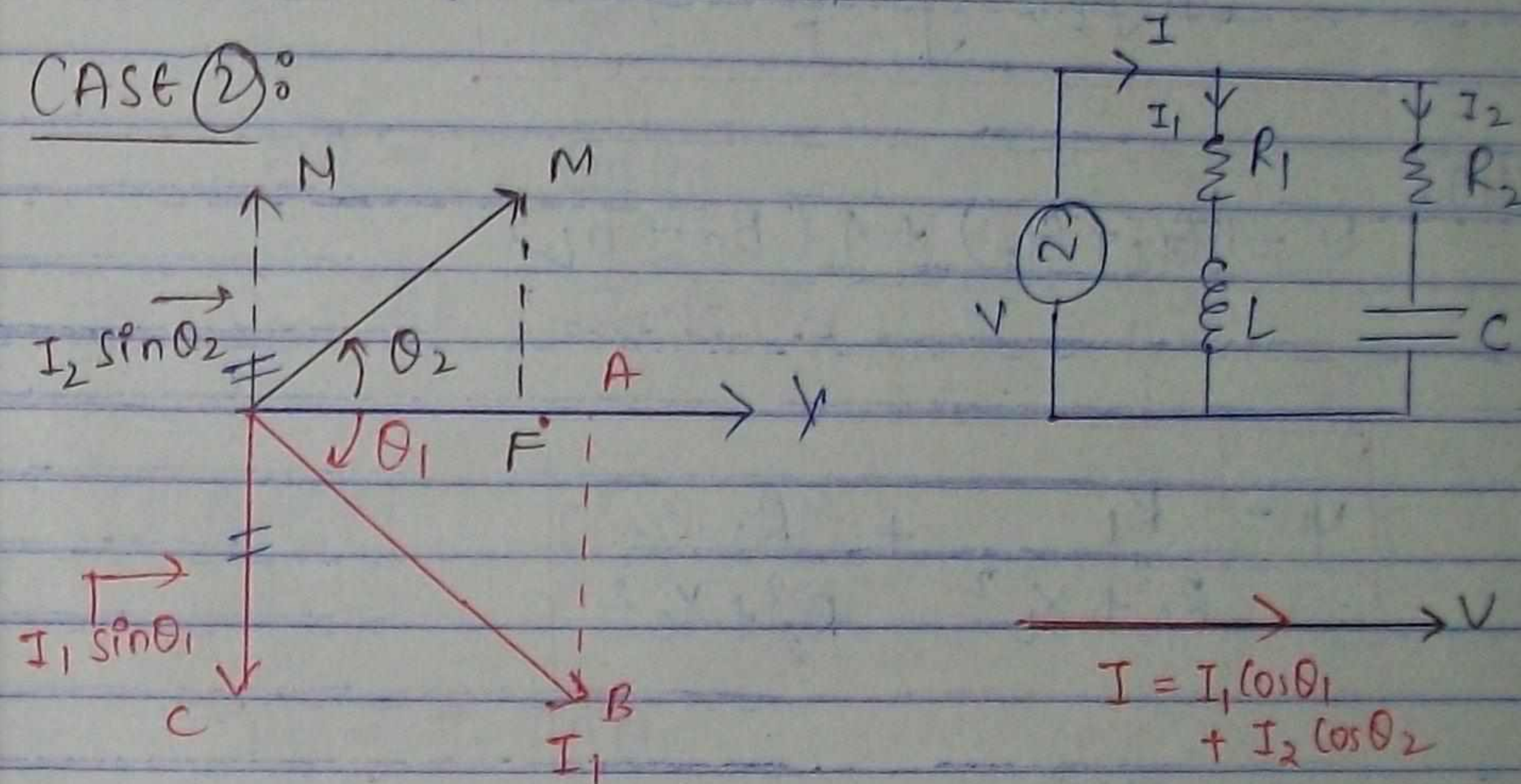
(6) Net reactive current is equal to zero.

(7) Current flow into inductor (or) current flow into capacitor is greater than source current. This phenomenon is called as CURRENT MAGNIFICATION.

(8) Parallel resonance circuit is also known as ANTI-RESONANT CIRCUIT (or) REJECTOR CIRCUIT.

(9) Series resonance circuit is also called as ACCEPTOR CIRCUIT.

CASE (2):



$$OA = I_1 \cos \theta_1$$

$$OF = I_2 \cos \theta_2$$

$$AB = I_1 \sin \theta_1$$

$$FM = ON = I_2 \sin \theta_2$$

Now at resonance

$$B_L = B_C$$

$$\frac{X_L}{R_1^2 + X_L^2} = \frac{X_C}{R_2^2 + X_C^2}$$

$$\frac{\omega L}{R_1^2 + (\omega L)^2} = \frac{1/\omega C}{R_2^2 + (1/\omega C)^2}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \sqrt{\frac{R_1^2 - L/C}{R_2^2 - L/C}}$$

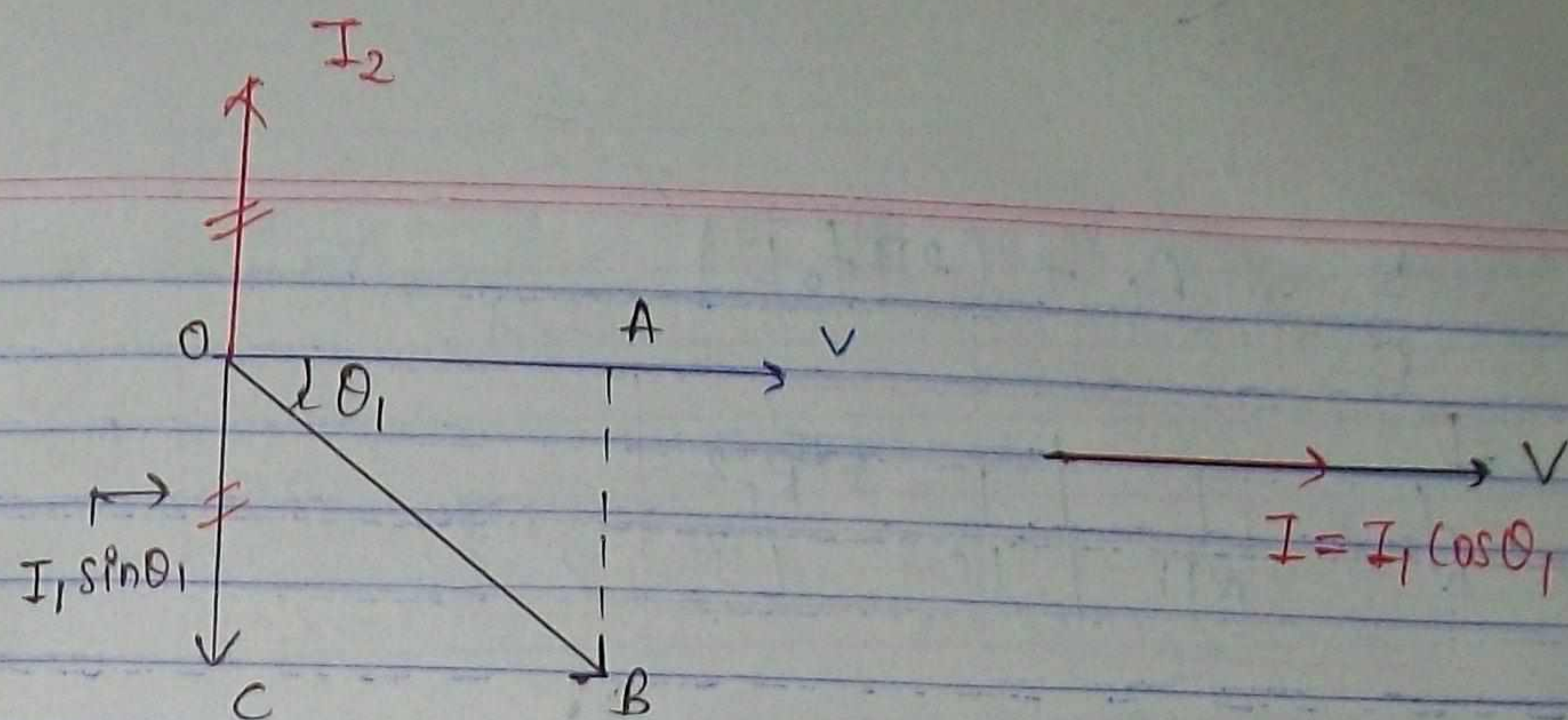
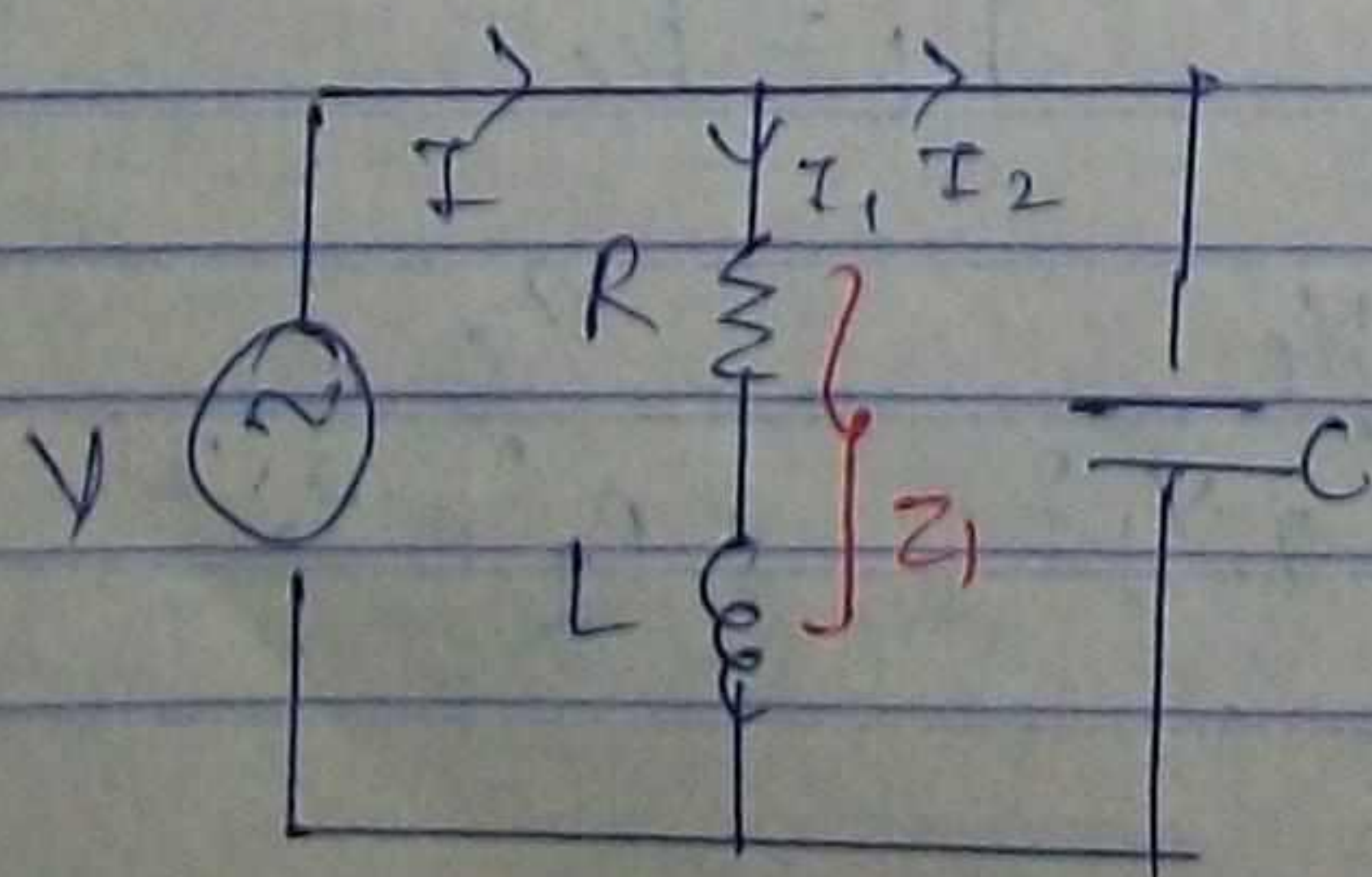
also, $Y = (G_1 + G_2) + j(B_C - B_L)$

$$Y = \frac{R_1}{R_1^2 + X_L^2} + \frac{R_2}{R_2^2 + X_C^2}$$

$$I = YV$$

$$I = V \left[\frac{R_1}{R_1^2 + X_L^2} + \frac{R_2}{R_2^2 + X_C^2} \right]$$

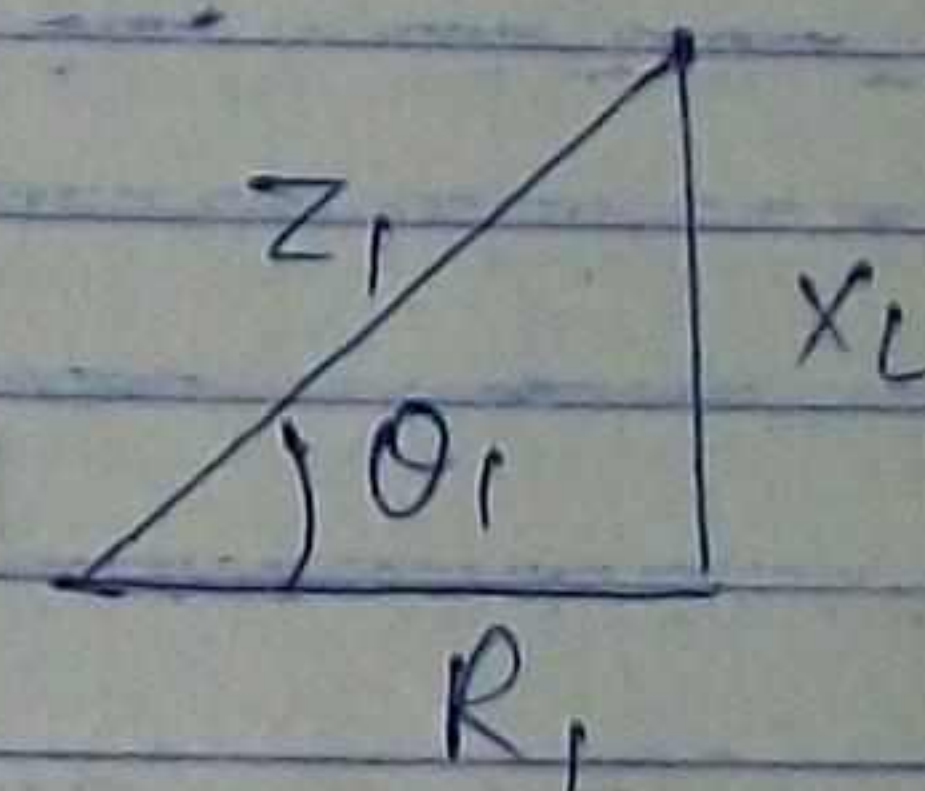
③ CASE ③ -



$$OA = I_1 \cos \theta_1$$

$$OC = AB = I_1 \sin \theta_1$$

$$\cos \theta_1 = \frac{R_1}{Z_1}, \quad \sin \theta_1 = \frac{X_L}{Z_1}$$



$$I_2 = I_1 \sin \theta_1$$

$$\frac{V}{X_C} = \frac{V}{Z_1} \times \frac{X_L}{Z_1}$$

$$Z_1^2 = X_L X_C = \frac{\omega L}{\omega C}$$

④

$$Z_1 = \sqrt{\frac{L}{C}}$$

$$Z_1^2 = R_1^2 + X_L^2$$

$$\frac{L}{C} = R_1^2 + (2\pi f_0 L)^2$$

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R_1^2}{L^2}}$$

$$I = I_1 \cos \theta_1$$

$$I = \frac{V}{Z_1} \times \frac{R}{Z_1}$$

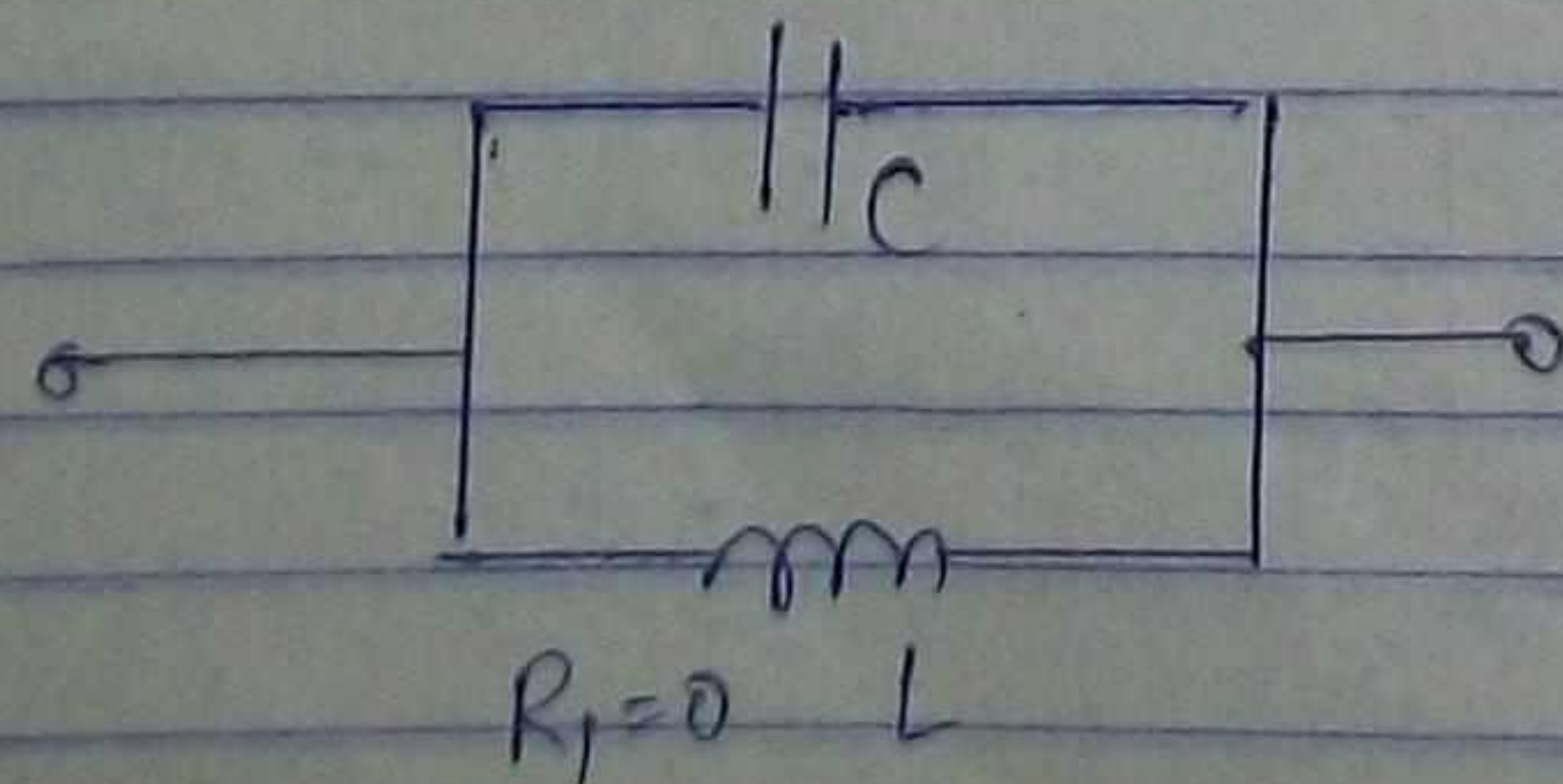
$$I = \frac{VR_1}{L/C}$$

$$I = \frac{V}{L/R_1 C}$$

$$Z_{dy} = \frac{L}{R_1 C} \Omega$$

DYNAMIC
IMPEDANCE

FOR IDEAL TANK CIRCUIT -



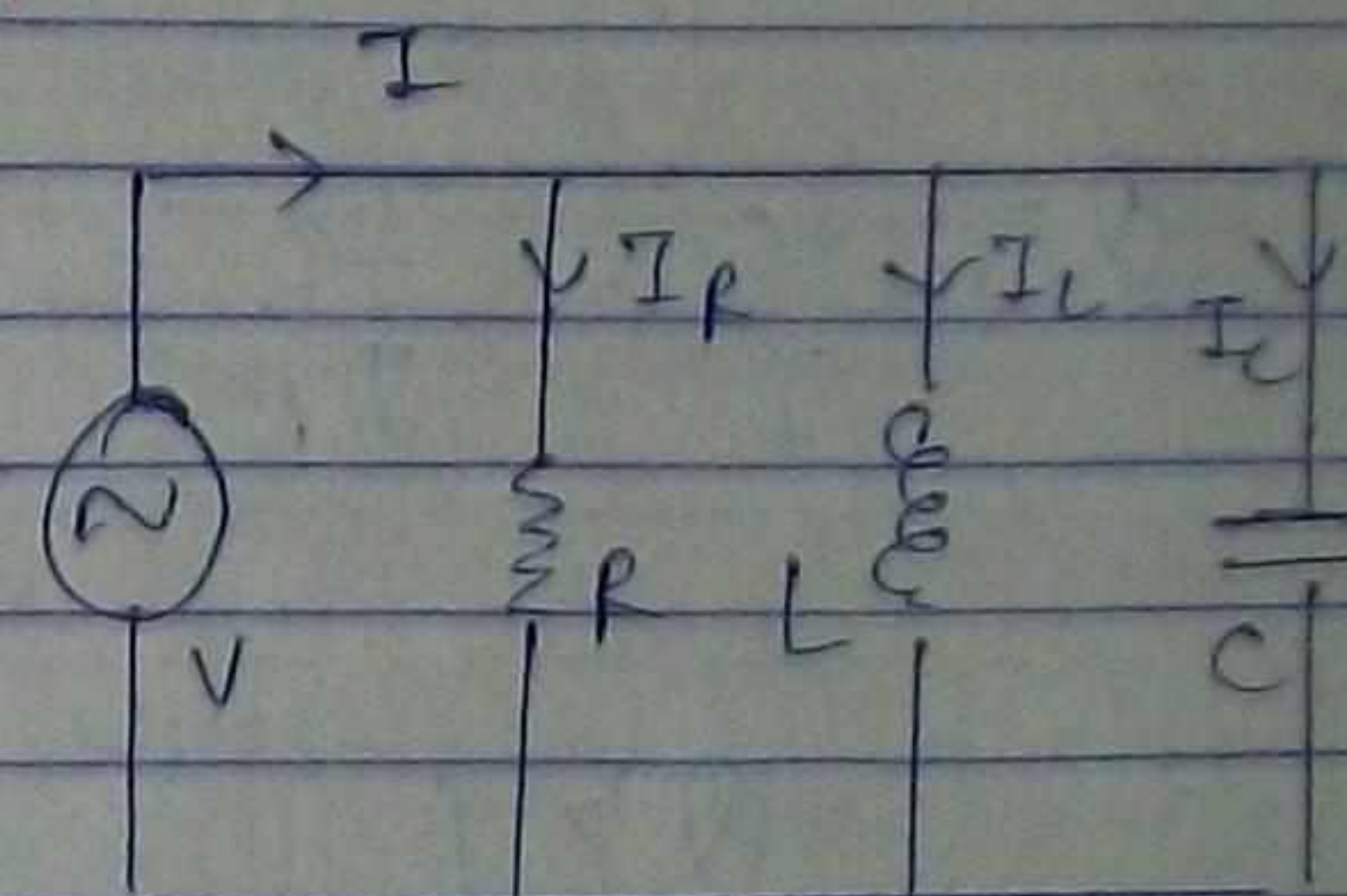
$$Z_{dy} = \frac{L}{R_1 C} \approx \infty$$

$$f_0 = \frac{1}{2\pi \sqrt{LC}} \text{ Hz}$$

QUALITY FACTOR -

$$Q = \frac{V_L}{V} = \frac{V_C}{V}$$

SERIES
COND



$$Q = \frac{I_L}{I} = \frac{I_C}{I}$$

PARALLEL
COND

PR

$$Q = \frac{I_L}{I} = \frac{I_L}{I_R} = \frac{\text{REACTIVE COMPONENT OF CURRENT}}{\text{ACTIVE COMPONENT OF CURRENT}}$$

FOR CASE (1):

$$Q = \frac{I_L}{I_R} = \frac{V/X_L}{V/R} = \frac{R}{X_L} = \frac{R}{\omega L}$$

$$\text{also, } \omega = \frac{1}{\sqrt{LC}}$$

$$Q = R \sqrt{\frac{C}{L}}$$

$$Q = \frac{1/X_L}{1/R} = \frac{B_L}{G}$$

AA

$$Q = \frac{B_L}{G} = \frac{B_C}{G}$$

$$(B_L = B_C)$$

$$Q = \frac{I_C}{I} = \frac{I_C}{I_R} = \frac{R}{X_C} = R\omega C$$

$$Q > 1$$

$$R > X_L$$

$$R > X_C$$

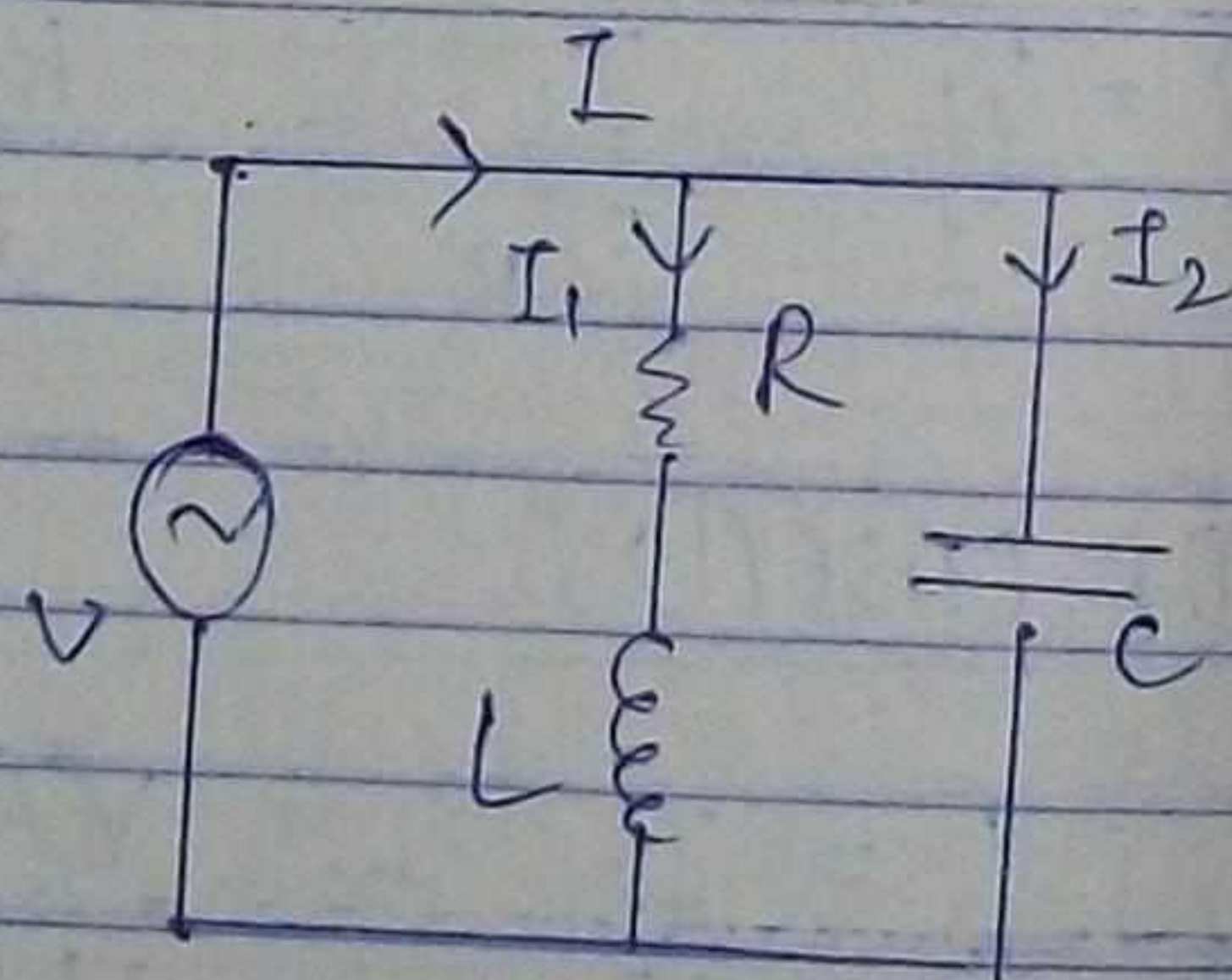
CONDITION FOR
11th RESONANCE

FOR CASE (3) -

$$Q = \frac{I_1 \sin \theta_1}{I} = \frac{I_2}{I}$$

AA

$$Q = \frac{I_2}{I} = \frac{V/X_C}{V'} = \frac{1/\omega C}{L/R} = \frac{\omega L}{R} = \frac{X_L}{R}$$



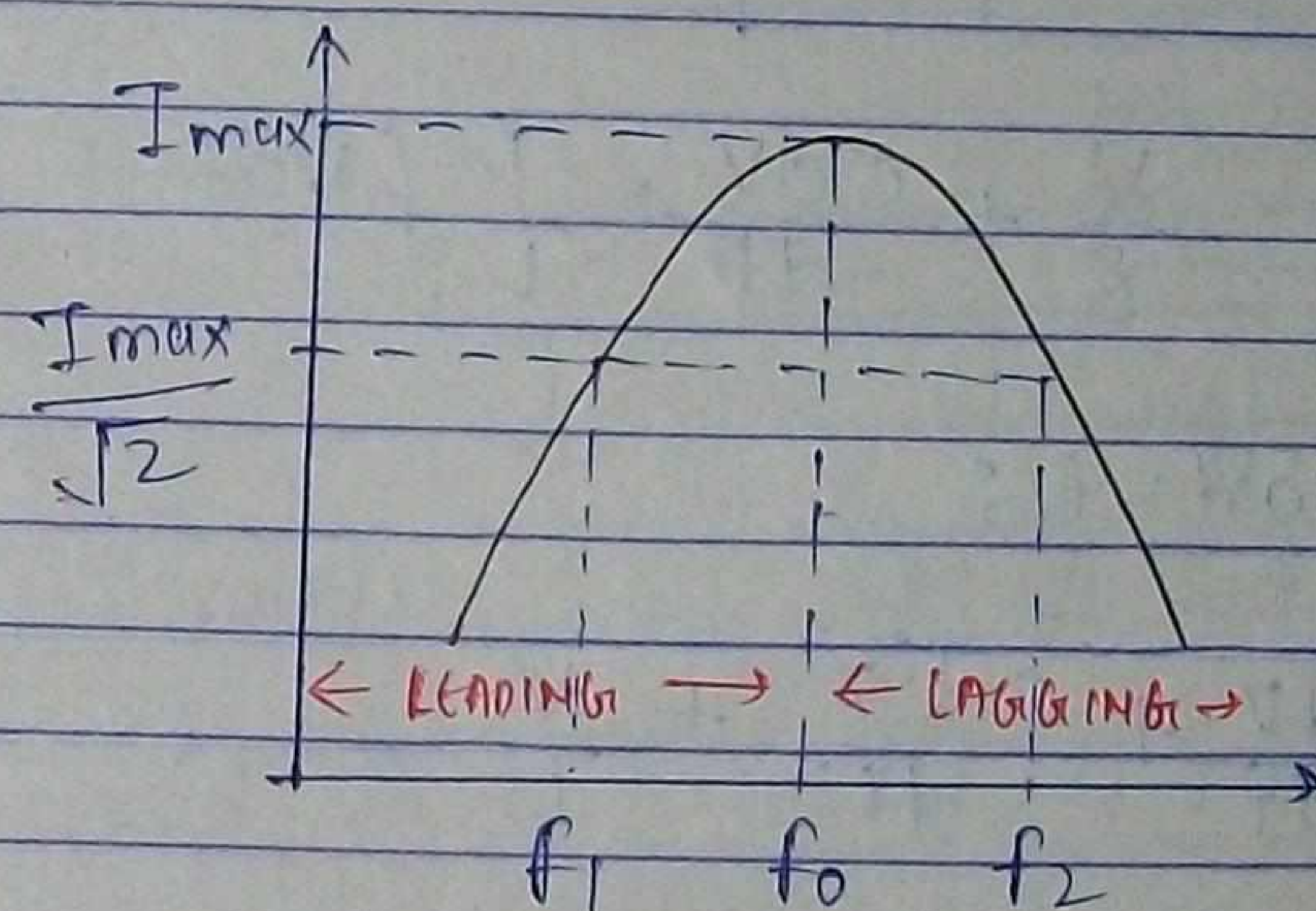
$$Q = \frac{X_L}{R}$$

$$Q > 1$$

$$X_L > R$$

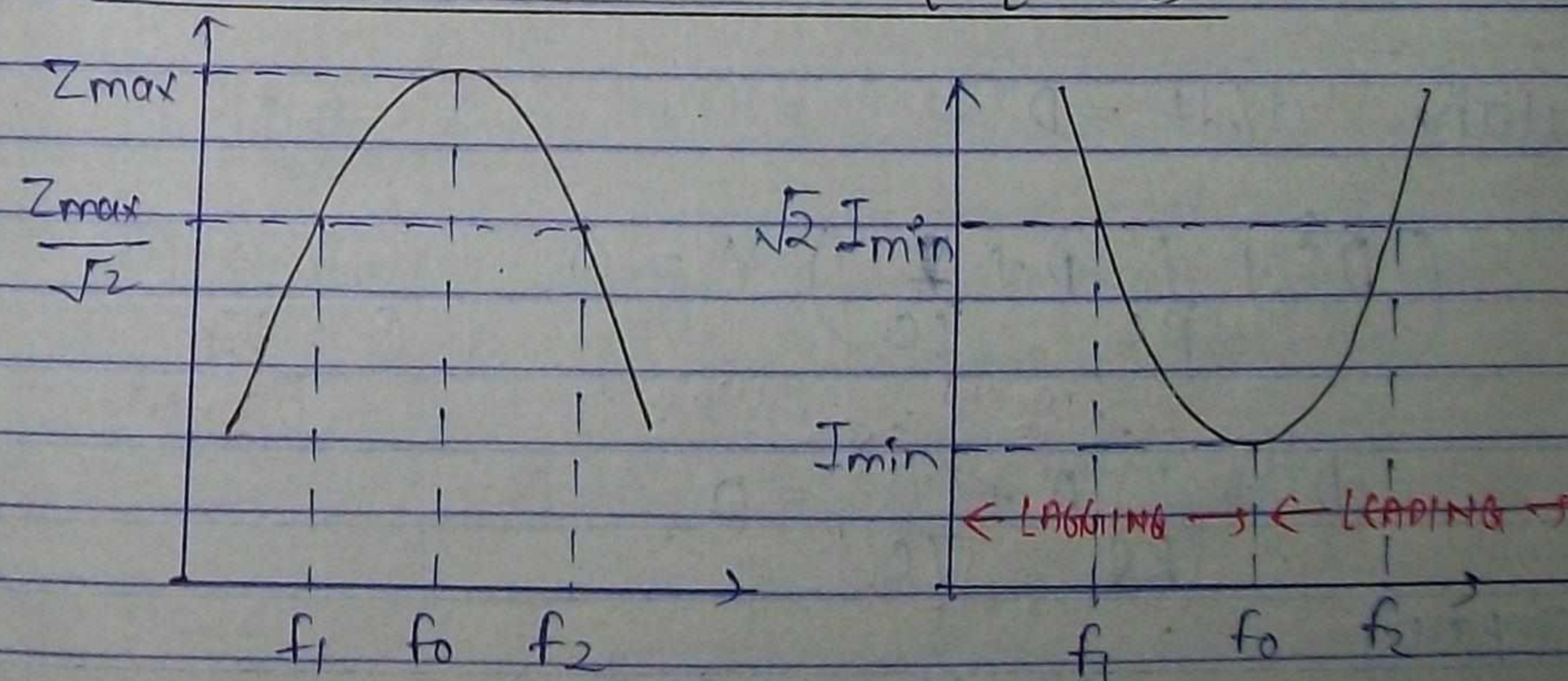
RESONANCE CONDⁿ
FOR TANK CIRCUIT

FOR SERIES RESONANCE - ($X_L = X_C$)



$$BW = f_2 - f_1$$

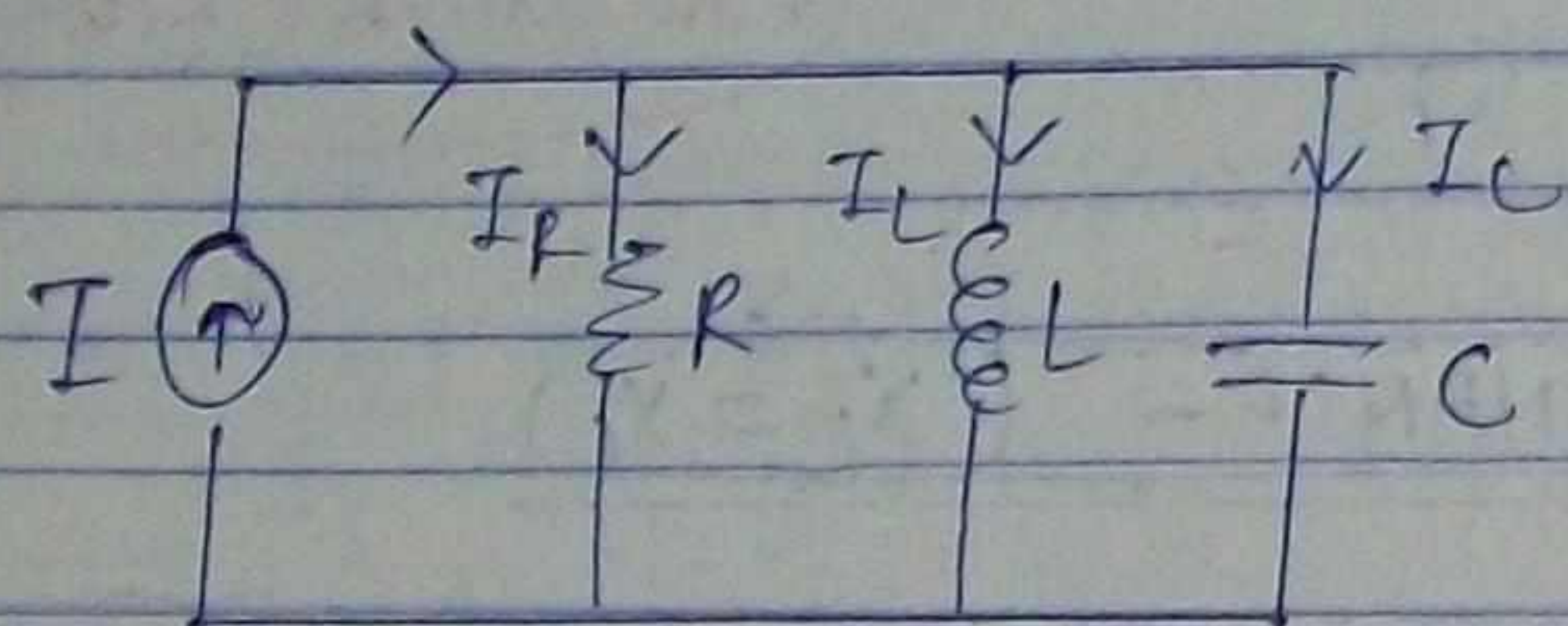
FOR PARALLEL RESONANCE - ($B_L = B_C$)



$$BW = f_2 - f_1$$

$$BW = f_2 - f_1$$

⇒ RELⁿ B/W Q-FACTOR AND DAMPING RATIO -



By KCL, $I = \frac{V}{R} + C \frac{dV}{dt} + \frac{1}{L} \int V dt$

Differentiating wrt t :

$$I' = \frac{1}{R} \frac{dV}{dt} + C \frac{d^2V}{dt^2} + \frac{V}{L}$$

$$\frac{d^2V}{dt^2} + \frac{1}{RC} \frac{dV}{dt} + \frac{V}{LC} = \frac{I'}{C}$$

Putting $d/dt = D$

$$\left(D^2 + \frac{1}{RC} D + \frac{1}{LC} \right) V = 0$$

$$D^2 + \frac{1}{RC} D + \frac{1}{LC} = 0$$

$\xrightarrow{\omega_2 - \omega_1}$ $\xrightarrow{\omega_0}$
 $\frac{1}{RC}$ $\frac{1}{LC}$

$$Q = \frac{\omega_0}{\omega_2 - \omega_1}$$

Comparing with

$$s^2 + 2\zeta \omega_n s + \omega_n^2 = 0$$

$$\omega_n = \frac{1}{\sqrt{LC}}$$

Ans

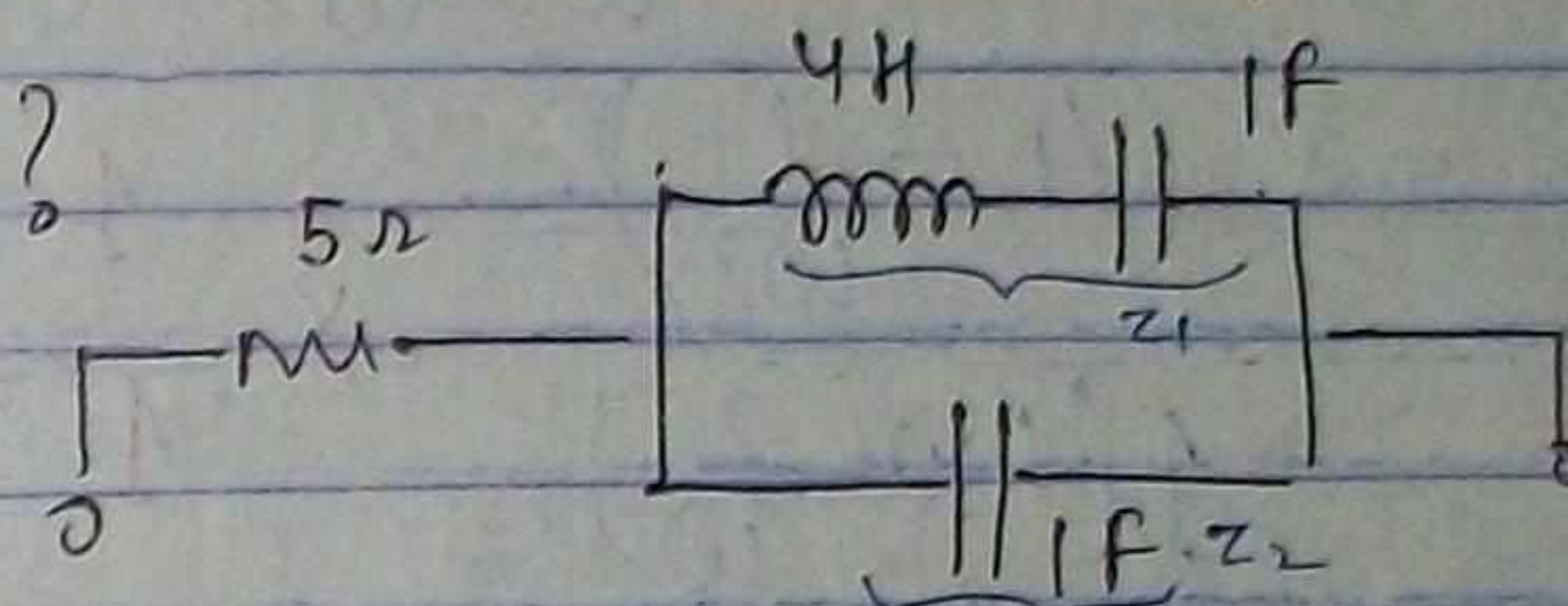
$$\zeta = \frac{1}{2R} \sqrt{\frac{L}{C}}$$

$$Q = R \sqrt{\frac{C}{L}}$$

Ans

$$\zeta = \frac{1}{2Q}$$

Ques find $f_0 = ?$



Ⓐ 0.5 rad/sec

Ⓑ 2 rad/sec

Soln Ⓐ 1 rad/sec

Ⓒ 4 rad/sec.

Ans

— To find resonant freq for any combⁿ of the circuit:

① find Z_{eq} of the circuit

② Equate $Z_{img} = 0$.

$$Z_1 = j(X_L - X_C) = j\left(4\omega - \frac{1}{\omega}\right)$$

$$Z_2 = \frac{-j}{\omega C_2} = \frac{-j}{\omega}$$

$$Z_{eq} = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$Z_{eq} = \frac{j(4\omega - \frac{1}{\omega}) \left(-\frac{1}{\omega}\right)}{j4\omega - \frac{1}{\omega} - \frac{1}{\omega}}$$

$$Z_{eq} = \frac{\left(4 - \frac{1}{\omega^2}\right)}{j\left(4\omega - \frac{2}{\omega}\right)} \times \frac{j}{j}$$

$$Z_{eq} = \frac{j\left(4 - \frac{1}{\omega^2}\right)}{\left(\frac{2}{\omega} - 4\omega\right)}$$

$$Z_{eq} \approx 4 - \frac{1}{\omega^2} = 0 \quad \omega = 0.5 \text{ rad/sec}$$

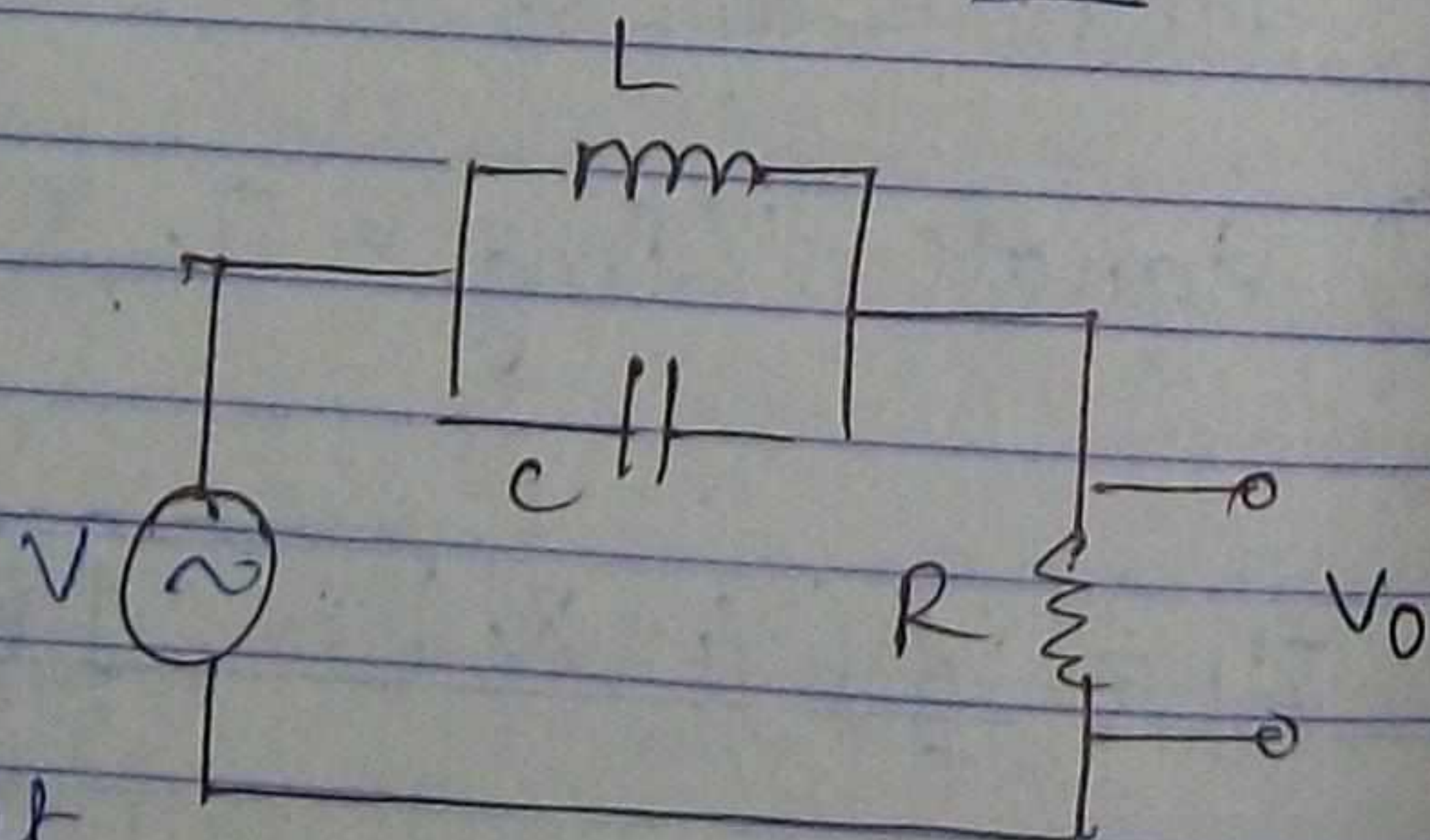
Ques. find V_o at resonance

solⁿ:

ideal tank circuit

$$Z_{DY} = \infty$$

$$\therefore V_o = 0 \quad \checkmark$$



Ques. A parallel RLC circuit have $C = 0.1 \mu F$
 $BW = 1 \text{ kHz}$ find effective resistance of the circuit.

solⁿ:

$$BW = \omega_2 - \omega_1 = \frac{1}{RC} \text{ rad/s}$$

$$f_2 - f_1 = \frac{1}{2\pi RC} \text{ Hz}$$

$$1 \times 10^3 = \frac{1}{2\pi RC (0.1 \times 10^{-6})}$$

$$R = 1.59 \text{ k}\Omega$$

THEOREMS

— When network is having more no. of meshes and nodes, response can be easily obtained by using theorems.

⇒ SUPER POSITION THEOREM —

"In any linear bidirectional circuit having more than one independent source the response in any one of the element is equal to algebraic sum of the response caused by individual sources, while the rest of the sources are replaced by its internal resistances."

Ques. Find I by super position theorem

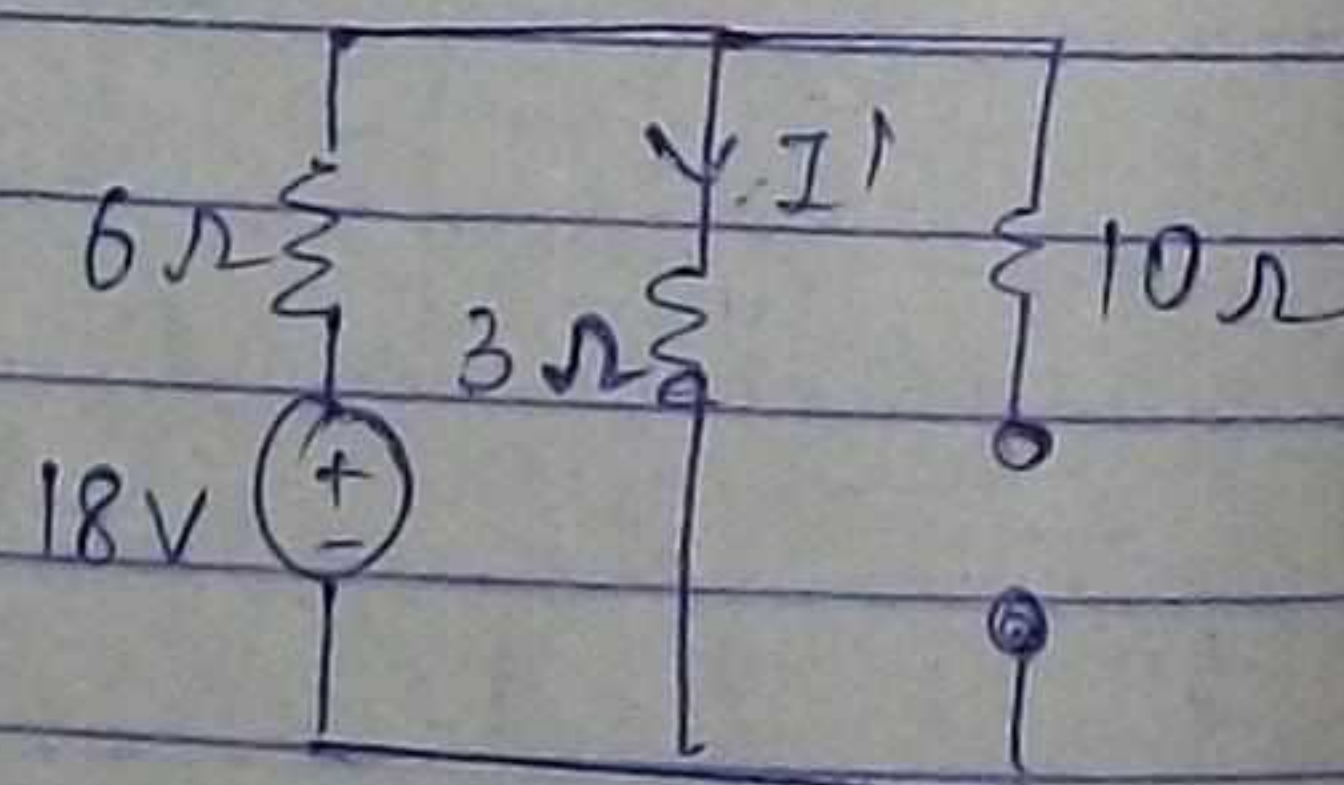
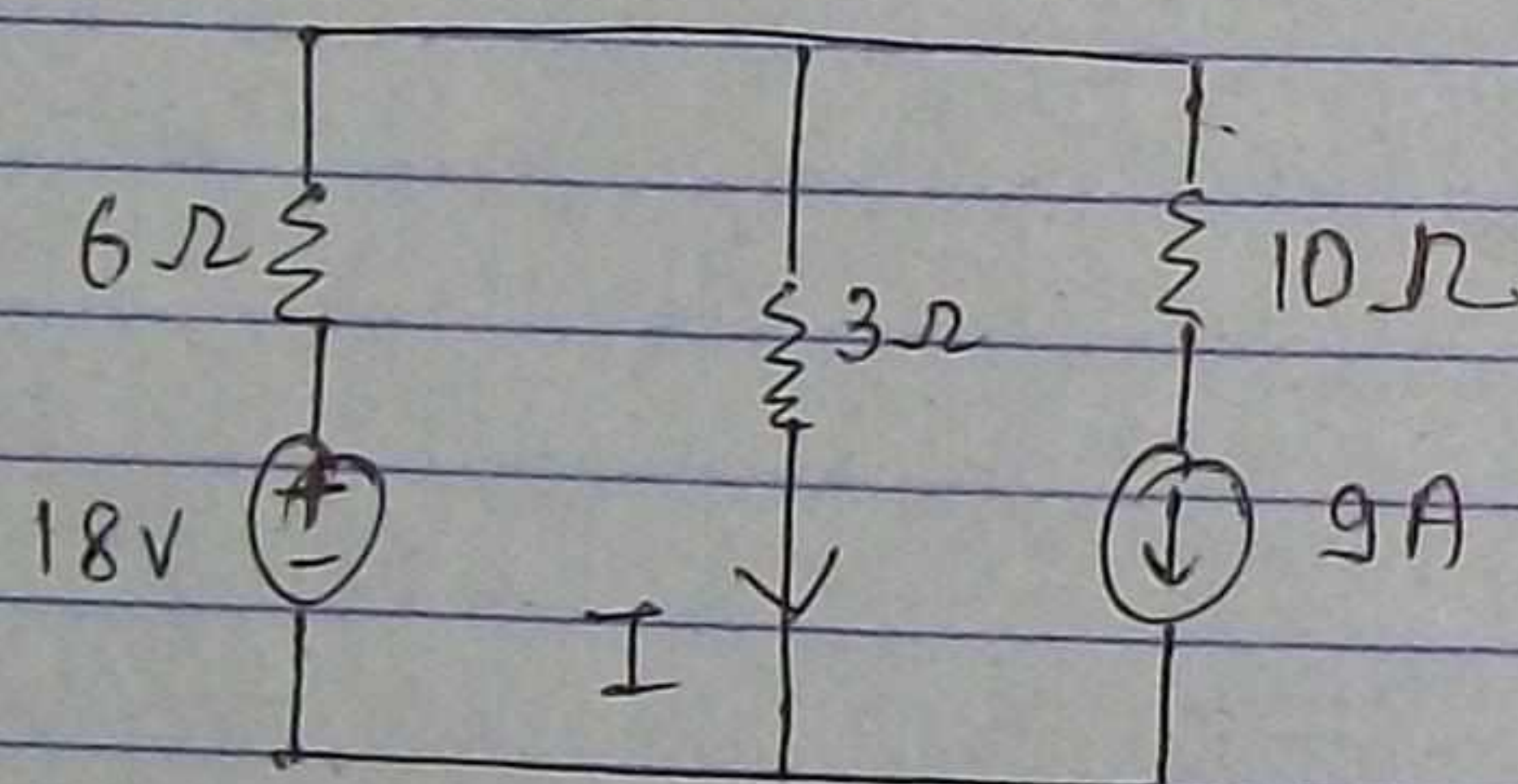
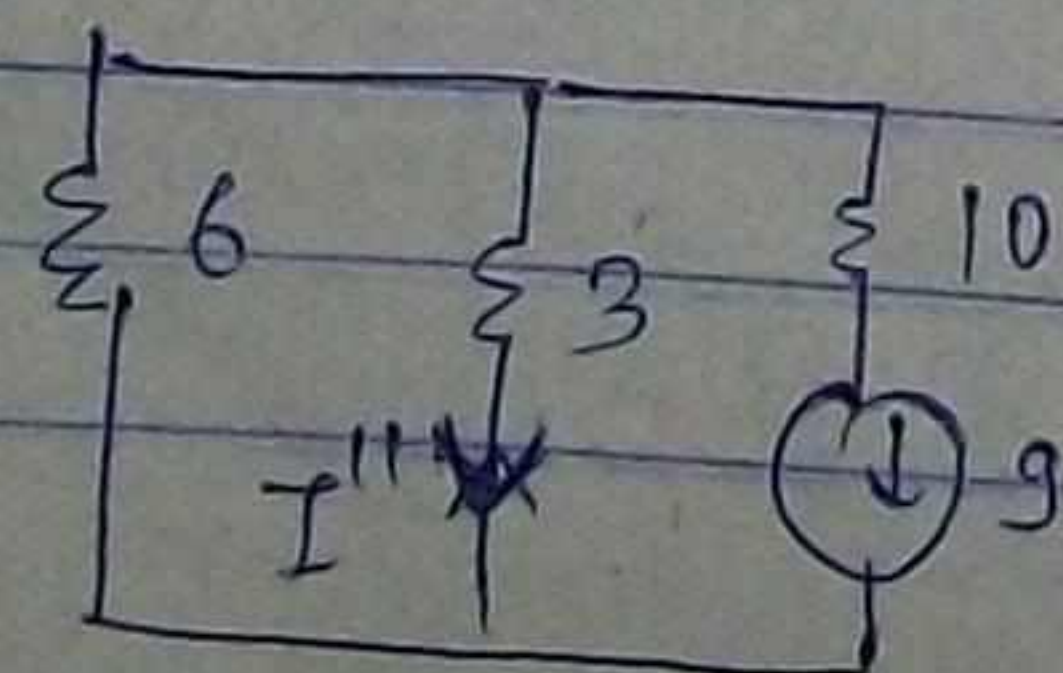
Solⁿ:

Case (1):

$$I' = \frac{18}{6+3}$$

$$I' = 2A$$

Case (2):

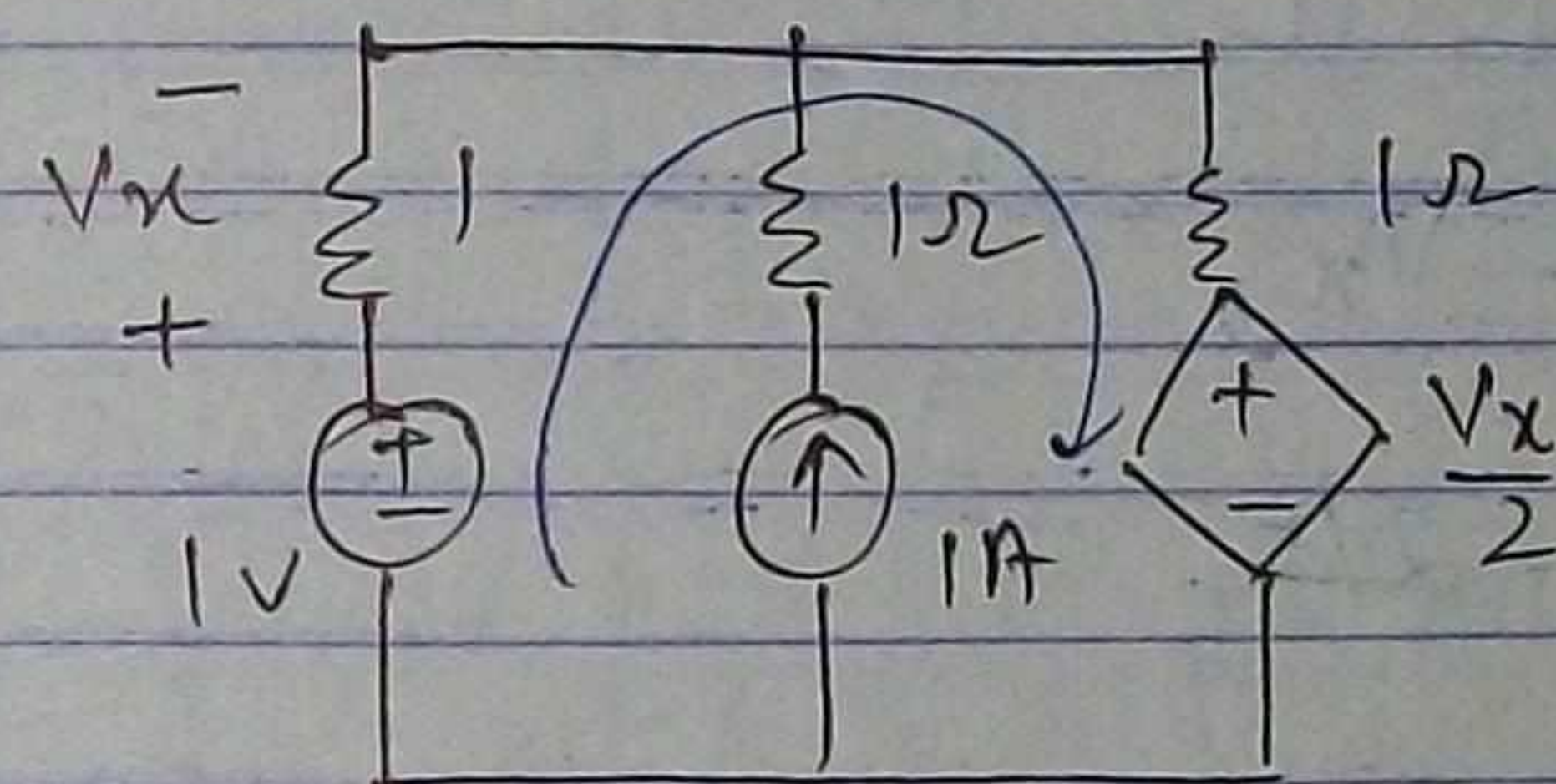


$$I'' = -\frac{6}{6+3} \times 9 = -6A$$

Total current, $I = I' + I''$

$$I = 2 - 6 = -4A$$

Ques.



find V_x using super position

Solⁿ:

Case (1)

$$1 = V_x + V_x + \frac{V_x}{2}$$

$$1 = \frac{5V_x}{2}$$

$$V_{x1} = \frac{2}{5}$$

While applying super position theorem dependent source cannot be replaced by either open circuit or short circuit.

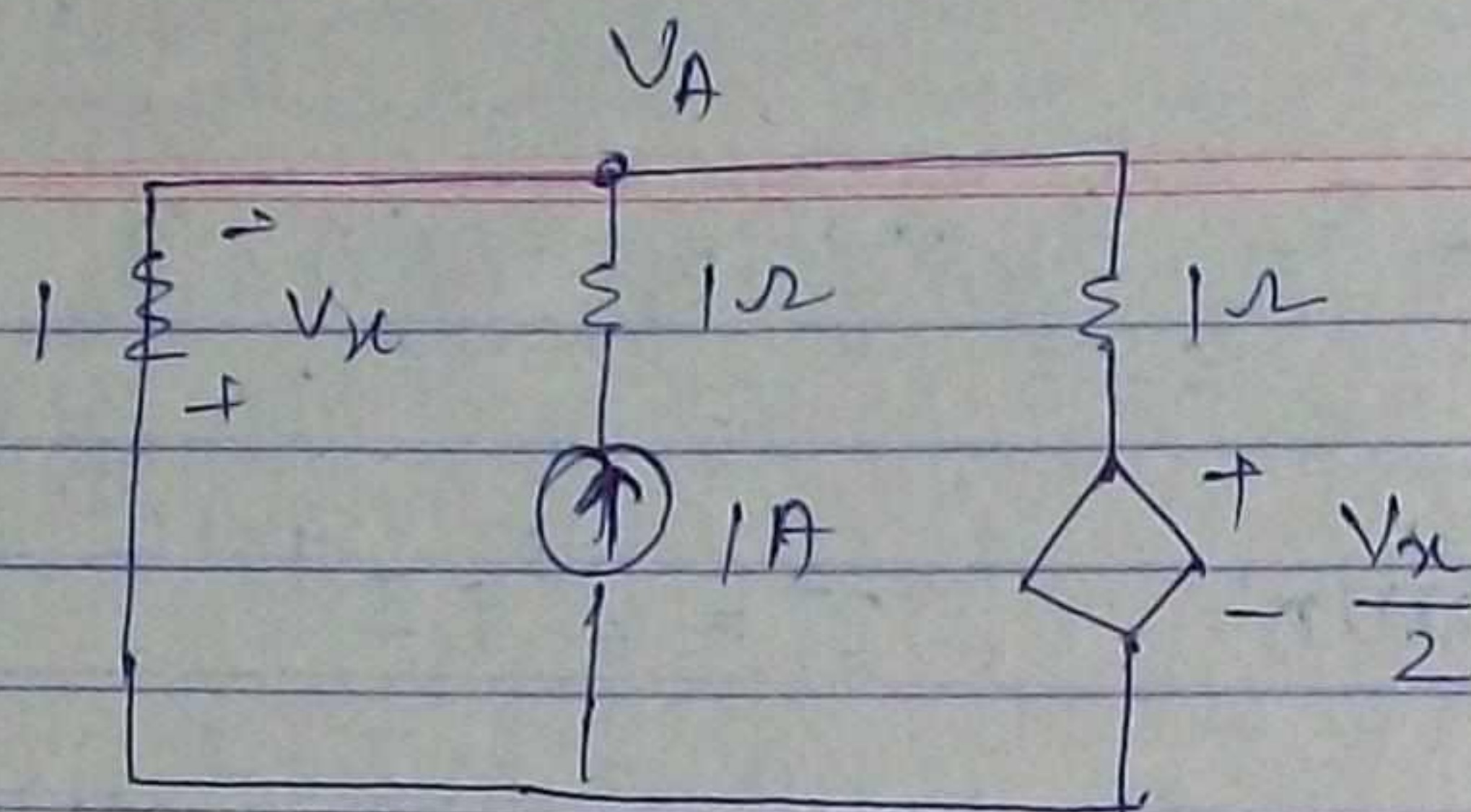
$$I = \frac{1 - V_x/2}{1+1}$$

$$V_{x1}' = 1 \times I = I$$

$$2V_x \neq \frac{V_x}{2} = 1$$

$$V_{x1} = \frac{2}{5}$$

Case ②:



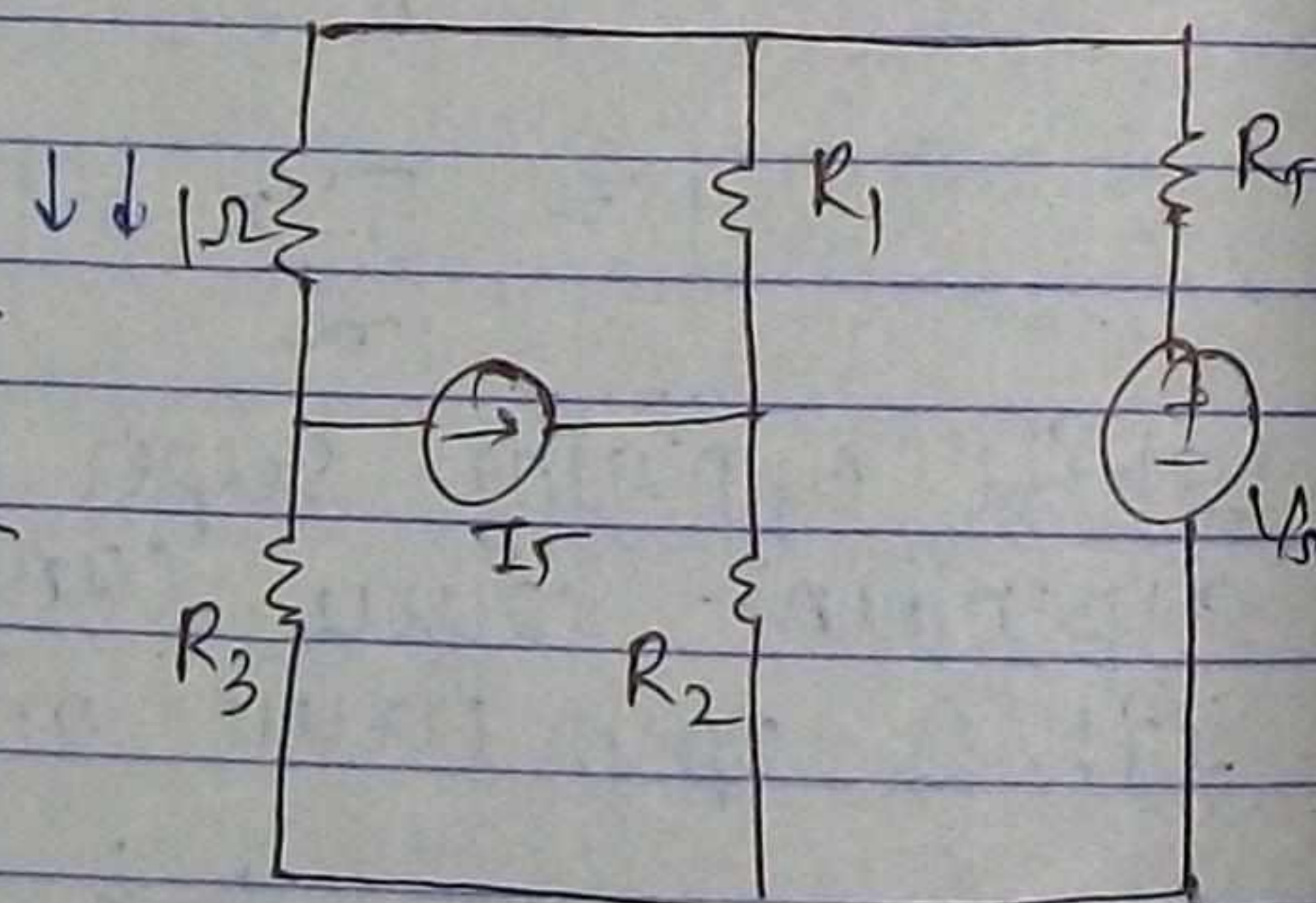
$$\frac{V_A}{1} + \frac{V_A - V_{x2}/2}{1} = 1$$

$$V_A = -V_{x2}$$

$$V_{x2} = -2/5$$

$$\text{So, } V_x = V_{x1} + V_{x2} = \frac{2}{5} - \frac{2}{5} = 0$$

Ques. Power dissipation in 1Ω resistor is 576 W when current source is acting alone and power dissipation is 1 W when voltage source is acting alone. find total P.D in 1Ω resistor.



Solⁿ:

$$I = \pm I' \pm I''$$

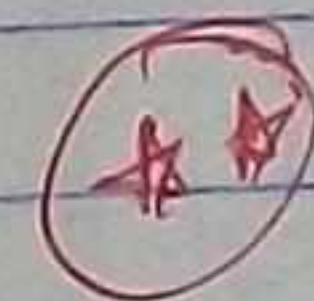
$$V = \pm V' \pm V''$$

$$P = \pm P' \pm P'' \quad \times$$

$$\text{Since } I = \pm I' \pm I''$$

$$I = \pm \sqrt{\frac{P'}{R}} \pm \sqrt{\frac{P''}{R}}$$

$$P = \left[\pm \sqrt{\frac{P'}{R}} \pm \sqrt{\frac{P''}{R}} \right]^2 R$$



$$P = \left[\pm \sqrt{P'} \pm \sqrt{P''} \right]^2$$

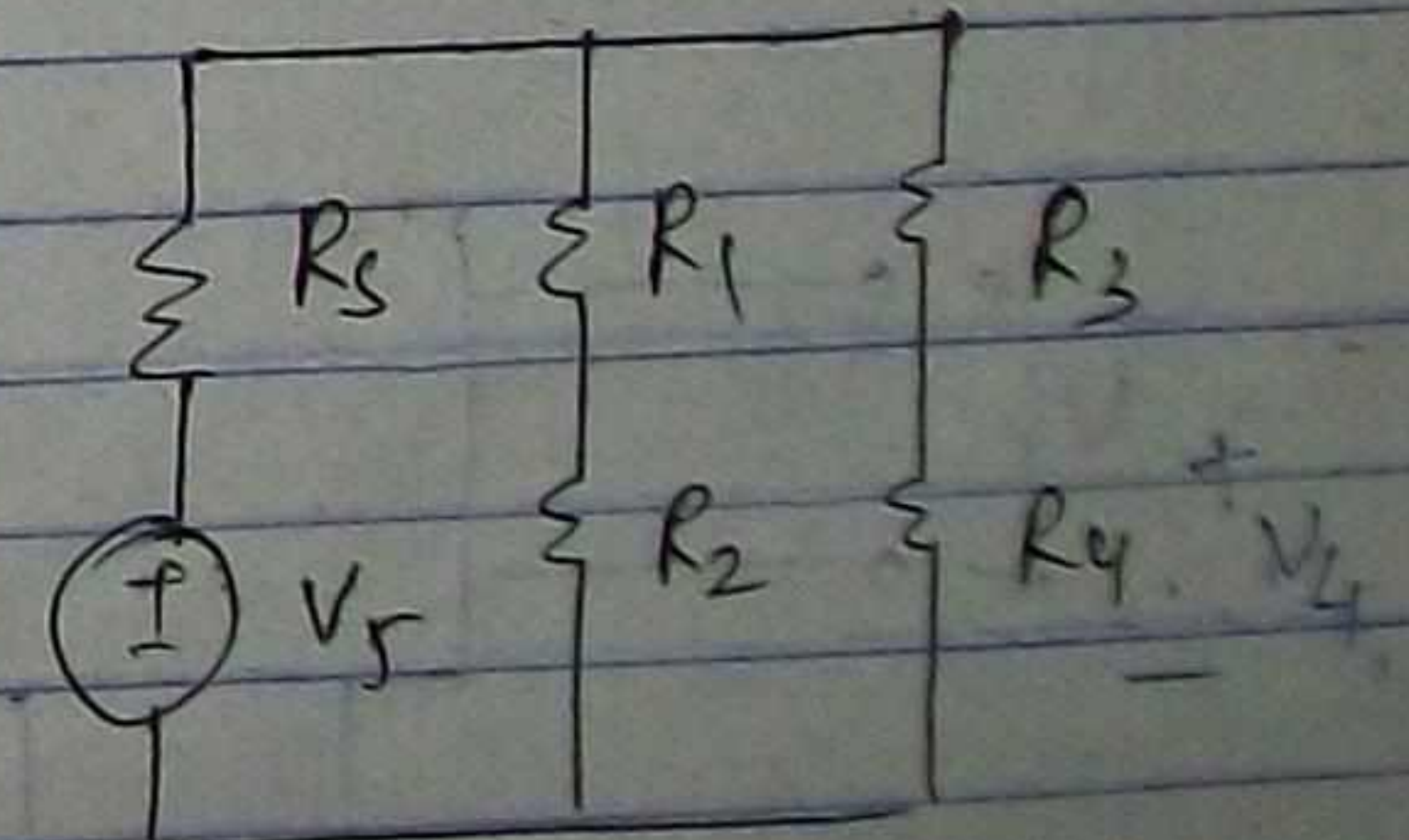
for above quesⁿ

$$P = \left[\pm \sqrt{P'} \pm \sqrt{P''} \right]^2$$

$$P = \left[\sqrt{576} + \sqrt{1} \right]^2$$

$$P = \underline{\underline{625\text{ W}}}$$

Ques. In the circuit shown if source voltage increase by 10% find variation of the power in the R_4 resistor.



- ① 10% ② 20% ③ 21% ④ 19%

$$P_y = \frac{V_y^2}{R_y}$$

$$P_y = \frac{(1.1 V_y)^2}{R_y} \quad 10\% \uparrow$$

$$P_y' = 1.21 \cdot \frac{V_y^2}{R_y}$$

$$P_y' = 1.21 P_y \quad \underline{\text{Ans.}} \quad 21\%$$

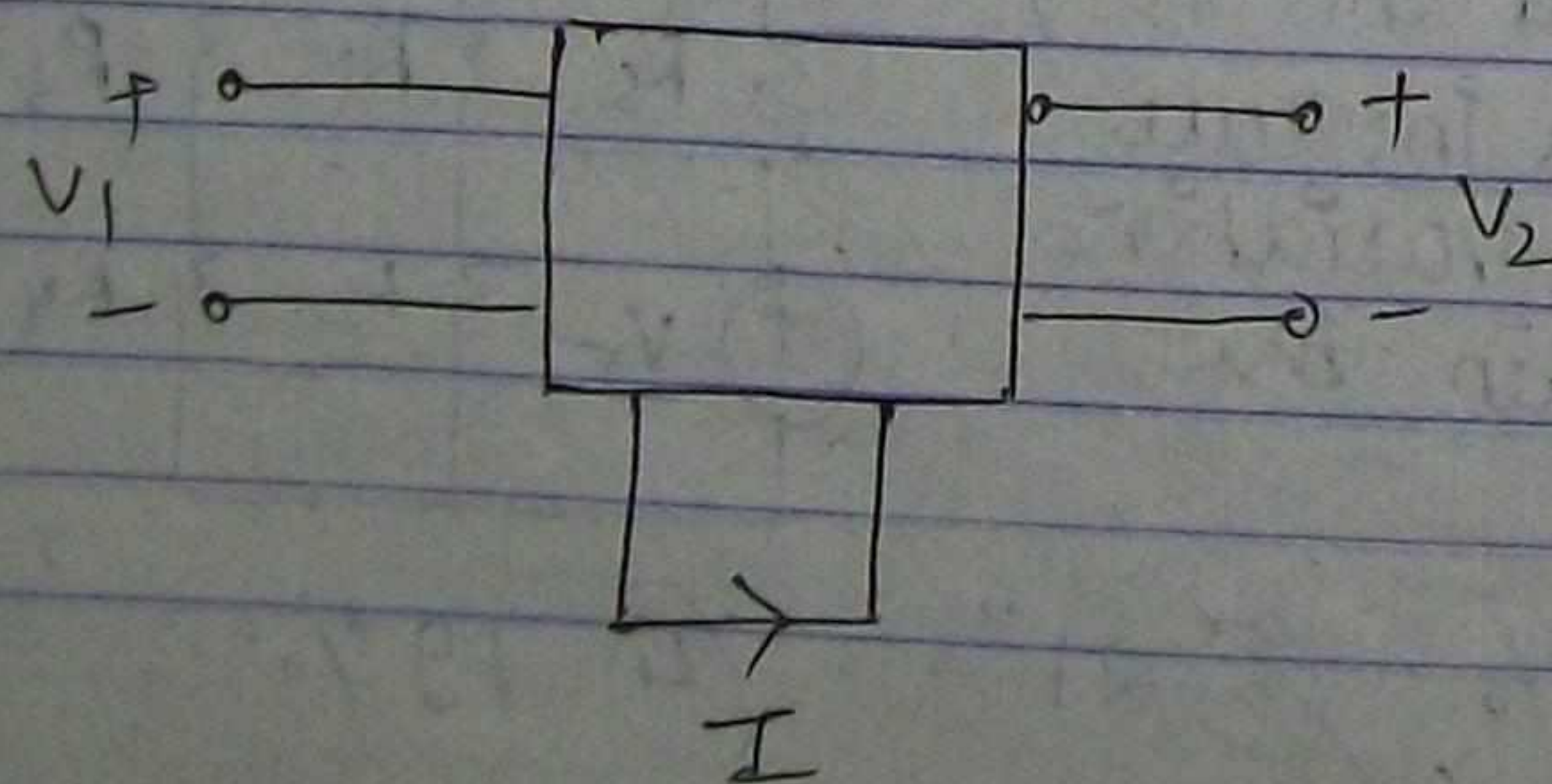
- when network is having linear bilateral elements based on the homogeneity principle if excitation is multiplied with constant 'k' response of each element also multiplied with constant 'k'.

Ques. Find I when

$$V_1 = 10V$$

$$V_2 = -12V$$

V_1	V_2	I
2	0	3
0	4	-2



15 + 6

$$\text{Sol}^n: \quad V_1 = 2 \quad I = 3A$$

$$V_1 = 10 \quad I' = 15A$$

$$V_2 = 4 \quad I = -2A$$

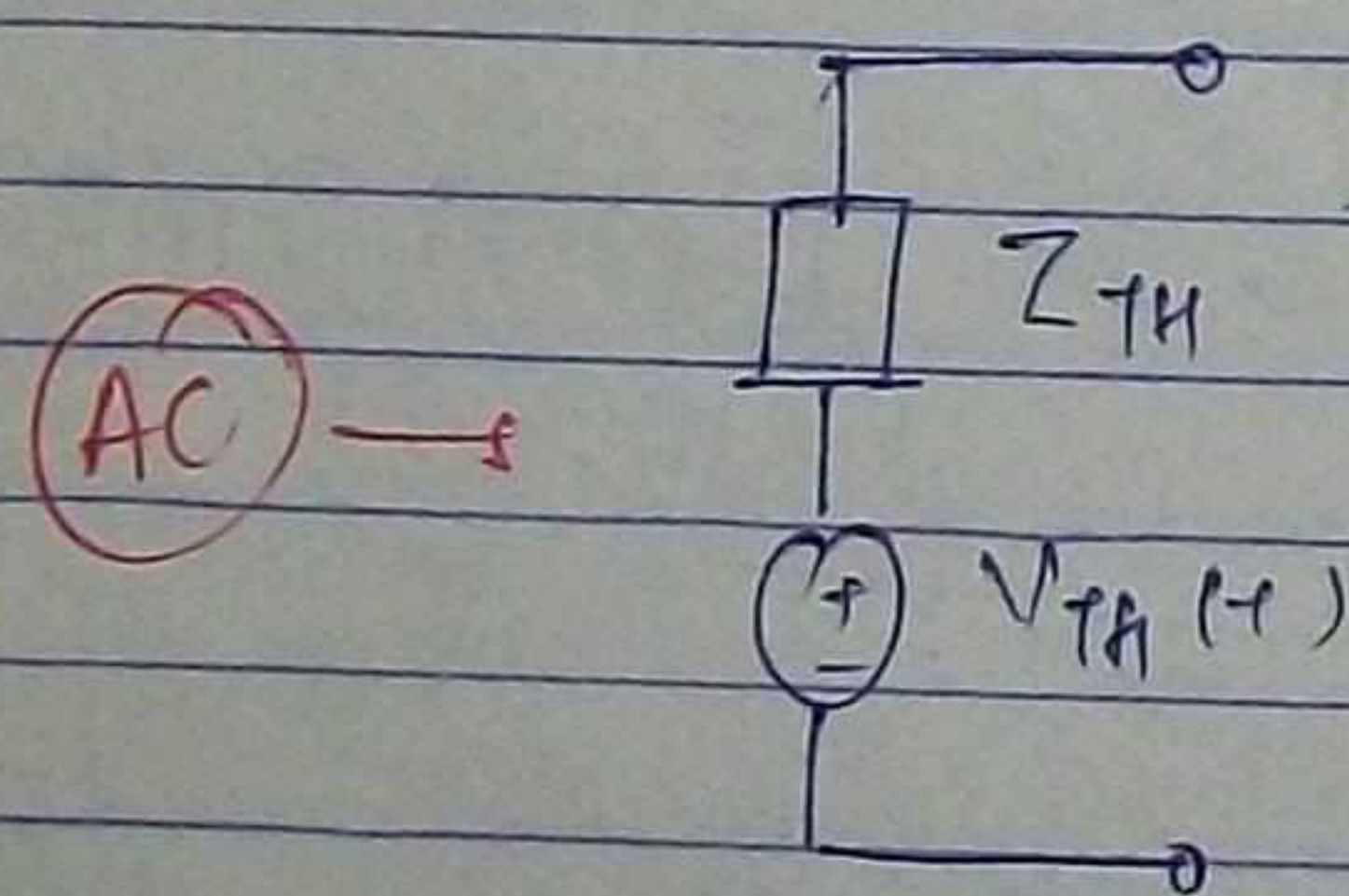
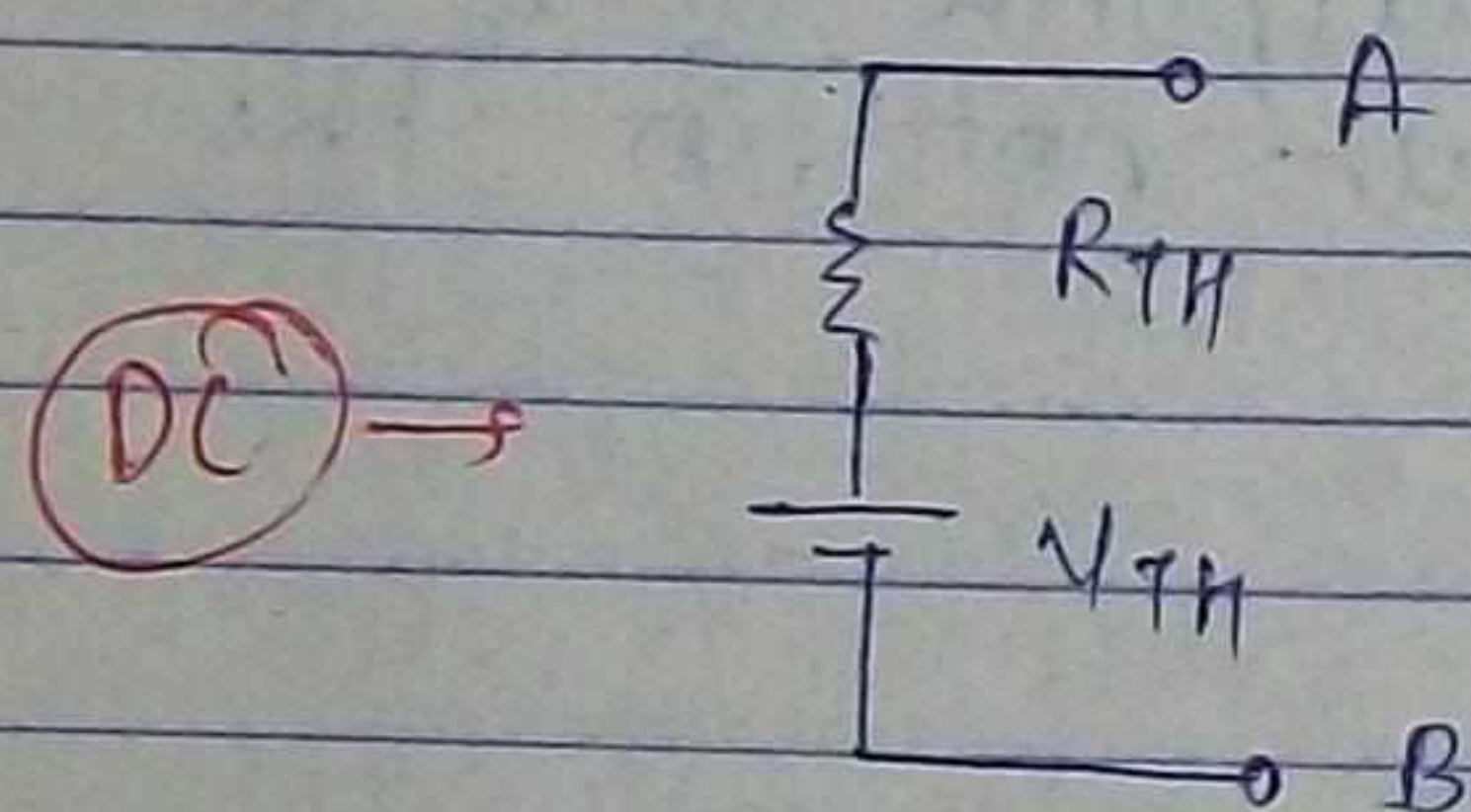
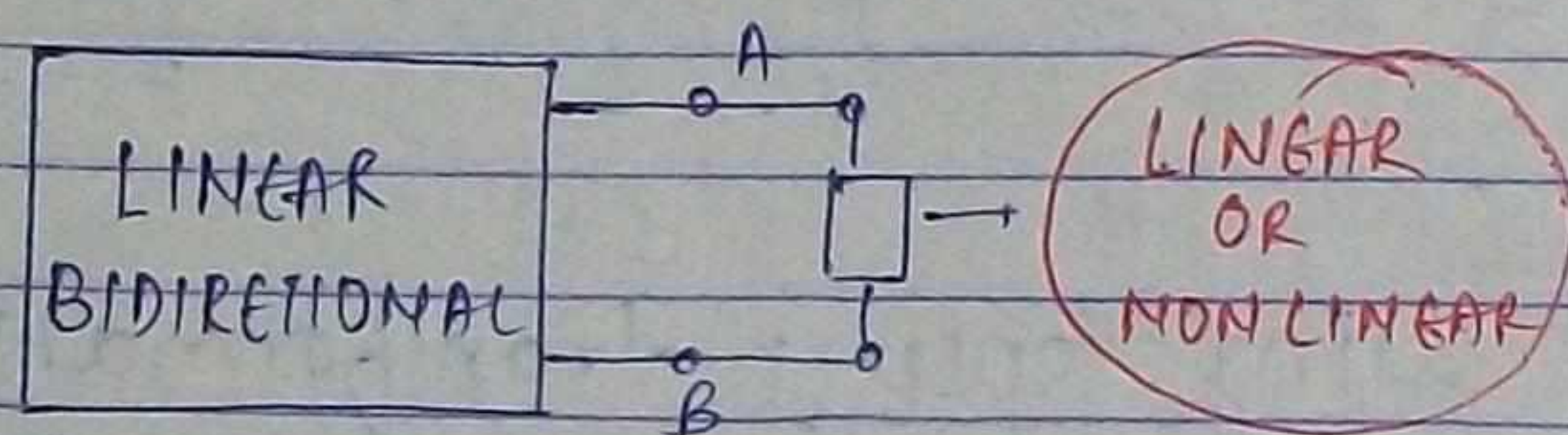
$$V_2 = 12 \quad I'' = -6A$$

$$I = I' + I'' = 15 + 6 = \underline{21 \text{ Amp}}$$

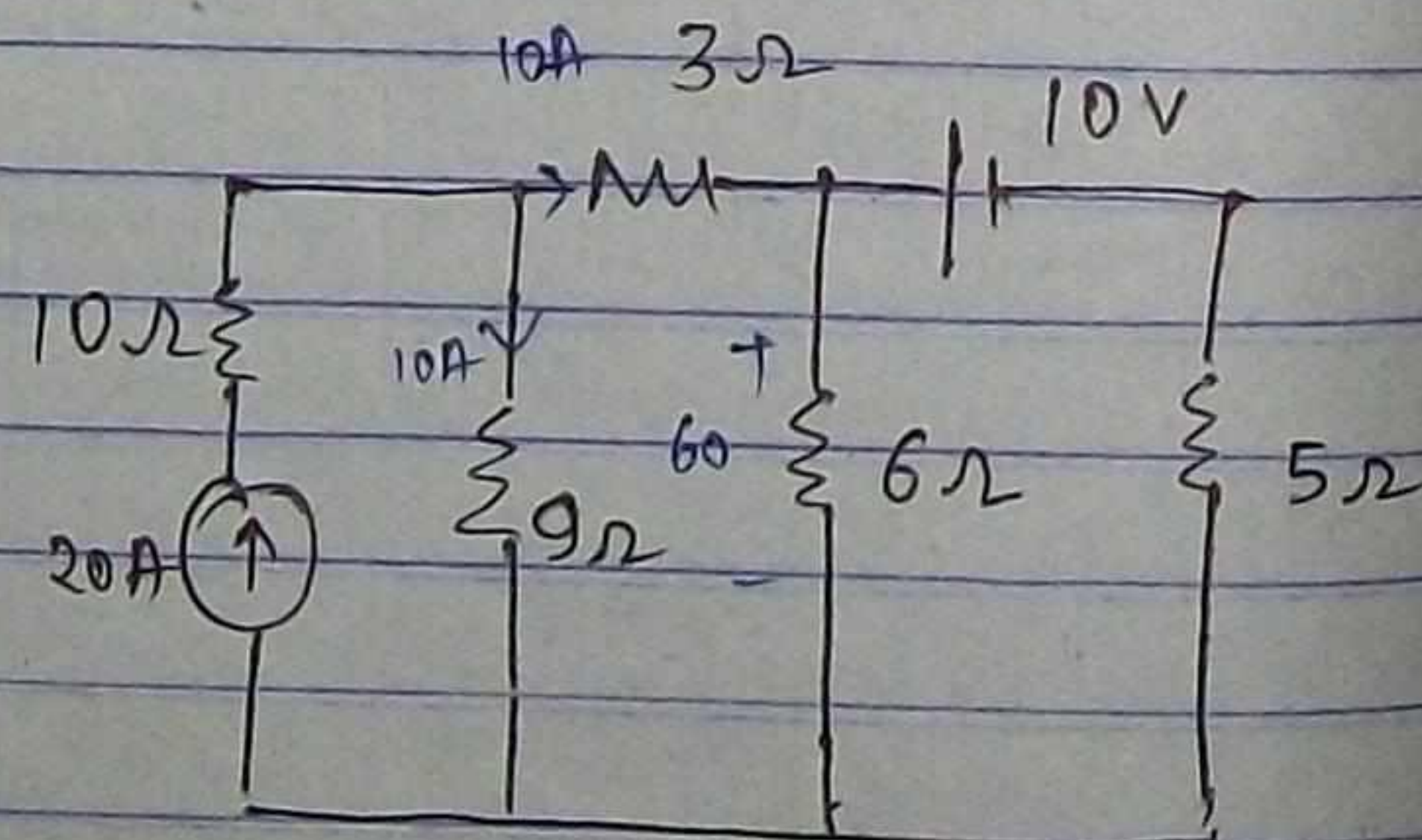
⊗ while using source transformation it is possible to obtain response only in the load resistance but not in the internal resistance.

⇒ THEVENIN'S THEOREM -

- In any linear bidirectional circuit having more number of active and passive elements, it can be replaced by single equivalent circuit consisting of equivalent voltage source (V_{TH}) in series with equivalent resistance (R_{TH}).

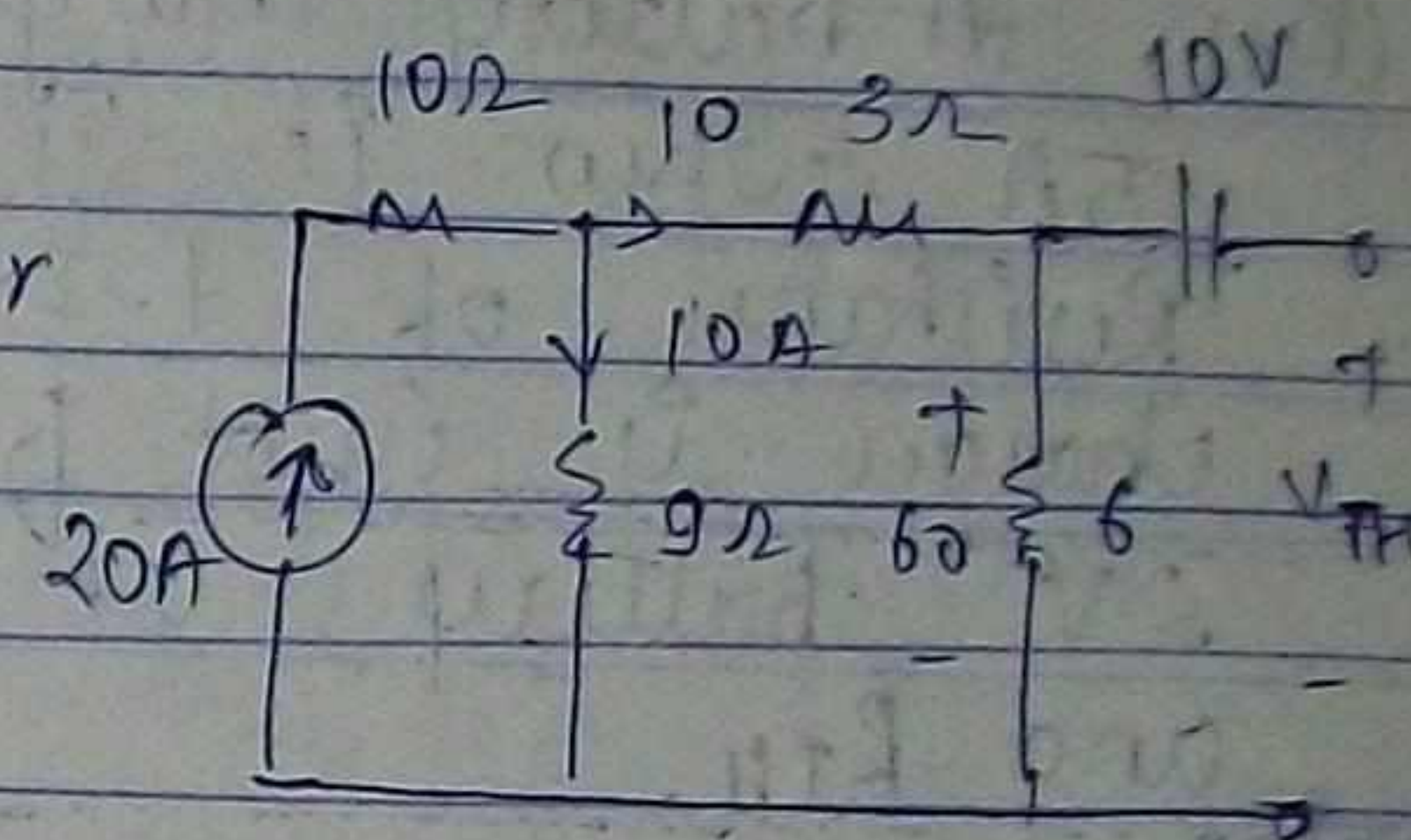


Ques: Find current flow in 5Ω resistor by using thevenin's theorem



Solⁿ: ① for V_{TH} :

Disconnect the load resistor and find open circuit voltage w.r.t load terminals.

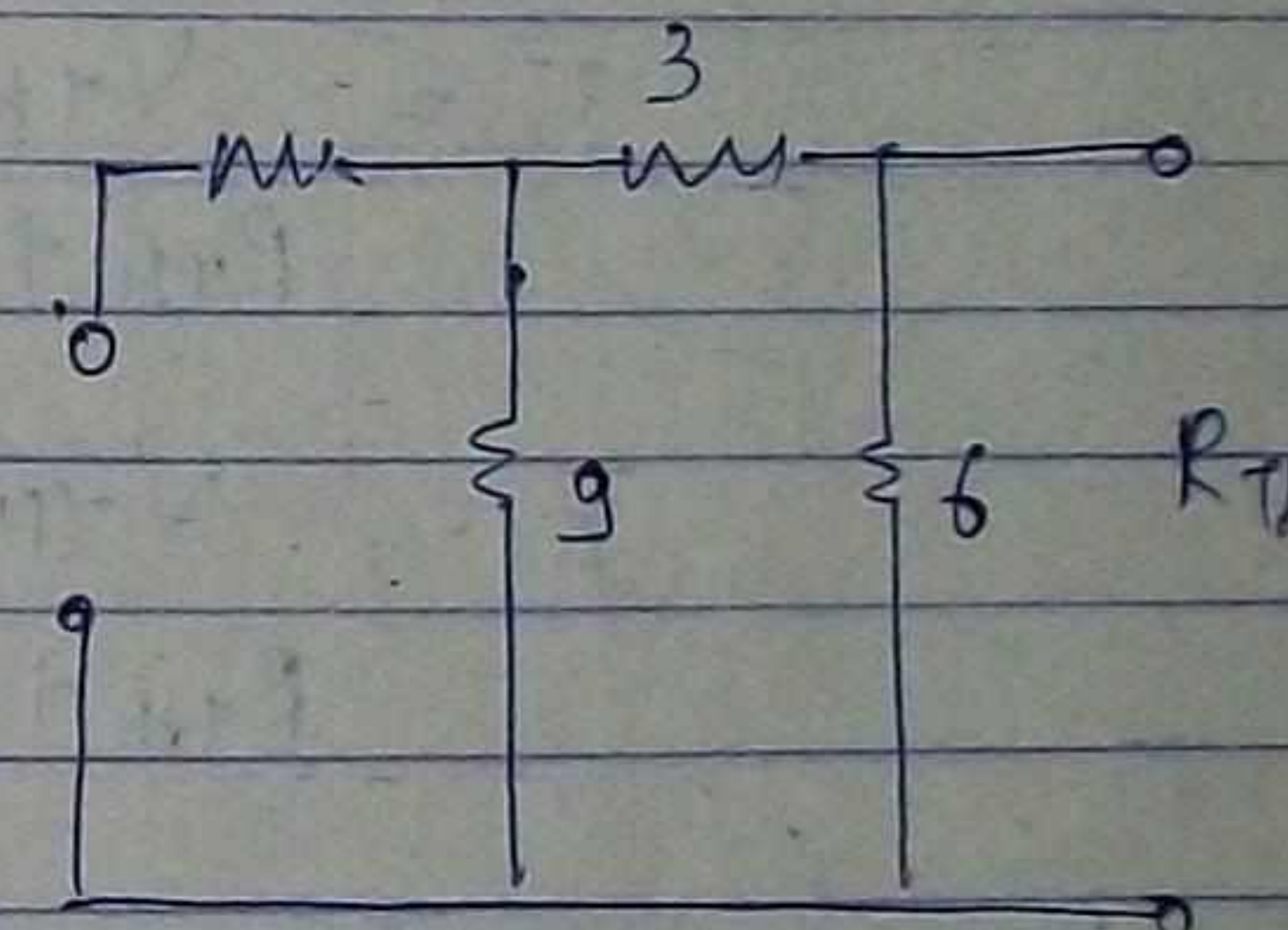


$$60 = 10 + V_{TH}$$

$$V_{TH} = 50$$

② for R_{TH} :

Deactivate all the independent sources and find equivalent resistance w.r.t load terminals.

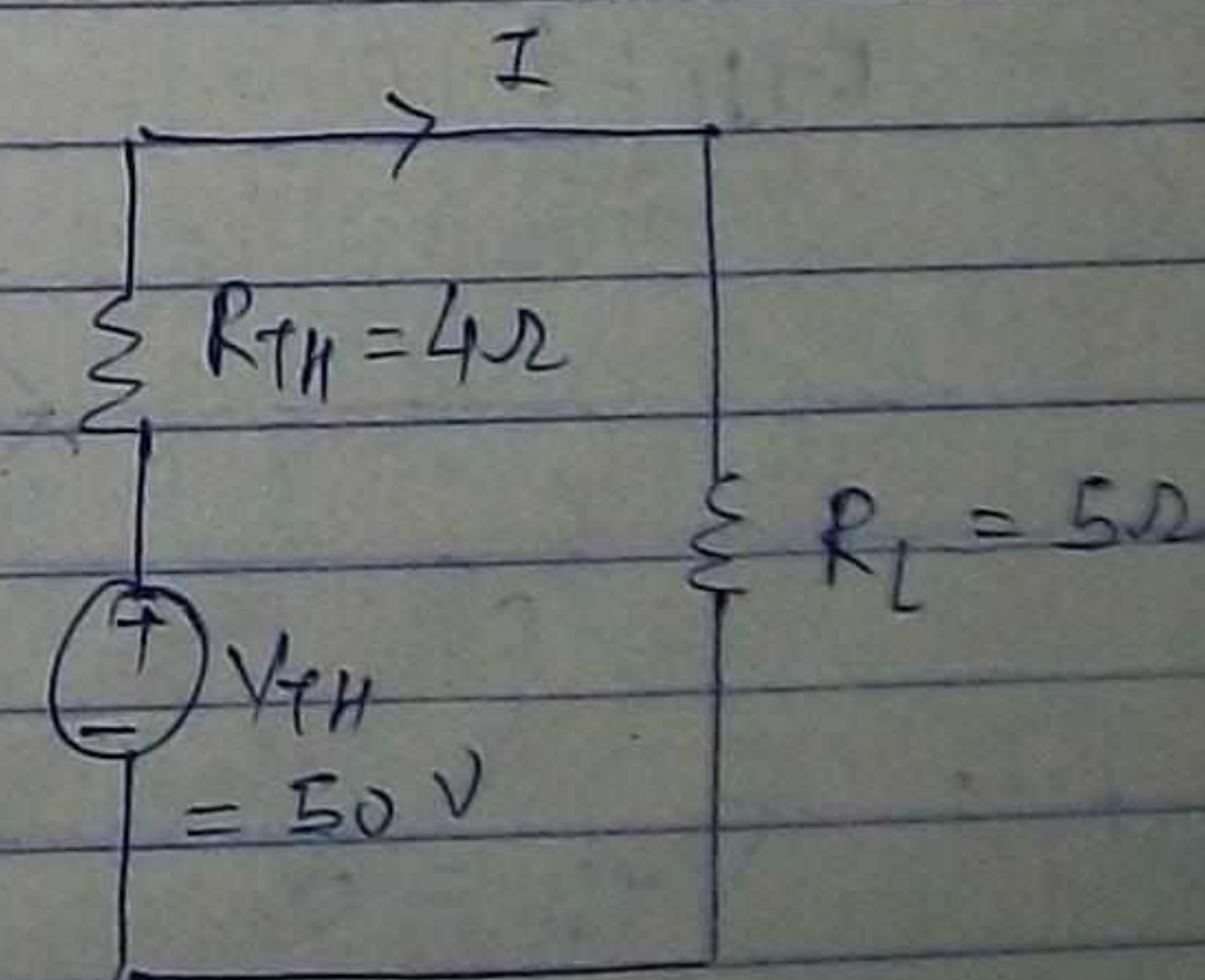


$$R_{TH} = (9 + 3) \parallel 6 = 4\Omega$$

Thevenin's eqⁿ circuit

$$I = \frac{V_{TH}}{R_{TH} + R_L}$$

$$I = \frac{50}{9} = 5.5 \text{ A}$$



Ques. A battery charger draws a current of 5A when it is connected to load resistance of 1Ω when the same battery charger is used for charging of ideal 2V battery at 7A rate find V_{TH} and R_{TH} .

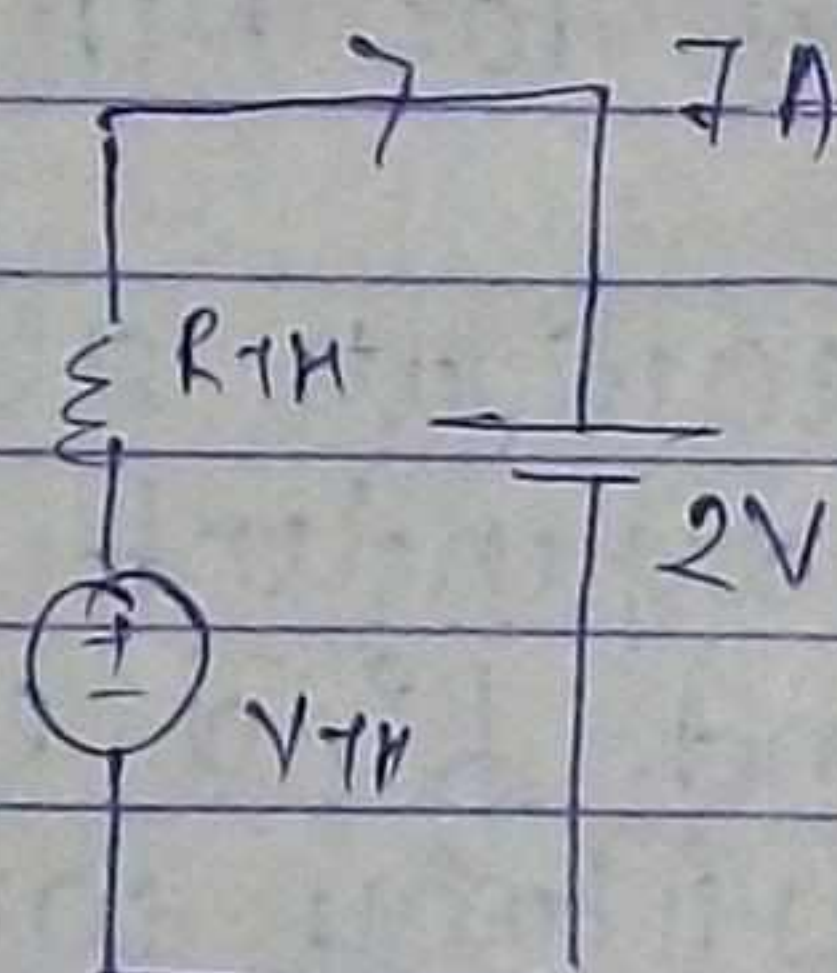
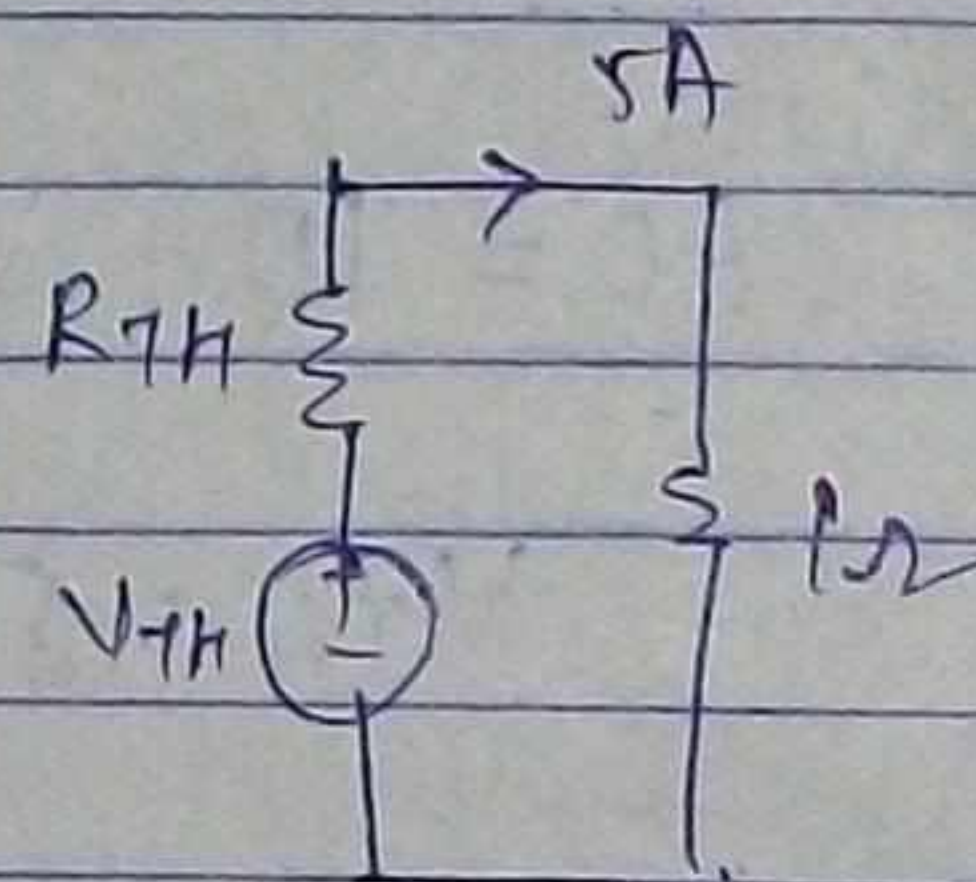
Soln:

$$I = \frac{V_{TH}}{R_{TH} + R_L}$$

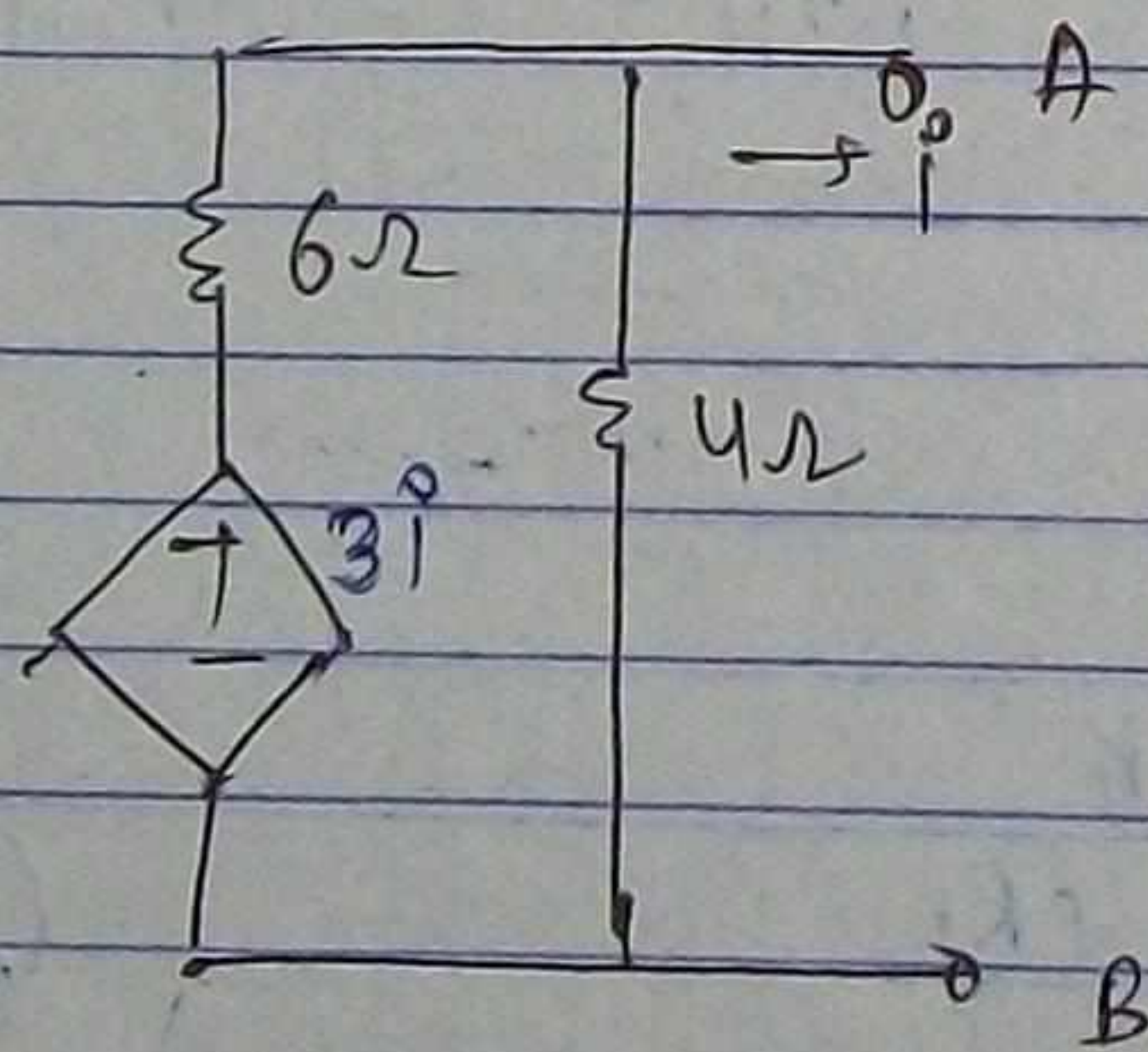
$$5 = \frac{V_{TH}}{R_{TH} + 1} \quad \text{--- (1)}$$

$$7 = \frac{V_{TH} - 2}{R_{TH} + 1} \quad \text{--- (2)}$$

$$R_{TH} = 1.5\Omega \quad V_{TH} = 12.5V$$



Ques. $V_{TH} = ?$
 $R_{TH} = ?$

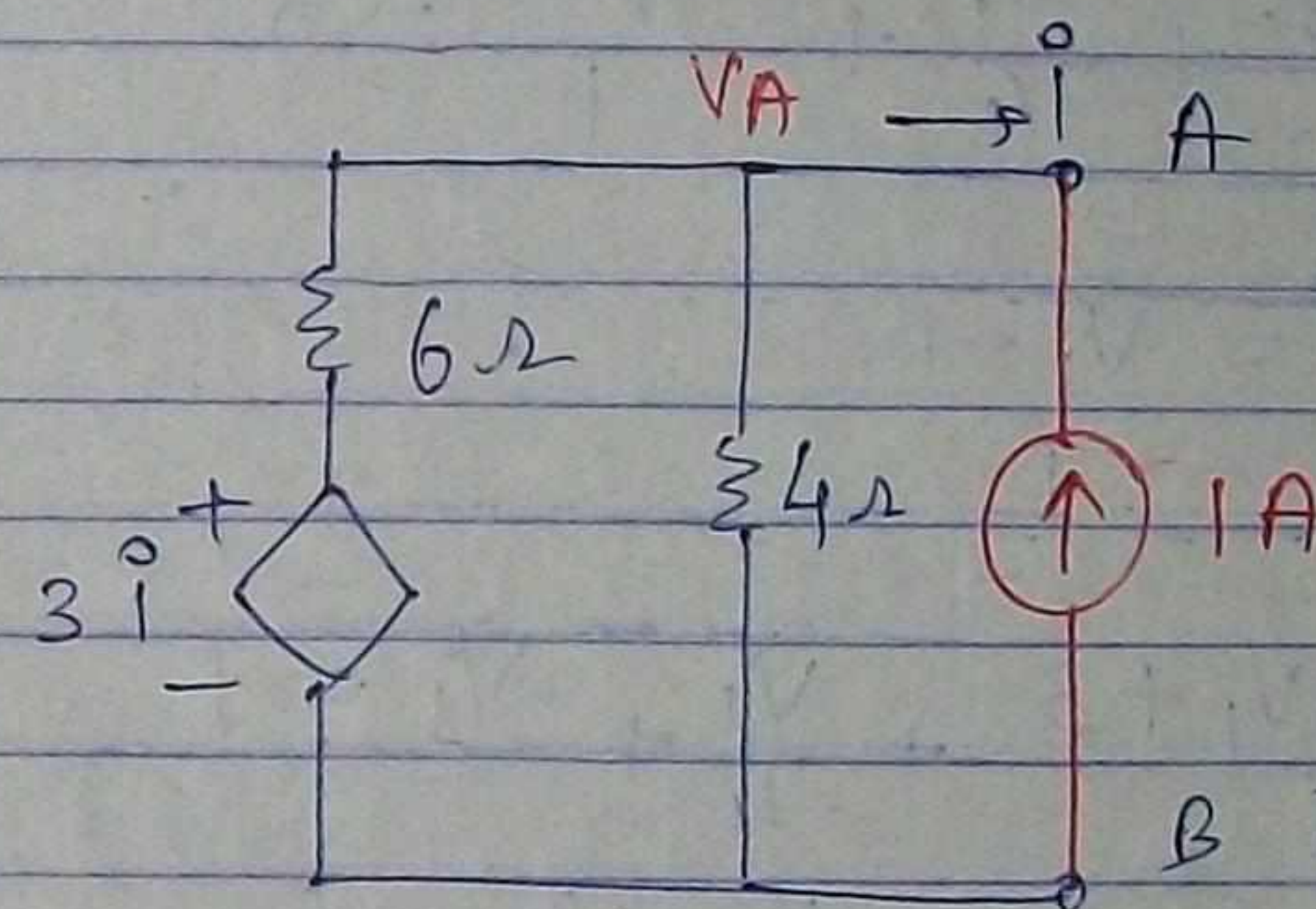


Soln: $V_{TH} = 0$

$\Rightarrow$ THEVENIN'S MAX. POWER THEOREM -

⊗ When independent source is not present in the given circuit $V_{TH} = 0$.

for R_{TH} :



$$\frac{V_A - 3i}{6} + \frac{V_A}{4} = 1 \quad i = -1$$

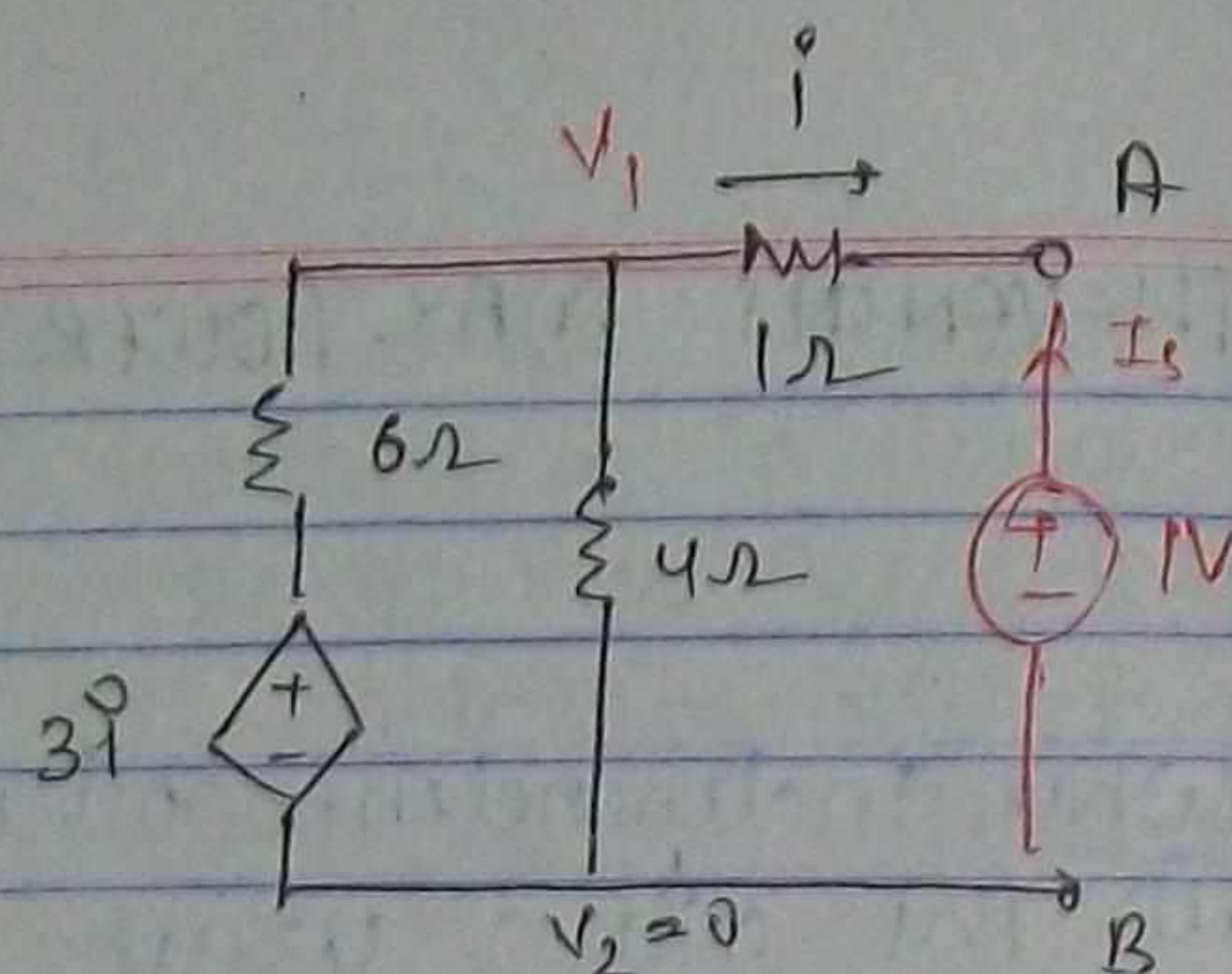
$$\frac{V_A}{6} + \frac{V_A}{4} = 1 - \frac{3}{6} = 0.5$$

$$V_A = 6/5 = 1.2, \quad V_B = 0$$

$$V_{AB} = V_A - V_B = 1.2$$

$$R_{TH} = \frac{V_{AB}}{I_s} = \frac{1.2}{1} = 1.2\Omega$$

Ques. find R_{TH} ?



Solⁿ:

$$\frac{V_1 - 3}{6} + \frac{V_1}{4} + \frac{V_1 - 1}{1} = 0$$

$$i = \frac{V_1 - 1}{1}$$

$$\frac{V_1 - 3V_1 + 3}{6} + \frac{V_1}{4} + V_1 = 1$$

$$-\frac{2V_1}{6} + \frac{1}{2} + \frac{V_1}{4} + V_1 = 1$$

$$V_1 = 6/11$$

$$i = -5/11$$

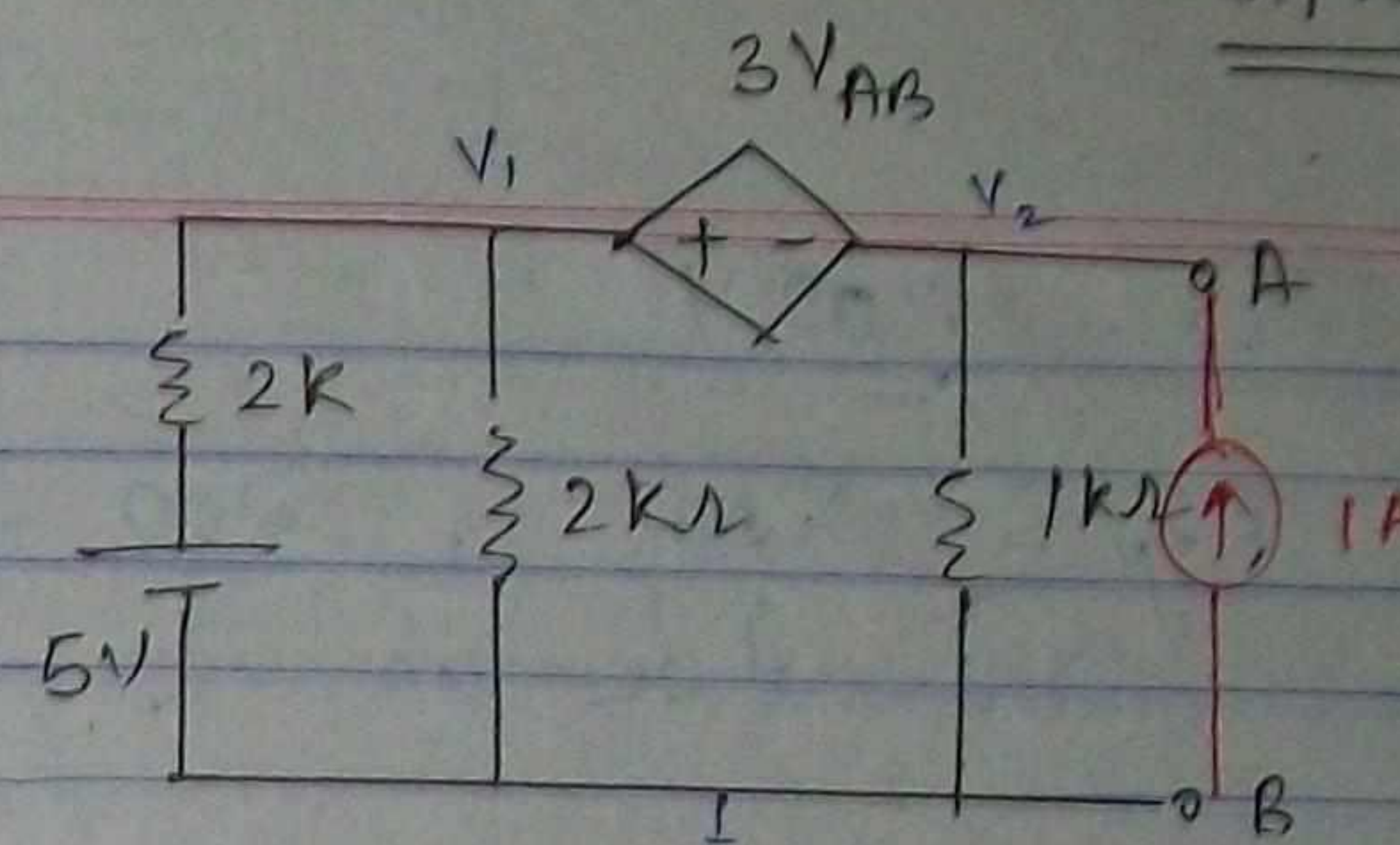
$$I_s = -i = \underline{5/11}$$

$$R_{TH} = \frac{V_{AB}}{I_s} = \frac{1}{5/11} = \underline{\underline{11/5}}$$

$$R_{TH} = 2.2\Omega$$

31/10/11

Ques. find V_{TH} and R_{TH} ?



Solⁿ:

$$V_{TH} -$$

$$\frac{V_1 - 5}{2k} + \frac{V_1}{2k} + \frac{V_2}{1k} = 0 \quad \text{--- (1)}$$

$$V_1 - V_2 = 3V_{TH} \quad \text{--- (2)}$$

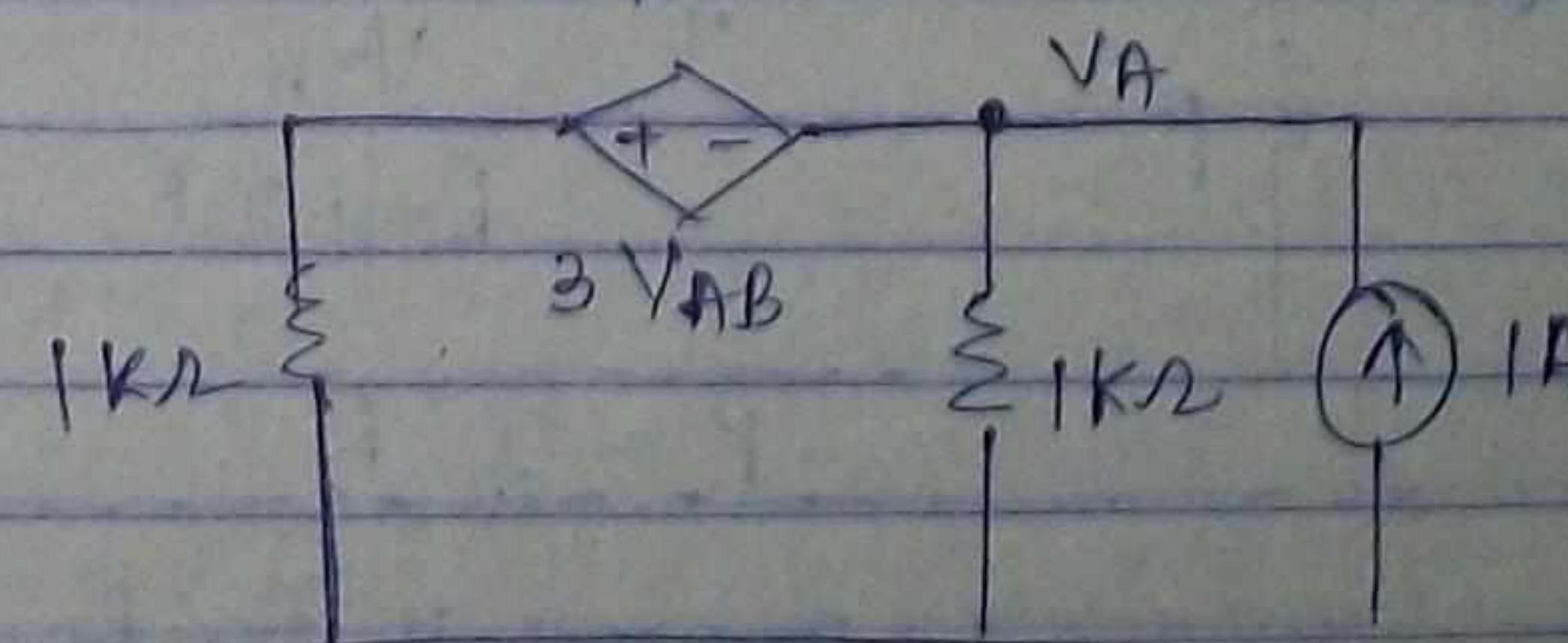
$$V_2 = V_{TH}, \quad V_1 = 4V_{TH}$$

from (1) $\frac{4V_{TH} - 5}{2} + \frac{4V_{TH}}{2} + \frac{V_{TH}}{1} = 0$

$$10V_{TH} = 5$$

$$V_{TH} = \underline{\underline{0.5V}}$$

$$R_{TH} -$$



$$\frac{V_A + 3V_{AB}}{1k} + \frac{V_A}{1k} = 1$$

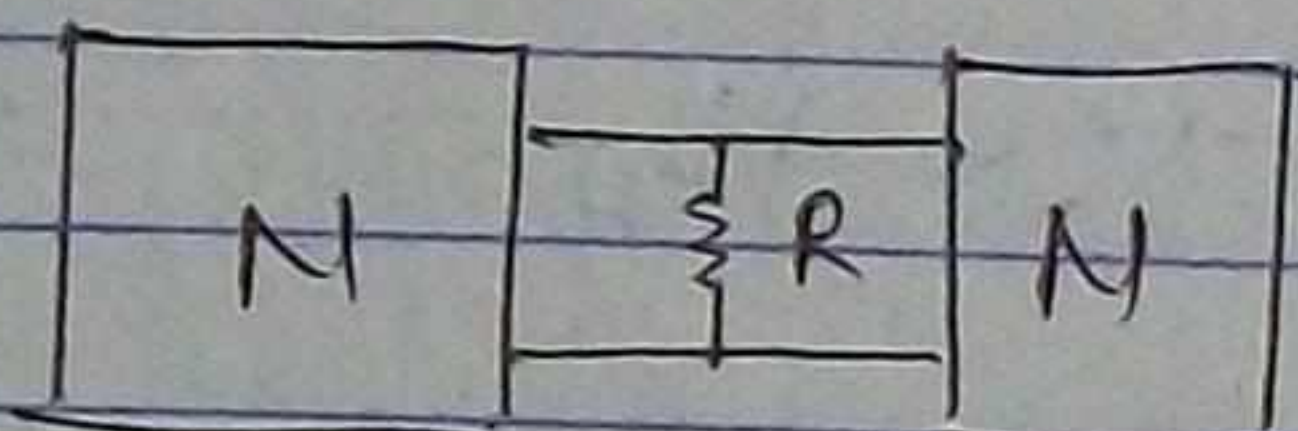
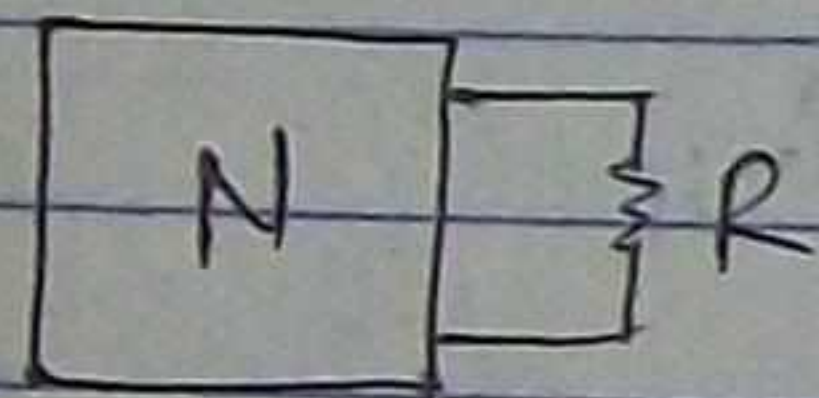
also, $V_{AB} = V_A - V_B = V_A - 0 = V_A$

$$V_A = \underline{200\text{ V}}$$

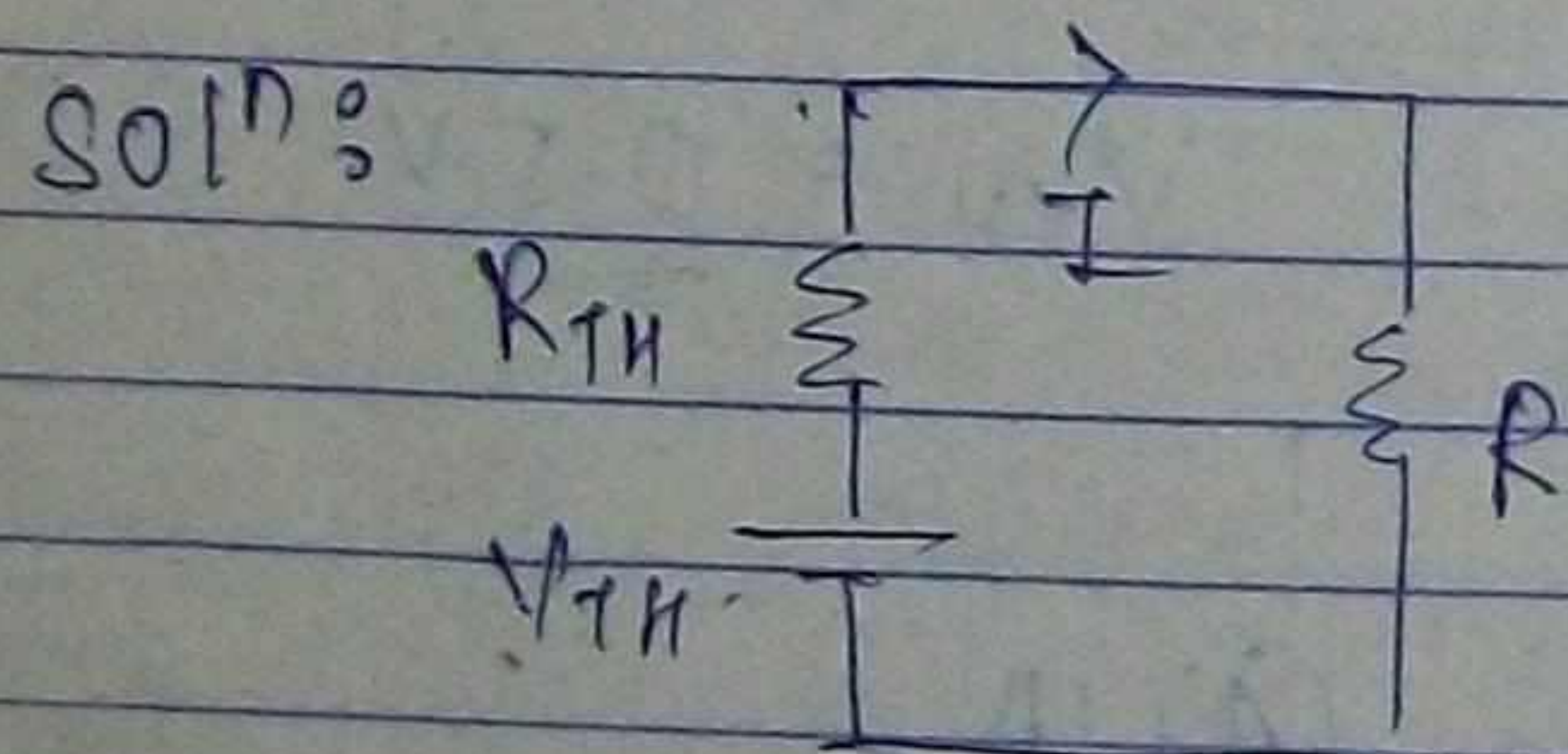
$$R_{TH} = \frac{V_{AB}}{I_{sc}} = \frac{200}{1} = \underline{200\Omega}$$

Ques.

When a complex N/w 'N' is connected to load resistance $R\Omega$, power dissipation in the resistor is P watts. When two identical complex N/w of 'N' are connected to load resistor of $R\Omega$ find power dissipation in the load resistor.



