

IES - Radar
satellite commn
optical fibre
wave prop.

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- Analog communication - modulation.

Am $\left\{ \begin{array}{l} \text{DSB FC} \\ \text{DSB} \\ \text{SSB} \\ \text{FDM} \end{array} \right\} \begin{array}{l} \text{Generation, Demodulation,} \\ \text{power caln, spectrum} \\ \text{BW} \end{array}$

VSB
ISB
QAM $\left\{ \right.$ **OBJECTIVE**

ANGLE MODN $\left\{ \begin{array}{l} \text{FM} \left\{ \begin{array}{l} \text{NB FM} \\ \text{WB FM} \end{array} \right. \\ \text{PM} \left\{ \begin{array}{l} \text{NB PM} \\ \text{WB PM} \end{array} \right. \end{array} \right.$ Transmitter

Noise, Receiver (super heterodyne Rx), noise analysis

DSB
SSB
Am
fm
pm $\left\{ \right.$ SNR
&
FDM

- Digital communication -

Sampling theorem - types & analysis

PAM, PWM, PPM

Pem, DPCM, Delta, Adaptive Δ.

ASK, PSK, FSK

✓ Matched filter

✓ Information theory

Basic of FDMA, TDMA, CDMA, GSM

**RANDOM
VARIABLE
THEORY**

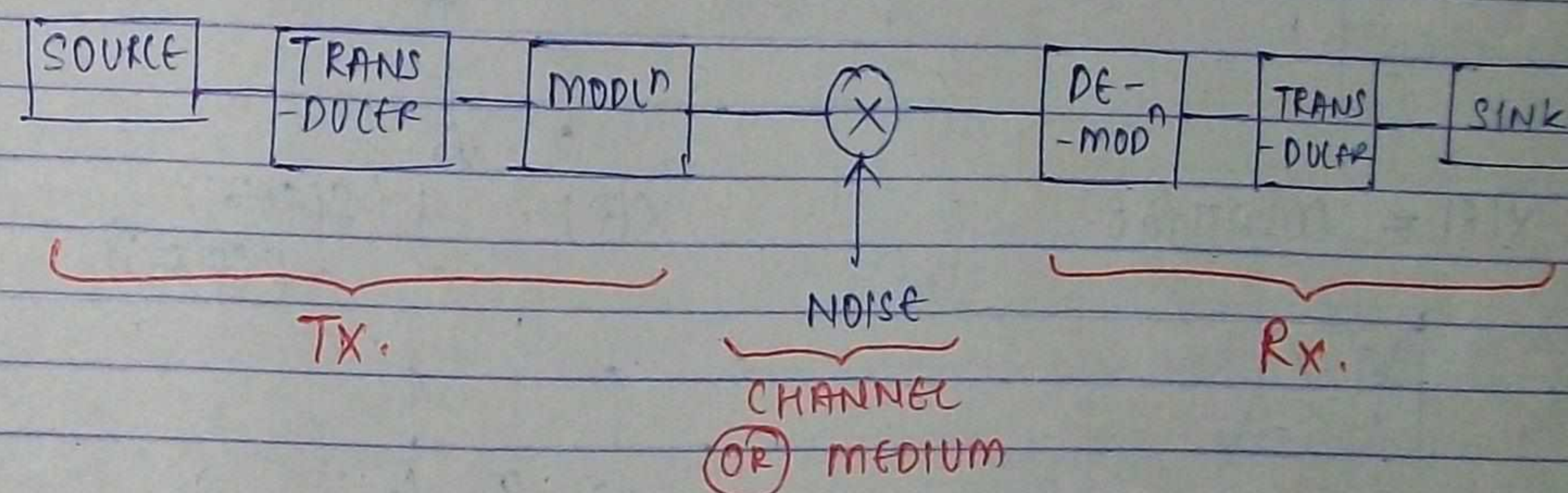
ANALOG COMMUNICATION

25/11/11

Voice $\rightarrow$ 300 Hz - 3.4 kHz
video $\rightarrow$ DC - 5 MHz
audio $\rightarrow$ 20 Hz - 20 kHz
Data $\rightarrow$ TX rate (depend on BW).

SIGNAL

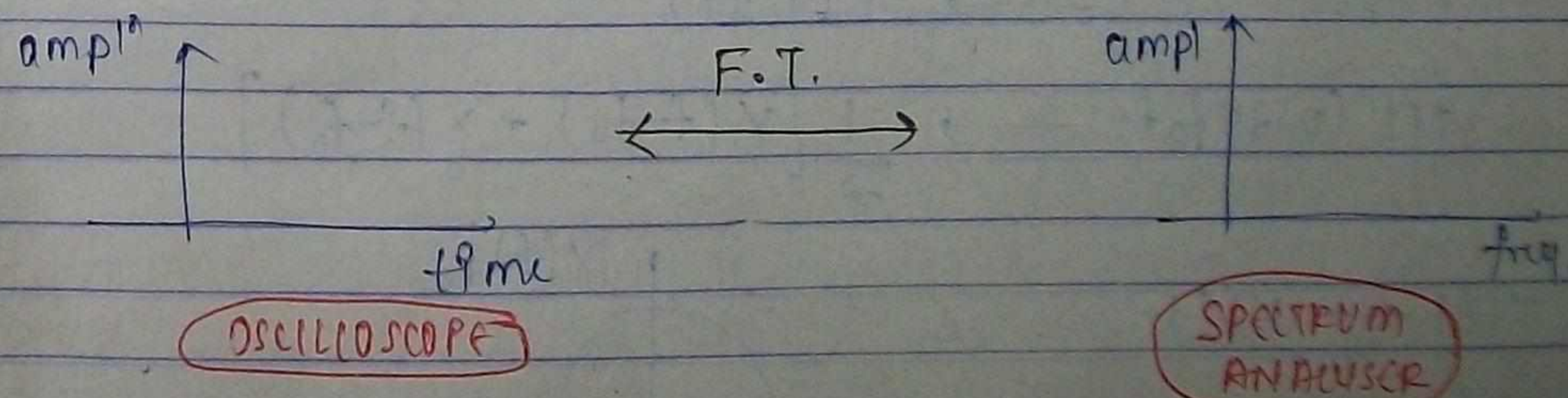
COMMUNICATION SYSTEM -



BANDWIDTH -

$$BW = f_H - f_L$$

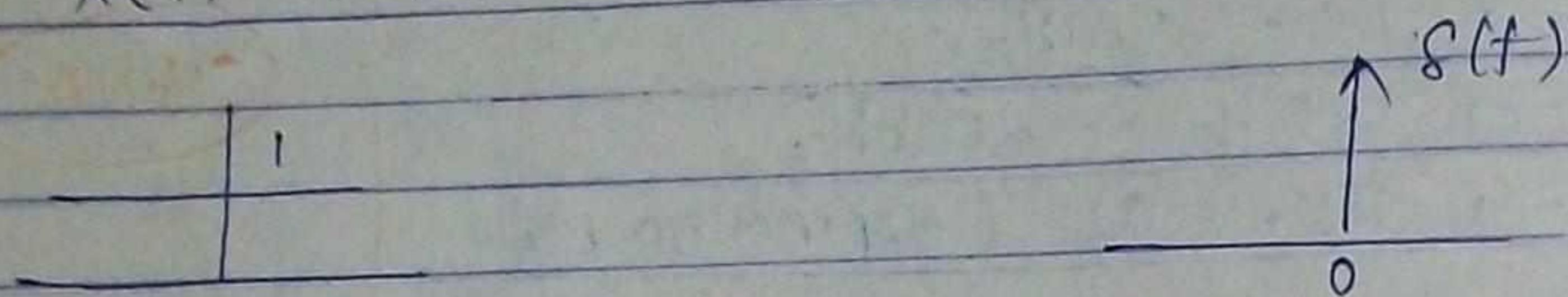
BW $\rightarrow$ voice $\rightarrow$ 3.1 kHz
BW $\rightarrow$ video $\rightarrow$ 5 MHz
audio $\rightarrow$ 20 kHz.



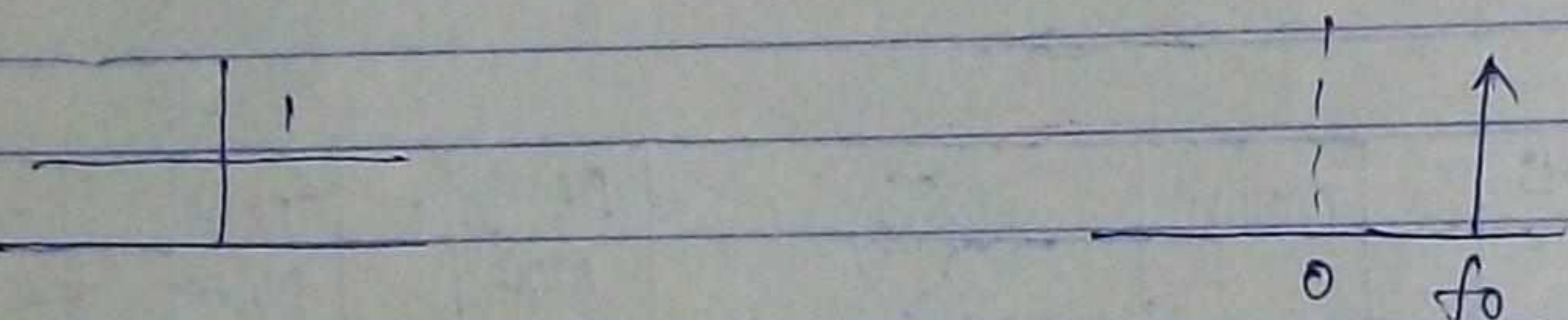
$$\sin 2\pi f_0 t = \frac{1}{2j} [e^{j2\pi f_0 t} - e^{-j2\pi f_0 t}] = \frac{1}{2j} [\delta(f-f_0) - \delta(f+f_0)]$$

- Practically - we freq does not exist.

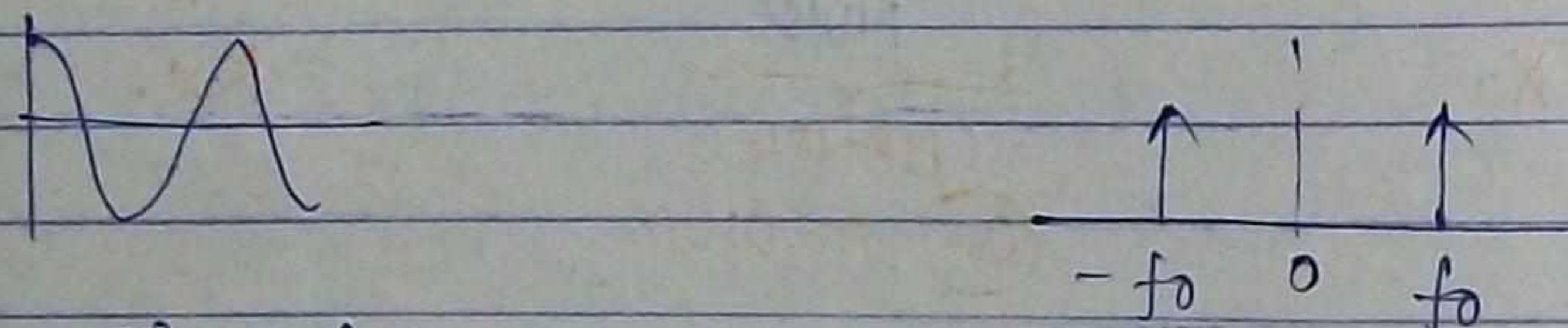
① $x(t) \rightarrow 1$ $x(f) = \delta(f)$



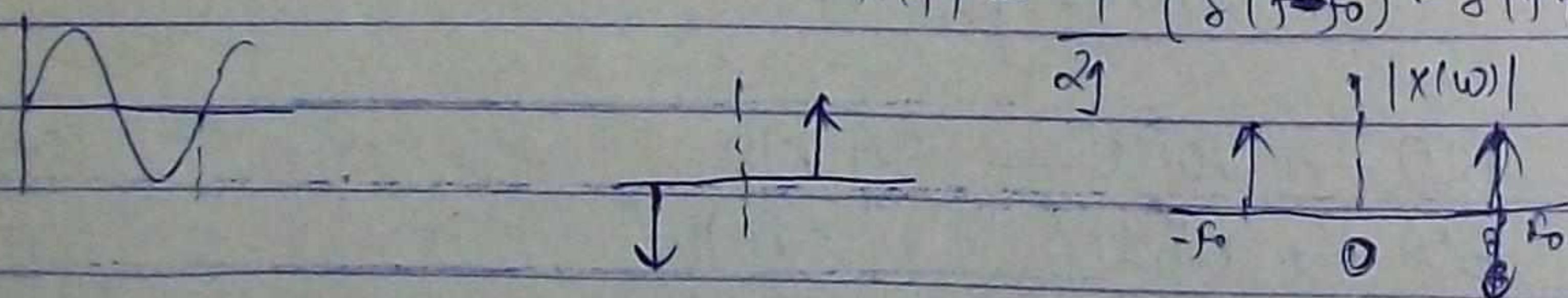
② $x(t) \rightarrow 1 \cdot e^{j2\pi f_0 t}$ $x(f) = \delta(f-f_0)$



③ $x(t) = \cos 2\pi f_0 t$ $x(f) = \frac{1}{2} (\delta(f-f_0) + \delta(f+f_0))$

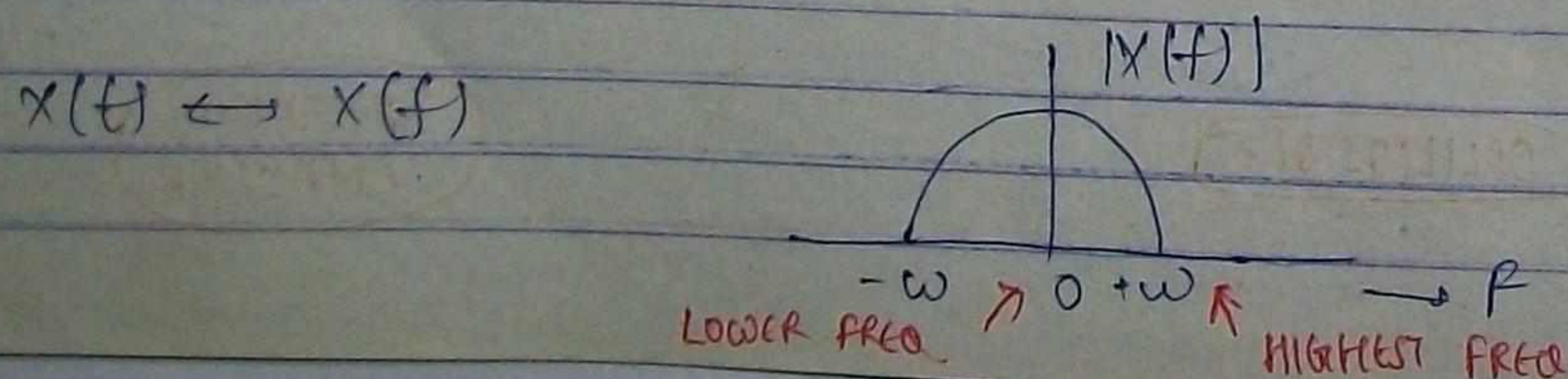


④ $x(t) = \sin 2\pi f_0 t$ $x(f) = \frac{1}{2j} (\delta(f-f_0) - \delta(f+f_0))$



⑤ $x(t) e^{j2\pi f_0 t} \rightarrow x(f-f_0)$

$x(t) \cos 2\pi f_0 t \rightarrow \frac{1}{2} [x(f-f_0) + x(f+f_0)]$



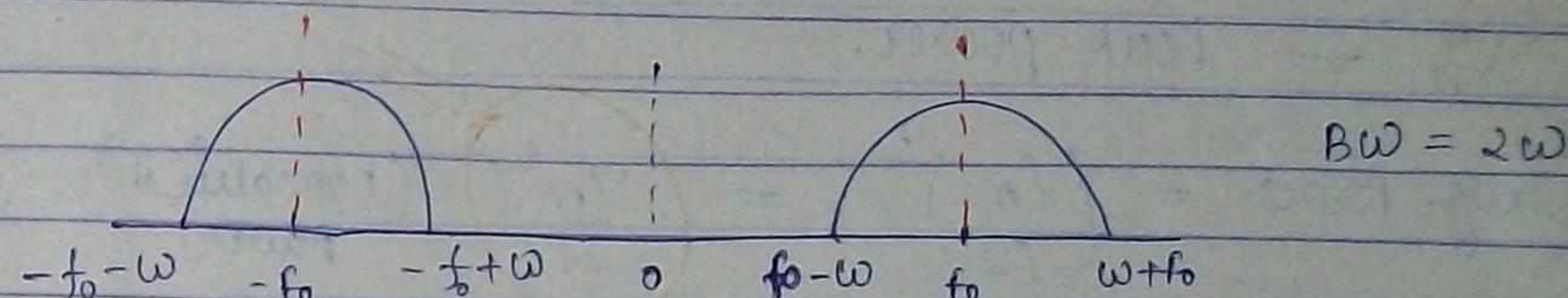
$$PSD = \frac{\text{Power}}{BW}$$

message BW

$$BW = w$$

$$(w-0)$$

$x(t) \cdot \cos 2\pi f_0 t = y(t)$ $y(f) = \frac{1}{2} [x(f-f_0) + x(f+f_0)]$



$\Rightarrow$ POWER CALN -

Periodic; $P = \frac{1}{T} \int_T x^2(t) dt$

Non periodic (random); $P = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T x^2(t) dt$

$x(t) \rightarrow x(f) \rightarrow PSD \rightarrow S_x(f)$

$$P = \int_{-\infty}^{\infty} PSD \cdot df$$

$$P = \int_{-\infty}^{\infty} S_x(f) df$$

$R_{xx}(\tau) \xleftrightarrow{F.T.} S_x(f)$

$\uparrow$
AUTO
CORRELATION
FUNⁿ

$\uparrow$
PSD

$$P = \int_{-\infty}^{\infty} S_x(f) df = R_{xx}(0)$$

$$m(t) = V_m \cos 2\pi f t.$$

Peak, amp, or max.

$$V_m^2 \rightarrow \text{Peak power.}$$

$$\text{avg. Power} = \frac{(V_m)^2}{R} = \frac{V_m^2}{2} \quad (\text{normalized power})$$

$$m(t) = 10 \cos 2\pi f t + 5 \sin 2\pi f t$$

$$P = \frac{10^2}{2} + \frac{5^2}{2} = 50 + 12.5 = 62.5 \text{ watt}$$

$$m(t) = 5 + 10 \cos 2\pi f t$$

$$P = 5^2 + \left(\frac{10}{\sqrt{2}}\right)^2 = 75 \text{ watt.}$$

⑧ If $m(t) \rightarrow P_m$ (power)

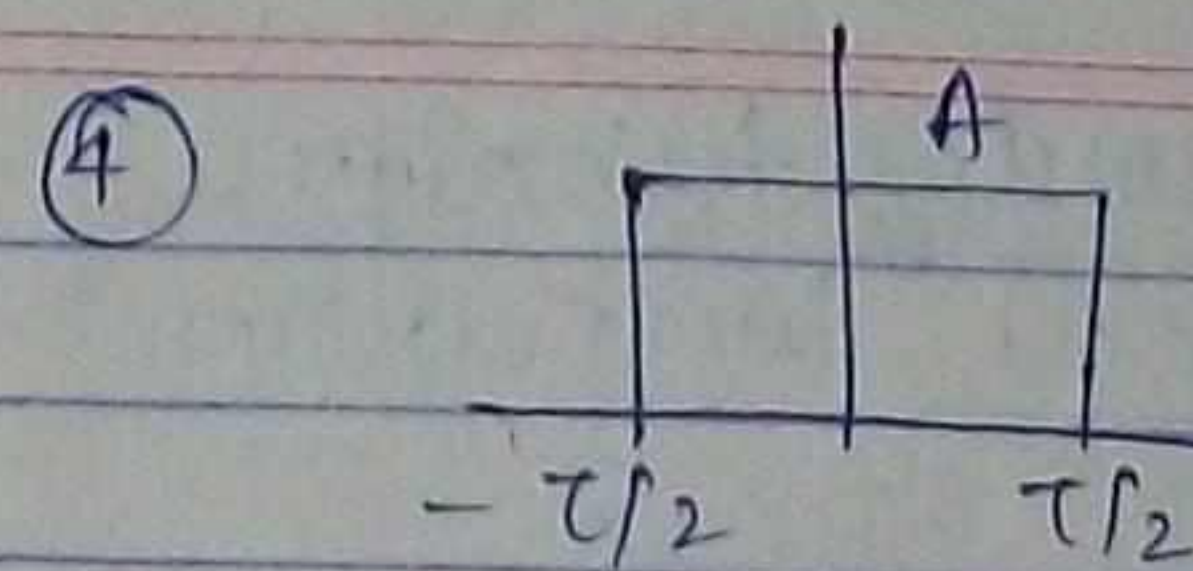
then $K m(t) \rightarrow K^2 P_m$

Ex $x(t) = 5 \times 10 \cos 2\pi f t$

$$\downarrow \quad \downarrow$$

$$25 \times \frac{10^2}{2} \quad \text{or} \quad \frac{50^2}{2}$$

$$K^2 P_m$$

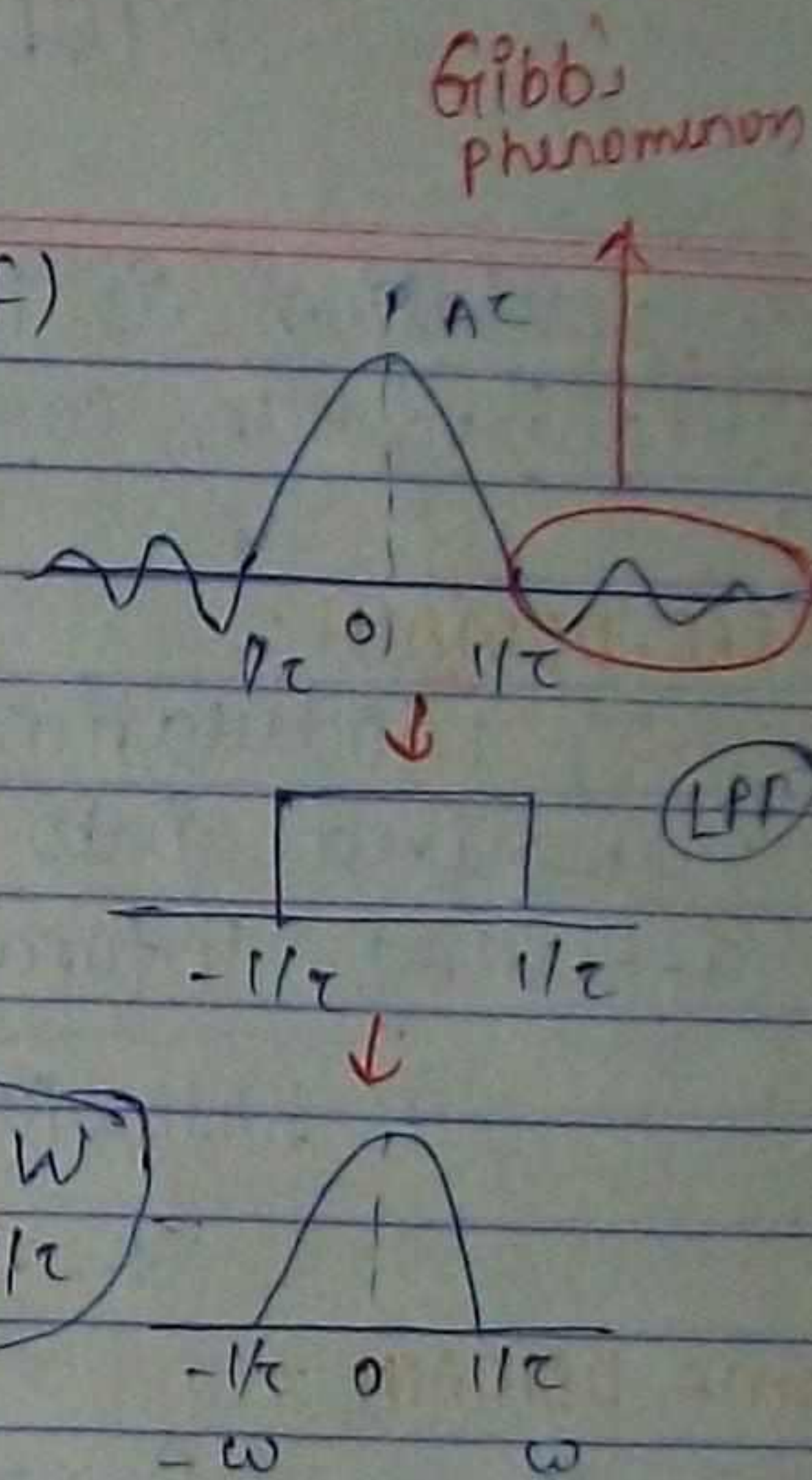


$$A T \text{sinc}(\pi f T)$$

absolute $BW = \infty$

zero cross $BW = \frac{2}{T}$

absolute $BW = W = 1/T$

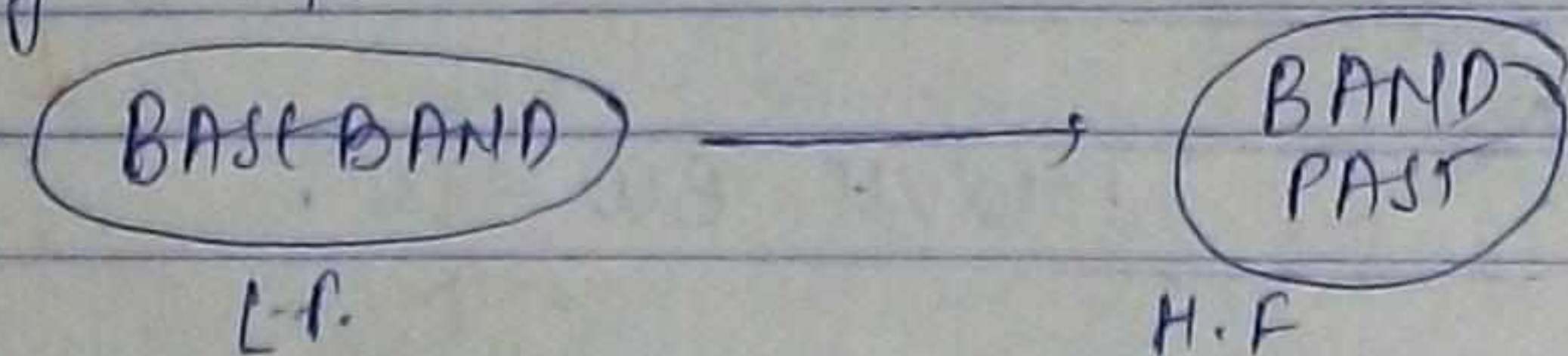


MODULATION

- Modulation is the basic signal processing technique required in commⁿ to transmit signal long distance

FREQ. DOMAIN -

In modulation low freq. ^{Information} modⁿ signal is transferred into high freq. carrier signal without affecting information content



TIME DOMAIN -

In modⁿ one of the parameter of carrier signal (amp., freq, phase) is changed in accordance with instantaneous amplitude of message signal.

$$c(t) = V_c \cos(2\pi f_c t) \quad \text{Carrier signal}$$

Annotations: V_c is Amplitude, $2\pi f_c t$ is Phase.

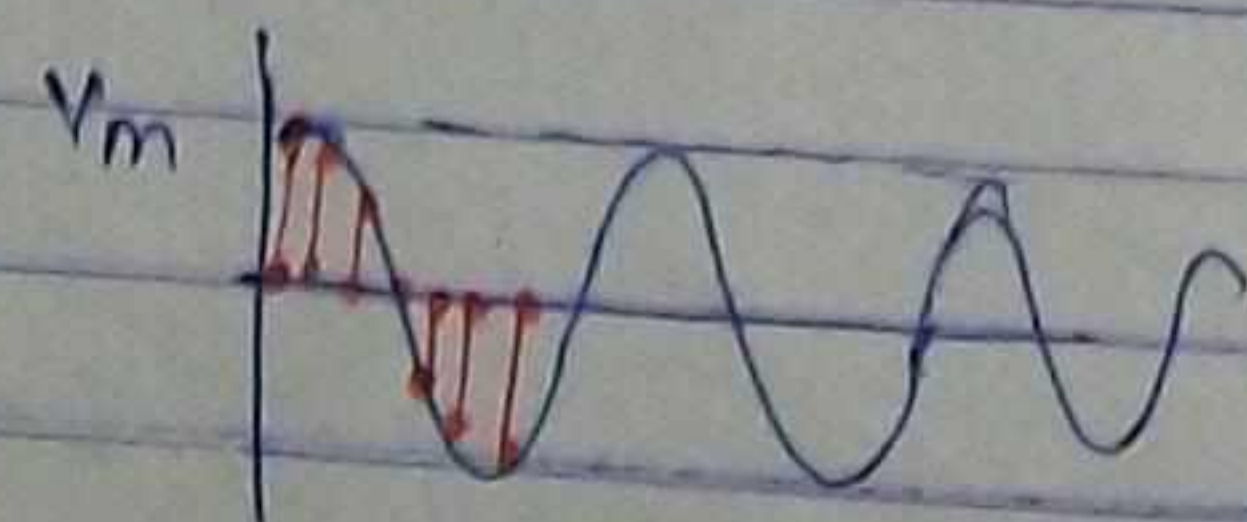
$$\text{also } c(t) = V_c \cos(2\pi f_c t + \phi_0)$$

Annotations: $2\pi f_c t$ is Instantaneous phase, ϕ_0 is Initial phase.

message signal

$$m(t) = V_m \cos 2\pi f_m t$$

Annotations: V_m is Instantaneous amp (varies with time)



⇒ Need of modⁿ -

1. To provide long distance commⁿ.
2. To construct practical antenna ($h = \lambda/4$) ($\lambda = c/f$)
ex - voice $f = 3 \text{ kHz}$
(L.F) $\lambda = \frac{3 \times 10^8 \text{ m/s}}{3 \times 10^3} = 10^5 = 100 \text{ km}$
 $h = 100/4 = 25 \text{ km}$
3. To avoid interference. (same freq.)
4. To ~~avoid~~ provide multiplexing. (cable wire - 100 channel)
5. To reduce noise.
6. To provide freq. allocation.
7. To make narrow banding.

$$\frac{f_H}{f_L} \gg 1 \quad \text{wide band} \quad \frac{f_H}{f_L} \approx 1 \quad \text{Narrow banding}$$

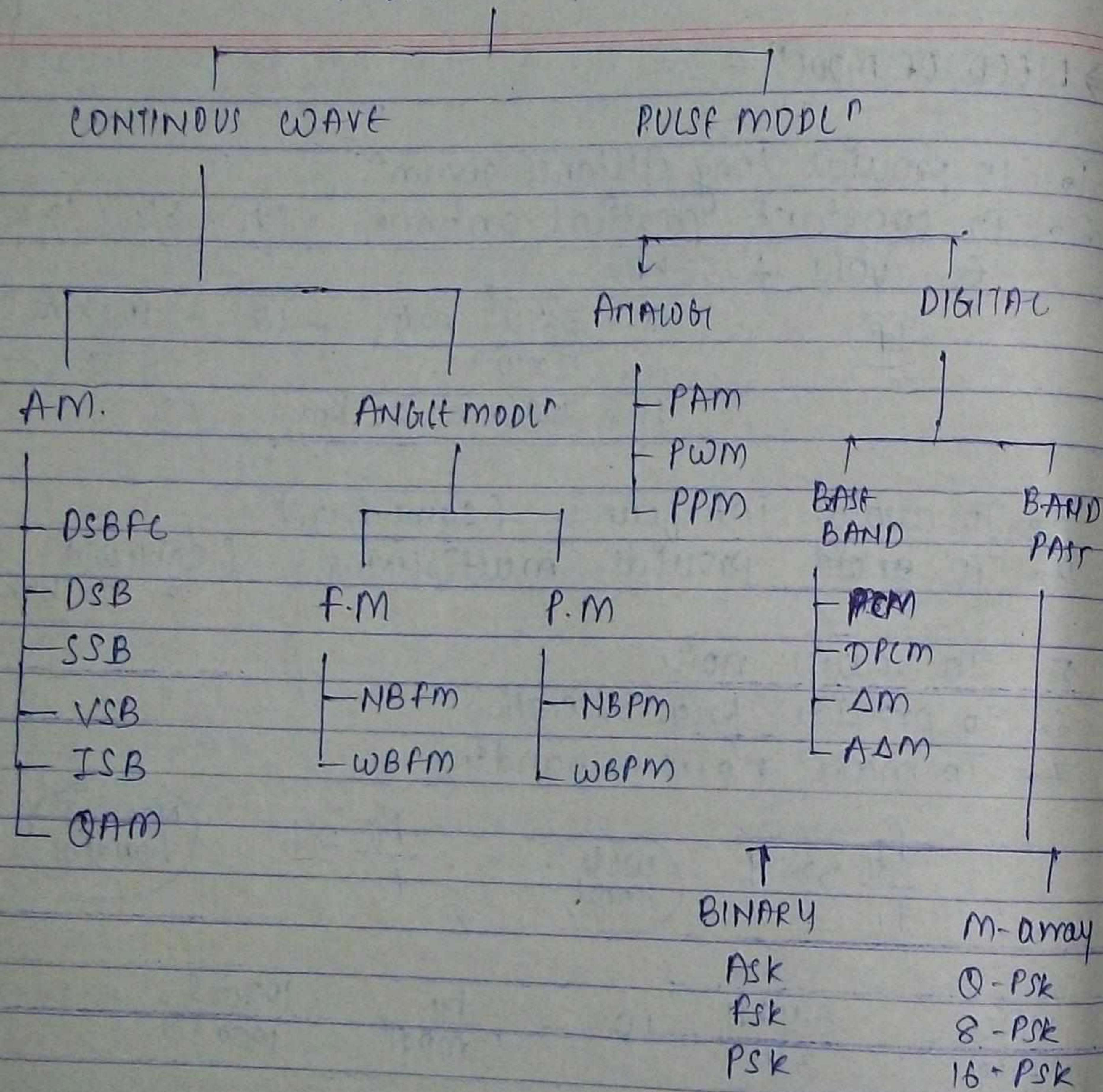
$$\text{voice freq} = \frac{3.4 \text{ kHz}}{3 \text{ Hz}} = 10 \quad \text{by mod}^n \quad \frac{1000+3}{1000-3} \approx 1$$

DISADVANTAGE -

- Transmitted signal BW increases.

⇒ CLASSIFICATION OF MODULATION -

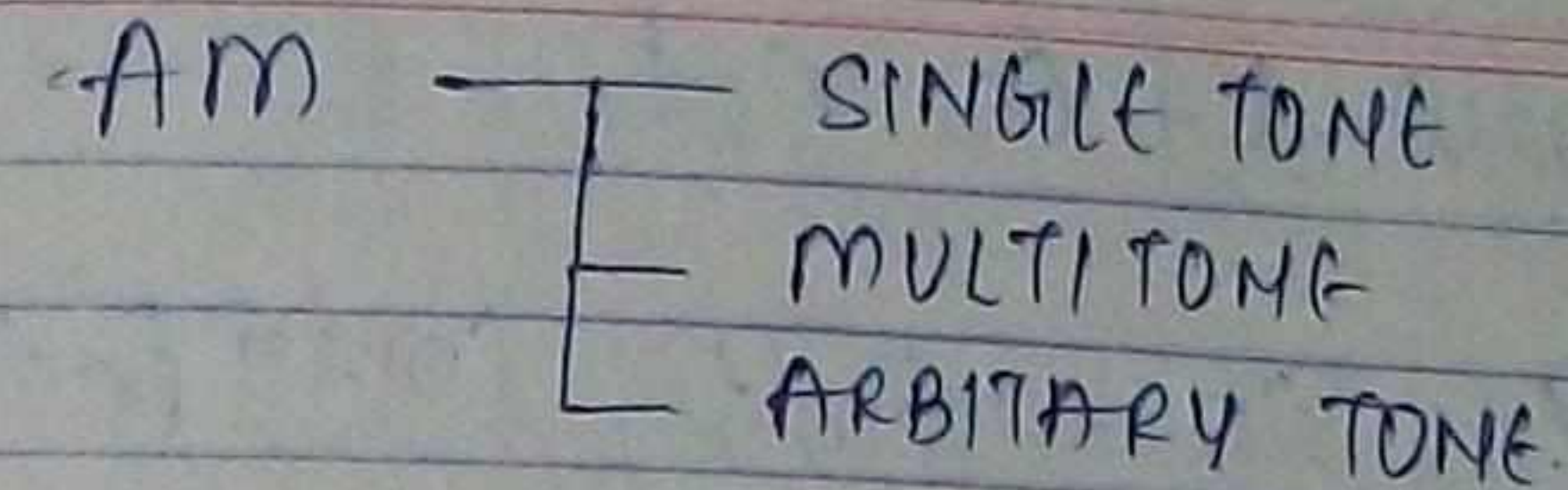
MODULATION (Based on carrier type)



⇒ AMPLITUDE MODULATION —

Defⁿ — In A.M amplitude of carrier signal is changed in accordance with instantaneous amplitude of message signal.

tone — single sinusoidal.



① SINGLE TONE A.M —

$$m(t) = V_m \cos 2\pi f_m t \quad (\text{message signal})$$

max, let $m(t)$ be a sinusoidal message signal where f_m is message signal freq.

$$f_{\max} = 5 \text{ kHz} \quad (\text{max. modⁿ freq. using in A.M.})$$

$$c(t) = V_c \cos 2\pi f_c t \quad (\text{carrier signal})$$

$f_c \rightarrow$ carrier freq.

$$f_c \text{ range} \rightarrow 550 \text{ kHz} - 1650 \text{ kHz}$$

AM BAND.

modulate signal $\rightarrow S(t)_{\text{Am}}$

$$S(t)_{\text{Am}} = (V_c + m(t)) \cos \omega_c t$$

$$= [V_c + m(t)] \cos 2\pi f_c t$$

$$S(t)_{AM} = [V_c + V_m \cos \omega_m t] \cos \omega_c t$$

$$= V_c \left[1 + \frac{V_m}{V_c} \cos 2\pi f_m t \right] \cos 2\pi f_c t$$

where $\frac{V_m}{V_c} = m$ modulation index.

$$\% \text{ mod } = m \times 100$$

$$S(t)_{AM} = V_c [1 + m \cos 2\pi f_m t] \cos 2\pi f_c t$$

$V_c \rightarrow$ carrier amp. , $f_m \rightarrow$ message freq.
 $m \rightarrow$ mod index , $f_c \rightarrow$ carrier freq.

$$S(t)_{AM} = V_c \cos 2\pi f_c t + m V_c \cos 2\pi f_m t \cdot \cos 2\pi f_c t$$

CARRIER

SIDE BAND

$$S(t)_{AM} = \underbrace{V_c \cos 2\pi f_c t}_{\text{CARRIER}} + \underbrace{\frac{m V_c}{2} \cos 2\pi (f_c + f_m) t}_{\text{UPPER SIDE BAND}} + \underbrace{\frac{m V_c}{2} \cos 2\pi (f_c - f_m) t}_{\text{LOWER SIDE BAND}}$$

∴ AM contains carrier and 2 side bands

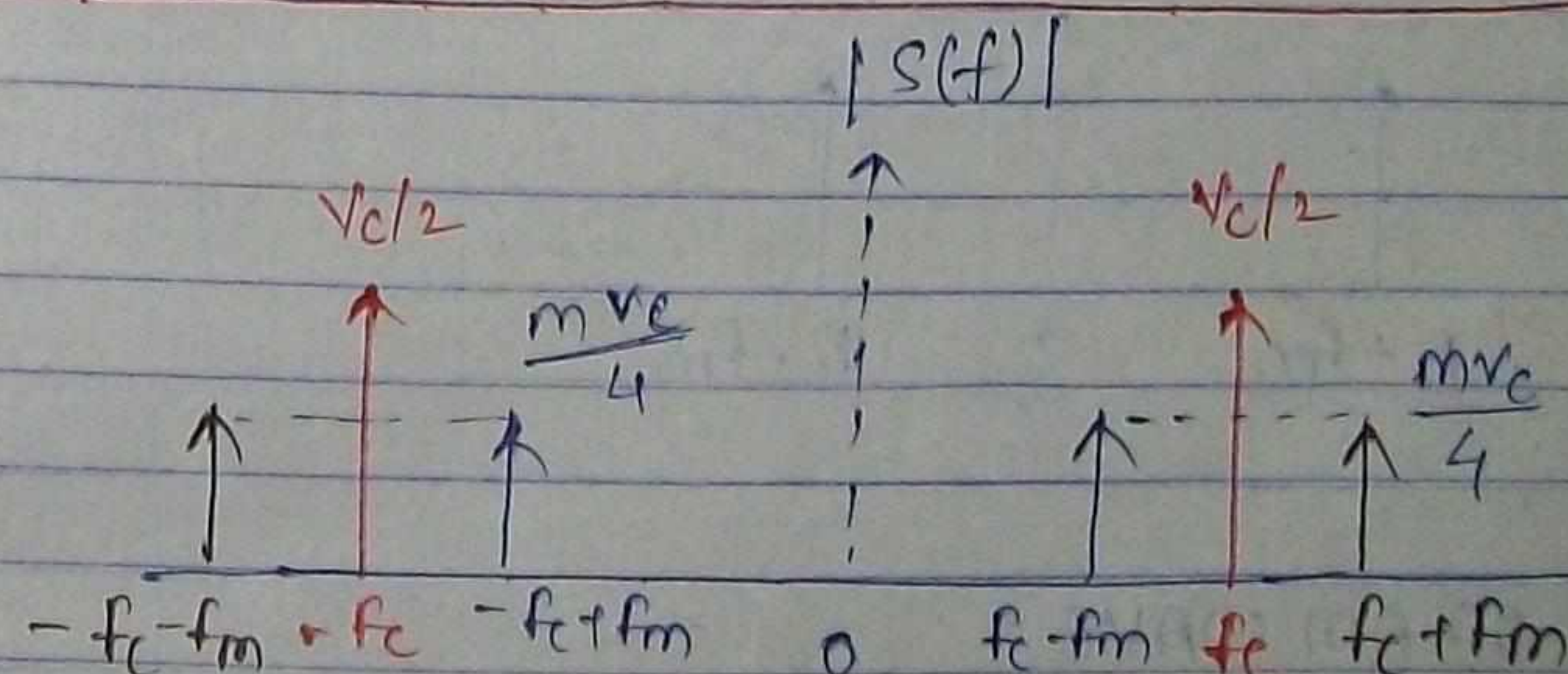
- In this information lies on side band; both the side band contains same information.

CARRIER + 2 SIDED BAND $\rightarrow$ AM.
DSBFC.

$$S(f) = \frac{V_c}{2} [S(f - f_c) + S(f + f_c)] +$$

$$+ \frac{m V_c}{4} [S(f - (f_c + f_m)) + S(f + (f_c + f_m))] +$$

$$+ \frac{m V_c}{4} [S(f - (f_c - f_m)) + S(f + (f_c - f_m))]$$

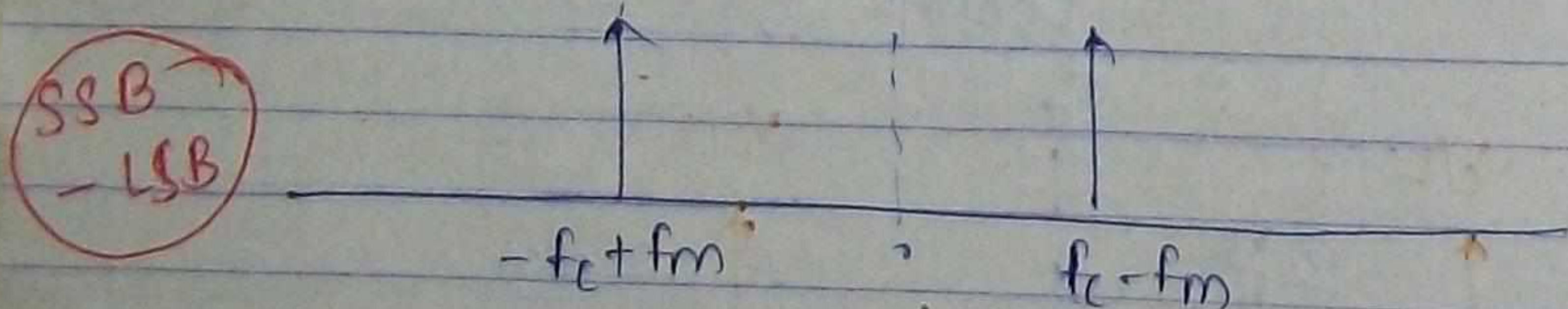
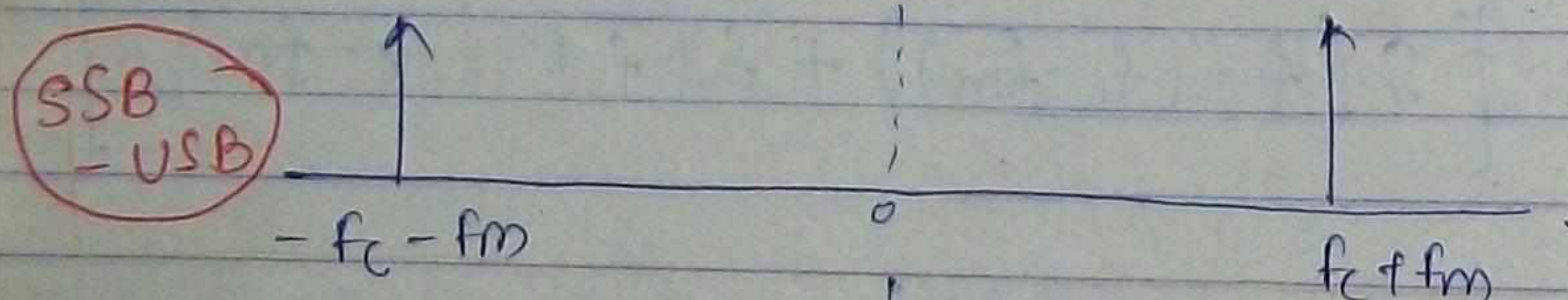
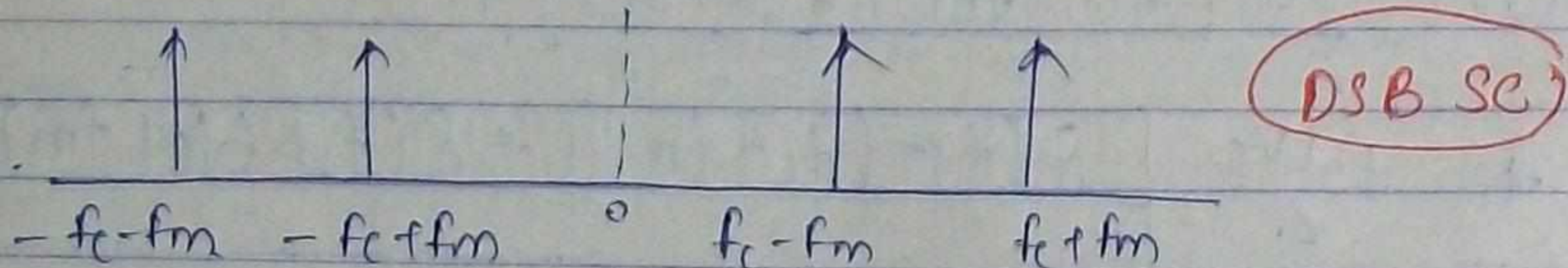
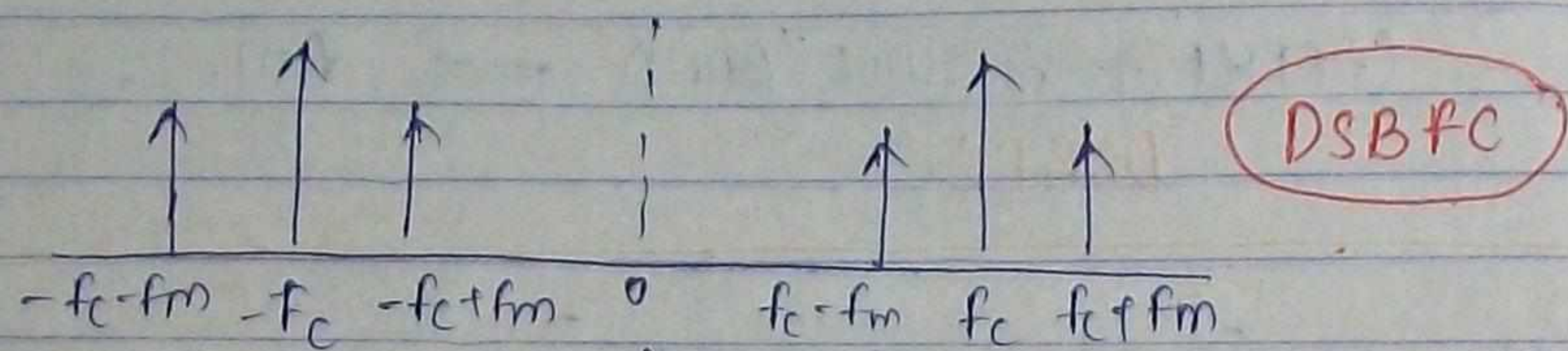


$$BW = (f_c + f_m) - (f_c - f_m)$$

$$BW = 2f_m$$

∴ max. BW required for AM is 10 kHz.

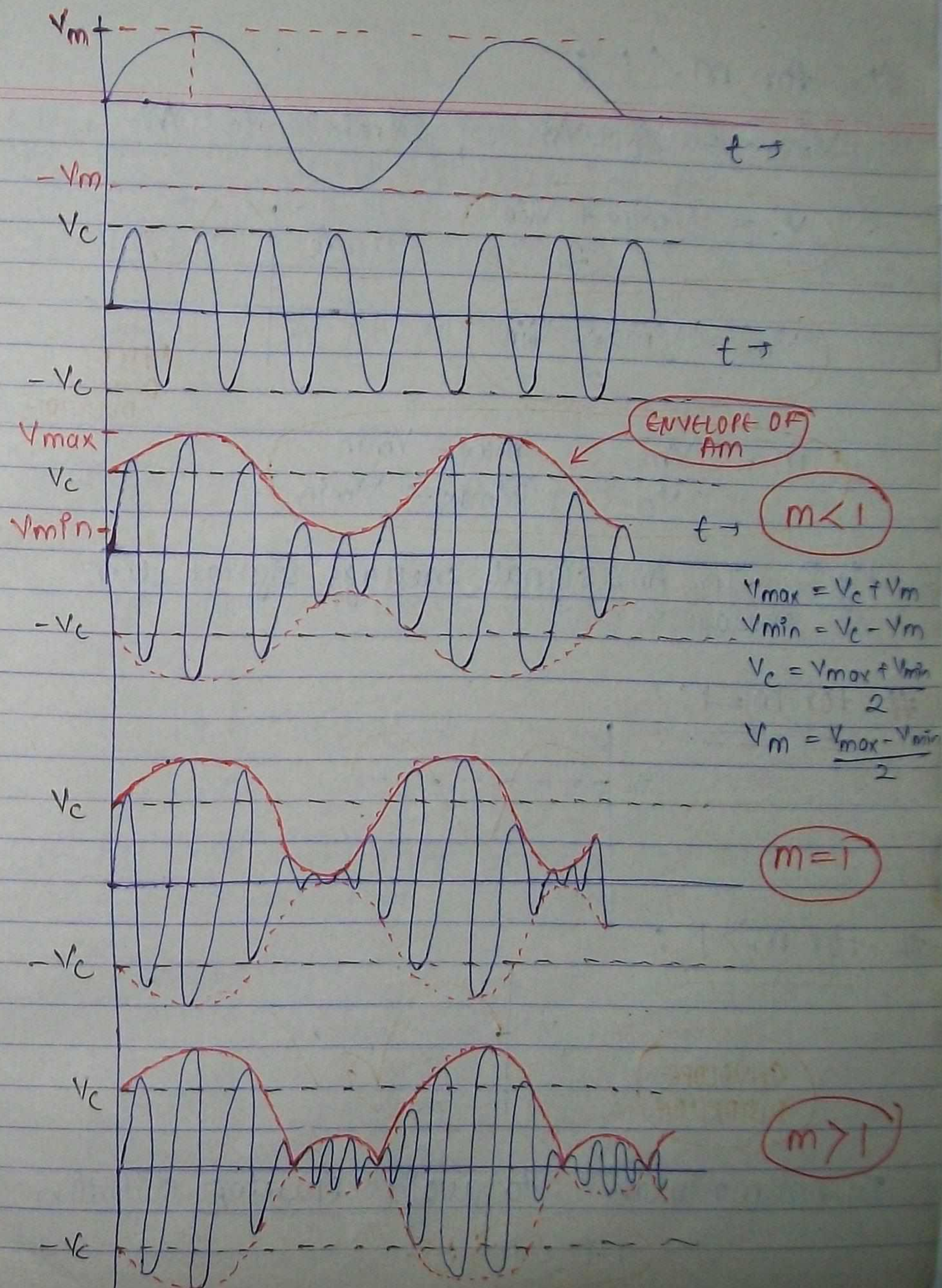
$$BW = 2f_m = 10 \text{ kHz}$$



WAVE FORM OF AM WAVE -

$$m(t) = V_m \cos 2\pi f_m t$$

$$= V_m \sin 2\pi f_m t$$



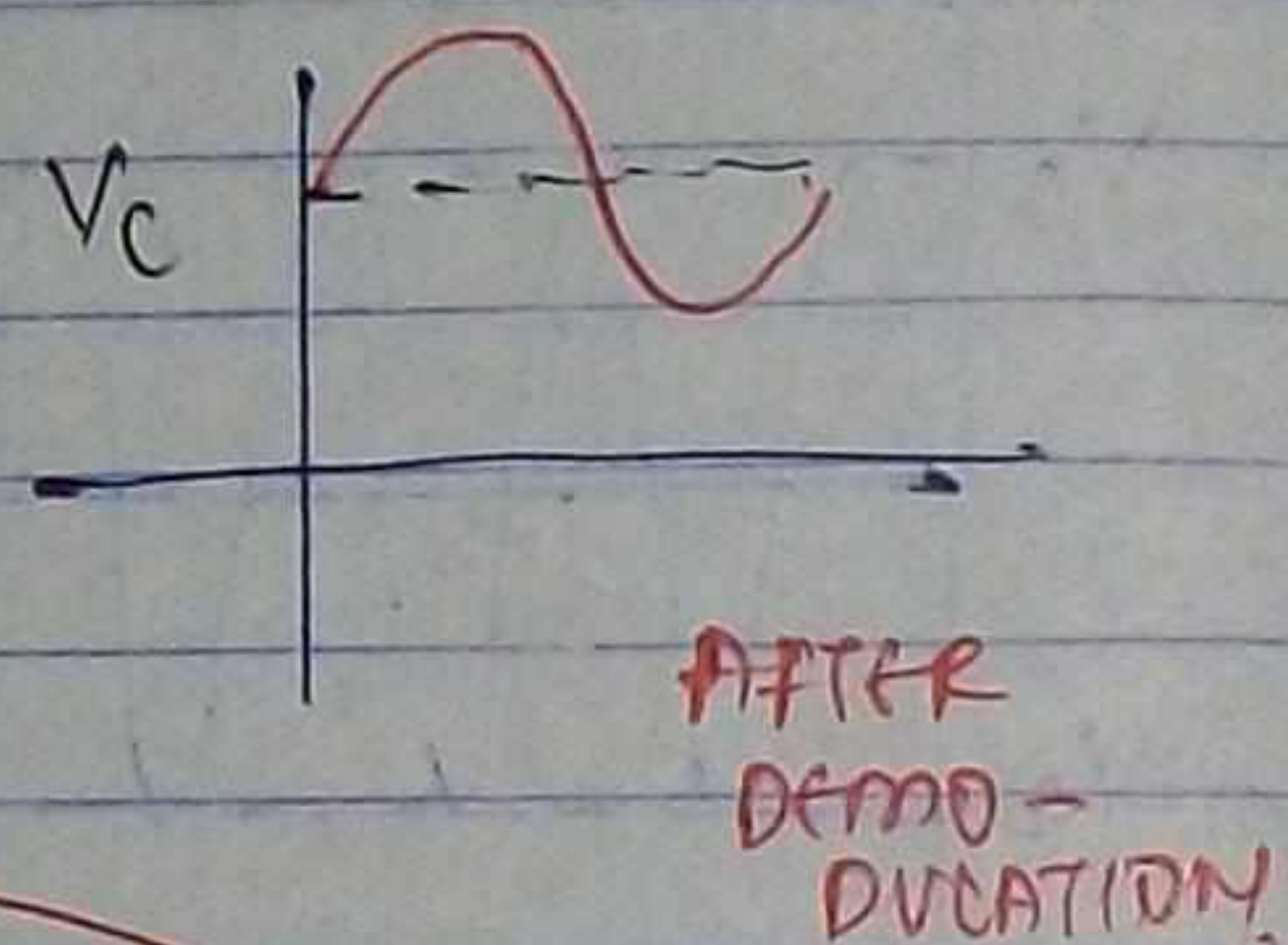
for $m < 1$:

$$V_{\max} = V_c + V_m, \quad V_{\min} = V_c - V_m$$

$$V_c = \frac{V_{\max} + V_{\min}}{2}$$

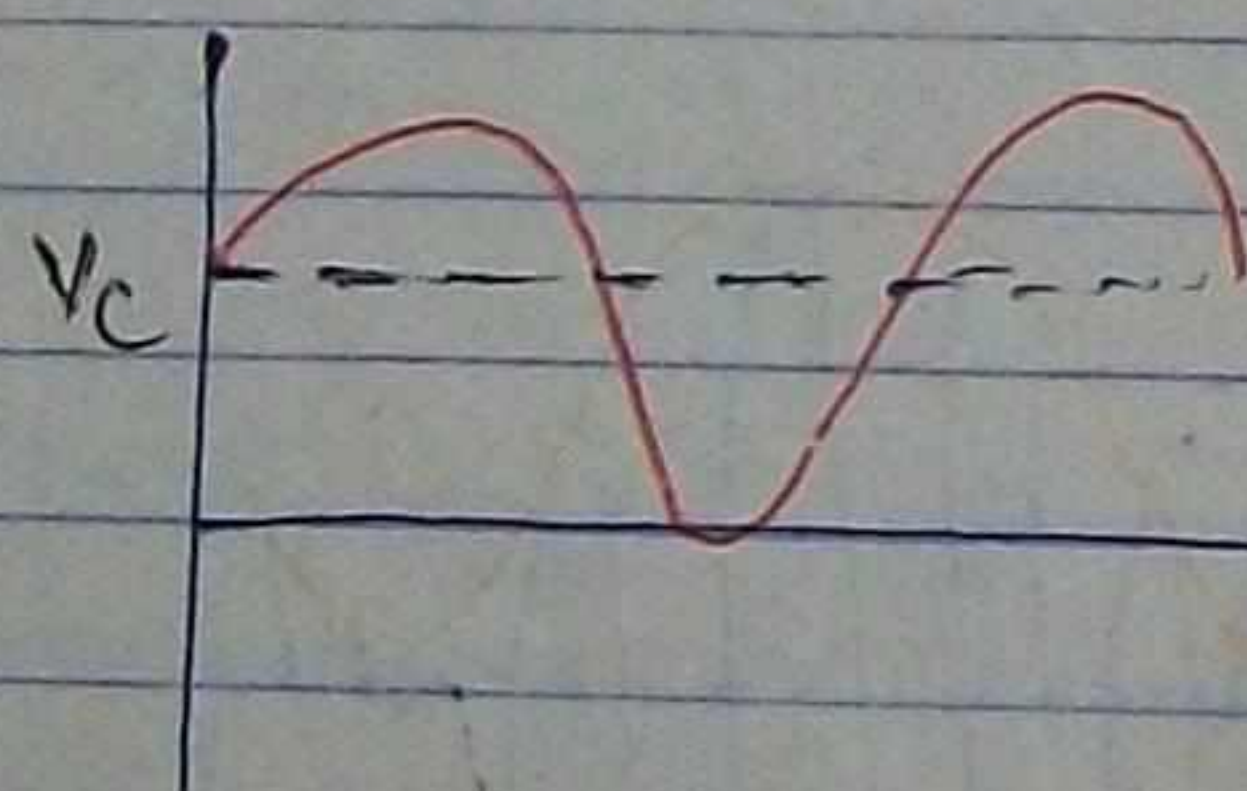
$$V_m = \frac{V_{\max} - V_{\min}}{2}$$

$$m = \frac{V_m}{V_c} = \frac{V_{\max} - V_{\min}}{V_{\max} + V_{\min}}$$

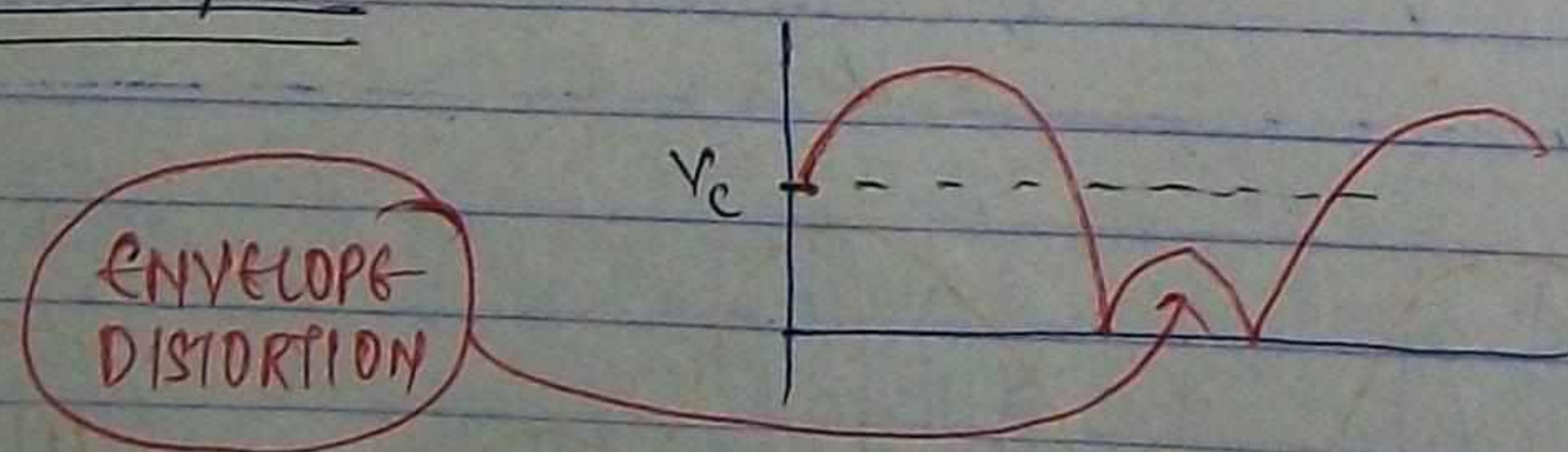


- fig 3 In AM signal message signal lies on envelope.

for $m = 1$:



for $m > 1$:



In AM modulation to avoid envelope distortion

$$m \leq 1$$

- when AM signal is applied to an envelope detector then it will detect envelope of AM.

- In single tone modulated signal envelope is:

$$\text{envelope} = V_c [1 + m \cos 2\pi f_m t] \quad \text{when } m \leq 1$$

$$\text{envelope} = |V_c [1 + m \cos 2\pi f_m t]|, \quad m > 1$$

- In AM phase reversal (180° phase shift) occurs when

$$V_c [1 + m \cos 2\pi f_m t] \text{ crosses zero axis.}$$

Q. In AM modⁿ the max. amp is 10V and min amp. is 5V then % modⁿ is :

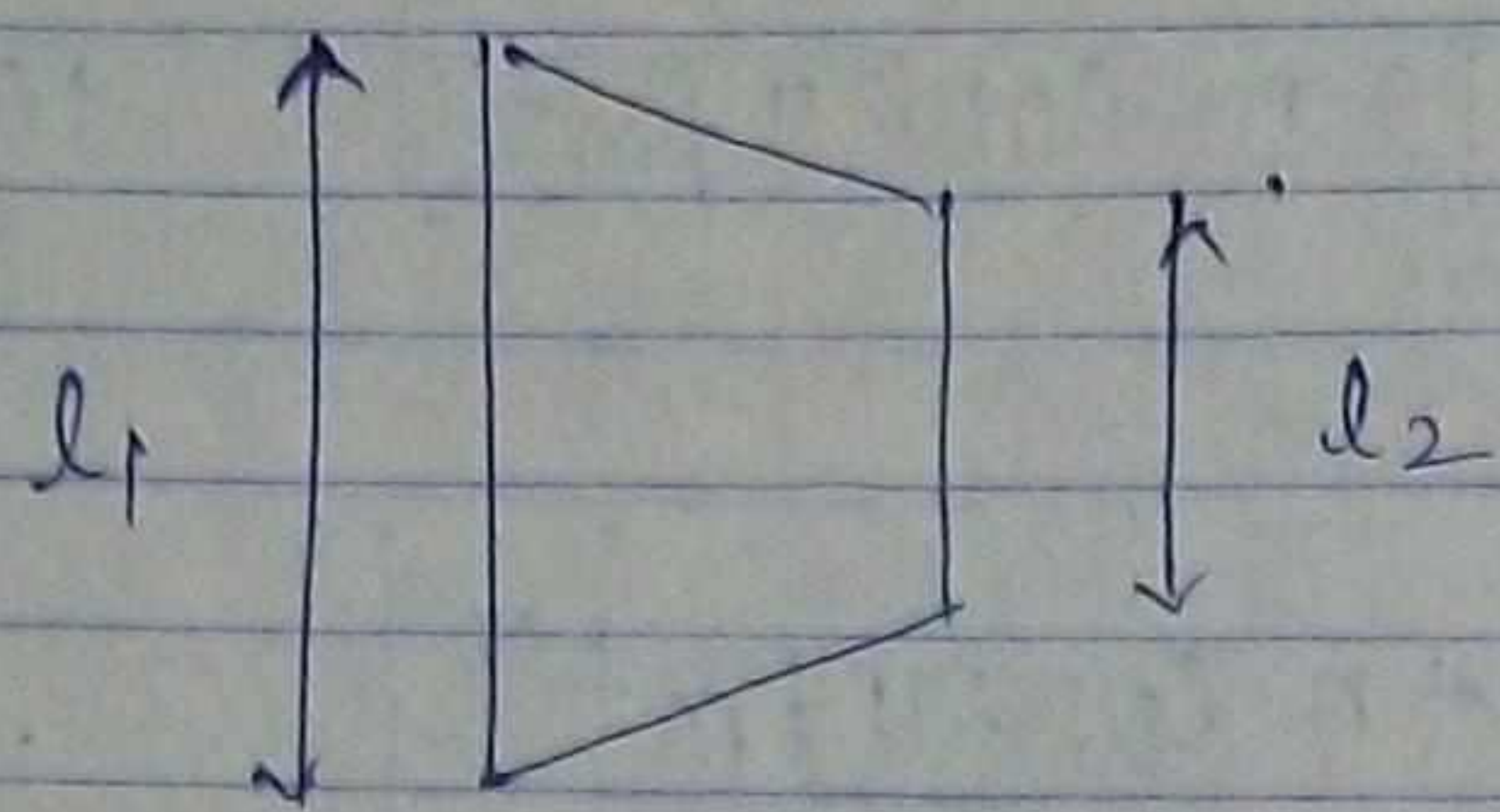
Solⁿ:

$$m = \frac{10 - 5}{10 + 5} = \frac{1}{3}$$

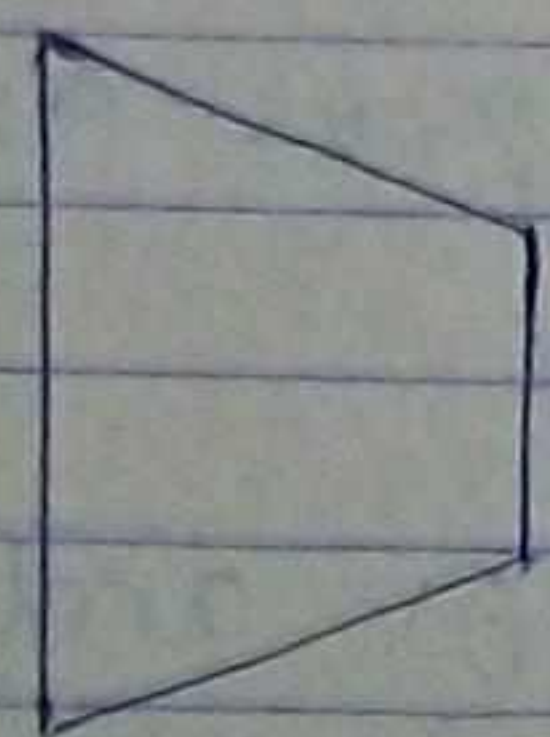
$$\begin{aligned} \% m &= \frac{1}{3} \times 100 \\ &= 33.33\% \end{aligned}$$

NOTE -

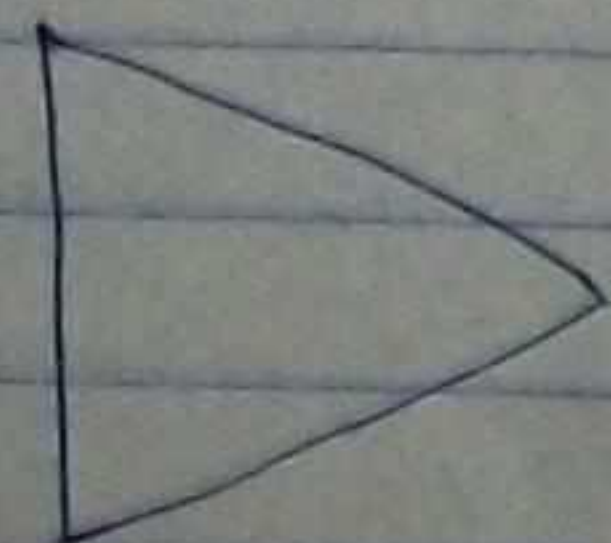
- In oscilloscope if message signal is applied to X-channel and AM signal is applied to Y-channel then trapezoidal pattern is form as shown below.



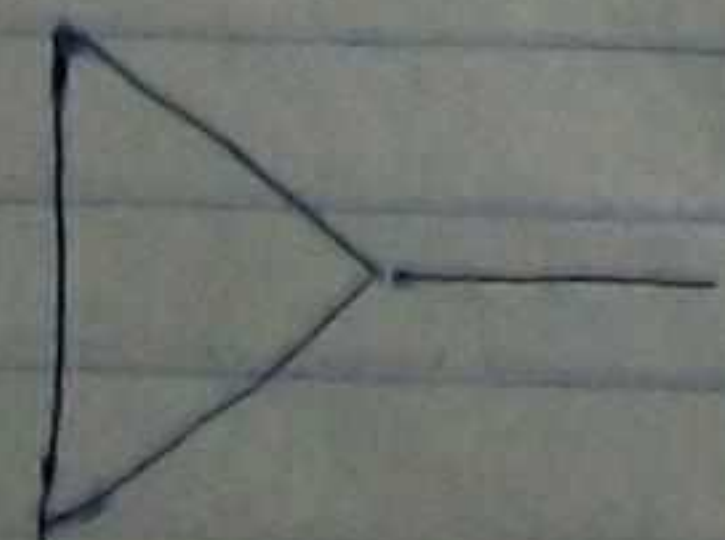
$$m = \frac{l_1 - l_2}{l_1 + l_2}$$



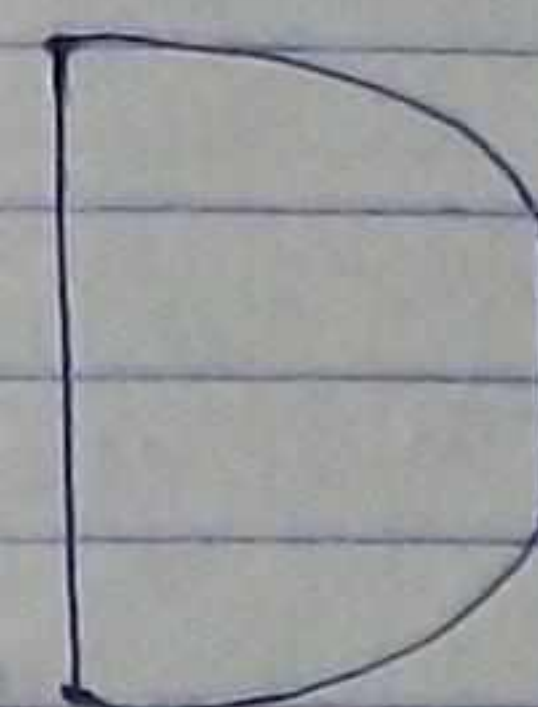
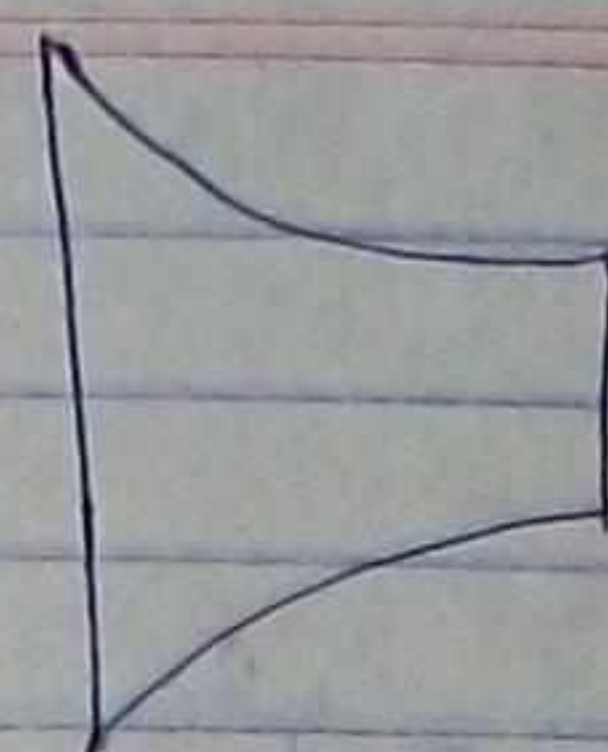
$$m < 1$$



$$m = 1$$



$$m > 1$$



NON-LINEAR
DISTORTION

⇒ POWER CALCULATION -

$$s(t)_{AM} = V_c [1 + m \cos 2\pi f_m t] \cdot \cos 2\pi f_c t$$

$$= V_c \cos 2\pi f_c t + \frac{m V_c}{2} \cos 2\pi (f_c + f_m) t$$

CARRIER

USB

$$+ \frac{m V_c}{2} \cos 2\pi (f_c - f_m) t$$

LSB

amp. of carrier $\rightarrow V_c$

amp. of USB $\rightarrow \frac{m V_c}{2}$

amp of LSB $\rightarrow \frac{m V_c}{2}$

$$P_T = P_c + P_{SB}$$

also, $P_{SB} = P_{USB} + P_{LSB}$

avg. carrier power $\Rightarrow P_c = \frac{(V_c/\sqrt{2})^2}{R} = \frac{V_c^2}{2R}$

avg. sideband power $\Rightarrow P_{SB} =$

$$P_{SB} = \left(\frac{mV_c}{2\sqrt{2}} \right)^2 + \left(\frac{mV_c}{2\sqrt{2}} \right)^2 = \frac{m^2 V_c^2}{4R}$$

$$= \frac{m^2}{2} \cdot \frac{V_c^2}{2R}$$

Total avg.
sideband power

$$P_{SB} = \frac{m^2}{2} \cdot P_c$$

Total
power

$$P_T = P_c + P_{SB}$$

$$= P_c + P_c \cdot \frac{m^2}{2}$$

$$P_T = P_c \left(1 + \frac{m^2}{2} \right)$$

FOR
SINGLE TONE

- In AM carrier power is independent of m , mod index, whereas side band power and total power depends on mod index (m).

% CARRIER POWER -

$$\% \text{ Carrier power} = \frac{P_c}{P_T} \times 100$$

$$\% \text{ Carrier power} = \frac{2}{2+m^2} \times 100$$

Ques. In 100% AM mod index % carrier power is?
Solⁿs

$$\% \text{ Carrier power} = \frac{2}{2+1} \times 100$$

$$= 66.66\%$$

$$(m=1)$$

for $m=0.5$

$$\% \text{ Carrier power} = \frac{2}{2+(0.5)^2} \times 100 = 88.89\%$$

EFFICIENCY OF AM -

$$\% \eta_{AM} = \frac{P_{SB}}{P_T} \times 100$$

$$= \frac{\text{useful power}}{\text{Total power}} = \frac{m^2}{2+m^2} \times 100$$

$$\% \eta_{AM} = 1 - \% \text{ Carrier power}$$

- max. efficiency of AM for single tone is 33.33% at $m=1$

$$\eta_{\max} \% = \frac{1}{1+2} \times 100 = 33.33\%$$

- If $m=1$ $\% P_{SB} = 11.11$

- If $m=0$ $\% P_c = 100\%$

$$\% P_{SB} = 0\%$$

- In AM carrier have no information and if it is eliminated most of the power can be saved.

- In 100% AM modⁿ if carrier is eliminated (DSBSC)

$$\% \text{ power saved} \rightarrow 66.67\%$$

- In 100% AM modⁿ if carrier & one side band is eliminated (SSB)

$$P_T = \underbrace{66.67}_{\% P_c} + \underbrace{\frac{33.33}{2}}_{\% P_{SB}} = 83.3\%$$

Ques: In 50% AM if SSB is used % power saved is Solⁿ:

$$P_c = \frac{2}{2+10.5^2} \times 100$$

$$\% P_c = 88.89$$

$$\text{one side band} = 5.56$$

$$94.45\%$$

- Efficiency of DSB and SSB is 100% possible.

Clue: In AM modⁿ, m changed from 0 to 1 then total transmitted power,

(1) remain constant

(2) $\uparrow 50\%$

(3) $\downarrow 50\%$

(4) $\uparrow 25\%$

$$\text{Solⁿ: } P_T = P_c \left[1 + \frac{m^2}{2} \right]$$

$$m=0$$

$$P_T = P_c$$

$$m=1$$

$$P_T = \frac{3}{2} P_c = 1.5 P_c$$

$\therefore$ Increased by 50%

$$\frac{P_{Tf} - P_{Ti}}{P_{Ti}} \times 100\%$$

$$P_T = P_C \left[1 + \frac{m^2}{2} \right]$$

$$m = \sqrt{2 \left(\frac{P_T}{P_C} - 1 \right)}$$

Ques In AM modⁿ $P_C = 1 \text{ kWatt}$ and P_T is 1.125 kW then m ?

Solⁿ:

$$m = \sqrt{2 \left(\frac{1.125}{1000} - 1 \right)} = \sqrt{0.25} = \underline{\underline{0.5}}$$

$$* m = \sqrt{2 \left[\left(\frac{I_T}{I_C} \right)^2 - 1 \right]}$$

Ques In AM the antenna current without modⁿ is 4 Amp and current increases to 4.5 Amp then % modⁿ is.

Solⁿ:

$$m = \sqrt{2 \left(\left(\frac{4.5}{4} \right)^2 - 1 \right)}$$

$$= 72.8\%$$

(2) MULTITONE A.M. —

$$\left. \begin{array}{l} m_1(t) \\ m_2(t) \\ m_3(t) \\ \vdots \end{array} \right\} c(t) \text{ (same)}$$

$$m_1(t) = V_{m1} \cos 2\pi f_{m1} t$$

$$m_2(t) = V_{m2} \cos 2\pi f_{m2} t$$

$\vdots$

$$c(t) = V_c \cos 2\pi f_c t$$

$$s(t) = [V_c + m_1(t) + m_2(t) + \dots] \cos 2\pi f_c t$$

$$= [V_c + V_{m1} \cos 2\pi f_{m1} t + V_{m2} \cos 2\pi f_{m2} t + \dots] \cos 2\pi f_c t$$

$$= V_c [1 + m_1 \cos 2\pi f_{m1} t + m_2 \cos 2\pi f_{m2} t + \dots] \cos 2\pi f_c t$$

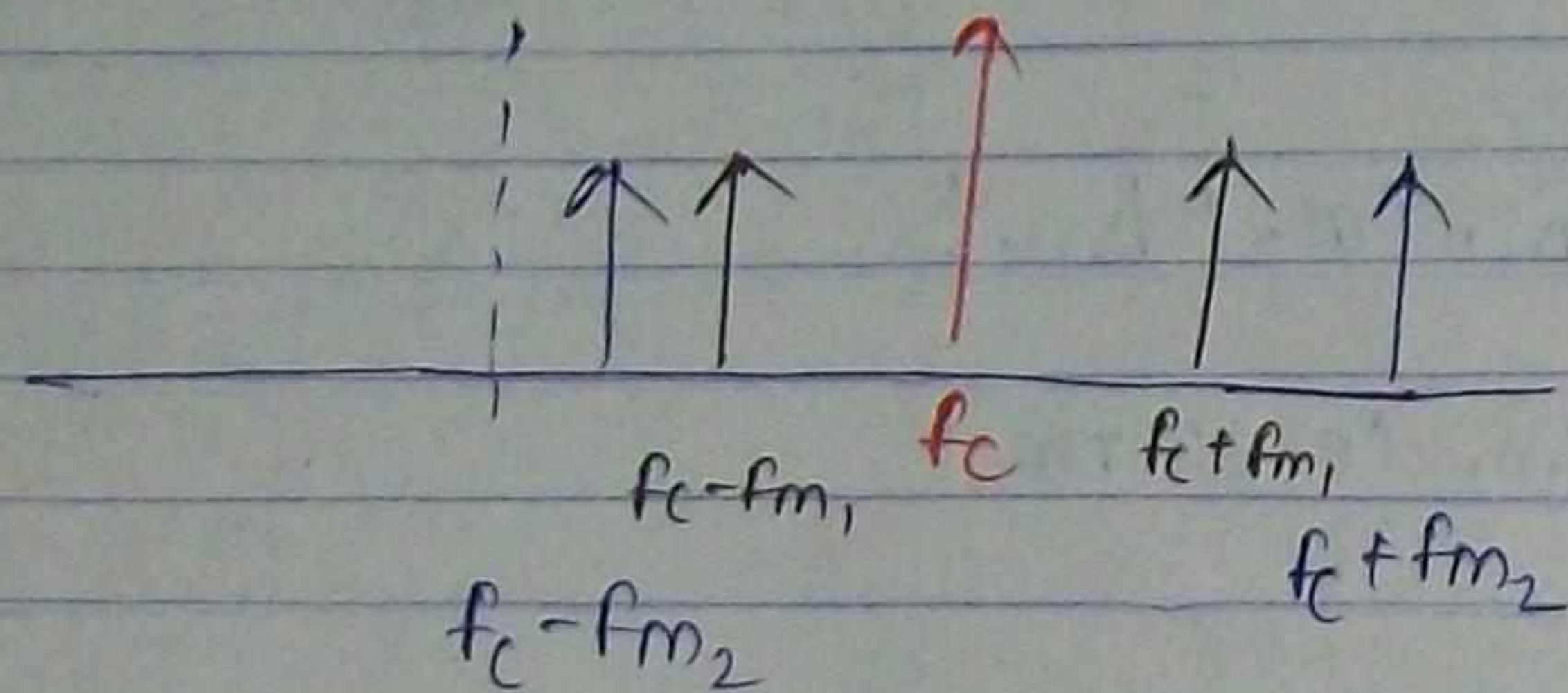
TOTAL POWER —

$$P_T = P_C \left[1 + \frac{m^2}{2} \right]$$

$$m = \sqrt{m_1^2 + m_2^2 + \dots}$$

not
considering
-ve freq.

assuming $f_{m2} \neq f_{m1}$



$$BW = (f_c + f_{m2}) - (f_c - f_{m2})$$

$$BW = 2f_{m2}$$

$$BW = 2f_{max}$$

Q. In AM modulated signal is given by

$$s(t) = 10 [1 + 0.5 \cos 2\pi \times 3000t] \cos 2\pi \times 10^4 t$$

then can

- | | | | |
|------------------|----------------|--------------|-------|
| 1. Carrier power | 3. P_T | 5. f_{USB} | 7. BW |
| 2. P_{SB} | 4. η_{AM} | 6. f_{LSB} | |

Soln:

$$\textcircled{1} \% P_c = \frac{2}{2 + (0.5)^2} \times 100$$

$$P_c = 10, m = 0.5, f_m = 3 \text{ kHz}, f_c = 10000 \text{ kHz}$$

$R = 1 \text{ kHz}$

$$\textcircled{1} P_c = \frac{V_c^2}{2} = \frac{10^2}{2} = 50 \text{ W}$$

$$\textcircled{2} P_{SB} = P_c \times \frac{m^2}{2} = 50 \times \frac{(0.5)^2}{2} = 6.25 \text{ W}$$

$$\textcircled{3} P_T = 50 + 6.25 = 56.25 \text{ W}$$

$$\textcircled{4} \eta_{AM} = \frac{11.11\%}{\frac{m^2}{2 + m^2} \times 100}$$

$$\textcircled{5} f_{USB} = f_c + f_m = 1003 \text{ kHz}$$

$$\textcircled{6} f_{LSB} = f_c - f_m = 997 \text{ kHz}$$

$$\textcircled{7} BW = 2f_m = 6 \text{ kHz}$$

Ques. A AM modulated signal is given by?

$$s(t) = 10 [2 + 1.6 \cos 2\pi \times 2000t + 1.2 \cos 2\pi \times 5000t] \cos 2\pi \times 10^4 t$$

$$\textcircled{1} P_c \quad \textcircled{2} P_{SB} \quad \textcircled{3} P_T \quad \textcircled{4} BW$$

Soln:

$$s(t) = 20 [1 + 0.8 \cos 2\pi \times 2000t + 0.6 \cos 2\pi \times 5000t] \cos 2\pi \times 10^4 t$$

$$m = \sqrt{m_1^2 + m_2^2} = 1$$

$$m = 1$$

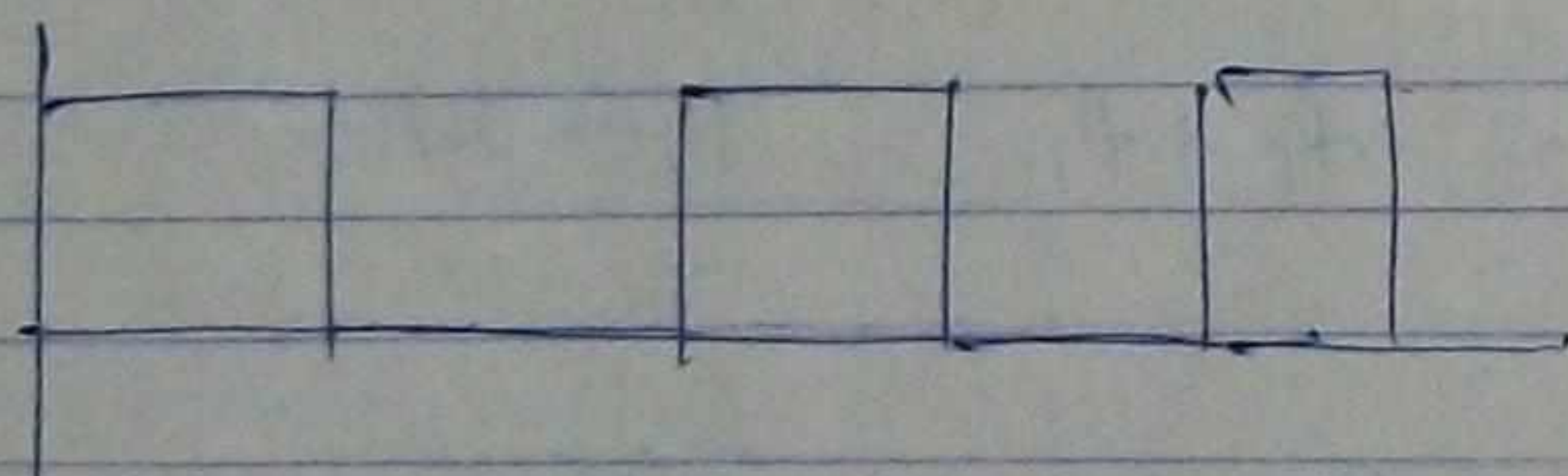
$$\textcircled{1} P_c = \frac{V_c^2}{2} = \underline{\underline{200 \text{ W}}}$$

$$\textcircled{2} P_{SB} = P_c \times \frac{m^2}{2} = \underline{\underline{100 \text{ W}}}$$

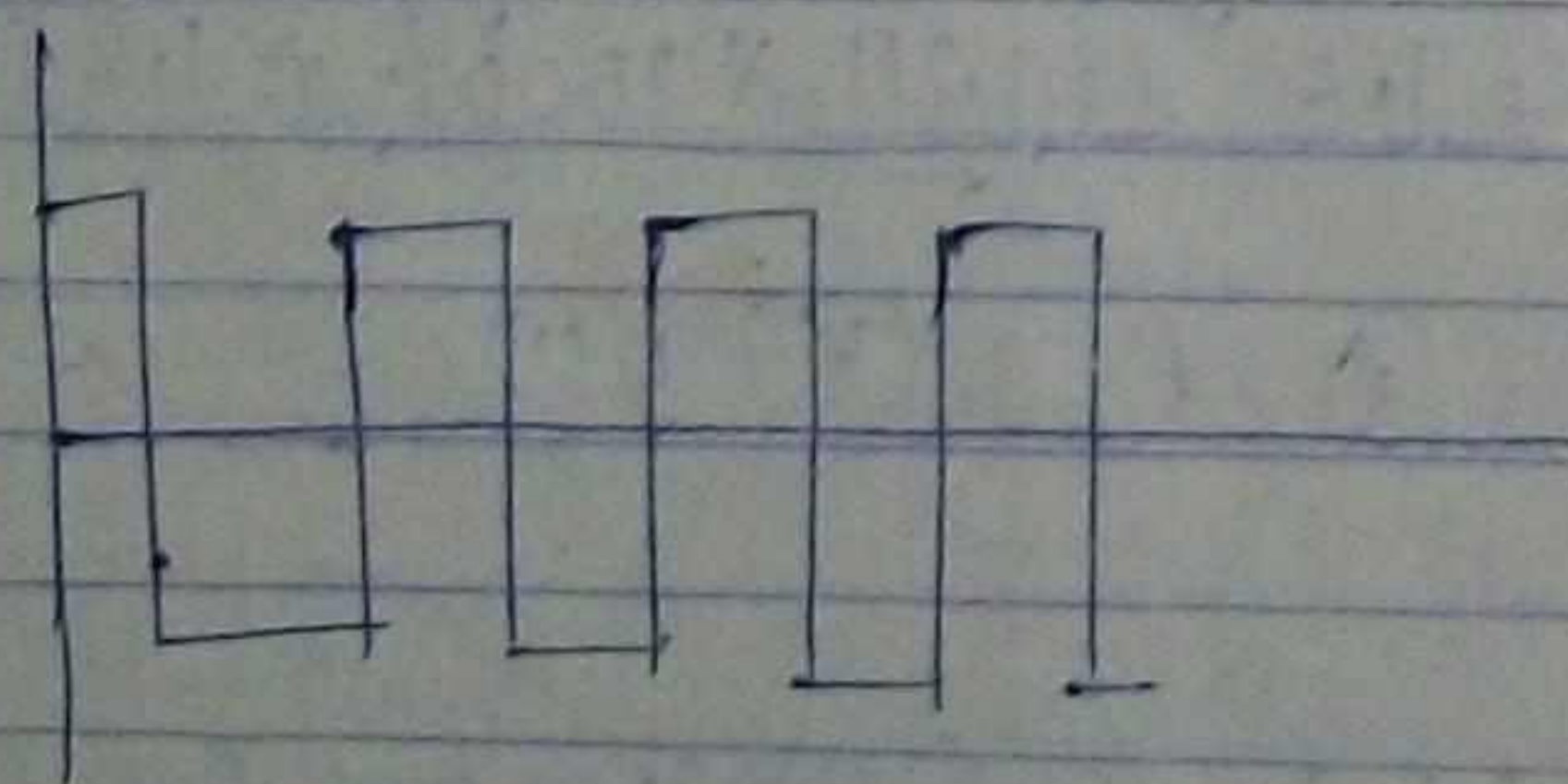
$$\textcircled{3} P_T = \underline{\underline{300 \text{ W}}}$$

$$\textcircled{4} BW = 2f_{max} = \underline{\underline{10 \text{ kHz}}}$$

NOTE-



$$x(t) = \frac{A}{2} + \frac{2A}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos 2\pi(2n-1)f_0 t$$



$$x(t) = \frac{4A}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos 2\pi(2n-1)f_0 t$$

- In multitone AM modⁿ for each message signal two side band will present.

$\textcircled{3}$ ARBITRARY SIGNAL AM -

$m(t)$ $\rightarrow$ arbitrary signal, $m(t) \xleftrightarrow{F.T} M(f)$

$$c(t) = V_c \cos 2\pi f_c t$$

$$s(t) = V_c [1 + K_a m(t)] \cos 2\pi f_c t$$

$K_a \rightarrow$ amplitude sensitivity of modⁿ.

$$m_a = |K_a m(t)| = \text{mod}^n \text{ index}$$

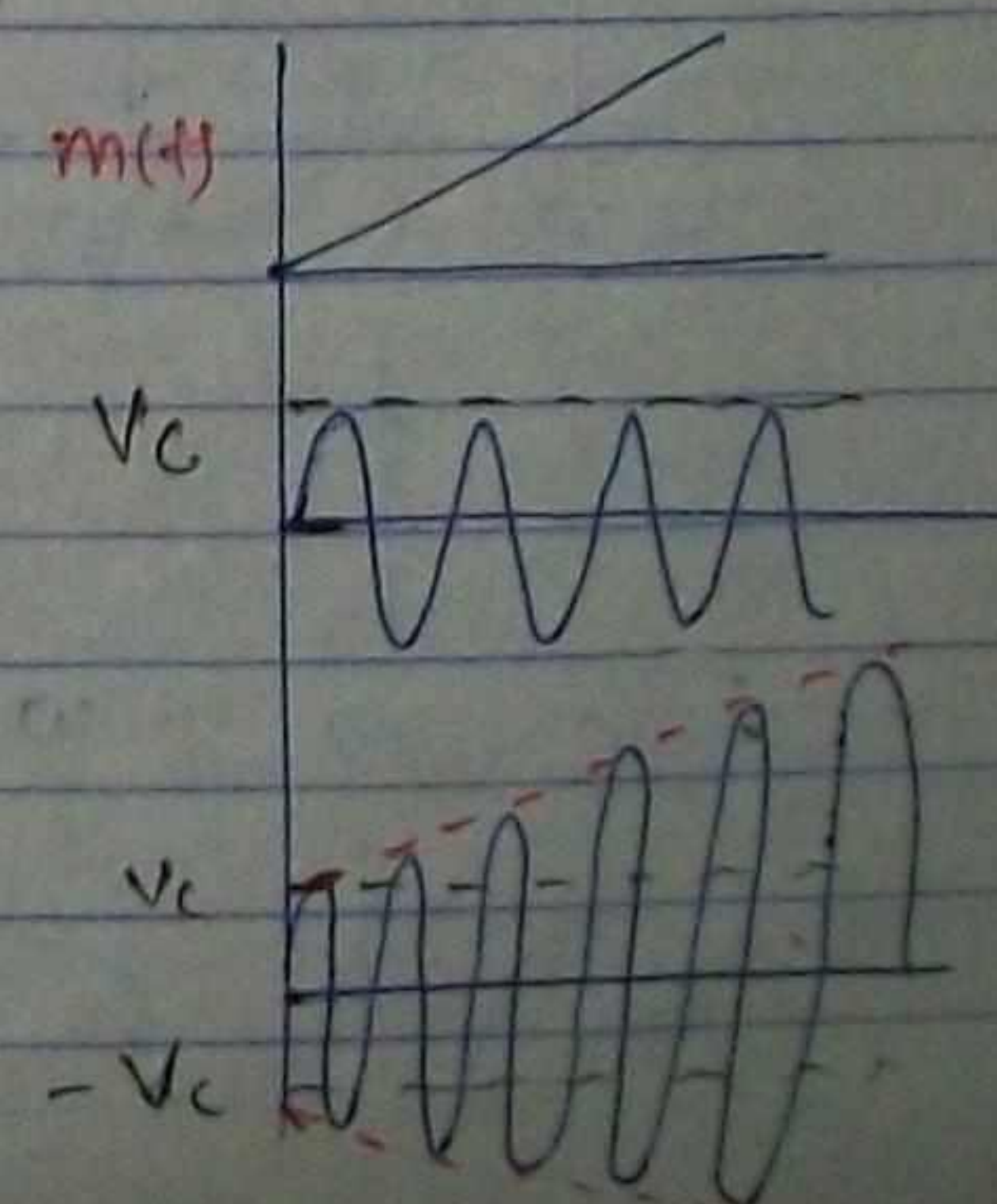
$$|K_a m(t)| \leq 1. \quad \text{To avoid envelope distortion.}$$

Ques. Draw a AM waveform if $m(t) = t, \forall t \geq 0$
with $c(t) = V_c \cos 2\pi f_c t$

Soln:

$$m(t) = t$$

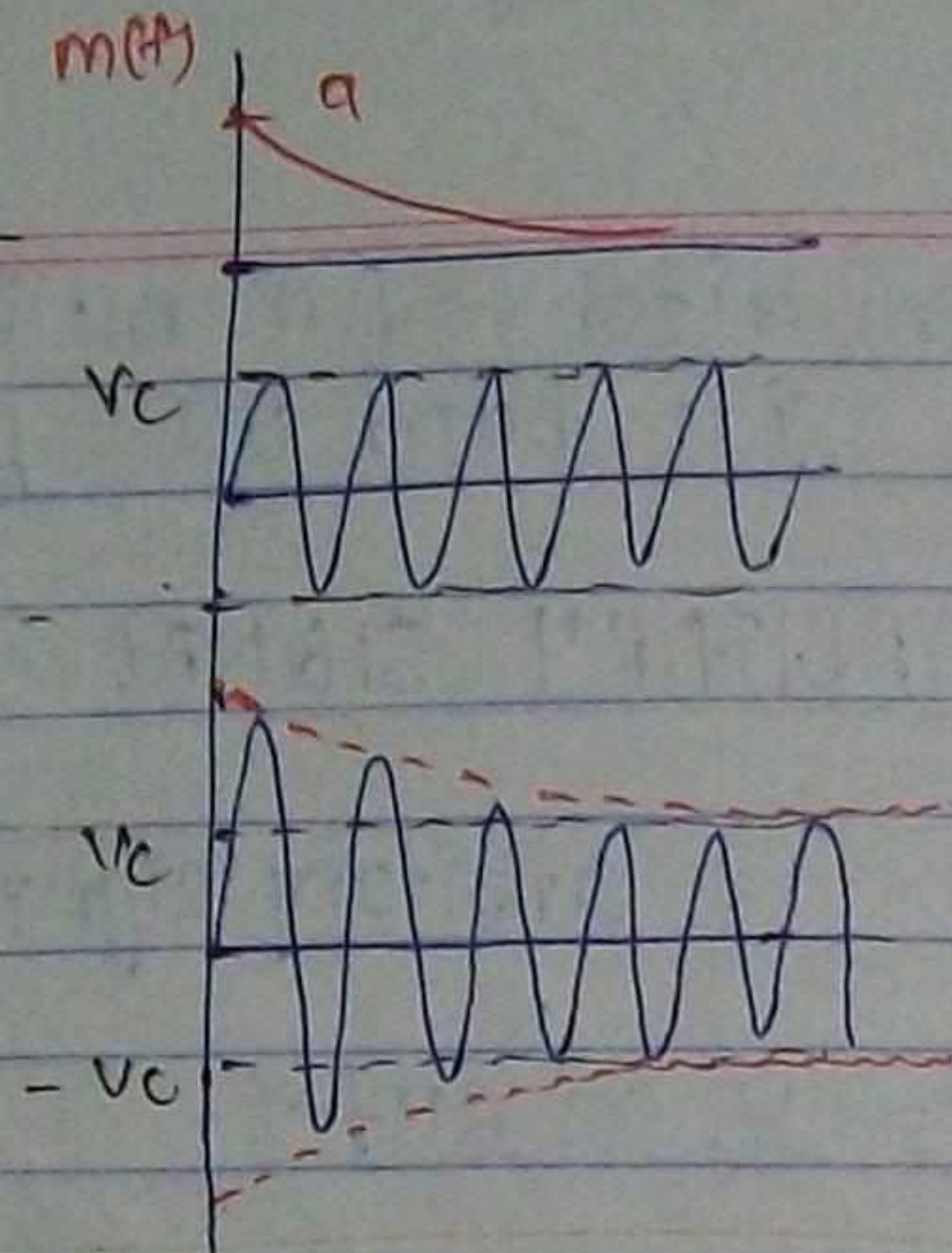
DIVERGENT



$m=0$ then only carrier

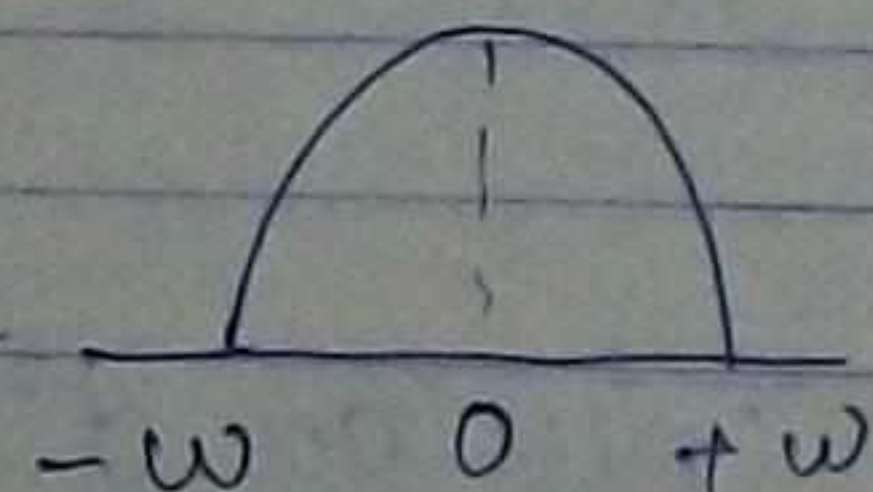
② Let $m(t) = a e^{-t}$

CONVERGENT

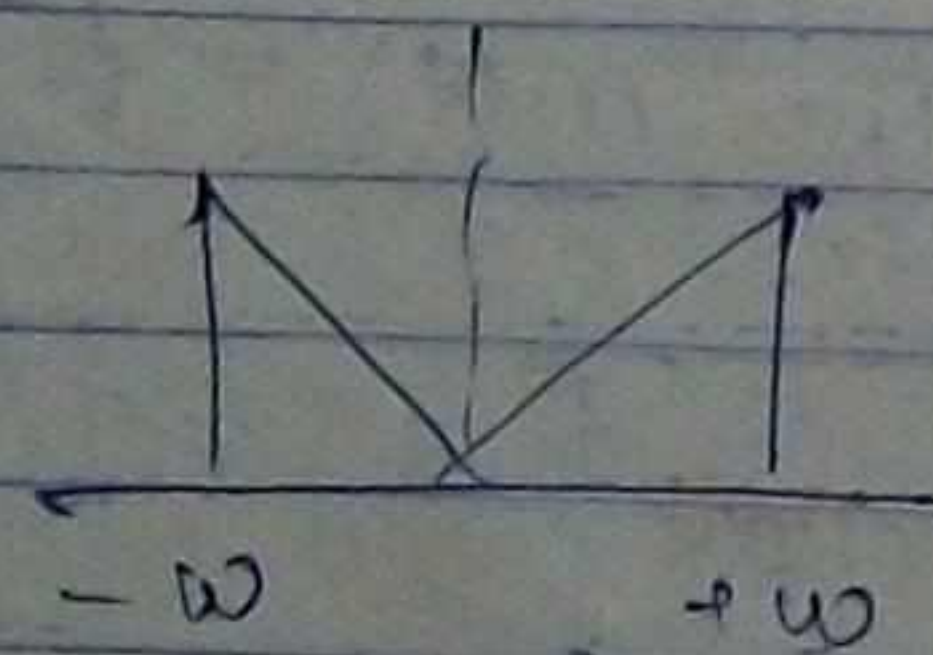


NOTE-

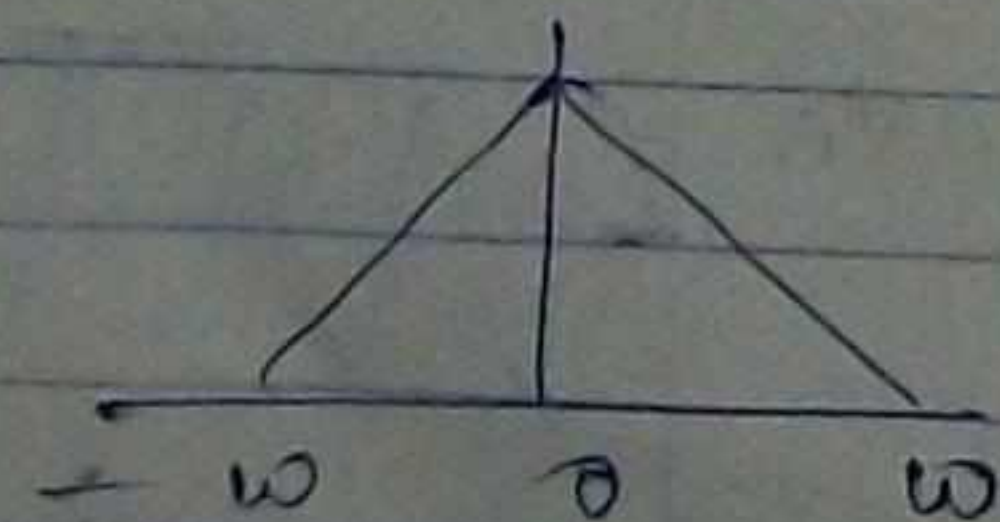
$m(t) \longleftrightarrow m(f)$
 $\Downarrow$



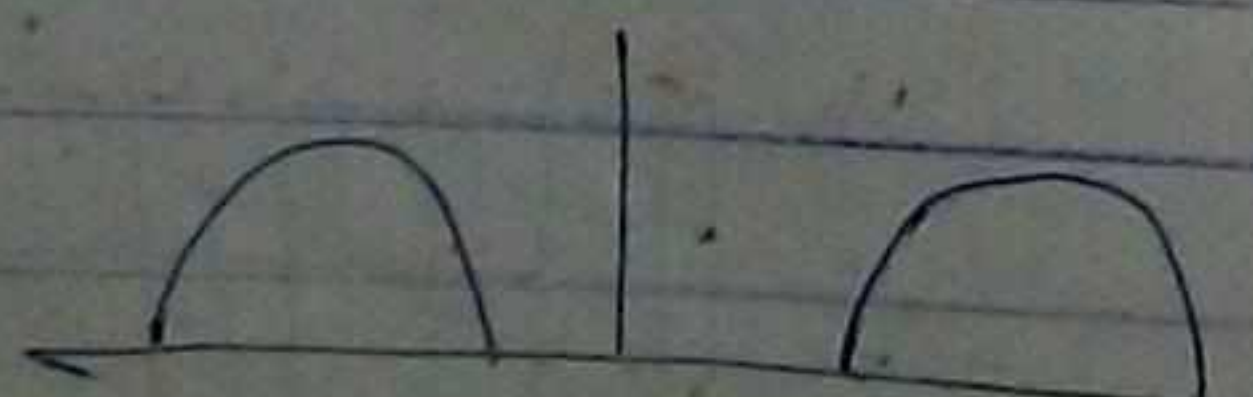
BW = w



BW = w



BW = w

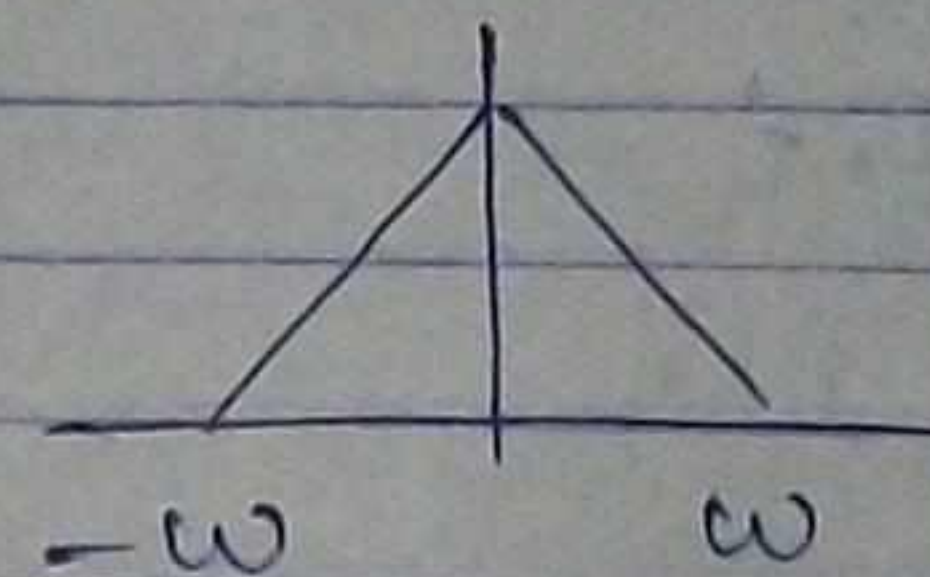


BW = w

#

- let $m(t)$ have message signal spectrum $m(f)$

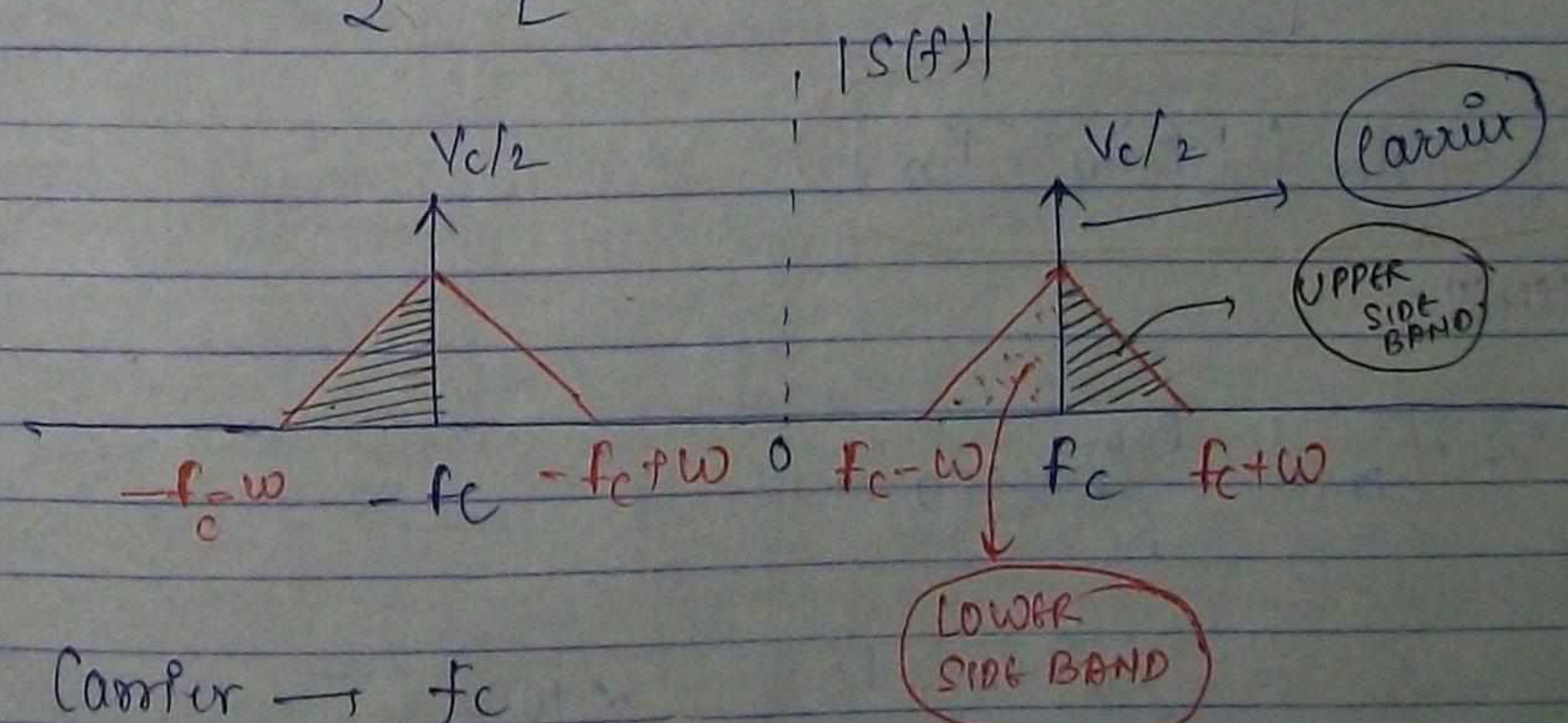
then draw AM signal spectrum.



Solⁿs $S(t) = V_c [1 + K_a m(t)] \cdot \cos 2\pi f_c t$

$S(t) = V_c \cos 2\pi f_c t + K_a \cdot V_c \cdot m(t) \cdot \cos 2\pi f_c t$

$S(f) = \frac{V_c}{2} [S(f-f_c) + S(f+f_c)]$
 $+ \frac{K_a \cdot V_c}{2} [m(f-f_c) + m(f+f_c)]$



Carrier $\rightarrow f_c$

USB $\rightarrow f_c$ to $f_c + w$

LSB $\rightarrow f_c - w$ to f_c

w $\rightarrow$ message signal BW

AM SIGNAL BW = 2w

$$P_c =$$

POWER CALCULATIONS -

$$S(t) = V_c \cos 2\pi f_c t + k_a m(t) \cdot V_c \cos 2\pi f_c t$$

CARRIER

SIDEBAND

$$m(t) \rightarrow P_m \text{ (given Power)}$$

$$P_T = P_c + P_{SB}$$

$$P_c = \frac{V_c^2}{2R} = \frac{V_c^2}{2} \quad (R=1\Omega)$$

$$P_{SB} = k_a^2 \cdot P_m \cdot P_c = k_a^2 \cdot P_m \cdot P_c$$

$$P_T = P_c [1 + k_a^2 P_m]$$

$$P_T = P_c (1 + k_a^2 P_m)$$

EFFICIENCY -

$$\% \eta_{AM} = \frac{P_{SB}}{P_T} \times 100\%$$

$$= \frac{k_a^2 P_m \cdot P_c}{P_c (1 + k_a^2 P_m)} \times 100$$

$$\% \eta_{AM} = \frac{k_a^2 P_m}{1 + k_a^2 P_m} \times 100$$

* If msg. signal is a square wave then.

$$k_a^2 P_m = 1$$

$$\eta_{max} = 50\%$$

⇒ GENERATION OF AM SIGNAL -

$$S(t)_{AM} = V_c [1 + m \cos 2\pi f_m t] \cdot \cos 2\pi f_c t$$

$$= V_c [1 + k_a m(t)] \cdot \cos 2\pi f_c t$$

(i) Square law modulator.

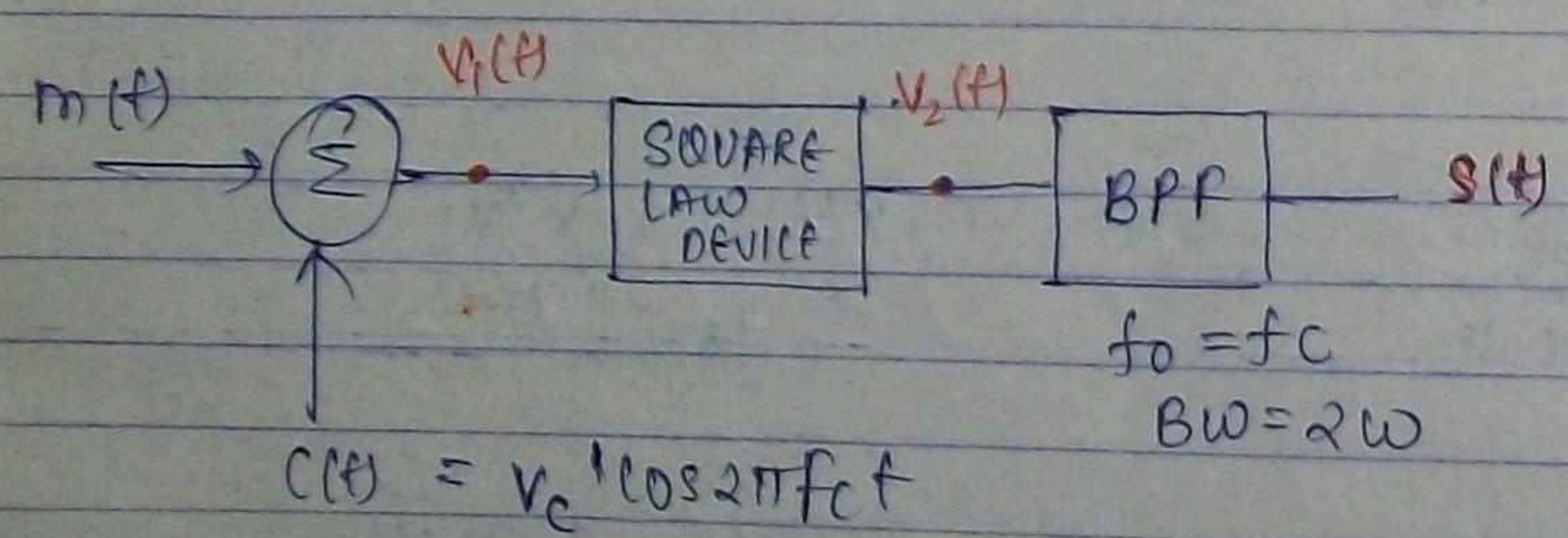
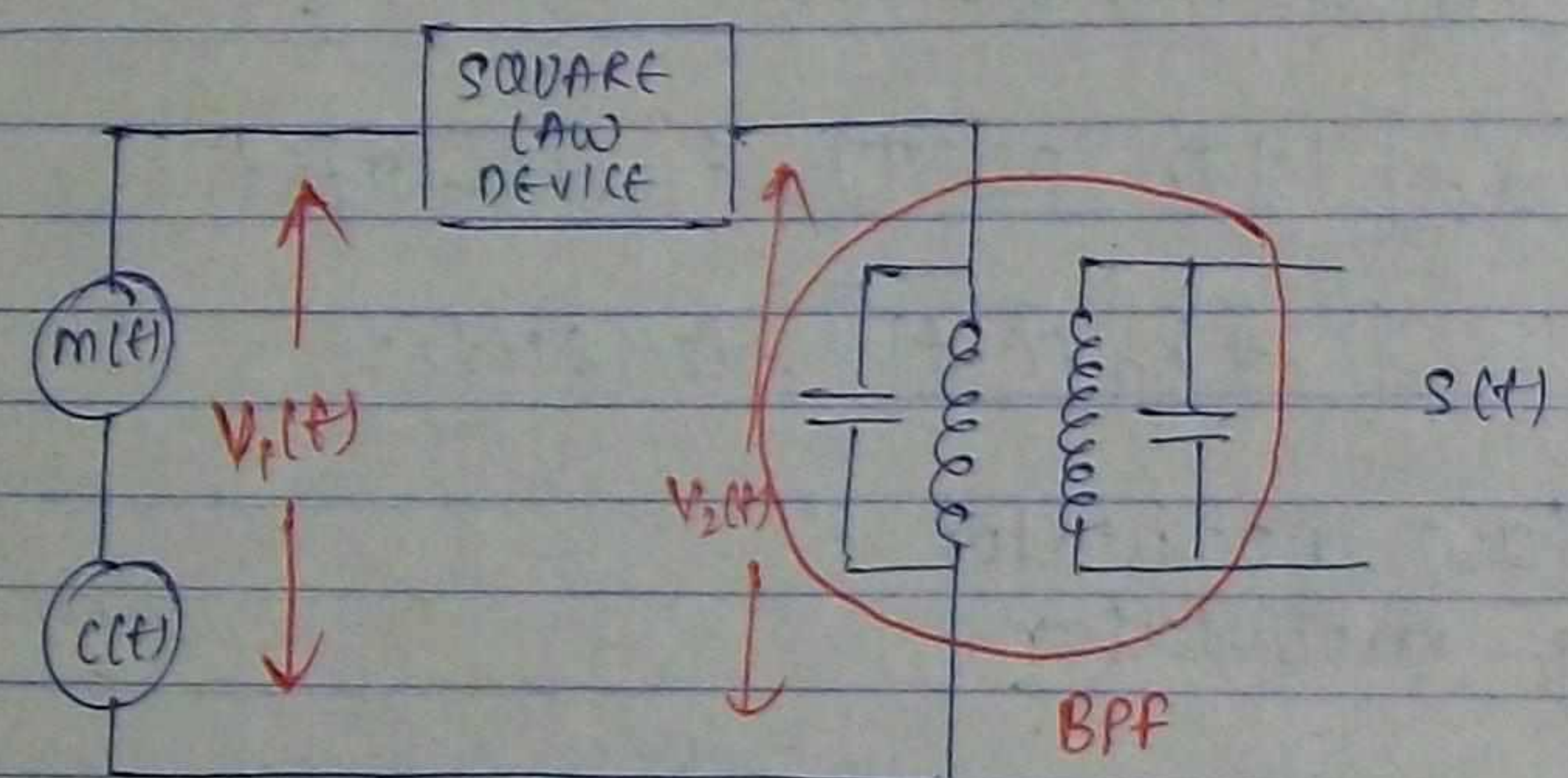
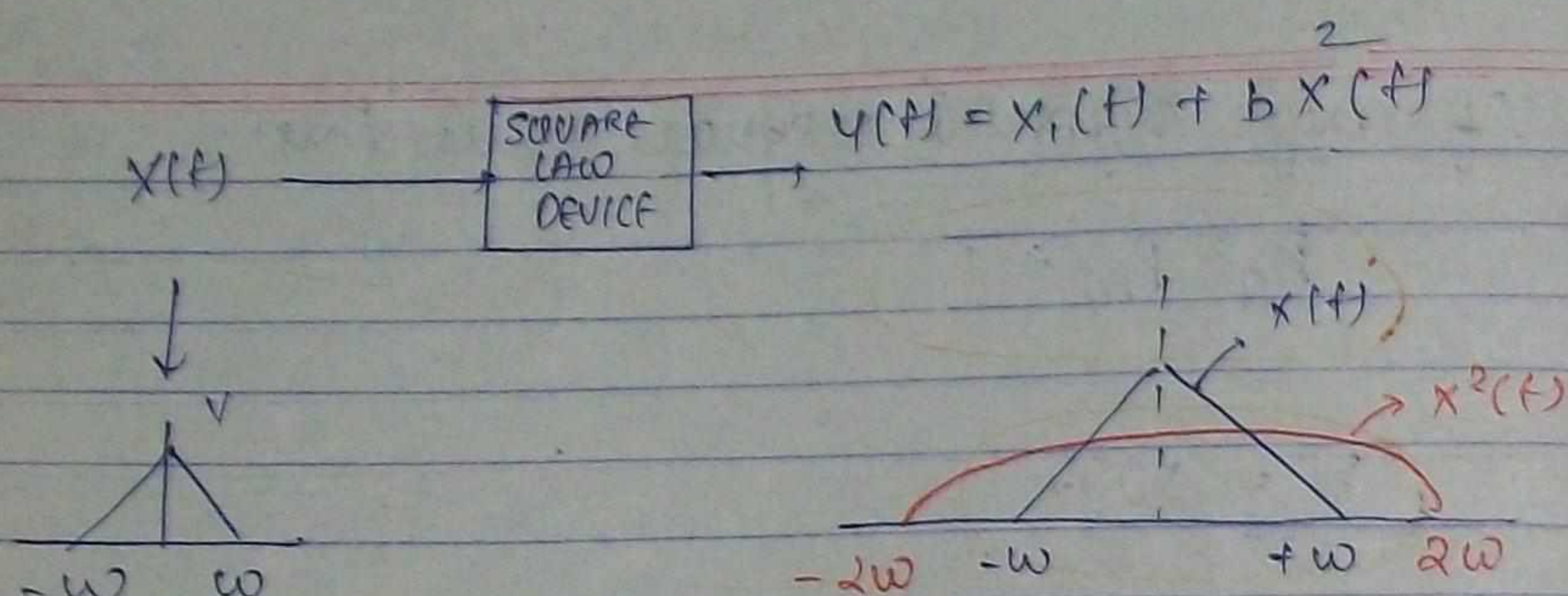
(ii) Switching modulator.

(i) SQUARE LAW MODULATOR - (NON-LINEAR DEVICE)

$$X(t) \rightarrow \boxed{\text{NON LINEAR DEVICE}} \rightarrow Y(t) = a_1 X(t) + a_2 X^2(t) + a_3 X^3(t) + \dots$$

$$X(t) \rightarrow \boxed{\text{SQUARE LAW}} \rightarrow Y(t) = a_1 X(t) + a_2 X^2(t)$$

$$X(t) \rightarrow \boxed{\text{CUBE LAW DEVICE}} \rightarrow Y(t) = a_1 X(t) + a_2 X^3(t)$$

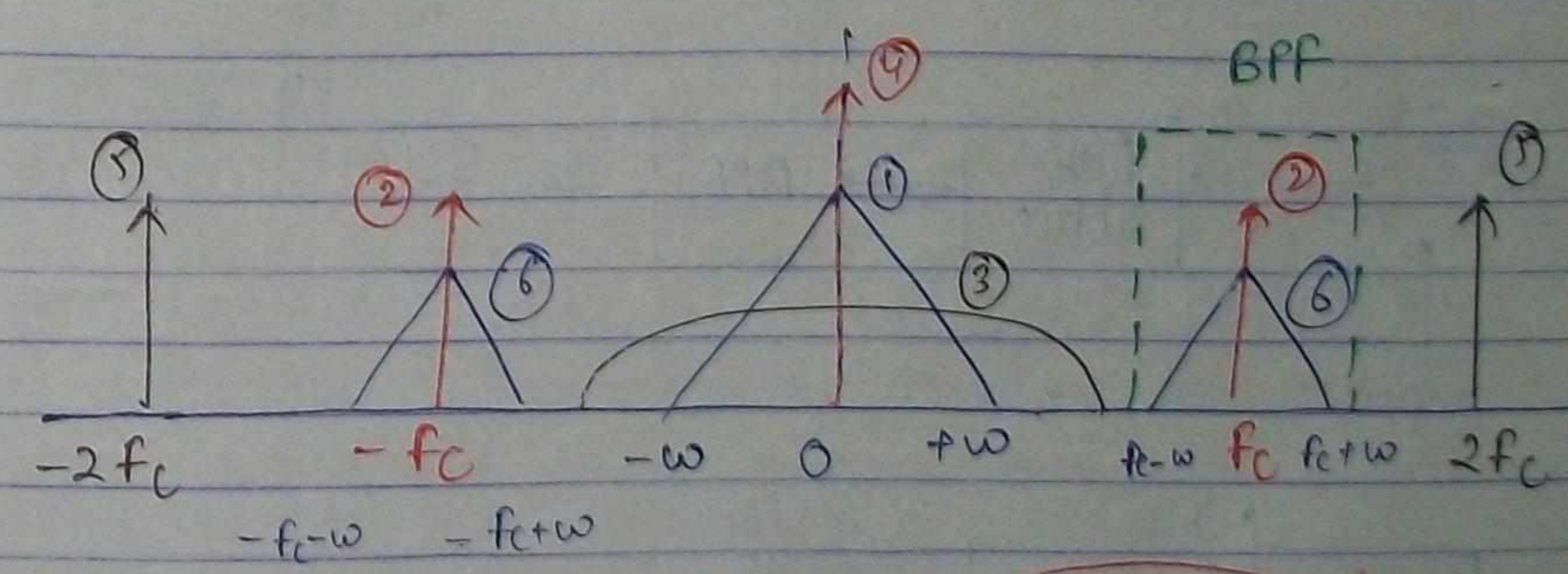
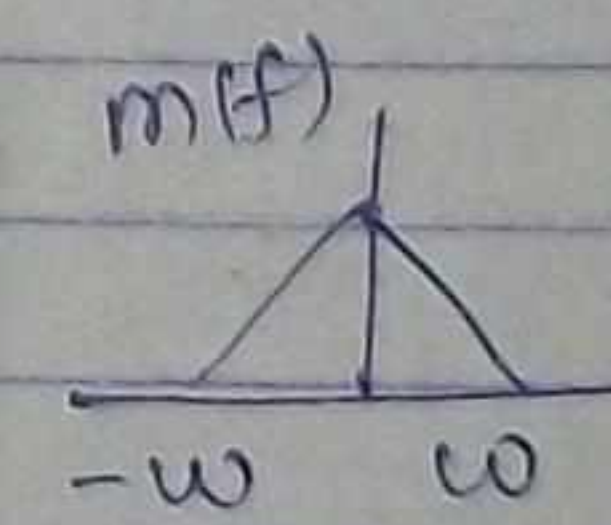


$$V_1(t) = m(t) + V_c \cos 2\pi f_c t$$

$$V_2(t) = a_1 V_1(t) + a_2 V_1^2(t)$$

$$V_2(t) = a_1 [m(t) + V_c \cos 2\pi f_c t] + a_2 [m^2(t) + 2m(t)V_c \cos 2\pi f_c t + (V_c \cos 2\pi f_c t)^2]$$

$$V_2(t) = \underbrace{a_1 m(t)}_{(1)} + \underbrace{a_1 V_c \cos 2\pi f_c t}_{(2)} + \underbrace{a_2 m^2(t)}_{(3)} + \underbrace{2a_2 m(t)V_c \cos 2\pi f_c t}_{(4)} + \underbrace{a_2 \frac{V_c^2}{2}}_{(5)} + \underbrace{a_2 \frac{V_c^2}{2} \cos 2\pi(2f_c)t}_{(6)}$$



BPF at $f_0 = f_c$

$BW = 2w$

after BPF:

$$S(t) = a_1 V_c \cos 2\pi f_c t + 2a_2 V_c m(t) \cos 2\pi f_c t$$

$$s(t) = a_1 v_c \left[1 + \frac{2a_2}{a_1} m(t) \right] \cos 2\pi f_c t$$

$$s(t) = V_c [1 + k_a \cdot m(t)] \cos 2\pi f_c t$$

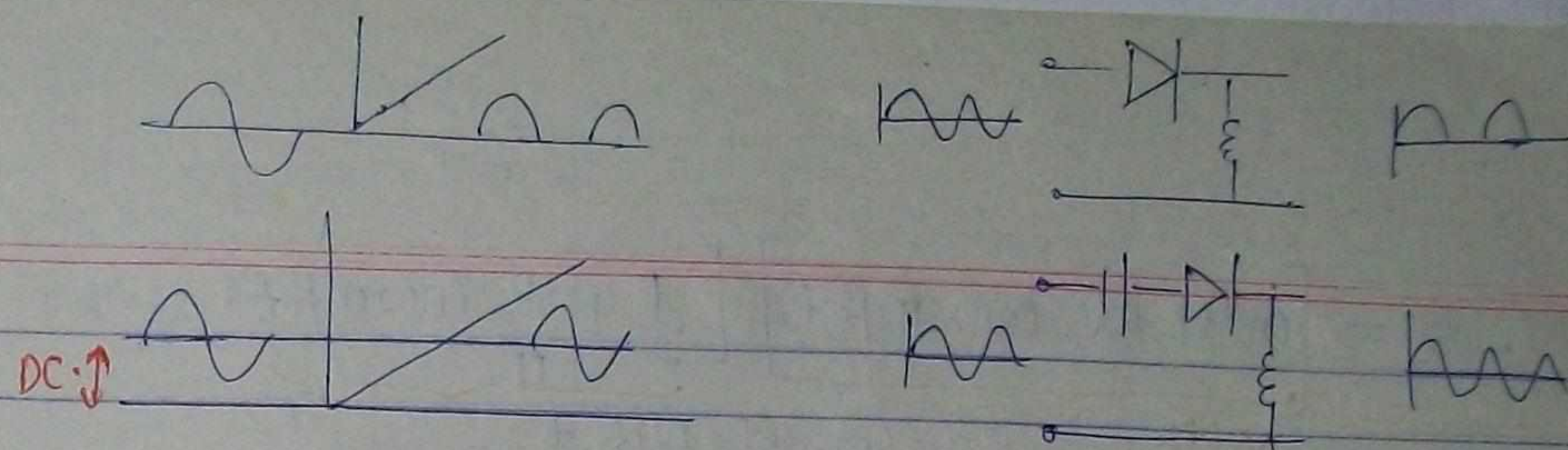
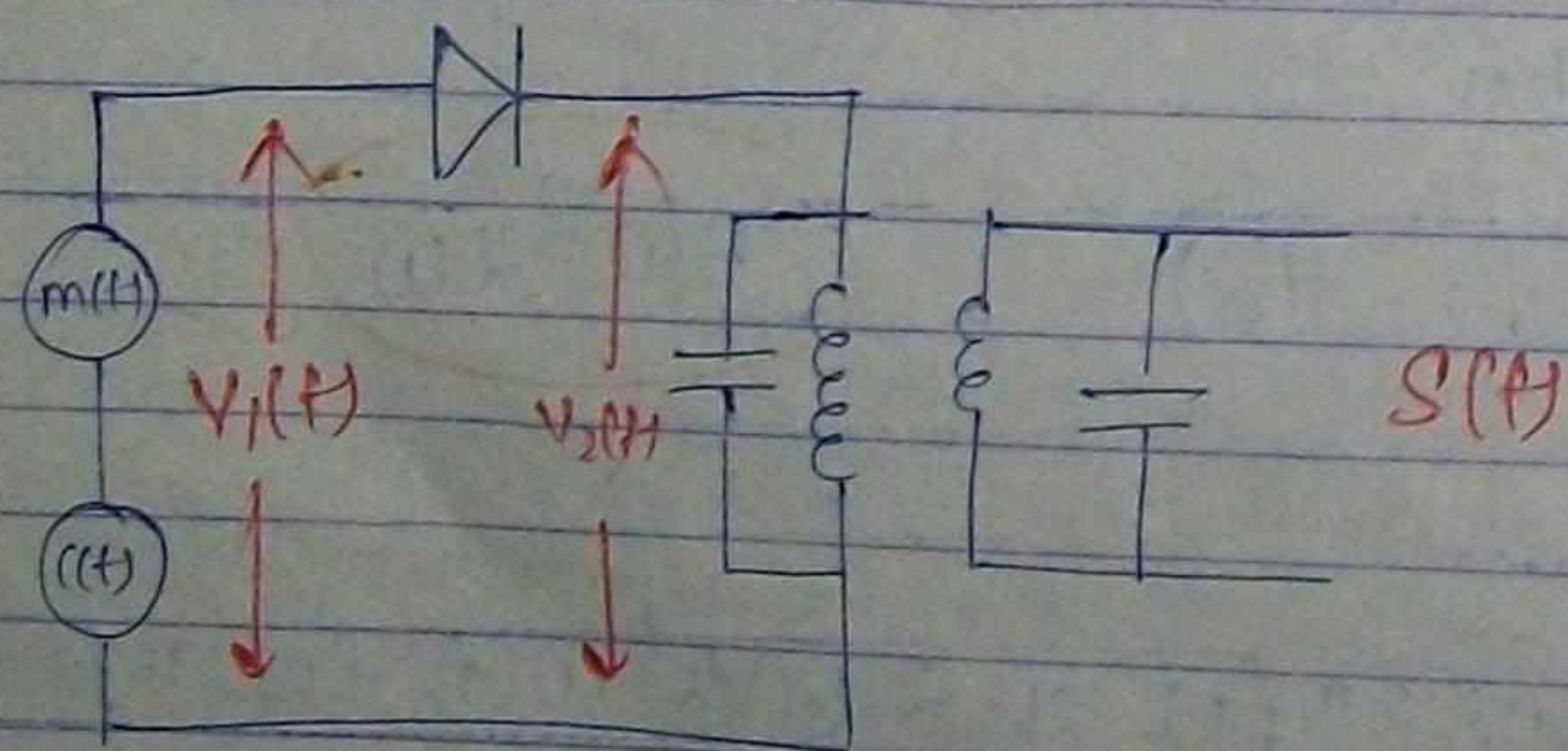
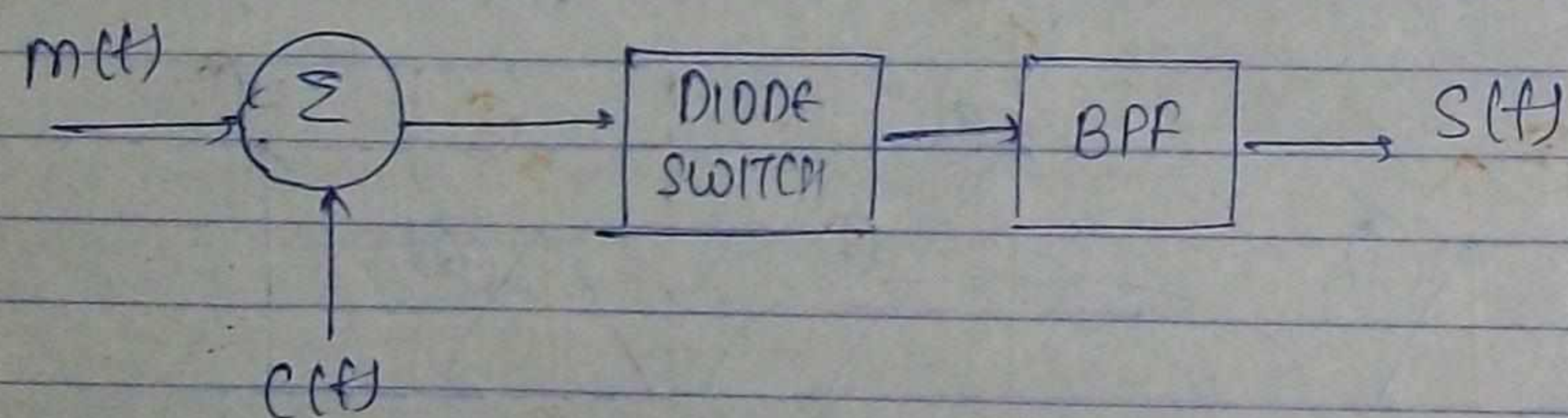
$$f_0 = f_c$$

$$BW = 2\omega$$

$$f_c > 3\omega$$

In square law modⁿ to generate AM signal
a BPF with $f_0 = f_c$, at a $BW = 2\omega$ is
used with $f_c > 3\omega$.

② SWITCHING MODULATOR -

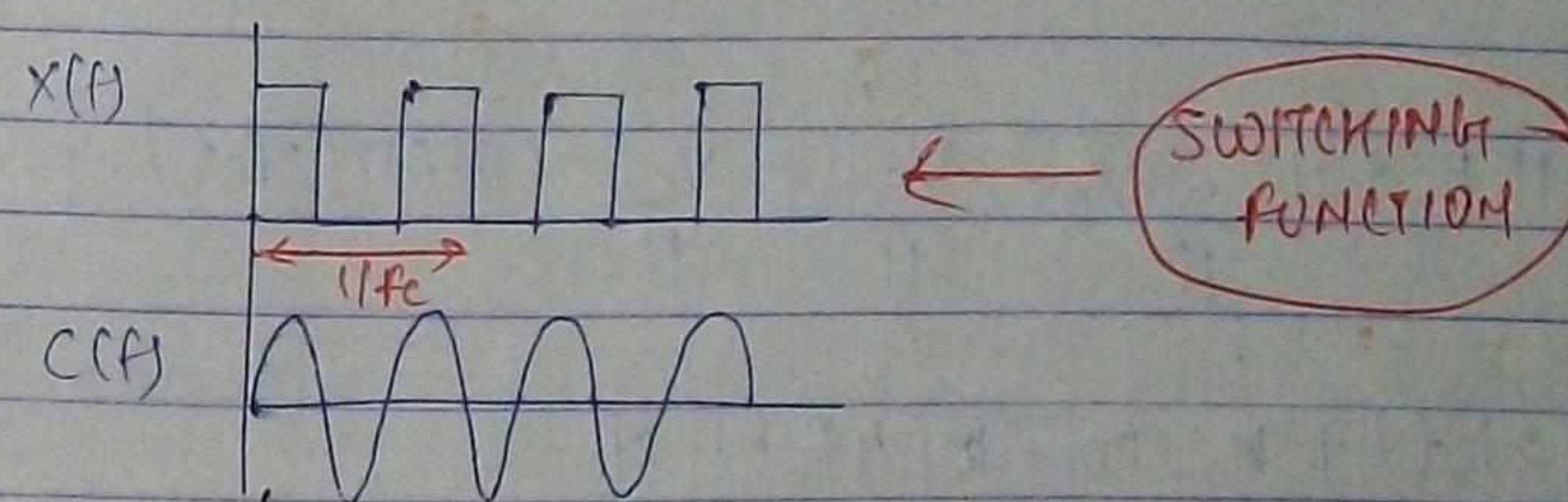


$$|c(t)| > |m(t)|$$

$$v_2(t) = v_1(t) \quad c(t) > 0$$

$$v_2(t) = 0 \quad c(t) < 0$$

$$v_2(t) = v_1(t) x(t)$$



- In switching modulator the diode switch is controlled with the amplitude of carrier, when

$c(t) > 0$ acts as on switch

$c(t) < 0$ acts as off switch

This action of diode can be represented as a switching function as shown above.

$$v_2(t) = v_1(t) \left[\frac{1}{2} + \frac{2}{\pi} \cos 2\pi f_c t - \frac{2}{3\pi} \cos 2\pi 3f_c t + \frac{2}{5\pi} \cos 2\pi 5f_c t + \dots \right]$$

$$V_2(t) = \underbrace{[m(t) + V_c \cos 2\pi f_c t]}_{V_1(t)} \underbrace{\left[\frac{1}{2} + \frac{2}{\pi} \cos 2\pi f_c t - \frac{2}{3\pi} \cos 2\pi (3f_c)t + \dots \right]}_{x(t)}$$

$$V_2(t) = \frac{1}{2} m(t) + \frac{2}{\pi} m(t) \cos 2\pi f_c t - \frac{2}{3\pi} m(t) \cos 2\pi (3f_c)t + \frac{2}{5\pi} m(t) \cos 2\pi (5f_c)t - \dots$$

$$+ \frac{V_c}{2} \cos 2\pi f_c t + \frac{2}{\pi} \frac{V_c}{\pi} \cos 2\pi (2f_c)t + \frac{V_c}{\pi} \cos 2\pi (4f_c)t + \dots$$

$$+ k_1 \cos 2\pi (2f_c)t + k_2 \cos 2\pi (4f_c)t + k_3 \cos 2\pi (6f_c)t + \dots$$

after BPF -

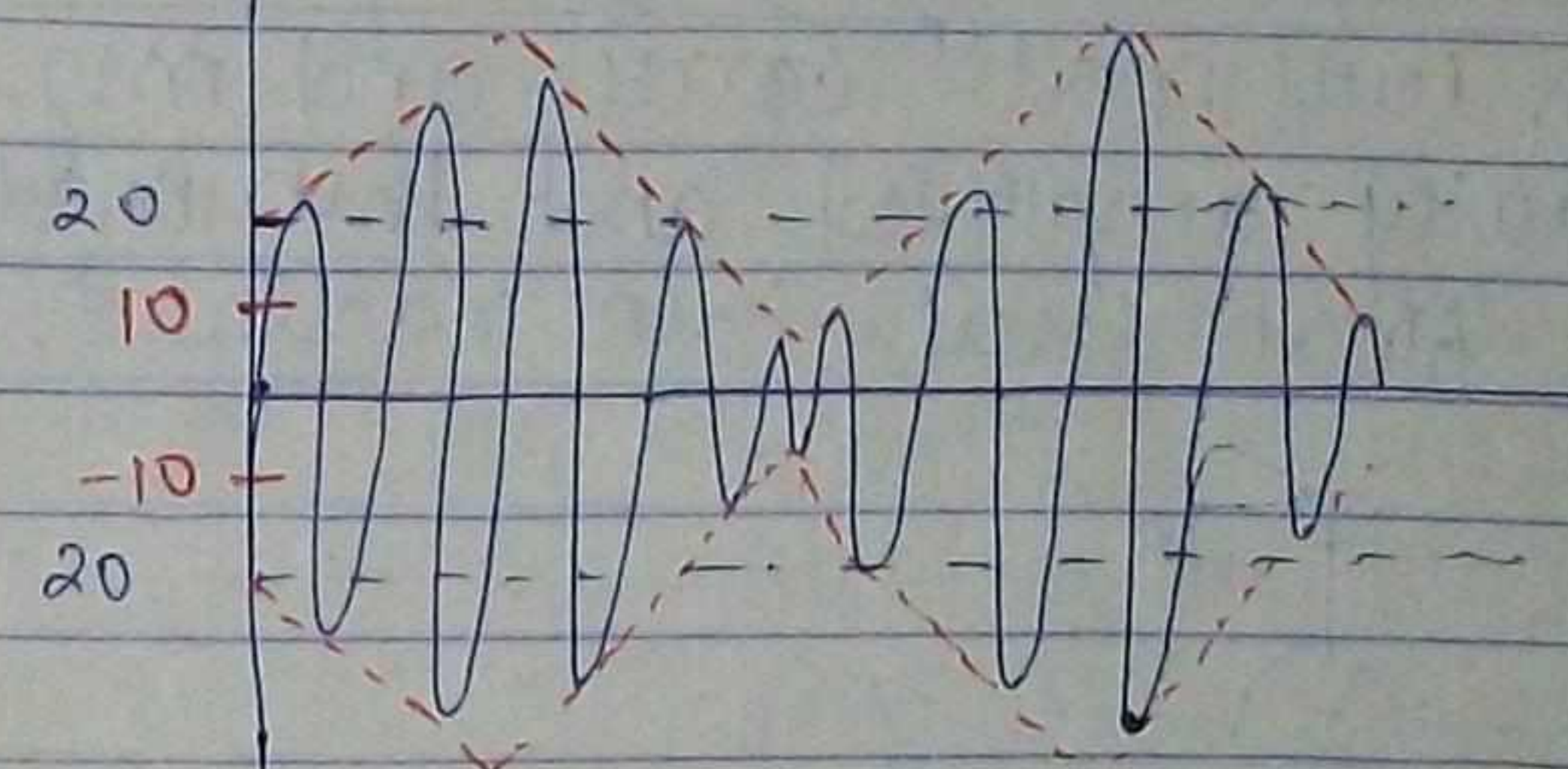
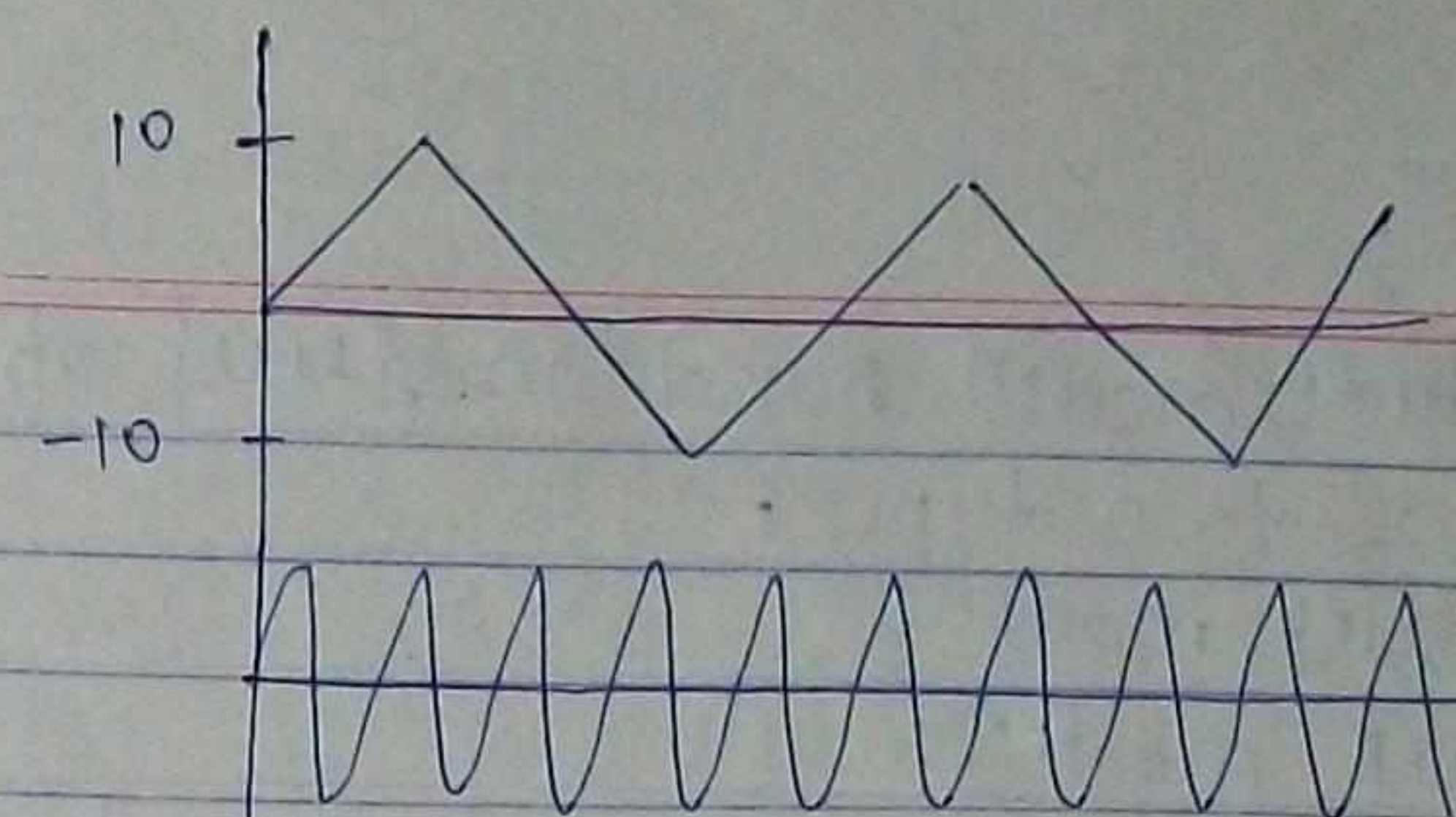
$$= \frac{V_c}{2} \cos 2\pi f_c t + \frac{2}{\pi} m(t) \cos 2\pi f_c t$$

$$= \frac{V_c}{2} \left[1 + \frac{4}{\pi} m(t) \right] \cos 2\pi f_c t$$

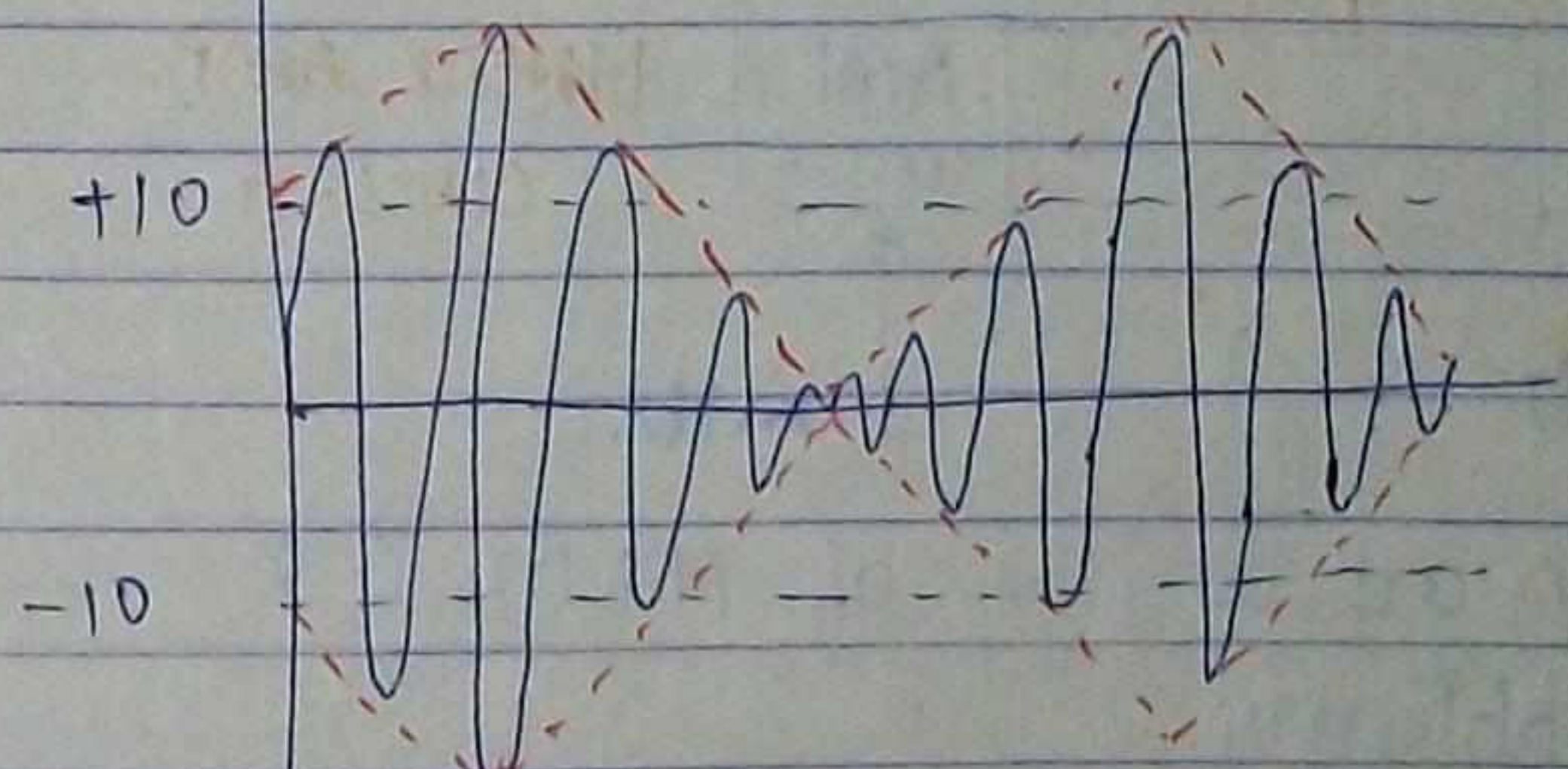
$$S(t) = V_c [1 + k_a m(t)] \cos 2\pi f_c(t)$$

Que. 1. P-11

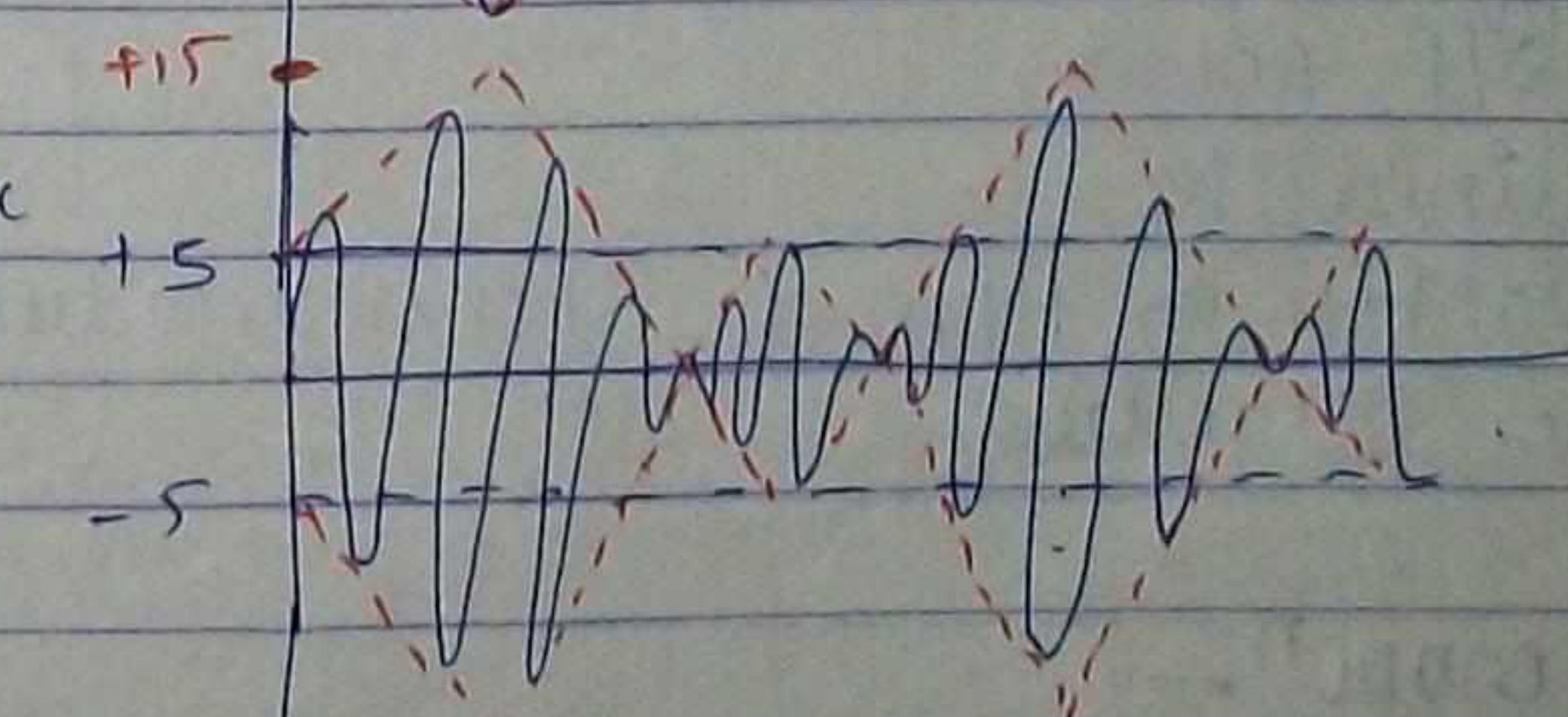
① $\mu = 0.5$
 $= \frac{V_m}{V_c}$
 $V_c = 20$



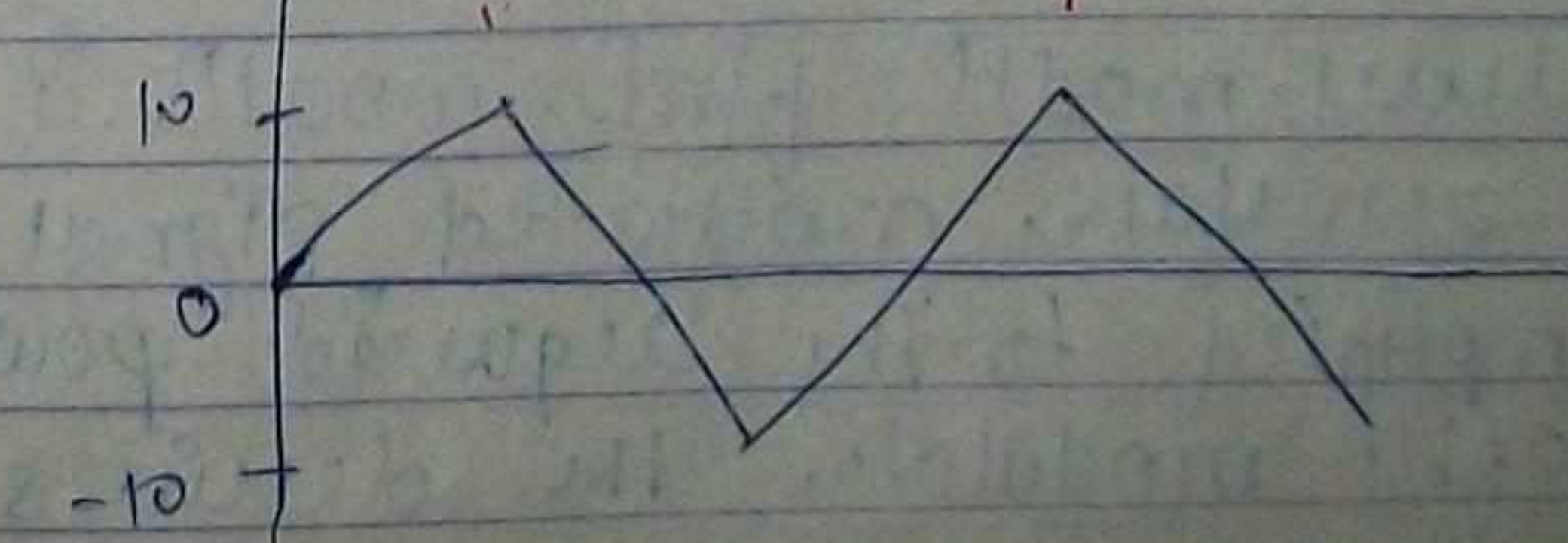
② $\mu = 1$
 $V_c = 10$
 $V_m = V_c$



③ $\mu = 2 = V_m/V_c$
 $V_c = 5V$



④ $\mu = \infty$
 $V_c = 0$

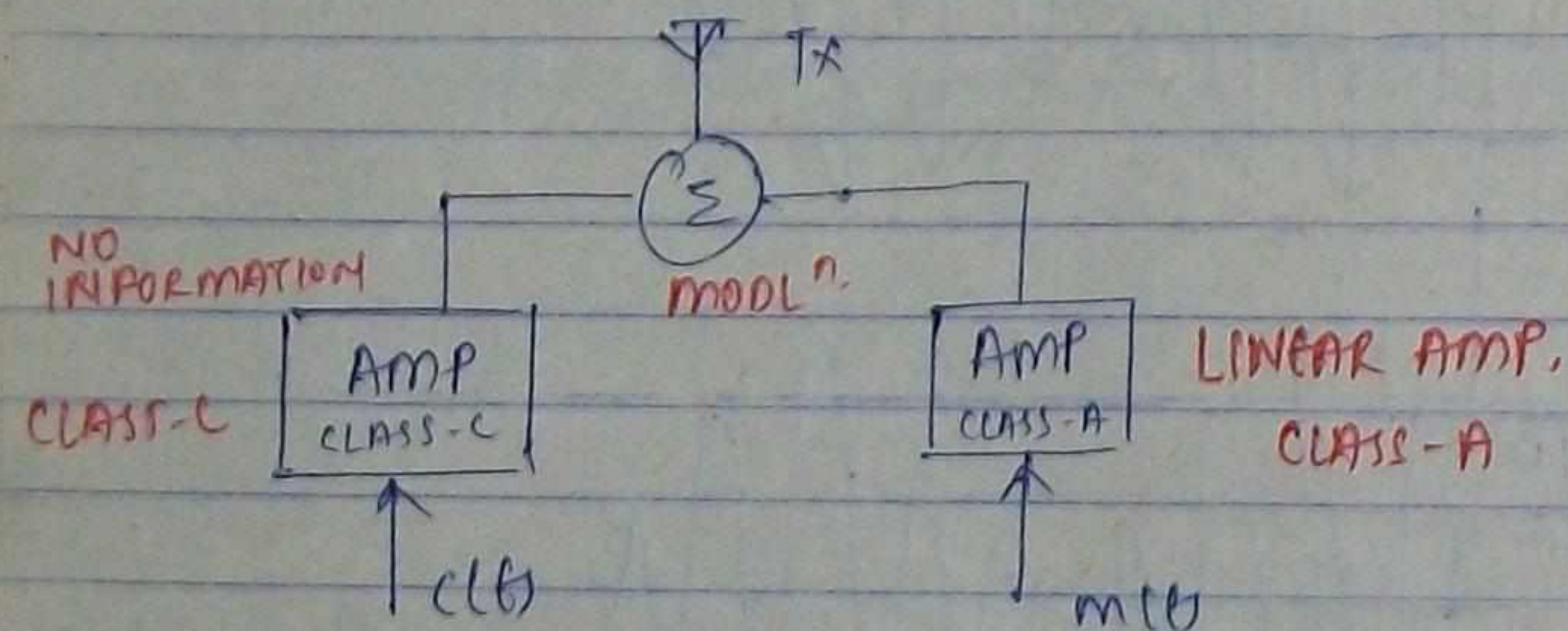


- In amplitude modⁿ based on level of modⁿ they are of two types:

- ① High level modⁿ.
- ② Low level modⁿ.

