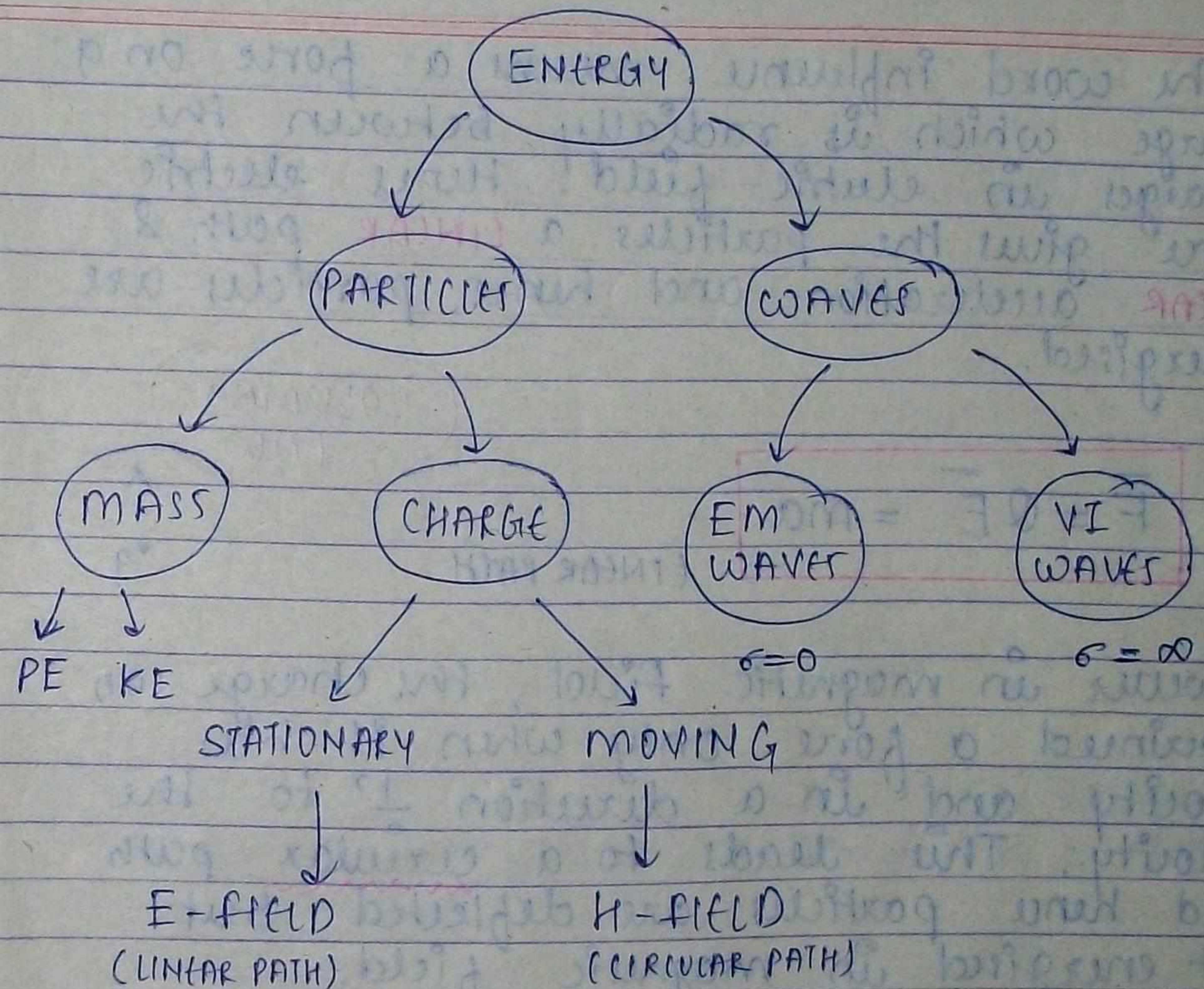


STATIC ELECTROMAGNETIC FIELDS

12/03/1



ELECTRIC FIELD (E-FIELD) -

E-field is an energy format that is all around a ^{stat} charge and influences any other charge in that region.

MAGNETIC FIELD (H-FIELD) -

H-field is an energy format that is all around a moving charge and influences any other moving charge in that region.

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- The word influence means a force on a charge which is radially between the charges in electric field. Hence electric force gives the particles a **LINEAR** path & **LINEAR** acceleration and hence particles are energised.

$$\vec{F} = Q\vec{E} = ma$$

LINEAR PATH

COLUMB'S LAW

↑
q

- However in magnetic field, the charge can experience a force only when it has velocity and in a direction $\perp$ to the velocity. This leads to a circular path and hence particles are deflected but not energised in magnetic field.

$$\vec{F} = Q(\vec{v} \times \vec{B}) = \frac{mv^2}{r}$$

CIRCULAR PATH.

LORENTZ LAW



SUMMARY :

- ① q → stationary
q → accumulated

CAUSE - VOLTAGE
EFFECT - ELECTRIC FIELD

ENERGY

②

q → moving.

[without acceleration
with constant velocity.]

q → linearly changes with time.

$$q = kt$$

CAUSE -

$$\frac{dq}{dt} = k = I$$

D.C. CURRENT

EFFECT -

H-FIELD

ENERGY

③

Both this field (E and H) are called static field because they are time independent.

④

q → moving

[with acceleration]

E(t)

H(t)

NOT ENERGY

TIME VARYING FIELDS

POWER

OR

RADIATION

⇒ ANALOGY'S BETWEEN E & H-FIELDS -

- The strength of any field at any point in space can be expressed as INTENSITY in per unit length analysis and as DENSITY in per unit area analysis.

$\vec{E}$ - Electric field intensity (Volts/m)

$\vec{H}$ - Magnetic field intensity (Amp/m)

$\vec{B}$ - Magnetic flux density (weber/m²)

$\vec{D}$ - Electric flux density (Coulombs/m²)

- # ϵ = permittivity of the material/medium for E-field (ability to allow or permit or hold E field)

$$\epsilon = \epsilon_0 \epsilon_R$$

$\epsilon_R \geq 1$ for any medium.

ϵ_0 = the least possible value for any medium in the world.

$$C = \frac{\epsilon A}{d}$$

- ∴ ϵ_0 = permittivity of free space.

$$\epsilon_0 = 8.85 \times 10^{-12} = \frac{1}{36\pi \times 10^9} \text{ farads/m}$$

- # μ = permeability of material/medium for H field (ability to allow or hold H field).

$$\mu = \mu_0 \mu_R$$

$\mu_R \geq 1$ (for any medium)

μ_0 = the least possible value for any medium in the world.

- ∴ μ_0 = permeability of free space.

$$\mu_0 = 4\pi \times 10^{-7} \text{ Henry/m}$$

#

$$C = \frac{Q}{V}$$

$$Q = C \times V$$

Coulombs
m²
(Farads)
m
Volts
m

$$C = \frac{\epsilon A}{d}$$

$$C = \epsilon \frac{A}{d}$$

$$D = \epsilon E$$

$$\frac{C}{A} = \epsilon$$

$$\# \quad L = \frac{\Psi_m}{I} \quad \begin{matrix} \text{(flux)} \\ \text{(current)} \end{matrix}$$

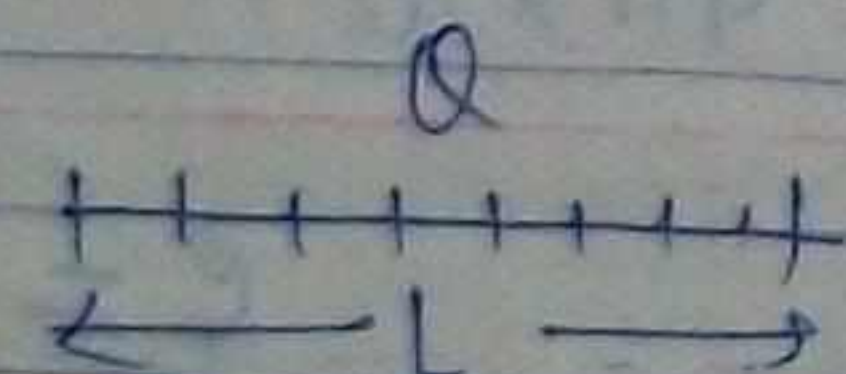
$$\Psi_m = L \times I$$

$$\frac{\text{webers}}{\text{m}} = \frac{\text{Henry}}{\text{m}} \times \frac{\text{ampres}}{\text{m}}$$

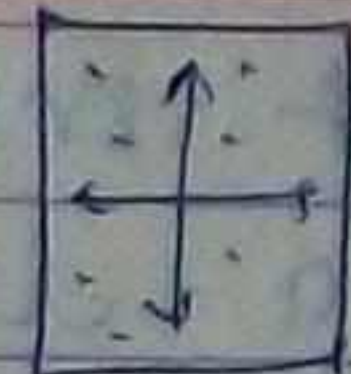
$$B = \mu H$$

⇒ BASIC CHARGE DISTRIBUTION -

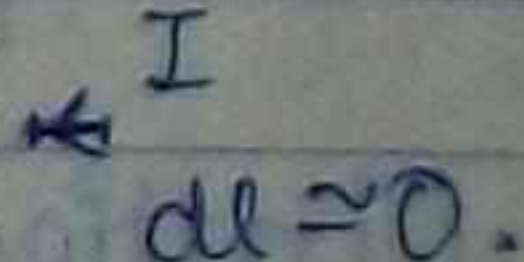
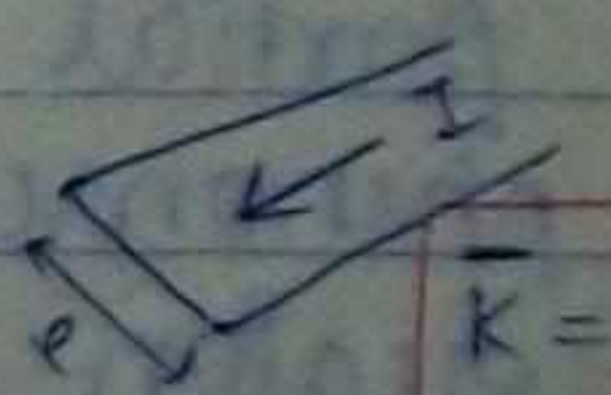
- charge exist in four forms, all scalar format. It involves simple point to a solid volume.

Q	NAME	UNIT	EXAMPLES
① Q	POINT CHARGE	COLOMB	• ELECTRON
② ρ_L	LINE CHARGE	Colomb/m	 $\rho_L = \frac{dQ}{dl}$ (TRANSMISSION) LINE

(Q, ρ_L , ρ_s , ρ_v all are scalars.)

③ ρ_s	SURFACE CHARGE	C/m ²	 $\rho_s = \frac{dQ}{ds}$ (capacitor plate)
④ ρ_v	VOLUME CHARGE	C/m ³	 $\rho_v = \frac{dQ}{dv}$ ionosphere, depletion region of diode.

⇒ BASIC CURRENT DISTRIBUTION -

CURRENT	NAME	UNIT	EXAMPLE
① $I d\vec{l}$	CURRENT ELEMENT	Amp-m	 $I d\vec{l} \approx 0$
② I	LINE CURRENT	Amp	
③ $\vec{K}$	SURFACE CURRENT	Amp/m	 $\vec{K} = \frac{dI}{dl}$
④ $\vec{J}$	VOLUME CURRENT (SOLID CONDUCTOR)	Amp/m ²	 $\vec{J} = \frac{dI}{ds} = \frac{\text{Amp}}{\text{m}^2}$

- Current most commonly occur in a wire along a line but can also be defined on a surface as $\vec{K}$ and also defined on solid conductor as $\vec{J}$.

- But when current flows in a wire of zero length i.e. $dl \rightarrow 0$, it is called by current element $I dl$.

$(I dl, \vec{K}, \vec{J} \rightarrow \text{vector})$
 $I \rightarrow \text{scalar}$

⇒ VECTOR CALCULAS -

Directional cumulative effect - Directional integration.

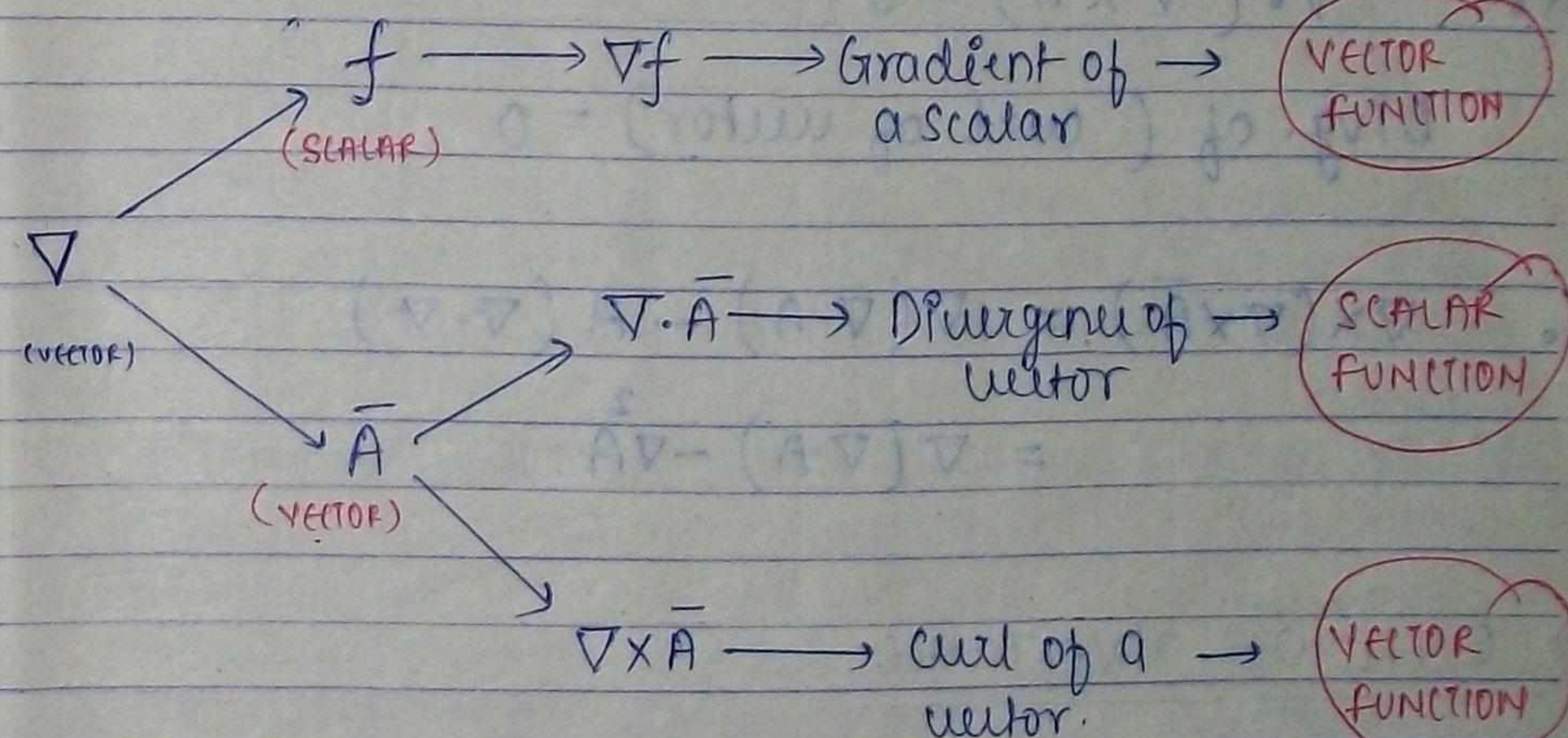
Directional Instantaneous effect - Directional derivatives.

- Vector calculus is study of total or partial or instantaneous effect of a phenomenon at a point in space or a region in space.

- ∇ is a vector operator and is used for spatial derivatives. When operated on a space varying quantity it gives us

|| The rate of change of quantity along with the direction of the change.

$$\nabla = \frac{\partial}{\partial x} u_x + \frac{\partial}{\partial y} u_y + \frac{\partial}{\partial z} u_z$$



- ∇ is a vector and hence can be operated on scalar functions and vector functions with the operation resulting in scalar or vector.

$$\nabla \rightarrow \nabla \rightarrow \nabla \cdot \nabla \rightarrow \nabla^2 \rightarrow \text{SCALAR LAPLACIAN OPERATOR}$$

⇒ VECTOR IDENTITY -

1. $\nabla \times (\nabla f) = 0$

curl of (grad. of scalar) = 0

2. $\nabla \cdot (\nabla \times \vec{A}) = 0$

Divg. of (curl of vector) = 0.

3. $\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$
 $= \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$

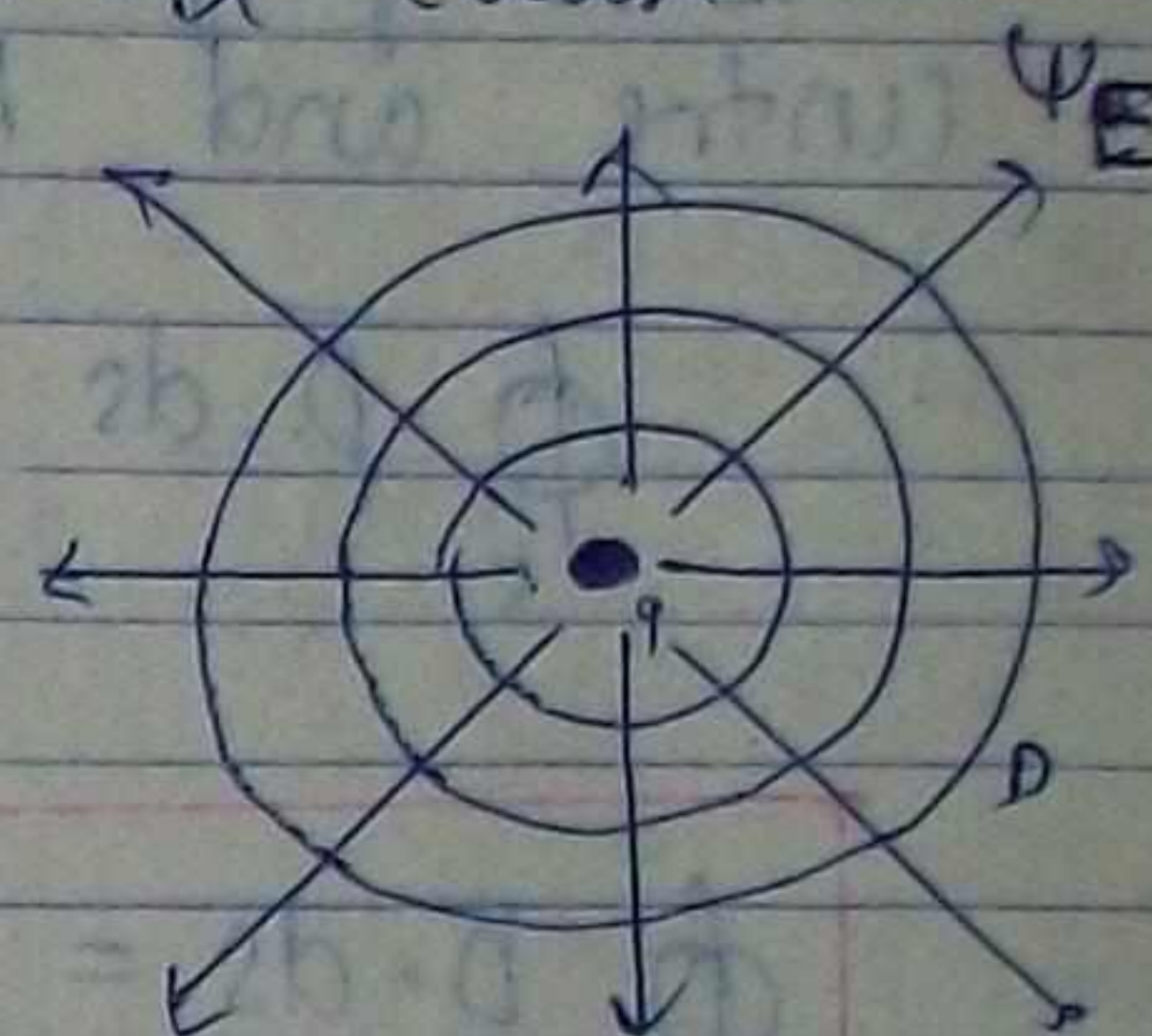
⇒ OUTFLOW & DIVERGENCE -

- Consider a cause which has effect - outward or radial from the cause.
 ex - air velocity from a punctured tyre.
 Light from a bulb
 Electric force from a charge.

- For all such phenomenon the strength of the effects decreases as the area of expansion increases, hence,

Strength $\downarrow$ x area $\uparrow$ = CONSTANT $\propto$ Cause

- The total outflow crossing any closed surface around the cause will be constant for any surface and the constant value depends on the cause at the centre.



- Hence the strength for such effect is any constant per unit area, or cause per unit area called as density function.

Strength = $\frac{\text{Const. or cause}}{\text{area}}$ - density function

- If the cause is a charge of Q colombs, then the effects is called as electric flux ψ_e and the strength is called as displacement flux density D and hence total outflow is $\psi_{e \text{ TOTAL}}$

$\psi_{e \text{ TOTAL}} = \oint_S D \cdot ds$

- This is proportional to the charge at the centre and hence

$$\oint_S \vec{D} \cdot d\vec{s} = \psi_{e \text{ TOTAL}} \propto Q \quad (k=1) \text{ proportionality const.}$$

GAUSS'S LAW

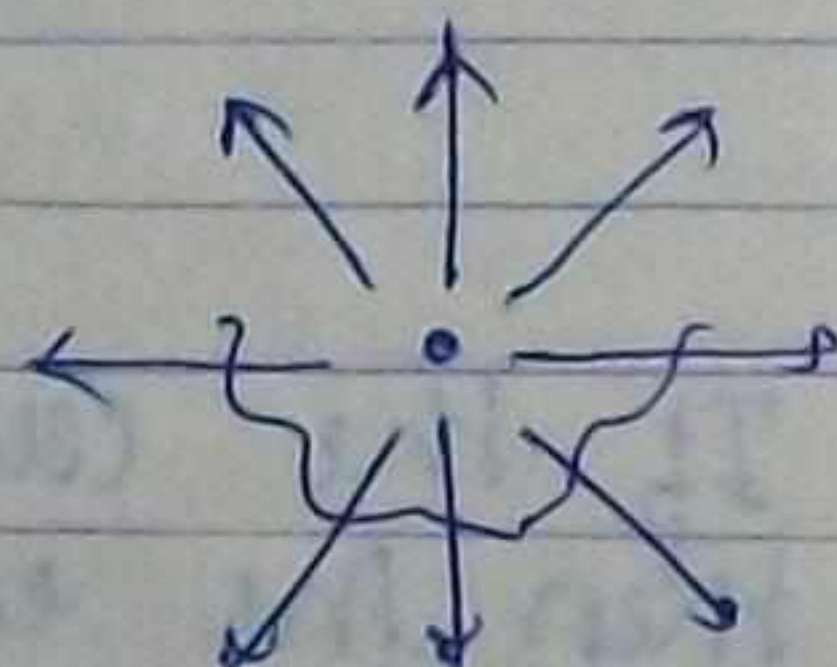
$$\oint_S \vec{D} \cdot d\vec{s} = Q$$

MAXWELL 1ST EQUATION

- If the same analysis is carried on a open surface then the outflow through the surface is the flux crossing that surface it is partial effects of that cause.

$$\int \vec{D} \cdot d\vec{s} = \psi_e$$

NOT GAUSS'S LAW



= FLUX CROSSING THAT SURFACE

NOTE -

A closed surface always has a finite volume enclosed.

VECTOR

SCALAR

ex - Sphere, cube, cylinder

$4\pi r^2$	$\frac{4}{3}\pi r^3$	$6a^2$	a^3	$2\pi rh$	$\pi r^2 h$
------------	----------------------	--------	-------	-----------	-------------

⇒ CIRCULATION AND CURL -

- Consider a cause which has effect circulatory around the cause.

ex - air velocity under the fan.

✓ Magnetic field around a current ck.

- for all such phenomenon the strength decreases as the length of circulation increases.

$$\text{Strength} \downarrow \times \text{Length} \uparrow = \text{CONSTANT} \propto \text{CAUSE}$$

- The total circulation in any closed line around the cause will be constant for any length and this constant has a value that depends on the central cause.



- If cause - Current I amp.

then, effect - magnetic field and

strength of effect - magnetic field intensity (H)

Hence circulation is

$$\oint_{\text{closed}} \vec{H} \cdot d\vec{l} = I$$

AMPER'S CIRCUAL LAW

MAXWELL 4th EQUATION

NOTE -

VECTOR

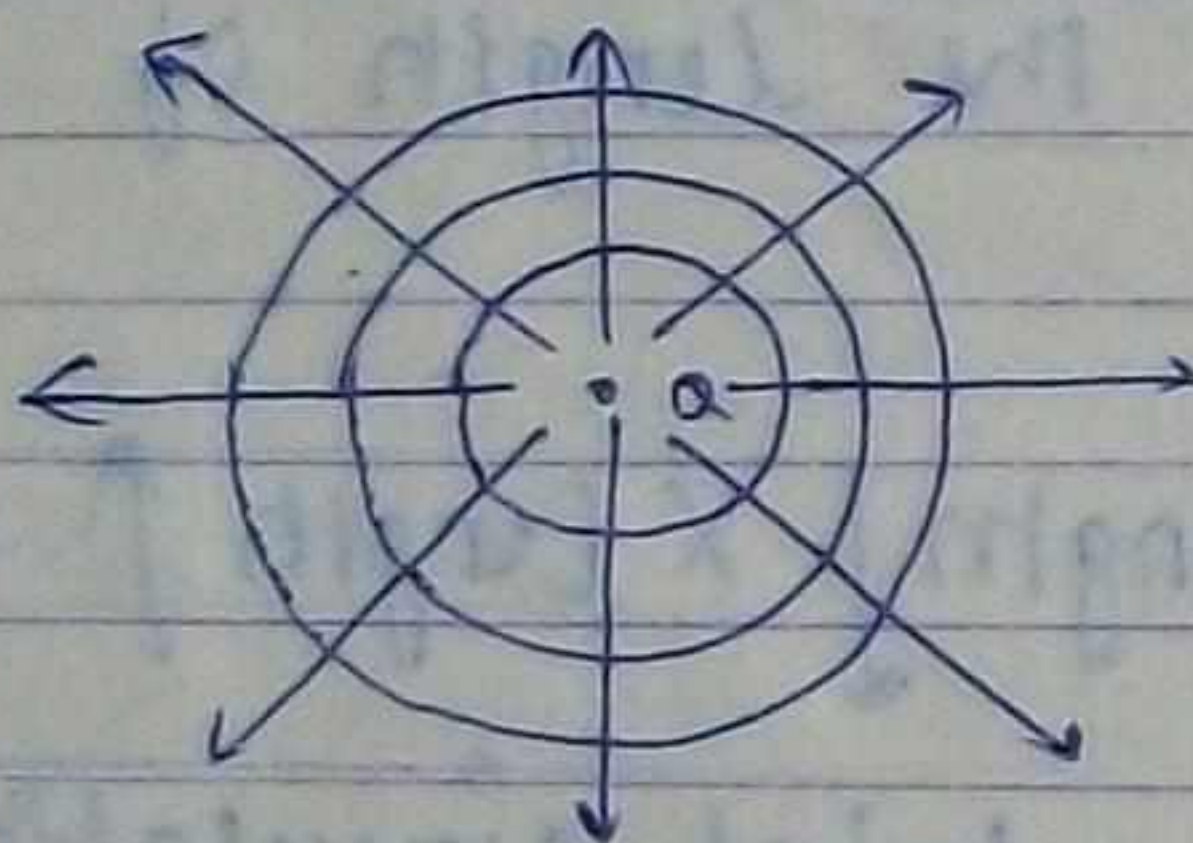
A closed line is identified with a finite area enclosed.

Ex - Circle, Square

$$\begin{array}{cc} \swarrow & \searrow \\ 2\pi r & \pi r^2 \end{array} \quad \begin{array}{cc} \swarrow & \searrow \\ 4a & a^2 \end{array}$$

⇒ SUMMARY (1) :

- Q is the cause i.e. the total outflow, then D is the cause/Area i.e. strength of effect around the cause. Then $\nabla \cdot D$ is the derivative of density i.e. cause/vol or outflow/vol i.e. dQ/dV i.e. strength of the effect at the cause.



$Q \rightarrow$ cause $\rightarrow$ outflow

$D \rightarrow$ cause/area $= \frac{dQ}{ds} = \frac{d\psi_e}{ds}$ Strength of the effect around the cause

$\nabla \cdot D \rightarrow$ cause/vol $= \frac{dQ}{dV}$ Strength of the effect at the cause
- outflow/vol

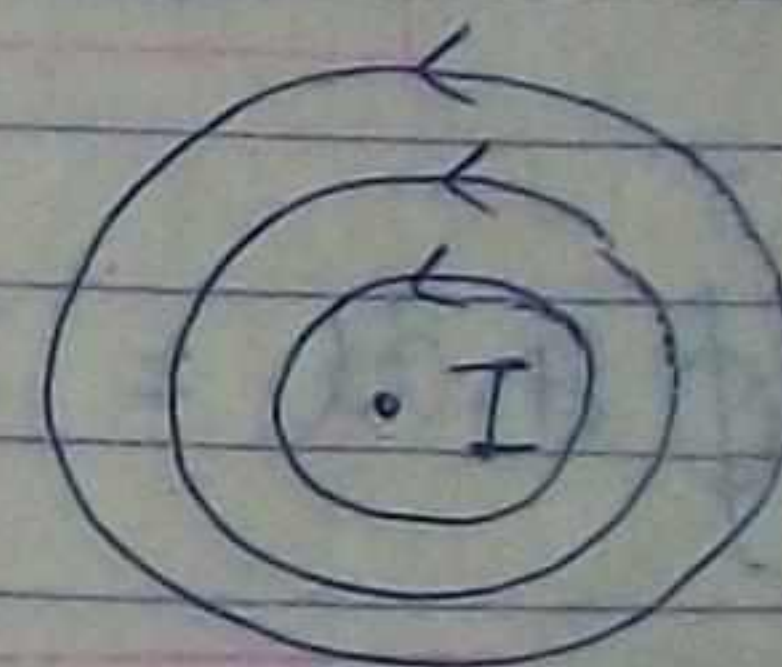
$$\therefore \nabla \cdot D = \int_V$$

GAUSS'S LAW IN POINT FORM

"The divergence depends on charge density."

- $I \rightarrow$ cause - circulation

$H \rightarrow$ cause - $\frac{dI}{dl}$
length (strength)



$\nabla \times H \rightarrow$ cause/area $= \frac{dI}{ds}$
- circulation/area

curl of $H = J$

$$\nabla \times H = J$$

"curl depends on current density"

SUMMARY (2) :

Divergence and Stokes's theorem

$\oint_{\text{closed}} \mathbf{D} \cdot d\mathbf{s} = Q = \int \rho_v dv = \int (\nabla \cdot \mathbf{D}) dv$

$\oint_{\text{closed}} \mathbf{D} \cdot d\mathbf{s} = \int (\nabla \cdot \mathbf{D}) dv$

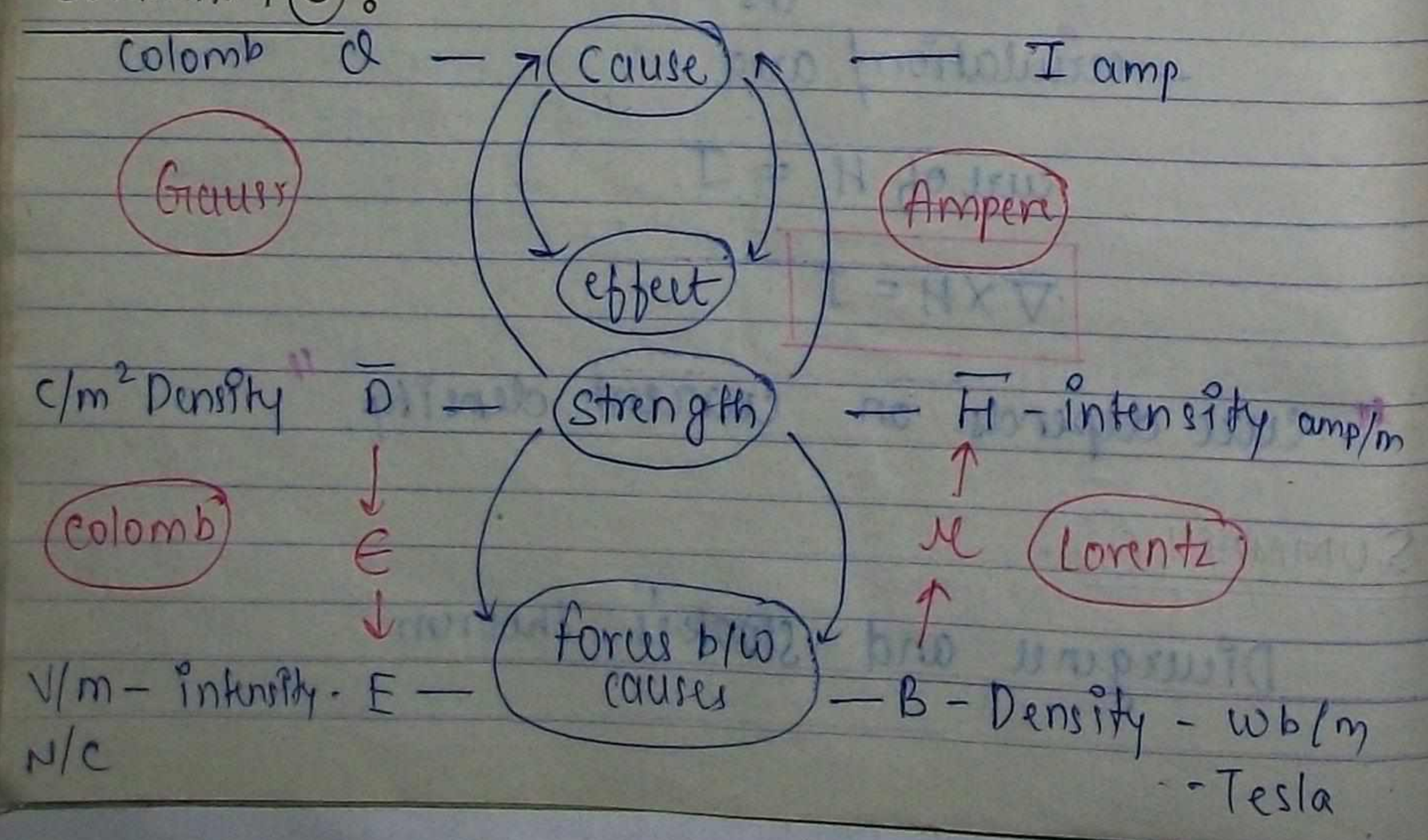
DIVERGENCE THEOREM

$\oint \mathbf{H} \cdot d\mathbf{l} = I = \int \mathbf{J} \cdot d\mathbf{s} = \int (\nabla \times \mathbf{H}) \cdot d\mathbf{s}$

$\oint_{\text{closed}} \mathbf{H} \cdot d\mathbf{l} = \int (\nabla \times \mathbf{H}) \cdot d\mathbf{s}$

STROKES THEOREM

SUMMARY (3):



SUMMARY (4): MAXWELL'S EQⁿ FOR STATIC E/H FIELDS

INTEGRAL FORM

POINT FORM

- | | | | | |
|---|--|---------------------|---|----------------------------|
| ① | $\oint \mathbf{D} \cdot d\mathbf{s} = Q$ | Gauss law | $\nabla \cdot \mathbf{D} = \rho_v$ | |
| ② | $\oint \mathbf{E} \cdot d\mathbf{l} = 0$ | irrotational vector | $\nabla \times \mathbf{E} = 0$ | validity check for E field |
| ③ | $\oint \mathbf{B} \cdot d\mathbf{s} = 0$ | solenoidal vector | $\nabla \cdot \mathbf{B} = 0$ | validity check for H field |
| ④ | $\oint \mathbf{H} \cdot d\mathbf{l} = I$ | Ampere's law | $\nabla \times \mathbf{H} = \mathbf{J}$ | |

- The vector whose variations are radially outwards and has divergent nature cannot have variations in the angular direction i.e. its curly nature is always zero. Hence $(\nabla \times \mathbf{E} = 0)$ applying Stokes theorem

$\oint \mathbf{E} \cdot d\mathbf{l} = \int (\nabla \times \mathbf{E}) \cdot d\mathbf{s} = 0$

- mathematically,

$\nabla \times \mathbf{E} = 0$
 $\nabla \times \frac{\rho}{\epsilon} = 0$

$$\frac{1}{\epsilon} \nabla \times D = 0$$

$$\therefore \nabla \times D = 0$$

($\nabla \times D = 0$) is also possible correct and it apply when ϵ is constant and the material is homogenous and isotropic.

- when the cross product or curl is zero it means the vector never takes curvature anywhere, it is called as irrotational vector. This is a way of checking validity of electric field.

- mathematically,

$$\nabla \cdot B = 0$$

$$\nabla \cdot (\mu H) = 0$$

$$\mu \nabla \cdot H = 0$$

$$\nabla \cdot H = 0$$

$\mu = \text{constant}$

NOTE -

$$\textcircled{1} \oint E \cdot dl = 0 \quad \text{is KVL}$$

② Ψ_e or electric flux — units — coulomb.

Ψ_m or magnetic flux — unit — weber.

③ Faraday's law,

$$\frac{d\Psi_m}{dt} = \frac{\text{weber}}{\text{second}} = \text{volt}$$

$$\frac{d\Psi_e}{dt} = \frac{\text{coulomb}}{\text{second}} = \text{amp}$$

COORDINATE SYSTEMS

- A way of addressing/locating point in a 3D space from a known reference, is called coordinate system.

⇒ CARTESIAN COORDINATE SYSTEM -

- The reference is a system of three infinite mutually orthogonal plane.
- The cartesian coordinate system is suited for all sheet symmetry distribution.
Ex - sheet charge, uniform plane wave, Rectangular waveguide

Parameter - x, y, z

unit vector - a_x, a_y, a_z

⇒ CYLINDRICAL COORDINATE SYSTEM -

- The reference is a infinite line along the z-axis.
- cylindrical coordinate system is suited for axial symmetry distribution.
Ex - line charge, coaxial cable, current carrying using cylindrical waveguide

Parameter - ρ, ϕ, z

unit vector - a_ρ, a_ϕ, a_z

⇒ SPHERICAL COORDINATE SYSTEM -

- Here the reference is 1 point in origin.
- Best suited for point symmetry.
Ex - point charge, antenna.

Parameters - r, θ, ϕ .

unit vectors - a_r, a_θ, a_ϕ .

NOTE -

All coordinate systems are :

unit orthogonal, orthonormal right handed systems.

ORTHOGONALITY - The dot product of any two different unit vector of the same coordinate system is always zero.

$$a_x \cdot a_y = a_y \cdot a_z = a_z \cdot a_x = 0$$

$$a_x \cdot a_x = a_y \cdot a_y = a_z \cdot a_z = 1$$

ORTHONORMALITY - The cross product of any 2 different unit vector of the same coordinate system is always the third unit vector in a right handed order.

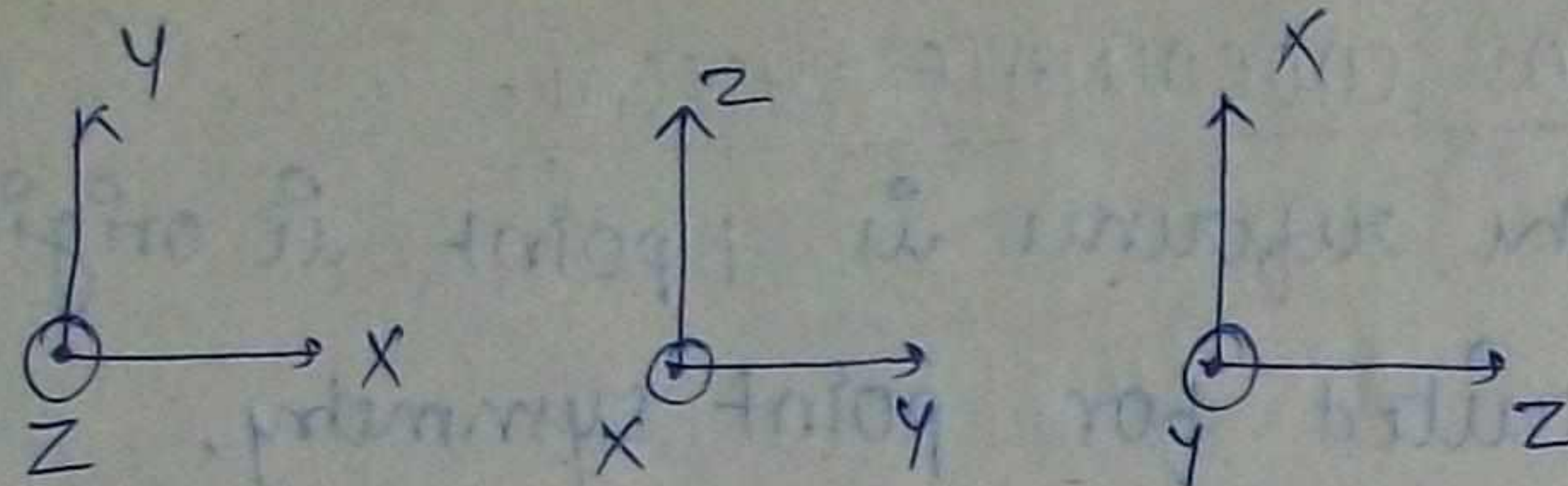
$$a_x \times a_y = a_z$$

$$a_y \times a_z = a_x$$

$$a_z \times a_x = a_y$$

$$a_x \times a_x = a_y \times a_y = a_z \times a_z = 0$$

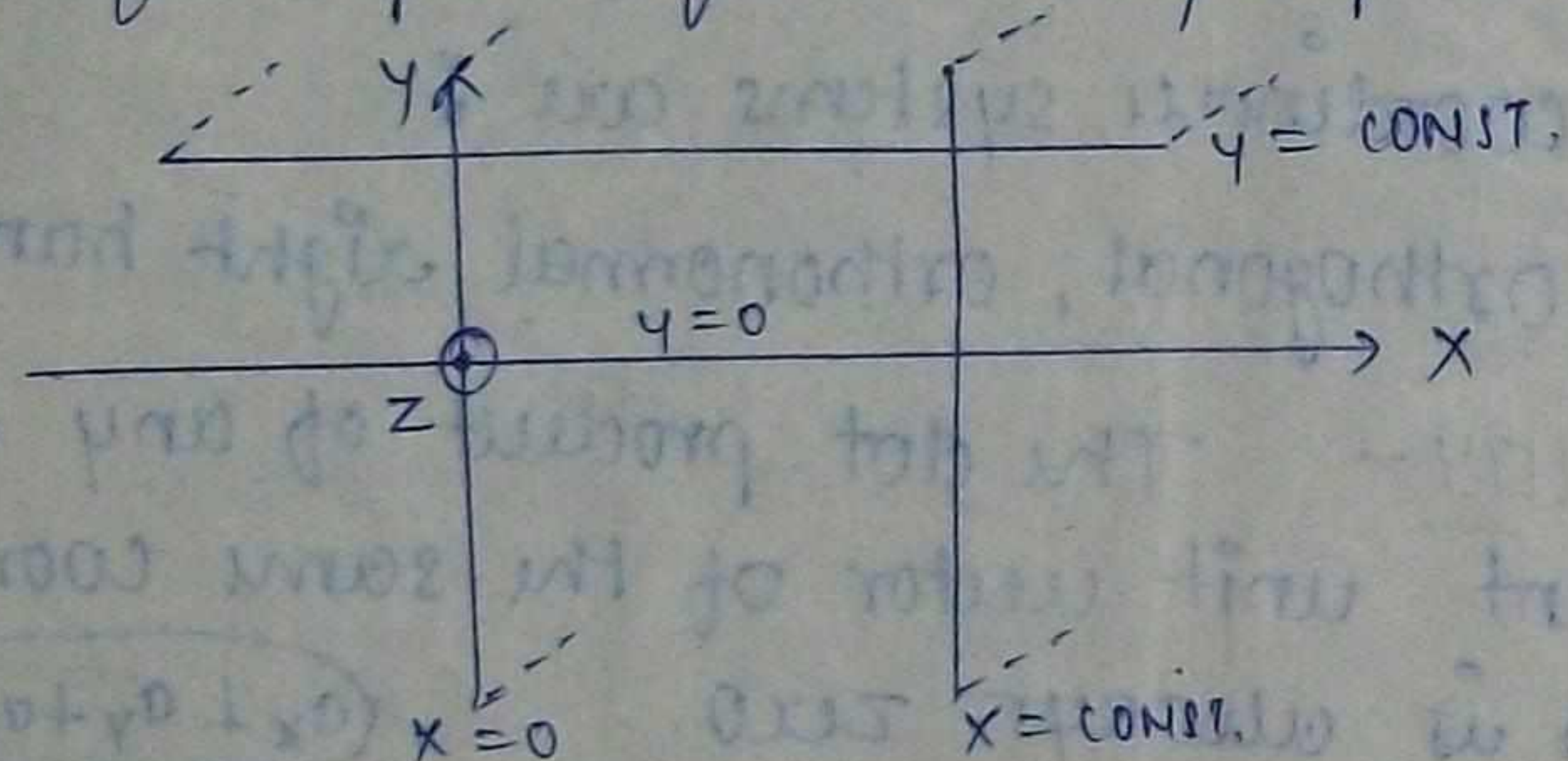
also, $a_x \times a_z = -a_y$



⇒ CARTESIAN COORDINATE SYSTEM -

Parameters:

x = The shortest distance or the $\perp^r$ distance of the point from the yz plane.



- The locus of all the points with the same x is $x = \text{const.}$ is a plane $\parallel^r$ to yz -plane.

- range of x is from $(-\infty, \infty)$ similarly for y and z .

- The locus of all the points with $x = \text{const.}$ & $y = \text{const.}$ is an infinite line parallel to z -axis.

$x = \text{const.} \Rightarrow yz \text{ plane} \quad \left. \begin{array}{l} x = \text{const.} \\ y = \text{const.} \end{array} \right\} \Rightarrow \text{LINE } \parallel^r z \text{ axis}$

$$\left. \begin{array}{l} x = \text{const.} \\ y = \text{const.} \\ z = \text{const.} \end{array} \right\} \Rightarrow \text{ORIGIN.}$$

⇒ CYLINDRICAL COORDINATE SYSTEM -

ρ = The radial distance or shortest distance or $\perp^r$ distance of the point from the reference line.

- The locus of all the points with the same ρ is $\rho = \text{const.}$ is a cylindrical surface concentric with the reference line.

range $[0, \infty)$

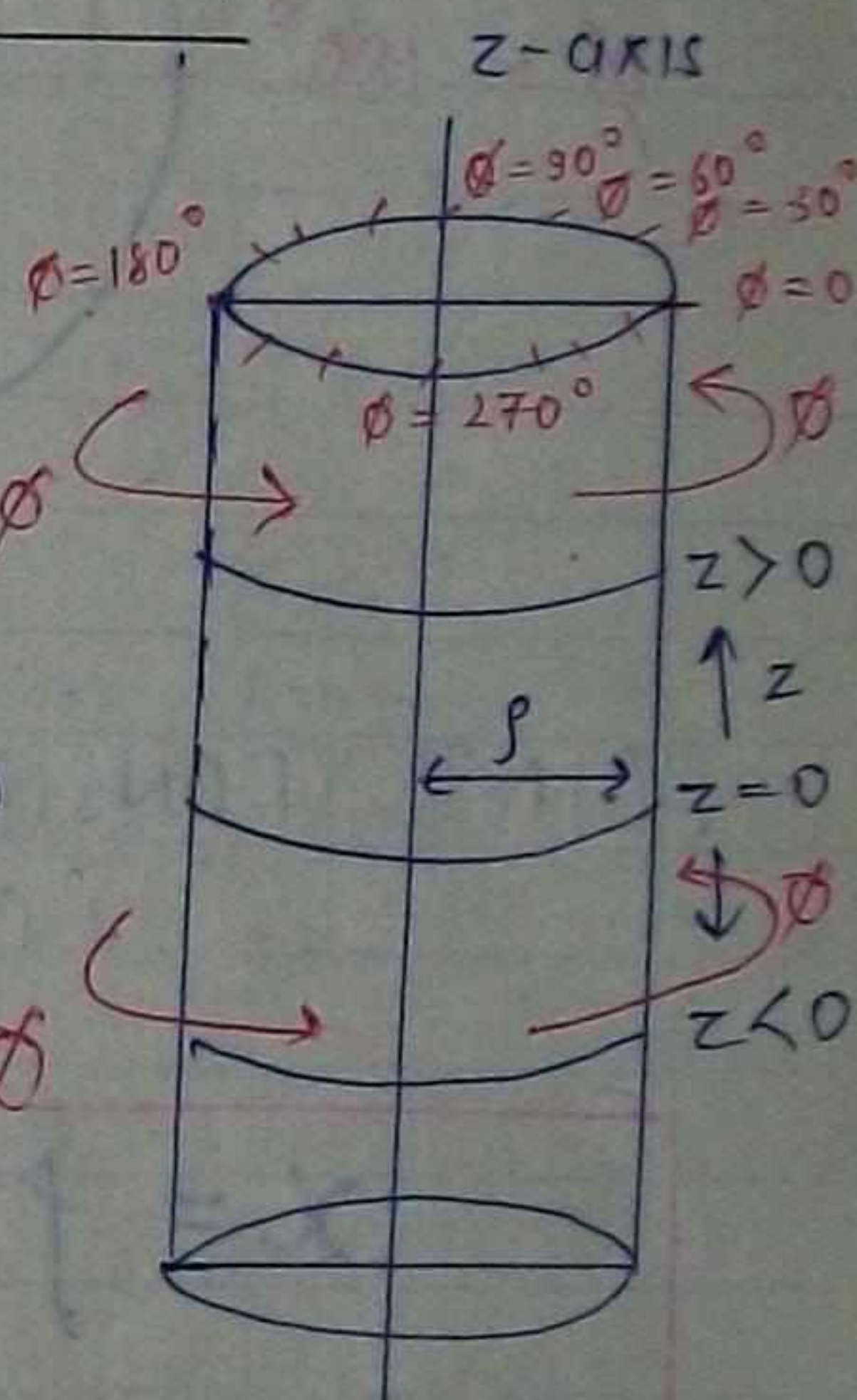
z = The height of the point along the reference line.

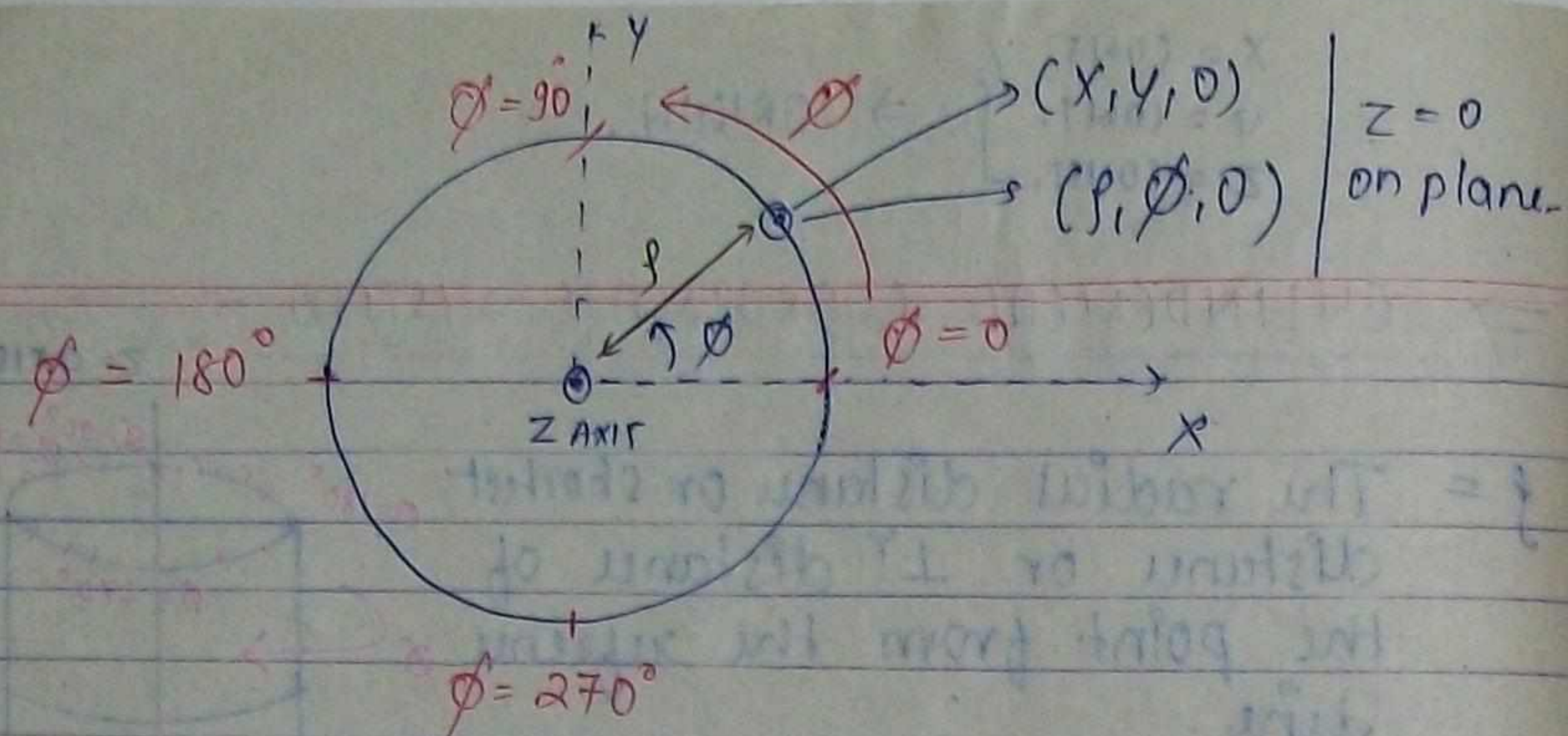
- The locus of all the points with the $\rho = \text{const.}$ and the $z = \text{const.}$ is the circle around the line.

range of $z \rightarrow (-\infty, \infty)$

ϕ = The orientation angle of the point around the reference line.

range of $\phi \rightarrow [0, 2\pi)$





POINT TRANSFORMATION -

(CARTESIAN $\leftrightarrow$ CYLINDRICAL)

$$x = r \cos \phi$$

$$y = r \sin \phi$$

$$z = z$$

$$r = \sqrt{x^2 + y^2}$$

$$\phi = \tan^{-1}\left(\frac{y}{x}\right)$$

$\Rightarrow$ SPHERICAL COORDINATE SYSTEM -

r = The radial distance or shortest distance of the point from the reference point (origin)

- The locus of all the point with same r is a concentric sphere around the origin.
range $[0, \infty)$

θ = The angle of the cone or the angle with the imaginary axis whose base is a circle on a $r = \text{const.}$ sphere.

range of $\theta = [0, \pi]$

ϕ = The orientation angle of the point around the vertical imaginary axis.

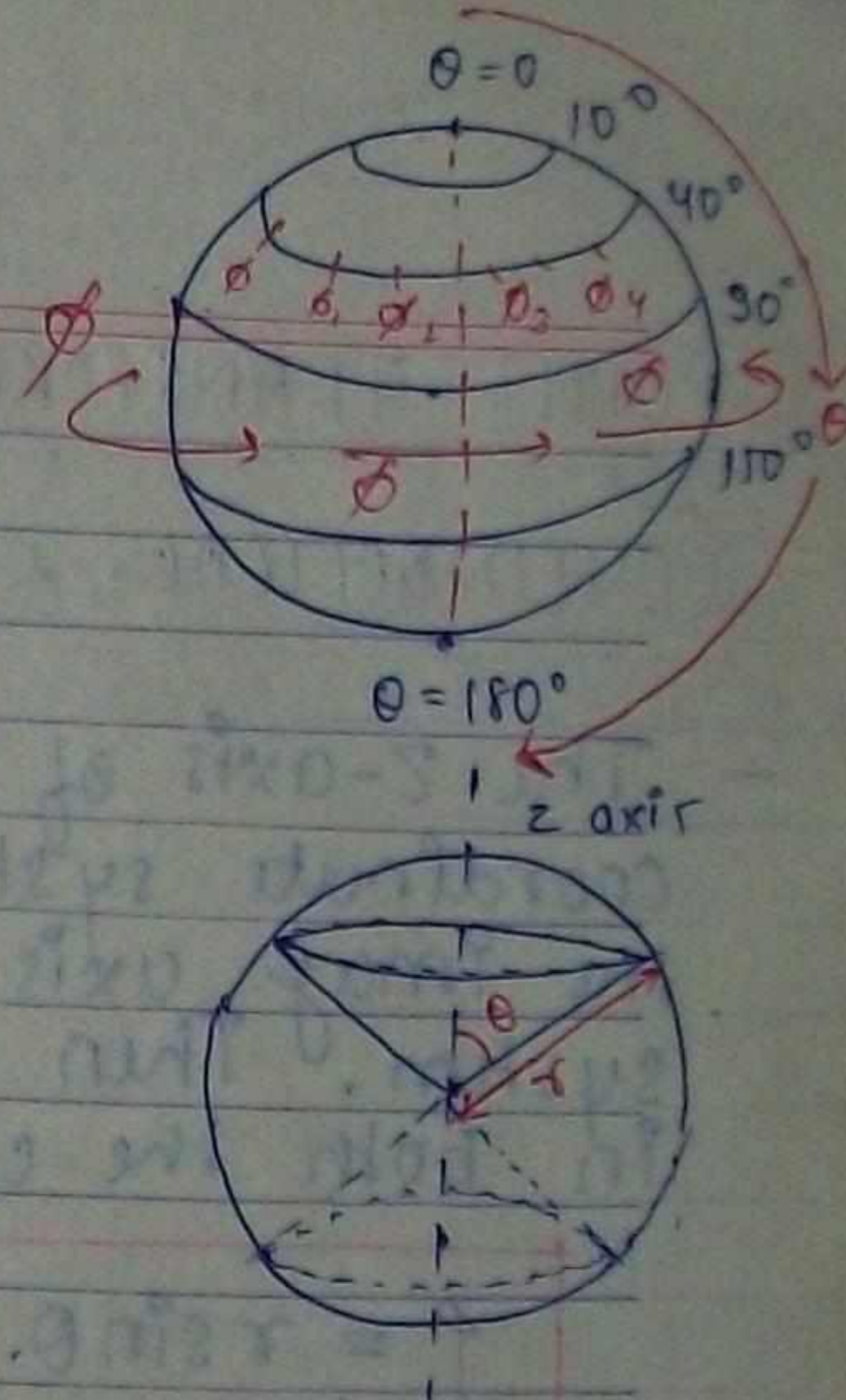
range of $\phi = [0, 2\pi]$

ϕ always follows right hand rule along z-axis (imag).

IDENTIFICATION OF θ -

- Consider an imaginary axis vertically through the centre of sphere, take an angle θ with this axis and rotate a radial segment around the imaginary axis.

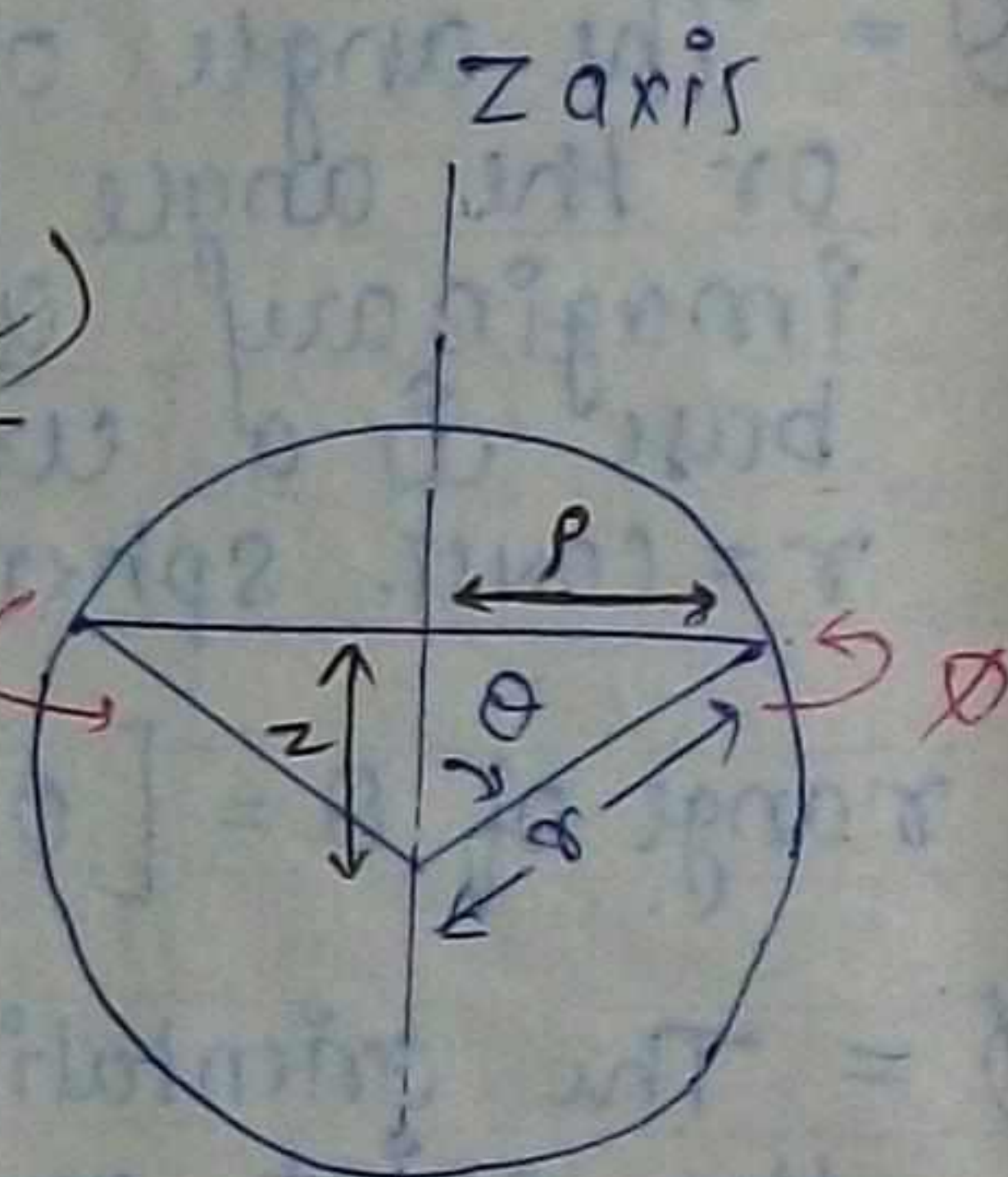
- The resultant shape is a cone whose base is a circle on the sphere, that circle is θ circle.



POINT TRANSFORMATION —

(CYLINDRICAL $\longleftrightarrow$ SPHERICAL)

- The z-axis of cylindrical coordinate system is the imag. axis of spherical system. Then ϕ is same in both the coordinates.



$$\rho = r \sin \theta$$

$$\phi = \phi$$

$$z = r \cos \theta$$

(CARTESIAN $\longleftrightarrow$ SPHERICAL)

$$x = r \sin \theta \cos \phi$$

$$y = r \sin \theta \sin \phi$$

$$z = r \cos \theta$$

$$r = \sqrt{x^2 + y^2 + z^2} \quad \theta = \cos^{-1}(z/r)$$

$$\phi = \tan^{-1}(y/x)$$

$\Rightarrow$ UNIT VECTORS IN CARTESIAN COORDINATE SYSTEM —

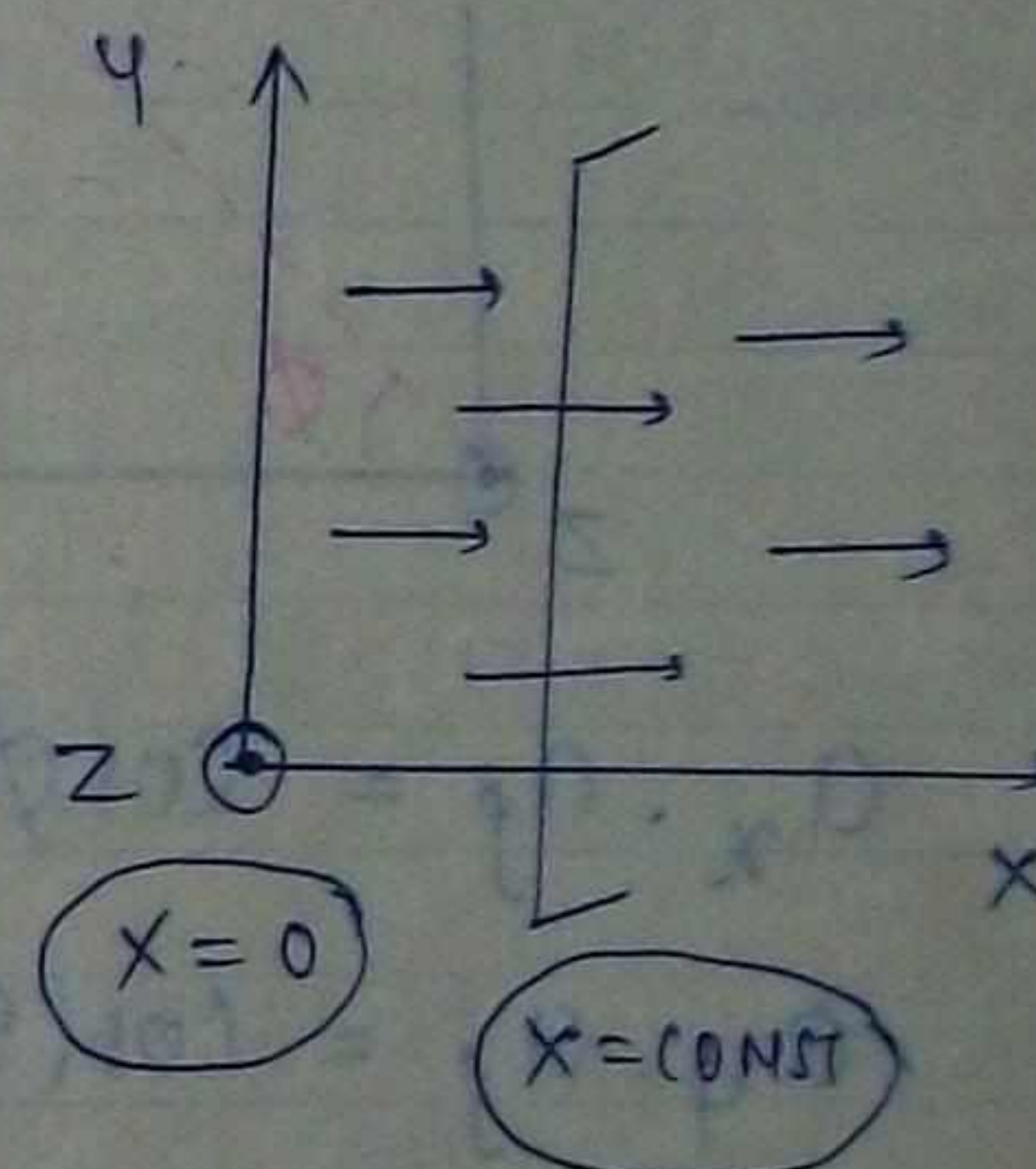
$$\hat{a}_x \perp \hat{a}_y \perp \hat{a}_z \perp \hat{a}_x$$

UNIT VECTOR :

It has a unit magnitude & a direction in which the parameter increases.

$\hat{a}_x$ - Has direction along x-axis

$\hat{a}_x$ - Direction is always normal to the $x = \text{const.}$ surface.



Similarly $\hat{a}_y, \hat{a}_z$.

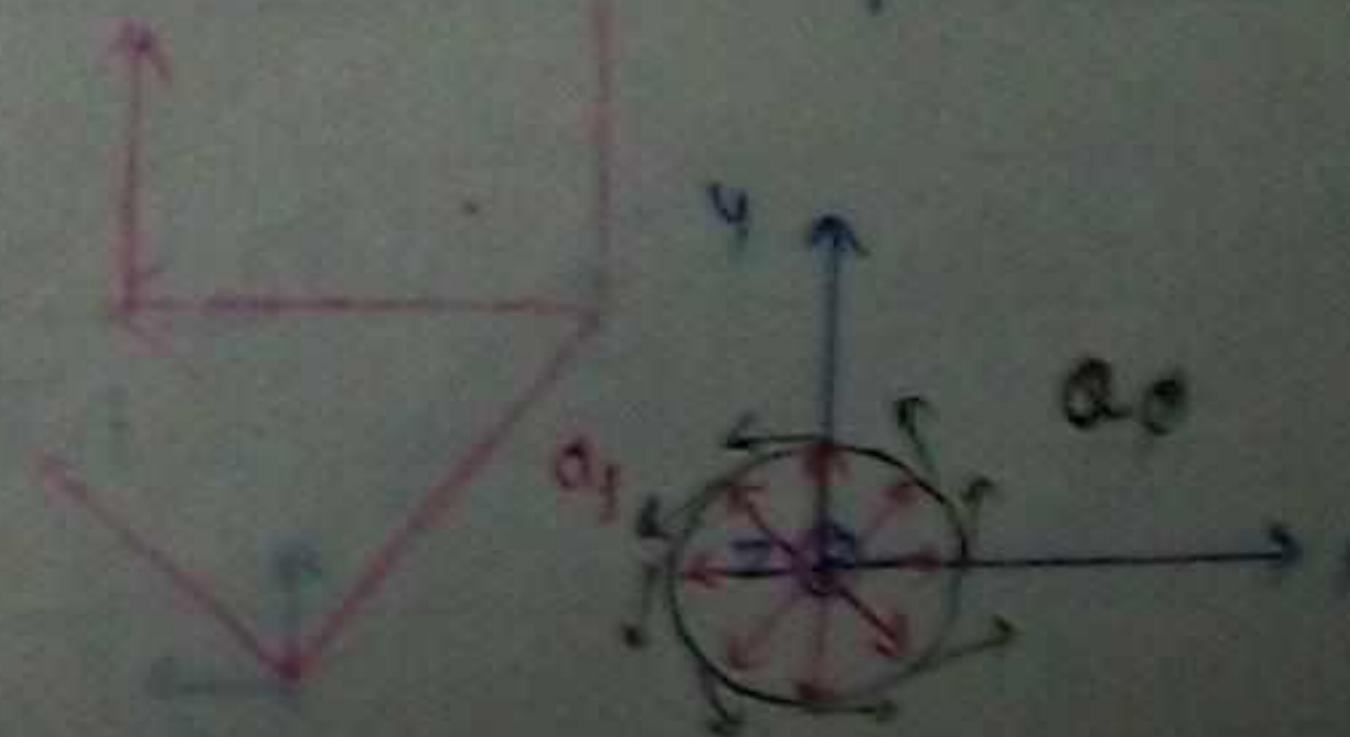
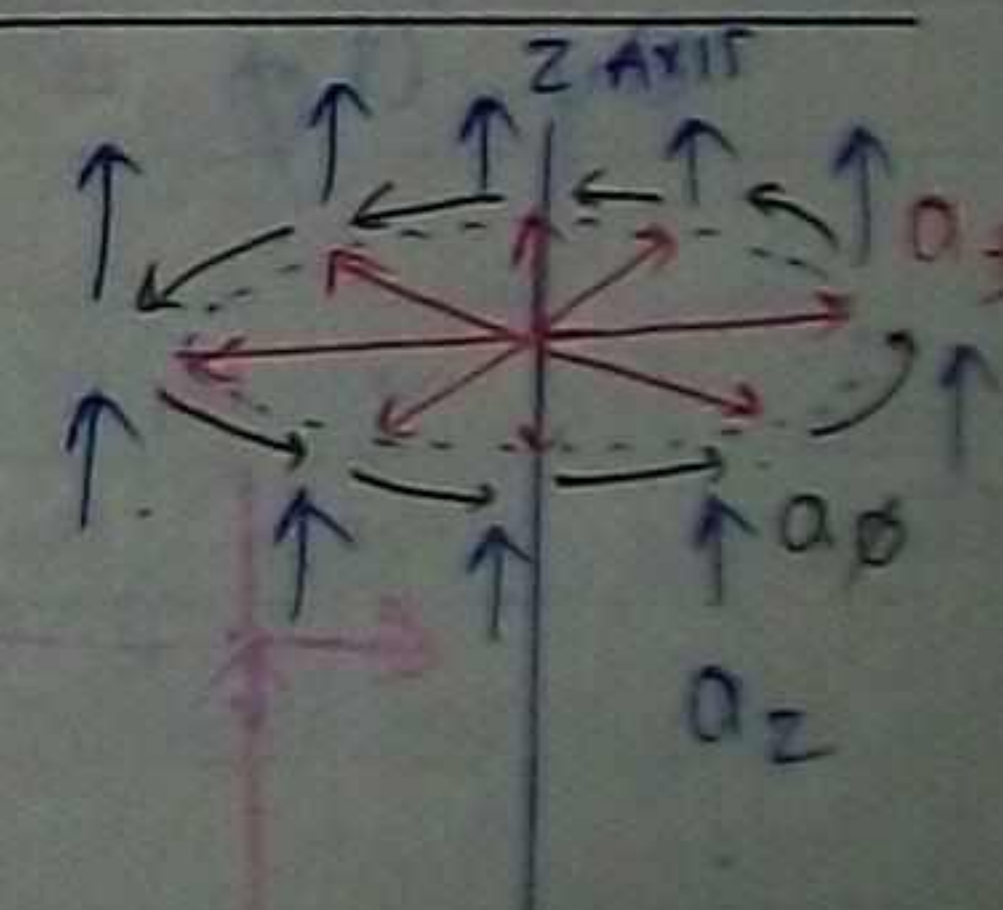
$\Rightarrow$ UNIT VECTORS IN CYLINDRICAL COORDINATE SYSTEM —

$\hat{a}_\rho$ = Radially outwards from z-axis and $\perp$ to z-axis.

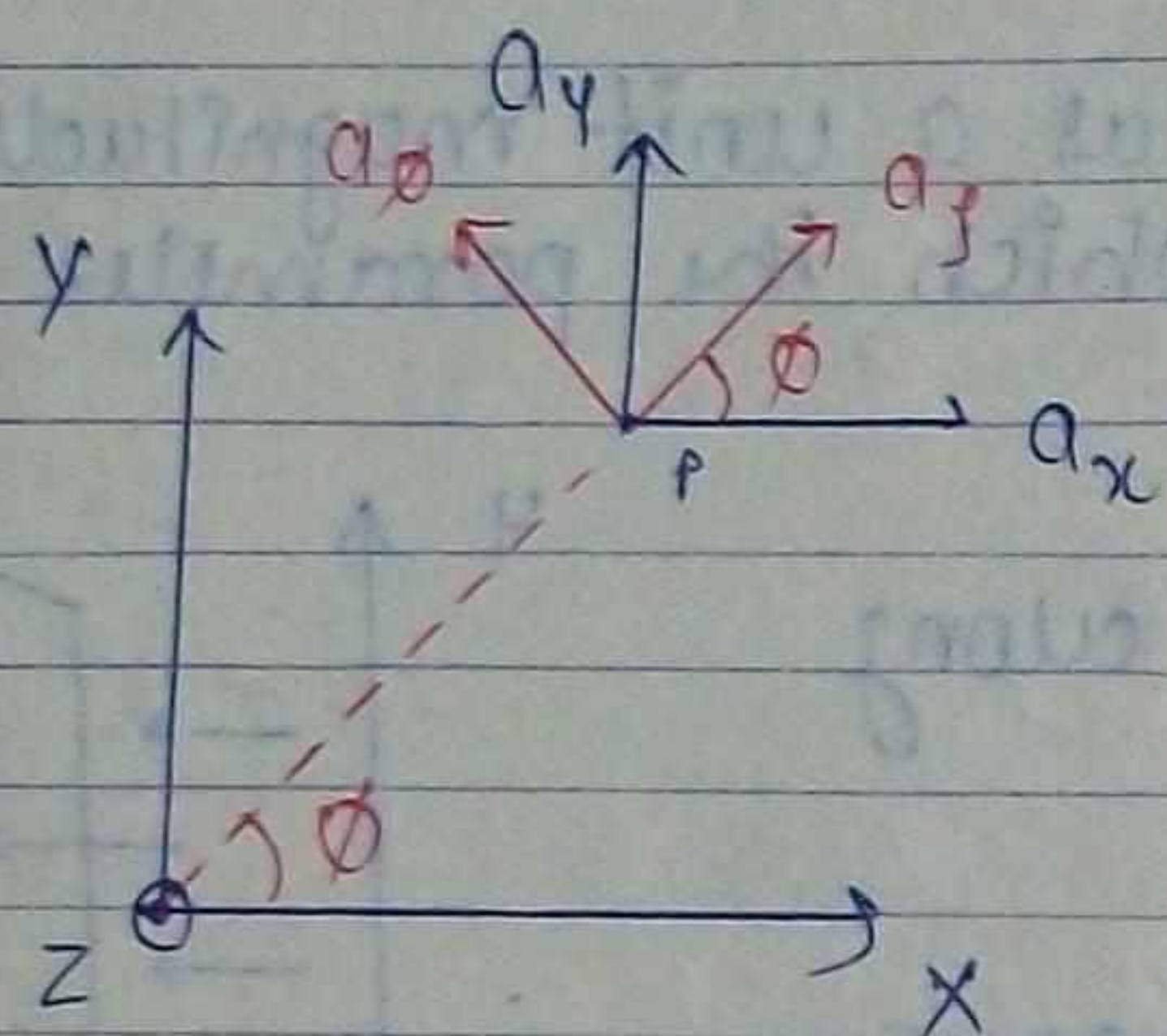
$\hat{a}_\phi$ = Tangentially around the z-axis.

$\hat{a}_z$ = linearly along the z-axis

$$\hat{a}_\rho \perp \hat{a}_\phi \perp \hat{a}_z \perp \hat{a}_\rho$$



⇒ VECTOR TRANSFORMATION -

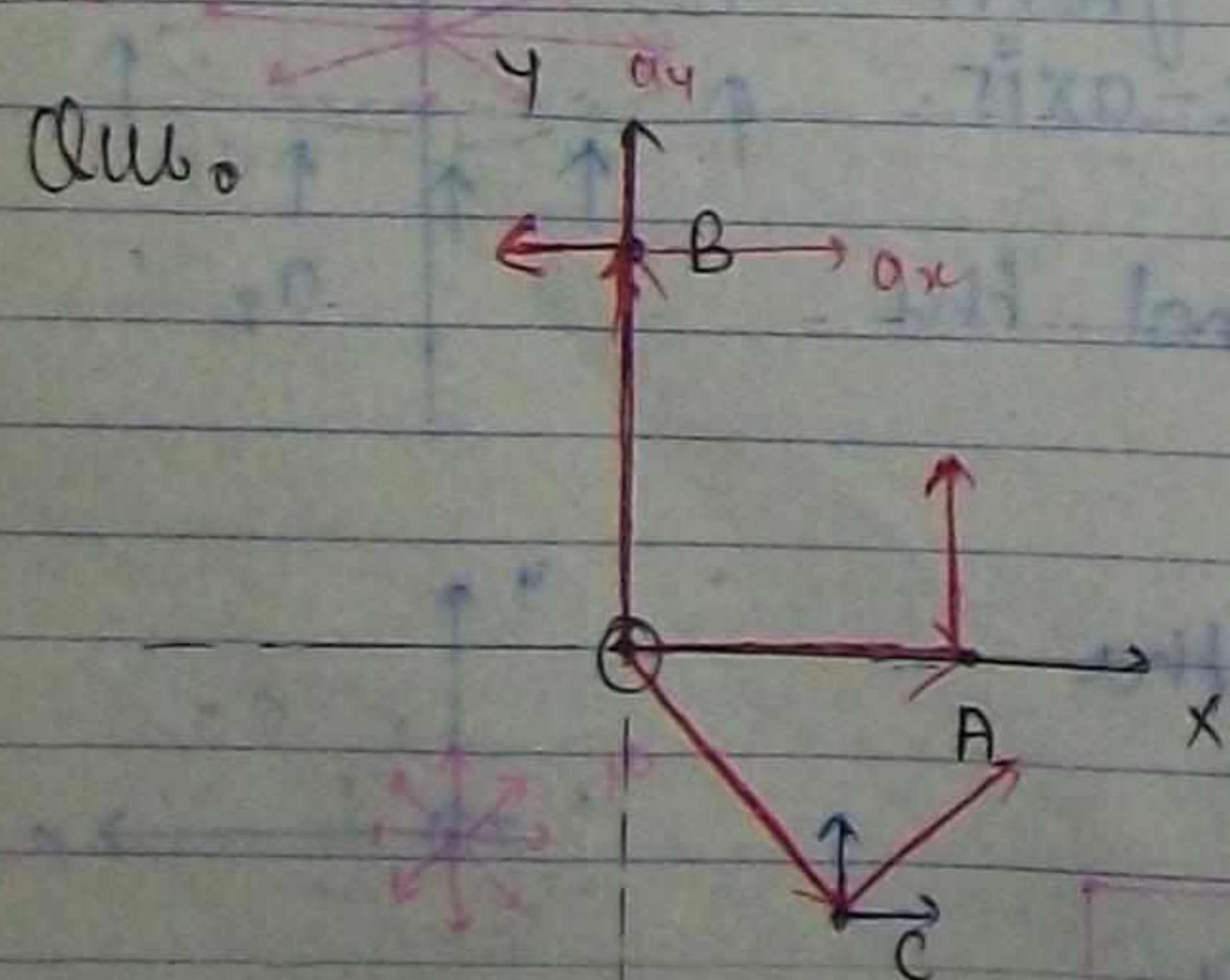


$$a_x \cdot a_\phi = \cos \phi$$

$$a_y \cdot a_\phi = \cos(90 - \phi) = \sin \phi$$

$$a_x \cdot a_\phi = -\sin \phi$$

$$a_y \cdot a_\phi = \cos \phi$$



Identify a_r, a_θ
at the point
A, B, C.

Solⁿ: At point A :

$$a_r = a_x, \quad a_\theta = a_y$$

At point B :

$$a_r = a_y, \quad a_\theta = -a_x$$

At point C :

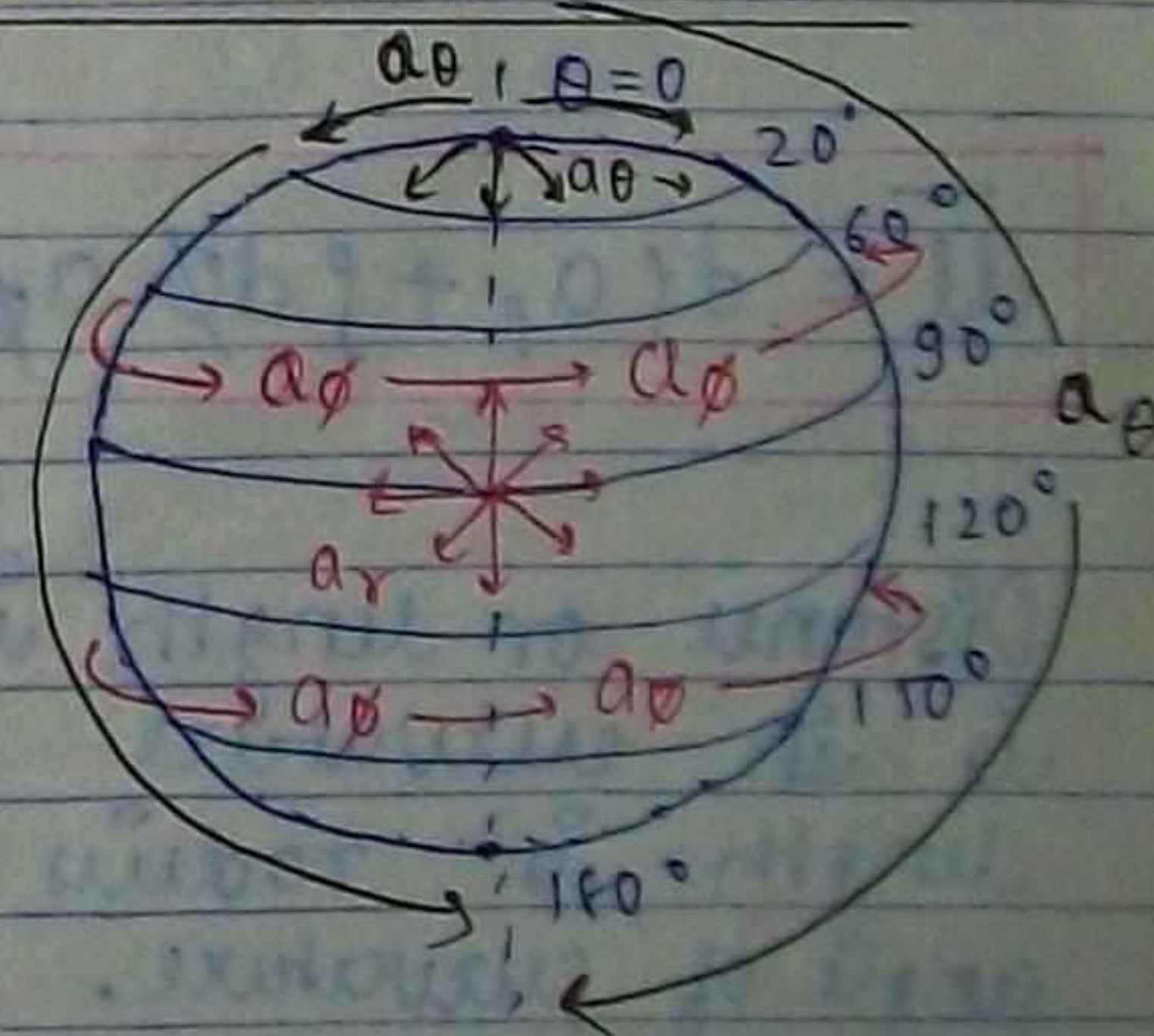
$$a_r = a_x, -a_y, \quad a_\theta = a_x, a_y$$

⇒ UNIT VECTORS IN SPHERICAL COORDINATE SYSTEM -

a_r = radially outward
from the origin.

a_θ = vertically from the
top to the bottom
of the sphere

a_ϕ = Horizontally around the sphere at
various heights (i.e. at dist θ).



$$a_r \perp a_\theta \perp a_\phi \perp a_r$$

21/09/

⇒ LINE, SURFACE & VOLUME INTEGRAL -

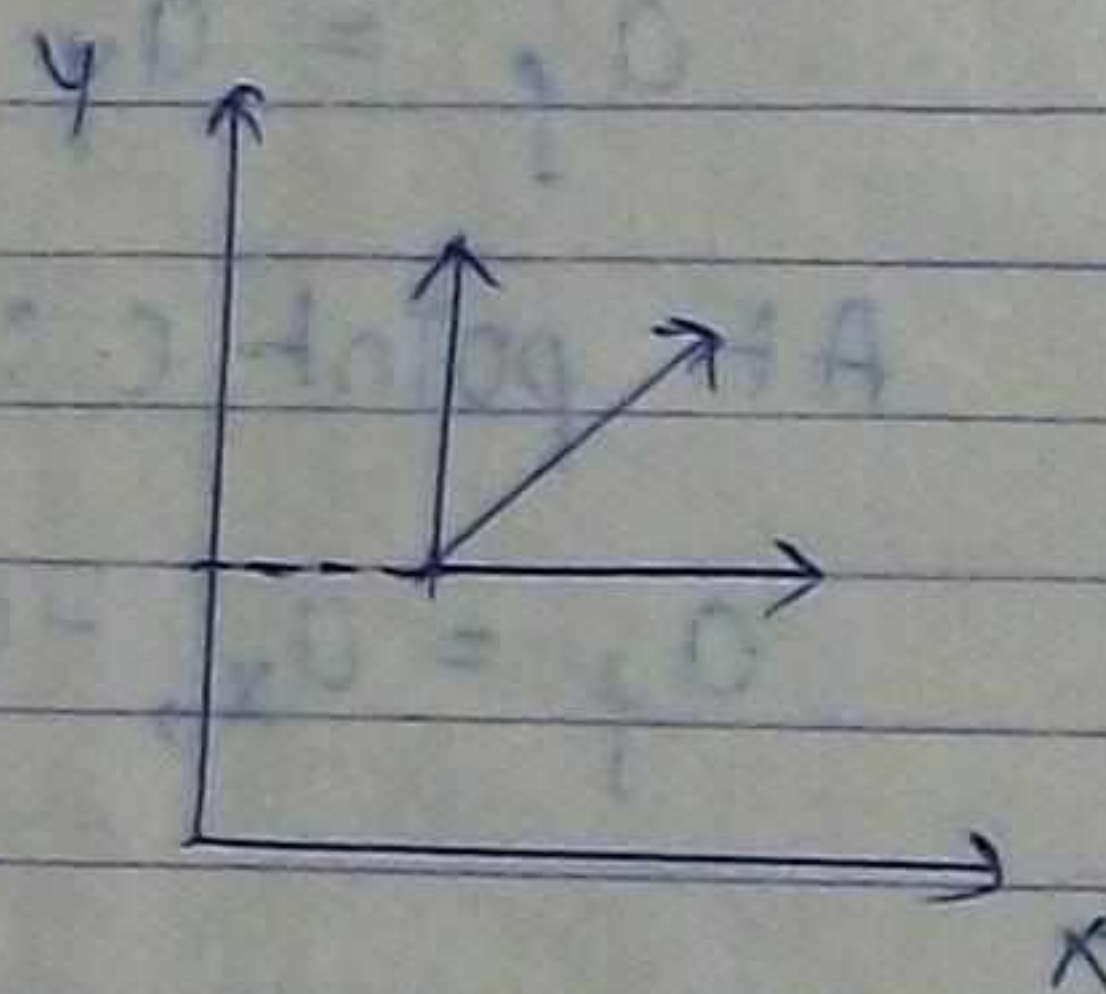
① LINE INTEGRAL -

- Line as a vector:

Line has a direction in which the parameter of the length increases.

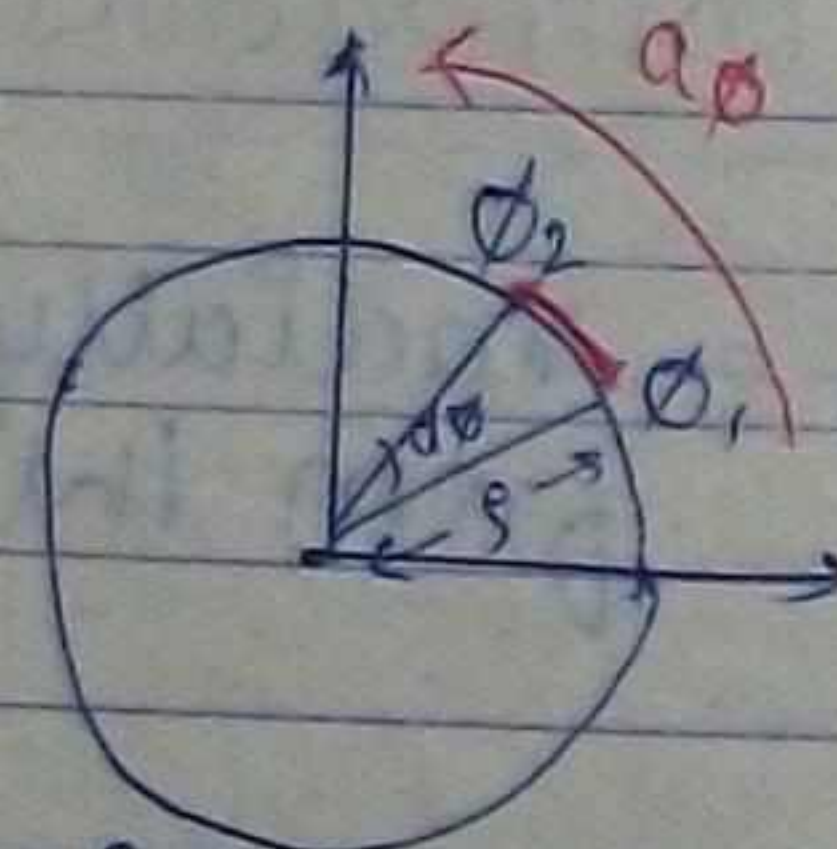
FOR CARTESIAN -

$$d\vec{l} = dx\vec{a}_x + dy\vec{a}_y + dz\vec{a}_z$$



FOR CYLINDRICAL -

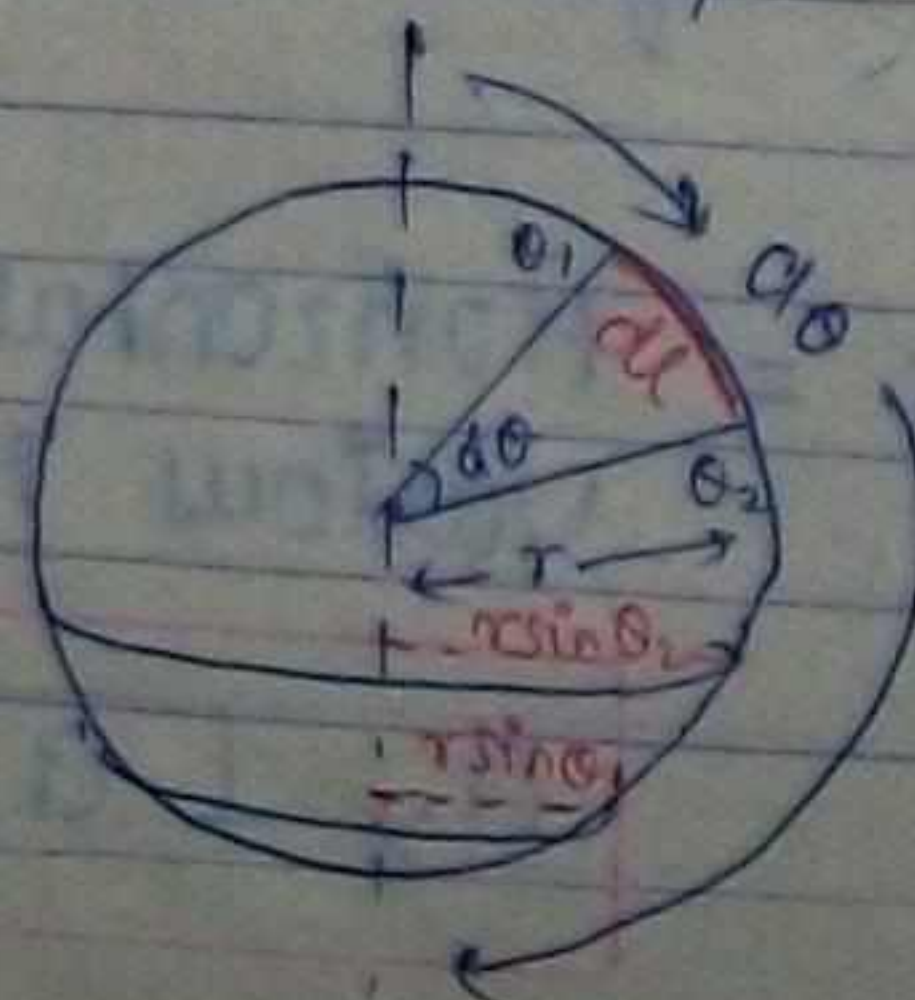
$$d\vec{l} = ds\vec{a}_s + s d\phi\vec{a}_\phi + dz\vec{a}_z$$



Distance or length in angular direction is in curvature and hence curvature length is radius of curvature multiply by angle of curvature.

FOR SPHERICAL -

$$d\vec{l} = dr\vec{a}_r + r d\theta\vec{a}_\theta + r \sin\theta d\phi\vec{a}_\phi$$



- The curvature radius in the θ direction is always r . And the curvature radius in the ϕ direction depends on the height of the circle on the sphere hence $r \sin\theta$.

REMEMBER -

	x	y	z		1	1	1
	ρ	ϕ	z		1	ρ	1
	r	θ	ϕ		1	r	$r \sin\theta$
PARAMETERS							
	u	v	w		h_1	h_2	h_3
SCALING FACTORS							

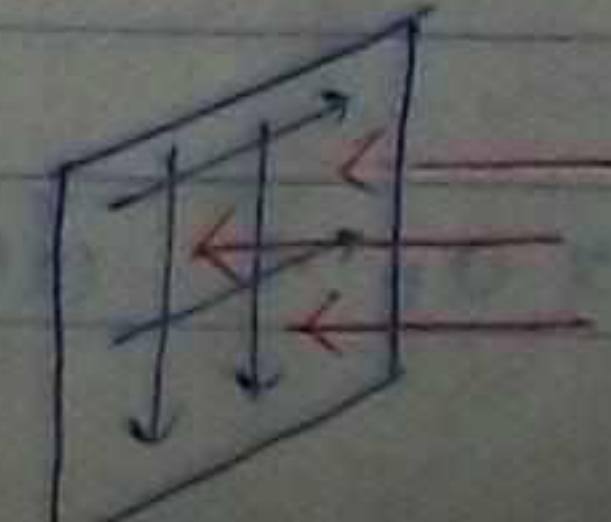
GENERALISED -

$$d\vec{l} = h_1 du\vec{a}_u + h_2 dv\vec{a}_v + h_3 dw\vec{a}_w$$

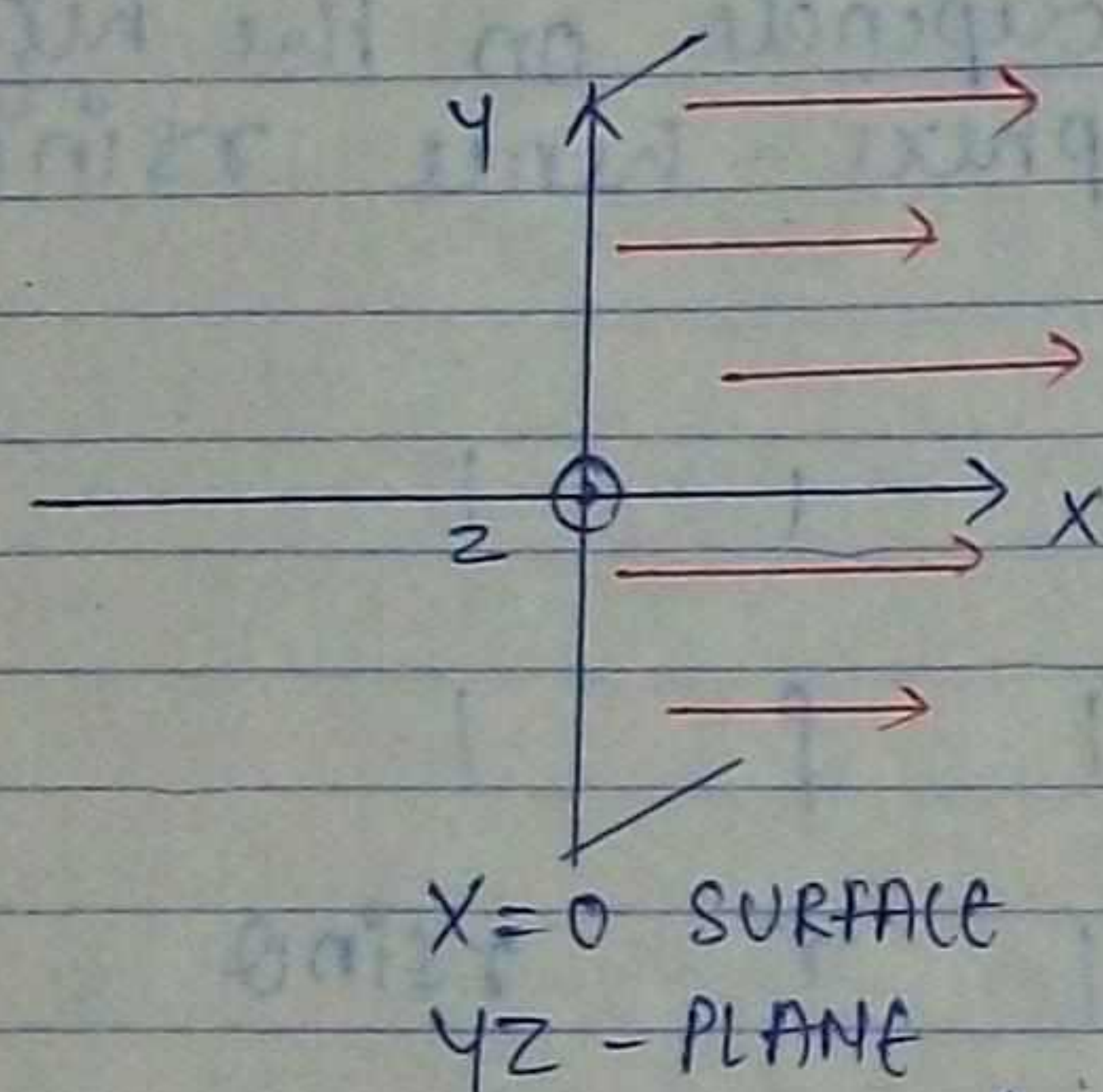
② SURFACE INTEGRAL -

Surface as a vector:

Surface is a vector and has a direction through which any physical quantity can cut through or pass through it.

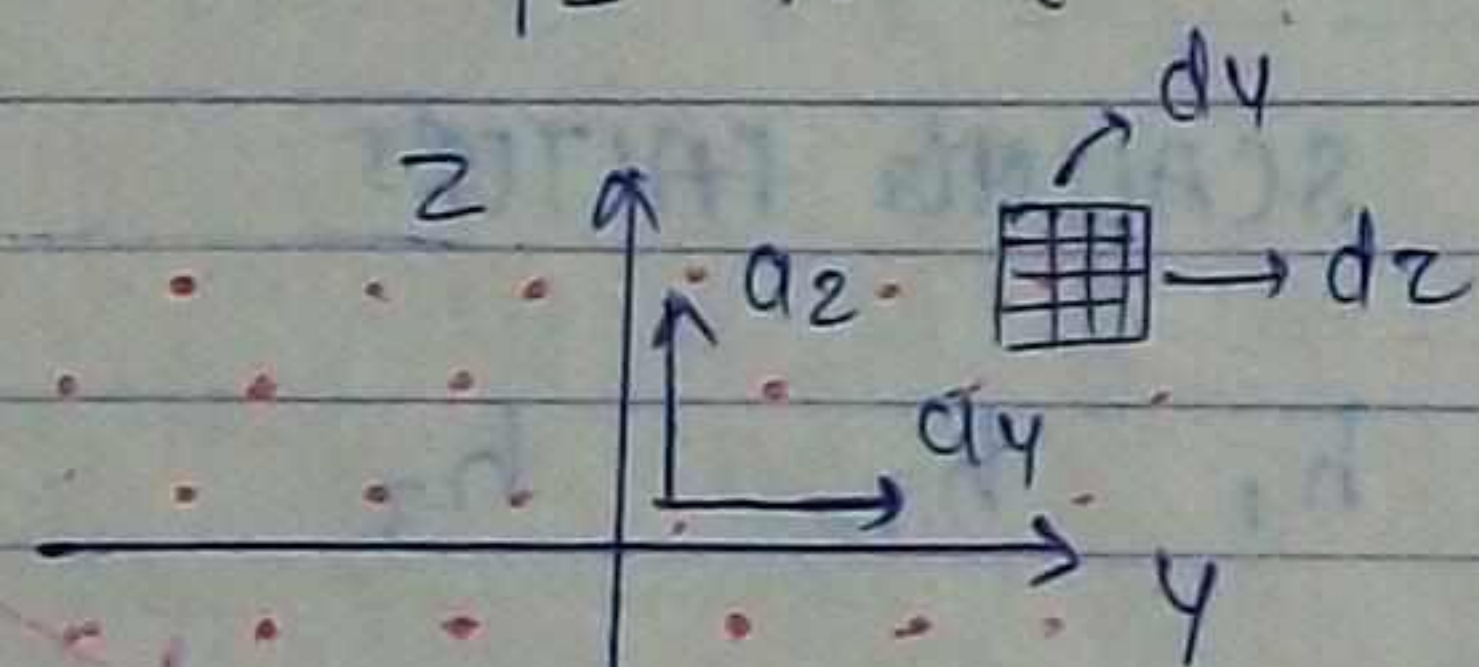


- Surface direction is the direction which is normal to the plane of the surface.



$x = \text{CONST.}$

a_x - normal surface direction.



a_y, a_z - tangential.

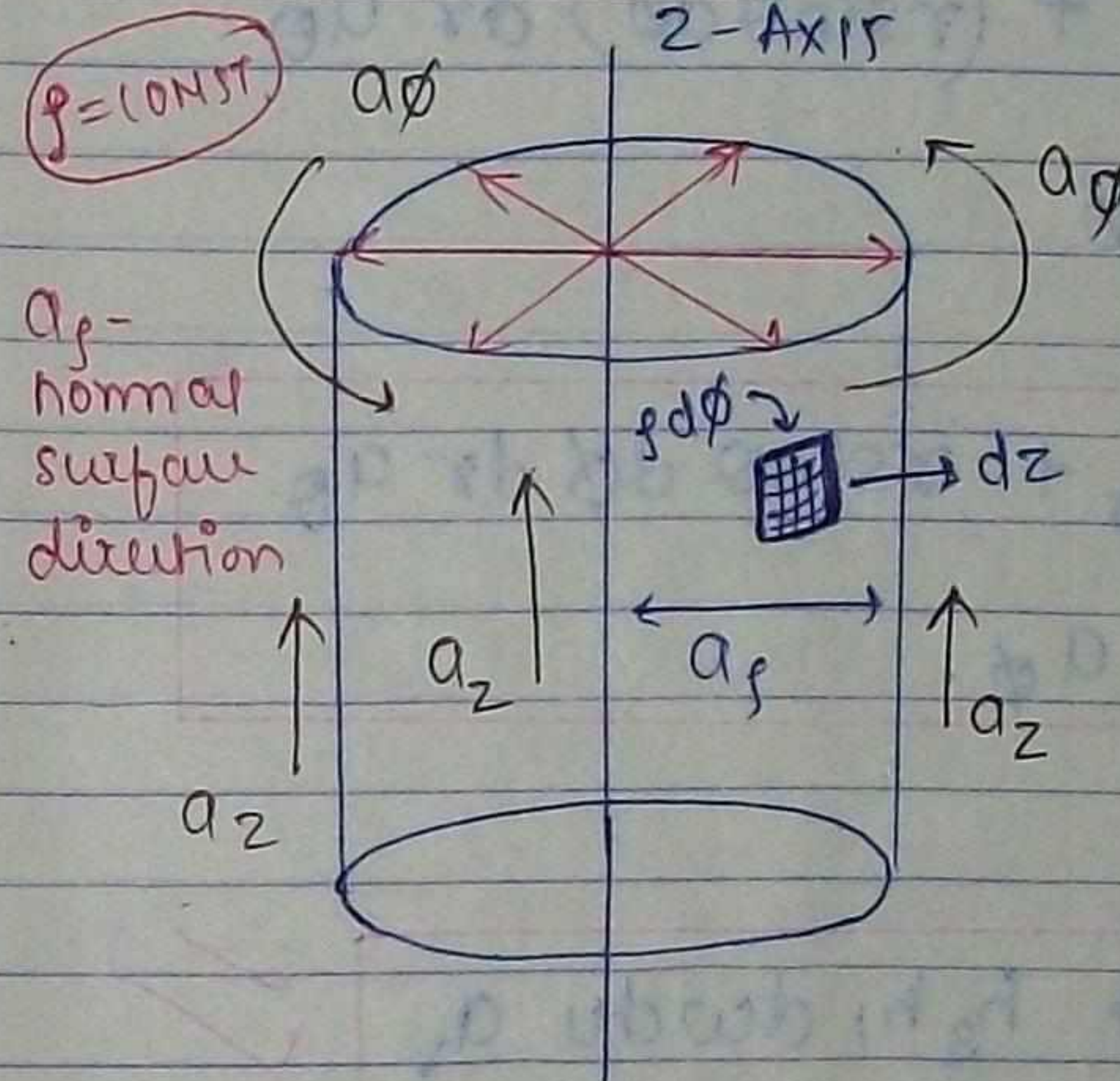
FOR CARTESIAN -

$$d\vec{s} = dydz a_x + dzdx a_y + dx dy a_z$$

surface normal

$$\therefore d\vec{s} = dydz a_x + dzdx a_y + dx dy a_z$$

FOR CYLINDRICAL -



a_ρ - normal surface direction

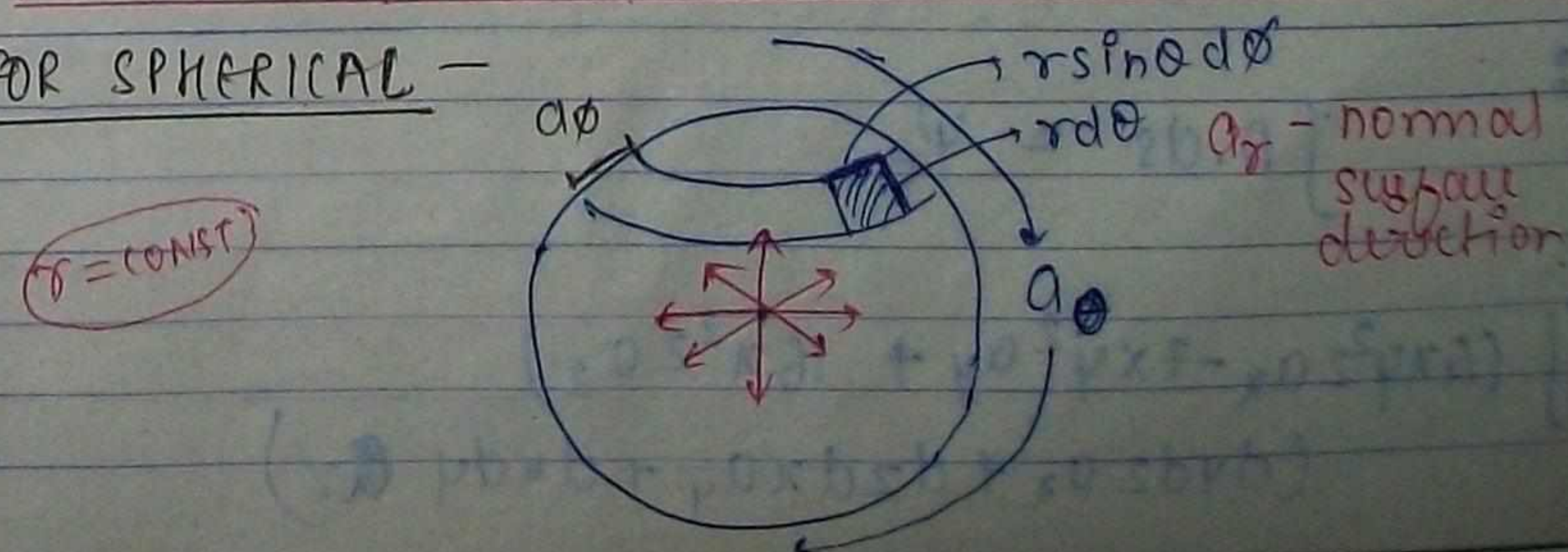
a_ϕ, a_z - tangential to $\rho = \text{CONST.}$ surface.

$$d\vec{s} = (\rho d\phi) dz a_\rho + dz d\rho a_\phi + d\rho (\rho d\phi) a_z$$

product of two surfaces normal remaining surface.

$$d\vec{s} = \rho d\phi dz a_\rho + dz d\rho a_\phi + \rho d\rho d\phi a_z$$

FOR SPHERICAL -



$r = \text{CONST.}$

a_r - normal surface direction

product of surface
normal

$$d\vec{s} = r^2 \sin\theta d\theta d\phi \hat{a}_r + (r \sin\theta d\phi) dr \hat{a}_\theta + (r d\theta) dr \hat{a}_\phi$$

$$\therefore d\vec{s} = r^2 \sin\theta d\theta d\phi \hat{a}_r + r \sin\theta d\phi dr \hat{a}_\theta + r dr d\theta \hat{a}_\phi$$

GENERALISED -

$$d\vec{s} = h_2 h_3 dv dw \hat{a}_u + h_3 h_1 dw du \hat{a}_v + h_1 h_2 du dv \hat{a}_w$$

Ques. Given an electric field whose $\vec{D} = 4xy^2z \hat{a}_x - 7xy^2 \hat{a}_y + 16x^2z \hat{a}_z$ C/m². Find the flux cross through the surface $y=2$, $0 < x < 1$, $0 < z < 2$.

Solⁿ:

$$\int \vec{D} \cdot d\vec{s} = \Psi_e$$

$$\iint (4xy^2z \hat{a}_x - 7xy^2 \hat{a}_y + 16x^2z \hat{a}_z) (dydz \hat{a}_x + dzdx \hat{a}_y + dx dy \hat{a}_z)$$

$$\Psi_e = \iiint (4xy^2z) dy dz + (-7xy^2) dx dz + (16x^2z) dx dy$$

Since $y=2$ surface $dy=0$

$$\Psi_e = \int_{x=0}^1 \int_{z=0}^2 (-7xy^2) dx dz \quad (\text{ay, ay})$$

$$\Psi_e = \int_{x=0}^1 \int_{z=0}^2 (-28x) dx dz = -28 \text{ C}$$

↓
passing through

Ques. Repeat the same quesⁿ if $\vec{D} = 5(\rho-3)^2 \hat{a}_\rho$ C/m² and surface $\rho=4$, $0 < \phi < \pi$, $-5 < z < 5$.

Solⁿ:

$$\int \vec{D} \cdot d\vec{s} = \Psi_e$$

$$= \int_{\phi=0}^{\pi} \int_{z=-5}^5 5(\rho-3)^2 \rho d\phi dz$$

$$= \int_{\phi=0}^{\pi} \int_{z=-5}^5 20 d\phi dz = 200\pi$$

III VOLUME INTEGRAL -

Volume as a scalar:

It is the scalar triple product of the lengths in all the three direction.

FOR CARTESIAN -

$$dV = dx dy dz$$

FOR CYLINDRICAL -

$$dV = r^2 \sin \theta dr d\theta d\phi = \rho d\rho d\phi dz$$

FOR SPHERICAL -

$$dV = r^2 \sin \theta dr d\theta d\phi$$

GENERALISED -

$$dV = h_1 h_2 h_3 du dv dw$$

$\int dV = \text{volume of sphere} = \int_{r=0}^R \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} r^2 \sin \theta dr d\theta d\phi$

$$= \frac{4}{3} \pi R^3$$

$\Rightarrow$ DIVERGENCE & CURL OF A VECTOR -

$$\vec{A} = A_u a_u + A_v a_v + A_w a_w$$

$$\nabla \cdot \vec{A} = \frac{1}{h_1 h_2 h_3} \left[\frac{\partial}{\partial u} (h_2 h_3 A_u) + \frac{\partial}{\partial v} (h_3 h_1 A_v) + \frac{\partial}{\partial w} (h_1 h_2 A_w) \right]$$

FOR CARTESIAN -

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

FOR CYLINDRICAL -

$$\nabla \cdot \vec{A} = \frac{1}{\rho} \left[\frac{\partial}{\partial \rho} (\rho A_\rho) + \frac{\partial}{\partial \phi} A_\phi + \frac{\partial}{\partial z} A_z \right]$$

$$= \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho A_\rho) + \frac{1}{\rho} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

FOR SPHERICAL -

$$\nabla \cdot \vec{A} = \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial r} (r^2 \sin \theta A_r) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} (r \sin \theta A_\theta) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi} (r A_\phi)$$

$$\nabla \cdot \bar{A} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 A_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta A_\theta) + \frac{1}{r \sin \theta} \frac{\partial A_\phi}{\partial \phi}$$

Ques. If $\bar{D} = f z \cos^2 \phi \mathbf{a}_\phi$ C/m² find the volume charge density at $(1, \pi/4, 1)$

Solⁿ: $\bar{D} = f z \cos^2 \phi \mathbf{a}_\phi$

$$\nabla \cdot \bar{D} = \frac{1}{f} \frac{d}{d\phi} (f^2 z \cos^2 \phi)$$

$$\nabla \cdot \bar{D} = \frac{1}{f} \times z \cos^2 \phi \times 2f$$

$$\nabla \cdot \bar{D} = 2 z \cos^2 \phi = 2 \times 1 \cdot \cos^2(\pi/4) = 1 \text{ C/m}^3$$

Ques. $\bar{D} = f z \cos^2 \phi \mathbf{a}_z$ C/m² at $(1, \pi/4, 1)$

Solⁿ: $\nabla \cdot \bar{D} = f \cos^2 \phi$

$$= 1 \times \left(\frac{1}{\sqrt{2}} \right)^2 = \frac{1}{2} \text{ C/m}^3$$

Ques. $\oint 5r \mathbf{a}_r \cdot d\mathbf{s} \mathbf{a}_r$

Solⁿ:

$$= \int \nabla \cdot (5r \mathbf{a}_r) dv$$

$$\nabla \cdot (5r \mathbf{a}_r) = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 5r) = \frac{5 \cdot 3r^2}{r^2} = 15$$

$$= \int 15 dv = 15V$$

⇒ CURL OF A VECTOR -

$\nabla \times \bar{A} = \frac{1}{h_1 h_2 h_3}$	$\begin{vmatrix} h_1 \mathbf{a}_u & h_2 \mathbf{a}_v & h_3 \mathbf{a}_w \\ \frac{\partial}{\partial u} & \frac{\partial}{\partial v} & \frac{\partial}{\partial w} \\ h_1 A_u & h_2 A_v & h_3 A_w \end{vmatrix}$
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⇒ GAUSS LAW -

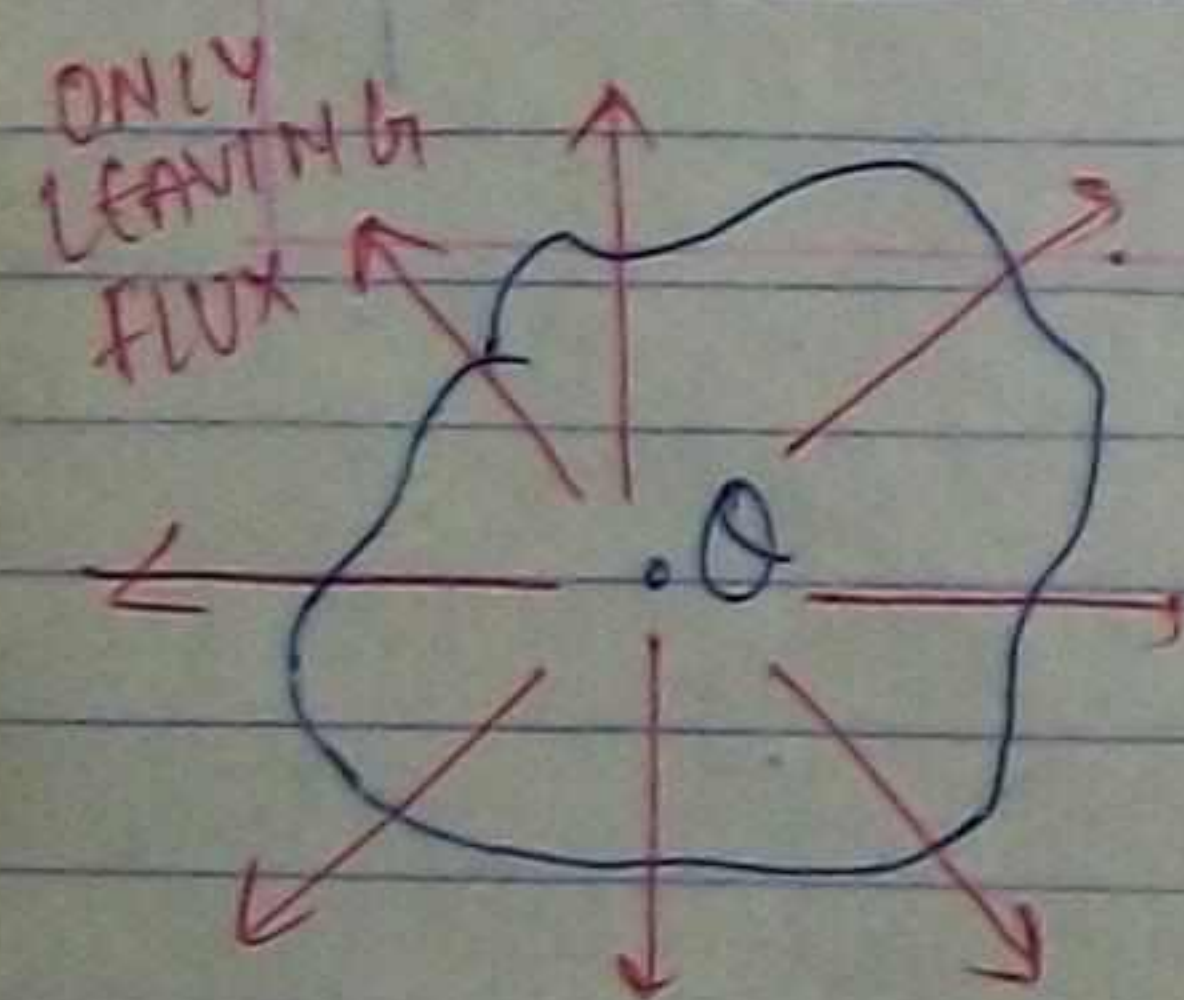
The net flux (electric flux) leaving any closed surface is equal to the charge enclosed in that volume.