- ① P watts ② $2P$ ③ $4P$ ④ P to $4P$



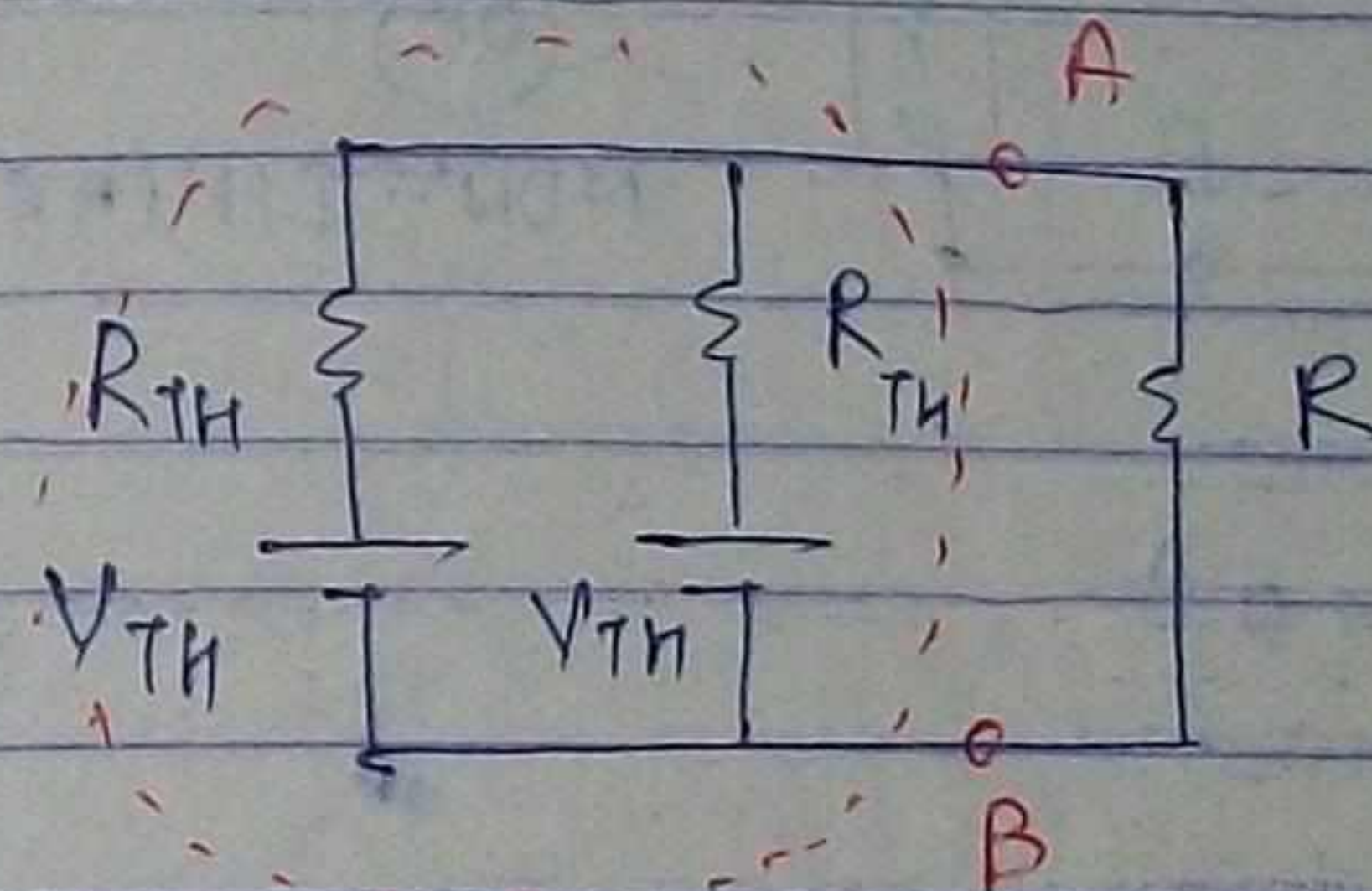
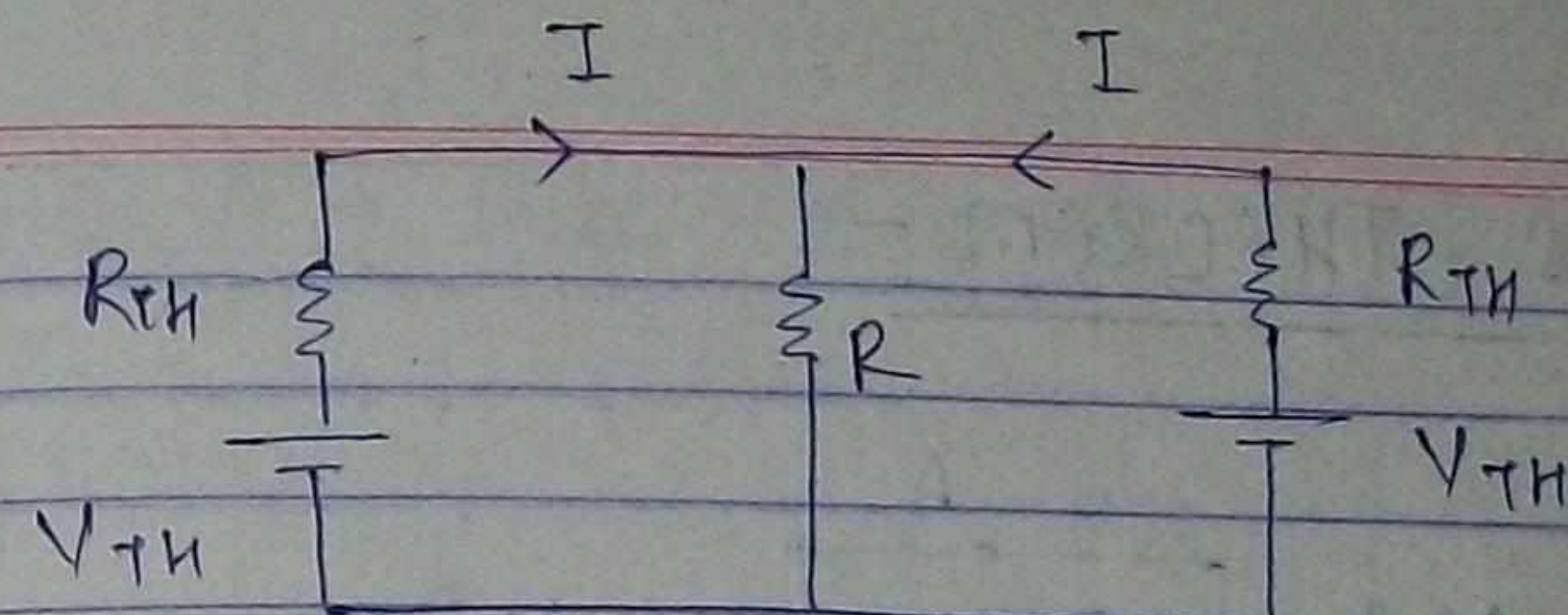
$$I = \frac{V_{TH}}{R_{TH} + R}$$

$$P = I^2 R$$

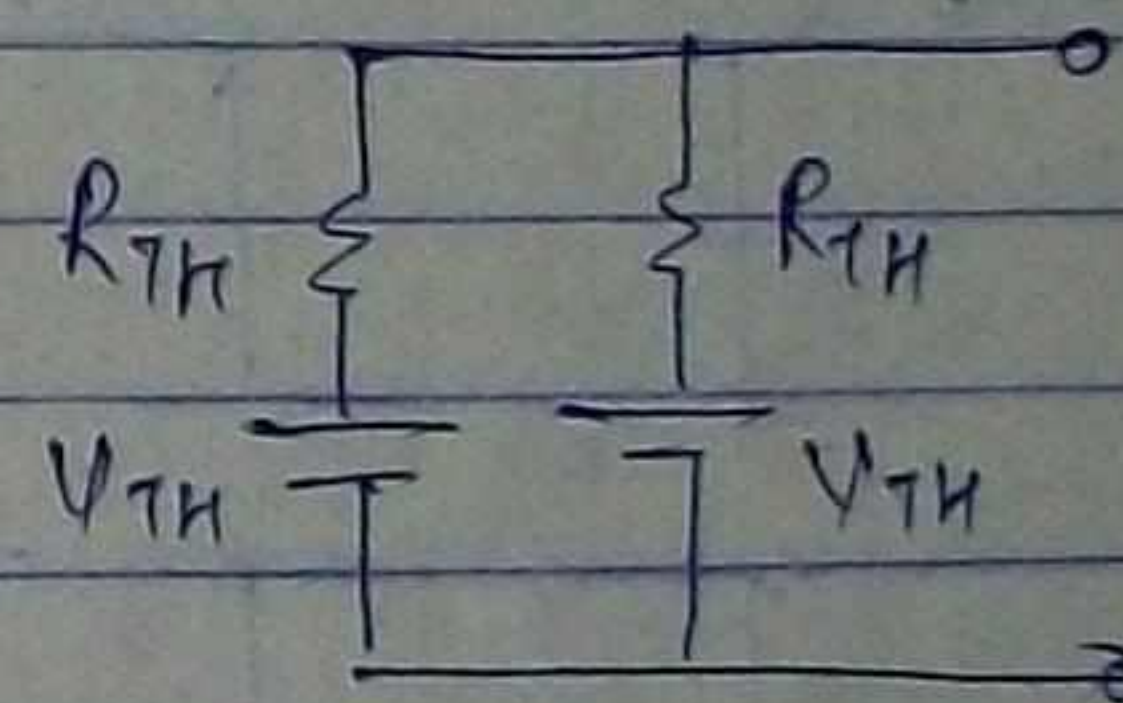
$$P = \left(\frac{V_{TH}}{R_{TH} + R} \right)^2 R$$

Now,

⌞ ①

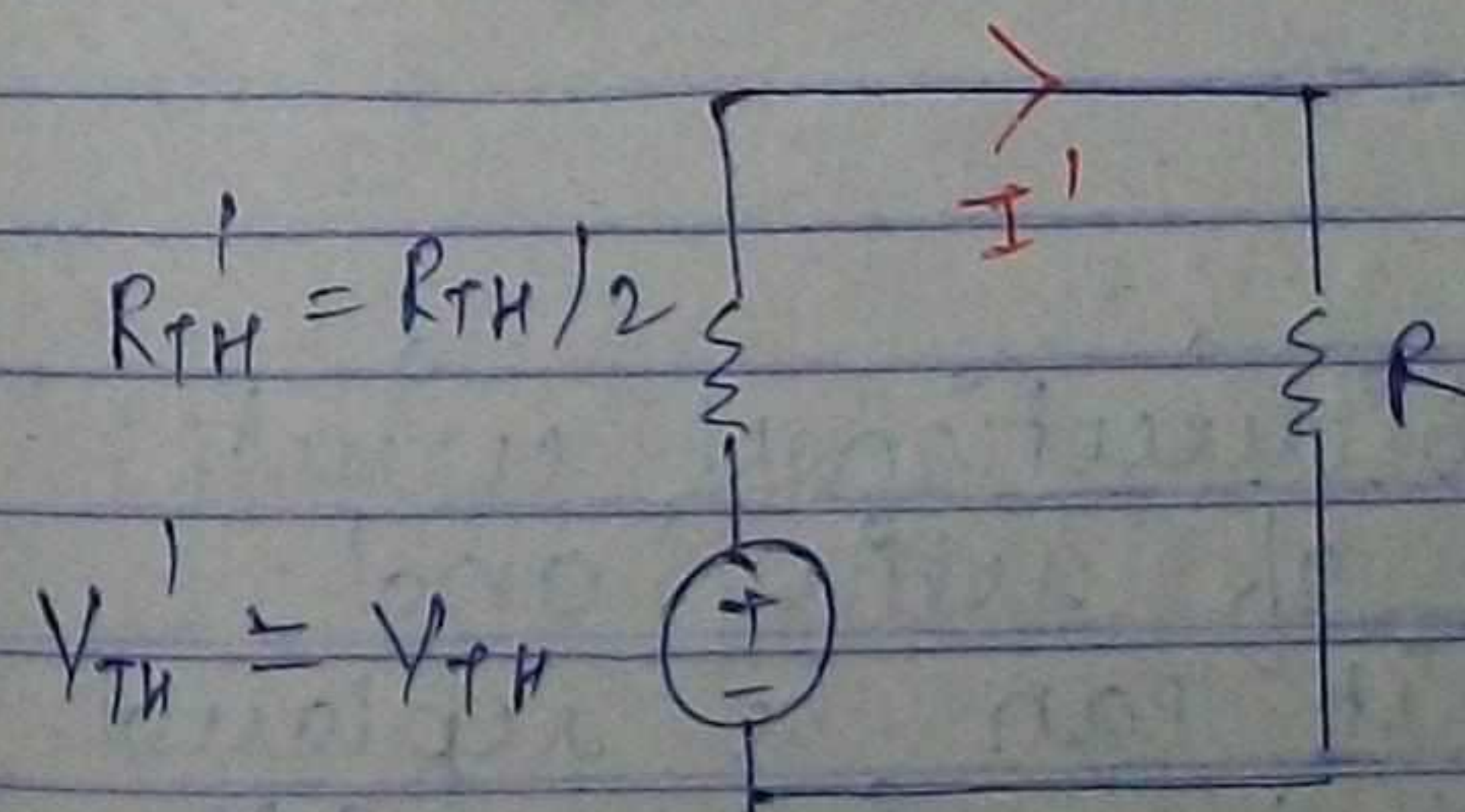


THEVENIN'S EQUIVALENT



$$V_{TH}' = V_{TH}$$

$$R_{TH}' = R_{TH}/2$$



$$I' = \frac{V_{TH}}{\frac{R_{TH}}{2} + R}$$

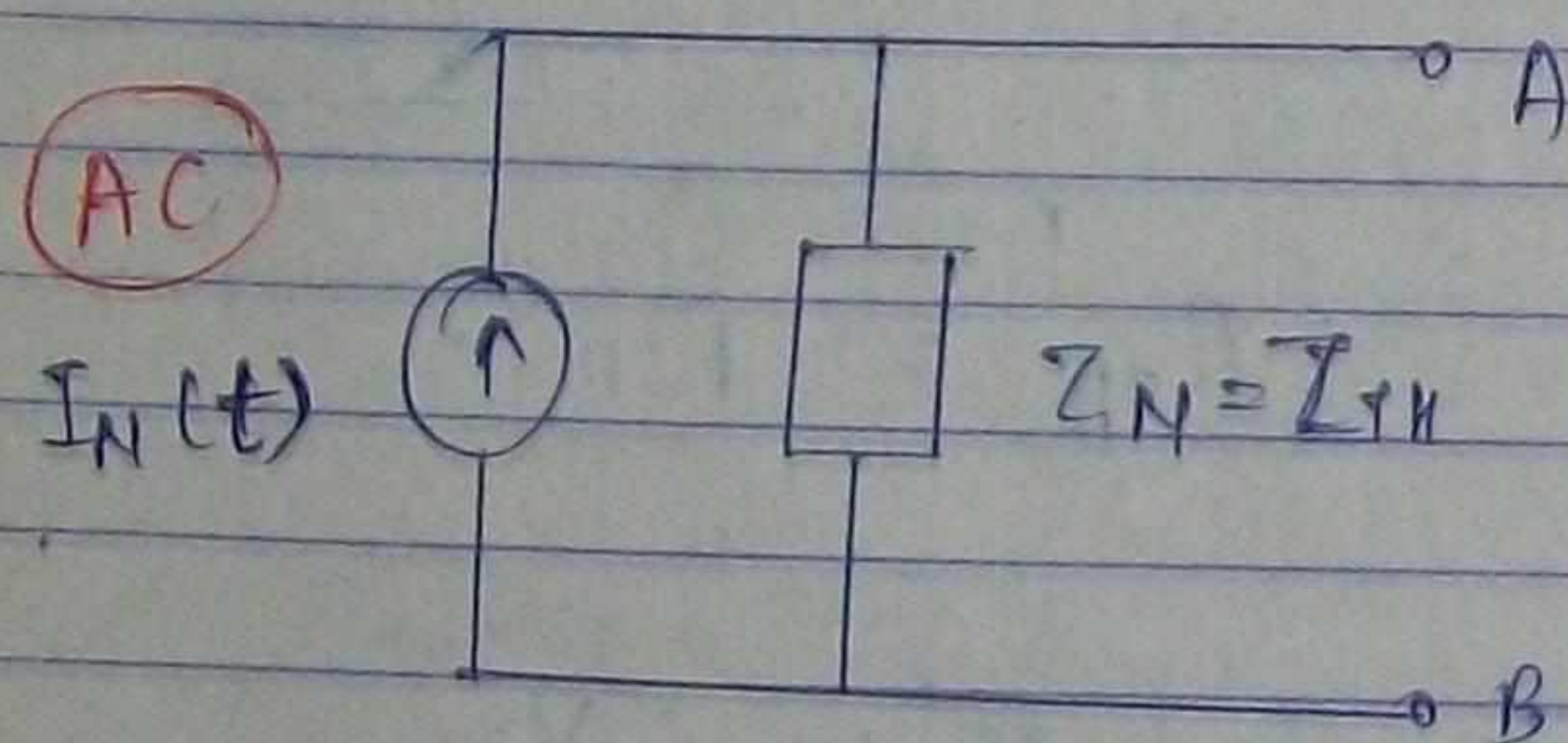
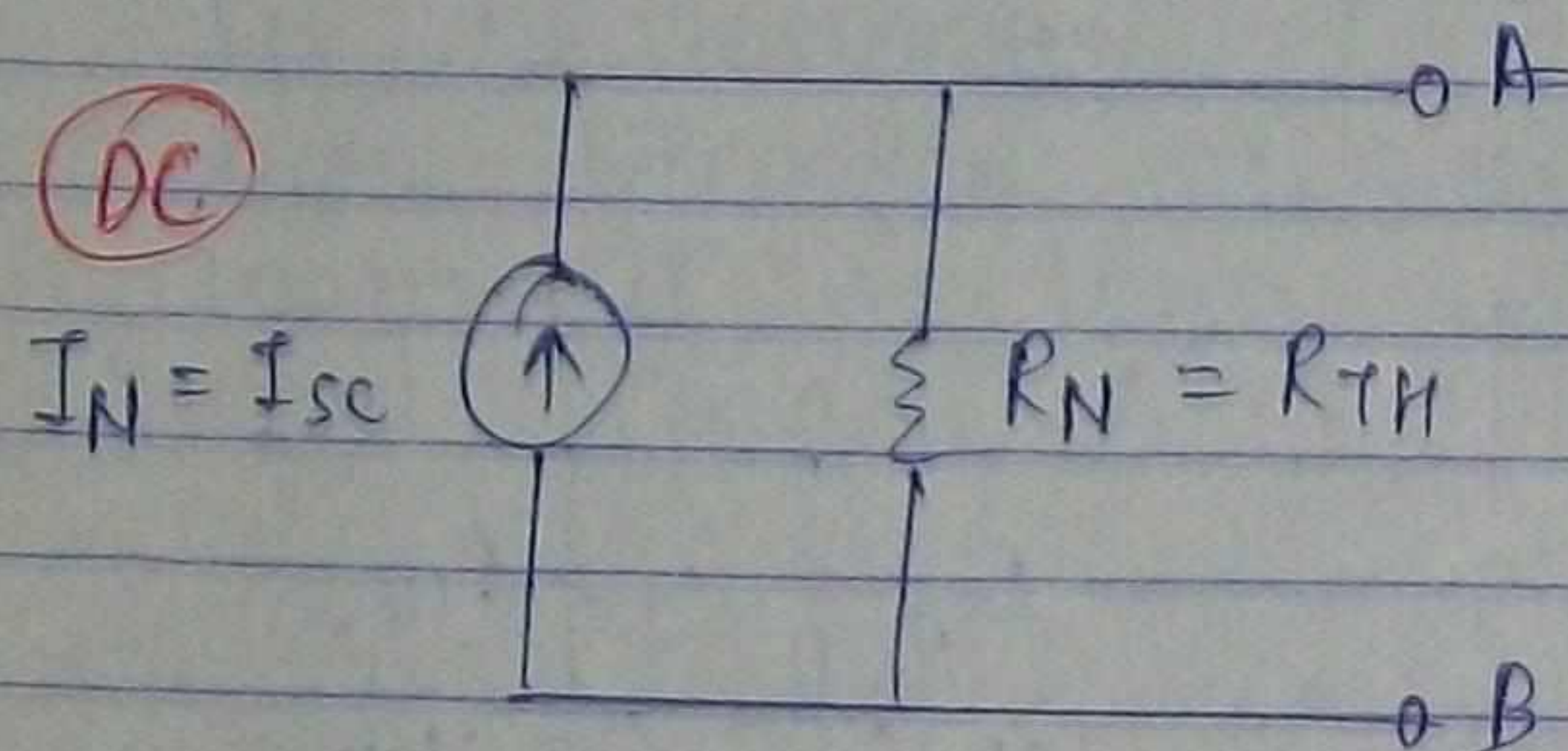
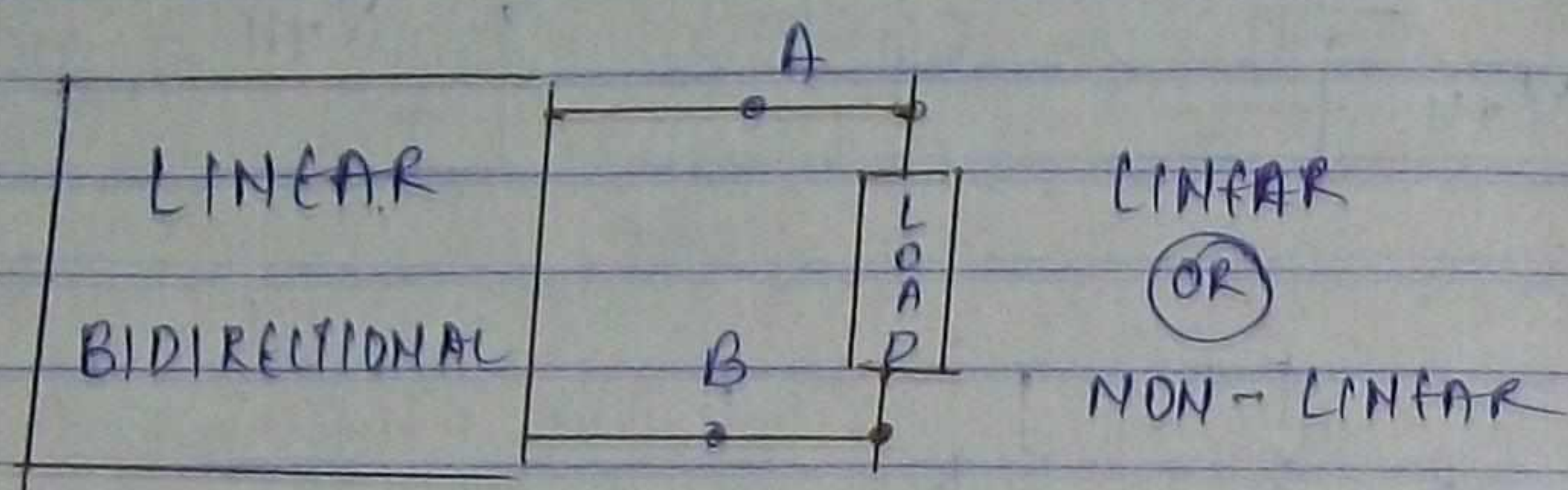
$$P' = I'^2 R$$

$$P' = \left(\frac{2V_{TH}}{2R + R_{TH}} \right)^2 R$$

Ans. (D)

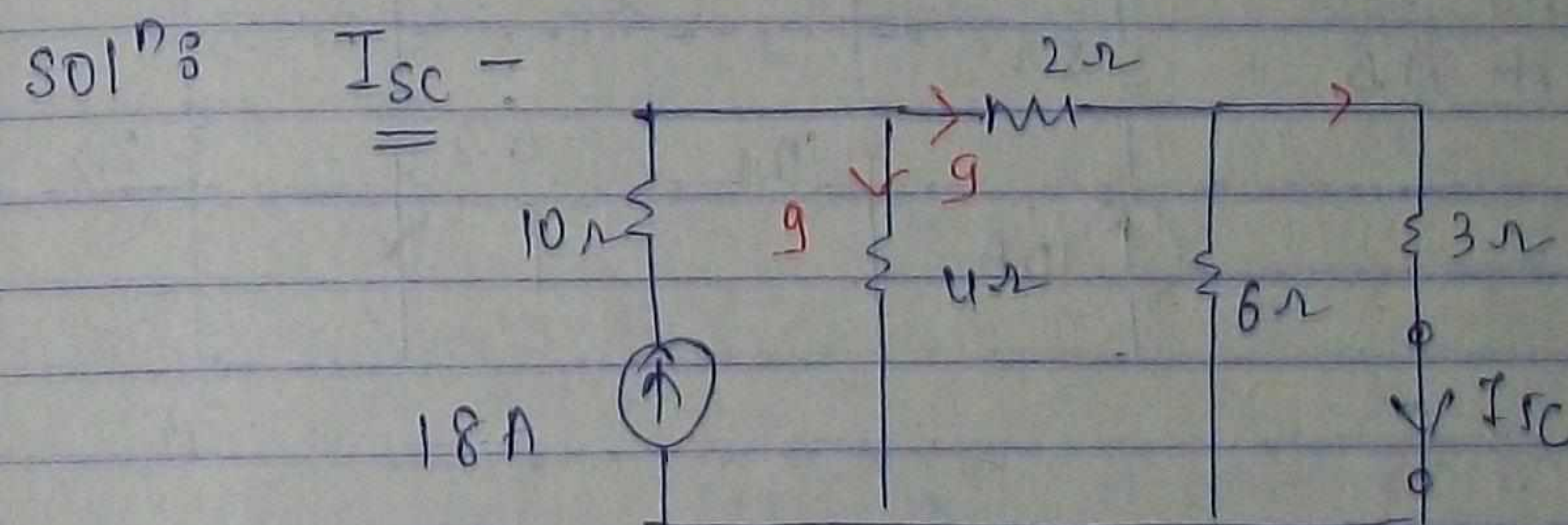
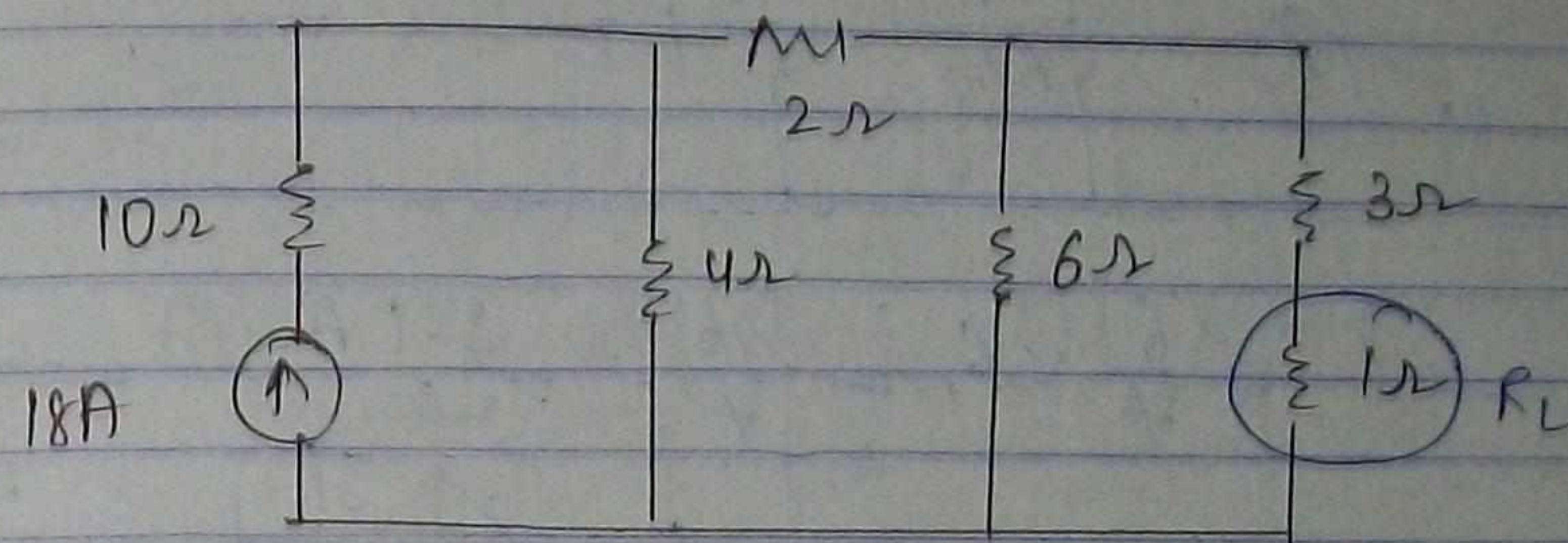
⌞ ②

⇒ NORTON'S THEOREM -



- In any linear bidirectional circuit having more no. of active and passive elements, it can be replaced by single equivalent circuit consisting of equivalent current source (I_N) in parallel with equivalent resistance (R_N).

Ques. Find I in 1Ω resistor by Norton theorem



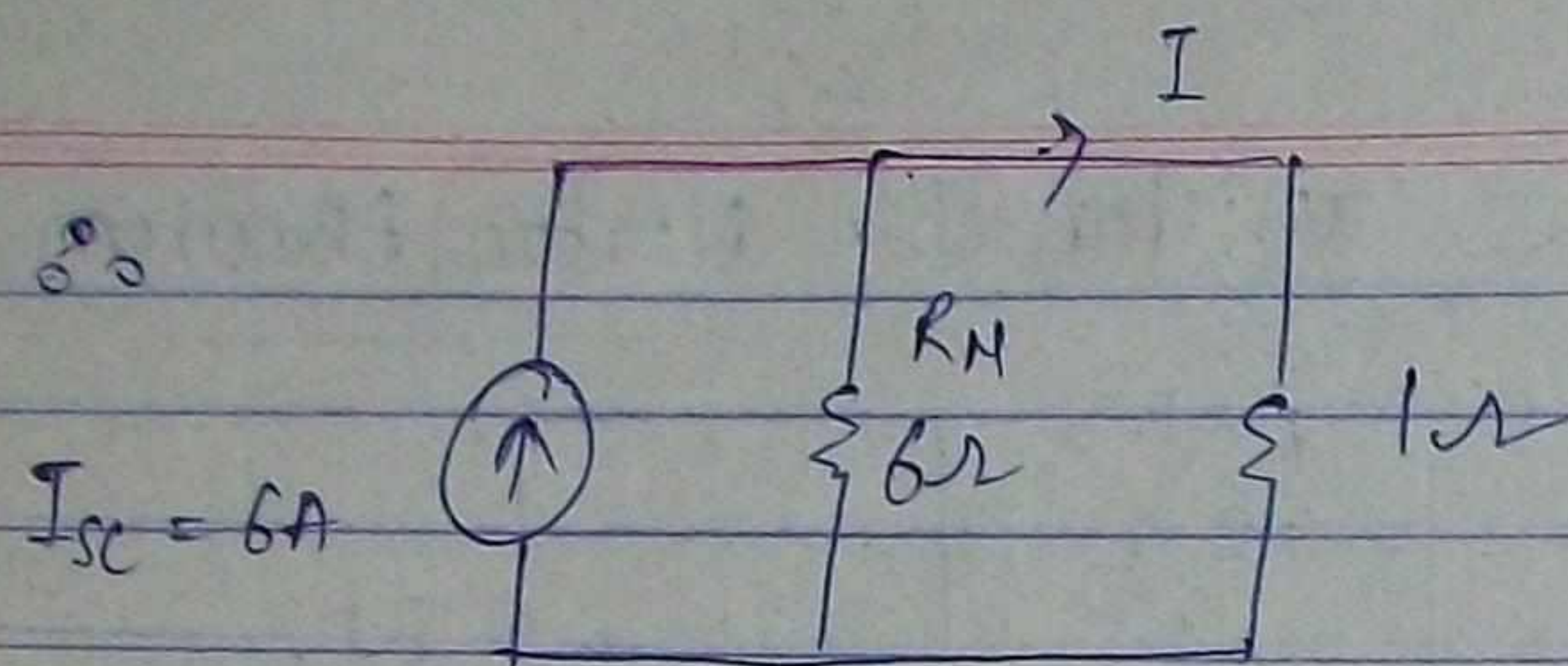
- Replace the load resistor by short circuit and find short circuit current.

$$I_{sc} = \frac{6}{6+3} \times 9 = 6A = I_N$$

$$R_N = \left(\left[\left(\frac{10 \parallel 4}{1} \right) + 2 \right] \parallel 6 \right) + 3$$

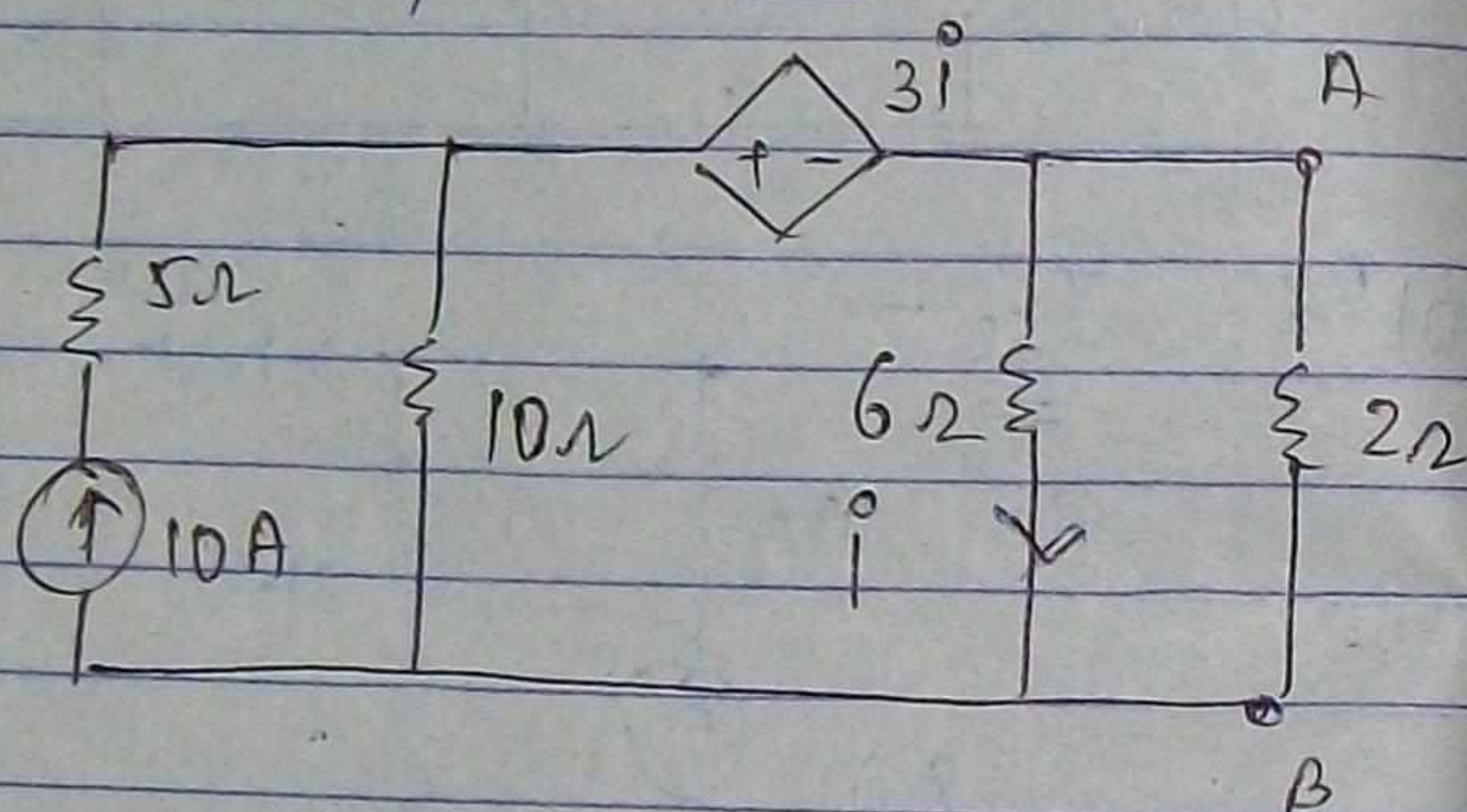
$$R_N = 6\Omega$$

current source $\rightarrow$ O.C.
voltage source $\rightarrow$ S.C.

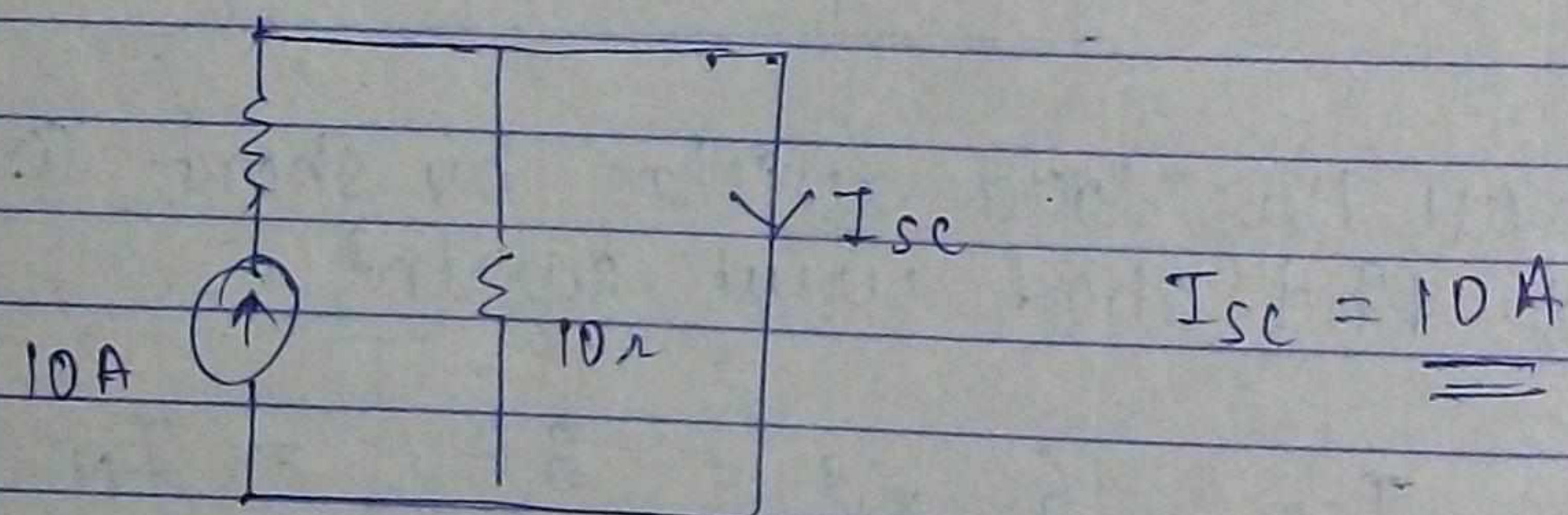


$$I = \frac{6}{7} \times 6 = \frac{36}{7} = 5.1 \text{ A}$$

Que. find I_{sc} w.r.t A.B



Solⁿ:



$$I_{sc} = 10 \text{ A}$$

Que. find current flow using 4Ω resistor using following data.

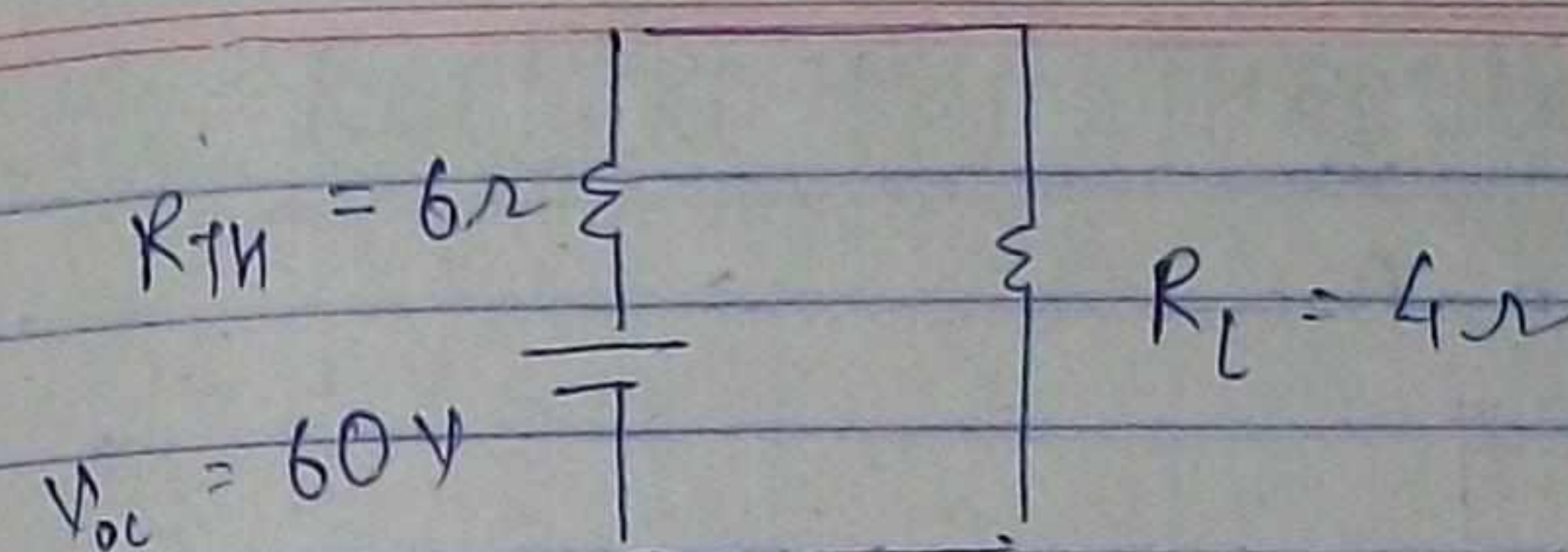
V	0	60
I	10	0

Solⁿ:

$$I_{sc} = 10$$

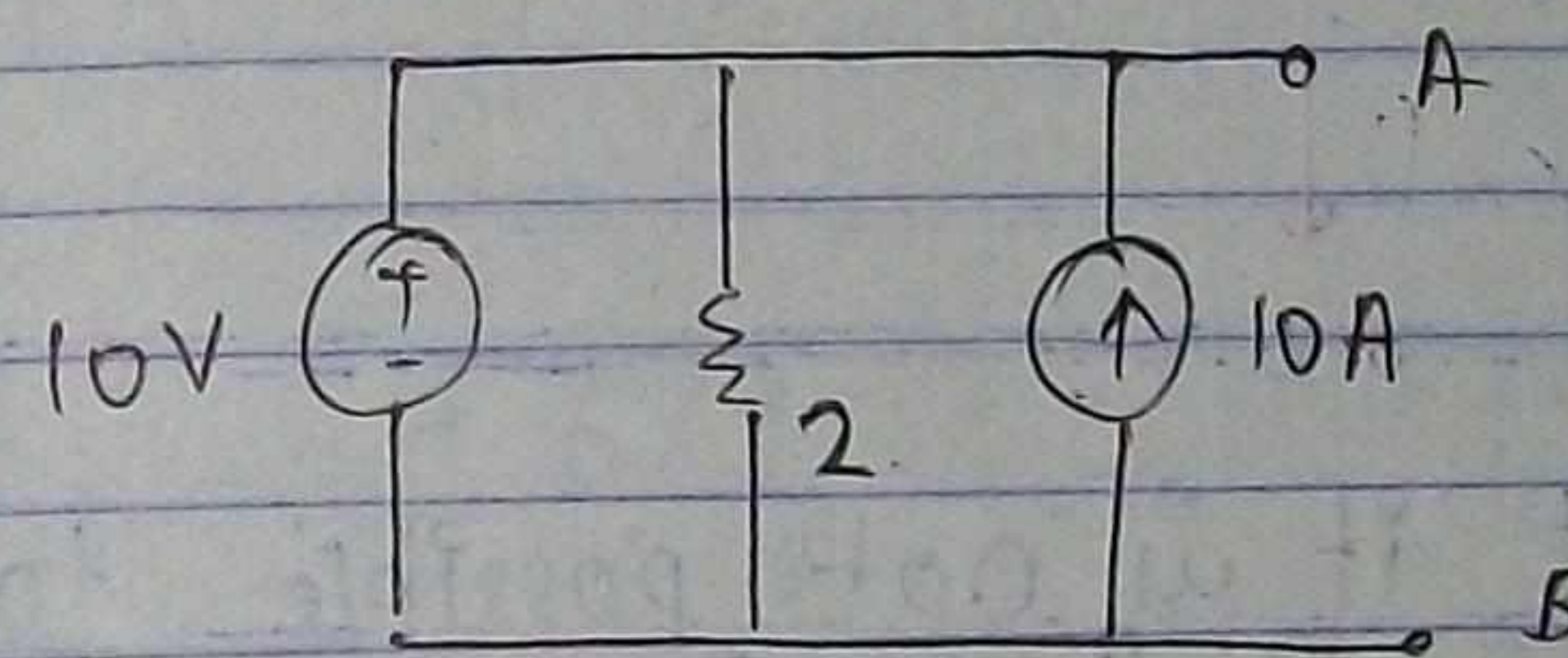
$$V_{oc} = 60$$

$$R_{TH} = \frac{V_{oc}}{I_r} = 6\Omega$$



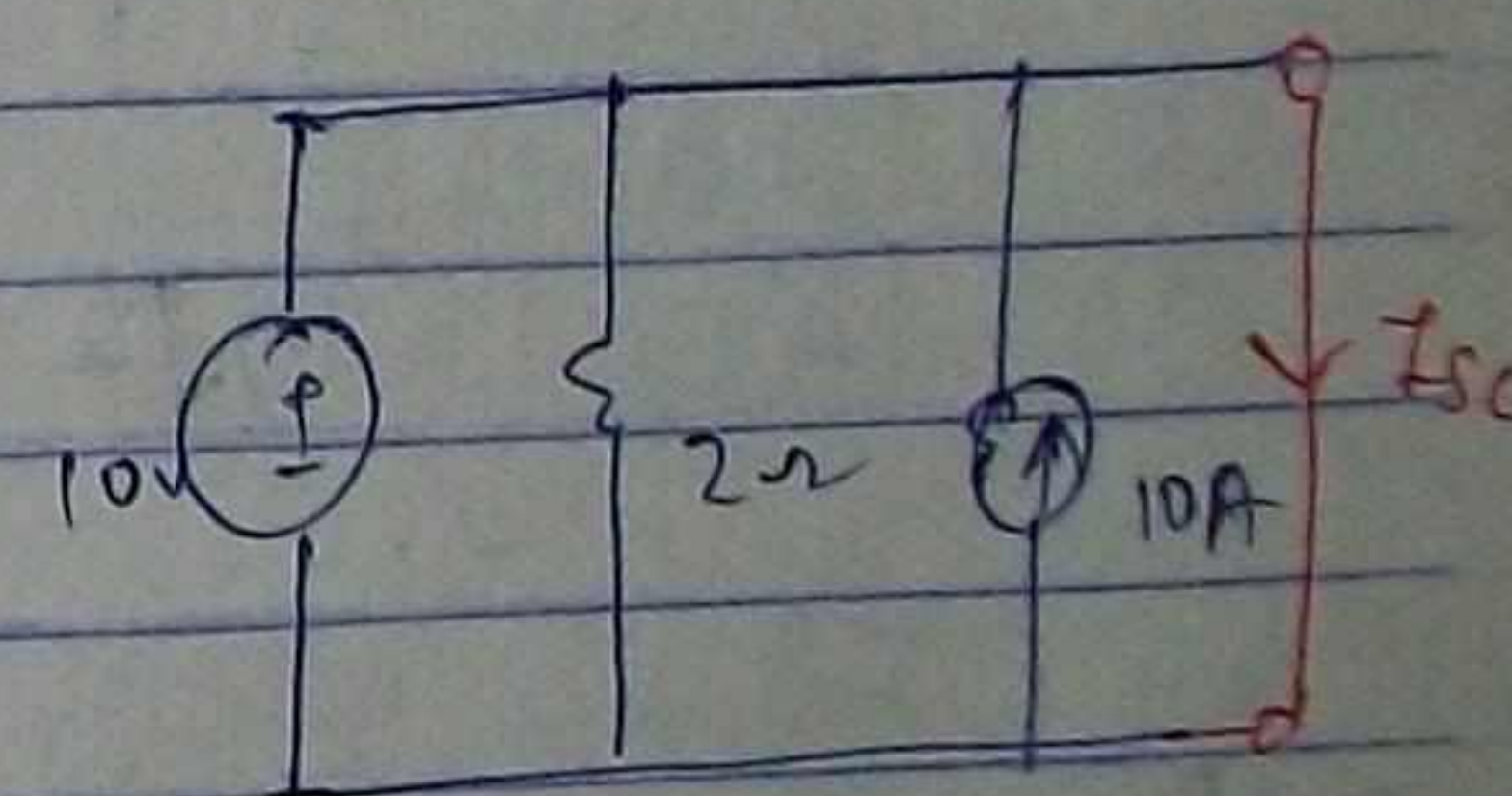
$$I = \frac{60}{6+4} = 6 \text{ A}$$

Que. find V_{oc} and I_{sc} w.r.t A and B

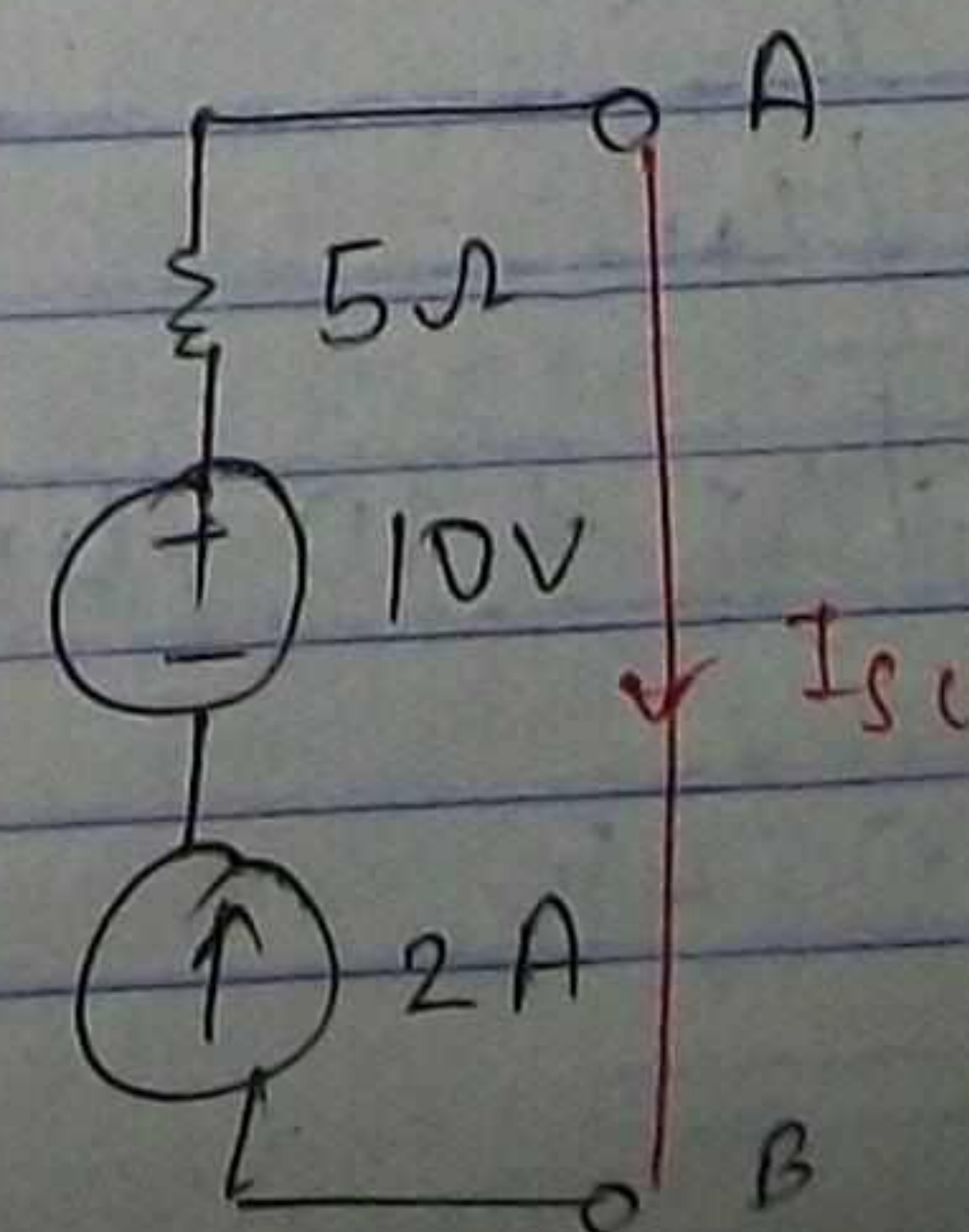


Solⁿ: $V_{TH} = 10 \text{ V}$

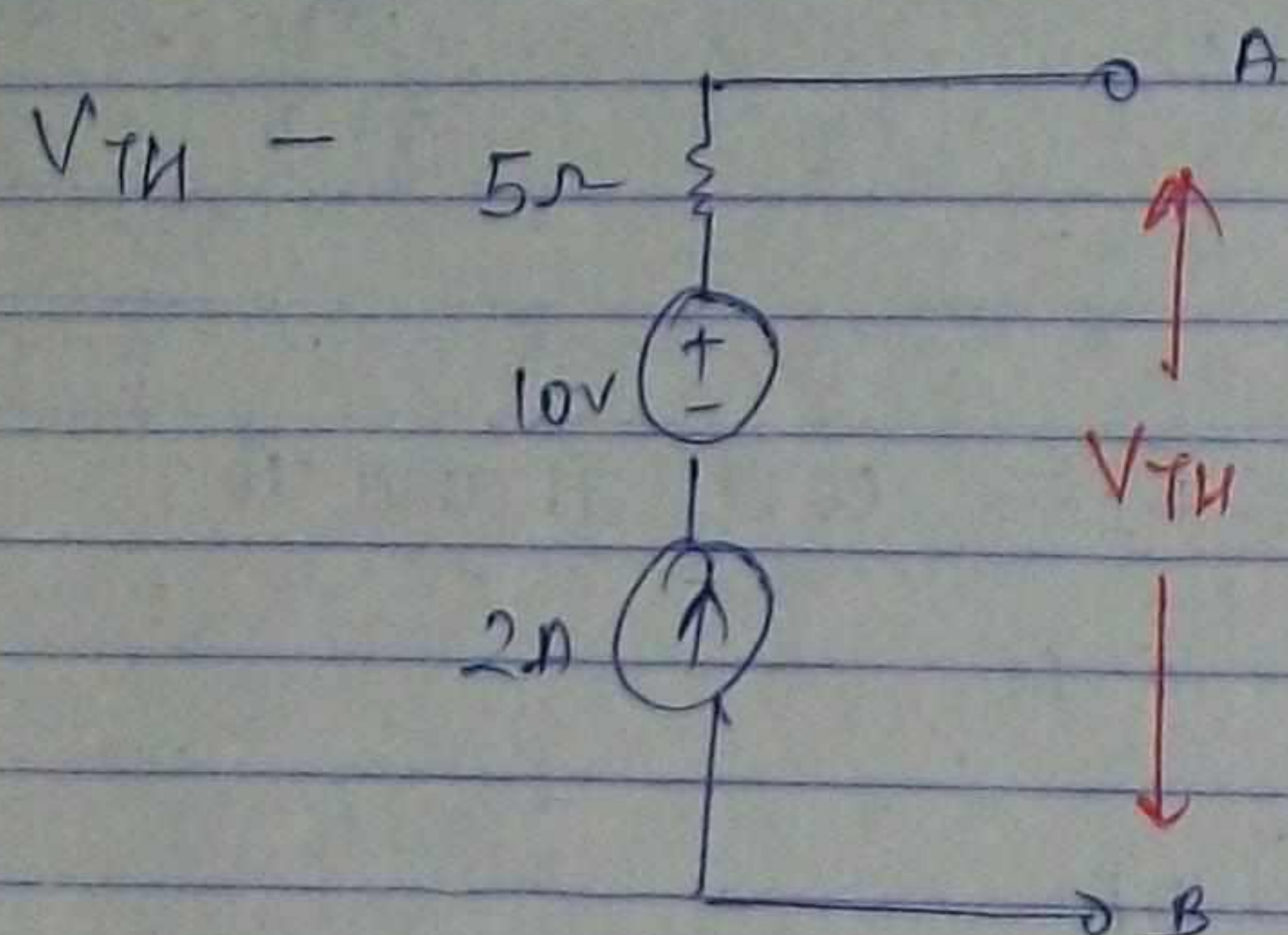
* for this circuit it is not possible to determine I_{sc} because Non-feasible circuit, not satisfying KVL.



Que. find V_{oc} and I_{sc} w.r.t A and B

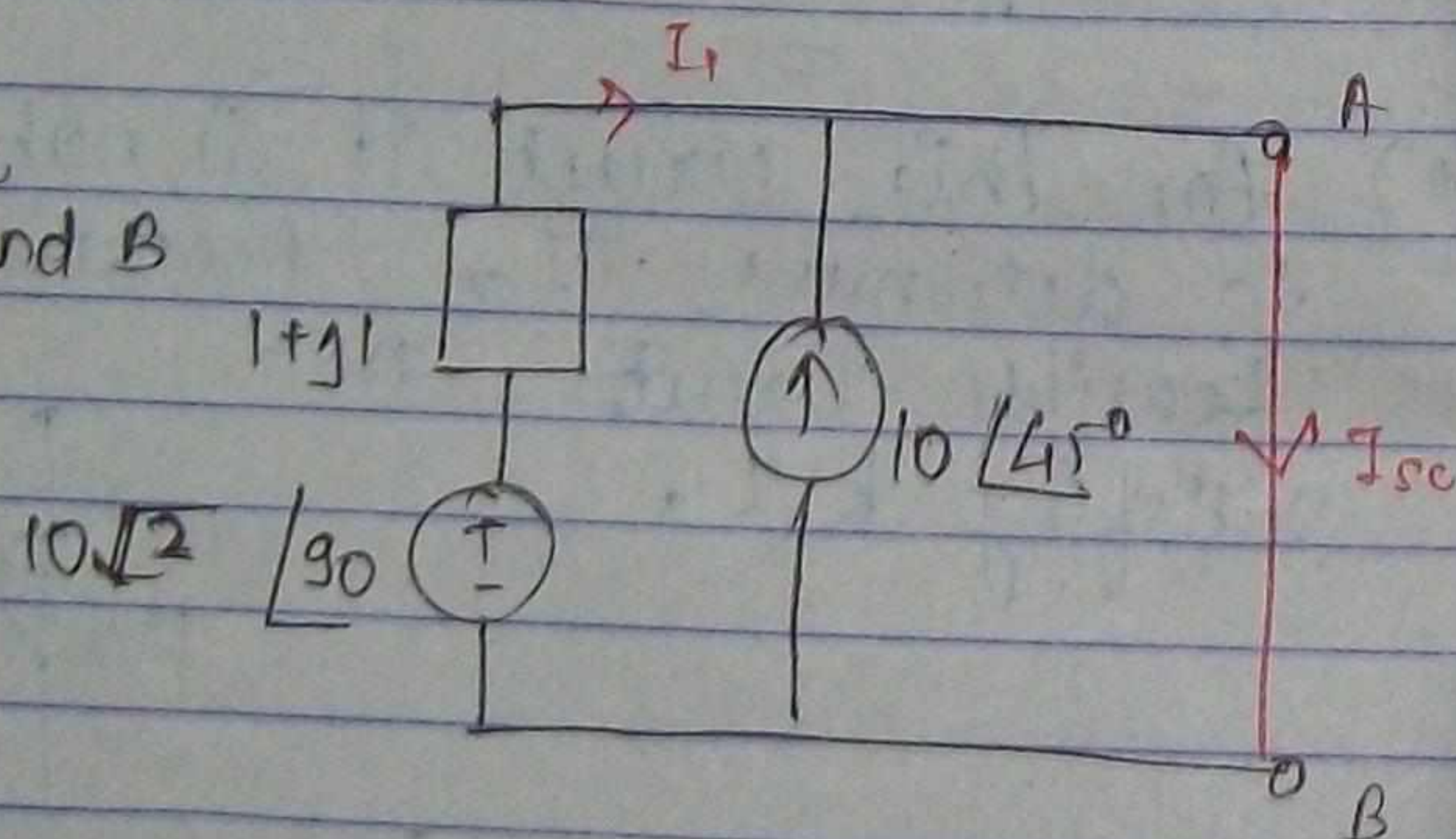


Solⁿ: $I_{sc} = 2A$



- In above circuit it is not possible to find V_{TH} since it is not satisfying KCL.

Qw. find I_{sc} wrt A and B

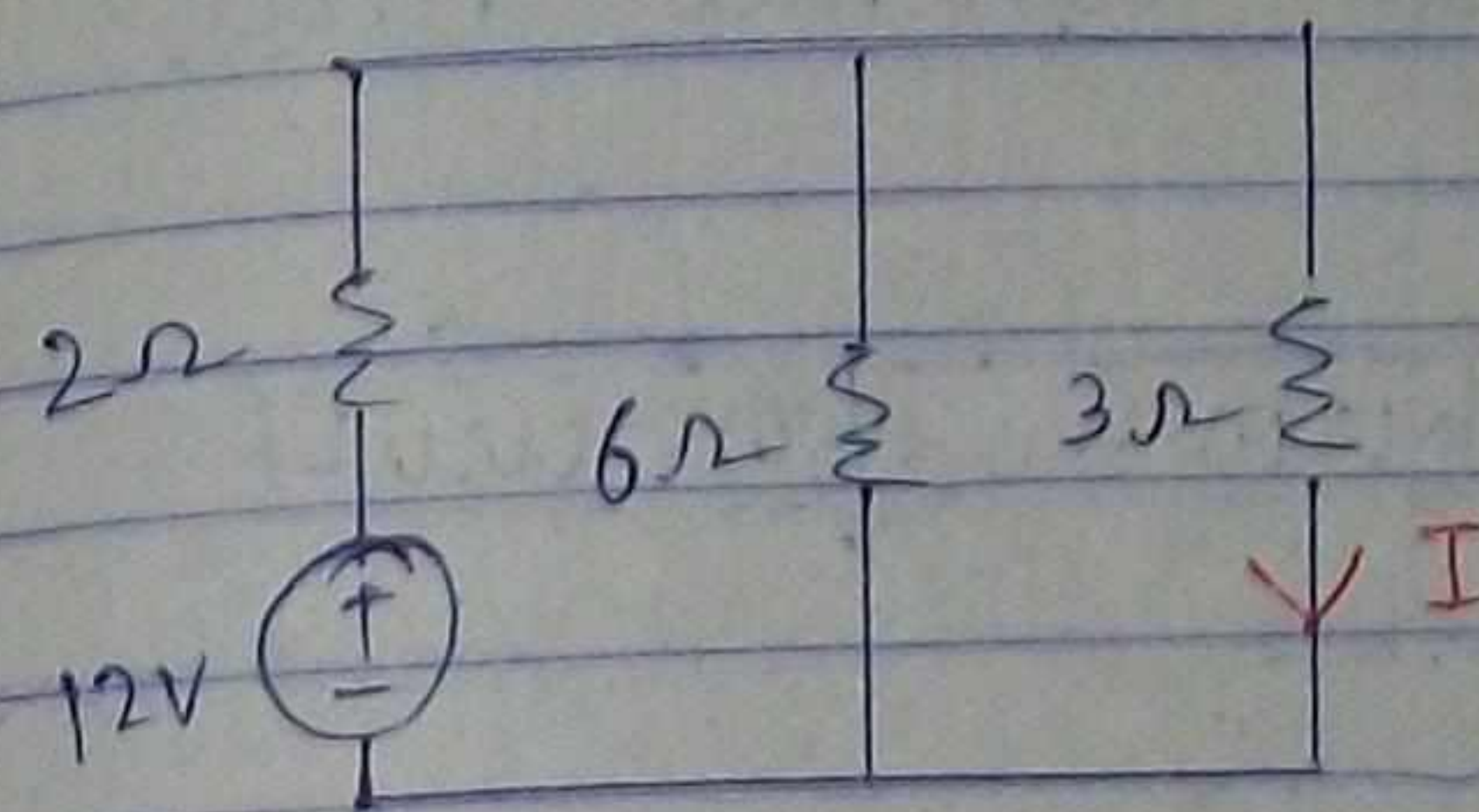


Solⁿ:

$$I_1 = \frac{10\sqrt{2} \angle 90^\circ}{\sqrt{2} \angle 45^\circ} = 10 \angle 45^\circ$$

$$I_{sc} = 10 \angle 45^\circ + 10 \angle 45^\circ = 20 \angle 45^\circ$$

⇒ RECIPROCITY THEOREM -

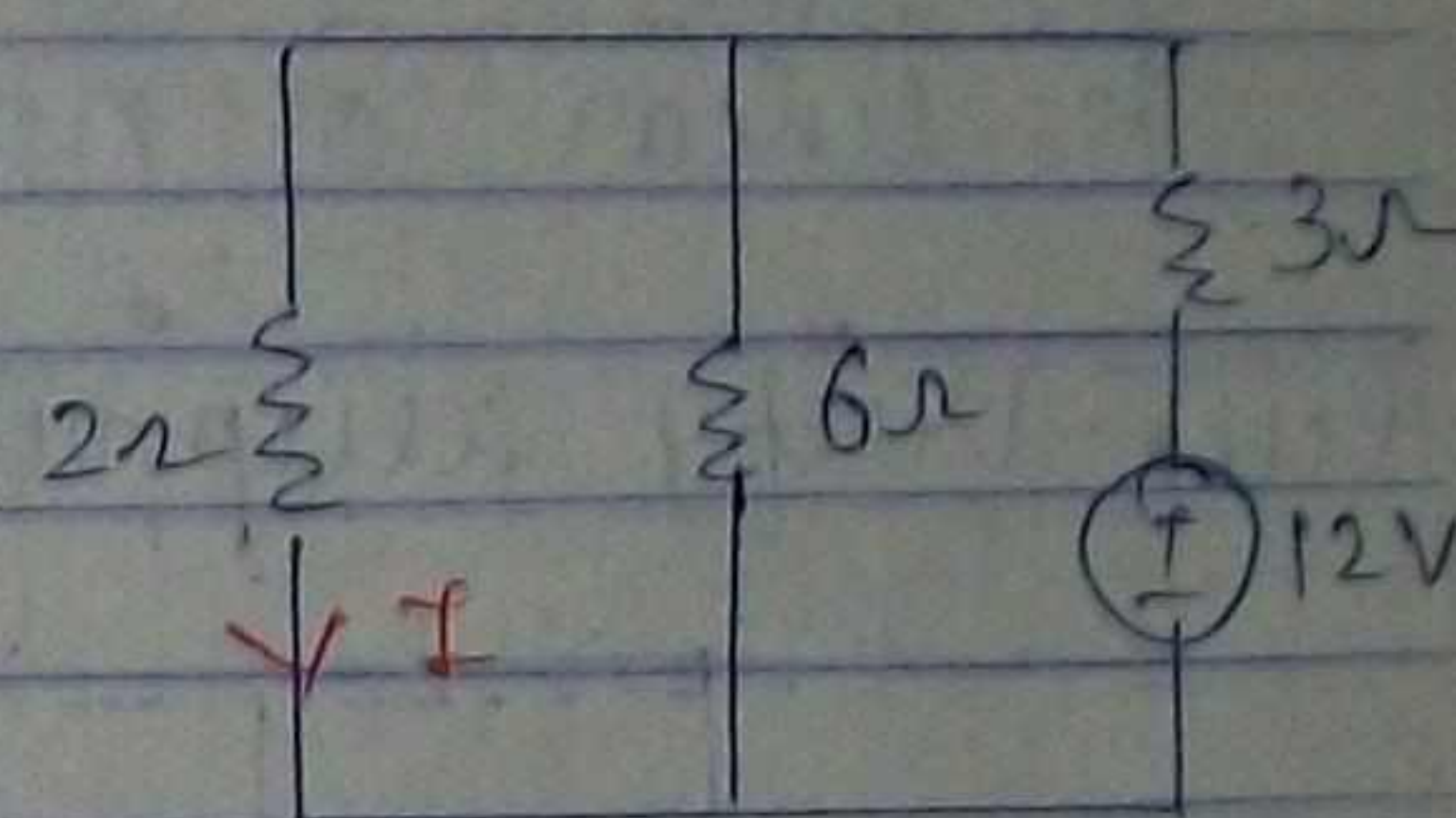


$$R_{eq} = 2 + \frac{6 \times 3}{6+3} = 4 \Omega$$

$$I_T = \frac{12}{4} = 3$$

$$I = 3 \times \frac{6}{6+3} = 2A$$

$$\frac{\text{RESPONSE}}{\text{EXCITATION}} = \frac{2}{12} = \frac{1}{6}$$



$$R'_{eq} = 3 + \frac{6 \times 2}{6+2} = 3 + \frac{12}{8} = \frac{9}{2}$$

$$I_T = \frac{12}{R'_{eq}} = \frac{12 \times 2}{9} = \frac{8}{3}$$

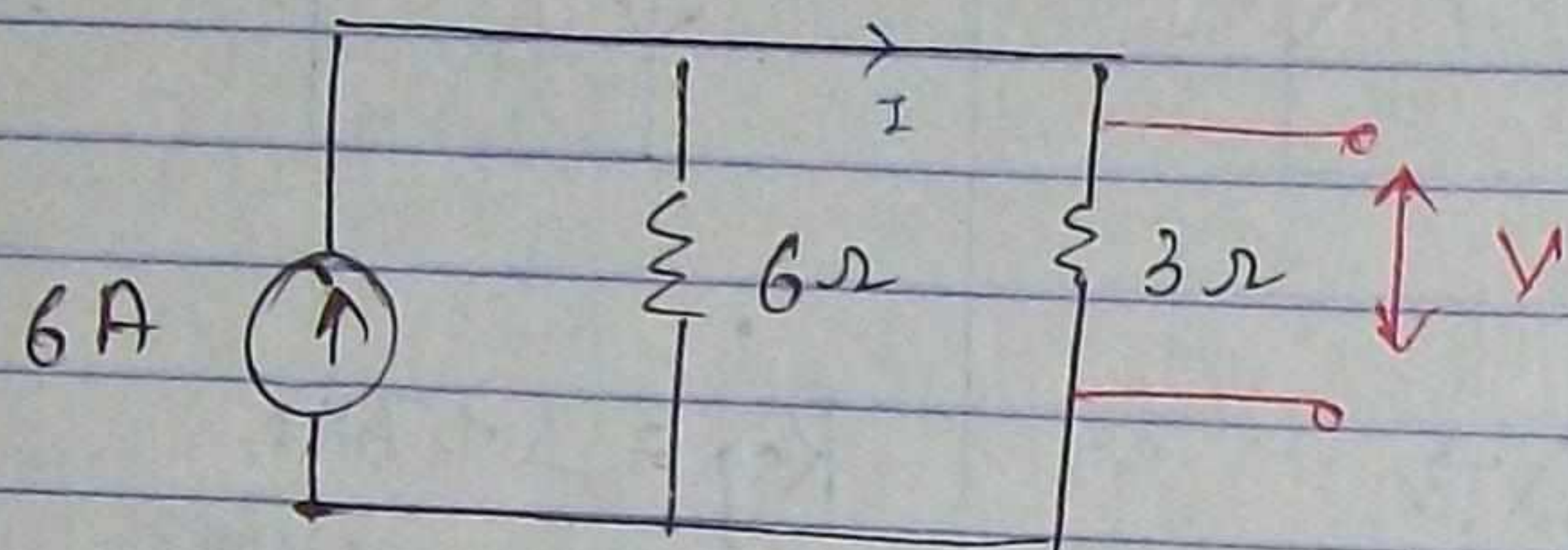
$$I = I_T \times \frac{6}{6+2} = 2A$$

$$\frac{\text{RESPONSE}}{\text{EXCITATION}} = \frac{2}{12} = \frac{1}{6}$$

- In above network after interchanging the position of response and excitation the ratio of response to excitation is constant hence given N/w satisfy the reciprocity. (LINEAR N/w)

- By using reciprocity theorem it is possible to conclude whether N/w is linear or not.

Ques. Verify reciprocity theorem for circuit



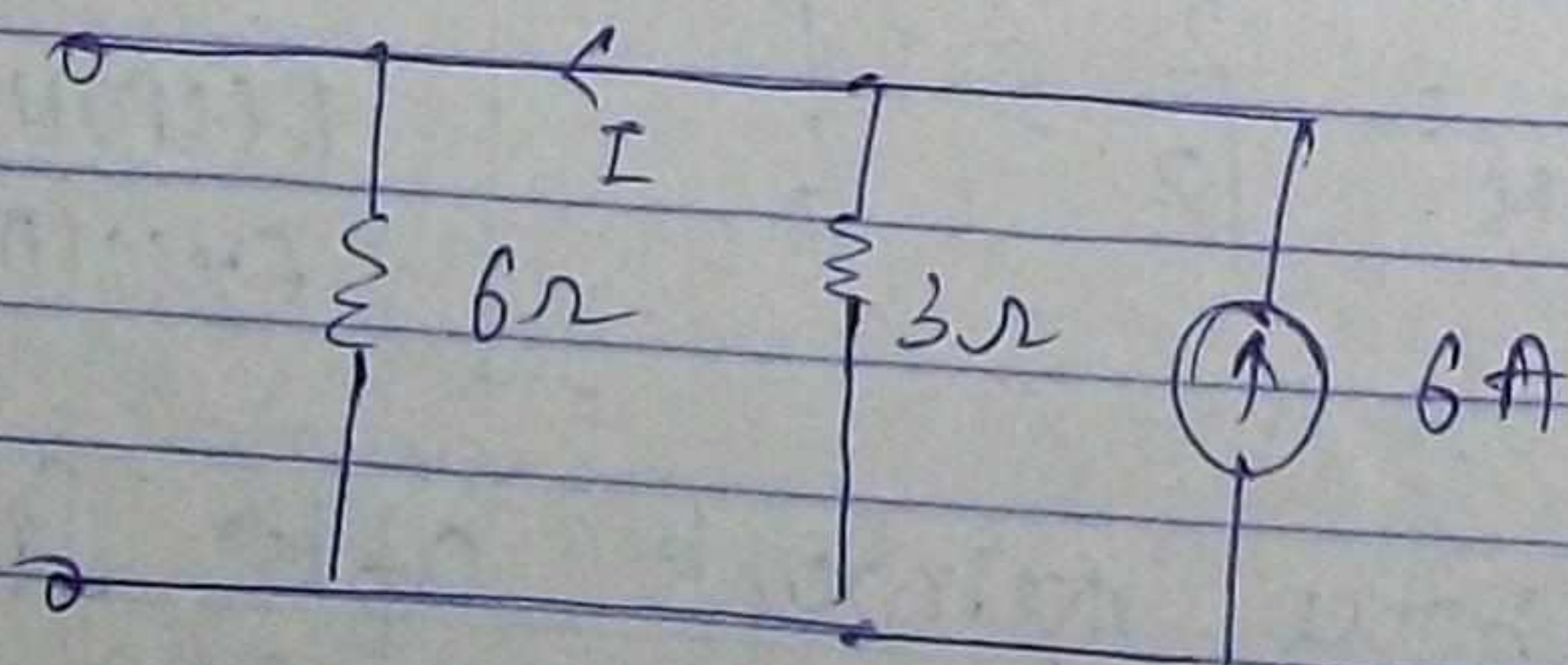
Solⁿ: Case (1):

$$I = \frac{6}{6+3} \times 6 = \frac{36}{9} = 4A$$

$$V = 12V$$

$$\frac{\text{Response}}{\text{Excitation}} = \frac{12}{6} = 2$$

Case (2):

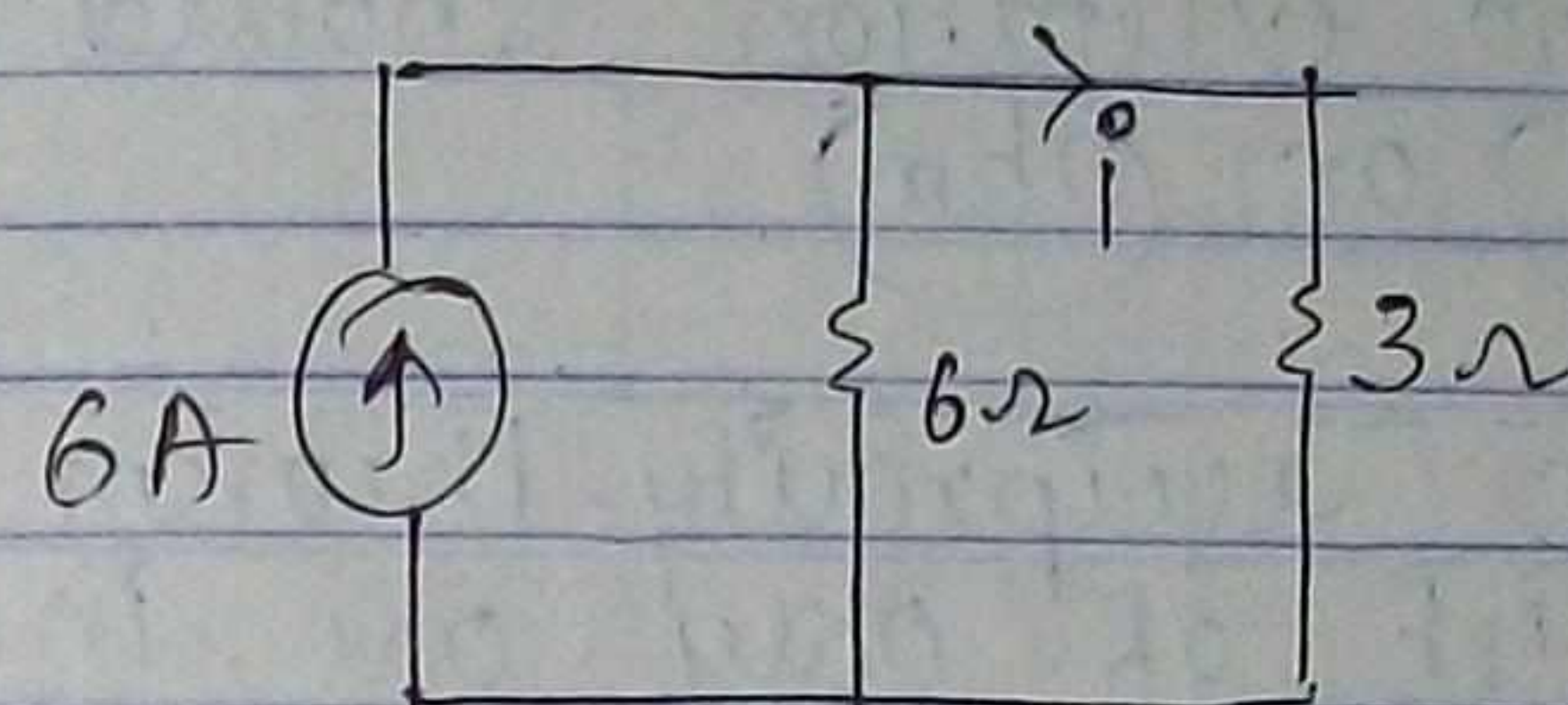


$$I = \frac{3}{9} \times 6 = 2A$$

$$V = 12V$$

$$\frac{\text{Response}}{\text{Excitation}} = \frac{12}{6} = 2$$

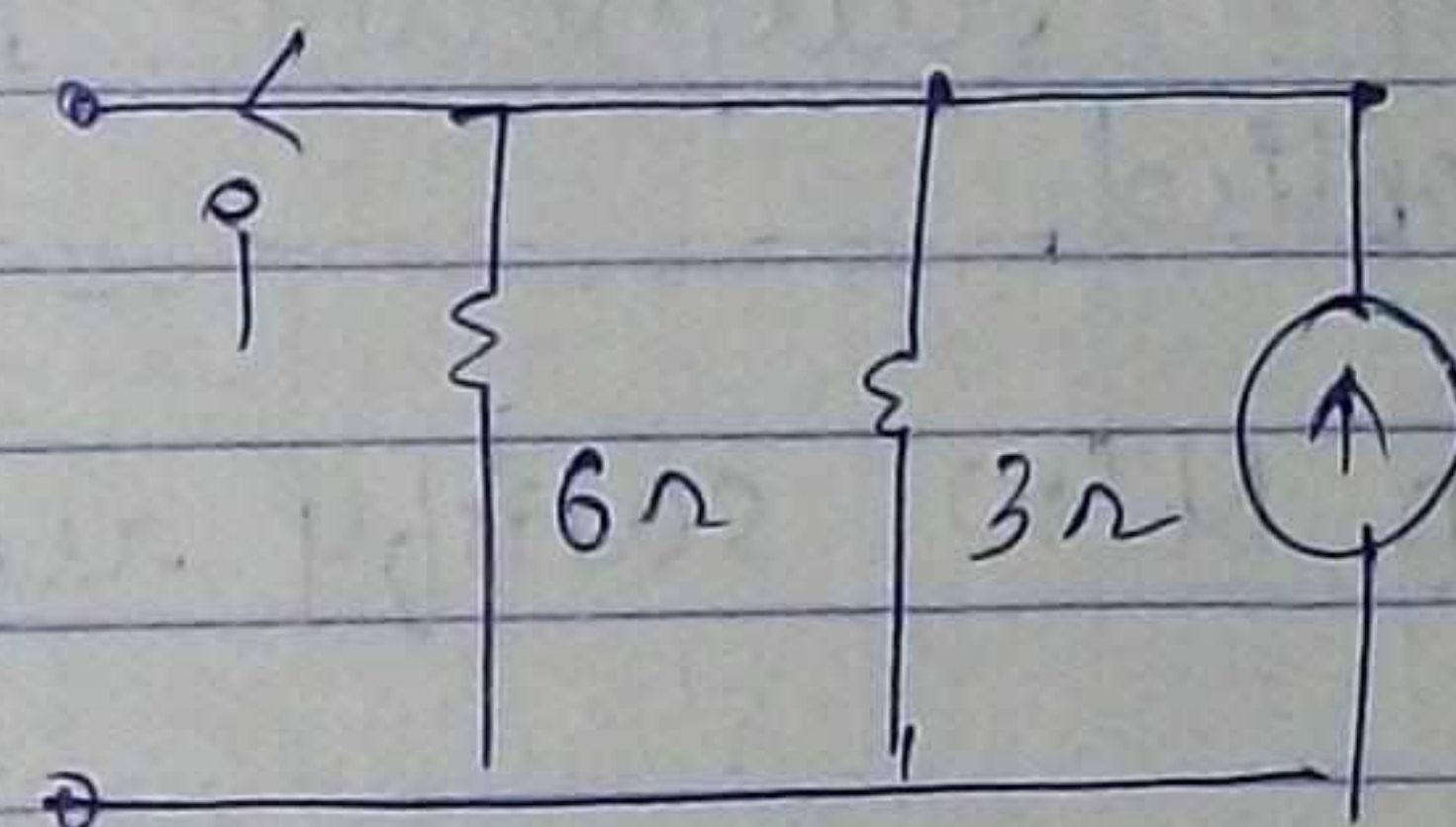
Ques. Verify reciprocity theorem for circuit



Solⁿ:

$$I = \frac{6}{6+3} \times 6 = 4A \quad (\text{answer})$$

Case (2)



$$I = 0 \quad (\text{answer})$$

NOTE:-

$$1 \quad \frac{R_{es}}{EXCT} = \frac{I}{V_s} \quad (\text{mho})$$

$$2 \quad \frac{R_{es}}{EXC} = \frac{V_s}{I_s} \quad (\text{ohm})$$

$$3 \quad \frac{R_{es}}{EXC} = \frac{I}{I_r} \quad (\text{NO UNIT})$$

$$4 \quad \frac{R_{es}}{EXC} = \frac{V}{V_s}$$

FOLLOW RECIPROCALITY

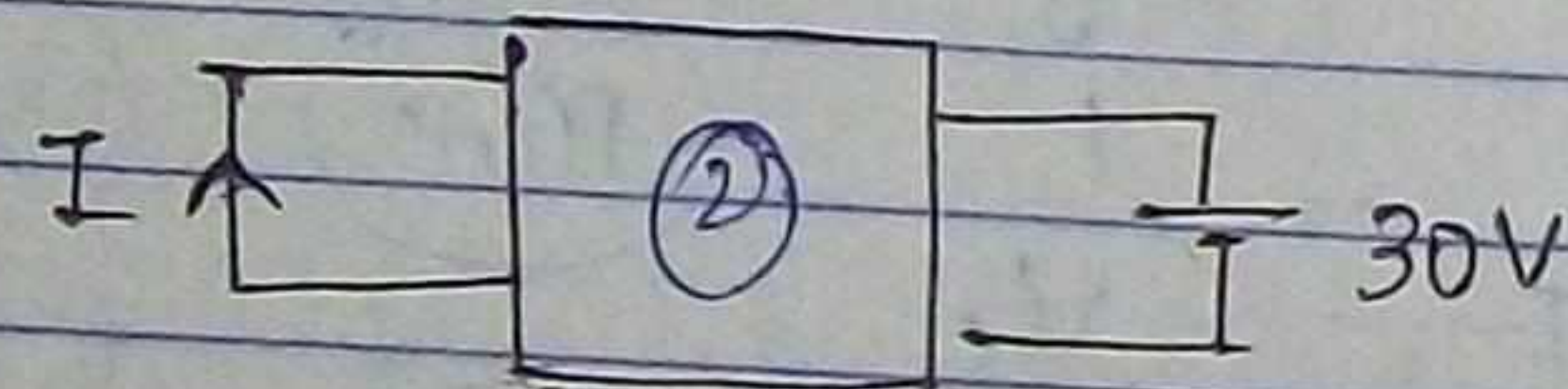
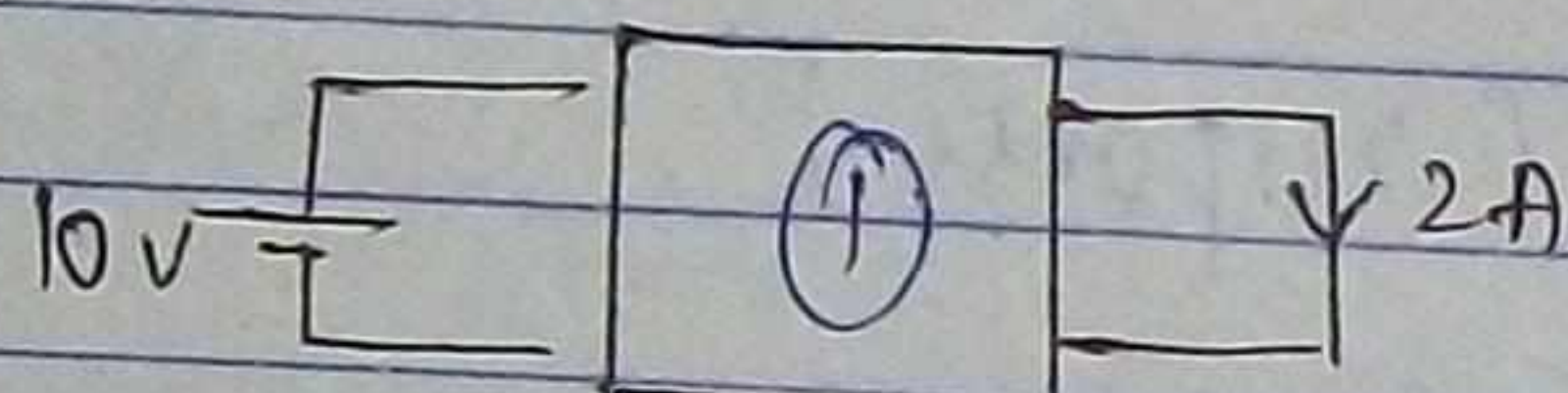
NOT FOLLOW RECIPROCALITY

- To obtain the reciprocity theorem unit of response to excitation should be either mho or ohm.

- while applying reciprocity theorem circuit should consist of only one independent source.

- when dependent source are present in the given N/w reciprocity theorem cannot be applied.

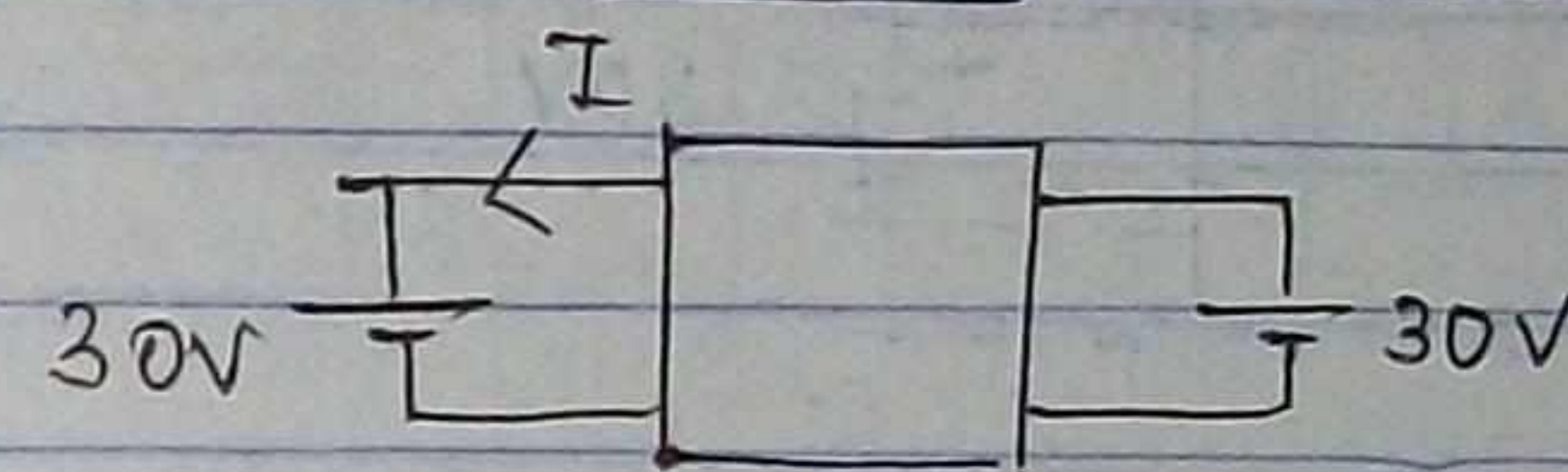
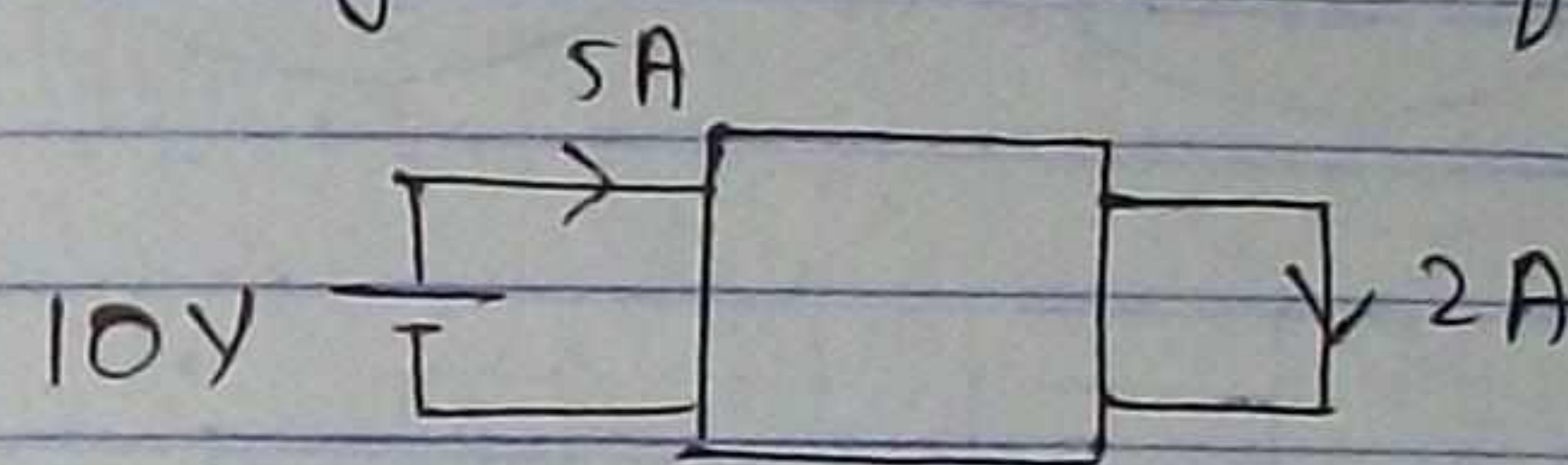
Ques. when given N/w satisfy reciprocity find I.



Solⁿ: $\text{CONSTANT} = \frac{\text{RESP}}{\text{EXCIT}} = \frac{2}{10} = -\frac{I}{30}$

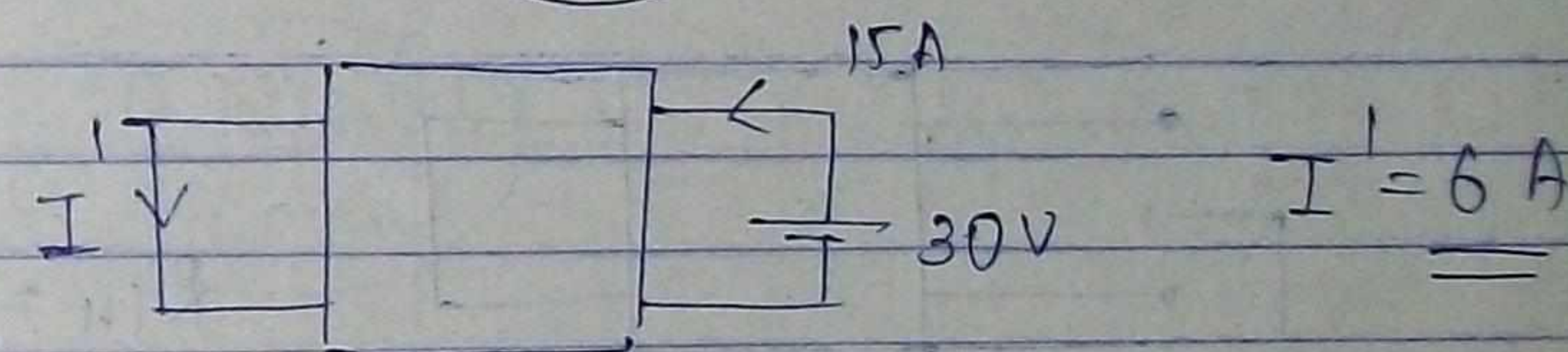
$I = -6A$

Ques. when given N/w satisfy reciprocity find I

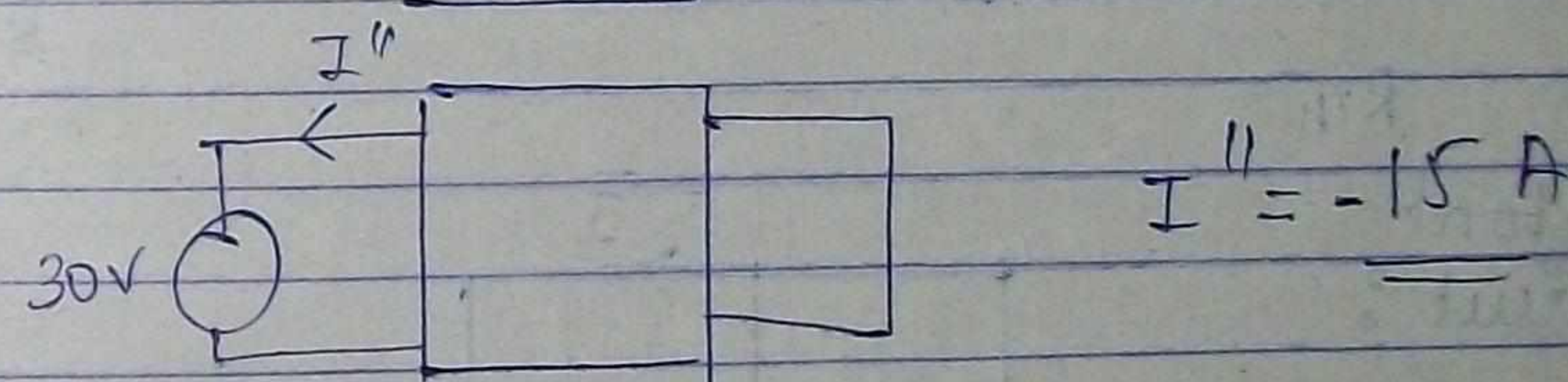


Solⁿ: Applying Superposition & Reciprocity:

①

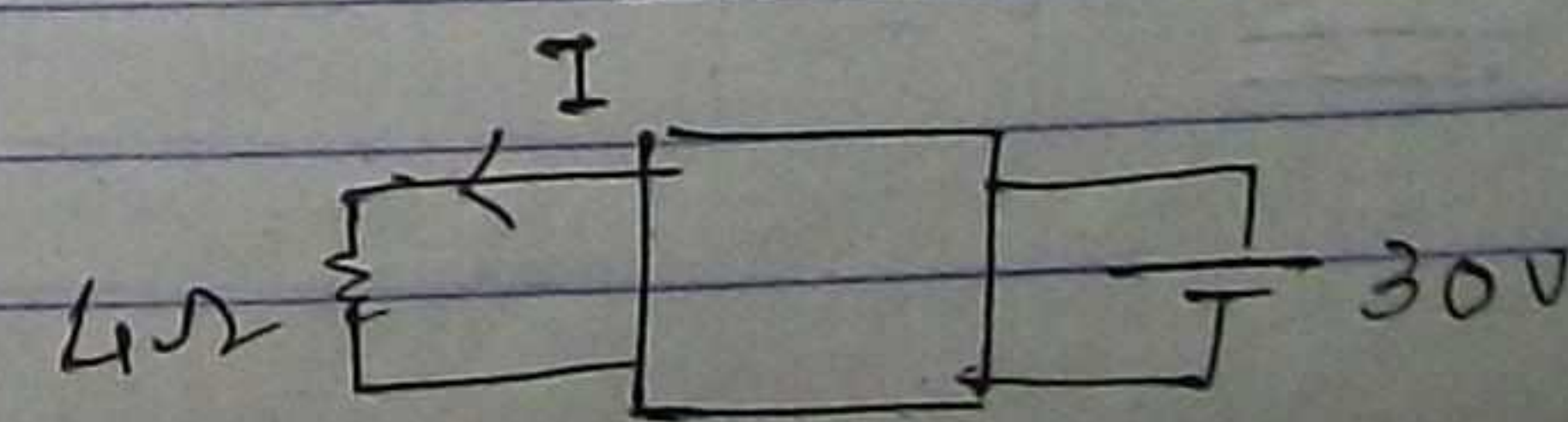
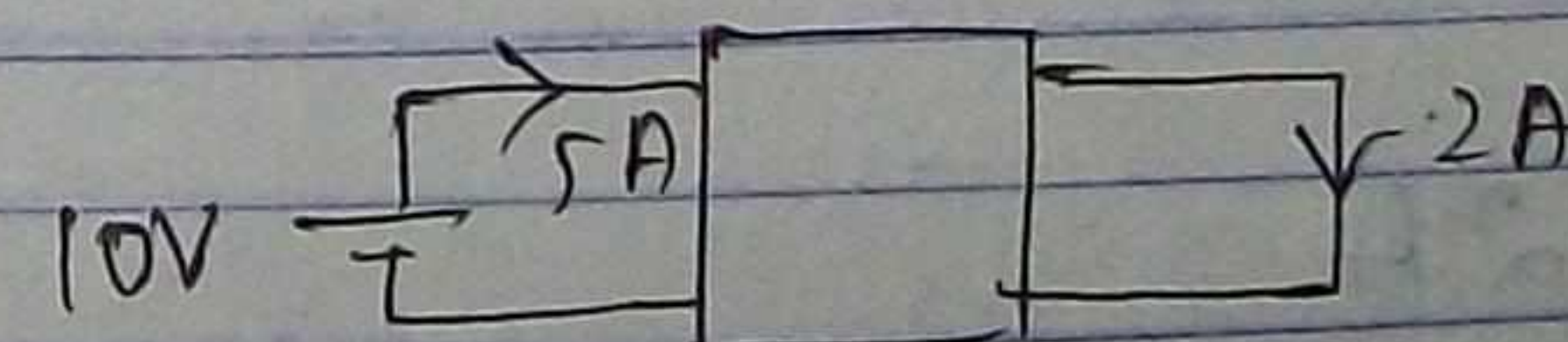


②



$\therefore I = I' + I'' = + (6 + 15) = -9A$

Ques. when given N/w satisfy reciprocity find I

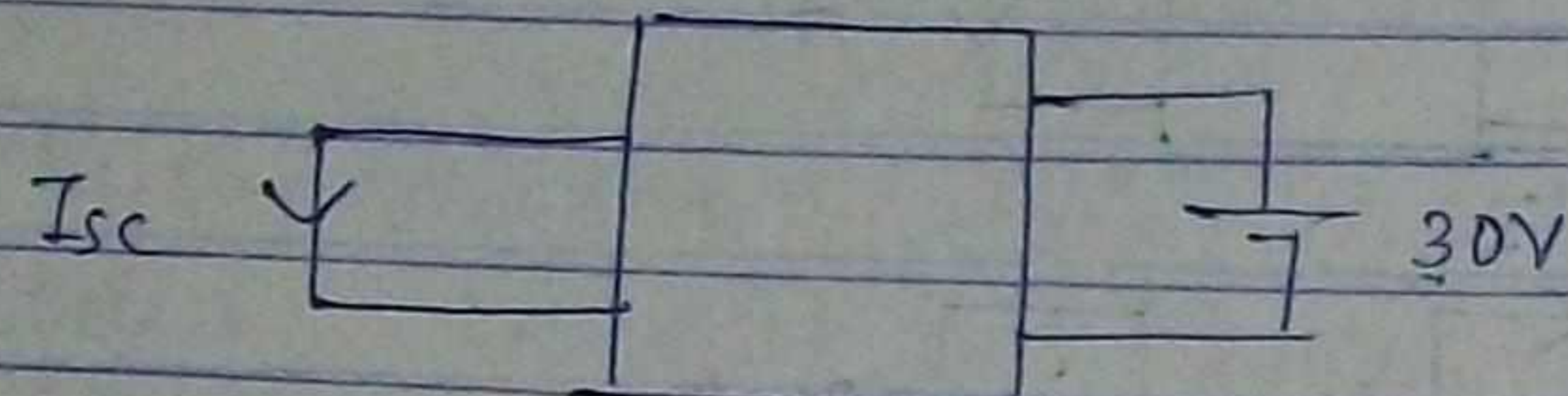


Solⁿs Applying

NORTON
THEOREM

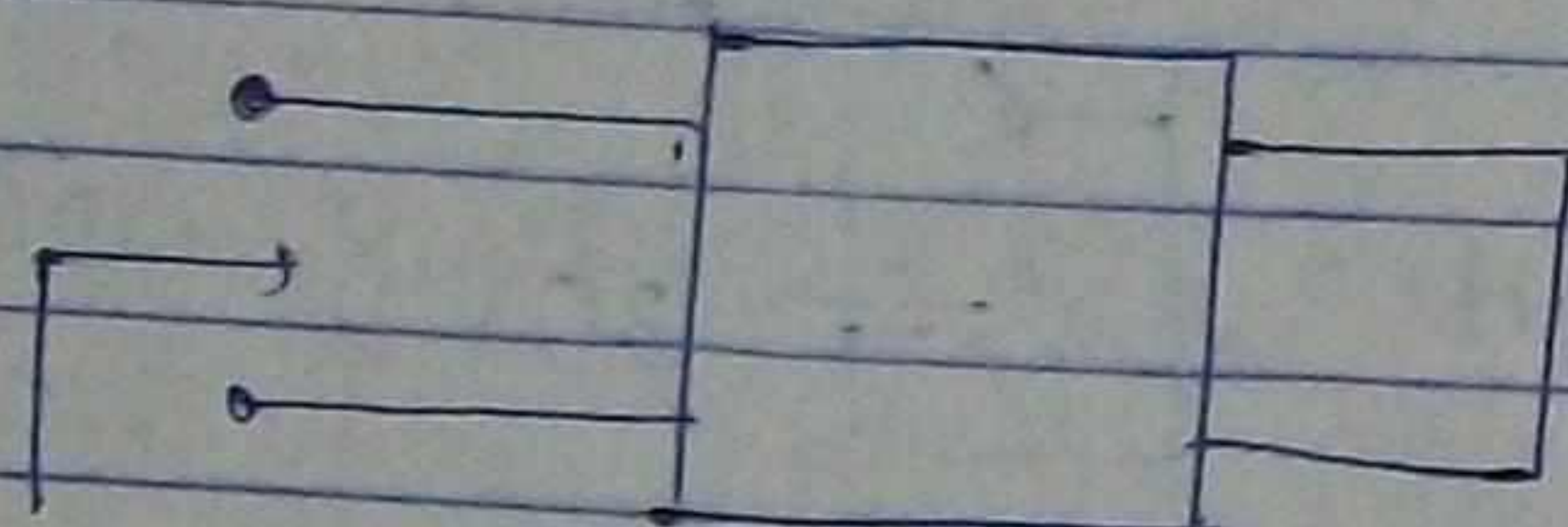
RECIPROCITY

① I_{sc} -



$$I_{sc} = \underline{6}$$

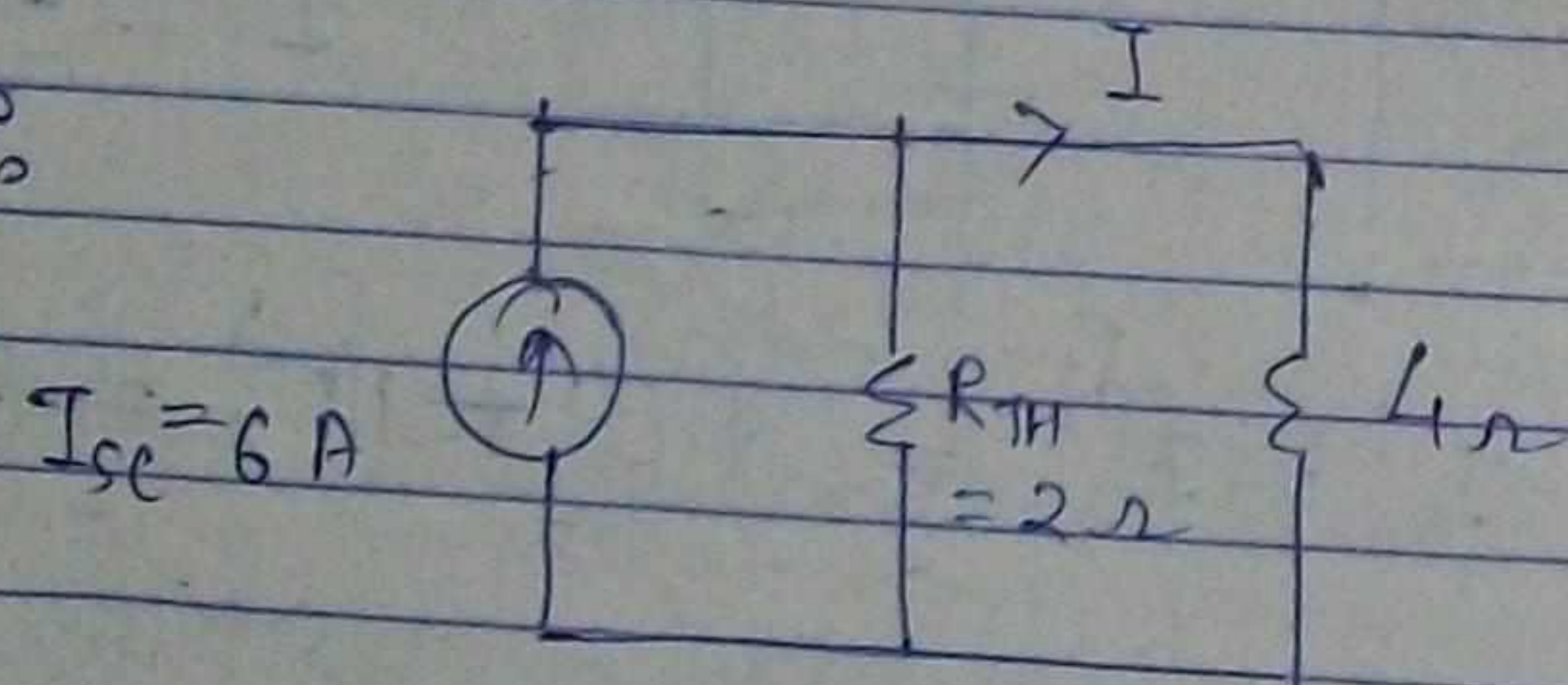
② R_{TH} - deactivate all independent sources



R_{TH}

$$R_{TH} = \frac{10}{5} = \underline{2\Omega}$$

Norton's circuit:

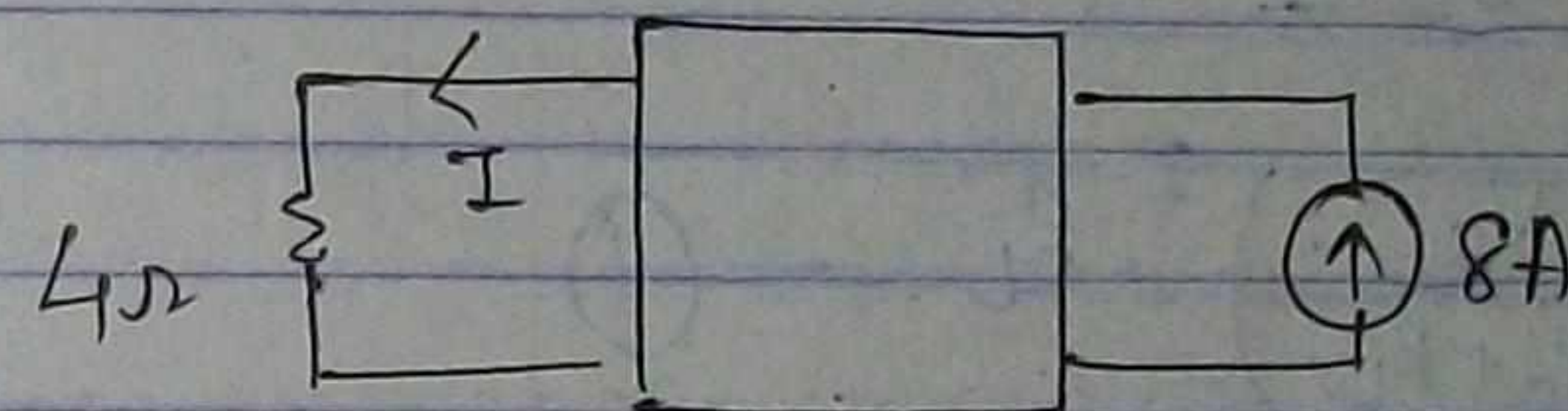
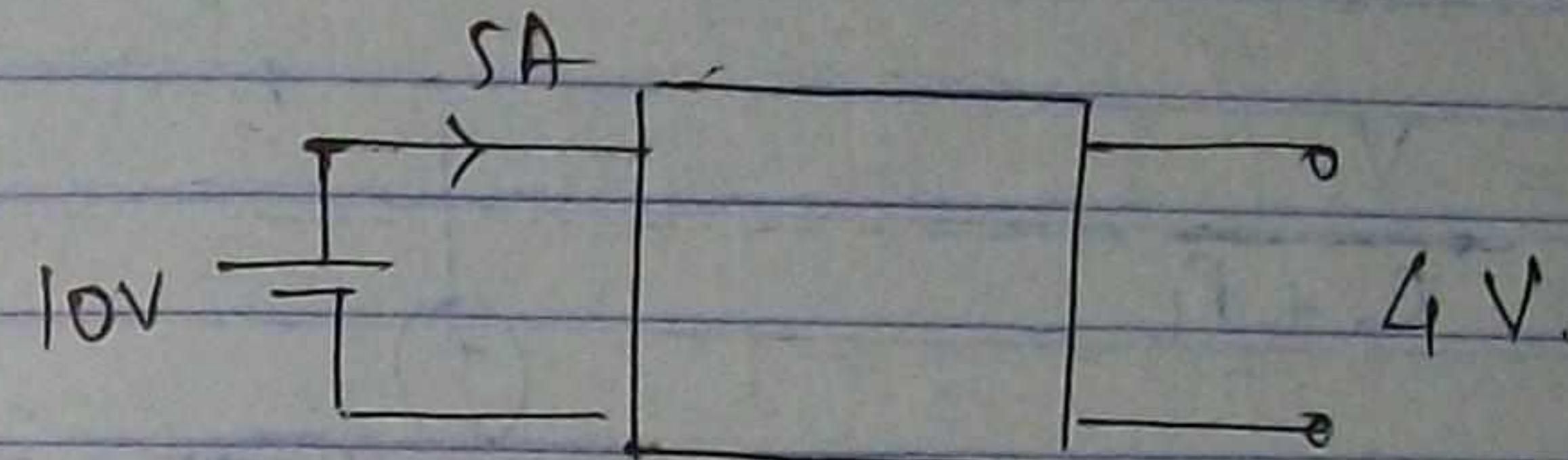


$$I = 6 \times \frac{2}{2+4}$$

$$I = \underline{2A}$$

How

Clue: when given N/w satisfy reciprocity find I



$$\text{Ans. } \underline{\frac{16}{15} A}$$

Solⁿs:

Thevenin
Theorem

+

Reciprocity

⇒ MAX. POWER TRANSFER THEOREM -

$$I = \frac{V_s}{R_s + R_L}$$

$$P_L = I^2 R_L$$

$$P_L = \left(\frac{V_s}{R_s + R_L} \right)^2 R_L \quad \text{--- (1)}$$

Diff. eq (1) wrt R_L and equate to zero.

$$R_L = R_s$$

MAX. POWER -

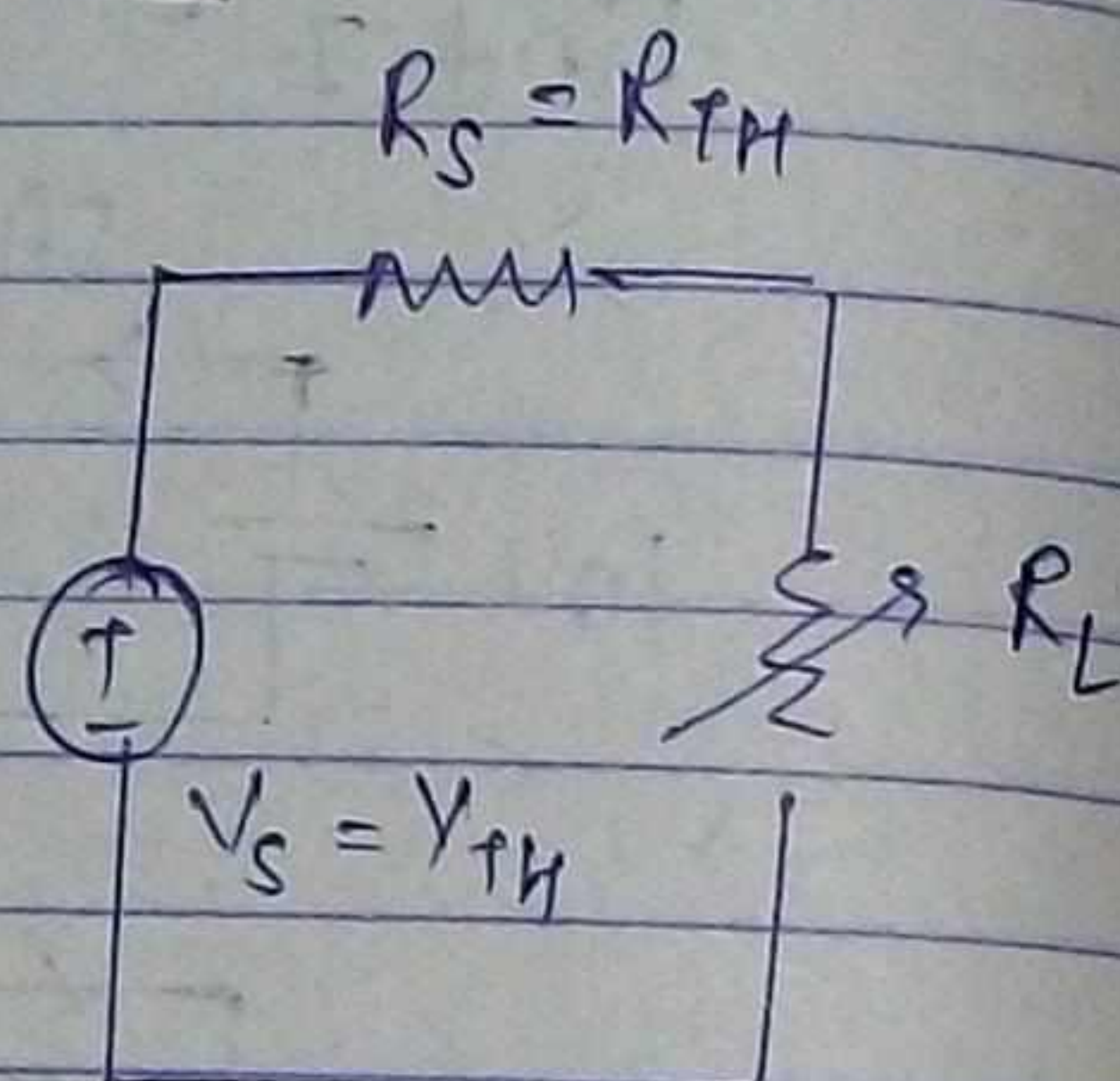
$$P_{\max} = \frac{V_s^2}{(R_s + R_L)^2} \cdot R_L$$

$$P_{\max} = \frac{V_s^2}{4 R_L}$$

Efficiency -

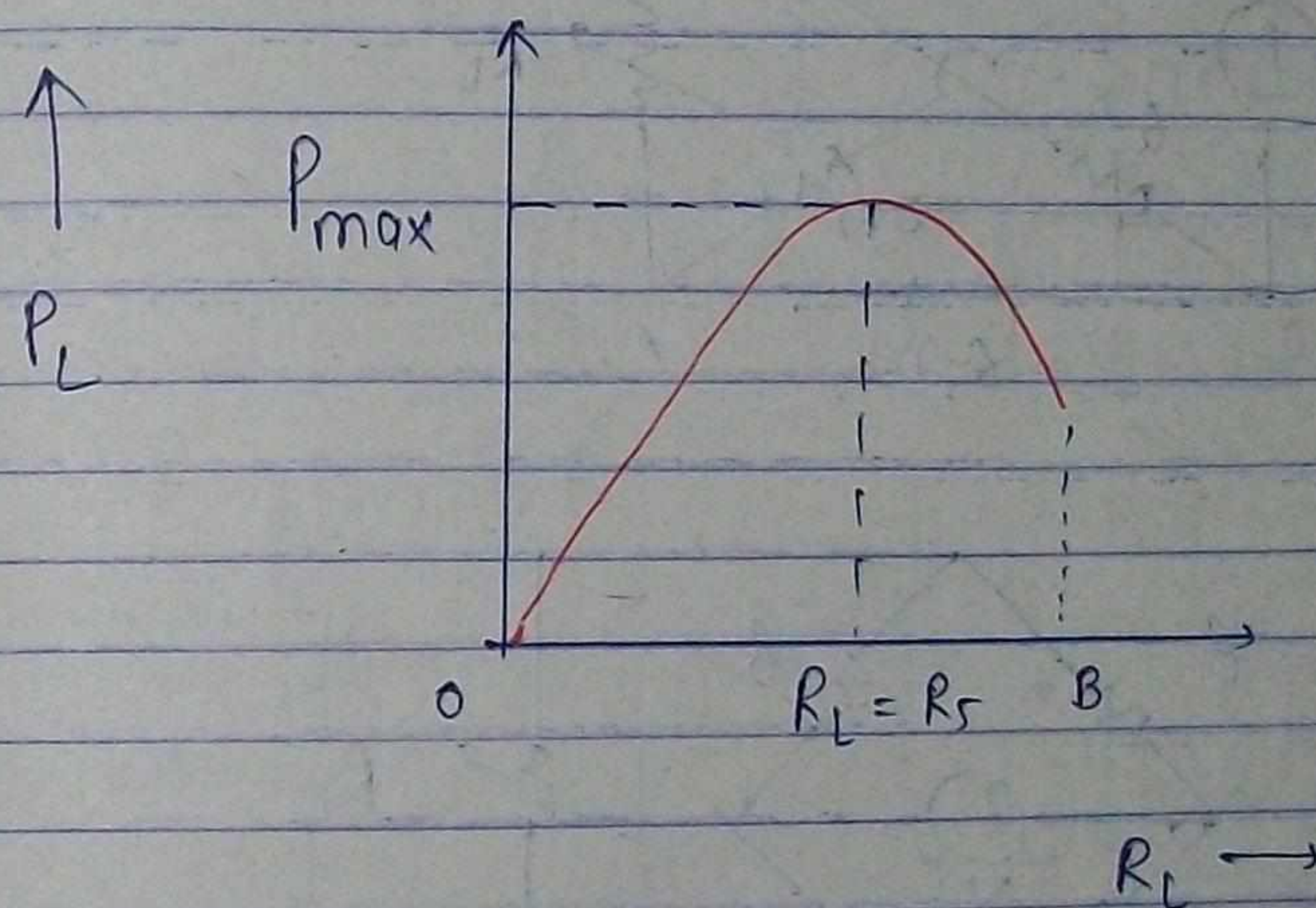
$$\eta = \frac{O/P}{I/P} \times 100$$

$$\eta \% = \frac{I^2 R_L}{I^2 (R_s + R_L)} \times 100 \%$$



$$\eta = \frac{R_L}{R_s + R_L} \times 100$$

$$\eta = 50\%$$



① AT OA,

$$R_s > R_L$$

$$\eta < 50\%$$

② AT A,

$$R_s = R_L$$

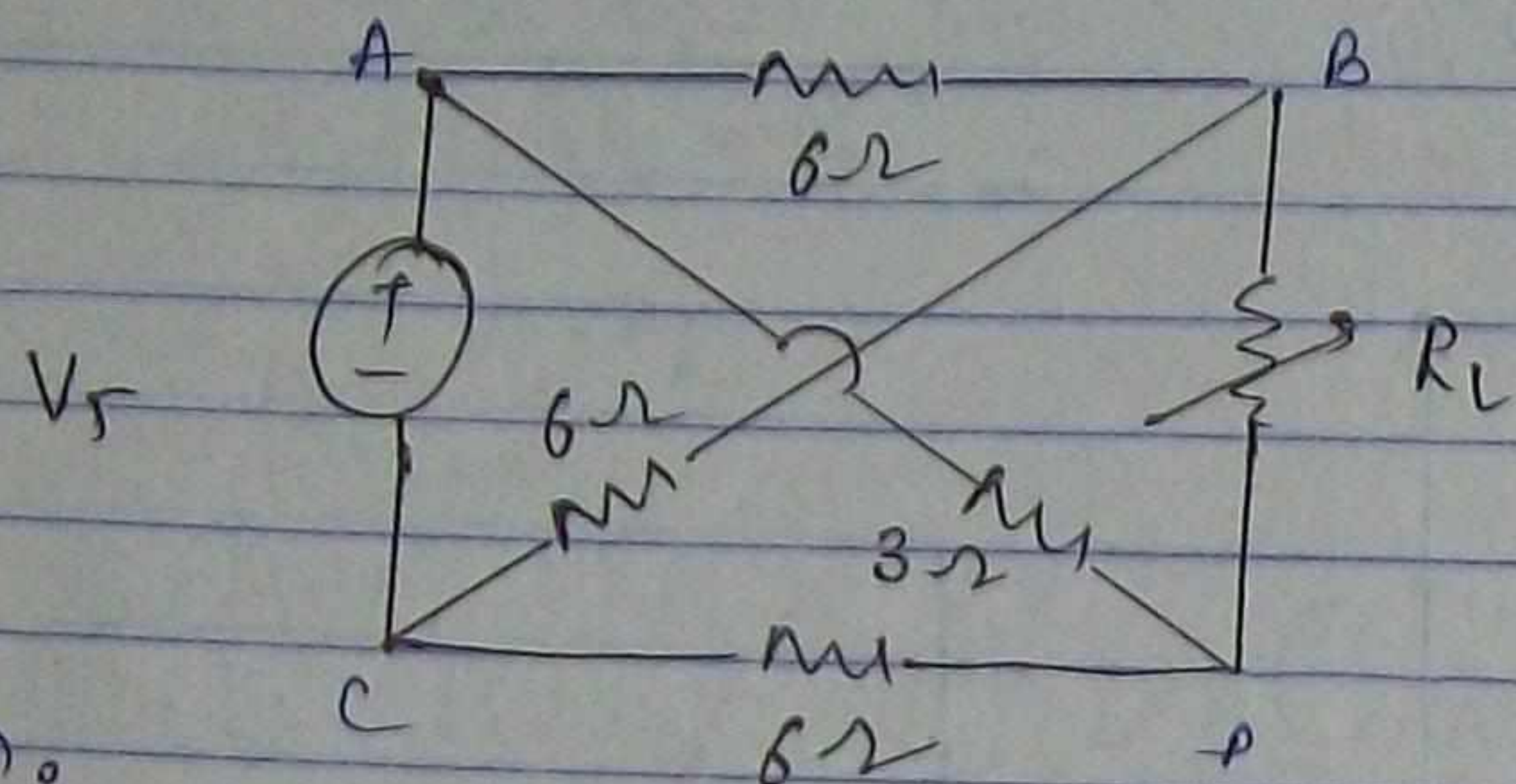
$$\eta = 50\%$$

③ AT AB,

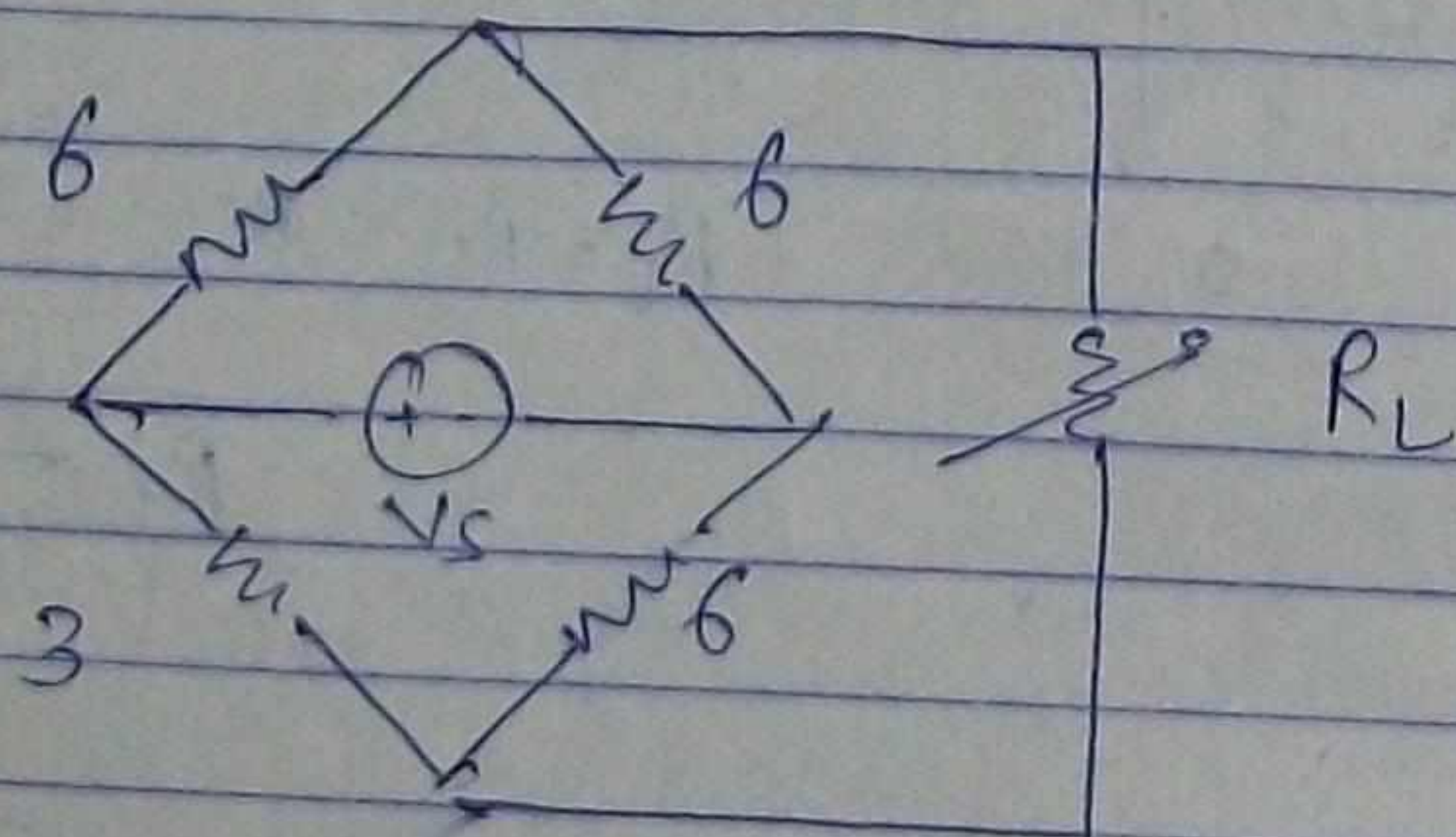
$$R_s < R_L$$

$$\eta > 50\%$$

Ques. In the circuit shown at what value of R_L power delivered from source to load is max?



Soln:



$$R_{TH} = \quad V_s = 0$$

$$R_{TH} = (6 \parallel 6) + (3 \parallel 6) = 5\Omega$$

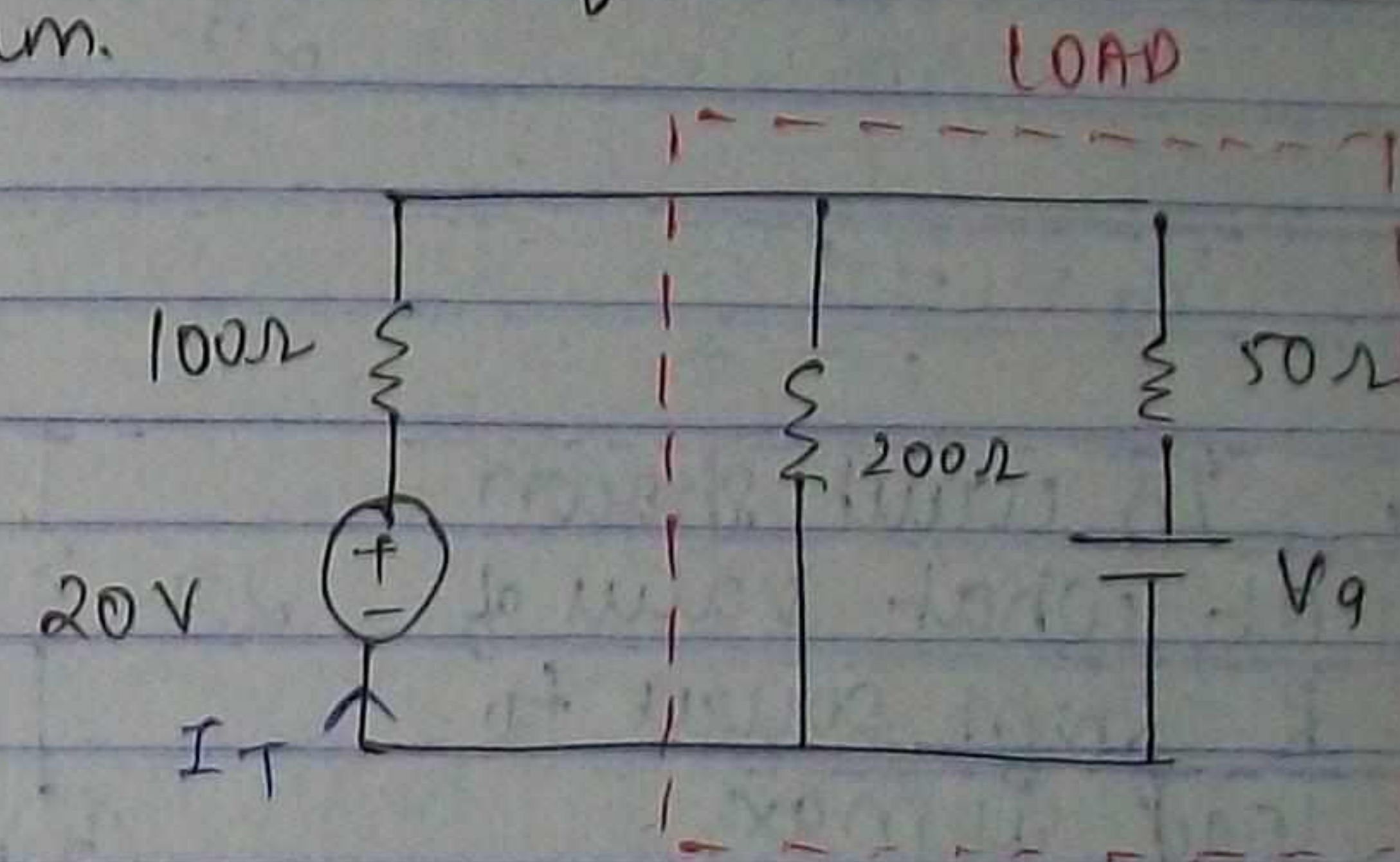
$$\therefore R_L = R_{TH}$$

$$R_L = 5\Omega$$

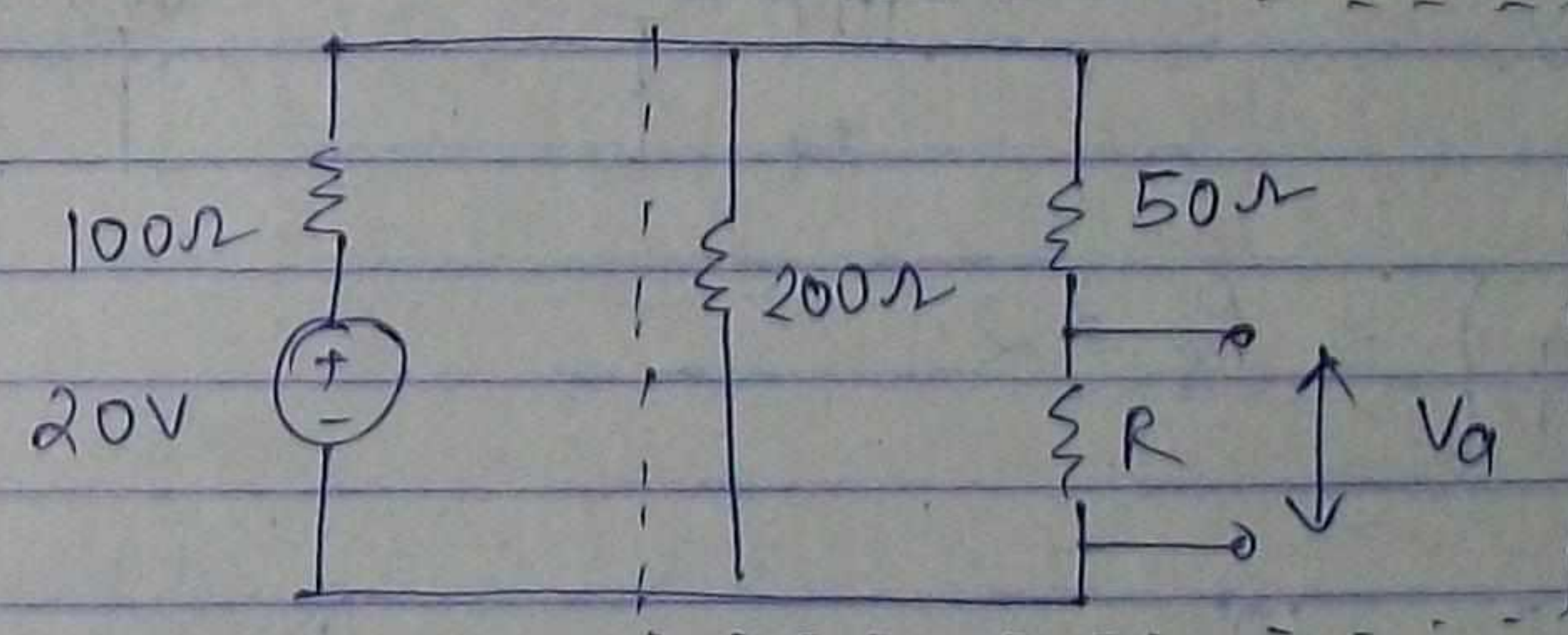
44

Ques. In the circuit shown at what value of V_a power delivered from source to load is maximum.

- (1) 10V
- (2) 7.5V
- (3) 0V
- (4) 20V



Soln:



$$(R_{eq})_L = \frac{200(50 + R)}{200 + 50 + R}$$

$$100 = \frac{(200)(50 + R)}{(250 + R)}$$

$$R = 150\Omega$$

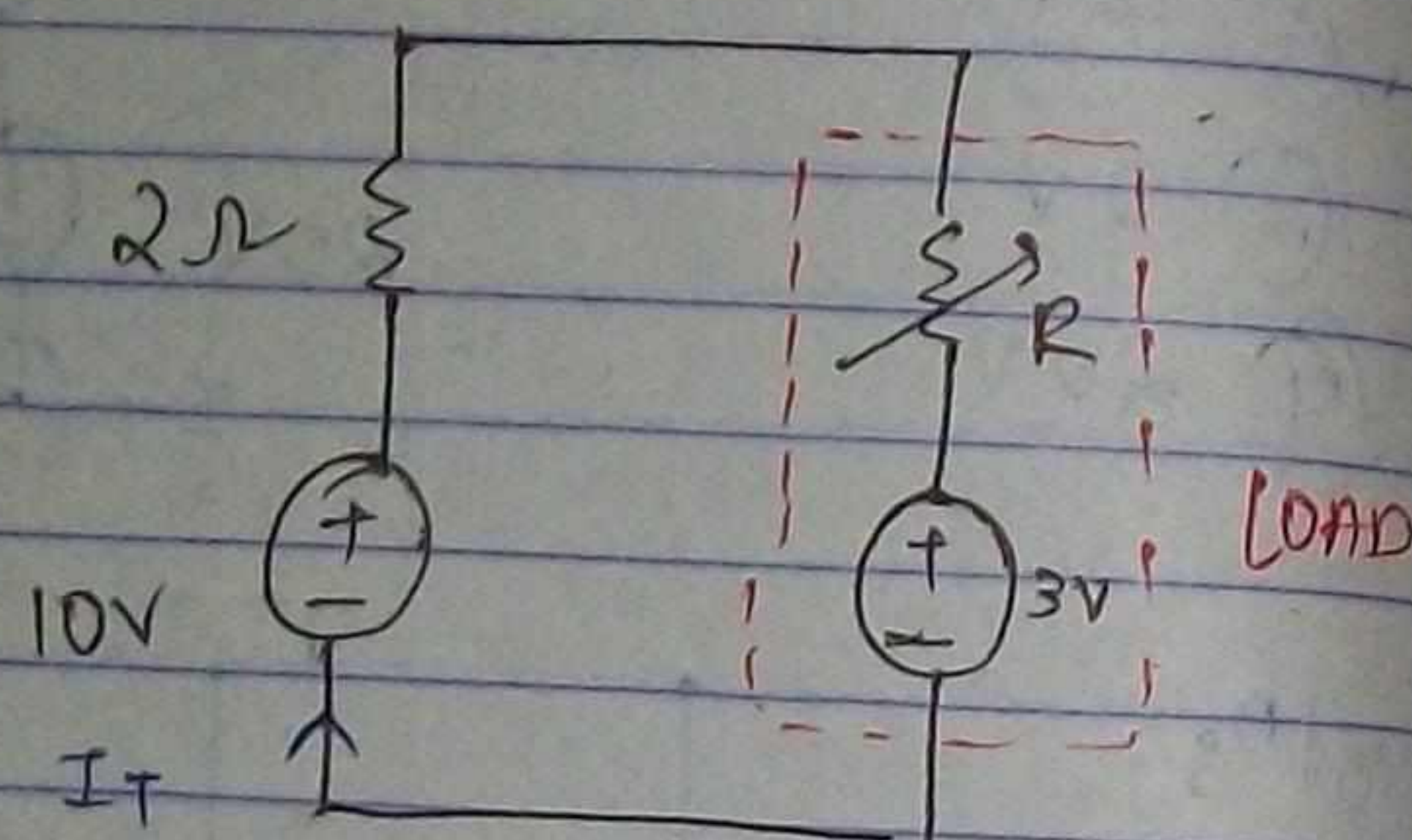
$$(R_{eq})_L = R_s = 100$$

$$I_T = \frac{20}{R_s + (R_{eq})_L} = \frac{20}{100 + 100} = \frac{1}{10}$$

$$V_a = \frac{I_T \times R}{2} = \frac{1}{20} \times 150$$

$$V_a = 7.5 \text{ V}$$

Ques. In circuit shown at what value of R from source to load is max.



Solⁿ:

$$(R_{eq})_L = R_S = 2$$

$$I_T = \frac{10}{R_S + (R_{eq})_L} = \frac{10}{2+2} = 2.5 \text{ A}$$

$$I_T = \frac{10-3}{2+R}$$

$$2.5 = \frac{7}{2+R}$$

$$R = 0.8 \Omega$$

⇒ MAX. POWER TRANSFER THEOREM (AC) -

$$I = \frac{V_{TH}}{(R_{TH} + R_L) + j(X_L + X_{TH})}$$

$$I = \frac{V_{TH}}{\sqrt{(R_{TH} + R_L)^2 + (X_L + X_{TH})^2}}$$

$$P_L = I^2 R_L$$

ACTIVE POWER

$$P_L = \frac{V_{TH}^2 \cdot R_L}{(R_{TH} + R_L)^2 + (X_{TH} + X_L)^2} \quad \text{--- (1)}$$

CASE (1): BOTH R_L & X_L ARE VARIABLE -

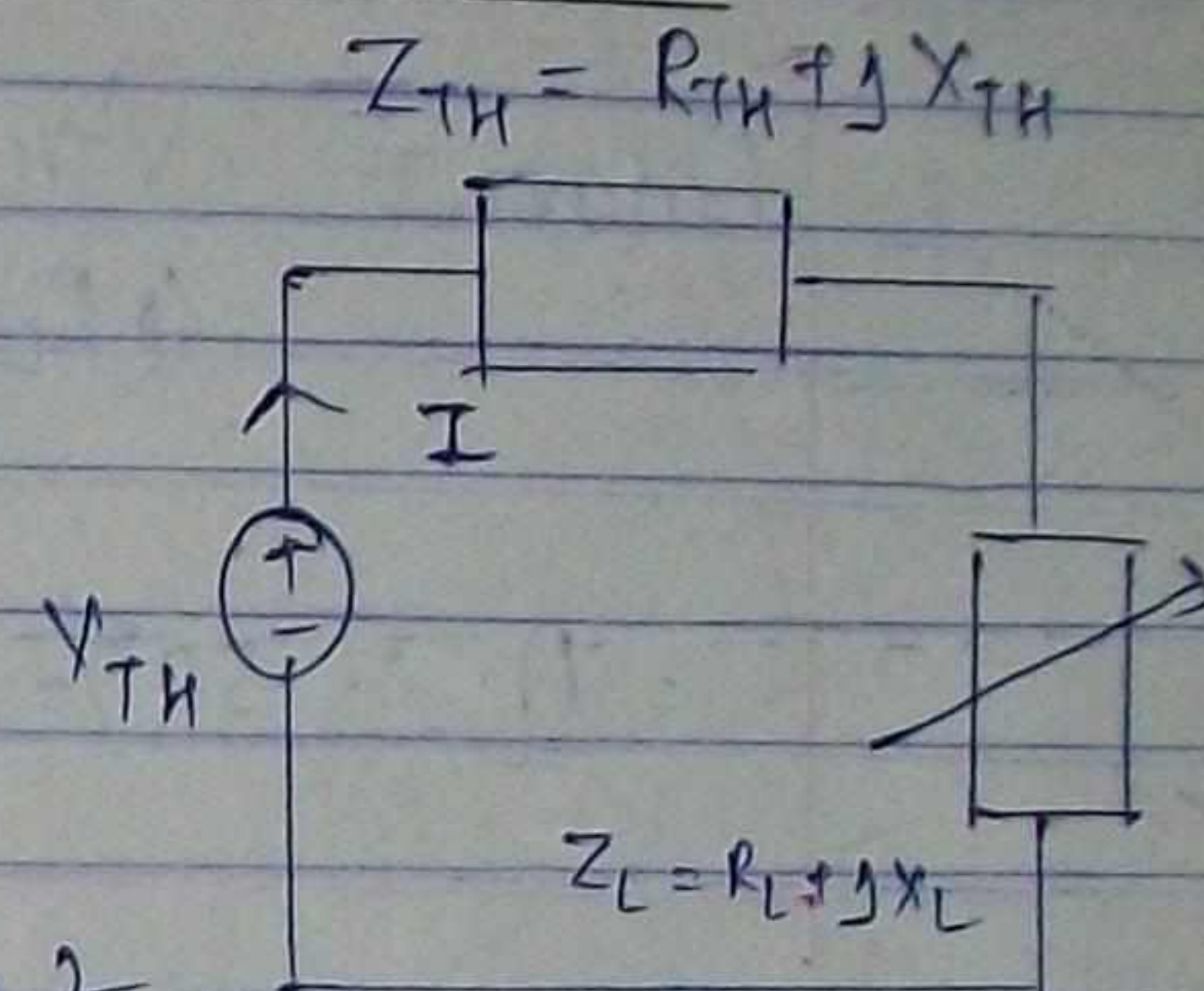
→ Diff. eqⁿ (1) wrt R_L and equated to 0.