① HIGH LEVEL MODⁿ -

- In high level modⁿ carrier and msg. signal are separately amplified and then modⁿ is done at final level then radiated.



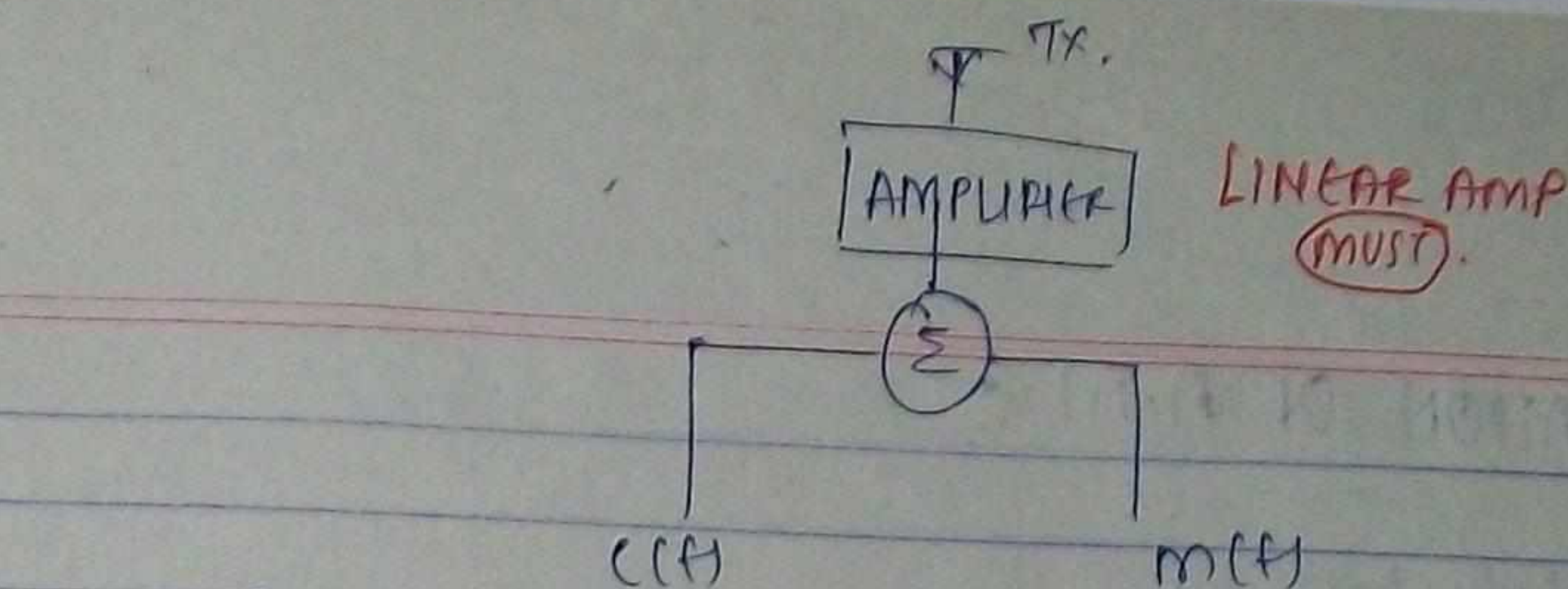
- The main advantage of HLM is:

1. Higher efficiency.
2. Higher o/p power.
3. Higher linearity.

Disadvantage of HLM is higher modulating signal power required.

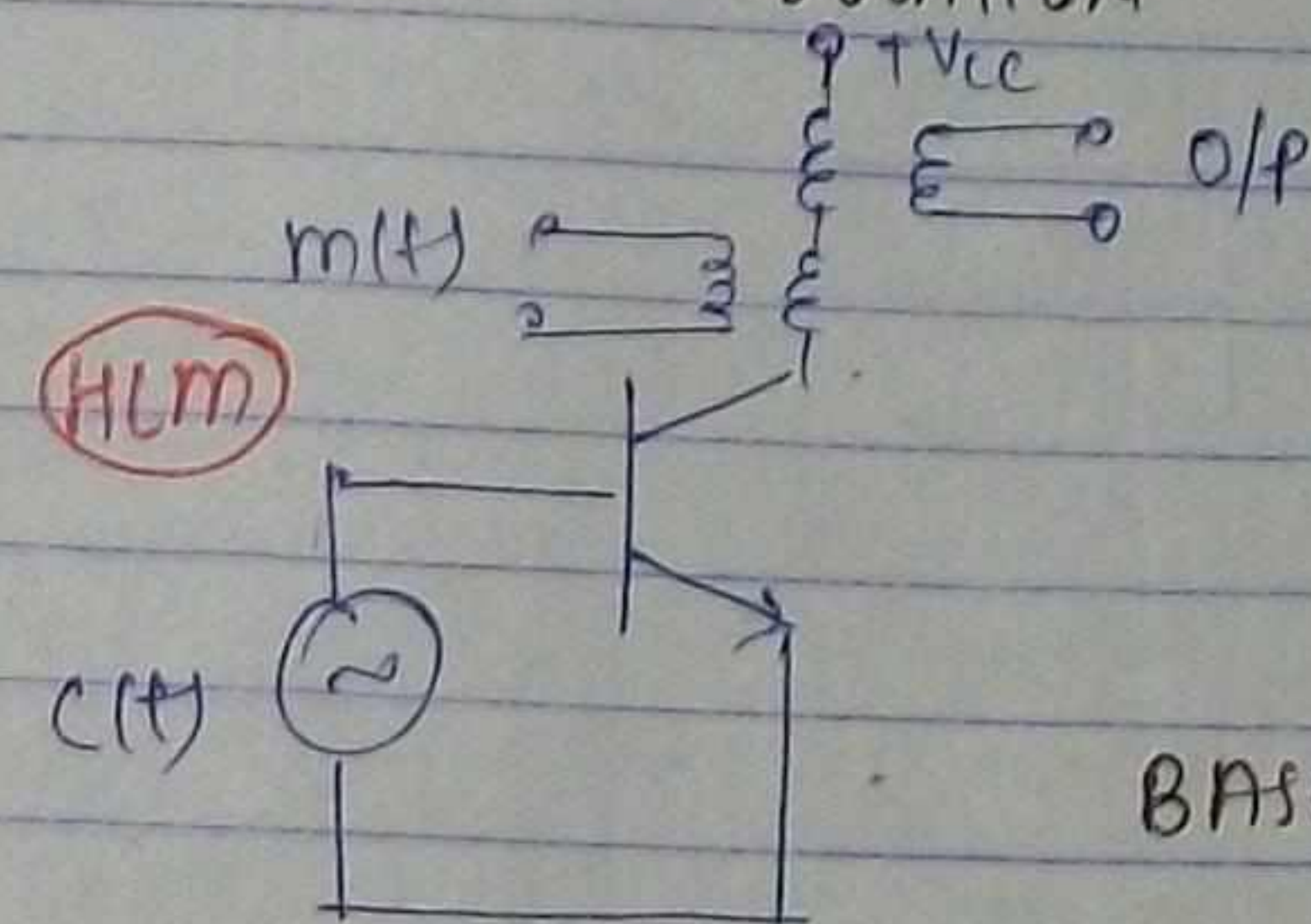
② LOW LEVEL MODⁿ -

- In low level modⁿ first modⁿ is done then the resultant modulated signal is amp. amplified to the required power level. In this after modulator the device should be linear device.



- Main advantage of Low level modⁿ is:
1. Less modulating power required.

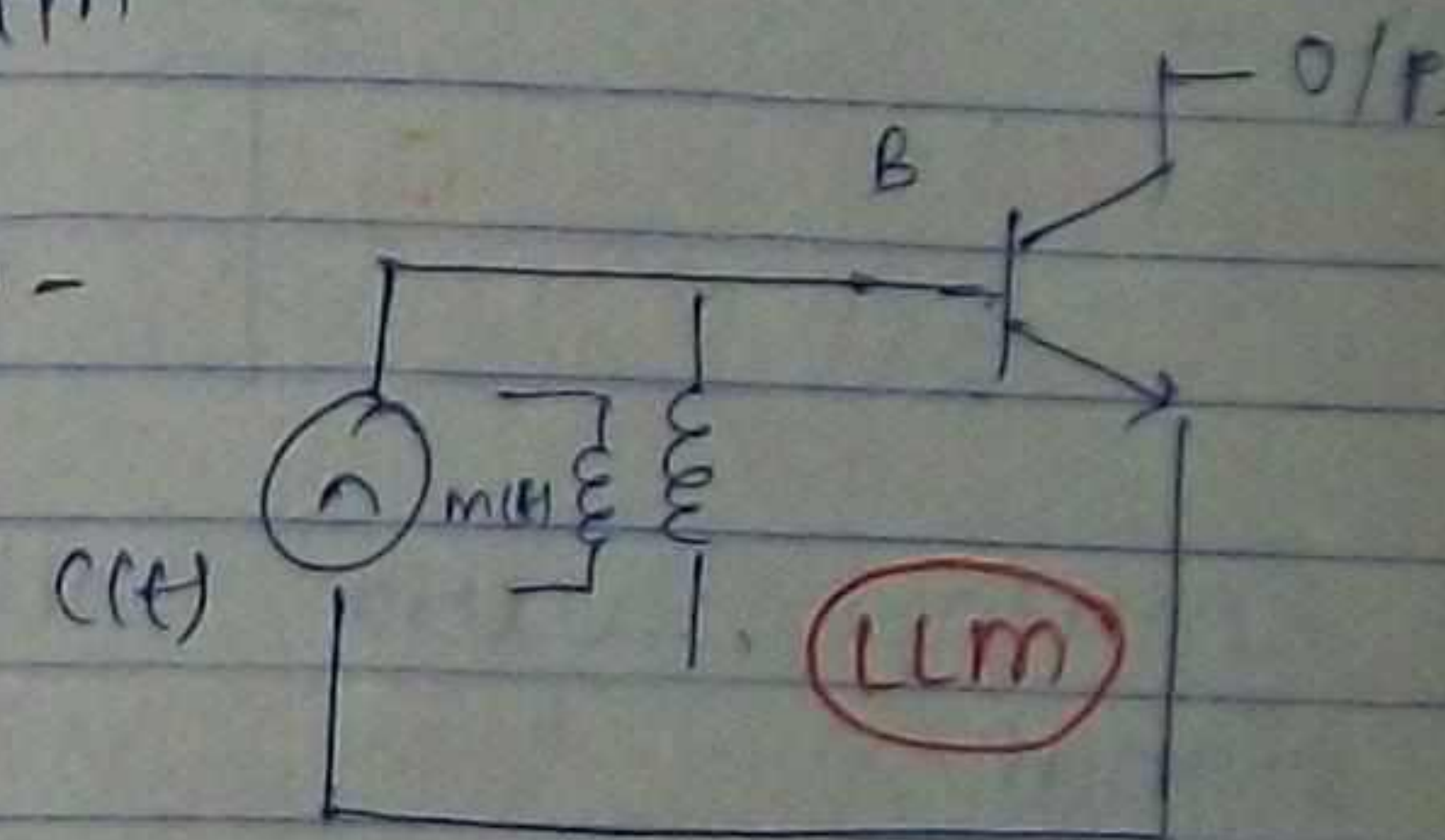
COLLECTOR MODULATION -



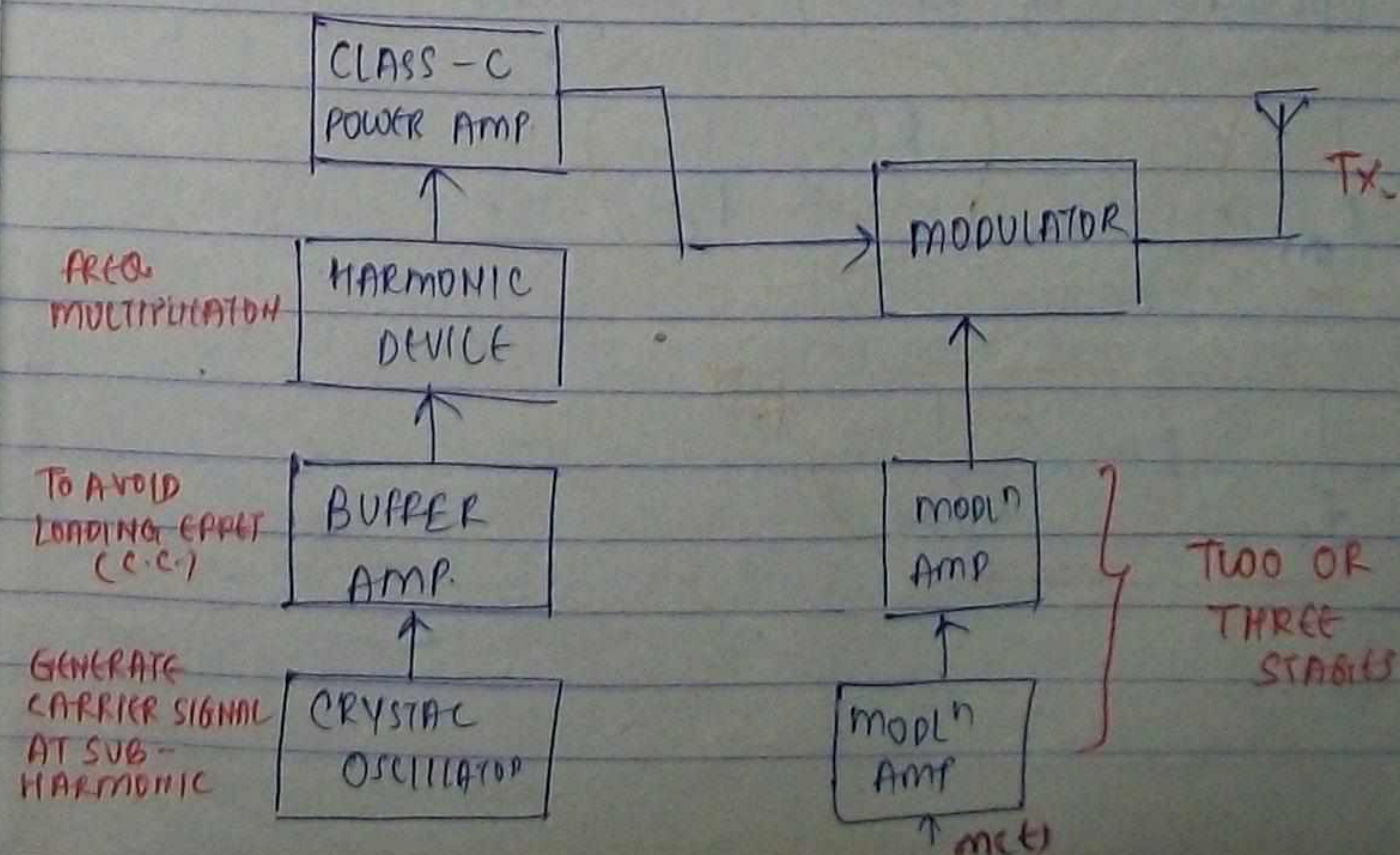
- collector modⁿ is a high level modⁿ.

- Base modⁿ is low level modⁿ.

BASE MODULATION -



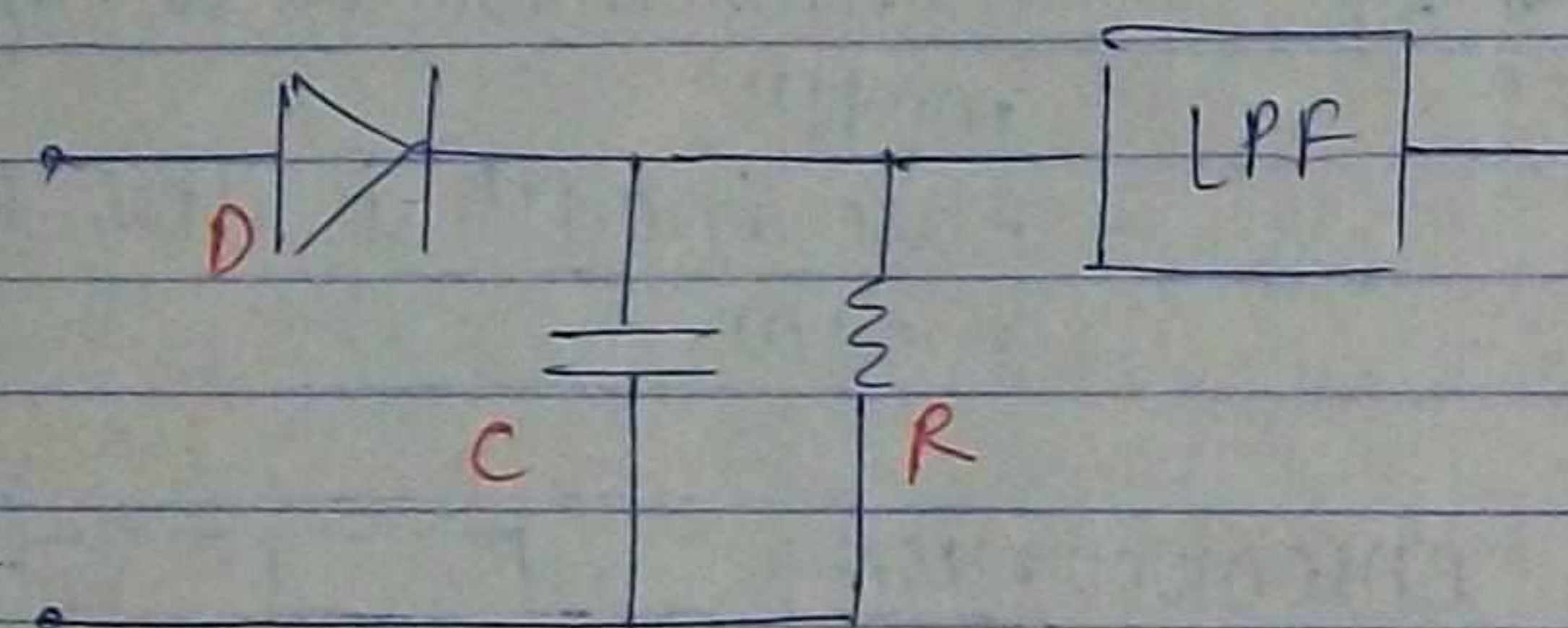
HIGH LEVEL TX. -



⇒ DEMODULATION OF A.M -

- ① Envelope detector
- ② Square law demodulator
- ③ Coherent detector / synchronous detector.

① ENVELOPE DETECTOR -



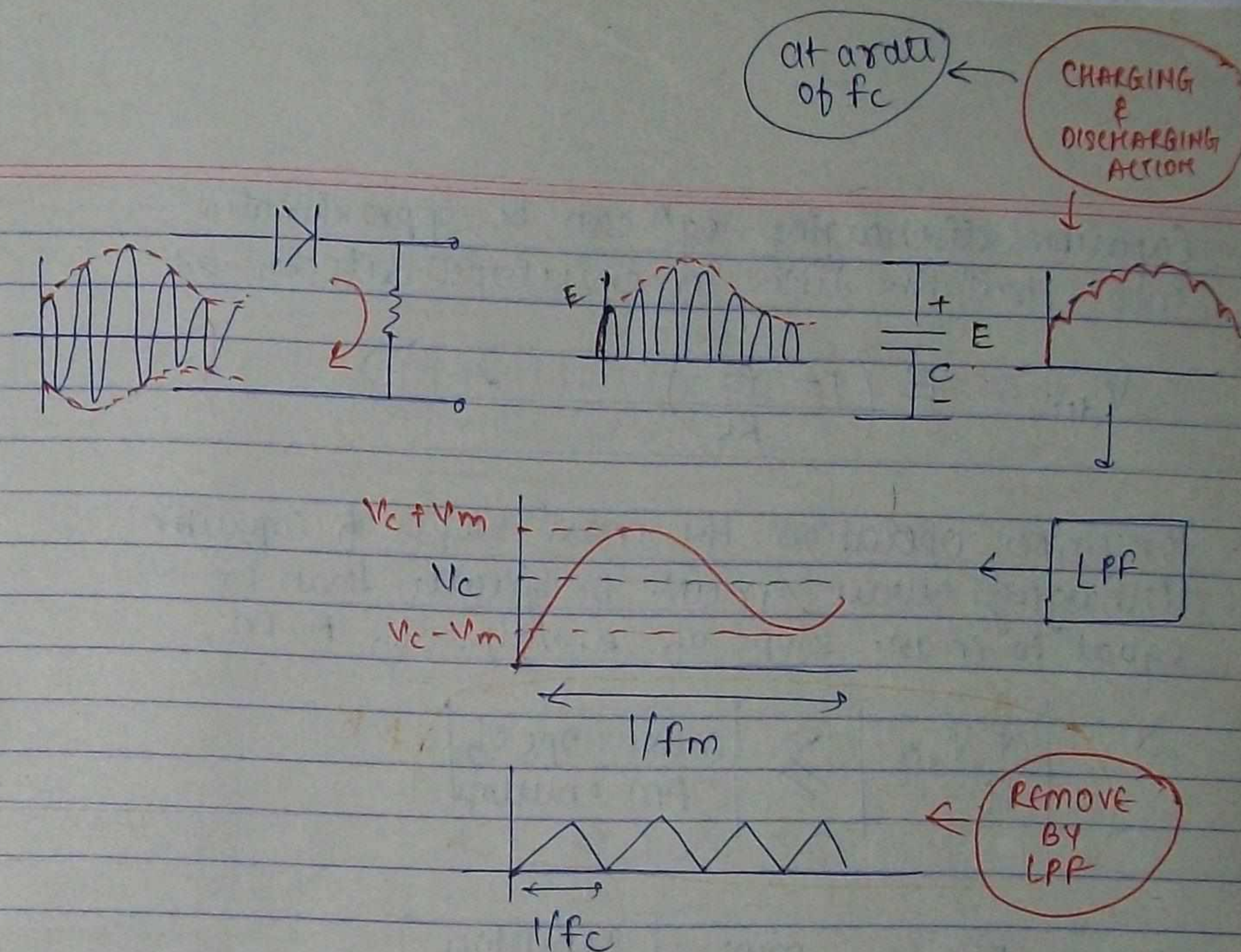
- Envelope detector will detect envelope of AM signal.
- for proper operation of envelope detector:

$$RC < \frac{1}{f_m}$$

$$RC > \frac{1}{f_c}$$

thus,

$$\frac{1}{f_c} < RC < \frac{1}{f_m}$$

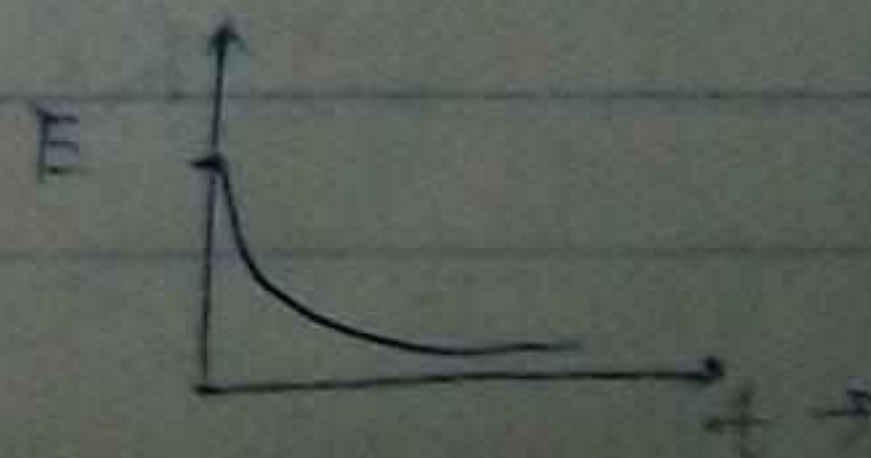


- In envelope detector when AM signal is applied then at the o/p it have dc signal and msg. signal will present, in this dc signal is due to transmitted carrier signal

DERIVATION FOR TIME CONSTANT ON ENVELOPE DETECTOR -

- let us consider capacitor charge to a peak value 'E'.
- when diode is not conducting capacitor will discharge by:

$$V_{015} = E \cdot e^{-t/RC}$$



- Capacitor discharging eqⁿ can be approximated into straight line in envelope detector as

$$V_{dis} \approx E \left(1 - \frac{t}{RC}\right) \quad \text{--- (1)}$$

- for proper operation the max. slope of capacitor discharging curve must be greater than or equal to max. slope of envelope of A.M.

$$\left| \frac{dV_{dis}}{dt} \right| \geq \left| \text{max. slope of AM envelope} \right|$$

$$\frac{E}{RC} \geq \text{max. } \frac{d}{dt} (\text{envelope})$$

$$\text{envelope} = V_c (1 + m \cos 2\pi f_m t)$$

$$\frac{E}{RC} \geq \text{max. } \frac{d}{dt} (1 + m \cos 2\pi f_m t)$$

$$\frac{E}{RC} \geq V_c \cdot m \cdot 2\pi f_m \sin 2\pi f_m t$$

$$\frac{V_c (1 + m \cos 2\pi f_m t)}{RC} \geq V_c m \cdot 2\pi f_m \cdot \sin 2\pi f_m t$$

$$\frac{1 + m \cos 2\pi f_m t}{m \cdot \omega_m \sin 2\pi f_m t} \geq RC$$

$$RC \leq \frac{1 + m \cos 2\pi f_m t}{m \omega_m \sin 2\pi f_m t}$$

for worst case

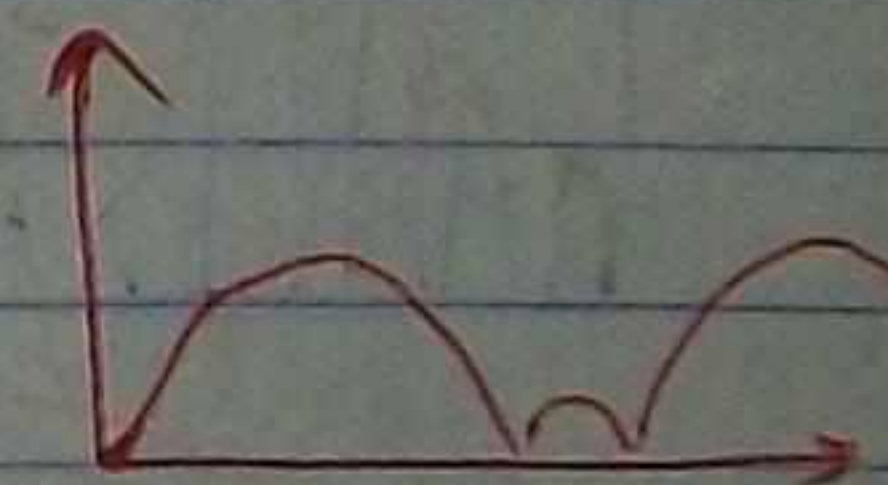
$$\begin{aligned} \cos 2\pi f_m t &= -m \\ \sin 2\pi f_m t &= \sqrt{1 - m^2} \end{aligned}$$

$$RC \leq \frac{1 - m^2}{m \omega_m \sqrt{1 - m^2}}$$

$$RC \leq \frac{\sqrt{1 - m^2}}{m \omega_m}$$

DISTORTION IN ENVELOPE DETECTOR -

- ① Negative peak clipping
- ② Diagonal clipping.



① NEGATIVE PEAK CLIPPING

- Negative peak clipping occurs in envelope detector when DC impedance of circuit is greater than AC impedance of circuit.

$$m_d = \text{modulation index of detector} = \frac{I_{ac}}{I_{dc}} \quad \begin{matrix} \text{(due to mod)} \\ \text{(due to det)} \end{matrix}$$

$$I_{ac} = \frac{V_m}{Z_m}, \quad I_{dc} = \frac{V_c}{R_{dc}}$$

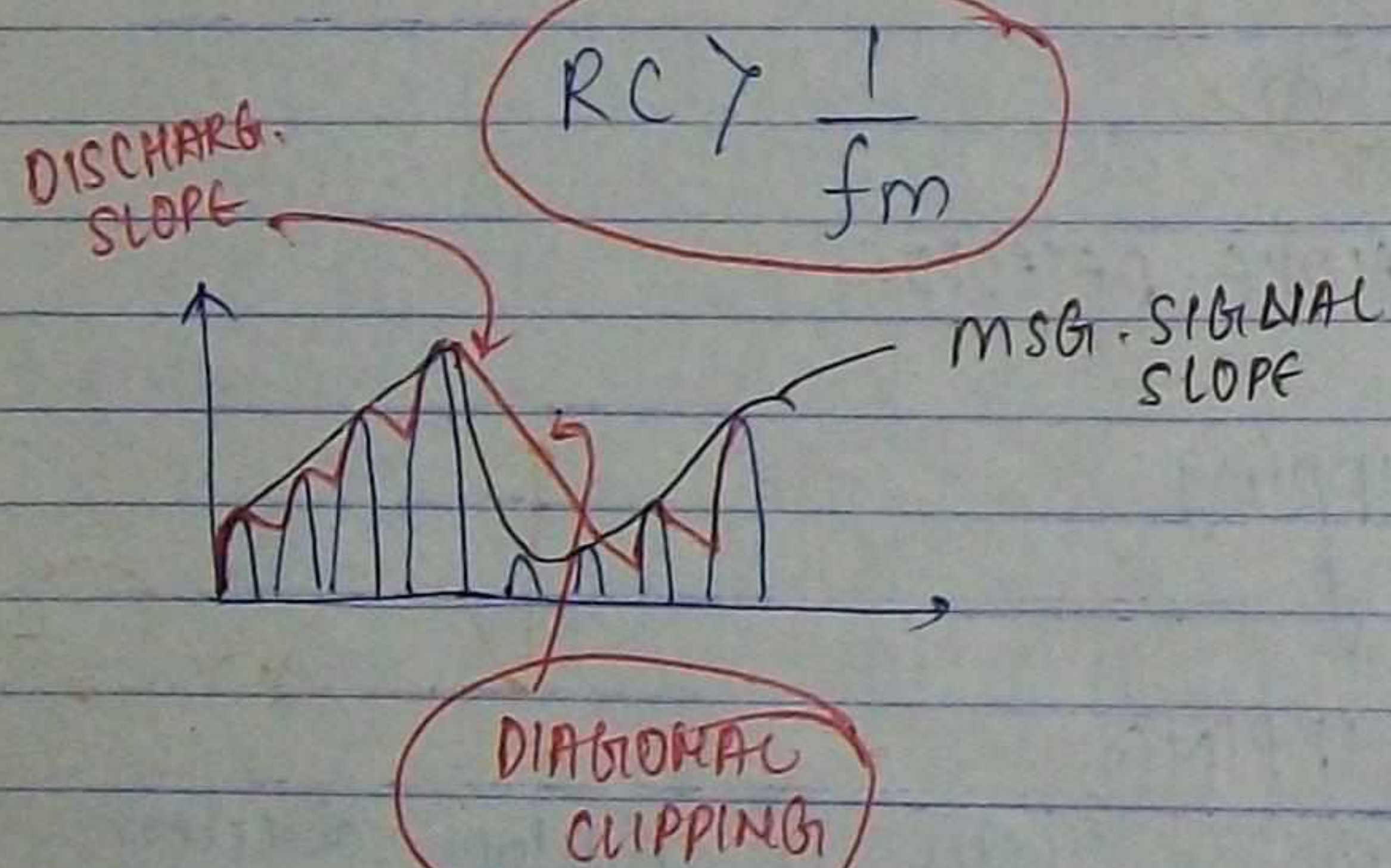
$$m_d = m \times \frac{R_{dc}}{Z_m}$$

To avoid -ve peak clipping 'm' is less than 0.7

$$m < 0.7$$

① DIAGONAL CLIPPING -

- Diagonal clipping will occur when RC time constant is greater than $1/f_m$

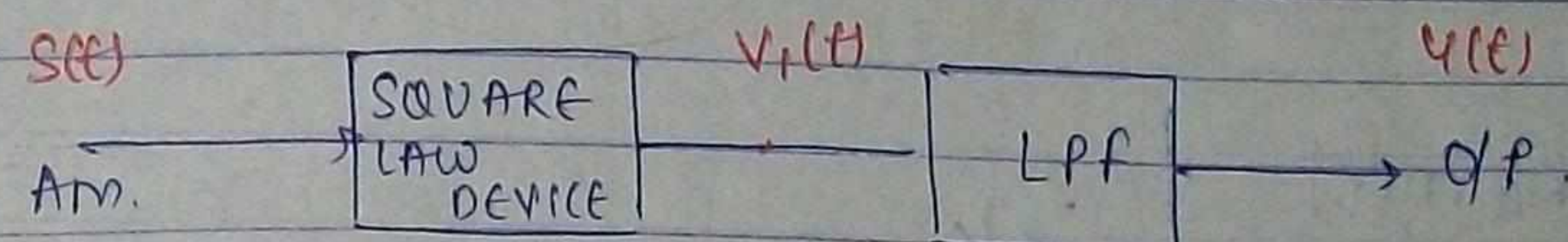


- To avoid diagonal clipping

$$RC < \frac{1}{f_m}$$

$$RC \leq \frac{\sqrt{1-m^2}}{m\omega_m}$$

② SQUARE LAW DEMODULATOR -



$$S(t) = V_c [1 + k_a m(t)] \cdot \cos \omega_c t$$

$$V_1(t) = a_1 S(t) + a_2 S^2(t)$$

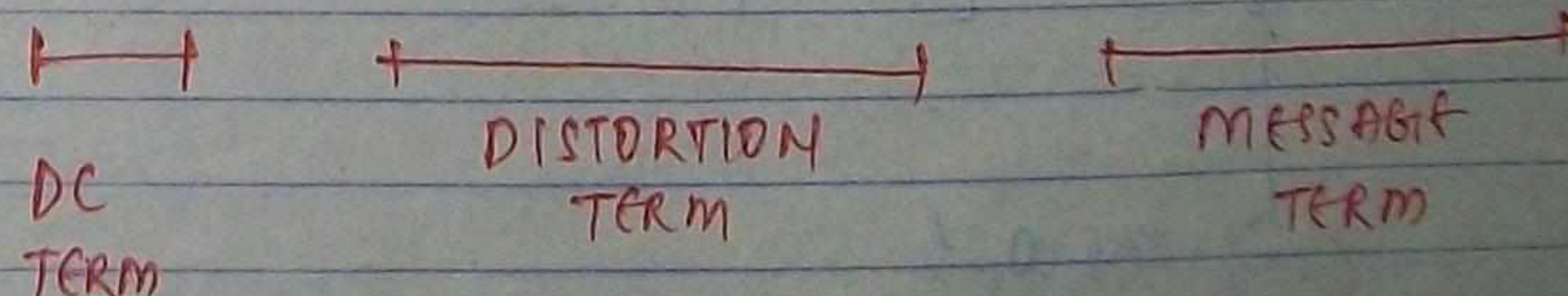
$$= a_1 S(t) + a_2 [V_c^2 (1 + k_a m(t))^2 \cos^2 \omega_c t]$$

$$= a_1 S(t) + \frac{a_2}{2} [V_c^2 (1 + k_a^2 m^2(t) + 2 k_a m(t)) (1 + \cos 2\omega_c t)]$$

$$V_1(t) = \cancel{a_1 S(t)} + \frac{a_2 V_c^2}{2} + \frac{a_2 V_c^2}{2} k_a^2 m^2(t) +$$

$$\frac{a_2 V_c^2}{2} k_a m(t) + \cancel{\frac{a_2 V_c^2}{2} (1 + k_a^2 m^2(t) + 2 k_a m(t)) \cos 2\omega_c t}$$

$$y(t) = \frac{a_2 V_c^2}{2} + \frac{a_2 V_c^2}{2} k_a^2 m^2(t) + \frac{a_2 V_c^2}{2} k_a m(t)$$



$$\left| \frac{\text{MESSAGE SIGNAL}}{\text{DISTORTION TERM}} \right| \geq 10.$$

Generally
To
avoid
distortion

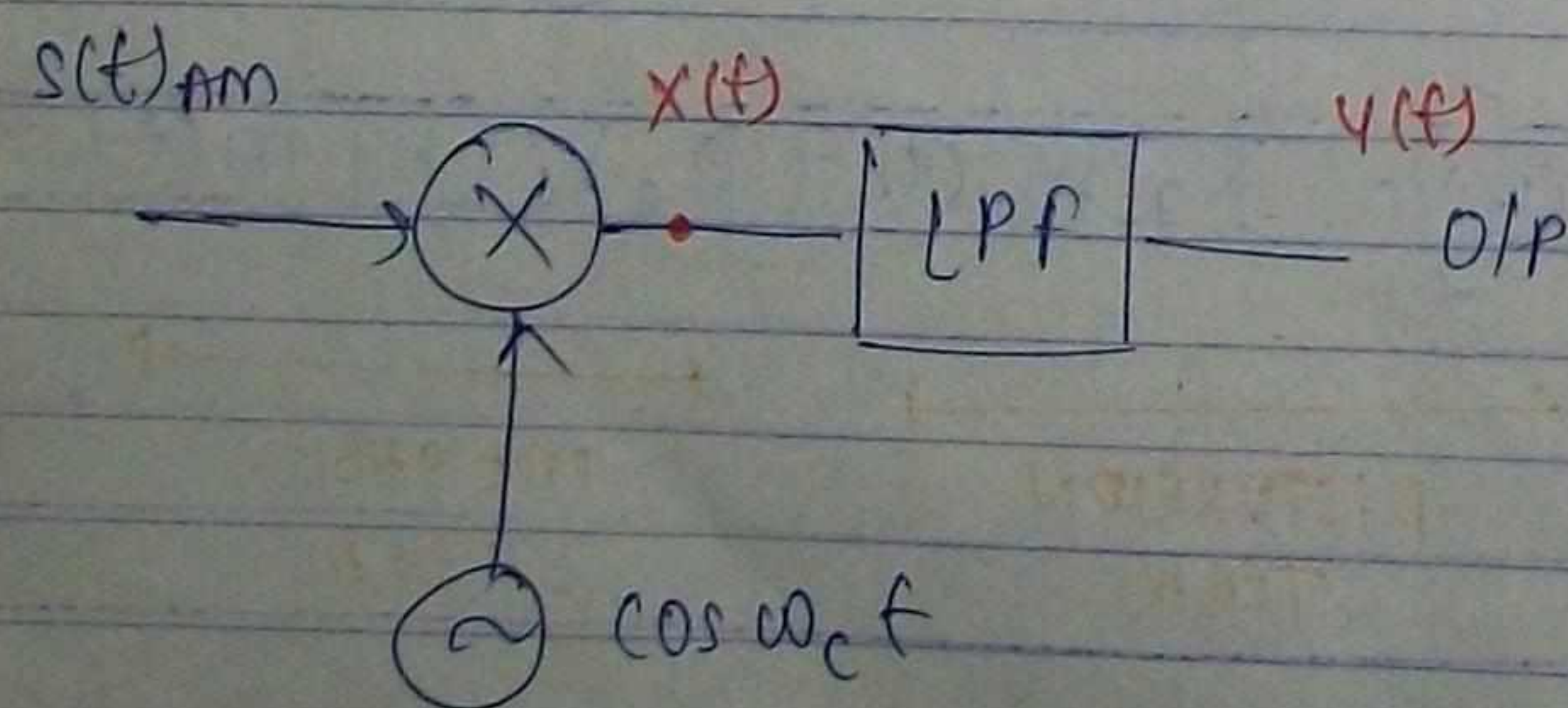
$$\left| \frac{a_2^0 V_c^2 K_a m(t)}{a_2 \frac{V_c^2}{2} K_a^2 m^2(t)} \right| \geq 10$$

$$\left| \frac{2}{K_a m(t)} \right| \geq 10$$

$$\frac{2}{|K_a m(t)|} \geq 10$$

- Square law demodulator is used when modulation index is very low ($m \leq 0.2$)

③ COHERENT DETECTOR -



$$s(t) = V_c [1 + K_a m(t)] \cos \omega_c t$$

$$x(t) = s(t) \cos \omega_c t$$

$$x(t) = V_c [1 + K_a m(t)] \cos^2 \omega_c t$$

$$x(t) = \frac{V_c (1 + K_a m(t))}{2} + \frac{V_c [1 + K_a m(t)] \cos 2\omega_c t}{2}$$

$$x(t) = + \frac{V_c}{2} + \frac{V_c K_a m(t)}{2} + \frac{V_c \cos 2\omega_c t}{2} + \frac{V_c \cos 2\omega_c t}{2} K_a m(t)$$

after LPF

$$y(t) = \frac{V_c}{2} + \frac{V_c K_a m(t)}{2}$$

$$O/P \propto m(t)$$

- Coherent detector can be used for any modulation index.

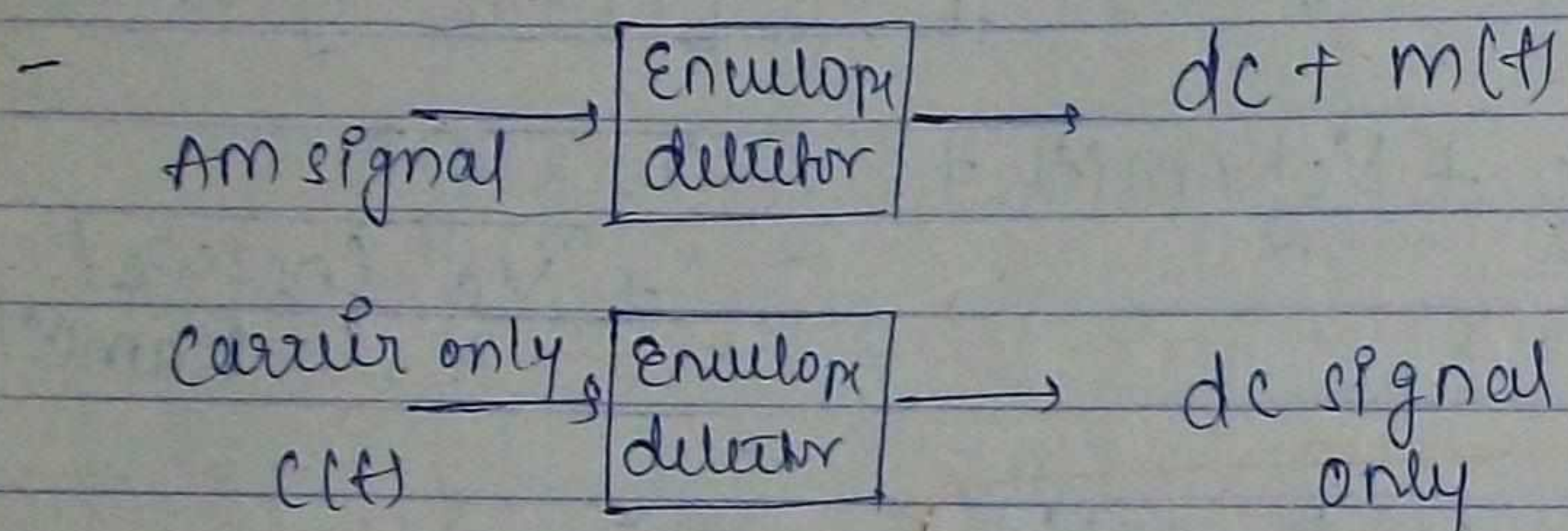
APPLICATION -

- AM is mostly used in Radio broadcasting.

- Advantage of AM is receiver complexity is less due to use of envelope detector.

DISADVANTAGE OF AM-

- most of power will present in carrier which have no. information.
- AM is mostly affected by noise.



DSB-SC

- In AM signal if carrier signal is eliminated then the resultant signal is known as DSBSC or DSB SIGNAL. (Linear system)

- In DSB system only side bands are transmitted due to elimination of carrier, most of power can be ^{saved} same. The percentage power saved in sinusoidal modulating signal is

$$\% \text{ Power saved} = \frac{2}{2+m^2} \times 100$$

$$m=1 \quad \% P_{\text{saved}} = 66.67 \%$$

$$m=0.5 \quad \% P_{\text{saved}} = 88.89 \%$$

EXPRESSION FOR DSBSC -

$$S(t)_{\text{Am}} = V_c [1 + K_a m(t)] \cos \omega_c t$$

$$= V_c \cos \omega_c t + K_a m(t) \cdot V_c \cos \omega_c t$$

$$= \underbrace{\quad}_{\text{CARRIER}} + \underbrace{\quad}_{\text{SIDE BAND}}$$

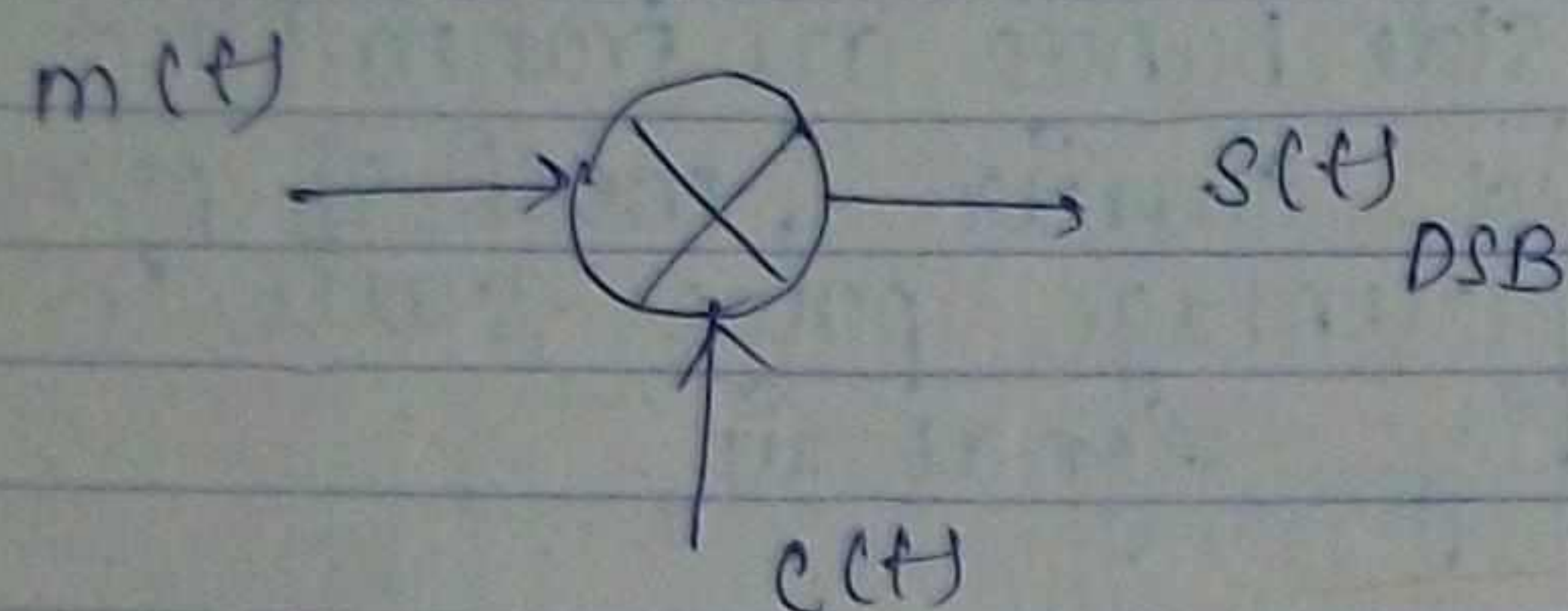
$\uparrow$
ELIMINATE

$$S(t)_{\text{DSB}} = m(t) \cdot c(t)$$

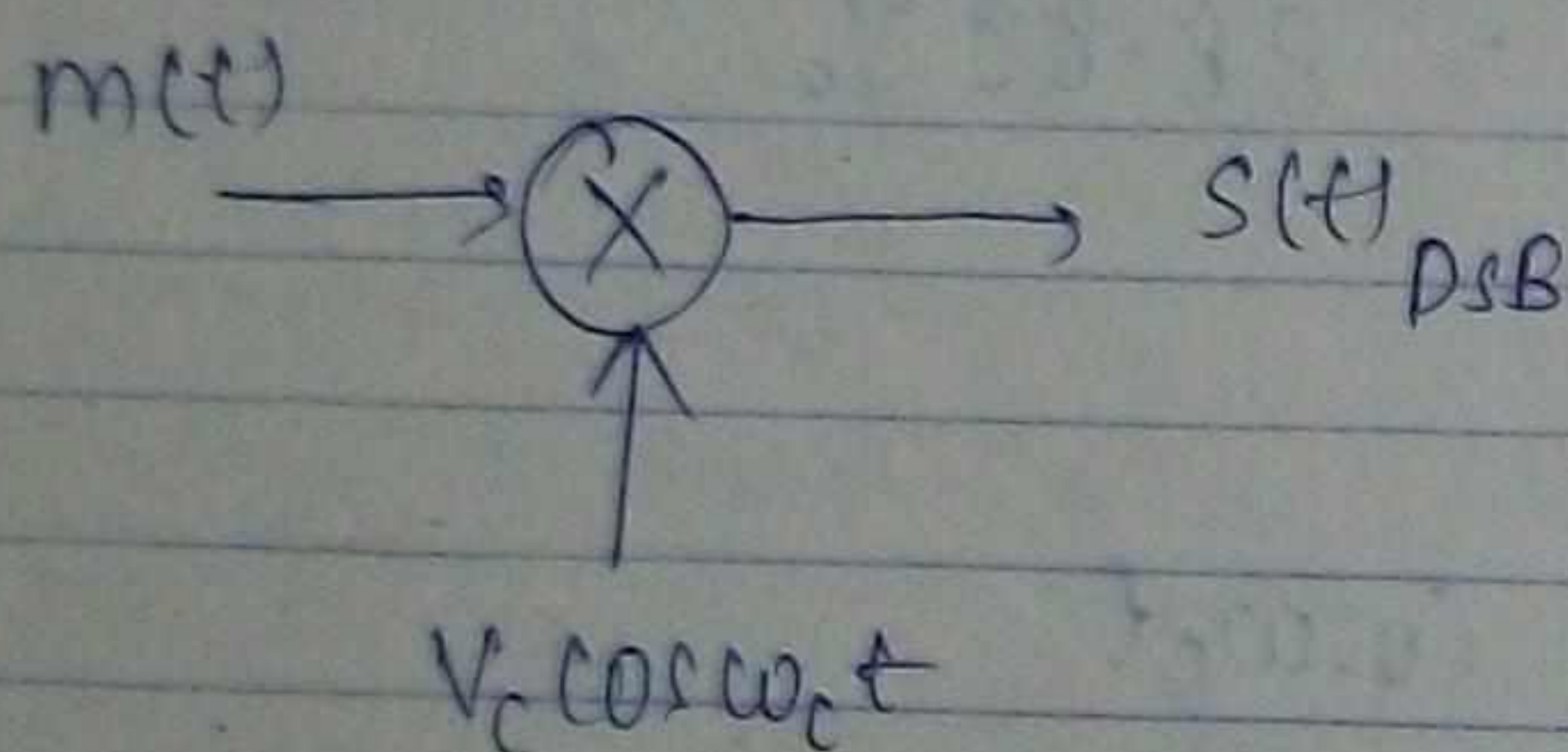
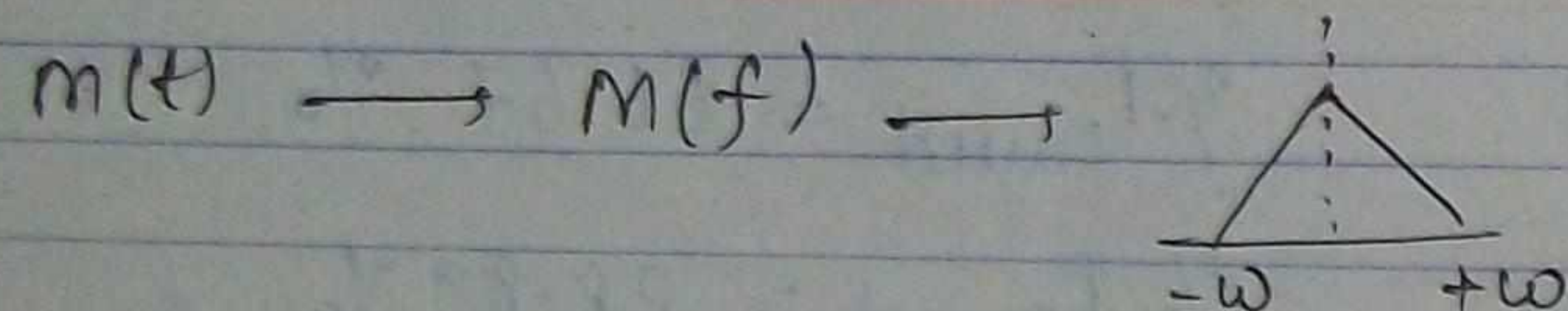
GENERAL
EXPRESSION

$$S(t)_{DSB} = m(t) \cdot c(t)$$

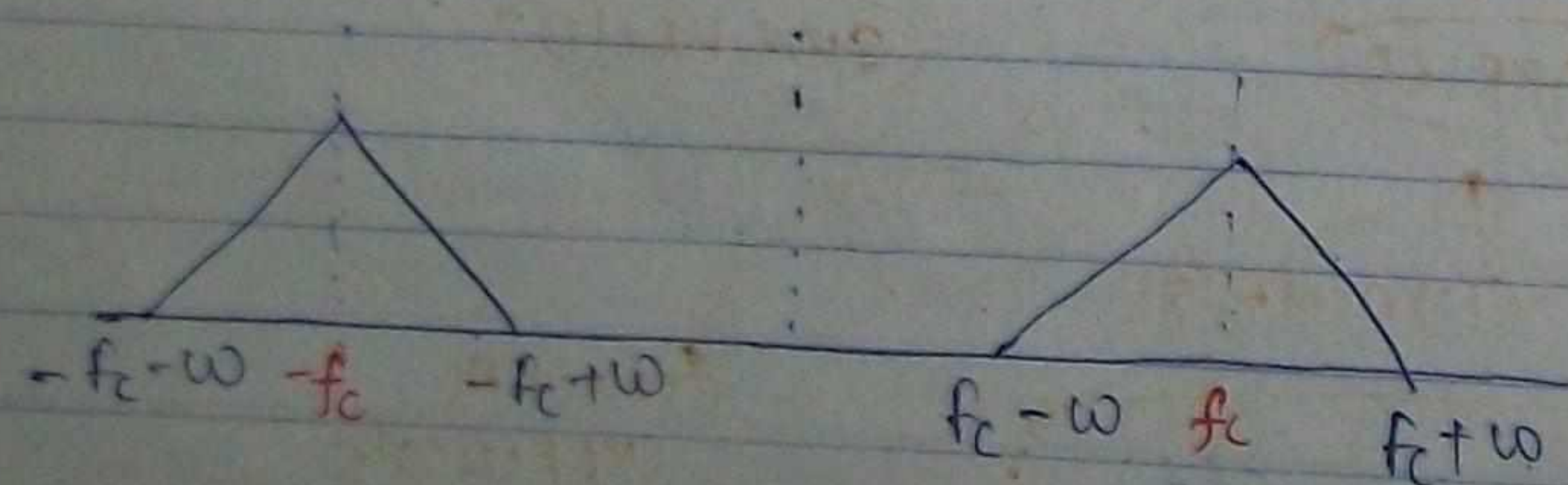
$$= V_c \cdot m(t) \cdot \cos \omega_c t$$



Let $m(t) \rightarrow$ arbitrary signal



$$S(f) = \frac{V_c}{2} [m(f-f_c) + m(f+f_c)]$$

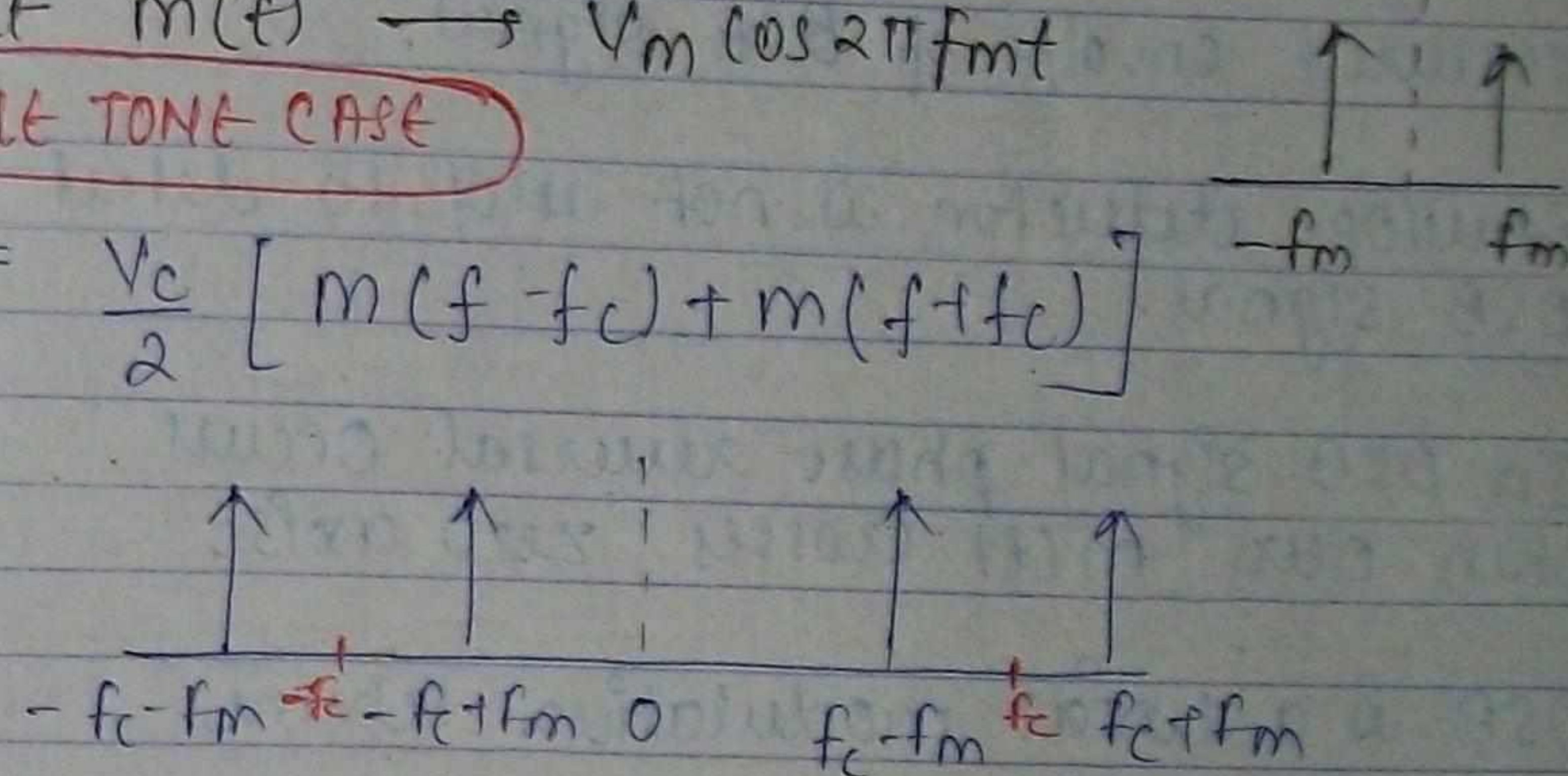


$$BW = 2w$$

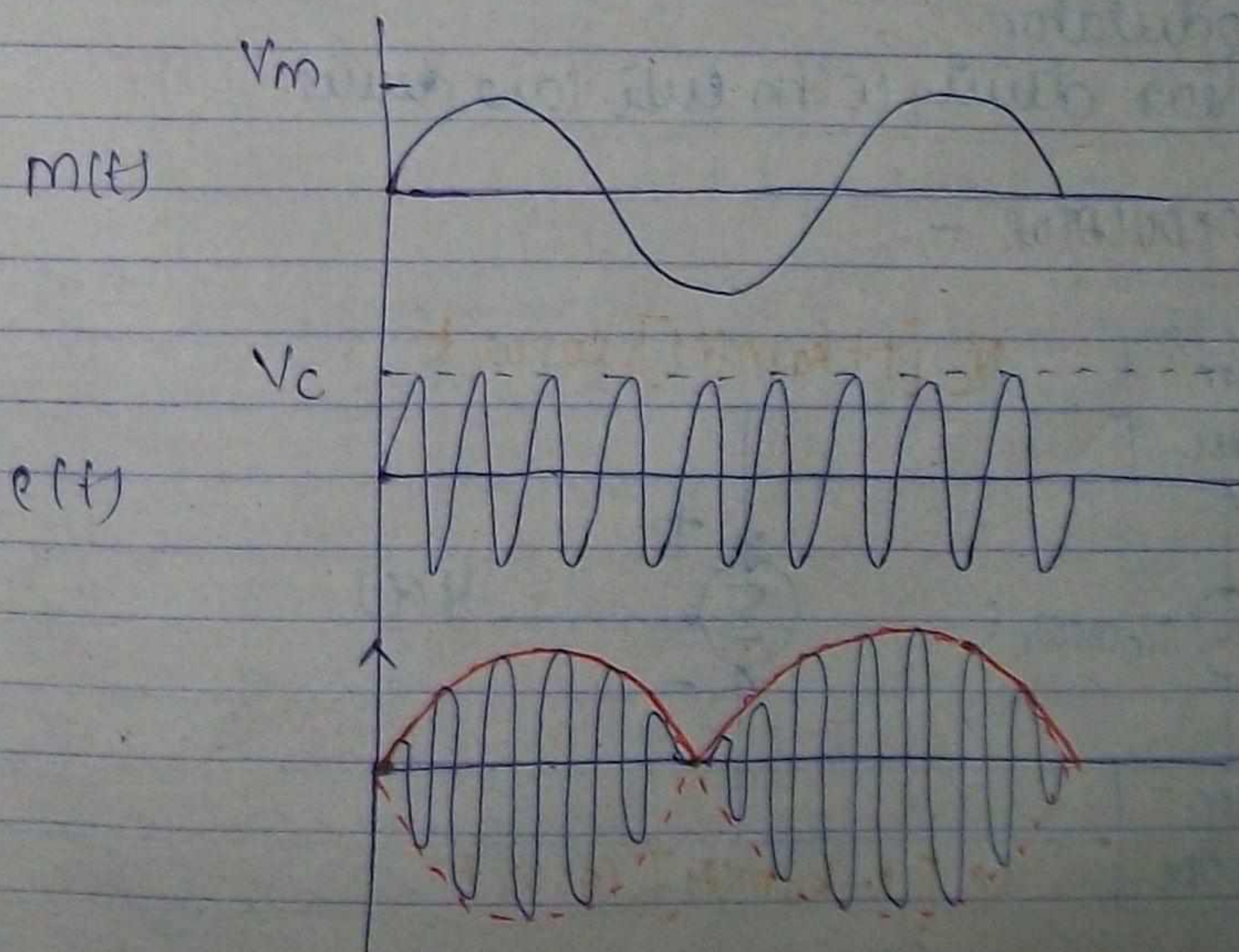
Let $m(t) \rightarrow V_m \cos 2\pi f_m t$

SINGLE TONE CASE

$$S(f) = \frac{V_c}{2} [m(f-f_c) + m(f+f_c)]$$



$$BW = 2f_m$$



- DSB $\rightarrow$ envelope $\rightarrow V_c |m(t)|$

Required envelope of DSB signal.

- Envelope detector is not used to detect DSB signal.

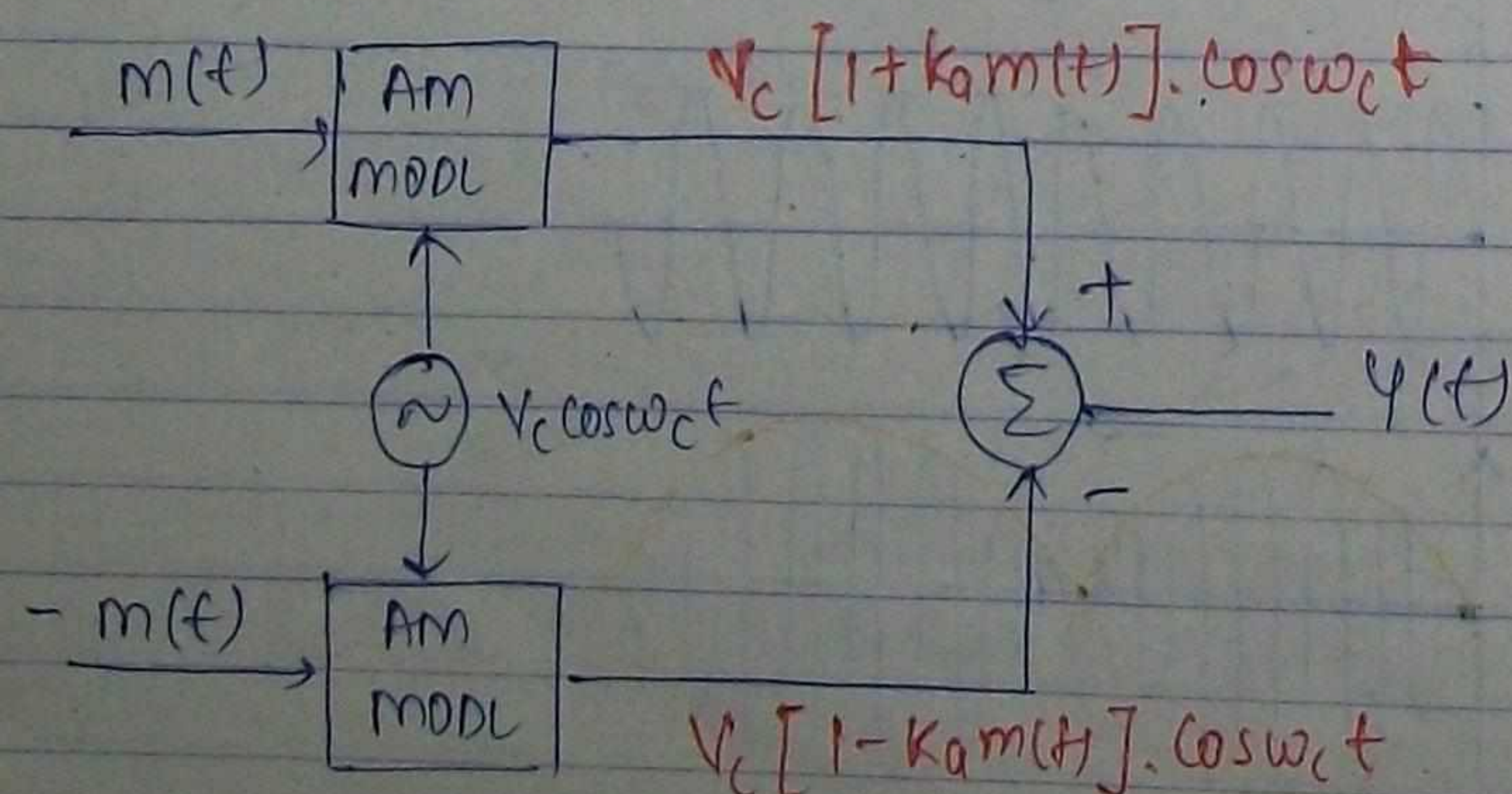
• In DSB signal phase reversal occurs when our $m(t)$ crosses zero axis.

- DSB is a linear modulation scheme

$\Rightarrow$ GENERATION OF DSB SIGNAL -

- ① Balanced modulator,
- ② Ring modulator.
- ③ Non-linear device with cube law device

① BALANCED MODULATOR -



$$y(t) = V_c \cos \omega_c t + K_a m(t) V_c \cos \omega_c t - V_c \cos \omega_c t + K_a m(t) V_c \cos \omega_c t$$

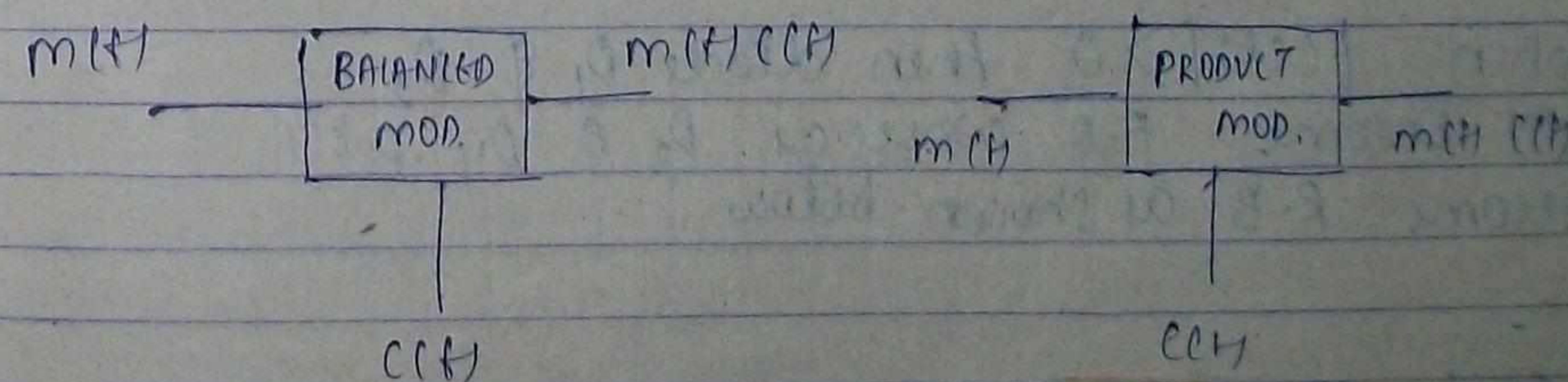
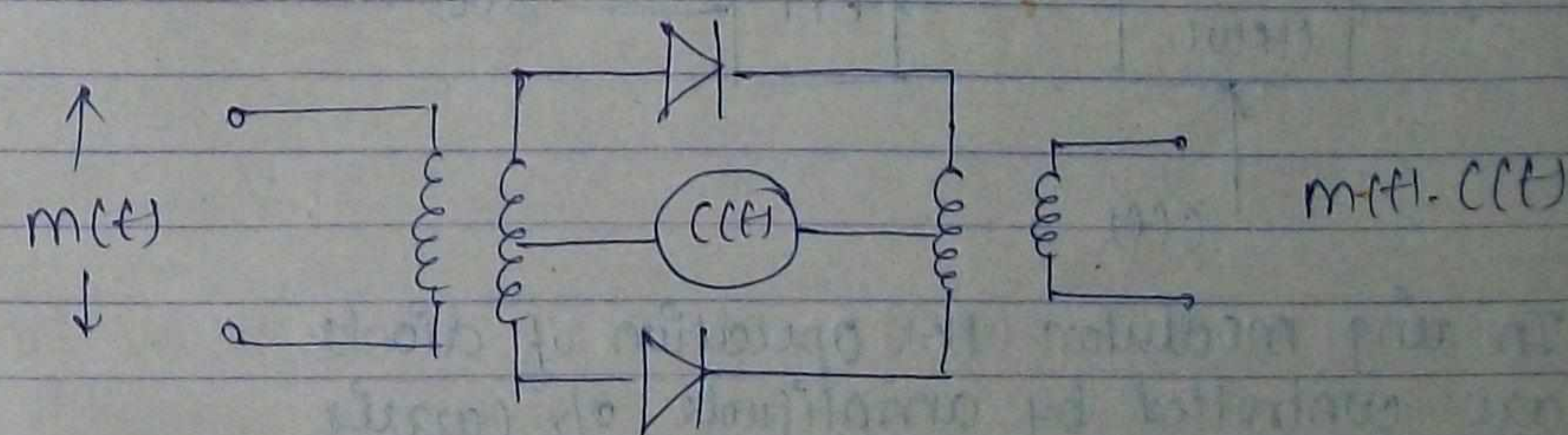
$$y(t) = 2 K_a m(t) V_c \cos \omega_c t$$

When $K_a = 1/2$

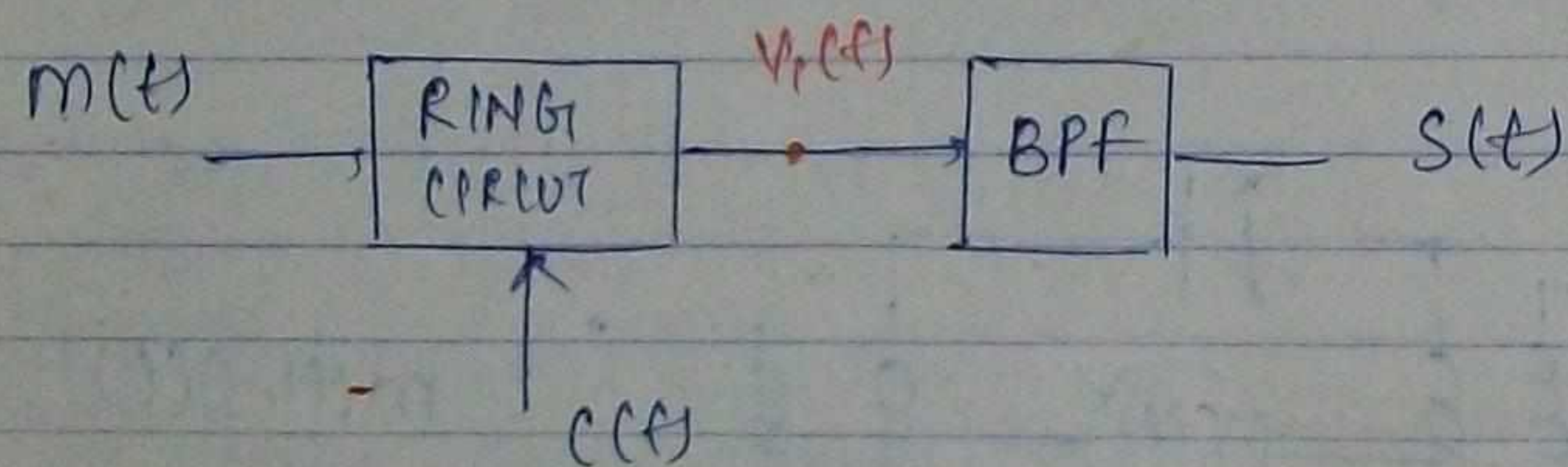
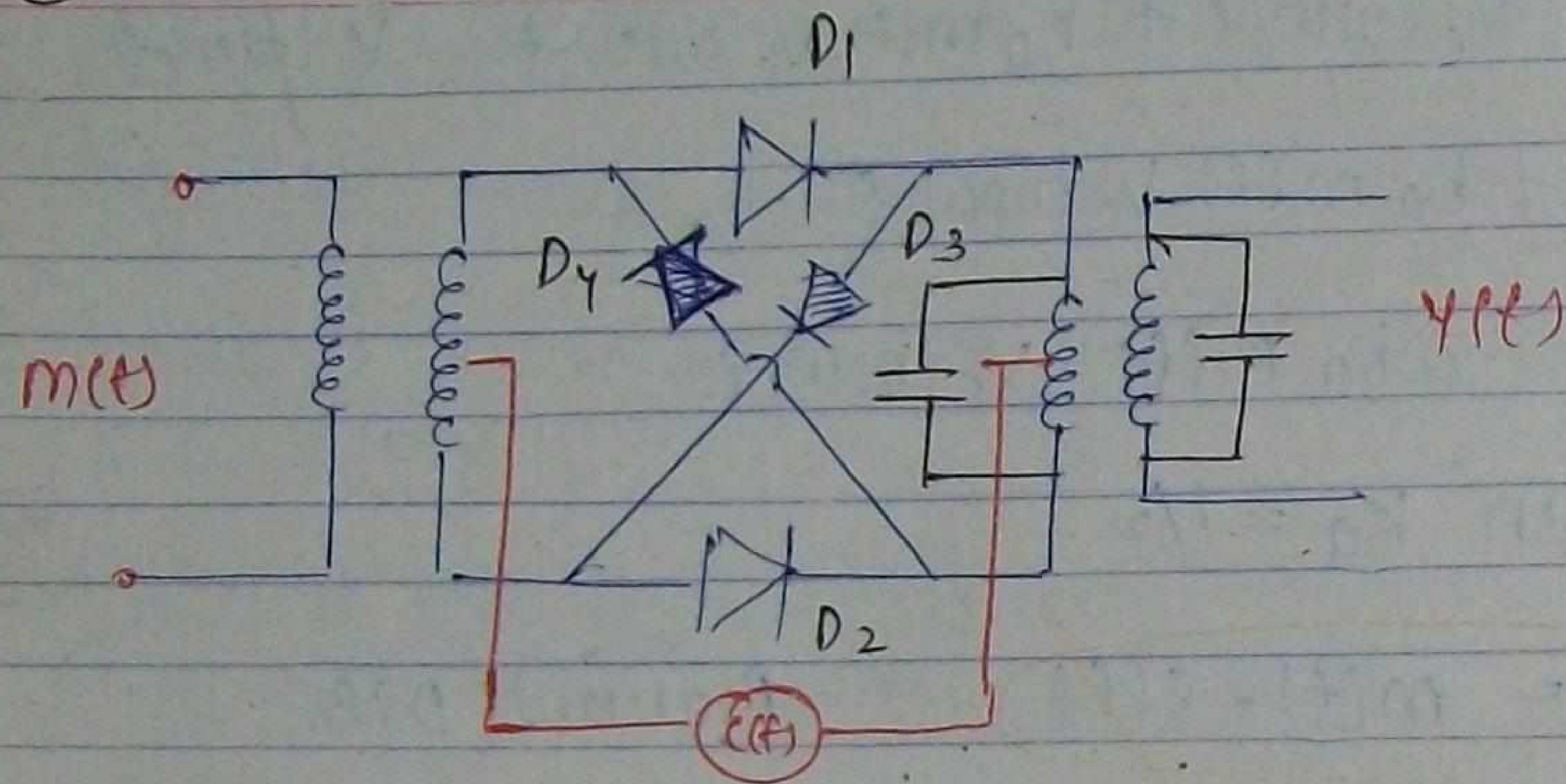
$$y(t) = m(t) \cdot c(t)$$

Required DSB.

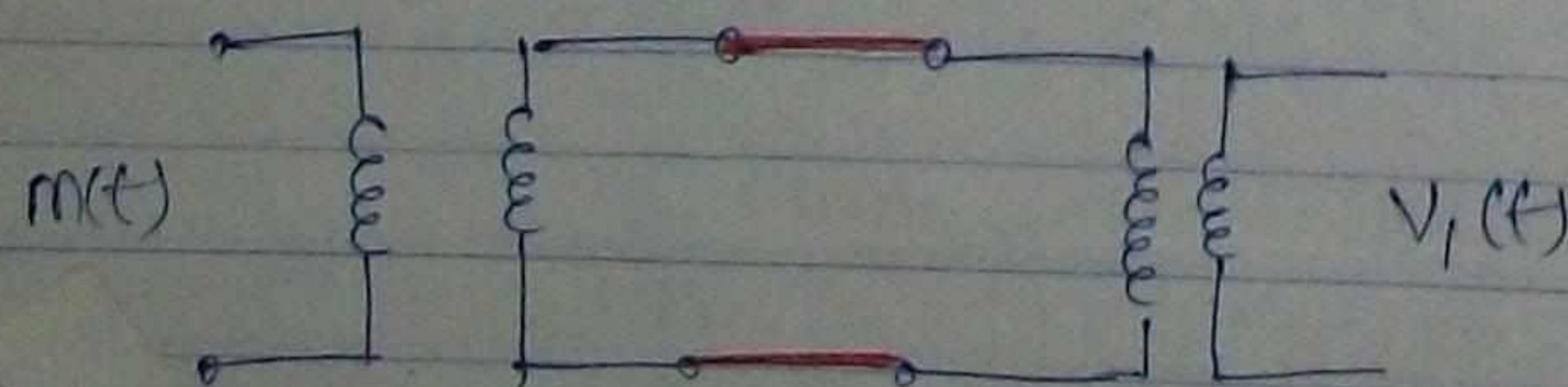
EQUIVALENT
CIRCUIT -



② RING MODULATOR -

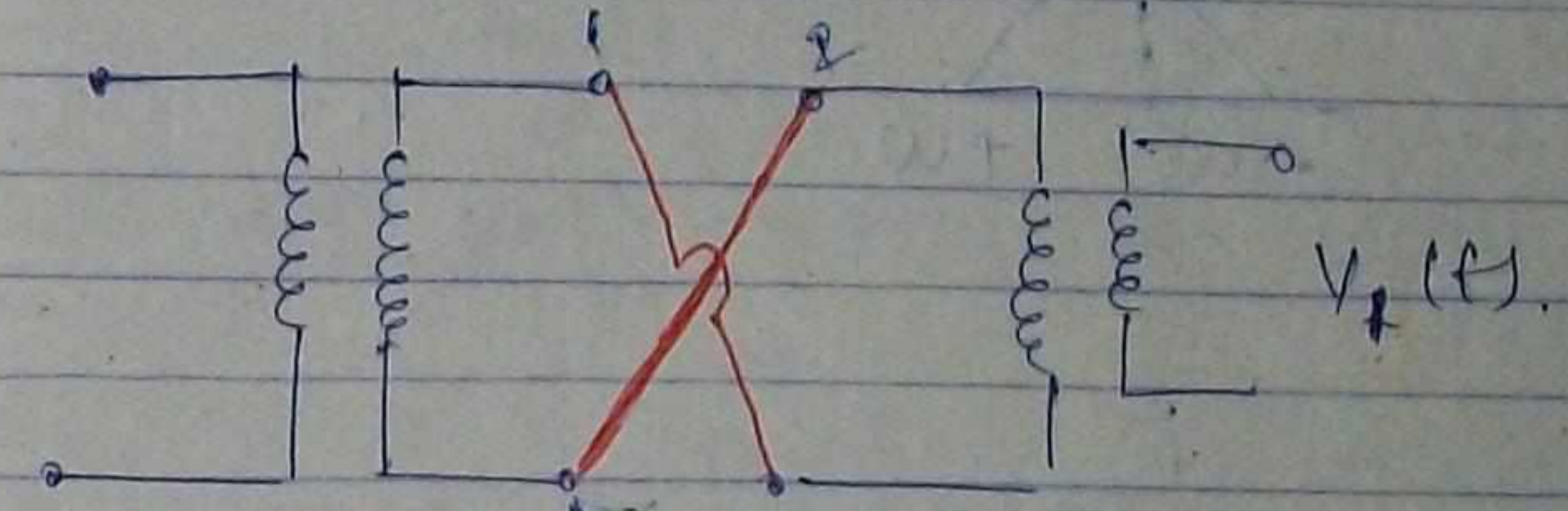


- In ring modulator the operation of diode are controlled by amplitude of carrier.
- when $|c(t)| > 0$ then diode D_1 & D_2 will become F.B whereas D_3 & D_4 will become R.B as shown below



$$\therefore V_1(t) = m(t), \text{ when } c(t) > 0.$$

- During -ve half cycle of carrier D_1 and D_2 are R.B whereas D_3 and D_4 are F.B as shown below.

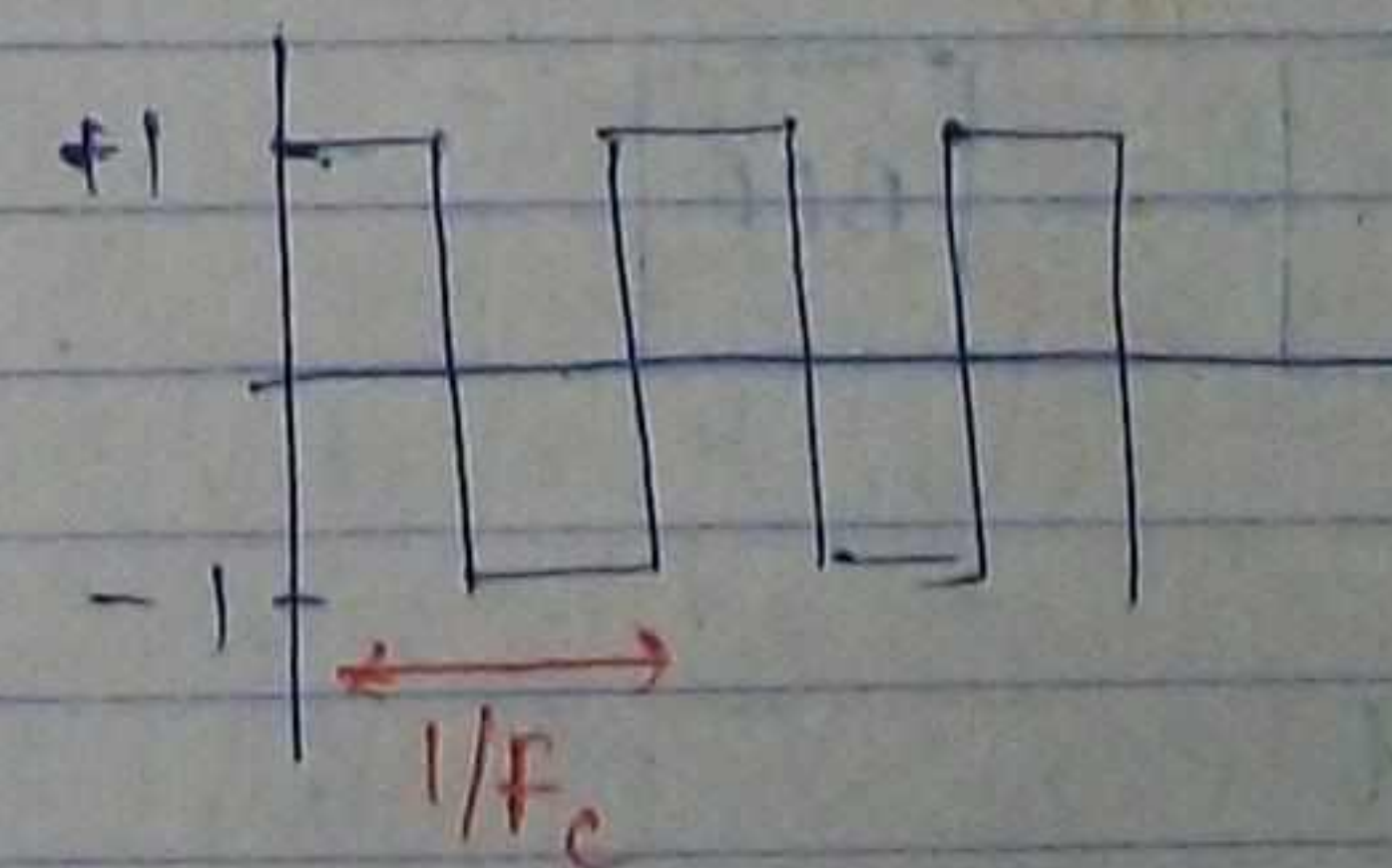


$$\therefore V_1(t) = -m(t), \text{ when } c(t) < 0$$

$$\therefore V_1(t) = m(t) \cdot X(t)$$

$$X(t) = \begin{cases} 1 & c(t) > 0 \\ -1 & c(t) < 0 \end{cases}$$

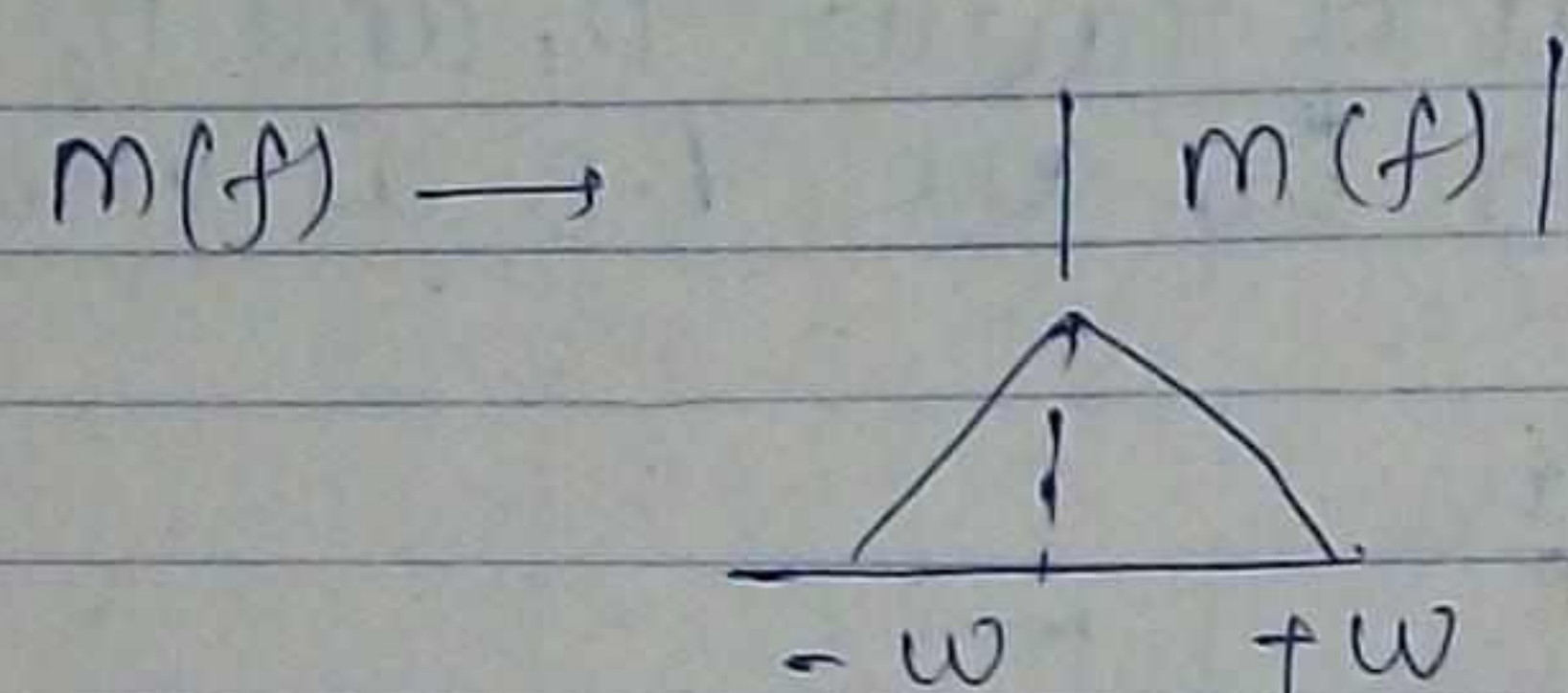
The operation of ring circuit can be represented with a switching function $X(t)$ as shown below



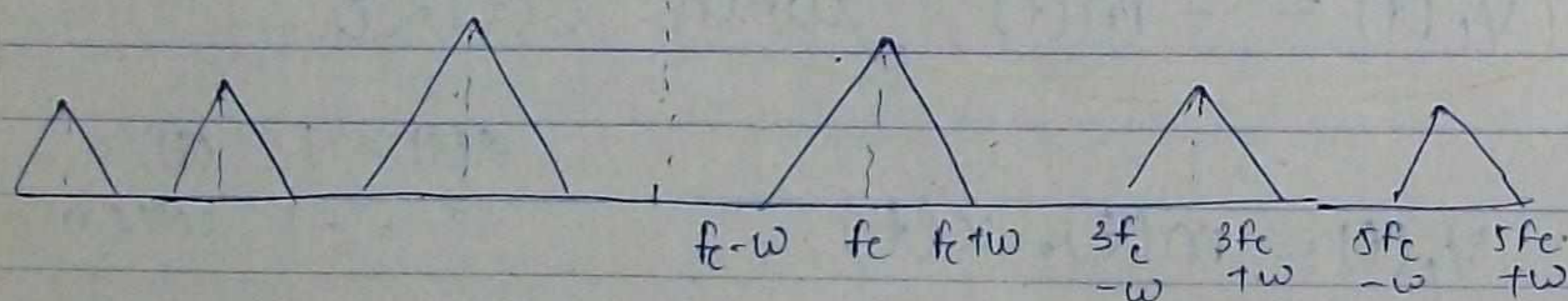
$$\therefore V_1(t) = m(t) \cdot X(t)$$

$$= m(t) \cdot \left[\frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos 2\pi(2n-1)f_c t \right]$$

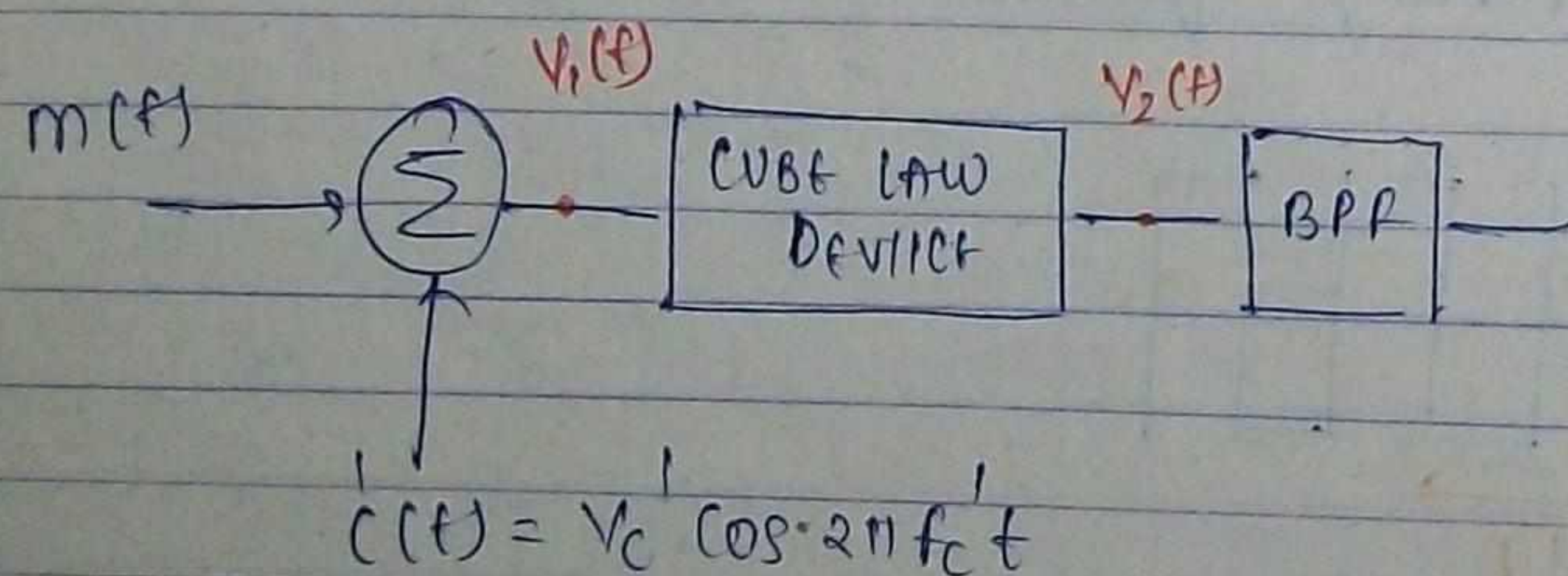
$$V_1(t) = m(t) \left[\frac{2}{\pi} \cos \omega_c t - \frac{2}{3\pi} \cos 3\omega_c t + \frac{2}{5\pi} \cos 5\omega_c t \right]$$



$|V_1(f)|$



③ NON-LINEAR (CUBE LAW DEVICE)



$$V_1(t) = m(t) + V_c' \cos 2\pi f_c' t$$

$$V_2(t) = a_0 V_1(t) + a_1 V_1^3(t)$$

$$V_2(t) = a_0 V_1(t) + a_1 [m^3(t) + (V_c' \cos 2\pi f_c' t)^3 + 3m^2(t) V_c' \cos 2\pi f_c' t + 3m(t) (V_c' \cos 2\pi f_c' t)^2]$$

$$V_2(t) = 3m(t) \frac{V_c'^2}{2} [1 + \cos 2\pi (2f_c' t)]$$

$$= 3m(t) \frac{V_c'^2}{2} + 3m(t) \frac{V_c'^2}{2} \cos 2\pi (2f_c' t)$$

$$= \frac{3}{2} m(t) V_c'^2 \cos 2\pi (2f_c' t)$$

$$V_2(f) = V_c \cdot m(f) \cdot \cos 2\pi f_c' t$$

$$f_c = 2f_c'$$

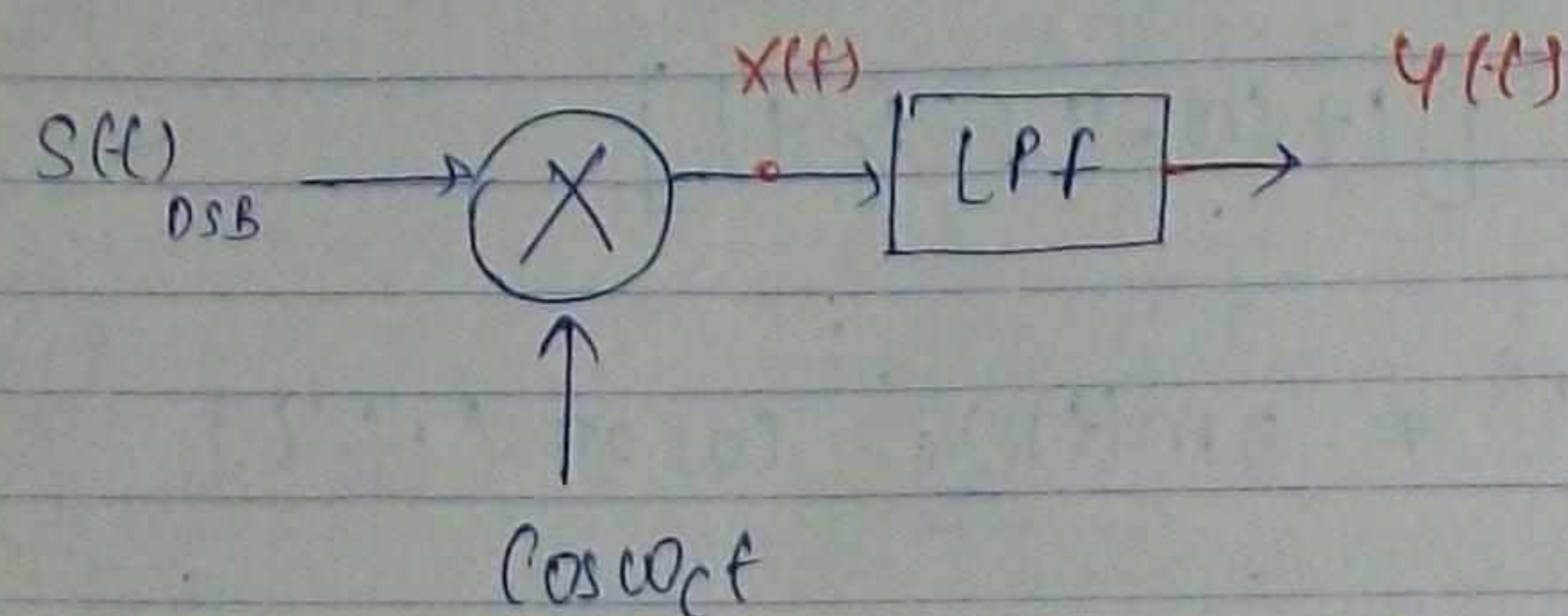
- In DSB generation using non-linear device having cube law device is used when a band pass filter have centre frequency (f_c)

$$f_c = 2f_c'$$

where f_c' is the generated carrier frequency,

⇒ DSB DEMODULATION -

COHERENT DETECTION -



$$x(t) = S(t) \cdot \cos \omega_c t$$

$$\begin{aligned} x(t) &= (V_c m(t) \cdot \cos \omega_c t) \cos \omega_c t \\ &= \frac{V_c m(t)}{2} + \frac{V_c m(t)}{2} \cos 2\omega_c t \end{aligned}$$

after LPF -

$$y(t) = \frac{V_c}{2} m(t)$$

$$\text{o/p} \propto m(t)$$

$$\propto m(t) \rightarrow P_m \text{ (power)}$$

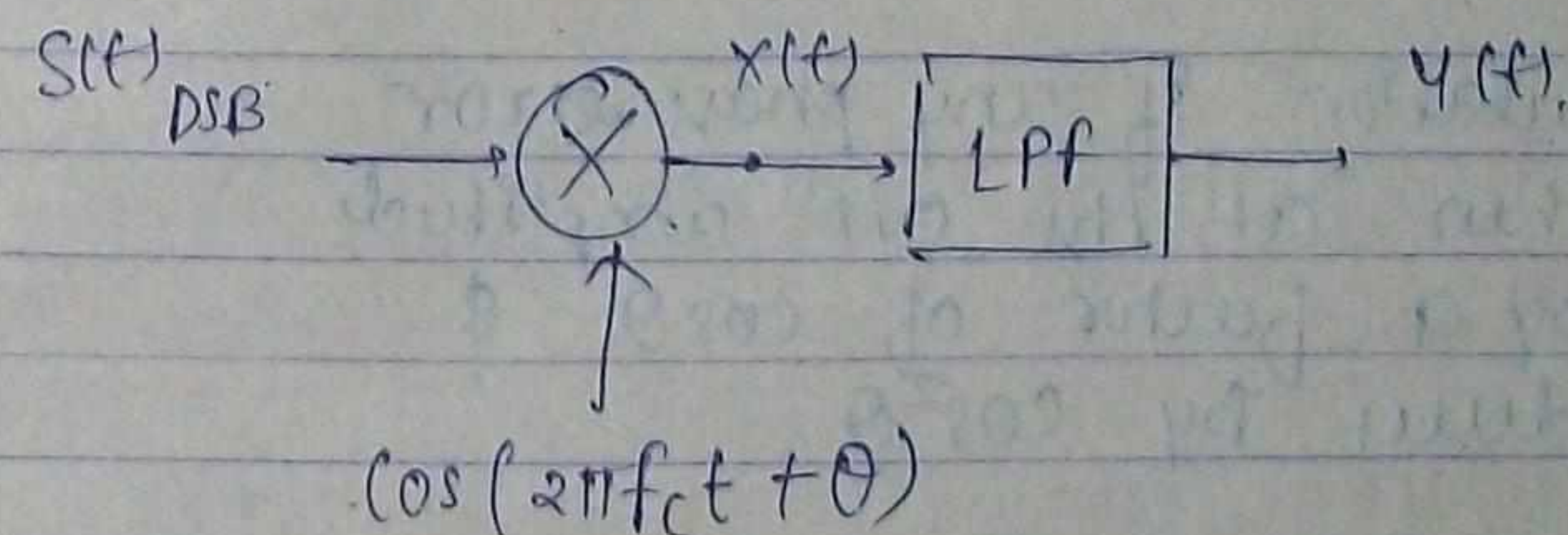
$$V_c \cdot m(t) \cos \omega_c t \rightarrow \frac{V_c^2}{2} P_m$$

$$\therefore \text{at o/p} \quad y(t) = \frac{V_c}{2} m(t)$$

$$\text{o/p power} = \frac{V_c^2}{4} P_m$$

- In DSB demodulation coherent carrier is required to provide message signal at the o/p.

PHASE DISCREPANCY ($\Delta f = 0, \theta \neq 0$).
(ERROR)



$$\begin{aligned} x(t) &= (V_c m(t) \cdot \cos 2\pi f_c t) \cdot \cos(2\pi f_c t + \theta) \\ &= \frac{V_c}{2} m(t) [\cos(2\pi (2f_c) t + \theta) + \cos \theta] \end{aligned}$$

after LPF

$$y(t) = \frac{V_c}{2} m(t) \cdot \cos \theta$$

$$\frac{1}{2} \rightarrow -3 \text{ dB}$$

$$\frac{2}{2} \rightarrow 3 \text{ dB}$$

θ	O/P SIGNAL AMP.	O/P SIGNAL POWER
$\theta = 0$	$\frac{V_c}{2} m(t)$	$\frac{V_c^2}{4} P_m$
$\theta = \pm \frac{\pi}{4}$	$\frac{1}{\sqrt{2}} \cdot \frac{V_c}{2} m(t)$	$\frac{1}{2} \cdot \frac{V_c^2}{4} P_m$
$\theta = \pm \frac{\pi}{2}$	0	0

QUADRATURE
NULL EFFECT

★

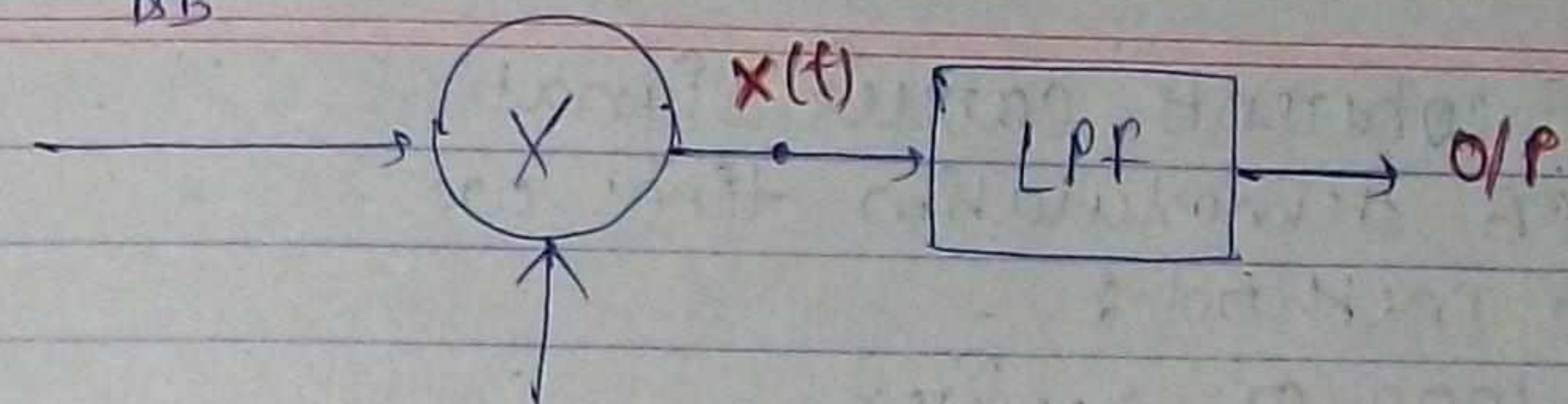
- In DSB - demodulation if any phase error θ is present, then at the o/p amplitude will reduce by a factor of $\cos \theta$ & power will reduce by $\cos^2 \theta$.

- If θ varies randomly then o/p amplitude also varies randomly resulting FADING of signal.

$$\therefore \text{O/P power} = \frac{V_c^2}{4} P_m \cos^2 \theta$$

FREQUENCY DESCREPENCY
(ERROR)

$S(f)_{\text{DSB}}$



$$\cos 2\pi(f_c + \Delta f)t$$

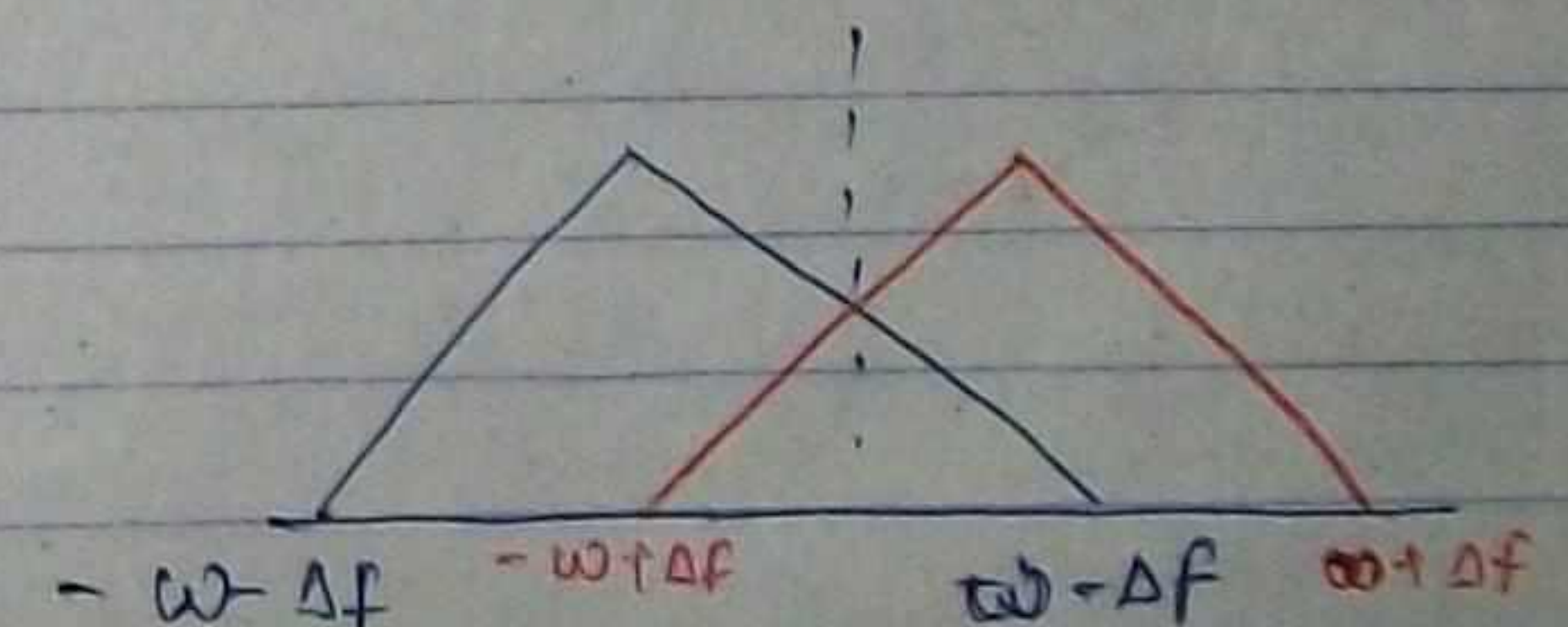
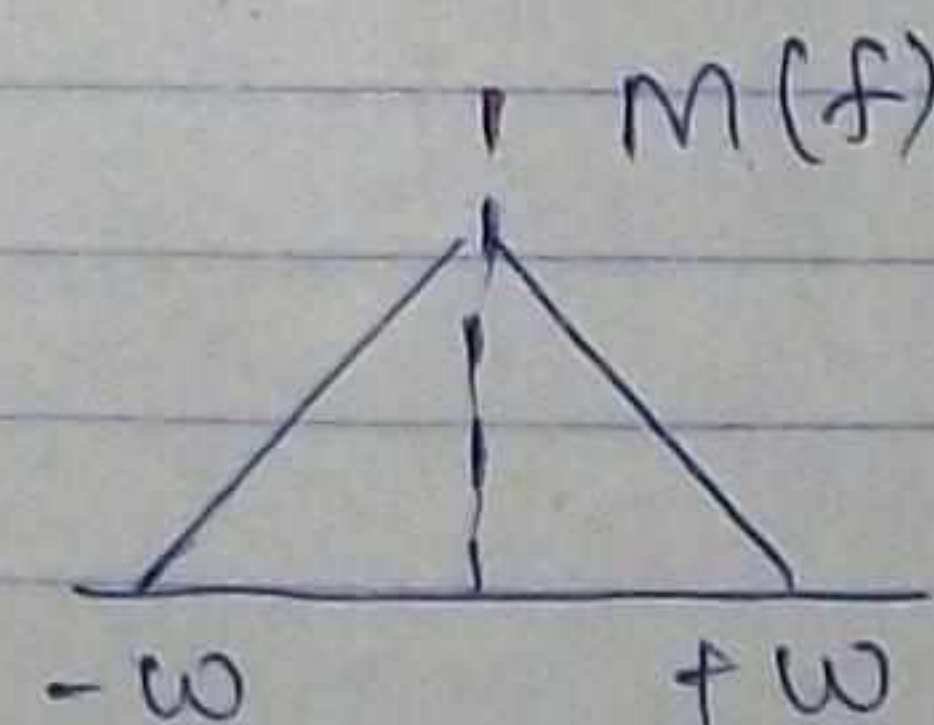
$$x(t) = V_c m(t) \cdot \cos 2\pi f_c t \cdot \cos 2\pi(f_c + \Delta f)t$$

$$x(t) = \frac{V_c}{2} m(t) \left[\cos 2\pi(2f_c + \Delta f)t + \cos 2\pi(\Delta f)t \right]$$

after LPF -

$$y(t) = \frac{V_c}{2} m(t) \cdot \cos 2\pi(\Delta f)t$$

$$y(f) = \frac{V_c}{4} [m(f - \Delta f) + m(f + \Delta f)]$$



- In DSB demodulation if any freq. error Δf is present then each demodulated freq. will change by $\pm \Delta f$.

To generate coherent carrier signal during DSB demodulation time we have two methods

- ① Costas's loop or receiver
- ② Square loop.

- main advantage of DSB is most of the transmitted power can be saved.

- Disadvantage of DSB is receiver complexity is more.

APPLICATION -

1. used in point to point communication.

SSB-SC

- In AM modulation is carrier and one of sideband is eliminated the resultant system is known as SSB.

ADVANTAGE -

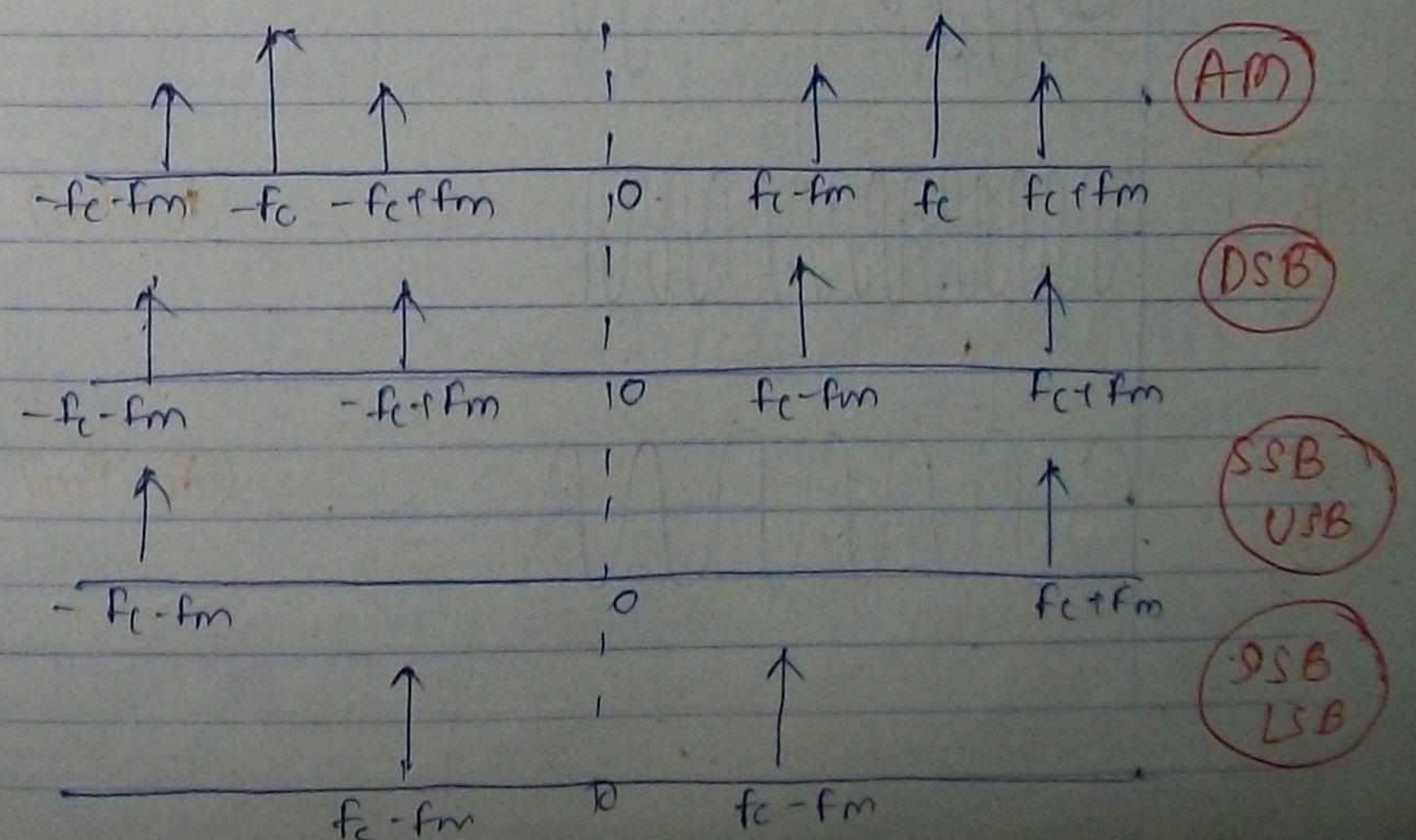
1. Most of power is saved
2. Bandwidth will decrease ✓
3. Less affected by noise.
4. No fading.

Q. which of the following is less affected by noise

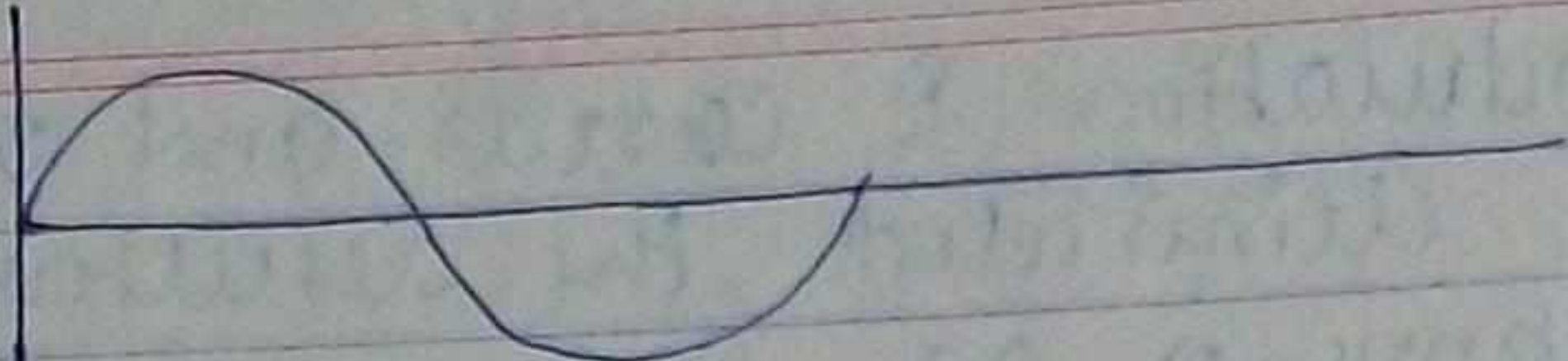
(A) AM (B) DSB ~~(C) SSB~~ (D) VSB.

(A) AM (B) DSB (C) SSB ~~(D) FM~~

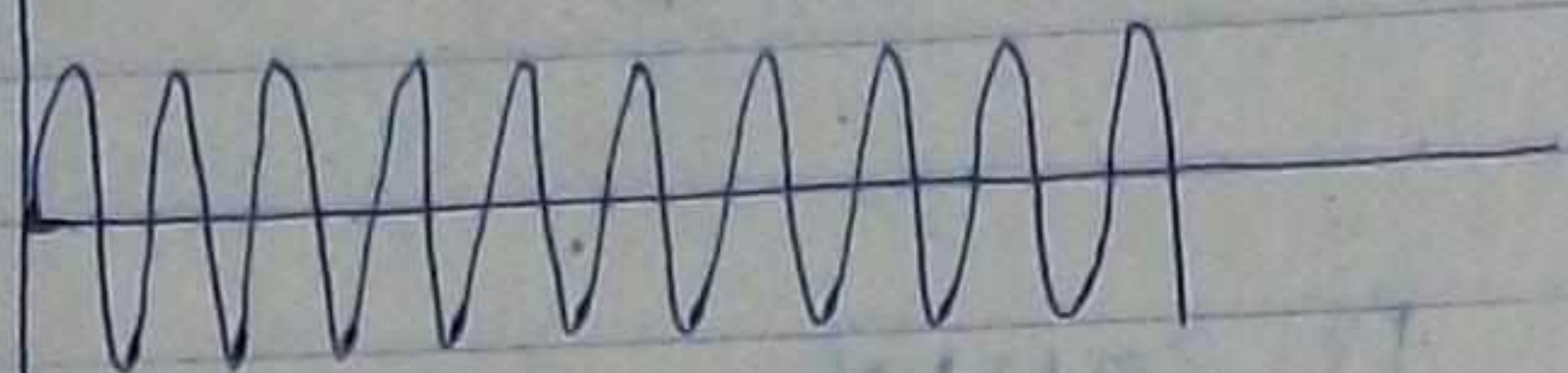
(A) DSB (B) SSB (C) FM ~~(D) PCM~~



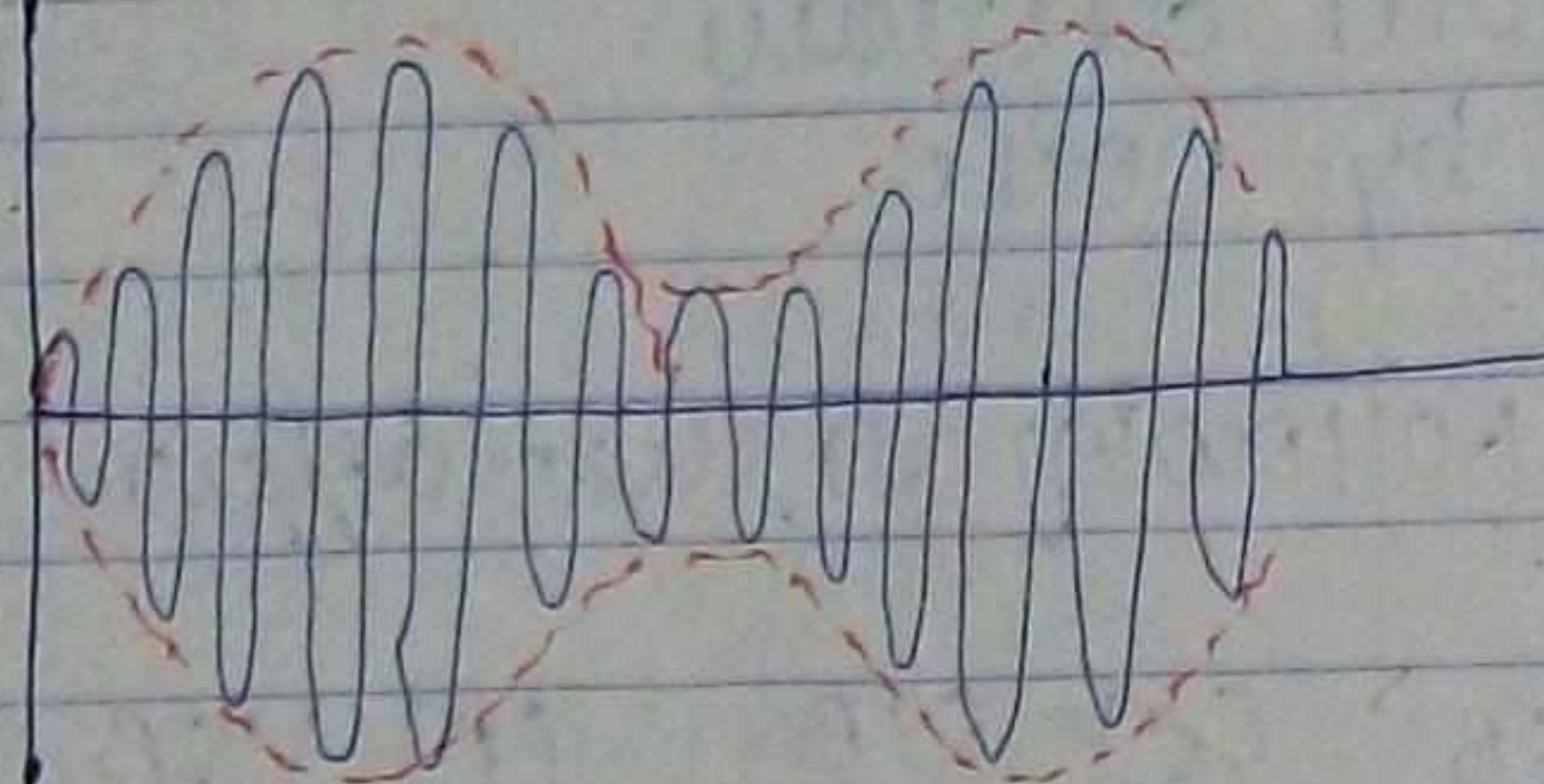
$m(t)$



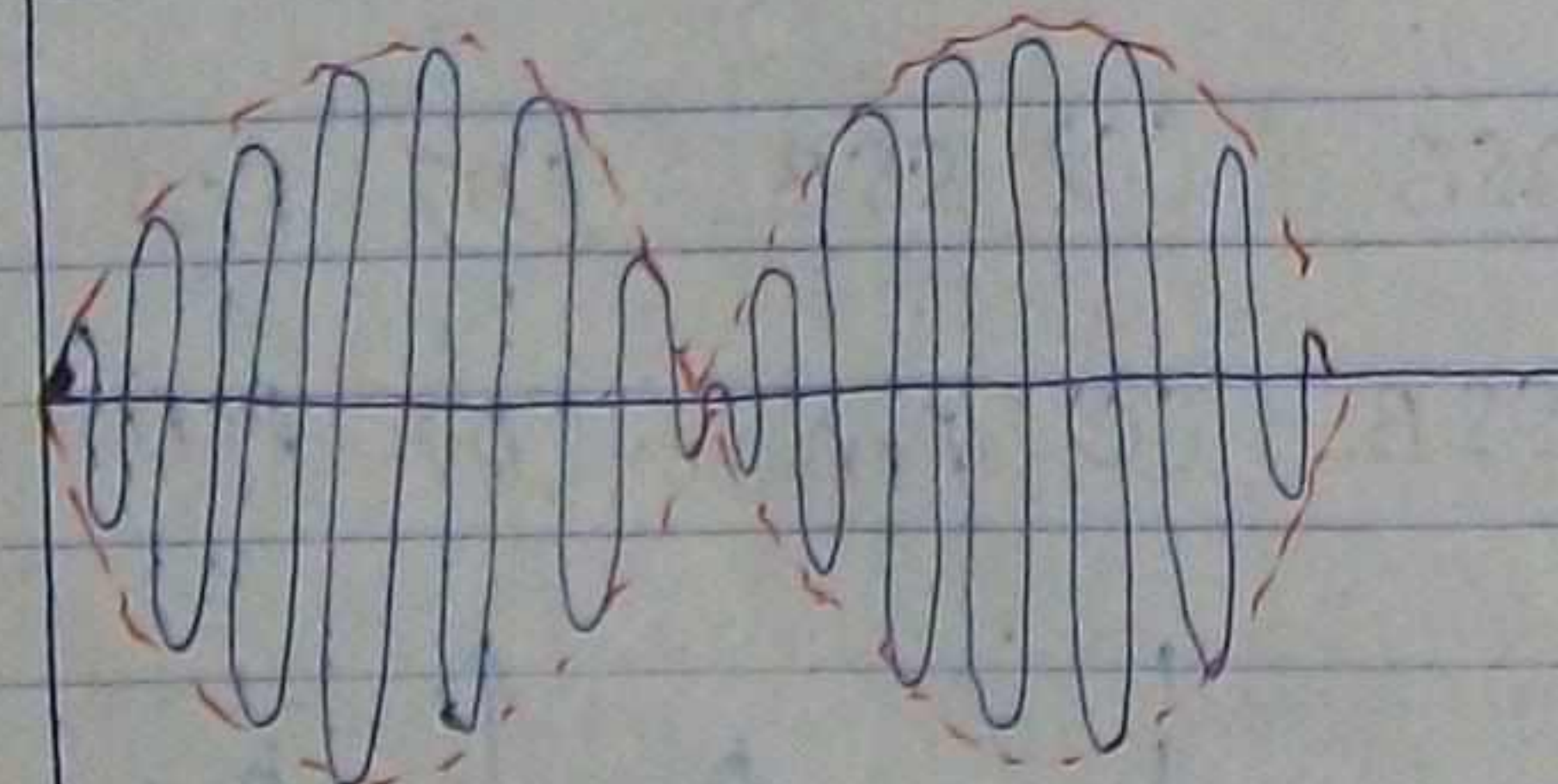
$c(t)$



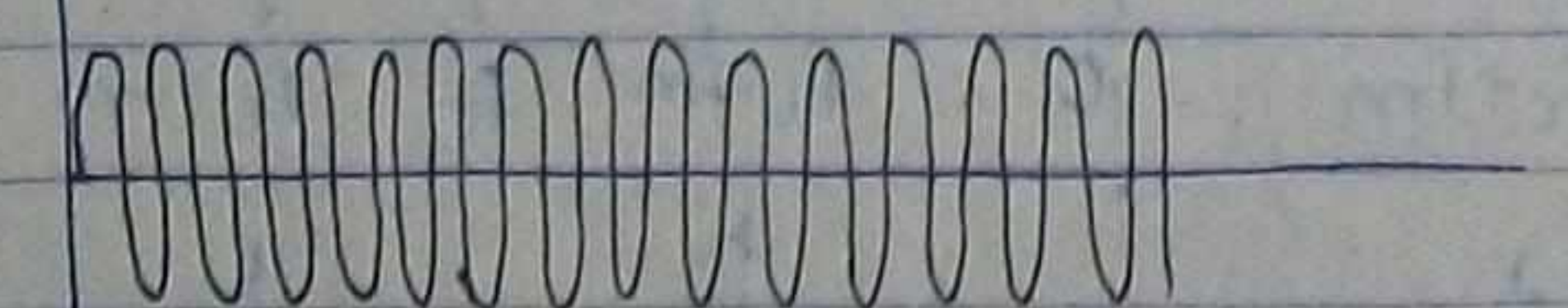
A-m



DSB

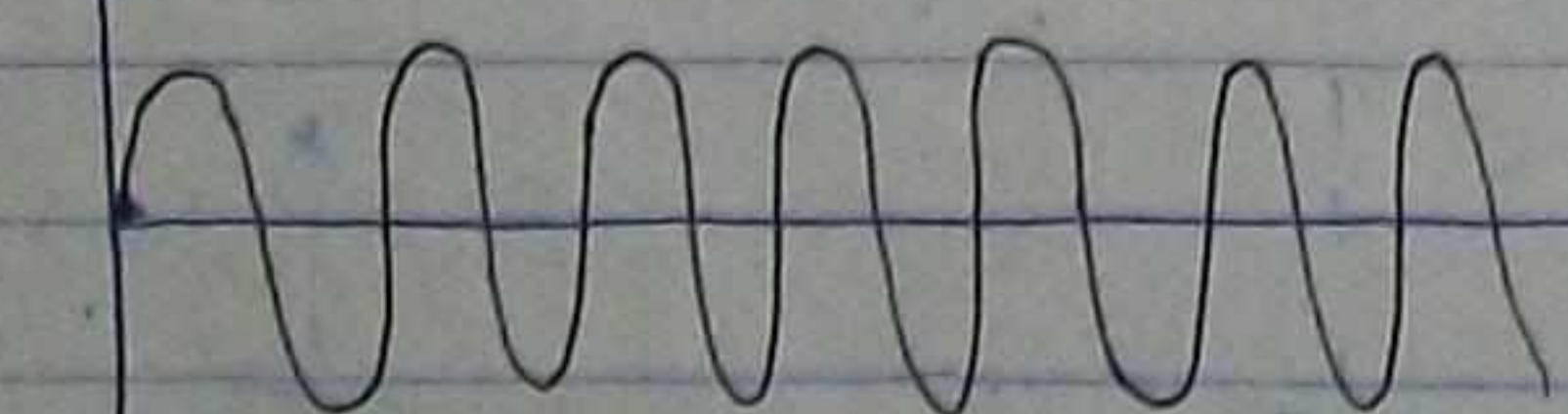


SSB
USB



$(f_c + f_m)$

SSB
LSB



$(f_c - f_m)$

USB + LSB $\rightarrow$ DSB

- The envelope of SSB remains constant.

$\Rightarrow$ HILBERT'S TRANSFORM -

Occurs 90° phase shift at o/p.

$$S(t)_{Am} = V_c \cos \omega_c t + \frac{m V_c}{2} \cos 2\pi (f_c + f_m) t + \frac{m V_c}{2} \cos 2\pi (f_c - f_m) t$$

$K_a = 1$

$$S(t) = \frac{V_m}{V_c} \times \frac{V_c}{2} \cos 2\pi (f_c + f_m) t$$

$$= \frac{V_c}{2} V_m \cos 2\pi (f_c + f_m) t$$

$$= \frac{V_c}{2} V_m \left[\cos 2\pi f_m t \cdot \cos 2\pi f_c t - \sin 2\pi f_m t \cdot \sin 2\pi f_c t \right]$$

$$SSB_{USB} = \frac{V_c}{2} m(t) \cdot \cos 2\pi f_c t - \frac{V_c}{2} \hat{m}(t) \sin 2\pi f_c t$$

$$SSB_{LSB} = \frac{V_c}{2} m(t) \cdot \cos 2\pi f_c t + \frac{V_c}{2} \hat{m}(t) \sin 2\pi f_c t$$

HILBERT TRANSFORM -

- Hilbert transform is used to provide, 90° phase shift to each i/p signal freq.

$$x(t) \xrightarrow{H.T.} \hat{x}(t)$$

$$\hat{x}(t) = x(t) * \frac{1}{\pi t}$$

- In Hilbert transform each +ve freq. will undergo -90° phase shift and each -ve freq. will undergo $+90^\circ$ phase shift.

$$(1) \cos \theta \xrightarrow{H.T.} \cos(\theta - 90) = \sin \theta$$

$$(2) \sin \theta \xrightarrow{H.T.} \sin(\theta - 90) = -\cos \theta$$

$$(3) \cos 2\pi f_1 t \xrightarrow{H.T.} \sin 2\pi f_1 t$$

$$(4) \sin 2\pi f_2 t \xrightarrow{H.T.} -\cos 2\pi f_2 t$$

$$\begin{aligned} \text{sgn}(f) &= 1 \quad f > 0 \\ &= -1 \quad f < 0 \end{aligned}$$

$$x(t) \longleftrightarrow X(f)$$

$$\frac{1}{\pi t} \longleftrightarrow -j \text{sgn}(f)$$

$$\hat{x}(t) \longleftrightarrow \hat{X}(f)$$

$$\hat{X}(f) = -j \text{sgn}(f) \cdot X(f)$$

FOR CAL'N
I.F.T OF
HILBERT
TRANSFORM

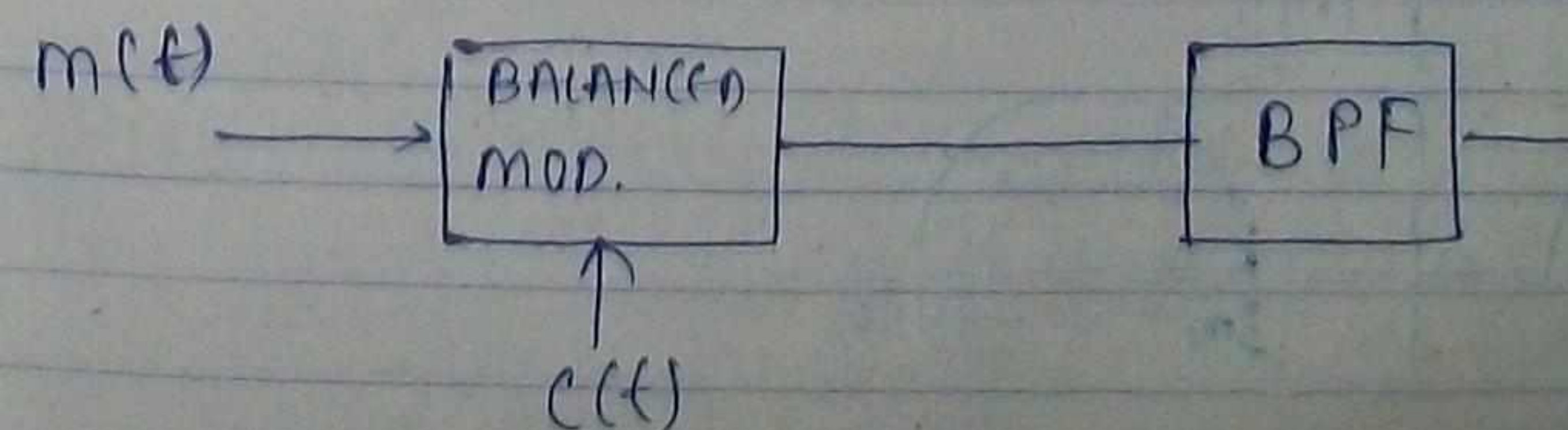
→ $x(t)$ and $\hat{x}(t)$ amplitude spectrum remains same i.e.

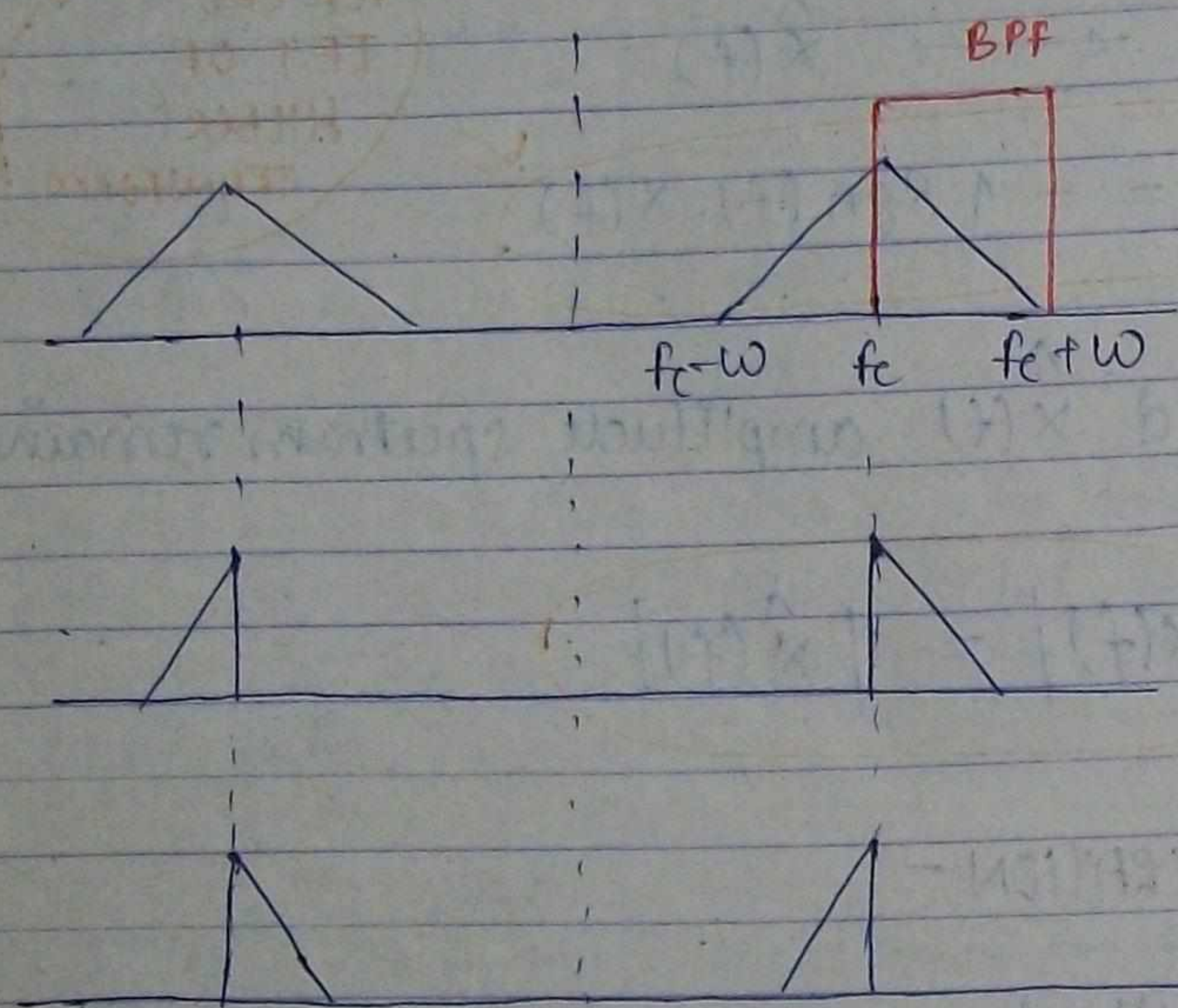
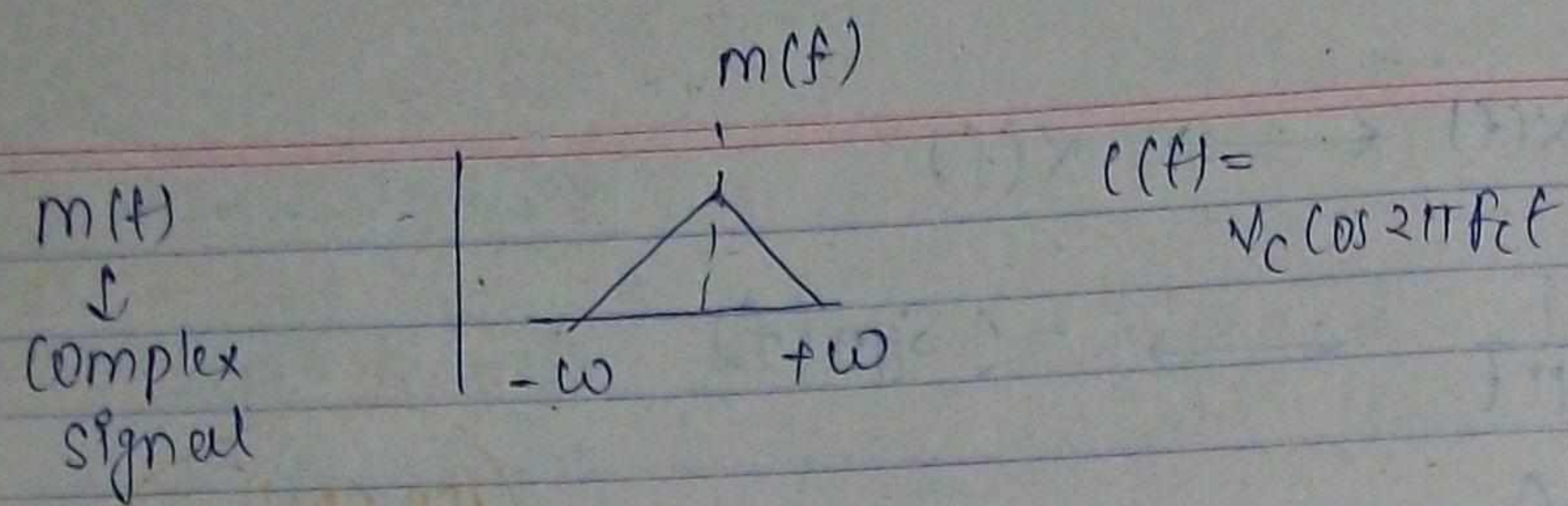
$$|X(f)| = |\hat{X}(f)|$$

⇒ SSB GENERATION -

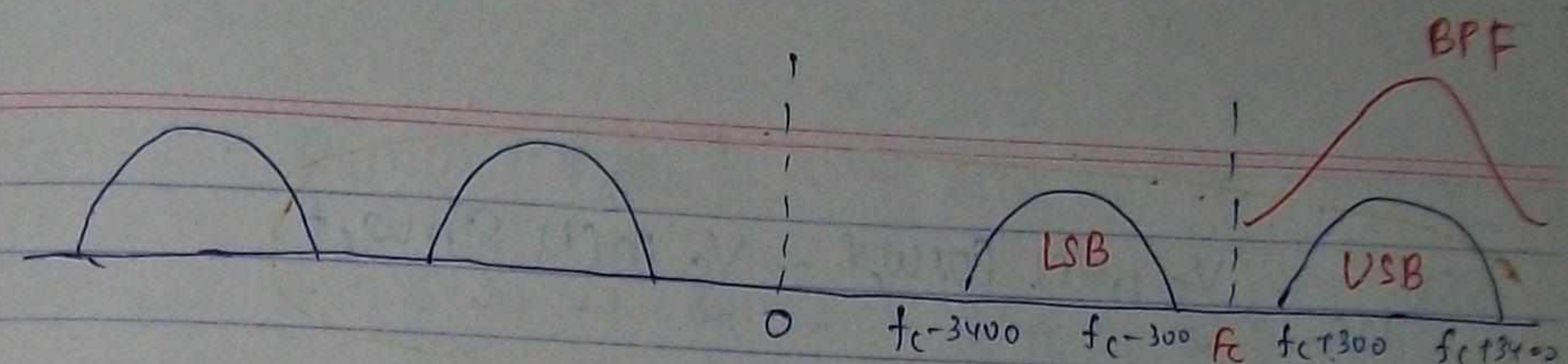
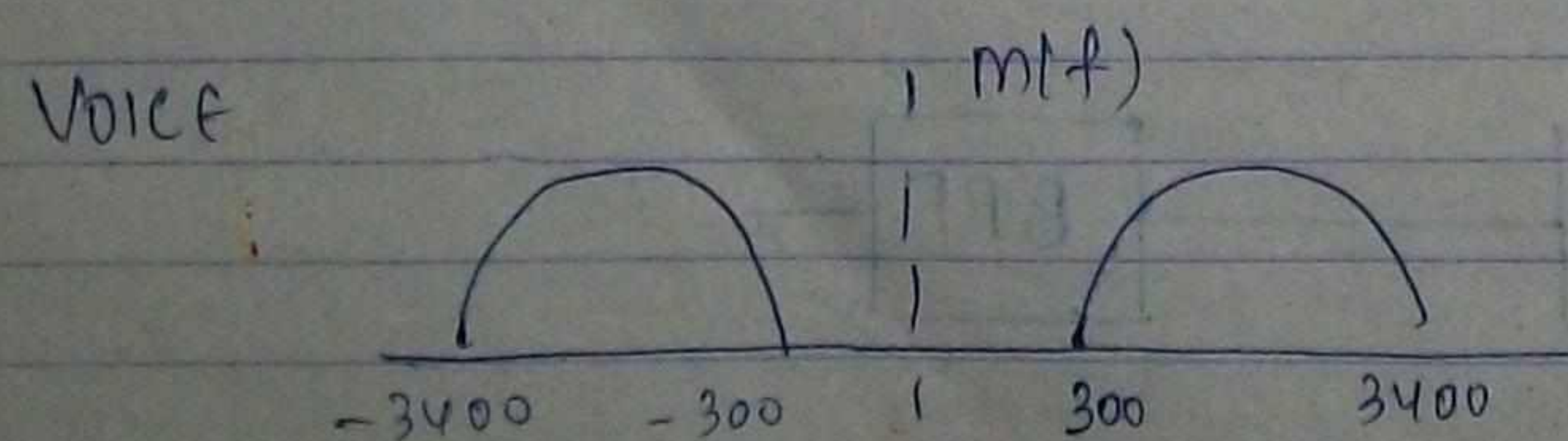
- (1) filter method
- (2) phase shift method.
- (3) Third method / weaver's method.

(1) FILTER METHOD OF SSB GENERATION -



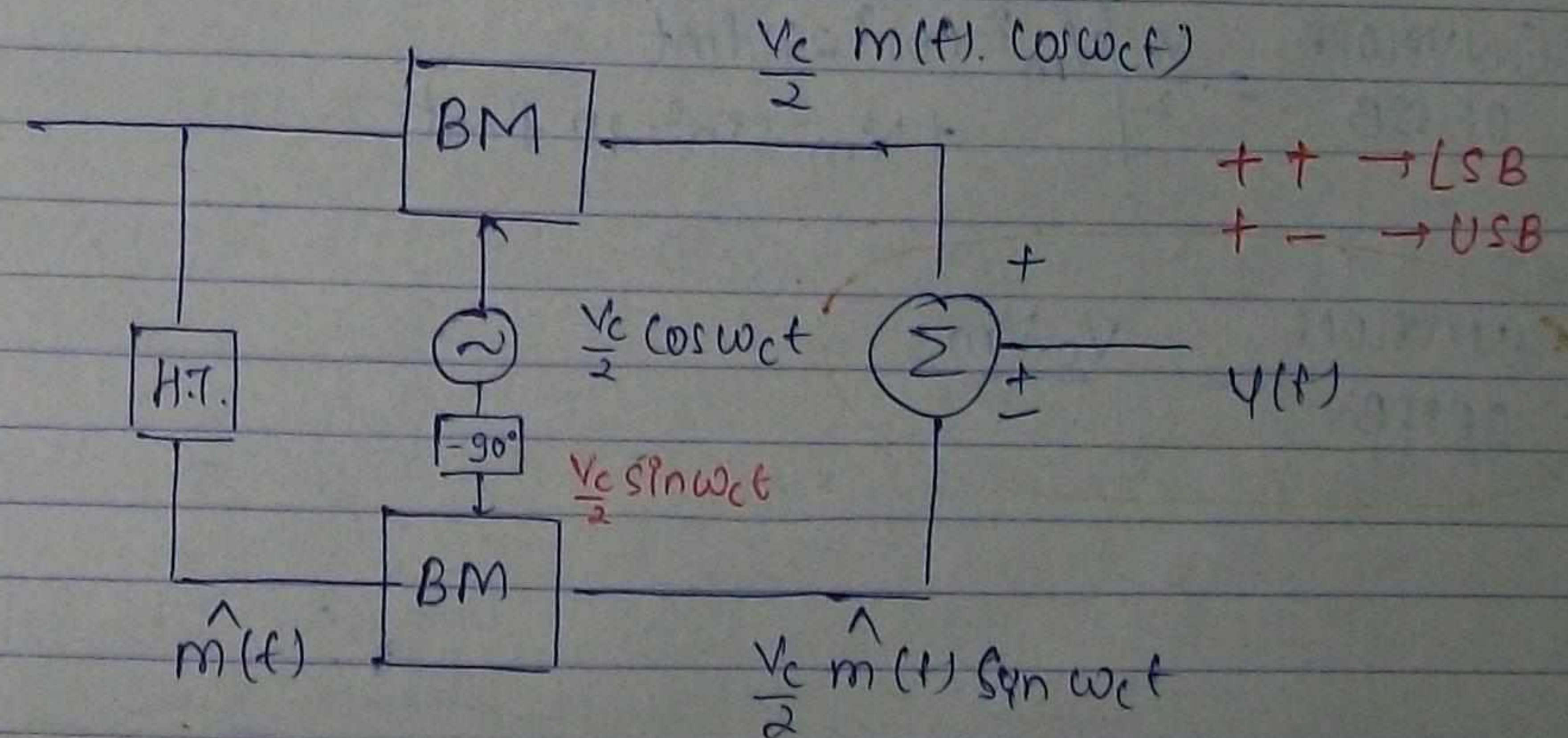


- In filter method of SSB generation, video signals are not transmitted due to ideal BPF requirement.



- filter method is mostly used to transmit voice frequency signals.
- filter used mostly in SSB generation is MECHANICAL FILTER. (made from Tx. line).