CAUSE

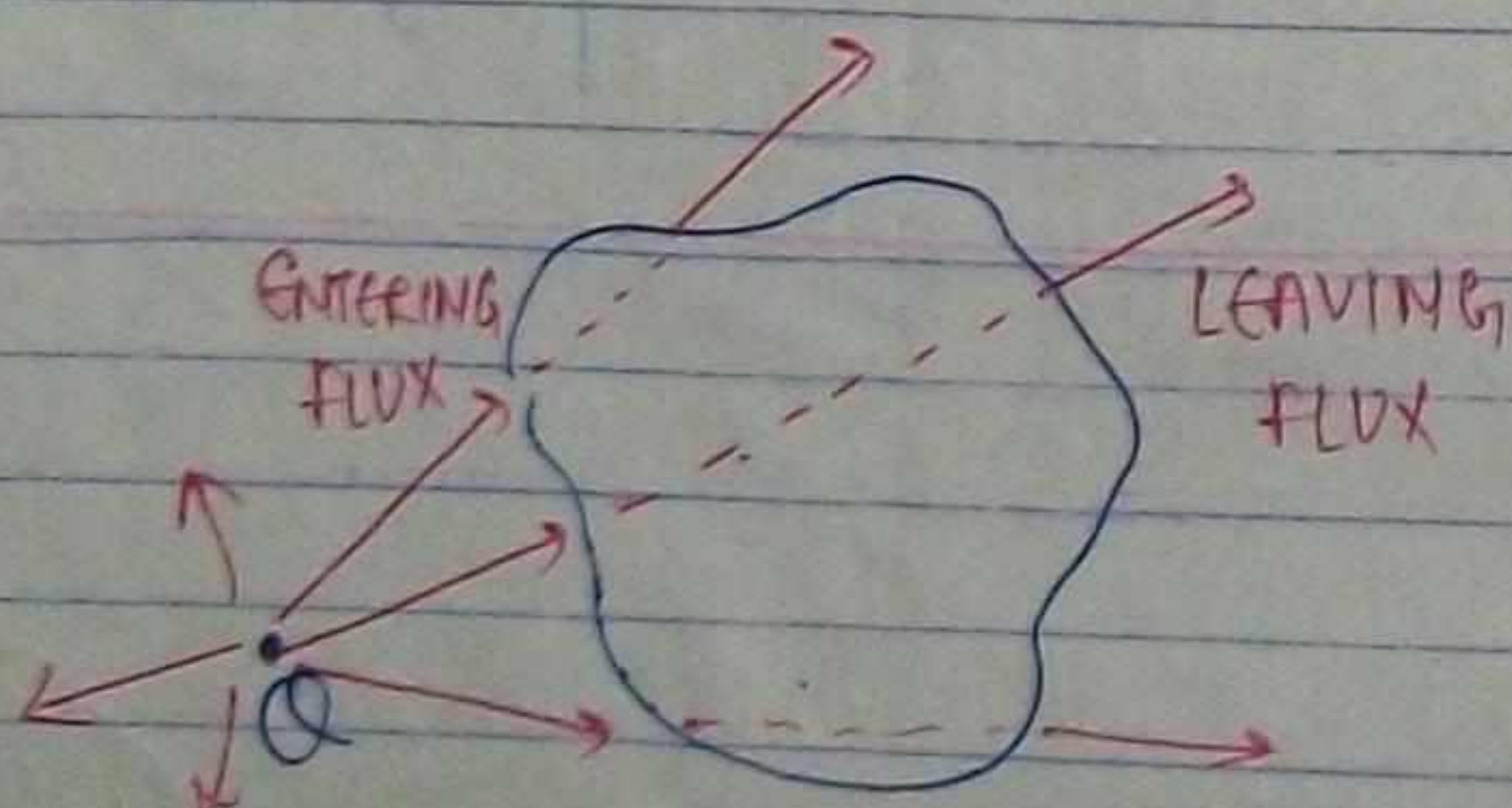
$$\oint \mathbf{D} \cdot d\mathbf{s} = Q \quad (\text{integral form})$$

$$\nabla \cdot \mathbf{D} = \rho_v \quad (\text{point form})$$

- The word electric flux means the repulsive force from a charge on a charge. and hence electric field direction or flux direction is the direction in which the positive charge experiences a linear force.



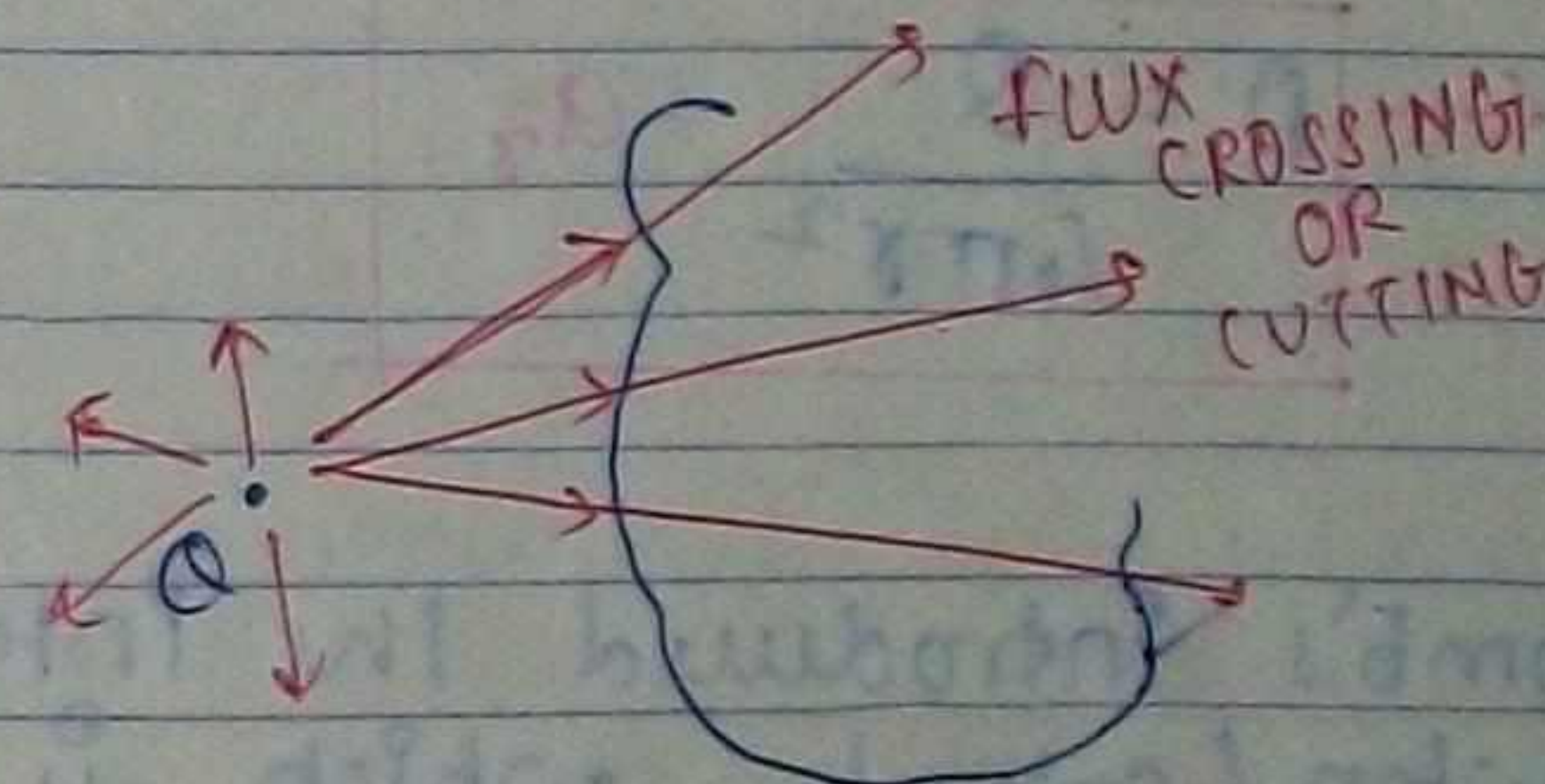
$$\oint \mathbf{D} \cdot d\mathbf{s} = Q$$



$$\oint \mathbf{D} \cdot d\mathbf{s} = 0$$

- Gauss states that every charge is a source of electric flux line. If it is a positive charge. Similarly, it is a sink for electric flux if it is a negative charge.

When surface is not closed



$$\oint \mathbf{D} \cdot d\mathbf{s} = \Psi_e$$

This is not Gauss law.

⇒ GAUSS LAW APPLICATION (1) :

- Consider a Gaussian surface which is a concentric, symmetric sphere around the charge, the density D is same everywhere and hence

$$\oint \mathbf{D} \cdot d\mathbf{s} = Q$$

$$D \cdot S = Q$$

$$D = \frac{Q}{4\pi r^2}$$

Gaussian spherical surface.

The surface has a_r direction and hence D also has radially outward direction

$$\oint D \cdot ds = Q$$

$r = \text{CONST.} \quad (a_r)$

Density $\leftarrow$ $D = \frac{Q}{4\pi r^2} a_r$
 C/m^2

- Colomb's introduced the intensity measure Newton/colomb which is force experienced by a test charge and successfully related to density. Hence

$$\vec{E} = \frac{\vec{F}}{Q_t} = \frac{\vec{D}}{\epsilon} = \frac{Q}{4\pi\epsilon r^2} a_r$$

INTENSITY

(Newton/colomb)

test charge

$$\vec{F} = \frac{Q_1 Q_2}{4\pi\epsilon r^2}$$

$$D \cdot E = \frac{C N}{m^2 C} = \frac{N}{m^2}$$

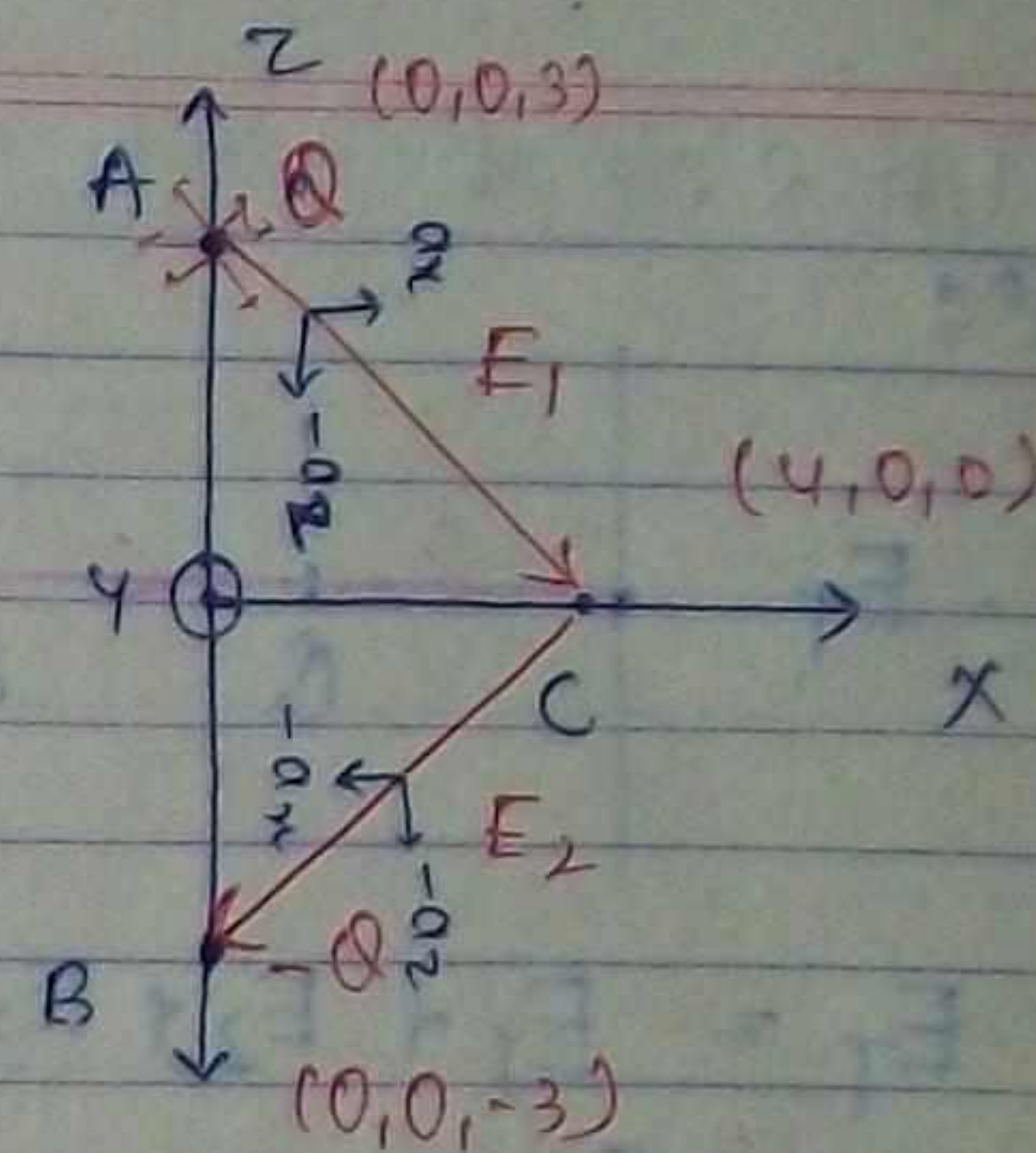
Q. No. 1 P-3

Solⁿ:

$$E_1 \rightarrow a_x, -a_z$$

$$E_2 \rightarrow -a_x, -a_z$$

$$E = E_1 + E_2 \rightarrow \text{direction } -a_z$$



(or) by force method.

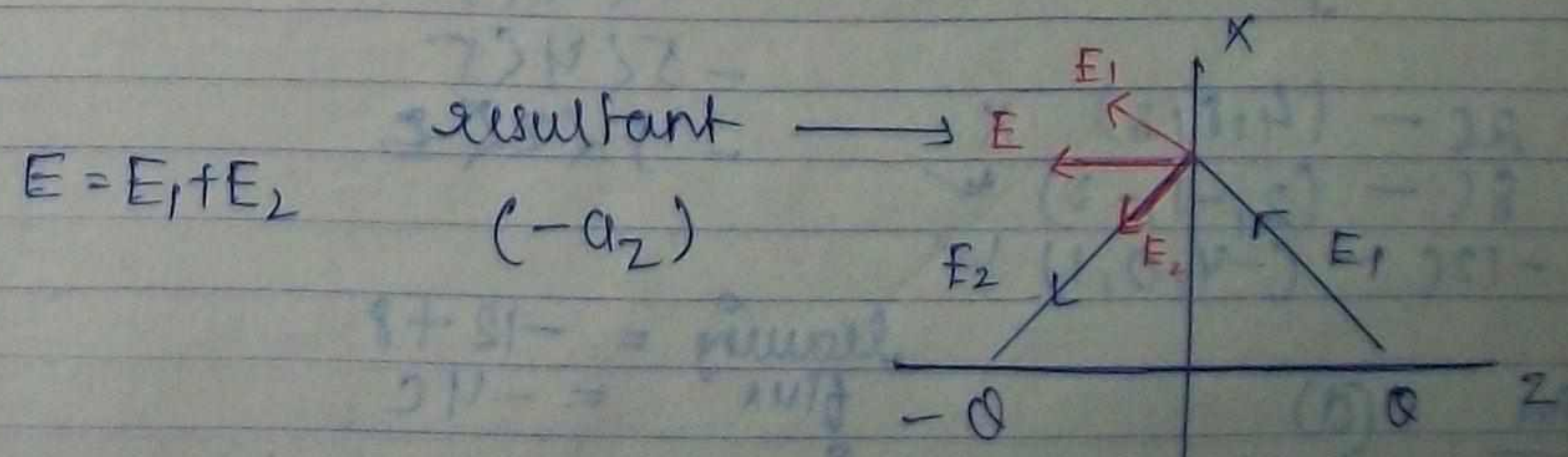
$$E_1 = \frac{Q}{4\pi\epsilon (\sqrt{4^2+3^2})^2} (4a_x - 3a_z)$$

$$a_r = \frac{\vec{r}}{|\vec{r}|} = \frac{4a_x - 3a_z}{5}$$

$(x_2 - x_1) + (y_2 - y_1) + (z_2 - z_1)$

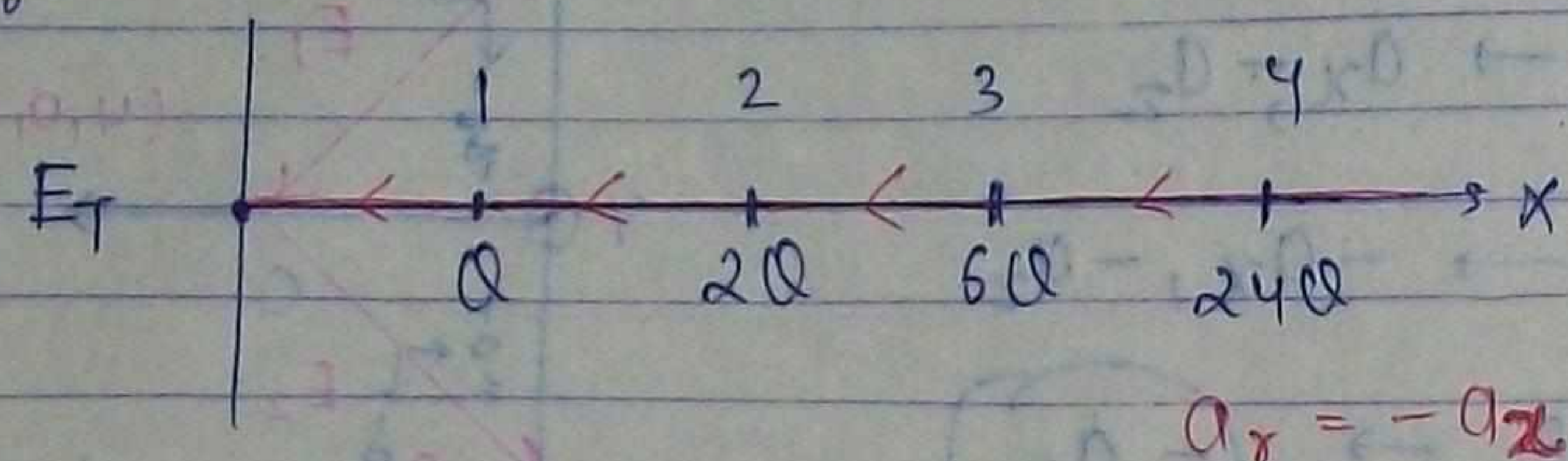
$$E_2 = \frac{-Q}{4\pi\epsilon (\sqrt{4^2+3^2})^2} (+4a_x + 3a_z)$$

(point coordinate) - (charge coordinate) = coordinate



Que. 2. P-3

Solⁿ:



$$E_T = E_1 + E_2 + E_3 + E_4$$

$$= \left[\frac{Q}{4\pi\epsilon_0 1^2} + \frac{2Q}{4\pi\epsilon_0 2^2} + \frac{6Q}{4\pi\epsilon_0 3^2} + \frac{24Q}{4\pi\epsilon_0 4^2} \right] - a_x$$

$$= \frac{Q}{4\pi\epsilon_0} \left(1 + \frac{1}{2} + \frac{2}{3} + \frac{3}{2} \right) (-a_x)$$

strongest (4) weakest (2)

Que. 4 P-3

Solⁿ:

$$\oint \mathbf{D} \cdot d\mathbf{s} = Q$$

Net flux = charge enclosed = -4C

$$-5 < x < 5$$

$$-5 < y < 5$$

$$-5 < z < 5$$

$$2C - (4, 8, 3) \times$$

$$8C - (2, -1, -3) \checkmark$$

$$-12C - (-4, 0, 1) \checkmark$$

Ans (d)

leaving flux = -12 + 8 = -4C

entering flux = 4C

$$\text{Que 5 } 2C - (4, 8, 3)$$

$$d_1 = \sqrt{2^2 + 8^2 + 3^2} > 6$$

$$8C - (2, -1, -3)$$

$$d_2 = \sqrt{2^2 + 1^2 + 3^2} < 6 \checkmark$$

$$-12C - (-4, 0, 1)$$

$$d_3 = \sqrt{4^2 + 1^2} < 6 \checkmark$$

∴ (D)

(-4) coulomb (-12 + 8)

Que 6.

$$d_1 = \sqrt{2^2 + 1^2 + (-1)^2} > 6$$

$$d_2 = \sqrt{0^2 + 2^2 + 5^2} < 6$$

$$d_3 = \sqrt{6^2 + 3^2 + 1^2} > 6$$

ans (8) coulomb

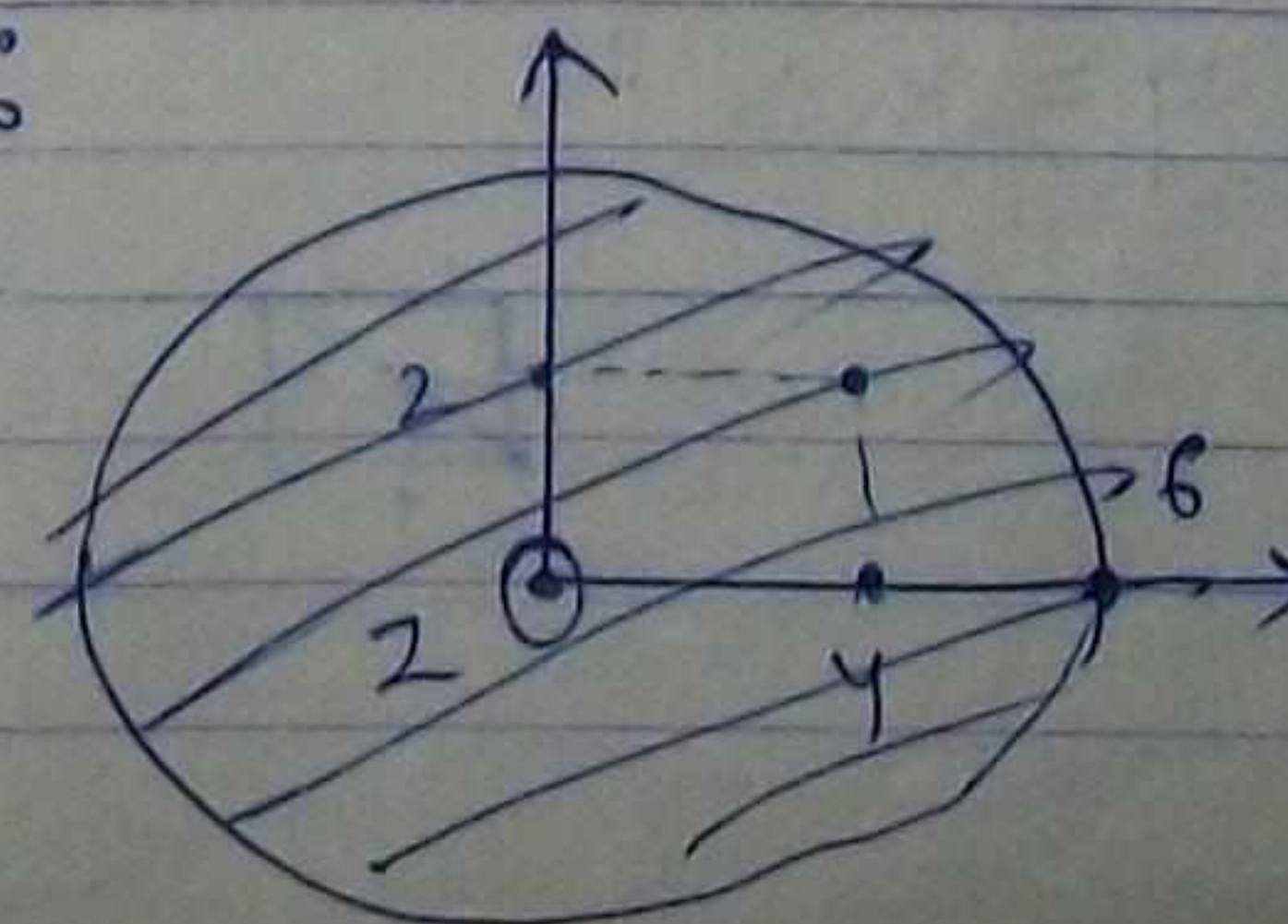
Que. Repeat the same ques. if the charge is a lin charge of $\rho_L = 10 \text{ nC/m}$ located at $x=4, y=2$ for all z . and the surface is $-5 < x < 5$

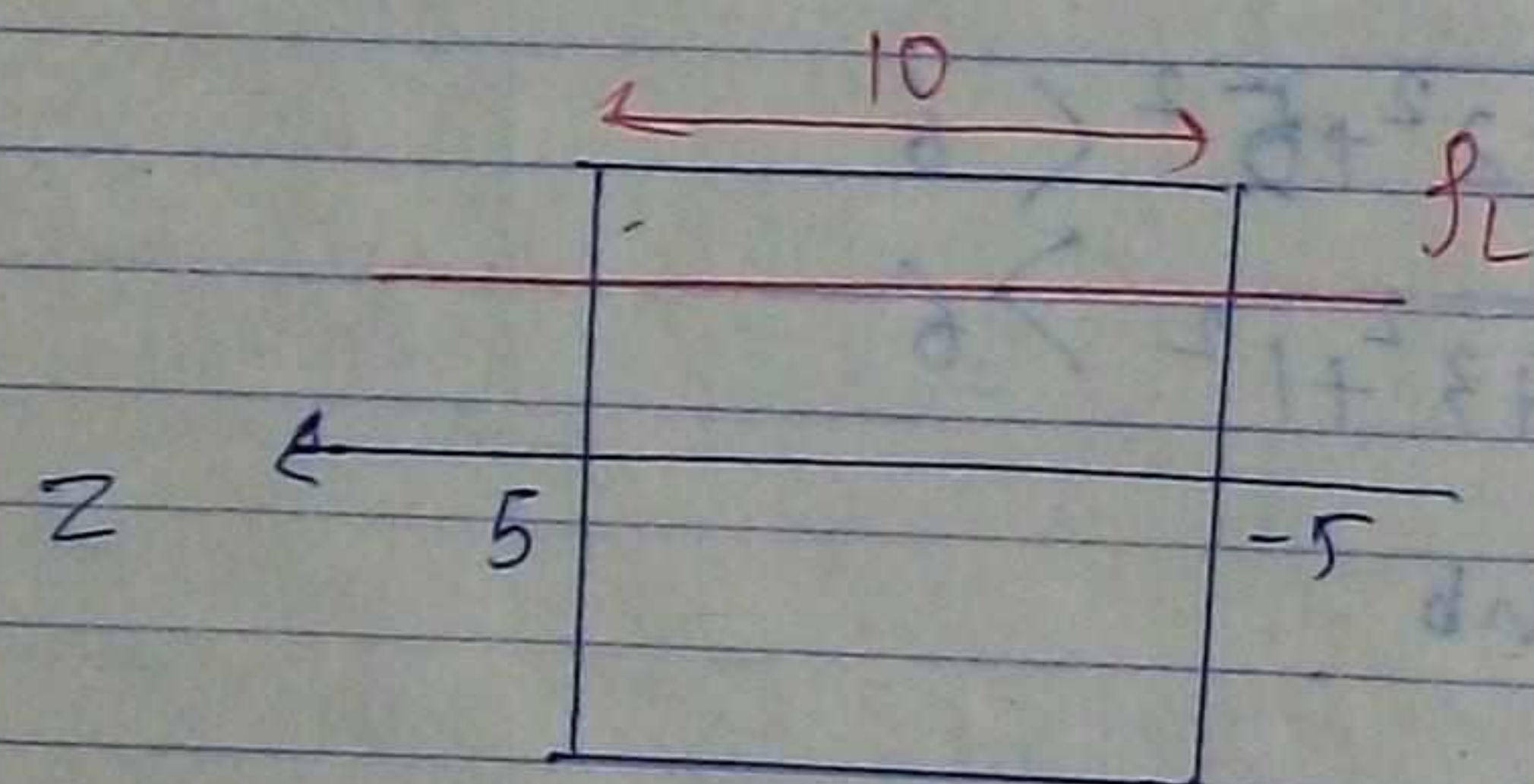
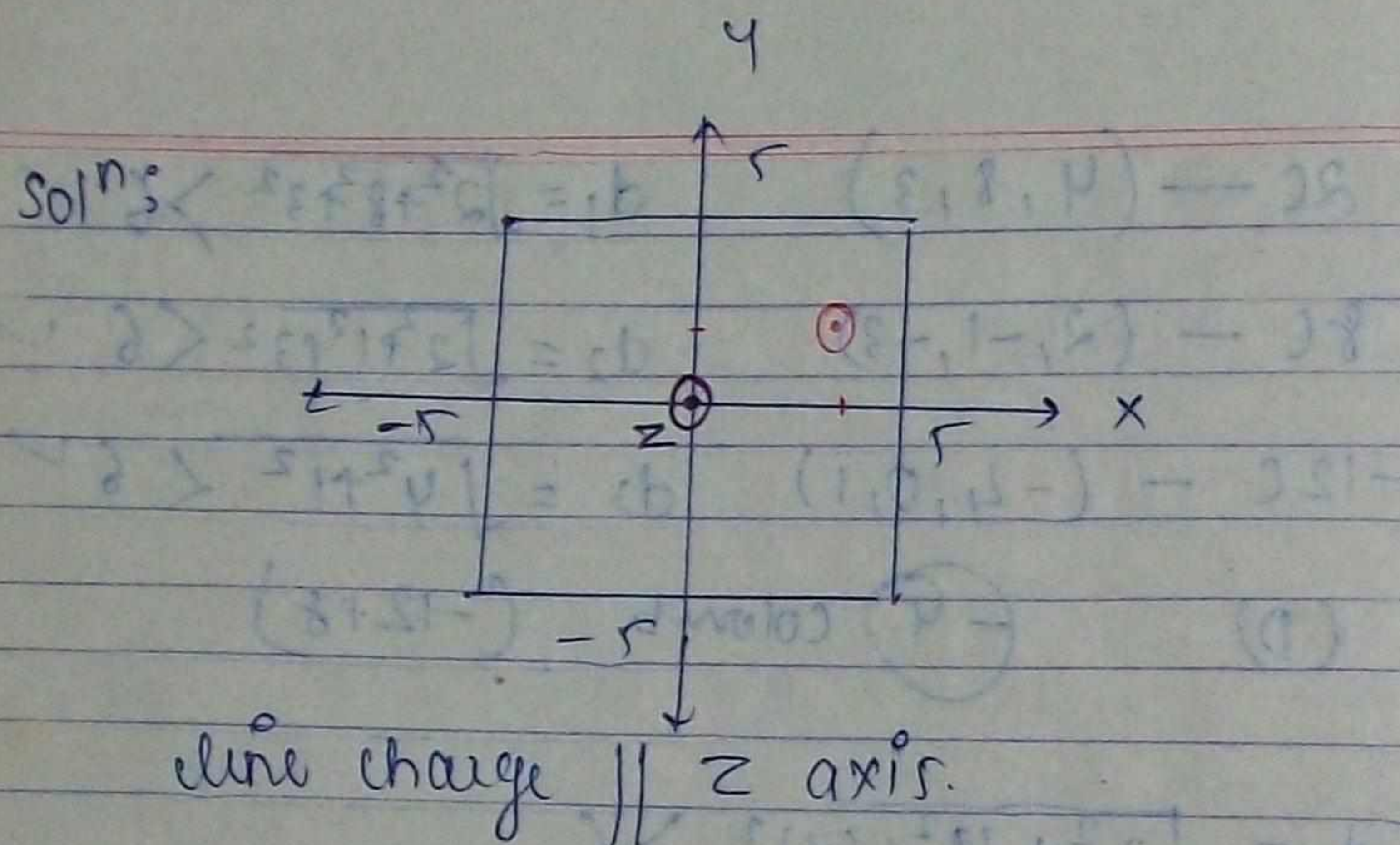
$$-5 < x < 5$$

$$-5 < y < 5$$

$$-5 < z < 5$$

Solⁿ:

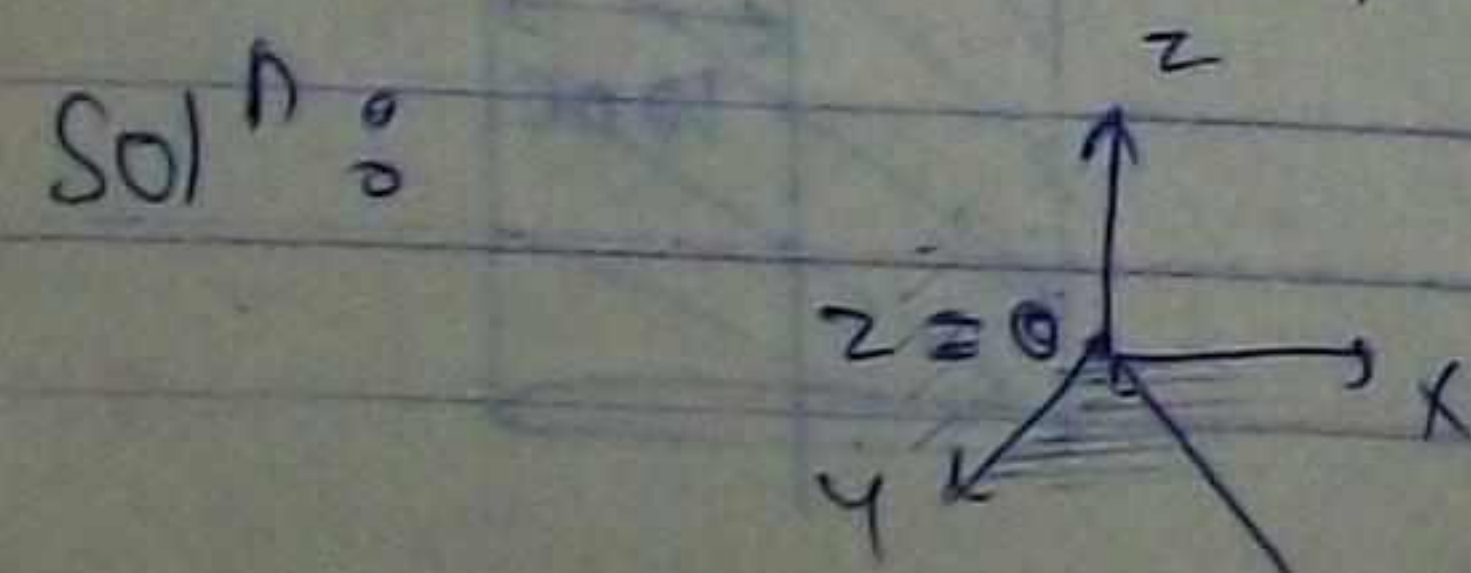




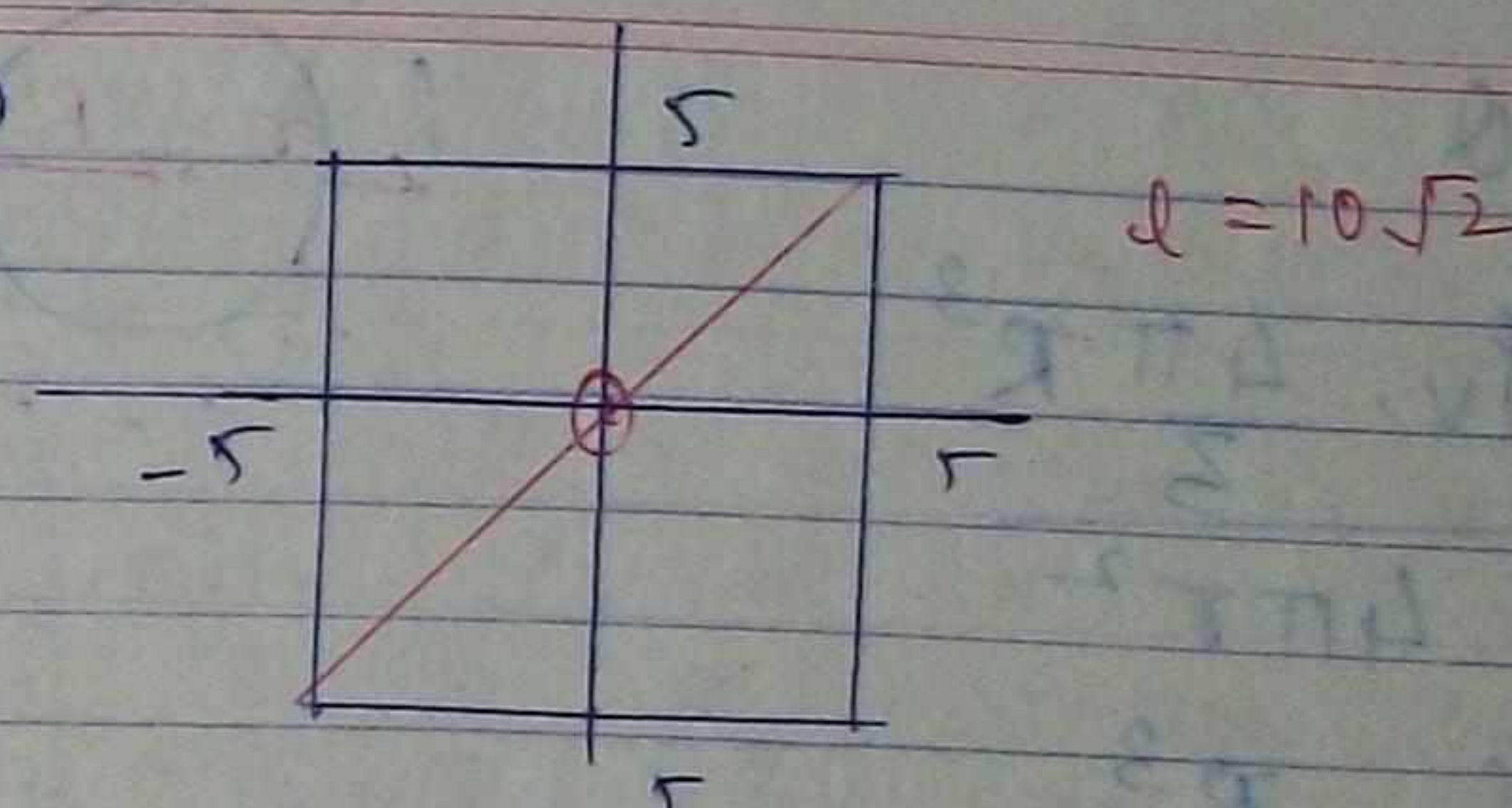
$$Q_{\text{enclosed}} = \int_L \lambda \times dl$$

$$= 10 \times 10 = \underline{\underline{100 \text{ nC}}}$$

Ques repeat same question when line is located at $y=x$ in the $Z=4$ plane.



at $z=0$



$$Q = \int_L \lambda = 10 \times 10\sqrt{2} = 100\sqrt{2} \text{ nC}$$

Ques. 8 P-4

Solⁿs: $\oint \mathbf{D} \cdot d\mathbf{s} = Q$

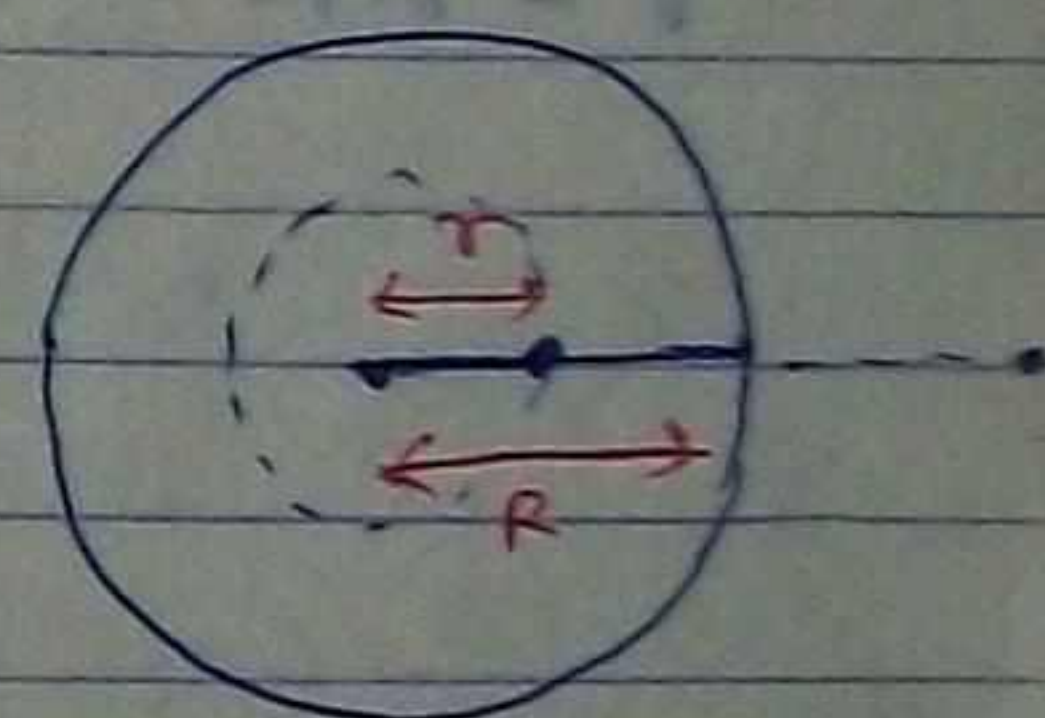
$$\mathbf{D} \cdot \mathbf{S} = Q$$

$$D = \frac{\int_V \frac{4}{3} \pi r^3}{4\pi r^2}$$

$$D = \int_V \frac{r}{3}$$

$$E = \frac{\int_V r}{3\epsilon}$$

$$r=R/2 \Rightarrow E = \frac{\int_V R}{6\epsilon}$$



Gaussian surface

$$Q = \int_V \rho$$

$$D = \frac{\int_V \frac{4}{3} \pi r^3}{4\pi R^2} \quad r=R/2$$

$$= \frac{\int_V r^3}{3R^2} \quad \frac{R/2}{3R^2}$$

$$= \frac{\int_V R^3}{3} \quad \frac{R^3}{3}$$

$$D \cdot S = Q$$

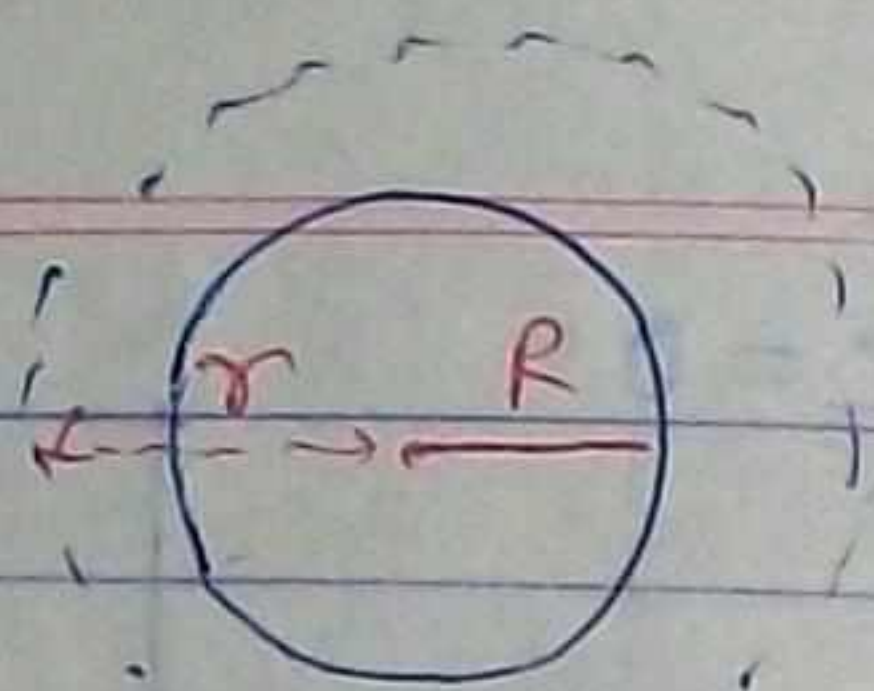
$$D = \frac{\int_V \frac{4\pi R^3}{3}}{4\pi r^2}$$

$$D = \int_V \frac{R^3}{3r^2}$$

$$r = 2R$$

$$D = \int_V \frac{R}{12}$$

$$E = \frac{\int_V R}{12\epsilon}$$



STATIC MAGNETIC FIELDS

22/09/14

⇒ BIOT-SAVART'S LAW -
(AMPERE LAW FOR CURRENT ELEMENTS)

- The law identifies a current in a small zero length wire as the basic cause of magnetic fields and expresses the field strength as intensity ($\vec{H}$) as:

intensity $\leftarrow$ Amp/m $\boxed{\vec{H} = \frac{I d\vec{l}}{4\pi r^2} \times \vec{a}_r}$

- It identifies the $\vec{H}$ direction at any point is equals the current direction (cross) the radial direction from the current to the point.

$\vec{H} = \frac{I}{\text{direction}} \times \frac{1}{\text{direction (cross)}}$
at any point

Radial direction from the current to the point.

- Lorentz had a different measure of field strength called as flux density (weber/m²) which is

$$\vec{F} = q(\vec{v} \times \vec{B})$$

$$d\vec{F} = dq \left(\frac{d\vec{l}}{dt} \times \vec{B} \right)$$

$$B = \frac{f}{Idl}$$

$$d\vec{F} = I d\vec{l} \times \vec{B}$$

LORENTZ FORCE EQ.

$$[F = BIl \sin \theta]$$

Density

$$\vec{B} = \mu H = \frac{\vec{F}}{I d\vec{l}} = \frac{\mu I d\vec{l} \times \vec{r}}{4\pi r^2}$$

weber
m²

or

Newton
Amp-m

PHYSICALLY

Hence physically flux density $\vec{B}$ is force experienced / current element i.e. Newton / (Amp-m)

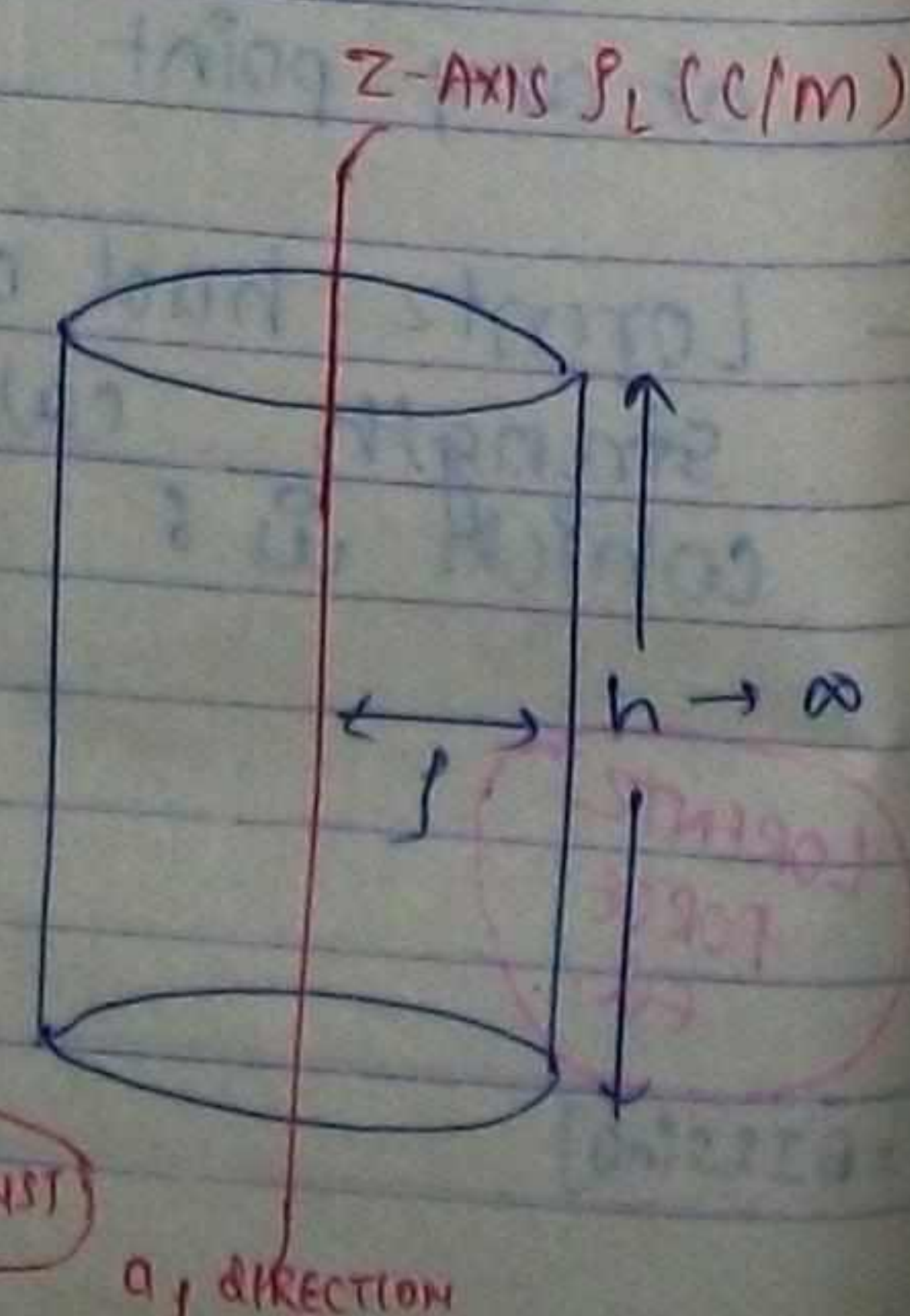
NOTE:

$\oint \vec{D} \cdot d\vec{s}$ in electric field is analogous to $\oint I d\vec{l}$ in magnetic fields.

⇒ GAUSS LAW: APPLICATION NO ②:
E-FIELD STRENGTH OF AN INFINITE LENGTH-
 ρ_L (C/m) LINE CHARGE

- Consider a gaussian surface which is a concentric symmetric cylinder of ρ radius and height $h \rightarrow \infty$

$$\oint \vec{D} \cdot d\vec{s} = Q$$



$$\oint \vec{D} \cdot d\vec{s} = Q$$

$$D \cdot s = Q$$

$$D = \frac{\rho_L h}{2\pi \rho h}$$

$$D = \frac{\rho_L}{2\pi \rho}$$

- All the point on the surface have the same strength hence 'D' is constant.

$$\therefore D = \frac{\rho_L}{2\pi \rho} \hat{a}_\rho$$

- The surface considered is a ρ constant $\hat{a}_\rho$ directed cylinder, hence direction of $\vec{D}$ is also $\hat{a}_\rho$:

$$\vec{D} = \frac{\rho_L}{2\pi \rho} \hat{a}_\rho$$

- Also, $E = \frac{\rho_L}{2\pi \epsilon \rho} \hat{a}_\rho$ ($D = \epsilon E$)

⇒ H-FIELD STRENGTH OF AN INFINITE LENGTH CURRENT CARRYING WIRE -

- Consider a small length $d\vec{L}$ at z -height anywhere around the z -axis. The incremental strength because of this length $d\vec{L}$ is calculated dH

$$dH = \frac{I d\vec{L} \times \vec{a}_r}{4\pi r^2}$$

$$d\vec{L} = dz \vec{a}_z$$

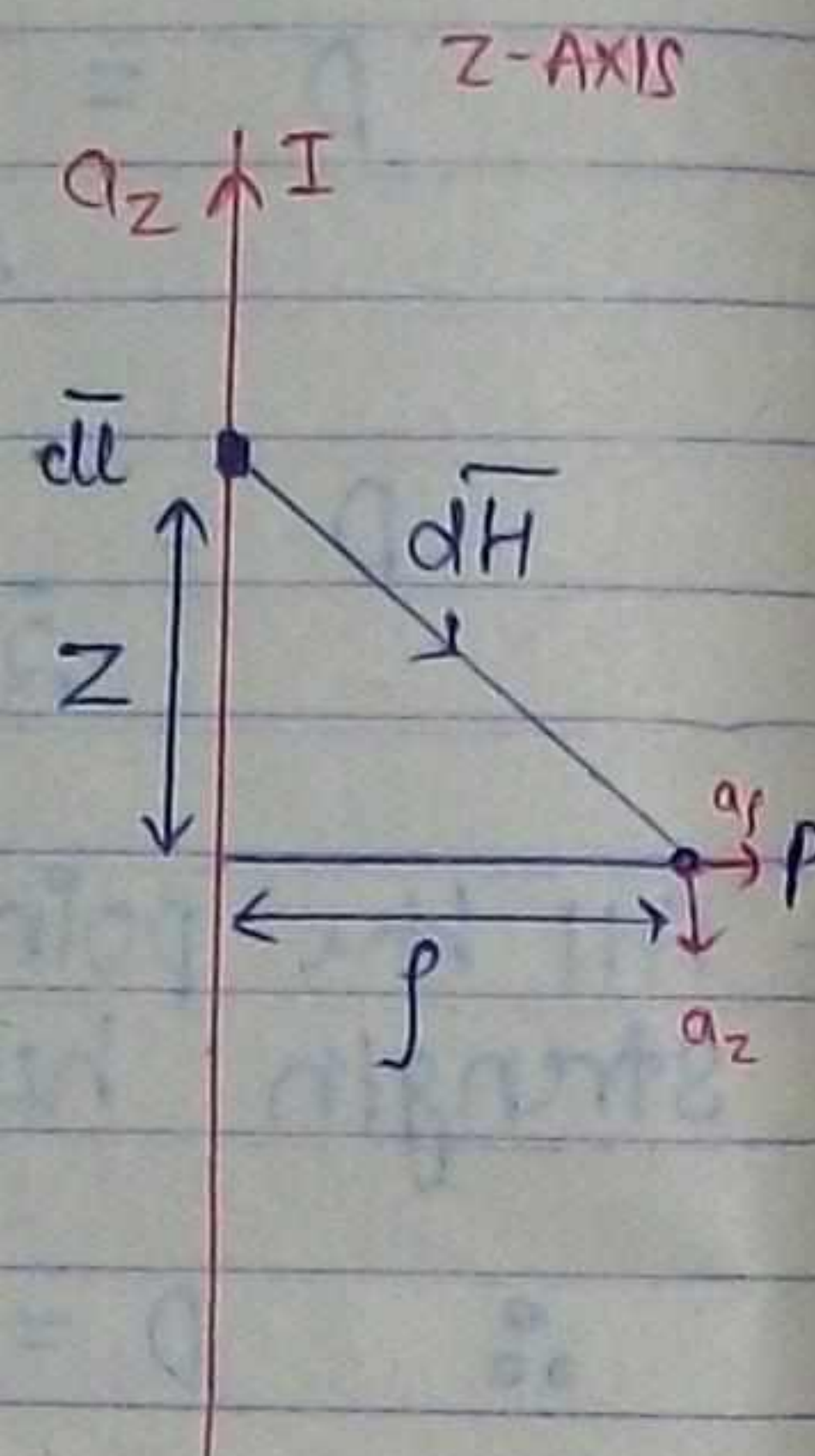
$$r = \sqrt{f^2 + z^2}$$

$$\vec{a}_r = \frac{\vec{r}}{|\vec{r}|} = \frac{f\vec{a}_f - z\vec{a}_z}{\sqrt{f^2 + z^2}}$$

$$\therefore dH = \frac{I dz \vec{a}_z}{4\pi (\sqrt{f^2 + z^2})^2} \times \frac{f\vec{a}_f - z\vec{a}_z}{\sqrt{f^2 + z^2}} \quad \text{direction.}$$

$$H = \int_{z=-\infty}^{\infty} dH$$

$$H = \frac{If}{4\pi} \int_{z=-\infty}^{\infty} \frac{dz}{(f^2 + z^2)^{3/2}} \quad (\vec{a}_z \times \vec{a}_f)$$



Putting $z = f \tan \theta$

$$\theta = -\pi/2 \text{ to } \pi/2$$

$$dz = f \sec^2 \theta d\theta$$

$$(f^2 + z^2) = f^2 + f^2 \tan^2 \theta = f^2 \sec^2 \theta$$

$$H = \frac{If}{4\pi} \int_{-\pi/2}^{\pi/2} \frac{f \sec^2 \theta d\theta}{f^3 \sec^3 \theta}$$

$$H = \frac{I}{4\pi f} \int_{-\pi/2}^{\pi/2} \cos \theta d\theta$$

$$\therefore H = \frac{I}{2\pi f} (\vec{a}_z \times \vec{a}_f) = \frac{I}{2\pi f} (\vec{a}_\phi)$$

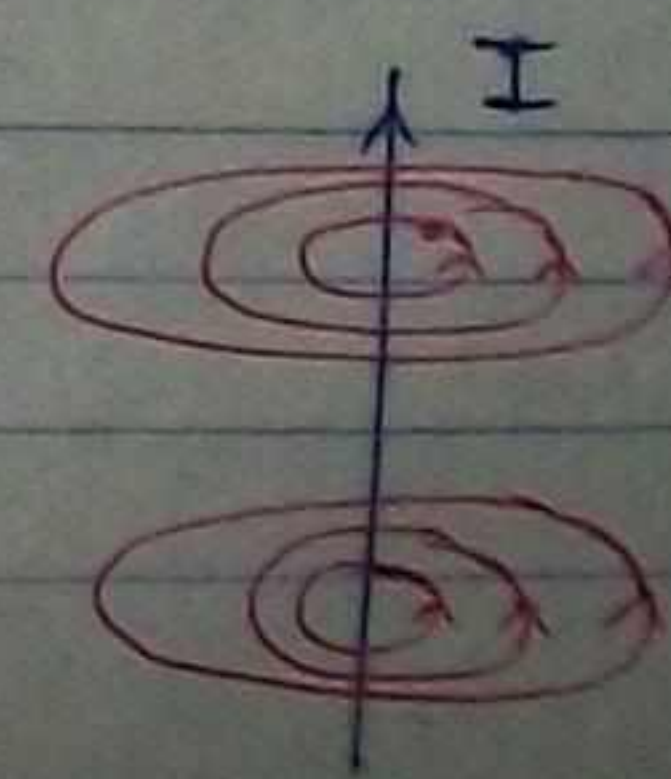
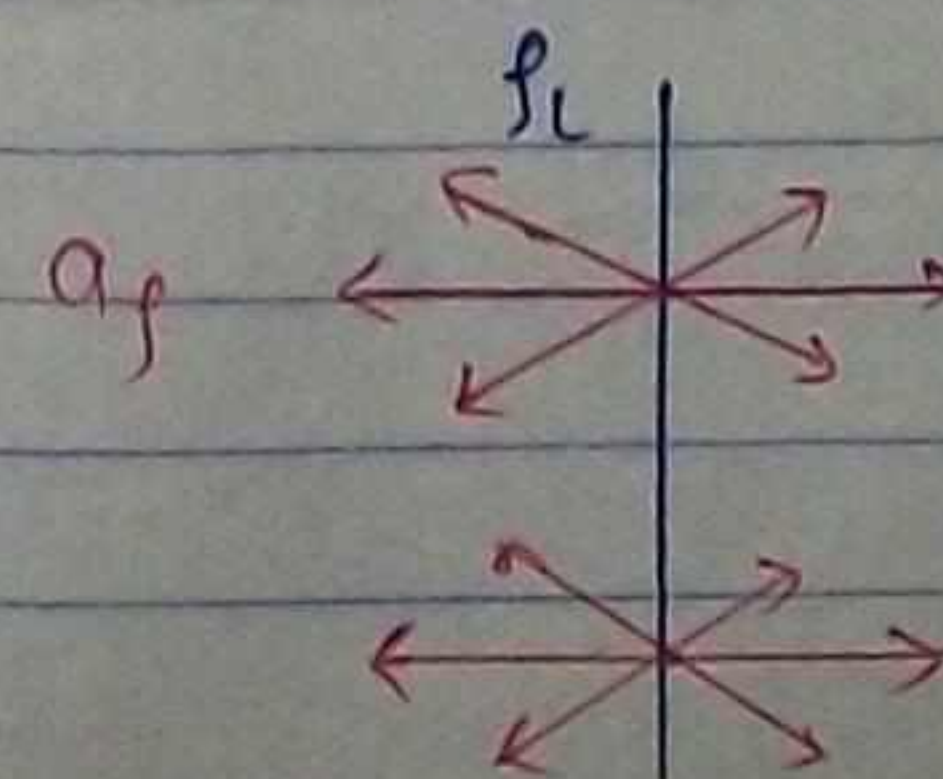
NOTE:

f_L - line charge (E-fields)

I - current carrying wire (H-fields)

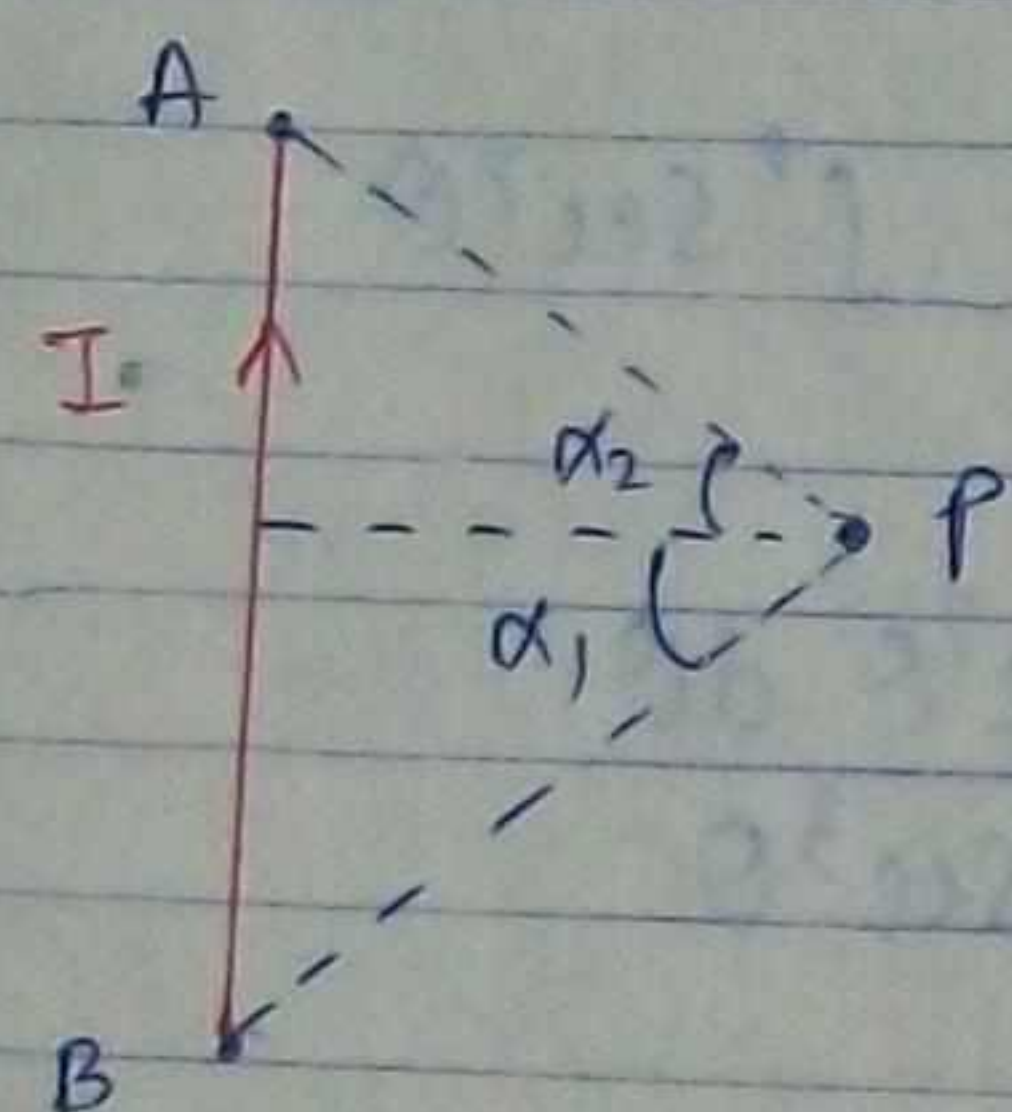
$$E = \frac{f_L}{2\pi \epsilon f} \vec{a}_f$$

$$H = \frac{I}{2\pi f} \vec{a}_\phi$$



EXTENSION :

H - DUE TO AN FINITE LENGTH CURRENT WIRE



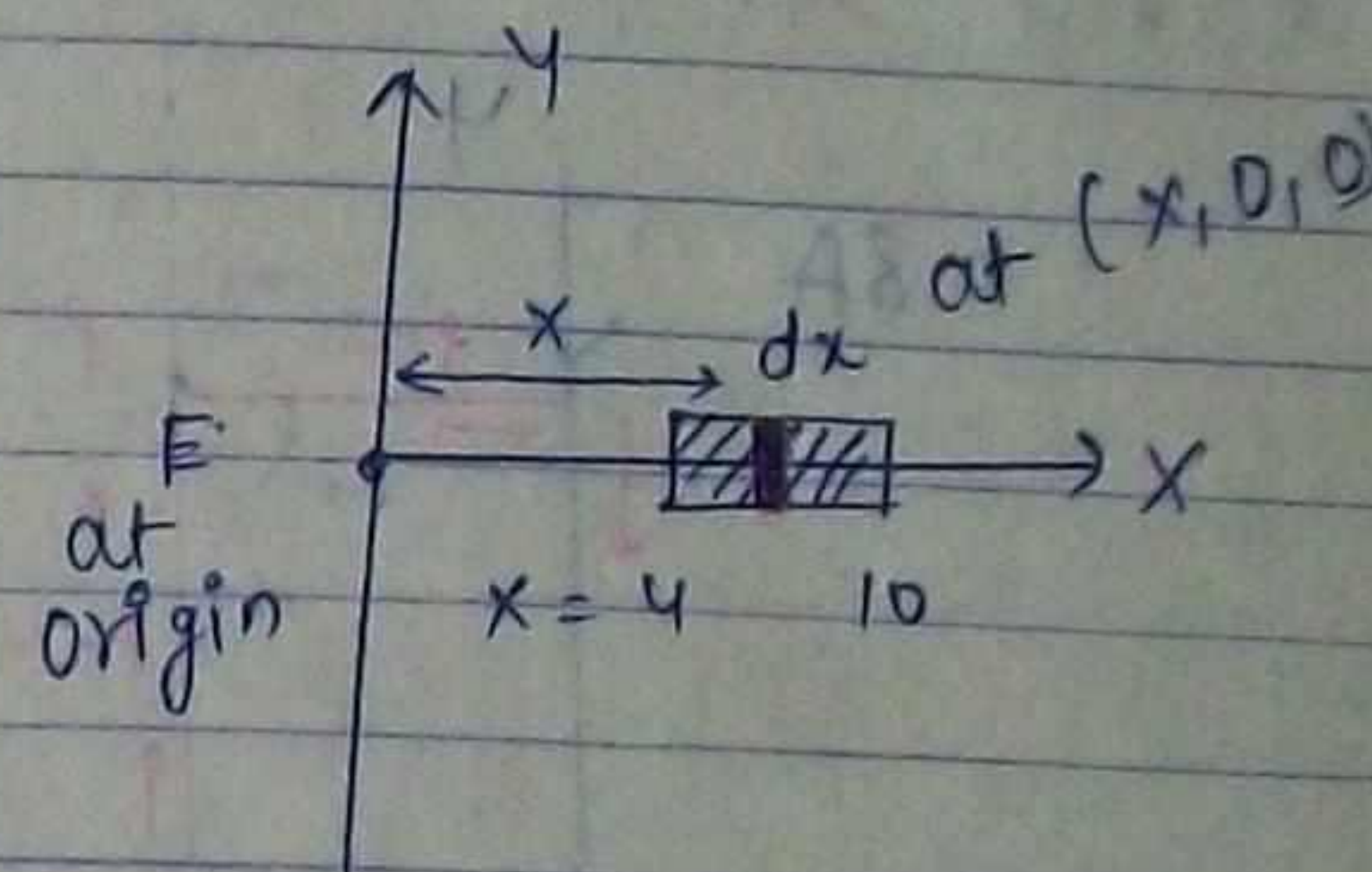
$$H = \frac{I}{4\pi\epsilon_0} \int_{\alpha_1}^{\alpha_2} \cos\theta d\theta \quad a_\phi$$

$$H = \frac{I}{4\pi\epsilon_0} (\sin\alpha_1 + \sin\alpha_2) \quad a_\phi$$

Ques 3. P-3

Solⁿ:

Consider a small length dx and since $dx \rightarrow 0$ then at $(x, 0, 0)$, then calculate the field due to charge with in the small length using the point charge formula.



$$Q = \int_L dx, \quad r = x, \quad a_r = -a_x$$

$$\int dE = \int_{x=4}^{10} \frac{f_L dx}{4\pi\epsilon_0 x^2} (-a_x) \quad dE = 0 \quad \frac{f_L dx}{4\pi\epsilon_0 x^2}$$

$$E = \frac{f_L}{4\pi\epsilon_0} \left(\frac{-1}{x} \right)_4^{10} (-a_x)$$

$$E = \frac{f_L}{4\pi\epsilon_0} \cdot \frac{3}{20} (-a_x)$$

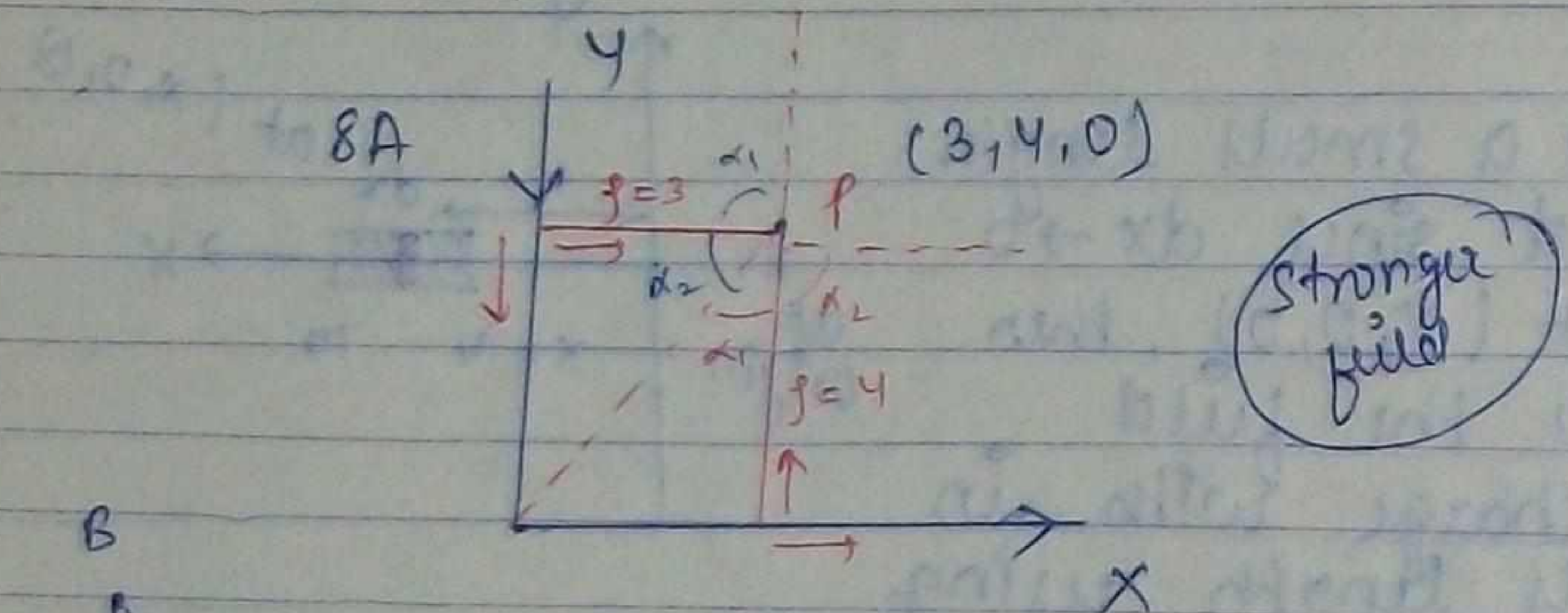
If E at (2, 3, 4)

$$Q = \int_L dx$$

$$a_r = \frac{(2-x)a_x + 3a_y + 4a_z}{\sqrt{(2-x)^2 + 3^2 + 4^2}}$$

$$r = \sqrt{(x-2)^2 + 3^2 + 4^2}$$

Que. 11. P-4.



$$H = \frac{8}{4\pi \cdot 3} (\sin \alpha_1 + \sin \alpha_2)$$

(due to y axis)

$$\alpha_1 = 90^\circ \quad \sin \alpha_2 = 4/5$$

Direction

$$H = -a_y \times a_x = a_z$$

$$H_{(y \text{ axis})} = \frac{8}{4\pi \cdot 3} \left(1 + \frac{4}{5}\right) a_z$$

$$H_{(x \text{ axis})} = \frac{8}{4\pi \cdot 3} \left(\frac{3}{5} + 1\right) (a_x \times a_y) = a_z$$

$$H = \frac{8}{4\pi} = \frac{4}{2\pi} a_z \quad (H_{x \text{ axis}} + H_{y \text{ axis}})$$

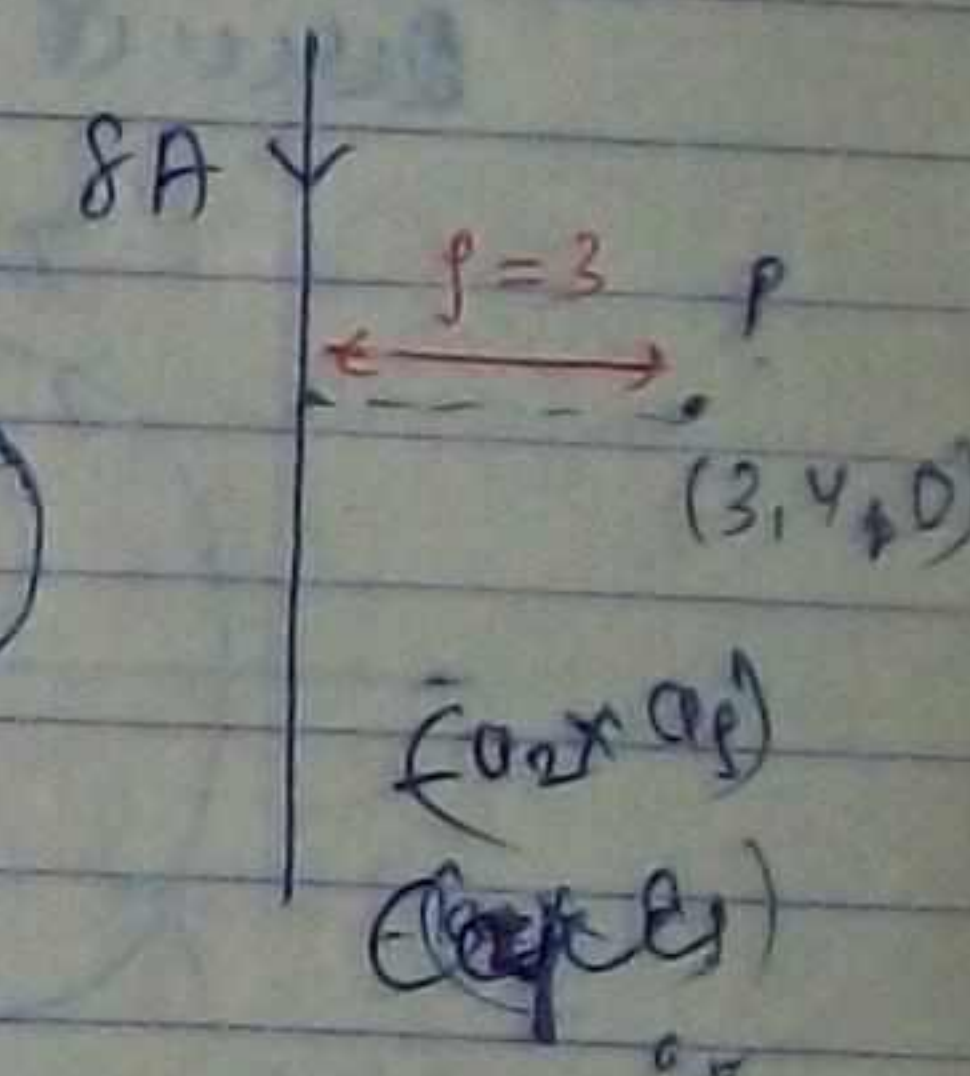
$$= \frac{2}{\pi} a_z$$

Que. Repeat same que. if current is only on the y-axis completely.

Solⁿ: $H = \frac{I}{2\pi r} a_z$

$$H = \frac{8}{2\pi \cdot 3} a_z = \frac{1.33}{\pi} a_z$$

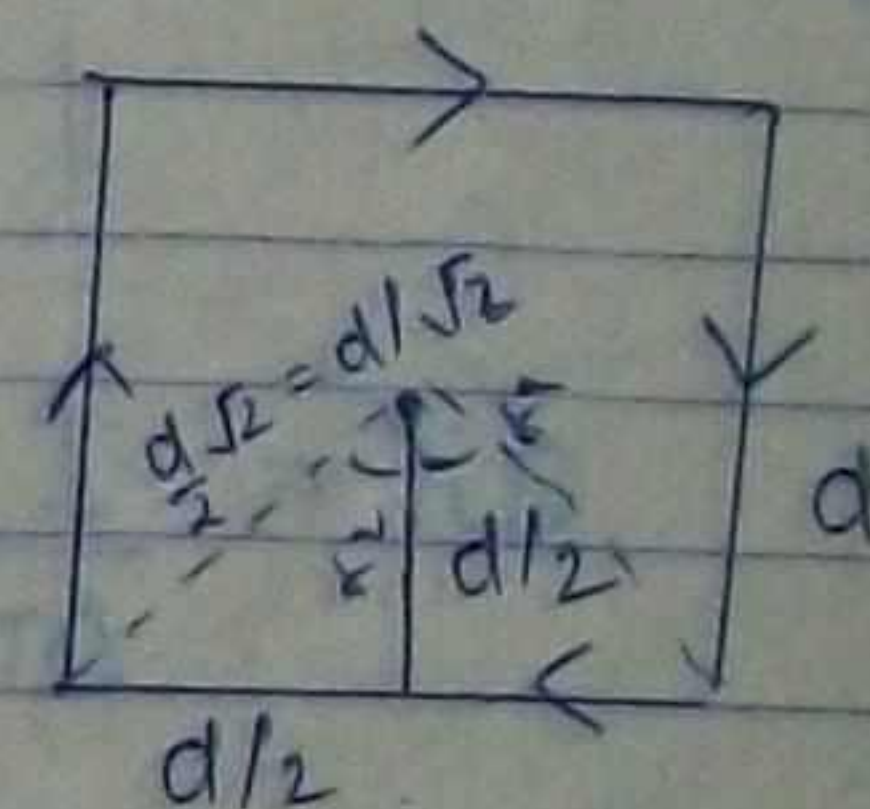
weaker field



Que. 12 P-5

$$H = \frac{I}{4\pi r} (\sin \alpha_1 + \sin \alpha_2)$$

$$H_1 = \frac{I}{4\pi \cdot (d/2)} \left(\frac{\pi}{2}\right)$$



$$H = \frac{4 \cdot I \times 2}{4\pi d} \left(\frac{\pi}{2}\right) = \frac{I}{\pi d}$$

$$= \frac{2\sqrt{2}}{\pi} \frac{I}{d} (-a_z)$$

$$\sin \alpha_1 = \frac{d/2}{d/2} = 1$$

$$= \frac{\pi}{2} = \frac{\pi}{2}$$

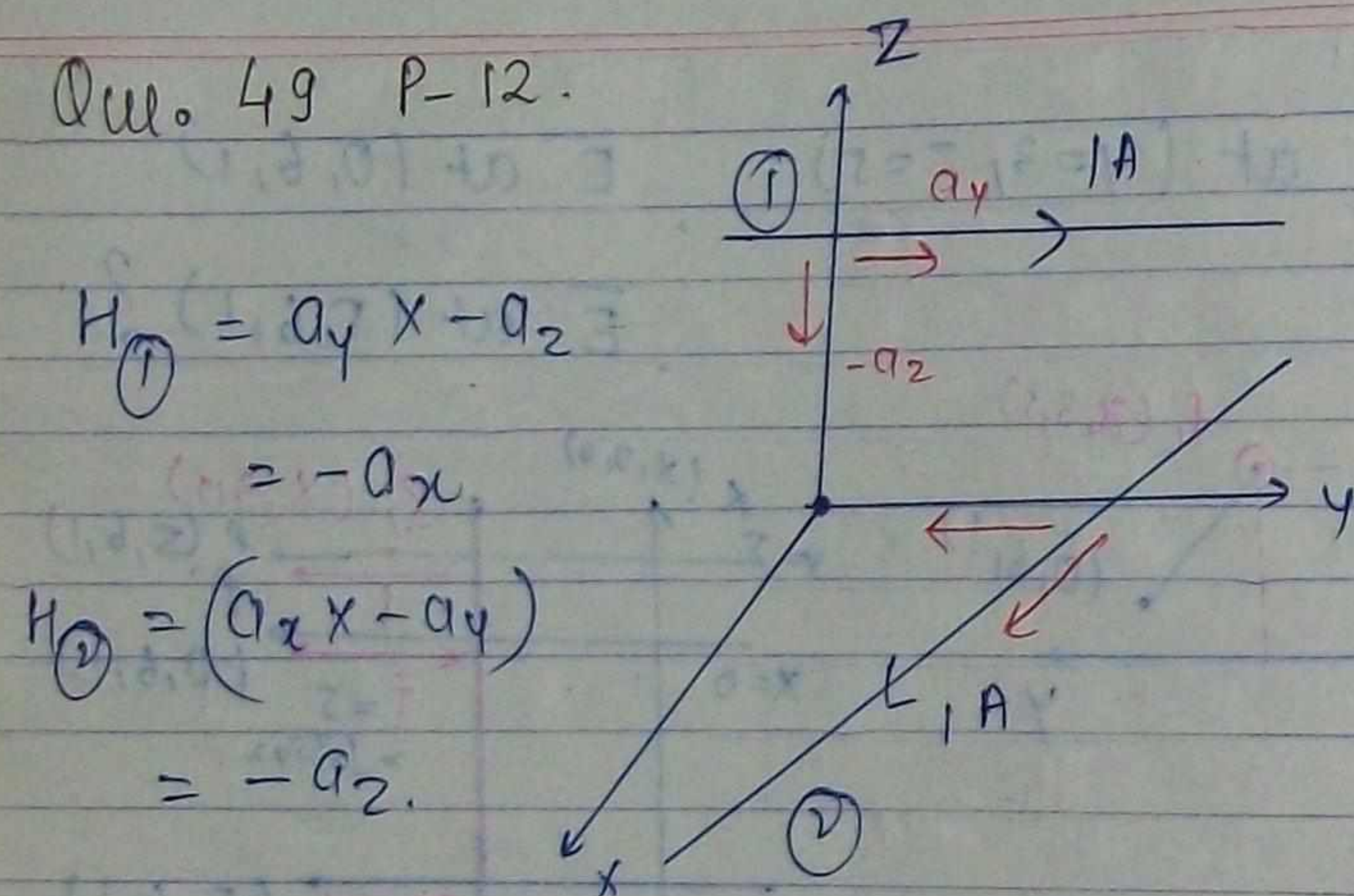
$$\frac{d\sqrt{2}}{2} = \frac{d}{2}$$

* Magnetic field at the centre of symmetric current distribution is always maximum.

* The electric field at the centre of symmetric charge distribution is always zero.

$$H = \frac{2\sqrt{2}}{\pi} \frac{I}{d} (-a_z)$$

Ques. 49 P-12.



$$H_1 = a_y \times -a_z$$

$$= -a_x$$

$$H_2 = (a_z \times -a_y)$$

$$= -a_z$$

$$H = (-a_x \times -a_z)$$

Ques. 50 P-13

$$\vec{V} = \nabla \times \vec{A}$$

Closed line - C
Surface - Sc

$$\oint_C \vec{A} \cdot d\vec{l} = \int_{Sc} (\nabla \times \vec{A}) \cdot d\vec{r}$$

$$= \int_{Sc} \vec{V} \cdot d\vec{r}$$

SUMMARY:

E-FIELD

① Q - scalar
- point charge

② $\vec{E}/\vec{D}$ - direction is
radially outward from the
charge

③ $\vec{D}$ - density C/m²
Strength - charge

It is always
independent of ϵ

$\vec{E}$ - intensity V/m
Strength - force

It is always $1/\epsilon$
dependent.

④ $f \propto E \propto 1/\epsilon$
(Strongest)

H-FIELD

① $I d\vec{l}$ - vector
- current element

② $\vec{B}/\vec{H}$ - direction
is tangentially
around the
current.

③ $\vec{B}$ - density wb/m²
Strength - force.

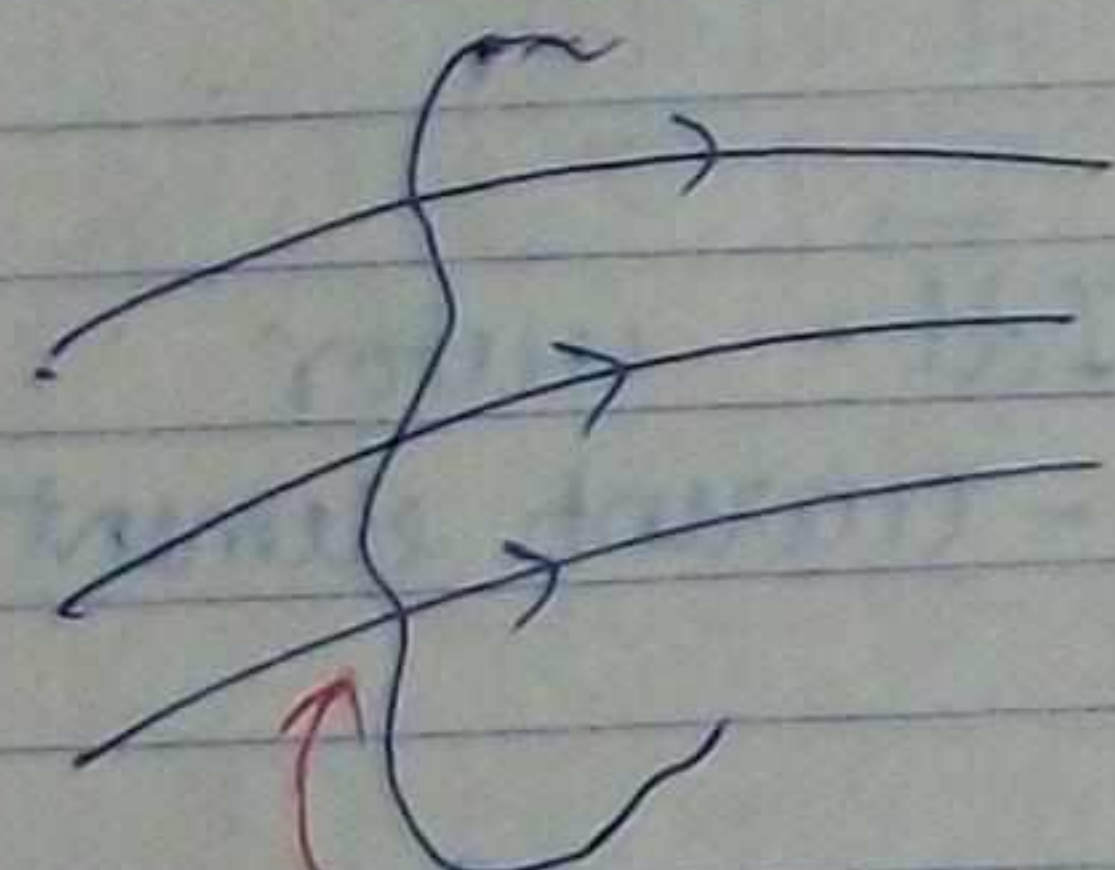
It is always
dependent

$\vec{H}$ - intensity A/m
Strength - current

It is always
independent of μ .

④ $f \propto B \propto \mu$
(weakest)

⇒ MAXWELL'S (3) EQⁿ: CLOSED SURFACE INTEGRAL OF B -



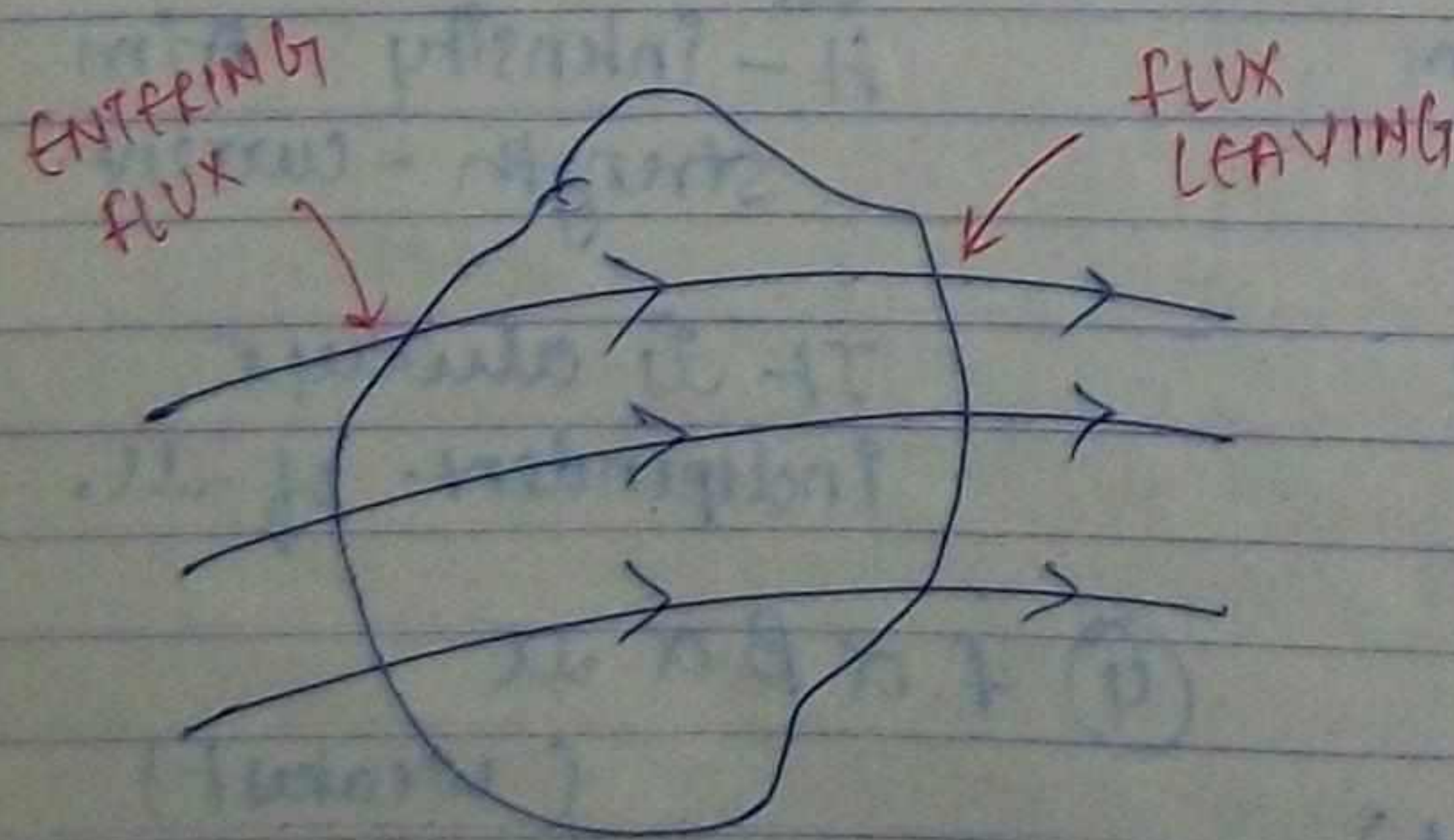
$$\int B \cdot ds = \Psi_m = \text{weber}$$

Not Maxwell's eqⁿ.

- The total flux through a open surface depends on the surface area and the strength of the magnetic field and hence flux cutting is

$$\int B \cdot ds = \Psi_m \text{ (webers)}$$

This is not Maxwell's eqⁿ.



- As magnetic flux is always closed in nature flux entering is equal to flux leaving in any closed surface. hence magnetic flux has no source point or sink point.

∴

$$\oint B \cdot ds = 0$$

$$\nabla \cdot B = 0$$

Maxwell's (3)
eqⁿ

CONCLUSION

- for electric fields:

$\oint D \cdot ds = 0$ is possible in two condition

(a) $Q = 0$

(b) $+Q$ and $-Q$ exist (dipole)

- In magnetic fields $\oint B \cdot ds = 0$ always.

CONCLUSION (1):

Magnetic monopole or equivalent of electric charge does not exist.

CONCLUSION (2):

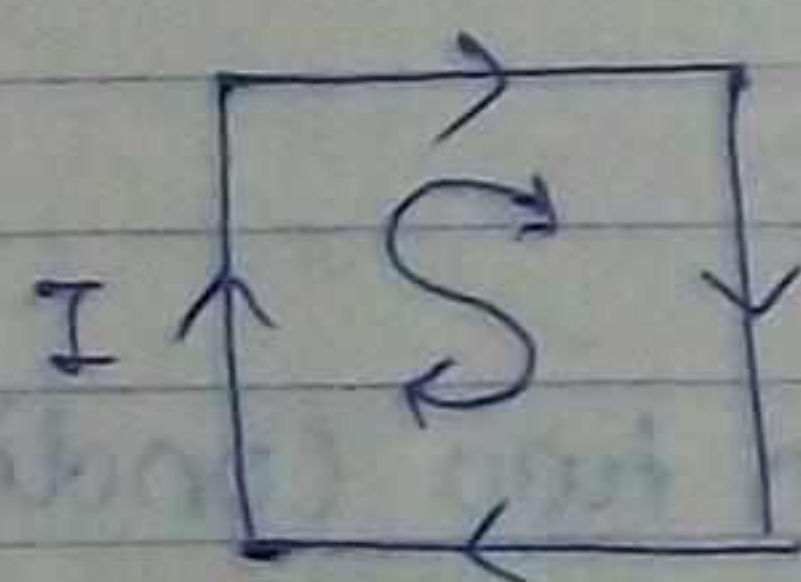
Every cause of magnetic field is a dipole. Ex - bar magnet or any current wire.

CONCLUSION (3):

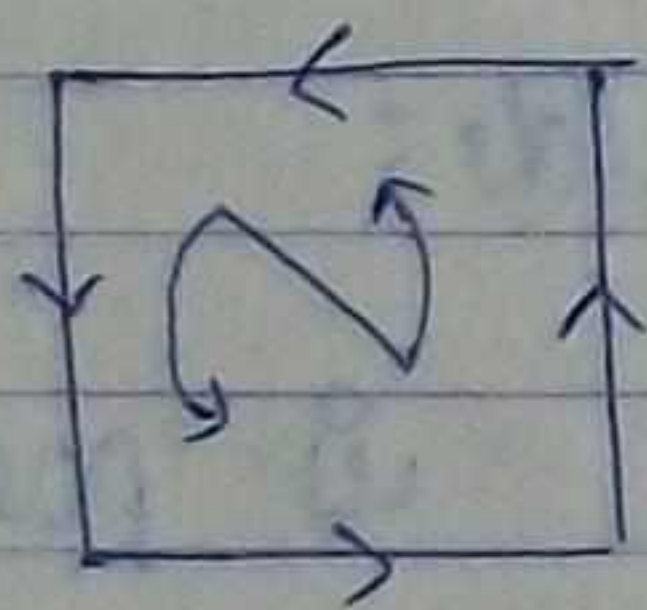
All current carrying wire are closed in nature and starts or end at the same point.

CONCLUSION (4):

Closed current carrying wires are magnetic dipoles.



South pole
(clockwise I)



North pole
(anti-clockwise I)

⇒ POTENTIAL GRADIENT AND CLOSED LINE INTEGRAL OF E

POTENTIAL:

A scalar measure of E field strength in terms of energy at that point in the field.

Voltage = Work done by a charge to move to that point
Charge

$$V = \frac{W}{Q} \quad \left(\frac{\text{Joules}}{\text{Coulomb}} \right)$$

— Potential at any point is a workdone by the charge or the energy expended by the charge per unit charge.

$$\text{work} = \text{force} \times \text{displacement}$$

$$dW = F \cdot dl$$

$$dW = -Q \vec{E} \cdot d\vec{l}$$

— work is done by the charge only when it moves opposite to the field. however in the field direction it moves away due to the repulsive force and acquires energy getting accelerated. Hence -ve sign

$$W = -Q \int \vec{E} \cdot d\vec{l}$$

① $V = \frac{W}{Q} = - \int \vec{E} \cdot d\vec{l}$ → Scalar potential function

② $V_{AB} = - \int_{ref B}^{at A} \vec{E} \cdot d\vec{l} = \text{potential diff B/w A and B}$

③ if $V_B = 0$

$V_A = - \int_{ref B}^{at A} \vec{E} \cdot d\vec{l} = \text{absolute potential at A}$

23/09/11

Q. No. 20 P-6.

Solⁿ: $V(4,1,0)$ $\vec{E} = y a_x + x a_y$

$$V = - \int \vec{E} \cdot d\vec{l} = - \int (y a_x + x a_y) (dx a_x + dy a_y)$$

$$V = - \int_{(0,0,0)}^{(4,1,0)} (y dx + x dy) = - \int_{(0,0,0)}^{(4,1,0)} d(xy)$$

$$V = - [xy]_{(0,0,0)}^{(4,1,0)} = - [4 \times 1 - (0 \times 0)]$$

IInd method $V = -4V$

(OR)

$$V = - \int_{x=0}^4 y dx - \int_{y=0}^1 x dy$$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

Egn of straight line

$$\frac{y-1}{1-0} = \frac{x-4}{4-0}$$

$x = 4y$

$$V = - \int_{x=0}^4 \frac{x}{4} dx - \int_{y=0}^1 y dy = -2 - 2 = -4V$$

NOTE -

In line integrals $\int f(x, y, z) dx$

or $\int g(x, y, z) dy$ may be frequently used

then, convert all other variable into single variable using the path of integration.

i.e. $\int f(x) dx$ or $\int g(y) dy$.

Ques. Repeat same question using the parabolic path $x = 4y^2$ in the $z = 0$ plane.

Soln:

$$V = - \left[\int_{x=0}^4 y dx + \int_{y=0}^1 x dy \right]$$

$$= \int_0^4 \frac{x^{1/2}}{2} dx + \int_0^1 4y^2 dy$$

$$= \frac{1}{2} \cdot 2 \left(\frac{x^{3/2}}{3} \right)_0^4 + 4 \cdot \left(\frac{y^3}{3} \right)_0^1$$

$$= \left(\frac{1}{3} \cdot 2^3 + \frac{4}{3} \right) = \frac{-12}{3} = \underline{\underline{-4V}}$$

Ques. 21. P-6.

Soln:

$$\vec{A} = 2f \cos \phi \vec{a}_f$$

$$\oint \vec{A} \cdot d\vec{l}$$

$$\int 2f \cos \phi \vec{a}_f \cdot (df \vec{a}_f + f d\phi \vec{a}_\phi + dz \vec{a}_z)$$

for ①: $dl = df \vec{a}_f$

$$= \int_{f=0}^1 2f \cos \phi \vec{a}_f \cdot df \vec{a}_f$$

$$= \int_{f=0}^1 2f df = 1 \quad (\phi = 0)$$

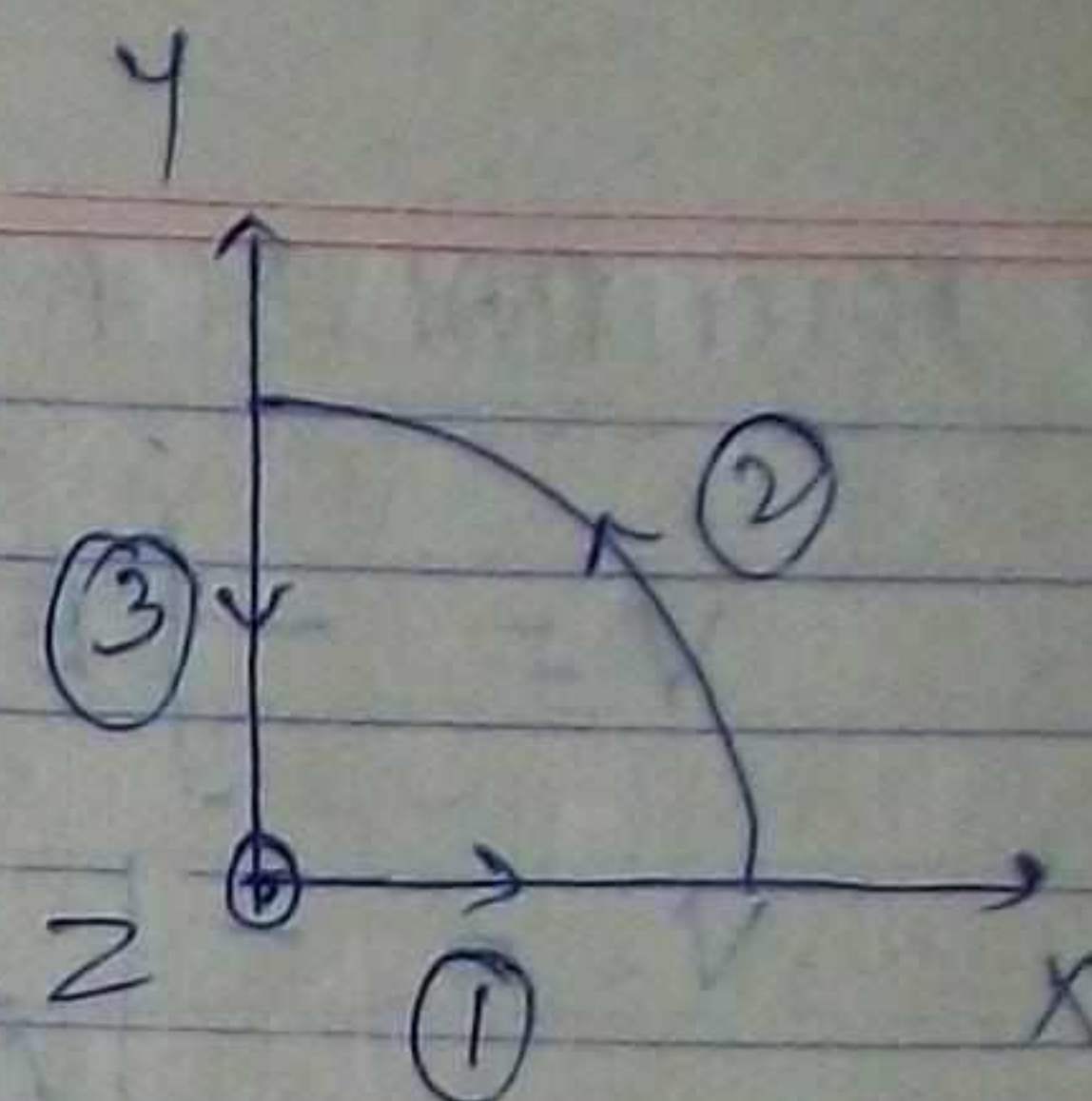
for ②: $dl = f d\phi \vec{a}_\phi$

$$\int_0^{\pi/4} \vec{A} \cdot d\vec{l} = 0$$

for ③: $dl = df \vec{a}_f$ $f = 1$ to 0

$$\vec{A} = 0 \quad \sin \phi = 90^\circ$$

$$\therefore \oint \vec{A} \cdot d\vec{l} = \underline{\underline{1}}$$



⇒ POTENTIAL OF A POINT CHARGE -

$$V = - \int E \cdot dl$$

$$V = - \int \frac{Q}{4\pi\epsilon r^2} a_r (dr a_r)$$

Spherical
coordinates

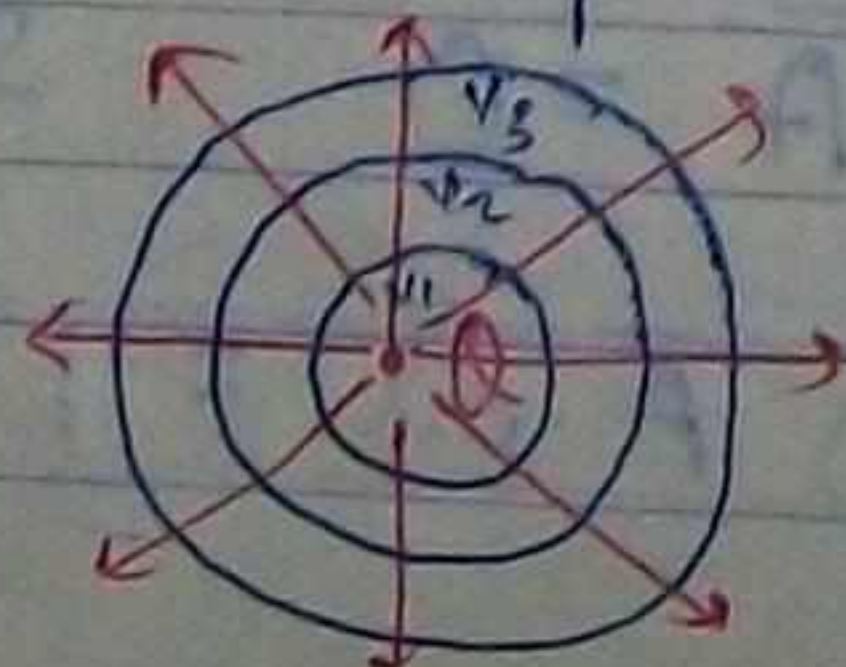
$$V = - \frac{Q}{4\pi\epsilon} \int \frac{dr}{r^2}$$

$$V = \frac{Q}{4\pi\epsilon r}$$

$E \rightarrow$ decreases as $1/r^2$ — a_r direction

$V \rightarrow$ decreases as $1/r$ — scalar
(no direction)

- The locus of all the points with $r = \text{const.}$ is a concentric sphere around the charge and on all these points the voltage is same hence the equipotential surfaces for a point charge are concentric spheres around the charge.



- The family of equipotential surfaces represent a means of showing the variation of potential (scalar) in a given region. They are similar to vector intensity line for E .

⇒ POTENTIAL OF A LINE CHARGE -

$$V = - \int E \cdot dl$$

$$V = - \int \frac{\rho_L}{2\pi\epsilon \rho} a_\rho (d\rho a_\rho)$$

$$V = - \frac{\rho_L}{2\pi\epsilon} \int \frac{d\rho}{\rho}$$

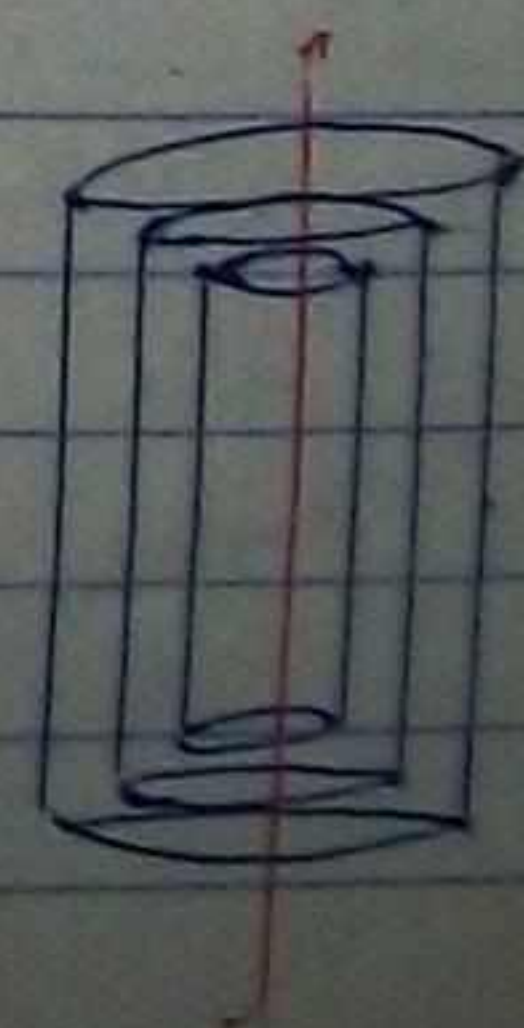
$$V = \frac{\rho_L}{2\pi\epsilon} \ln\left(\frac{1}{\rho}\right)$$

$E \rightarrow$ decreases $\rightarrow 1/\rho$

$V \rightarrow$ decreases — logarithmic — $\ln(1/\rho)$

$\rho = \text{CONST.}$ $V = \text{CONST.}$

∴ Equipotential surface is concentric cylinder



⇒ POTENTIAL OF A UNIFORM FIELD -

uniform → space independent

$$V = - \int E \cdot dl$$

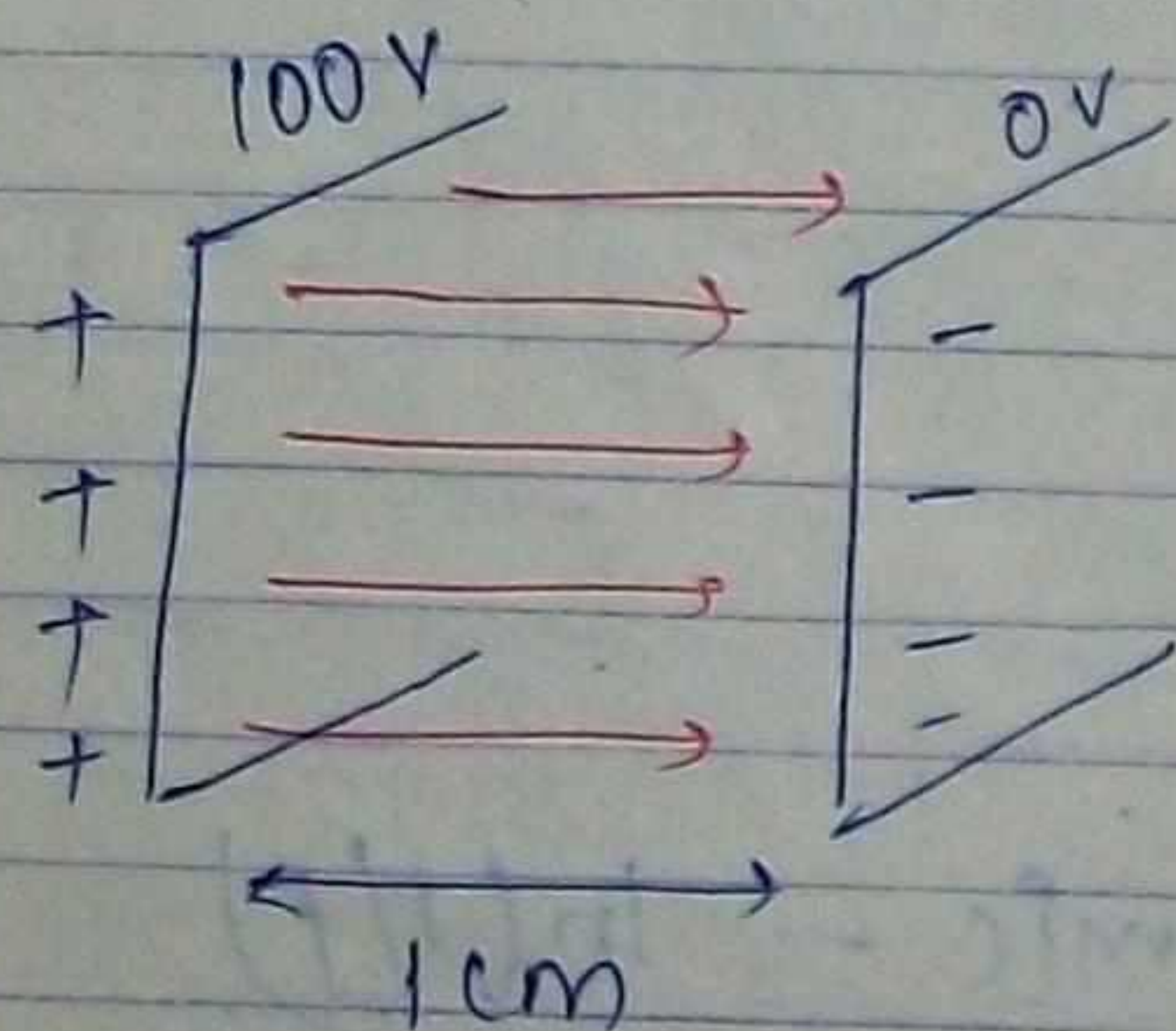
$$V = -E \cdot l$$

$$E = -\frac{V}{l}$$

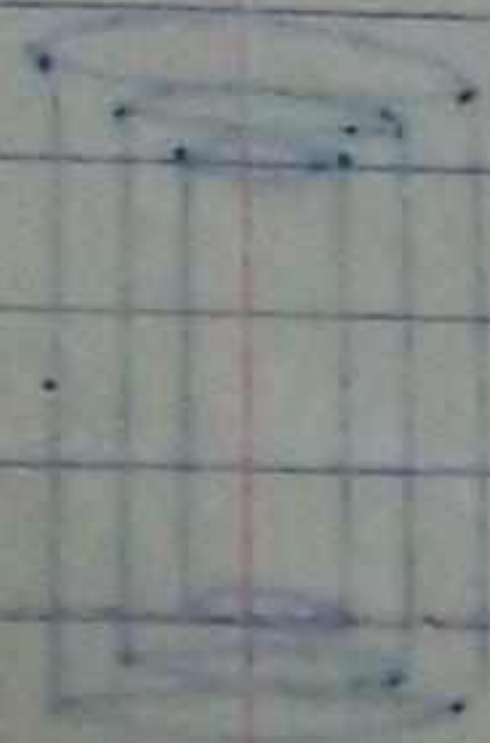
field b/w
Ex = capacitor
plate

$E \rightarrow$ CONST. $\rightarrow$ UNIFORM

$V \rightarrow$ linearly decrease



$$E = \frac{100 \text{ V}}{1 \text{ cm}} = 10 \text{ KV/m}$$



⇒ POTENTIAL GRADIENT -

Scalar surface
Eqⁿ f

∇f

Vector direction of
the surface
(normal)

- Consider a scalar surface eqⁿ f, gradient of this surface results in vector direction of the surface. which is normal to the surface every where.

LINEAR SURFACE

NON LINEAR SURFACE

$$f = 3x - 4y + 8z = 15$$

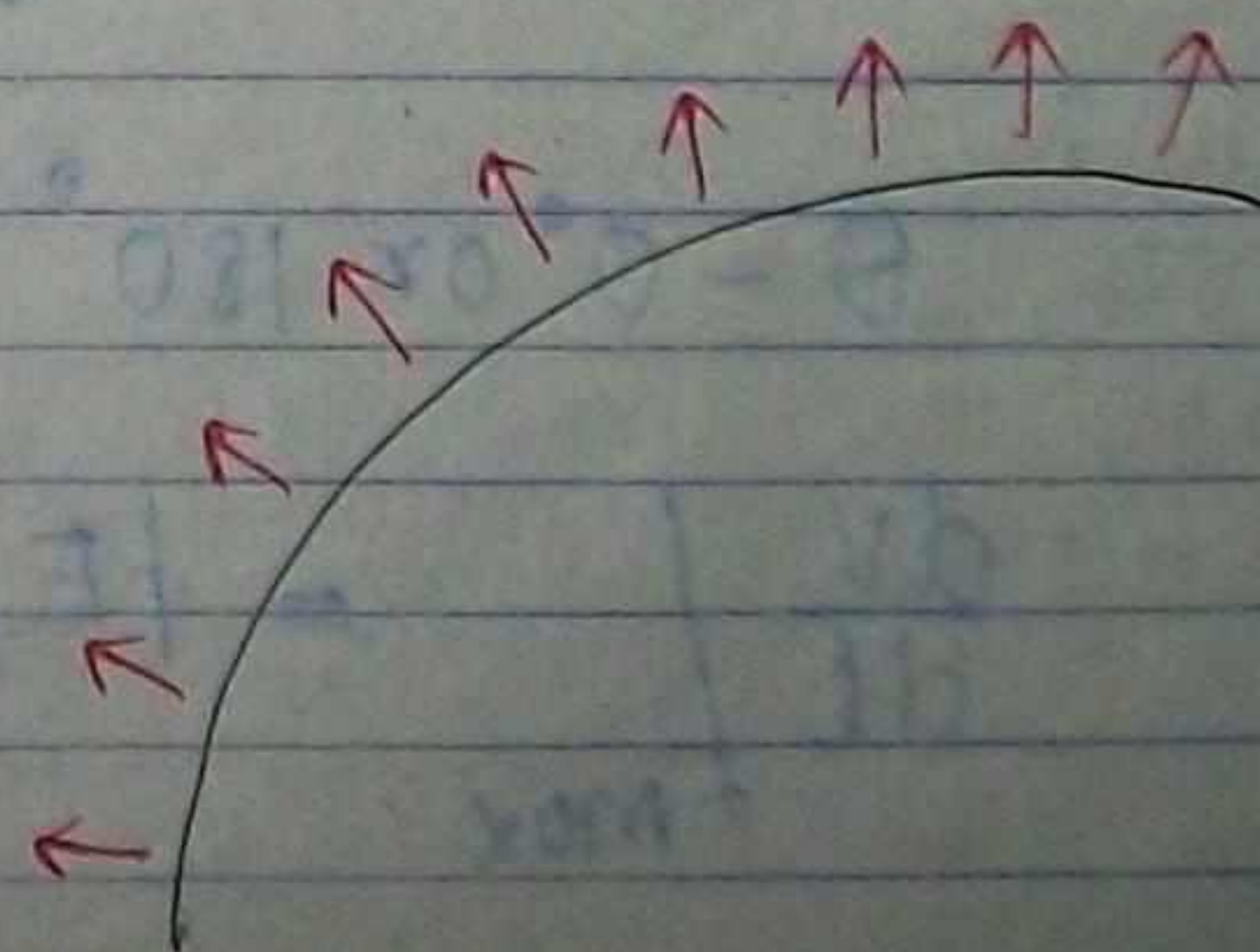
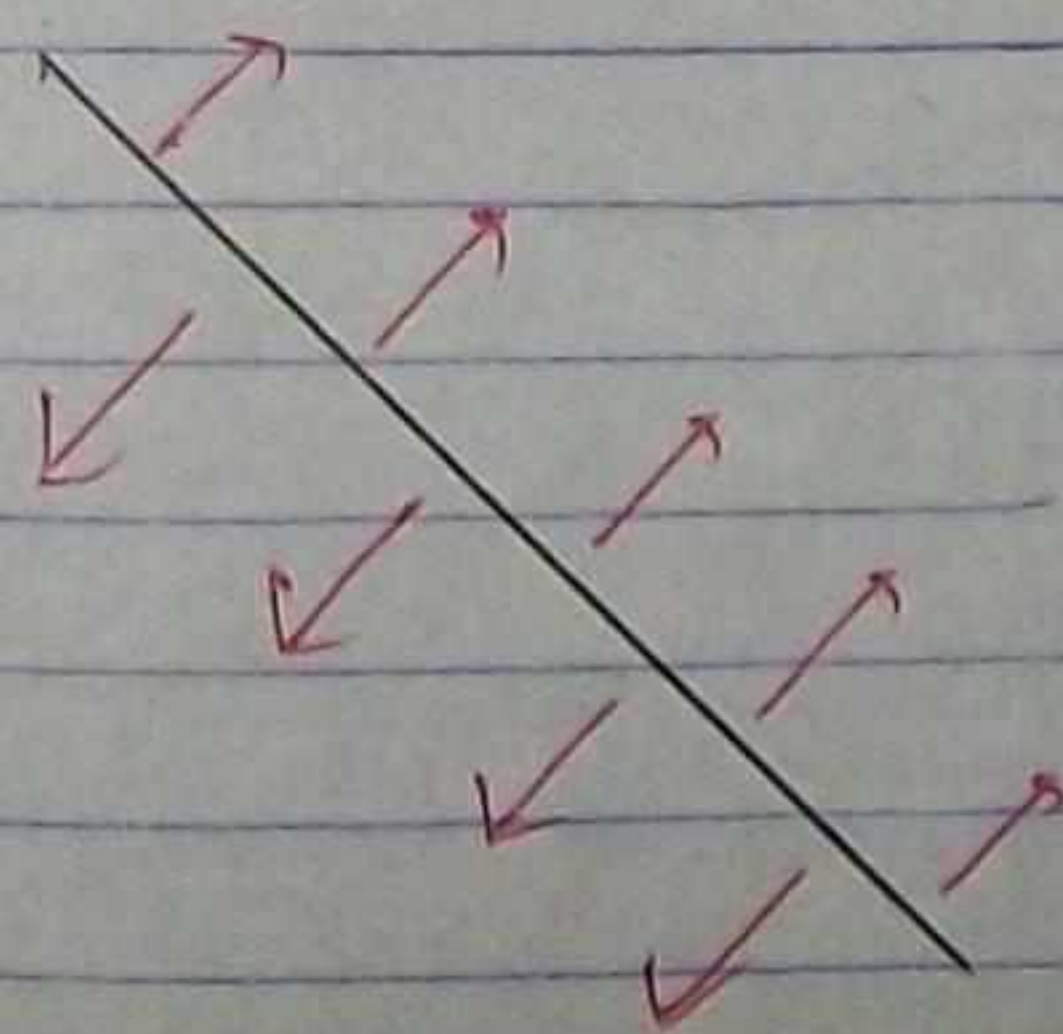
$$f = 3xy - 4x^2y = 12$$

$$\nabla f = 3a_x - 4a_y + 8a_z$$

$$\nabla f = (3y - 8xy)a_x + (3x - 4x^2y)a_y$$

CONSTANT
DIRECTION

Space dependent
direction



$$V = - \int \vec{E} \cdot d\vec{l}$$

$$\vec{E} = - \nabla V$$

$$\# \quad V = - \int \vec{E} \cdot d\vec{l}$$

$$dV = - \vec{E} \cdot d\vec{l}$$

$$= - E \cdot dl \cos \theta$$

$$\frac{dV}{dl} = - E \cos \theta$$

CASE (1):

$$\theta = 90^\circ \quad V = \text{CONST}$$

The $\vec{E}$ field direction is always $\perp$ to equipotential surfaces.