→ Diff. eq (1) wrt X_L and equated to 0.

$$R_L + jX_L = R_{TH} - jX_{TH}$$

AA

$$Z_L = Z_{TH}^*$$



Substituting in eq ①:

$$P_{max} = \frac{V_{TH}^2}{4R_L}$$

$$\eta = 50\%$$

CASE ②: ONLY R_L IS VARIABLE -
($X_L = \text{CONSTANT}$)

- Diff. eqn ① wrt R_L and equate to 0.

$$R_L = \sqrt{R_{TH}^2 + (X_L + X_{TH})^2}$$

$$\eta > 50\%$$

$$\eta = \frac{R_L}{R_{TH} + R_L} \times 100\%$$

Since, $R_L > R_{TH}$

CASE ③: LOAD IMPEDANCE IS ONLY RESISTIVE -
($X_L = 0$)

$$P_L = \frac{V_{TH}^2 R_L}{(R_L + R_{TH})^2 + X_{TH}^2} \quad - (2)$$

Diff. eq ② wrt R_L and equate to zero

$$R_L = \sqrt{R_{TH}^2 + X_{TH}^2}$$

AA

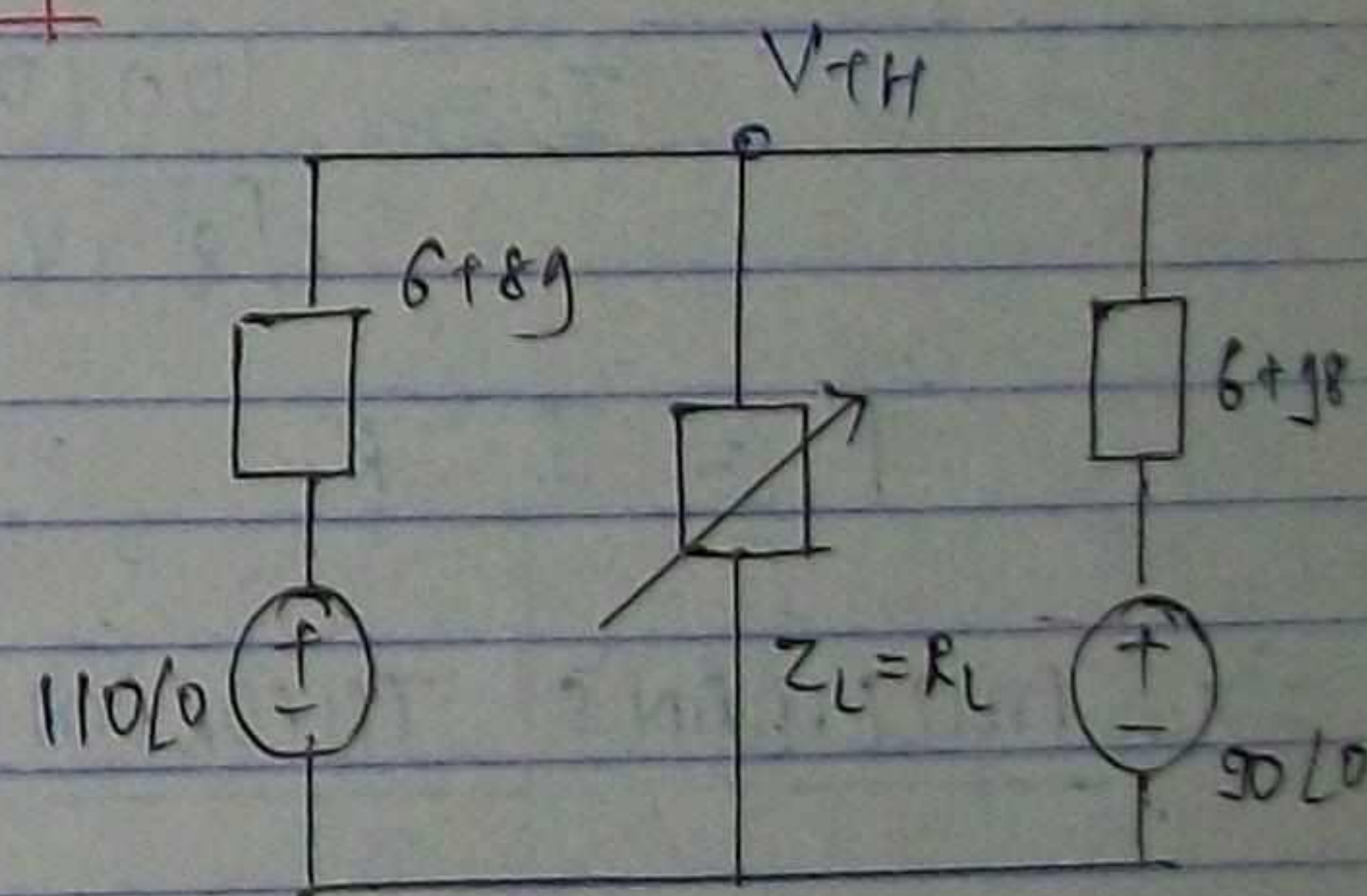
$$R_L = Z_{TH}$$

$$\eta > 50\%$$

$$R_L > R_{TH}$$

Que.

find max Power dissipation in the load impedance



Soln:

$$V_{TH} =$$

$$\frac{V_{TH} - 110}{6 + j8} + \frac{V_{TH} - 90\angle 0}{6 + j8} = 0$$

$$V_{TH} = \frac{200}{2} = 100\angle 0$$

$$R_{TH} = \frac{(6 + j8)}{2} = 3 + j4$$

$$R_L = \sqrt{R_{TH}^2 + X_{TH}^2}$$

$$= 3 + j4$$

$$= \sqrt{3^2 + 4^2}$$

$$= 5$$

$$=$$

$$I = \frac{100 \angle 0}{3 + j4 + 5}$$

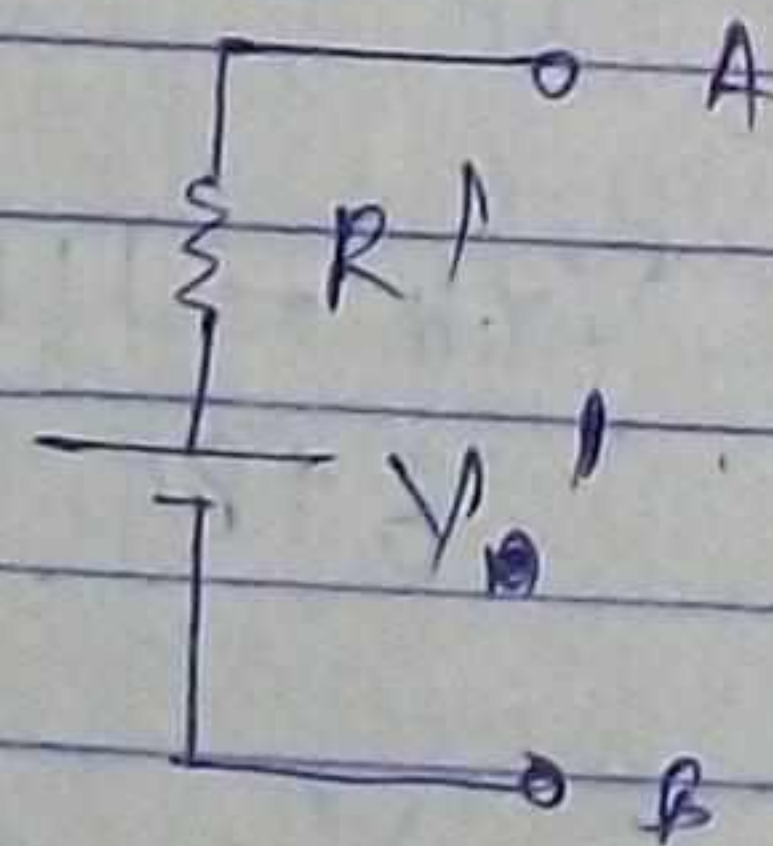
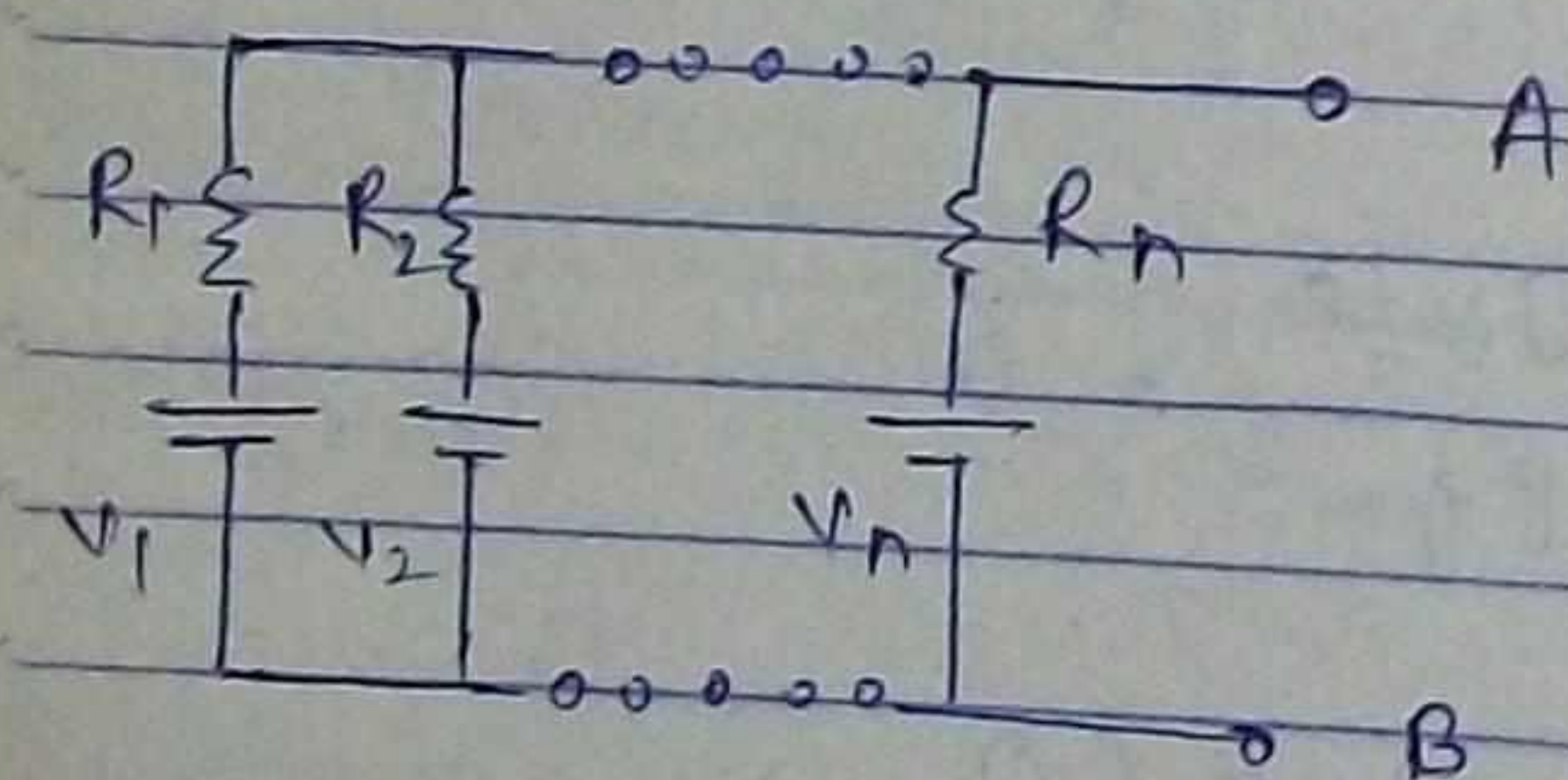
$$3 + j4 + 5$$

$$I = \frac{100 \angle 0}{\sqrt{8^2 + 4^2}}$$

$$\sqrt{8^2 + 4^2}$$

$$P_L = I^2 R_L = 625 \text{ W}$$

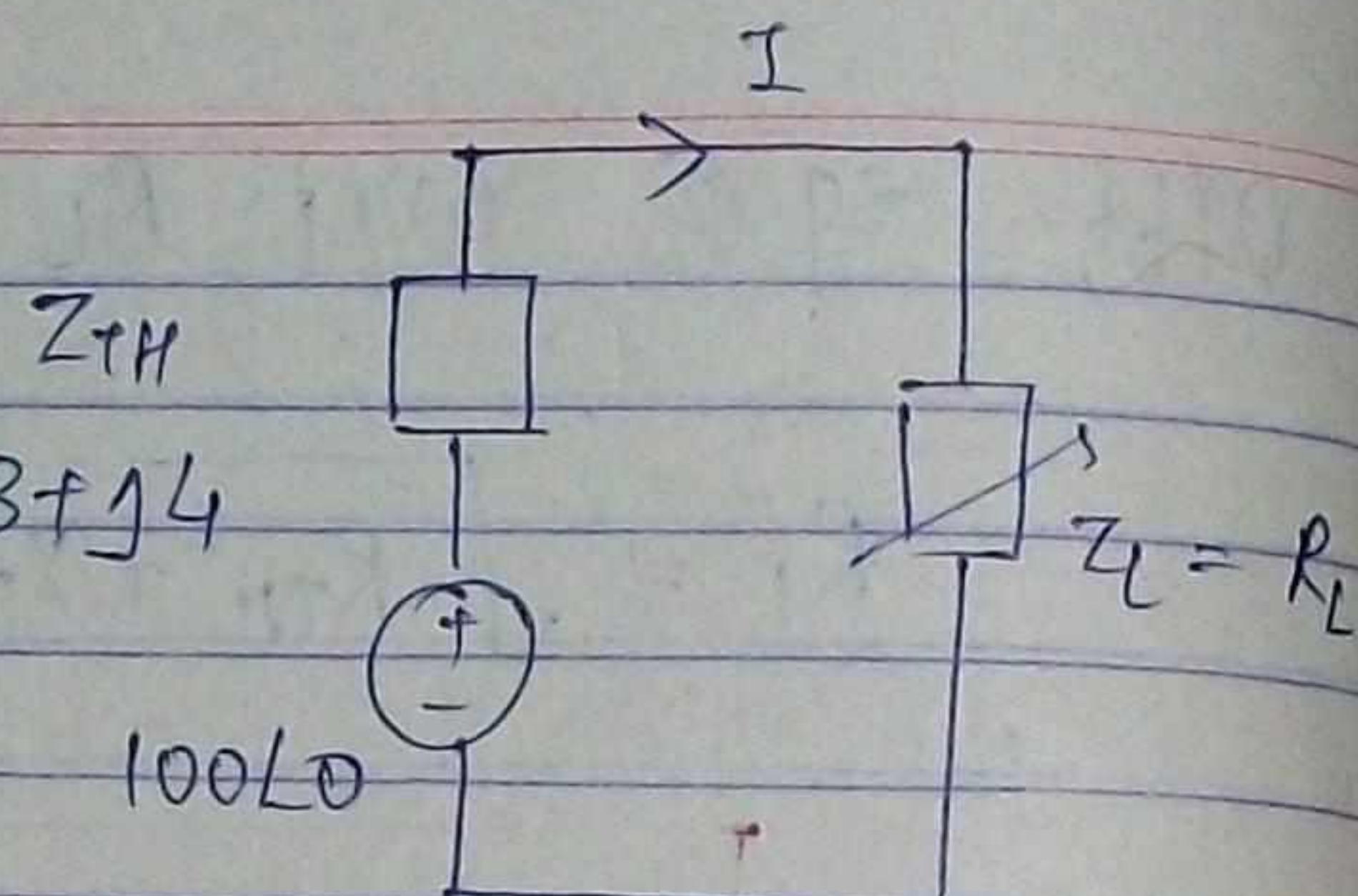
⇒ MILLMAN'S THEOREM -



$$\frac{1}{R'} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_n}$$

$$R' =$$

$$\frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_n}}$$



$$R' =$$

$$\frac{1}{G_1 + G_2 + G_3 + \dots + G_n}$$

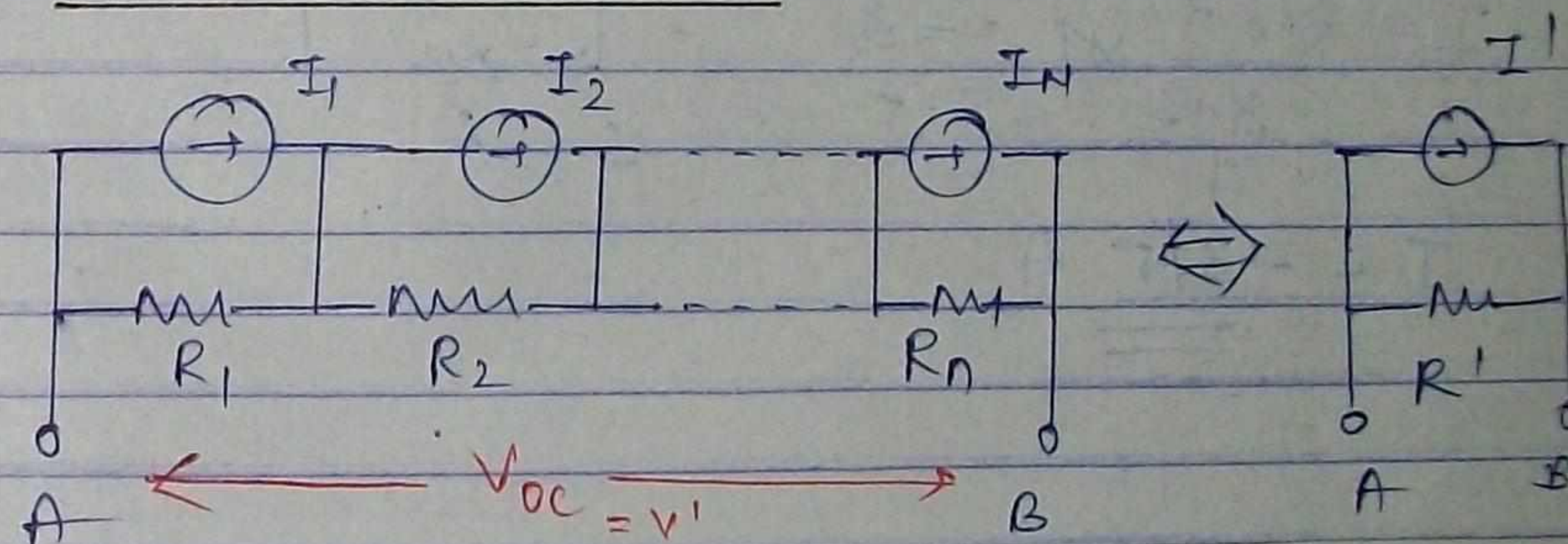
$$V' =$$

$$\frac{\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} + \dots + \frac{V_n}{R_n}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_n}}$$

$$V' =$$

$$\frac{V_1 G_1 + V_2 G_2 + \dots + V_n G_n}{G_1 + G_2 + \dots + G_n}$$

⇒ DUAL OF MILLMAN'S -



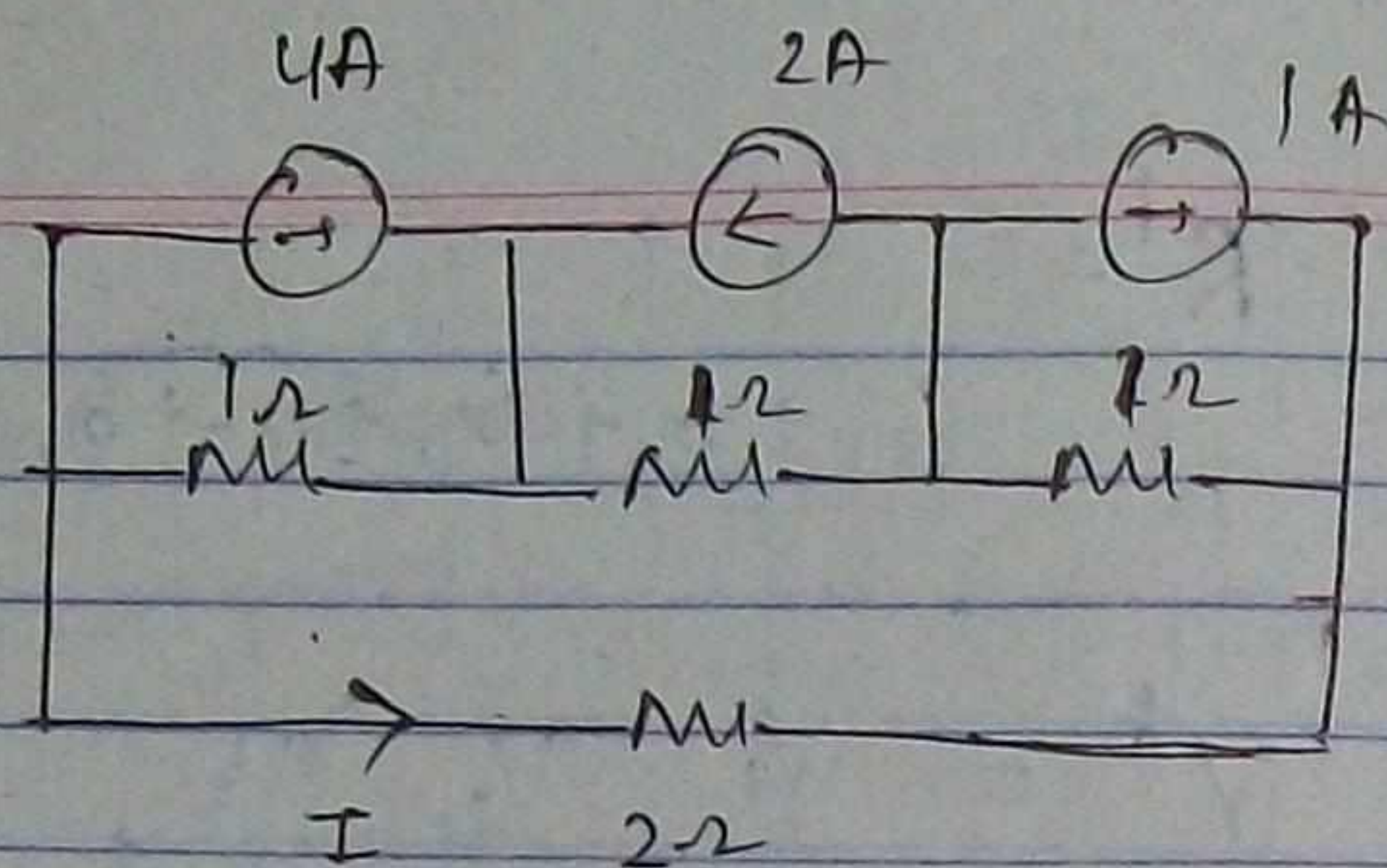
$$R' =$$

$$R_1 + R_2 + R_3 + \dots + R_n$$

$$I' =$$

$$\frac{I_1 R_1 + I_2 R_2 + \dots + I_n R_n}{R_1 + R_2 + R_3 + \dots + R_n}$$

Ques. Find I



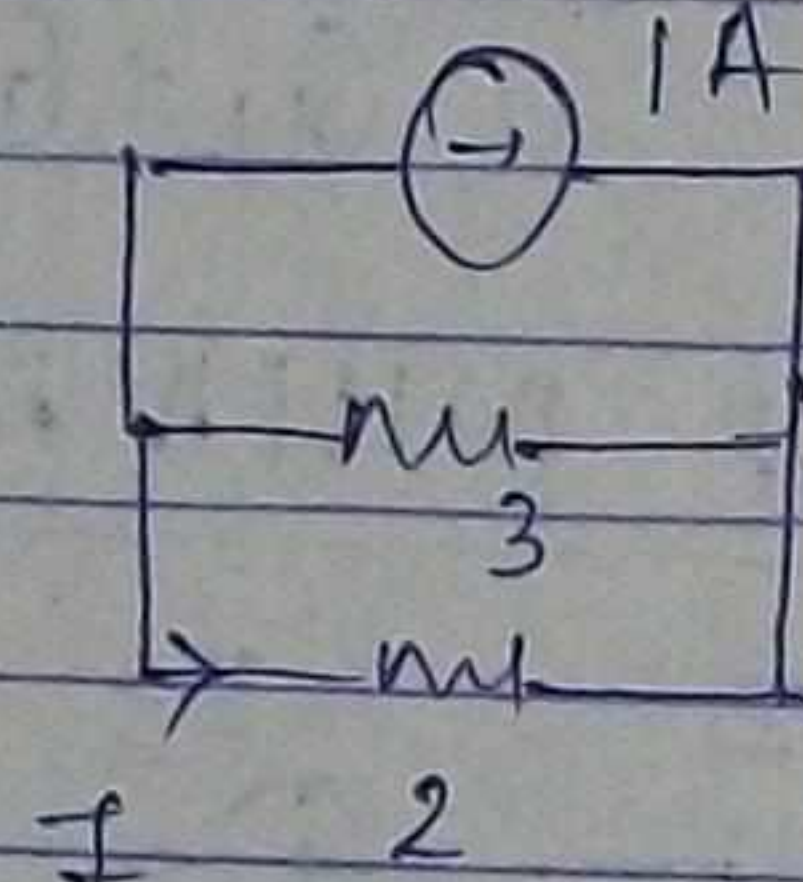
Solⁿ:

$$I' = \frac{(4 \times 1) - (2 \times 1) + (1 \times 1)}{1 + 1 + 1} = 1$$

$$R' = 3$$

$$I = -\frac{3}{3+2} \times 1 = -\frac{3}{5}$$

$$I = -\frac{3}{5} \text{ A.}$$



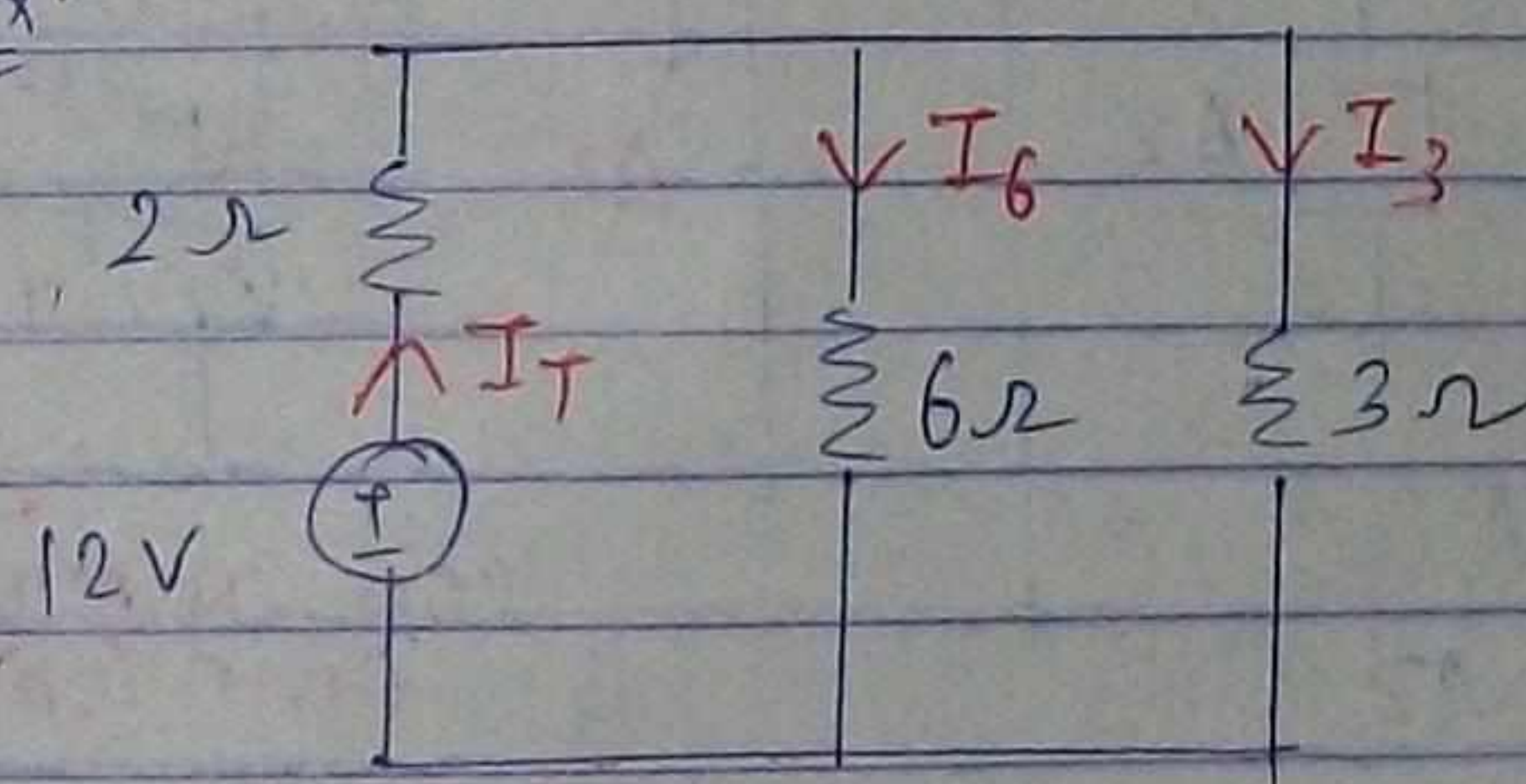
-1/11/11

⇒ TELLEGEN'S THEOREM -

- Tellegen's theorem states that algebraic sum of the powers in any circuit at any instant is equal to zero. (linear, non linear, unidirectional, bidirectional, time variant and invariant elements)

$$\sum_{k=1}^n V_k I_k = 0$$

Ex-



$$R_{eq} = 2 + \frac{6 \times 3}{6+3} = 4\Omega$$

$$I_T = 3 \text{ A}$$

$$I_6 = 1 \text{ A}$$

$$I_3 = 2 \text{ A}$$

$$V_2 = 3 \times 2 = 6 \text{ V}$$

$$V_6 = 6 \times 1 = 6 \text{ V}$$

$$V_3 = 3 \times 2 = 6 \text{ V}$$

$$\text{Now, } \sum V_k I_k = V_2 I_T + V_6 I_6 + V_3 I_3 - V_s I_T = 0$$

$$= 6 \times 3 + 6 \times 1 + 6 \times 2 - 12 \times 3 = 0$$

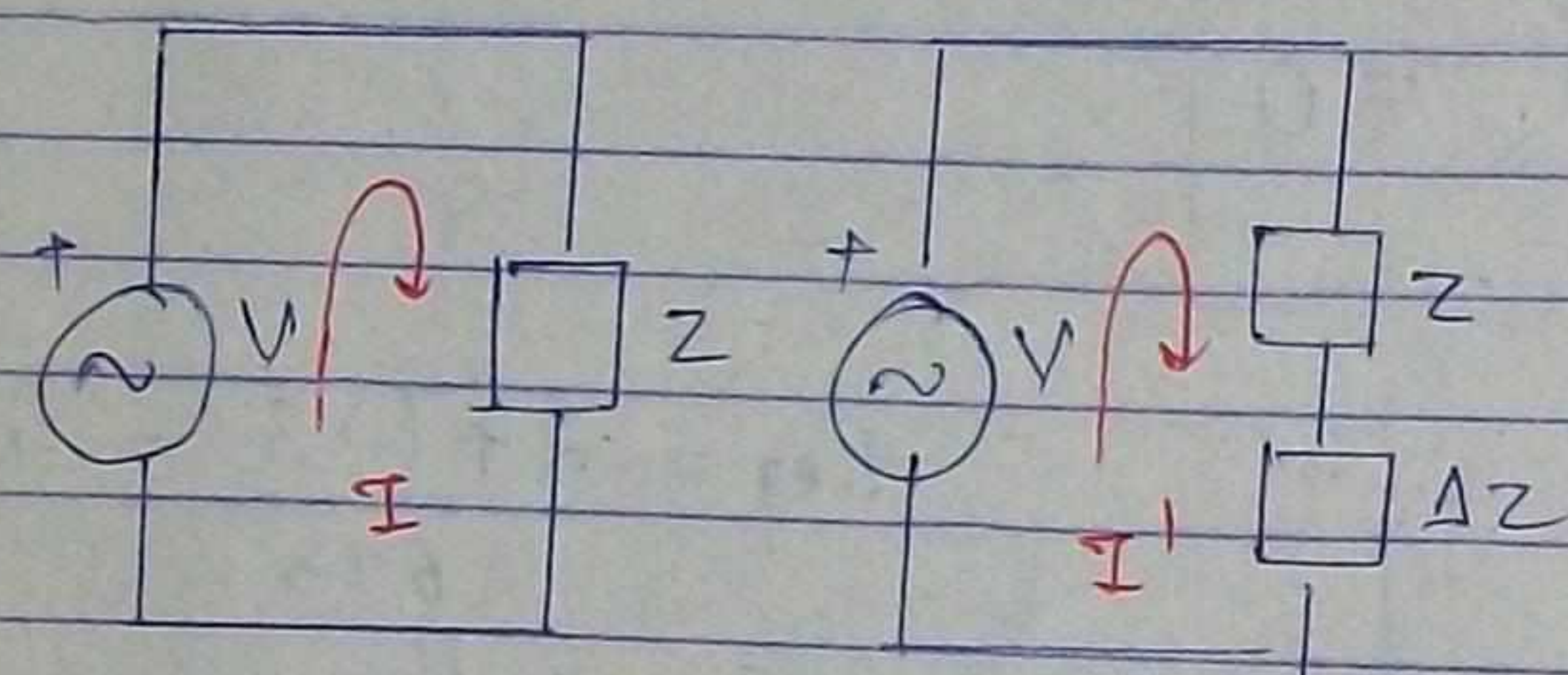
$$18 + 6 + 12 - 36 = 0$$

$$= 0$$

- for verification of the Tellegen's theorem KVL and KCL equations are used.

- Tellegen's theorem works based on the principle of law of conservation of energy.

⇒ COMPENSATION THEOREM -



$$I = \frac{V}{Z}$$

$$I' = \frac{V}{Z + \Delta Z}$$

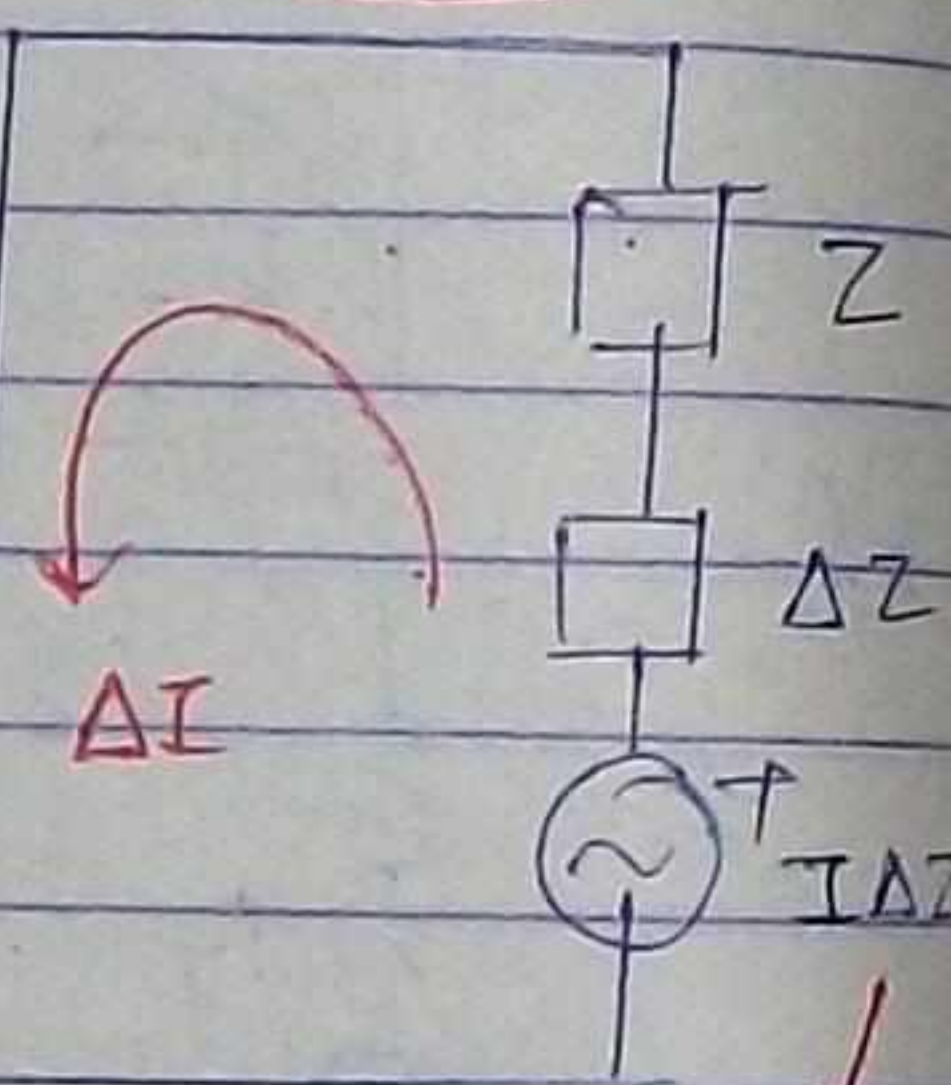
$$\Delta I = |I - I'|$$

$$\Delta I = \left| V \left[\frac{1}{Z} - \frac{1}{Z + \Delta Z} \right] \right|$$

$$\Delta I = \left| \frac{V \Delta Z}{Z(Z + \Delta Z)} \right|$$

$$\Delta I = \left| \frac{I \Delta Z}{Z + \Delta Z} \right|$$

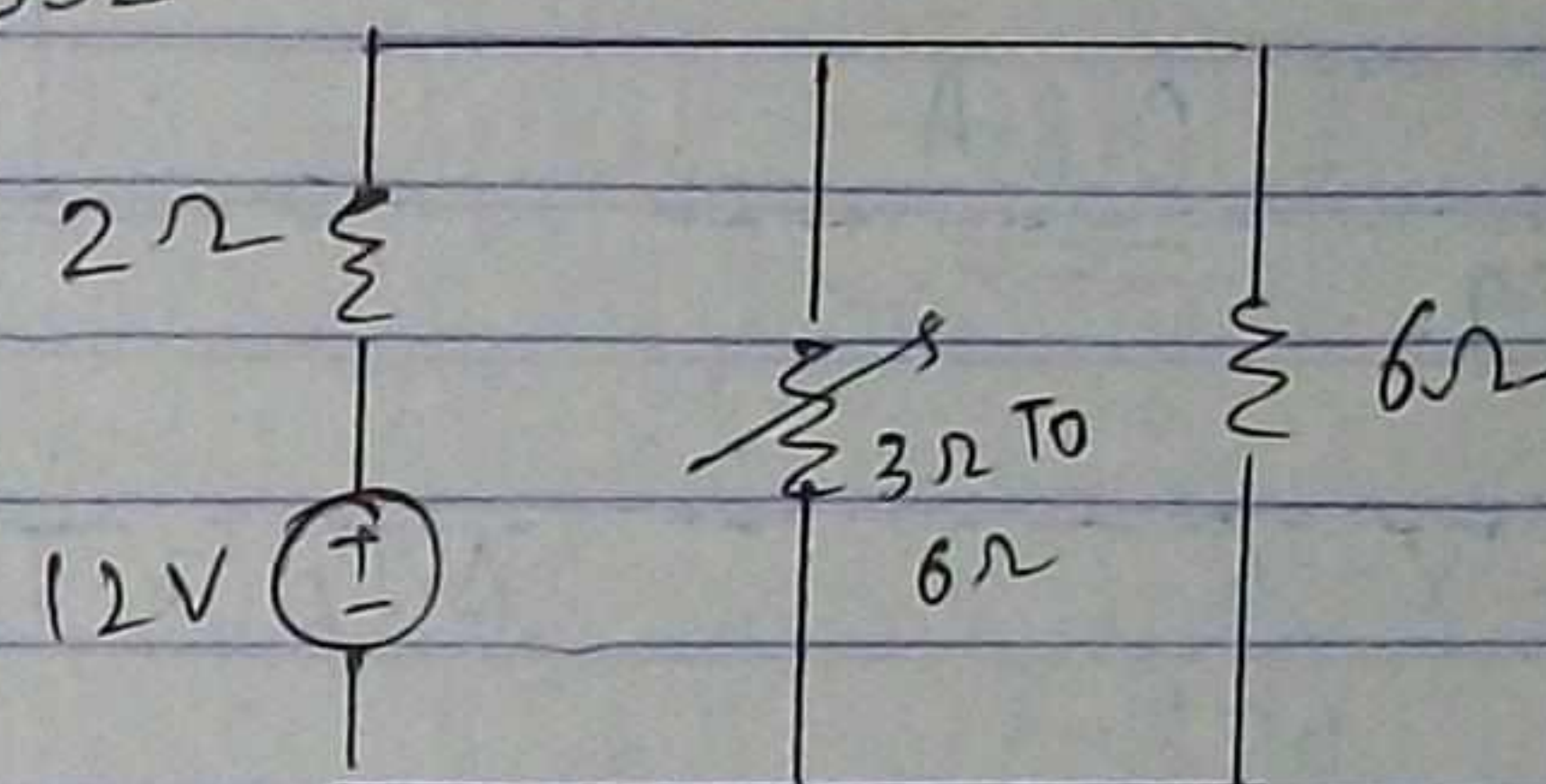
MODIFIED CIRCUIT



COMPENSATION
EMF (OR)
OPPOSING EMF

$$\Delta I = - \frac{I \Delta Z}{Z + \Delta Z}$$

Ques. find change in current in 2Ω and 6Ω when resistance of variable branch is change from $3\Omega \rightarrow 6\Omega$



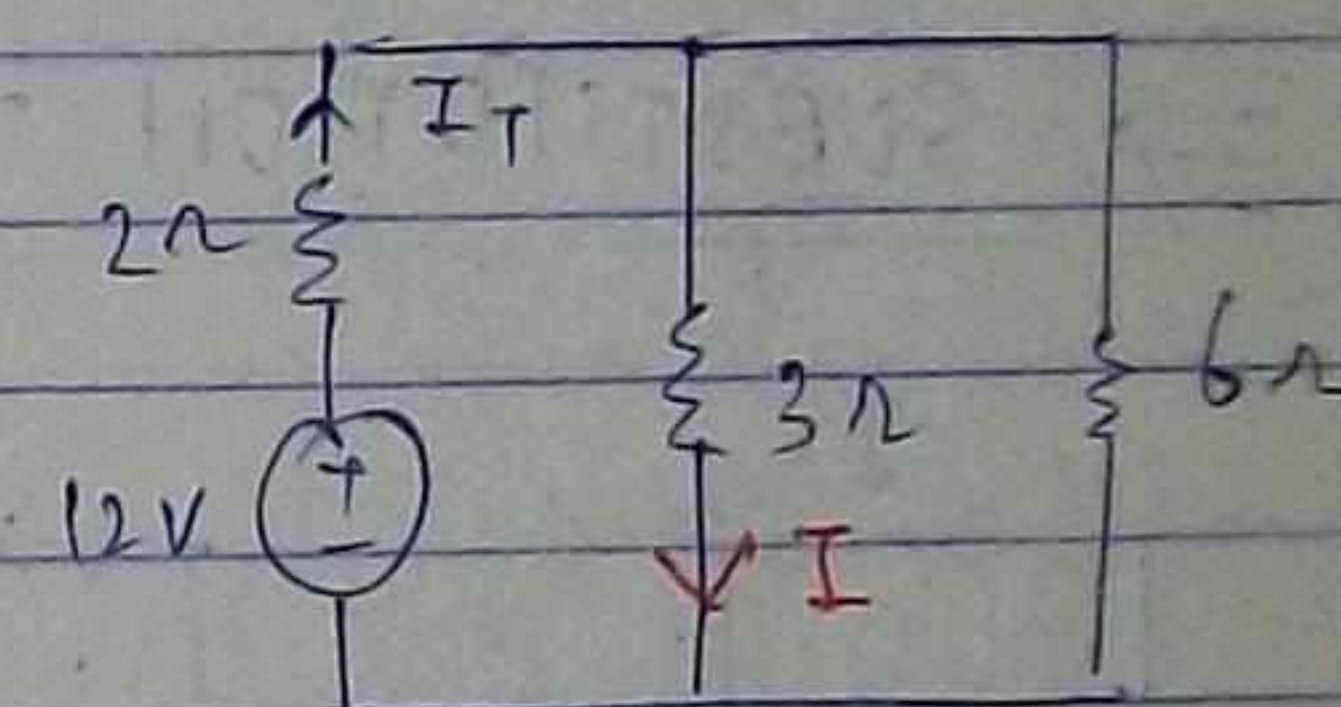
Solⁿ:

STEP ①: find original current in the variable branch.

$$R_{eq} = 4\Omega$$

$$I_T = 3A$$

$$I = 2A$$



STEP ②: find compensation emf.

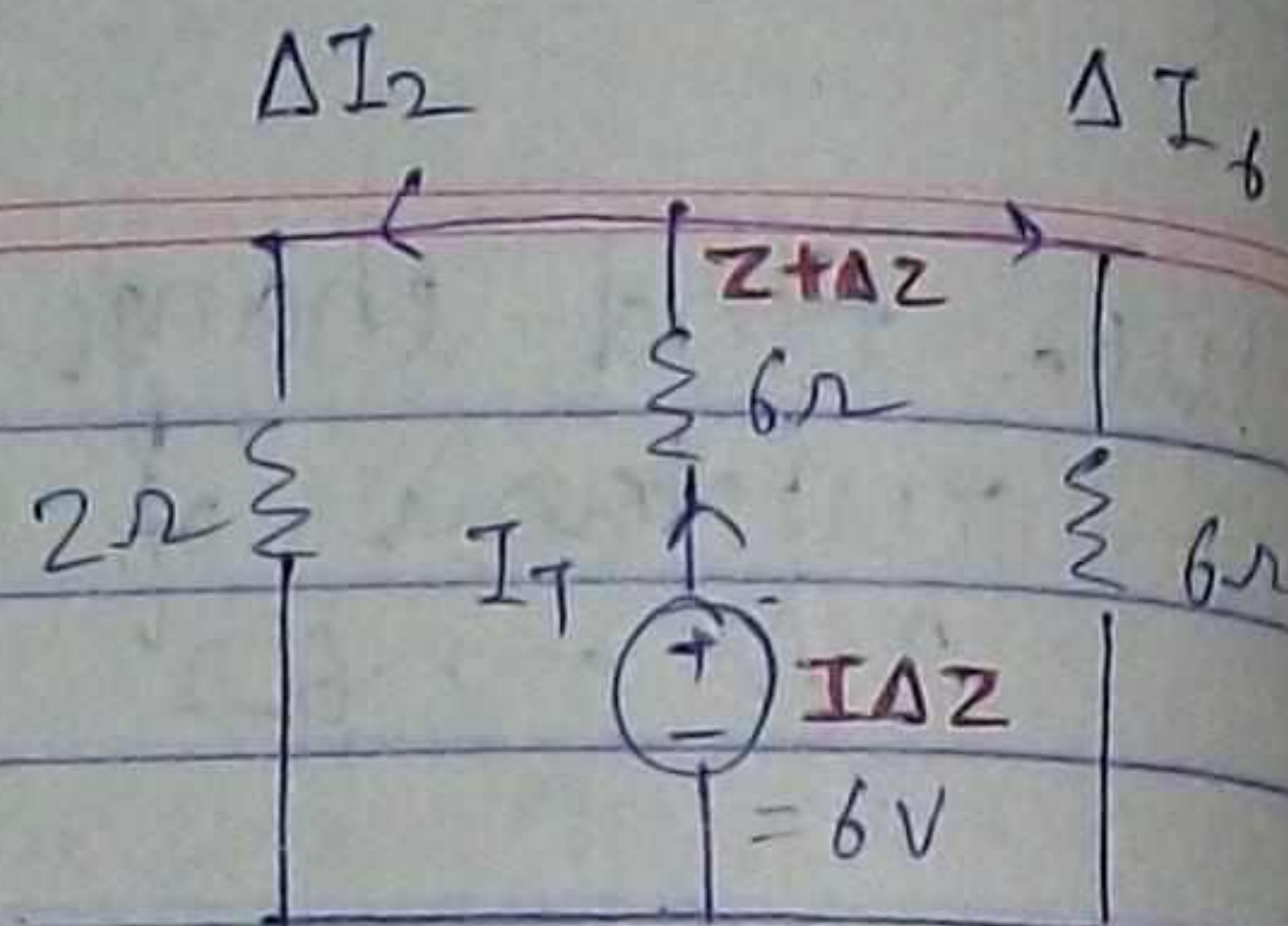
$$= I \Delta Z$$

$$= 2(6-3) = 6V$$

STEP ③: - Develop modified circuit.
- while developing modified circuit deactivate all the original source and connect compensation emf in series to variable branch.

$$R_{eq} = \frac{2 \times 6}{2+6} + 6 = 7.5 \Omega$$

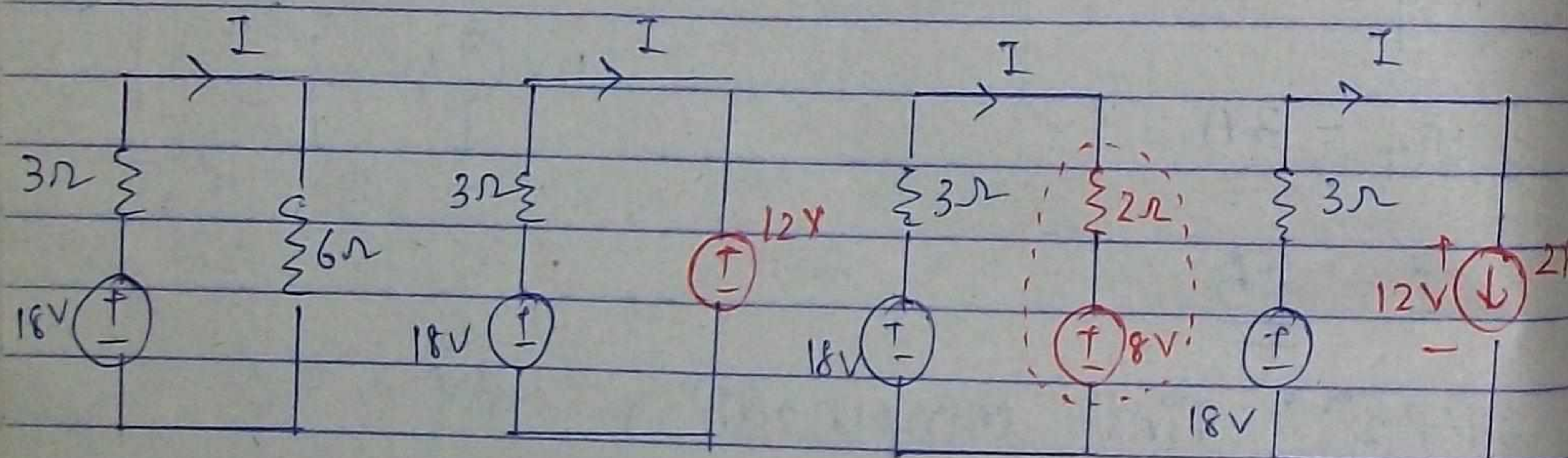
$$I_T = \frac{6}{R_{eq}} = 0.8 \text{ A}$$



$$\Delta I_2 = I_T \times \frac{6}{6+2} = 0.6 \text{ A}$$

$$\Delta I_6 = I_T \times \frac{2}{6+2} = 0.2 \text{ A}$$

⇒ SUBSTITUTION THEOREM—



$$I = \frac{18}{9} = 2 \text{ A}$$

$$I = \frac{18-12}{3} = 2 \text{ A}$$

$$I = \frac{18-8}{3+2} = 2 \text{ A}$$

$$I = 2 \text{ A}$$

TRANSIENTS

- Transients are present in the circuit when the circuit is subjected to any change either by changing source magnitude or by changing any circuit element provided circuit should consist of any energy storage elements.

- when the N/w is having only resistive elements no transients are present in the circuit since resistor allows sudden change of current and voltage and it does not store any energy.

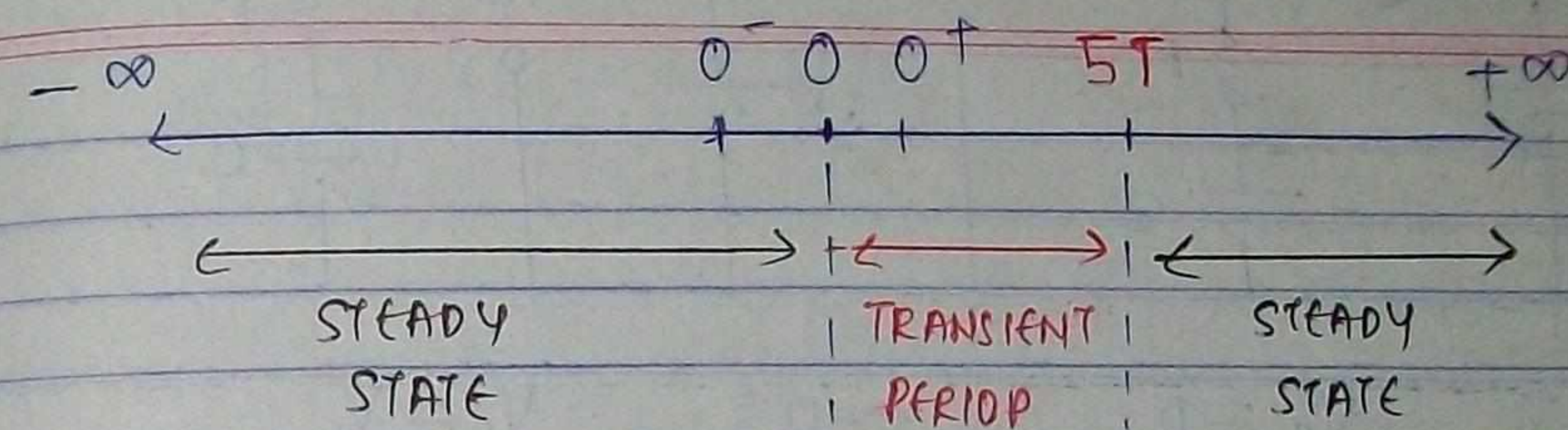
- Transients are present in the circuit when circuit is having any energy storage elements. Since:

① Inductor does not allow sudden change of current and it stores energy in the form of magnetic field

② Capacitor does not allow sudden change of voltage and it stores energy in the form of electric field.

- In the transient period voltage and current magnitude depends on time.

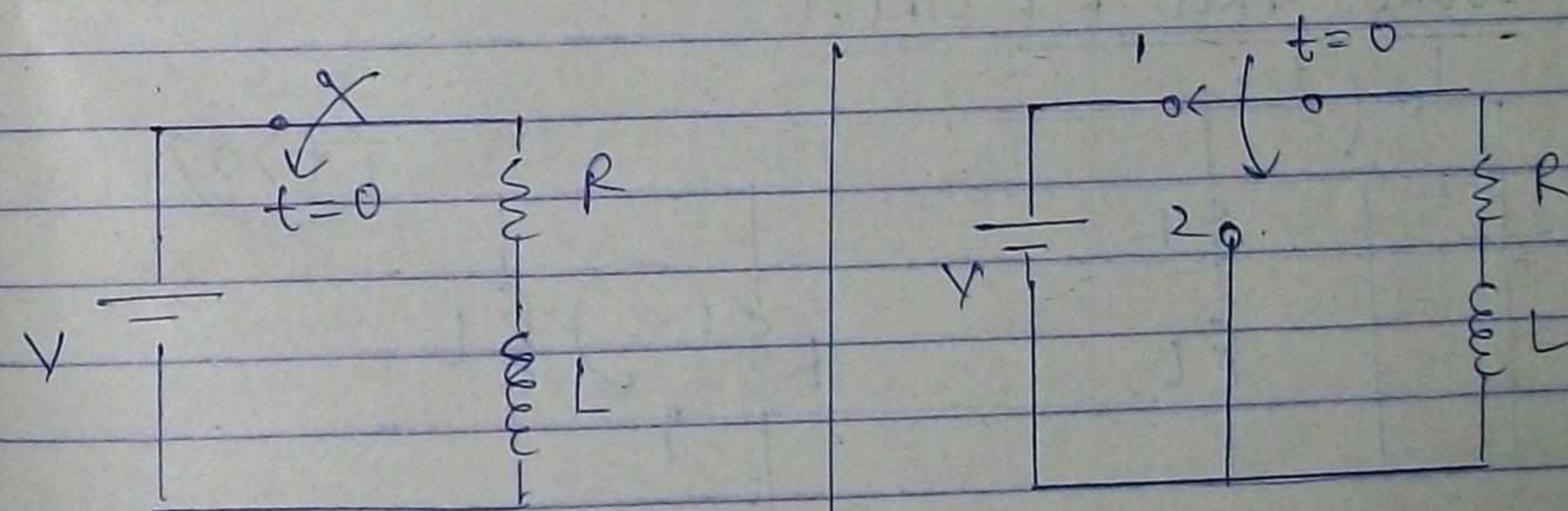
$T = \text{TIME CONST.}$



- $T = 0^-$ indicates immediately before operating the switch.

- $T = 0^+$ indicates immediately after operating the switch.

- $T = \infty$ indicates steady state conditions.



$$t = 0^- \quad i = 0$$

$$t = 0^+ \quad i = 0 \quad (\text{O.C.})$$

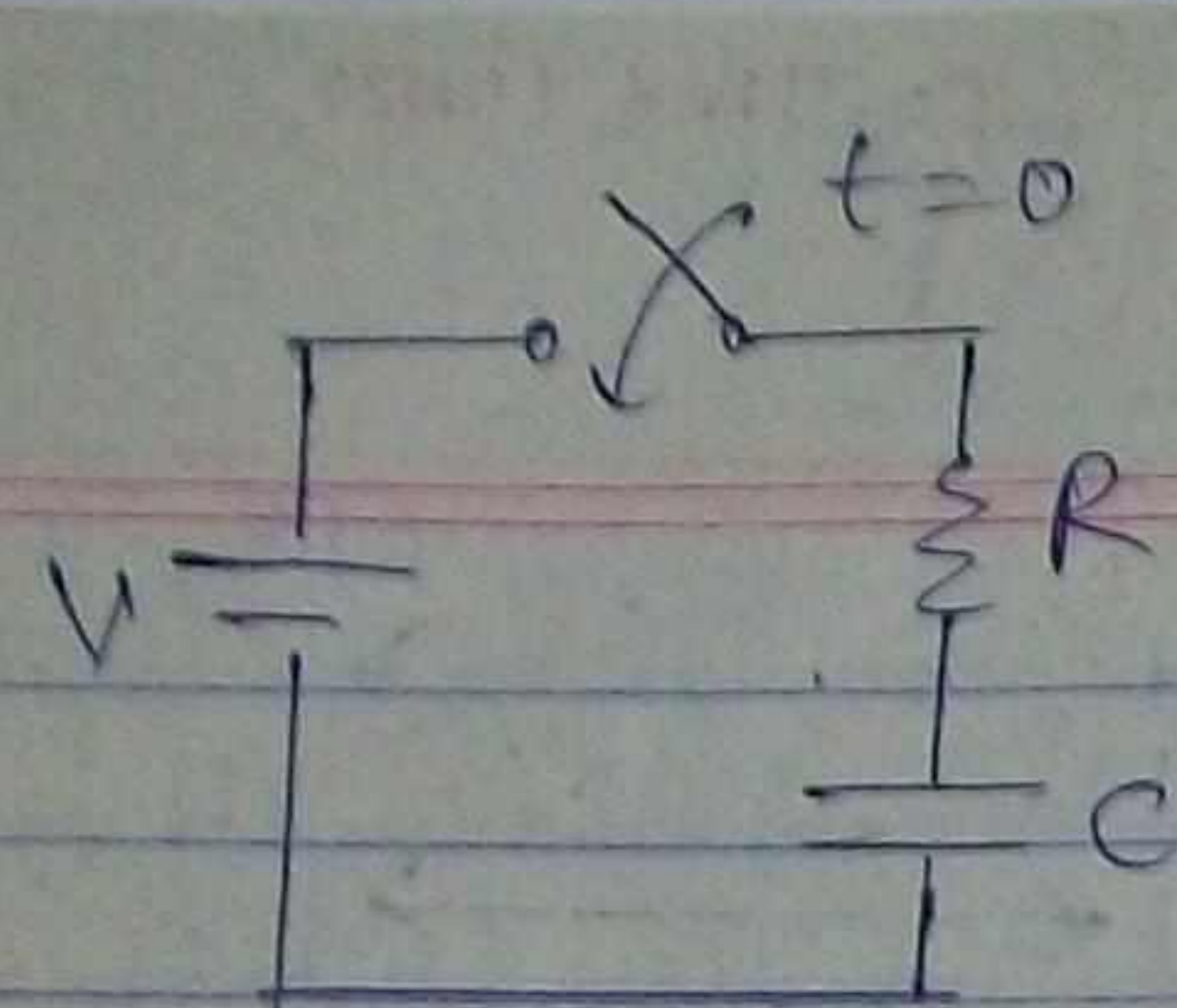
$$t = \infty \quad V_L = L \frac{di}{dt} = 0 \quad (\text{S.C.})$$

$$i = \frac{V}{R}$$

$$t = 0^- \quad i = I_0$$

$$t = 0^+ \quad i = I_0 \quad (\text{CURRENT SOURCE})$$

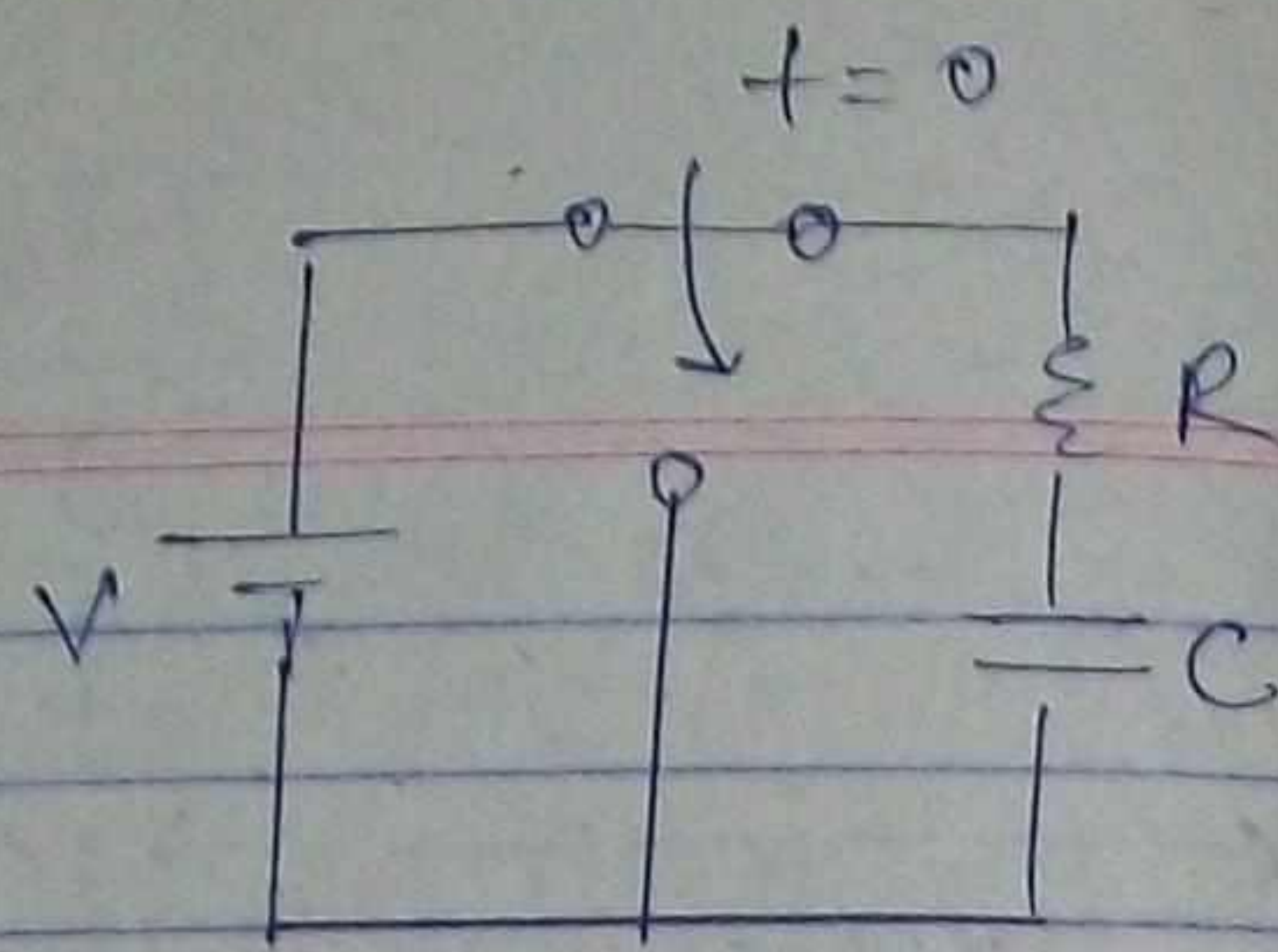
$$t = \infty \quad i = 0$$



$$t=0^-, V_C = 0$$

$$t=0^+, V_C = 0 \quad (\text{S.C.})$$

$$t=\infty, V_C = V, i=0 \quad (\text{O.C.})$$

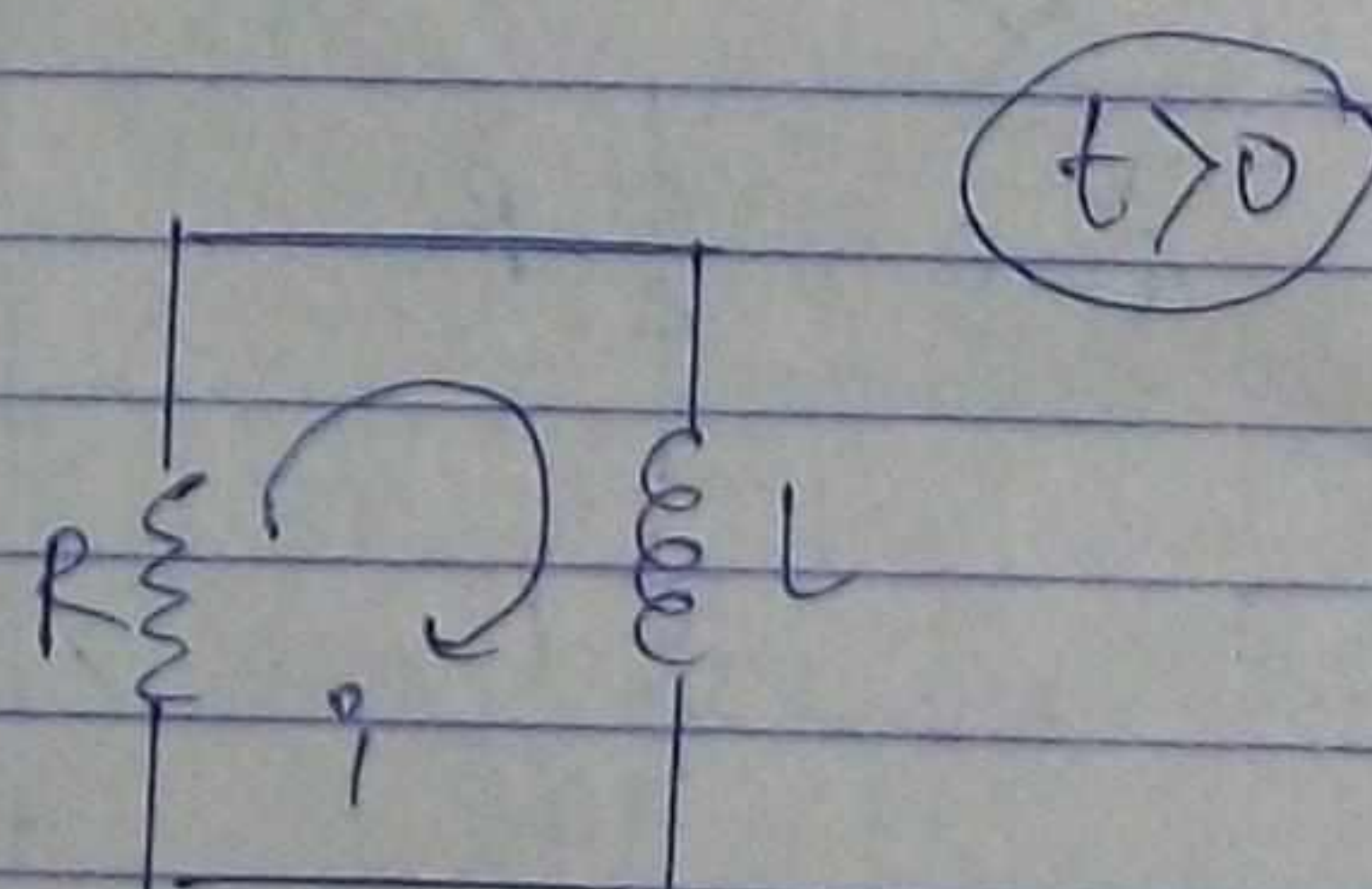
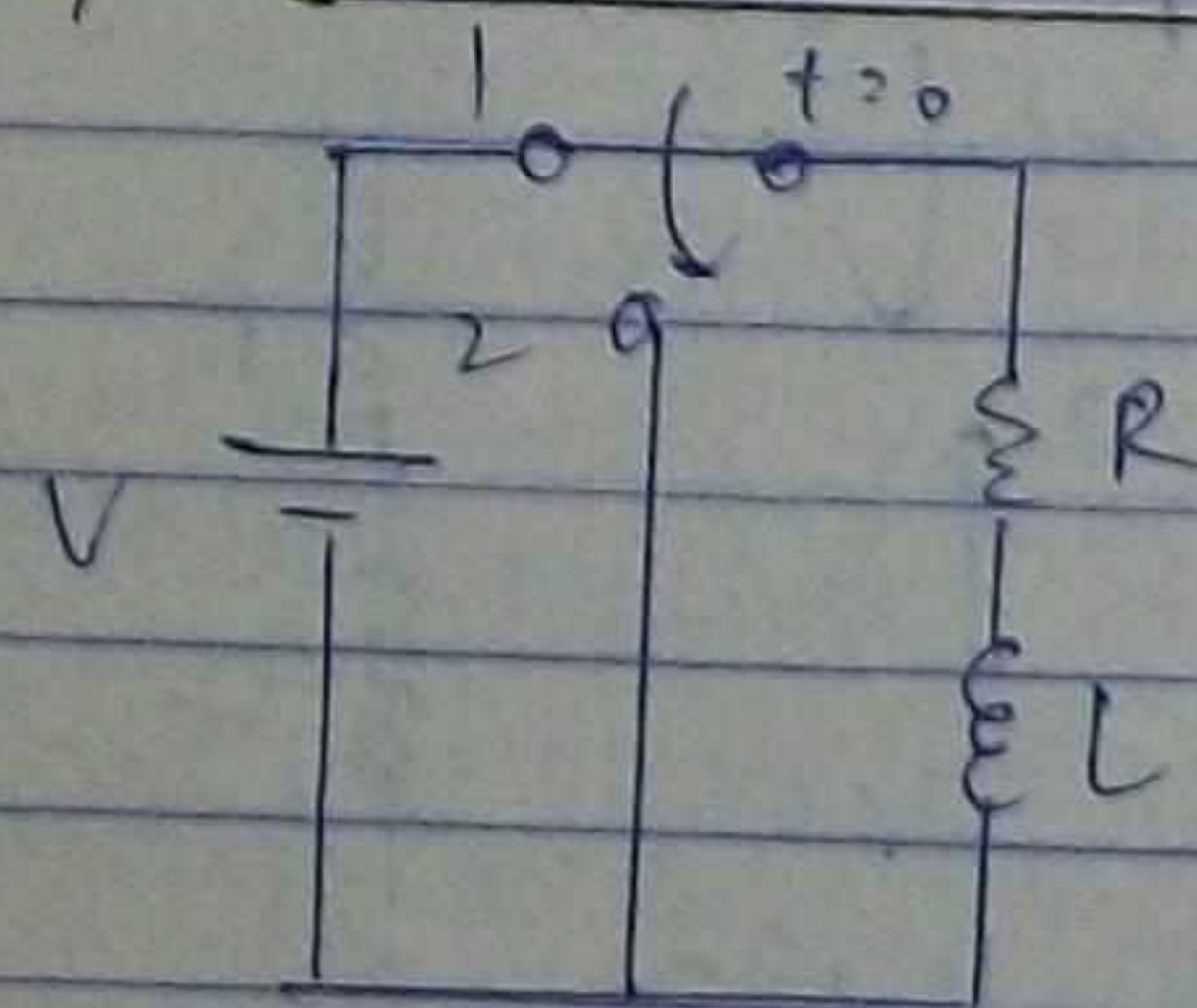


$$t=0^-, V_L = V_0$$

$$t=0^+, V_L = V_0 \quad (\text{VOLTAGE SOURCE})$$

$$t=\infty, V_L = 0$$

⇒ SOURCE FREE R-L CIRCUIT -



$$t \neq 0^-, i = I_0$$

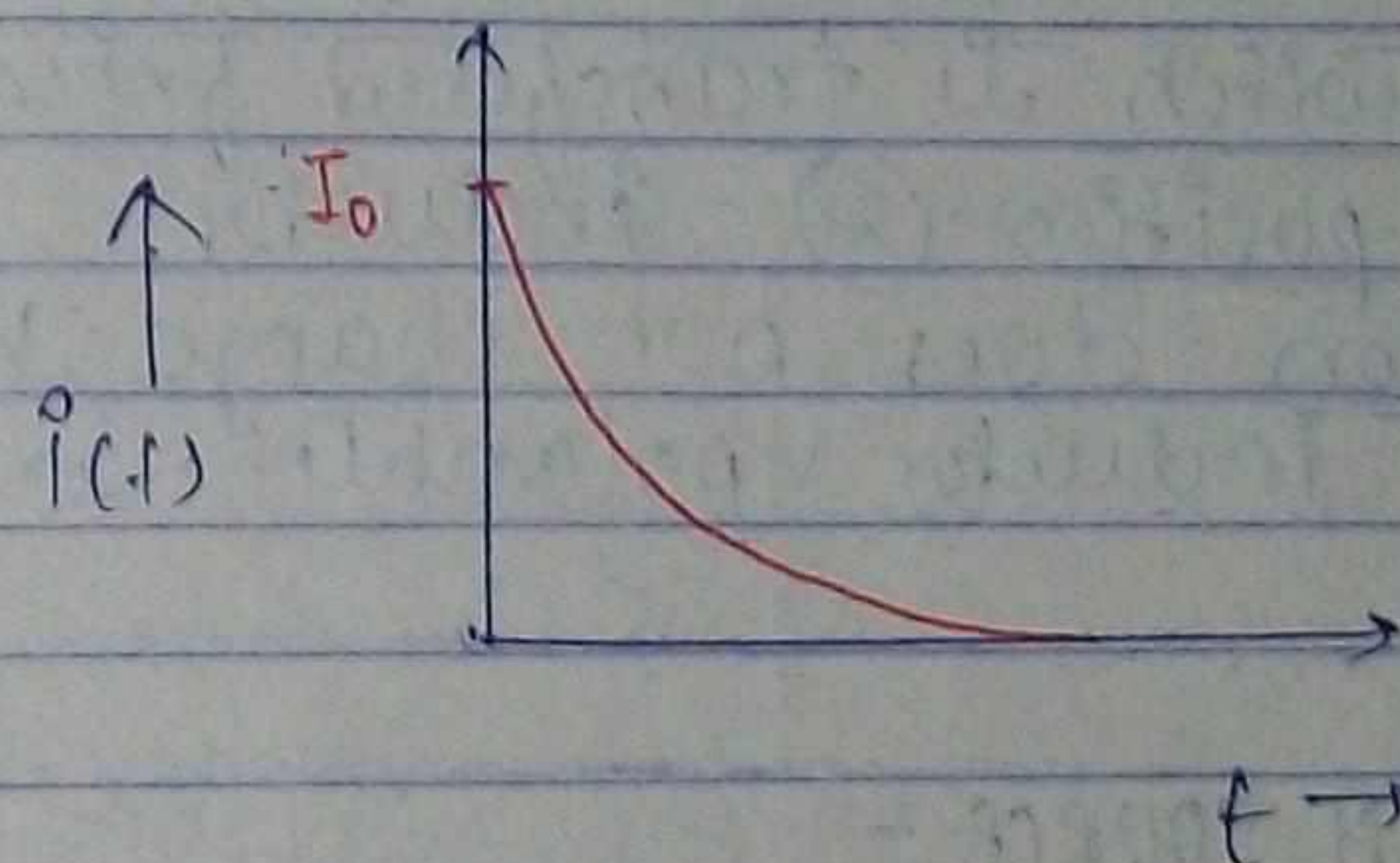
$$t=0^+, i = I_0$$

$$\text{at } t>0, \quad iR + L \frac{di}{dt} = 0$$

$$iR = -L \frac{di}{dt}$$

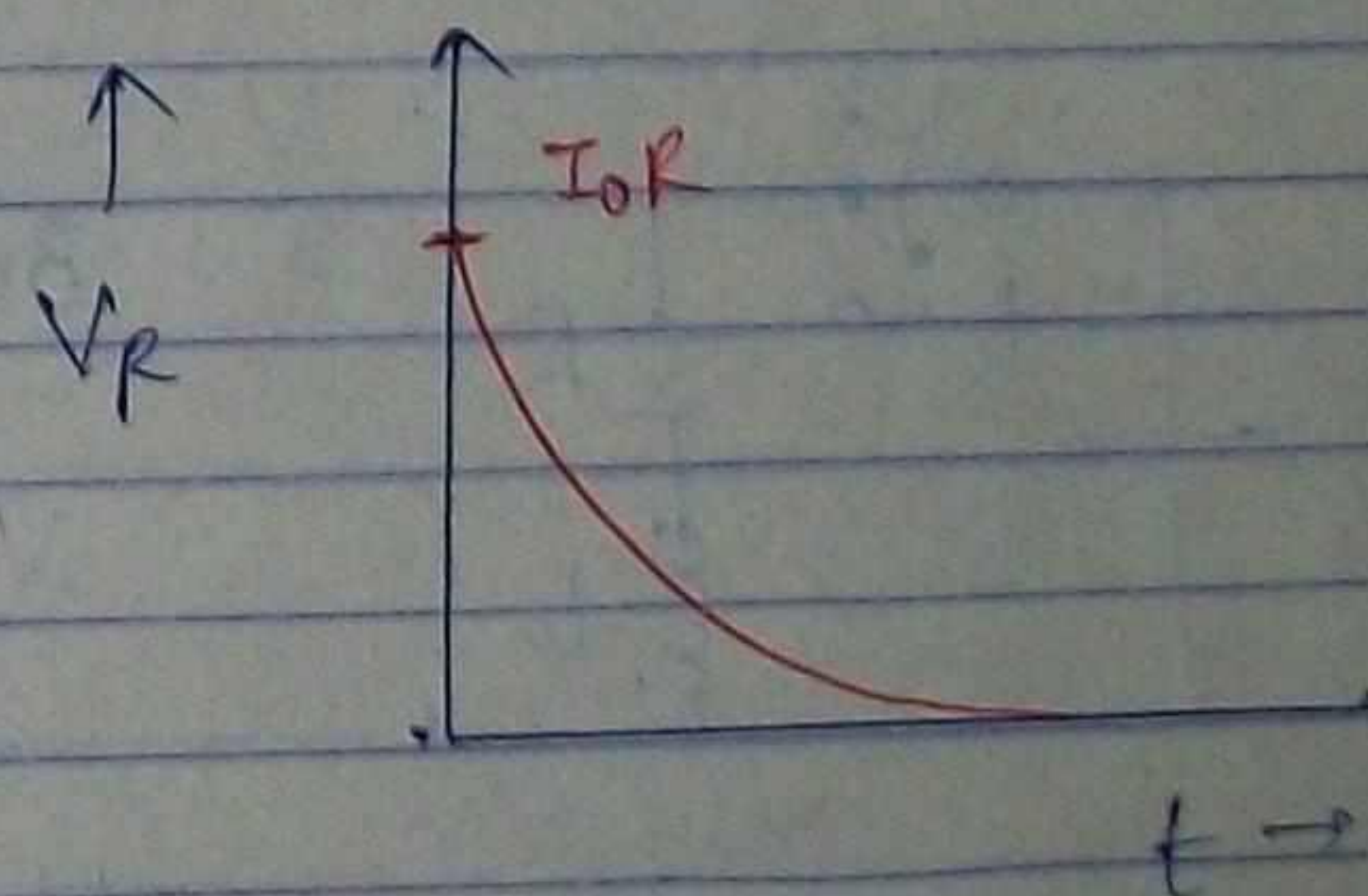
$$\int_0^t -\frac{R}{L} dt = \int_0^t \frac{di}{i}$$

$$i(t) = I_0 e^{-\frac{Rt}{L}}$$



$$V_R = iR$$

$$V_R = I_0 R e^{-\frac{Rt}{L}}$$

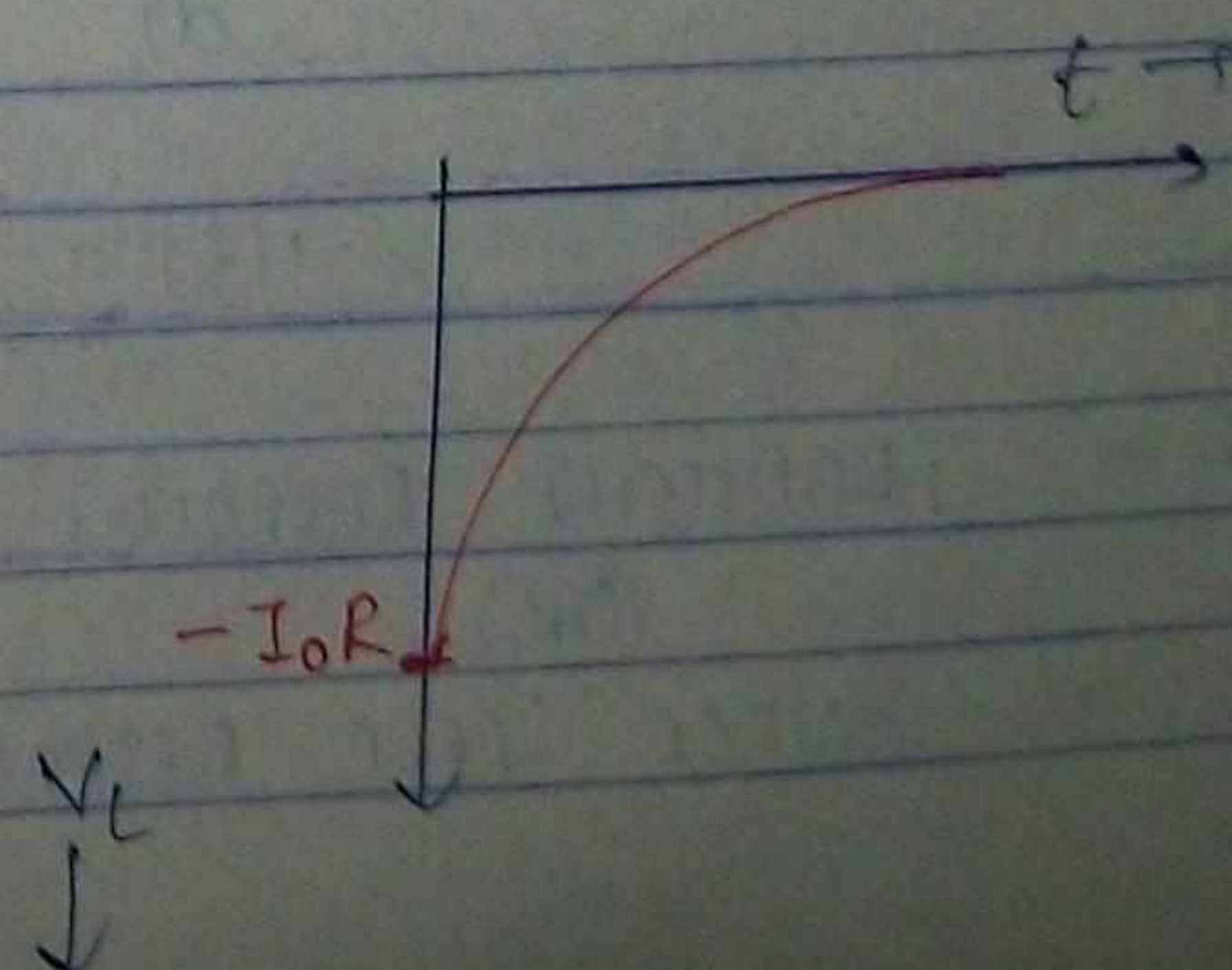


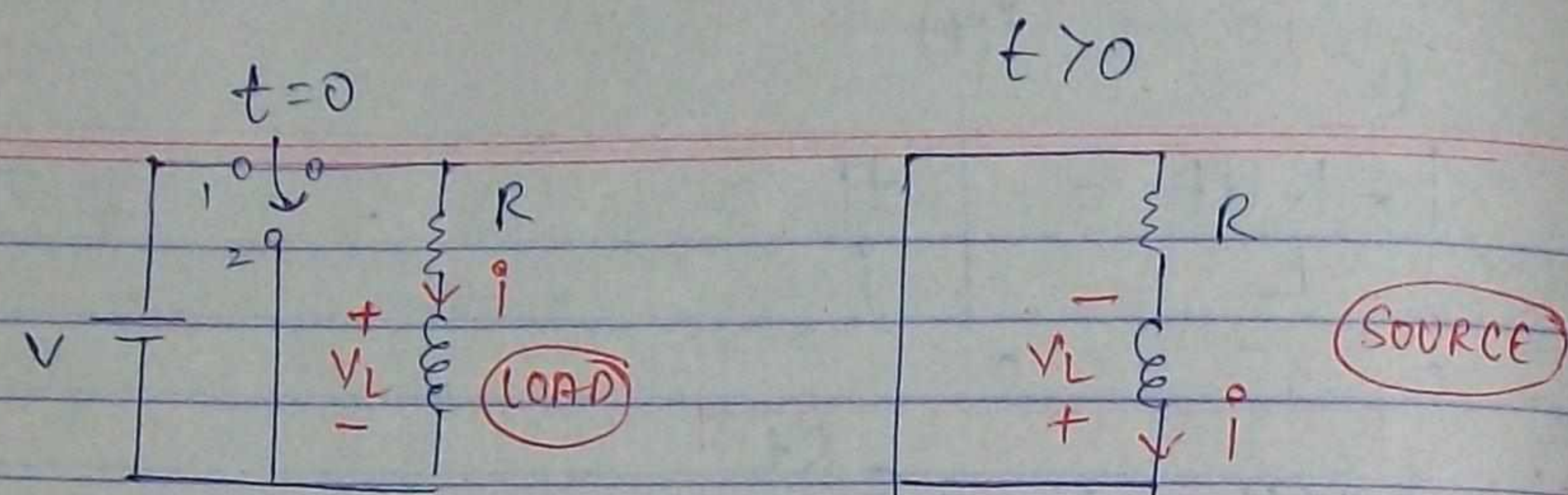
$$V_L = L \frac{di}{dt}$$

$$= L \frac{d(I_0 e^{-\frac{Rt}{L}})}{dt}$$

$$= -I_0 R e^{-\frac{Rt}{L}}$$

$$V_L = -I_0 R e^{-\frac{Rt}{L}}$$

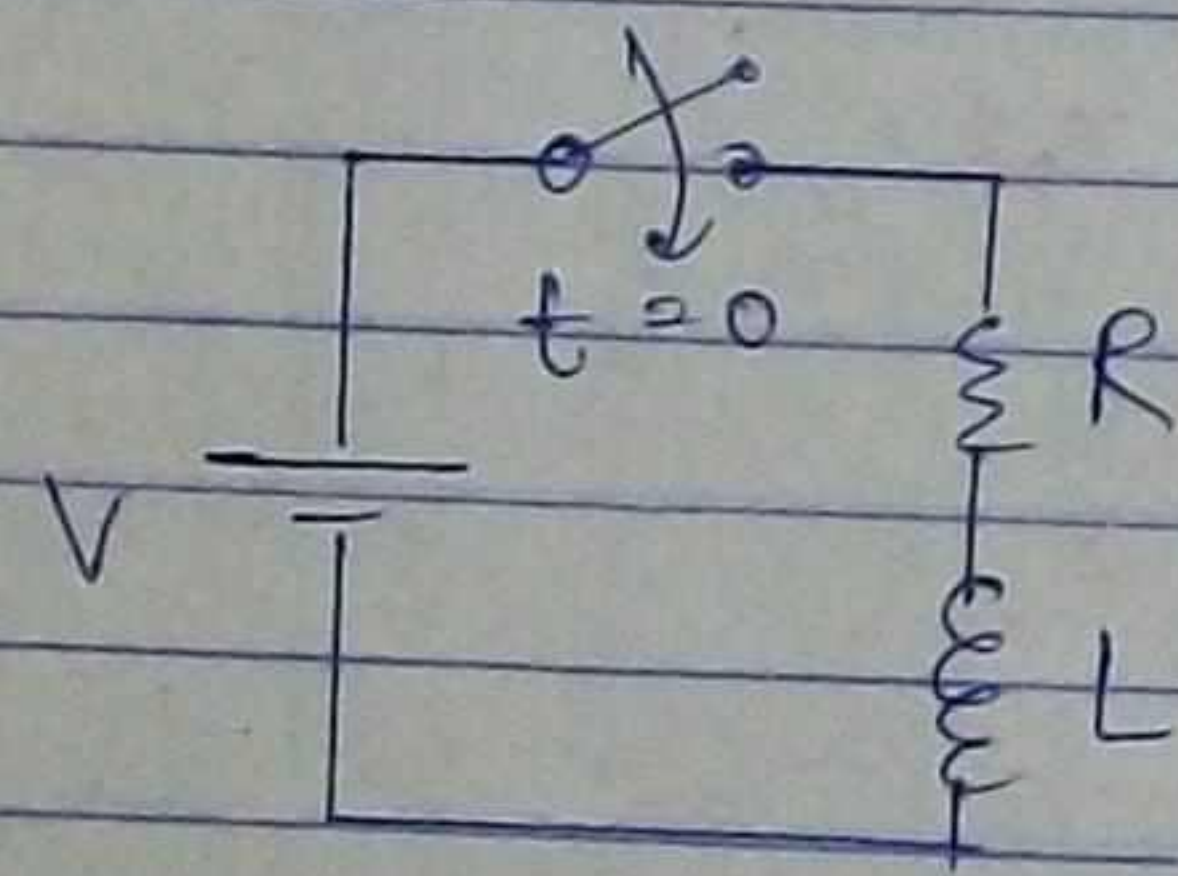




NOTE -

When the switch is transferred from position (1) to position (2) inductor current direction does not change but voltage across inductor polarities are reversed.

⇒ RL CIRCUIT WITH SOURCE -



By KVL at $t=0$

$$V = iR + L \frac{di}{dt}$$

$$\frac{di}{dt} + \frac{R}{L} i = \frac{V}{L}$$

$$i(t) = Cf + PI$$

Cf → TRANSIENT RESPONSE

(OR)

SOURCE FREE RESPONSE

$$\frac{di}{dt} + \frac{R}{L} i = 0$$

$$i(t) = A e^{-\frac{R}{L}t}$$

PI → STEADY STATE RESPONSE

(OR)

FINAL VALUE

$$i(t) = \frac{V}{R} \quad [t=\infty]$$

Now, $i(t) = Cf + PI$

$$i(t) = A e^{-R/Lt} + \frac{V}{R}$$

$$t=0^-, i=0$$

$$t=0^+, i=0$$

$$0 = A + V/R$$

$$A = 0 - \frac{V}{R}$$

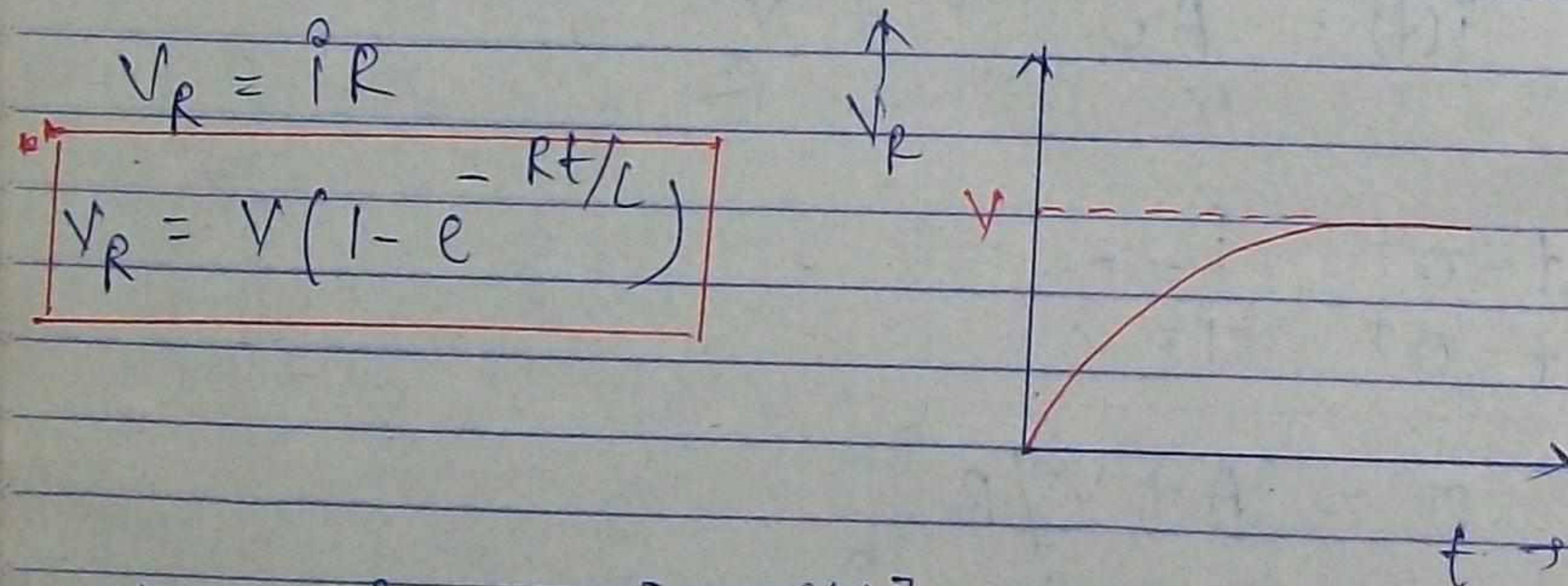
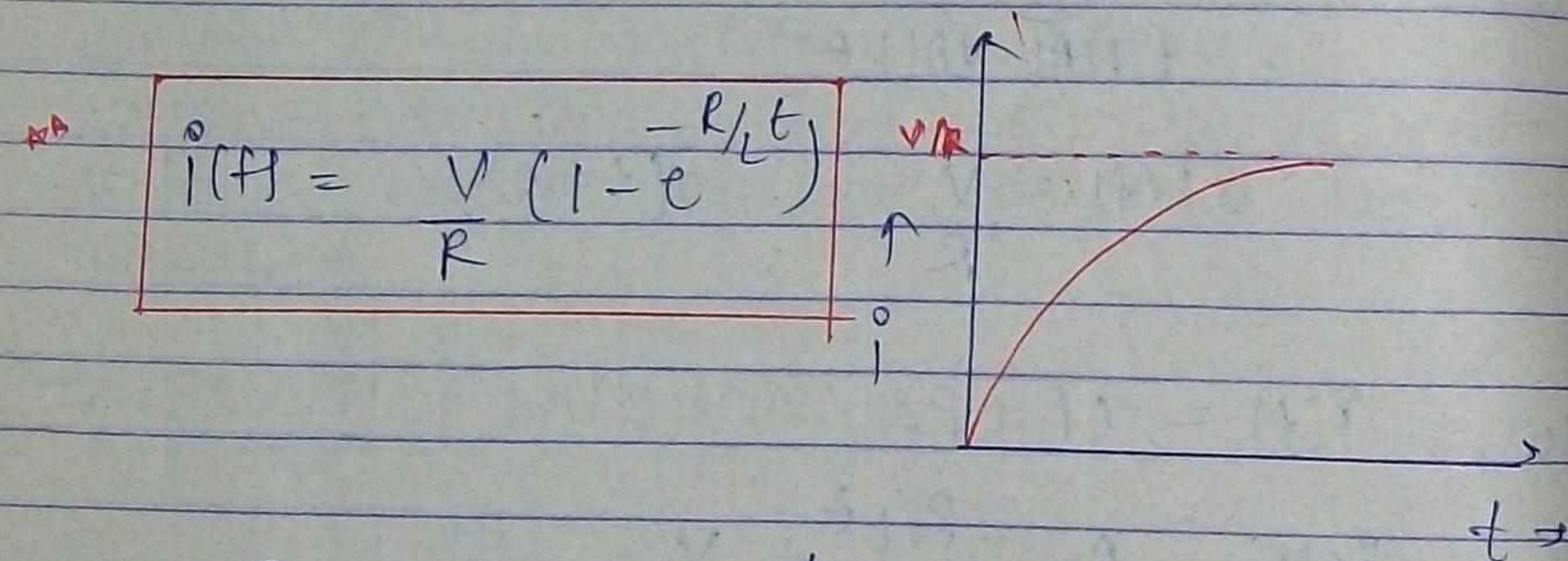
(*)

$$A = i(0^+) - i(\infty)$$

INITIAL - FINAL

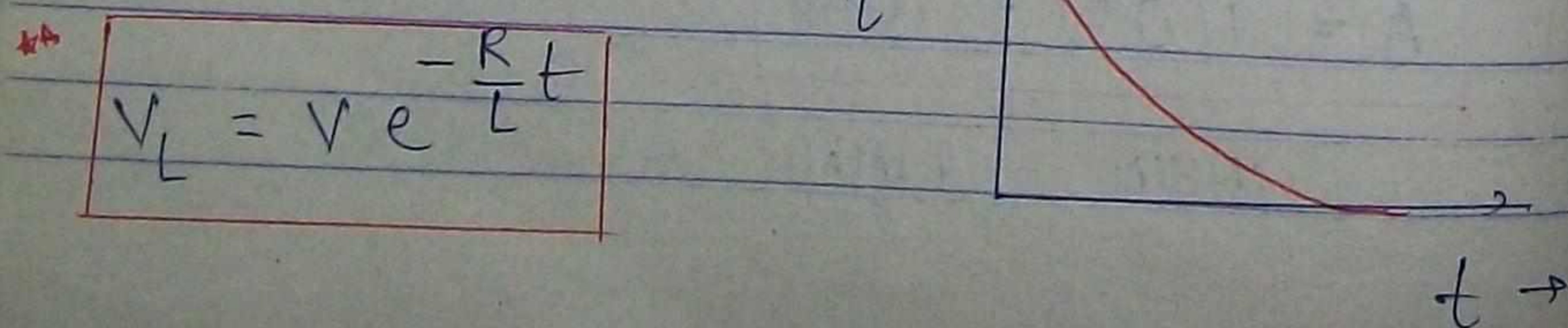
$$i(t) = -\frac{V}{R} e^{-\frac{R}{L}t} + \frac{V}{R}$$

$$i(t) = \left[\underset{\substack{\uparrow \\ \text{INITIAL} \\ \text{VALUE}}} i(0^+) - \underset{\substack{\uparrow \\ \text{FINAL} \\ \text{VALUE}}} i(\infty) \right] e^{-\frac{R}{L}t} + \underset{\substack{\uparrow \\ \text{FINAL} \\ \text{VALUE}}} i(\infty)$$

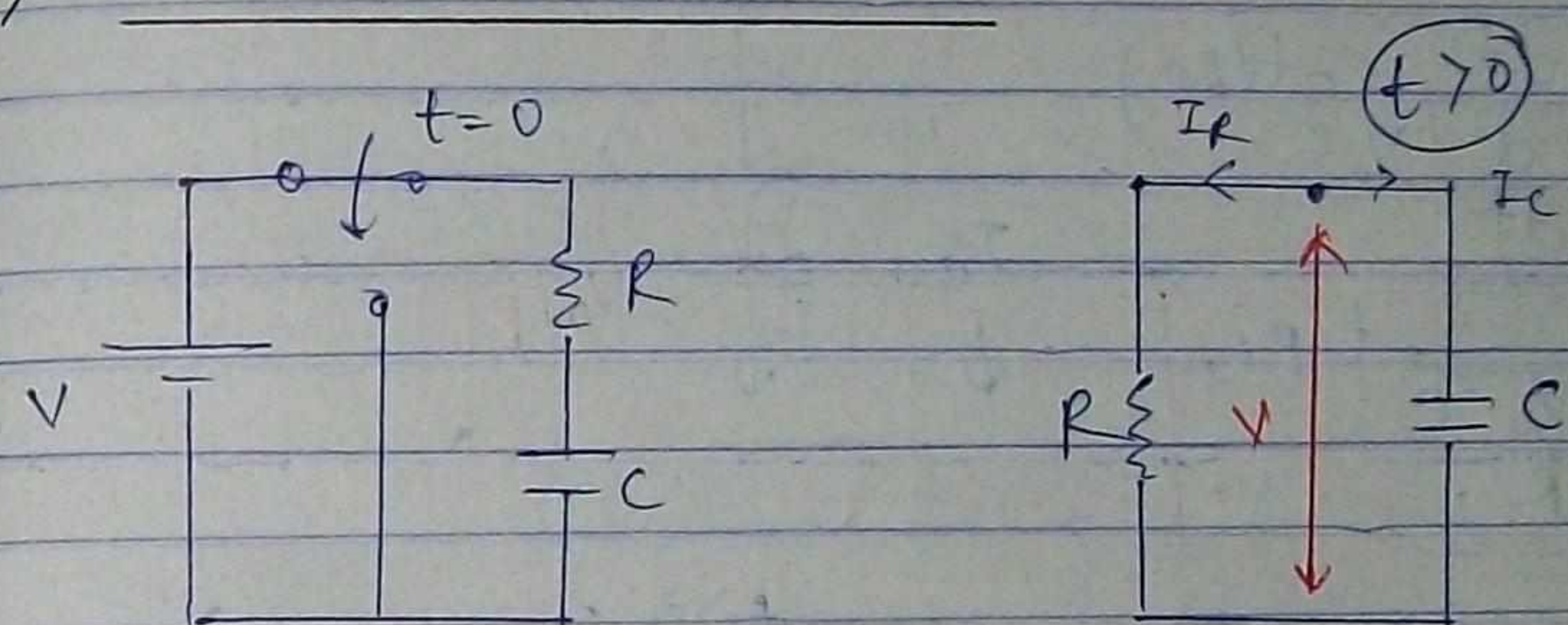


$$V_L = L \frac{di}{dt} = L \frac{d}{dt} \left[\frac{V}{R} (1 - e^{-\frac{R}{L}t}) \right]$$

$$= V e^{-\frac{R}{L}t}$$



⇒ SOURCE FREE RC CIRCUIT -



$$t=0^-, V_C = V_0$$

$$t=0^+, V_C = V_0$$

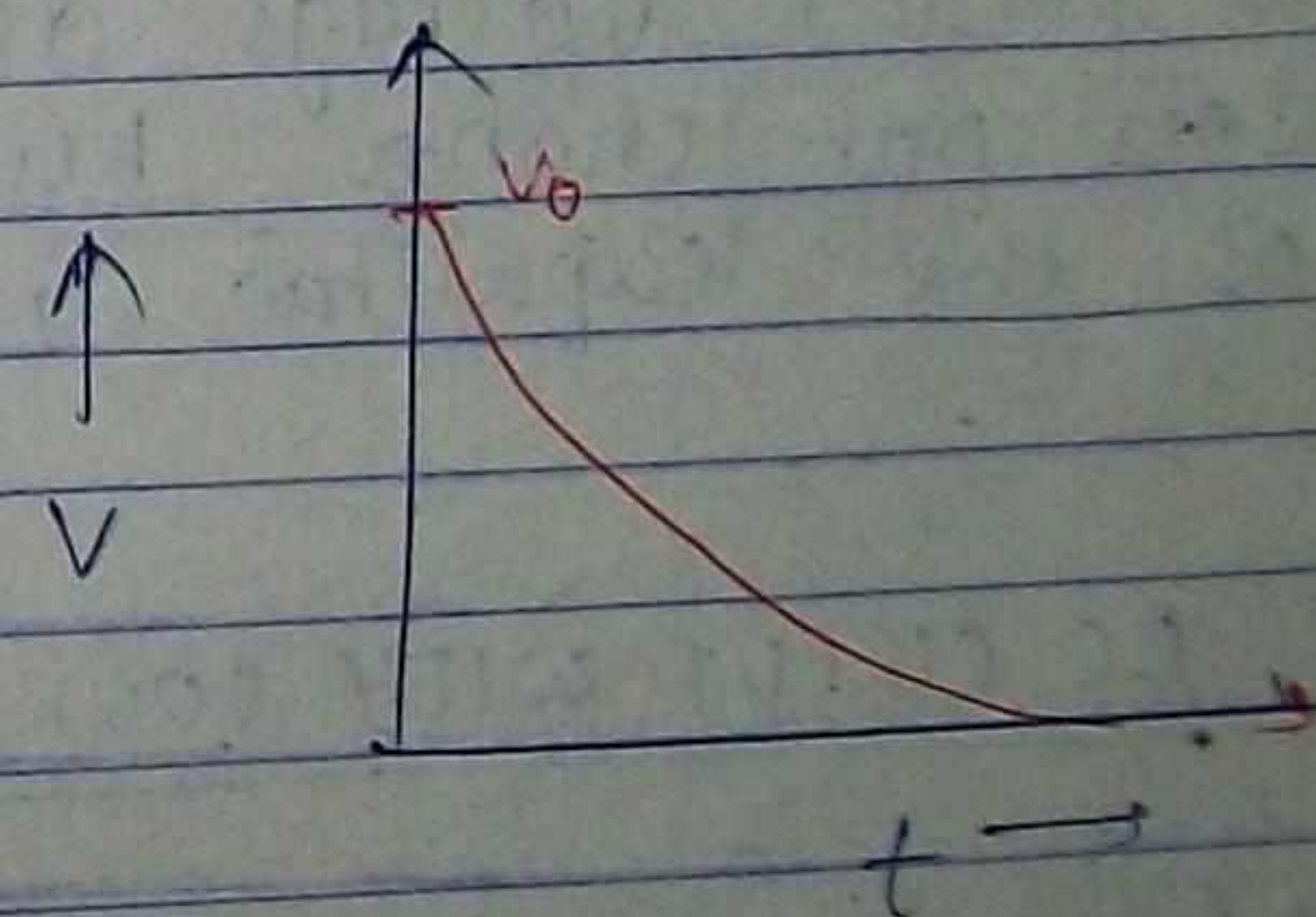
$$I_R + I_C = 0$$

$$\frac{V}{R} + C \frac{dV}{dt} = 0$$

$$\int_0^t -\frac{1}{RC} dt = \int_{V_0}^{V(t)} \frac{dV}{V}$$

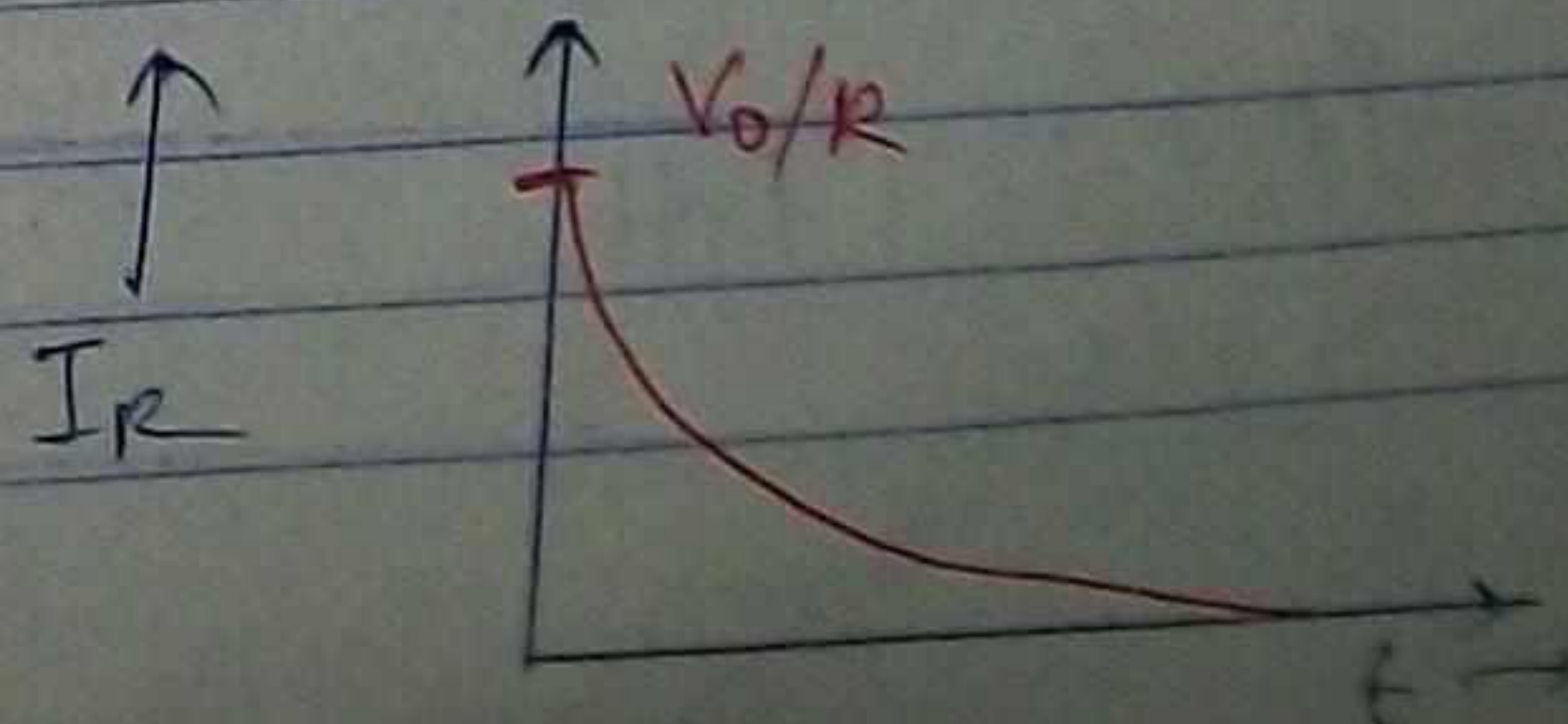
On solving:

$$V(t) = V_0 e^{-t/RC}$$



$$I_R = \frac{V}{R}$$

$$I_R = \frac{V_0}{R} e^{-t/RC}$$

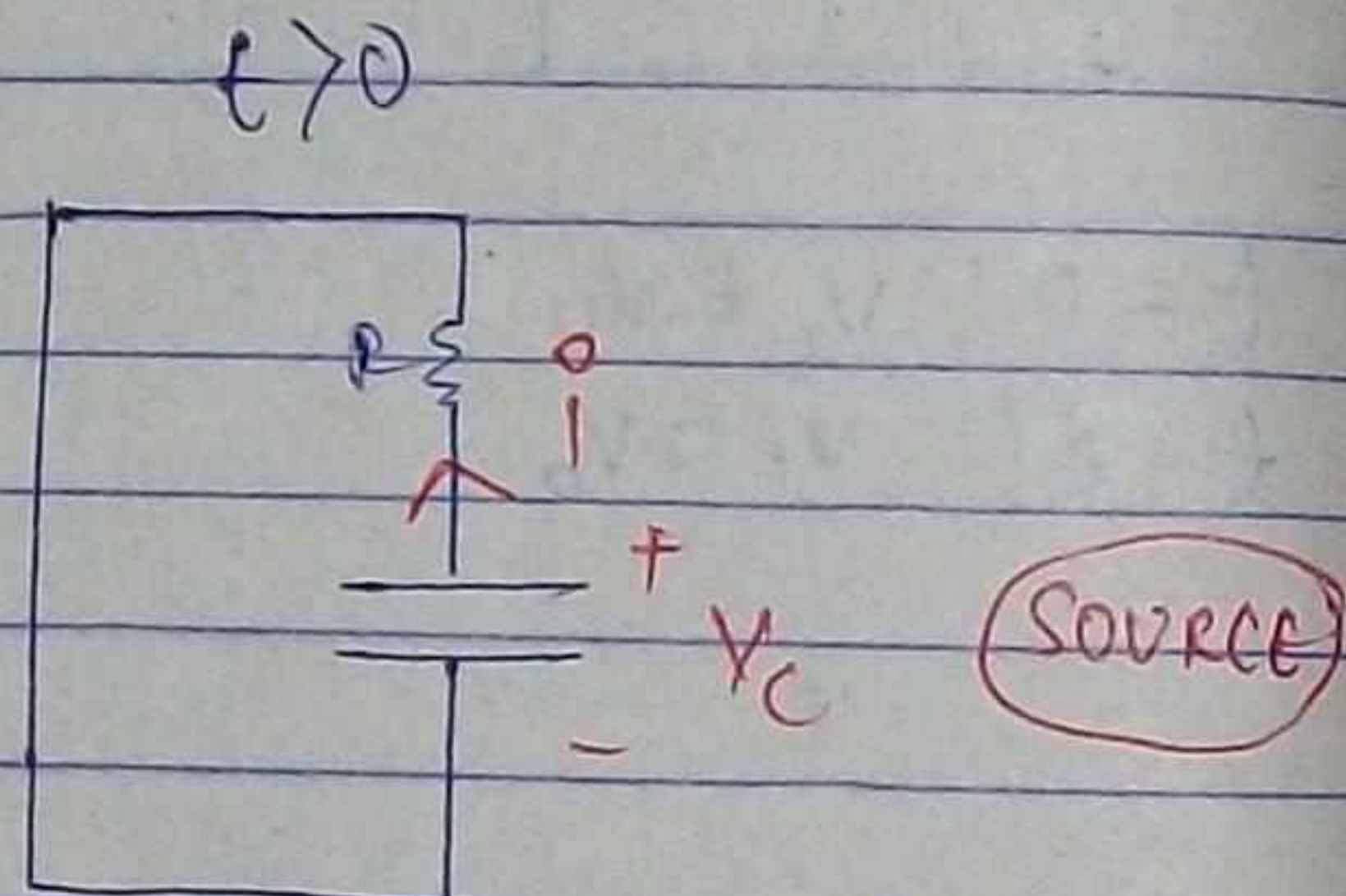
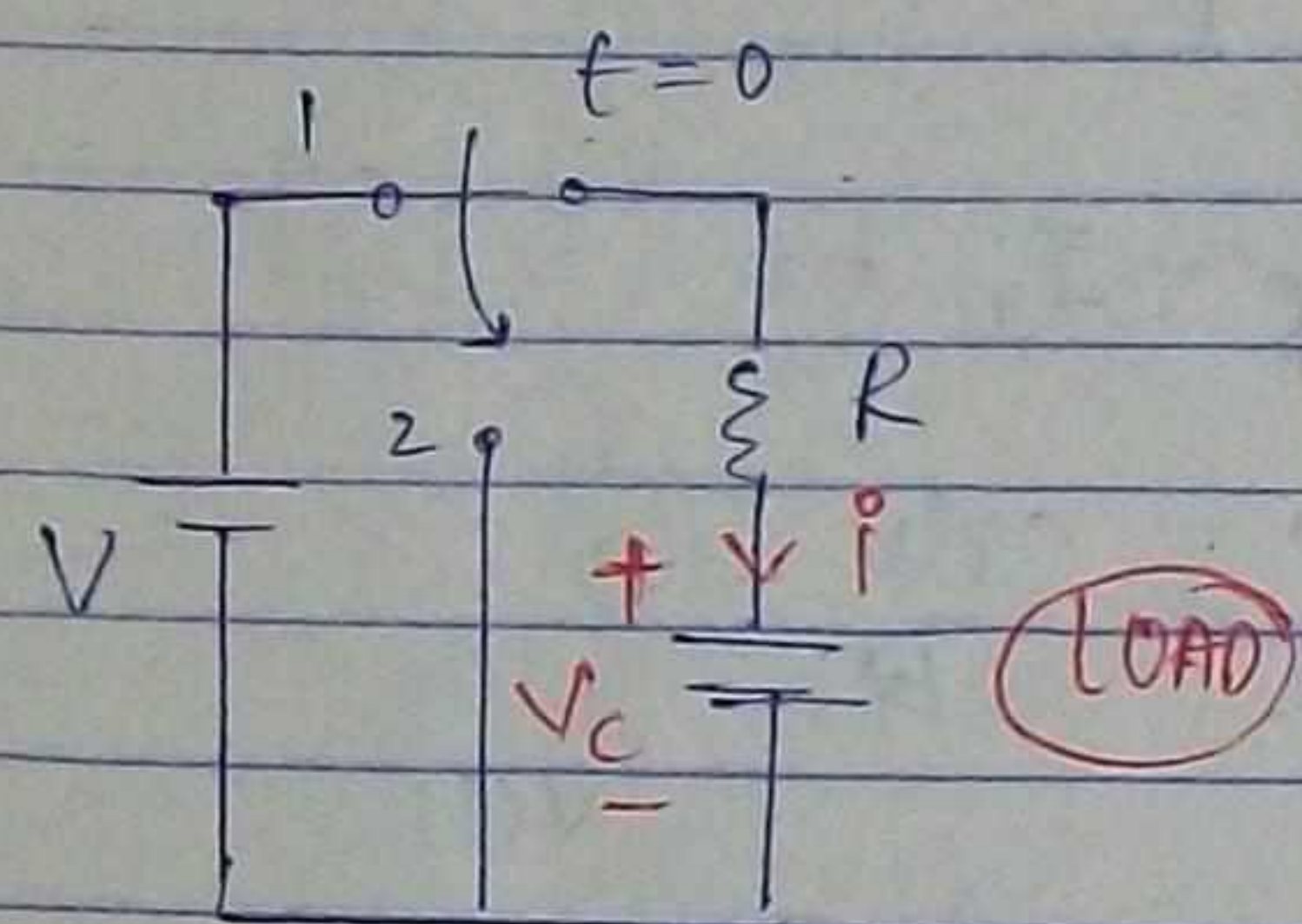
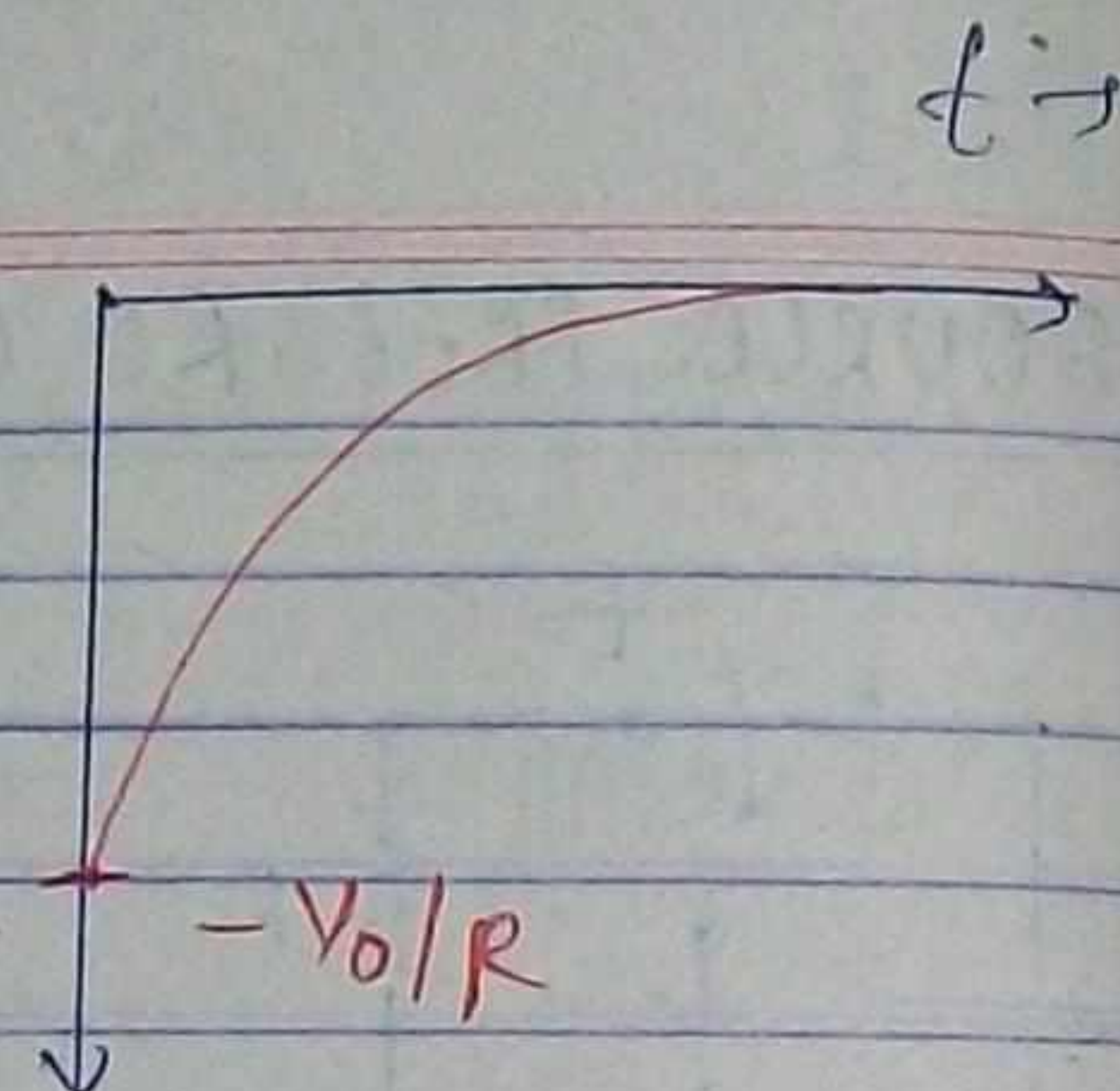


$$I_c = C \frac{dv}{dt} \quad (-t/RC)$$

$$= C \frac{d}{dt} (V_0 e^{-t/RC})$$

$$I_c = -\frac{V_0}{R} e^{-t/RC}$$

I_c
↓



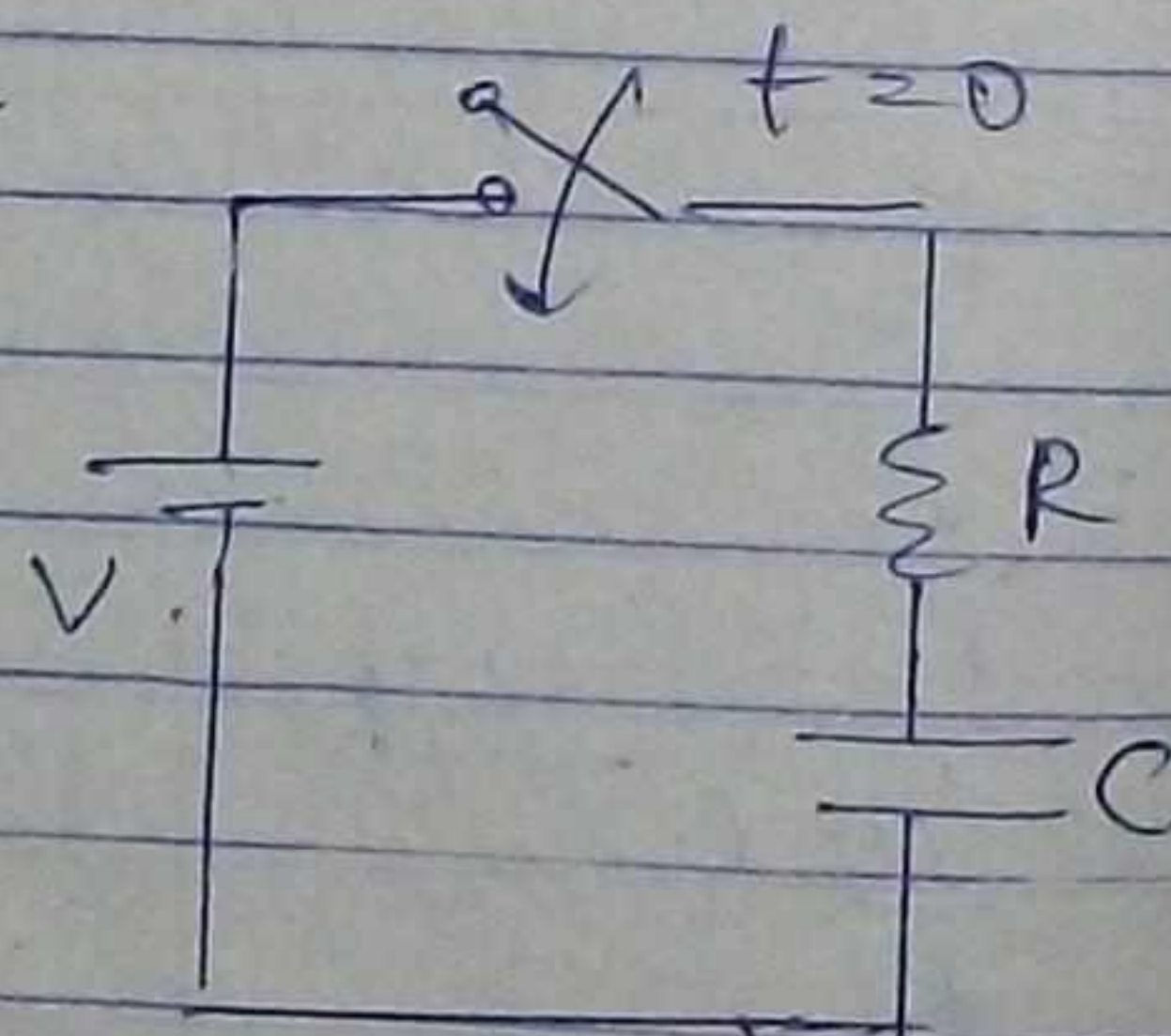
NOTE -

When switch is changed from pos (1) to pos (2) voltage across capacitor polarity does not change but current direction of the capacitor is reversed.

⇒ RC CIRCUIT WITH SOURCE -

By KVL,

$$V = iR + \frac{1}{C} \int i dt$$



diff. wrt t $0 = R \frac{di}{dt} + \frac{i}{C}$

$$\frac{di}{dt} + \frac{i}{RC} = 0$$

$$i(t) = C.F + P.I.$$

C.F → Transient response.

$$\frac{di}{dt} + \frac{i}{RC} = 0$$

$$i(t) = A e^{-t/RC}$$

$$A = i(0^+) - i(\infty)$$

$$A = \frac{V}{R} - 0$$

P.I → STEADY STATE RESPONSE -

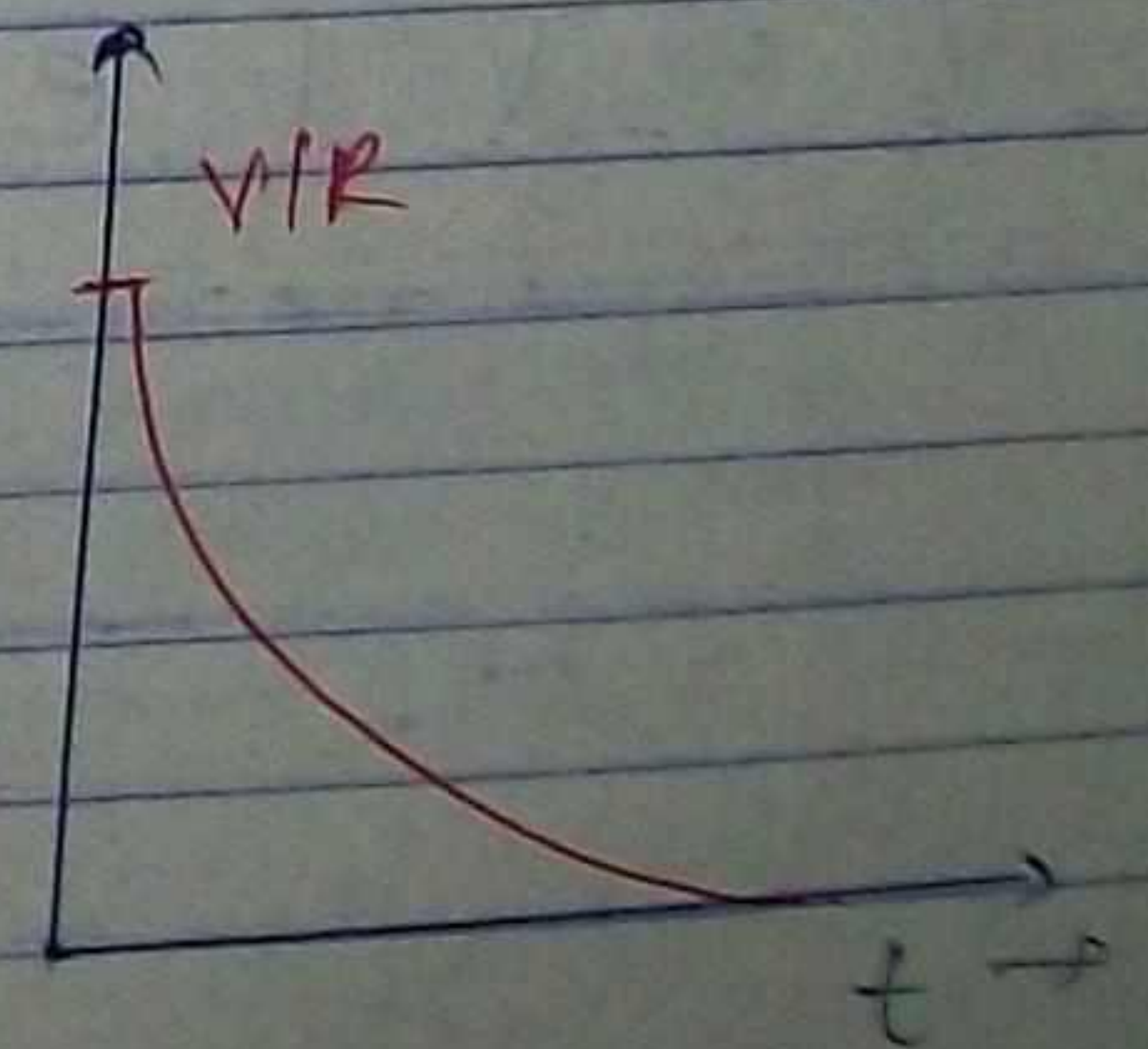
$$C \rightarrow 0, i(\infty) = 0.$$

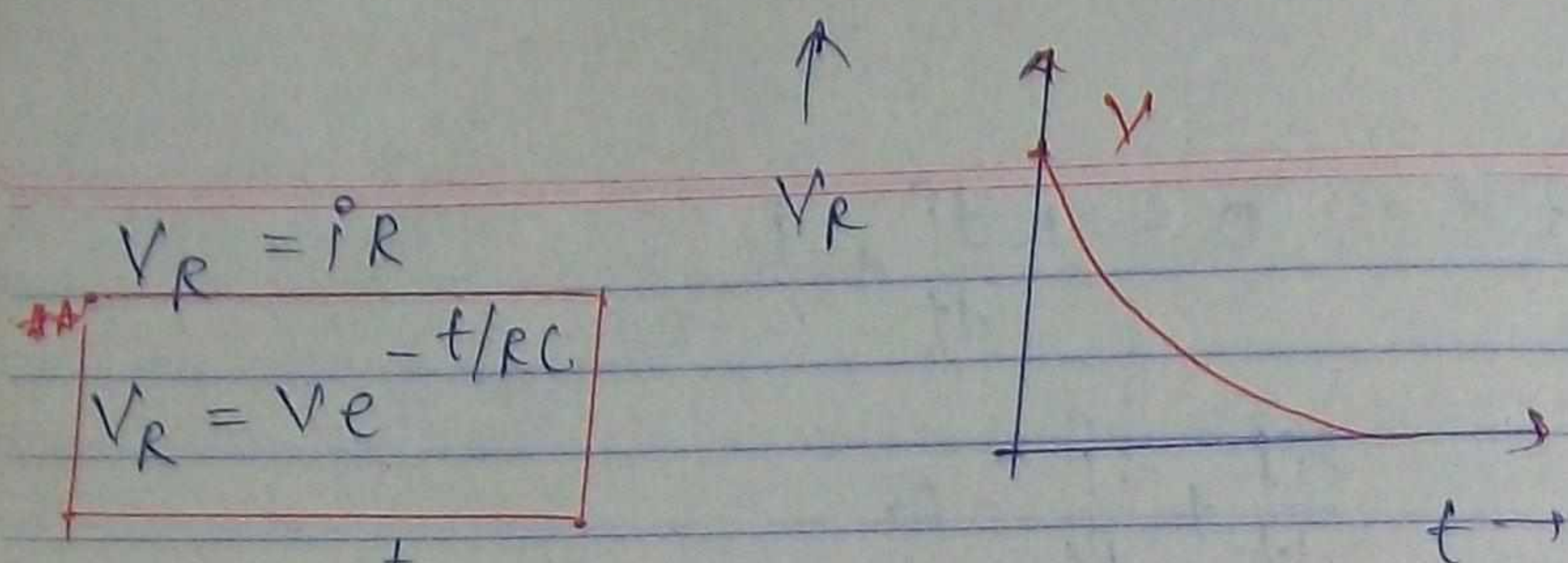
$$i(t) = C.F + P.I$$

$$i(t) = A e^{-t/RC} + 0$$

$$i(t) = \frac{V}{R} e^{-t/RC}$$

$i(t)$
↑





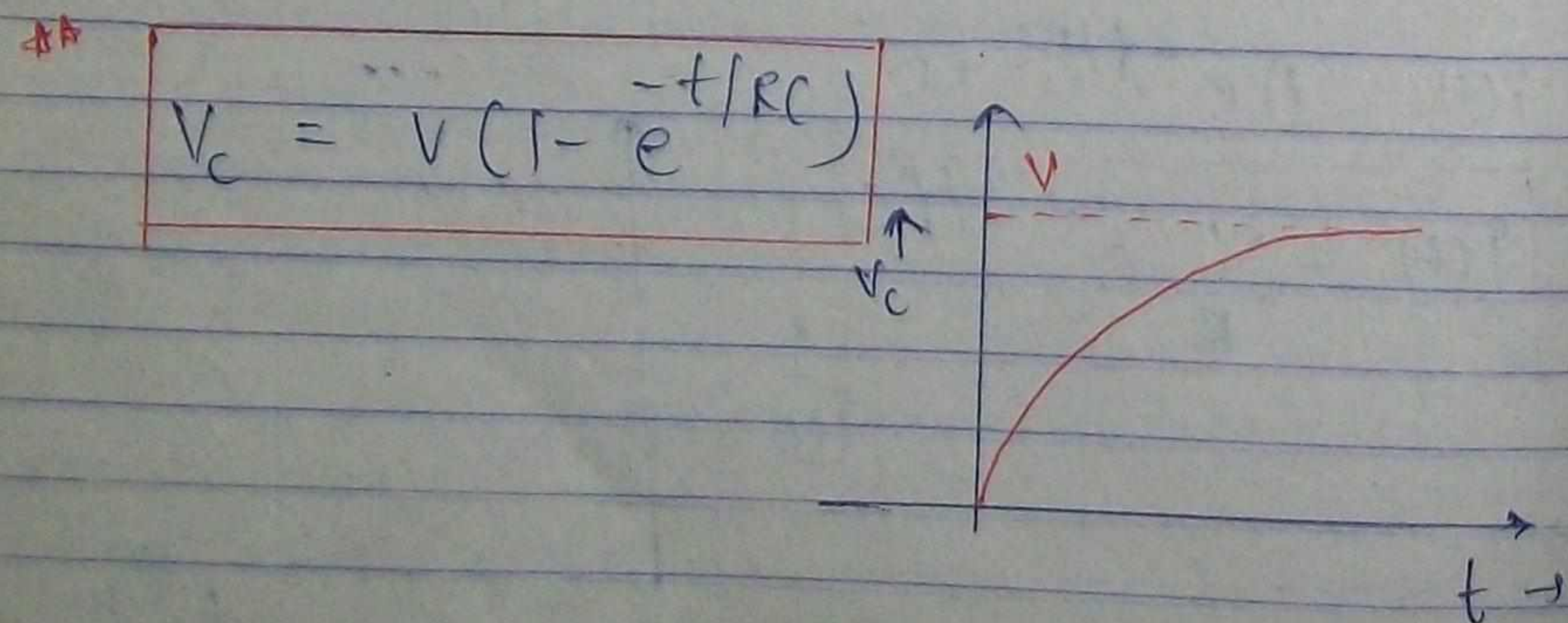
$$V_C = \frac{1}{C} \int i dt$$

$$V_C = \frac{1}{C} \int_0^t \frac{V}{R} e^{-t/RC} dt$$

$$V_C = -V e^{-t/RC} + V$$

Valid for DC
source and
RL or RC
circuit only

$$V_C(t) = \left[\underset{\substack{\downarrow \\ \text{INITIAL} \\ \text{VALUE}}}{V_C(0^+)} - \underset{\substack{\downarrow \\ \text{FINAL} \\ \text{VALUE}}}{V_C(\infty)} \right] e^{-t/RC} + \underset{\substack{\downarrow \\ \text{FINAL} \\ \text{VALUE}}}{V_C(\infty)}$$



⇒ TIME CONSTANT -

- Time constant can be defined either w.r.t charging or discharging action of energy storage element

- Time constant is a time taken for response to rise 63.2% of the maximum value and is given by:

$$\tau = \frac{L}{R} \text{ sec}$$

$$\tau = RC \text{ sec.}$$

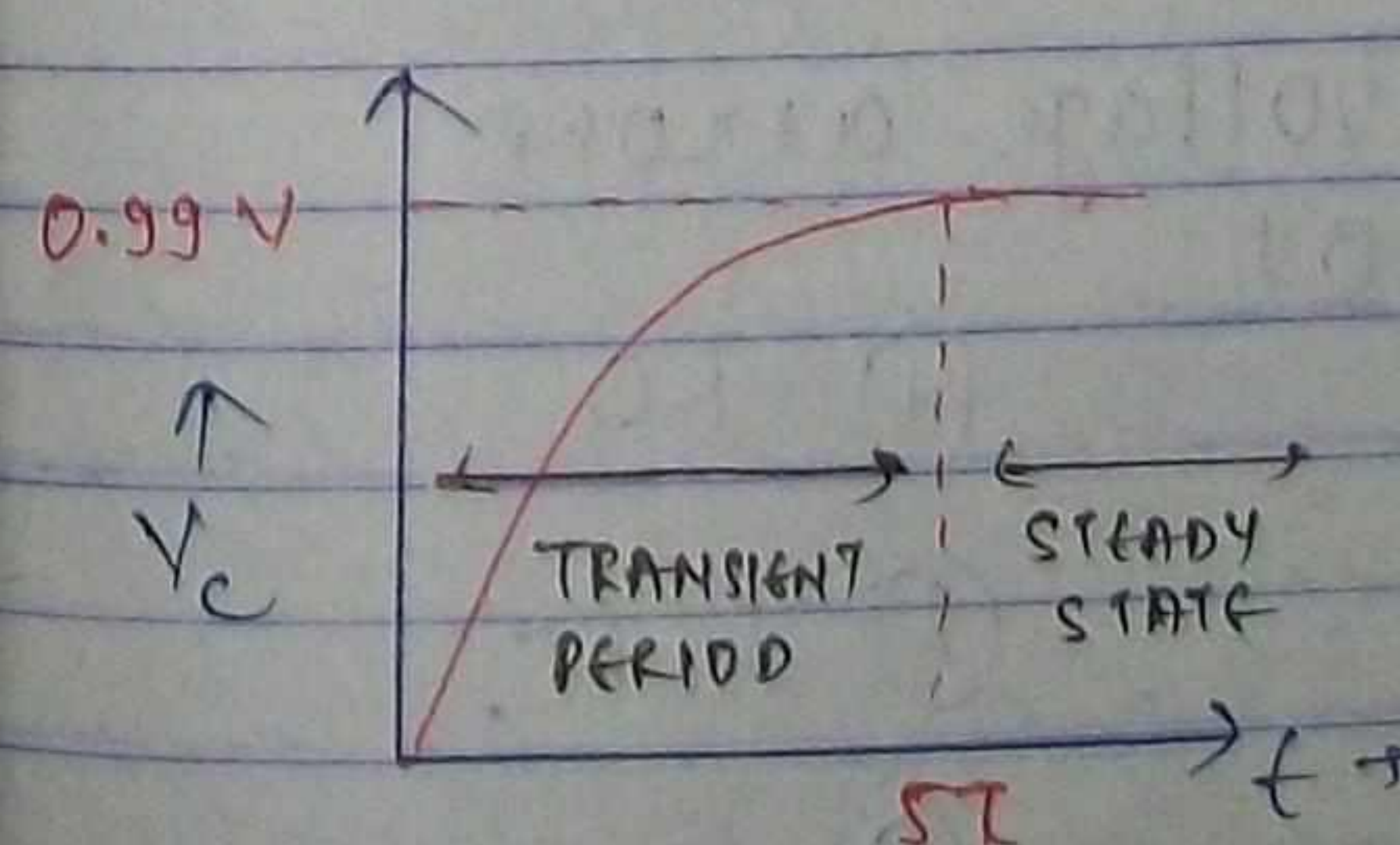
$$V_C = V(1 - e^{-t/\tau})$$

$$t = \tau \quad V_C = V(1 - e^{-1}) = 0.63V$$

⋮

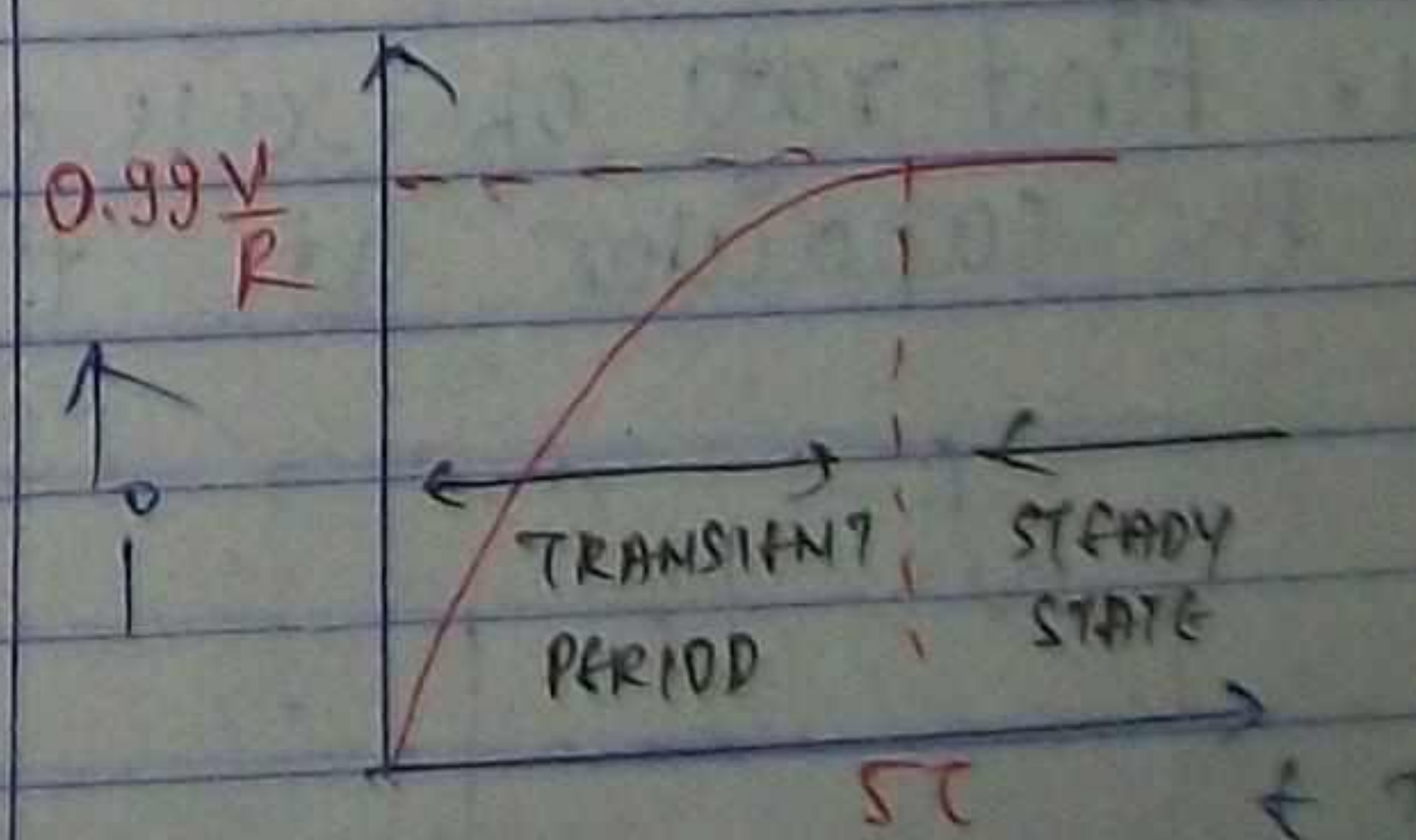
$$t = 5\tau \quad V_C = V(1 - e^{-5}) = 0.99V$$