② PHASE SHIFT METHOD. - (Phase discrimination method)
OR (Hartley method).



$$y(t) = \frac{V_c}{2} m(t) \cos \omega_c t + \frac{V_c}{2} \hat{m}(t) \sin \omega_c t$$

SSB LSB

$$SSB_{USB} = \frac{V_c}{2} m(t) \cos \omega_c t - \frac{V_c}{2} \hat{m}(t) \sin \omega_c t$$

ENVELOPE OF SSB SIGNAL -

$$\text{ENVELOPE OF SSB} = \sqrt{\left(\frac{V_c}{2} m(t)\right)^2 + \left(\frac{V_c}{2} \hat{m}(t)\right)^2}$$

If $m(t) = V_m \cos 2\pi f_m t$

$$\text{ENVELOPE OF SSB} = \frac{V_c}{2} \sqrt{V_m^2 \cos^2 2\pi f_m t + V_m^2 \sin^2 2\pi f_m t}$$

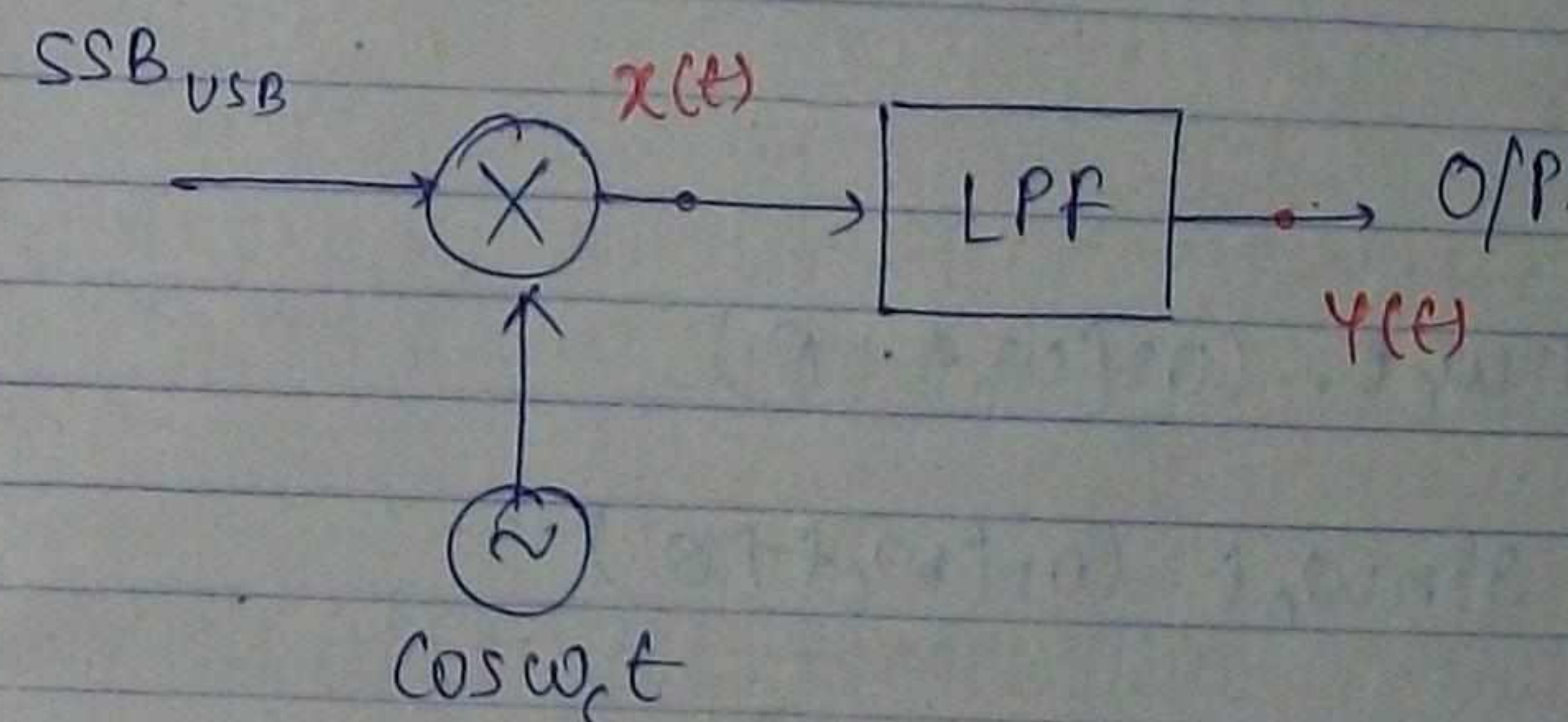
$$\text{ENVELOPE OF SSB} = \frac{V_c V_m}{2}$$

27/11/11

⇒ SSB DEMODULATION -

$$S(t)_{SSB_{USB}} = \frac{V_c}{2} m(t) \cos \omega_c t - \frac{V_c}{2} \hat{m}(t) \sin \omega_c t$$

$$S(t)_{SSB_{LSB}} = \frac{V_c}{2} m(t) \cos \omega_c t + \frac{V_c}{2} \hat{m}(t) \sin \omega_c t$$



COHERENT DETECTION

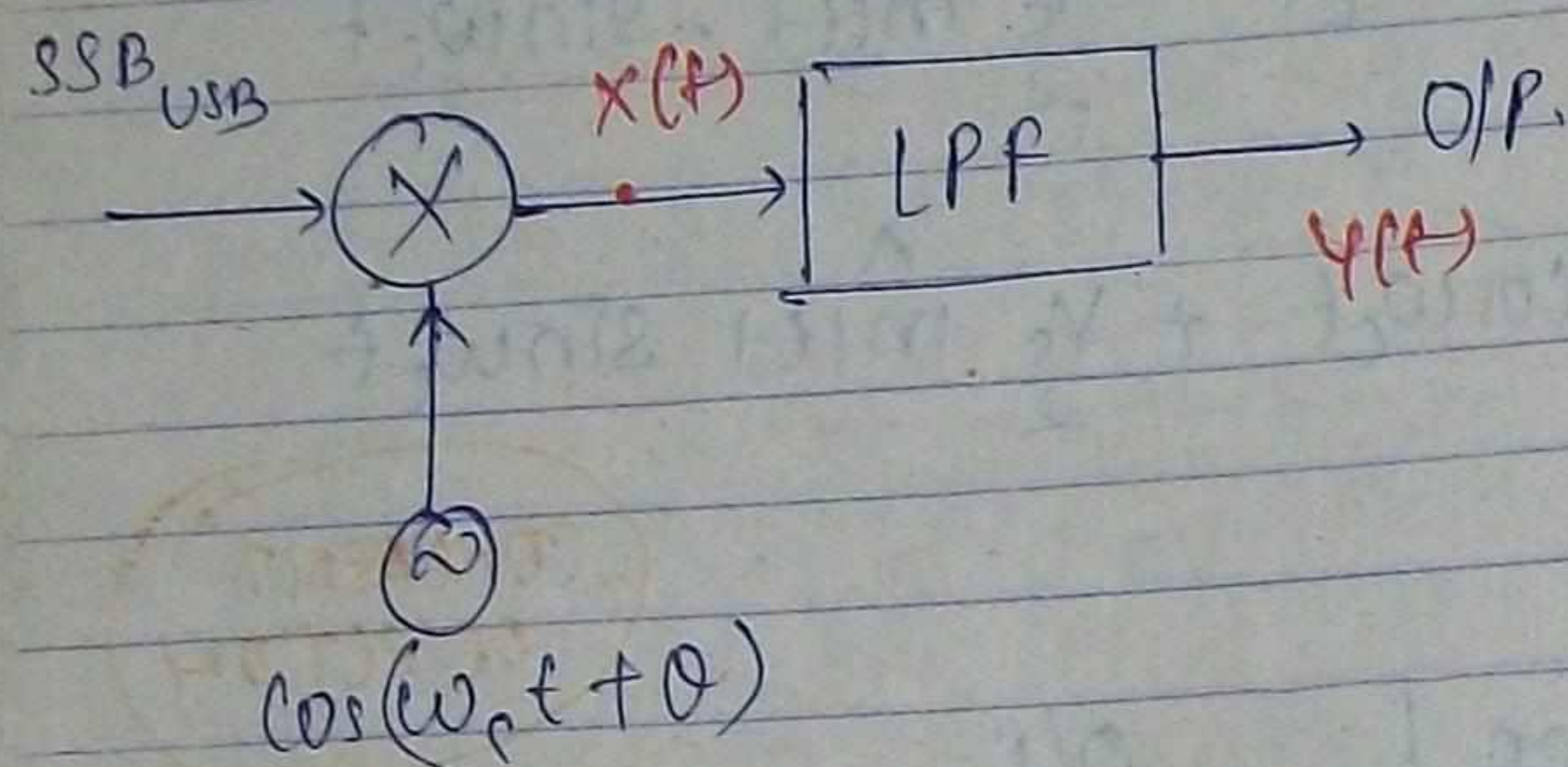
$$X(t) = \frac{V_c}{2} m(t) \cos^2 \omega_c t - \frac{V_c}{2} \hat{m}(t) \sin \omega_c t \cdot \cos \omega_c t$$

$$X(t) = \frac{V_c}{4} m(t) + \frac{V_c}{4} m(t) \cos 2\omega_c t - \frac{V_c}{4} \hat{m}(t) \sin 2\omega_c t$$

after LPF.

$$Y(t) = \frac{V_c}{4} m(t)$$

PHASE DISCREPANCY -



$$x(t) = \frac{V_c}{2} m(t) \cos \omega_c t \cdot \cos(\omega_c t + \theta) - \frac{V_c}{2} \hat{m}(t) \sin \omega_c t \cdot \cos(\omega_c t + \theta)$$

After LPF -

$$y(t) = \frac{V_c}{4} m(t) \cos \theta + \frac{V_c}{4} \hat{m}(t) \sin \theta$$

$$y(f) = \frac{V_c}{4} \cos \theta \cdot M(f) + \frac{V_c}{4} \hat{m}(f) \sin \theta$$

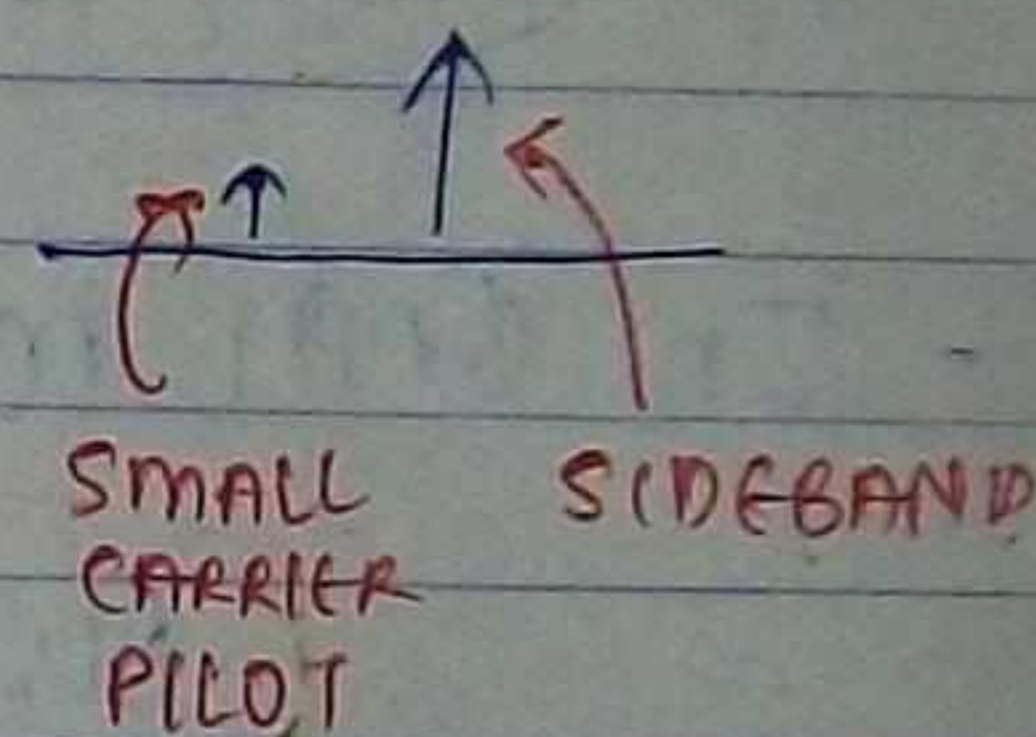
where, $\hat{m}(f) = -j \operatorname{sgn}(f) m(f)$

$$y(f) = \frac{V_c}{4} m(f) [\cos \theta - j \sin \theta \cdot \operatorname{sgn}(f)]$$

- There is no quadrature null effect in SSB demod!

- PILOT CARRIER SYSTEM is used in SSB to provide freq. stability at receivers.

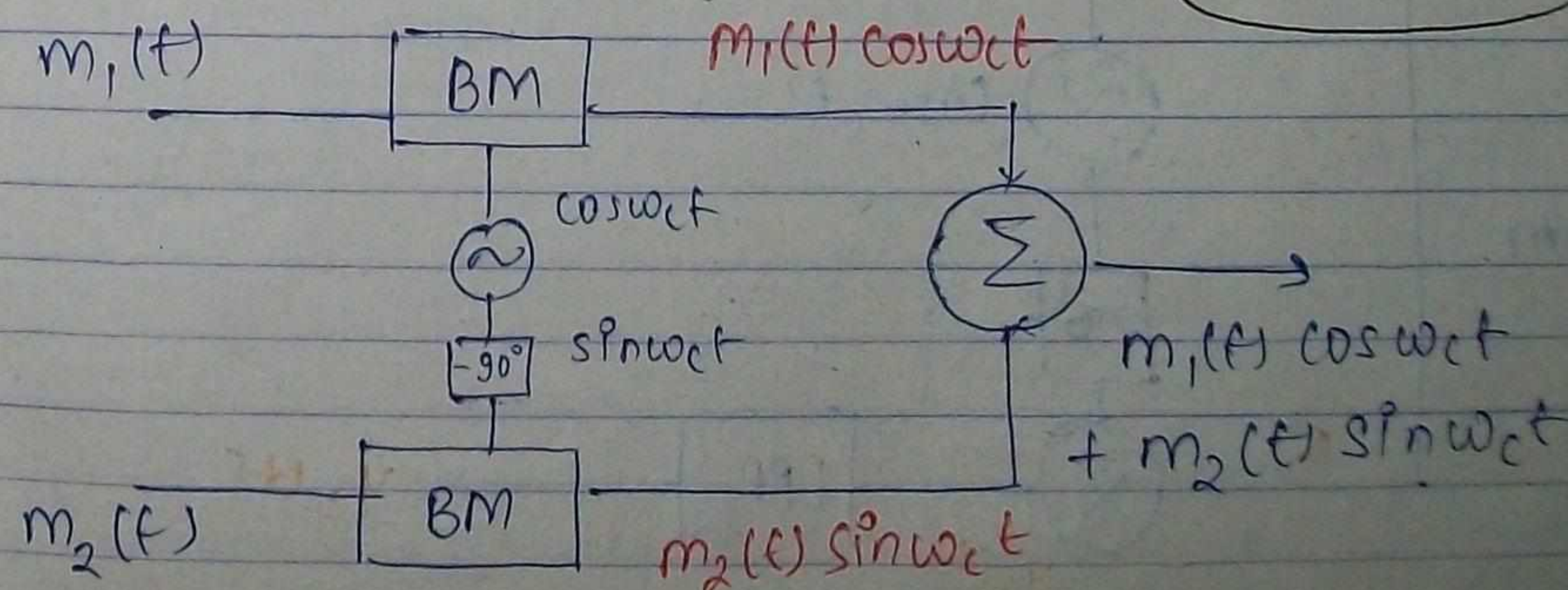
- In pilot carrier SSB system small trace of carrier is transmitted to improve freq. signal stability.

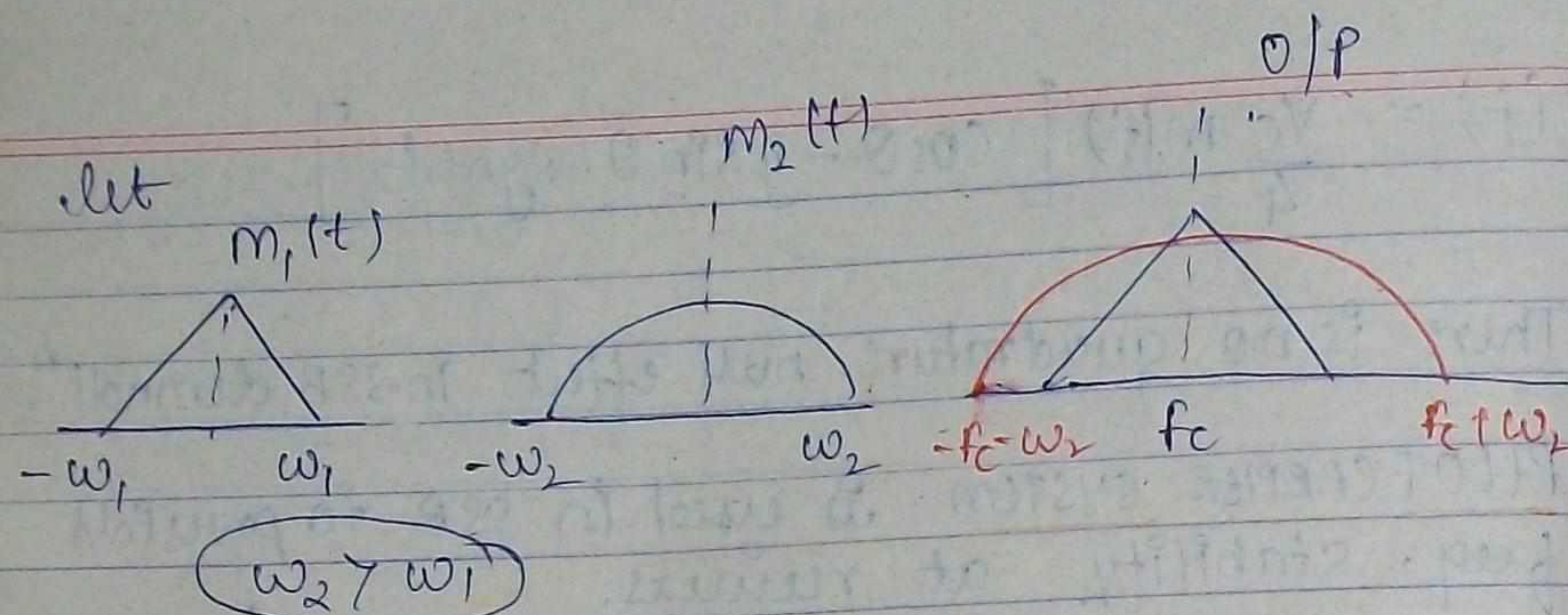


⇒ QUADRATURE AMPLITUDE MOD (QAM) -
(QUADRATURE CARRIER MULTIPLEXING (QCM))

- In QAM two msg. signals are transmitted using DSB with a bandwidth of $2W$. (W = msg. signal b/w).

TRANSMITTER



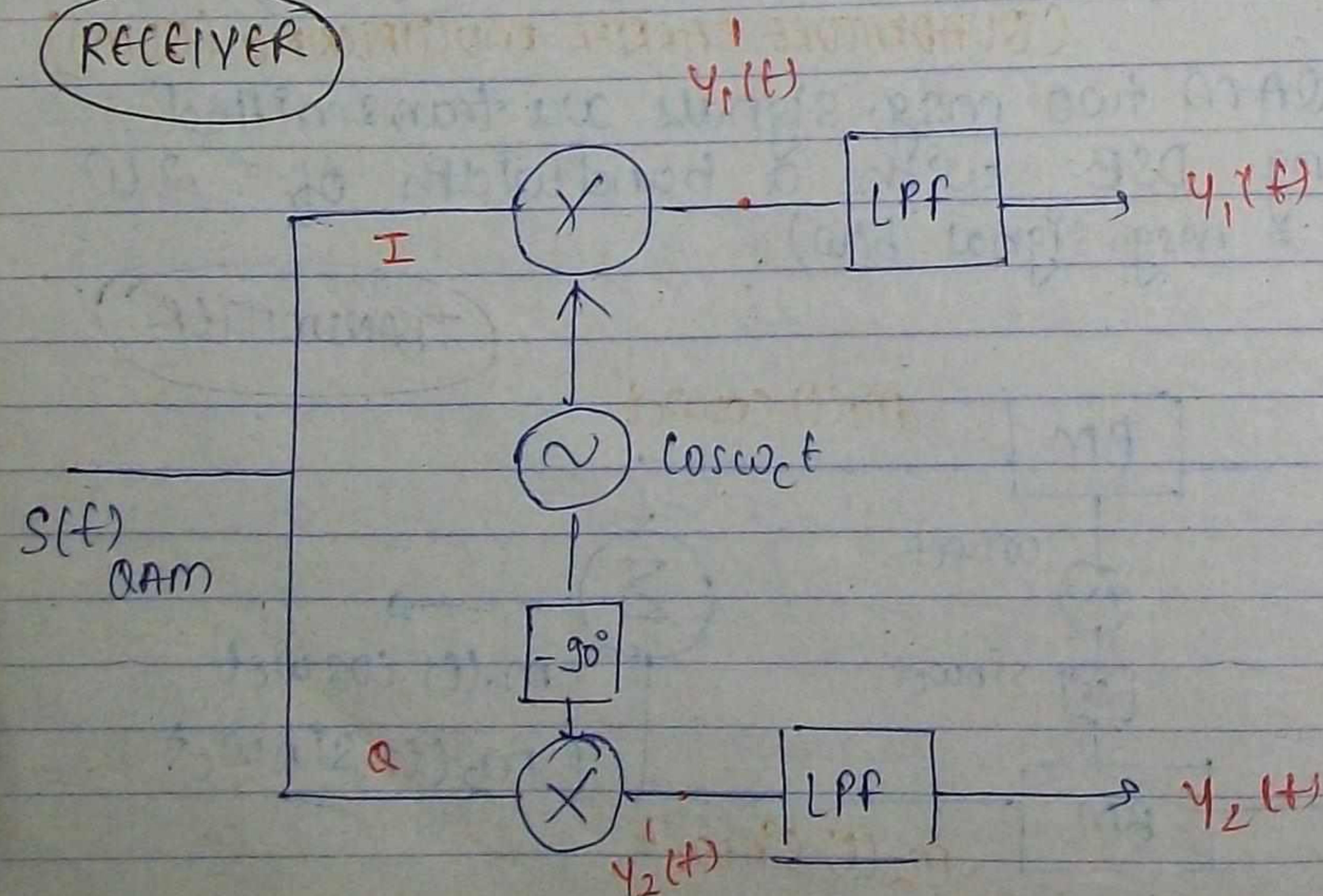


- In QAM max. bandwidth required is

$$BW_{max} = 2w_{max}$$

where, w_{max} = max. msg. bandwidth in given signal.

RECEIVER



$$y_1(t) = m_1(t) \cos^2 w_c t + m_2(t) \sin w_c t \cos w_c t$$

after LPF

$$y_1(t) = \frac{1}{2} m_1(t)$$

Similarly,

$$y_2(t) = \frac{1}{2} m_2(t)$$

⇒ INDEPENDENT SIDE BAND (ISB) -

- In ISB two or more voice signals are transmitted with same carrier frequency.

TRANSMITTER

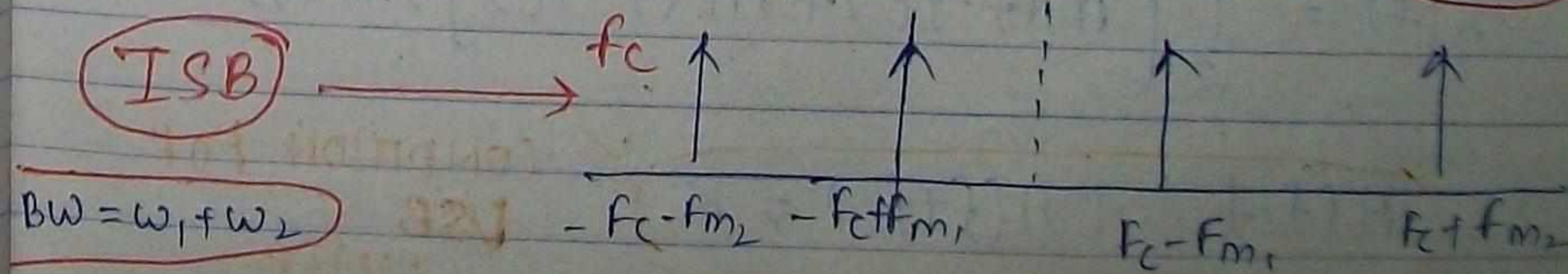
$$m_1(t) \xrightarrow{DSB} f_c$$

f_{m1}

$$m_2(t) \xrightarrow{DSB} f_c$$

f_{m2}

ISB

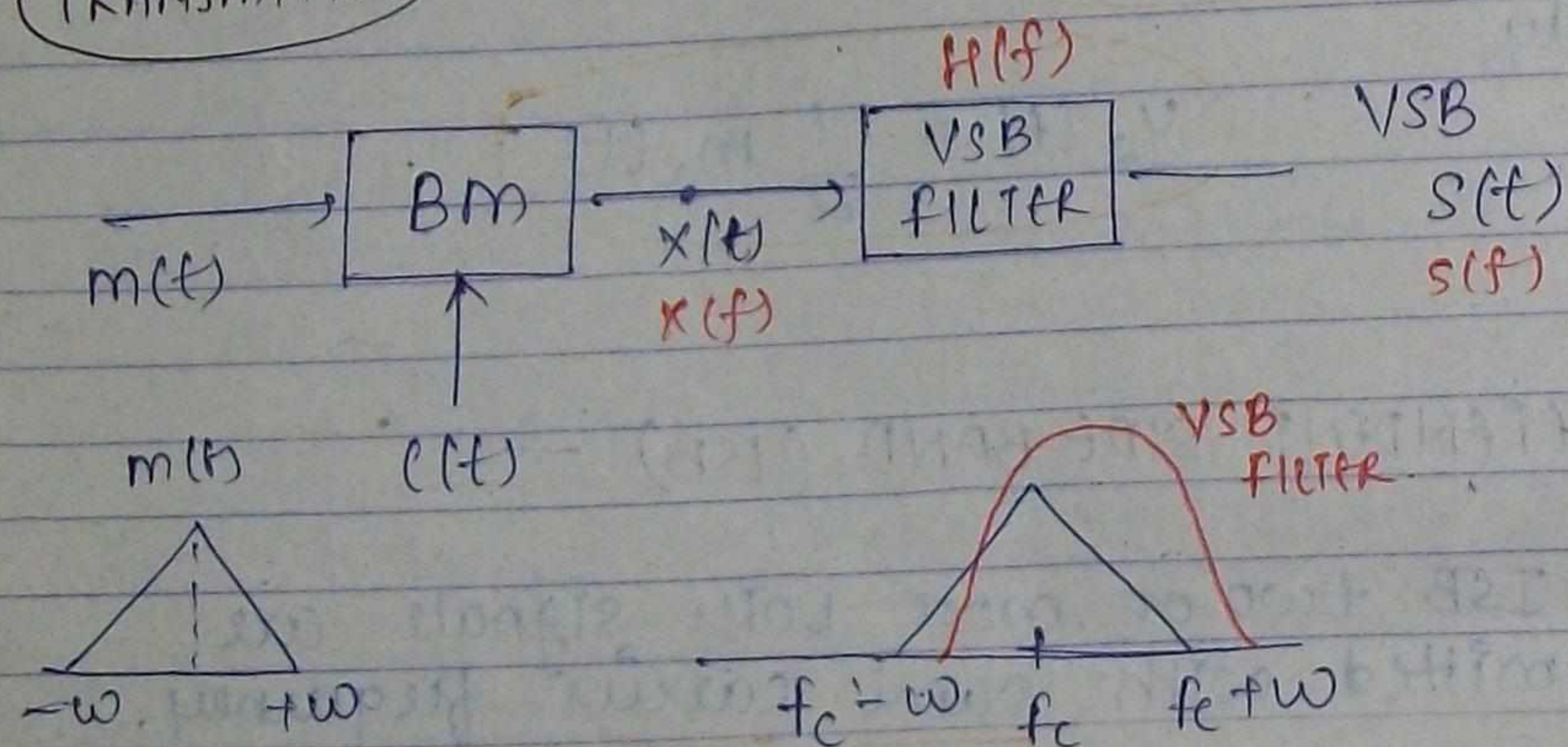


VIDEO
Tx.

⇒ VESTIGIAL SIDE BAND (VSB) —

— In VSB one full side band and part of second side band is transmitted.

TRANSMITTER



— $w < BW < 2w$ Bandwidth of VSB.

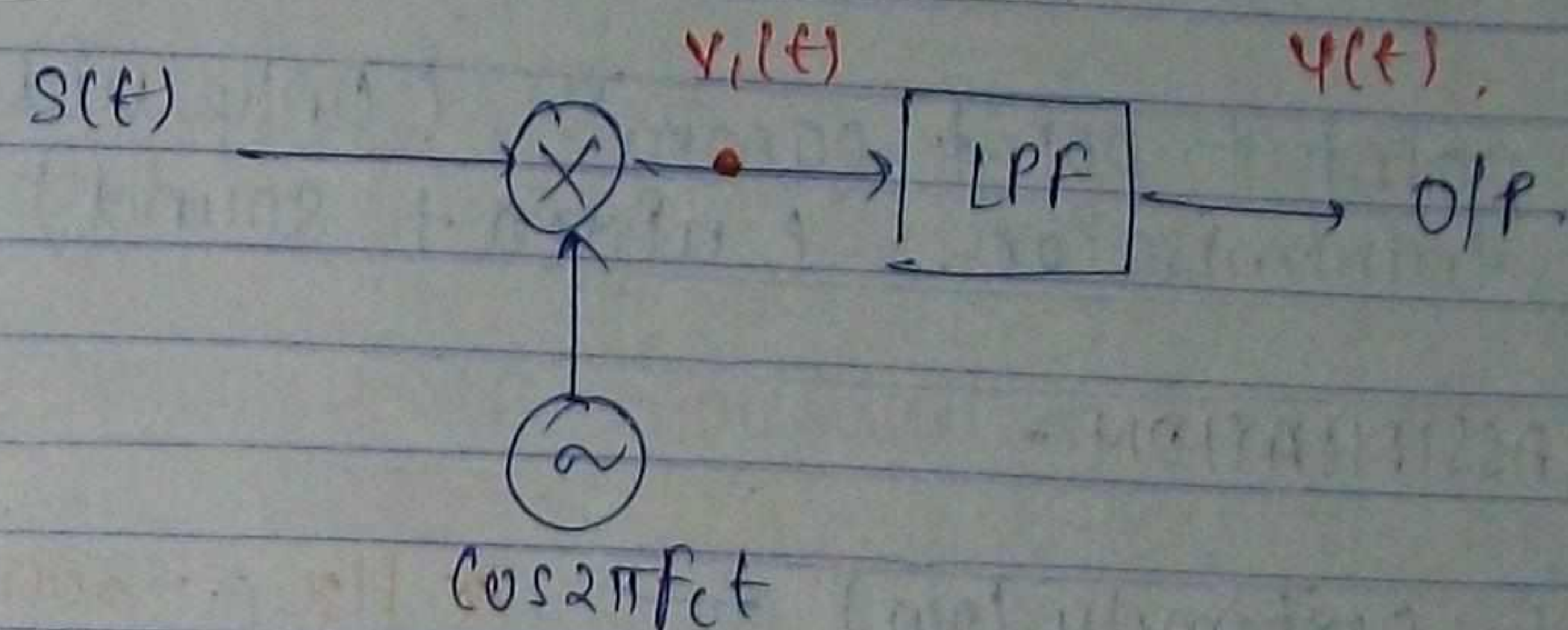
Sinu $x(t) = m(t) \cdot \cos \omega_c t$

$$x(f) = \frac{1}{2} [m(f-f_c) + m(f+f_c)]$$

$$S(f) = \frac{1}{2} [m(f-f_c) + m(f+f_c)] \cdot H(f)$$

$H(f-f_c) + H(f+f_c) = K$ ✓ CONDITION FOR VSB FILTER

RECEIVER



$$V_1(t) = S(t) \cdot \cos 2\pi f_c t$$

$$V_1(f) = \frac{1}{4} [m(f-2f_c) + m(f)] H(f-f_c) + \frac{1}{4} [m(f) + m(f+2f_c)] H(f+f_c)$$

after LPF

$$y(f) = \frac{1}{4} m(f) \cdot H(f-f_c) + \frac{1}{4} m(f) H(f+f_c)$$

$$y(f) = \frac{1}{4} m(f) [H(f-f_c) + H(f+f_c)]$$

CONSTANT.

APPLICATION -

1. HF point to point commⁿ. (Only video)
2. TV. transmission. (video + sound)

FREQ. CLASSIFICATION -

ELF (extremely low) $\rightarrow$ 30 Hz - 300 Hz

VF (voice) $\rightarrow$ 300 Hz - 3 kHz

VLF (very low) $\rightarrow$ 3 kHz - 30 kHz

LF (low) $\rightarrow$ 30 kHz - 300 kHz

AM BROAD. MF (medium) $\rightarrow$ 300 kHz - 3 MHz

commⁿ HF (high) $\rightarrow$ 3 MHz - 30 MHz

VHF (very high) $\rightarrow$ 30 MHz - 300 MHz

CELLULAR UHF (ultra high) $\rightarrow$ 300 MHz - 3 GHz

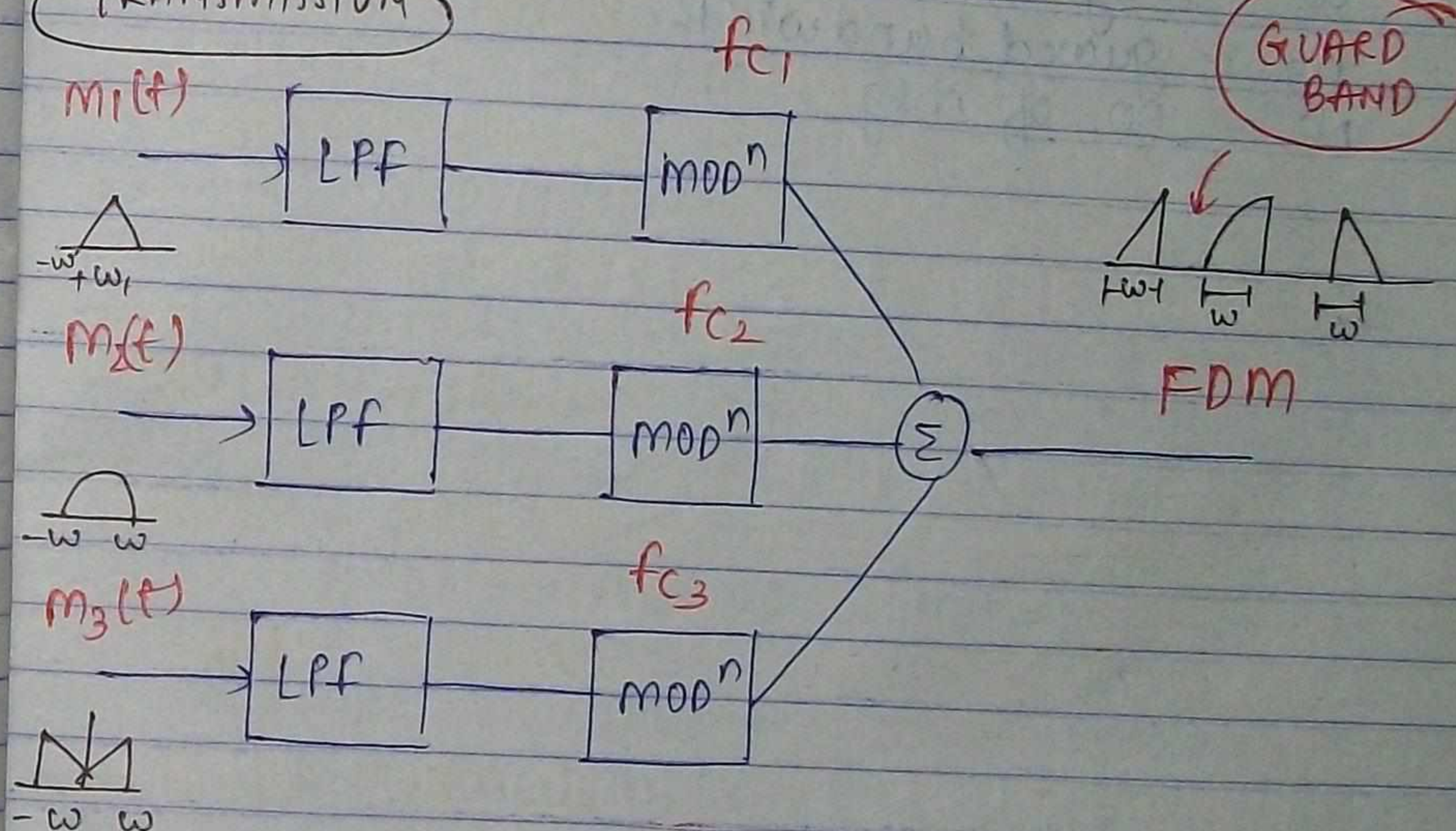
MF (micro) $\rightarrow$ greater than 1 MHz

$\Rightarrow$ FREQ. DIVISION MULTIPLEXING (FDM) -

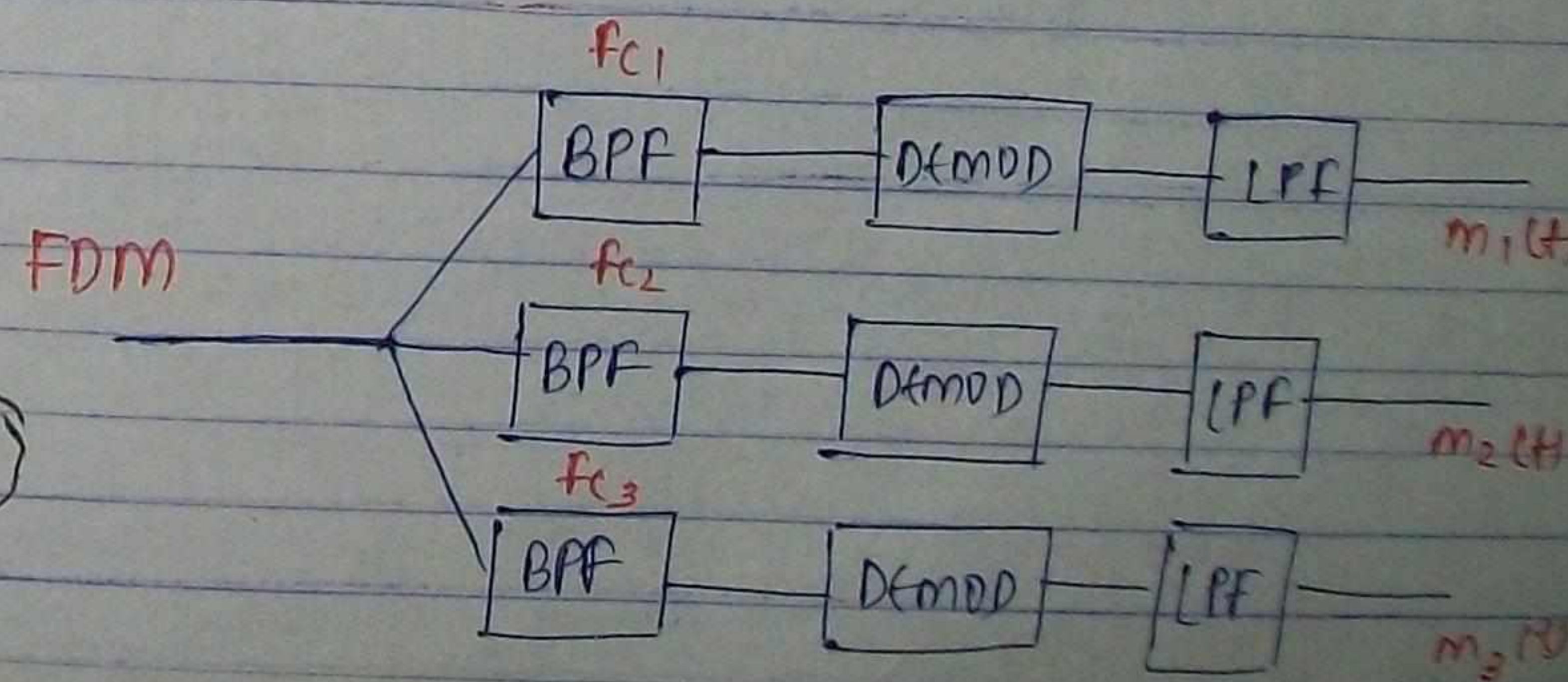
- In multiplexing no. of signals are transmitted through a single channel.

- In FDM, SSB modulation is used.

TRANSMISSION



RECEIVER



- If SSB used in FDM then BW required is :

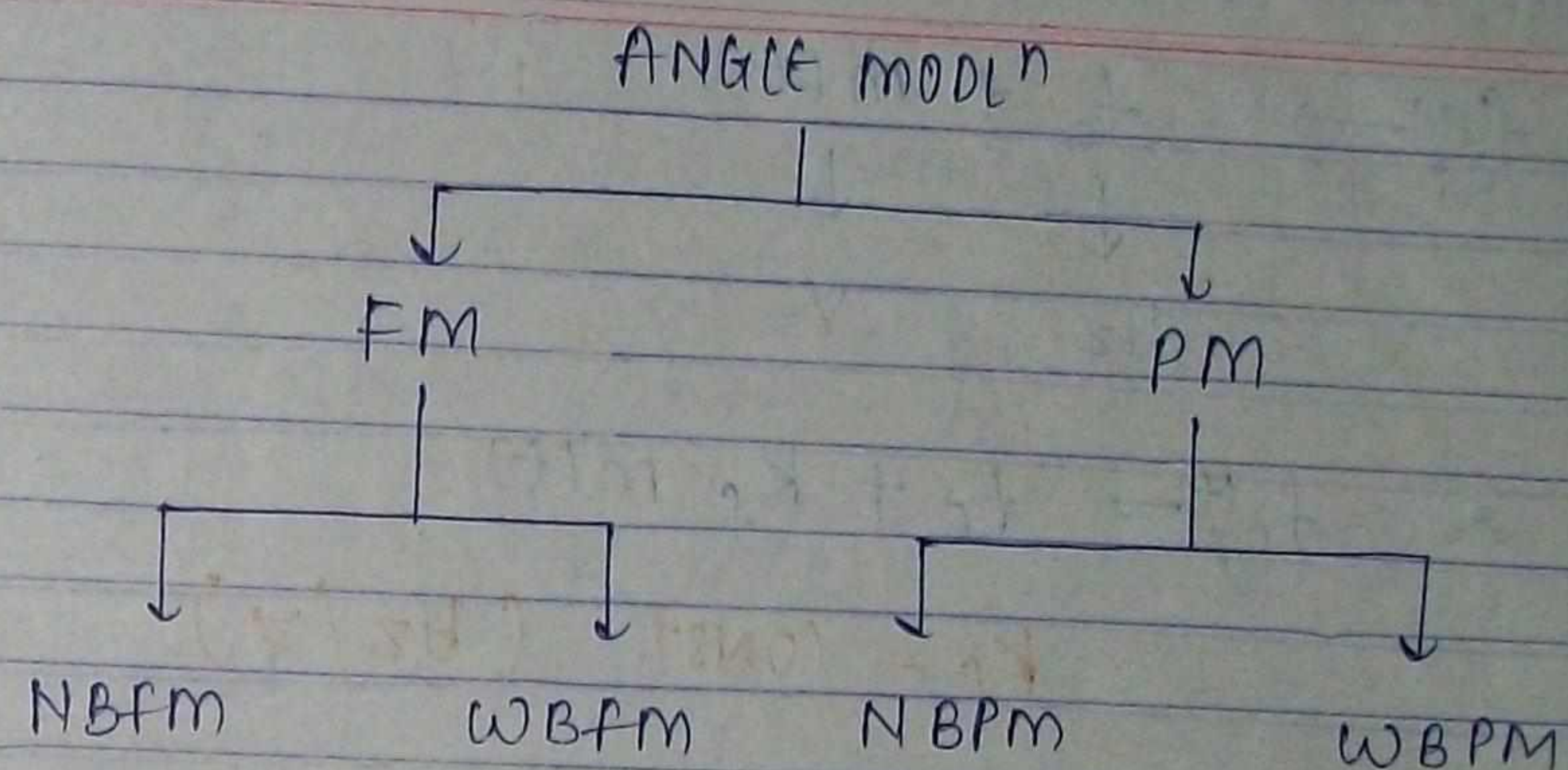
$$BW_{SSB} = Nw + (N-1)B_g$$

w = msg BW

B_g = guard bandwidth

N = no. of msg.

ANGLE MODULATION



⇒ FREQ. MODULATION -

- In F.M. freq. of carrier signal is varied in accordance with instantaneous amplitude of msg. signal.

Let, $m(t)$ → message signal,

max. modulating freq. used in FM → 15 KHz.

Carrier → $c(t) = V_c \cos 2\pi f_c t$

f_c → 88 MHz - 108 MHz

FM
BW

Let,

$$s(t)_{FM} = V_c \cos \theta_i(t)$$

GENERAL
FORM

INSTANTANEOUS
CARRIER FREQ.

$$f_{ci}^{(t)} \rightarrow \begin{array}{cc} f_c + m(t) \\ \downarrow \quad \downarrow \\ \text{Hz} \quad \text{V.} \end{array}$$

$$\therefore f_{ci}^{(t)} \rightarrow f_c + K_f m(t)$$

$$K_f = \text{CONST. (Hz/V)}$$

$$\begin{aligned} \omega_i(t) &= 2\pi f_{ci}(t) \\ &= 2\pi f_c + 2\pi K_f m(t) \end{aligned}$$

$$\begin{aligned} \theta_i(t) &= \int \omega_i(t) dt \\ &= \int (2\pi f_c + 2\pi K_f m(t)) dt \end{aligned}$$

$$\theta_i(t) = 2\pi f_c t + 2\pi K_f \int_{-\infty}^t m(t) dt$$

$$s(t)_{\text{FM}} = V_c \cos \left[2\pi f_c t + 2\pi K_f \int_{-\infty}^t m(t) dt \right]$$

GENERAL
FORM

FREQ. DEVIATION -

$$f_{ci}(t) = f_c + K_f m(t)$$

$$\begin{aligned} \Delta f_{ci}(t) &= f_{ci}(t) - f_c \\ &= K_f m(t) \end{aligned}$$

$$\Delta f = |\Delta f_{ci}(t)| = K_f |m(t)|$$

PHASE DEVIATION -

$$\theta_i(t) = 2\pi f_c t + 2\pi K_f \int_{-\infty}^t m(t) dt$$

$$\Delta \theta_i(t) = \theta_i(t) - 2\pi f_c t$$

$$\Delta \phi = |\Delta \theta_i(t)|$$

$$\Delta \phi = 2\pi K_f \left| \int m(t) dt \right|$$

CASE (I): SINGLE TONE MESSAGE -

$$m(t) = V_m \cos 2\pi f_m t$$

$$c(t) = V_c \cos 2\pi f_c t$$

$$s(t) = V_c \cos \theta_i(t)$$

$$f_{ci}(t) = f_c + K_f \cdot V_m \cos 2\pi f_m t$$

$$f_{ci}(t) = f_c + \Delta f \cdot \cos 2\pi f_m t$$

where $\Delta f = K_f \cdot V_m$
 $\Delta f \propto V_m$

FOR SINGLE
TONE F.M,

$$\omega_i(t) = 2\pi f_{ci}(t)$$

$$= 2\pi f_c + 2\pi \Delta f \cdot \cos 2\pi f_m t$$

$$\theta_i(t) = \int \omega_i(t)$$

$$\theta_i(t) = 2\pi f_c t + 2\pi \Delta f \frac{\sin 2\pi f_m t}{2\pi f_m}$$

$$\theta_i(t) = 2\pi f_c t + \frac{\Delta f}{f_m} \cdot \sin 2\pi f_m t$$

where

mod'n
index, $\beta = \frac{\Delta f}{f_m}$ rad.

$\beta > 1$ WBFM
 $\beta < 1$ NBFM

$$\theta_i(t) = 2\pi f_c t + \beta \sin 2\pi f_m t$$

$$s(t)_{FM} = V_c \cos [2\pi f_c t + \beta \sin 2\pi f_m t]$$

SINGLE
TONE

$$\theta_i(t) = 2\pi f_c t + \beta \sin 2\pi f_m t$$

$$\Delta \theta_i(t) = \beta \sin 2\pi f_m t$$

$$\Delta \phi = |\Delta \theta_i(t)| = \beta$$

NOTE -

$$m(t) = V_m \cos 2\pi f_m t$$

$$c(t) = V_c \cos 2\pi f_c t$$

$$s(t) = V_c \cos(2\pi f_c t + \beta \sin 2\pi f_m t)$$

$$\beta = \frac{\Delta f}{f_m}$$

$$\Delta f = K_f \cdot V_m, \quad \Delta \phi = \frac{K_f \cdot V_m}{f_m}$$

$$\Delta f \propto V_m, \quad \Delta \phi \propto V_m \propto \frac{1}{f_m}$$

STANDARD -

$$f_m = 15 \text{ kHz}$$

$$\Delta f_{\max} = 75 \text{ kHz}$$

$$BW = 200 \text{ kHz}$$

$$f_c \rightarrow 88 - 108 \text{ MHz}$$

FOR
F.M.

$$\% \text{ MODULATION} = \frac{\Delta f_{\text{ACTUAL}}}{\Delta f_{\max}} \times 100$$

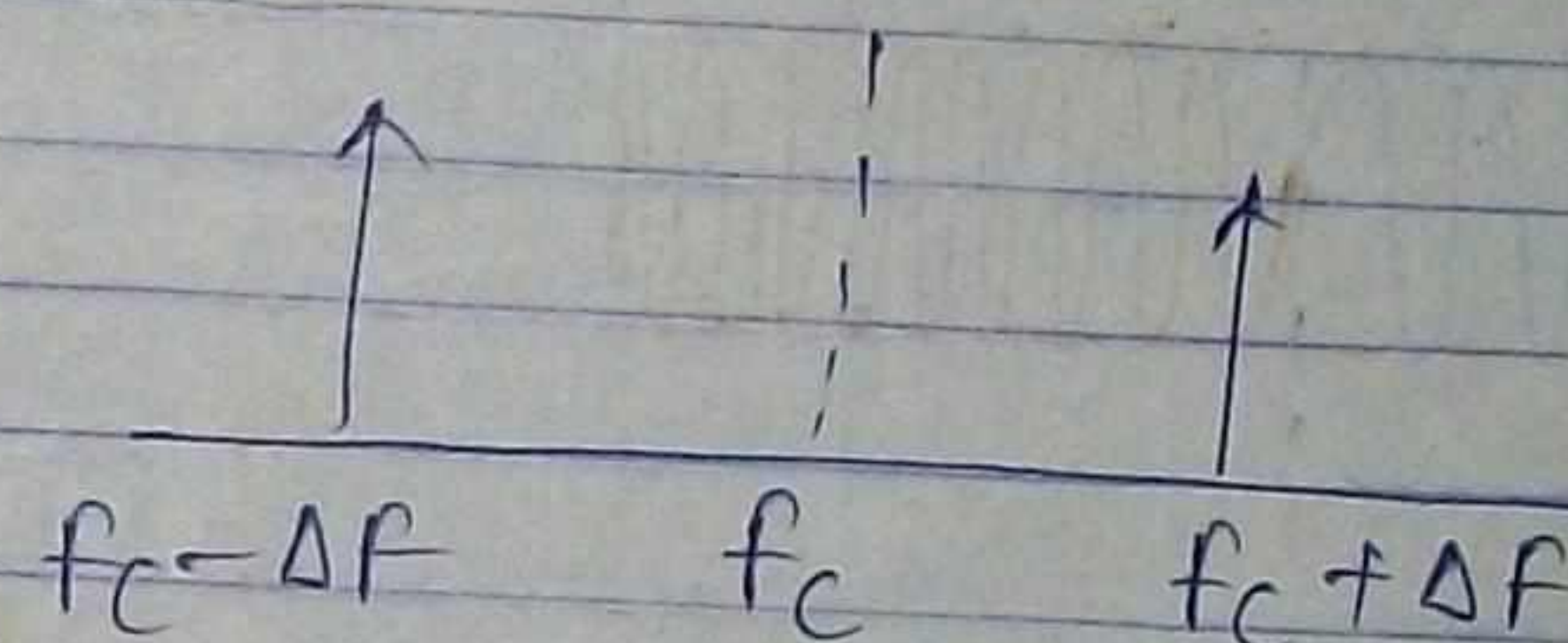
$$\Delta f_{\max} \rightarrow 75 \text{ kHz} \rightarrow \text{radio broadcasting}$$

$$\Delta f_{\max} \rightarrow 25 \text{ kHz} \rightarrow \text{T.V.}$$

Ques. In FM modⁿ freq deviation is 50 kHz
then % modⁿ

$$\text{sol}^n: \% \text{ mod}^n = \frac{50 \text{ kHz}}{75 \text{ kHz}} \times 100 = \underline{\underline{66.6 \%}}$$

$$f_c(t) = f_c + \Delta f \cdot \cos 2\pi f_m t$$



$$f_{c \max} = f_c + \Delta f$$

$$f_{c \min} = f_c - \Delta f$$

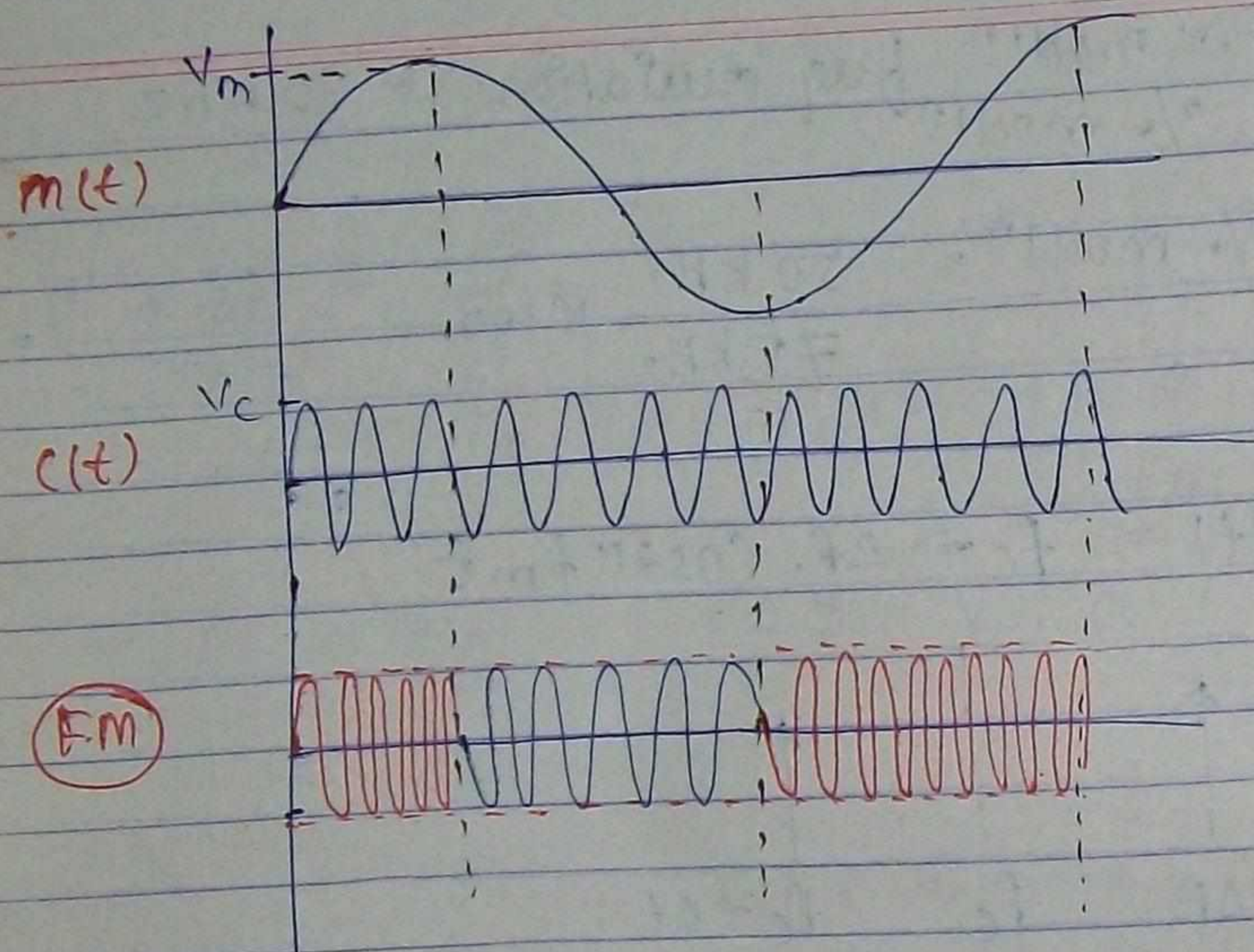
Δf — peak or max

$$\text{CARRIER SWING} = 2\Delta f$$

Ques. In FM modⁿ carrier swing is 125 kHz
then % modⁿ is

$$\text{sol}^n: 125 = 2\Delta f \quad \Delta f = 62.5 \text{ kHz}$$

$$\% \text{ mod}^n = \frac{62.5}{75} \times 100 = \underline{\underline{83.3 \%}}$$



Envelope of FM signal $\rightarrow V_c$.

DEVIATION
RATIO

$$\text{DEVIATION RATIO} = \frac{\text{Peak } \Delta f}{\text{max } f_m}$$

MULTITONE
SIGNAL

$\Rightarrow$ EXPRESSION OF NBFM -

$$S(t)_{FM} = V_c \cos \left[\underbrace{2\pi f_c t}_A + \underbrace{\beta \sin 2\pi f_m t}_B \right]$$

$$= V_c \left[\cos 2\pi f_c t \cdot \cos (\beta \sin 2\pi f_m t) - \sin 2\pi f_c t \cdot \sin (\beta \sin 2\pi f_m t) \right]$$

WBFM -

$$\text{for } \beta > 1 \quad \left. \begin{array}{l} \cos(\sin \theta) \\ \& \sin(\sin \theta) \end{array} \right\} \text{NO SOL}$$

for NBFM, -

$$\beta < 1$$

$$\theta \rightarrow 0 \quad \begin{array}{l} \cos \theta \rightarrow 1 \\ \sin \theta \rightarrow \theta \end{array}$$

$$\therefore \cos (\beta \sin 2\pi f_m t) \approx 1$$

$$\sin (\beta \sin 2\pi f_m t) \approx \beta \sin 2\pi f_m t$$

$$S(t)_{NBFM} = V_c \cos 2\pi f_c t - \beta V_c \sin 2\pi f_m t \cdot \sin 2\pi f_c t$$

NBFM

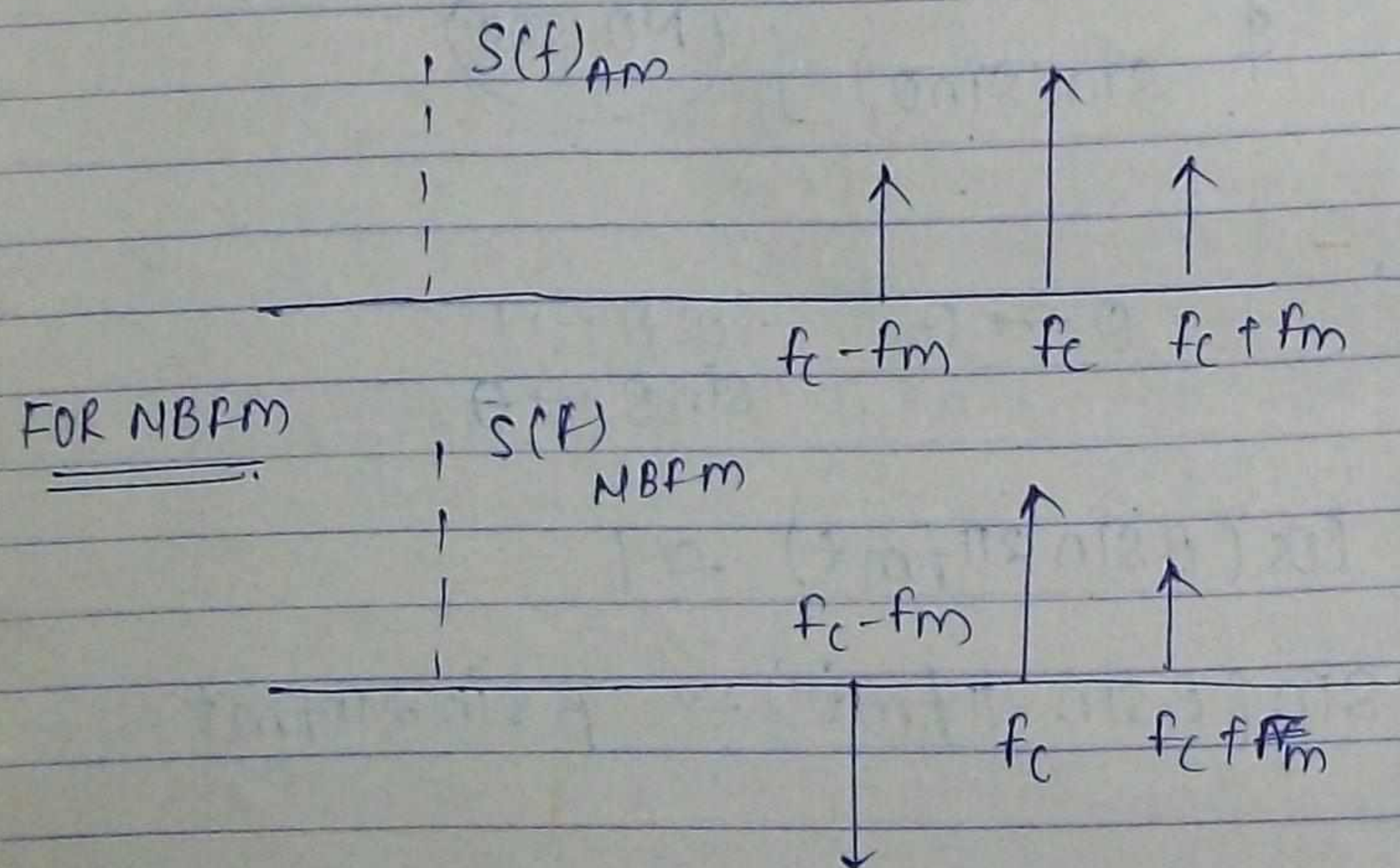
$$S(t)_{NBFM} = V_c \cos 2\pi f_c t - \frac{\beta V_c}{2} \cos (2\pi (f_c - f_m) t) + \frac{\beta V_c}{2} \cos (2\pi (f_m + f_c) t)$$

CARRIER + 2 SIDEBAND

$$BW_{NBPM} = 2F_m$$

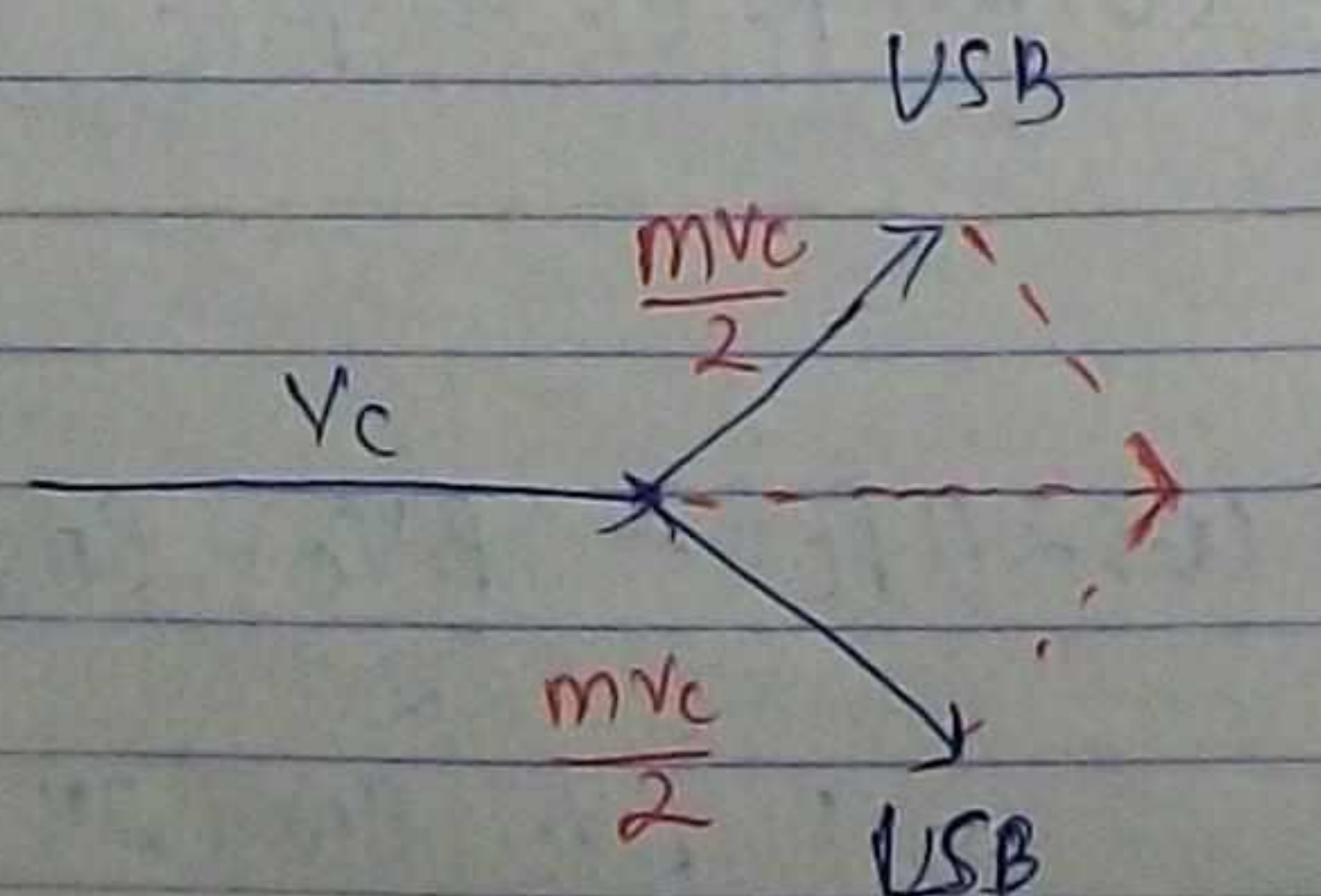
FOR AM -

$$S(t)_{AM} = V_c \cos 2\pi f_c t + \frac{mV_c}{2} \cos 2\pi (f_c + f_m)t + \frac{mV_c}{2} \cos 2\pi (f_c - f_m)t$$



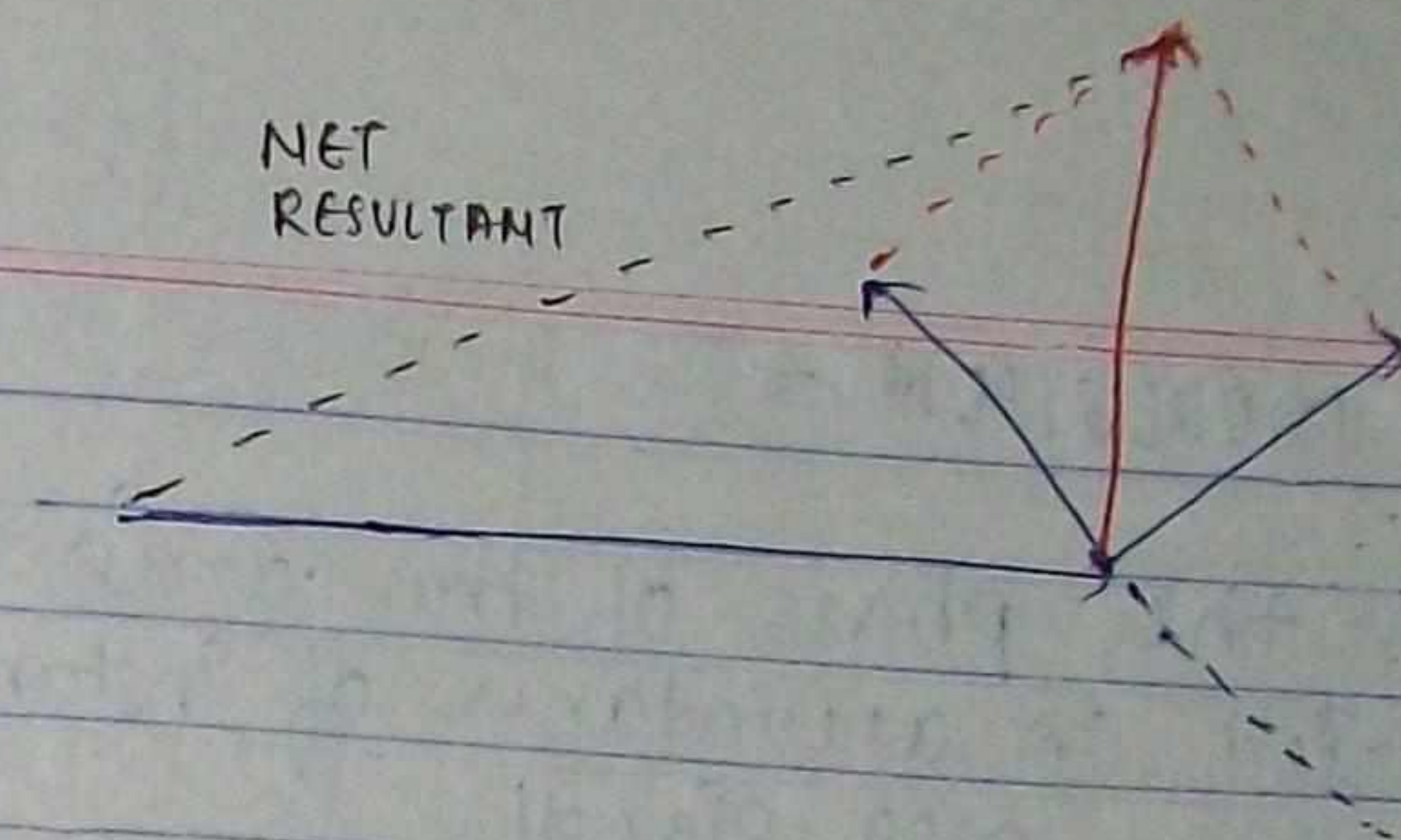
FOR AM

PHASOR DIAGRAM



FOR NBPM -

Phasor diag.



ENVELOPE OF NBPM -

$$\text{Envelope} = \sqrt{V_c^2 + (\beta V_c \sin 2\pi f_m t)^2}$$

$$= V_c \sqrt{1 + \beta^2 \sin^2 2\pi f_m t}$$

POWER IN NBPM -

$$(P_T)_{AM} = P_c \left(1 + \frac{m^2}{2} \right)$$

$$(P_T)_{NBPM} = P_c \left(1 + \frac{\beta^2}{2} \right)$$

⇒ PHASE MODULATION -

- In PM the phase of the carrier signal is varied in accordance of instantaneous amplitude of msg signal.

$m(t) \rightarrow$ message signal.

$$c(t) = V_c \cos 2\pi f_c t$$

$$s(t) = V_c \cos \theta_i(t)$$

$$\text{phase} \rightarrow 2\pi f_c t + k_p m(t)$$

$$k_p = \text{CONST} \quad (\text{rad/V})$$

$$\theta_i(t) = 2\pi f_c t + k_p m(t)$$

$$s(t)_{\text{PM}} = V_c \cos [2\pi f_c t + k_p m(t)]$$

GENERAL FORM

PHASE DEVIATION

$$\Delta \theta_i(t) = \theta_i(t) - 2\pi f_c t = k_p m(t)$$

$$B_p = \Delta \phi = |\Delta \theta_i(t)| = k_p |m(t)|$$

FREQ. DEVIATION -

$$\omega_i(t) = \frac{d}{dt} \theta_i(t)$$

$$\omega_i(t) = 2\pi f_c + k_p \frac{d}{dt} (m(t))$$

$$f_{ci} = \frac{\omega_i(t)}{2\pi}$$

$$\Delta f = |\Delta f_{ci}(t)| = \frac{k_p}{2\pi} \left| \frac{d}{dt} (m(t)) \right|$$

CASE (1): SINGLE TONE MESSAGE SIGNAL.

$$s(t)_{\text{PM}} = V_c \cos [2\pi f_c t + k_p V_m \cos 2\pi f_m t]$$

$$s(t)_{\text{PM}} = V_c \cos [2\pi f_c t + \beta_p \cos 2\pi f_m t]$$

PHASE MODULATION INDEX

$$\beta_p = k_p V_m$$

single tone.

$$\Delta\phi = B_p = k_p V_m$$

$$\Delta\phi \propto V_m$$

$$\Delta f = \frac{k_p}{2\pi} \left| \frac{d}{dt} (V_m \cos 2\pi f_m t) \right|$$

$$\Delta f = \frac{k_p}{2\pi} \cdot |V_m \sin 2\pi f_m t \cdot 2\pi f_m|$$

$$\Delta f = \frac{k_p}{2\pi} V_m \cdot 2\pi f_m$$

$$\Delta f = k_p \cdot V_m \cdot f_m$$

$$\Delta f = B_p \cdot f_m$$

different
— fm & pm waveform look similarly except definition of modulation index.

⇒ COMPARISON —

FM

$$\Delta f = k_f \cdot V_m$$

$$\Delta f \propto V_m$$

$$\Delta\phi = \frac{k_f V_m}{f_m}$$

$$\Delta\phi \propto V_m \propto \frac{1}{f_m}$$

EXPRESSION FOR PM —

$$\begin{aligned} S(t) &= V_c \cos [2\pi f_c t + B_p \cos 2\pi f_m t] \\ &= V_c \cos 2\pi f_c t \cdot \cos (B_p \cos 2\pi f_m t) \\ &\quad - V_c \sin(2\pi f_c t) \cdot \sin(B_p \cos 2\pi f_m t) \end{aligned}$$

NBPM —

$$B_p < 1$$

$$S(t)_{\text{NBPM}} = V_c \cos 2\pi f_c t - B_p V_c \cos 2\pi f_m t \cdot \sin 2\pi f_c t$$

PM

$$\Delta f = k_p \cdot V_m \cdot f_m$$

$$\Delta f \propto V_m \propto f_m$$

$$\Delta\phi = k_p V_m$$

$$\Delta\phi \propto V_m$$

Bessel's eqn

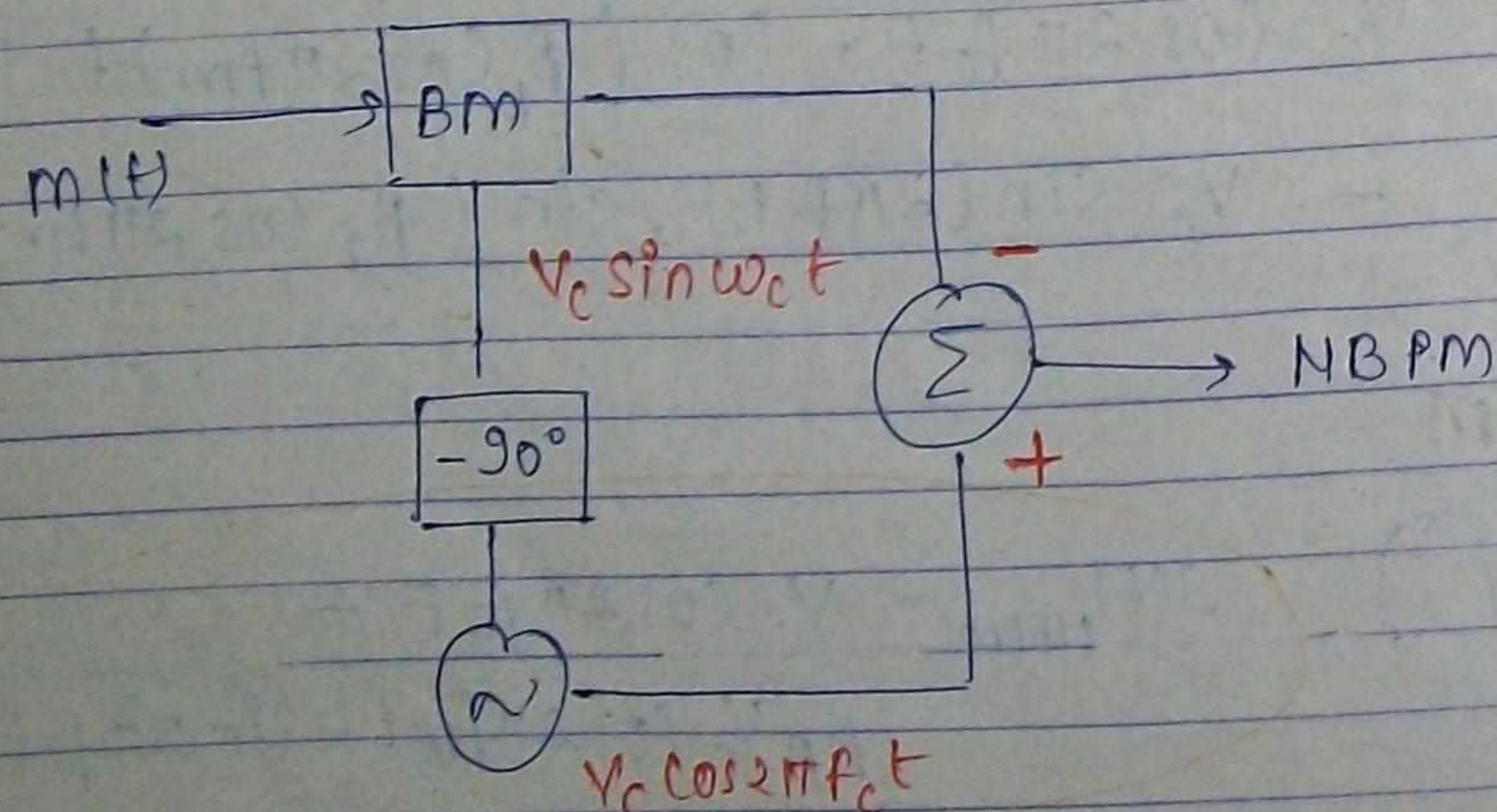
$$S(t)_{NBPM} = V_c \cos 2\pi f_c t - \frac{V_c \beta_p}{2} \sin 2\pi (f_c + f_m) t - \frac{V_c \beta_p}{2} \sin 2\pi (f_c - f_m) t$$

ENVELOPE OF NBPM -

Envelope of NBPM is not constant.

$$\text{Envelope} = V_c \sqrt{1 + \beta_p^2 \cos^2 2\pi f_m t}$$

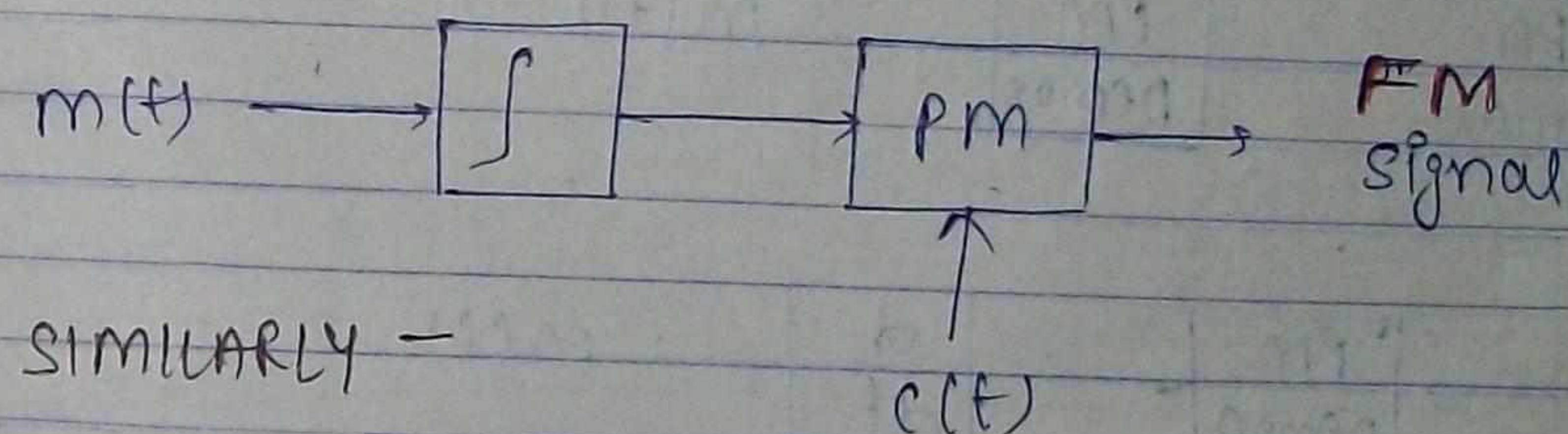
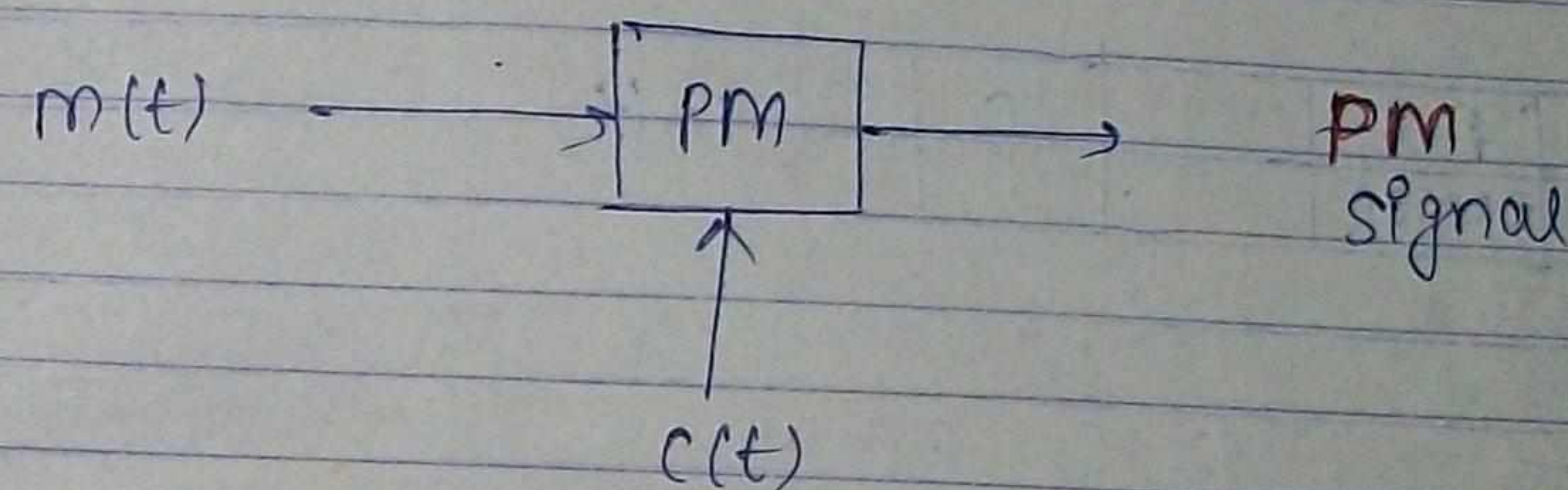
GENERATION OF NBPM -



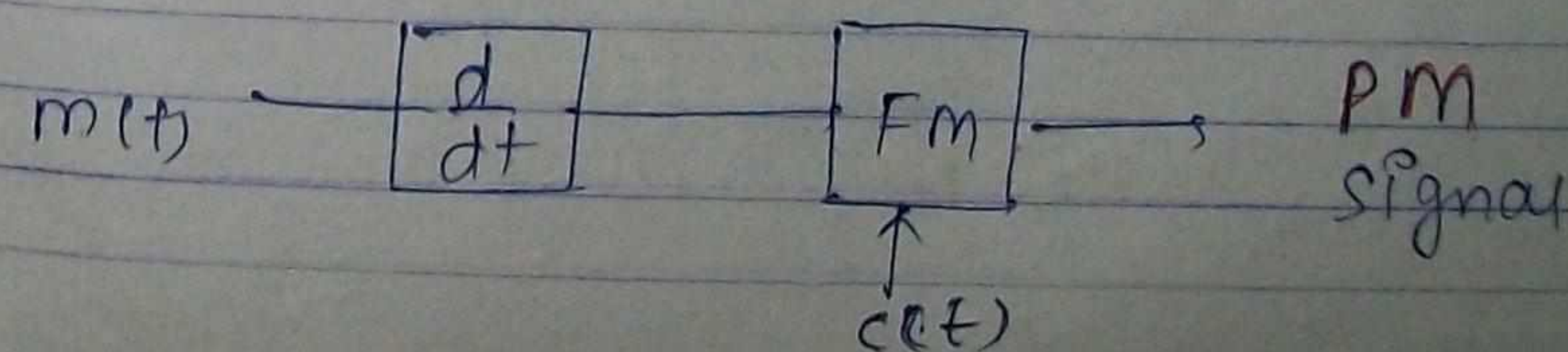
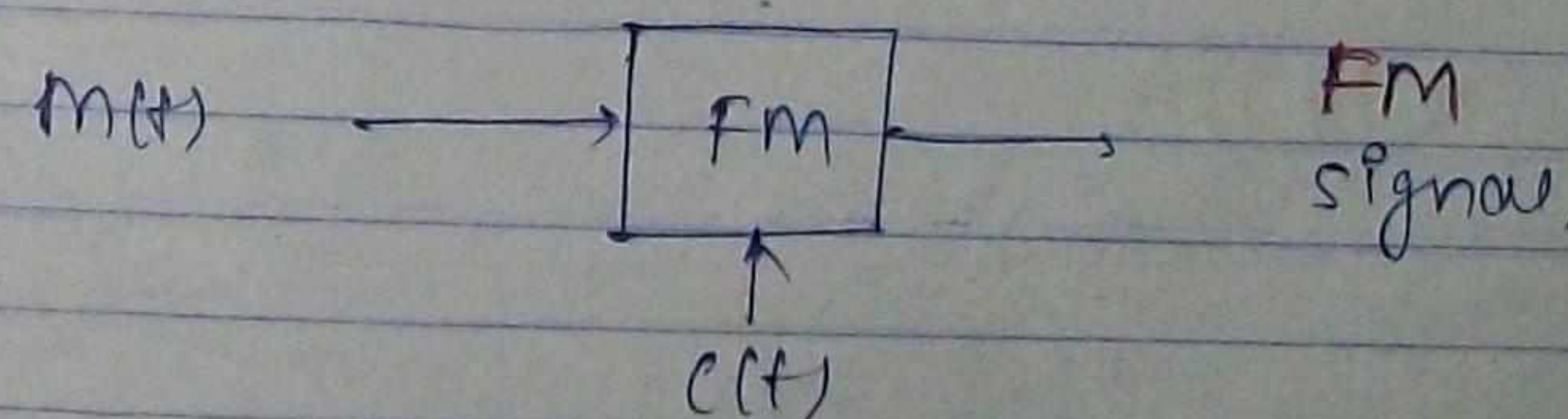
⇒ GENERATION OF FM USING PM MODULATOR -

$$S(t)_{FM} = V_c \cos \left[2\pi f_c t + 2\pi K_f \int m(t) dt \right]$$

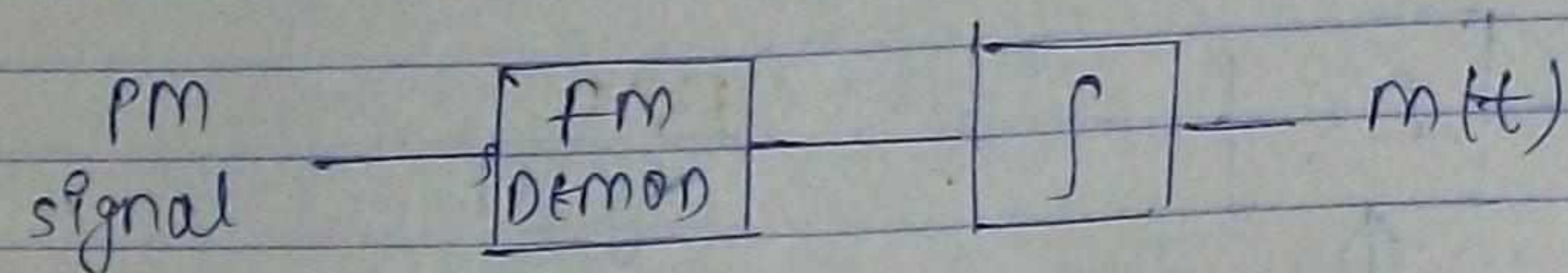
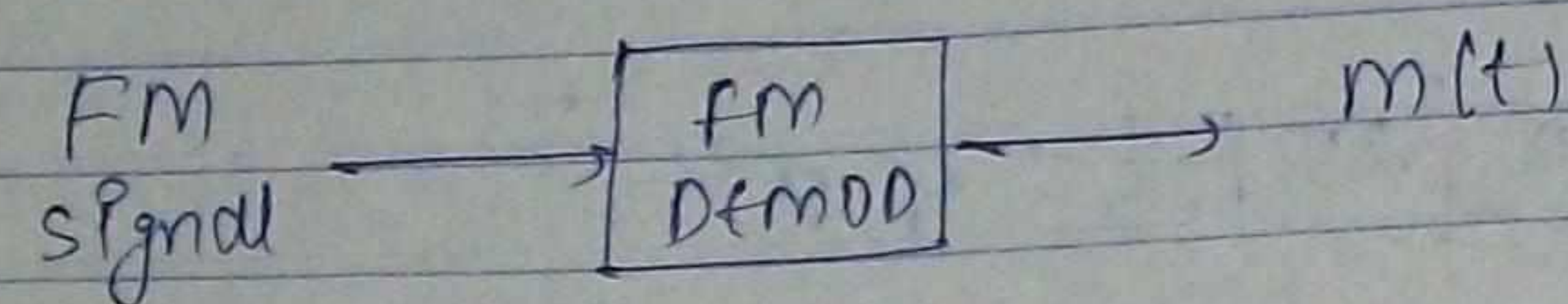
$$S(t)_{PM} = V_c \cos [2\pi f_c t + k_p m(t)]$$



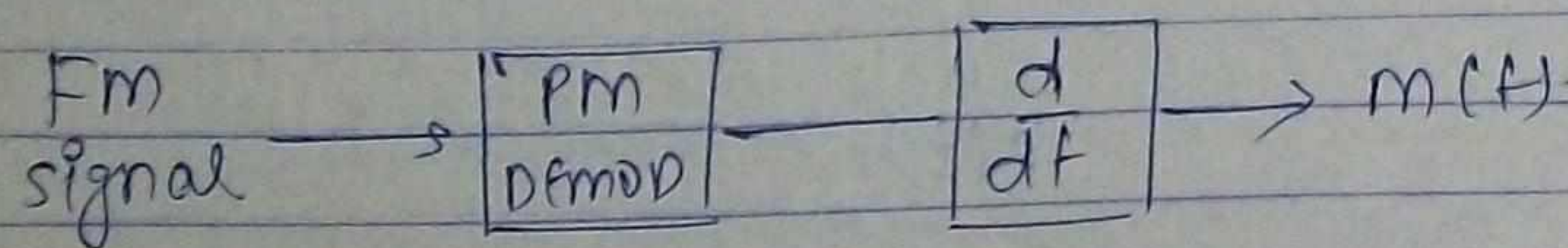
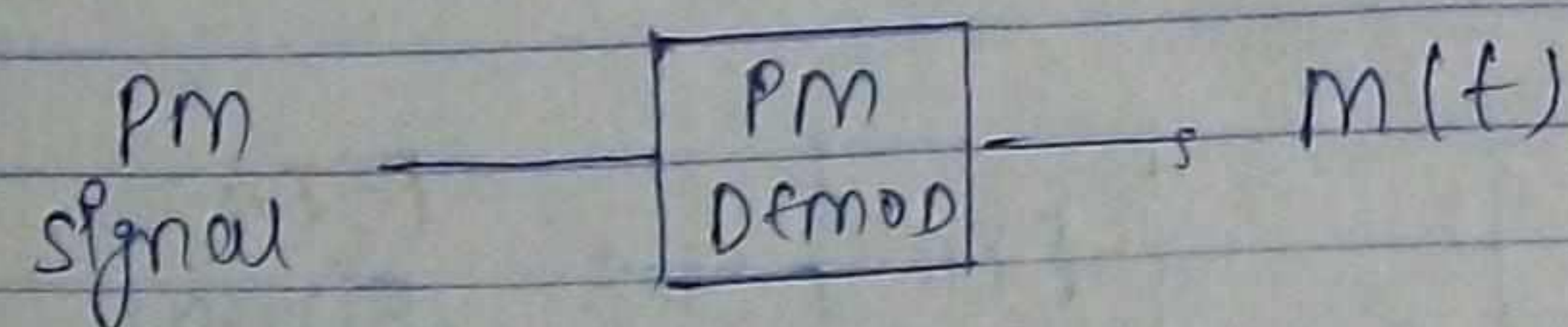
SIMILARLY -



⇒ DEMODULATION OF PM USING FM DEMODULATOR —



SIMILARLY —



⇒ ANALYSIS OF WBFM —

① FOR SINGLE TONE —

$$s(t)_{FM} = V_c \cos [2\pi f_c t + \beta \sin 2\pi f_m t]$$

$$s(t)_{FM} = V_c \operatorname{Re} \left[e^{j(2\pi f_c t + \beta \sin 2\pi f_m t)} \right]$$

$$= V_c \operatorname{Re} \left[e^{j2\pi f_c t} \cdot e^{j\beta \sin 2\pi f_m t} \right]$$

$$= \operatorname{Re} \left[V_c e^{j2\pi f_c t} \cdot e^{j\beta \sin 2\pi f_m t} \right]$$

$$= \operatorname{Re} \left[e^{j2\pi f_c t} \cdot \underbrace{V_c e^{j\beta \sin 2\pi f_m t}}_{x(t)} \right] \quad \text{--- ①}$$

Complex envelope

$$= \tilde{s}(t) = V_c e^{j\beta \sin 2\pi f_m t}$$

$$x(t) = e^{j\beta \sin 2\pi f_m t}$$

$x(t)$ is a periodic with period = $1/f_m$

fourier series

$$c_n = \frac{1}{T} \int_0^T x(t) e^{-j2\pi \frac{n}{T} t} dt$$

$$c_n = f_m \int_0^{1/f_m} \frac{1}{f_m} e^{-j2\pi n f_m t} \cdot e^{jB \sin 2\pi f_m t} dt$$

$$= f_m \int_0^{1/f_m} \frac{1}{f_m} e^{j(B \sin 2\pi f_m t - 2\pi n f_m t)} dt$$

$$\text{let } u = 2\pi f_m t \quad du = 2\pi f_m dt$$

$$c_n = \frac{1}{2\pi} \int_0^{2\pi} e^{j(B \sin u - nu)} du$$

$$c_n = J_n(\beta)$$

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n t/T}$$

$$x(t) = \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi n f_m t}$$

BESSEL'S EQUATION

from (1)

$$s(t)_{fm} = V_c \cdot \text{Re} \left[e^{j2\pi f_c t} \sum_{n=-\infty}^{\infty} J_n(\beta) e^{+j2\pi n f_m t} \right]$$

$$s(t)_{fm} = V_c \sum_{n=-\infty}^{\infty} J_n(\beta) \left[e^{j2\pi (f_c + n f_m) t} \right]$$

$$s(t)_{fm} = V_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(2\pi (f_c + n f_m) t)$$

PROPERTY OF BESSEL FUNⁿ -

$$(1) \quad n \uparrow \quad J_n(\beta) \downarrow$$

$$(2) \quad \sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$$

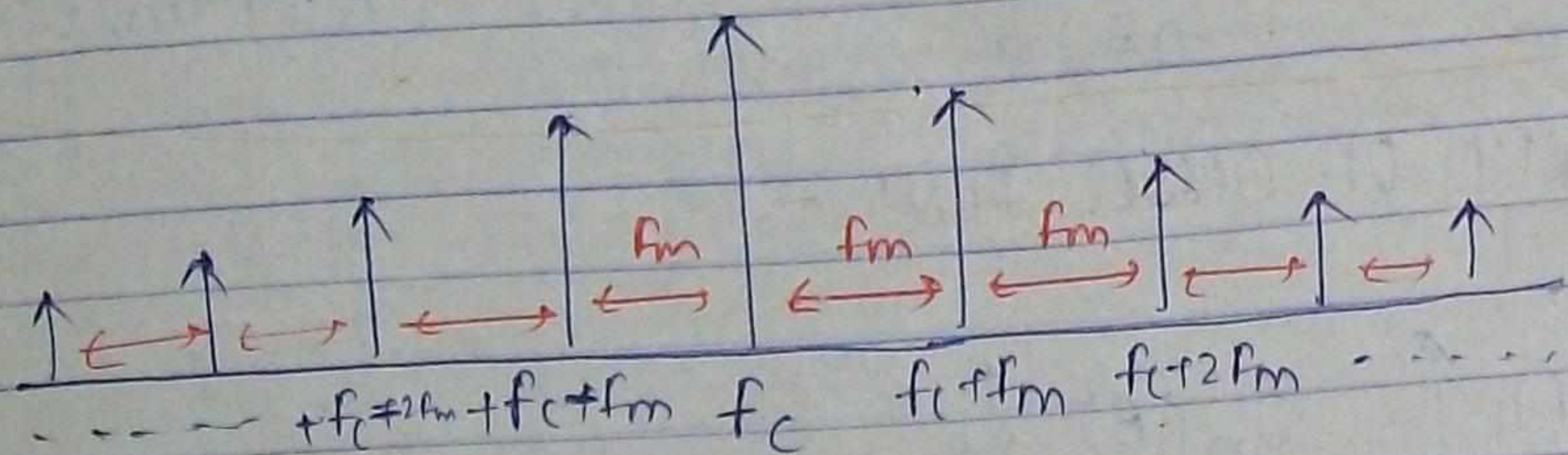
$$(3) \quad \beta < 1 \quad \left. \begin{array}{l} J_0(\beta) = 1 \\ J_1(\beta) = \frac{\beta}{2} \end{array} \right\}$$

Remaining all coefficient will be less than zero

$$(4) \quad J_{-n}(\beta) = (-1)^n J_n(\beta)$$

$$\begin{aligned}
 s(t)_{FM} = & V_c J_0(\beta) \cdot \cos 2\pi f_c t \\
 & + V_c J_1(\beta) \cdot \cos 2\pi (f_c + f_m) t \\
 & + V_c J_2(\beta) \cos 2\pi (f_c + 2f_m) t + \dots \\
 & + V_c J_{-1}(\beta) \cdot \cos 2\pi (f_c - f_m) t \\
 & + V_c J_{-2}(\beta) \cdot \cos 2\pi (f_c - 2f_m) t + \dots
 \end{aligned}$$

|s(t)|



- WBfm contain ∞ sidebands.
- Max ~~theoretical~~ theoretical BW = ∞ .
- In WBfm carrier and sideband amplitude depends on modulation index (β).
- In this only significant sidebands are transmitted

- A significant sideband have amplitude $\geq 1\%$ unmodulated carrier amplitude.
- Let N is number of significant sideband then bandwidth of wideband FM is $2Nf_m$

$$BW_{WBfm} = 2Nf_m \quad \text{PRACTICAL BW}$$

$$BW_{WBfm} = 2(\Delta f + f_m) \quad \text{CARSON'S BW}$$

$$BW_{WBfm} = 2f_m(1 + \beta)$$

Then, $N \approx (1 + \beta)$ no. of sideband.

- Max. BW used in WBfm = 200 kHz (PRACTICALLY)
- $= \infty$ (THEORETICALLY)

- In WBfm

$$\text{guard Band BW} = 20 \text{ kHz.}$$

- Due to this no adjacent channel interference present in FM.

POWER IN WBFM -

$$P_T = P_C + P_{SB}$$

$$P_C = \frac{V_c^2}{2} J_0^2(B)$$

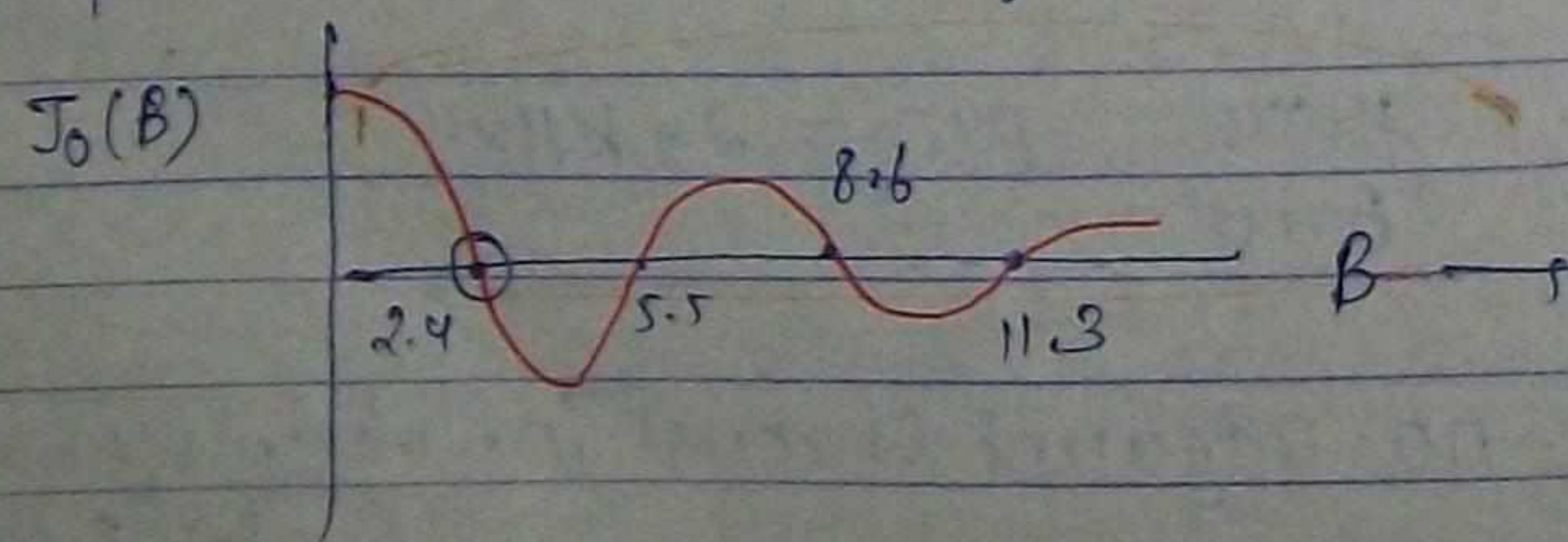
$$P_{SB} = \frac{V_c^2}{2} \left[J_1^2(B) + J_2^2(B) + \dots + J_n^2(B) + J_{-1}^2(B) + J_{-2}^2(B) + \dots + J_{-n}^2(B) \right]$$

$$P_T = \frac{V_c^2}{2} \left[\sum_{n=-\infty}^{\infty} J_n^2(B) \right]$$

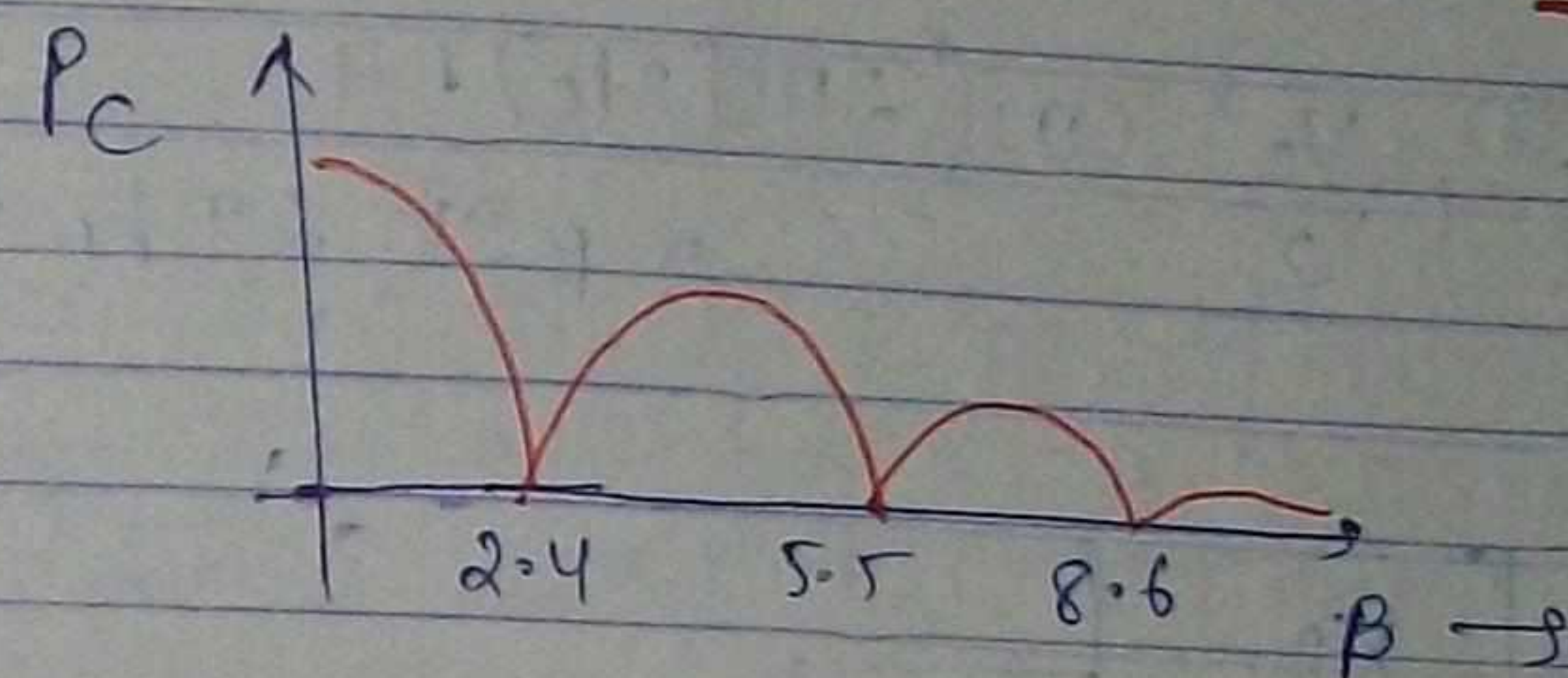
$$P_T = \frac{V_c^2}{2} \quad \text{CONSTANT}$$

- $B \uparrow \quad P_S \uparrow \quad P_C \downarrow \quad P_T = \text{CONST.}$

P_T is independent of B .



- In WBFM at certain modⁿ index, carrier power will become zero. The corresponding modⁿ index are known as eigen value.



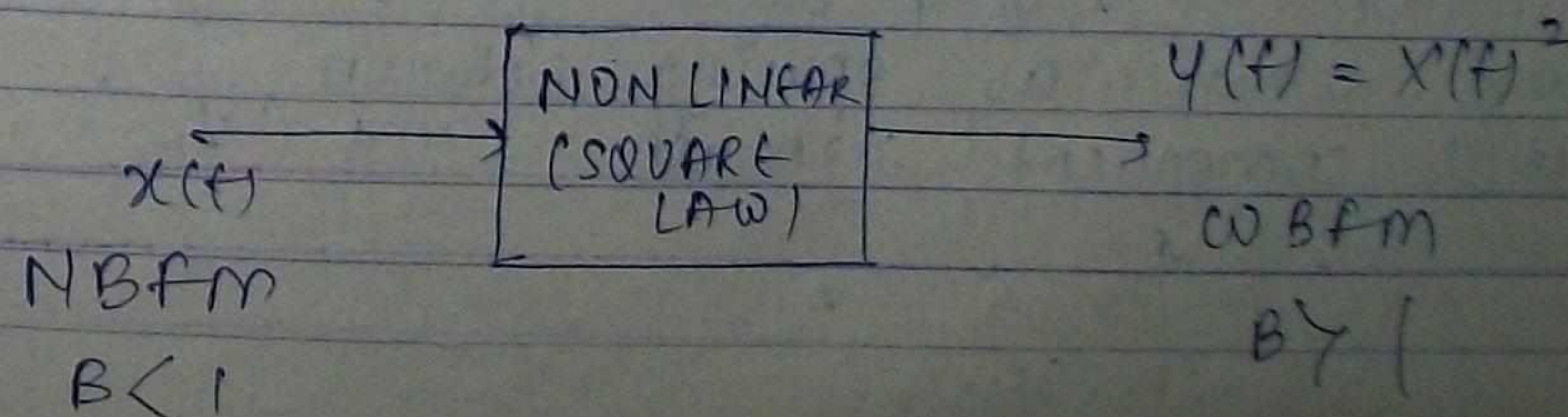
$$\text{amplitude of } k^{\text{th}} \text{ sideband} = V_c J_k(B)$$

- WBFM is used in radio broadcasting.

- NBFM is used in wireless (police) commⁿ.

- To convert NBFM into WBFM freq. multipliers are used.

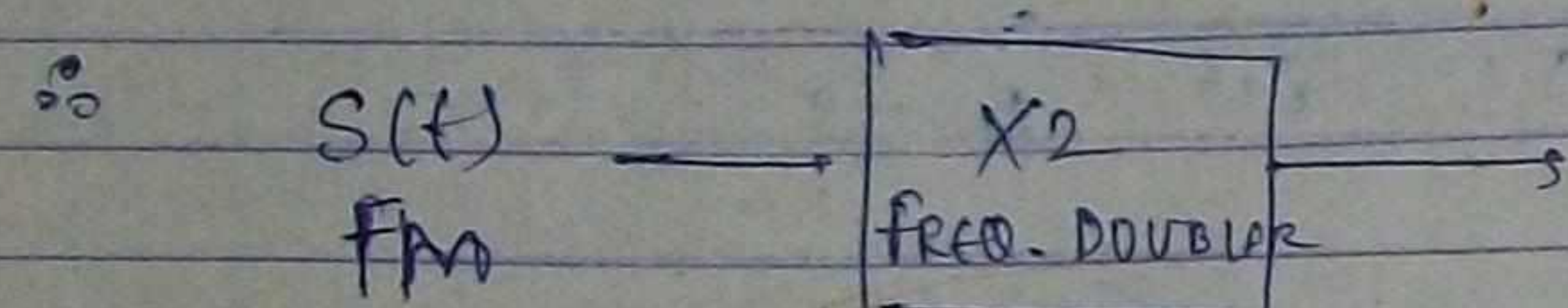
⇒ CONVERSION OF NBFM TO WBFM -



$$y(t) = \frac{2}{S(t)}$$

$$= V_c^2 \cos^2(2\pi f_c t + \beta \sin 2\pi f_m t)$$

$$= \frac{V_c^2}{2} + \frac{V_c^2}{2} \cos(2\pi(2f_c)t + 2\beta \sin 2\pi f_m t)$$



$$f_m \longrightarrow f_m$$

$$f_c \longrightarrow 2f_c$$

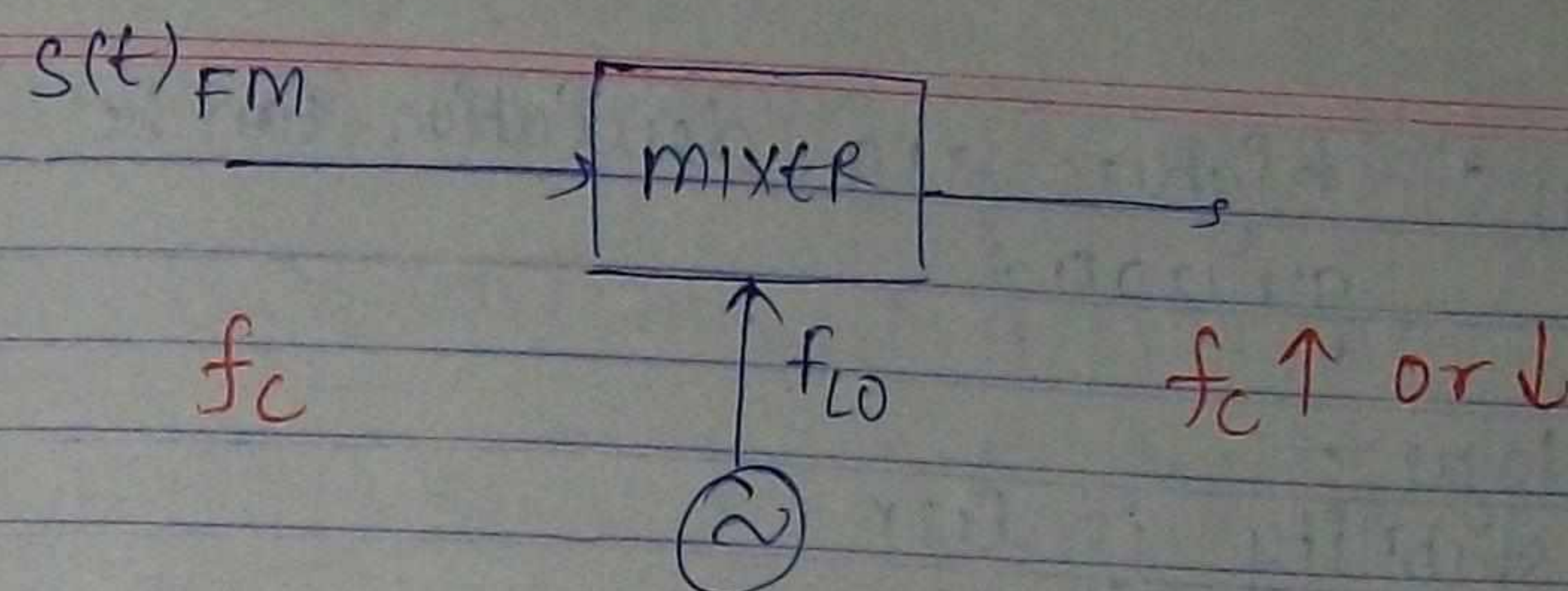
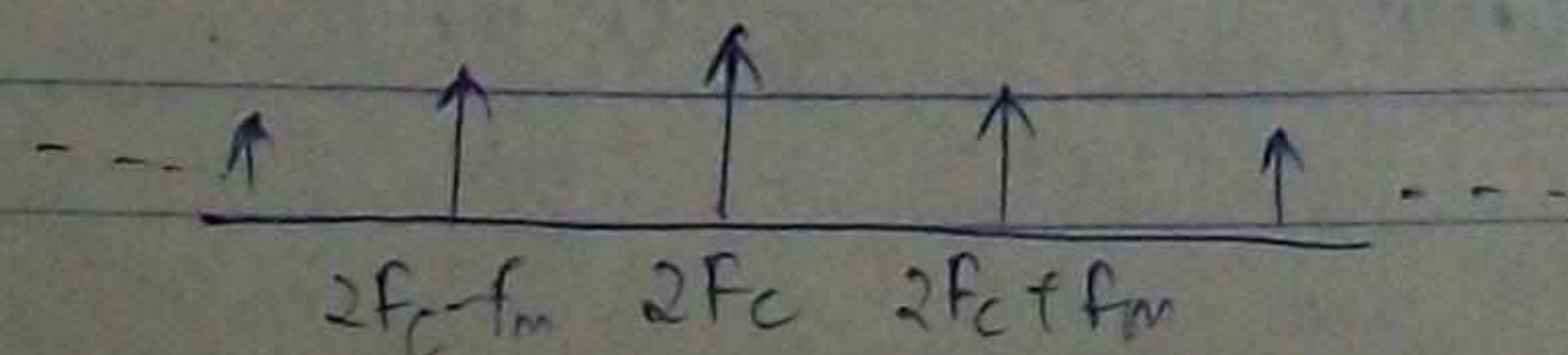
$$\beta \longrightarrow 2\beta$$

$$\Delta f \longrightarrow 2\Delta f$$

$$BW \longrightarrow \approx 2BW$$

Spectrum of O/P —

- When FM signal is passed through a freq. multiplier then only carrier freq. will change where as spacing between spectral component remain same (f_m)



- When FM signal is passed through a mixer only carrier freq. will change ($\uparrow$ or $\downarrow$) but other parameters remain same.

⇒ GENERATION OF FM —

① Direct method.

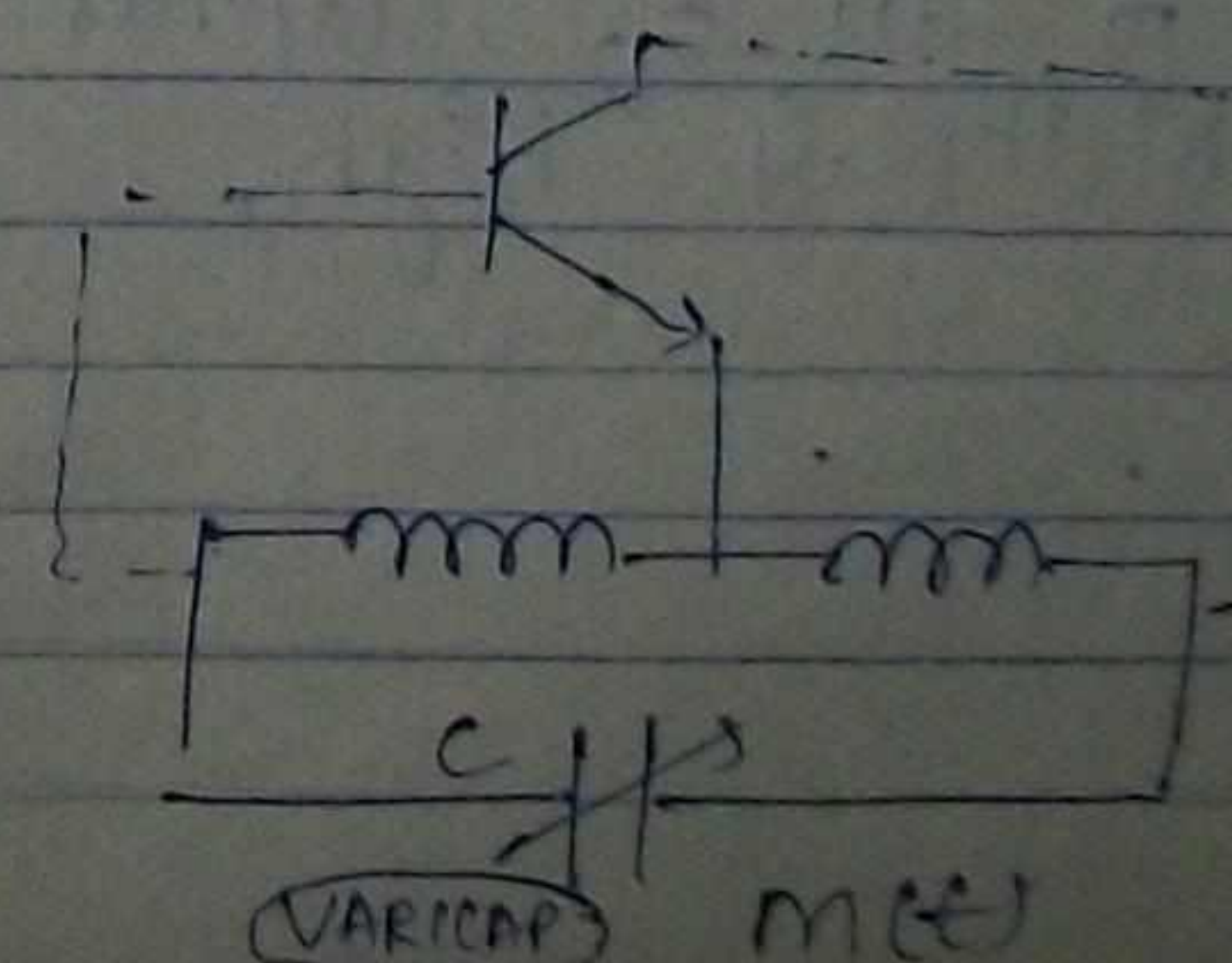
② Indirect method. (or) Armstrong method.

① DIRECT METHOD — (REACTANCE TUBE METHOD)

- In this method LC oscillator is used to generate FM signal

- A varactor diode is used to change carrier freq. when msg. signal amplitude is changing.

$$f_c = \frac{1}{2\pi \sqrt{L_{eq} \cdot C}}$$

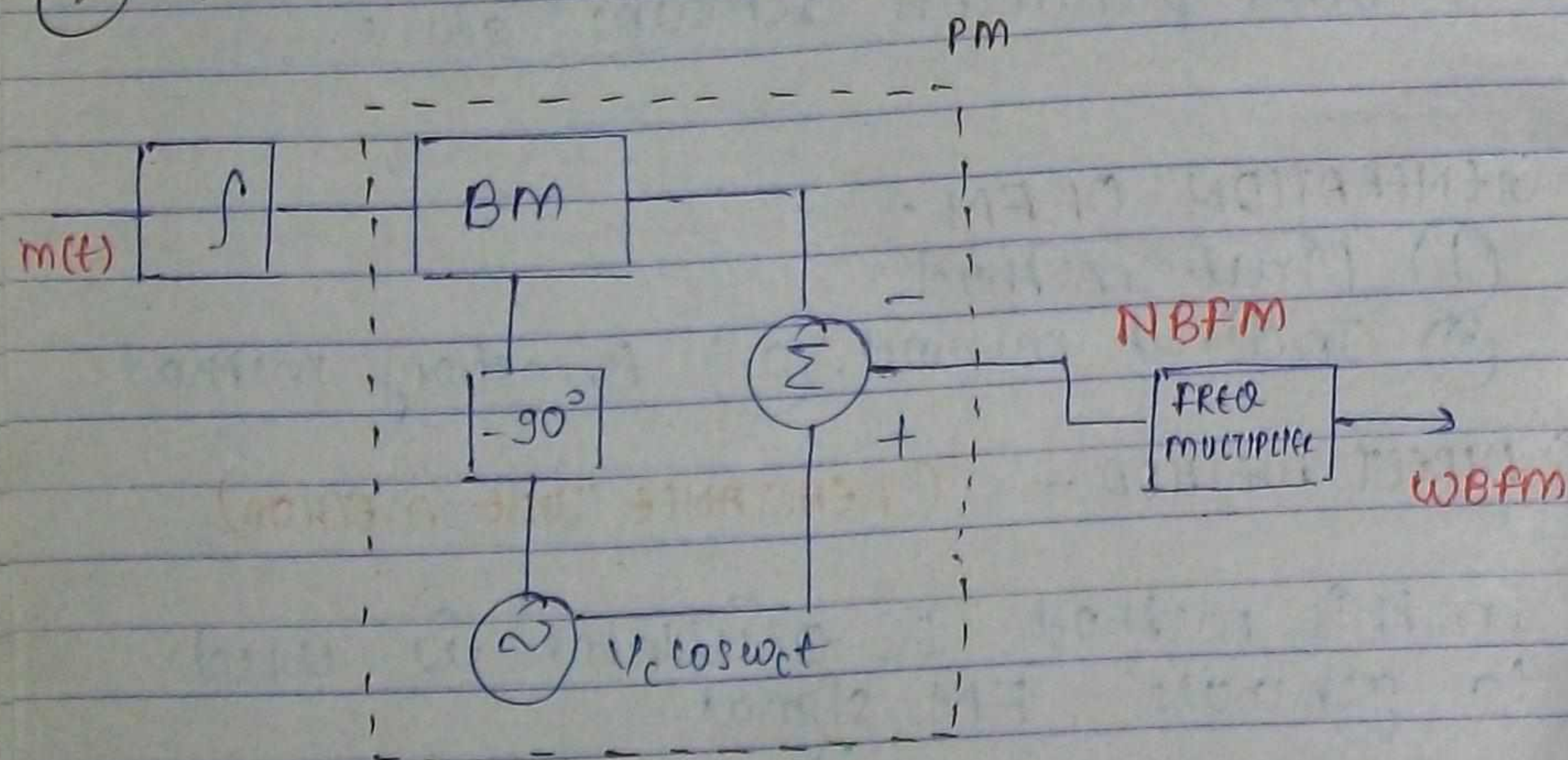


- Advantages: higher freq. deviation can be generated.

- Disadvantages -

1. freq. stability is less
2. Non linearity is more.

② INDIRECT METHOD -



- Advantage:

1. linearity is more
2. Due to use of crystal oscillator freq. stability is high.

Direct method.

$$S(t) = V_c \cos [2\pi f_c t + \beta \sin 2\pi f_m t]$$

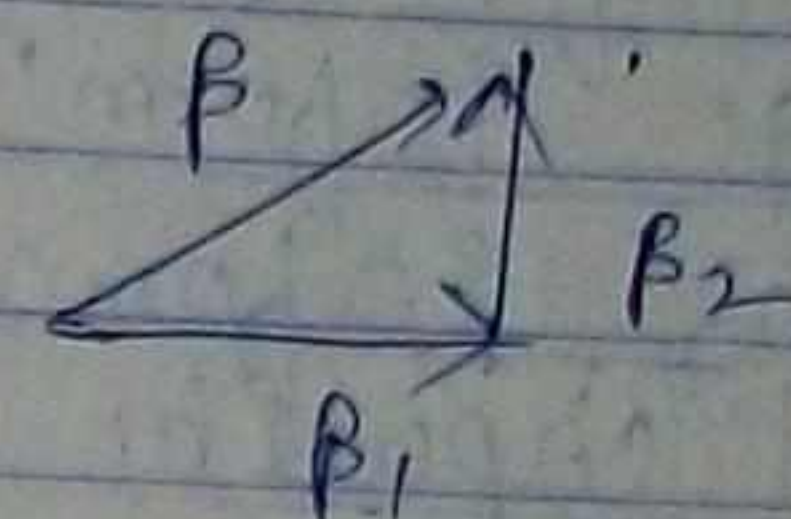
SINGLE TONE

$$S(t) = V_c \cos [2\pi f_c t + \beta_1 \cos 2\pi f_{m1} t + \beta_2 \sin 2\pi f_{m2} t]$$

MULTI TONE

$$\beta = \Delta\phi = \sqrt{\beta_1^2 + \beta_2^2}$$

$$\Delta f = \beta f_m$$

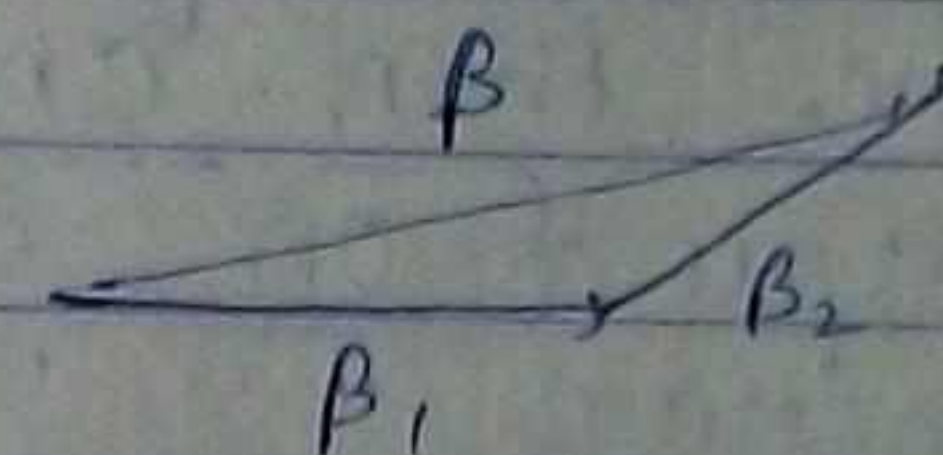


MULTITONE

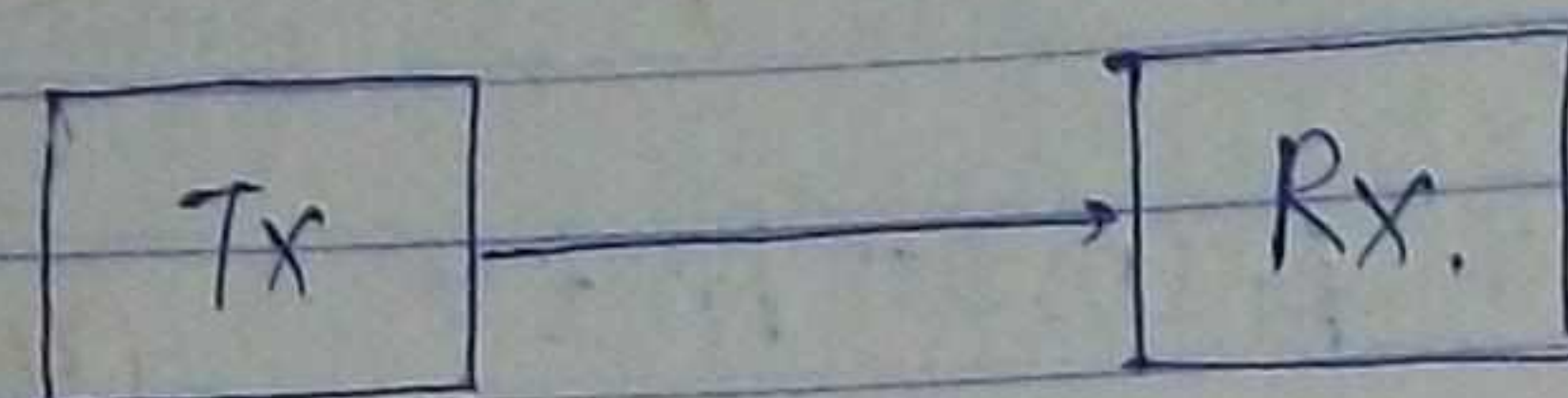
$$S(t) = V_c \cos (2\pi f_c t + \beta_1 \cos 2\pi f_{m1} t + \beta_2 \cos 2\pi f_{m2} t)$$

$$\Delta\phi = \beta \approx \beta_1 + \beta_2$$

$$\Delta f = \beta \cdot f_{\max}$$



INFORMATION THEORY



- ① Amount of information
- ② Total information
- ③ average informⁿ or entropy
- ④ Rate of informⁿ
- ⑤ channel capacity theorem

① AMOUNT OF INFORMATION -

- when a msg. signal is transmitted from a Tx. then amount of information received depend on probability of transmitting message.

let probability of msg = P_k

then, amount of information received from msg = I_k

$$I_k = \log_2 \frac{1}{P_k} \text{ bits}$$

$$= \log_{10} \frac{1}{P_k} \text{ Decit}$$

$$= \log_e \frac{1}{P_k} \text{ nat}$$

$$I_k = 3.32 \log_{10} \frac{1}{P_k} \text{ bits}$$

Ex. A msg is transmitted with $P_k = 1/2$ then
① amount of informⁿ is?

Solⁿ:

$$I = \log_2 \frac{1}{1/2} \text{ bits} = \underline{1} \text{ bit}$$

when $P_k = 1/4$

$$I = \log_2 \frac{1}{1/4} = \underline{2} \text{ bits}$$

when $P_k = 1/16$

$$I = \log_2 \frac{1}{1/16} = \underline{4} \text{ bits}$$

Ex. 2

② TOTAL INFORMATION -

$L \rightarrow$ no. of message.

$P_1, P_2, P_3, \dots, P_n$ probability of msg.

$$\text{TOTAL INFO} = L \sum_{i=1}^L P_i \log_2 \frac{1}{P_i}$$

$$P_1 = 1/2 \longrightarrow I_1 = 1$$

$$P_2 = 1/4 \longrightarrow I_2 = 2$$

$$P_3 = 0 \longrightarrow I_3 = \infty$$

$$P_4 = 1 \longrightarrow I_4 = 0$$

$$\# \text{ If } P_1 \longrightarrow I_1$$

$$P_2 \longrightarrow I_2$$

$$P_1 > P_2 \longrightarrow I_1 < I_2$$

(3) AVERAGE INFORMATION (OR) ENTROPY (H) -

$$\text{ENTROPY (H)} = \frac{\text{TOTAL INFORMATION}}{\text{NO. OF MSG. TX.}}$$

$$= \sum_{i=1}^L P_i \log_2 \frac{1}{P_i}$$

bits/msg

bits/symbol

Ques. 3 msg. are Tx. with prob. of $1/2, 1/4, 1/4$
then entropy is?

Solⁿ:

$$H = \sum_{i=1}^3 P_i \log_2 \frac{1}{P_i}$$

$$H = \frac{1}{2} \log \frac{1}{1/2} + \frac{1}{4} \log \frac{1}{1/4} + \frac{1}{4} \log \frac{1}{1/4}$$

$$H = 3/2 \text{ bits/msg.}$$

Ques. 4 symbols are Tx. with prob of $1/2, 1/4, 1/8, 1/8$
then H is?

Solⁿ:

$$H = \frac{1}{2} \times 1 + \frac{1}{4} \times 2 + \frac{1}{8} \times 3 + \frac{1}{8} \times 3$$

$$H = 7/4 \text{ bits/symbol.}$$

Ques. 2 msg are transmitted which are equiprobable
then max H is?

Solⁿ:

$$H = \frac{1}{2} \log \frac{1}{1/2} + \frac{1}{2} \log \frac{1}{1/2}$$

$$= 1 \text{ bit/msg.}$$

Ques. If M no. of msg. are transmitted with equiprob. then max. prob. is: H is?

Soln:

$$H_{\max} = \log_2 M$$

— for maximum entropy all msg. should be transmitted with equiprobability.

* Ques. 2 msg are Tx which are mutually exclusive then max. entropy is.

Soln:

$$\left. \begin{array}{l} \text{let } P_1 \rightarrow p \\ P_2 \rightarrow 1-p \end{array} \right\}$$

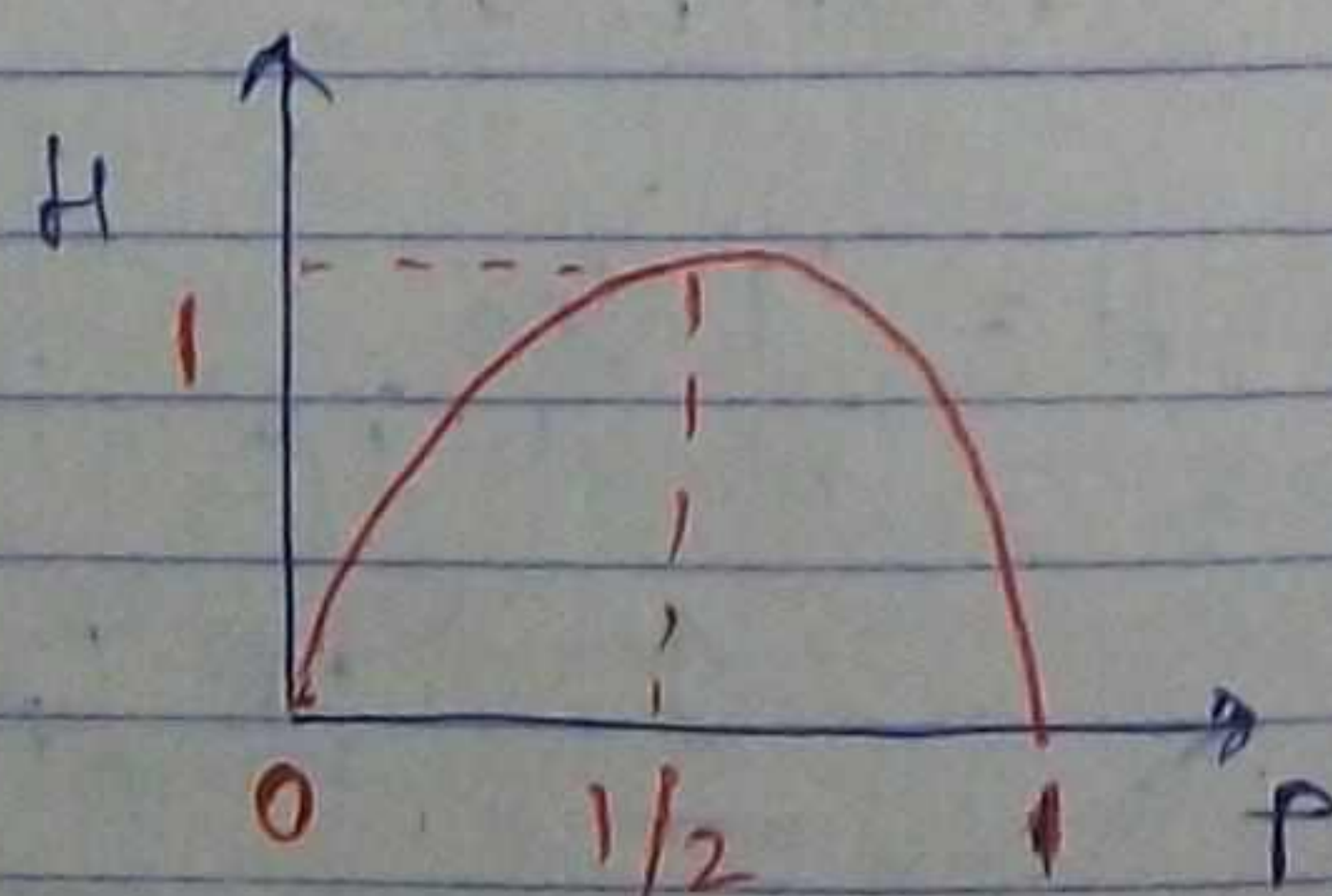
BINARY
SYMMETRICAL
CHANNEL

$$H = p \log_2 \frac{1}{p} + (1-p) \log_2 \frac{1}{1-p}$$

$$\frac{dH}{dp} = 0$$

$$p = \frac{1}{2}$$

$$\therefore H_{\max} = 1$$



(4) RATE OF INFORMATION (R) —

BIT RATE

DATA RATE

$$R = \gamma H$$

bits/sec (bps)

γ = no. of msg Tx. second
(or)
symbol.

28/11/11

RECEIVERS

- Radio receiver are used to received transmitted signal and amplifies then demodulated the received signal.

mainly two types:

- ① TRF receiver
- ② Superheterodyne Rx.

RECEIVER CHARACTERISTICS -

- ① Sensitivity
- ② Selectivity
- ③ Fidelity
- ④ Image freq. rejection.

① SENSITIVITY -

It is the ability of radio receiver to respond weakest signal present at the receiver I/P.

In radio receivers sensitivity depends on RF amplifier.

② SELECTIVITY -

Selectivity is the ability of receiver to select required signal and to reject unwanted signal.

Selectivity is mostly depends on IF amplified.

③ FIDELITY -

fidelity represent faithful reproduction of I/P signals.
fidelity mostly depends on AF amp.

④ IMAGE FREQ. REJECTION -

In superheterodyne receiver image freq. may present due to poor selectivity

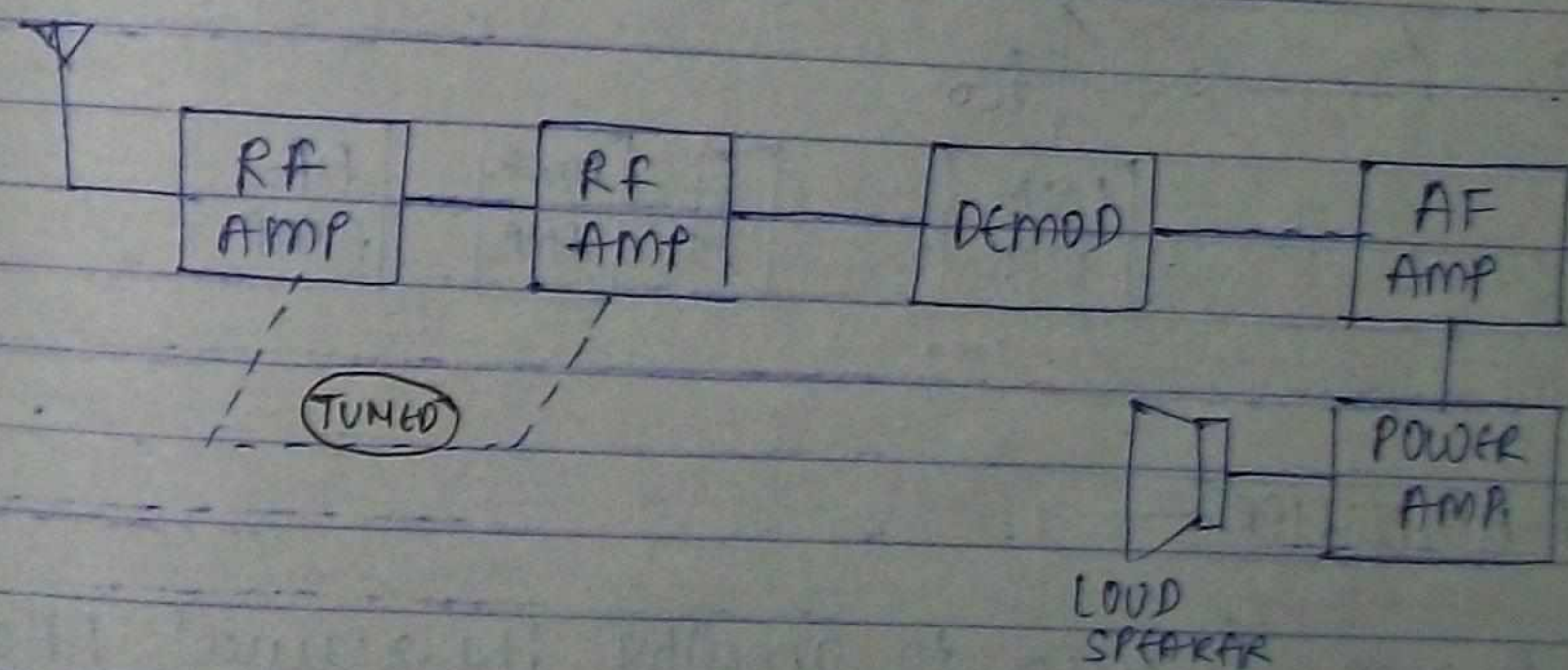
message freq $\rightarrow f_s \rightarrow I_f$ (intermediate freq)

image freq.

$$f_{si} = f_s + 2IF$$

$$f_{si} = f_{LO} + I_f$$

$\Rightarrow$ TRF RECEIVERS -

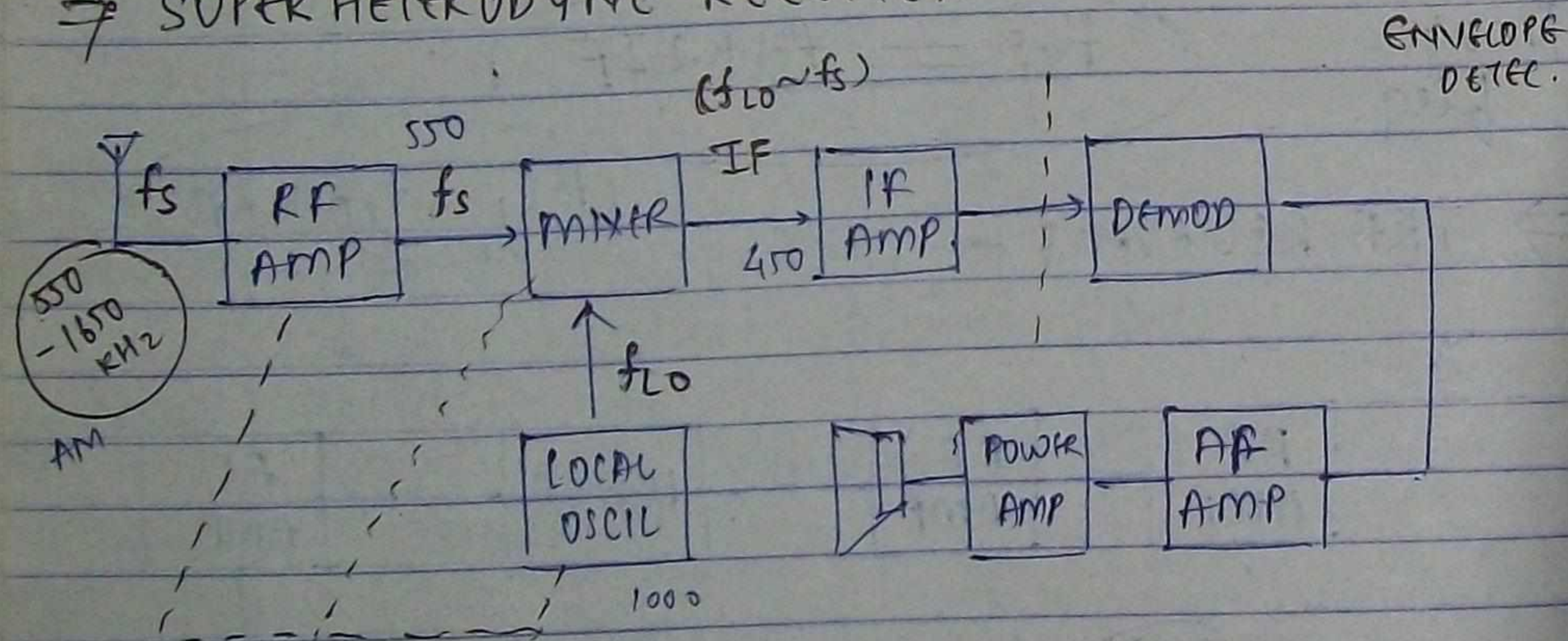


- The basic principle of TRF receiver is amplified no. of stages then demodulate the msg. signal.

- Disadvantage of PRF receiver is variable BW. and gain and stability is less.
- To overcome this disadvantages superheterodyne receiver is used.

- advantages:
 - ① RX design is simple.
 - ② No image freq.
 - ③ sensitivity is good.

⇒ SUPER HETERODYNE RECEIVER —



① RF AMPLIFIER —

- RF amp. is used to amplify the received RF signal.
- RF amp. have tuned circuit which is used to select required freq.

Tuned amp are class C (Narrowband) amp.

advantages of Rfamp:

- ① Sensitivity will improve.
- ② Selectivity will improve.
- ③ Image frequency are rejected.
- ④ Adjacent channel interference is avoided.
- ⑤ Local oscillator re-radiation is avoided.

$$C \propto \frac{1}{f_0^2}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$\frac{C_{\max}}{C_{\min}} = \left(\frac{f_{\max}}{f_{\min}} \right)^2$$

IN A TUNED CIRCUIT

for Rf amplifier

$$f_s \min = 550 \text{ kHz}$$

$$f_s \max = 1650 \text{ kHz}$$

$$\frac{C_{\max}}{C_{\min}} = \left(\frac{f_s \max}{f_s \min} \right)^2 = \left(\frac{1650}{550} \right)^2$$

$$\frac{C_{\max}}{C_{\min}} = \frac{9}{1}$$

② MIXER —

Mixer is used to down convert incoming signal freq. to a fixed freq. known as intermediate freq (IF).

- mixer is a non-linear device, which will provide difference b/w ($f_{LO} \sim f_s$)

③ LOCAL OSCILLATOR -

LO is a LC oscillator which will provide f_{LO} freq. to mixer depending on incoming signal freq.

In most of radio receivers LO frequency is greater than incoming signal freq f_s to avoid tracking difficulties.

The intermediate freq. used in AM receiver is 450 kHz or 455 kHz.

CASE ①: $f_{LO} > f_s$

$$f_{LO} = f_s + IF$$

$$f_s \rightarrow 550 - 1650 \text{ kHz}$$

$$IF = 450 \text{ kHz}$$

$$f_{LO(\min)} = f_{s(\min)} + IF \\ = 550 + 450 = \underline{1000 \text{ kHz}}$$

$$f_{LO(\max)} = f_{s(\max)} + IF = \underline{2100 \text{ kHz}}$$

$$\frac{C_{\max}}{C_{\min}} = \left(\frac{f_{LO(\max)}}{f_{LO(\min)}} \right)^2 = \left(\frac{2100}{1000} \right)^2$$

$$\frac{C_{\max}}{C_{\min}} = \frac{4.41}{1} \checkmark$$

CASE ②: $f_{LO} < f_s$

$$f_{LO} = f_s - IF$$

$$f_{LO(\min)} = f_{s(\min)} - 450 = 100 \text{ kHz}$$

$$f_{LO(\max)} = f_{s(\max)} - 450 = 1200 \text{ kHz}$$

$$\frac{C_{\max}}{C_{\min}} = \left(\frac{f_{LO(\max)}}{f_{LO(\min)}} \right)^2 = \left(\frac{12}{1} \right)^2$$

$$\frac{C_{\max}}{C_{\min}} = \frac{144}{1} \checkmark$$

- In radio receiver $f_{LO} > f_s$ to cover entire freq. range with a single capacitor.

- In radio receiver 3-point tracking is achieved with PADDER capacitor.

(c) IF AMPLIFIER -

IF amp. is used to amplify fixed freq. known as IF. It is a double tuned amp. which will provide most of selectivity in radio receiver.

- If amp. tuned circuit will not reject image freq.

- Due to image freq. double spotting is present in radio receiver.

- Image freq. will present due to poor front end selectivity of the receiver. (R.F. amp)

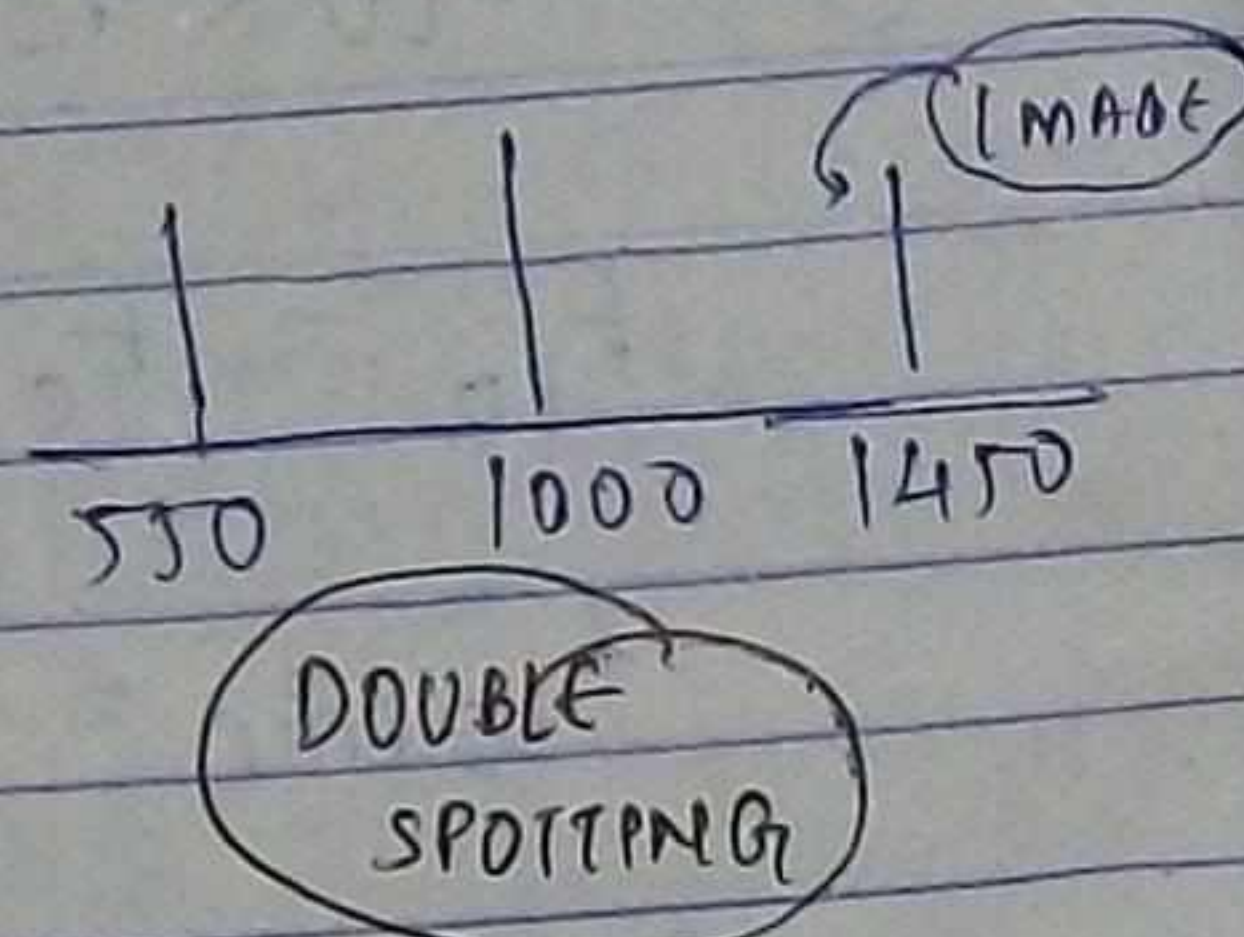


IMAGE
FREQ.
REJECTION
RATIO

$$\alpha = \sqrt{1 + Q^2 f^2}$$

Q = quality factor

$$f = \frac{f_{si}}{f_s} - \frac{f_s}{f_{si}}$$

SELECTION OF IF -

(i) WHEN IF IS VERY HIGH -

Selection of IF depends on:

(*) If avoid image freq but selectivity will become poor.

(ii) WHEN IF IS VERY LOW -

If IF is ~~very~~ low then selectivity will improve. image freq rejection will decrease.

If IF is very low then side band will cut off.

SELECTED IF -

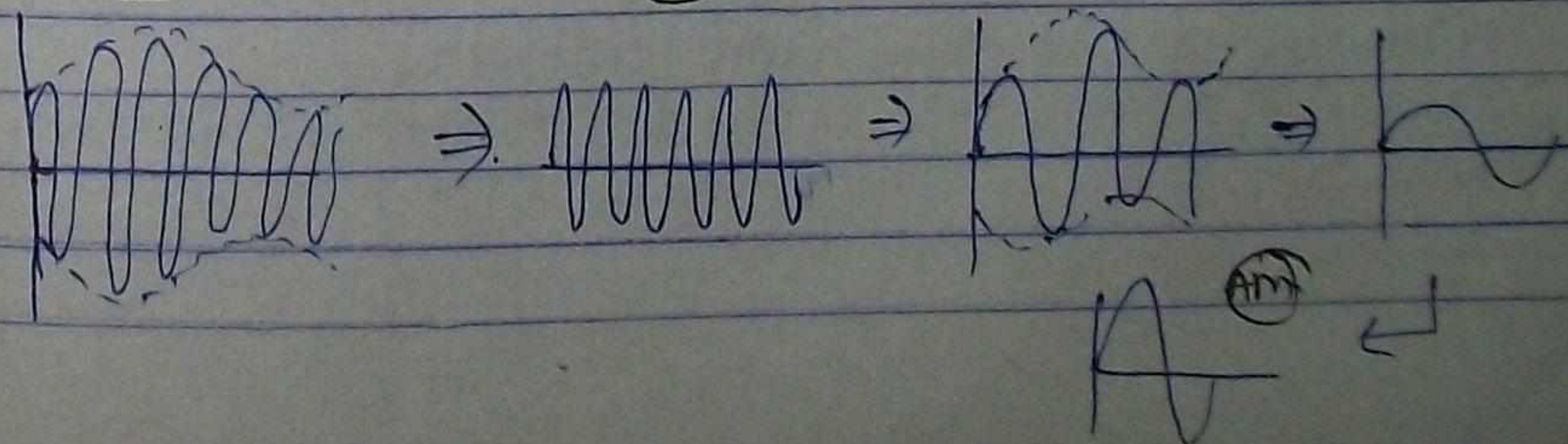
Selected IF must not fall in the tuning range of receiver.

(Am)

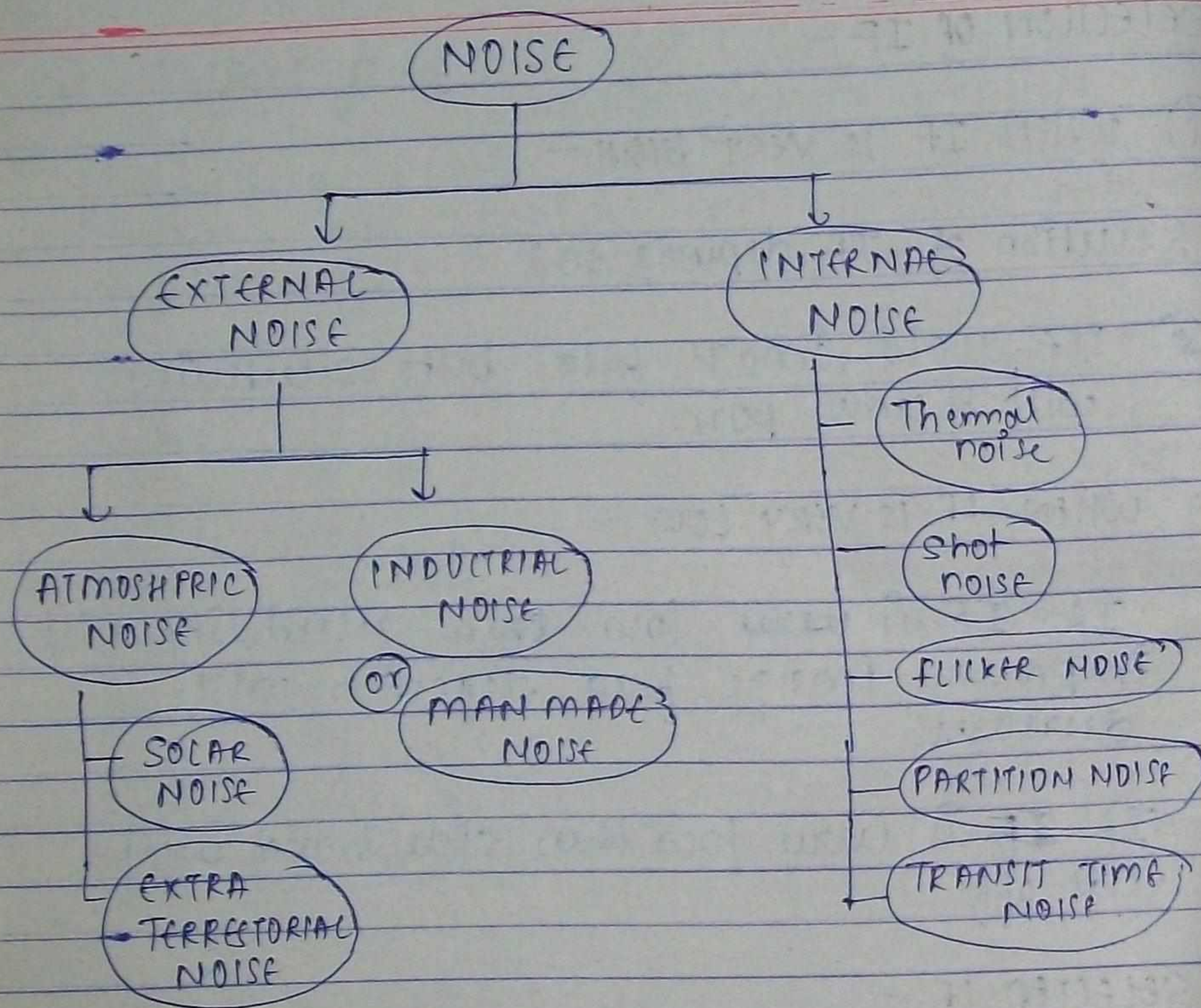
(LO)

(IF)

(DEMOD)



NOISE



- man made noise is vertically polarised.
- for communication ~~the~~ ~~use~~ horizontal polarisation is used.

① THERMAL NOISE -

Thermal noise is due to random motion of electrons.

$$P_n = KTB$$

$B \rightarrow BW$

② SHOT NOISE -

Due to DC current present in active device.

$$i_n^2 = 2 I_0 \Delta f$$

$\Delta f \rightarrow BW$
 $I_0 \rightarrow$ dc current

③ FLICKER NOISE -

Due to surface irregularity of the active devices.

$$\text{flicker noise} \propto \frac{1}{\text{freq.}}$$

flicker noise will present upto 500 Hz.

④ PARTITION NOISE -

Partition noise is due to junction present in active device such as diode, BJT.

⑤ TRANSIT TIME NOISE -
Transit time noise will present in higher frequencies.