CASE (2):

$$\theta = 0^\circ \text{ or } 180^\circ$$

$$\left. \frac{dV}{dl} \right|_{\text{max}} = |E|$$

The magnitude of $\vec{E}$ -field intensity is the maximum rate of change of potential per unit length.

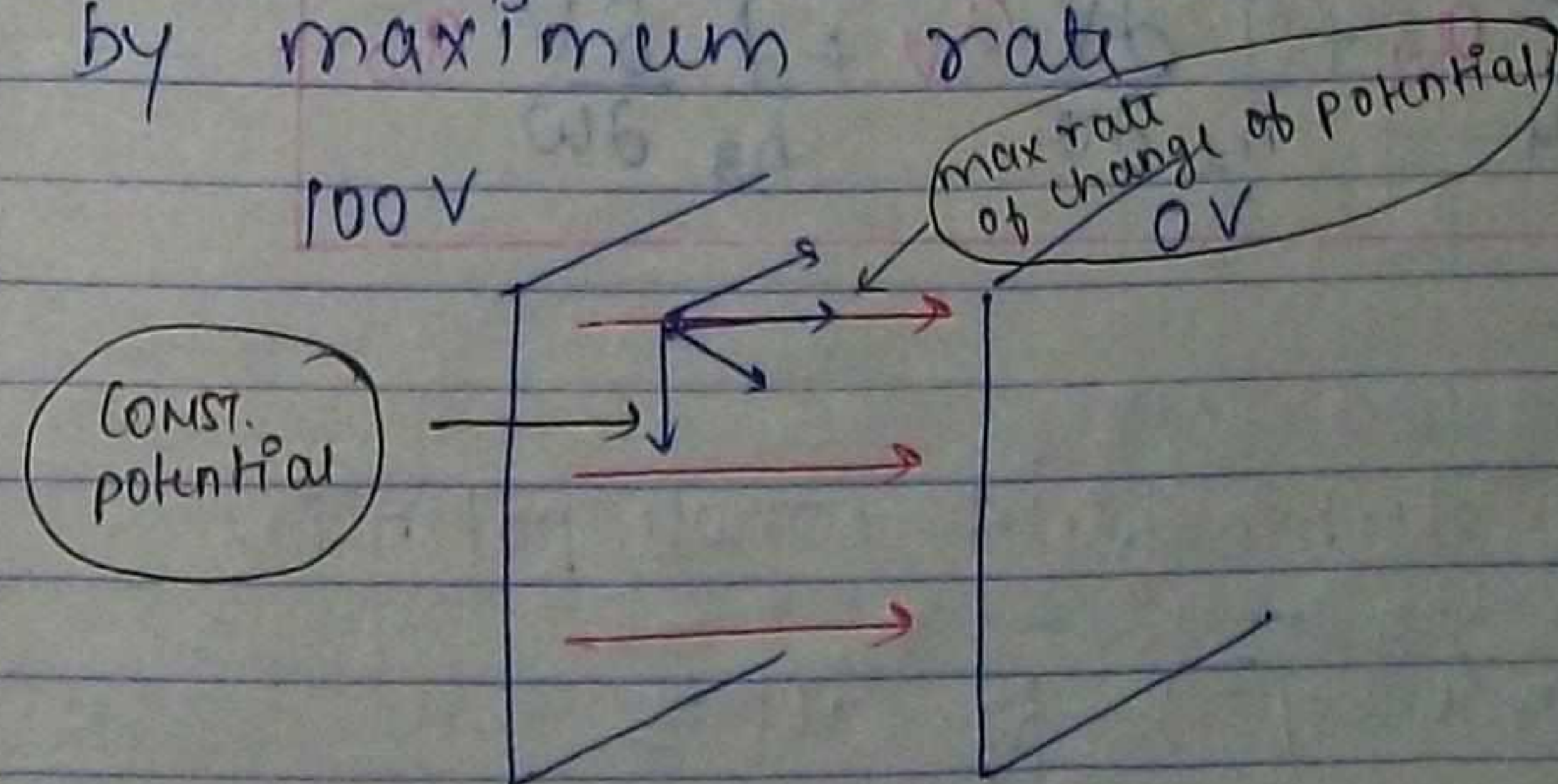
CASE (3):

$$\theta = 0^\circ$$

$$\left. \frac{dV}{dl} \right|_{\text{max}} = -E$$

$$\text{or } - \left. \frac{dV}{dl} \right|_{\text{max}} = |E|$$

The direction of $\vec{E}$ -field intensity is the direction in which potential decreases by maximum rate.



- This operation results in a vector quantity and it is called as gradient of the scalar hence gradient is the max rate of change of scalar along with the direction of the change.

- If the scalar is potential function (V) then the vector is E-field intensity (E) hence potential gradient is E-field intensity.

potential funⁿ $\xrightarrow{\quad}$ field intensity
V $\xrightarrow{\quad}$ $\vec{E}$

$$\therefore \vec{E} = -\nabla V$$

$\Rightarrow$ MATHEMATICALLY GRADIENT -

$$\nabla V = \frac{1}{h_1} \frac{\partial V}{\partial u} \hat{a}_u + \frac{1}{h_2} \frac{\partial V}{\partial v} \hat{a}_v + \frac{1}{h_3} \frac{\partial V}{\partial w} \hat{a}_w$$

Que. Given an electric field whose potential funⁿ $V = 100(x^2 - y^2)$ for all z

① find $\vec{E}$ at (2, 1, 2)

② find the equipotential surface passing through (2, 1, 2)

Solⁿ: ① $\vec{E} = -\nabla V$

$$\vec{E} = -\nabla V$$

$$\vec{E} = -[2x \hat{a}_x + -2y \hat{a}_y] \times 100$$

$$\vec{E} = -100[2x \hat{a}_x - 2y \hat{a}_y]$$

$$\vec{E} = -100[4 \hat{a}_x - 2 \hat{a}_y]$$

$$\vec{E} = -400 \hat{a}_x + 200 \hat{a}_y$$

② for equipotential surface $V = \text{CONST}$

$$V = 100(x^2 - y^2) = \underline{\underline{300}}$$

i.e. $x^2 - y^2 = 3$ is the equipotential surface.

Que. Given $V = \frac{4 \cos \theta}{r^2}$ find the $\vec{E}$ at $(2, \pi/4, \pi)$

Solⁿ:

$$\vec{E} = \left(\frac{\partial V}{\partial r} + \frac{1}{r} \frac{\partial V}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial V}{\partial \phi} \right) \left(\frac{4 \cos \theta}{r^2} \right)$$

$$= 4 \cos \theta \left(\frac{-2}{r^3} \right) + \frac{4}{r^3} (-\sin \theta) + \frac{1}{r \sin \theta} \cdot 0$$

$$= \frac{2x - 4\cos\theta}{r^3} \hat{a}_r - \frac{4\sin\theta}{r^3} \hat{a}_\theta$$

$$= \frac{2x - 4\cos\pi/2}{2^3} \hat{a}_r - \frac{4\sin\pi/4}{(2)^3} \hat{a}_\theta$$

$$= \frac{4x - 2}{2\sqrt{2}} \hat{a}_r - \frac{1}{2\sqrt{2}} \hat{a}_\theta$$

$$= -\frac{4\sqrt{2}}{2\sqrt{2}} \hat{a}_r - \frac{1}{2\sqrt{2}} \hat{a}_\theta$$

$$= -2\sqrt{2} \hat{a}_r - \frac{1}{2\sqrt{2}} \hat{a}_\theta$$

Qu. 23 P-7

Soln: (1) $V = 3x^2y - yz$

$$\nabla V = (6xy - 0) \hat{a}_x + (3x^2 - z) \hat{a}_y + (0 - y) \hat{a}_z$$

$$\begin{aligned} \nabla V \Big|_{(1,0,-1)} &= 0 \hat{a}_x + (3+1) \hat{a}_y + 0 \hat{a}_z \\ &= 4 \hat{a}_y + \hat{a}_z = \underline{4 \hat{a}_y} \end{aligned}$$

(2) $x^2y = 1$

Since at xy plane $z=0$ and $x=y=1$

$$\begin{aligned} V &= 3 \\ 3x^2y &= 3 \\ x^2y &= 1 \end{aligned}$$

(3) $V = 3(2)^2(-1) - (-1)(4) = -8$

(4)
$$\begin{aligned} \nabla V &= \frac{-12\hat{a}_x + 8\hat{a}_y + \hat{a}_z}{\sqrt{12^2 + 8^2 + 1}} \\ &= -0.83 \hat{x} + 0.55 \hat{y} + 0.07 \hat{z} \end{aligned}$$

⇒ MAXWELL'S (2) EQUATION -

CLOSED LINE INTEGRAL OF E

$$\oint E \cdot dl = 0, \quad \nabla \times E = 0$$

CONCLUSION (1) -

The potential at point is always same at a given time.

CONCLUSION (2) -

The work done in moving a charge in any closed loop is always zero.

CONCLUSION (3) -

E-field is a conservative field i.e. work done or energy is conserved in a closed path i.e.

$$\text{Energy acquired} = \text{Energy lost}$$

CONCLUSION (4) -

E-field never forms closed loop i.e. E-field is irrotational in nature

ii

$$\oint E \cdot dl = 0$$

$$\nabla \times E = 0$$

CONCLUSION (5) -

work done in moving a charge is always independent of path of consideration.

CONCLUSION (6) -

KVL

⇒ POTENTIAL, VECTOR POTENTIAL & MAXWELL'S (4) EQⁿ -
(AMPERE'S LAW)

POTENTIAL (IN MAGNETIC FIELD):

A scalar measure of magnetic field strength at any point in the field. It is called as M.M.F (V_m) (magneto-motive force)

$$V_m = \int \vec{H} \cdot d\vec{l}$$

(ampere)

$$H = \nabla V_m$$

MMF not current

$$\oint H \cdot dl = I \text{ (amp)} \leftarrow \text{current}$$

AMPERE
CIRCUTIAL
LAW

$$\oint H \cdot dl = I$$

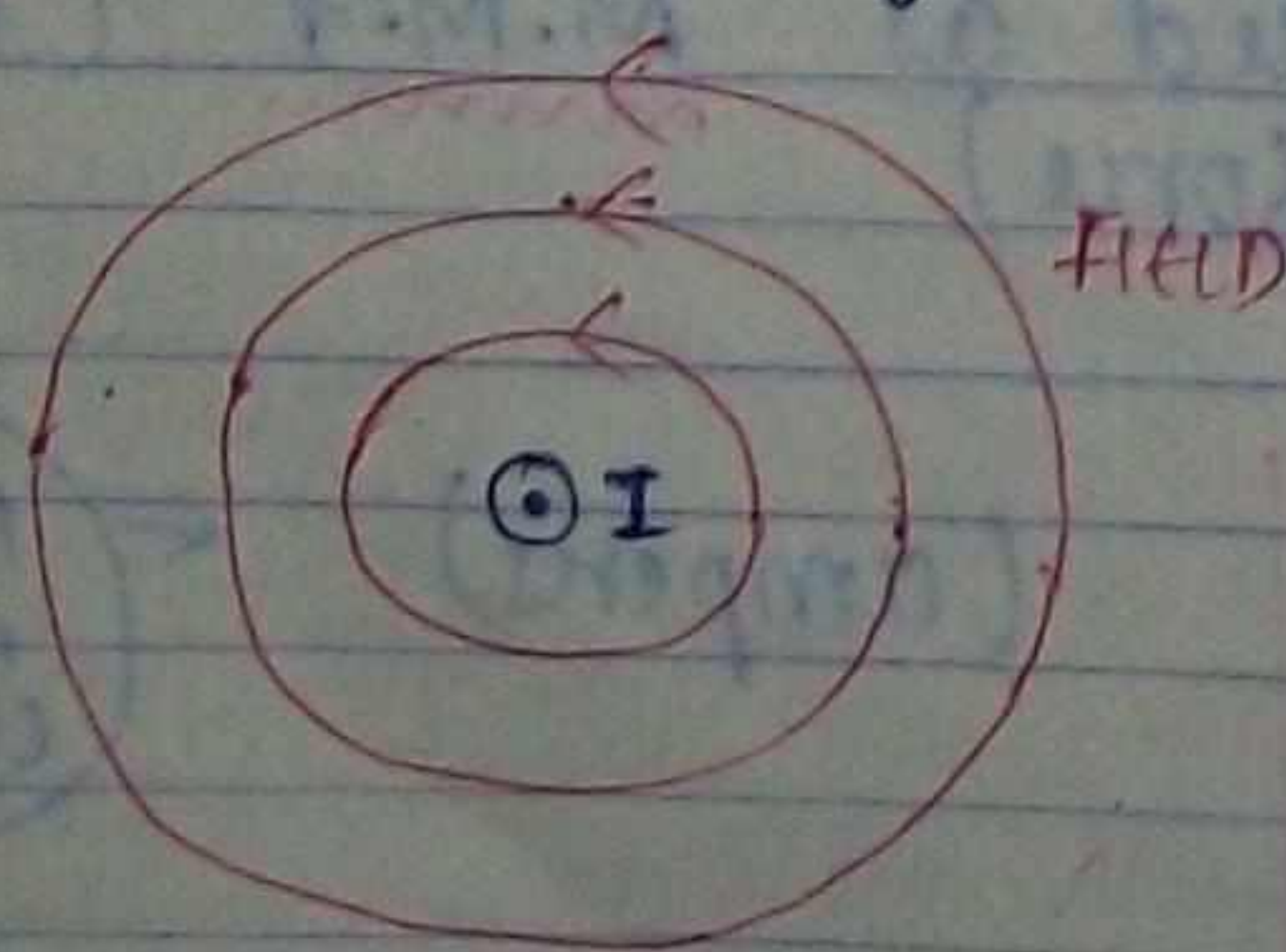
(integral form)

$$\nabla \times H = J$$

(point form)

STATEMENT (AMPERE'S LAW) -

The ^{EFFECT} circulation of magnetic field in any closed path is equal to the ^{CAUSE} current crossing the surface enclosed.



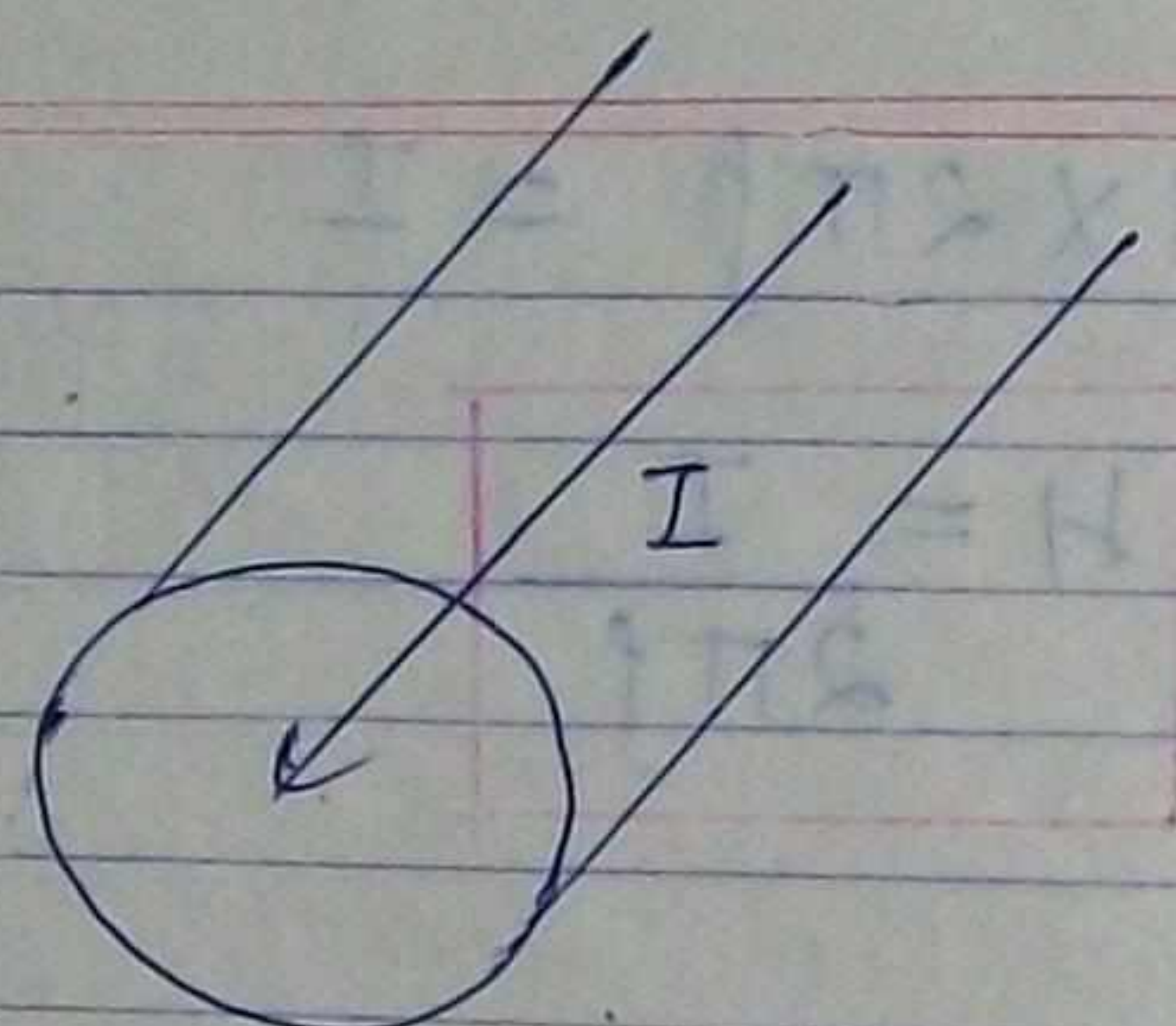
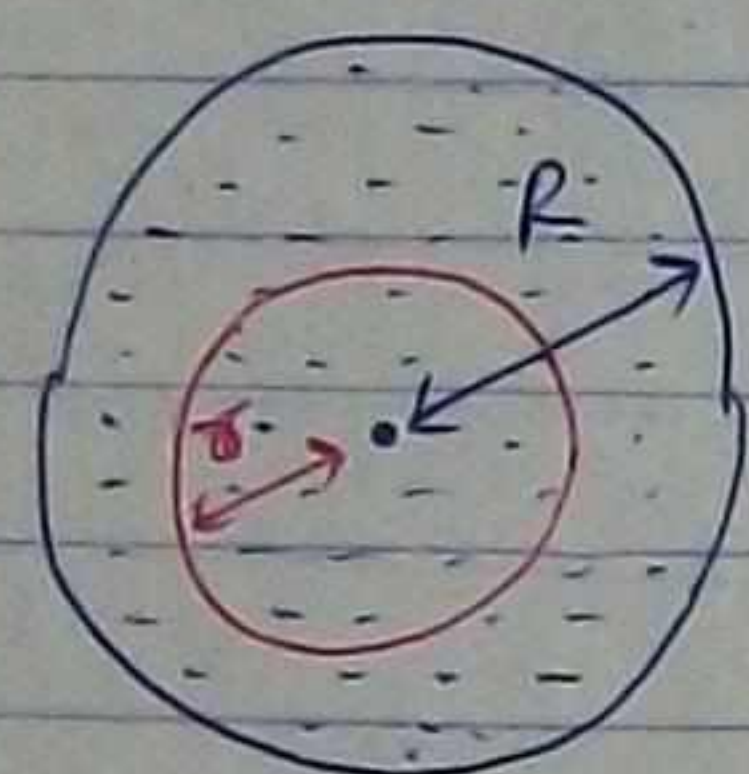
$$\text{STRENGTH} \times \text{LENGTH} = I$$

$$H \times 2\pi r = I$$

$$H = \frac{I}{2\pi r}$$

26/09/11

Ques. 26 P-7



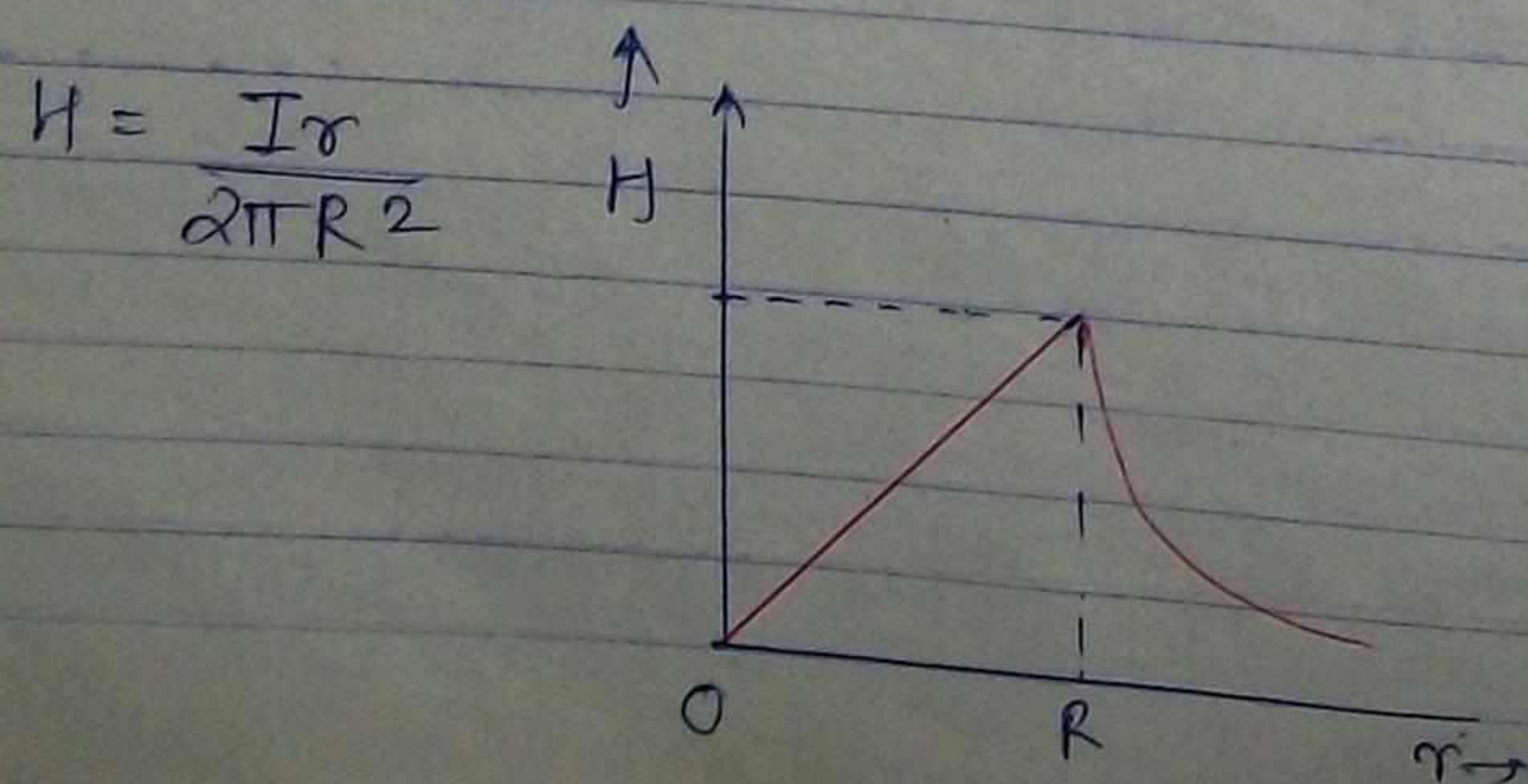
Consider a closed line which is a circle, symmetric with the current flow and concentric with the cross-section. Hence

$$\oint H \cdot dl = I$$

$$H \times \text{length} = I$$

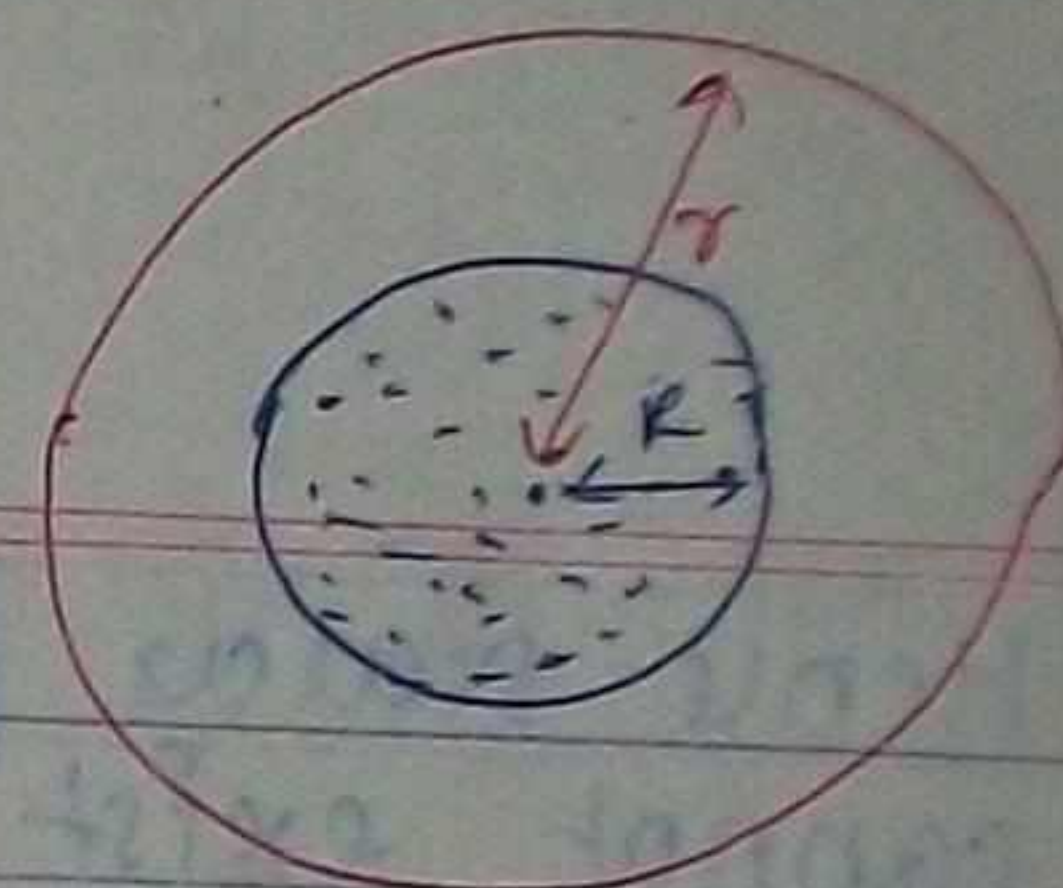
current area $(r < R)$

$$H = \frac{\frac{I}{\pi R^2} \times \pi r^2}{2\pi r} = \frac{I r}{2\pi R^2}$$



for $(r > R)$

$$H = \frac{I}{2\pi r}$$



⇒ VECTOR POTENTIAL ($\vec{A}$)

- In electric fields scalar potential V always satisfies the basic nature of electric field as shown:

$$E = -\nabla V$$

$$\nabla \times E = 0$$

satisfies as $\nabla \times (-\nabla V) = 0$

Since curl of (grad of scalar) = 0

- However in magnetic fields the same expression cannot exist where $J \neq 0$

$$H = \nabla V_m$$

$$\nabla \times H = J$$

$$\nabla \times (\nabla V_m) = 0$$

= J (actually)

Hence scalar potential (MMF) ∇ cannot exist in conductor or current flowing materials, it can be defined for free space and current free region.

Ex - MMF in the air-gaps between the machines

- VECTOR POTENTIAL ($\vec{A}$) -

The definition of vector potential is in accordance to the basic nature of magnetic fields i.e.

$$\nabla \cdot \vec{B} = 0$$

Hence $\vec{B} = \nabla \times \vec{A}$

where $\vec{A}$ = vector potential

Hence $\nabla \cdot (\nabla \times \vec{A}) = 0$

Divergence of (curl of vector) = 0

- The units of $\vec{A}$ are $\frac{\text{weber}}{\text{m}}$ which physically means $\frac{\text{Joules}}{\text{Amp} \cdot \text{m}}$

which means, potential is work done per basic cause i.e.

$$\vec{A} = \frac{\omega}{I d\vec{l}}$$

as $I d\vec{l}$ is a vector, potential is a vector

Faraday's law

$$\frac{\text{weber}}{\text{sec}} = \text{Volts} = \frac{\text{Joules}}{\text{Coulombs}}$$

$$\frac{\text{weber}}{\text{Amp}} = \frac{\text{Joules}}{\text{Amp}}$$

$$\therefore \frac{\text{weber}}{\text{meter}} = \frac{\text{Joules}}{\text{Amp} \cdot \text{m}}$$

NOTE -

$$\vec{E} = \frac{\vec{F}}{Q}$$

$$V = \frac{\omega}{Q}$$

$$\therefore \vec{E} = -\nabla V$$

$$\vec{B} = \frac{\vec{F}}{I d\vec{l}}$$

$$\vec{A} = \frac{\omega}{I d\vec{l}}$$

$$\vec{B} = \nabla \times \vec{A}$$

- formulae for vector potentials

$$\vec{A} = \frac{\mu I d\vec{l}}{4\pi r}$$

$$V = \frac{Q}{4\pi r \epsilon}$$

$\vec{A}$ direction is always I flow direction and always forms closed loop.

Ques. The closed line integral of vector magnetic potential is ____.

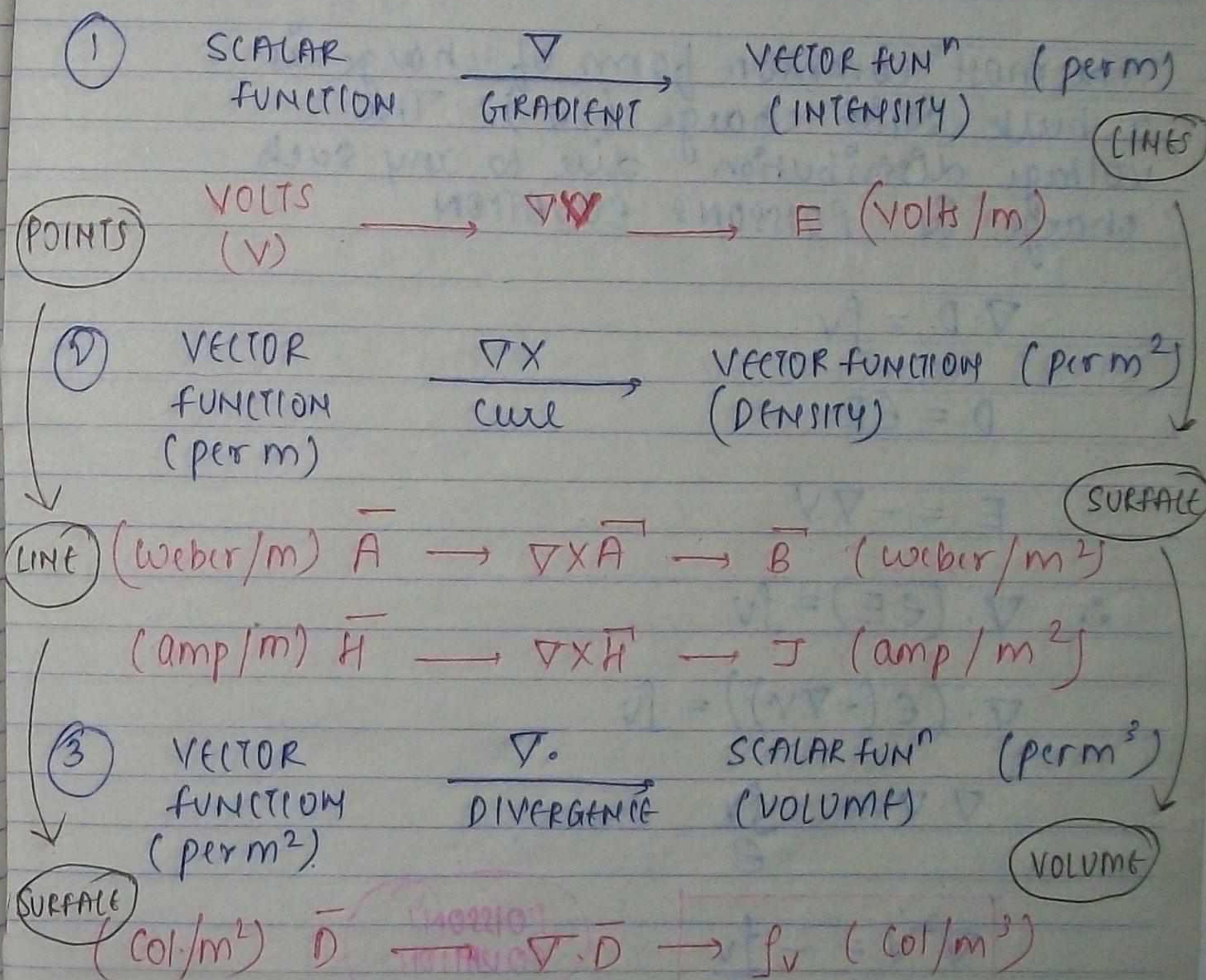
Solⁿ:

$$\oint \vec{A} \cdot d\vec{l} = \frac{\text{weber}}{\text{m}} \times \text{m} \\ = \text{weber} (\Psi_m)$$

also,

$$\oint \vec{A} \cdot d\vec{l} = \int (\nabla \times \vec{A}) \cdot d\vec{s} \\ = \int \vec{B} \cdot d\vec{s} = \Psi_m$$

SUMMARY -



- Directional integration (effect over a region)
- Directional derivative (effect at a point)

⇒ LAPLACE & POISSON'S EQUATIONS —

- The most common form of charge is a bulk solid charge (ρ_v). The voltage distribution due to any such charge is POISSON'S EQUATION

$$\nabla \cdot D = \rho_v$$

$$D = \epsilon E$$

$$E = -\nabla V$$

$$\therefore \nabla \cdot (\epsilon E) = \rho_v$$

$$\nabla \cdot (\epsilon (-\nabla V)) = \rho_v$$

$$\nabla \cdot \nabla V = -\frac{\rho_v}{\epsilon}$$

$$\boxed{\nabla^2 V = -\frac{\rho_v}{\epsilon}}$$

POISSON'S
EQUATION

for charge free region or neutral materials
($\rho_v = 0$)

$$\boxed{\nabla^2 V = 0}$$

LAPLACE
EQUATION

NOTE —

The equations are second order diff. equation which always gives single solution i.e. voltage-function is always a single valued function for valid electric fields.

EXTENSION: FOR MAGNETIC FIELDS —

$$\nabla \times H = J$$

$$B = \mu H$$

$$\nabla \times A = B$$

$$\therefore \nabla \times \left(\frac{B}{\mu} \right) = J$$

$$\nabla \times (\nabla \times A) = \mu J$$

for magnetic
field
 $\nabla \cdot A = 0$

$$0 = \nabla(\nabla \cdot A) - (\nabla \cdot \nabla) A = \mu J$$

$$\boxed{\nabla^2 A = -\mu J}$$

for current free region:

$$\boxed{\nabla^2 A = 0}$$

$\nabla^2 = \nabla \cdot (\nabla \text{ scalar})$ LAPLACIAN OPERATOR
 $= \text{Div. (grad of scalar)}$

$$\nabla^2 V = \frac{1}{h_1 h_2 h_3} \left[\frac{\partial}{\partial u} \left(\frac{h_2 h_3}{h_1} \frac{\partial V}{\partial u} \right) + \frac{\partial}{\partial v} \left(\frac{h_3 h_1}{h_2} \frac{\partial V}{\partial v} \right) + \frac{\partial}{\partial w} \left(\frac{h_1 h_2}{h_3} \frac{\partial V}{\partial w} \right) \right]$$

Que. 30 P-8
 Solⁿ:
 $V = -\frac{6r^5}{\epsilon_0}$

① $\nabla^2 V = -\frac{\rho_v}{\epsilon}$

② $Q = \int \rho_v dV$

$$\begin{aligned} \nabla^2 V &= \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial r} \left(r^2 \sin \theta \frac{\partial V}{\partial r} \right) = -\frac{\rho_v}{\epsilon} \\ &= \frac{1}{r^2} \frac{\partial}{\partial r} \left(-\frac{6r^2}{\epsilon} \frac{\partial}{\partial r} (-r^5) \right) = -\frac{\rho_v}{\epsilon} \end{aligned}$$

$$\rho_v = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \cdot 6 \cdot 5 \cdot r^4)$$

$$\rho_v = \frac{1}{r^2} \cdot 30 \cdot 6r^5$$

$$\rho_v = 180 r^3$$

Now, $Q = \int_0^1 \int_0^\pi \int_0^{2\pi} 180 r^3 r^2 \sin \theta dr d\theta d\phi$
 $r=0 \quad \theta=0 \quad \phi=0$

$$= 180 \cdot \left(\frac{r^6}{6} \right)_0^1 \cdot (-\cos \theta)_0^\pi \cdot (\phi)_0^{2\pi}$$

$$= \underline{\underline{120 \pi}} \text{ Coulomb}$$

Que. 31. P-8
 Solⁿ:

$$\rho_v = -10^{-8} (1 + 10r) \text{ C/m}^3$$

V on $r=5$ wrt $r=2$

V and E = 0 for both inner cylinder f=2
 initial condⁿ = zero

$$\nabla^2 V = -\frac{\rho_v}{\epsilon}$$

$$\frac{1}{r} \left(\frac{\partial}{\partial r} r \frac{\partial V}{\partial r} \right) = -\frac{\rho_v}{\epsilon} = \frac{-(-10^{-8}(1+10j))}{36\pi \times 10^{-9}}$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial V}{\partial r} \right) = 360\pi (1+10j)$$

$$\frac{\partial}{\partial r} \left(r \frac{\partial V}{\partial r} \right) = 360\pi (r + 10r^2)$$

$$r \frac{\partial V}{\partial r} = 360\pi \int (r + 10r^2) dr$$

$$r \frac{\partial V}{\partial r} = 360\pi \left[\frac{r^2}{2} + \frac{10r^3}{3} \right]$$

$$\frac{\partial V}{\partial r} = 360\pi \left[\frac{r}{2} + \frac{10}{3} r^2 \right]$$

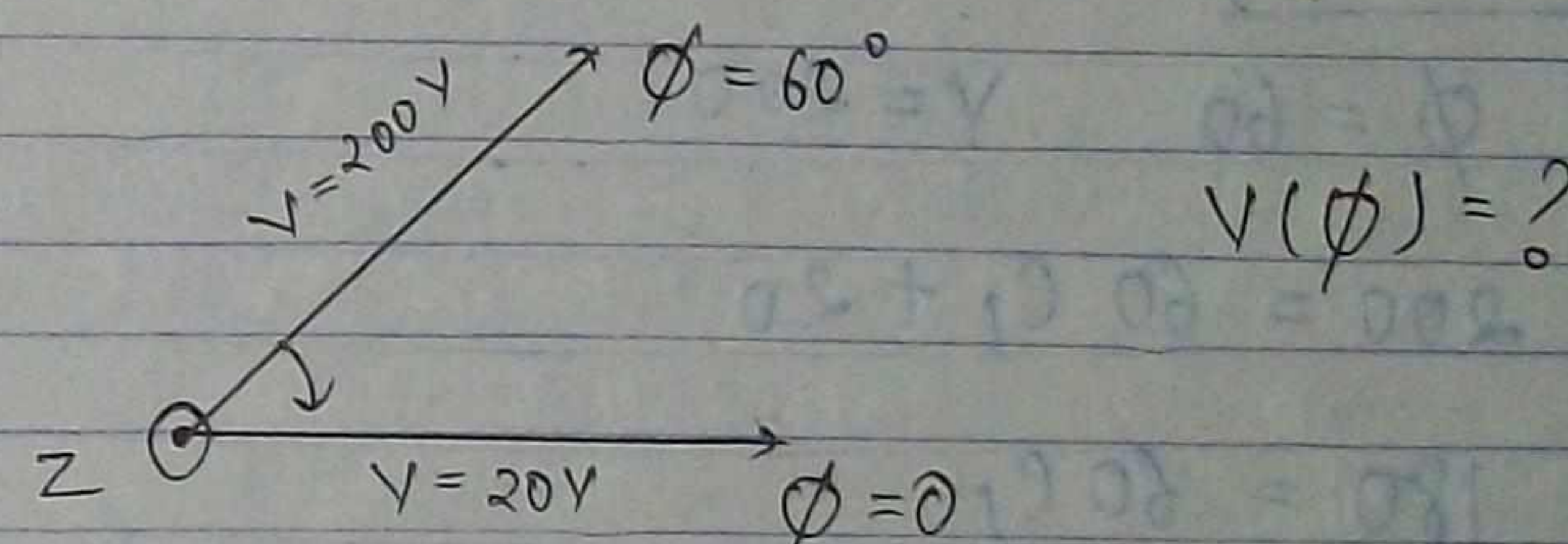
$$V = 360\pi \int \left[\frac{r}{2} + \frac{10}{3} r^2 \right] dr$$

$$V = 360\pi \left[\frac{r^2}{4} + \frac{10}{3} \times \frac{r^3}{3} \right]$$

$$V = 0.5 V$$

$$2 \times 10^{-3}$$

Que. In the diag. shown below, the two plates are infinitely large in shape of V. find the voltage b/w the plates $V(\phi)$.



Soln: free space $\nabla^2 V = 0$

$$\left. \begin{array}{l} V(60^\circ) = 200 \text{ V} \\ V(0^\circ) = 20 \text{ V} \end{array} \right\} V - \text{fun}^n \text{ of } \phi$$

$$\nabla^2 V = 0$$

$$\frac{1}{r} \frac{\partial}{\partial \phi} \left(\frac{1}{r} \frac{\partial V}{\partial \phi} \right) = 0$$

$$\frac{1}{r^2} \frac{\partial^2 V}{\partial \phi^2} = 0 \Rightarrow \int \frac{\partial^2 V}{\partial \phi^2} = 0$$

$$\int \frac{\partial V}{\partial \phi} = C_1$$

again, $V = C_1 \phi + C_2$

at $\phi = 0$ $V = 20$

$$\boxed{20 = C_2}$$

at $\phi = 60$ $V = 200$

$$200 = 60 C_1 + 20$$

$$180 = 60 C_1$$

$$\boxed{C_1 = 3}$$

$$\therefore V(\phi) = 3\phi + 20.$$

Ques 32. P-8

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2}$$

$$V = 10y^4 + 20x^3$$

$$\nabla^2 V = \frac{\partial}{\partial x} (60x^2) + \frac{\partial}{\partial y} (40y^3)$$

$$\nabla^2 V = 120x + 120y^2 = -\frac{f_v}{\epsilon}$$

$$f_v = 240\epsilon$$

at $(2, 0)$

Ques 34 P-9

Soln:

$$\nabla^2 \phi = 0$$

$\phi = \text{linear fun}^n$

$$\frac{\partial^2 \phi}{\partial x^2} = 0, \quad \frac{\partial \phi}{\partial x} = C_1$$

$$\phi = C_1 x + C_2$$

$\phi = \text{has a constant rate of change.}$

$$\therefore \frac{\phi_2 - \phi_1}{d} = \frac{\phi_3 - \phi_2}{2d}$$

$$\frac{\phi_2}{d} + \frac{\phi_2}{2d} = \frac{\phi_3}{2d} + \frac{\phi_1}{d}$$

$$\frac{3\phi_2}{2d} = \frac{2\phi_1 + \phi_3}{2d}$$

$$\phi_2 = \frac{2\phi_1 + \phi_3}{3}$$

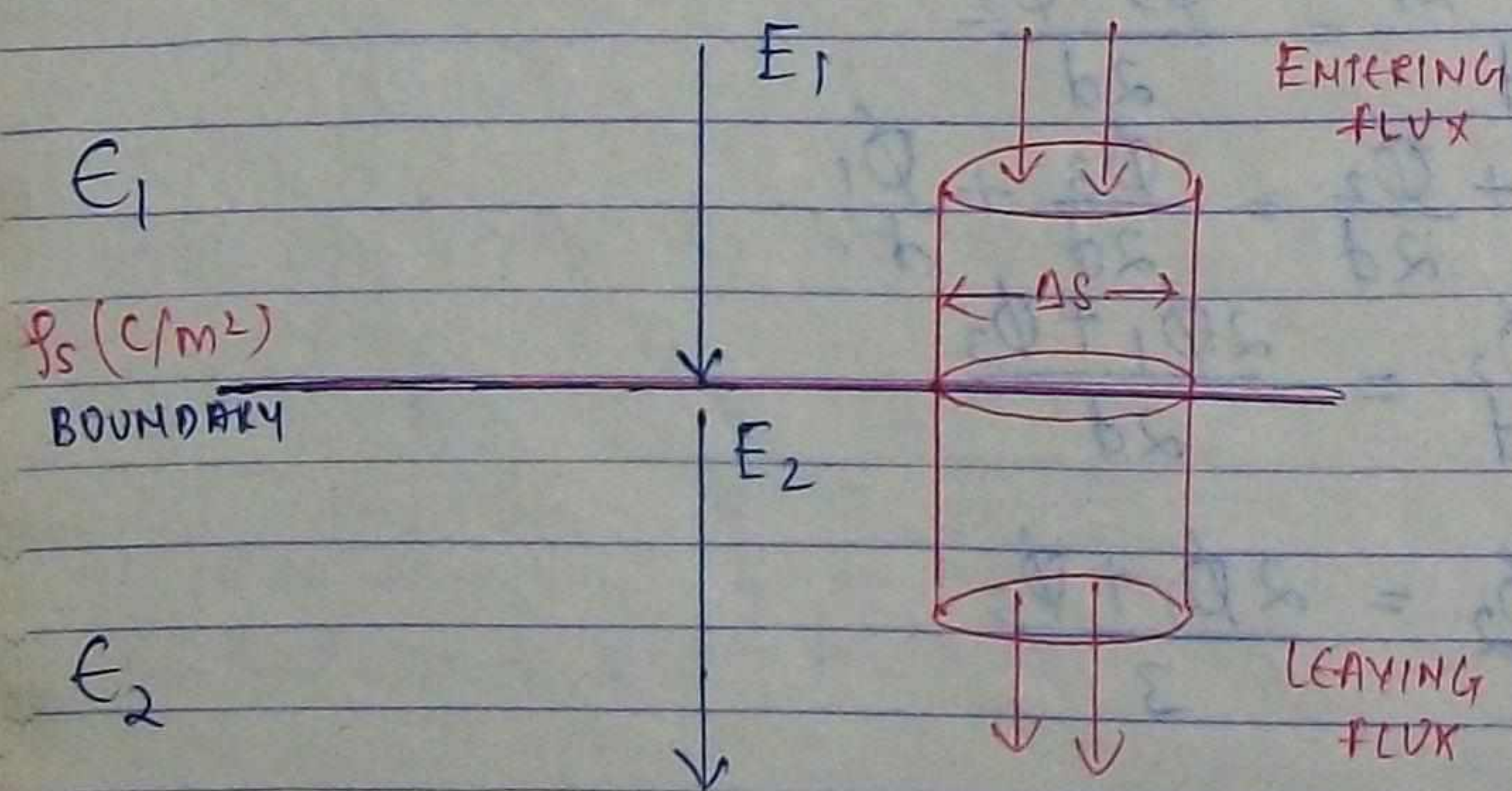
⇒ ELECTRIC BOUNDARY CONDITIONS -

DIELECTRIC - DIELECTRIC MEDIUM -

CASE (1): DIELECTRIC - DIELECTRIC MEDIUM -

If the field in one region dielectric ϵ_1 is known, the field in the adjacent region can be calculated using boundary conditions.

CASE (1) : E-FIELD IS NORMAL TO THE BOUNDARY



Consider a surface spread in both the regions and applying gauss's law:

$$\oint \mathbf{D} \cdot d\mathbf{s} = Q$$

$$D_2 \cdot \Delta S - D_1 \cdot \Delta S = 0$$

$$D_1 = D_2$$

If the boundary have a surface charge of ρ_s (C/m^2) then,

$$D_2 \Delta S - D_1 \cdot \Delta S = \rho_s \Delta S$$

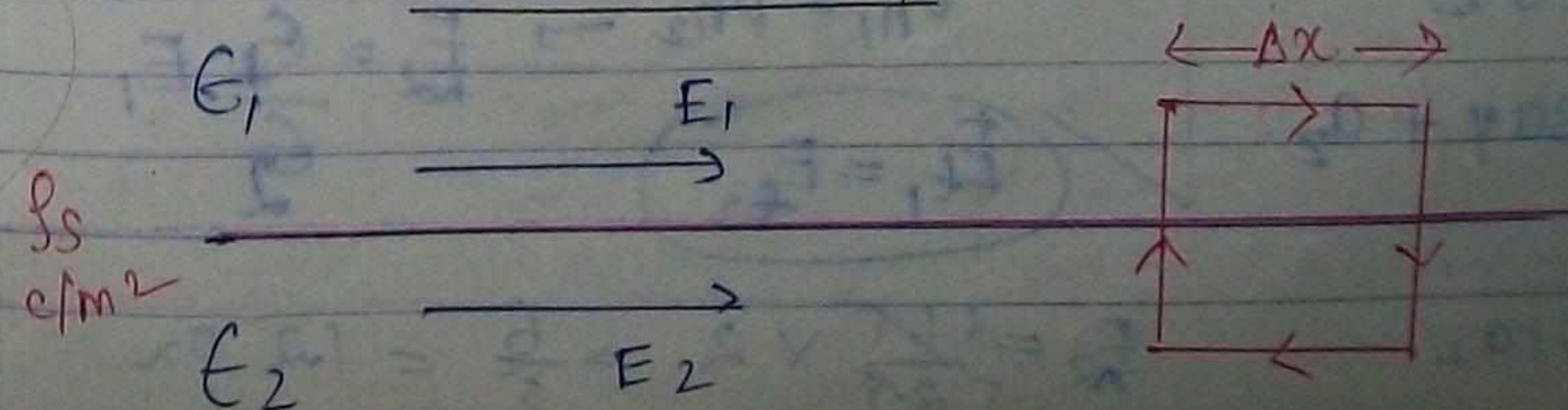
$$D_2 - D_1 = \rho_s$$

STATEMENTS:

$$\begin{aligned} D_{n1} &= D_{n2} \\ D_{n2} - D_{n1} &= \rho_s \end{aligned}$$

The normal component of flux density is continuous on both the sides, when there is no surface charge, hence discontinuity by an amount equal to surface charge density.

CASE (2) : E-FIELD IS TANGENTIAL (PARALLEL) TO THE BOUNDARY



Consider a closed line in both the medias and applying

$$\oint E \cdot dl = 0$$

$$E_1 \Delta x - E_2 \Delta x = 0$$

$$E_1 = E_2$$

STATEMENT:

$$E_{t1} = E_{t2}$$

Tangential component of field intensity is always continuous.

Ques 36 P-9

$$x < 0$$

$$\epsilon_1 = 1.5 \epsilon_0$$

$$E_1 = 2a_x - 3a_y + a_z$$

$$E_{n1} = 2a_x$$

$$E_{t1} = -3a_y + a_z$$

$$E_2 = 1.2a_x - 3a_y + a_z$$

$$x=0$$

$$x > 0$$

$$\epsilon_2 = 2.5 \epsilon_0$$

$$E_2$$

$$D_{n1} = D_{n2} \rightarrow$$

$$E_{n2} = \frac{\epsilon_1}{\epsilon_2} \cdot E_{n1}$$

$$E_{t1} = E_{t2}$$

$$E_{n2} = \frac{1.5}{2.5} \times 2 = \frac{6}{5} = 1.2 a_x$$

yz plane

$$D_1 = \epsilon_0 (3a_x - 4a_y + 1.5a_z) \quad D_2 = \epsilon_0 (3a_x - 7.5a_y + 2.5a_z)$$

$$D_1 \neq D_2$$

$$\text{Since, } D_{n1} = D_{n2}$$

$$\# \text{ for surface } 3x + 4y - 5z = 9$$

$$S = 3x + 4y - 5z$$

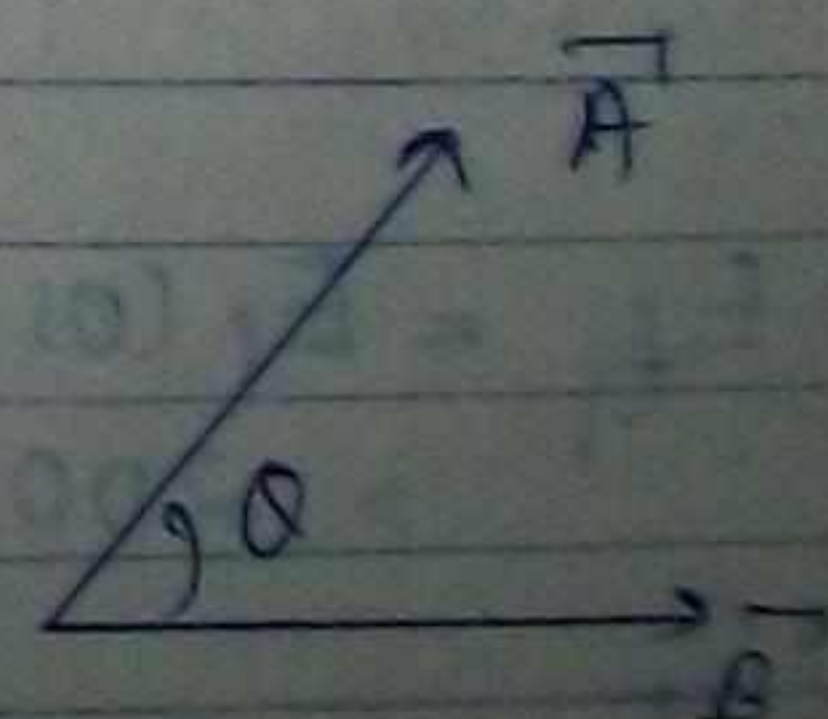
$$\nabla S = 3a_x + 4a_y - 5a_z \quad (\text{gradient})$$

NOTE - (1) If the surface is any general surface get the normal vector to the surface by taking the gradient and get the vector projection of E_1 in the direction of gradient i.e. E_{n1}

$$\text{and } E_{t1} = E_1 - E_{n1}$$

(2) vector projection of $\vec{A}$ on $\vec{B}$

$$A \cos \theta = \text{Scalar projection of } A \text{ and } B$$



$$\frac{A(A \cdot B)}{|A||B|} = \frac{A \cdot B}{|B|}$$

$$\therefore \text{vector projection of A on B} = \frac{(A \cdot B)}{|B|} \hat{B}$$

$$= \frac{(A \cdot B) \vec{B}}{|B|^2}$$

- ③ In the given question the field is shifted away from the normal comparatively as seen with decreased ϵ_r terms comparing E_1, E_2 . Similarly comparing D_1, D_2 the tangential component is increased in D_2 .

Que. 39 P-10.

$|E| = 400 \text{ V/m}$

ϵ_1 $\epsilon_2 = 20$ air die

30°

$E_{t1} = E_1 \cos 30$
 $= 200\sqrt{3} = E_{t2}$

$E_{n1} = E_1 \sin 30 = 200$

$$E_{n2} = \frac{200\epsilon_0}{20\epsilon_0} = 10, \quad D_{n1} = 200\epsilon_0 = D_{n2}$$

$$E_2 = \sqrt{(200\sqrt{3})^2 + (10)^2} \approx \underline{\underline{200\sqrt{3}}}$$

EXTENSION: MAGNETIC BOUNDARY CONDITION.

STATEMENT ①: from $\oint B \cdot ds = 0$

$$B_{n1} = B_{n2} \quad (\text{always})$$

STATEMENT ②: $\oint H \cdot dl = I$

$$H_{t1} = H_{t2}$$

But if there is a surface current density exist K (Amp/m)

$$H_{t2} - H_{t1} = K$$

Ques. 38 P-10

$z=0$

$\mu_1 = 2$	$\mu_2 = 1$
$B_{n1} = 0.4a_z$	$B_{n2} = B_{n1}$
$B_{H1} = 1.2ax + 0.8ay$	$B_{n2} = 0.4a_z$ $H_{n2} = \frac{1}{\mu_0} (0.4a_z)$

$$H_{t1} = \frac{B_1}{\mu_1}$$

$$H_{t1} = \frac{1}{2} (1.2ax + 0.8ay)$$

$$= (0.6ax + 0.4ay) / \mu_0$$

$$H_{t2} = H_{t1} = (0.6ax + 0.4ay) / \mu_0$$

$$\therefore E_2 = (0.6ax + 0.4ay + 0.4a_z) / \mu_0$$

$\Rightarrow$ ENERGY, ENERGY DENSITY OF AN ELECTRIC FIELD-

Consider 'n' point charges $Q_1, Q_2, \dots, Q_n$

Total energy of E-field of n charge = Energy expended in assembling 'n' charge

$$W_1 = 0$$

$$W_2 = -Q_2 V_{21}$$

(V_{21} = potential at 2 due to 1st charge)

$$W_3 = -Q_3 V_{31} - Q_3 V_{32}$$

$$W_n = -Q_n V_{n1} - Q_n V_{n2} - \dots - Q_n V_{nn-1}$$

Sum of all energies is W_E - (1)

$$\text{since, } Q_2 V_{21} = Q_2 \cdot \frac{Q_1}{4\pi\epsilon_0 r_{21}} = Q_1 \cdot \frac{Q_2}{4\pi\epsilon_0 r_{12}} = Q_1 V_{12}$$

$$\therefore W_1 = 0$$

$$W_2 = -Q_1 V_{12}$$

$$W_3 = -Q_1 V_{13} - Q_2 V_{23}$$

$$W_n = -Q_1 V_{1n} - Q_2 V_{2n} - \dots - Q_{n-1} V_{n-1,n}$$

Here also sum is also W_E - (2)

Now adding (1) and (2):

$$2W_E = -Q_1 V_1 - Q_2 V_2 - Q_3 V_3 \dots - Q_n V_n$$

$$W_E = -\frac{1}{2} \sum_{i=1}^n Q_i V_i$$

for a continuous charge distribution:

$$W_E = -\frac{1}{2} \int \rho_v \cdot V \, dv$$

$$W_E = -\frac{1}{2} \int (\nabla \cdot D) V \, dv$$

$$W_E = \frac{1}{2} \int D \cdot (-\nabla V) \, dv$$

$$W_E = \frac{1}{2} \int (D \cdot E) \, dv$$

$$\frac{\partial W_E}{\partial V} = \frac{1}{2} (D \cdot E) = \frac{1}{2} \epsilon E^2 \quad \left(\approx \frac{1}{2} \epsilon V^2 \right)$$

Similarly for magnetic fields

$$\frac{\partial W_H}{\partial V} = \frac{1}{2} \mu H^2 \quad \left(= \frac{1}{2} \mu I^2 \right)$$

27/03/11

⇒ OHM'S LAW & CONTINUITY EQUATION -

- $I = \frac{dQ}{dt}$ applied for a bulk charge movement

$$I = \frac{dQ}{dt} = \frac{d}{dt} \int \rho_v \, dv = \int \frac{\partial \rho_v}{\partial t} \, dv$$

By considering the close surface of the volume

$$I = \oint J \cdot ds = \int (\nabla \cdot J) \, dv$$

$$\nabla \cdot J = \frac{\partial \rho_v}{\partial t}$$

- Outflow of current depends on the rate of movement of volume charge density.

- Current has divergence if a continuous supply of charges can exist.

Practically,

$$\oint J \cdot ds = 0$$

LAW OF
CONSERVATION
OF CHARGE

ENTERING CURRENT = LEAVING CURRENT

- Applying for a unidirectional flow,

$$\frac{\partial J}{\partial x} = \frac{\partial f_v}{\partial t}$$

$$\partial J = \partial f_v \frac{\partial x}{\partial t} \quad \text{--- (1)}$$

where, $\frac{\partial x}{\partial t} = v_d = \text{drift velocity}$

$v_d \propto E \propto \text{potential gradient}$

$v_d = \mu E$, $\mu = \text{ability of carrier to move}$
(MOBILITY)

from (1)

$$J = f_v v_d$$

$$J = f_v \mu E = \sigma E$$

$$J = \sigma E$$

OHM'S LAW
IN POINT FORM

also,

$$\sigma = \mu E$$

$\sigma = \text{conductivity}$, ability to allow
current into that medium.
(mho/m²)

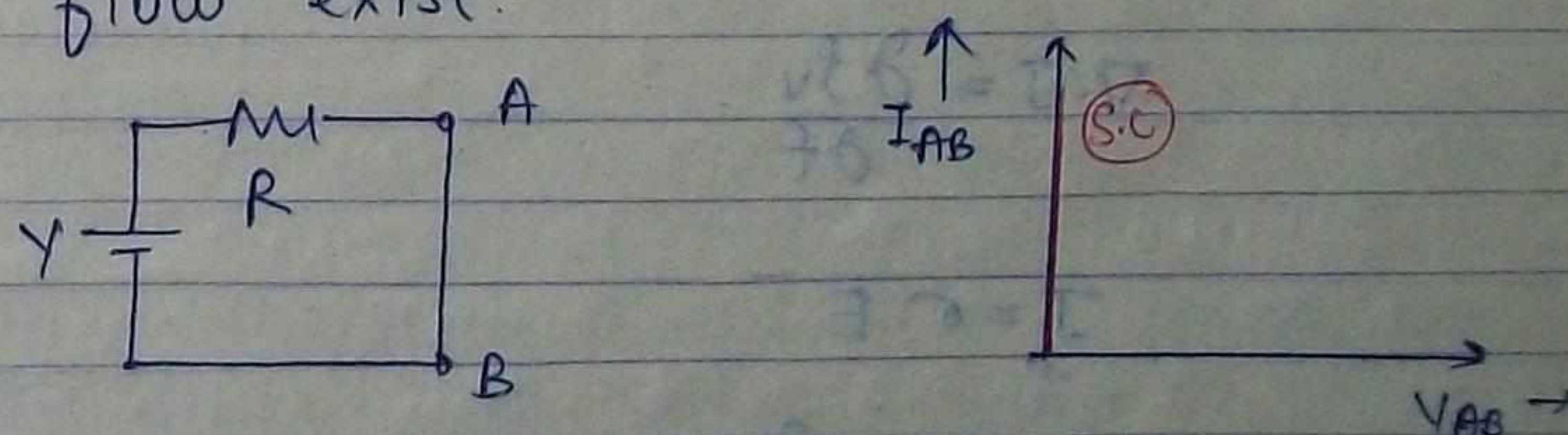
CASE (1): $\sigma = \infty$ very good conductor

$$\frac{J}{\sigma} = E = 0$$

i.e. E-field inside a good conductor is
always zero.

$$V = \int E \cdot dl = \int 0 \cdot dl = \text{CONST.}$$

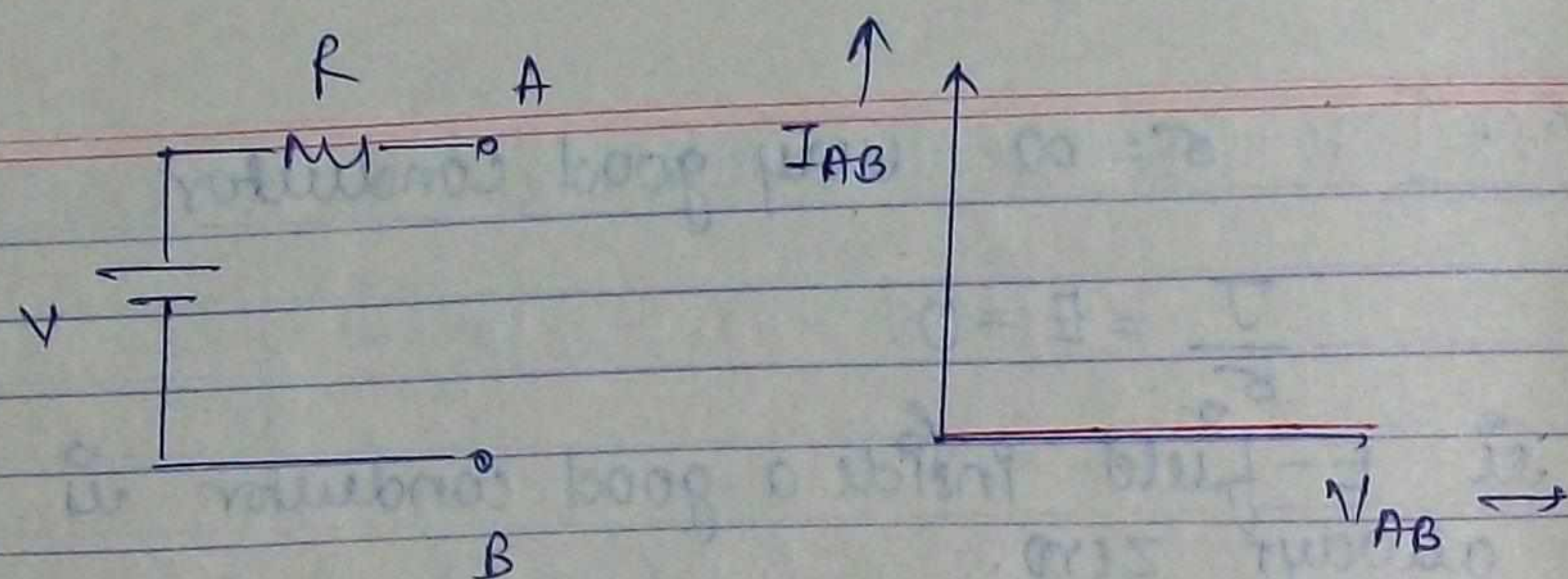
- i.e. Potential is same everywhere.
- A conductor is an equipotential surface.
- Potential diff. b/w any two points is always zero.
- Accumulation cannot exist in conductors
only flow exist.



CASE (2): $\sigma = 0$ very good dielectric.

$$J = \sigma E$$

- I (current) or flow never exist in dielectric
- Only accumulation or E-field exist



- Any charge exponentially spreads over any material with the time constant $\epsilon/\sigma = \text{ohm} \times \text{farads} = \text{seconds}$

This constant (ϵ/σ) is called as Relaxation time which is zero for conductor, and ∞ for dielectric.

$$\nabla \cdot \mathbf{J} = \frac{\partial \rho_v}{\partial t}$$

$$\mathbf{J} = \sigma \mathbf{E}$$

$$\nabla(\sigma \mathbf{E}) = \frac{\partial \rho_v}{\partial t}$$

$$\sigma \nabla \cdot \left(\frac{\mathbf{D}}{\epsilon} \right) = \frac{\partial \rho_v}{\partial t}$$

$$\frac{\partial \rho_v}{\partial t} = \left(\frac{\sigma}{\epsilon} \right) \rho_v$$

$$\rho_v(t) = \rho_{v0} e^{-t\sigma/\epsilon}$$

$\frac{\epsilon}{\sigma} =$ Relaxation time.

$\Rightarrow$ BOUNDARY CONDITIONS FOR CONDUCTORS -

① $E_t = E_{\text{tangential}} = 0$

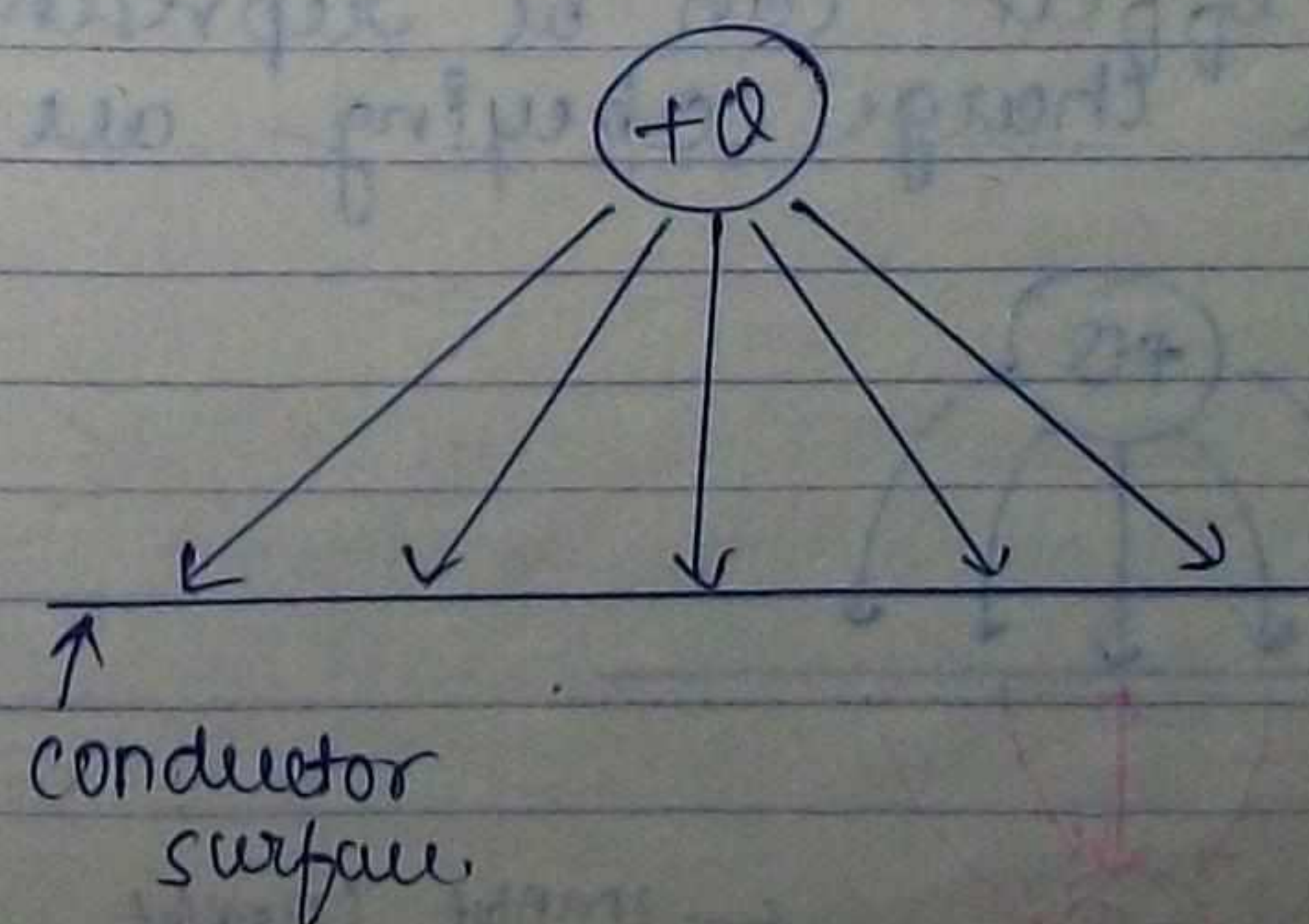
as $\sigma = \infty$ along the surface.

② $D_n = \rho_s$

Normal E field can exist which depends on the charge on the conductor.

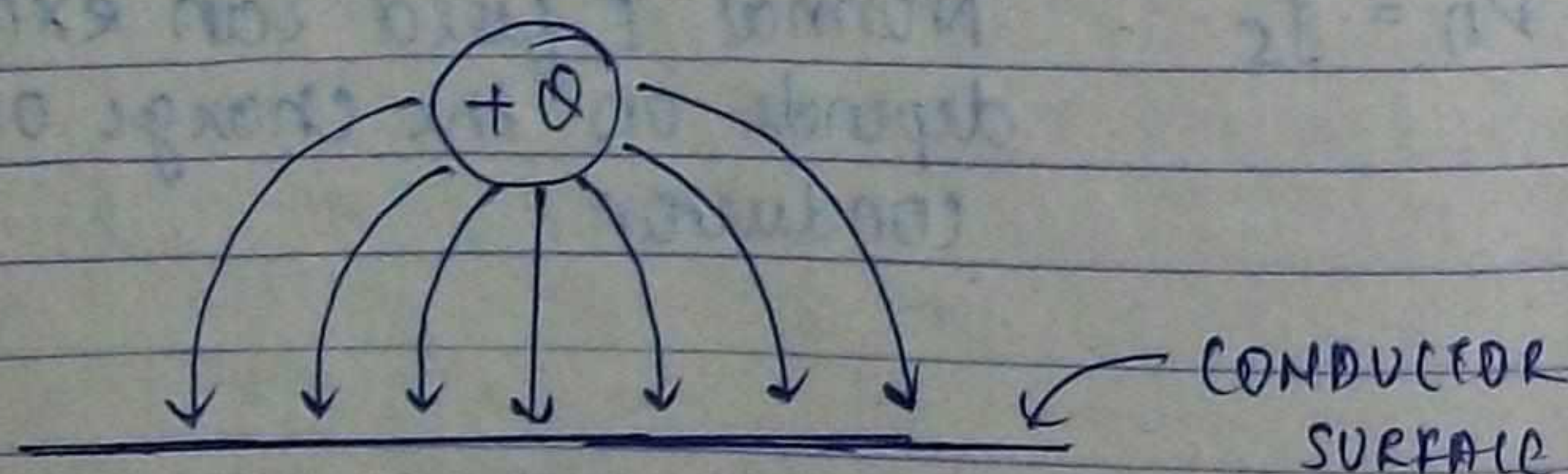
$\Rightarrow$ INDUCED CHARGE & METHOD OF IMAGES -

- When a charge is placed near a conductor surface there is an electric field as shown.

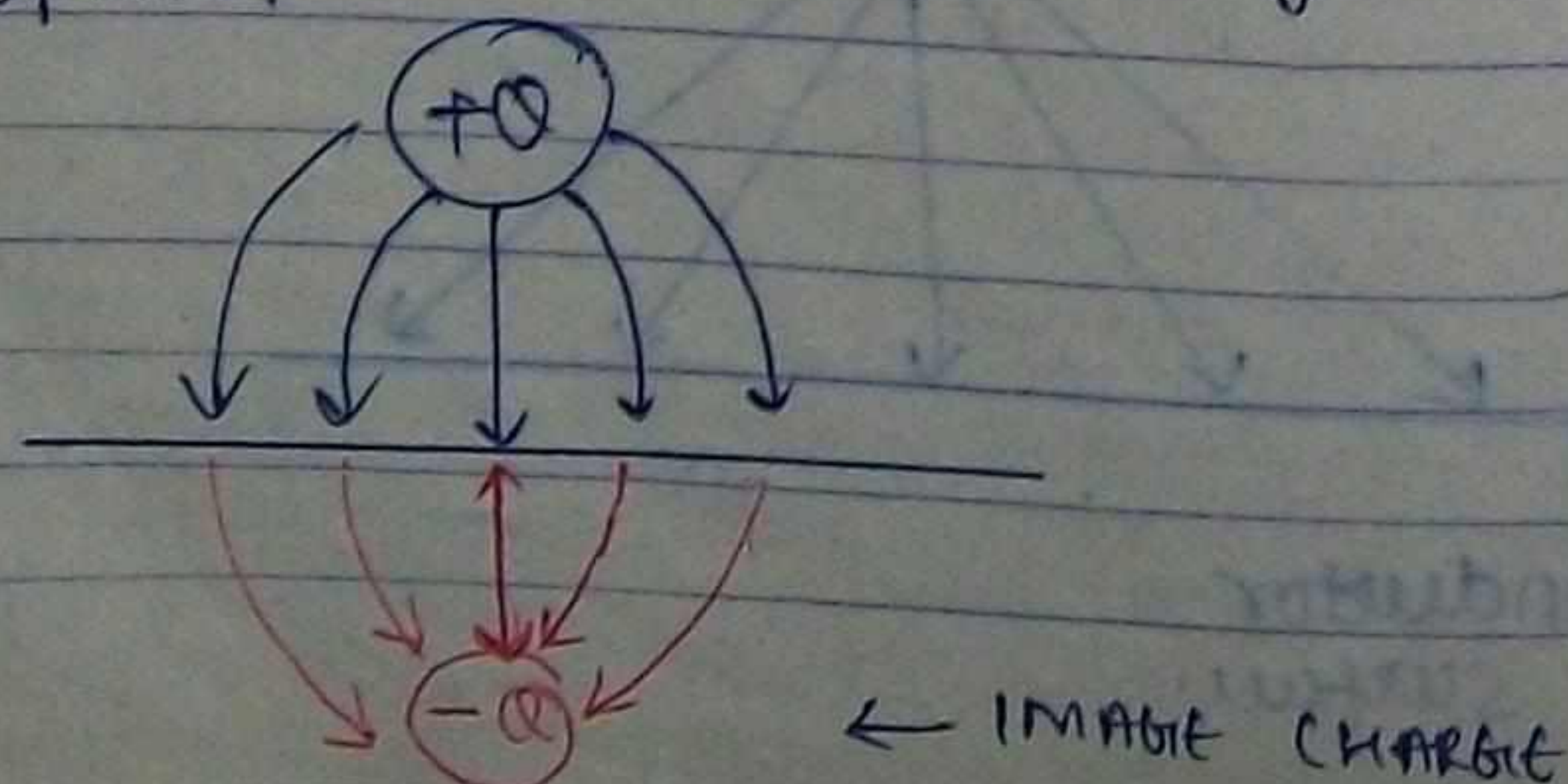


- The field have tangential and normal components both of which the tangential components displace the charges such that the charges are periodically accumulated at various point such that this displaced charges are called as induced charges.

- The field at any point 'P' is due to the actual charge and the induced charge as shown below:



- The field is similar to the field of a dipole with the conductor having only normal field. Hence the induced charge and its effect can be represented with an image charge obeying all rules of optics.



★

Ques 40 P-10

Soln:

$$\vec{D} = 2(a_x - \sqrt{3}a_z) \text{ C/m}^2$$

$$f_s = |D_n| = 2\sqrt{1+3} = 4 \text{ C/m}^2$$

Ques 41 P-11

Soln:

$$f_s = D_n = \epsilon E_n$$

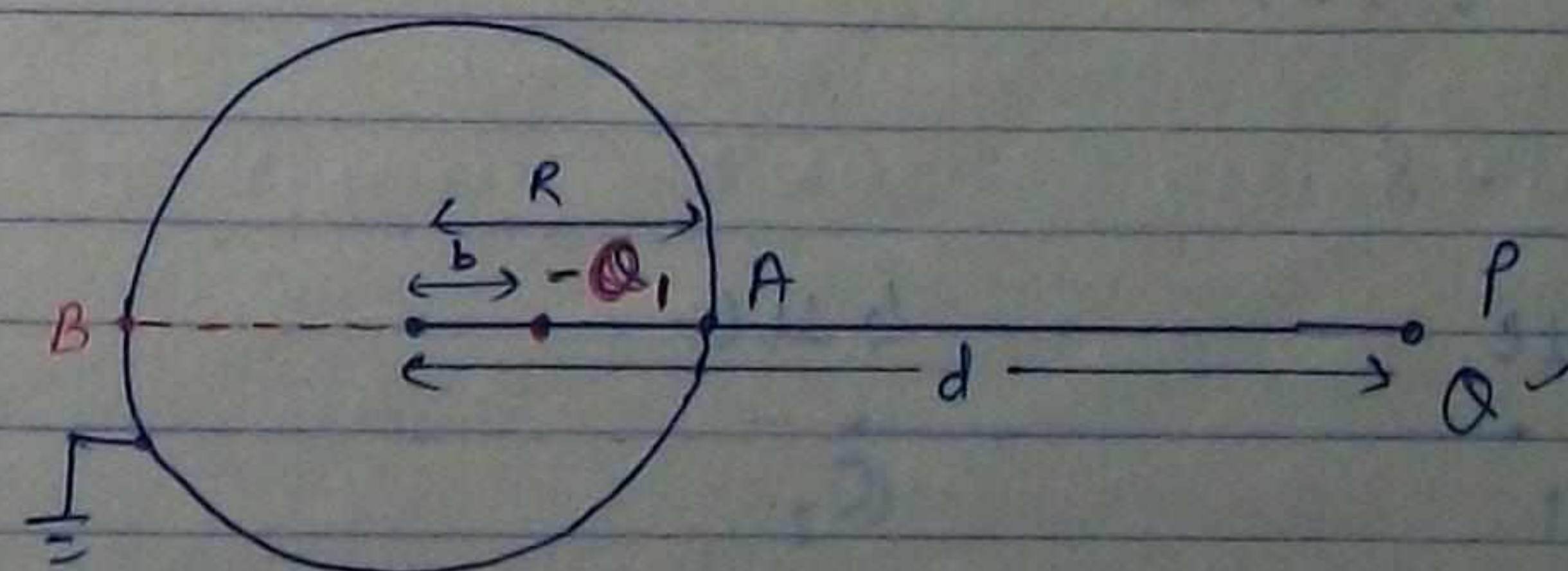
$$D_n = 80 \times 60 \times 2$$

$$= 80 \times 10^{-9} (36\pi) \times 2$$

$$= 1.41 \times 10^{-9} \text{ C/m}^2$$

Ques 42 P-12

Soln:



The induced charge on the curvature is represented with an image charge arbitrary coaxial with the charge.

$$V_A = \frac{Q}{4\pi\epsilon(d-R)} - \frac{Q_1}{4\pi\epsilon(R-b)} = 0$$

$$V_B = \frac{Q}{4\pi\epsilon(d+R)} - \frac{Q_1}{4\pi\epsilon(R+b)} = 0$$

from both eqⁿs

$$\frac{Q}{Q_1} = \frac{d-R}{R-b} = \frac{d+R}{R+b} \quad \text{--- (1)}$$

$$dR - R^2 + db - Rb = dR + R^2 - db - bR$$

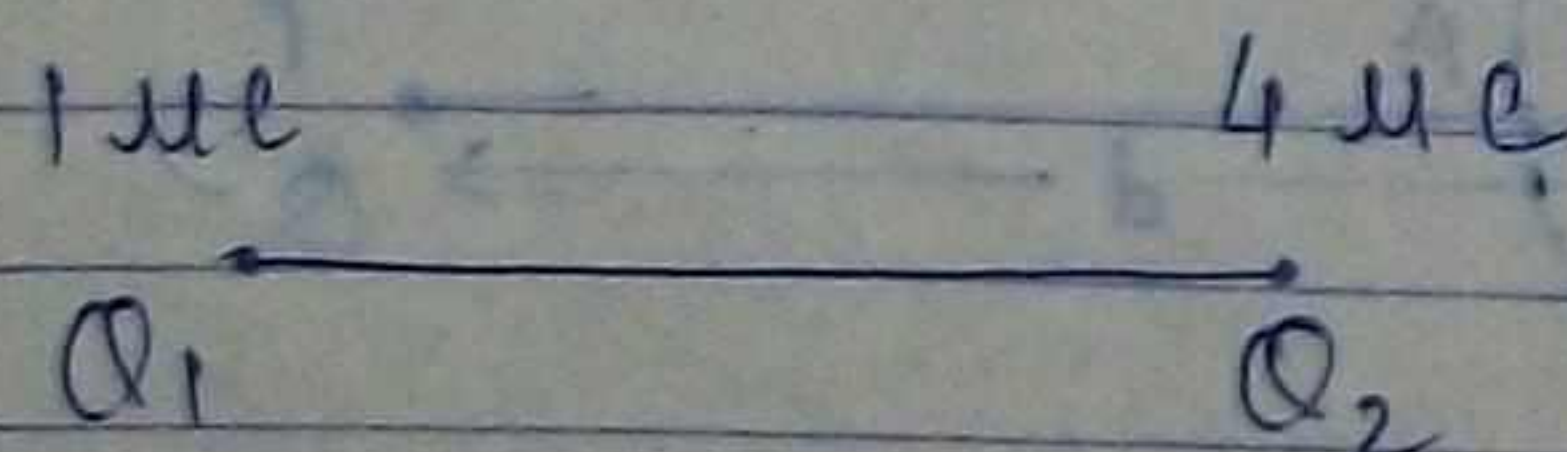
$$\therefore b = \frac{R^2}{d}$$

Putting value in (1)

$$Q_1 = -(R/d)Q$$

Ques 28 P-8

Solⁿ:



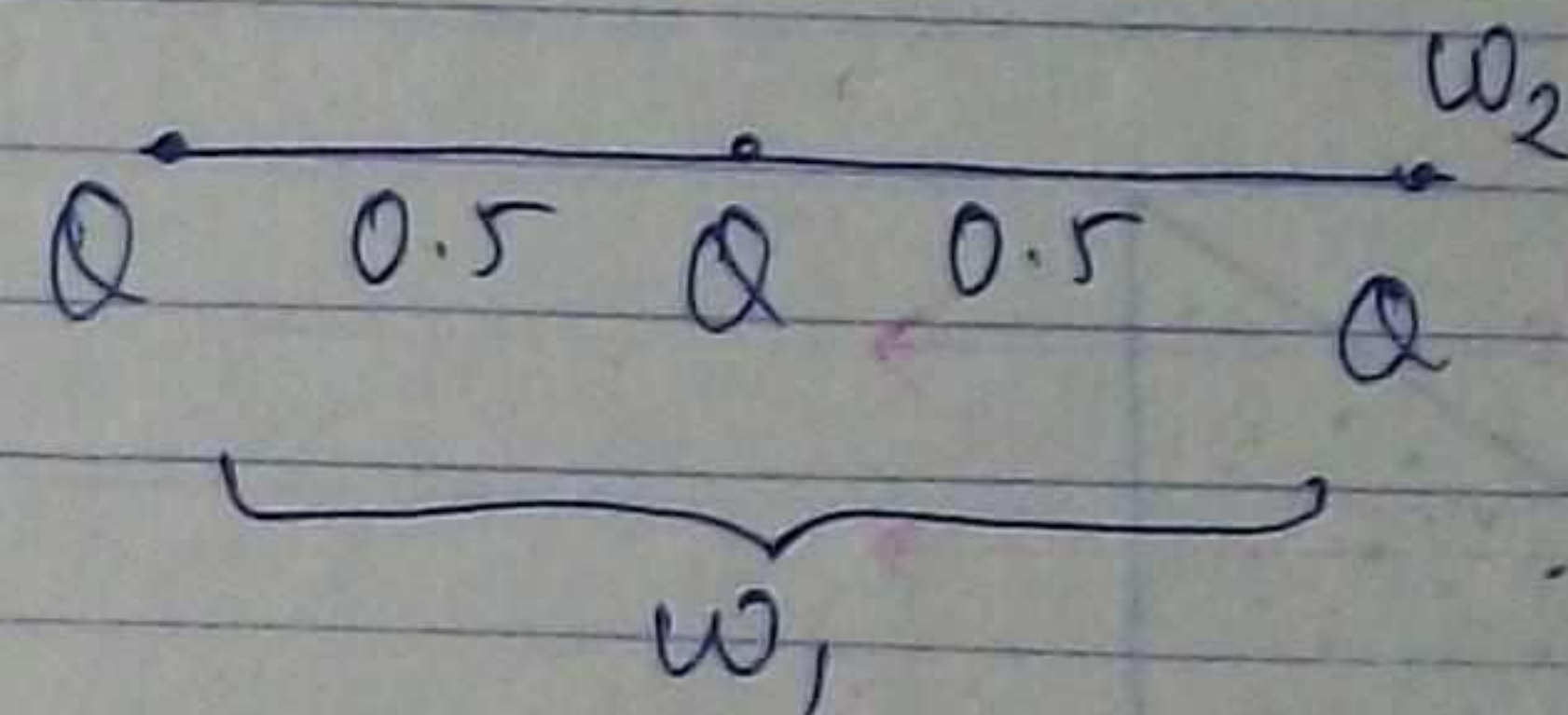
$$W_F = W_1 + W_2$$

$$= 0 + Q_2 V_{21}$$

$$= \frac{4 \times 10^{-6} \times 1 \times 10^{-6}}{4\pi\epsilon \sqrt{3^2 + 2^2 + 6^2}} = 5.14 \text{ mJ}$$

Ques 29 P-8

Solⁿ:

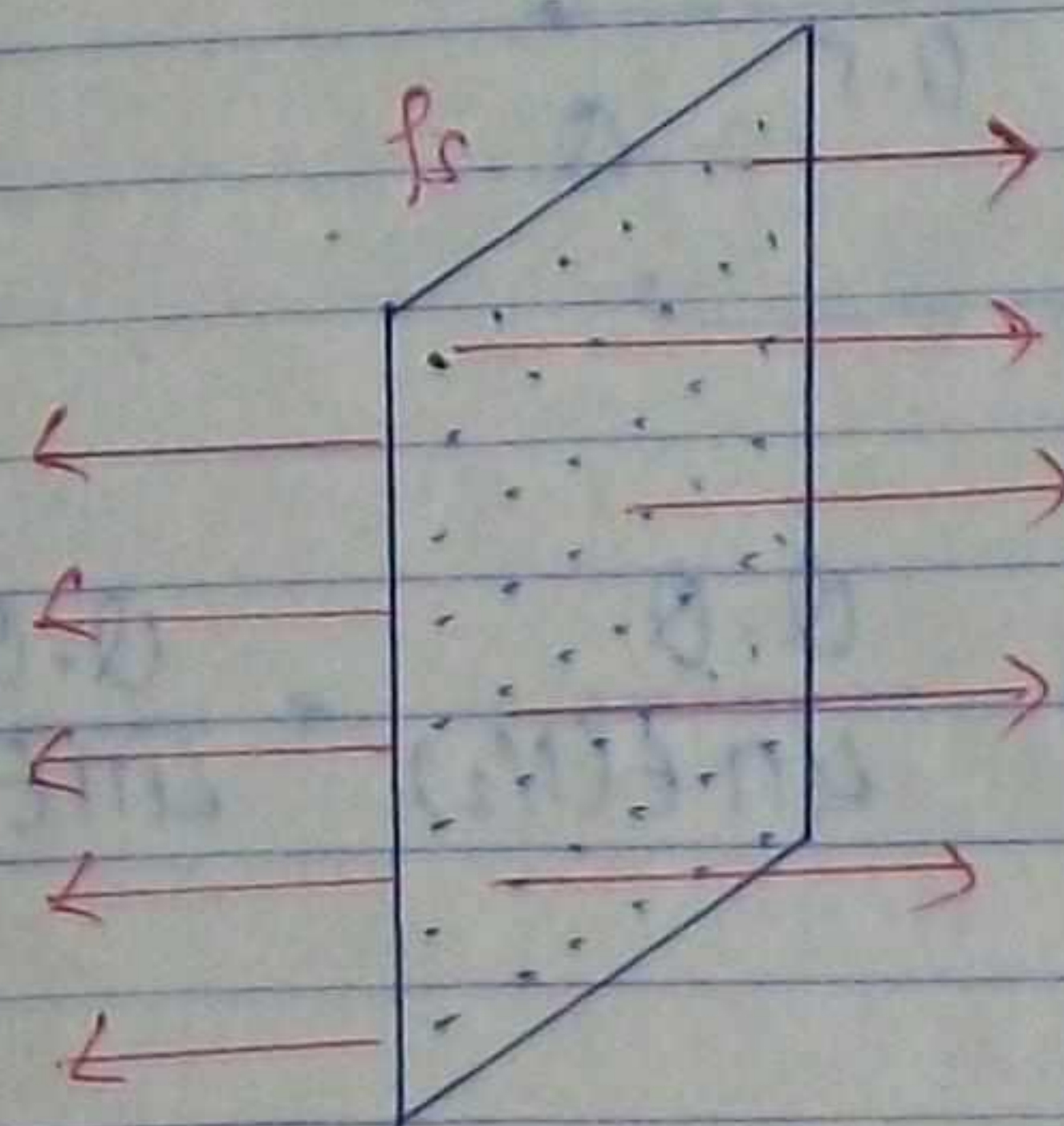


$$W_1 = 0 + \frac{Q \cdot Q}{4\pi\epsilon(1/2)} + \frac{Q \cdot Q}{4\pi\epsilon(1/2)} + \frac{Q \cdot Q}{4\pi\epsilon(1)^2}$$

$$W_1 = \frac{5Q^2}{4\pi\epsilon}$$

$$W_2 = \frac{5Q^2}{8\pi\epsilon}$$

⇒ (INFINITE) SHEET CHARGE & UNIFORM FIELDS -



- The field lines are completely normal (E-direction $\mathbf{a}_N$) to the sheet (assume a large sheet).

- The E-field lines are parallel to themselves.

∴ lines/area = const everywhere

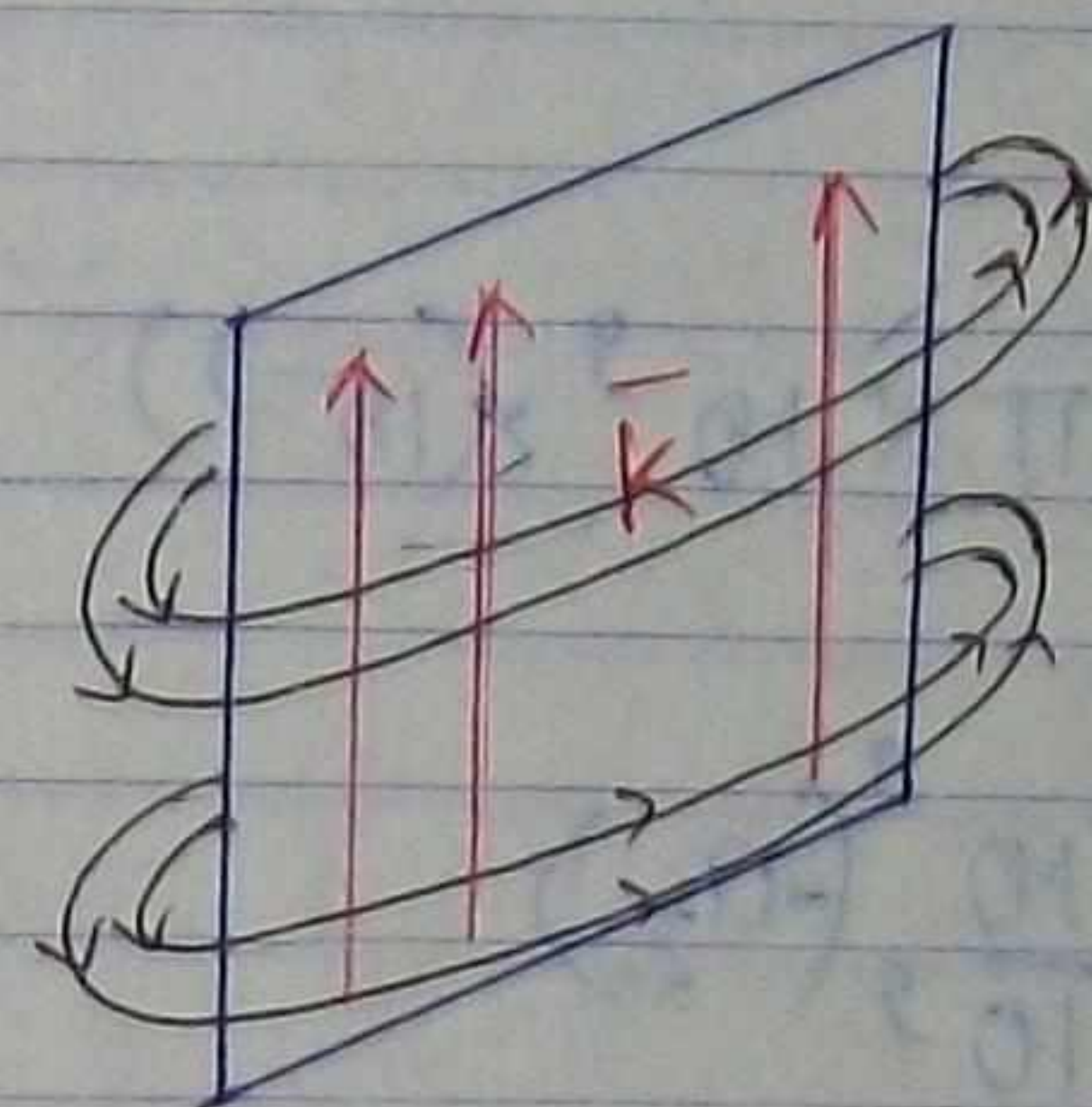
Flux × density = const.

E field is uniform.

- The flux density anywhere away from the sheet is

$$\boxed{D = \frac{\rho_s}{2} \mathbf{a}_N}, \quad \boxed{E = \frac{\rho_s}{2\epsilon} \mathbf{a}_N}$$

EXTENSION: (INFINITE) SHEET OF CURRENT DENSITY ($\mathbf{K}$ A/m) AND UNIFORM FIELDS



$$\mathbf{K} = \frac{dI}{dl}$$

- The H-direction is perpendicular to the current but parallel to the surface of the sheet.

- The H-field lines are uniform for an infinite sheet.

$$\boxed{H = \frac{K}{2} \times \mathbf{a}_N}$$

$$\boxed{B = \frac{\mu_0 K}{2} \times \mathbf{a}_N}$$

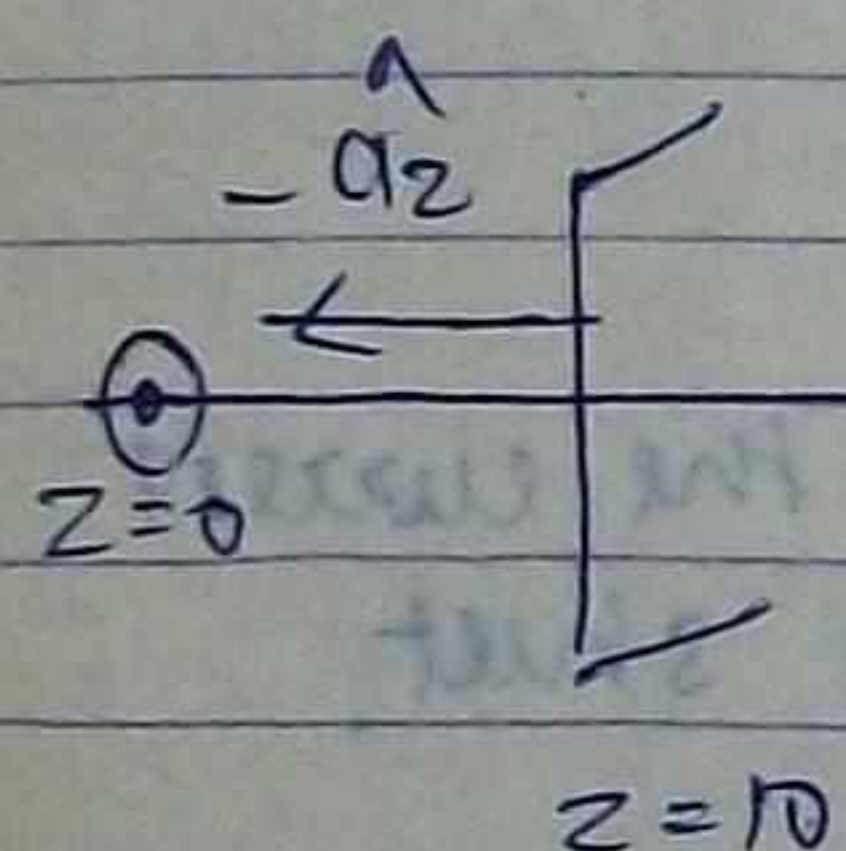
Que. 18 P-6
Soln:
 $\rho_s = 20 \text{ nC/m}^2$

$$E = \frac{\rho_s}{2\epsilon} \cdot a_z$$

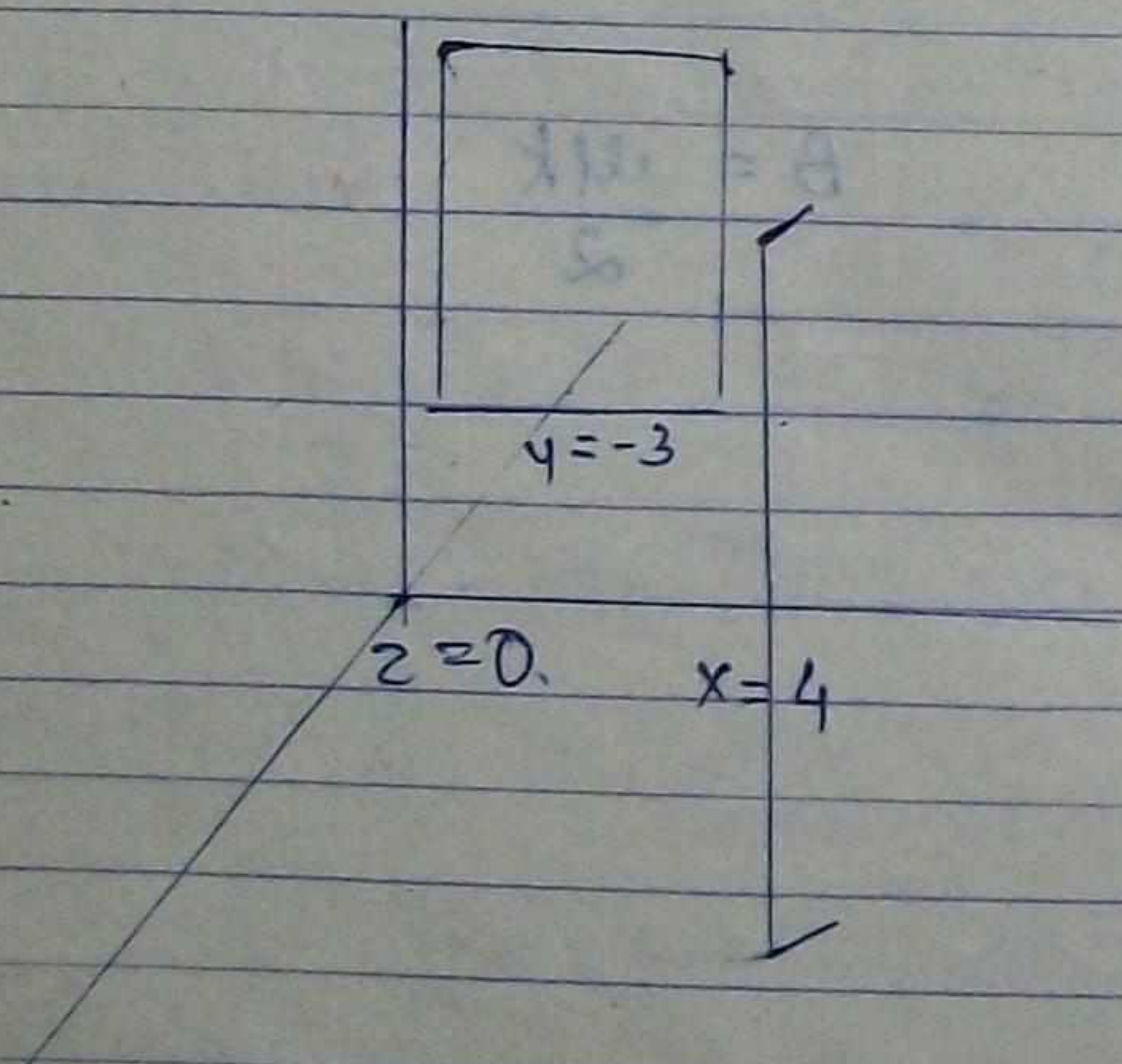
$$= \frac{20 \times 36\pi \times 10^9 \times (10^{-9})}{2 \times 1}$$

$$= 360\pi \times 10^9 \left(-a_z \right)$$

$$= -360\pi a_z$$



Que. 17 P-6
Soln:



$$18 \text{ nC/m}^2 \rightarrow \rho_{s1} \rightarrow x=4 \rightarrow E_1 = \pm a_x$$

$$9 \text{ nC/m}^2 \rightarrow \rho_{s2} \rightarrow y=-3 \rightarrow E_2 = \pm a_y$$

$$-24 \text{ nC/m}^2 \rightarrow \rho_{s3} \rightarrow z=0 \rightarrow E_3 = \pm a_z$$

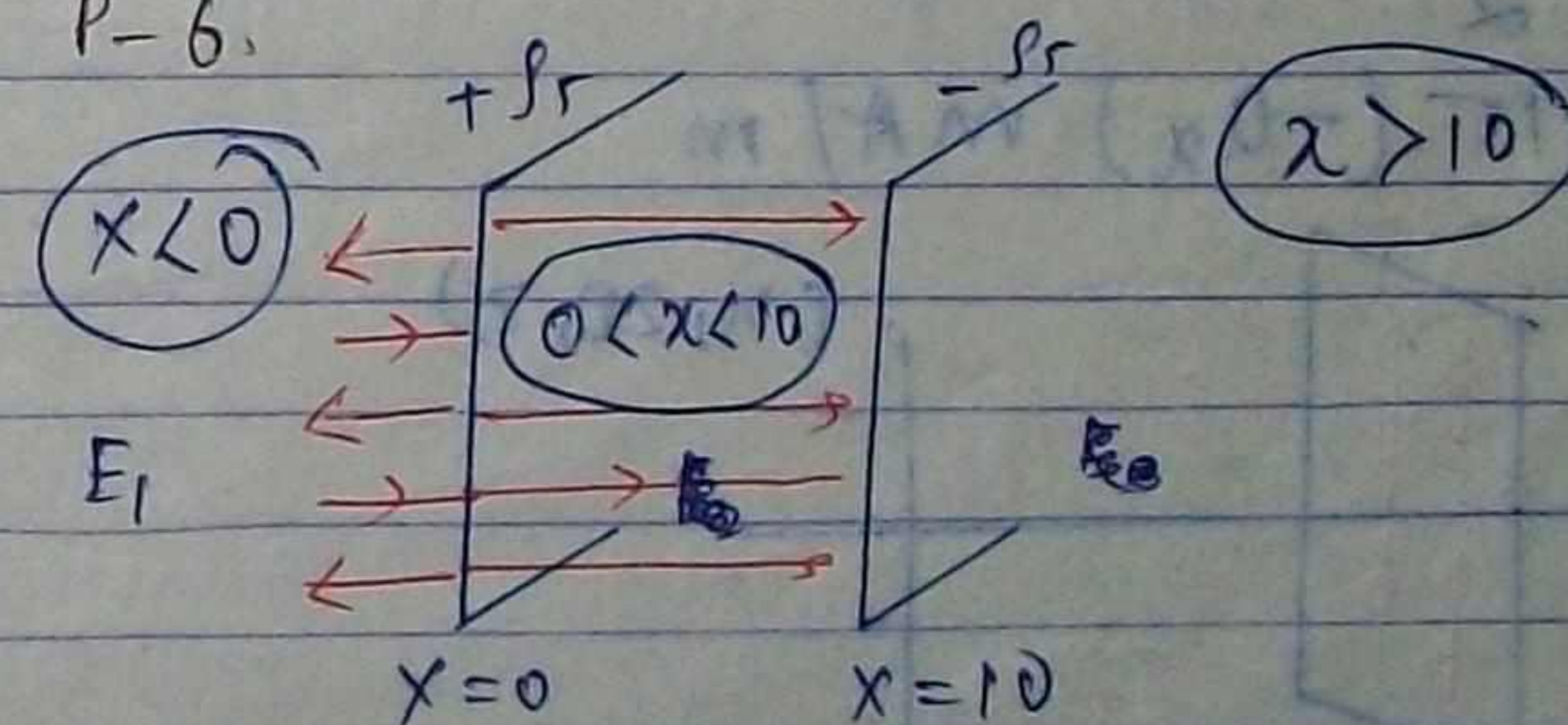
$$E_1 = \frac{18}{2\epsilon} a_x \text{ nN/C}$$

$$E_2 = \frac{9}{2\epsilon} a_y \text{ nN/C}$$

$$E_3 = \frac{-24}{2\epsilon} a_z = \frac{24}{2\epsilon} (-a_z) \text{ nN/C}$$

$$E = \frac{3}{2\epsilon} [6a_x + 3a_y - 8a_z] \text{ nN/C.}$$

Que. 16 P-6.
Soln:



at $x < 0$

$$E_1 = \frac{\rho_s}{2\epsilon} (-a_x)$$

$$E_2 = \frac{-\rho_s}{2\epsilon} (-a_x) = \frac{\rho_s}{2\epsilon} a_x$$

$$E = 0 = E_1 + E_2$$

Similarly for $x > 10$ $E = 0$

for $0 < x < 10$

$$E_1 = \frac{\rho_s}{2\epsilon} a_x \quad E_2 = \frac{\rho_s}{2\epsilon} a_x$$

$$E_T = 2 \frac{\rho_s}{2\epsilon} a_x = \frac{\rho_s}{\epsilon} a_x$$

$$E_T = \frac{\rho_s}{\epsilon} a_x$$

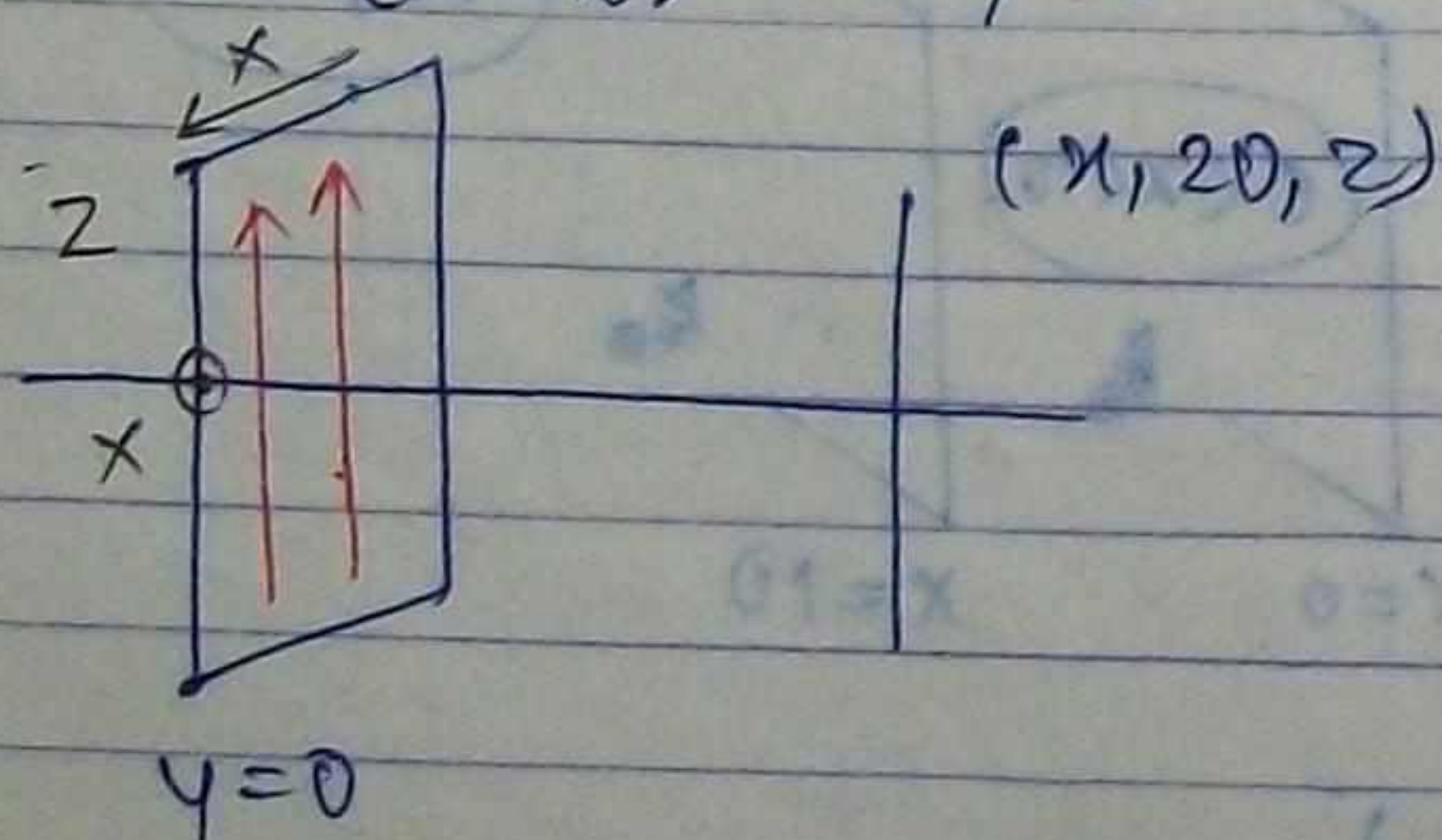
Ques 190 P-6

Solⁿ: $K = 30 \hat{k} \text{ mA/m} = 30 a_z \text{ mA/m}$

$$H = K/2 (a_2 \times a_4)$$

$$H = \frac{K}{2} (-a_x)$$

$$H = 15 (-a_x) \text{ mA/m}$$



⇒ CAPACITORS AND INDUCTORS -

- Capacitance is the ability to hold E-field confined in a finite region.

SPECIAL GEOMETRY -

- ① Parallel plate capacitor.
- ② Concentric cylinder capacitor
- ③ Concentric spherical capacitor

$$C = \frac{\oint D \cdot d\mathbf{s}}{\int E \cdot d\mathbf{l}} = \frac{\epsilon \oint E \cdot d\mathbf{s}}{\int E \cdot d\mathbf{l}} = \frac{Q}{V}$$

(Forced)

- It is the ratio of charge utilized and the potential developed due to that charge.

INDUCTANCE -

- Inductance is the ability to hold H-field confined in a finite region.

SPECIAL GEOMETRY —

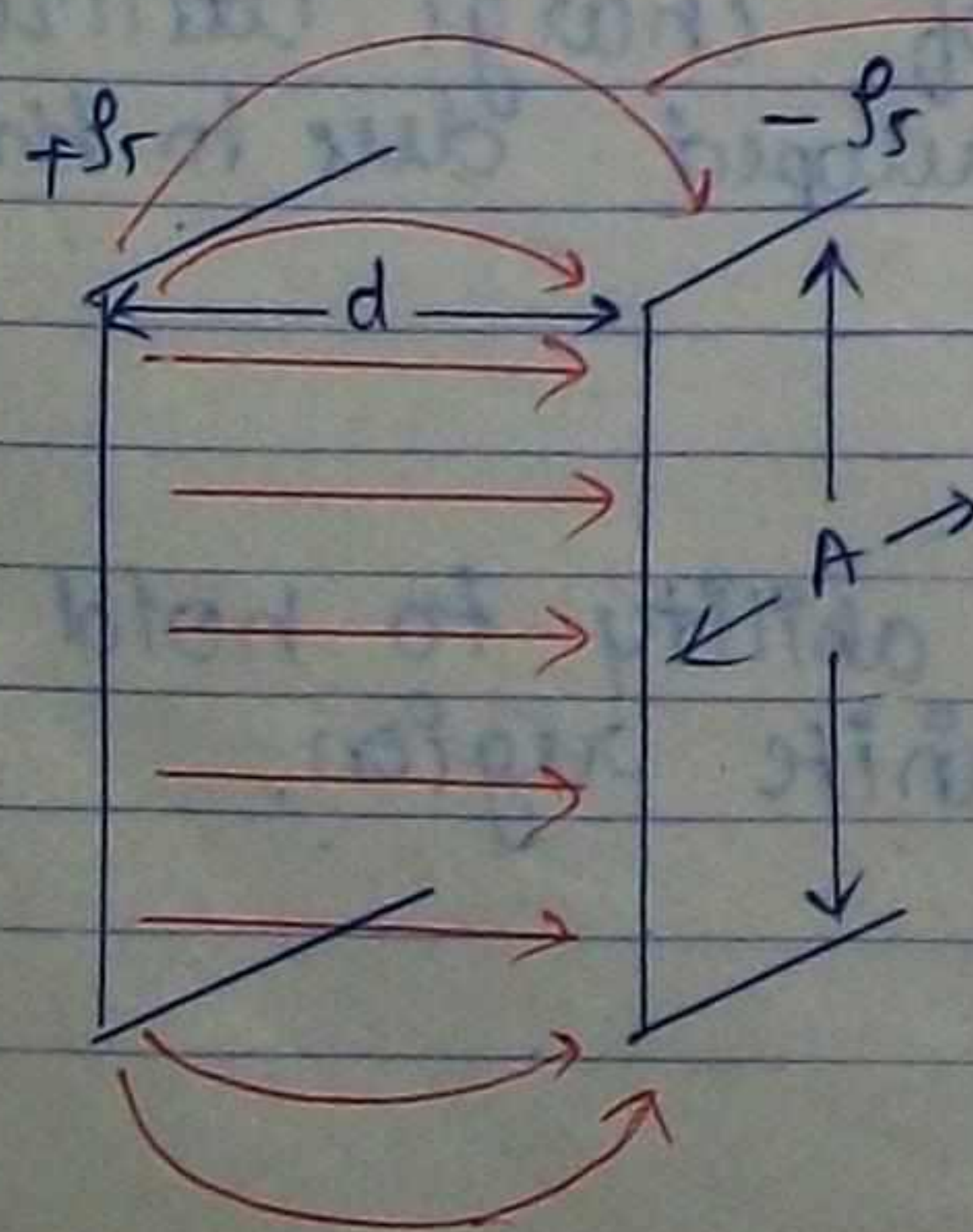
- ① Solenoids
- ② Concentric cylinder
- ③ Torroids

$$L = \frac{\int B \cdot ds}{\oint H \cdot dl} = \frac{\mu \int H \cdot dr}{\oint H \cdot dl} = \frac{\Psi_m}{I}$$

(Henry)

- It is the ratio of flux confined and the current utilized to hold the flux.

⇒ PARALLEL PLATE CAPACITANCE—



$$E = \frac{P_s}{\epsilon}$$

$$C = \frac{Q}{V} = \frac{P_s A}{\frac{P_s \cdot d}{\epsilon}}$$

$$C = \epsilon \frac{A}{d}$$

- Capacitance is independent of charge, voltage, E etc and depends only the physical dimensions.

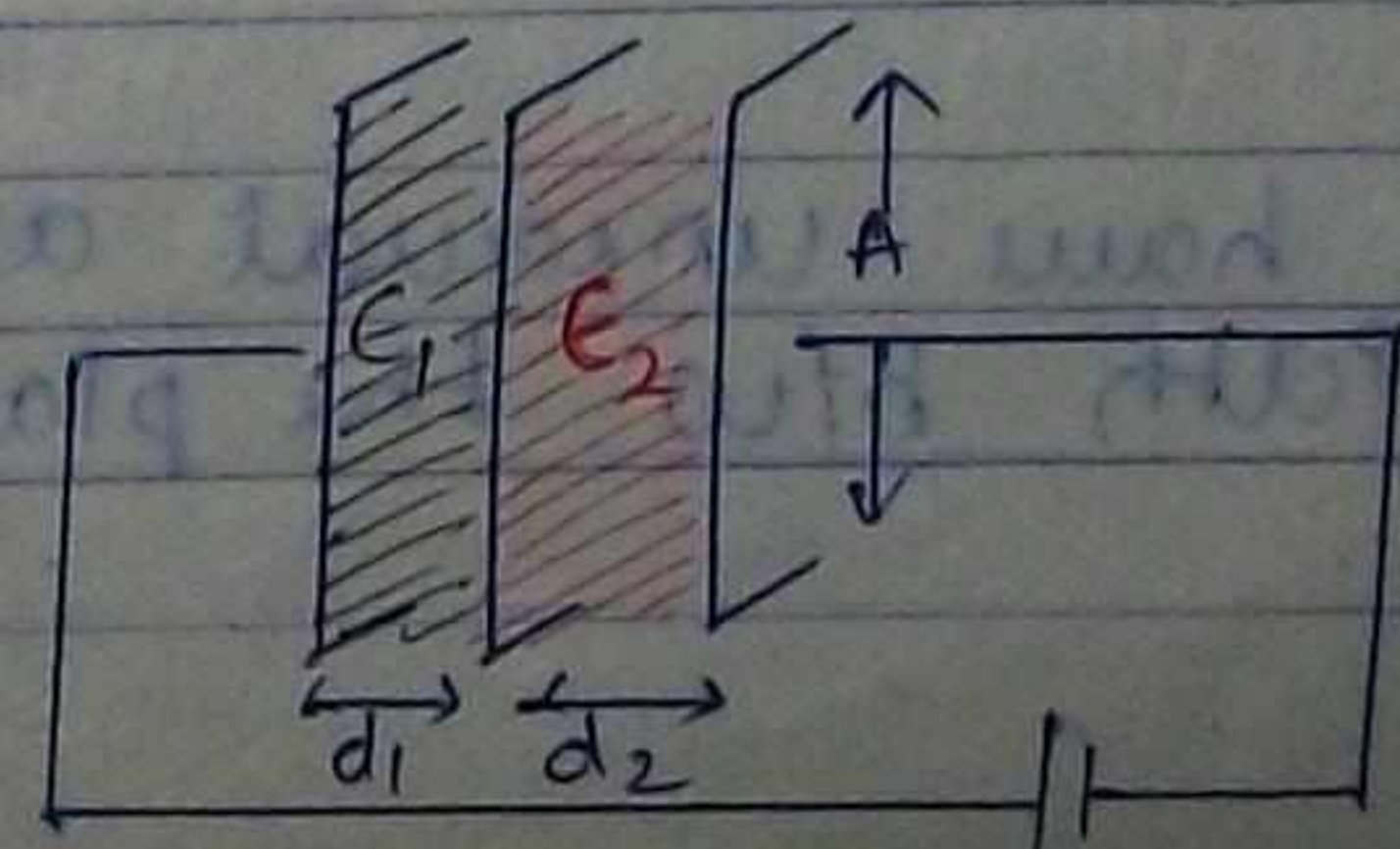
$$W_E = \frac{1}{2} \epsilon E^2 \cdot A d$$

$$W_E = \frac{1}{2} \epsilon \frac{A}{d} (Ed)^2$$

$$W_E = \frac{1}{2} C V^2 = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} QV$$

⇒ MULTIPLE DIELECTRIC IN ||^r PLATE CAPACITOR —

CASE ①: Dielectric have equal areas but unequal widths b/w the plates.