RC WITH SOURCE



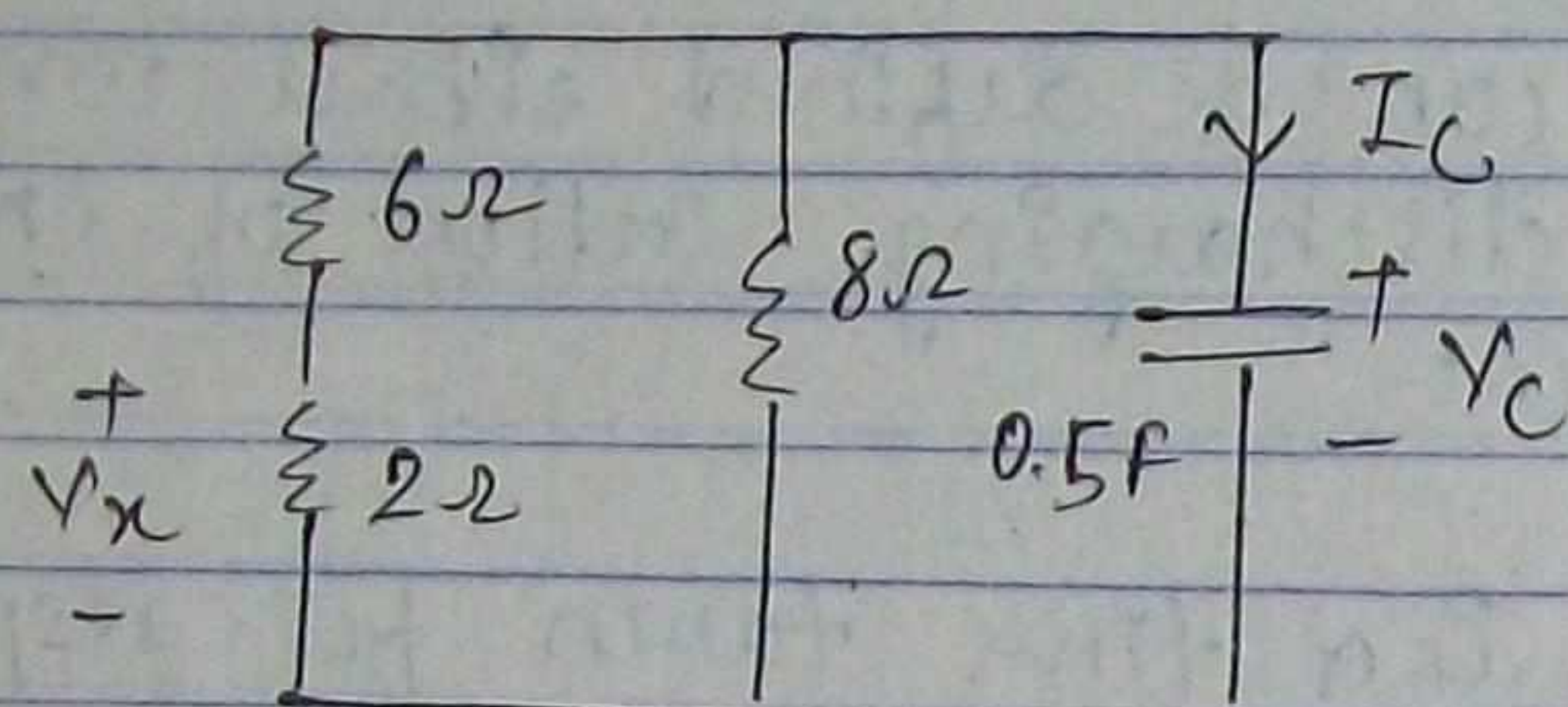
$$V_C = V(1 - e^{-t/RC})$$

RL WITH SOURCE

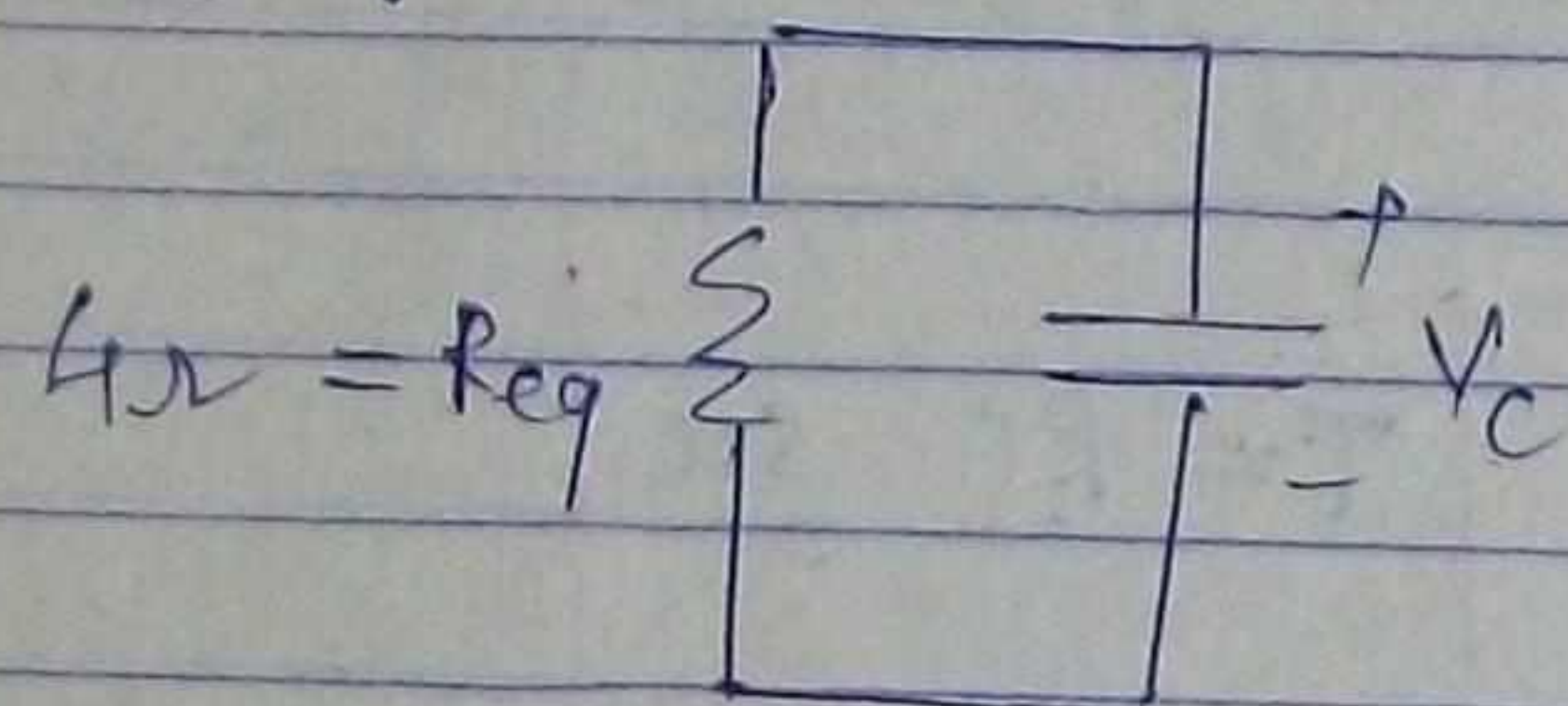


$$i(t) = \frac{V}{R}(1 - e^{-R/L t})$$

Ques. find V_c, I_c, V_x when $V_c(0^-) = 3V$



Soln:



$$V_c = V_0 e^{-t/\tau}$$

$$V_c = 3 e^{-t/2}$$

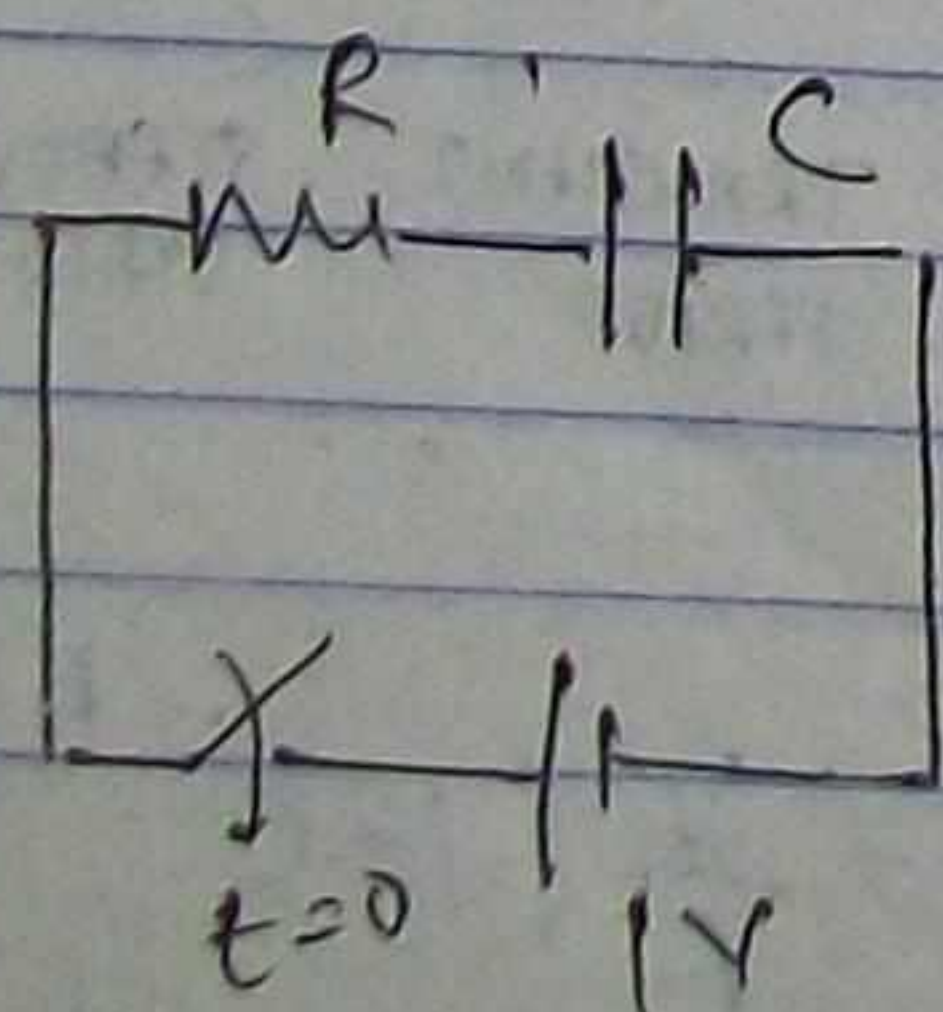
also, $V_x = \frac{2}{2+6} V_c$

$$V_x = \frac{3}{4} e^{-t/2}$$

$$I_c = C \frac{dV_c}{dt}$$

$$I_c = -\frac{3}{4} e^{-t/2}$$

Ques. find rate of rise of voltage across the capacitor at $t=0^+$



(1) RC

(2) $1/RC$

(3) 0

(4) 1V

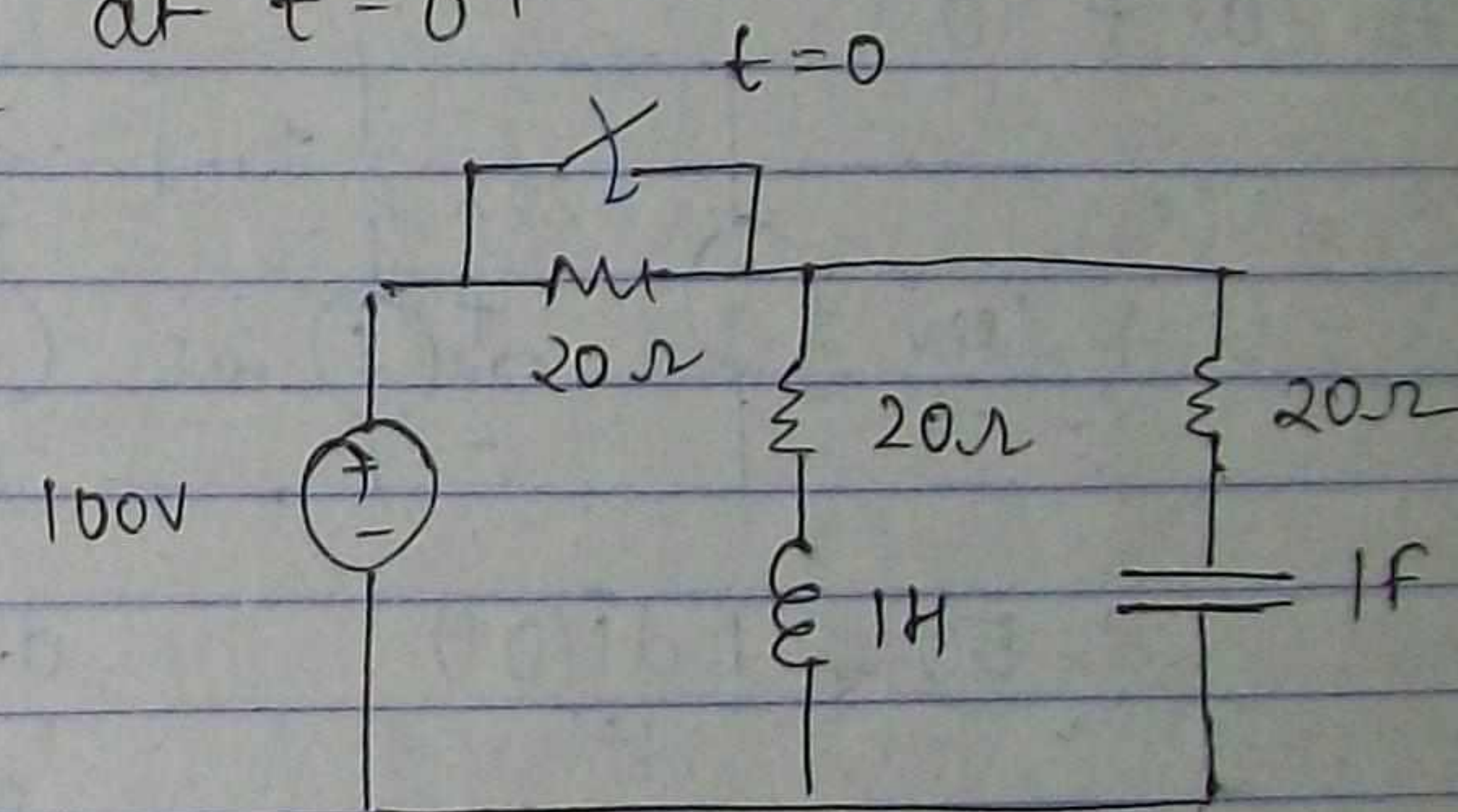
$$V_c = -V e^{-t/RC} + V$$

$$\frac{dV_c}{dt} = (-V) \left(\frac{-1}{RC} \right) e^{-t/RC}$$

$$\frac{dV_c}{dt} = \frac{V}{RC} e^{-t/RC}$$

$$\frac{dV_c(0^+)}{dt} = \frac{V}{RC} = \frac{1}{RC}$$

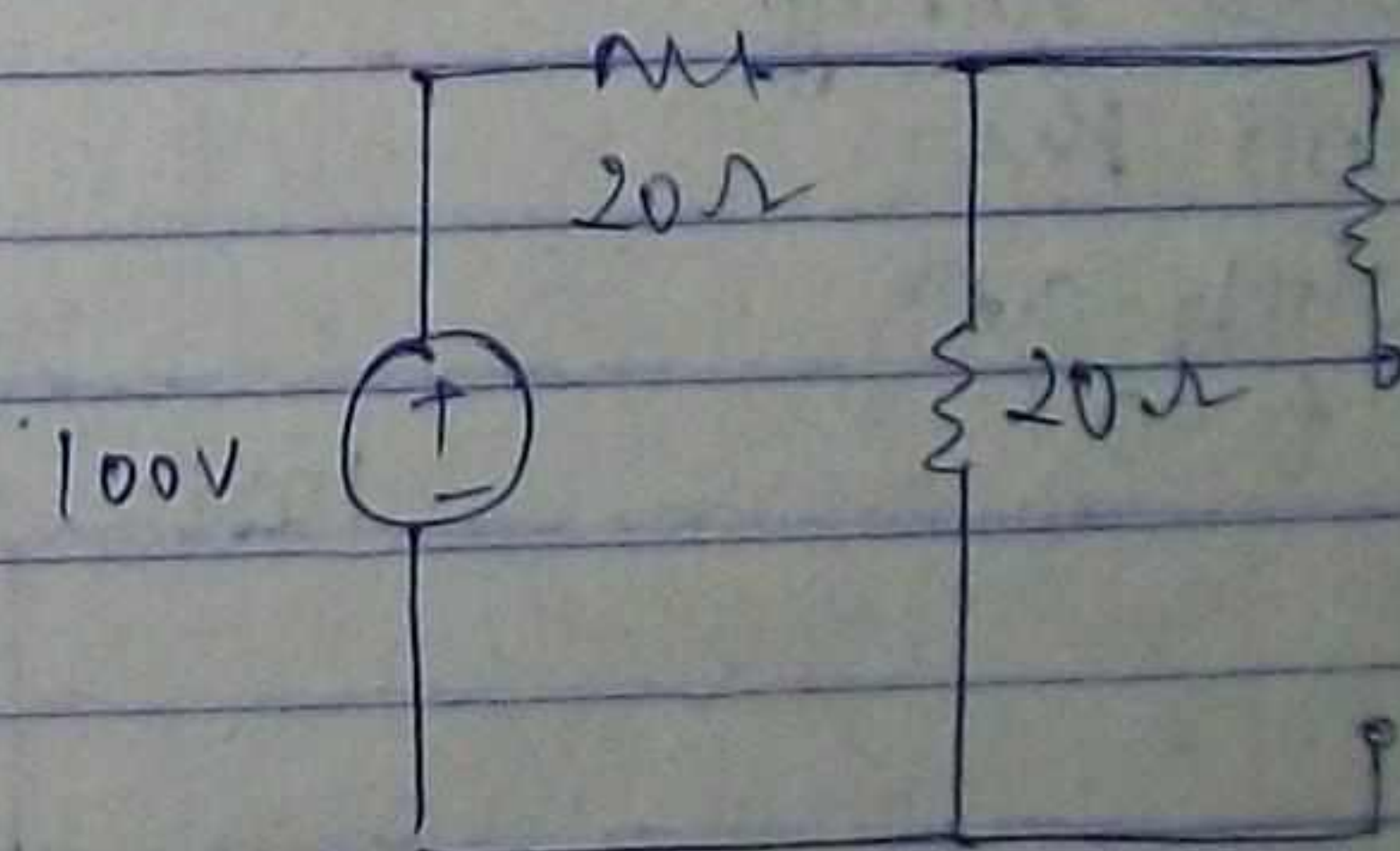
Ques. find $\frac{di}{dt}$ at $t=0^+$



Soln: at $t=0^-$

$$\frac{i(0^-)}{L} = \frac{100}{40} = 2.5$$

$$V_c(0^-) = 50V$$



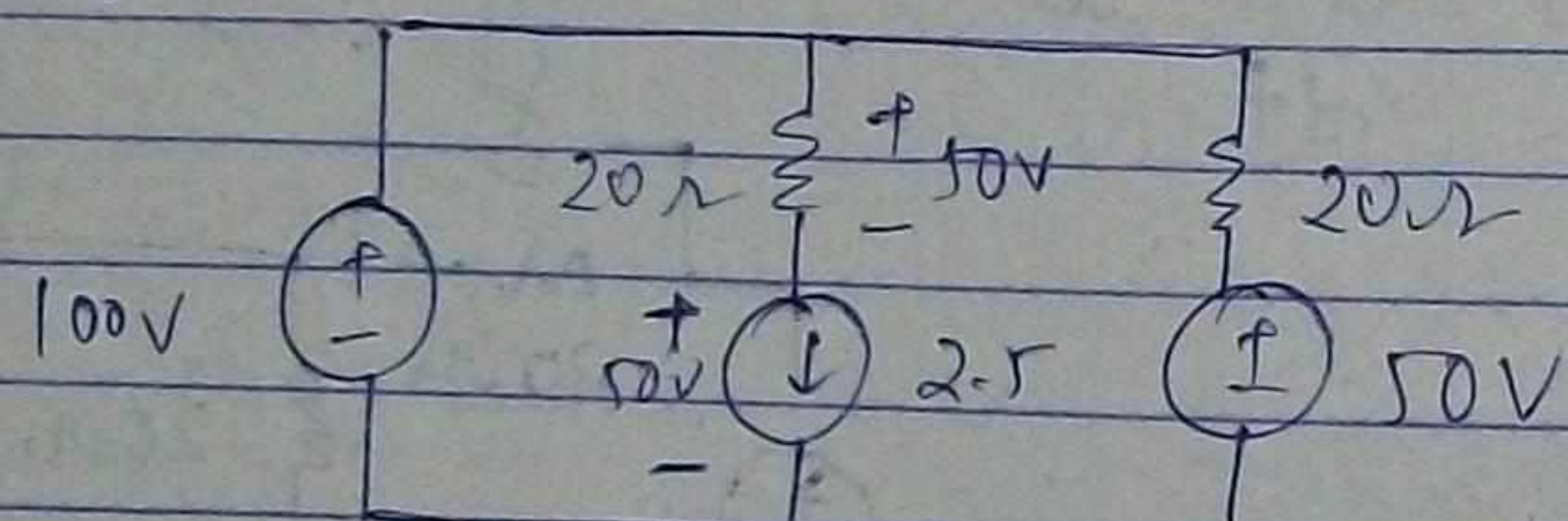
at $t=0^+$ $i_L(0^+) = i_L(0^-) = 2.5 \text{ A}$

$V_C(0^+) = V_C(0^-) = 50 \text{ V}$

$L \frac{di_L(0^+)}{dt} + R i_L(0^+) = 100$

$1 \frac{di_L(0^+)}{dt} = 100 - 20(2.5) = 50 \text{ A/s}$

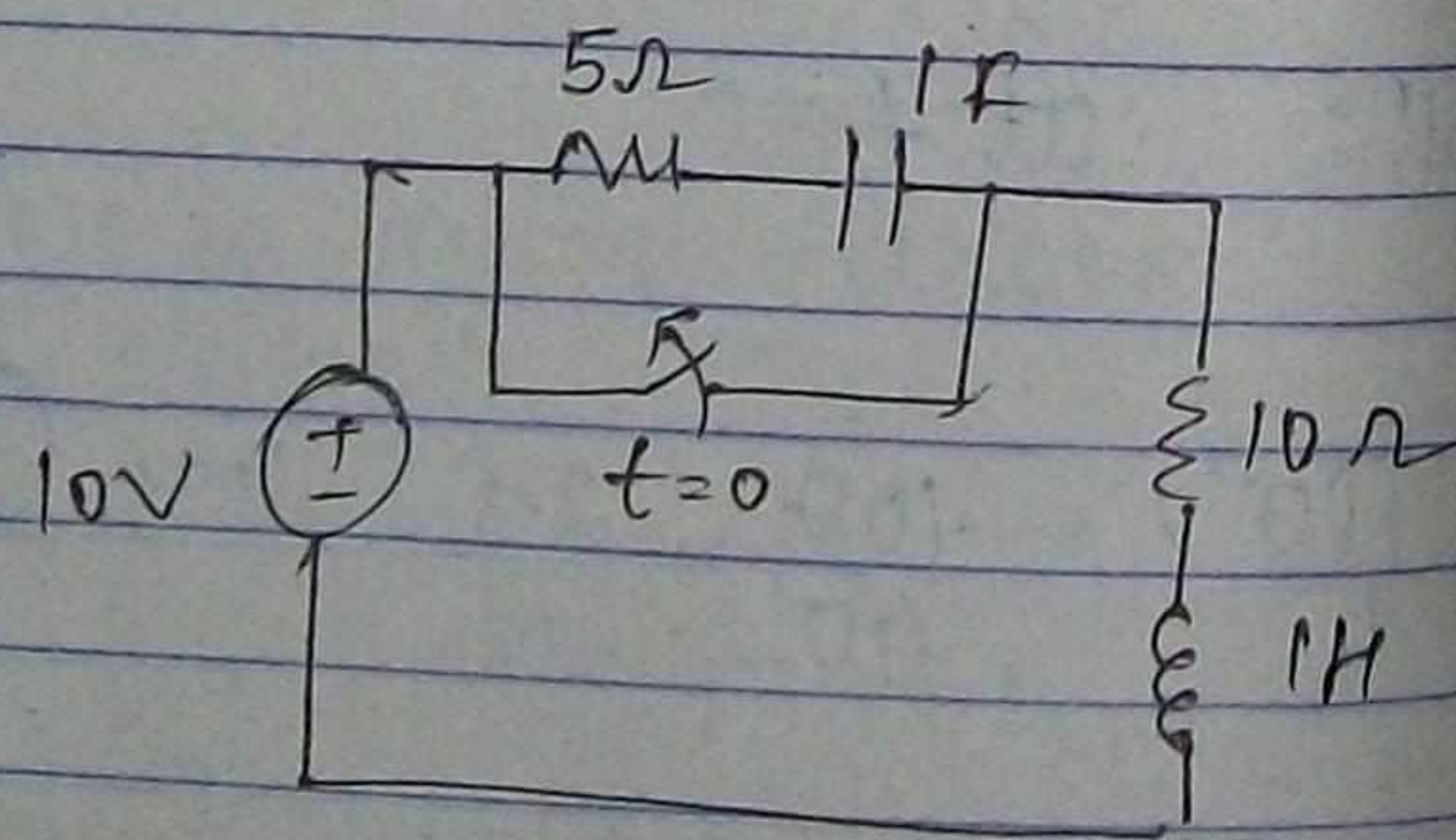
at $t=0^+$



$50 = L \frac{di(0^+)}{dt}$

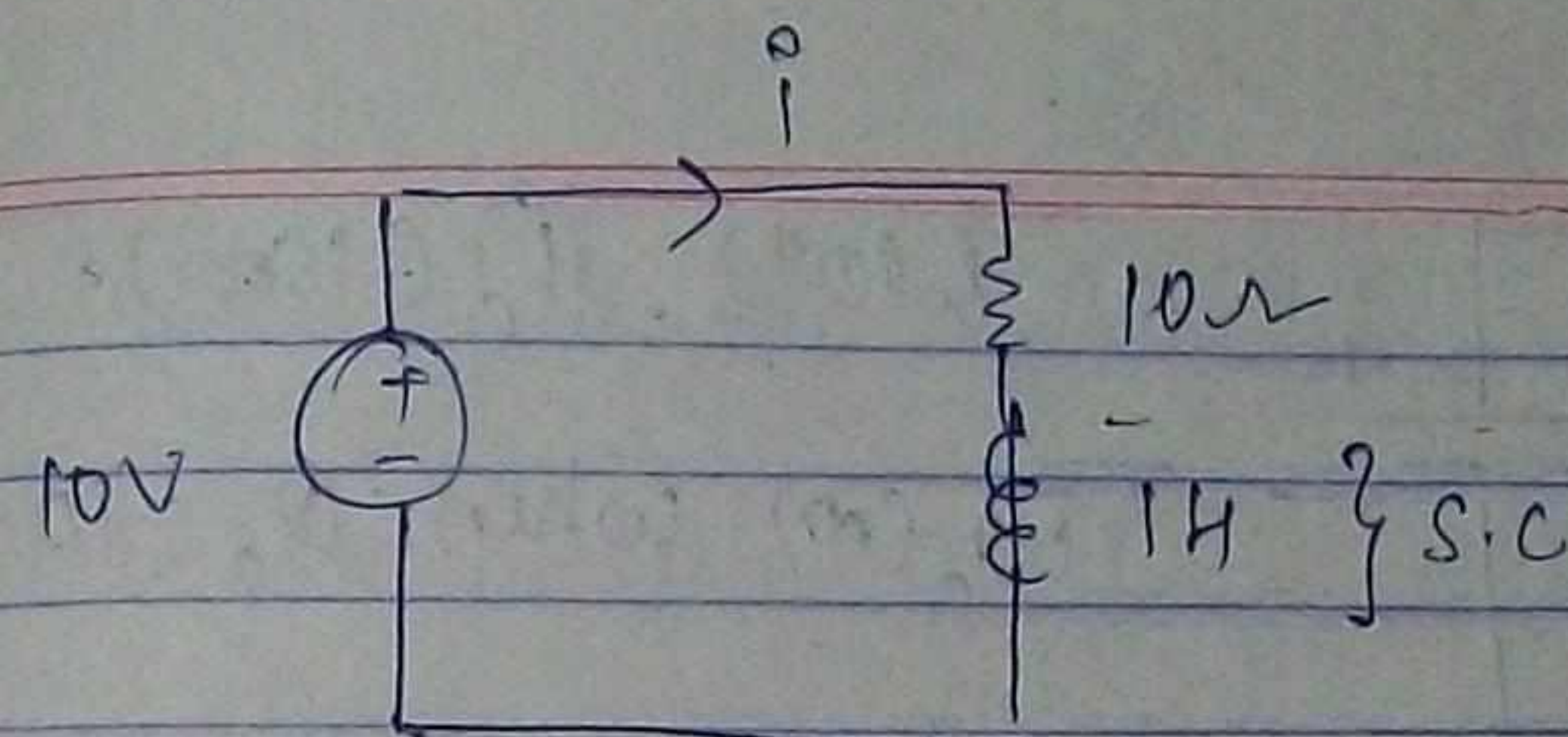
$\frac{di(0^+)}{dt} = 50 \text{ A/s}$

Ques. find voltage across the switch at $t=0^+$



Solⁿ:

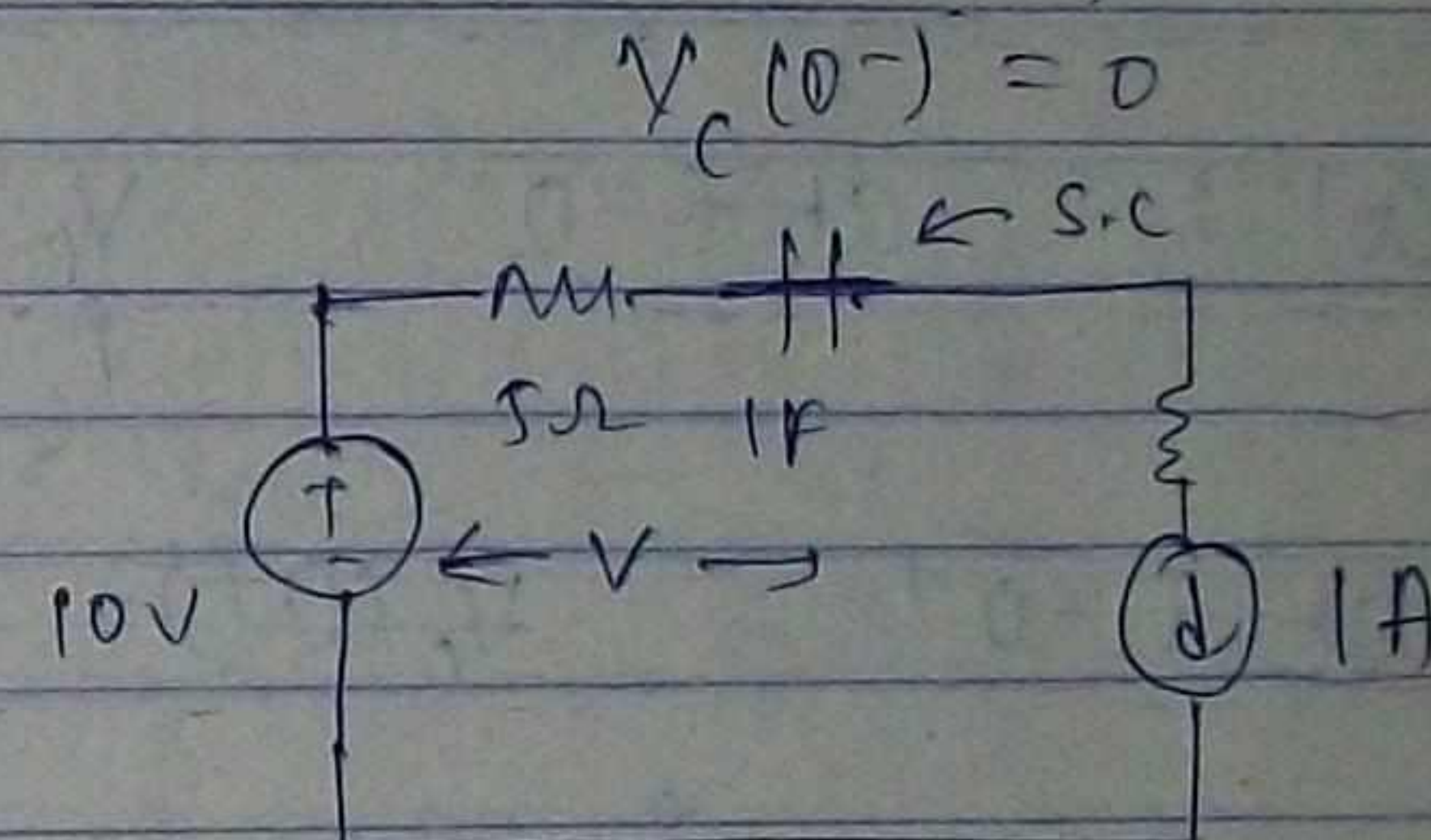
at $t=0^-$



$i_L(0^-) = \frac{10}{10} = 1 \text{ A}$

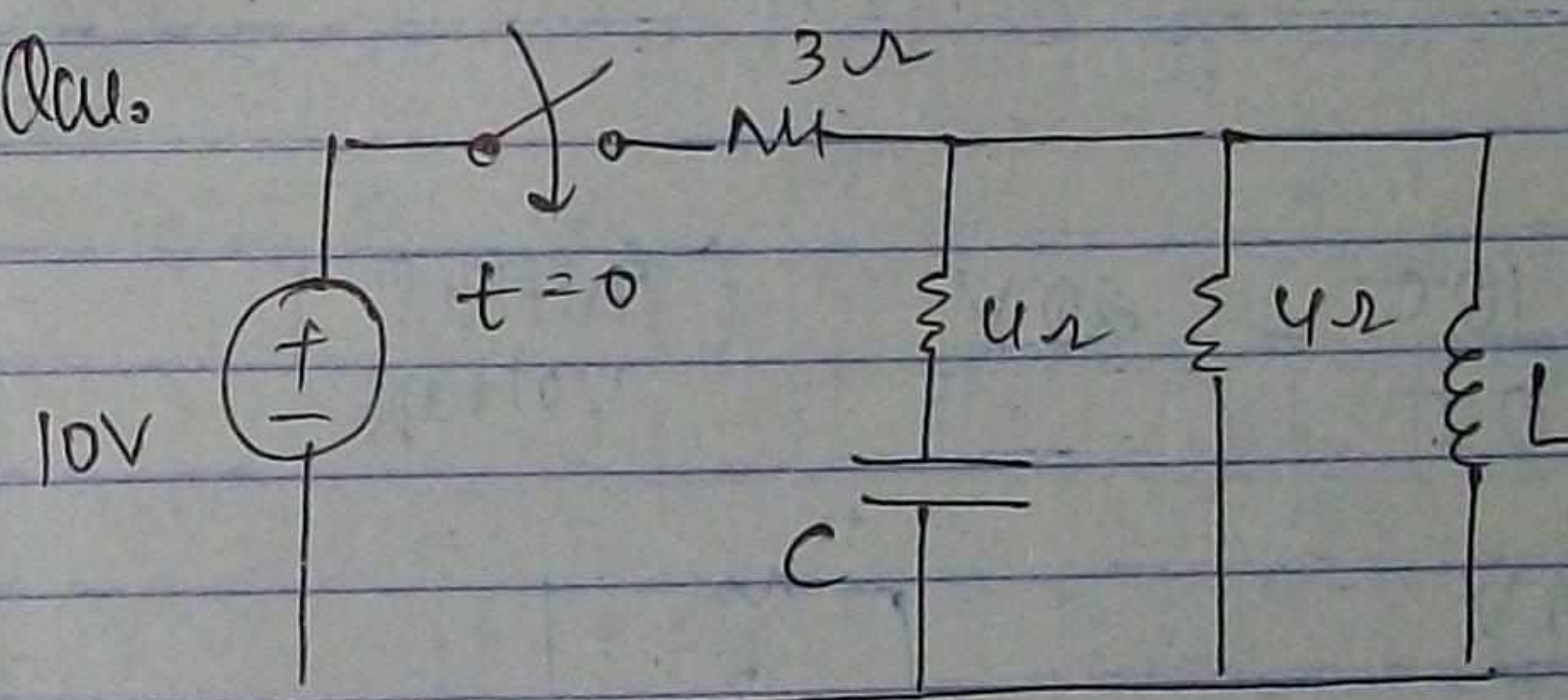
at $t=0^+$

$V = 5 \text{ V}$



$V_C(0^-) = 0$

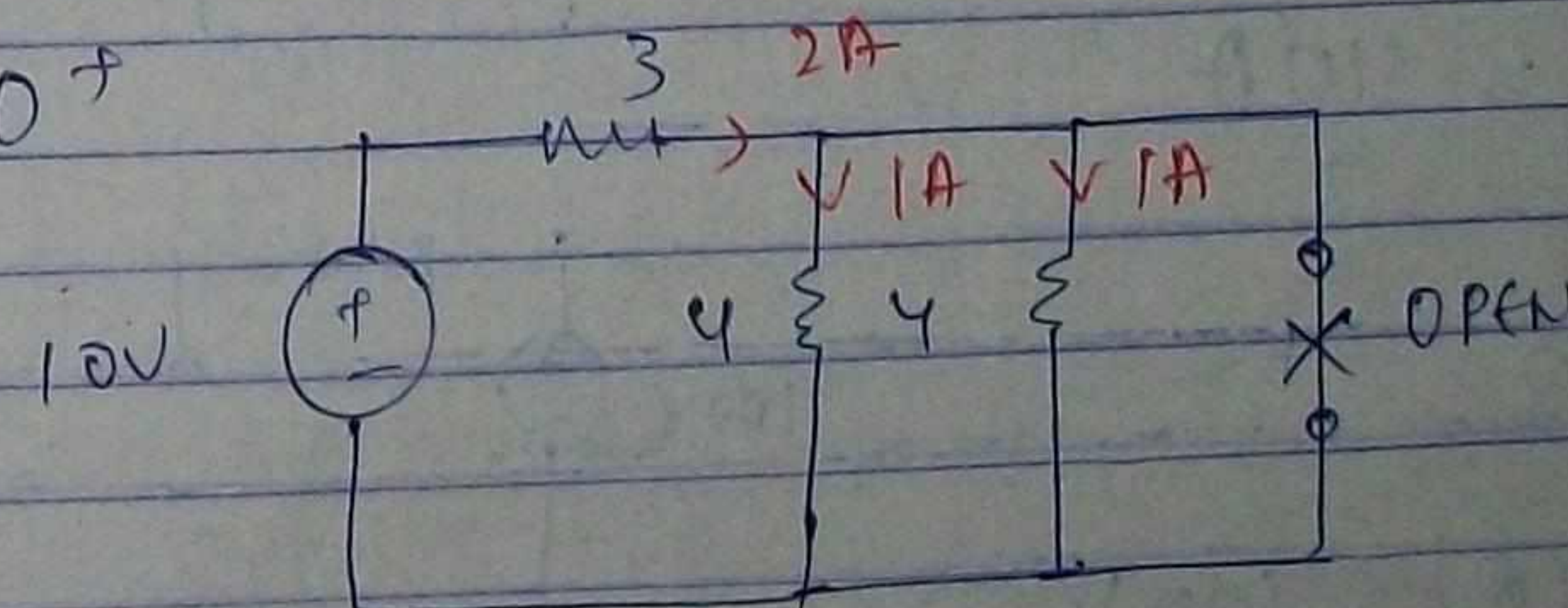
Ques.



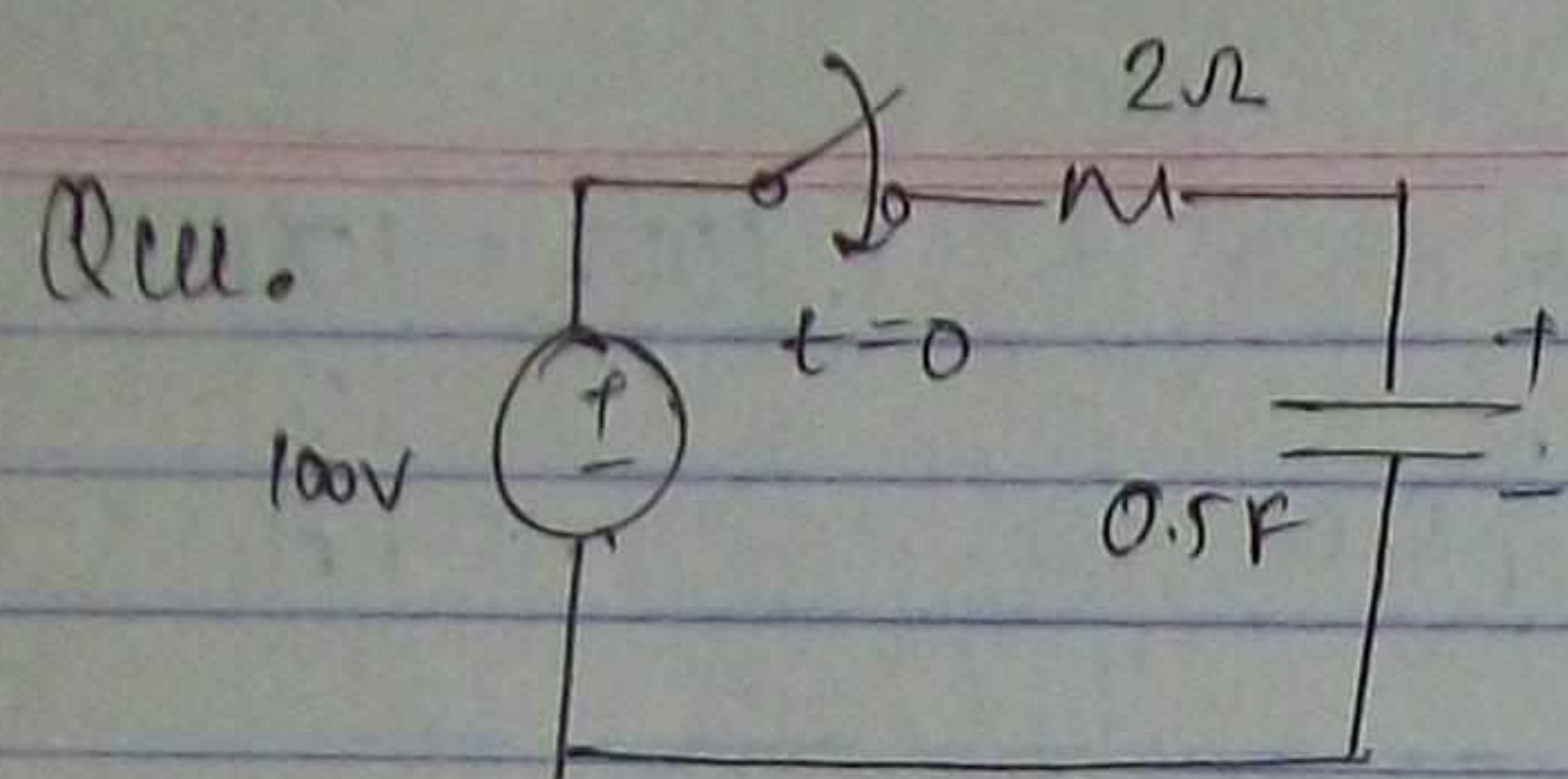
$i_L(0^+) = ?$
 $V_L(0^+) = ?$

Solⁿ: at $t=0^-$ $V_C(0^-) = 0$ $i_L(0^-) = 0$

at $t=0^+$



$V_L(0^+) = 4 \text{ V}$ $i_C(0^+) = 1 \text{ A}$



$i_c(0^+)$, $i_c(1\text{sec})$
 $w_c(\infty)$ when $Q_0 = 10C$

Solⁿ: at $t=0^-$ $V_c(0^-) = 0$
 $i_c(0^-) = 0$

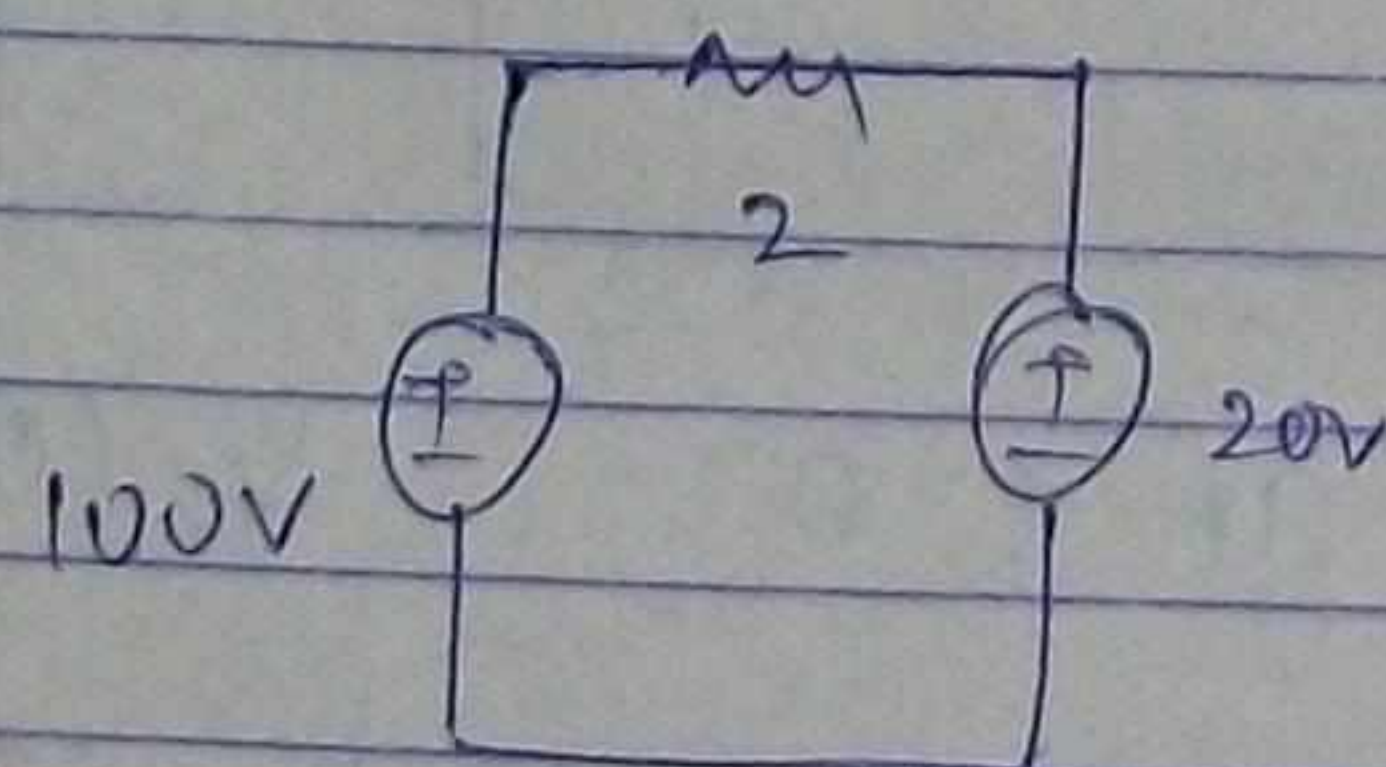
at $t=0^+$ $V_c(0^+) = V_c(0^-) = 0$

$$i_c(0^+) = \frac{100}{2} = 50 \text{ A}$$

$$V_0 = \frac{Q_0}{C} = \frac{10C}{0.5} = 20 \text{ V} \quad (\text{initial voltage})$$

$$V_c(0^+) = 20 \text{ V}$$

$$i_c(0^+) = \frac{100 - 20}{2} = 40 \text{ A}$$



at $t=\infty$

$$V_c(\infty) = 100 \text{ V}$$

$$w_c(\infty) = \frac{1}{2} C (V_c(\infty))^2 = \frac{1}{2} \times \frac{1}{2} (100)^2 = 2500 \text{ J}$$

$$i(t) = [i_c(0^+) - i_c(\infty)] e^{-t/\tau} + i_c(\infty)$$

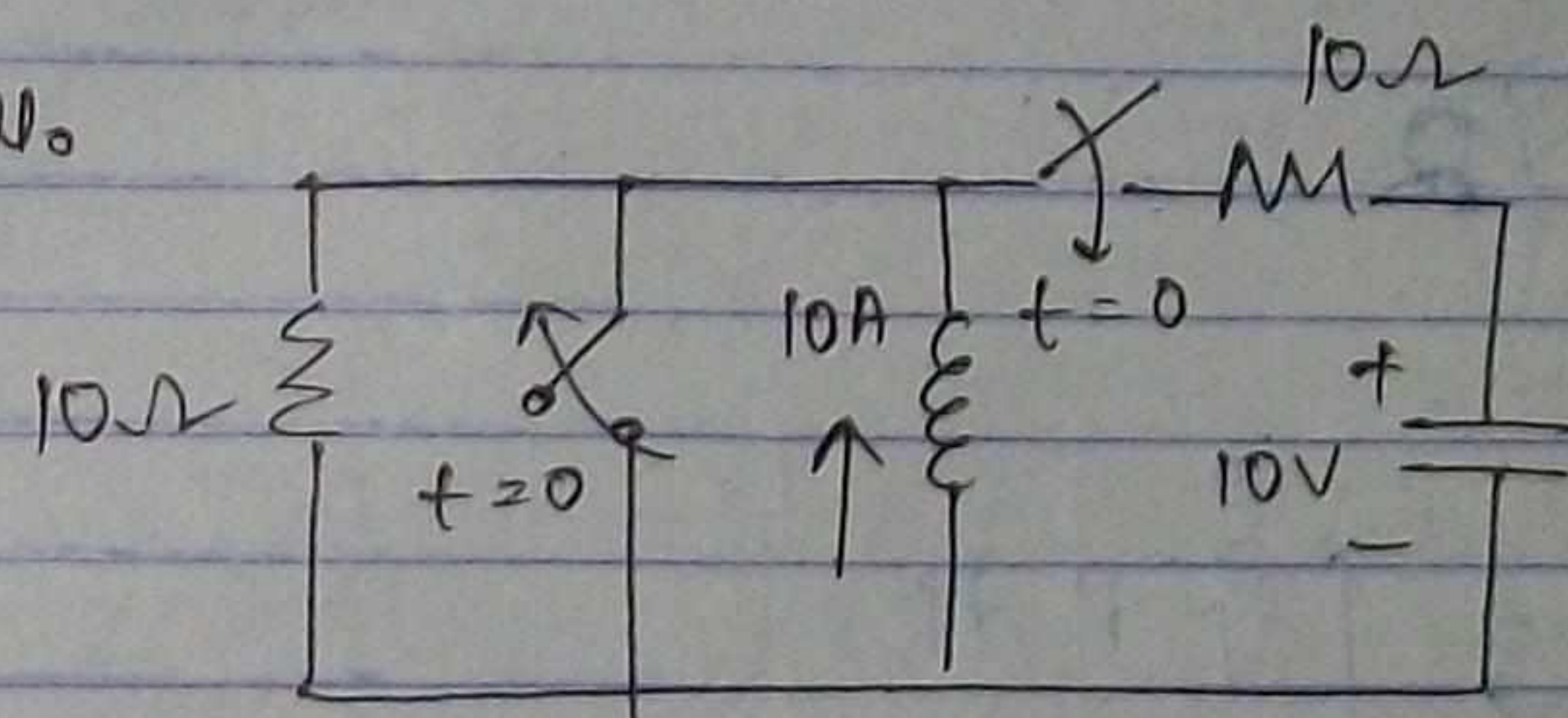
$$i(t) = [40 - 0] e^{-t} + 0$$

$$i(t) = 40 e^{-t}$$

at $t=1\text{s}$

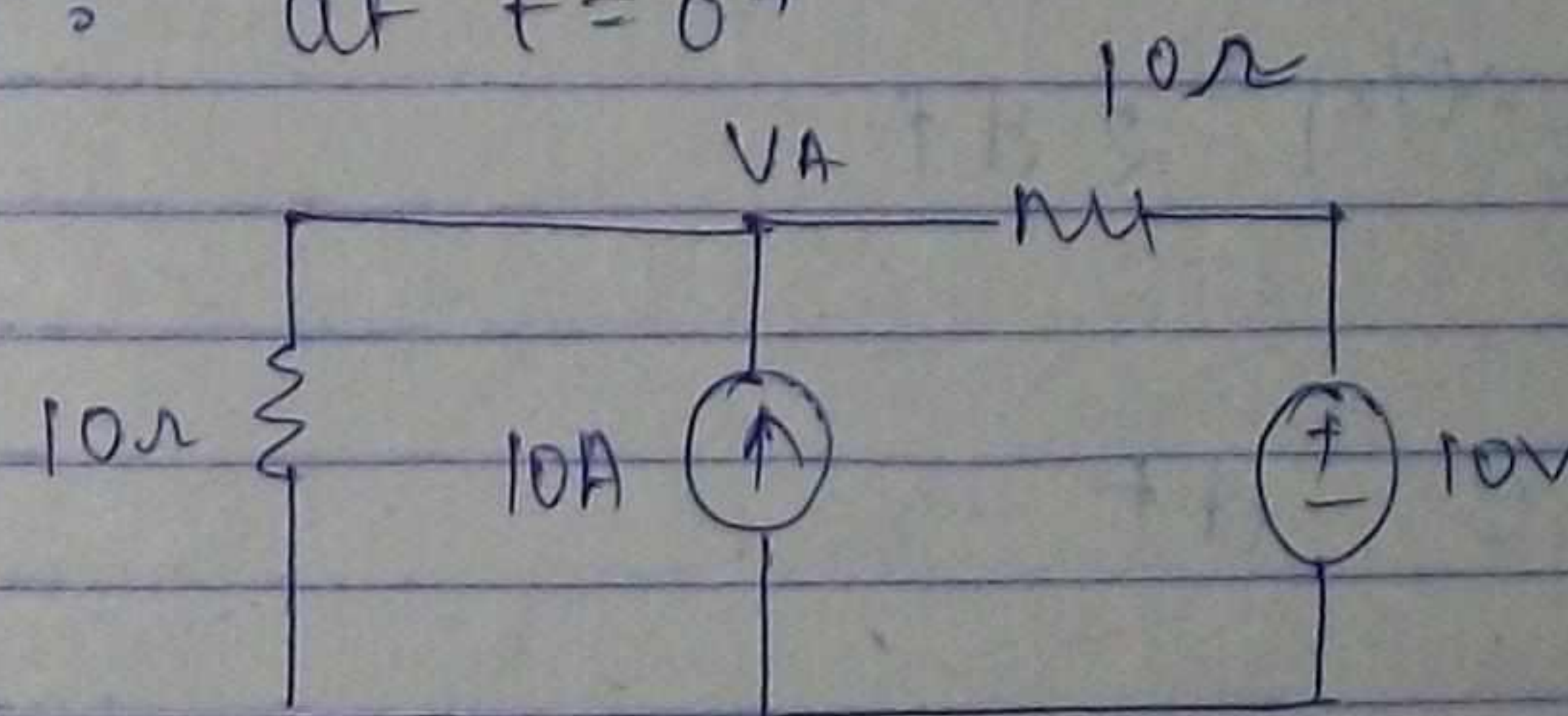
$$i(1) = 40 e^{-1} = \frac{40}{2.73}$$

Ques.



find voltage across inductor and current flow in cap at $t=0^+$

Solⁿ: at $t=0^+$



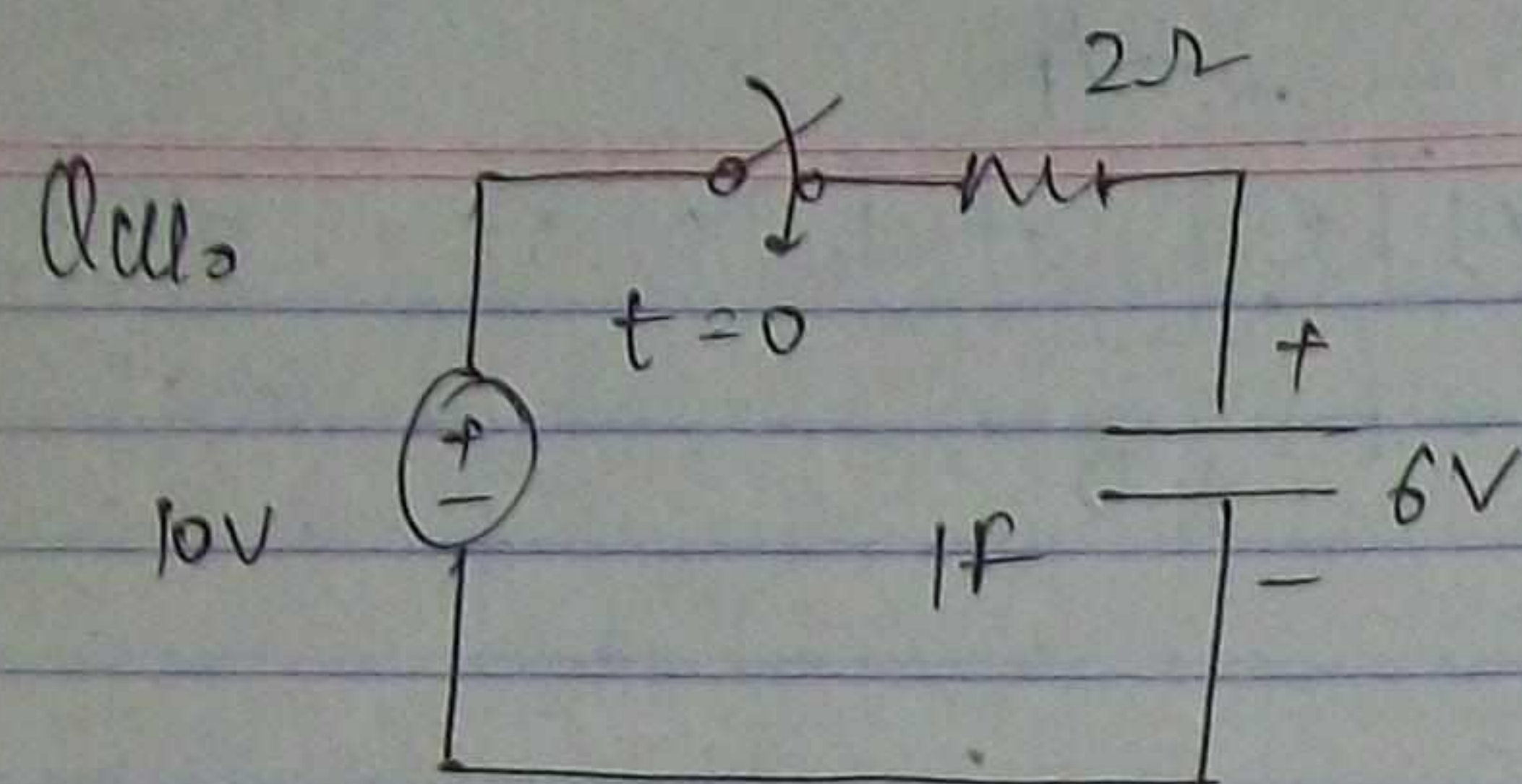
$$\frac{V_A}{10} + \frac{V_A - 10}{10} = 10$$

$$2V_A = 110$$

$$V_A = 55$$

$$V_c(0^+) = 55 \text{ V}$$

$$i_c(0^+) = \frac{55 - 10}{10} = 4.5 \text{ A}$$



find energy of the resistor from 0 to ∞ interval.

Solⁿ:

$$i(t) = i(\infty) + [i(0^+) - i(\infty)] e^{-t/RC}$$

$$i(0^+) = \frac{10-6}{2} = 2A \quad i(\infty) = 0$$

$$i(t) = 2e^{-t/2}$$

$$W = \int_0^{\infty} P dt = \int_0^{\infty} i^2 R dt$$

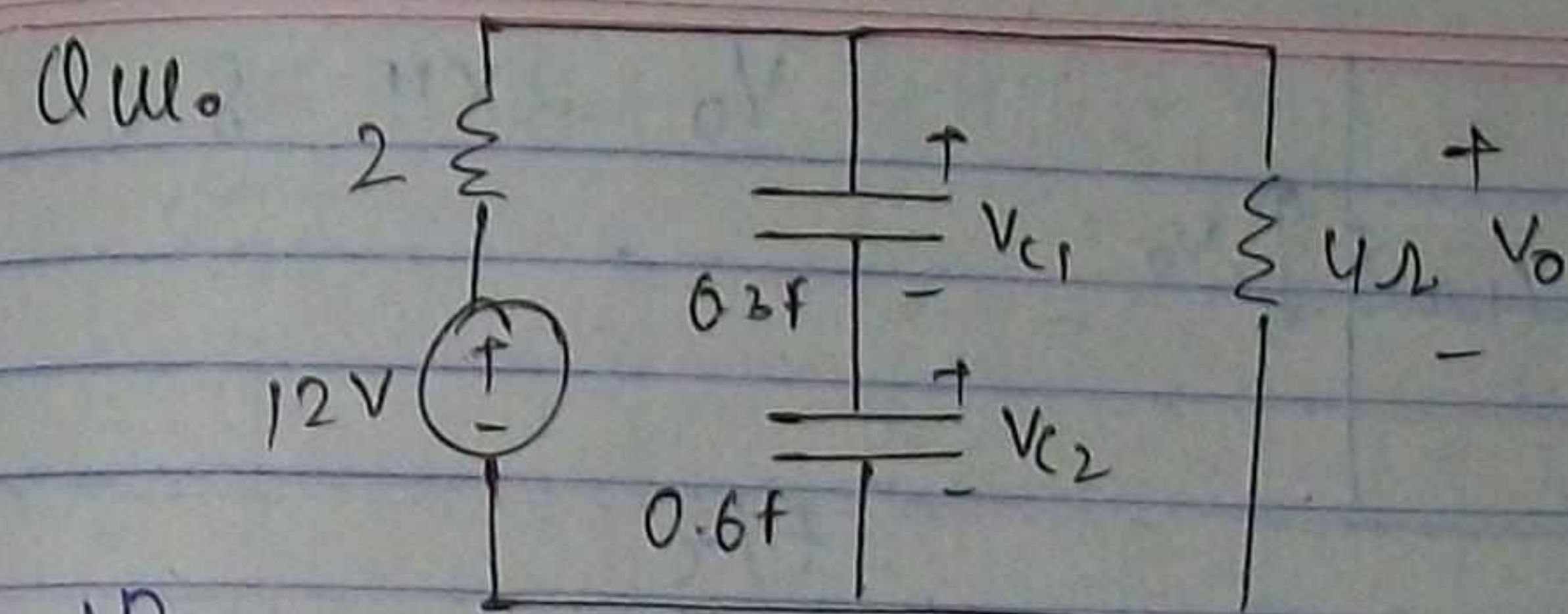
$$= \int_0^{\infty} (2e^{-t/2})^2 \cdot 2 dt$$

$$= 8 \int_0^{\infty} e^{-t} dt$$

$$= 8 (0 - 1) = -8$$

$$= 8 \text{ Joules (delivery)}$$

2/11/11



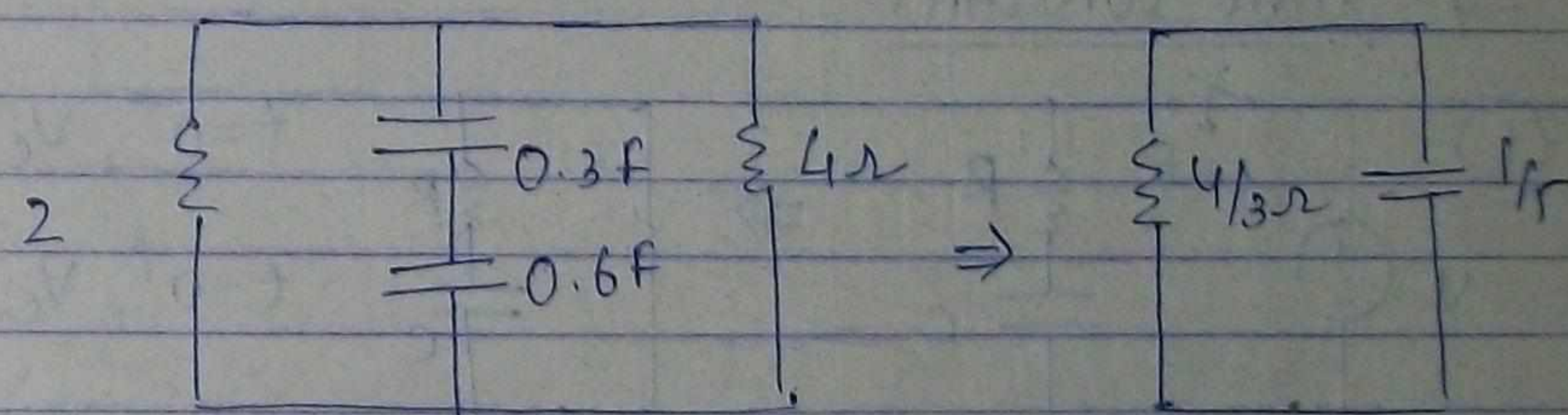
find V_o
 Total response
 $V_{C1}(0) = 24V$
 $V_{C2}(0) = 6V$

Solⁿ:

NOTE -

While finding time constant deactivate all the independent sources.

$$\begin{aligned} & 8 + 22e^{-3.75t} \\ & 8 + 22e^{-t/3.75} \\ & 8 + 22e^{-1.2t} \\ & 8 + 22e^{-t/1.2} \end{aligned}$$



$$\tau = RC = \frac{4}{15} = 0.26$$

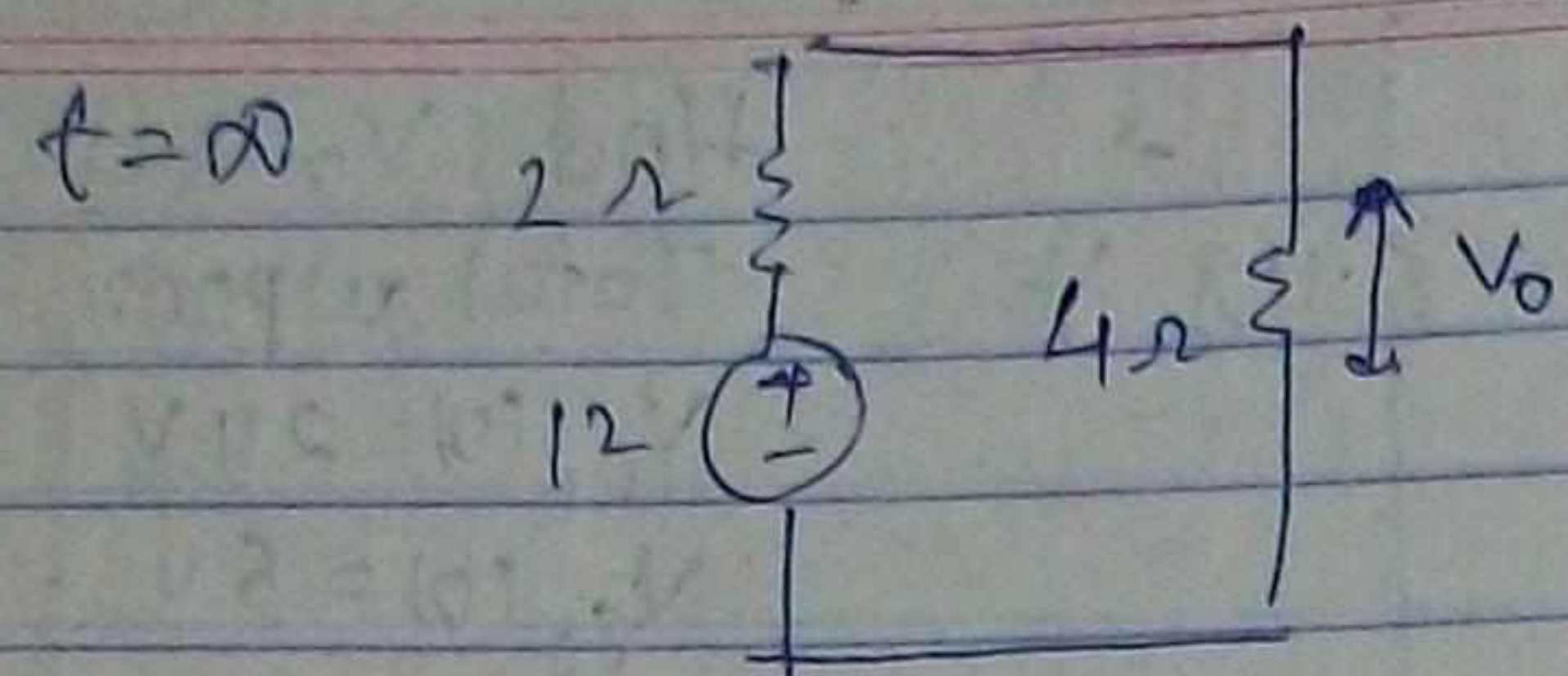
$$1/\tau = 3.75$$

$$V_o = e^{-3.75t}$$

$$V_o = V_{C1} + V_{C2}$$

$$t=0^+ \quad V_o(0^+) = V_{C1}(0^+) + V_{C2}(0^+)$$

$$V_o(0^+) = 24 + 6 = 30V$$



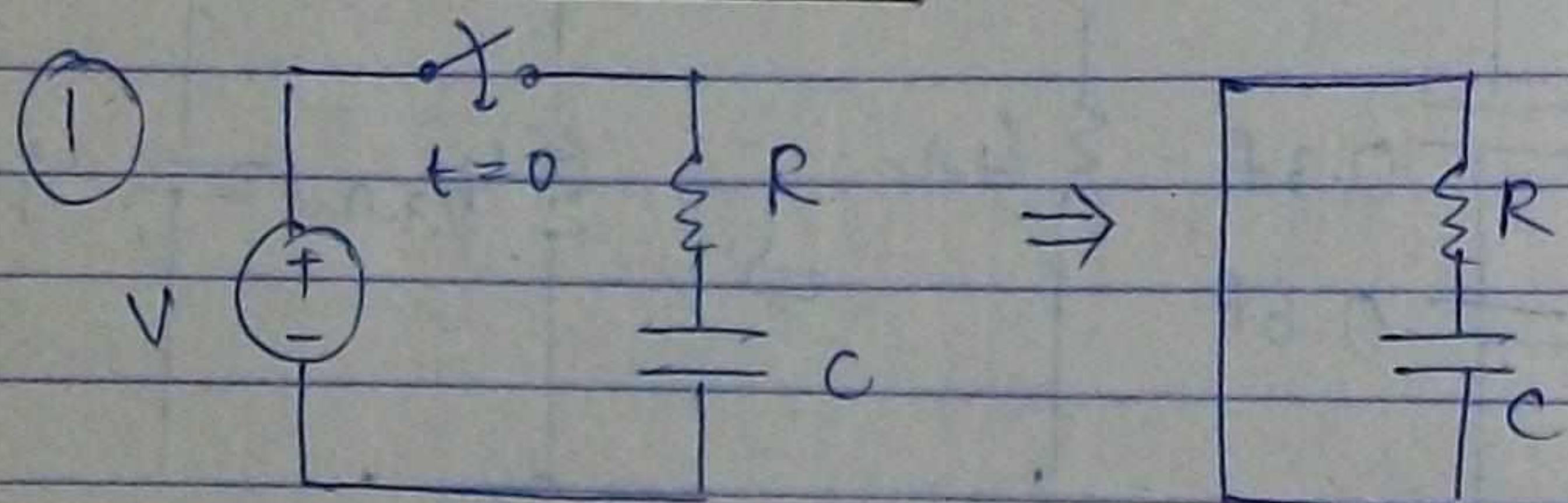
$$V_0 = 2 \times 4 = 8 \text{ V}$$

$$V(t) = V_f + (V_i - V_f) e^{-t/\tau}$$

$$V(t) = 8 + (30 - 8) e^{-t(3.75)}$$

$$V(t) = 8 + 22 e^{-3.75t}$$

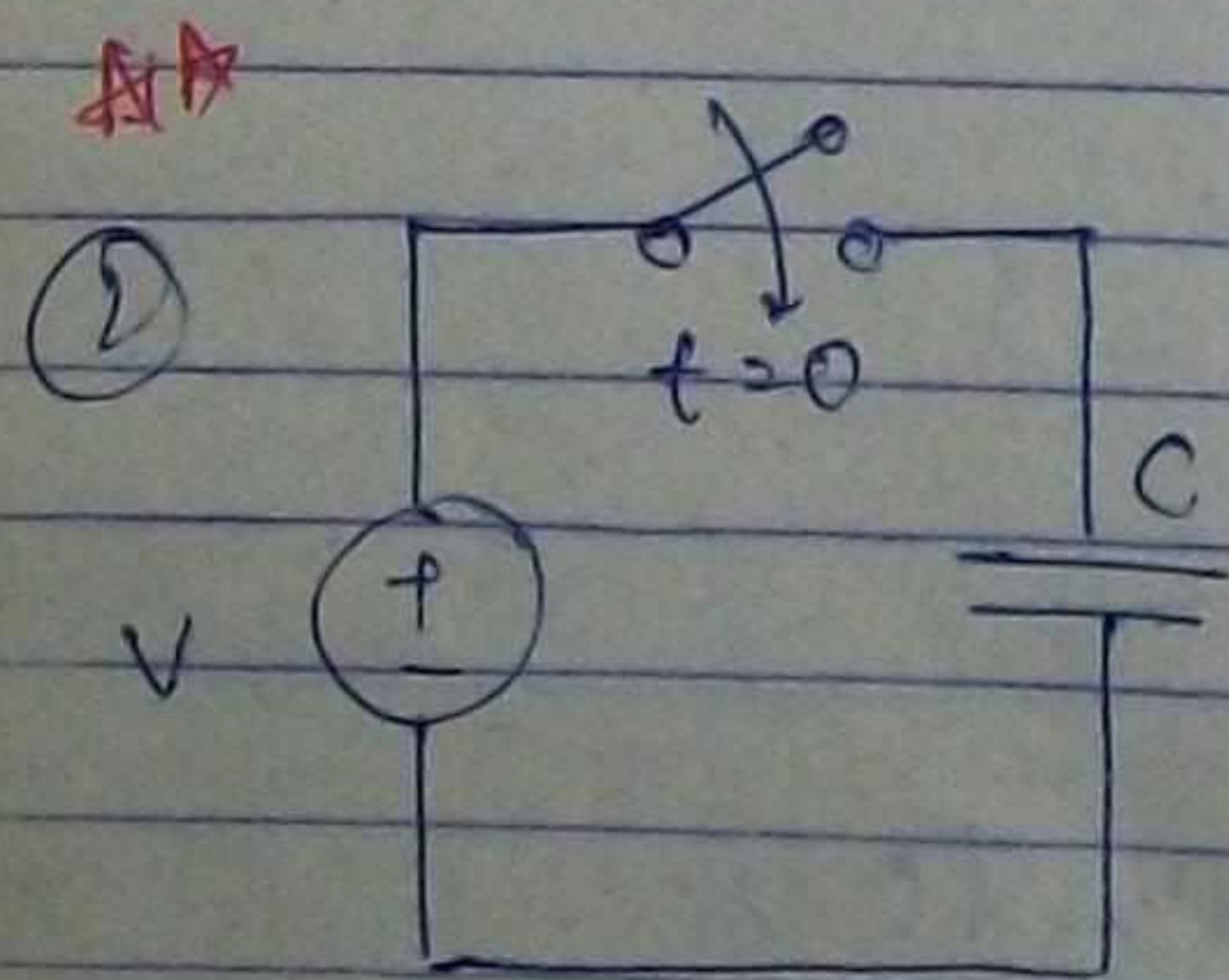
$\Rightarrow$ TIME CONSTANT -



$$t = 0^-, V_C = 0$$

$$t = 0^+, V_C = 0$$

$$\tau = RC$$

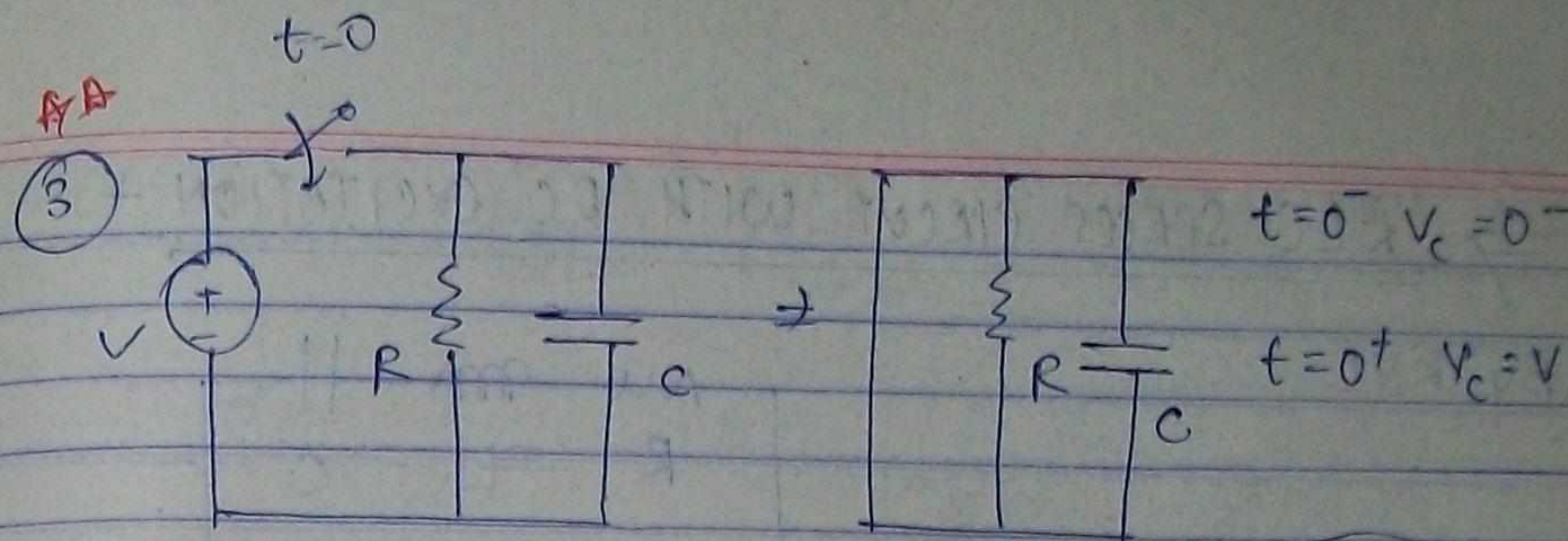


$$t = 0^-, V_C = 0$$

$$t = 0^+, V_C = V$$

$$\tau = 0$$

$$R = 0$$

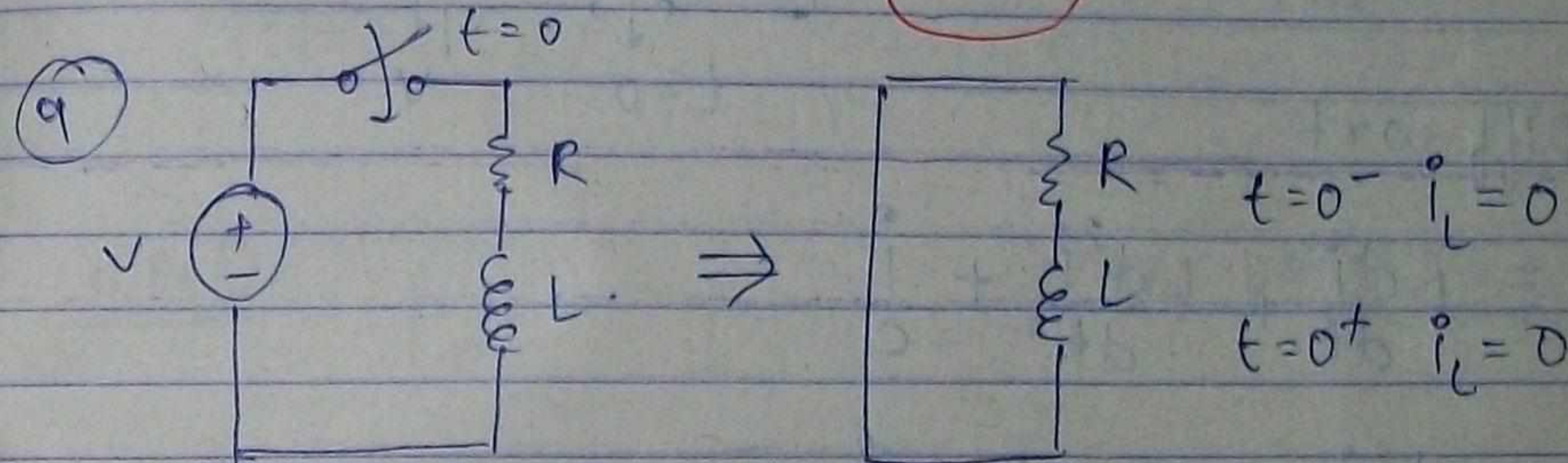


$$t = 0^-, V_C = 0$$

$$t = 0^+, V_C = V$$

$$R_{eq} = 0$$

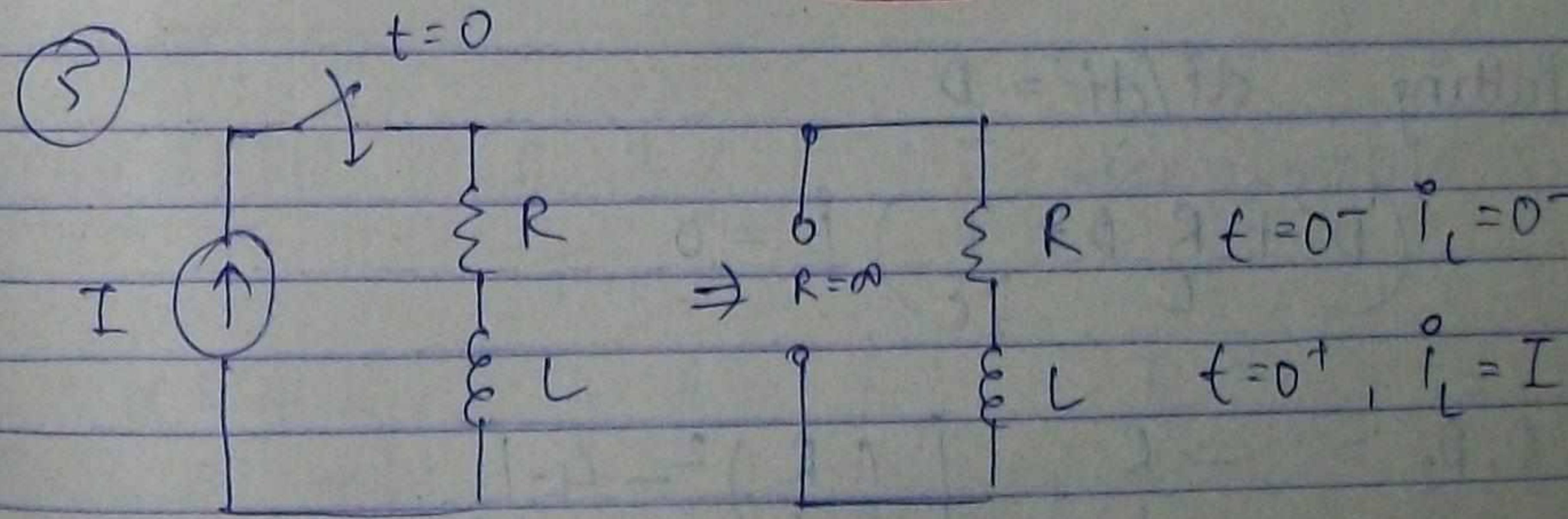
$$\tau = 0$$



$$t = 0^-, i_L = 0$$

$$t = 0^+, i_L = 0$$

$$\tau = L/R$$



$$t = 0^-, i_L = 0$$

$$t = 0^+, i_L = I$$

$$\tau = \infty$$

$$\tau = \frac{L}{R_{eq}} = \frac{L}{\infty}$$

$$\tau = 0$$

⇒ RLC SERIES CIRCUIT WITH DC EXCITATION -

By KVL

$$V = iR + L \frac{di}{dt} + \frac{1}{C} \int i dt$$

Diff wrt

$$0 = R \frac{di}{dt} + L \frac{d^2 i}{dt^2} + \frac{i}{C}$$

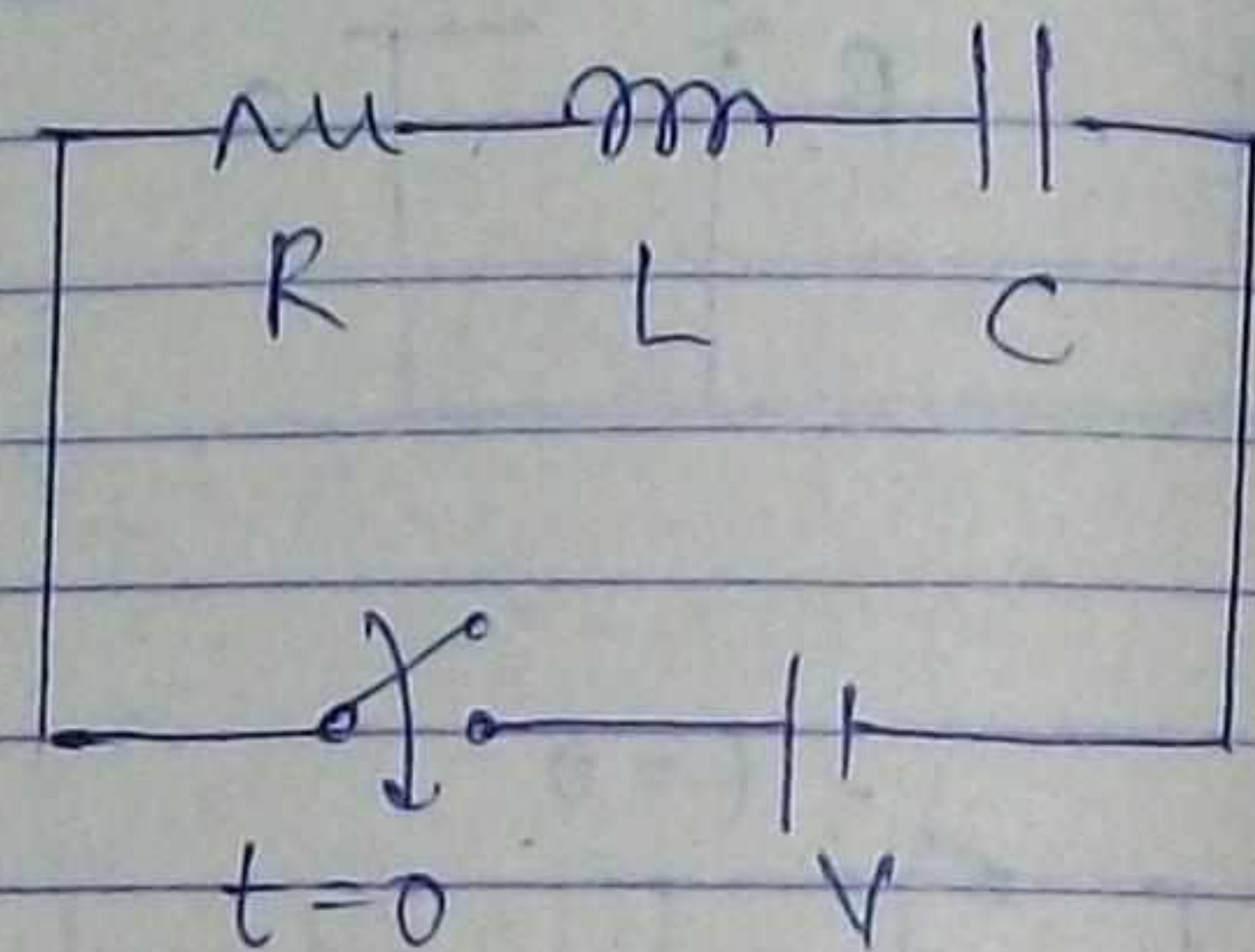
$$\frac{d^2 i}{dt^2} + \frac{R}{L} \frac{di}{dt} + \frac{i}{LC} = 0$$

Putting $di/dt = D$

$$\left(D^2 + \frac{R}{L} D + \frac{1}{LC}\right) i = 0$$

$$D_1, D_2 = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - 4 \cdot \frac{1}{LC}}}{2}$$

$$D_1, D_2 = \frac{-\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}}{1}$$



CASE (1): $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$ [OVERDAMPING]

$$\alpha = -\frac{R}{2L} \quad \beta = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$i(t) = C_1 e^{+(\alpha+\beta)t} + C_2 e^{(\alpha-\beta)t}$$

CASE (2): $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$ [CRITICAL DAMPING]

$$i(t) = (C_1 + C_2 t) e^{\alpha t}$$

CASE (3): $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$ [UNDER DAMPING]

$$\alpha = -\frac{R}{2L} \quad \beta = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

$$i(t) = [C_1 \cos \beta t + C_2 \sin \beta t] e^{\alpha t}$$

$$\text{DAMPING COEFFICIENT} = \frac{R}{2L}$$

$$\text{TIME CONSTANT OF RLC CIRCUIT} = \frac{1}{\text{DAMPING COEFF}} = \frac{2L}{R}$$

$$\text{CASE (4)}: R=0 \quad [\text{UNDAMPED}]$$

$$\alpha = 0, \quad \beta = \frac{1}{\sqrt{LC}}$$

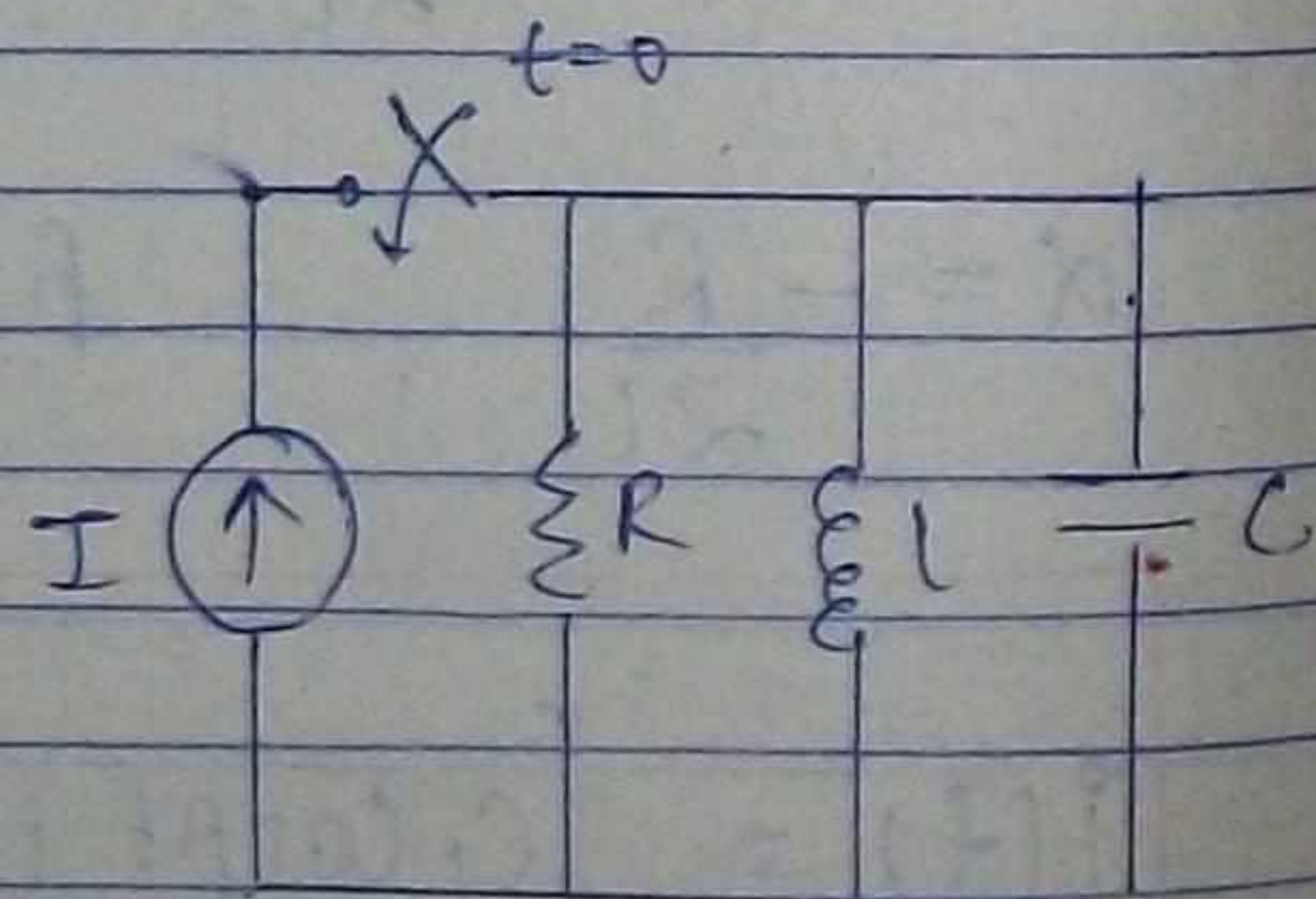
$$i(t) = C_1 \cos \beta t + C_2 \sin \beta t$$

⇒ RLC PARALLEL CIRCUIT WITH DC EXCITATION -

By KCL,

$$I = \frac{V}{R} + C \frac{dv}{dt} + \frac{1}{L} \int v dt$$

$$0 = \frac{1}{R} \frac{dv}{dt} + C \frac{d^2 v}{dt^2} + \frac{v}{L}$$



$$\frac{d^2 v}{dt^2} + \frac{1}{RC} \frac{dv}{dt} + \frac{v}{LC} = 0$$

Putting $d/dt = D$

$$\left(D^2 + \frac{1}{RC} D + \frac{1}{LC} \right) v = 0$$

$$D_1, D_2 = -\frac{1}{RC} \pm \sqrt{\left(\frac{1}{RC}\right)^2 - \frac{4}{LC}} / 2$$

$$D_1, D_2 = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

$$\text{CASE (1)}: \left(\frac{1}{2RC}\right)^2 > \frac{1}{LC} \quad [\text{OVERDAMPED}]$$

$$\alpha = -\frac{1}{2RC}, \quad \beta = \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

$$v(t) = C_1 e^{(\alpha+\beta)t} + C_2 e^{(\alpha-\beta)t}$$

$$\text{CASE (2)}: \left(\frac{1}{2RC}\right)^2 = \frac{1}{LC} \quad [\text{CRITICALLY DAMPED}]$$

$$v(t) = (C_1 + C_2 t) e^{\alpha t}$$

CASE (3): $\left(\frac{1}{2RC}\right)^2 < \frac{1}{LC}$ [UNDER DAMPED]

$$\alpha = \frac{-1}{2RC} \quad \beta = \sqrt{\frac{1}{LC} - \left(\frac{1}{2RC}\right)^2}$$

$$V(t) = (C_1 \cos \beta t + C_2 \sin \beta t) e^{\alpha t}$$

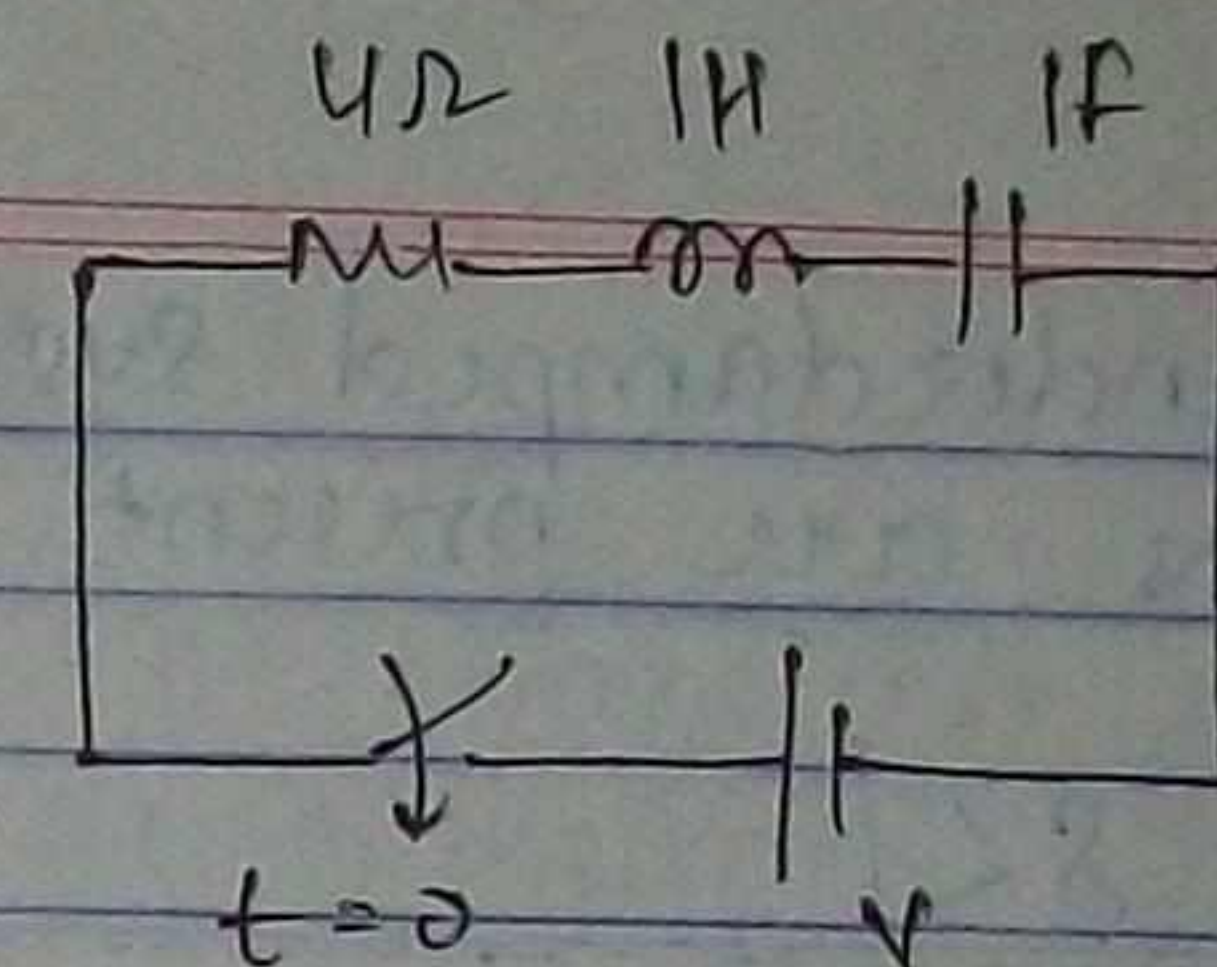
$$\text{DAMPING COEFFICIENT} = \frac{1}{2RC}$$

$$\text{TIME CONSTANT OF RLC // CT} = \frac{1}{\text{DAMPING COEFF}} = 2RC$$

CASE (4): $G=0$ (or) $\frac{1}{R}=0$ [UNDAMPED]
(or) $R=\infty$.

$$V(t) = (C_1 \cos \beta t + C_2 \sin \beta t)$$

Ques. find current response.



Soln:

- In RLC series circuit with dc excitation steady state current response is zero.

$$s = \frac{R}{2} \sqrt{\frac{C}{L}}$$

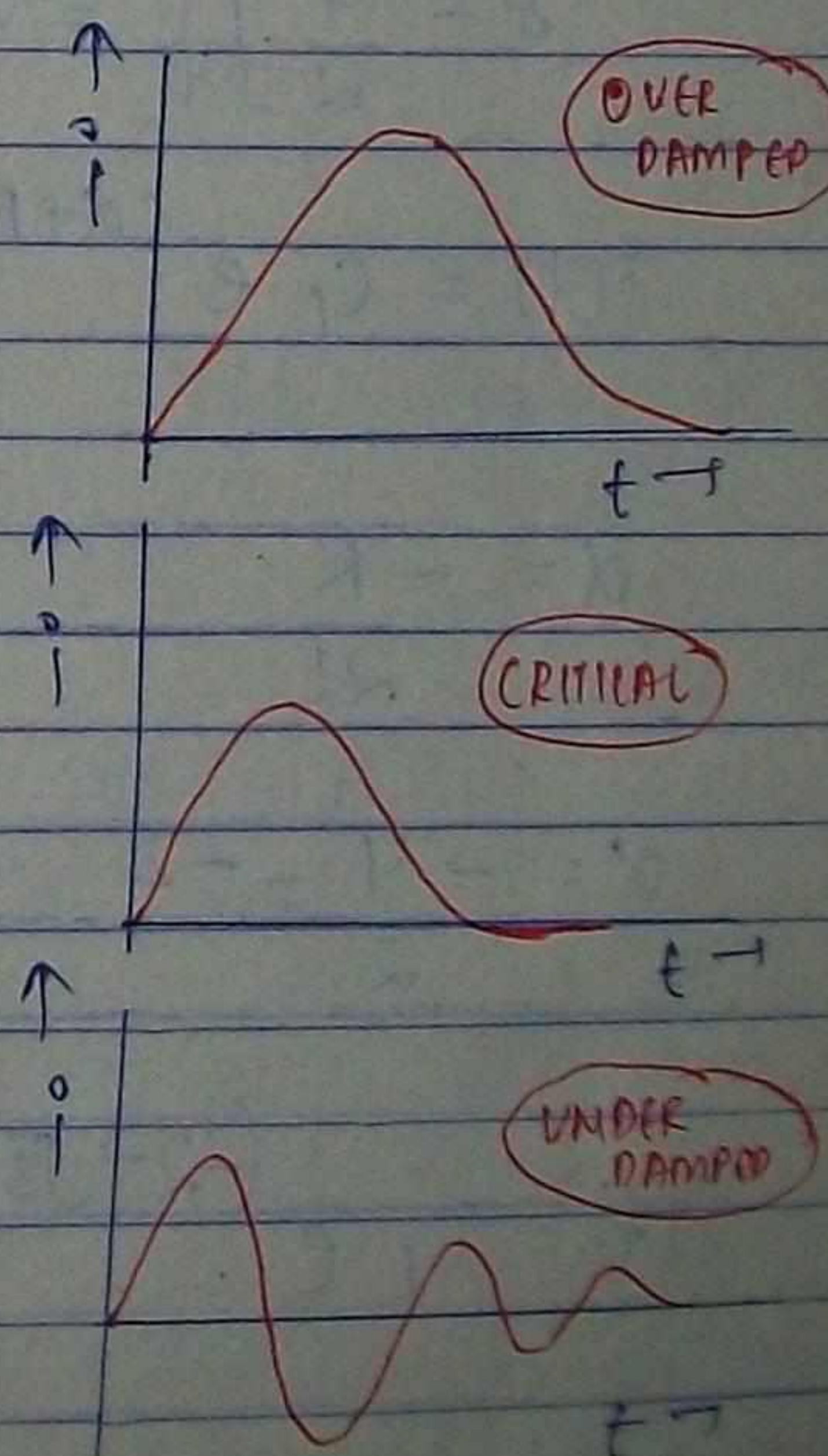
$s > 1$ overdamped
 $s = 1$ critically
 $s < 1$ underdamped
 $s = 0$ undamped.

- Condition for no oscillation

$$s \geq 1$$

$$\frac{R}{2} \sqrt{\frac{C}{L}} \geq 1$$

In over and critical damping no oscillation are present.

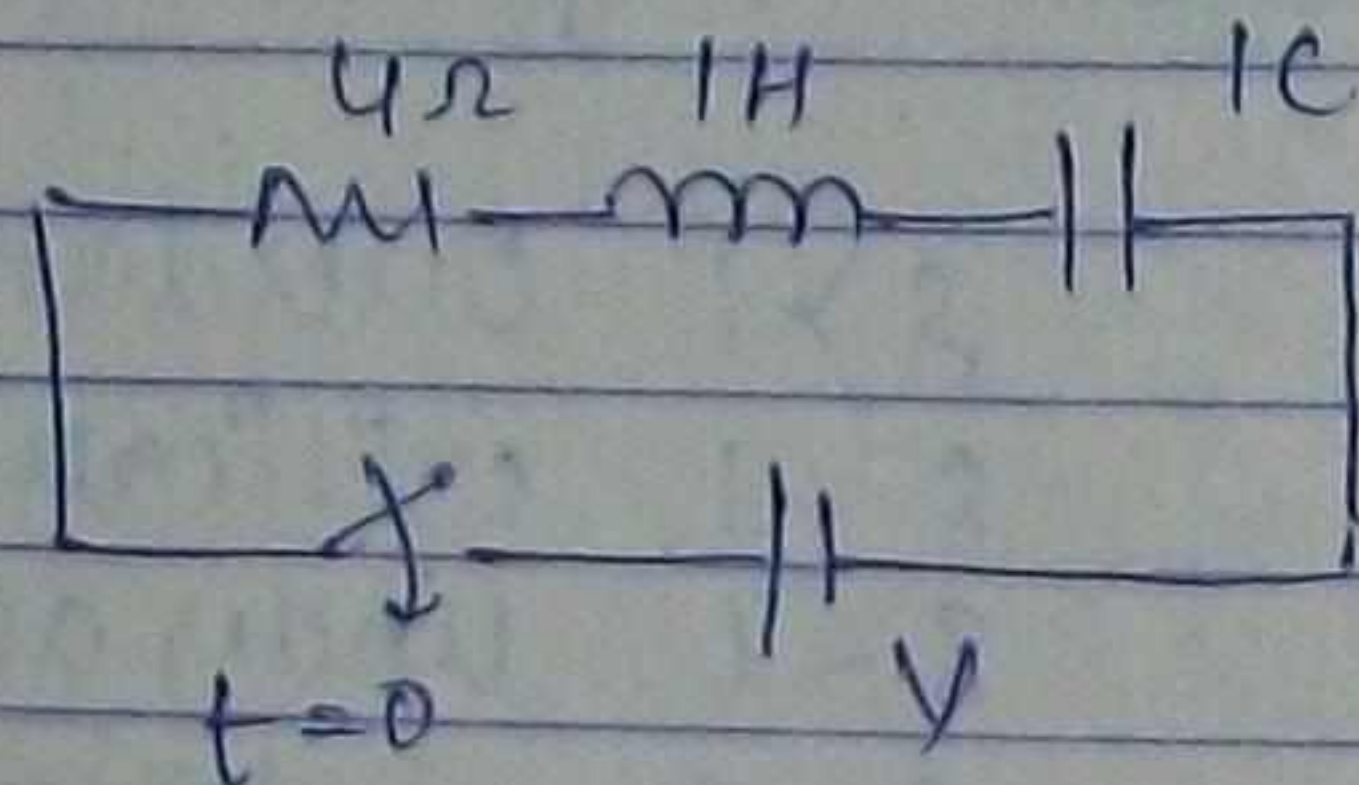


- In the underdamped system more than one oscillations are present.

$$\delta < 1$$

$$\frac{R}{2} \sqrt{\frac{C}{L}} < 1$$

Soln:



$$\delta = \frac{4}{2} \sqrt{\frac{1}{1}} = 2$$

$$(\delta > 1)$$

overdamped,

$$i(t) = C_1 e^{(\alpha + \beta)t} + C_2 e^{(\alpha - \beta)t}$$

$$\alpha = -\frac{R}{2L}$$

$$\beta = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$\alpha = -\frac{4}{2} = -2$$

$$\beta = \sqrt{4 - 1} = \sqrt{3}$$

$$i(t) = C_1 e^{(-2 + \sqrt{3})t} + C_2 e^{-(2 + \sqrt{3})t}$$

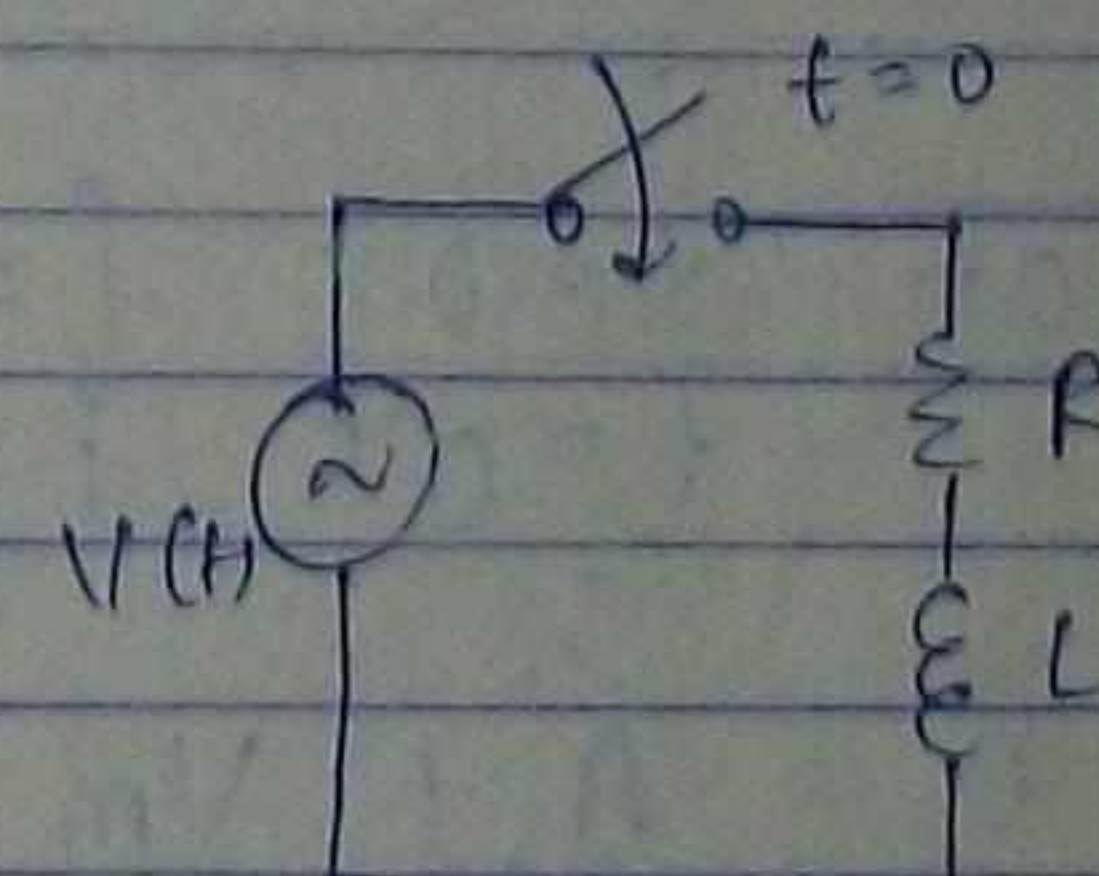
⇒ AC TRANSIENTS -

- Intensity of the DC transients are more than intensity of the AC transients.
- In AC circuits based on selection of circuit parameters, operating freq. and switching operation it is possible to design transient free circuit. But in the D.C. system it is not possible to obtain transient free response.

By KVL,

$$V = iR + L \frac{di}{dt}$$

$$\frac{di}{dt} + \frac{R}{L} i = \frac{V}{L}$$



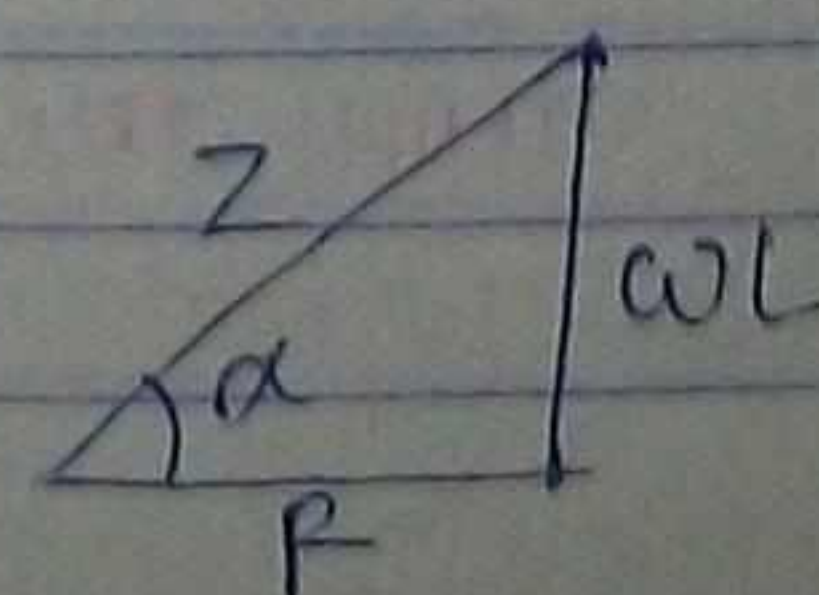
$$V(t) = V_m \sin(\omega t + \theta)$$

CF → Transient response

$$\frac{di}{dt} + \frac{R}{L} i = 0$$

$$i(t) = A e^{-\frac{R}{L}t}$$

PI → Steady state response



$$\alpha = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$i = \frac{V}{Z/\alpha} = \frac{V}{Z} \angle -\alpha$$

$$i(t) = \frac{V_m}{Z} \sin(\omega t + \theta - \alpha)$$

$$i(t) = C.F + P.I$$

$$i(t) = A e^{-R/L t} + \frac{V_m}{Z} \sin(\omega t + \theta - \alpha)$$

$$\text{at } t=0^- \quad i=0$$

$$t=0^+ \quad i=0$$

$$0 = A + \frac{V_m}{Z} \sin(\theta - \alpha)$$

$$A = - \frac{V_m}{Z} \sin(\theta - \alpha)$$

$$i(t) = - \frac{V_m}{Z} \sin(\theta - \alpha) e^{-Rt/L} + \frac{V_m}{Z} \sin(\omega t + \theta - \alpha)$$

TRANSIENT

STEADY STATE

CASE (1) : $V(t) = V_m \sin(\omega t + \theta)$, $t=0$

$$\theta - \alpha = 0$$

$$\theta = \alpha$$

$$\theta = \tan^{-1} \frac{\omega L}{R}$$

CONDⁿ FOR
TRANSIENT FREE
RESPONSE

CASE (2) : $V(t) = V_m \cos(\omega t + \theta)$, $t=0$

$$\theta = \tan^{-1} \frac{\omega L}{R} + \frac{\pi}{2}$$

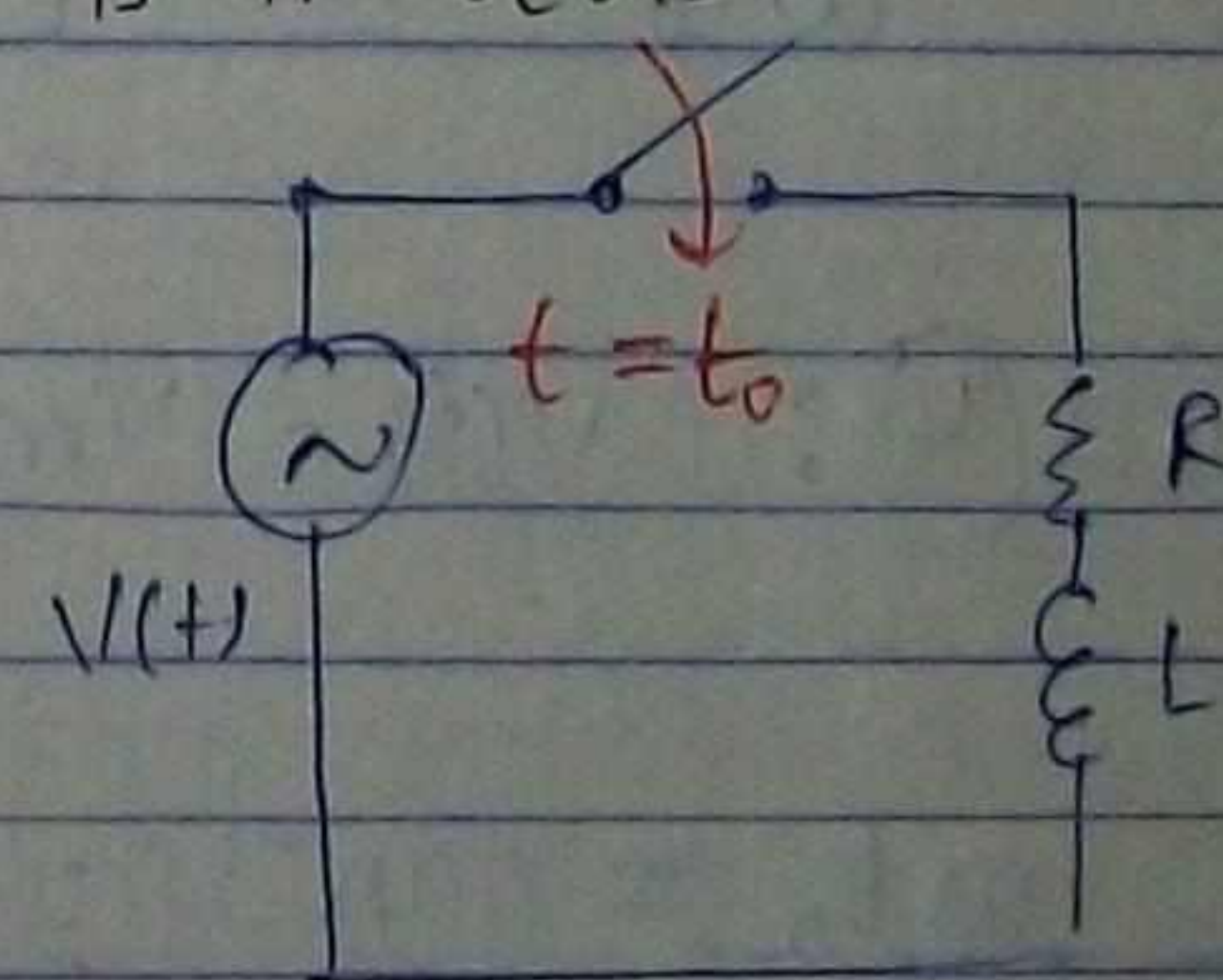
FOR
TRANSIENT FREE
RESPONSE

CASE (3) : WHEN SWITCHING TIME IS INVOLVED -

$$i(t) = C.F + P.I$$

$$i(t) = A e^{-R/L (t-t_0)}$$

$$+ \frac{V_m}{Z} \sin(\omega t + \theta - \alpha)$$



$$t = t_0^- \quad i = 0$$

$$t = t_0 \quad i = 0$$

$$0 = A + \frac{V_m}{Z} \sin(\omega t_0 + \theta - \alpha)$$

$$A = -\frac{V_m}{Z} \sin(\omega t_0 + \theta - \alpha)$$

$$i(t) = -\frac{V_m}{Z} \sin(\omega t_0 + \theta - \alpha) e^{-\frac{R}{L}(t-t_0)} + \frac{V_m}{Z} \sin(\omega t + \theta - \alpha)$$

CASE (3) : $v(t) = V_m \sin(\omega t + \theta)$ $t = t_0$

Now, $\omega t_0 + \theta - \alpha = 0$

$$\omega t_0 = \alpha - \theta$$

$$\omega t_0 = \tan^{-1}\left(\frac{\omega L}{R}\right) - \theta$$

CASE (4) : $v(t) = V_m \cos(\omega t + \theta)$ $t = t_0$

$$\omega t_0 = \tan^{-1}\left(\frac{\omega L}{R}\right) - \theta + \frac{\pi}{2}$$

CONDⁿ FOR
TRANSIENT FREE
RESPONSE

RL
CIRCUIT

GENERATED

RC
CIRCUIT

① $v(t) = V_m \sin(\omega t + \theta)$ $t = 0$	$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$ (ωt_0)	$\theta = \tan^{-1}(\omega \tau)$	$\theta = \tan^{-1}(\omega RC)$
---	---	-----------------------------------	---------------------------------

② $v(t) = V_m \cos(\omega t + \theta)$ $t = 0$	$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right) + \pi/2$	$\theta = \tan^{-1}(\omega \tau) + \pi/2$	$\theta = \tan^{-1}(\omega RC) + \pi/2$
---	---	---	---

③ $v(t) = V_m \sin(\omega t + \theta)$ $t = t_0$	$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$ (ωt_0) $-\theta$	$\theta = \tan^{-1}(\omega \tau)$ $-\theta$	$\theta = \tan^{-1}(\omega RC)$ $-\theta$
---	--	--	--

④ $v(t) = V_m \cos(\omega t + \theta)$ $t = t_0$	$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$ $-\theta + \pi/2$	$\theta = \tan^{-1}(\omega \tau)$ $-\theta + \pi/2$	$\theta = \tan^{-1}(\omega RC)$ $-\theta + \pi/2$
---	--	--	--

CONDITIONS FOR
TRANSIENT FREE
RL AND RC
SERIES CIRCUIT

for parallel RL
and RC circuit
instead of $v(t)$
we use

$$i(t) = I_m \sin(\omega t + \theta)$$

expression remain
same

⇒ NOTE -

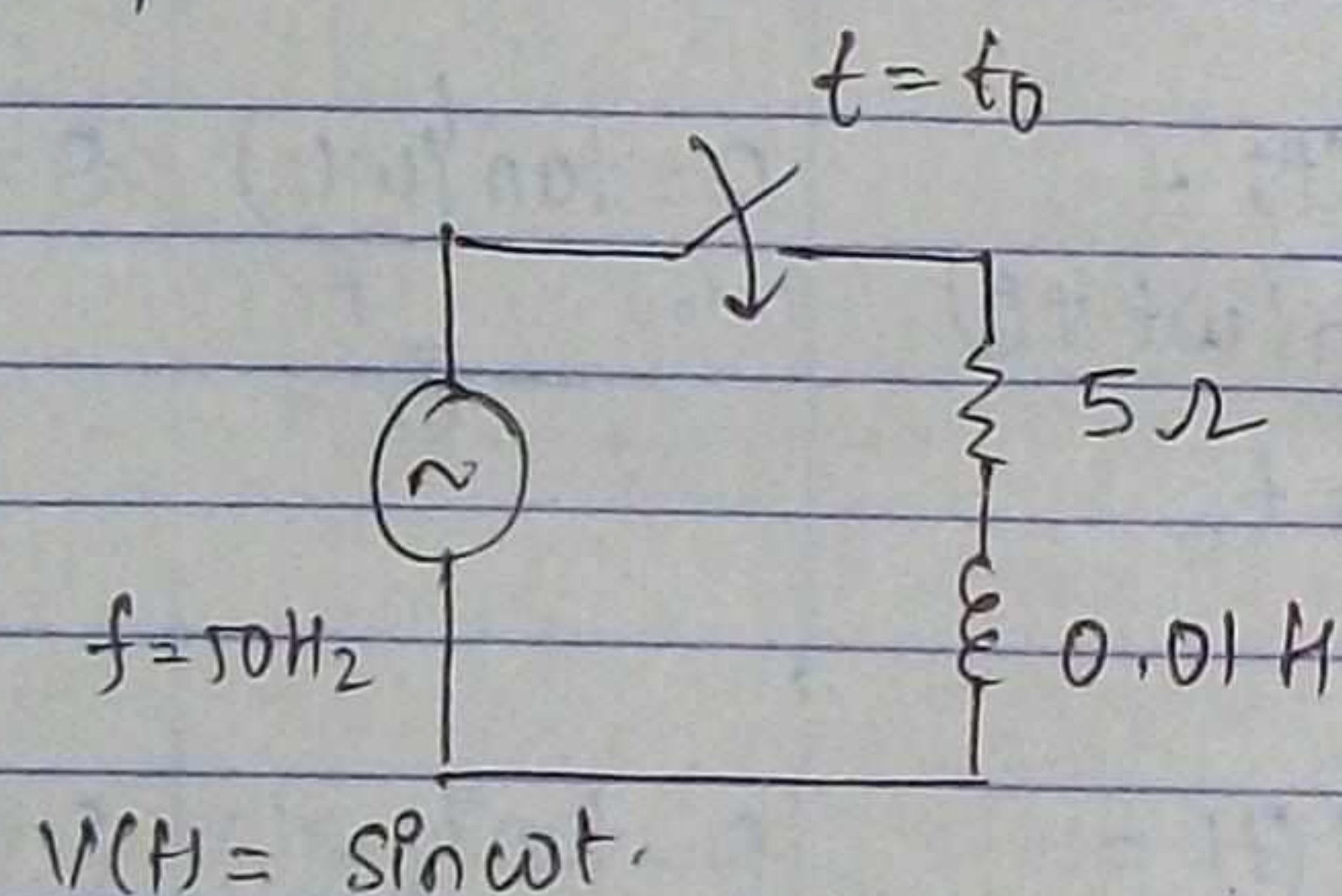
- In RLC circuit it is not possible to obtain transient free response since circuit is having two energy storage elements.

Ex - $i(t) = (C_1 \cos \beta t + C_2 \sin \beta t) e^{\alpha t}$

UNDER DAMPING

Both cannot be simultaneously zero

Ques. In circuit at what value of t_0 transient free response is obtain



Solⁿ:

$$\omega t_0 = \tan^{-1} \left(\frac{\omega L}{R} \right) - \theta$$

$$\theta = 0$$

$$t_0 = \frac{\tan^{-1} \left(\frac{\omega L}{R} \right)}{\omega}$$

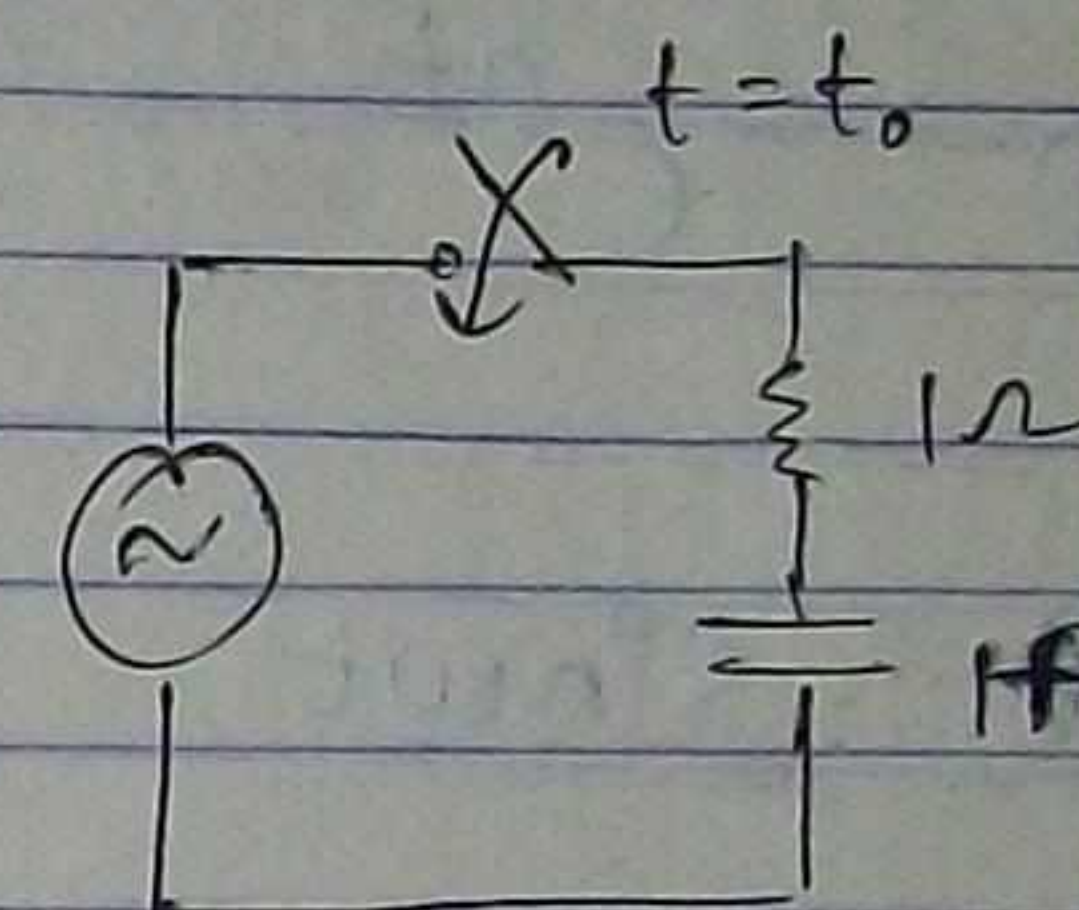
$$t_0 = \frac{\tan^{-1} \left(\frac{2\pi \times 50 \times 0.01}{5} \right)}{2\pi \times 50}$$

$$t_0 = \frac{32.1^\circ \times \pi / 180}{100\pi} \text{ radians}$$

$$t_0 = 1.78 \text{ ms}$$

Ques. At what value of t_0 transient free response is obtain

$$V(t) = \cos t$$



Solⁿ:

$$\omega t_0 = \tan^{-1} (\omega RC) - \theta + \pi/2$$

$$t_0 = \frac{\tan^{-1} (1) + \pi/2}{\omega}$$

$$t_0 = \frac{\pi}{4} + \frac{\pi}{2} = \frac{3\pi}{4}$$

LAPLACE TRANSFORM

$$(1) \quad A \longrightarrow \frac{A}{s}$$

$$(2) \quad e^{at} \longrightarrow \frac{1}{s-a}$$

$$(3) \quad \sin \omega t \longrightarrow \frac{\omega}{s^2 + \omega^2}$$

$$(4) \quad \cos \omega t \longrightarrow \frac{s}{s^2 + \omega^2}$$

$$(5) \quad e^{at} \sin \omega t \longrightarrow \frac{\omega}{(s-a)^2 + \omega^2}$$

$$(6) \quad e^{-at} \cos \omega t \longrightarrow \frac{s}{(s+a)^2 + \omega^2}$$

$$(7) \quad \frac{d f(t)}{dt} \longrightarrow s F(s) - F(0^+)$$

$$(8) \quad u(t) \longrightarrow \frac{1}{s}$$

$$(9) \quad \delta(t) \longrightarrow 1$$

INITIAL VALUE THEOREM -

$$f(0) = \lim_{s \rightarrow \infty} s F(s) = \lim_{t \rightarrow 0} f(t)$$

Que. find initial value of

$$F(s) = \frac{(2s+1)(s+3)}{s(s+2)(s+4)}$$

$$\begin{aligned} \text{Sol}^n: f(0) &= \lim_{s \rightarrow \infty} s F(s) \\ &= \lim_{s \rightarrow \infty} \frac{s(2+1/s)(1+3/s)}{s(1+2/s)(1+4/s)} \\ &= \frac{2}{1} = 2 \end{aligned}$$

Que. find initial value of

$$f(t) = 5 + \delta(t)$$

$$\text{Sol}^n: f(s) = \frac{5}{s} + 1$$

Cannot be determined

(*) Initial value theorem cannot be applied for above funⁿ. since to apply the initial value theorem

$\left. \begin{array}{l} \text{denominator} \\ \text{power} \end{array} \right\} \begin{array}{l} \text{Numerator} \\ \text{power} \end{array}$

⇒ FINAL VALUE THEOREM -

$$f(\infty) = \lim_{s \rightarrow 0} s F(s) = \lim_{t \rightarrow \infty} f(t)$$

Ques. find final value of

$$F(s) = \frac{(s+1)(s+3)}{s(s+2)(s+4)}$$

Solⁿ:

$$f(\infty) = \lim_{s \rightarrow 0} s F(s) = \underline{\underline{\frac{3}{8}}}$$

Ques. find final value of

$$f(t) = 6 + e^{2t}$$

Solⁿ:

$$f(s) = \frac{6}{s} + \frac{1}{s-2}$$

* for the above funⁿ final value theorem cannot be applied since pole is present in RHS of the plane. (UNSTABLE SYSTEM).

Ques. find final value of

$$f(s) = \frac{\omega}{s^2 + \omega^2}$$

- ① 0 ② 1 ③ ∞ ④ ~~NONE~~

Solⁿ: $f(t) = \sin t$

lies b/w -1 and +1

* for marginally stable system also final value theorem cannot be applied.

Ques. Current flow in 4H inductor is given by $I(s) = \frac{10}{s(s+2)}$, find initial voltage of inductor.

Solⁿ:

$$V = L \frac{di}{dt} \cdot \frac{10}{s(s+2)}$$

$$V = L \frac{di}{dt}$$

$$V(s) = L [s I(s) - I(0^+)]$$

$$V(s) = 4 \left[\frac{s \times 10}{s(s+2)} - I(0^+) \right]$$

$$V(s) = 4 \left[\frac{10}{s+2} - I(0^+) \right]$$

$$V(s) = \frac{40}{s+2}$$

$$I(s) = \frac{10}{s(s+2)}$$

$$I(0^+) = \lim_{s \rightarrow \infty} s I(s) = \frac{10}{\infty} = 0$$

$$V(0^+) = \lim_{s \rightarrow \infty} sV(s) = \lim_{s \rightarrow \infty} \frac{40s}{(s+2)} = 40$$

⇒ CURRENT IN INDUCTOR -

$$i_L = \frac{1}{L} \int_{-\infty}^t v dt$$

$$i_L = \frac{1}{L} \int_{-\infty}^{0^-} v dt + \frac{1}{L} \int_{0^-}^t v dt$$

$$i_L(t) = i_L(0^-) + \frac{1}{L} \int_{0^-}^t v dt$$

at $t = 0^+$

$$i_L(0^+) = i_L(0^-) + \frac{1}{L} \int_{0^-}^{0^+} v dt$$

$$i_L(0^+) = i_L(0^-)$$

From above relation it is concluded that inductor does not allow instantaneous changes from given input.

$$i_L(0^+) = i_L(0^-) + \frac{1}{L} \int_{0^-}^{0^+} v dt$$

when $v \rightarrow \delta(t)$

$$i_L(0^+) = i_L(0^-) + \frac{1}{L} \int_{0^-}^{0^+} \delta(t) dt$$

area = 1

$$i_L(0^+) = \frac{1}{L}$$

ENERGY -

$$w_L(0^+) = \frac{1}{2} L i_L^2(0^+)$$

$$w_L(0^+) = \frac{1}{2} L \left(\frac{1}{L} \right)^2$$

$$w_L(0^+) = \frac{1}{2L}$$

* Inductor allows instantaneous change for voltage impulse function.

⇒ VOLTAGE ACROSS CAPACITOR -

$$V_c = \frac{1}{C} \int_{-\infty}^t i dt$$

$$V_c = \frac{1}{C} \int_{-\infty}^{0^-} i dt + \frac{1}{C} \int_{0^-}^t i dt$$

$$V_c(t) = V_c(0^-) + \frac{1}{C} \int_{0^-}^t i dt$$

at $t = 0^+$

$$V_c(0^+) = V_c(0^-) + \frac{1}{C} \underbrace{\int_{0^-}^{0^+} i dt}_0$$

$$V_c(0^+) = V_c(0^-)$$

⊛ from the above relation it is concluded that capacitor does not allow instantaneous change for given I/P.

$$V_c(0^+) = V_c(0^-) + \frac{1}{C} \int_{0^-}^{0^+} i dt$$

when $i \rightarrow \delta(t)$

$$V_c(0^+) = V_c(0^-) + \frac{1}{C} \int_{0^-}^{0^+} \delta(t) dt$$

AREA = 1

⊛

$$V_c(0^+) = \frac{1}{C}$$

ENERGY -

$$W_c(0^+) = \frac{1}{2} C V_c^2(0^+)$$

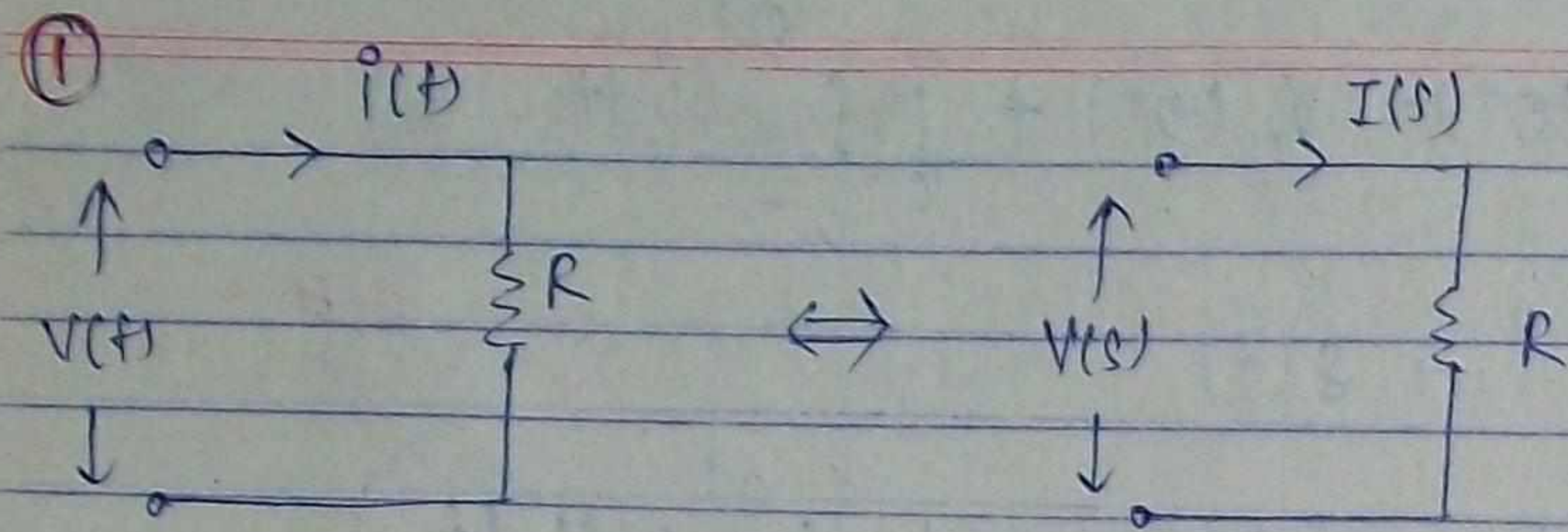
$$W_c(0^+) = \frac{1}{2} C \left(\frac{1}{C}\right)^2$$

⊛

$$W_c(0^+) = \frac{1}{2C}$$

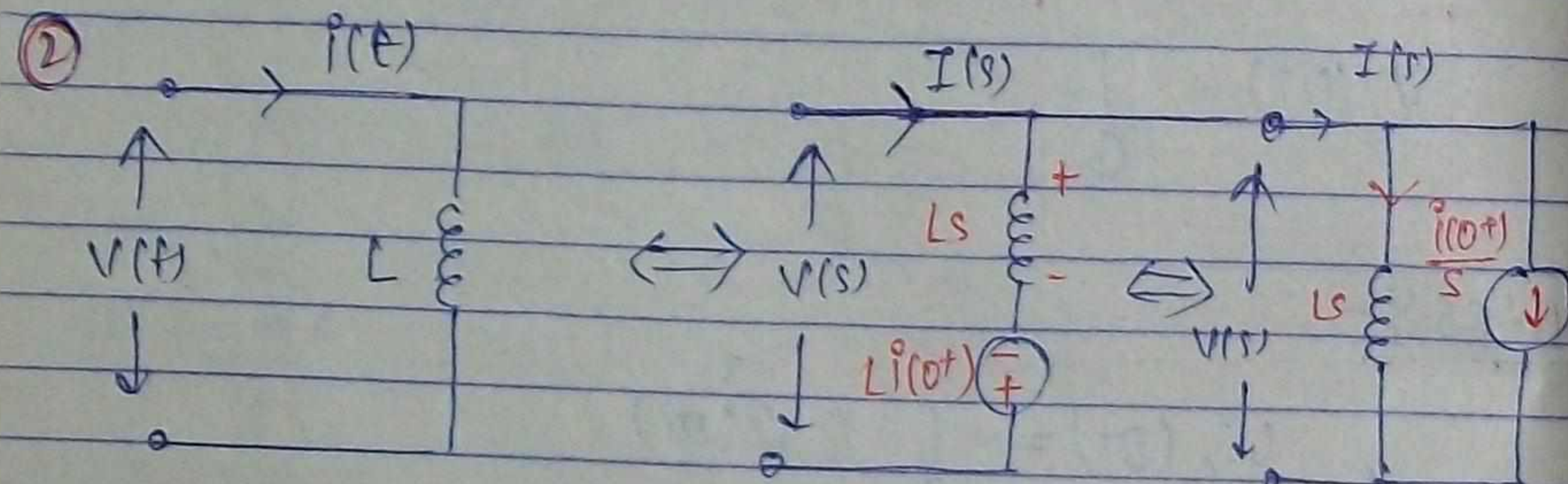
⊛ from the above relation it is concluded that capacitor allow instantaneous change for current impulse function.