⇒ THERMAL NOISE -

- mostly affect in communication system

500 Hz — 10^{13} Hz.

- also called white Gaussian noise,
Johnson's noise

PSD (watts/Hz)

$$\text{PSD} = \frac{\text{Power}}{\text{BW}}$$

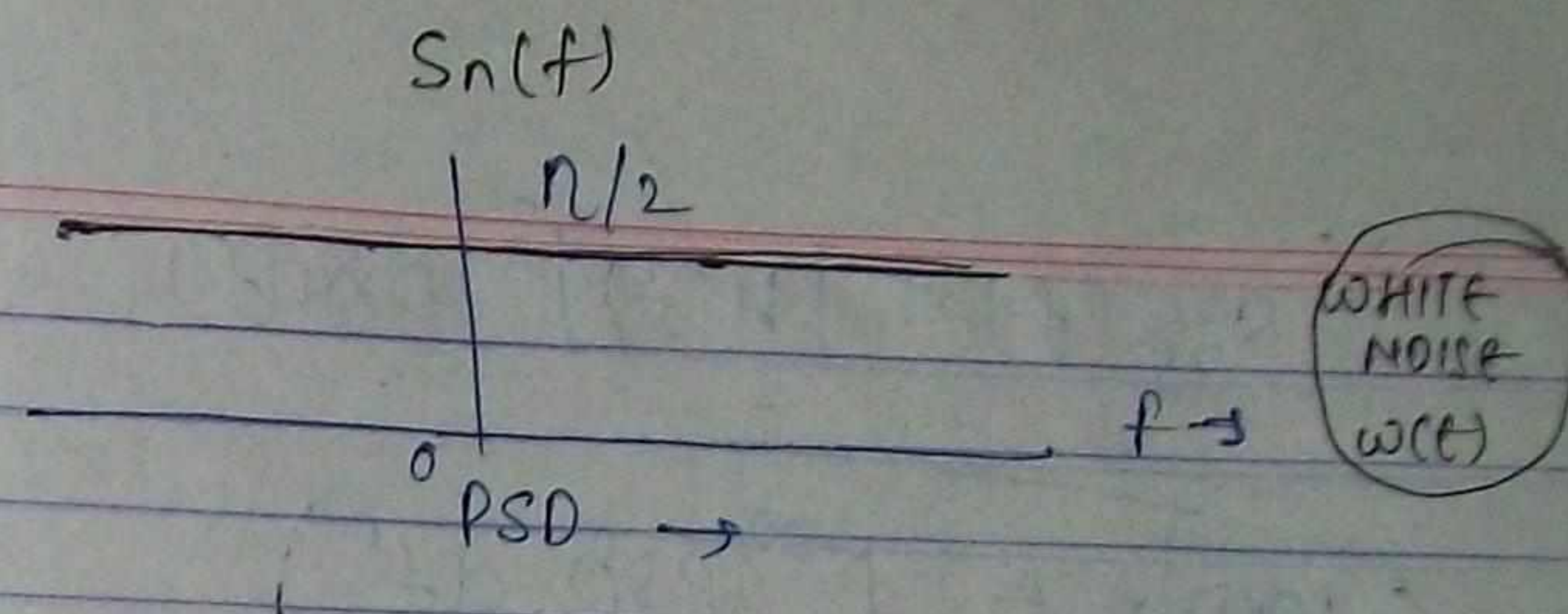
Thermal Noise PSD

$$\eta = \frac{KT B}{B}$$

$$\eta = KT$$

$$N_0 = \eta = \text{one sided PSD}$$

$$N_0 = \eta/2 = \text{two sided PSD}$$



$$N_0 = \frac{\eta}{2} = \frac{KT}{2}$$

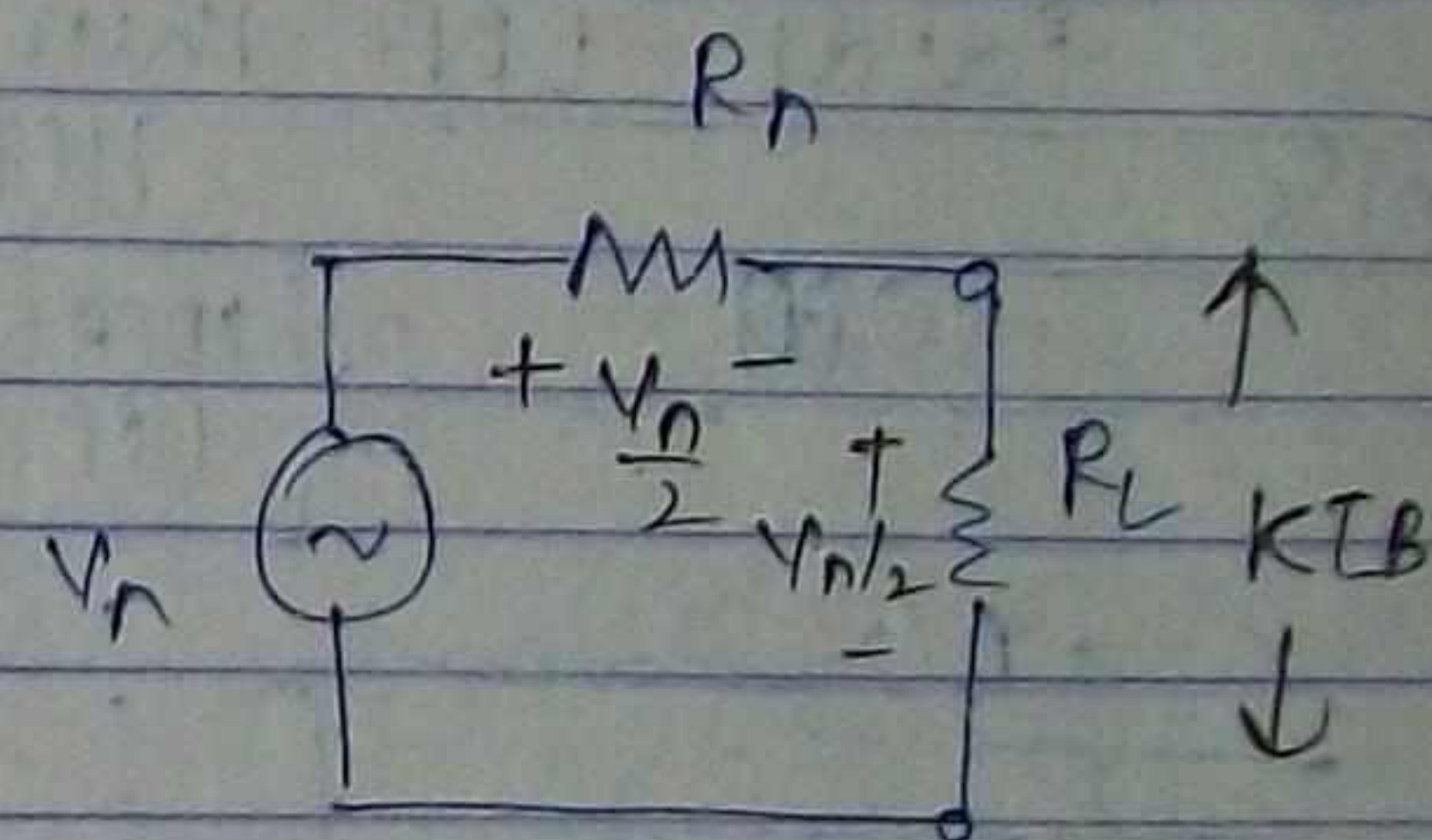
Two sided
PSD of thermal noise.

NOISE VOLTAGE -

$$\left(\frac{V_n}{2}\right)^2 = KTB$$

$$\frac{V_n^2}{4} = KTB$$

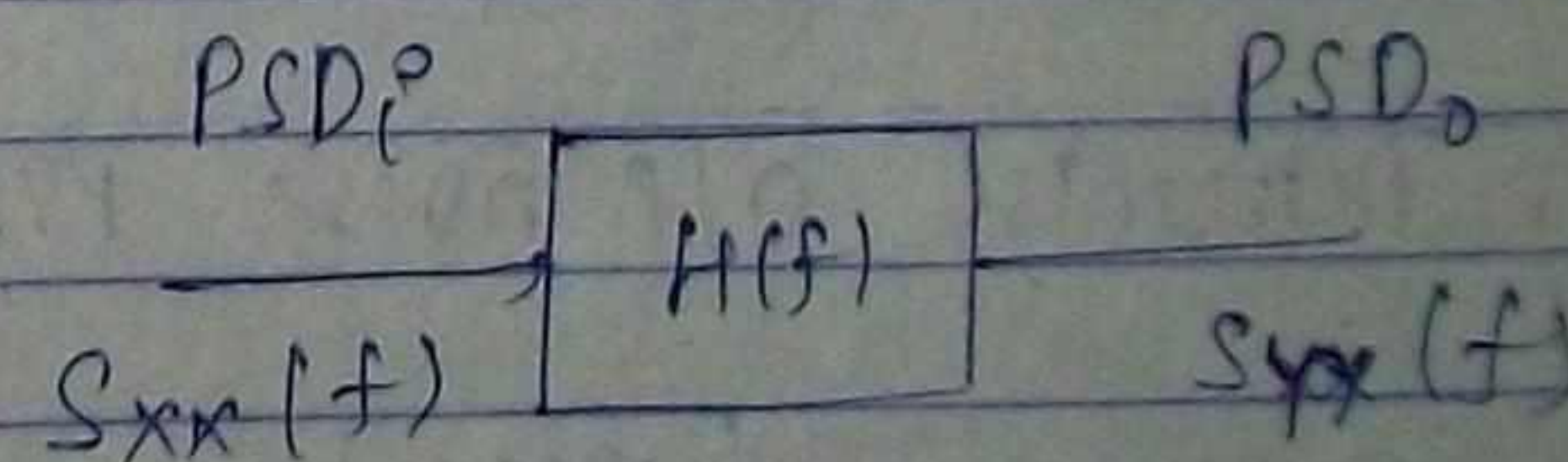
$$V_n = \sqrt{4KTB R}$$



B → BW

R → resistance

NOISE RESPONSE -



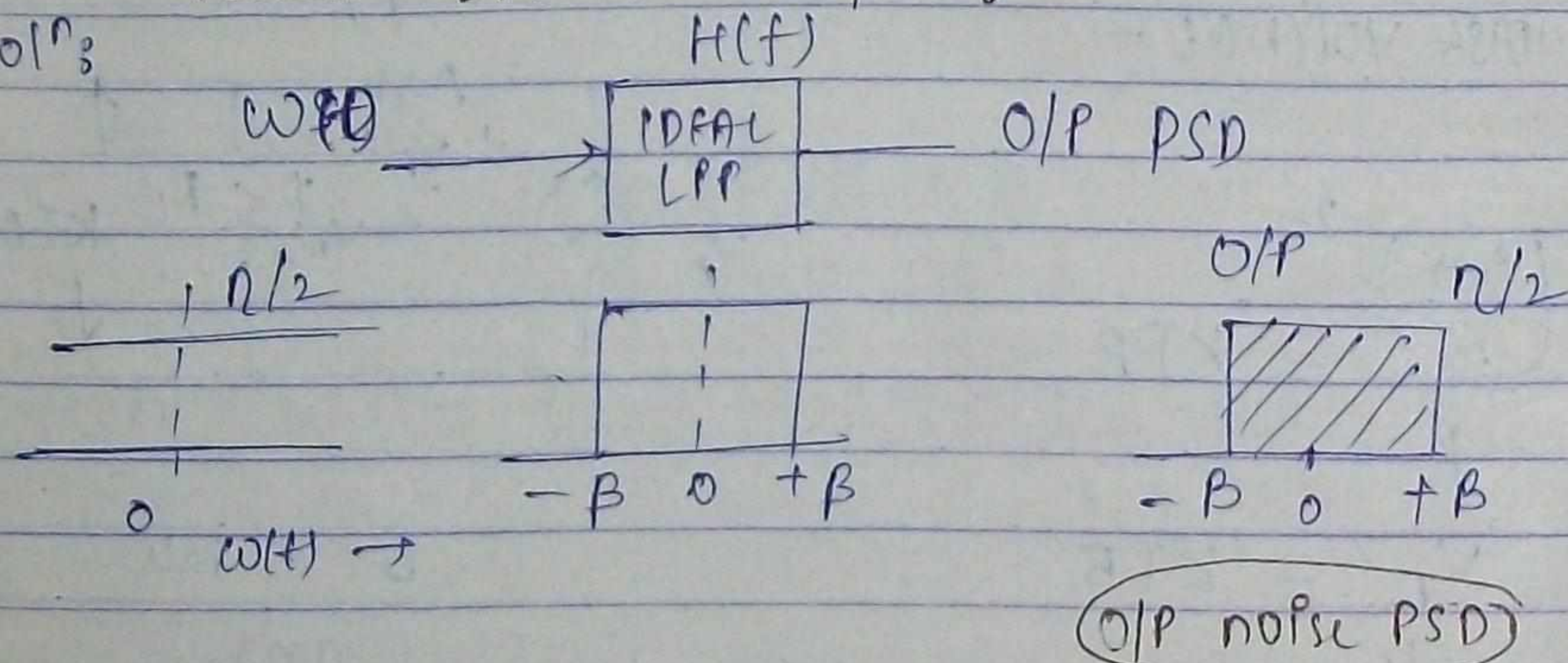
$$\text{PSD}_o = |H(f)|^2 \text{PSD}_i$$

$$S_{yy}(f) = |H(f)|^2 S_{xx}(f)$$

$$\text{power} = \int_{-\infty}^{\infty} \text{PSD } df$$

Ques. Let white noise $w(t)$ be passed through ideal LPF then O/P = ?

Soln:

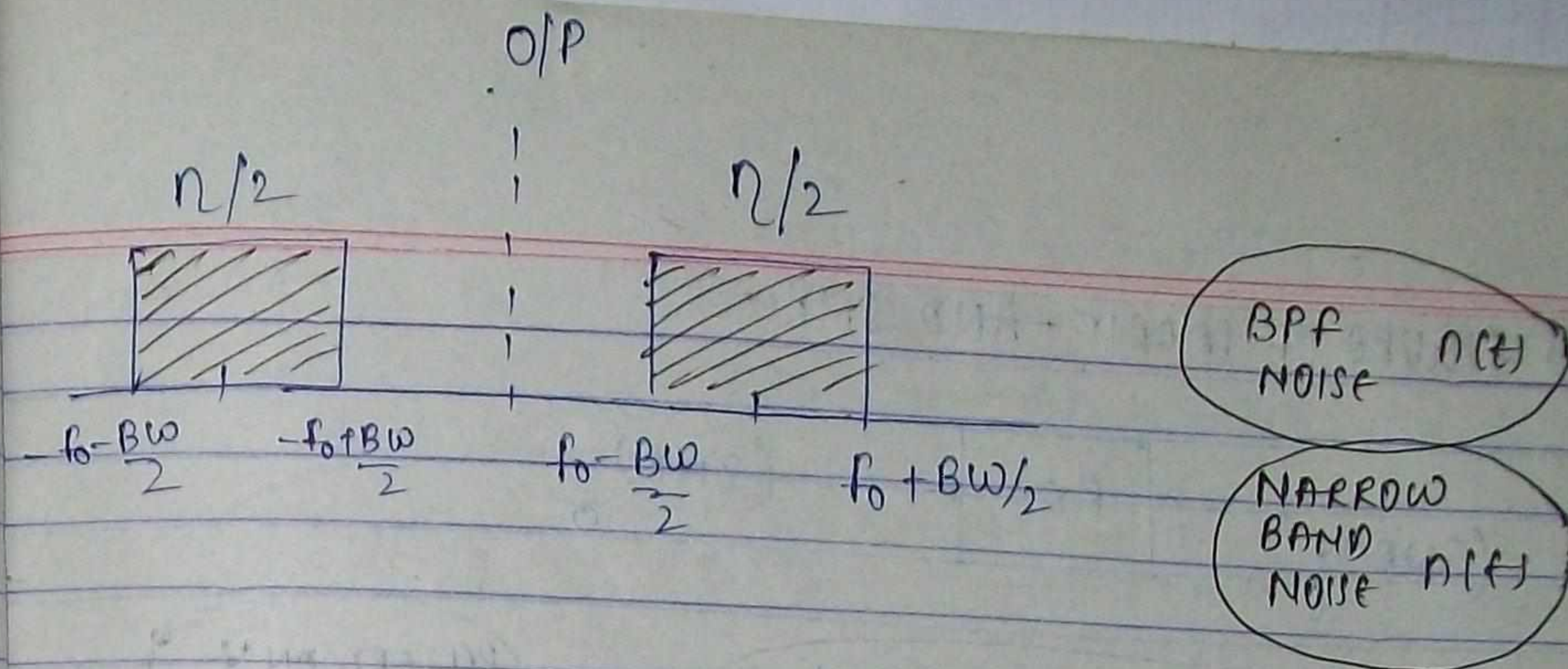
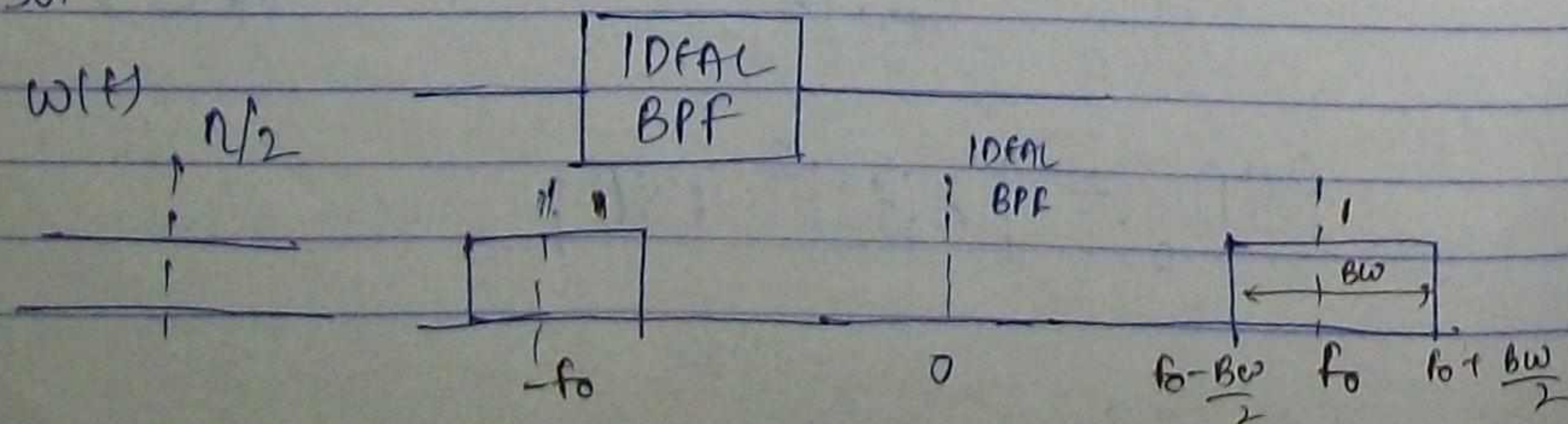


O/P noise power = area = $\frac{n}{2} \times 2B = nB$

(LPF NOISE)

Ques. Determine O/P noise PSD of ideal BPF?

Soln:



$$n(t) = n_c(t) \cos 2\pi f_0 t - n_s(t) \sin 2\pi f_0 t$$

where,

$$n_c(t) = n_I(t) = \text{inphase noise.}$$

$$n_s(t) = n_Q(t) = \text{quadrature noise.}$$

Low Pass Filter Noise.

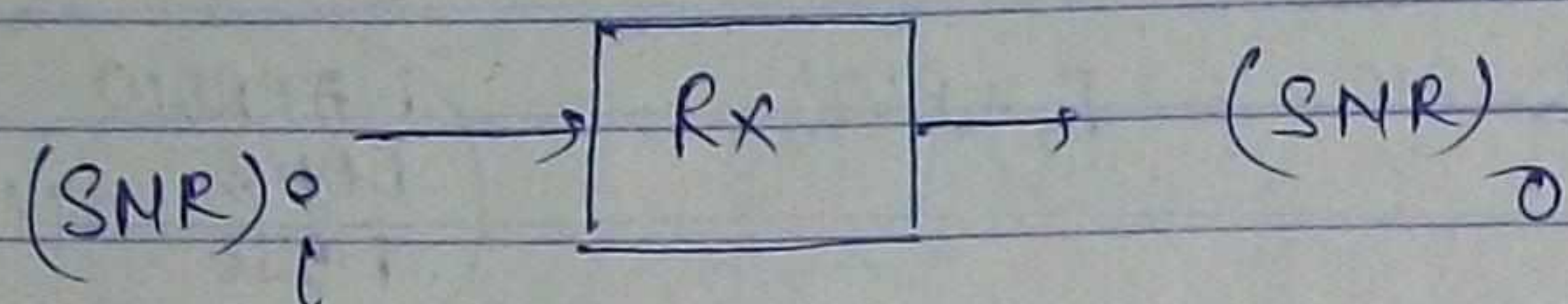
$$n(t) = \gamma(t) \cdot \cos(2\pi f_0 t + \psi(t))$$

where,

$$\gamma(t) = \sqrt{n_c^2(t) + n_s^2(t)}$$

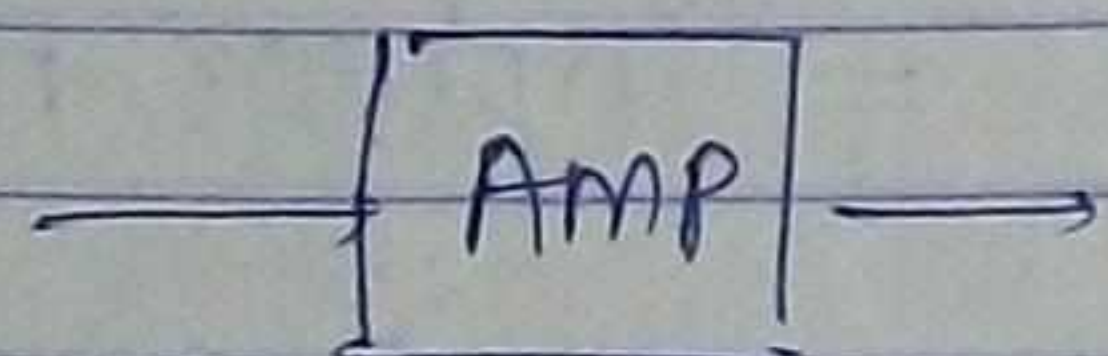
$$\psi(t) = -\tan^{-1} \left(\frac{n_s(t)}{n_c(t)} \right)$$

⇒ FIGURE OF MERIT AND SNR -



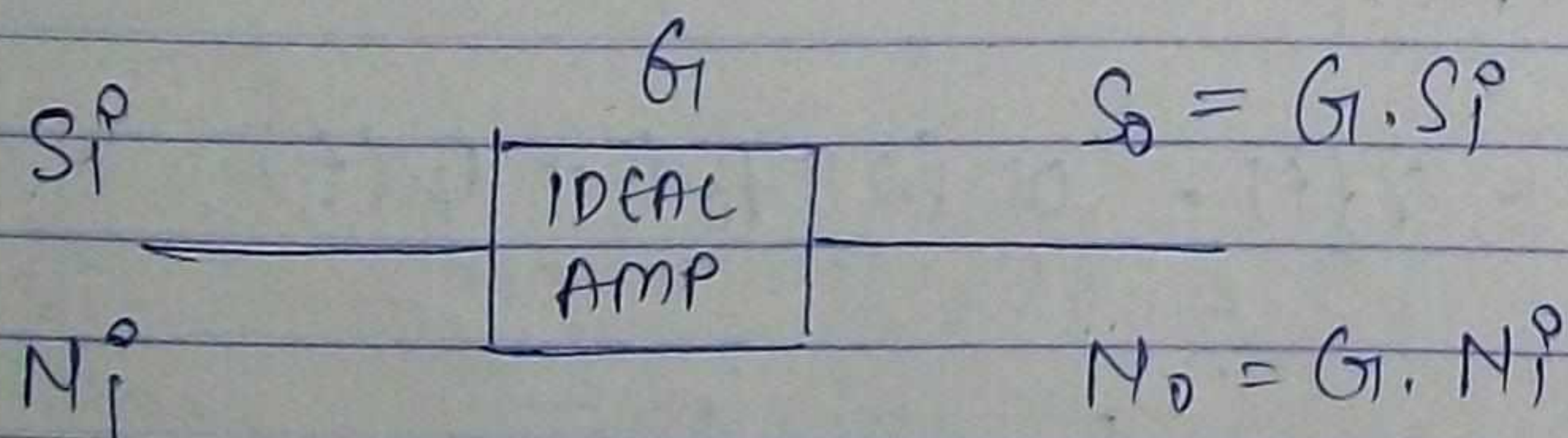
$$\text{FIG. OF MERIT} = \frac{(SNR)_o}{(SNR)_i}$$

PERFORMANCE OF RX.



$$\text{NOISE FIG} = \frac{(SNR)_i}{(SNR)_o}$$

PERFORMANCE OF AMP.

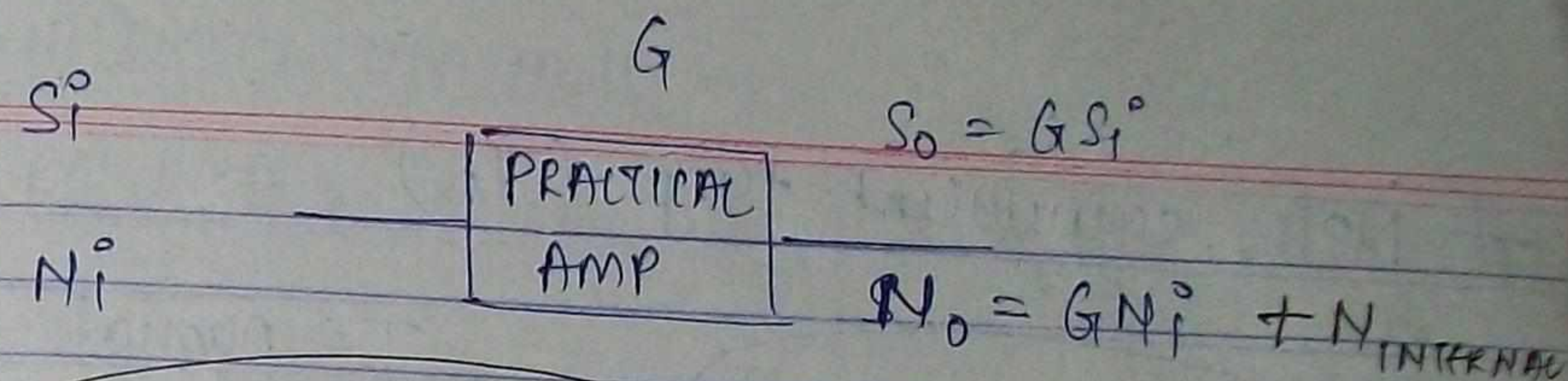


$$\text{Noise figure} = \frac{(SNR)_i}{(SNR)_o} = \frac{S_i/N_i}{G S_i / G N_i} = 1$$

NOISE FIGURE = 1

$$(SNR)_{o/p} = (SNR)_{i/p}$$

FOR IDEAL AMP.



$$(SNR)_i > (SNR)_o$$

FOR PRACTICAL AMP.

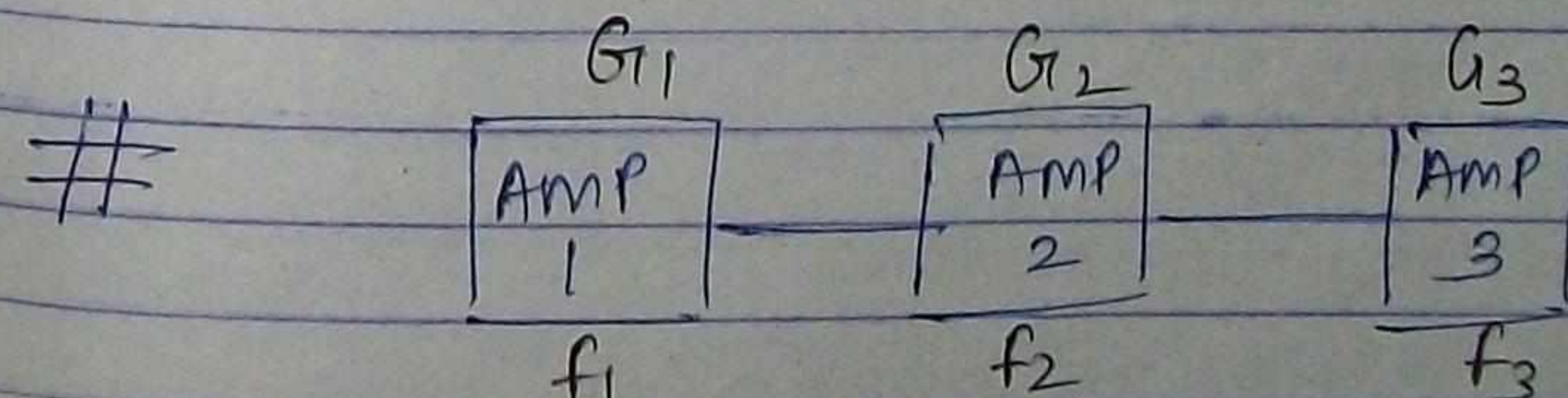
NOISE FIG > 1

NF = NF.
(NF)_{db} = noise factor

$$\text{NOISE FIGURE } F = \frac{(SNR)_i}{(SNR)_o}$$

$$(F)_{db} = 10 \log_{10} F$$

$$(F)_{db} = 0 \quad \text{IDEAL AMP.}$$



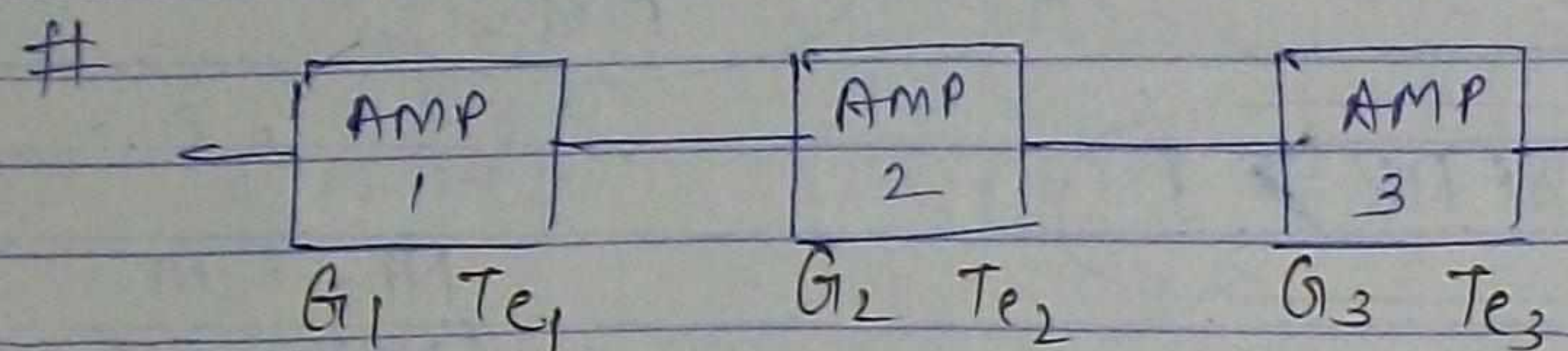
$$F_{eq} = F_1 + \frac{(F_2 - 1)}{G_1} + \frac{(F_3 - 1)}{G_1 G_2}$$

MAX. NOISE IS INTRODUCED BY (AMP 1)

Noise equivalent temp (T_e)

$$T_e = (F-1) T$$

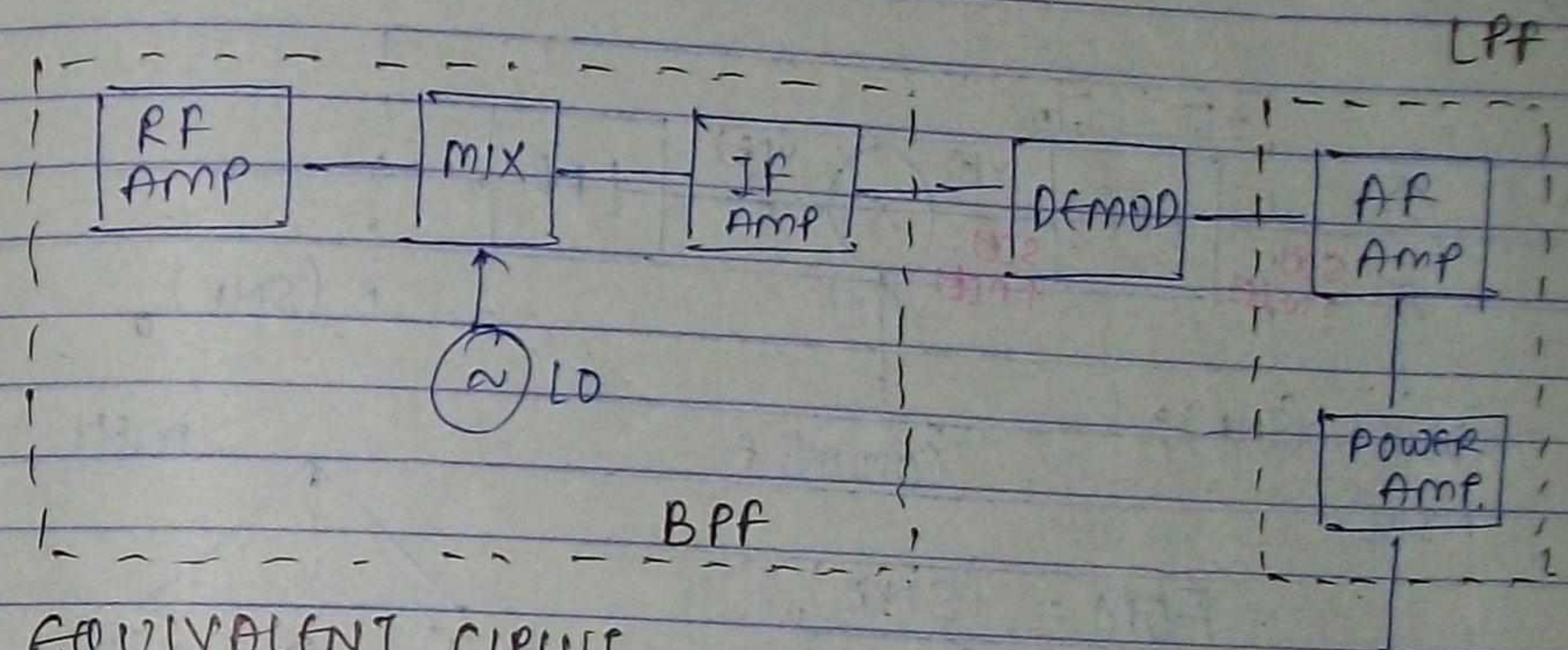
$T = \text{normal}$
 $= \text{temp}$



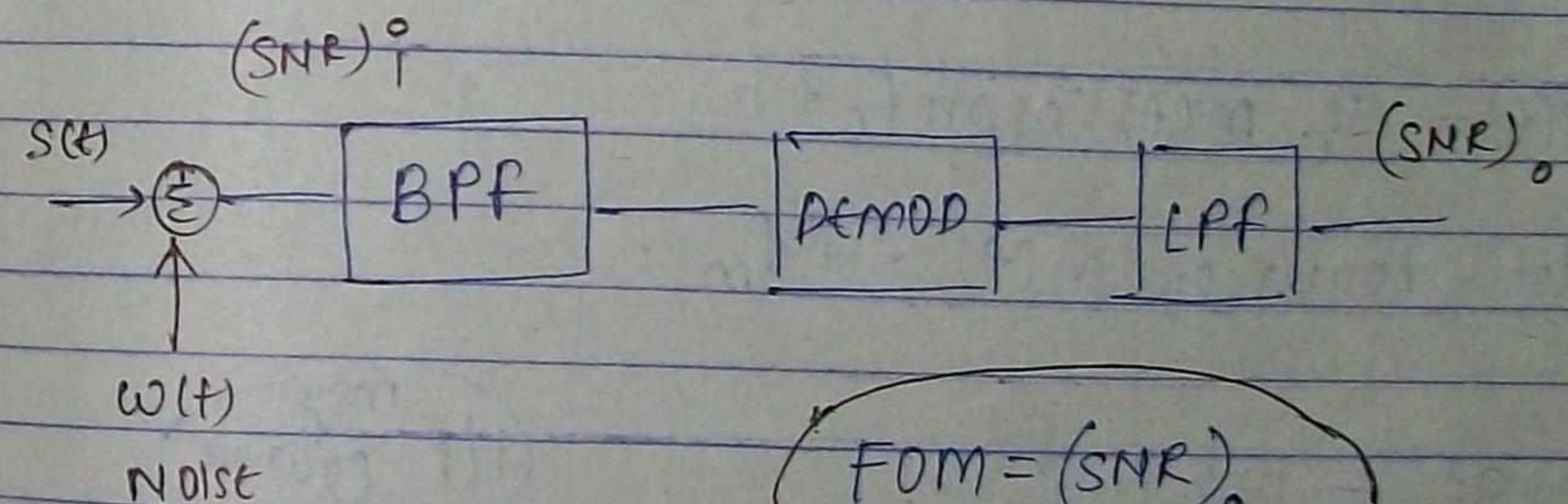
$$T_{eq} = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1 G_2}$$

⇒ NOISE CALCULATIONS -

① AM RECEIVER -

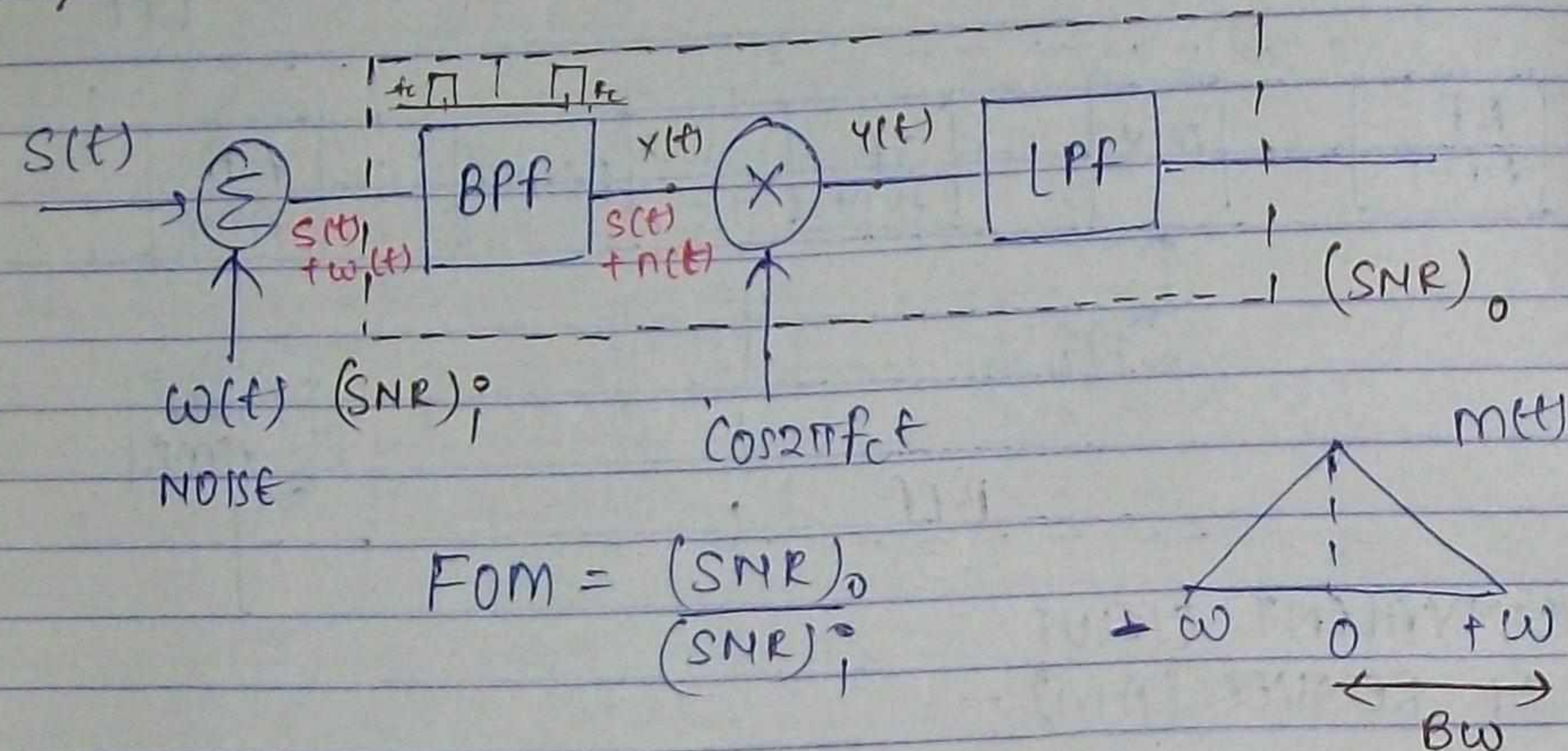


EQUIVALENT CIRCUIT
OF RECEIVER (AM) -



$$FOM = \frac{(SNR)_o}{(SNR)_i}$$

⇒ SNR CALCULATION OF (DSB) SYSTEM -



$$FOM = \frac{(SNR)_o}{(SNR)_i}$$

$$S(t) = V_c m(t) \cos(2\pi f_c t)$$

$$\text{Let Power of } m(t) \rightarrow P_m$$

$$S_i^p = \frac{V_c^2}{2} \cdot P_m$$

msg
1/P power

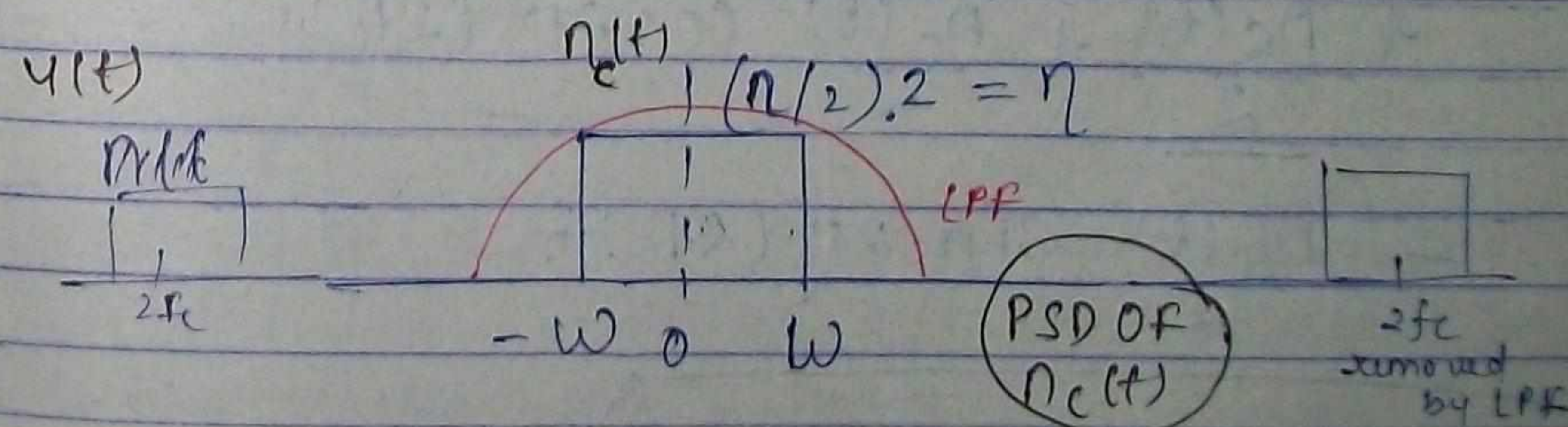
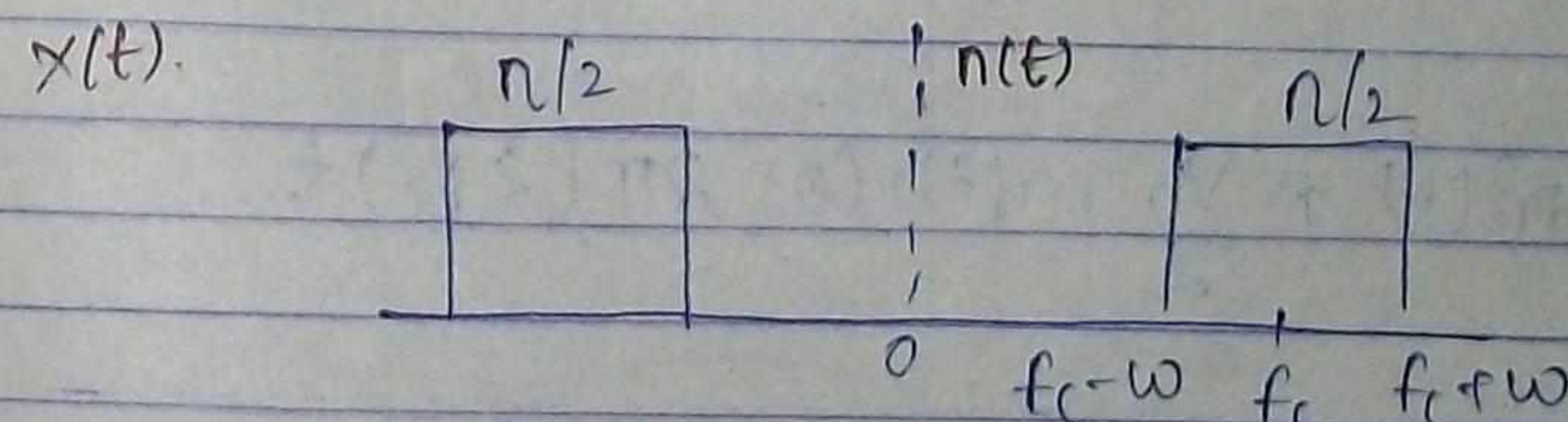
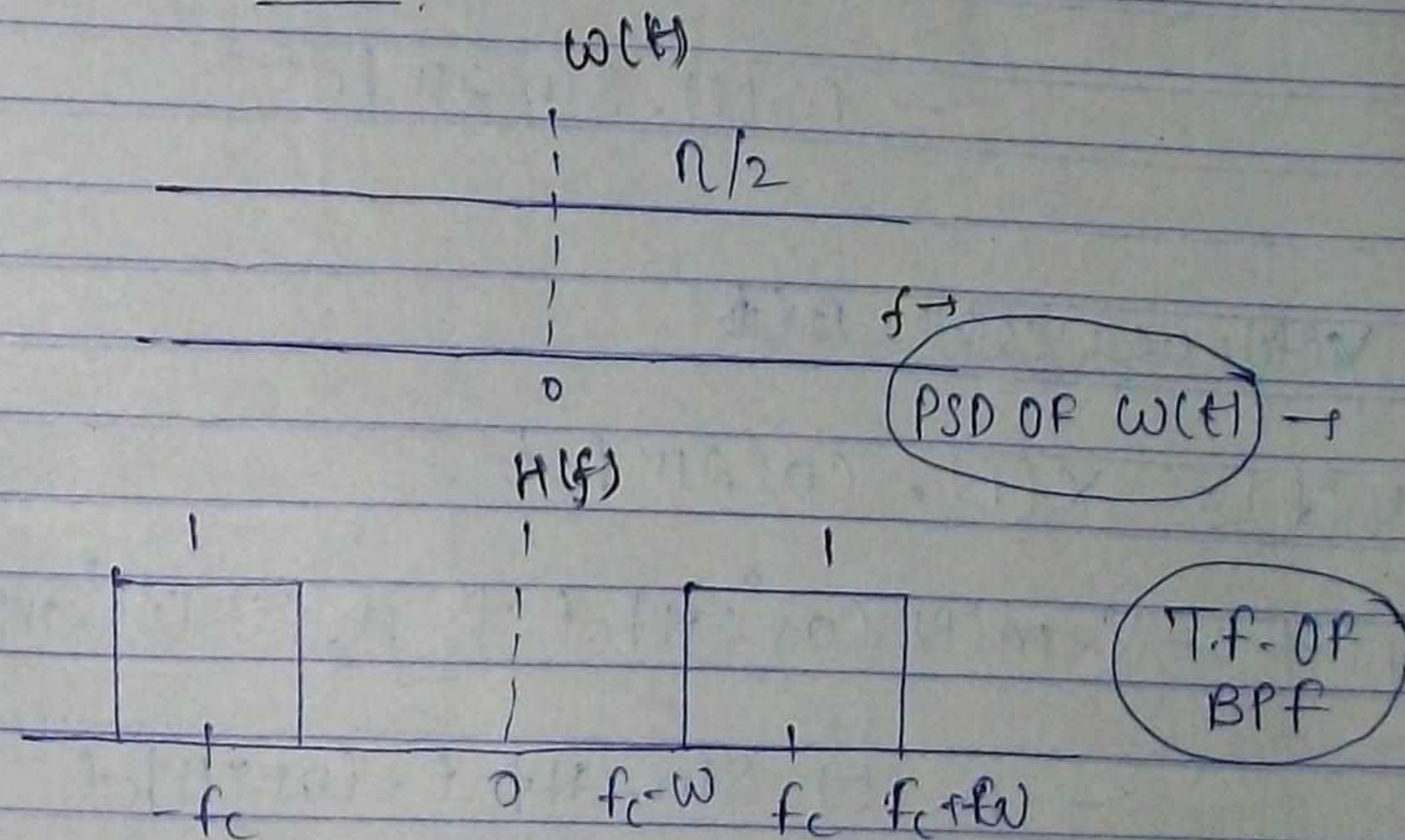
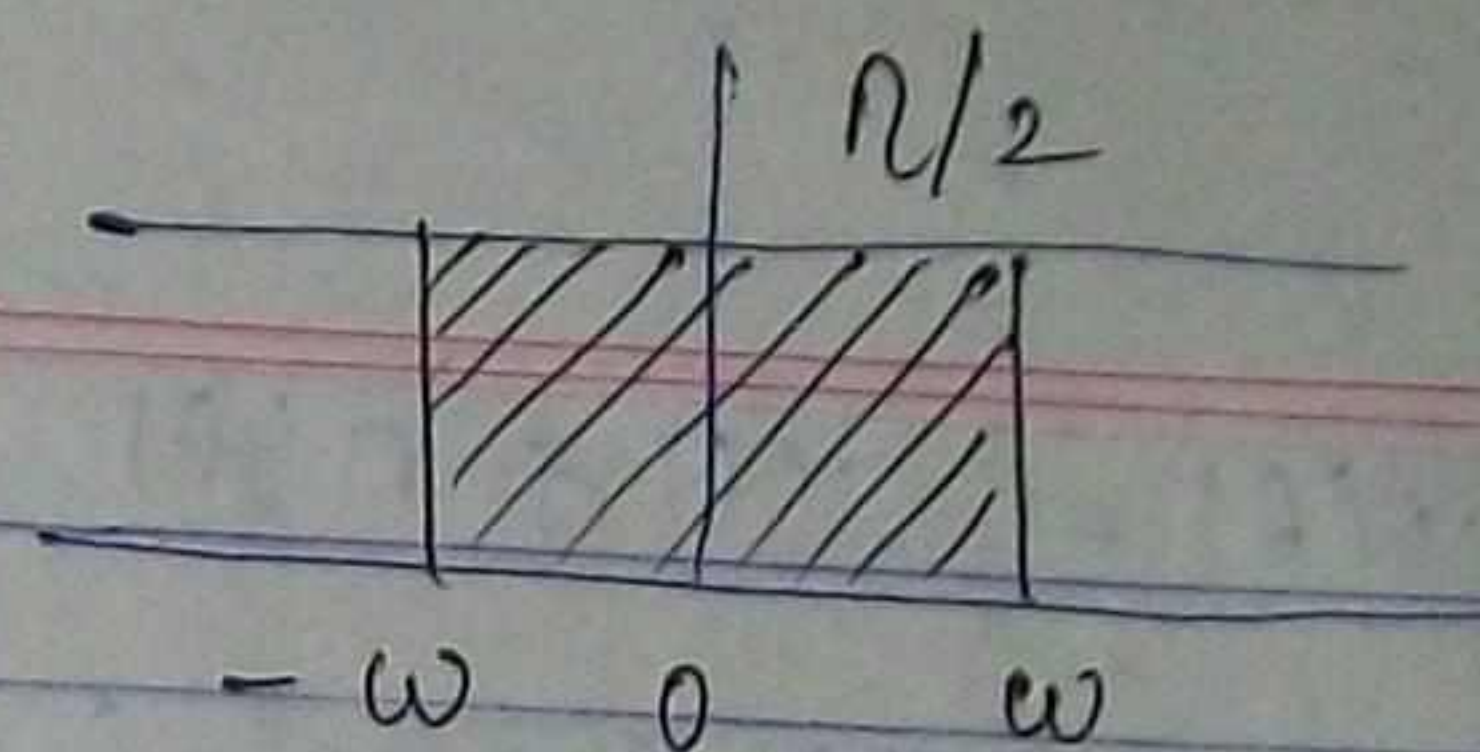
$$N_i^p = kT \cdot \omega = \eta \cdot \omega$$

noise
1/P power

$$(SNR)_i^p = \frac{V_c^2 \cdot P_m}{2 \eta \omega} \quad \text{--- (1)}$$

$$N_i^p = \frac{\eta}{2} \times 2\omega$$

$$N_i^p = \eta \omega$$



$$n(t) = n_c(t) \cdot \cos(2\pi f_c t) - n_s(t) \cdot \sin(2\pi f_c t)$$

after BPF

$$x(t) = s(t) + n(t)$$

$$= V_c m(t) \cos 2\pi f_c t + n_c(t) \cos 2\pi f_c t - n_s(t) \sin 2\pi f_c t$$

~~Let us assume~~

$$y(t) = x(t) \cdot \cos 2\pi f_c t$$

$$y(t) = V_c m(t) \cos^2 2\pi f_c t + n_c(t) \cos^2 2\pi f_c t - n_s(t) \sin 2\pi f_c t \cdot \cos 2\pi f_c t$$

$$y(t) = \frac{V_c m(t)}{2} + \frac{V_c m(t)}{2} \cos 2\pi (2f_c) t$$

$$+ \frac{n_c(t)}{2} + \frac{n_c(t)}{2} \cos 2\pi (2f_c) t$$

$$- \frac{n_s(t)}{2} \sin 2\pi (2f_c) t$$

$$y(t) = \frac{V_c m(t)}{2} + \frac{1}{2} n_c(t)$$

After LPF $O/P = \frac{V_c m(t)}{2} + \frac{1}{2} n_c(t)$

O/P signal power $S_o = \frac{V_c^2 \cdot P_m}{4}$

noise power $N_o = \frac{1}{4} \cdot P_n$

$$N_o = \frac{\eta W}{2}$$

let $n_c(t) = P_n$ power

$$\therefore P_n = \eta \times 2W = 2\eta W$$

$$(SNR)_o = \frac{S_o}{N_o} = \frac{V_c^2 \cdot P_m}{2\eta W} \quad (2)$$

$$FOM = \frac{(SNR)_o}{(SNR)_i}$$

$$FOM = 1$$

- In DSB demodlⁿ if coherent detector is used then $(SNR)_o$ of O/P ~~sum~~ and $(SNR)_i$ of ~~sum~~ are equal.

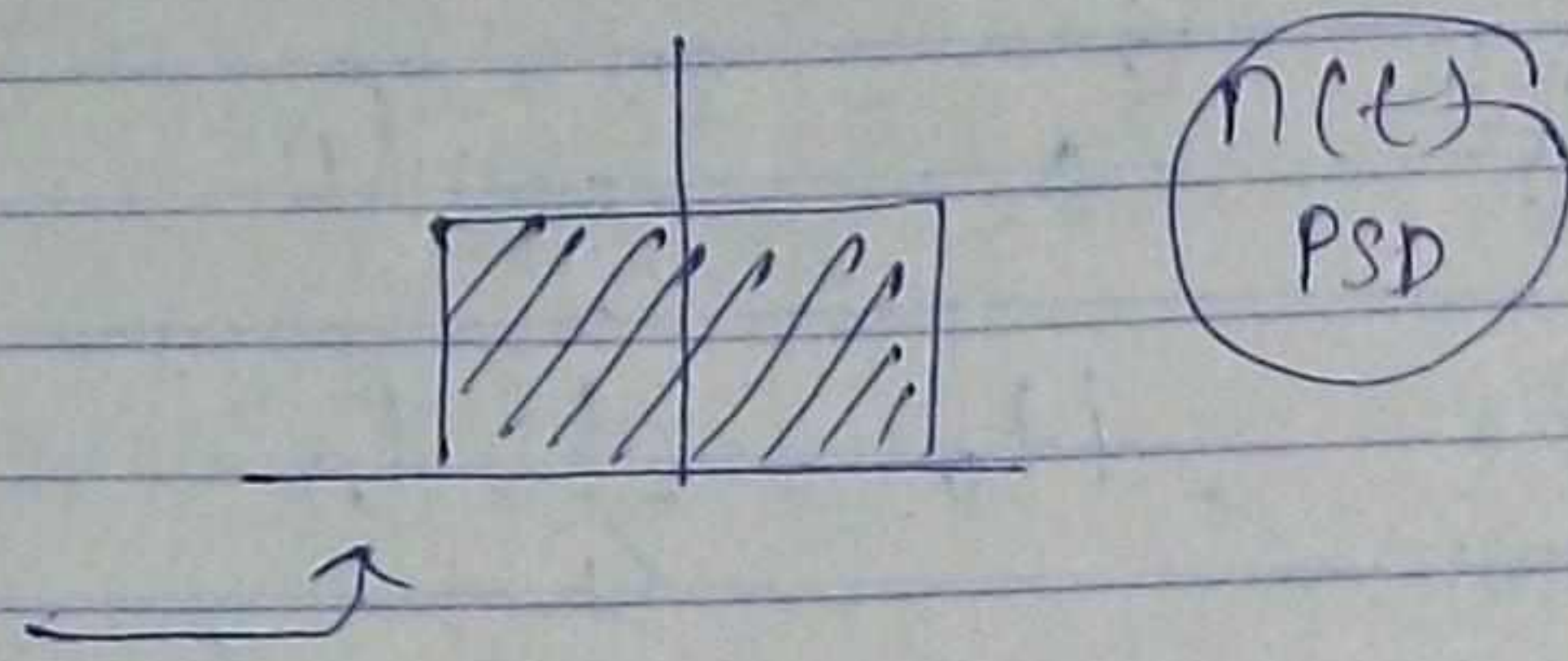
- If DSB signal & noise signal is applied to coherent detector then at the O/P only msg and in phase component of noise will present whereas

quadrature component of noise is completely eliminated.

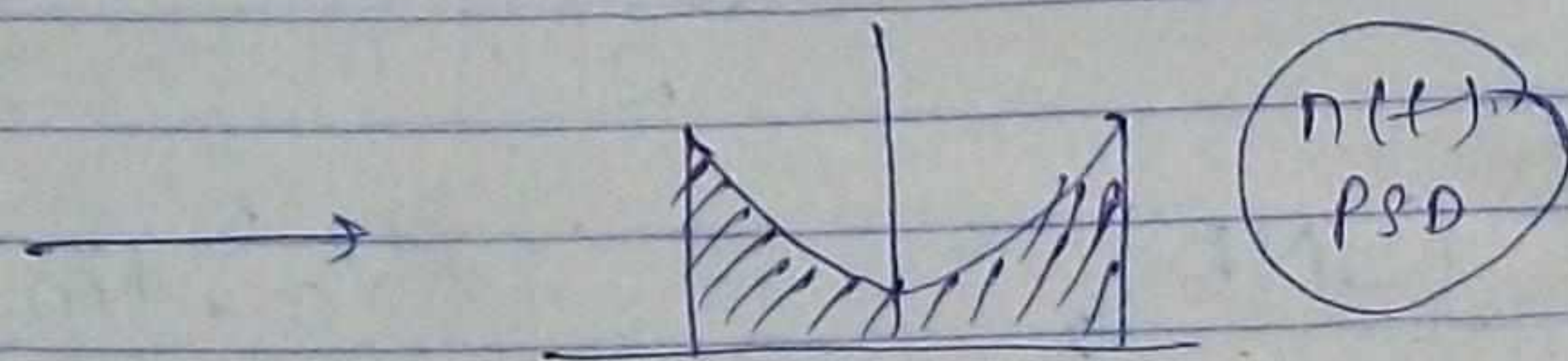
O/p noise PSD

for

AM, DSB,
SSB, PM



FM signal



pre-emphasis

& de-emphasis

are used.

Fom

AA

DSB $\rightarrow 1$

AM $\rightarrow 1/2$

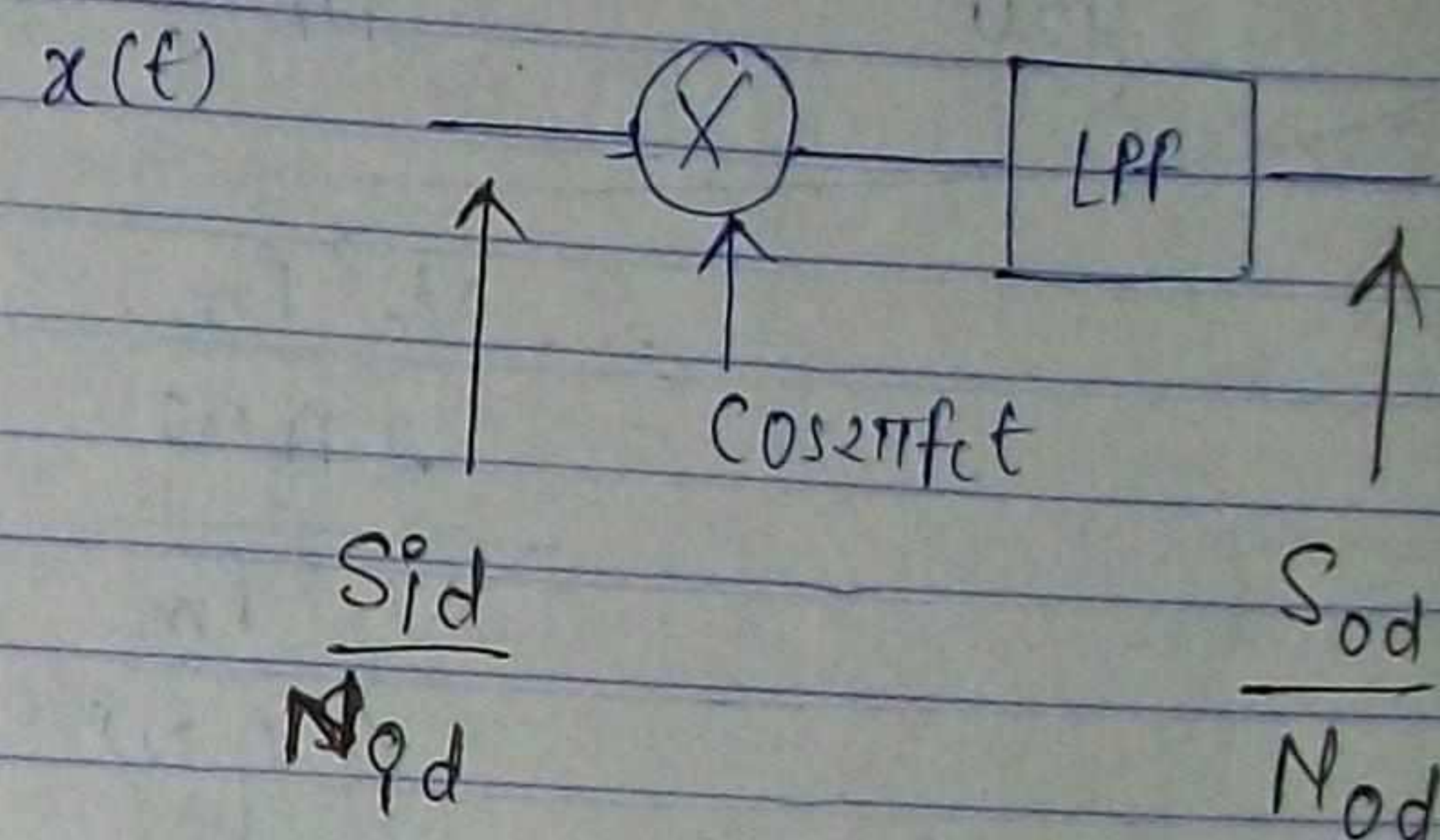
SSB $\rightarrow 1$

PM $\rightarrow \frac{1}{2} B_p^2$

FM $\rightarrow \frac{3}{2} B^2$

d = detector

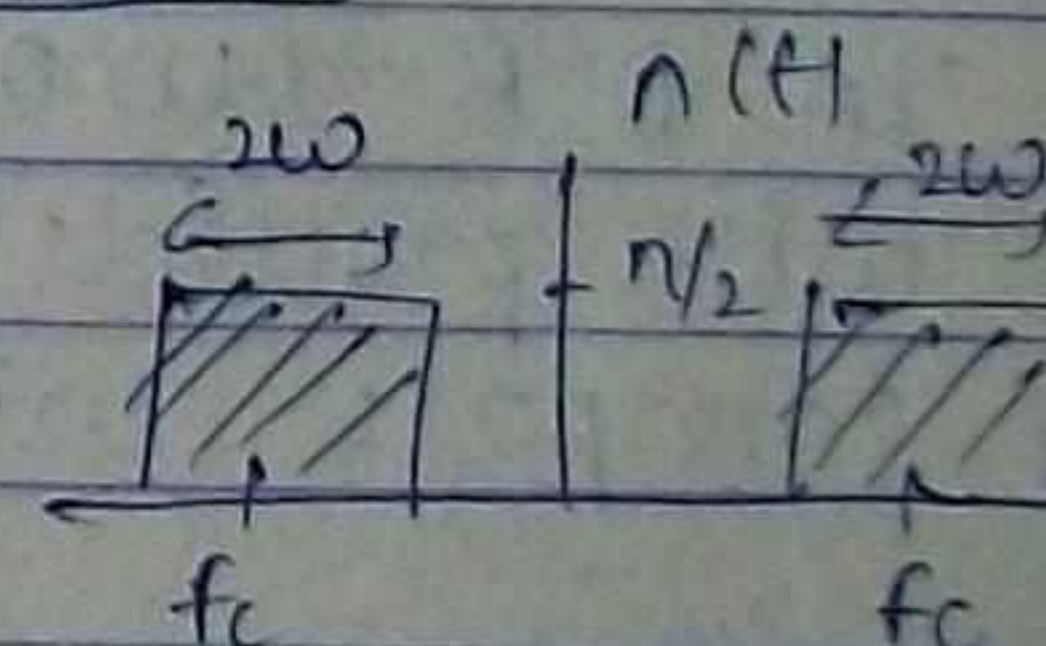
$\Rightarrow$ CALCULATION OF COHERENT DETECTOR GAIN -



$$S_{id} = S_i = \frac{V_c^2}{2} P_m$$

$N_{id} =$ noise power in $n(t)$

$$N_{id} = \frac{\eta}{2} \times 2W + \frac{\eta}{2} \times 2W = 2\eta W$$



$$(SNR)_i = \frac{V_c^2 P_m}{4\eta W} \quad \text{--- (1)}$$

$$(SNR)_o = \frac{V_c^2 P_m}{2\eta W} \quad \text{--- (2)}$$

$$\text{Coherent detector gain} = \frac{(SNR)_o}{(SNR)_i}$$

$$= \frac{V_c^2 P_m}{2 \eta W} \div \frac{V_c^2 P_m}{4 \eta W} = 2$$

$$\text{Coherent detector gain} = 2$$

- In DSB demodulation coherent detector will provide a gain of 2, when compare with 1/2 of detector.

INFORMATION THEORY

(CONTD)...

(5) CHANNEL CAPACITY THEOREM (SHANNON'S THEOREM) -

- Channel capacity provides no. of bits transmitted per second through the channel.

(A) SHANNON'S LAW -

$$R \leq C$$

WITHOUT NOISE

$R \rightarrow$ Data rate

$C \rightarrow$ Channel capacity

(B) HARTLEY LAW -

$$C = 2BT$$

$B \rightarrow$ BW

$T \rightarrow$ Time.

(C) SHANNON-HARTLEY -

$$C = B \log_2 \left(1 + \frac{S}{N} \right) \text{ bps}$$

$$= 3.32 B \log_{10} \left(1 + \frac{S}{N} \right) \text{ bps}$$

Ques. In a commⁿ channel with a BW of 3 kHz a signal is Tx. with SNR of 9 then channel capacity is.

Solⁿ:

$$\begin{aligned} C &= 3.32 \times 3000 \times \log_{10}(1+9) \\ &= 9.96 \text{ kbps} \approx 10 \text{ kbps} \\ &= 9960 \text{ bps} \end{aligned}$$

Ques. In commⁿ BW = 4 kHz, SNR = 30 dB is Tx. then channel capacity.

Solⁿ:

$$\begin{aligned} C &= 3.32 \times 4 \times \log_{10}(1+10^3) \\ C &\approx 40 \text{ kbps} \end{aligned}$$

Ques. A commⁿ channel has capacity of 10 kbps with a BW = 10 kHz then SNR is

$$10 = 3.32 \times 10 \times \log_{10}(1+SNR)$$

$$1 = \log_{10}(1+SNR)$$

$$\text{SNR} = 10$$

(*)

for same channel capacity

BW $\uparrow$ SNR $\downarrow$

BW $\uparrow$ then C $\uparrow$

BW $\rightarrow \infty$ then C = ?
max.

MAX. CHANNEL CAPACITY —

$$\begin{aligned} C &= B \log_2 \left(1 + \frac{S}{N}\right) \\ &= B \times \frac{S}{N} \times \frac{1}{S} \times \log_2 \left(1 + \frac{S}{N}\right) \end{aligned}$$

$$= B \times \frac{S}{N} \times \frac{1}{S} \times \log_2 \left(1 + \frac{S}{NB}\right)$$

$$= \frac{S}{N} \log_2 \left(1 + \frac{S}{NB}\right)^{NB/S}$$

$$= \frac{S}{N} \log(1+x)^{1/x}$$

$$\begin{aligned} \frac{NB}{S} &= x \\ B &\rightarrow \infty \\ x &\rightarrow 0 \end{aligned}$$

$$\lim_{x \rightarrow 0} (1+x)^{1/x} = e$$

$$x = \frac{\log_e e}{\log_e 2}$$

$$\log_e 2 = \frac{1}{x}$$

$$x = (\log_e 2)^{-1}$$

$$C_{\max} = \frac{S}{\eta} \log_e e$$

$$C_{\max} = 1.44 \frac{S}{\eta}$$

amount of information

$$I = \log_2 \frac{1}{p}$$

Entropy.

$$H = \sum_{i=1}^k p_i \log_2 \frac{1}{p_i}$$

$H_{\max}$ → when all mszs are equiprobable

DIGITAL COMMUNICATION

- Sampling theorem
- PAM, PWM, PPM
- Random variable
- PCM, DPCM, ΔM, ADM
- ASK, FSK, PSK
- Matched filter
- GSM, FDMA, TDMA, CDMA

SAMPLING THEOREM

⇒ SAMPLING THEOREM -

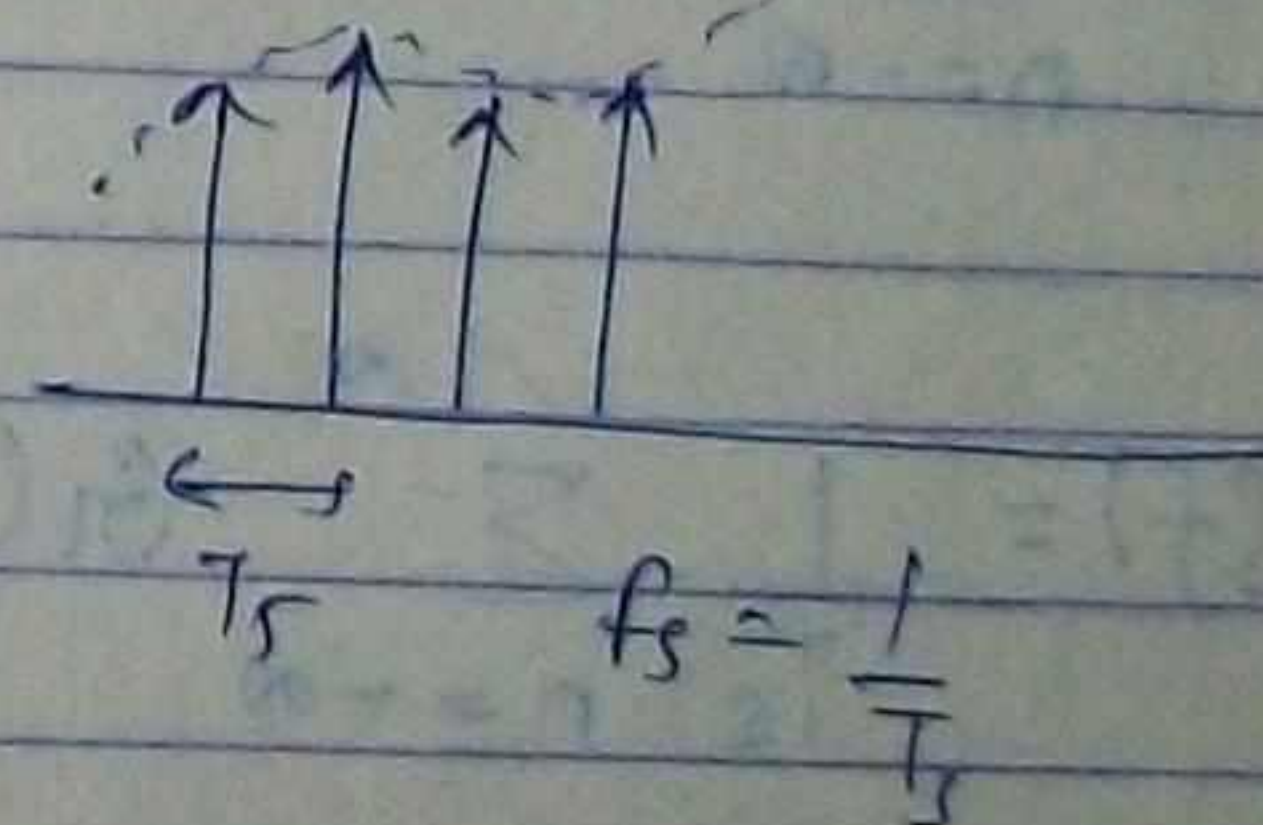
According to this theorem 'Any continuous wave signal is sample such that, if these sample are transmitted then in receiver original signal can be reconstructed with a minimum distortion when.

$$f_s \geq 2 f_m$$

or

$$f_s \geq 2 \omega$$

ω = msz bandwidth



⇒ LOW PASS SAMPLING -

$$\delta_{T_s}(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT_s)$$

where $f_s = 1/T_s$

$$s(t) = g(t) \times \delta_{T_s}(t)$$

$$= g(t) \times \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

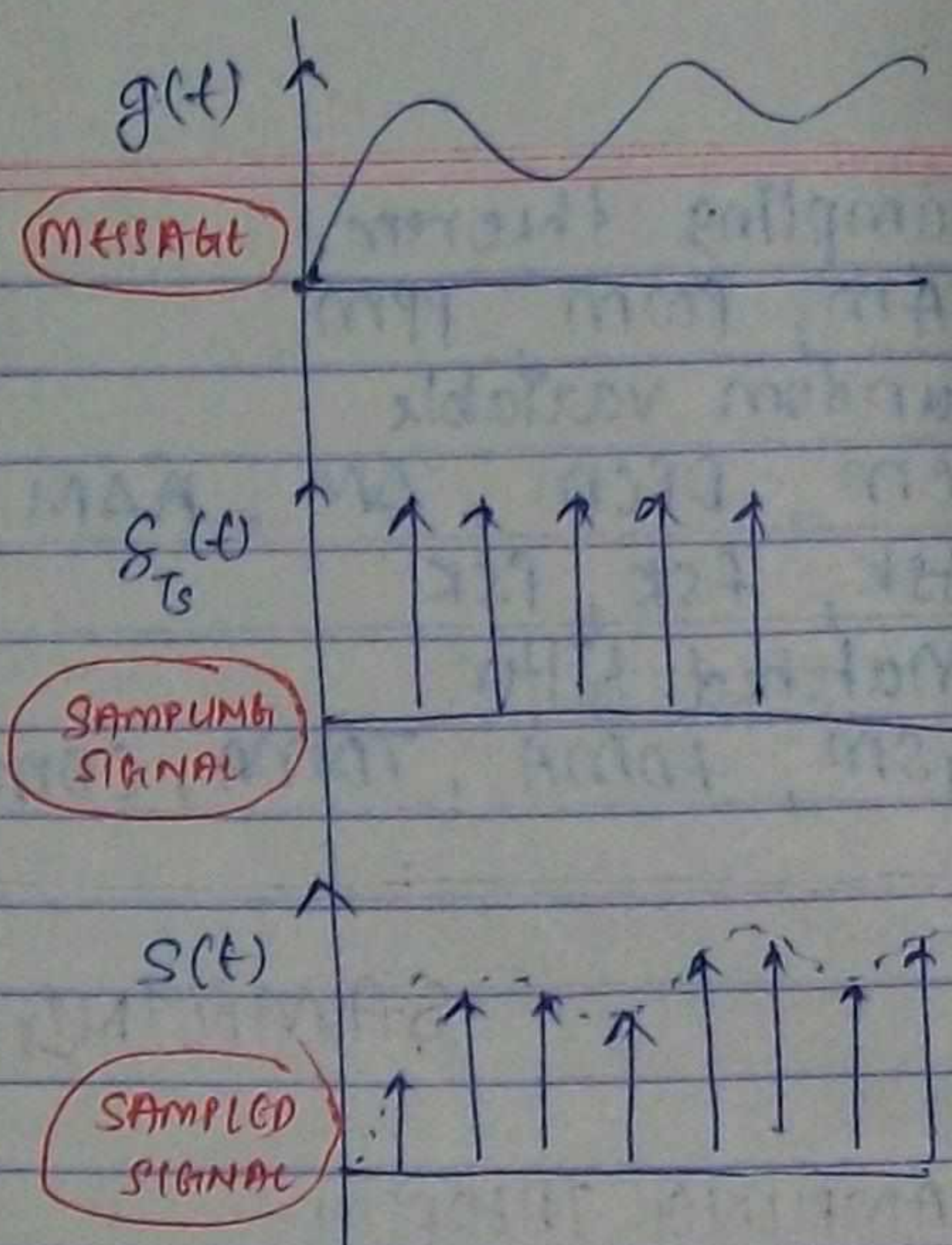
also. $s(f) = g(f) * \frac{1}{T_s} \sum_{n=-\infty}^{\infty} \delta(f - \frac{n}{T_s})$

sinu $\sum_{n=-\infty}^{\infty} \delta(t - nT_s) \xrightarrow{f.T.} \frac{1}{T_s} \sum_{n=-\infty}^{\infty} \delta(f - \frac{n}{T_s})$

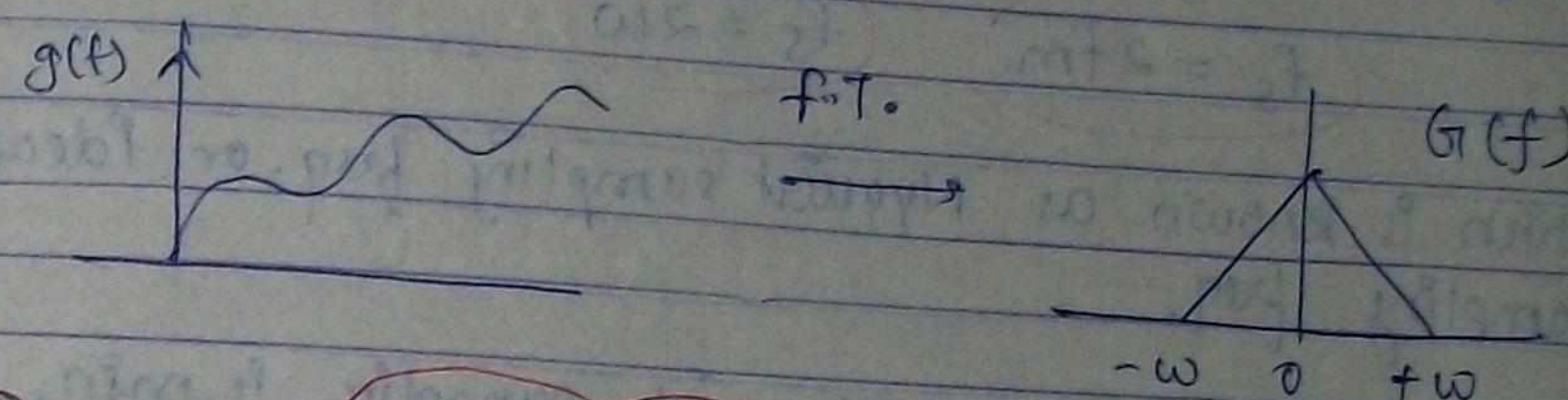
$$s(f) = \frac{1}{T_s} \sum_{n=-\infty}^{+\infty} G(f) * \delta(f - n f_s)$$

$$= \frac{1}{T_s} \sum_{n=-\infty}^{\infty} G(f - n f_s)$$

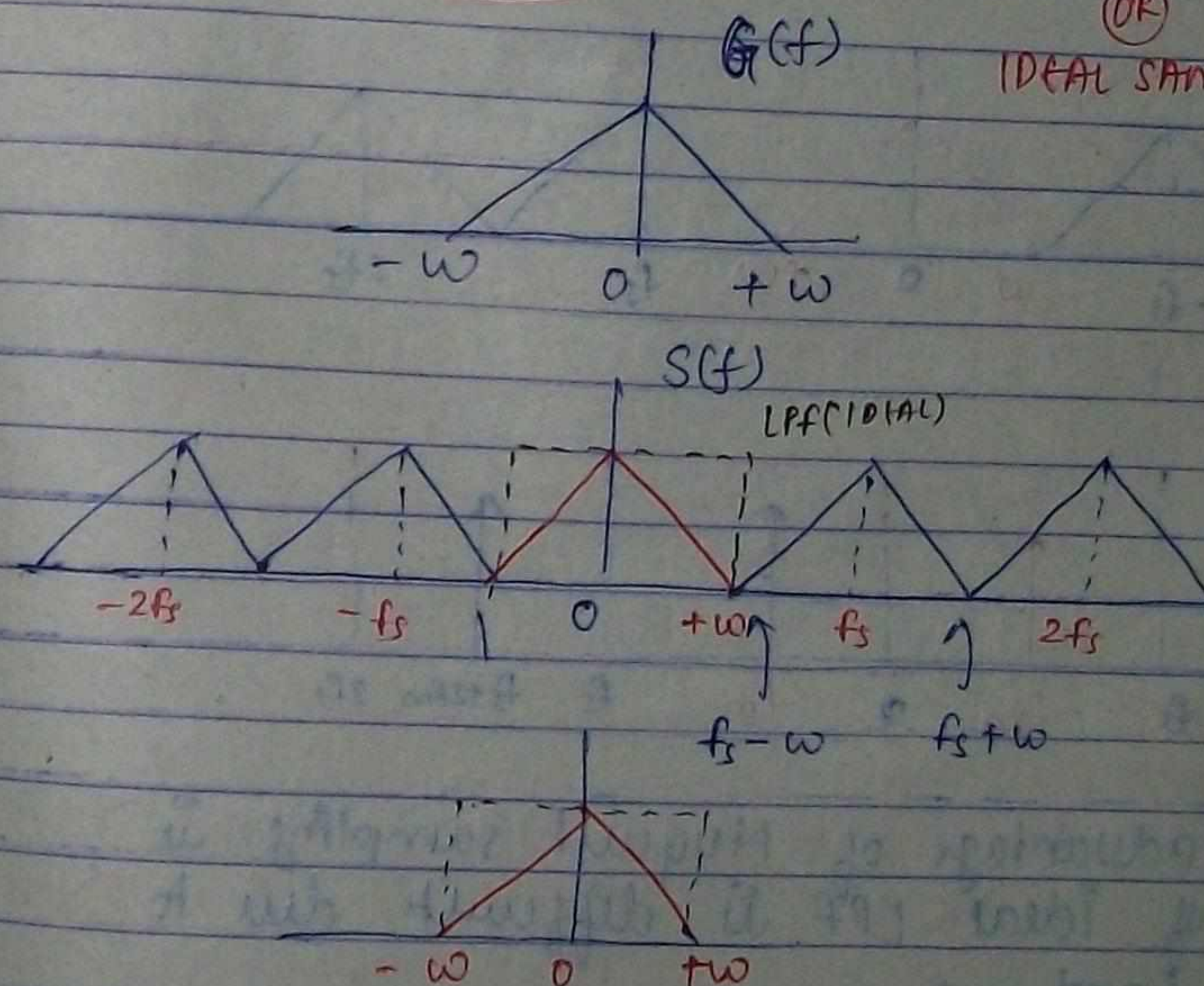
Now according to sampling theorem when a



signal $g(t)$ is uniformly sampled in time domain leads to periodicity in freq. domain with freq f_s



① WHEN $f_s = 2\omega$: NYQUIST SAMPLING (OR) IDEAL SAMPLING



To recover original signal sampled signal is passed through an ideal LPF with $\omega_c = \omega$

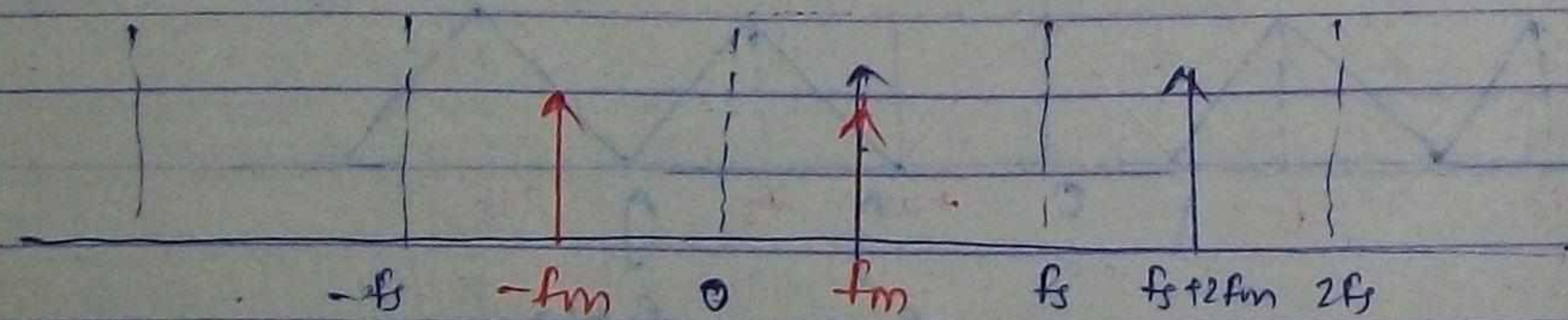
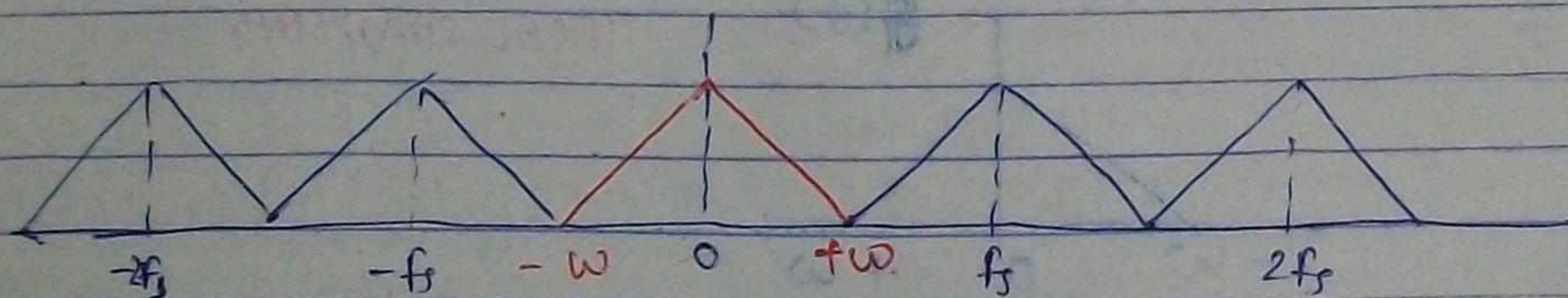
- the min. sampling freq. required to recover original signal is:

$$f_s = 2f_m$$

$$f_s = 2\omega$$

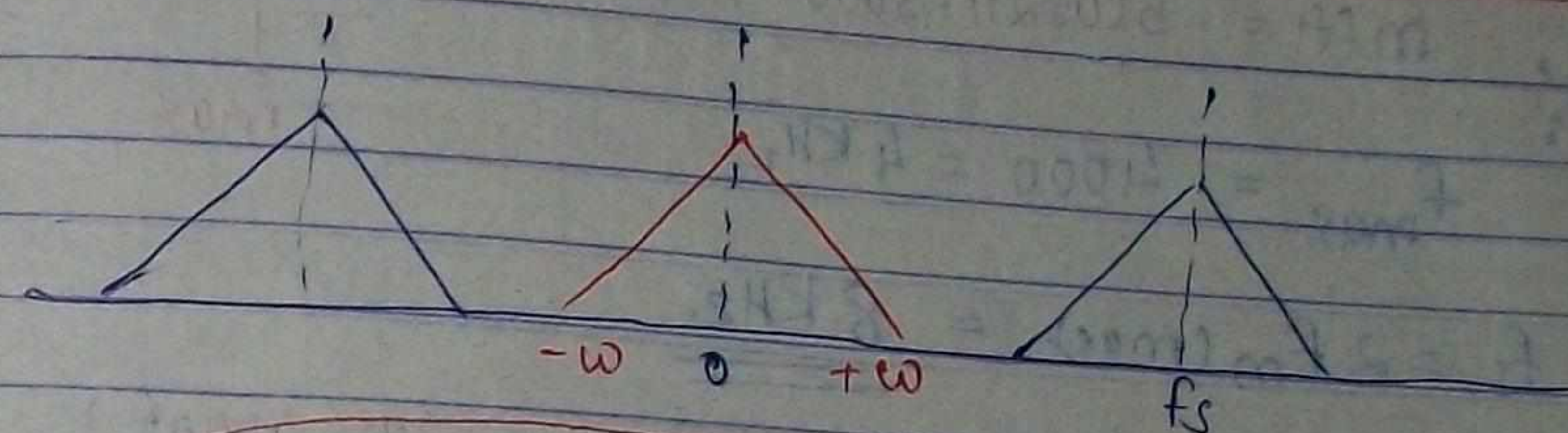
which is known as Nyquist sampling freq. or Ideal sampling freq.

- Main advantage of Nyquist sampling is min. BW of ω (msz bw) (f_m for single tone).



- The disadvantage of Nyquist sampling is design of ideal LPF is difficult due to Guard band = 0.

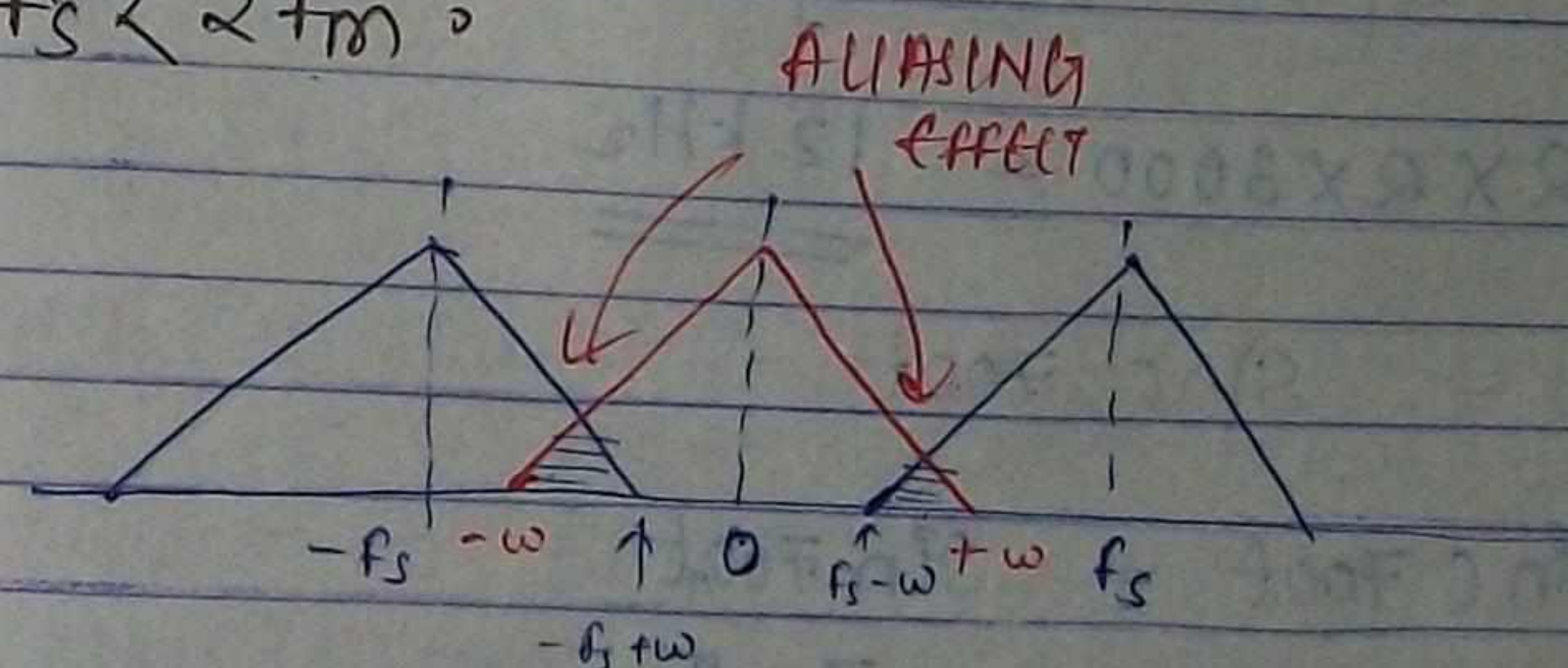
② $f_s > 2f_m$:



$$\text{guard band} = (f_s - w) - w = f_s - 2w$$

- In practical telephone system to transmit voice, the sampling freq used is 8 kHz

③ $f_s < 2f_m$:



Ques. $m(t) = 5 \cos 2\pi \times 3000t$
Soln: $f_m = 3000 = 3 \text{ kHz}$

$$f_s = 2f_m = 6 \text{ kHz}$$

Ques. $m(t) = 5 \cos 2\pi \cdot 3000t + 10 \sin 2\pi \cdot 4000t$
 Solⁿs

$$f_{\max} = 4000 = 4 \text{ kHz}$$

$$f_s = 2 f_{m(\max)} = 8 \text{ kHz}$$

Ques. $m(t) = 5 \cos 2\pi (3000t) \cdot \cos 2\pi (4000t)$

Solⁿs $\Rightarrow \frac{5}{2} [\cos 2\pi (7000t) + \cos 2\pi (1000t)]$

$$f_s = 2 f_m = 2 \times 7 \text{ kHz} = 14 \text{ kHz}$$

Ques. $m(t) = 5 [\cos 2\pi (3000t)]^2$

Solⁿs: $f_s = 2 \times 2 \times 3000 = 12 \text{ kHz}$

Ques. $m(t) = \text{sinc } 700t$

Solⁿs: $\text{sinc } 700t = \frac{\sin 700\pi t}{700\pi}$

$$= \frac{1}{K} \times \sin 2\pi \cdot 350t$$

$$f_s = 2 \times 350 = 700 \text{ Hz}$$

$$\text{Guard band} = f_s - 2f_m$$

GENERAL FORMULA -

1. $m_1(t) \rightarrow 2f_m$

2. $m_1(t) \cdot m_2(t) \rightarrow 2(f_{m_1} + f_{m_2})$

3. $m_1(t) + m_2(t) \rightarrow 2f_{\max}$

4. $m_1^n(t) \rightarrow n \cdot 2f_m$

5. $\text{sinc } kt \rightarrow k$

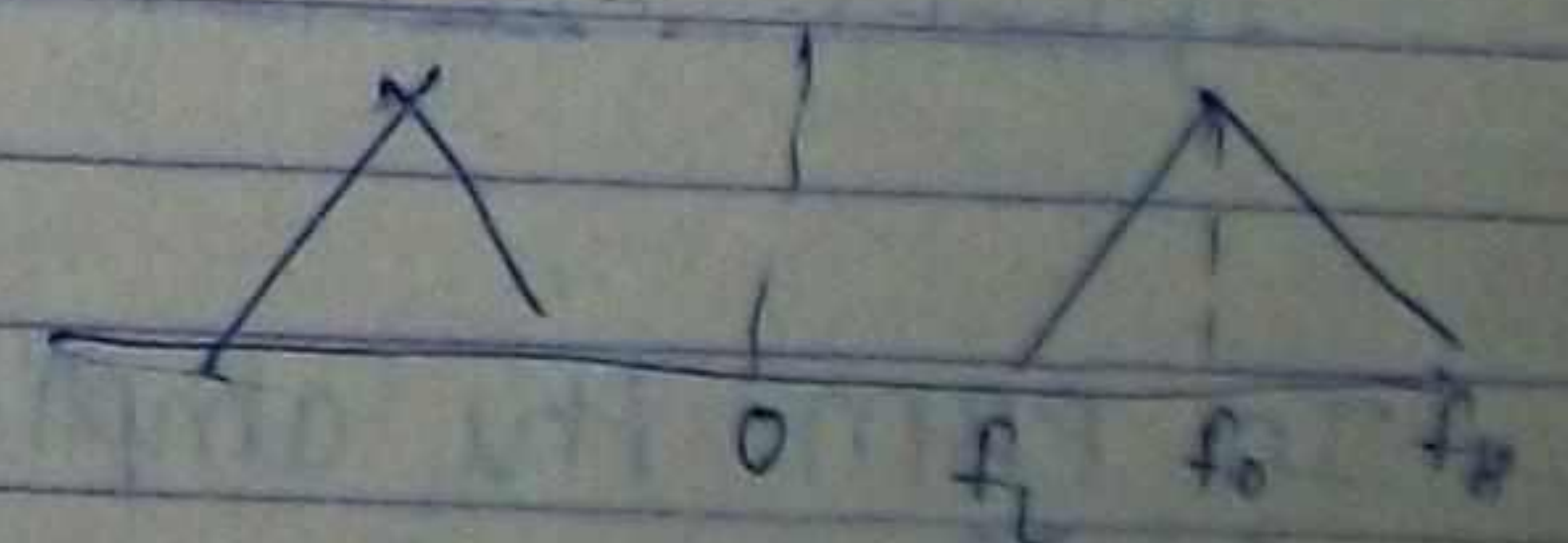
6. $\text{sinc } k_1 t \cdot \text{sinc } k_2 t \Rightarrow k_1 + k_2$

7. $[\text{sinc } kt]^n \rightarrow nk$

$\Rightarrow$ BAND PASS SAMPLING THEOREM -

According to BPS.T, "any signal to recover at receive the sampling freq."

$$f_s \geq \frac{2f_H}{K}$$

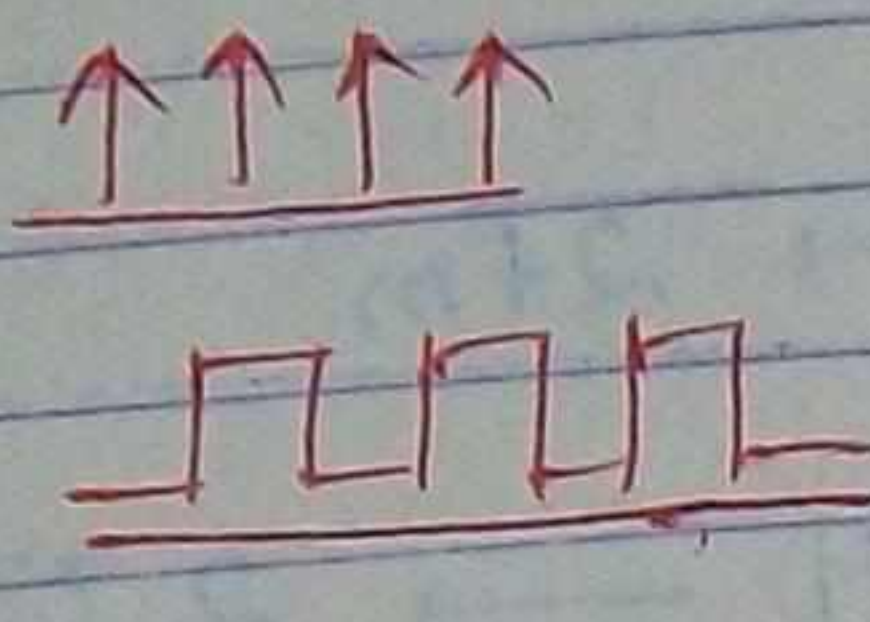


$$K = \text{Integer} \left(\frac{f_H}{B} \right)$$

$$B = f_H - f_L$$

⇒ SAMPLING -

- ① Impulse sampling
- ② Natural sampling
- ③ flat top sampling.

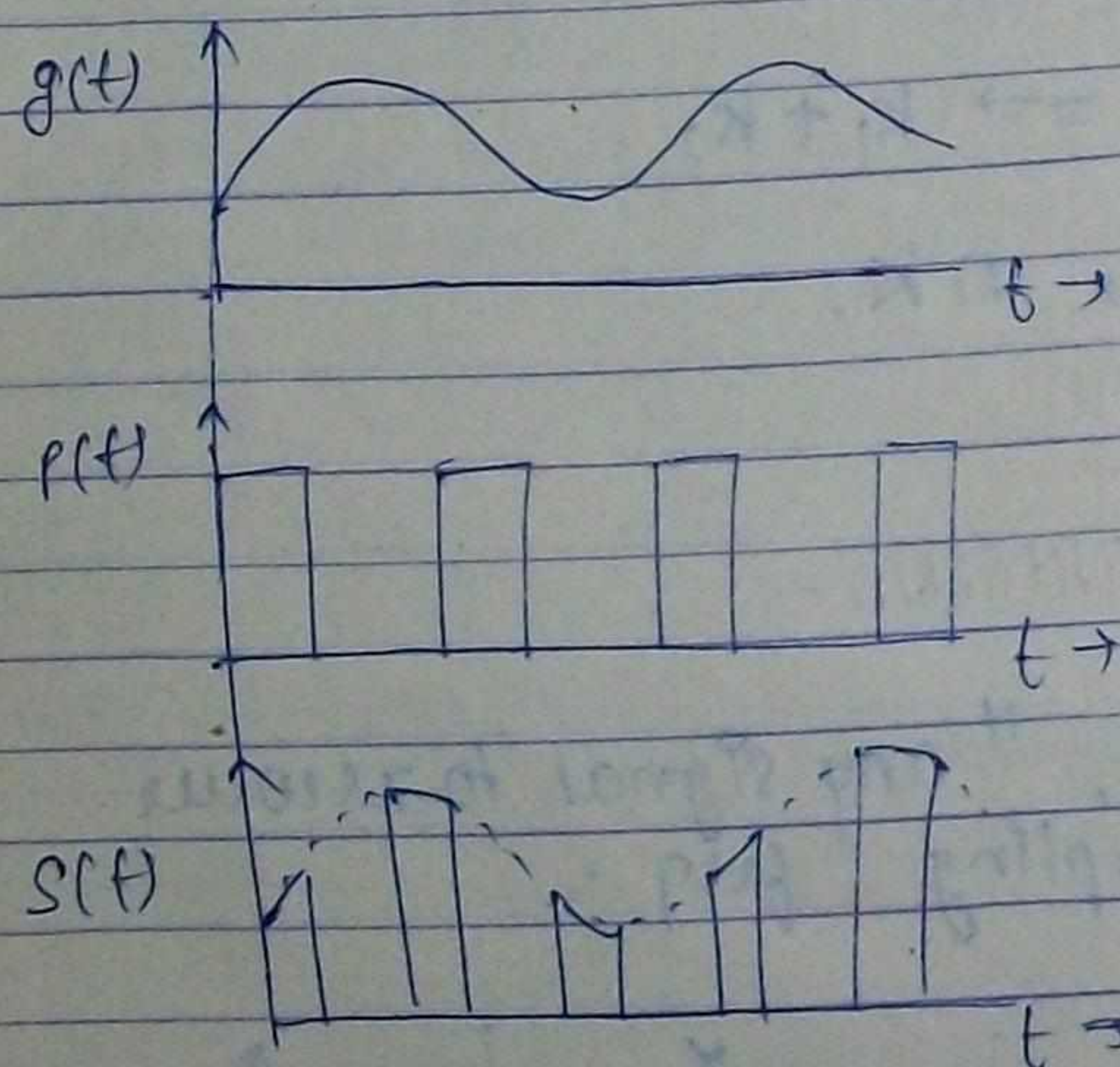


① IMPULSE SAMPLING -

In impulse sampling the disadvantage is generation of sampling signal.

② NATURAL SAMPLING -

$$S(f) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} G(f - nf_s)$$

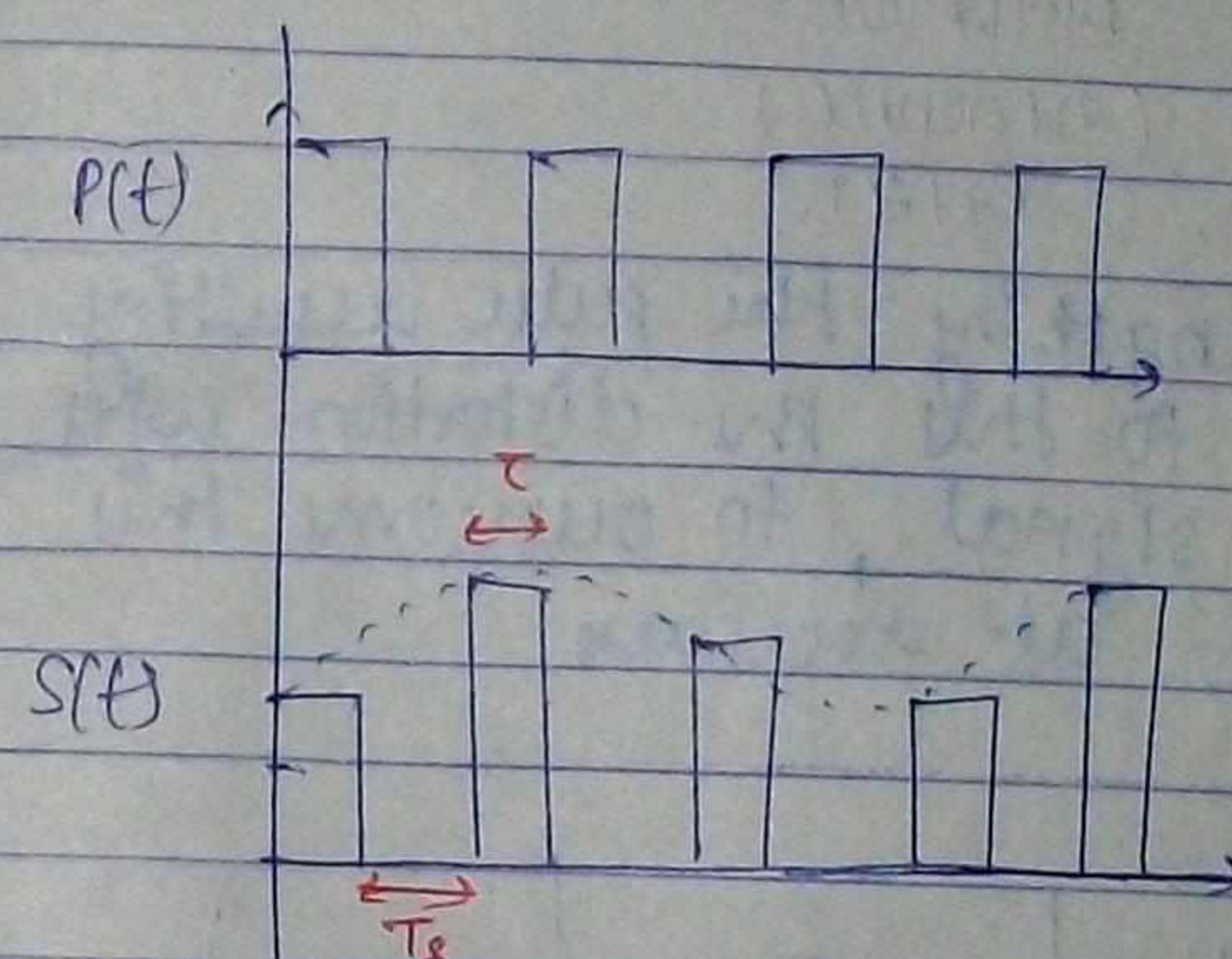


(PAM)

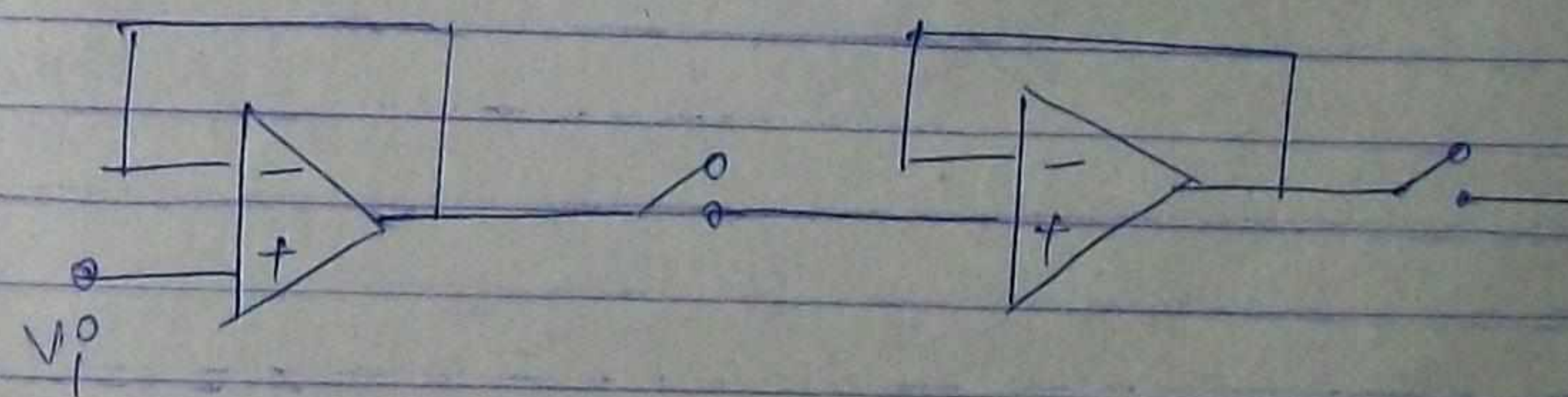
- In PAM the amplitude of msg signal is varied in accordance with instantaneous amplitude of msg signal.

- In natural sampling the effect of noise is more to overcome this the top portion of signal is made flat and it is known as FLAT-TOP SAMPLING.

③ FLAT-TOP SAMPLING -



By using sample & hold circuit



$$S(f) = \frac{1}{T_s} \sum_{n=-\infty}^{+\infty} G(f - nf_s) \times H(f)$$

$$H(f) = \tau \text{sinc}(f\tau) \cdot e^{-j\pi f\tau}$$

all pass filter = equalizer (dsp)

$$G(f) = \frac{1}{T_s} \sum_{n=-\infty}^{+\infty} G(f - n f_s) \cdot \underbrace{T \text{sinc}(fT)}_{\text{DISTORTION (APERTURE) EFFECT}} \cdot \underbrace{e^{-j\pi f T}}_{\text{DELAY}}$$

Due to artificially lengthening the pulse resulting Aperture effect. Due to this the distortion will present in received signal, to overcome this Equalizer is used at receiver.

Equalizer funⁿ $H_e(f) = \frac{1}{|H(f)|} = \frac{1}{T \text{sinc}(fT)}$

DIGITAL COMMⁿ

BAND BAND MODⁿ

BAND PASS MODⁿ

ANALOG

DIGITAL

BINARY

M-ARRAY

PAM

PTM

PWM

PPM

PCM

DPCM

DM

ADM

ASK

PSK

FSK

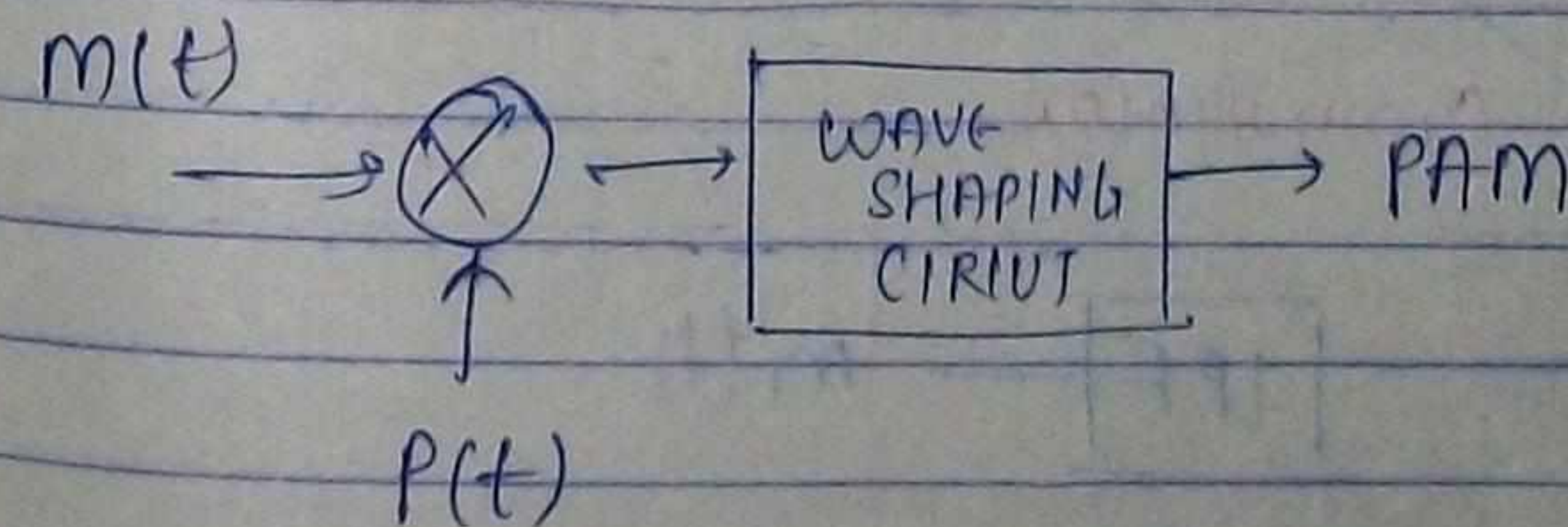
QPSK

8PSK

16PSK

⇒ PAM -

GENERATION -



Pulse train

To demodulate PAM signal LPF is used at receiver

- The min. BW required for PAM signal is $\frac{f_s}{2}$

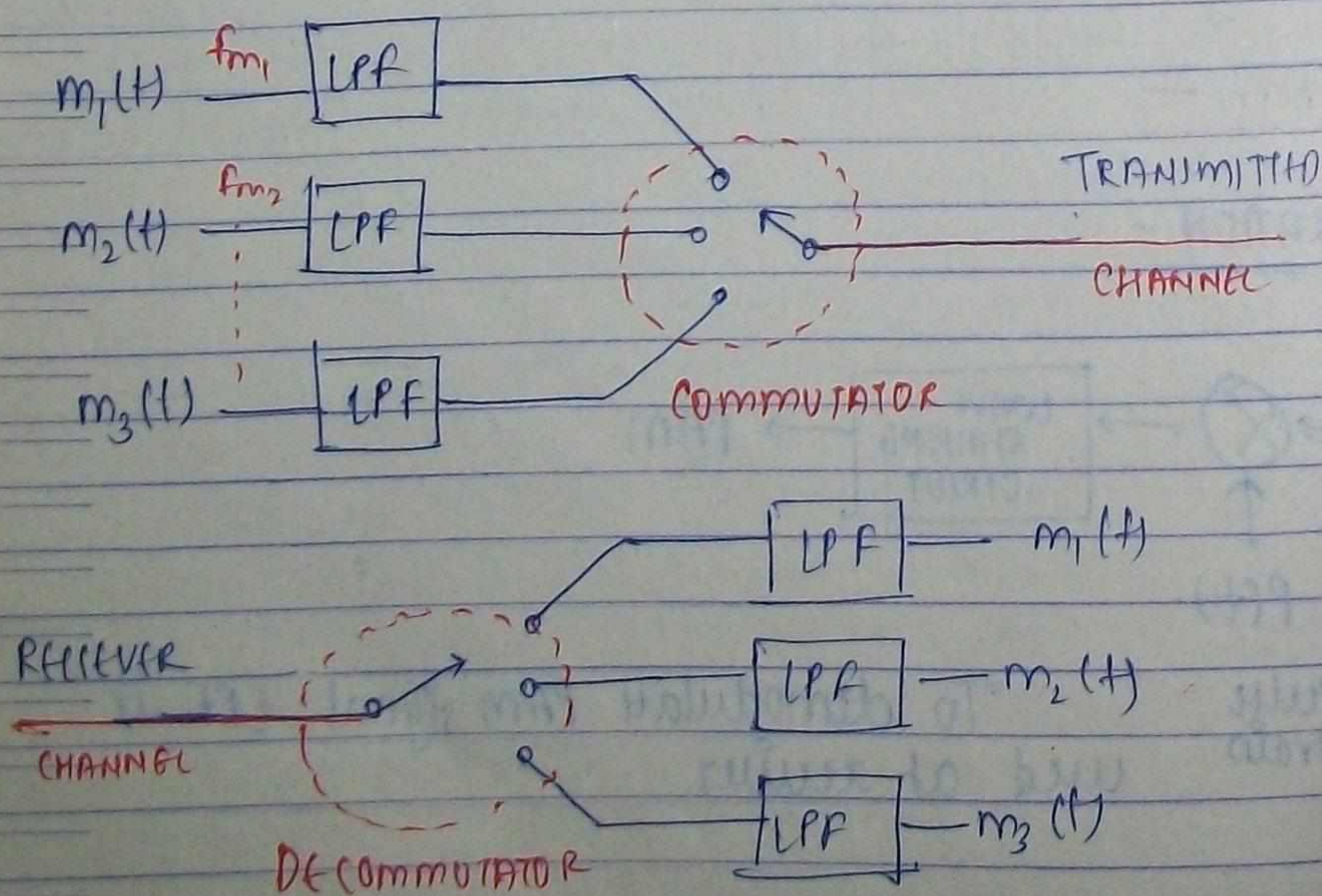
$$BW = \frac{f_s}{2}$$

$$= f_m$$

f_s = sampling freq.

⇒ TIME DIVISION MULTIPLEXING -

- In TDM no. of signals are multiplexed and transmitted through a single channel. So during this each signal is passed through a low pass filter to avoid aliasing then signals are multiplexed by passing through one commutator switch.



$$\text{Sampling rate} = N \cdot f_s$$

$$BW = \frac{1}{2} N \cdot f_s$$

- In TDM To provide synchronisation, synch. samples are transmitted b/w each frame. then,

$$\text{Sampling rate} = N f_s + f_s$$

↓
due to frame ↓
due to synch.

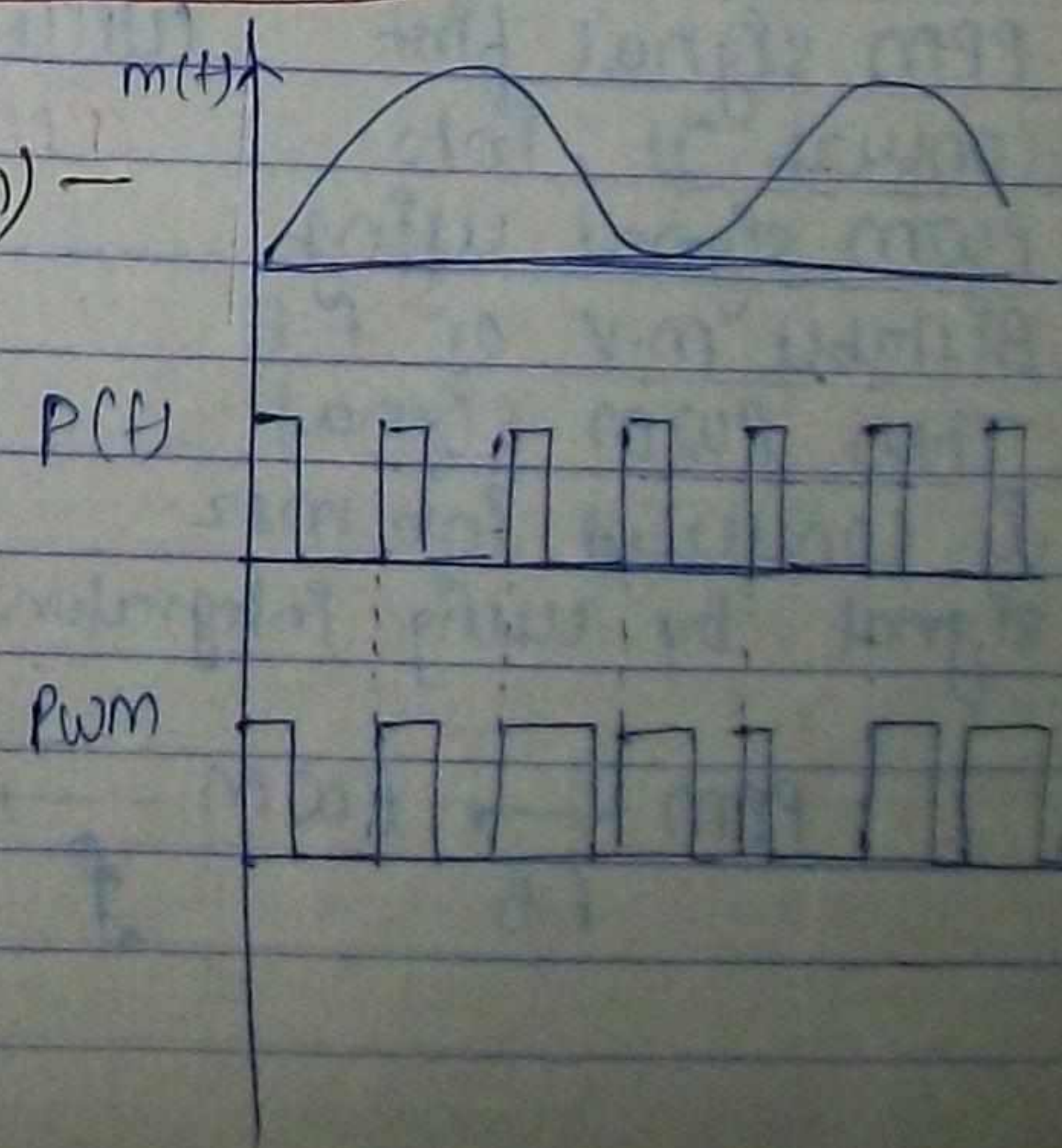
- If signals are sampled at individual rates then combined, then sampling rate will become

$$\text{Sampling rate} = f_{s1} + f_{s2} + f_{s3} + \dots$$

⇒ PULSE WIDTH MODULATION (PWM) -

PW = Pulse width

$$BW \propto \frac{1}{PW}$$

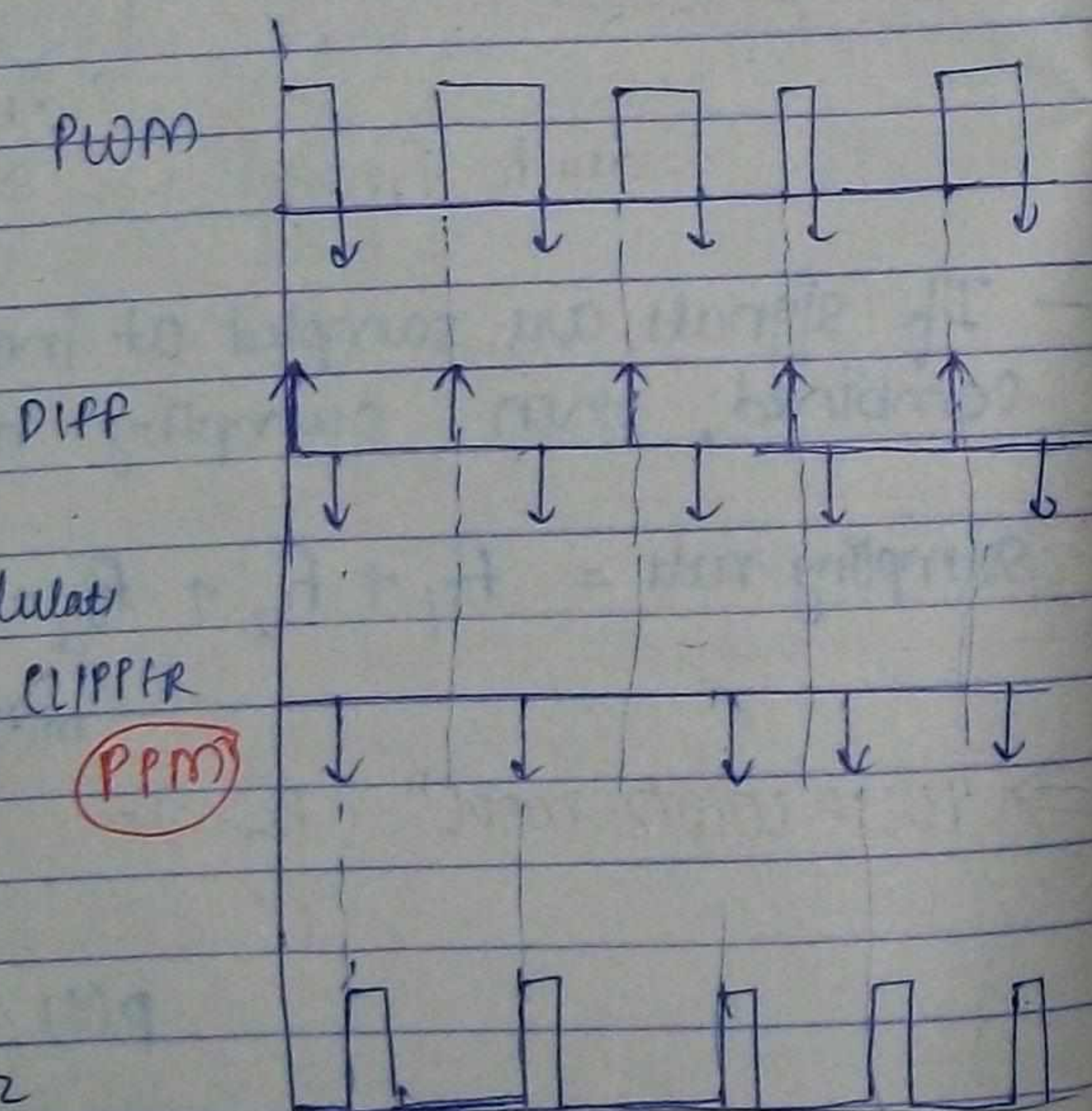


- To generate PWM $\rightarrow$ monostable M.V
- To demodulate PWM $\rightarrow$ integrator unit is used
- Disadvantage of PWM is BW and Tx. power will vary with Tx. msg signal. To overcome this PPM signal is used.

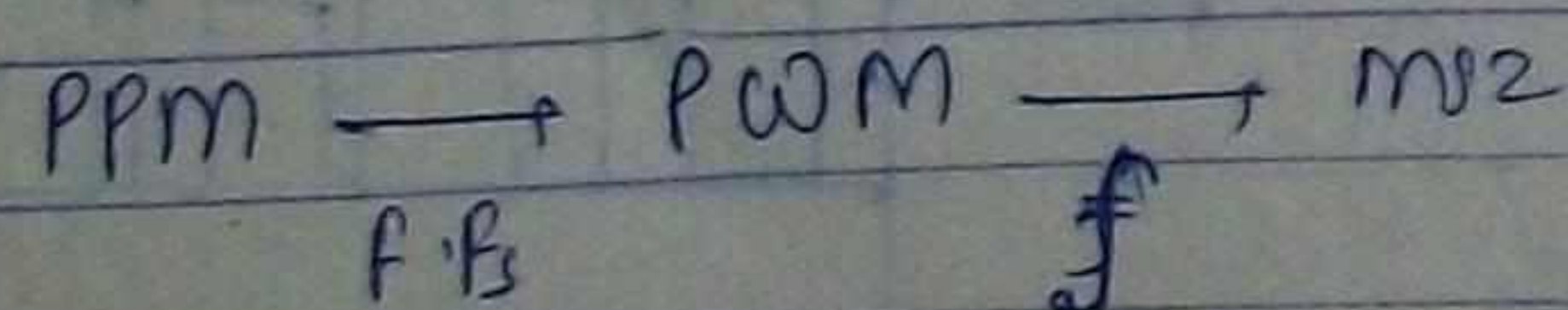
$$BW \propto \frac{1}{\text{Pulse width}}$$

$\Rightarrow$ PULSE POSITION MODⁿ (PPM) -

- To generate PPM signal, differentiator followed by clipper circuit is used.



- To convert or demodulate PPM signal first convert it into PWM signal using Bistable M.V or F.F. Then PWM signal is converted into msg signal by using integration.



PROBABILITY & RANDOM VARIABLE

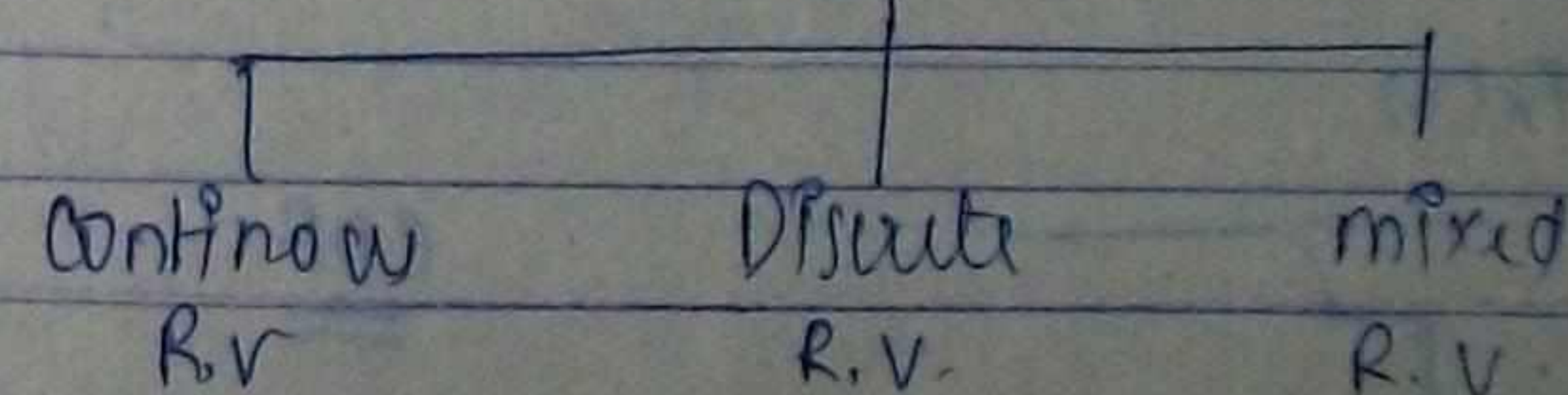
- $\rightarrow$ Random variable
- $\rightarrow$ CDF, PDF
- $\rightarrow$ Probability
- $\rightarrow$ Statistical values. (Mean, msv, RMS, DC power, variance, s.d.)
- $\rightarrow$ Various distribution. — Gaussian, poisson, uniform, exp. Binomial, Bernoulli
- $\rightarrow$ autocorrelation, PSD
- $\rightarrow$ Cross correlation, covariance
- $\rightarrow$ Joint distribution
- $\rightarrow$ Random process

- Experiment is conducted to get some outcome.
- Outcome is a result obtained after conducting the experiment.
- Event is totality of all outcomes
- Sample space is sum of contains all possible events

$\Rightarrow$ RANDOM VARIABLE -

R.V. is a mathematical funⁿ which will assign numerical value to each event in sample space

Random variable



⇒ CUMMULATIVE DISTRIBUTION FUNCTION (CDF)

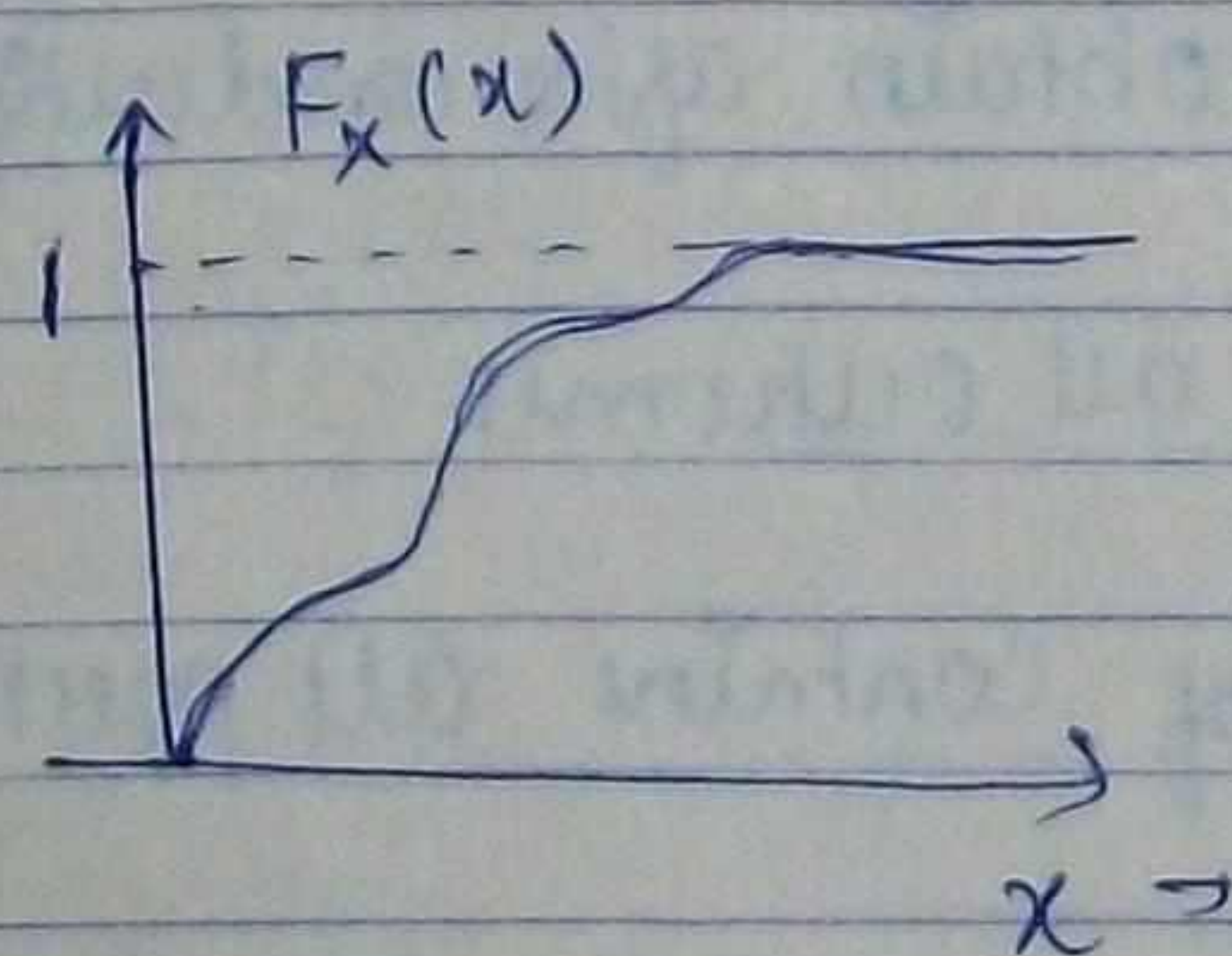
Represented by $\rightarrow F_X(x)$

where $x \rightarrow$ R.V.
 $x \rightarrow$ fixed value.

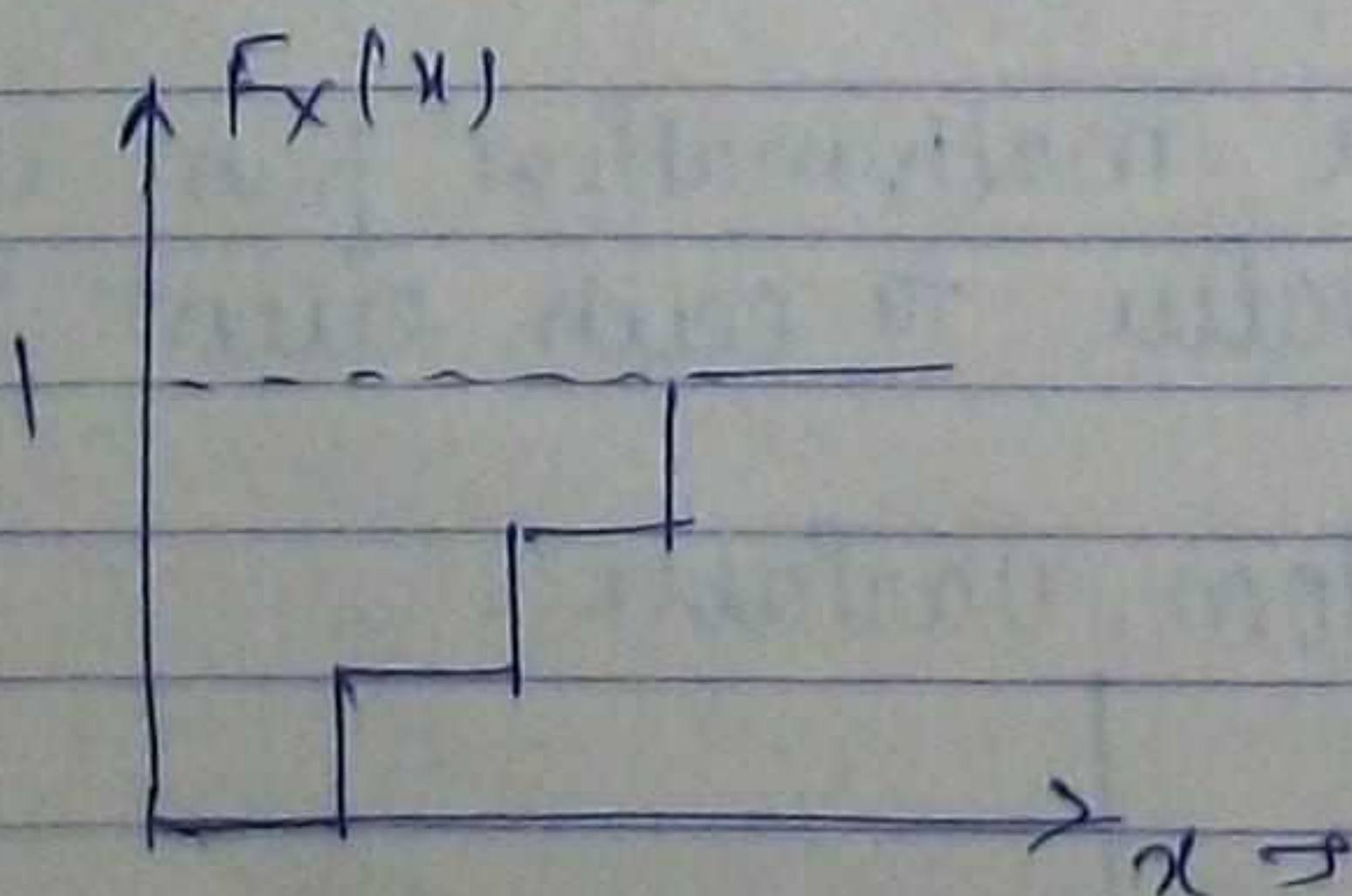
- CDF $F_X(x)$ is probability that an event $X \leq x$

$$F_X(x) = P(X \leq x)$$

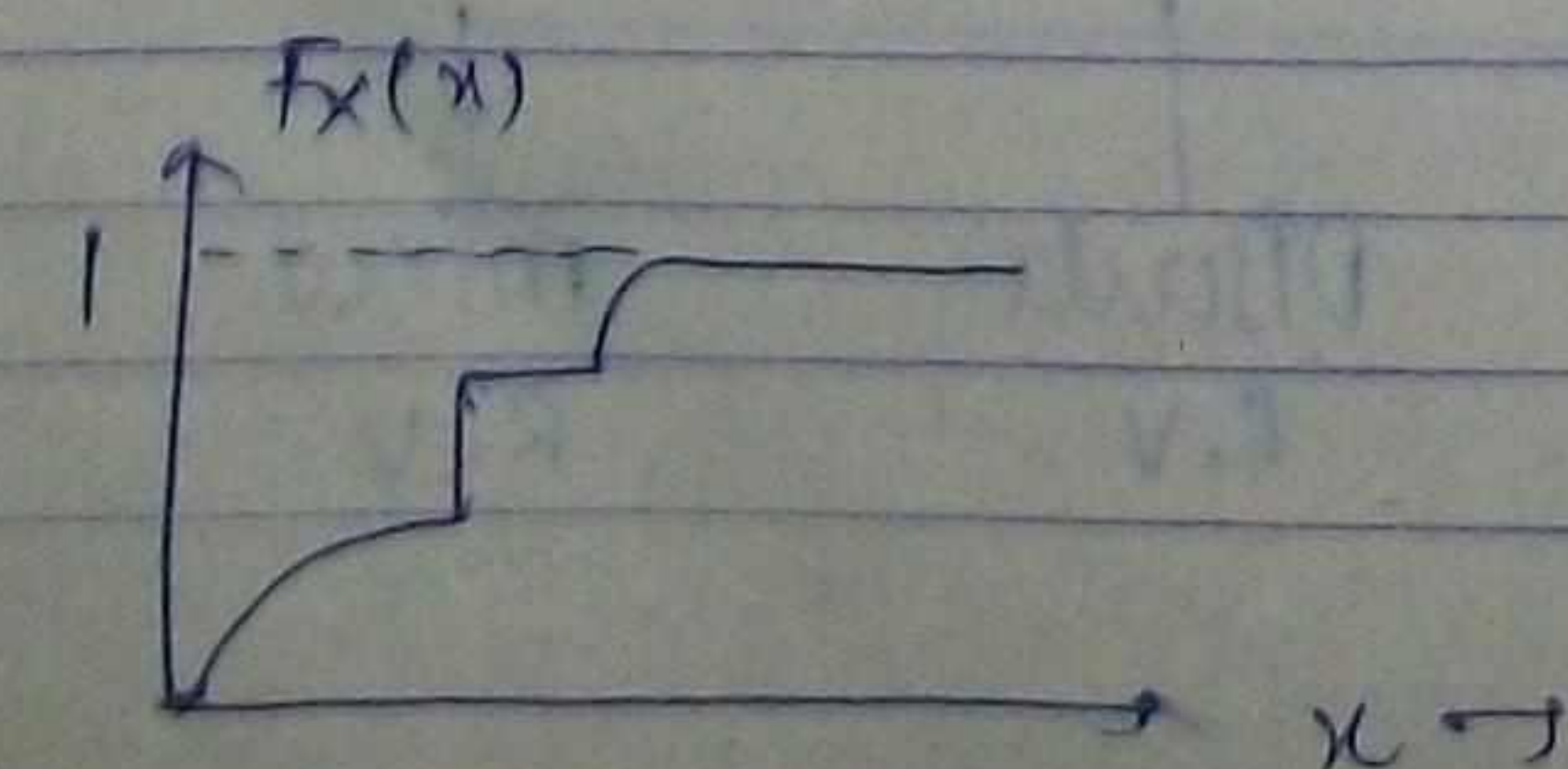
CONTINUOUS
R.V.



DISCRETE
R.V.



MIXED
R.V.



PROPERTIES -

$$① F_X(x) = P(X \leq x)$$

$$② 0 \leq F_X(x) \leq 1$$

$$③ F_X(-\infty) = 0, F_X(+\infty) = 1$$

$$④ P(x_1 < X < x_2) = F_X(x_2) - F_X(x_1)$$

⑤ CDF, $F_X(x)$ is non decreasing funⁿ of 'x'

⑥ If $x_2 > x_1$ then

$$F_X(x_2) \geq F_X(x_1)$$

FOR CONTINUOUS R.V. -

$$① P(x_1 < X < x_2)$$

$$P(x_1 \leq X < x_2)$$

$$P(x_1 < X \leq x_2)$$

$$P(x_1 \leq X \leq x_2)$$

$$= F_X(x_2) - F_X(x_1)$$

$$② P(X = x) = 0.$$

FOR DISCRETE R.V. -

$$① P(X = x) = F_X(x) - F_X(x^-)$$

$$(2) P(x_1 < X \leq x_2) = F_X(x_2) - F_X(x_1)$$

$$(3) P(x_1 \leq X \leq x_2) = F_X(x_2) - F_X(x_1^-)$$

↓

$$= P(x_1 < X \leq x_2) + P(X = x_1)$$

$$= F_X(x_2) - F_X(x_1) + F_X(x_1) - F_X(x_1^-)$$

$$= F_X(x_2) - F_X(x_1^-)$$

$$(4) P(x_1 < X < x_2) = F_X(x_2^-) - F_X(x_1)$$

↓

$$= P(x_1 < X \leq x_2) - P(X = x_2)$$

$$= F_X(x_2) - F_X(x_1) - F_X(x_2) + F_X(x_2^-)$$

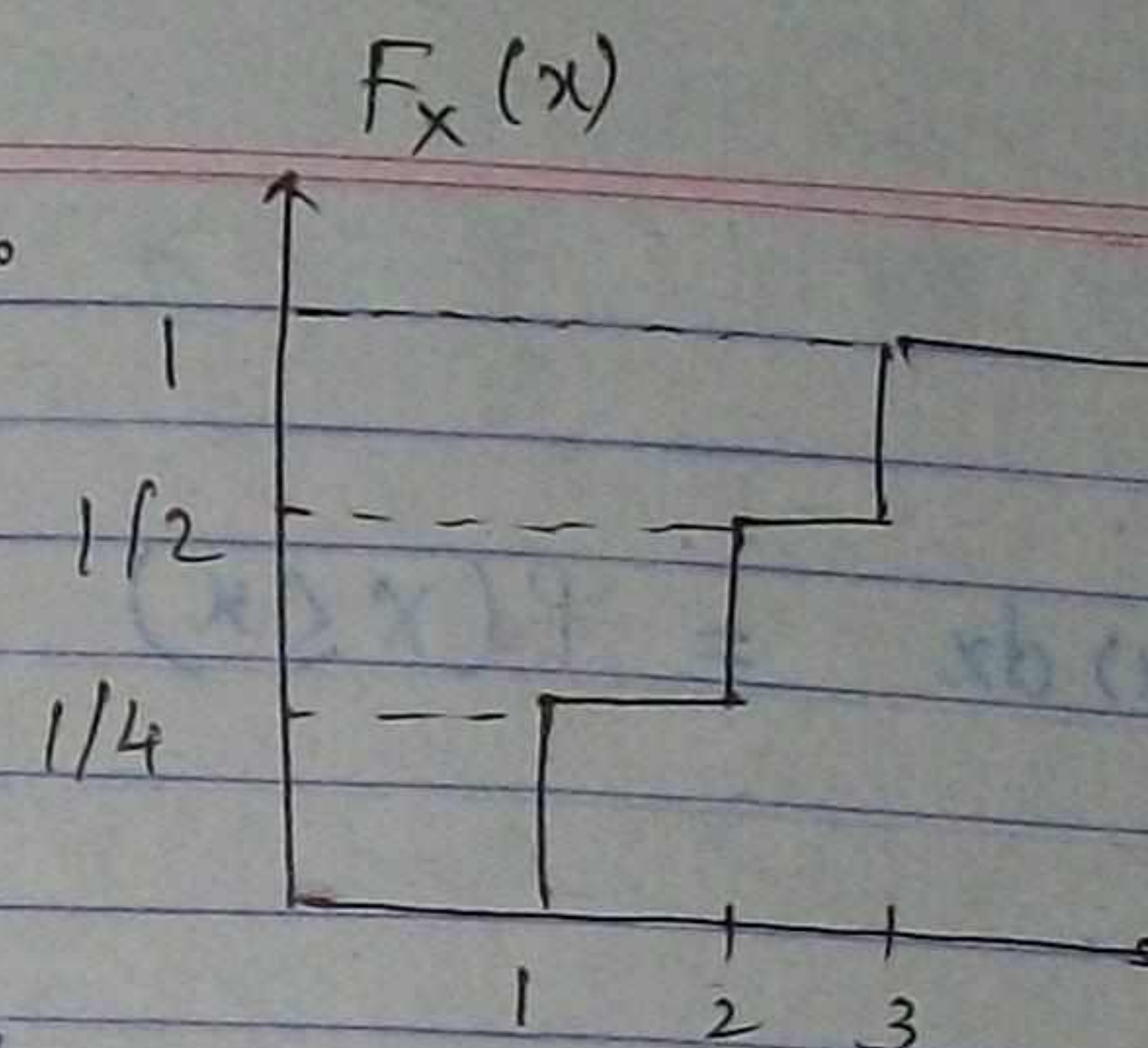
$$= F_X(x_2^-) - F_X(x_1)$$

$$(5) P(x_1 \leq X < x_2) = F_X(x_2^-) - F_X(x_1^-)$$

$$(6) P(X \leq x) = F_X(x)$$

$$(7) P(X > x) = 1 - F_X(x)$$

Ques.



DRV

- (1) $P(X=1)$
- (2) $P(1 < X \leq 3)$
- (3) $P(1 \leq X < 3)$
- (4) $P(1 \leq X \leq 3)$
- (5) $P(1 < X < 3)$
- (6) $P(X \leq 3)$
- (7) $P(X > 3)$

Solⁿs

$$(1) P(X=1) = F_X(1) - F_X(1^-) = 1/4 - 0 = 1/4$$

$$(2) P(1 < X \leq 3) = F_X(3) - F_X(1) = 1 - 1/4 = 3/4$$

$$(3) P(1 \leq X < 3) = F_X(3) - F_X(1^-) = 1 - 0 = 1$$

$$(4) P(1 \leq X \leq 3) = F_X(3^-) - F_X(1^-) = 1/2 - 0 = 1/2$$

$$(5) P(1 < X < 3) = F_X(3^-) - F_X(1) = 1/2 - 1/4 = 1/4$$

$$(6) P(X \leq 3) = F_X(3) = 1$$

$$(7) P(X > 3) = 1 - 1 = 0$$

⇒ PROBABILITY DENSITY FUNCTION (PDF)

$$f_X(x) = \rightarrow \text{CRV}$$

$$f_X(x) \rightarrow \text{slope of } F_X(x)$$

$$① f_x(x) = \frac{d}{dx} F_x(x)$$

$$② F_x(x) = \int_{-\infty}^x f_x(x) dx = P(X \leq x)$$

$$\left. \begin{array}{l} P(x_1 < X < x_2) \\ P(x_1 \leq X < x_2) \\ P(x_1 < X \leq x_2) \\ P(x_1 \leq X \leq x_2) \end{array} \right\} = \int_{x_1}^{x_2} f_x(x) dx$$

$$④ F_x(\infty) = 1$$

$$⑤ F_x(\infty) = \int_{-\infty}^{\infty} f_x(x) dx = 1$$

∴ Total area under PDF = 1.

Ques. Given $f_x(x) = k\sqrt{x}$ $0 < x < 1$

① determine k value

② $P(0.3 < X < 0.6)$

Soln: ① $\int_0^1 k\sqrt{x} = 1$

$$K \cdot \left(2 \frac{x^{1/2+1}}{3} \right) \Big|_0^1 = 1$$

$$K = 3/2$$

$$② P(0.3 < X < 0.6) = \int_{0.3}^{0.6} K\sqrt{x} dx \quad (K=3/2)$$

$$= K \left(\frac{x^{3/2}}{3/2} \right) \Big|_{0.3}^{0.6} \approx 0.3$$

⇒ EXPECTED VALUES (OR) STATICAL VALUES -

① n^{th} moment -

$$n^{\text{th}} \text{ moment} = E[X^n] = \int_{-\infty}^{\infty} x^n f_x(x) dx$$

for $n=1$ → mean (or) avg. = dc value

DC. VALUE $E[X] = \int_{-\infty}^{\infty} x \cdot f_x(x) dx = m_x = \mu_x$

$$E[X] = m_x = \mu_x$$

for $n=2$ → mean square value (msv)
= total power.

$$E[X^2] = \int_{-\infty}^{\infty} x^2 f_x(x) dx$$

PROPERTIES -

$$① E[X] = \mu_x$$

$$② E[KX] = K \cdot \mu_x$$

$$③ E[K] = K$$

$$④ E[aX + b] = a\mu_x + b$$

$$⑤ E[X_1 + X_2] = E[X_1] + E[X_2] = \mu_{x_1} + \mu_{x_2}$$

$$⑥ E[X_1 \cdot X_2] = E[X_1] \cdot E[X_2] \quad \text{when } x_1 \& x_2 \text{ are independent}$$

$$⑦ \text{DC power} = (E[X])^2 = \mu_x^2$$

⑧ n^{th} CENTRAL MOMENT -

$$E[(X - \mu_x)^n] = \int_{-\infty}^{\infty} (x - \mu_x)^n f_x(x) dx$$

for $n=1$, first central moment

$$E(X - \mu_x) = \int_{-\infty}^{\infty} (x - \mu_x) f_x(x) dx = 0$$

$$E[X - \mu_x] = E[X] - E[\mu_x]$$

$$= \mu_x - \mu_x = 0$$

for $n=2$ (2^{nd} central moment) Variance

$$E[(X - \mu_x)^2] = E[X^2] + E[\mu_x^2] - 2E[X \cdot \mu_x]$$

$$= E[X^2] + \mu_x^2 - 2\mu_x \cdot \mu_x$$

$$= E[X^2] - \mu_x^2$$

$$\text{Variance} = \sigma_x^2 = \text{ac power} = E[(X - \mu_x)^2] = E[X^2] - \mu_x^2$$

$$\text{Variance} = E[X^2] - \mu_x^2$$

$$\text{Variance} = \text{Mean square value} - (\text{mean})^2$$

$$\text{ac power} = \text{Total power} - \text{DC power}$$

$$\text{STANDARD DEVIATION} = \sqrt{\text{VARIANCE}}$$

Ques Evaluate ① mean ② msv ③ variance
④ SD ⑤ RMS ⑥ DC power.

given $f_x(x) = 2(x-1) \quad 1 < x < 2$

Solⁿ: ① $E[X] = \int_{-\infty}^{+\infty} x \cdot f_x(x) \cdot dx$
 $= \int_1^2 x \cdot 2(x-1) dx = 5/3$

② $E[X^2] = \int_{-\infty}^{+\infty} x^2 \cdot f_x(x) dx$
 $= \int_1^2 x^2 \cdot 2(x-1) dx = 17/6$

③ Variance.

$$E[(X - \mu_x)^2] = E[X^2] - \mu_x^2$$

$$= \frac{17}{6} - \left(\frac{5}{3}\right)^2 = \frac{1}{18}$$

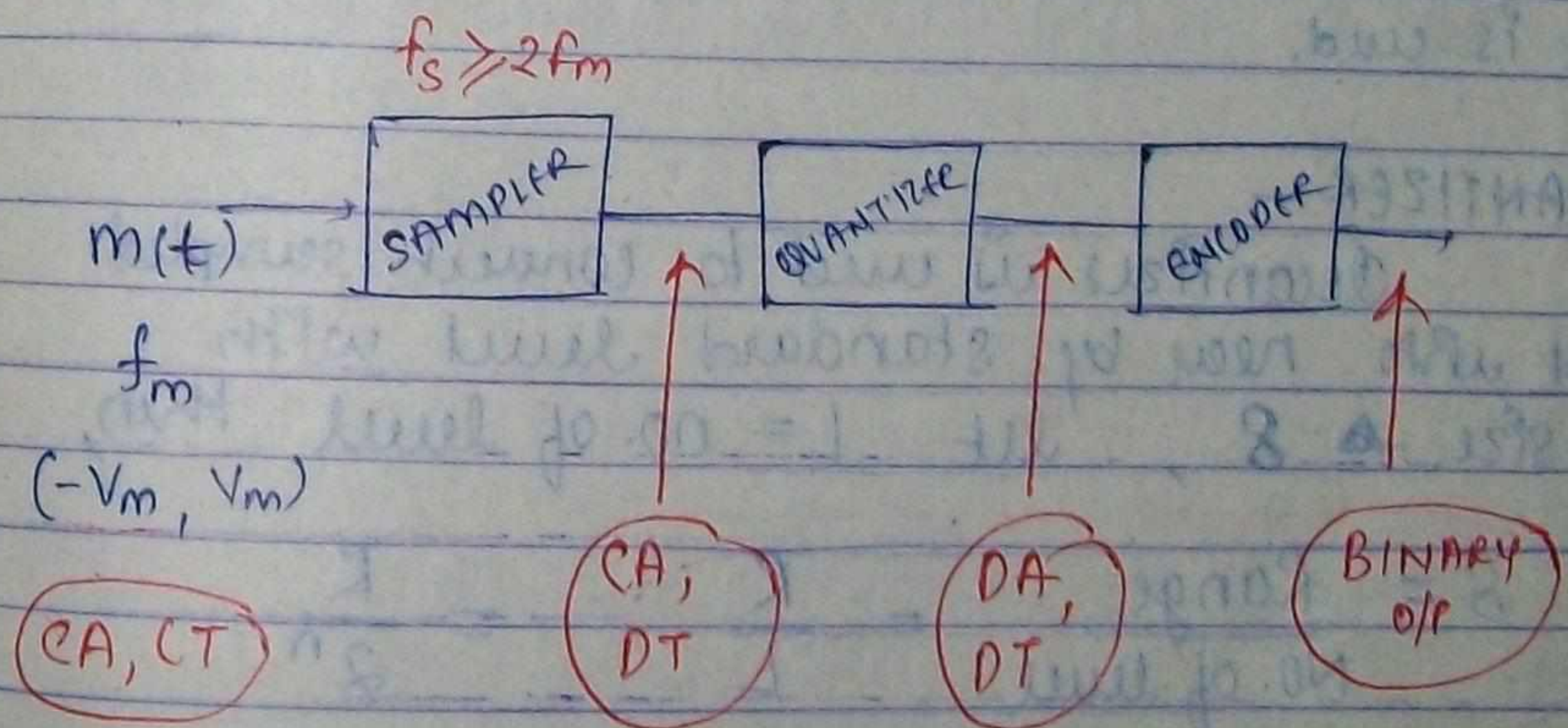
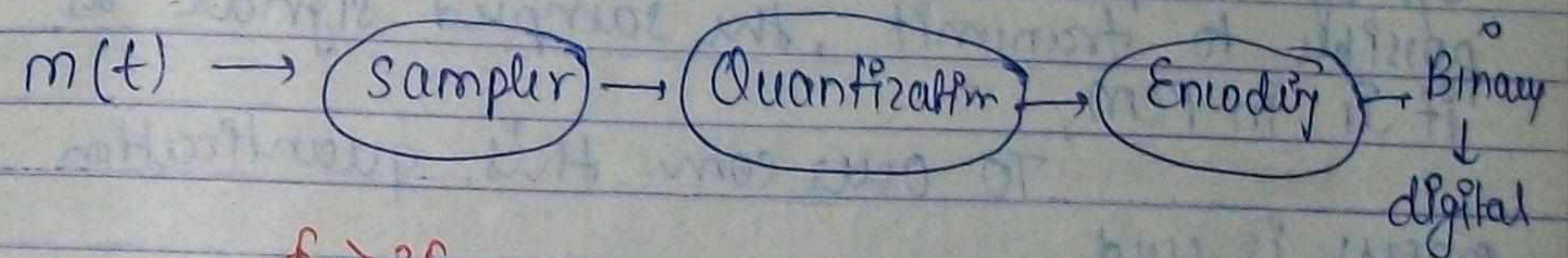
④ $SD = \sqrt{\text{variance}} = \sqrt{\frac{1}{18}}$

$$= \frac{1}{3} \frac{51-50}{18} \quad \frac{3}{3} \frac{16,9}{2,1} \quad \frac{217}{5}$$

⑤ $RMS = \sqrt{MSV} = \sqrt{\frac{17}{6}}$

⑥ DC. power = $\mu_x^2 = \left(\frac{5}{3}\right)^2 = \frac{25}{9}$

⇒ PULSE CODE MODULATION (PCM) -



- In PCM, the msg signal $m(t)$ is sampled, quantized and converted into n -bit binary bits then its transmitted, to provide PCM it basically contain three blocks.