Thus, there are two capacitors in series as voltage divider.

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{\frac{\epsilon_1 A}{d_1} \cdot \frac{\epsilon_2 A}{d_2}}{\frac{\epsilon_1 A}{d_1} + \frac{\epsilon_2 A}{d_2}}$$

for voltage division

① $C_1 V_1 = C_2 V_2$

$$\frac{V_1}{V_2} = \left(\frac{\epsilon_2}{\epsilon_1} \right) \frac{d_1}{d_2}$$

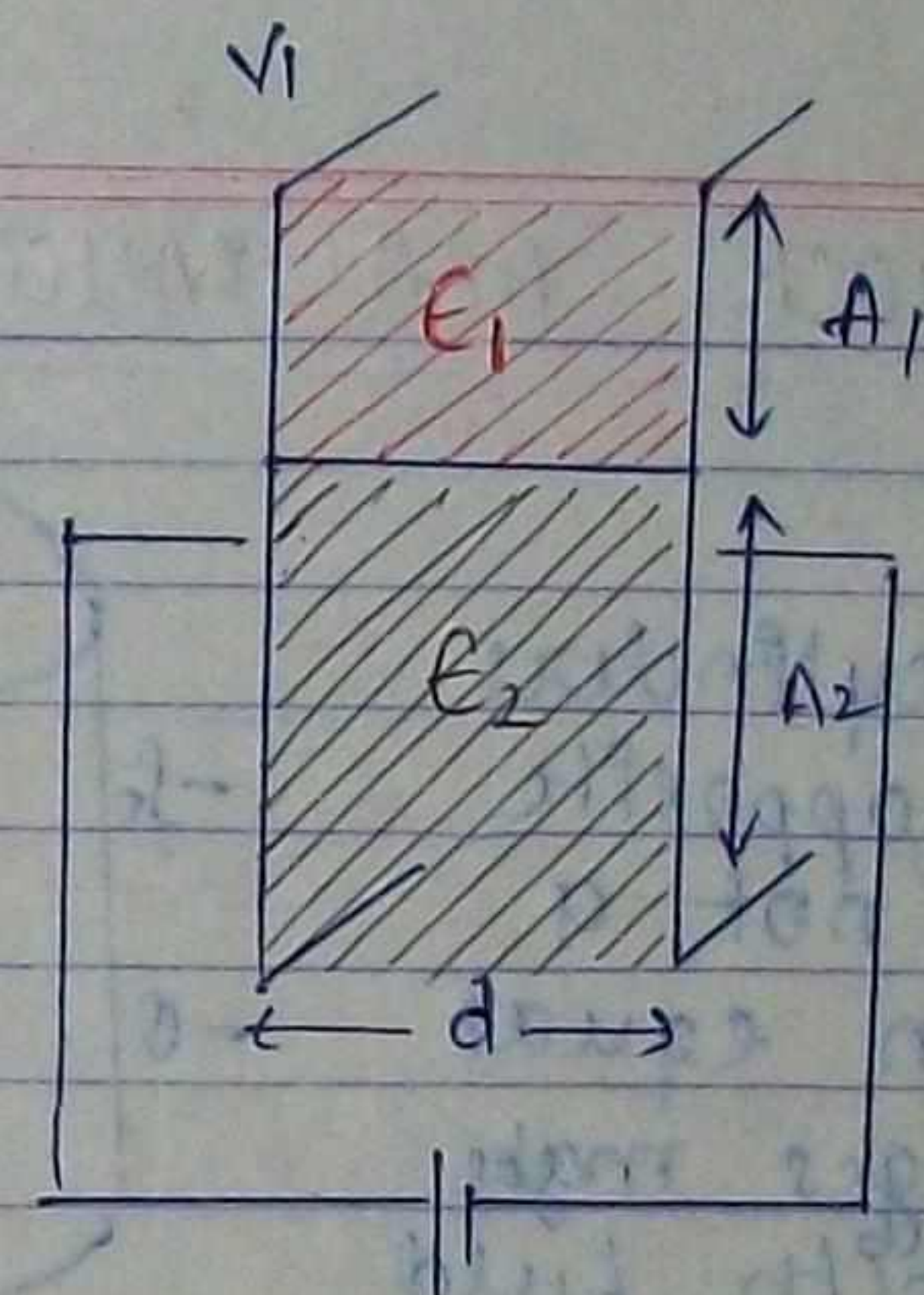
or ② $d_1 = d_2$

$$\epsilon_1 E_1 = \epsilon_2 E_2$$

$$\epsilon_1 \frac{V_1}{d_1} = \epsilon_2 \frac{V_2}{d_2}$$

$$\frac{V_1}{V_2} = \left(\frac{\epsilon_2}{\epsilon_1} \right) \frac{d_1}{d_2}$$

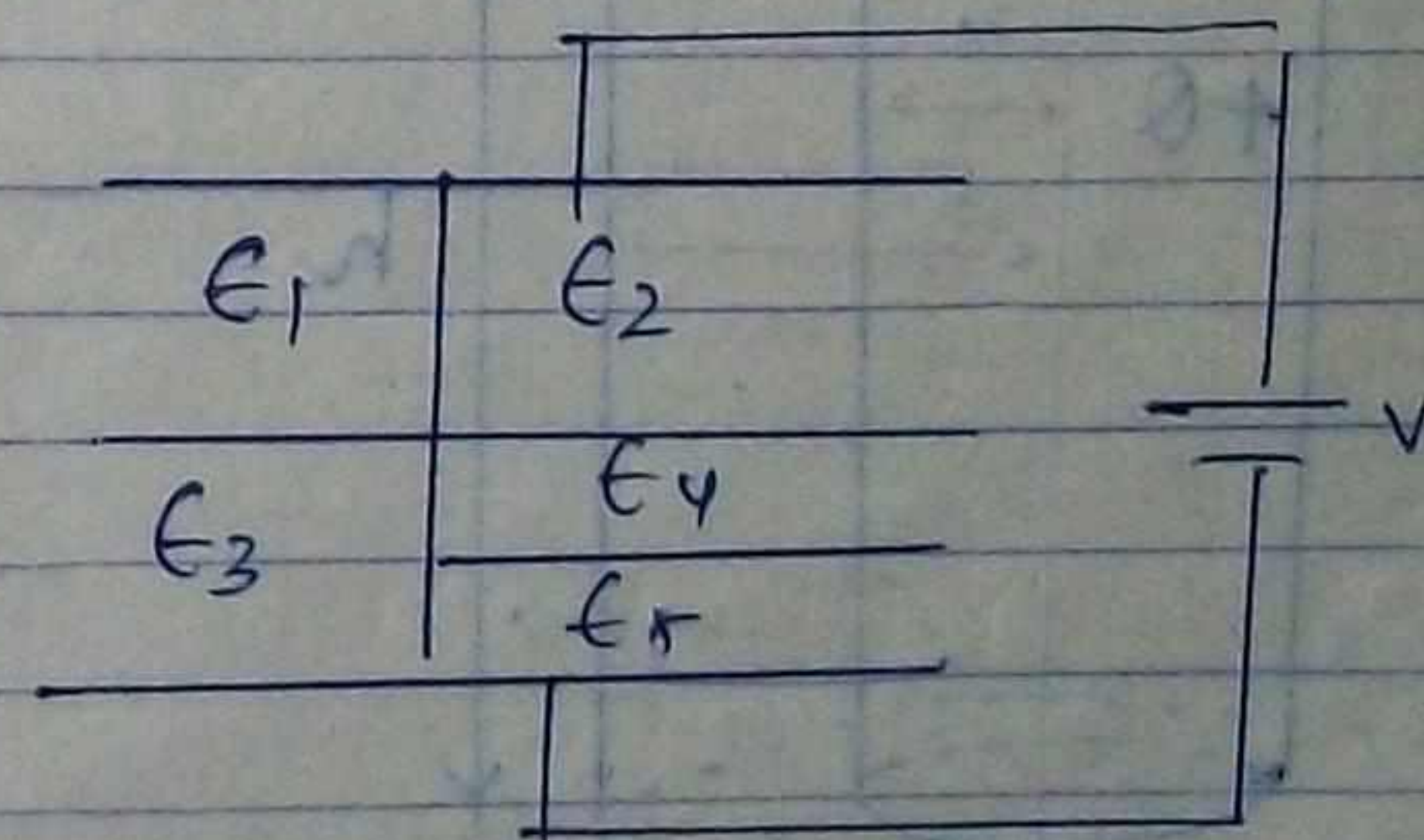
CASE ②: Dielectrics have unequal area but equal width b/w the plates.



$$C_{eq} = C_1 + C_2$$

$$C_{eq} = \frac{\epsilon_1 A_1 + \epsilon_2 A_2}{d}$$

#

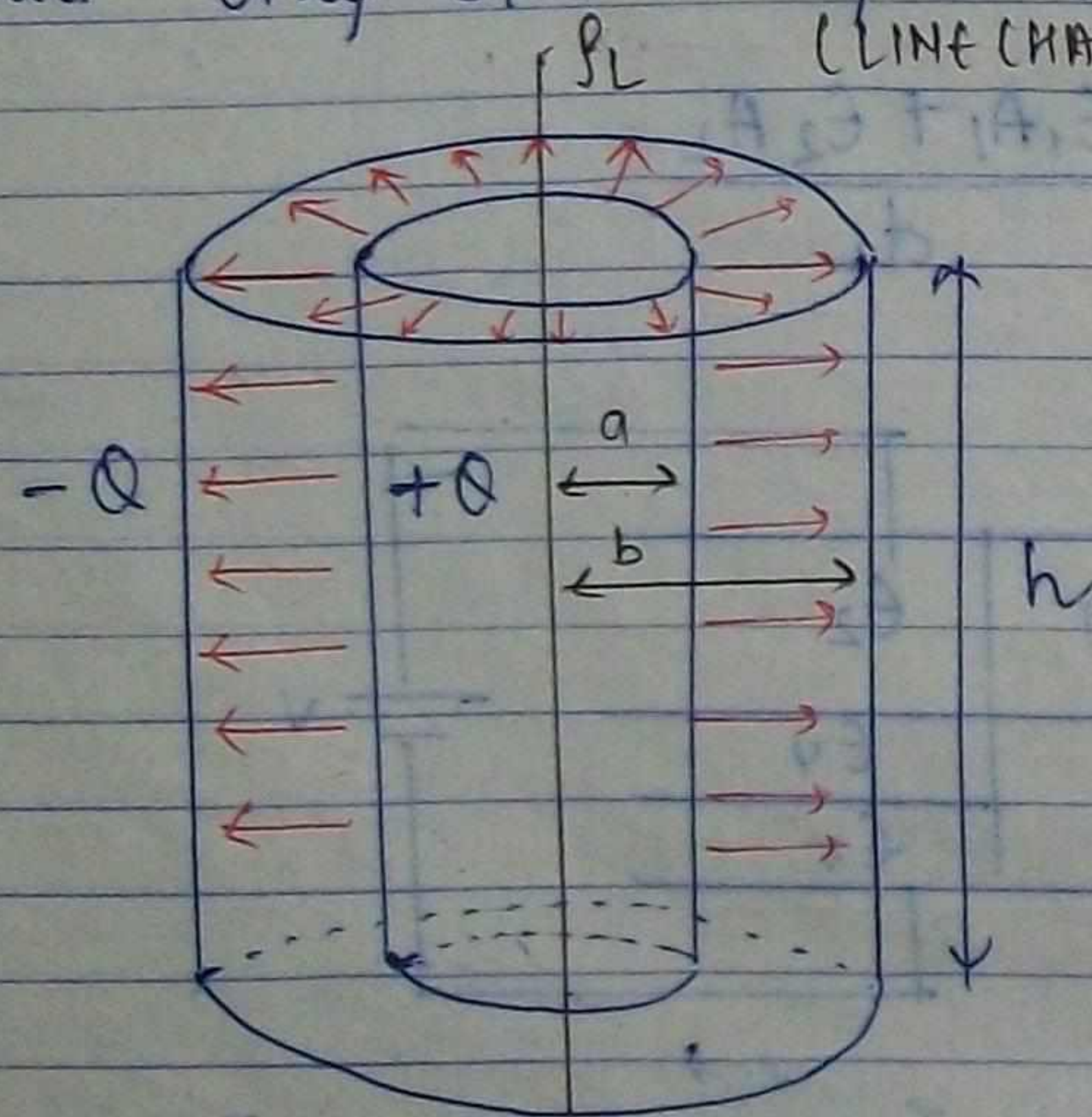
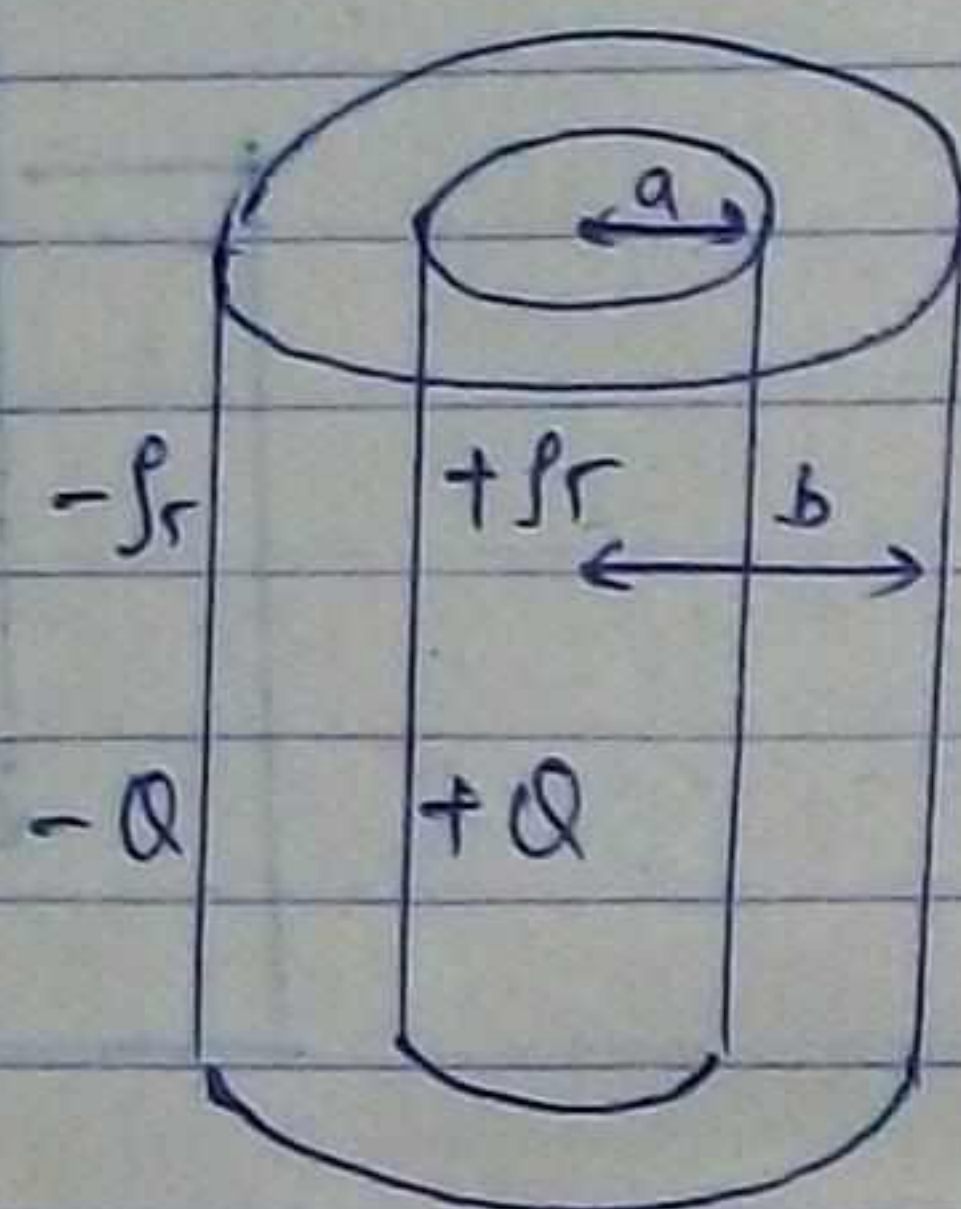


$$C_{eq} = (C_1 || C_2) \text{ series } (C_3 || C_4 \text{ series } C_5)$$

⇒ CONCENTRIC CYLINDER CAPACITANCE -

NOTE -

Two concentric cylinders having equal and opposite charge densities is not a good capacitor, an equal and opposite charges make a good capacitor with field confined only b/w the cylinder. (LINE CHARGE)



The field is identical to the field of a line charge and hence

$$D \propto \frac{1}{r} a_f$$

$$E \propto \frac{1}{r} a_f$$

$$C = \frac{Q}{V} = \frac{\lambda_L h}{\frac{\lambda_L}{2\pi\epsilon} \ln\left(\frac{b}{a}\right)}$$

$$C = \frac{2\pi\epsilon h}{\ln(b/a)}$$

⇒ CONCENTRIC SPHERE CAPACITANCE -

$$C = \frac{Q}{V} = \frac{Q}{\frac{Q}{4\pi\epsilon_0} \left(\frac{1}{a} - \frac{1}{b}\right)} \quad (\text{point charge})$$

$$C = \frac{4\pi\epsilon}{\left(\frac{1}{a} - \frac{1}{b}\right)}$$

21/03/11

E-FIELD

Q - charge
monopole

+Q - Q
Hole separation
'd' (Dipole)

Dipole moment

$$P = Q \cdot d$$

Torque

$$T = P \times E$$

P's direction is
always from -ve
charge to +ve charge

Ex - Atom

H-FIELD

→ x →

I - (current) with
Hole area 'A'
(Dipole)

Dipole moment

$$M = I \text{ area}$$

Torque

$$T = M \times B$$

M's direction is area
direction which is outward
normal following
Right handed rule
with current

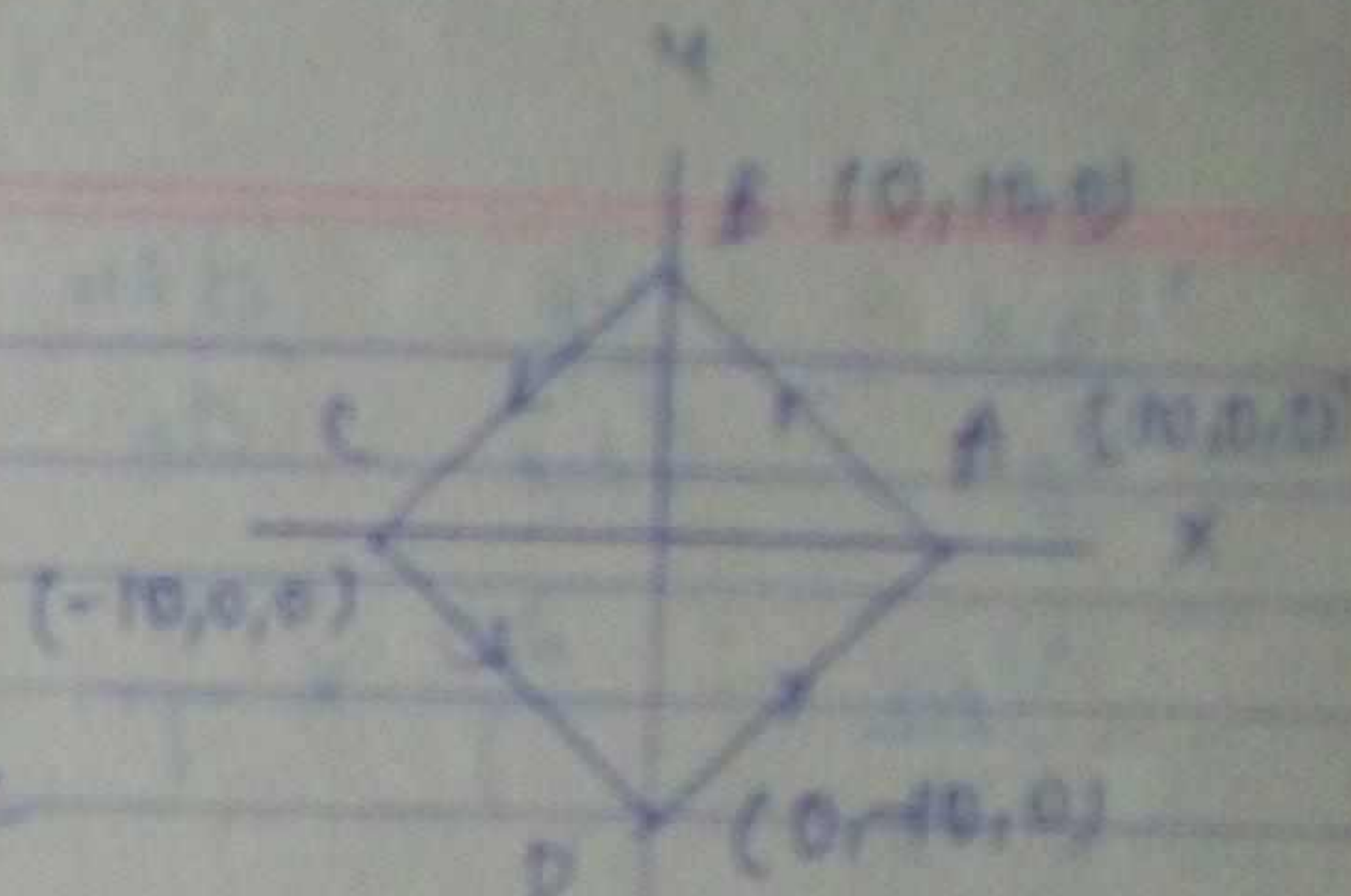
Ex - Atom

Ques 46 P-12

$$M = I \times \text{area}$$

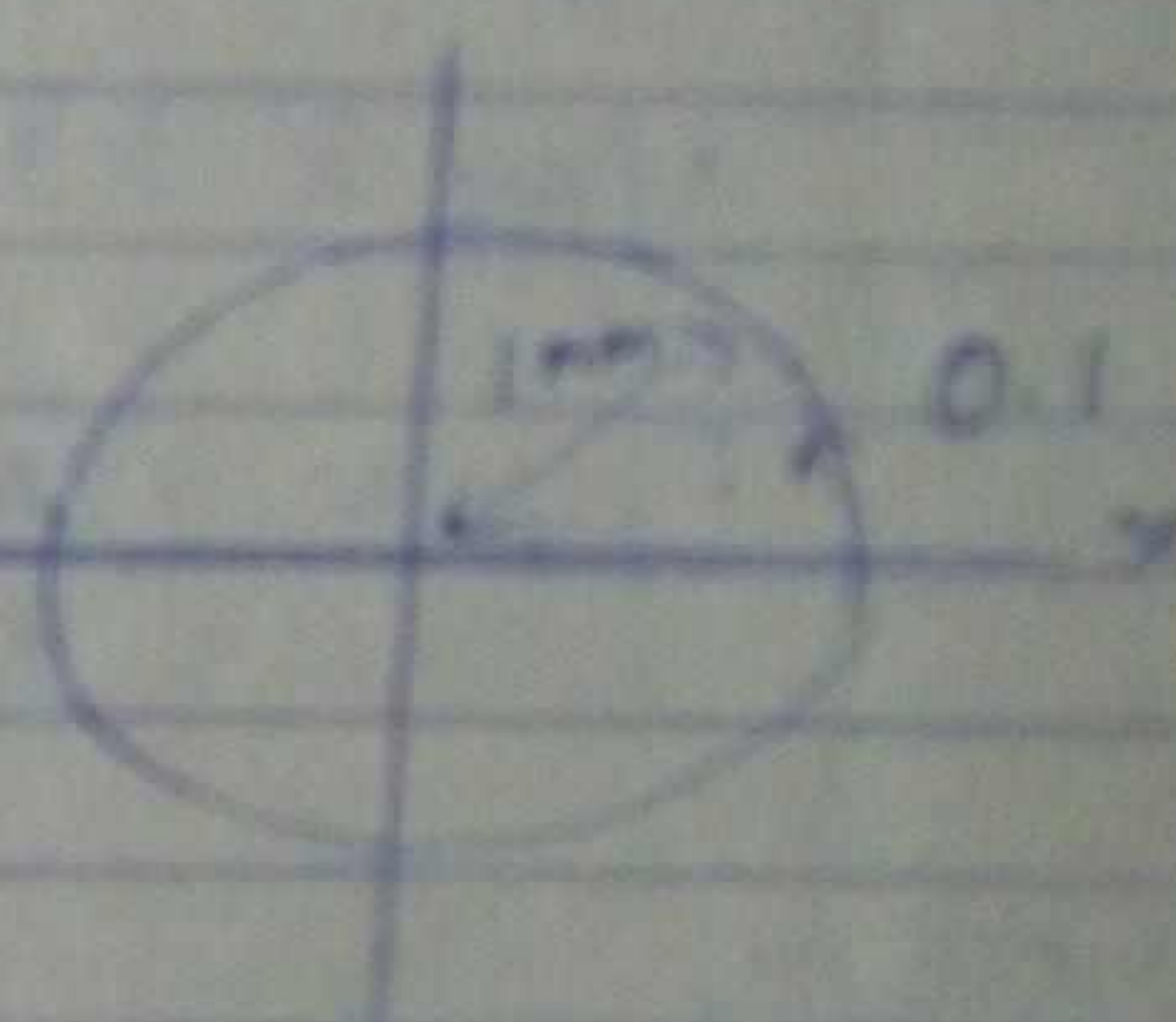
$$= 0.01 \times (10 \pi) \hat{a}_z$$

$$= 2 \hat{a}_z$$



Ques 45 P-11

$$M = 0.1 \times [1 \times (10^{-3})^2] \hat{a}_z$$



$$T = M \times B$$

$$T = 10^{-5} (2 \hat{a}_x - 2 \hat{a}_y + \hat{a}_z) \times 0.1 \times [1 \times (10^{-3})^2] \hat{a}_z$$

$$T = 0.2 \times 10^{-12} (\hat{a}_x + \hat{a}_y)$$

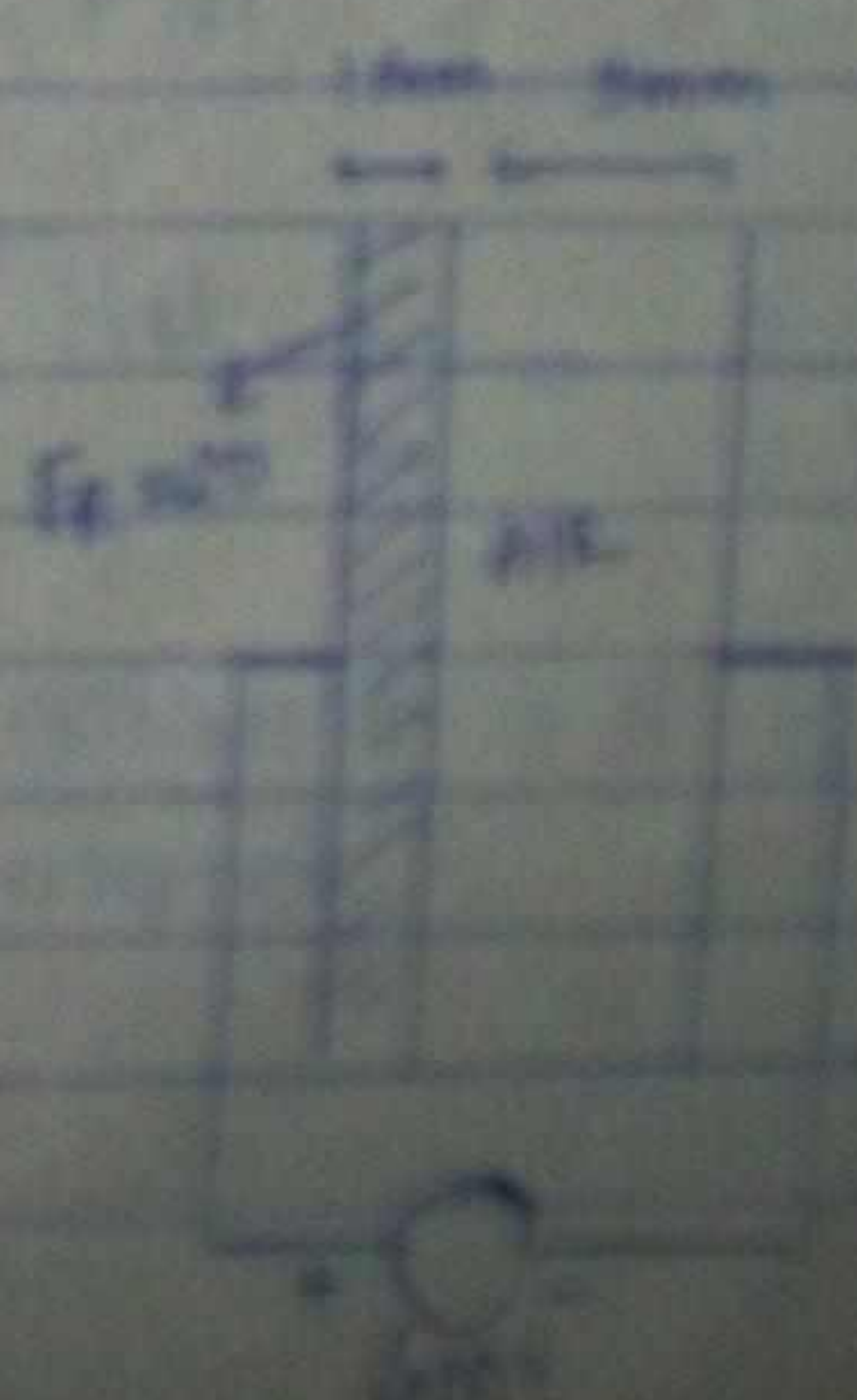
Ques 48 P-12

$$E_{AIR} = \frac{V_{AIR}}{9 \text{ mm}}$$

$$V_{AIR} + V_D = 400$$

$$C_1 V_1 = C_2 V_2$$

$$\frac{V_1}{V_2} = \left(\frac{C_2}{C_1} \right) \left(\frac{d_1}{d_2} \right)$$



$$\frac{V_{AIR}}{V_D} = \frac{\epsilon_D}{\epsilon_{AIR}} \cdot \frac{d_{AIR}}{d_D}$$

$$\frac{V_{AIR}}{V_D} = \frac{20}{1} \times \frac{9}{1} = 180$$

$$V_{AIR} = 180 V_D$$

$$V_{AIR} + V_D = 400$$

$$V_{AIR} \approx 397 V$$

$$E_{AIR} = \frac{V_{AIR}}{9m} = \frac{397}{9 \times 10^{-3}} = \underline{\underline{44 \text{ kV/m}}}$$

NOTE -

$$\frac{dW_E}{dv} = \frac{1}{2} \epsilon E^2 = \frac{1}{2} D \cdot E$$

$$D \cdot E = \frac{\text{Coulomb}}{m^2} \times \frac{\text{Newton}}{\text{Coulomb}} = \frac{\text{Newton}}{m^2} \times \frac{m}{m} = \frac{\text{Joule}}{m^3}$$

$$\frac{dW_H}{dv} = \frac{1}{2} \mu H^2 = \frac{1}{2} B \cdot H$$

$$B \cdot H = \frac{\text{Newton}}{\text{Ampm}} \times \frac{\text{Amp}}{m} = \frac{\text{Newton}}{m^2}$$

TIME VARYING FIELDS - MAXWELL'S EQUATIONS

⇒ MAXWELL'S EQUATIONS - (SUMMARY)

NOT MAXWELL'S
EQNS.

MAXWELL
EQNS.

OPEN
INTEGRAL

INTEGRAL
FORM

POINT
FORM

$$\int D \cdot ds = \Psi_e$$

$$\textcircled{1} \oint D \cdot ds = Q$$

$$\nabla \cdot D = \rho_v$$

Not modified
even in $E(t)$

$$\int E \cdot dl = V \text{ (Volts)}$$

$$\textcircled{2} \oint E \cdot dl = 0$$

$$\nabla \times E = 0$$

Modified due
to Faraday's
law

$$\int B \cdot ds = \Psi_m$$

$$\textcircled{3} \oint B \cdot ds = 0$$

$$\nabla \cdot B = 0$$

Not modified
even in $H(t)$

$$\int H \cdot dl = I_m \text{ (MMF)}$$

$$\textcircled{4} \oint H \cdot dl = I$$

$$\nabla \times H = J$$

modified by
Maxwell's

$$\textcircled{5} \oint E \cdot dl = 0 \text{ at a fixed time}$$

$$\Psi = \text{Density} \times \text{area}$$

NOTE -

Among the four Maxwell eqns, the surface integrals are not modified as they are consistent in time varying fields also.

The line integrals are modified to explain ac voltages and ac currents.

⇒ FARADAY'S LAW & MAXWELL'S (2) EQUATION -

An emf is produced even in a closed conductor when the magnetic flux through the area changes with time.

$$\oint E \cdot dl = - \frac{d\Phi_m}{dt}$$

LENZ'S LAW -

The produced voltage always opposes or opposes the basic cause.

- Potential varies at the same point as time varies and hence the modification is made.

INTEGRAL
FORM

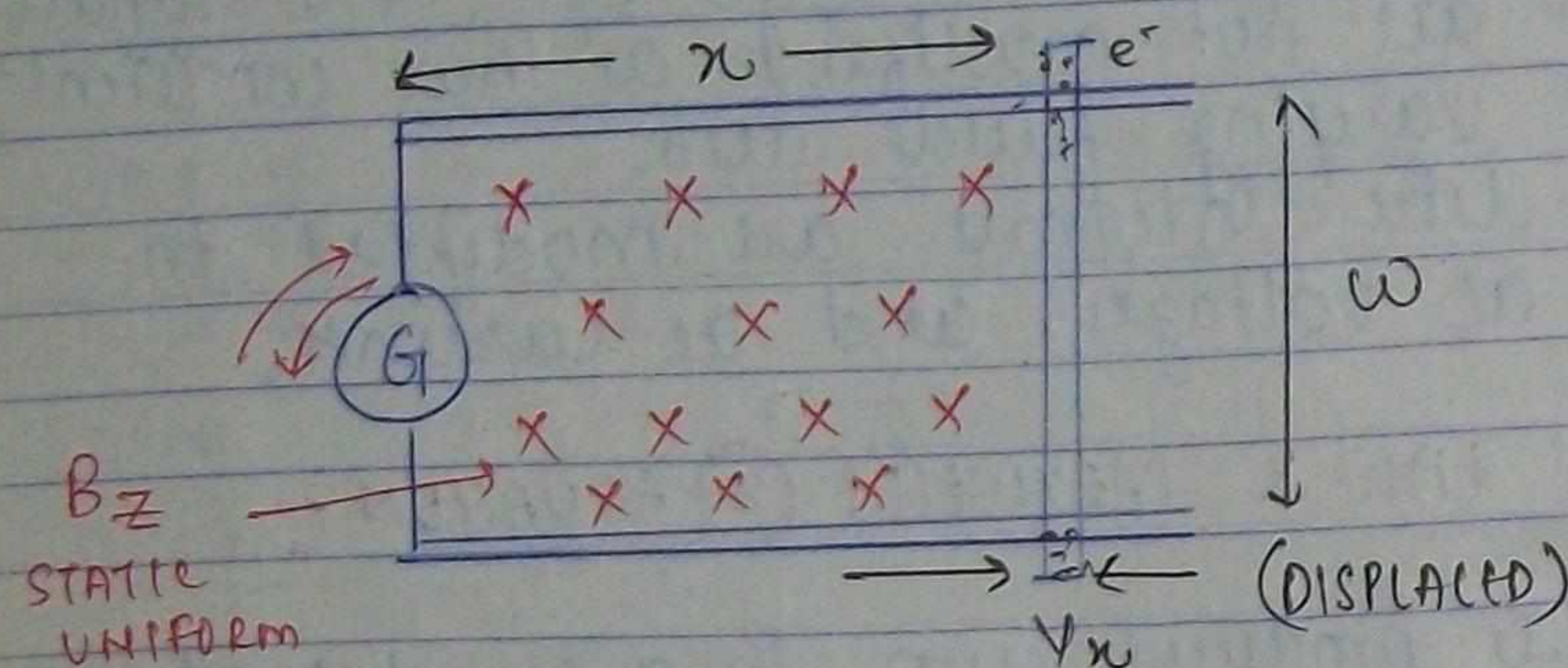
$$\oint E \cdot dl = - \frac{d}{dt} \int B \cdot ds = \int - \frac{\partial B}{\partial t} \cdot ds$$

POINT
FORM

$$\nabla \times E = - \frac{\partial B}{\partial t}$$

(from Stokes' theorem)

⇒ SLIDING RAILS EXPERIMENT -



$$V = - \frac{d}{dt} (B \cdot A) = - B l \frac{dx}{dt}$$

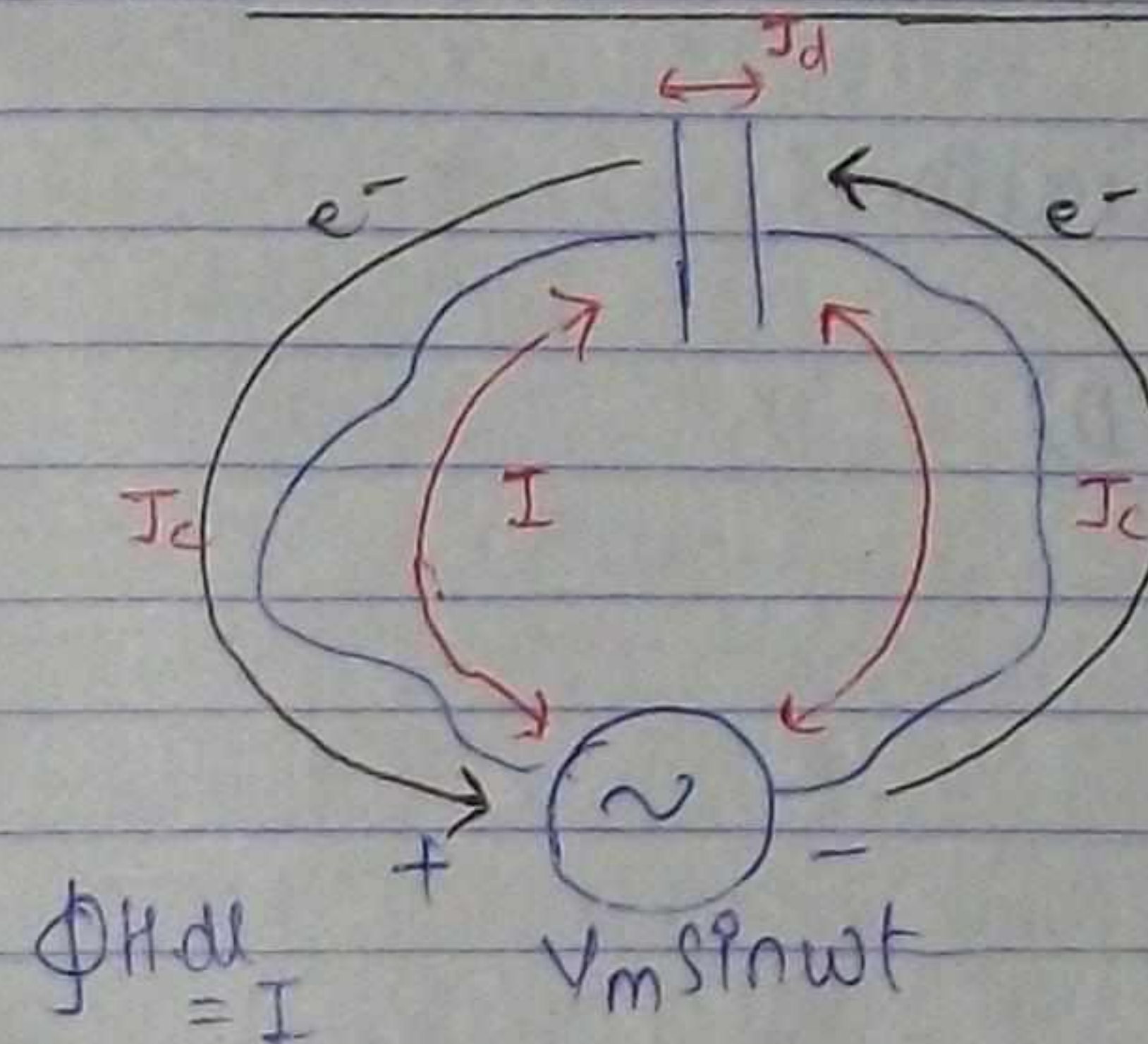
$$V = - B l v_r \quad f_y = q (v_x \times B_z)$$

- When the conducting rod is physically moved in the magnetic field, the free e^- in the rod experience a force as per Lorentz force eqn. Hence they are displaced to the edges of the rod. This is called as induced voltage.

- The displaced e^- oppose the further coming e^- , This is called as Lenz's law.

v_r = velocity of rod

⇒ INCONSISTENCY OF AMPERE'S LAW & MAXWELL (4) eqn -



$$I = c \frac{dV}{dt}$$

$$I = c \omega V_m \sin(\omega t + 90^\circ)$$

$$V_m \sin(\omega t + 90^\circ) = \frac{1}{c \omega} I$$

$$\nabla \times H = J$$

$$V_m \sin \omega t = V = I \left(\frac{1}{c \omega} \right)$$

- There is a current in the wires of the capacitor obeying Ohm's law, when applied with sinusoidal ac voltage. There is no current b/w the plates as there is no physical movement of e^- b/w the plates of the capacitor. Hence the circuit is not closed. But current is flowing. This is the inconsistency of Ampere's law, that is it fails to explain capacitor as a continuity element. This is modified by Maxwell as shown.

$$\oint H \cdot dl = I_c + I_d, \quad \nabla \times H = J_c + J_d$$

$$(e^{90^\circ} = j)$$

J_c - conduction current density

Applying the continuity equation:

$$\nabla \cdot J_d = \frac{\partial \rho_v}{\partial t} = \frac{\partial}{\partial t} (\nabla \cdot D)$$

$$\nabla \cdot J_d = \nabla \cdot \frac{\partial D}{\partial t}$$

$$J_d = \frac{\partial D}{\partial t}$$

$$I_d = \int \frac{\partial D}{\partial t} \cdot d\mathbf{s}$$

∴ Maxwell's eqn become

$$\nabla \times H = J_c + \frac{\partial D}{\partial t}$$

POINT
FORM

$$\oint H \cdot d\mathbf{l} = I_c + \int \frac{\partial D}{\partial t} \cdot d\mathbf{s}$$

INTEGRAL
FORM

- when an ac voltage is applied to the plate of the capacitor. It alternately charges & discharges and hence there is a current flowing in the wires, this is CONDUCTION CURRENT, the charging or discharging leads to change in charge density on the plate and hence change in flux density between the plate which is also a form of current. This is called as 'DISPLACEMENT CURRENT DENSITY'.

$$J_d = \frac{\partial D}{\partial t} = \frac{C/m^2}{\text{sec}} = \text{Amp}/m^2$$

SUMMARY (1):

Time varying magnetic flux is a form of voltage and time varying electric flux is a form of current.

$$\oint E \cdot d\mathbf{l} = - \int \frac{\partial B}{\partial t} \cdot d\mathbf{s}$$

$$\oint H \cdot d\mathbf{l} = \int \frac{\partial D}{\partial t} \cdot d\mathbf{s} + I_c$$

CAUSE

VOLTAGE

↑ Time

EFFECT

H FIELD

E FIELD

↓ Time

CURRENT

EFFECT

CAUSE

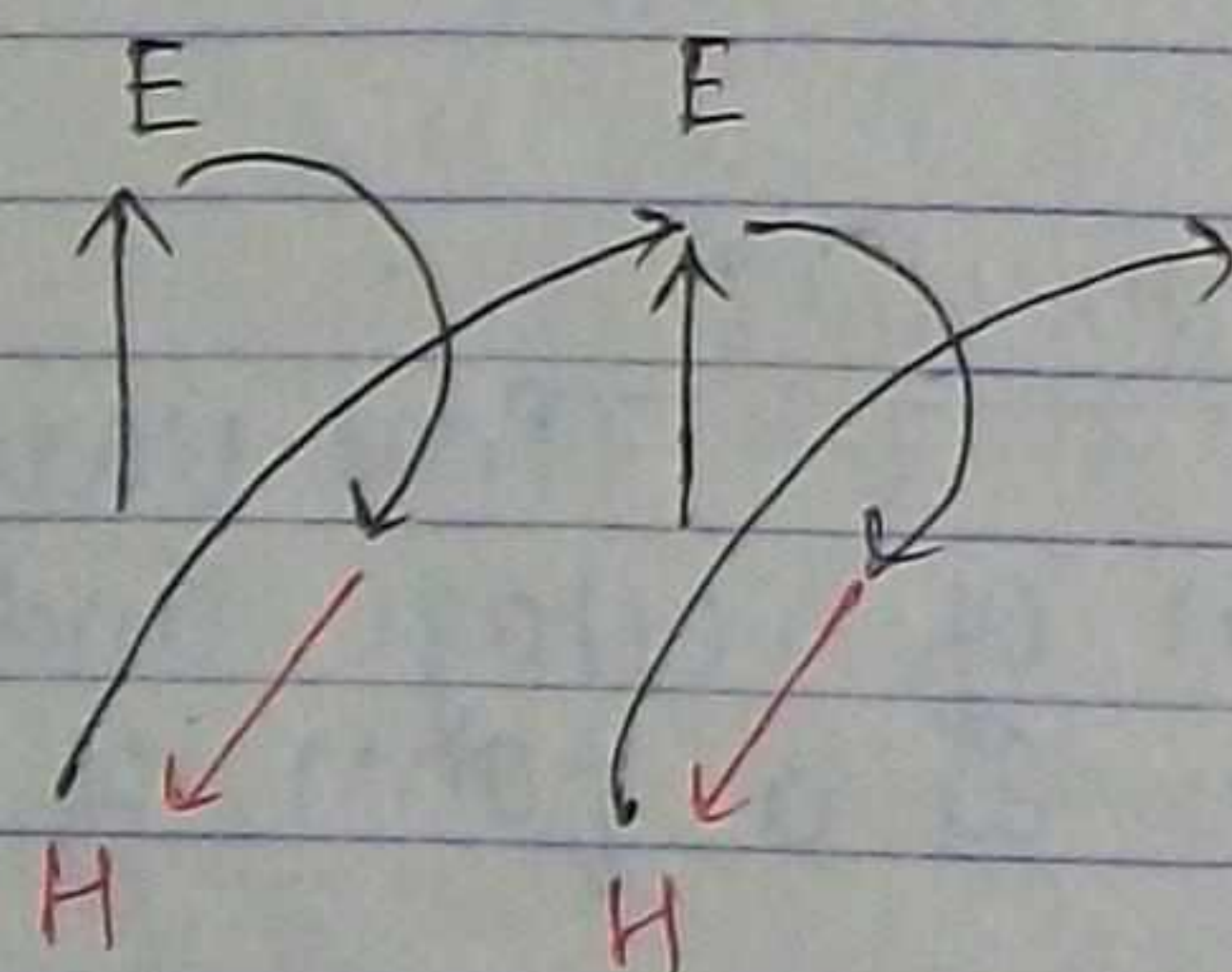
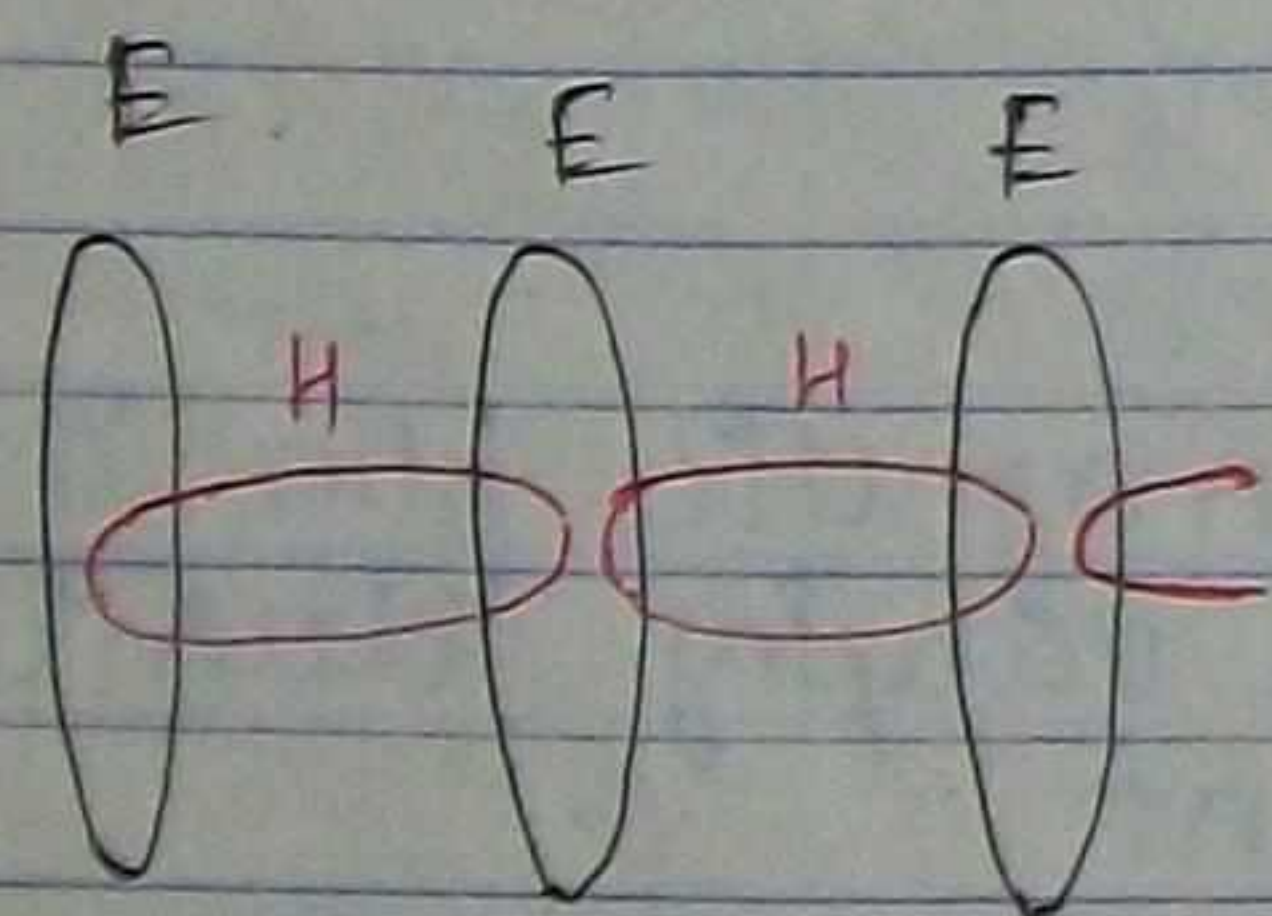
SUMMARY (2) :

Time varying magnetic fields always produce a space varying orthogonal E-field and vice versa.

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times H = \frac{\partial D}{\partial t}$$

Accumulation and flow help each other and give rise to self sustained phenomenon and energy propagation.



SUMMARY (3) :

$$\nabla \times H = \epsilon \frac{\partial E}{\partial t} + \sigma E$$

$$\nabla \times E = -\mu \frac{\partial H}{\partial t}$$

$$(E, \mu, \sigma)$$

MATERIAL
CONSTANT

$$\frac{f}{m}, \frac{H}{m}, \frac{Z}{m}$$

The energy propagation and sustained phenomenon all depend on the materials permittivity for energies. They are called as material constants.

$$(\sigma, \mu, \epsilon)$$

NOTE -

for sustained oscillation the time variations should be such that its derivative is back the same function. Such a function is a harmonic function.

⇒ HARMONIC FUNCTIONS -

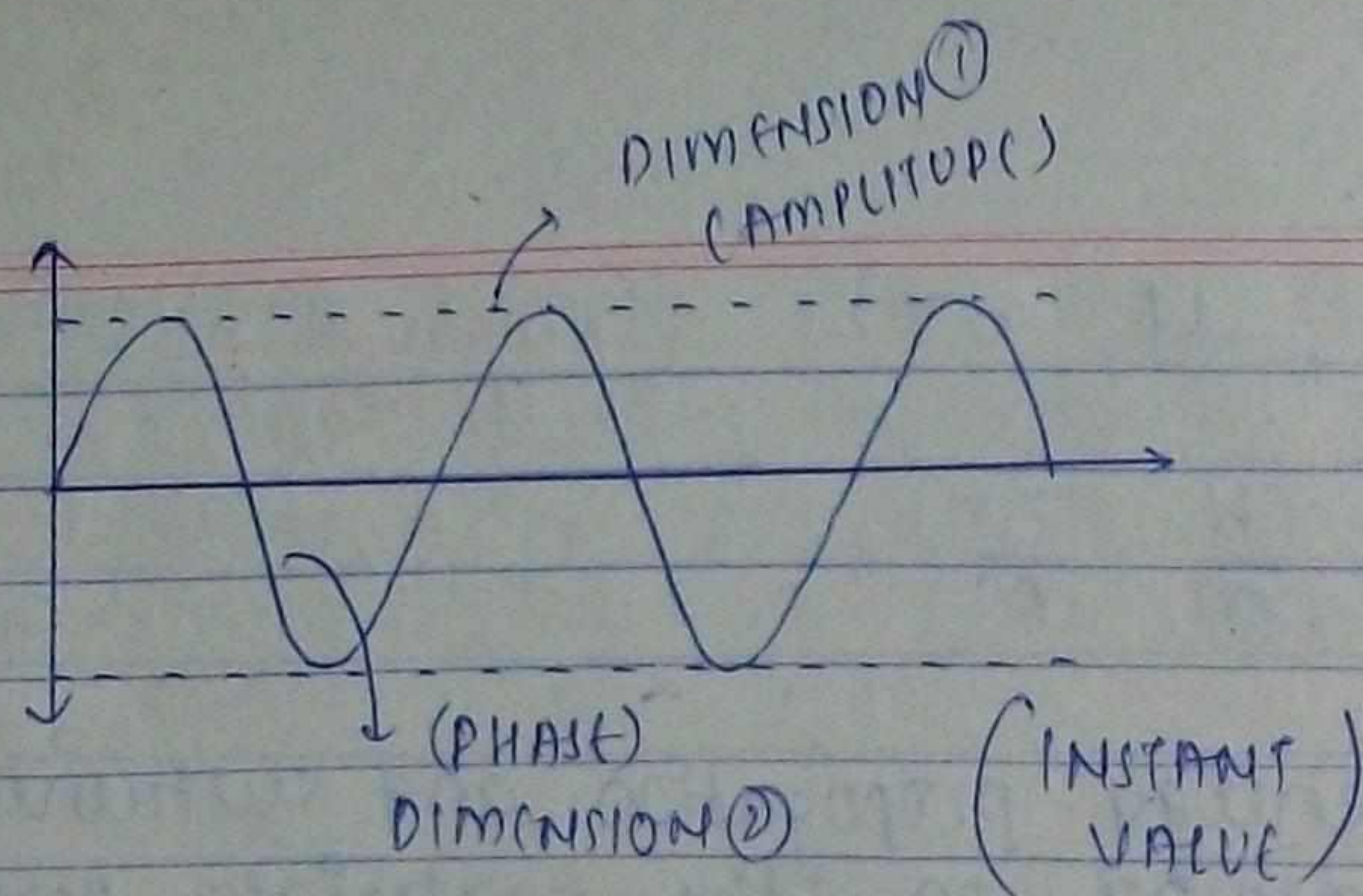
- Every harmonic function is a 2 dimensional quantity and have three format

2 dimensions → Amplitude (A) & phase (θ)

3 formats → $A \sin \theta$

$$A \cos \theta$$

$$A e^{j\theta}$$



PROPERTY (1): The phase should be a linear function of the variable.

$\theta \propto t$ (time harmonic $\rightarrow$ Amp & θ)

$\theta = \omega t$ (ω = phase shift const. per unit time)

$$\omega = \frac{\theta}{t} = \frac{2\pi}{T} \quad (\text{rad/sec})$$

$\theta \propto z$ (space harmonic)

$\theta = \beta z$ (β = phase shift const. per unit length)

$$\beta = \frac{2\pi}{\lambda} \quad (\text{rad/m})$$

$$j = e^{j90^\circ} \quad -1 = e^{j180^\circ} \quad -j = e^{j270^\circ}$$

PROPERTY (2): The derivative is always the same function with an orthogonal shift of 90° .

$$A \sin \omega t \xrightarrow{\text{I deriv}} \omega A \sin(\omega t + 90^\circ) \xrightarrow{\text{II deriv}} \omega^2 A \sin(\omega t + 180^\circ)$$

Similarly for $A \cos \omega t$.

$$A e^{j\omega t} \xrightarrow{\text{I deriv}} j\omega A e^{j(\omega t + 90^\circ)} \xrightarrow{\text{II deriv}} -\omega^2 A e^{j(\omega t + 180^\circ)} \xrightarrow{\text{III deriv}} -j\omega^3 A e^{j(\omega t + 270^\circ)}$$

NOTE -

The shift is essential for advancement and propagation. The secondary derivative back again the same function with negation is a most common property of oscillatory phenomenon.
Ex: LC circuits.

$$V = -L \frac{dI}{dt} = -L \frac{d}{dt} \left(C \frac{dV}{dt} \right) = -LC \frac{d^2V}{dt^2}$$

$$\frac{d^2 I}{dt^2} = -\frac{1}{LC} I$$

By comparing with $\frac{d^2 V}{dt^2} = -\frac{1}{LC} V$

$$\omega = \frac{1}{\sqrt{LC}}$$

#

Free space
Maxwell's eq.

$$\nabla \times E = -\mu \frac{\partial H}{\partial t}$$

$$\nabla \times H = \epsilon \frac{\partial E}{\partial t}$$

$$V = -L \frac{dI}{dt}$$

$$I = C \frac{dV}{dt}$$

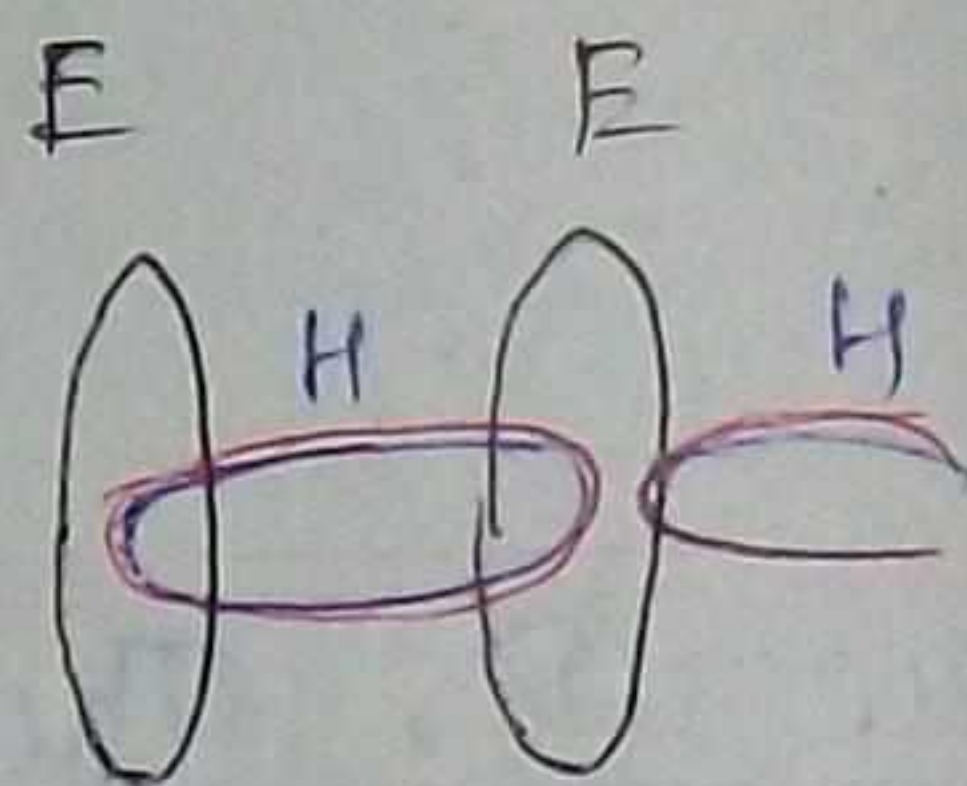
$$\nabla \times (\nabla \times E) = \nabla \times \left(-\mu \frac{\partial H}{\partial t} \right)$$

$$\nabla(\nabla \cdot E) - \nabla^2 E = -\mu \frac{\partial}{\partial t} (\nabla \times H)$$

$$\nabla^2 E = \mu \epsilon \frac{\partial^2 E}{\partial t^2}$$

Similarly,

$$\nabla^2 H = \mu \epsilon \frac{\partial^2 H}{\partial t^2}$$



NOTE -

Every EM wave is a time and space harmonic.

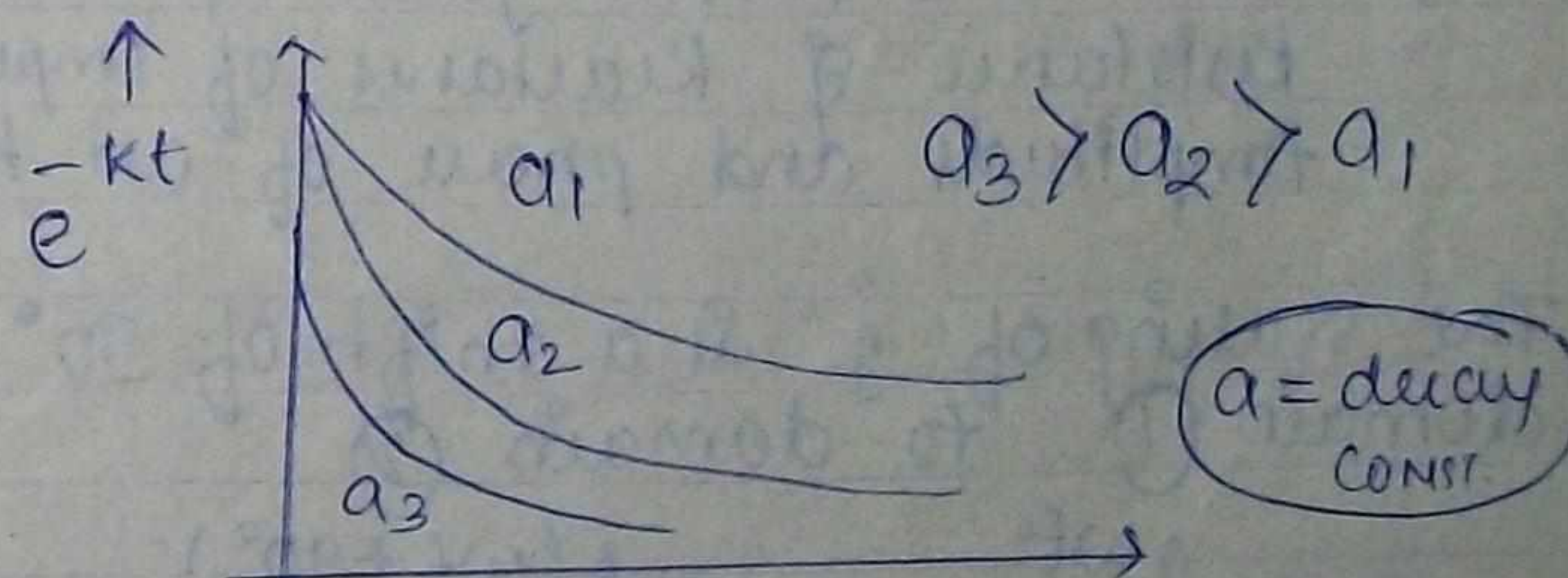
Two differentiation in ∇ is back the same function as.

Two derivations in time.

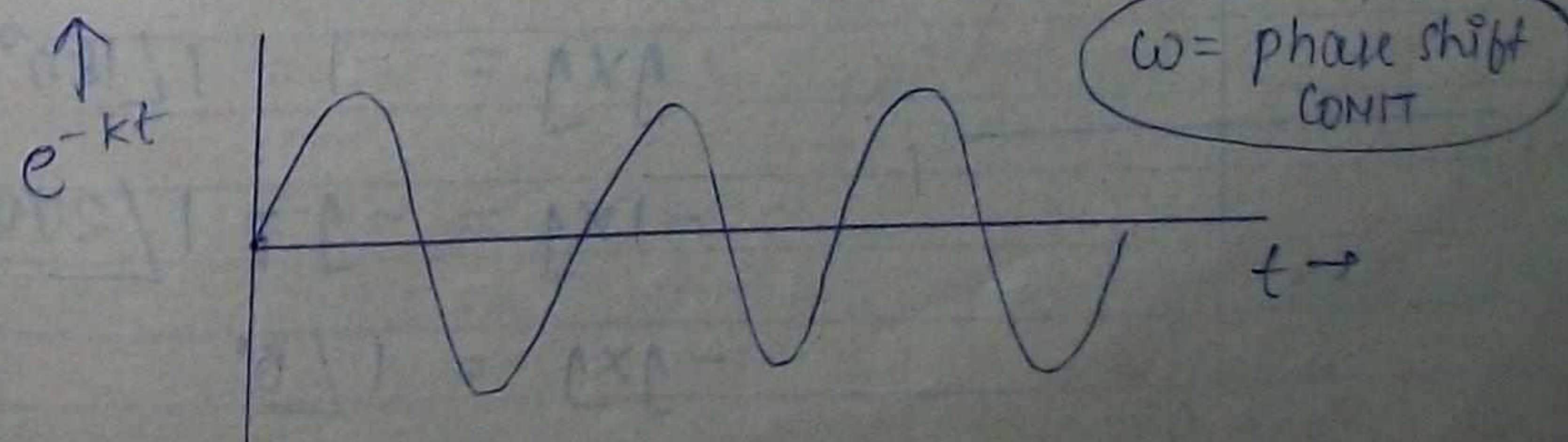
⇒ TYPES OF EXPONENTIAL FUNCTION -

$$e^{-kt} \text{ vs } t$$

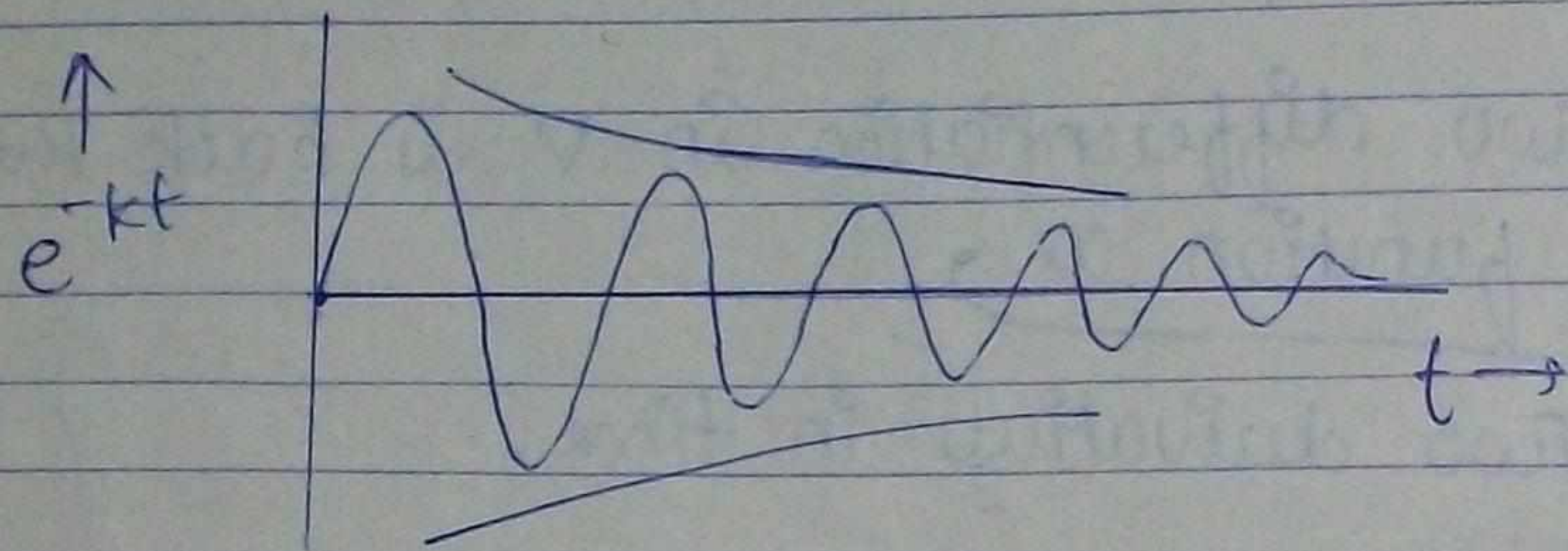
CASE (1): $k = a = +ve$ real no.



CASE (2): $k = j\omega = \text{purely imaginary no.}$



CASE (3): $K = \alpha + j\omega = \text{complex no.}$

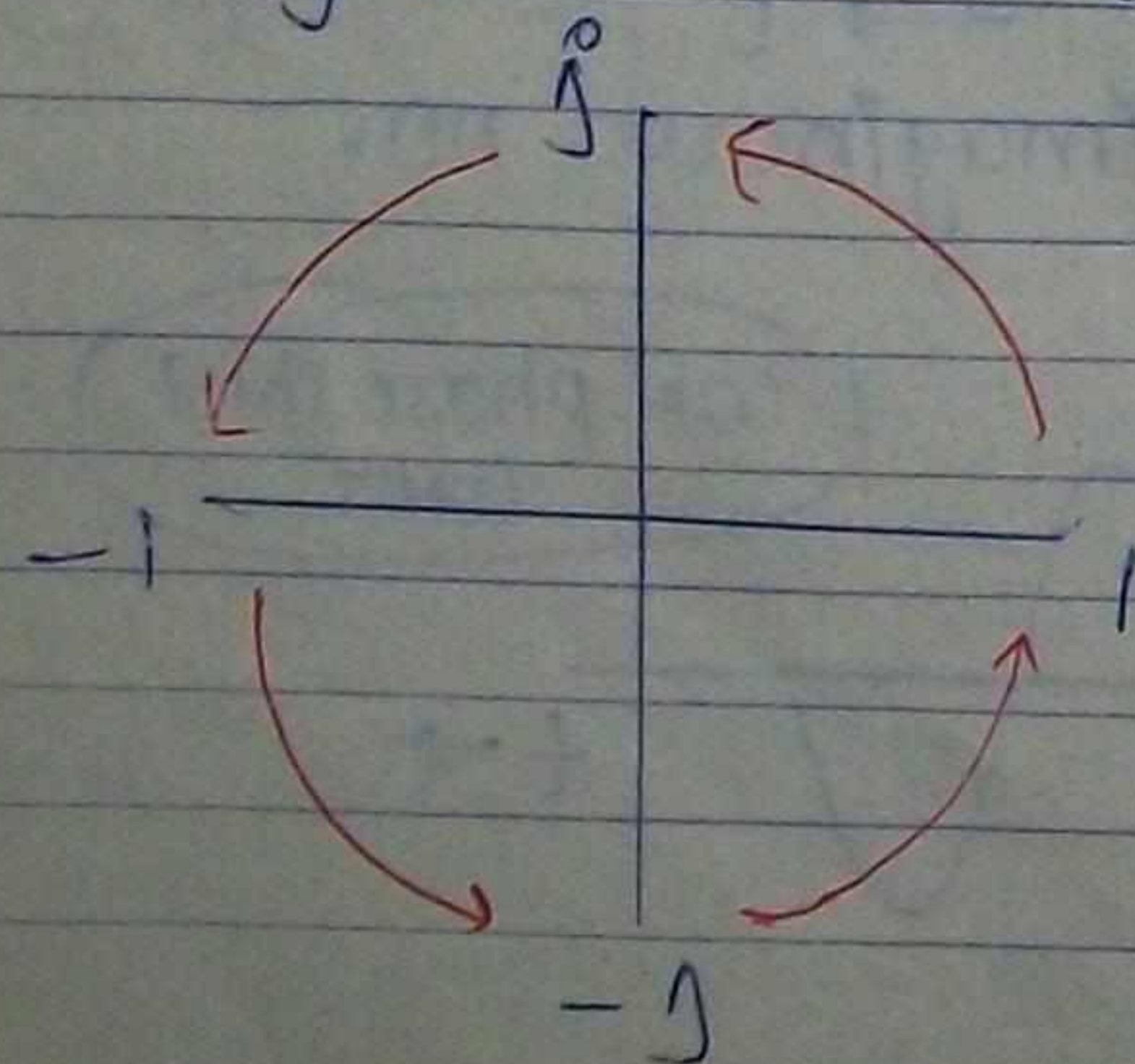


- j represents an orthogonal shift from one dimension to a second independent orthogonal dimension.

Ex - East and North journey
Resistance & Reactance of Impedance
Amplitude and phase of an Harmonic.

- The scaling of j is a shift of 90° from domain (1) to domain (2)

$$j\omega e^{j\omega t} = \omega e^{j(\omega t + 90^\circ)}$$



$$1 \times j = 1 \angle 90^\circ = j$$

$$j \times j = -1 = 1 \angle 180^\circ$$

$$-1 \times j = -j = 1 \angle 270^\circ$$

$$-j \times j = 1 \angle 0^\circ$$

$\Rightarrow$ EM WAVE EQUATIONS & PROPAGATION IN GENERAL MATERIALS -
(changing σ, ϵ, μ)

$$\nabla \times H = \epsilon \frac{\partial E}{\partial t} + \sigma E \quad - (1)$$

$$\nabla \times E = -\mu \frac{\partial H}{\partial t} \quad - (2)$$

Assume a time harmonic source at one end

$$E_s = E_0 e^{j\omega t}$$

$$H_s = H_0 e^{j\omega t}$$

$\omega = \text{source frequency}$

ω — Hz/KHz —> Af waves

MHz —> RF waves

GHz- (10^{12}) —> microwave

$10^{11} - 10^{18}$ —> Light

$10^{20} - 10^{22}$ —> X rays, Gamma rays

10^{25} —> Cosmic rays.

- for an EM wave generation a time harmonic source is needed at one end of the medium where the source is any oscillatory device of any frequency.

Ex - crystals, piezoelectric, hartley osc. X-ray guns etc.

from (1):

$$\nabla \times H = \epsilon j\omega E_0 e^{j\omega t} + \sigma E_0 e^{j\omega t}$$

$$\nabla \times H = (\sigma + j\omega\epsilon) E \quad \text{--- (A)}$$

from (2):

$$\nabla \times E = -\mu j\omega H_0 e^{j\omega t}$$

$$\nabla \times E = (-j\omega\mu) H. \quad \text{--- (B)}$$

But H from (B) and substitute in (A):

$$H = \frac{\nabla \times E}{-j\omega\mu}$$

$$\nabla \times \left(\frac{\nabla \times E}{-j\omega\mu} \right) = (\sigma + j\omega\epsilon) E$$

$$\frac{\nabla \cdot (\nabla E) - \nabla^2 E}{-j\omega\mu} = (\sigma + j\omega\epsilon) E$$

$$\nabla \cdot (\nabla E) = 0 \quad (\text{charge free region})$$

$$\nabla^2 E = j\omega\mu(\sigma + j\omega\epsilon) E \quad \text{--- (C)}$$

HELMHOLTZ EQUATION

Similarly,

$$\nabla^2 H = j\omega\mu(\sigma + j\omega\epsilon) H \quad \text{--- (D)}$$

NOTE -

Eq (C) and (D) prove that if the source is time harmonic, the effect has to be space harmonic propagation.

Consider a unidirectional propagation in z, i.e.:

$$\nabla \rightarrow \frac{\partial}{\partial z} a_z$$

$$E \rightarrow a_x$$

$$H \rightarrow a_y$$

uniform plane wave

Let $\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$

$\therefore \frac{\partial^2 E_x}{\partial z^2} = \gamma^2 E_x$ — (C)

$\frac{\partial^2 H_y}{\partial z^2} = \gamma^2 H_y$ — (D)

Solution of above eqn are:

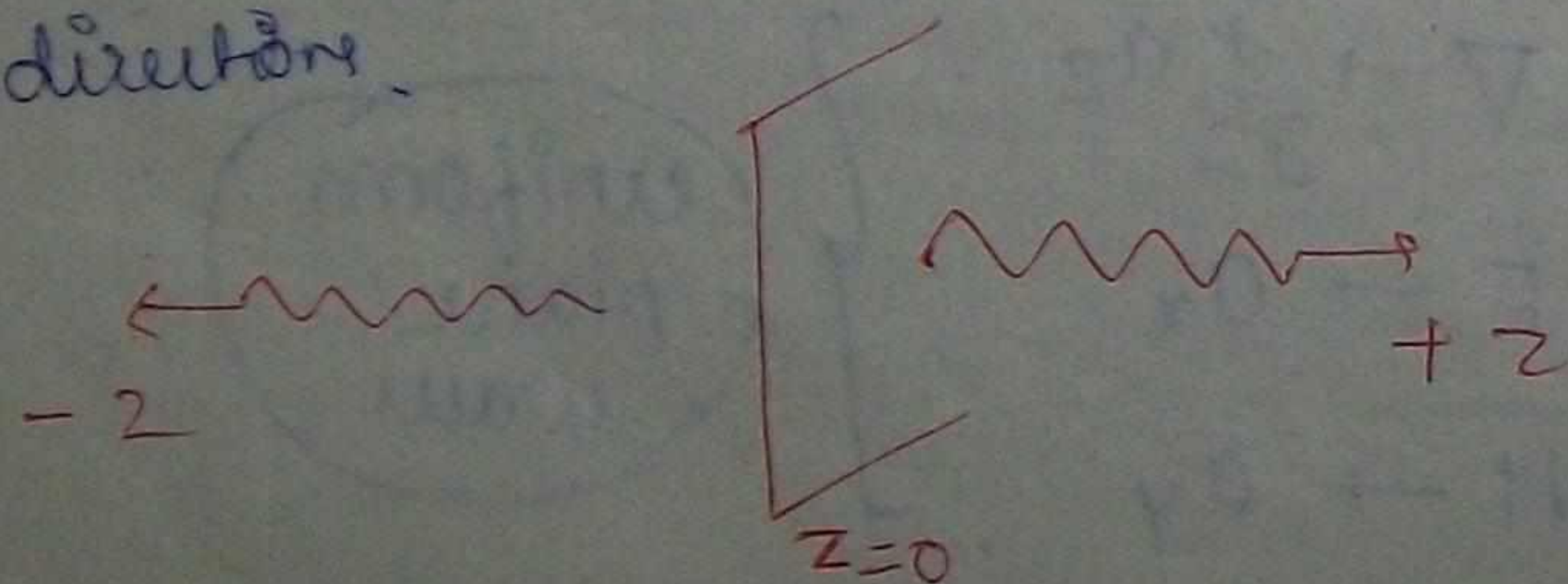
$E(z)_x = (C_1 e^{-\gamma z} + C_2 e^{\gamma z}) a_x$ — (E)

$H(z)_y = (C_3 e^{-\gamma z} + C_4 e^{\gamma z}) a_y$

$\downarrow$ $\downarrow$
 $+z$ direction $-z$ direction (decay)

NOTE -

The final solution represent two possible waves leaving a planar source both exponentially decaying in two directions.

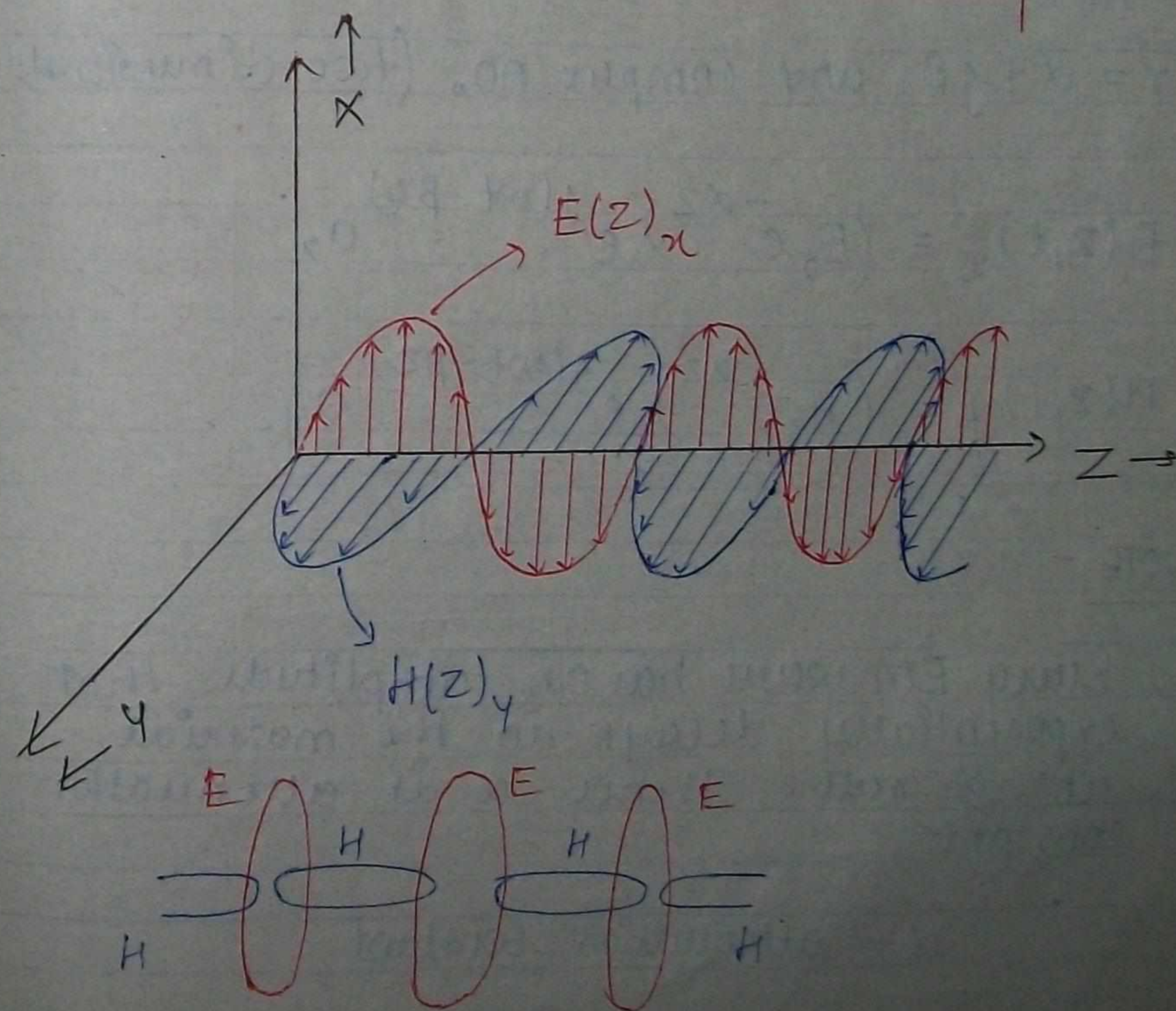


23/03/11

- Considering the +ve z solution in the material to be located on the right of the source, the final EM wave solution is the product solution of independent domains of time and space.

$E(z, t)_x = (E_0 e^{j\omega t} \cdot e^{-\gamma z}) a_x$

$H(z, t)_y = (H_0 e^{j\omega t} \cdot e^{-\gamma z}) a_y$



PROPAGATION CONSTANT (γ) -

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

γ is that constant of the exponential with z that decides the shape of the exponential as the z varies.

It γ decide the course of propagation of the wave.

$\gamma = \alpha + j\beta$ any complex no. (two dimensional)

$$E(z, t)_x = (E_0 e^{-\alpha z}) e^{j(\omega t - \beta z)} a_x$$

$$H(z, t)_y = (H_0 e^{-\alpha z}) e^{j(\omega t - \beta z)} a_y$$

NOTE -

- ① Every EM wave has an amplitude that exponentially decays in the material at α rate. Hence α is attenuation constant

α = attenuation constant

- ② Every EM wave has a phase that linearly varies with time and space at ω and β rate.

- ③ The E and H fields have their direction obeying the transverse nature to propagation such that

$$\begin{array}{ccc} E - & \times & H - \\ \text{direction} & & \text{direction} \end{array} = \text{Propagation direction}$$

$$\text{It } a_x \times a_y = a_z$$

$\Rightarrow$ INTRINSIC WAVE IMPEDANCE (η) -

- When E is converted into H and H into E the rate of transformation or the slope of transformation is η intrinsic impedance of the medium offered to the wave. It is a complex number having two dimension

$$\eta = R + jX$$

R is a cause of amplitude loss in α and X is the cause of phase shift in β

$$\eta = \frac{E}{H} \quad \begin{array}{l} \text{volt/m} \\ \text{amp/m} \end{array}$$

$$\eta = R + jX$$

EXPRESSION FOR η -

$$\nabla \times E = -\mu \frac{\partial H}{\partial t}$$

$$\frac{\partial}{\partial z} a_z \times E_{ax} = -j\omega\mu H_{ay}$$

$$\frac{\partial E}{\partial z} a_y = -j\omega\mu H_{ay}$$

$$\frac{d^2 E}{dz^2} = \gamma^2 E$$

$$-\gamma^2 E = -j\omega\mu H$$

$$\frac{E}{H} = \frac{j\omega\mu}{\gamma} = \frac{j\omega\mu}{\sqrt{j\omega\mu(\sigma + j\omega\epsilon)}}$$

$$\eta = R + jX = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

Ques 1 P-15
Soln:

$$V = -\frac{\partial \psi}{\partial t} = -\frac{d}{dt} (100(t^3 - 2t))$$

$$V = -0.1(3t^2 - 2) \Big|_{t=2} = -1V$$

Ques 3 P-15

ii $I_c = I_d$ only one exist at a time.

$$I_c \left(\frac{1}{j\omega\epsilon} \right) = V$$

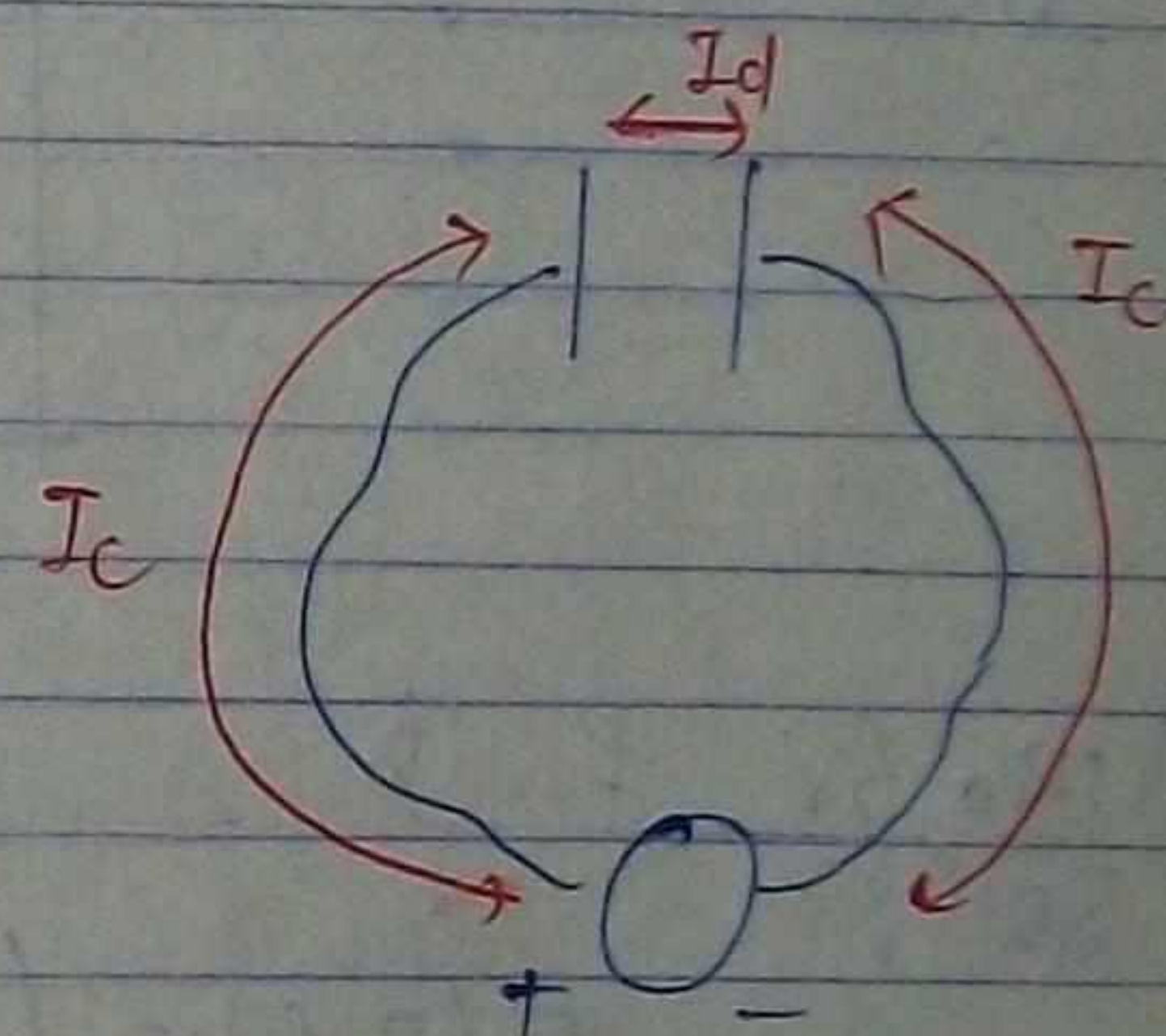
$$I_c = \frac{j\omega\epsilon A}{d} V = 10mA$$

$$I_d = \epsilon \frac{\partial E}{\partial t} = \frac{I_d}{A}$$

$$I_d = \epsilon A \frac{\partial E}{\partial t} = \epsilon A j\omega E$$

$$= j\omega \epsilon \frac{A}{d} V$$

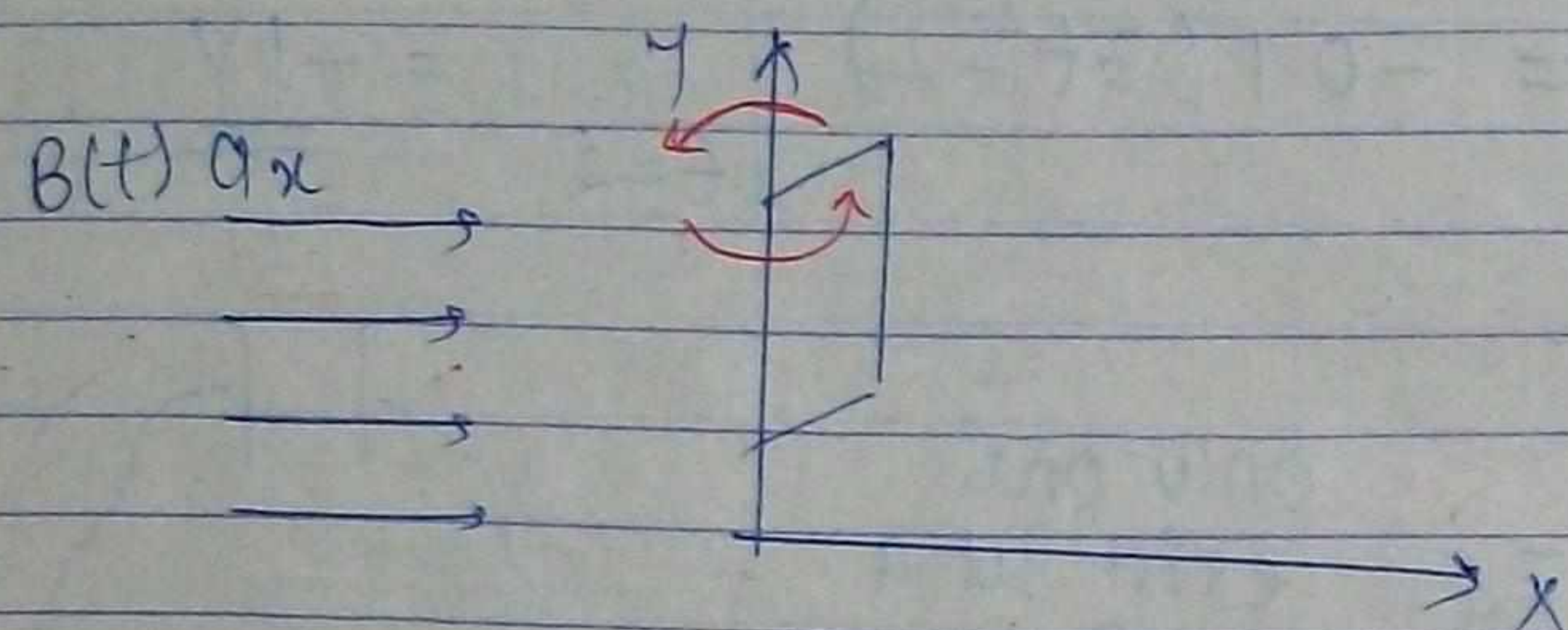
$$= 10mA$$



Que. 4 P-16

Solⁿ:

transformer = time varying H field involved



Que. 6. P-16

Solⁿ:

$$E(z, t) = (E_0 e^{-\alpha z}) e^{j(\omega t - \beta z)} a_x$$

$$|E| = E_0 e^{-\alpha z} = \frac{E_0}{e} \quad \alpha z = 1$$

at $z = 20$

$$\alpha = 1/20$$

$$\beta = \frac{\theta}{z} = \frac{\pi/6}{20} = \frac{\pi}{120}$$

$$\gamma = \left(\frac{1}{20} + \frac{j\pi}{120} \right)$$

or

$$e^{j\omega t - j\beta z} = e^{j\omega t - j\frac{\pi}{120}z} = 30^\circ$$

$$\beta z = \pi/6$$

$$\beta = \pi/120^\circ$$

or

$$\frac{30^\circ}{360^\circ} = \frac{20 \text{ m}}{\lambda \text{ m}}$$

$$\lambda = 240^\circ$$

$$\beta = \frac{2\pi}{\lambda} = \frac{\pi}{120}$$

Que. 7. P-16

Solⁿ:

decays to 20% — 5m

$$\begin{array}{lcl} \text{at } 100 & \longrightarrow & 20 \text{ after } 5 \text{ m} \\ 100 & \longrightarrow & 40 \text{ after } x \text{ m} \end{array}$$

$$20 = 100 e^{-\alpha 5} \quad \alpha = \ln 5/5$$

$$40 = 100 e^{-\alpha z}$$

$$e^{\alpha z} = 5/2$$

$$\alpha z = \ln(5/2), \quad z = \frac{\ln(5/2)}{(\ln 5)/5} = \frac{5 \ln(5/2)}{\ln 5}$$

$$z = \underline{\underline{2.85}}$$

$e = 2.71$

$$\left[\begin{array}{l} \frac{2}{\alpha} \\ \frac{3}{\alpha} \end{array} \right] \left\{ \begin{array}{l} \frac{1}{\alpha} \downarrow \frac{E_0}{e} (100) \leftarrow \frac{1}{e} \\ \frac{1}{\alpha} \downarrow \frac{E_0}{e^2} (37) \leftarrow \frac{1}{e} \\ \frac{1}{\alpha} \downarrow \frac{E_0}{e^3} (13) \leftarrow \frac{1}{e} \end{array} \right.$$

$y = e^x$
 $\frac{dy}{dx} = e^x$

$\Rightarrow$ EM WAVE PROPAGATION -

CASE (1) : EM WAVE PROPAGATION IN FREE SPACE

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} \quad (\sigma = 0, \epsilon = \epsilon_0, \mu = \mu_0)$$

$$= j\omega\sqrt{\mu_0\epsilon_0} = 0 + j\beta$$

ii $\alpha = 0$, No attenuation in free space
 only propagation in free space

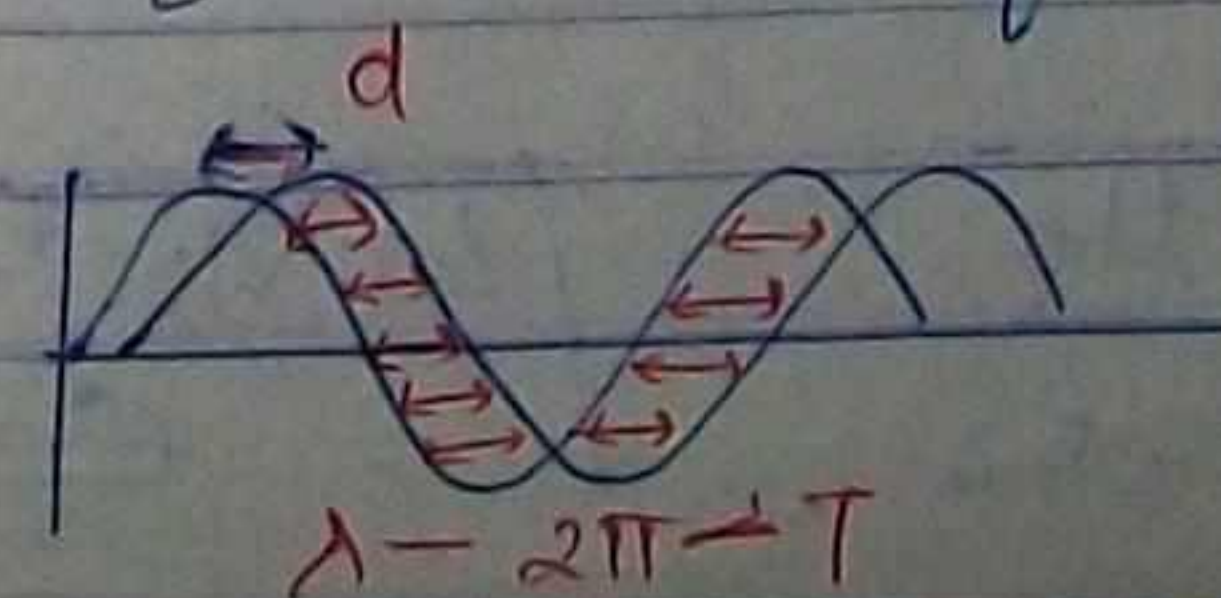
$$\beta = \omega\sqrt{\mu_0\epsilon_0}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \sqrt{\frac{\mu_0}{\epsilon_0}} = \sqrt{4\pi \times 10^{-7} \times 36\pi \times 10^9}$$

$$\eta = 120\pi = 377\Omega$$

PHASE VELOCITY (V_p) -

$$V_p = \frac{d}{t} = \text{The distance an in phase point shift per unit time}$$



$$V_p = \frac{\lambda}{T} = \lambda f$$

$$= \frac{2\pi\lambda}{2\pi T} = \frac{\omega}{\beta} = \frac{\omega}{\omega \sqrt{\mu_0 \epsilon_0}}$$

$$V_p = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \frac{1}{\sqrt{4\pi \times 10^{-7} \times \frac{1}{36\pi \times 10^9}}}$$

$$V_p = 3 \times 10^8 \text{ m/s}$$

CASE (2): EM WAVE PROPAGATION IN IDEAL (OR) PERFECT (OR) LOSSLESS DIELECTRIC

$$(\sigma = 0, \epsilon = \epsilon_0 \epsilon_r, \mu = \mu_0)$$

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

$$= j\omega \sqrt{\mu_0 \epsilon_0 \epsilon_r} = 0 + j\beta$$

$\alpha = 0$; (No attenuation)

$$\beta = \omega \sqrt{\mu_0 \epsilon_0 \epsilon_r}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \sqrt{\frac{\mu_0}{\epsilon_0 \epsilon_r}} = \sqrt{\frac{4\pi \times 10^{-7} \times 36 \times 10^9}{\epsilon_r}}$$

$$\eta = \frac{120\pi}{\sqrt{\epsilon_r}} = \frac{377\Omega}{\sqrt{\epsilon_r}}$$

PHASE VELOCITY -

$$V_p = \frac{1}{\sqrt{\mu_0 \epsilon_0 \epsilon_r}} = \frac{1}{\sqrt{4\pi \times 10^{-7} \times \frac{\epsilon_r}{36\pi \times 10^9}}}$$

$$V_p = \frac{3 \times 10^8}{\sqrt{\epsilon_r}} \text{ m/s}$$

Que. 11 P-17

Solⁿ:

$$E(x,t) = 25 \sin(\omega t + 4x) a_y$$

$e^{j(\omega t - \beta z)}$ → wave travelling in +z

$e^{j(\omega t + 4x)}$ → -a_x direction

$$\beta = 4 = \frac{2\pi}{\lambda}$$

$$\lambda = \pi/2$$

$$v = \lambda f$$

$$f = \frac{3 \times 10^8}{\pi/2}$$

$$f = \frac{6}{\pi} \times 10^8 \text{ Hz}$$

(1) method:
for conventional:

$$\nabla \times E = -\mu \frac{\partial H}{\partial t}$$

$$H = \frac{-1}{\mu} \int (\nabla \times E) dt$$

(2) method:

STEP 1: $\frac{E \text{ (amplitude)}}{H \text{ (amplitude)}} = |\eta|$

STEP 2: E and H have the same harmonic and phase.

STEP 3: $\begin{matrix} E \\ \text{dir}^n \\ (+a_y) \end{matrix} \times \begin{matrix} H \\ \text{dir}^n \\ (-a_z) \end{matrix} = \begin{matrix} \text{Propagation} \\ \text{dir}^n \\ (-a_x) \end{matrix}$

$$H(x,t) = \frac{25}{(120\pi)} \sin(\omega t + 4x) (-a_z)$$

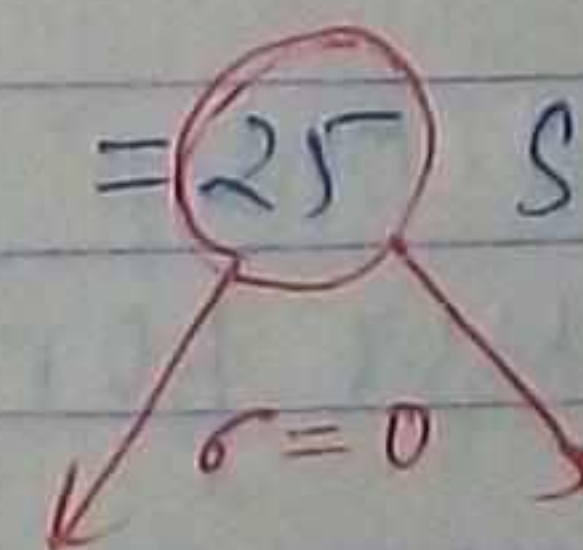
$$= -\frac{25}{120\pi} \sin(\omega t + 4x) a_z$$

Que. 12. P-17

Solⁿ:

$$E(y,t) = 25 \sin(10^8 t - y) a_z$$

$$E = 25 \sin(10^8 t - y) a_z$$



free space

Ideal dielectric

$$v_p = \frac{\omega}{\beta} = \frac{10^8}{1} = 10^8$$

$$v_p < 3 \times 10^8 \text{ hence (Ideal dielectric)}$$

$$a_z \times a_x = a_y$$

$$V_p = \frac{3 \times 10^8}{\sqrt{\epsilon_r}} = 10^8$$

$$\therefore \epsilon_r = 9$$

$$f = \frac{\omega}{2\pi} = \frac{10^8}{2\pi}$$

$$\beta = 1 = \frac{2\pi}{\lambda} \quad \lambda = 2\pi$$

$$H(y, t) = \frac{25}{120\pi} \sin(10^8 t - y) a_x$$

⊛ Polarization is only for E-field direction.

wave polarization is defined only for the E-field orientation, never for H-field.

$$V_p = \frac{\omega}{\beta} = \frac{c}{\sqrt{\epsilon_r}}$$

Que 18. P-18

Solⁿ: $\sigma = 0 \quad \mu_r = 2 \quad \epsilon_r = 8$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \sqrt{\frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r}} = 188 \Omega$$

Que. 21. P-21

Solⁿ:

$$\beta = \omega \sqrt{\mu_0 \epsilon_0 \epsilon_r} = \frac{2\pi}{\lambda}$$

$$\lambda \propto \frac{1}{\sqrt{\epsilon_r}}$$

$$\frac{2}{1} = \frac{\lambda_0}{\lambda} = \sqrt{\frac{\epsilon_r \epsilon_0}{\epsilon_0}}$$

$$\epsilon_r = 4$$

Que. 22. P-22

Solⁿ:

$$E = 20 e^{-5jz} a_x - 25 e^{-5jz} a_y$$

$$H = \left(\frac{20}{120\pi} \right) e^{-5jz} a_y - \left(\frac{25}{120\pi} \right) e^{-5jz} (-a_x)$$

Que 24. P-19.

Solⁿ:

3mm — 90° phase shift

$$\beta = \frac{\pi/2}{3\text{mm}} = \frac{\pi}{6\text{mm}} = \frac{2\pi}{\lambda}$$

$$\lambda = 12\text{mm}$$

$$V_p = \lambda f = \frac{3 \times 10^8}{\sqrt{\epsilon_r}}$$

$$\lambda f = v_p = \frac{3 \times 10^8}{\sqrt{\epsilon_r}}$$

$$\sqrt{\epsilon_r} = \frac{3 \times 10^8}{12 \times 10^{-3} \times 10 \times 10^9}$$

$$\epsilon_r = 6.25$$

Ques. 25 P-20.

Soln:

$$E = A \cos\left(\omega t - \frac{\omega}{c} z\right) a_y \quad \frac{\omega}{c} = \beta$$

$$H = \frac{-A}{\sqrt{\mu_0}} \cos\left(\omega t - \frac{\omega}{c} z\right) a_x$$

$$= -j A \sqrt{\frac{\epsilon_0}{\mu_0}} \sin\left(\omega t - \frac{\omega}{c} z\right) a_x$$

Ques. 26. P-20.

Soln:

$$\nabla^2 E_x - \gamma^2 E_x = 0$$

$$E = E_0 e^{-\gamma z}$$

$$\gamma = j\beta$$

$$\nabla^2 E_x + \beta^2 E_x = 0$$

$$E = E_0 e^{-j\beta z}$$

AA

Ques. 13. P-17

Soln:

$$E(x, z, t) = 25 \sin(\omega t - 3x + 4z) a_y$$

$$= e^{j\omega t} \cdot e^{-j\beta' z} \cdot e^{-j\beta'' x}$$

$$\text{prop. direction} = 3a_x - 4a_z$$

$$\text{unit prop dir}^n = \frac{3a_x - 4a_z}{5}$$

$$\beta = \sqrt{\beta_x^2 + \beta_z^2} = 5$$

$$H(x, z, t) = \frac{25}{120\pi} \sin(\omega t - 3x + 4z) (3a_z + 4a_x)$$

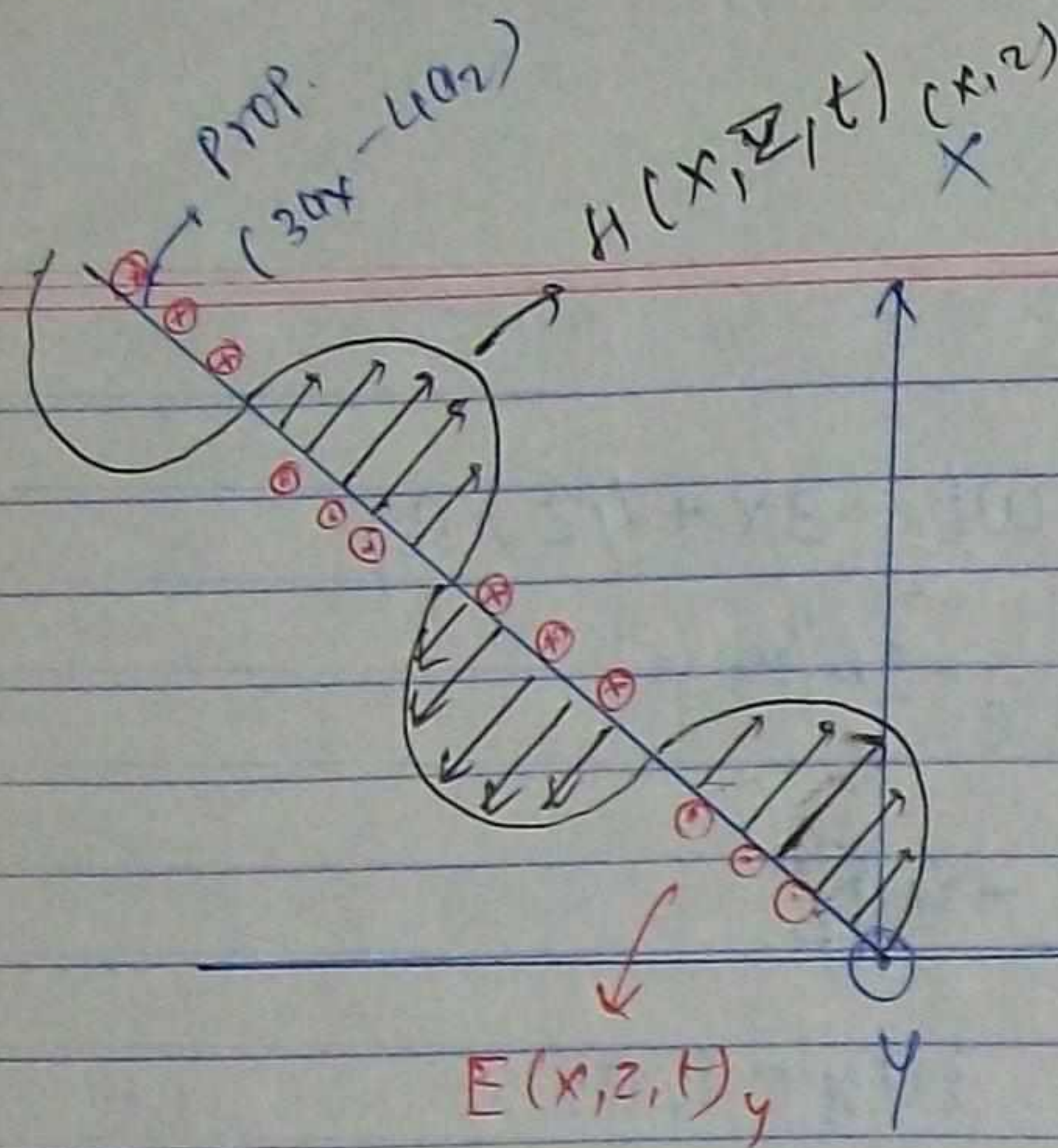
$$\text{for dir}^n \quad a_y \times (?) = 3a_x - 4a_z$$

$$? = 3a_z + 4a_x$$

$$\therefore H(x, z, t) = \frac{5}{120\pi} \sin(\omega t - 3x + 4z) (3a_z + 4a_x)$$

$$E \perp H \perp \text{Prop} \perp E$$

$$a_y \perp (4a_x + 3a_z) \perp (3a_x - 4a_z)$$



Ques. 19, P-18.
Solⁿs:

$$\beta = \frac{2\pi}{\lambda}$$

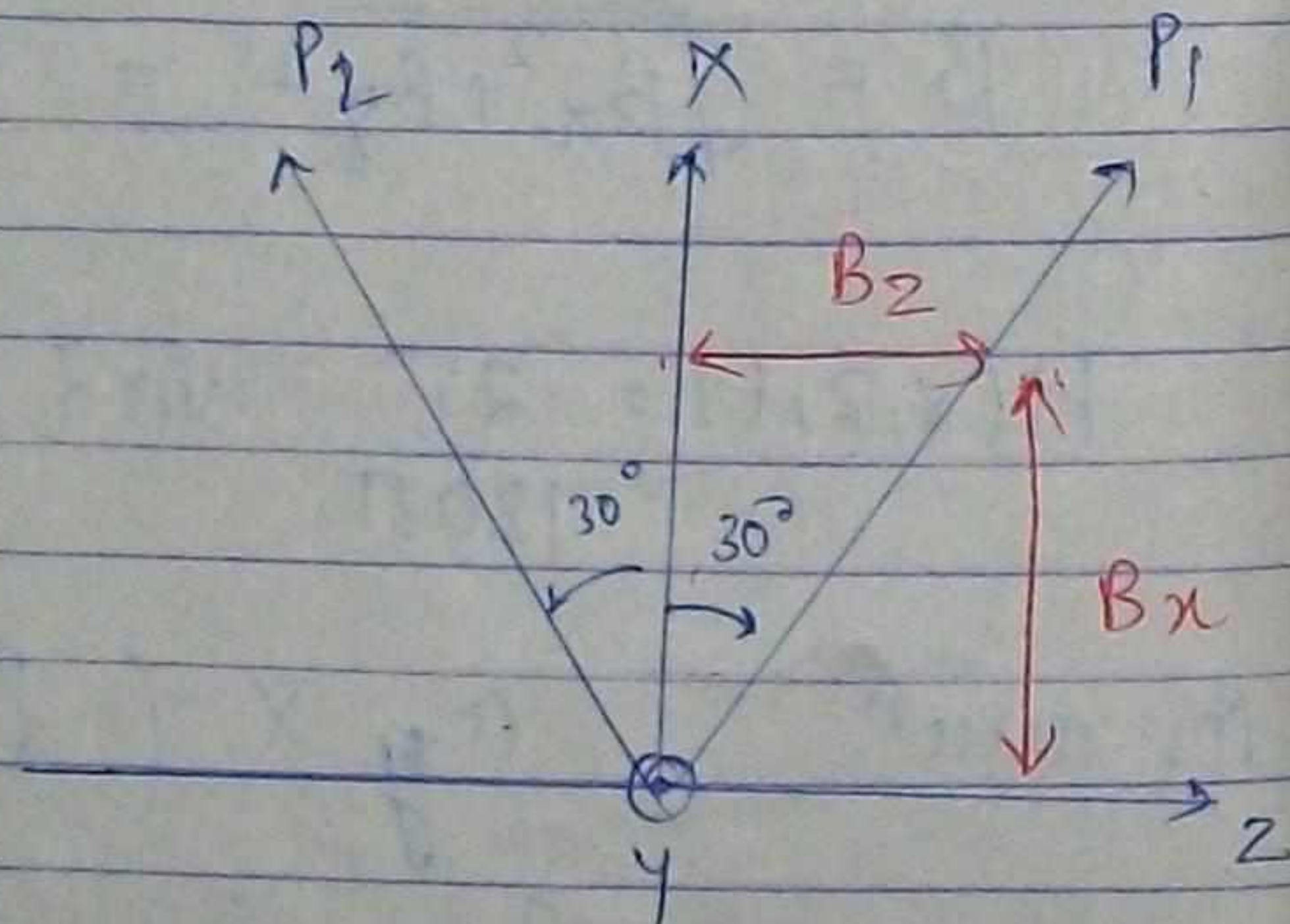
$$\tan 30^\circ = \frac{B_z}{B_x}$$

$$B_x = \sqrt{3} B_z$$

$$\beta = \frac{2\pi}{\lambda} =$$

$$E = \frac{1}{\sqrt{2}} E_0 e^{j(\omega t - \frac{\sqrt{3}\pi x}{\lambda} - \frac{\pi}{2})}$$

first quad



$$\beta = \sqrt{B_x^2 + B_z^2}$$

$$\beta = \sqrt{3B_z^2 + B_z^2} = 2B_z$$

#

$J_b \rightarrow \mu$ (ferromagnetic material only)
 $\left. \begin{array}{l} \rightarrow \text{spinning / revolving } e^- \\ \rightarrow \text{Bound current density} \end{array} \right\}$

Cause of temporary / permanent magnet

$$J_c \rightarrow \sigma$$

$$J_d \rightarrow \epsilon$$

AA

NOTE -

$$(1) \eta = 120\pi \text{ or } \frac{120\pi}{\sqrt{\epsilon_r}} \Rightarrow \text{Real}$$

$$\eta = \frac{E}{H} \Rightarrow \text{real}$$

E_x and H_y are inphase in time.

E_x and H_y are orthogonal in direction. (space)

#

$$\text{when } \eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = |\eta| \angle \theta = \text{complex}$$

E_x and H_y have a phase shift in time that is phase of η

$$E_x = E_0 \sin(\omega t - \beta z) a_x$$

$$H_y = \frac{E_0}{|\eta|} \sin(\omega t - \beta z - \theta) a_y$$

(2) In transmission line.

$R, L, G, C \rightarrow$ primary constant $\rightarrow$ cause V/I waves on Tx line.

$\mu, \sigma, \epsilon \rightarrow$ material constant $\rightarrow$ cause E/H waves in materials.

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\eta = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$\gamma = \sqrt{j\omega \mu (\sigma + j\omega \epsilon)}$$

$$\eta = \sqrt{\frac{j\omega \mu}{\sigma + j\omega \epsilon}}$$

γ and η are called secondary constant

$$\gamma \cdot \eta = j\omega \mu$$

$$\mu = \frac{|\gamma \cdot \eta|}{\omega}$$

$$\gamma/\eta = \sigma + j\omega \epsilon$$

$$\sigma = \text{Real}[\gamma/\eta] \quad \epsilon = \frac{\text{Imag}[\gamma/\eta]}{\omega}$$

(3) $\gamma = \sqrt{j\omega \mu (\sigma + j\omega \epsilon)} = \alpha + j\beta$

$$\alpha^2 - \beta^2 + 2\alpha\beta j = -\omega^2 \mu \epsilon + j\omega \mu \sigma$$

$$\alpha^2 - \beta^2 = -\omega^2 \mu \epsilon$$

$$2\alpha\beta = \omega \mu \sigma$$

$$\alpha = \omega \sqrt{\frac{\mu \epsilon}{2} \left(\sqrt{1 + \left(\frac{\sigma}{\omega \epsilon}\right)^2} - 1 \right)}$$

$$\beta = \omega \sqrt{\frac{\mu \epsilon}{2} \left(\sqrt{1 + \left(\frac{\sigma}{\omega \epsilon}\right)^2} + 1 \right)}$$

30/09/11

⇒ LOSS TANGENT (OR) DISSIPATION FACTOR -

$$\alpha = \omega \sqrt{\frac{\mu\epsilon}{2} \left(\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1 \right)}$$

① The value of α and β always depend on the term $\left(\frac{\sigma}{\omega\epsilon}\right)$ which is called as loss tangent.

$$\text{Loss TANGENT} = \frac{\sigma}{\omega\epsilon}$$

for very good conductor $\left(\frac{\sigma}{\omega\epsilon}\right) \gg 1$

hence α is very large and hence EM waves cannot propagate in good conductor.

② The ability to propagate is dependent on σ of the material and frequency of the wave and hence $\left(\frac{\sigma}{\omega\epsilon}\right)$ is crucial.

Ex- $\left(\frac{\sigma}{\omega\epsilon}\right) \gg 1$ for frequency upto light (10^{15}) only.

but for X-rays $\left(\frac{\sigma}{\omega\epsilon}\right) \ll 1$ and

$$\frac{\sigma}{\omega\epsilon} = \left| \frac{J_c}{J_d} \right|$$

hence X-rays can pass through human body similarly $\left(\frac{\sigma}{\omega\epsilon}\right) \gg 1$ for earth upto radio frequency (MHz) only.

③ $\left(\frac{\sigma}{\omega\epsilon}\right)$ is the ratio of $|J_c/J_d|$ and hence a unit less quantity.

$$\frac{\sigma}{\omega\epsilon} = \left| \frac{J_c}{J_d} \right| = \left| \frac{\sigma E}{j\omega\epsilon E} \right| = \frac{\sigma}{\omega\epsilon}$$

⇒ EM WAVE PROPAGATION -

CASE ③ : EM WAVE PROPAGATION IN GOOD CONDUCTORS.

$$(\sigma \gg \omega\epsilon)$$

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = \sqrt{j\omega\mu\sigma}$$

$$\gamma = \sqrt{\omega\mu\sigma} \angle 90^\circ = \sqrt{\omega\mu\sigma} \angle 45^\circ$$

$$\gamma = \sqrt{\frac{\omega\mu\sigma}{2}} + j\sqrt{\frac{\omega\mu\sigma}{2}} = \alpha + j\beta$$

$$\therefore \alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \sqrt{\frac{j\omega\mu}{\sigma}} \quad (\sigma \gg j\omega\epsilon)$$

$$\eta = \sqrt{\frac{\omega\mu}{\sigma}} \angle 90^\circ = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ$$

$$\eta = \sqrt{\frac{\omega\mu}{2\sigma}} + j \sqrt{\frac{\omega\mu}{2\sigma}} = R + jX$$

$$\therefore R = X = \sqrt{\frac{\omega\mu}{2\sigma}}$$

NOTE -

- ① The impedance resistance is equal to the reactance and hence the rate of amplitude decay is equal to the rate of phase shift

$$R = X \quad \text{ie} \quad \alpha = \beta$$

- ② η phase is 45° it means E and H are out of phase by 45° in good conductor.