⇒ LAPLACE TRANSFORMATION OF ELEMENTS ~



$$V(t) = R i(t)$$

$$V(s) = R I(s)$$



$$V = L \frac{di}{dt}$$

$$V(s) = L [s I(s) - i(0^+)]$$

$$V(s) = LS I(s) - L i(0^+)$$

(KVL)

Amp/sec

TRANSFORMED
IMPEDANCE

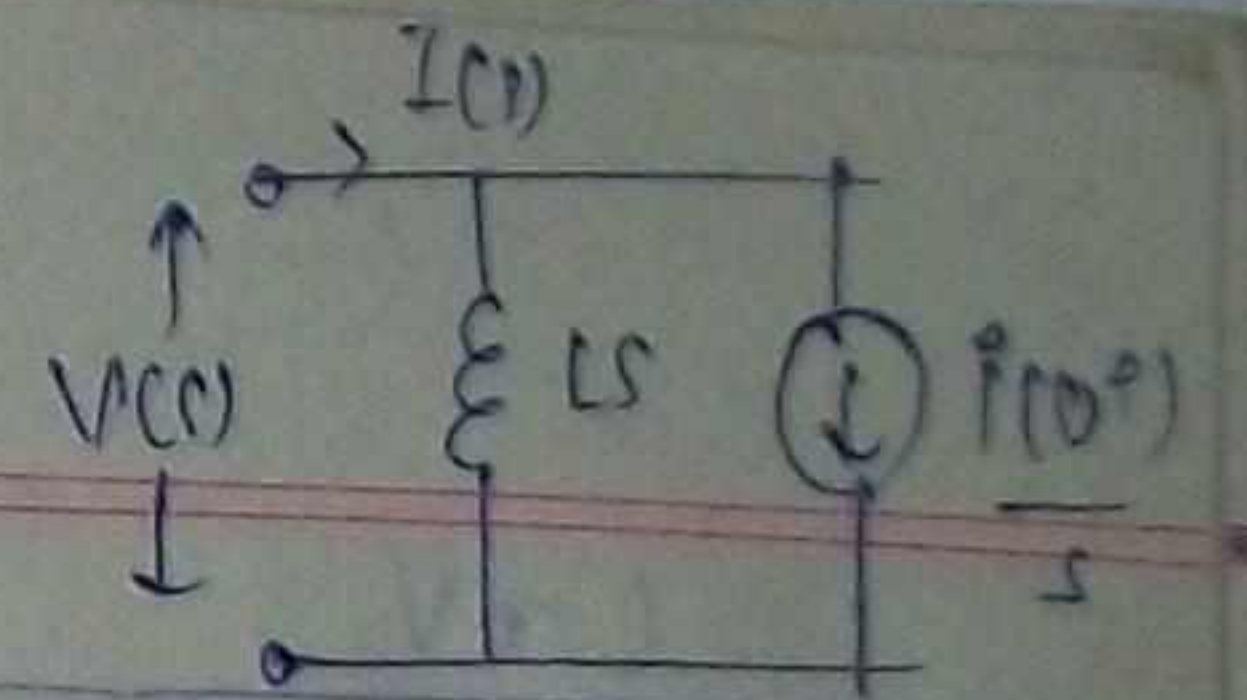
$$Z(s) = \frac{V(s)}{I(s)}$$

$$Z(s) = LS - \frac{L i(0^+)}{I(s)}$$

$$LS I(s) = V(s) + L i(0^+)$$

$$I(s) = \frac{V(s)}{LS} + \frac{i(0^+)}{s}$$

(KCL)

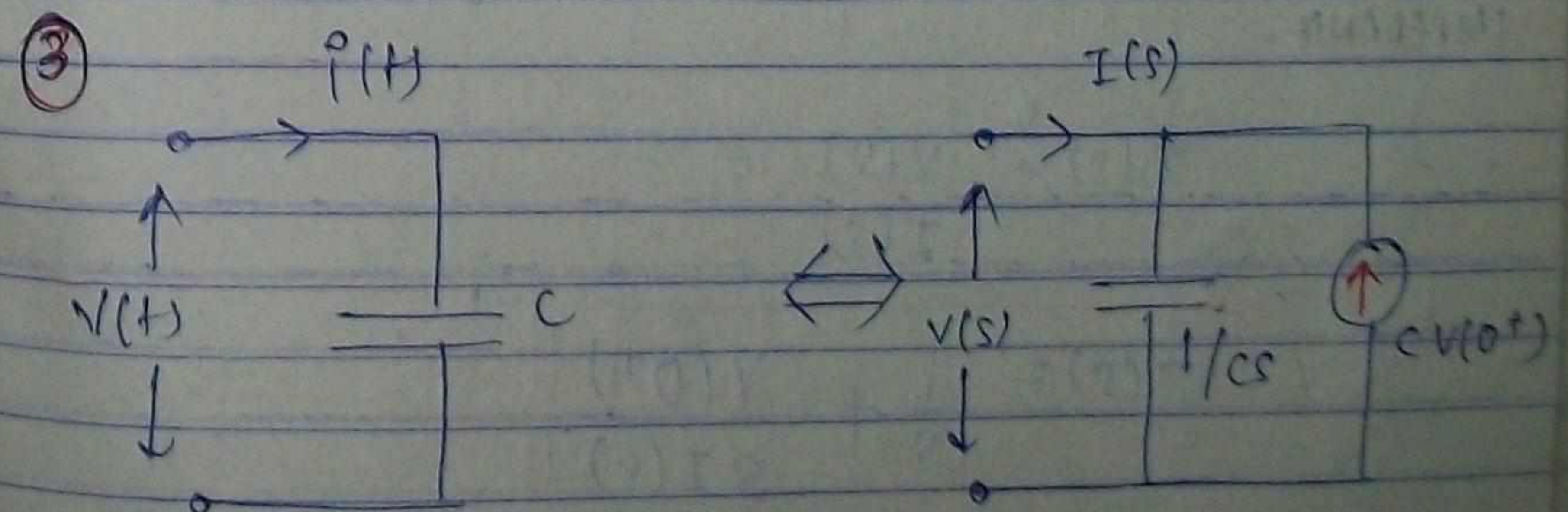
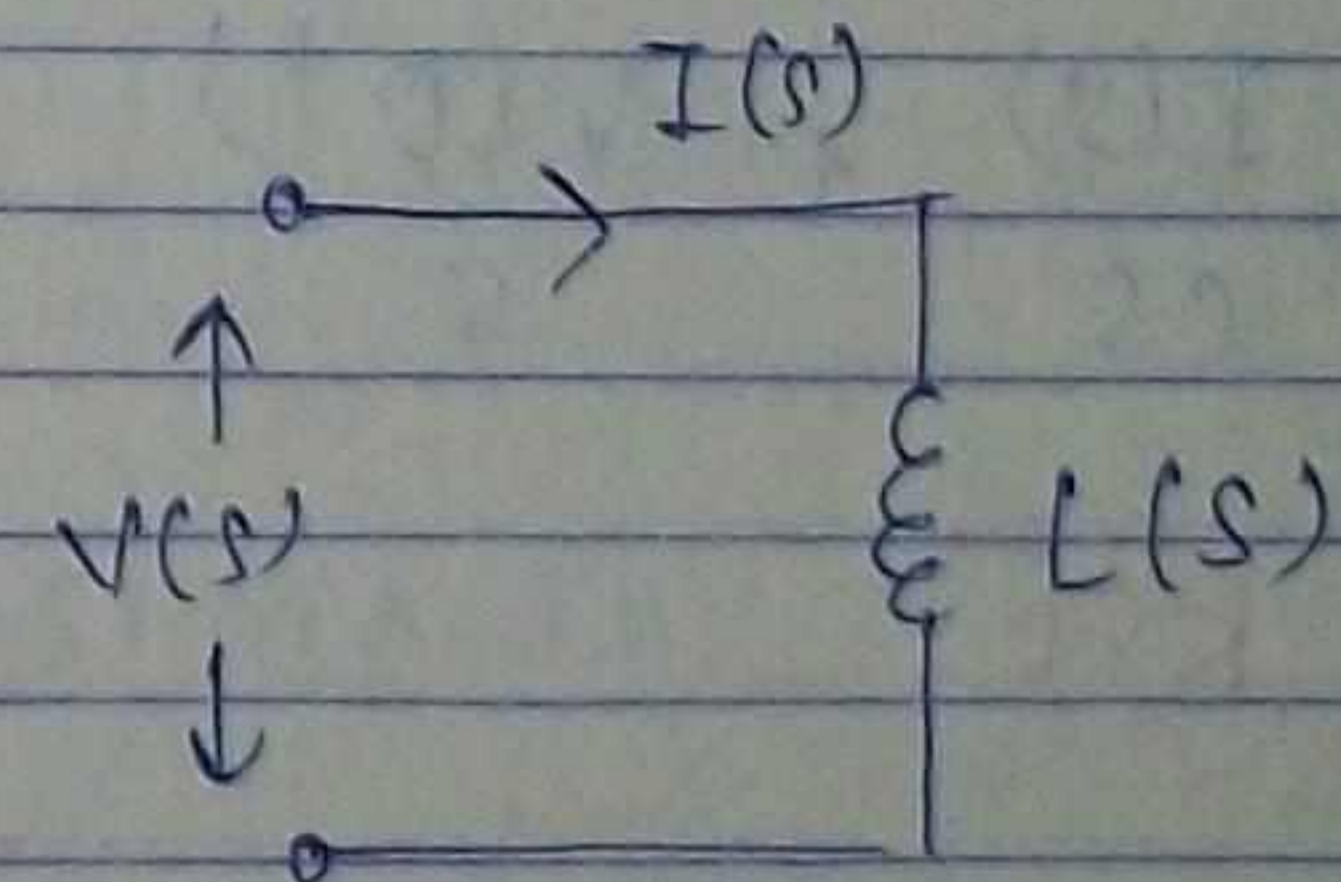


TRANSFORMED
ADMITTANCE

$$Y(s) = \frac{I(s)}{V(s)}$$

$$Y(s) = \frac{1}{LS} + \frac{i(0^+)}{s V(s)}$$

for $i(0^+) = 0$



$$i = C \frac{dv}{dt}$$

$$I(s) = C [sV(s) - v(0^+)]$$

$$I(s) = \frac{V(s)}{1/Cs} - C v(0^+)$$

(KCL)

$$\frac{V(s)}{1/Cs} = I(s) + C v(0^+)$$

$$V(s) = \frac{I(s)}{Cs} + \frac{v(0^+)}{s}$$

(KVL)

TRANSFORMED
IMPEDANCE -

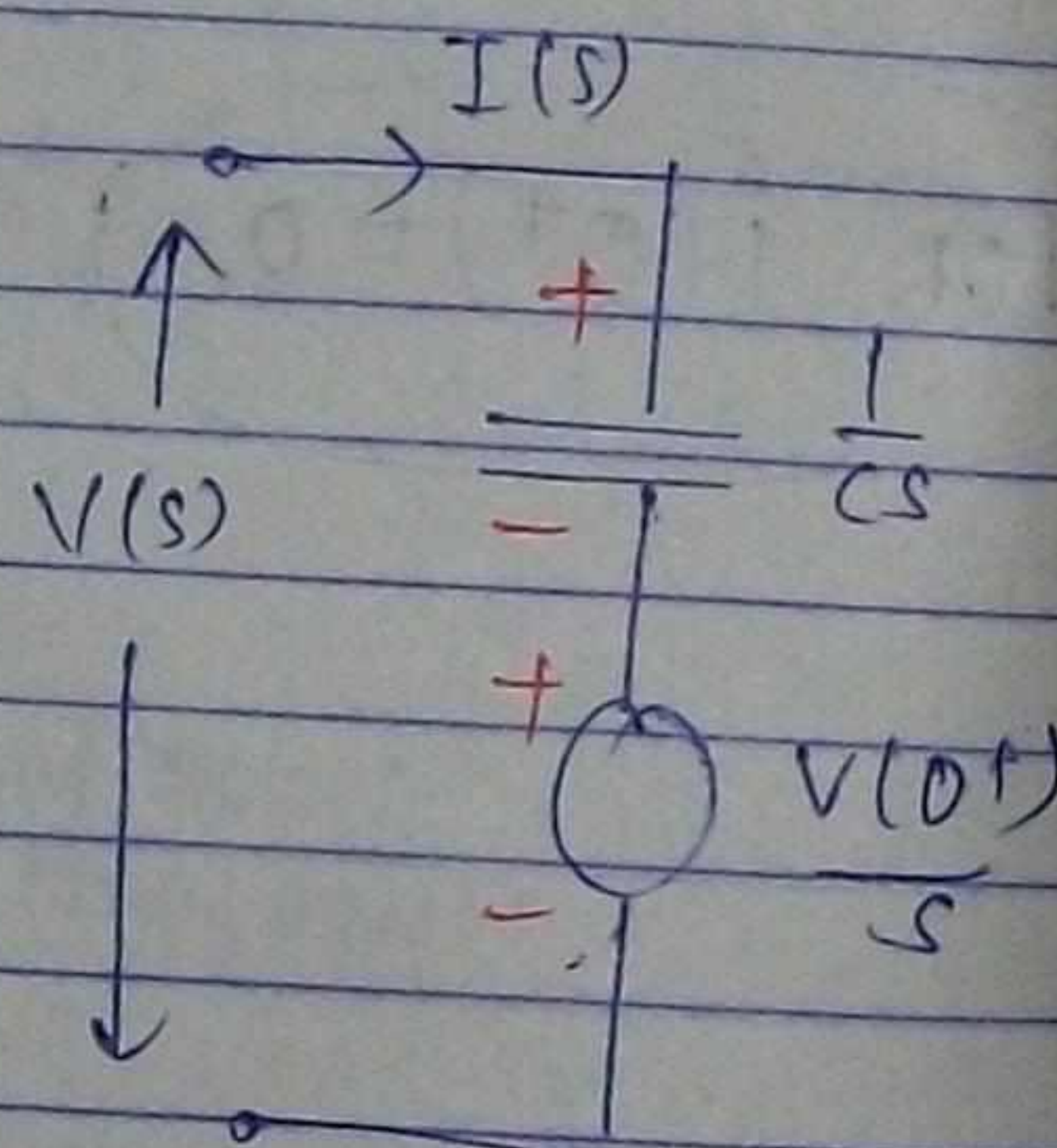
$$Z(s) = \frac{V(s)}{I(s)}$$

$$Z(s) = \frac{1}{Cs} + \frac{v(0^+)}{s I(s)}$$

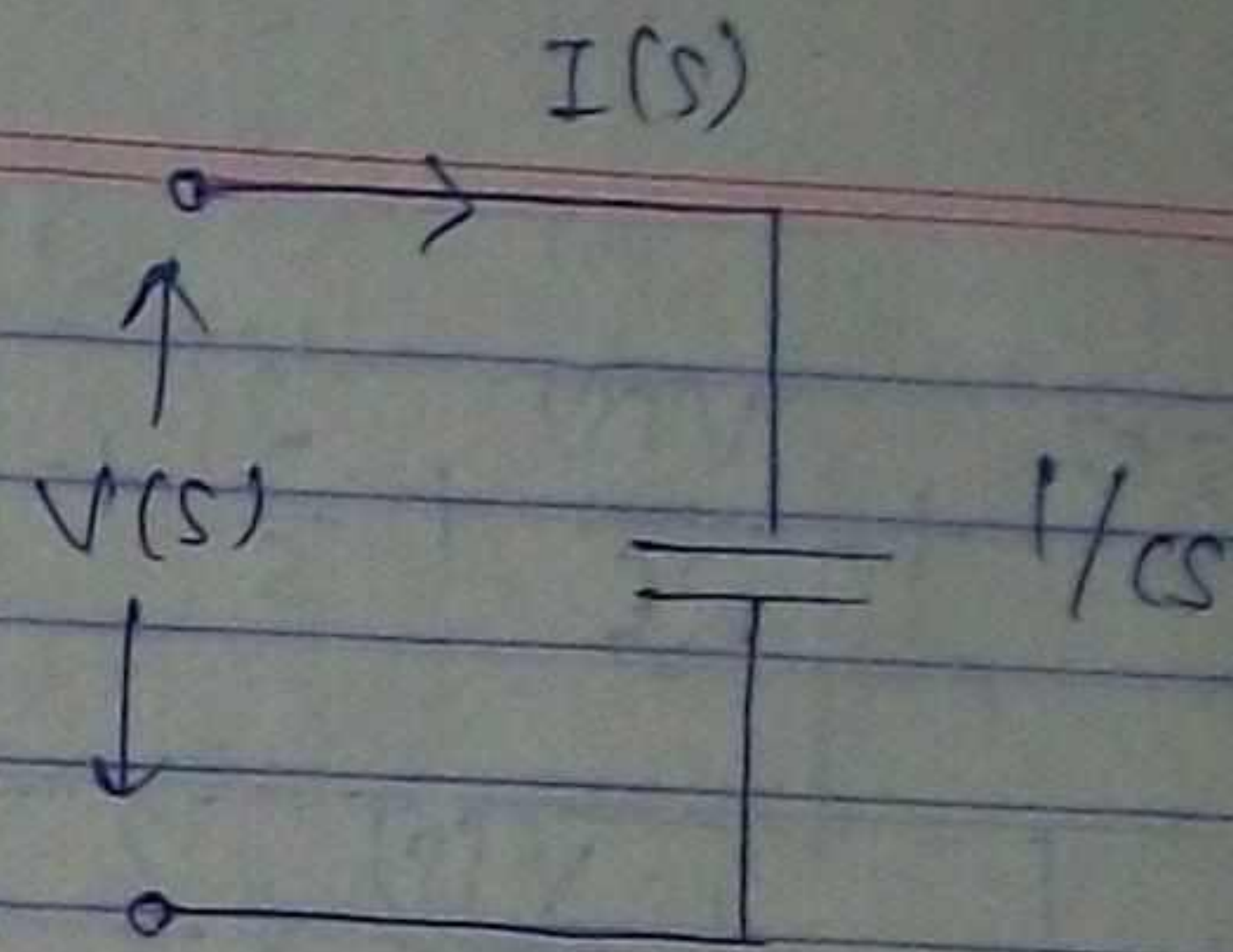
TRANSFORMED
ADMITTANCE

$$Y(s) = \frac{I(s)}{V(s)}$$

$$Y(s) = Cs - \frac{C v(0^+)}{V(s)}$$

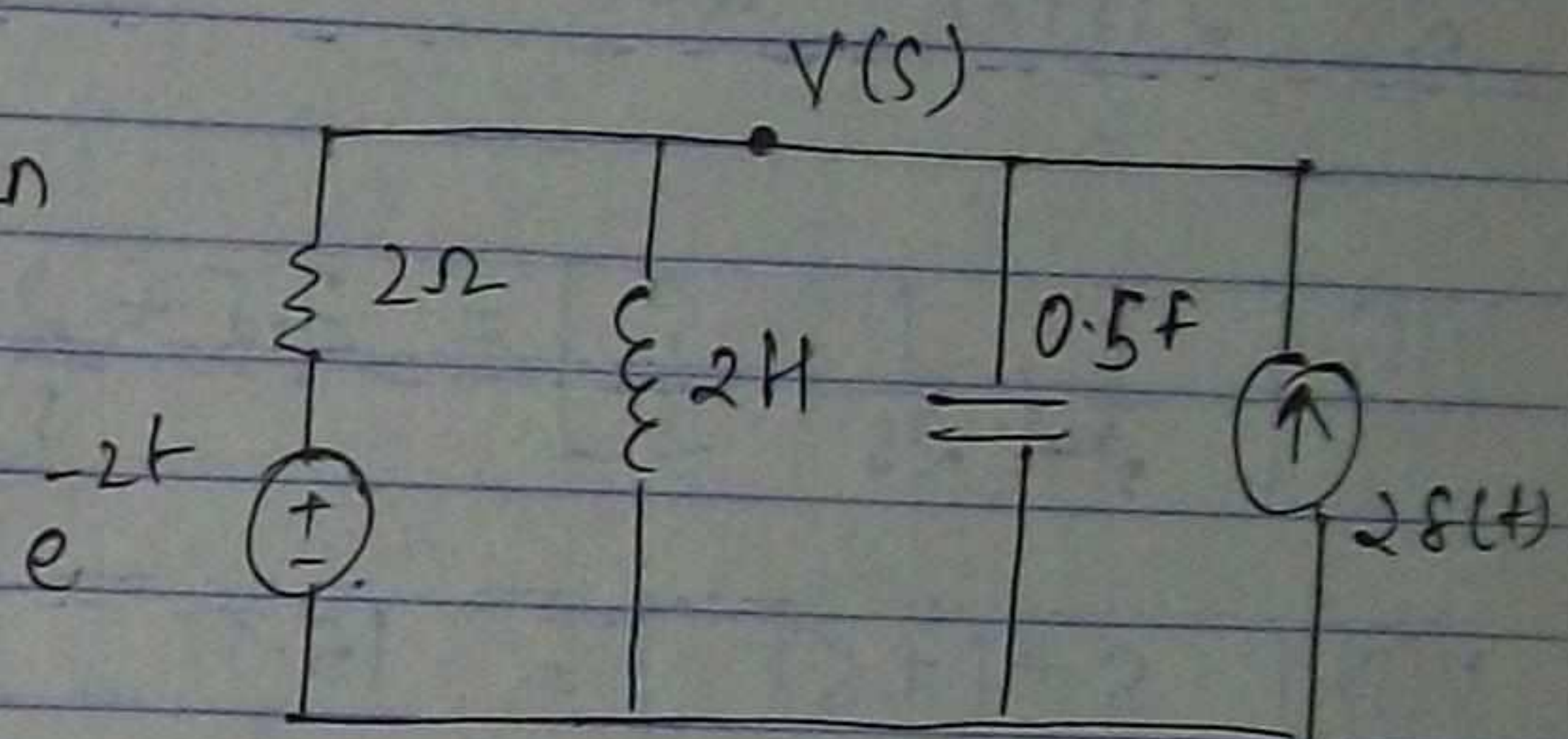


for $v(0^+) = 0$



Ques. find $V(s)$ when
 $i_L(0^-) = 2A$

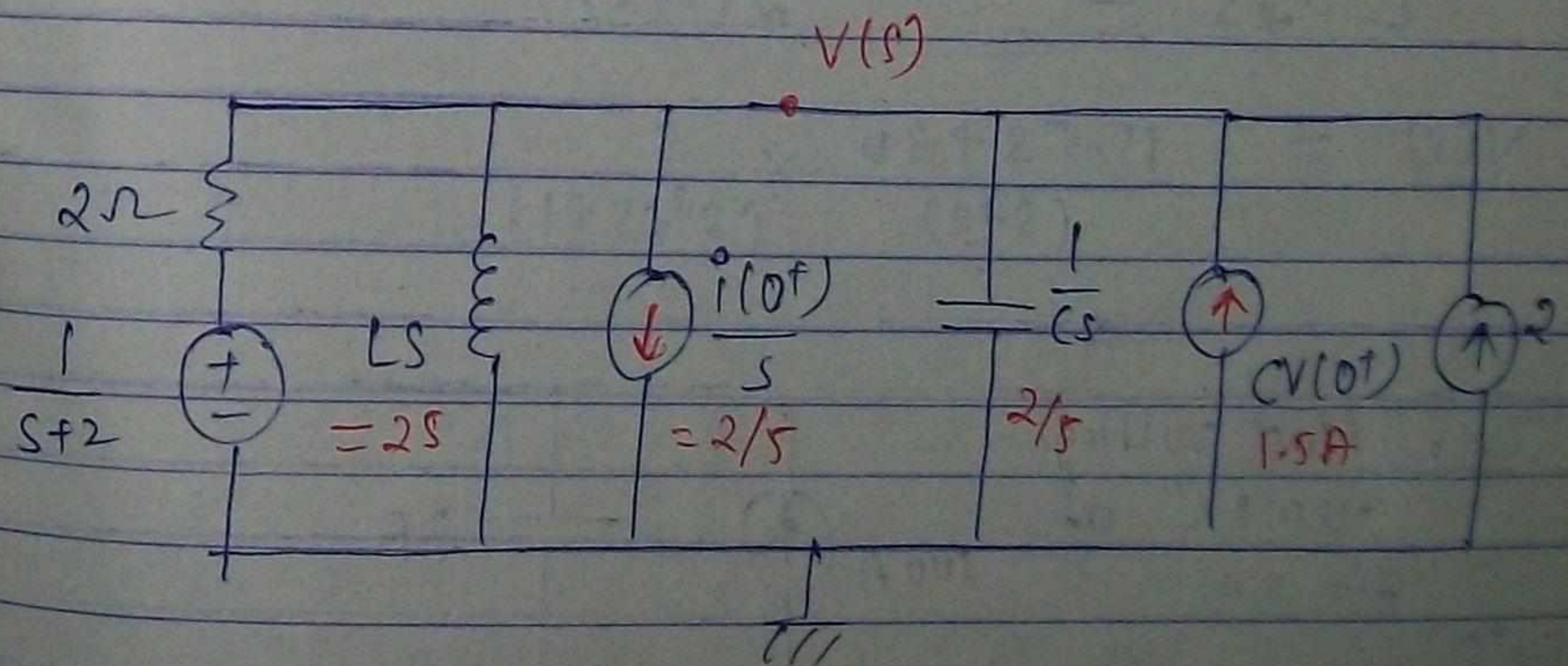
$V_C(0^-) = 3V$



Soln:

$$\frac{V(t) - e^{-2t}}{2} + \frac{1}{2} \int_{-\infty}^t V(t) dt + 0.5 \frac{dV(t)}{dt} = 2 \delta(t)$$

(or) converting into s-domain.



$$\frac{V(s) - \frac{1}{s+2}}{2} + \frac{V(s)}{2s} + \frac{2}{s} + \frac{V(s)}{2/s} = 1.5 + 2$$

$$\frac{V(s)}{2} - \frac{1}{2(s+2)} + \frac{V(s)}{2s} + \frac{2}{s} + \frac{V(s)}{2/s} = 3.5$$

$$V(s) \left[\frac{1}{2} + \frac{1}{2s} + \frac{s}{2} \right] = 3.5 - \frac{2}{s} - \frac{1}{2(s+2)}$$

$$V(s) \left[\frac{s+1+s^2}{2s} \right] = \frac{15.5}{2} - \frac{1}{2(s+2)}$$

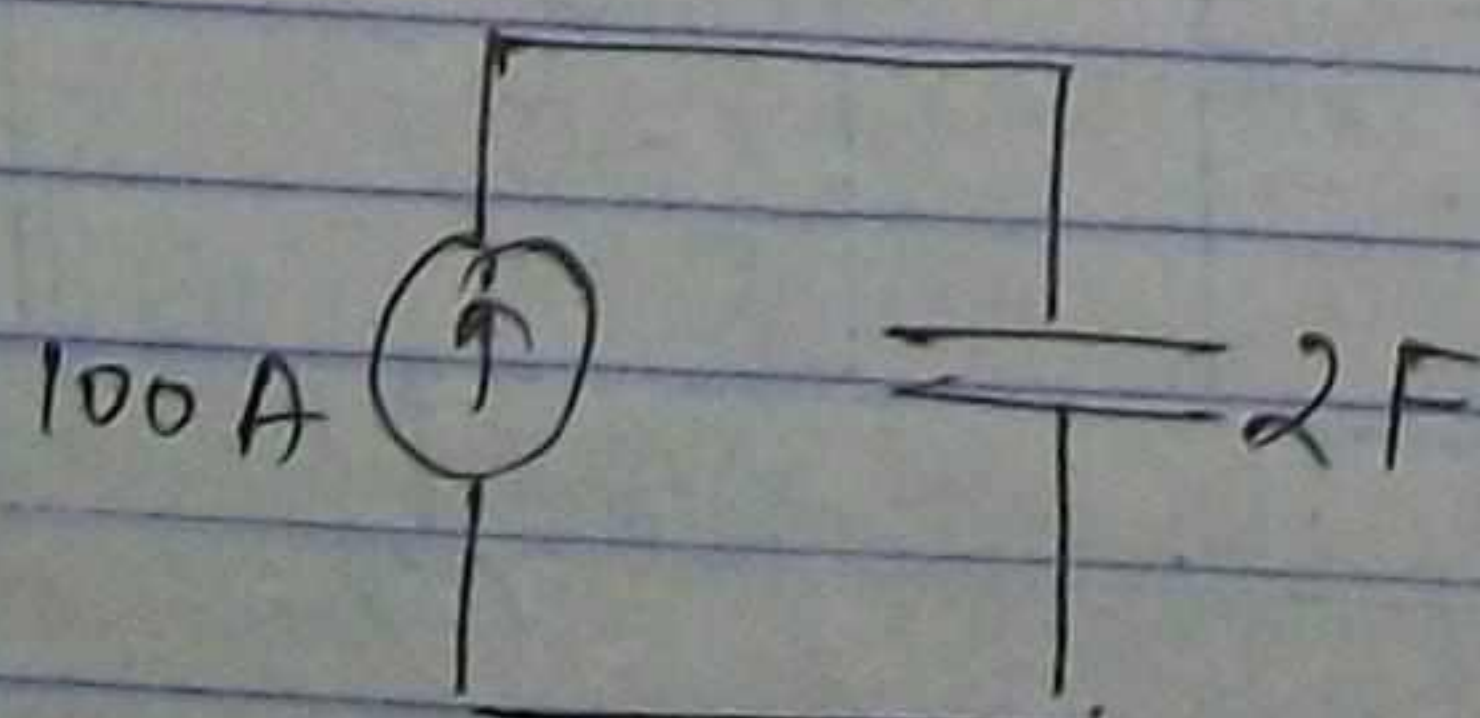
$$V(s) \left[\frac{s^2+s+1}{2s} \right] = \frac{15.5(s+2) - 1}{2(s+2)}$$

$$V(s) \left[\frac{s^2+s+1}{2s} \right] = \frac{15.5s + 31 - 1}{2(s+2)}$$

$$V(s) = \frac{15.5s + 30}{(s+2)} \times \frac{s}{(s^2+s+1)}$$

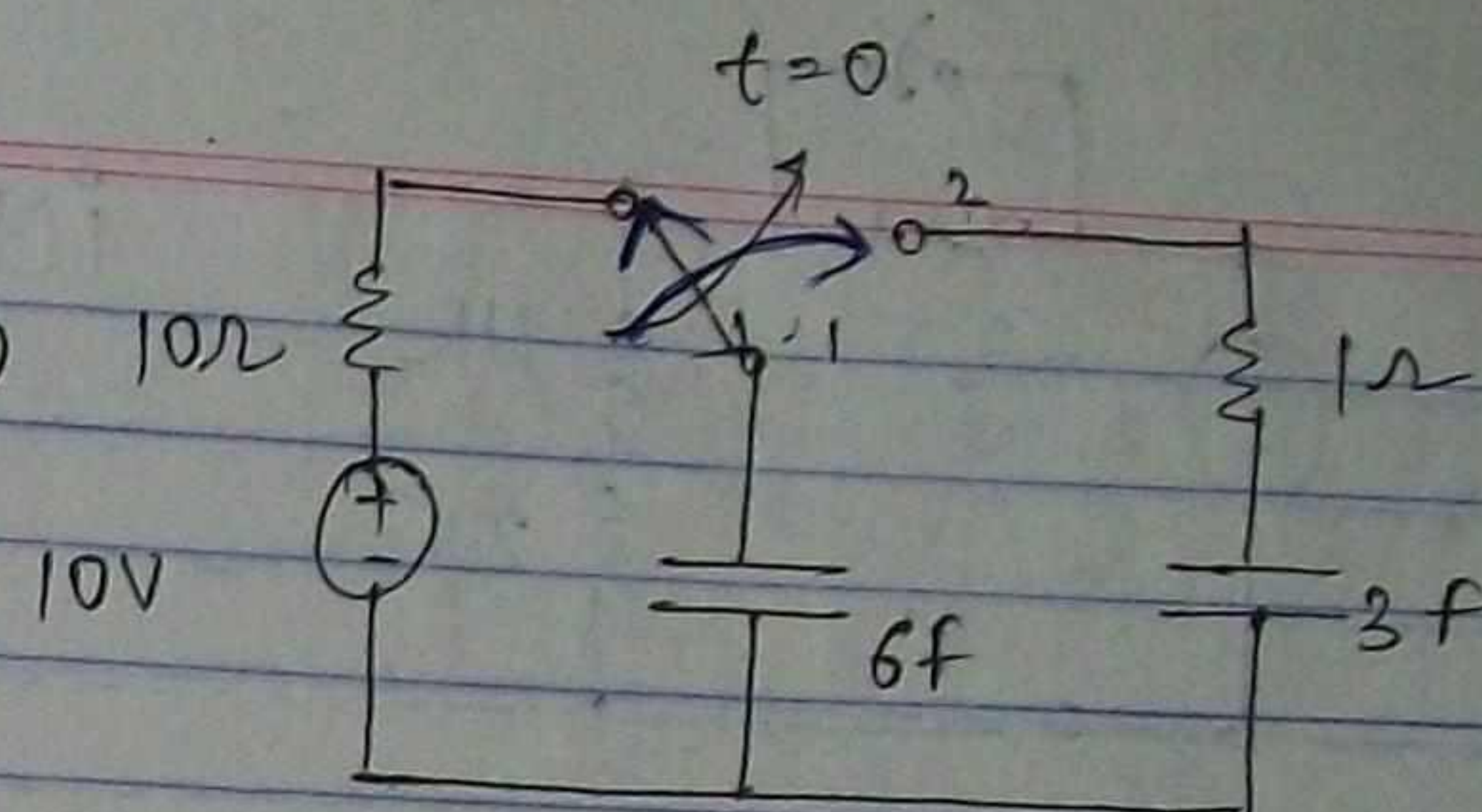
Ques. find voltage across C at $t = 3s$

Solⁿ:



Ques. find

$i(t), t > 0$

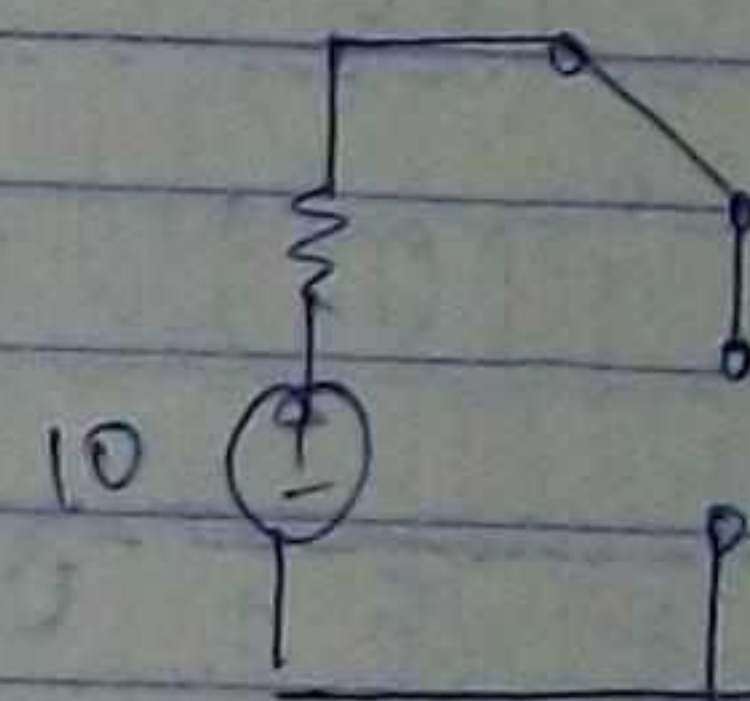


Solⁿ:

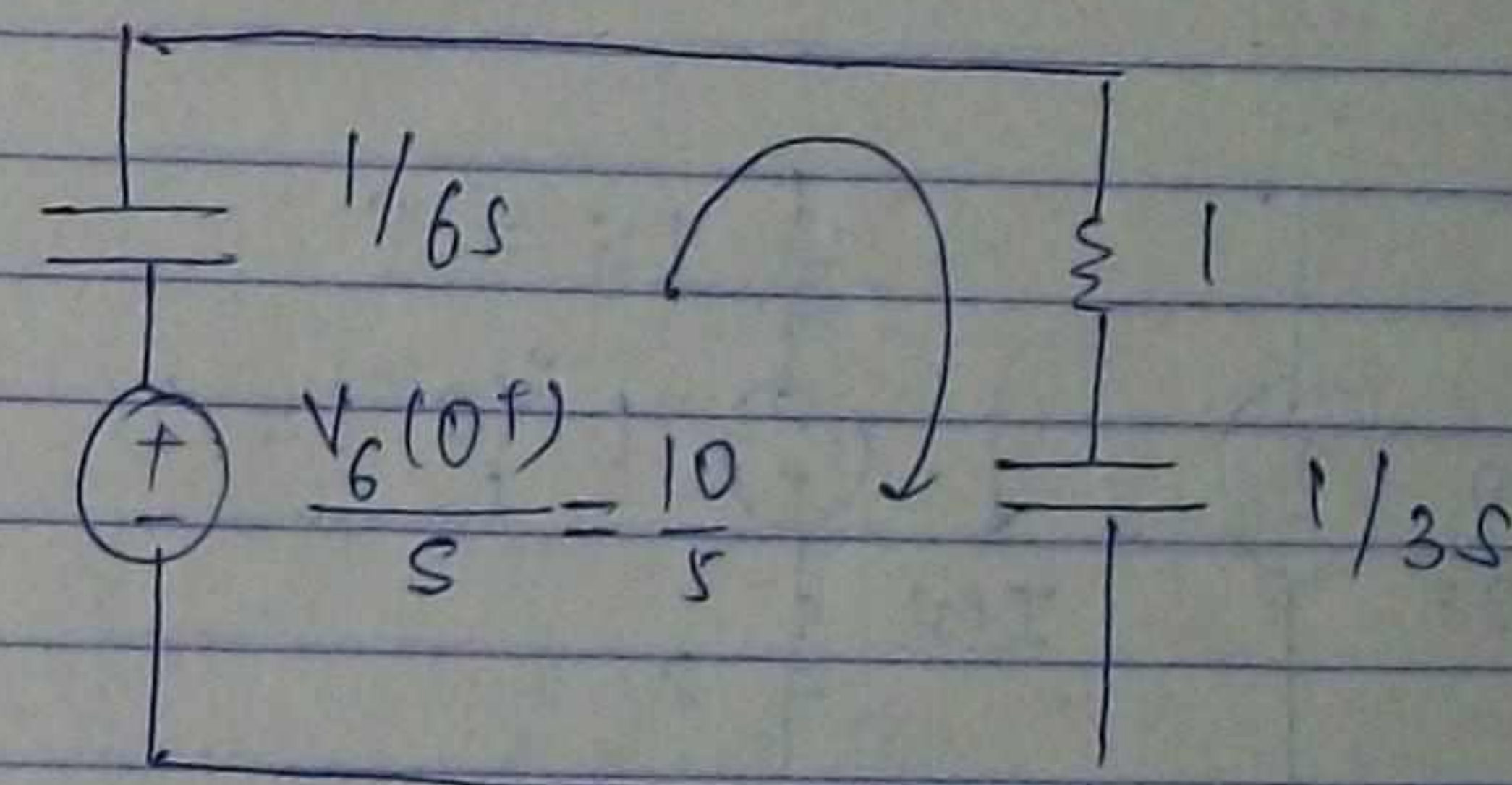
at $t = 0^-$

$$V_C(0^-) = 10V$$

$$V_C(0^-) = 0V$$



$t = 0^+$

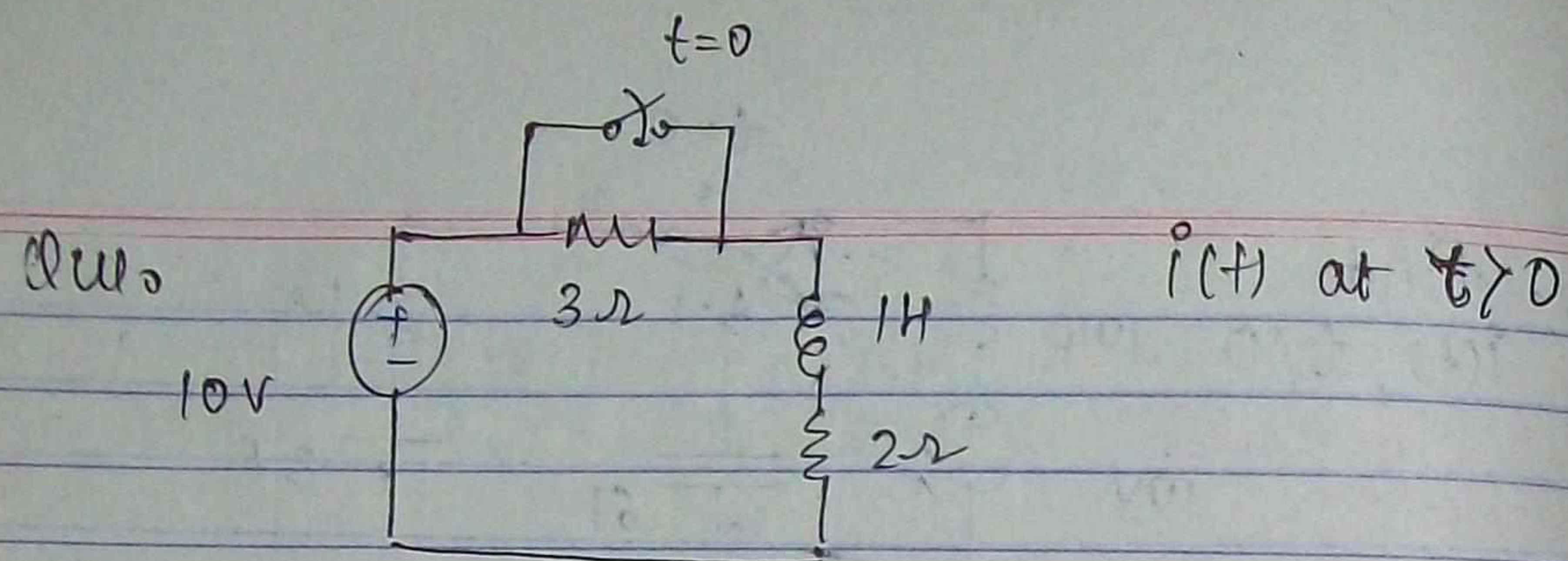


$$i(t) = \mathcal{L}^{-1} \left[I(s) = \frac{10/s}{1/3s + 1/6s + 1} \right]$$

$$= 10e^{-t/2}$$

(or)

$$i(t) = \frac{V_0}{R} e^{-t/RC} = \frac{10}{1} e^{-t/1 \times 2} = 10e^{-t/2}$$



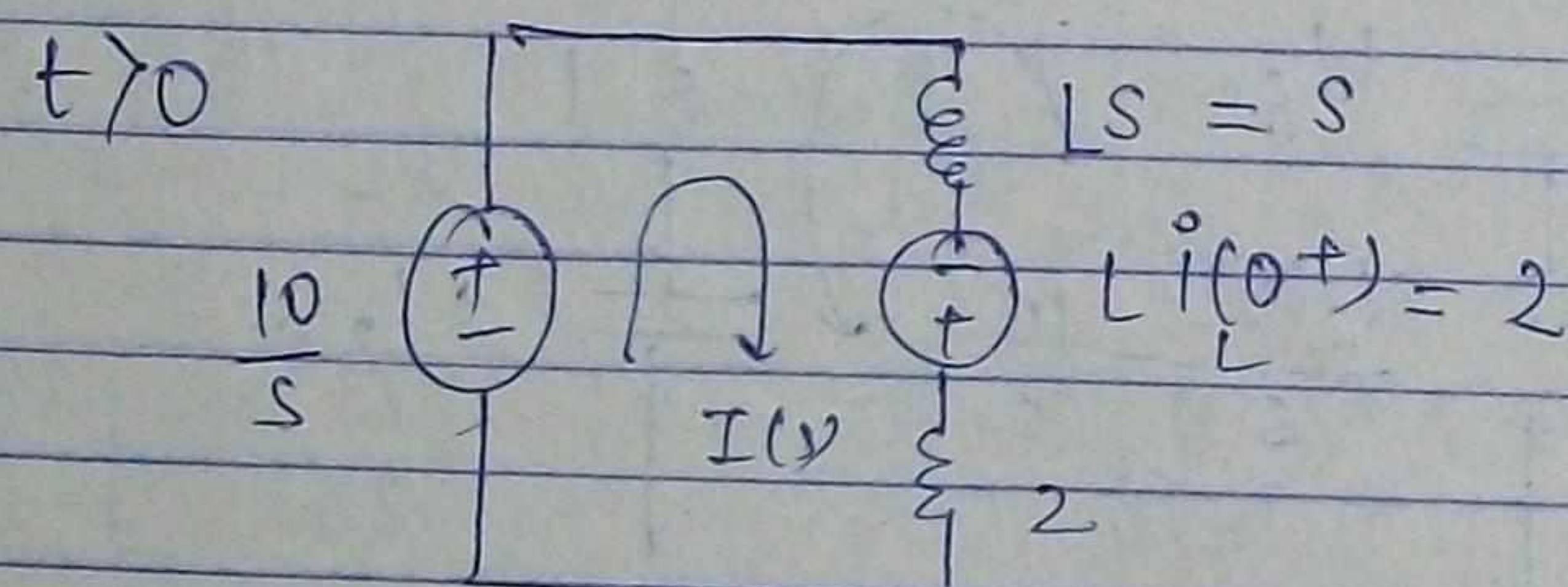
Solⁿs $i(0^-) = 0$

$i(0^+) = 0$ $i_1 = 2$ $i_f = 5$

$i(t) = 5 + (0-5)e^{-2t}$

$i(t) = 5(1 - e^{-2t})$ $5 - 5e^{-2t}$

OP

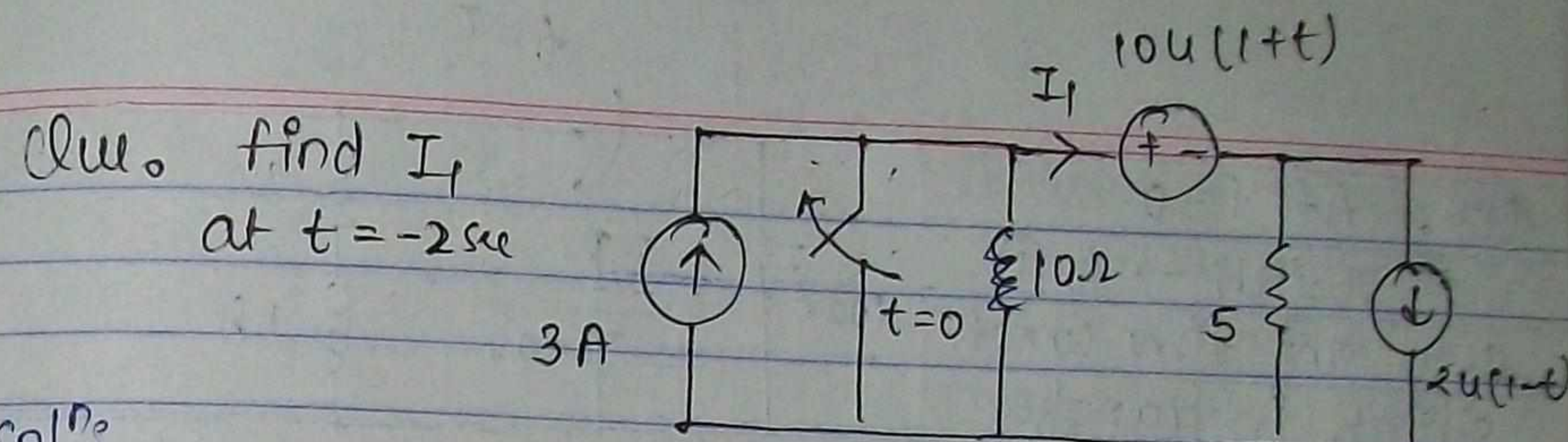


$I(s) = \frac{10}{s} + 2$
 $= \frac{10 + 2s}{s(s+2)}$

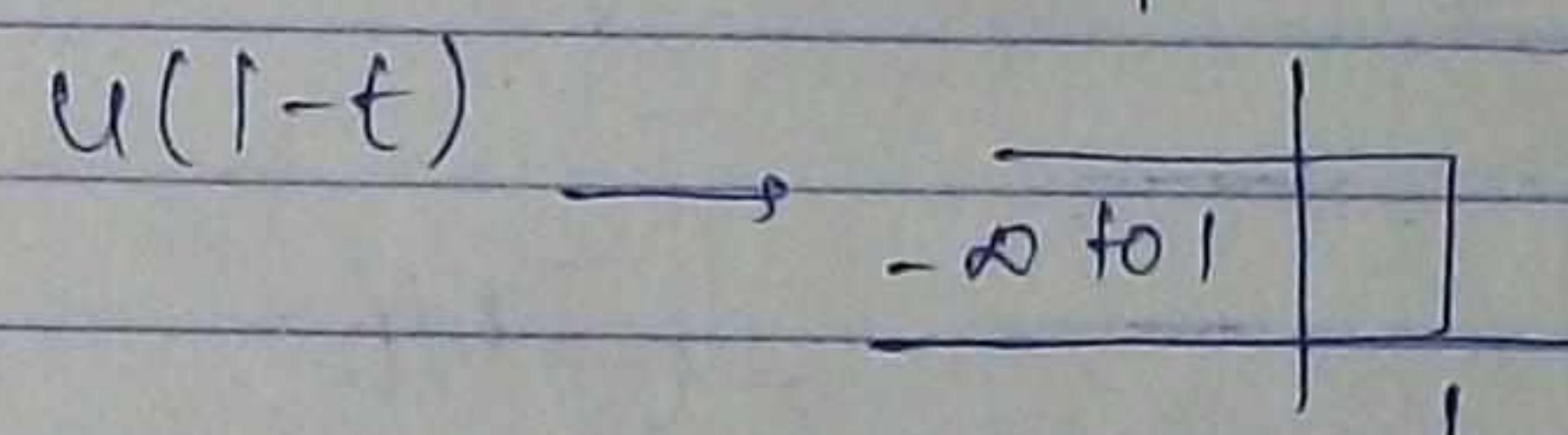
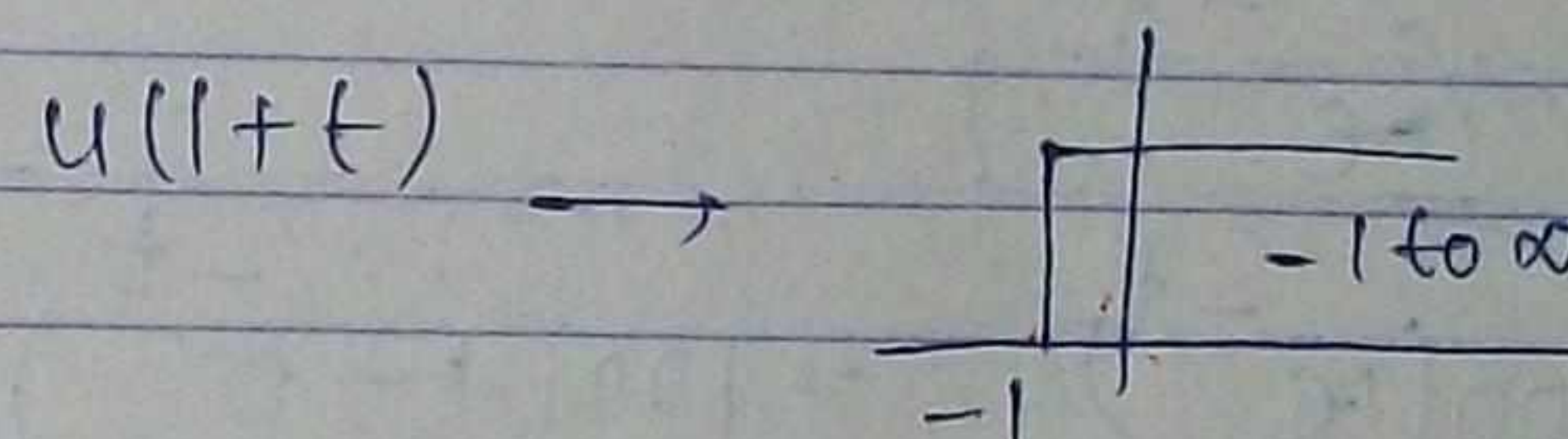
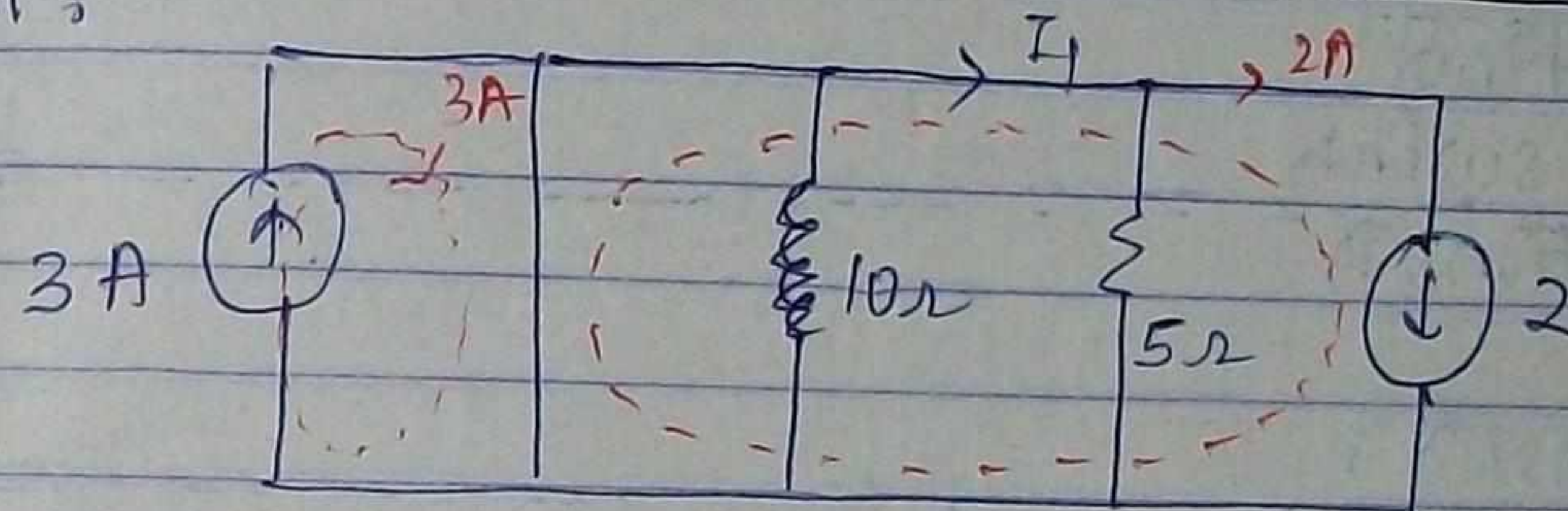
$i(t) = L^{-1} \left(\frac{5}{s} - \frac{3}{s+2} \right)$

$= 5 - 3e^{-2t}$

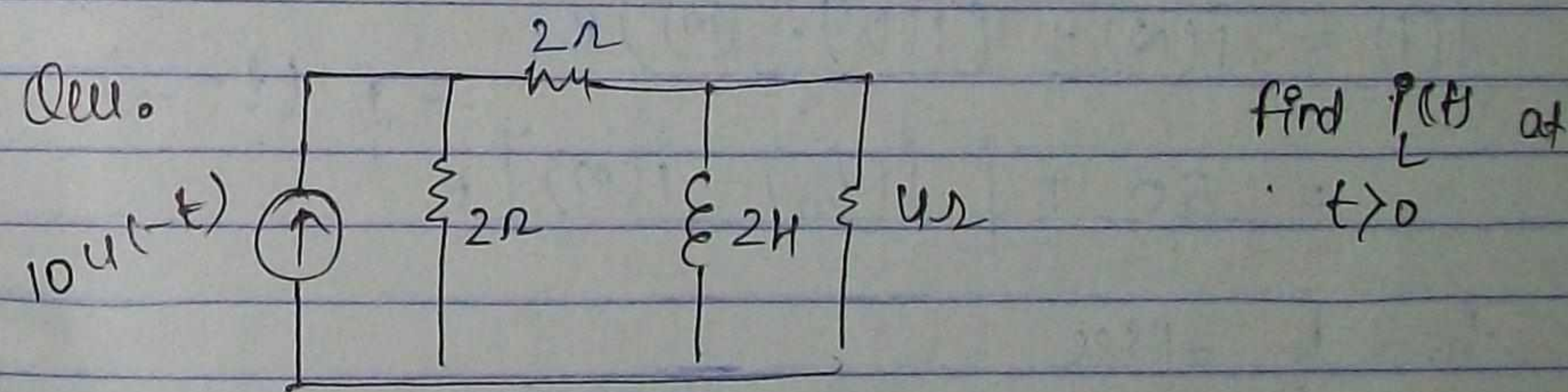
$\frac{V_c}{V} = \frac{V_L}{V} =$



Solⁿs



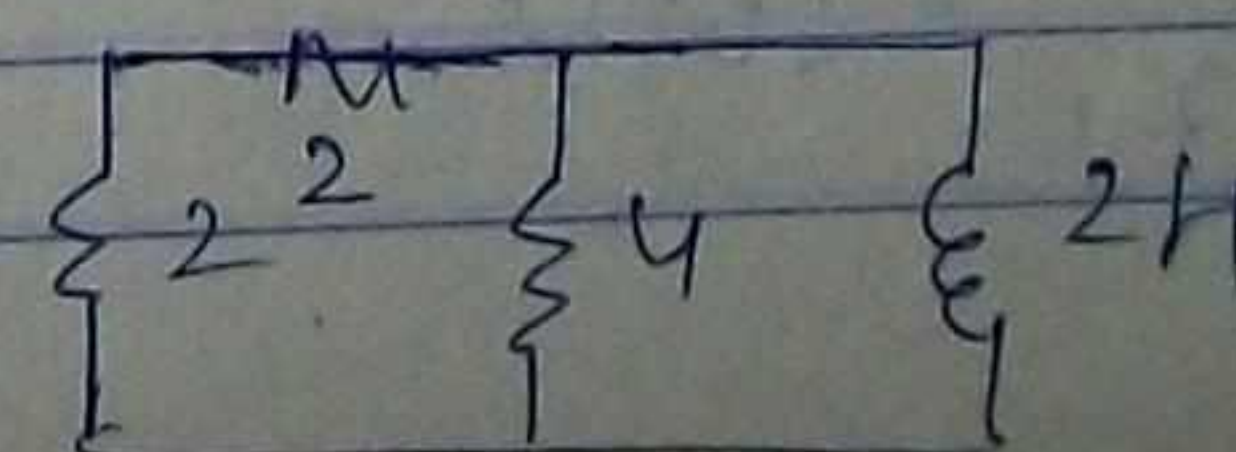
$I_1 = 2A$



$i_L(t) = 5e^{-t}$

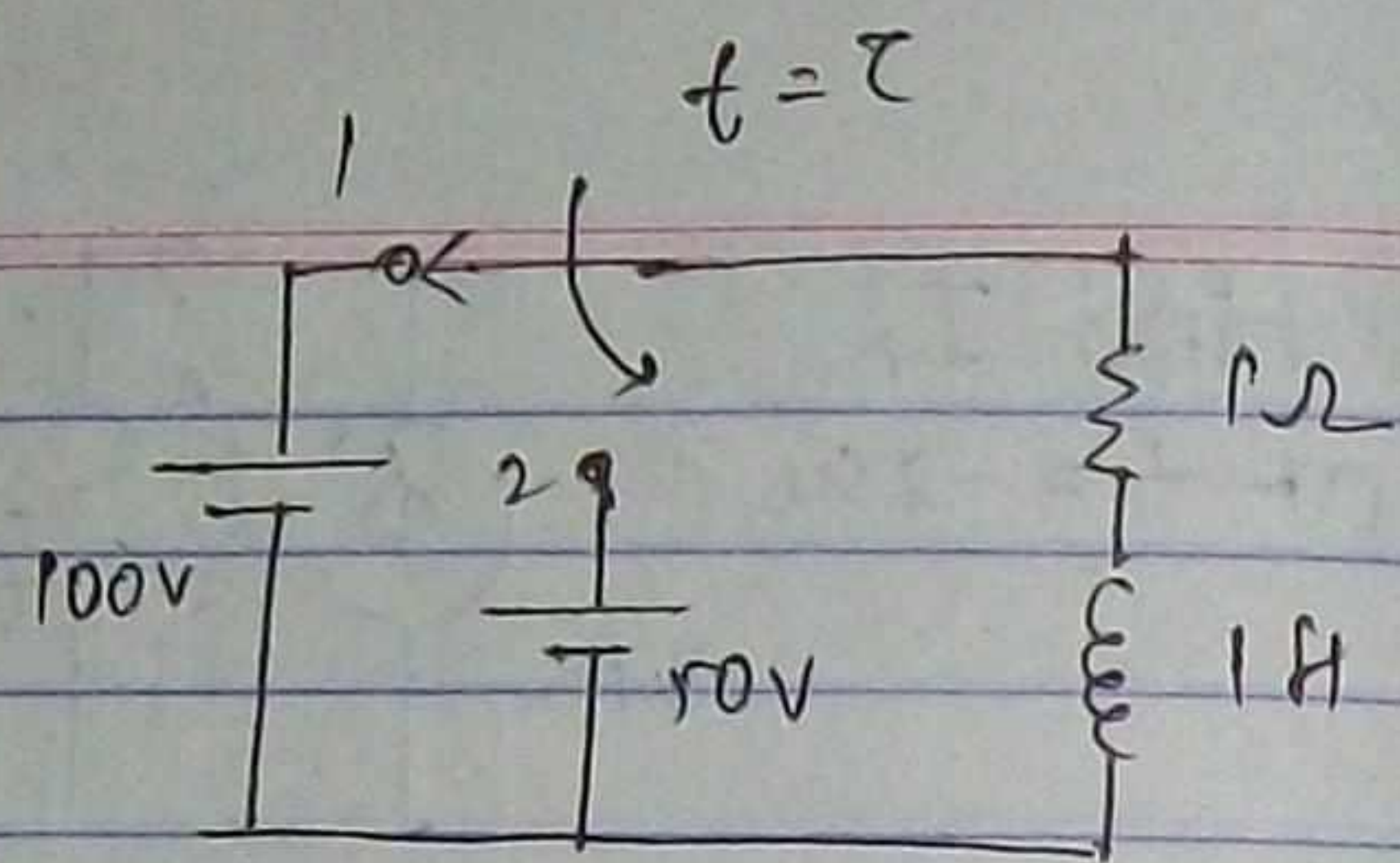
at $t = 0^-$ $i_L(0^-) = 5A$

at $t = 0^+$



★

Ques. At $t=0$
 Switch $\rightarrow$ Pos (1)
 after one time const.
 switch is transfer
 to Pos (2). find
 $i(t)$ when switch
 is at position (2)



Solⁿ: At Pos (1):

$$i_L(t) = 100 \left(1 - e^{-\frac{R}{L}t} \right)$$

$$i_L(t) = 100(1 - e^{-t}) = 100(1 - e^{-t})$$

At Pos (2):

$$i(t) = i(\infty) + (i(0^+) - i(\infty)) e^{-\frac{R}{L}(t-t_0)}$$

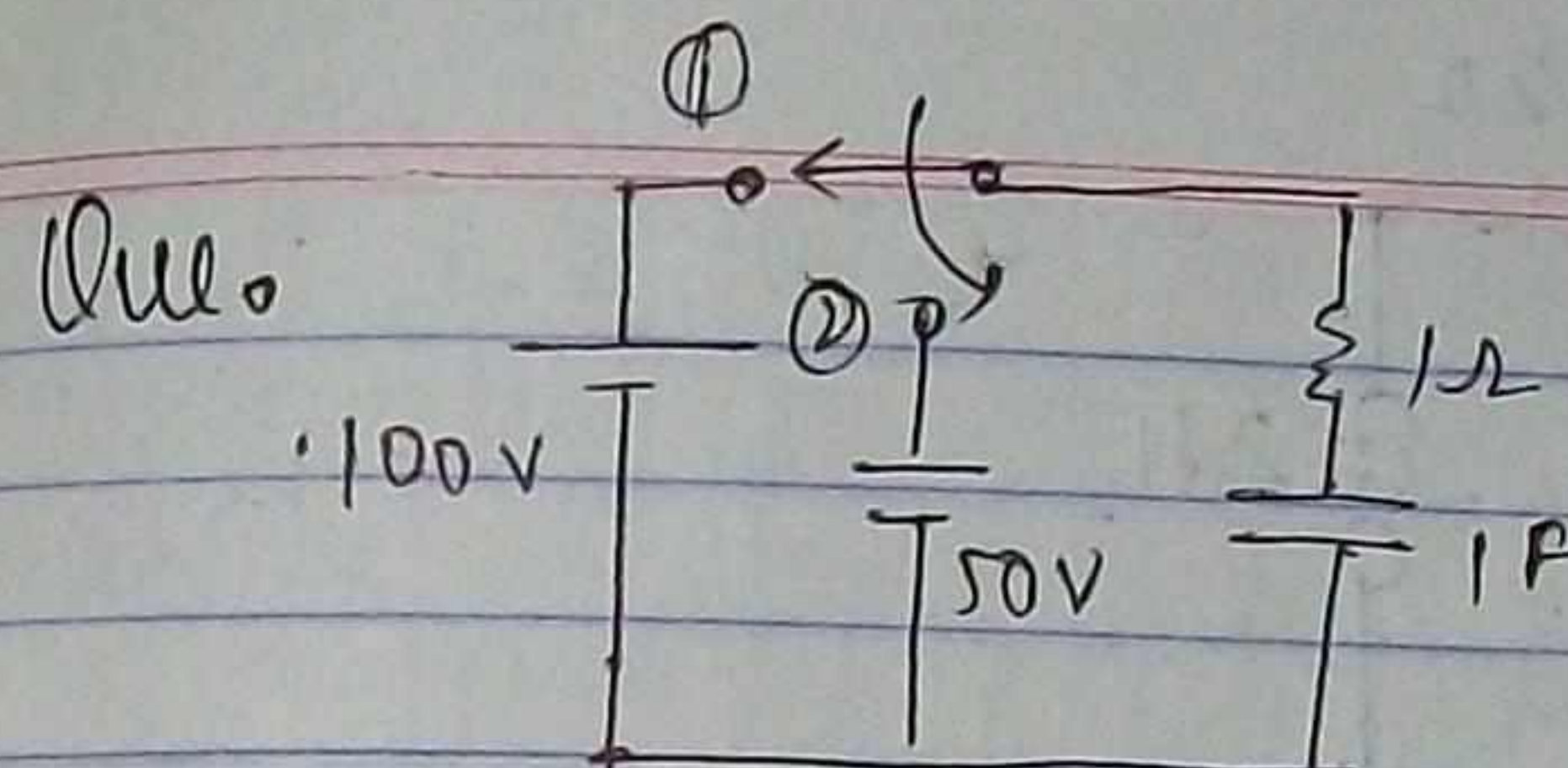
$$= 50 + [i(t_0) - i(\infty)] e^{-\frac{R}{L}(t-t_0)}$$

$$t_0 = \frac{L}{R} = 1 \text{ sec}$$

$$i_L(t_0) = 100(1 - e^{-1}) = 63.2 \text{ A}$$

$$i(t) = 50 + (63.2 - 50) e^{-(t-1)}$$

$$= 50 + 13.2 e^{-(t-1)}$$

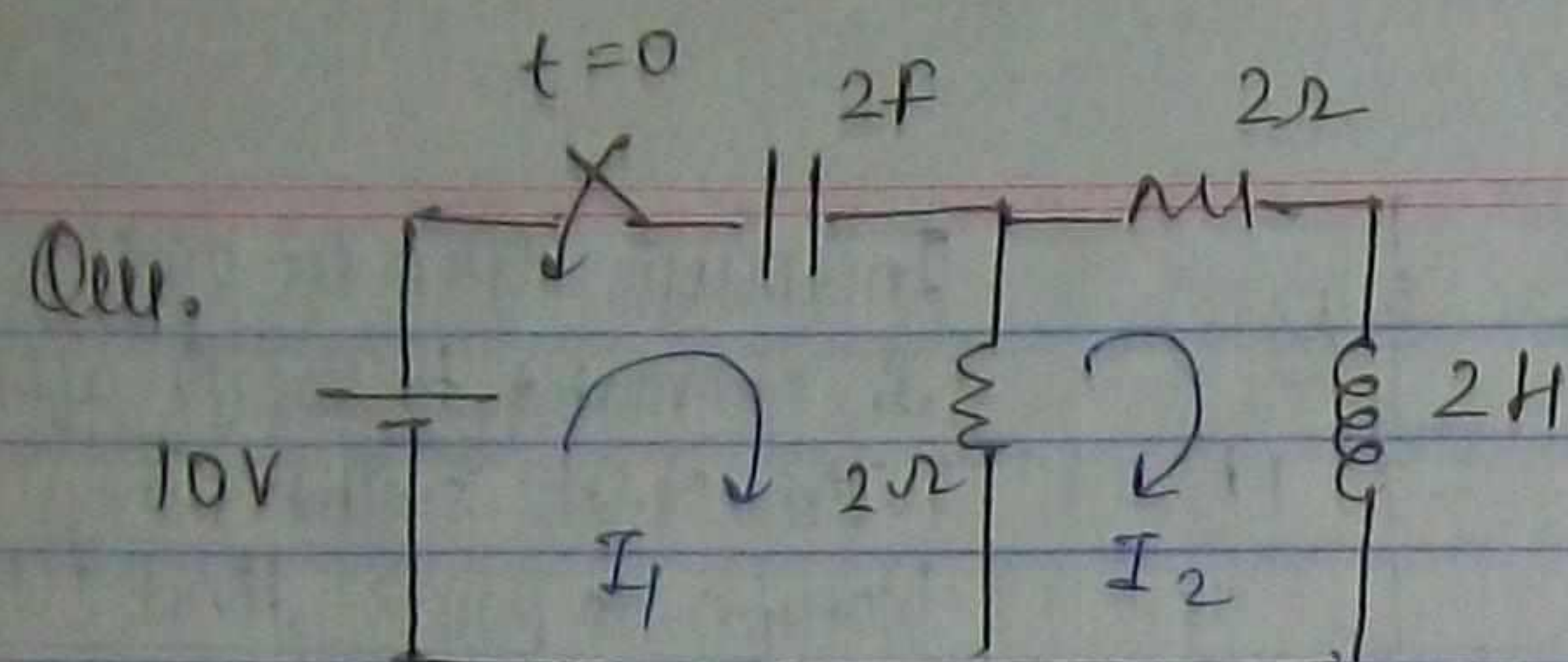


Ques.

In circuit $t=0$ see switch
 is connected to Pos (1) after
 1 Time const switch is
 transfer to pos (2) find $i(t)$
 when switch at pos (2)

Ans.

$$i(t) = 113.2 e^{-(t-1)}$$



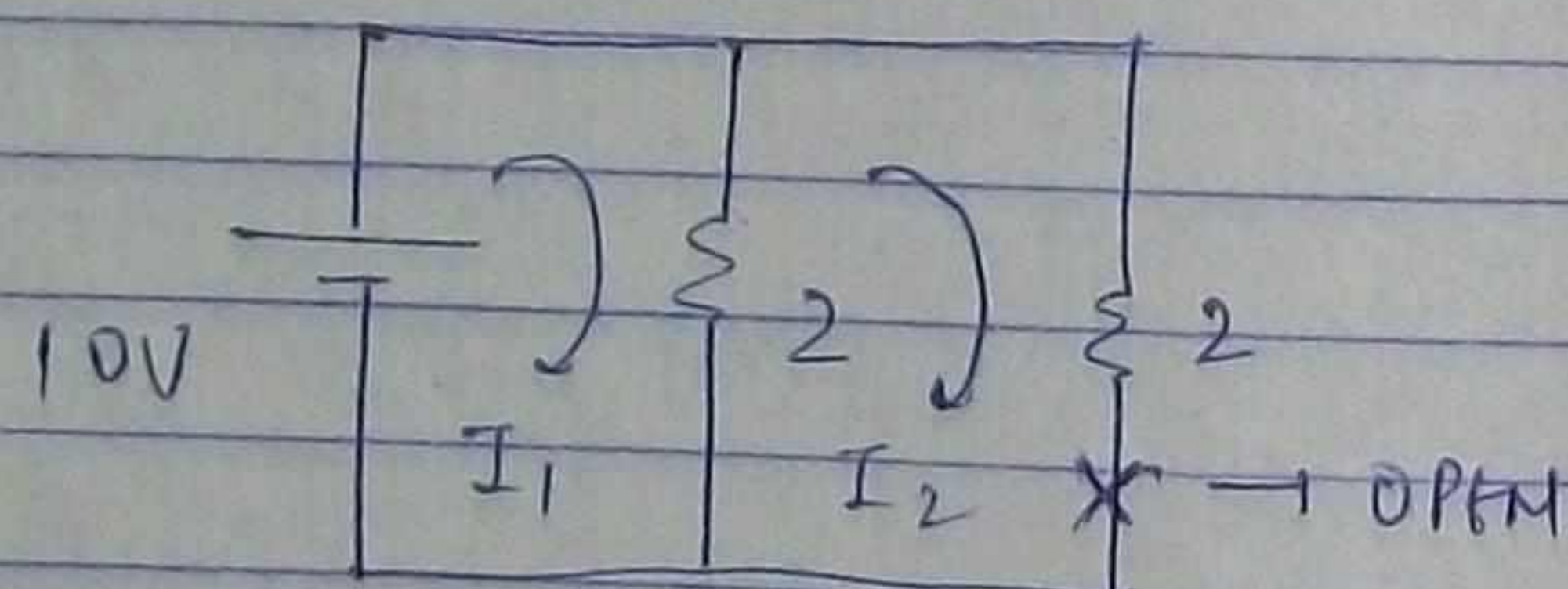
Find $i_1(0^+)$, $i_2(0^+)$, $V_C(0^+)$

$\frac{di_1(0^+)}{dt}$, $\frac{di_2(0^+)}{dt}$, $\frac{d^2 i_1(0^+)}{dt^2}$, $\frac{d^2 i_2(0^+)}{dt^2}$

Solⁿ: at $t=0^-$

$V_C(0^-) = 0$ $i_L(0^-) = 0$

at $t=0^+$



$i_1(0^+) = 5A$, $i_2(0^+) = 0A$, $V_C(0^+) = 0$

At loop ②:

$4I_2(0^+) + 2 \frac{dI_2(0^+)}{dt} - 2I_1(0^+) = 0$ — (1)

$2 \frac{dI_2(0^+)}{dt} = 2I_1(0^+) - 4I_2(0^+)$

$\frac{dI_2(0^+)}{dt} = 5 \text{ A/s}$

loop ①:

$10 = \frac{1}{C} \int I_1(0^+) dt + 2(I_1(0^+) - I_2(0^+))$ — (2)

$0 = \frac{I_1(0^+)}{C} + 2 \frac{dI_1(0^+)}{dt} - 2 \frac{dI_2(0^+)}{dt}$

$0 = \frac{5}{2} + 2 \frac{dI_1(0^+)}{dt} - 10$

$(10 - \frac{5}{2}) \frac{1}{2} = \frac{dI_1(0^+)}{dt}$

$\frac{dI_1(0^+)}{dt} = \frac{15}{4} = 3.75 \text{ A/s}$

diff ①

$4 \frac{dI_2(0^+)}{dt} + 2 \frac{d^2 I_2(0^+)}{dt^2} - 2 \frac{dI_1(0^+)}{dt} = 0$

$\frac{d^2 I_2(0^+)}{dt^2} = \frac{1}{2} \left[\frac{2 \times 15}{4} - 4 \times 5 \right]$
 $= \frac{1}{2} \left[\frac{15 - 40}{2} \right] = -\frac{25}{4}$

$\frac{d^2 I_2(0^+)}{dt^2} = -6.25 \text{ A/s}^2$

diff ②

$$\frac{1}{2} \frac{d I_1(0^+)}{dt} + 2 \frac{d^2 I_1(0^+)}{dt^2} - 2 \frac{d^2 I_2(0^+)}{dt^2} = 0$$

$$\frac{15}{8} + 2 \frac{d^2 I_1(0^+)}{dt^2} + 2 \times \frac{25}{4} = 0$$

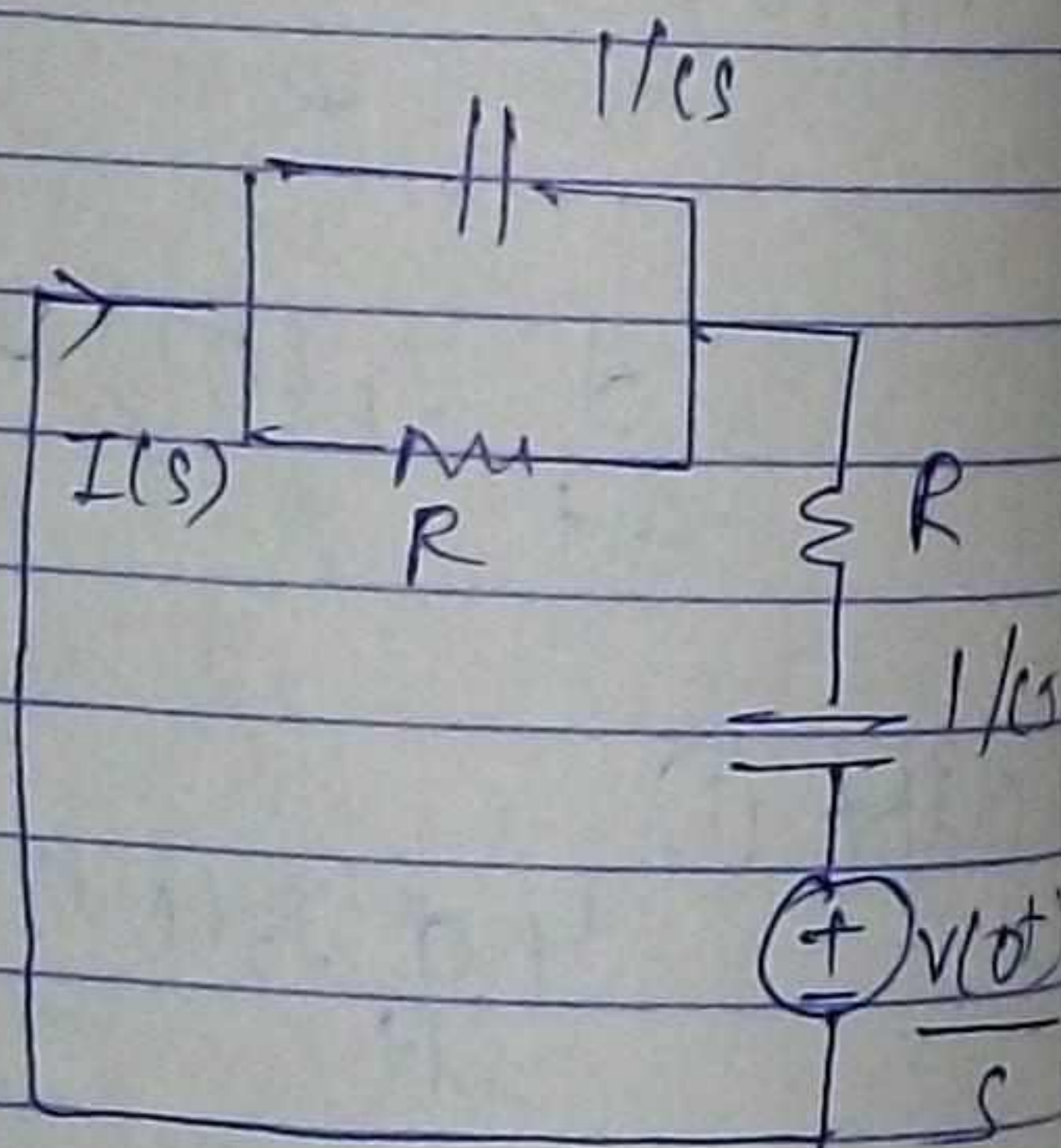
$$\frac{d^2 I_1(0^+)}{dt^2} = -\frac{25}{2} - \frac{15}{8} = -\frac{115}{8}$$

$$\frac{d^2 I_1(0^+)}{dt^2} = -14.3 \text{ A/s}^2$$

Que P-34 Q-1
Solⁿ:

$$I(s) = -V(0^+)/s$$

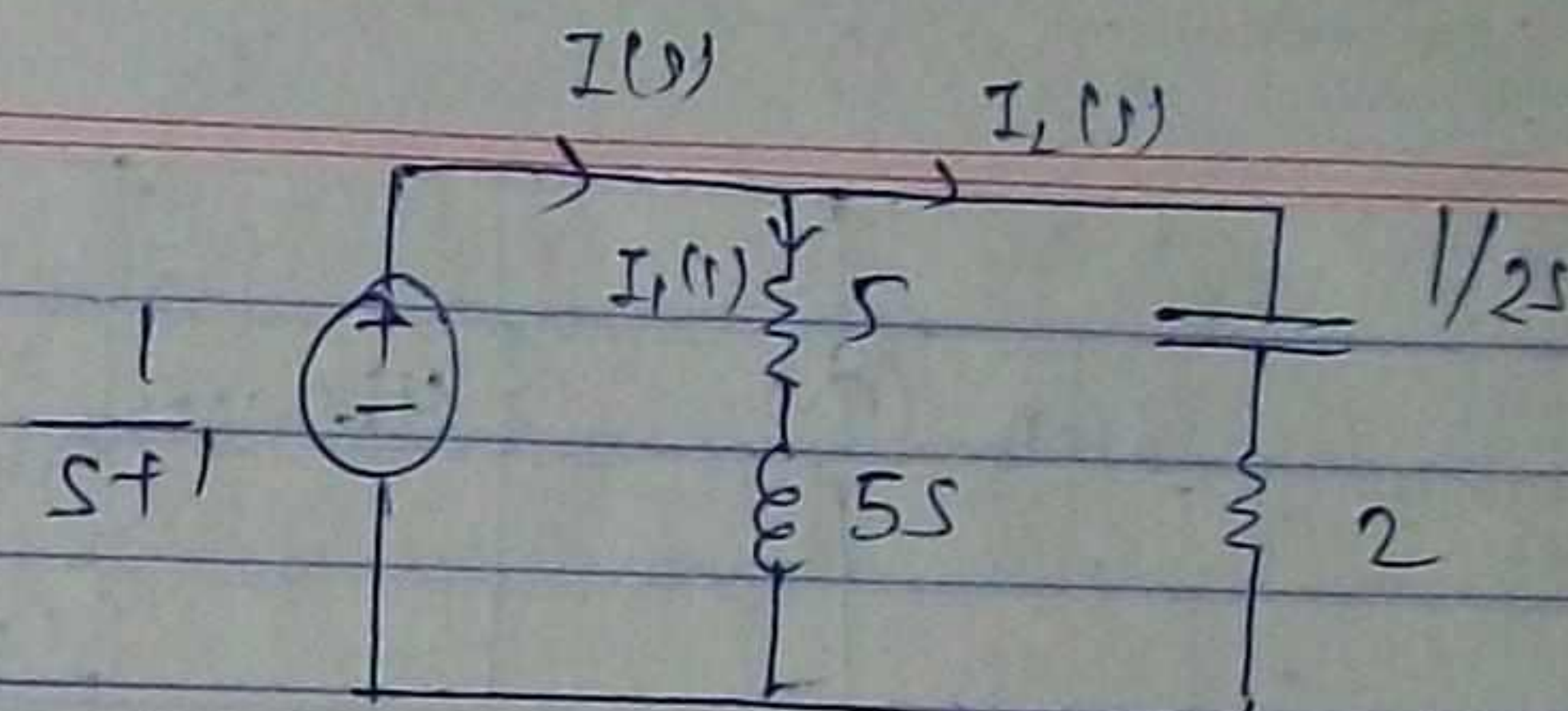
$$R + \frac{1}{Cs} + \frac{R \times 1/Cs}{R + 1/s}$$



$$I(t) = \mathcal{L}^{-1} I(s) = - \left[14.46 e^{-2.62t} + 5.54 e^{-0.38t} \right] \text{ mA}$$

14.

Que.-3 P-34.



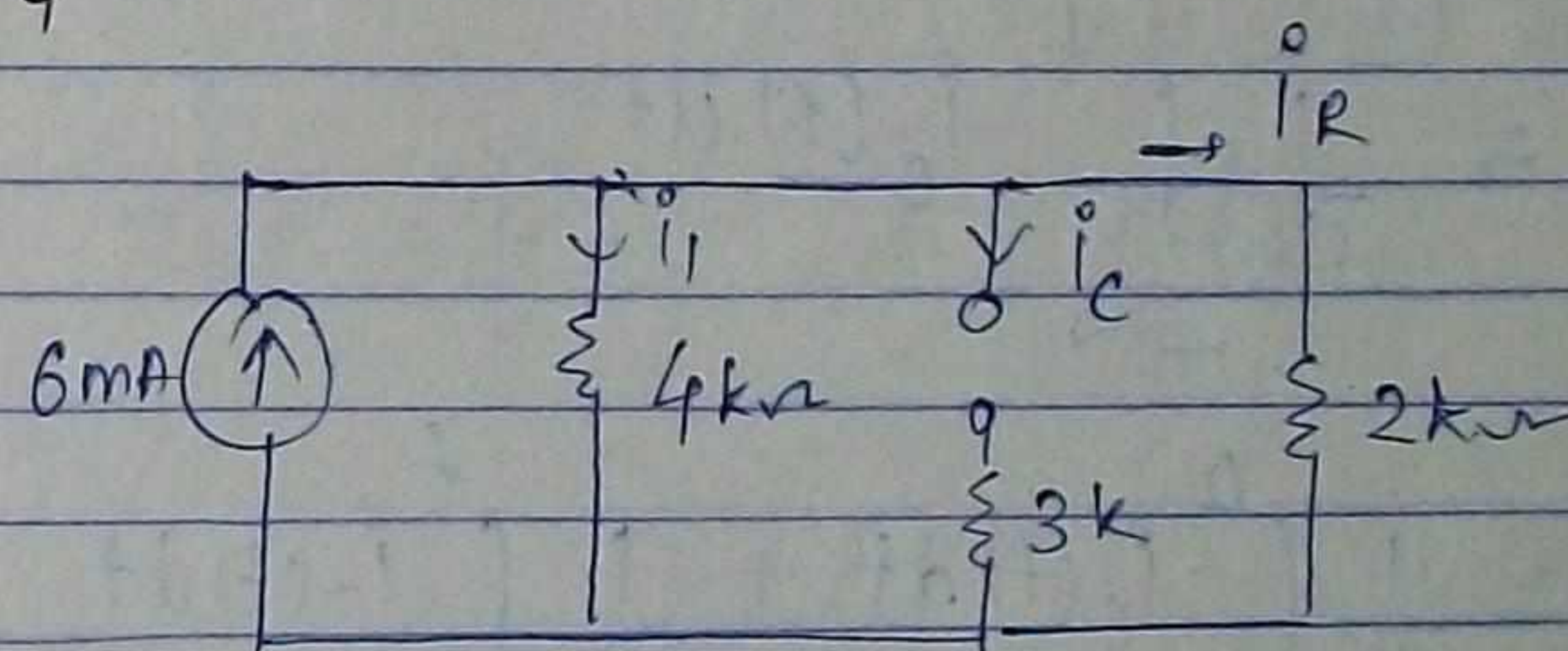
$$I(s) = I_1(s) + I_2(s)$$

$$I(s) = \frac{1/(s+1)}{5+5s} + \frac{1/(s+1)}{(2+1/2s)}$$

$$i(t) = \frac{2}{3} e^{-t} + \frac{1}{5} t e^{-t} - \frac{1}{6} e^{-t/4}$$

Que 4 P-34
Solⁿ:

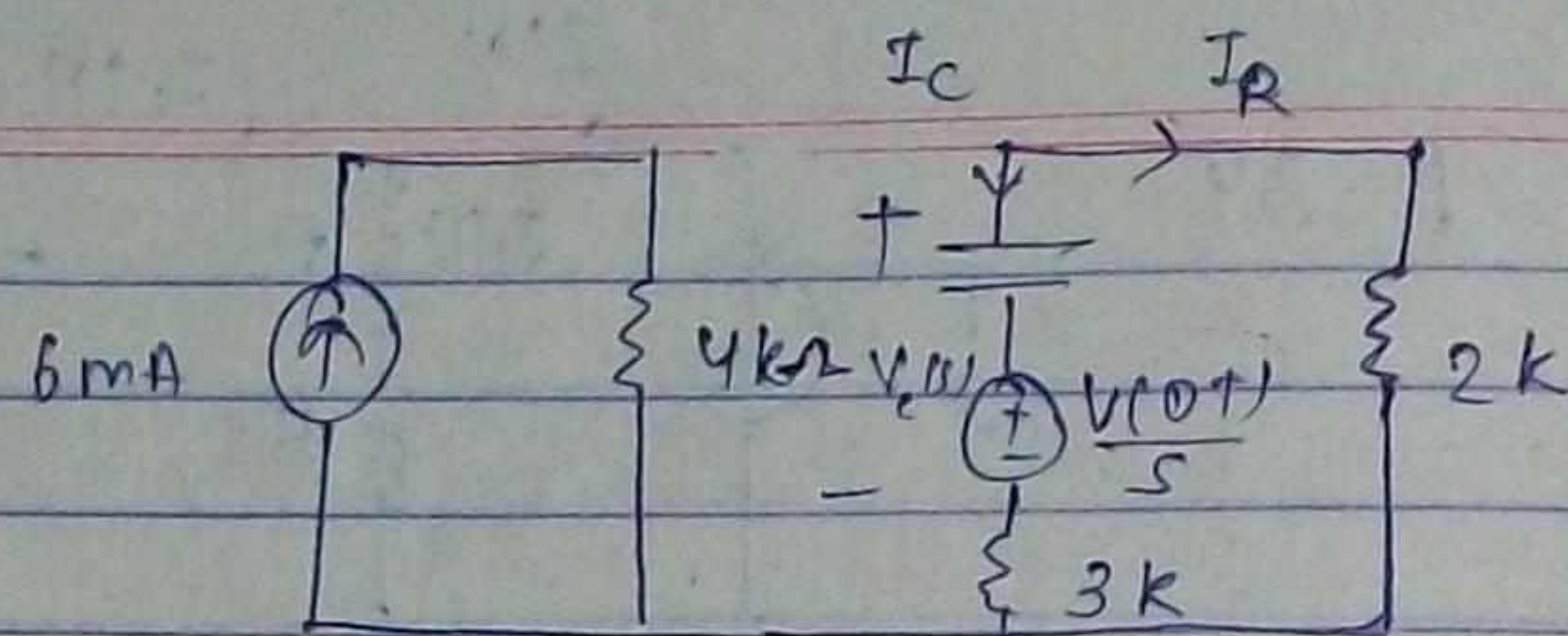
$t=0^-$



$$i_1 = \frac{2}{2+4} \times 6 = 2 \text{ mA}$$

$$V_c(0^-) = 4 \times 2 = 8 \text{ V}$$

Now at $t=0^+$



$$V_s = 4 \times 6 = 24V$$

$$V_s(0^+) = 24V$$

$$I_R(s) = \frac{V(0^+)/s}{\frac{1}{Cs} + 2k + 3k} = \frac{8/s}{\frac{1}{Cs} + 2k + 3k}$$

$$i_R(t) = 1.6e^{-100t} \text{ mA}$$

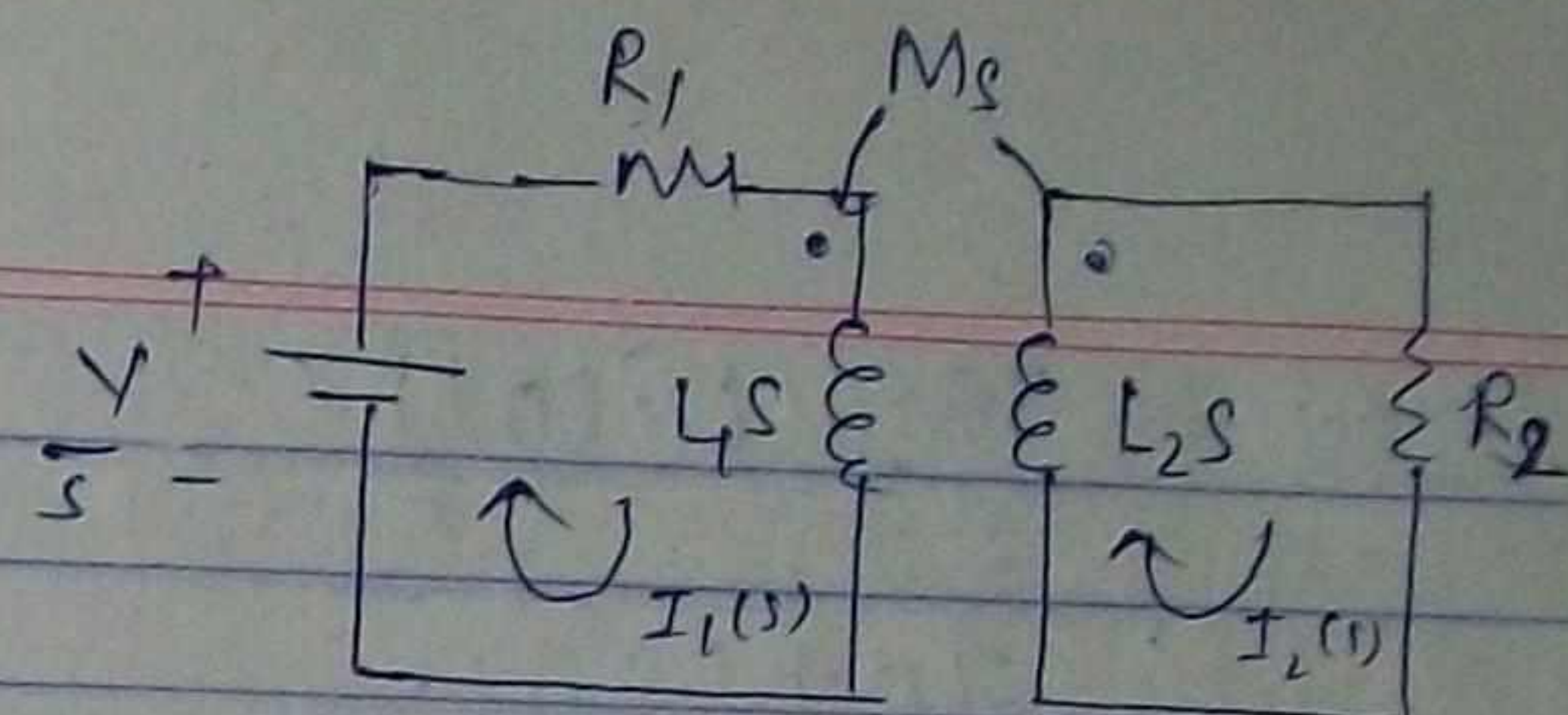
$$i_R(t) = -i_C(t)$$

$$V_c = \frac{1}{C} \int_{-\infty}^t i_C(t) dt$$

$$= \frac{1}{C} \int_{-\infty}^0 i_C(t) dt + \frac{1}{C} \int_0^t i_C(t) dt$$

$$V_c = 8 + \frac{1}{C} \int_0^t -1.6e^{-100t} dt$$

Ques 5. P-35



solⁿ

$$(R_1 + L_1 s) I_1(s) - M s I_2(s) = \frac{V}{s} \quad \text{--- (1)}$$

$$(R_2 + L_2 s) I_2(s) - M s I_1(s) = 0 \quad \text{--- (2)}$$

$$I_2(s) = -\frac{M s I_1(s)}{(R_2 + L_2 s)}$$

$$I_1(s) = -\frac{M s}{(R_1 + L_1 s)} \cdot \frac{M s}{(R_2 + L_2 s)} + \frac{V}{s(R_2 + L_2 s)}$$

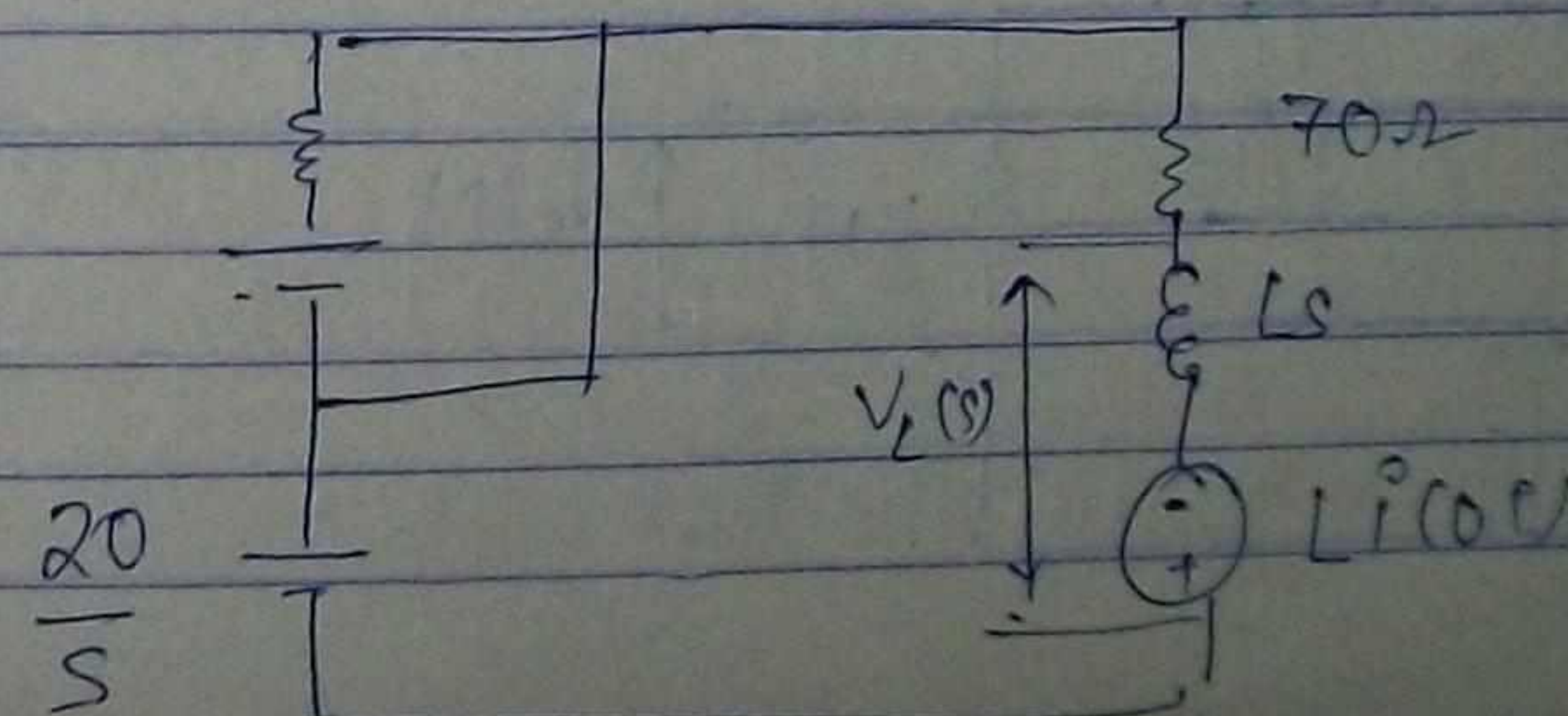
onsolving

$$I_1(t) = [5e^{-t/5}] u(t)$$

Ques 6. P-35

$$t=0^- \quad V_c(0^-) = \frac{20 + 19}{60 + 70}$$

t=0⁺



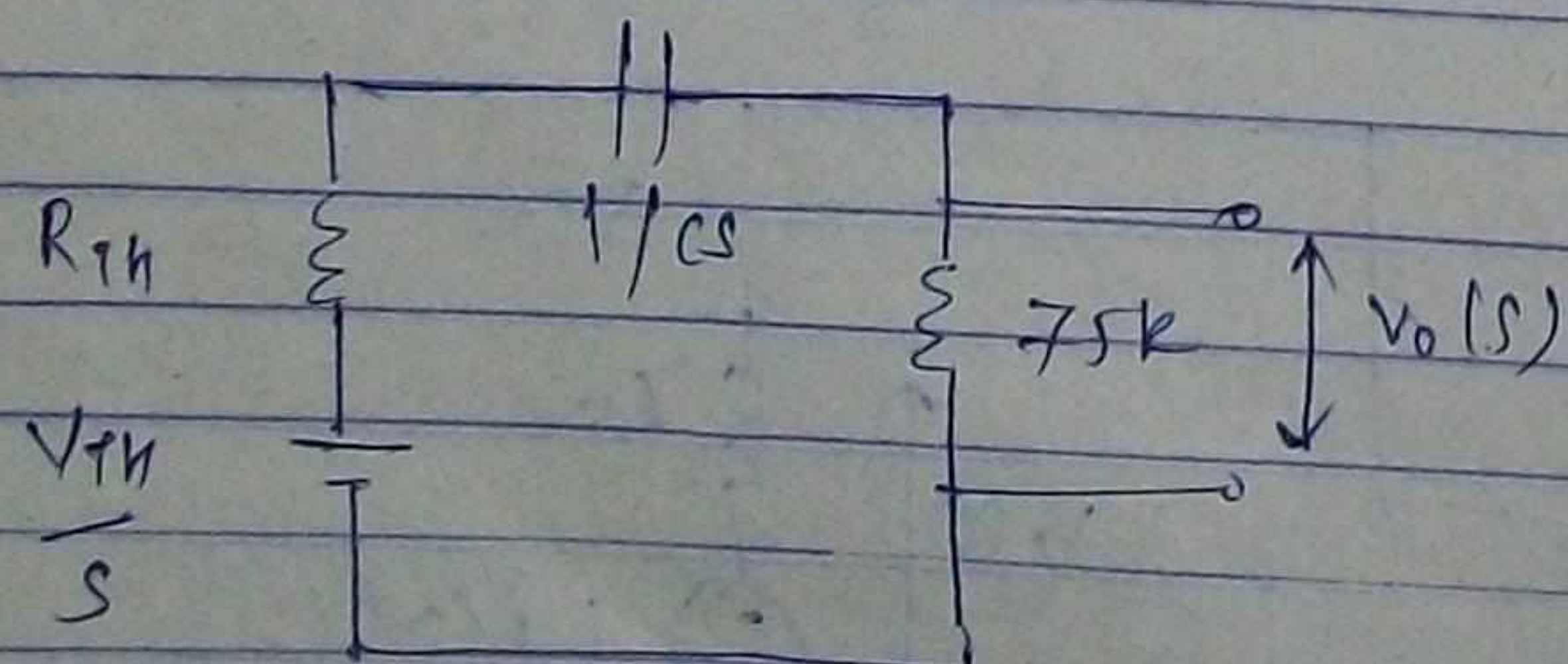
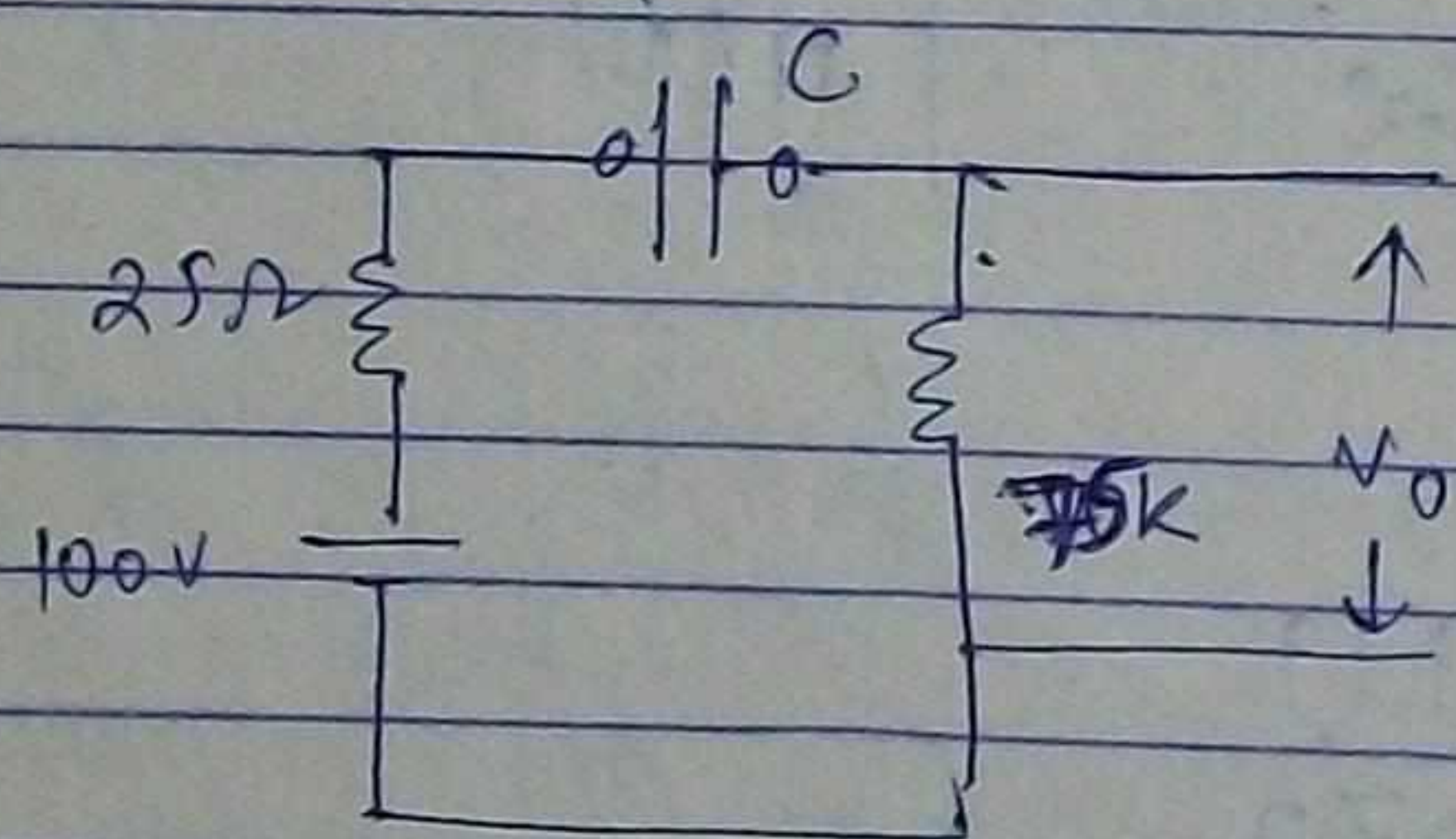
$$I(s) = \frac{20}{s} + \frac{L i(0^+)}{70 + Ls}$$

$$i(t) = \left[\frac{2}{7} + \frac{1}{70} e^{-210t} \right] u(t)$$

$$V_L = L \frac{di}{dt} = \frac{1}{3} - \frac{210}{70} e^{-210t} = -3 \text{ V} \quad t=0$$

Que. 7 P-35

Solⁿ: Replacing by Thevenin equivalent



$$V_o(s) = \frac{75}{75 + 1/s + R_{th}} \times \frac{V_{th}}{s}$$

$$V_o(t) = 75 e^{-10t}$$

$$V_o(t_0) = 75 e^{-10(t-t_0)}$$

① at $t = t_0$ $V_o = 75 \text{ V}$

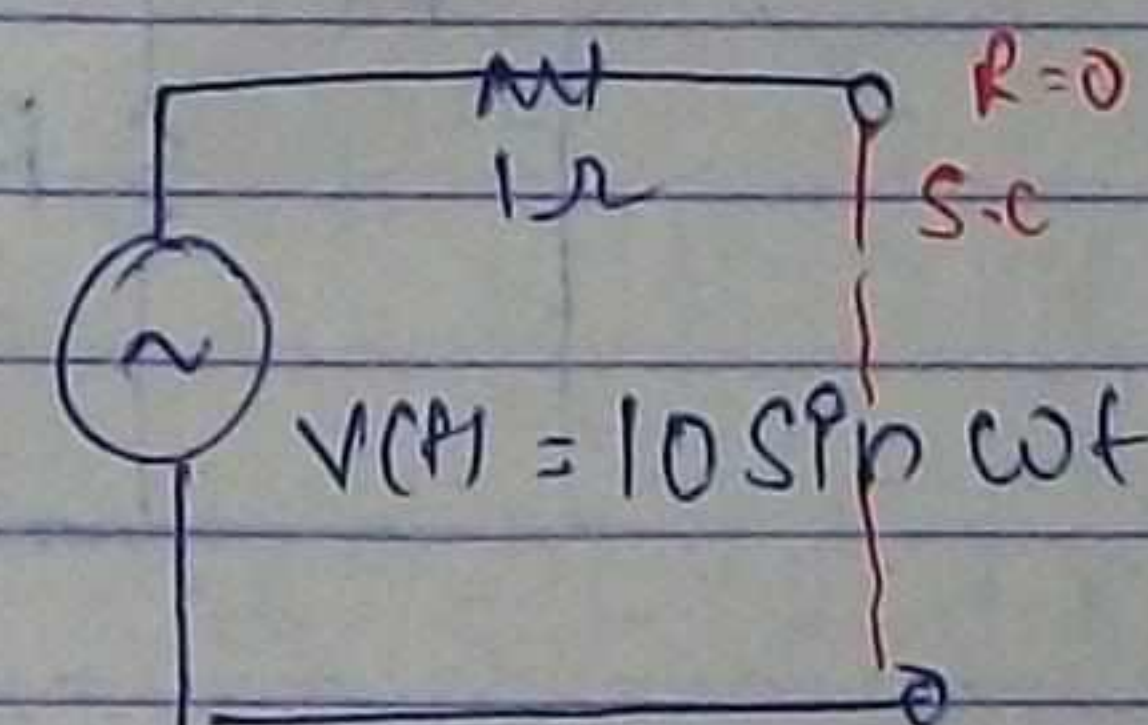
② at $t = 25 \text{ ms}$ $V_o(t) = 75 e^{-10(25 \times 10^{-3} - t_0)}$

③ at $t = \infty$ $V_o(t) = 0$

Que - 5 P-57

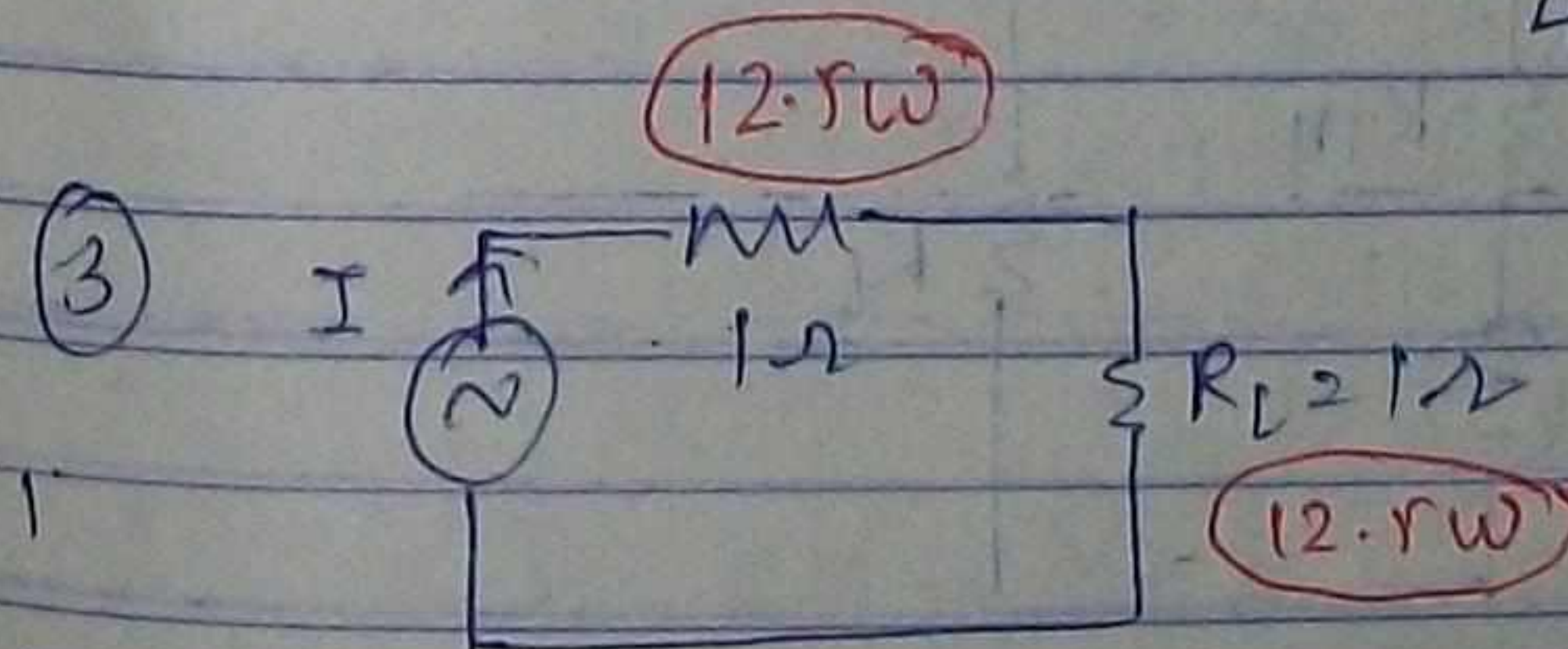
Solⁿ:

①



$$P = \frac{V_s^2}{R_s} = \frac{(10/\sqrt{2})^2}{1} = 50 \text{ W}$$

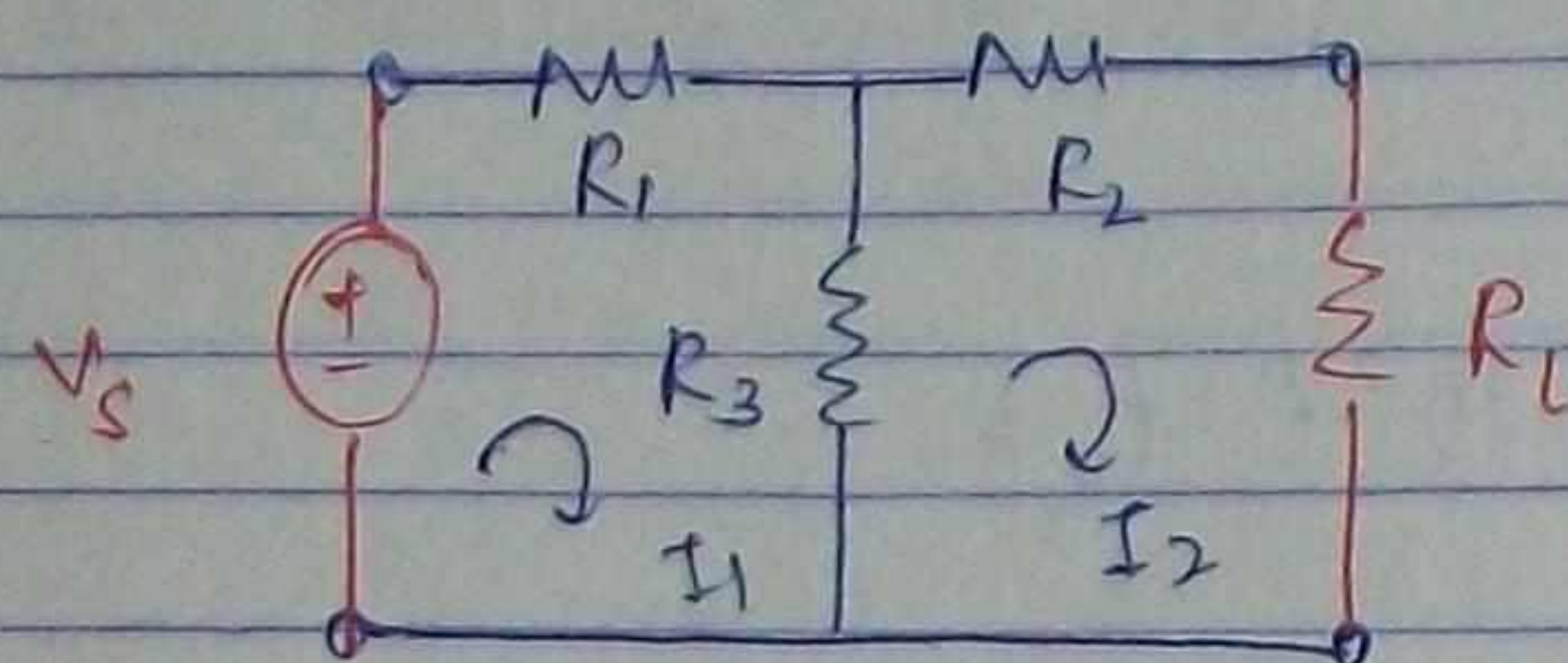
② $R_L = R_s = 1$ $P_{max} = \frac{V_s^2}{4R_L} = \frac{(10/\sqrt{2})^2}{4 \times 1} = 12.5 \text{ W}$



$$P_T = 12.5 + 12.5 = 25 \text{ W}$$

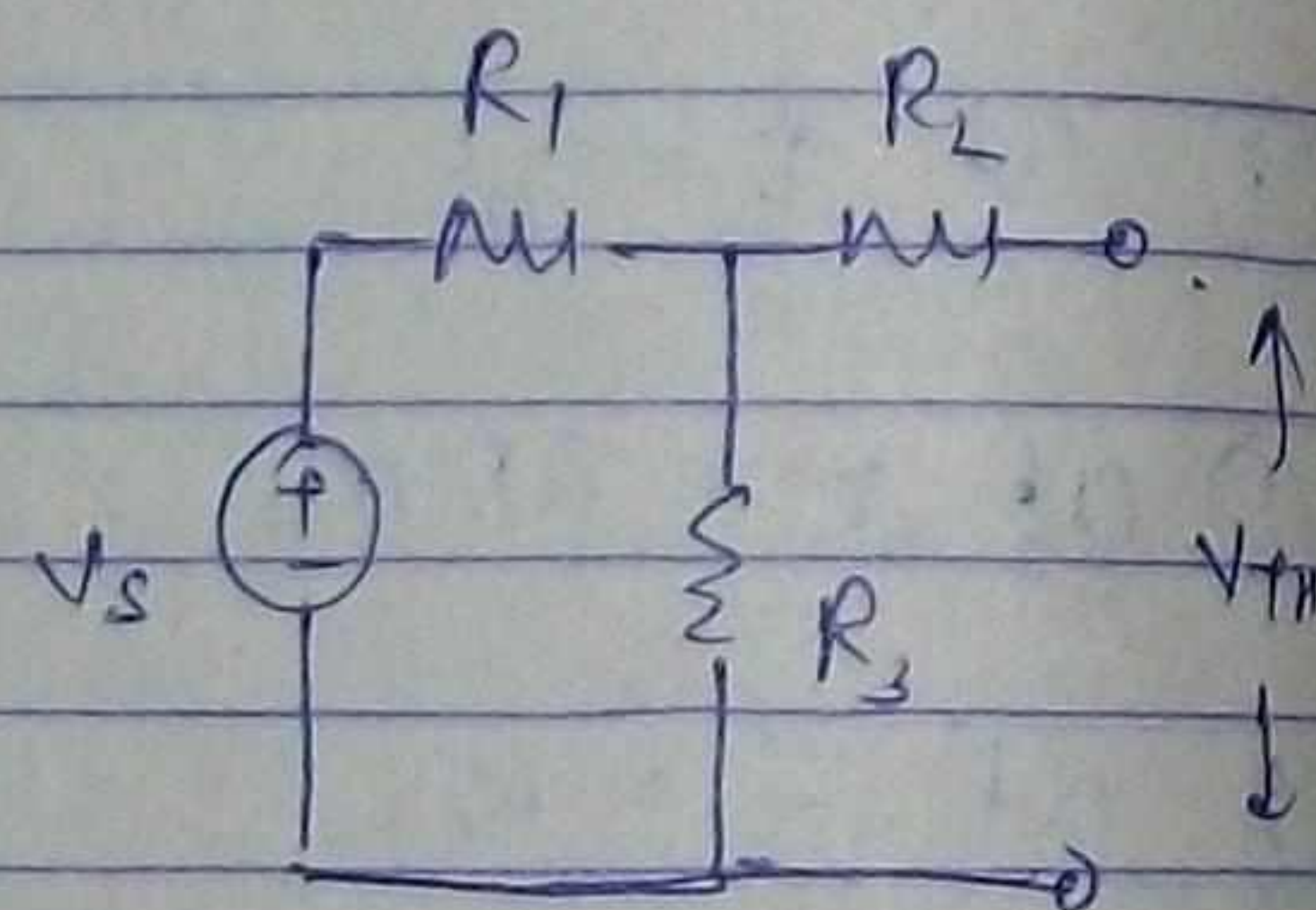
4/11/11

⇒ PROOF OF THEVENIN'S THEOREM -



FOR V_{TH} -

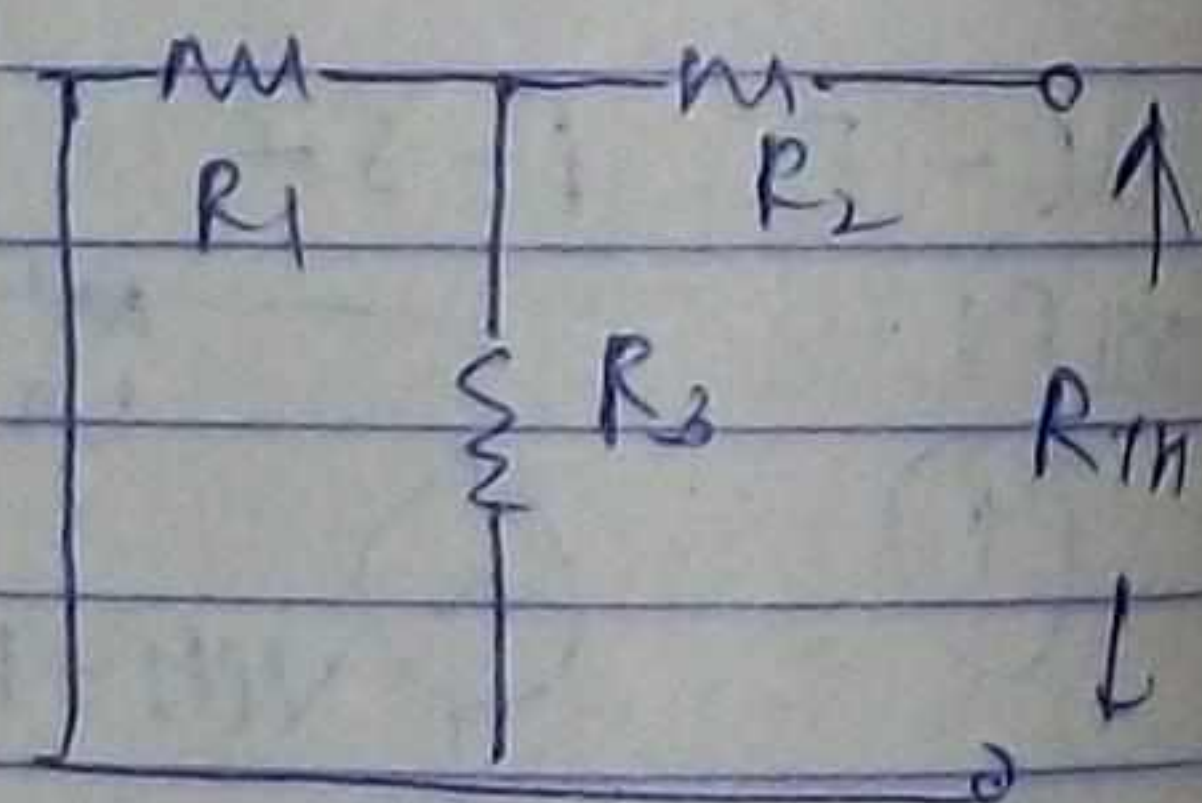
$$V_{TH} = \frac{R_3}{R_1 + R_3} \times V_s$$



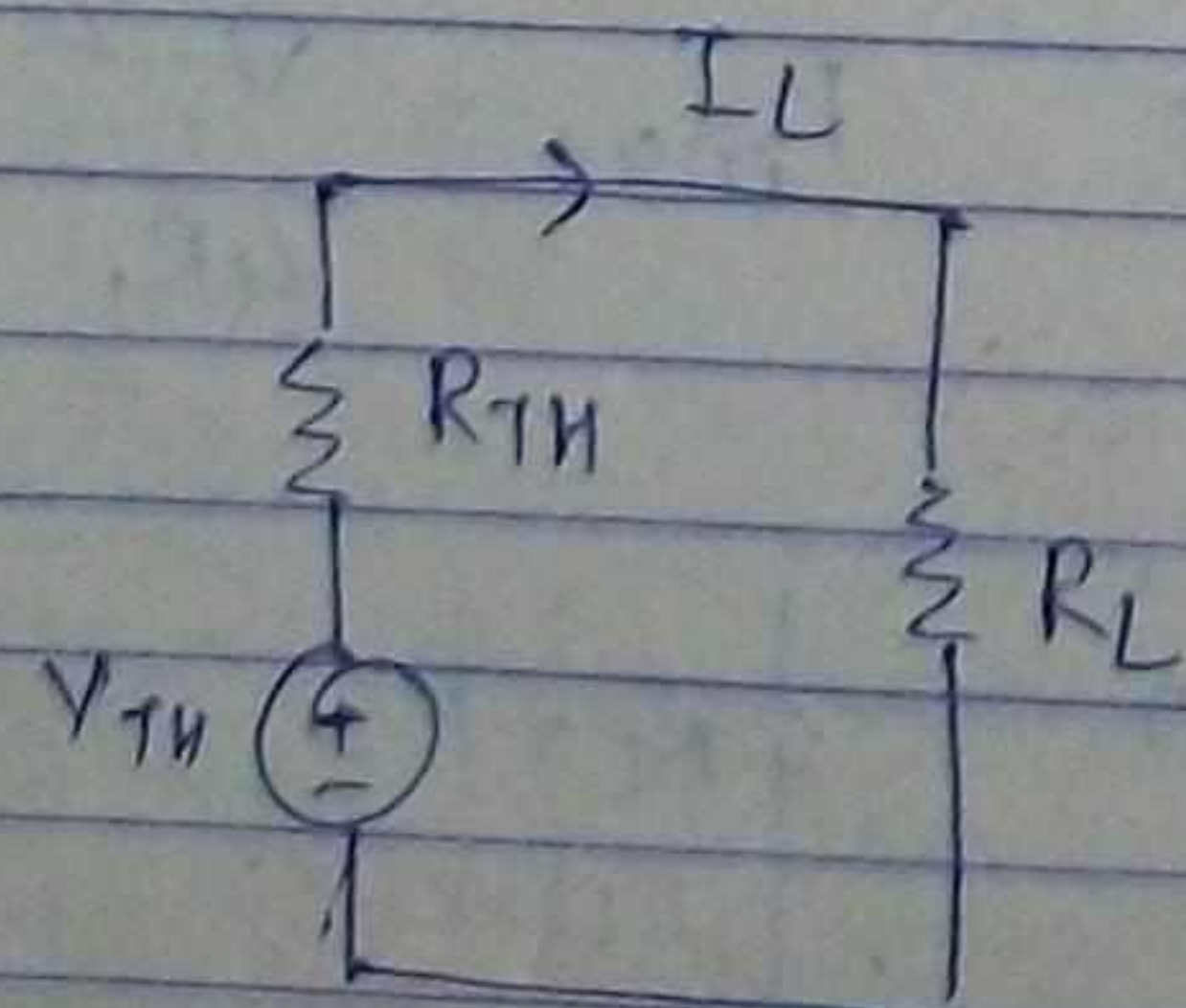
FOR R_{TH} -

$$R_{TH} = R_1 \parallel R_3 + R_2$$

$$= \frac{R_1 R_3}{R_1 + R_3} + R_2$$



THEVENIN'S EQUIVALENT CIRCUIT -



$$I_L = \frac{V_{TH}}{R_{TH} + R_L}$$

Substituting value:

$$I_L = \frac{V_s \times R_3}{\frac{R_1 R_3}{R_1 + R_3} + R_2 + R_L} = \frac{V_s R_3}{R_1 R_3 + R_2 R_1 + R_2 R_3 + R_L (R_1 + R_3)}$$

$$I_L = \frac{V_s R_3}{R_1 R_2 + R_2 R_3 + R_3 R_1 + R_L (R_1 + R_3)} \quad \text{--- (A)}$$

Now from mesh analysis,

$$V_s = (R_1 + R_3) I_1 - R_3 I_2 \quad \text{--- (1)}$$

$$(R_2 + R_3 + R_L) I_2 - I_1 R_3 = 0 \quad \text{--- (2)}$$

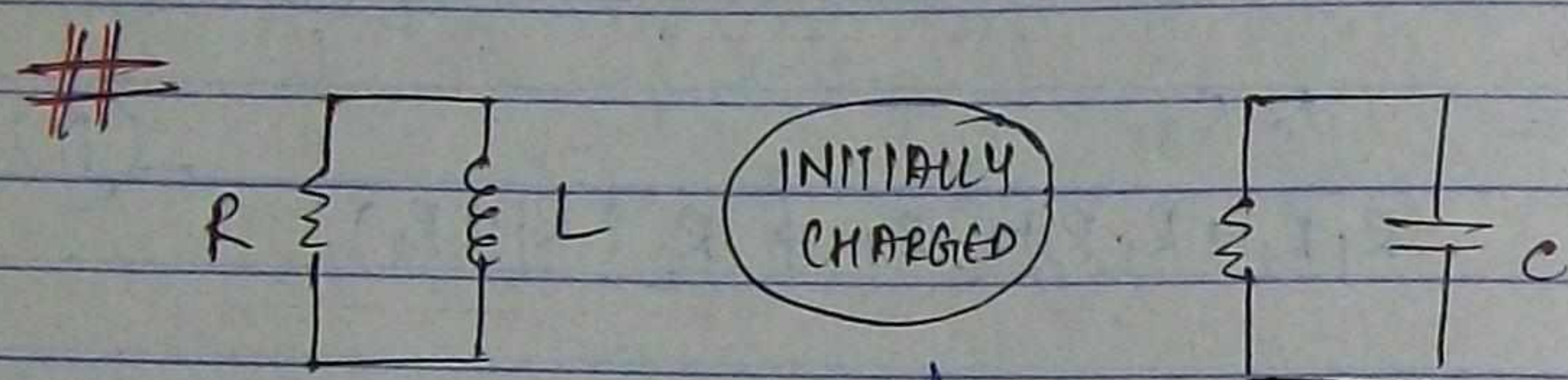
$$I_1 = \frac{R_2 + R_3 + R_L}{R_3} I_2$$

from (1) $V_s = \frac{(R_1 + R_3)(R_2 + R_3 + R_L) I_2 - R_3 I_2}{R_3}$

$$I_2 = \frac{V_s R_3}{R_1 R_2 + R_3 R_1 + R_1 R_L + R_2 R_3 + R_3^2 + R_3 R_L - R_3^2}$$

$$I_s = \frac{V_s R_3}{R_1 R_2 + R_2 R_3 + R_3 R_1 + R_L (R_1 + R_3)} \quad \text{--- (B)}$$

from the above calculation it is concluded that load current of eq (A) and eq (B) are equal hence theorem is proved.



$$R_1 = 10\Omega \quad P_1 = I^2 R_1 \quad t = 15 \text{ sec}$$

$$R_2 = 100\Omega \quad P_2 = I^2 R_1 \quad t = 10 \text{ sec}$$

$$t \propto \frac{1}{R}$$

$$T = \frac{L}{R}$$

$$R_1 = 10\Omega \quad P_1 = \frac{V^2}{R_1} \quad t = 10 \text{ sec}$$

$$R_2 = 100\Omega \quad P_2 = \frac{V^2}{R_2} \quad t = 15 \text{ sec}$$

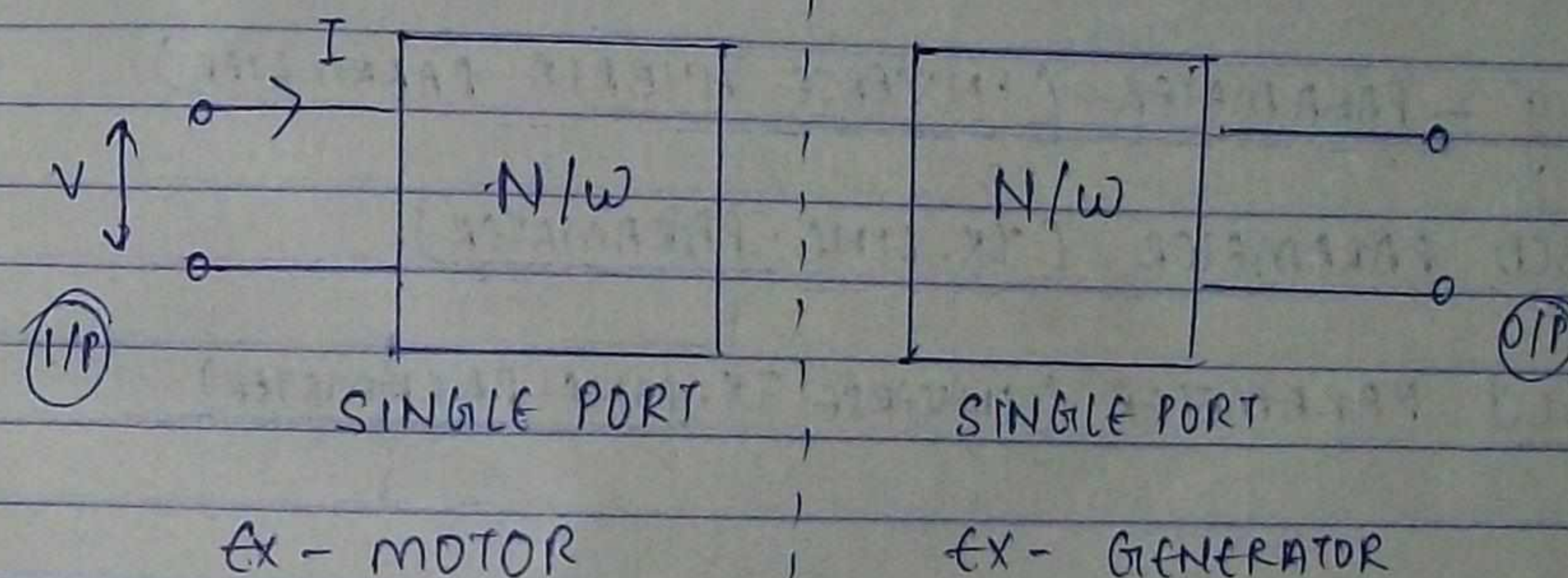
$$t \propto R$$

$$T = RC$$

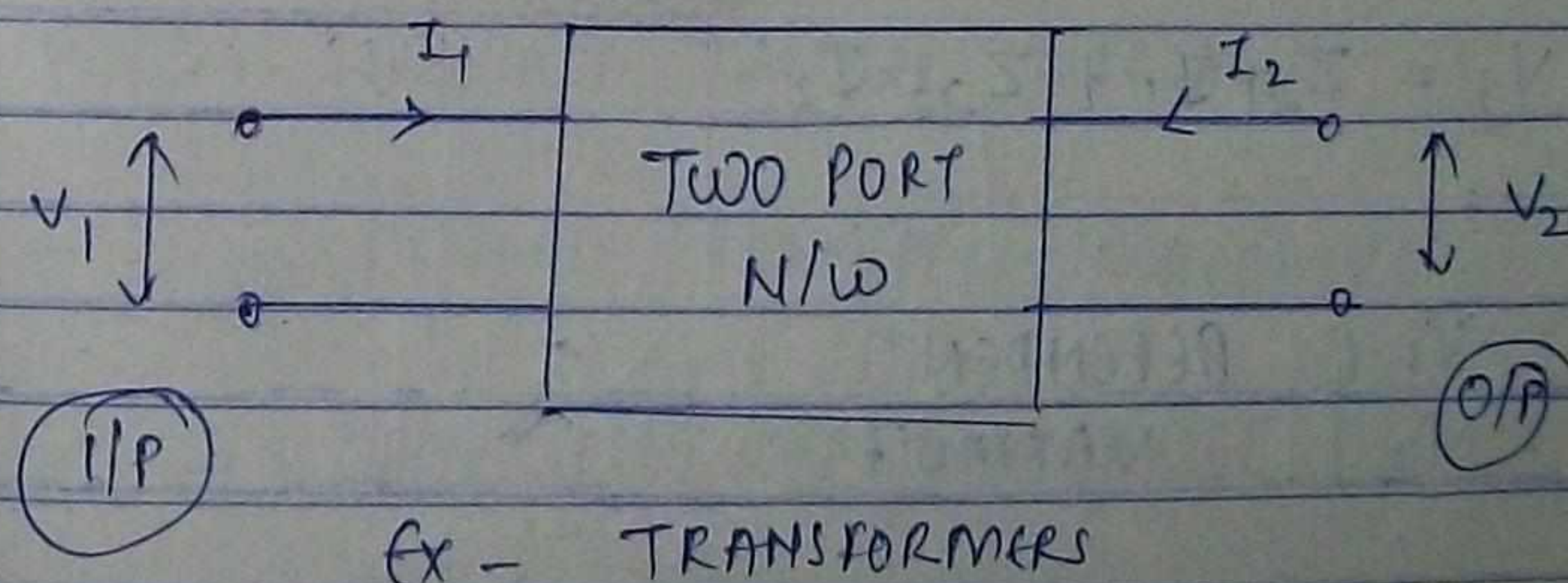
TWO PORT NETWORK

- A pair of terminals at which signal may enter or leave from the N/w is called as port.

- when the N/w is having only one pair of terminal then the N/w is called as single port N/w.



- when N/w is having two pair of terminals then the N/w is called as two port N/w.



⇒ CLASSIFICATION OF PARAMETERS -

- ① Z-PARAMETERS (OPEN CIRCUIT PARAMETERS)
- ② Y-PARAMETERS (SHORT CIRCUIT PARAMETER)
- ③ H-PARAMETERS (HYBRID PARAMETERS)
- ④ g-PARAMETER (INVERSE HYBRID PARAMETER)
- ⑤ ABCD PARAMETER (TX. LINE. PARAMETER)
- ⑥ ABCD PARAMETER (INVERSE TX. LINE PARAMETER)

⇒ Z-PARAMETER -

$$\begin{aligned} V_1 &= Z_{11} I_1 + Z_{12} I_2 & \text{(KVL)} \\ V_2 &= Z_{21} I_1 + Z_{22} I_2 & \text{(KVL)} \end{aligned}$$

$\left. \begin{matrix} V_1 \\ V_2 \end{matrix} \right\}$ DEPENDENT VARIABLE

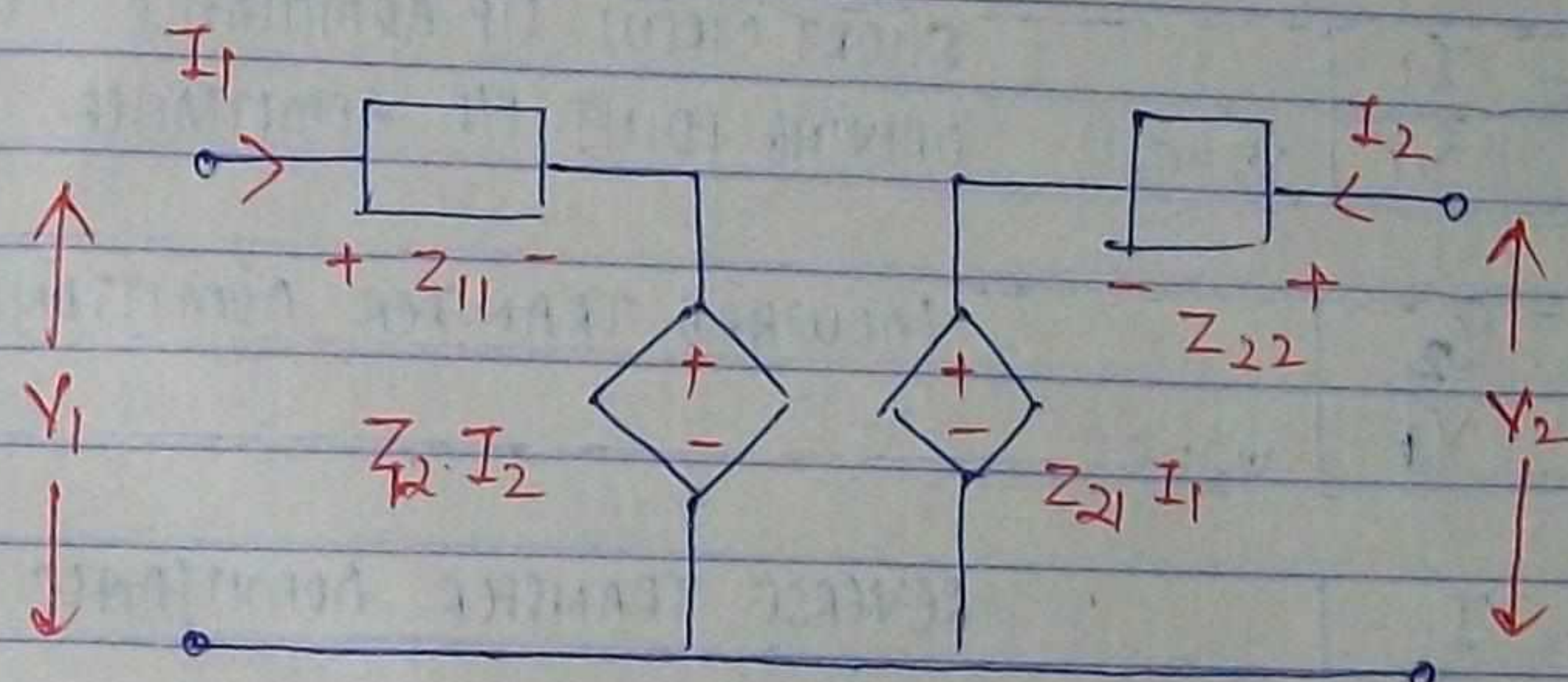
$\left. \begin{matrix} I_1 \\ I_2 \end{matrix} \right\}$ INDEPENDENT VARIABLE
(SOURCE)

$$Z_{11} = \frac{V_1}{I_1} \bigg|_{I_2=0}$$

$$Z_{12} = \frac{V_1}{I_2} \bigg|_{I_1=0}$$

$$Z_{21} = \frac{V_2}{I_1} \bigg|_{I_2=0}$$

$$Z_{22} = \frac{V_2}{I_2} \bigg|_{I_1=0}$$



$Z_{11} \rightarrow$ OPEN CIRCUIT I/P IMPEDANCE
(OR)
DRIVING POINT I/P IMPEDANCE

$Z_{21} \rightarrow$ FORWARD TRANSFER IMPEDANCE

$Z_{12} \rightarrow$ REVERSE TRANSFER IMPEDANCE

$Z_{22} \rightarrow$ OPEN CIRCUIT O/P IMPEDANCE
(OR)
DRIVING POINT O/P IMPEDANCE

⇒ Y-PARAMETER -

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad (\text{KCL})$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \quad (\text{KCL})$$

$$Y_{11} = \frac{I_1}{V_1} \bigg|_{V_2=0}$$

SHORT CIRCUIT I/P ADMITTANCE (OR)
DRIVING POINT I/P ADMITTANCE

$$Y_{21} = \frac{I_2}{V_1} \bigg|_{V_2=0}$$

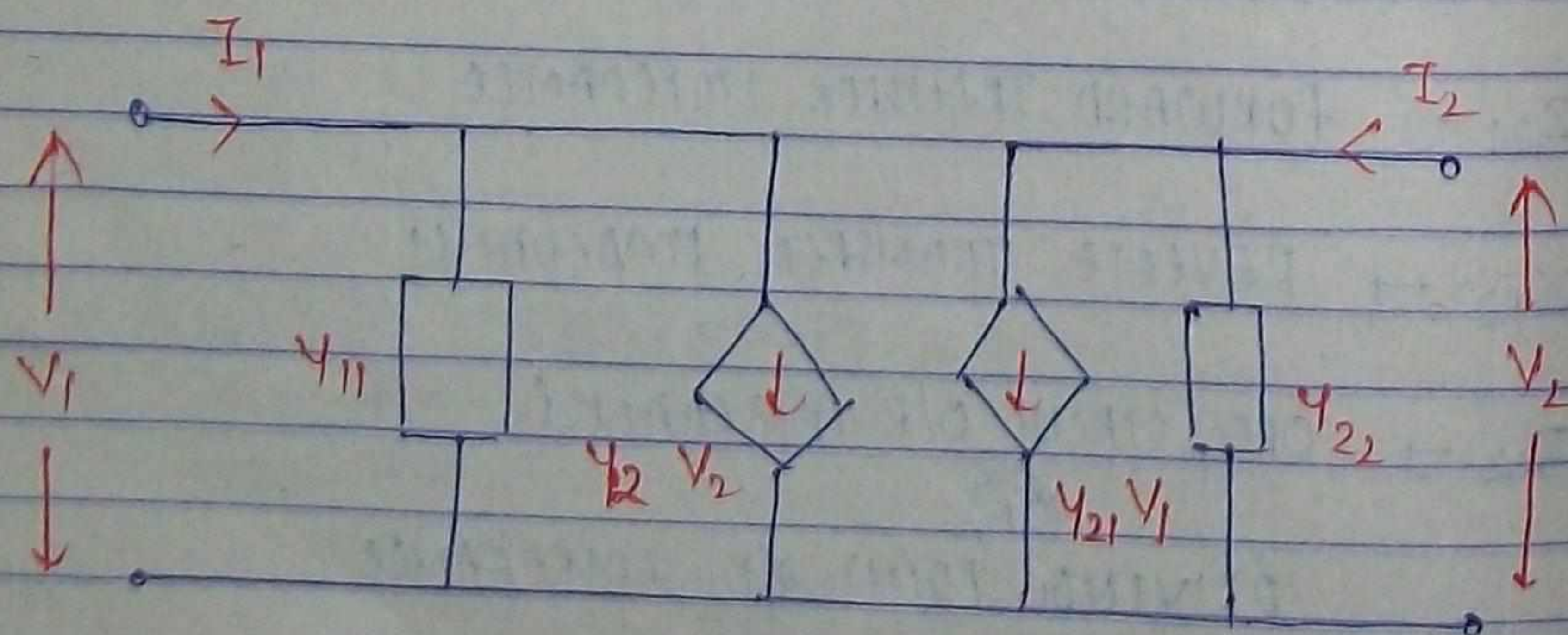
FORWARD TRANSFER ADMITTANCE

$$Y_{12} = \frac{I_1}{V_2} \bigg|_{V_1=0}$$

REVERSE TRANSFER ADMITTANCE

$$Y_{22} = \frac{I_2}{V_2} \bigg|_{V_1=0}$$

SHORT CIRCUIT O/P ADMITTANCE (OR)
DRIVING POINT O/P ADMITTANCE



⇒ h-PARAMETER -

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad (\text{KVL})$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad (\text{KCL})$$

$$h_{11} = \frac{V_1}{I_1} \bigg|_{V_2=0}$$

($h_{11} \neq Z_{11}$) ($h_{11} = 1/Y_{11}$) (ohms)

$$h_{21} = \frac{I_2}{I_1} \bigg|_{V_2=0}$$

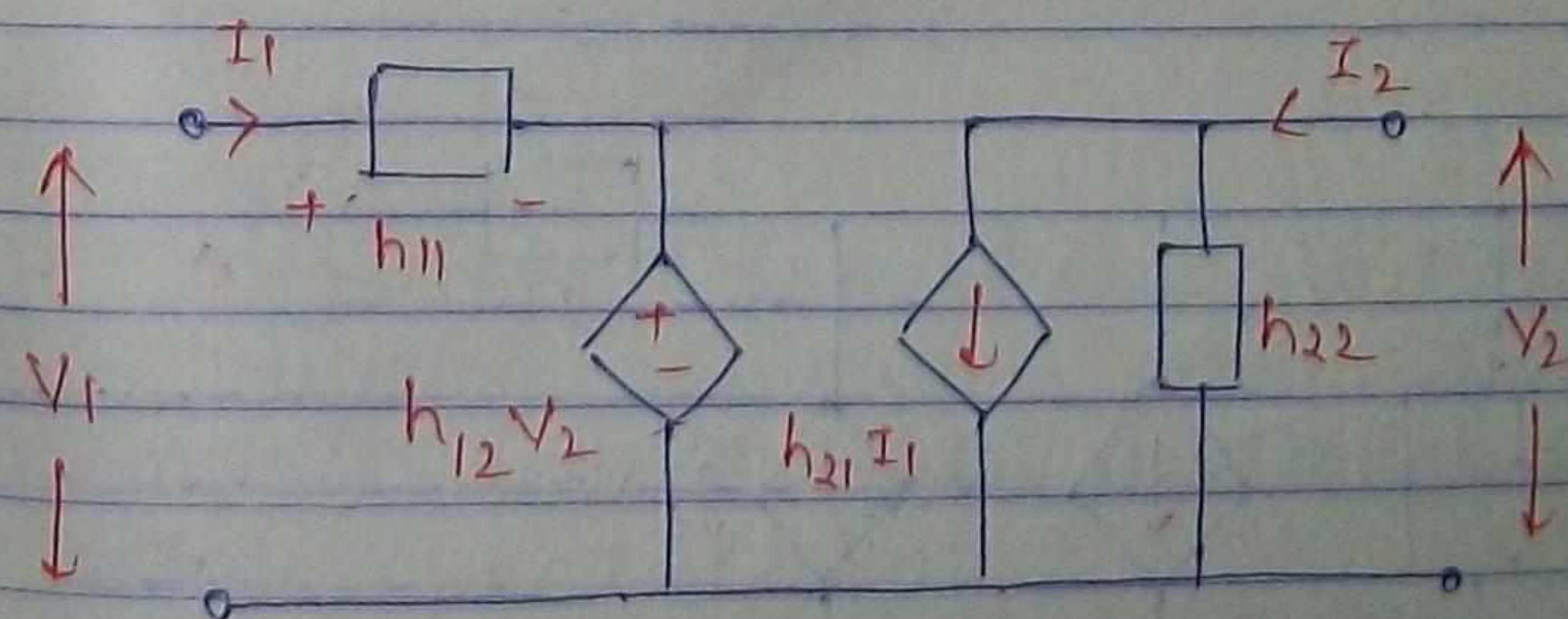
($h_{21} = \frac{Y_{21}}{Y_{11}}$) (NO UNIT)

$$h_{12} = \frac{V_1}{V_2} \bigg|_{I_1=0}$$

($h_{12} = \frac{Z_{12}}{Z_{22}}$) (NO UNIT)

$$h_{22} = \frac{I_2}{V_2} \bigg|_{I_1=0}$$

($h_{22} \neq Y_{22}$) ($h_{22} = \frac{1}{Z_{22}}$) (mho)



⇒ g - PARAMETER -

$$I_1 = g_{11} V_1 + g_{12} I_2$$

(KCL)

$$V_2 = g_{21} V_1 + g_{22} I_2$$

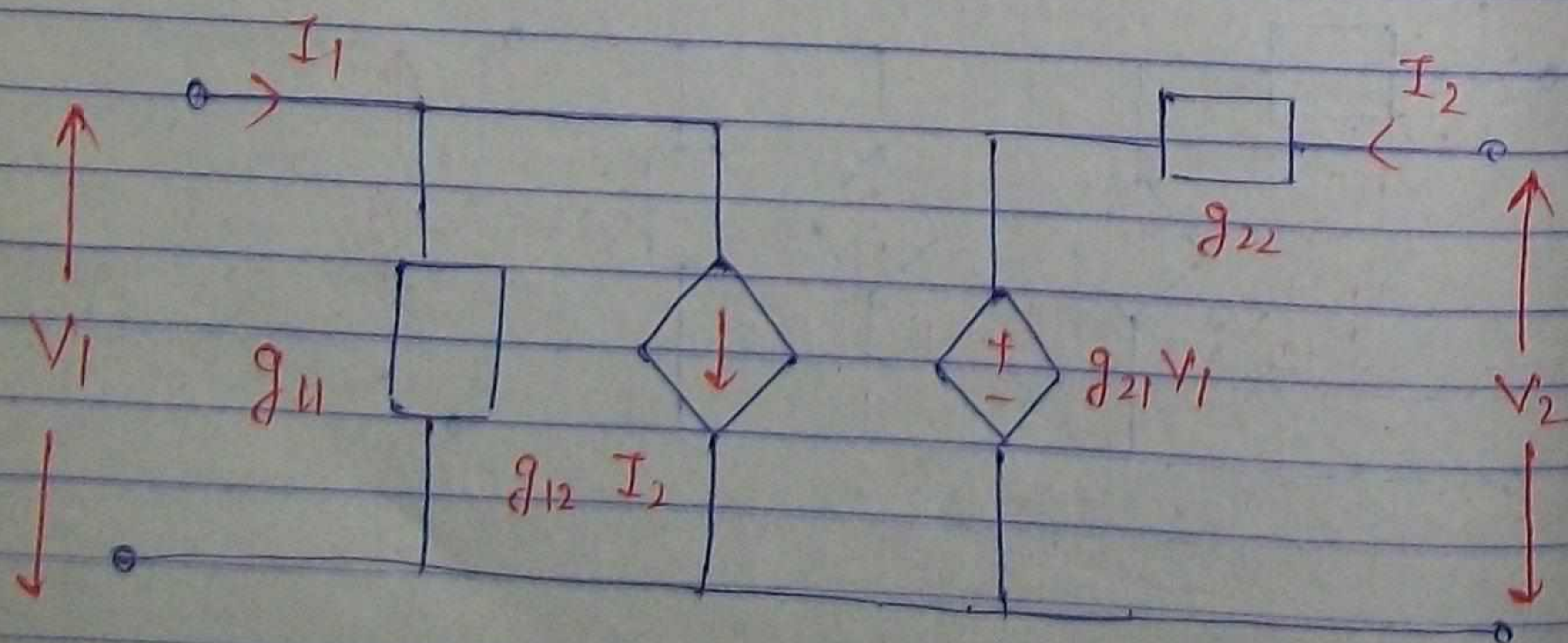
(KVL)

$$g_{11} = \frac{I_1}{V_1} \Big|_{I_2=0} \quad (g_{11} \neq y_{11}) \quad (g_{11} = 1/Z_{11}) \quad (\text{mho})$$

$$g_{21} = \frac{V_2}{V_1} \Big|_{I_2=0} \quad (g_{21} = \frac{Z_{21}}{Z_{11}}) \quad (\text{NO UNIT})$$

$$g_{12} = \frac{I_1}{I_2} \Big|_{V_1=0} \quad (g_{12} = \frac{y_{12}}{y_{22}}) \quad (\text{NO UNIT})$$

$$g_{22} = \frac{V_2}{I_2} \Big|_{V_1=0} \quad (g_{22} \neq Z_{22}) \quad (g_{22} = \frac{1}{y_{22}}) \quad (\text{ohm})$$



(-I₂) due to outward current in Tx. 1st

⇒ ABCD - PARAMETER -

$$V_1 = A V_2 - B I_2$$

KVL

$$I_1 = C V_2 - D I_2$$

KCL

$$A = \frac{V_1}{V_2} \Big|_{I_2=0} \quad (A = \frac{Z_{11}}{Z_{21}}) \quad (\text{NO UNITS})$$

$$C = \frac{I_1}{V_2} \Big|_{I_2=0} \quad (C = \frac{1}{Z_{21}}) \quad (\text{mho})$$

$$B = -\frac{V_1}{I_2} \Big|_{V_2=0} \quad (B = -\frac{1}{y_{21}}) \quad (\text{ohm})$$

$$D = -\frac{I_1}{I_2} \Big|_{V_2=0} \quad (D = -\frac{y_{11}}{y_{21}}) \quad (\text{NO UNITS})$$

NOTE -

for ABCD parameter it is not possible to develop equivalent circuit since both eqn ① and ② are developed w.r.t input.