① Sampler ② quantizer ③ Encoder

(no. of bits) $n = \log_2 L$

① SAMPLER -

Sampler is used to convert a continuous signal (CA, CT) into discrete time signal. The msg signal $m(t)$ have message signal freq f_m and range $= R = V_{max} - V_{min}$.

The o/p of sampler may sometime any amplitude b/w V_{min} & V_{max} . Due to this infinite no. of levels may present at the o/p of sampler. Due to this practically not possible to transmit, the sampled signal as it is in PCM.

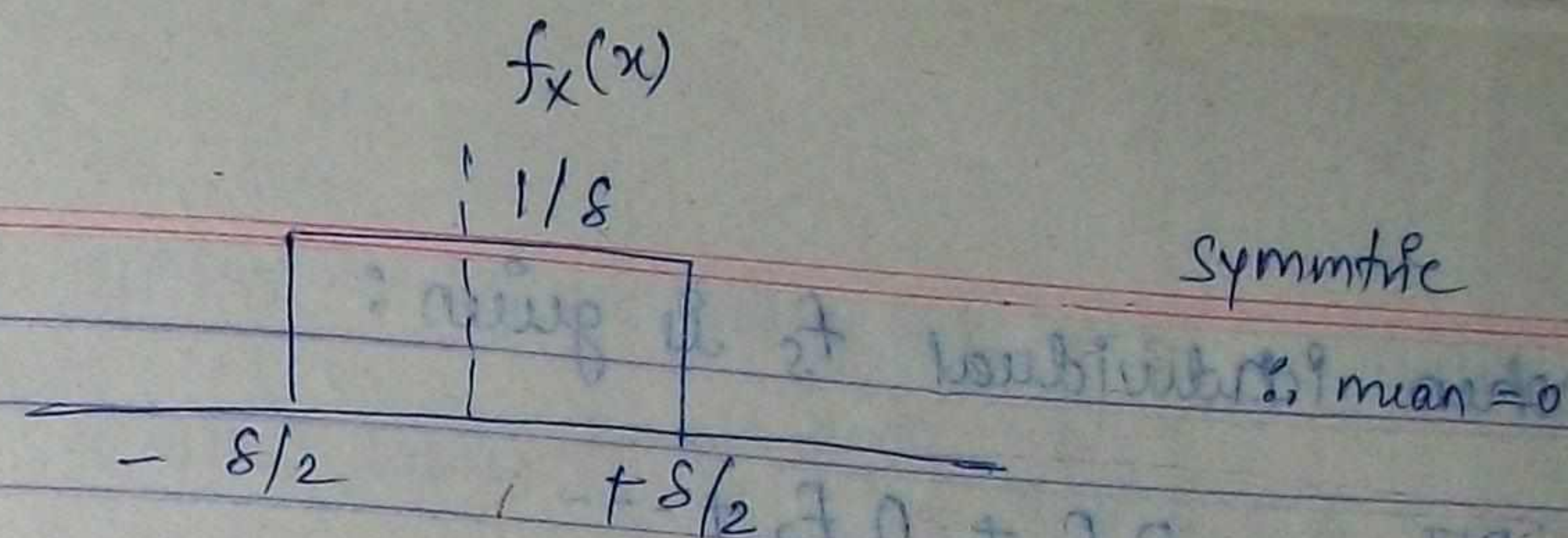
To overcome this quantization process is used.

② QUANTIZER -

Quantizer is used to convert sampled signal into near by standard level with step size Δ , let $L = \text{no. of level}$ then,

$$\Delta = \frac{\text{Range}}{\text{No. of level}} = \frac{R}{L} = \frac{R}{2^n}$$

- Due to quantization error or noise will present at the o/p of quantizer, this quantization noise is uniformly present b/w $-\Delta/2$ to $+\Delta/2$ with PDF as show below:



③ ENCODER -

Encoder is used to convert each quantized value into n -bit binary o/p.

$$L = 2^n$$

$$n = \log_2 L$$

① BIT RATE OF PCM -

In PCM if each bit is transmitted for a bit interval of t_b , then bit rate is given by:

$$\text{BIT RATE} = r_b = \frac{1}{t_b}$$

for one signal only

$$\text{BIT RATE} = r_b = n \cdot f_s \quad \text{bps}$$

$n = \text{no. of bits}$

for two signal $r_b = 2 \cdot n \cdot f_s$

If 'N' no. of signals are TDMed then bit rate

$$r_d = N \cdot n \cdot f_s$$

- when individual f_s is given:

$$\text{BIT RATE} = n f_1 + n f_2 + \dots$$

- when synchronisation bit is considered

$$\text{BIT RATE} = r_b = \underbrace{N \cdot n f_s}_{\text{data}} + \underbrace{n f_s}_{\text{synchronisation signalling}}$$

- In standard telephone channel to transmit voice signal 8-bit PCM with 8 kHz sampling freq. is used

(2) BW of PCM -

The min. BW required for PCM system is:

$$\text{BW} = \frac{r_b}{2} = \frac{n \cdot f_s}{2}$$

$$\frac{r_b}{2} \leq \text{BW}_{\text{PCM}} \leq r_b$$

(3) $(\text{SNR})_q$ of PCM -

(SIGNAL TO QUANTIZATION NOISE)

let $m(t) = V_m \cos 2\pi f_m t$

$$\text{signal power} = \frac{V_m^2}{2} \quad \text{Range} = R = 2V_m$$

Quantization noise power -

- In PCM it is less affected by thermal noise but it is mostly affected by quantization noise.

- The quantization noise is uniformly distributed b/w $-\delta/2$ to $+\delta/2$ with PDF = $\frac{1}{\delta}$

$$\text{Quantization noise power} = \text{msv} = \langle q_e^2 \rangle$$

$$= \int_{-\infty}^{+\infty} x^2 \cdot f_x(x) \cdot dx$$

$$\text{Quantization noise power} = \int_{-\delta/2}^{+\delta/2} x^2 \cdot \frac{1}{\delta} dx = \frac{\delta^2}{12}$$

$$\delta = \frac{\text{Range}}{\text{No. of levels}} = \frac{2V_m}{2^n}$$

$$\therefore \text{quantization noise power} = \frac{1}{3} \cdot \frac{V_m^2}{2^{2n}}$$

$$(SNR)_q = \frac{\text{signal power}}{(\text{Noise power})_q} = \frac{V_m^2/2}{V_m^2/3 \cdot 2^{2n}}$$

$$(SNR)_q = \frac{3}{2} \cdot 2^{2n}$$

$$(SNR)_q = 3 \cdot 2^{(2n-1)}$$

$$(SNR)_q \text{ (db)} = 10 \log_{10} \left(\frac{3}{2} \cdot 2^{2n} \right)$$

$$(SNR)_q \text{ (db)} = (6n + 1.76) \text{ db}$$

$$\approx (6n + 1.8) \text{ db}$$

$$\approx 6n \text{ db}$$

Ques. An 8-bit PCM has $(SNR)_q$ is 36 db

Soln: $48 + 1.8 = 49.8 \text{ db}$

① 49.8 db

② 54.8 db

③ 48 db

Ques. In PCM system no. of bits are increased from 6 to 7 bits then $(SNR)_q$ will be

① ↑ 6 db ② ↓ 6 db ③ ↑ 12 db ④ const

Soln: $\frac{42 - 36}{36} = \frac{6}{36} = \frac{1}{6}$

Ques. In PCM

① In PCM each 1 bit increment will provide 6 db increment in $(SNR)_q$.

② In PCM if no. of bits are changed from $n \rightarrow n+m$ then $(SNR)_q$ will be improved by 6m db by factor = 2^{2m}

Ques. In PCM no. of bits are changed from 10 to 12 then $(SNR)_q$ will improve by a factor of

① 12 ② 16 ③ 4 ④ 8

Soln: 10 to 12 → 2 bit ↑

↑ by → 12 db

2 bit → 12 db

3 db → 2

6 db → 4

9 db → 8

12 db → 12

$10 \log_{10} 12 = 16$

Ques. In PCM no. of bits are changed from 8 to 9 then quantization noise power will be

① ↑ 6 db ② ↓ 6 db ③ ↑ 12 db ④ const

$N_q = \frac{1}{3} \times \frac{V_m^2}{2^{2n}}$

↑ $N_q \propto \frac{1}{2^{2n}}$ ↓

Ques. No of bits change from 10 to 11 then signal power in PCM
 (1) $\uparrow$ 6db (2) $\downarrow$ 6db (3) $\uparrow$ 12db (4) remain const

- Advantage of PCM is:

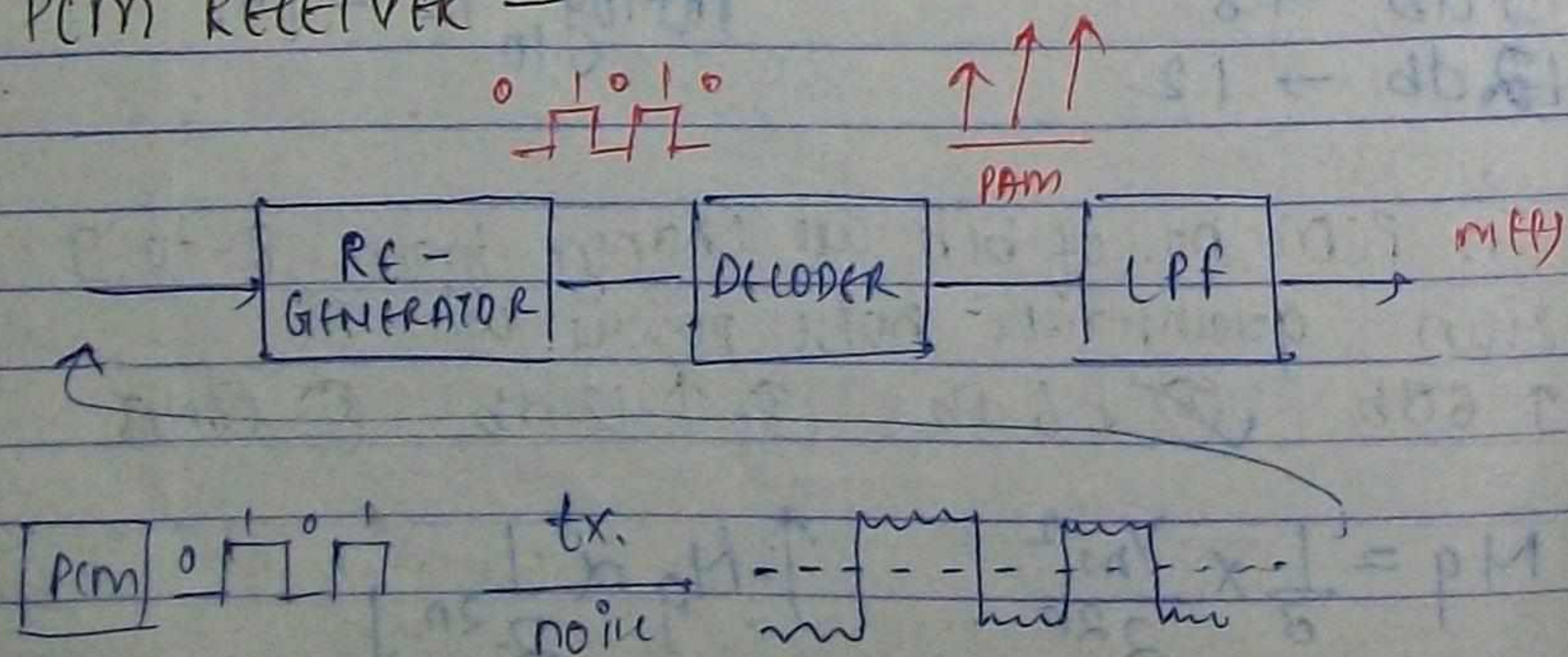
- (1) It is less affected by thermal noise or external noise.
- (2) It is less complex.

(3) - Disadvantage:

1. BW required is more.
2. Quantization noise will present.

$q \downarrow, L \uparrow, n \uparrow, BW \uparrow$

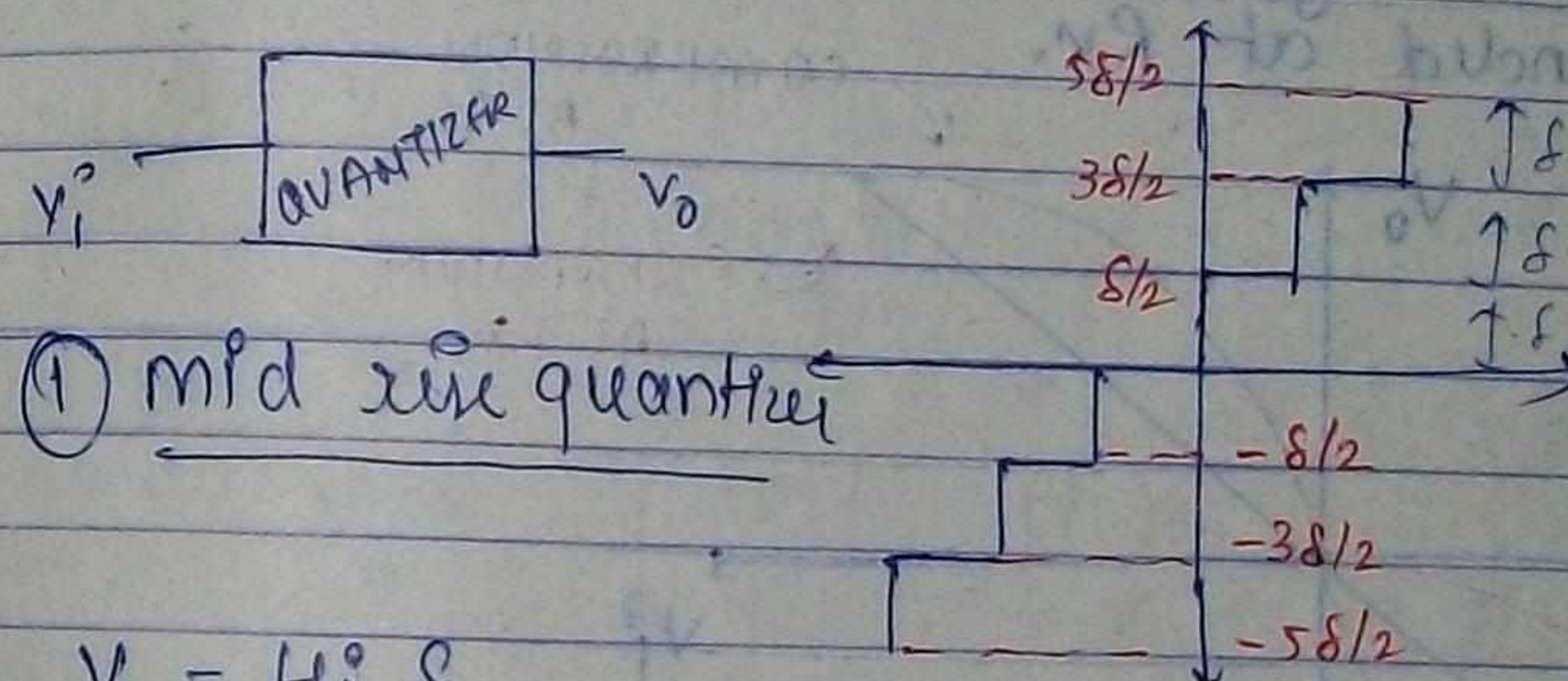
PCM RECEIVER -



$\Rightarrow$ QUANTIZERS -

Quantization -
 - uniform quantization
 - Non-uniform quantization (Companding)

uniform quantization -
 - mid rise quantizer
 - mid tread quantizer



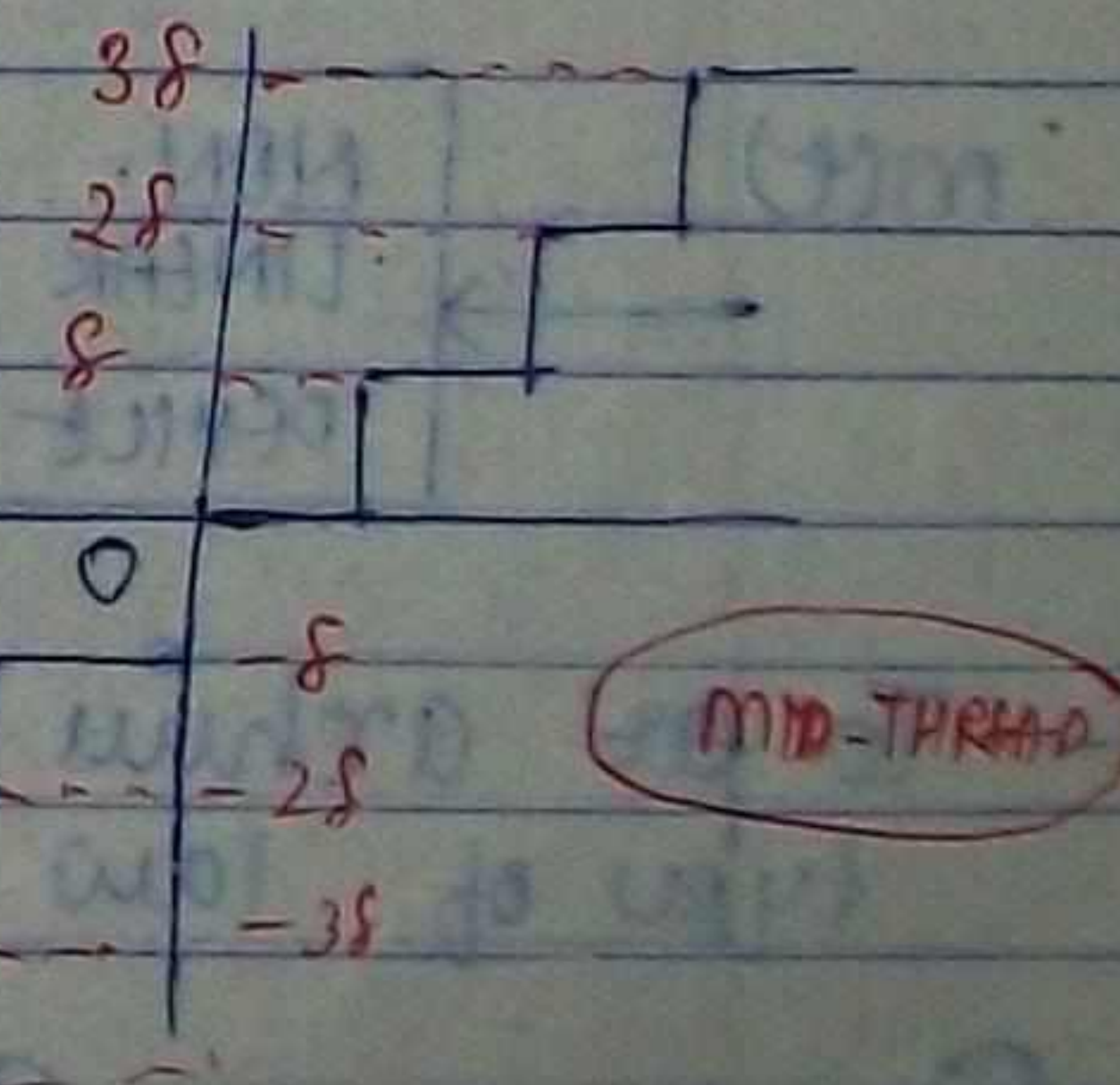
(1) mid rise quantizer

$$V_o = H_i \frac{\delta}{2}$$

$$H_i \rightarrow \pm (1, 3, 5, 7)$$

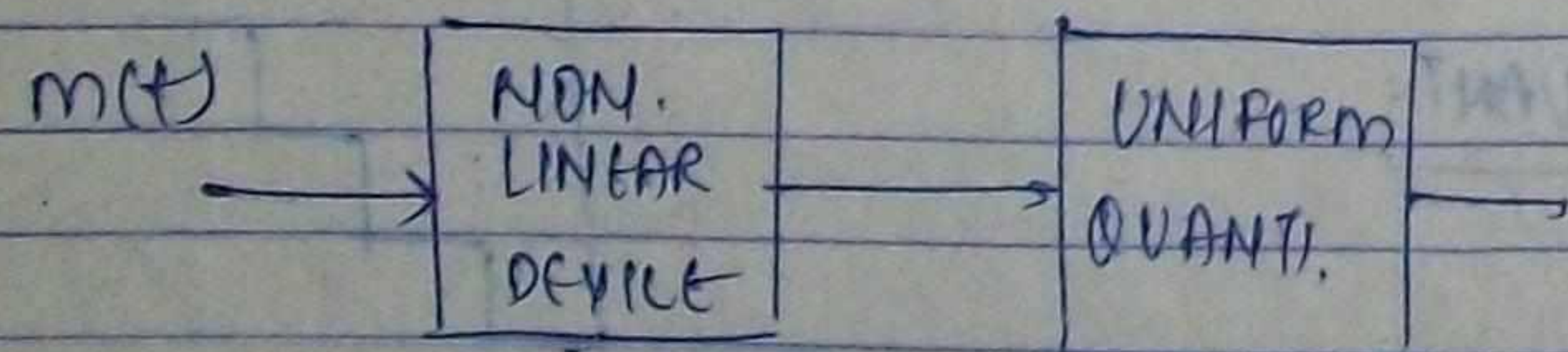
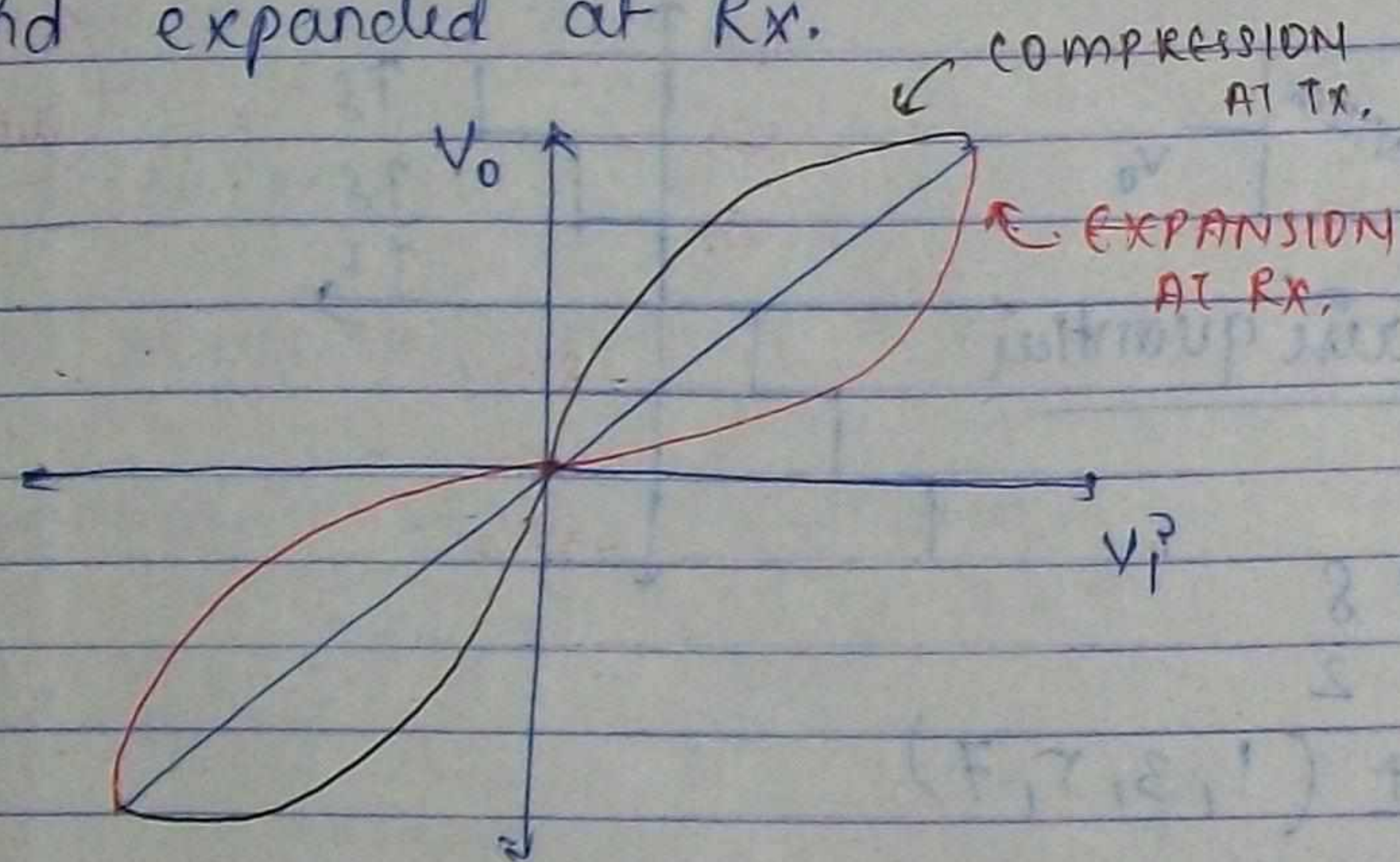
(2) MID THREAD QUANT:

(linear)



NON-UNIFORM QUANT. (COMPANDING)

- In uniform quant. the low amplitude signals are mostly affected by quantization noise. In order to protect low amplitude signal companding technique is used in PCM.
- Companding is a non uniform quantⁿ in which high amplitude signals are compressed at Tx. and expanded at Rx.



- To achieve companding there are two types of law:

- ① μ -law
- ② A-law

① μ -LAW -

$$V_0 = \frac{\log(1 + \mu |V_1|)}{\log(1 + \mu)} \quad |V_1| = \frac{V_0}{V_{max}}$$

μ varies 1-255

② A-LAW -

$$V_0 = \frac{A |V_1|}{1 + \log A} \quad 0 \leq |V_1| \leq \frac{1}{A}$$

$$V_0 = \frac{1 + \log A |V_1|}{1 + \log A} \quad \frac{1}{A} \leq |V_1| \leq 1$$

- In μ law μ varies from 1-255

- In A law mostly $A \rightarrow 87.6$

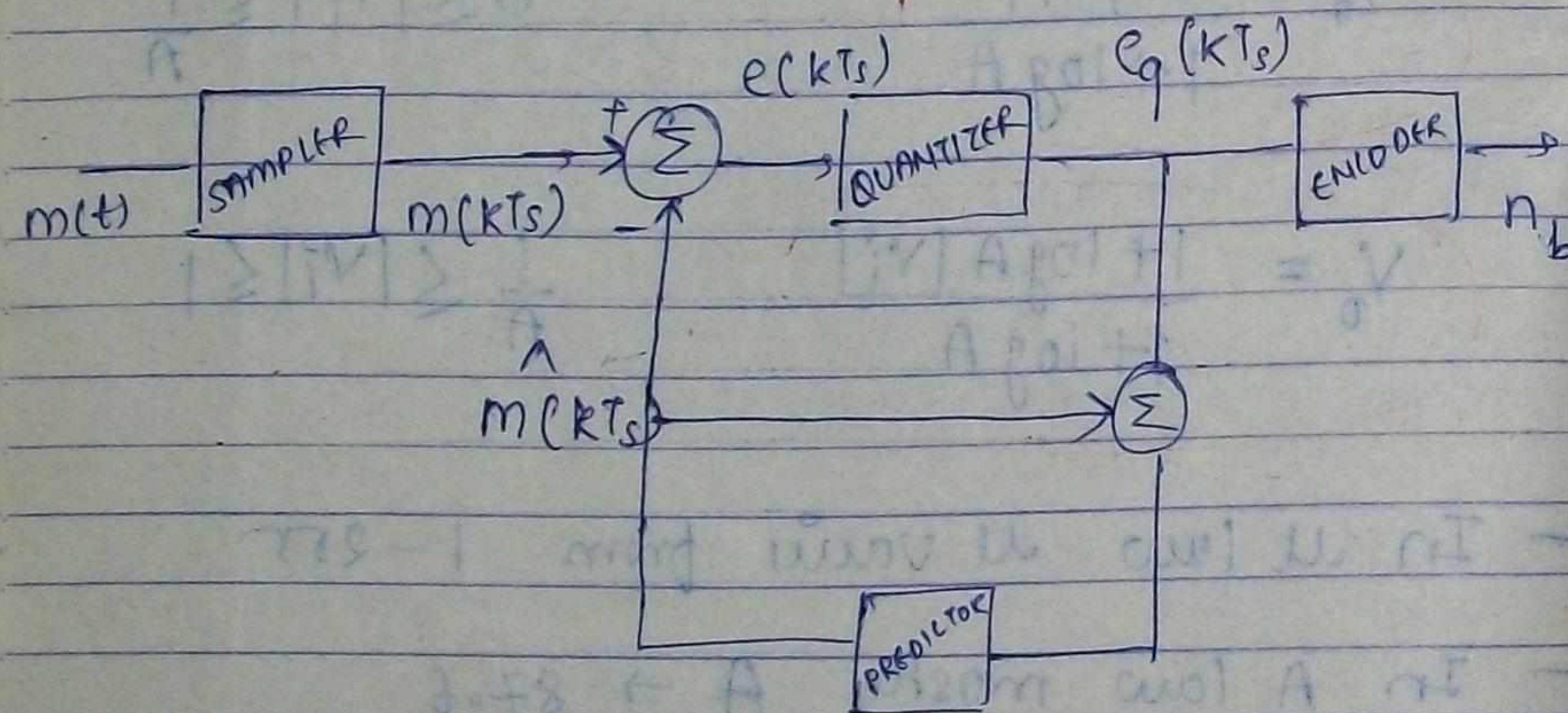
- To reduce BW in PCM following steps are used:

- ① DPCM
- ② Δm
- ③ $A\Delta m$

⇒ DPCM (DIFFERENTIAL PCM) -

- In DPCM instead of transmitting sampled signal as it is, the difference b/w present sample & previous sample is Tx'd. Instead of Tx. absolute value, relative value is Tx'd.

- In ^{some} DPCM system difference b/w previous sample & expected or predicted sample is Tx.

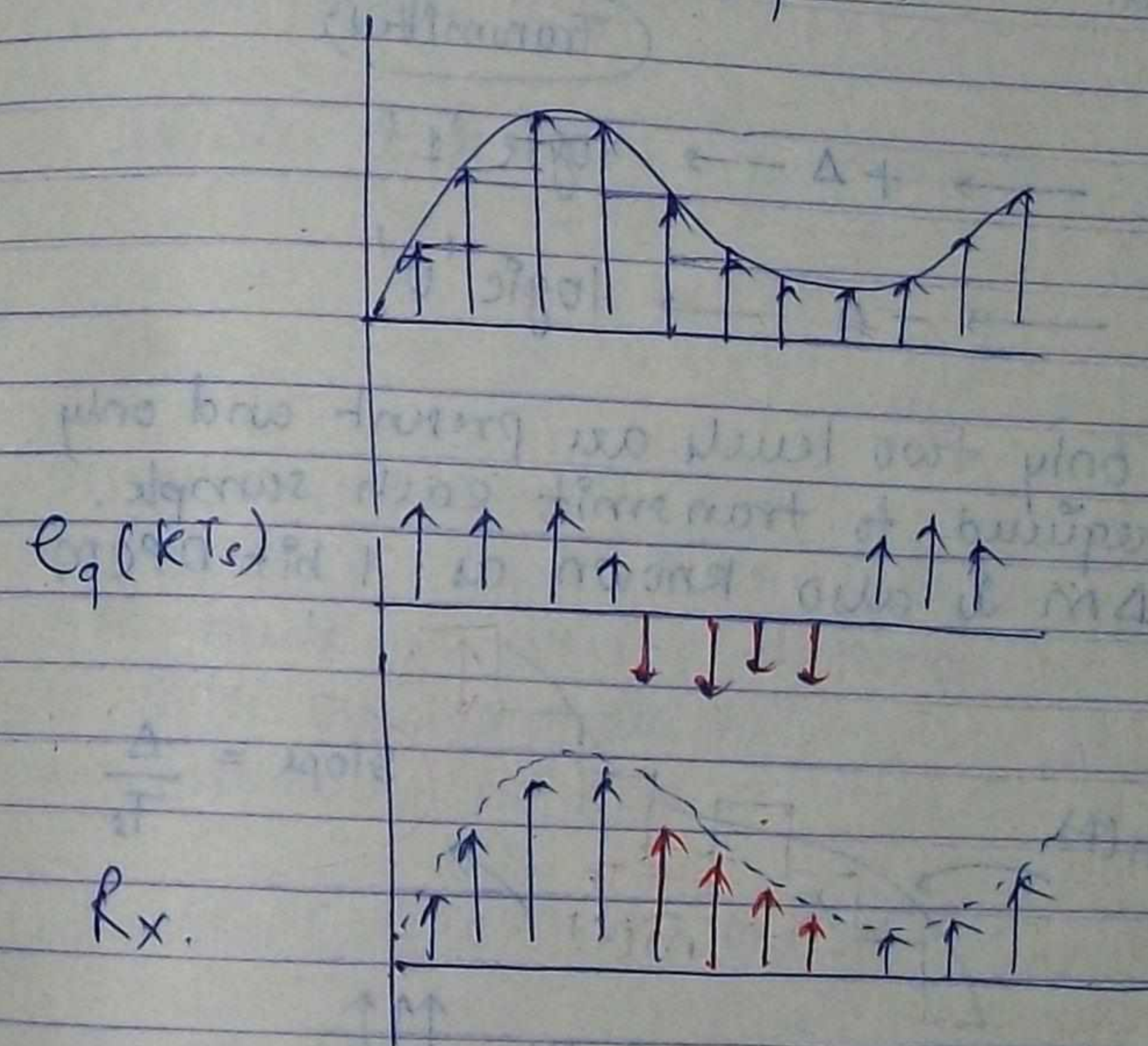


- In DPCM correlation is required b/w successive samples to recover original signal. To provide correlation the sampling freq f_s

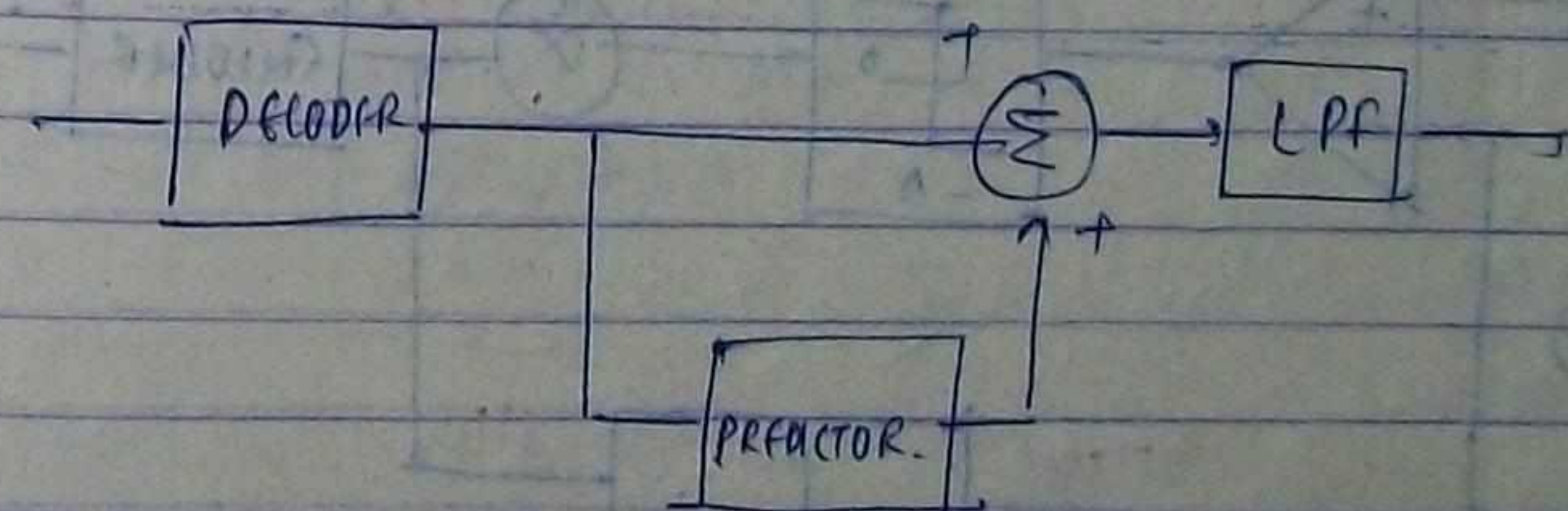
$f_s \ggg 2F_m$ is used in DPCM system

- In DPCM main advantage is:

① no. of bits required to Tx. each sample will be decreased, due to this BW & bit rate will decrease. In DPCM system.



RECEIVER -



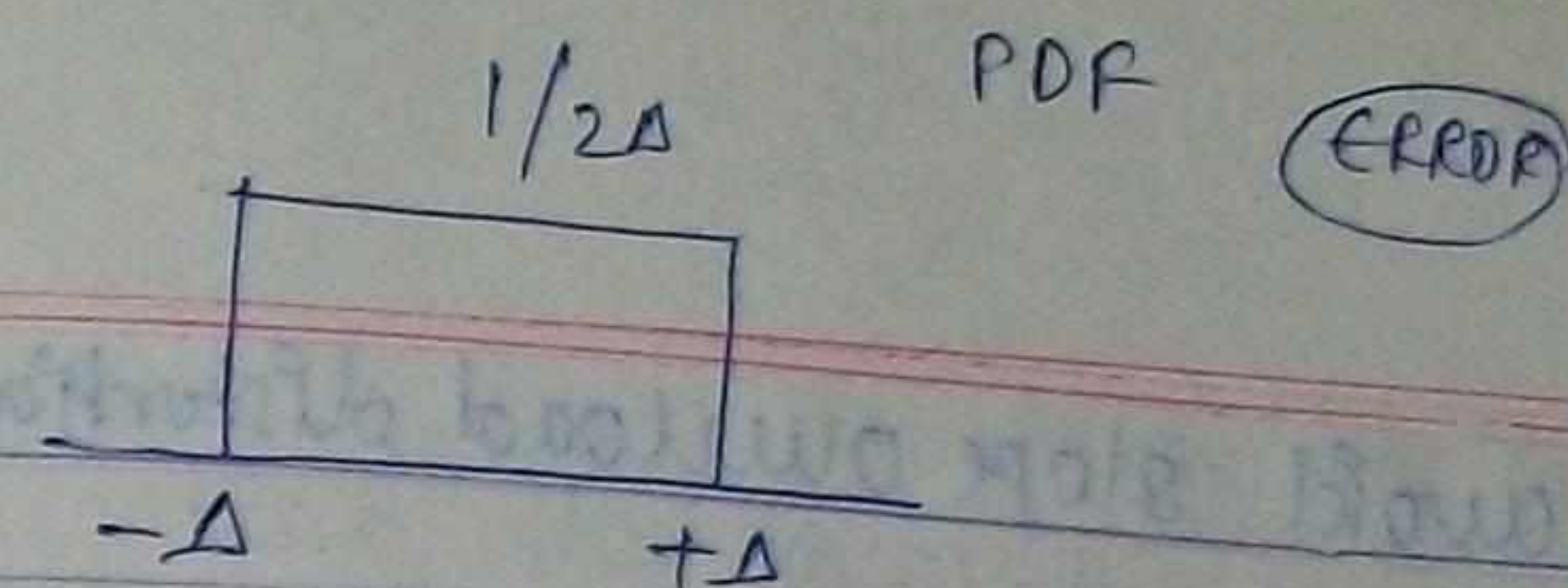
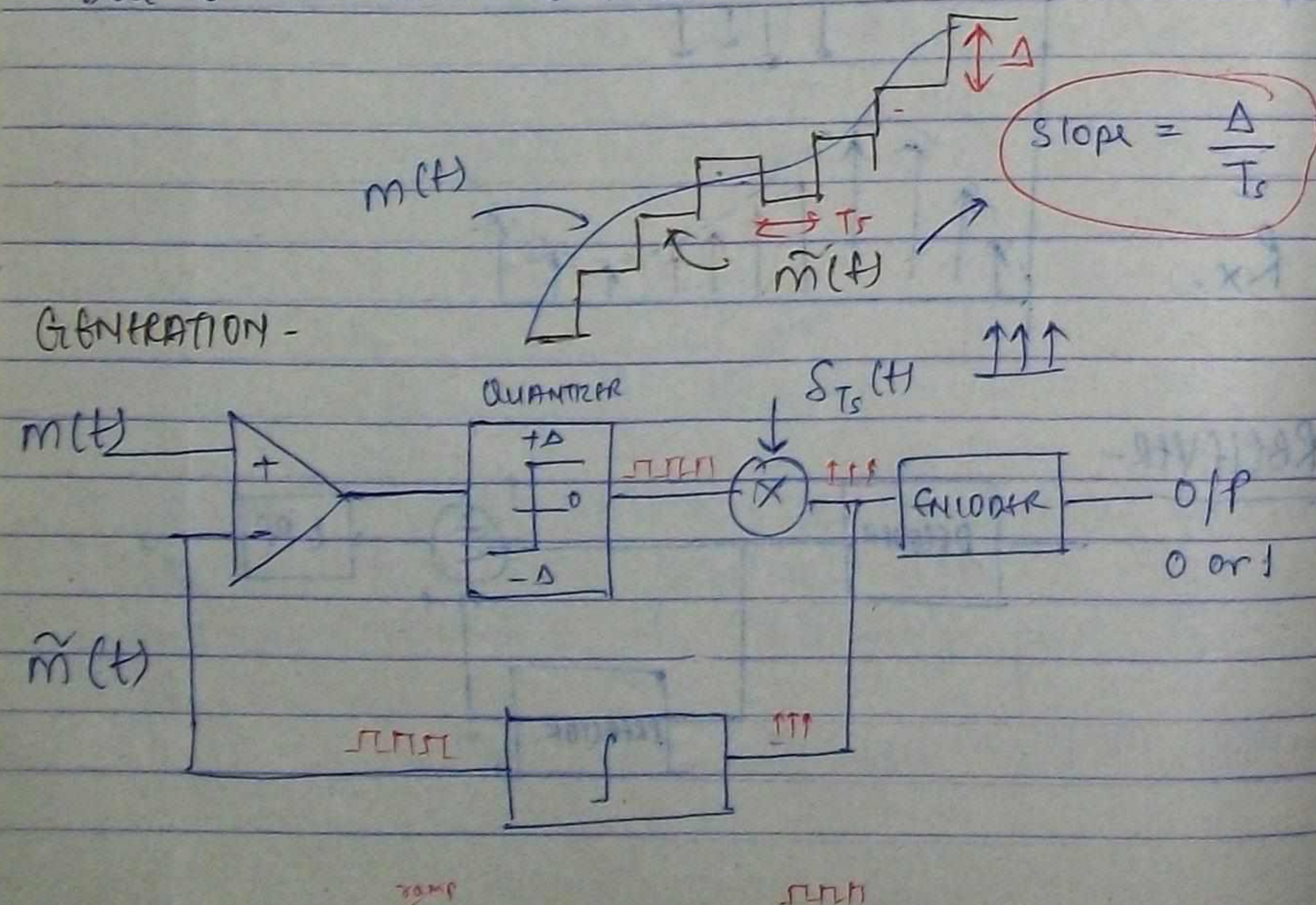
⇒ DELTA MODULATION -

- In ΔM the msg signal $m(t)$ and internally generated staircase waveform $\tilde{m}(t)$ is compared and when,

$$m(t) > \tilde{m}(t) \longrightarrow +\Delta \longrightarrow \text{logic '1'}$$

$$m(t) < \tilde{m}(t) \longrightarrow -\Delta \longrightarrow \text{logic '0'}$$

Hence in ΔM only two levels are present and only one bit is required to transmit each sample. Due to this ΔM is also known as 1 bit DPCM.



- In ΔM the quantization error is uniformly present b/w $-\Delta$ to $+\Delta$ with PDF $1/2\Delta$ as shown above.

$$\text{In } \Delta M \text{ Bit rate } r_b = f_s$$

$$BW = \frac{f_s}{2}$$

⇒ DISTORTION PRESENT IN ΔM -

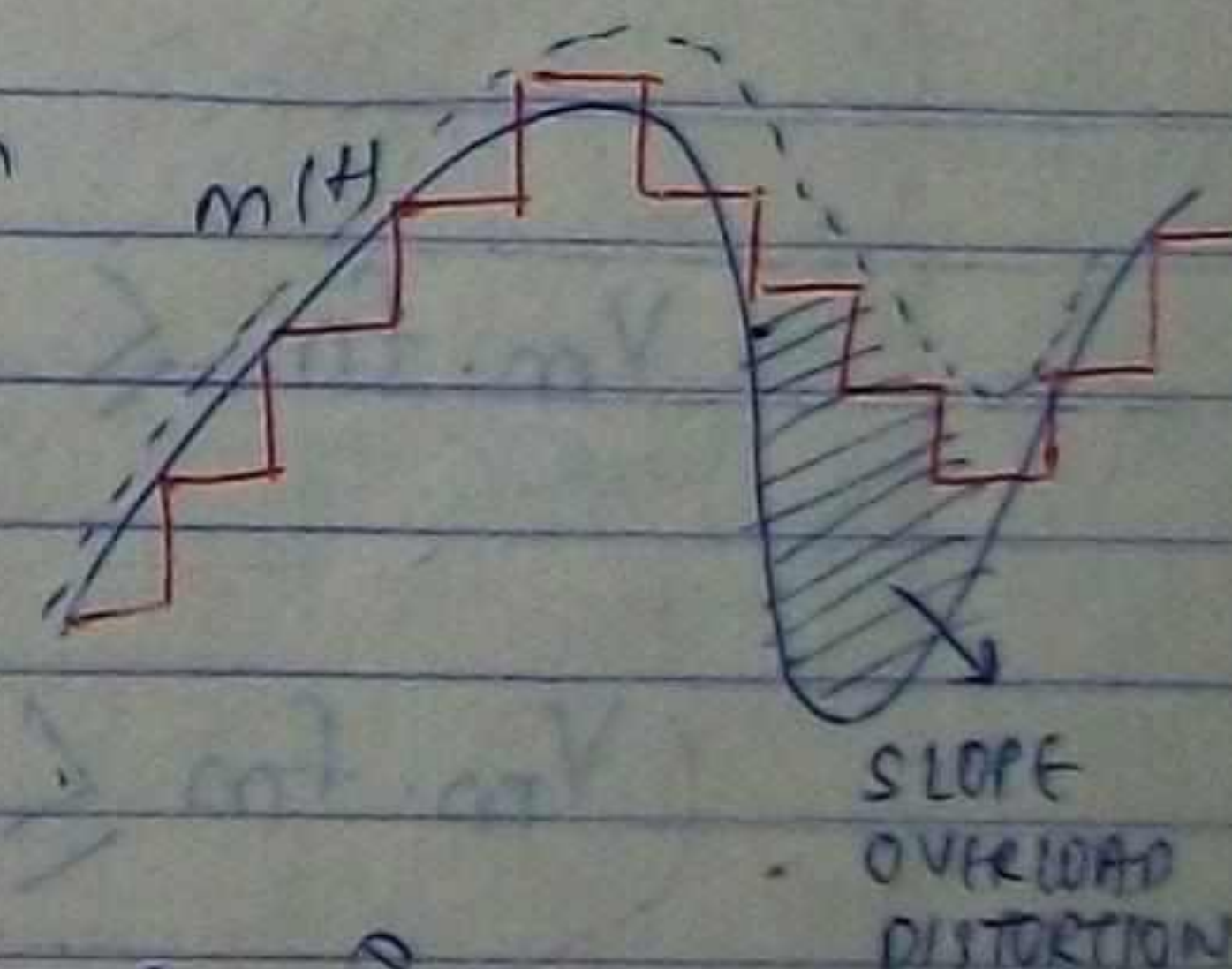
- ΔM have two types of distortion:
 1. Slope overload distortion.
 2. Granular noise.

① SLOPE OVERLOAD DISTORTION -

- In ΔM Slope overload distortion occurs when slope of msg signal $m(t)$ is greater than slope of staircase waveform $\tilde{m}(t)$ i.e.

$$\left| \frac{dm(t)}{dt} \right| > \frac{d\tilde{m}(t)}{dt}$$

Required condⁿ



Condition to avoid slope overload distortion is:

$$\left| \frac{d m(t)}{dt} \right| \leq \frac{d \tilde{m}(t)}{dt}$$

Let us consider $m(t) = V_m \cos 2\pi f_m t$

$$\frac{d m(t)}{dt} = -V_m \cdot 2\pi f_m \cdot \sin 2\pi f_m t$$

$$\left| \frac{d m(t)}{dt} \right| = V_m \cdot 2\pi f_m$$

In sinusoidal signal to avoid slope overload,

$$V_m \cdot 2\pi f_m \leq \frac{d \tilde{m}(t)}{dt}$$

$$V_m \cdot 2\pi f_m \leq \frac{\Delta}{T_s}$$

$$V_m \cdot f_m \leq \frac{\Delta}{2\pi} \times f_s$$

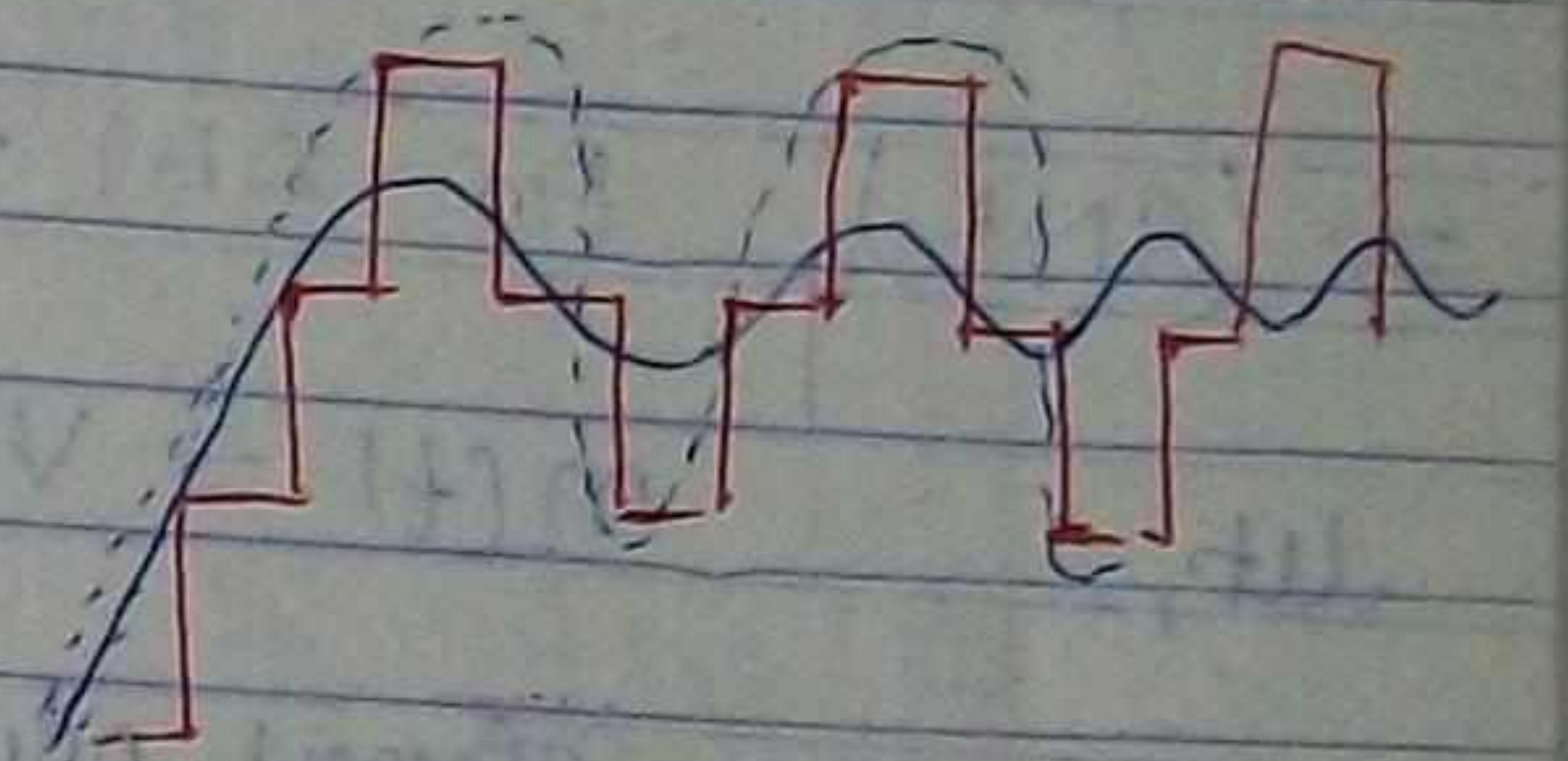
$$V_m \cdot f_m \leq \frac{\Delta}{2\pi} \times f_s$$

$$V_m \leq \frac{\Delta}{2\pi} \times \frac{f_s}{f_m}$$

- To overcome slope overload distortion step size Δ is required to increase.

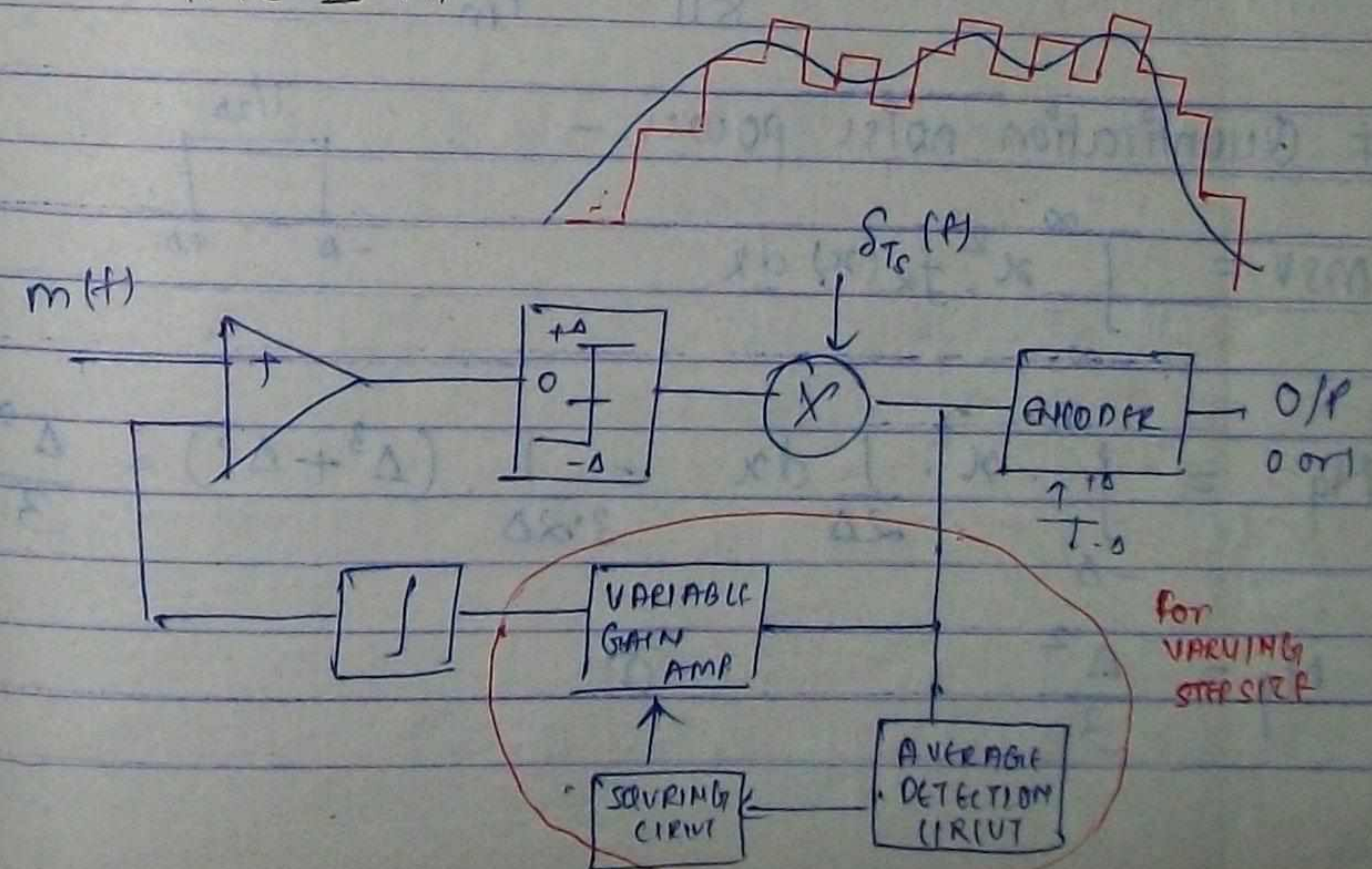
① GRANULAR NOISE -

- Granular noise will present in Δm when msz signal varies with in step size. To overcome granular noise step size is required to reduce. (but due to this slope overload distortion will increase).



- To overcome both slope overload distortion & granular noise variable stepsize is used in Δm and resulting system is known as ADM (adaptive).

⇒ ADAPTIVE Δ -M -



- In ADM due to variable step size it is mostly affected by channel noise or error.

$\Rightarrow (SNR)_q$ for AM -

Let, $m(t) = V_m \cos 2\pi f_m t$

signal power = $V_m^2/2$

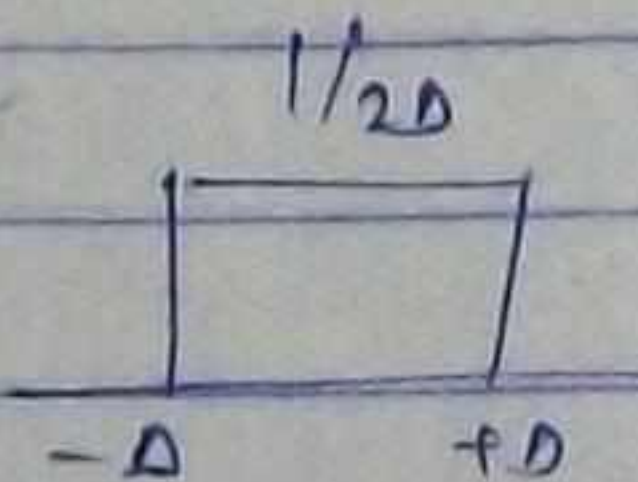
Sinusoidal signal amplitude V_m must be

$V_m \leq \frac{\Delta}{2\pi} \times \frac{f_s}{f_m}$ to avoid slope overload distortion.

$\therefore$ signal power = $\frac{\Delta^2}{8\pi} \times \frac{f_s^2}{f_m^2}$ - (1)

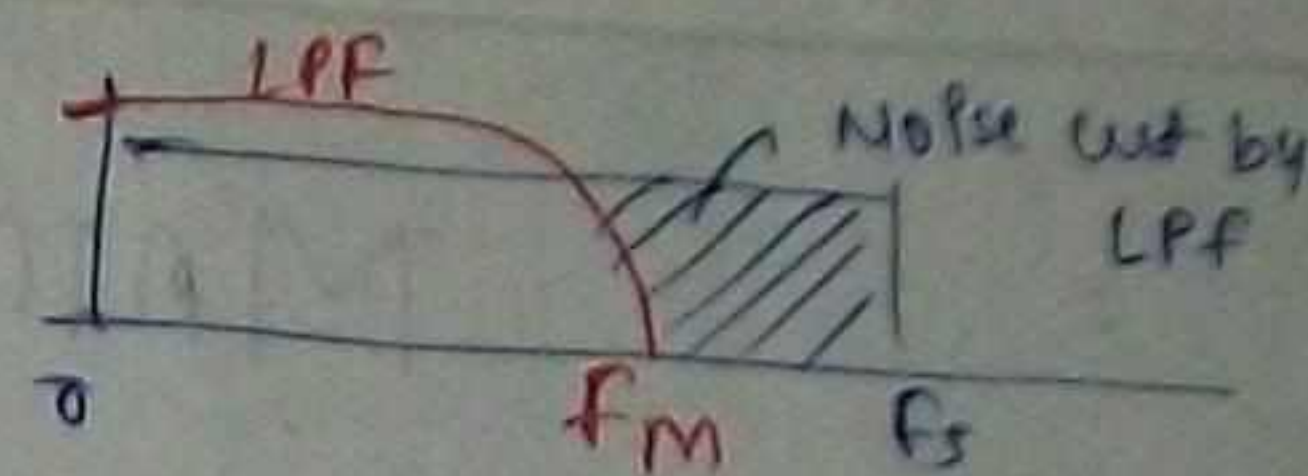
Quantization noise power -

$MSV = \int_{-\infty}^{\infty} x^2 f_x(x) dx$



$N_q = \int_{-\Delta}^{\Delta} x^2 \cdot \frac{1}{2\Delta} dx = \frac{1}{3 \cdot 2\Delta} (\Delta^3 + \Delta^3) = \frac{\Delta^2}{3}$

$N_q = \frac{\Delta^2}{3}$ - (2)



Let f_M = cutoff freq of LPF then o/p noise power in Δm is given by

$N_q = \frac{\Delta^2}{3} \times \frac{f_m}{f_s}$ - (3)

$(SNR)_q = \frac{\frac{\Delta^2}{8\pi} \times \frac{f_s^2}{f_m^2}}{\frac{\Delta^2}{3} \times \frac{f_m}{f_s}} = \frac{3}{8\pi} \times \frac{f_s^3}{f_M \cdot f_m^2}$

$(SNR)_q = \frac{3}{8\pi} \times \frac{f_s^3}{f_M \cdot f_m^2}$

when $f_M = f_m$

$(SNR)_q = \frac{3}{8\pi} \times \left(\frac{f_s}{f_m}\right)^3$

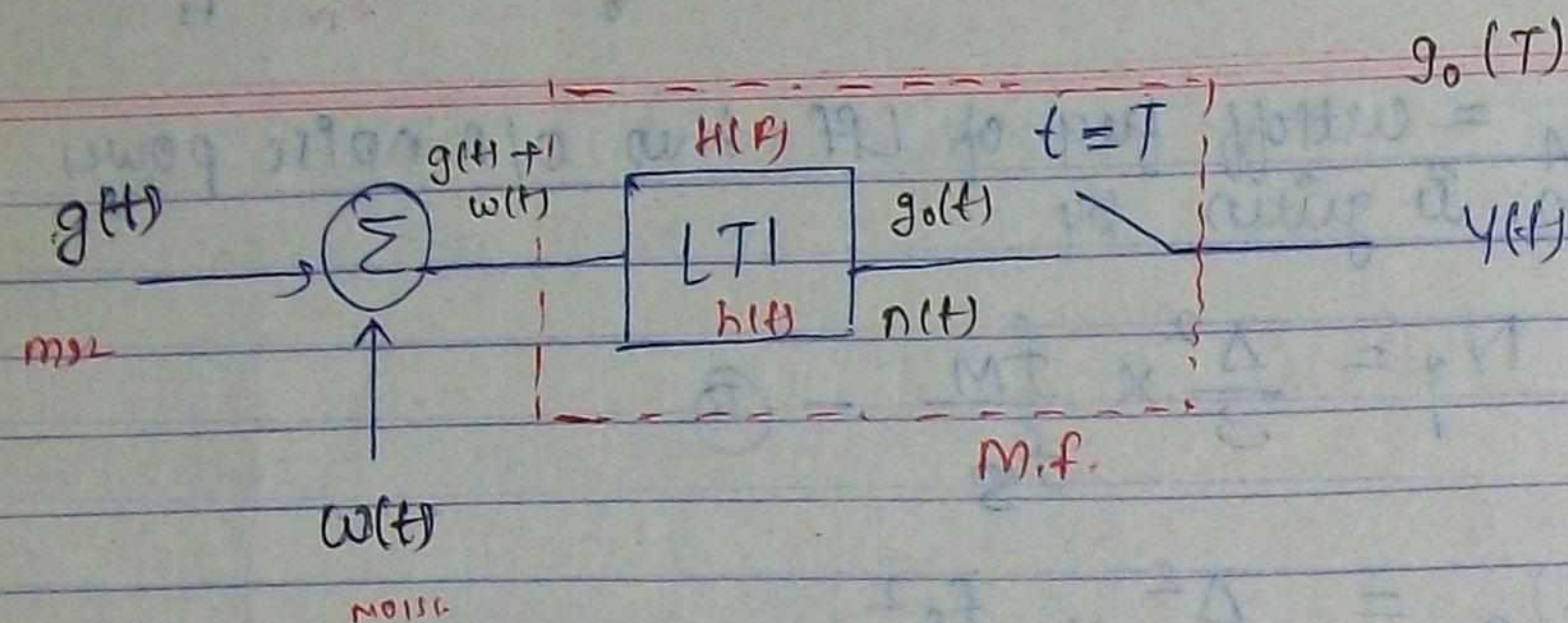
Ques. A msg signal $m(t) = at$ is applied to a Δm with step size = Δ condⁿ to occur slope overload distortion is:

Solⁿ: $\frac{dm(t)}{dt} > \frac{\Delta}{T_s}$

$a > \frac{\Delta}{T_s}$

$a T_s > \Delta$

MATCHED FILTER

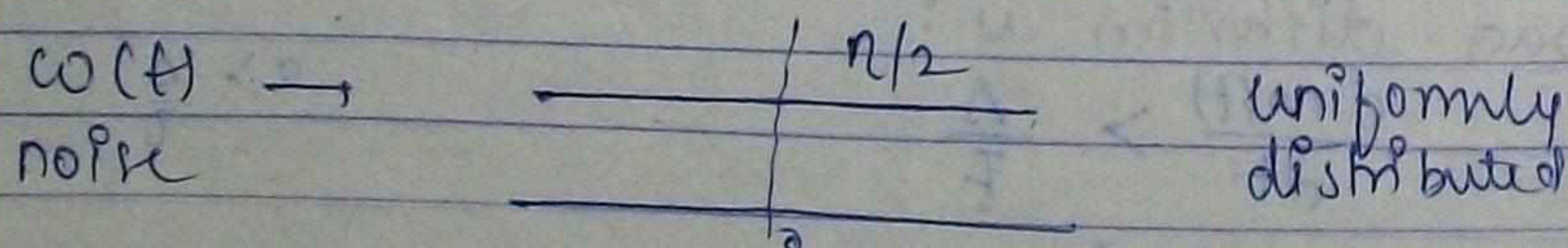


- In digital commⁿ when binary data is transmitted in baseband as pulse transmission matched filter is used to maximize (SNR) under given noise condition.

- In matched filter a sampler or switch is used at o/p to convert I/P signal for the interval of $t = T$, due to this matched filter will provide peak signal power to noise ratio compared to other systems.

$$SNR = \frac{\text{Peak signal power}}{\text{Noise power}}$$

$$SNR = \frac{|g_o(T)|^2}{\text{Noise power}} \quad \text{--- (1)}$$



$$\text{o/p noise PSD} = |H(f)|^2 \times \frac{n}{2}$$

$$\text{Noise power} = \frac{n}{2} \int_{-\infty}^{+\infty} |H(f)|^2 df \quad \text{--- (2)}$$

$$\text{let } g(t) \xrightarrow{\text{F.T.}} G(f)$$

$$h(t) \xrightarrow{\text{F.T.}} H(f)$$

$$g_o(t) \xrightarrow{\text{F.T.}} G_o(f)$$

$$H(f) = \frac{G_o(f)}{G(f)}, \quad G_o(f) = H(f) \cdot G(f)$$

$$g_o(t) = \int_{-\infty}^{+\infty} G_o(f) \cdot e^{j2\pi ft} df$$

$$g_o(t) = \int_{-\infty}^{+\infty} H(f) \cdot G(f) \cdot e^{j2\pi ft} df$$

$$g_o(T) = \int_{-\infty}^{+\infty} H(f) \cdot G(f) \cdot e^{j2\pi fT} df$$

$$|g_o(T)|^2 = \left| \int_{-\infty}^{+\infty} \underbrace{H(f)}_{\phi_1(f)} \cdot \underbrace{G(f)}_{\phi_2(f)} \cdot e^{j2\pi fT} df \right|^2$$

$$|\phi_1(f) \cdot \phi_2(f)| \leq |\phi_1(f)| \cdot |\phi_2(f)|$$

SCHWARTZ
CONDITION

$$\phi_1(f) = \phi_2^*(f) \quad (\text{Schwartz Cond}^n)$$

$$|g_o(T)|^2 \leq \int_{-\infty}^{\infty} |H(f)|^2 df \cdot \int_{-\infty}^{\infty} |G(f)|^2 df \quad (2)$$

$$\text{SNR} \leq \frac{\int_{-\infty}^{\infty} |H(f)|^2 df \cdot \int_{-\infty}^{\infty} |G(f)|^2 df}{\frac{n}{2} \int_{-\infty}^{\infty} |H(f)|^2 df}$$

$$\text{SNR} \leq \frac{\int_{-\infty}^{\infty} |G(f)|^2 df}{n/2}$$

$$|G(f)|^2 \rightarrow \text{ESD}, \quad \int_{-\infty}^{\infty} |G(f)|^2 df = \text{Energy of bit}$$

$$\text{SNR} \leq \frac{2 \cdot E_b}{n}$$

from Schwartz inequality

$$\phi_1(f) = H(f)$$

$$\phi_2(f) = G(f) \cdot e^{j2\pi fT}$$

⇒ CALCULATION OF IMPULSE RESPONSE -

- According to Schwartz inequality.

$$\phi_1(f) = H(f), \quad \phi_2(f) = G(f) e^{j2\pi fT}$$

$$\text{Since } \phi_1(f) = \phi_2^*(f)$$

$$\therefore H(f) = (G(f) \cdot e^{j2\pi fT})^*$$

$$= G(f)^* \cdot e^{-j2\pi fT}$$

for real valued

$$G(f)^* = G(-f)$$

$$H(f) = G(-f) \cdot e^{-j2\pi fT}$$

$$\downarrow$$

$$H(f) = \int_{-\infty}^{+\infty} G(-f) \cdot e^{-j2\pi fT} \cdot e^{j2\pi f \cdot t} \cdot df$$

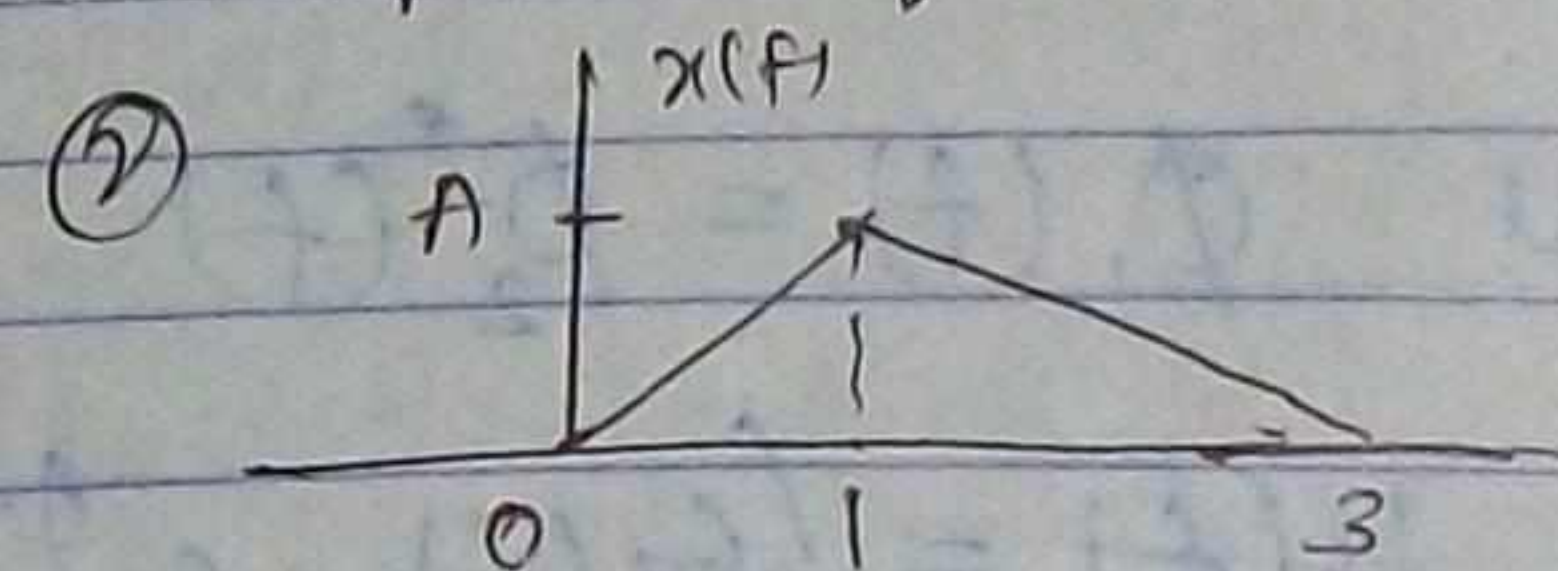
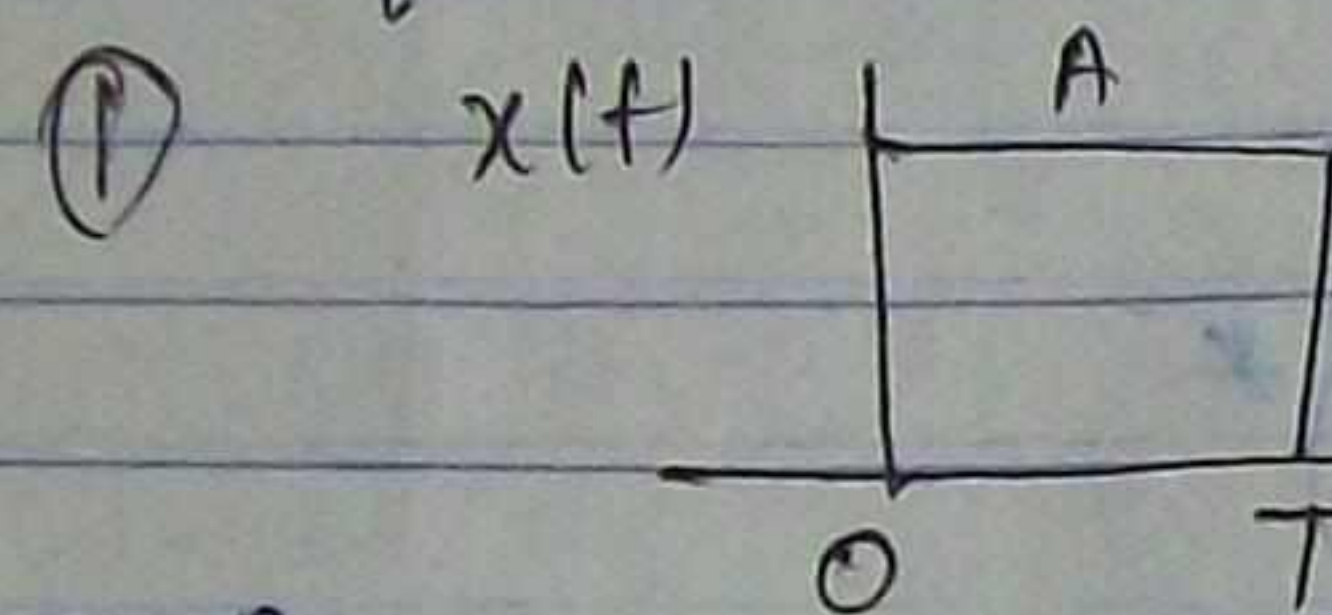
$$h(t) = \int_{-\infty}^{\infty} G(-f) \cdot e^{j2\pi(T-t) \cdot (-f)} df$$

$$h(t) = g(T-t)$$

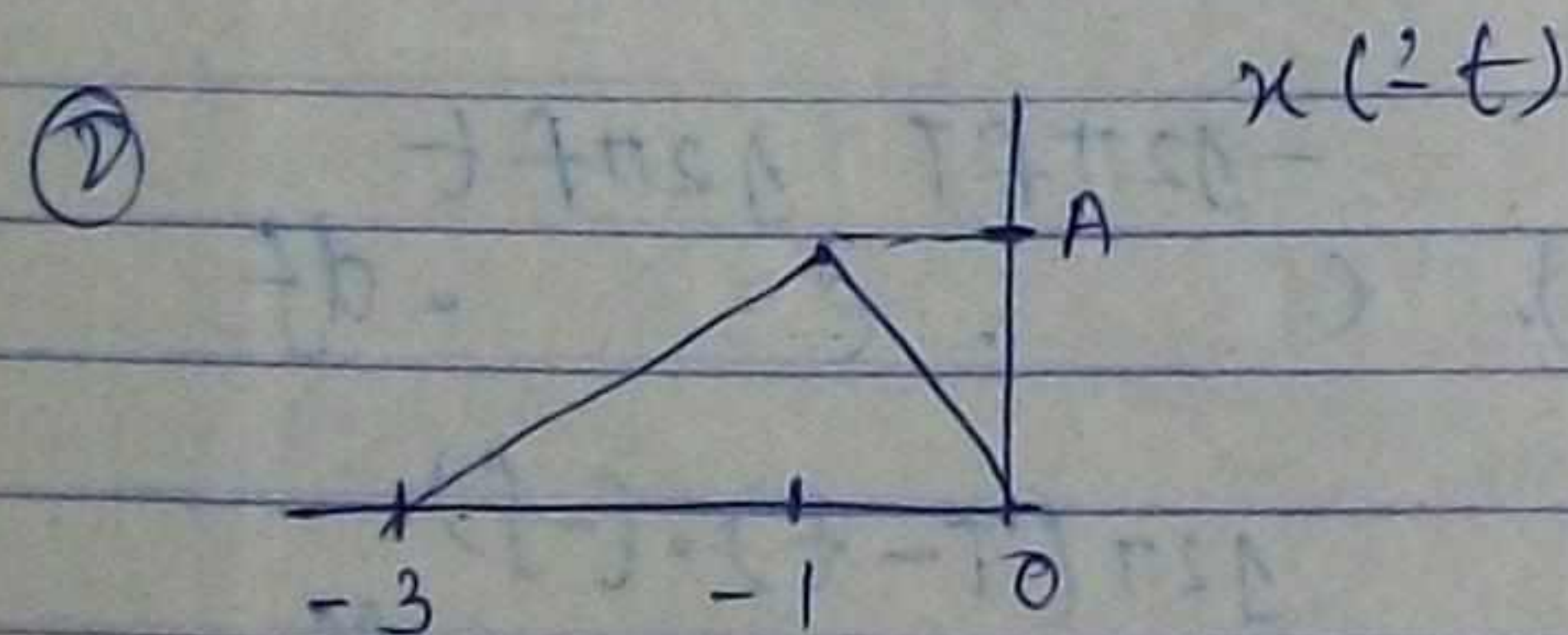
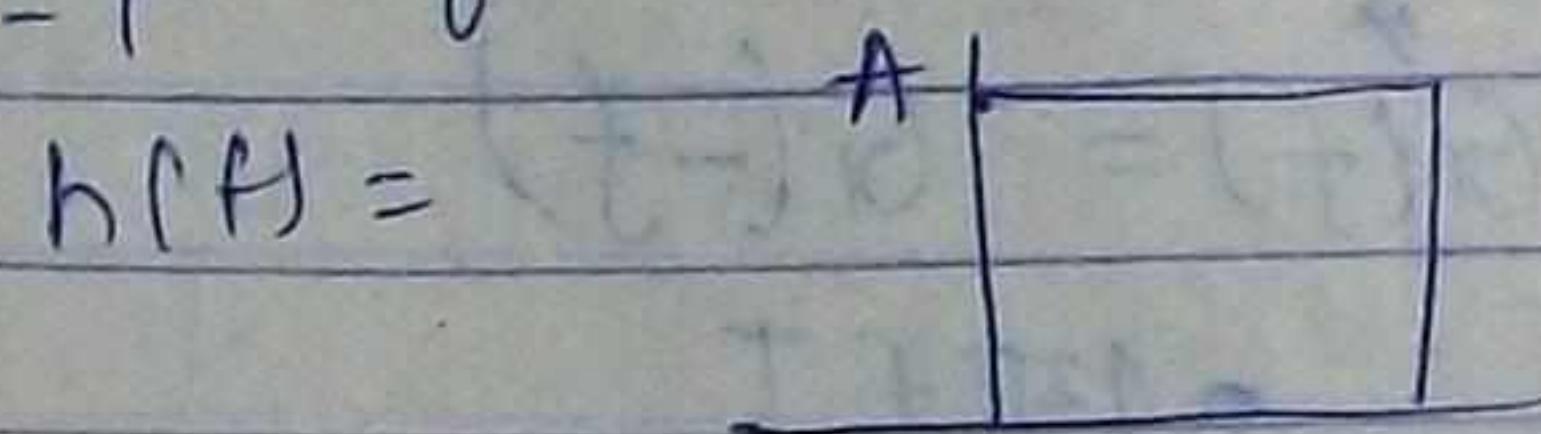
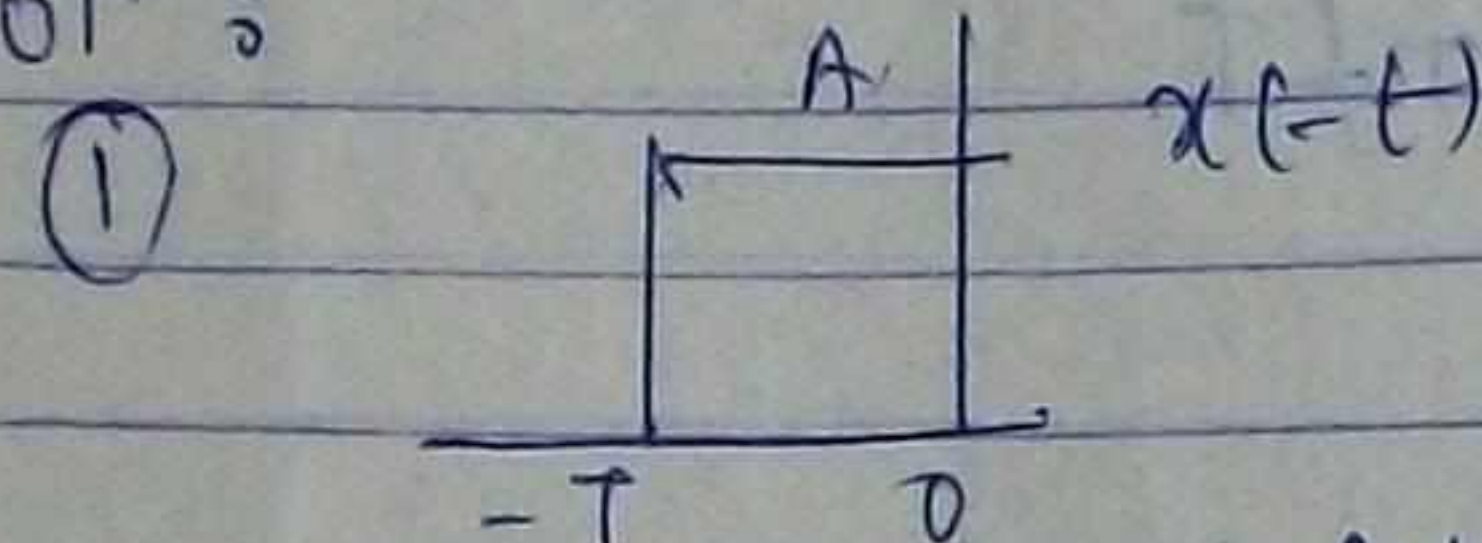
$$h(t) = g(-(t-T))$$

- In matched filter the impulse response is time reversal and shifted version of I/P signal

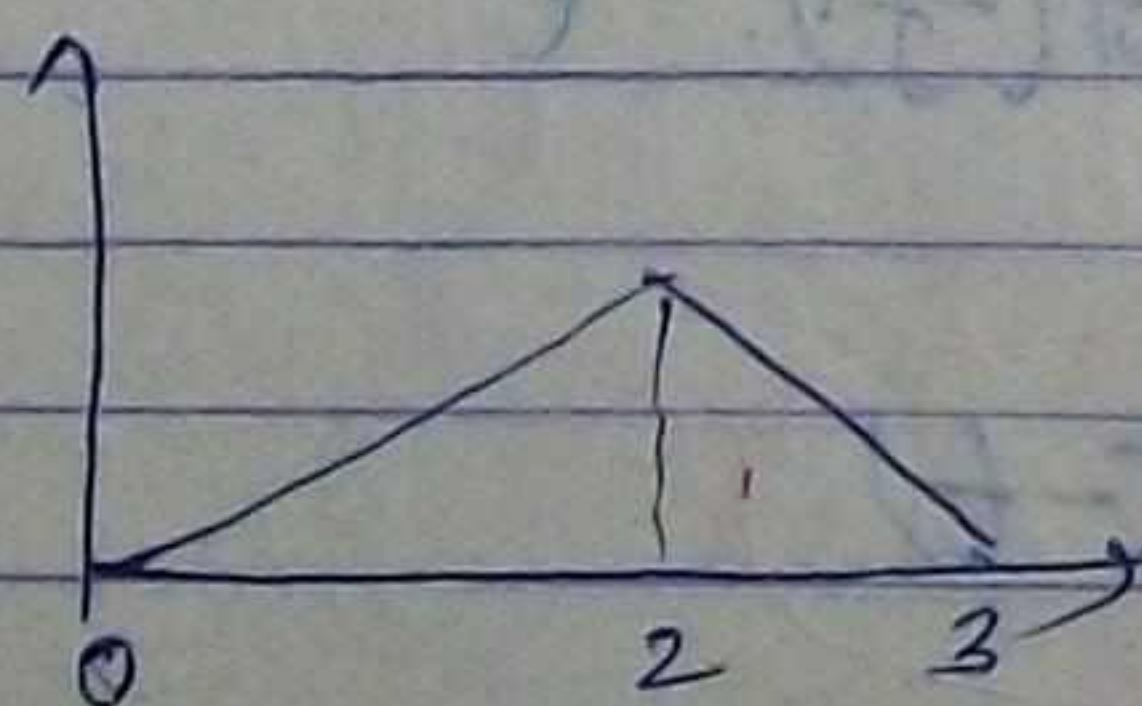
Ques. signal $x(t)$ is applied to a matched filter then impulse response of matched filter



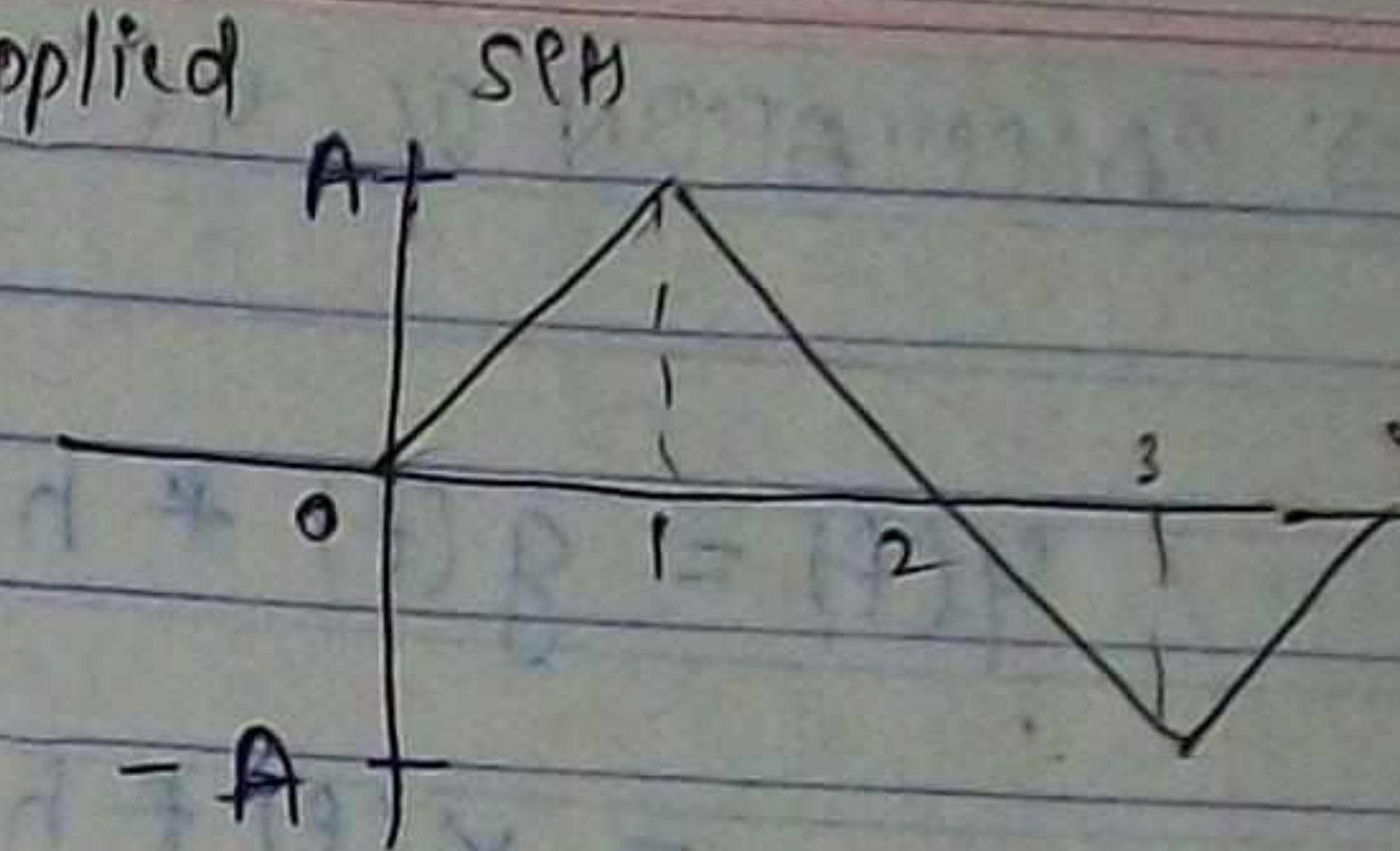
Soln:



$h(t) = g(-(t-T))$

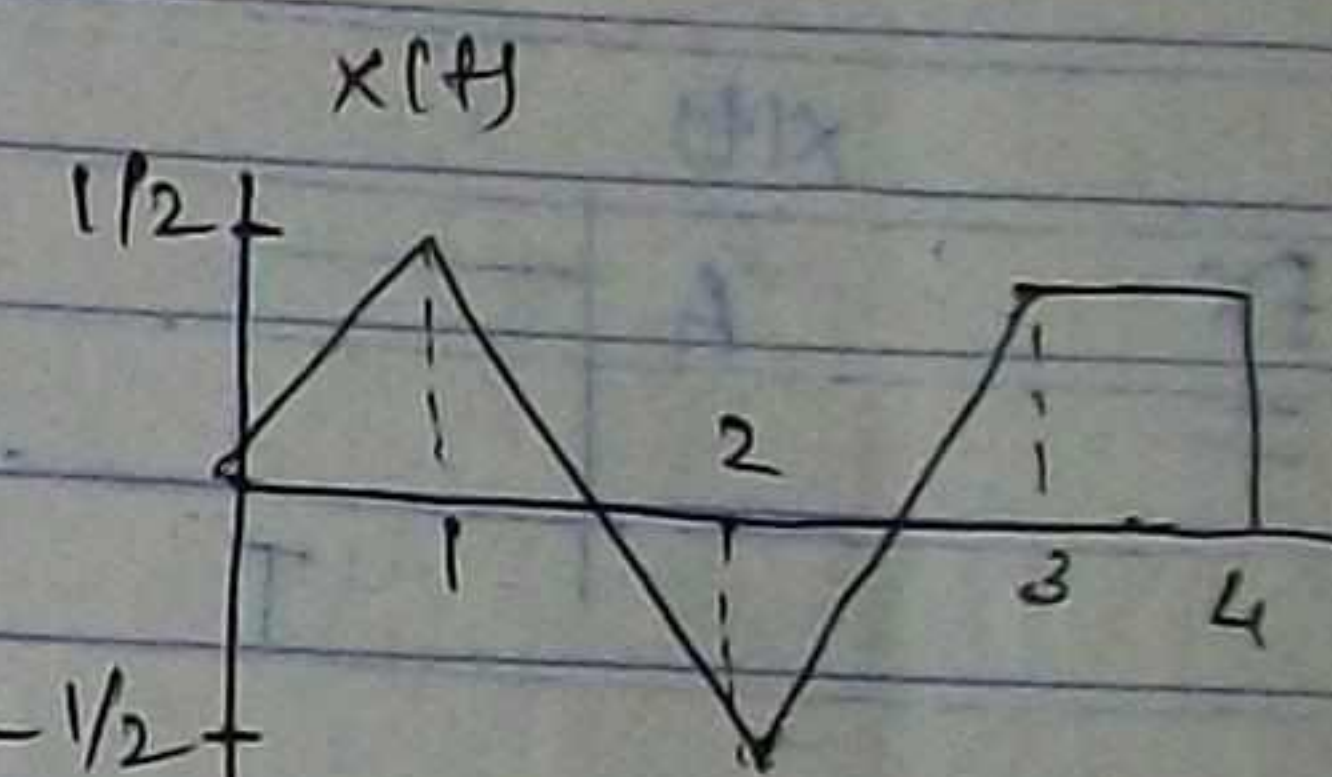


Ques. A signal $s(t)$ shown is applied to matched filter then impulse response is



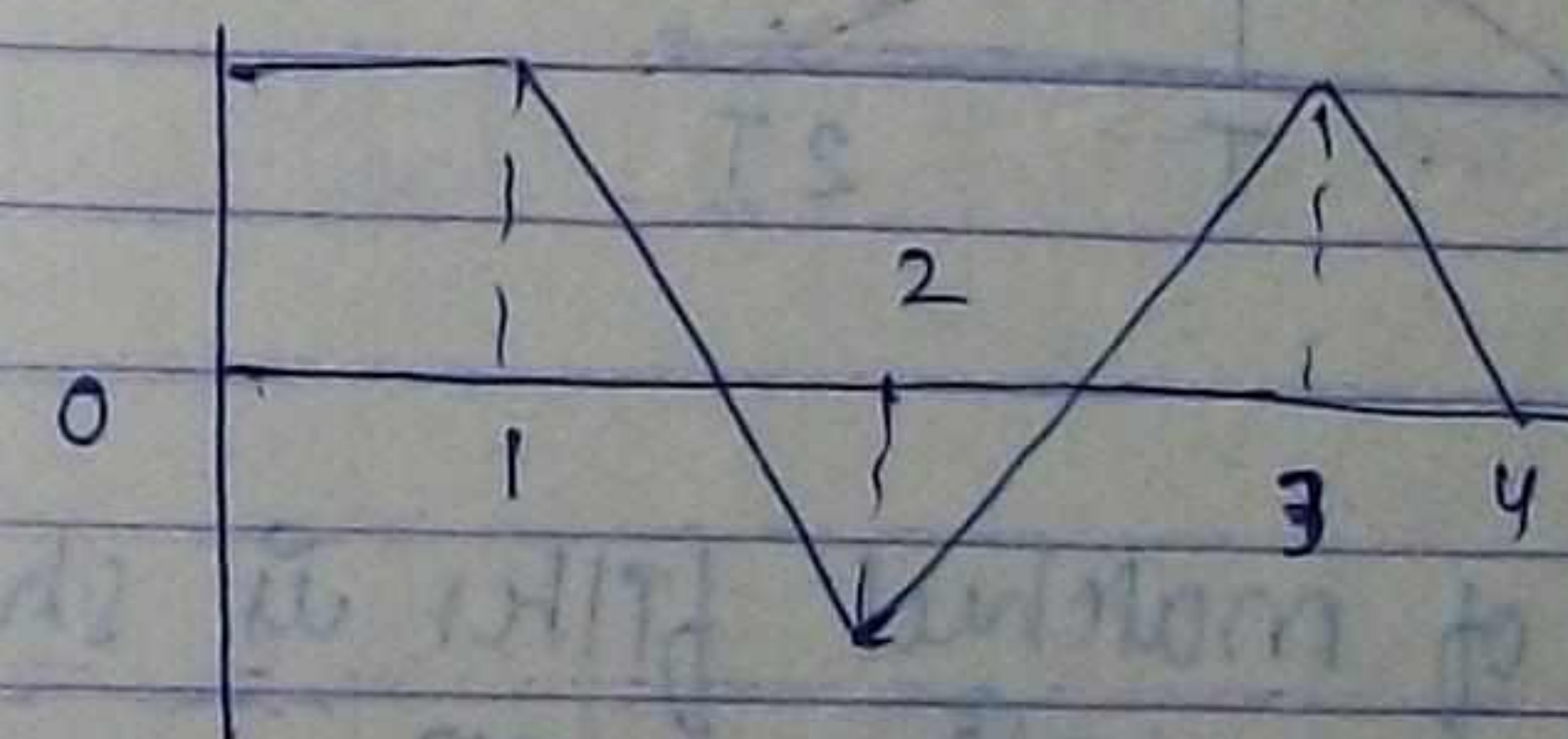
- ① $s(t)$
- ② $-s(t)$
- ③ $s(t-4)$
- ④ $s(t+4)$

Ques. A signal $x(t)$ is shown in fig is applied to a matched filter, $h(t)$ is the impulse response of matched filter matched to input signal $x(t)$ in which $h(t) \neq 0$ $0 < t < 4$ then slope of $h(t)$ in interval $3 < t < 4$ sec.



- ① $-1/2 \text{ sec}^{-1}$
- ② -1 sec^{-1}
- ③ $1/2 \text{ sec}^{-1}$
- ④ 1 sec^{-1}

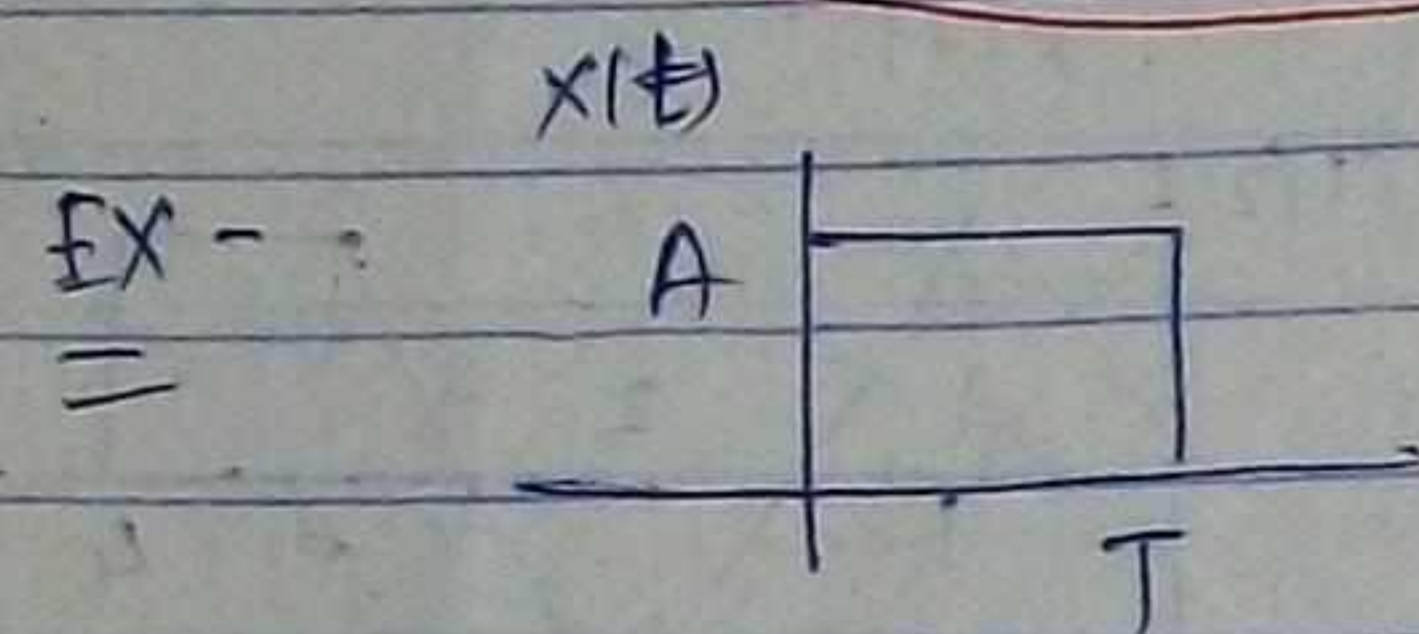
Soln:



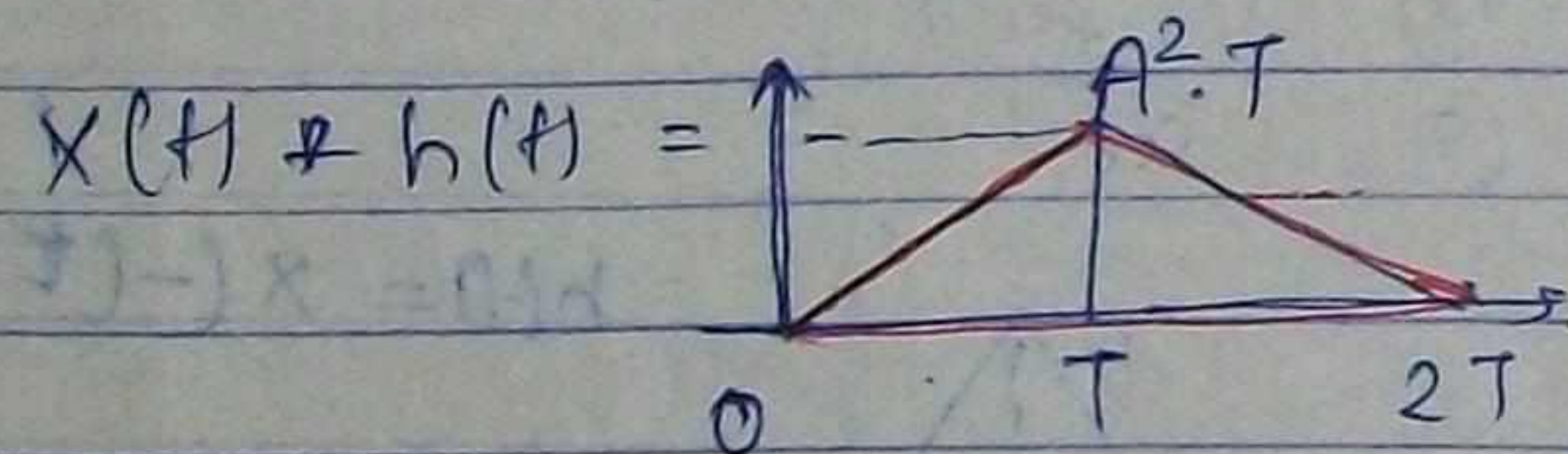
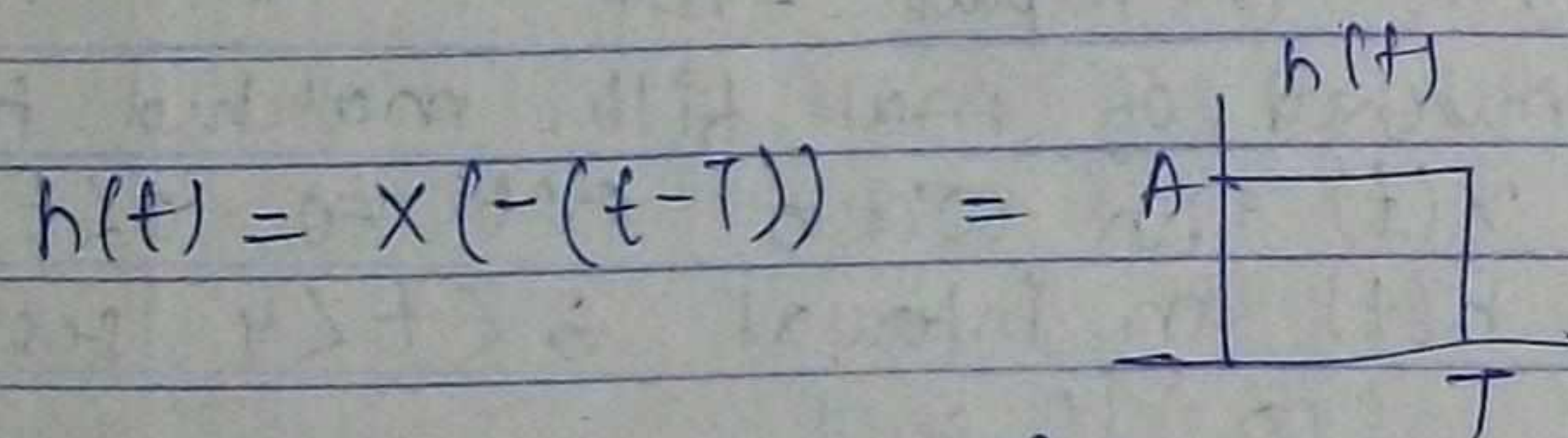
$h(t) = x(-(t-T))$

⇒ CALCULATION OF $y(t)$ —

$$y(t) = g(t) * h(t) \\ = x(t) * h(t)$$



is applied to matched filter
then o/p response of
matched filter is.



— The o/p response of matched filter is shifted
version of auto correlation funn.

$$y(t) = R_{xx}(t-T)$$

RANDOM VARIABLE

CONTD. ~

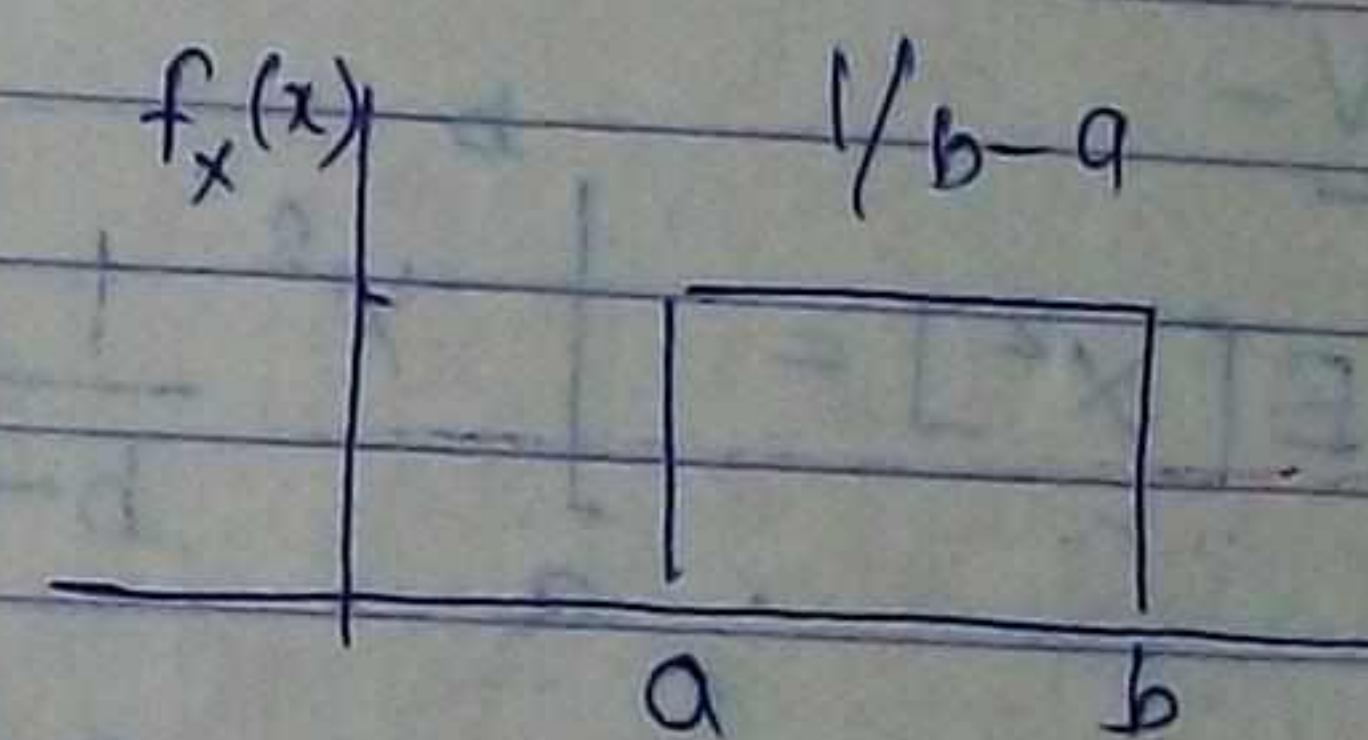
⇒ DISTRIBUTION —

- | | | |
|---------------|---|-----------------|
| ① UNIFORM | } | CONTINUOUS R.V. |
| ② EXPONENTIAL | | |
| ③ GAUSSIAN | | |
| ④ BERNOULLI'S | } | DISCRETE R.V. |
| ⑤ BINOMIAL | | |
| ⑥ POISSON'S | | |

⇒ UNIFORM DISTRIBUTION —

$$f_x(x) = \frac{1}{b-a}$$

$$a \leq x \leq b$$

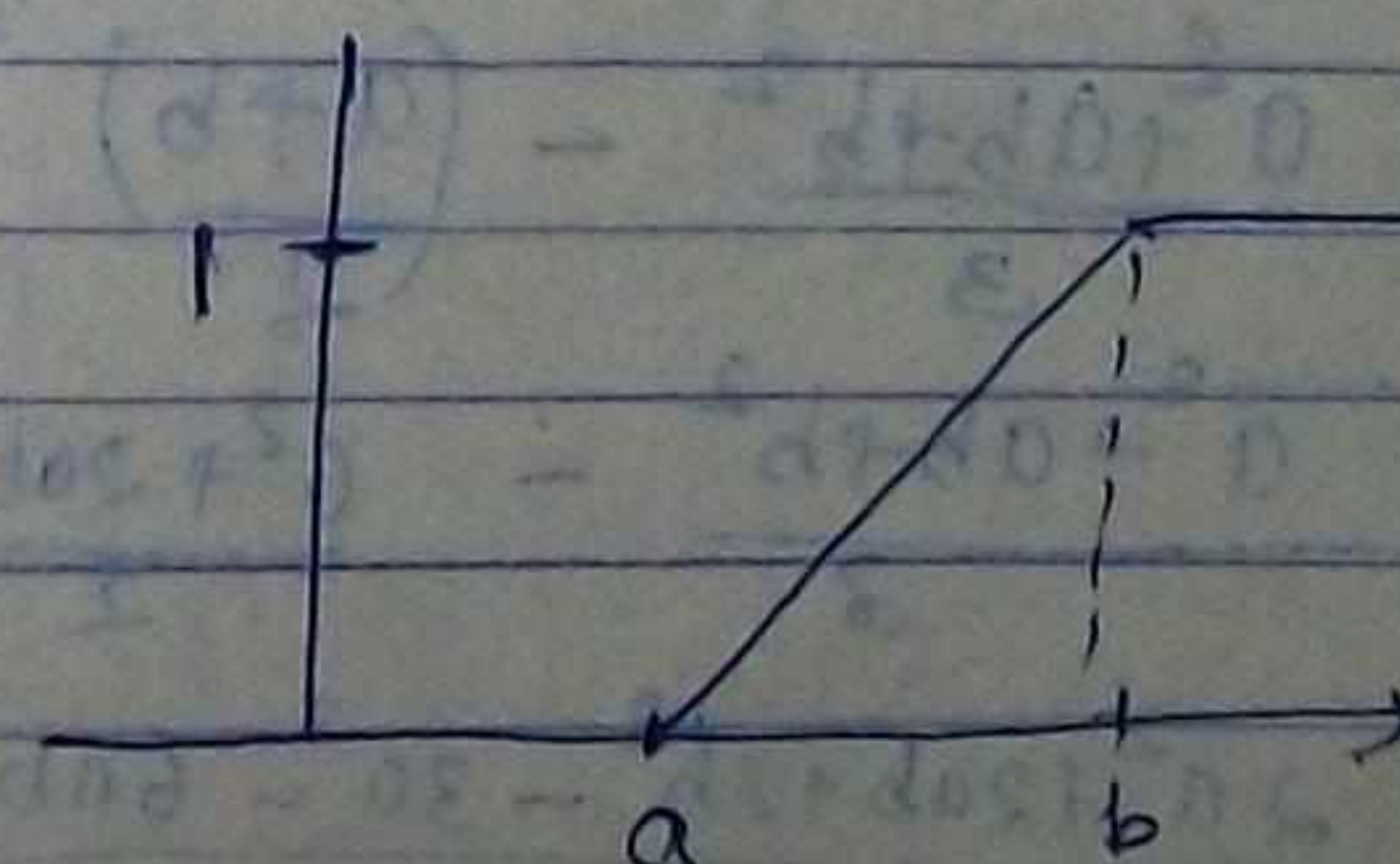


① CDF —

$$F_x(x) = \int_{-\infty}^x f_x(x) \cdot dx$$

$$= \frac{1}{b-a} \int_a^x dx = \frac{x-a}{b-a}$$

$F_x(x)$

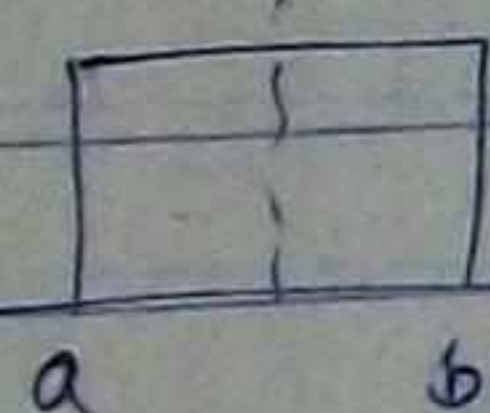


$$F_x(x) = \frac{x-a}{b-a}$$

② MEAN -

$$E[X] = \int_a^b x \cdot \frac{1}{b-a} \cdot dx$$

$1/(b-a)$



$$= \frac{1}{2(b-a)} (b^2 - a^2) = \frac{a+b}{2}$$

$$E[X] = \frac{a+b}{2}$$

③ MSV -

$$E[X^2] = \int_a^b x^2 \cdot \frac{1}{b-a} \cdot dx$$

$$= \frac{1}{3(b-a)} (b^3 - a^3) = \frac{(a^2 + ab + b^2)}{3}$$

$$E[X^2] = \frac{a^2 + ab + b^2}{3}$$

④ VARIANCE -

$$E[(X - \mu_x)^2] = E[X^2] - \mu_x^2$$

$$= \frac{a^2 + ab + b^2}{3} - \left(\frac{a+b}{2}\right)^2$$

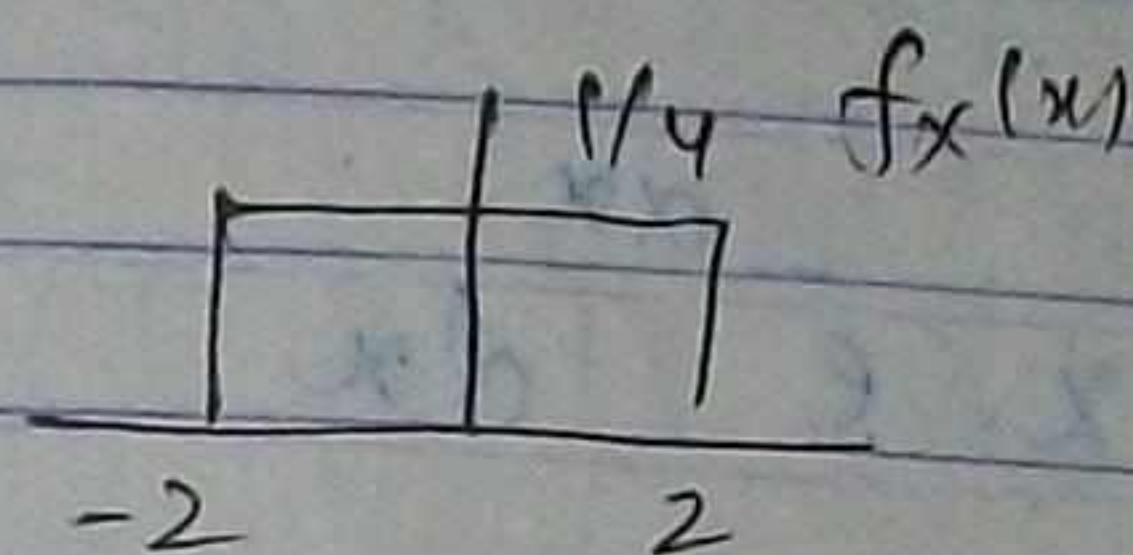
$$= \frac{a^2 + ab + b^2}{3} - \frac{a^2 + 2ab + b^2}{2}$$

$$= \frac{2a^2 + 2ab + 2b^2 - 3a^2 - 6ab - 3b^2}{6}$$

$$= \frac{2a^2 + 2ab + 2b^2 - 3a^2 - 6ab - 3b^2}{6}$$

$$E[(X - \mu_x)^2] = \sigma_x^2 = \frac{(b-a)^2}{12}$$

Ques.



Calculate mean, msv, variance

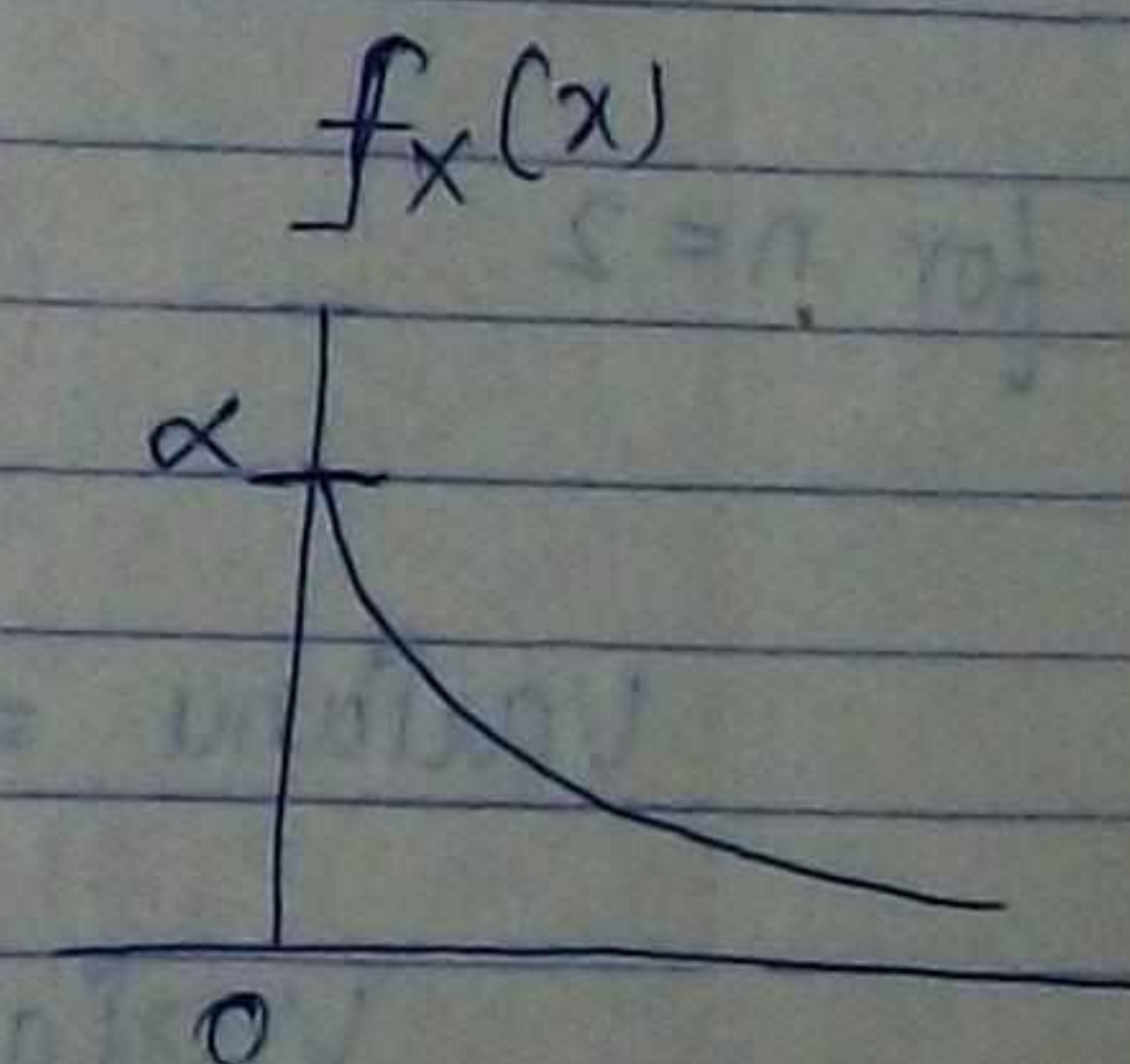
Solⁿ: ① mean = 0

$$② \text{msv} = \frac{4+4+4}{3} = \frac{4}{3}$$

$$③ \text{varian} = \frac{16}{12} = \frac{4}{3}$$

⇒ EXPONENTIAL DISTRIBUTION -

$$f_x(x) = \alpha \cdot e^{-\alpha x}; x \geq 0$$

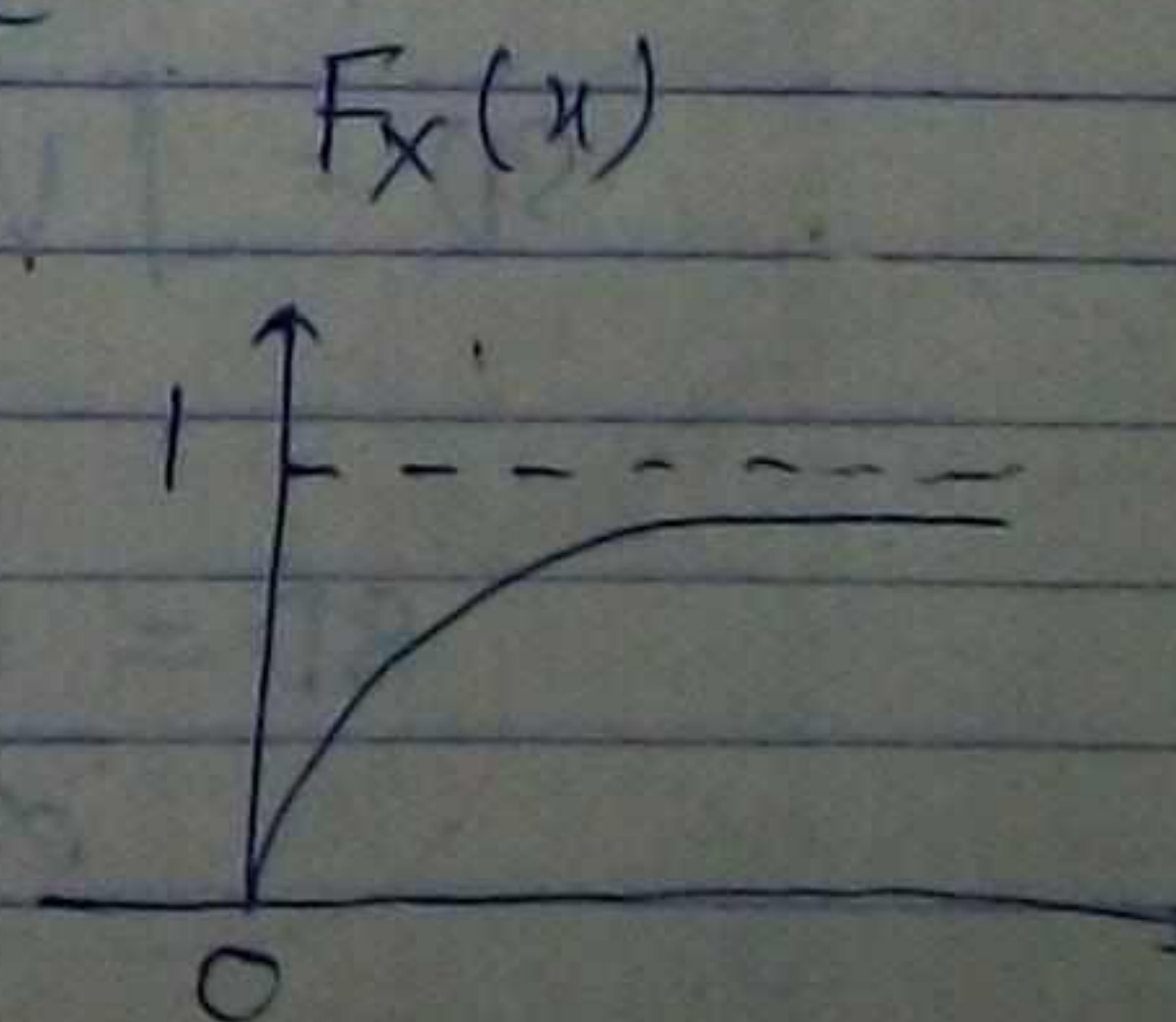


① CDF -

$$F_x(x) = \int_{-\infty}^x f_x(x) \cdot dx$$

$$= \int_0^x \alpha \cdot e^{-\alpha x} \cdot dx$$

$$F_x(x) = 1 - e^{-\alpha x}; x \geq 0$$



② MEAN -

$$E[X^n] = \int_{-\infty}^{\infty} x \cdot \alpha \cdot e^{-\alpha x} dx$$

$$= \alpha \int_{-\infty}^{\infty} x \cdot e^{-\alpha x} dx$$

$$E[X^n] = \frac{n!}{\alpha^n}$$

for $n=1$

$$\text{mean} = \frac{1}{\alpha}$$

for $n=2$

$$\text{msv} = \frac{2}{\alpha^2}$$

$$\text{Variance} = \text{msv} - (\text{mean})^2$$

$$\text{Variance} = \frac{1}{\alpha^2}$$

$$\text{SD} = \sqrt{\text{Variance}}$$

$$\text{SD} = \frac{1}{\alpha}$$

$$x \cdot e^{-\alpha x} = -e^{-\alpha x} \int$$

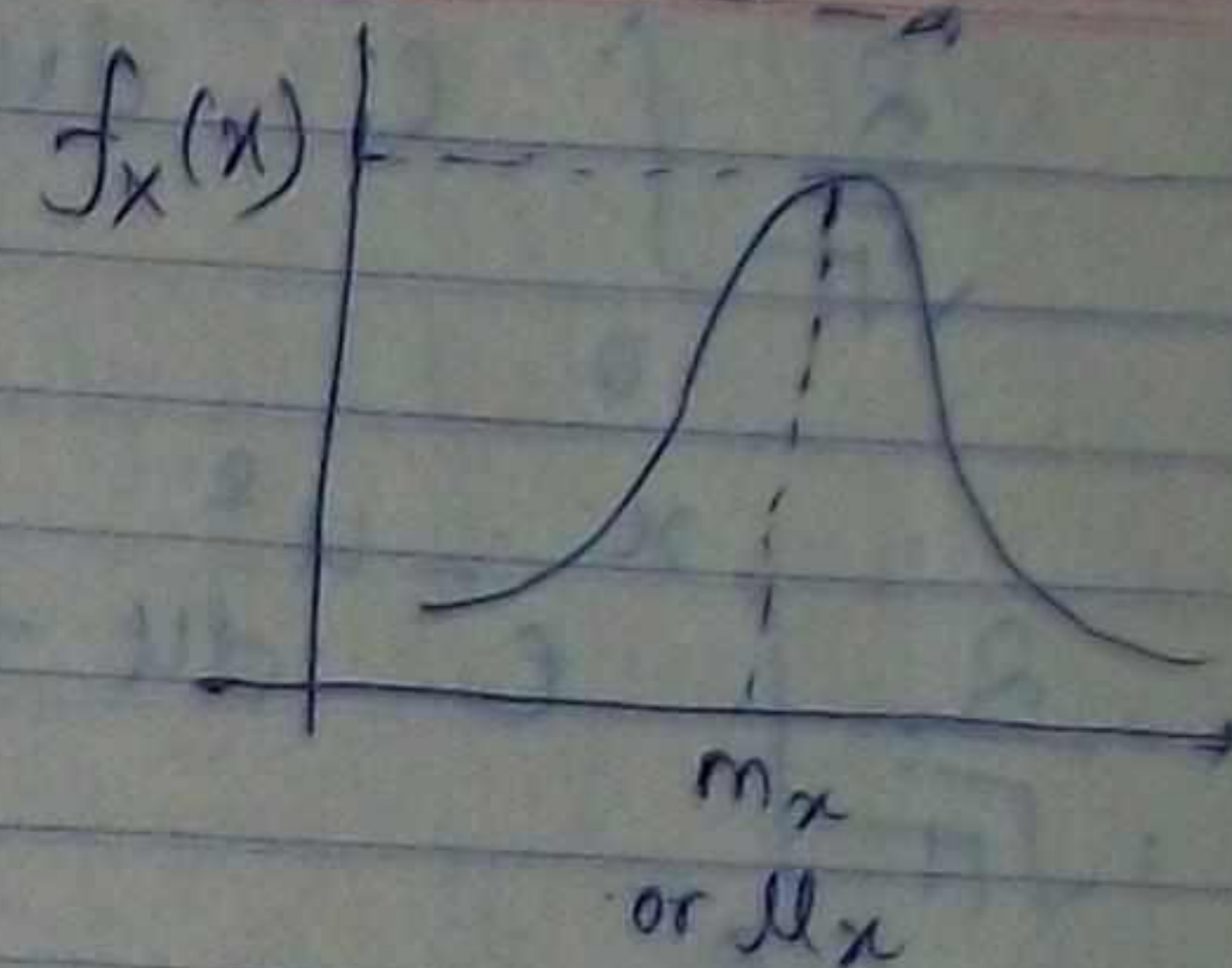
$$x \cdot \frac{e^{-\alpha x}}{-\alpha} = -\frac{1}{\alpha} \cdot \frac{e^{-\alpha x}}{-\alpha}$$

$$x \cdot \frac{e^{-\alpha x}}{-\alpha} + \frac{1}{\alpha} \cdot \int e^{-\alpha x} = x \cdot \frac{e^{-\alpha x}}{-\alpha} + \frac{e^{-\alpha x}}{-\alpha}$$

$$= \left[\frac{e^{-\alpha x}}{-\alpha} (1-x) \right]_{-\infty}^{+\infty}$$

⇒ GAUSSIAN DISTRIBUTION -

$$f_x(x) = \frac{1}{\sqrt{2\pi\sigma_x^2}} e^{-\frac{(x-\mu_x)^2}{2\sigma_x^2}}$$

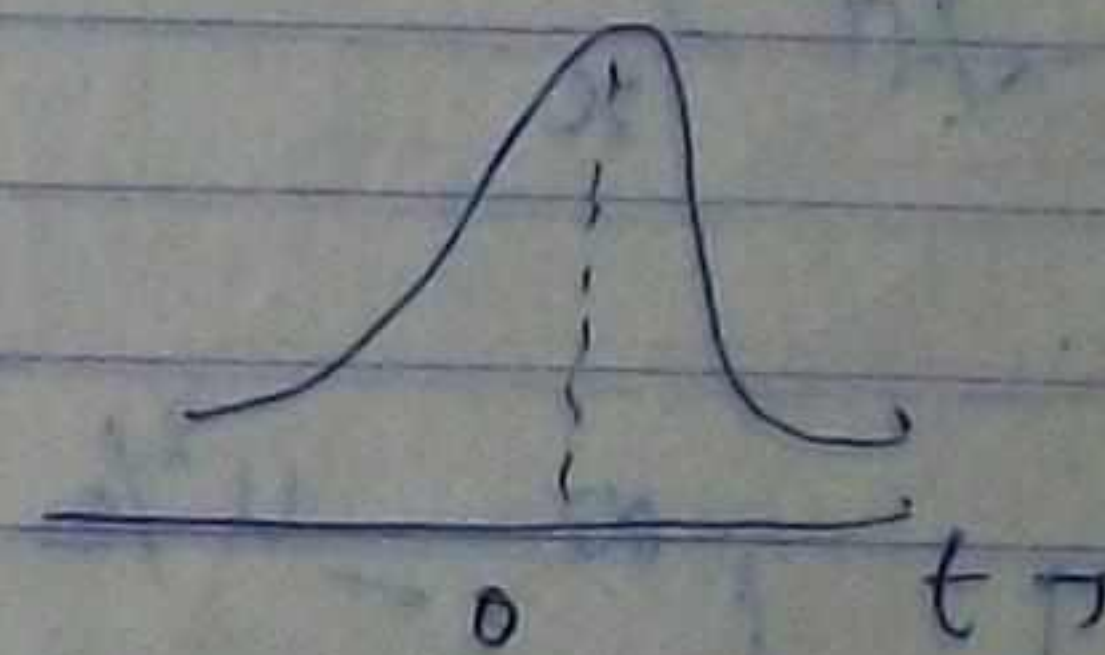


① CDF -

$$F_x(x) = \frac{1}{\sqrt{2\pi}\sigma_x} \int_{-\infty}^x e^{-\frac{(x-\mu_x)^2}{2\sigma_x^2}} \cdot dx$$

PROPERTY -

① $e^{-\pi t^2} \rightarrow$



② $\int_{-\infty}^{+\infty} e^{-\pi t^2} dt = 1$ (area)

$$= 2 \int_0^{\infty} e^{-\pi t^2} dt = 1$$

$$2 \cdot \frac{2}{\sqrt{\pi}} \int_0^{\infty} e^{-u^2} du = 1$$

$$\text{let } \pi t^2 = u^2$$

$$2\pi t dt = 2u du$$

$$dt = \frac{u}{\pi t} du$$

$$= \frac{u \cdot \sqrt{\pi} du}{\pi u} = \frac{du}{\sqrt{\pi}}$$

$$\frac{2}{\sqrt{\pi}} \int_0^{\infty} e^{-u^2} du = 1$$

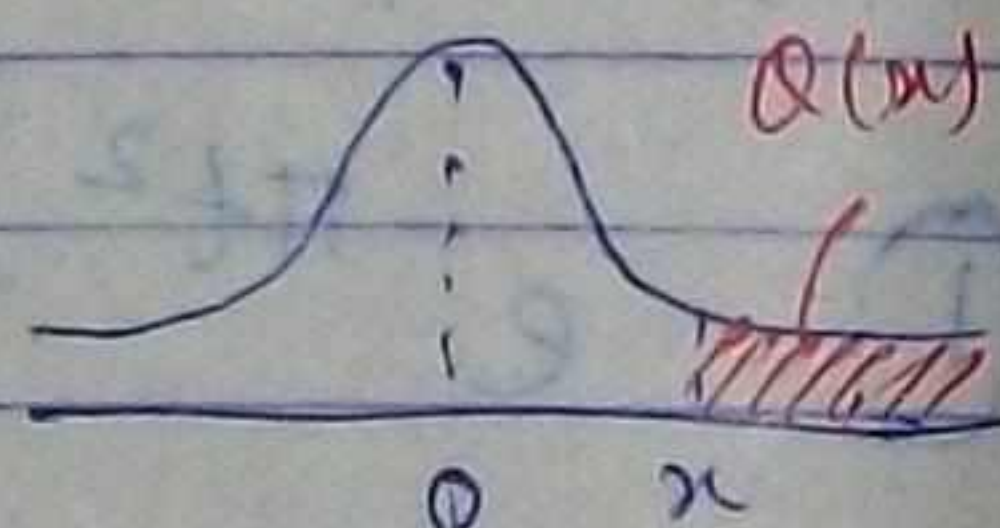
$$\frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du + \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-u^2} du = 1$$

erf(x)

erfc(x)

$$\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-u^2} du$$

COMPLEMENTARY
ERROR
FUNCTION



TAIL AREA
OF GAUSSIAN
FUNN

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-u^2/2} du$$

$Q(x) \rightarrow$ Complementary error funⁿ

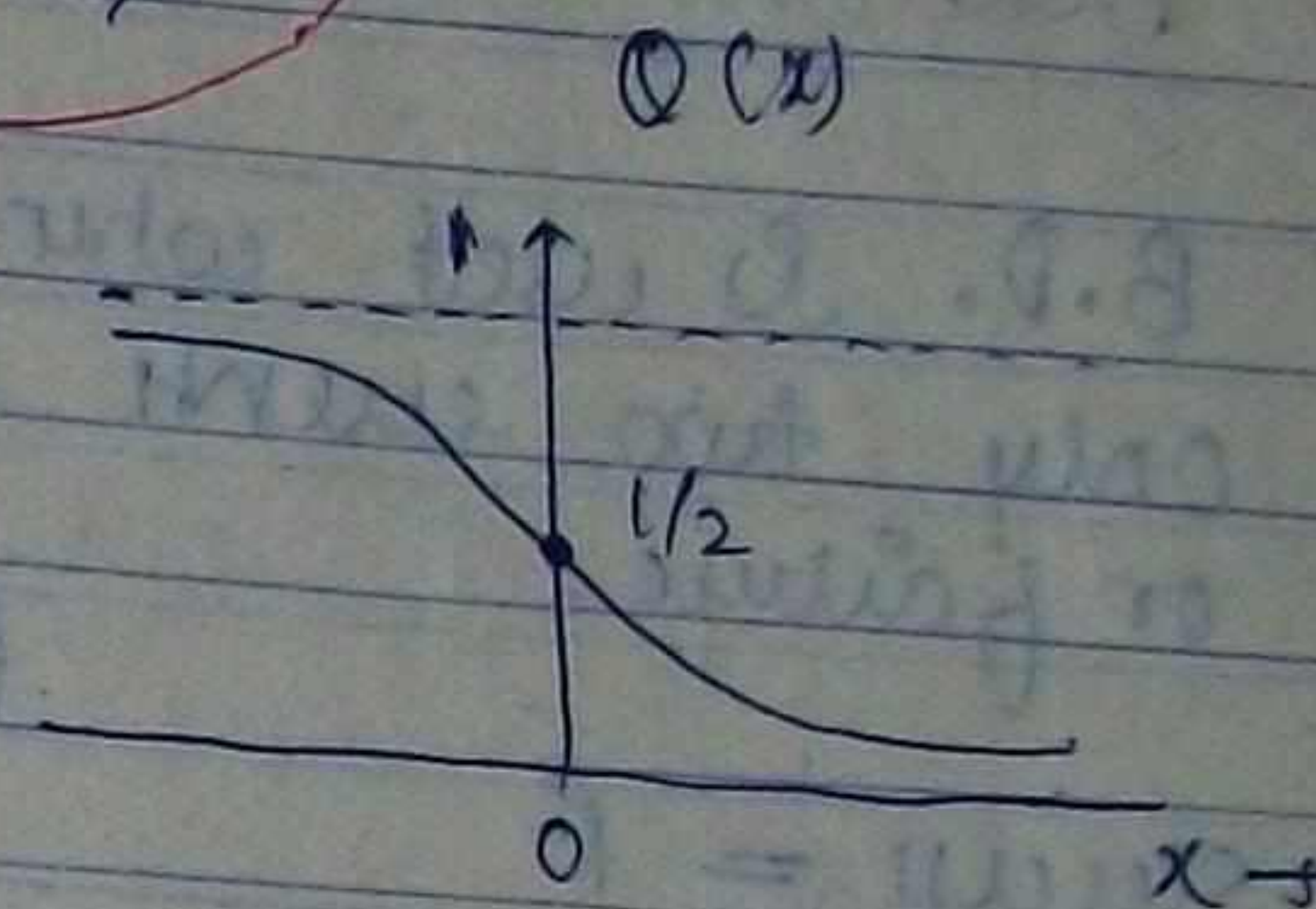
$$Q(x) = \frac{1}{2} \text{erfc}\left(\frac{x}{\sqrt{2}}\right)$$

$$Q(x) \leq \frac{1}{2} e^{-x^2/2}; x \geq 0$$

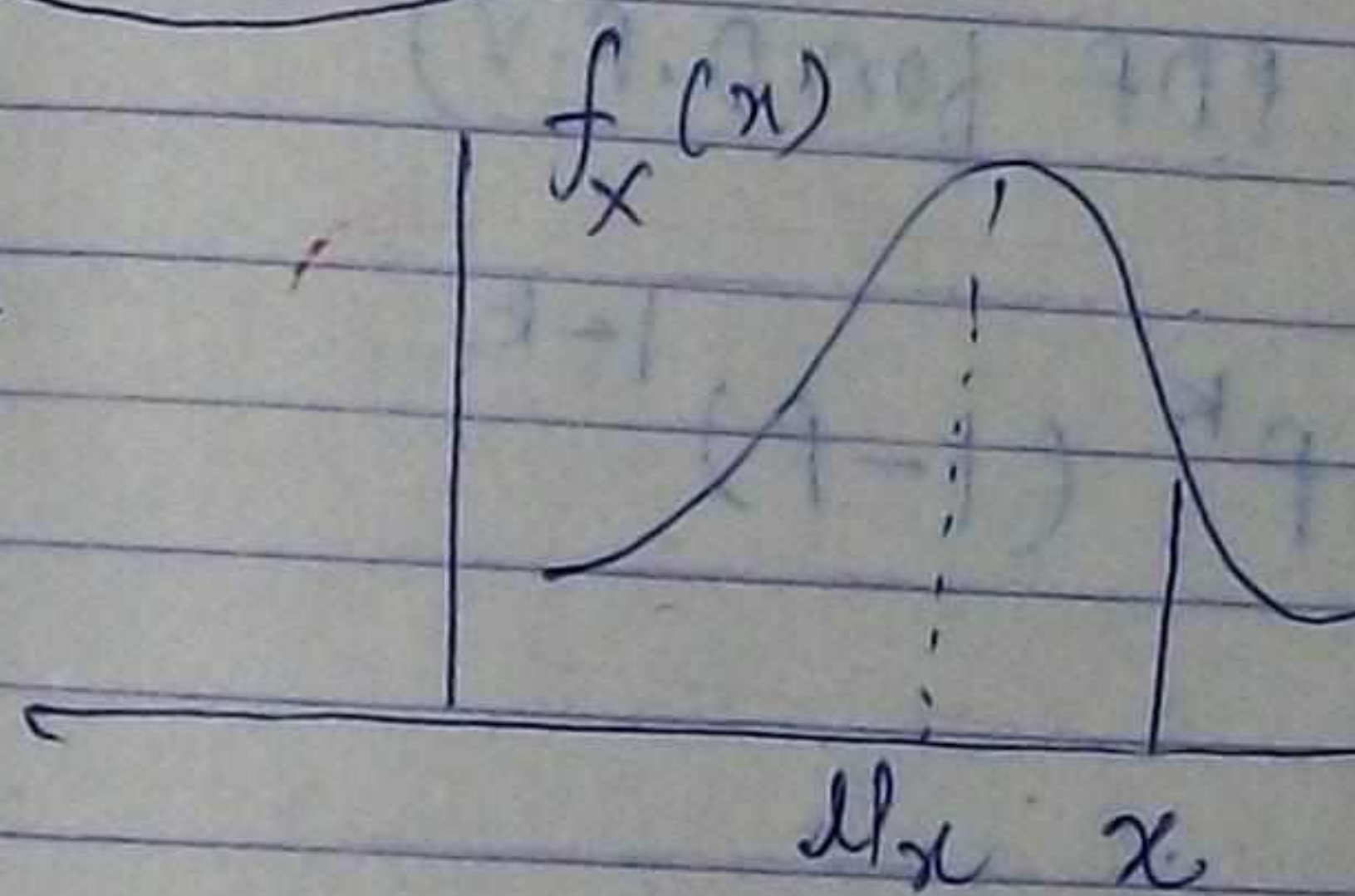
$$x \uparrow \quad Q(x) \downarrow$$

$$Q(-x) = 1 - Q(x)$$

$$Q(0) = \frac{1}{2}$$



#



$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma_x} e^{-\frac{(x-\mu_x)^2}{2\sigma_x^2}}$$

$$P(X \geq x) = Q\left(\frac{x - \mu_x}{\sigma_x}\right)$$

- In Gaussian distribution:

$$P(X \leq x) = 1 - Q\left(\frac{x - \mu_x}{\sigma_x}\right)$$

1 bit

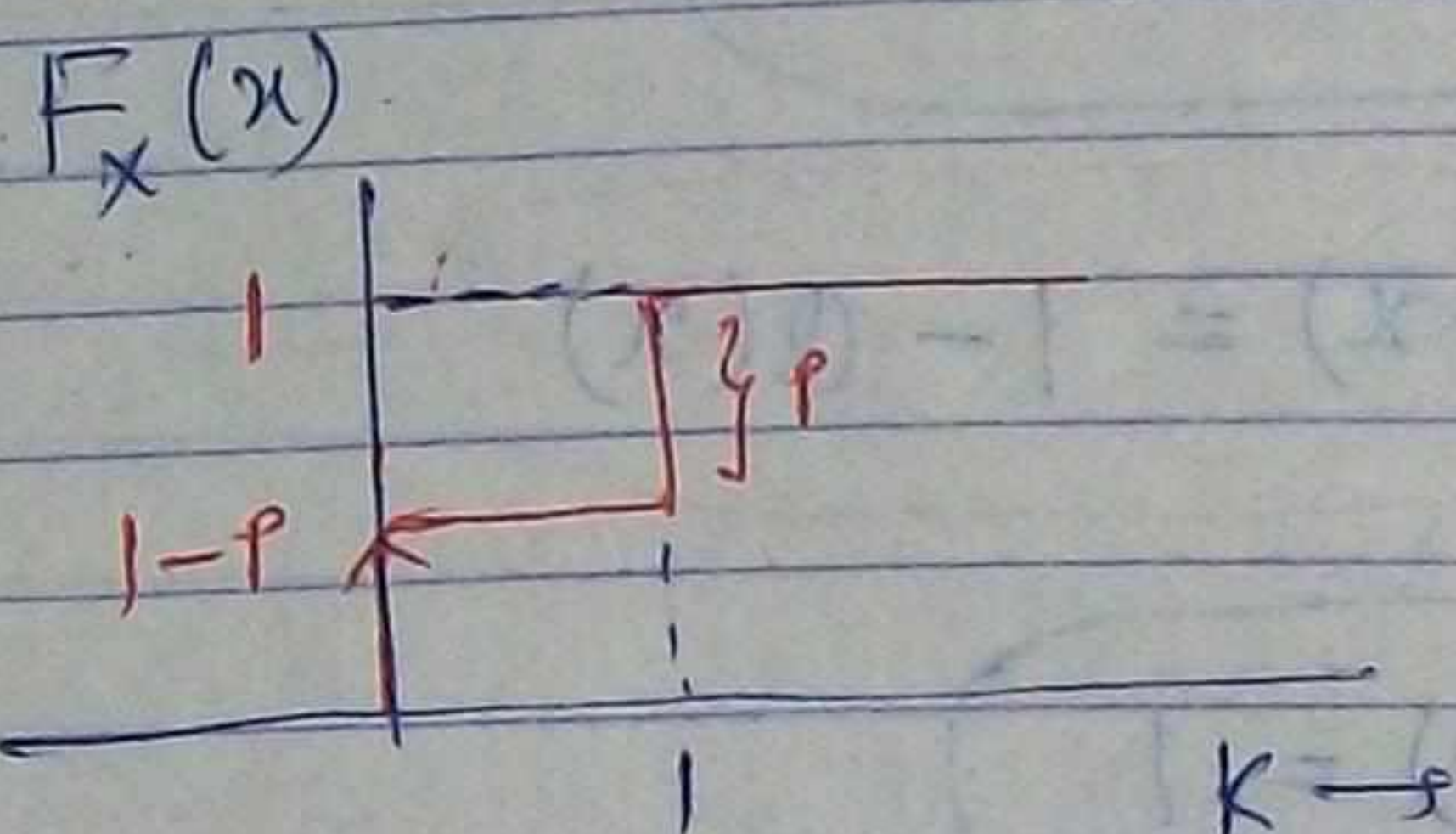
(nT)

(single event)

⇒ BERNOULLI'S DISTRIBUTION -

- B.D. is used when a random variable x have only two events with outcome either success or failure.

- success = p
- failure = $1-p$

 p = probabilityProbability mass funⁿ (PDF for D.R.V)

$$P_X(k) = P(X=k) = p^k \cdot (1-p)^{1-k}$$

PMF mean = p

$$\text{variance} = (1-p)p$$

CDF

$$F_X(x) = \sum_{k=0}^1 p^k (1-p)^{1-k}$$

$$MSV = \text{variance} + (\text{mean})^2$$

(n bit)

(same event occur repeatedly)

⇒ BINOMIAL DISTRIBUTION -

PMF -

$$P_X(k) = \frac{n!}{k!(n-k)!} p^k \cdot (1-p)^{n-k}$$

 n = no. of experiment

$$\text{mean} = np$$

$$\text{variance} \rightarrow np(1-p)$$

CDF -

$$F_X(x) = \sum_{k=0}^n \frac{n!}{k!(n-k)!} p^k \cdot (1-p)^{n-k}$$

But according to majority rule

P-34

Q-12

011	100	111
↓	↓	↓
1	0	1

$$F_X(x) = \sum_{k=\frac{N+1}{2}}^N \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k}$$

Q-7
P-33 During digital comm'n in n -bit data if $\frac{1}{n}$, then chance of getting k -bit error is

$$= (1-p)^n + \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k}$$

⇒ POISSON'S DISTRIBUTION -

- In P.D with parameter ' λ ' is given by

$$P_x(k) = e^{-\lambda} \cdot \frac{\lambda^k}{k!}$$

PMF
=

$$\text{CDF} \quad F_x(x) = \sum_{k=0}^{\lambda} e^{-\lambda} \frac{\lambda^k}{k!}$$

$$\text{mean} = \lambda$$

$$\text{variance} = \lambda$$

$$\text{msv} = \lambda + \lambda^2$$

⇒ JOINT DISTRIBUTION -

$$F_x(x) = P(X \leq x)$$

$$F_y(y) = P(Y \leq y)$$

$$F_{xy}(x, y) = P(X \leq x, Y \leq y)$$

JOINT
DISTRIBUTION
of x, y

$$f_{xy}(x, y) = \text{Joint density funn.}$$

$$F_{xy}(x, y) = \int_{-\infty}^x \int_{-\infty}^y f_{xy}(x, y) dy \cdot dx$$

$$- F_{xy}(\infty, \infty) = 1$$

$$- F_{xy}(-\infty, -\infty) = 0$$

$$- F_{xy}(-\infty, y) = 0$$

$$- F_{xy}(x, -\infty) = 0$$

$$- F_{xy}(x, \infty) = F_x(x)$$

$$- F_{xy}(\infty, y) = F_y(y)$$

marginal
distribution

$$R_{xy} =$$

$$E[xy] = \int_{-\infty}^{+\infty} \int_{-\infty}^{\infty} xy f_{xy}(x, y) dy dx$$

R_{xy} = cross-correlation funⁿ.

⇒ COVARIANCE-

$$\text{Covariance} = E[(x - m_x)(y - m_y)]$$

$$= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (x - m_x)(y - m_y) f_{xy}(x, y) dy dx$$

$$= E[xy - xm_y - ym_x + m_x m_y]$$

$$= E[xy] - m_y E[x] - m_x E[y] + m_x m_y$$

$$= R_{xy} - m_x m_y - m_x m_y + m_x m_y$$

$$\text{Covariance} = R_{xy} - m_x m_y$$

$\text{Cov}(x, y)$

if x and y are independent then

$$\text{Cov}(x, y) = 0$$

If x and y are orthogonal then

$$R_{xy} = 0$$

$$\text{CORRELATION COEFFICIENT } (\rho) = \frac{\text{Cov}(x, y)}{\sigma_x \cdot \sigma_y} = \frac{\text{Cov}(x, y)}{\sqrt{\text{var}(x)} \cdot \sqrt{\text{var}(y)}}$$

- If x and y are independent then

$$\text{CORREL}^n \text{ COFF} = 0$$

- If $x = y$

$$\rho = 1$$

$$-1 \leq \rho \leq 1$$

- If $x = -y$

$$\rho = -1$$

$$\# \quad x \rightarrow \mu_x, \sigma_x^2, f_x(x)$$

$$y \rightarrow \mu_y, \sigma_y^2, f_y(y)$$

$$\text{for } z = x + y$$

$$f_z = f_x(x) * f_y(y)$$

$$\mu_z = \mu_x + \mu_y$$

$$\sigma_z^2 = \sigma_x^2 + \sigma_y^2$$

$$\text{for } z = x - y$$

$$f_z(z) = f_x(x) * f_y(y)$$

$$\mu_z = \mu_x - \mu_y$$

$$\sigma_z^2 = \sigma_x^2 + \sigma_y^2$$

$\Rightarrow$ CENTRAL LIMIT THEOREM -

According to (CLT) when no. of independent R.V. are added then, it leads to gaussian distribution.

$$\# \quad \text{Var}(x) = \sigma_x^2$$

$$\text{Var}(cx) = c^2 \cdot \text{Var}[x] = c^2 \sigma_x^2$$

$$\text{Var}[k] = 0 \quad k = \text{const.}$$

$$\text{Var}[ax+b] = a^2 \text{Var}[x] = a^2 \sigma_x^2$$

$\Rightarrow$ AUTO CORRELATION FUNCTION -

$$R_{xx}(\tau) = E[x(t) \cdot x(t+\tau)]$$

$$= \int_{-\infty}^{+\infty} x(t) \cdot x(t+\tau) \cdot f_x(x) \cdot dt$$

$$- \quad R_{xx}(-\tau) = R_{xx}(\tau) \quad [\text{even fun}]$$

$$- \quad R_{xx}(0) = \text{Total power} = \text{msv}$$

$$- \quad R_{xx}(0) \geq R_{xx}(\tau)$$

$$- R_{xx}(\tau) \xrightarrow{\text{F.T.}} S_{xx}(f)$$

(PSD)

(WINNER KINCHIN THEOREM)

$$S_{xx}(f) = \int_{-\infty}^{\infty} R_{xx}(\tau) \cdot e^{j2\pi f \tau} \cdot d\tau$$

$$- S_{xx}(-f) = S_{xx}(f) \rightarrow \text{Real \& even}$$

$$- S_{xx}(f) \geq 0$$

$$- \int_{-\infty}^{\infty} S_{xx}(f) df = \text{Power} = R_{xx}(0)$$

Ques. A R.V. have autocorrelation funⁿ

$$R_{xx}(\tau) = 5 + 5 \cos \omega \tau + 10 e^{-5|\tau|}$$

then calculate ① total power ② ac power

③ DC power ④ mean ⑤ RMS ⑥ SD.

Solⁿ:

① Total power, $\tau=0$

$$R_{xx}(0) = 5 + 5 + 10 = 20 = \text{msv}$$

$$\textcircled{1} \text{ ac power} = 15$$

$$\textcircled{3} \text{ DC power} = 5$$

$$\textcircled{4} \sigma_x^2 = \text{msv} - (\text{mean})^2 \quad \text{mean} = \sqrt{5}$$

$\downarrow$
ac
power

$\downarrow$
Total
power

$\downarrow$
dc
power

$$\textcircled{5} \text{ RMS} = \sqrt{\text{msv}} = \sqrt{20}$$

$$\textcircled{6} \text{ SD} = \sqrt{\text{var.}} = \sqrt{15}$$

$\Rightarrow$ RANDOM PROCESS - (Randⁿ variable varies with time)

- Stationary process
- wide sense stationary process
- cyclo stationary process
- Ergodic process

① STATIONARY PROCESS -

In stationary process statistical value of any random variable is independent of time.

② WIDE SENSE S.P -

In wss process mean & autocorrelation funⁿ are independent of time.

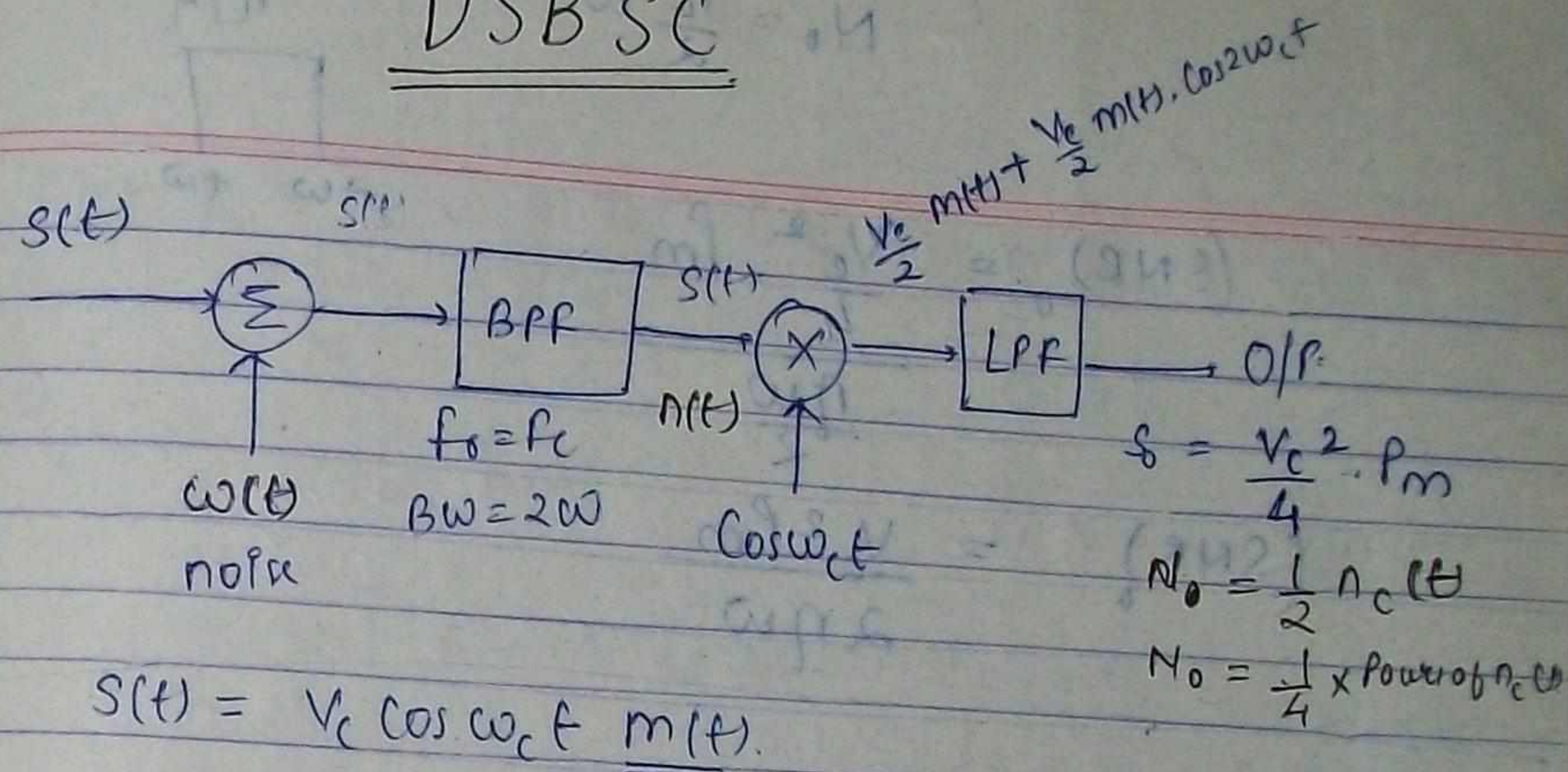
③ CYCLO STATIONARY P. -

In CSP, mean and autocorrelⁿ are periodic with time.