↓
delay in time

⇒ SKIN DEPTH (OR) DEPTH OF PENETRATION -

- The exponentially decaying amplitude never reaches an exact zero value but reaches at threshold after travelling for $1/\alpha$ distance. This distance is called as SKIN DEPTH. Hence skin depth is defined as:

"The distance travelled by the wave where the amplitude reaches $1/e$ times of initial value".

$$\delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\sigma}}$$

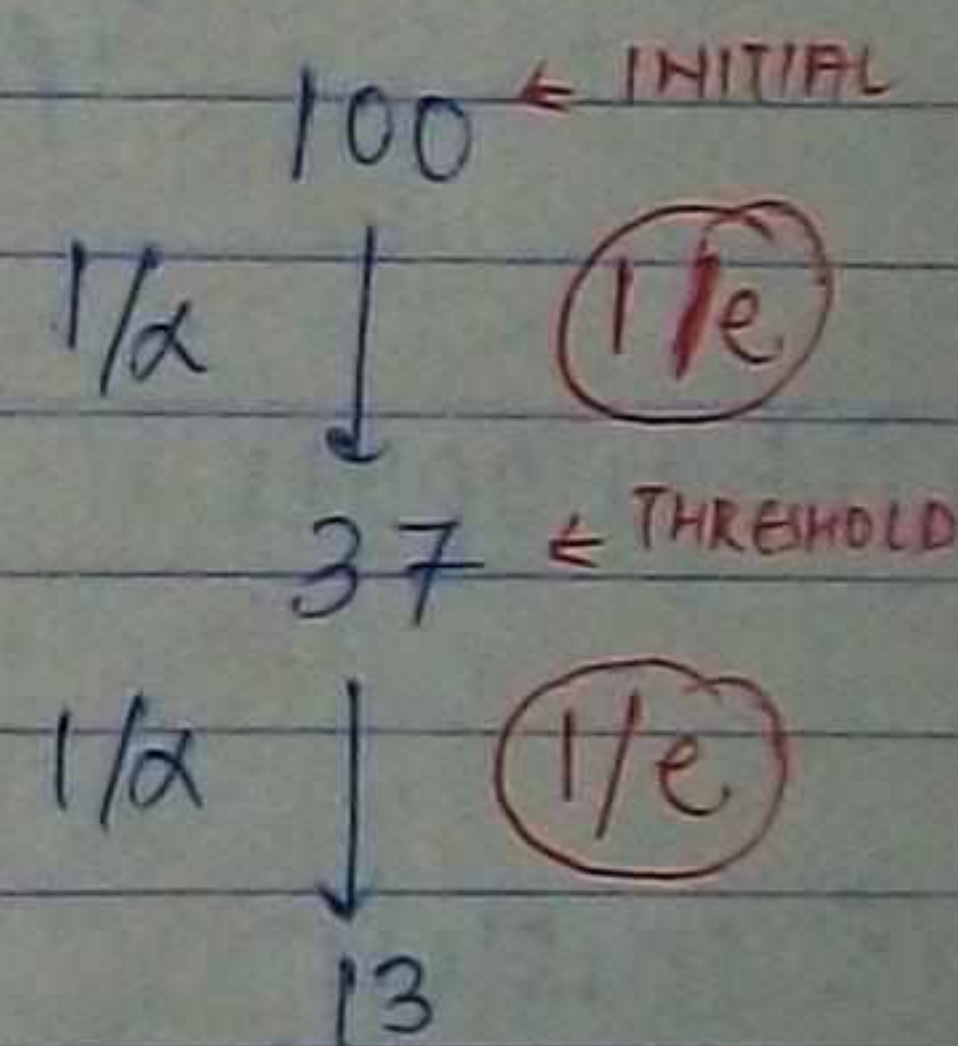
$$\text{also } \delta = \sqrt{\frac{2\sigma}{\omega\mu\sigma^2}} = \frac{1}{\sigma} \sqrt{\frac{2\sigma}{\omega\mu}}$$

$$\delta = \frac{(1/R_s)}{\sigma} = \frac{1}{\sigma R_s}$$

- Most conductor support the EM waves almost on the surface and hence the word 'SKIN DEPTH' and 'SKIN RESISTANCE'.

$$\delta = \frac{1}{\sigma R_s}$$

$R_s = \text{SKIN RESISTANCE}$



Que. 27 P-20

Soln: $\delta = 4 \text{ mm}$ $f = 200 \text{ kHz}$

$$V_p = \frac{\omega}{\beta} = \frac{\omega}{\alpha} \quad (\beta = \alpha) \text{ good cond}^n$$

$$\alpha = \frac{1}{\delta}$$

$$V_p = \omega \delta$$

$$= 2\pi \times 2 \times 10^5 \times 4 \times 10^{-6} = \underline{5 \text{ m/s}}$$

$$V_p = \frac{\omega}{\beta}$$

① free space	$V_p = 3 \times 10^8$	$\frac{\sigma}{\omega\epsilon} = 0$
② lossless dielectric	$V_p = \frac{3 \times 10^8}{\sqrt{\epsilon_r}}$	$\frac{\sigma}{\omega\epsilon} = 0$
③ General material	$V_p = 10^5 - 10^6$	$\frac{\sigma}{\omega\epsilon} \uparrow$
④ conductor	$V_p = \text{few m/s}$	$\frac{\sigma}{\omega\epsilon} \gg \gg 1$

Simu,

$$V_p = \frac{1}{\sqrt{\frac{\mu\epsilon}{2} \left(\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1 \right)}}$$

Que 28. P-20

Soln: E/H phase (η' phase)

$$\frac{\sigma}{\omega\epsilon} = 1.73 \quad (\text{general material})$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \sqrt{\frac{\omega\mu}{k} \frac{190^\circ}{\tan^{-1}(\omega\epsilon/\sigma)}}$$

$$\eta = \sqrt{k' [90 - \tan^{-1}(\omega\epsilon/\sigma)]}$$

$$\angle \eta = \frac{90 - \tan^{-1}(\omega\epsilon/\sigma)}{2}$$

$$= \frac{90 - \tan^{-1}(1/\sqrt{3})}{2} = \underline{30^\circ}$$

① free space	$\angle \eta = 0$	$\frac{\sigma}{\omega\epsilon} = 0$
② lossless dielectric	$\angle \eta = 0$	$\frac{\sigma}{\omega\epsilon} = 0$
③ General material	$\angle \eta = 20^\circ - 30^\circ$	$\frac{\sigma}{\omega\epsilon} \uparrow$
④ Conductor	$\angle \eta = 45^\circ$	$\frac{\sigma}{\omega\epsilon} \gg \gg 1$ ($\sigma \approx \infty$)

Ques 29. P-20

Solⁿ:

$$\sigma = 10^{-2} \text{ mho/m}$$

$$\left| \frac{J_c}{J_d} \right| = \frac{\sigma}{\omega \epsilon} = 1$$

$$= \frac{10^{-2}}{2\pi f \times 36\pi \times 10^9} = 1$$

$$f = 45 \text{ MHz}$$

Ques 30. P-20

Solⁿ:

$$\delta = \sqrt{\frac{2}{\omega \mu \sigma}}$$

$$\omega' = 4\omega \quad \delta' = \delta/2 = 12.5 \text{ cm}$$

Ques 31 P-20

Solⁿ:

$$\frac{J_c}{J_d} = \frac{I_c}{I_d} = \frac{\sigma}{\omega \epsilon}$$

$$I_d = \frac{\omega \epsilon I_c}{\sigma}$$

$$= \frac{2\pi \times 50 \times 8.8 \times 10^{-12}}{58} \times 1$$

$$= 4.8 \times 10^{-11} \text{ A}$$

⇒ POWER, POWER DENSITY & POYNTING VECTOR -

$$\nabla \times H = J_c + J_d = \sigma E + \epsilon \frac{\partial E}{\partial t}$$

multiply ($\cdot E$) on both sides;

$$(\nabla \times H) \cdot E = \sigma E^2 + \epsilon \frac{\partial E \cdot E}{\partial t}$$

$$-\nabla \cdot (E \times H) + H \cdot (\nabla \times E) = \sigma E^2 + \epsilon \frac{\partial E \cdot E}{\partial t}$$

$$-\nabla \cdot (E \times H) - \sigma E^2 = \epsilon \frac{\partial E \cdot E}{\partial t} + \mu \frac{\partial H \cdot H}{\partial t}$$

$$\text{Since } x dx = d(x^2/2)$$

$$-\nabla \cdot (E \times H) - \sigma E^2 = \frac{\partial}{\partial t} \left(\frac{1}{2} \epsilon E^2 + \frac{1}{2} \mu H^2 \right)$$

integrate both sides with dv .

$$\int \nabla \cdot (E \times H) dv + \int \sigma E^2 dv = -\frac{\partial}{\partial t} \int \left(\frac{1}{2} \epsilon E^2 + \frac{1}{2} \mu H^2 \right) dv$$

$$\oint (E \times H) ds + \underbrace{\int \sigma E^2 dv}_{\text{loss}} = -\frac{\partial}{\partial t} \int \left(\frac{1}{2} \epsilon E^2 + \frac{1}{2} \mu H^2 \right) dv$$

outward power (power crossing closed surface)

decrease in power

The RHS term denotes the decreasing power of the electric and magnetic fields inside the volume, this is due to the EM wave and its power crossing out of a surface. Hence

$$\begin{aligned} EXH &= \text{power crossing the closed surface per unit area} \\ &= \text{power density} = \text{strength of the power of EM wave.} \\ &= \text{Poynting vector of the EM wave.} \end{aligned}$$

$$E(z,t)_x \times H(z,t)_y = P(z,t)_z$$

= Instantaneous Poynting vector.

$$(E_0 e^{-\alpha z}) e^{j(\omega t - \beta z)} \times (H_0 e^{-\alpha z}) e^{j(\omega t - \beta z)}$$

$$\text{Time averaged Poynting vector} = \frac{E_0 e^{-\alpha z}}{\sqrt{2}} \times \frac{H_0 e^{-\alpha z}}{\sqrt{2}}$$

- EXH expresses power as a harmonic

and hence given, the power at an instant of time and at a instant in space. for most calculation we need the time independent or average power of a harmonic which is called as rms value. Hence.

$$\left\{ \begin{aligned} \text{Time average Poynting vector} &= \frac{E_0 e^{-\alpha z}}{\sqrt{2}} \times \frac{H_0 e^{-\alpha z}}{\sqrt{2}} \\ &= E_{rms} \times H_{rms} \end{aligned} \right\}$$

$$P(z)_{avg} = \frac{1}{2} \frac{E_0^2}{\eta} e^{-2\alpha z} = \frac{1}{2} \eta H_0^2 e^{-2\alpha z}$$

NOTE -

① Power decays in the material exponentially at 2α rate. This is the power given to the material by EM wave and hence

σE^2 or $J \cdot E$ is ohmic power dissipated to the material per unit volume.

$$J \cdot E = \frac{\text{Amp}}{\text{m}^2} \times \frac{\text{volt}}{\text{m}} = \text{watts/m}^3$$

② Alternately the same expression can also

be written as:

$$P(z)_{avg} = \frac{1}{2} (E \times H^*)$$

where * operation means the average power is independent of harmonic and phase

(3) In free space however it is P_{avg} is constant power independent of z

$$P_{avg} = \frac{1}{2} E_0 H_0 = \frac{1}{2} \frac{E_0^2}{\eta} = \frac{1}{2} \eta H_0^2$$

Ques 33

P-21

Soln:

$$\vec{H} = 0.1 \sin(10^8 \pi t + \beta y) \hat{a}_x$$

$$P_{avg} = \frac{1}{2} \eta H_0^2$$

$$= \frac{1}{2} \times 120\pi \times (0.1)^2 = 1.88 \text{ W/m}^2$$

Ques 34

P-21

Soln:

$$W = \int P_{avg} d\vec{r}$$

$$= \int P_{avg} \cdot \hat{a}_x \cdot d\vec{r} \cdot \hat{a}_x$$

$$= P_{avg} \times \text{area}$$

$$= \frac{60^2}{120\pi \times 2} \times \pi (4)^2 = 240 \text{ W}$$

Ques 40. P-22

$$W = \int P_{avg} \cdot \hat{a}_z \cdot d\vec{r} \cdot \hat{a}_z$$

$$= \text{Power density} \times \text{area}$$

$$= \frac{1}{2} (E_0 \times H_0) \times \pi r^2$$

$$= 250 \text{ W}$$

AA

Que. 36

P-21.

Soln:

$$P_{avg}(z) = \frac{1}{2} \frac{E_0^2}{\eta} e^{-2\alpha z}$$

$$= \frac{1}{2} \frac{(E_0 e^{-\alpha z})^2}{\eta}$$

(i) Total power lost = (Initial power density - final power density) $\times$ area

$$= \left[\frac{1}{2} \times \frac{200^2}{80} - \frac{1}{2} \frac{(200 e^{-0.4 \times 0.6})^2}{80} \right] \times 4 \times 10^{-4}$$

(ii) η = not given.
 power dissipation to medium at an instance

$$\text{Total power lost} = \int \sigma E^2 dv = \int \sigma (E_0 e^{-\alpha z})^2 dA dz$$

$$= \int \sigma E_0^2 e^{-2\alpha z} dA dz$$

$$= \sigma E_0^2 A \int_{z=0}^{0.6} e^{-2\alpha z} dz$$

$$= (0.8)^2 (200)^2 (4)$$

AA

Que 37. P-22

Soln: (i) $H = 50 \sin(\omega t - \beta z) a_x + 150 \sin(\omega t - \beta z) a_y$

$$= 50 \sin(\omega t - \beta z) [a_x + 3a_y]$$

↓
not unit vector

$$= 50 \sqrt{10} \sin(\omega t - \beta z) \frac{(a_x + 3a_y)}{\sqrt{10}}$$

$$P_{avg} = \frac{1}{2} \eta H_0^2$$

$$= \frac{1}{2} \times 120\pi (50\sqrt{10})^2$$

(ii) $H = 50 \sin(\omega t - \beta z) a_x + 150 \cos(\omega t - \beta z) a_x$

$$= 50 \sin(\omega t - \beta z) a_x (1 + 3j)$$

$$= 50 (1 + 3j) \sin(\omega t - \beta z) a_x$$

$$H_0 = 50 \sqrt{10}$$

- Two individual components orthogonal in space are represented as a_x, a_y and the same component orthogonal in time are represented as $\sin$ and $\cos$ or $(1 \& j)$

$$\begin{aligned}
 \textcircled{3} \quad H &= 50 \sin(\omega t - \beta z) a_x + 150 \cos(\omega t - \beta z) a_y \\
 &= 50 \sin(\omega t - \beta z) (a_x + 3j a_y) \\
 &= \frac{50\sqrt{10}}{\sqrt{10}} \sin(\omega t - \beta z) (a_x + 3j a_y)
 \end{aligned}$$

2nd method.

$$H = 50 \sin(\omega t - \beta z) (a_x + 3j a_y)$$

$$H = 50 \sin(\omega t - \beta z) a_x + 150 \cos(\omega t - \beta z) a_y$$

So

$$E = 50\eta \sin(\omega t - \beta z) (-a_y) + 150\eta \cos(\omega t - \beta z) (a_x)$$

$$E = 50\eta \sin(\omega t - \beta z) (3ja_x - a_y)$$

$$P_{avg} = \frac{1}{2} (E \times H^*)$$

$$= \frac{1}{2} (50)^2 \eta [(3ja_x - a_y) \times (a_x - 3ja_y)]$$

$$= \frac{1}{2} \times 50 \times 50 \times 120\pi (0 + a_z + 9a_z + 0)$$

$$= \frac{1}{2} (50)^2 \times 120\pi \times 10 a_z$$

Ques. 30 P-22
Solⁿ:

$$H = \frac{8\sqrt{3}}{\eta_0} \cos(\omega t - \beta z) a_x + \frac{5}{\eta_0} \sin(\omega t - \beta z + \frac{\pi}{2}) a_y$$

$$\begin{aligned}
 P_{avg} &= \frac{1}{2} (E \times H^*) = \frac{1}{2} (a_x + ja_y) \times \frac{k}{\omega\mu} (a_y - ja_x) \\
 &= \frac{k}{2\omega\mu} (a_z + 0 + 0 - a_z) \\
 &= \underline{\underline{0}}
 \end{aligned}$$

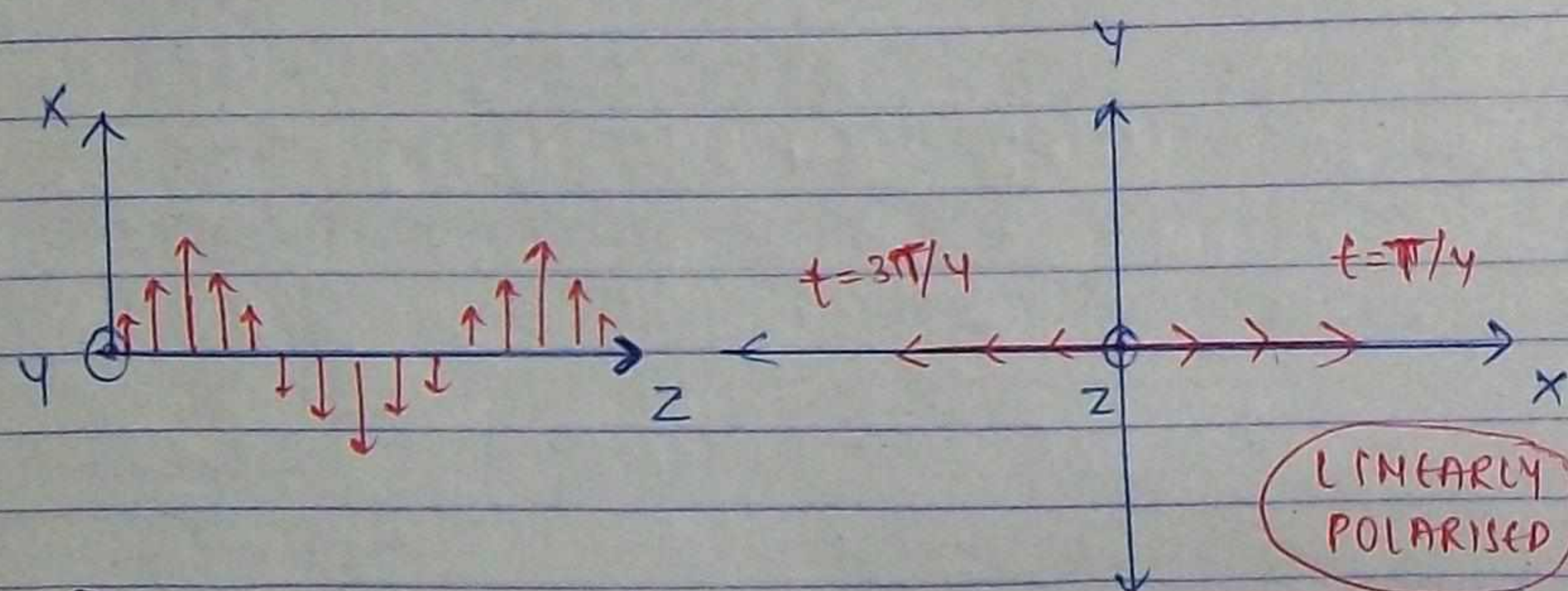
⇒ WAVE POLARIZATION -

- Polarization is defined by the electric field's orientation and the possible planar component of the E field, satisfying the basic transverse nature.

⊗ propagation is along the z direction (assume)

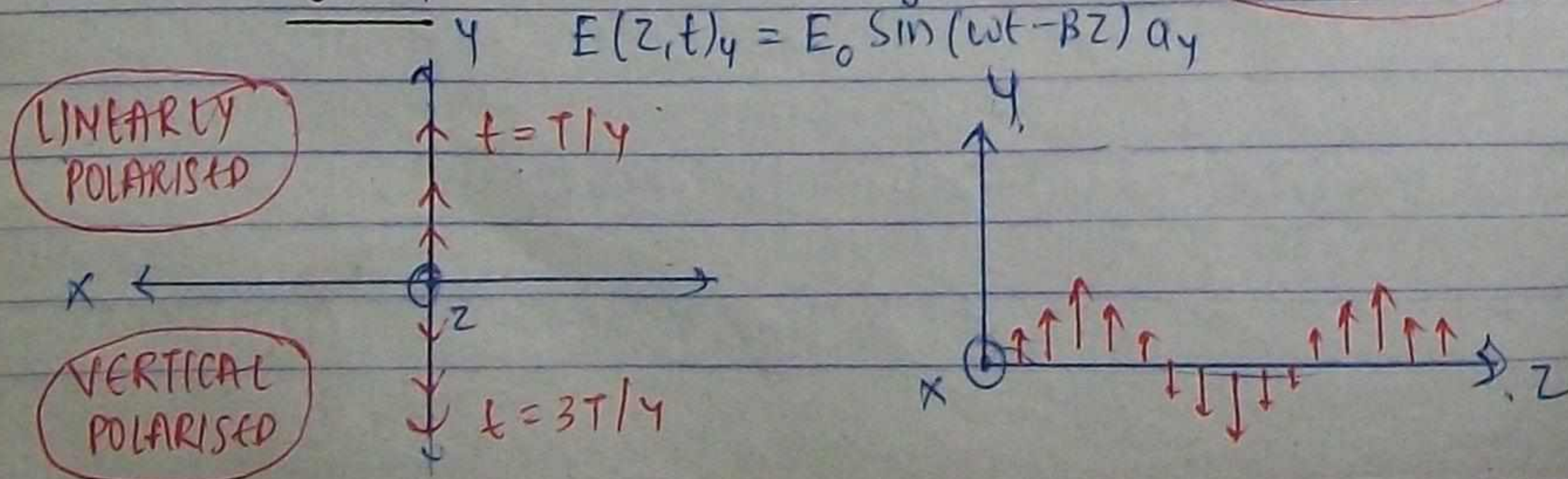
CASE (1): E-FIELD IS IN X-DIRECTION ONLY -

$$E(z, t)_x = E_0 \sin(\omega t - \beta z) a_x$$



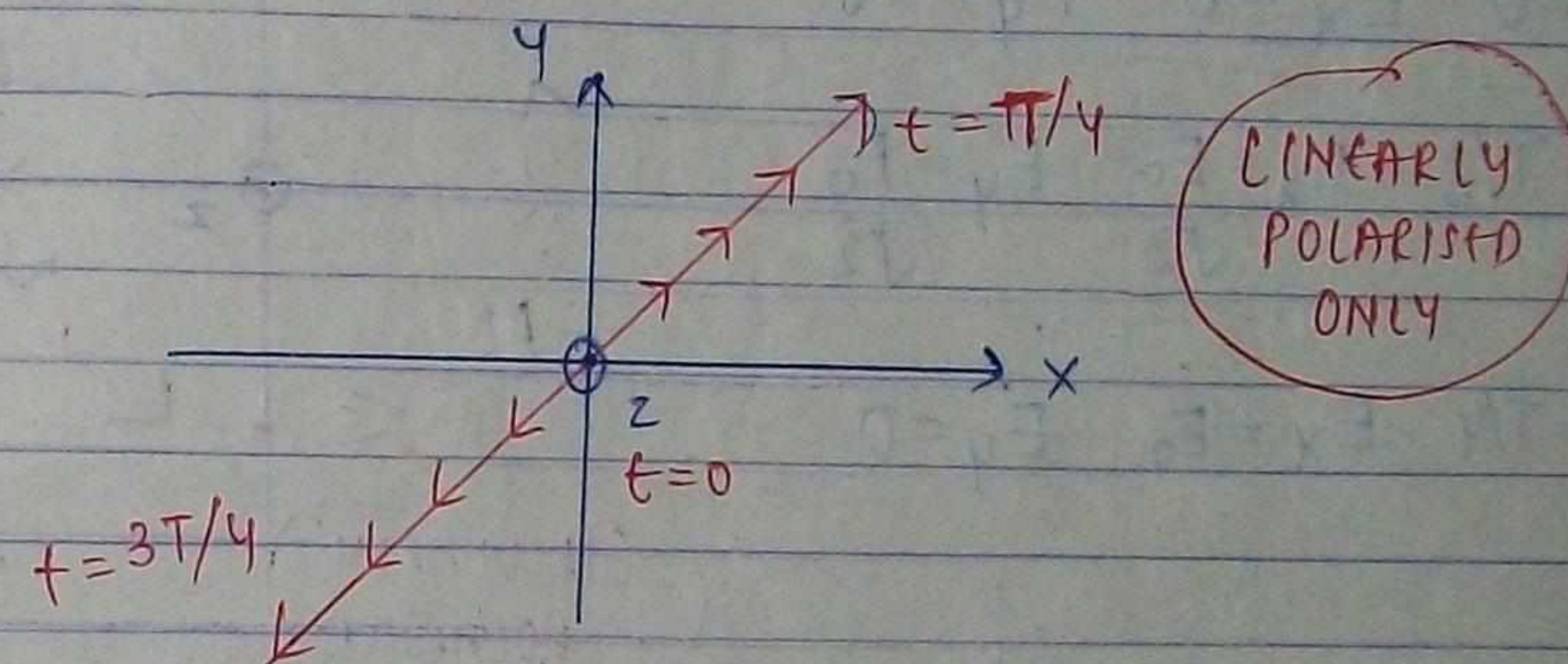
CASE (2): E-FIELD IN Y-DIRECTION ONLY -

$$E(z, t)_y = E_0 \sin(\omega t - \beta z) a_y$$



CASE (3): E-FIELD IN X & Y-DIRECTION -

$$E(z, t) = E_{x0} \sin(\omega t - \beta z) a_x + E_{y0} \sin(\omega t - \beta z) a_y$$



SUMMARY OF LINEAR POLARISATION -

- The EM wave has a single E-field component or two components both in phase, the wave is linearly polarised.

POLARISATION -

Polarization is identified by studying the trace of the E field on the $z = \text{const}$ plane for various advancing times.

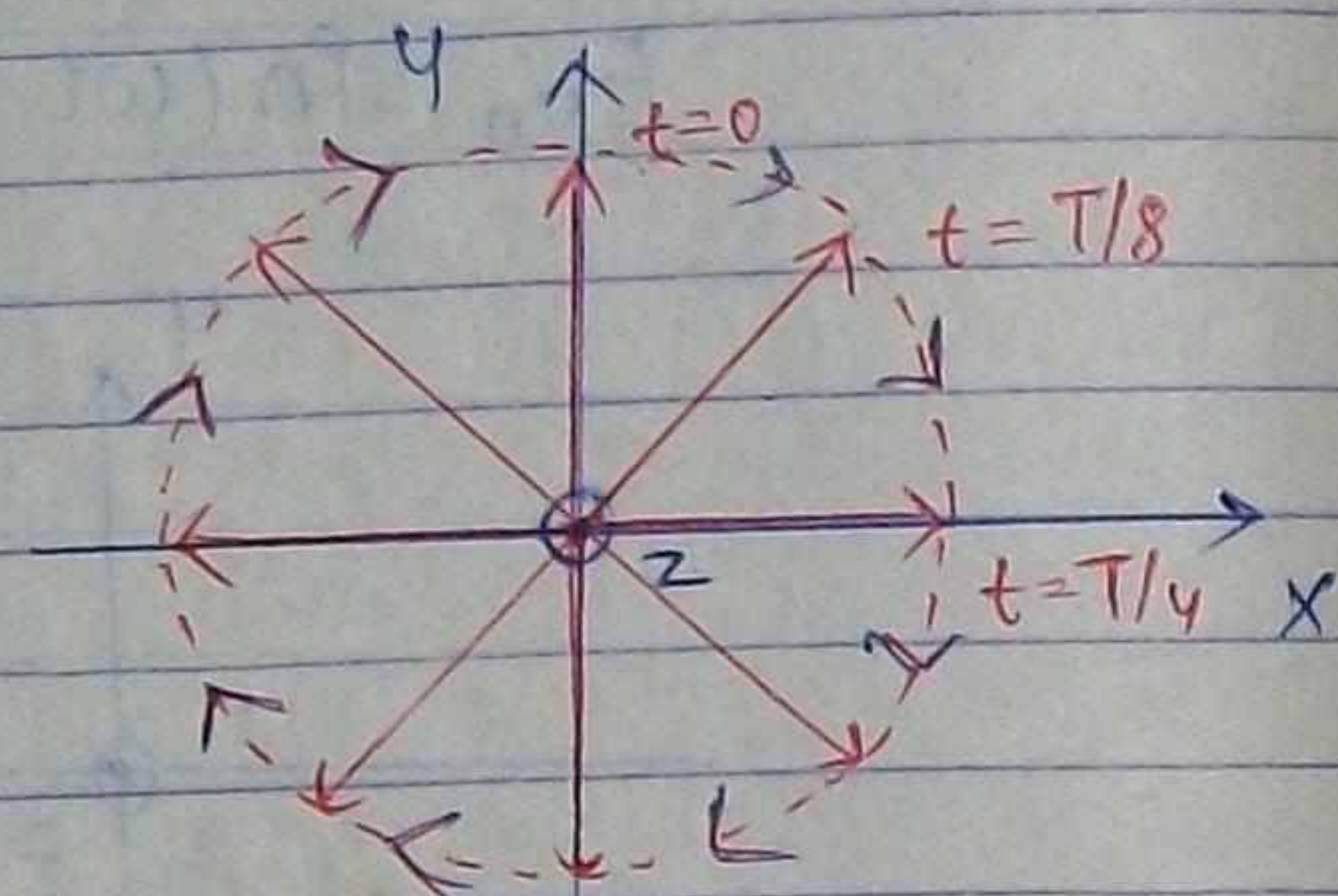
CASE (4): E-field is in x and y direction -

$$E(z, t) = E_0 \sin(\omega t - \beta z) a_x + E_0 \cos(\omega t - \beta z) a_y$$

$$t=0 \quad E_x=0 \quad E_y=E_0$$

$$t=T/8 \quad E_x = \frac{E_0}{\sqrt{2}} \quad E_y = \frac{E_0}{\sqrt{2}}$$

$$t=T/4 \quad E_x=E_0 \quad E_y=0$$



CIRCULARLY POLARISED

SUMMARY:

If the electric field have two planar components both equal amplitude and out of phase by 90° , the wave is circularly polarised.

- SENSE OF ROTATION LEFT OR RIGHT -

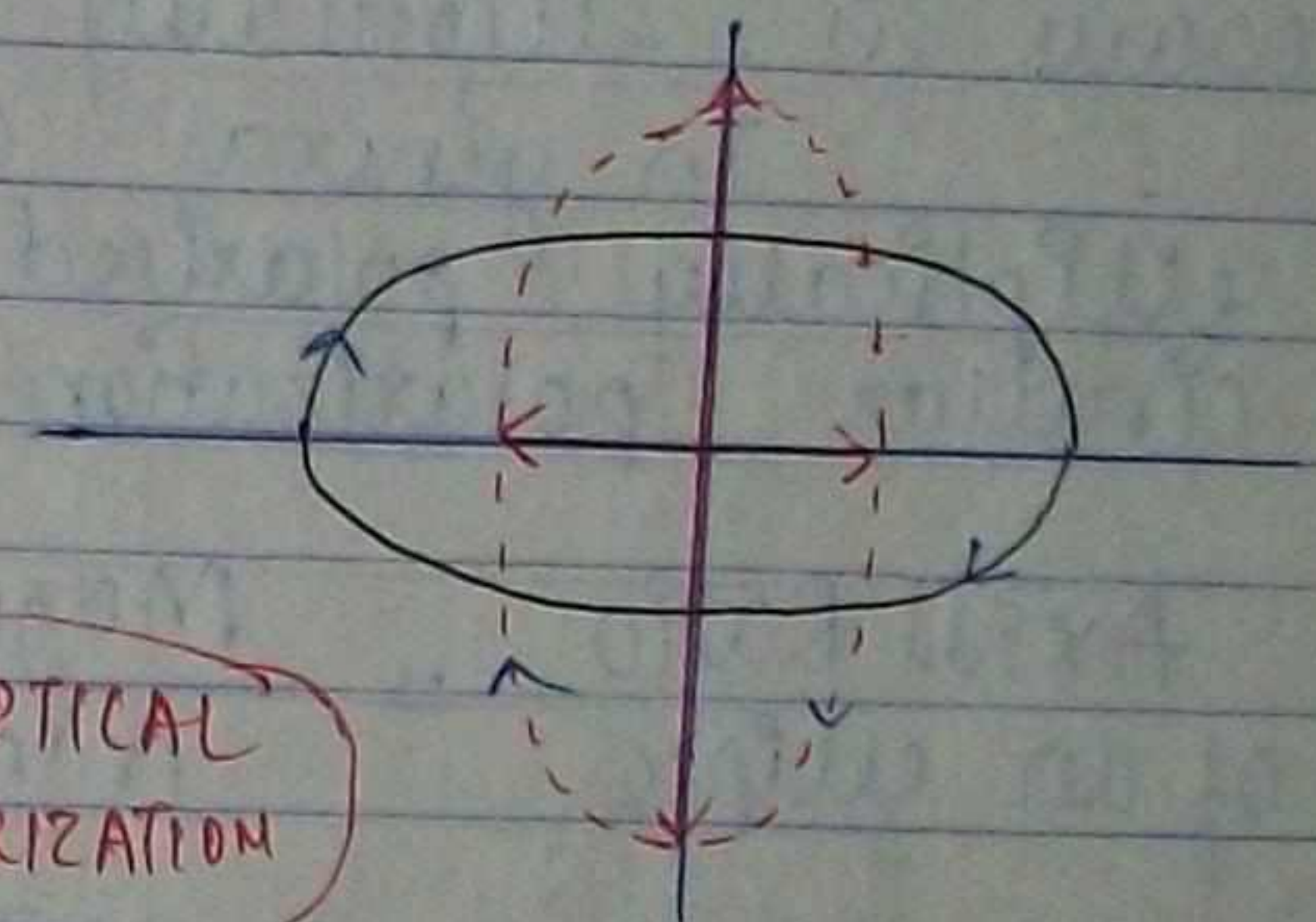
If the left hand thumb points towards propagation direction and the closed fingers along advancing time the wave is left circularly polarised.

Ex - out of the paper propagation & clockwise time advancement
(Left circular polarised)

CASE (5): E-field is in x & y direction -

$$E(z, t) = E_{x_0} \sin(\omega t - \beta z) a_x + E_{y_0} \cos(\omega t - \beta z) a_y$$

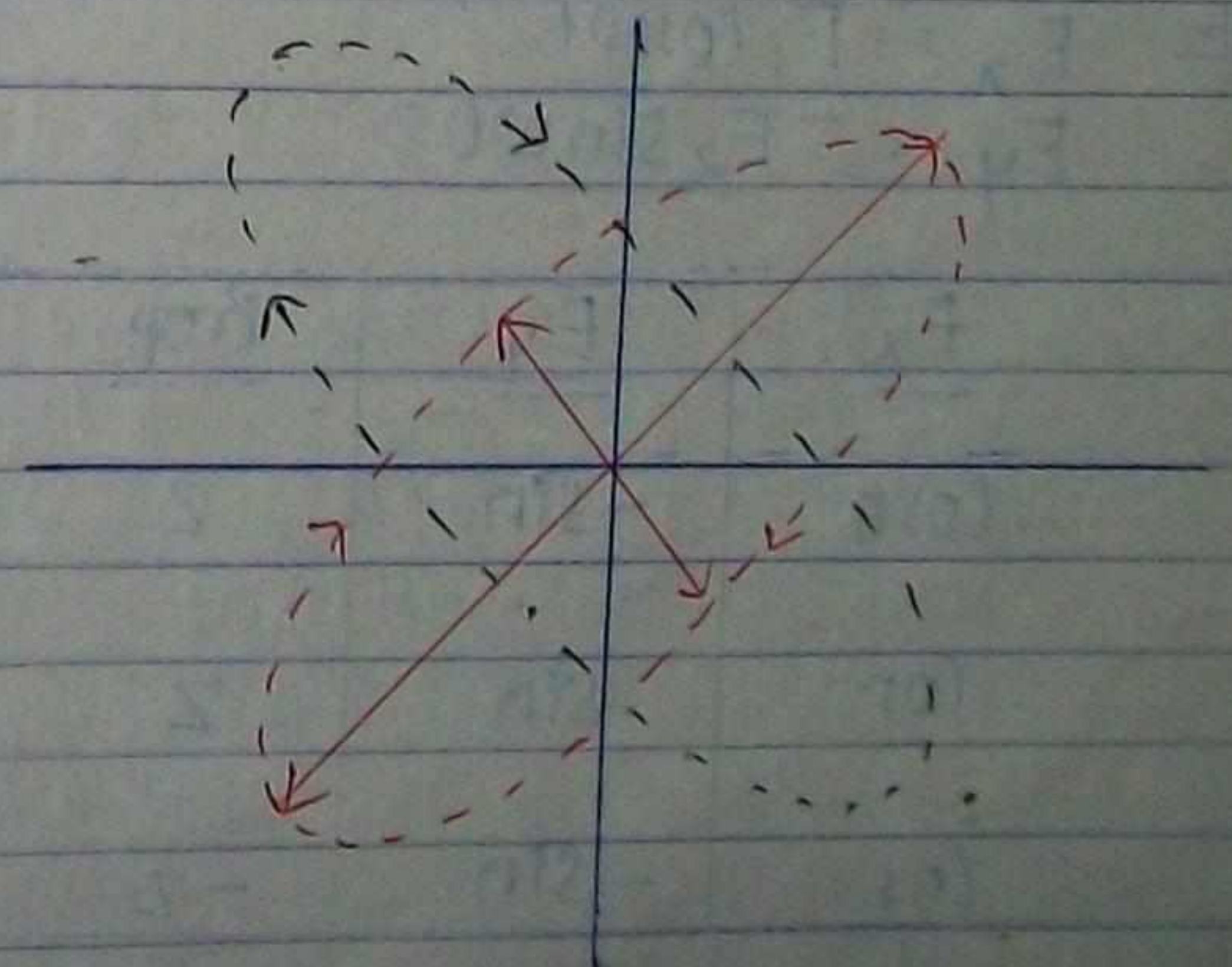
$$\left(\frac{x}{E_{x_0}}\right)^2 + \left(\frac{y}{E_{y_0}}\right)^2 = 1$$



ELLIPTICAL POLARIZATION

CASE (6): E-field is in x & y direction -

$$E(z, t) = E_{x_0} \sin(\omega t - \beta z) a_x + E_{y_0} \sin(\omega t - \beta z + \theta) a_y$$



ALSO ELLIPTICAL POLARIZATION

INCLINED AT θ°

1/10/11

SUMMARY OF ELLIPTICAL POLARIZATION -

If the E field planar components have unequal amplitude and any phase difference the wave is elliptically polarised.

In general all EM waves are elliptically polarised except linear and circular polarization as,

$$\text{Axial Ratio of an ellipse} = \frac{\text{Major axis}}{\text{minor axis}}$$

$$AR = \left[\begin{array}{cc} 1 & \infty \\ \swarrow & \searrow \\ \text{Circular} & \text{Linear} \end{array} \right]$$

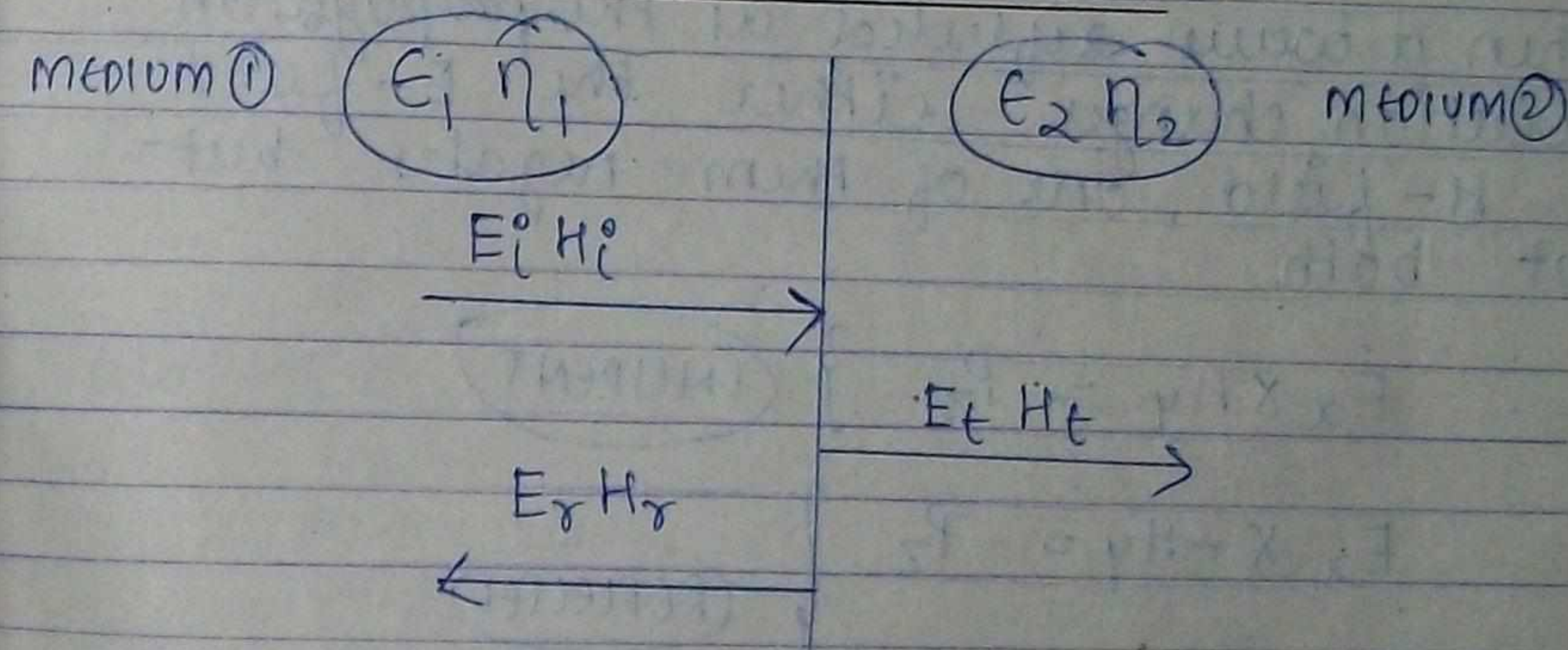
$E_x = E_1 \cos \omega t$
 $E_y = \pm E_2 \sin \omega t$

E_x	E_y	Prop.	Polarization
$\cos \theta$	$-\sin$	z	LEFT
$\cos$	$\sin$	z	RIGHT
$\cos$	$-\sin$	$-z$	RIGHT
$-\sin$	$\cos$	$+z$	RIGHT

⇒ REFLECTION AND TRANSMISSION OF UNIFORM PLANE WAVE (EM-WAVE) -

(A) NORMAL INCIDENCE -

(1) DIELECTRIC - DIELECTRIC MEDIUM -



(1) $E_t > E_i$ (or) $E_t < E_i$

that depends on ϵ_1 and ϵ_2

(2) $H_t > H_i$ (or) $H_t < H_i$

$\downarrow$ (1b) $\downarrow$ (1b)
 $E_t < E_i$ $E_t > E_i$

(3) $P_t < P_i$ always.

$$E_i^o = \eta_1 H_i^o$$

$$E_t = \eta_2 H_t$$

$$E_r = -\eta_1 H_r$$

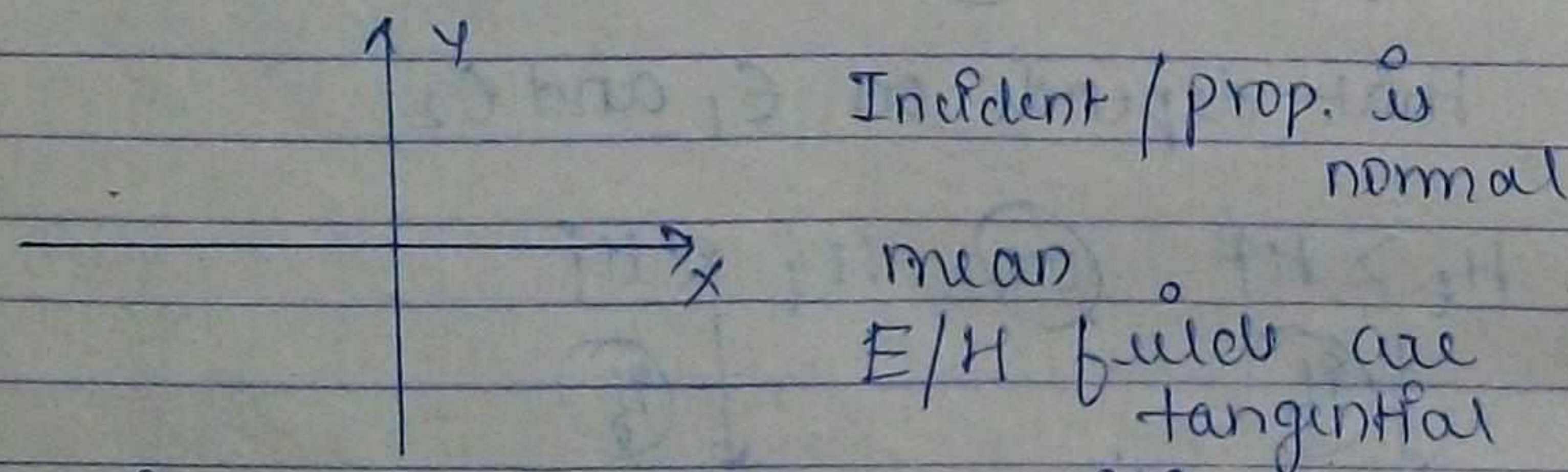
- When a wave is reflected at the propagation direction changes either the E-field or H-field, one of them negates but not both

$$\underline{\text{Ex}} - E_x \times H_y = P_z \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{INCIDENT}$$

$$E_x \times -H_y = -P_z \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{REFLECTED}$$

$$-E_x \times H_y = -P_z$$

#



Applying E/H boundary conditions.

$$H_{t1} = H_{t2}$$

$$H_i^o + H_r = H_t$$

$$E_{t1} = E_{t2}$$

$$E_i^o + E_r = E_t$$

from,

$$E_i^o + E_r = E_t$$

$$1 + \frac{E_r}{E_i} = \frac{E_t}{E_i}$$

$\frac{E_r}{E_i}$ = It is a measure of the reflected E field to the total incident E field.

$$\therefore \frac{E_r}{E_i} = \Gamma \quad \text{for E field}$$

REFLECTION COEFFICIENT

$$\text{also } \frac{E_t}{E_i} = \tau \quad \text{for E field}$$

TRANSMISSION COEFFICIENT

$$1 + \Gamma = \tau$$

-(1)

from magnetic boundary conditions

$$H_i^o + H_r = H_t$$

$$\frac{E_i^o}{\eta_1} - \frac{E_r}{\eta_1} = \frac{E_t}{\eta_2} \times \frac{\eta_1}{\eta_2}$$

$$\frac{E_i^o}{\eta_1} - \frac{E_r}{\eta_1} = \frac{E_t}{\eta_2}$$

$$E_i^o - E_r = \frac{\eta_1}{\eta_2} E_t$$

$$1 - \Gamma = \frac{\eta_1}{\eta_2} \tau \quad - (2)$$

from (1) and (2)

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \Gamma_E$$

for dielectrics $\eta_1 = \sqrt{\frac{\mu_0}{\epsilon_1}}$ $\eta_2 = \sqrt{\frac{\mu_0}{\epsilon_2}}$

$$\Gamma = \frac{\sqrt{\epsilon_1} - \sqrt{\epsilon_2}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}} = \Gamma_E$$

NOTE -

$$(1) \quad \Gamma_H = -\Gamma_E = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2}$$

$$(2) \quad \Gamma_P = \Gamma_E \cdot \Gamma_H = -\Gamma_E^2 = -\Gamma_H^2 \quad (\text{Power})$$

$$(3) \quad 1 + \Gamma = \tau \quad (\text{always})$$

$$(4) \quad \text{If } \epsilon_1 > \epsilon_2 \quad \Gamma_E = +ve$$

$$\tau_E > 1$$

$$\Gamma_H = -ve$$

$$\tau_H < 1$$

$$\tau_P < 1 \quad (\text{always})$$



Ques 45 P-23
SOLNS

$$\Gamma_E = \frac{\sqrt{1} - \sqrt{4}}{\sqrt{1} + \sqrt{4}} = -\frac{1}{3} \quad (\text{reflected})$$

$$\Gamma_H = \frac{1}{3}$$

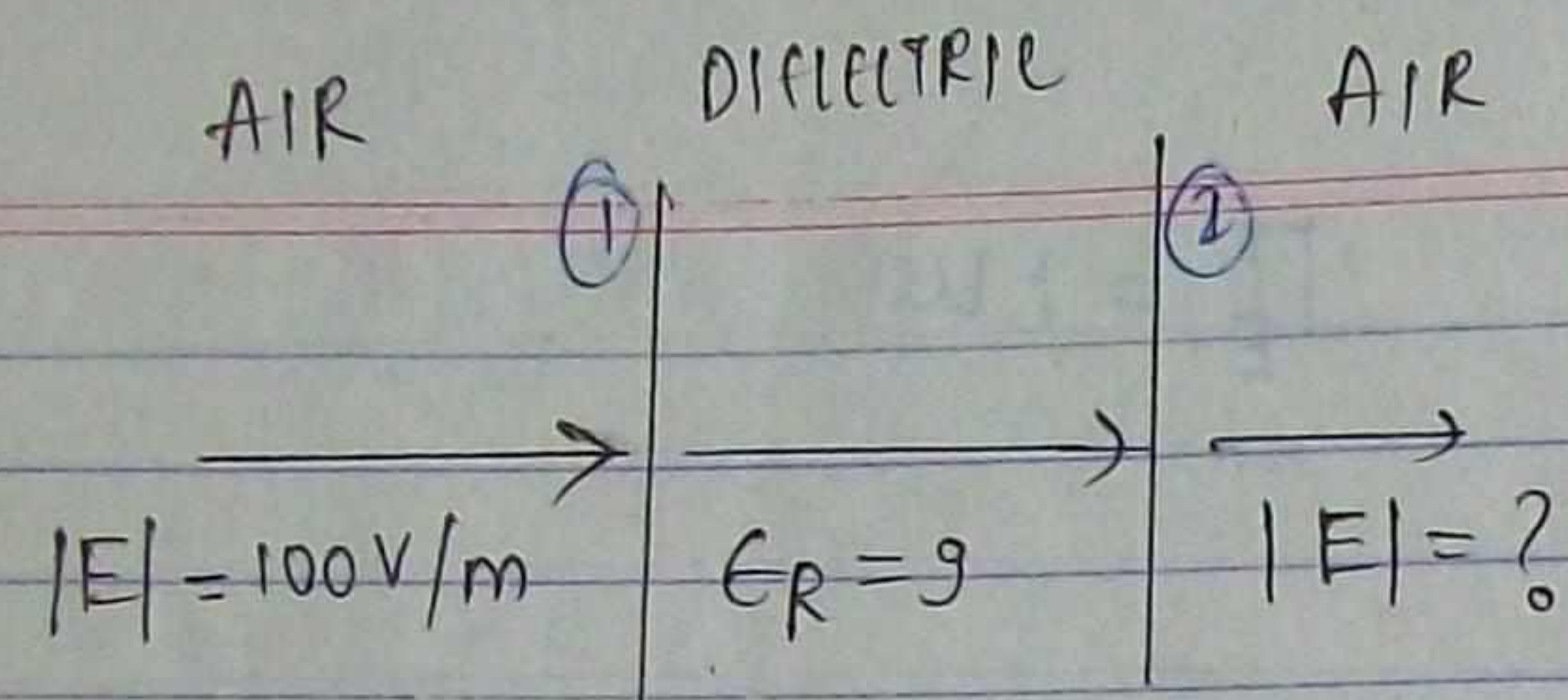
$$\Gamma_P = \frac{1}{3} \times -\frac{1}{3} = -\frac{1}{9}$$

$$1 + \Gamma_P = \tau_P = 1 - \frac{1}{9} = \frac{8}{9}$$

$$\tau_P = \frac{8}{9}$$



Ques. In the diagram show below find the electric field strength in the air outside the dielectric.



Solⁿ: $\Gamma_E = \frac{\sqrt{1} - \sqrt{9}}{\sqrt{1} + \sqrt{9}} = \frac{-2}{4} = -\frac{1}{2}$

Boundary ①:

$\Gamma_{E_i} = -\frac{1}{2}$ $\Gamma_H = \frac{1}{2}$

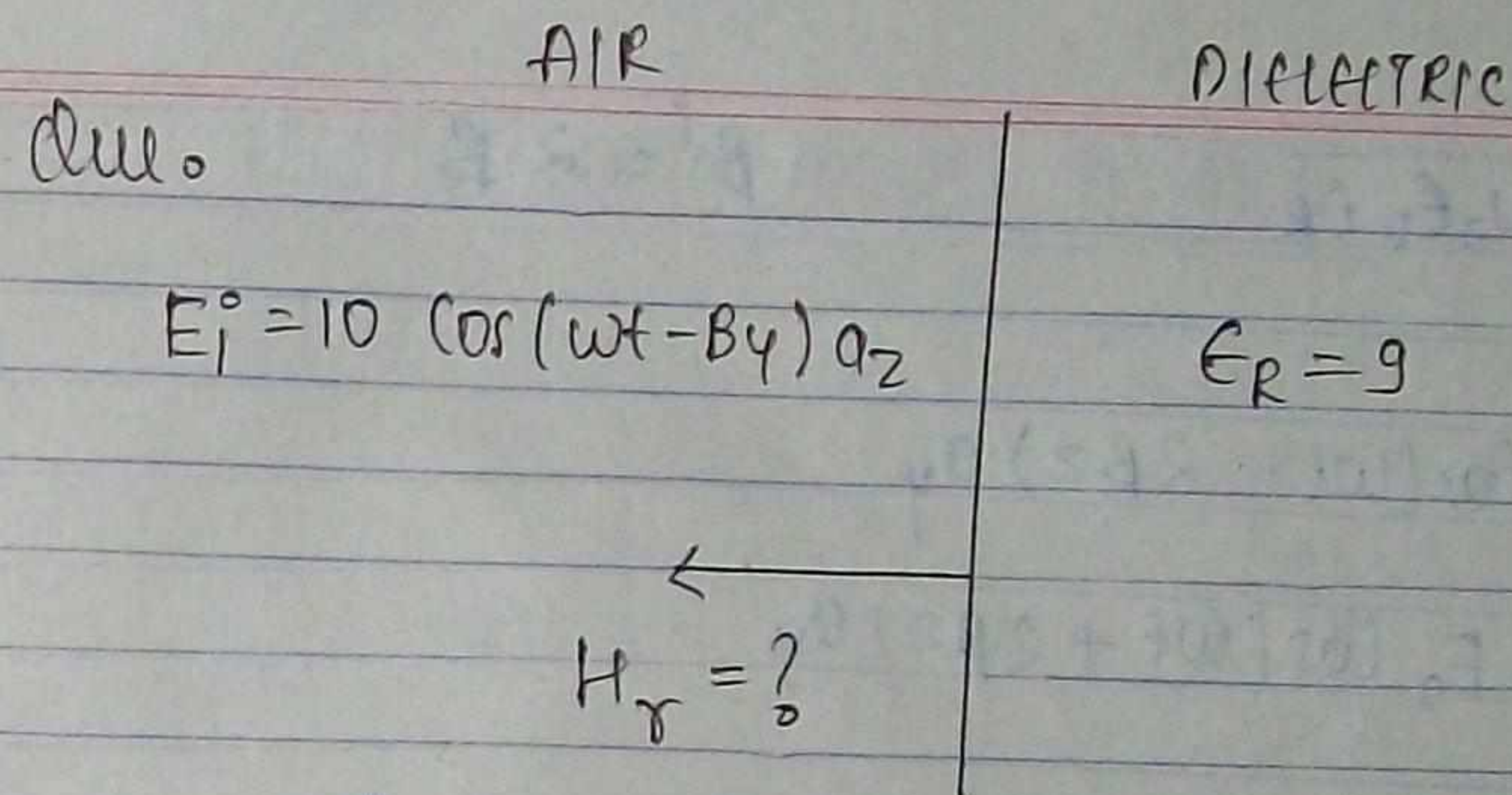
transmitted $E_{\text{field}} = \tau_E = 1 - \frac{1}{2} = \frac{1}{2}$
 $= \tau_E = \frac{1}{2} = \frac{E_t}{E_i}$

$E_t = \frac{1}{2} \times E_i = \frac{100}{2} = 50 \text{ V/m}$

for boundary ②: become E_i for boundary ②.

$\Gamma_E = \frac{\sqrt{9} - \sqrt{1}}{\sqrt{9} + \sqrt{1}} = \frac{1}{2}$

$\tau_E = 1 + \frac{1}{2} = \frac{3}{2}$ $E_t = \frac{3}{2} \times 50 = 75 \text{ V/m}$



Solⁿ: ①st method:

$E_i \xrightarrow{\Gamma_E} E_r \xrightarrow{-\eta_1} H_r$

②nd method:

$E_i \xrightarrow{\eta_1} H_i \xrightarrow{\Gamma_H} H_r$

Ques. 46 P-24.

Solⁿ:

$E_i = E_0 \cos(wt - \beta z) a_y$

$\Gamma_E = \frac{1 - \sqrt{4}}{1 + \sqrt{4}} = -\frac{1}{3}$

$\tau_E = 1 - \frac{1}{3} = \frac{2}{3} = \frac{E_t}{E_i}$

$\alpha, \beta, \gamma, \eta, \lambda, v_p \rightarrow$ material property $\rightarrow$ Change from medium to medium

ω (frequency) $\rightarrow$ source property $\rightarrow$ Independent of material

$$\beta = \omega \sqrt{\mu_0 \epsilon_0 \epsilon_r}$$

$$\beta' = 2\beta$$

$$\therefore E_t = \frac{2}{3} E_0 \cos(\omega t - 2\beta z) a_y$$

$$\text{also } E_r = \frac{2}{3} E_0 \cos(\omega t + 2\beta z) a_y$$

Ques 47 P-24.

Soln:

$$\frac{E_t}{E_r} = -2 = \frac{E_t}{E_i \cdot \frac{E_r}{E_i}} = \frac{\Gamma}{1}$$

$$-2 = \frac{1 + \Gamma}{\Gamma} \quad \therefore \Gamma = -1/3$$

$$\Gamma = \frac{\sqrt{\epsilon_1} - \sqrt{\epsilon_2}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}} = \frac{1 - \sqrt{\epsilon_2/\epsilon_1}}{1 + \sqrt{\epsilon_2/\epsilon_1}} = -\frac{1}{2}$$

$$\frac{\epsilon_2}{\epsilon_1} = 4$$

$\Rightarrow$ REFLECTION AT CONDUCTOR SURFACES -

$$\Gamma_E = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$\eta_1 = 120\pi = 377 \Omega$$

$$\eta_2 = \sqrt{\frac{j\omega\mu}{\sigma}} \approx 0$$

$$\Gamma_E \approx -1$$

$$\Gamma_H \approx +1$$

$$\tau_E = 1 - 1 = 0 \quad \text{ie } E\text{-field does not exist inside a conductor.}$$

EM wave completely reflected back from conductor but don't propagate into conductor.

$$\rightarrow E_i + E_r = E_t = 0 \quad \therefore E\text{-field is minimum}$$

$$\rightarrow H\text{-field is maximum.}$$

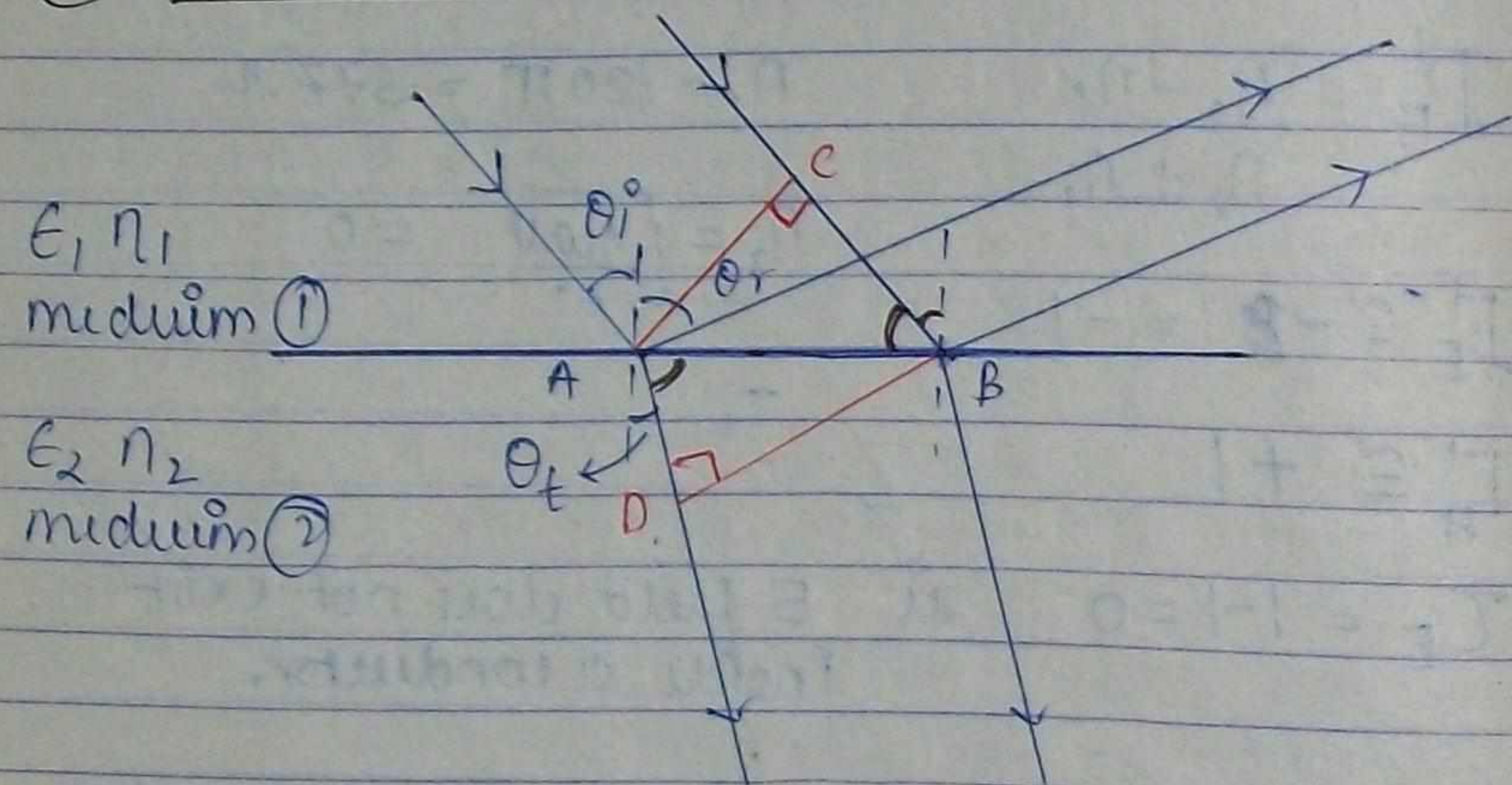
Ques 48 P-24

Soln:

$$\eta = E \quad \eta = \sqrt{\frac{\mu}{\epsilon}} \approx 1$$

$$\Gamma_E = \frac{\eta_2 - 377}{\eta_2 + 377} \approx -1$$

(B) OBLIQUE INCIDENCE -



CB - Extra distance travelled by the wave in medium ①.

AD - extra distance travelled by the wave in medium ②.

A/C are in phase, B/D are in phase.

$$\frac{CB}{AD} = \frac{v_1}{v_2} = \text{Refractive Index of med ② wrt med ①}$$

$$\frac{AB \cos(90 - \theta_i)}{AB \cos(90 - \theta_t)} = \frac{v_1}{v_2}$$

$$\frac{\sin \theta_i}{\sin \theta_t} = \frac{1}{\sqrt{\mu_0 \epsilon}} \times \sqrt{\mu_0 \epsilon_2}$$

$$\frac{\sin \theta_i}{\sin \theta_t} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

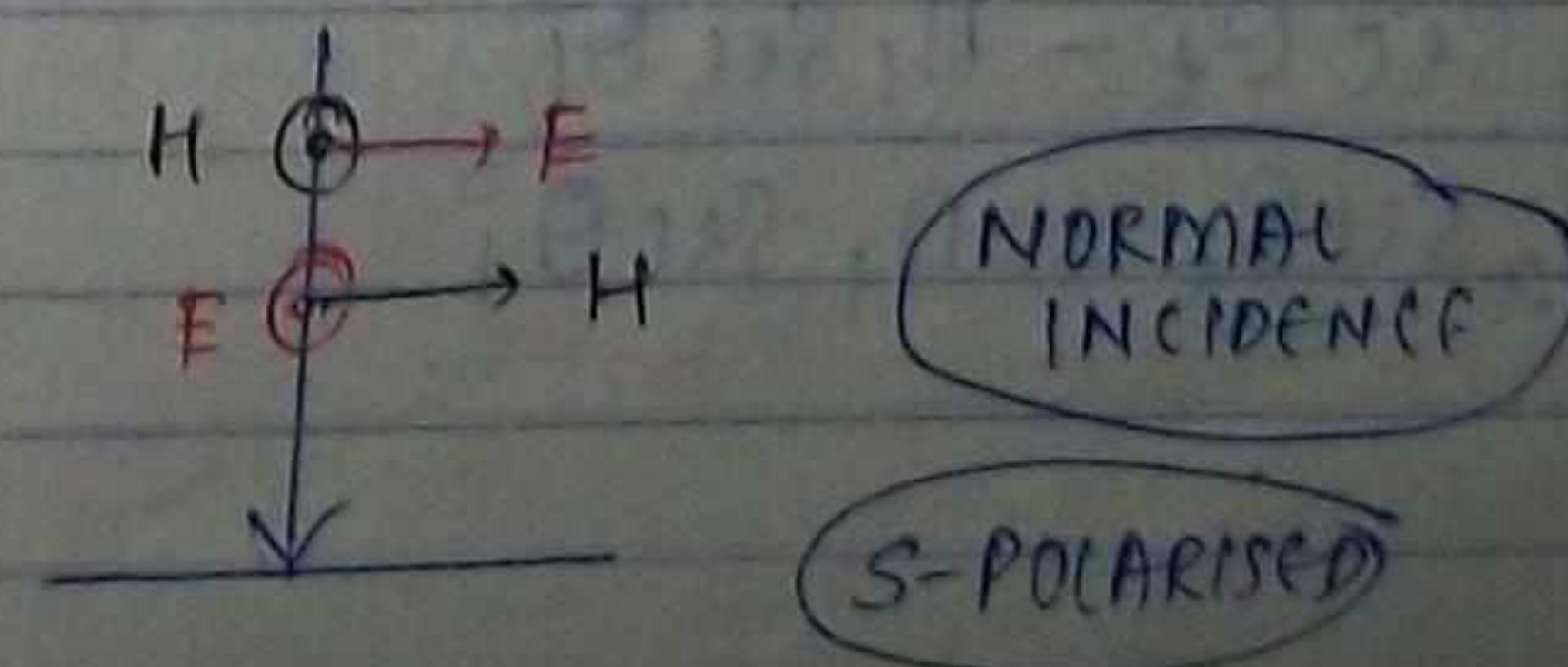
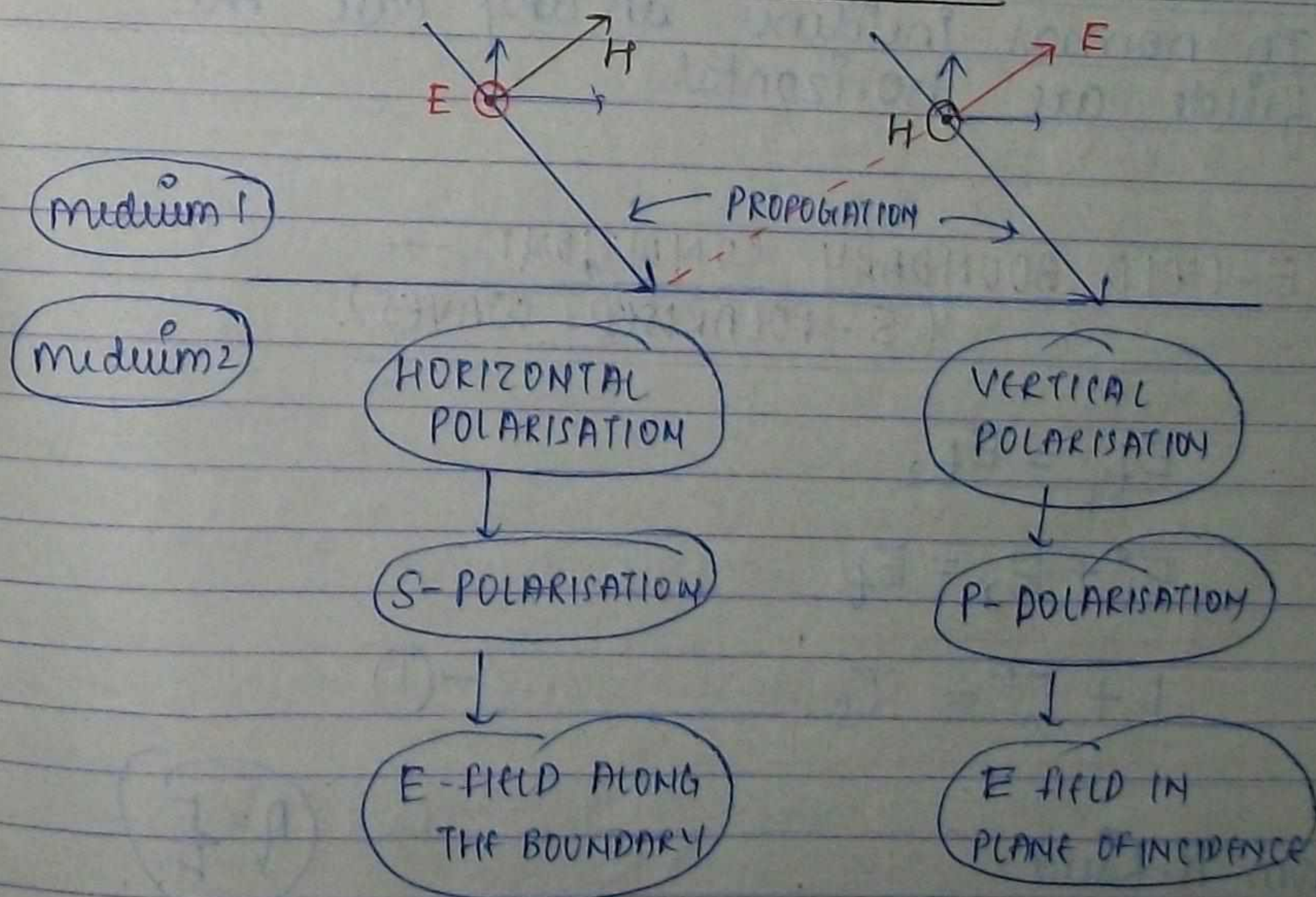
SNELL'S II LAW

also,

$$\theta_i = \theta_r$$

SNELL'S I LAW

⇒ SURFACE & PLANE POLARIZATION -



① In oblique incidence the E-field may be along the surface or horizontally polarised or surface polarised. If it is not along the surface, it is in the plane of incidence and is vertically polarised but it is physically inclined with the surface.

② In normal incidence in any case the fields are horizontal

⇒ E-FIELD BOUNDARY CONDITIONS —
(S-POLARISED WAVES).

$$E_{t1} = E_{t2}$$

$$E_i + E_r = E_t$$

$$1 + \Gamma_s = T_s$$

— (1)

from H-field

$$\Gamma_s = \frac{n_2 \sec \theta_t - n_1 \sec \theta_i}{n_2 \sec \theta_t + n_1 \sec \theta_i}$$

$$\eta = \frac{E}{H}$$

⇒ E-FIELD BOUNDARY CONDITION —
(P-POLARISED WAVES)

$$E_{t1} = E_{t2}$$

$$E_i \cos \theta_i + E_r \cos \theta_r = E_t \cos \theta_t$$

$$1 + \Gamma_p = T_p \frac{\cos \theta_t}{\cos \theta_i}$$

$$\eta = E/H$$

$$\Gamma_p = \frac{n_2 \cos \theta_t - n_1 \cos \theta_i}{n_2 \cos \theta_t + n_1 \cos \theta_i}$$

from H field

$$\# \quad \Gamma_s \neq \Gamma_p$$

Γ_s is never equal to Γ_p .

except in normal incidence where

$$\Gamma_s = \Gamma_p = \frac{n_2 - n_1}{n_2 + n_1}$$

when $\theta_i = \theta_r = \theta_t = 0^\circ$

for dielectrics;

$$\eta = \sqrt{\frac{\mu_0}{\epsilon}}$$

$$\frac{\sin \theta_i}{\sin \theta_t} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

$$\sin \theta_t = \sqrt{\frac{\epsilon_1}{\epsilon_2}} \sin \theta_i$$

$$\cos \theta_t = \sqrt{1 - \frac{\epsilon_1}{\epsilon_2} \sin^2 \theta_i}$$

$$\Gamma_s = \frac{\eta_2 \sec \theta_t - \eta_1 \sec \theta_i}{\eta_2 \sec \theta_t + \eta_1 \sec \theta_i}$$

$$\Gamma_s = \frac{\sqrt{\epsilon_1} \cos \theta_i - \sqrt{\epsilon_2} \cos \theta_t}{\sqrt{\epsilon_1} \cos \theta_i + \sqrt{\epsilon_2} \cos \theta_t}$$

$$= \frac{\sqrt{\epsilon_1} \cos \theta_i - \sqrt{\epsilon_2} \left(\sqrt{1 - \frac{\epsilon_1}{\epsilon_2} \sin^2 \theta_i} \right)}{\sqrt{\epsilon_1} \cos \theta_i + \sqrt{\epsilon_2} \left(\sqrt{1 - \frac{\epsilon_1}{\epsilon_2} \sin^2 \theta_i} \right)}$$

$$\Gamma_s = \frac{\cos \theta_i - \sqrt{\frac{\epsilon_2}{\epsilon_1} - \sin^2 \theta_i}}{\cos \theta_i + \sqrt{\frac{\epsilon_2}{\epsilon_1} - \sin^2 \theta_i}}$$

Similarly,

$$\Gamma_p = \frac{\frac{\epsilon_2}{\epsilon_1} \cos \theta_i - \sqrt{\frac{\epsilon_2}{\epsilon_1} - \sin^2 \theta_i}}{\frac{\epsilon_2}{\epsilon_1} \cos \theta_i + \sqrt{\frac{\epsilon_2}{\epsilon_1} - \sin^2 \theta_i}}$$

Consider $\Gamma_p = 0$

$$\frac{\epsilon_2}{\epsilon_1} \cos \theta_i = \sqrt{\frac{\epsilon_2}{\epsilon_1} - \sin^2 \theta_i}$$

$$\left(\frac{\epsilon_2}{\epsilon_1}\right)^2 \cos^2 \theta_i = \frac{\epsilon_2}{\epsilon_1} - \sin^2 \theta_i$$

$$\left(\frac{\epsilon_2}{\epsilon_1}\right)^2 (1 - \sin^2 \theta_i) = \frac{\epsilon_2}{\epsilon_1} - \sin^2 \theta_i$$

$$\sin^2 \theta_i \left[1 - \left(\frac{\epsilon_2}{\epsilon_1}\right)^2 \right] = \frac{\epsilon_2}{\epsilon_1} - \left(\frac{\epsilon_2}{\epsilon_1}\right)^2$$

$$\sin^2 \theta_i = \frac{\frac{\epsilon_2}{\epsilon_1} \left(1 - \frac{\epsilon_2}{\epsilon_1} \right)}{\left(1 + \frac{\epsilon_2}{\epsilon_1} \right) \left(1 - \frac{\epsilon_2}{\epsilon_1} \right)}$$

$$\sin^2 \theta_i = \frac{\epsilon_2}{\epsilon_1 + \epsilon_2}, \quad \sin \theta_i = \frac{\sqrt{\epsilon_2}}{\sqrt{\epsilon_1 + \epsilon_2}}$$

$$\text{ii } \sin \theta_i = \frac{\sqrt{\epsilon_2}}{\sqrt{\epsilon_1 + \epsilon_2}}$$

$$\text{also, } \tan \theta_i = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

$\Gamma_P = 0$ means zero reflection & complete transmission into the second medium are possible for a P-polarised wave at a single angle called Brewster angle θ_B .

$$\tan \theta_B = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

BREWSTER'S ANGLE FOR P-POLARISED WAVE

for S-polarised waves

$$\text{consider } \Gamma_S = 0$$

$$\cos^2 \theta_i = \frac{\epsilon_2}{\epsilon_1} - \sin^2 \theta_i$$

$$\text{ii } \epsilon_1 = \epsilon_2 \quad (\text{means same medium})$$

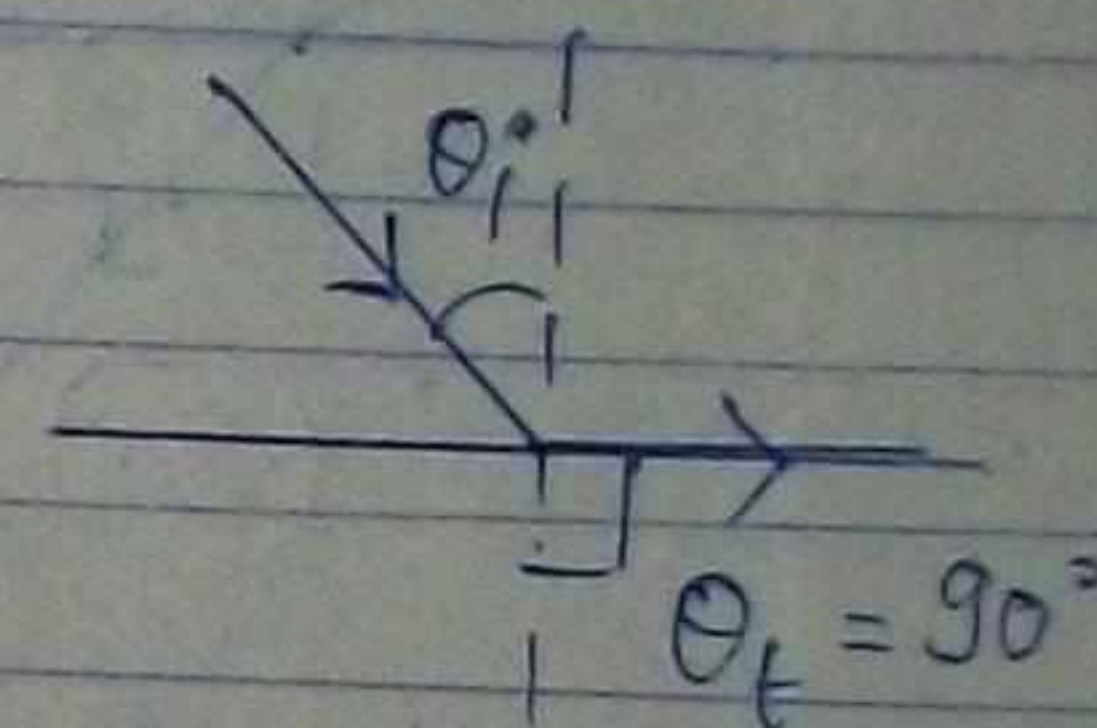
Brewster angle is undefined or $\Gamma_S = 0$ is impossible for S-polarised wave between two different medium.

⇒ CRITICAL ANGLE & TOTAL INTERNAL REFLECTION -

$$\text{if } \theta_t = 90^\circ \text{ \& } \theta_i = \theta_c$$

$$\frac{\sin \theta_c}{\sin 90} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

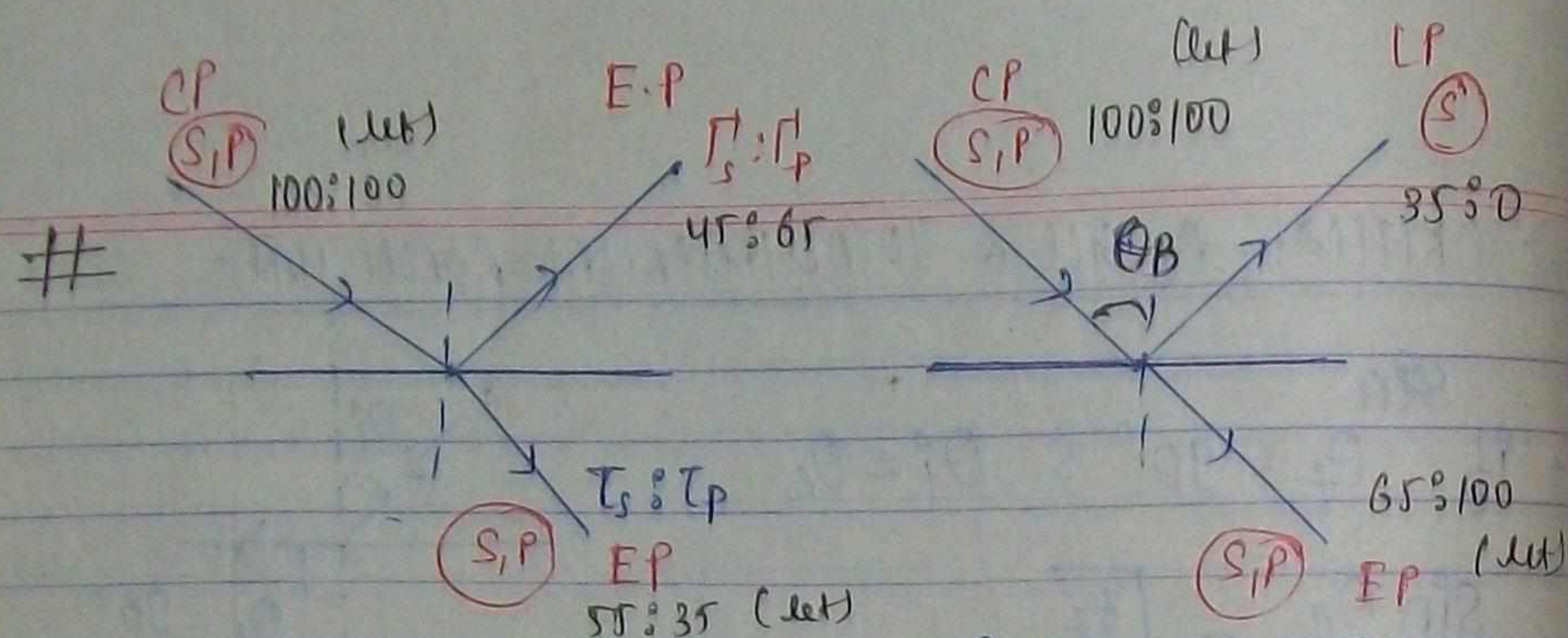
$$\sin \theta_c = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$



- when $\theta_i > \theta_c$ the wave is reflected back into the medium.

ii zero transmission and complete reflection,

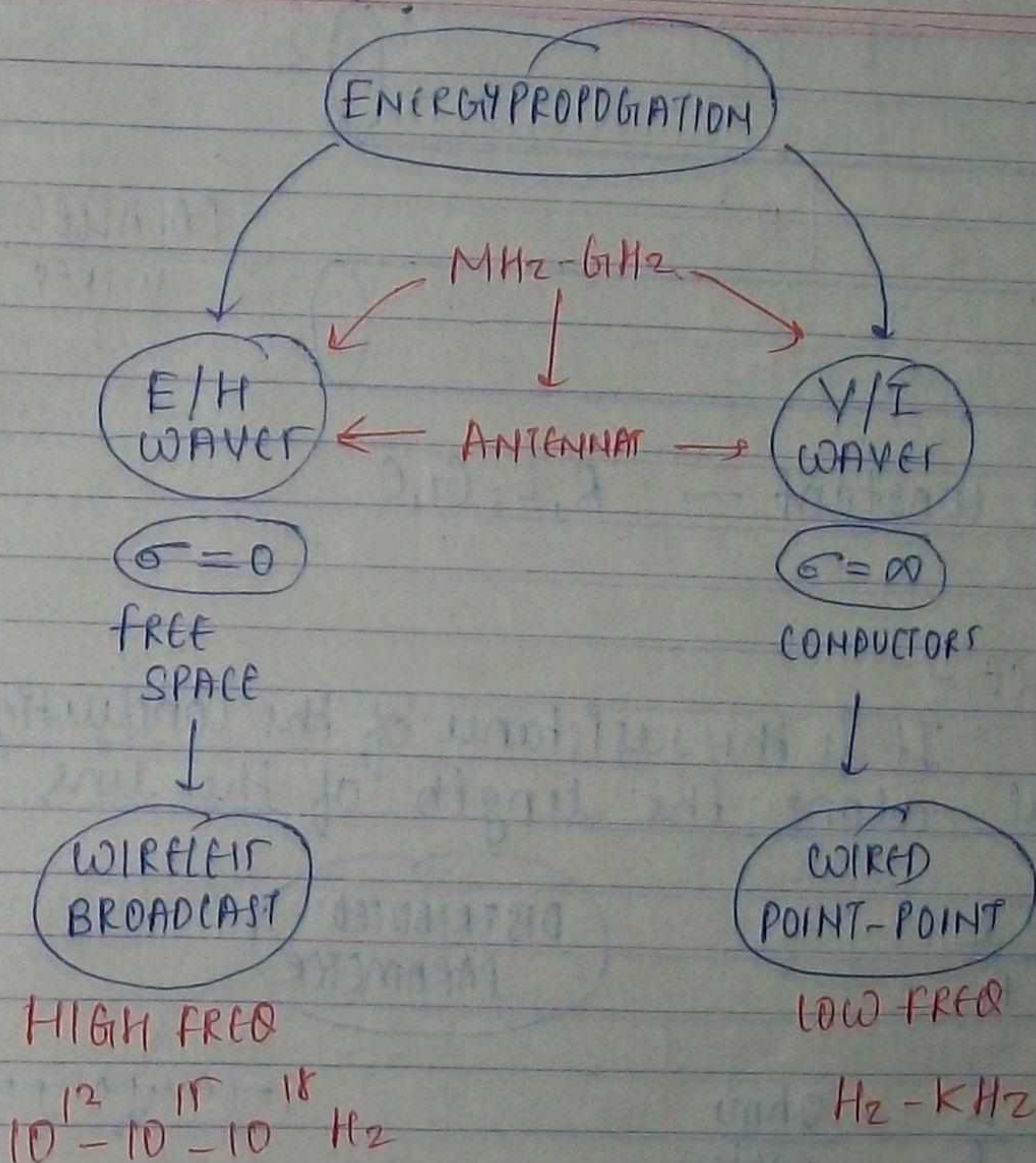
#	θ_B - BREWSTER ANGLE ZERO REFLECTION COMPLETE TRANSMIT	θ_c - CRITICAL ANGLE ZERO TRANSMISSION COMPLETE REFLECTION
①	$\tan \theta_B = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$	① $\sin \theta_c = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$
②	Only for P-polarised wave	② Polarization independent
③	Any two media can have θ_B .	③ $\epsilon_1 > \epsilon_2$ is must
④	Exactly occur at single angle $\theta_i = \theta_B$	④ occur for all angle $\theta_i > \theta_c$



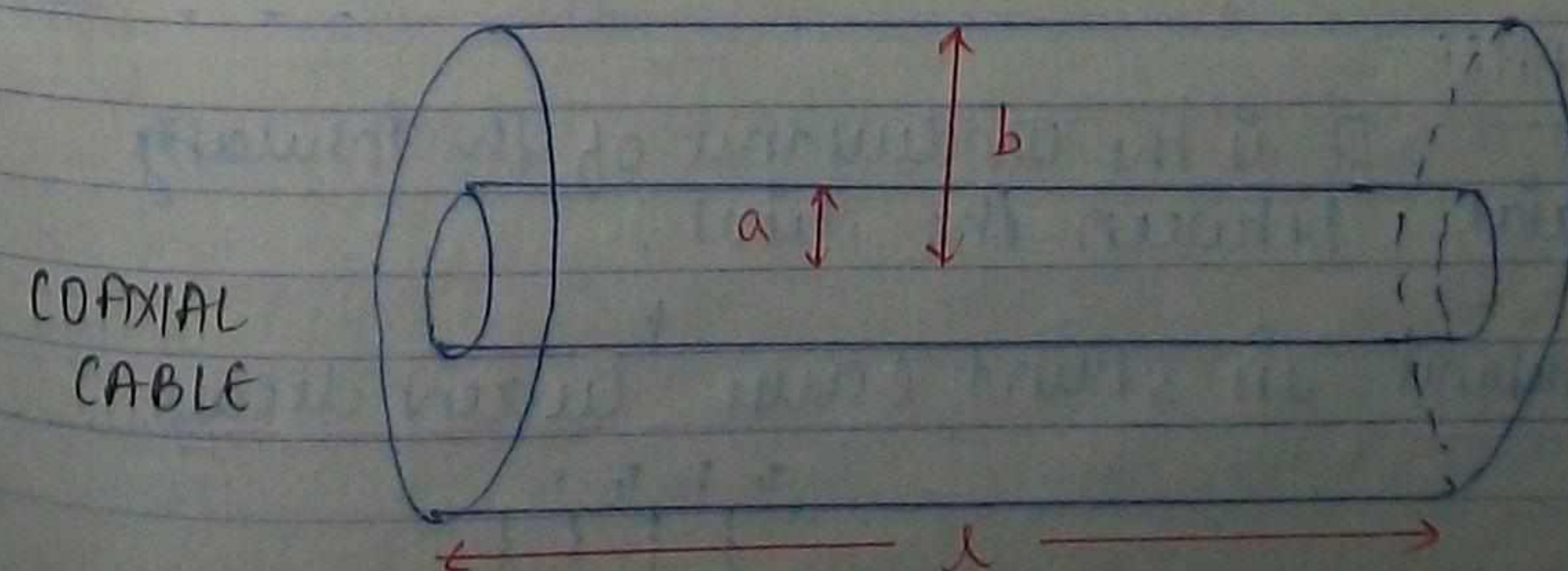
- A generally circular polarised wave is a combination of s and p polarisation of equal amount and when such a wave reflects it is elliptically polarised due to unequal reflection coefficient and the transmitted wave is also elliptically polarised.

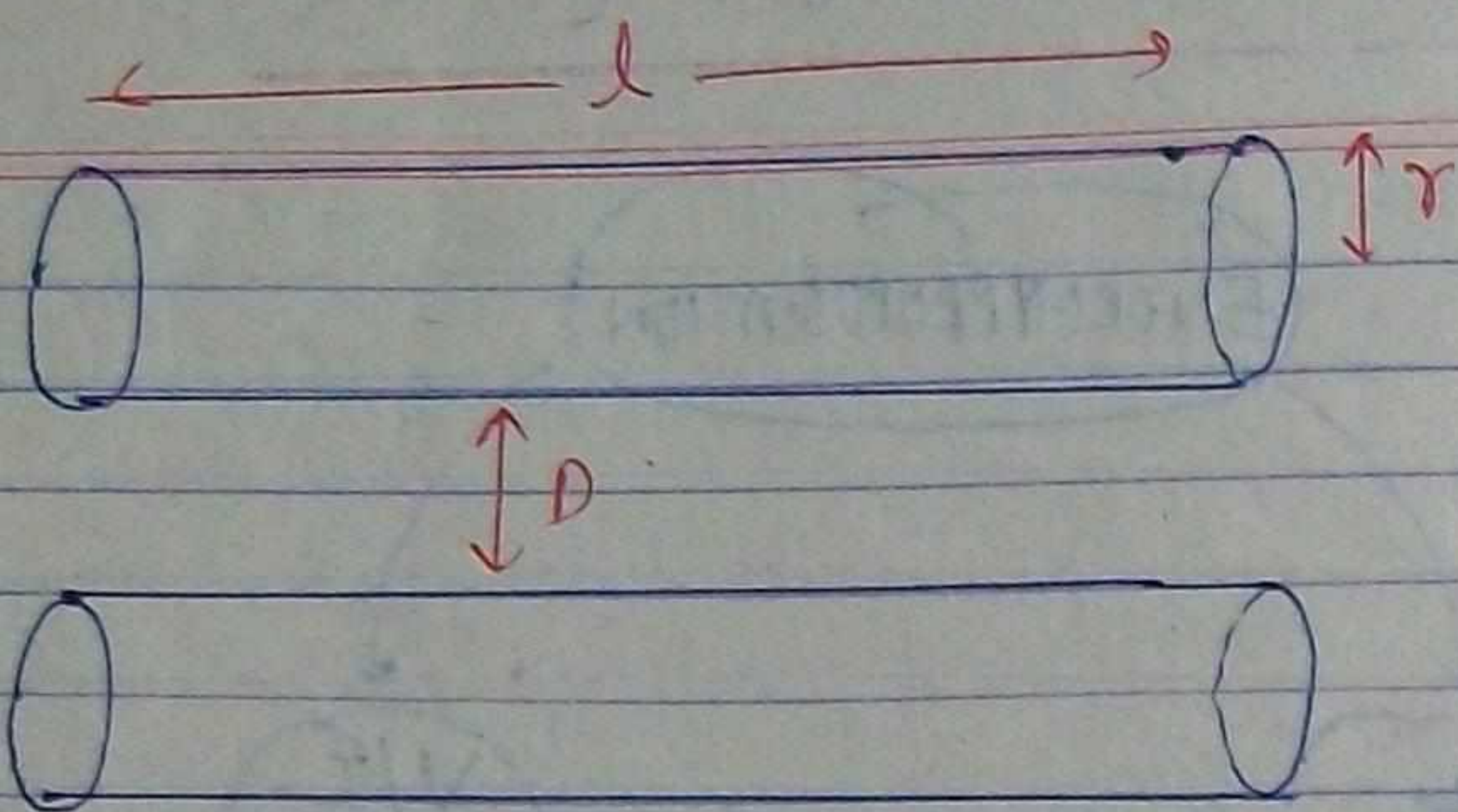
- However at Brewster's angle of incidence the reflected wave is only s-polarised and hence linearly polarised but the transmitted wave is always elliptically polarised.

TRANSMISSION LINES



⇒ PRIMARY CONSTANT OF A TX. LINE -





PARALLEL WIRES

primary constant $\rightarrow R, L, G, C$

RESISTANCE -

It is the resistance of the conducting material along the length of the line.

$$R = \frac{\rho l}{A}$$

DISTRIBUTED PARAMETER

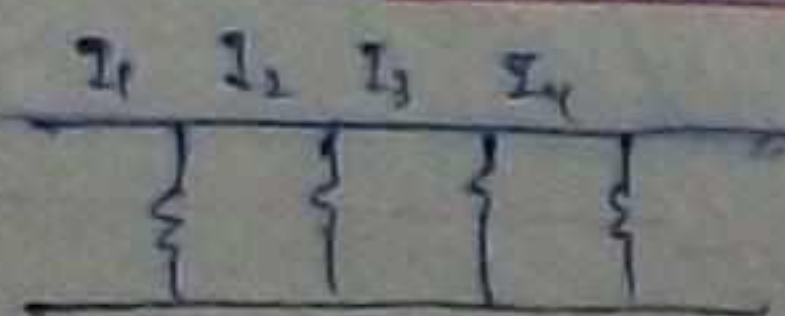
$$\therefore R = \frac{R}{l} = \frac{\text{ohm}}{\text{meter}}$$

- Resistance in series cause voltage decay!

CONDUCTANCE -

It is the conductance of the insulating material between the lines.

- Conductance in shunt cause current decay.



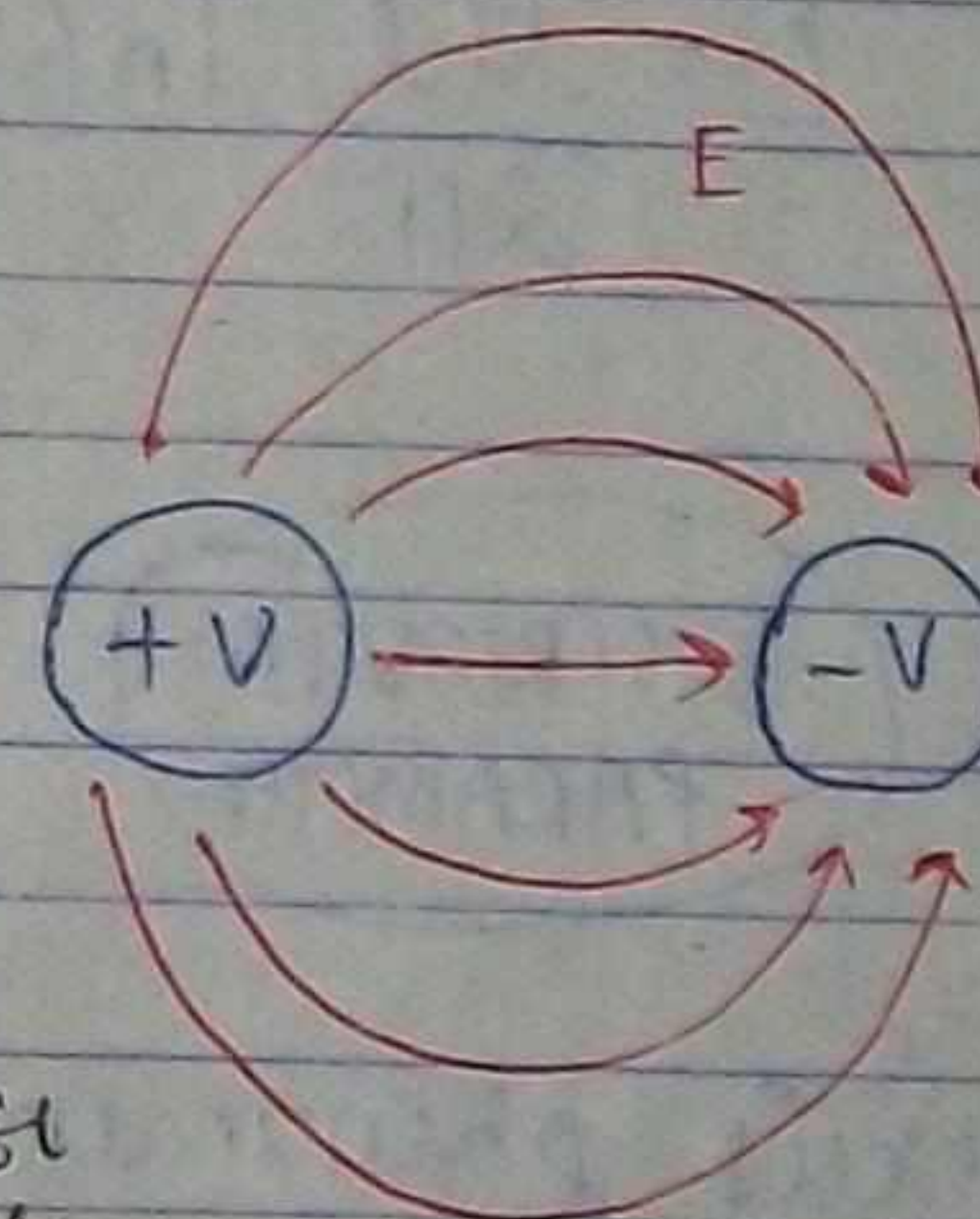
$$(CAUSE) \propto \begin{matrix} R & VLE & L \\ \swarrow & \searrow & \swarrow \\ G & ILE & C \end{matrix} \quad B (CAUSE)$$

$$G = \frac{G}{l} = \frac{\text{mho}}{\text{meter}}$$

DISTRIBUTED PARAMETER

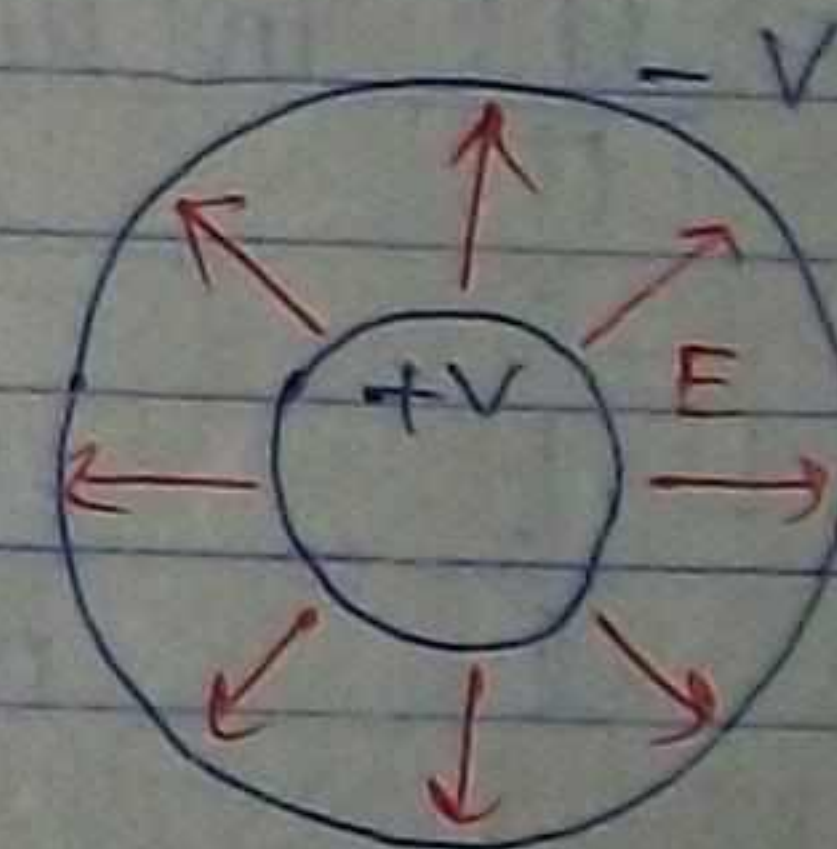
CAPACITANCE -

It is due to the voltage on the line there is a surrounded E-field and hence capacitance.



PARALLEL WIRE

$$C = \frac{2\pi\epsilon l}{\ln(D/r)}$$



COAXIAL CABLE

$$C = \frac{2\pi\epsilon l}{\ln(b/a)}$$

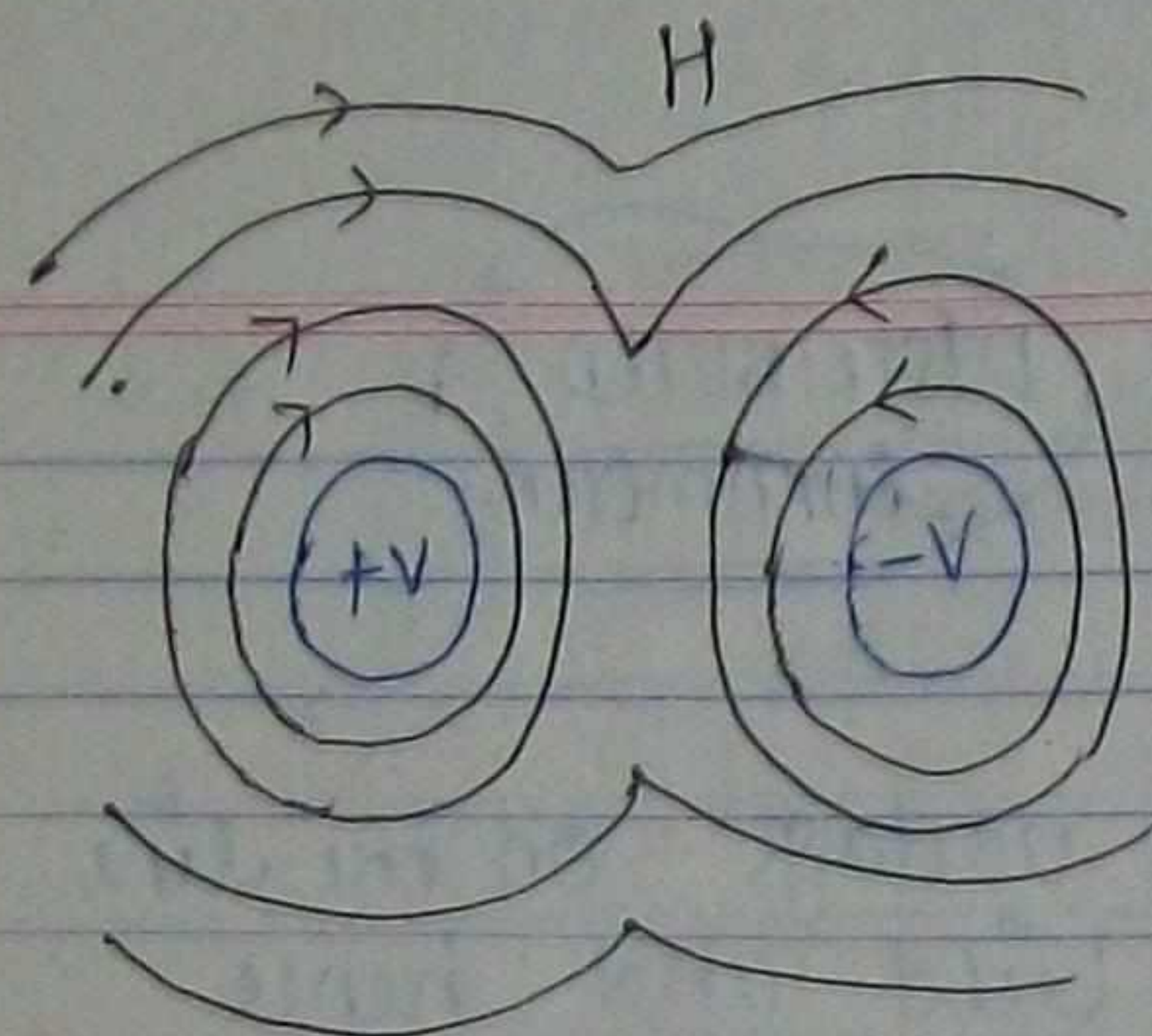
DISTRIBUTED PARAMETER

$$C = \frac{C}{l} = \frac{\text{farad}}{\text{meter}}$$

Capacitance always occurs in shunt.

INDUCTANCE -

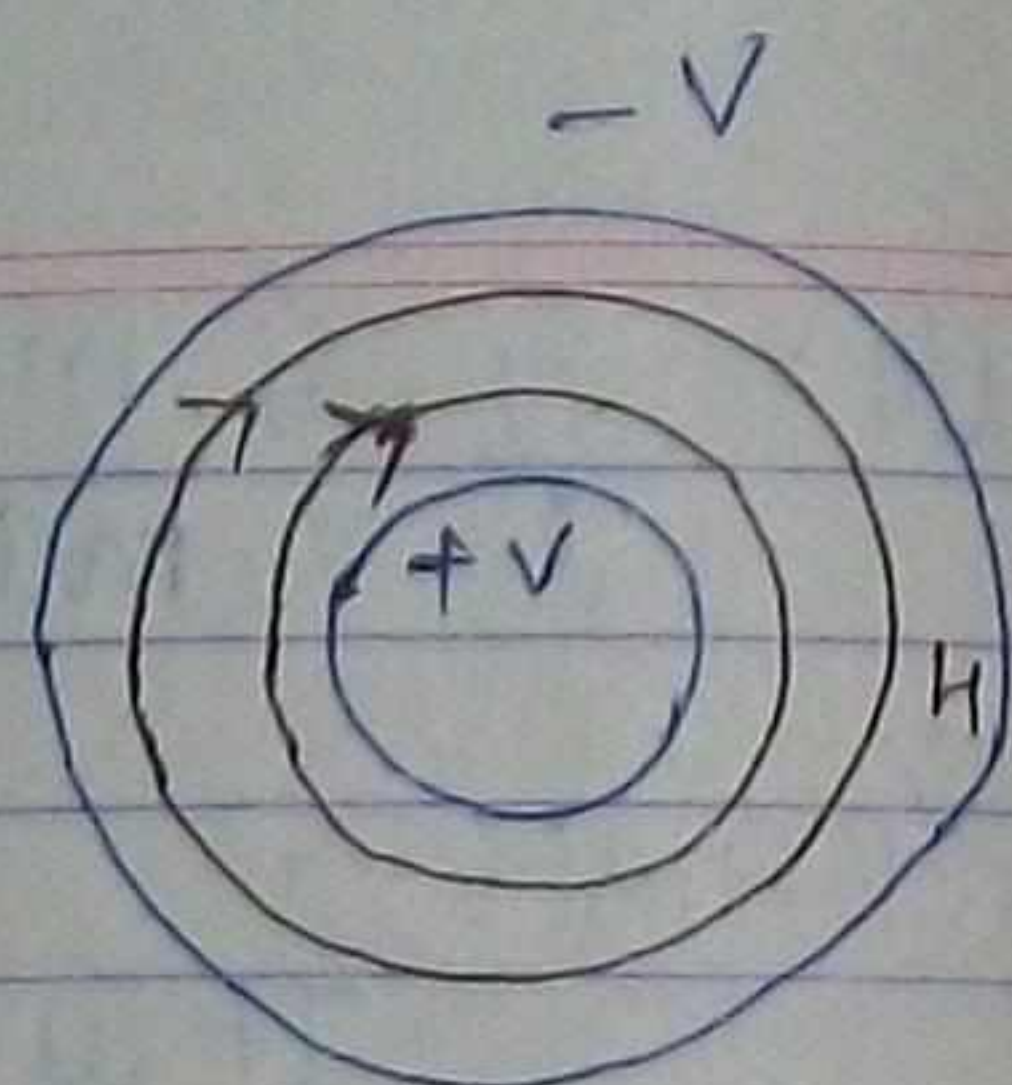
It is due to the current on the line there is a surrounded H-field and hence inductance.



PARALLEL WIRE

$$L = \frac{\mu l}{\pi} \ln(D/r)$$

$$L = \frac{L}{l} = \frac{\text{Henry}}{\text{meter}}$$



COAXIAL CABLE

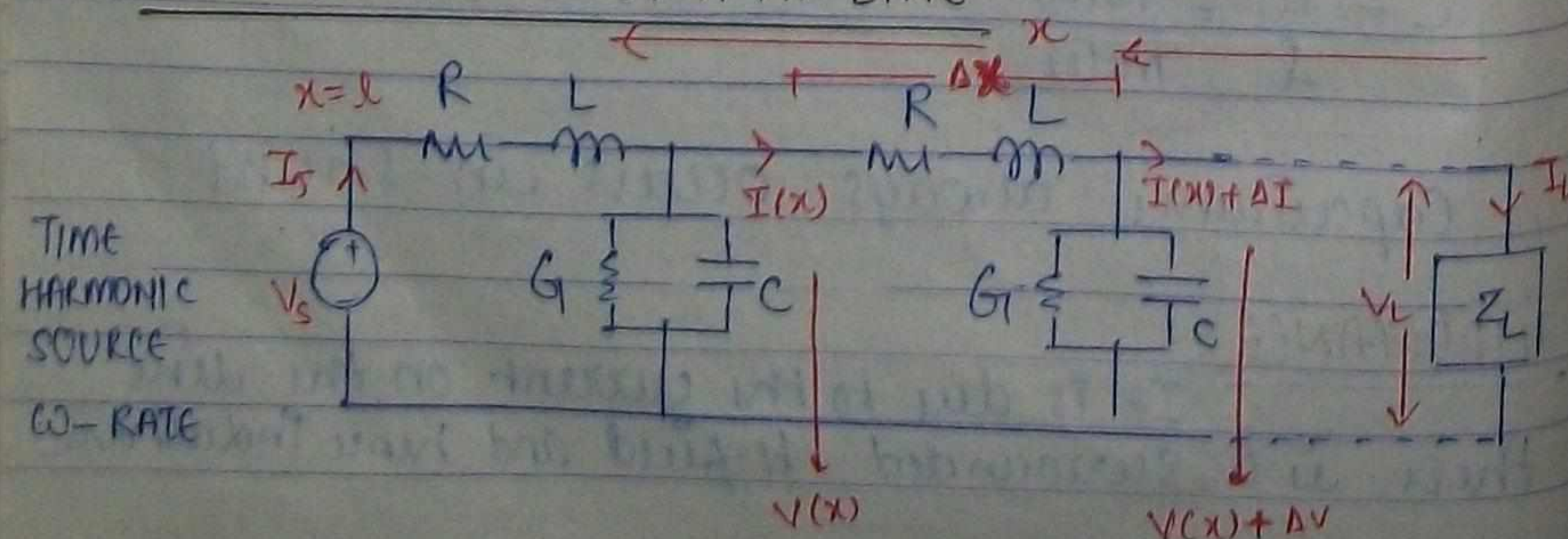
$$L = \frac{\mu l}{2\pi} \ln(b/a)$$

DISTRIBUTED
PARAMETER

- Inductance is always a series phenomena.

$$\# \quad LC = \mu\epsilon$$

⇒ VI EQUATIONS OF A TX. LINE -



Series impedance of the line = $Z_{\text{series}} = (R + j\omega L) \Delta x$ for Δx length

Shunt admittance of line = $Z_{\text{shunt}} = Y_{\text{shunt}} = (G + j\omega C) \Delta x$ for Δx length

$$\Delta V = I \cdot (R + j\omega L) \Delta x$$

$$\frac{\Delta V}{\Delta x} = \frac{dV}{dx} = (R + j\omega L) I \quad - (1)$$

$$\Delta I = V \cdot (G + j\omega C) \Delta x$$

$$\frac{dI}{dx} = V (G + j\omega C) \quad - (2)$$

Take I from (1) and substitute in (2):

$$I = \left(\frac{1}{R + j\omega L} \right) \frac{dV}{dx}$$

$$\frac{d}{dx} \left(\frac{1}{R + j\omega L} \frac{dV}{dx} \right) = V \cdot (G + j\omega C)$$

$$\frac{d^2 V}{dx^2} = (R + j\omega L)(G + j\omega C) V \quad - (3)$$

Similarly,

$$\frac{d^2 I}{dx^2} = (R + j\omega L)(G + j\omega C) I \quad - (4)$$

NOTE -

The equations clearly shows that if the source is time harmonic in one end, V and I are harmonic along the length. Such that

$$(R + j\omega L)(G + j\omega C) = \gamma^2$$

unit of $\gamma \rightarrow$ (per meter)

$$\frac{d^2 V}{dx^2} - \gamma^2 V = 0 \quad - (3)$$

$$\frac{d^2 I}{dx^2} - \gamma^2 I = 0 \quad - (4)$$

Soln of above eqn are:

$$V(x) = C_1 e^{-\gamma x} + C_2 e^{\gamma x} \quad - (5)$$

$$I(x) = C_3 e^{-\gamma x} + C_4 e^{\gamma x} \quad - (6)$$

using initial conditions:

at $x=0$, $V = V_L$, $I = I_L$

$$V_L = C_1 + C_2 \quad - (7)$$

$$I_L = C_3 + C_4 \quad - (8)$$

$$-\frac{dV}{dx} = (R + j\omega L)I$$

$$\frac{d}{dx} (C_1 e^{-\gamma x} + C_2 e^{\gamma x}) = (R + j\omega L)I$$

$$-\gamma C_1 e^{-\gamma x} + \gamma C_2 e^{\gamma x} = (R + j\omega L)I$$

$$-\gamma C_1 + \gamma C_2 = (R + j\omega L)I_L$$

$$C_2 - C_1 = \frac{R + j\omega L}{\sqrt{(R + j\omega L)(G + j\omega C)}} \times I_L$$

$$C_2 - C_1 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \times I_L$$

$$C_2 - C_1 = Z_0 I_L \quad - (9)$$

$$\text{Similarly } C_4 - C_3 = \frac{V_L}{Z_0} \quad - (10)$$

where,

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \text{ Ohm.}$$

from (7), (8), (9), (10):

$$C_1 = \frac{V_L - I_L Z_0}{2}, \quad C_3 = \frac{I_L - V_L / Z_0}{2}$$

$$C_2 = \frac{V_L + I_L Z_0}{2}, \quad C_4 = \frac{I_L + V_L / Z_0}{2}$$

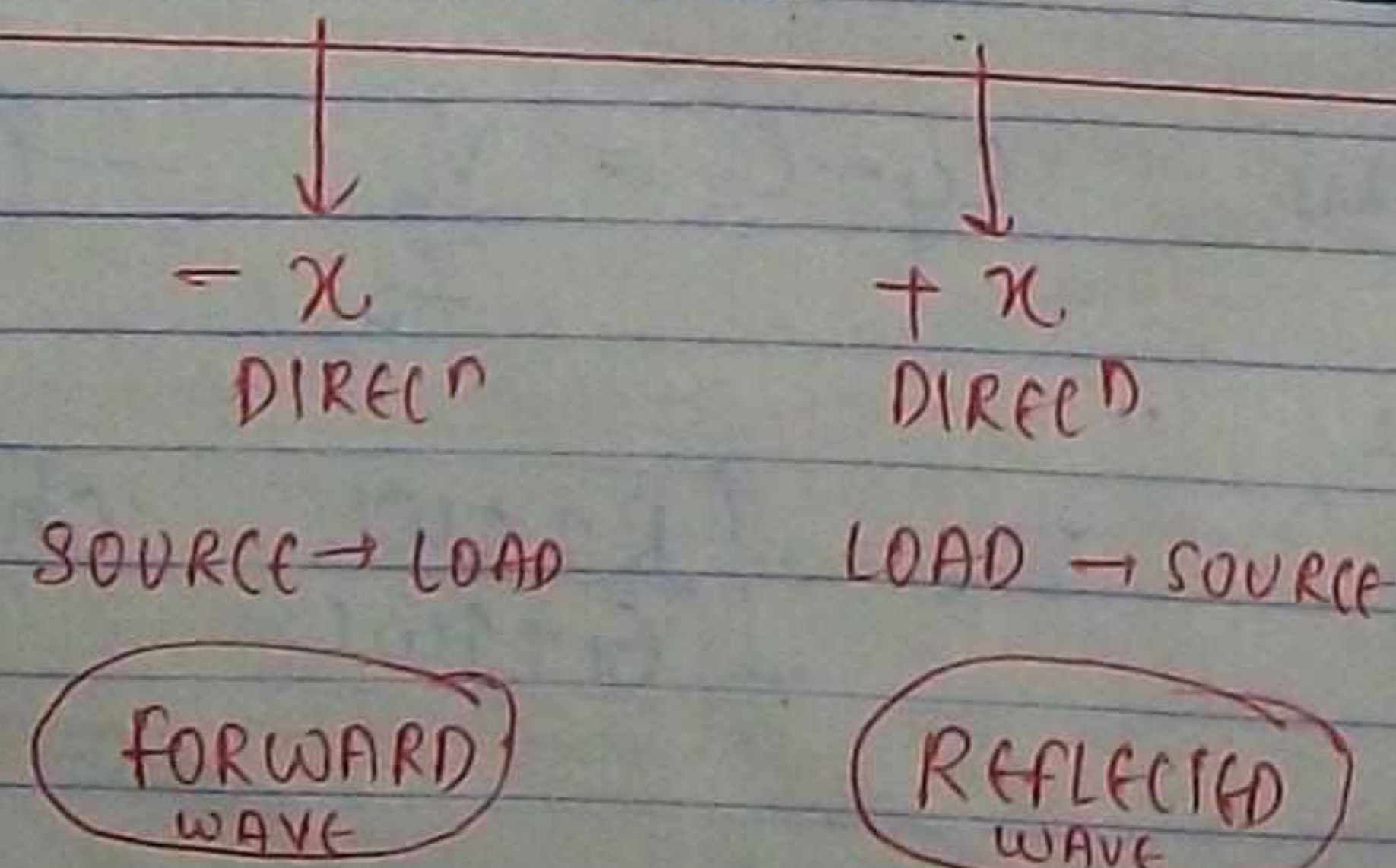
Substituting values in (5) and (6):

$$V(x) = \frac{V_L}{2} \left[\left(1 - \frac{Z_0}{Z_L}\right) e^{-\gamma x} + \left(1 + \frac{Z_0}{Z_L}\right) e^{\gamma x} \right]$$

$$V(x) = \frac{V_L (Z_L + Z_0)}{2 Z_L} \left[e^{\gamma x} + \left(\frac{Z_L - Z_0}{Z_L + Z_0} \right) e^{-\gamma x} \right]$$

Similarly,

$$I(x) = \frac{I_L (Z_L + Z_0)}{2 Z_0} \left[e^{\gamma x} + \left(\frac{Z_0 - Z_L}{Z_0 + Z_L} \right) e^{-\gamma x} \right]$$



NOTE -

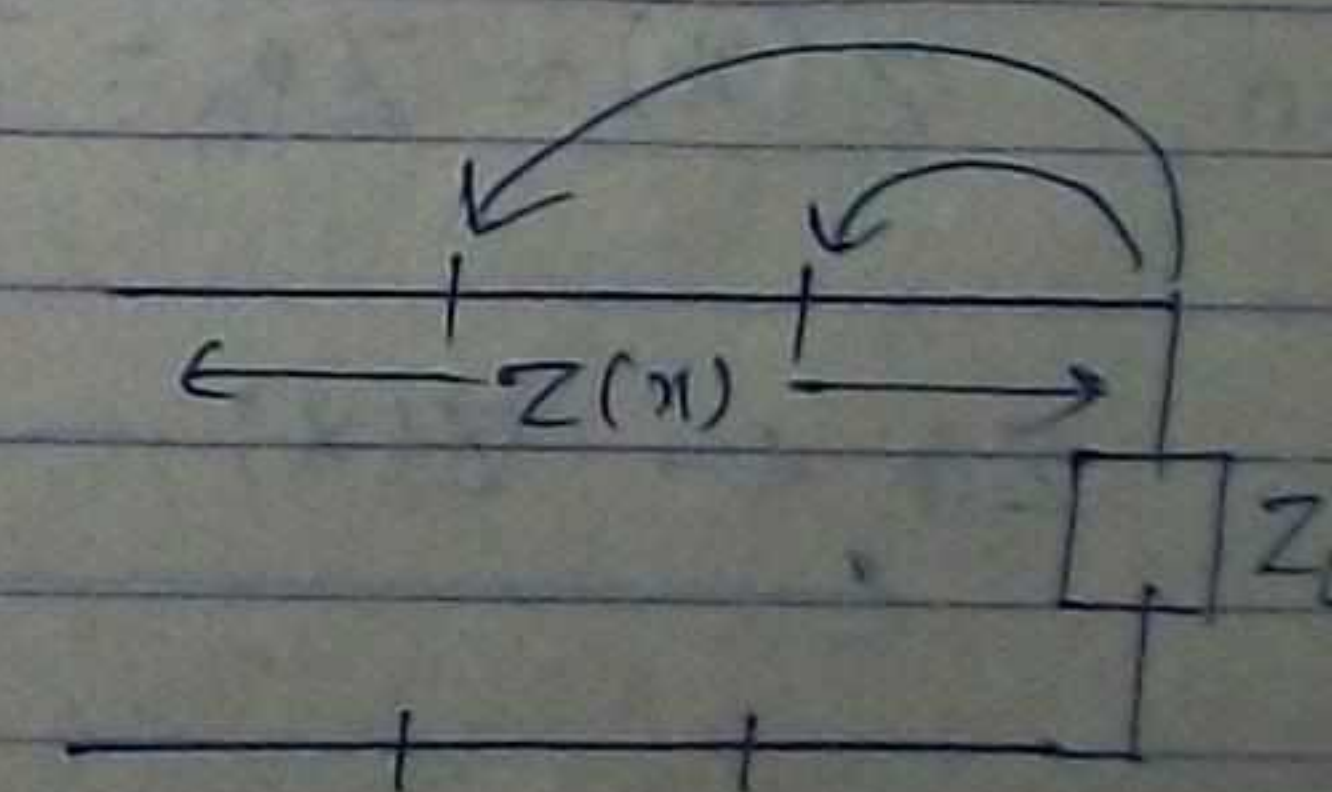
- Every Tx. line has a forward waveform and a reflected wave such that, the reflected wave is due to the unabsorbed power in the load.
- The only condition for the complete power to be absorbed is $Z_L = Z_0$. Hence Z_0 is a unique property of the line with which if the load is terminated the power delivered to the load is maximum. and a line where $Z_L = Z_0$ is called MATCHED LINE.