⇒ abcd - PARAMETER -

$$V_2 = a V_1 - b I_1$$

$$I_2 = c V_1 - d I_1$$

$$a = \left. \frac{V_2}{V_1} \right|_{I_1=0}$$

(NO UNIT)

$$c = \left. \frac{I_2}{V_1} \right|_{I_1=0}$$

(mho)

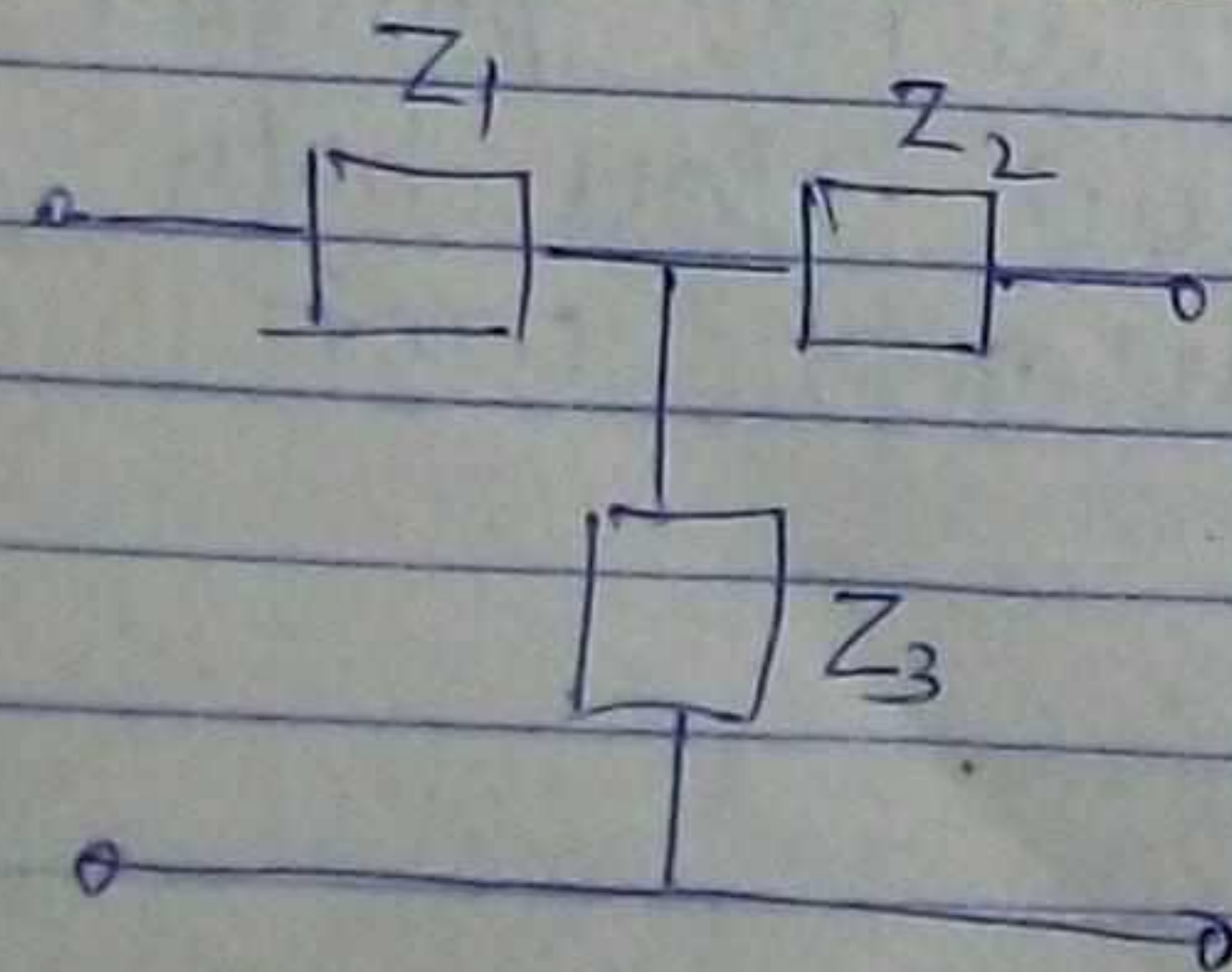
$$b = \left. \frac{V_2}{I_1} \right|_{V_1=0}$$

(ohm)

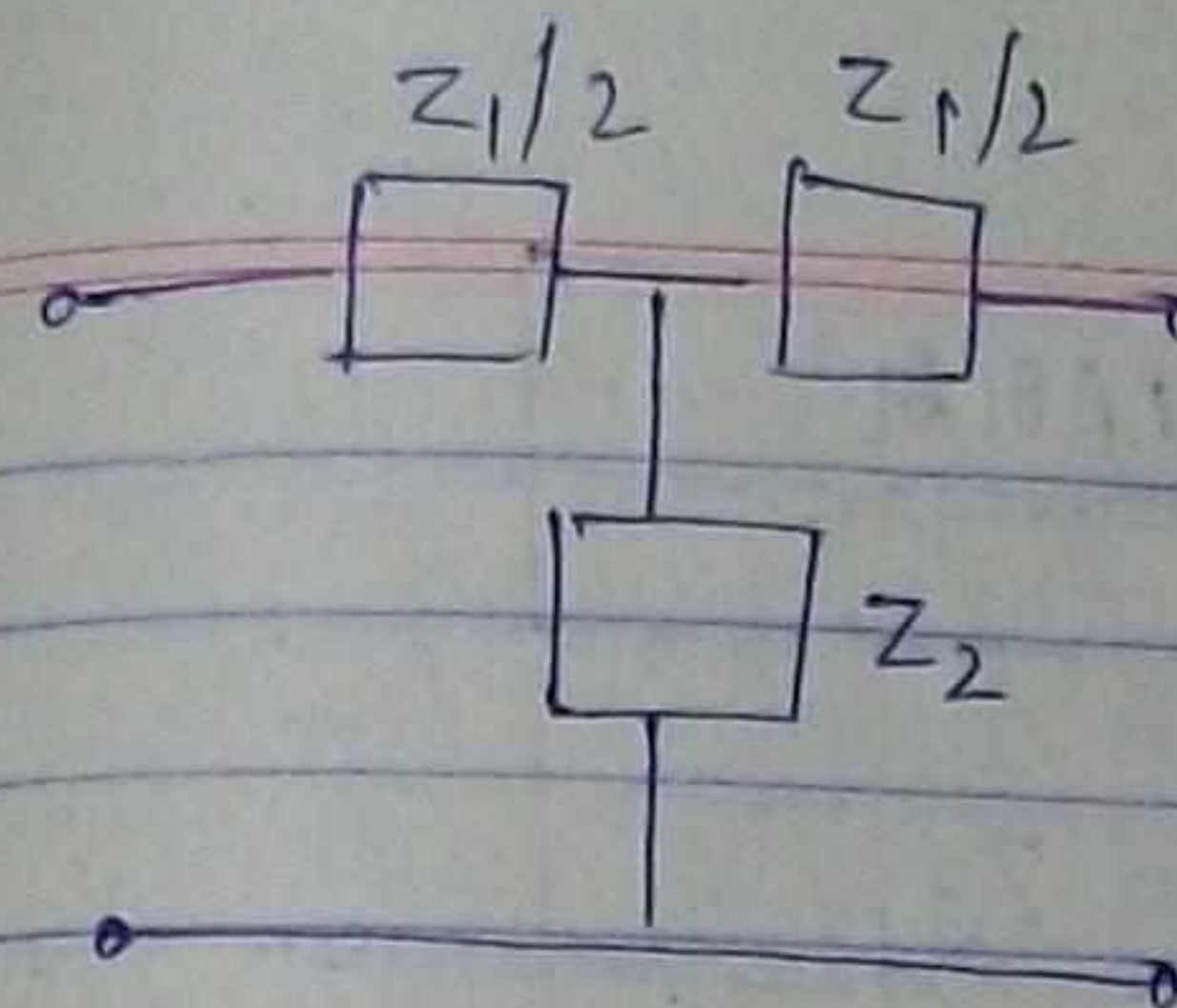
$$d = \left. \frac{I_2}{I_1} \right|_{V_1=0}$$

(NO UNIT)

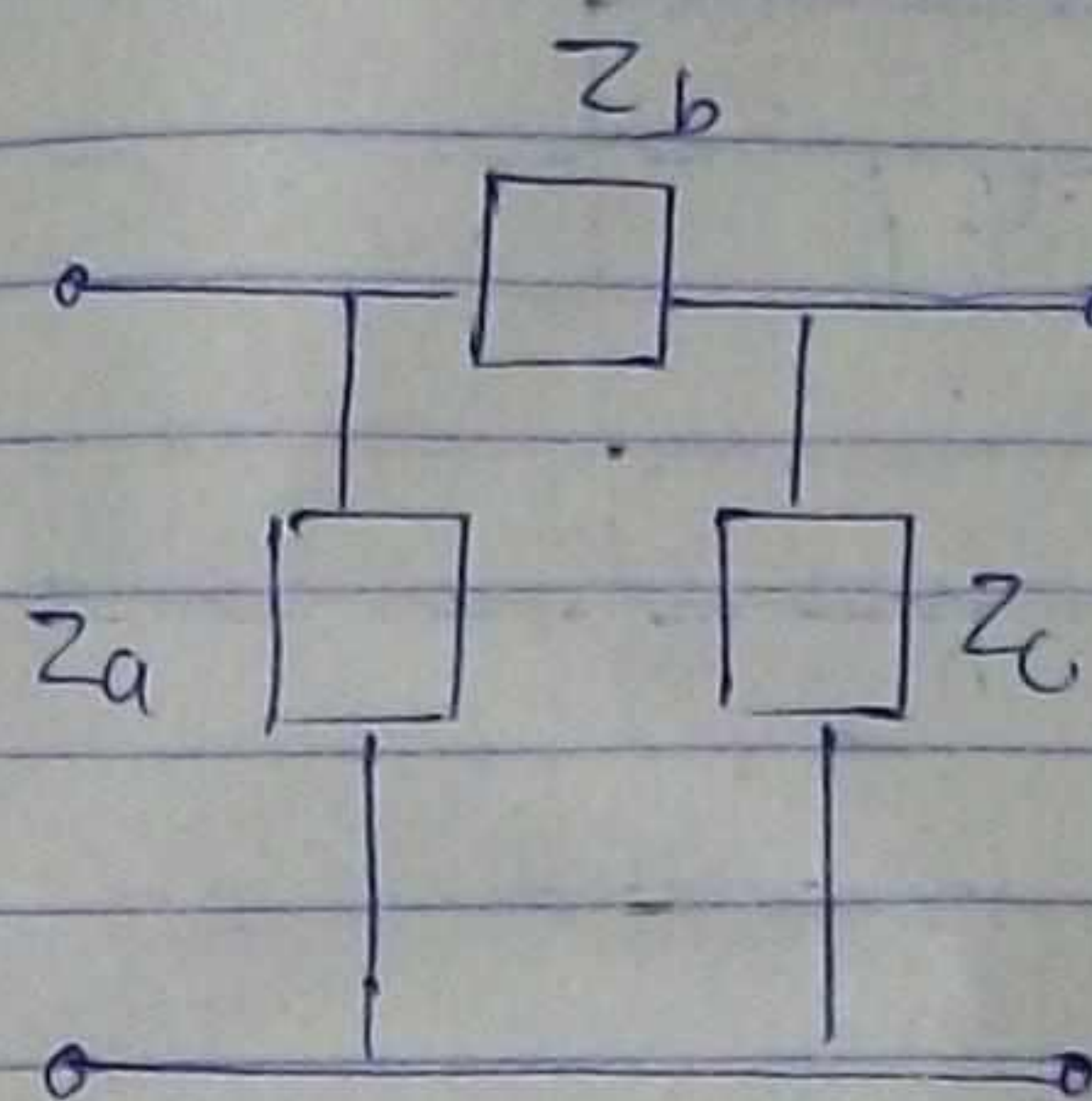
⇒ SYMMETRICAL & UNSYMMETRICAL N/W -



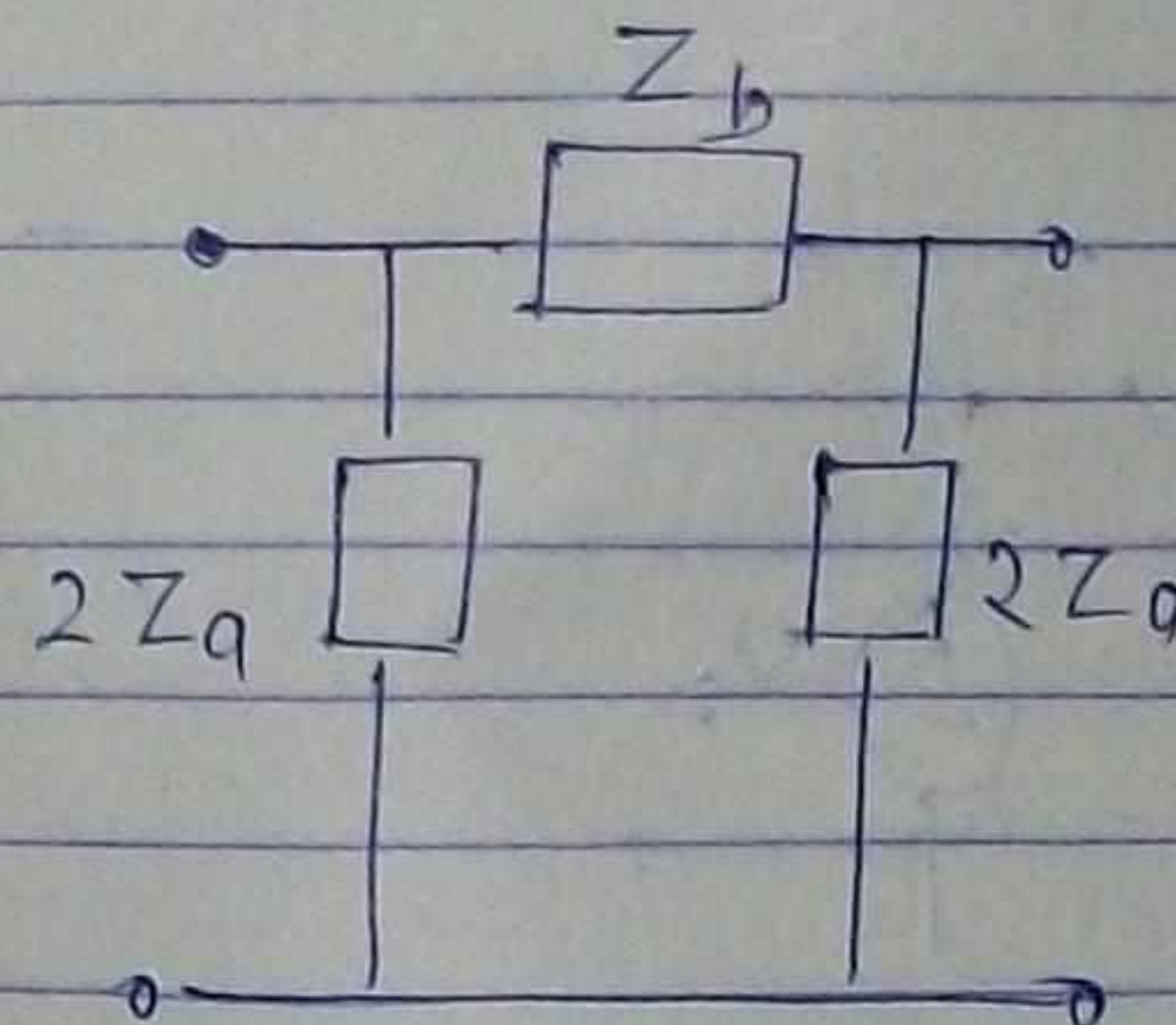
UNSYMMETRICAL
T-N/W



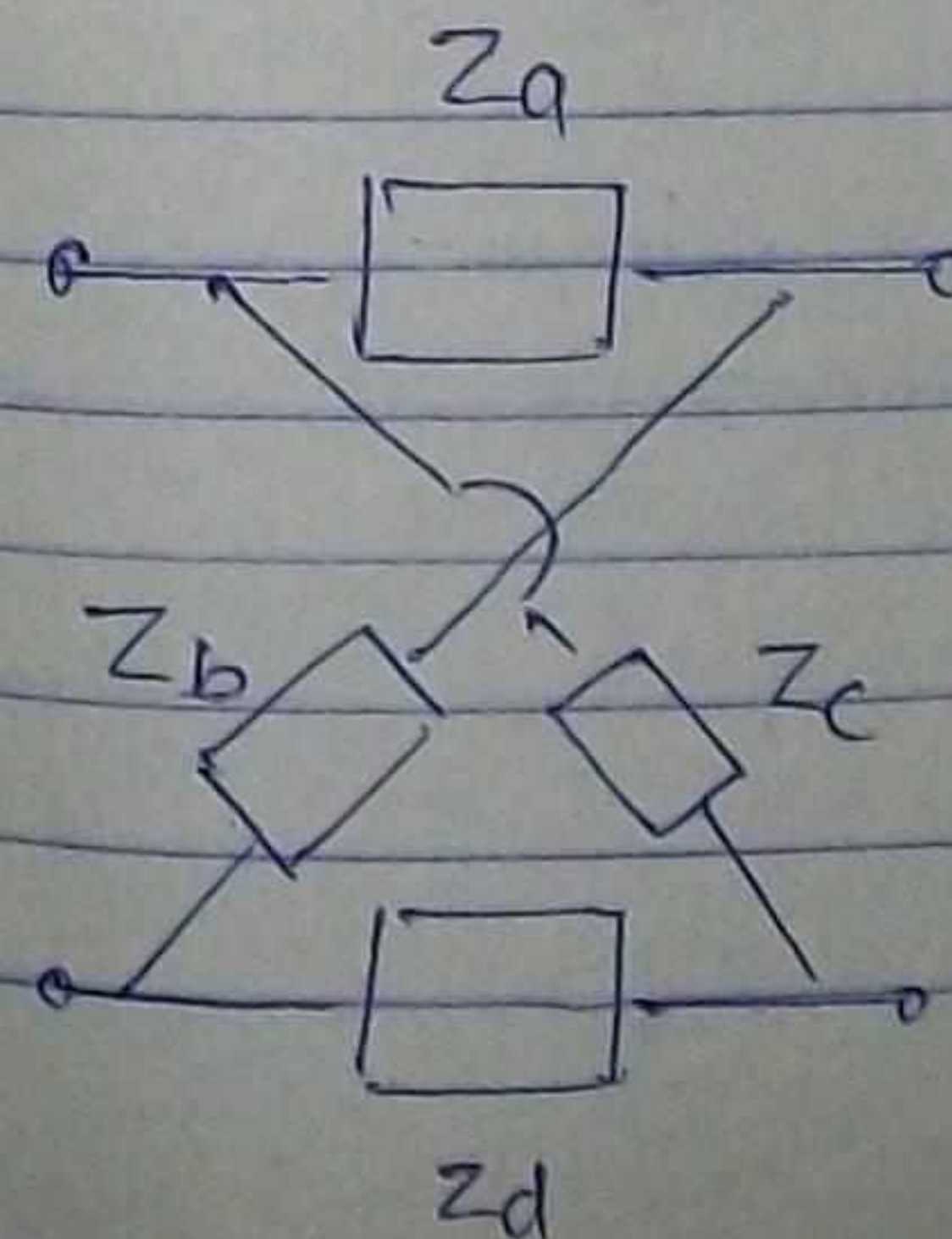
SYMMETRICAL
T-N/W



UNSYMMETRICAL
π-N/W



SYMMETRICAL
π-N/W

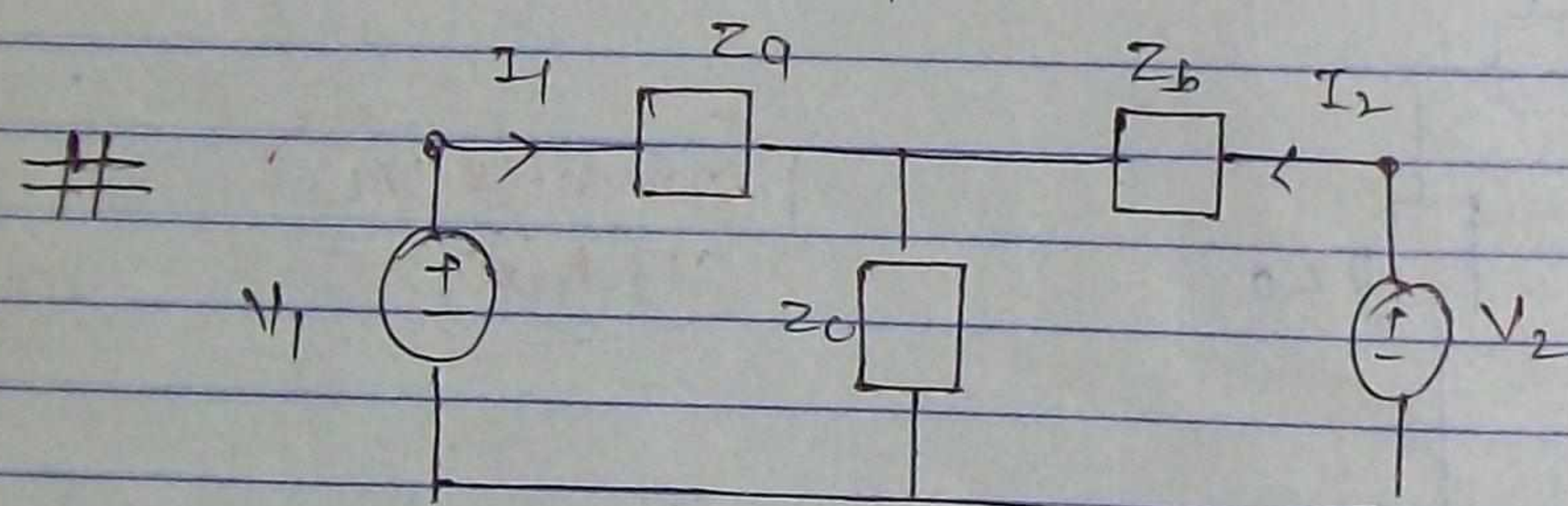


$Z_a = Z_d$
 $Z_b = Z_c$ } SYMMETRICAL
LATTICE N/W

$Z_a \neq Z_d$
 $Z_b \neq Z_c$ } UNSYMMETRICAL
LATTICE N/W

AA

<u>SYMMETRICAL</u>	<u>RECIPROCAL</u>
$Z_{11} = Z_{22}$	$Z_{12} = Z_{21}$
$Y_{11} = Y_{22}$	$Y_{12} = Y_{21}$
$h_{11}h_{22} - h_{12}h_{21} = 1$	$h_{12} = -h_{21}$
$A = D$	$AD - BC = 1$



$$V_1 = (Z_q + Z_c) I_1 + Z_c I_2 \quad \text{--- (1)}$$

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{--- (2)}$$

$$\left. \begin{aligned} Z_{11} &= Z_q + Z_c \\ Z_{12} &= Z_{21} = Z_c \end{aligned} \right\}$$

$$V_2 = Z_c I_1 + (Z_b + Z_c) I_2 \quad \text{--- (3)}$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{--- (4)}$$

$$\left. \begin{aligned} Z_{21} &= Z_c \\ Z_{22} &= Z_b + Z_c \end{aligned} \right\}$$

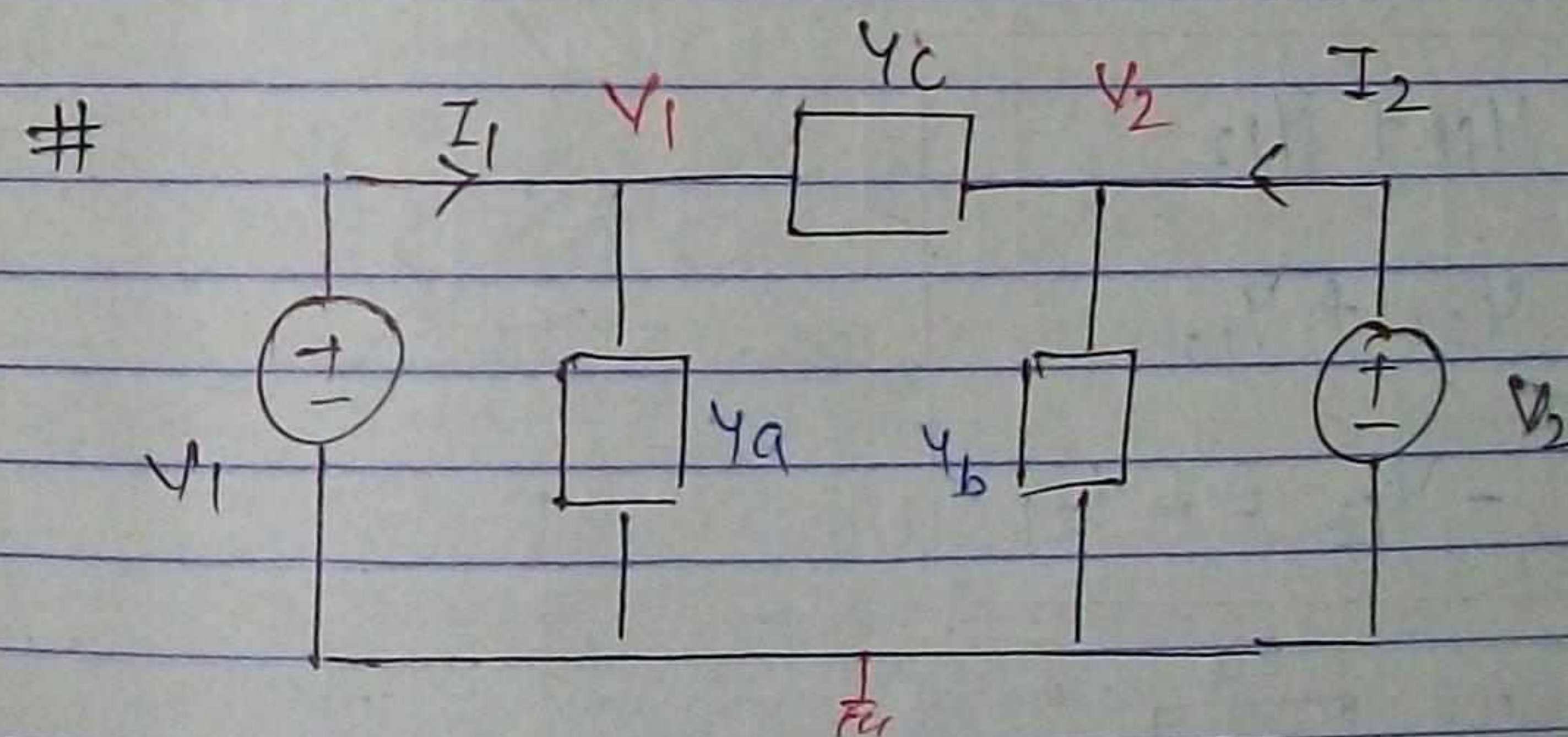
fhw

AA

$$Z_a = Z_{11} - Z_{12}$$

$$Z_b = Z_{22} - Z_{12}$$

$$Z_c = Z_{21} = Z_{12}$$



$$I_1 = V_1 Y_a + (V_1 - V_2) Y_c$$

$$I_1 = (Y_a + Y_c) V_1 - Y_c V_2 \quad \text{--- (1)}$$

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad \text{--- (2)}$$

5/11/11

$$Y_{11} = Y_a + Y_c$$

$$Y_{12} = -Y_c$$

$$I_2 = V_2 Y_b + (V_2 - V_1) Y_c$$

$$I_2 = -Y_c V_1 + (Y_b + Y_c) V_2 \quad - (3)$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \quad - (4)$$

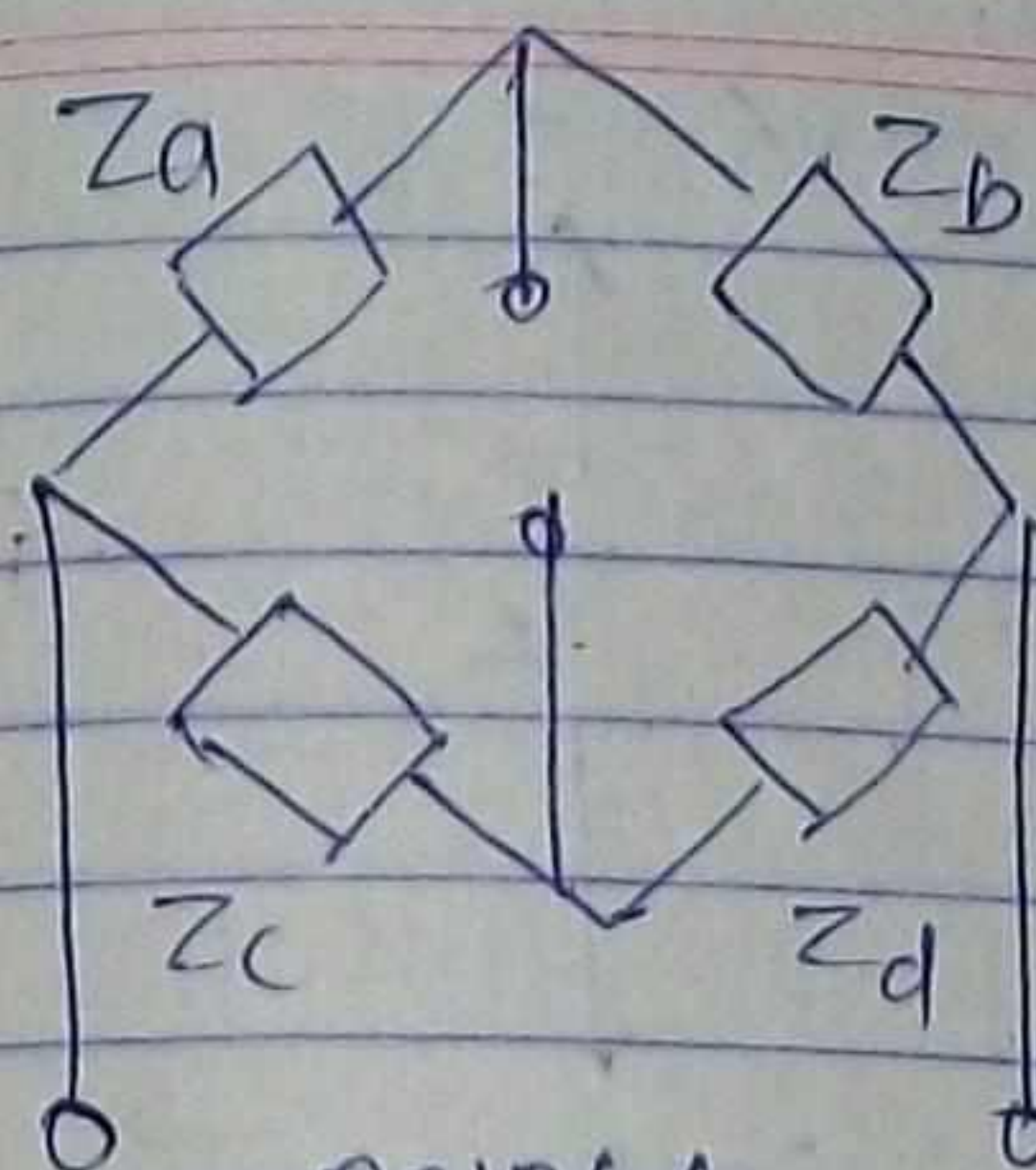
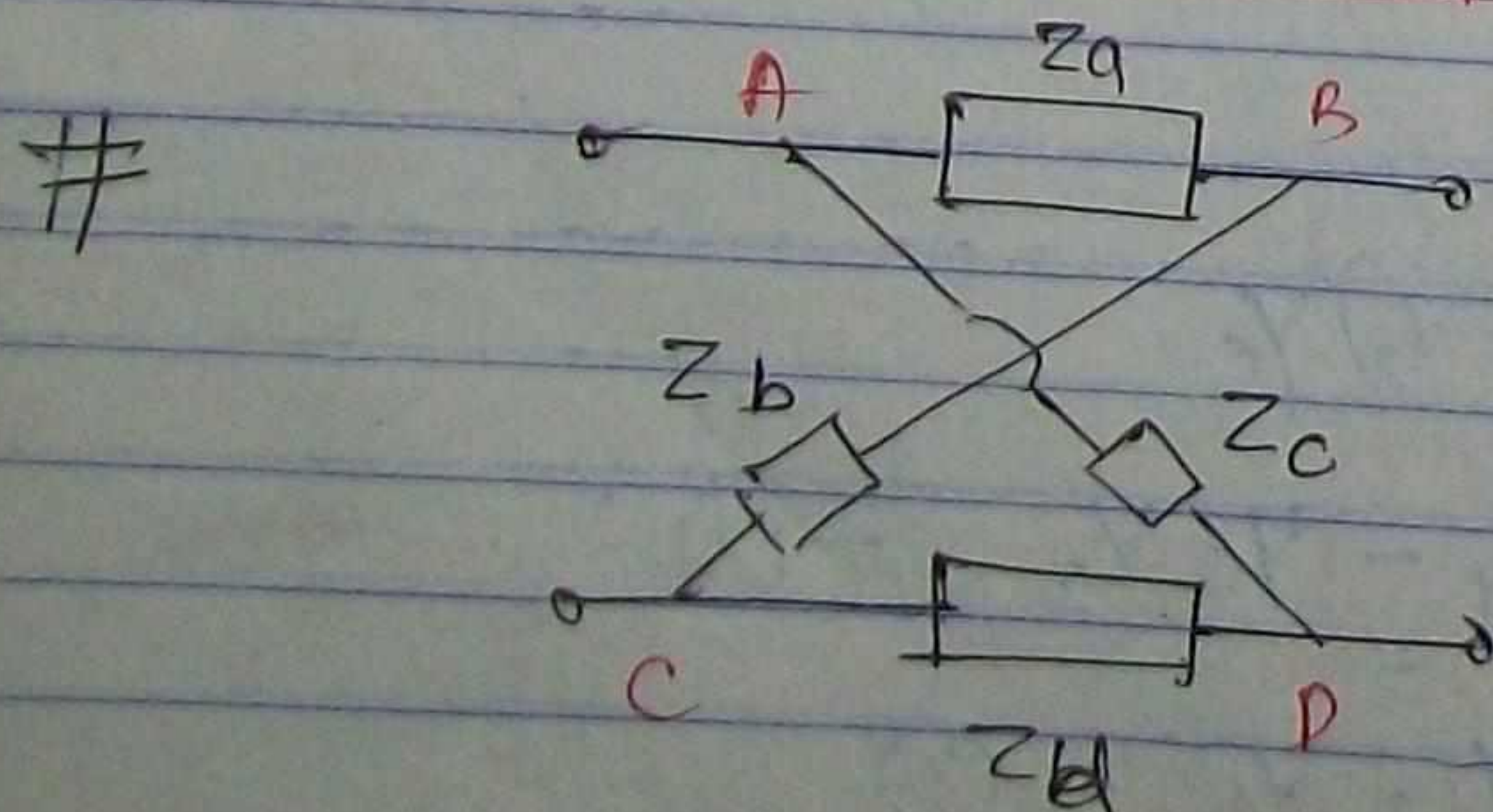
$$Y_{22} = Y_b + Y_c$$

$$Y_{21} = -Y_c$$

$$Y_a = Y_{11} + Y_{12}$$

$$Y_b = Y_{22} + Y_{12}$$

$$Y_c = -Y_{12} = -Y_{21}$$



BRIDGE
N/W

$$Z_a = Z_d$$

$$Z_b = Z_c$$

SYMMETRICAL
LATTICE

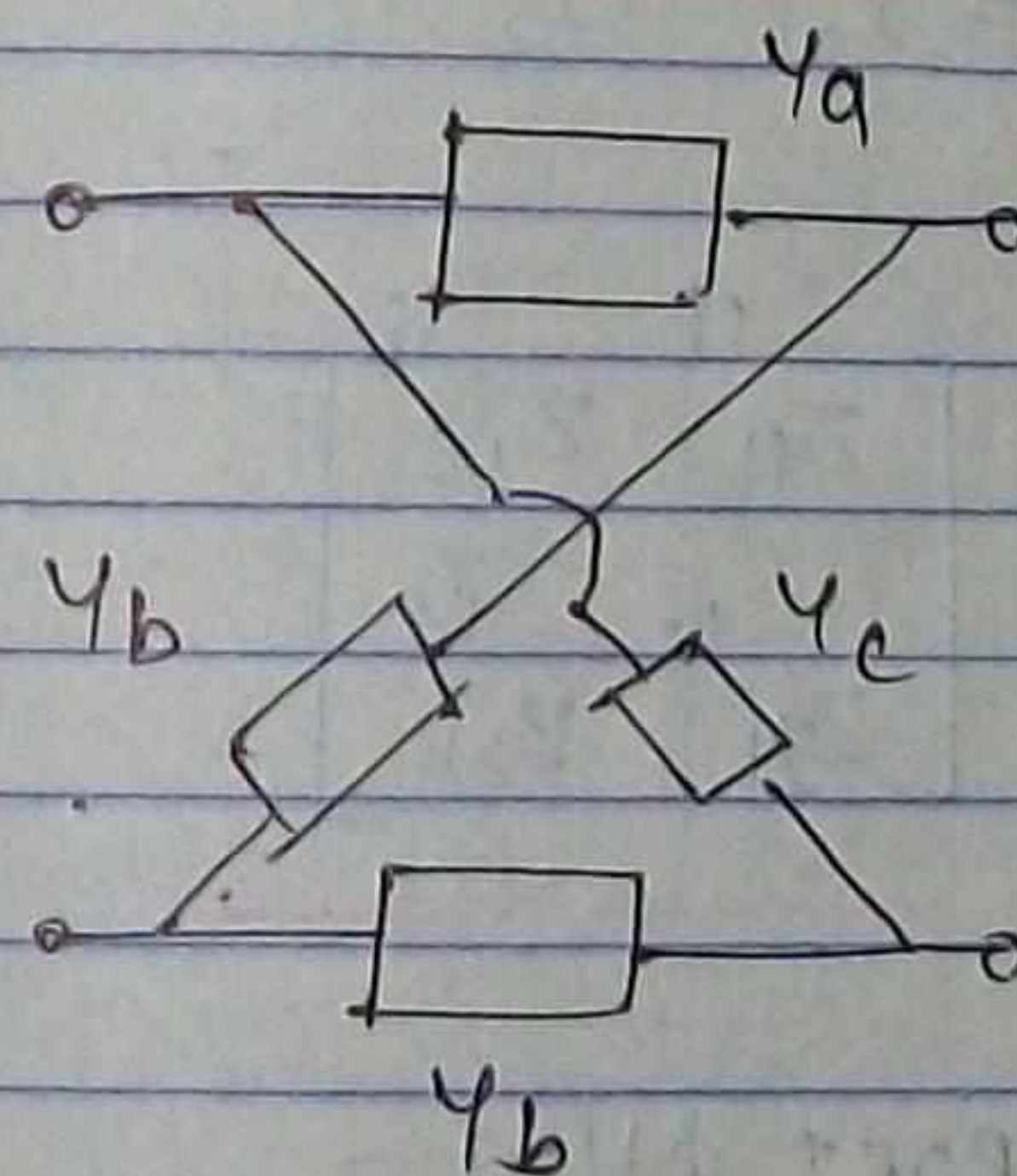
$$Z_{11} = Z_{22} = \frac{Z_b + Z_a}{2}$$

$$Z_{12} = Z_{21} = \frac{Z_b - Z_a}{2}$$

$$Z_b = Z_{11} + Z_{12}$$

$$Z_a = Z_{11} - Z_{12}$$

#



$$Y_a = Y_d$$

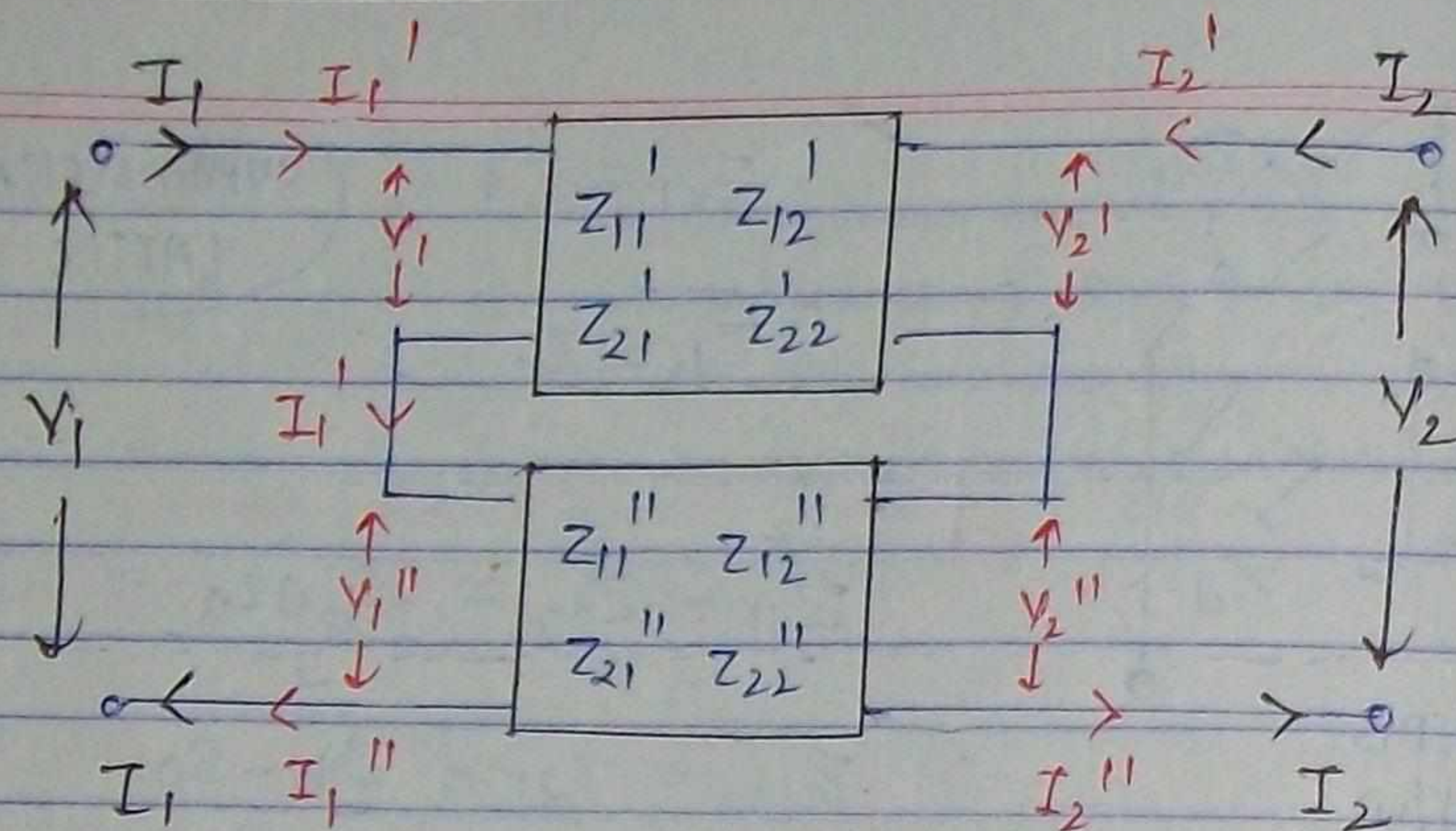
$$Y_b = Y_c$$

SYMMETRICAL
LATTICE

$$Y_{11} = Y_{22} = \frac{Y_b + Y_a}{2}$$

$$Y_{12} = Y_{21} = \frac{Y_b - Y_a}{2}$$

⇒ SERIES CONNECTION OF 2 PORT N/W -



$$V_1 = V_1' + V_1''$$

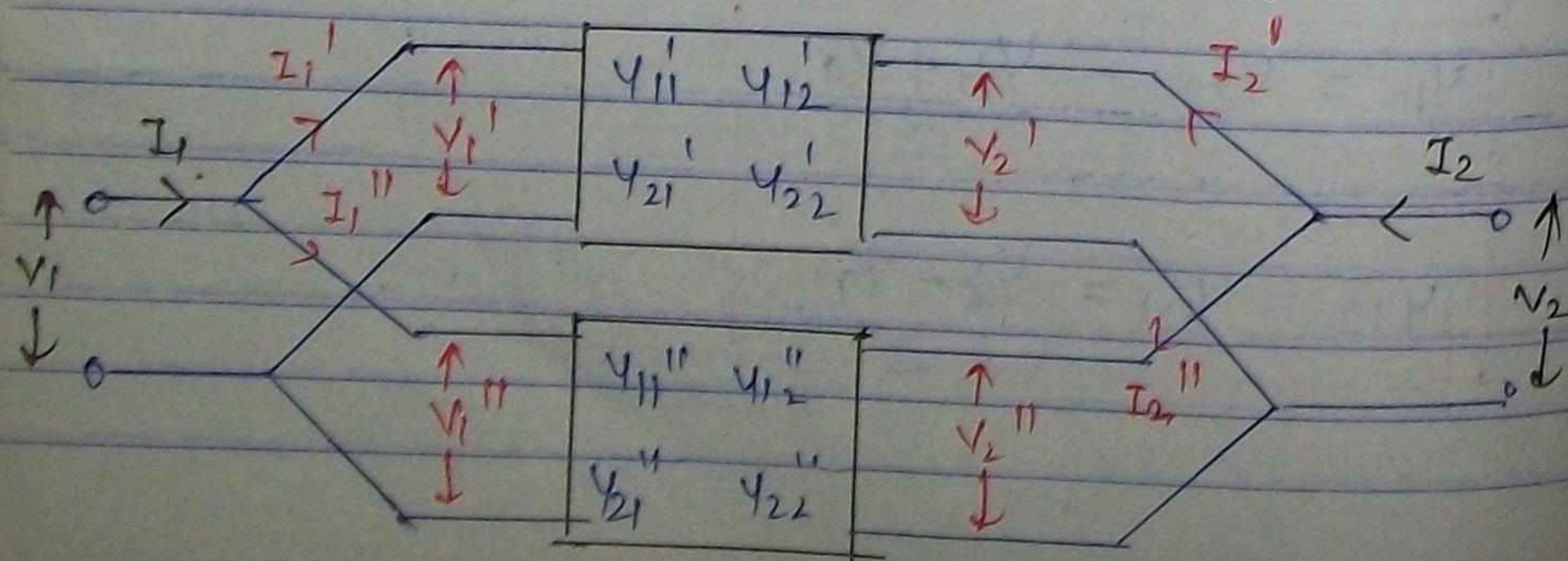
$$V_2 = V_2' + V_2''$$

$$I_1 = I_1' = I_1''$$

$$I_2 = I_2' = I_2''$$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Z_{11}' & Z_{12}' \\ Z_{21}' & Z_{22}' \end{bmatrix} + \begin{bmatrix} Z_{11}'' & Z_{12}'' \\ Z_{21}'' & Z_{22}'' \end{bmatrix}$$

⇒ PARALLEL CONNECTION OF 2 PORT N/W -



$$V_1 = V_1' + V_1''$$

$$I_2 = I_2' + I_2''$$

$$I_1 = I_1' = I_1''$$

$$V_2 = V_2' = V_2''$$

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} h_{11}' & h_{12}' \\ h_{21}' & h_{22}' \end{bmatrix} + \begin{bmatrix} h_{11}'' & h_{12}'' \\ h_{21}'' & h_{22}'' \end{bmatrix}$$

$$I_1 = I_1' + I_1''$$

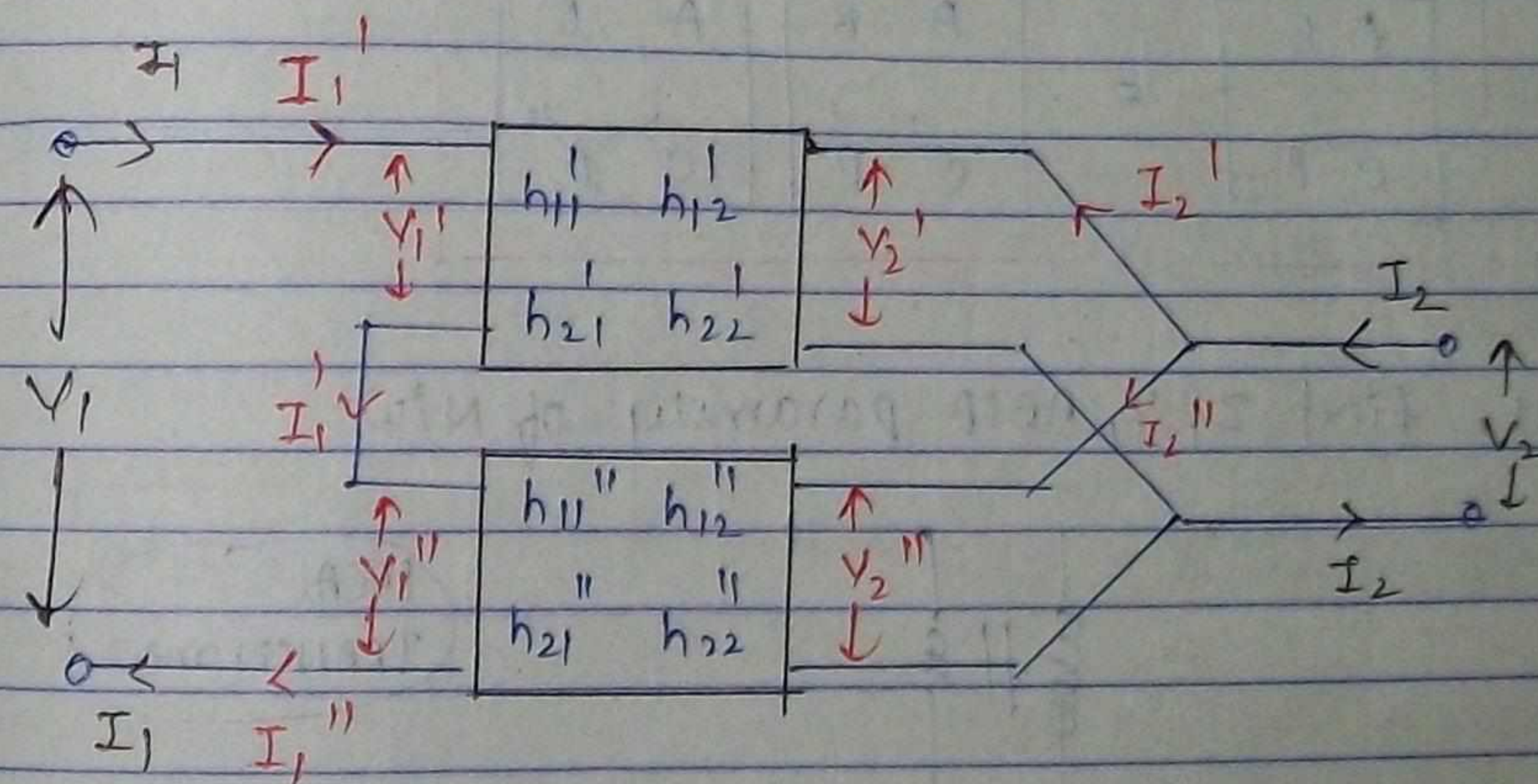
$$I_2 = I_2' + I_2''$$

$$V_1 = V_1' = V_1''$$

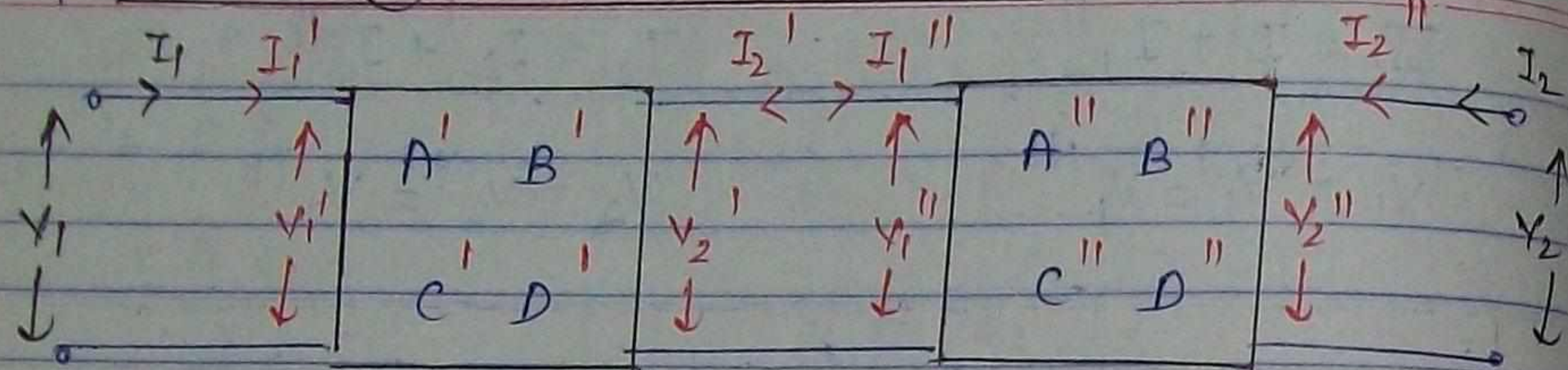
$$V_2 = V_2' = V_2''$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Y_{11}' & Y_{12}' \\ Y_{21}' & Y_{22}' \end{bmatrix} + \begin{bmatrix} Y_{11}'' & Y_{12}'' \\ Y_{21}'' & Y_{22}'' \end{bmatrix}$$

⇒ COMPLEX CONNECTION -



⇒ CASCADE (OR) TANDUM CONNECTION -



$$V_1 = V_1'$$

$$V_2' = V_1''$$

$$V_2 = V_2''$$

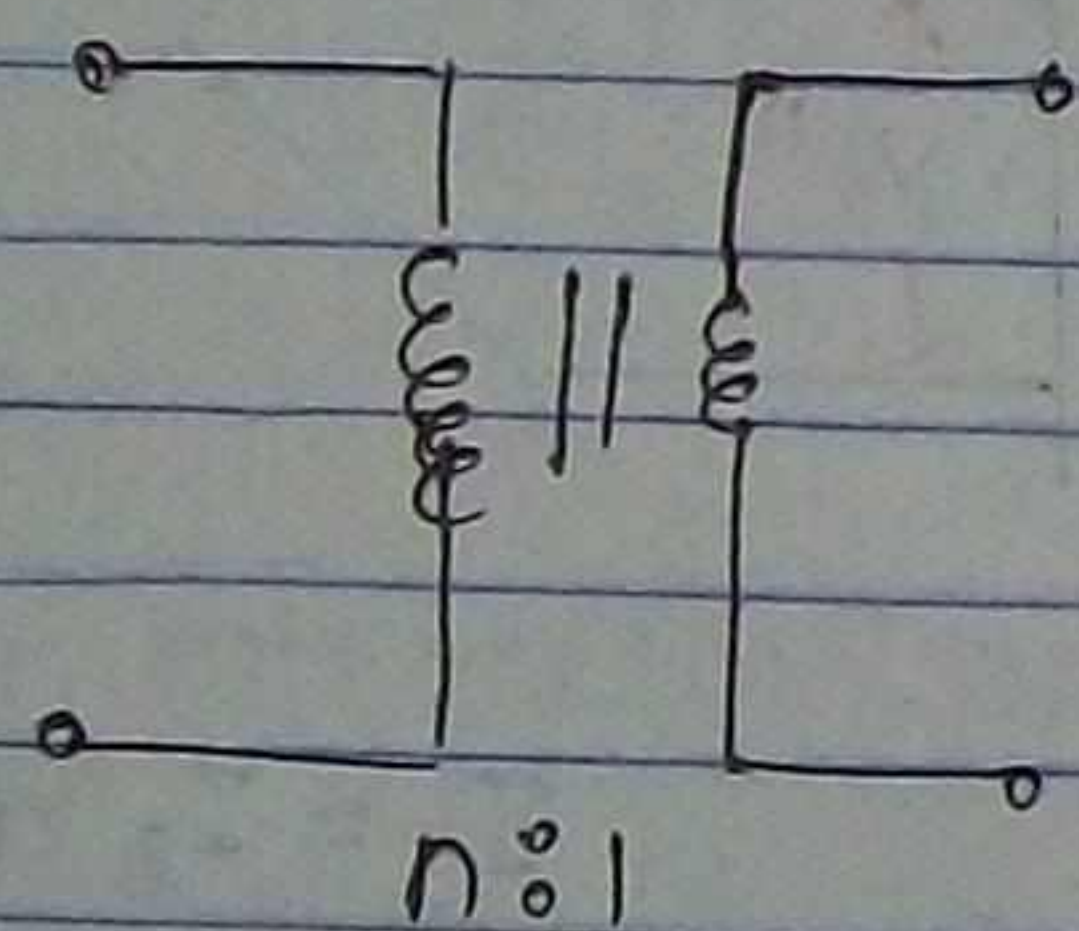
$$I_1 = I_1'$$

$$I_1'' = -I_2'$$

$$I_2 = I_2''$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} A'' & B'' \\ C'' & D'' \end{bmatrix}$$

Ques. find Z, Y, ABCD parameters of N/w



IDEAL TRANSFORMERS

Solⁿ

- for ideal T/F it is not possible to find impedance and admittance value since self and mutual inductance of ideal T/F are infinite.

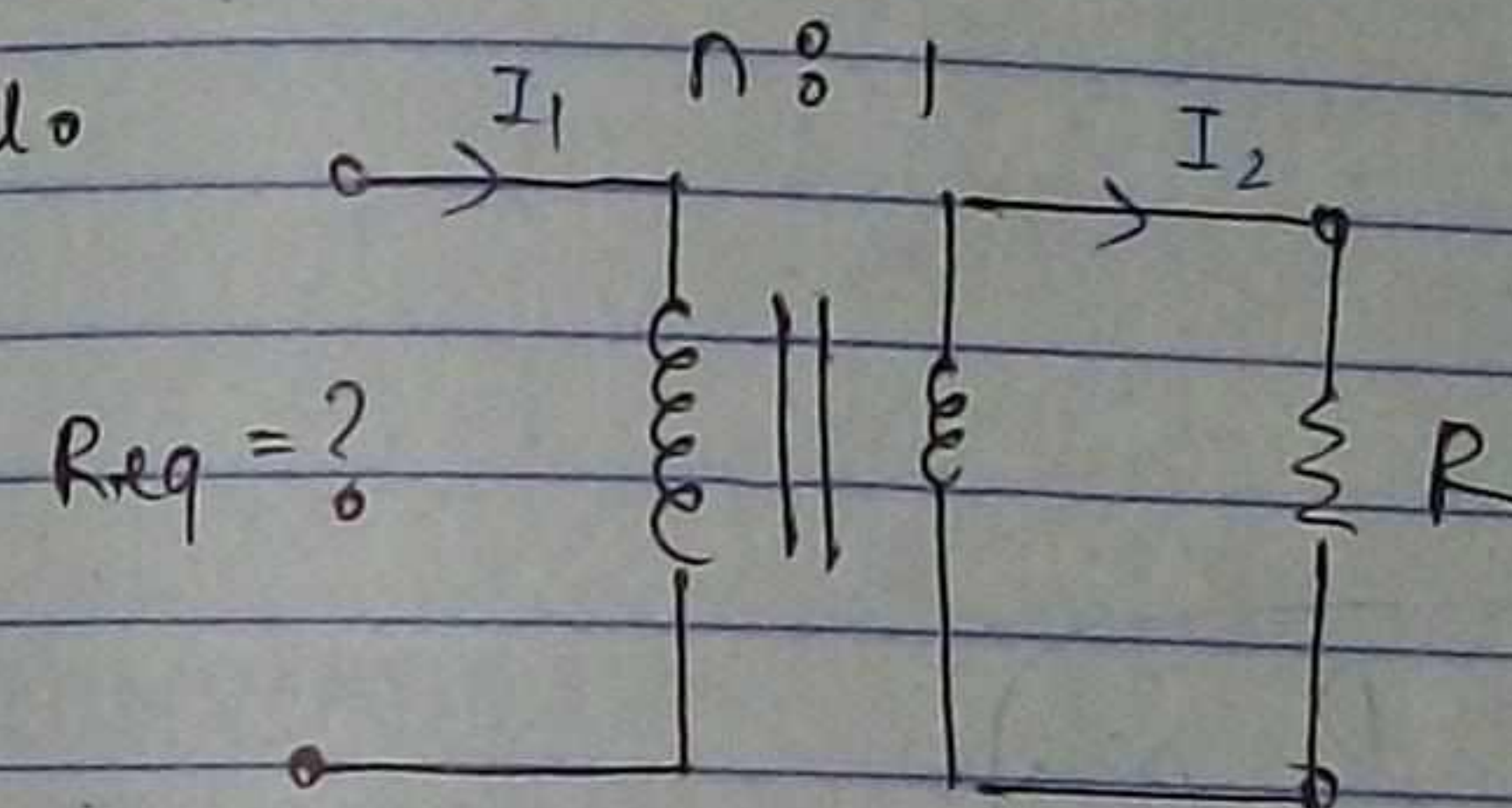
$$A = \frac{V_1}{V_2} = n$$

$$D = -\frac{I_1}{I_2} = +\frac{1}{n}$$

$$\frac{I_2}{I_1} = \frac{V_1}{V_2} = \frac{N_1}{N_2}$$

$$\frac{I_2}{I_1} = \frac{V_1}{V_2} = \frac{n}{1}$$

Ques.



IDEAL TIP

Solⁿ:

$$I_1^2 R' = I_2^2 R$$

$$R' = \left(\frac{I_2}{I_1}\right)^2 R$$

similarly,

$$L' = n^2 L$$

$$Z' = n^2 Z$$

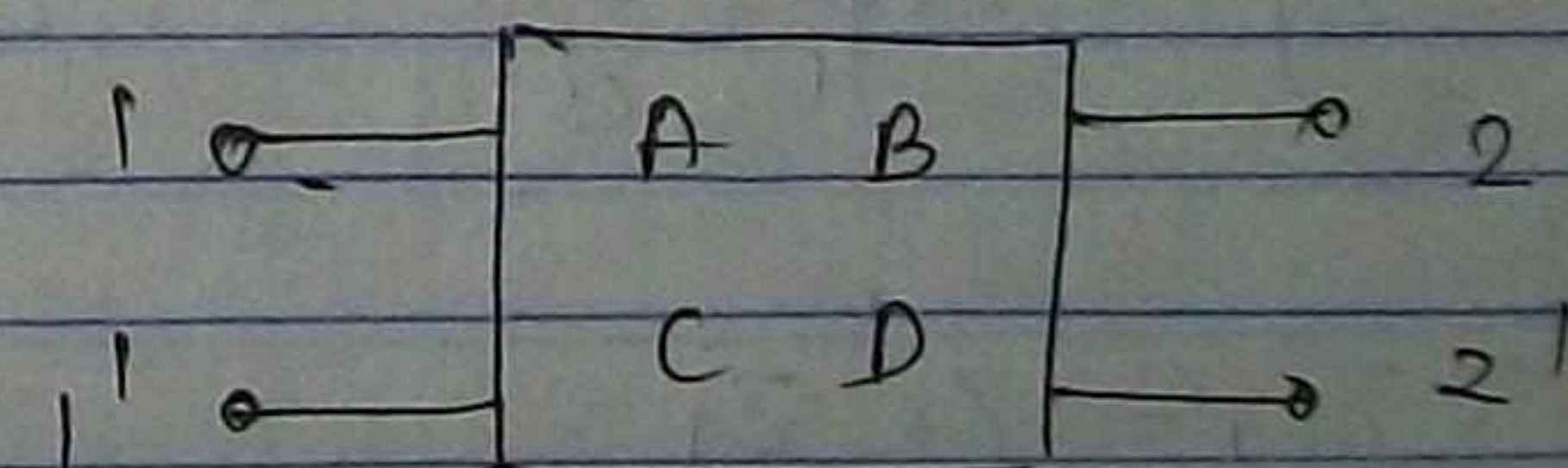
$$C' = \frac{C}{n^2}$$

$$R' = n^2 R$$

Ques. find Z_{eq}

① wrt 1-1'

② 2-2'



Solⁿ: ① 1-1'

$$Z_{eq} = \frac{V_1}{I_1} = \frac{AV_2 - BI_2}{CV_2 - DI_2} \Big|_{I_2=0}$$

$$Z_{eq} = \frac{A}{C} \Omega$$

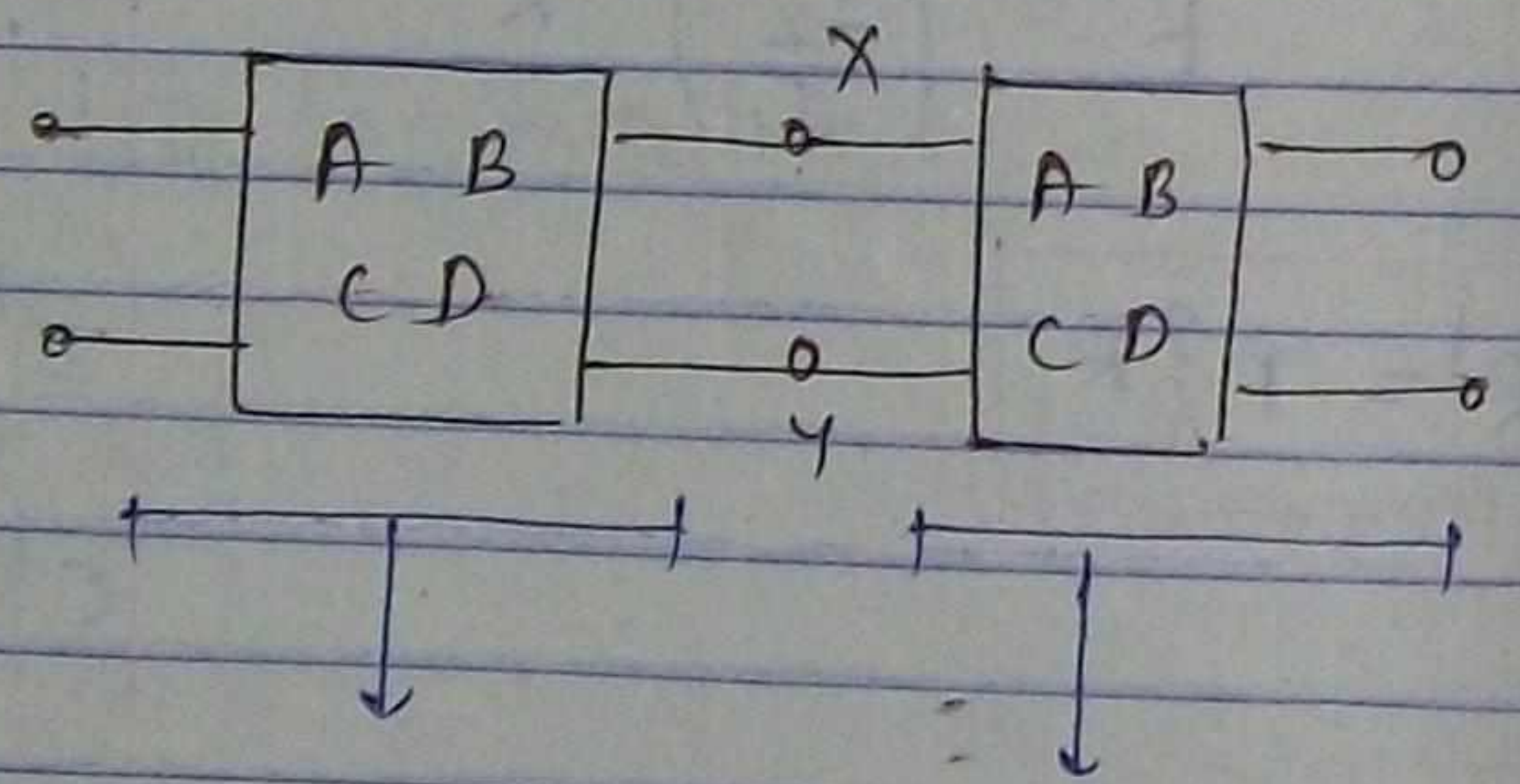
Q. wrt 2-2' - $Z_{eq} = \frac{V_2}{I_2} \Big|_{I_1=0}$

$$I_1 = CV_2 - DI_2$$

$$0 = CV_2 - DI_2$$

$$Z_{eq} = \frac{V_2}{I_2} = \frac{D}{C} \Omega$$

Q. Find Z_{eq} wrt X and Y

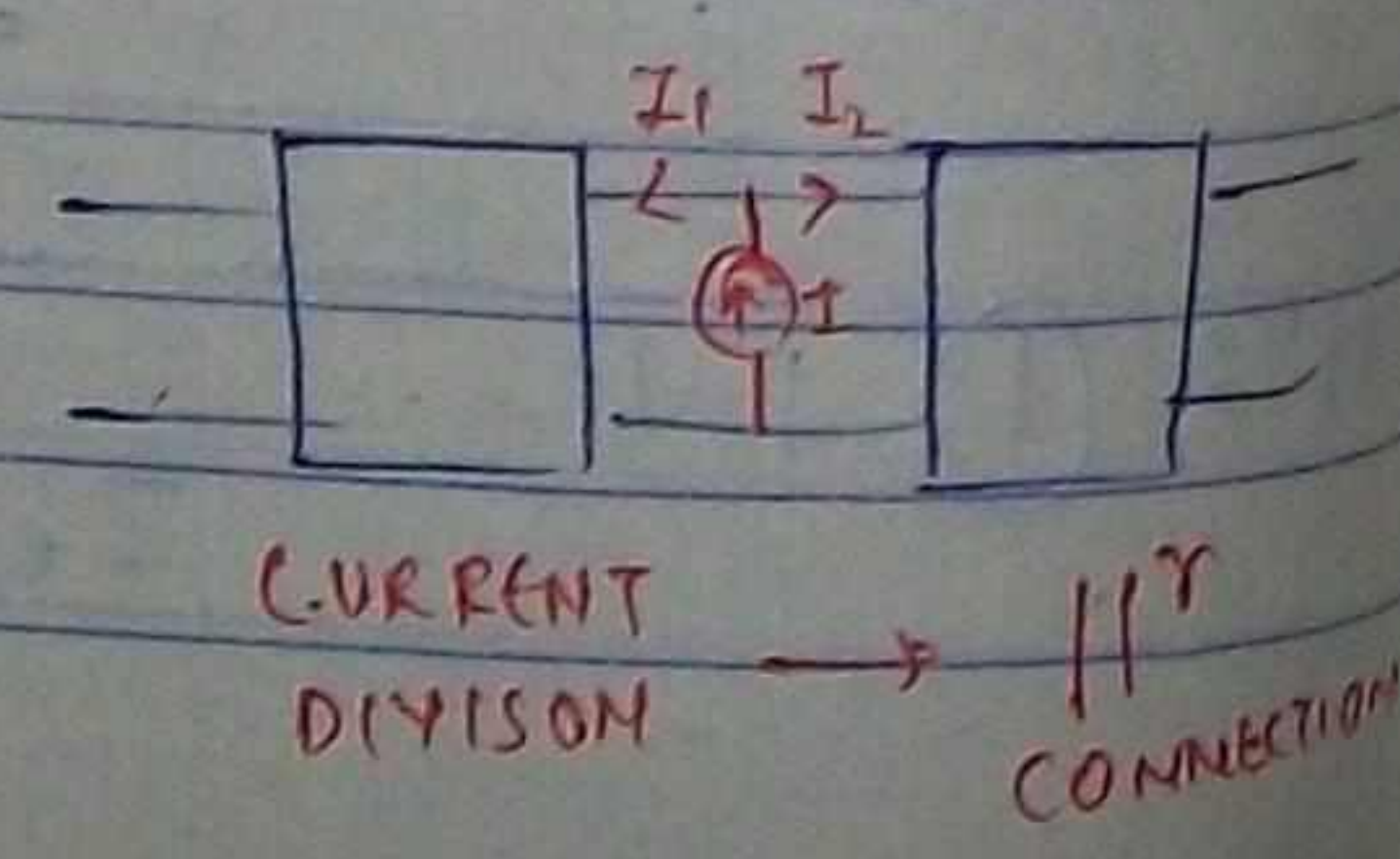


Solⁿ:

$$Z_1 = Z_{eq1} = \frac{D}{C}$$

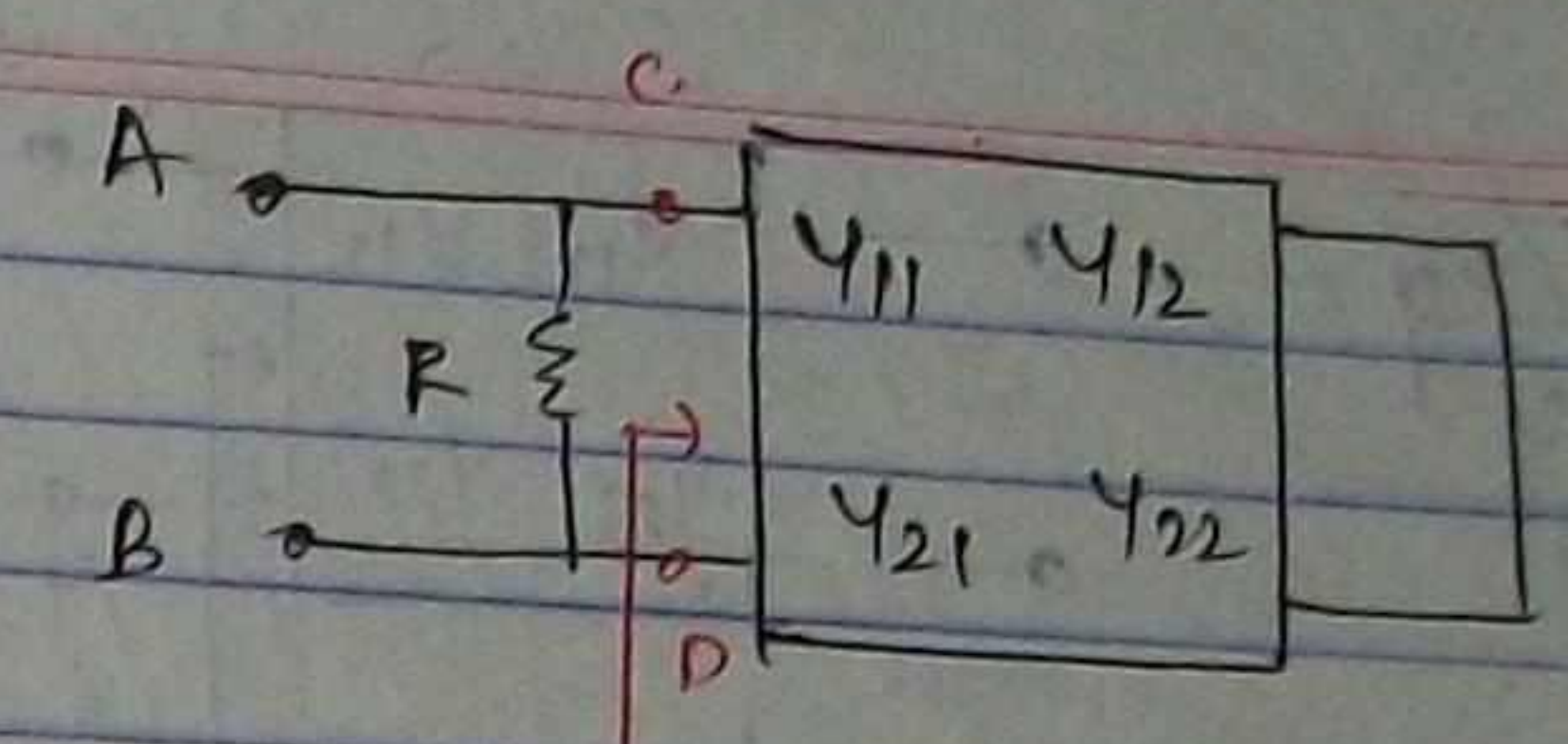
$$Z_1' = \frac{A}{C}$$

$$Z_{eq} = \frac{Z_1' Z_1}{Z_1 + Z_1'}$$

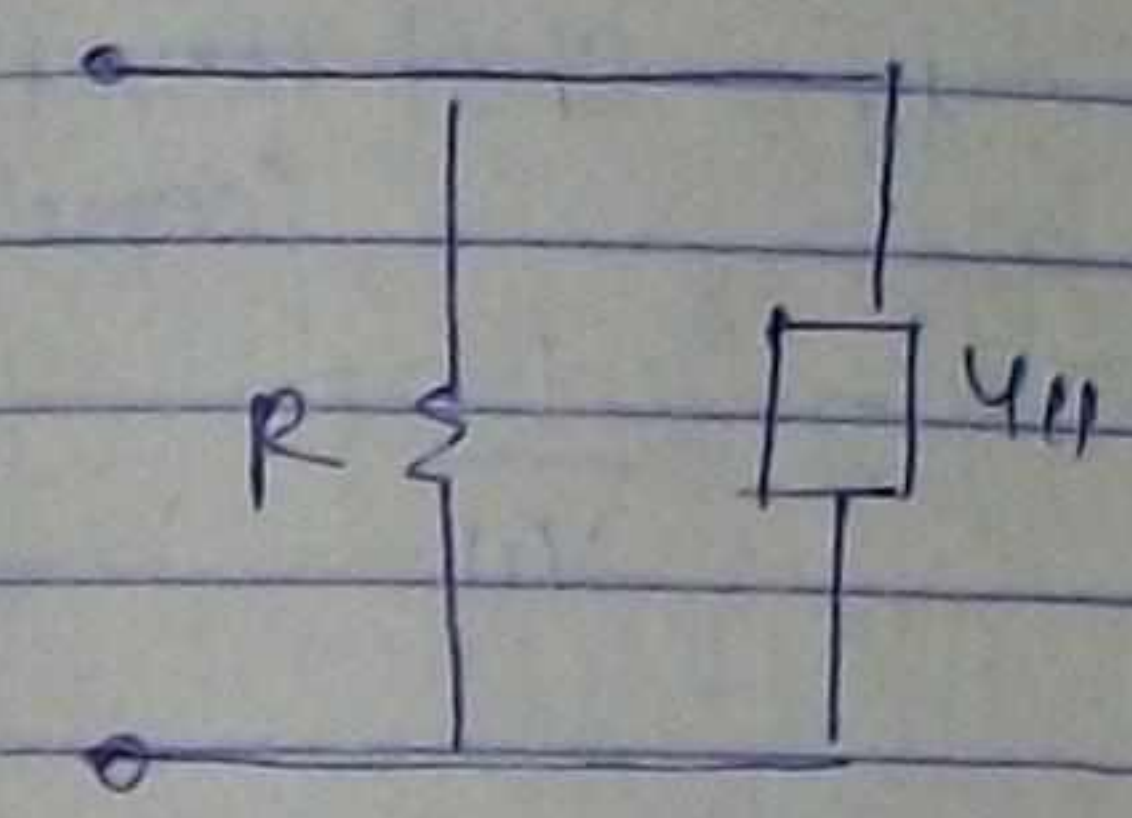


Q. find Y_{eq} wrt A, B

Solⁿ:



Y_{11} $\rightarrow$ equivalent admittance



$$Y_{eq} = Y_{11} + \frac{1}{R}$$

$\Rightarrow$ CONVERSIONS -

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 1 & Z_{22}-Z_{12} \\ Z_{11}Z_{22}-Z_{12}Z_{21} & -Z_{21}Z_{11} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1 & Y_{22}-Y_{12} \\ Y_{11}Y_{22}-Y_{12}Y_{21} & -Y_{21}Y_{11} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$Z_{11} = \frac{Y_{22}}{Y_{\Delta}}, \quad Z_{12} = \frac{-Y_{12}}{Y_{\Delta}}$$

$$Y_{11} = \frac{Z_{22}}{Z_{\Delta}}, \quad Y_{12} = \frac{-Z_{12}}{Z_{\Delta}}$$

$$Z_{21} = \frac{-Y_{21}}{Y_{\Delta}}, \quad Z_{22} = \frac{Y_{11}}{Y_{\Delta}}$$

$$Y_{21} = \frac{-Z_{21}}{Z_{\Delta}}, \quad Y_{22} = \frac{Z_{11}}{Z_{\Delta}}$$

$$Y_{\Delta} = Y_{11}Y_{22} - Y_{12}Y_{21}$$

$$Z_{\Delta} = Z_{11}Z_{22} - Z_{12}Z_{21}$$

Ques. find Z_{eq}
wrt x4

Solⁿ:

$$Z_{22} = Y_{11}$$

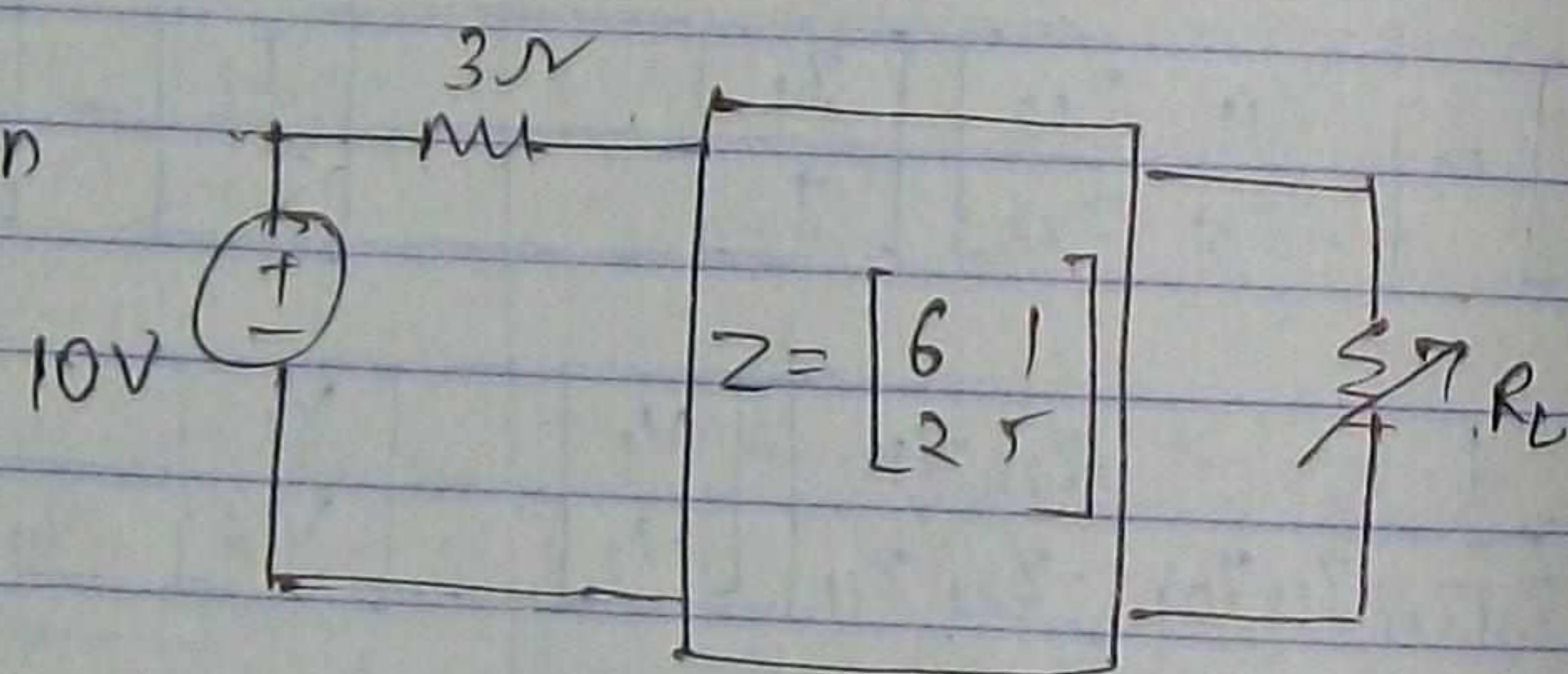
$$Y_{11} Y_{22} - Y_{12} Y_{21}$$

Y_{11} = Short circuit I/P
Imped

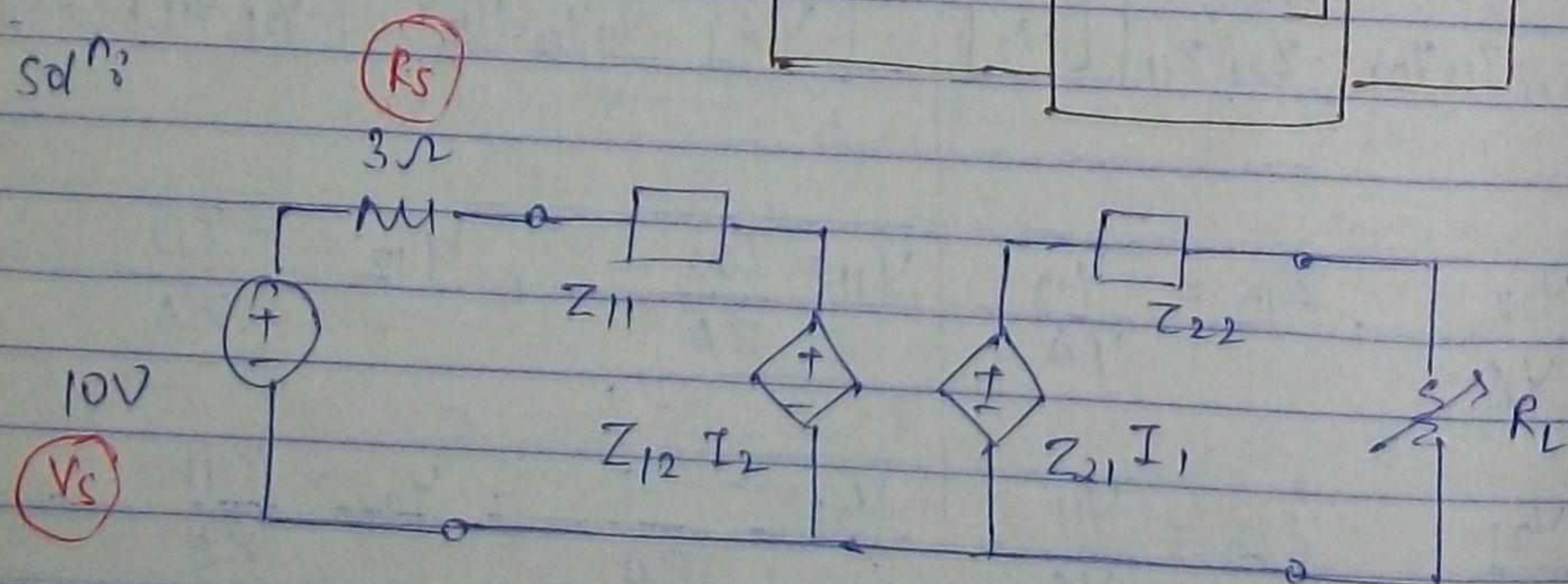
$$Z' = \frac{1}{Y_{11}}$$

$$Z_{eq} = \frac{Z_{22} Z'}{Z' + Z_{22}}$$

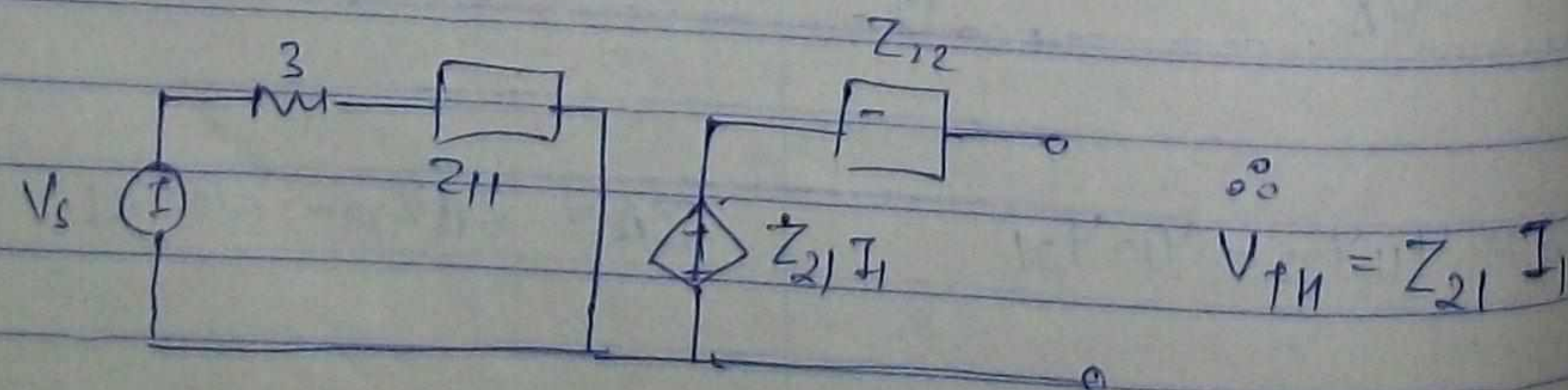
Ques. find max
power dissipation



Solⁿ:



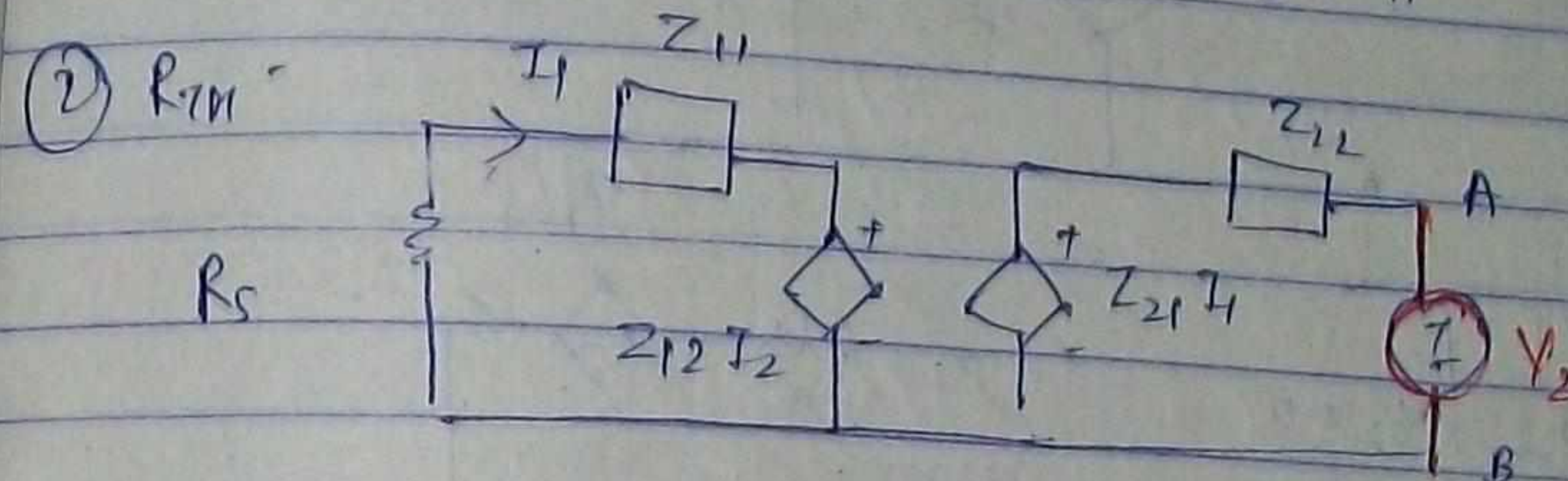
① V_{TH}



$$V_{TH} = Z_{21} I_1$$

$$\frac{5}{30-3} \quad \left(\frac{27}{5}\right)^2$$

$$V_{TH} = Z_{21} I_1 = Z_{21} \cdot \frac{V_s}{R_s + Z_{11}} = \frac{2 \times 10}{3 + 6} = \frac{20}{9}$$



$$I_1 = \frac{-Z_{12} I_2}{R_s + Z_{11}}$$

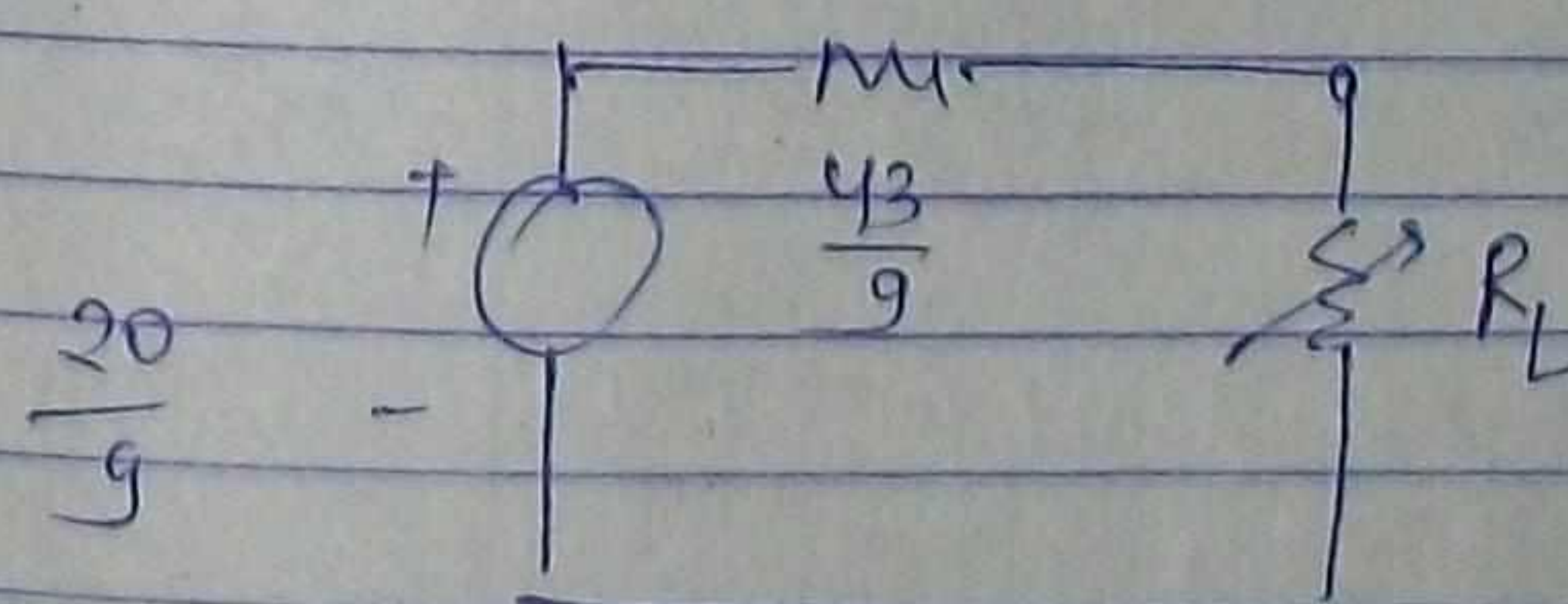
$$V_2 = I_2 Z_{22} + Z_{21} I_1$$

$$V_2 = I_2 Z_{22} - \frac{Z_{21} Z_{12} I_2}{R_s + Z_{11}}$$

$$\frac{V_2}{I_2} = \frac{Z_{22} - \frac{Z_{21} Z_{12}}{R_s + Z_{11}}}{1} = \frac{5 - \frac{2 \times 1}{3 + 6}}{1} = \frac{5 - \frac{2}{9}}{1}$$

$$Z_{TH} = \frac{43}{9} \Omega$$

③



$$R_{TH} = R_L$$

$$P_m = \frac{V^2}{4R_L} = \frac{20^2 \times 9}{9^2 \times 4 \times 43} = \frac{400}{387}$$

Ques find
Band D
of circuit

Soln:

$$B = -\frac{V_1}{I_2} \Big|_{V_2=0}$$

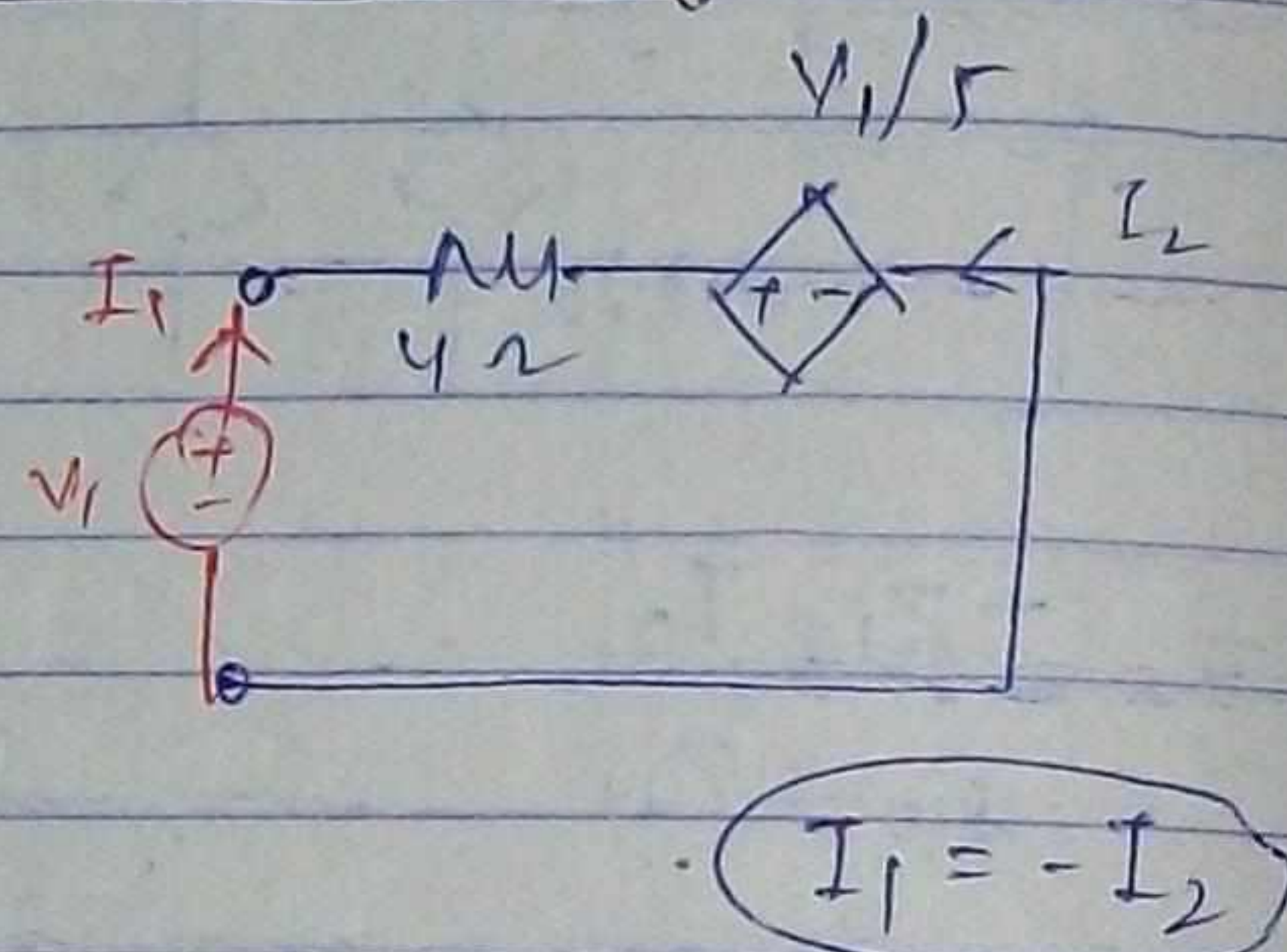
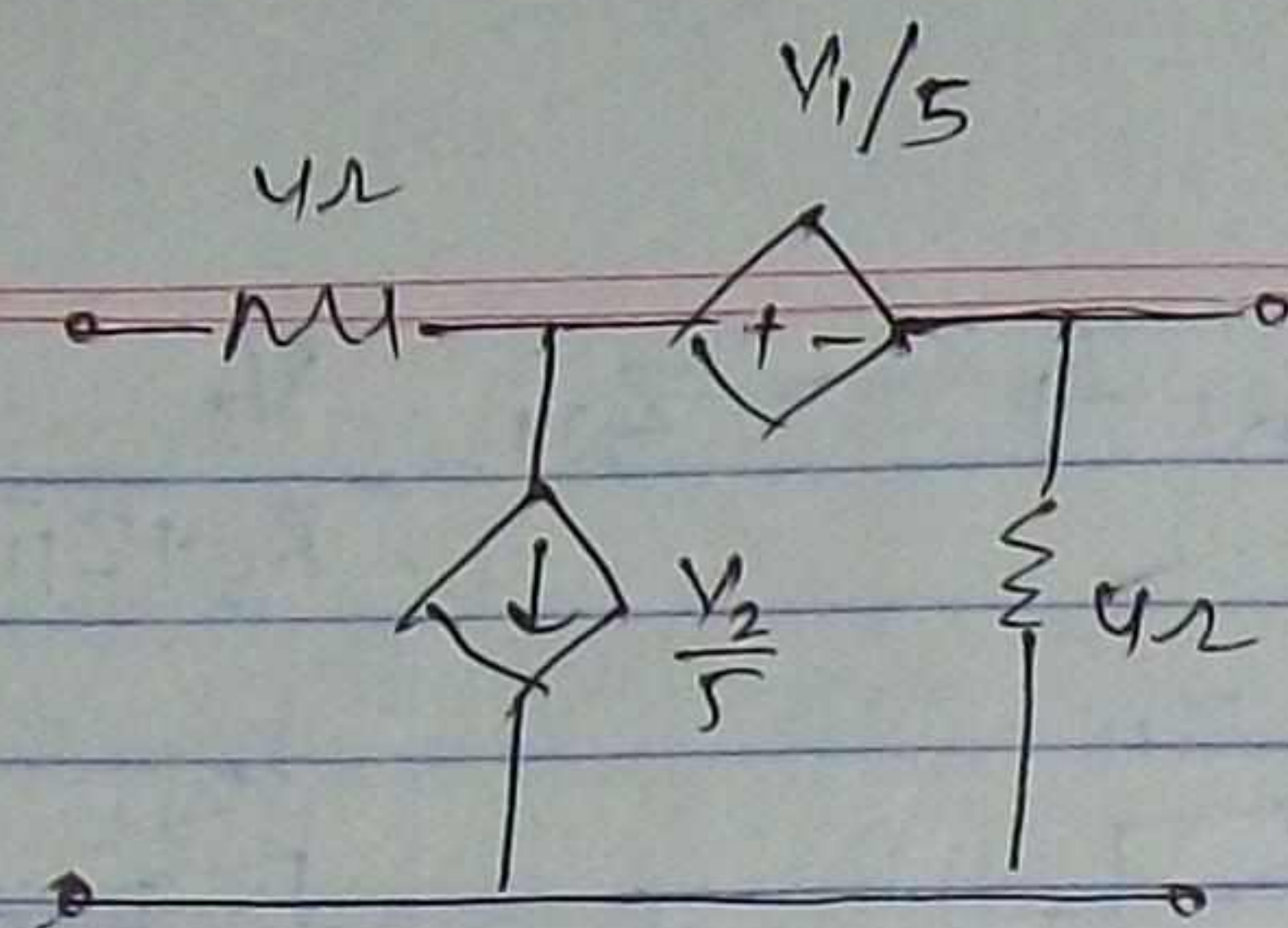
$$D = -\frac{I_1}{I_2} \Big|_{V_2=0}$$

$$\textcircled{1} \quad B = -\frac{V_1}{I_2} = \frac{V_1}{I_2} = \frac{V_1 \times 4}{(V_1 - \frac{V_1}{5})}$$

$$\textcircled{B = 5}$$

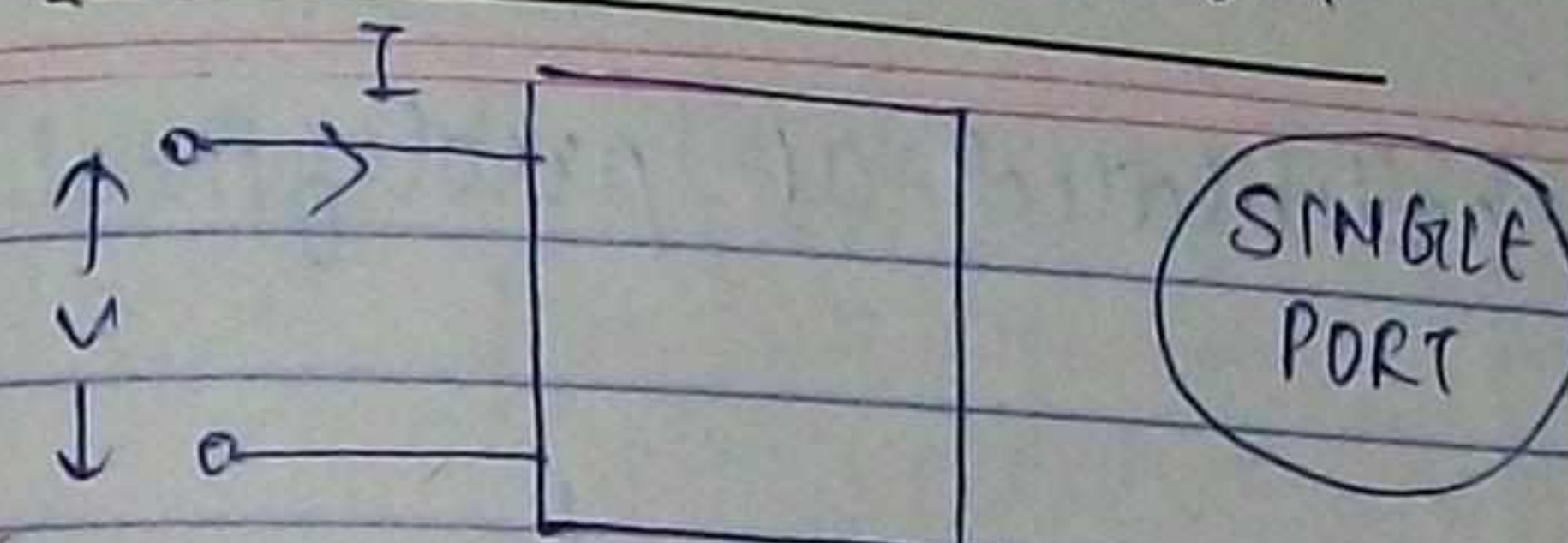
$$\textcircled{2} \quad D = \frac{I_1}{-I_2} = \frac{I_1}{I_1} = \left(\frac{V_1 - \frac{V_1}{5}}{\frac{V_1}{5}} \right) = 1$$

$$\textcircled{D = 1}$$



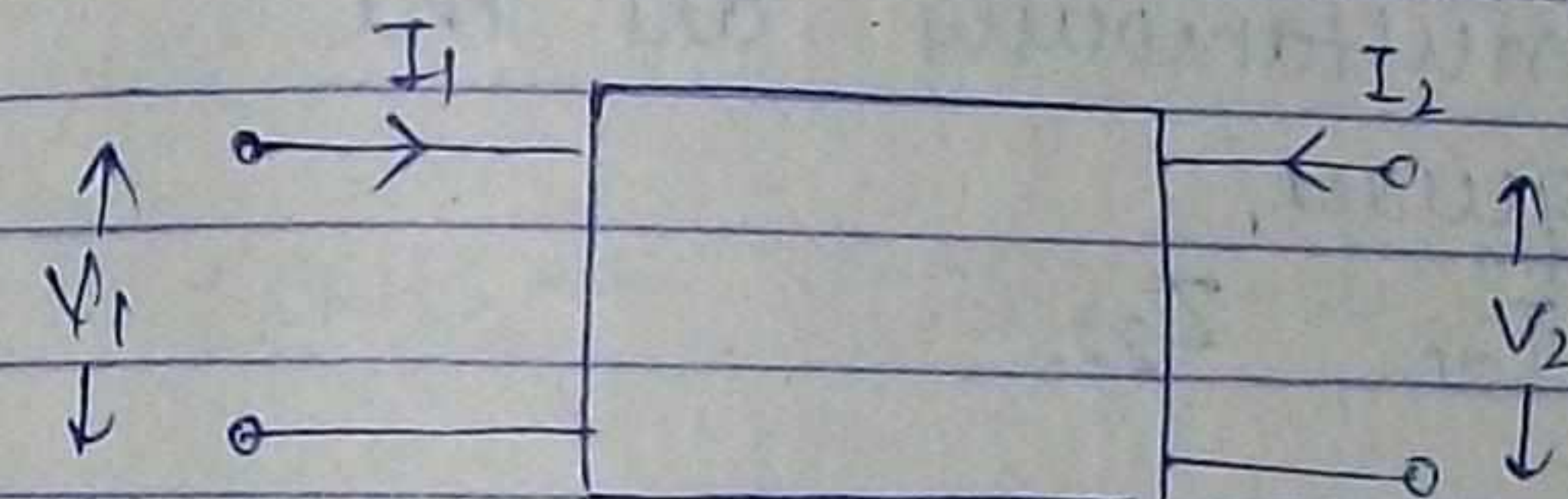
NETWORK SYNTHESIS

⇒ NETWORK FUNCTION -



IMMITTANCE

$$\begin{cases} Z(s) = \frac{V(s)}{I(s)} \\ Y(s) = \frac{I(s)}{V(s)} \end{cases}$$



$$Z_{11} = \frac{V_1}{I_1} \Big|_{I_2=0}$$

$$Y_{11} = \frac{I_1}{V_1} \Big|_{V_2=0}$$

$Z_{11}(s) = \frac{V_1(s)}{I_1(s)}$	$Y_{11}(s) = \frac{I_1(s)}{V_1(s)}$	$Z_{12}(s) = \frac{V_1(s)}{I_2(s)}$	$Y_{12}(s) = \frac{I_1(s)}{V_2(s)}$	$G_{12} = \frac{V_1(s)}{V_2(s)}$
$Z_{22}(s) = \frac{V_2(s)}{I_2(s)}$	$Y_{22}(s) = \frac{I_2(s)}{V_2(s)}$	$Z_{21}(s) = \frac{V_2(s)}{I_1(s)}$	$Y_{21}(s) = \frac{I_2(s)}{V_1(s)}$	$G_{21} = \frac{V_2(s)}{V_1(s)}$
DRIVING POINT Z FUNN	DRIVING POINT Y FUNN	TRANSFER IMPEDANCE RATIO	TRANSFER Y RATIO	TRANSFER VOLTAGE RATIO

$$\alpha_{12} = \frac{I_1(s)}{I_2(s)}$$

$$\alpha_{21} = \frac{I_2(s)}{I_1(s)}$$

TRANSFER
CURRENT
RATIO

- Network parameter is calculated at predefined condition.

$$Z_{11} = \frac{V_1}{I_1} \quad \text{with } I_2 = 0$$

$$Z_{11} = \frac{I_1}{V_1} \bigg|_{V_2=0} \quad \text{S.O.C}$$

- To find N/w funn no predefined condⁿ is required
- To represent the N/w simultaneously all the four parameters are required.
 $Z_{11}, Z_{12}, T_{21}, Z_{22}$
- By using only one N/w funn it is possible to represent the N/w.

- To represent the N/w simultaneously all the four parameters are required.

$$Z_{11}, Z_{12}, T_{21}, Z_{22}$$

- By using only one N/w funn it is possible to represent the N/w.

⇒ NETWORK SYNTHESIS -

- In the N/w synthesis for a given funn N/w is designed.
- In the N/w synthesis by using the following funn it is possible to design the N/w.

- In the M/W synthesis by using the following form it is possible to design the M/W.

SINGLE PORT
N/W

$$Z(s), Y(s)$$

IMMITTANCE

TWO PORT N/W

1. $Z_{11}(s), Z_{22}(s), Y_{11}(s), Y_{22}(s)$
 2. $Z_{12}(s), Z_{21}(s), Y_{12}(s), Y_{21}(s), G_{12}(s)$
 $G_{21}(s), \alpha_{12}, \alpha_{21}$ } T.F.

2. $Z_{12}(s)$, $Z_{21}(s)$, $Y_{12}(s)$, $Y_{21}(s)$, $G_{12}(s)$
 $G_{21}(s)$, α_{12} , α_{21} } T.F.

- In the N/w synthesis for a given funn it is possible to design the following N/w's:

- ① SERIES $\rightarrow$ FORSTER - I form
- ② PARALLEL $\rightarrow$ FORSTER - II form
- ③ LADDER $\rightarrow$ CAVER - I form
- ③ LADDER $\rightarrow$ CAVER - II form
- PARTIAL FRACTION OF EXPANSION
- CONTINUED FRACTION OF EXPANSION.

- ⑦ PARALLEL $\rightarrow$ FOSTER-II form

- (5) LADDER $\left\{ \begin{array}{l} \text{CAVER-I form} \\ \text{CAVER-II form} \end{array} \right\}$ CONTINUED FRACTION OF EXPANSION.

CAVER - II form

CONDITIONS -

- ① $f(s)$ should be PRF (positive real function)

$$R \geq 0 \quad L \geq 0 \quad C \geq 0$$

- ⑦ If $f(s)$ is PRF then $\frac{1}{f(s)}$ is also PRF

- ③ If $f_1(s)$ and $f_2(s)$ are PRF then

$$(1) f(s) = f_1(s) + f_2(s) \quad \text{PRF}$$

$$(2) f(s) = f_1(s) - f_2(s)$$

$$a) f_1(s) > f_2(s) \quad \text{PRF}$$

$$b) f_1(s) < f_2(s) \quad \text{-ve}$$

- All the poles of the function should be present in left half of the plane.

- Imp poles and zeros should be CONJUGATE PAIR.

- Total no. of poles should be equal to total no. of zeros.

- In the partial fraction of expansion Residue should be positive real.

- Numerator & denominator polynomials should satisfy HURWITZ.

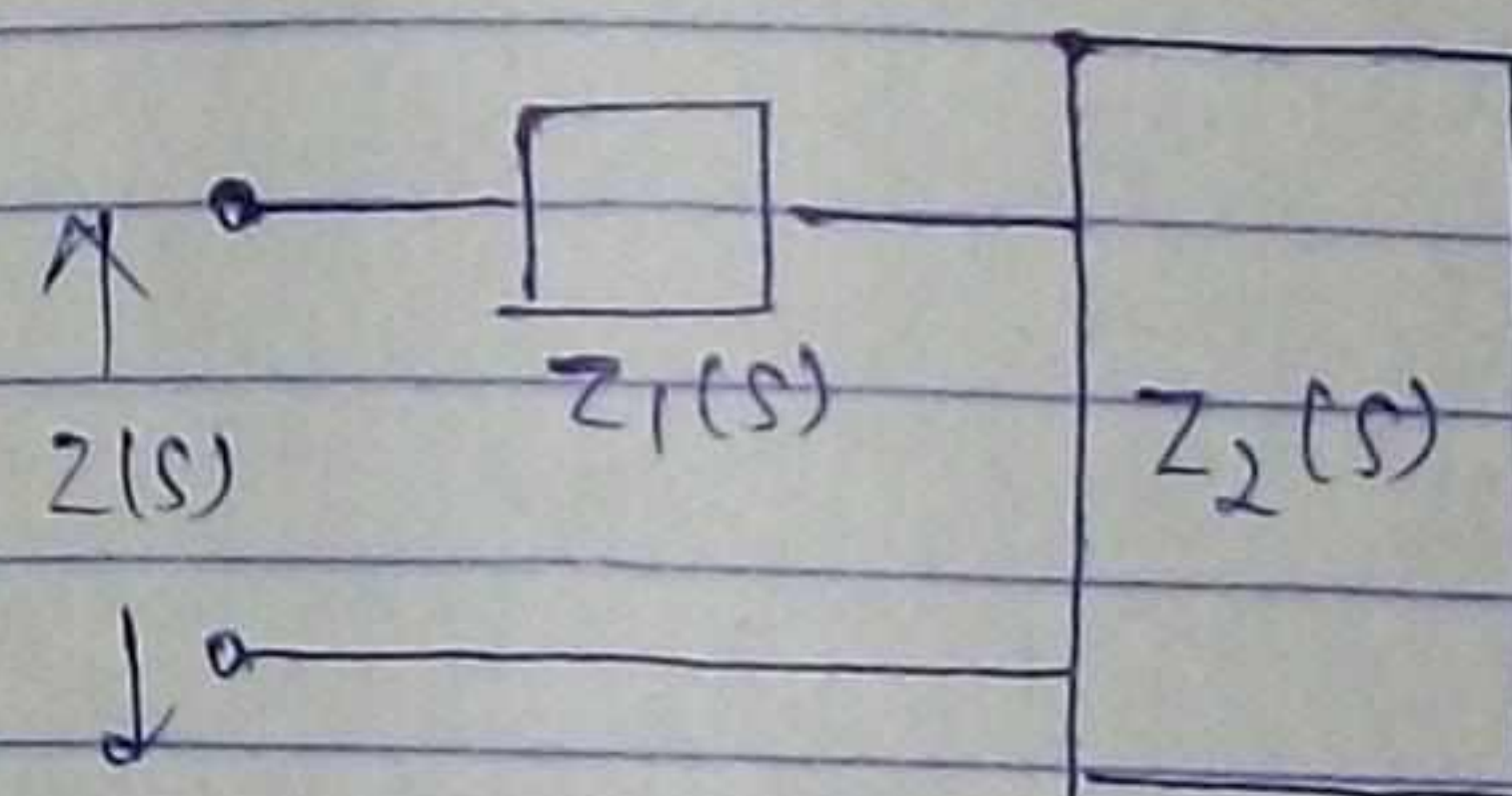
- The highest power of Numerator & denominator polynomials should be differ by almost unity. This condⁿ prohibits multiple poles and zeros at infinite.

- Lower power of Numerator and denominator

polynomial should be differ by almost unity. This condⁿ prohibits multiple poles and zeros at origin.

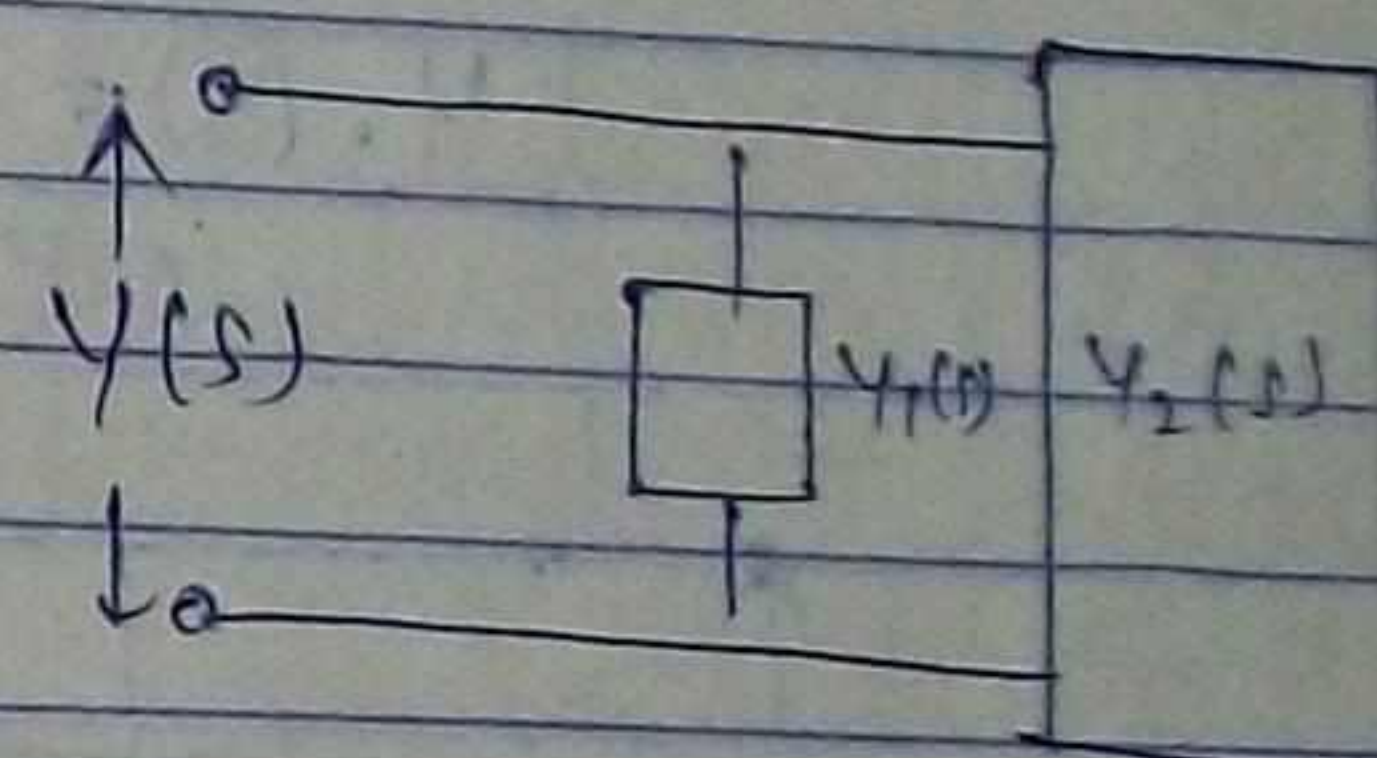
$$Z(s) = Z_1(s) + Z_2(s)$$

$$Z_2(s) = Z(s) - Z_1(s)$$



$$Y(s) = Y_1(s) + Y_2(s)$$

$$Y_2(s) = Y(s) - Y_1(s)$$



(1) REMOVAL OF POLE AT ∞ -

FOR IMPEDANCE FUNCTION

$$Z(s) = \frac{b_{n+1}s^{n+1} + b_ns^n + \dots + b_0}{a_ns^n + a_{n-1}s^{n-1} + \dots + a_0}$$

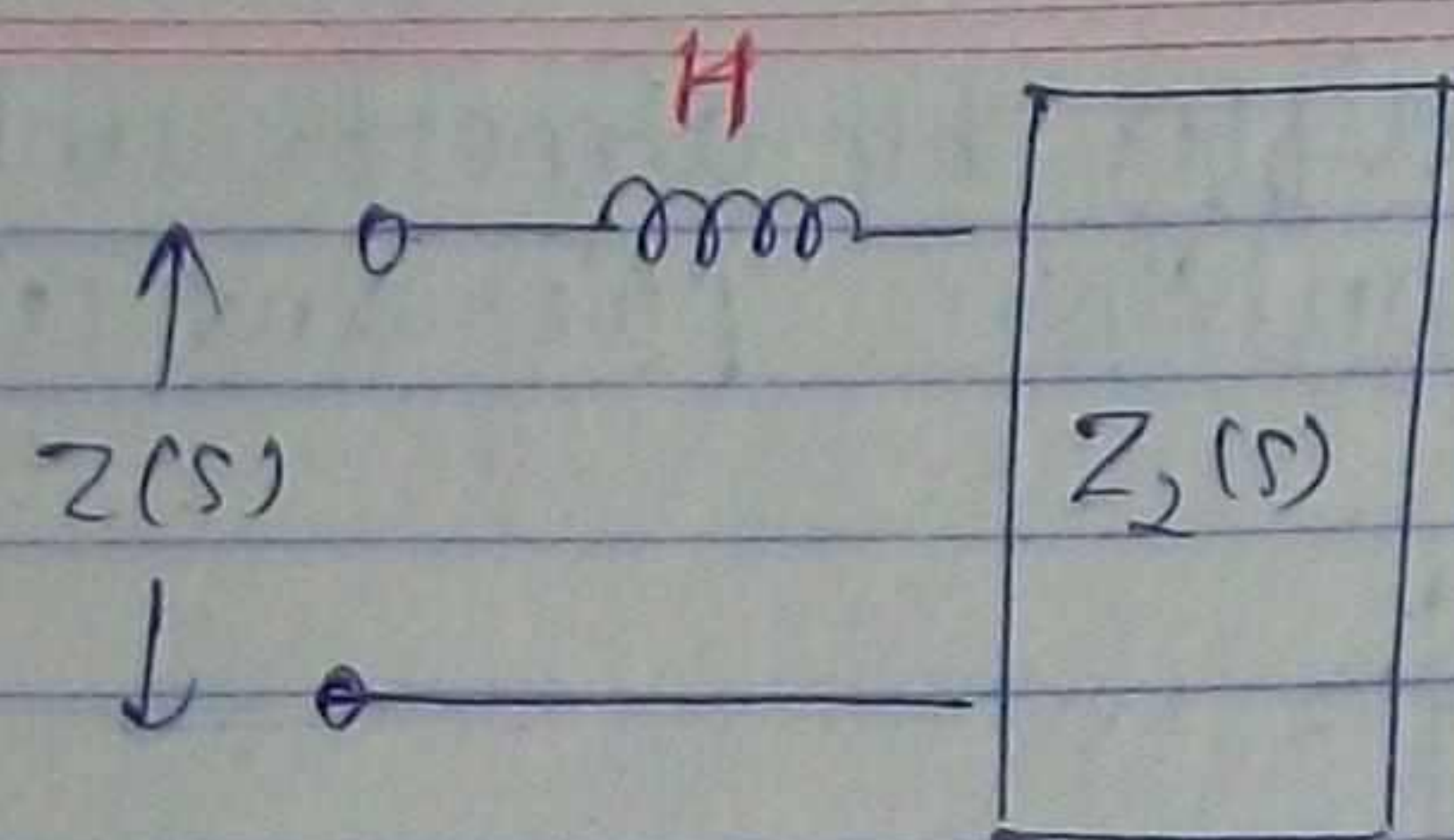
By long division method.

$$Z(s) = \frac{b_{n+1}s^{n+1}}{a_ns^n} + Z_2(s) \quad \left(\frac{b_{n+1}}{a_n} = H \right)$$

$$Z(s) = Hs + Z_2(s)$$

$$Z_2(s) = Z(s) - Hs$$

$$X_L = Ls$$

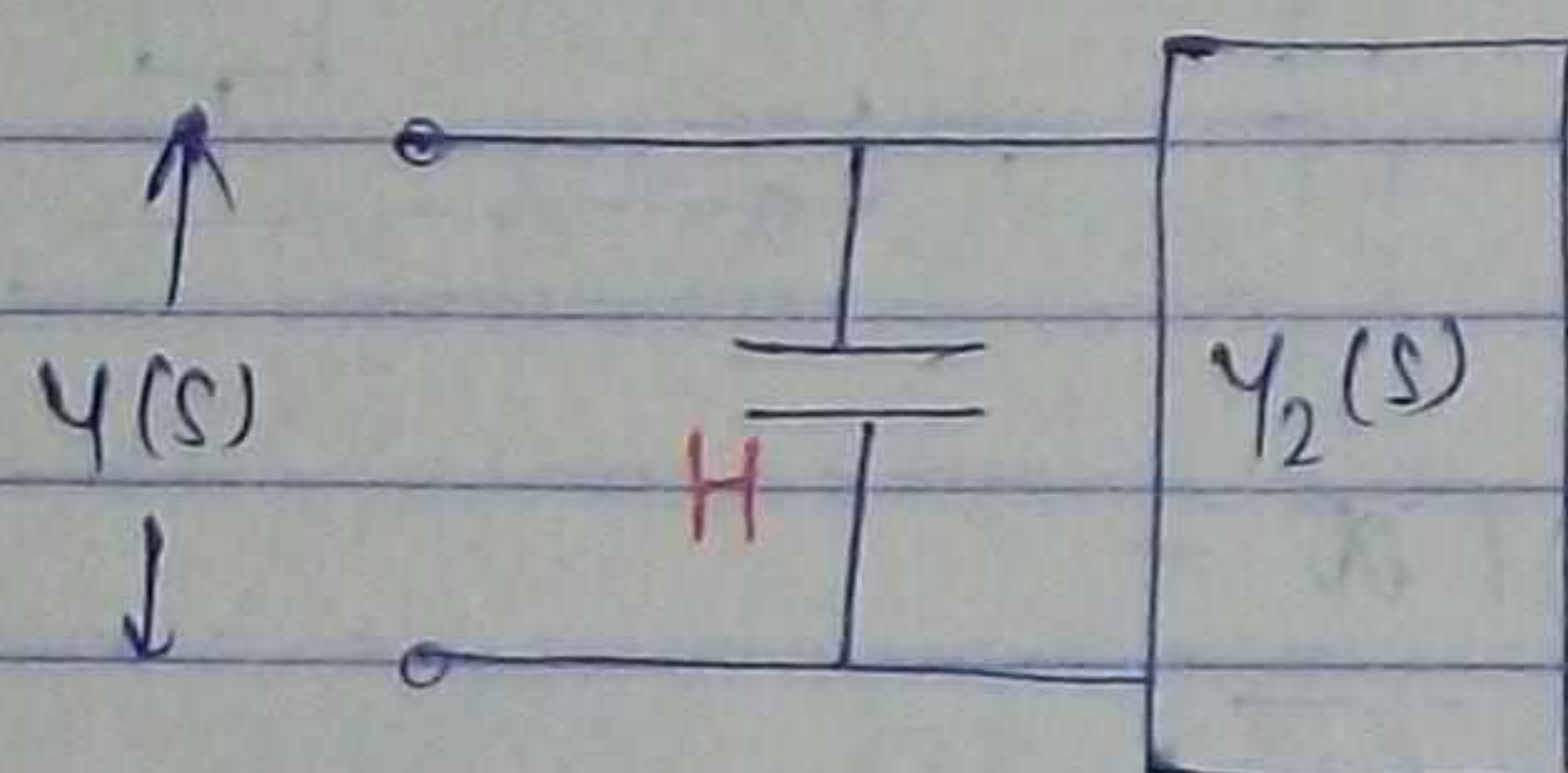


SIMILARY FOR
ADMITTANCE
FUNCTION.

$$Y(s) = H(s) + Y_2(s)$$

$$B_c = C_s$$

$$Y_2(s) = Y(s) - H(s)$$



② REMOVAL OF POLE AT ORIGIN -

FOR IMPEDANCE
FUNCTION

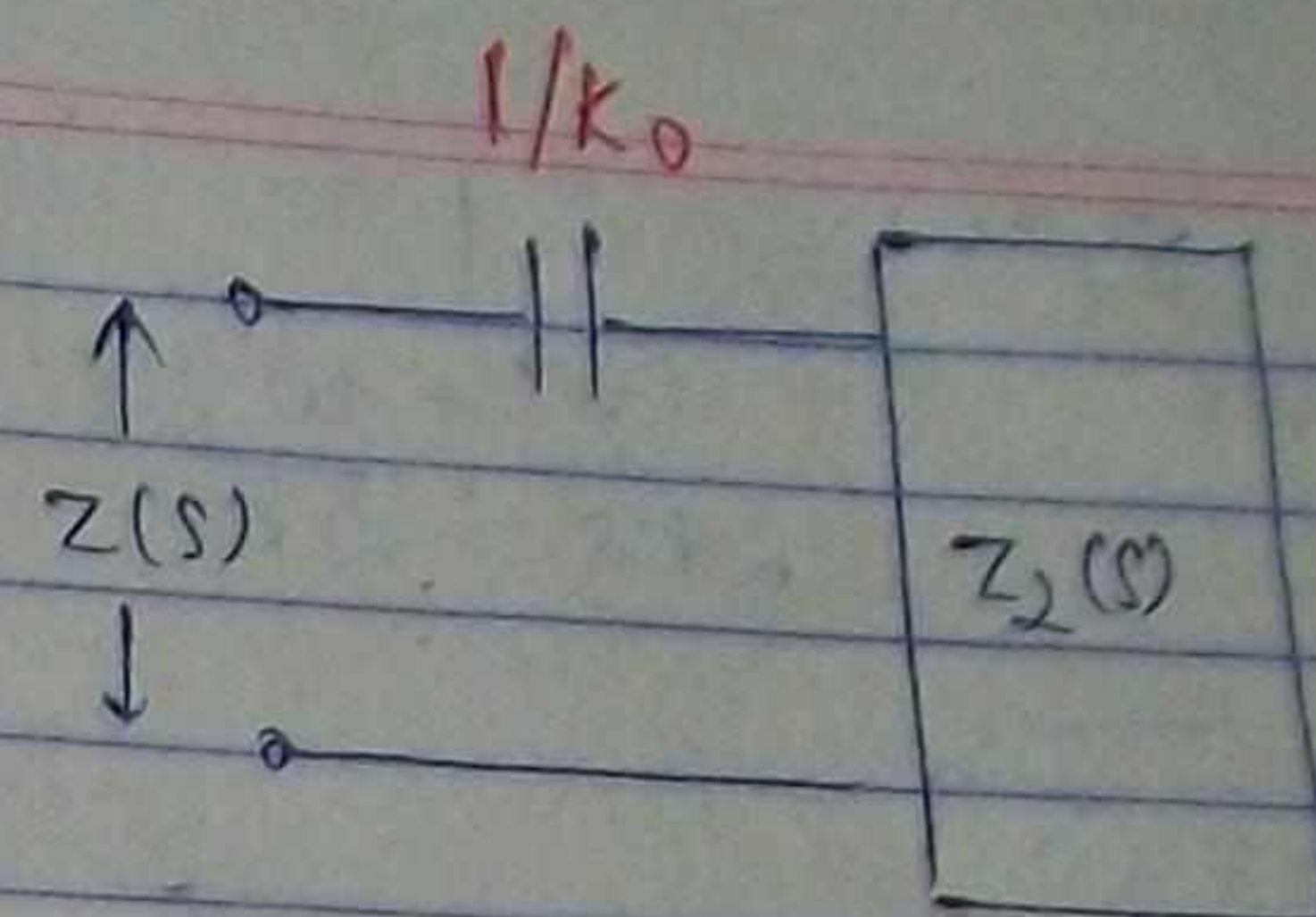
$$Z(s) = \frac{b_0 + \dots + b_{n-1}s^{n-1} + b_n s^n}{s(a_0 + \dots + a_{n-1}s^{n-1} + a_n s^n)}$$

By long division method

$$Z(s) = \frac{b_0}{s a_0} + Z_2(s) \quad \left(\frac{b_0}{a_0} = k \right)$$

$$Z_2(s) = \frac{k_0}{s} + Z_2(s)$$

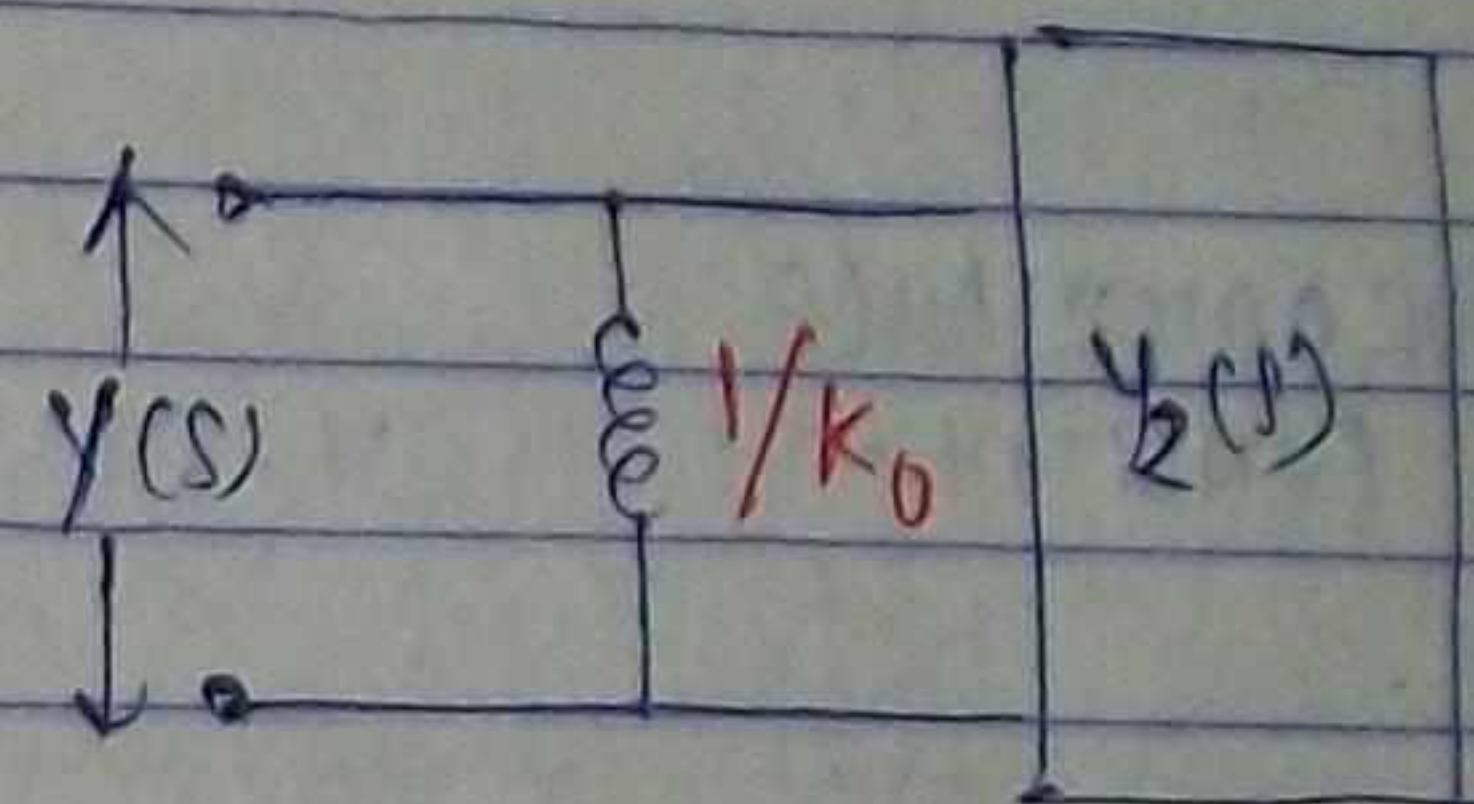
$$Z_2(s) = Z(s) - \frac{k_0}{s}$$



FOR
ADMITTANCE
FUNCTION -

$$Y(s) = \frac{k_0}{s} + Y_2(s)$$

$$Y_2(s) = Y(s) - \frac{k_0}{s}$$



③ REMOVAL OF CONJUGATE PAIR OF POLES -

FOR IMPEDANCE
FUNCTION

$$Z(s) = Z_1(s) + Z_2(s)$$

$$Z_2(s) = Z(s) - Z_1(s)$$

$$Z_1(s) \rightarrow \text{POLES} \rightarrow \pm j\omega \quad (\text{assume complex pair})$$

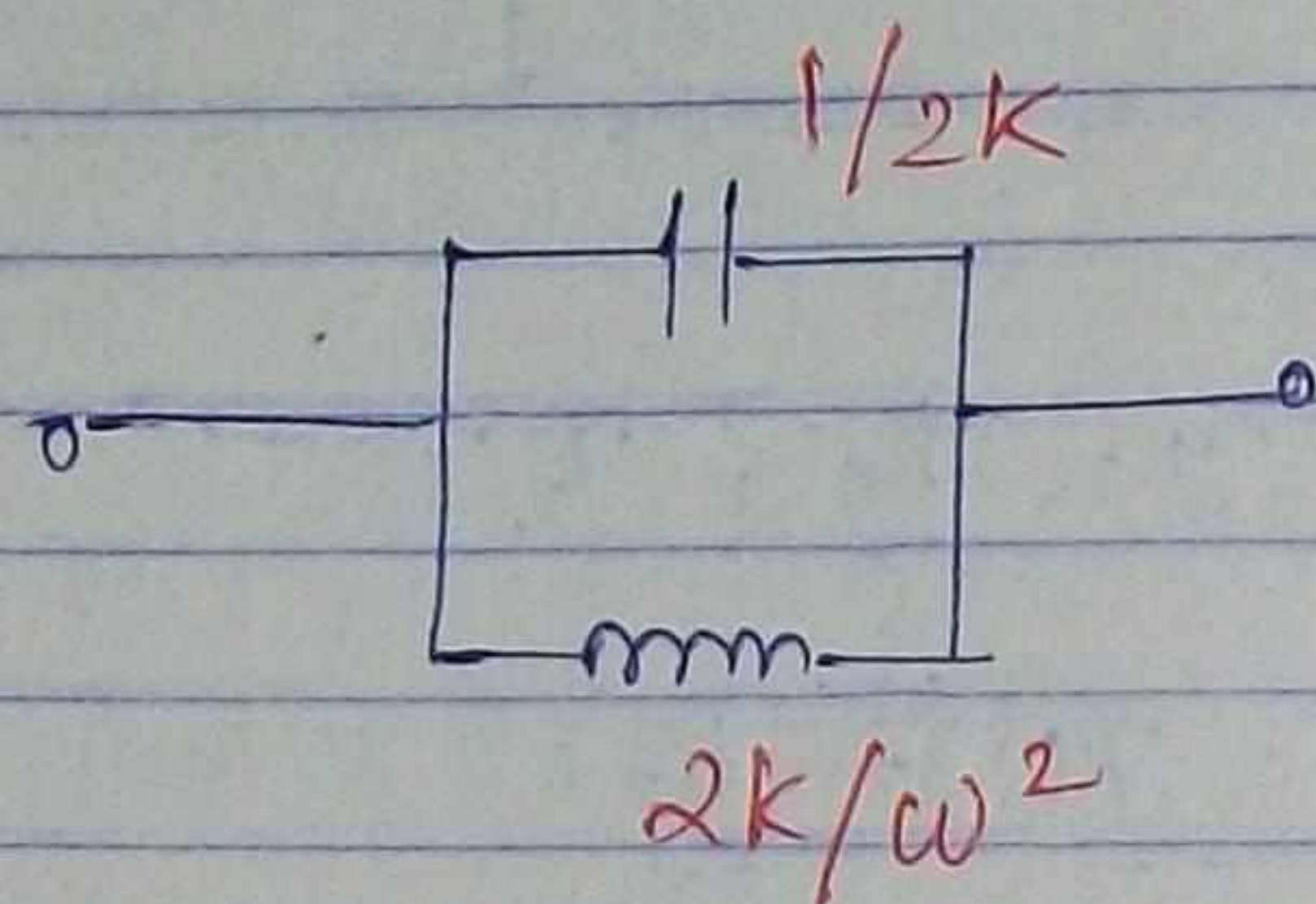
$$Z_1(s) = \frac{k_1}{s+j\omega} + \frac{k_2}{s-j\omega} \quad (k_1 = k_2 = k)$$

$$Z_1(s) = \frac{k}{s+j\omega} + \frac{k}{s-j\omega} = \frac{2ks}{s^2 + \omega^2}$$

$$Z_1(s) = \frac{1}{\frac{s^2 + \omega^2}{2ks}} = \frac{1}{Y_a + Y_b}$$

$$B_c = Cs$$

$$B_c = 1/Ls$$

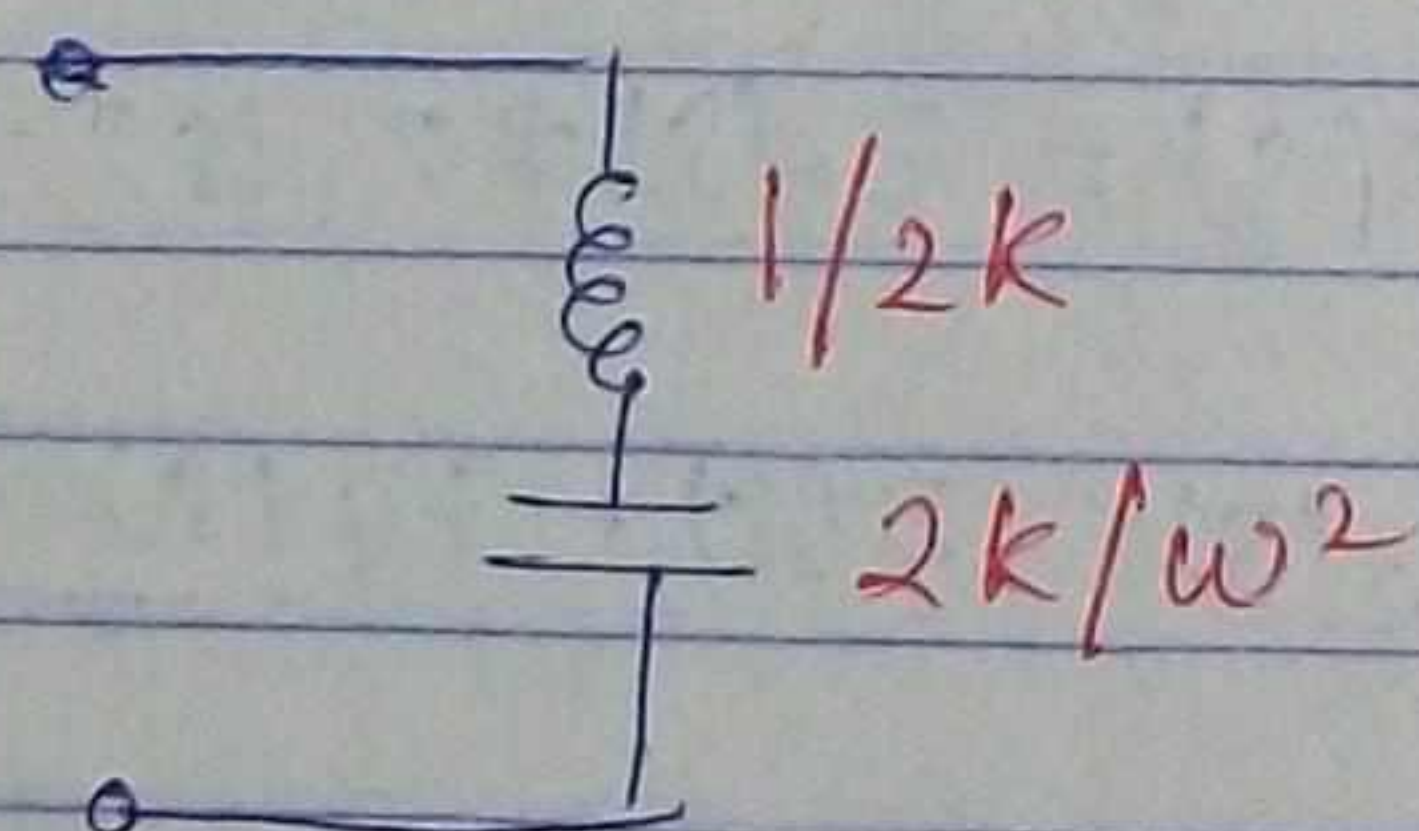


FOR ADMITTANCE
FUNCTION -

$$Y_1(s) = \frac{1}{\frac{s}{2k} + \frac{\omega^2}{2ks}} = \frac{1}{Z_a + Z_b}$$

$$X_L = Ls$$

$$X_C = 1/sc$$

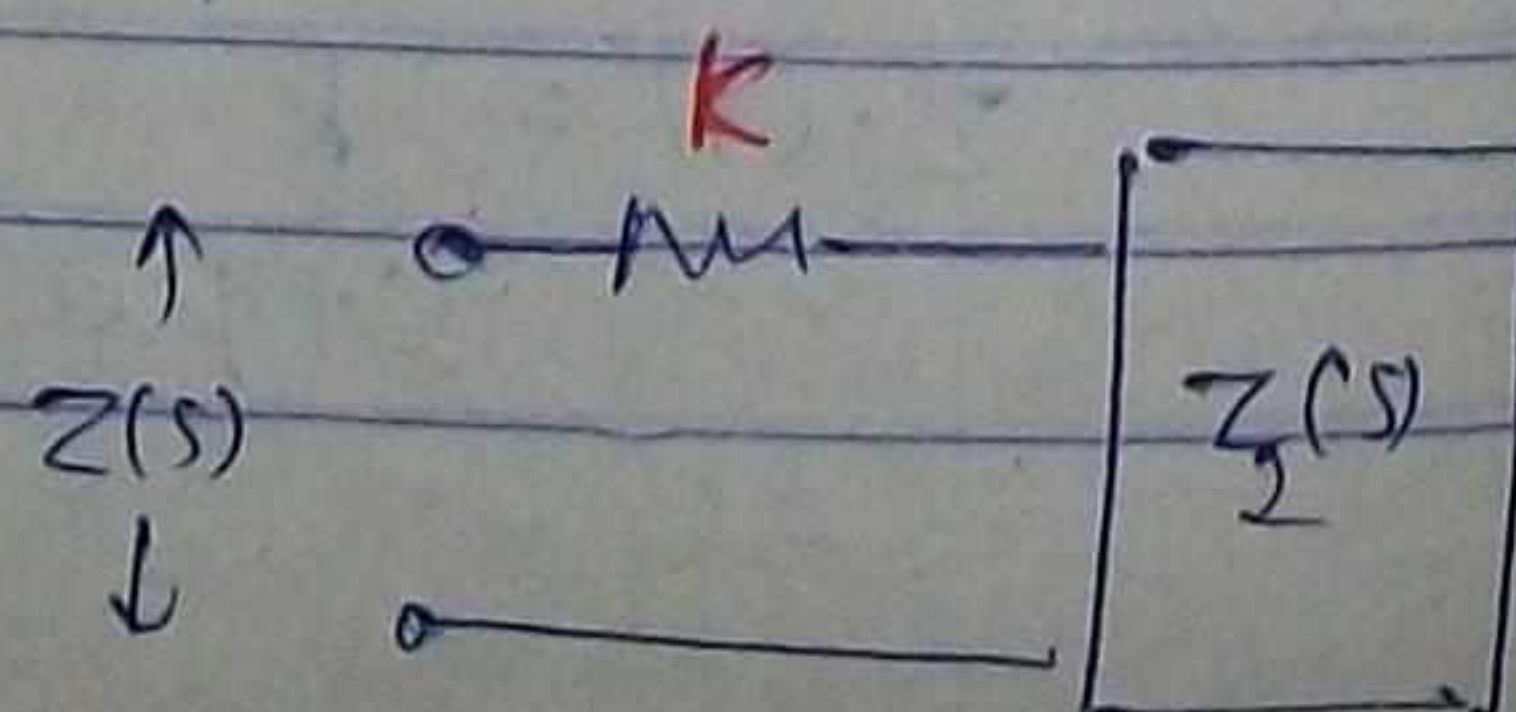


④ REMOVAL OF CONSTANT -

FOR
IMPEDANCE

$$Z(s) = k + Z_2(s)$$

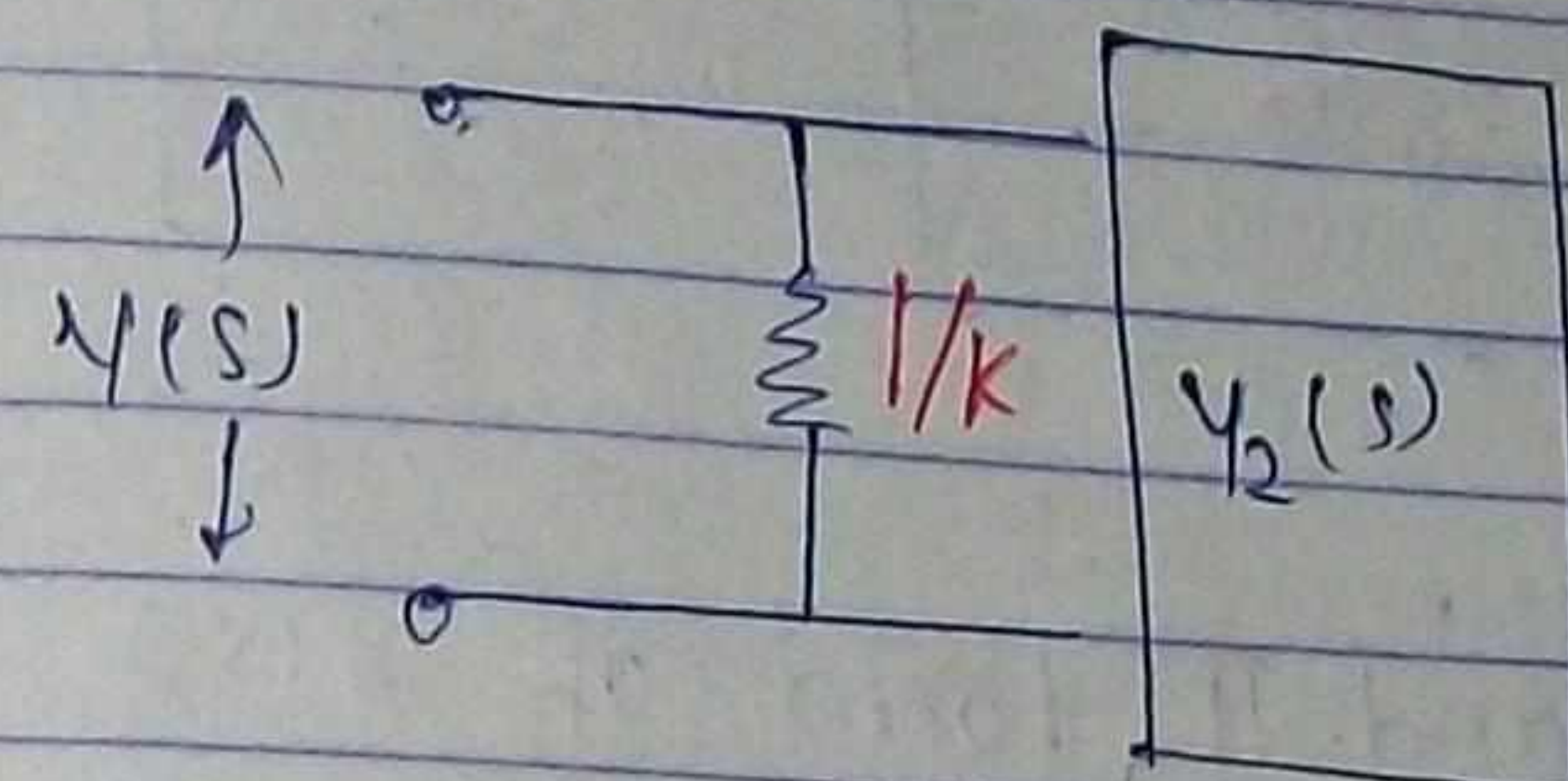
$$Z_2(s) = Z(s) - k$$



FOR
ADMITTANCE

$$Y(s) = k + Y_2(s)$$

$$Y_2(s) = Y(s) - k$$

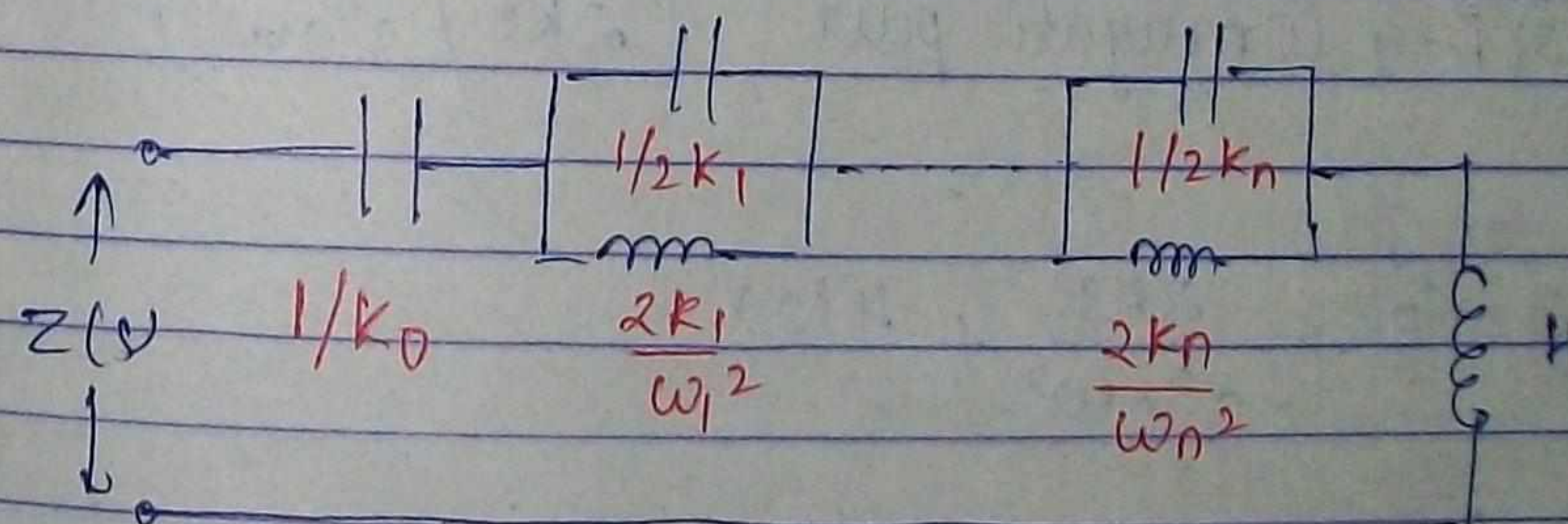


⇒ LC NETWORK -

① FOSTER - I FORM -

SERIES
NETWORK

$$Z(s) = \frac{k_0}{s} + \sum_{i=1}^n \frac{2k_i s}{s^2 + \omega_i^2} + Hs$$



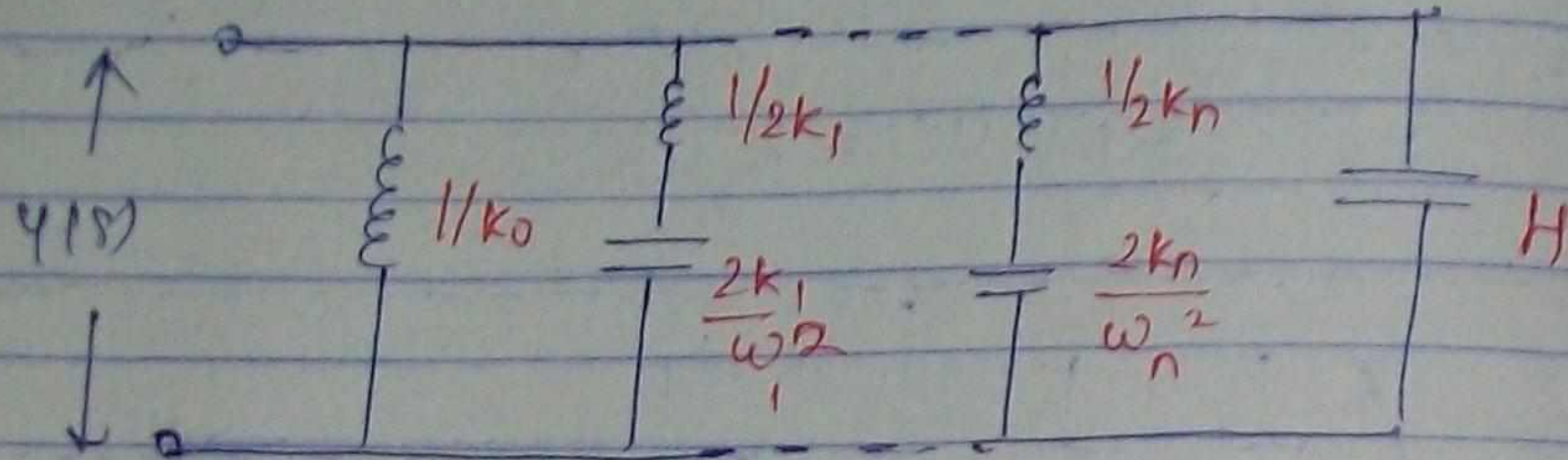
② FOSTER - II FORM -

PARALLEL
N/W

$$Y(s) = \frac{k_0}{s} + \sum_{i=1}^n \frac{2k_i s}{s^2 + \omega_i^2} + Hs$$

$$B_L = 1/LS$$

$$B_C = sC$$



Ques. Obtain Foster-I and II form of $Z(s)$

$$Z(s) = \frac{(s^2+2)(s^2+4)}{s(s^2+3)}$$

(A) BY FOSTER-I form -

Solⁿ: ① $P \rightarrow \infty$ present (H_s)

② $P \rightarrow$ origin present (k_0/s)

③ $P \rightarrow$ conjugate pair ($2ks/s^2+w^2$)

$$\therefore Z(s) = \frac{k_0}{s} + \frac{2ks}{s^2+w^2} + H(s)$$

$$\frac{(s^2+2)(s^2+4)}{s(s^2+3)} = \frac{k_0}{s} + \frac{2ks}{s^2+w^2} + H(s)$$

$$k_0 \rightarrow 8/3$$

$$2k \rightarrow 1/3$$

$$H \rightarrow 1$$

6/11/11

$$Z(s) = \frac{8/3}{s} + \frac{s/3}{s^2+3} + s$$

$$Z(s) = \frac{1}{\frac{3}{8}s} + \frac{1}{\frac{s^2}{3} + \frac{3}{s/3}} + s$$

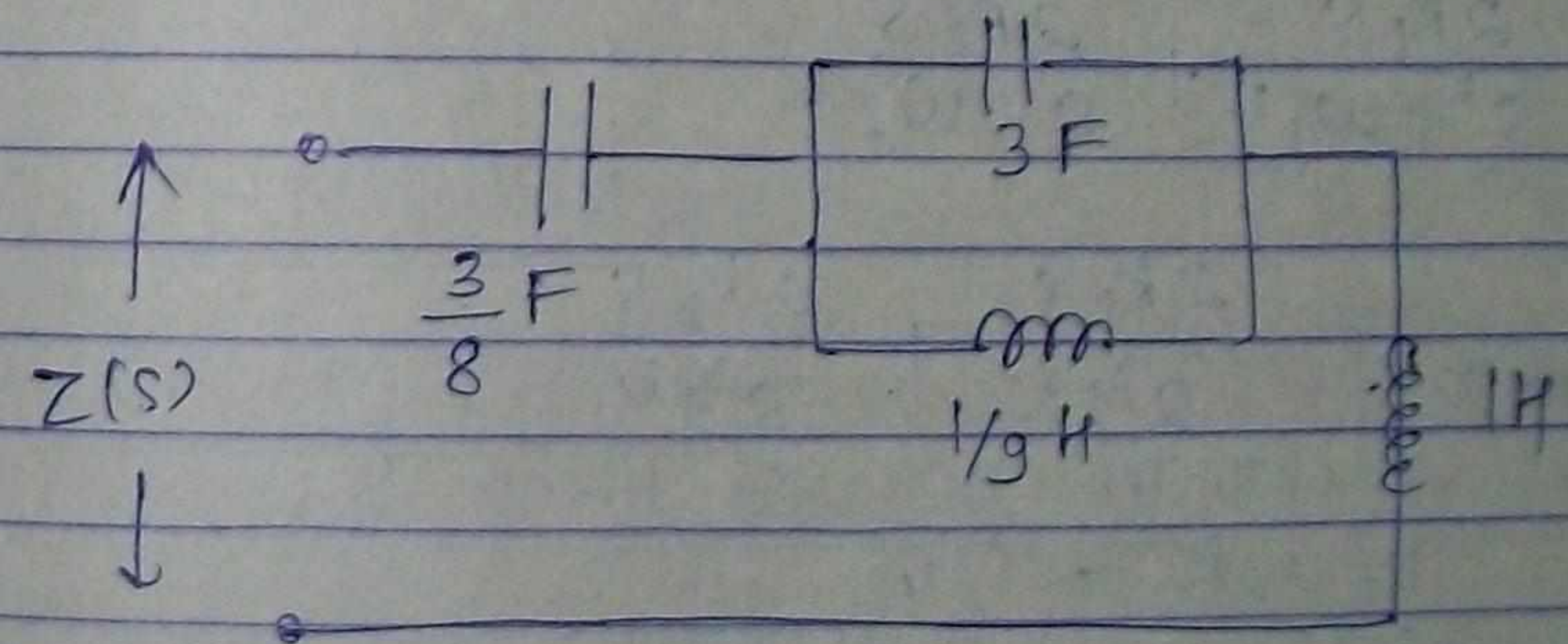
$$Z(s) = \frac{1}{\frac{3}{8}s} + \frac{1}{\frac{3s+9}{s}} + s$$

$$\downarrow \quad \downarrow$$

$$X_C = 1/sc \quad \frac{1}{Y_a + Y_b}$$

$$\downarrow \quad \downarrow$$

$$B_C = sC \quad B_L = 1/Ls$$



(B) BY FOSTER-II form -

$$Y(s) = \frac{1}{Z(s)} = \frac{s(s^2+3)}{(s^2+2)(s^2+4)}$$

⇒ NOTE -

- If $f(s)$ is given same funⁿ is utilized to obtain FOSTER - I and II form.
- If $Z(s)$ is given to obtain FOSTER II form $1/Z(s)$ function is used.
- If $Y(s)$ is given to obtain FOSTER I form $1/Y(s)$ funⁿ is used.