④ ERGODIC PROCS -

In ergodic process time averages & expected or statistical (ensemble) averages are same. Thus it is called ergodic process.

DSB SC

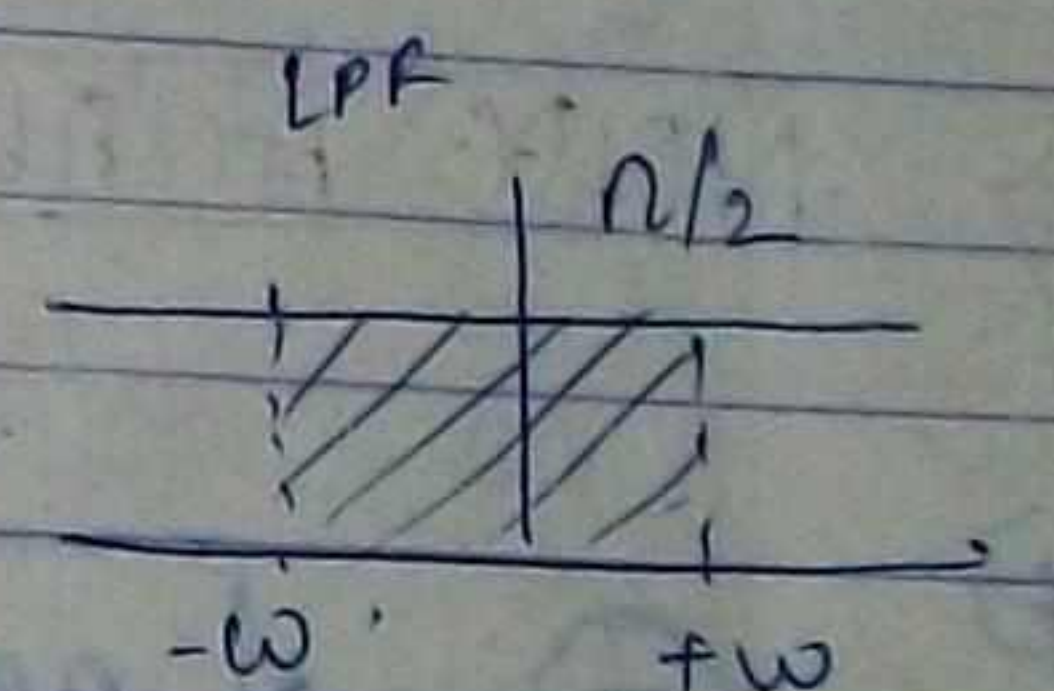


$$S(t) = V_c \cos \omega_c t \underline{m(t)}$$

$$S_i = \frac{V_c^2}{2} \cdot P_m$$

$$N_i = \eta W$$

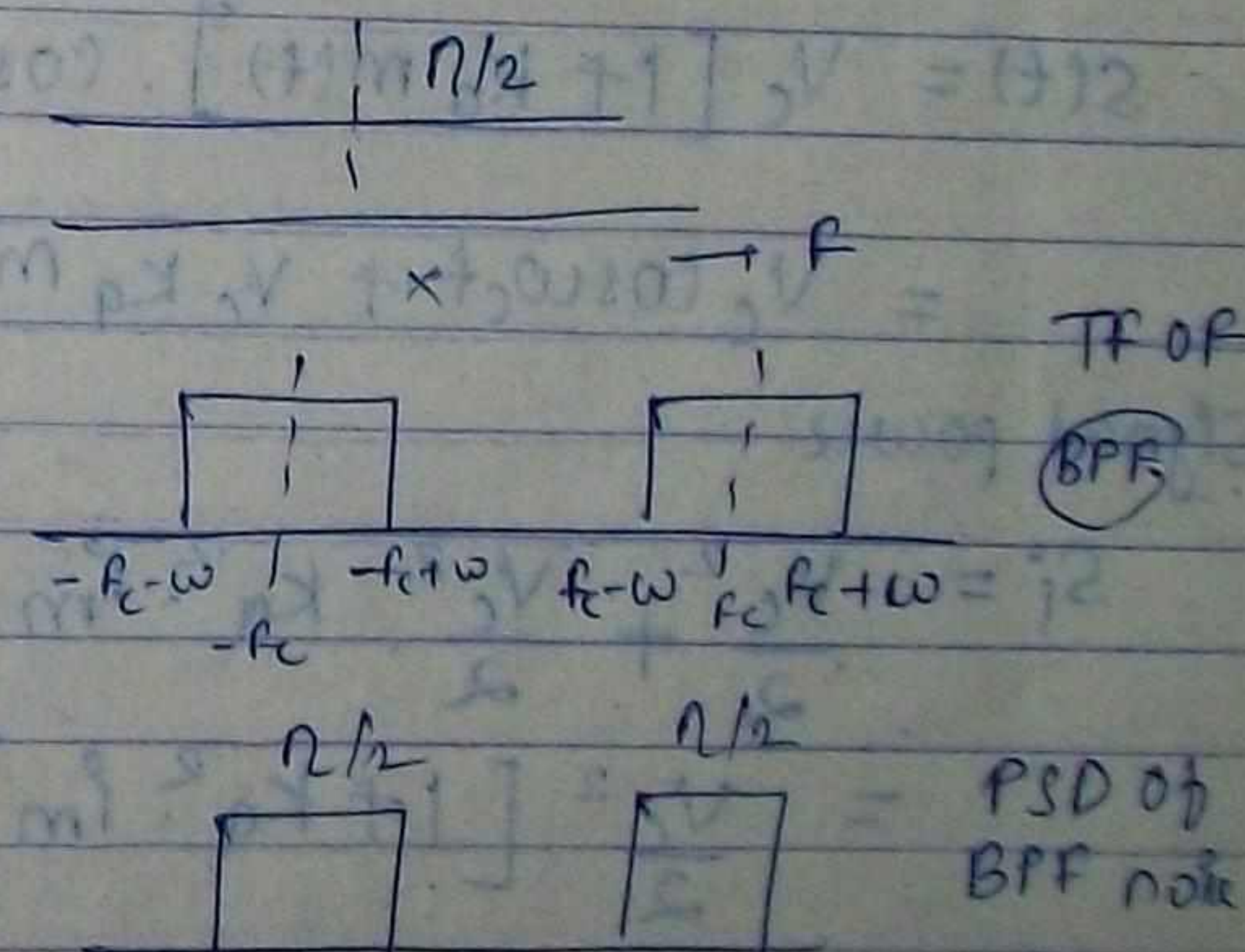
$$(SNR)_i = \frac{V_c^2 \times P_m}{2 \times \eta W}$$



$$N_f = n_f \times 2W = \eta W$$

for noise at O/P.

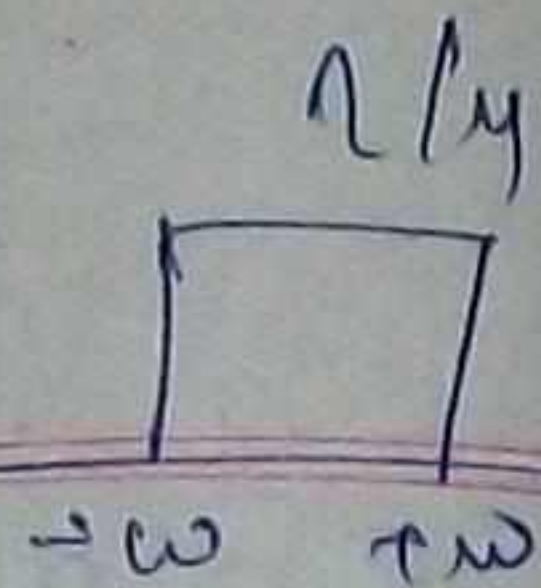
$n(t)$ = narrowband filter noise.



$$n(t) = n_c(t) \cos \omega_c t - n_s(t) \sin \omega_c t$$

LPF noise

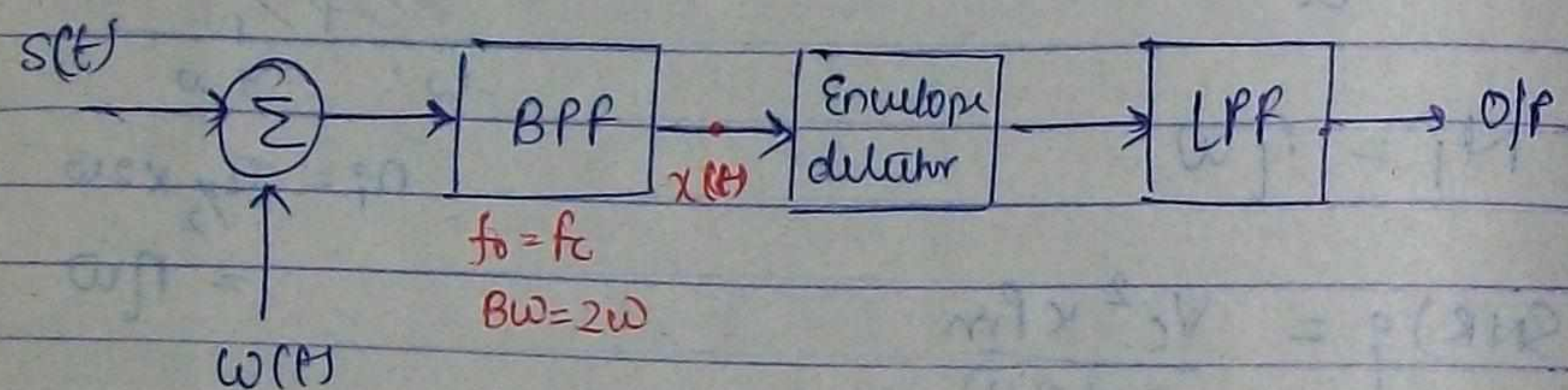
$$N_0 = \frac{N}{2} \omega$$



$$(SNR)_0 = \frac{V_c^2 \cdot P_m}{\frac{\eta \omega}{2}}$$

$$(SNR)_0 = \frac{V_c^2 P_m}{2 \eta \omega}$$

⇒ NOISE ANALYSIS OF AM -



$$s(t) = V_c [1 + k_a m(t)] \cdot \cos \omega_c t$$

$$= V_c \cos \omega_c t + V_c k_a m(t) \cdot \cos \omega_c t$$

Signal power

$$S_i = \frac{V_c^2}{2} + \frac{V_c^2}{2} k_a^2 P_m$$

$$= \frac{V_c^2}{2} [1 + k_a^2 P_m]$$

$$N_i = \eta \omega$$

$$(SNR)_i = \frac{V_c^2 (1 + k_a^2 P_m)}{2 \eta \omega} \quad \text{--- (1)}$$

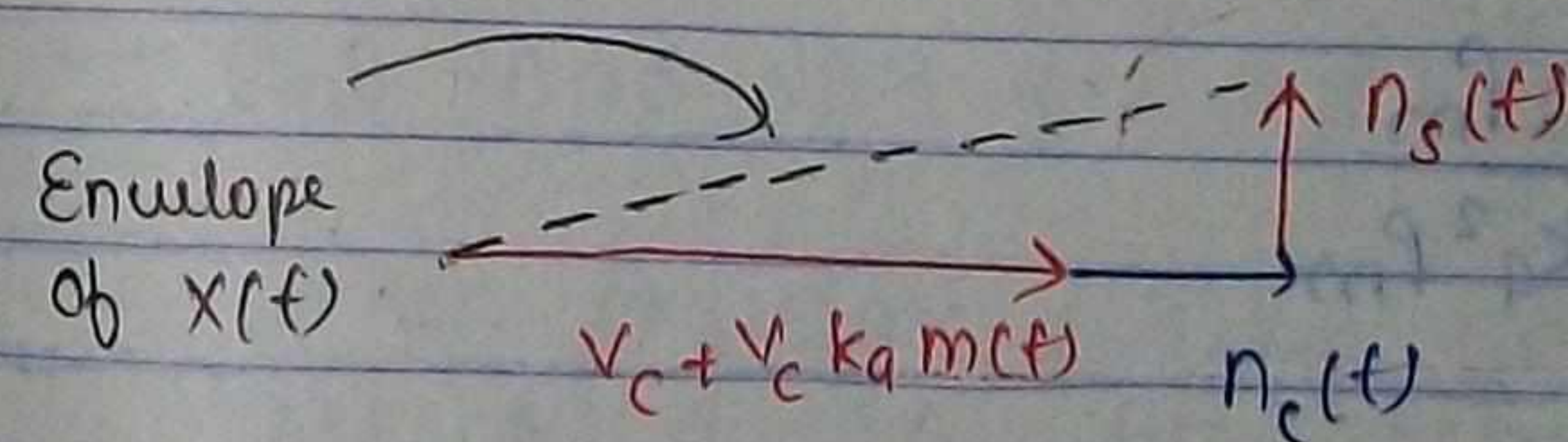
Now,

$$x(t) = s(t) + n(t)$$

$$= V_c [1 + k_a m(t)] \cos \omega_c t + n_c(t) \cos \omega_c t - n_s(t) \sin \omega_c t$$

$$x(t) = [V_c + V_c k_a m(t) + n_c(t)] \cos \omega_c t - n_s(t) \sin \omega_c t$$

assumption $|s(t)| > |n(t)|$



$$\text{Envelope of } x(t) \approx \underbrace{V_c}_{\text{reject d.c.}} + \underbrace{V_c k_a m(t)}_{\text{o/p signal}} + \underbrace{n_c(t)}_{\text{noise}}$$

$$\text{o/p signal power} = V_c^2 \cdot k_a^2 \cdot P_m$$

$$\text{o/p noise power} = \text{power in } n_c(t)$$

∴ O/p noise PSD $\eta_c(f)$

$$= 2\eta\omega$$

$$\text{noise power} = 2\eta\omega$$

$$(SNR)_o = \frac{V_c^2 \cdot k_a^2 \cdot P_m}{2\eta\omega} \quad \text{--- (2)}$$

$$\# \text{ FOM} = \frac{(SNR)_o}{(SNR)_i}$$

$$= \frac{V_c^2 \cdot k_a^2 \cdot P_m}{2\eta\omega} \times \frac{2\eta\omega}{V_c^2 (1 + k_a^2 P_m)}$$

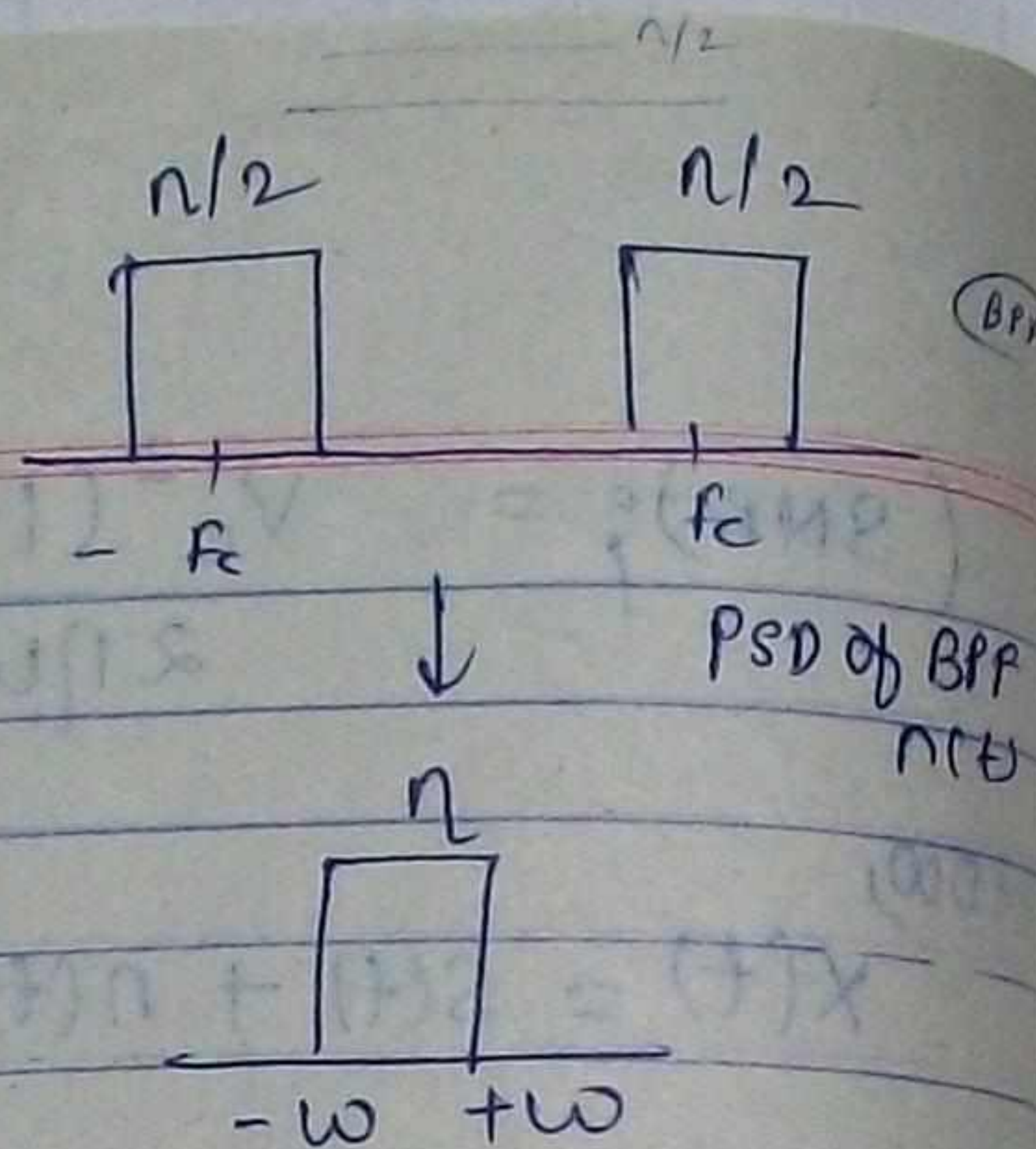
$$\text{FOM} = \frac{k_a^2 \cdot P_m}{1 + k_a^2 P_m}$$

Comparing with single tone:

$$s(t) = V_c [1 + k_a m(t)] \cos \omega_c t$$

$$s(t) = V_c [1 + (k_a \cdot V_m) \cos \omega_m t] \cos \omega_c t$$

$$m = k_a \cdot V_m = \text{modulation index}$$



$$m(t) = V_m \cos 2\pi f_m t, \quad P_m = \frac{V_m^2}{2} \quad \text{--- (1)}$$

$$k_a = \frac{m}{V_m} \quad \text{--- (2)}$$

then

$$\text{FOM} = \frac{m^2}{V_m^2} \times \frac{V_m^2}{2} \times \frac{1}{1 + \frac{m^2}{V_m^2} \times \frac{V_m^2}{2}}$$

$$\text{FOM} = \frac{m^2}{2 + m^2} = \text{efficiency of single tone Am}$$

$$= \eta_{\text{max}} \% = 33.33\%$$

$$(SNR)_o = \frac{m^2}{2 + m^2} (SNR)_i$$

— DSB and SSB are more efficient than Am.

— when envelope detector and square law demod are used in Am demodulation then threshold effect will occur when $s(t)$

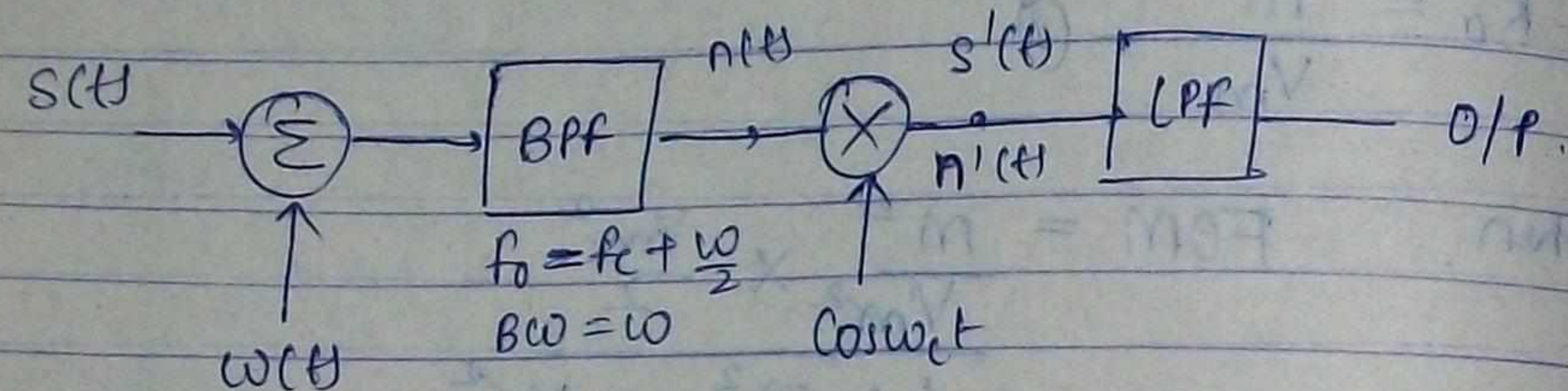
$$|s(t)| \ll |n(t)|$$

due to this msg signal is completely lost at the o/p of receiver.

THRESHOLD EFFECT

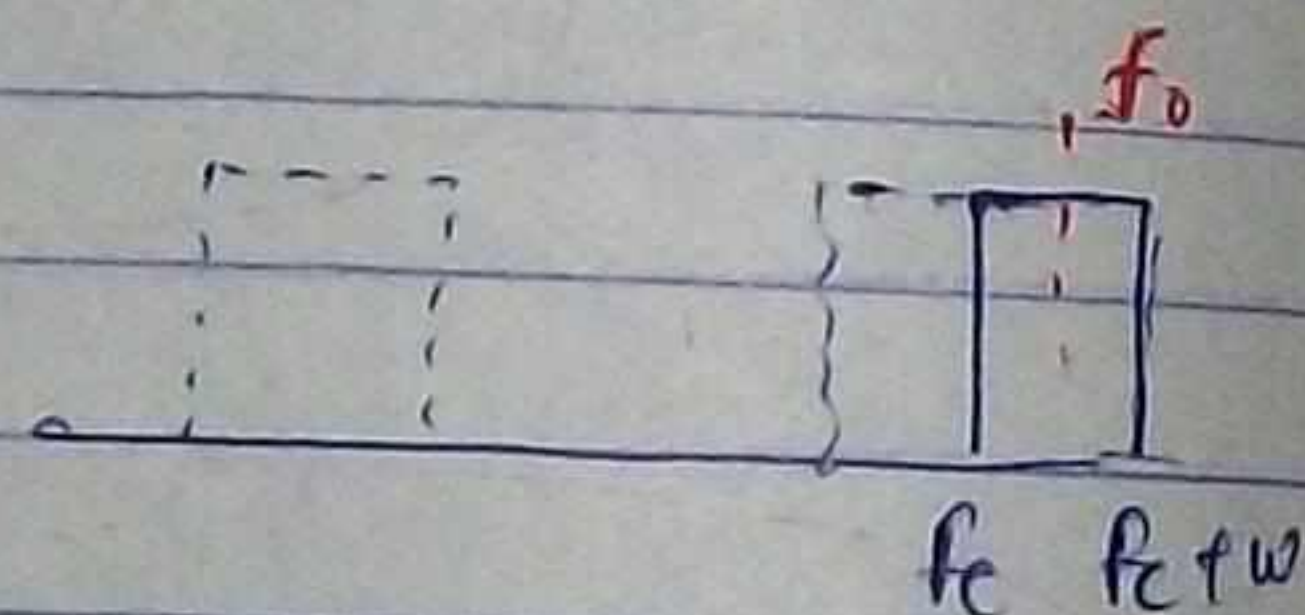
Square law
Envelope
Discriminator } Threshold
Signal effect
is present

⇒ NOISE ANALYSIS OF SSB -



$$S(t) = V_c m(t) \cos \omega_c t - V_c \hat{m}(t) \sin \omega_c t$$

SSB
USB



$$s'(t) = V_c m(t) \cos \omega_c t - \frac{V_c}{2} \hat{m}(t) \cos \omega_c t \sin \omega_c t$$

$$= \frac{V_c}{4} m(t) + \frac{V_c}{4} \cos 2\omega_c t - \frac{V_c}{4} \hat{m}(t) \sin 2\omega_c t$$

$$S_o(t) = \frac{V_c}{4} m(t)$$

$$(SNR)_i = \frac{V_c^2 \times P_m \times \frac{1}{2}}{4} + \frac{V_c^2 \times P_m \times \frac{1}{2}}{4}$$

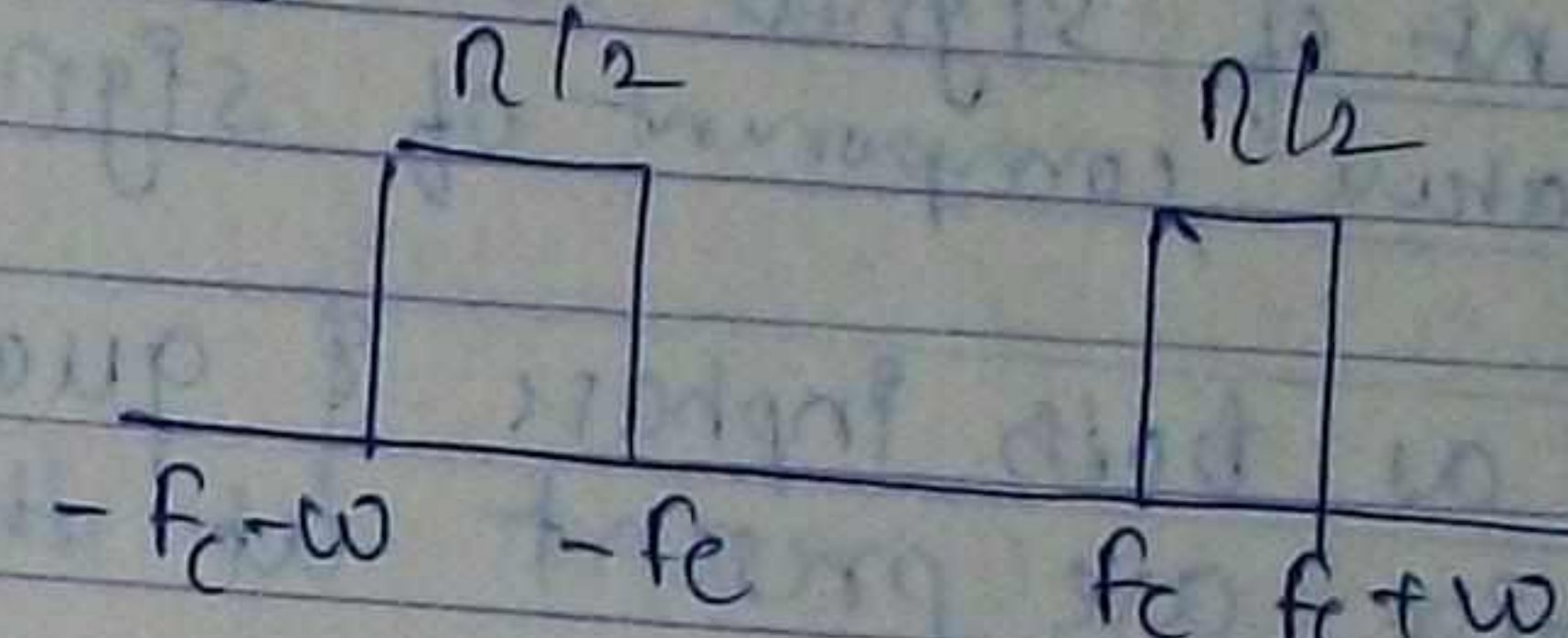
$\eta \omega$

$$(SNR)_i = \frac{V_c^2 P_m}{4 \eta \omega}$$

(11)

$$O/P \text{ Signal power} = \frac{V_c^2}{16} P_m$$

for noise power



$$n(t) = n_c(t) \cos 2\pi f_c t - n_s(t) \sin 2\pi f_c t$$

$$= n_c(t) \cos 2\pi \left(f_c + \frac{\omega}{2}\right) t - n_s(t) \sin 2\pi \left(f_c + \frac{\omega}{2}\right) t$$

$$n'(t) = n(t) \cos \omega_c t$$

$$= n_c(t) \cos 2\pi \left(\omega_c t + \omega t\right) \cos \omega_c t$$

$$- n_s(t) \sin 2\pi \left(f_c + \frac{\omega}{2}\right) t \cos 2\pi f_c t$$

$$= \frac{1}{2} n_c(t) \left[\cos 2\pi \left(2f_c + \frac{\omega}{2}\right) t + \cos 2\pi \left(\frac{\omega}{2}\right) t \right]$$

$$- \frac{n_s(t)}{2} \left[\sin 2\pi \left(2f_c + \frac{\omega}{2}\right) t + \sin 2\pi \left(\frac{\omega}{2}\right) t \right]$$

$$N_o(t) = \frac{1}{2} n_c(t) \cos 2\pi \left(\frac{\omega}{2}\right) t - \frac{n_s(t)}{2} \sin 2\pi \left(\frac{\omega}{2}\right) t$$

$A \cos \omega t$
 $\left(\frac{A}{\sqrt{2}}\right)^2 = \frac{A^2}{2}$

SSB = $\eta \omega$

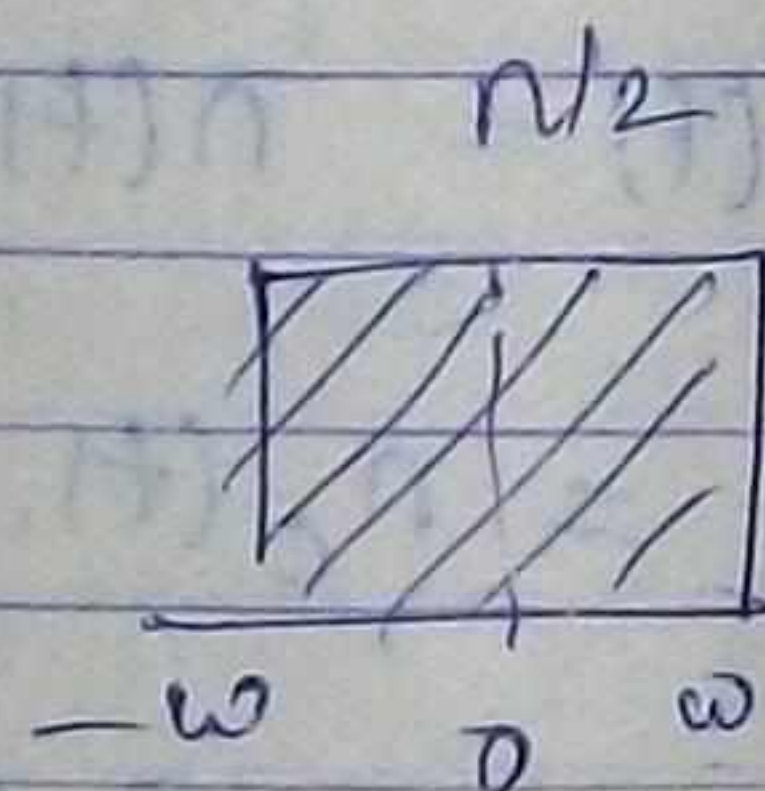
- In SSB demodulation (coherent detection), only inphase component of signal will present where as quadrature component of signal is eliminated
- where as both inphase & quadrature component of noise are present at the O/P of coherent detector.

$$N_o(f) = \frac{1}{2} n_c(f) \cos 2\pi \left(\frac{\omega}{2}\right)t - \frac{1}{2} n_s(f) \sin 2\pi \left(\frac{\omega}{2}\right)t$$

$$= \left(\frac{1}{2\sqrt{2}}\right)^2 P_n + \left(\frac{1}{2\sqrt{2}}\right)^2 \times P_n$$

$$= \frac{1}{8} P_n + \frac{1}{8} P_n = \frac{1}{4} P_n$$

$$= \frac{1}{4} \times \eta \omega$$



$$(SNR)_o = \frac{V_c^2 P_m}{16 \frac{\eta \omega}{4}}$$

$$(SNR)_o = \frac{V_c^2 P_m}{4 \eta \omega} \quad \text{--- (2)}$$

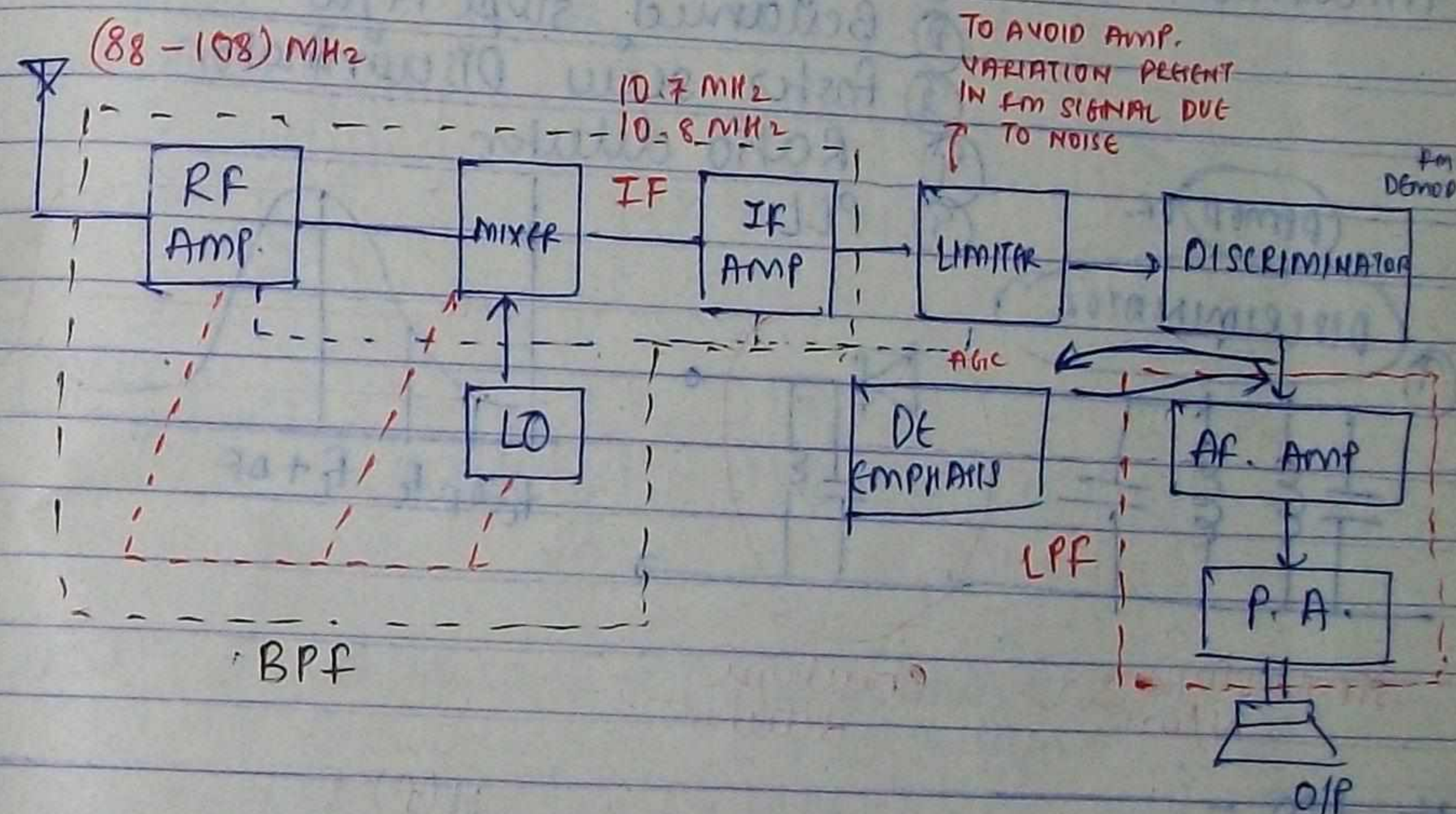
FDM = 1

mean $\eta\% = 100\%$

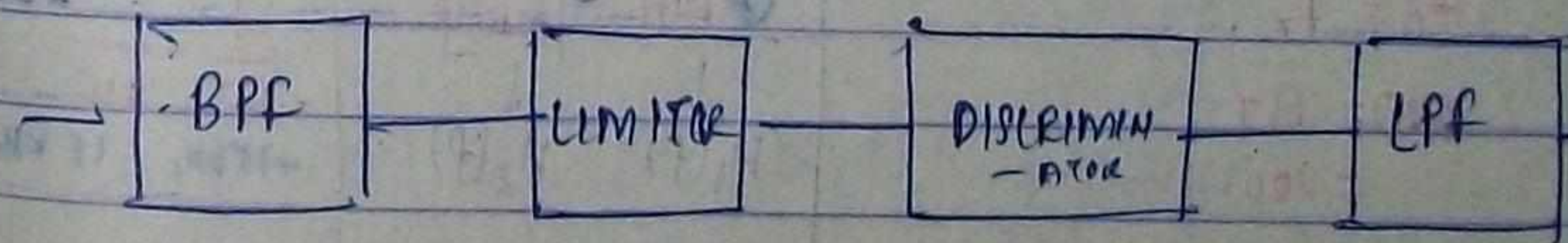
form

DSB $\rightarrow 1$
 SSB $\rightarrow 1$
 AM $\rightarrow 1$
 3

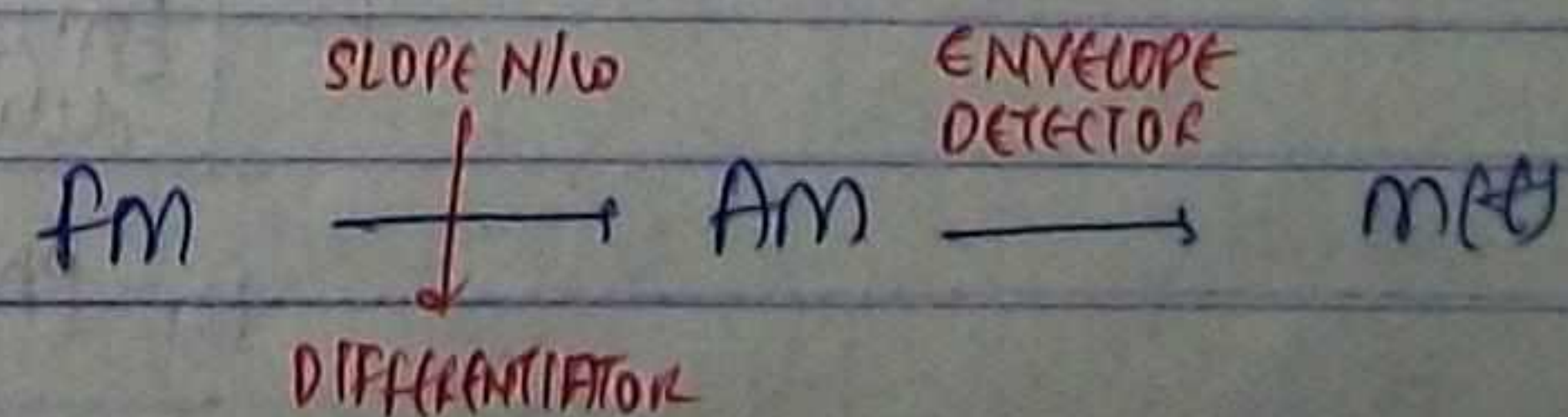
NOISE ANALYSIS OF ANGLE MODULATED SIGNAL



Equivalent circuit for FM



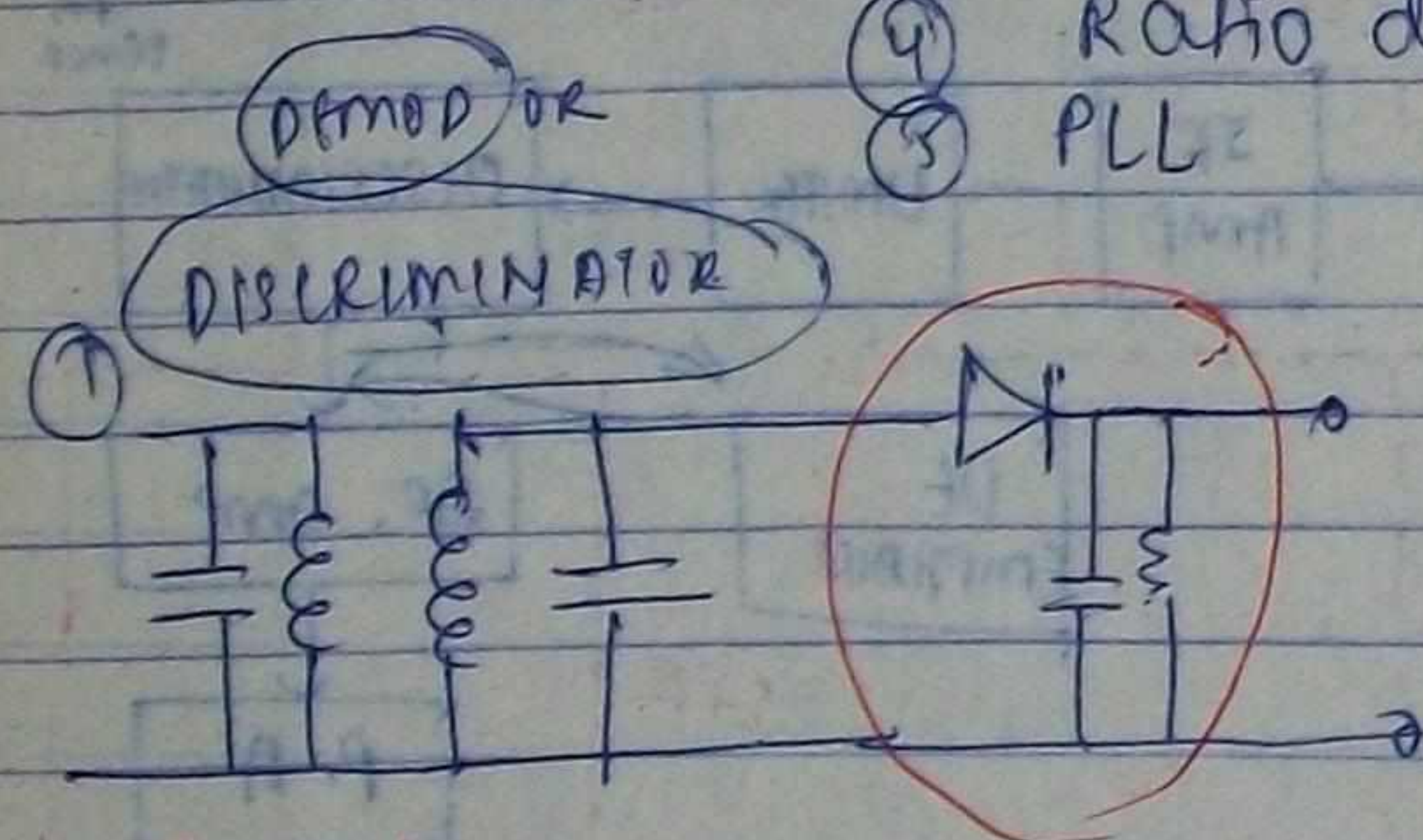
FM demod.



- The basic principle of FM demodulation is any FM signal is first converted into amplitude variations then an envelope detector is used to demodulate the resultant signal to provide msg. signal.

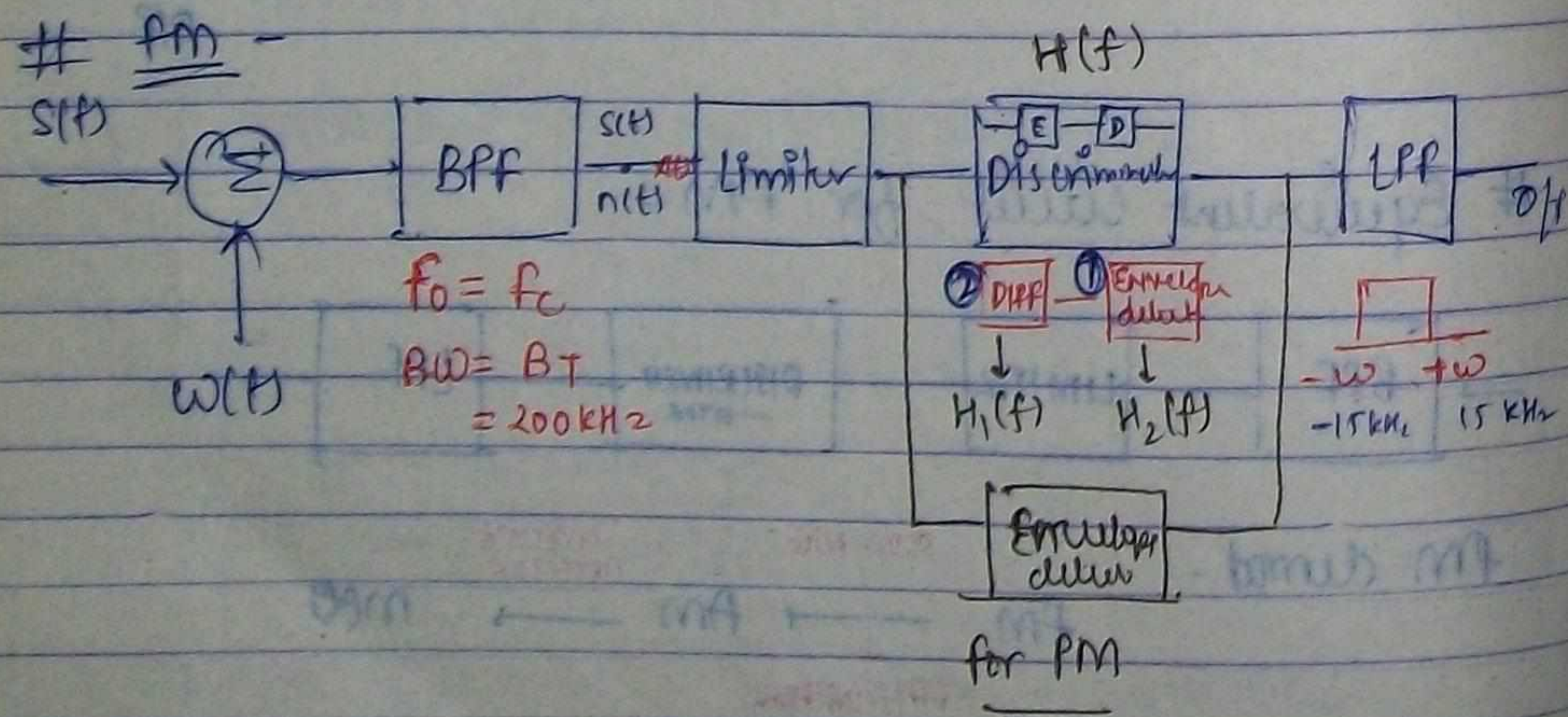
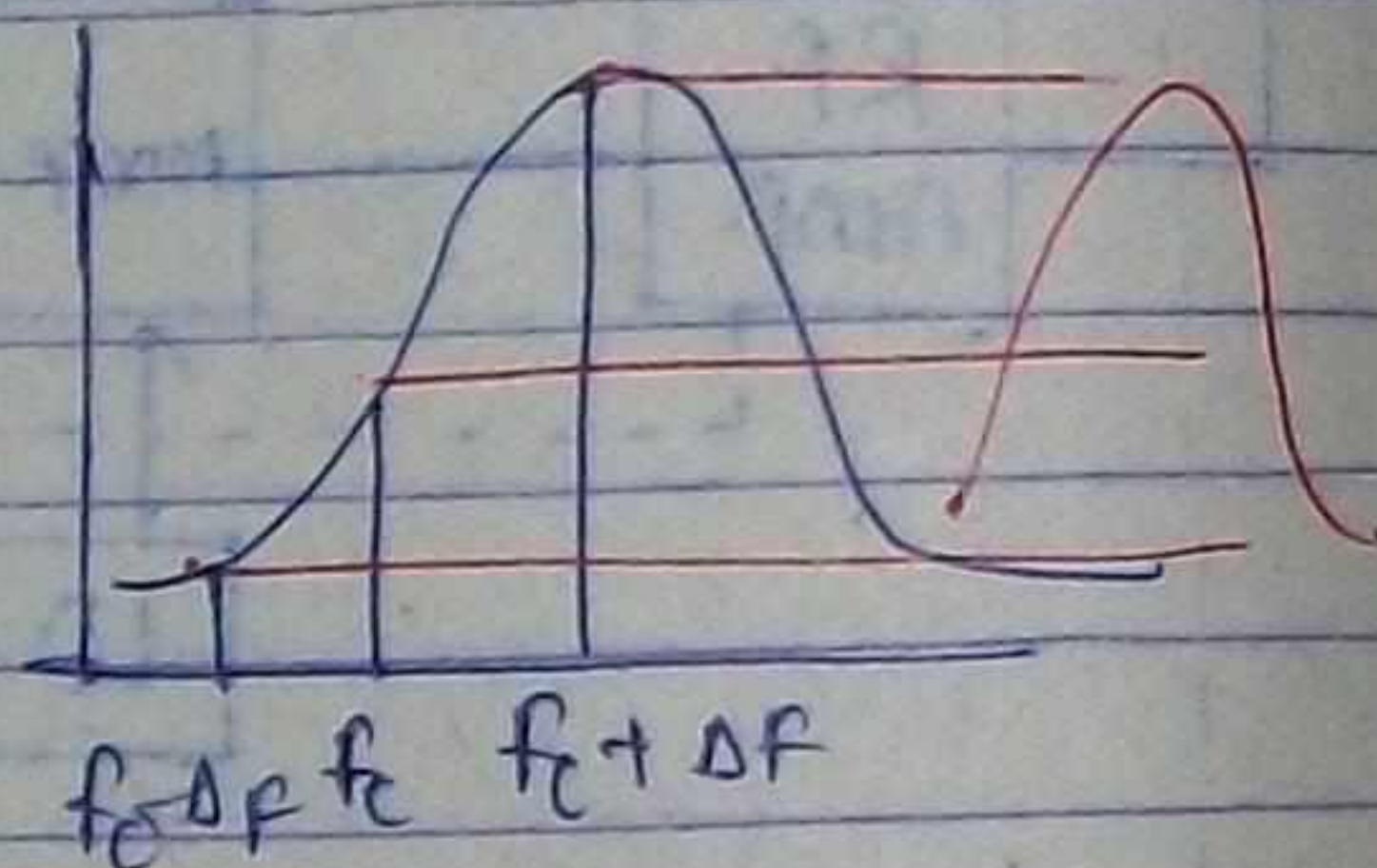
FM demodulators are:

- 1) Single slope N/w
- 2) Balanced slope N/w
- 3) Foster see Discriminator.
- 4) Ratio detector
- 5) PLL



Single slope
N/w

envelope
detector



Let us consider.

$$s(t)_{fm} = V_c \cos [2\pi f_c t + 2\pi k_f \int m(t) dt]$$

$$s(t)_{pm} = V_c \cos [2\pi f_c t + k_p m(t)]$$

$$s(t)_{angle\ mod\ in} = V_c \cos [2\pi f_c t + \phi(t)]$$

$$S_i = \frac{V_c^2}{2}, \quad N_f = \eta \omega$$

$$(SNR)_f = \frac{V_c^2}{2\eta\omega} \quad \text{--- (1)}$$

$$r(t) = n_c(t) \cos \omega_c t - n_s(t) \sin \omega_c t$$

$$= r(t) \cdot \cos [\omega_c t + \psi(t)]$$

$$r(t) = \sqrt{n_c^2(t) + n_s^2(t)}$$

$$\psi(t) = \tan^{-1} \left(\frac{-n_s(t)}{n_c(t)} \right)$$

$$x(t) = V_c \cos [2\pi f_c t + \phi(t)]$$

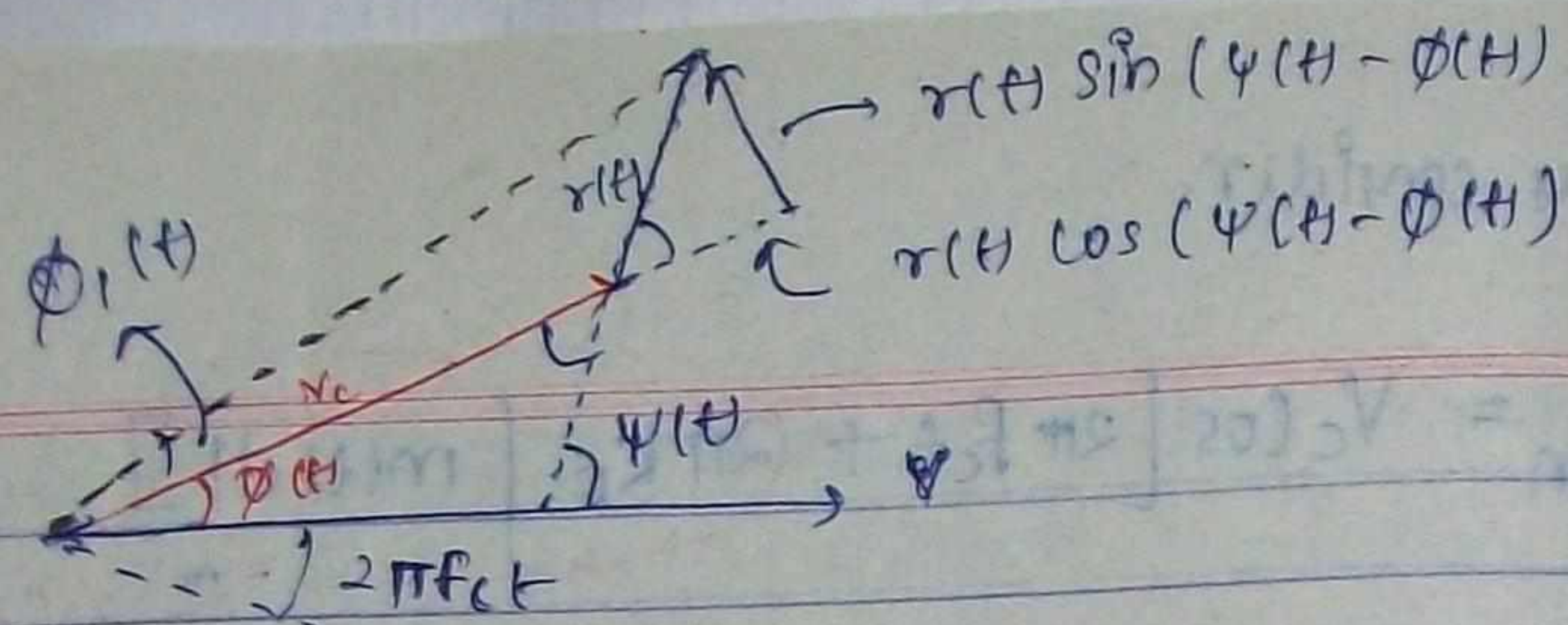
$$+ r(t) \cdot \cos [\omega_c t + \psi(t)]$$

Assumption

$$|s(t)| \gg |r(t)|$$

Threshold
effect

phaser
diagram



$$x(t) = \underbrace{\left[V_c + r(t) \cos(\psi(t) - \phi(t)) \right]}_{\text{remove by limiter}} \cdot \cos \left[2\pi f_c t + \phi(t) \right] + \tan^{-1} \left[\frac{r(t) \sin(\psi(t) - \phi(t))}{r(t) \cos(\psi(t) - \phi(t)) + V_c} \right]$$

↑ ↑
negligible very large

$$x(t) = \cos \left[2\pi f_c t + \phi(t) + \frac{n_s(t)}{V_c} \right] \approx 0$$

↓
remove by envelope detector

① for PM signal after demodulation o/p is

$$x_o(t) = \phi(t) + \frac{n_s(t)}{V_c}$$

$$= k_p m(t) + \frac{n_s(t)}{V_c}$$

O/P signal power is given by $k_p^2 \cdot P_m$

envelope detector - remove carrier

$$\text{o/p noise power} = \left(\frac{1}{V_c} \right)^2 \times \text{power in } n_s(t)$$

$$= \left(\frac{1}{V_c} \right)^2 \times P_n$$

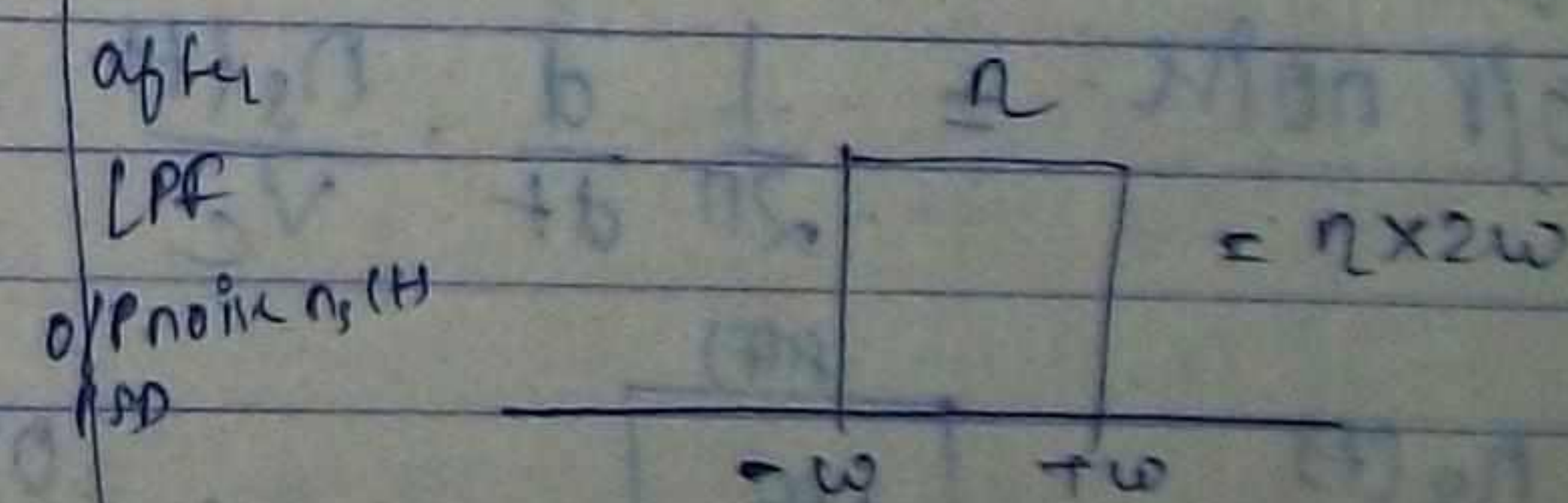
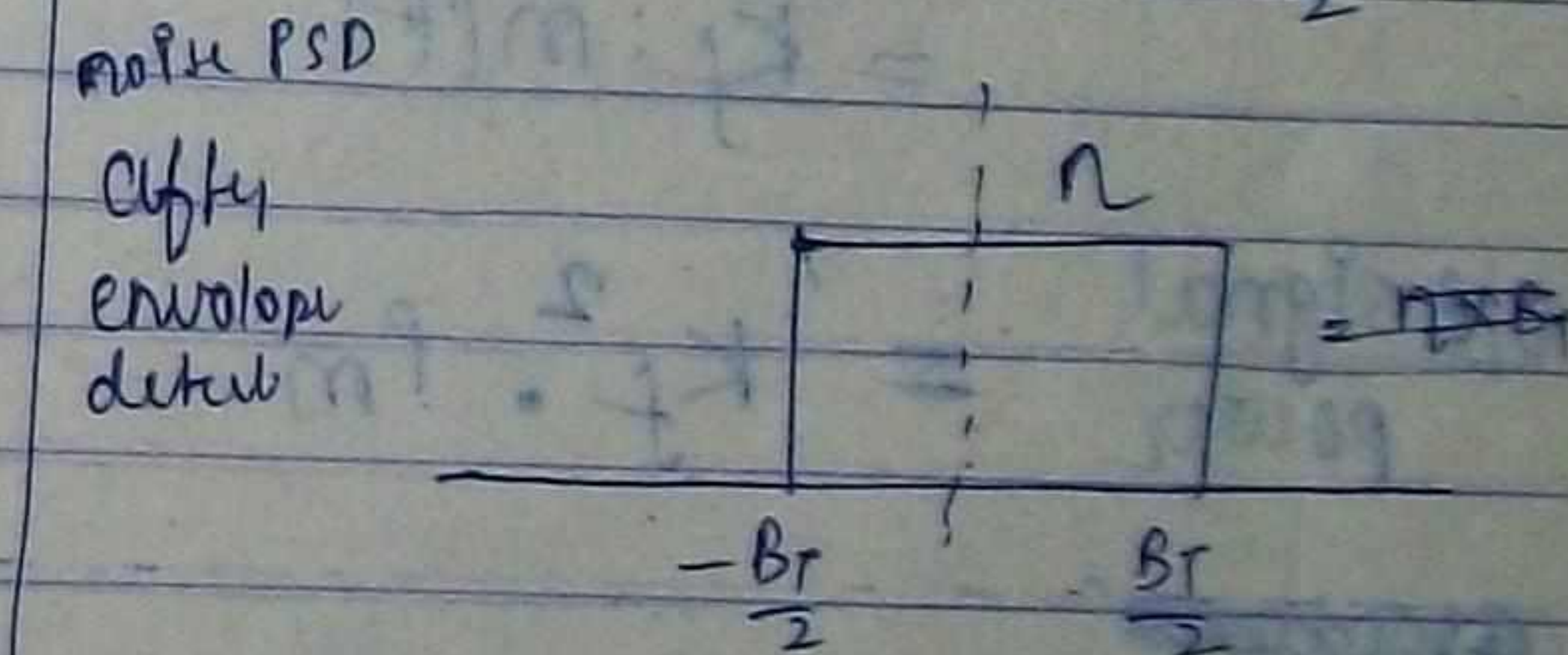
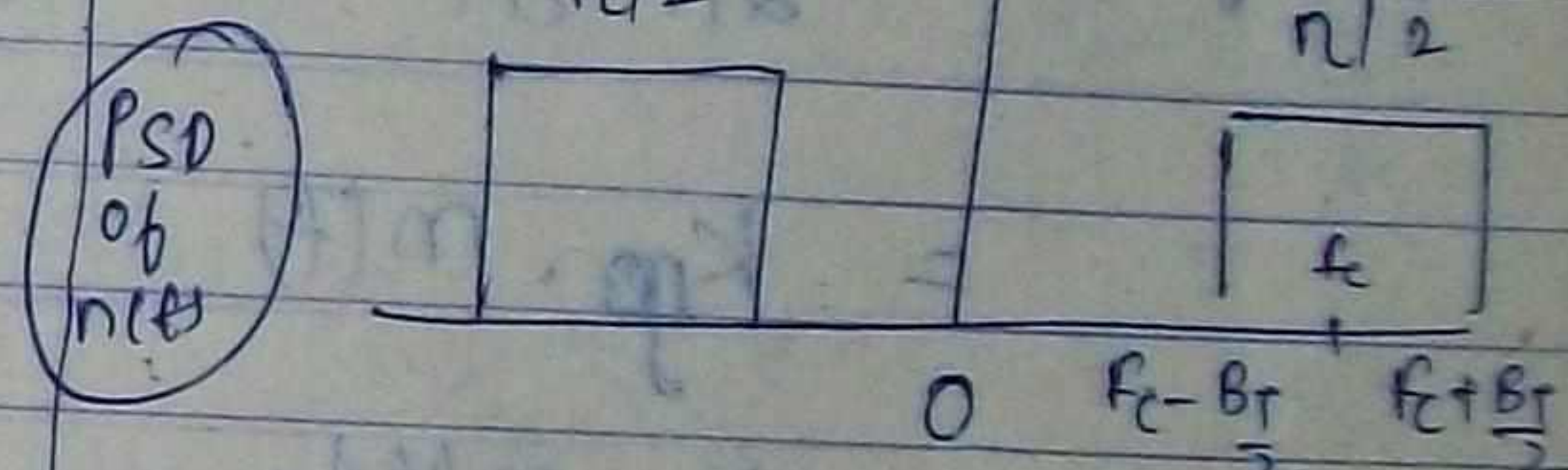
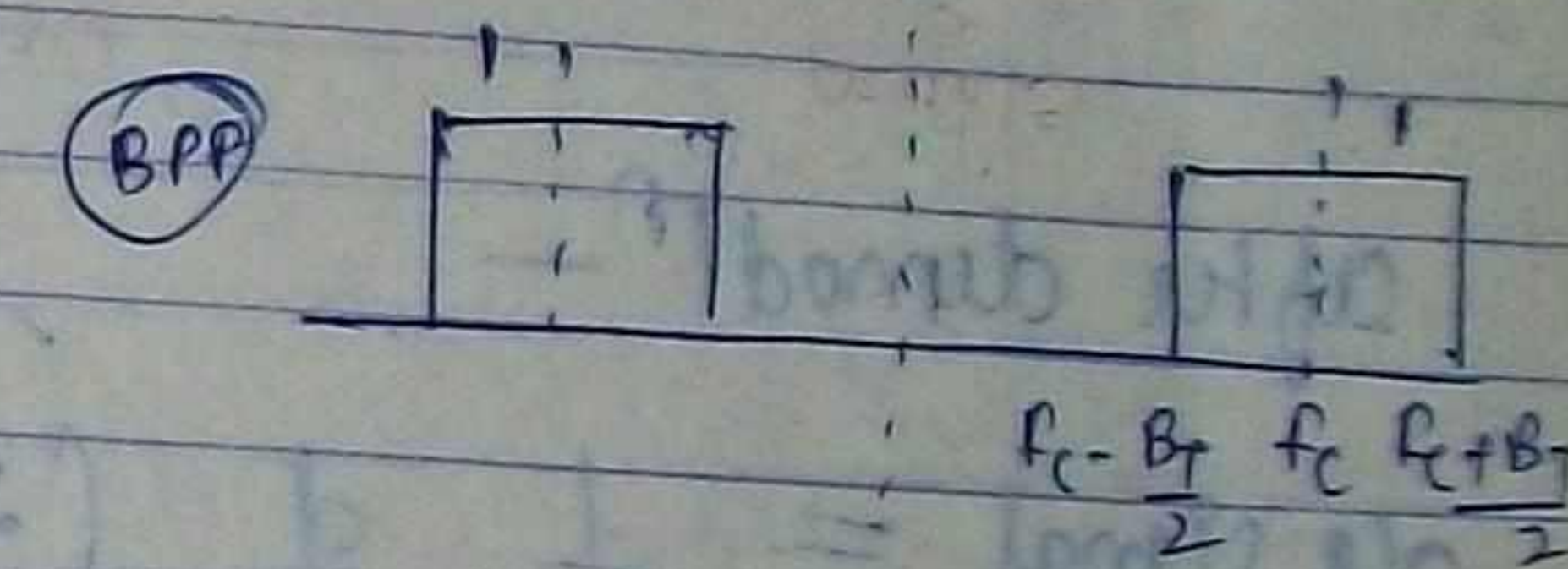
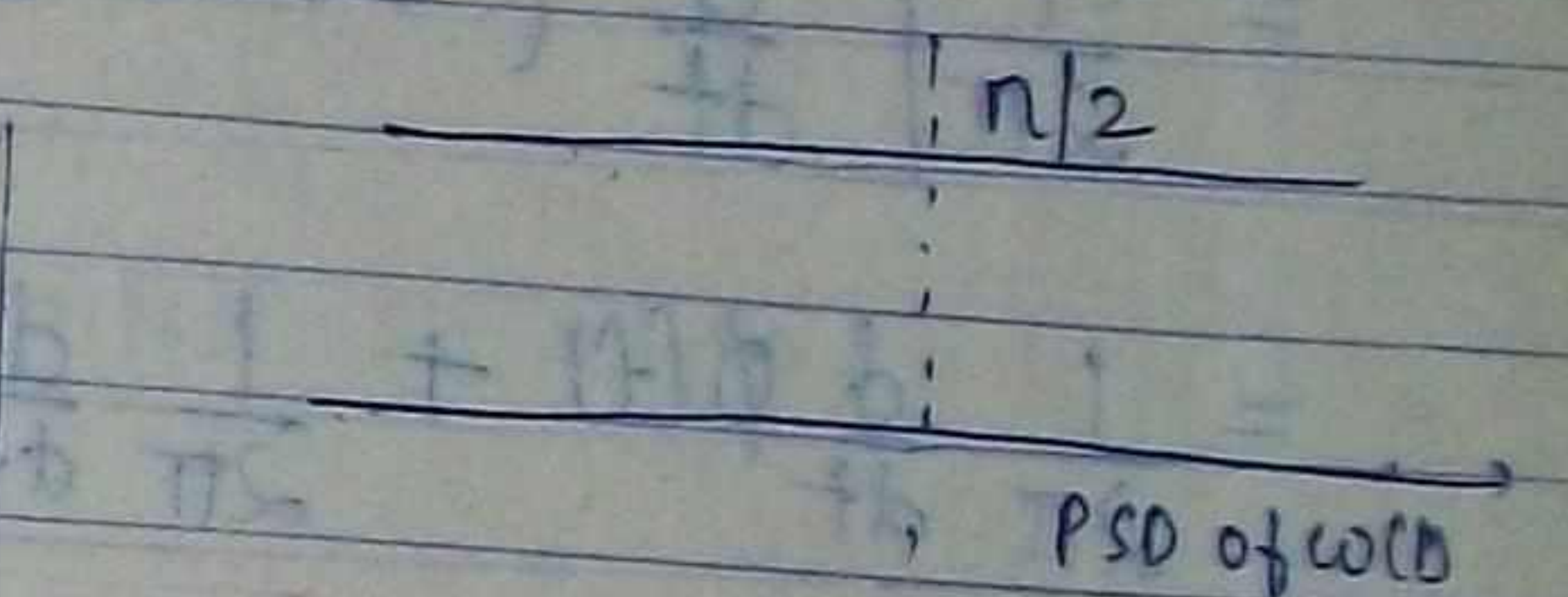
$$= \left(\frac{1}{V_c} \right)^2 \times 2\eta\omega$$

$$(\text{SNR})_o = \frac{k_p^2 \cdot P_m \cdot V_c^2}{2\eta\omega}$$

(2)

$$\text{form} = \frac{k_p^2 \cdot P_m \cdot V_c^2}{2\eta\omega} \times \frac{2\eta\omega}{V_c^2}$$

$$\text{form} = k_p^2 \cdot P_m$$



② for FM demodulation:

Now o/p of differentiator is

$$= \frac{1}{2\pi} \left[\frac{d}{dt} \left(\phi(t) + \frac{n_s(t)}{V_c} \right) \right]$$

$$= \frac{1}{2\pi} \frac{d}{dt} \phi(t) + \frac{1}{2\pi} \frac{d}{dt} \frac{n_s(t)}{V_c}$$

signal

noise

after demodulation

$$\text{o/p signal} = \frac{1}{2\pi} \frac{d}{dt} (2\pi k_f \int m(t) dt)$$

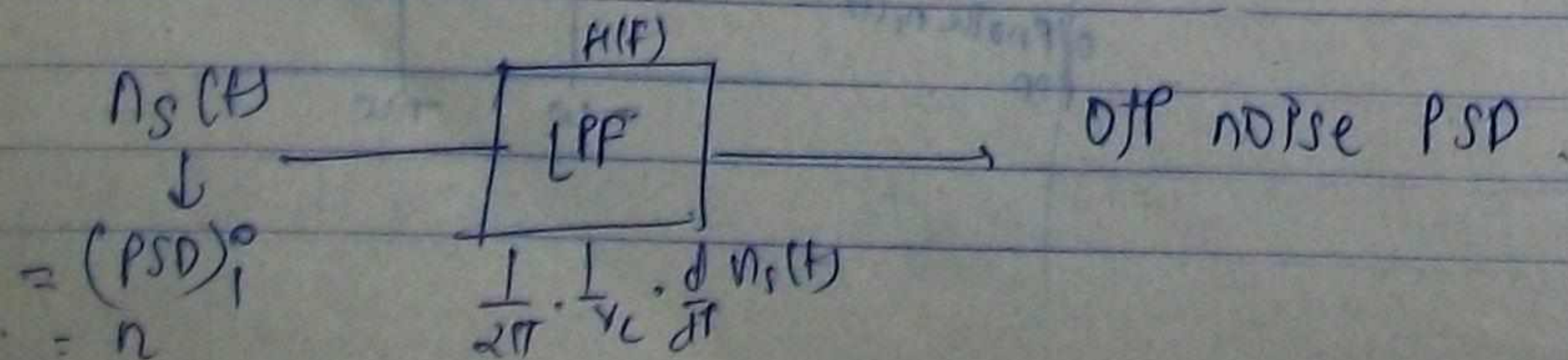
$$= k_f \cdot m(t)$$

$$= k_f \cdot m(t)$$

$$\text{o/p signal power} = k_f^2 \cdot P_m$$

after demodulation

$$\text{o/p noise} = \frac{1}{2\pi} \frac{d}{dt} \frac{n_s(t)}{V_c}$$



$$n \rightarrow \left[\frac{1}{V_c} \cdot \frac{1}{2\pi} \times j\omega f \right]$$

$$n \rightarrow \left[\frac{jf}{V_c} \right]$$

o/p noise PSD

$$\frac{f^2}{V_c^2} \times n$$

after demodulation (Envelope detector + Diff. T.C)

$$\text{o/p noise PSD} = \frac{f^2}{V_c^2} \times n$$

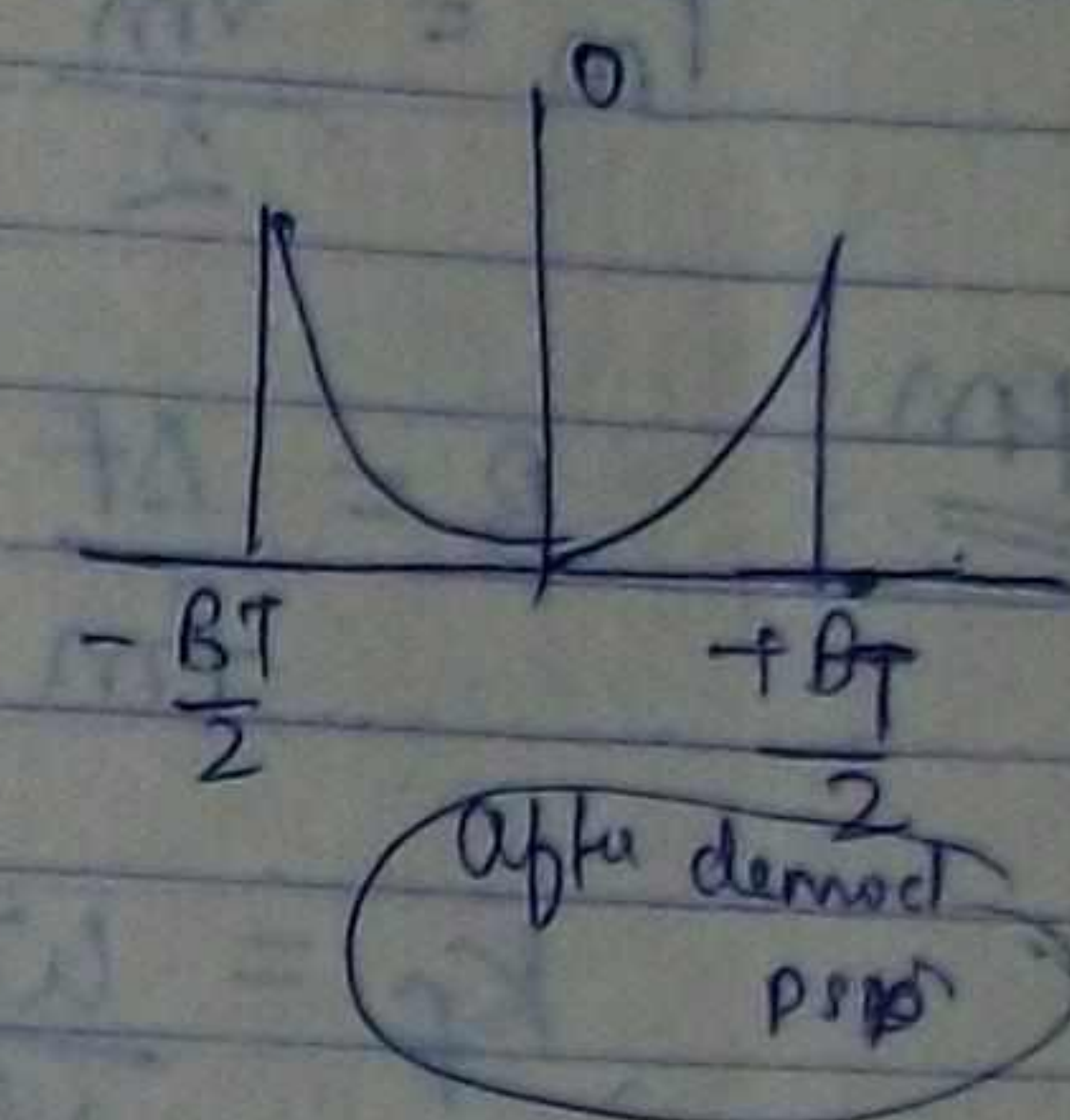
o/p noise power

$$= \int_{-\omega}^{\omega} \frac{n}{V_c^2} \cdot f^2 \cdot df$$

$$= \frac{n}{V_c^2} \cdot \left(\frac{f^3}{3} \right)_{-\omega}^{\omega}$$

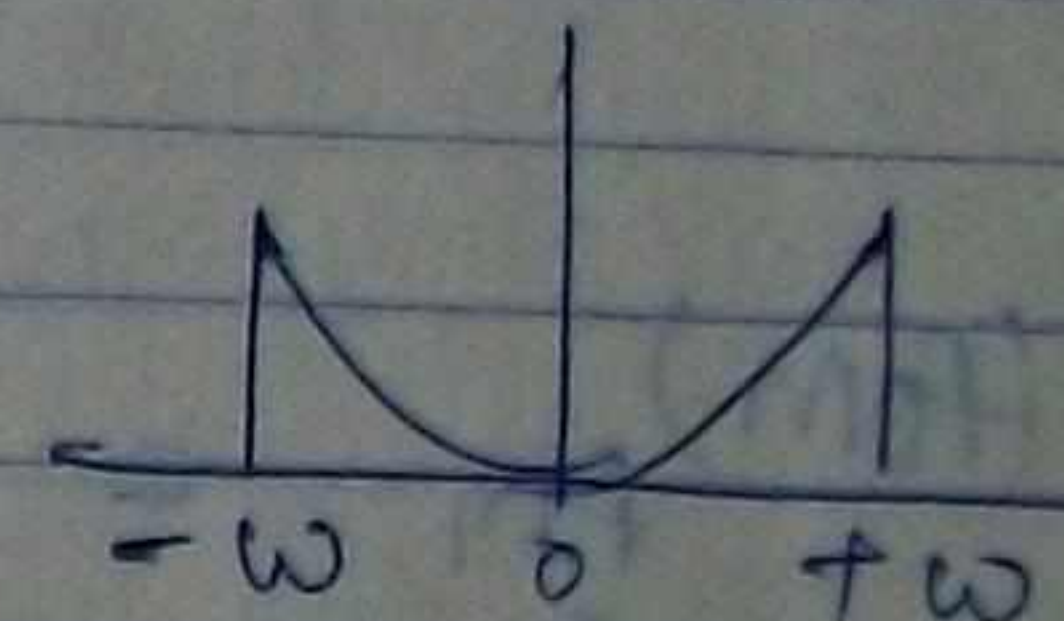
Noise power o/p

$$= \frac{2\omega^3 \cdot n}{3 V_c^2}$$



after LPP

o/p



$$(SNR)_o = \frac{k_f^2 \cdot P_m \cdot (3 V_c^2)}{2 \omega^3 n} = \frac{3}{2} \frac{k_f^2 P_m V_c^2}{\omega^3 n}$$

$$FOM = \frac{3}{2} \frac{k_f^2 P_m V_c^2}{\omega^3 n} \times \frac{2\pi\omega}{V_c^2}$$

$$(FOM)_{fm} = 3 P_m \left(\frac{K_f}{\omega} \right)^2$$

for
single tone mod'n -

$$m(t) = V_m \cos 2\pi f_m t$$

$$P_m = \frac{V_m^2}{2}$$

$$\beta = \frac{\Delta f}{f_m} = \frac{\Delta f}{\omega} = \frac{K_f \cdot V_m}{\omega}$$

$$K_f = \frac{\omega \beta}{V_m}$$

$$(FOM)_{fm} = 3 \times \frac{V_m^2}{2} \times \frac{\omega^2 \beta^2}{V_m^2 \cdot \omega^2}$$

$$(FOM)_{fm} = \frac{3 \cdot \beta^2}{2}$$

for single tone.

$$(SNR)_o = \frac{3}{2} \beta^2 (SNR)_i$$

$\Rightarrow PM$ -

$$(FOM) = \frac{(SNR)_o}{(SNR)_i} = K_p^2 P_m$$

for single tone

$$P_m = \frac{V_m^2}{2} \quad (m(t) \text{ power})$$

$$B_p = K_p \cdot V_m$$

$$(FOM) = \frac{B_p^2}{V_m^2} \cdot \frac{V_m^2}{2}$$

$$(FOM) = \frac{B_p^2}{2}$$

SUMMARY OF FOM -

① DSB - 1

② SSB - 1

③ AM - $1/3$

④ FM - $\frac{3}{2} \beta^2$

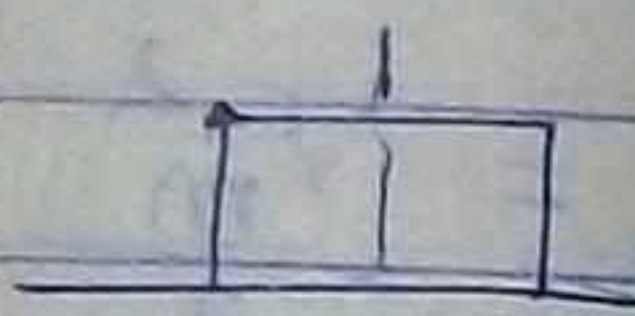
⑤ PM - $\frac{1}{2} B_p^2$

} for single tone

- In fm receiver o/p noise PSD follows parabolic shape

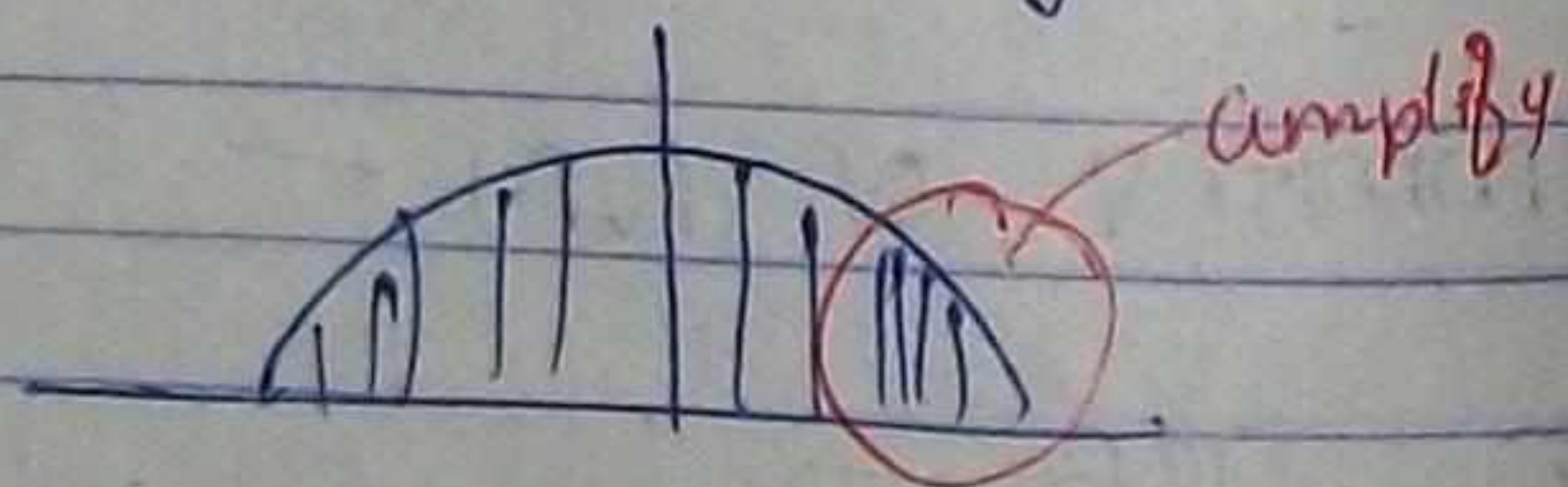


- Other modⁿ schemes (AM, PM, SSB, DSB) o/p noise PSD follows uniform shape in band pass freq. range



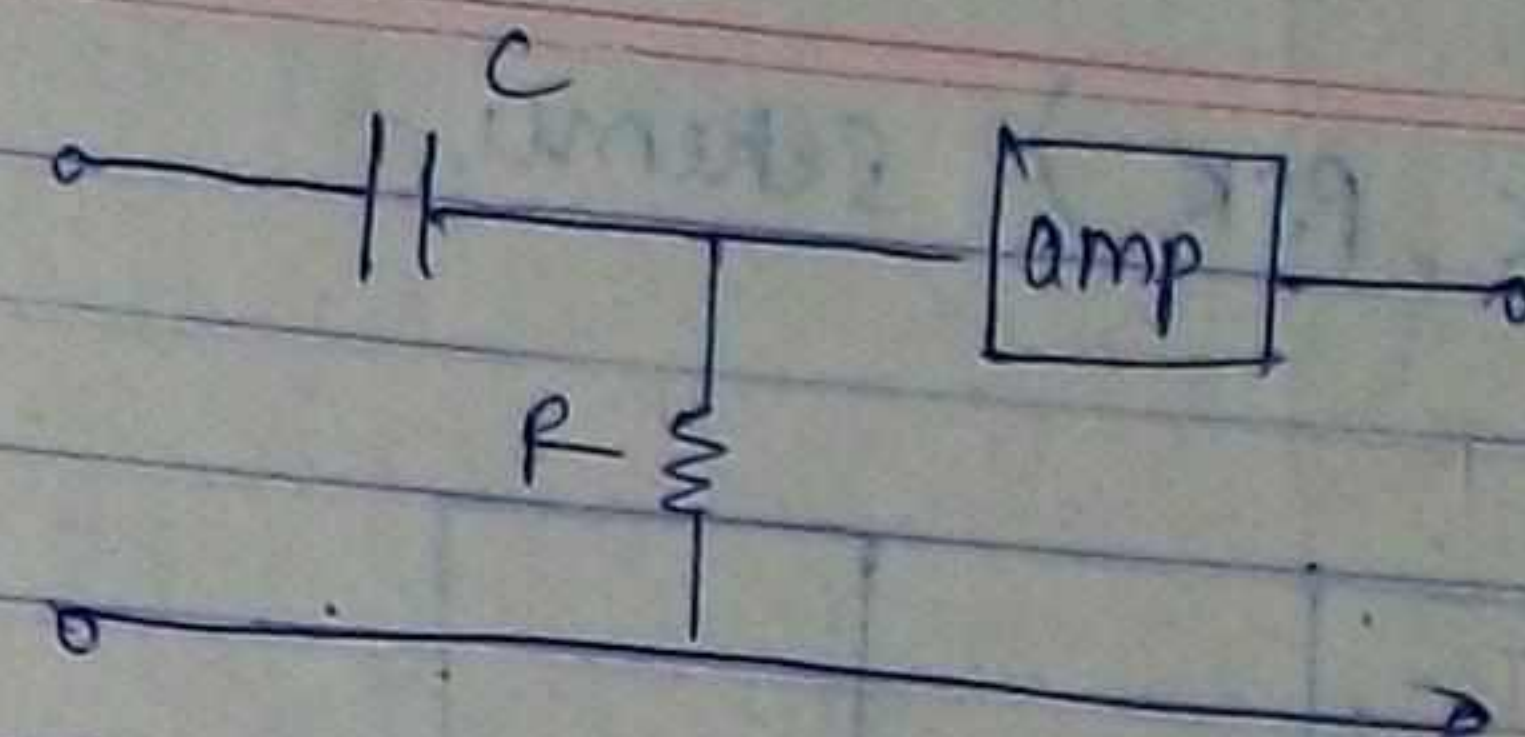
- due to parabolic noise PSD in fm (SNR) can be further improved by using **PRE-EMPHASIS** & **DE-EMPHASIS** in fm.

- **Pre-emphasis** - is a active high pass filter which is used before modulator to amplify high frequency msg signal

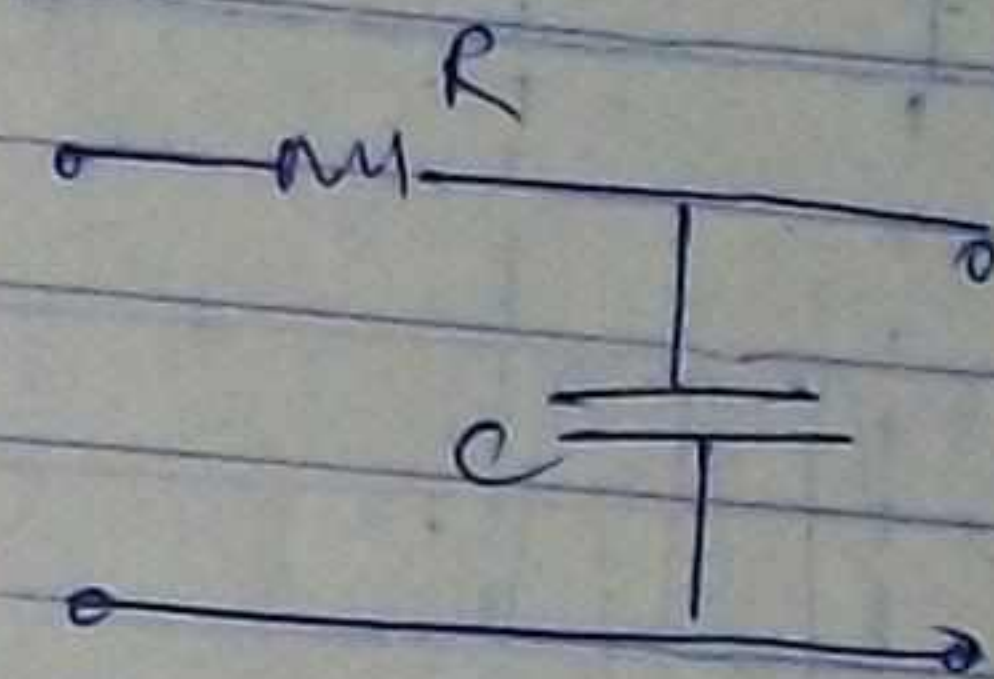


- **De-emphasis** - is used at receiver after demod to compensate pre-emphasis at transmitter. (low pass filter)

Preemphasis -



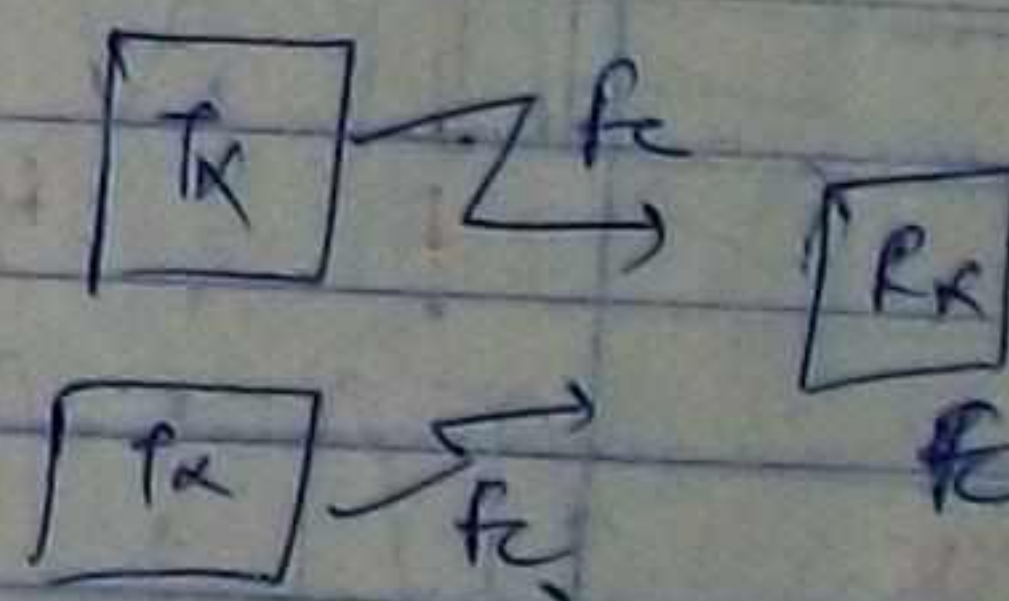
De emphasis



$$T = RC = 75 \mu\text{sec}$$

⇒ Pre emphasis and deemphasis is used in fm system then it will provide the (SNR) improvement of factor of $(22 \approx 13 \text{ dB})$

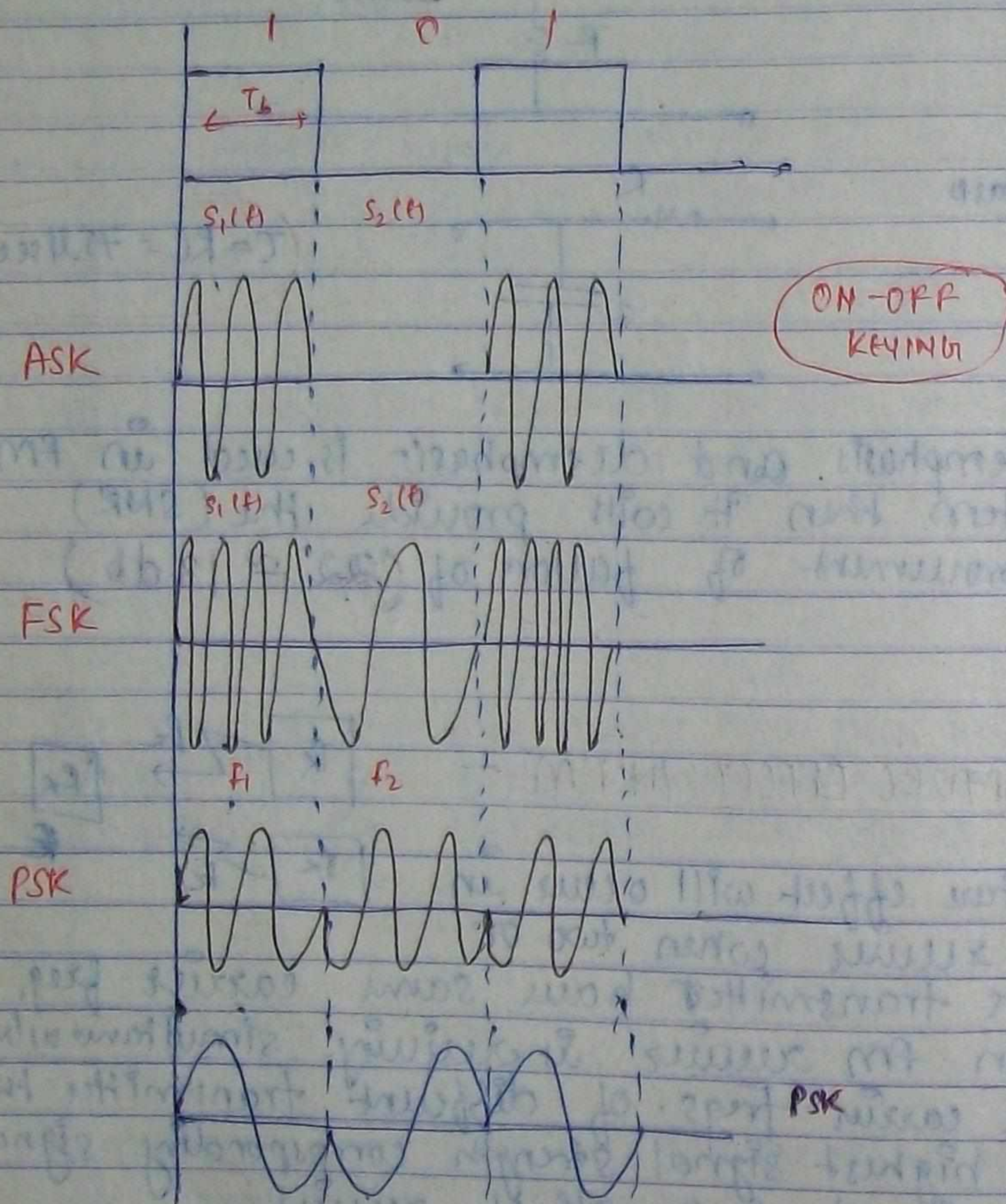
⇒ CAPTURE EFFECT IN FM -



Capture effect will occur in fm receiver when two or more transmitter have same carrier freq. when fm receiver is receiving simultaneously same carrier freqs. of different transmitter then the highest signal strength corresponding signal is received at the o/p of receiver

DIGITAL MODULATION SCHEMES.

⇒ (ASK, FSK, PSK) schemes.



P_e, E_b, γ_b , Band rate, BW, BW efficiency,

Probability of error

$$P_e = Q \left[\sqrt{\frac{E_d}{2\eta}} \right]$$

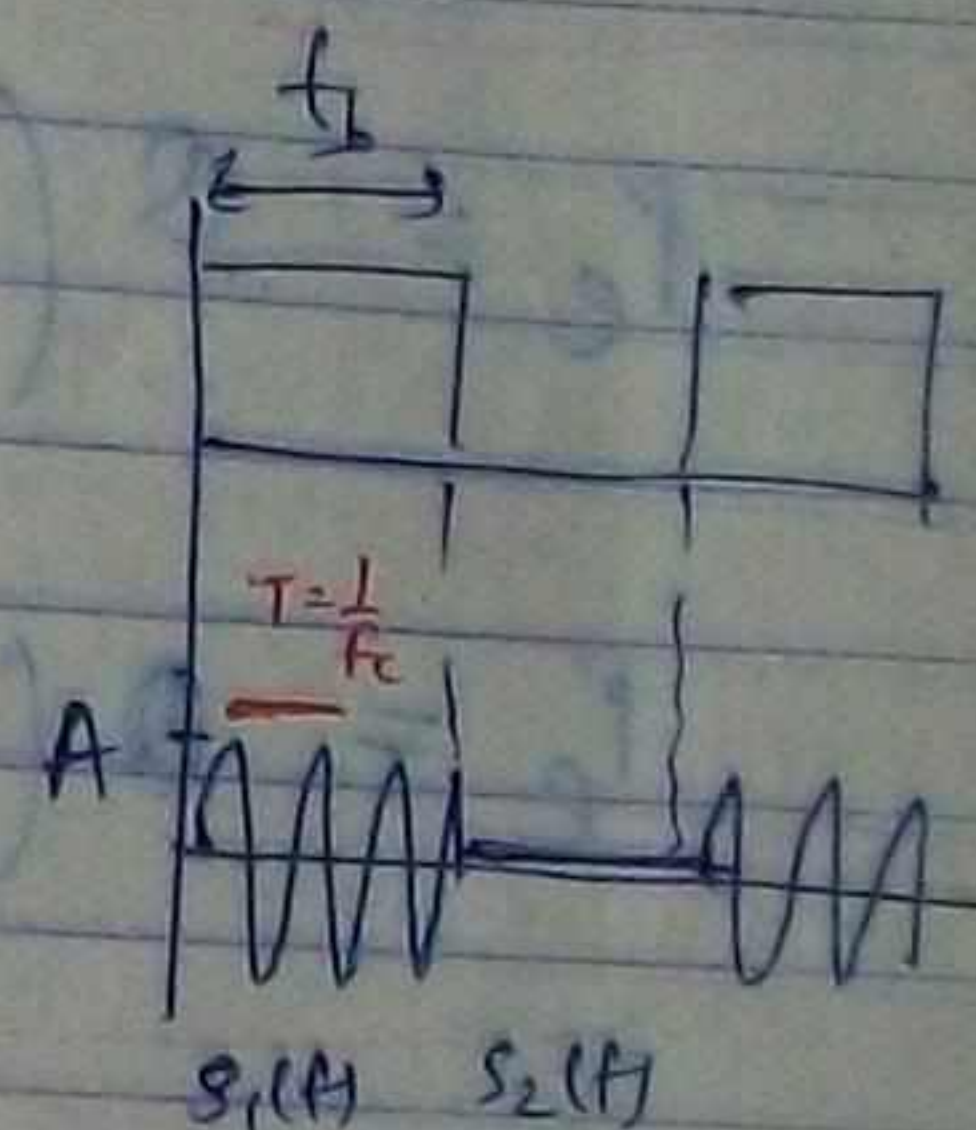
$$s(t) \rightarrow E = \int_0^{t_b} (s(t))^2 \cdot dt \quad (\text{energy})$$

$$E_d = \int_0^{t_b} (s_1(t) - s_2(t))^2 dt$$

⇒ ASK scheme:-

$$s_1(t) = A \cos 2\pi f_c t \quad ; 0 \leq t \leq t_b$$

$$s_2(t) = 0 \quad ; 0 \leq t \leq t_b$$



$$\textcircled{1} P_e = Q \left(\sqrt{\frac{E_d}{2\eta}} \right)$$

$$E_d = \int_0^{t_b} (s_1(t) - s_2(t))^2 dt$$

$$E_d = \int_0^{t_b} s_1^2(t) \cdot dt = \int_0^{t_b} (A^2 \cos^2 2\pi f_c t) dt$$

$$= \frac{A^2}{2} \int_0^{t_b} (1 + \cos 2\pi (2f_c) t) dt$$

$$f_c = \frac{k}{t_b}$$

$$t_b = \frac{k}{f_c}$$

$$E_d = \frac{A^2}{2} \left[t_b + \frac{\sin 2\pi(2f_c)t}{4\pi f_c} \right]_0^{t_b}$$

$$E_b = \frac{A^2 t_b}{2} + \left[\frac{\sin(2\pi 2f_c t_b) - \sin 0}{4\pi f_c} \right]$$

$$\sin(k \times 2\pi) = 0$$

$$E_b = \frac{A^2 t_b}{2}$$

$$P_e = Q \left(\sqrt{\frac{A^2 t_b}{4\eta}} \right)$$

$$E_b = \frac{A^2 t_b}{2}$$

$$P_e = Q \left(\sqrt{\frac{E_b}{4\eta}} \right)$$

$$P_e = \frac{1}{2} \operatorname{erfc} \left(\sqrt{\frac{E_b}{4\eta}} \right)$$

① Bit rate

$$r_b = \frac{1}{T_b}$$

③ Band rate
or
Symbol rate

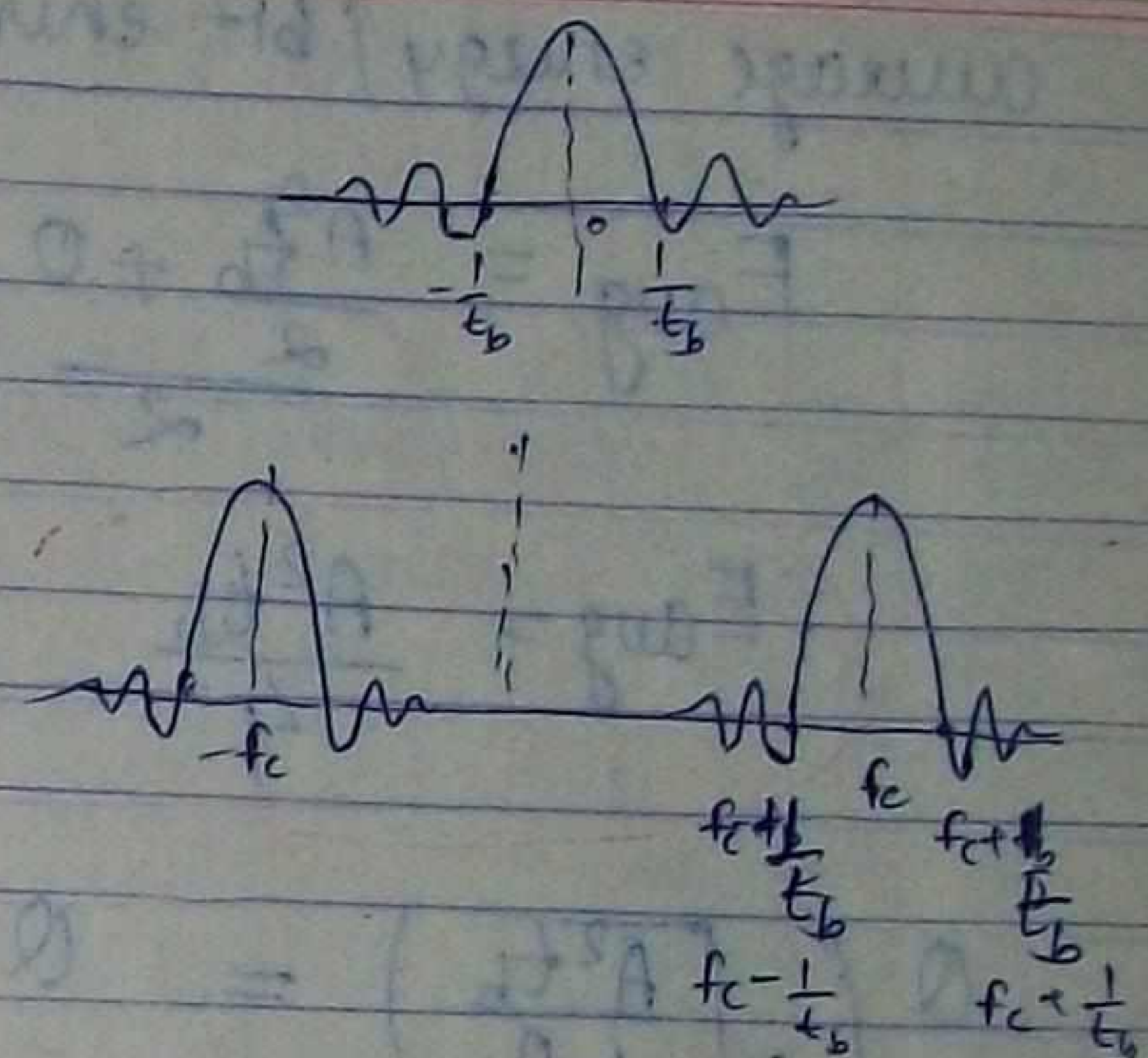
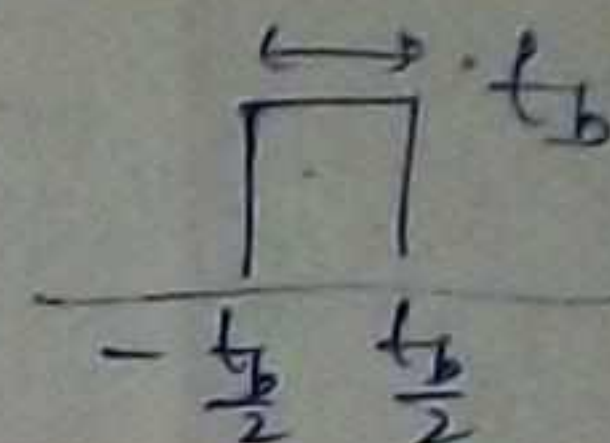
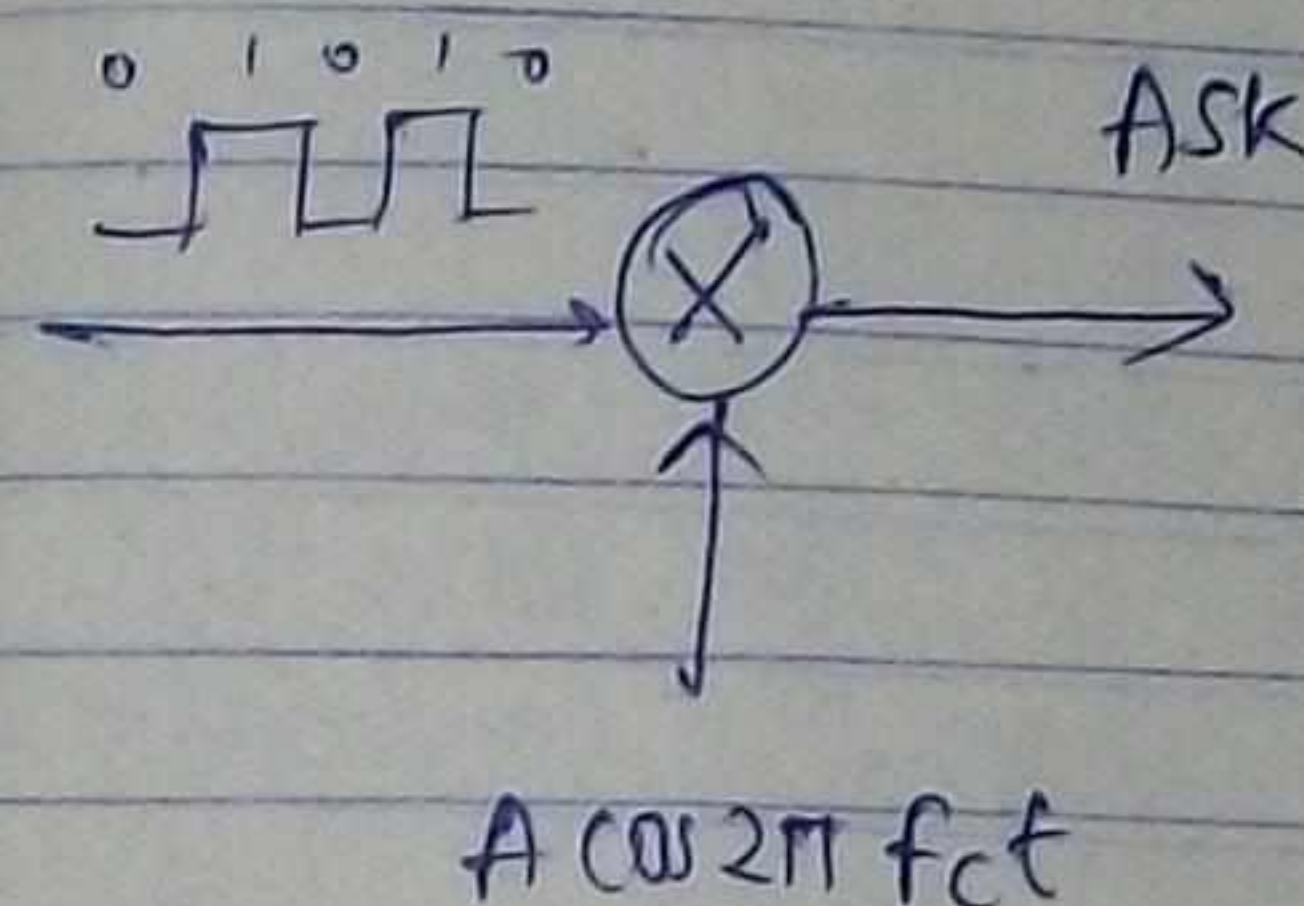
= No. of symbols transmitted
per second

$$= \frac{1}{t_b} \times r_b$$

$$R = r_b H$$

Bit rate Band rate entropy

GENERATION OF ASK -



④ Bandwidth

$$BW = f_c + \frac{1}{t_b} - f_c + \frac{1}{t_b} = \frac{2}{t_b}$$

$$BW = \frac{2}{t_b} = 2r_b$$

⑤ BW efficiency -

$$BW \text{ efficiency} = \frac{r_b}{BW} = \frac{r_b}{2r_b} = \frac{1}{2} = 50\%$$

for non-coherent ASK -

$$P_e = \frac{1}{2} e^{-E_b/2\eta}$$

average energy/bit energy

$$E_{avg} = \frac{A^2 t_b}{2} + 0$$

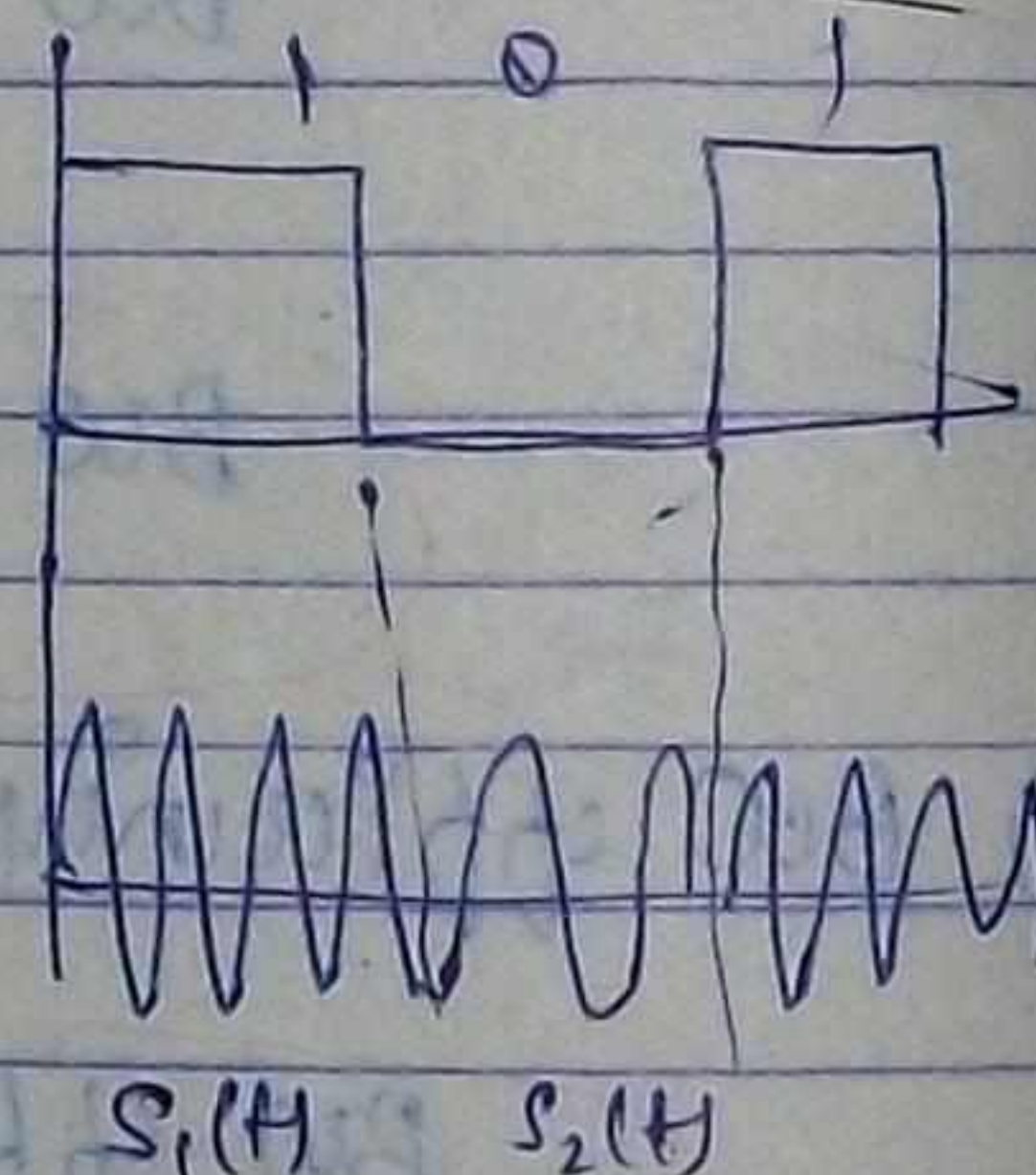
$$E_{avg} = \frac{A^2 t_b}{4}$$

$$Q\left(\sqrt{\frac{A^2 t_b}{4n}}\right) = Q\left(\sqrt{\frac{E_{avg}}{n}}\right)$$

⇒ FSK -

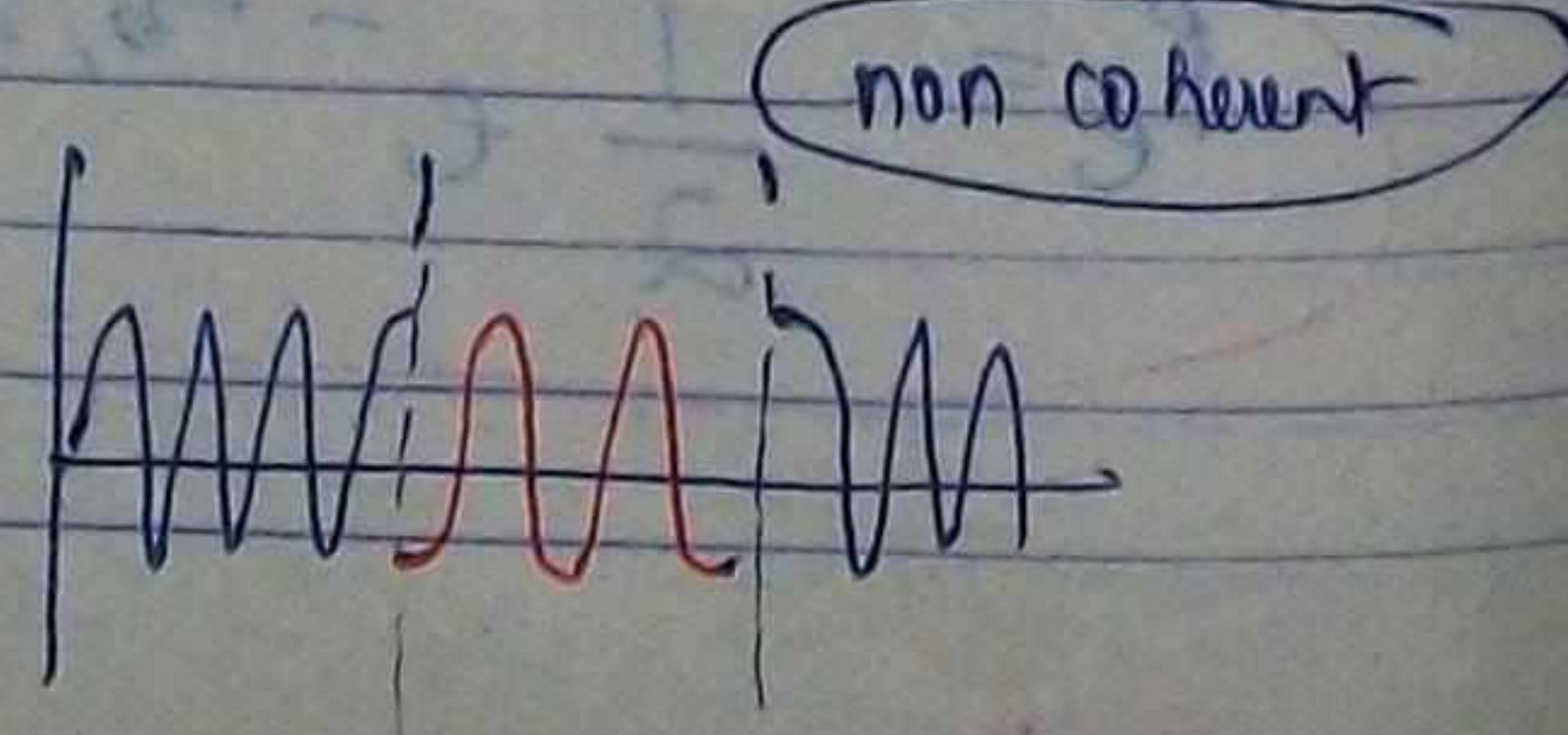
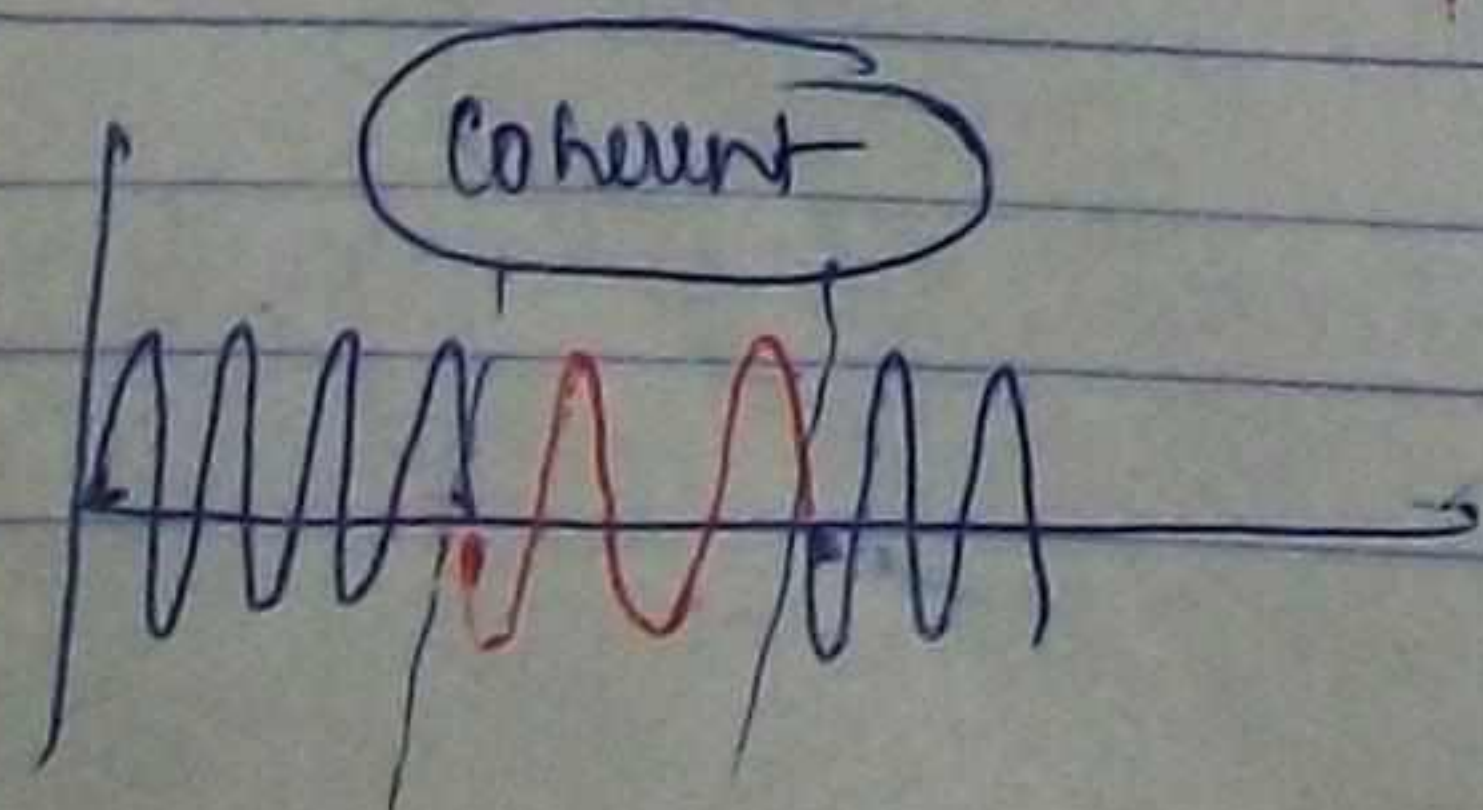
$$s_1(t) = A \cos(2\pi f_1 t + \phi_1); 0 \leq t \leq t_b$$

$$s_2(t) = A \cos(2\pi f_2 t + \phi_2)$$



$\phi_1 = \phi_2 = 0 \rightarrow$ coherent FSK scheme.

$\phi_1 \neq \phi_2 \rightarrow$ non coherent scheme.



∴ for coherent scheme

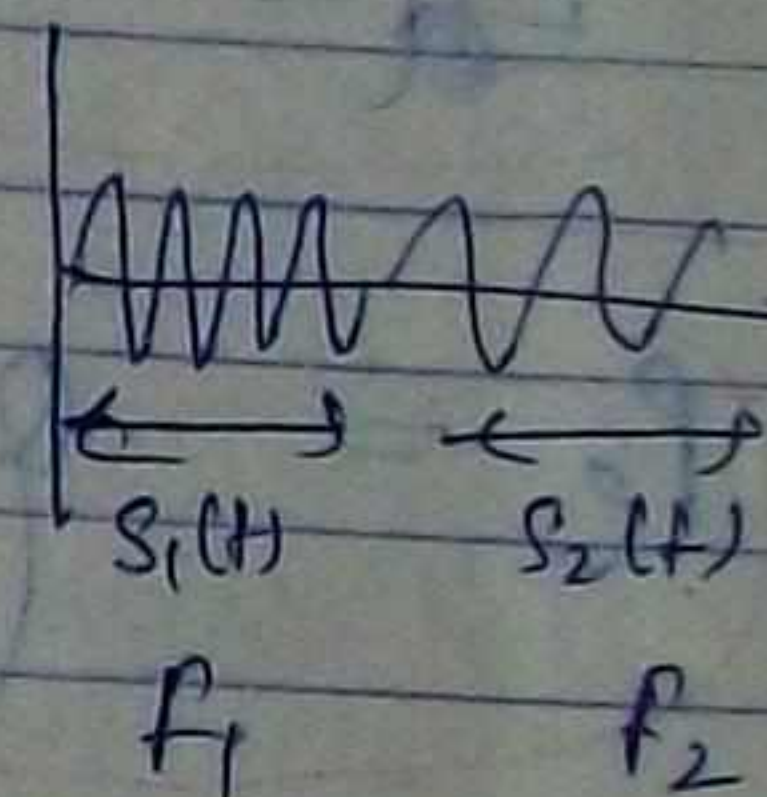
$$\int_0^{t_b} s_1(t) \cdot s_2(t) dt = 0$$

Orthogonal
coherent
FSK

- In coherent FSK to provide orthogonality in bit interval t_b , f_1 and f_2 frequencies must be integer multiple of γ_b

$$f_1 = \frac{k_1}{t_b} = k_1 \gamma_b$$

$$f_2 = \frac{k_2}{t_b} = k_2 \gamma_b$$



$$f_1 - f_2 = \frac{k \cdot \gamma_b}{2}$$

where $k = \text{integer}$

$$f_1 - f_2 = \frac{k \cdot \gamma_b}{2}$$

for coherent FSK scheme

$$s_1(t) = A \cos(2\pi f_1 t)$$

$$s_2(t) = A \cos(2\pi f_2 t)$$

$$P_e = Q\left(\sqrt{\frac{E_d}{2n}}\right)$$

$$E_d = \int_0^{t_b} (s_1(t) - s_2(t))^2 dt$$

$$= \int_0^{t_b} (A \cos(2\pi f_1 t) - A \cos(2\pi f_2 t))^2 dt$$

Orthogonal.

$$= \int_0^{t_b} A^2 \cos^2 2\pi f_1 t + \int_0^{t_b} A^2 \cos^2 2\pi f_2 t - 2 \int_0^{t_b} A \cos 2\pi f_1 t \cdot \cos 2\pi f_2 t$$

$$= \frac{A^2 t_b}{2} + \frac{A^2 t_b}{2} = A^2 t_b$$

$$E_b = A^2 t_b$$

$$P_e = Q \left(\sqrt{\frac{A^2 t_b}{2\eta}} \right)$$

$$E_{avg} = \frac{A^2 t_b}{2} + \frac{A^2 t_b}{2} = A^2 t_b$$

bit energy $E_b = E_{avg} = \frac{A^2 t_b}{2}$

$$P_e = Q \left(\sqrt{\frac{A^2 t_b}{2\eta}} \right) = Q \left(\sqrt{\frac{E_b}{\eta}} \right)$$

$$P_e = \frac{1}{2} \operatorname{erfc} \left(\sqrt{\frac{E_b}{2\eta}} \right)$$

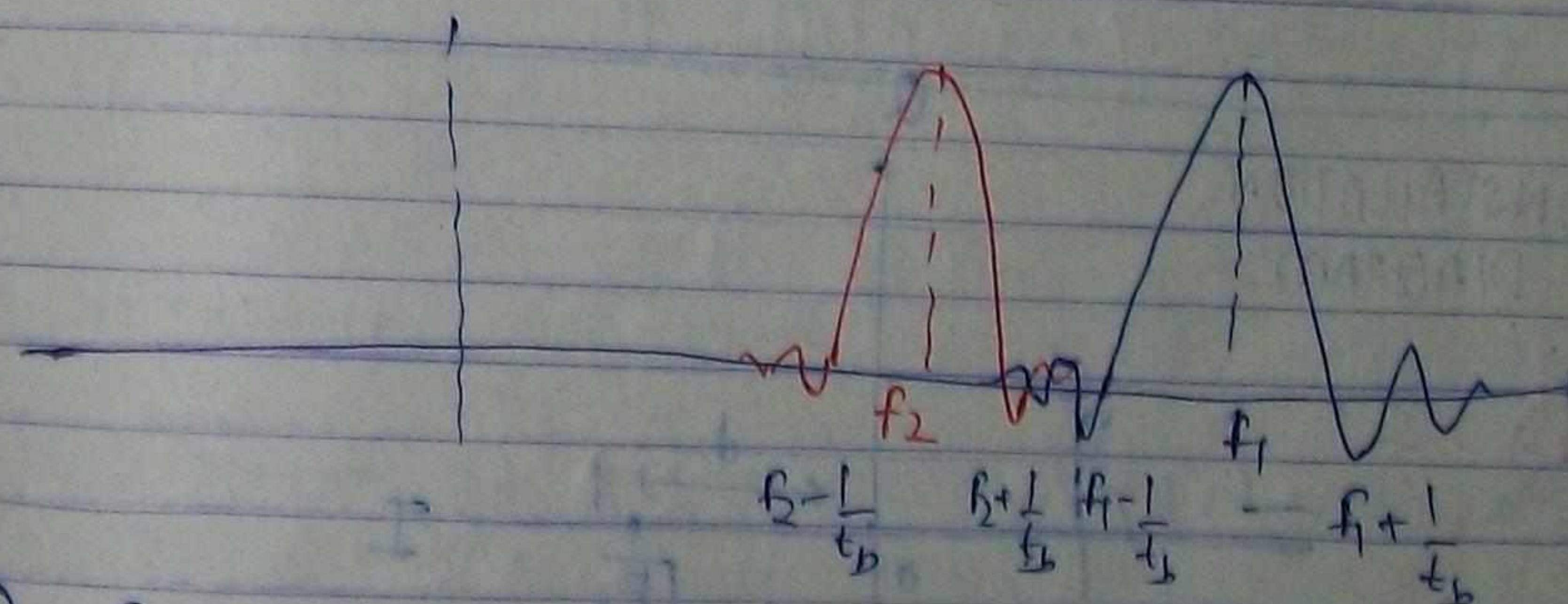
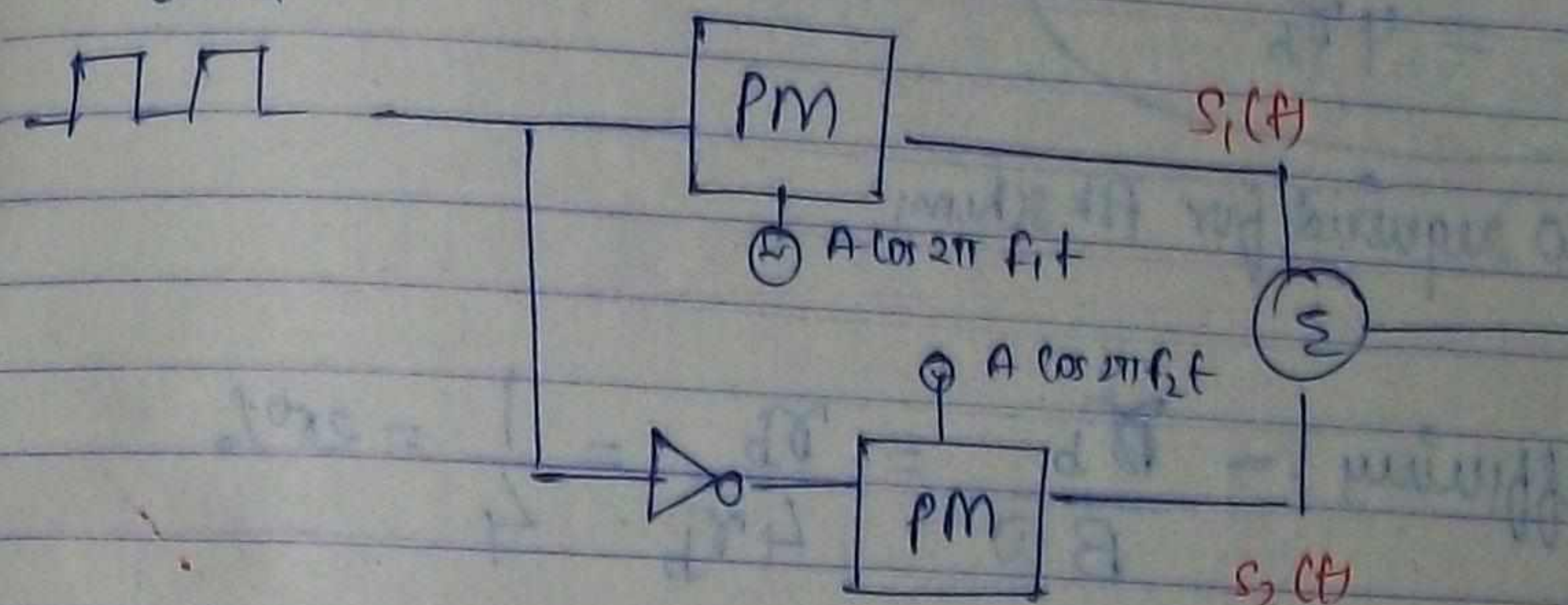
Bit rate -

$$r_b = \frac{1}{t_b}$$

$$\text{Baud rate} = \frac{1}{t_b} = r_b$$

Generate

Binary data



$$BW = f_1 + \frac{1}{t_b} - f_2 + \frac{1}{t_b}$$

$$BW = f_1 - f_2 + \frac{2}{t_b}$$

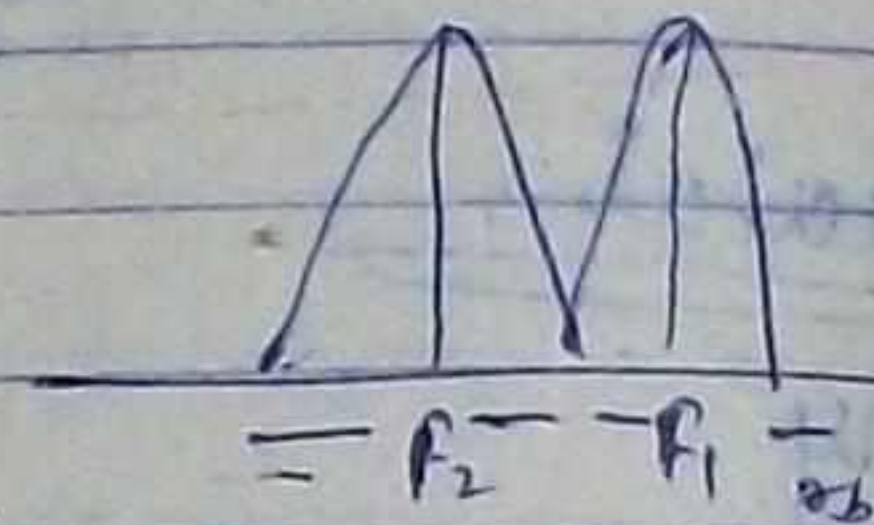
$$BW = f_1 - f_2 + 2r_b = 2\Delta F + 2r_b =$$

$\Delta f = \text{deviation}$

$$BW = 2(\Delta f + \gamma_b)$$

$$BW_{min} = 2\gamma_b + 2\gamma_b = 4\gamma_b$$

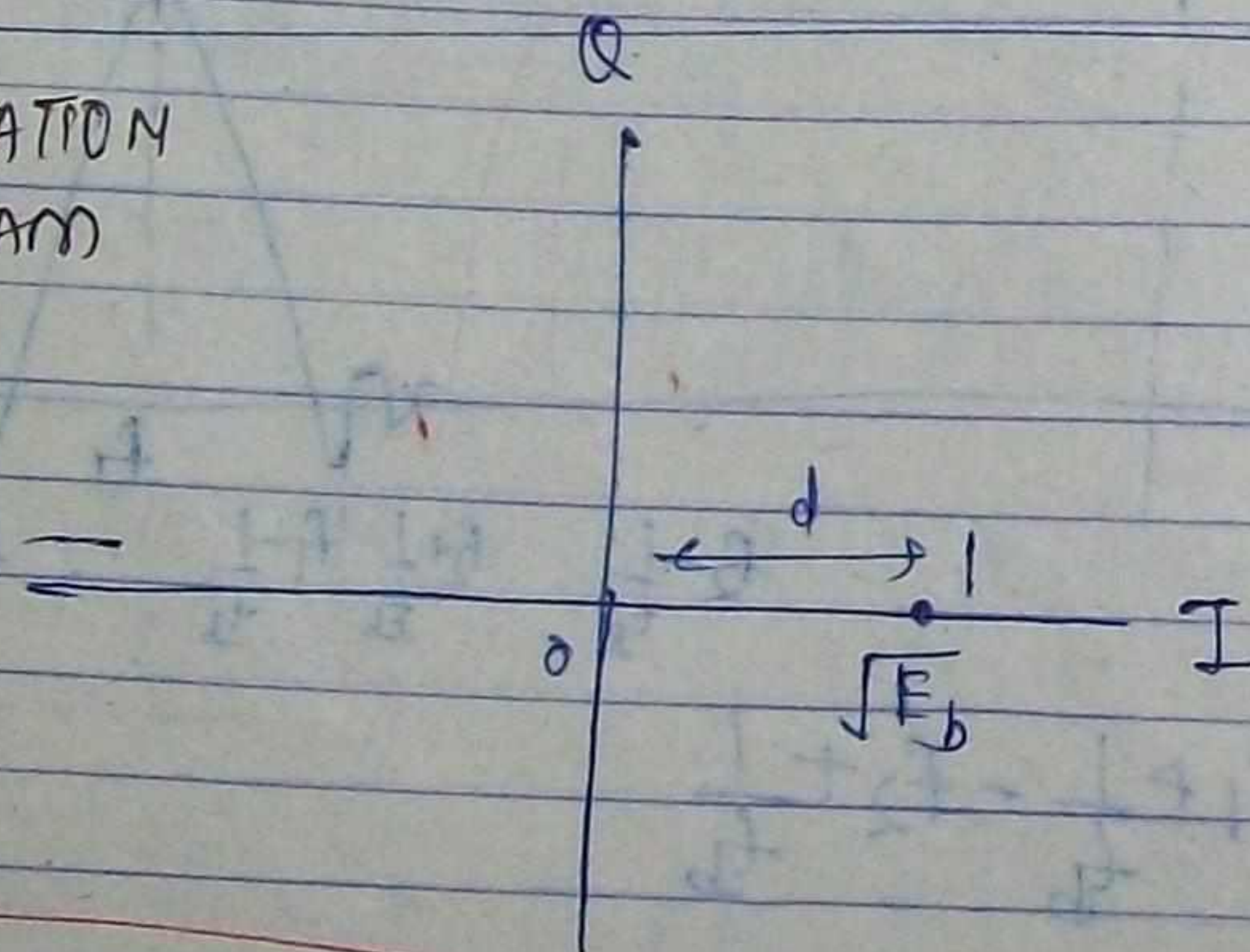
min. BW required for FSK scheme



$$\text{BW efficiency} = \frac{\gamma_b}{BW} = \frac{\gamma_b}{4\gamma_b} = \frac{1}{4} = 25\%$$

CONSTELLATION DIAGRAM

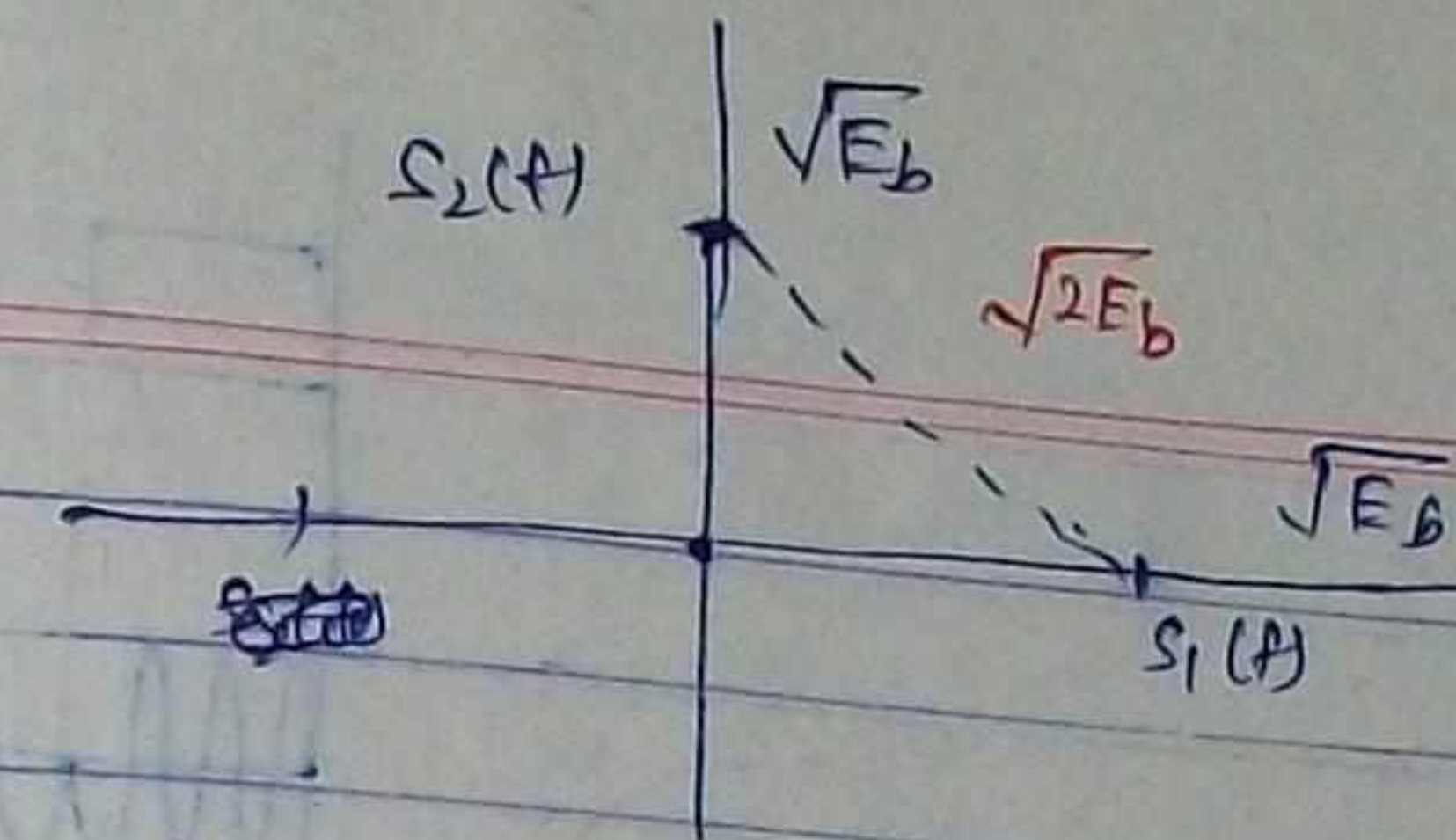
ASK



$$P_e = Q\left(\sqrt{\frac{d^2}{2\eta}}\right)$$

$$d \uparrow P_e \downarrow$$

FSK



$$P_e = Q\left(\sqrt{\frac{E_d}{2\eta}}\right) = Q\left(\sqrt{\frac{2E_b}{2\eta}}\right)$$

$$P_e = Q\left(\sqrt{\frac{E_b}{\eta}}\right)$$

$\Rightarrow$ NON-COHERENT FSK -

$$P_e = \frac{1}{2} e^{-E_b/2\eta}$$

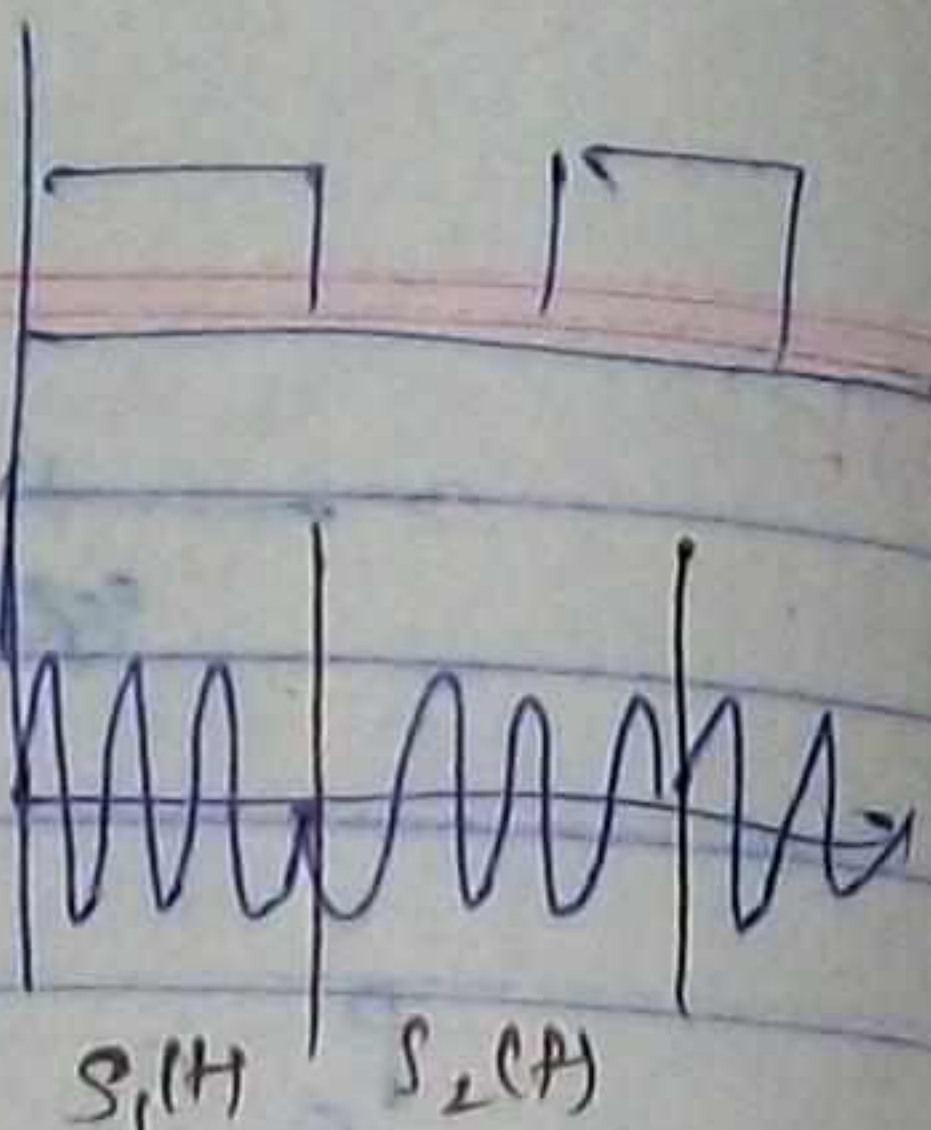
- In coherent FSK scheme if $F_1 - F_2 = \frac{\gamma_b}{2}$ then that scheme is called MSK (mini shift keying) or (CPFSK), (continuous phase-FSK).

⇒ BPSK -

$$s_1(t) = A \cos 2\pi f_c t ; 0 \leq t \leq t_b$$

$$s_2(t) = A \cos(2\pi f_c t + 180^\circ) ; 0 \leq t \leq t_b$$

$$= -A \cos 2\pi f_c t$$



$$\textcircled{1} P_e = Q\left(\sqrt{\frac{E_d}{2\eta}}\right)$$

$$E_d = \int_0^{t_b} (s_1(t) - s_2(t))^2 dt$$

$$= \int_0^{t_b} (2 \cos 2\pi f_c t)^2 dt$$

$$= 4 \cdot \frac{A^2 t_b}{2} = 2 A^2 t_b$$

$$E_d = 2 A^2 t_b$$

$$P_e = Q\left(\sqrt{\frac{A^2 t_b}{\eta}}\right)$$

$$E_b = E_1 = E_2 = \frac{A^2 t_b}{2}$$

$$P_e = Q\left(\sqrt{\frac{2 E_b}{\eta}}\right) = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{\eta}}\right)$$

SUMMARY -

$$P_{e \text{ FSK}} = Q\left(\sqrt{\frac{E_b}{2\eta}}\right)$$

$$P_{e \text{ ASK}} = Q\left(\sqrt{\frac{E_b}{\eta}}\right)$$

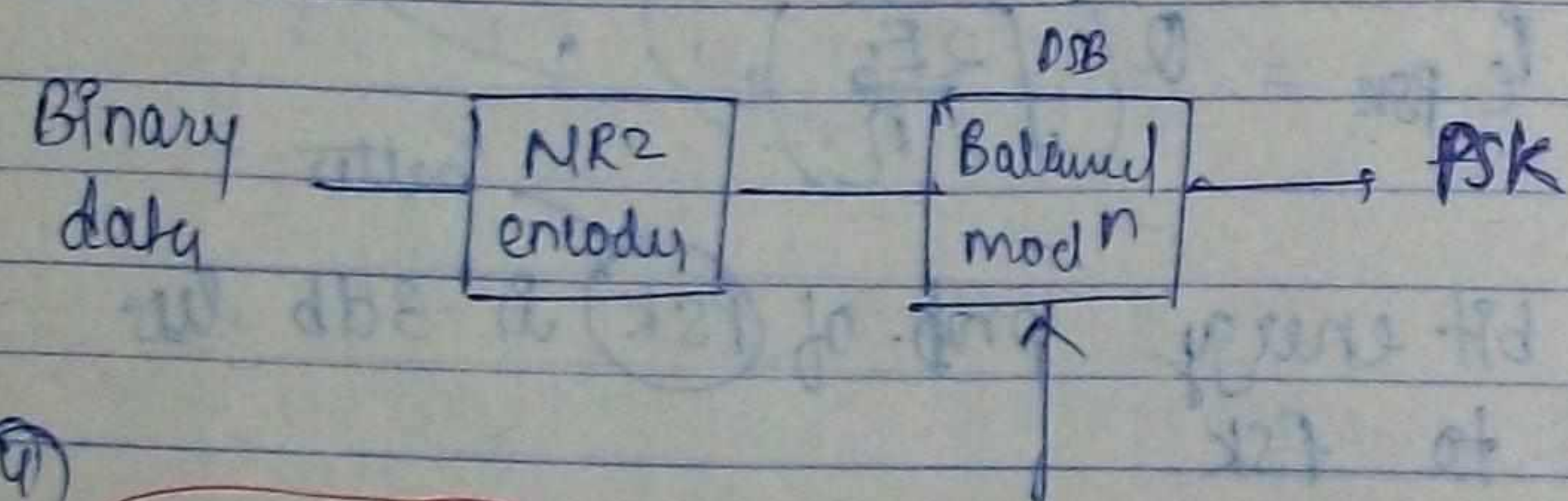
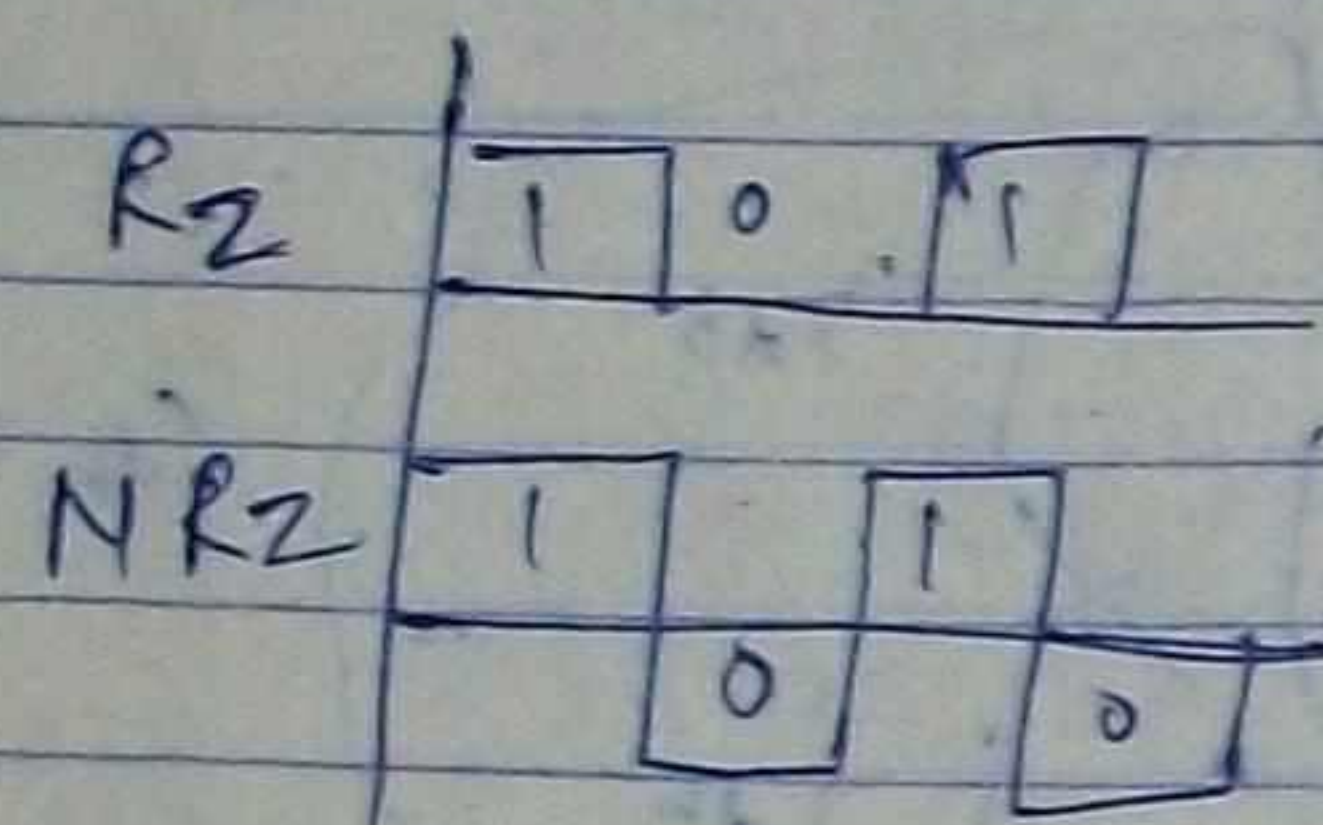
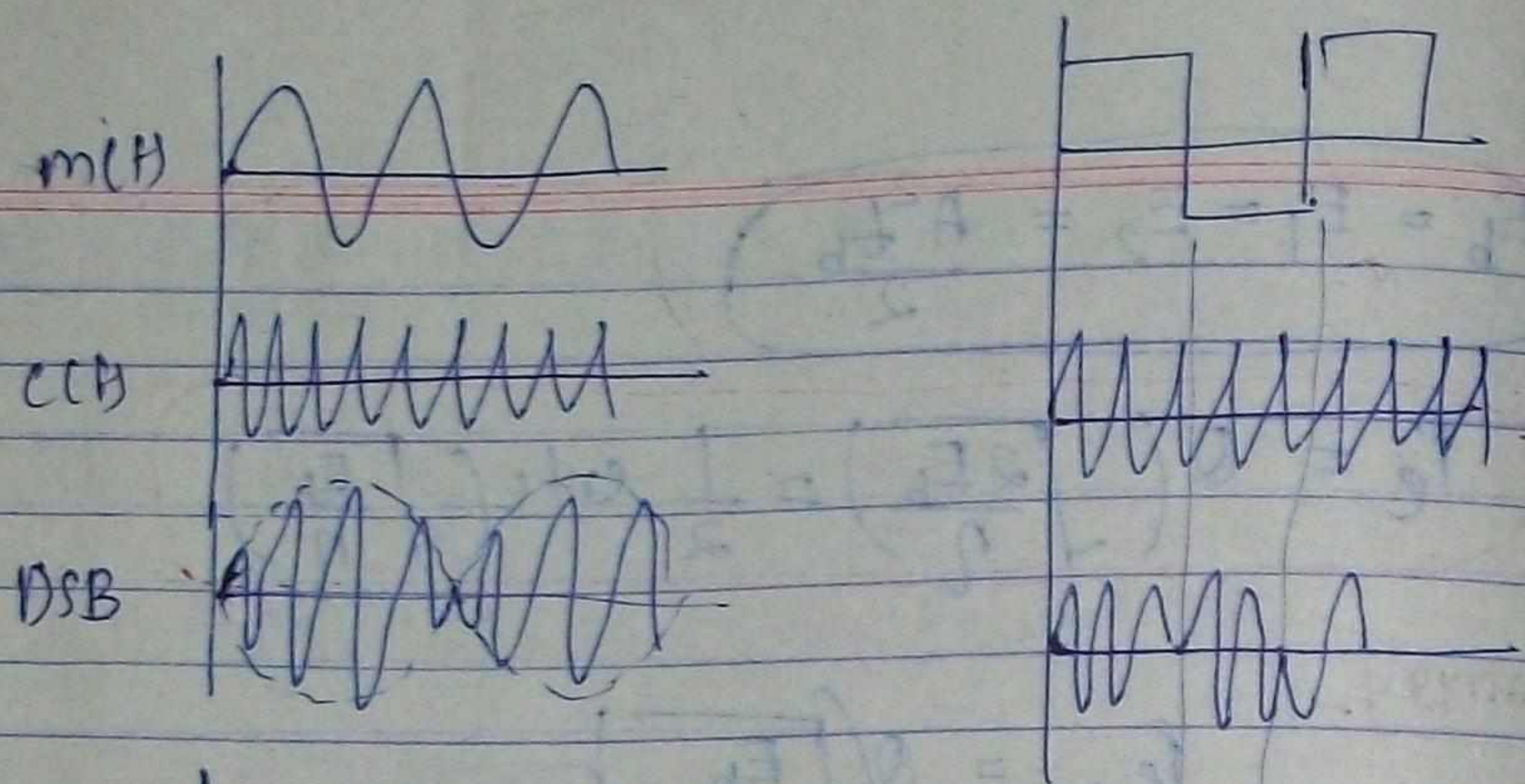
$$P_{e \text{ PSK}} = Q\left(\sqrt{\frac{2 E_b}{\eta}}\right)$$

- for same bit energy prob. of PSK is 3db less compared to FSK

$$\textcircled{2} \text{ Bit rate } = r_b = \frac{1}{t_b}$$

$$\textcircled{3} \text{ Baud rate } = \frac{1}{t_b} = r_b$$

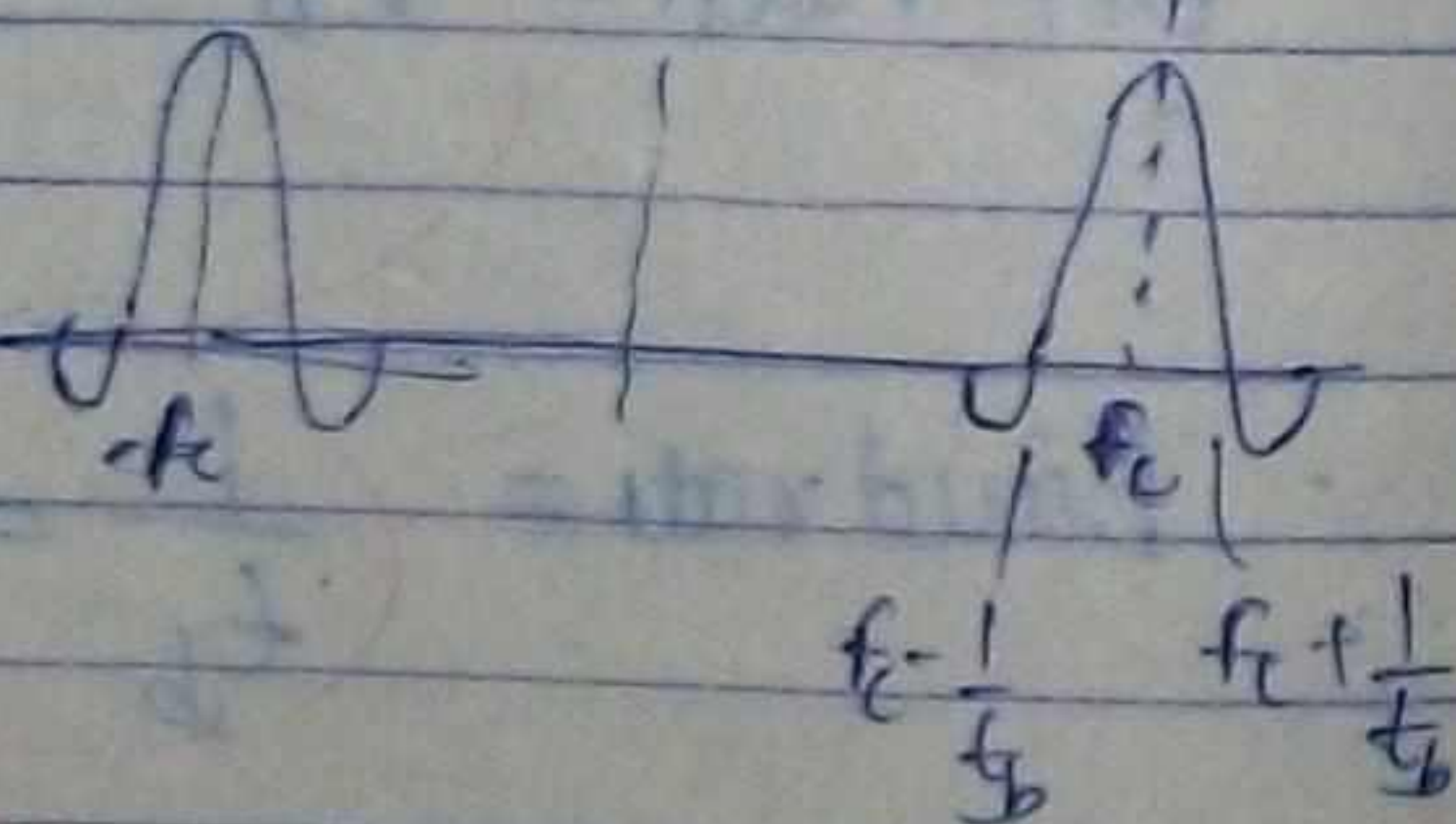
Generation of PSK



④ $BW = 2\gamma_b$

⑤ BW efficiency
 $= \frac{\gamma_b}{2\gamma_b} = \frac{1}{2} = 50\%$

Also see



- In PSK demodulation, coherent detection is used. During demodulation if any phase error θ exists then SNR will decrease by $\cos^2 \theta$.