$\Rightarrow$ IMPEDANCE ON THE LINE & INPUT IMPEDANCE -

- A tx. line having a load at one end & primary constant all along, as a length dependent impedance anywhere on the line which is:

$$Z(x) = \frac{V(x)}{I(x)}$$

$$Z_{IN} = Z(x) \Big|_{x=l}$$



$$V(x) = \frac{V_L (Z_L + Z_0)}{2Z_L} \left[e^{\gamma x} + \left(\frac{Z_L - Z_0}{Z_L + Z_0} \right) e^{-\gamma x} \right]$$

$$V(x) = \frac{V_L}{Z_L} \left[\frac{(Z_L + Z_0)e^{\gamma x}}{2} + \frac{(Z_L - Z_0)e^{-\gamma x}}{2} \right]$$

$$V(x) = V_L \cosh(\gamma x) + I_L Z_0 \sinh(\gamma x)$$

Similarly

$$I(x) = I_L \cosh(\gamma x) + \frac{V_L}{Z_0} \sinh(\gamma x)$$

$$\therefore Z(x) = Z_0 \left[\frac{Z_L \cosh(\gamma x) + Z_0 \sinh(\gamma x)}{Z_0 \cosh(\gamma x) + Z_L \sinh(\gamma x)} \right]$$

$$\text{If } Z_L = Z_0$$

$$\text{Then, } Z(x) = Z_{in} = Z_0 = Z_L$$

$$Z_{in} \leftarrow Z(x) \rightarrow Z_L$$

MISMATCH
LINE

$$Z_0 \leftarrow Z_0 \rightarrow Z_0$$

MATCHED
LINE

- Characteristic impedance is that impedance with which if the line is terminated the impedance anywhere in the line and at the I/P is also the same.

⇒ SHORT CIRCUIT AND OPEN CIRCUIT LINES -

- If $Z_L = 0$ (short circuit line)

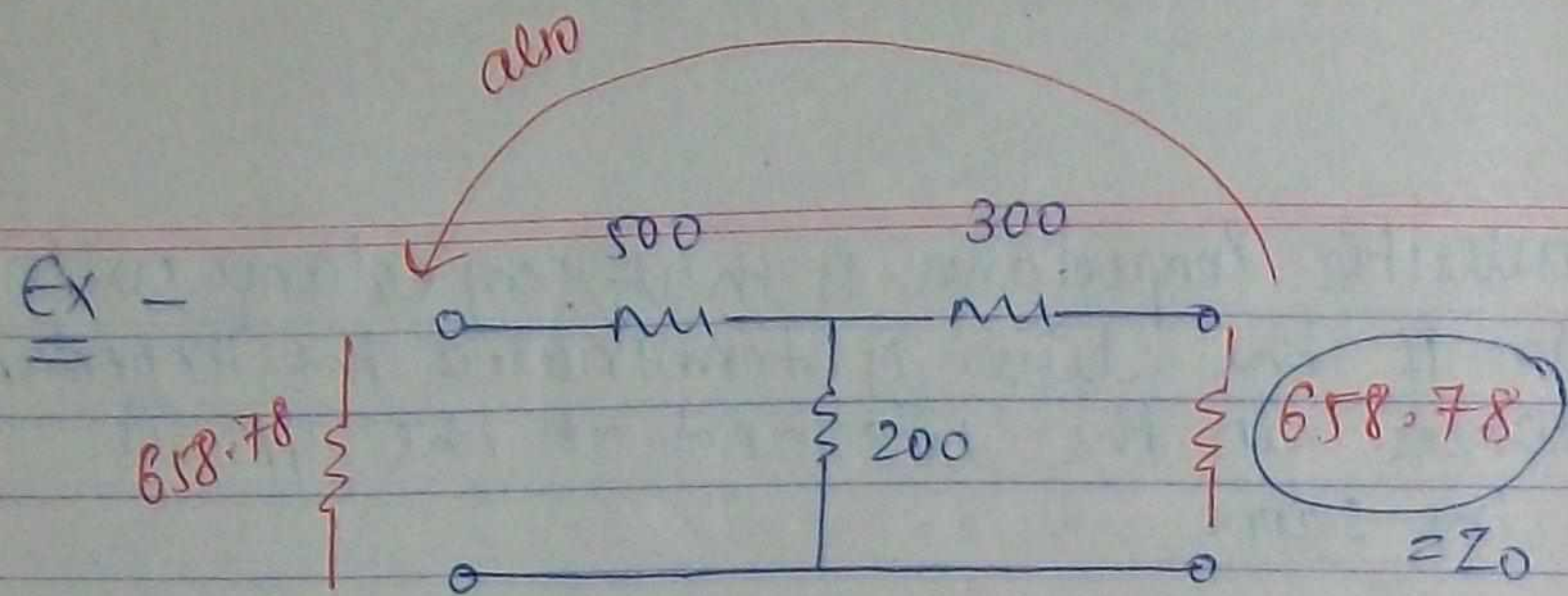
$$Z_{in} = Z_{sc} = Z_0 \tanh(\gamma x)$$

- If $Z_L = \infty$ (open circuit line)

$$Z_{in} = Z_{oc} = Z_0 \coth(\gamma x)$$

$$\therefore Z_0 = \sqrt{Z_{oc} \times Z_{sc}}$$

$Z_0 \rightarrow$ Geometric mean of short circuit and open circuit I/P impedance.



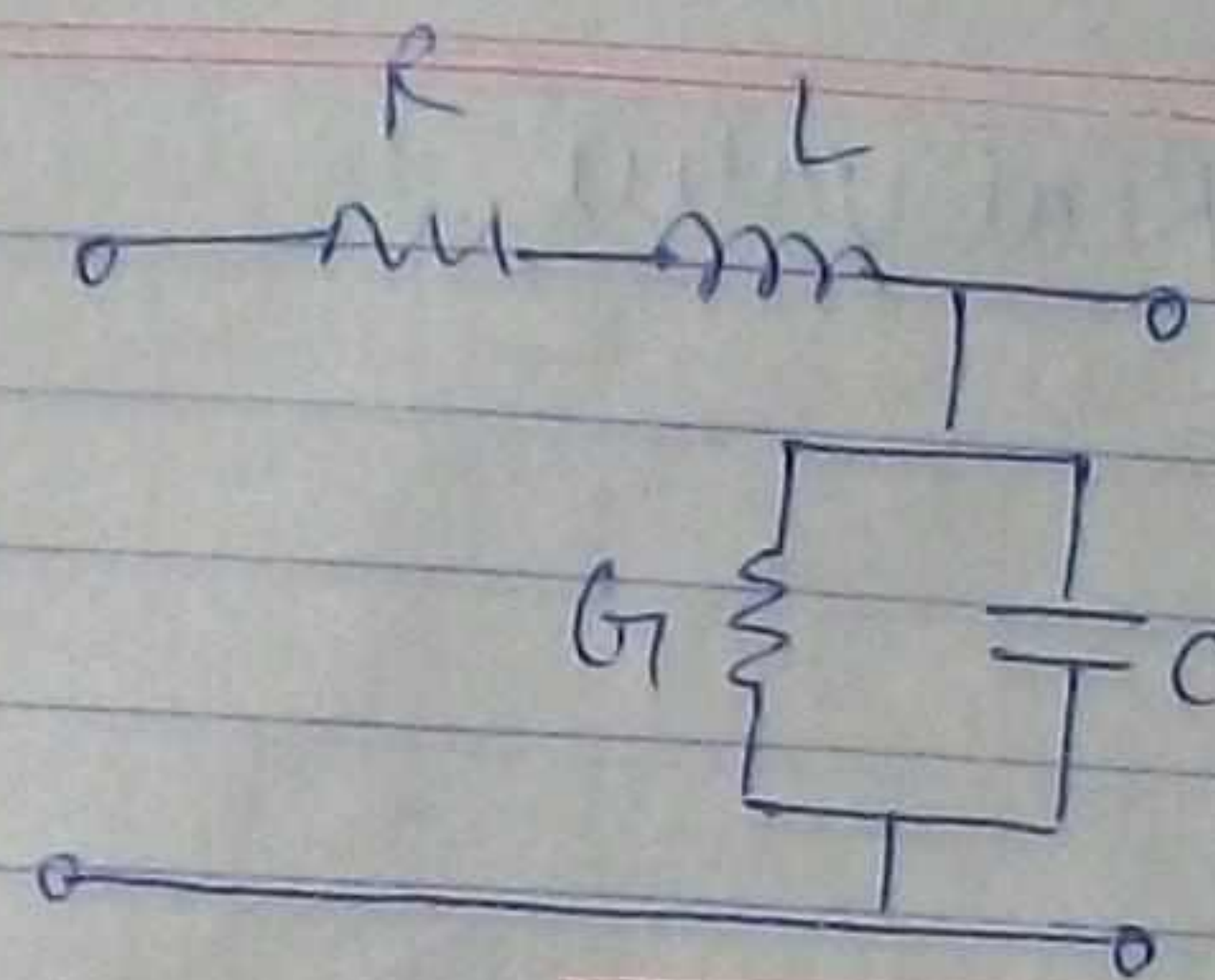
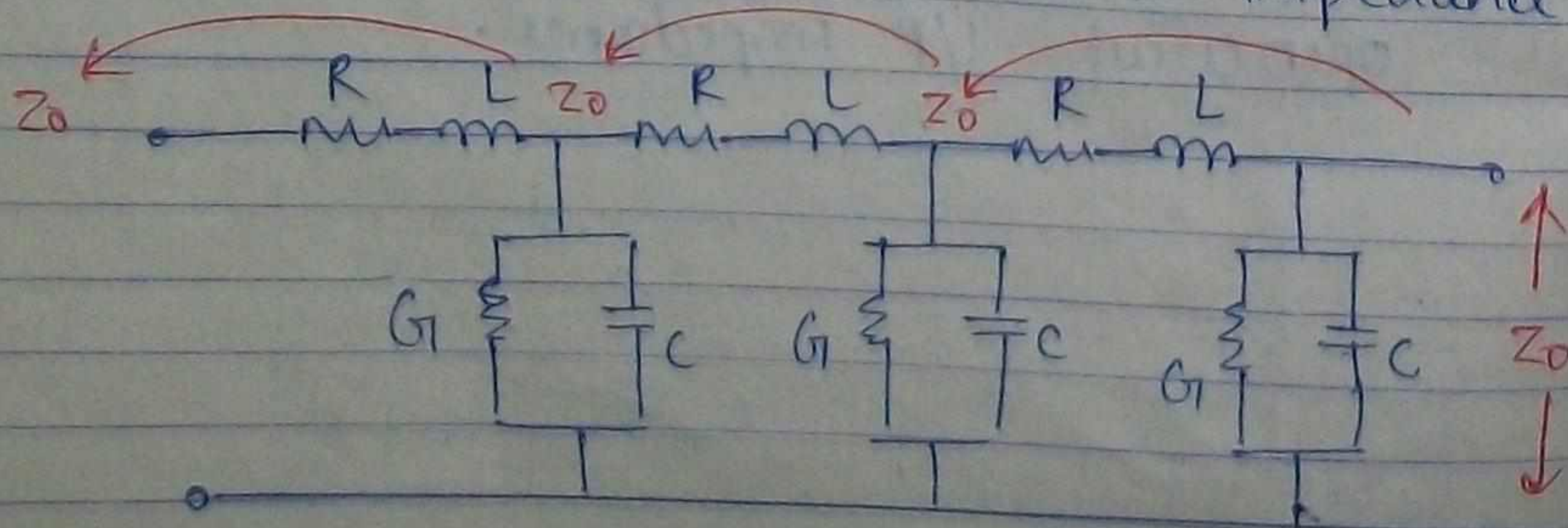
$$Z_{oc} = 700$$

$$Z_{sc} = 500 + 300 \parallel 200 = 620$$

$$Z_0 = \sqrt{700 \times 620}$$

$$Z_0 = 658.78 \Omega$$

- Every network discrete or continuous when terminated with its characteristic impedance the other end also has the same impedance hence for a tx. line where the R, L, G, C repeats the loading effect anywhere on the line is just same when it is terminated with characteristic impedance



$$Z_{sc} = R + j\omega L$$

$$Z_{oc} = \frac{1}{G + j\omega C}$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$\Rightarrow$ LOSSLESS AND DISTORTIONLESS LINE -

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

= propagation constant

$$= \alpha + j\beta$$

when, $\alpha = 0$ No attenuation on the line the line is lossless.

$$(1) R = G = 0$$

$$\gamma = j\beta = j\omega \sqrt{LC}$$

$$(2) \omega L \gg R \text{ and } \omega C \gg G$$

High freq line

or

inductive/cap. line

$$\beta = \omega \sqrt{LC}$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{L}{C}} = \text{Real value}$$

$$Z_0 = \sqrt{\frac{\mu_0 \ln(b/a)}{2\pi} \frac{\ln(b/a)}{2\pi \epsilon_0 \epsilon_r}}$$

$$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0 \epsilon_r} \frac{1}{2\pi} \ln(b/a)}$$

FOR
COAXIAL
CABLE

$$Z_0 = \frac{60 \ln(b/a)}{\sqrt{\epsilon_r}}$$

$$Z_0 = \frac{120 \ln(D/a)}{\sqrt{\epsilon_r}}$$

FOR
PARALLEL
WIRE

Z_0 is independent of primary constants and frequency of operation and depends only on the physical dimension.

VELOCITY -

$$V_p = \frac{\omega}{\beta} = \frac{\omega}{\omega \sqrt{LC}} = \frac{1}{\sqrt{LC}}$$

$$V_p = \frac{1}{\sqrt{\mu_0 \epsilon_0 \epsilon_r}} = \frac{3 \times 10^8}{\sqrt{\epsilon_r}} \text{ m/s}$$

2/10/11

DISTORTIONLESS LINE -

RISE TIME = FALL TIME

CHARGING SERIES ARMS TIME CONSTANT = DISCHARGING SHUNT ARM TIME CONSTANT

- If the phase shift with time and the phase shift with space are linear to each other that is called distortionless line

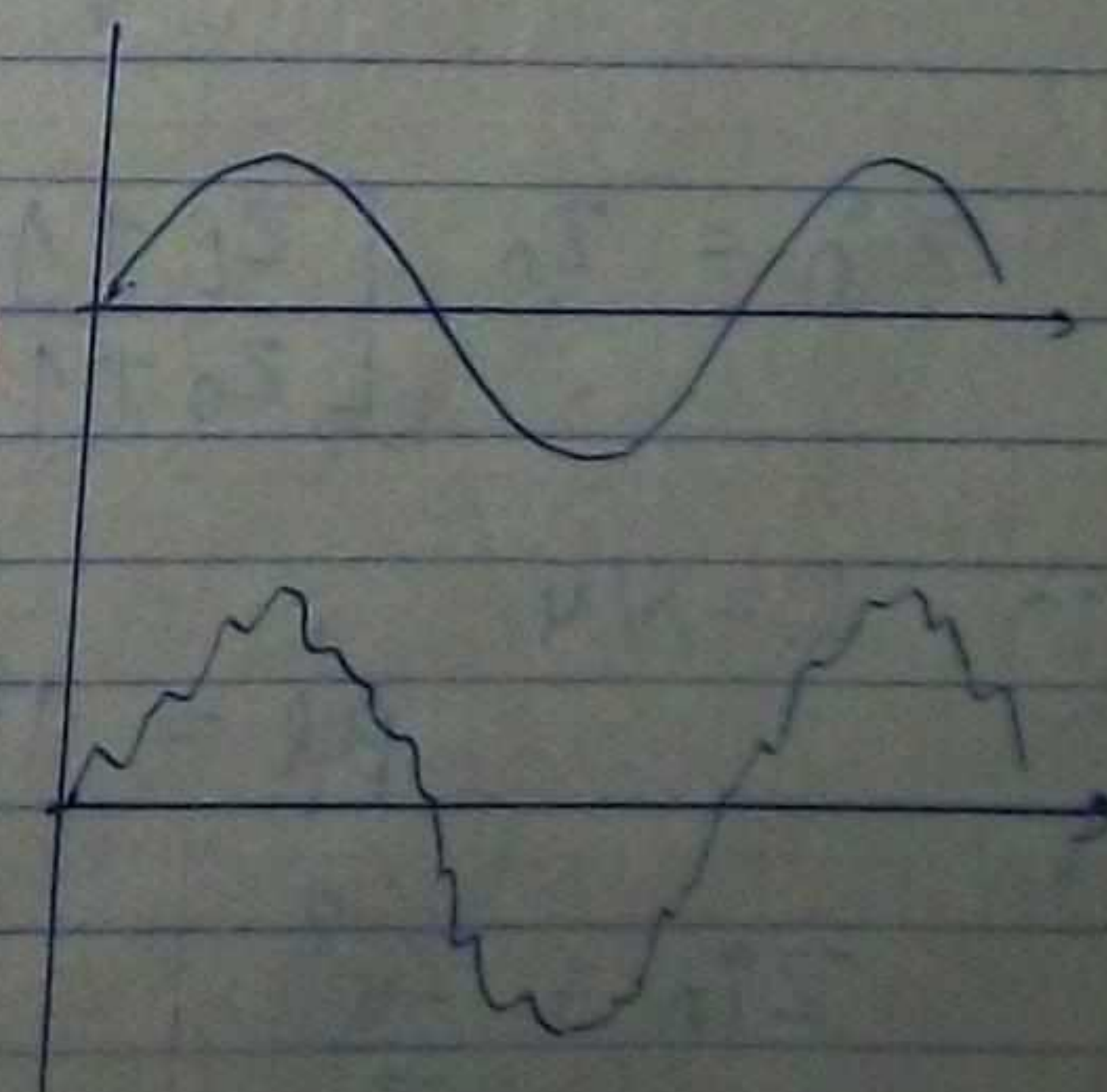
$$\omega \propto \beta, \quad \beta \propto \omega$$

$$\frac{L}{R} = \frac{C}{G}$$

$$LG = RC$$

SIMPLE
HARMONIC

DISTORTED
HARMONIC



NOTE -

All lossless lines are distortionless but the viceversa is not true.

Ques. 1 P-27.

Solⁿ:

$$Z_{in} = Z_0 \left[\frac{Z_L \cosh(\gamma B l) + Z_0 \sinh(\gamma B l)}{Z_0 \cosh(\gamma B l) + Z_L \sinh(\gamma B l)} \right]$$

$$\frac{e^{j\theta} + e^{-j\theta}}{2} = \cos \theta, \quad \frac{e^{\theta} + e^{-\theta}}{2} = \cosh \theta$$

$$Z_{in} = Z_0 \left[\frac{Z_L \cos \beta l + j Z_0 \sin \beta l}{Z_0 \cos \beta l + j Z_L \sin \beta l} \right]$$

(1) $l = \lambda/8$

$$\beta l = \frac{2\pi}{\lambda} \times \frac{\lambda}{8} = \frac{\pi}{4}$$

$$Z_{in} = Z_0 \left[\frac{Z_L + j Z_0}{Z_0 + j Z_L} \right]$$

(2) $l = \lambda/4$

$$\beta l = \pi/2$$

$$Z_{in} = \frac{Z_0^2}{Z_L}$$

IMPEDANCE
INVERTER

CAPACITIVE

IMPEDANCE
REFLECTOR

INDUCTIVE

(3) $l = \lambda/2$

$$\beta l = \pi$$

$$Z_{in} = Z_L$$

$\beta l =$ ELECTRICAL
LENGTH OF
LINE (radian
or degree)

(4) $l = \lambda$

$$\beta l = 2\pi$$

$$Z_{in} = Z_L$$

NOTE -

The performance of a line always depend on the length to wavelength relationship and hence the term βl is crucial and is called as electrical length of the line.

Ques. 2 P-27

Solⁿ:

$$Z_L = j50 \quad Z_0 = 50$$

$$Z_{in} = 50 \left[\frac{j50 + j50}{50 + j(j50)} \right] = \infty$$

(b) $\lambda/4$

$$Z_{in} = \frac{(50)^2}{j50} = -j50 \text{ (capacitive)}$$

(c) $\lambda/2$

$$Z_{in} = j50 \text{ (inductive)}$$

Ques 5. P-27

Solⁿ:

$$Z_0 = R_0 + j0$$

$$Z_L = (Z_{in2} \parallel Z_{in3})$$

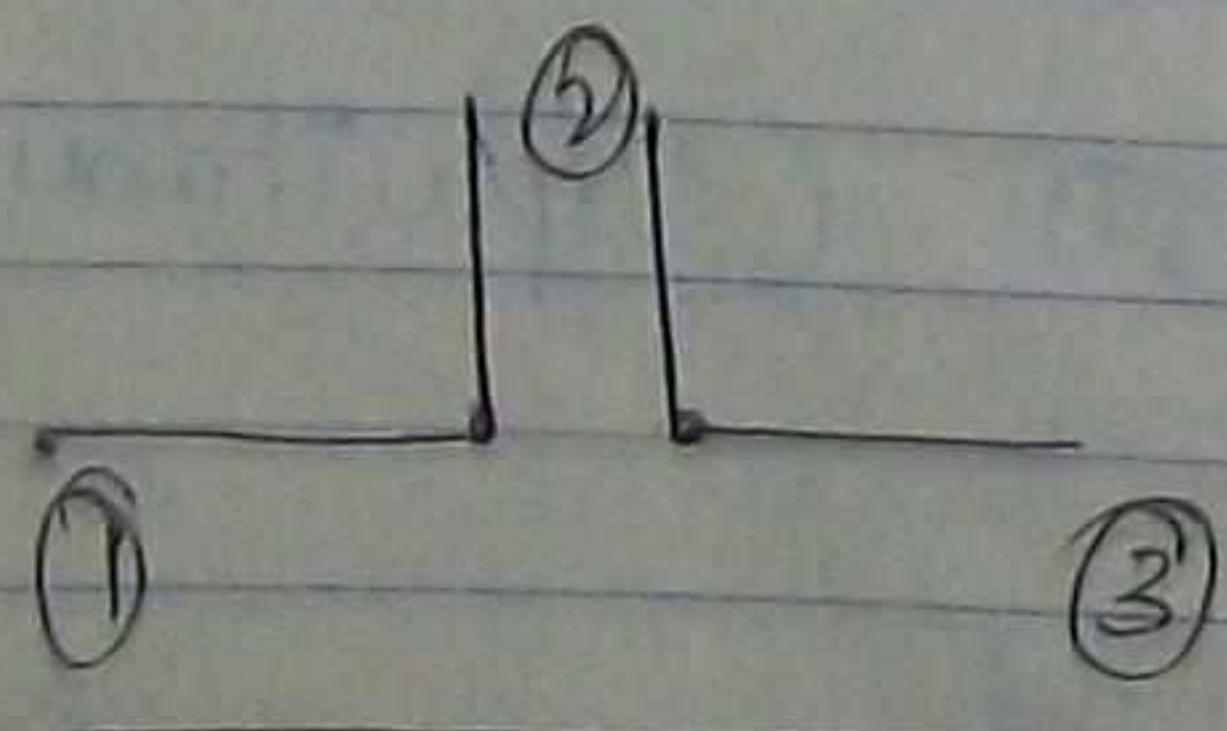
$$Z_{in2} = \frac{Z_0^2}{Z_L} = \frac{R_0^2}{R_0/2} = 2R_0$$

$$Z_{in3} = 2R_0$$

$$Z_L = 2R_0 \parallel 2R_0 = \underline{R_0}$$

$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0}{Z_0 + jZ_L} \right]$$

$$Z_{in} = R_0 \left[\frac{R_0 + jR_0}{R_0 + jR_0} \right] = \underline{R_0}$$



$$Z_L = Z_{in2} + Z_{in3}$$

$$\frac{100 + j50}{10 + j100} = \frac{219}{172j}$$

Repeat for load interchanged.

$$Z_{in2} = \frac{Z_0^2}{Z_L} = \frac{R_0^2}{2R_0} = \frac{R_0}{2}$$

$$Z_{in3} = R_0/2$$

$$Z_{in2} \parallel Z_{in3} = Z_L = R_0/2 \parallel R_0/2 = R_0/4$$

$$Z_{in} = Z_0 \left(\frac{Z_0 + jZ_0}{Z_0 + jZ_L} \right)$$

$$= R_0 \left(\frac{1 + j4}{4 + j} \right) = R_0 \left(\frac{15}{17} + \frac{15}{17}j \right)$$

Ques 8 P-30.

Solⁿ:

$$\frac{50}{100 + j50} = \frac{50 + j50}{50 + j100}$$

$$Z_{in1} = Z_0 \left(\frac{Z_L + jZ_0}{Z_0 + jZ_L} \right) = jZ_0 = j50$$

$$Y_{in1} = \frac{1}{j50} = -\frac{j}{50}$$

$$Z_{in2} = 100$$

$$Y_{in2} = \frac{1}{100} = 0.01$$

$$Y = 0.01 - j0.02$$

*

$$Z_{sc} = Z_0 \tanh(\gamma l)$$

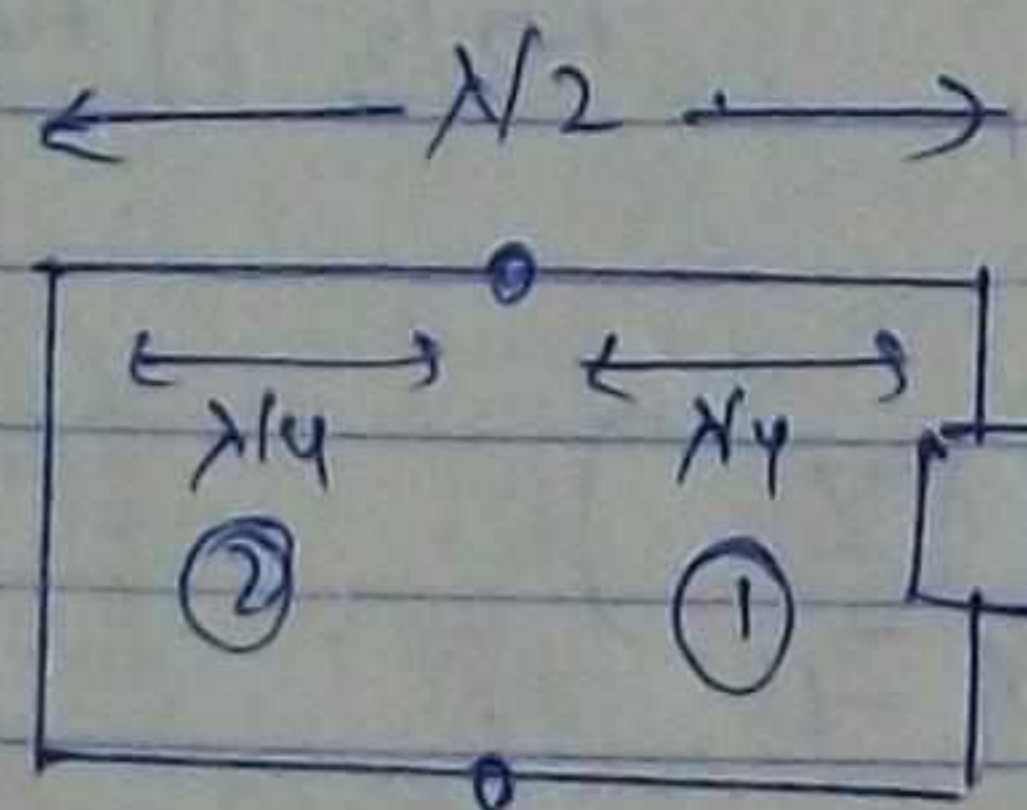
$$Z_{sc} = j Z_0 \tan \beta l$$

$$Z_{sc} = j Z_0$$

FOR SHORT
CIRCUIT

Que. 7 P-28
Solⁿ:

①



$$l = \lambda/2$$

$$l'_1 = l'_2 = \lambda/4$$

Impedance
at centre

= Impedance
due to R_0
side ①
= $\infty \parallel R_0$

Impedance
due to
s/c side ②

$$\approx R_0$$

②

$$l = \lambda/4$$

$$l' = \lambda/8$$

$$= R_0 (0 + j R_0) \parallel R_0$$

$$= j R_0 \parallel R_0$$

$$= R_0 \frac{j}{(1+j)}$$

$$\frac{j R_0}{\infty + R_0} = R_0$$

$$\frac{Z_0}{Z_L} = \frac{Z_0}{\infty} = 0$$

$$\frac{j R_0 R_0}{j R_0 R_0} = \frac{j R_0}{(1+j)} = R_0 \frac{j}{(1+j)} \times \frac{1-j}{1-j} = \frac{R_0 (j+1)}{1+1} = \frac{R_0 (1+j)}{2}$$

③

$$l = \lambda$$

$$l' = \lambda/2$$

$$Z = R_0 \parallel 0 = 0$$

Que. 8 P-28.

Solⁿ:

$$l = 10 \text{ m}$$

$$f = 10 \text{ MHz}$$

$$v = 2 \times 10^8 \text{ m/s}$$

$$\lambda = \frac{v}{f}$$

$$= \frac{2 \times 10^8}{10 \times 10^6} = 20$$

$$l = \lambda/2$$

$$Z_{in} = Z_L = (30 - j40) \Omega$$

(b)

$$\lambda = \frac{2 \times 10^8}{5 \times 10^6}$$

$$= \frac{200}{5} = 40$$

$$l = \lambda/4$$

$$Z_{in} = \frac{Z_0^2}{Z_L}$$

Que. 9 P-28

Solⁿ:

$$Z_{in} = j60$$

s/c line

$$\beta l = \pi/4$$

$$l = \lambda/8$$

$$Z_{sc} = j Z_0 \tan \beta l$$

$$= 60 \Omega$$

$$\lambda = \lambda/8$$

$$\lambda' = \lambda_2 = \lambda/16$$

Que. 11. P-28.

Soln: $l = \lambda/8$

$$f = 12 \text{ MHz}$$

$$\lambda' = \lambda/2$$

$$f' = 24 \text{ MHz}$$

$$\lambda = 2\lambda'$$

$$l = \lambda'/4$$

$$Z_{in} = \frac{Z_0^2}{Z_L} = \infty$$

(also)

$$Z_{sc} = j Z_0 \tan \beta l$$

$$= j Z_0 \tan \pi/2$$

$$= \infty$$

$$\beta l = \pi/4$$

$$\frac{2\pi}{\lambda} \cdot l$$

$$\beta' l = \pi/2$$

Que 12. P-28.

Soln:

$$Z_{sc} = Z_{in} = j60 \Omega$$

at 12 MHz

$$j60 = j\omega L$$

$$L = \frac{60}{2\pi \times 12 \times 10^6}$$

$$= \frac{2.5}{\pi} \text{ nH}$$

⇒ NOTE -

$$j \tan \beta l \begin{cases} +ve \rightarrow \text{Cap} \\ -ve \rightarrow \text{Induct.} \end{cases}$$

SHORT CIRCUIT
LINE

$$Z_{sc} = j Z_0 \tan \beta l$$

OPEN CIRCUIT
LINE

$$Z_{oc} = -j Z_0 \tan \beta l$$

$$(1) 0 < \beta l < \pi/2$$

$$0 < l < \lambda/4$$

INDUCTIVE
NATURE
AT I/P

CAPACITIVE

$$(2) \pi/2 < \beta l < \pi$$

$$\lambda/4 < l < \lambda/2$$

CAPACITIVE
NATURE

INDUCTIVE

(*) Open and short circuit lines are always reactive

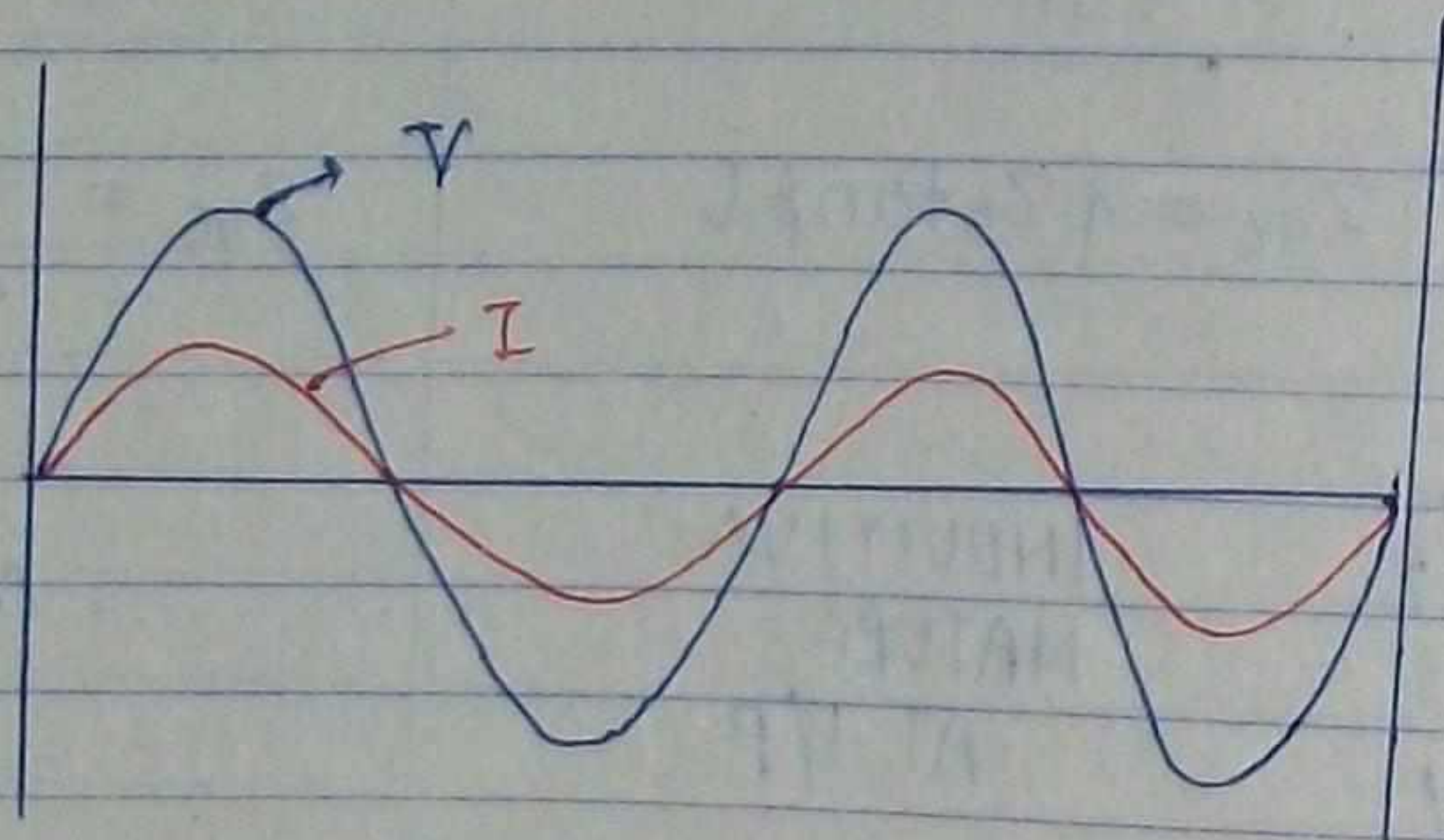
Que. 13. P-29

Soln:

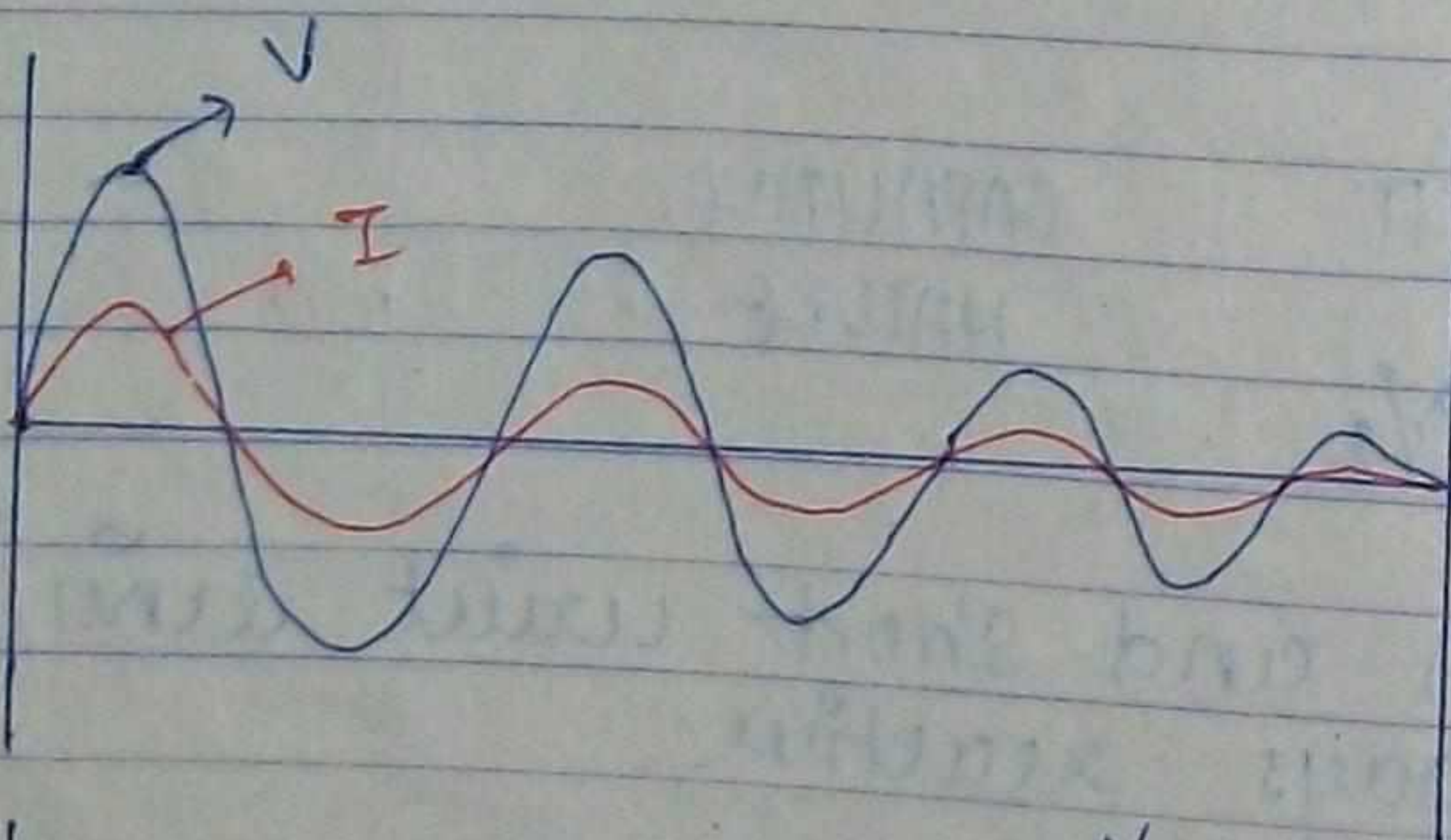
$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{LC}{C^2}} = \sqrt{\frac{\mu_0 \epsilon_0}{C}} = \sqrt{\frac{\epsilon_r}{v \cdot c}}$$

Que. 19. P-30

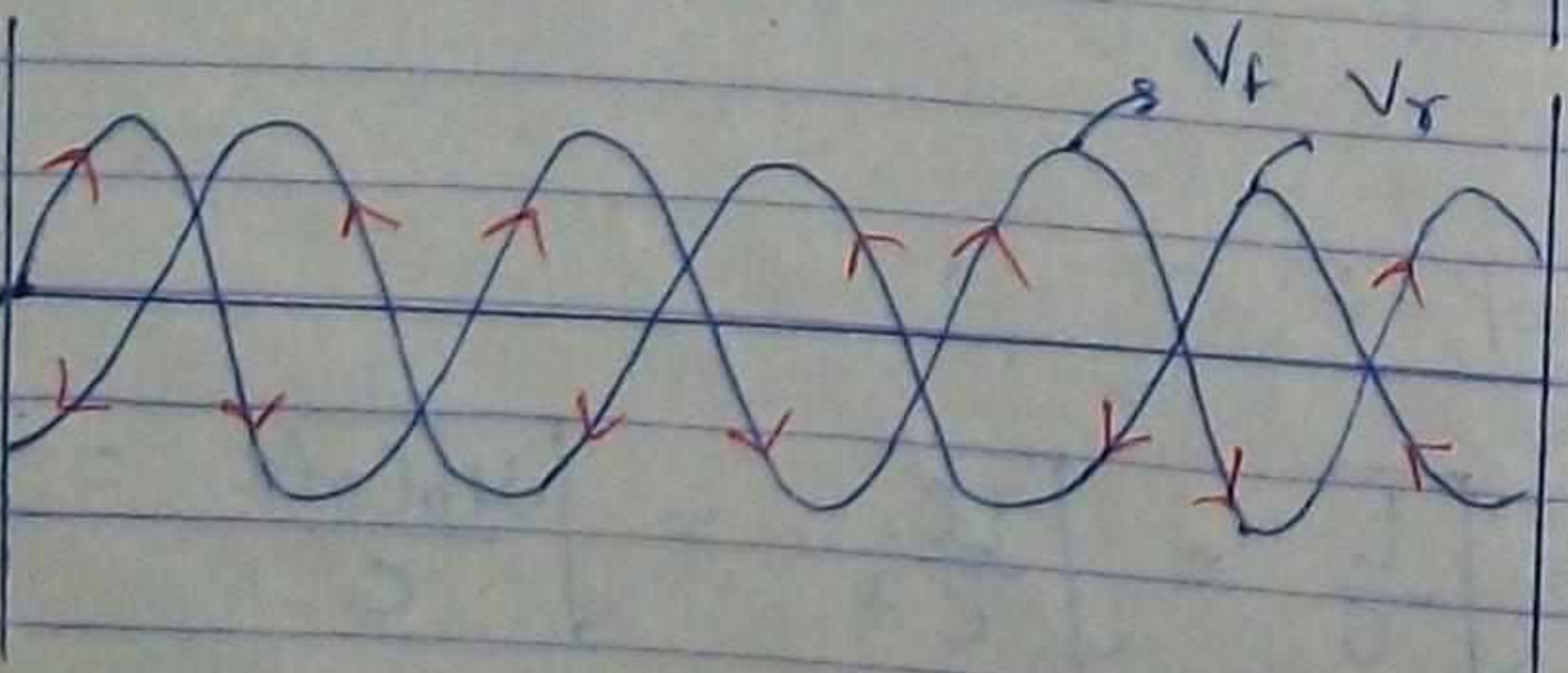
LOSSLESS
MATCHED



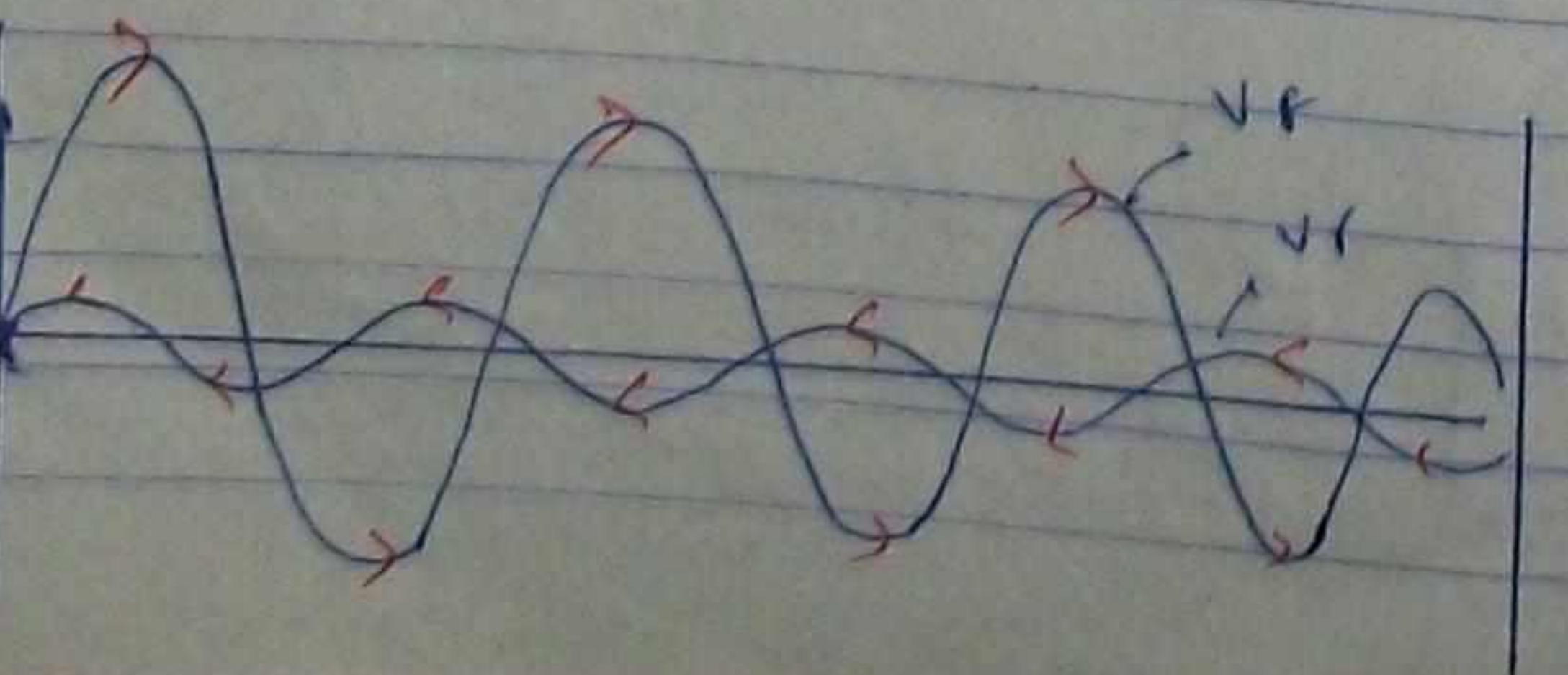
LOSSY
MATCHED



LOSSLESS
MISMATCH



LOSSY
MISMATCH



⇒ REFLECTION COEFFICIENT -

Since

$$V(x) = V_0 e^{\gamma x} + V_0 \left(\frac{Z_L - Z_0}{Z_L + Z_0} \right) e^{-\gamma x}$$

where,

$$V_0 = \frac{V_L (Z_L + Z_0)}{2 Z_L}$$

$$V(x) = V_0 e^{j\beta x} + V_0 \left(\frac{Z_L - Z_0}{Z_L + Z_0} \right) e^{-j\beta x}$$

V_f
(FORWARD V)

V_r
(REFLECTED V)

$\frac{V_r}{V_f}$ = REFLECTION COEFFICIENT FOR THE VOLTAGE
ANYWHERE ON THE LINE ($\Gamma(x)$)

$$\Gamma(x) = \left(\frac{Z_L - Z_0}{Z_L + Z_0} \right) e^{-2j\beta x}$$

① At $x=0$:

Γ at the load.

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = |\Gamma| \angle \theta = \left| \frac{V_r}{V_f} \right| \angle \theta \quad \text{ANY COMPLEX VALUE}$$

$|\Gamma|$ = Ratio of the amplitude of the reflected voltage to the forward voltage.

$\angle \theta$ = The phase difference between the reflected and forward voltage.

$\therefore \Gamma$ is a measure of mismatch between the expected impedance Z_0 and the actual impedance Z_L at the load.

$$\Gamma(x) = \frac{Z(x) - Z_0}{Z(x) + Z_0}$$

- In generally $\Gamma(x)$ is a measure of mismatch between the expected impedance Z_0 and the actual impedance $Z(x)$ at that point on the line.

NOTE -

$$(1) \quad I(x) = I_0 e^{j\beta x} + I_0 \left(\frac{Z_0 - Z_L}{Z_0 + Z_L} \right) e^{-j\beta x}$$

$$\therefore \quad \Gamma_I = -\Gamma_V$$

(2) The reflected voltage and reflected current are out of phase by 180° .

$$(3) \quad 0 \leq |\Gamma| \leq 1$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$\Rightarrow$ CASES OF COMPLETE MISMATCH ON A LINE -
($|\Gamma| = 1$)

CASE (1):

$$Z_L = jR_0 = \text{inductive load}$$

$$Z_0 = R_0 = \text{lossless line}$$

$$\Gamma = \frac{jR_0 - R_0}{jR_0 + R_0} = \frac{j-1}{j+1} = \frac{\sqrt{2} \angle 135^\circ}{\sqrt{2} \angle 45^\circ} = 1 \angle 90^\circ = j$$

CASE (2):

$$Z_L = -jR_0 = \text{capacitive load}$$

$$Z_0 = R_0 = \text{lossless line}$$

$$\Gamma = \frac{-jR_0 - R_0}{-jR_0 + R_0} = \frac{-j-1}{-j+1} = 1 \angle -90^\circ = -j$$

* Reactive element don't dissipate any real power.

CASE (3):

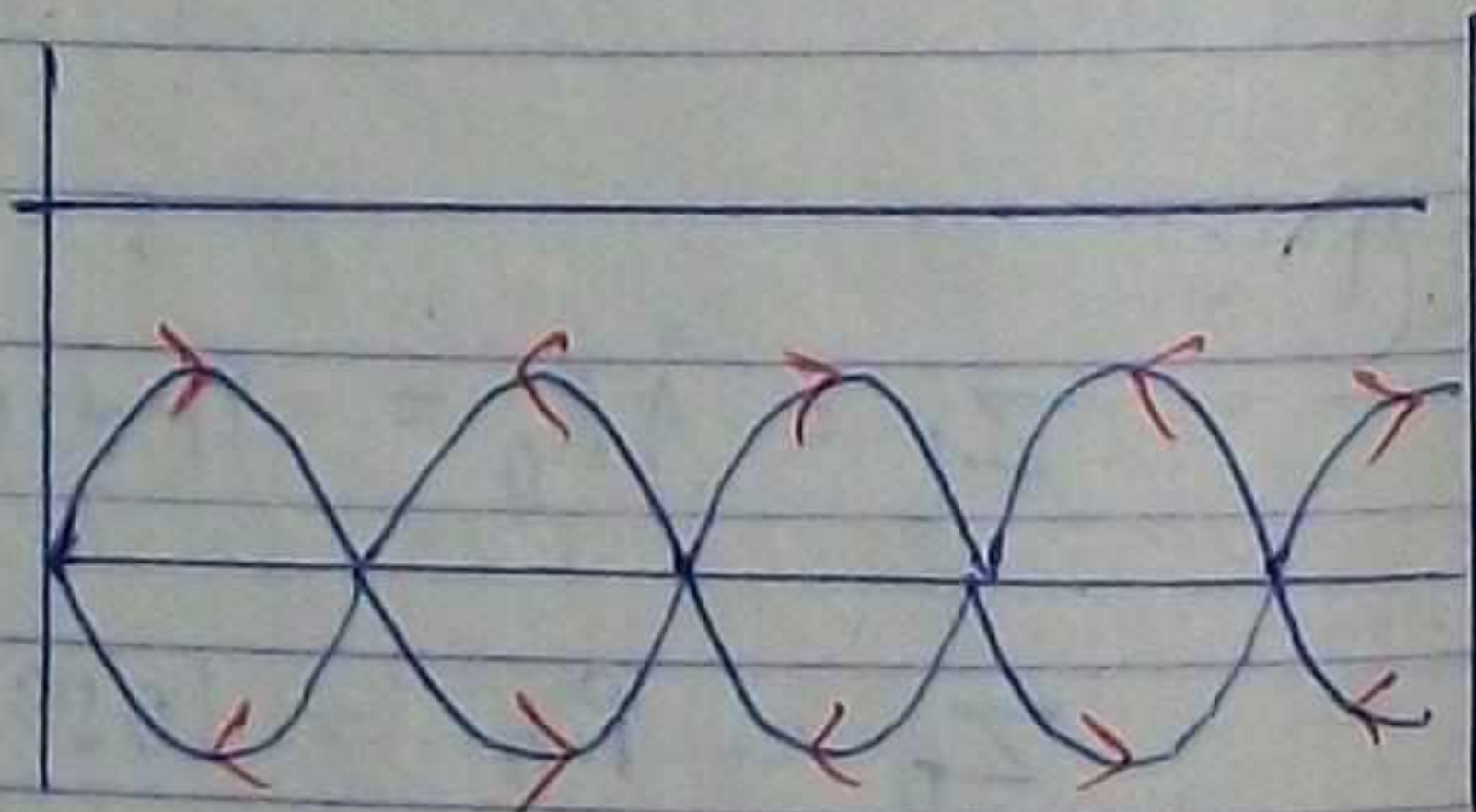
$Z_L = 0$ short circuit line

$Z_0 = R_0 = \text{lossless line}$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = -1 = 1 \angle 180^\circ$$

ie $V_f \rightarrow \sin$

$V_r \rightarrow -\sin$



CASE (4):

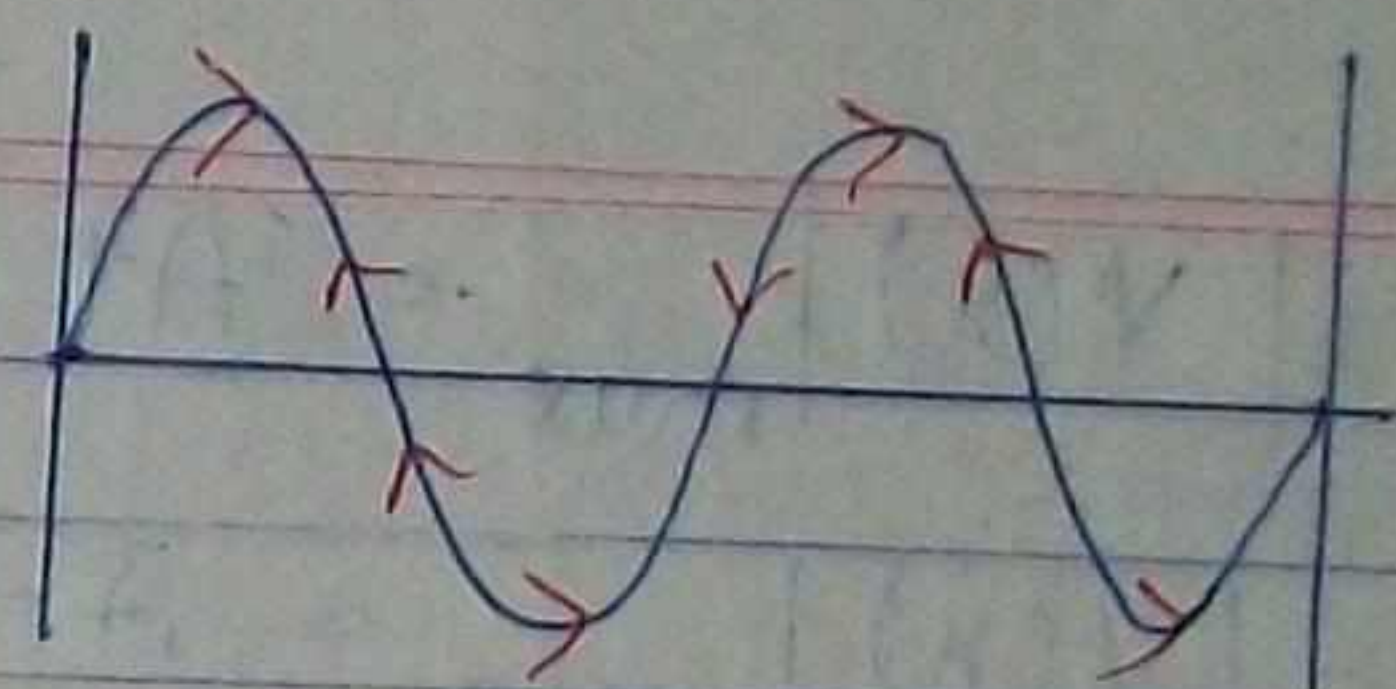
$Z_L = \infty$ (open circuit line)

$Z_0 = R_0 = \text{lossless line}$

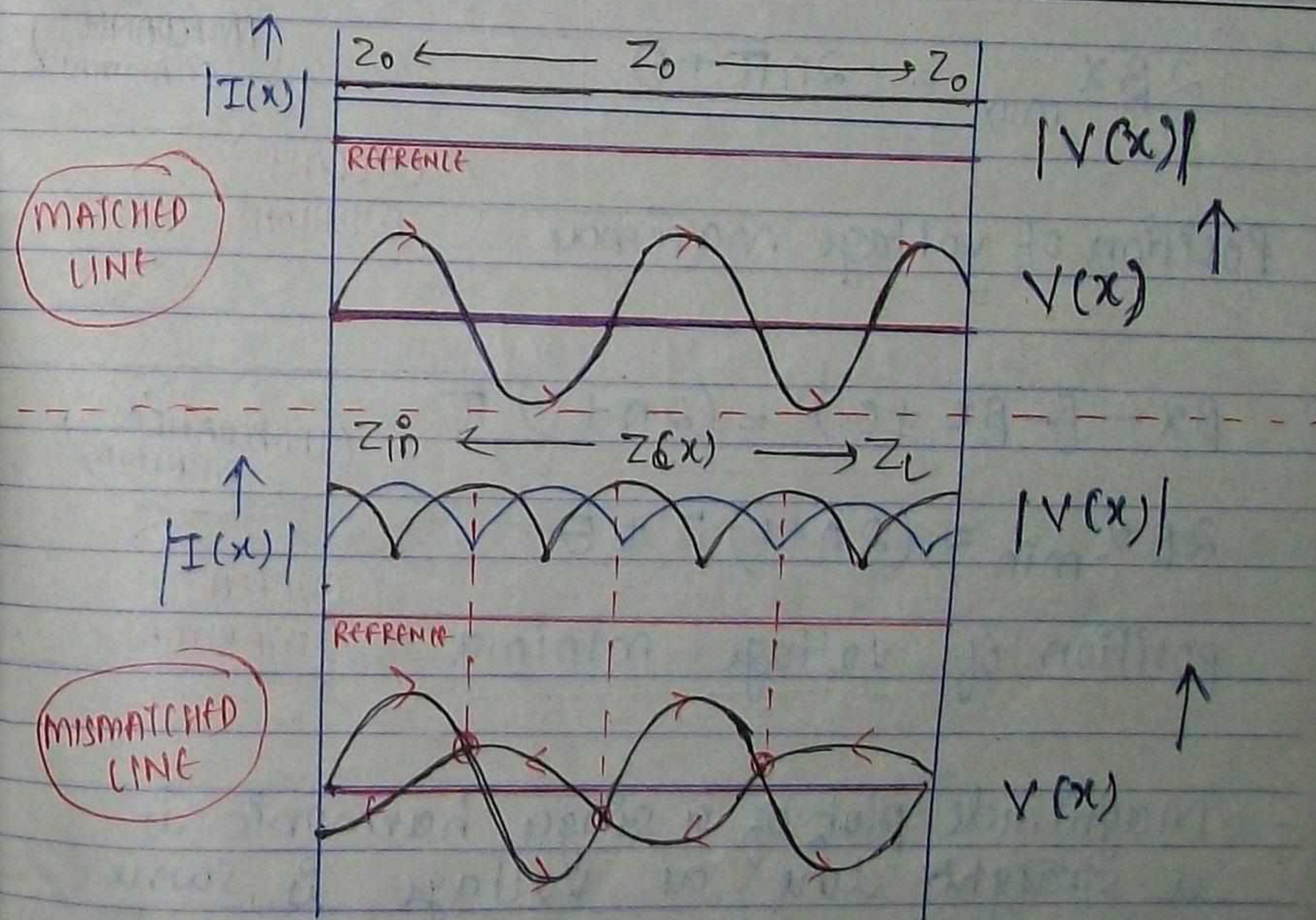
$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{1 - Z_0/Z_L}{1 + Z_0/Z_L} = 1 \angle 0^\circ$$

ie $V_f \rightarrow \sin$

$V_r \rightarrow \sin$



⇒ STANDING WAVE & STANDING WAVE RATIO (SWR)



$$\begin{aligned} V(x) &= V_0 e^{j\beta x} + V_0 |\Gamma| e^{j\theta} e^{-j\beta x} \\ &= V_0 e^{j\beta x} + V_0 |\Gamma| e^{j(-\beta x + \theta)} \\ &= A \sin \psi_1 + B \sin \psi_2 \end{aligned}$$

$$|V(x)|_{\max} = A + B \quad \psi_1 - \psi_2 = 2n\pi$$

$$|V(x)|_{\min} = A - B \quad \psi_1 - \psi_2 = (2n+1)\pi$$

$$(1) \quad \beta x - (-\beta x + \theta) = 2n\pi \quad n = 0, 1, 2, 3, \dots$$

$$2\beta x_{\max} = 2n\pi + \theta$$

IMPEDANCE
MAXIMAL

Position of voltage maxima

CURRENT
MINIMAS

$$(2) \quad \beta x - (-\beta x + \theta) = (2n+1)\pi$$

$$2\beta x_{\min} = (2n+1)\pi + \theta$$

IMPEDANCE
MINIMA

position of voltage minima

CURRENT
MAXIMAS

- Magnitude plot of a single harmonic is a straight line as voltage is same everywhere.

- Magnitude plot of two harmonics travelling in opposite directions has periodic additions and cancellations due to interference. This results in voltage maxima and

minima all along the length of the line, this is called as standing wave pattern.

STANDING WAVE (DEFINITION) -

The magnitude plot of unevenly distributed voltage due to interference of two waveforms is called as standing wave formation.

NOTE -

(1) Between two successive maxima or minima of a standing wave the gap is $(\lambda/2)$

βx is periodic with 2π (FOR HARMONIC)

$2\beta x_{\max}$ is periodic with 2π (FOR STANDING WAVE)

∴ $x_{\max}$ is periodic with $\lambda/2$

(2) The current standing waves also identically formed with the voltage maxima coinciding with the current minima and vice versa.

$\frac{2\pi}{2\pi}$

③ As impedance is unevenly distributed on an mismatched line, the voltage and current are also uneven everywhere with periodic accumulations and flows.

$$Z_{\max} = \frac{V_{\max}}{I_{\min}}, \quad Z_{\min} = \frac{V_{\min}}{I_{\max}}$$

STANDING WAVE RATIO (SWR) -

$$= V_{\text{SWR}} \text{ or } C_{\text{SWR}}$$

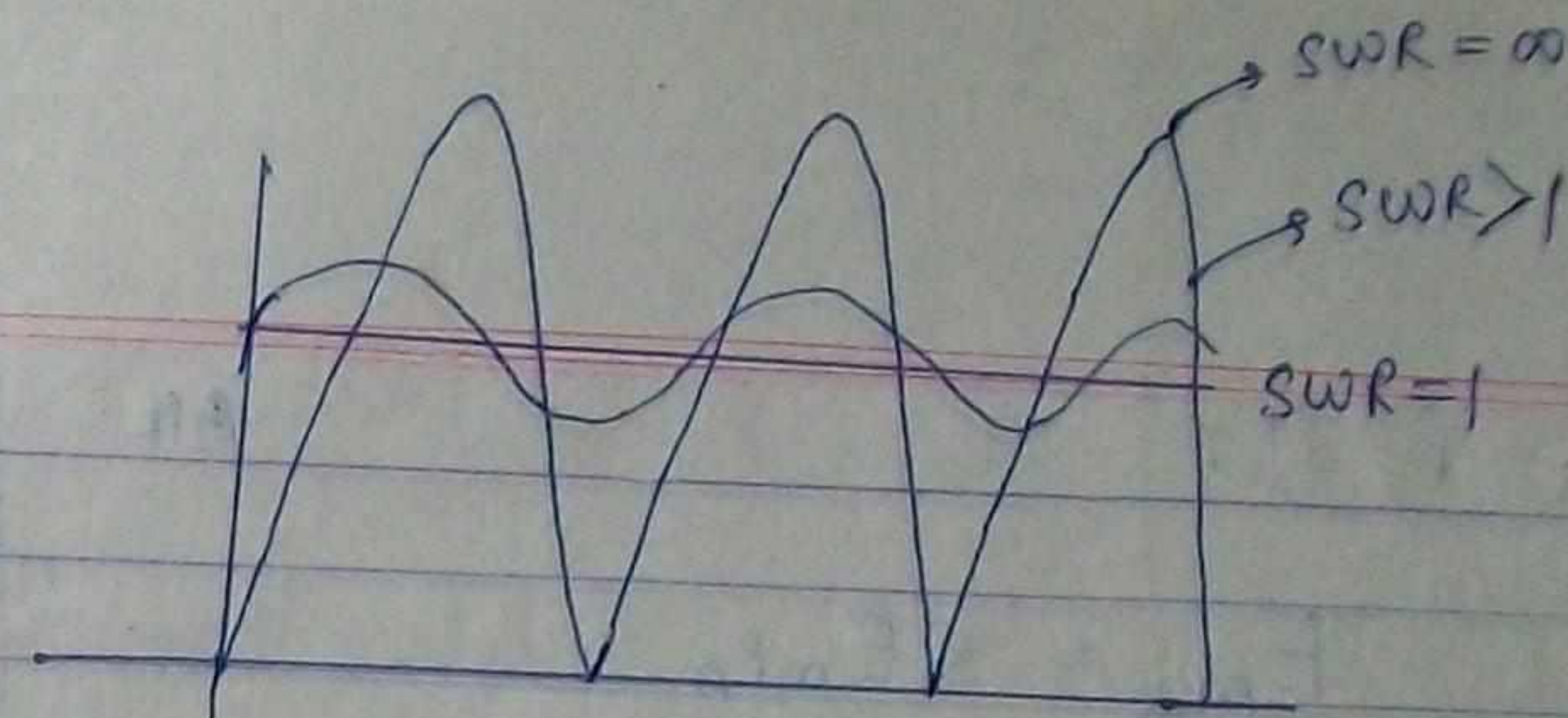
$$= \frac{|V_{\max}|}{|V_{\min}|} \text{ or } \frac{|I_{\max}|}{|I_{\min}|}$$

$$\boxed{\text{SWR} = \frac{1 + |\Gamma|}{1 - |\Gamma|}}$$

$$\text{SWR} [1, \infty)$$

MATCHED
LINE

COMPLETE
MISMATCH



If $\text{SWR} = \infty$

$$V_{\min} = 0 \text{ or } I_{\min} = 0$$

$$|\Gamma| = 1$$

SWR $\rightarrow$ measure of mismatch on the line.

Ques. 25, P-31.
Soln:

$$E_{\max} = 5 E_{\min}$$

$$ESWR = 5$$

$$SWR = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$|\Gamma| = \frac{SWR - 1}{SWR + 1} = \frac{5 - 1}{5 + 1} = \frac{2}{3}$$

$$|\Gamma| = 2/3$$

$$\frac{2}{3} \neq \Gamma = \frac{n_2 - n_1}{n_2 + n_1} \neq \frac{n_2 - 120 \pi}{n_2 + 120 \pi}$$

$$n_2 \pi$$

$$|\Gamma| = 2/3$$

$$\Gamma = 2/3 \angle \theta^\circ \text{ real}$$

$$= \frac{2}{3} \angle 0^\circ \text{ or } 180^\circ = \pm 2/3$$

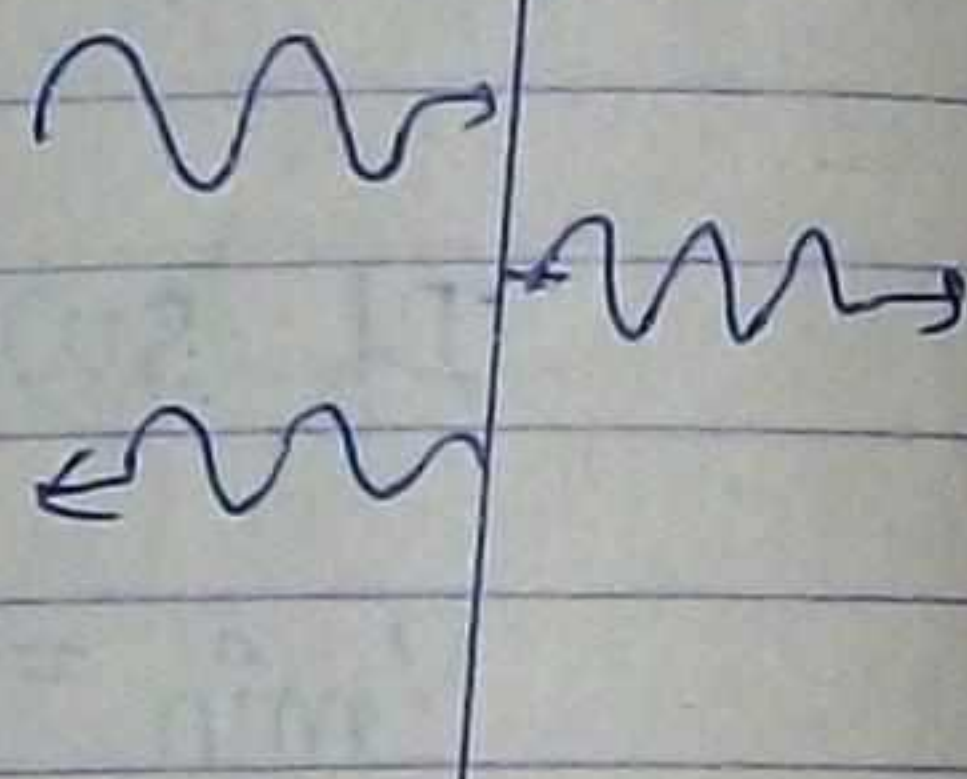
$$n_1 = 120 \pi \text{ real}$$

$$n_2 = 120 \pi / \sqrt{\epsilon_r} \text{ real}$$

AIR

DIELECTRIC

$$\epsilon > \epsilon_0$$



$$n_2 < n_1 \Rightarrow \Gamma = -ve$$

$$-\frac{2}{3} = \frac{n_2 - 120 \pi}{n_2 + 120 \pi}$$

$$n_2 = \underline{\underline{24 \pi}}$$

Ques. 26.
Soln:

$$SWR = 4$$

$$|\Gamma| = \frac{4 - 1}{4 + 1} = \frac{3}{5}$$

$$\Gamma = \pm 3/5$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = |\Gamma| \angle 0^\circ \text{ or } 180^\circ$$

$$= \left| \frac{3}{5} \right| \angle 0^\circ \text{ or } 180^\circ$$

$\therefore V_{\min}$ is at the load

$Z_{\min}$ is at the load

$$(R_L \text{ or } Z_L) < Z_0 \quad \Gamma = -ve$$

$$-\frac{3}{5} = \frac{Z_L - 50}{Z_L + 50}$$

$$Z_L = \underline{\underline{12.5 \Omega}}$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

↓
real ↓
 ↓
 real ↓
 real

NOTE -

① If Z_L and Z_0 are real (resistive load & lossless line)

⇒ If $Z_L > Z_0$

Γ is positive & real.

$V_{\max}$ occurs at the load.

ex - o/c line $Z_L = \infty$

$$\Gamma = +1$$

⇒ If $Z_L < Z_0$

Γ is real and negative.

$V_{\min}$ occurs at the load

ex - s/c line $Z_L = 0$

$$\Gamma = -1$$

②

$$V_{\text{SWR}} = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + \left| \frac{Z_L - Z_0}{Z_L + Z_0} \right|}{1 - \left| \frac{Z_L - Z_0}{Z_L + Z_0} \right|}$$

If Z_L, Z_0 are real.

⇒ $Z_L > Z_0$

$$V_{\text{SWR}} = \frac{1 + \frac{Z_L - Z_0}{Z_L + Z_0}}{1 - \frac{Z_L - Z_0}{Z_L + Z_0}}$$

$$V_{\text{SWR}} = \frac{Z_L}{Z_0}$$

⇒ If $Z_L < Z_0$

$$V_{\text{SWR}} = \frac{Z_0}{Z_L}$$

③

$$Z_{\max} = \frac{V_{\max}}{I_{\min}} = \frac{V_{\max} \times I_{\max}}{I_{\max} \times I_{\min}}$$

$$Z_{\max} = Z_0 \cdot (\text{SWR})$$

$$Z_{\min} = \frac{Z_0}{(\text{SWR})}$$

Que 47, P-35

Soln:

$$Z_0 = 50$$

$$VSWR = 5$$

$$|\Gamma| = \frac{4}{6} = \frac{2}{3}$$

$V_{max} \rightarrow \lambda/4$ distance from load

$V_{min} \rightarrow$ at load

$$\Gamma = -ve$$

$$\Gamma = -\frac{2}{3} = \frac{Z_L - 50}{Z_L + 50} \Rightarrow Z_L = 10 \Omega$$

Que 37, P-33

Soln:

$$VSWR = 5$$

$$\Gamma = |\Gamma| \angle \theta$$

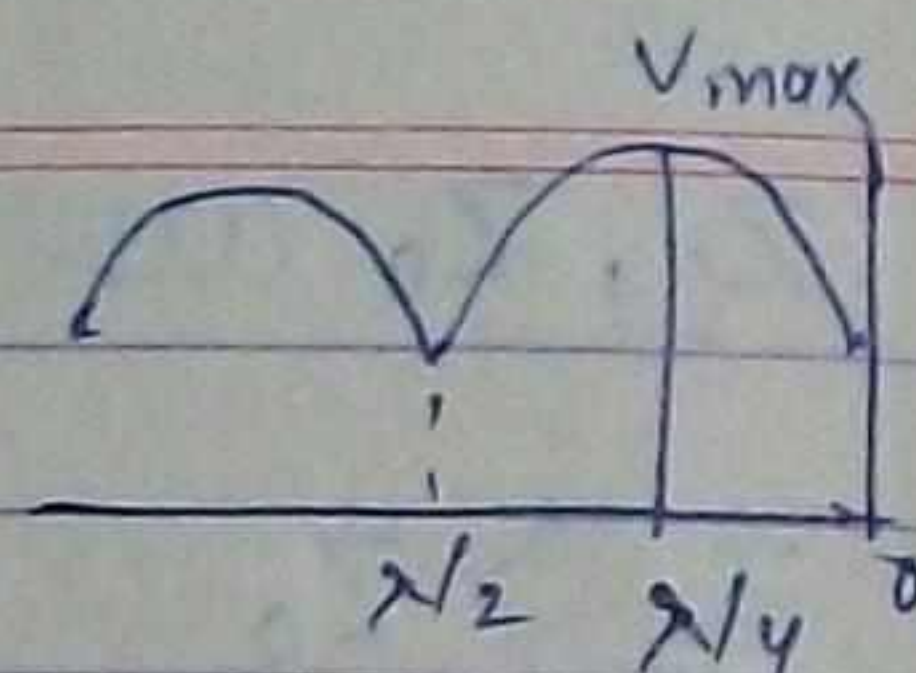
$$|\Gamma| = 4/6 = 2/3$$

$\lambda_{min} \rightarrow 17 \text{ cm} \text{ \& } 27 \text{ cm}$

$$2\beta x_{min} = (2n+1)\pi + \theta$$

$$\frac{\lambda}{2} = 10 \quad (\lambda = 20 \text{ cm})$$

Successive minima



$$n=0$$

$$7 \text{ cm}$$

$$n=1$$

$$17 \text{ cm}$$

$$n=2$$

$$27 \text{ cm}$$

$$2\beta x_{min} = (2n+1)\pi + \theta$$

$$2 \times \frac{2\pi}{20} \times 7 = \pi + \theta$$

$$\theta = \frac{8\pi}{20} = \frac{2\pi}{5} = 72^\circ$$

$$\Gamma = \frac{2}{3} \angle 72^\circ$$

Que 51, P-36

Soln:

$$\Gamma = \}$$

$$l = 500 \text{ m}$$

$$\theta = -150$$

$$\lambda = 150 \text{ m}$$

$$(n=0)$$

$$2\beta x_{max} = (2n\pi) - 150^\circ$$

$$2 \times \frac{2\pi}{150} \times x_{max} = -150^\circ$$

$$x_{max} = \frac{-150}{\pi} = -ve$$

$$(n=1)$$

$$2\beta x_{max} = 2\pi - 150^\circ = 2\pi - 5\pi/6 = 7\pi/6$$

$$x_{max} = \frac{7\pi}{6} \times 150 \times \frac{1}{2 \times 2\pi} = 43 \text{ m}$$

$$1^{\text{st}} V_{\text{max}} \longrightarrow 43 \text{ m.}$$

$$2^{\text{nd}} V_{\text{max}} \longrightarrow 43 + \left(\frac{\lambda}{2}\right) = 118 \text{ m}$$

$$3^{\text{rd}} V_{\text{max}} \longrightarrow 118 + 75 = 193 \text{ m}$$

$$4^{\text{th}} \longrightarrow 193 + 75 = 268 \text{ m}$$

$$5^{\text{th}} \longrightarrow 268 + 75 = 343 \text{ m}$$

$$6^{\text{th}} \longrightarrow 343 + 75 = 418 \text{ m}$$

$$7^{\text{th}} \longrightarrow 418 + 75 = \underline{\underline{493}}$$

∴ 7 maximas

7/10/11

Ques. 27 P-32

Solⁿ:

$$\Gamma_{\text{IN}} = \frac{Z_{\text{IN}_1} - Z_0}{Z_{\text{IN}_1} + Z_0}$$

$$Z_{\text{IN}_2} = \frac{50^2}{100} = 25$$

$$Z_{\text{IN}_1} = \frac{Z_0^2}{Z_L} = \frac{50^2}{25/3}$$

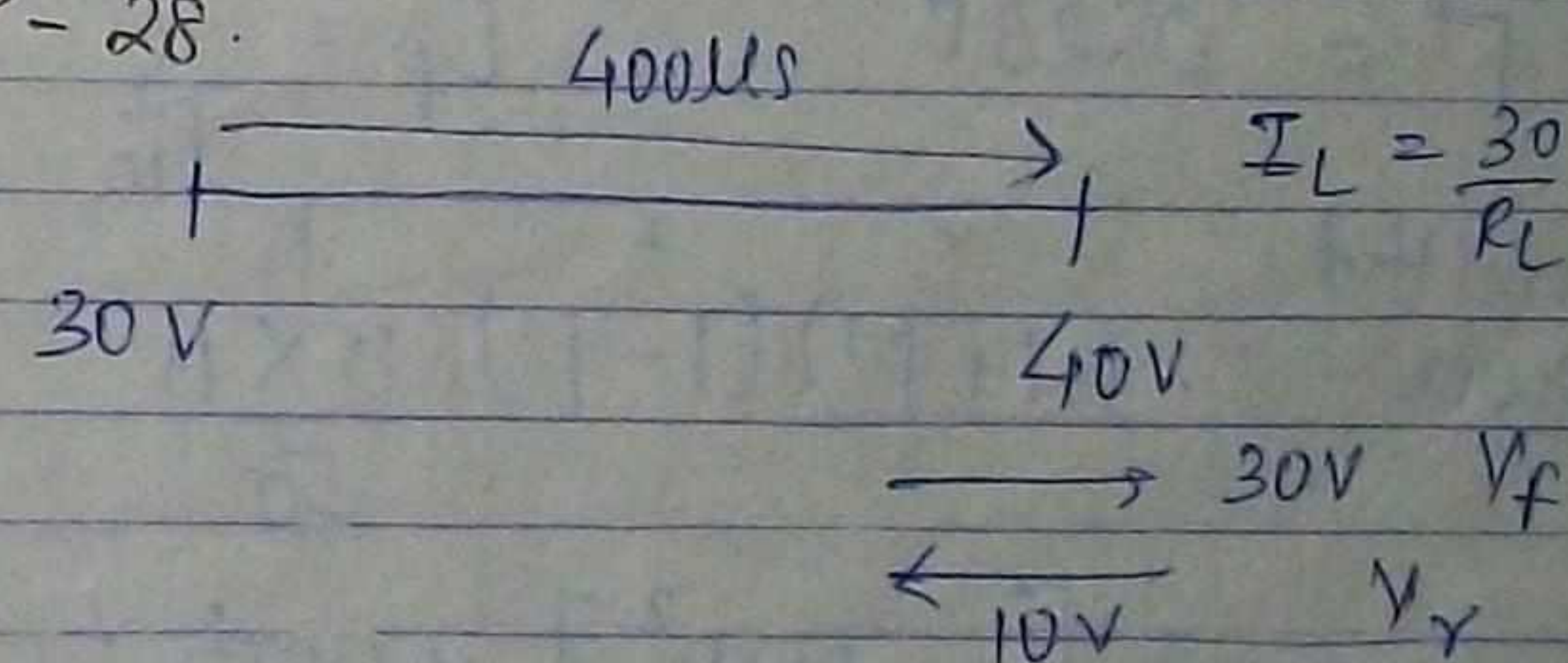
$$Z_{\text{IN}_2} = \frac{50^2}{200} = \frac{25}{2}$$

$$Z_{L1} = (Z_{\text{IN}_2} \parallel Z_{\text{IN}_3}) = 25 \parallel \frac{25}{2} = \frac{25}{3}$$

$$\Gamma_{\text{IN}} = \frac{300 - 50}{300 + 50} = \frac{5}{7}$$

Ques. 28 P-28.

Solⁿ:



$$\Gamma = \frac{V_r}{V_f} = \frac{10}{30} = \frac{R_L - R_0}{R_L + R_0} = \frac{R_L - 50}{R_L + 50}$$

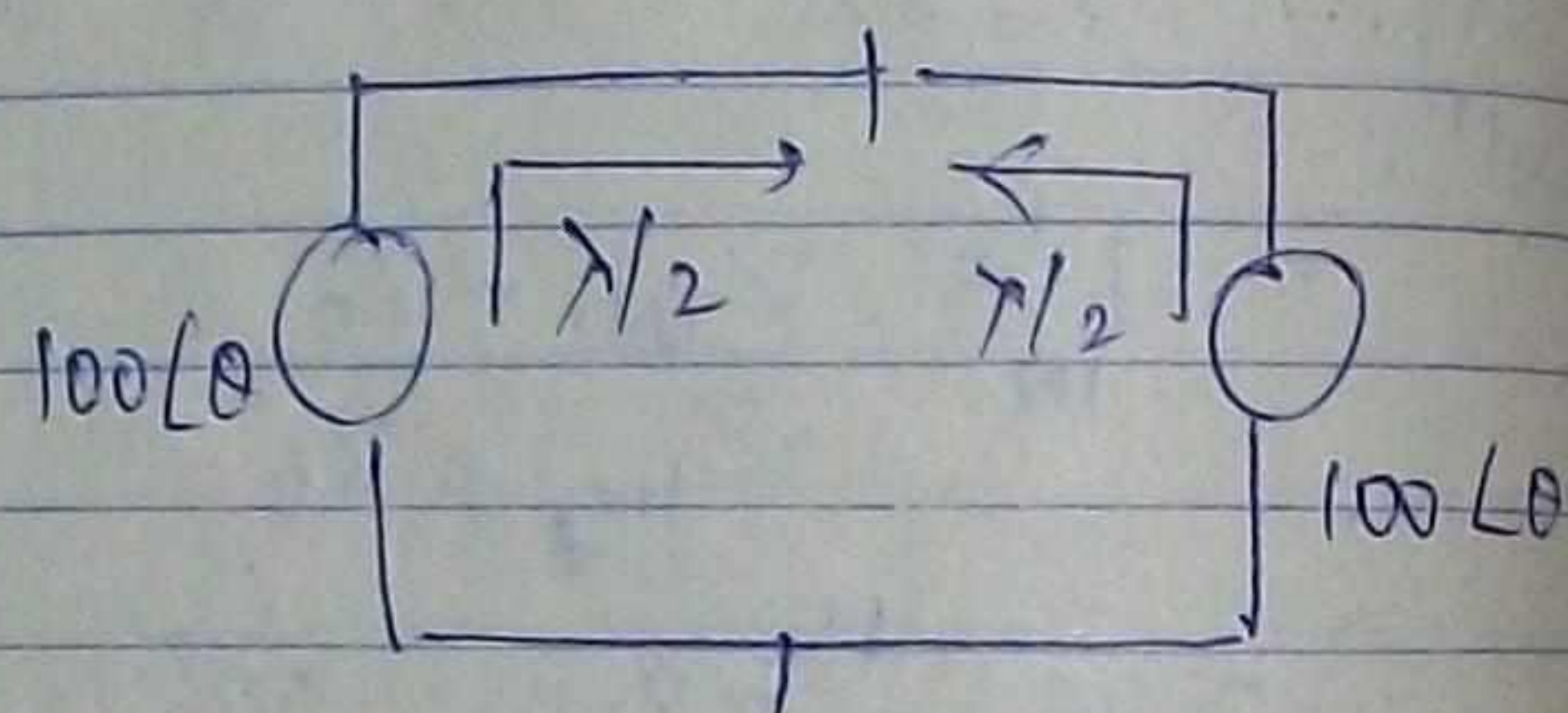
$$R_L = \underline{\underline{100 \Omega}}$$

$$I_L = \frac{V_L}{R_L} = \frac{30}{100} = \frac{30}{100} = 0.3 \text{ Amp}$$

★

Ques. 29. P-32.

$V_1 \leftrightarrow V_2$
in phase



Add up. V_{max}

I cancel, $I_{min} = 0$
Impedance $= \infty$
VSWR $= \infty$

Ques. 31 P-32

Soln:

$$\Gamma = 0.268$$

$$\Gamma_p = \frac{P_t}{P_{in}}$$

$$\begin{aligned} \text{Transmitted power} &= (1 + \Gamma)(1 - \Gamma) \frac{P_{in}}{50} \\ &= [1 - |\Gamma|^2] \times \frac{(15)^2}{50} \end{aligned}$$

Ques. 33. P-33

Soln:

VSWR = 3

$$|\Gamma| = \frac{3-1}{3+1} = \frac{1}{2}$$

$$\begin{aligned} \Gamma_p &= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \\ &= 25\% \end{aligned}$$

Ques. 34 P-33

Soln:

$$\begin{aligned} \Gamma(x) &= \frac{V_r}{V_f} = \frac{Z_L - Z_0}{Z_L + Z_0} e^{-j2\beta x} = \frac{Z(x) - Z_0}{Z(x) + Z_0} \\ &= 0.6 e^{j60} e^{-j2 \times \frac{2\pi}{\lambda} \times \frac{\lambda}{8}} \\ &= 0.6 \angle -30^\circ \end{aligned}$$

★

Ques. 35 P-33

Soln:

$$\Gamma(x) = \frac{V_r}{V_f} \quad \Gamma = 0.6 \angle -30^\circ$$

$$V_f = 25 \sin(\beta x - 75^\circ)$$

$$\left| \frac{V_r}{V_f} \right| = |\Gamma|$$

$$|V_r| = 0.6 \times 25 = 15$$

$$V_{r \text{ phase}} - V_{f \text{ phase}} = \Gamma_{\text{phase}}$$

$$V_{r \text{ phase}} = -30 - 75^\circ = -105^\circ$$

$$V_r = 15 \sin(-\beta x - 105^\circ)$$

due to direction change

Ques 36 P-33
Soln:

$$2\beta x_{\min} = (2n+1)\pi + 0$$

$$2 \times \frac{2\pi}{\lambda} \times x_{\min} = (2n+1)\pi \Rightarrow \frac{\pi}{6}$$

for $n=0$

$$x_{\min} = \frac{5\lambda}{24} \quad 1^{\text{st}} \text{ minima}$$

④ $\frac{P_1}{P_2}$ or $\log \frac{P_1}{P_2} = \underline{\text{bels unit}}$

$$10 \log \frac{P_1}{P_2} = \underline{\text{decibels unit}}$$

for 3 db $P_1/P_2 = 2$

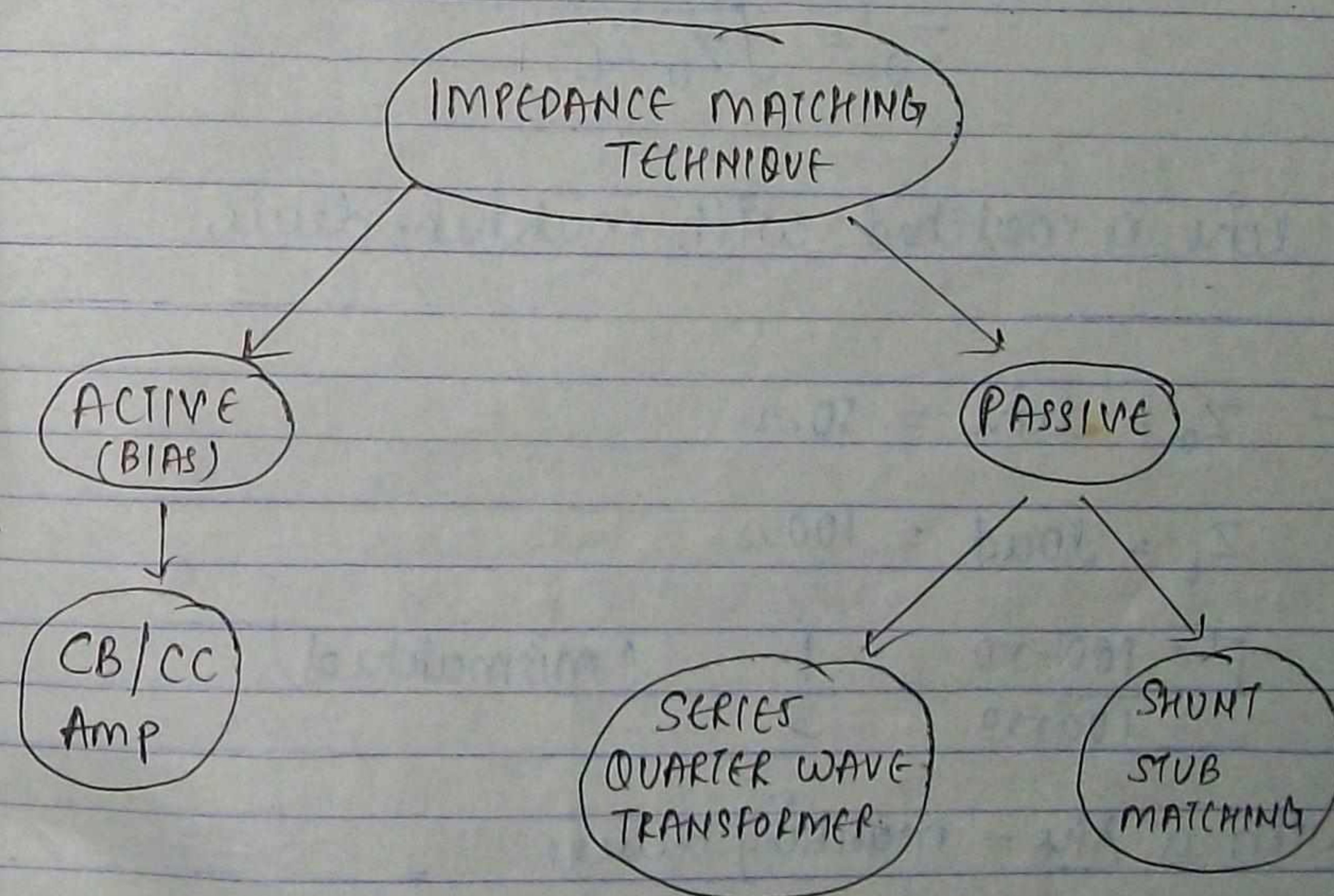
$$1/2 = -3 \text{ db}$$

$$30 \text{ db} = 1000$$

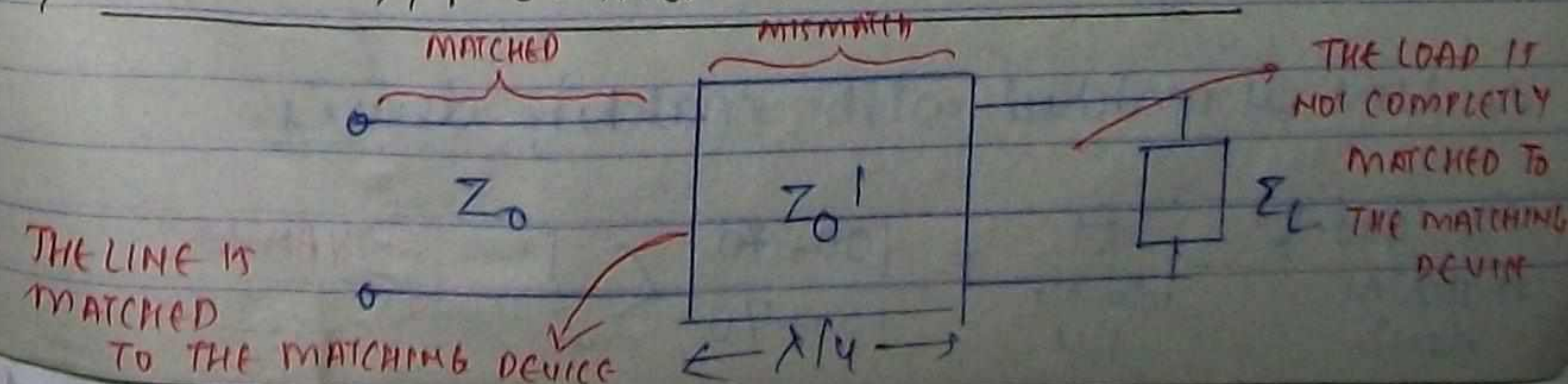
$$\ln\left(\frac{P_1}{P_2}\right) = \frac{P_1}{P_2} \quad \underline{\text{Nepers}}$$

⇒ IMPEDANCE MATCHING TECHNIQUES -

- Typical tx. lines have a very small impedance of the order of few Ω -100 Ω where as loads are high impedances and hence devices are connected in b/w for impedance matching.



⇒ SERIES - $\lambda/4$ QUARTER WAVE TRANSFORMER -



Z_0 line has its termination as Z_{in} of the $\lambda/4$ line.

$$Z_0 \text{ lines termination} = \frac{Z_0'^2}{Z_L} \quad (\lambda/4 \text{ line})$$

for matching $Z_0 = \frac{Z_0'^2}{Z_L}$

$$Z_0' = \sqrt{Z_0 \cdot Z_L}$$

$\therefore$ line is matched with matching device

Ex - $Z_0 = \text{line} = 50 \Omega$

$$Z_L = \text{load} = 100 \Omega$$

$$\Gamma = \frac{100-50}{100+50} = \frac{1}{3} \quad (\text{mismatched})$$

with a $\lambda/4$ - matching device

$$Z_0' = \sqrt{50 \times 100} \approx 70 \Omega$$

$\therefore$ line is matched with matching device

for load & matching device

$$\Gamma_{\text{with } \lambda/4 \text{ line}} = \frac{100-70}{100+70} < \frac{1}{3}$$

ADVANTAGE

NOTE -

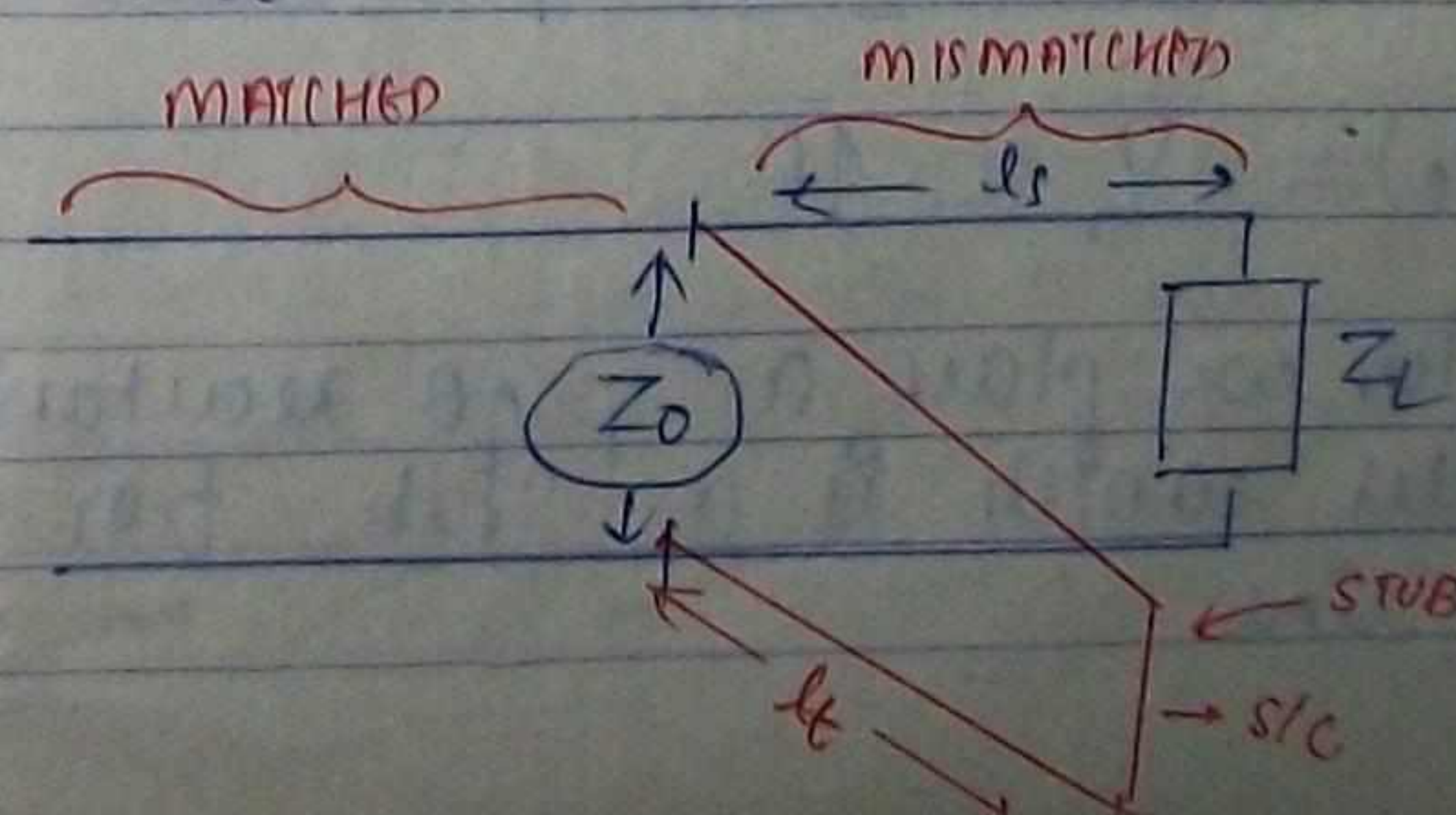
- The technique cannot be used for a wide range of frequencies as the length of the matching device is λ dependent.

- The technique is used only for resistive load as Z_0' is real and lossless, with Z_L real and Z_0 real.

(*) The technique is excellent for matching the line but for the load it is an improvement over the original mismatch.

$\Rightarrow$ SHUNT-STUB MATCHING -

- A stub is a short circuit line of an appropriate length placed at a precalculated distance, so that the junction impedance is Z_0 at that point and the line is matched towards the source.



DESIGN OF A STUB -

- Identify a length on a line from load side where the impedance has real part Z_0 i.e.

$$Z(x) = Z_0 \left[\frac{Z_L \cos \beta x + j Z_0 \sin \beta x}{Z_0 \cos \beta x + j Z_L \sin \beta x} \right]$$

$$\text{Real} \left[\frac{Z_L \cos \beta l_s + j Z_0 \sin \beta l_s}{Z_0 \cos \beta l_s + j Z_L \sin \beta l_s} \right] = 1$$

l_s = position of stub

$$l_s = \frac{\lambda}{2\pi} \tan^{-1} \left(\frac{Z_L}{Z_0} \right)$$

- The admittance at this point is

$$Y(l_s) = Y_0 \pm jB$$

To which we place a $-jB$ reactance in parallel which is a stub for a short circuit line.

$$Y(l_s) = Y_0 \pm jB \quad \boxed{-jB} \text{ - STUB.}$$

- A stub is a l_s length short circuit line such that

$$Z_{sc} = j Z_0 \tan \beta l_s = -\text{Imag} [Z(l_s)]$$

So, that,

$$l_s = \frac{\lambda}{2\pi} \tan^{-1} \left(\frac{\sqrt{Z_L Z_0}}{Z_L - Z_0} \right)$$

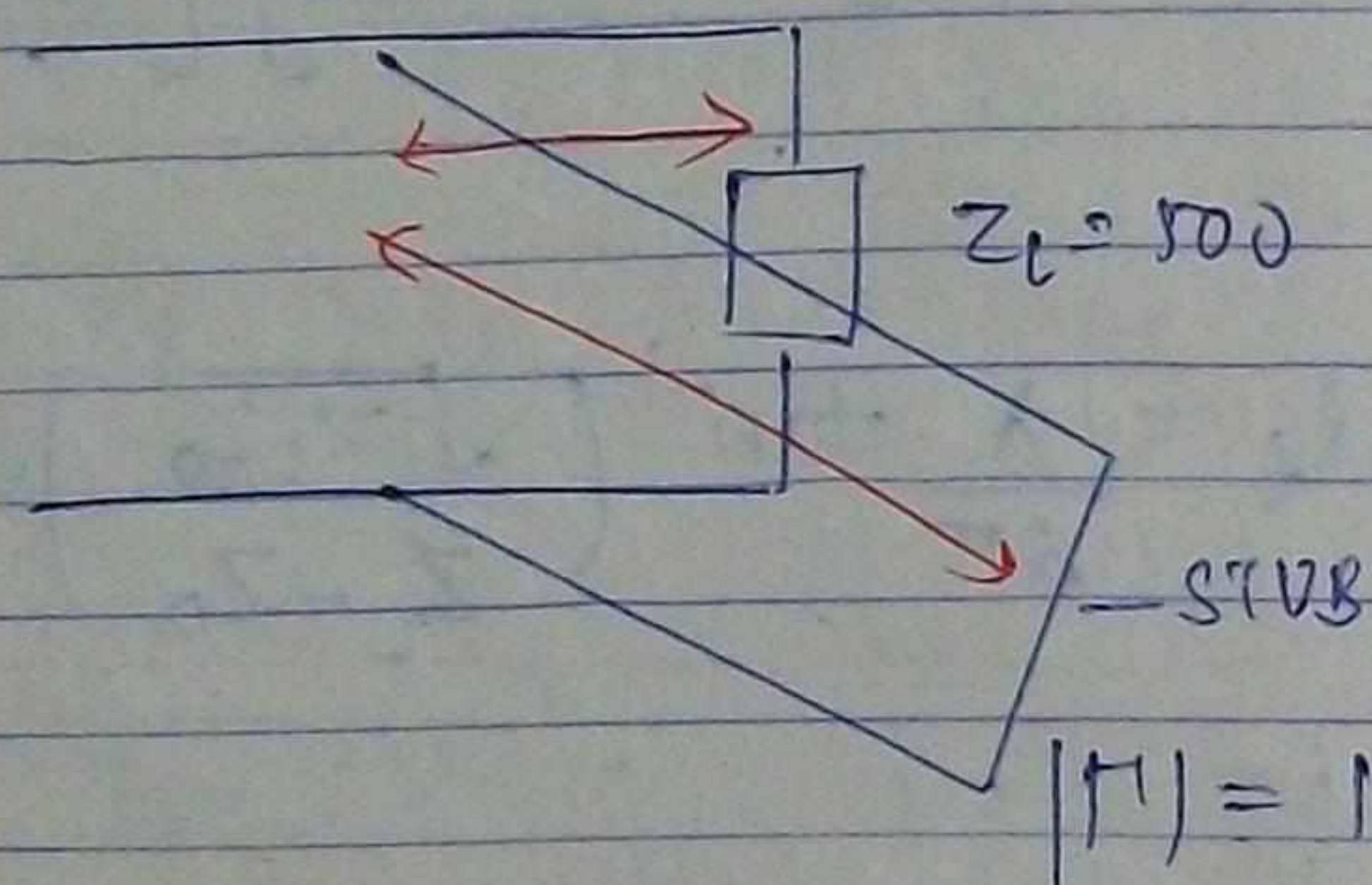
DISADVANTAGE -

- The length and position of stub are both frequency dependent and hence moving the stub or adjusting the stub length as the frequency changes is mechanically inconvenient, hence this technique is used for limited frequencies.
- l_s adjustment is simple by using movable shorts. Hence short circuit stubs are preferred to open circuit stub and double stub matching is preferred for wide range of frequencies.

DOUBLE STUB MATCHING -

- Double stub matching involves two fixed stubs at l_{s1} and l_{s2} with a spacing of $(0.18\lambda - 0.24\lambda)$ and l_{e1} and l_{e2} both varied with frequency.

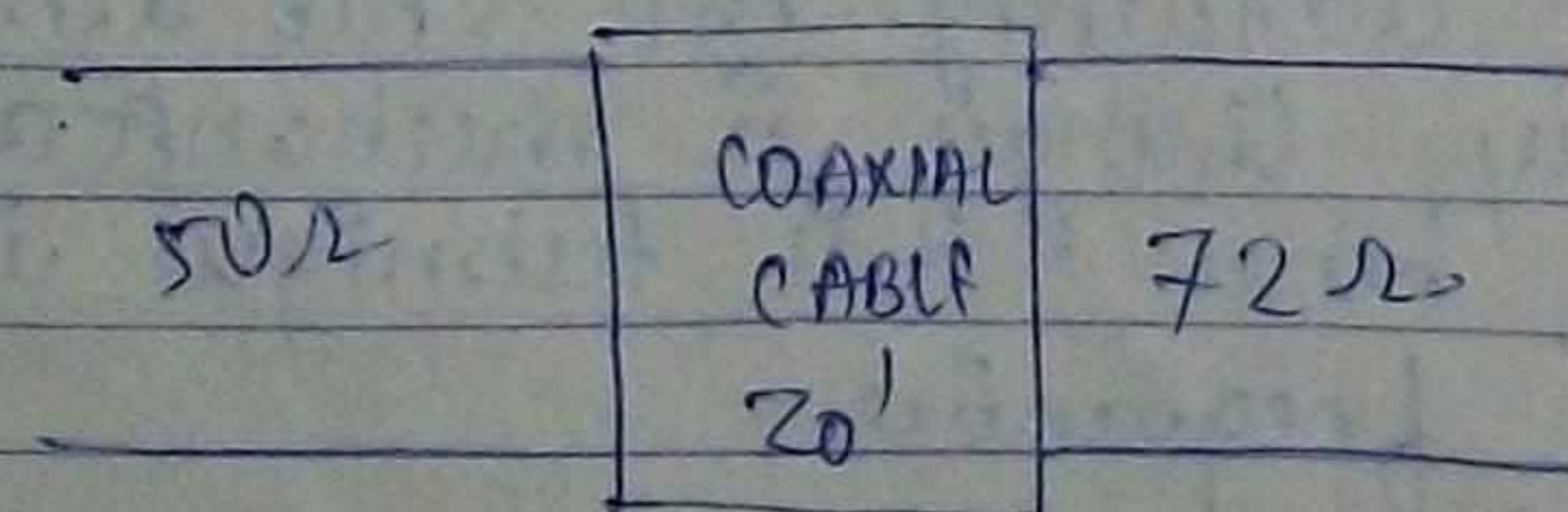
Que 32. P-33
solⁿ:



$$V_{SWR} = \frac{Z_L}{Z_0} = \frac{500}{100} = 5$$

$$SWR = \infty$$

Que 43. P-34,
solⁿ:



$$Z_0' = \sqrt{50 \times 72} = 60$$

$$60 = 60 \ln(b/a)$$

$$\sqrt{\epsilon_r} = 1$$

$$\ln(b/a) = 1$$

$$b = 2.7 \times 10$$

$$b = 27 \text{ mm}$$

⇒ SCHMITH - CHART -

- To calculate Γ and VSWR

from known the Z_L and Z_0

after getting $\frac{Z_L}{Z_0} = \text{normalised load impedance.}$

$$\frac{Z_L}{Z_0} = R + jX$$

- It is a rectangular graph (polar plot)

$$\Gamma_r \text{ vs } \Gamma_i \text{ or } |\Gamma| \text{ vs } \theta.$$

and contains 2 families of circles

- Constant R circles
- Constant X circles

$$\begin{aligned} \Gamma_r + j\Gamma_i &= \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{Z_L/Z_0 - 1}{Z_L/Z_0 + 1} \\ &= \frac{R + jX - 1}{R + jX + 1} = \frac{(R-1) + jX}{(R+1) + jX} \\ &= \Gamma_r + j\Gamma_i \end{aligned}$$

$$\left(\Gamma_r - \frac{R}{R+1}\right)^2 + \Gamma_i^2 = \left(\frac{1}{R+1}\right)^2$$

CONSTANT
'R' CIRCLES

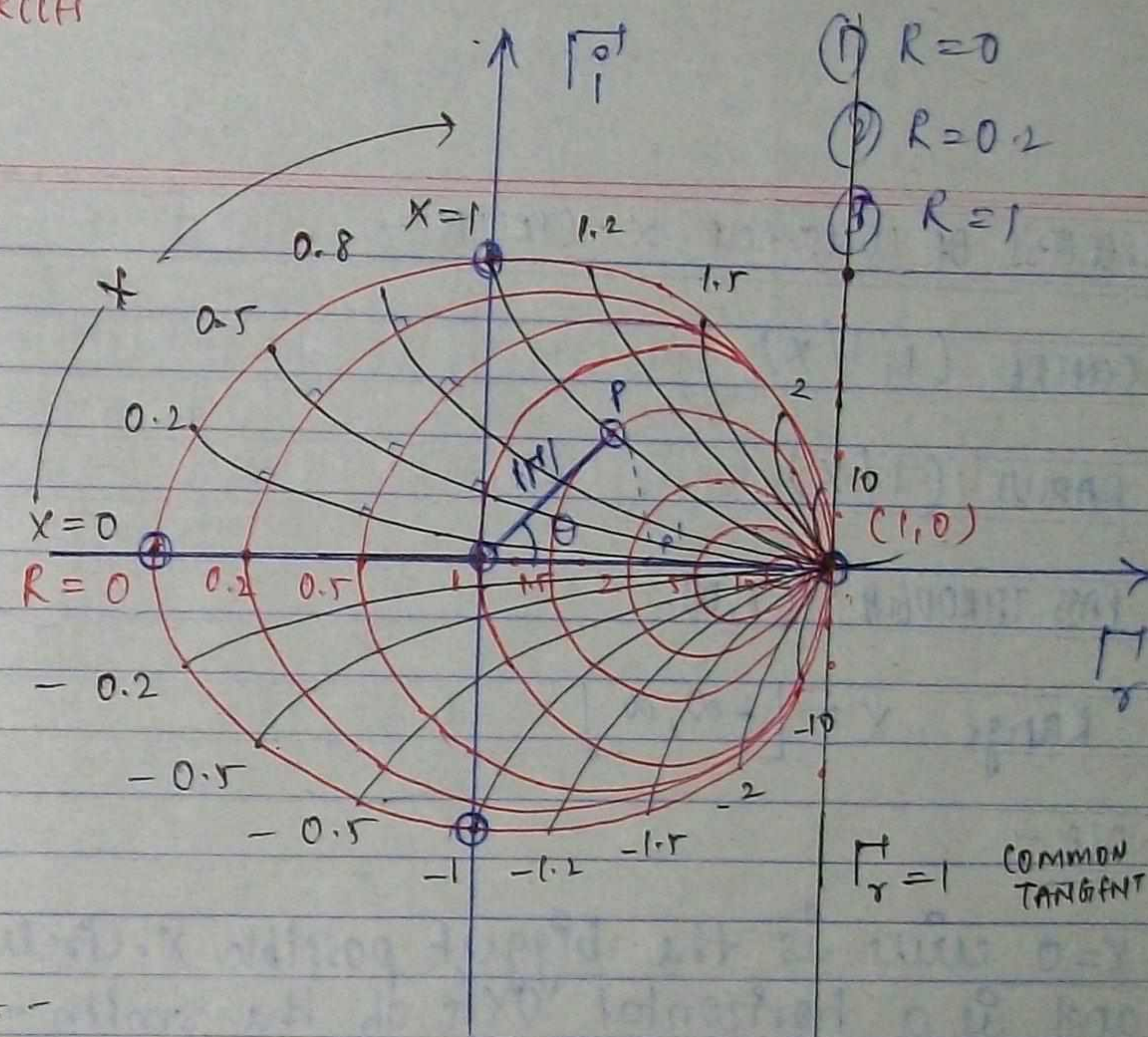
$$\left(\Gamma_r - 1\right)^2 + \left(\Gamma_i - \frac{1}{X}\right)^2 = \left(\frac{1}{X}\right)^2$$

CONSTANT
X - CIRCLE

PROPERTIES OF CONSTANT R CIRCLES -

- ① CENTRE $\left(\frac{R}{R+1}, 0\right)$
- ② RADIUS $\left(\frac{1}{R+1}\right)$
- ③ ALL PASS THROUGH $= (1, 0)$
- ④ Range of R, $R = [0, \infty]$

- X CIRCLE
- R CIRCLE



NOTE -

- The $R=0$ circle is the biggest possible R-circle and has Radius 1 and it is the boundary of the Smith chart.
- All circles with $R \rightarrow 0$ to 1 lie in the four quadrant, but circle with $R > 1$ lie in Ist and IVth quadrant only.
- All the R-circles are concurrent and have a common tangent at $\Gamma_r = 1$ axis.

PROPERTIES OF CONSTANT X CIRCLES -

- ① CENTRE $(1, 1/X)$
- ② RADIUS $(1/X)$
- ③ PASS THROUGH $(1, 0)$
- ④ Range $X \in [-\infty, \infty]$

NOTE -

- $X=0$ circle is the biggest possible X -circle and is a horizontal axis of the Smith-chart i.e. Γ_r axis.
 - The circles with $X = 0$ to 1 lie in the first and second quadrants, but circles with $X > 1$ lie in the first quadrant only.
 - All the X circles are concurrent and have a common tangent Γ_r axis.
- ⊛ As we horizontally moved from left to right, the R values increase on the $X=0$ circle and as we move clockwise

$X=0$ CIRCLE $R \uparrow$

$R=0$ CIRCLE $X \uparrow$

On the $R=0$ circle the X -values increase.

- ⊛ The centre of R -circles are on the common-tangent of X -circles and vice versa. Hence R and X constitute a pair of orthogonal circles.

EXERCISE -

① $Z_L = jR_0 \quad Z_0 = R_0$

$$\frac{Z_L}{Z_0} = j = 0 + j1 \quad (R=0, X=1)$$

$$\Gamma = j \quad (\text{Inductive load lossless line})$$

② $Z_L = -jR_0 \quad Z_0 = R_0$

$$\frac{Z_L}{Z_0} = 0 - j1 \quad (R=0, X=-1)$$

$$\Gamma = -j \quad (\text{Capacitive load lossless line})$$

③ $Z_L = 0 \quad Z_0 = R_0$

$$Z_L/Z_0 = 0 + j0 \quad (R=0, X=0)$$

$$\Gamma = -1 \quad (\text{Short circuit line})$$

$$(4) \quad Z_L = \infty \quad Z_0 = R_0$$

$$Z_L/Z_0 = \infty + j\infty \quad (R=\infty, X=\infty)$$

$$\Gamma = 1 \quad (\text{open circuit line})$$

$$(5) \quad Z_L = R_0 \quad Z_0 = R_0$$

$$Z_L/Z_0 = 1 + j0 \quad (R=1, X=0)$$

$$\Gamma = 0$$

PROCEDURE TO CALCULATE Γ -

- Identify the impedance as the intersection of R and X circles on the Smith chart (P). Join the point (P) with the origin the segment OP length is $|\Gamma|$.
- The angle made by the segment with the Γ real axis is the phase of Γ .

PROCEDURE TO CALCULATE VSWR -

- with the known OP length identify P' on the Γ real axis with $OP = OP'$. The R -circle value at P' is VSWR.