$P \rightarrow \infty$ (not present)

$P \rightarrow \text{origin}$ (not present)

$P \rightarrow \text{conjugate pair present}$

$$Y(s) = \frac{2k_1 s}{s^2 + \omega_1^2} + \frac{2k_2 s}{s^2 + \omega_2^2}$$

$$\frac{s(s^2 + 3)}{(s^2 + 2)(s^2 + 4)} = \frac{2k_1 s}{s^2 + 2} + \frac{2k_2 s}{s^2 + 4}$$

$$k_1 = k_2 = 1/4$$

$$Y(s) = \frac{s/2}{s^2 + 2} + \frac{s/2}{s^2 + 4}$$

$$Y(s) = \frac{1}{\frac{s^2}{s/2} + \frac{2}{s/2}} + \frac{1}{\frac{s^2}{s/2} + \frac{4}{s/2}}$$

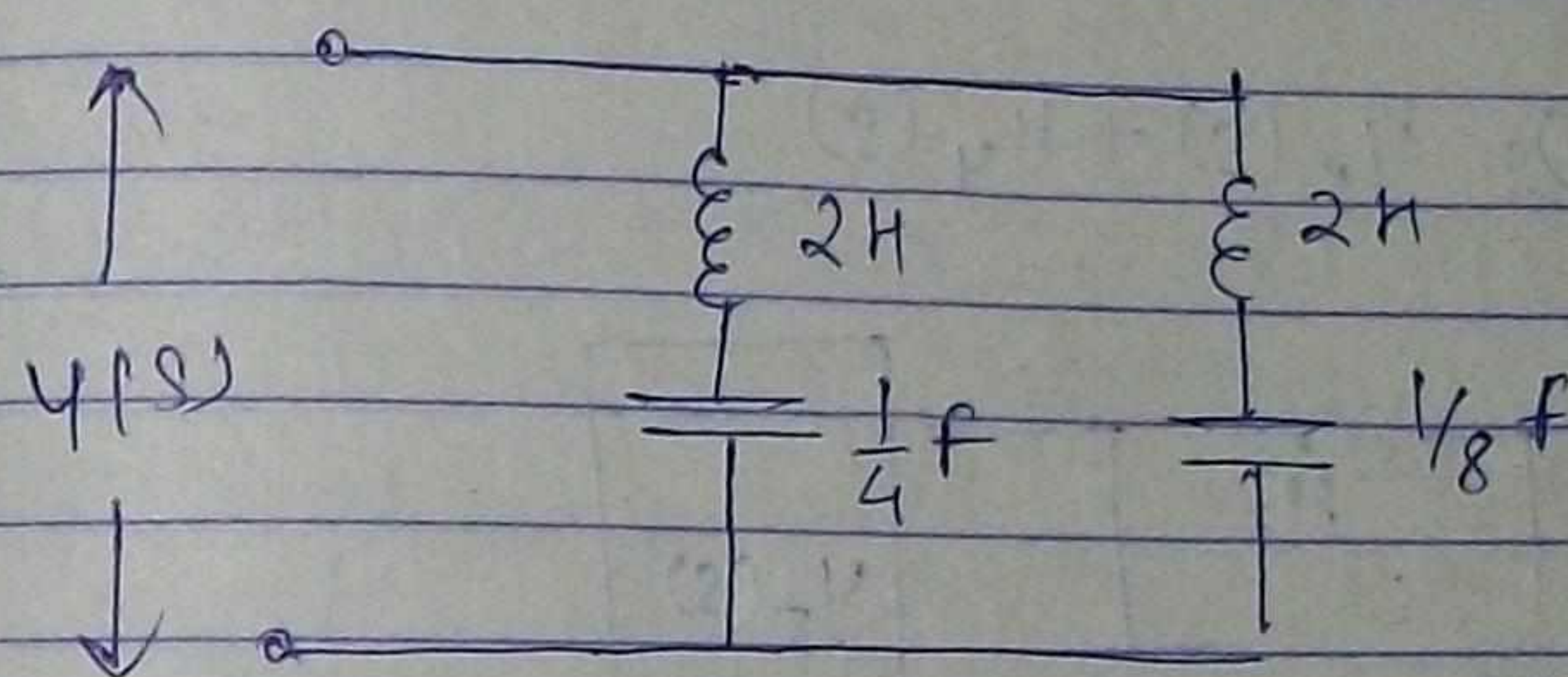
$$Y(s) = \frac{1}{2s + 4/s} + \frac{1}{2s + 8/s}$$

$$\downarrow \quad \downarrow$$

$$\frac{1}{Z_a + Z_b} \quad \frac{1}{Z_c + Z_d}$$

$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$

$$X_L = Ls \quad X_C = \frac{1}{sc} \quad X_L = Ls \quad X_C = \frac{1}{sc}$$



⇒ LC-NETWORK -

① CAVER - I FORM -

REMOVAL OF POLE AT ORIGIN INFINITY

$$Z_2(s) = Z(s) - H_1(s) \quad X_L = Ls$$

$$Y_2(s) = \frac{1}{Z_2(s)} \quad B_C = sc$$

$$Y_3(s) = Y_2(s) - H_2(s)$$

LADDER NETWORK

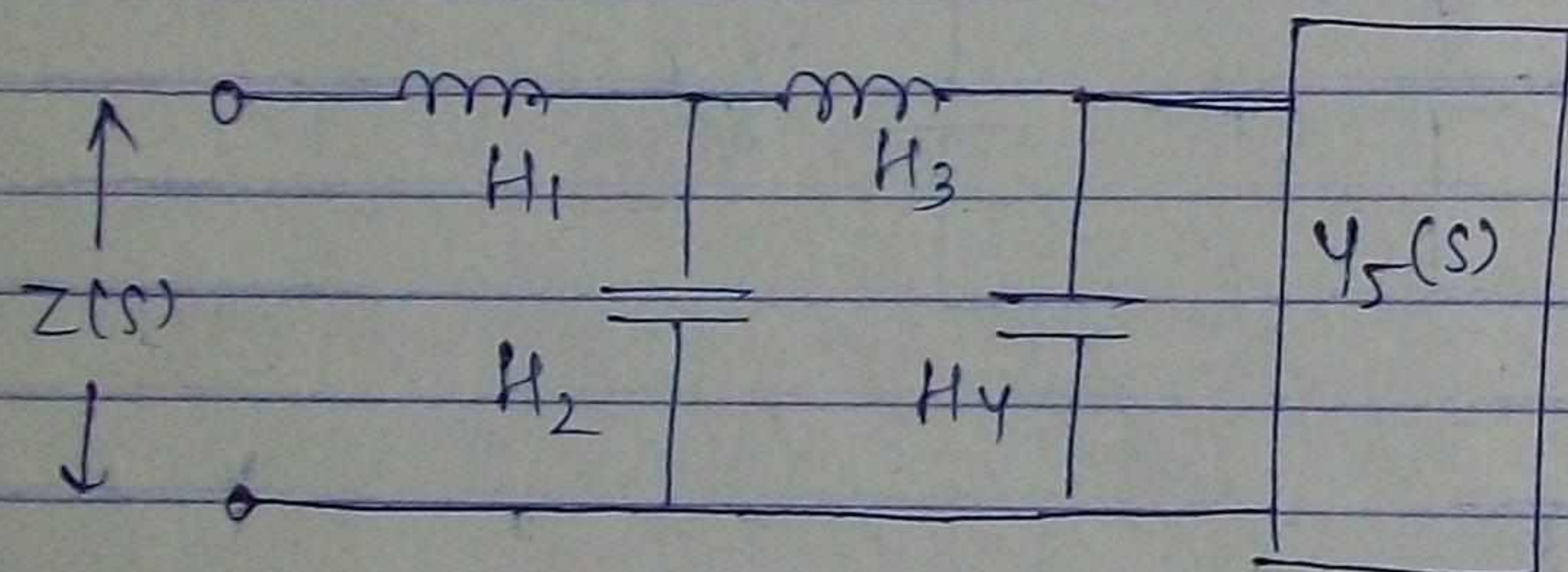
$$Z_3(s) = \frac{1}{Y_3(s)}$$

$$Z_4(s) = Z_3(s) - H_3 s$$

$$Y_4(s) = \frac{1}{Z_4(s)}$$

$$Y_5(s) = Y_4(s) - H_4 s$$

$$Y_4(s) = Y_5(s) + H_4 s$$



② CAUER - II FORM -

REMOVAL OF POLE AT ORIGIN

$$Z_2(s) = Z(s) - \frac{K_{01}}{s}$$

$$X_c = 1/s$$

$$Y_2(s) = \frac{1}{Z_2(s)}$$

$$B_c = 1/s$$

$$Y_3(s) = Y_2(s) - \frac{K_{02}}{s}$$

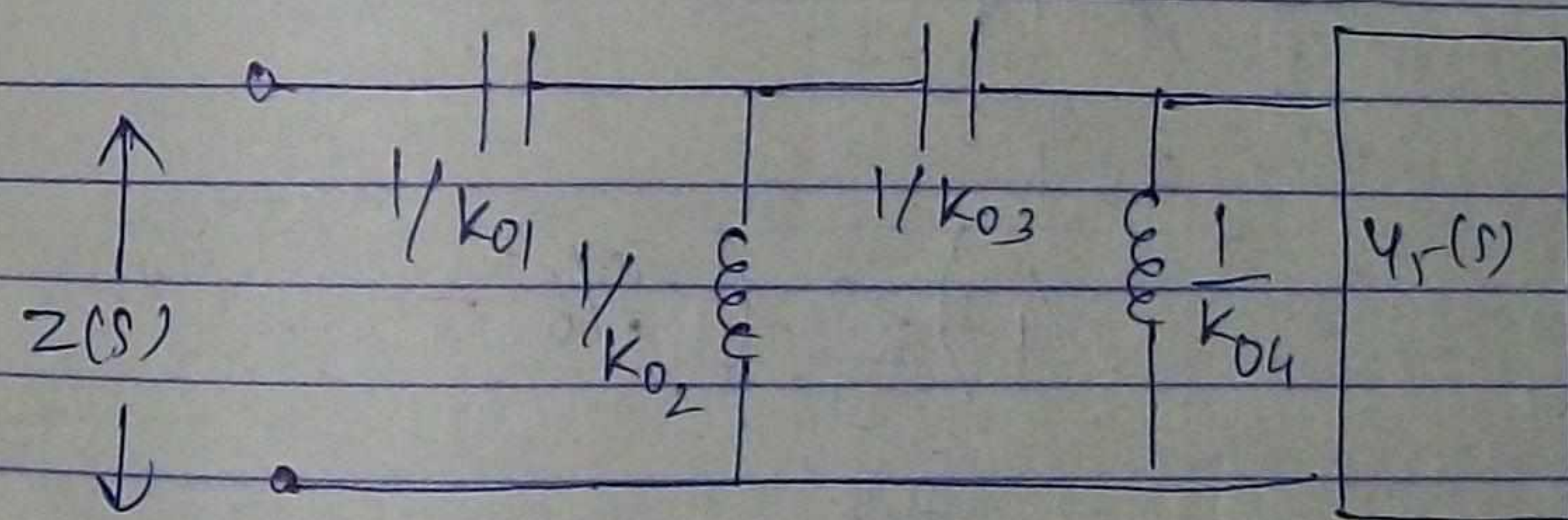
$$Z_3(s) = \frac{1}{Y_3(s)}$$

$$Z_4(s) = Z_3(s) - \frac{K_{03}}{s}$$

$$Y_4(s) = \frac{1}{Z_4(s)}$$

$$Y_5(s) = Y_4(s) - \frac{K_{04}}{s}$$

$$Y_4(s) = Y_5(s) + \frac{K_{04}}{s}$$



Ques. Obtain Cauer

I and II form
of $Z(s)$

$$Z(s) = \frac{s^3 + 2s}{s^4 + 5s^2 + 4}$$

Soln: CAUER - I Form -

$$\frac{s^3 + 2s}{s^4 + 5s^2 + 4} = Z(s)$$

poles in not present
at ∞ .

$$Y(s) = \frac{s^4 + 5s^2 + 4}{s^3 + 2s}$$

$$\begin{array}{r} s + \\ s^3 + 2s \overline{) s^4 + 5s^2 + 4} \\ \underline{s^4 + 2s^2} \\ 3s^2 + 4 \end{array}$$

$$s^3 + 2s \overline{) s^4 + 5s^2 + 4} \quad s \rightarrow Y \quad (R_c = sL)$$

$$3s^2 + 4 \overline{) s^2 + 2s} \quad (s/3 \rightarrow Z \quad (X_L = \omega L))$$

$$3s^2 + 4/3 s$$

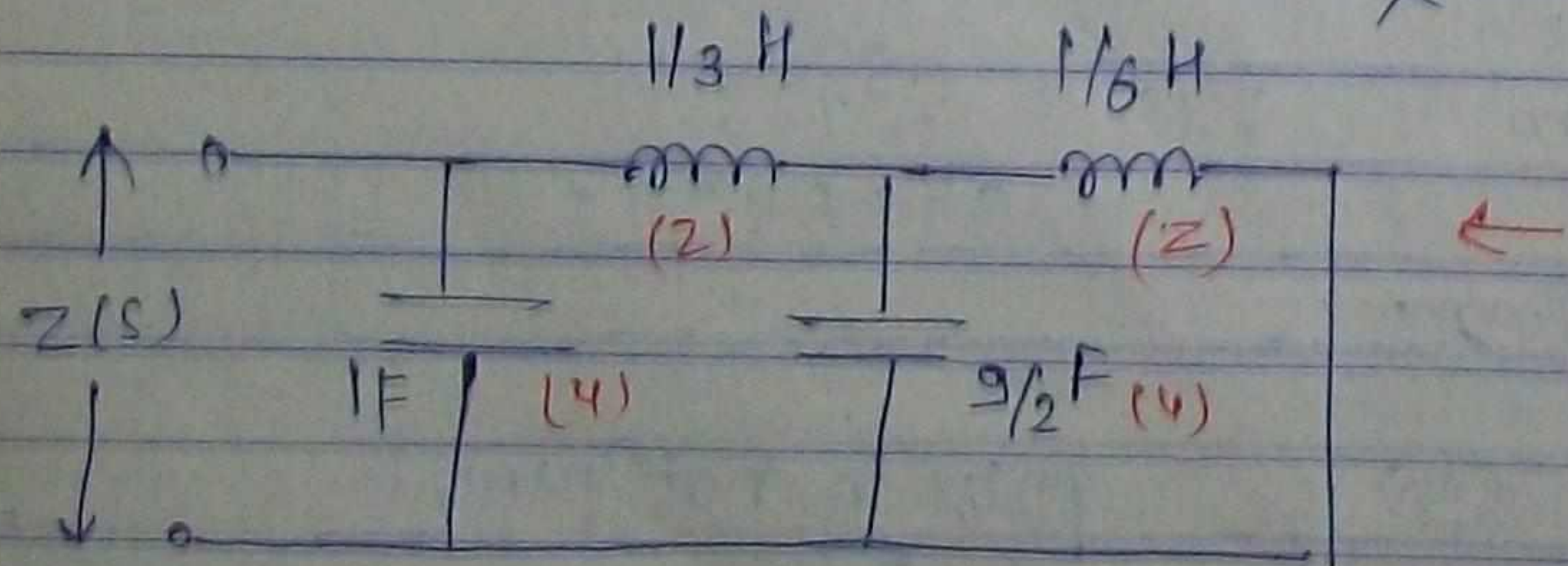
$$\frac{2s}{3} \overline{) 3s^2 + 4} \quad (9s/2 \rightarrow Y \quad (R_c))$$

$$3s^2 +$$

$$4 \overline{) \frac{2s}{3}} \quad (s/6 \rightarrow Z \quad (X_L))$$

$$\frac{2s}{3}$$

$$x$$



Start from end of N/W

mathematical expression -

$$Z(s) = \frac{1}{s} + \frac{1}{\frac{s}{3} + \frac{1}{\frac{9s}{2} + \frac{1}{\frac{s}{6}}}}$$

BY COVER II FORM -

$$Y(s) = \frac{s^4 + 5s^2 + 4}{s^3 + 2s}$$

here pole is not present at origin

$$Z(s) = \frac{s^3 + 2s}{s^4 + 5s^2 + 4} = \frac{s(s^2 + 2s)}{s^4 + 5s^2 + 4}$$

$$Y(s) = \frac{s^4 + 5s^2 + 4}{s^3 + 2s}$$

mathematical expression -

$$Z(s) = \frac{2}{s} + \frac{1}{\frac{2}{3s} + \frac{1}{\frac{9}{s} + \frac{1}{3s}}}$$

$$2s + s^3 \int \frac{4 + 5s^2 + s^4}{4 + 2s^2} \left(\frac{2}{5} \rightarrow 4 \right) (B_L = 1/s)$$

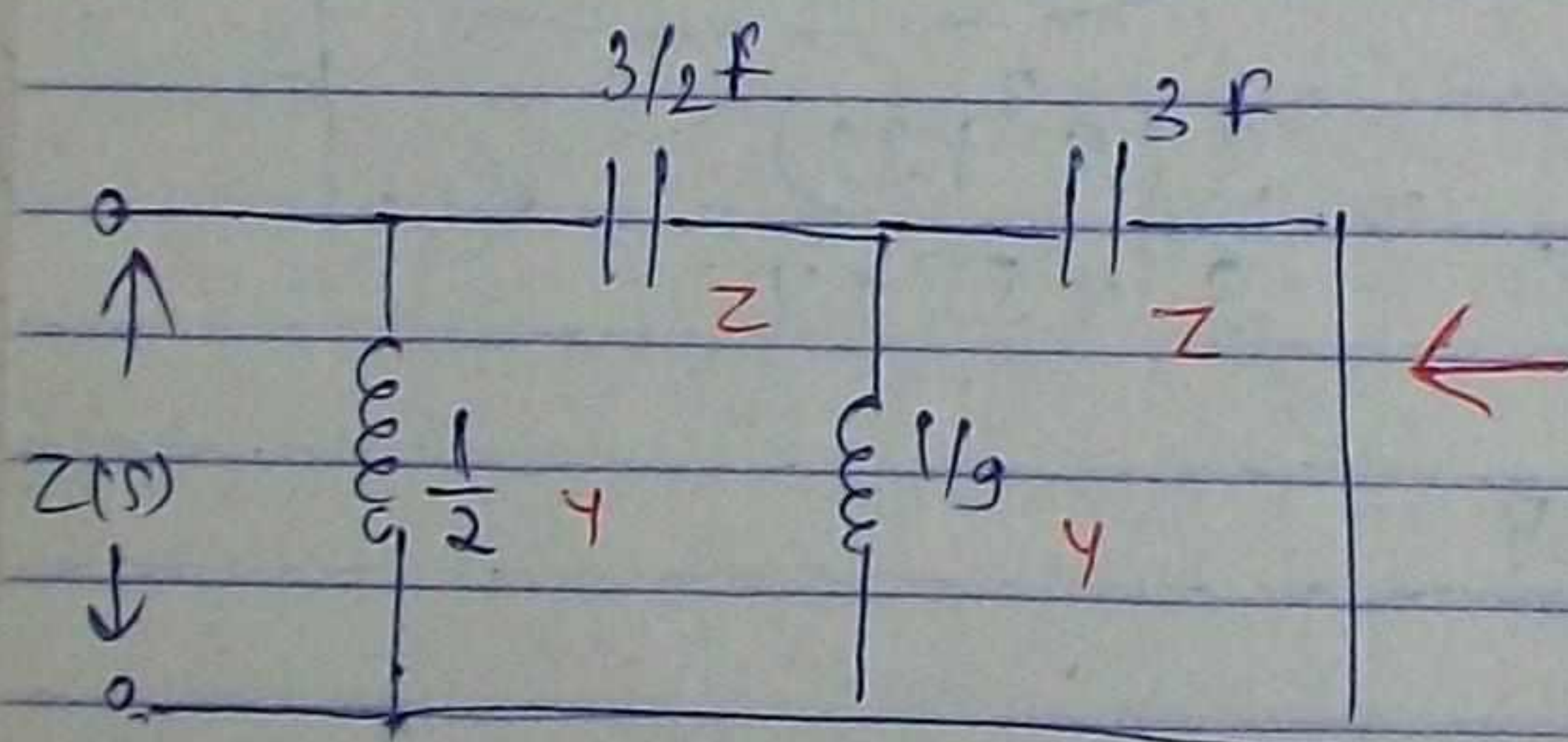
$$\frac{3s^2 + s^4}{2s + s^3} \left(\frac{2}{3s} \rightarrow Z \left(X_c = \frac{1}{Cs} \right) \right)$$

$$\frac{s^3}{3} \Big) 3s^2 + s^4 \left(\frac{g}{s} \rightarrow y \right)$$

B_L

$$s^4 \left) \frac{s^3}{3} \left(\frac{1}{3s} \rightarrow Z \right.$$

$$\frac{S^3}{3}$$



⇒ RC-NETWORK -

① Foster-I form -

~~SERIES~~
N/W

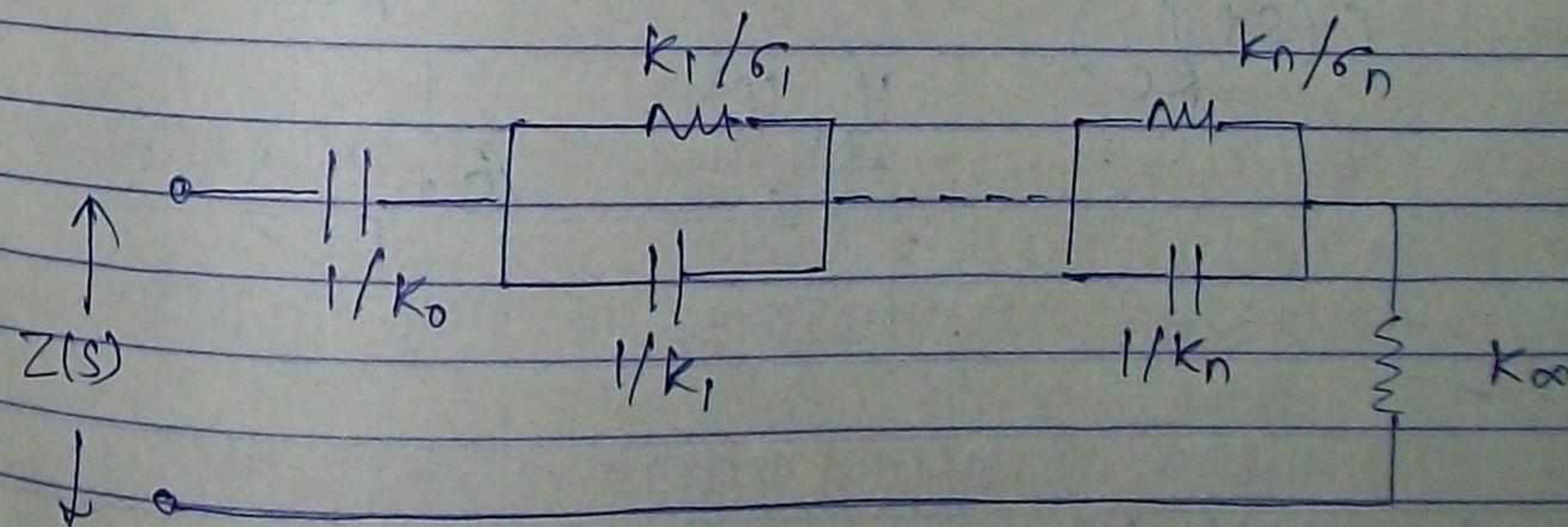
$$Z_{Le}(s) = \frac{k_0}{s} + \sum_{i=1}^n \frac{2k_i s}{s^2 + \omega_i^2} + h(s)$$

for RC network —

$$Z_{RC}(s) = \frac{k_0}{s} + \sum_{i=1}^n \frac{k_i}{s + \sigma_i} + k_\infty$$

here,

$$\frac{K}{s + \sigma} = \frac{1}{\frac{s}{K} + \frac{\sigma}{K}} = \frac{1}{\frac{1}{B_c} + \frac{1}{B_b}}$$

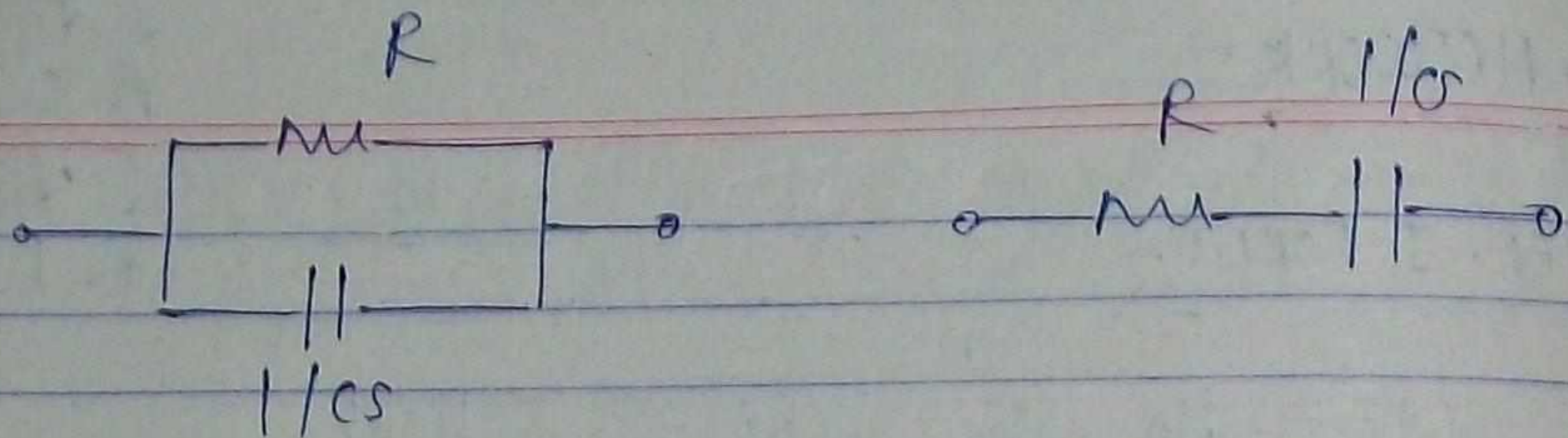


② Foster - II form -

PARALLEL
N/w

7/11/11

LC
↓ ↓
RC
↓ ↓
RL



$$Z_{RC}(s) = \frac{R \cdot 1/cs}{R + 1/cs}$$

$$Y_{RC}(s) = \frac{1}{R + 1/cs}$$

$$Z_{RC}(s) = \frac{R}{RCs + 1}$$

$$Y_{RC}(s) = \frac{Cs}{RCs + 1}$$

$$Z_{RC}(s) = \frac{R}{RC(s + 1/RC)}$$

$$Y_{RC}(s) = \frac{Cs}{RC(s + 1/RC)}$$

$$Z_{RC}(s) = \frac{1}{C(s + 1/RC)}$$

$$Y_{RC}(s) = \frac{s}{R(s + 1/RC)}$$

$$\frac{Y_{RC}(s)}{s} = \frac{1}{(s + 1/RC)}$$

NOTE -

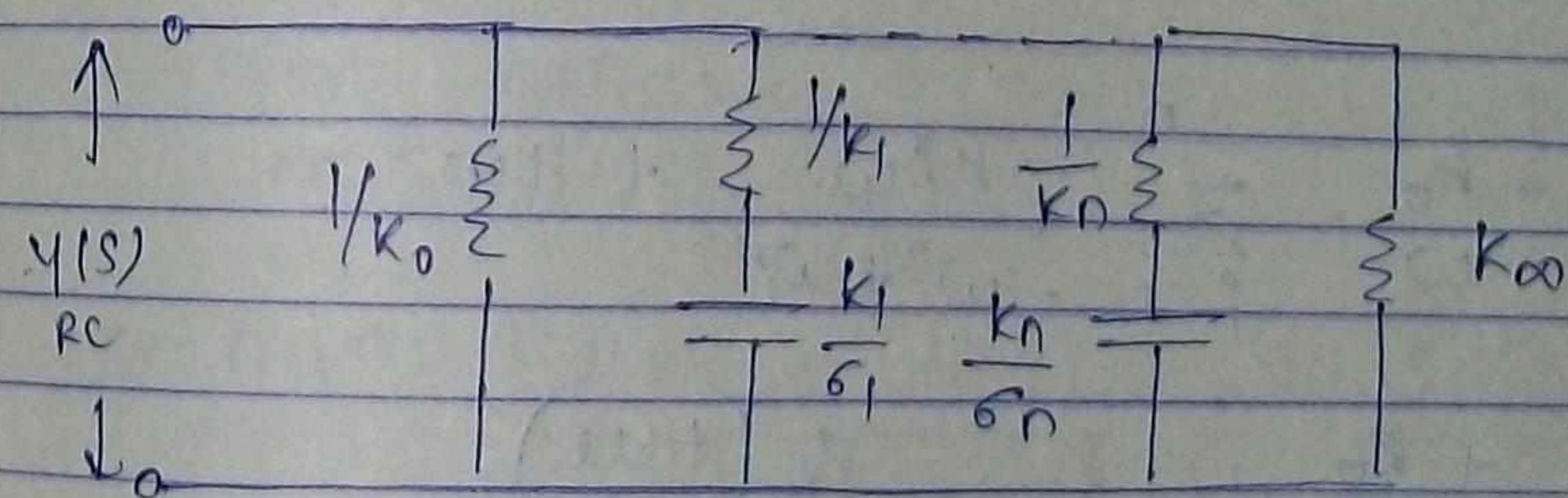
$Z_{RC}(s)$ and $\frac{Y_{RC}(s)}{s}$ pole zero pattern are identical. Hence same mathematical eqn is used for both functions.

$$Y_{RC}(s) = \frac{K_0}{s} + \sum_{n=1}^n \frac{k_i}{s + \sigma_i} + K_\infty$$

where,

$$\frac{Ks}{s + \sigma} = \frac{1}{\frac{s}{Ks} + \frac{\sigma}{Ks}} = \frac{1}{Z_a + Z_b}$$

$x_c = 1/sc$



⇒ PROPERTIES OF LC NETWORK -

1. Either pole or zero should be present at origin.
2. Either pole or zero should be present at infinity.
3. Poles and zeros are arranged alternately on jw axis.

$$f(-s) = f(s) \text{ even}$$

$$f(-s) = -f(s) \text{ odd}$$

4. In the partial fraction of expansion residue should be positive real,

5. $\frac{dX}{d\omega} > 0$ (slope is +ve)

$$Z(s) = \frac{k_0}{s} + \sum_{i=1}^n \frac{2k_i s}{s^2 + \omega_i^2} + Hs$$

$$jX = \frac{k_0}{j\omega} + \sum_{i=1}^n \frac{2k_i j\omega}{-\omega^2 + \omega_i^2} + Hj\omega$$

$$X = -\frac{k_0}{\omega} + \sum_{i=1}^n \frac{2k_i \omega}{-\omega^2 + \omega_i^2} + H\omega$$

$$\frac{dX}{d\omega} = +\frac{k_0}{\omega^2} + \dots \therefore (+ve)$$

$$6. \frac{m_1 \text{ (even)}}{n_2 \text{ (odd)}} = \frac{n_1 \text{ (odd)}}{m_2 \text{ (even)}}$$

$$F(s) = \frac{P(s)}{Q(s)} = \frac{m_1(s) + n_1(s)}{m_2(s) + n_2(s)}$$

$$F(s) = \frac{m_1 + n_1}{m_2 + n_2} \times \frac{m_2 - n_2}{m_2 - n_2}$$

$$(1-1)^2 = 1-1-2+1 = -1$$

$$F(s) = \frac{m_1 m_2 - n_1 n_2}{m_2^2 - n_2^2} + \frac{n_1 m_2 - m_1 n_2}{m_2^2 - n_2^2}$$

AA

$$F(s) = \text{Even } F(s) + \text{Odd } F(s)$$

$$F(s) = \text{Real } F(s) + \text{Imag } F(s)$$

$$\frac{m_1 m_2 - n_1 n_2}{m_2^2 - n_2^2} \gg 0$$

$$\frac{m_1 m_2 - n_1 n_2}{m_2^2 - n_2^2} = 0$$

$$m_1 m_2 = n_1 n_2$$

AA

$$\frac{m_1 \text{ (even)}}{n_2 \text{ (odd)}} = \frac{n_1 \text{ (odd)}}{m_2 \text{ (even)}}$$

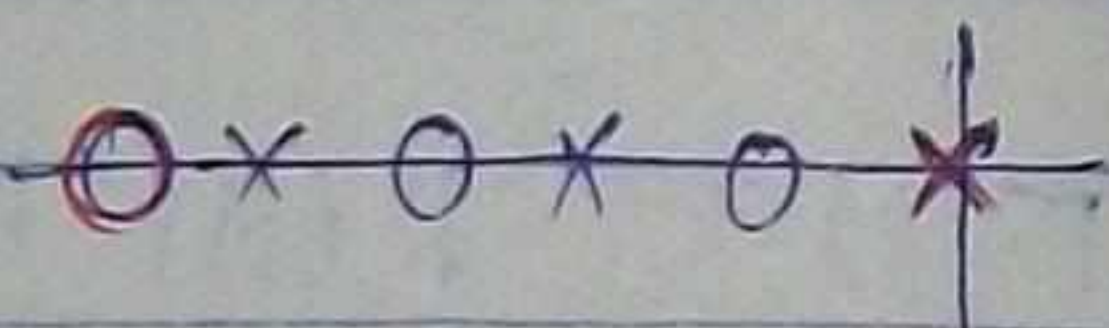
⇒ PROPERTIES OF $Z_{RC}(s)$ and $Y_{RL}(s)$ -

(10) lowest critical frequency (1st critical freq) is due to pole, it may present either at origin or nearer to origin.

$$Z_{RC}(s) = \frac{k_0}{s} + \sum_{i=1}^n \frac{k_i}{s + \sigma_i} + k_{\infty}$$

(2) Poles and zero are arranged alternately on -ve real axis.

(3) Last critical freq. is due to zero, it may present either at ∞ or nearer to ∞ .



(4) Slope is -ve.

$$\frac{dZ_{RC}}{ds} < 0$$

$$\frac{dY_{RL}}{ds} < 0$$

by diff $Z_{RC}(s) = \frac{k_0}{s} + \sum_{i=1}^n \frac{k_i}{s+\sigma_i} + k_\infty (-ve)$

$$Z_{RC}(0) > Z_{RC}(\infty)$$

$$Y_{RL}(0) > Y_{RL}(\infty)$$

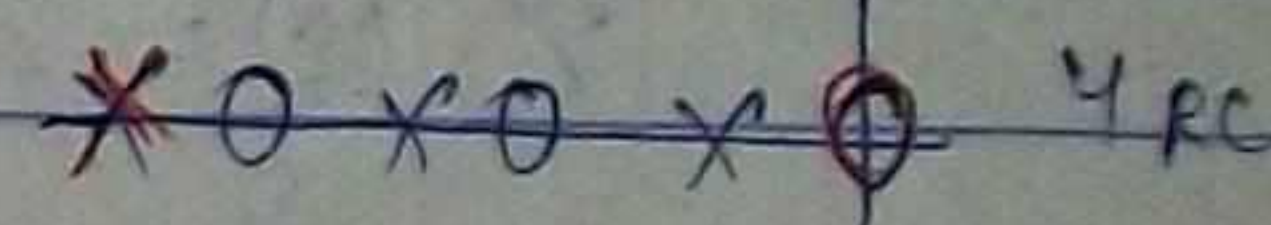
$s=0$	$s=\infty$
$X_c = \frac{1}{s_c}$	
$X_c = \infty$	$X_c = 0$

$\Rightarrow$ PROPERTIES OF $Z_{RL}(s)$ and $Y_{RC}(s)$ -

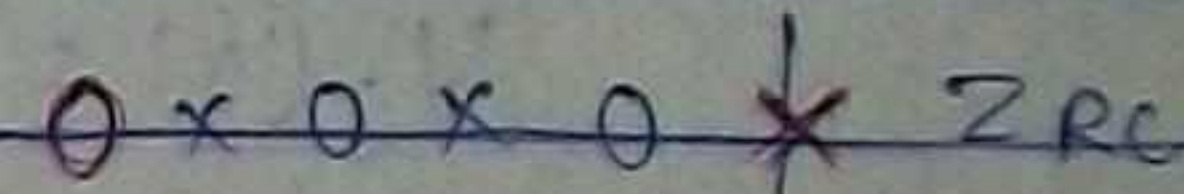
(1) Highest critical freq. is due to pole, it may present either at ∞ or nearer to ∞ .

(2) Poles and zeros are arranged alternately on -ve real axis.

(3) Lowest critical freq. (1st critical freq) is due to zero, it may present either at origin or nearer to origin



(4) Slope is +ve.



$$\frac{dY_{RC}}{ds} > 0$$

$$\frac{dZ_{RL}}{ds} > 0$$

also by diff $Y_{RC}(s) = k_0 + \sum_{i=1}^n \frac{k_i s}{s+\sigma_i} + k_\infty s$

$$Z_{RL}(0) < Z_{RL}(\infty)$$

$$Y_{RC}(0) < Y_{RC}(\infty)$$

$s=0$	$s=\infty$
$X_L = \frac{1}{s}$	$X_L = \infty$
$X_L = 0$	

⇒ RC NETWORK -

① CAVER - I FORM -

LADDER
N/W

$$LC \rightarrow Z_2(s) = Z(s) - H_1(s), \quad Y_2(s) = \frac{1}{Z_2(s)}$$

$$Y_3(s) = Y_2(s) - H_2(s)$$

for RC N/W →

STEP ①: Removal of constant from $Z(s)$

$$Z_2(s) = Z(s) - k_1$$

$$Y_2(s) = \frac{1}{Z_2(s)}$$

STEP ②: Removal of pole at ∞ from $Y(s)$

$$Y_3(s) = Y_2(s) - H_1(s) \quad (B_c = sC)$$

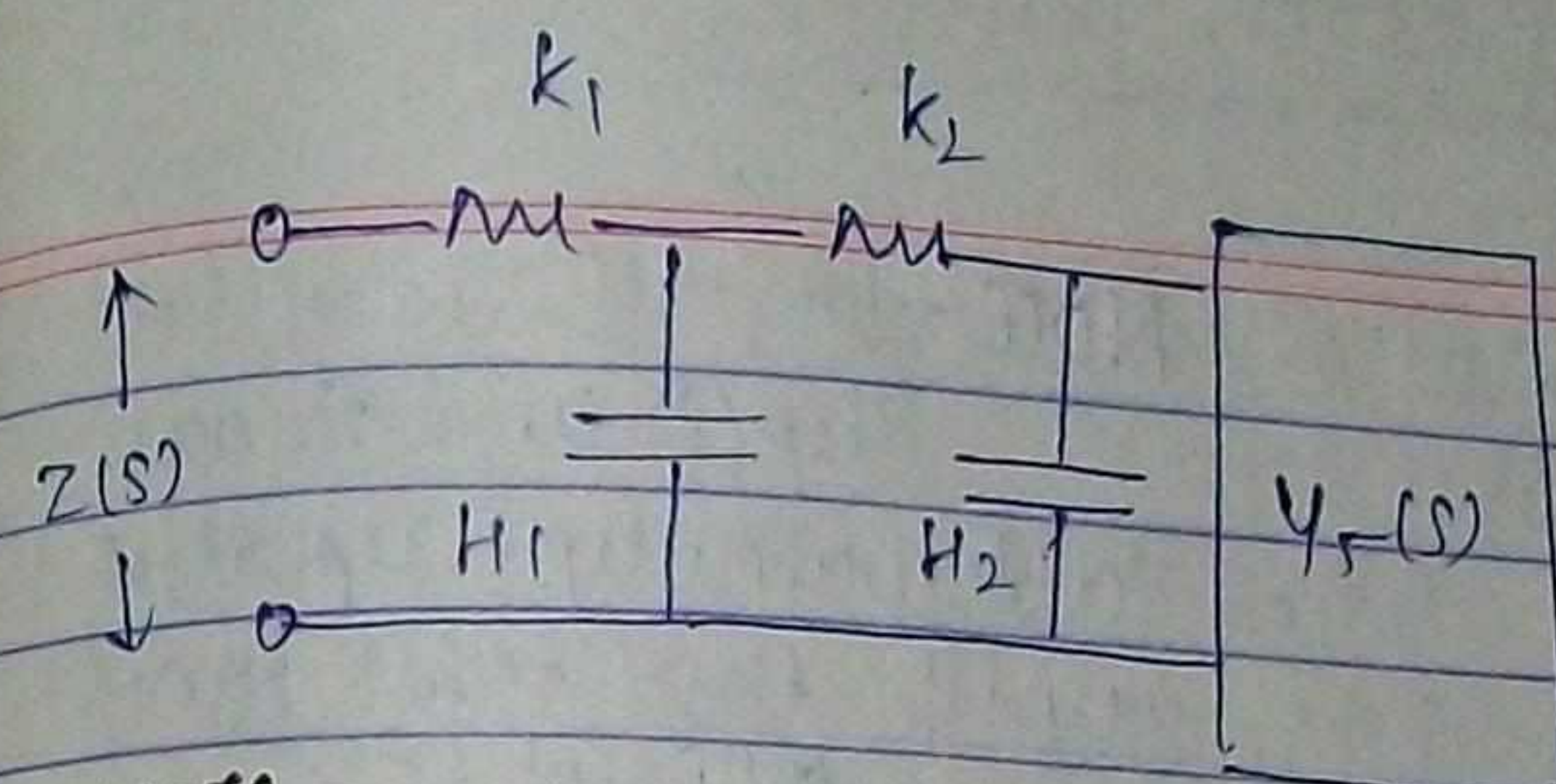
$$Z_3(s) = \frac{1}{Y_3(s)}$$

$$Z_4(s) = Z_3(s) - k_2$$

$$Y_4(s) = 1/Z_4(s)$$

$$Y_5(s) = Y_4(s) - H_2(s)$$

$$Y_4(s) = Y_5(s) + H_2(s)$$



NOTE -

Step ① and step ② are alternately repeated until the total funn is realised.

② CAVER - II FORM -

LADDER
N/W

$$LC \rightarrow Z_2(s) = Z(s) - \frac{k_{01}}{s}, \quad Y_2(s) = \frac{1}{Z_2(s)}$$

$$Y_3(s) = Y_2(s) - \frac{k_{02}}{s}$$

RC Network →

STEP ①: Removal of pole at origin from $Z(s)$

$$Z_2(s) = Z(s) - \frac{k_{01}}{s}$$

$$Y_2(s) = \frac{1}{Z_2(s)} \quad (X_c = \frac{1}{sC})$$

STEP ②: Removal of constant from $Y(s)$

$$Y_3(s) = Y_2(s) - k_1$$

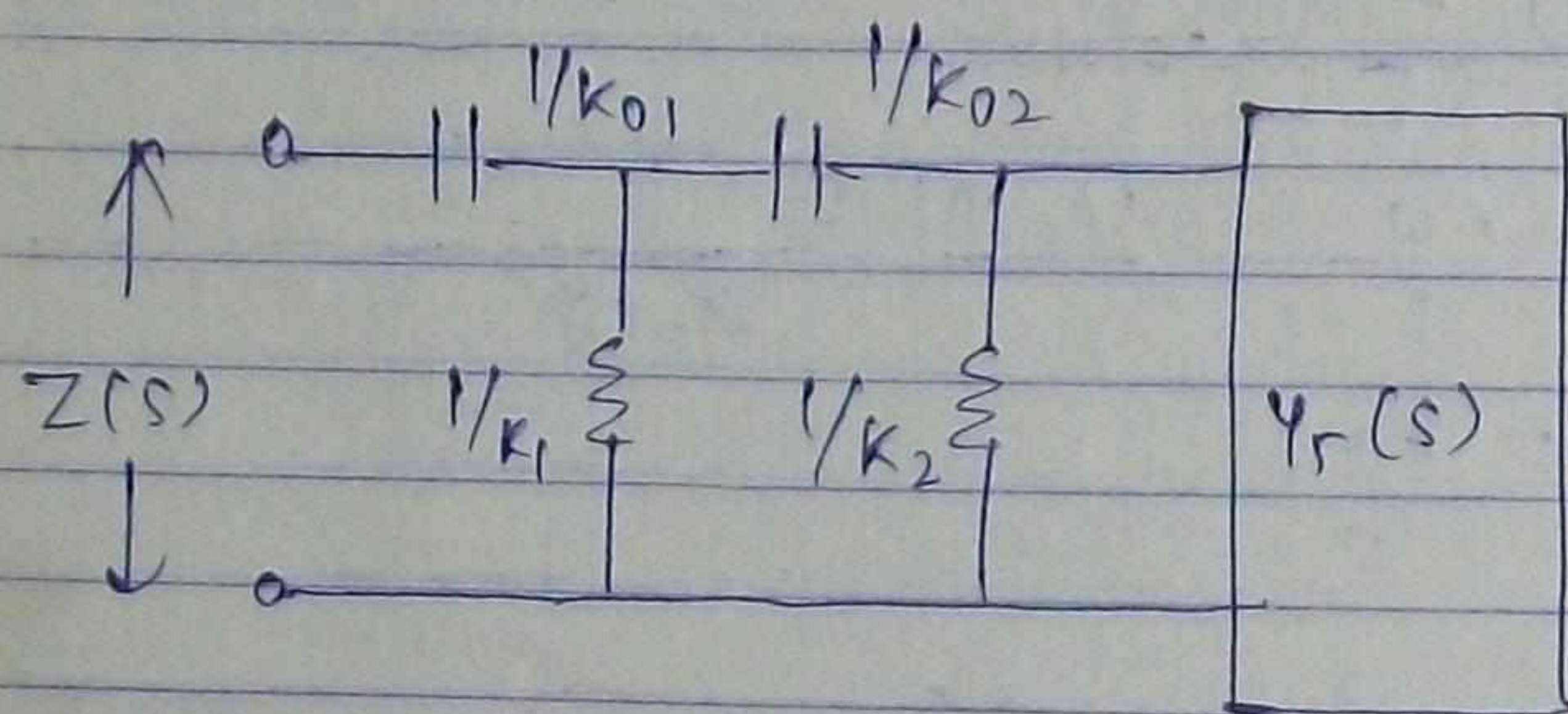
$$Z_3(s) = \frac{1}{Y_3(s)}$$

$$Z_4(s) = Z_3(s) - \frac{k_{02}}{s}$$

$$Y_4(s) = \frac{1}{Z_4(s)}$$

$$Y_5(s) = Y_4(s) - k_2$$

$$Y_4(s) = Y_5(s) + k_2$$



NOTE -

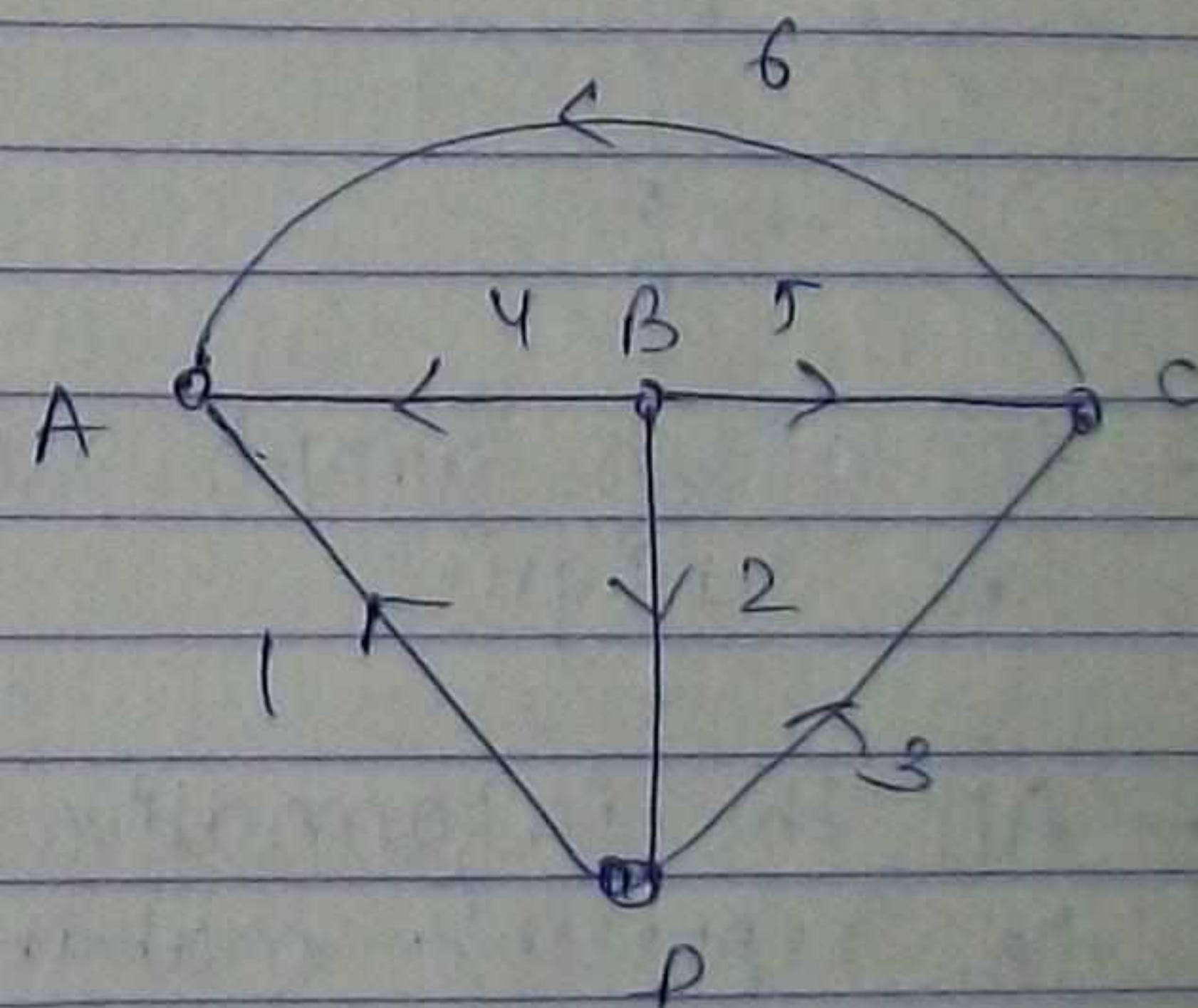
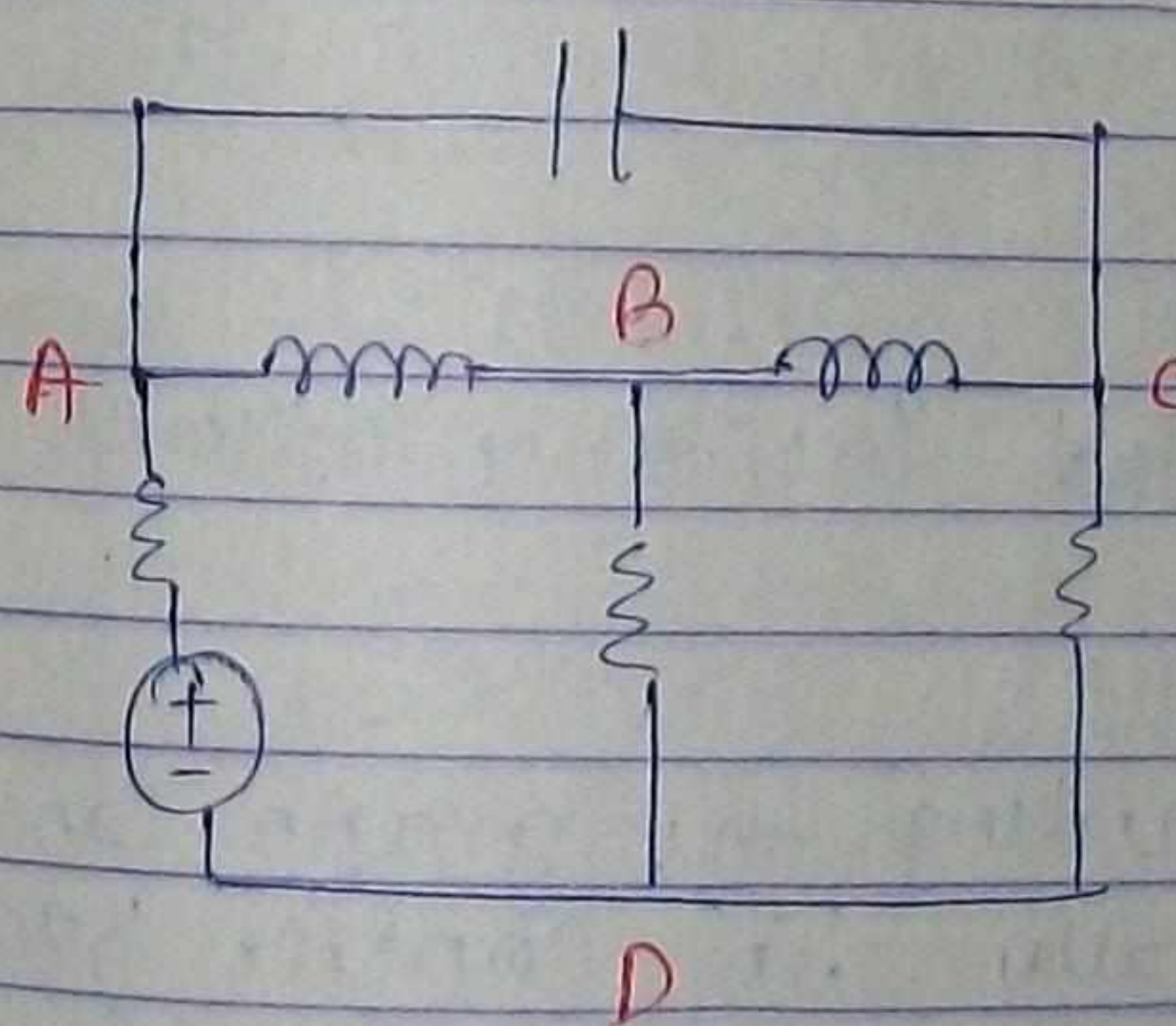
Step (1) and (2) are alternatively repeated until the total funn is realised.

GRAPH THEORY

8/11/11

- Network topology is the study of the N/w properties by investigating interconnections b/w branches and nodes. It mainly concentrates on geometry of the network.

- In the N/w topology any N/w is replaced by graph, to develop the graph each element is replaced by either straight line or arc of the semicircle, voltage source is replaced by short circuit, current source is replaced by open circuit and graph retains all the nodes of the original N/w.



NO. OF
BRANCHES
OF N/W

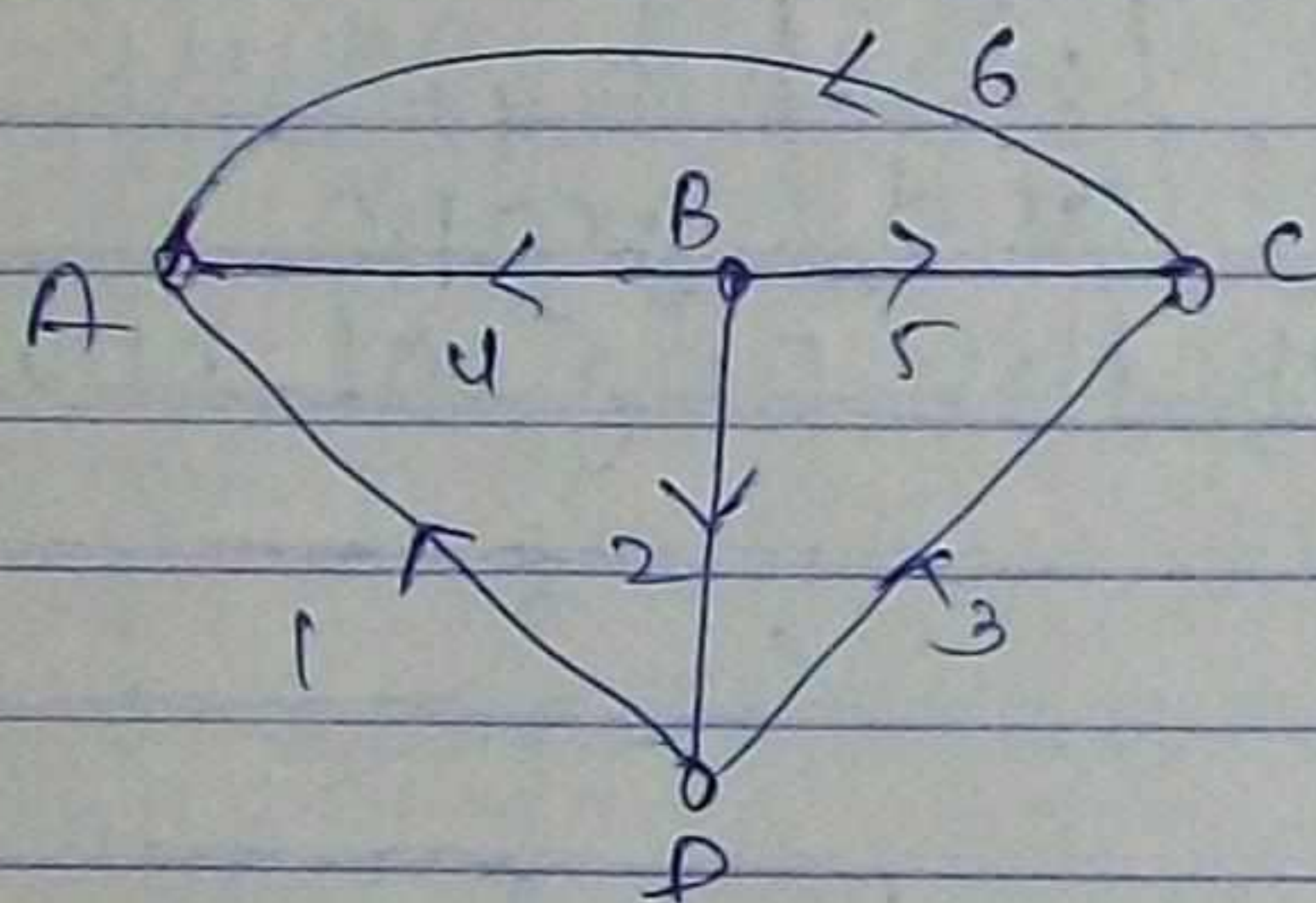
$\geq$

NO. OF
BRANCHES
OF GRAPH

AUGMENTED INCIDENCE MATRIX

$[A_a] =$

	1	2	3	4	5	6
A	1			1		1
B		-1		-1	-1	
C			+1		1	-1
D	-1	+1	-1			



- for given graph augmented incidence matrix is unique.

- All the information regarding the graph can be represent mathematically in concise form is called as incidence matrix.

⇒ TREE -

Tree is a connected subgraph it connects all the nodes of the NW by it does not consist of any closed path.

REDUCED INCIDENCE MATRIX

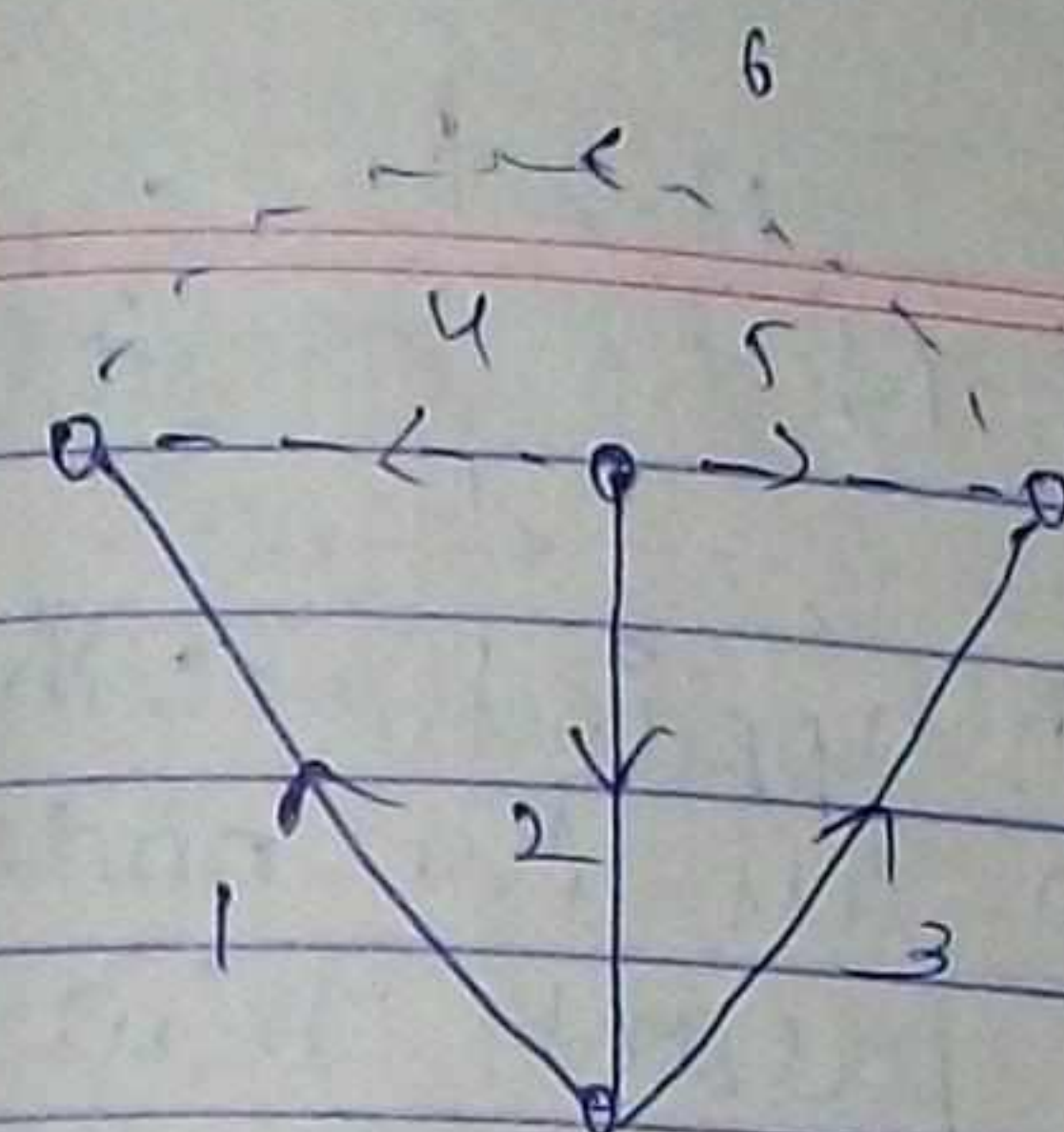
$[A] =$

	1	2	3	4	5	6
A	1		+1	+1		+1
B		-1		-1	-1	
C			+1		+1	-1

D → REF. NODE

(OR)

DATUM - NODE



TREE {1, 2, 3}

⇒ CO-TREE -

The set of branches which are disconnected to form a tree is called as co-tree. (Complementary tree)

CO Tree {4, 5, 6}

⇒ TREE BRANCH (TWIG) -

The branch which forms a tree is called as tree branch (twig). Generally it is represented by solid lines.

TOTAL NO. OF TREE BRANCH	$= N - 1$	$= 3 (4 - 1)$
--------------------------	-----------	---------------

⇒ LINK (CHORD) -

The branch which is disconnected to form a tree is called as link (chord). Generally it is indicated by dotted line.

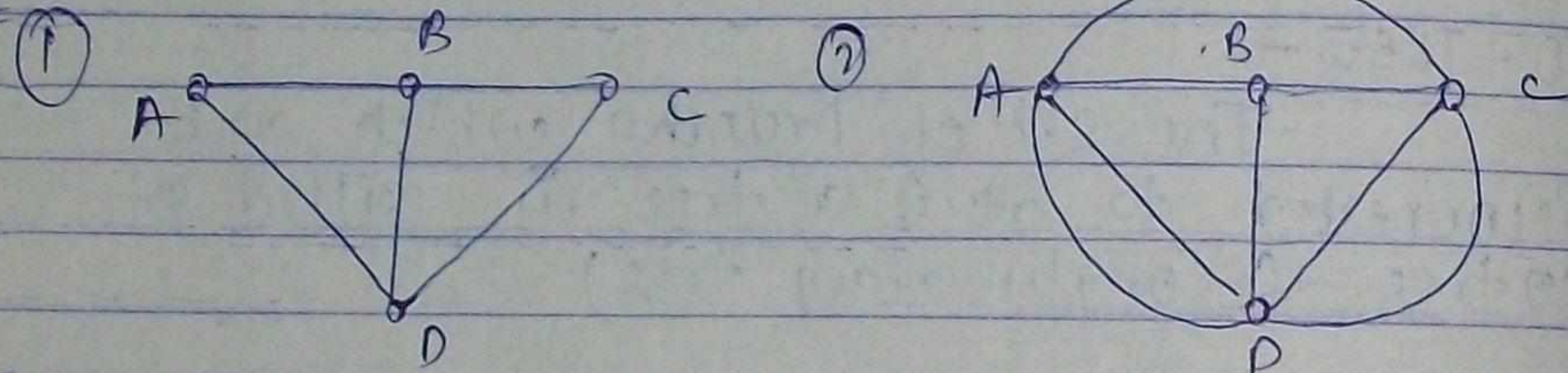
TOTAL NO. OF LINK	$= B - (N - 1)$	$= 6 - (4 - 3)$
-------------------	-----------------	-----------------

- Total no. of possible trees = N^{N-2} $(4^{4-2} = 16)$

NOTE - The above formula is applied when:

- ① connection is present b/w all the nodes
- ② NO repeated branch is present b/w the nodes

Ex -



for these two graphs formula cannot be applied

- Total no. of possible trees = $\text{Det}[A \cdot A^T]$ for any graph

where, A = reduced incidence matrix.

⇒ NODE PAIR VOLTAGE -

$$\text{TOTAL NO. OF NODE-PAIR VOLTAGES} = \frac{N(N-1)}{2} = N C_2$$

for given graph

$$\{V_{AB} \ V_{AC} \ V_{AD} \ V_{BC} \ V_{BD} \ V_{CD}\}$$

$$= \frac{4(4-1)}{2} = 6$$

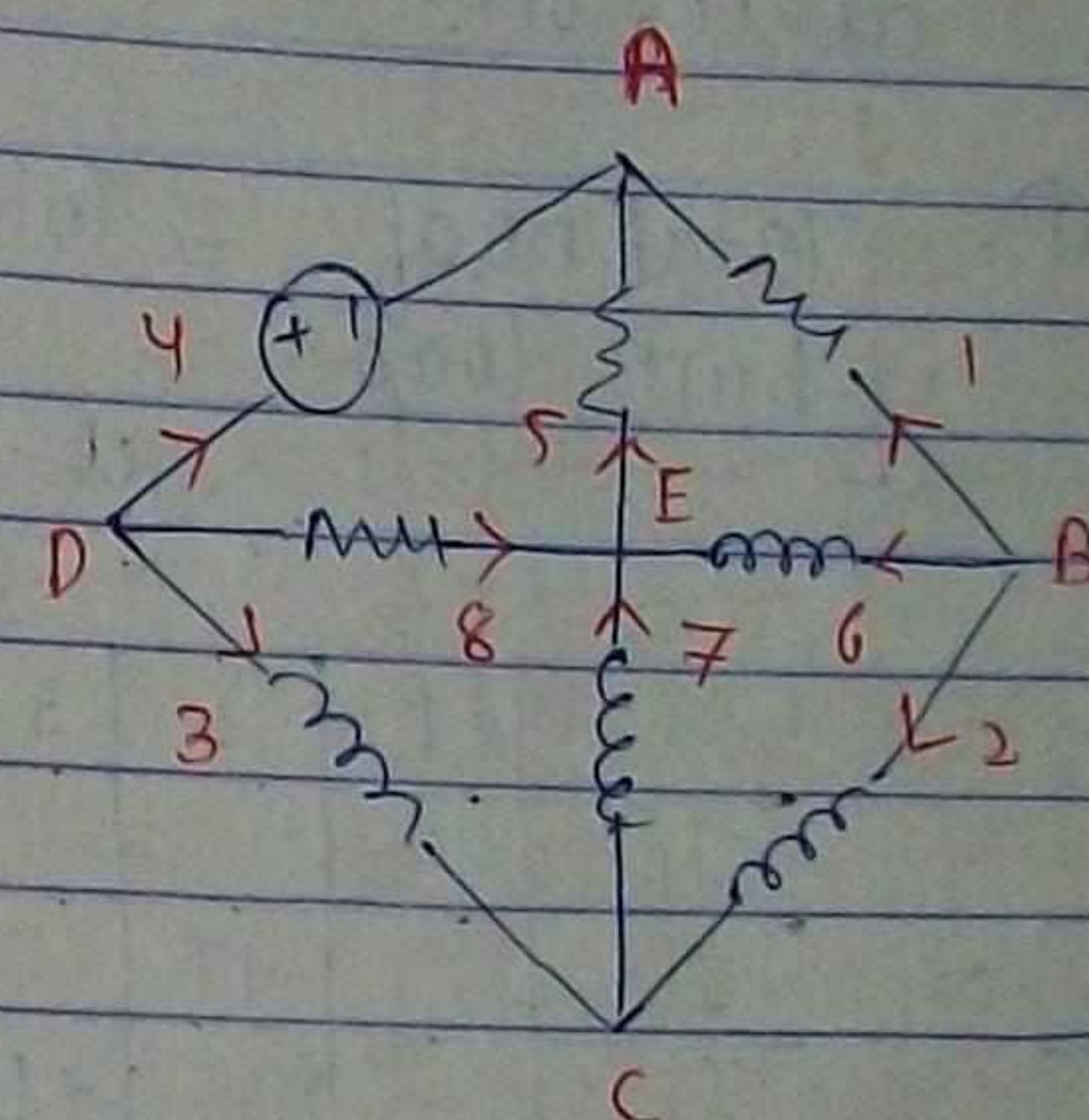
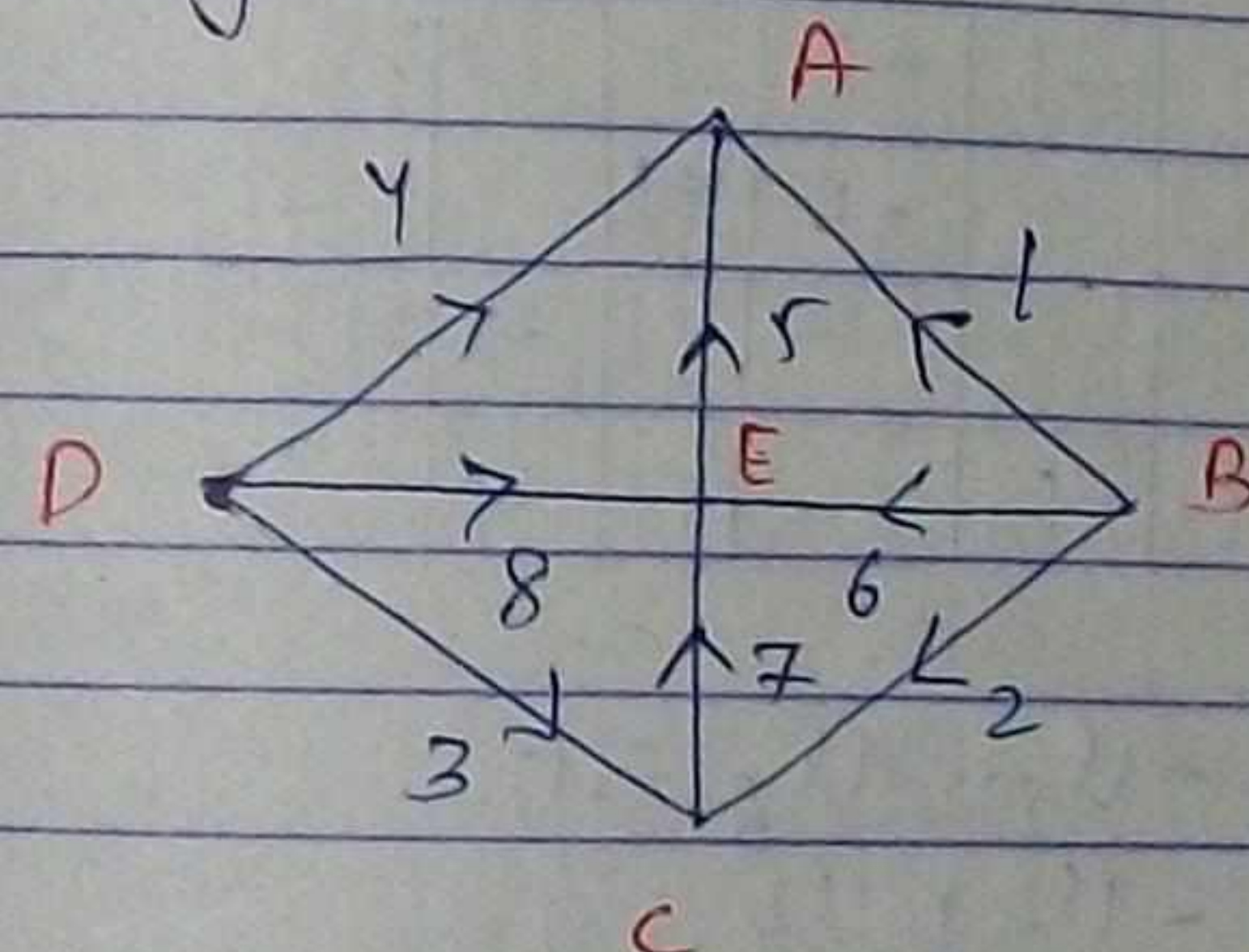
- TOTAL NO. OF EDGES (BRANCHES) = $\frac{N(N-1)}{2}$

$$\frac{4 \times 3}{2} = 6$$

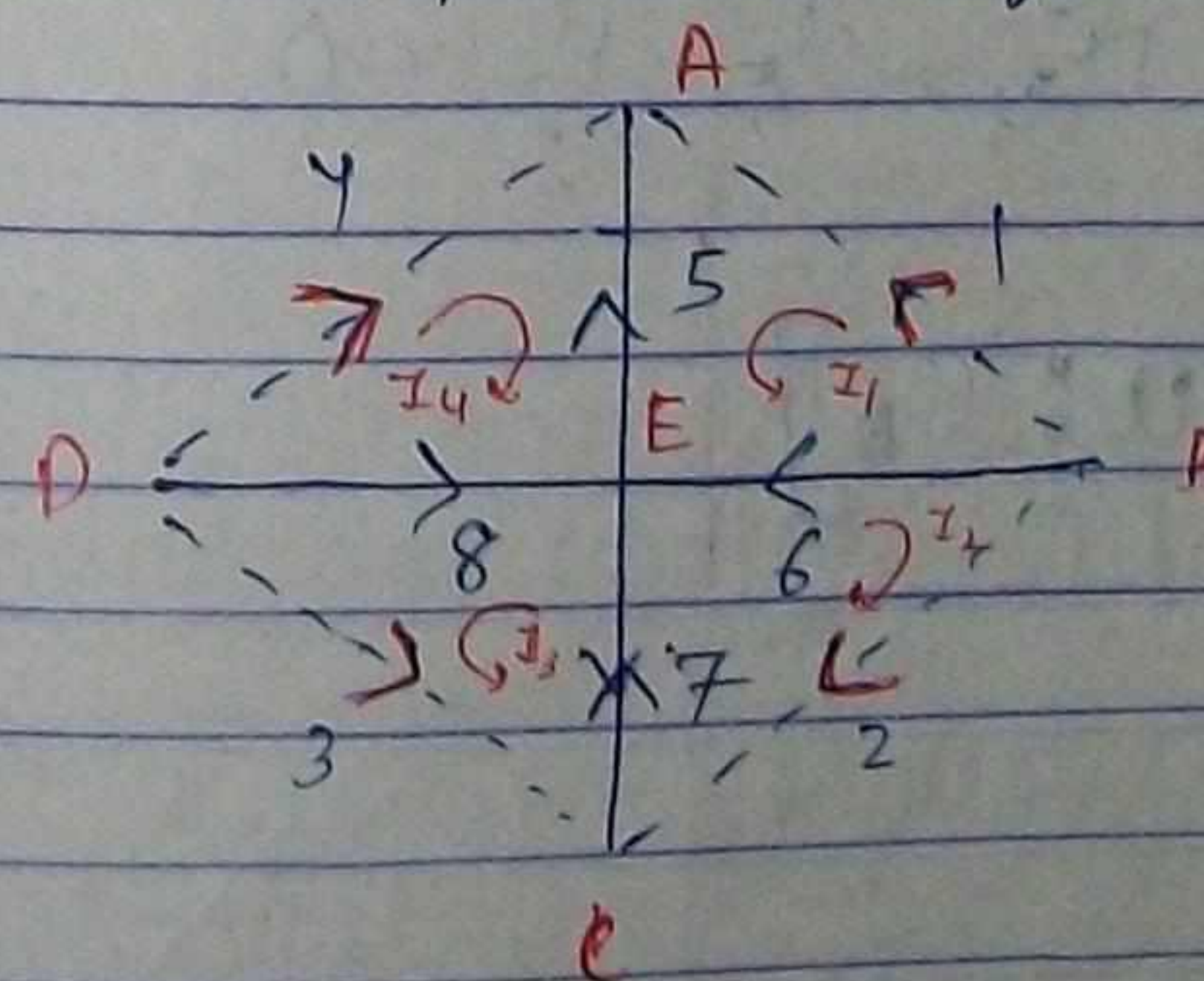
⇒ TIE-SET MATRIX -

Ex -

STEP ①: Develop a graph for given N/w



STEP ②: Develop a tree for the graph.



STEP ③:

- ① Identify total no. of basic loop (or) fundamental loops (or) f-loop (or) independent loop

② Basic loop should consist of only one link.

③ Basic loop direction is same as link current direction.

④ $\boxed{\text{Total no. of basis loop} = \text{Total no. of links} = b - (N - 1)}$

	u								C_B	
	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8		
I_1	+1				-1	-1				
I_2		+1				-1	+1			
I_3			+1				+1	-1		
I_4				+1	-1			-1		

KVL equation :

$$u_1 - u_5 - u_6 = 0$$

$$u_2 - u_6 + u_7 = 0$$

$$u_3 + u_7 - u_8 = 0$$

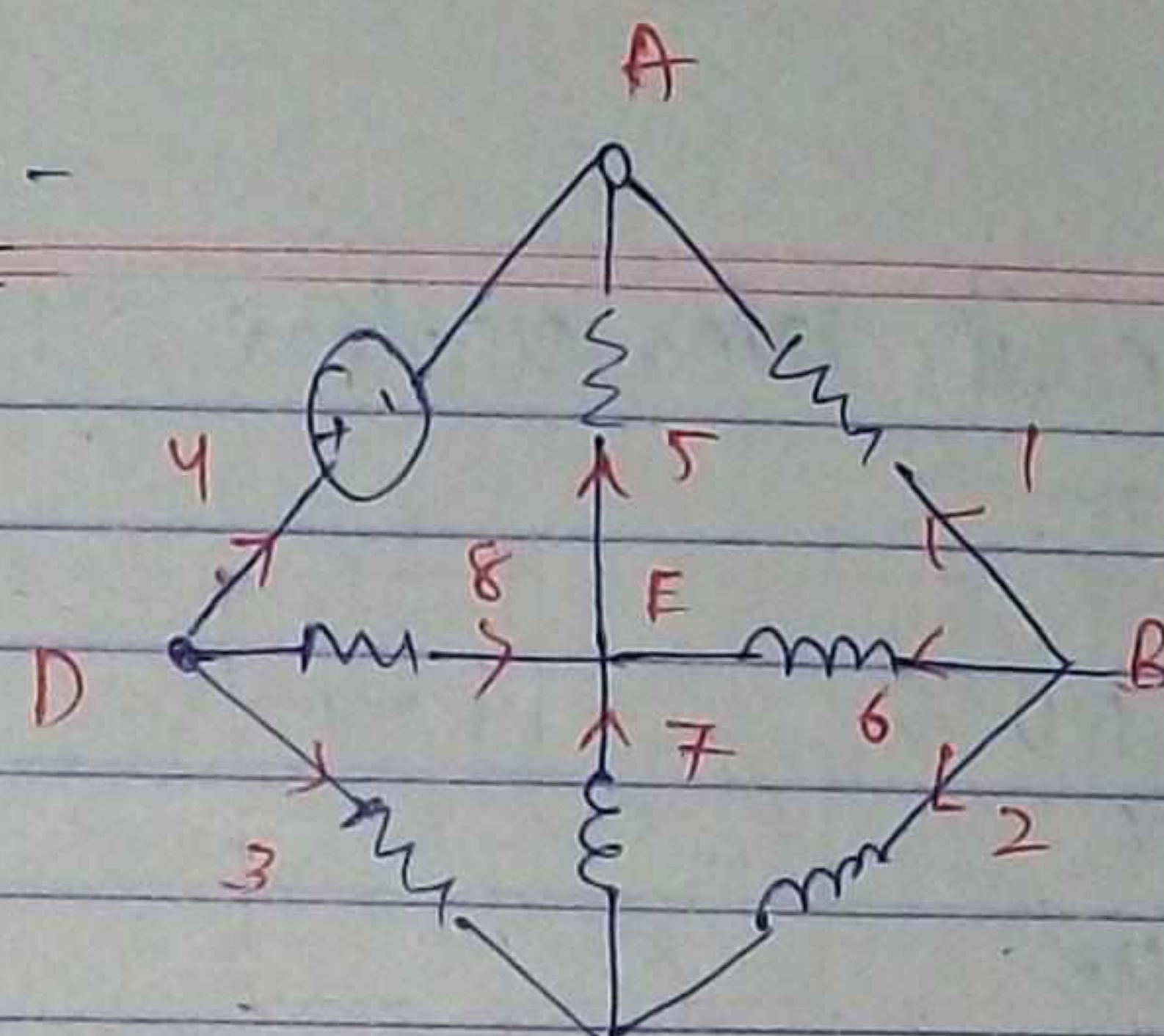
$$u_4 - u_5 - u_8 = 0$$

To identify cut set matrix.

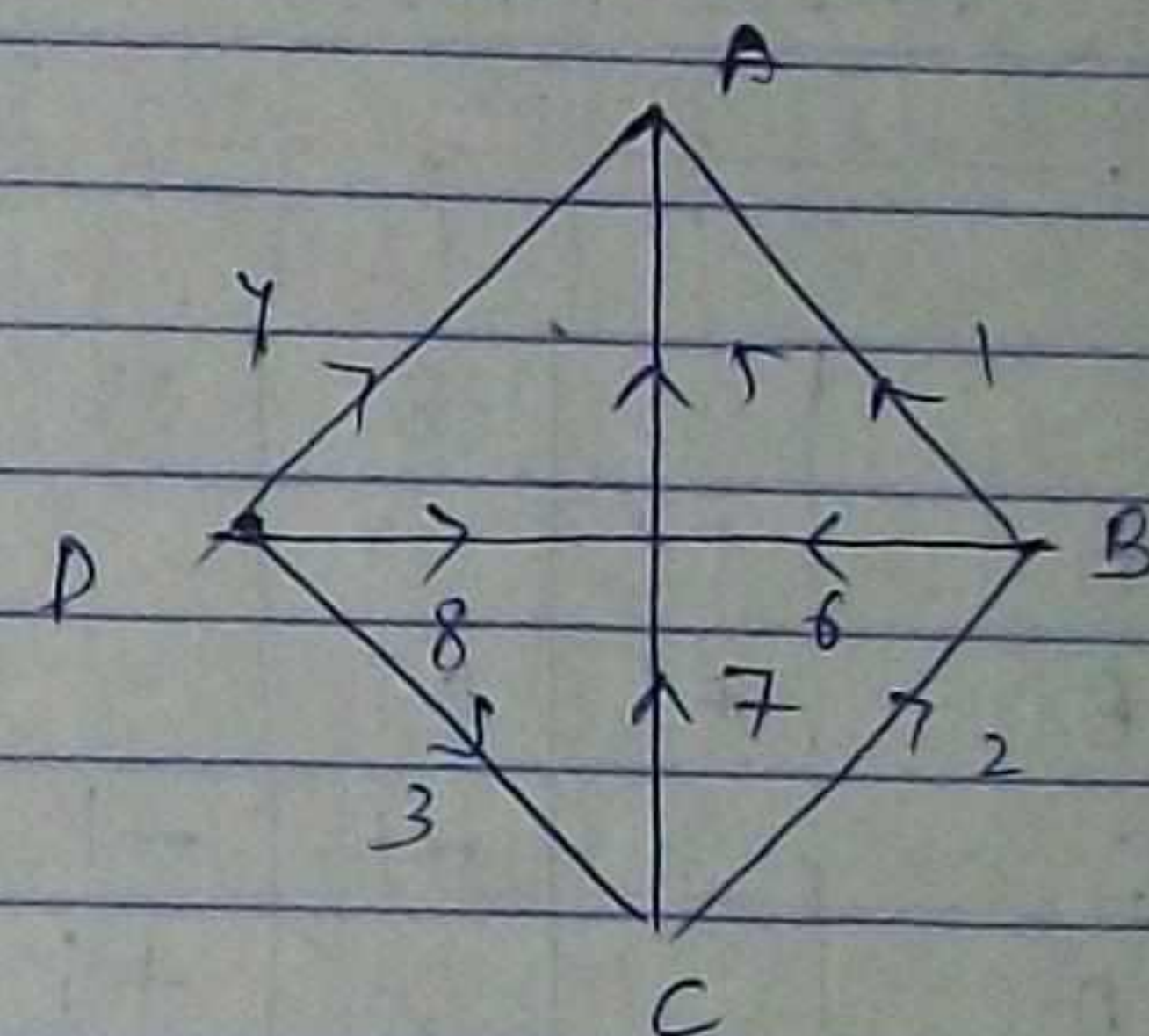
$\boxed{[C] = [u : C_B]}$

⇒ CUT-SET MATRIX -

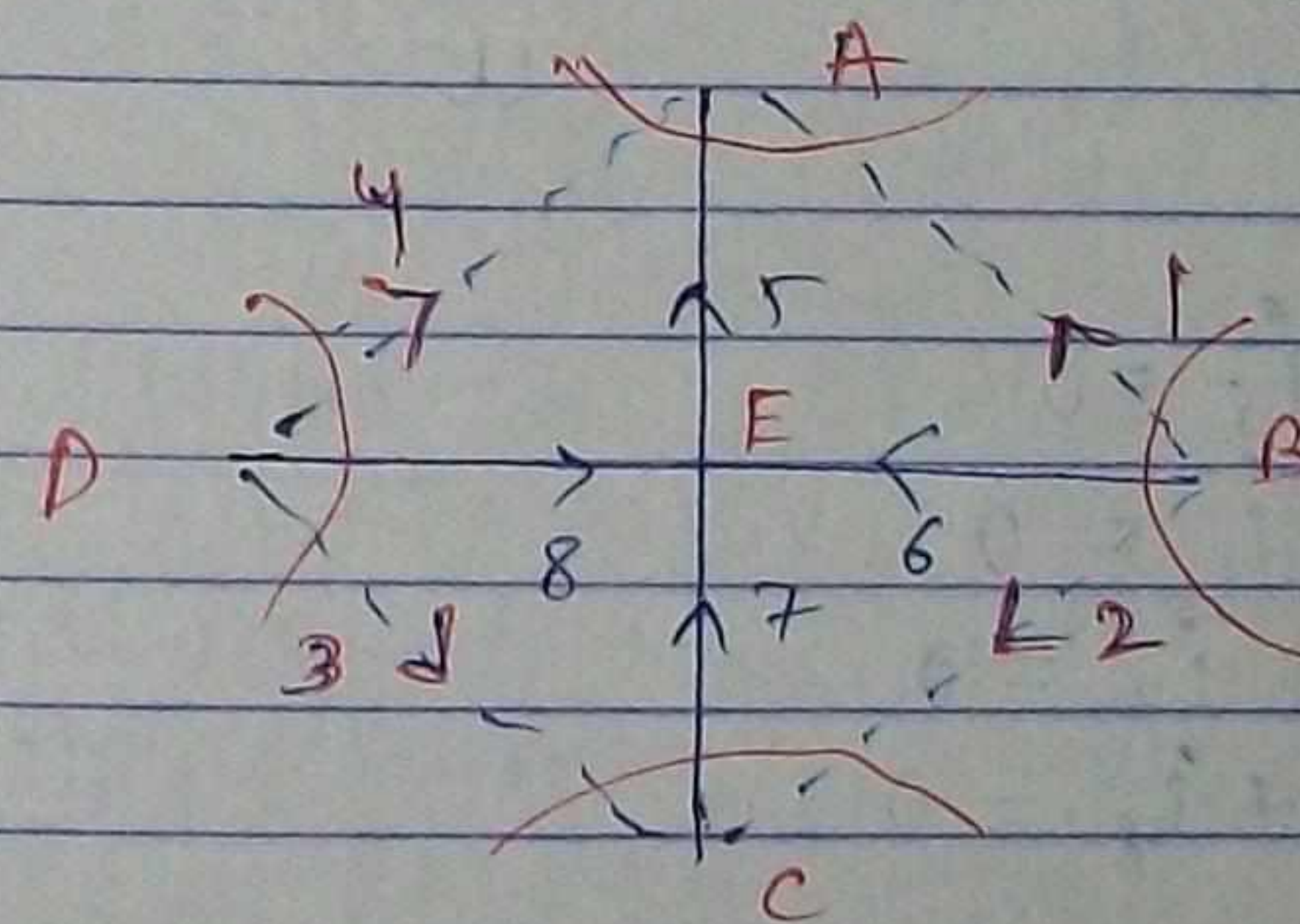
Ex -



STEP ①: Develop a graph for given N/w.



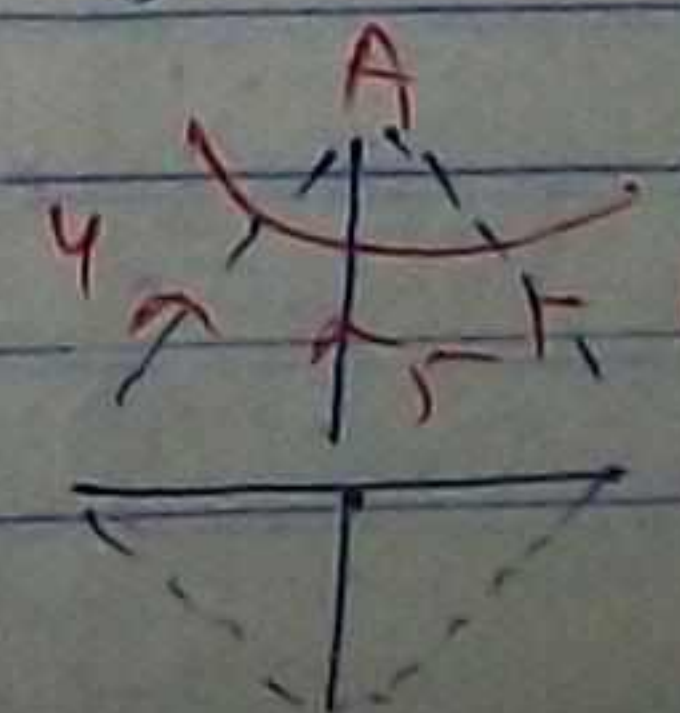
STEP ②: Develop a tree for the graph.



STEP ③:

① Identify total no. of Basic cut-set or fundamental cut set or f-cutset

② Basic cut set should consist of only one tree branch.



③ Basic cut set direction is same as tree branch current direction.

④ NO. TOTAL NO. OF BASIC CUTSET = TOTAL NO. OF TREE BRANCHES = $N-1$

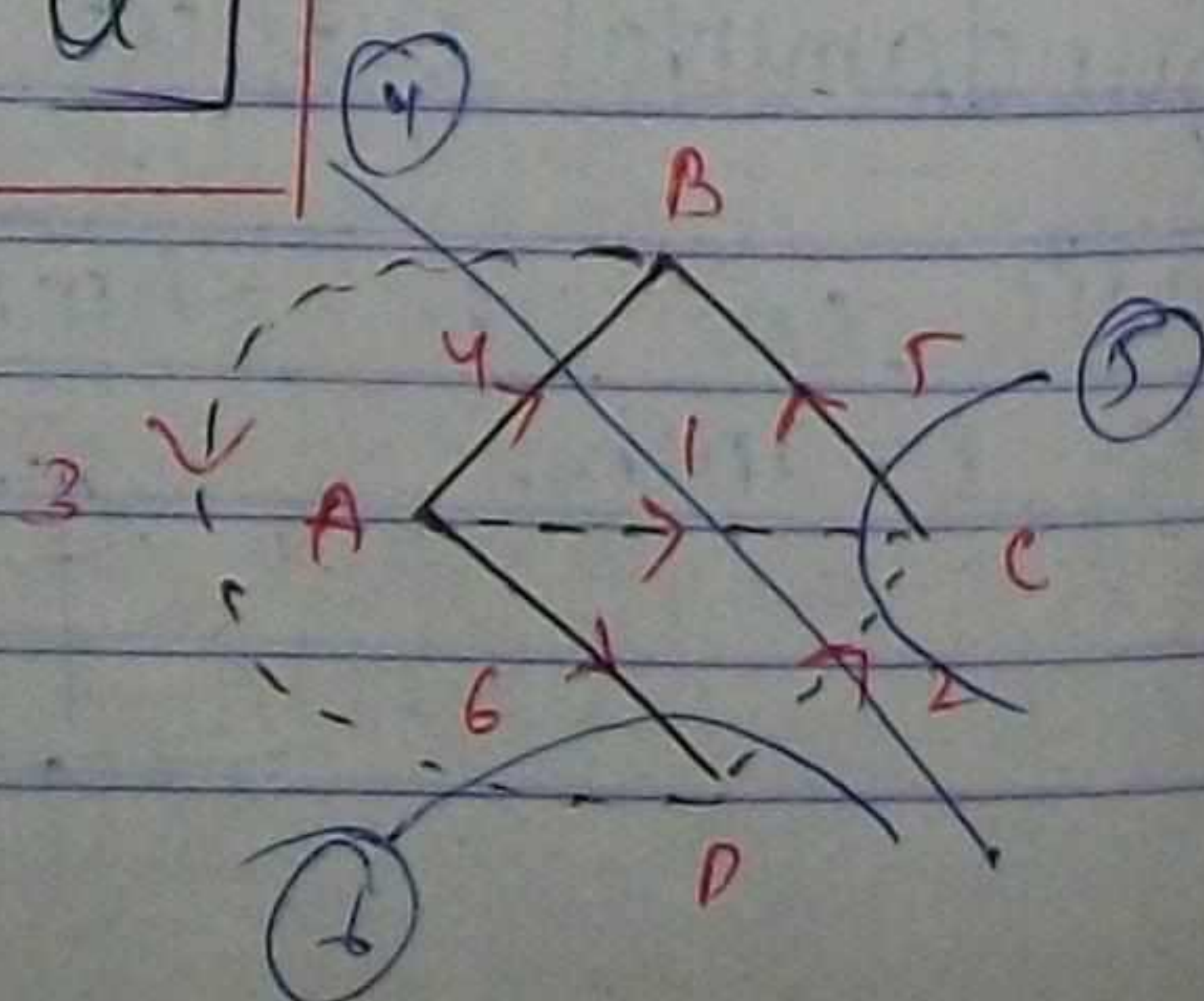
	1	2	3	4	5	6	7	8
A	+1			+1	+1			
B	+1	+1				+1		
C		-1	-1				+1	
D			+1	+1				+1

KCL Equation - B_1 u

$$\begin{aligned} i_1 + i_4 + i_5 &= 0 \\ i_1 + i_2 + i_6 &= 0 \\ -i_2 - i_3 + i_7 &= 0 \\ i_3 + i_4 + i_8 &= 0 \end{aligned}$$

$$[B] = [B_1 : u]$$

Ques Develop cutset matrix for

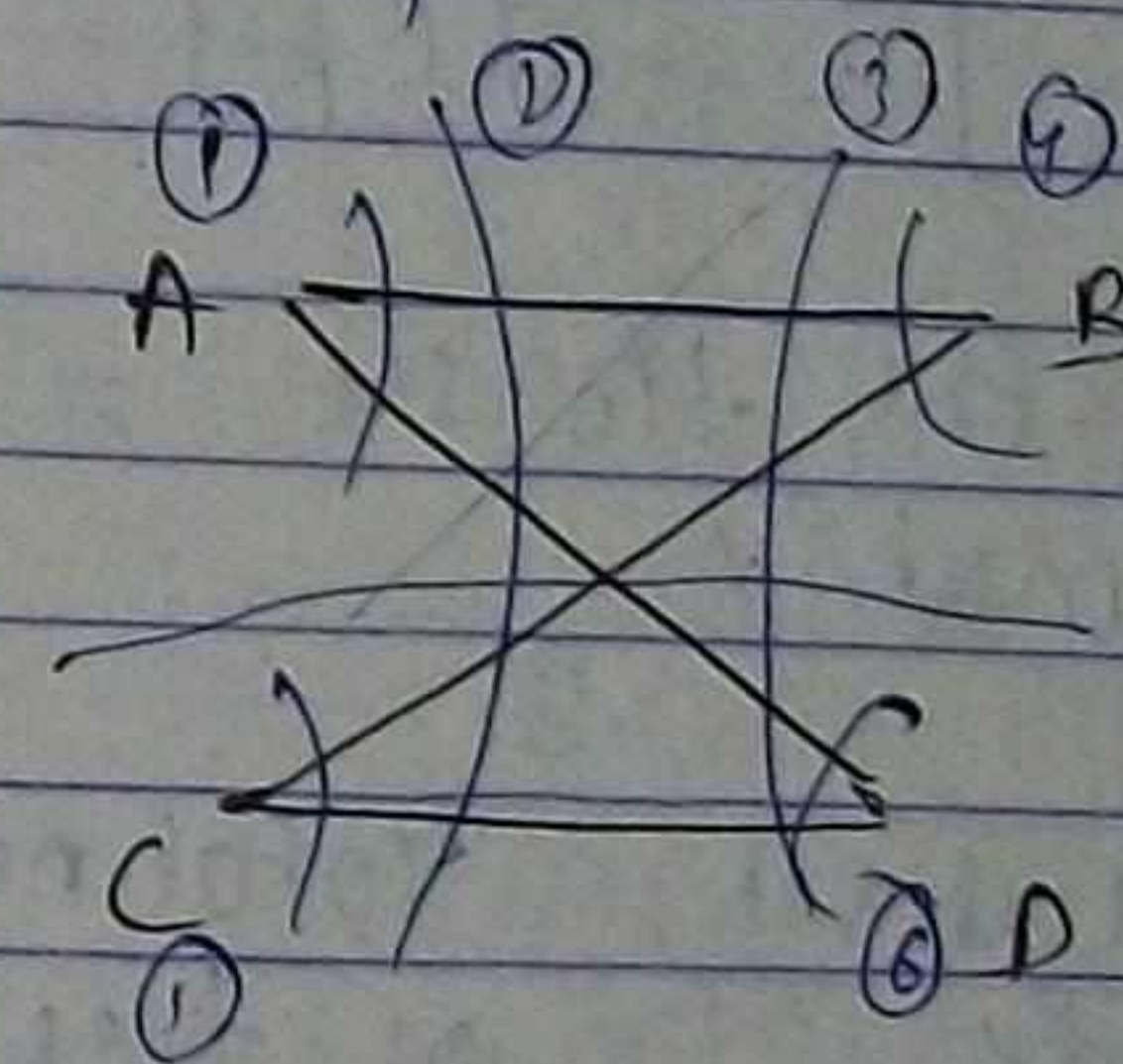


Solⁿ: Total Basic cutset = $N-1 = 3$

	1	2	3	4	5	6
①	+1	+1	-1	+1		
②	-1	-1			+1	
③		-1	+1			+1

Ques Identify total No. of cutset for graph

- ① 3 ② 4
③ 5 ④ 6



⇒ CONCLUSIONS -

- ① Total no. of possible trees = N^{N-2}
- ② Tie set matrix is not unique, total no. of possible tie set matrix is equal to N^{N-2} .
- ③ Cut set matrix is not unique, total no. of possible cut set matrix is equal to N^{N-2} .

(4) $[C] = [u; C_B]$ Tie-set

$[B] = [B_L; u]$ Cut-set

$[C_B] = -[B_L]^T$

$[B_L] = -[C_B]^T$

(5) Rank of the tie-set matrix = No of links = $b - (N - 1)$ Total

(6) Rank of cut set matrix = Total no of tree branches = $N - 1$

(7) Rank of the incidence matrix = $N - 1$

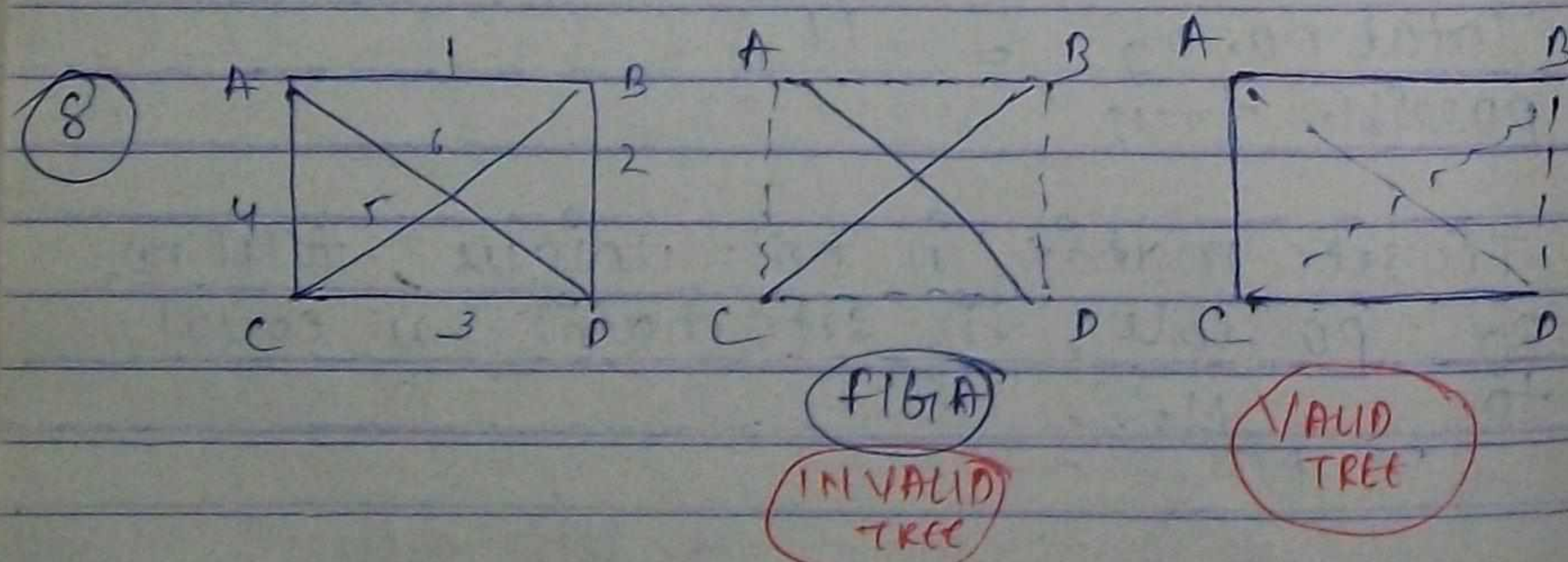


fig A is invalid tree since no connection is present b/w the tree branches.

$\Rightarrow$ DUALITY -

$R \longleftrightarrow G$

$L \longleftrightarrow C$

$V \longleftrightarrow I$

LOOP $\longleftrightarrow$ NODE

KVL $\longleftrightarrow$ KCL

O.C $\longleftrightarrow$ S.C

Thevenin $\longleftrightarrow$ Norton

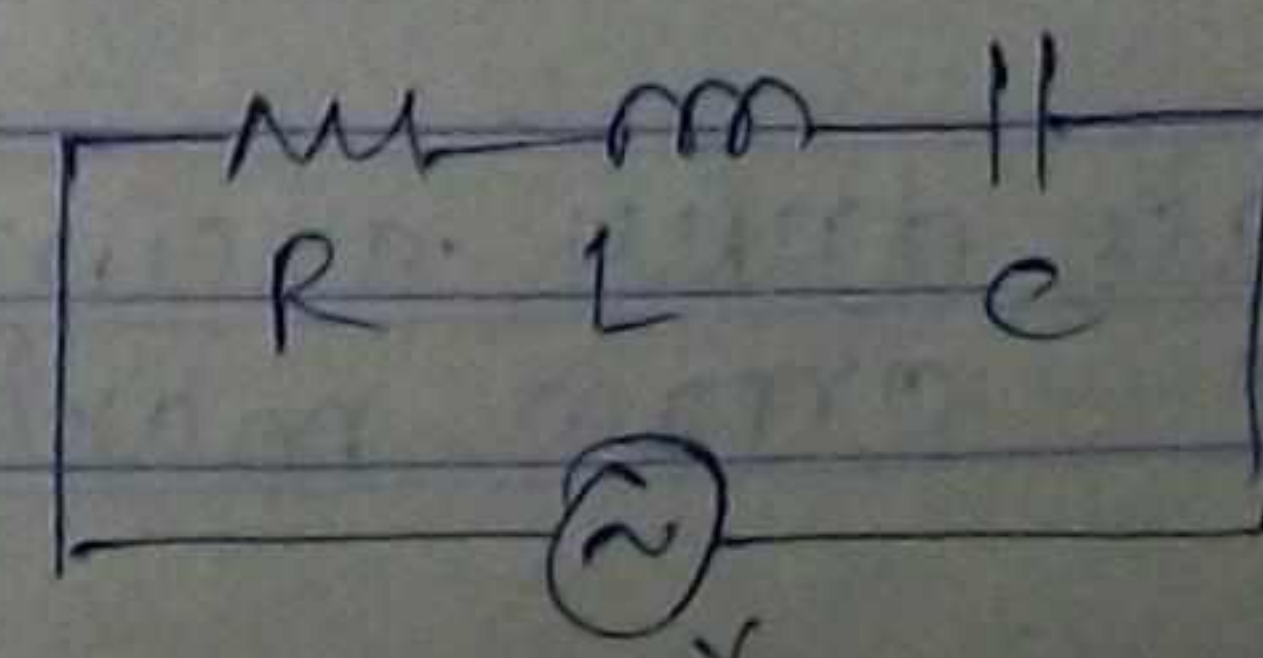
Tie set $\longleftrightarrow$ Cut set

foster I form (series) $\longleftrightarrow$ foster II form (parallel)

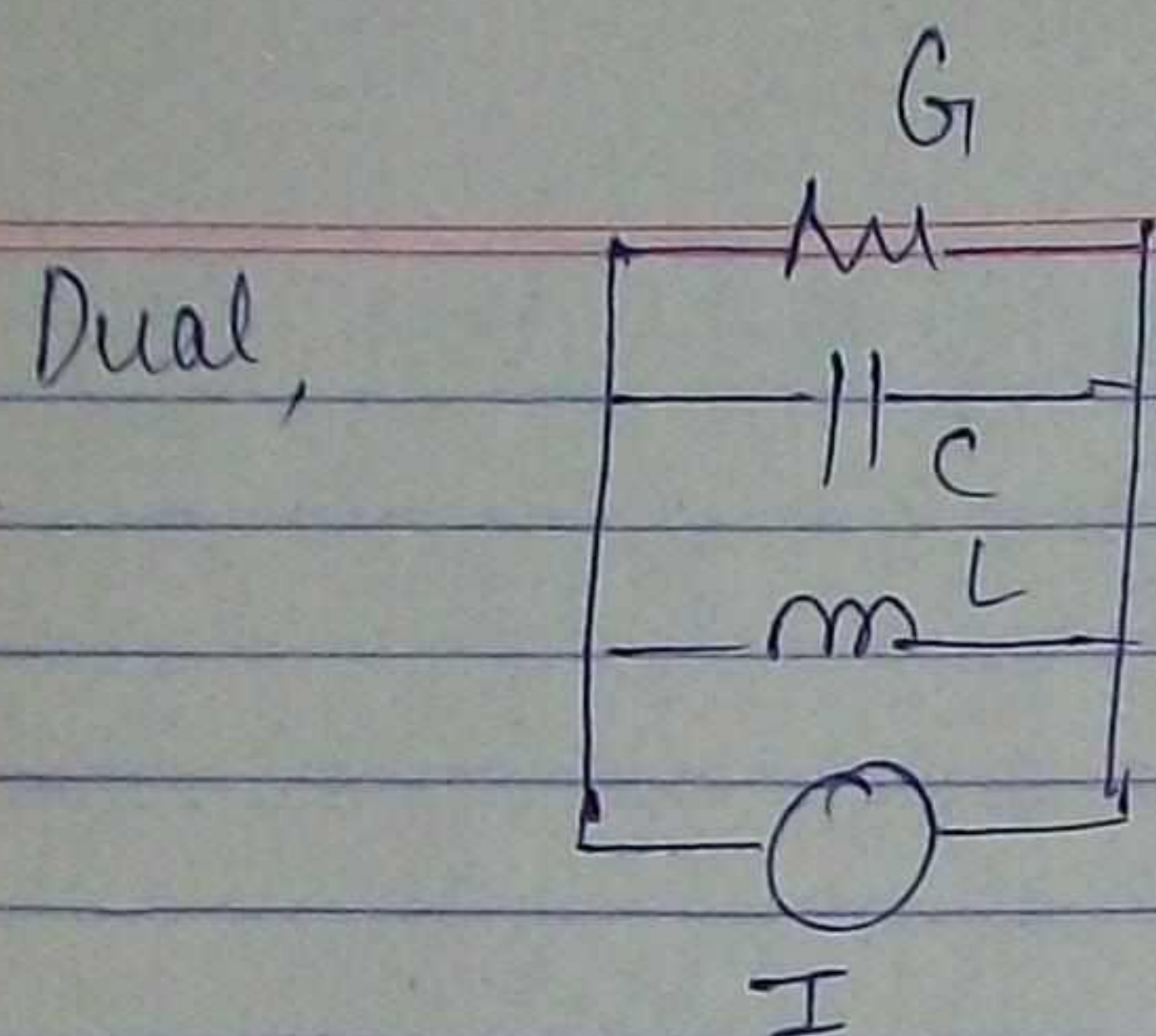
$\frac{di}{dt} \longleftrightarrow \frac{dv}{dt}$

$\int i dt \longleftrightarrow \int v dt$

Ex -



$V = iR + L \frac{di}{dt} + \frac{1}{C} \int i dt$ (KVL)

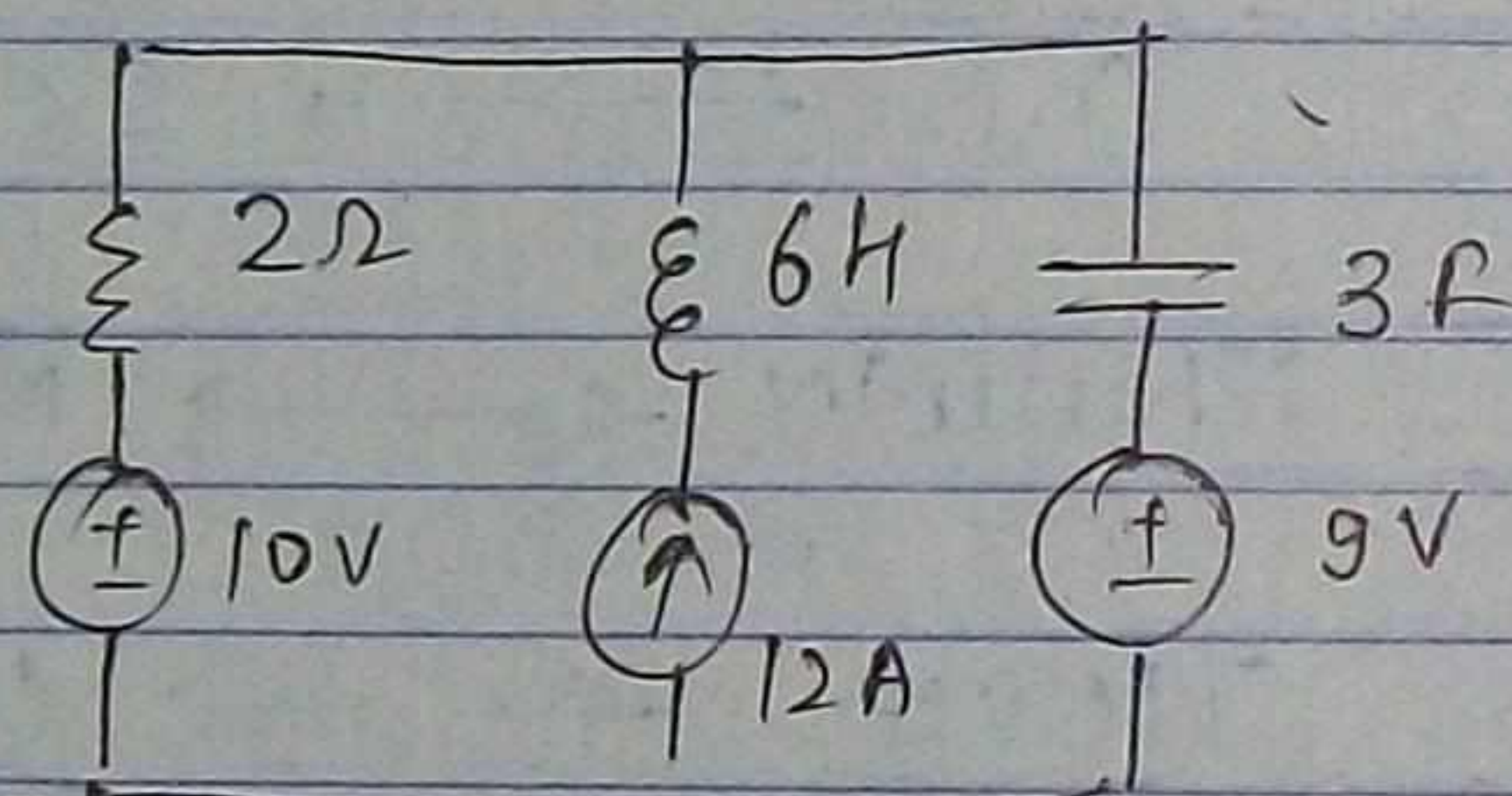


(kel)

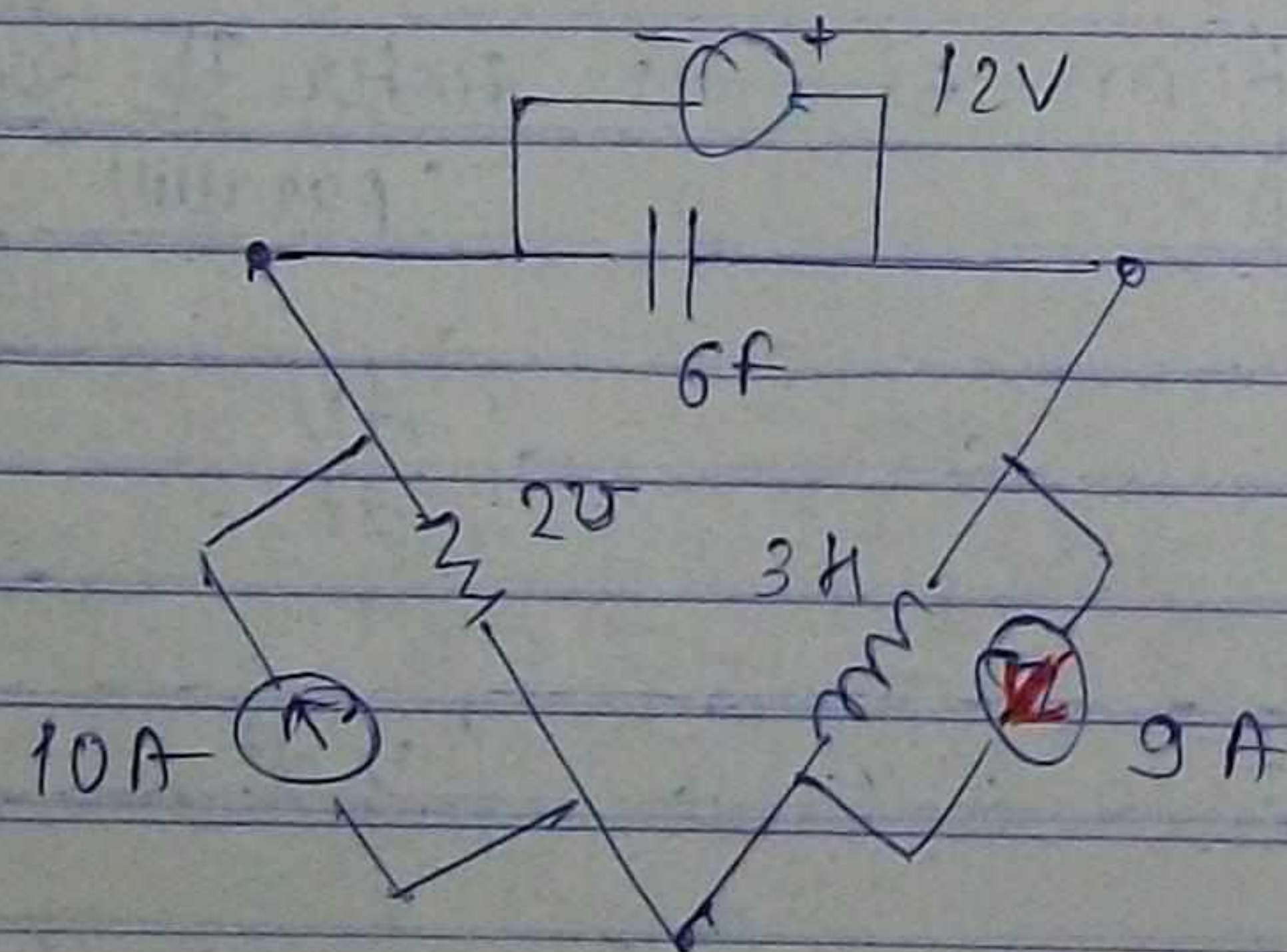
$$I = VG + C \frac{dv}{dt} + \frac{1}{L} \int v dt$$

- Duality does not mean that equivalent but it means that mathematical representation of the both N/w are identical.

Ques. Draw the dual

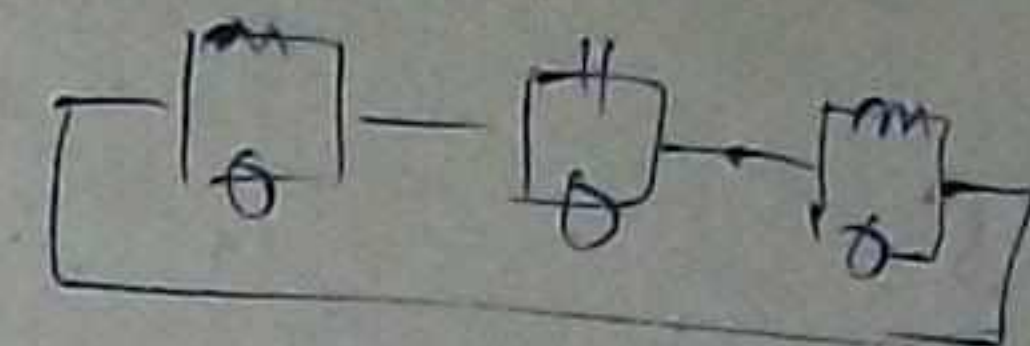


Soln:



NOTE-

- ① When voltage source drives a current in clockwise direction, arrow mark of the



current source is indicated towards respective node.

- ② When current source drives a current in clockwise direction positive sign is assign to respective node.

FILTERS

- Based on the components present in the filter the filters are classified as:

1. active filter
2. passive filter.

- Active filter consist of OPAMP and capacitor

- In the active filter it is possible to increase gain of the signal.

- Generally in the active filter inductor is not used since size of inductor is bulky and cost is high.

- Passive filter consist of series parallel LC sections (reactive N/w).

- Based on operating freq. filter are classified as:

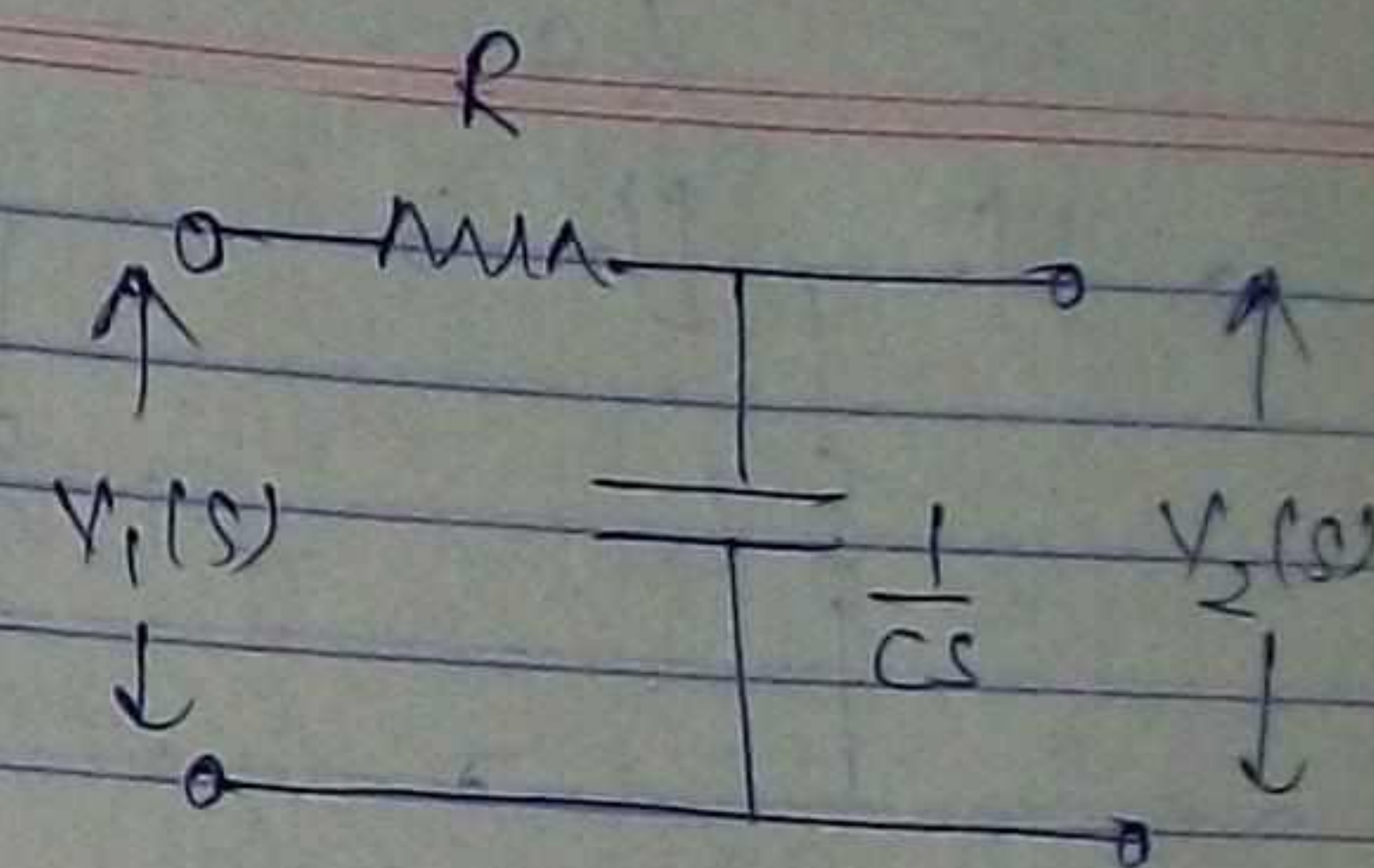
1. Low pass filter
2. High pass filter
3. Band pass filter
4. Band elimination filter (Band stop filter)
5. All pass filter.