	<u>ASK</u>	<u>FSK</u>	<u>PSK</u>
P_e	$Q\left(\sqrt{\frac{A^2 t_b}{4\eta}}\right)$	$Q\left(\sqrt{\frac{A^2 t_b}{2\eta}}\right)$	$Q\left(\sqrt{\frac{A^2 t_b}{\eta}}\right)$
γ_b	$1/t_b$	$1/t_b$	$1/t_b$
Baud rate	γ_b	γ_b	γ_b
BW	$2\gamma_b$	$f_1 - f_2 + 2\gamma_b$	$2\gamma_b$
Constellation			

area of constellation = energy

⇒ M-ary PSK -

M - different symbols are points

$m = 2 \rightarrow$ BPSK

$m = 4 \rightarrow$ QPSK

$m = 8 \rightarrow$ 8 PSK

$$\text{phase shift b/w symbols} = \frac{2\pi}{M}$$

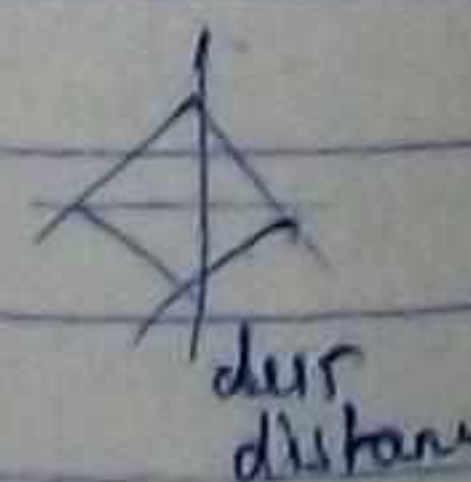
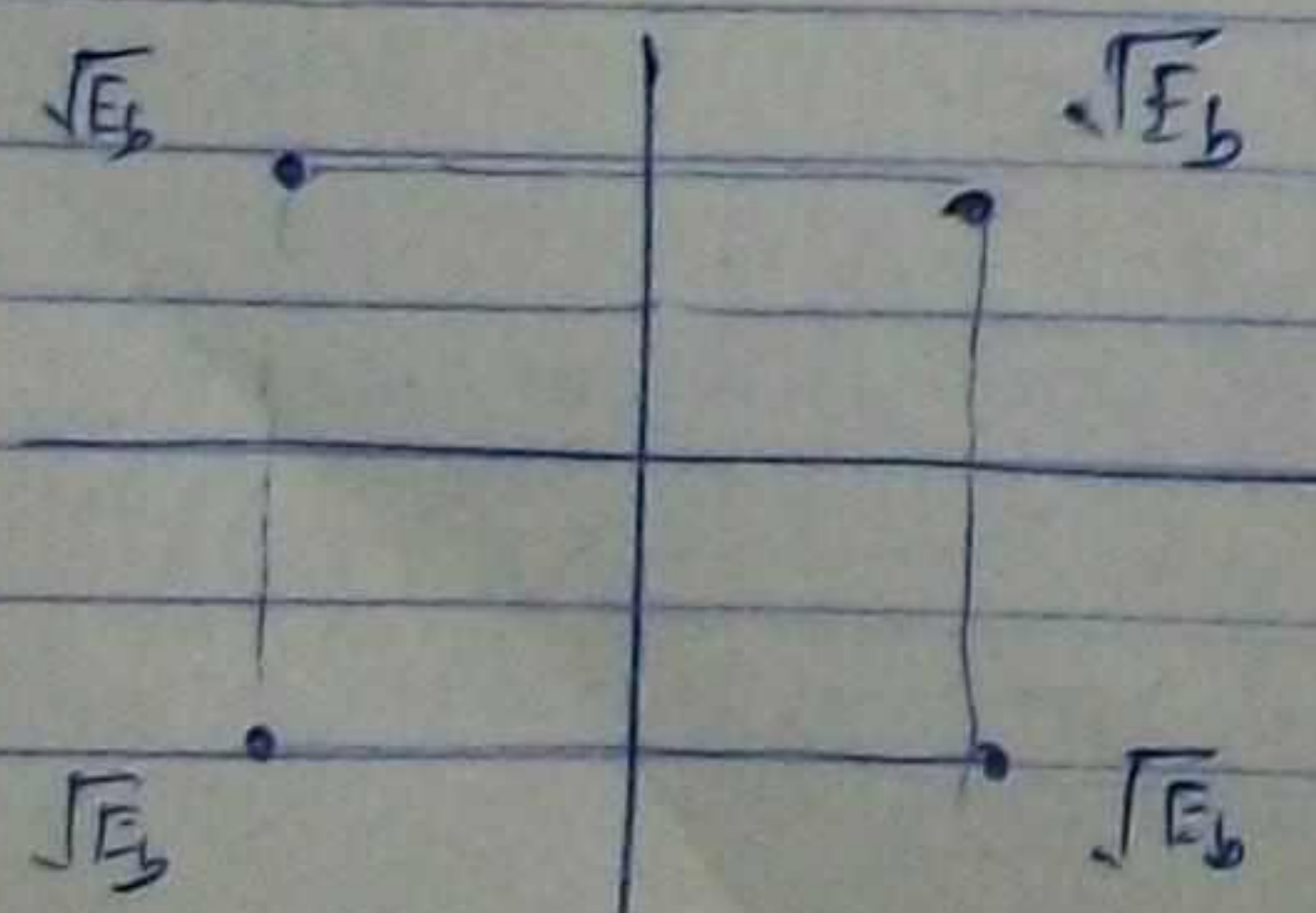
QPSK

10 ← $S_1(t) = A \cos(2\pi f_c t + \pi/4)$

00 ← $S_2(t) = A \cos(2\pi f_c t + \frac{3\pi}{4})$

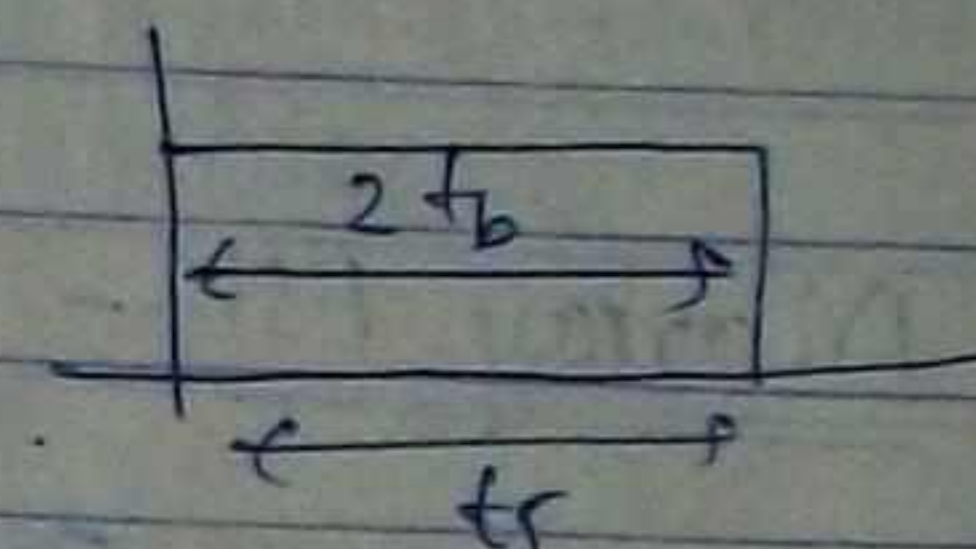
01 ← $S_3(t) = A \cos(2\pi f_c t + \frac{5\pi}{4})$

11 ← $S_4(t) = A \cos(2\pi f_c t + \frac{7\pi}{4})$



- QPSK has same prob. of error compared to BPSK

$$\text{Baud rate} = \text{Symbol rate} = \frac{1}{T_s} = \frac{1}{2T_b} = \frac{r_b}{2}$$



$$\text{QPSK BW} = \frac{2r_b}{2} = r_b$$

$$\text{BW}\eta\% = \frac{r_b}{r_b} = 1 = 100\%$$

Generalized -

$m \uparrow \quad P_e \uparrow$

for M-ary PSK -

$$\text{Baud rate} = \frac{r_b}{\log_2 M} = \frac{r_b}{n}$$

$n = \text{no. of bits transmitted}$

$$n = \log_2 M$$

$$\text{BW} = \frac{2r_b}{\log_2 M} = \frac{2r_b}{n}$$

- In M-ary PSK as 'M' value increased BW and Baud rate will decrease while as P_e will increase

$\Rightarrow$ QAM - Combⁿ of (PSK and ASK)

$\Rightarrow$ M-ary FSK -

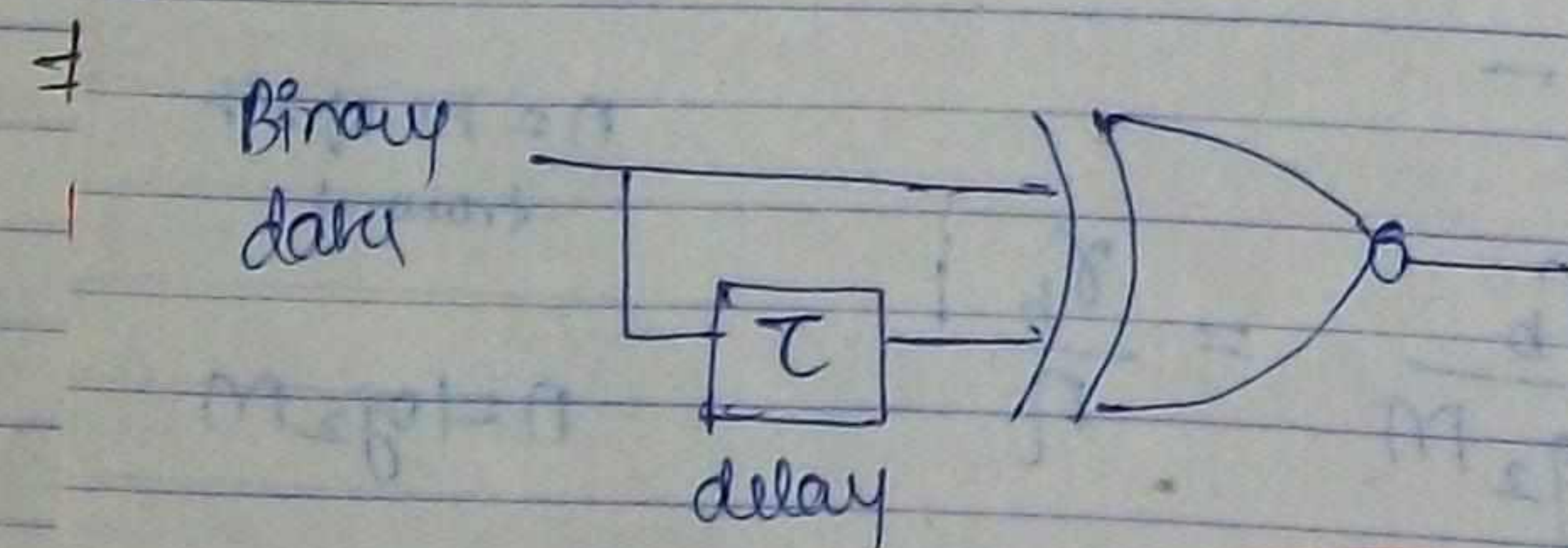
- In M-ary FSK as 'M' value increases BW will increase whereas P_e will decrease.

$$BW = 2 \frac{n+1}{n} \tau_b$$

$n = \text{no of bits}$

$$n = \log_2 M$$

$\Rightarrow$ DPSK -



$$P_e = \frac{1}{2} e^{-E_b/n}$$

Channel Capacity

$$C = 332 \cdot B \log_{10} \left(1 + \left(\frac{S}{N} \right) \right)$$

BASICS OF FDMA, TDMA, CDMA, GSM

- Multiple user techniques are used to share common resources such as satellite, cellular towers.

$\Rightarrow$ FDMA -

- In FDMA scheme users are separate in freq. domain. In this available BW is divided into small subband and assigned to each user.

Let 'B' is the total BW available which is required to allocate 'K' no. of users, then BW of each user is

$$B_k = \frac{B}{K}$$

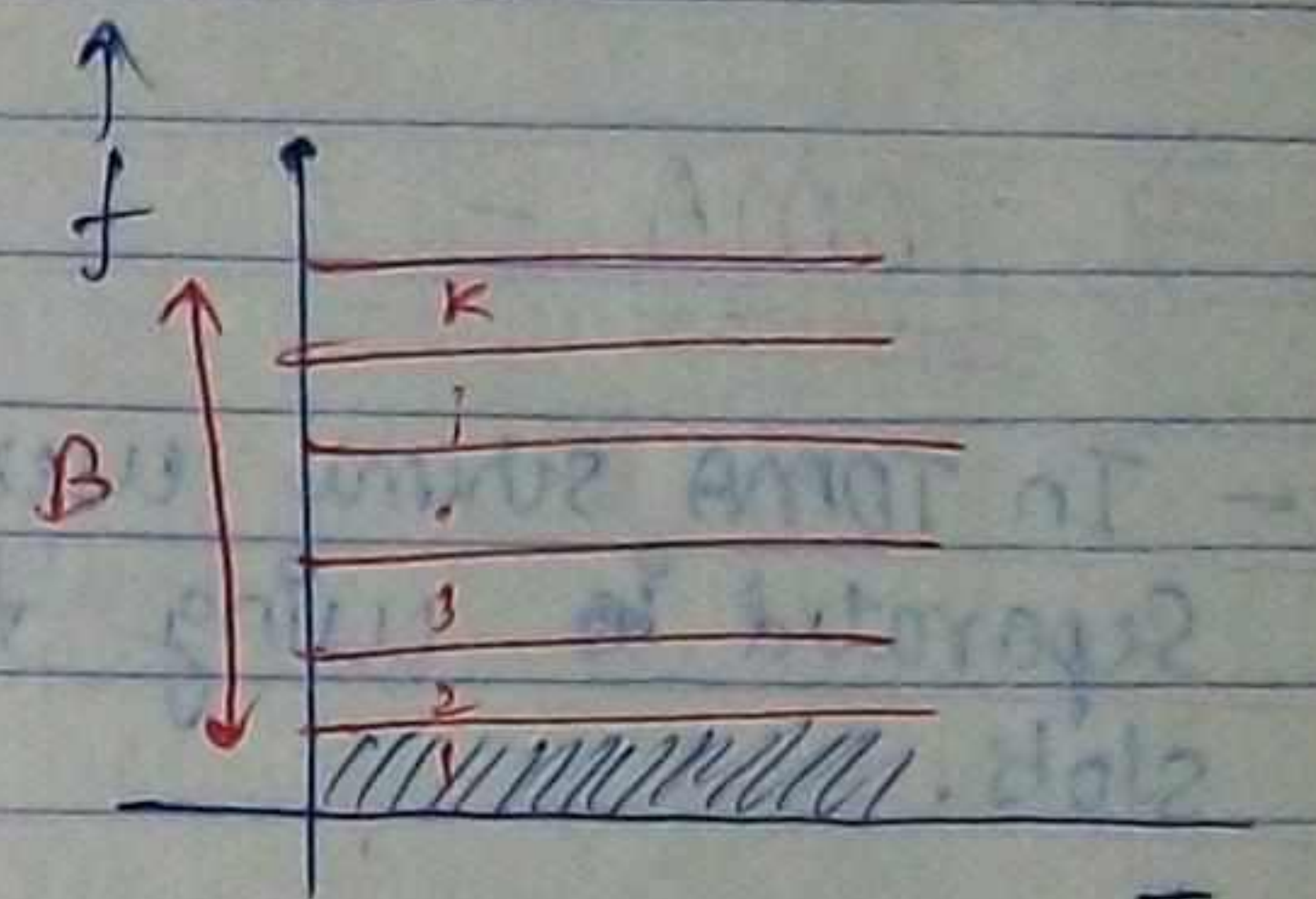
- channel capacity of one user in FDMA scheme is given by

$$C_k = B_k \log_2 \left(1 + \frac{S}{\eta B_k} \right)$$

$$C_k = \frac{B}{K} \log_2 \left(1 + \frac{S \cdot K}{\eta \cdot B} \right)$$

- Total channel Capacity

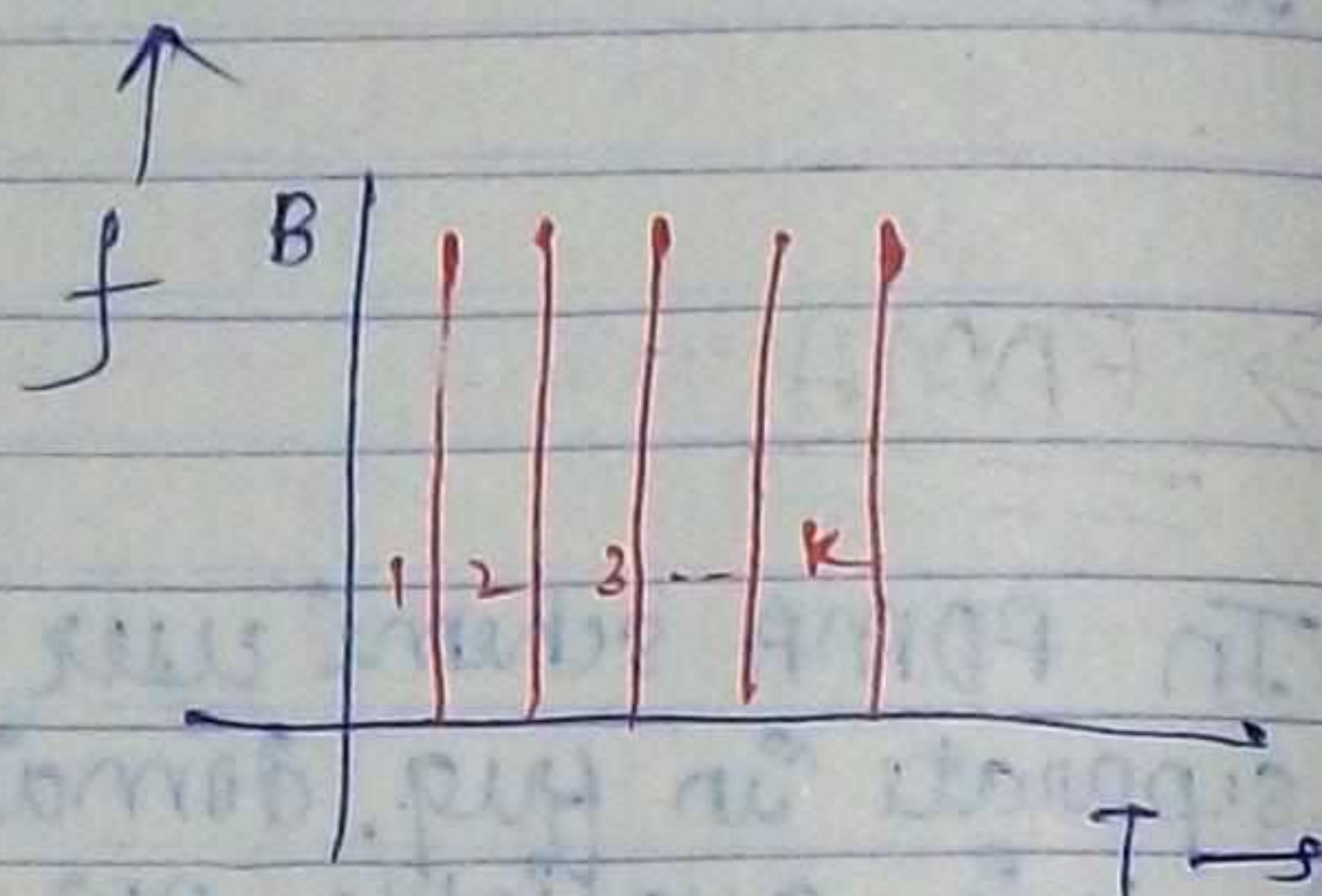
$$C = K C_k = B \log_2 \left(1 + \frac{S \cdot K}{\eta \cdot B} \right)$$



- In FDMA scheme when all users are continuously use the corresponding band then only efficiency will increase.

⇒ TDMA -

- In TDMA scheme users are separated using time slots.
- Here in the given time slot the corresponding user can use the entire available band.



$$C_k = \frac{1}{K} B \log_2 \left(1 + \frac{K S}{\eta B} \right)$$

Total capacity

$$C = B \log_2 \left(1 + \frac{K S}{\eta B} \right)$$

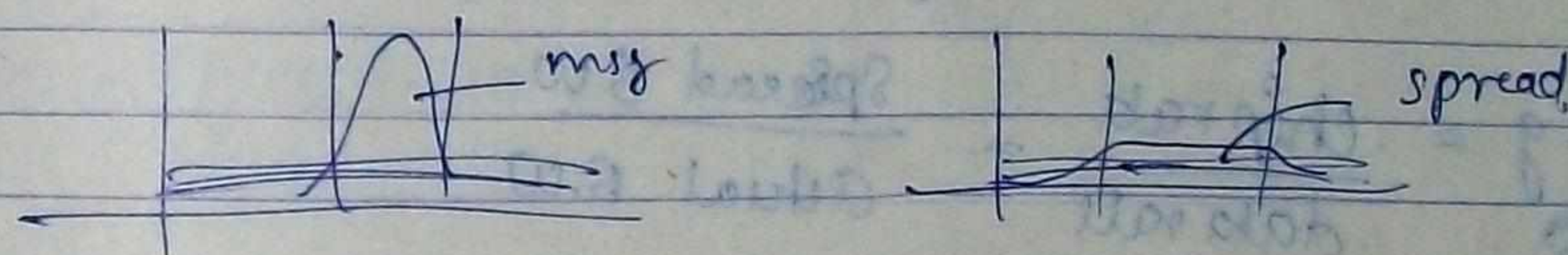
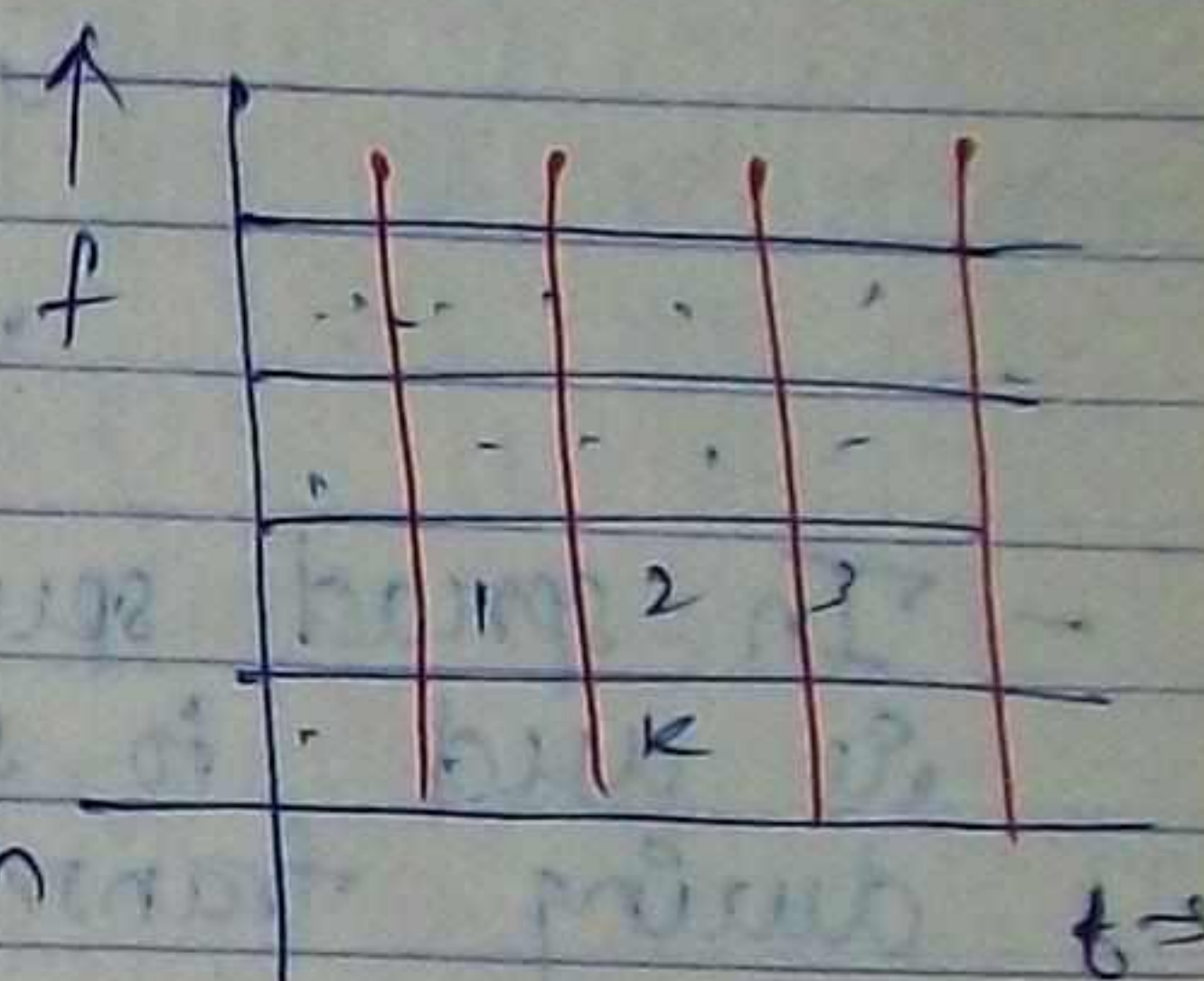
T_f = total frame slot

T_s = synchronization time

$$\begin{aligned} \% \eta &= \frac{T_f - T_s}{T_f} \times 100 \\ &= 1 - \frac{T_s}{T_f} \times 100 \end{aligned}$$

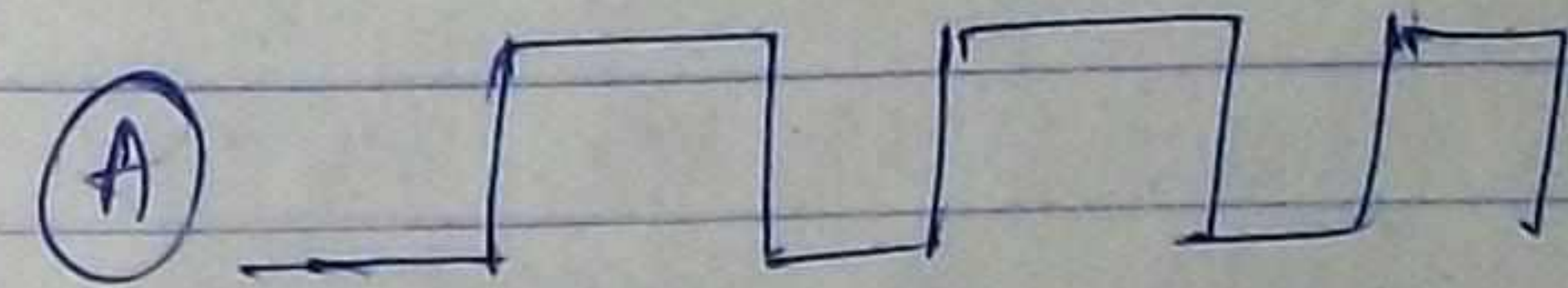
⇒ CDMA - (code division multiple access)

- In CDMA scheme users are separated by assigning each user with separate code different code.
- In CDMA scheme spread spectrum technique is used.

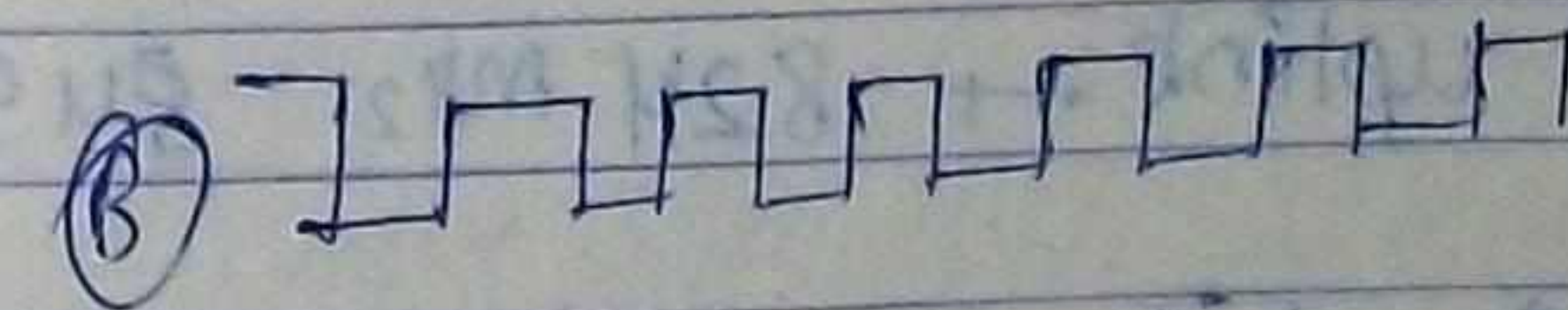


pseudo-random code

Data

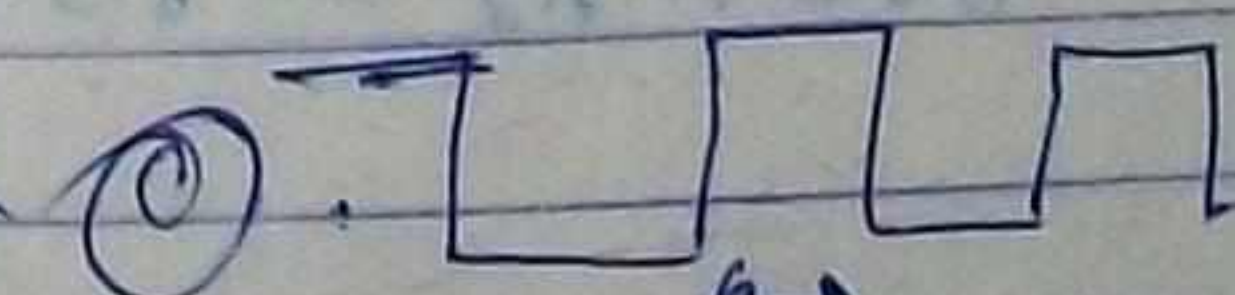


PN code



EXOR

Data



$$C = A \oplus B$$

t_c

$$\text{Chip rate} = \frac{1}{t_c}$$

$$\text{Processing gain} = \frac{\text{Chip rate}}{\text{data rate}} = \frac{\text{Spread BW}}{\text{actual data BW}}$$

at receiver $Y = C \oplus B$

$Y = A \oplus B \oplus B$

$Y = A$

- In spread spectrum direct sequence PN code is used to spread the actual data during transmission, the performance of CDMA scheme is given by processing gain

P.g = $\frac{\text{chip rate}}{\text{data rate}} = \frac{\text{Spread B.W}}{\text{Actual B.W}}$

or

spread factor ≥ 10

- In CDMA

uplink $\rightarrow 824 \text{ MHz} - 849 \text{ MHz}$

Downlink $\rightarrow 869 \text{ MHz} - 894 \text{ MHz}$

Channel capacity

$C = B \log_2 \left(1 + \frac{S}{\eta B + (K-1)S} \right)$

900 - 1800

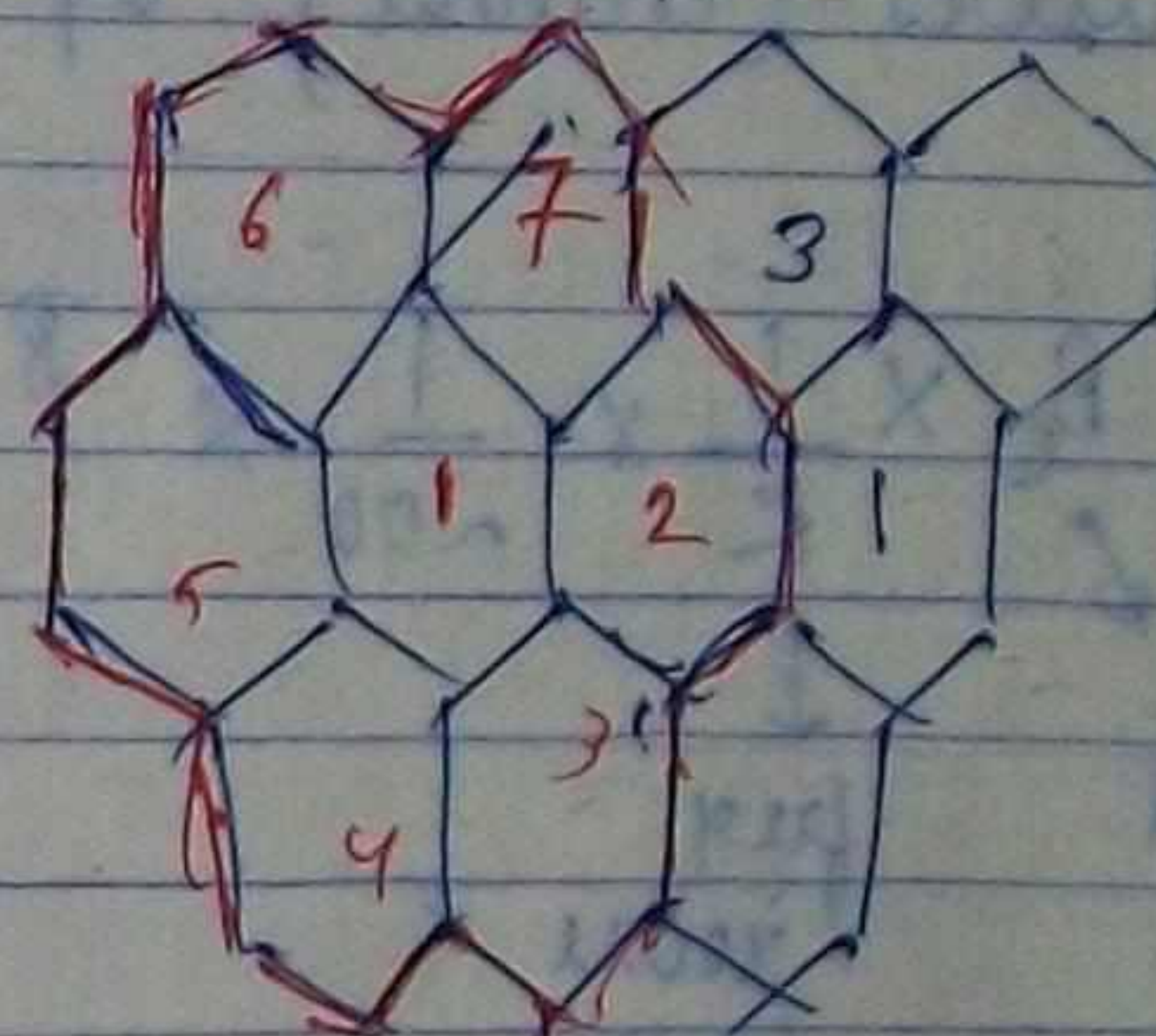
$\Rightarrow$ GSM - (Global system for mobile comm)

- GSM is basically standard for cellular operation.

- Cellular concept is based on freq. reuse

Minimum cluster size -

$K = 3, 4, 7, 12$



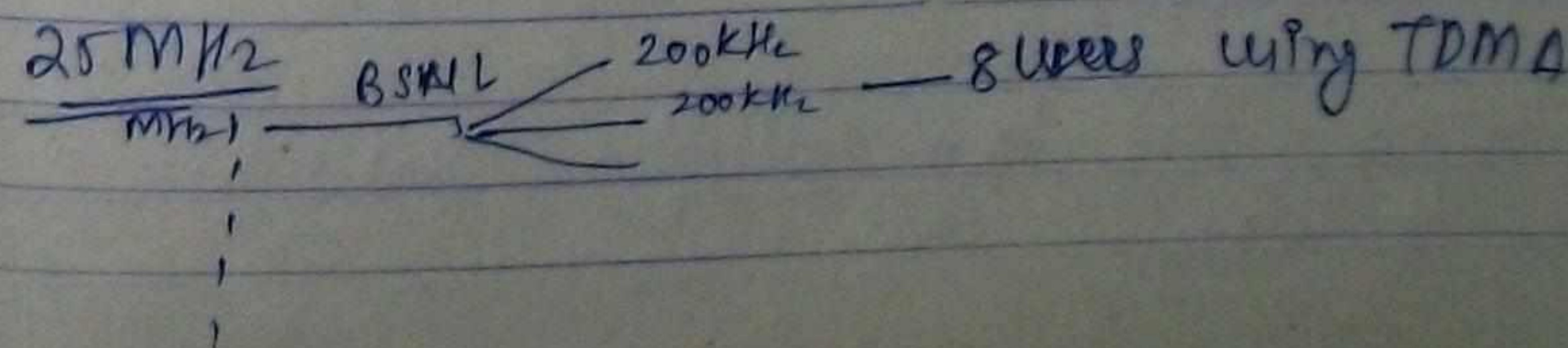
cluster size = 7 cells

$\frac{1}{K} = \text{freq reuse factor}$

- In GSM system the cells having different carrier frequency is known as cluster size.

Let K is cluster size then $1/K$ is known as freq reuse factor.

GSM $\overset{25 \text{ MHz}}{890 - 915 \text{ MHz}} - \text{uplink}$
 $935 - 960 \text{ MHz} - \text{Downlink}$



- The available 25 MHz band in GSM is subdivided into small bands and assign to each cell operator and the corresponding cell operator will further subdivide the available sub band into 200 kHz using FDMA scheme. Each 200 kHz is assign to 8 users (channel) by using TDMA.

$$B_c \times \frac{1}{K} \times \frac{1}{200} \times 8 = \text{Different no. max channel can exist}$$

Sub band $\downarrow$ freq reuse factor

RADARS

6/04/12

EMT FUNDAMENTAL -

Circuit theory $\rightarrow V, I, P \rightarrow$ Low frequency

field theory $\rightarrow \vec{E}, \vec{H}, \vec{P}$
(m/A) minimum int. to polarization = 0, all. 0.11 = 0.11

$$V \rightarrow V = \frac{\omega}{\phi} \rightarrow (\omega/c) \quad \text{m/A} \quad 0.1 \times 10^{-11} = 0.11$$

$$I = Q/t \rightarrow (C/s) \quad \text{m/A} \quad 0.1 \times 10^{-11} = 0.11$$

$$P = V \cdot I$$

$$E = \frac{V}{L} \quad (V/m)$$

$$\vec{E} = \vec{F} \quad (N/C)$$

$$\vec{D} = \epsilon \vec{E} \rightarrow (C/m^2)$$

$$\epsilon = \epsilon_0 \epsilon_r$$

$$\epsilon_0 \rightarrow \text{absolute permittivity} = 8.85 \times 10^{-12} \text{ f/m}^2$$

$$\epsilon_0 = \frac{1}{36\pi} \times 10^{-9} \text{ f/m}$$

$$\epsilon_r \rightarrow \text{relative permittivity} \Rightarrow \epsilon_r = \frac{E}{E_0}$$

$$\text{free space, } \epsilon_r = 1$$

$$\vec{H} = \frac{I}{L} \text{ A/m}, \quad \vec{B} = \mu H \text{ (wb/m}^2\text{)}$$

$\mu \rightarrow$ permeability of the medium (A/m)
 $\mu = \mu_0 \cdot \mu_r$

$$\mu_0 = 4\pi \times 10^{-7} \text{ A/m (absolute permeability)}$$

$\mu_r =$ relative permeability,
 $\mu_r = 1$ (free space)

$$V_p = \frac{1}{\sqrt{\mu\epsilon}} = \text{PHASE VELOCITY}$$

$$= \frac{1}{\sqrt{\mu_0\epsilon_0} \sqrt{\mu_r\epsilon_r}}$$

$$V_p = \frac{c}{\sqrt{\mu_r\epsilon_r}}$$

If not given $\mu_r = 1$

where, $\frac{1}{\sqrt{\mu_r\epsilon_r}} =$ velocity factor.

$$V_p \approx \frac{c}{\sqrt{\epsilon_r}}$$

when $\mu_r \approx 1$

$$V_p = \frac{1}{\sqrt{LC}} \text{ for transmission line}$$

$$Z = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r}}$$

for free space,

$$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = \frac{120\pi}{\text{ohm}} = 377\Omega$$

MAXWELL'S EQUATIONS -

① AMPERE'S LAW $\rightarrow$

$$\nabla \times \vec{H} = \vec{J}_c + \vec{J}_D$$

$$\vec{J}_c = \sigma \vec{E}$$

$$\vec{J}_D = \frac{\partial \vec{D}}{\partial t} = \epsilon \frac{\partial \vec{E}}{\partial t}$$

$$\nabla \times \vec{H} = \vec{J}_c + \vec{J}_D = \sigma \vec{E} + \epsilon \frac{\partial \vec{E}}{\partial t}$$

$$\nabla \times \vec{H} = \sigma \vec{E} + \epsilon \frac{\partial \vec{E}}{\partial t}$$

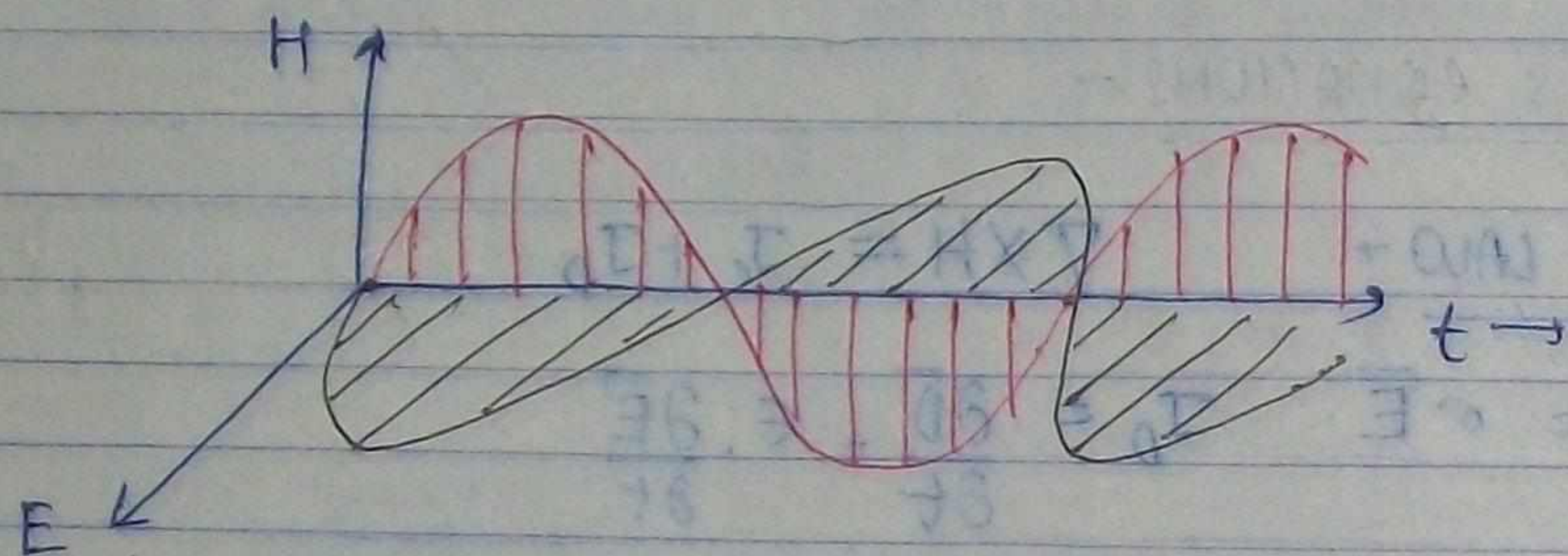
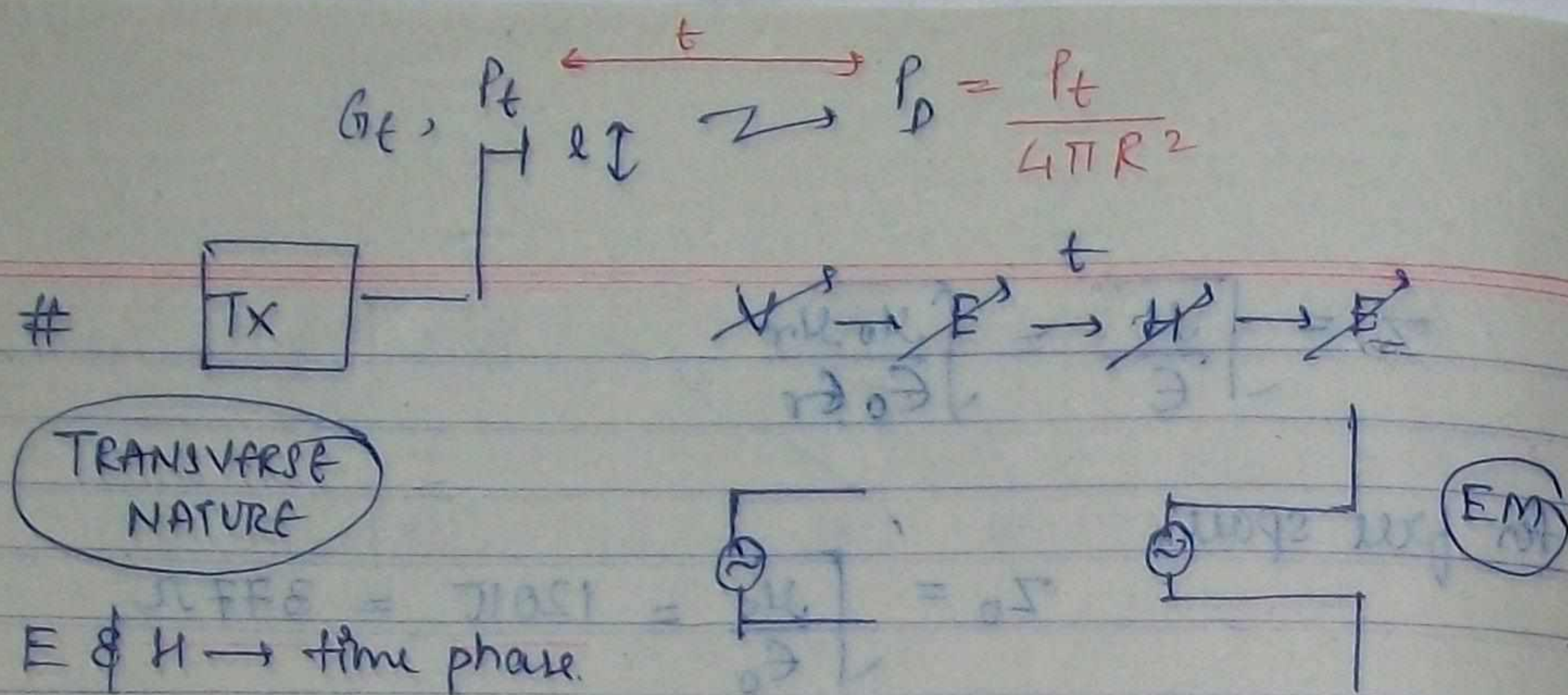
②

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

③

$$\nabla \cdot \vec{D} = \rho$$

$$\nabla \cdot \vec{B} = 0$$



Directive gain = $\frac{\text{Power density of a particular antenna}}{\text{Power density of isotropic antenna}}$
at the point when both are fed with same power.

$D_g = G = \text{Directive gain}$

Power density of practical antenna = $\frac{P_t}{4\pi R^2} \times G_t$

also, $P_t \times G_t \rightarrow \text{EIRP}$

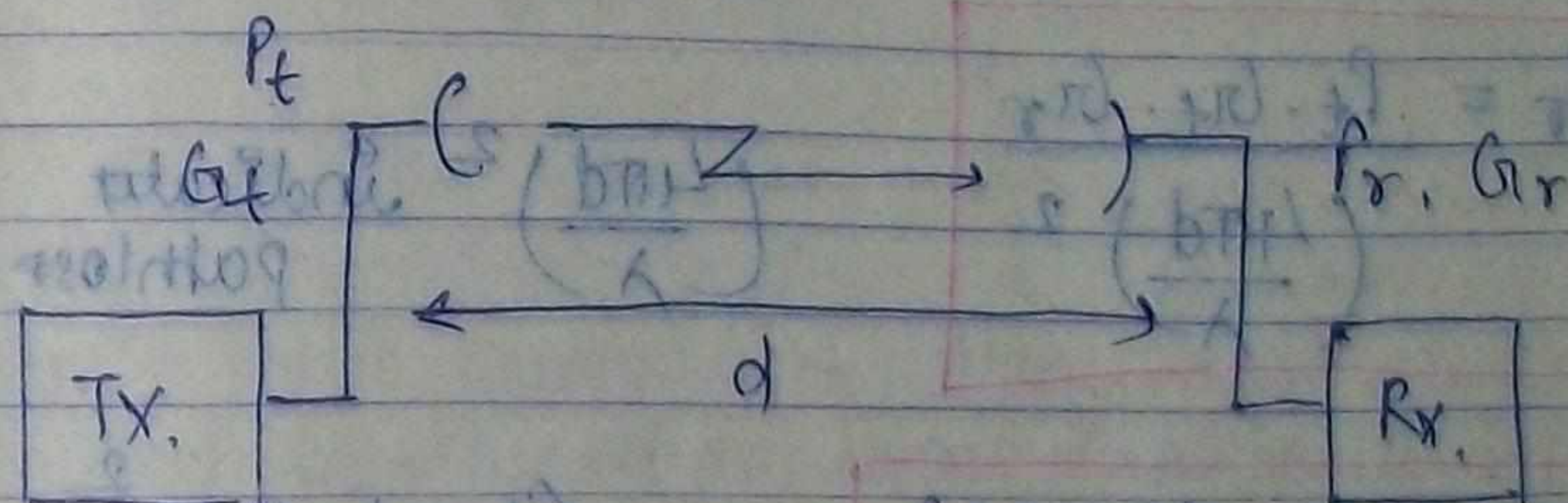
$P = \frac{V^2}{R}$, $P_D = \frac{E^2}{Z_0}$
In free space, E at a distance 'd' is given by:

$$E^2 = P_D \times Z_0 = \frac{P_t \cdot G_t}{4\pi d^2} \times 120\pi$$

$$E = \sqrt{\frac{30 \cdot P_t \cdot G_t}{d}}$$

$$E \propto \frac{1}{d}$$

⇒ PATH LOSS CALCULATION



let us consider a Tx.
with Tx. power P_t
and gain of Antenna G_t
then P_D is available at a distance 'd' is given by:

$$A_e = \frac{\lambda^2}{4\pi} \times D_g$$

$$P_D = \frac{P_t}{4\pi d^2} \times G_t, \quad \text{power density} = \frac{\text{Power}}{\text{Area}}$$

Power = Power density $\times$ Area
let A_{e_r} is the effective area of Rx. antenna. Then power received by receiver is given by:

$$\text{Power received by Rx} = \frac{P_t \cdot G_t \cdot A_{er}}{4\pi d^2}$$

$$\text{where, } A_{er} = \frac{\lambda^2}{4\pi} \cdot G_r$$

$$\begin{aligned} \text{Power received, } P_r &= \frac{P_t \cdot G_t \cdot G_r \cdot \lambda^2}{(4\pi d)^2} \\ &= \frac{P_t \cdot G_t \cdot G_r \cdot \lambda^2}{(4\pi)^2 \cdot d^2} \end{aligned}$$

$$P_r = \frac{P_t \cdot G_t \cdot G_r}{\left(\frac{4\pi d}{\lambda}\right)^2}$$

$$\left(\frac{4\pi d}{\lambda}\right)^2 \text{ Indicator Pathloss}$$

$$\text{PATH LOSS} = \left(\frac{4\pi d}{\lambda}\right)^2 = \left(\frac{4\pi d \cdot f}{c}\right)^2$$

$$\text{Path loss} \propto d^2$$

$$\propto \frac{1}{\lambda^2}$$

$$\propto f^2$$

$$\text{PATH LOSS} /_{db} = 10 \log \left(\frac{4\pi d f}{c} \right)^2$$

$$= 20 \log_{10} \left(\frac{4\pi}{c} \right) + 20 \log_{10} (d) + 20 \log_{10} (f)$$

where, $d \rightarrow m$

$f \rightarrow \text{Hz}$

by general $d \rightarrow \text{km}$

$f \rightarrow \text{MHz}$

$$= 20 \log_{10} \left(\frac{4\pi}{c} \right) + 20 \log_{10} (10^3) + 20 \log_{10} (d) + 20 \log_{10} (f)$$

$$\text{PATH LOSS} /_{db} = 32.5 + 20 \log_{10} d + 20 \log_{10} f \text{ dB}$$

$f \rightarrow \text{MHz}$ $d \rightarrow \text{km}$

Ques. In a radio comm. the distance b/w Tx & Rx is 100 km and freq. is 1 GHz then Pathloss in dB is?

Solⁿ

$$\text{Path loss} /_{db} = 32.5 + 20 \log 100 + 20 \log 10^3$$

$$= 32.5 + 100 \log 10 = \underline{\underline{132.5 \text{ dB}}}$$

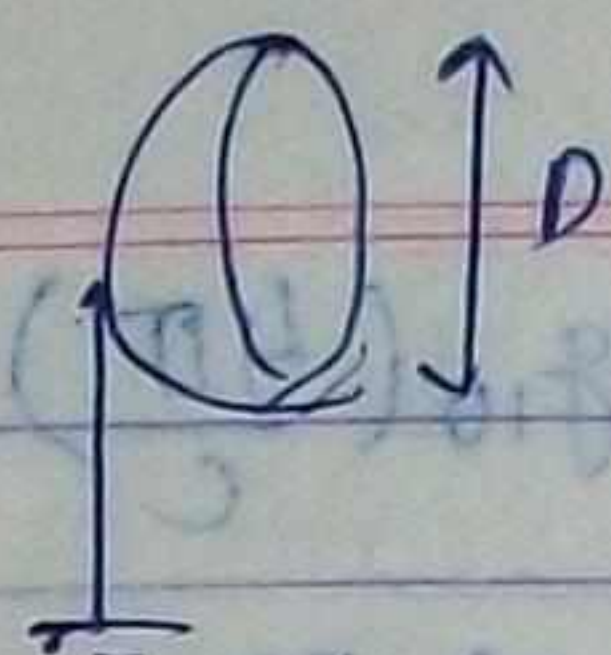
also,

$$\text{PATH LOSS} /_{db} = 32.5 + 20 \log_{10} (d) + 20 \log_{10} (f)$$

where

$d \rightarrow \text{km}$ $f \rightarrow \text{GHz}$

Parabolic reflector:



$$A = \frac{\pi D^2}{4}$$

$$A_e = \frac{1}{4\pi} \times G$$

$$A_e \propto G$$

$$G = 6 \times \left(\frac{D}{\lambda}\right)^2$$

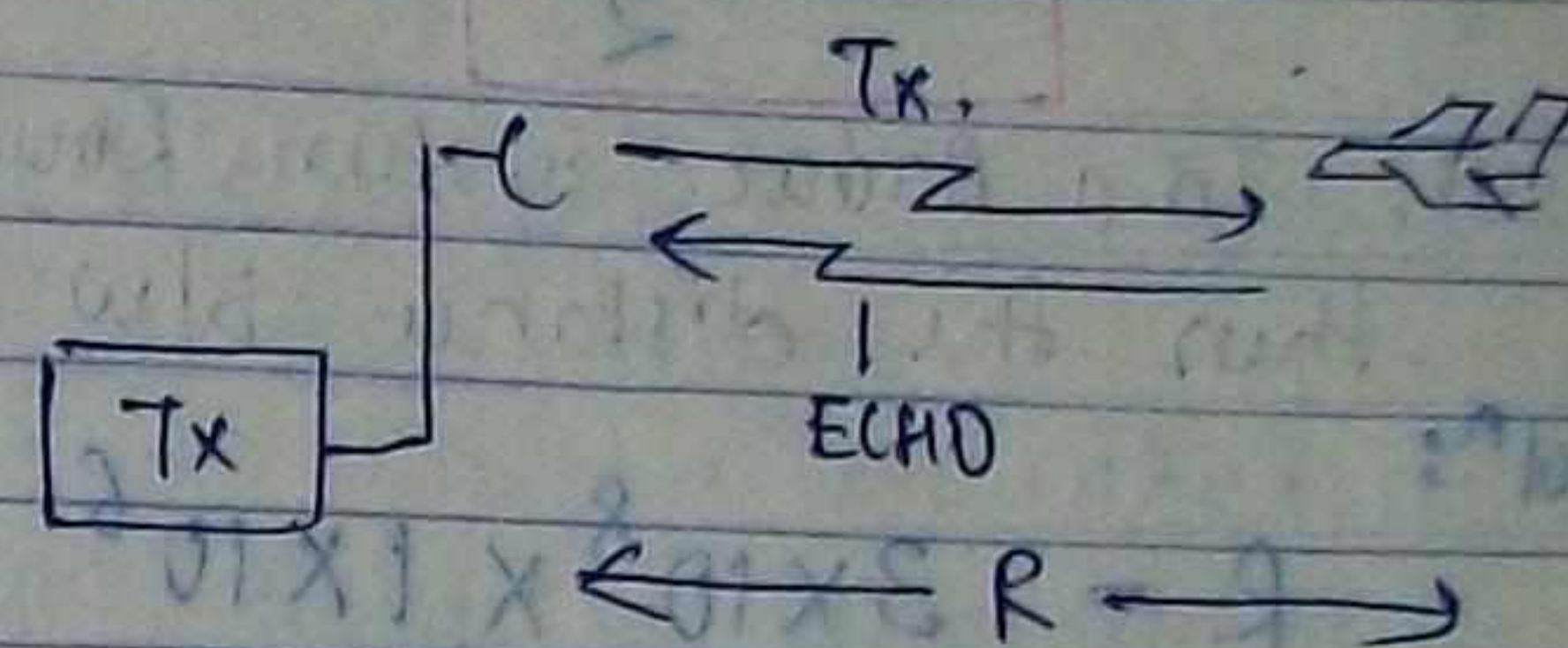
Parabolic reflector

RADAR

- Radio detection and Ranging (RADAR).

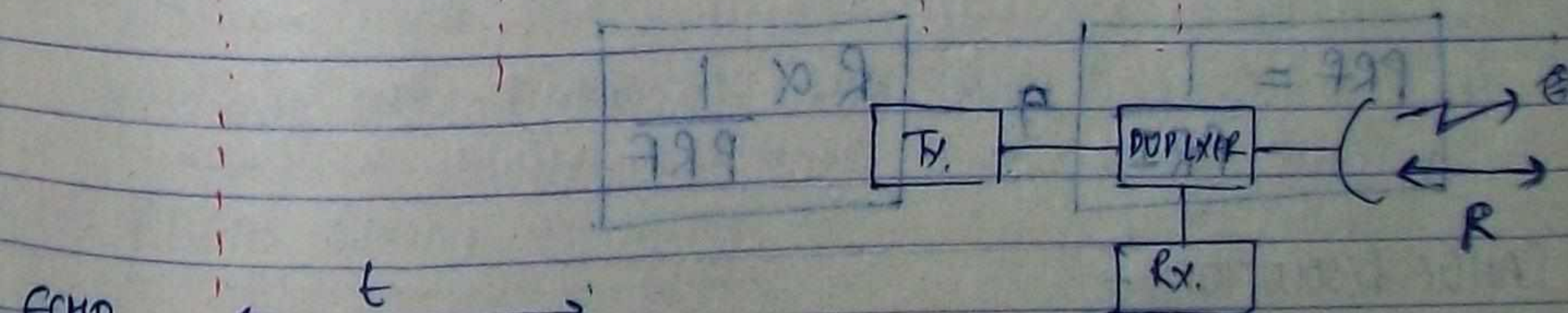
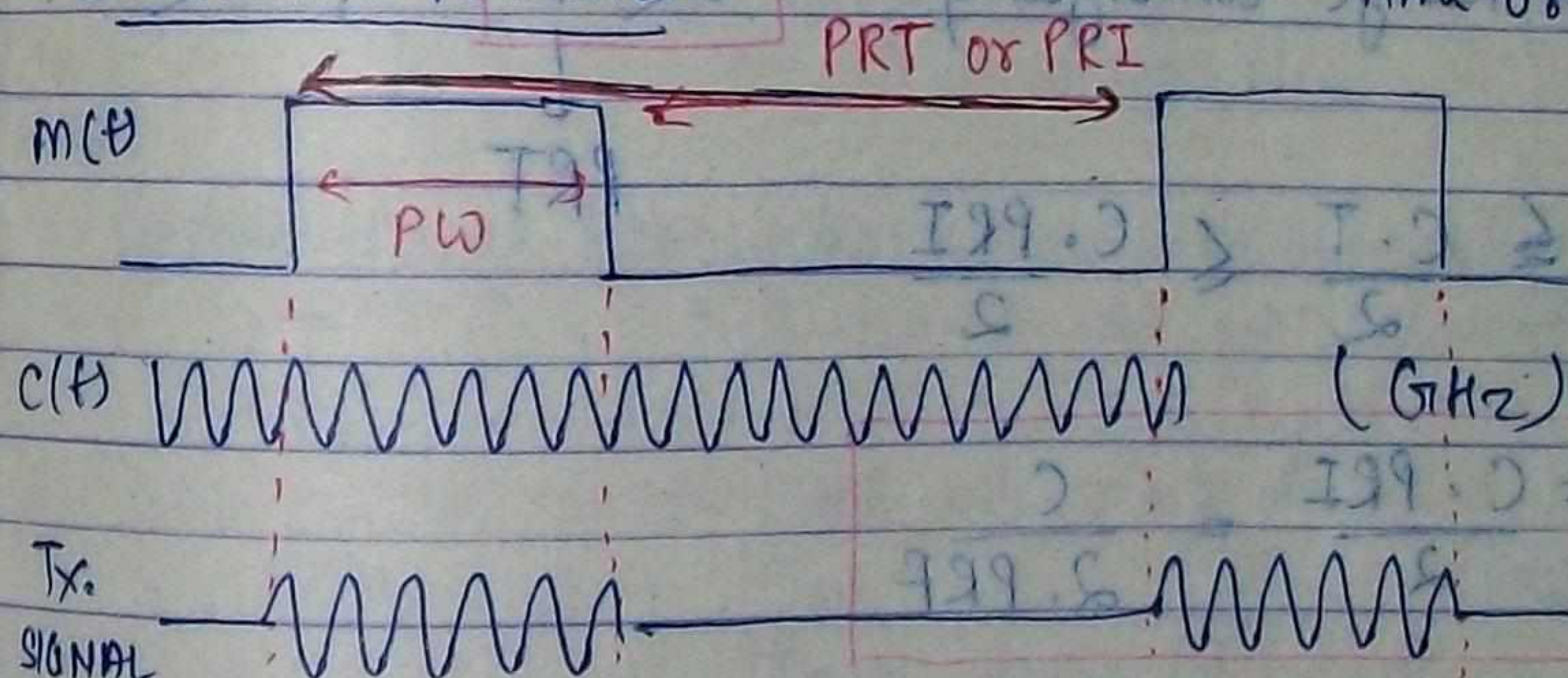
- principle: E.M. wave reflection

- In radar Radio signals are used to detect any object present in free space & to provide distance or Range of the object.



WORKING PRINCIPLE

Pulse repetition time or interval.



ECHO SIGNAL

$t \rightarrow$ Round trip travel time

$$2R = c \times t$$

$$R = \frac{ct}{2}$$

Ques. In a Radar system Round trip travel time is 1.1 sec then the distance b/w Radar & target is?

Solⁿ:

$$R = \frac{3 \times 10^8 \times 1.1}{2} = 165 \text{ km}$$

- To avoid Range ambiguity $t \leq T$, $t \leq C$

$$R_{\text{UNambiguous}} \leq \frac{c \cdot T}{2} \leq \frac{c \cdot \text{PRI}}{2}$$

$$R_{\text{max}} = \frac{c \cdot \text{PRI}}{2} = \frac{c}{2 \cdot \text{PRF}}$$

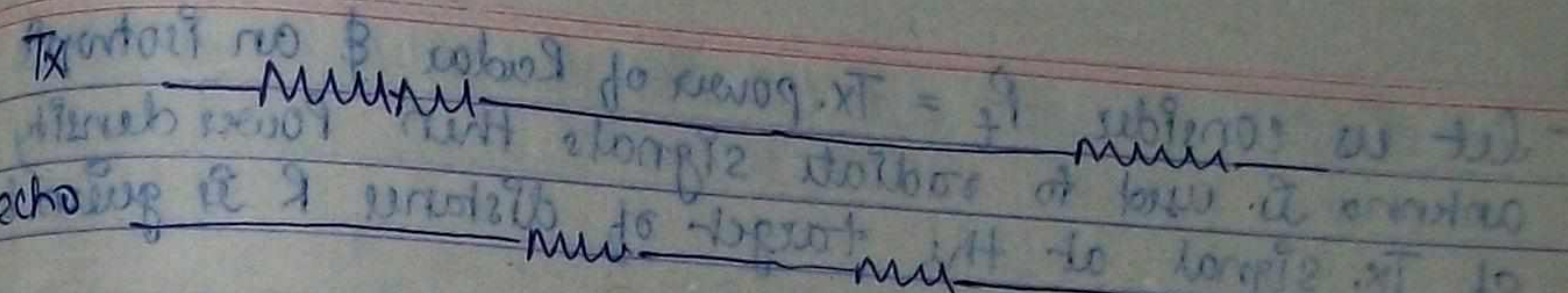
$$\text{PRF} = \frac{1}{\text{PRI}}$$

$$R \propto \frac{1}{\text{PRF}}$$

RANGE RESOLUTION -

Radar range resolution is the shortest distance (time difference) between which the radar can detect two echo. (discriminated) (target)

$$\text{Resol}_{\text{min}} = \text{PW}$$



- Radar Range resolution is the minimum distance over which radar can identify two targets.

- for higher range resolution Pulse width of Tx signal should be less.

$$\text{Resolution} \propto \frac{1}{\text{Pulsewidth}}$$

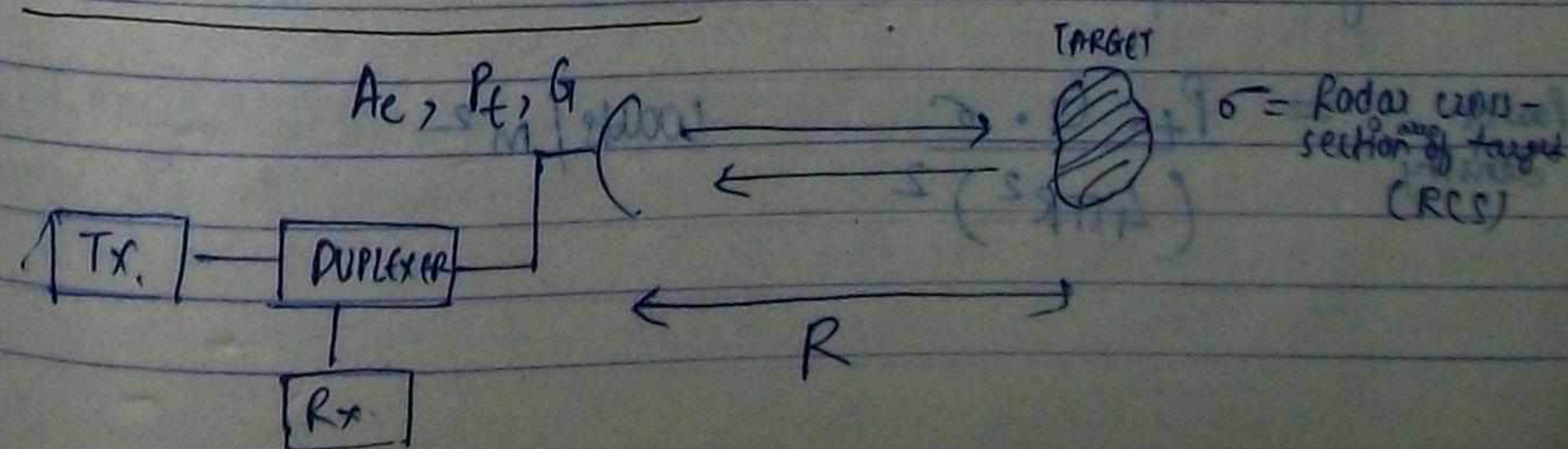
- Radar range also depends on pulse energy.

$$P_{\text{avg}} = \text{Peak Power} \times \text{duty cycle}$$

$$\text{Duty cycle} = \frac{T_{\text{on}}}{T_{\text{total}}} = \frac{\text{PW}}{\text{PRT}} = \text{PW} \times \text{PRF}$$

$$P_{\text{avg}} = \text{Peak power} \times \text{PW} \times \text{PRF}$$

★ $\Rightarrow$ RADAR RANGE EQUATION -



- Let us consider P_t = Tx. power of Radar & an isotropic antenna is used to radiate signals then Power density of Tx. signal at the target of distance R is given by

$$\text{Power density} = \frac{P_t}{4\pi R^2} \text{ W/m}^2$$

- In Radar directional antennas are used with direction gain G then power density available around target is given by:

$$\text{Power density} = \frac{P_t \times G}{4\pi R^2} \text{ W/m}^2$$

- If σ is radar cross section area of the target then power intercepted by the target is:

$$= \frac{P_t \times G \times \sigma}{4\pi R^2} \text{ (watts)}$$

- The target is a metallic object which will radiate the intercepted power due to which the echo or reflected signal power density available at radar is given by:

$$\text{Power density} = \frac{P_t \cdot G \cdot \sigma}{(4\pi R^2)^2} \text{ watts/m}^2$$

- If A_e is the effective area of Radar Rx. antenna then power received by Radar is (S) :

$$S = \frac{P_t}{(4\pi R^2)^2} \cdot G \cdot \sigma \cdot A_e$$

$$S = \frac{P_t \cdot G \cdot \sigma \cdot A_e}{(4\pi)^2 R^4}$$

Radar Range equation:

$$R = \left[\frac{P_t \cdot G \cdot \sigma \cdot A_e}{(4\pi)^2 \cdot S} \right]^{1/4}$$

$$R \propto [P_t]^{1/4}$$

- ie If $P_t \uparrow$ by 16 times then Range is $\uparrow$ by 2 times only.

- To increase range by 3 times Tx. power is $\uparrow$ by 81 times.

$$R \propto [P_t]^{1/4} = \left[\frac{1}{S} \right]^{1/4}$$

- If Received signal power is decreases by 16 times then range is doubled.

- In Radar if same antenna is used for both Tx and Rx then effective area is given by,

$$A_e = \frac{\lambda^2}{4\pi} \times G$$

$$R = \left[\frac{P_t \cdot G^2 \cdot \sigma \cdot \lambda^2}{(4\pi)^3 \cdot S} \right]^{1/4}$$

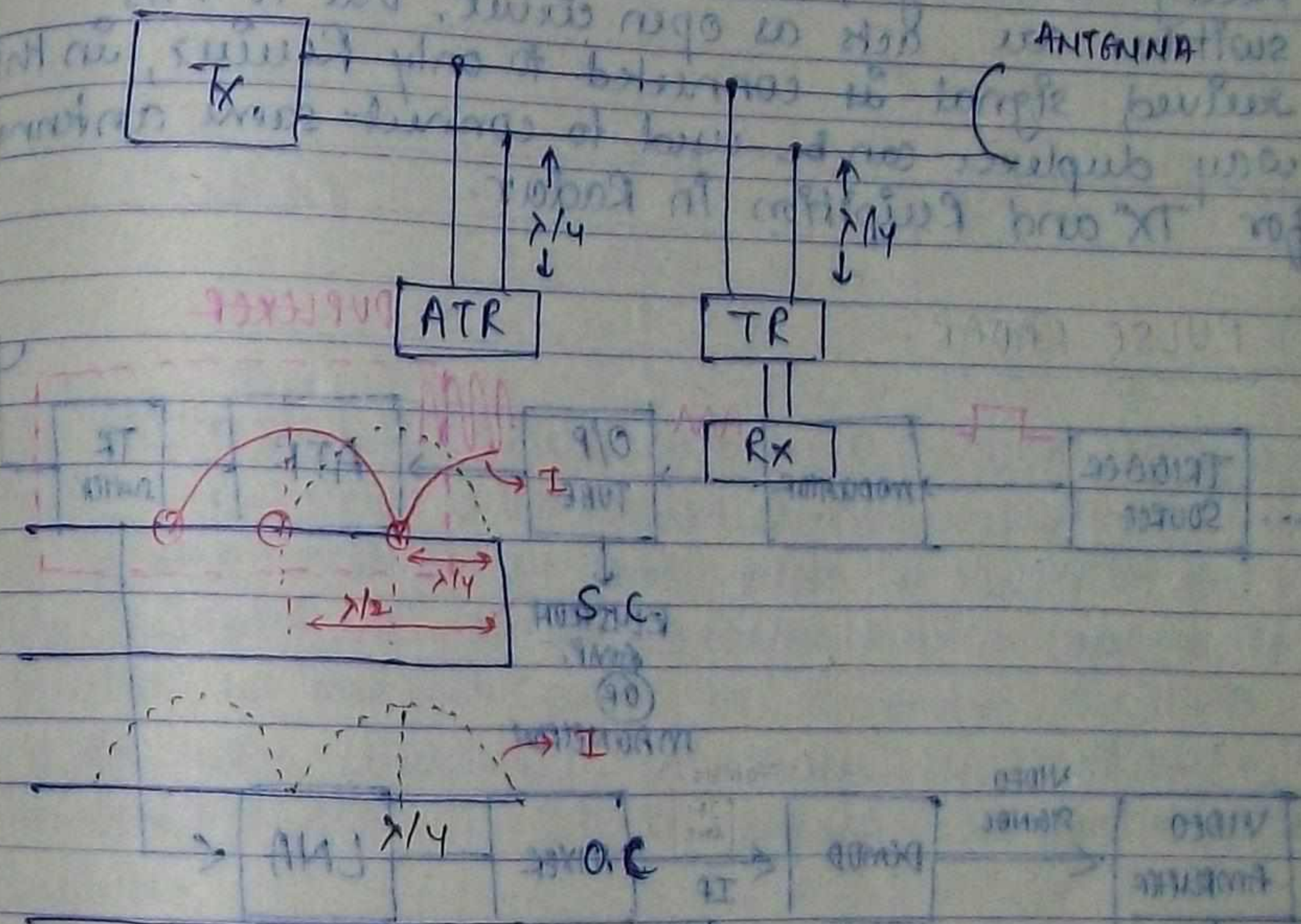
$$\left. \begin{aligned} R &\propto \sqrt{G} \\ R &\propto \sqrt{A_e} \\ R &\propto D \end{aligned} \right\}$$

⊛ $dBm = 10 \log \left(\frac{P}{1 \text{ mW}} \right)$

⇒ TYPES OF RADARS

1. Pulsed radar
2. CW radar
3. FMCW radar
4. MFL radar

DUPLEXER

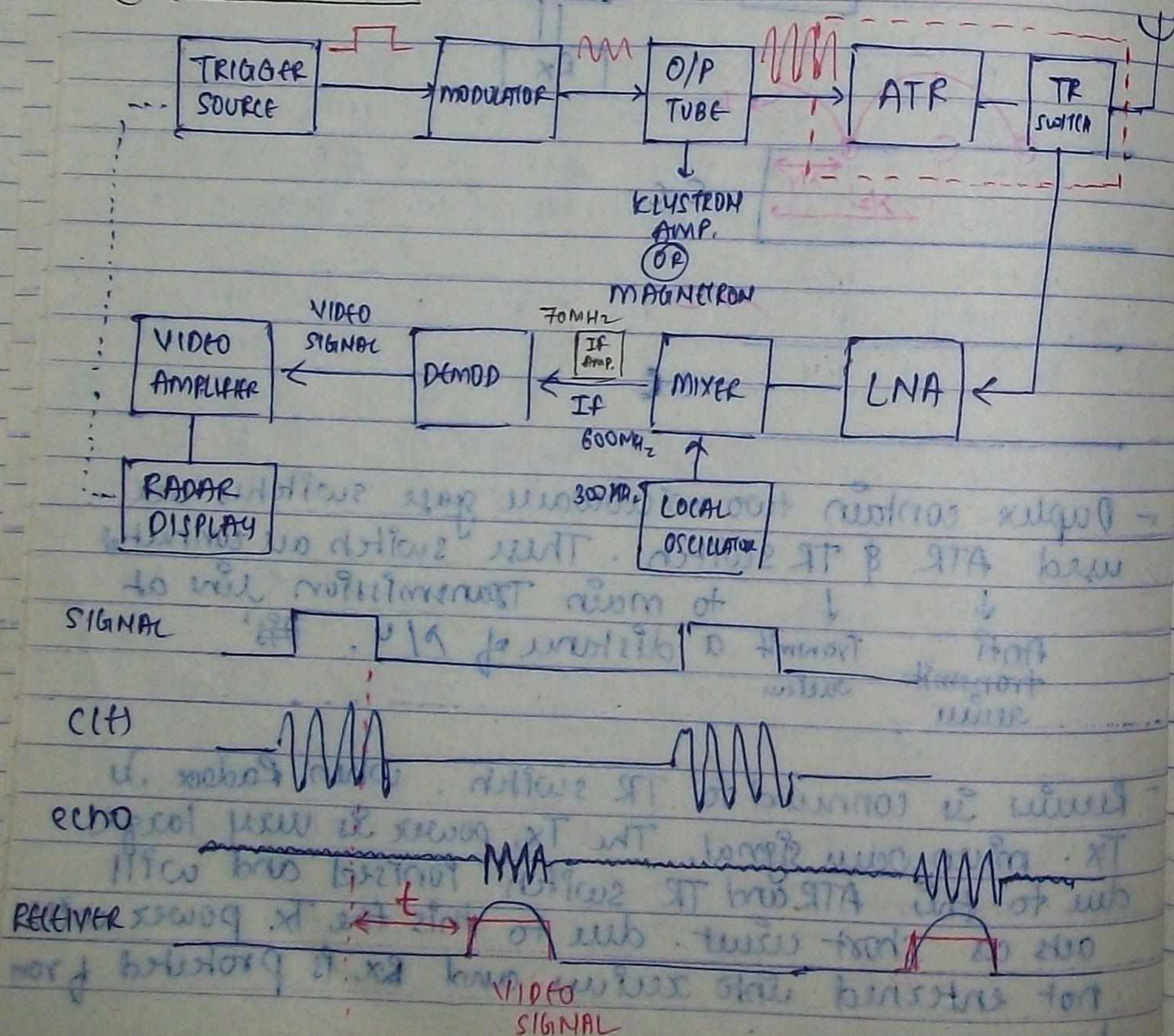


- Duplex contain two microwave gase switches are used ATR & TR switch. These switch are connected to main Transmission line at a distance of $\lambda/4$.
 ↓ ↓
 Anti transmit Transmit
 receive section

- Receiver is connected to TR switch. When Radar is Tx. microwave signal. The Tx. power is very large due to this ATR and TR switch is closed and will acts as short circuit. due to this the Tx. power will not entered into receiver and Rx. is protected from large Tx. power

- During Receiving power is very less and ATR & TR switches are acts as open circuit. Due to this received signal is connected to only Receiver, in this way duplexer can be used to connect same antenna for TX and Reception in Radar.

(1) PULSE RADAR -



$$S \gg (F-1) N_f$$

$$S \gg (F-1) KTB$$

$$\# R_{max} = \left[\frac{P_t G_o A_e}{(4\pi)^2 (F-1) KTB} \right]^{1/4}$$

- In pulse radar a pulse is generated using trigger source such as monostable multivibrator then it is pulse modulated and given to a o/p tube. The o/p tube is a microwave which can be a klystron or magnet amp. or magnetron oscillator which will provide Tx power by the Radar transmitted signal. It is connected to antenna via duplexer.

- During Receiving if any echo signal is present then it is connected to the Receiver the 1st stage in Radar receiver is LNA which is using Varactor diode or MESFET (metal-S.C FET) (Solid state device).

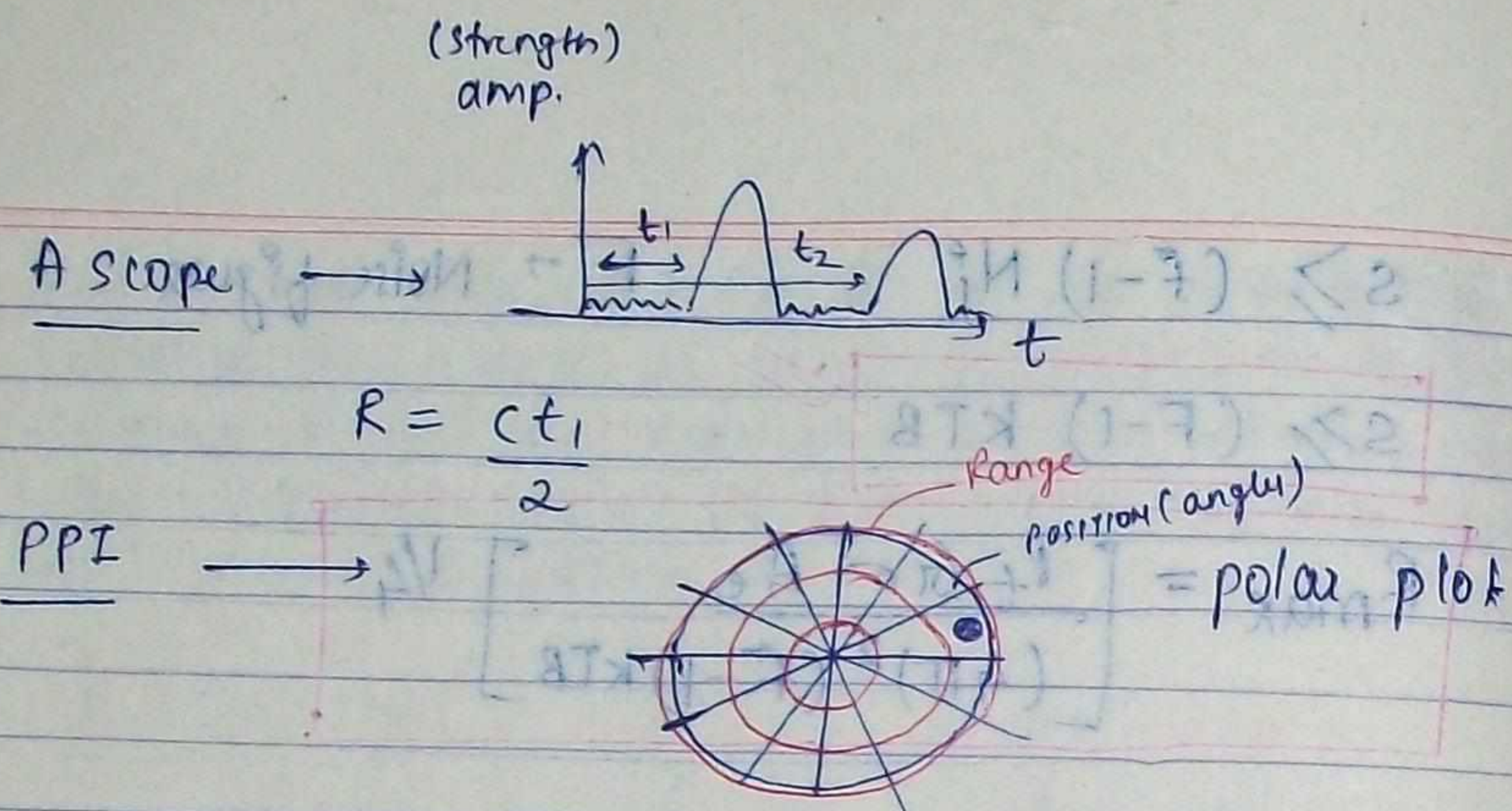
Radar Displays

A - scope

B - scope

P → PPI - plan position indicator

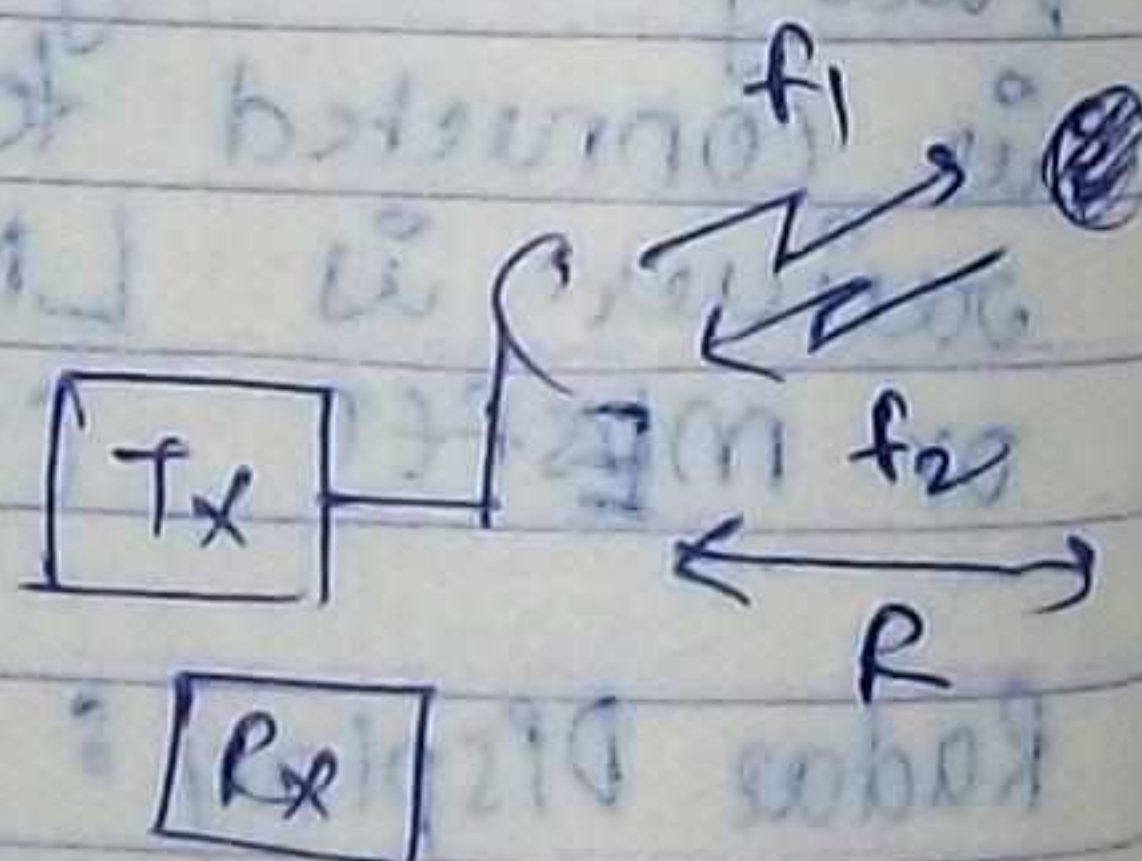
R → RHI - Range height indicator



- In pulse radar if A scope will used then it will provides only Range of the target and if PPI display is used then it will provides both range as well as position of the target.
- Disadvantage of pulse radar is it will not provides velocity of the target.
- To determine velocity of moving target Doppler effect is used.

$\Rightarrow$ DOPPLER'S EFFECT

- According to Doppler effect when Radar transmits radio signal with freq f_1 if any target is present it will reflect EM waves with different freq. (f_2) when target is moving,



- when target is moving towards the Radar then freq will increase ($f_2 > f_1$). If target is moving away from the radar then received freq will decrease ($f_2 < f_1$). If doppler freq ($\Delta f, f_d$) is calculated

$$f_d = \Delta f = f_2 - f_1$$

also, $2\pi f_d = \omega_d = \frac{d\theta}{dt}$

- let us consider a Radar is Tx. a signal with freq f_1 . If target is present at a distance of R , then it will reflect EM waves with freq. f_2 if target is moving. The distance travelled by EM wave is $2R$.

The distance in wavelength is given by $= \frac{2R}{\lambda}$

The phase shift that can be occurred in the received echo signal is $= \frac{2R}{\lambda} \times 2\pi$

$$\theta = \frac{4R\pi}{\lambda} = \frac{4\pi}{\lambda} \times R$$

- when target is moving then distance will change continuously phase will change and given by:

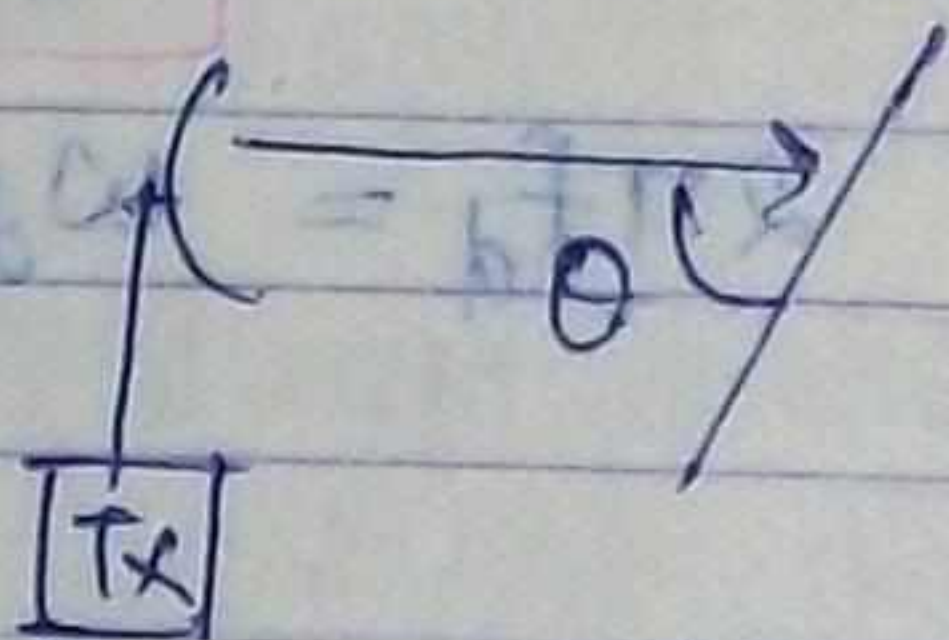
$$\omega_d = \frac{d\theta}{dt} = \frac{4\pi}{\lambda} \cdot \frac{dR}{dt} = \omega_d = 2\pi f_d$$

$$2\pi f_d = \frac{4\pi}{\lambda} V_r \quad (V_r = \text{Relative velocity})$$

$$f_d = \frac{2V_r}{\lambda}$$

$f_d = \text{Doppler's freq.}$

$$f_d = \frac{2V_r}{\lambda} \cdot \cos \theta$$



When target is reflected by a radial angle. Then Doppler freq is given by:

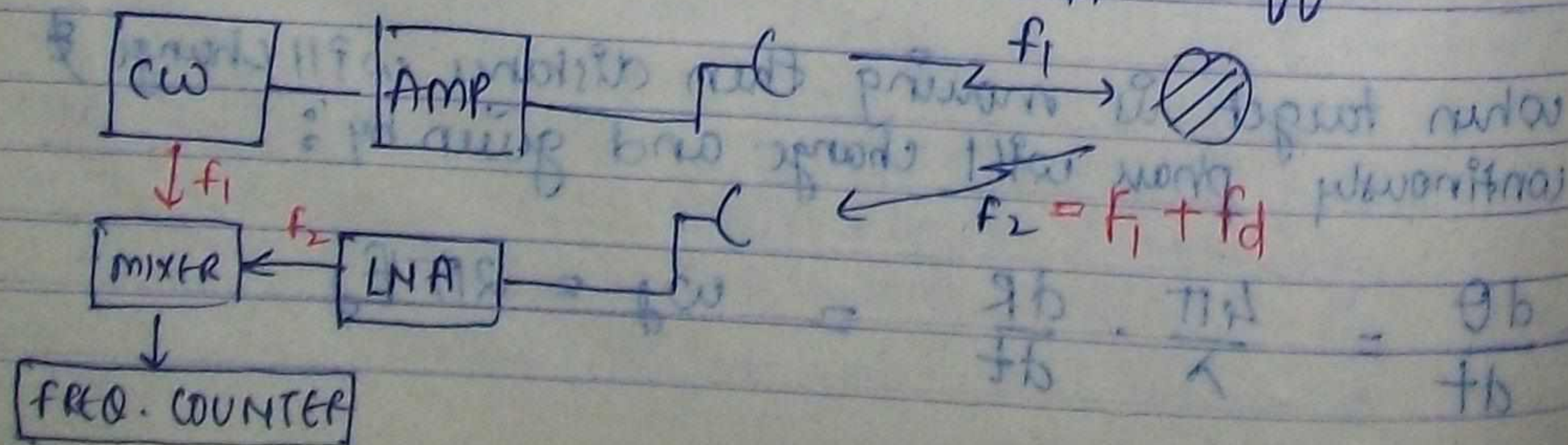
$$f_d = \frac{2V_r}{\lambda} \cos \theta$$

$\theta = \text{radial angle}$

$$V_r = \frac{f_d \cdot \lambda}{2}$$

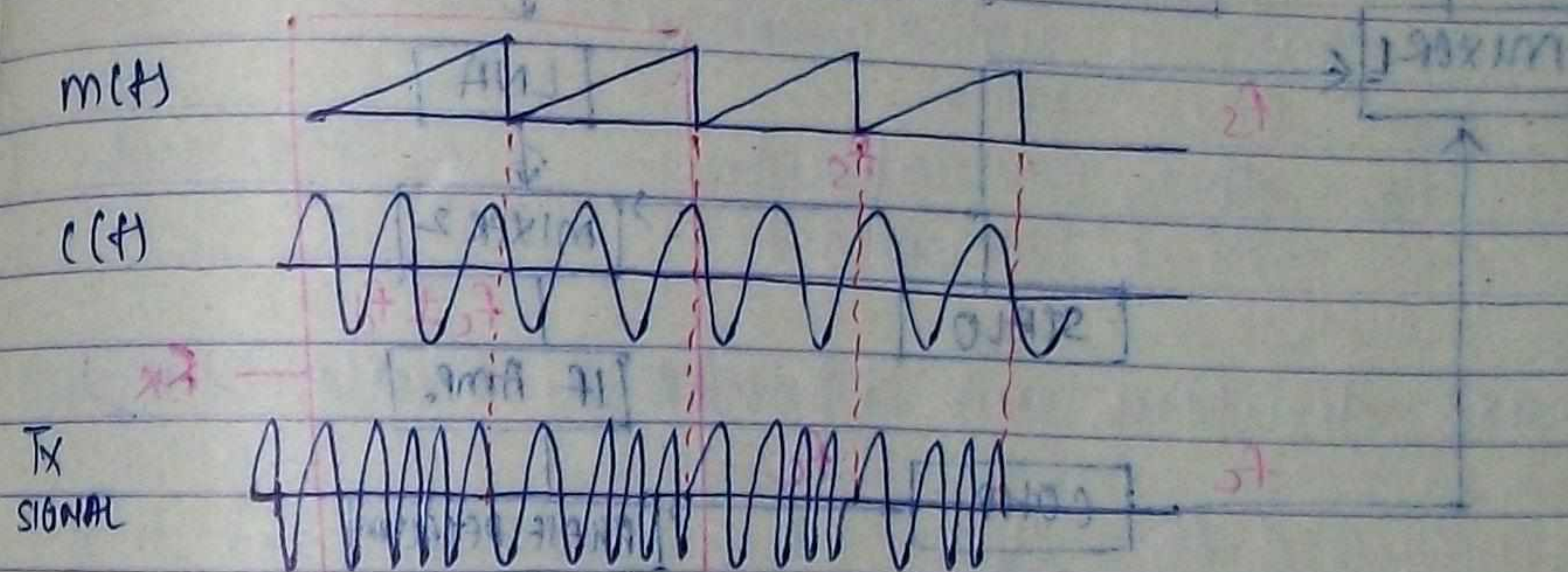
⇒ CONTINUOUS WAVE (CW-RADAR) -

- The principle of CW radar is Doppler effect.



- CW radar will provide velocity of the target but disadvantage is it will not provide Range of the target.

⇒ FM-CW RADAR -



- In fm cw radar a sawtooth wave signal is generated the fm modulated, due to this main advantage of fm cw radar is it will provide both range as well as velocity of the target.

RAM - Ram is a fm cw radar

- Ram is used to determine height of Aircraft

- Ram → Radio-Altitude meter

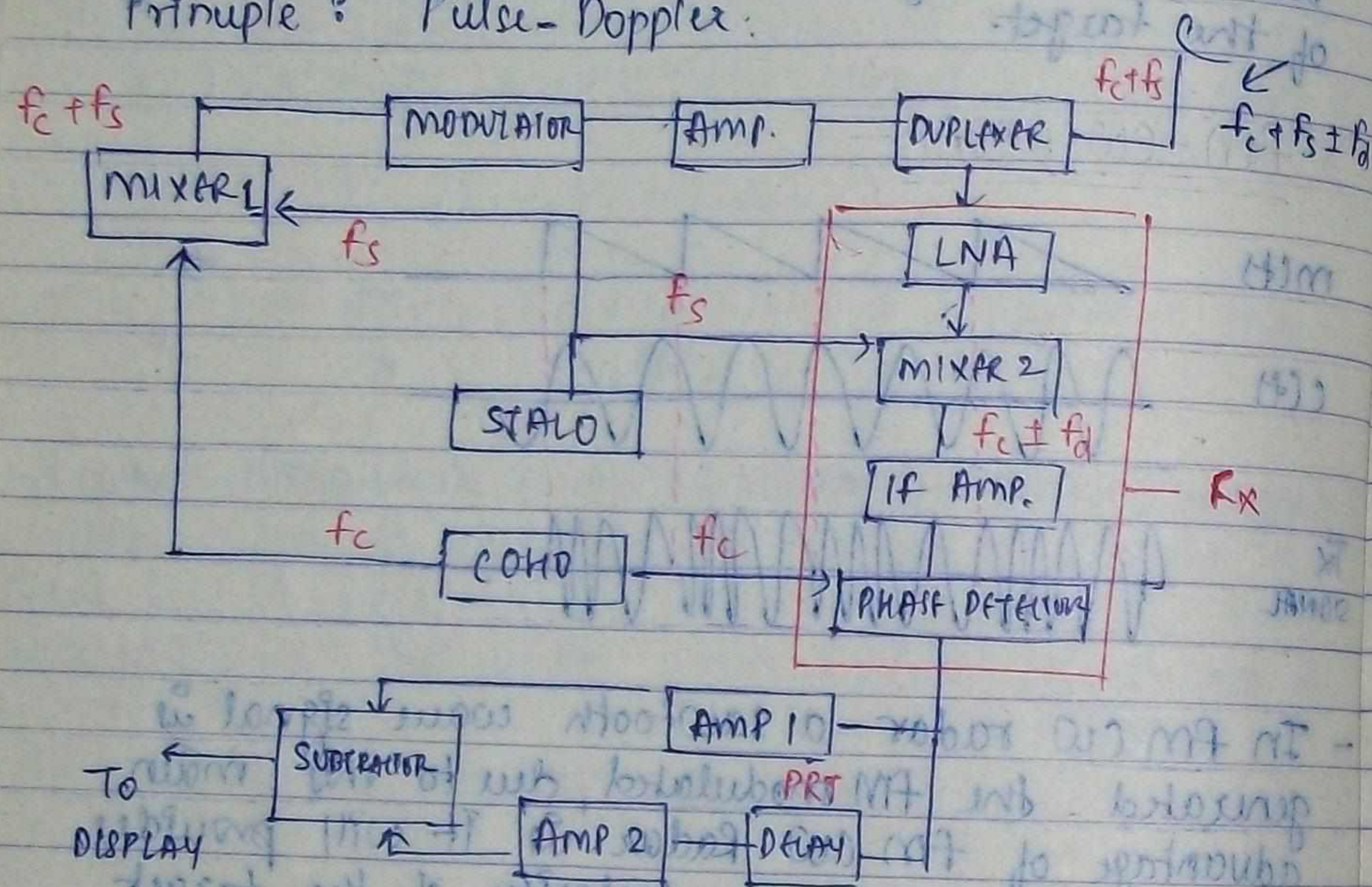
IFF - Identification of friend or foe.

TACAN - Tactical air navigation (both distance & angle position)

DME - Distance measuring equipment.

⇒ MTI RADAR - (MOVING TARGET INDICATOR)

Principle: Pulse-Doppler.



- MTI Radar are used to display only moving targets

- In this two oscillator one is STALO (stable local oscil) and COHO (coherent oscillator), COHO operated at IF frequency.

BLIND SPEED - $V_b = \frac{PRF \times \lambda}{2}$

The disadvantage of MTI radar is blind speed

To avoid blind speed, variable PRF or stagger freq. is used.

Que. Convention P-50 Q1. workbook.

$$R = \frac{ct}{2} \quad R_{max} = \frac{c}{2PRF} \quad (1)$$

$$f_d = \frac{2V_r}{\lambda} \quad V_r = \frac{f_d \lambda}{2} \quad (2)$$

for pulse radar, $PRF = f_d$

$$R_{max} = \frac{c}{2f_d}$$

$$V_r \times R_{max} = f_d \cdot \frac{\lambda}{2} \times \frac{c}{2f_d} = \frac{c}{4} \lambda = K\lambda$$

∴ product is independent of PRF.

⇒ SEARCH & TRACK METHOD -

Radar have two modes

1. Search or scanning.
2. Tracking.

- ① Horizontal
- ② Nodding
- ③ Helical
- ④ spiral

- ① Lobe switching
- ② conical switching
- ③ monopulse tracking

When target cross section area is changing then most efficient tracking method is monopulse tracking.

ANTENNA

- Power density is power radiated per unit area

isotropic, $P_D = \frac{P_t}{4\pi R^2}$

- Directive gain of isotropic antenna

$$D_g = \frac{P_D \text{ of practical ant.}}{P_D \text{ of isotropic ant.}}$$

max^m directive gain is known as Directivity (D_o)

$$0 \leq D_g \leq D_o$$

$$1 \leq D_o \leq \infty$$

- Efficiency :

$$\eta = \frac{P_{\text{radiated}}}{P_t}$$

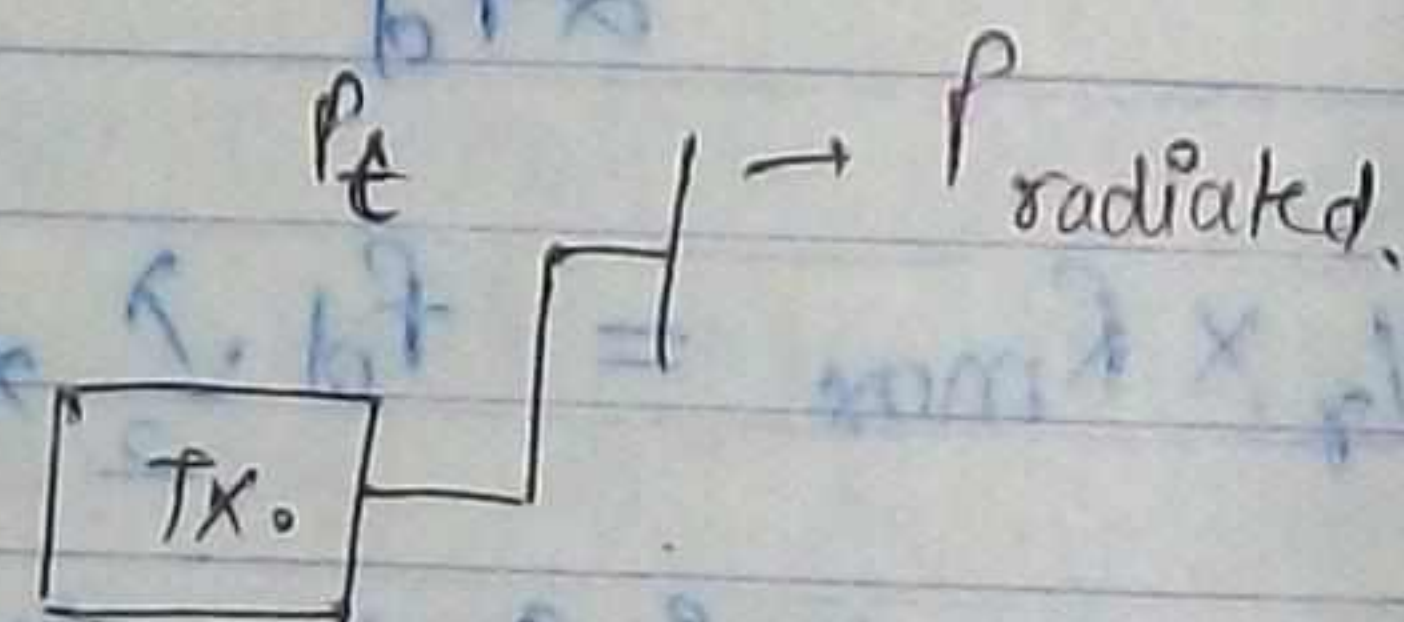
$$P_t = P_r + P_l$$

$$R_r = \frac{P_r}{I^2} = \frac{I^2 R_r}{I^2}$$

$$P_l = I^2 \times R_l$$

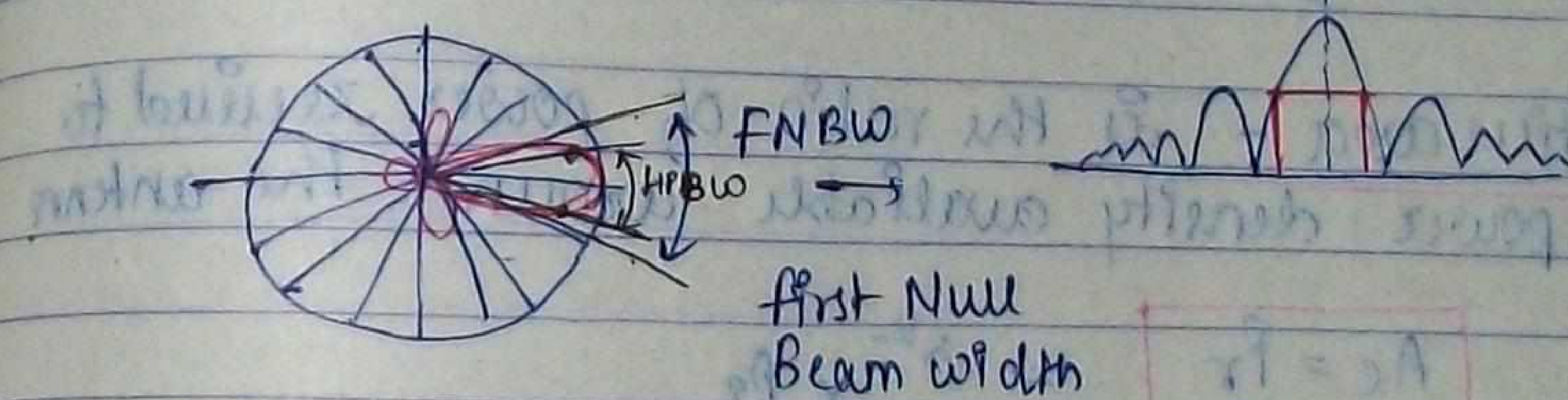
$$\eta = \frac{I^2 R_r}{I^2 R_r + I^2 R_l}$$

$$\eta = \frac{R_r}{R_r + R_l} \times 100$$



$$\text{Power gain} = \eta \times D_g$$

- Radiation pattern is the graphical representation of radiation characteristics of an antenna drawn on the polar plot.



$$\text{HPBW} \approx \frac{\text{FNBW}}{2} \text{ (generally)}$$

- Bandwidth is the frequency separation b/w two Half power points.

1. Narrow band. — Resonant
2. wide band. — Non-Resonant.

- Polarization is orientation of E-field w.r.t ground.

- ① Linear $\begin{matrix} \text{H} \\ \downarrow \end{matrix}$
- ② Circular $\longrightarrow$ Two E field angle $\pm 90^\circ$
- ③ Elliptical

(*) man made noise is vertically polarized.

In linear polarization either one or two E field components with phase shift 0 and 180

- In circular polarizⁿ have two E-field vector of same strength and $\phi = \pm 90^\circ$.

- In elliptical polarizⁿ have two E-field of equal or unequal strength and $\phi = \pm 90^\circ$.

- Effective area - is the ratio of power received to the power density available around the antenna

$$A_e = \frac{P_r}{P_d} = \frac{\lambda^2}{4\pi} \times \eta_g$$

* EFFECTIVE HEIGHT -

$$h_{eff} = \frac{V_{oc}}{E} = \frac{V}{V/m} = m$$

$$h_{eff} = \frac{I_{avg}}{I_{max}} \times l$$

l = physical length of antenna

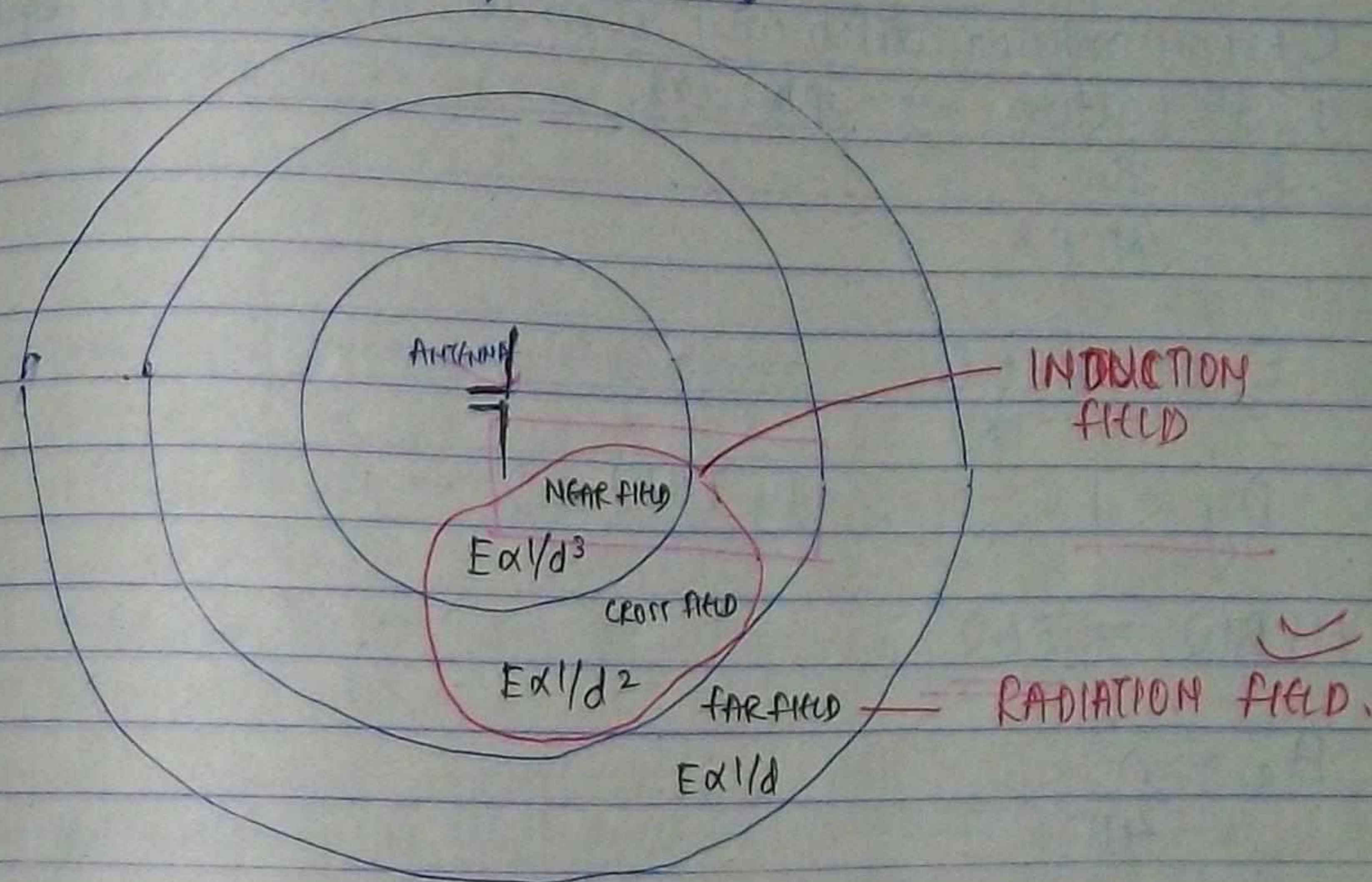
$$\text{Electrical length} = \beta l \text{ radian}$$

β radian/m

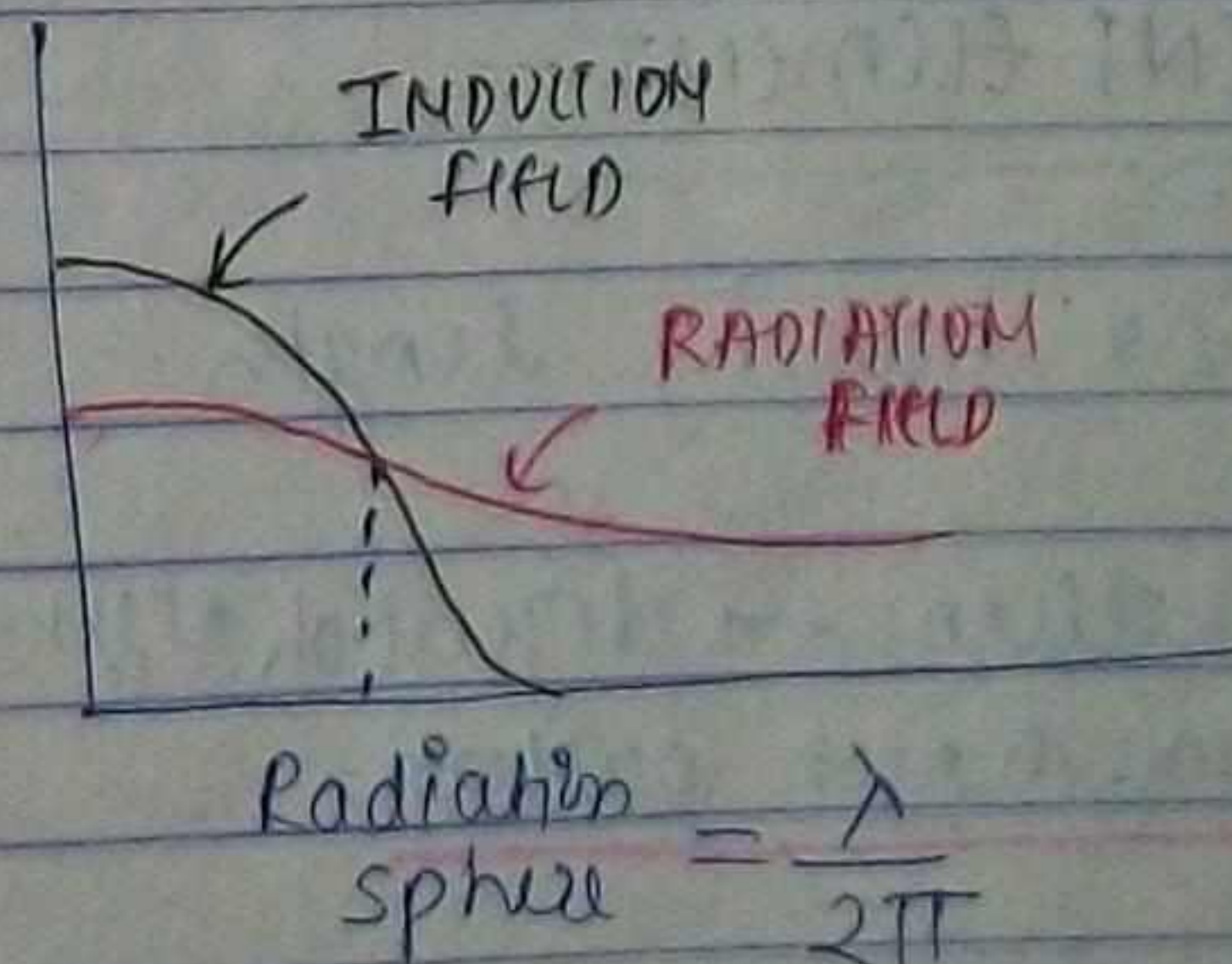
* TOP LOADING -

- Effective height of antenna can be changed by top loading. Top loading is bending of antenna by top side

- Electrical length of antenna is always greater than physical length of antenna.



- At a distance of $\lambda/2\pi$ Indⁿ field = Radⁿ field is called Radⁿ sphere.



- In antenna to make accurate parameter measurement the minimum distance is $\frac{2D^2}{\lambda}$.

⇒ ISOTROPIC ANTENNA - (hypothetical antenna)

- Reference antenna which will radiate in all dirⁿ equally
- Radⁿ pattern → spherical.
- $P_D = \frac{P_t}{4\pi R^2}$

$$E = \sqrt{30 P_t \frac{d}{r}}$$

- $D_g = 1$

- $D_g \text{ (dB)} = 0$

- HPBW = 360°

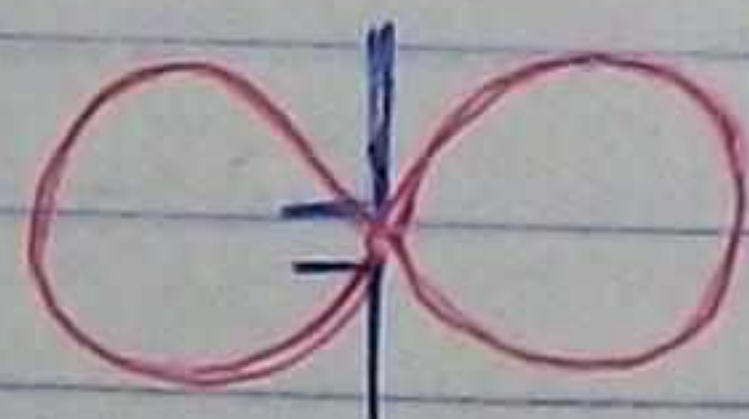
- $A_e = \frac{\lambda^2}{4\pi}$

⊛

⇒ CURRENT ELEMENT -

- Antenna having length $l \leq \lambda/50$

- Radⁿ pattern → figure of eight or Doughnut pattern



- In current element max. radn is $\perp^r$ to the antenna axis and the min. radn is along the axis.

- $D_g = 1.5$

- $A_e = \frac{\lambda^2}{4\pi} \times D_g = \frac{3\lambda^2}{8\pi} = 0.119\lambda^2 \approx 0.12\lambda^2$

- Radⁿ resistance

$$R_r = 80\pi^2 \left(\frac{l}{\lambda}\right)^2$$

- HPBW = 90°

- Current distribution is uniform.

⇒ SHORT DIPOLE -

- $\frac{\lambda}{50} \leq l \leq \frac{\lambda}{10}$

- $R_r = 20\pi^2 \left(\frac{l}{\lambda}\right)^2$

- Current distribution — triangular

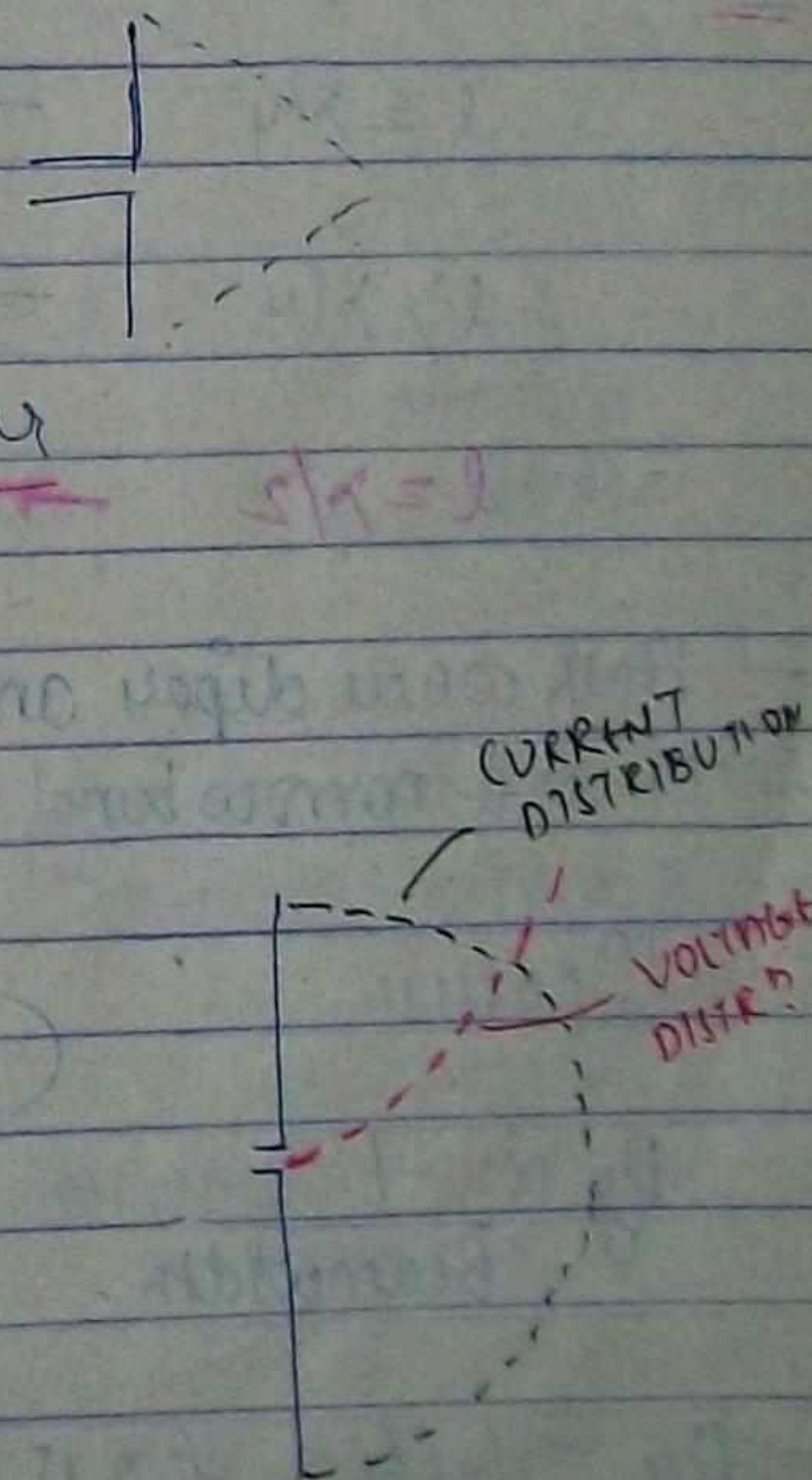
- $D_g = 1.5$ (1.76 dB)

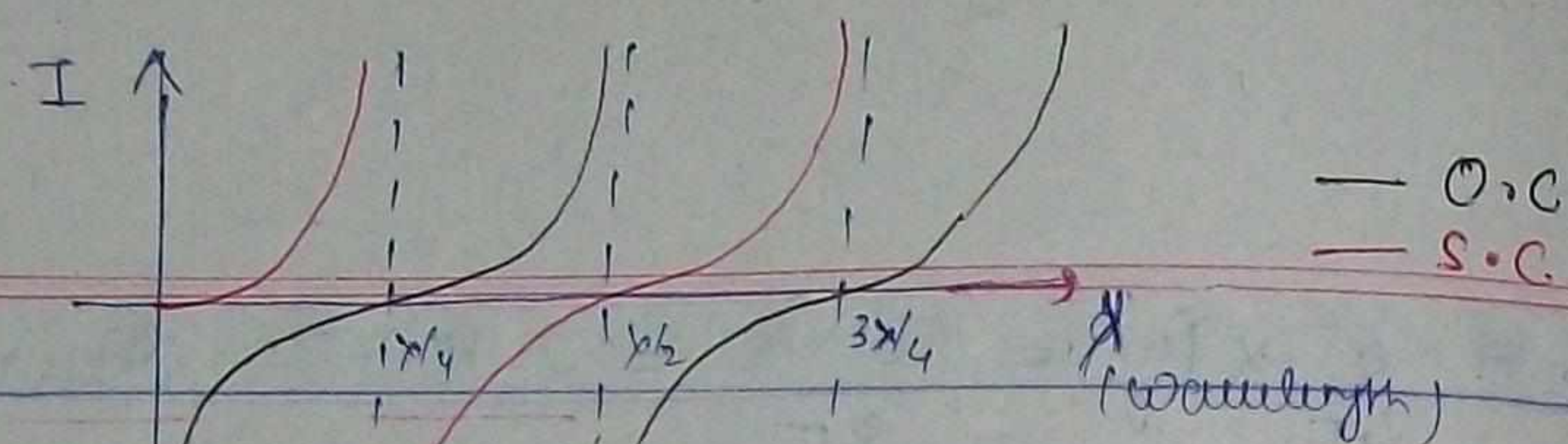
- $A_e = 0.119\lambda^2$

⇒ HALF WAVE DIPOLE -

- length $l = \lambda/2$

- Current distribution





S.C.

$$l = \lambda/2$$

$$l < \lambda/4$$

$$l = \lambda/4$$

$$l > \lambda/4$$

O.C.

$$l < \lambda/4$$

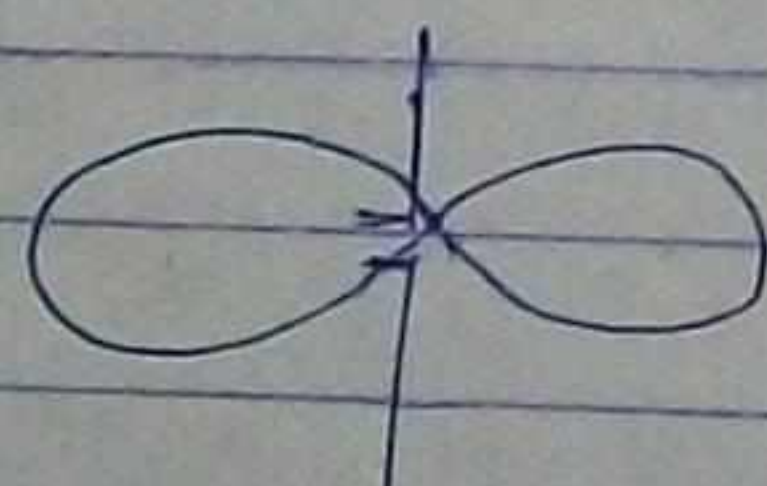
$$l = \lambda/4$$

$$l > \lambda/4$$

$$l = \lambda/2$$

- half wave dipole antenna is a resonant antenna
- It is a narrow band antenna

- Radⁿ pattern



$$D_g \propto \frac{1}{\text{Beamwidth}}$$

$$D_g = 1.64 \quad (2.15 \text{ dB})$$

7/04/12

$$A_e = \frac{\lambda^2}{4\pi} \times 1.64 = 0.13\lambda^2$$

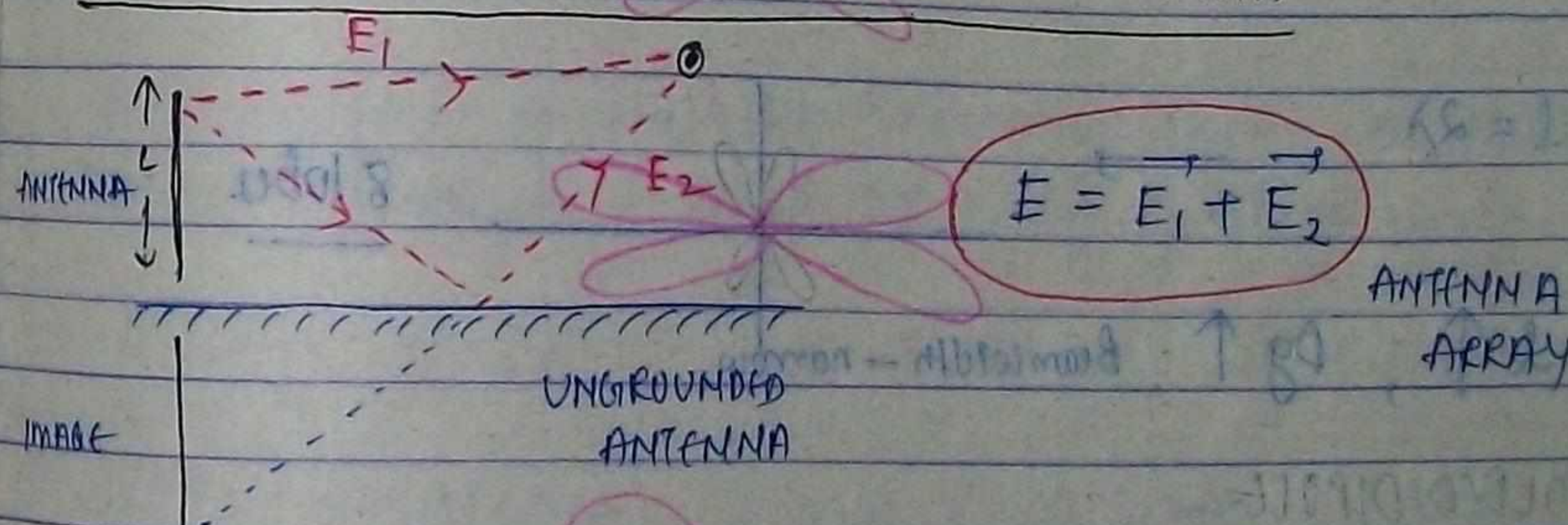
$$R_r = 73 \Omega$$

$$Z_{in} = 73 + j 42.5 \Omega$$

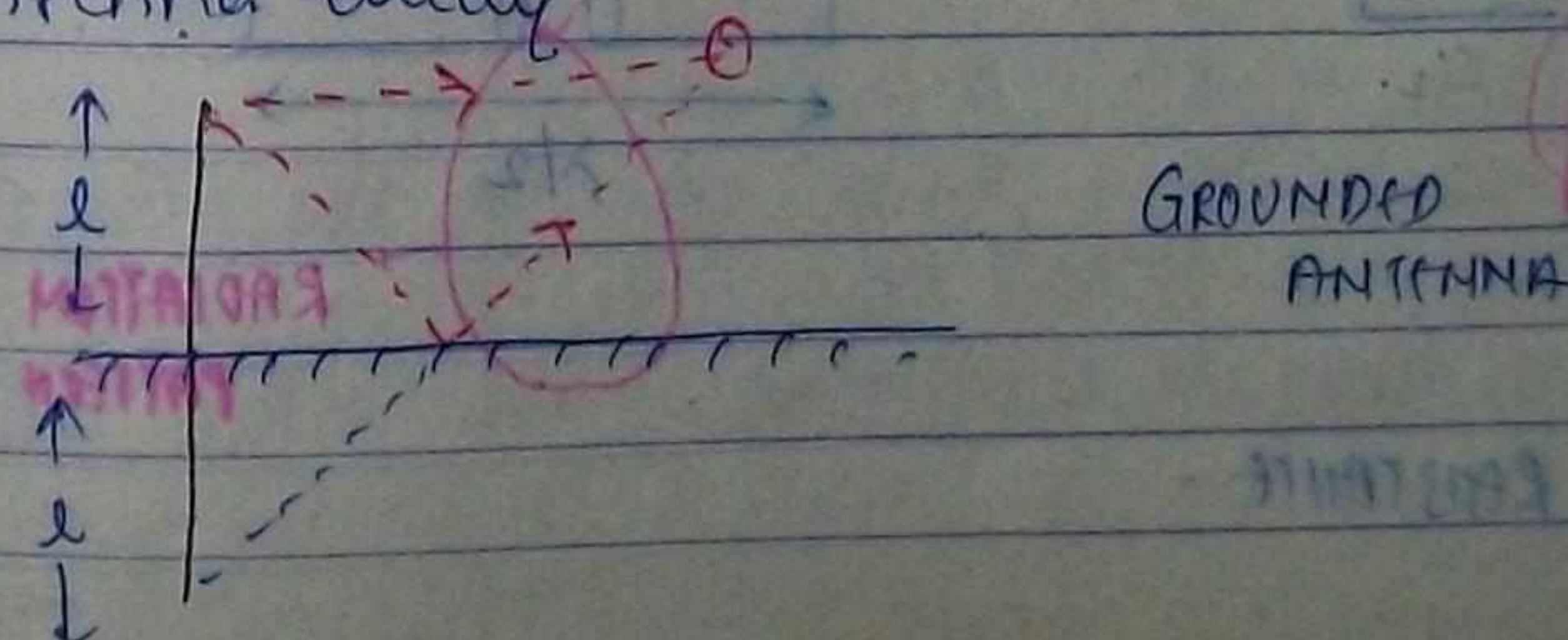
$$\text{HPBW} = 78^\circ$$

$$h_{eff} = \frac{I_{avg}}{I_{max}} \times l = \frac{2 I_m}{\pi} \times \frac{\lambda}{2} = \frac{\lambda}{\pi}$$

⇒ EFFECT OF GROUND ON RADIATION OF ANTENNA -

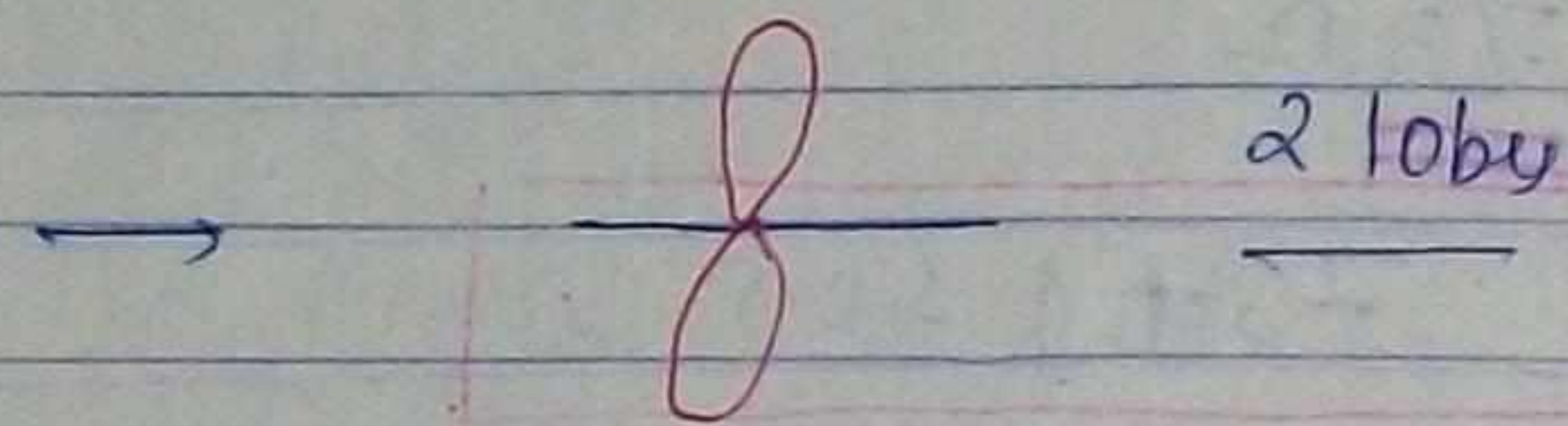


- When antenna is placed near to ground (ungrounded antenna) will act like two antenna element or antenna array

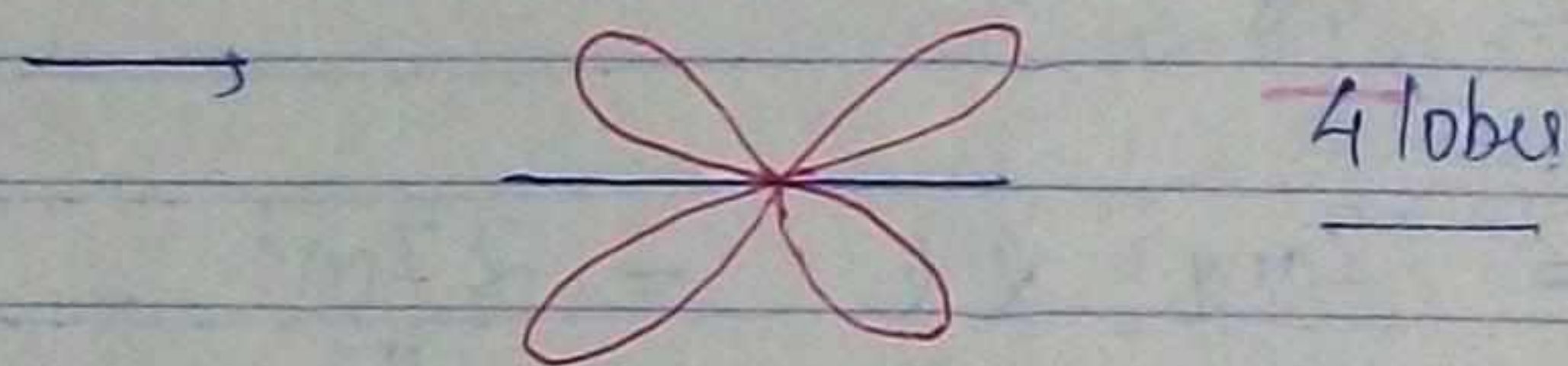


- when antenna is grounded it will act a single element but double the value.

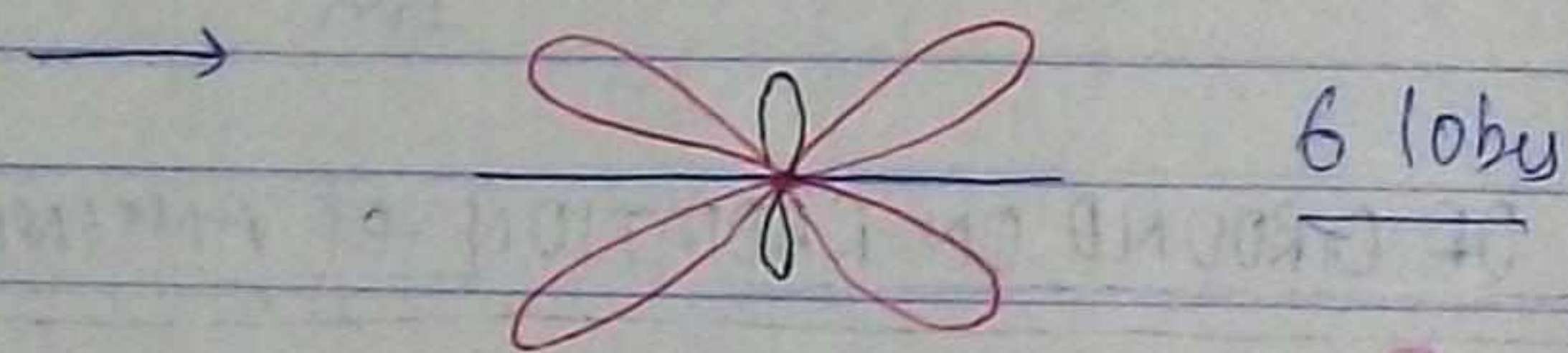
for $l = \frac{\lambda}{2}$



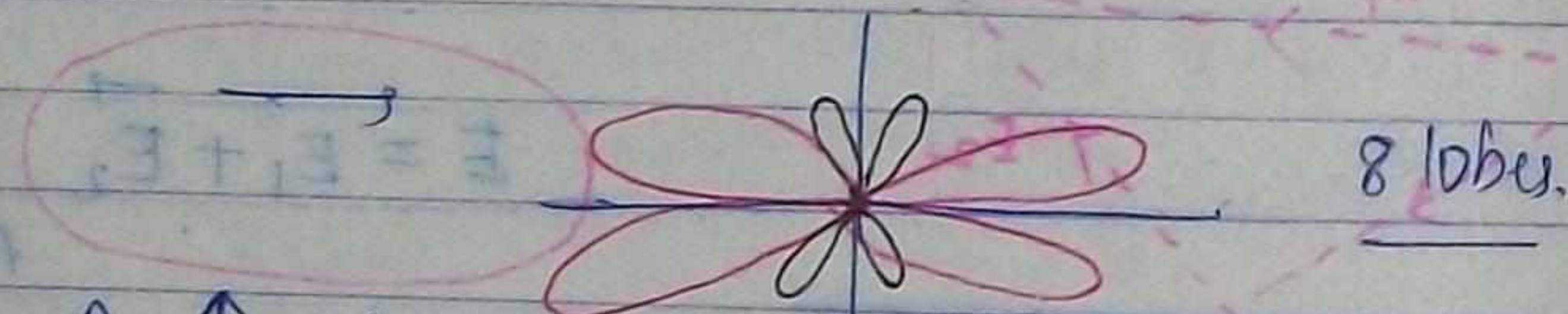
for $l = \lambda$



for $l = \frac{3\lambda}{2}$

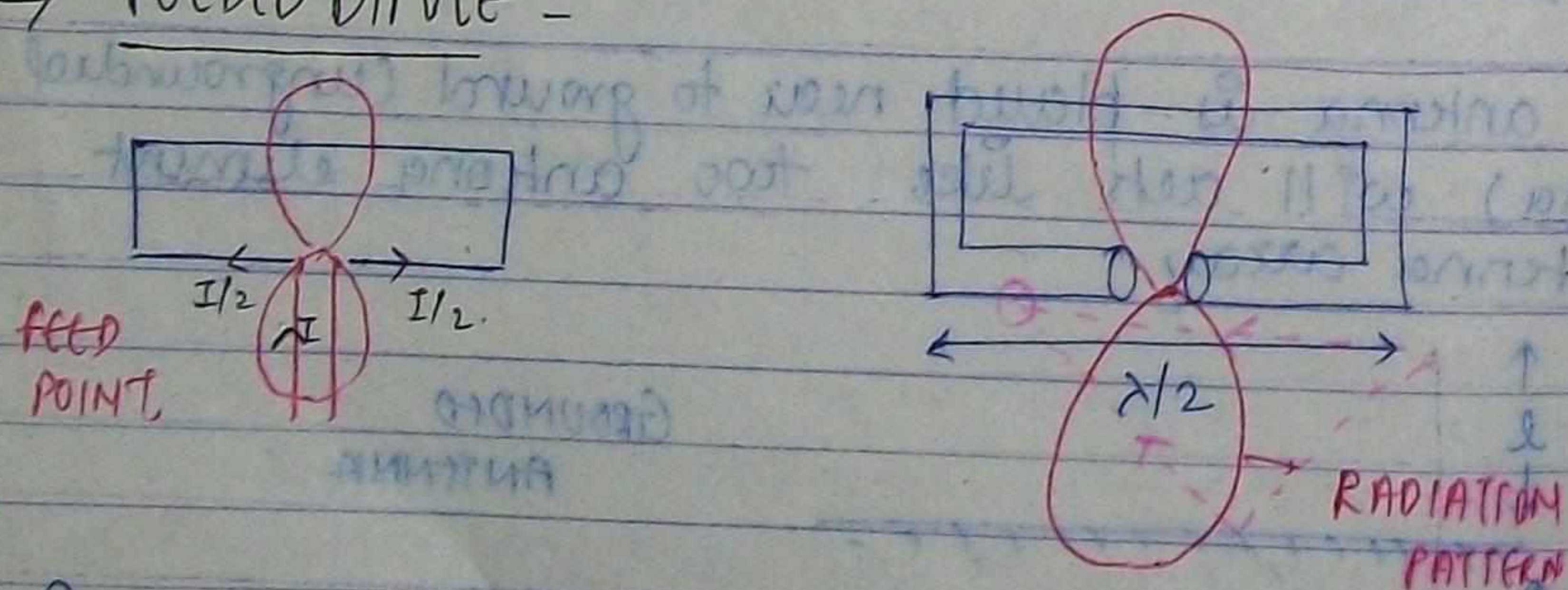


for $l = 2\lambda$



∴ $l \uparrow, R_r \uparrow$, Beamwidth $\rightarrow$ narrow

⇒ FOLDED DIPOLE -



- RADIATION RESISTANCE -

$$\text{Power radiated by folded dipole} = \text{Power radiated by Half wave dipole}$$

$$\left(\frac{I}{2}\right)^2 \times R_{rf} = I^2 \times 73$$

$$R_{rf} = 292 \Omega \approx 300 \Omega$$

- Radiation pattern is of figure of 8

$$R_r = 292 \Omega$$

- folded dipole is Broadband antenna.

- folded dipole characteristic impedance easily matches with characteristic impedance of feed wire.

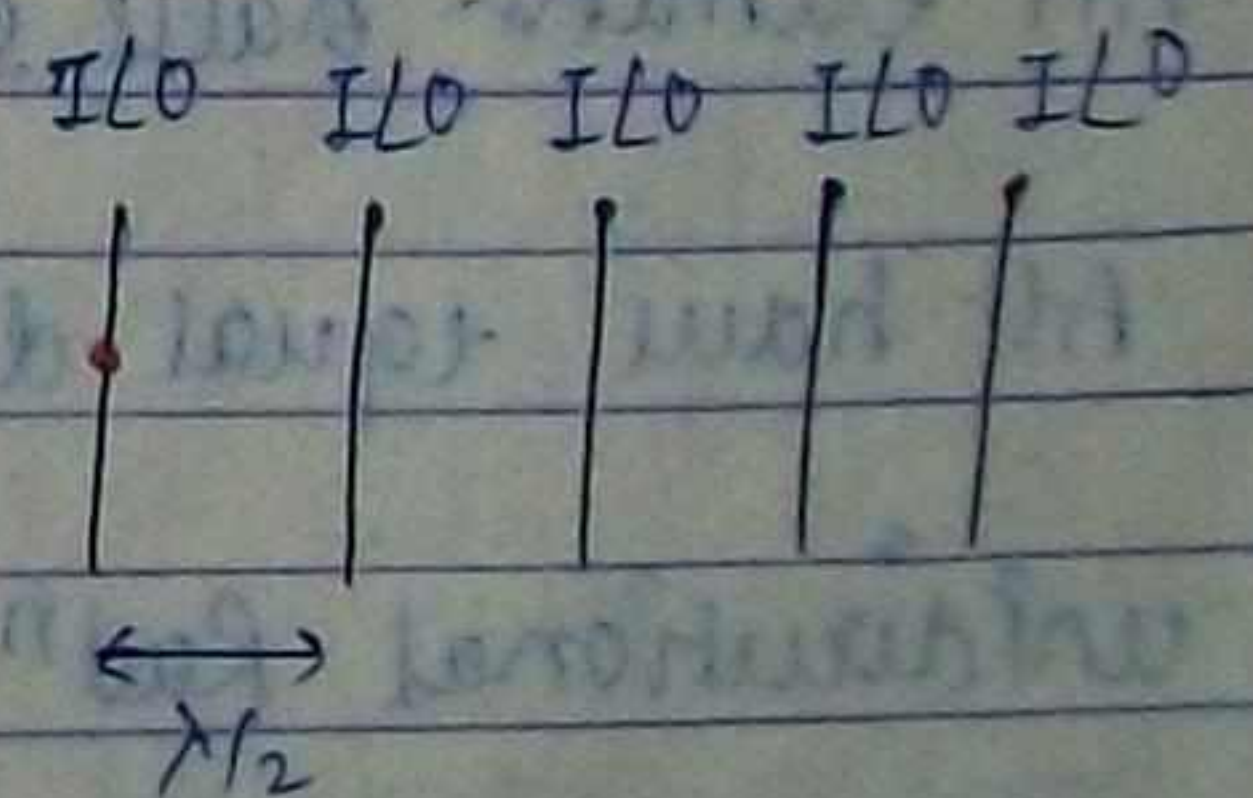
⇒ ANTENNA ARRAYS -

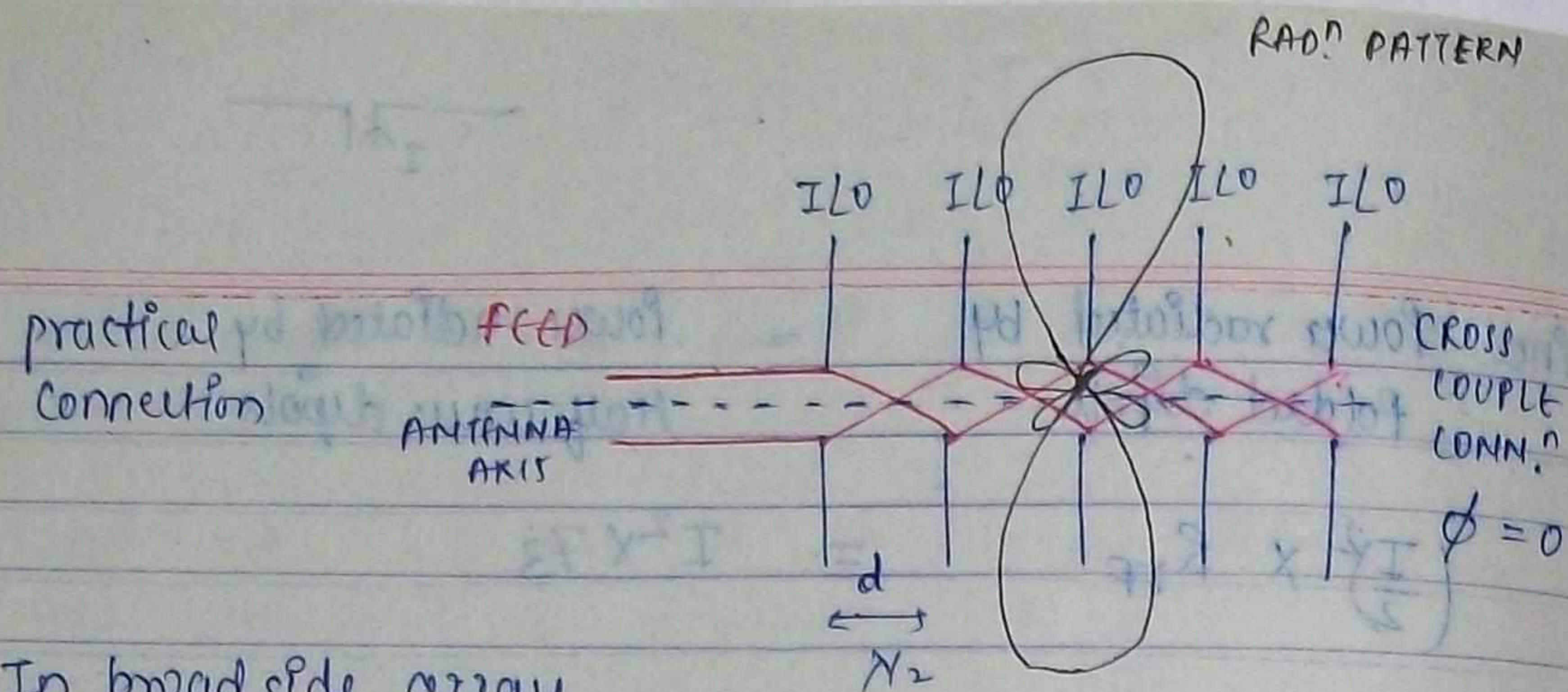
① BROAD SIDE ARRAY -

- all elements have equal length of $2\lambda - 8\lambda$

- same spacing b/w elements ($\lambda/2$)

- excited with same amplitude & phase current





- In broadside array Radⁿ pattern along the axis is minimum due to cancellation of Radⁿ of one element by another element

$$\text{length of array in wavelength} = \frac{n \cdot d}{\lambda}$$

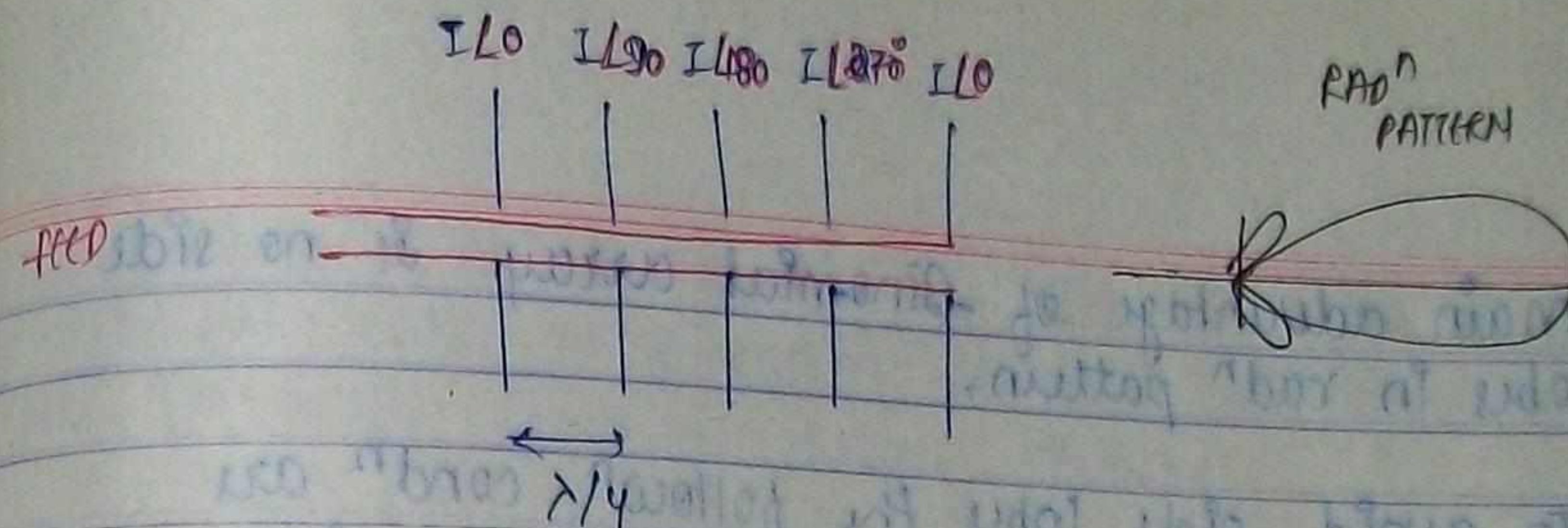
$$\text{FIRST NULL BEAM WIDTH} = 2 \times \frac{\text{LENGTH OF ARRAY IN WAVELENGTH}}{\lambda}$$

$$\text{HPBW} = \frac{\lambda}{n \cdot d}$$

$$\text{FNBW} = \frac{2\lambda}{n \cdot d}$$

② END FIRE ARRAY -

- All element have same length ($2\lambda - 8\lambda$)
- All have equal distance $d = \lambda/4$ (phase shift = 90°)
- unidirectional Radⁿ pattern
- Both Broadside & end fire antenna are used as transmitting antennas



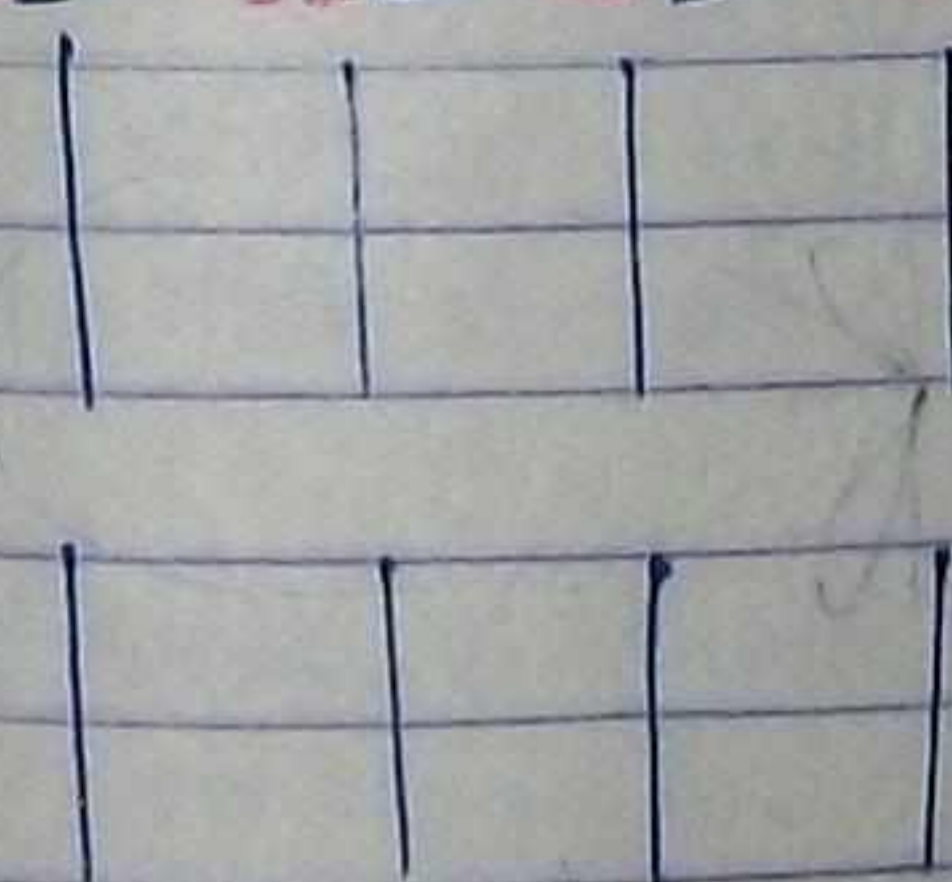
$$\text{FNBW} = 2 \times \sqrt{\frac{2 \times \text{length of array in wavelength}}{\lambda}}$$

$$\text{FNBW} = \sqrt{\frac{8\lambda}{n \cdot d}}$$

- The disadvantage of Broadside & end fire array is sidelobe present in the radⁿ pattern

③ BINOMIAL ARRAY -

ILO 3ILO 3ILO ILO



RADⁿ PATTERN.