OP length is $|\Gamma|$

if $X=0$

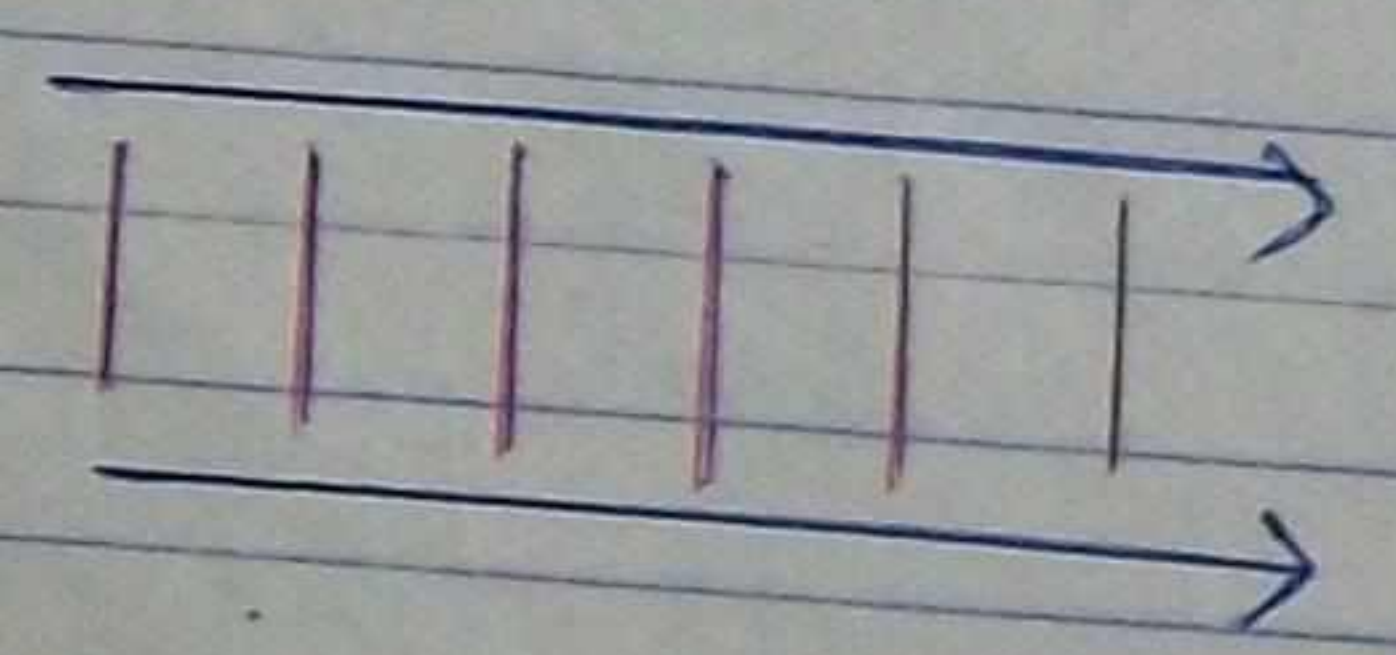
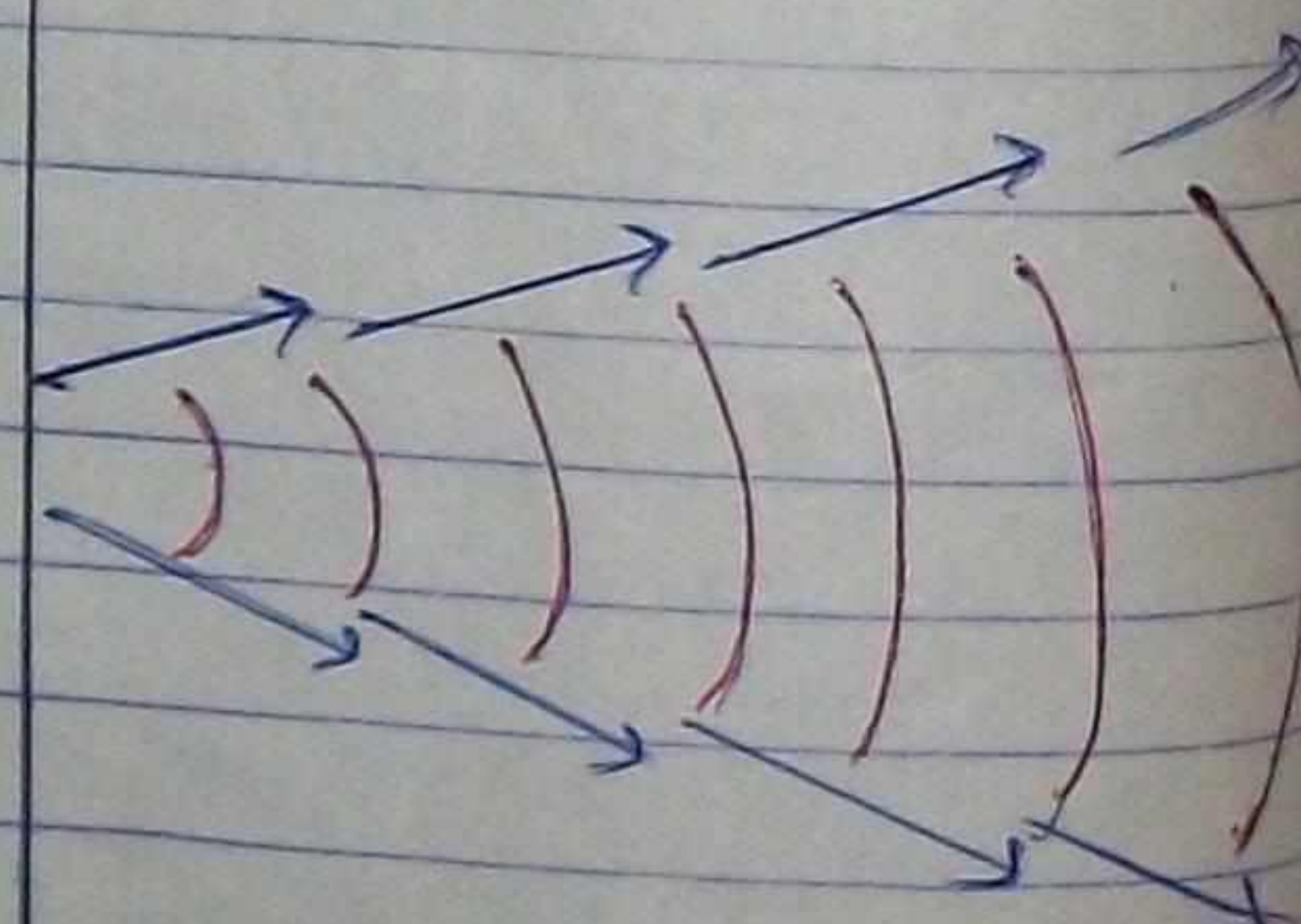
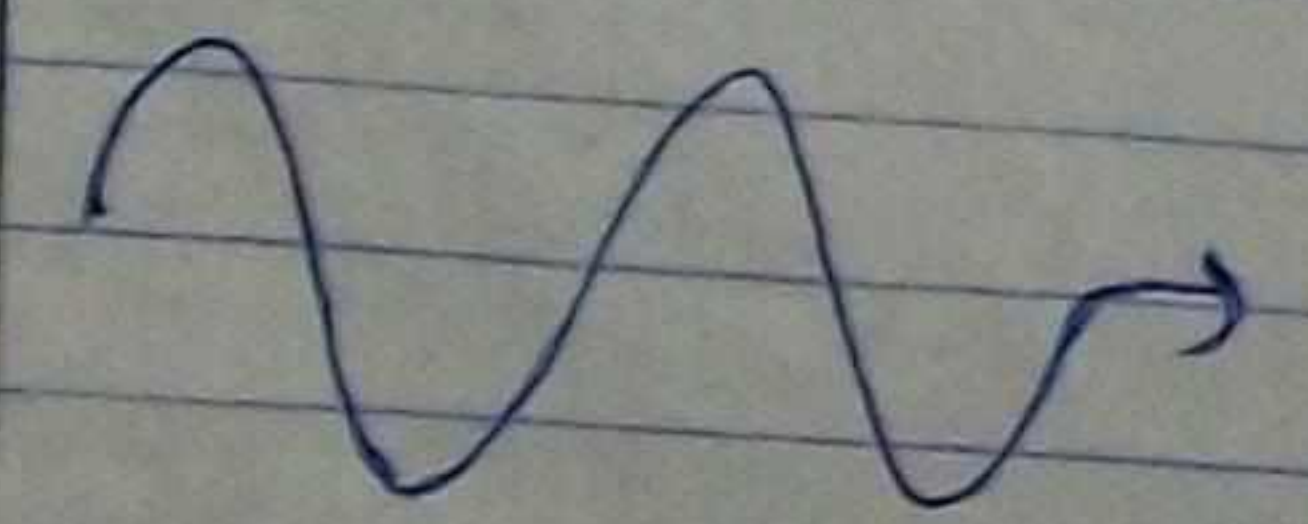
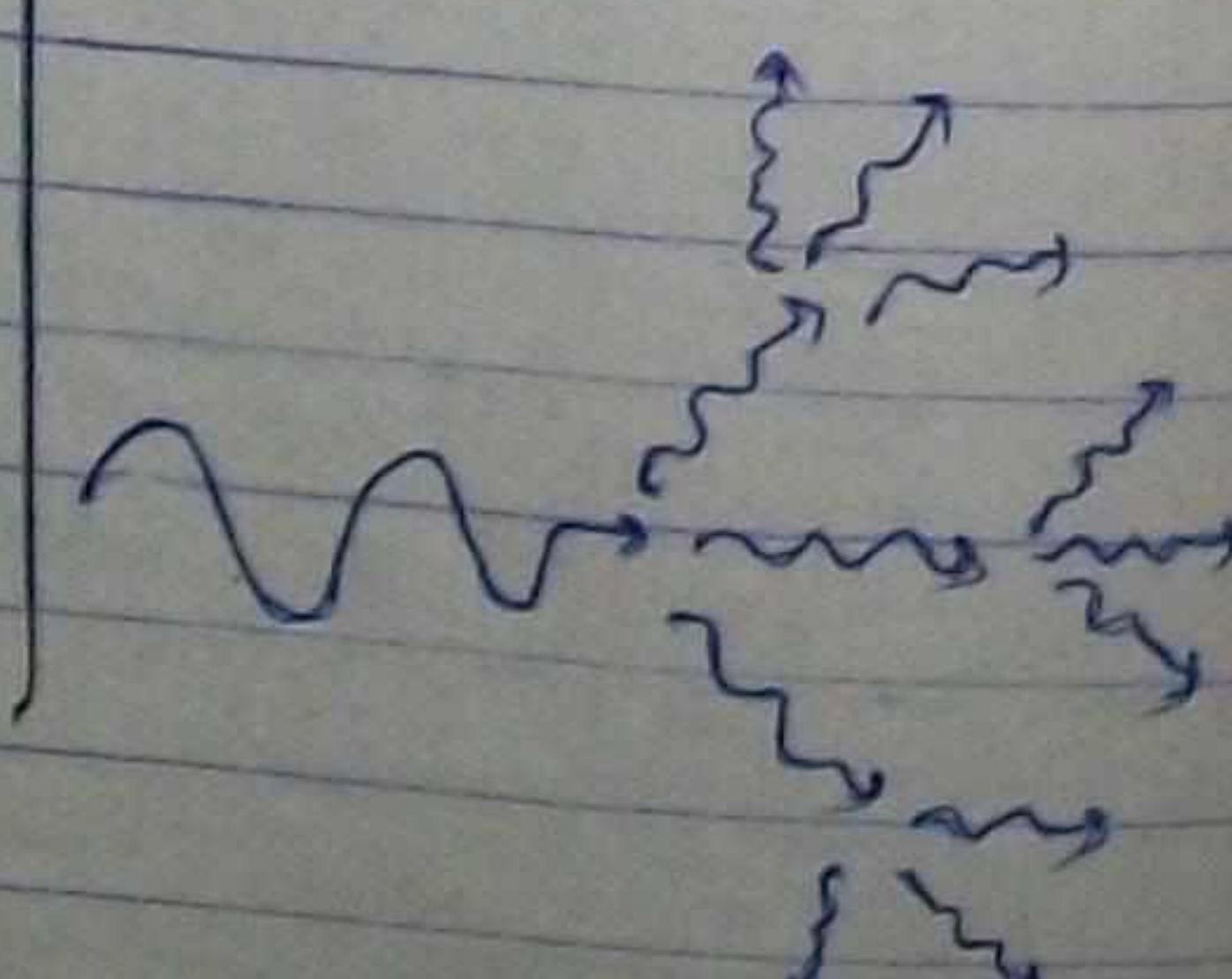
$$R = \frac{Z_L}{Z_0} = \text{VSWR}$$

- * Three scales on the Smith chart are: reactance X , phase θ and length l which are dependent quantity. Hence single scale is sufficient.

WAVEGUIDES

8/10/11

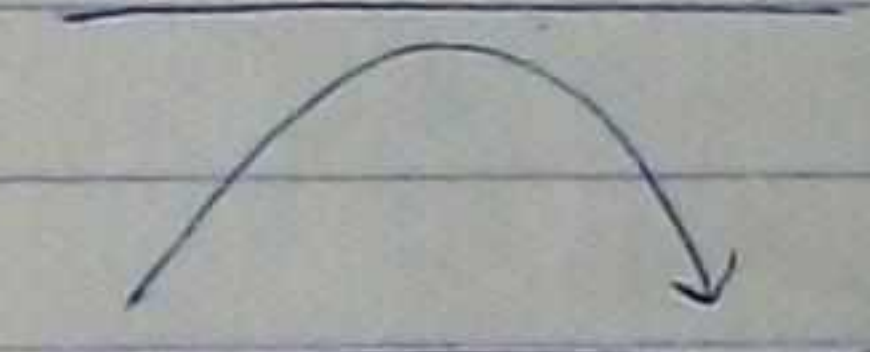
- Most electromagnetic waves are dispersive in nature and hence form spherical wavefronts as they travel forward. Every ray is a source of secondary emissions. This is HUYGENS WAVE PRINCIPLE. Hence all EM waves have diffraction and diffusion properties.

	UNIFORM PLANEWAVE	DISPERSIVE BEAM
MATHEMATICAL EQUATIONS	$E(z,t) \hat{x}$ $H(z,t) \hat{y}$	$E(x,y,z,t) \hat{x}$ $H(x,y,z,t) \hat{y}$
DIAGRAM		
PROPAGATION		

- Dispersive beams are very useful in broadcast application but have a serious disadvantage in point to point communication, hence waveguides are used to confine the wave within a specific bound

- PARALLEL PLANE WAVEGUIDE $\rightarrow E(x,z,t)$
 $H(x,z,t)$ $\rightarrow$ 1 dimensional confinement.

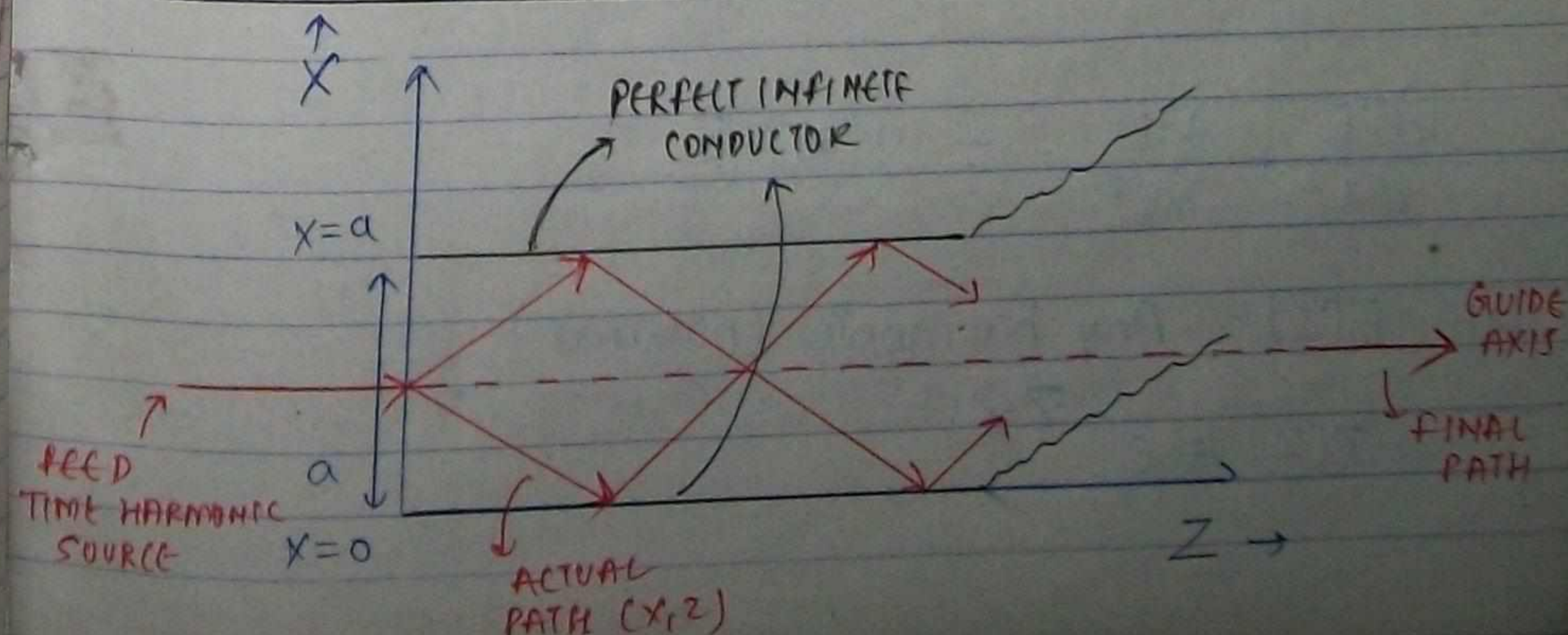
Ex -



Dual propagation Radio wave

- RECTANGULAR WAVEGUIDE $\rightarrow E(x,y,z,t)$
 $H(x,y,z,t)$ $\rightarrow$ 2 dimensional confinement.

$\Rightarrow$ PARALLEL PLANE WAVEGUIDE -



BOUNDARY CONDITION AT THE CONDUCTOR WALLS -

$$E_{\text{TANG}} = 0 - \text{E-FIELD AT THE SURFACE} = 0$$

$$E(x)_{\text{TANG}} = 0 \quad \text{at } x=0 \text{ and } x=a.$$

PROPAGATION ALONG THE GUIDE AXIS -

Let us consider Helmholtz's Equation

$$\nabla^2 E = \gamma^2 E$$

γ = propagation constant in free space between the guide walls.

$$\gamma = j\omega \sqrt{\mu\epsilon}$$

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial z^2} = -\omega^2 \mu\epsilon E$$

Let $E(z)$ = Any harmonic (natural)

$$E(z) = e^{-\gamma z}$$

(γ_z) $\bar{\gamma}$ = propagation constant in the z direction
= " " " " along guide walls.

$$\frac{\partial^2 E}{\partial x^2} + \bar{\gamma}^2 E = -\omega^2 \mu\epsilon E$$

$$\frac{\partial^2 E}{\partial x^2} = -(\bar{\gamma}^2 + \omega^2 \mu\epsilon) E$$

The wave has to be harmonic in x as seen but has

$$\gamma_x^2 = \bar{\gamma}^2 + \omega^2 \mu\epsilon$$

γ_x = propagation constant in x-direction.

$$\frac{\partial^2 E}{\partial x^2} = -\gamma_x^2 E$$

Soln of above eqn is:

$$E(x) = C_1 \sin(\gamma_x \cdot x) + C_2 \cos(\gamma_x \cdot x)$$

Applying the boundary conditions:

① at $x=0$ $E(x)_{\text{TANG}} = 0$

$$E(0)_{\text{tang}} = C_1 \cdot 0 + C_2 = 0$$

$$\therefore C_2 = 0$$

NOTE -

In the restricted side or between the guide walls the tangential E-field should be a 'SINE' harmonic only.

$$(2) E(a)_{\text{tang}} = C_1 \sin(\gamma_x \cdot a) = 0$$

$$\gamma_x \cdot a = m\pi$$

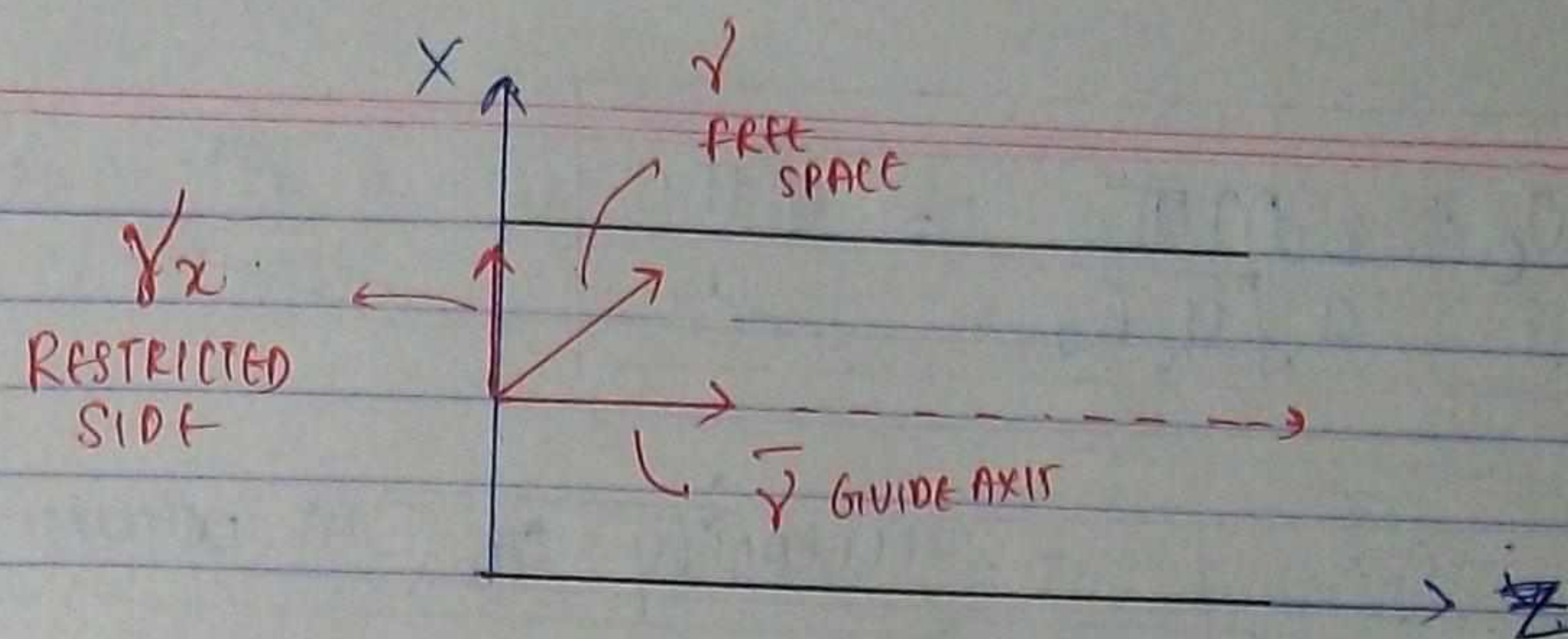
$$\gamma_x = \frac{m\pi}{a}$$

NOTE -

The 'x' dimension propagation constant has to be a discrete set of values

$$\text{i.e. } m = 0, 1, 2, 3, \dots$$

$$\therefore \bar{\gamma} = \sqrt{\left(\frac{m\pi}{a}\right)^2 - \omega^2 \mu_0 \epsilon_0}$$



CONCEPT (1) : CUTOFF FREQUENCY OF THE GUIDE -

$$- \text{If } \omega^2 \mu_0 \epsilon_0 < \left(\frac{m\pi}{a}\right)^2$$

$$\bar{\gamma} = \text{real} = \bar{\alpha} + j0$$

No propagation along guide axis.

It is removed by the guide walls.

$$- \text{If } \omega^2 \mu_0 \epsilon_0 > \left(\frac{m\pi}{a}\right)^2$$

$$\bar{\gamma} = 0 + j\bar{B} = j\text{mag.}$$

propagation exist, without attenuation.

(*) Hence every waveguide has a cutoff frequency below which there is no propagation along the axis i.e.

$$\omega_c = \frac{m\pi}{a\sqrt{\mu_0\epsilon_0}} = \frac{m\pi c}{a}$$

where,

$$c = \frac{1}{\sqrt{\mu_0\epsilon_0}} = \text{velocity of EM wave}$$

$$f_c = \frac{mc}{2a}$$

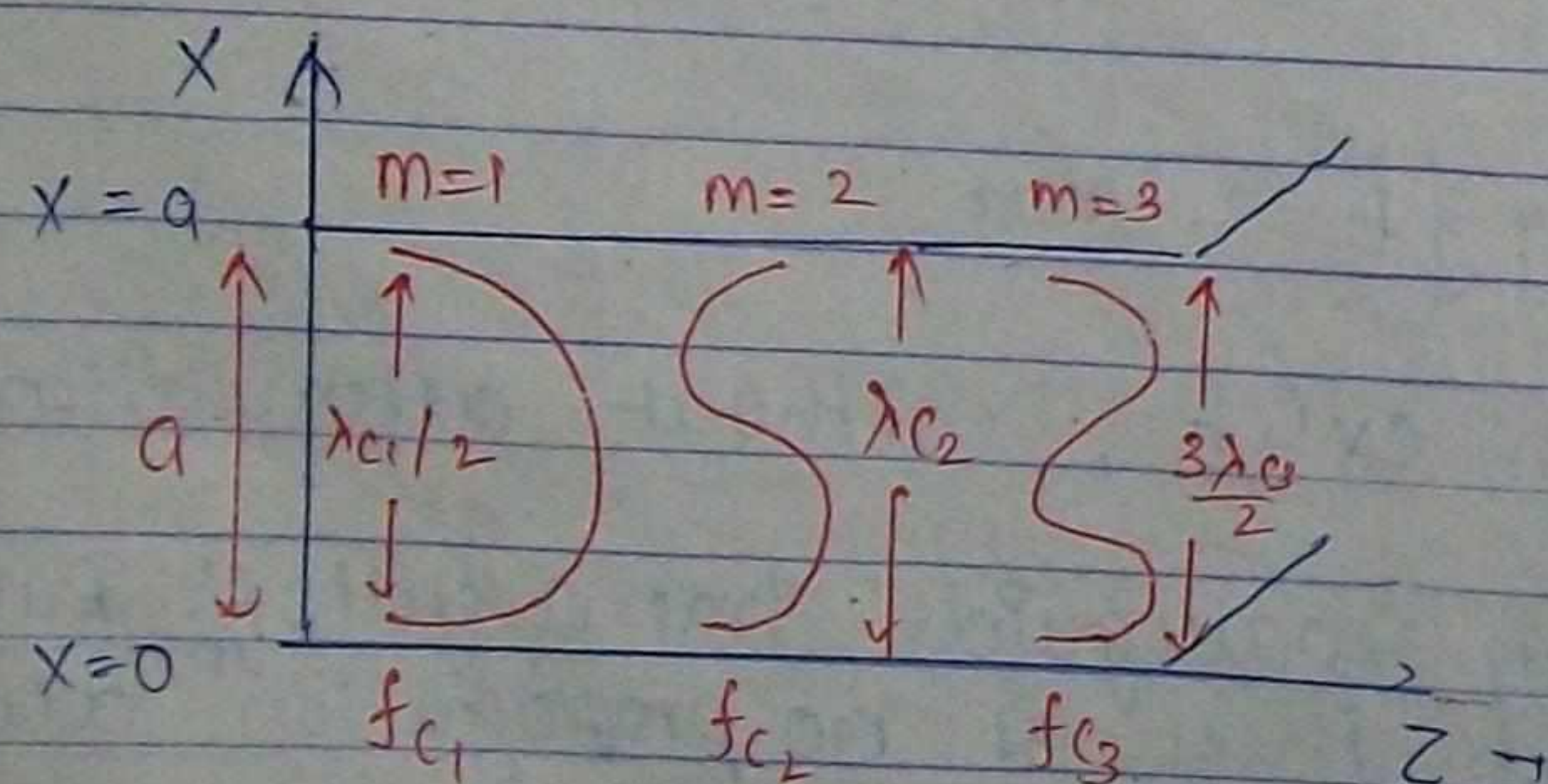
$$\frac{c}{\lambda_c} = \frac{mc}{2a}$$

$$\lambda_c = \frac{2a}{m}$$

$$f > f_c \text{ or } \lambda < \lambda_c$$

$$\lambda = \frac{c}{f}$$

At exact f_c of $m=1, 2, 3, \dots$
where $\gamma=0$.



ie the wave will oscillate between the conductor walls.

CONCEPT ②: WAVE ANGLE OR TILT ANGLE -

$$\bar{\gamma} = j \sqrt{\omega^2 \mu_0 \epsilon_0 - \left(\frac{m\pi}{a}\right)^2} = j \bar{\beta}$$

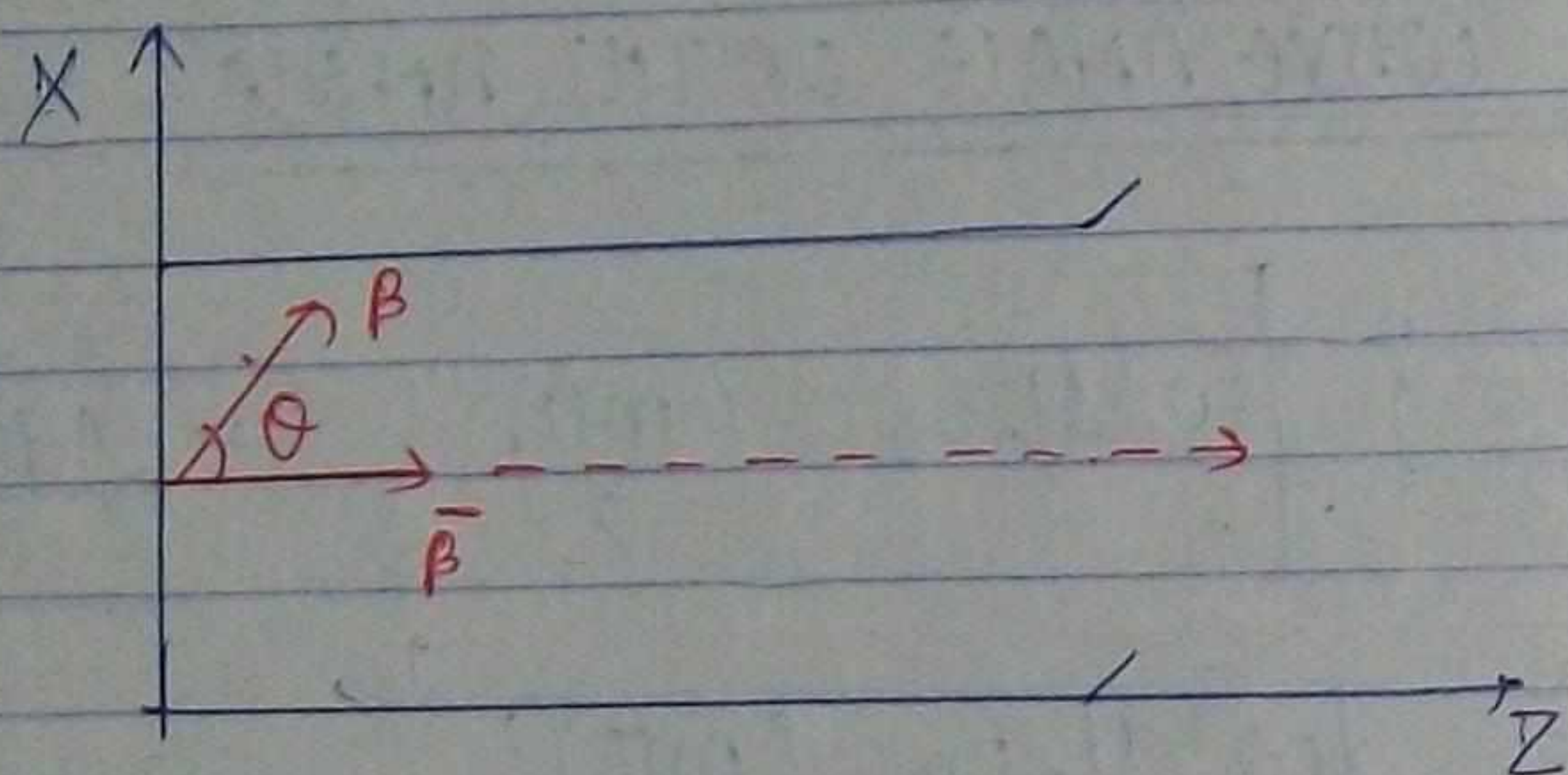
$$\bar{\beta} = \sqrt{\omega^2 \mu_0 \epsilon_0 - \left(\frac{m\pi}{a}\right)^2}$$

$$\bar{v}_p = \text{phase velocity along the guide axis} = \frac{\omega}{\bar{\beta}}$$

$$\begin{aligned} \bar{v}_p &= \frac{\omega}{\sqrt{\omega^2 \mu_0 \epsilon_0 - \left(\frac{m\pi}{a}\right)^2}} = \frac{1}{\sqrt{\mu_0 \epsilon_0 - \left(\frac{m\pi}{a\omega}\right)^2}} \\ &= \frac{1/\sqrt{\mu_0 \epsilon_0}}{\sqrt{1 - \left(\frac{m\pi}{a\omega\sqrt{\mu_0 \epsilon_0}}\right)^2}} = \frac{1/\sqrt{\mu_0 \epsilon_0}}{\sqrt{1 - \left(\frac{m\pi}{a\omega^2 \mu_0 \epsilon_0}\right)^2}} \end{aligned}$$

$$\bar{v}_p = \frac{c}{\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}} = \frac{c}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} \quad \text{--- (2)}$$

∴ Every wave has a phase velocity greater than c and the velocity is frequency dependent.



$$\bar{B} = B \cos \theta$$

$$\frac{2\pi}{\bar{\lambda}} = \frac{2\pi \cos \theta}{\lambda}$$

$$\bar{\lambda} = \frac{\lambda}{\cos \theta} \quad \text{--- (1)}$$

In the zig-zag path the wavelength along the guide axis appears to be longer than free space wavelength. Hence the velocity along the z -axis is apparently larger than the free space velocity.

$$\bar{v}_p = \frac{c}{\cos \theta}$$

By comparing (1) and (2) Tilt angle or wave angle is:

$$\sin \theta = \frac{f_c}{f}$$

TILT ANGLE
OR
WAVE ANGLE

- Every wave travels with a unique dispersion angle or tilt angle which depend on the frequency of the wave.
- Due to this unique tilt angle and unique velocity multiple frequencies can travel on the same guide and can be separated. (frequency division multiplexing).

CONCEPT (3): GROUP VELOCITY $\bar{v}_g$ ALONG THE GUIDE AXIS -

$$v_p = \frac{\omega}{\beta}$$

$\beta \propto \omega$

β is a linear function of ω ,

↓
Dispersionless beam
↓
uniform plane wave

$$\underline{\underline{\epsilon_x - \beta = \omega \sqrt{\mu_0 \epsilon_0}}}$$

$$\beta = \omega \sqrt{LC} \quad (\text{lossless trans. line})$$

In dispersive beams, β is a non linear function of ω .

$$\beta = \sqrt{\omega^2 \mu_0 \epsilon_0 - \left(\frac{m\pi}{a}\right)^2}$$

$$\boxed{v_g = \frac{d\omega}{d\beta} = \text{group velocity.}}$$

$$\frac{d\beta}{d\omega} = \frac{1}{2} \cdot \frac{2\omega\mu_0\epsilon_0}{\sqrt{\omega^2\mu_0\epsilon_0 - \left(\frac{m\pi}{a}\right)^2}}$$

$$\frac{d\beta}{d\omega} = \frac{\omega\mu_0\epsilon_0}{\omega \sqrt{\mu_0\epsilon_0 - \left(\frac{m\pi}{a\omega}\right)^2}}$$

$$\begin{aligned} \frac{d\beta}{d\omega} &= \frac{\sqrt{\mu_0\epsilon_0}}{\sqrt{1 - \left(\frac{m\pi}{a\omega\sqrt{\mu_0\epsilon_0}}\right)^2}} = \frac{1}{c \sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}} \\ &= \frac{1}{c \cos\theta} \end{aligned}$$

∴

$$\boxed{v_g = c \cos\theta}$$

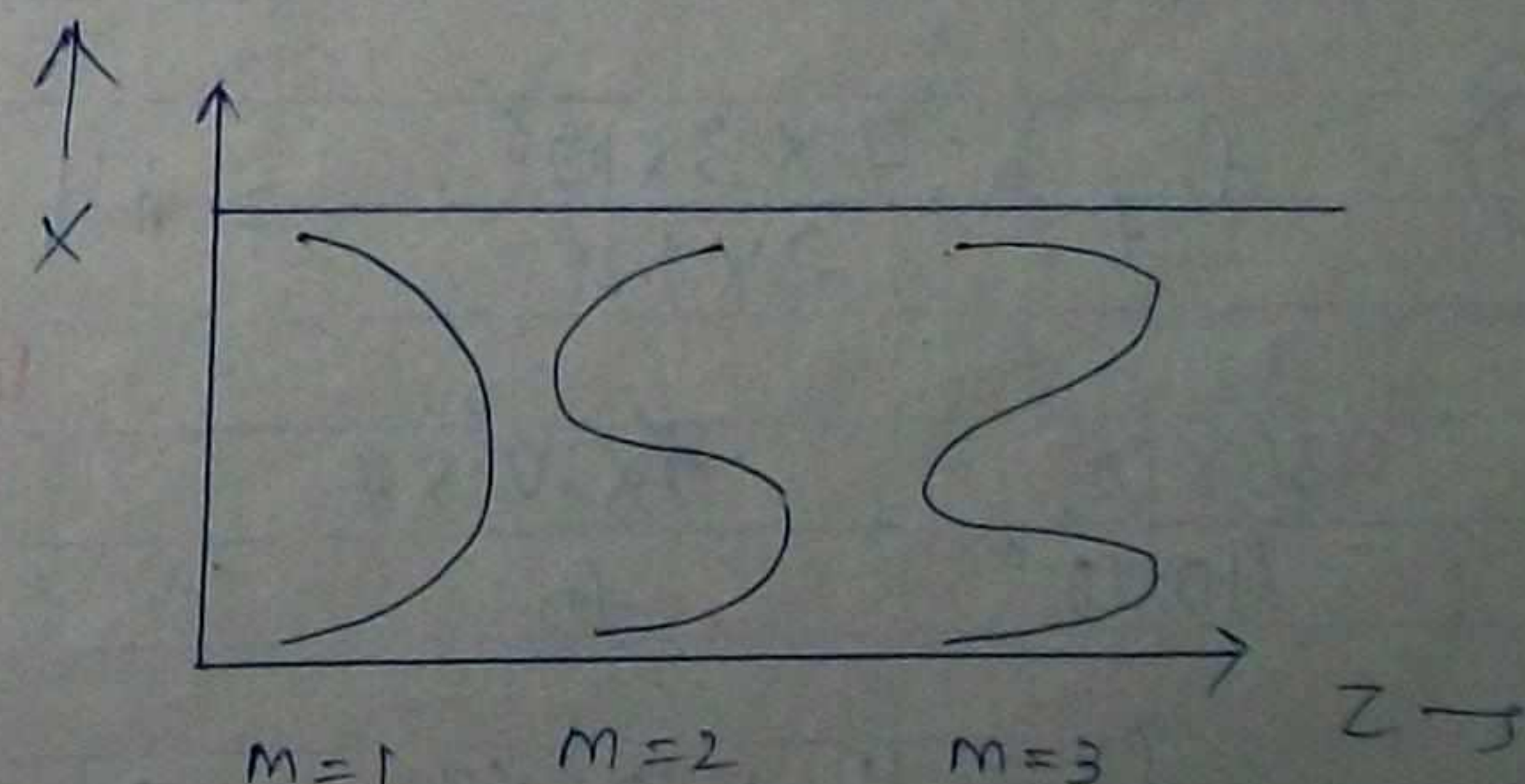
It is the physical rate of energy transfer along the axis (guide axis), due to zig-zag path.

∴

$$\boxed{v_p \cdot v_g = c^2}$$

CONCEPT (4): MODES OF OPERATIONS—

— modes stands for a physical connection mechanism of the feed to the guide. It decides the cutoff frequency and it depends on the no. of wired connections in the feed. However m — stands for no. of maximal for the wave between the guide walls, the no. of half cycles the wave completes between the guide walls.



Que 1. P-39.
Solⁿ: $a = 2.5$

$$f_{c1} = \frac{c}{2a} = \frac{3 \times 10^8}{2 \times 2.5 \times 10^{-2}} = 6 \text{ GHz}$$

$$\sin \theta = \frac{6}{22} = , \theta = \sin^{-1} \left(\frac{6}{22} \right)$$

$$f_{c3} = \frac{3c}{2a} = 18 \text{ GHz}$$

$$\theta = \sin^{-1} (18/22)$$

Que 4. P-39.

Solⁿ: (2) $f_{c3} = \frac{3 \times 3 \times 10^8}{2 \times a} = 36 \times 10^9$

$$a = 1.25 \text{ cm}$$

(4) $f_{c7} = \frac{7 \times 3 \times 10^8}{2 \times 1.25} = 84$

$$\frac{3 \times 12}{36 \times 10^9} = \frac{3 \times 10^8 \times 7}{f_0}$$

$$\frac{f_{c1}}{f_{01}} = \frac{f_{c7}}{f_{07}}$$

$$f_0 = \frac{10}{3} \times 10^8 \times 7 = \frac{84 \times 40}{36} = 93.3 \text{ GHz}$$

Que 3. P-38.

Solⁿ: $f_{c1} = \frac{3 \times 10^8 / \sqrt{9}}{2 \times 3 \times 10^{-2}}$

$$f_{c1} = \frac{5}{3} \text{ GHz} = 1.66 \text{ GHz}$$

$$\sin \theta = \frac{5/3}{2} = 5/6 \quad \theta = \sin^{-1} (5/6)$$

Summary:

1st mode $\rightarrow f_{c1} \rightarrow \infty$
 $1.66 \text{ GHz} \rightarrow \infty$ } support all
 freq. of remaining
 mode

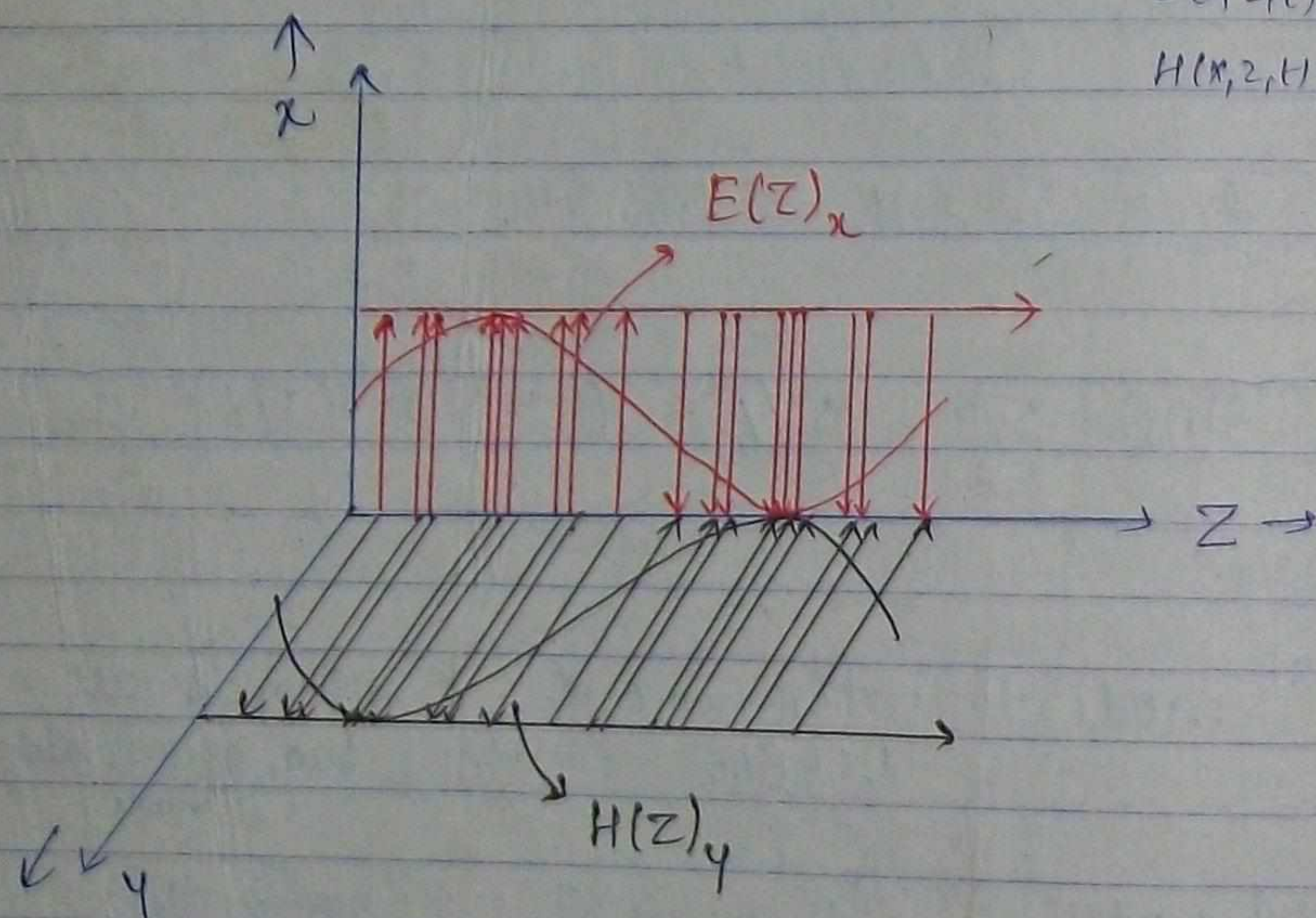
2nd mode $\rightarrow f_{c2} \rightarrow \infty$
 $3.34 \text{ GHz} \rightarrow \infty$ } support all
 freq. of 3, 4, ...
 mode

3rd mode $\rightarrow f_{c3} \rightarrow \infty$
 $5 \text{ GHz} \rightarrow \infty$

⇒ TRANSVERSE ELECTRIC (TE), TM & TEM MODES -

$$E(x, y, z, t)_{(x, y, z)}$$

$$H(x, y, z, t)_{(x, y, z)}$$



$$\nabla \times E = -j\omega\mu H = -\mu \frac{\partial H}{\partial t}$$

$$\nabla \times E = \begin{vmatrix} a_x & a_y & a_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x & E_y & E_z \end{vmatrix}$$

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -j\omega\mu H_x$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z$$

Similarly,

$$\nabla \times H = \epsilon \frac{\partial E}{\partial t} = j\omega\epsilon E$$

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = j\omega\epsilon E_x$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = j\omega\epsilon E_y$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\epsilon E_z$$

Since, All y derivatives are zero

and all z derivatives are $-\gamma$ the same

$$\frac{E(z)}{H(z)} = e^{-\gamma z}$$

$$\therefore \gamma E_y = -j\omega\mu H_x \quad - (4)$$

$$-\gamma E_x - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y \quad - (5)$$

$$\frac{\partial E_y}{\partial x} = -j\omega\mu H_z \quad - (6)$$

$$\text{also } \bar{\gamma} H_y = j\omega\epsilon E_x \quad - (1)$$

$$\bar{\gamma} H_x - \frac{\partial H_z}{\partial x} = j\omega\epsilon E_y \quad - (2)$$

$$\frac{\partial H_y}{\partial x} = j\omega\epsilon E_z \quad - (3)$$

$$\text{Now, } E_x = \frac{\bar{\gamma} H_y}{j\omega\epsilon} \quad \text{from (1)}$$

Putting in (2).

$$-\bar{\gamma} \left(\frac{\bar{\gamma} H_y}{j\omega\epsilon} \right) - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y$$

$$-\bar{\gamma}^2 H_y - j\omega\epsilon \frac{\partial E_z}{\partial x} = \omega^2 \mu \epsilon H_y$$

$$-j\omega\epsilon \frac{\partial E_z}{\partial x} = (\bar{\gamma}^2 + \omega^2 \mu \epsilon) H_y$$

$$\frac{\partial E_z}{\partial x} = -\frac{\bar{\gamma}^2}{j\omega\epsilon} H_y$$

Similarly,

$$\frac{\partial H_z}{\partial x} = -\frac{\bar{\gamma}^2}{j\omega\mu} E_y$$

$$\frac{\partial E_z}{\partial x} = -\frac{\bar{\gamma}^2}{\bar{\gamma}} E_x$$

$$\frac{\partial H_z}{\partial x} = \frac{\bar{\gamma}^2}{\bar{\gamma}} H_x$$

NOTE -

All the field components in any direction are interdependent such that the axial components along z decide the other components.

If $E_z = 0$

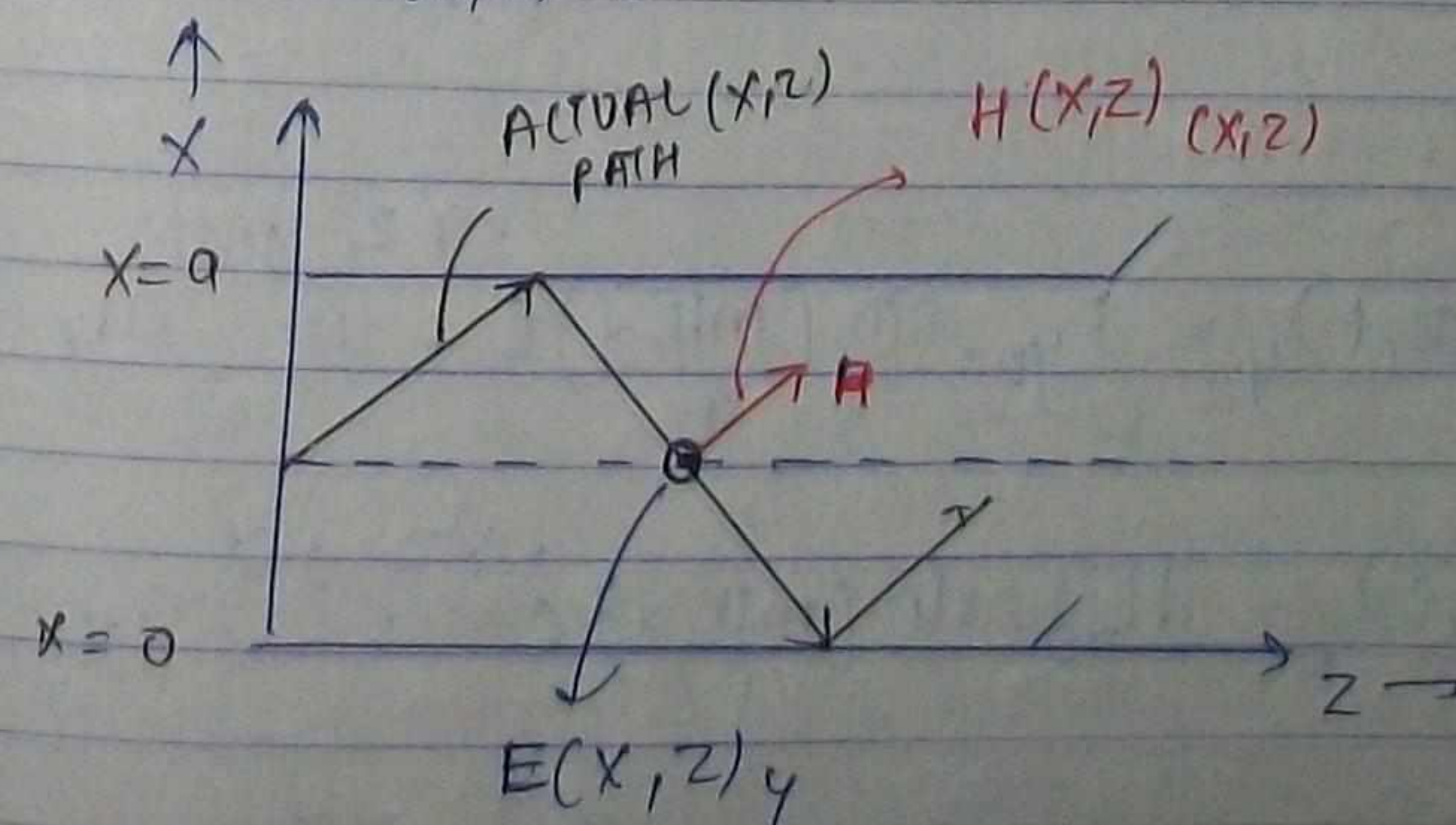
E_y, H_x, H_z are the existing components.

$$E(x, z, t) y$$

$$H(x, z, t)_{(x, z)}$$

TE WAVE

$E \perp H \perp \text{Prop} \perp E$



$E \perp \text{Guide axis}$

TRANSVERSE ELECTRIC WAVE

If $H_z = 0$, the wave

$$E(x, z, t)_{(x, z)}$$

$$H(x, z, t)_y$$

TRANSVERSE
MAGNETIC WAVE

⇒ TE WAVE SOLUTIONS -

$$E_z = 0$$

$$E(x, z, t)_y$$

$$H(x, z, t)_{(x, z)}$$

$$E(z)/H(z) \rightarrow e^{-\gamma z} \quad (\text{Natural harmonic})$$

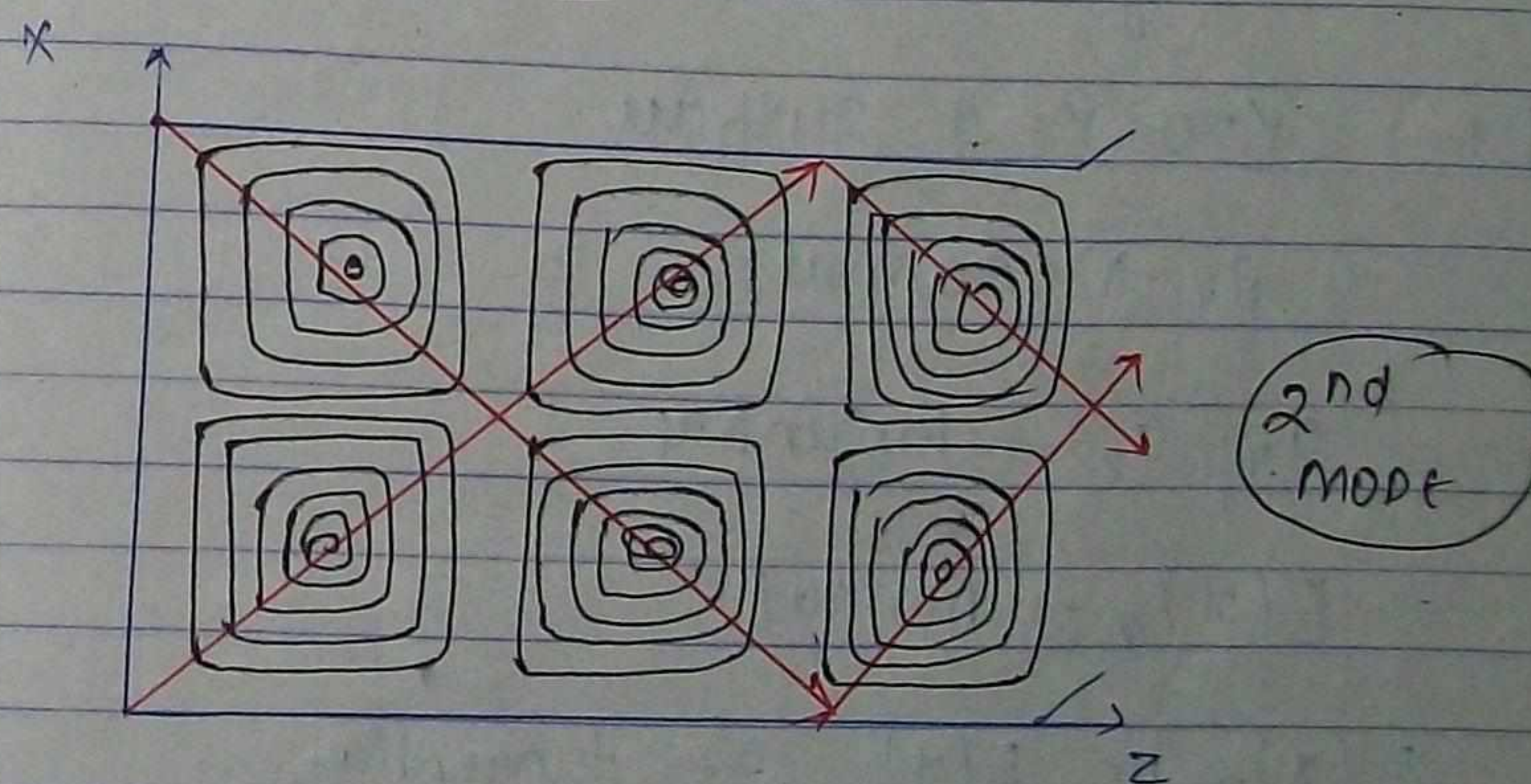
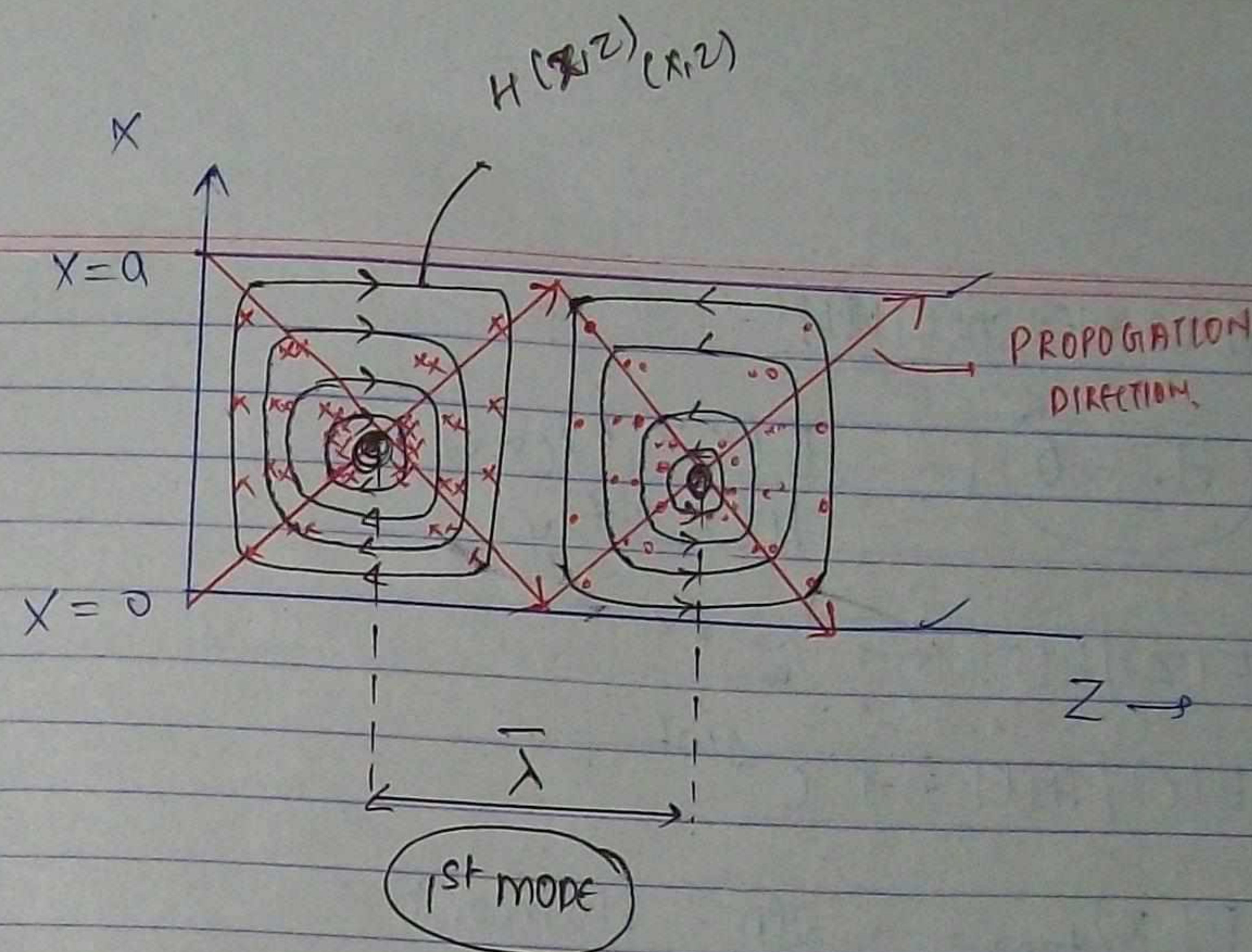
$$E(t)/H(t) \rightarrow e^{j\omega t} \quad (\text{Any source harmonic})$$

$$E(x)/H(x) \rightarrow \sin \quad (\text{Harmonic at } \gamma_x \text{ rate})$$

$$\therefore E_y = E(x, z, t)_y = E_{y0} \cdot \sin\left(\frac{m\pi x}{a}\right) e^{-\gamma z} e^{j\omega t} a_y$$

$$H_x = H(x, z, t)_x = H_{x0} \cdot \sin\left(\frac{m\pi x}{a}\right) e^{-\gamma z} e^{j\omega t} a_x$$

$$H_z = H(x, z, t)_z = H_{z0} \cos\left(\frac{m\pi x}{a}\right) e^{-\gamma z} e^{j\omega t} a_z$$



If $m=0$, TE does not exist.

TE modes are represented by :

$$TE_{10}, TE_{20}, TE_{30} \dots TE_{m0}$$

The second subscript '0' means no y-side guide.

⇒ TM WAVE SOLUTION -

$$\boxed{H_z = 0}, \quad \begin{matrix} E(x, z, t) \\ H(x, z, t) \end{matrix} \begin{matrix} (x, z) \\ y \end{matrix}$$

$$E(z)/H(z) \rightarrow e^{-\gamma z}$$

$$E(t)/H(t) \rightarrow e^{j\omega t}$$

$$E(x)_{\text{tang}} \rightarrow \sin - \text{Harmonic}$$

for $x=0$ $x=a$ surface

$a_x \rightarrow$ normal

$a_y, a_z \rightarrow$ tangential

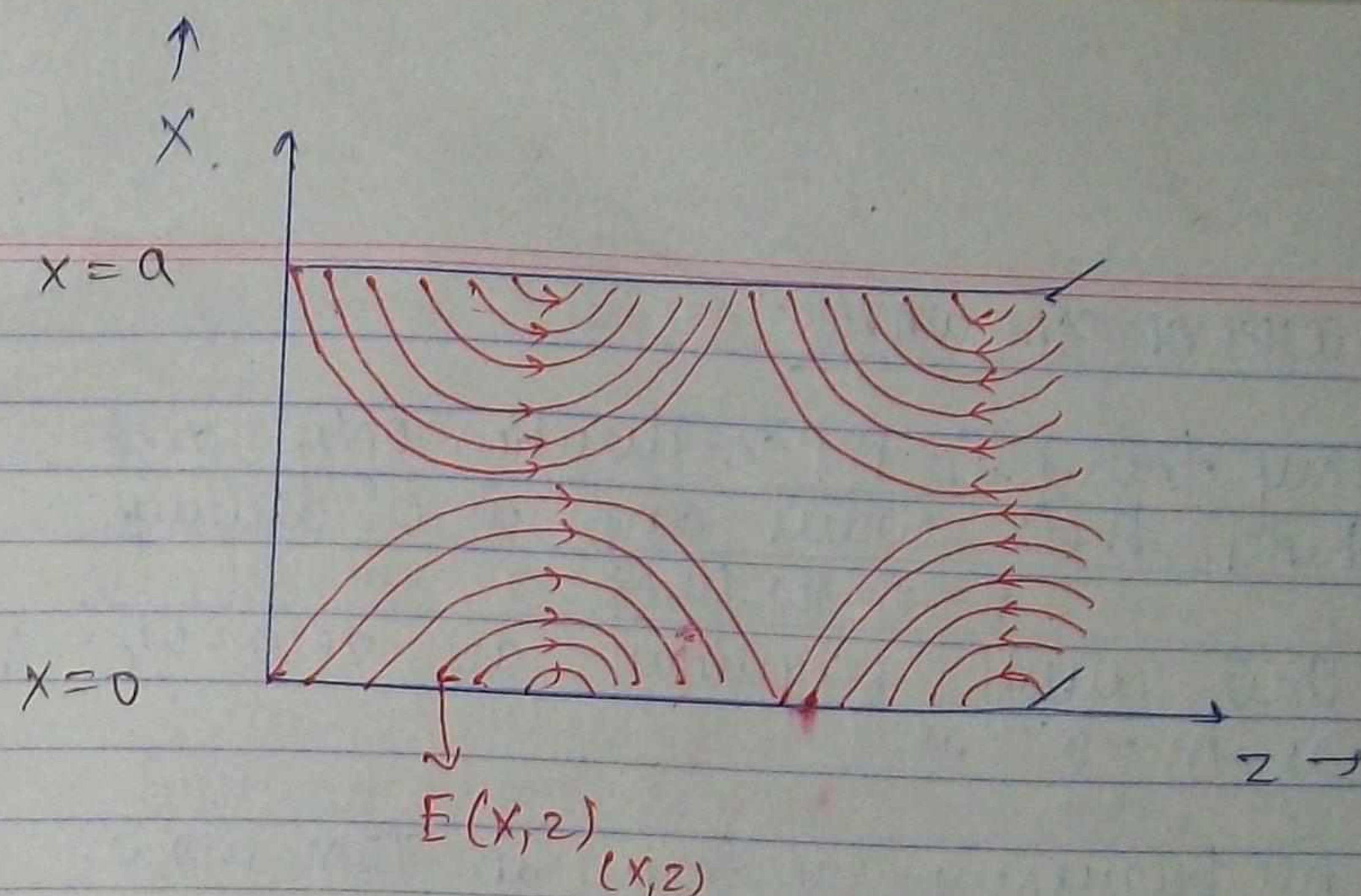
$E(x)_x \rightarrow$ normal

$E(x)_y$ or $E(x)_z \rightarrow$ tangential.

$$E(x, z, t)_x = E_{x0} \cos\left(\frac{m\pi x}{a}\right) e^{-\gamma z} e^{j\omega t} a_x$$

$$E(x, z, t)_z = E_{z0} \sin\left(\frac{m\pi x}{a}\right) e^{-\gamma z} e^{j\omega t} a_z$$

$$H(x, z, t)_y = H_{y0} \cos\left(\frac{m\pi x}{a}\right) e^{-\gamma z} e^{j\omega t} a_y$$

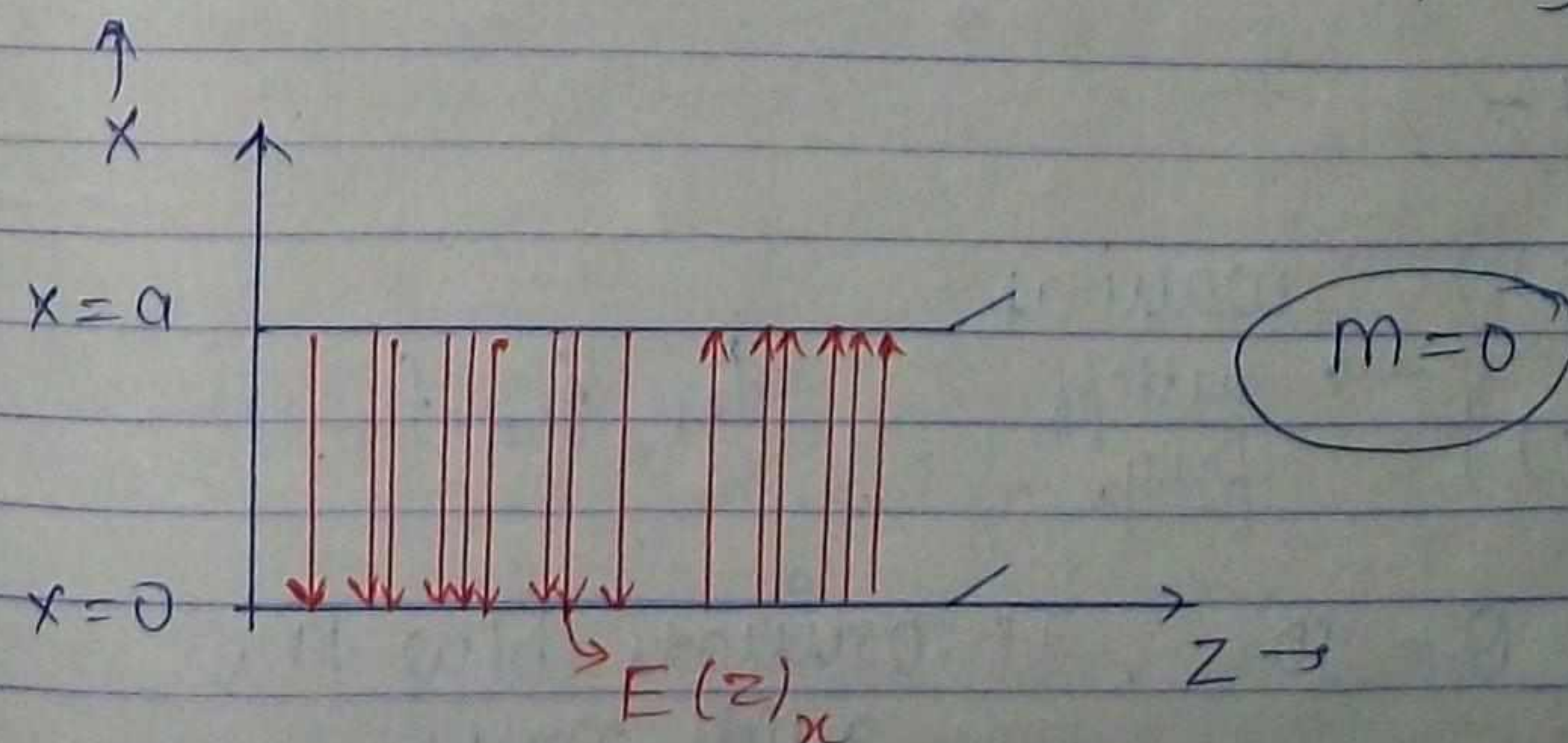


If $m=0$
IN TM
WAVE

$$E(z, t)_x = E_0 e^{-\gamma z} e^{j\omega t} a_x$$

$$H(z, t)_y = H_0 e^{-\gamma z} e^{j\omega t} a_y$$

TEM
WAVE



The TM waves mode representation

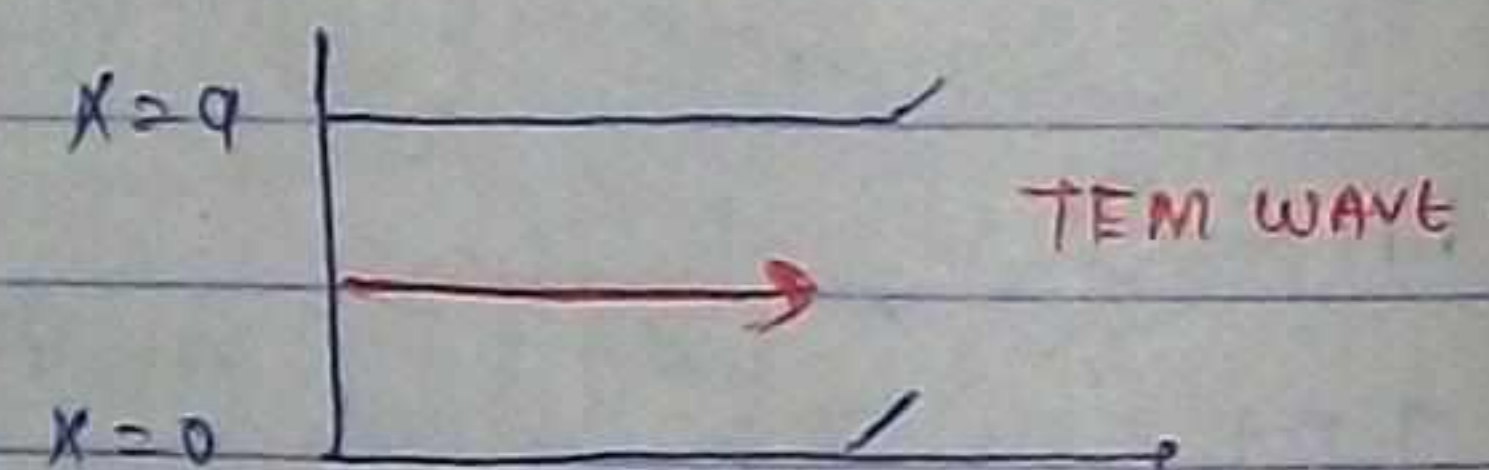
$TM_{10}, TM_{20}, TM_{30} \dots$

PROPERTIES OF TEM WAVES -

1. They travel in the z -direction only and hence their wave angle is 0° always.
 $\bar{\gamma} = \gamma = j\omega \sqrt{\mu_0 \epsilon_0}$

2. Their cutoff frequency are zero $\omega_c = 0$, as $m = 0$ in

All frequencies can travel in TEM mode.



3. They have no modes or multiple conduction mechanism.

SUMMARY -

① $E(x,t)$ } wave at
 $H(x,t)$ } → cutoff freq. $f_{c1}, f_{c2}, f_{c3} \dots$

$\bar{\gamma} = 0$ $\theta = 90^\circ$, If oscillate b/w the guide walls

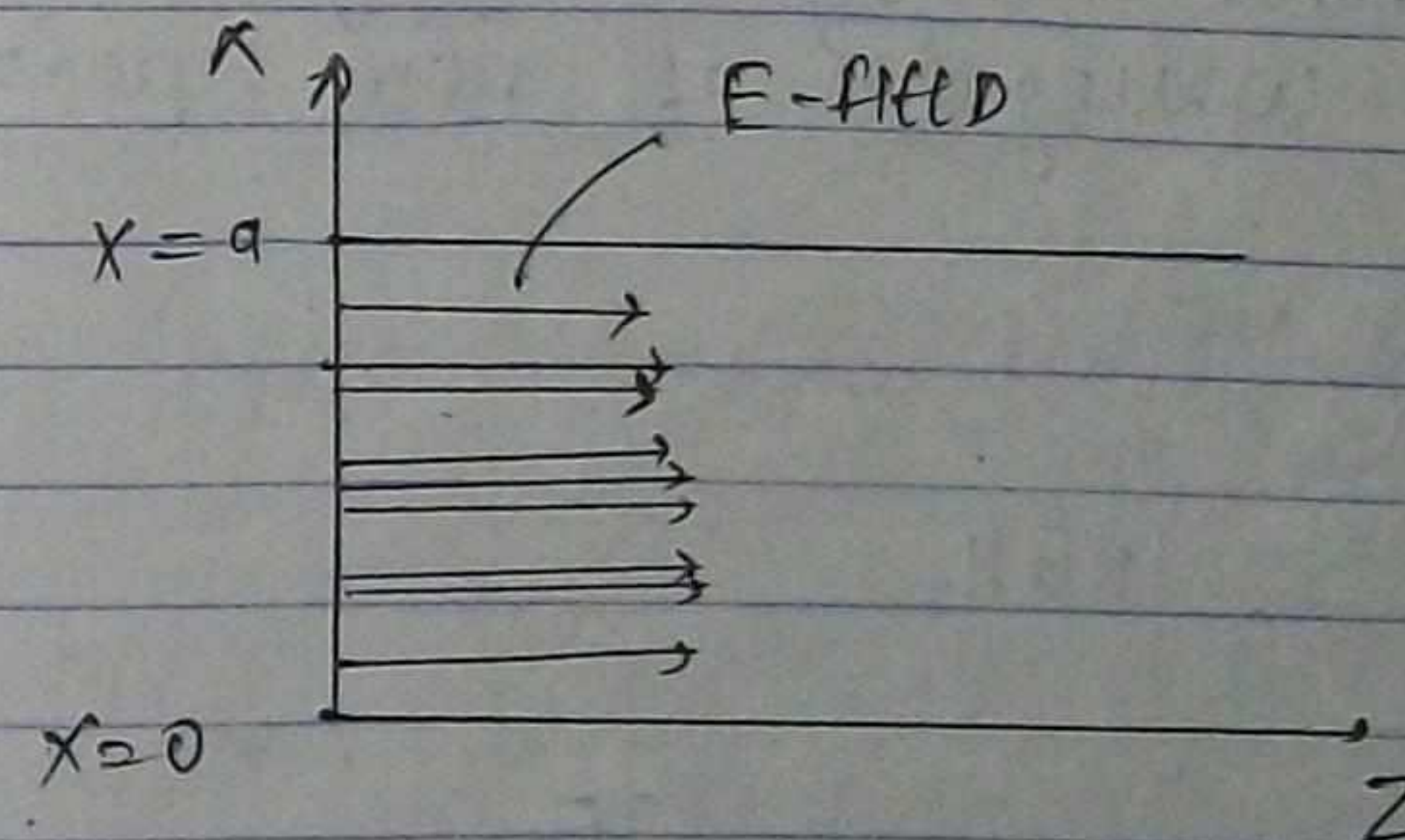
② $E(z,t)$ } → TEM wave
 $H(z,t)$ } $\bar{\gamma} = \gamma$, $\theta = 90^\circ$

All frequency can propagate in this mode,

③ $E(x,z,t)$ }
 $H(x,z,t)$ } $\left\{ \begin{array}{l} \text{TE} \\ \text{TM} \end{array} \right\} \rightarrow \theta, f_c, \text{mode}, v_p, v_g$

Ques. with all regular notation identify the field components belonging to _____ mode.

① TE_{10} ② TE_{20} ③ TM_{10} ④ TM_{20} ⑤ TEM.

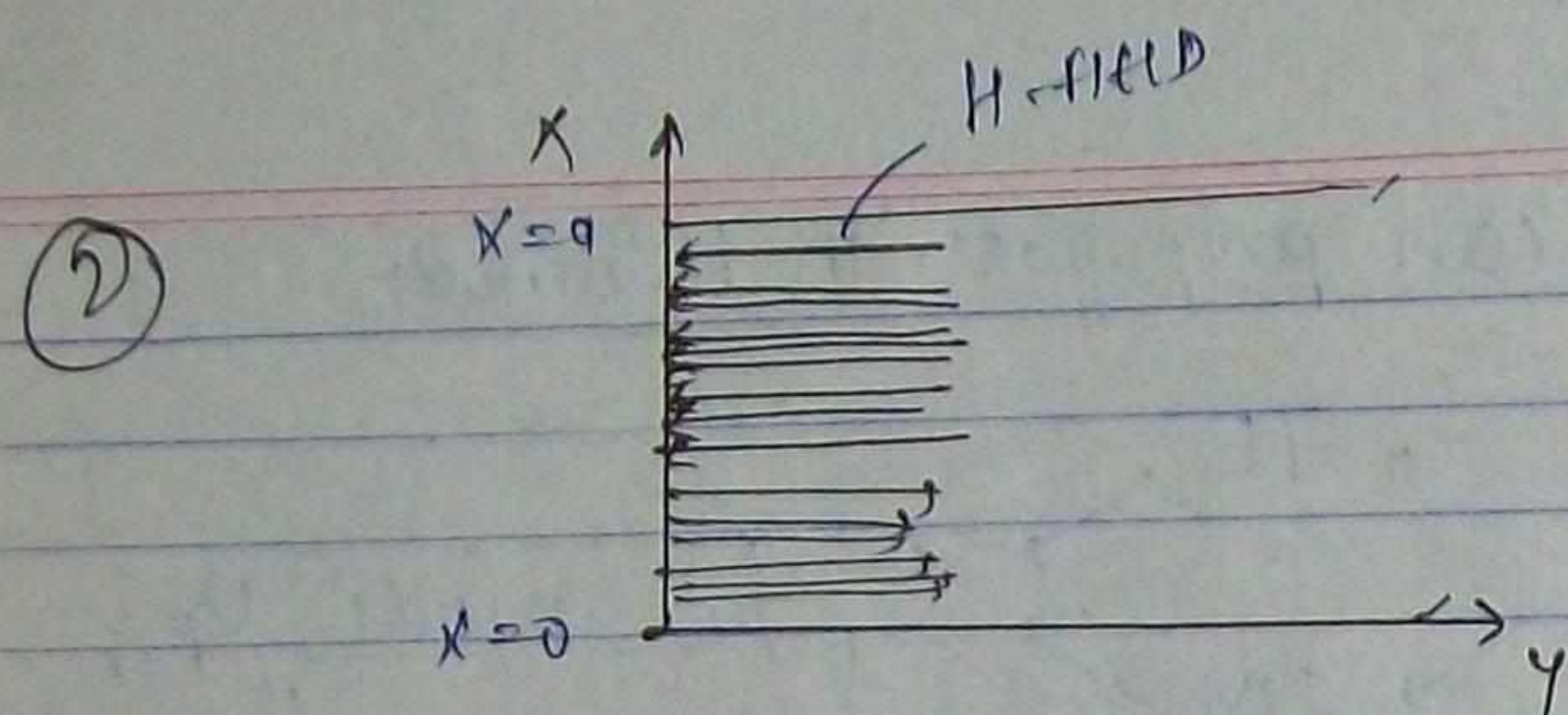


Soln's

for TE mode $E(x,z,t)_y$
 $H(x,z,t)_{(x,z)}$

✓ for TM mode $E(x,z,t)_{(x,z)}$
 $H(x,z,t)_y$

Since only single maxima Hence TM_{10}



(TM₂₀) E(x,z) H(y) plane

Ques. At a freq. of 36 GHz and less than that calculate the total no. of mode of energy transfer in a waveguide of 1 cm separation

Solⁿs

$$f_c = \frac{mc}{2a}$$

$$= \frac{c}{2a} = 15 \text{ GHz}$$

15 GHz $\rightarrow$ 36 GHz $\begin{cases} \rightarrow \text{TE}_{10} \\ \rightarrow \text{TM}_{10} \end{cases}$

30 GHz $\rightarrow$ 36 GHz $\rightarrow$ 2 $\begin{cases} \rightarrow \text{TE}_{20} \\ \rightarrow \text{TM}_{20} \end{cases}$

0 $\rightarrow$ 36 GHz $\rightarrow$ TEM

Total = 5 modes.

NOTE -

- The mode having the least cutoff frequency support all frequencies, that the guide supports. This is called as DOMINANT MODES.
- for parallel plane waveguide TEM mode is the default dominant mode.

$$\eta_{TE} = \frac{E_{trans}}{H_{trans}} = \frac{E_y}{H_x} = \frac{E_{TOTAL}}{H_{TOTAL} \cdot \cos \theta} = \frac{120\pi}{\cos \theta} = \frac{\eta_0}{\cos \theta}$$

$$\eta_{TE} = \frac{120\pi}{\cos \theta} = \frac{\eta_0}{\cos \theta}$$

Similarly,

$$\eta_{TM} = \frac{E_x}{H_y} = \frac{E_{TOTAL} \cos \theta}{H_{TOTAL}}$$

$$\eta_{TM} = 120\pi \cdot \cos \theta = \eta_0 \cos \theta$$

$$\eta_{TEM} = 120\pi$$

$$\bar{\lambda} = \lambda_g = \frac{\lambda}{\cos \theta}$$

λ_g = guide wavelength

$$= \frac{\lambda}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

$$\lambda_g = \bar{\lambda} = \frac{\lambda}{\sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2}}$$

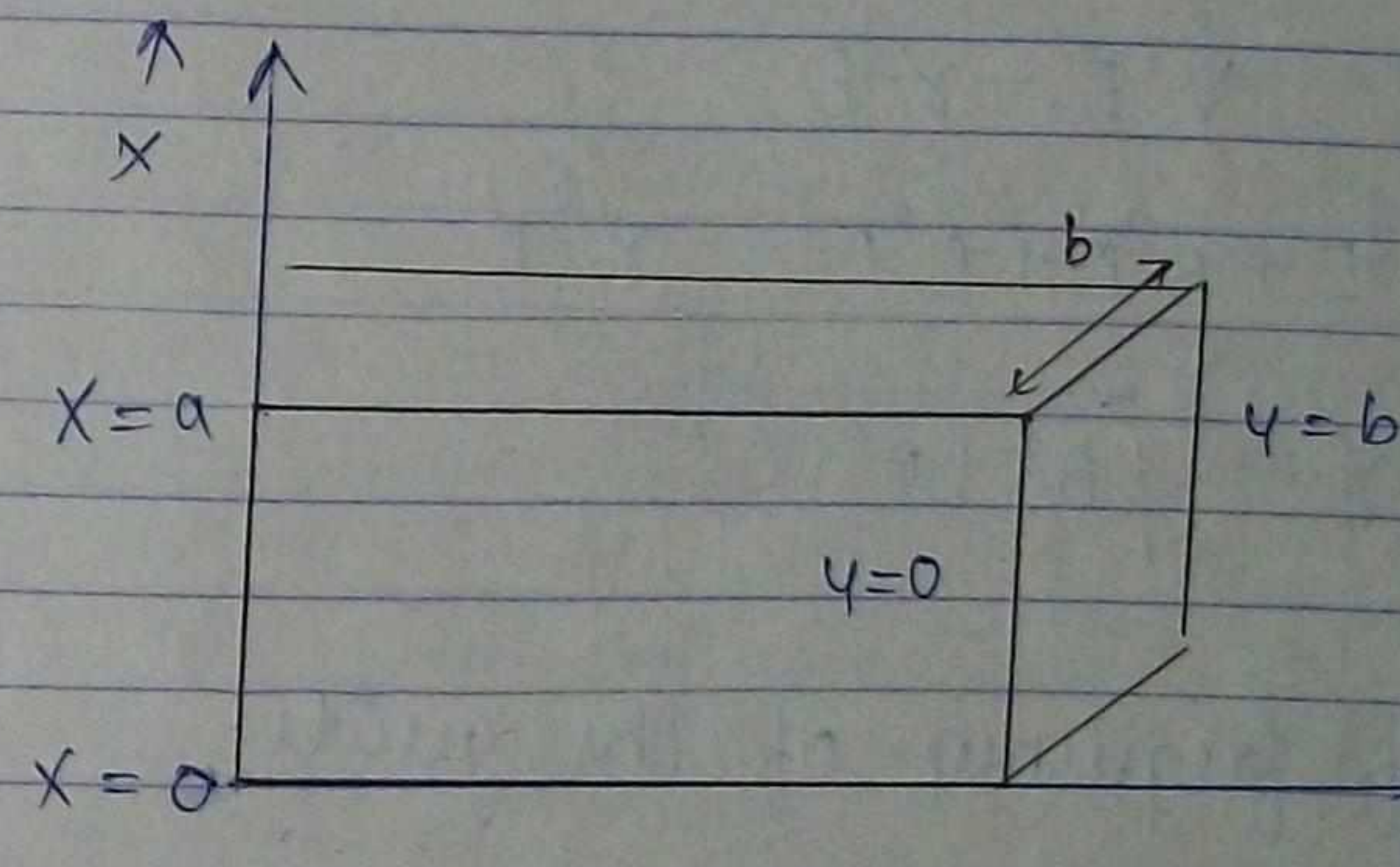
$$1 - \left(\frac{\lambda}{\lambda_c}\right)^2 = \left(\frac{\lambda}{\lambda_g}\right)^2$$

$$\frac{1}{\lambda^2} - \frac{1}{\lambda_c^2} = \frac{1}{\lambda_g^2}$$

FREE SPACE
WAVELENGTH
 $\lambda = c/f$

CUTOFF
WAVELENGTH
 $\lambda = \frac{2a}{m}$

⇒ RECTANGULAR WAVEGUIDE -



- A single conductor two dimensional confining structure with 4 walls.

- BOUNDARY
CONDITIONS

$$\begin{matrix} x=0 & , & x=a \\ y=0 & , & y=b \end{matrix}$$

$$\gamma_x = \frac{m\pi}{a}$$

$$\gamma_y = \frac{n\pi}{b}$$

RESTRICTION IN
TWO DIMENSION

$$\bar{\gamma} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} - \omega^2 \mu \epsilon$$

PROP. CONST.
ALONG
GUIDE
AXIS

$$\# E = (\sin Ax \cdot \sin By \cdot \sin cz)$$

$$\nabla^2 E = \gamma^2 E$$

$$A^2 E + B^2 E + C^2 E = \gamma^2 E$$

$$\gamma = \sqrt{A^2 + B^2 + C^2}$$

Cutoff frequency of the guide

$$\gamma = 0$$

$$\omega_c = \left(\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \right) c$$

$$f_c = \left(\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \right) \frac{c}{2}$$

wave angle

$$\sin \theta = \frac{f_c}{f}$$

$$v_p = \frac{c}{\cos \theta}$$

(phase velocity)

$$v_g = c \cdot \cos \theta$$

(group velocity)

$$\lambda_g = \frac{\lambda}{\cos \theta}$$

$$\eta_{TE} = \frac{\eta_0}{\cos \theta}$$

$$\eta_{TM} = \eta_0 \cos \theta$$

⇒ TE AND TM MODES IN RECTANGULAR WAVEGUIDES -

- If a vertical or horizontal field is given to the guide, there is a horizontal or vertical E-field but not axial E-field i.e. $E_z = 0$. This is called as TE field mechanism. The no. of such out of phase field collection decides the mode m and n.

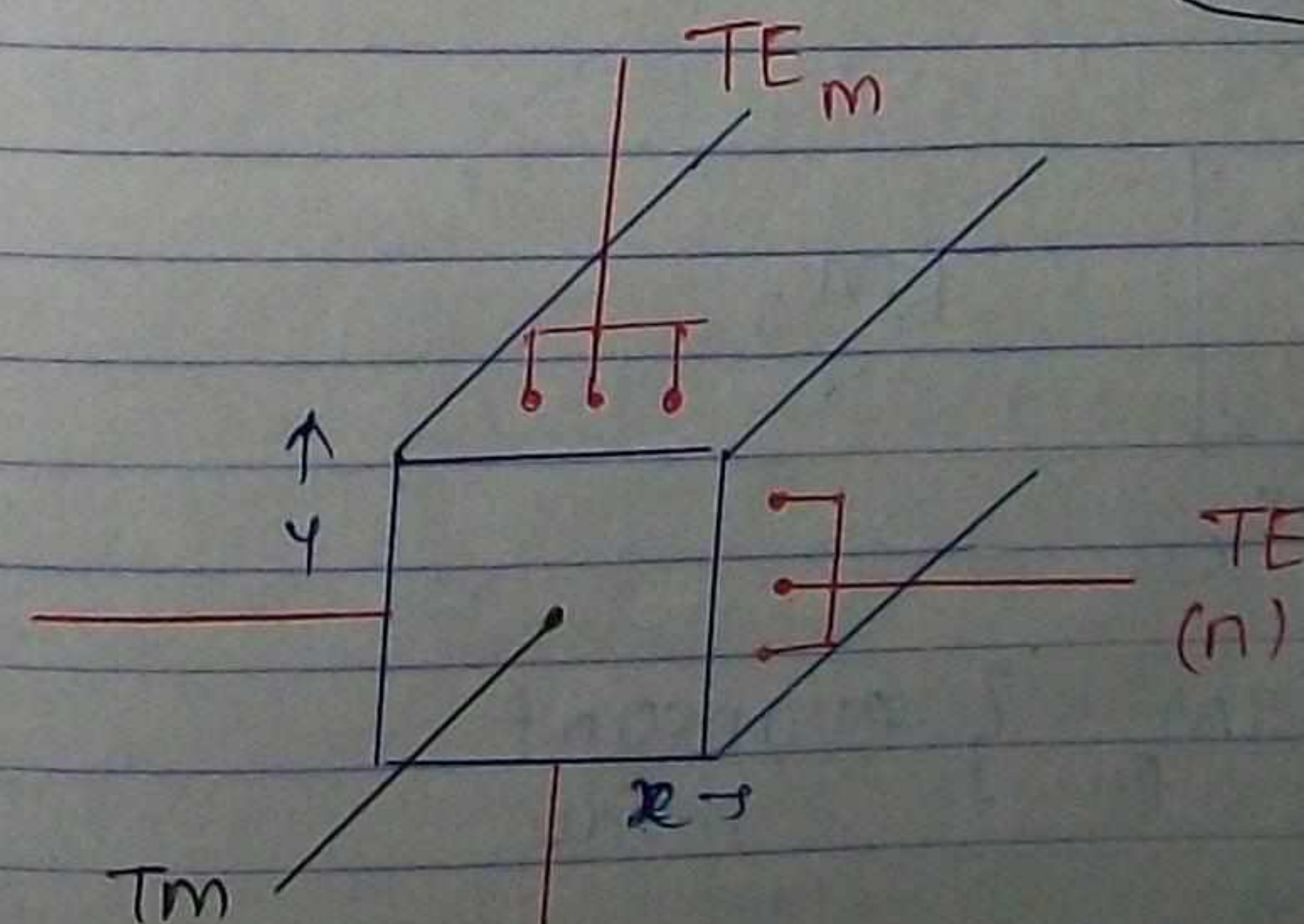
E_y and E_x will exist

$$E_z = 0$$

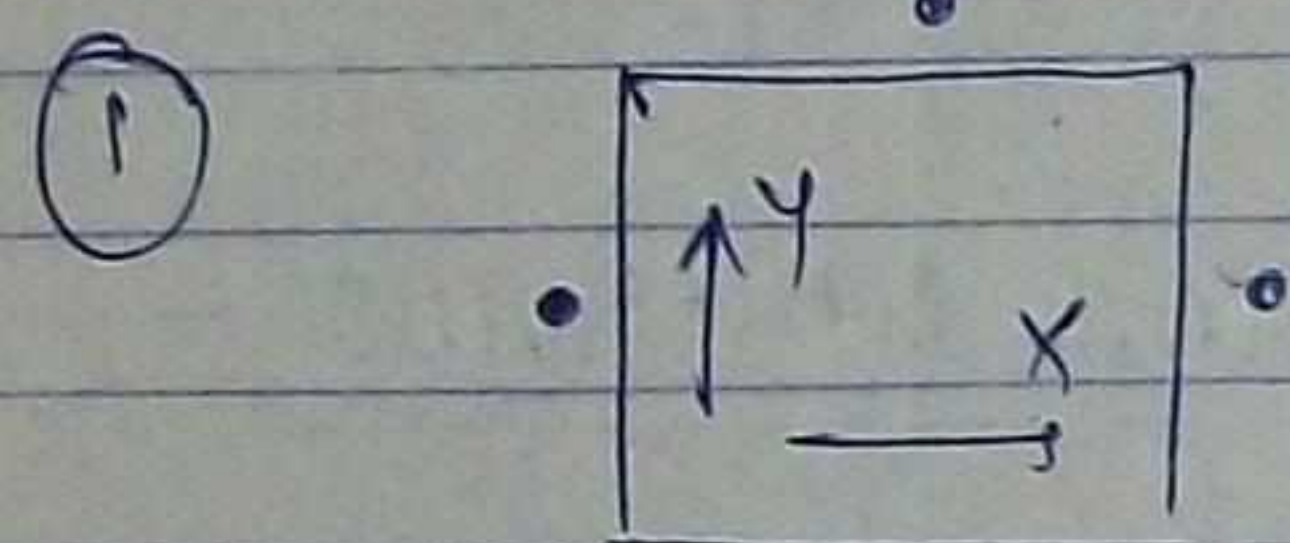
- An axial field never gives a H-field along the guide axis. This is TM field.

H_x and H_y exist

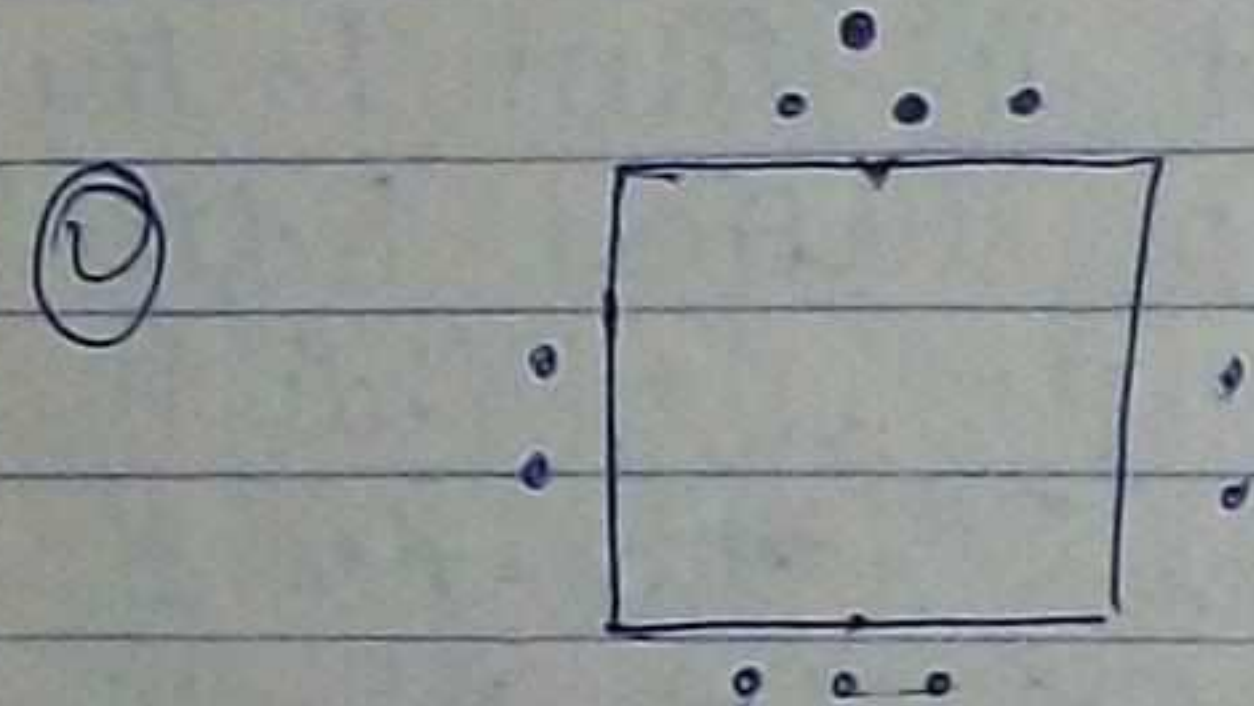
$$H_z = 0$$



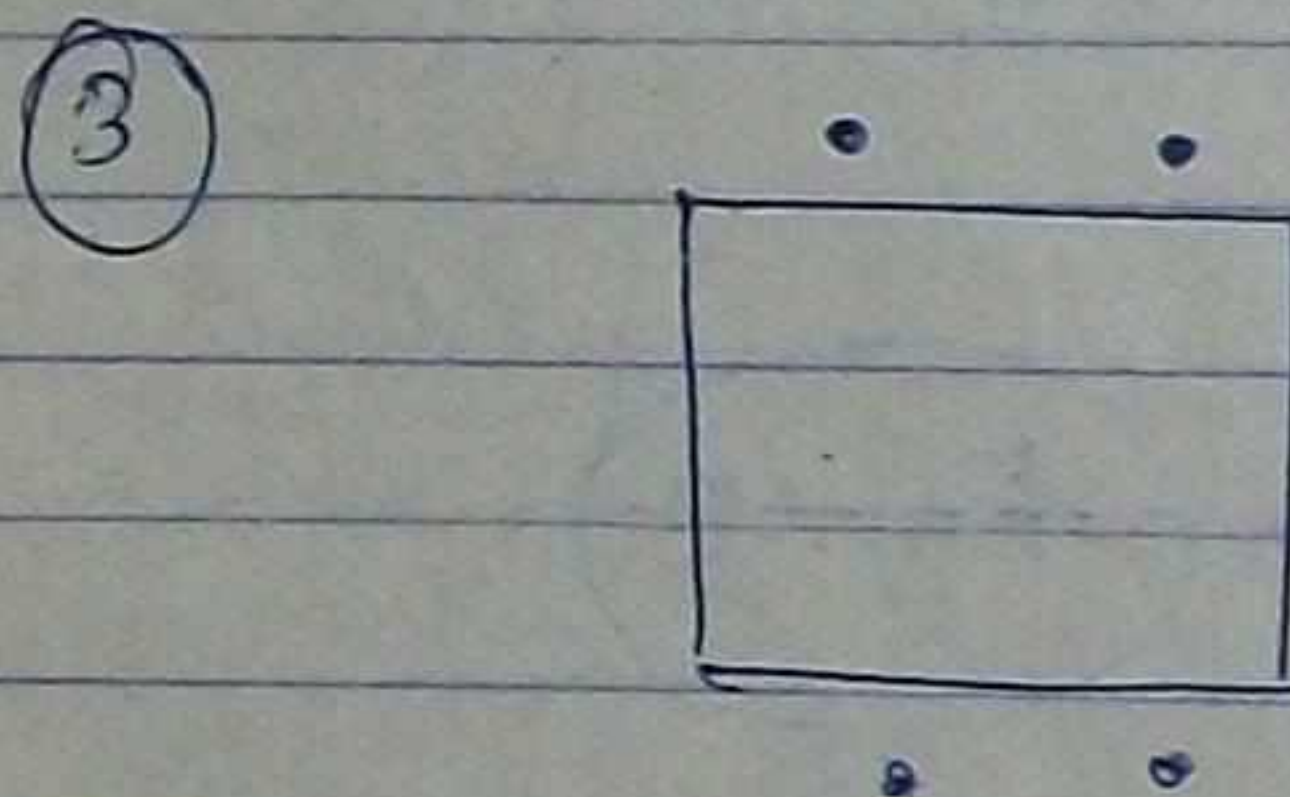
Ex - Identify the fields mode.



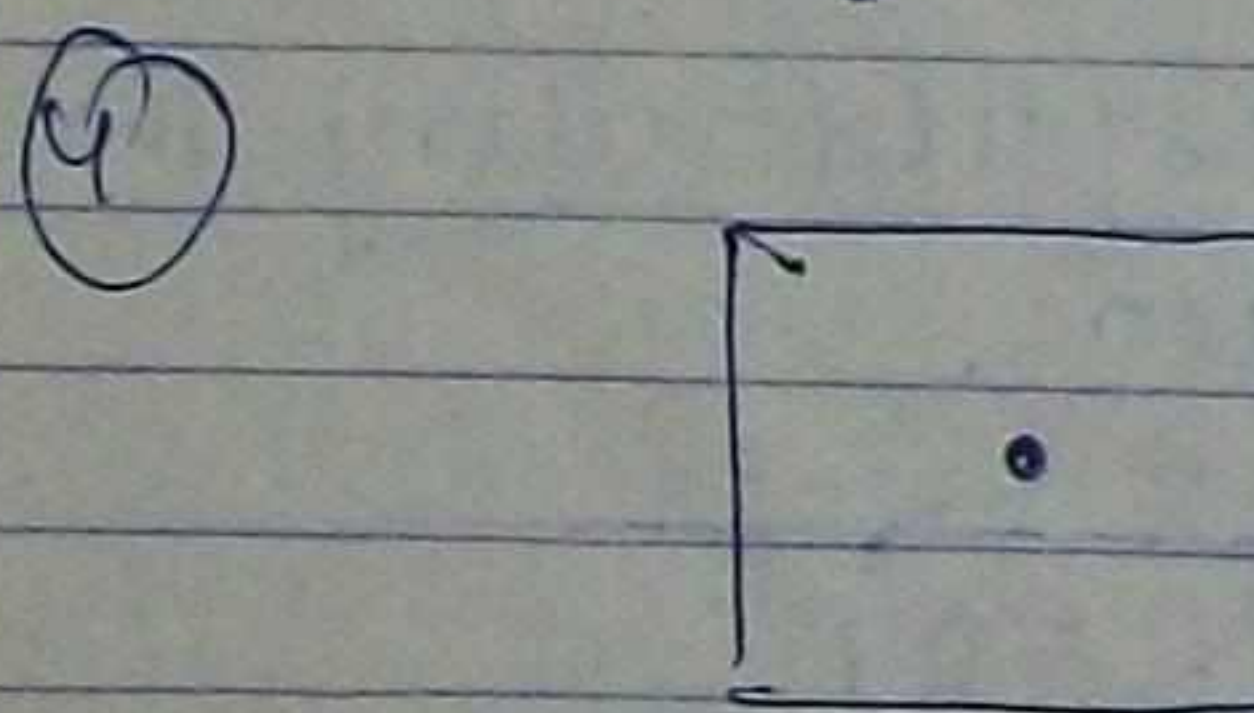
TE₁₁



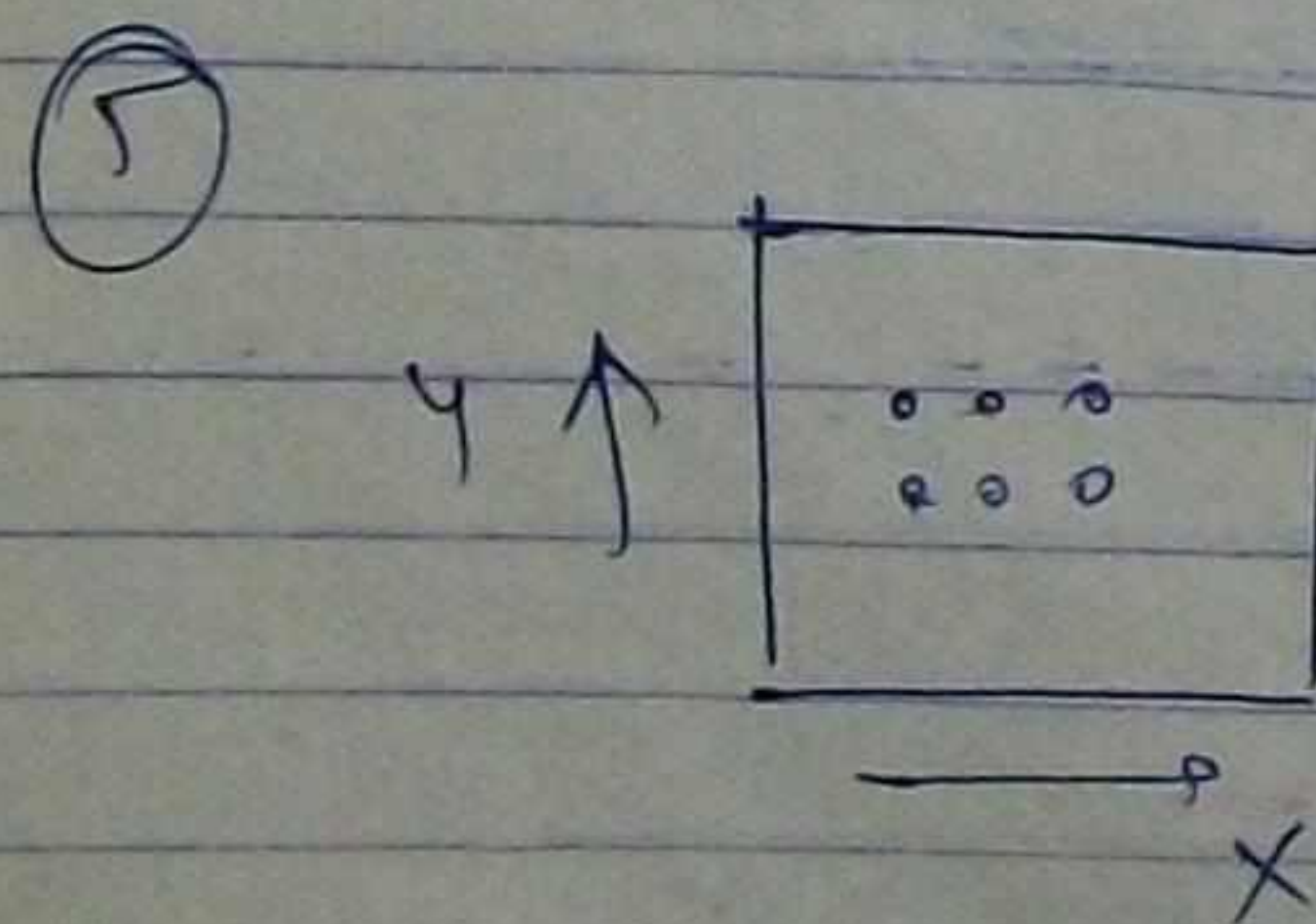
TE₃₂



TE₂₀



TM₁₁



TM₃₂

TM_{mo} & TM_{on} } evanescent modes

NOTE -

- The non existent connection mechanism are called as evanescent mode. In rectangular waveguide

TM_{mo} and TM_{on}

are evanescent modes.

- Two different connection mechanism having same cutoff freq. are called as degenerate modes.

Ex - TM_{mn} and TE_{mn}

⇒ TM WAVE SOLUTIONS IN RECTANGULAR W/G -

$$H_z = 0$$

$$E(x, y, z, t) \propto (x, y, z)$$

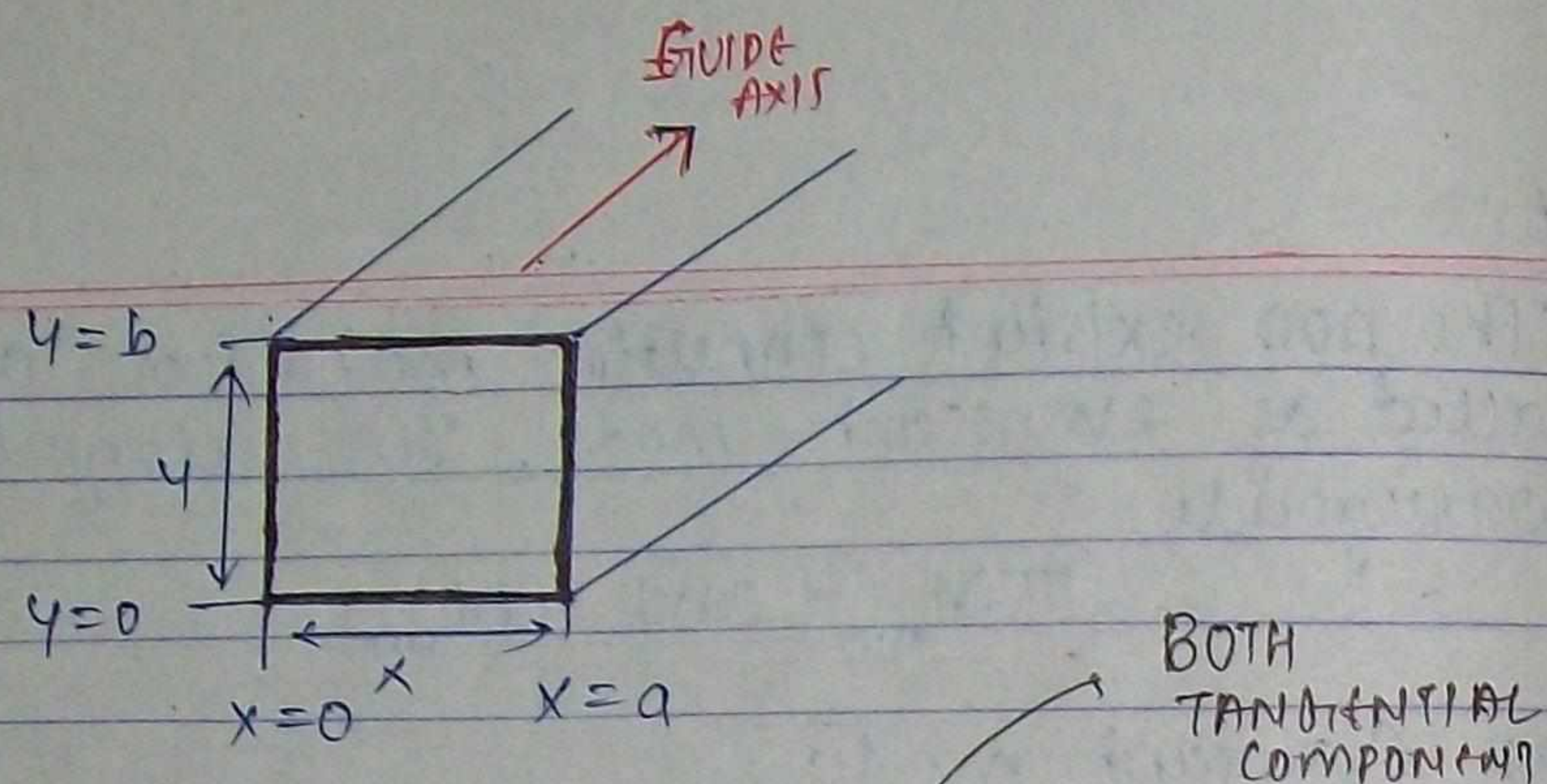
$$H(x, y, z, t) \propto (x, y)$$

$$E(x, y, z, t) \propto E(z)/H(z) \rightarrow e^{-\gamma z}$$

$$E(t)/H(t) \rightarrow e^{j\omega t}$$

$E(x, y)$ → Restricted dimension
→ Sin or Cos Harmonic

$E(x, y)$
tang → sin harmonic



$$E(x, y, z, t)_z = E_{z0} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z} e^{j\omega t} \hat{a}_z$$

BOTH TANGENTIAL COMPONENTS

$$E(x, y, z, t)_x = E_{x0} \left(\frac{m\pi}{a}\right) \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z} e^{j\omega t} \hat{a}_x$$

TANGENTIAL COMPONENT

$$H(x, y, z, t)_y = H_{y0} \left(\frac{m\pi}{a}\right) \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z} e^{j\omega t} \hat{a}_y$$

TANGENTIAL COMPONENT

$$E(x, y, z, t)_y = E_{y0} \left(\frac{n\pi}{b}\right) \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z} e^{j\omega t} \hat{a}_y$$

TANGENTIAL COMPONENT

$$H(x, y, z, t)_x = H_{x0} \left(\frac{n\pi}{b}\right) \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z} e^{j\omega t} \hat{a}_x$$

TANGENTIAL COMPONENT

If $m=0$ and $n \neq 0$

then all components vanish.

$\therefore TM_{0n} \rightarrow$ Does not exist.

If $m \neq 0$ and $n=0$

$\therefore TM_{m0} \rightarrow$ Does not exist.

$\Rightarrow$ TE WAVE SOLUTION IN RECTANGULAR W/G -

$$\boxed{E_z = 0} \quad \begin{matrix} H(x, y, z, t)_{(x, y, z)} \\ E(x, y, z, t)_{(x, y)} \end{matrix}$$

$\boxed{E_{\text{TANG}} = 0}$ } as magnetic field is never normal to a conductor.
 $\boxed{H_{\text{NORMAL}} = 0}$ } It neither starts or ends anywhere

$$H(x, y, z, t)_z = H_{z0} \cos\left(\frac{m\pi x}{a}\right) \cdot \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z} e^{j\omega t} a_z$$

$$E(x, y, z, t)_x = E_{x0} \left(\frac{n\pi}{b}\right) \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z} e^{j\omega t} a_x$$

$$E(x, y, z, t)_y = E_{y0} \left(\frac{m\pi}{a}\right) \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z} e^{j\omega t} a_y$$

$$H(x, y, z, t)_x = H_{x0} \left(\frac{m\pi}{a}\right) \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z} e^{j\omega t} a_x$$

$$H(x, y, z, t)_y = H_{y0} \left(\frac{n\pi}{b}\right) \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z} e^{j\omega t} a_y$$

If $m=0$ and $n \neq 0$

$$\left. \begin{array}{l} E(y, z, t)_x \\ H(y, z, t)_{(y, z)} \end{array} \right\} \text{TE}_{0n} \text{ WAVE EXIST}$$

If $m \neq 0$ and $n=0$

$$\left. \begin{array}{l} E(x, z, t)_y \\ H(x, z, t)_{(x, z)} \end{array} \right\} \text{TE}_{m0} \text{ WAVE EXIST}$$

TEM waves cannot exist in rectangular waveguide. ($m=0, n=0$)

In all single conductor structure TEM does not exist.

Ex - Rectangular w/G, Cylindrical w/G

- TEM waves exist in two distinct conductor structure.

Ex - Coaxial cable, parallel wires, parallel plate w/G

⇒ DOMINANT MODE IN RECTANGULAR w/G -

$$f_c = \left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 \right] \frac{c}{2}$$

$$\text{TE}_{10} \text{ where } f_c = \frac{c}{2a}$$

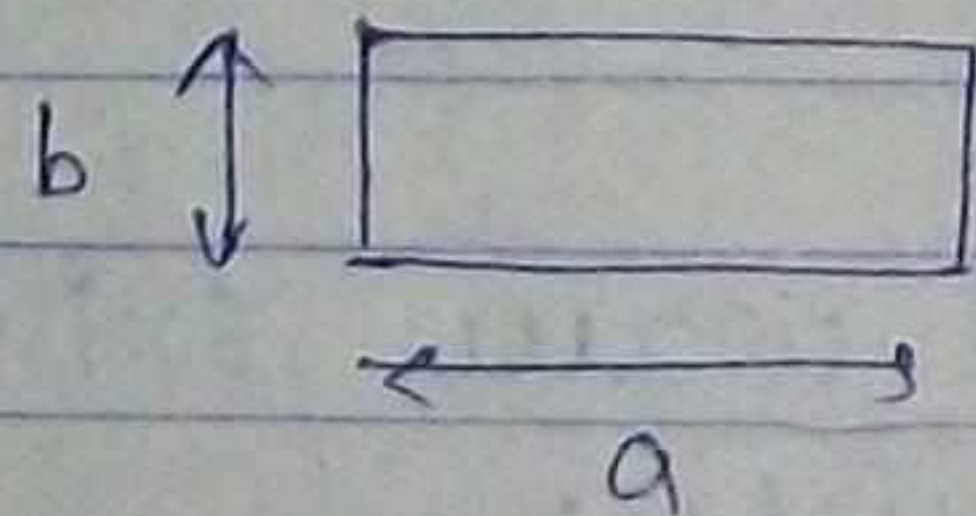
$$\text{TE}_{01} \text{ where } f_c = \frac{c}{2b}$$

If $a > b$ — TE₁₀ dominant mode
 $a < b$ — TE₀₁ dominant mode

9/10/11

- dominant mode always depends on broadside dimensions.
- voltage handling or power handling ability depends on narrow side dimensions.

$$E = \frac{V}{b}$$



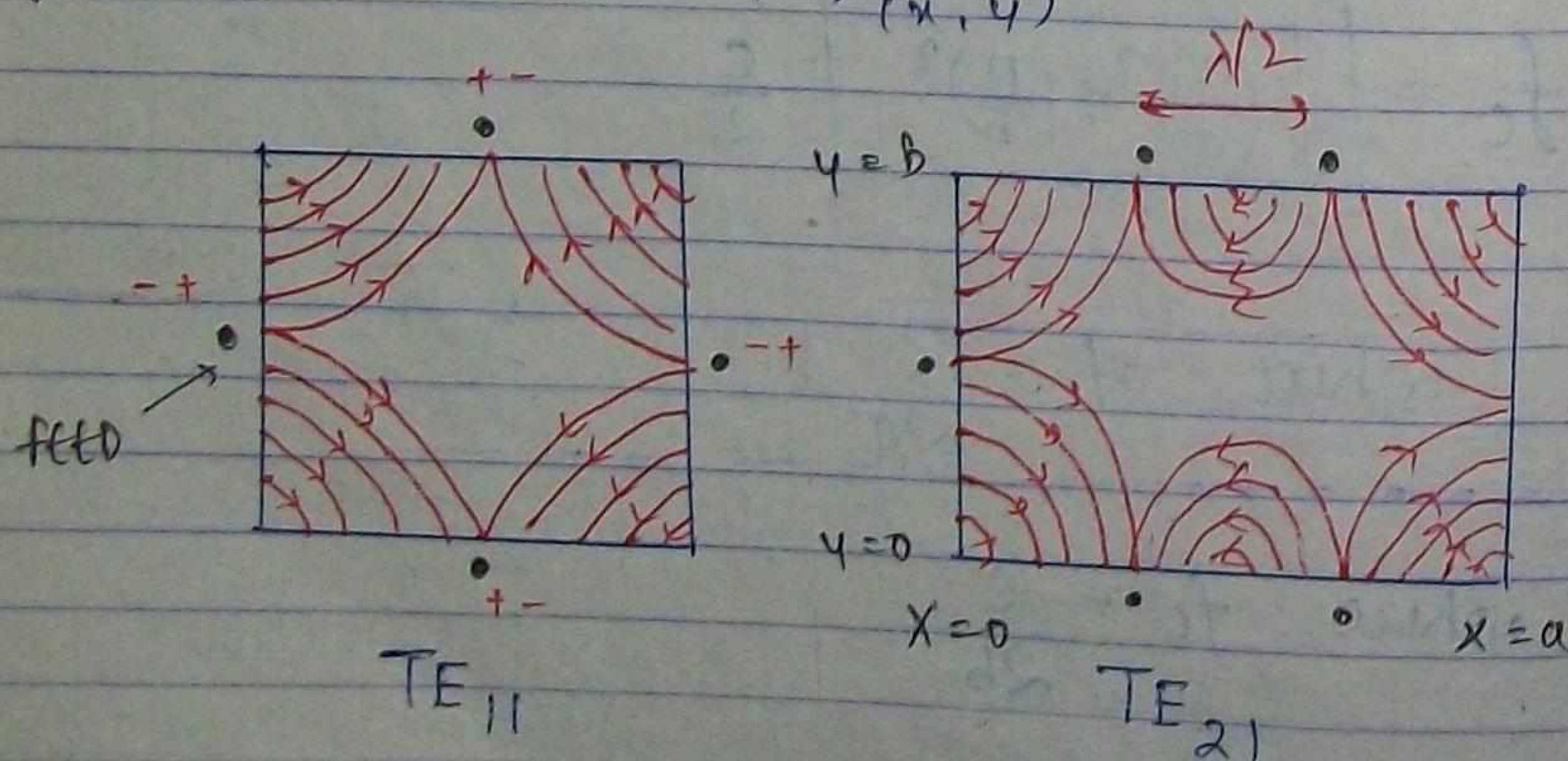
⇒ TE WAVES PROPAGATION IN W/G -

$$E_z = 0$$

E_x and E_y exist only

PROG. DIRECTION

$$E(x, y)$$



TE₁₁

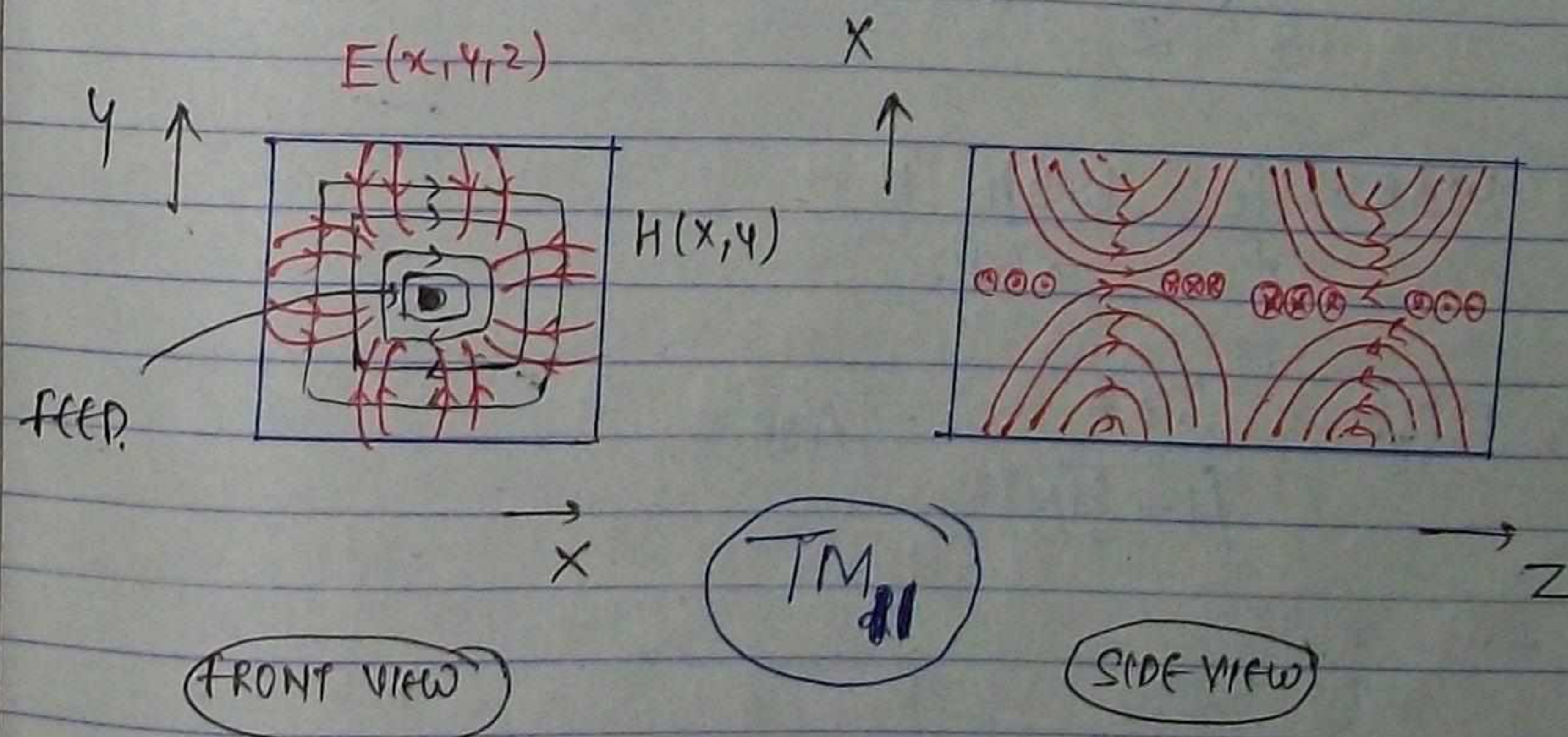
TE₂₁

(X-Y PLANE)

⇒ TM WAVES IN W/G -

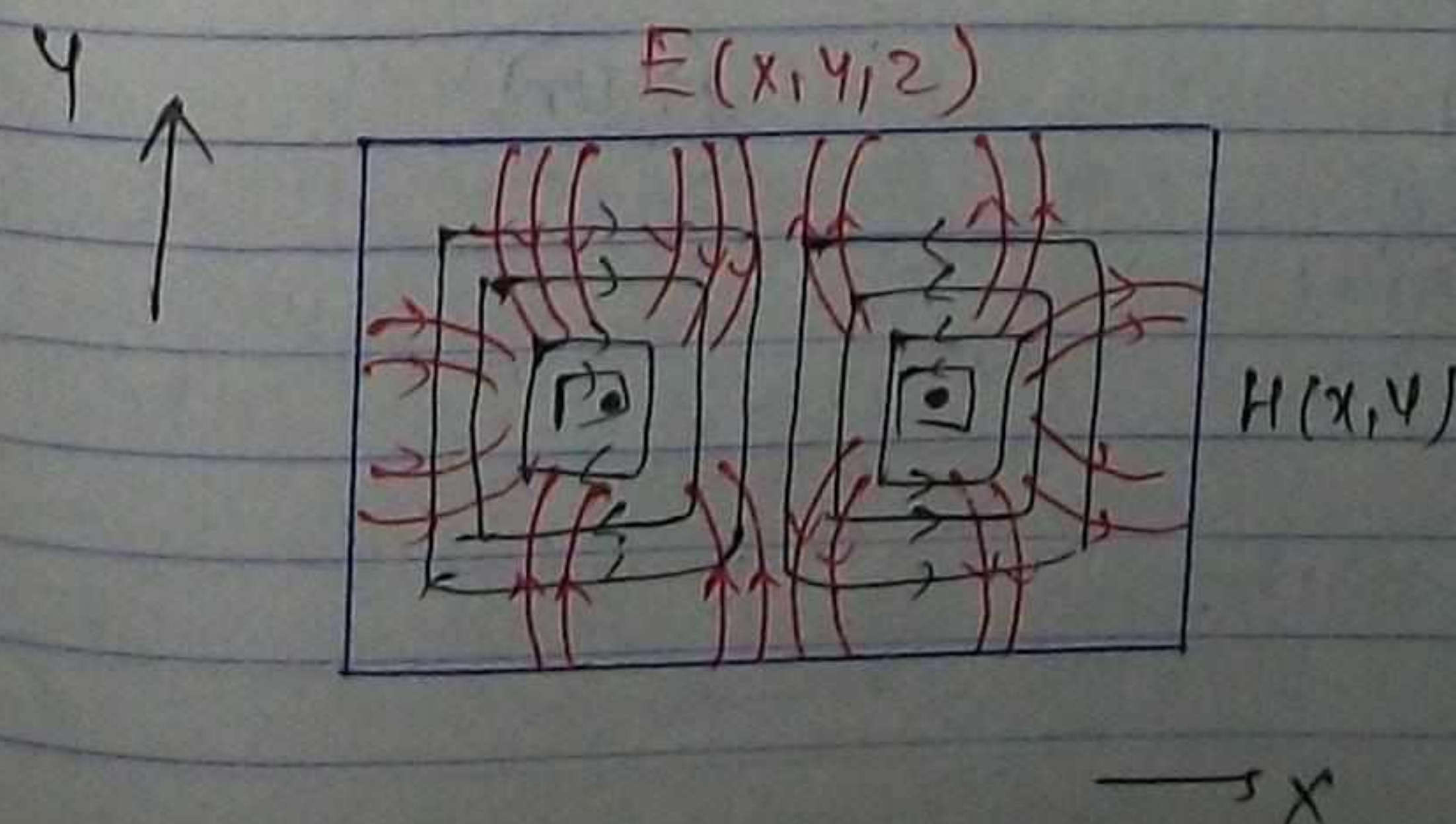
$$H_z = 0$$

E_x, E_y and E_z all exist



FRONT VIEW

SIDE VIEW



TM₂₁

Ques 5 P-39

Solⁿ: TE_{20} mode

$$\eta_{TE} = \frac{\eta_0}{\cos\theta} \quad \sin\theta = \frac{f_c}{f}$$

$$f_c = \left(\frac{m}{2a}\right) \cdot c = \frac{m}{a} \times \frac{c}{2}$$

$$f_c = \frac{2}{3} \times \frac{c}{2} = \underline{10 \text{ GHz}}$$

$$\sin\theta = \frac{f_c}{f} = \frac{10 \text{ GHz}}{30 \text{ GHz}}$$

$$\eta_{TE} = \frac{377}{\sqrt{1 - (1/3)^2}} = 400 \Omega$$

Ques 7. P-39

$$\text{Sol}^n: \frac{1}{\lambda^2} - \frac{1}{\lambda_c^2} = \frac{1}{\lambda_g^2}$$

$$\downarrow \quad \downarrow$$

$$\frac{c}{2.5 \times 10^9} \quad \frac{2a}{m=1}$$

(15 cm)

TE_{m0}
on

Ques 13. P-40

Solⁿ:

$$f_c = \left[\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \right] \frac{c}{2}$$

arranged mode

$$TE_{10} \rightarrow f_{c1} = \frac{m \cdot c}{2a} = (3.75 \text{ GHz} - \infty)$$

$$TE_{01} \rightarrow f_{c2} = \frac{nc}{2b} = (5 \text{ GHz} - \infty)$$

$$TE_{11}/TM_{11} \rightarrow f_{c3} = \left(\sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{1}{4}\right)^2} \right) \frac{c}{2} = 6.25 \text{ GHz} \rightarrow \infty$$

TE_{20}

TE_{02}

TE_{21}/TM_{21}

from (3.75 - 5 GHz)
guide can support only
single mode only
either TE_{10} or TE_{01}
(dominant mode)

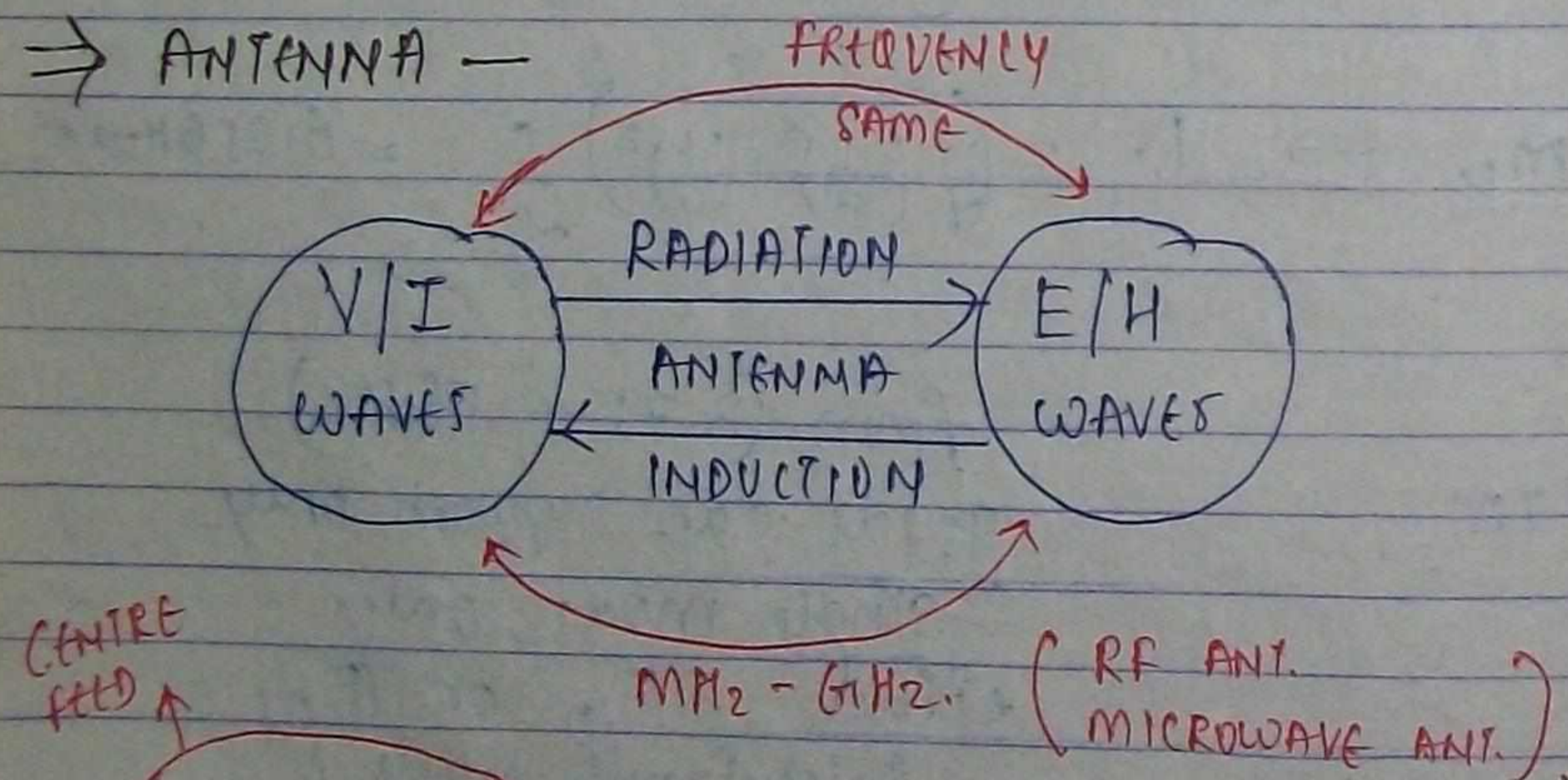
(*) If the walls are non-ideal the higher frequencies are not used in lower modes due to increased attenuation.