① LOW PASS FILTER - $X_L = 2\pi fL$ $X_L \propto f$
 $X_C = \frac{1}{2\pi fC}$ $X_C \propto 1/f$

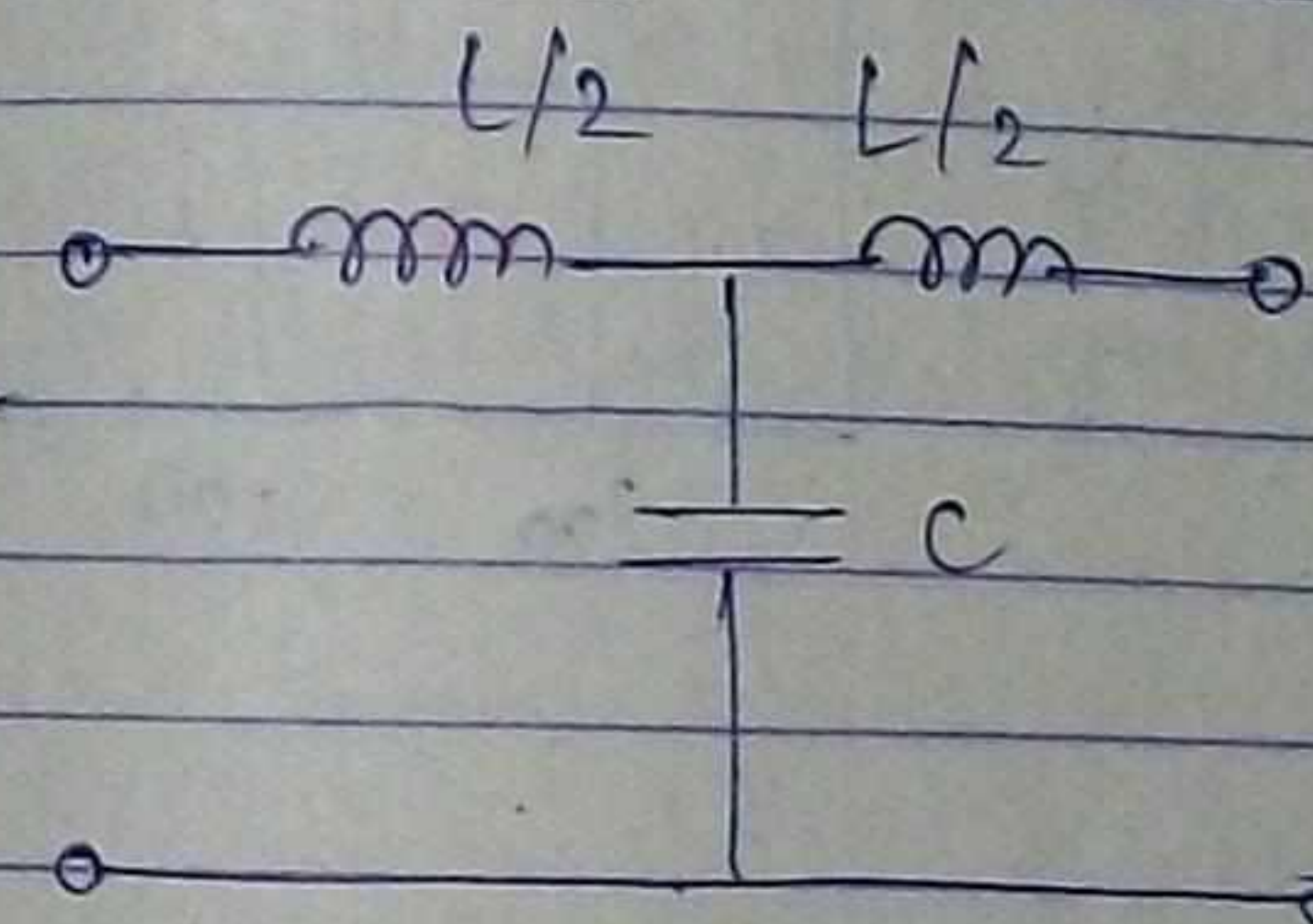
FIRST ORDER -

$$V_2(s) = V_1(s) \times \frac{1/c_s}{R + 1/c_s}$$

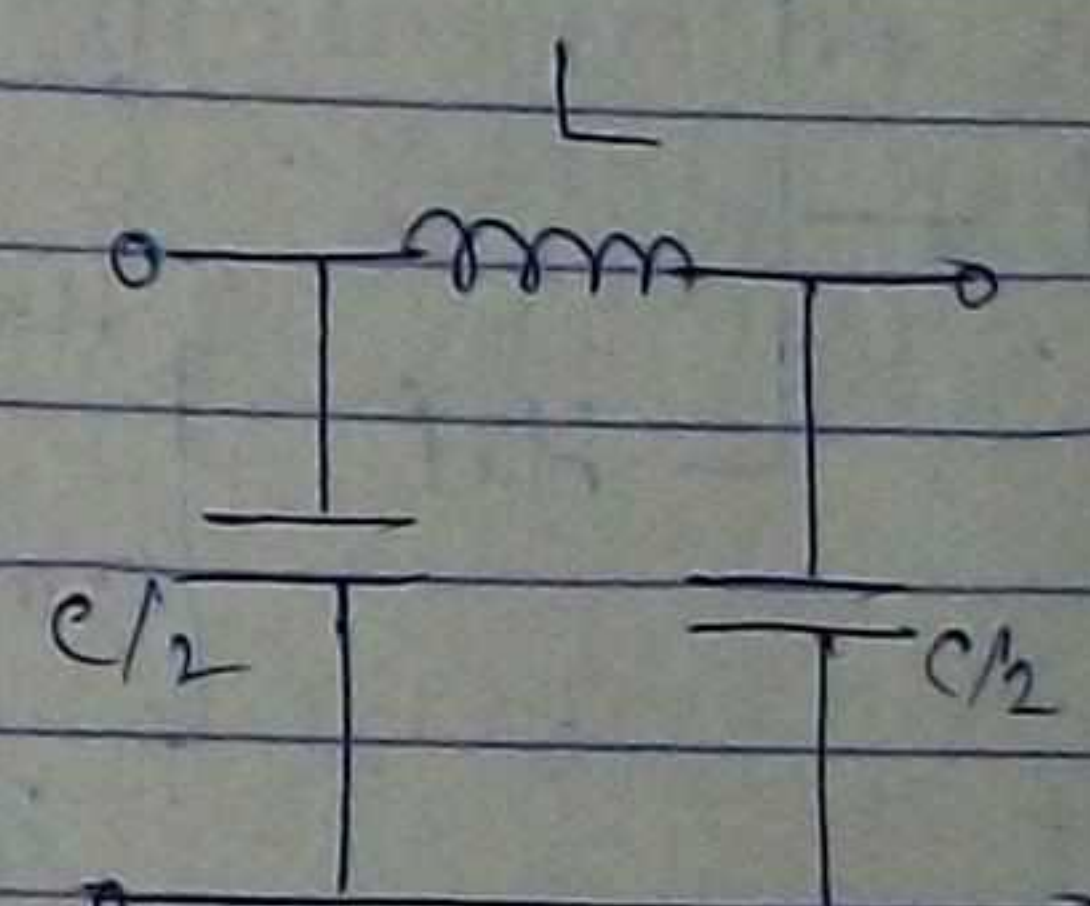
$$\frac{V_2(s)}{V_1(s)} = \frac{1}{RCs + 1}$$



SECOND ORDER -



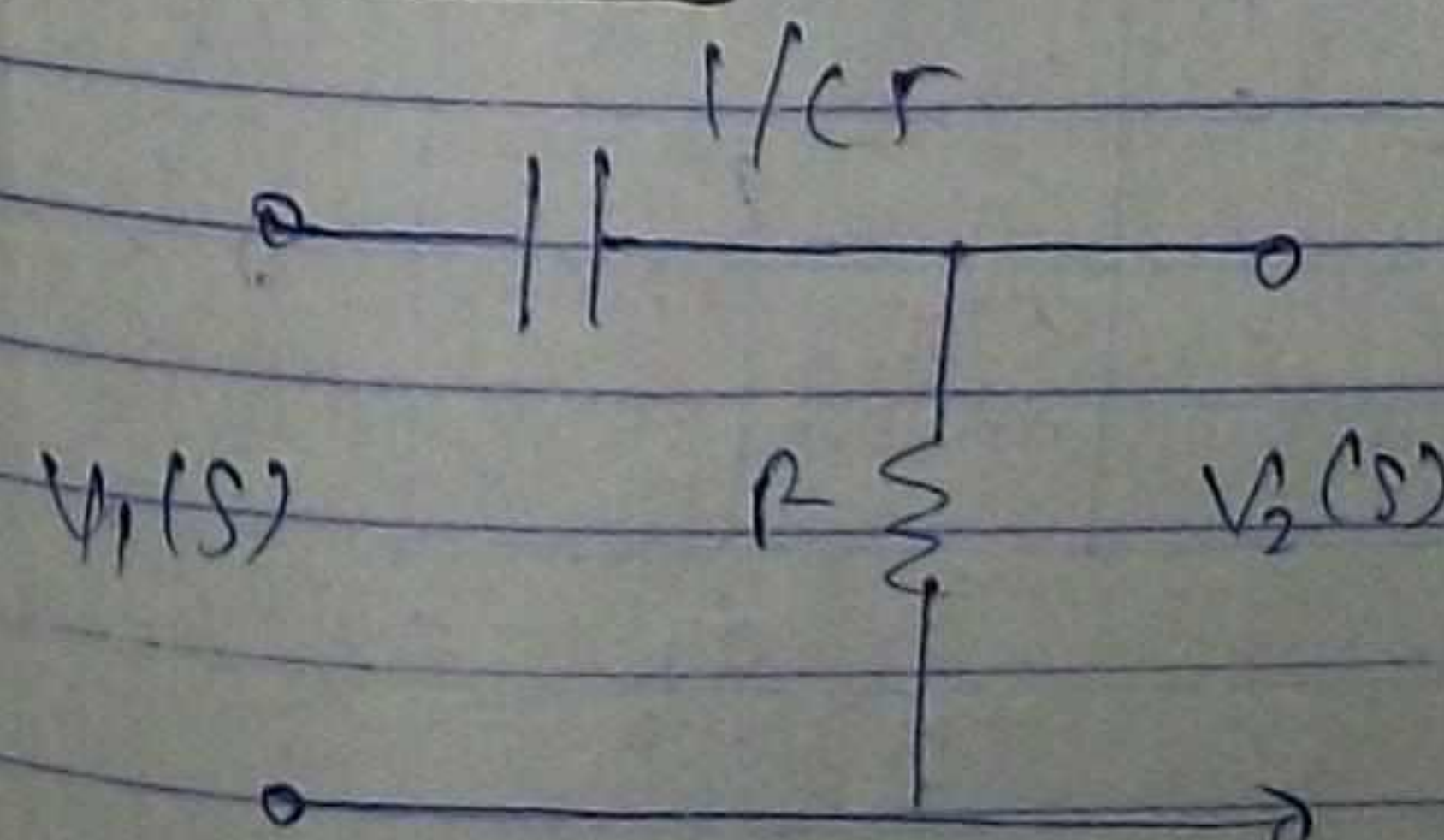
T-N/w



pi-N/w

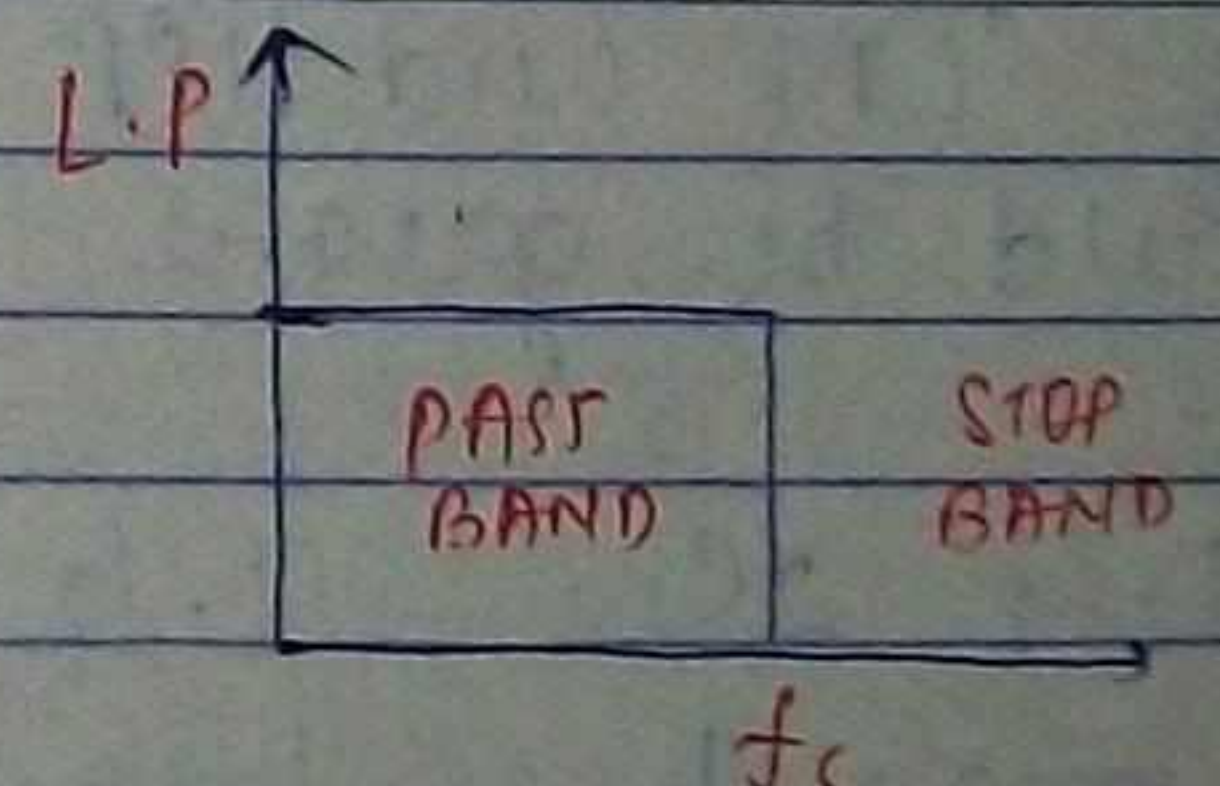
② HIGH PASS FILTER -

FIRST ORDER -

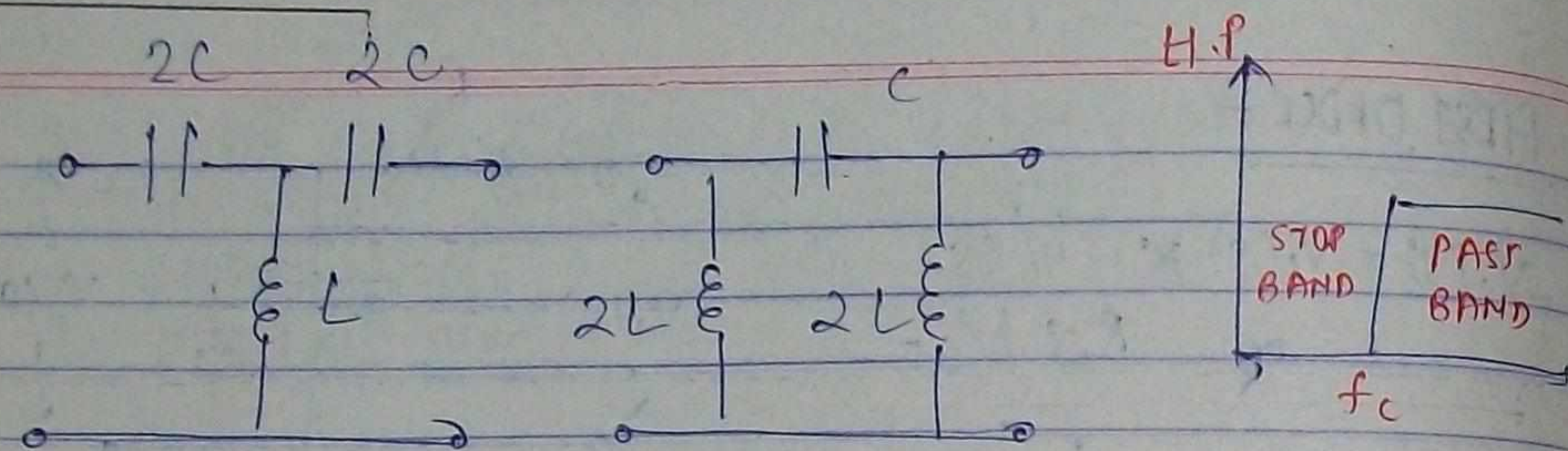


$$V_2(s) = V_1(s) = \frac{R}{R + 1/c_s}$$

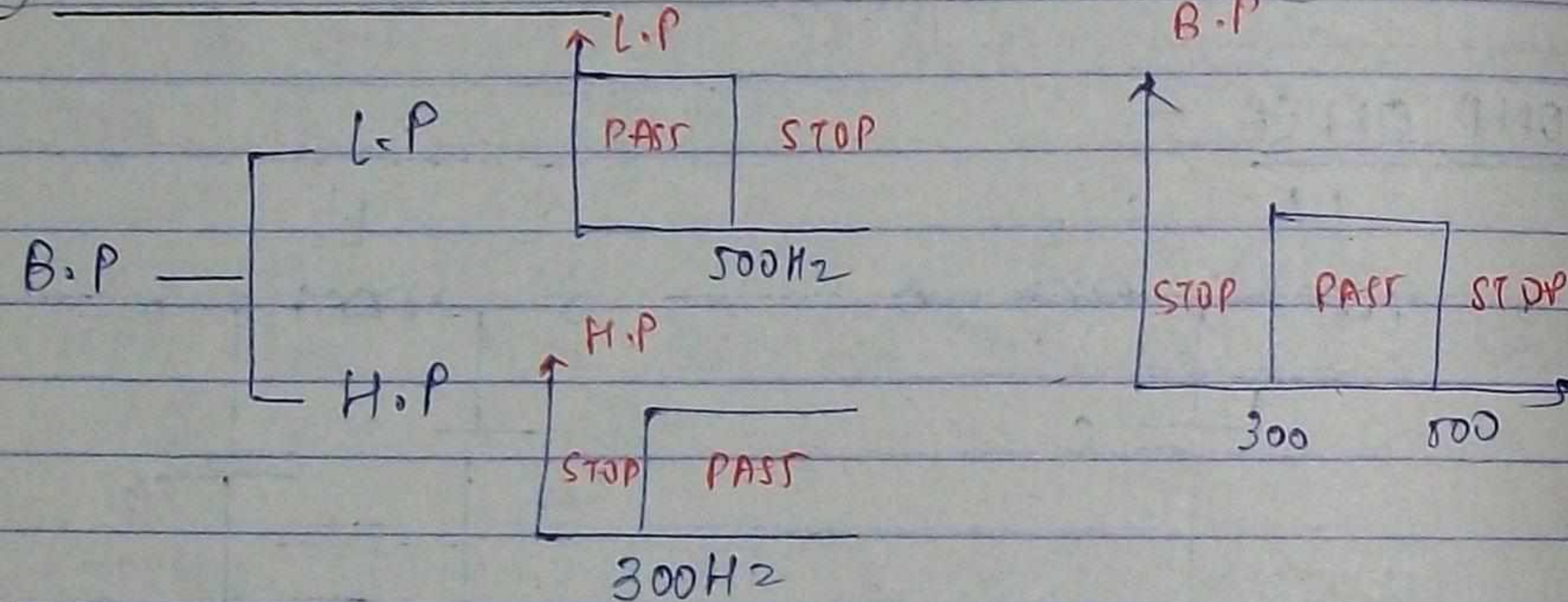
$$\frac{V_2(s)}{V_1(s)} = \frac{RCs}{RCs + 1}$$



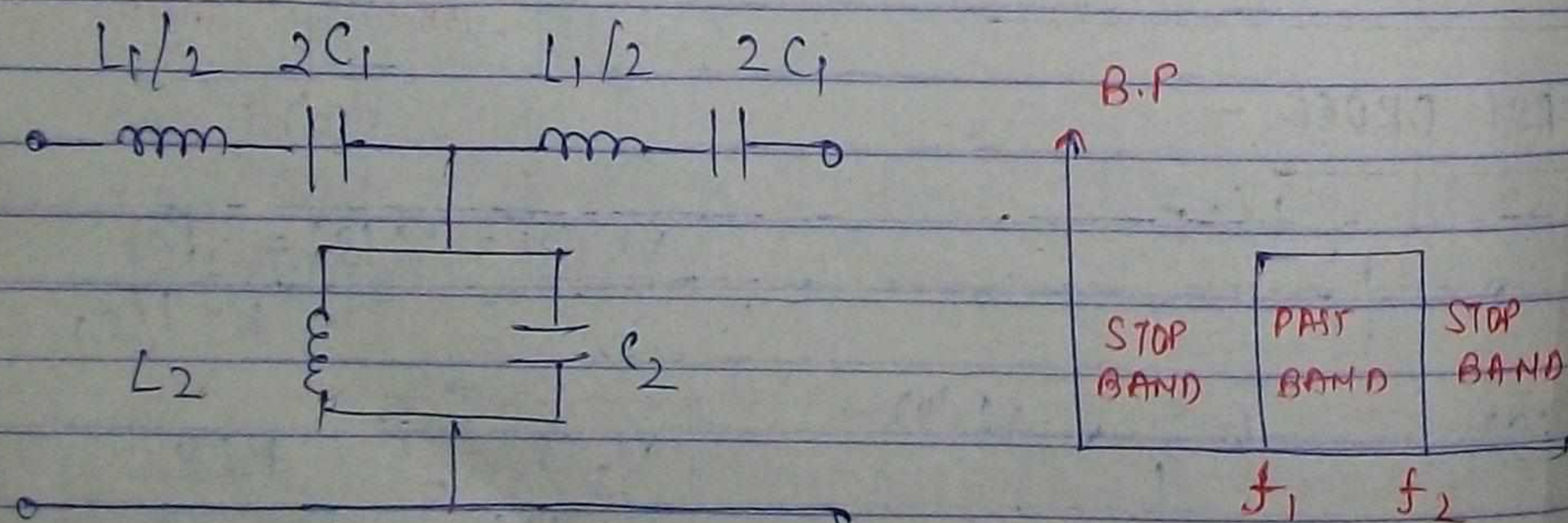
SECOND ORDER -



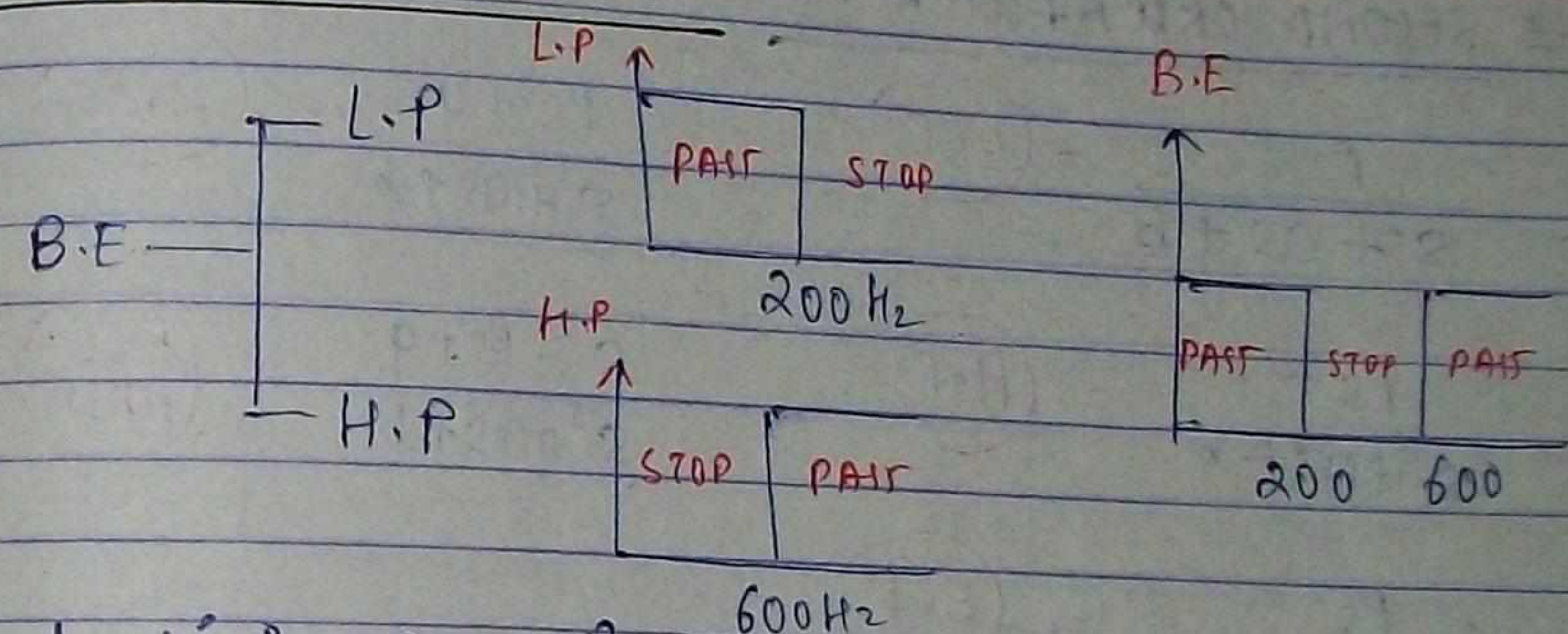
② BAND PASS FILTER -



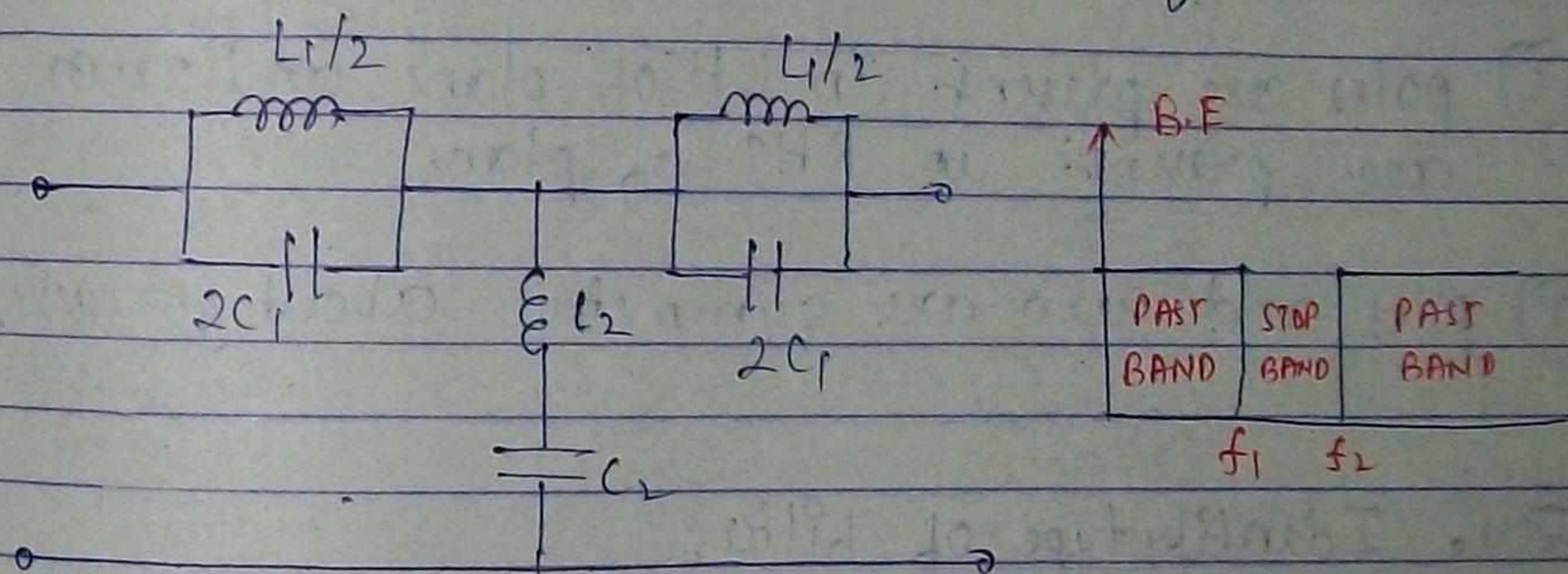
Band pass filter can be obtained with a combⁿ of LPF and HPF and cutoff freq of LPF should be greater than f_c of HPF



④ BAND ELIMINATE FILTER -



Band elimination filter is obtained with a combⁿ of LPF and HPF and cutoff freq of HPF should be greater than f_c of LPF.



⇒ FIRST ORDER FILTER T.F -

$$\frac{1}{1 + \tau s} = \text{L.P} \quad \frac{\tau s}{1 + \tau s} = \text{H.P} \quad \frac{1 - \tau s}{1 + \tau s} = \text{ALL PASS}$$

$$\tau = RC$$

⇒ SECOND ORDER FILTER T.F →

$$\frac{P}{s^2 + as + b} = \text{L.P.}$$

$$\frac{Ps^2 + q}{s^2 + as + b} = \text{B.E.}$$

$$\frac{Ps^2}{s^2 + as + b} = \text{H.P.}$$

$$\frac{s^2 - Ps + q}{s^2 + as + b} = \text{ALL PASS}$$

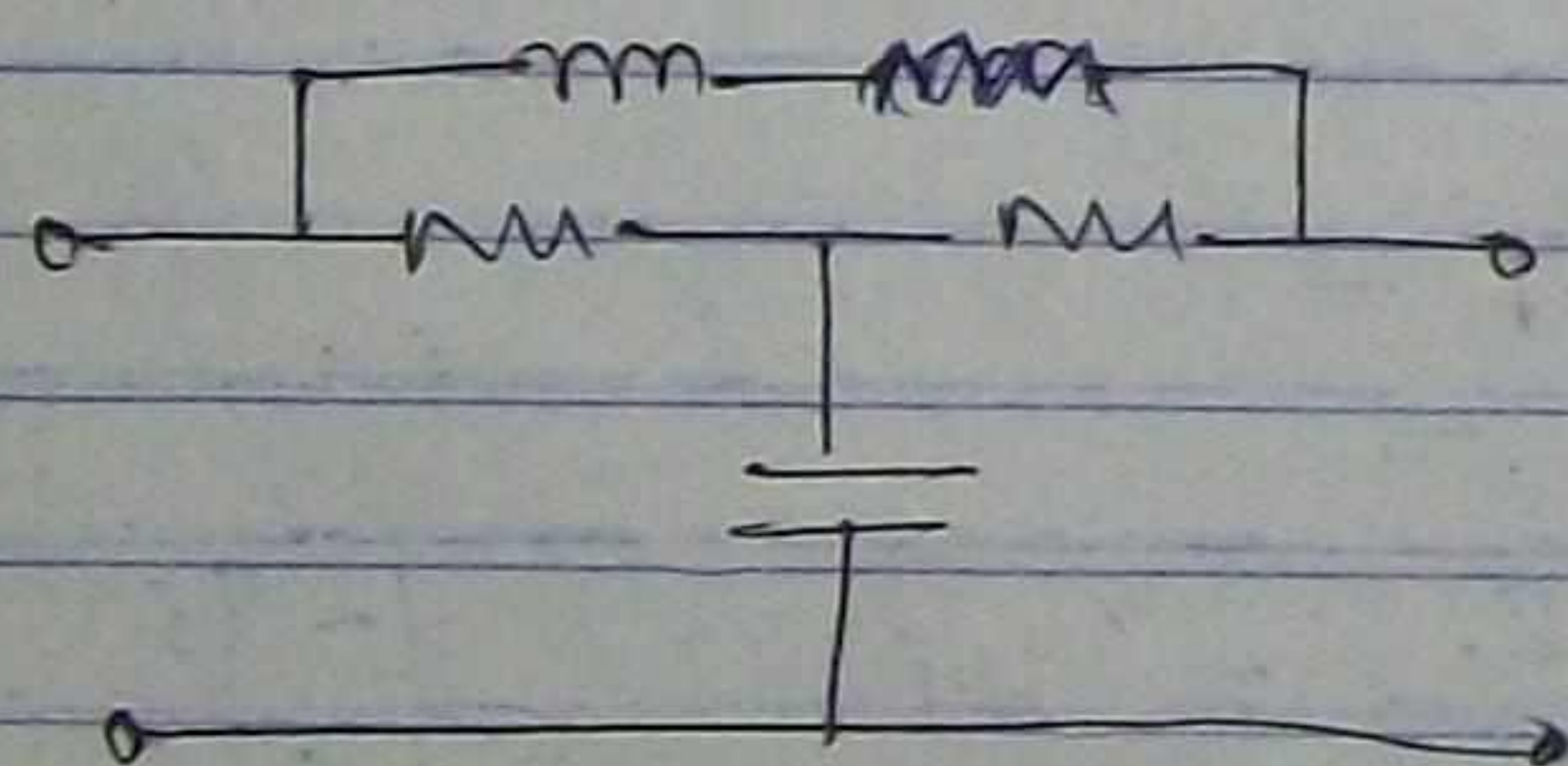
$$\frac{Ps}{s^2 + as + b} = \text{B.P.}$$

— In all pass filter (1st order) —

① poles are present in LHS of plane and zeros are present in RHS of plane.

② poles and zero are symmetry about jw axis.

Ques. Identify type of filter:



Solⁿ: at $f = 0$

$$X_C = \infty \quad C \rightarrow \text{O.C.}$$

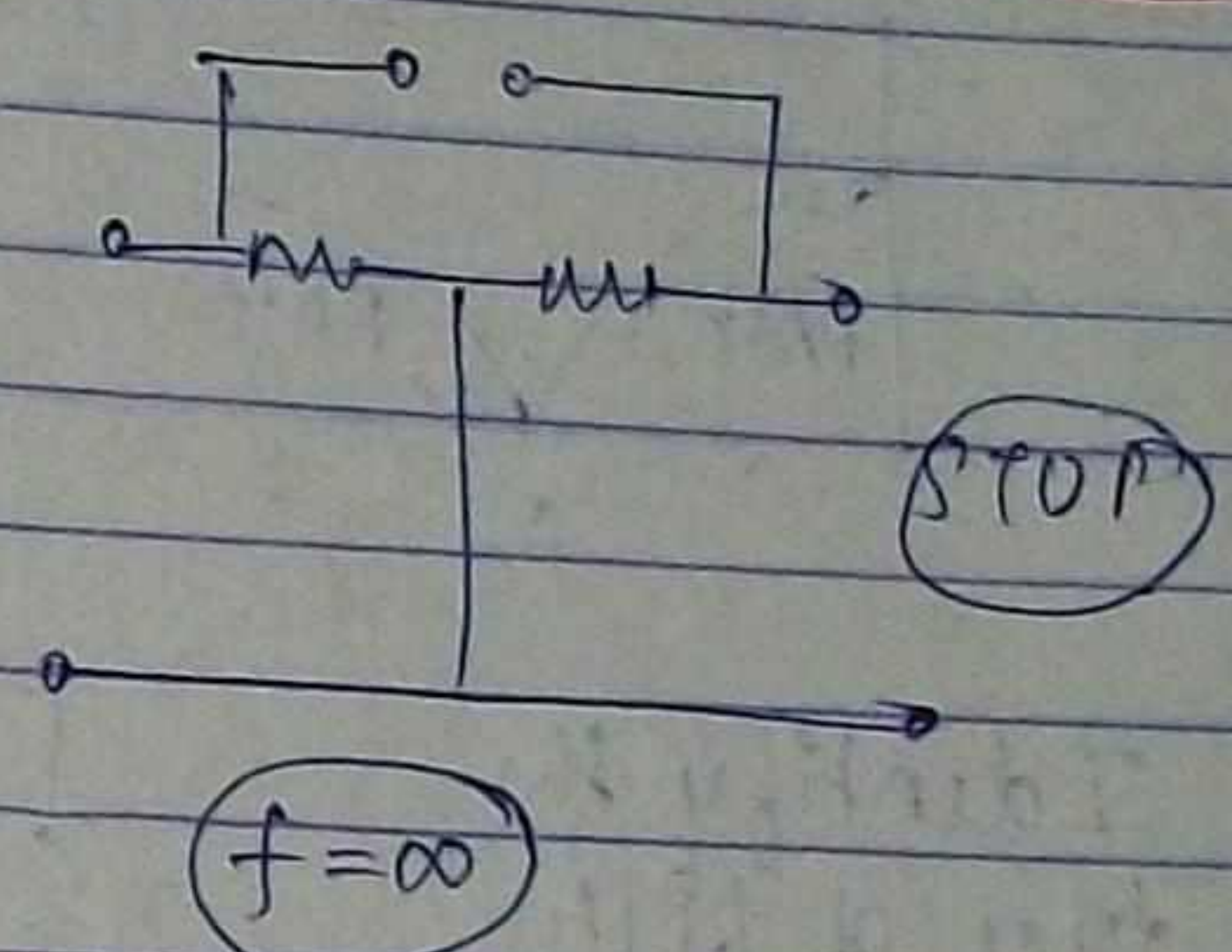
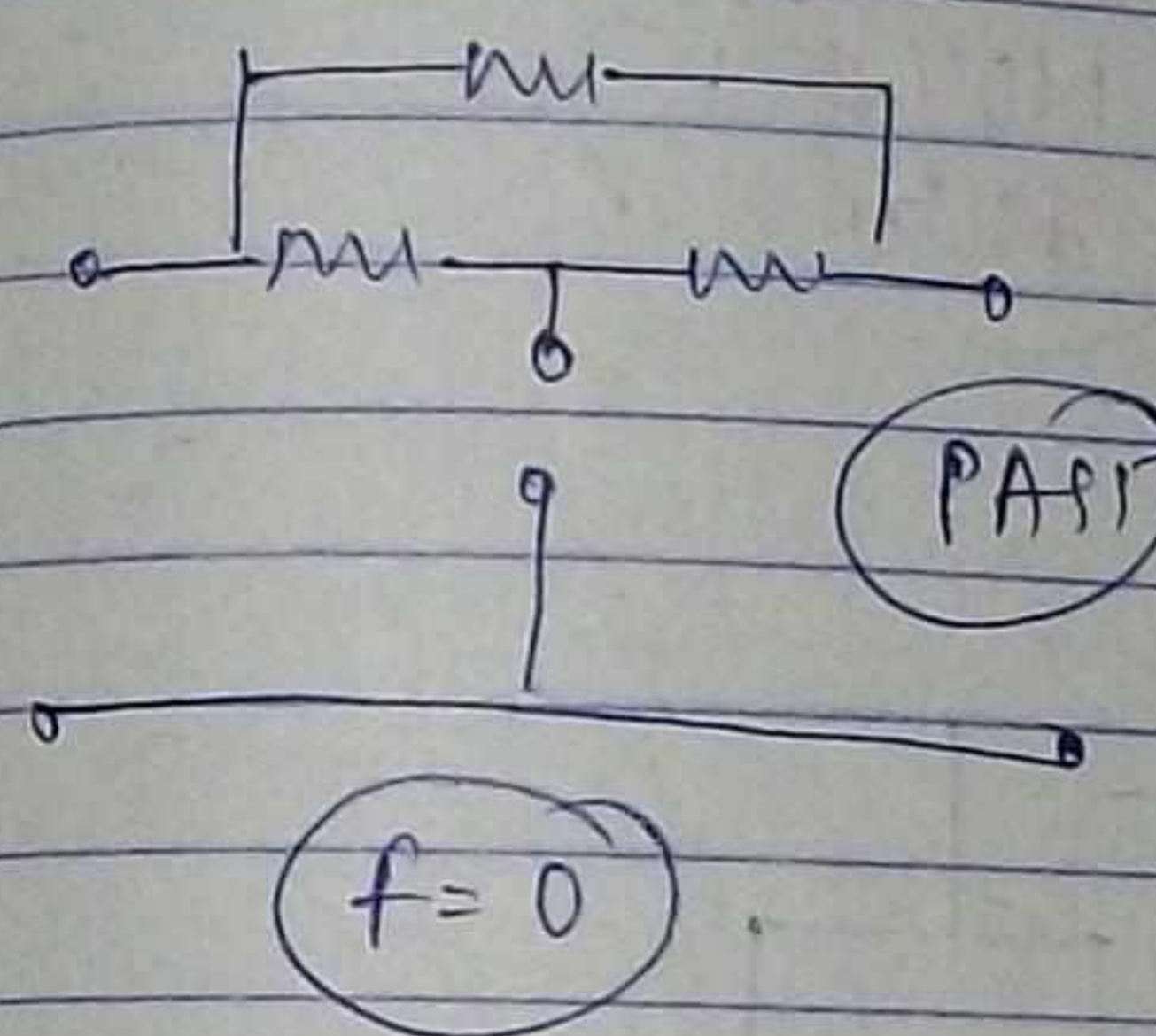
$$X_L = 0 \quad L \rightarrow \text{S.C.}$$

at $f = \infty$

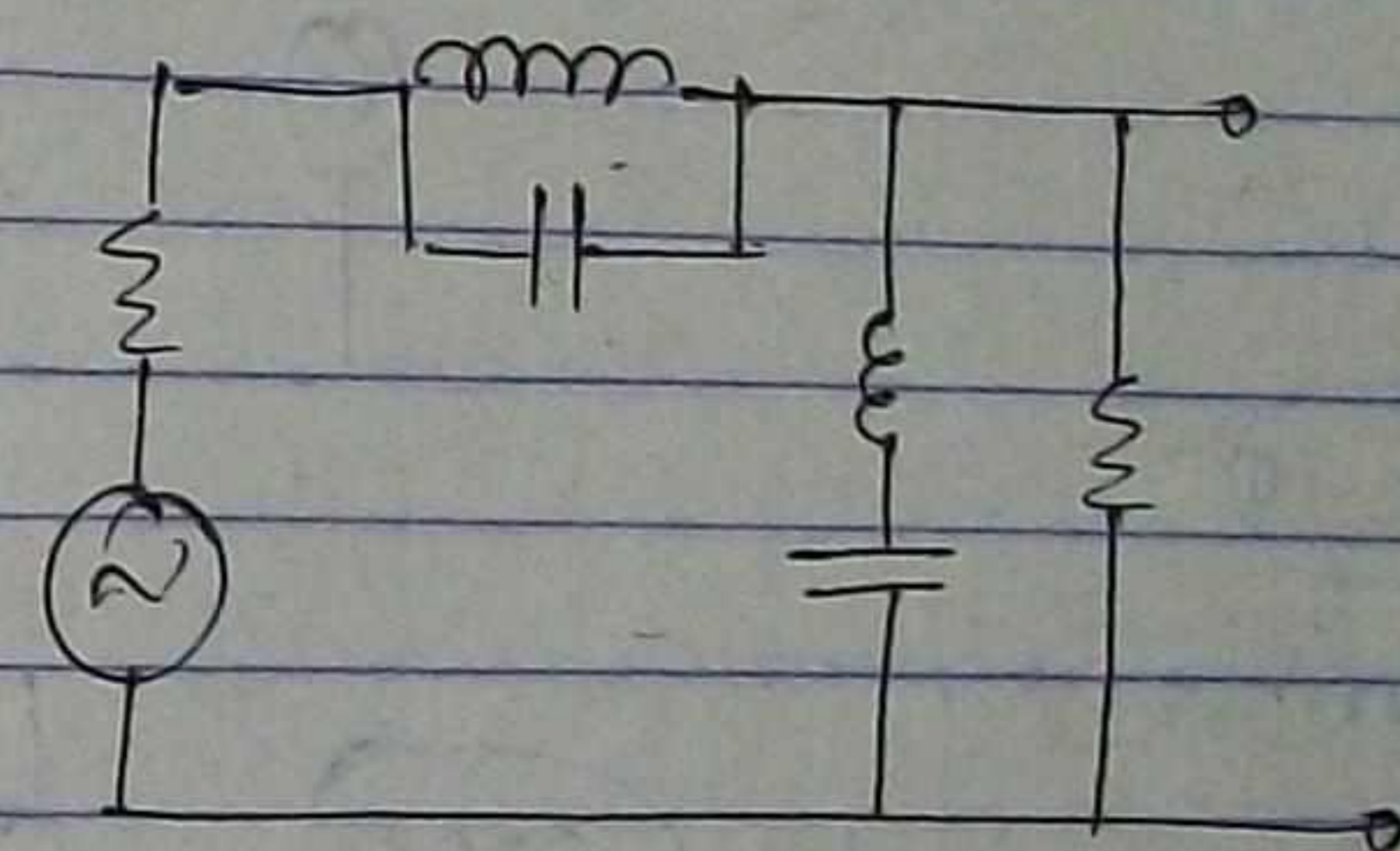
$$X_C = 0 \quad C \rightarrow \text{S.C.}$$

$$X_L = \infty \quad L \rightarrow \text{O.C.}$$

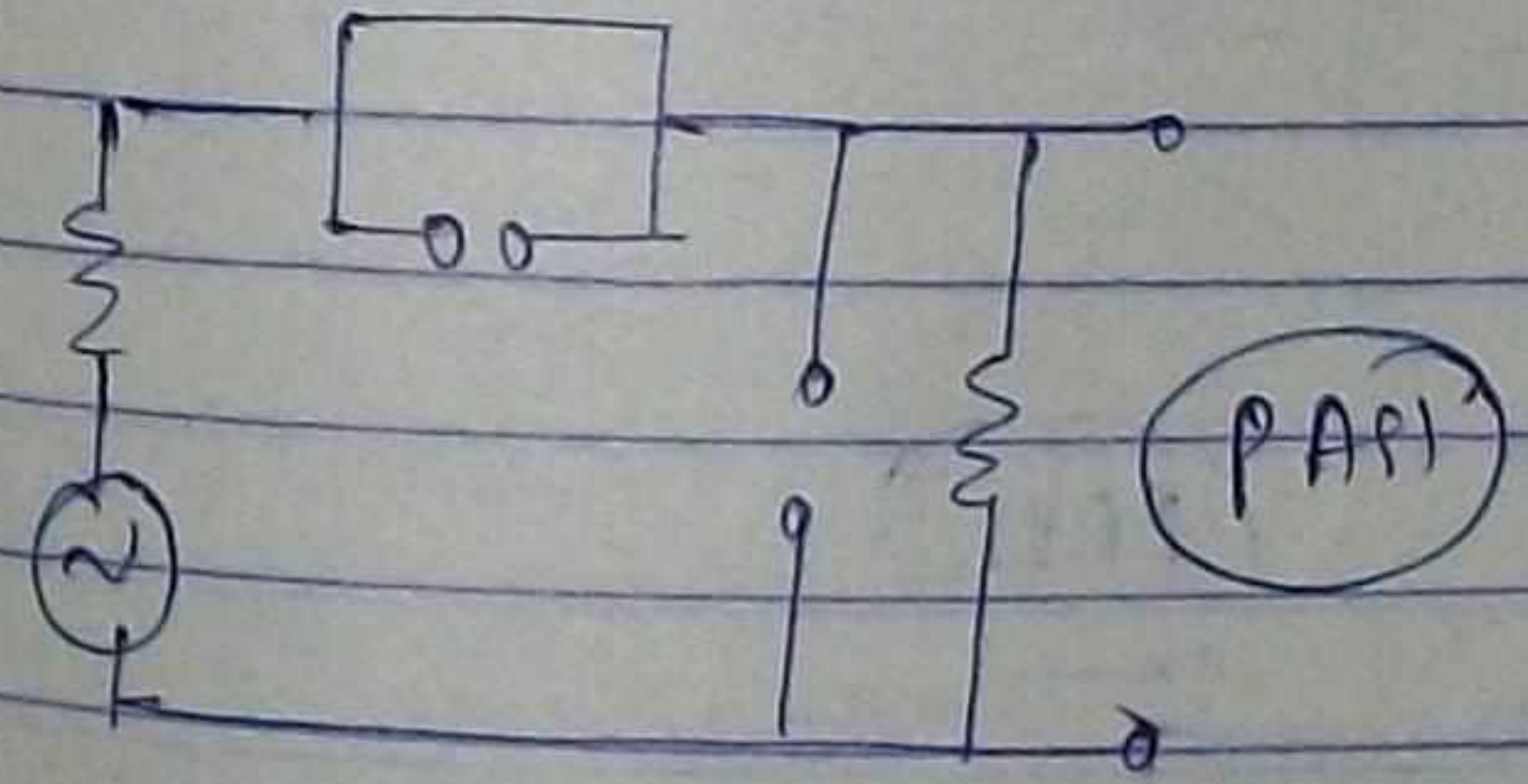
LOW PASS FILTER



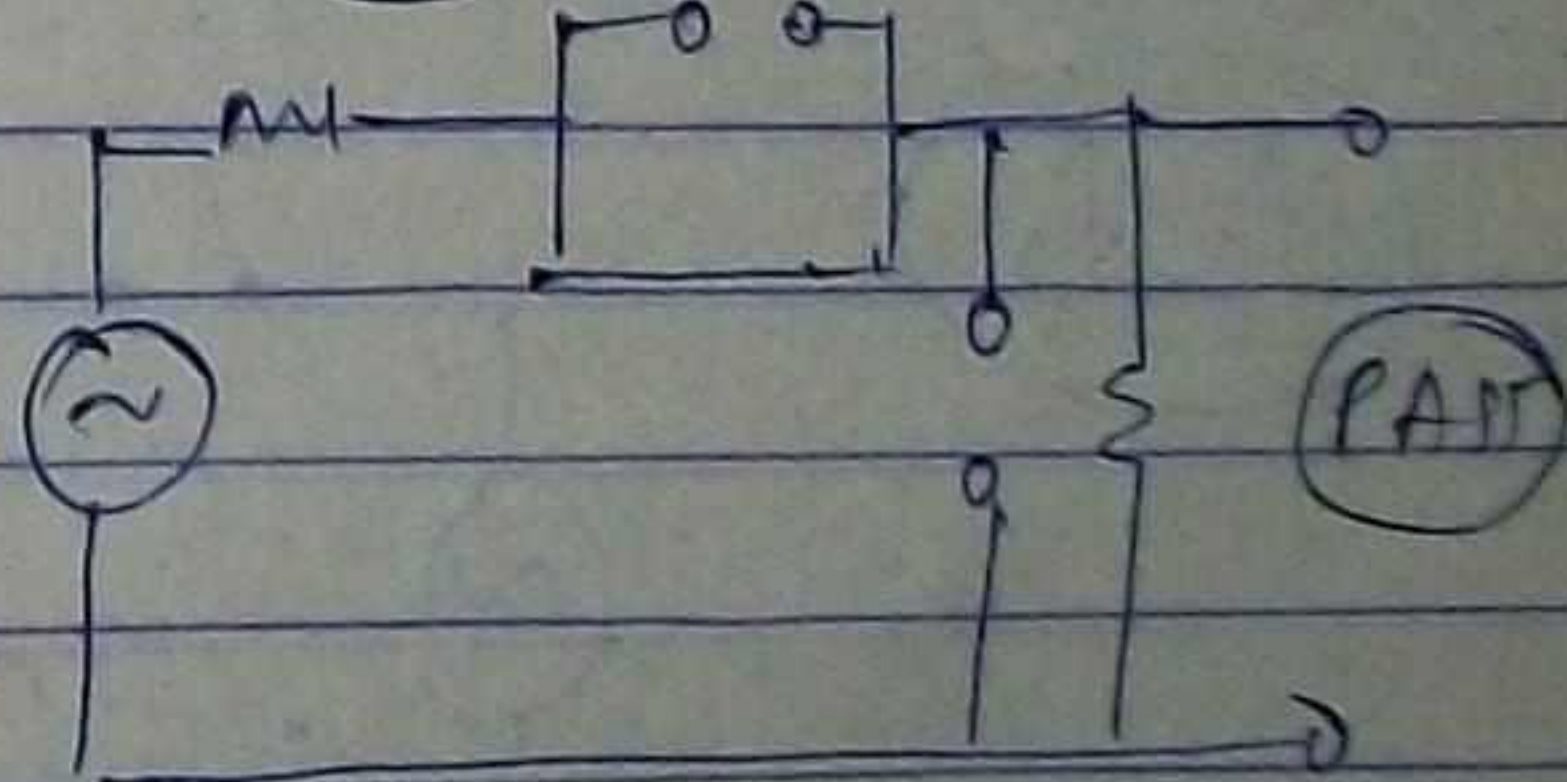
Ques. Identify type of filter



$f = 0$



$f = \infty$



$f = f_0$

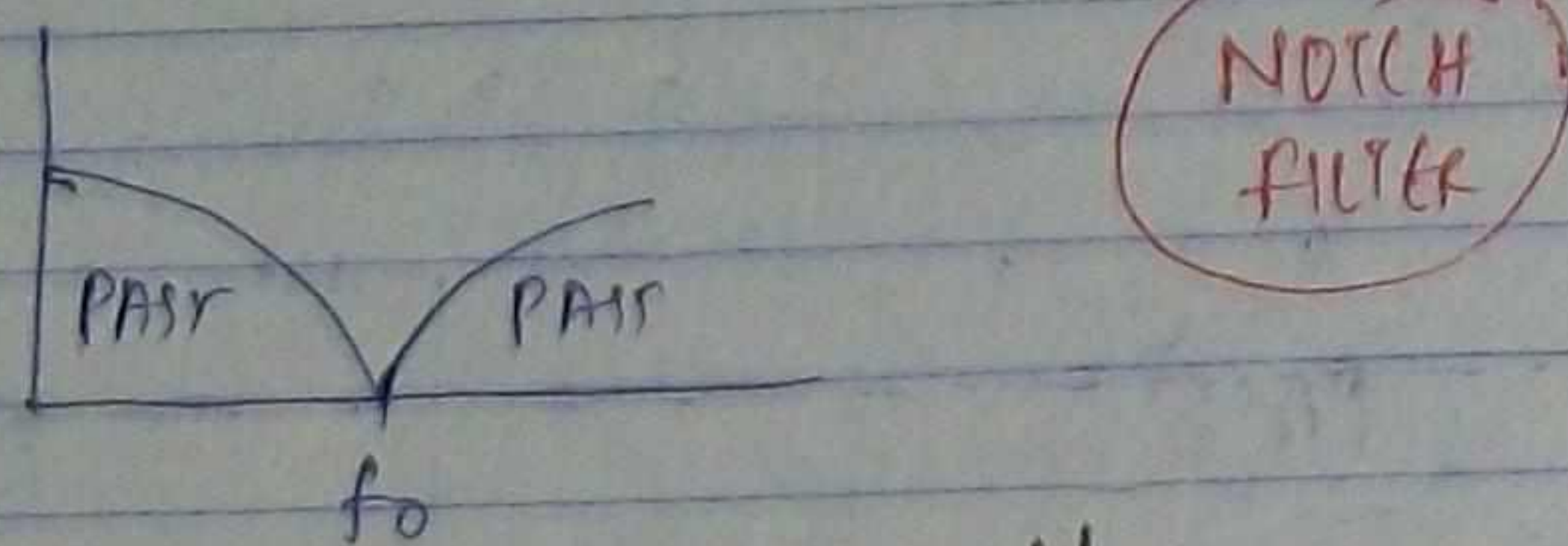
$$Z_{in} = \infty$$

$$Z = R$$

STOP

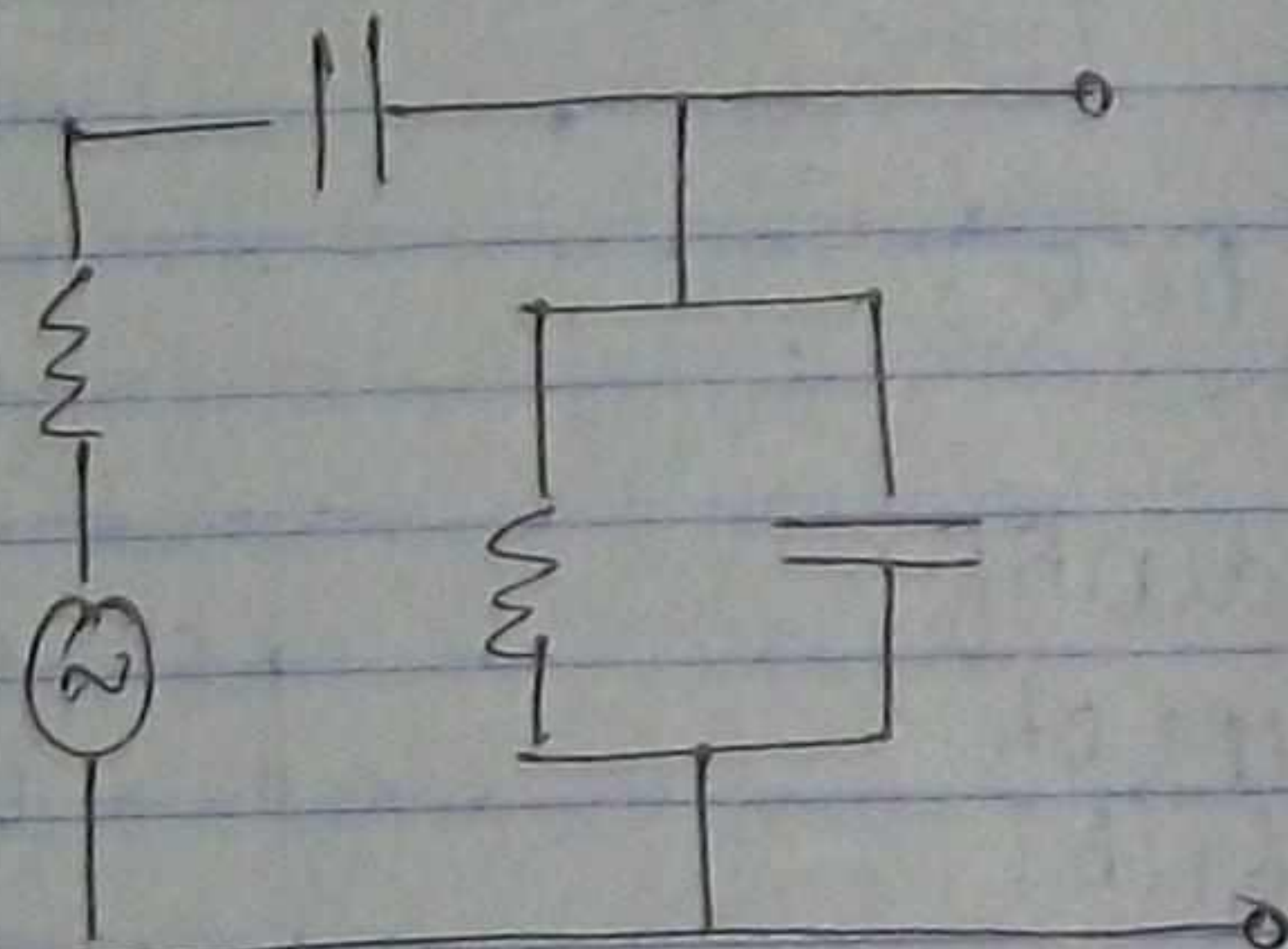
BAND ELIMINATION FILTER

— when band elimination filter eliminates only few freq. then it is also called as NOTCH filter



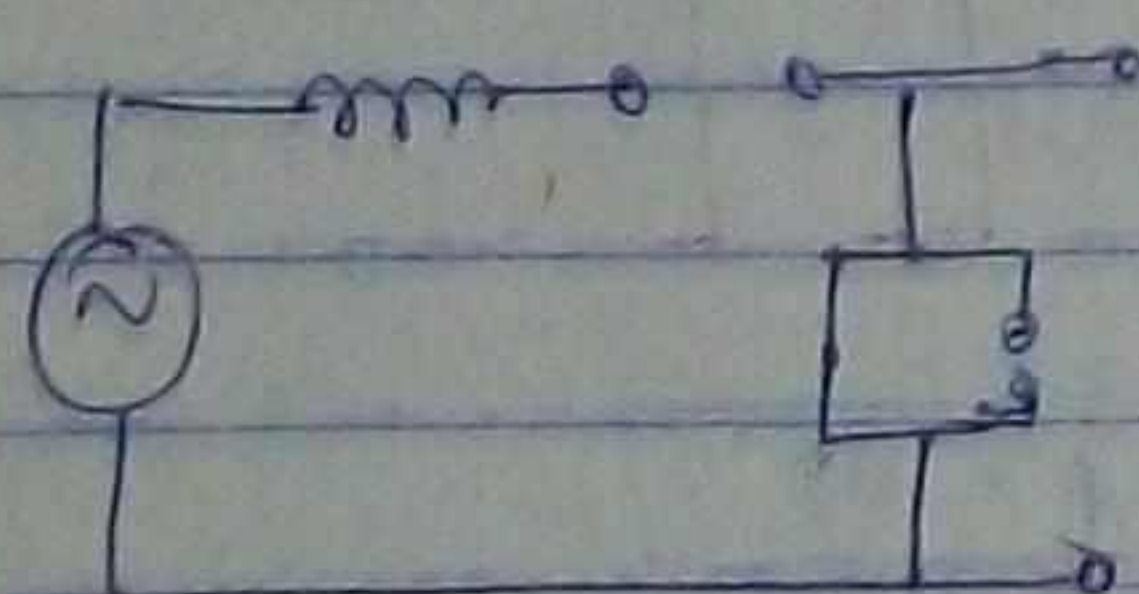
NOTCH
FILTER

Ques. Identify the type of filter for N/W



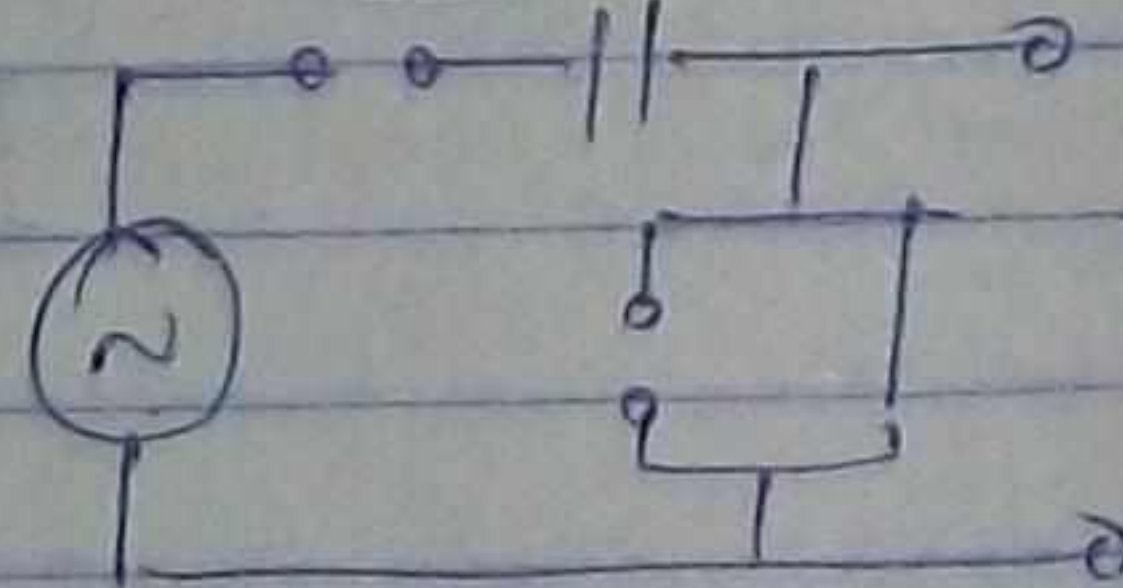
Solⁿs

$f = 0$



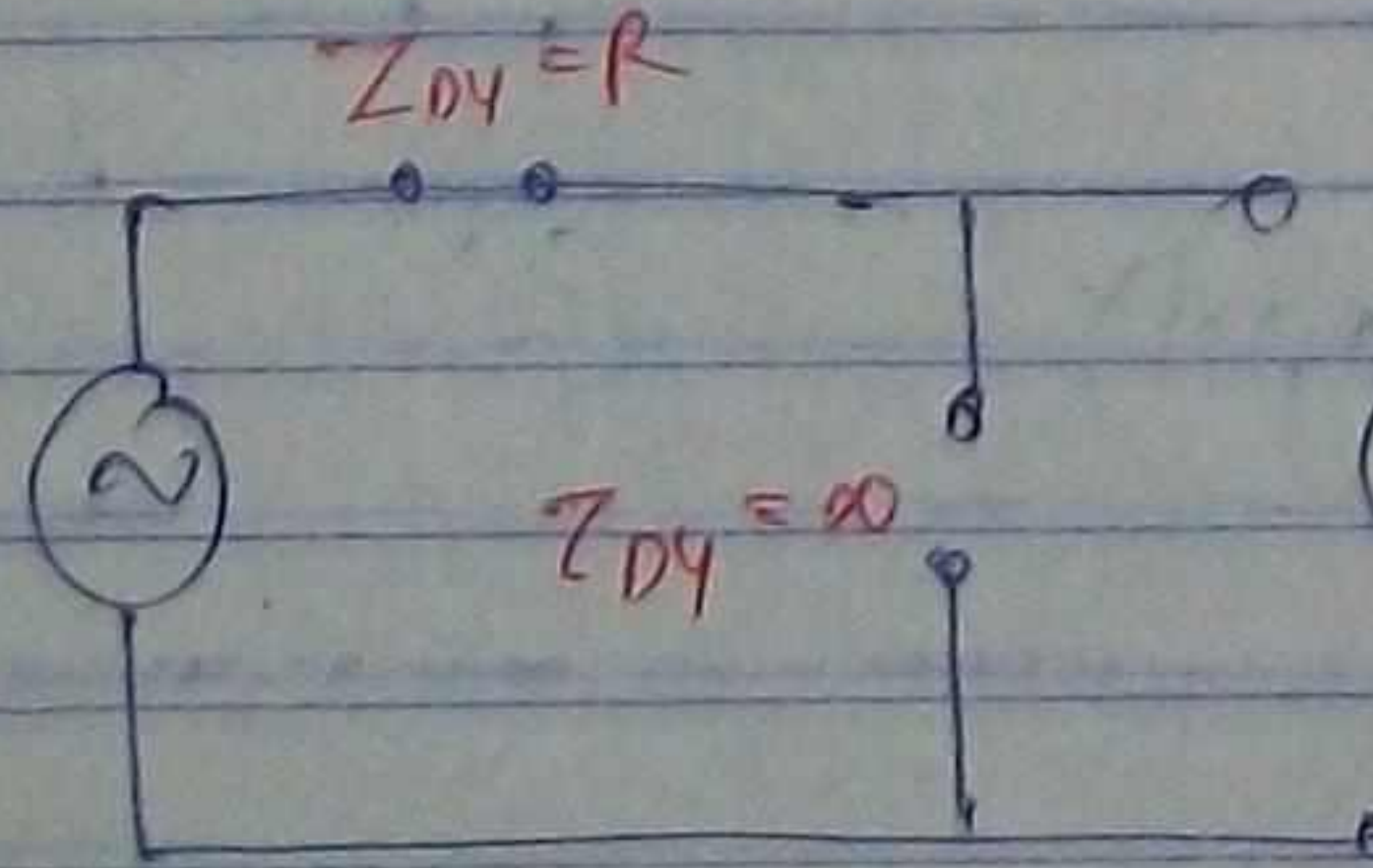
STOP

$f = \infty$



STOP

$f = f_o$

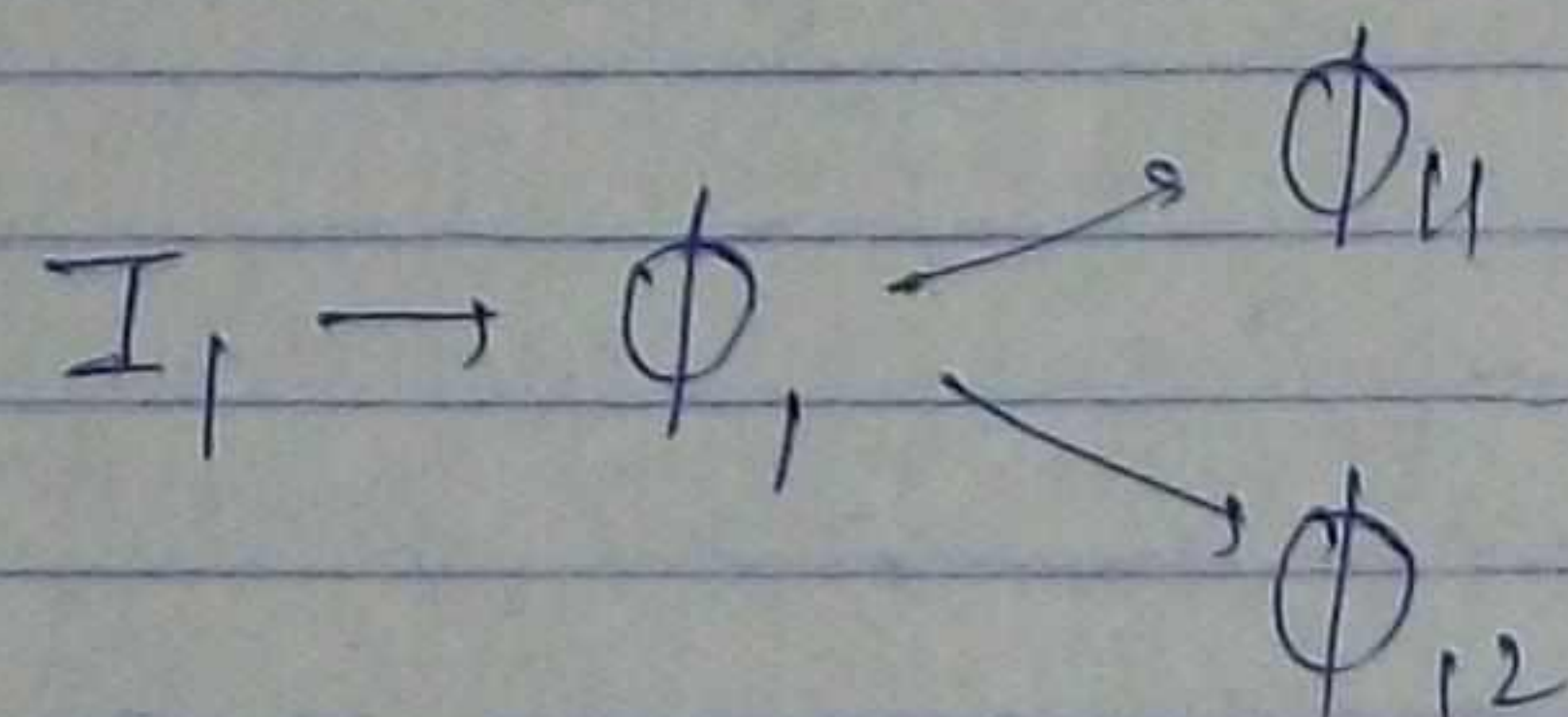
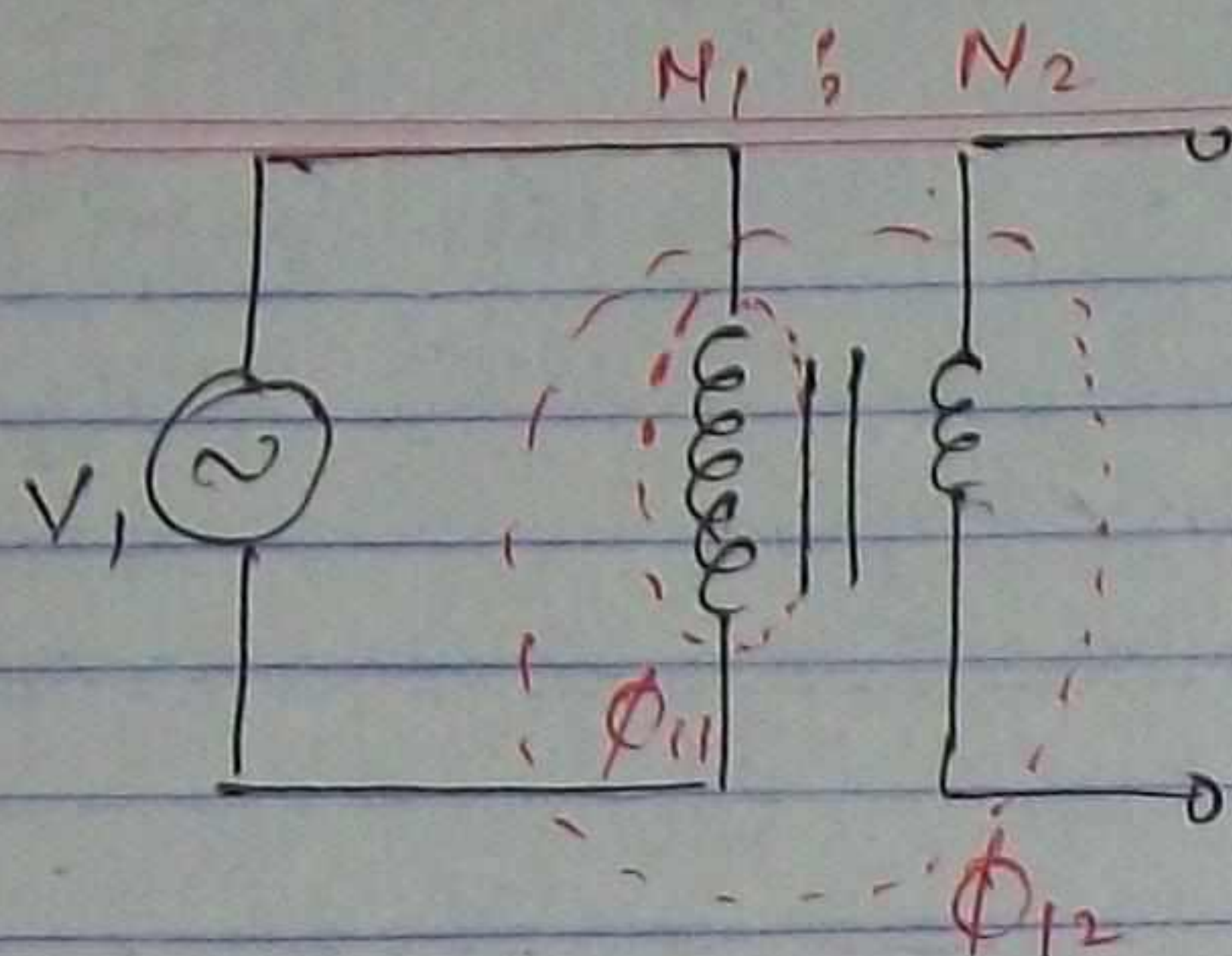


PASS

BAND
PASS
FILTER

MAGNETIC COUPLED CIRCUIT

CASE (1):



LEAKAGE FLUX

USEFUL FLUX OR MUTUAL FLUX

$$e_1 \propto \frac{d\Phi_1}{dt}$$

$$e_1 = -N_1 \frac{d\Phi_1}{dt}$$

$$e_1 = -N_1 \frac{d\Phi_1}{di_1} \cdot \frac{di_1}{dt}$$

$$e_1 = -L_1 \frac{di_1}{dt}$$

SELF INDUCED EMF

SELF INDUCTANCE

$$L_1 = N_1 \frac{d\Phi_1}{di_1}$$

$$e_2 \propto \frac{d\Phi_{12}}{dt}$$

$$e_2 = -N_2 \frac{d\Phi_{12}}{dt}$$

$$e_2 = -N_2 \frac{d\Phi_{12}}{di_1} \cdot \frac{di_1}{dt}$$

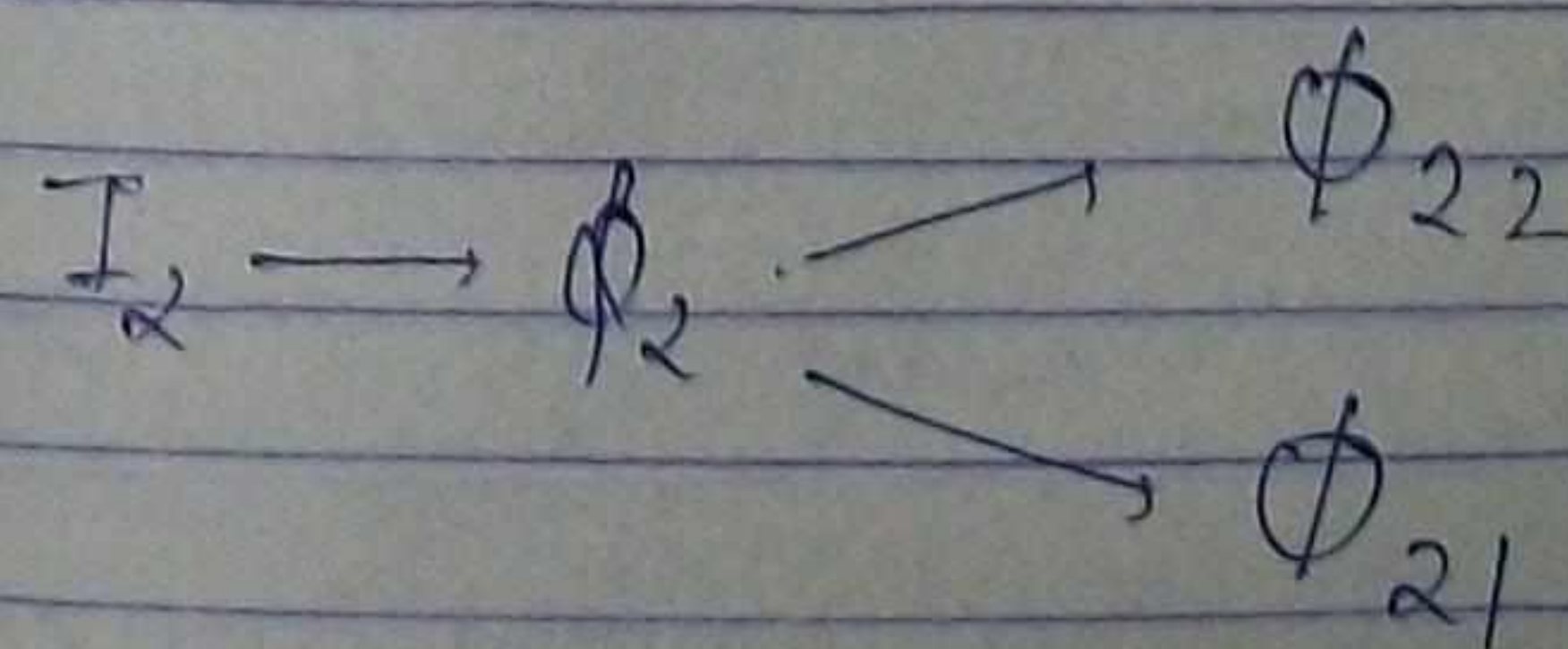
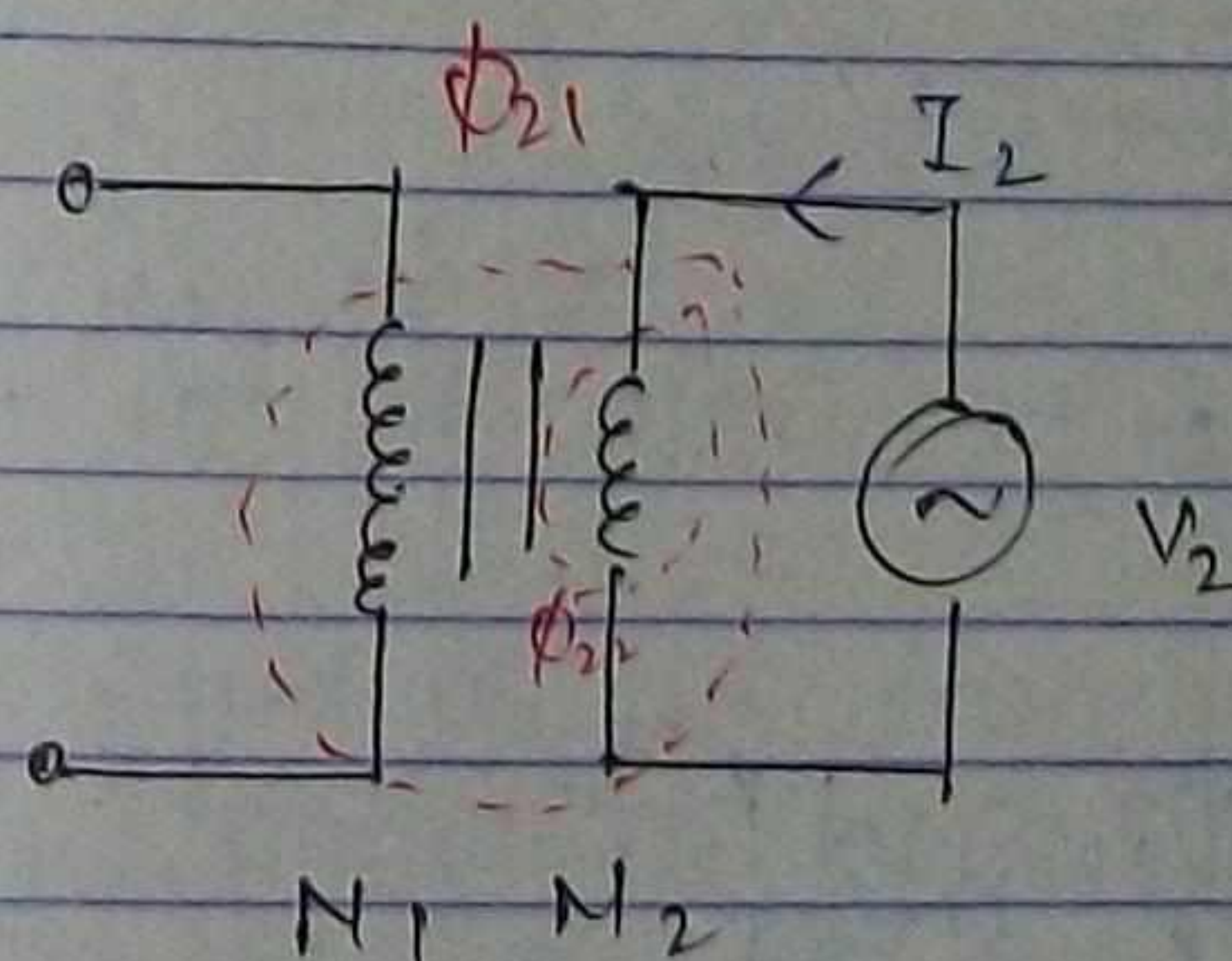
$$M_{21} = N_2 \frac{d\Phi_{12}}{di_1}$$

MUTUAL INDⁿ OF 2 INDUCTOR WRT 1st

$$e_2 = -M_{21} \frac{di_1}{dt}$$

MUTUALLY INDUCED EMF

CASE (2):



LEAKAGE FLUX

USEFUL FLUX OR MUTUAL FLUX

$$e_2 \propto \frac{d\phi_2}{dt}$$

$$e_2 = -N_2 \frac{d\phi_2}{dt}$$

$$e_2 = -N_2 \frac{d\phi_2}{di_2} \cdot \frac{di_2}{dt}$$

$$e_2 = -L_2 \frac{di_2}{dt}$$

SELF INDUCED
EMF

$$e_1 \propto \frac{d\phi_{21}}{dt}$$

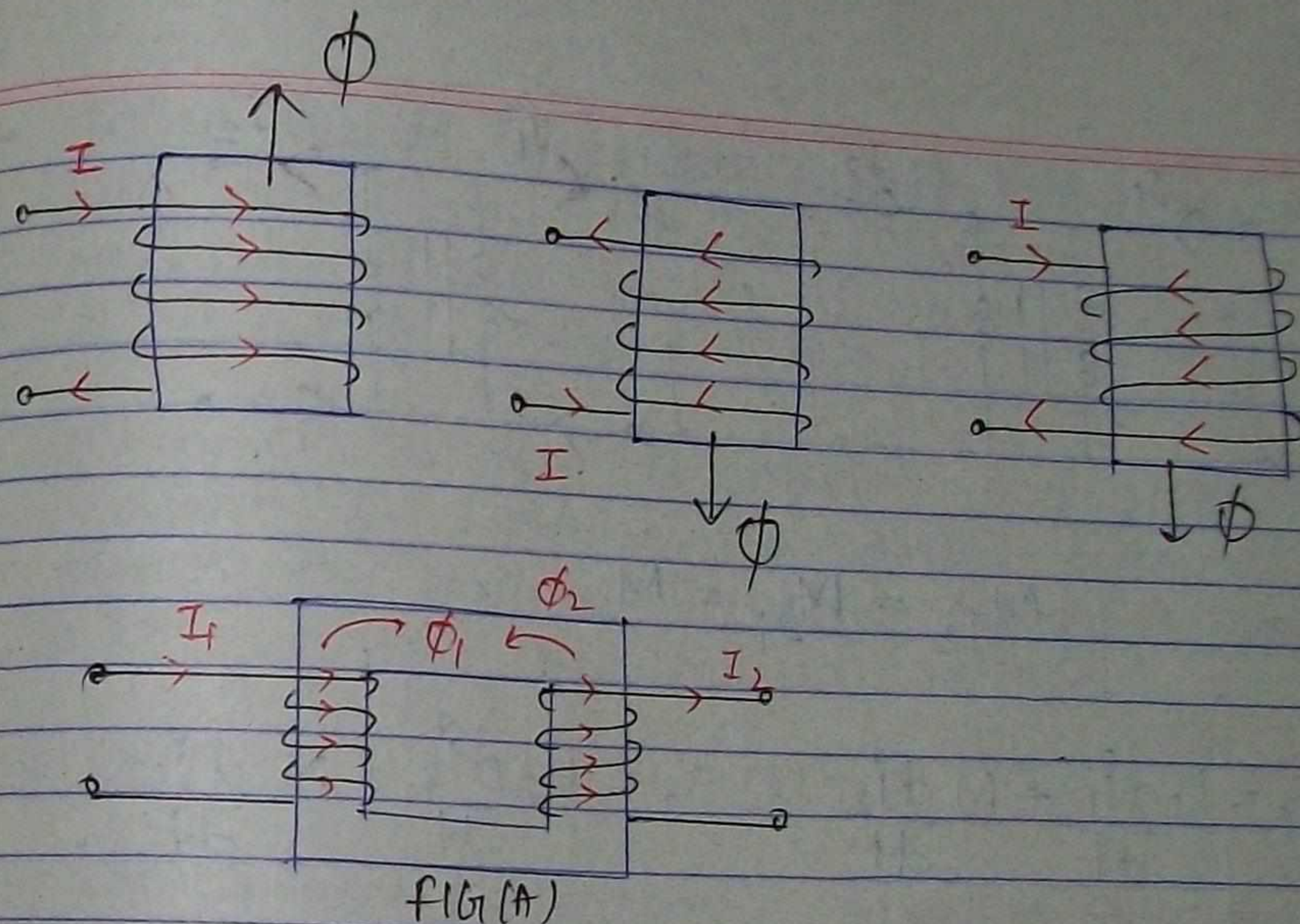
$$e_1 = -N_1 \frac{d\phi_{21}}{dt}$$

$$e_1 = -N_1 \frac{d\phi_{21}}{di_2} \times \frac{di_2}{dt}$$

$$M_{12} = N_1 \frac{d\phi_{21}}{di_2}$$

$$e_1 = -M_{12} \frac{di_2}{dt}$$

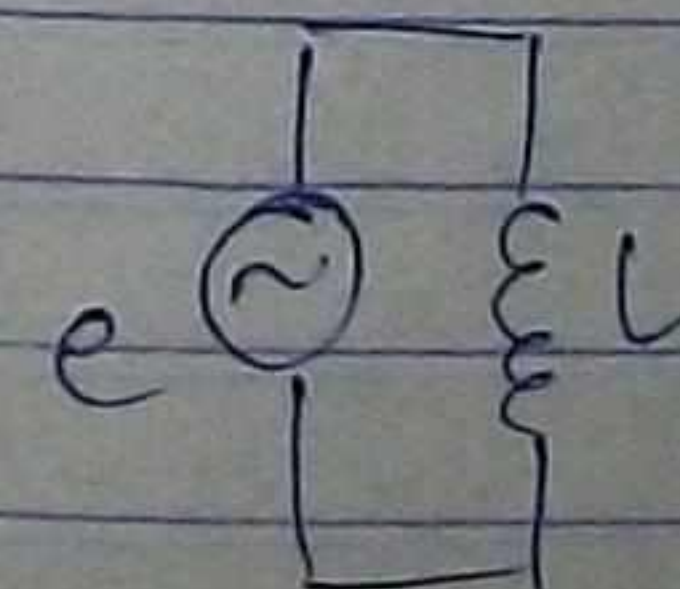
MUTUAL
INDUCED
EMF



- Direction of the flux depends on
 - ① Direction of current
 - ② arrangement of winding (sense of winding)

- In fig A flux of the two inductor are completing path in opposite direction. Hence sign of mutually induced voltage is opposite to sign of self induced voltage.

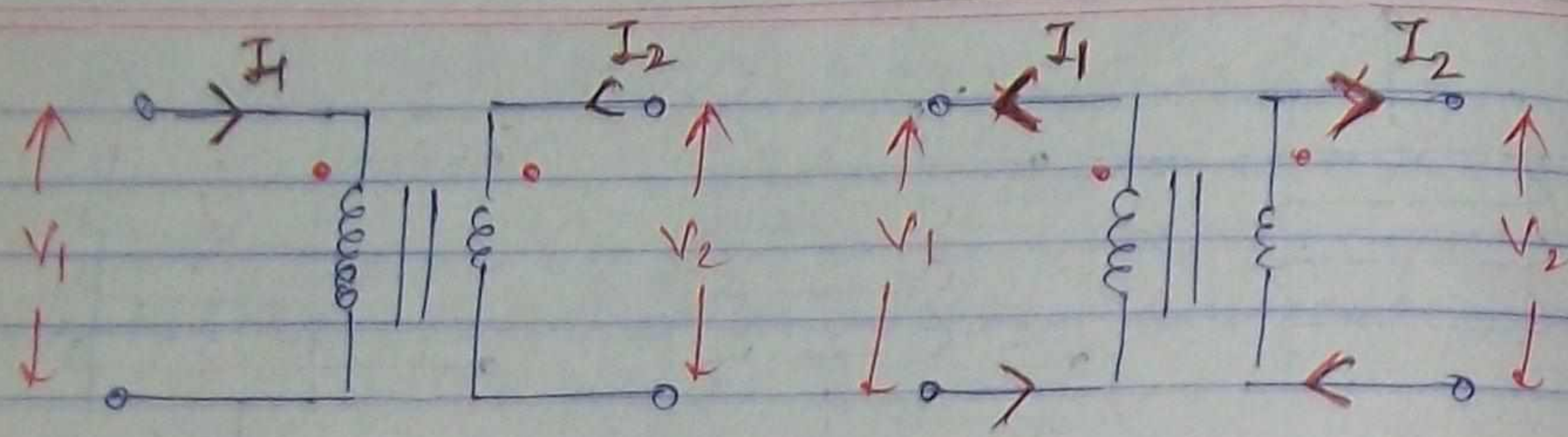
⇒ DOT CONVENTION -



$$e = -L \frac{di}{dt}$$

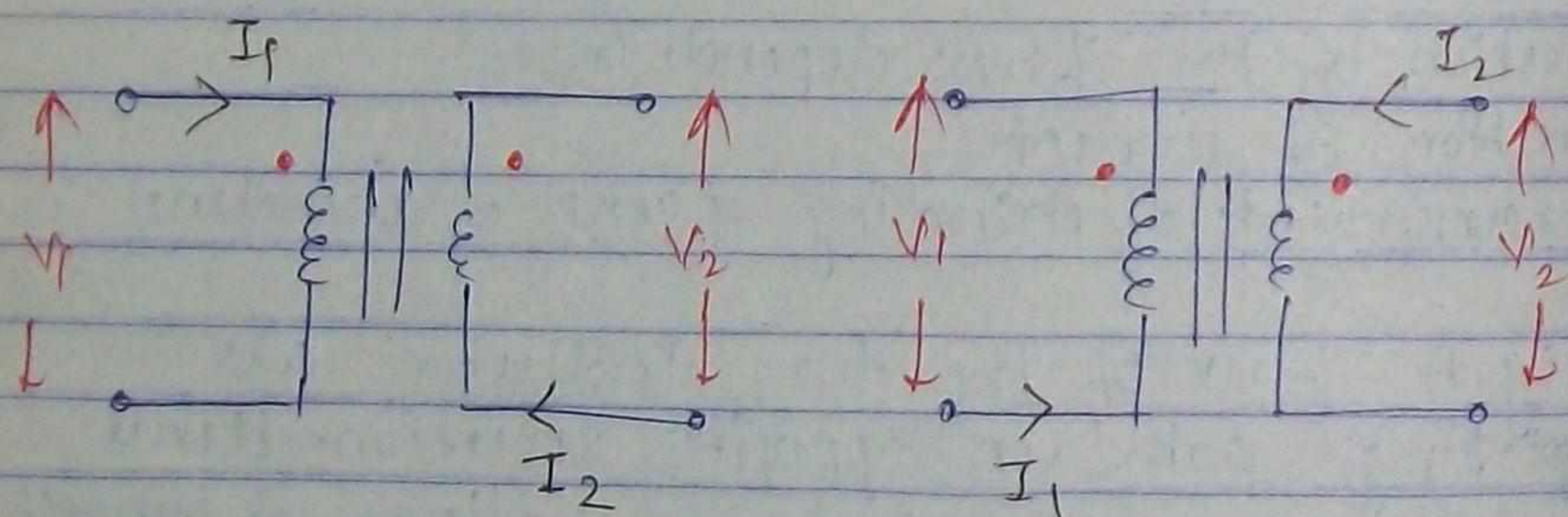
$$V = L \frac{di}{dt}$$

$V = -e$
LENZ
LAW



$$M_{12} = M_{21} = M$$

$$V_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \quad V_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$



$$V_1 = L_1 \frac{di_1}{dt} - m \frac{di_2}{dt}$$

$$V_2 = L_2 \frac{di_2}{dt} - m \frac{di_1}{dt}$$

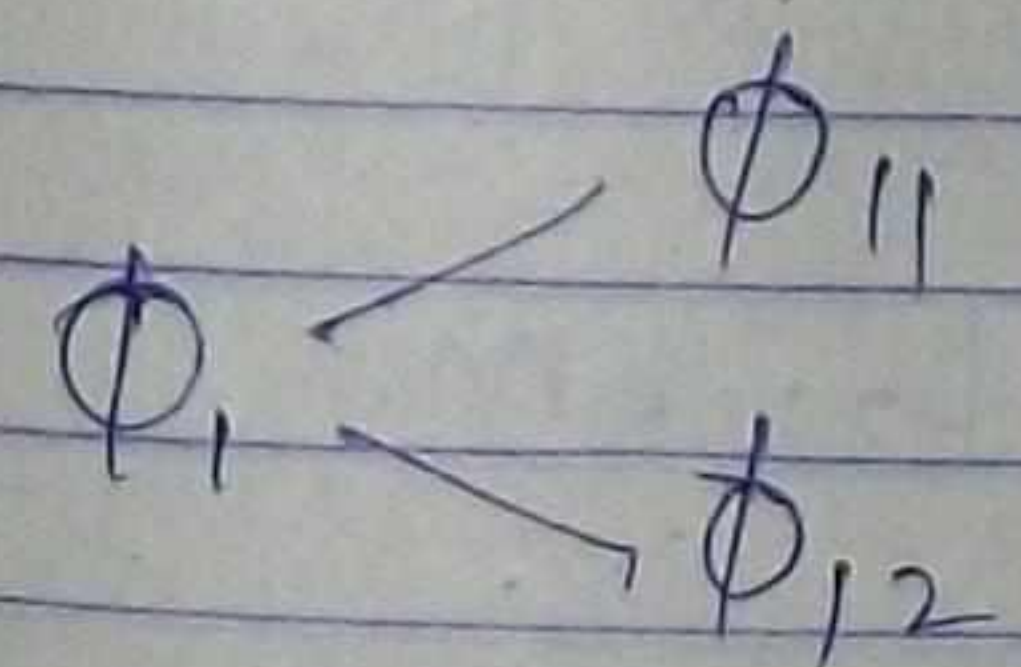
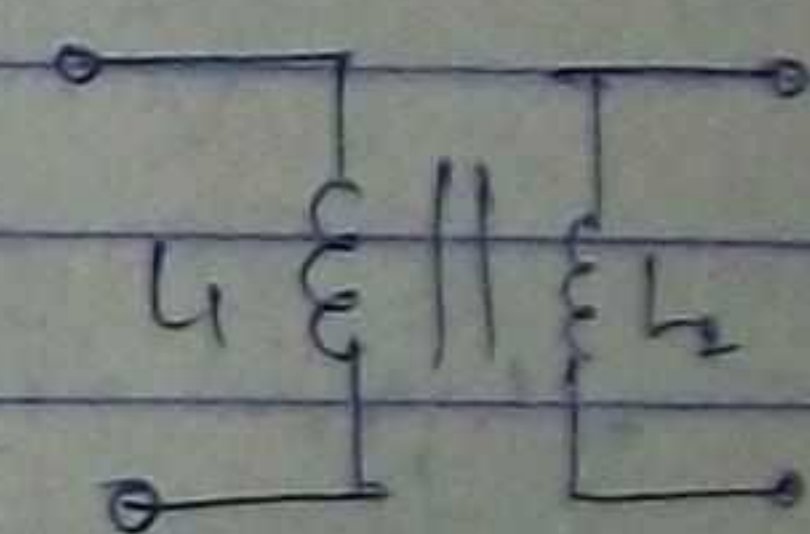
- when current is either entering or leaving at both dotted terminals sign of the mutual induced voltage is same the sign of the self induced voltage.

- when one current is entering and other current is leaving at dotted terminal sign of the mutual induced voltage is opposite to the sign of self induced voltage.

⇒ COEFFICIENT OF COUPLING (K) -

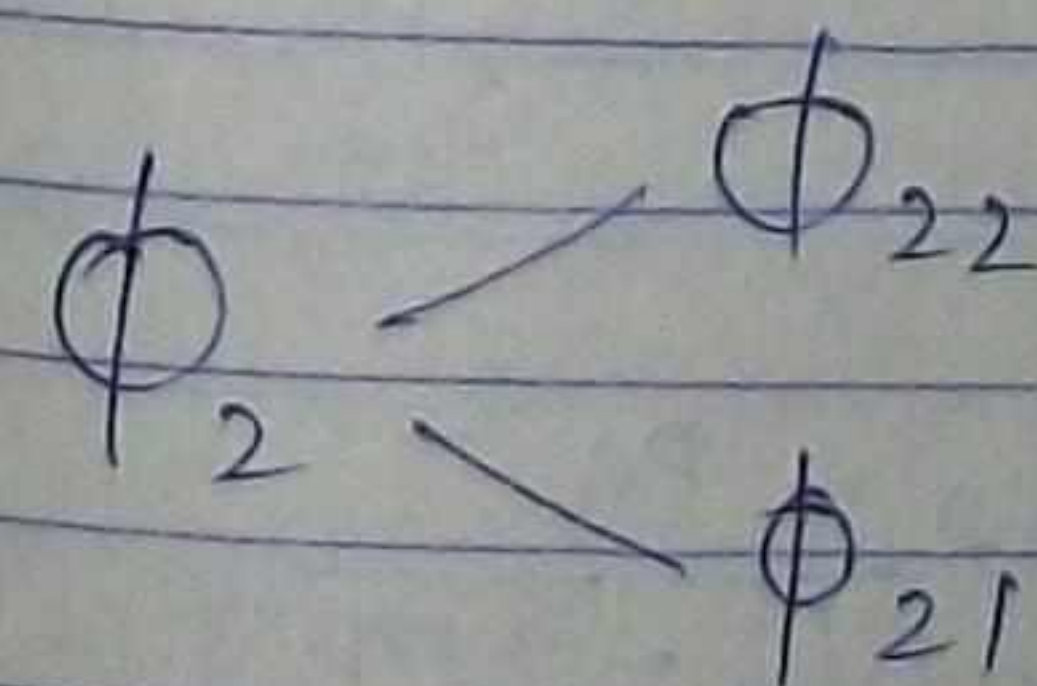
- The amount of magnetic coupling b/w two inductors is expressed by coefficient of coupling

$$K = \frac{\text{USEFUL FLUX}}{\text{TOTAL FLUX}}$$



$$K_1 = \frac{\Phi_{12}}{\Phi_1}$$

(COUPLING FACTOR OF L1)



$$K_2 = \frac{\Phi_{21}}{\Phi_2}$$

(COUPLING FACTOR OF L2)

$$K = \sqrt{K_1 K_2}$$

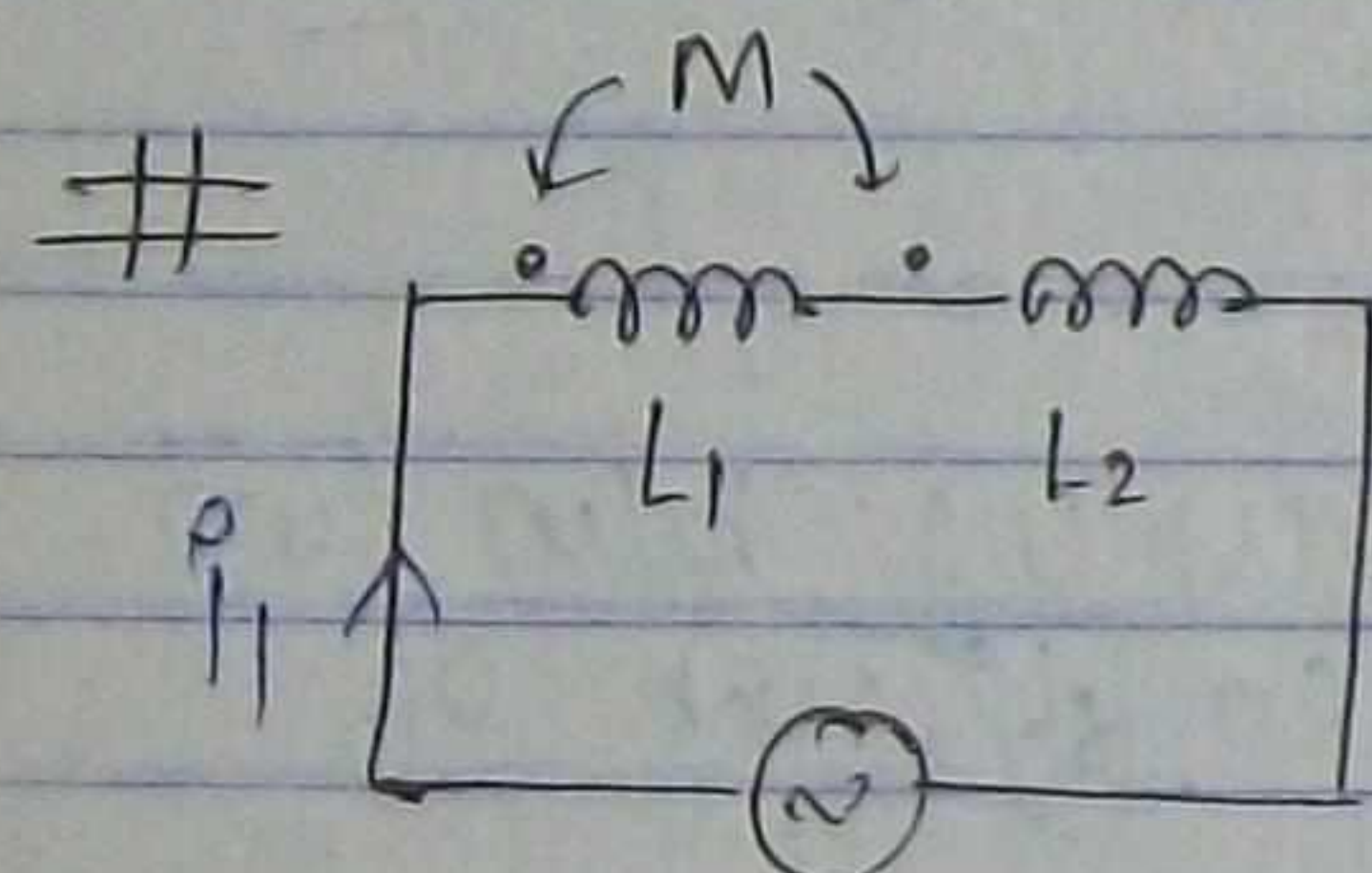
assumption ($K_1 \approx K_2$)

— for ideal system $k=1$

— for practical system $0 < k < 1$

⇒ RELATION B/W M, L_1, k —

$$M = k \sqrt{L_1 L_2}$$

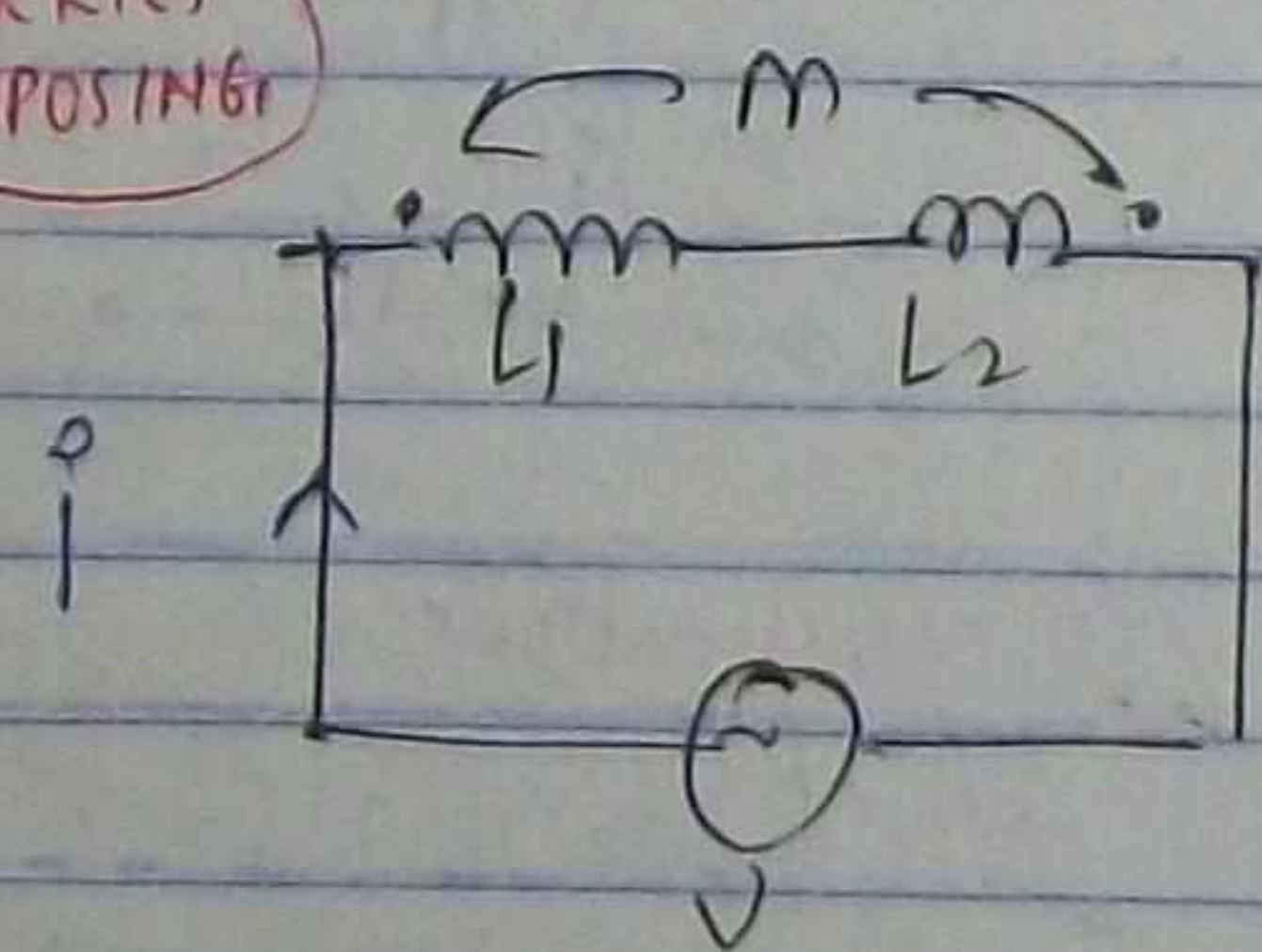


$$V_1 = L_1 \frac{di}{dt} + m \frac{di}{dt} + L_2 \frac{di}{dt} + m \frac{di}{dt}$$

$$L_{eq} \frac{di}{dt} = (L_1 + L_2 + 2m) \frac{di}{dt}$$

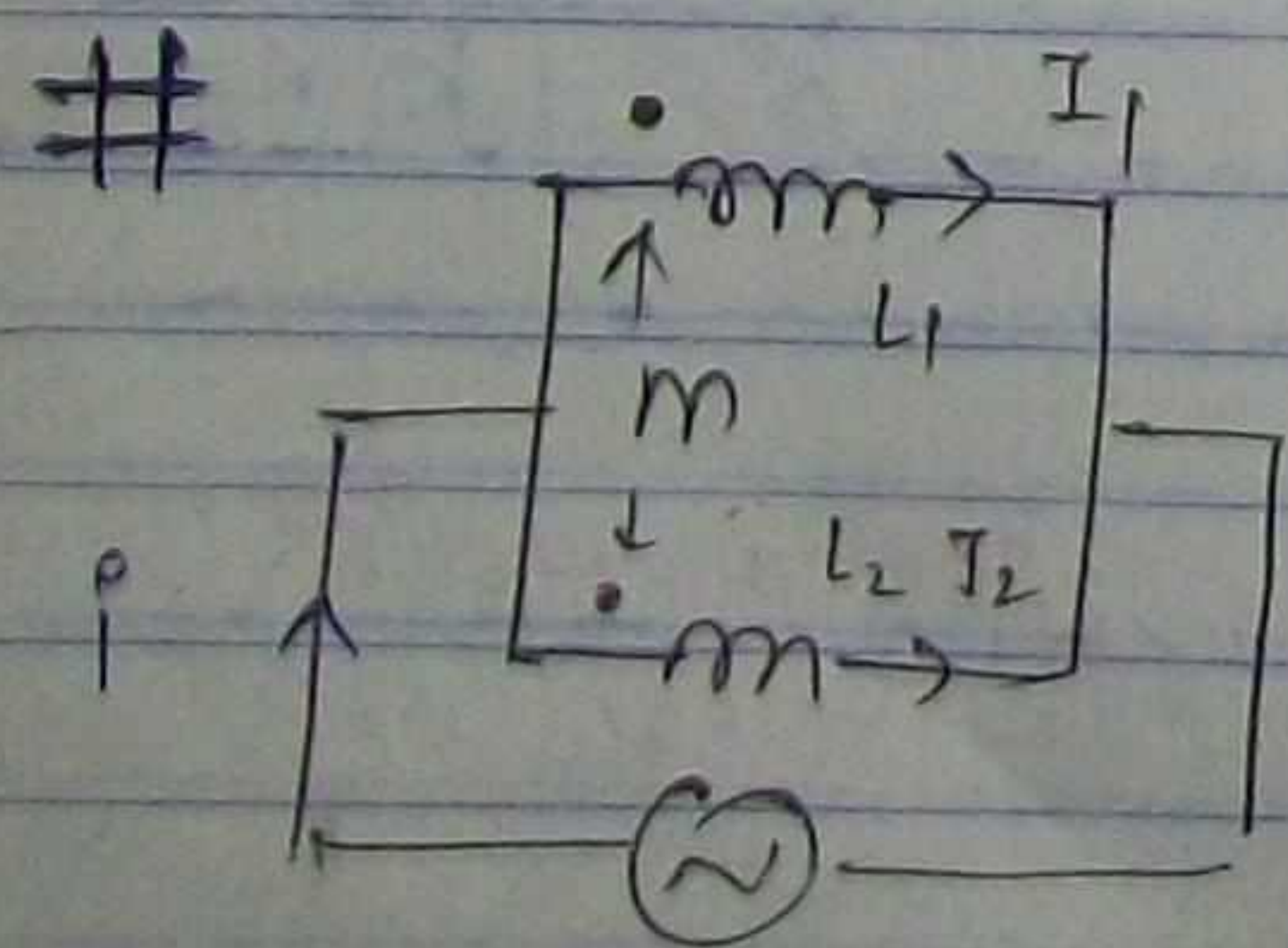
$$L_{eq} = L_1 + L_2 + 2m$$

SERIES
OPPOSING



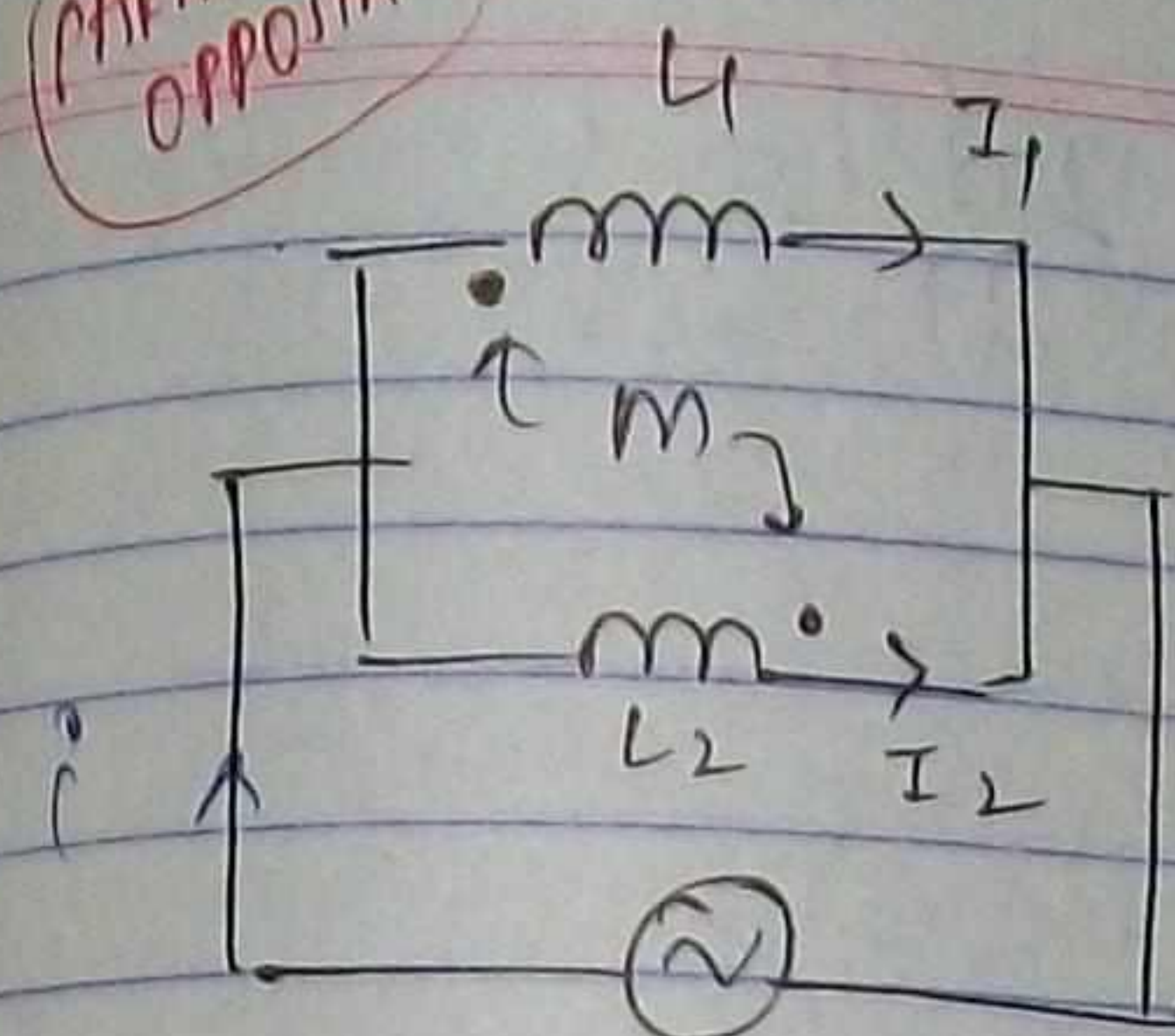
Similarity

$$L_{eq} = L_1 + L_2 - 2m$$



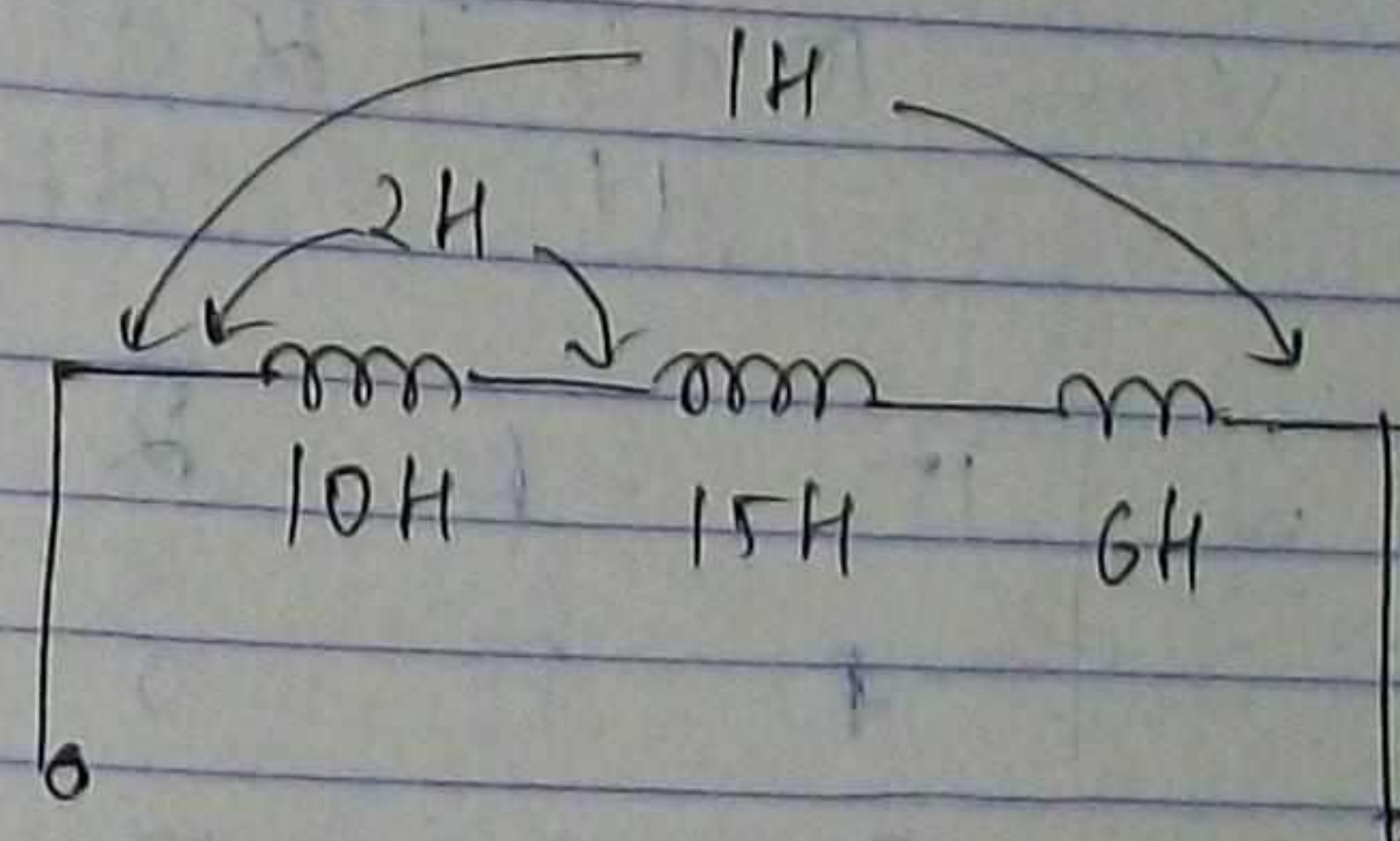
$$L_{eq} = \frac{L_1 L_2 - m^2}{L_1 + L_2 - 2m}$$

PARALLEL
OPPOSING



$$L_{eq} = \frac{L_1 L_2 - m^2}{L_1 + L_2 + 2m}$$

Ques. find L_{eq}



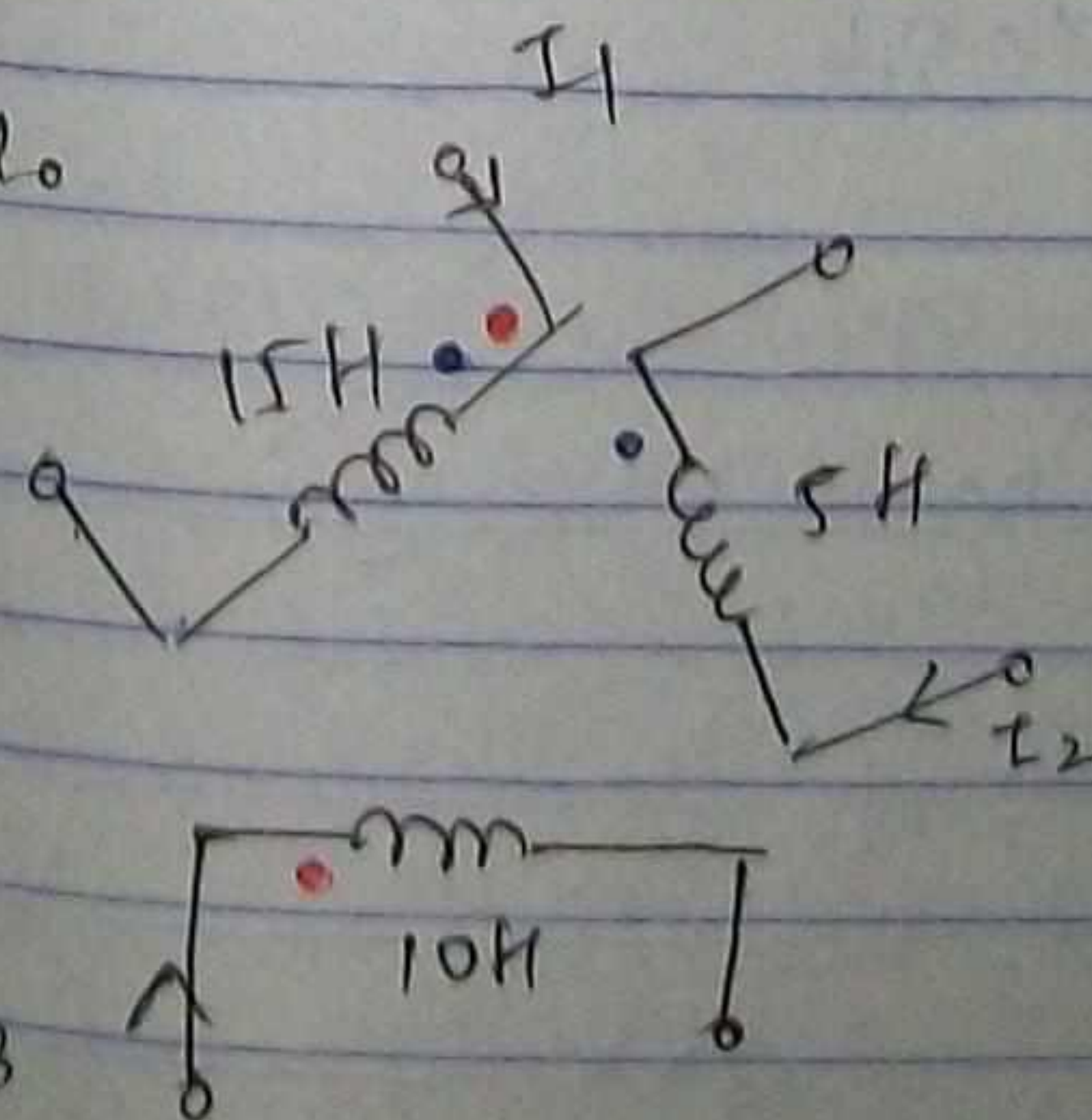
Soln:

$$L_{eq} = L_1 + L_2 + L_3 \pm 2M_1 \pm 2M_2 \pm 2M_3$$

$$L_{eq} = 10 + 15 + 6 + 2 \times 2 + 0 \neq 2(-1)$$

$$L_{eq} = 33H$$

Ques.



Developed Inductance
matrix.

$$\begin{bmatrix} 1H \\ 2H \end{bmatrix}$$

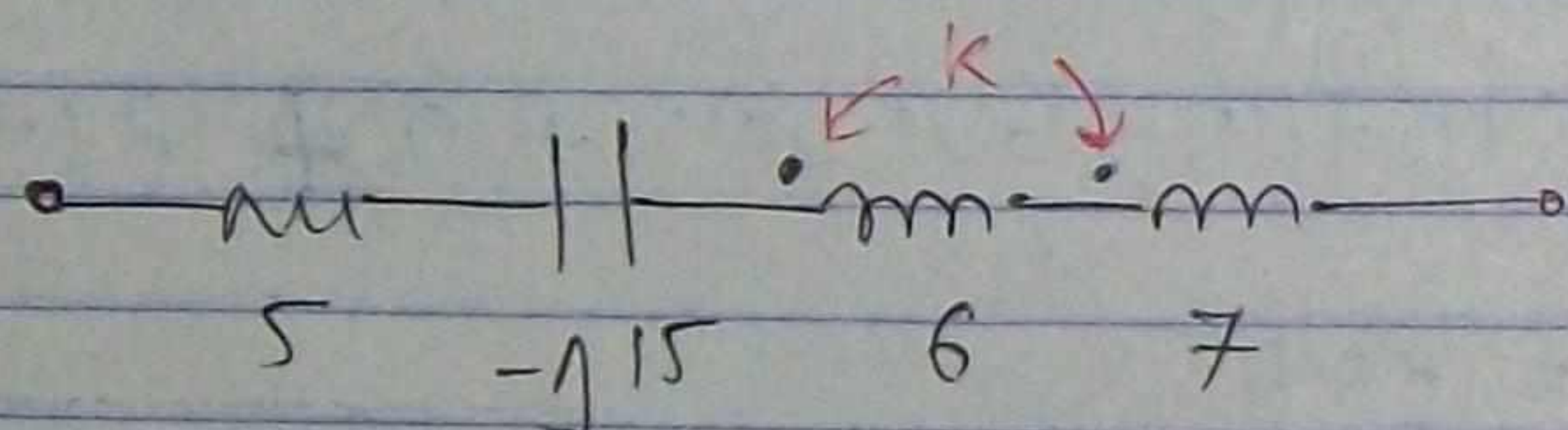
Solⁿ: $V_1 = 15 \frac{di_1}{dt} - 1 \frac{di_2}{dt} + 2 \frac{di_3}{dt}$

$V_2 = 5 \frac{di_2}{dt} - 1 \frac{di_1}{dt}$

$V_3 = 10 \frac{di_3}{dt} + 2 \frac{di_1}{dt}$

$L_{eq} = \begin{bmatrix} 15 & -1 & 2 \\ -1 & 5 & 0 \\ 2 & 0 & 10 \end{bmatrix}$

Ques. find k at resonance.



Solⁿ: $2\pi f(L_{eq}) = (L_1 + L_2 + 2m) \times 2\pi f$

$X_{eq} = X_1 + X_2 + 2X_m$ — (1)

$2\pi f(m = k\sqrt{L_1 L_2})$

$X_m = k\sqrt{X_1 X_2}$ — (2)

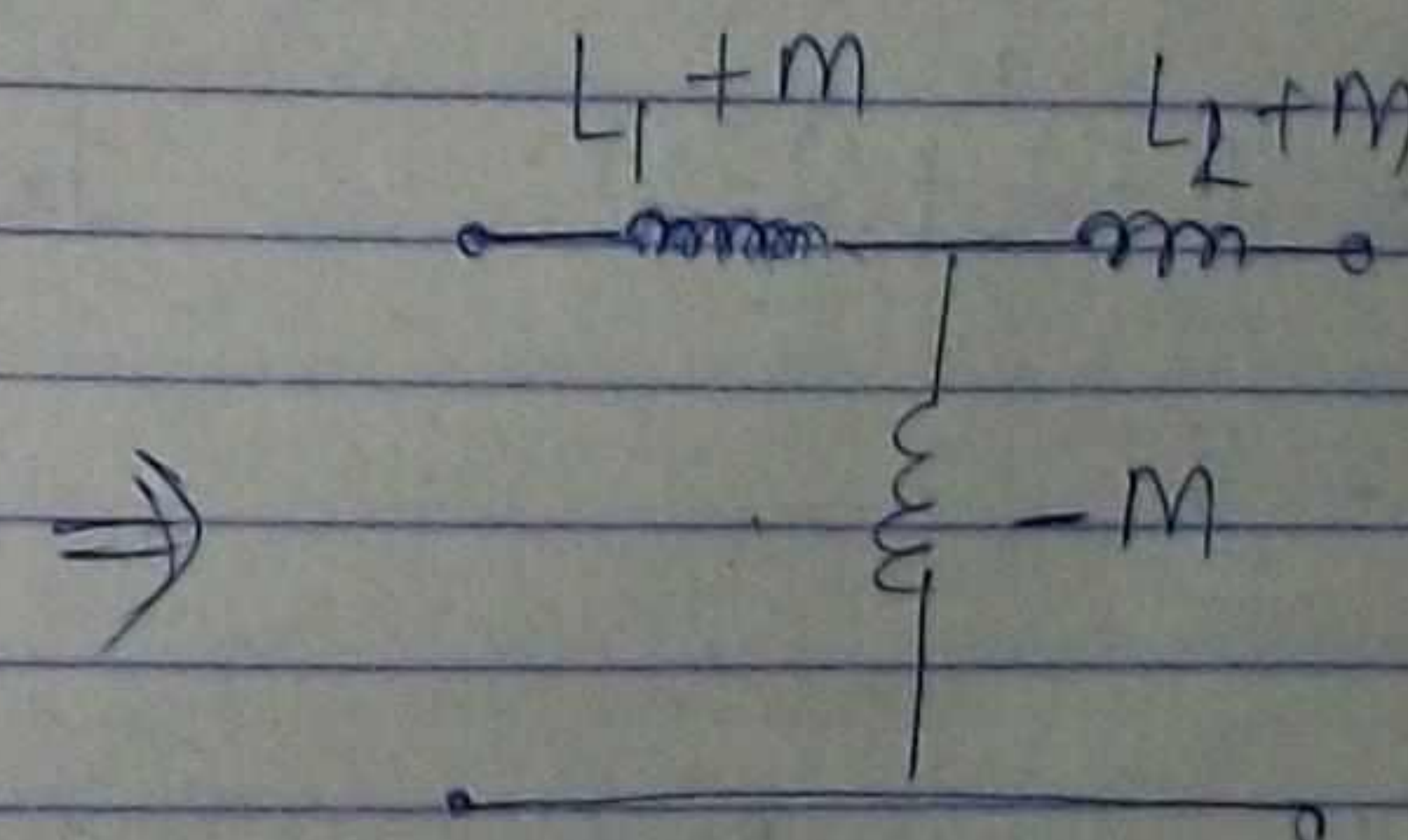
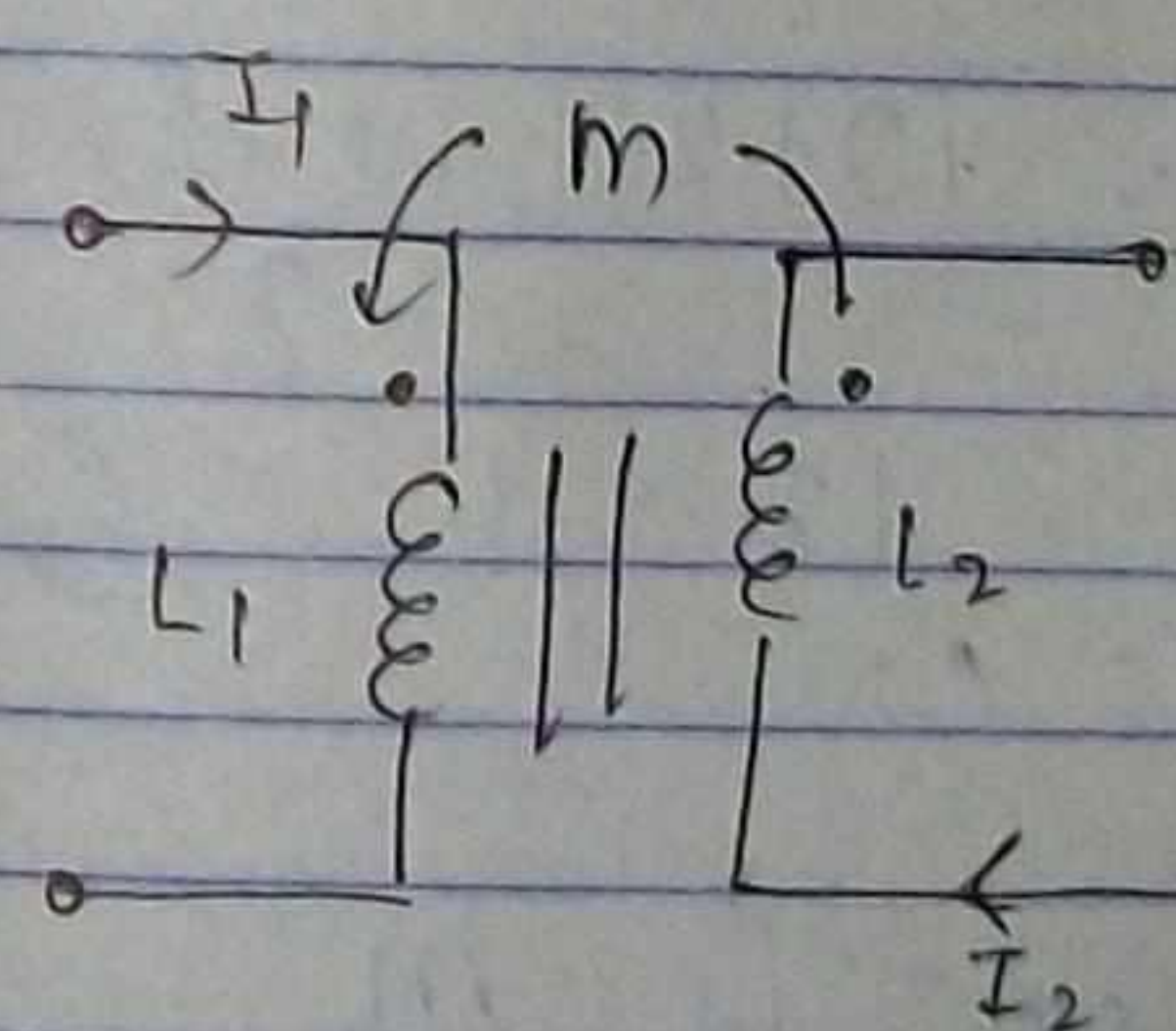
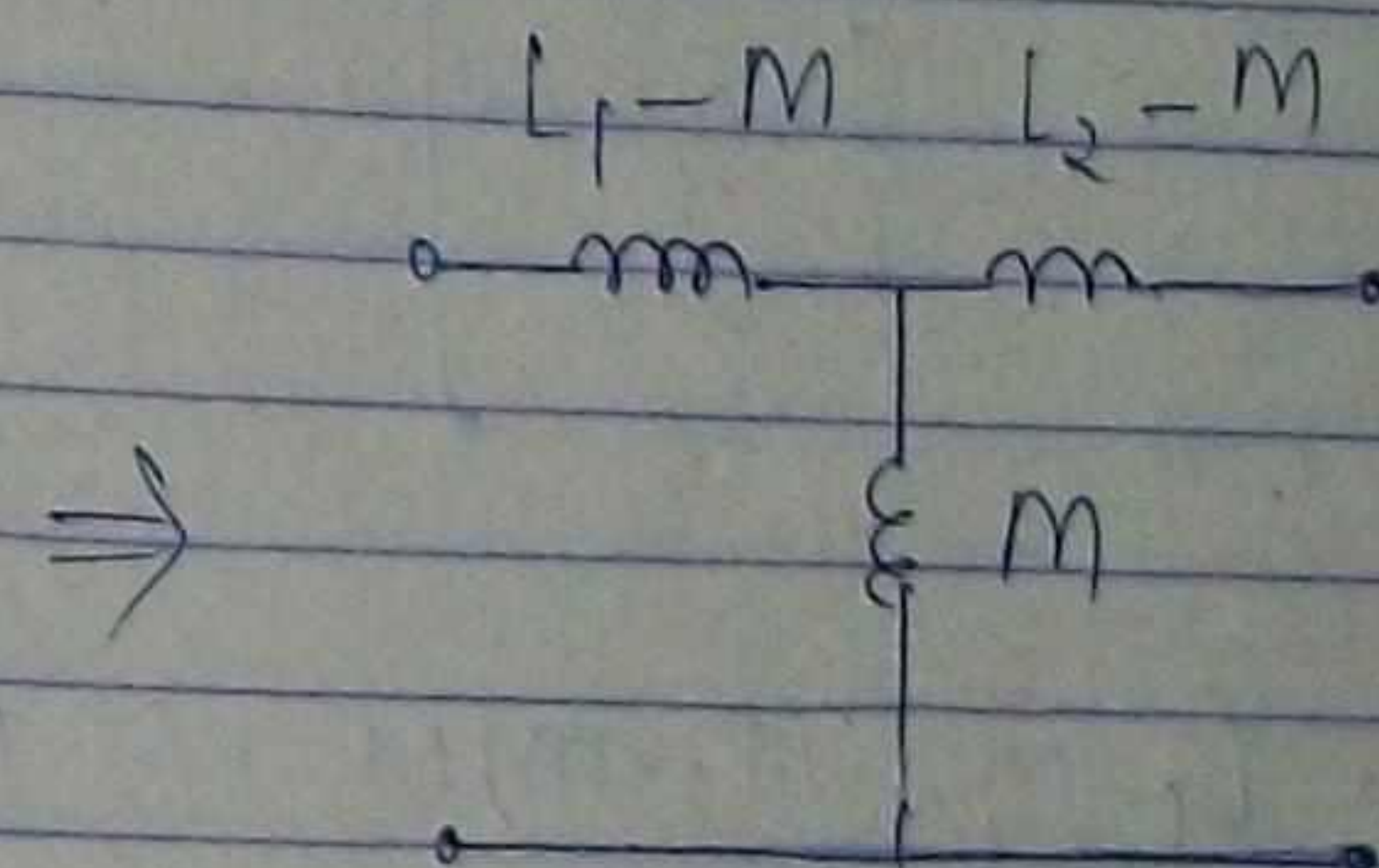
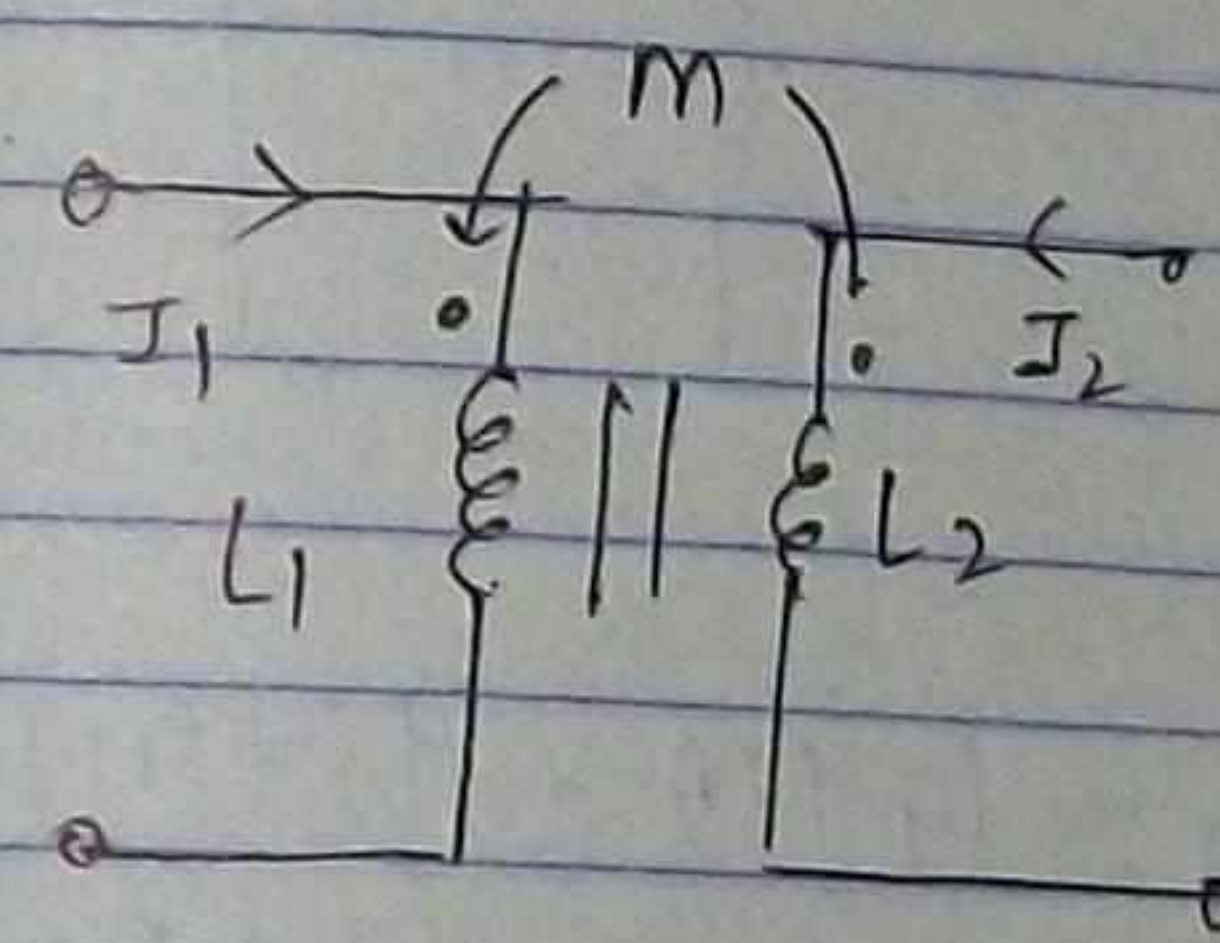
from (1)

$X_{eq} = X_1 + X_2 + 2k\sqrt{X_1 X_2}$

$15 = 6 + 7 + 2k\sqrt{7 \times 6}$

$k = \frac{1}{\sqrt{42}}$

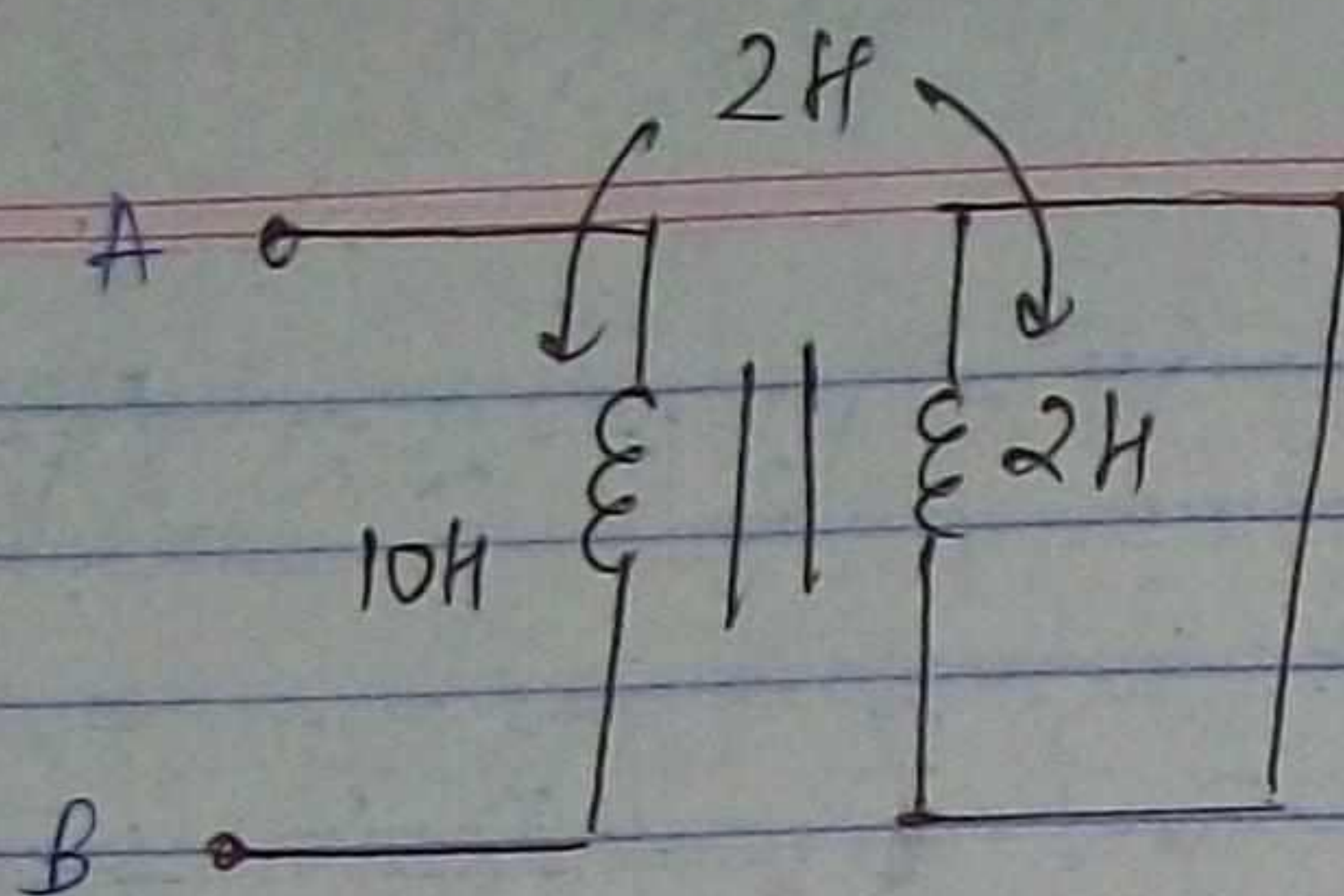
Ques.



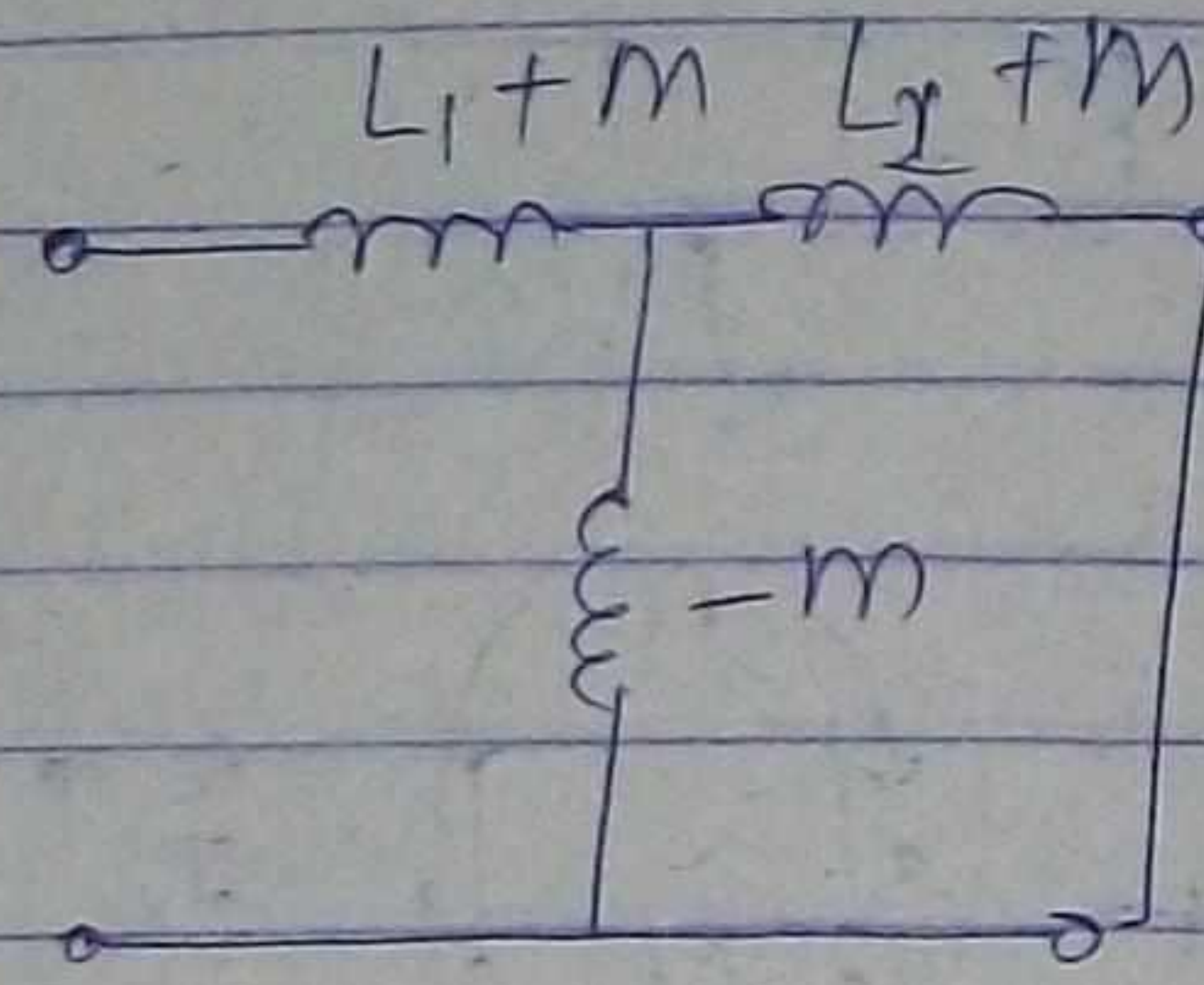
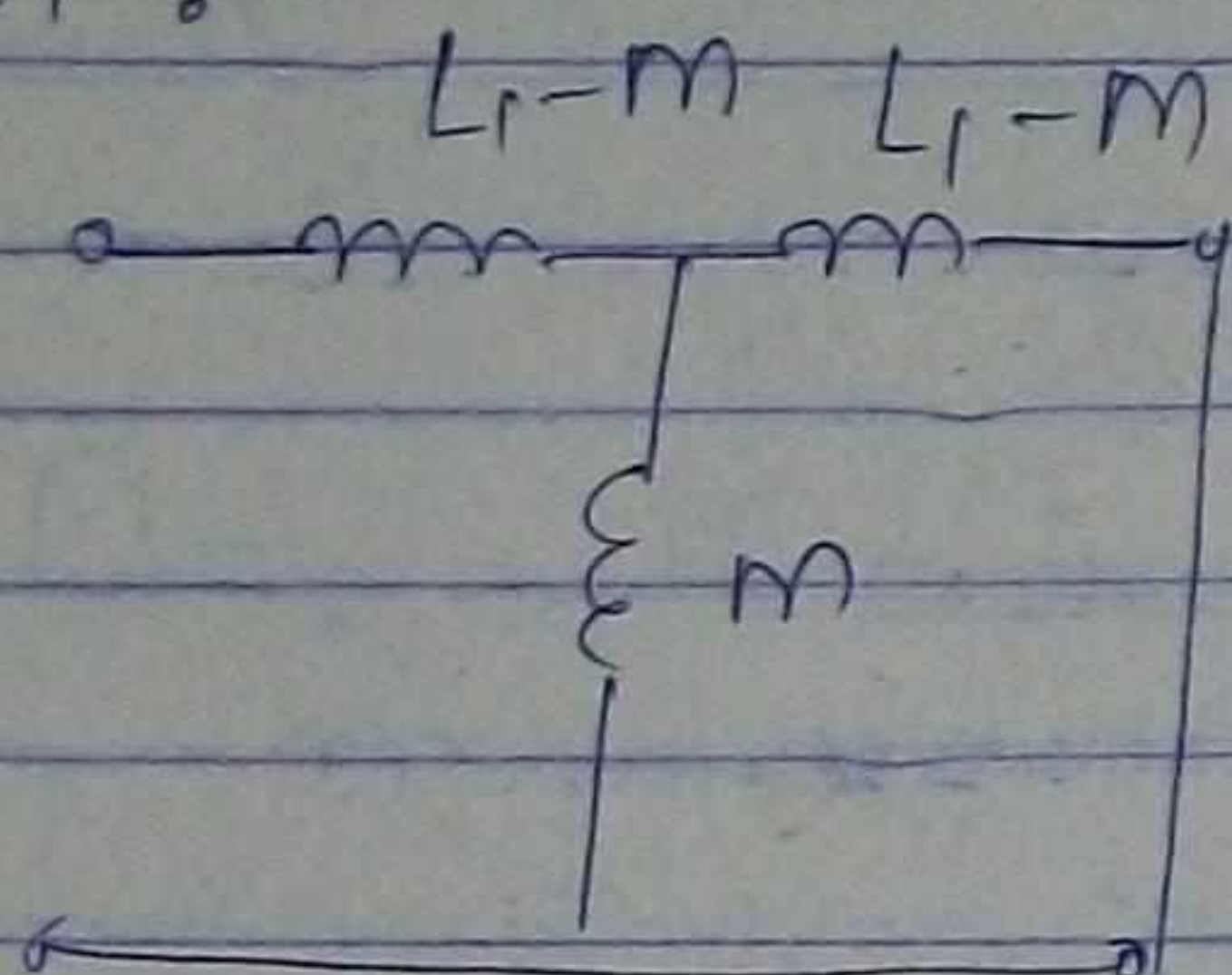
Draw equivalent T N/w^s

$V_1 = L_1 \frac{di_1}{dt} + m \frac{di_2}{dt}$
 $V_2 = m \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$

Ques. find L_{eq}



Solⁿ:



$$L_{eq} = L_1 - m + \frac{(L_2 - m)m}{L_2 - m + m}$$

$$= 10 - 2 + (2 - 2)$$

$$= \underline{\underline{8}}$$

$$L_{eq} = L_1 + m + \frac{(L_2 + m)(-m)}{L_2 + m - m}$$

$$L_{eq} = 10 + 2 + (2 + 2)(-2)$$

$$= 12 - 4 = \underline{\underline{8}}$$



$$L_{eq} = L_1 - \frac{m^2}{L_2}$$

$$L_{eq} = L_1 - \frac{m^2}{L_2}$$

$$V = L_1 \frac{di_1}{dt} + m \frac{di_2}{dt}$$

$$= L_2 \frac{di_2}{dt} + m \frac{di_1}{dt}$$