(P-41) workbook

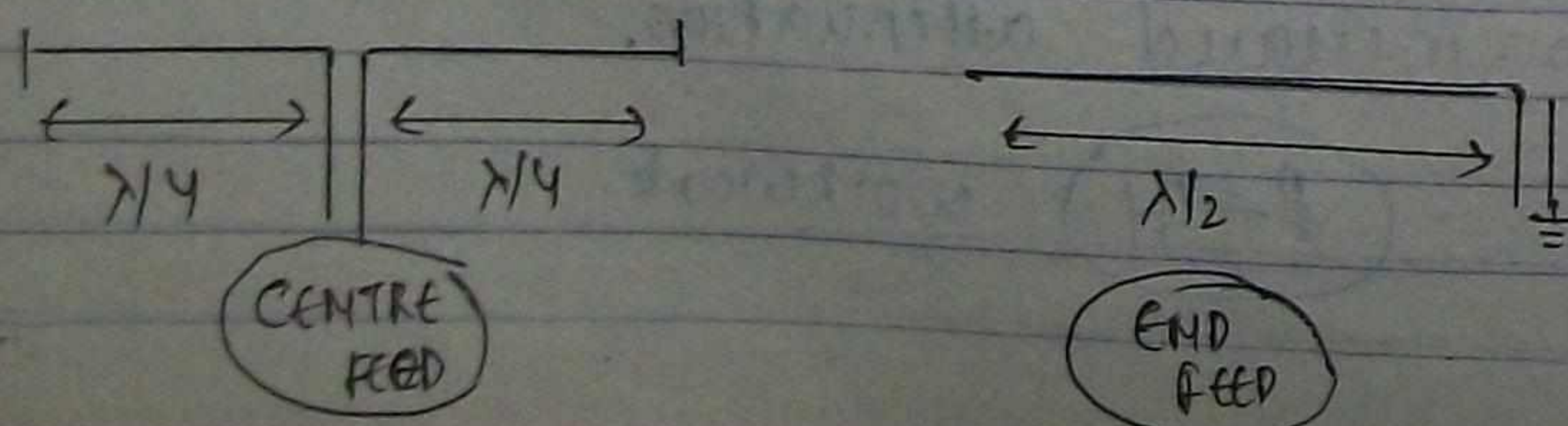
ANTENNAS

- HERTZIAN DIPOLE
HALFWAVE DIPOLE
- BASIC TERMS AND DEFINITION
- ARRAYS OF ANTENNA
- FRIS - FREE SPACE PROP. EQⁿ.
- CLASSIFICATION OF ANTENNAS.

⇒ ANTENNA —



⇒ HALF WAVE ANTENNA — DIPOLG AS A FUNDAMENTAL

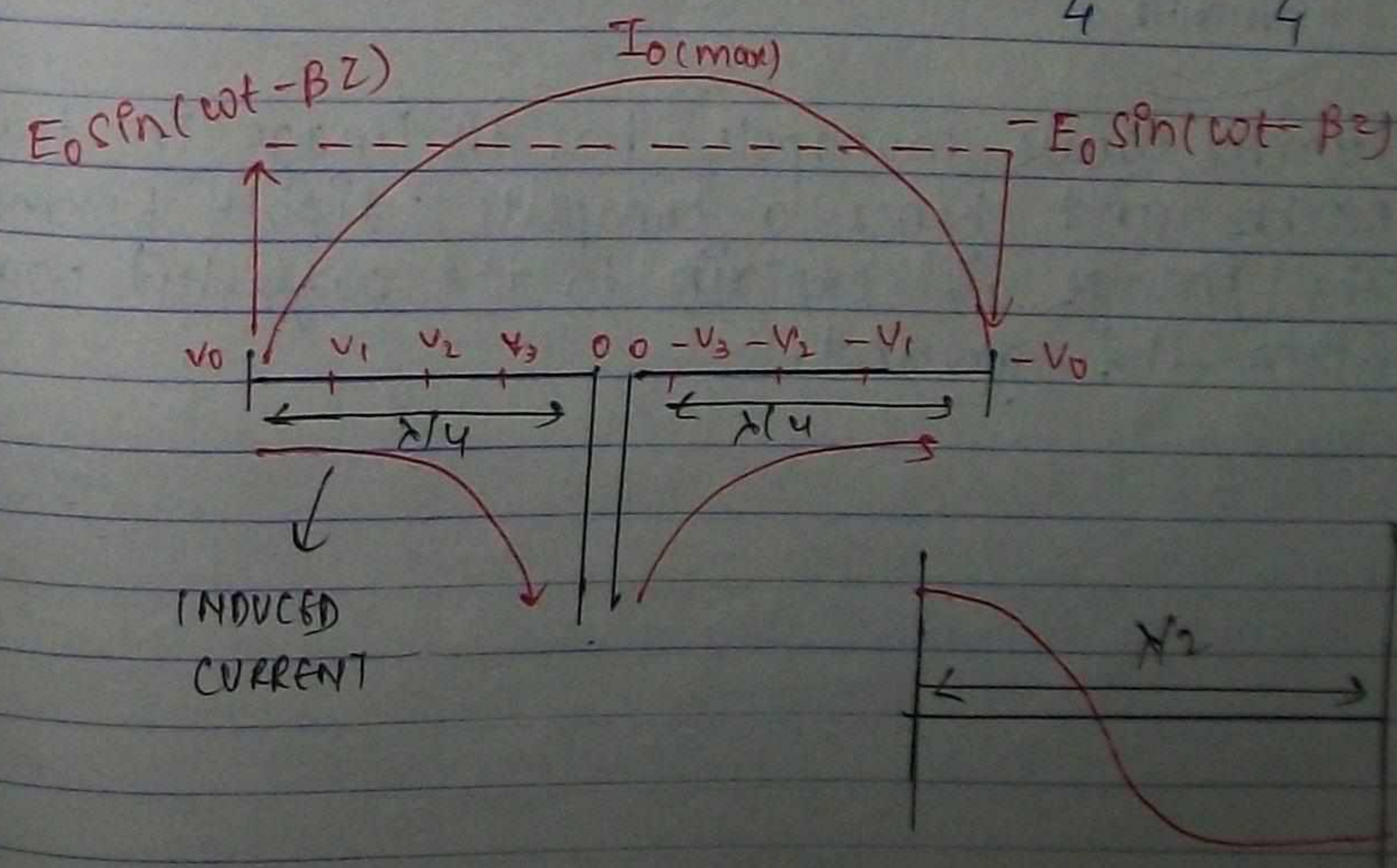


- A Tx. line opened out by $\lambda/4$ on either sides is called as centre feed half wave dipole. The length of the antenna is λ -dependent i.e. freq. - dependent.

- When a wave of the correct freq. correctly oriented its E-field induces V_0 voltage at one end and an equal and opposite voltage at the other end, due to a path difference of $\lambda/2$.

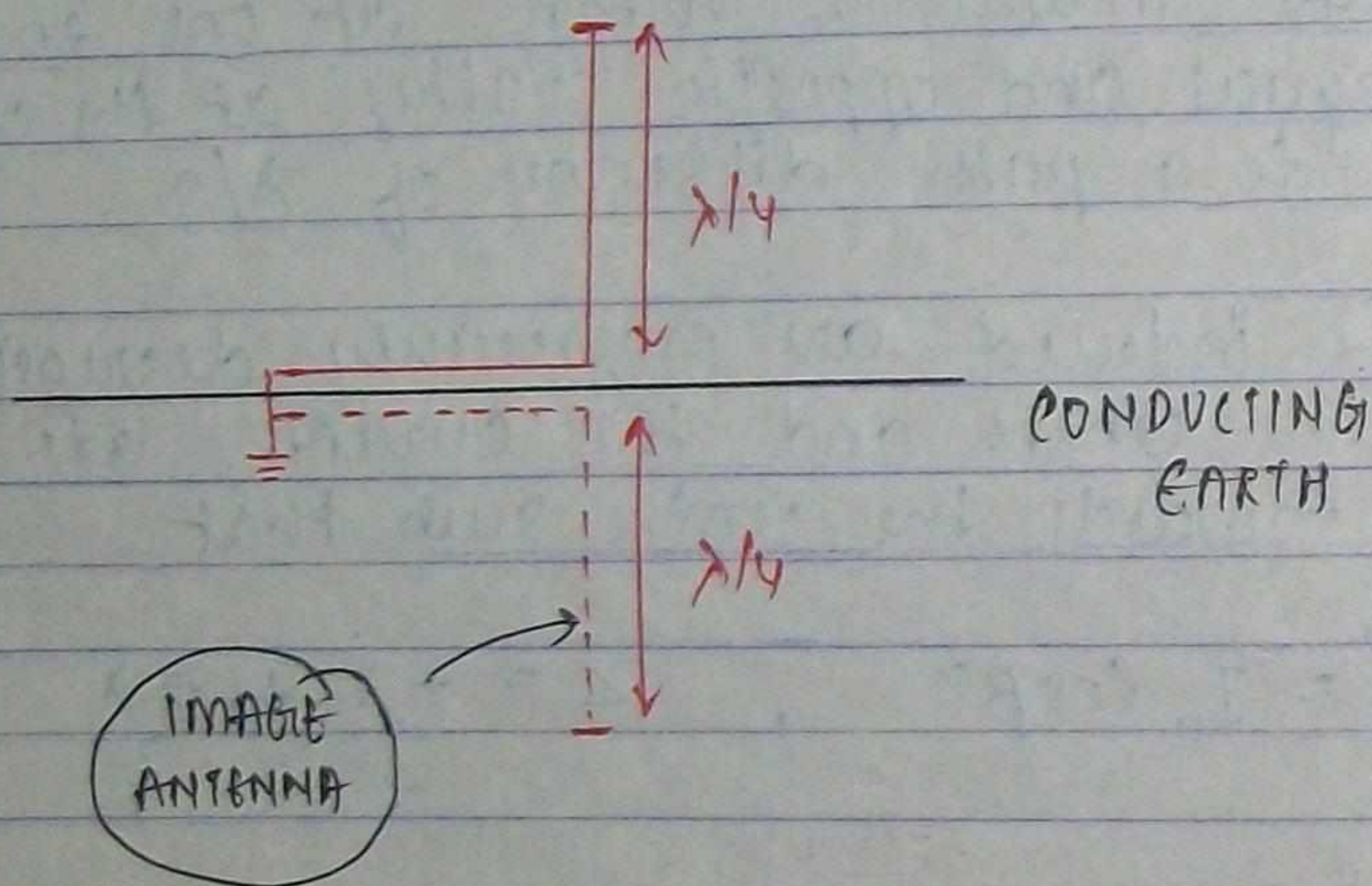
- The voltages induced are progressively decreasing towards the centre and the currents are increasing towards the centre such that

$$|I(z)| = I_0 \cos \beta z, \quad z = -\frac{\lambda}{4} \text{ to } +\frac{\lambda}{4}$$



⇒ QUARTERWAVE MONOPOLE AS A PRACTICAL ANTENNA -

- $\lambda/4$ - length - vertically grounded.
- low frequency - ground wave propagation.



- Quarter wave monopole has its image in the earth and hence a complete dipole forms. The image is explained to the reflected wave from the ground.

⇒ RADIATION OF A HERTZIAN DIPOLE -

$$I(t) \rightarrow dl \rightarrow \text{very small}$$

↓
 $I_m \sin \omega t$
 Time harmonic
 AC-oscillatory
 current element

$$dl \ll \lambda$$

- A time harmonic current flowing in a zero length wire is a source of EM-waves as shown. This is called as Radiation. Hence Radiation is defined as
 "The production of EM-waves all around a time harmonic current carrying wire."

$$1. \quad A(t) = \frac{\mu_0 I(t) dl}{4\pi r}$$

$$2. \quad \vec{B} = \nabla \times \vec{A}$$

$$3. \quad \nabla \times \vec{H} = \epsilon \frac{\partial \vec{E}}{\partial t}$$

$$4. \quad \frac{H(t)}{E(t)} \rightarrow \text{RADIATION}$$

$$\sin \omega t \rightarrow \text{message} \rightarrow m(t)$$

⇒ EXPRESSION FOR RADIATED WAVES -

$$E(r, \theta, \phi, t)_{\theta} = \left(\frac{-I_m d \sin \theta \cdot \omega}{4\pi \epsilon c^2 r} \right) \sin \omega t \cdot e^{-jBr} a_{\theta}$$

$$H(r, \theta, \phi, t)_{\phi} = \left(\frac{-I_m d \sin \theta \cdot \omega}{4\pi c r} \right) \sin \omega t \cdot e^{-jBr} a_{\phi}$$

$$E(z, t)_x = E_0 e^{j\omega t} e^{-\gamma z} \cdot a_x$$

$$H(z, t)_y = H_0 e^{j\omega t} e^{-\gamma z} \cdot a_y$$

RADIATED
WAVE

TEM
WAVE

⇒ PROPERTIES OF RADIATED WAVES -

- All radiated wave travel in radial directions from the point and hence their power density decreases as $1/r^2$ due to DISPERSION. Hence their amplitude decreases as $1/r$.

- They obey all fundamental rules of EM-wave propagation.

$$\frac{E_{\theta}}{H_{\phi}} = \frac{1}{\epsilon c} = \frac{1}{\epsilon \frac{1}{\sqrt{\mu \epsilon}}} = \sqrt{\frac{\mu}{\epsilon}} = \underline{\underline{120\pi}}$$

$$E_{\theta} \times H_{\phi} = \text{Propagation 'r' direction } (a_r)$$

- The amplitude of radiated wave is never the same in all directions, radiation is always directive in nature. (angle dependent $\sin \theta$)

- The amplitude of the radiated waves is always dependent on the length to wavelength relationships.

$$\frac{dl}{\lambda} = \underline{\underline{\text{is crucial}}}$$

⇒ TOTAL RADIATED POWER FROM A HERTZIAN DIPOLE -

$$W_r = \iint P_{avg} \cdot ds = \iint \frac{1}{2} \frac{E_0^2}{\eta} \cdot dr$$

$$= \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \frac{1}{2} \times \left(\frac{I_m d \sin \theta \cdot 2\pi c}{\lambda \cdot 4\pi \epsilon c^2 r} \right)^2 \times \frac{1}{120\pi} \times r^2 \sin \theta \cdot d\theta \cdot d\phi$$

$$W_r = \frac{2}{I_{rms}} \cdot 80\pi^2 \cdot \left(\frac{dl}{\lambda} \right)^2$$

where, $80\pi^2 \left(\frac{dl}{\lambda}\right)^2$ — Radⁿ Resistance

NOTE -

The final expression shows that every antenna dissipates power in the form of EM-waves, the terms

$$80\pi^2 \left(\frac{dl}{\lambda}\right)^2$$

determine the Radiated power for a given I/P current and is called as Radiation resistance. It should be as large as possible.

⇒ EXPRESSIONS FOR RADIATIONS OF HALF WAVE DIPOLE -

Half wave dipole

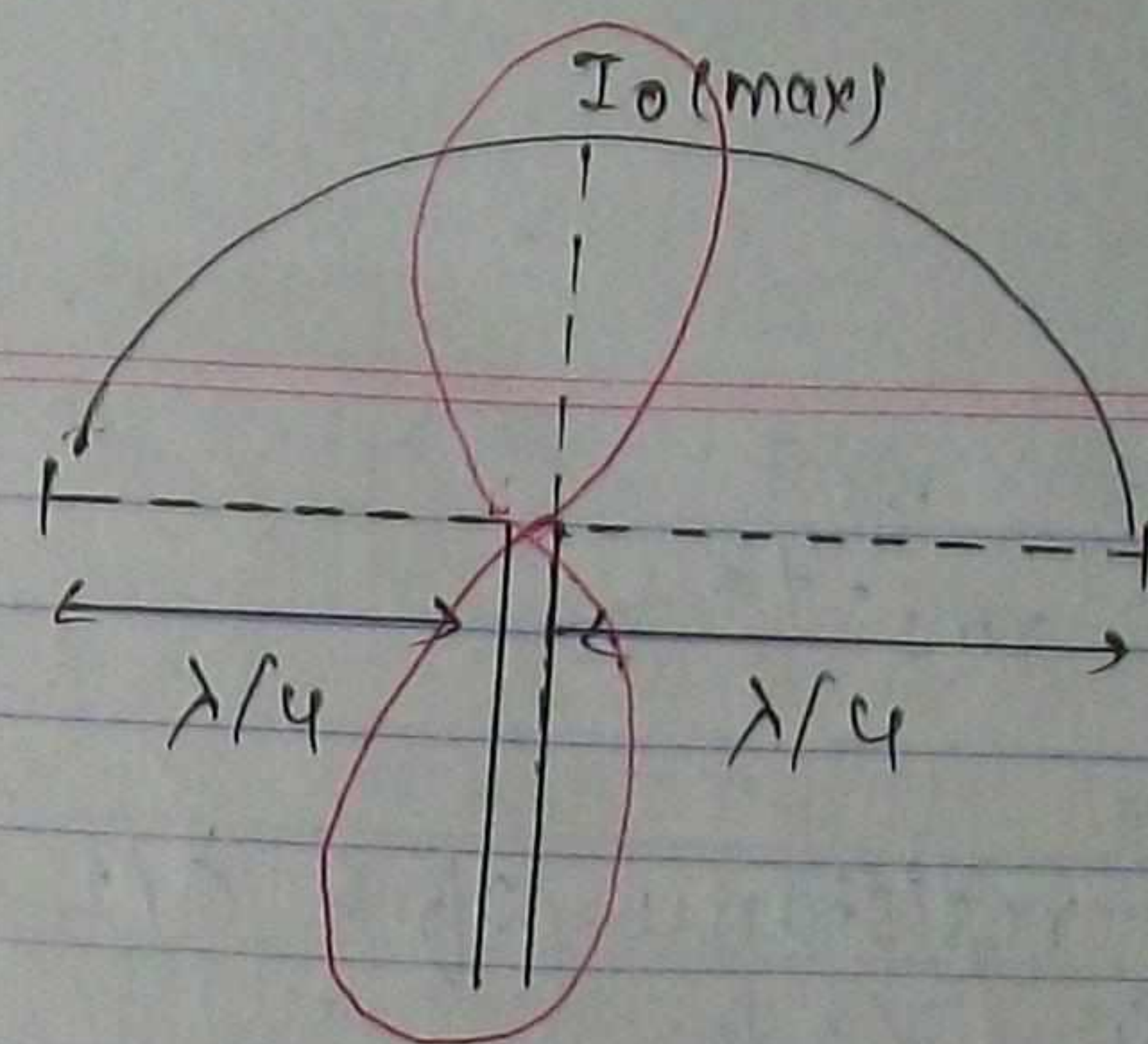


Non-uniform array of Hertzian dipole



$I_m = I_0 \cos \beta z$ (unequal current strength)

Asymptotic nature



- A half wave dipole is a non uniform array of hertzian dipoles of asymptotic current strength at various lengths.

- EM expression are given by :

$$E(r, \theta, \phi, t)_{\theta} = \left[\frac{60 I_m}{r} \frac{\cos(\pi/2 \cos \theta)}{\sin \theta} \right] e^{-j\beta r} \sin \omega t \quad a_{\theta}$$

$$H(r, \theta, \phi, t)_{\phi} = \left[\frac{I_m}{2\pi r} \frac{\cos(\pi/2 \cos \theta)}{\sin \theta} \right] e^{-j\beta r} \sin \omega t \quad a_{\phi}$$

⇒ TOTAL POWER RADIATED FROM AN HALF WAVE DIPOLE -

$$W_r = \iint P_{avg} \cdot d\Omega$$

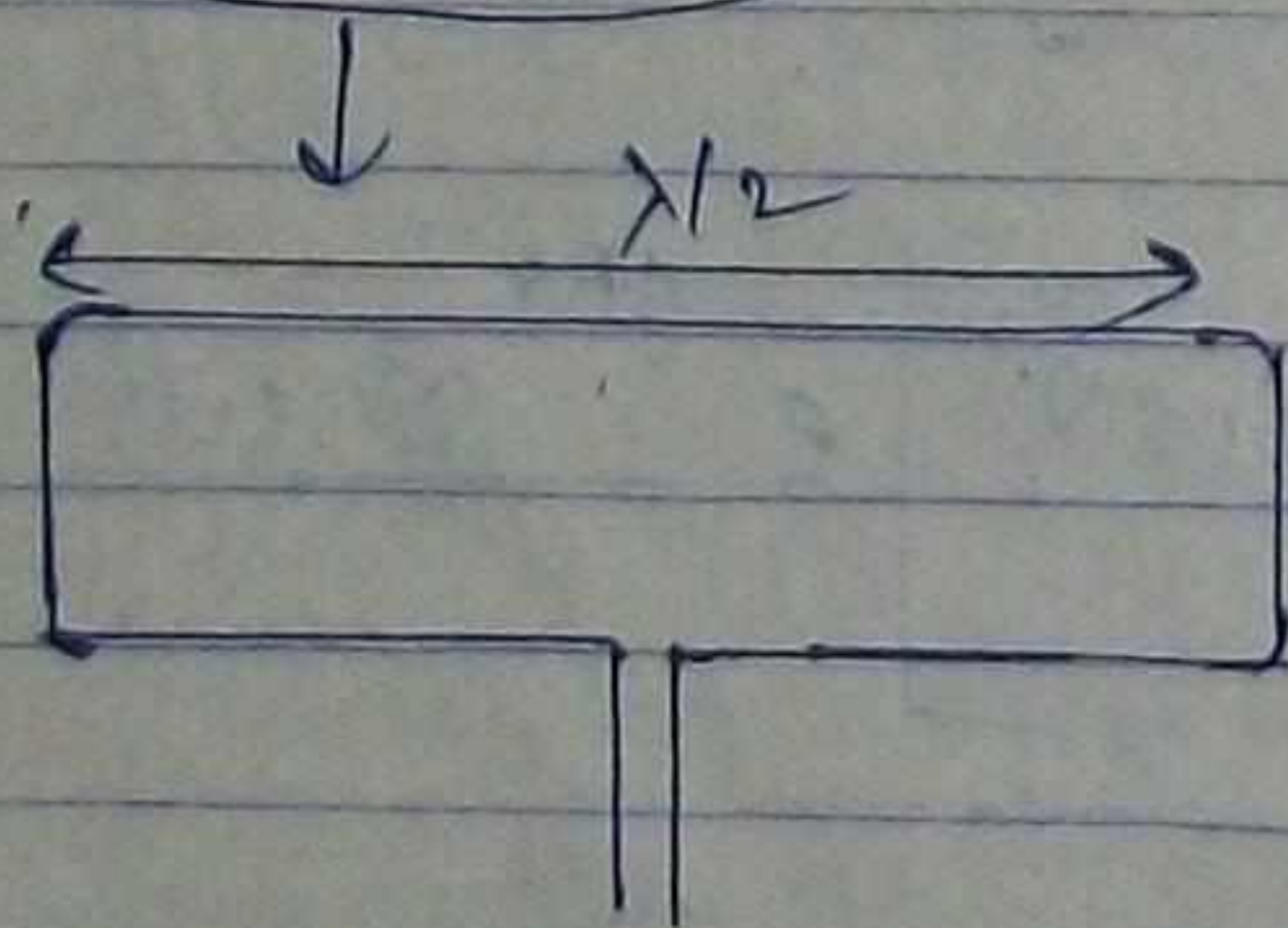
$$= \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \frac{1}{2} \left(\frac{60 I_m}{r} \right)^2 \left(\frac{\cos(\pi/2 \cos \theta)}{\sin \theta} \right)^2 \times \frac{1}{120\pi} r^2 \sin \theta d\theta d\phi$$

$$W_r = I_{rms}^2 \cdot 73$$

∴ Radiation resistance of a $\lambda/2$ wave dipole is 73Ω

Radiation resistance of a $\lambda/4$ - Monopole is 36.5Ω ($73/2$)

Radiation resistance of a $n^2 \times 73 = n^2 \times 73$ (folded dipole) $= 292 \Omega$



Radiation resistance of $\lambda/8$ - dipole is

$$\frac{36.5}{2} = \underline{18.25}$$

⇒ BASIC TERMS AND DEFINITION OF ANTENNA -

① ISOTROPIC ANTENNA - (OMNIDIRECTIONAL)

It is radiates power in all direction uniformly. This radiation fields are independent of θ and ϕ .

Broadcast applications needs this nature.

② RADIATION POWER DENSITY -

It is the strength of the power of the EM wave at any point around the antenna.

$$RPD = \frac{dW_r}{ds} = \frac{\text{watts}}{m^2} = \frac{\text{power}}{\text{area}}$$

= time averaged poynting vector of the radiated wave.

$$RPD = \frac{1}{2} \frac{E_0^2(r, \theta, \phi)}{\eta} = u(r, \theta, \phi)$$

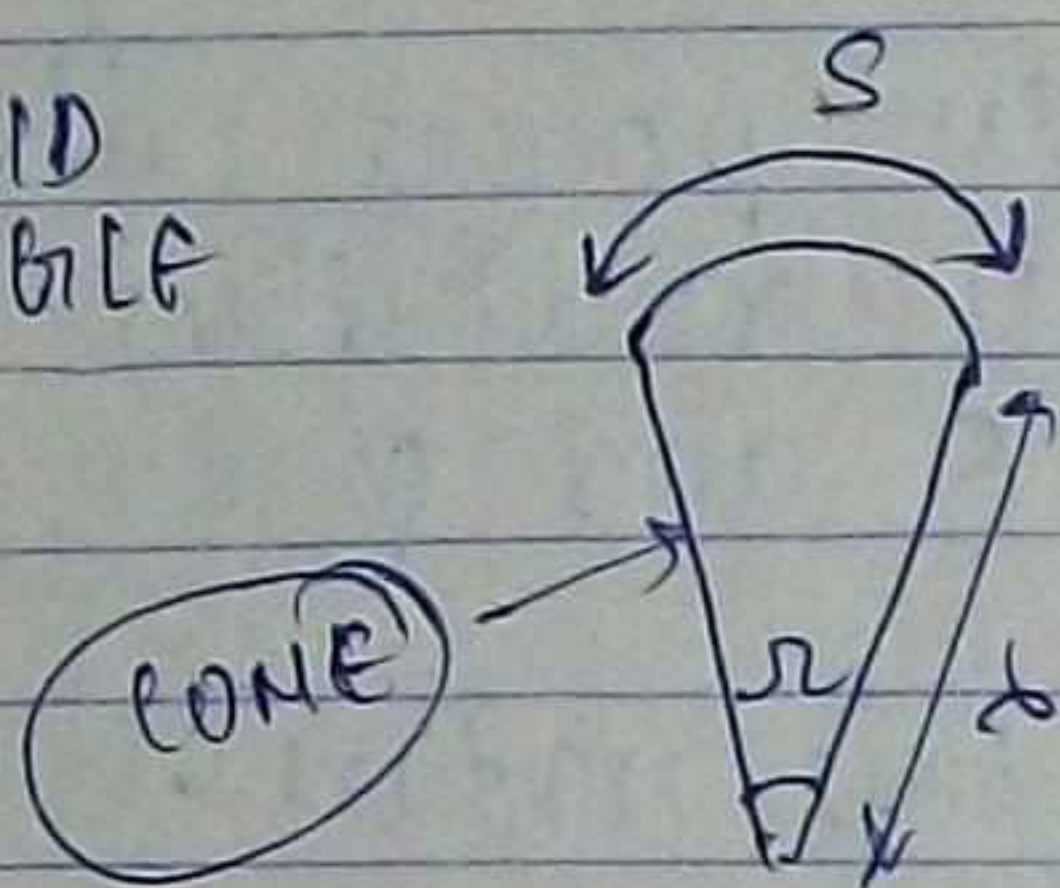
③ RADIATION POWER INTENSITY -

It is the strength of the power of the radiation in any direction from the antenna.

$$RPI = \frac{dw_r}{d\Omega} = \frac{\text{watts}}{\text{steradian}} = \frac{\text{Power}}{\text{solid angle}}$$

#

SOLID ANGLE



Ω = solid angle
 S = area of curvature

$$S = \Omega r^2$$

when $\Omega = 1$

$$S = r^2$$

unit $\rightarrow$ steradian = radian²

when $\Omega = 12.56$ steradian

$$TSA = 12.56 r^2$$

$$\frac{ds}{d\Omega} = r^2$$

— One steradian is a solid angle subtended by a cone whose base curvature area is r^2 .

$$\therefore RPI = \frac{dw_r}{d\Omega} = \frac{dw_r}{ds} \cdot \frac{ds}{d\Omega}$$

$$RPI = u(r, \theta, \phi) \cdot r^2 = \psi(\theta, \phi)$$

Ex: Isotropic antenna

(1) average power density —

$$\frac{dw_r}{ds} = \frac{w_r}{S} = U_{avg} = \frac{w_r}{4\pi r^2}$$

(2) average power intensity —

$$\frac{dw_r}{d\Omega} = \frac{w_r}{\Omega} = \psi_{avg} = \frac{w_r}{4\pi}$$

(4) GAIN OF AN ANTENNA —

(A) DIRECTIVE GAIN (G_D) —

Radiation intensity of the given antenna in a given direction watt Rad^n
 Intensity of isotropic antenna.

$$G_D = \frac{\text{Rad}^n \text{ Intensity of given antenna in a given direction}}{\text{Rad}^n \text{ Intensity of an isotropic antenna}}$$

$$G_D = \frac{\psi(\theta, \phi)}{\psi_{avg}} = \frac{\psi(\theta, \phi)}{\frac{w_r}{4\pi}}$$

$$= \frac{4\pi \psi(\theta, \phi)}{\omega_r} = \frac{4\pi \psi(\theta, \phi)}{\int \psi(\theta, \phi) \cdot d\Omega}$$

(B) POWER GAIN (G_p) ←

$$G_p = \frac{4\pi \cdot \psi(\theta, \phi)}{\omega_{IN}}$$

where,

ω_{IN} = total I/P power

$$G_p = \frac{4\pi \psi(\theta, \phi)}{\omega_r} \times \frac{\omega_r}{\omega_{IN}}$$

$$G_p = G_D \times \text{EFFICIENCY OF RADIATION}$$

$$G_p = G_D \times \frac{\omega_r}{\omega_r + \omega_l}$$

$$G_p = G_D \times \frac{R_r}{R_r + R_l}$$

where,

ω_l = loss power

R_r = radⁿ resistance

R_l = loss resistance.

(C) DIRECTIVITY -

$$\text{Directivity} = G_D / \max$$

D = max. possible value for G_D in the principle maxima.

$$D \geq 1 \quad (\text{Always})$$

$G_D \leq 1$ or $G_D \geq 1$, It depends on the direction.

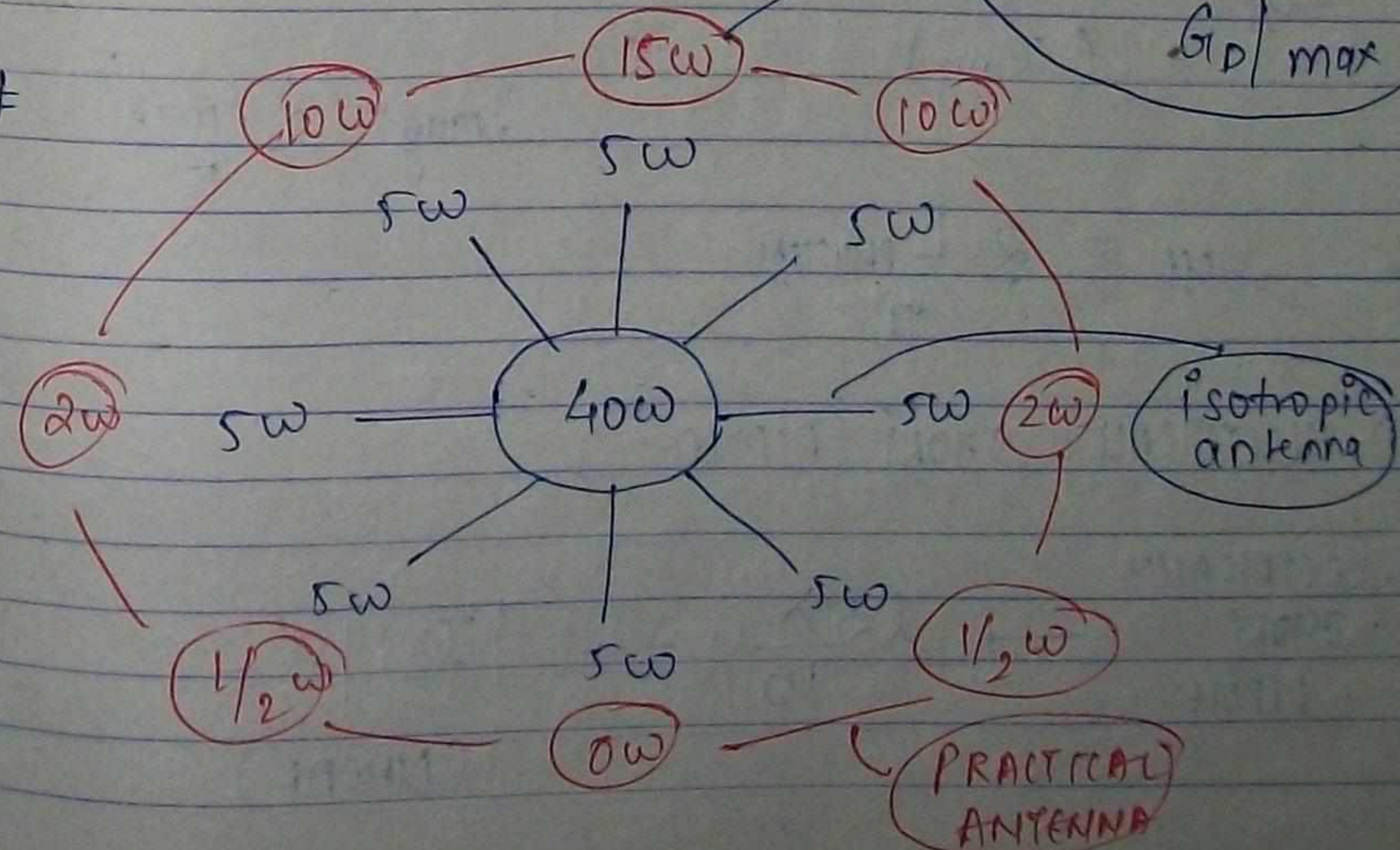
$$D = 1$$

Isotropic antenna

DIRECTIVITY = 3

$G_D / \max$

#

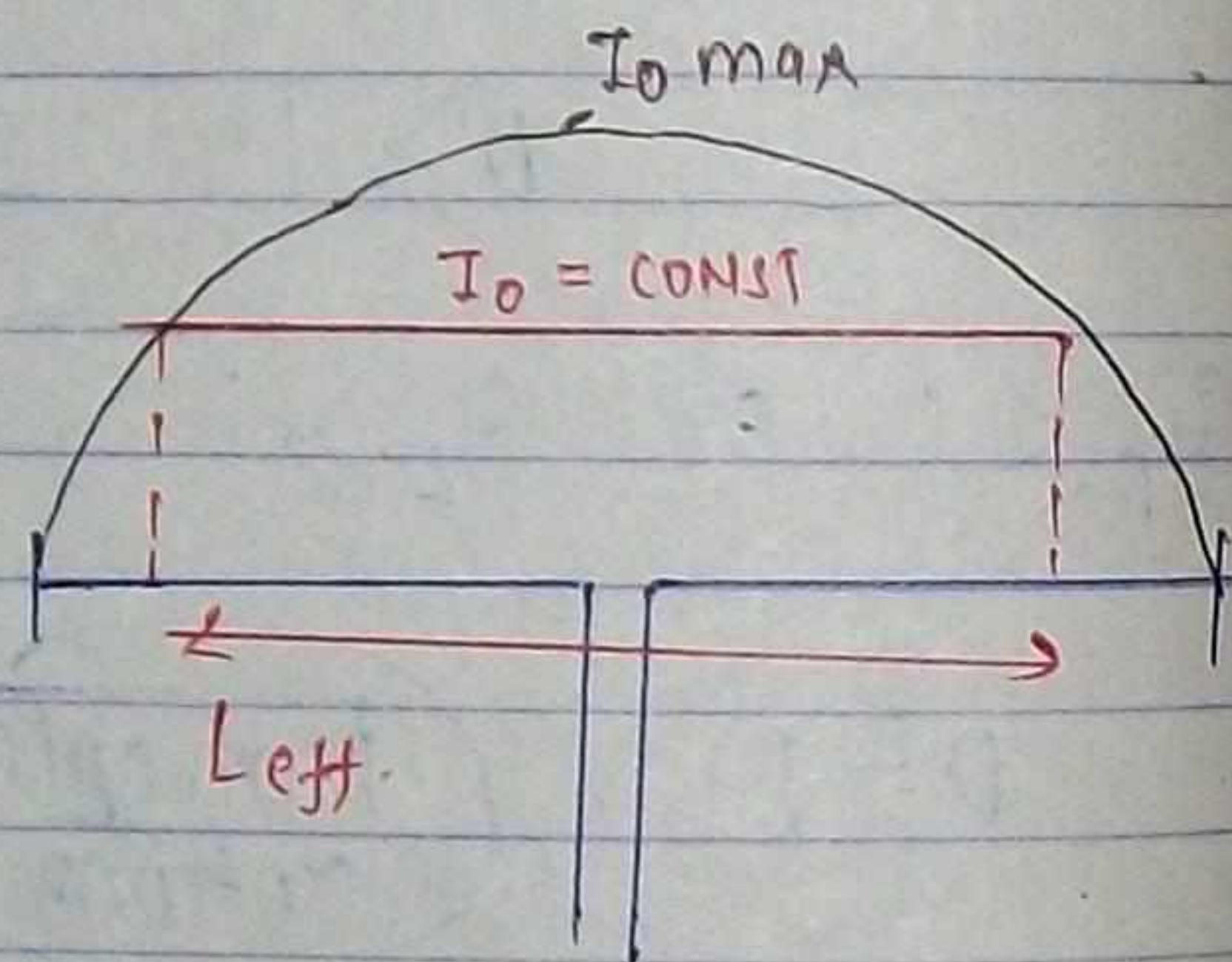


⇒ EFFECTIVE LENGTH OF AN ANTENNA -

① HALF WAVE DIPOLE -

- If an antenna radiates ω_r power over its physical length with non uniform current strengths, the length required to radiate the same power with averagely uniform current flowing everywhere is called effective length.

$\omega_r = L_{\text{physical}} = \text{Non uniform}$
 (0 - I_{max})
 asymptotic
 Same $\downarrow$
 $\omega_r = L_{\text{eff}} = \text{Uniform}$
 (I_{avg})



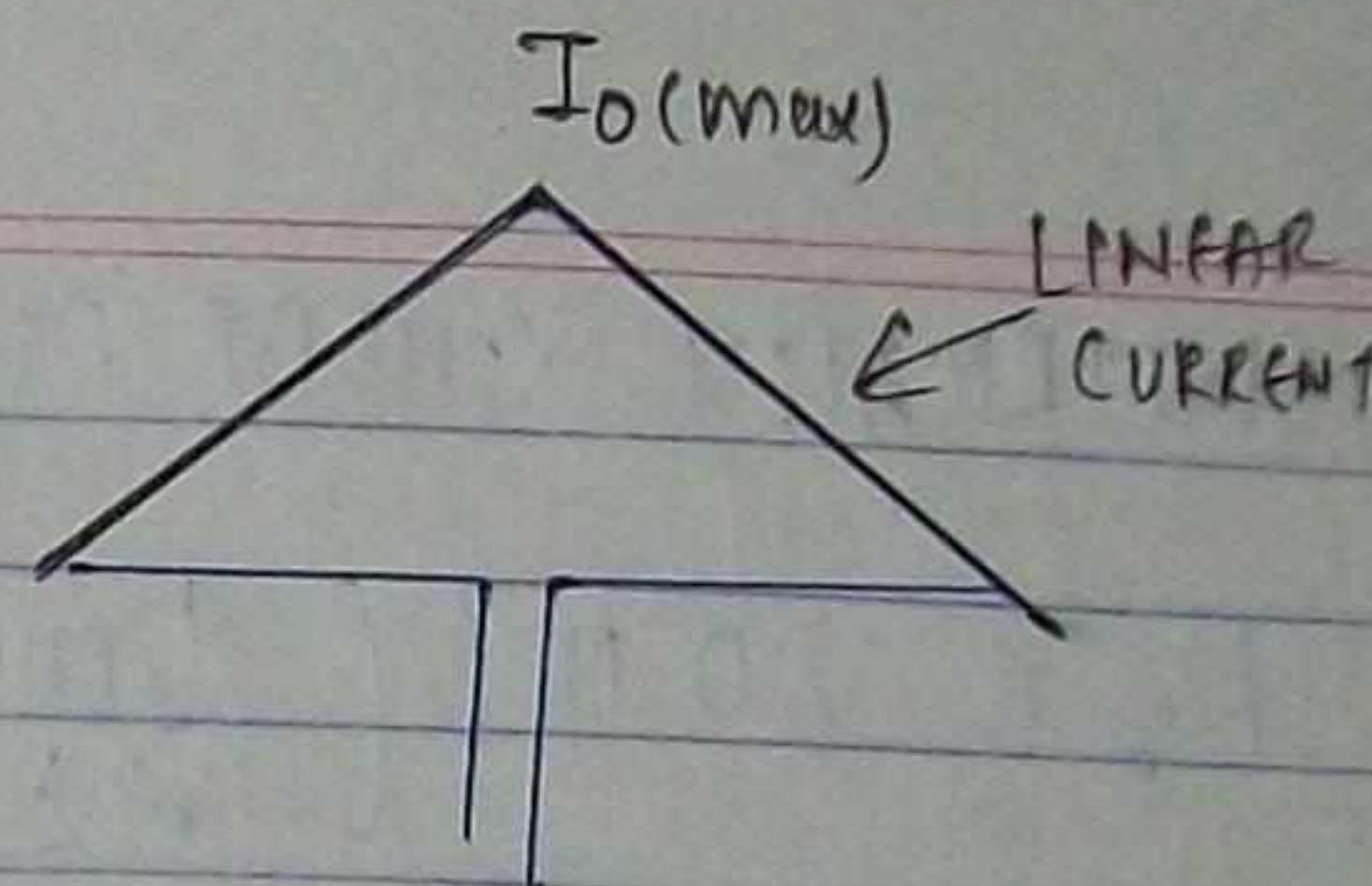
$$I_{\text{avg}} = \frac{2 I_{\text{max}}}{\pi}$$

$$\therefore L_{\text{eff}} = \frac{2 L_{\text{PHYSICAL}}}{\pi}$$

② ELECTRICALLY SHORT DIPOLE -

ELECTRICALLY SHORT DIPOLE $\rightarrow l < \frac{\lambda}{10} \rightarrow |I(z)| \propto z$
 (LINEAR)

$$I_{\text{avg}} = \frac{I_{\text{max}}}{2}$$



$$\therefore L_{\text{eff}} = \frac{L_{\text{PHY}}}{2}$$

③ HERTZIAN DIPOLE -

HERTZIAN DIPOLE $\rightarrow l < \frac{\lambda}{25}$

$\omega_r = L_{\text{PHY}} = \text{UNIFORM}$

$\omega_r = L_{\text{EFF}} = \text{UNIFORM}$

$$\therefore L_{\text{EFF}} = L_{\text{PHY}} = dl$$

⇒ RADIATION RESISTANCES -

① HERTZIAN DIPOLE -

$$R_r = 80 \pi^2 \left(\frac{dl}{\lambda} \right)^2$$

② FOR ANY DIPOLE -

$$R_r = 80 \pi^2 \left(\frac{L_{\text{EFF}}}{\lambda} \right)^2$$

FOR ELECTRICALLY SHORT DIPOLES -

$$R_r = 80 \pi^2 \left(\frac{L_{PHY}}{2\lambda} \right)^2$$

$$R_r = 20 \pi^2 \left(\frac{L_{PHY}}{\lambda} \right)^2$$

FOR ELECTRICALLY SHORT MONOPOLES -

$$R_r = 10 \pi^2 \left(\frac{L_{PHY}}{\lambda} \right)^2$$

for electrically short monopoles over conducting earth

$$R_r = 10 \pi^2 \left(\frac{2h}{\lambda} \right)^2 = 40 \pi^2 \left(\frac{h}{\lambda} \right)^2$$

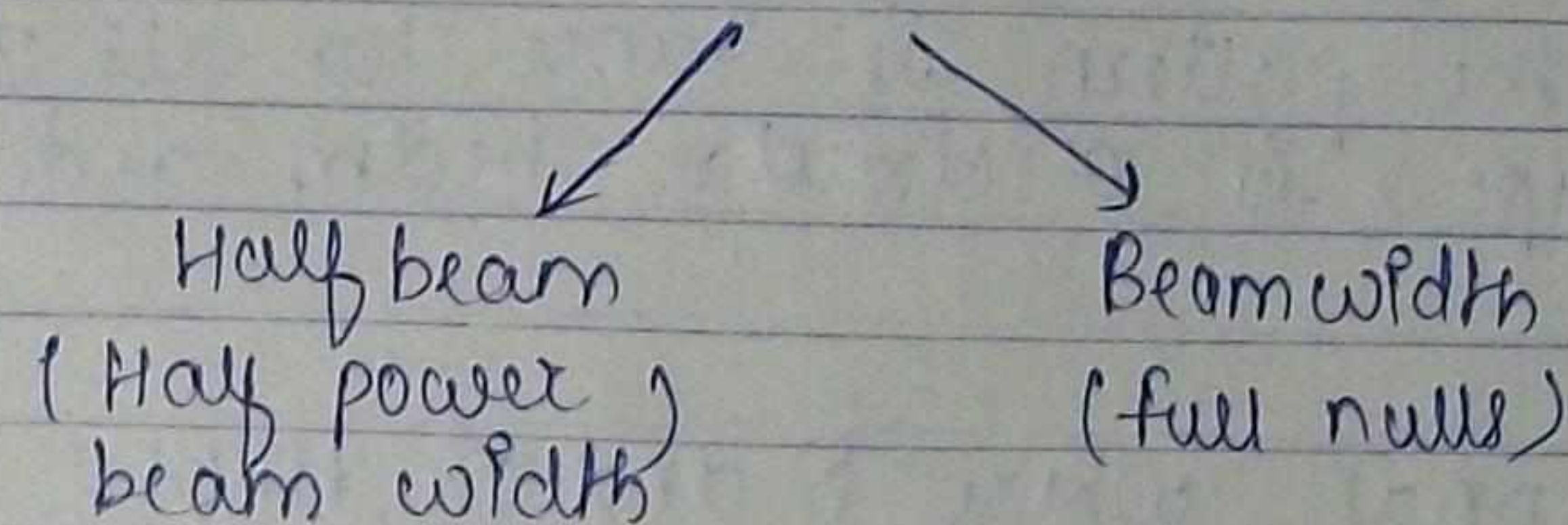
where, h = height of the monopole over the earth.

⇒ RADIATION PATTERN OF AN ANTENNA -

It is the polar plot of the radiation intensity around the antenna.

It indicates the direction where where radⁿ is finite. (angular)

finite



FOR HERTZIAN DIPOLE -

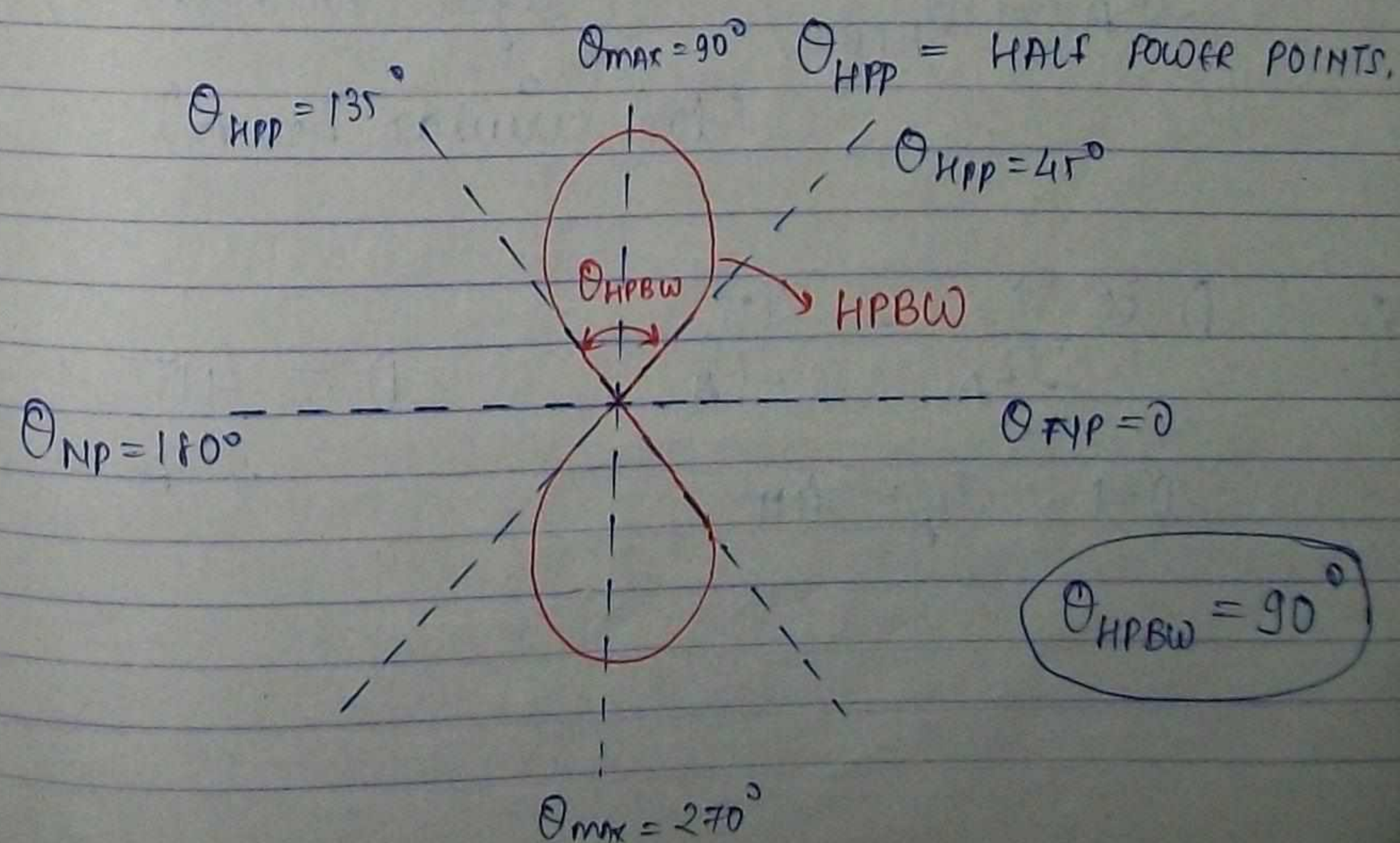
$$|E| = \frac{C_1 \sin \theta}{r^2}$$

$$\theta = 0^\circ / 180^\circ \quad \theta_{NP} = \text{NULL POINTS}$$

$$\psi = C_1' \sin^2 \theta$$

$$\theta = 90^\circ / 270^\circ \quad \theta_{max} = \text{MAX. RADIATION}$$

$$\theta = 45^\circ / 135^\circ / 225^\circ / 315^\circ$$



- If the antenna is has a angular dependency on θ only, the pattern is same for all ϕ hence the shape is a circular beam and a 3D plot.

- for other antennas where θ and ϕ HPBW are defined we defined beam solid angle (Ω)

$\Omega_A = \text{BEAM SOLID ANGLE}$ (STERADIAN)

$$\Omega_A = \theta_{\text{HPBW}} \times \phi_{\text{HPBW}}$$

$$\Omega_A = (\theta_{\text{HPBW}})^2 \quad (\text{degree}^2 \text{ or } \text{radian}^2)$$

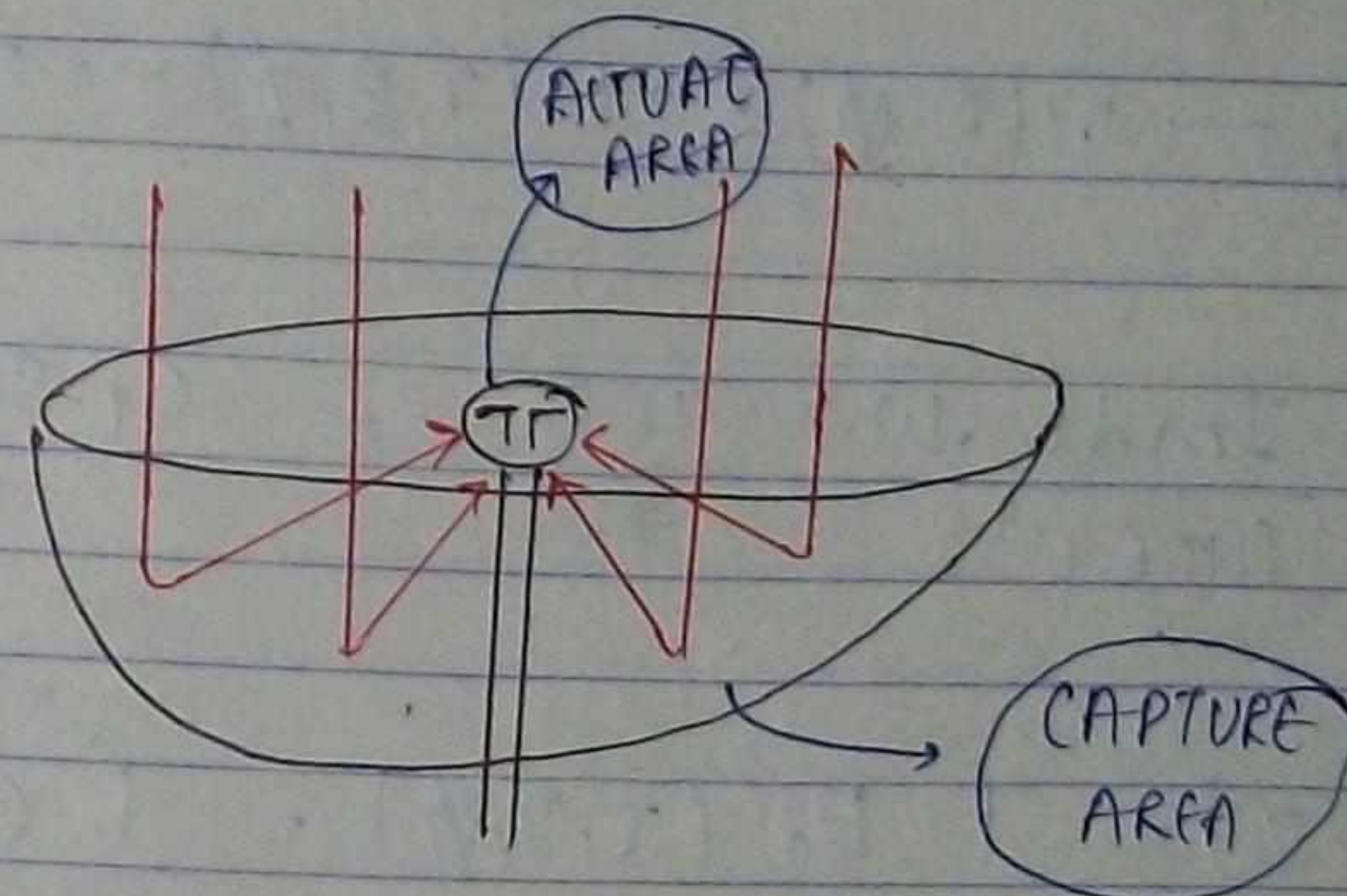
(for circular beam)

$$\therefore D \propto \frac{1}{\Omega_A} = \frac{4\pi}{\Omega_A}$$

$$D=1 \quad \Omega_A = 4\pi$$

$$D = \frac{4\pi}{\Omega_A}$$

$\Rightarrow$ EFFECTIVE AREA OR CAPTURE AREA -



$$\lambda - \text{MW} - 10^9 \text{ Hz} \rightarrow \text{cm}$$

$$x \rightarrow 1000 \text{ km}$$

$$A_e = \frac{\text{POWER INDUCED}}{\text{POYNTING VECTOR OF THE RADIATION}} = \frac{\text{WATT}}{\text{WATT/m}^2}$$

$$A_e = \frac{\lambda^2 \cdot D}{4\pi}$$

Ques 1. P-42

Soln:

$$D \rightarrow G_D \rightarrow \psi(\theta, \phi) \rightarrow U(r, \theta, \phi) \rightarrow E(r, \theta, \phi, t)$$

$$|E| = \frac{I_{\text{max}} \cdot \omega}{4\pi \epsilon c^2} \frac{\sin \theta}{r} = C_1 \frac{\sin \theta}{r}$$

RFD

$$U(r, \theta, \phi) = \frac{1}{2} \frac{E_0^2(r, \theta, \phi)}{\eta} = \frac{1}{2} \frac{C_1^2 \sin^2 \theta}{r^2 \eta}$$

RFD

$$\psi(\theta, \phi) = \frac{1}{2} \frac{C_1^2 \sin^2 \theta}{\eta}$$

$$G_D = \frac{4\pi \cdot \frac{1}{2} \frac{C_1^2 \sin^2 \theta}{\eta}}{\int_0^\pi \int_0^{2\pi} \frac{1}{2} \frac{C_1^2 \sin^2 \theta}{\eta} \sin \theta \cdot d\theta \cdot d\phi}$$

$$\int_0^\pi \int_0^{2\pi} \frac{1}{2} \frac{C_1^2 \sin^2 \theta}{\eta} \sin \theta \cdot d\theta \cdot d\phi$$

$$G_D = \frac{2 \sin^2 \theta}{\int_0^\pi \sin^3 \theta \cdot d\theta} = \frac{3 \sin^2 \theta}{2}$$

$$D = G_D|_{\text{max}} = \frac{3}{2}$$

Ques. Repeat same for HALF WAVE DIPOLE.

Soln:

$$|E| = \frac{60 I_m}{r} \frac{\cos(\pi/2 \cos \theta)}{\sin \theta}$$

$$= C_1 \frac{\cos(\pi/2 \cos \theta)}{r \cdot \sin \theta}$$

$$D = 1.63$$

FOR HALF WAVE DIPOLE

$$D = 1.5$$

FOR HERTZIAN DIPOLE

Ques. 2. P-42

Soln:

$$W_r = I_{\text{rms}}^2 \cdot 7.3$$

$$\begin{aligned} \text{Total power for 4 antenna} &= 4 \times \left(\frac{0.5}{\sqrt{2}}\right)^2 \times 7.3 \\ &= \underline{\underline{36.5 \text{ W}}} \end{aligned}$$

Ques. 3 P-42

Soln

$$44 = 10 \log D \quad D = (10)^{4.4}$$

$$D = \frac{4\pi}{\Omega_A} = 10^{4.4} = \frac{4\pi}{(\Theta_{HPBW})^2}$$

$$10^{4.4} = \frac{4 \times 3.14 \times (57 \times 57)}{(\Theta_{HPBW})^2}$$

$$\Theta_{HPBW} = 1.4456^\circ$$

Que. 4. P-42
Solⁿ:

$$\eta = \frac{R_r}{R_r + R_l} = \frac{18.25}{18.25 + 1.5} = 89.15\%$$

Que. 5. P-42
Solⁿ:

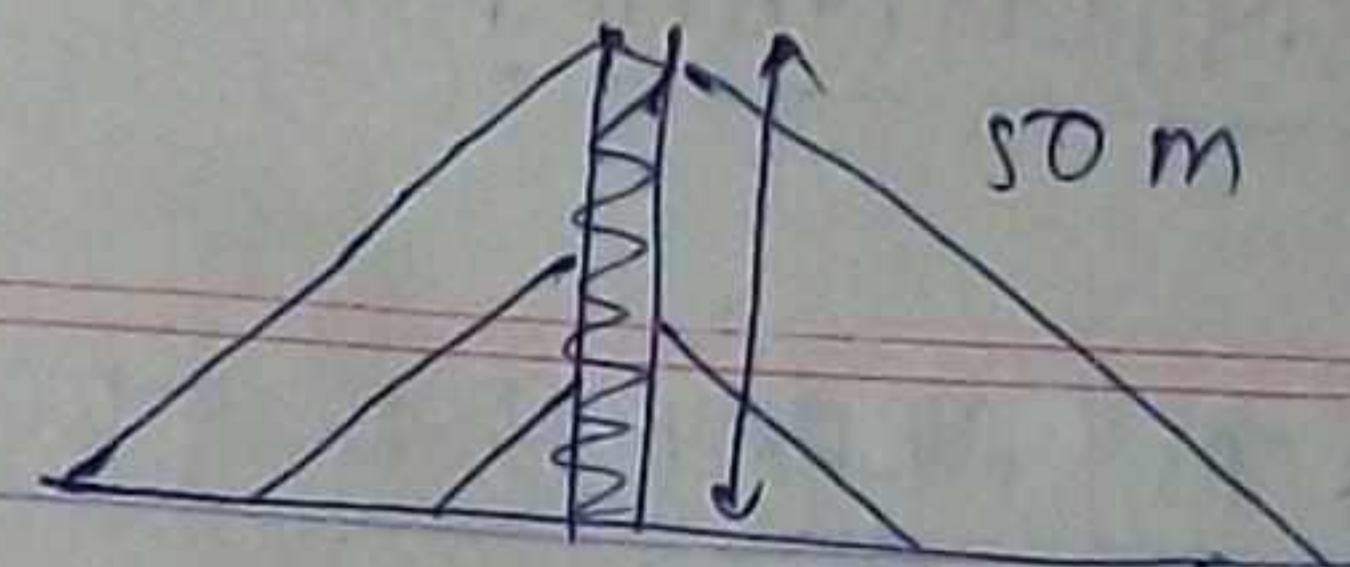
$$\lambda = 492 \text{ m} \quad l = 124 \text{ m} \quad \text{is } \lambda/4 \text{ antenna}$$

$$R_r \text{ for } \lambda/4 = 36.5 \Omega$$

Que. 7. P-43.

$$10 \log D = 6 \quad (D = 4)$$

Que. 9.



$$f = 600 \text{ kHz} \quad \lambda = 500 \text{ m} \quad h = \lambda/10$$

$$R_r = \frac{40\pi^2}{500} \left(\frac{50}{500}\right)^2 = \frac{2\pi^2}{5} \approx 4 \Omega$$

Que. 10.

$$\gamma_{avg} = \frac{100}{4\pi} = \frac{25}{\pi} = 7.96$$

$$V_{avg} = \frac{100}{4\pi (10 \times 10^3)^2} = 0.08 \text{ uW}$$

Que. 12.

$$\eta = 90\% \quad \psi(\theta, \phi)_{max} = 150 \text{ W/r}$$

$$W_{in} = 40\pi$$

$$D = G_D \Big|_{max} = \frac{4\pi \psi(\theta, \phi)_{max}}{W_r} = \frac{4\pi \times 150}{40 \times \frac{90}{100}}$$

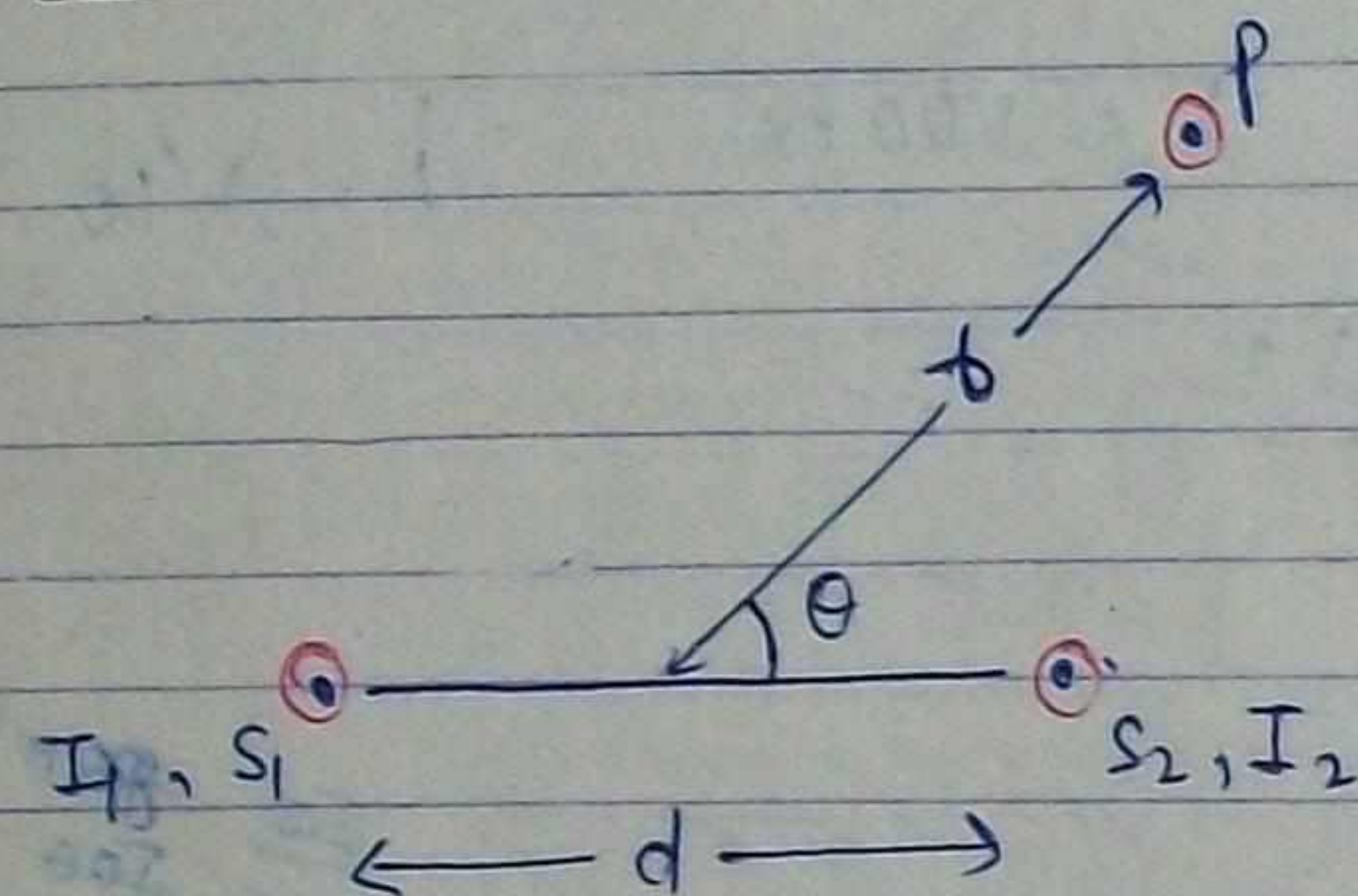
$$D = 16.67$$

$$D_{ndb} = 11.76 \text{ dB}$$

ANTENNA ARRAYS

4/4/12

⇒ 2 POINT SOURCES / ARRAY OF 2 ISOTROPIC ELEMENTS -



If $r \gg d$, means
far zone analysis.

Total electric field strength, $E_T = E_1 + E_2$

$$E_T = E_0 + K E_0 e^{j\psi}$$

where ψ = phase diffⁿ b/w the fields started from S_1 & S_2

ψ = Phase diffⁿ due to current in sources + Phase diffⁿ due to the path diffⁿ

∴ path diffⁿ = $d \cos \theta$ (d = max. path diffⁿ at $\theta = 0$)

$$\text{phase diff}^n = \frac{2\pi}{\lambda} \cdot d \cos \theta = \beta d \cos \theta$$

$$\psi = \alpha + \beta d \cos \theta \quad (\text{phase diff}^n)$$

If $|I_1| = |I_2|$, then $K=1$

$$E_T = E_0 + K E_0 e^{j\psi} = E_0 + E_0 e^{j\psi} \quad (K=1)$$

$$E_T = E_0 (1 + e^{j\psi}) = E_0 (1 + \cos \psi + j \sin \psi)$$

$$|E_T| = |E_0| \sqrt{(1 + \cos \psi)^2 + (\sin \psi)^2}$$

$$= |E_0| \sqrt{1 + \cos^2 \psi + 2 \cos \psi + \sin^2 \psi}$$

$$= |E_0| \sqrt{2 + 2 \cos \psi}$$

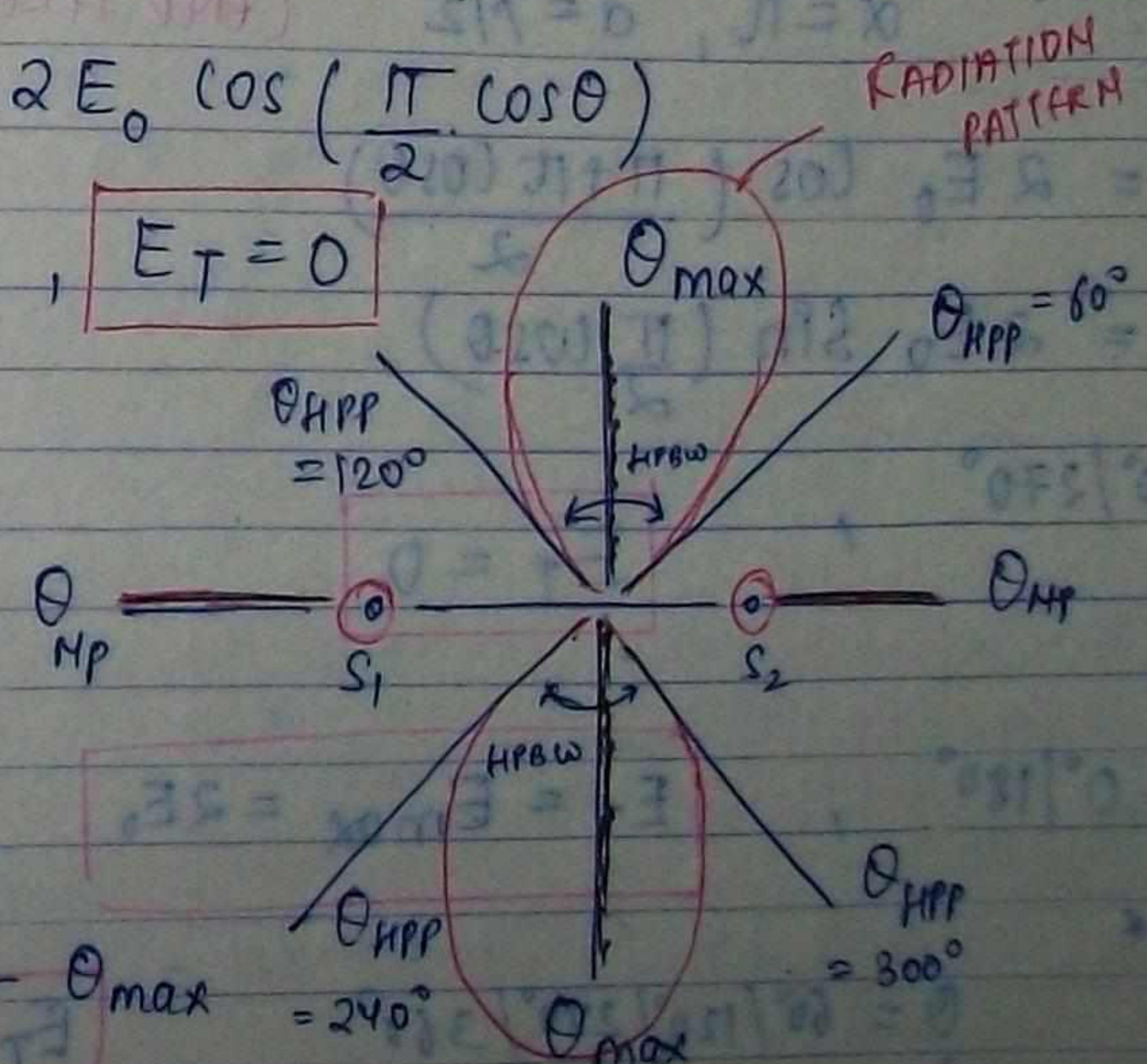
$$|E_T| = 2 E_0 \cos(\psi/2) \quad (\text{magnitude})$$

CASE (1): $\alpha = 0^\circ$, $d = \lambda/2$ (BROAD SIDE ARRAY)

$$E_T = 2 E_0 \cos\left(\frac{\pi}{2} \cos \theta\right)$$

At $\theta = 0^\circ/180^\circ$, $E_T = 0$

θ_{NP}
(null points)



At $\theta = 90^\circ/270^\circ$

$$E_T = E_{\max} = 2 E_0$$

At $\theta = 60^\circ / 120^\circ / 240^\circ / 300^\circ$

↓

θ_{HPP}

(Half power points)

$$E_T = \frac{E_{max}}{\sqrt{2}}$$

- This array is called as broad side array

whose $\theta_{max} = 90^\circ / 270^\circ$

- Half power beam width:

$$\theta_{HPBW} = 60^\circ$$

Case (2): $\alpha = \pi$, $d = \lambda/2$ (END FIRE ARRAY)

$$|E_T| = 2E_0 \cos\left(\frac{\pi + \pi \cos\theta}{2}\right)$$

$$= 2E_0 \sin\left(\frac{\pi}{2} \cos\theta\right)$$

$$\theta = 90^\circ / 270^\circ$$

↓
 θ_{NP}

$$E_T = 0$$

$$\theta = 0^\circ / 180^\circ$$

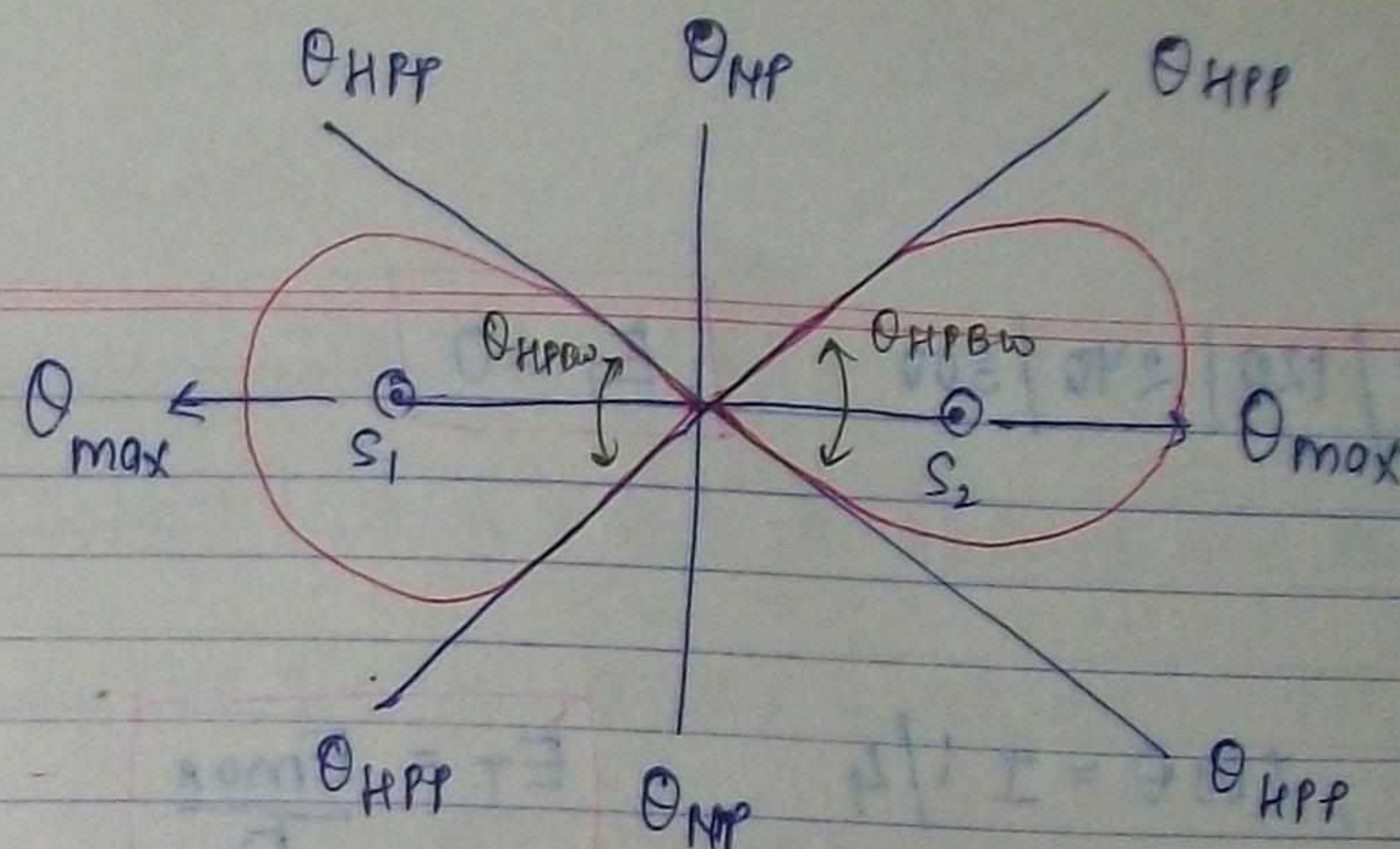
↓
 θ_{max}

$$E_T = E_{max} = 2E_0$$

$$\theta = 60^\circ / 120^\circ / 240^\circ / 300^\circ$$

↓
 θ_{HPP}

$$E_T = \frac{E_{max}}{\sqrt{2}}$$



- This is called as End-fire array.

whose $\theta_{max} = 0^\circ$ and 180°

Half power beam width

$$\theta_{HPBW} = 120^\circ$$

Case (3): $\alpha = 0$, $d = \lambda$

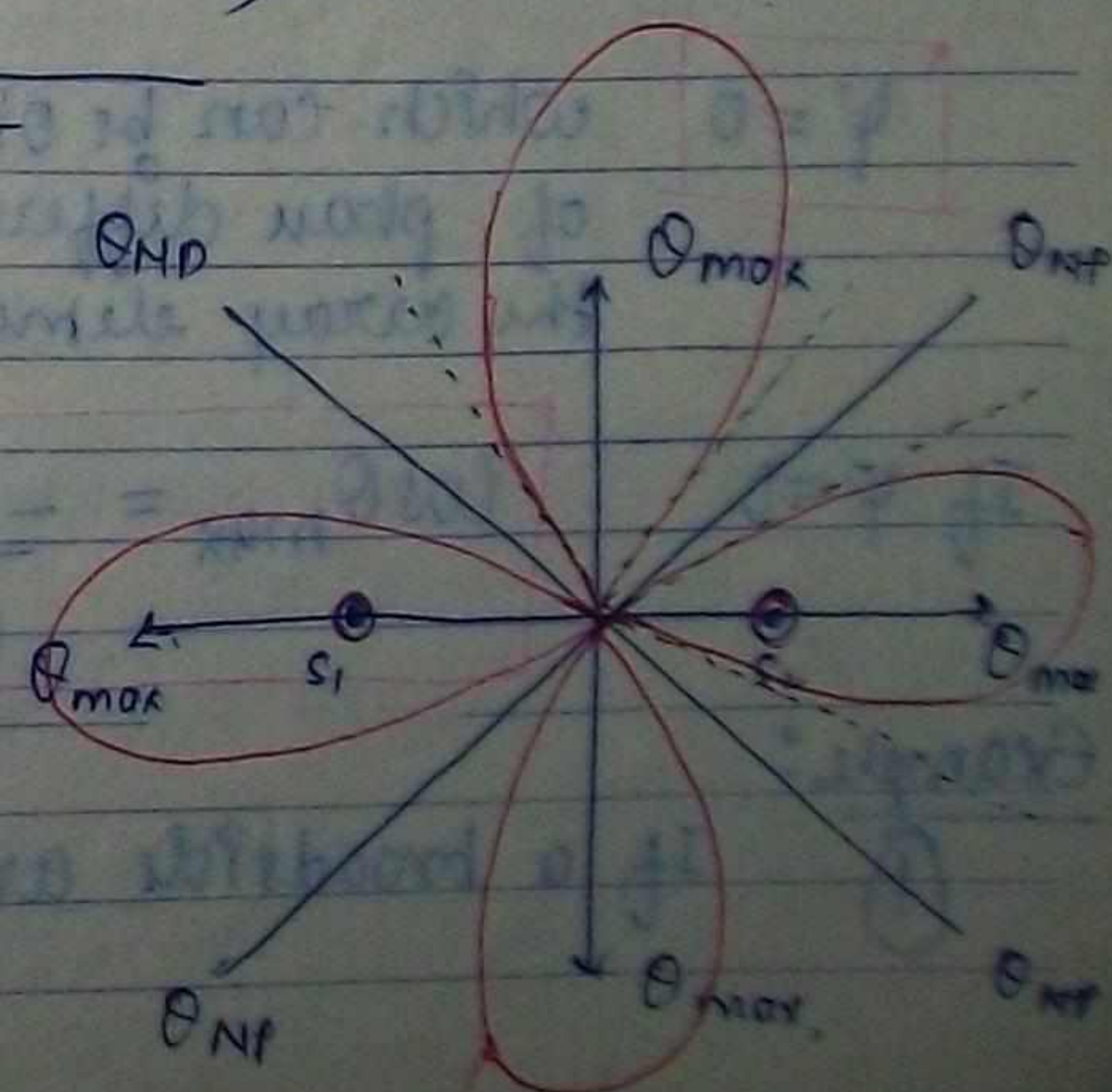
$$E_T = 2E_0 \cos\left(\frac{0 + \frac{2\pi}{\lambda} \lambda \cos\theta}{2}\right)$$

$$|E_T| = 2E_0 \cos(\pi \cos\theta)$$

$$\theta = 0^\circ / 180^\circ / 270^\circ$$

↓
 θ_{max}

$$E_T = E_{max}$$



at $\theta = 60^\circ / 120^\circ / 240^\circ / 300^\circ$

$\downarrow$
 θ_{NP}

$E_T = 0$

for HPP, $\cos \theta = \pm 1/4$

$\theta = 72^\circ, 108^\circ, 18^\circ, \dots$

$\downarrow$
 θ_{HPP}

$E_T = \frac{E_{max}}{\sqrt{2}}$

$\Rightarrow$ SCANNING ARRAYS -

$\psi = \alpha + \beta d \cdot \cos \theta$, $E_T = 2E_0 \cos(\psi/2)$

- In general for the radiation to be maximum in a desired direction, the criteria is:

$\psi = 0$

which can be obtained with right choice of phase difference in the currents of the array elements

If $\psi = 0$

$\cos \theta_{max} = \frac{-\alpha}{\beta d}$

Example:

① If a broadside array is designed:

ii $\theta_{max} = 90^\circ / 270^\circ$

ii $\alpha = 0$

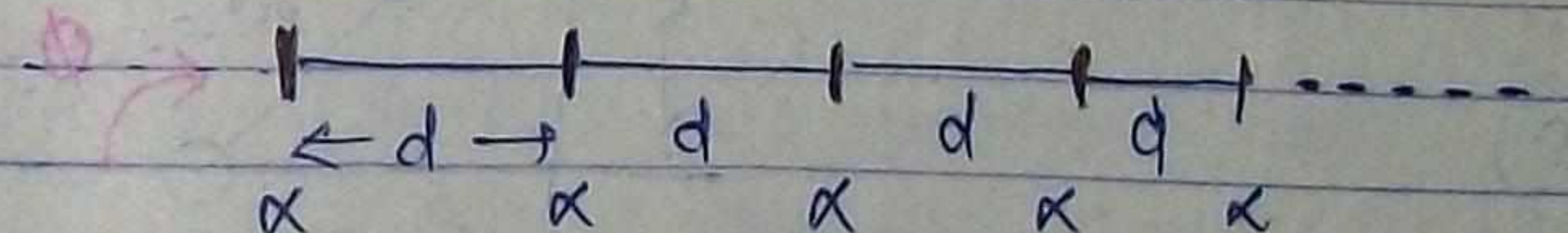
ii Inphase currents always give Broadside array.

② If an end fire array is designed

ii $\theta_{max} = 0^\circ / 180^\circ$

ii $\alpha = \pm \beta d$

$\Rightarrow$ EXTENSION: FOR N ELEMENT UNIFORM & LINEAR ARRAY -



where, d = spacing b/w any two consecutive elements
 α = progressive phase shift b/w elements

$E_T = E_0 (1 + e^{j\psi} + e^{j2\psi} + e^{j3\psi} + \dots + e^{j(N-1)\psi})$

It is G.P. with $a=1$ and $r = e^{j\psi}$

$E_T = E_0 \left(\frac{1 - e^{jN\psi}}{1 - e^{j\psi}} \right) = E_0 \frac{(1 - \cos N\psi - j \sin N\psi)}{(1 - \cos \psi - j \sin \psi)}$

$E_T = E_0 \frac{\sin(N\psi/2)}{\sin \psi/2}$

if $\psi \rightarrow 0$ $\sin \theta \approx \theta$, $E_T = NE_0$

Que-16. Two identical antennas are placed in $\theta = \pi/2$ plane, the elements have equal amplitude and 180° polarities diffⁿ, spaced at wavelength λ , the computed value of magnitude of far zone resultant E-strength normalized with that of single element both computed for $\phi = 0$ is:

(A) $2 \cos \left(\frac{2\pi s}{\lambda} \right)$

(B) $2 \cos \left(\frac{\pi s}{\lambda} \right)$

(C) $2 \sin \left(\frac{2\pi s}{\lambda} \right)$

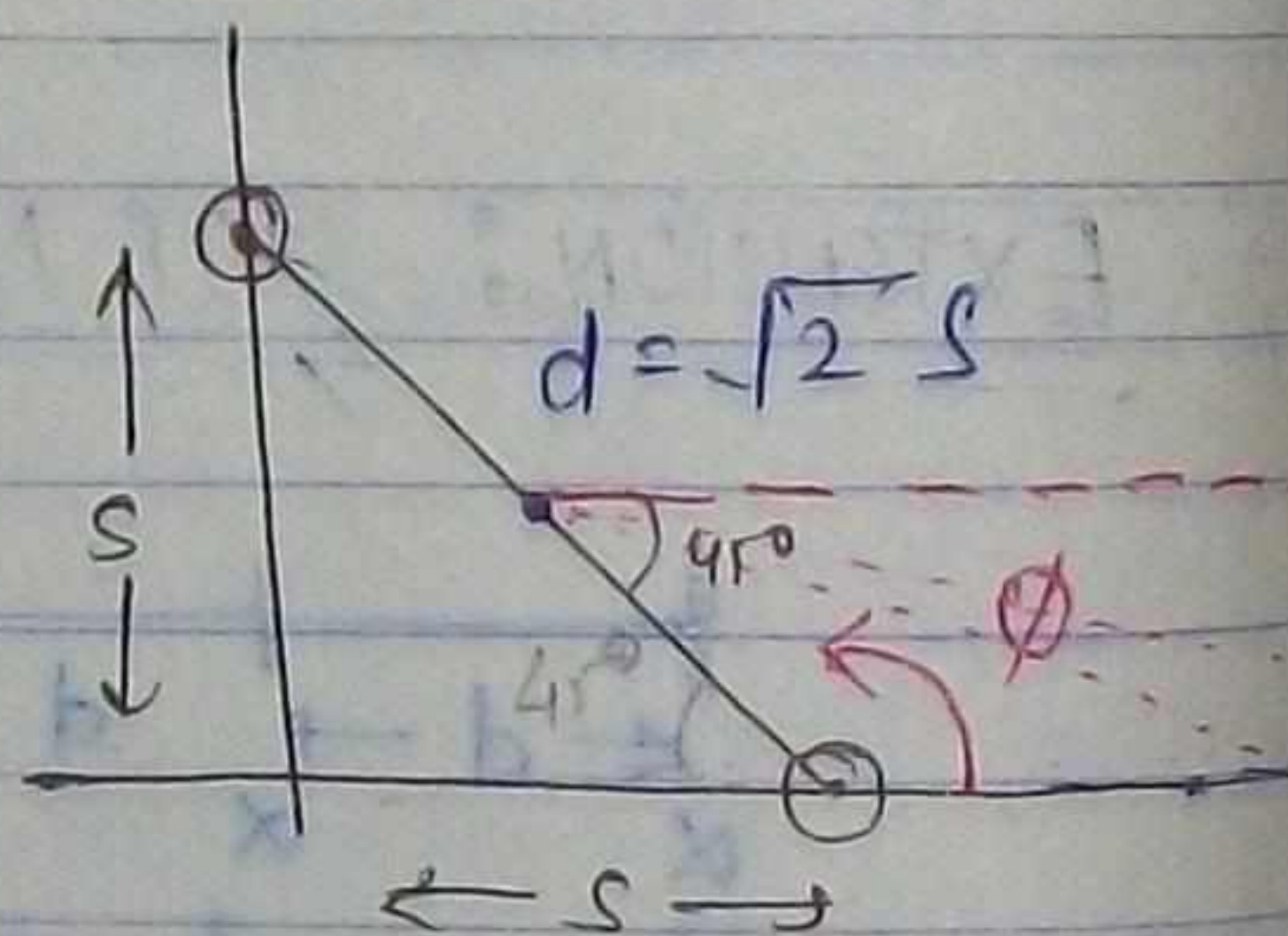
(D) $2 \sin \left(\frac{\pi s}{\lambda} \right)$

Solⁿ: $\theta = \pi/2$, $\rightarrow$ $Z=0$ plane $\rightarrow$ XY plane
 $\alpha = 180^\circ$, $E_T/E_0 = 2$, $\phi = 0$

$$\frac{E_T}{E_0} = 2 \cos \left(\frac{\alpha + \beta d \cos \theta}{2} \right)$$

$$= 2 \cos \left(\frac{\pi + \frac{2\pi}{\lambda} \cdot \sqrt{2}s \cos \left(\frac{\pi}{2} \right)}{2} \right)$$

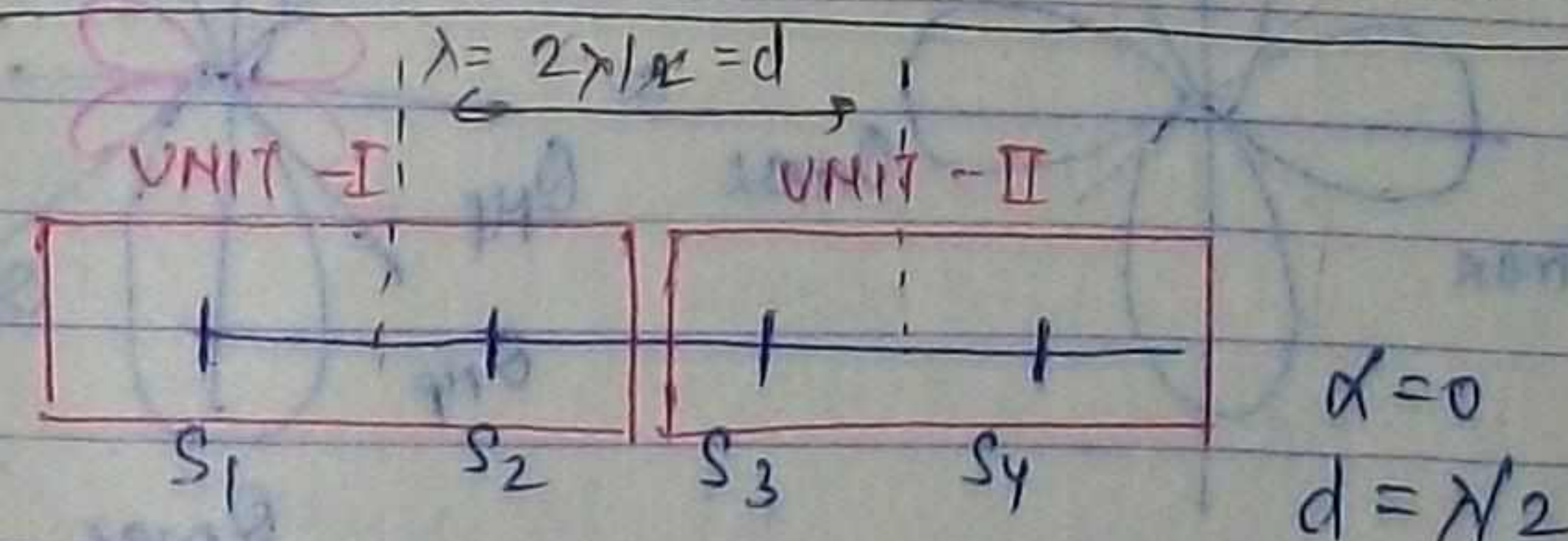
$$= 2 \sin \left(\frac{\pi s}{\lambda} \right)$$



Que. 14. A broadside array, 4 isotropic point sources of same amplitude & phase placed along the X -axis with spacing of $\lambda/2$, has a polar pattern consisting of the following:

1. ANS (c)

$\Rightarrow$ MULTIPLICATION TECHNIQUES OF PATTERNS (for symmetric array)



for an elements always even.

STEP 1: Identify two symmetric units in the group of array given

UNIT $\rightarrow$ A 2-element array of $\alpha = 0$, $d = \lambda/2$

$$E_T = 2 E_0 \cos \left(\frac{\pi}{2} \cos \theta \right)$$

STEP 2: Identify the group or the array factor of the two units

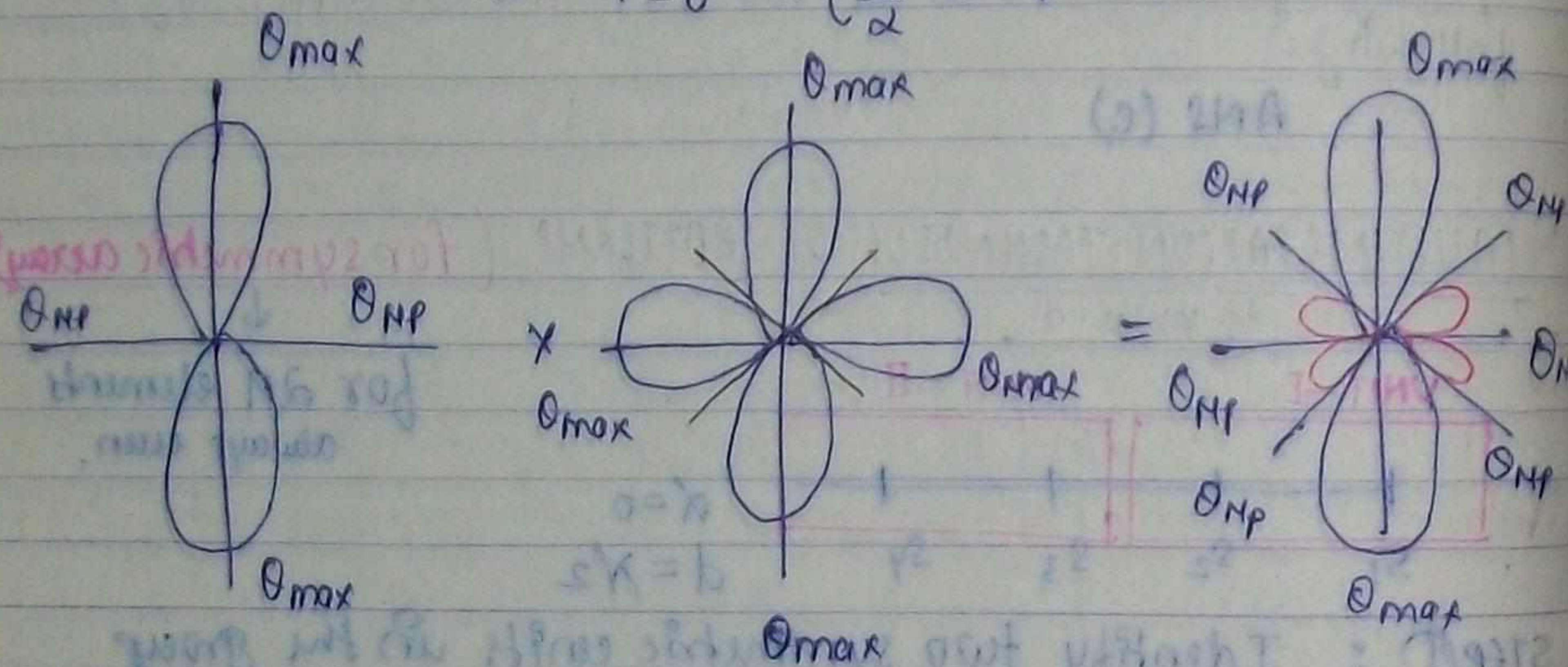
GROUP $\rightarrow$ $\alpha = 0$, $d = \lambda$ (b/w units)

$$E_T = 2 E_0 \cos (\pi \cos \theta)$$

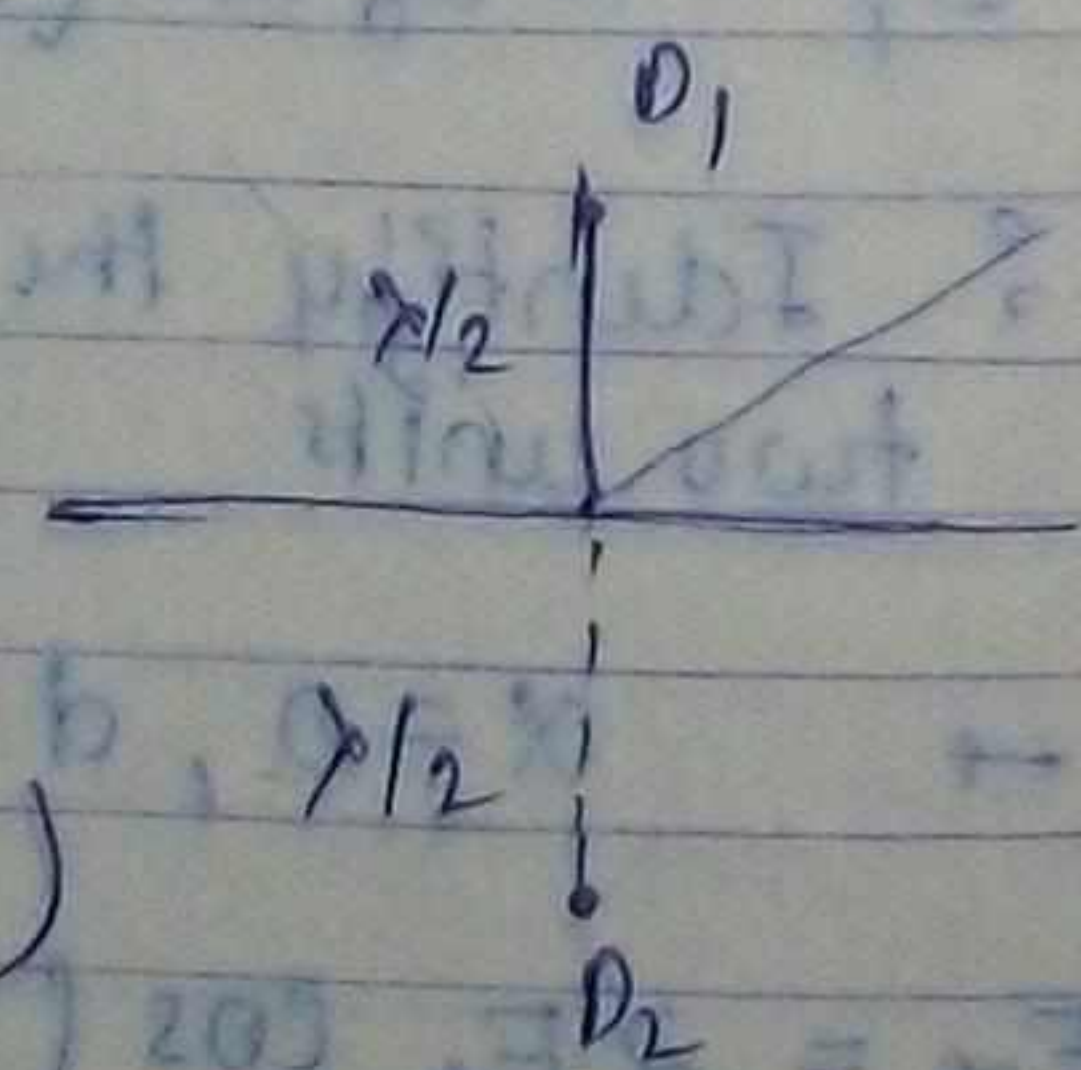
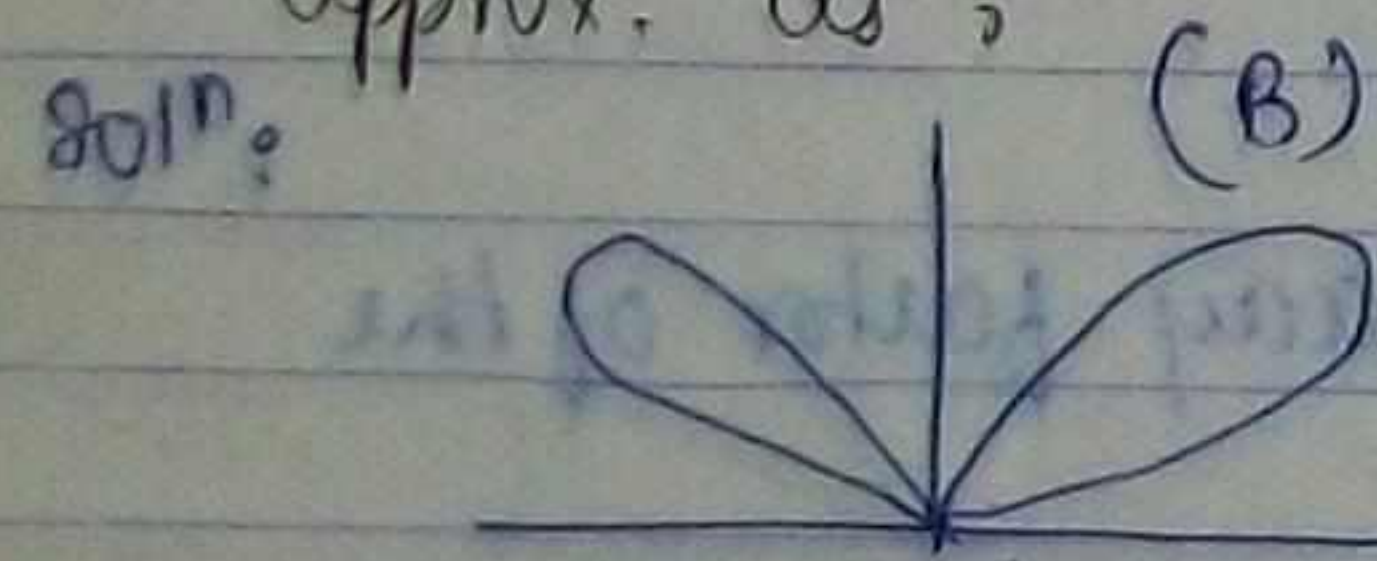
STEP 3: The resultant pattern = unit pattern $\times$ group pattern

Resultant pattern = unit pattern $\times$ Group pattern

$$= 4E_0 \cos\left(\frac{\pi}{2} \cos \theta\right) \cdot \cos(\pi \cos \theta)$$



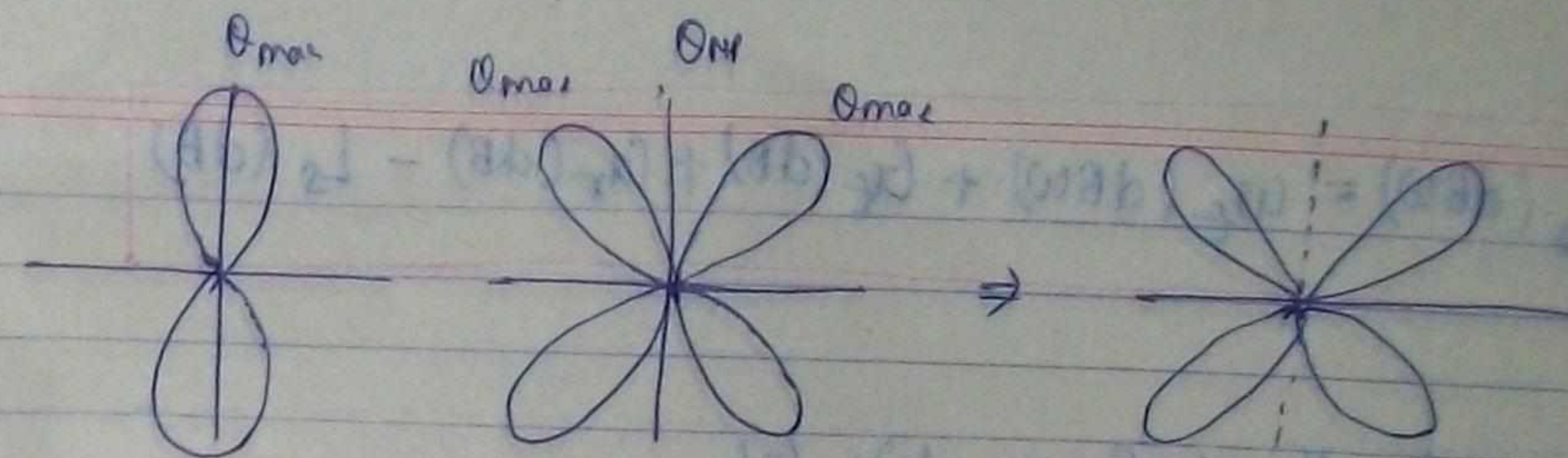
Ques 17. A $\lambda/2$ dipole is kept horizontally at a height of $\lambda/2$ above a perfectly conducting earth. The radⁿ pattern in the plane of the dipole look approx. as:



UNIT: Dipole - $\lambda/2$, $E_T = 2E_0 \cos\left(\frac{\pi}{2} \cos \theta\right)$

GROUP: $\alpha = \pi$, $d = \lambda$

$$E_T = 2E_0 \sin(\pi \cos \theta)$$

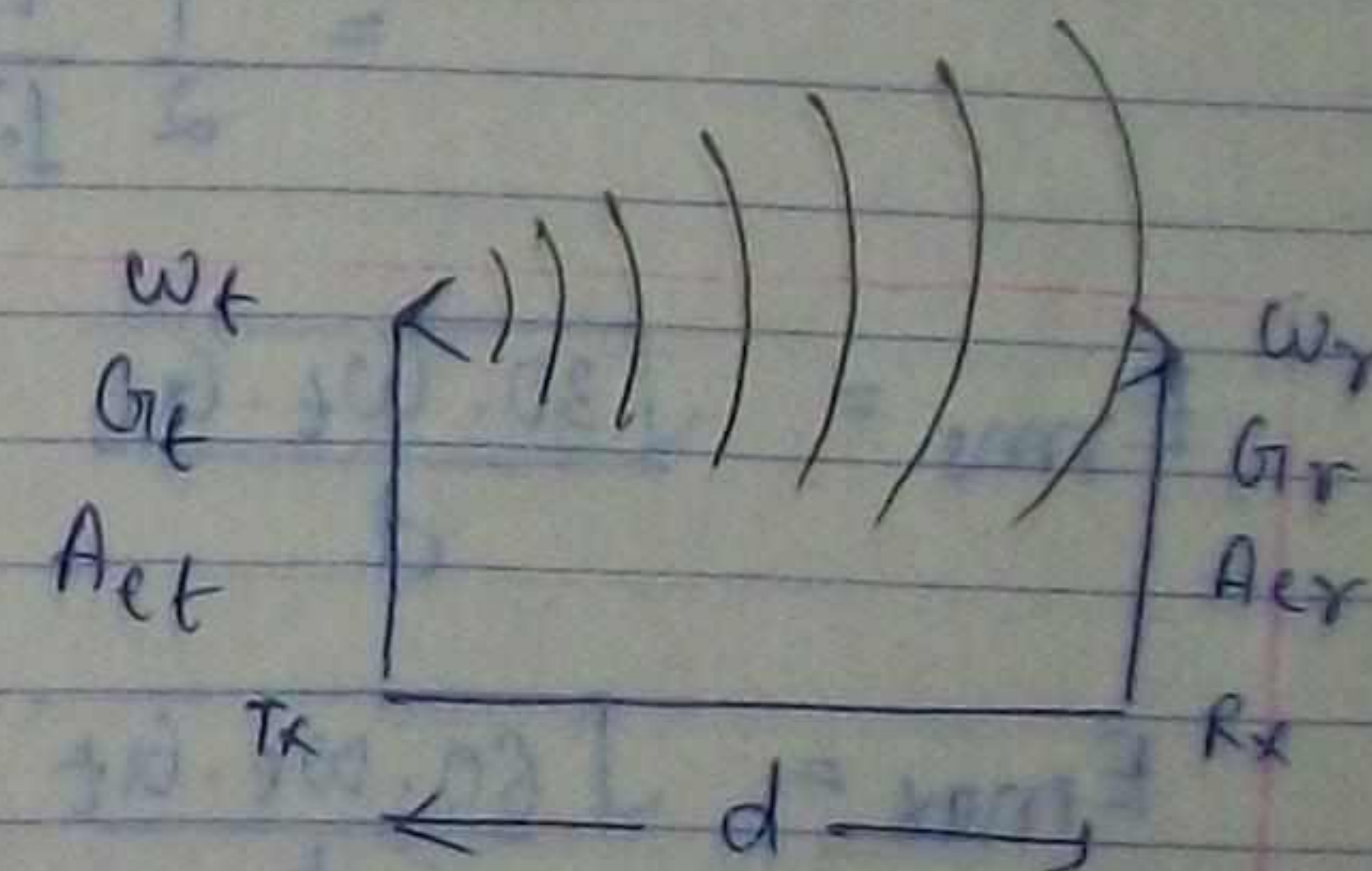


$\Rightarrow$ FRIS - FREE SPACE PROPAGATION EQUATION -

Power density = $\frac{W_t \cdot G_t}{4\pi d^2}$
at Rx.
(Tx.)

Power received at Rx

$$W_r = \frac{W_t \cdot G_t \cdot A_{er}}{4\pi d^2}$$



$$A_{er} = \frac{\lambda^2}{4\pi} G_r$$

$$W_r = \frac{W_t \cdot G_t \cdot G_r}{\left(\frac{4\pi d}{\lambda}\right)^2}$$

Received power.

$$W_r = \frac{W_t \cdot G_t \cdot A_{er}}{4\pi d^2}$$

$\left(\frac{4\pi d}{\lambda}\right)^2$ is called as L_s = loss due to spatial attenuation or expansion

$$L_s = \left(\frac{4\pi d}{\lambda}\right)^2$$

$$W_t = \text{EIRP}$$

$$W_r (\text{dBW}) = W_t (\text{dBW}) + G_t (\text{dB}) + G_r (\text{dB}) - L_s (\text{dB})$$

#

$$\text{Power density at Rx} = \frac{W_t \cdot G_t}{4\pi d^2}$$

$$= \frac{1}{2} \frac{E_0^2}{120\pi} = \frac{E_{\text{rms}}^2}{120\pi}$$

$$E_{\text{rms}} = \frac{\sqrt{30 \cdot W_t \cdot G_t}}{d}$$

$$E_{\text{max}} = \frac{\sqrt{60 \cdot W_t \cdot G_t}}{d}$$

Que 19.

$$\text{Soln: } G_t = 10, W_t = 1 \text{ W}, W_r = ?, A_e = 1 \text{ m}^2, d = 1 \text{ m}$$

$$W_r = \frac{W_t \cdot G_t \cdot A_e}{4\pi d^2} = 0.8 \text{ W}$$

Que 21. A person with a Rx. is 5 km away from the Tx. what is the distance this person must move further to detect a 3 db decrease in signal strength.

$$\text{Soln: } d = 5$$

$$E \propto \frac{1}{d}$$

(3db drop)

$$E_2 = \frac{E_1}{\sqrt{2}} \quad \frac{\sqrt{2} E_1}{E_1} = \frac{d_2}{5} \quad d_2 = 5\sqrt{2} \approx 7 \text{ km}$$

$$\text{Ans: } 7 - 5 = 2 \text{ km} \approx 2070 \text{ m}$$

$$\begin{aligned} \psi &= \int_0^{\pi/2} \int_0^{\pi} \frac{W_r}{4\pi r^2} \sin \theta \, d\theta \, d\phi \\ &= \frac{W_r}{4\pi} \int_0^{\pi/2} \int_0^{\pi} \frac{1}{r^2} \sin \theta \, d\theta \, d\phi \\ &= \frac{W_r}{4\pi} \int_0^{\pi/2} \left[-\cos \theta \right]_0^{\pi} d\phi \\ &= \frac{W_r}{4\pi} \int_0^{\pi/2} 2 \, d\phi \\ &= \frac{W_r}{2\pi} \left[\phi \right]_0^{\pi/2} \\ &= \frac{W_r}{2\pi} \cdot \frac{\pi}{2} \\ &= \frac{W_r}{4} \end{aligned}$$