Elements

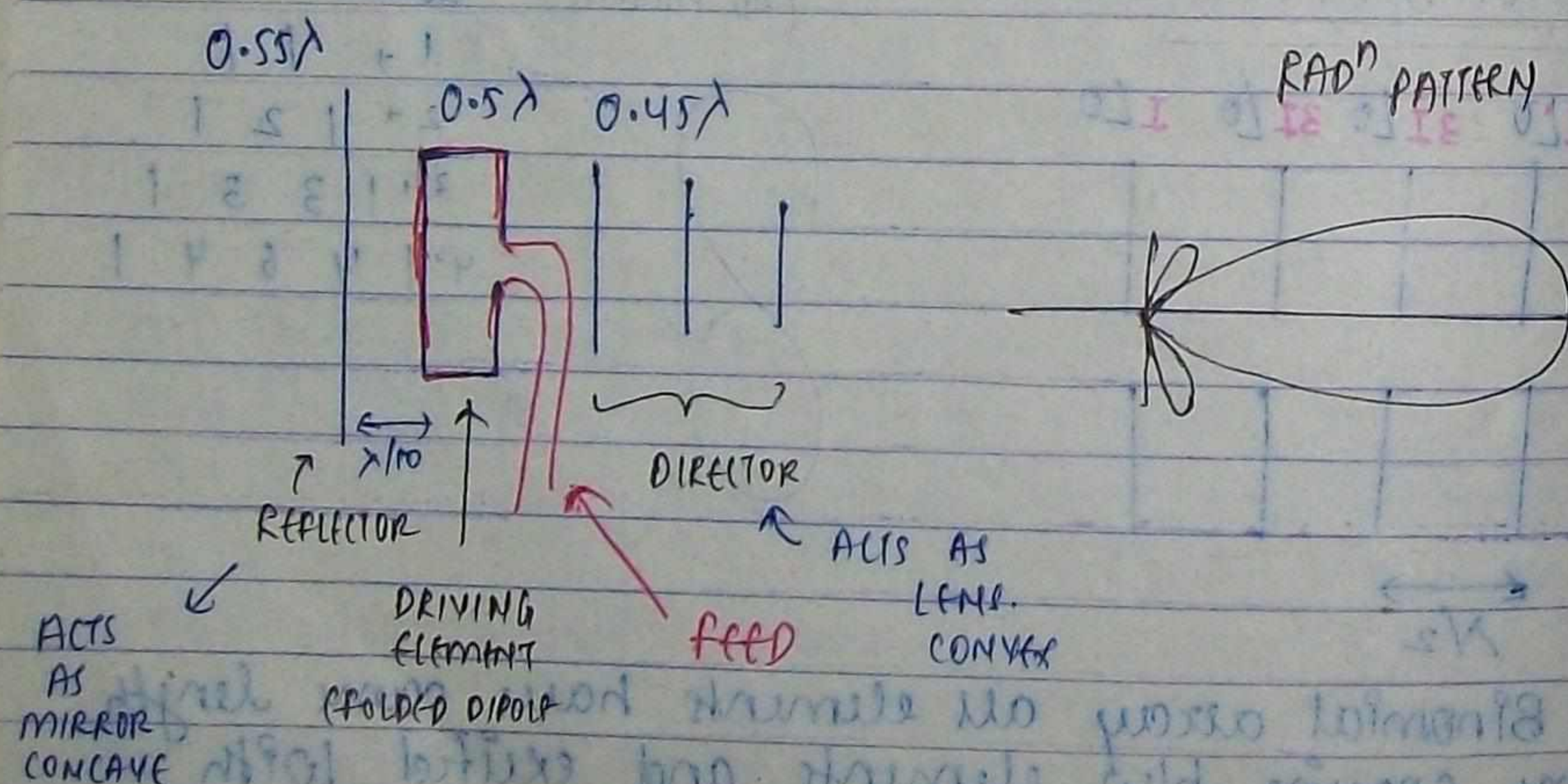
1	→	1	1	1		
2	→	1	2	1		
3	→	1	3	3	1	
4	→	1	4	6	4	1

- In Binomial array all elements have same length same spacing b/w elements and excited with same phase current but amplitudes are proportional to binomial coefficients

- Main advantage of Binomial array is no side lobes in radⁿ pattern.
- To avoid side lobes the followⁿ condⁿ are required to meet in binomial array:
 1. The spacing must not exceed $\lambda/2$.
 2. minimum no. of element should be three.
 3. Excited current are prop. to binomial coefficient.

④ YAGI-UDA ANTENNA ARRAY -

1. Driven element
2. Parasitic elements
 - Reflector
 - Directors.



- All elements have different lengths

- In yagi-uda array signal is applied only to the driven element, where as parasitic element will work under the induction field of driven element.

- Reflector will increase signal strength in opposite direction of EM wave where as directors will increase same direction of EM wave.

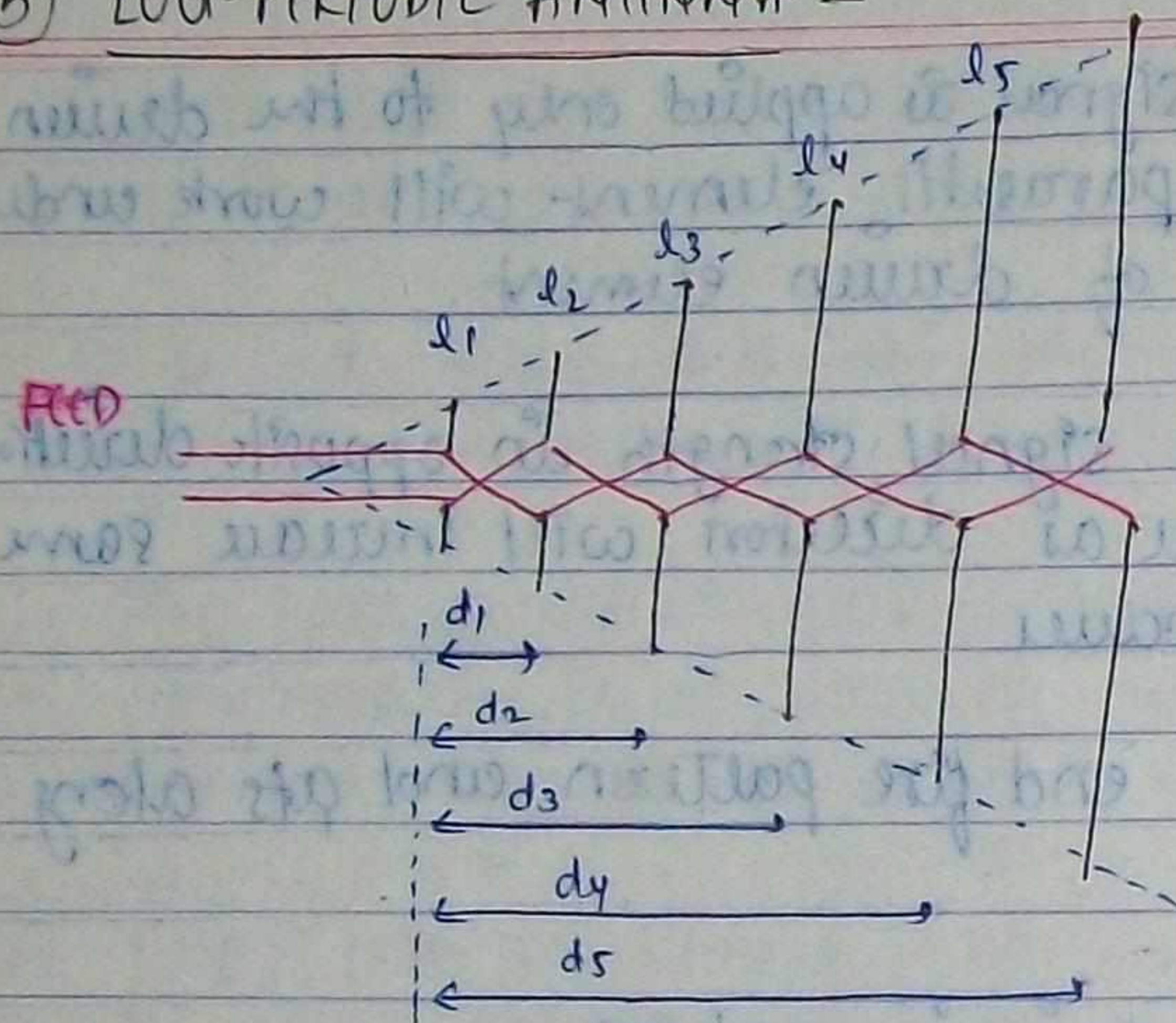
- The Radⁿ pattern is end fire pattern and pts along the axis.

- It is used as Tx receiving antenna, Yagi-uda antenna

HF → Tx. antenna
VHF, UHF → Receiving antenna

- It is also called super gain antenna. (5-7 dB)
- Yagi-uda array is a broadband antenna.
- Due to use of folded dipole it has higher I/P impedance
- The basic disadvantage of Yagi-uda antenna is low gain when compared to microwave antenna

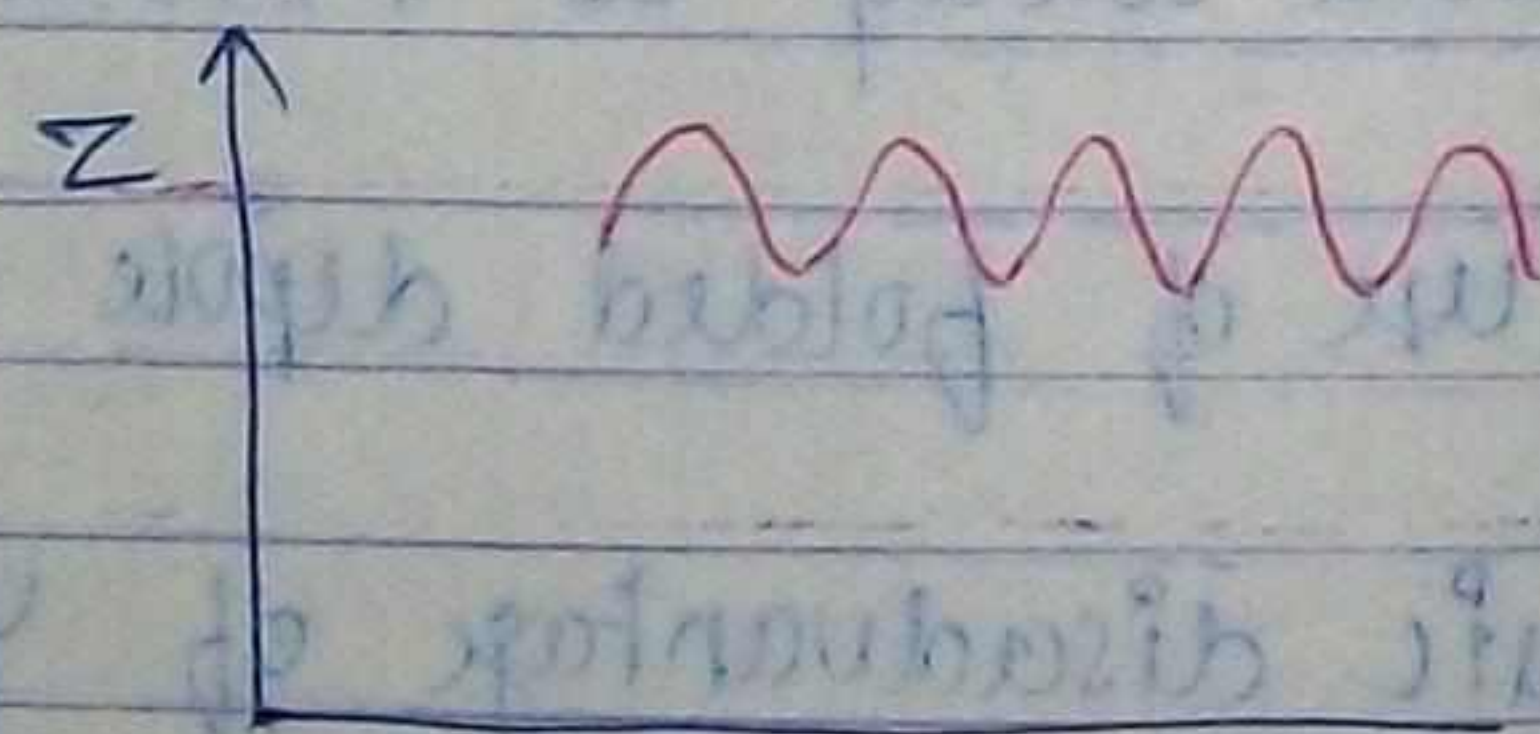
⑤ LOG-PERIODIC ANTENNA -



$$\frac{l_2}{l_1} = \frac{l_3}{l_2} = \frac{l_4}{l_3} = \dots = \frac{d_2}{d_1} = \frac{d_3}{d_2} = \frac{d_4}{d_3} = \dots$$

- log period antenna is broad band antenna.
- It is used as multichannel TV receiving antenna.

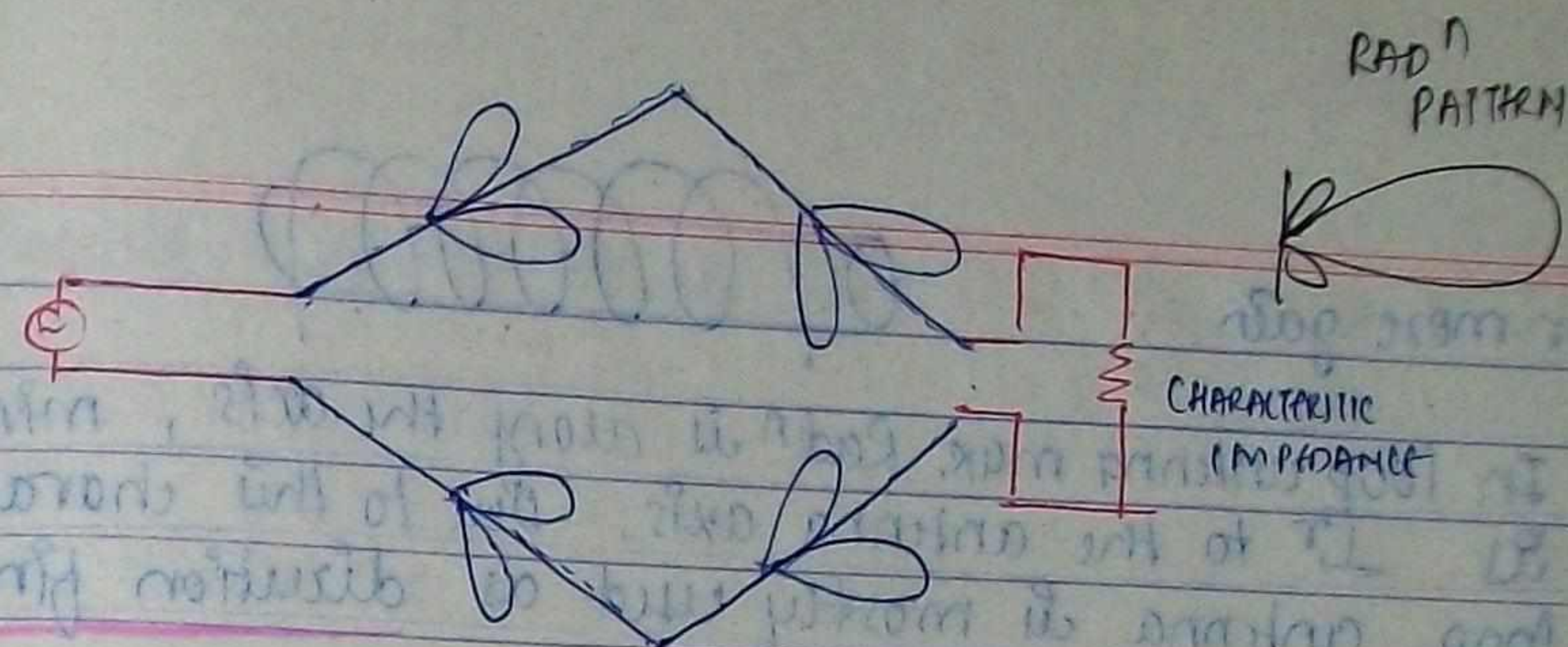
- Characteristic is of periodic nature hence it is called log periodic antenna.



⇒ RHOMBIC ANTENNA -

(EXCITING)

- In Rhombic antenna one end is provided with signal and other end is terminated with characteristic impedance.

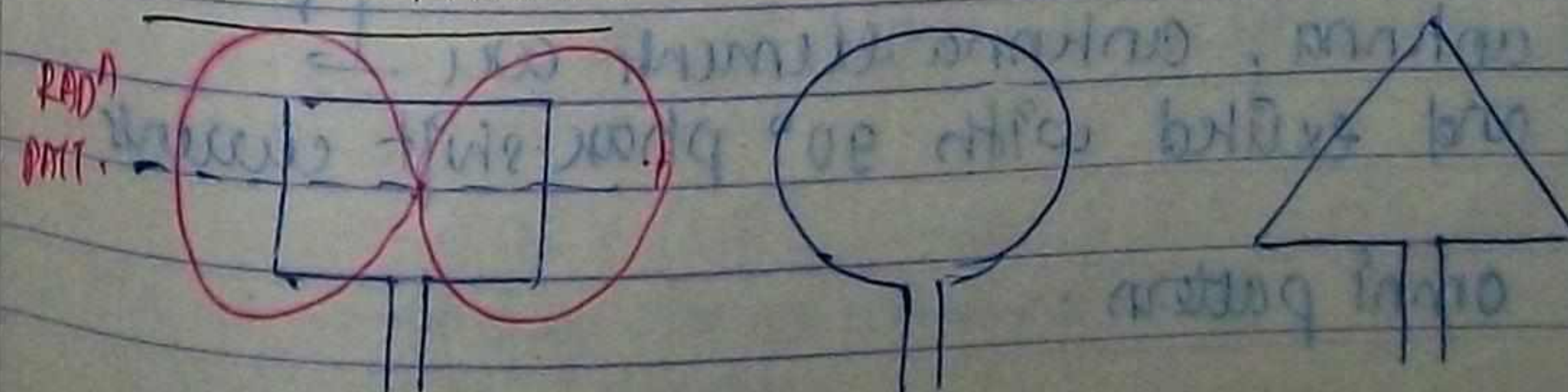


- It is a non-Resonant antenna.
- Due to this termination only forward travelling wave will present in antenna and there is no standing wave. Due to this radⁿ pattern is unidirectional and more directive.
- Rhombic antenna is broad band antenna.

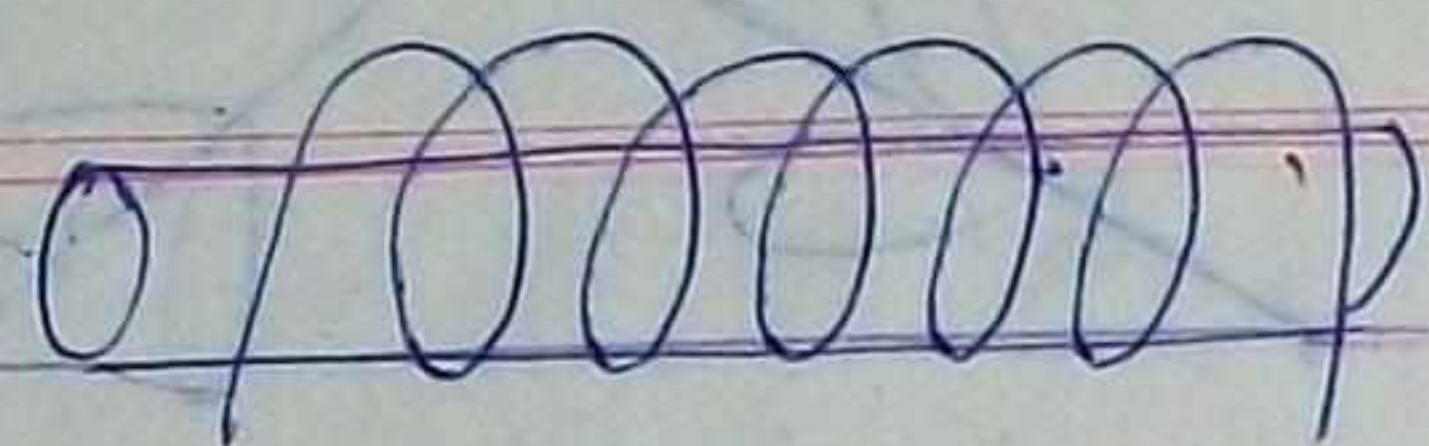
	$\lambda/2$ (m)	8λ
Reson. (Dg)	1.64	7.2
Non Reson. (Dg)	3.1	17.4

- Non Reson. antenna have more Dg compare to Resonant antenna of same length.

⇒ LOOP ANTENNA -



for more gain



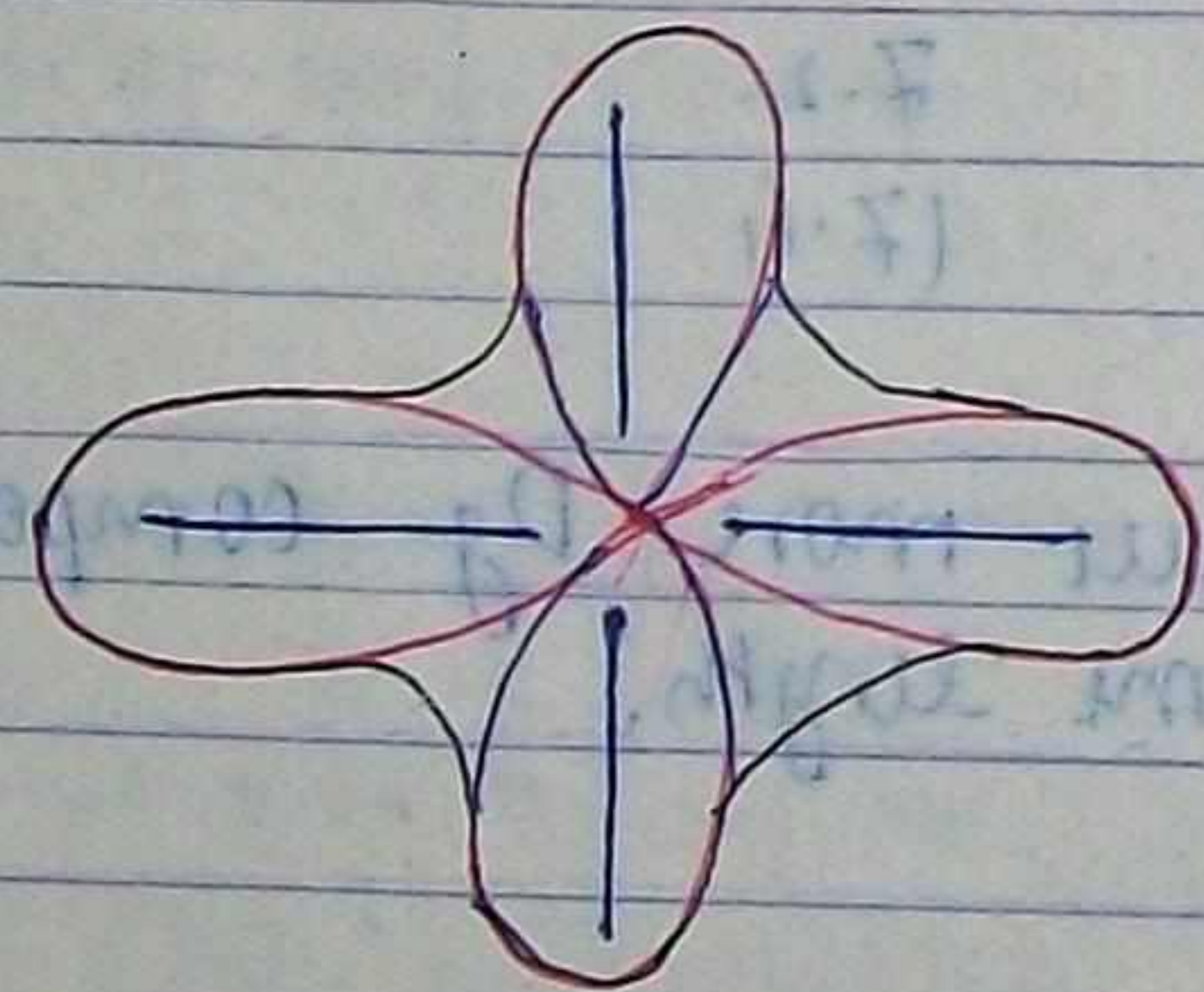
- In loop antenna max. Radⁿ is along the axis, min. radⁿ is $\perp$ to the antenna axis. Due to this characteristic loop antenna is mostly used as direction finding antenna.

- loop antenna can be also used as radio receiving antenna. (in medium freq.)

- HPBW = 90°

- If loop antenna is rotated 360° then 2 maxima and 2 minima will occur in radⁿ pattern.

⇒ TURNSTILE ANTENNA - (Omni antenna)



- In turnstile antenna, antenna elements are $\perp$ to each other and excited with 90° phase shift currents.

- Radⁿ pattern is omni pattern.

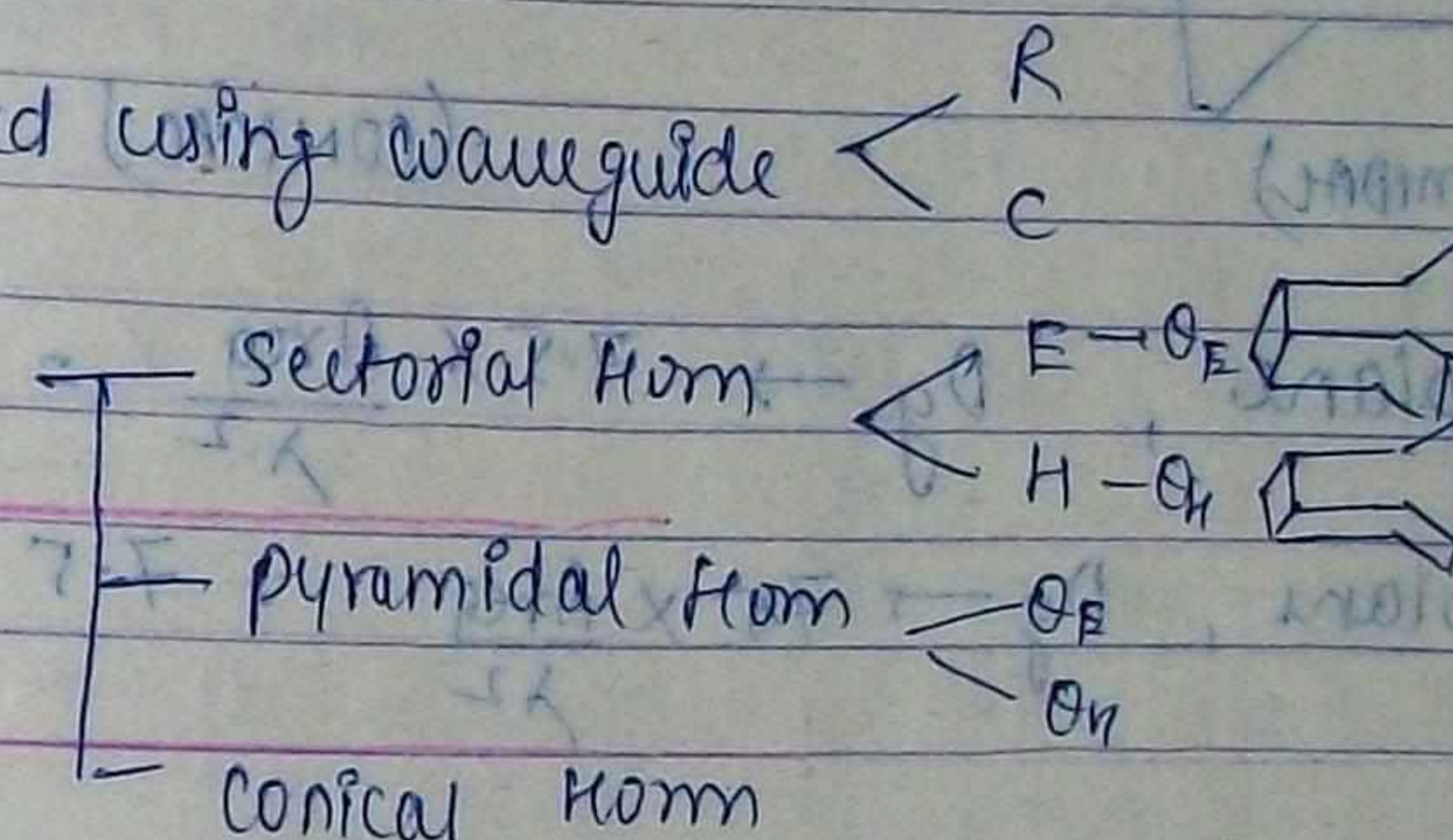
- It is mostly used as TV Tx. antenna.

⇒ MICROWAVE ANTENNA -

① HORN ANTENNA -

- It is constructed using waveguide

- Horn antenna



- In rectangular waveguide one end is excited with signal and other side is flared to get horn antenna. Based on the flaring of other end with rectangular w.g. either sectorial horn or pyramidal horn antenna can be constructed.

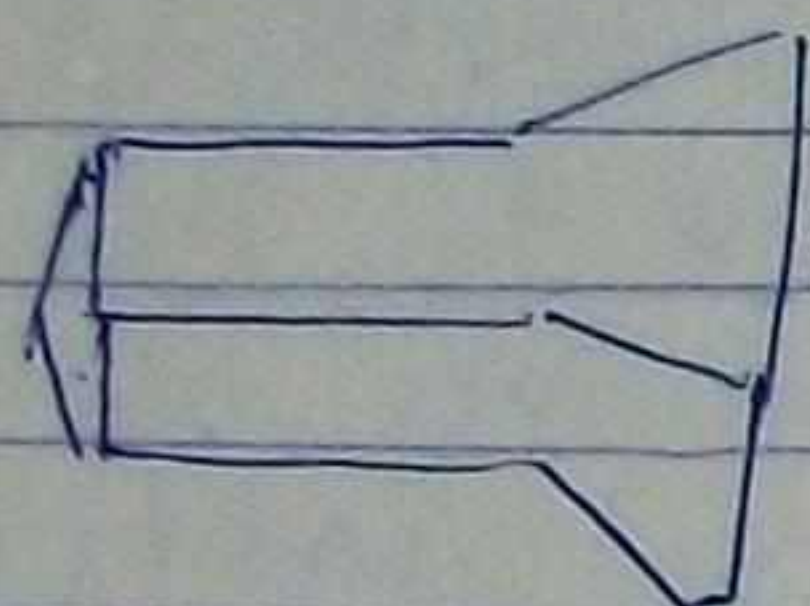
- In sectorial horn antenna there are two types:

① E-plane ② H-plane.

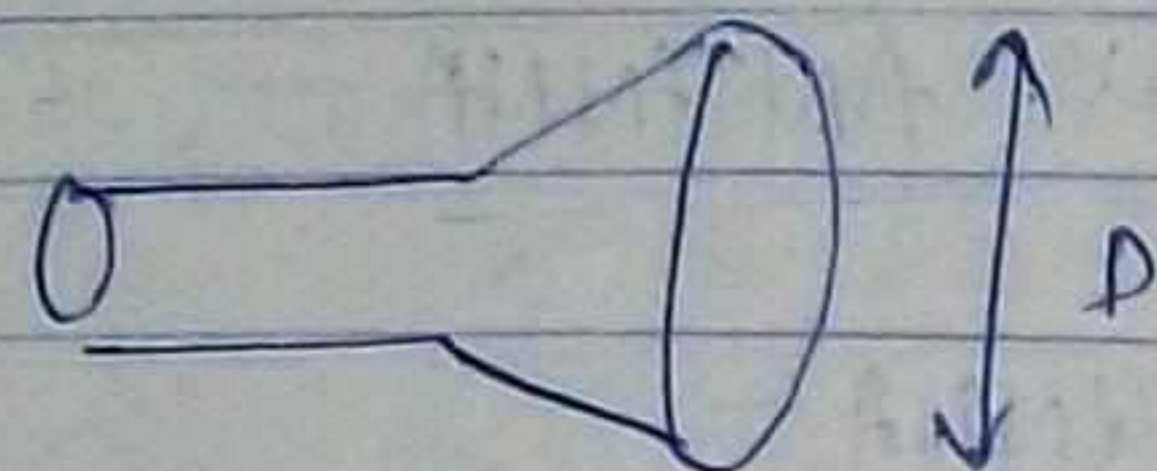
In E-plane Horn antenna b-dimension or height of the waveguide is increased. In H-plane Horn antenna a-dimension or width of waveguide is increased.

- In pyramidal horn antenna both height as well as width is increased.

- Conical horn antenna can be constructed using circular waveguide.



(PYRAMIDAL)



(CONICAL)

- E-plane, $D_g \rightarrow 7.5 \times \frac{\text{Area}}{\lambda^2} = 7.5 \times \frac{hw}{\lambda^2}$
- H-plane, $D_g \rightarrow 7.5 \times \frac{\text{Area}}{\lambda^2} = 7.5 \times \frac{hw}{\lambda^2}$

E plane, $\Theta_{HPBW} = 56 \times \frac{\lambda}{h}$

H plane, $\Theta_{HPBW} = 67 \times \frac{\lambda}{w}$

$D_g \propto 1/\lambda^2$, $D_g \propto A$

$HPBW \propto 1/\text{length}$, $HPBW \propto \lambda$

$f \uparrow$ $D_g \uparrow$ $HPBW \downarrow$

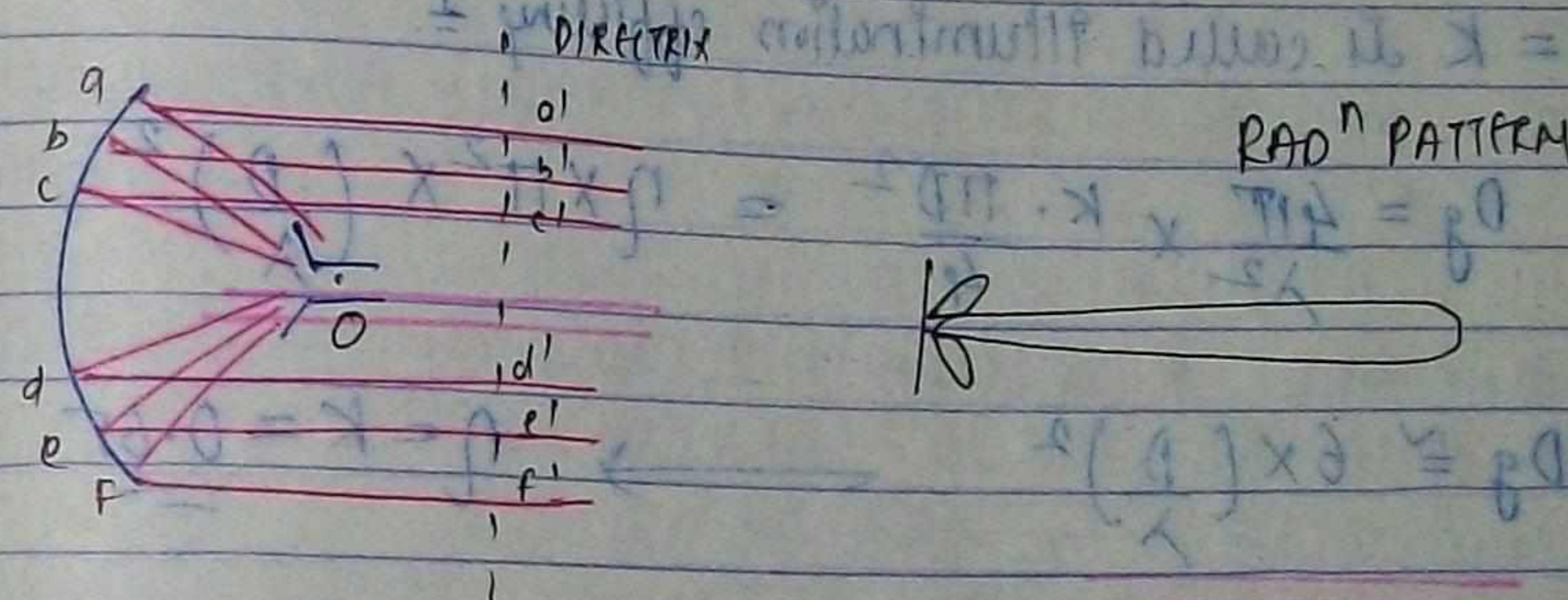
- Conical horn $\rightarrow$ HPBW $\rightarrow 61.5^\circ \times \frac{\lambda}{D}$

$D_g = 7.5 \times \frac{\text{Area}}{\lambda^2} = 7.5 \times \frac{\pi D^2}{4 \lambda^2}$

$D_g = (7.5 \times \frac{\pi}{4}) \cdot (\frac{D}{\lambda})^2 \approx 6 (\frac{D}{\lambda})^2$

② PARABOLIC REFLECTOR -

- It is a secondary antenna



$oa + aa' = ob + bb' = oc + cc' = od + dd'$

- The basic principle of parabolic reflector is travelling of EM wave equal path lengths from focal point to directrix, due to this all EM waves will reach same phase hence Radⁿ pattern is highly directive unidirectional and fire pattern.

- Parabolic reflector requires a primary antenna to radiated EM wave either Horn antenna or Half wave dipole is used as primary antenna in parabolic reflector.

- This can be used in both Tx and Receiving antenna.

$$A_e = \frac{\lambda^2}{4\pi} \times D_g \quad , \quad D_g = \frac{4\pi}{\lambda^2} \times A_e$$

$$D_g = \frac{4\pi}{\lambda^2} \times K \cdot A$$

$\eta = K$ is called illumination efficiency =

$$D_g = \frac{4\pi}{\lambda^2} \times K \cdot \frac{\pi D^2}{4} = \eta \times \pi^2 \times \left(\frac{D}{\lambda}\right)^2$$

$$D_g \approx 6 \times \left(\frac{D}{\lambda}\right)^2 \quad \longrightarrow \quad \eta = K = 0.65$$

$$D_g \propto D^2, \quad D_g \propto f^2, \quad D_g \propto 1/\lambda^2$$

Ques. In a parabolic reflector the $D = 10\text{m}$ and operating at 10GHz freq the D_g of parabolic reflector is?

Soln: $D_g \approx 6 \times \left(\frac{10}{0.03}\right)^2$ $\lambda = \frac{3 \times 10^8}{10 \times 10^9}$

$$D_g \approx \frac{6}{9} \times 10^6$$

$$\approx 0.666 \times 10^6$$

$$\approx 6.67 \times 10^5$$

30 m - longest diameter of parabolic reflector ever used

for parabolic reflector

$$\text{FNBW} = 140 \times \frac{\lambda}{D}$$

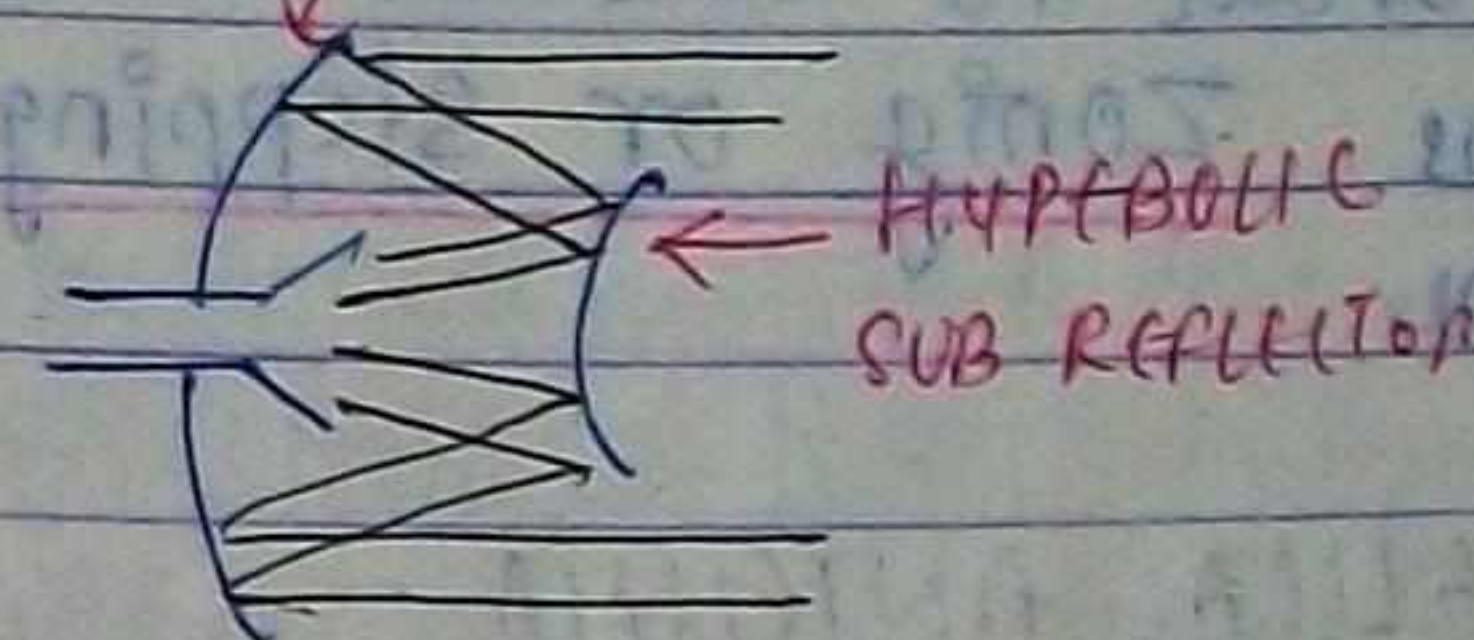
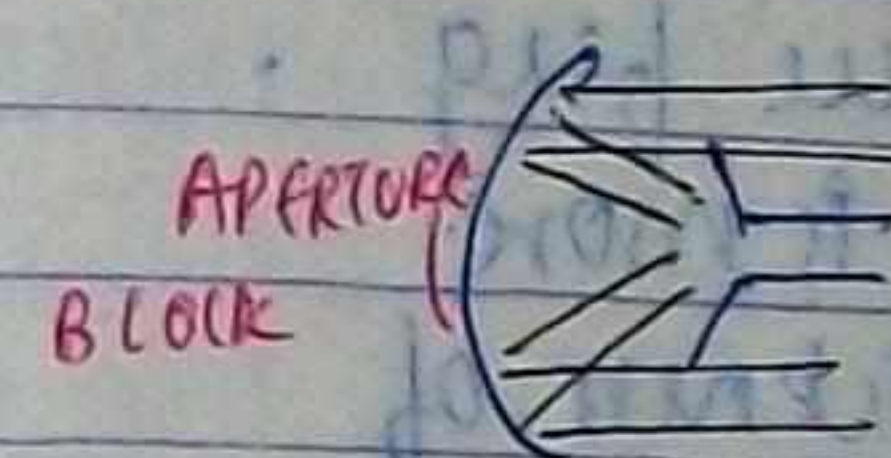
$$\text{HPBW} = 70 \times \frac{\lambda}{D}$$

⇒ FEEDS IN PARABOLIC REFLECTOR -

① Half wave dipole feed

② Horn feed

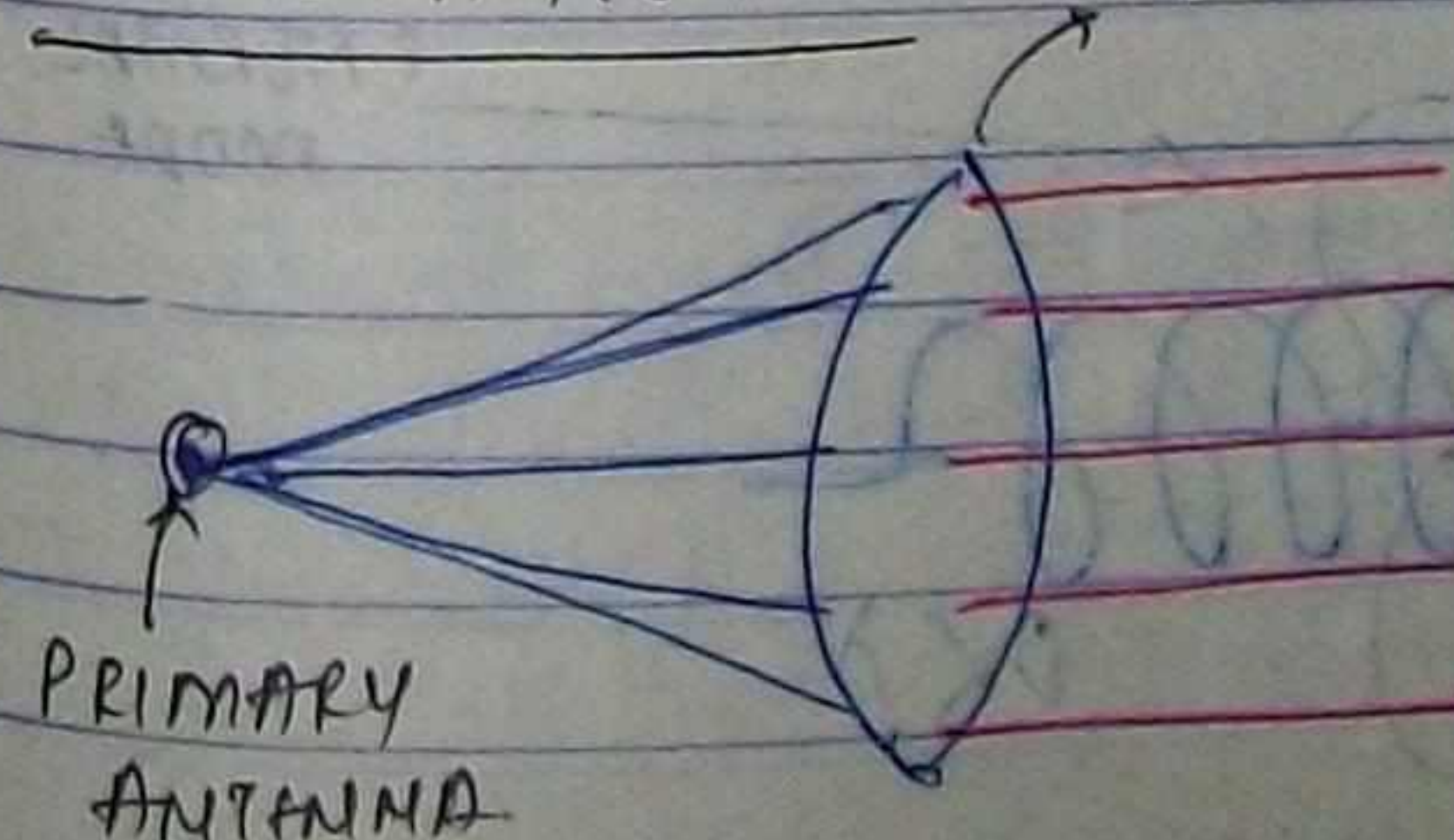
③ Cassegrain feed



- In parabolic reflector the advantage with Cassegrain feed is the feed point can be placed as per user requirement.

- Aperture blocking will present in parabolic reflector.

⇒ LENS ANTENNA - LENS ANTENNA

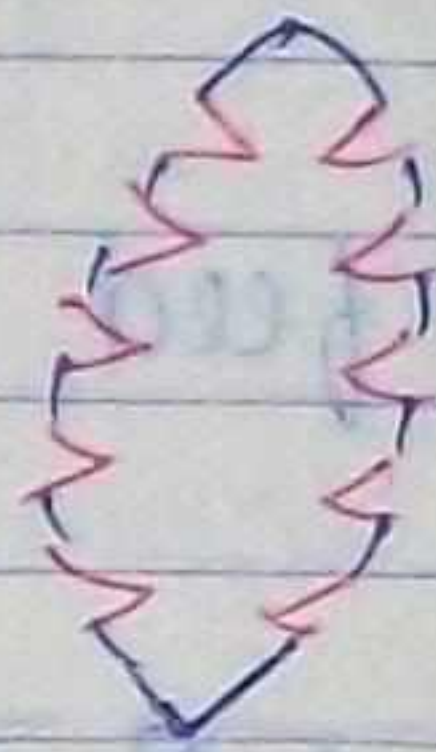


No aperture blocking

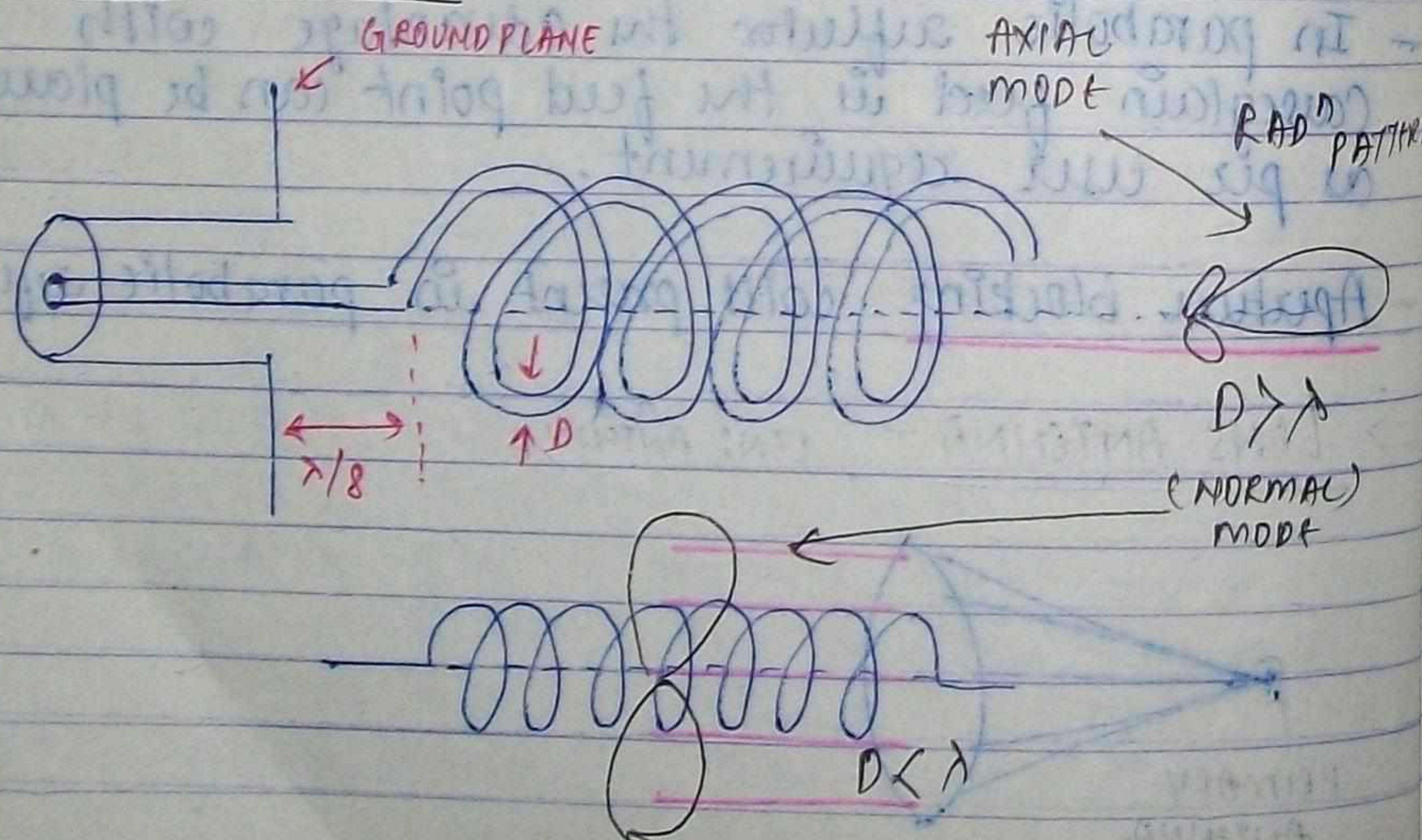
uses about 10 GHz

- To overcome aperture blocking lens antennas are used at microwave frequencies. It is also a secondary antenna.
- Lens antenna is constructed using a special material called POLYSTYRENE.
- Lens antenna is used only above 3 GHz.

ZONIG: At lower microwave freq. lens antenna thickness is more. In order to reduce thickness of lens Zonig or stepping is done.



⇒ HELICAL ANTENNA -



- Antenna is of Helical shape

- Based on dimension different modes are possible.
 1. Normal mode
 2. Axial mode or end fire mode.

- In end fire mode helical antenna will provide circular polarisation.

- Circular polarisation is used for satellite tracking.

- Helical antenna can be also used to overcome Faraday effect.

FARADAY EFFECT - Due to earth curvature and magnetic field the polarisation of EM wave will change which is known as Faraday effect or rotation.

8-11	8-11
8-12	8-12
8-13	8-13
8-14	8-14
8-15	8-15
8-16	8-16
8-17	8-17
8-18	8-18
8-19	8-19
8-20	8-20
8-21	8-21
8-22	8-22
8-23	8-23
8-24	8-24
8-25	8-25
8-26	8-26
8-27	8-27
8-28	8-28
8-29	8-29
8-30	8-30

MICROWAVE

⇒ CLASSIFICATION OF FREQUENCY

ELF (extremely low freq)	30 Hz - 300 Hz
VF (voice freq)	300 Hz - 3 kHz
VLf	3 kHz - 30 kHz
LF	30 kHz - 300 kHz
MF	300 kHz - 3 MHz
HF	3 MHz - 30 MHz
VHF	30 MHz - 300 MHz
UHF	300 MHz - 3 GHz
U wave	1 GHz - 40 GHz

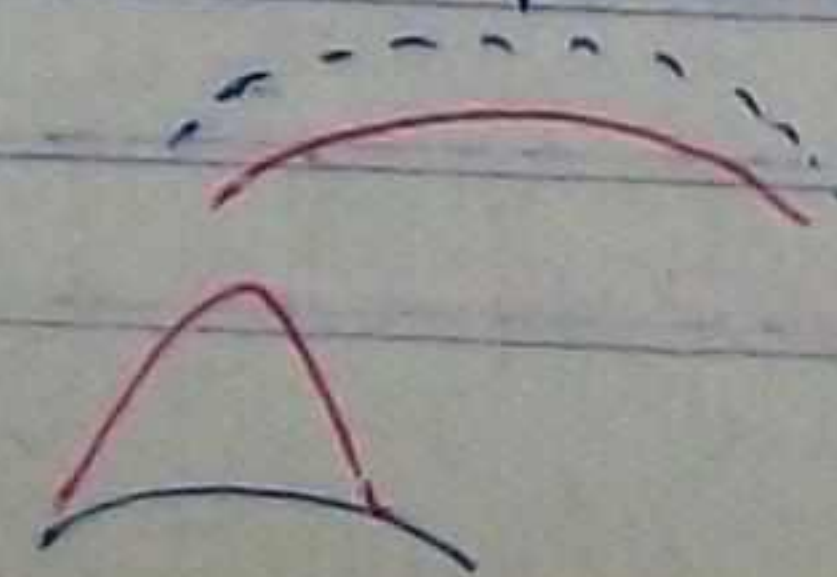
⇒ U waves -

L	1 - 2 GHz
S	2 - 4
C	4 - 8
X	8 - 12 (12.5)
Ku	12 - 18
K	18 - 27 (26.5)
Ka	27 - 40

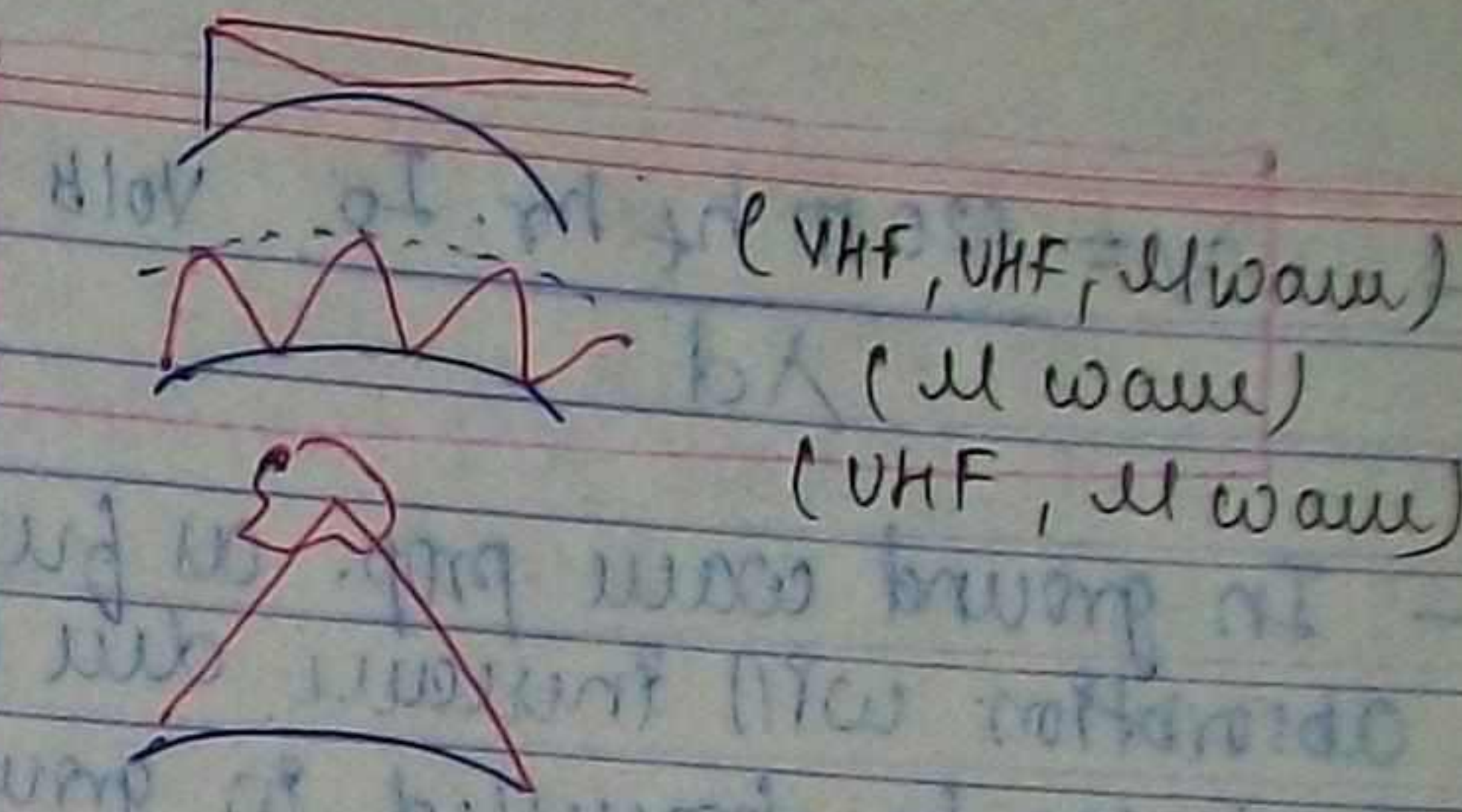
SATELLITE COMMUNICATION

⇒ WAVE PROPAGATION -

- In free space wave propagation depends on frequency of operation. The possible wave prop. are
- 1. Ground wave prop.
- 2. Sky wave prop.

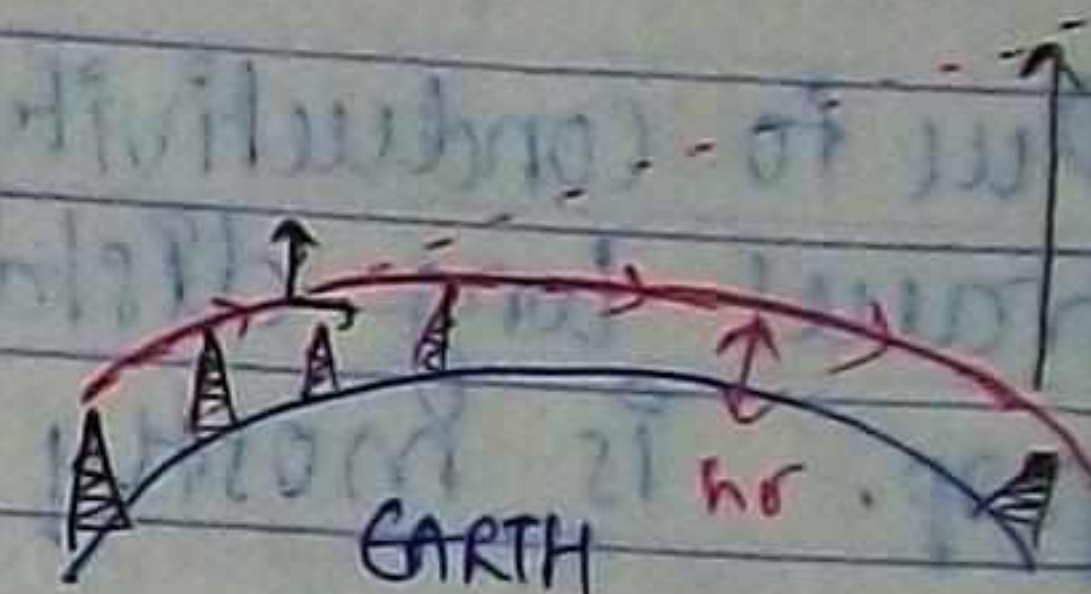


- 3. Space wave prop.
- 4. Duct wave prop.
- 5. Troposcatter prop.



① GROUND WAVE PROPAGATION -

- Ground wave prop. is mostly used in VLF, MF and LF frequency



- At lower freq earth will act as conductor due to this vertical polarizⁿ is used to overcome short circuit of horizontal polarized component.

- In ground wave propⁿ EM wave travels along the earth's curvature hence it is also known as surface wave propagation.

- TILTING - In ground wave propⁿ the EM wave will disappear after travelling certain distance due to tilting.

- Electric field at any point in ground wave is

$$E = \frac{120 \pi h_t I_0}{\pi d} \text{ V/m}$$

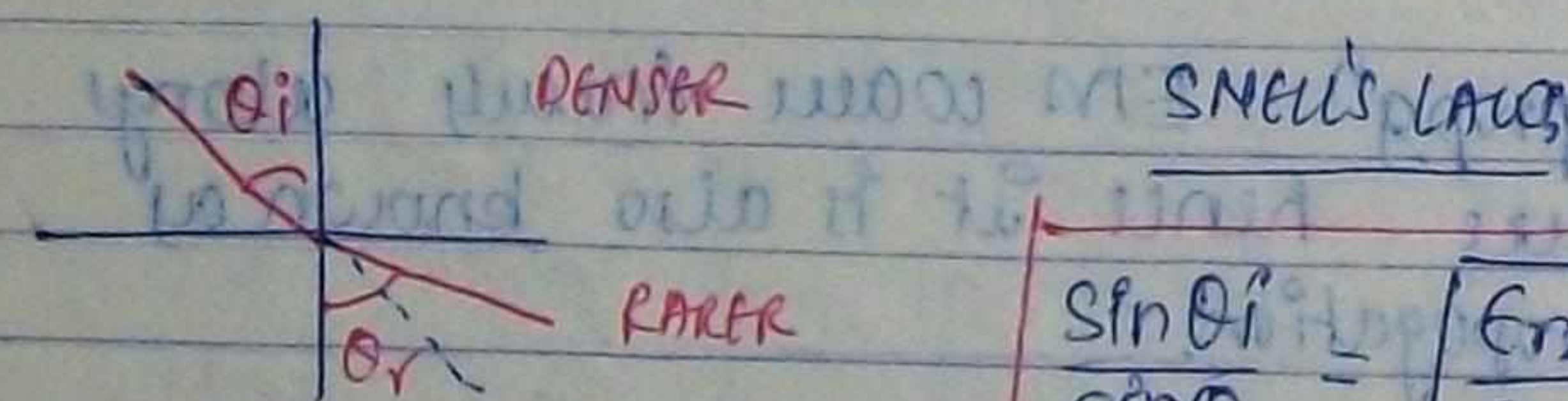
h_t = height of antenna

d = diameter of earth distance from tx. antenna

$$V = \frac{120 \pi h_f h_r I_o}{\lambda d} \text{ Volts}$$

- In ground wave prop. as freq. increase atmospheric absorption will increase due to this the max. freq that can be travelled in ground wave prop. is 2 MHz
- Due to conductivity of sea water Ground wave will travel long distance, due to this ground wave prop. is mostly used in ship to ship comm.
- Ground wave can be also used in AM broadcasting.

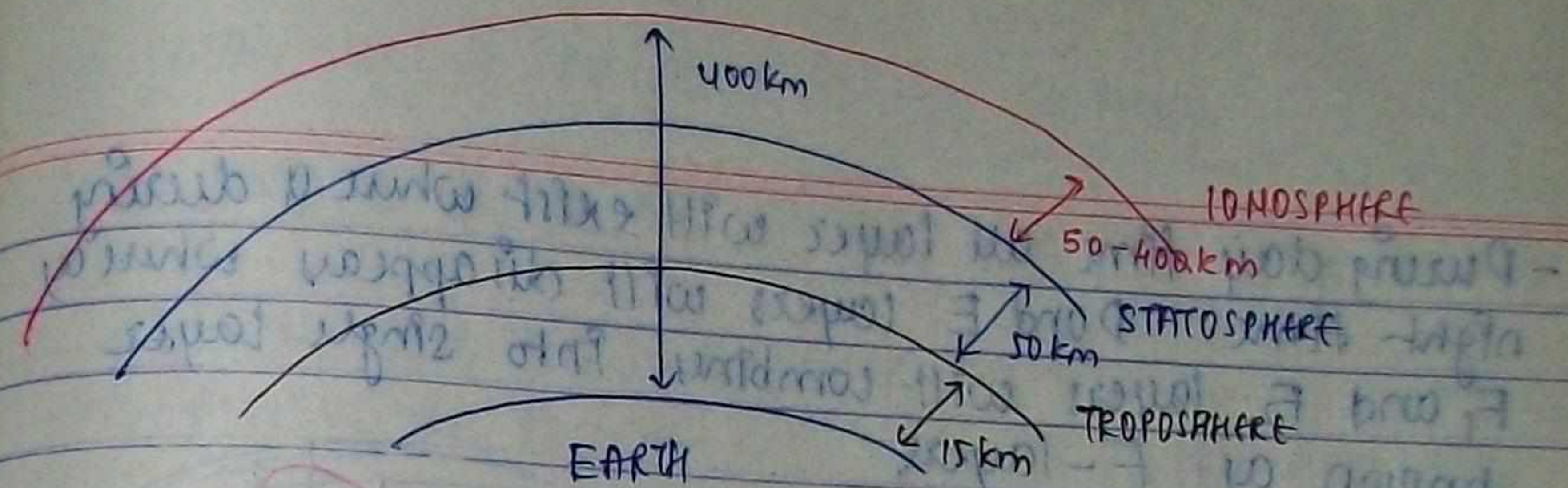
② SKY WAVE PROP.



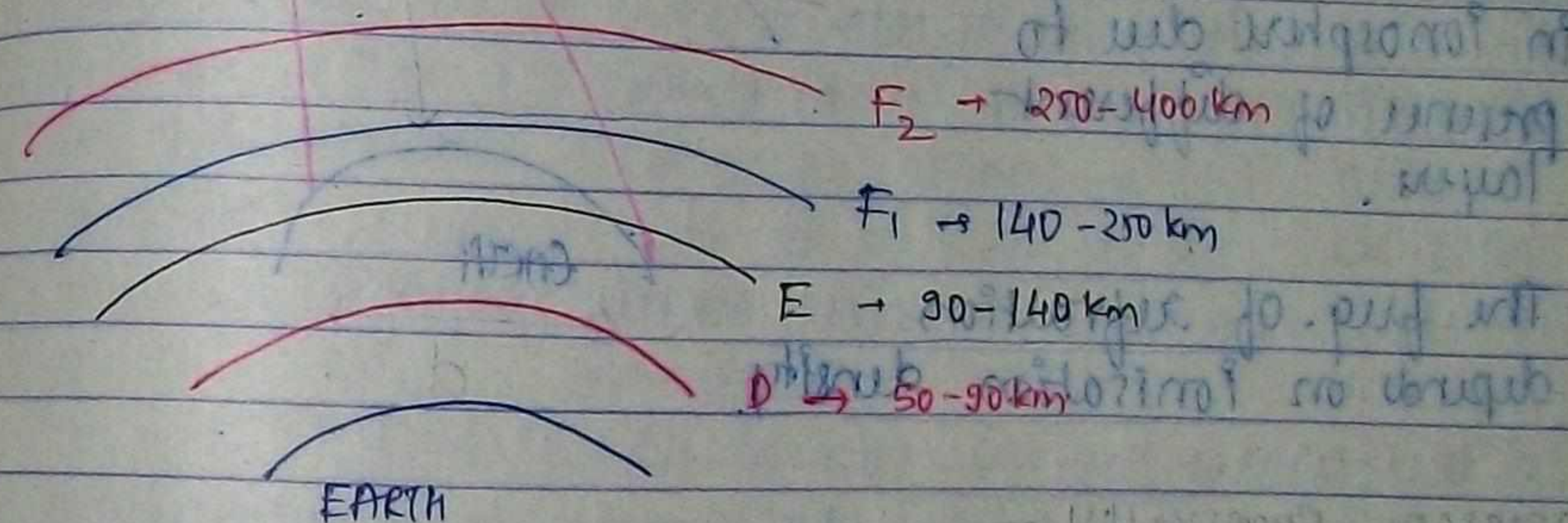
$$\frac{\sin \theta_i}{\sin \theta_r} = \frac{\sqrt{\epsilon_r}}{\sqrt{\epsilon_i}} = \frac{\mu_2}{\mu_1} = \frac{n_2}{n_1}$$

$\sqrt{\epsilon_r}$ = Refractive Index

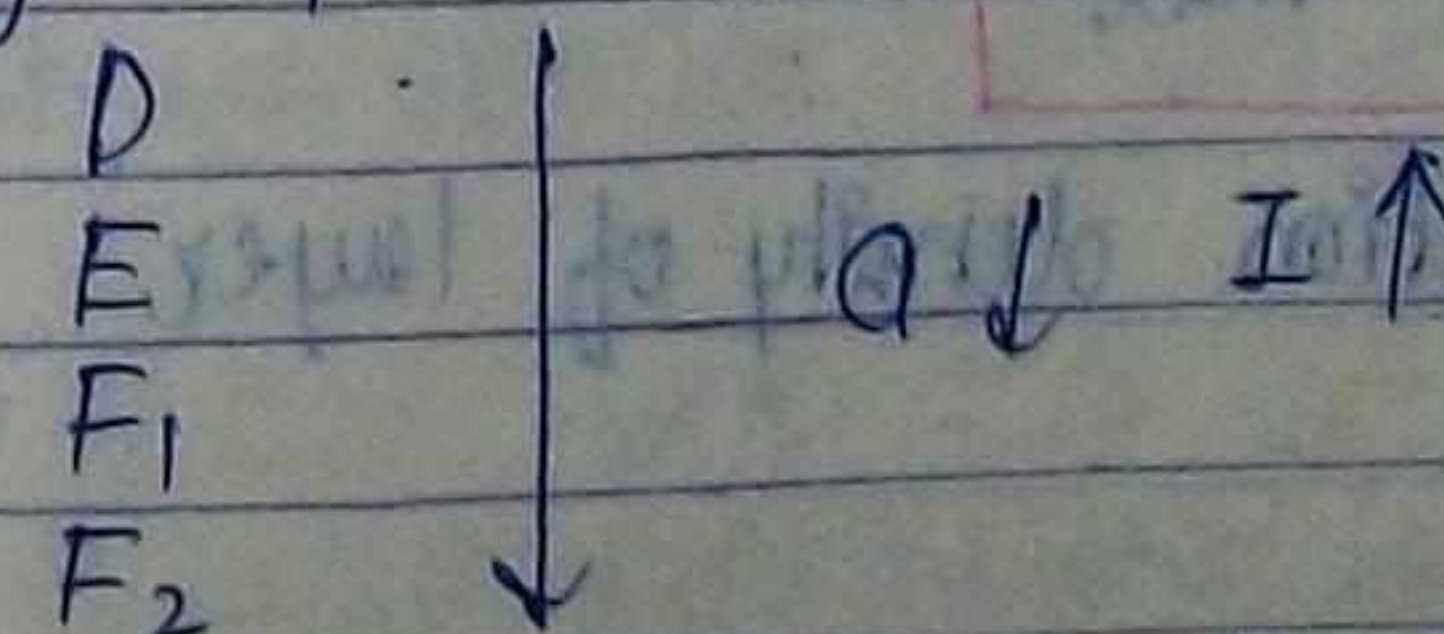
- The Basic principle of sky wave propn is refraction of EM waves in ionospheric layers. Hence it is also known as ionospheric wave propn.



Ionosphere is again classified as



- In Ionosphere due to sun energy the molecules or atoms are divided into ions or ionization will occur.
- Based on ionisation density Ionosphere can be divided into following layers

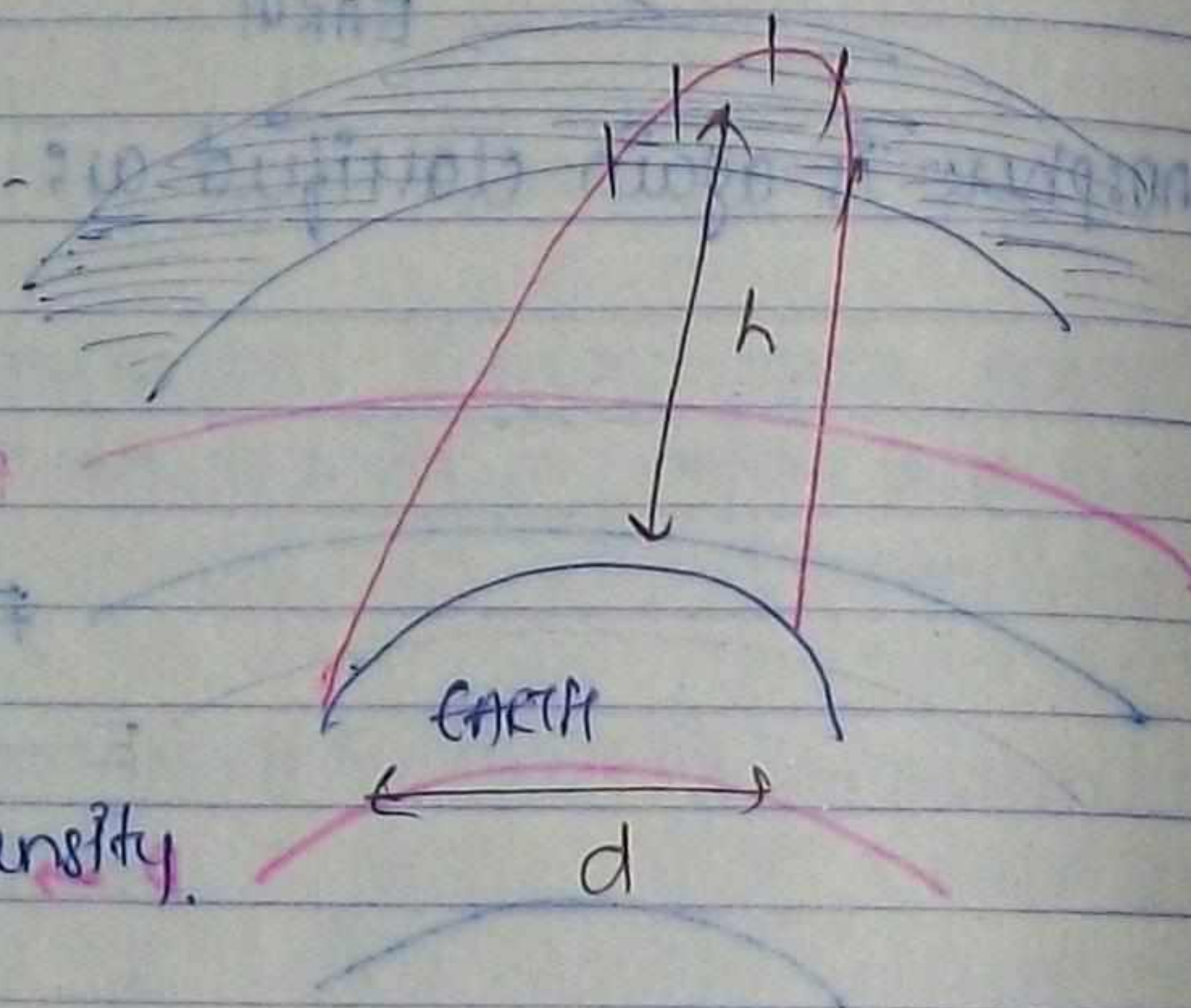


among these layers D-layer will have more atmospheric density where as the ionization density. F2 layer have highest ionization density.

- During day time all layer will exist where a during night time D and E layers will disappear where F_1 and F_2 layers will combine into single layer known as F-layers.

- In sky wave propn refraction will take place in ionosphere due to presence of different layers.

- The freq. of refraction depends on ionisation density.



CRITICAL FREQUENCY -

It is the max. frequency that can be reflected by a particular layer in ionosphere when vertically incident.

$$f_c = 9 \sqrt{N_{\max}}$$

where $N_{\max}$ = ionisation density of layer.

f_c	100 KHz
E	3-5 MHz
f_1	5-7 MHz
f_2	5-12 MHz

MAX. USEABLE FREQ (MUF) -

MUF is the max. useable freq which provides reliable comm. b/w two points on earth by a particular layer.

max. MUF used in sky wave is $\rightarrow 30 \text{ MHz}$

$$MUF = f_c \sec \theta$$

$$MUF = f_c \sqrt{1 + \left(\frac{D}{2h}\right)^2}$$

D = distance b/w two points
 h = height of the layer.

$$MUF > f_c$$

SKIP DISTANCE -

It is the min. distance from the transmitter where there is no sky wave transmission.

It is the distance b/w transmitter to the 1st received sky wave signal of the earth.

$$\frac{MUF}{f_c} = \sqrt{1 + \left(\frac{D}{2h}\right)^2}$$

D is given by

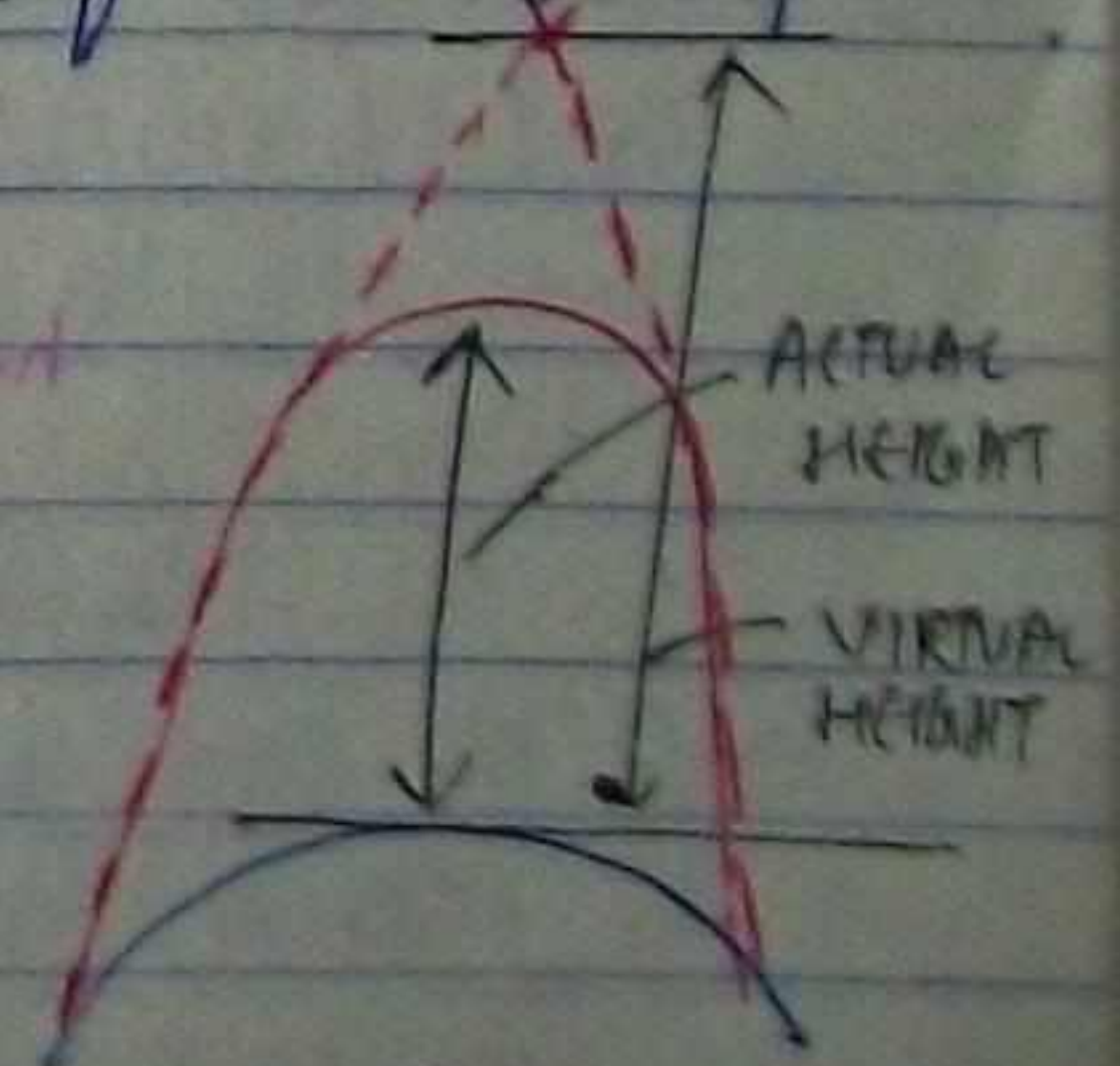
$$D = 2h \sqrt{\left(\frac{MUF}{f_c}\right)^2 - 1}$$

- The max. freq that can be used in sky wave prop. is 30 MHz

- The freq. above 30 MHz will not refract ionosphere.

VIRTUAL HEIGHT -

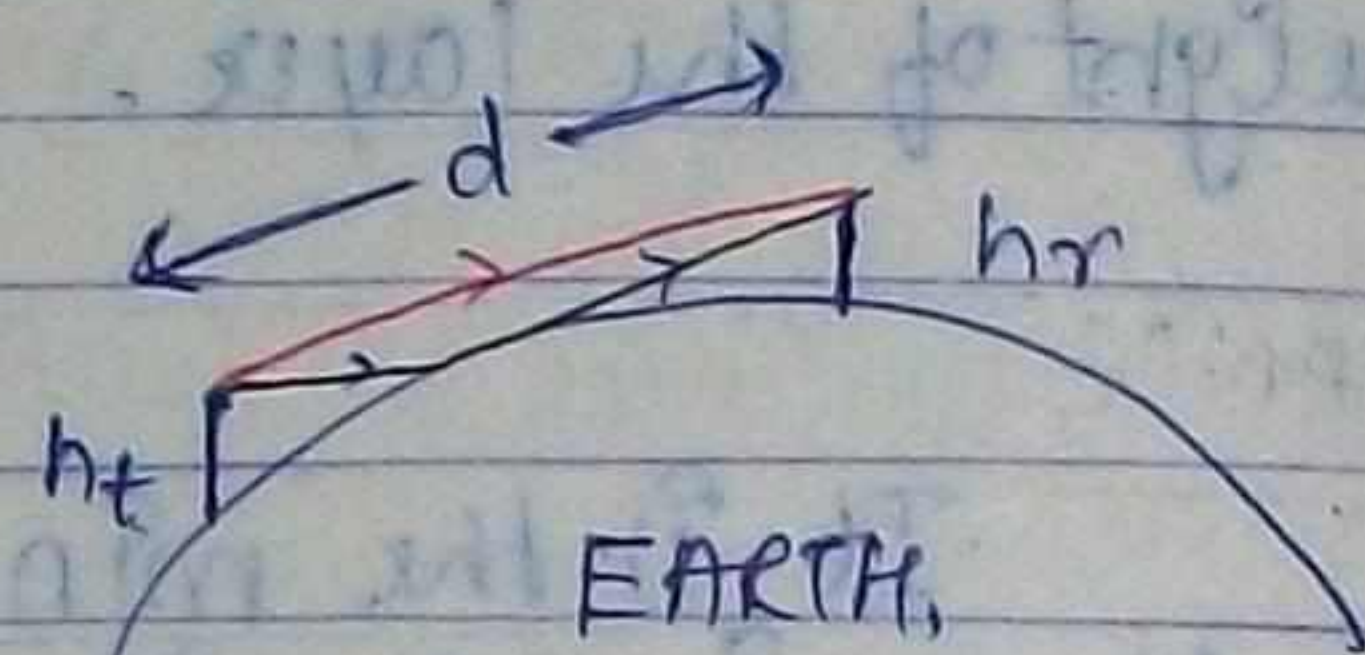
virtual height is always greater than actual height



③ SPACE WAVE PROPAGN - (Tropospheric wave)

- In space wave propn EM wave travels in straight line.

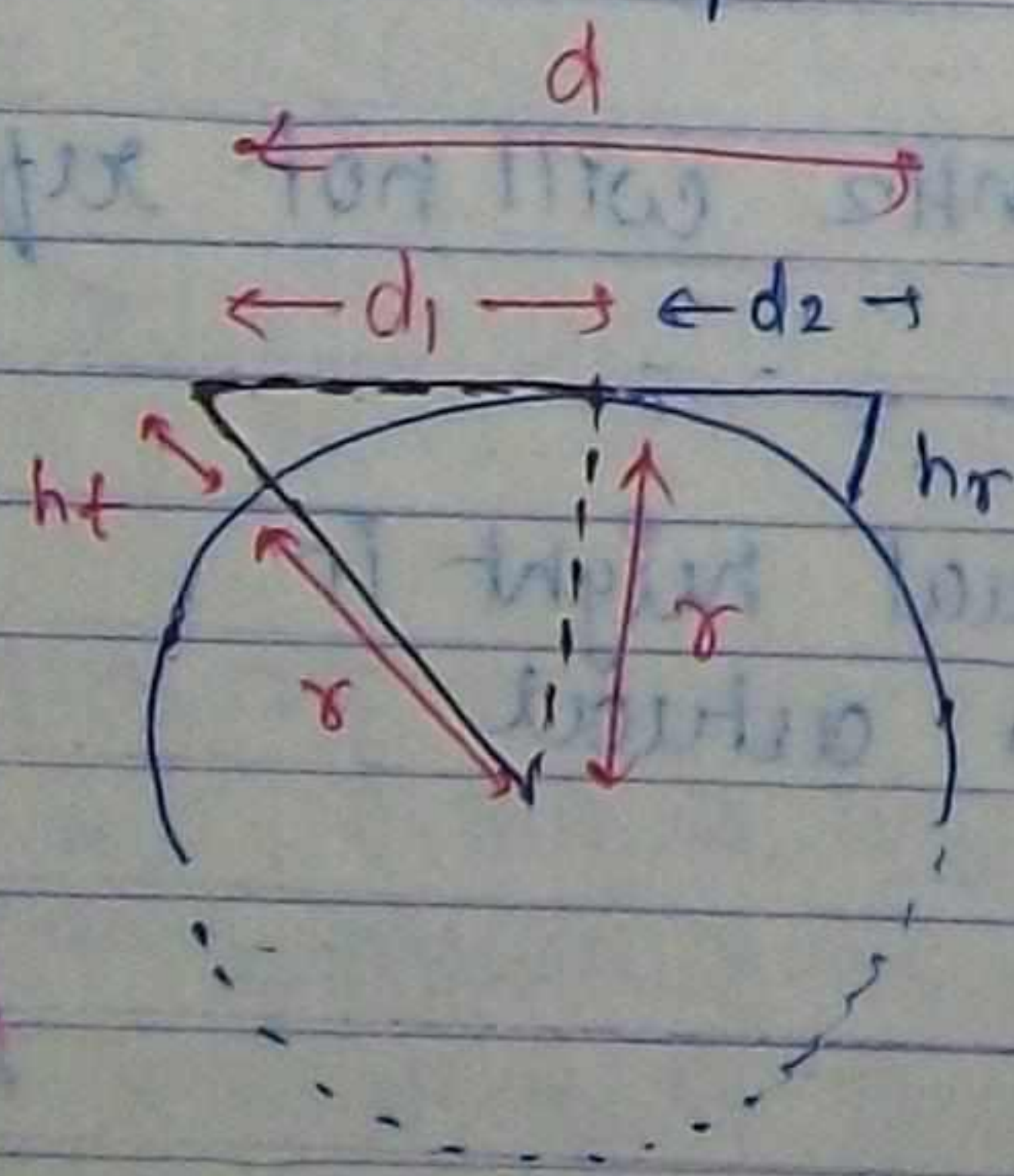
- In this wave propn the signal can be received in either due to direct wave or ground reflected wave.



- Most of propn is due to direct wave hence it is also known as direct wave propn.

- In this wave propn commⁿ is restricted to only line of sight (LOS) due to earth curvature hence it is known as LOS propn.

- Due to refraction of EM wave radio horizon is always greater than optical horizon.



$$d_1 = \sqrt{(h_t + r)^2 - r^2}$$

$$r \approx 6400 \text{ km}$$

$$h_t = \text{meter} = 100 \text{ m}$$

$$d_1 = \sqrt{h_t^2 + r^2 + 2h_tr - r^2}$$

$$d_1 \approx \sqrt{2h_tr}$$

- In radio waves due to refraction the radio horizon is always greater than optical horizon. To consider this effect the effective radius of earth is taken as:

$$R_e = \frac{4}{3} R$$

$$d_1 \approx \sqrt{2h_t \cdot \frac{4}{3} \times 6400 \times 10^3}$$

$$d_1 \approx 4.123 \sqrt{h_t} \text{ km}$$

$$\approx \sqrt{17h_t} \text{ km}$$

$$d_2 = \sqrt{17h_r} = 4.123 \sqrt{h_r} \text{ km}$$

$$d = \sqrt{17h_t} + \sqrt{17h_r}$$

$$d = 4.123 (\sqrt{h_t} + \sqrt{h_r}) \text{ km}$$

where
 h_t and h_r are
in meters.

Ques. In a sky wave propn $h_t = h_r = 100\text{m}$ then commⁿ distance possible is

Solⁿ:

$$d = 4.123 (\sqrt{100} + \sqrt{100}) = 16$$

$$d \approx 80 \text{ km}$$

④ In

FIELD STRENGTH IN LOS PROPⁿ -

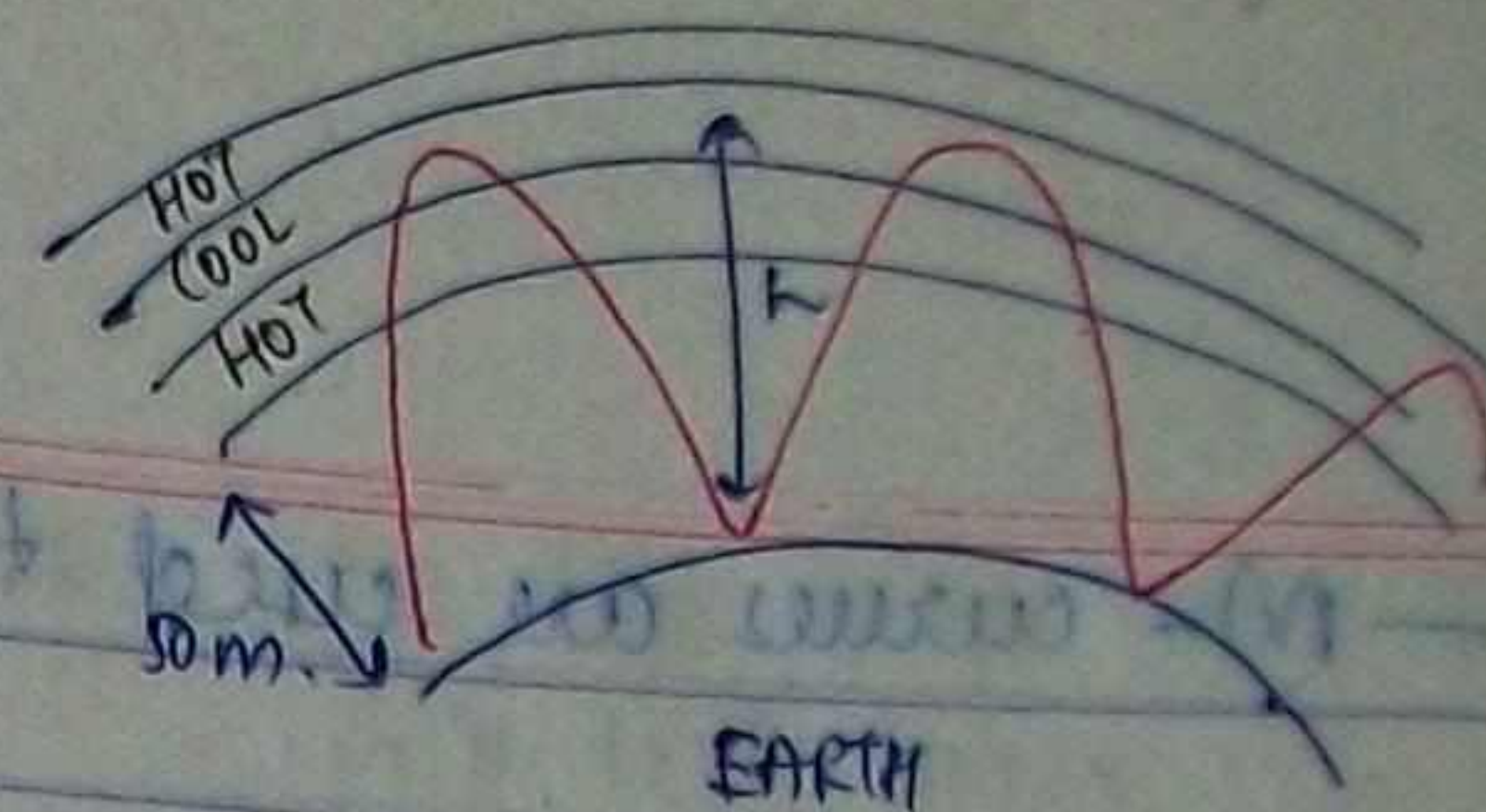
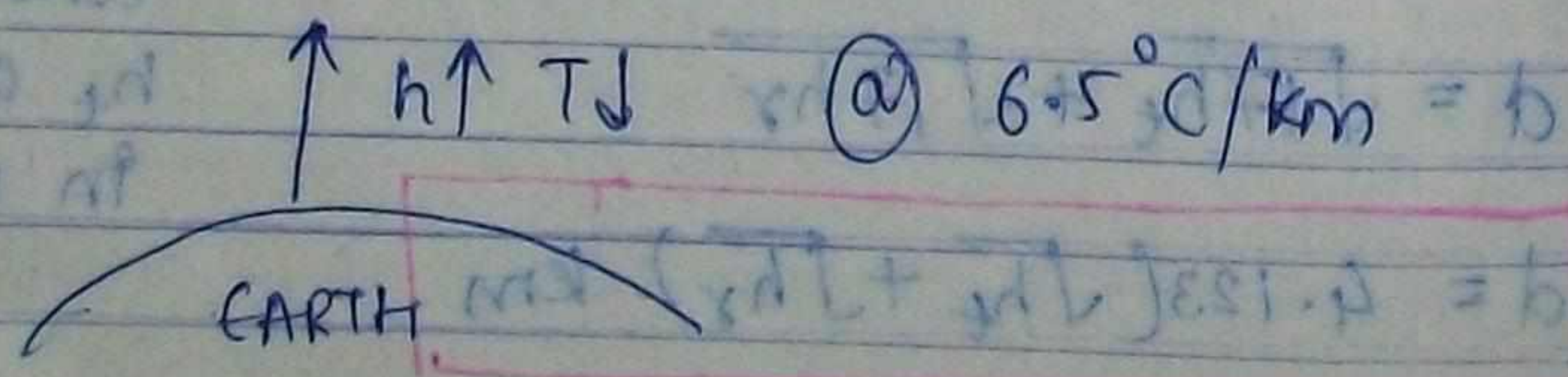
$$E = \frac{88 \sqrt{P_t \cdot h_t \cdot h_r}}{\lambda d^2}$$

DISTANCE TRAVELLED

Ground wave	→	1000 km
Sky wave	→	400 km - 4000 km
Space wave	→	40 - 60 km

④ DUCT PROPAGATION -

- microwave frequencies.
- temperature inversion layer (TIL) $\rightarrow h \uparrow T \uparrow$



- Due to temp. inversion layer super refraction occurs in microwave freq. and the coverage area or distance will increase in microwave frequency and it is known as Ducting.

- Once temp. inversion layer will form optical law will not work and it is required to use modified refractive index (M)

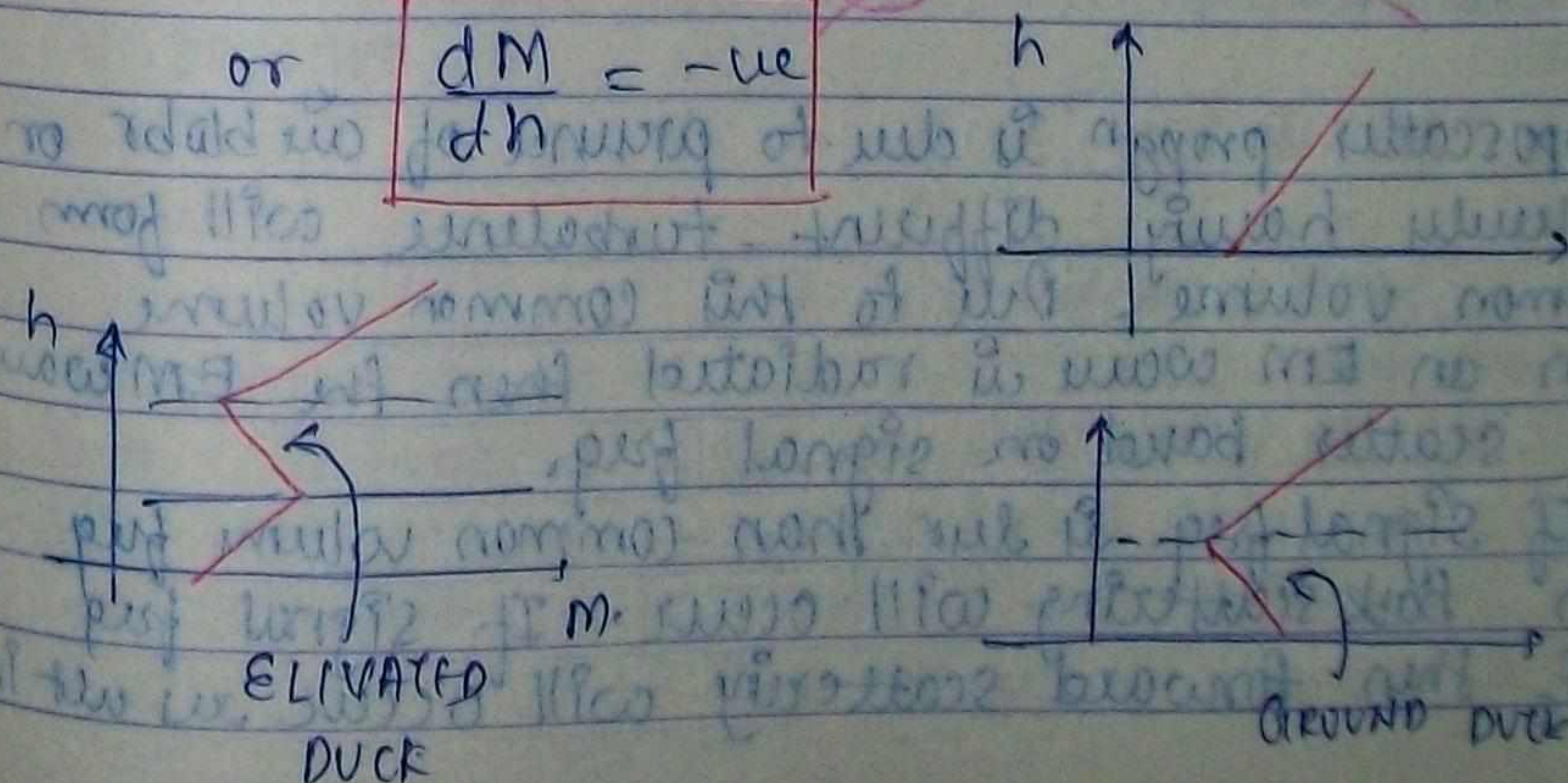
$$M = \left(\mu - 1 + \frac{h}{\gamma} \right) \times 10^6$$

μ = normal refractive index

If $\frac{dM}{dh} = -\mu$ then only duct prop. occurs.

- To provide duct wave propn. as $h \uparrow$ M is required to decrease

or $\frac{dM}{dh} = -\mu$



- M-curves are used to provide duct wave propn.

$$\lambda_{\max} = 2.5 \times h \sqrt{\Delta m \times 10^{-6}} \text{ cm}$$

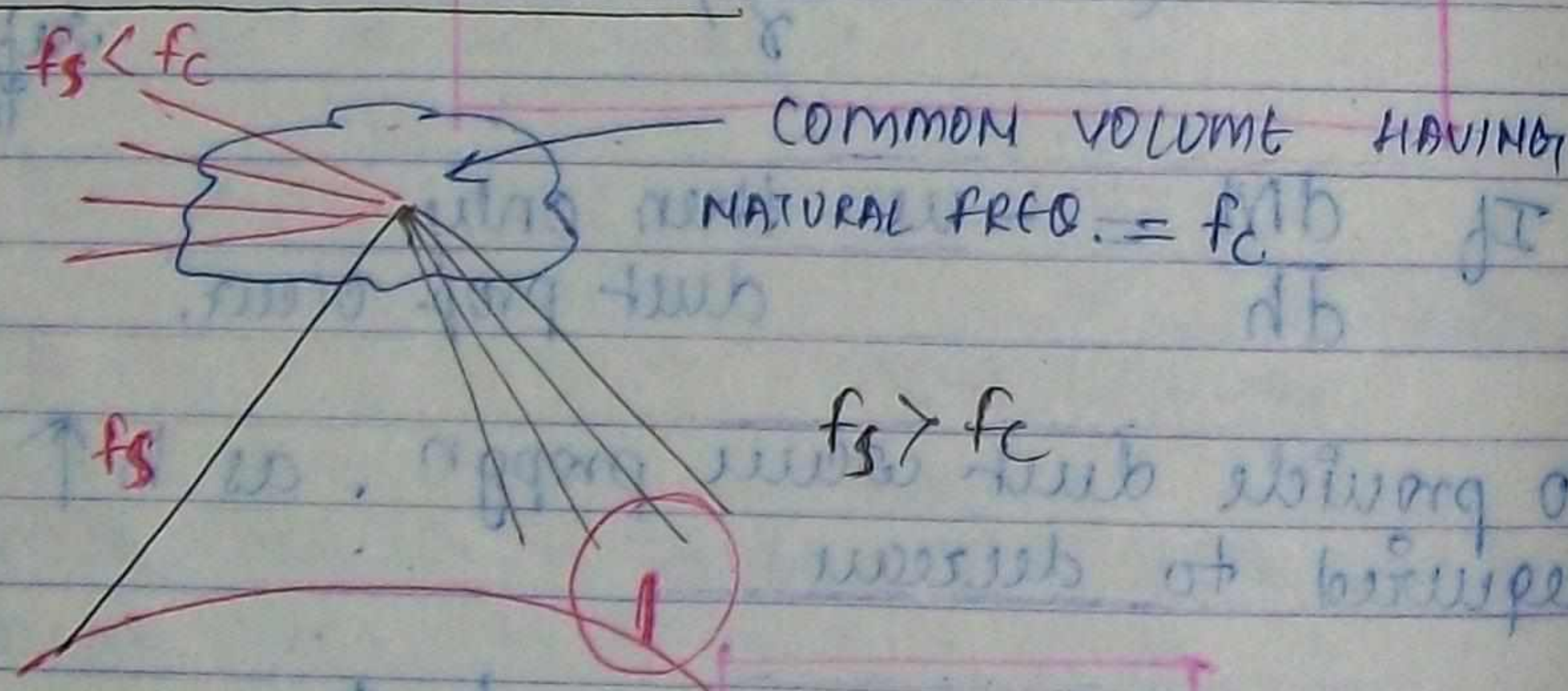
h = height of layer

Δm = change in modified refractive index

- coverage distance of duct propn is 1000 km.

⑤ TROPOSCATTER WAVE PROPⁿ →

Ex - AIR BLOBS



- Troposcatter propn is due to presence of air blobs or molecules having different turbulence will form common volume. Due to this common volume when an EM wave is radiated then the EM wave will scatter based on signal freq.

If signal freq. is less than common volume freq then back scattering will occur. If signal freq $f_s > f_c$ then forward scattering will occur result in

common b/w two points. in tropo scatter wave

- To provide troposcatter propn. high power transmitter & high gain Tx. and Rx. antennas are required.

- The freq.s 900 MHz, 2 GHz, 5 GHz will propagate through troposcatter propn.

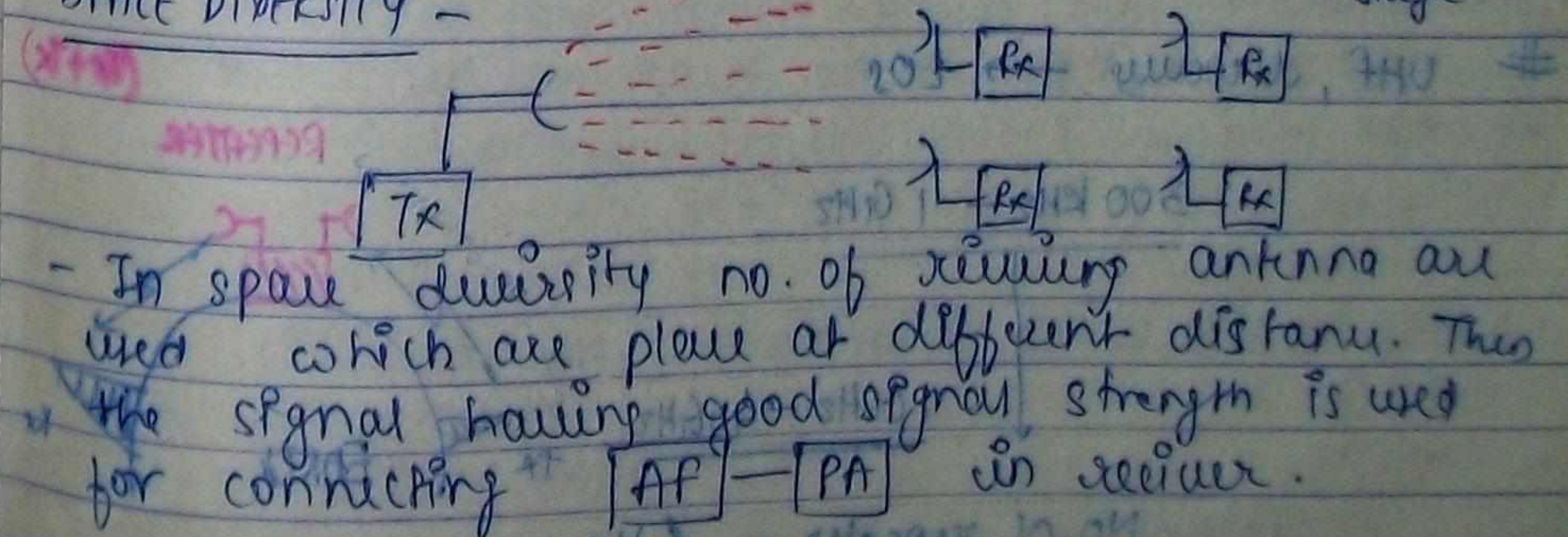
- In troposcatter propn, Rayleigh fading will present.

- In troposcatter propn to overcome fading Quadruple diversity reception system is used.

DIVERSITY RECEPTION - To overcome fading diversity reception system is used:

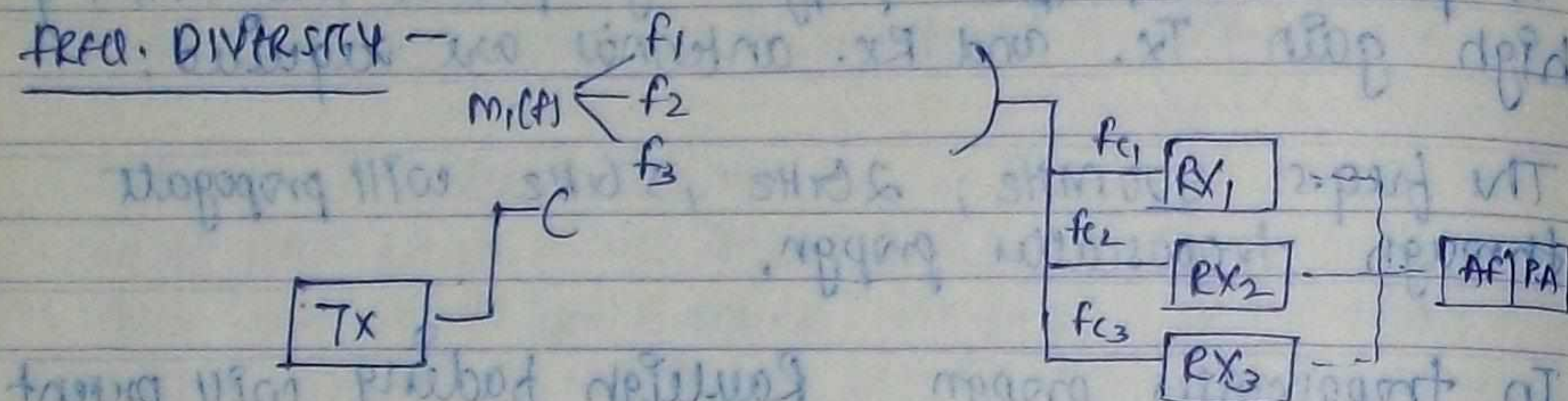
1. Space diversity,
2. frequency diversity.
3. polarization diversity.

SPACE DIVERSITY -

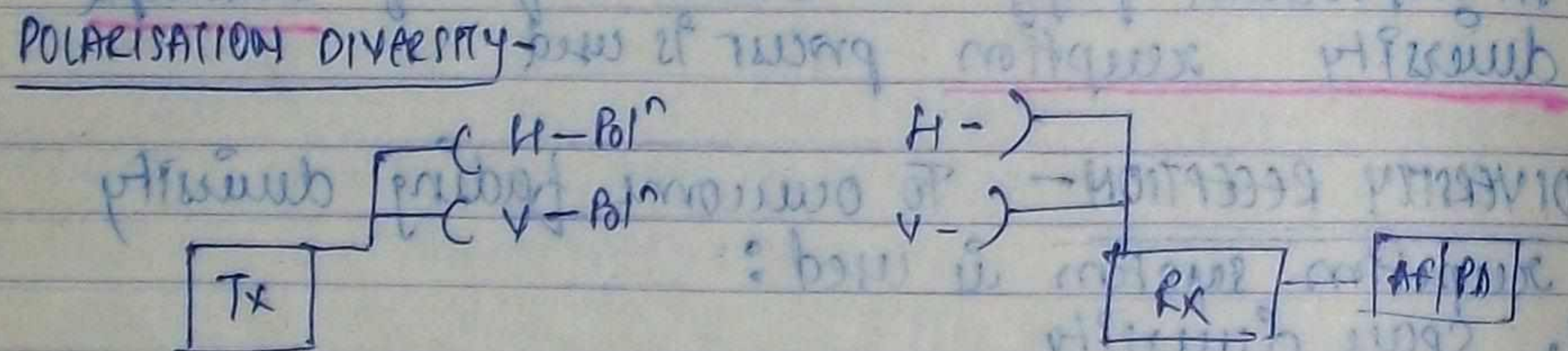


- In space diversity no. of receiving antenna are used which are placed at different distances. Then the signal having good signal strength is used for connecting [AP] - [PA] in receiver.

- space diversity system is mostly used diversity reception technique.



- freq. diversity is mostly used in ship to ship comm.



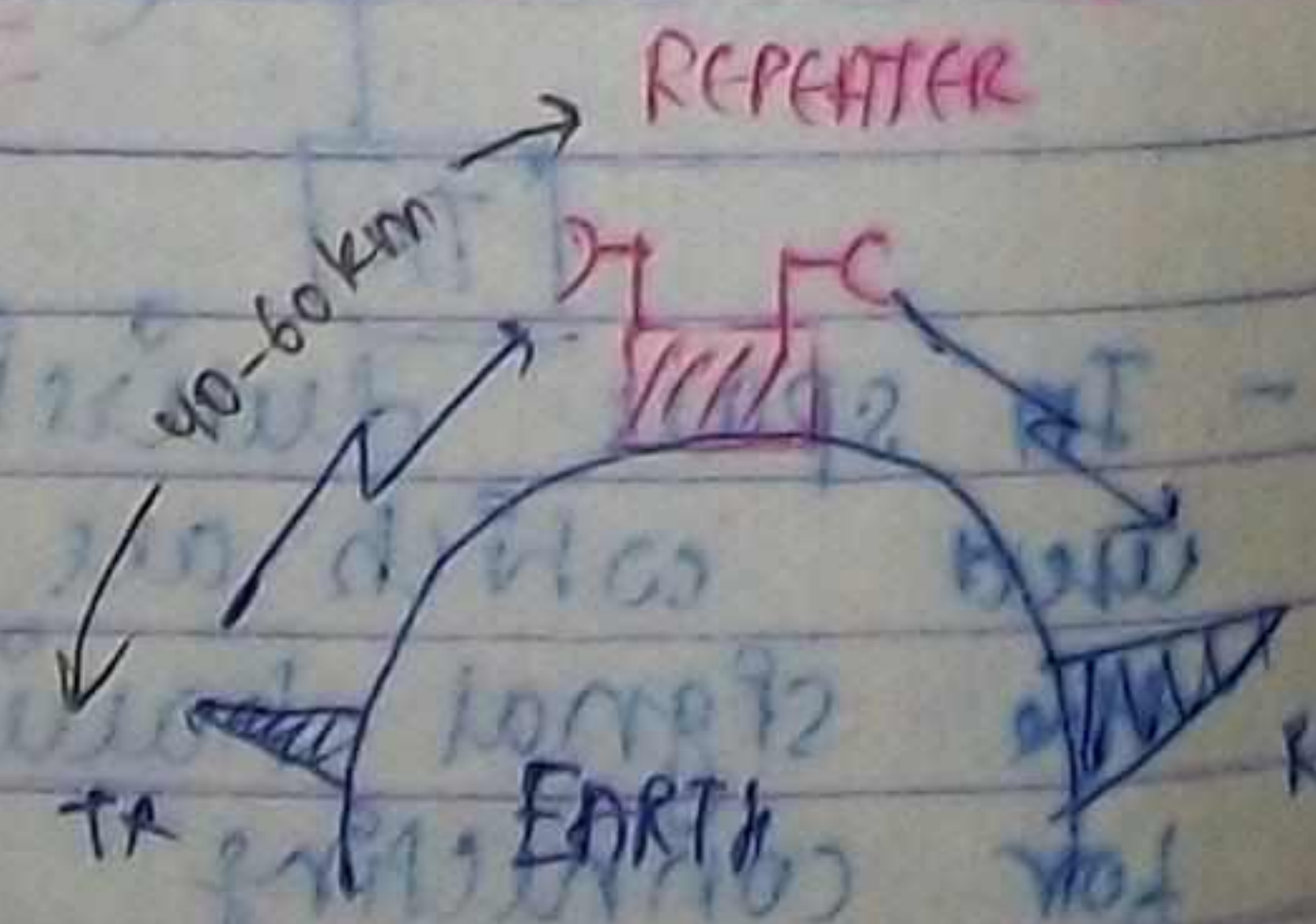
- In polarizⁿ diversity different polarizⁿ is used at Tx and Rx. at a time to over come fading.

UHF, Microwave $\rightarrow$ LOS

300 kHz - 1 GHz

1 GHz - 40 GHz

$$\text{No. of repeater} = \frac{8400}{50} =$$



SATELLITE COMMUNICATION

8/04/12

- In VHF and Microwave commⁿ is restricted to only 1-0.5 due to earth curvature. To overcome this microwave repeaters are placed every 40-60 km, but basic

1. No. of repeaters required is more.
2. maintenance of repeaters is difficult.

- satellite is a microwave repeater which is placed at certain height above the earth.

FOOT PRINT - coverage area of satellite



$\Rightarrow$ CLASSIFICATION -

① BASED ON PRINCIPLE OF OPERATION -

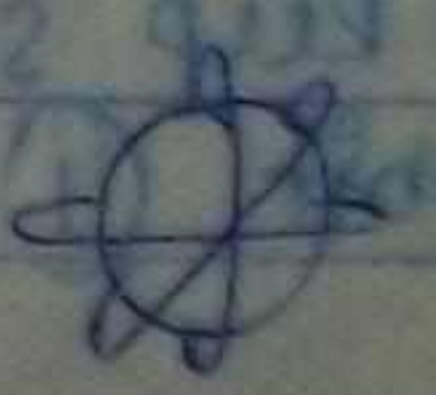
1. active satellite
2. passive satellite

② BASED ON ALTITUDES -

1. Low earth orbit (LEO) $\rightarrow$ 300 km - 1000 km
2. medium earth orbit (MEO) $\rightarrow$ 10,000 km
3. Geosynchronous earth orbit (GEO) $\rightarrow$ 36,000 km

③ BASED ON ORBIT USED -

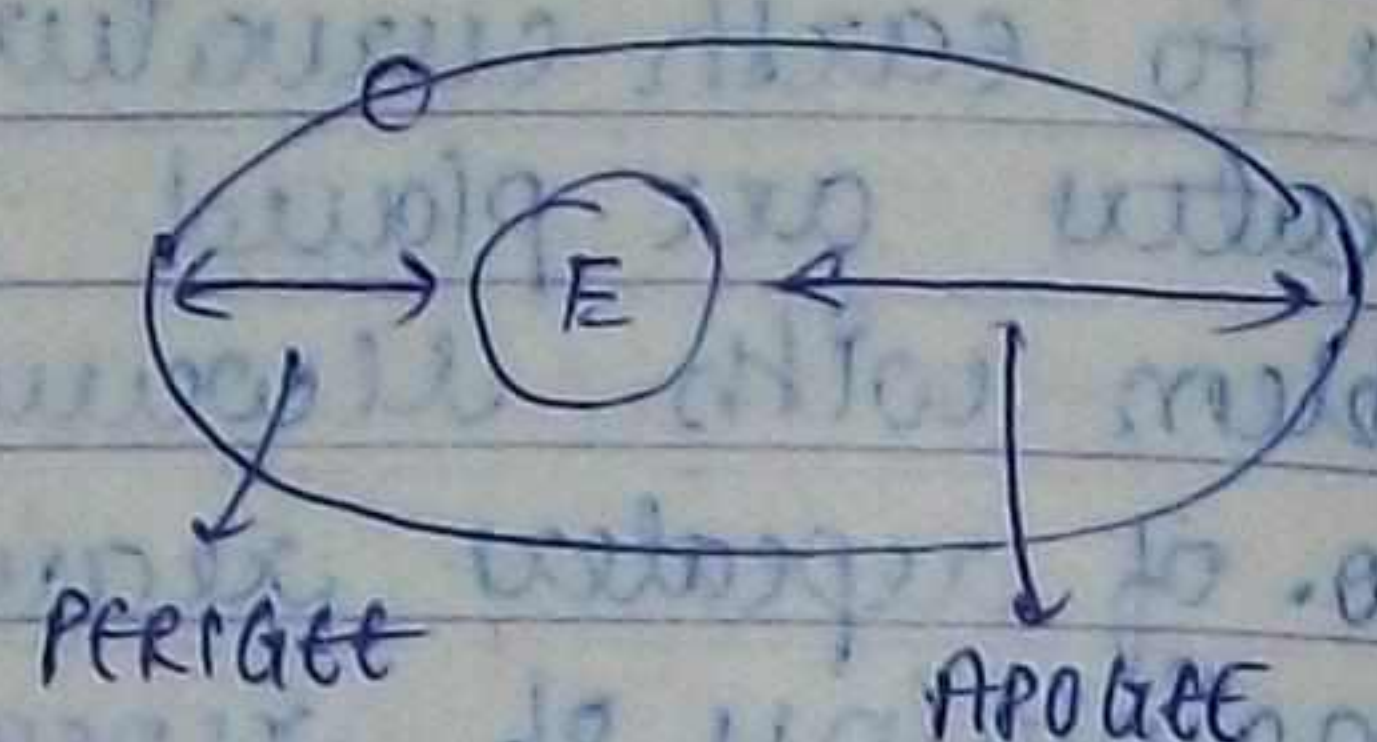
1. circular orbit Equatorial orbit
2. polar orbit
3. inclined orbit



IRS → Remote Sensing
INSAT → Comm. purpose
INTEL SAT → International satellite

④ BASED ON PATH TRACED -

1. Circular
2. Elliptical



⑤ BASED ON COVERAGE -

1. Domestic / National
2. Regional. (ex - ARAB satellite)
(two or more countries having same satellite)
3. International coverage (ex - Int'l)

⑥ BASED ON APPLICATION -

1. Communication satellite.
2. Remote sensing satellite.
3. Military satellite.

⇒ DERIVATION OF TIME PERIOD OF SATELLITE -

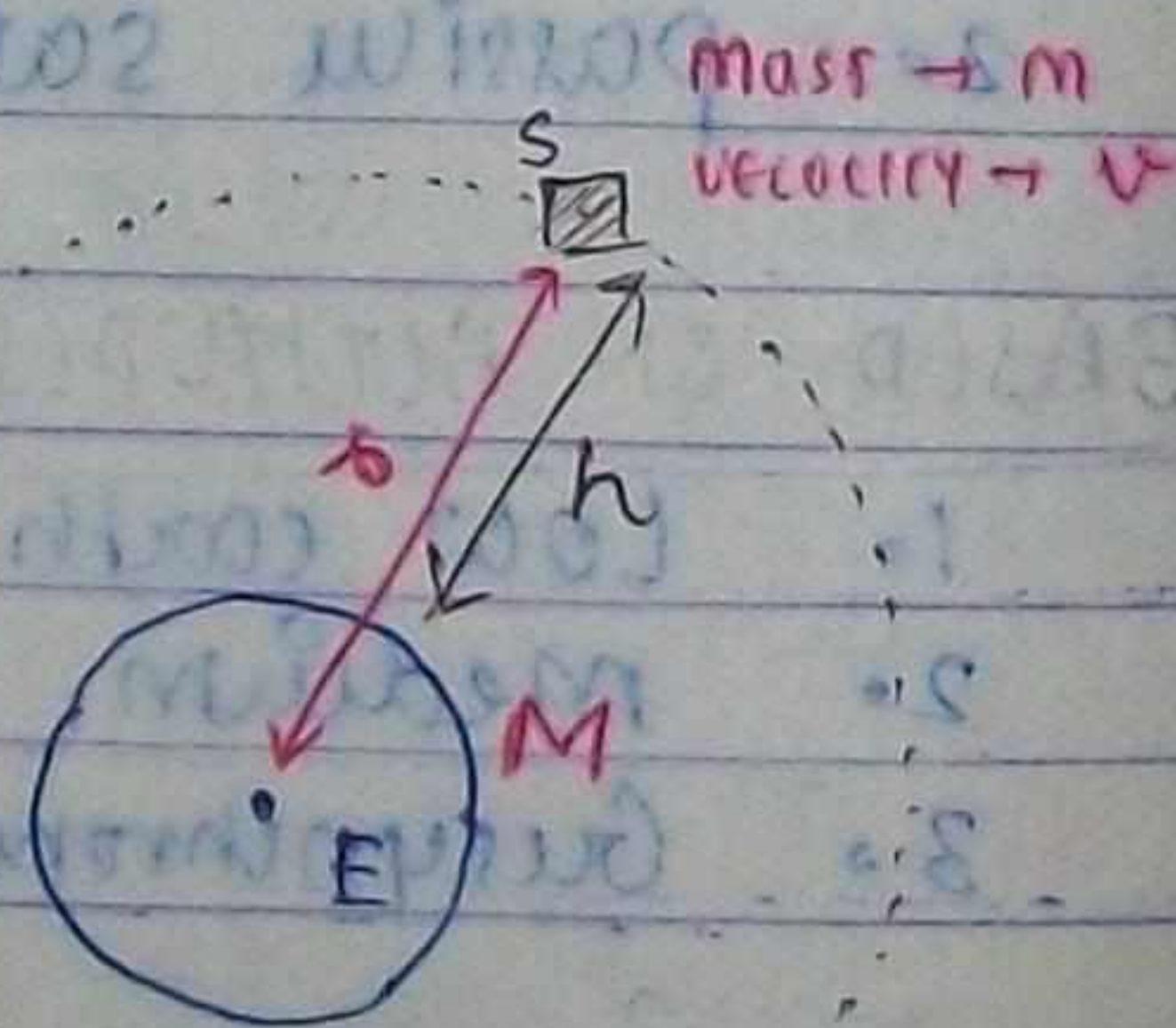
$$\text{Centripetal force} = \frac{mv^2}{r} \quad \text{--- (1)}$$

Gravitational force on satellite is given by

$$F_g = \frac{GMm}{r^2} \quad \text{--- (2)}$$

to keep satellite in same orbit (1) = (2)

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$



$$v^2 = \frac{GM}{r}$$

$$v = \sqrt{\frac{GM}{r}}$$

∴ velocity of satellite is independent of mass of satellite
 $G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$

$M \rightarrow$ mass of earth = $5.94 \times 10^{24} \text{ kg}$.

$$v \propto \frac{1}{\sqrt{r}}$$

- In circular orbit the distance travelled by satellite in one rotation is =

$$d = 2\pi r$$

Time period,

$$T = \frac{2\pi r}{v}$$

$$= \frac{2\pi}{\sqrt{\frac{GM}{r}}} \cdot r^{3/2}$$

$$T \propto r^{3/2} \quad \text{seconds.}$$

$$T = \frac{2\pi}{60} \cdot \frac{r^{3/2}}{\sqrt{GM}} \quad \text{minutes.}$$

$$T = 1.66 \times 10^{-4} \cdot r^{3/2} \quad \text{minutes.}$$

for Geosynchronous satellite:

$$T = 1.66 \times 10^{-4} \times r^{3/2} \text{ min}$$

↑
24 hr

$$r \approx 42218 \text{ km}$$

$$h = r - 6400 = 42218 - 6370$$

altitude

$$h = 35,850 \text{ kms}$$

$$\approx 36,000$$

⇒ LOW EARTH ORBIT SATELLITE -

- Placed at 300 km - 1000 km.

$$v = 28000 \text{ km/hr}$$

$$= 8 \text{ km/sec}$$

$$T = 1.5 \text{ hr.}$$

- Require 66 satellites to cover entire earth.

advantages:

1. Tx. power required is less.
2. Prop. delays are less.

3. Cost of satellite to placed is less.

Disadvantage -

1. No. of satellites required is more.
2. Complicated tracking equipment is required to track satellite.
2. Doppler freq. shift will present in received signal freq.

⇒ GEO SATELLITE -

- Altitude is around 36,000 km.

- Time period $T = 24 \text{ hr.}$

$$v = 10800 \text{ km/hr}$$

$$= 3 \text{ km/sec}$$

- Require min 3 satellites to cover entire earth.

- Each satellite will cover 40% of earth area.

advantage:

1. If satellite in space is in Geosynchronous equatorial orbit then satellite looks stationary from earth.
2. Due to this continuous coverage is possible in the given area.
3. No. of satellites required is less.
4. No tracking equipment is required.
5. No Doppler freq. shift is involved.

Hom antenna - satellite.

Disadvantages

1. In GEO satellite the delay involved in one way commⁿ is 240 ms ($72000/3 \times 10^8$)
2. In two commⁿ. the prop. delay is 480 ms
3. Pathloss is more hence more Tx. power is required in Geo. synchronous orbit.

1. Earth station → only

2. Ground station



Master control situated at Husan, Bangalore.

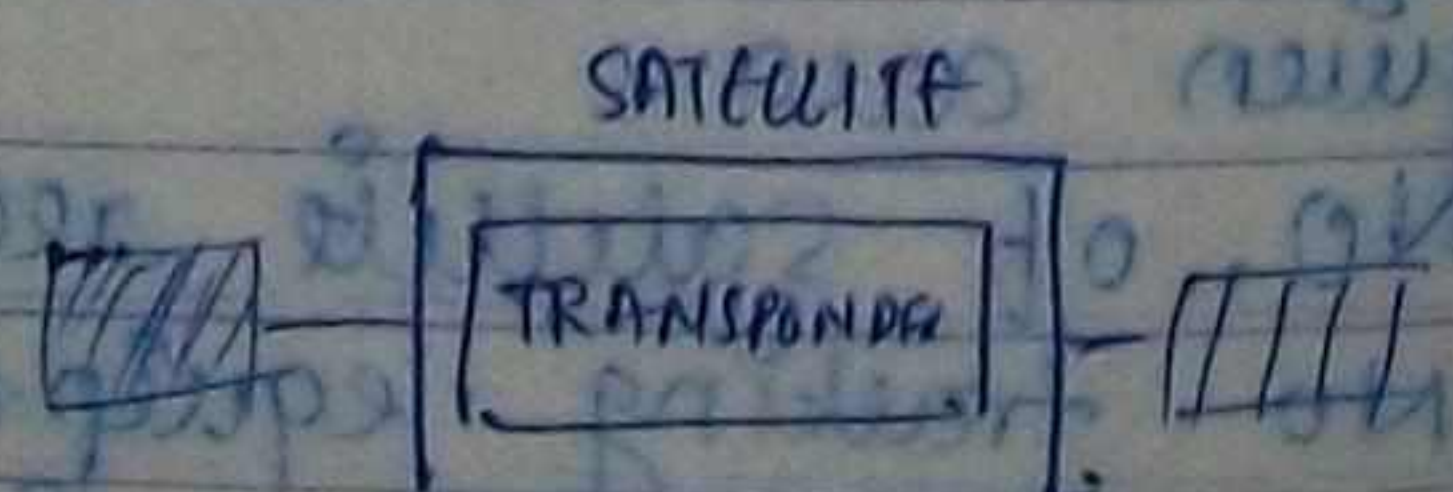
- Earth station will only transmit & receive signals from satellite.
- Ground station is used to Tx, Rx and controls the satellite.

⇒ SATELLITE COMMⁿ PROCS -

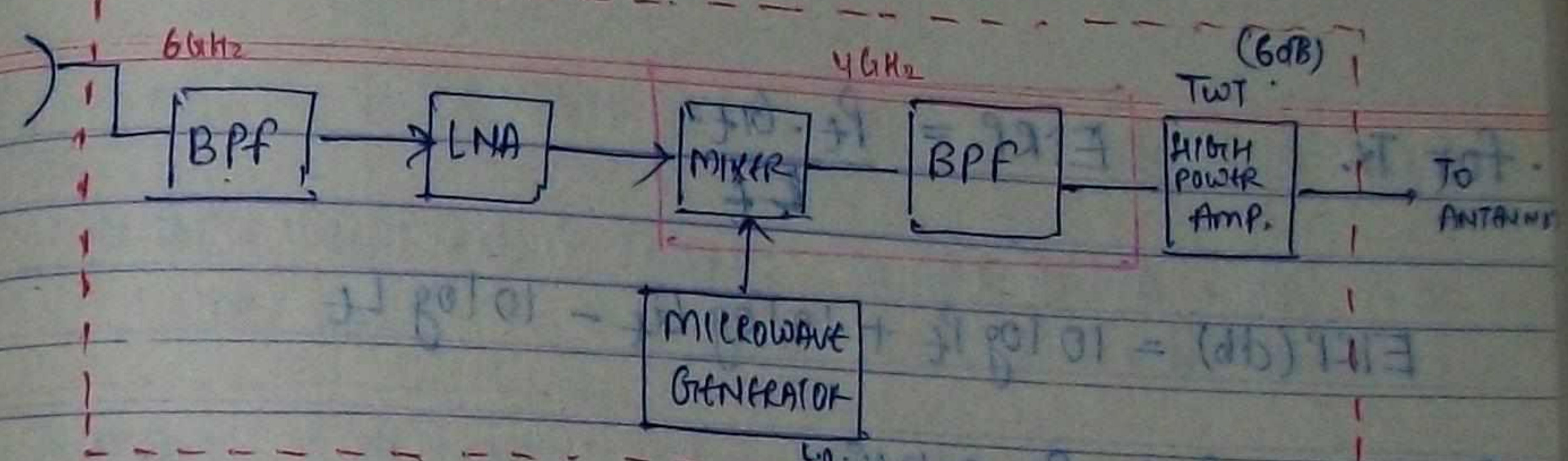
6/4 GHz → C band
14/12 GHz → Ku band

Uplink

Downlink



HORN ANTENNA



$$A - F = \frac{F_1 + F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \dots = (dB)_T$$

$$T = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2} + \dots$$

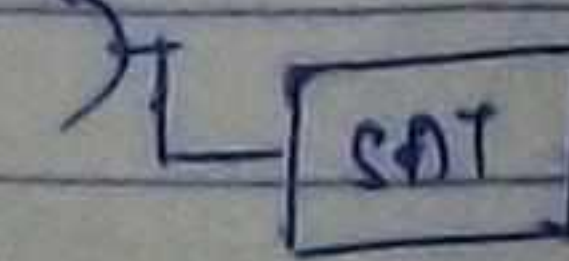
Total

- Bandwidth of each satellite is 500 MHz.

500 MHz divided into 12 channels by FDM. Each channel has BW = 36 MHz. Guardband = 5 MHz.

⇒ LINK BUDGET CALⁿ - (power required to Tx. for commⁿ.)

P_r, G_r, L_r



P_t

G_t

L_t

P_L = Path loss
 A_L = atmospheric loss

G_e

P_e

L_t

EARTH

R_e

L_r

P_r

G_r

L_r

P_r

G_r

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P_r

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Grain to temp $\frac{G_T}{T} = G(\text{db}) - T(\text{db})$

for Tx.

$$EIRP = \frac{P_t \cdot G_t}{L_t}$$

$$EIRP(\text{db}) = 10 \log P_t + 10 \log G_t - 10 \log L_t$$

Power received in satellite.

$$P_r(\text{db}) = P_t(\text{db}) + G_t(\text{db}) - L_t(\text{db}) - P_L(\text{db}) - A_L(\text{db}) - L_r(\text{db}) + G_r(\text{db})$$

C/T RATIO (Carrier to temp.)

$$\left(\frac{C}{T}\right) = \frac{P_r}{T} = \frac{EIRP(\text{db}) - P_L(\text{db}) - A_L(\text{db}) - L_r(\text{db}) + G_r(\text{db})}{T(\text{db})}$$

$$\left(\frac{C}{T}\right)_{\text{db}} = EIRP(\text{db}) - P_L(\text{db}) - A_L(\text{db}) - L_r(\text{db}) + \left(\frac{G_r}{T}\right)_{\text{db}}$$

G/T RATIO - (Grain to temp)

$$\left(\frac{G}{T}\right)_{\text{ratio}} \text{ db} = G(\text{db}) - T(\text{db})$$

C/N RATIO -

$$\left(\frac{C}{N}\right)_{\text{RATIO}}(\text{db}) = \frac{C}{T \cdot K B} = \left(\frac{C}{T}\right)_{\text{db}} - \frac{1}{KB} \text{ db}$$

$$\left(\frac{C}{N}\right)(\text{db}) = \left(\frac{C}{T}\right)(\text{db}) - 10 \log_{10} k - 10 \log_{10} B$$

here, $10 \log_{10} k = -228.6 \text{ dBW/K/Hz}$

FIBRE OPTICS COMMUNICATION

freq. of light $\rightarrow 10^{13} - 10^{16}$ Hz.

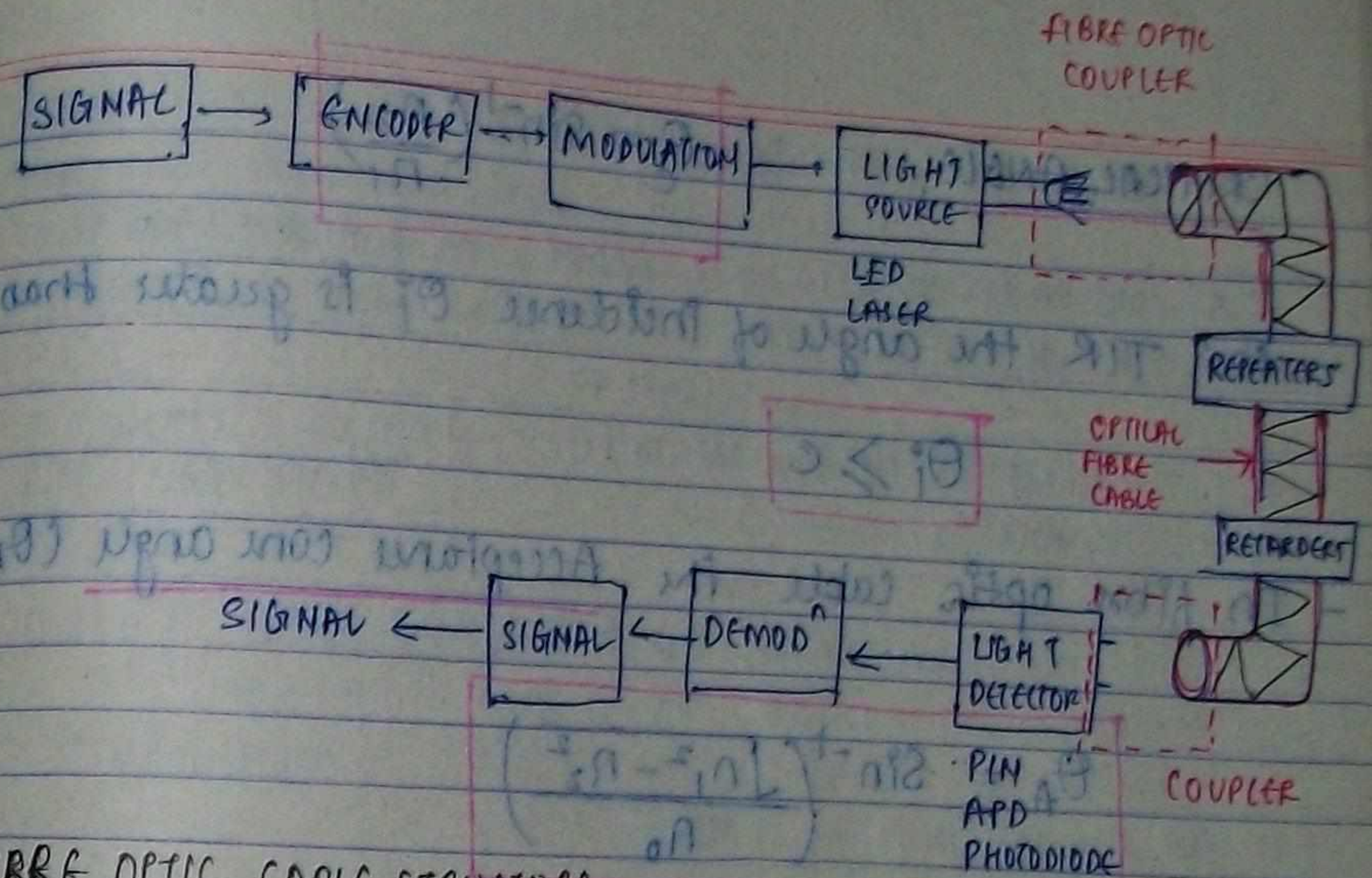
- In fibre optic commⁿ signal is transmitted in the form of light.
- The fibre optic cable is constructed using glass or silica material.

ADVANTAGES -

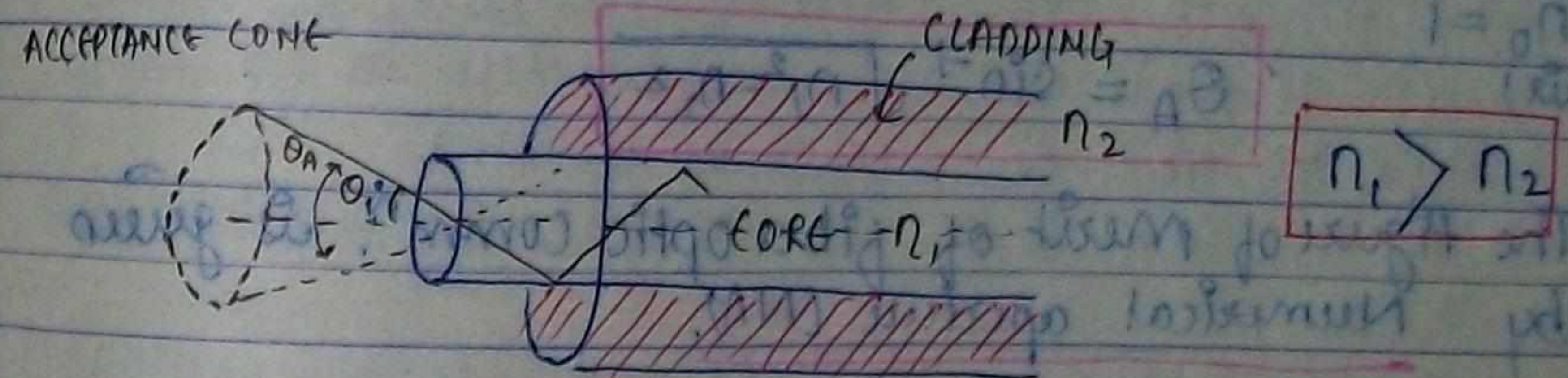
1. Operating freq. are very high ($10^{13} - 10^{16}$ Hz) BW available is more.
2. Low cost.
3. Thickness of cable are very less. (mm)
4. No cross-talk.
5. Signal security is more.
6. No fire hazards (accidents).
7. path loss is less.

DISADVANTAGE -

Initial cost is more.



FIBRE OPTIC CABLE STRUCTURE -



- Fibre optic cable contain basically core & cladding, for proper operation the refractive index of core n_1 is always greater the refractive index of cladding material n_2 .
- The basic principle of fibre optic commⁿ is TIR (total internal reflⁿ).

CRITICAL ANGLE,

$$\theta_c = \sin^{-1} \left(\frac{n_2}{n_1} \right)$$

- for TIR the angle of incidence θ_i is greater than θ_c

$$\theta_i \geq \theta_c$$

- In fibre optic cable the Acceptance cone angle (θ_A)

$$\theta_A = \sin^{-1} \left(\frac{\sqrt{n_1^2 - n_2^2}}{n_0} \right)$$

$n_0 = 1$
(air)

$$\theta_A = \sin^{-1} \sqrt{n_1^2 - n_2^2}$$

- The figure of merit of fibre optic commⁿ. is given by Numerical aperture (NA).

$$NA = \sin \theta_A = \sqrt{n_1^2 - n_2^2}$$

$$NA = n_1 \sqrt{2\Delta}$$

where Δ = Relative refractive index

$$\Delta = \frac{n_1^2 - n_2^2}{2n_1^2}$$

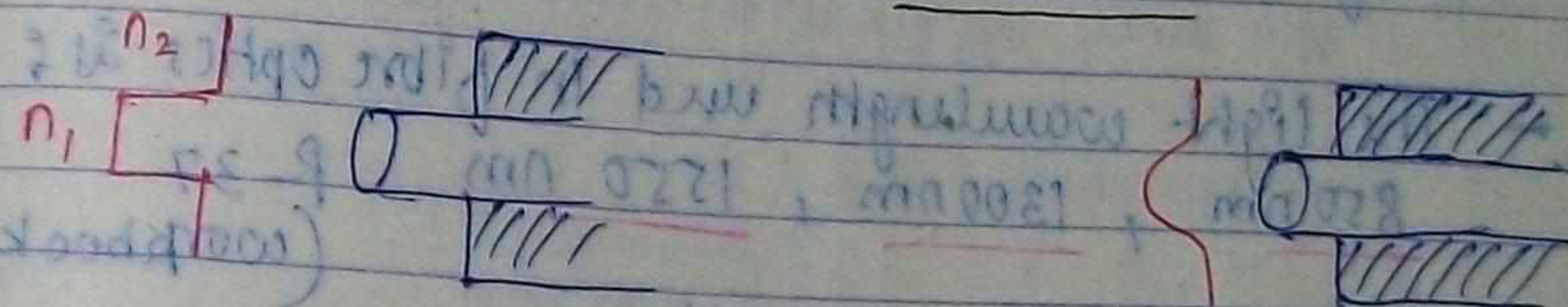
$$\frac{dI_2}{dI_1} = \sin \left(\frac{\theta_c}{\theta_0} \right) \frac{\theta_i}{\theta_r}$$

- Depending on basis of refractive index fibre optic cable is divided into two parts:

1. Step index fibre
 2. Graded index fibre
- single mode fibre
→ multimode fibre

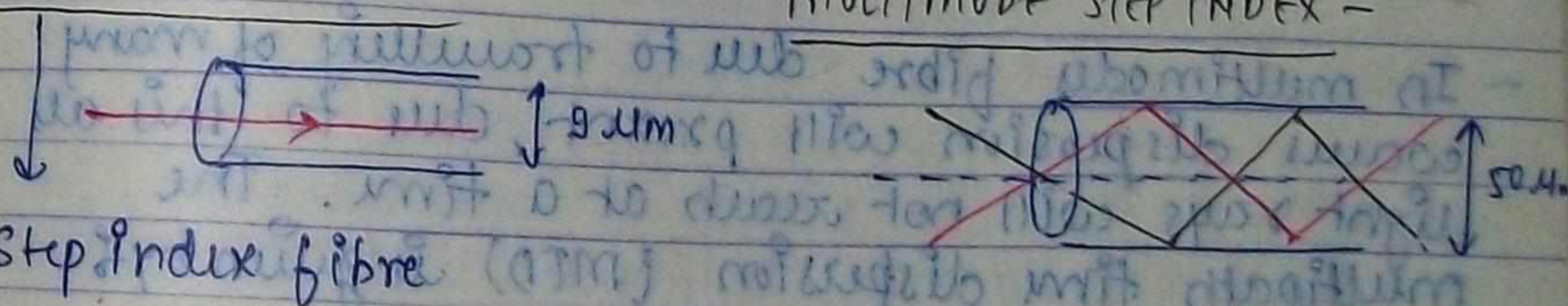
① STEP INDEX FIBRE -

② GRADED INDEX -

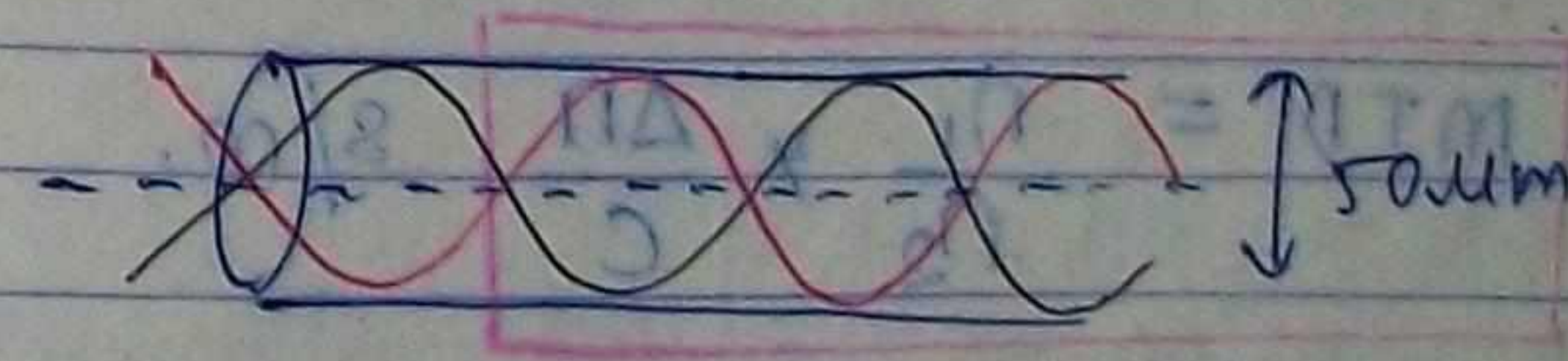


SINGLE MODE FIBRE -

MULTIMODE STEP INDEX -



GRADED INDEX FIBRE



- No. of modes present in fibre depend on V-number or Normalized freq.

$$V = \frac{2\pi a}{\lambda} (NA)$$

a = radius of core

$$V = \frac{\pi d}{\lambda} (NA)$$

d = diameter of core

- for single mode fibre

$$V = 2.405$$

- for step index multimode fibre
number of mode is

$$M_N = \frac{V^2}{2}$$

- for graded index multimode
no. of mode is

$$M_N = \frac{V^2}{4}$$

- The light wavelength used in fibre optics is
850 nm, 1300 nm, 1550 nm.

P-37
(workbook)

⇒ DISTORTION IN OPTICAL FIBRE -

- In multimode fibre due to travelling of many waves dispersion will present, due to this all light rays will not reach at a time. The multipath time dispersion (MTD) is given by

$$MTD = \frac{n_1}{n_2} \times \frac{\Delta n}{c} \text{ s/m.}$$

Seconds/meter

$$\Delta n = n_1 - n_2$$

- In single mode fibre dispersion is less when compare to multimode fibre.

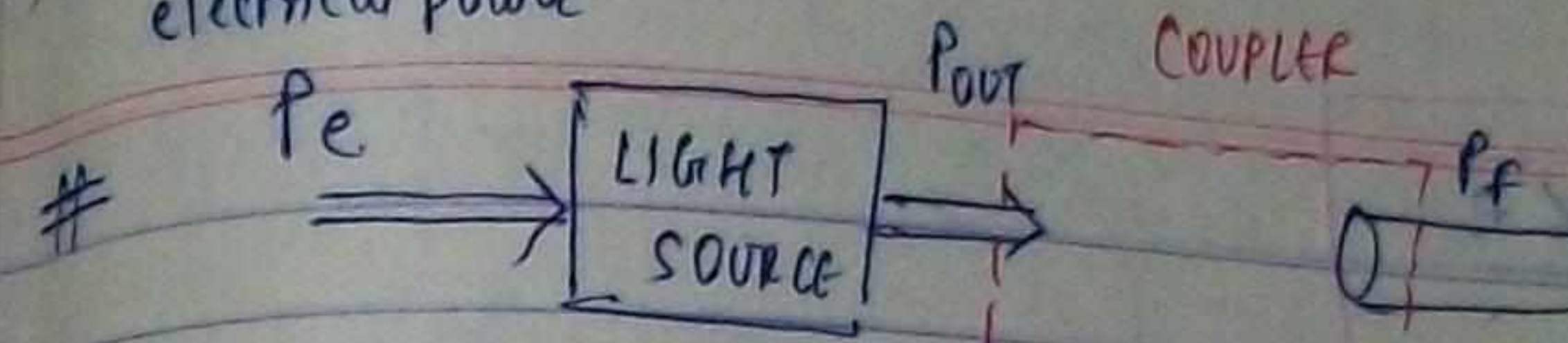
- The phase velocity of light signal propn in core is given by

$$V_p = \frac{c}{n_1}$$

$$(V_p = \frac{c}{\sqrt{\epsilon_r}})$$

⇒ EFFICIENCY -

electrical power

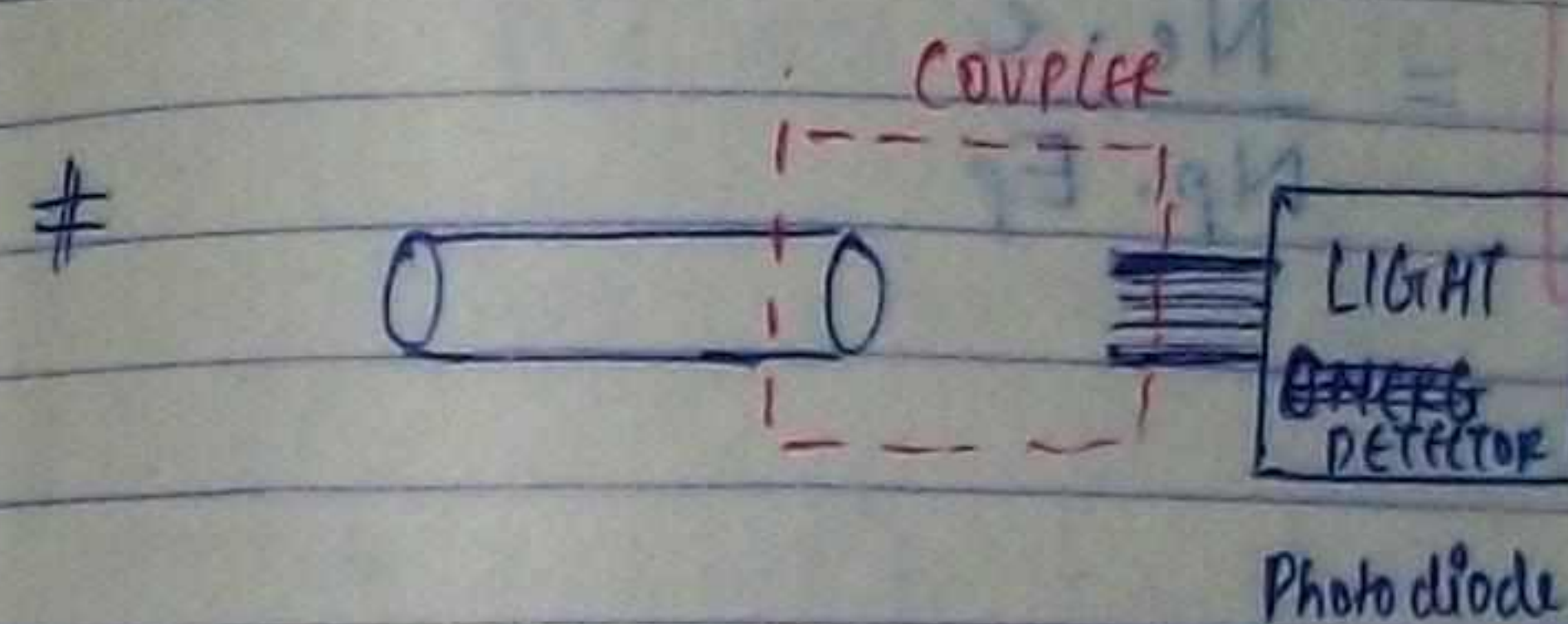


$$\eta_{ls} = \frac{P_{out}}{P_e}$$

$$\eta_c = \frac{P_f}{P_{out}}$$

$$\eta = \frac{P_{out}}{P_e} \times \frac{P_f}{P_{out}}$$

$$\eta = \frac{P_f}{P_e} = \eta_{ls} \times \eta_c$$



$$BW = \frac{0.35}{t_r}$$

E_p = photon energy

E_g = electron gap energy

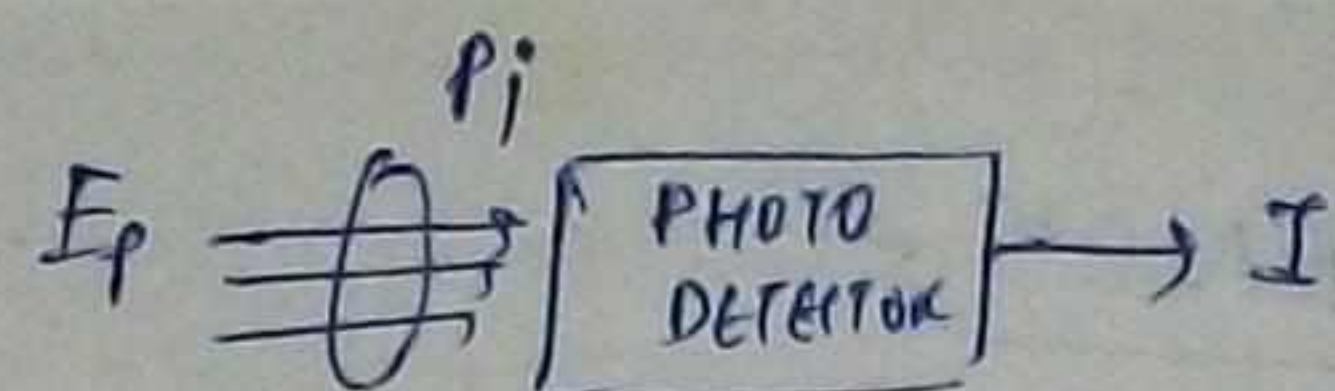
$$E_p \geq E_g$$

$$\lambda = \frac{1.24}{E_g \text{ (eV)}} \text{ nm}$$

⇒ PHOTO DETECTORS -

- Photodiode, APD, PIN diode are normally used as light detectors. When light energy is incident on to photodetector then depending on energy of incident light e^- are emitted and which will provides current in photodetector.

- Let N_e = no. of e^- emitted in photodetector. Then current flowing in PD is given by:



$$I = \frac{N_e \times e}{t}$$

$$P_i = \frac{N_p \times E_p}{t}$$

LIGHT
INCIDENT
POWER

- The efficiency of photo detector is given by responsivity (R)

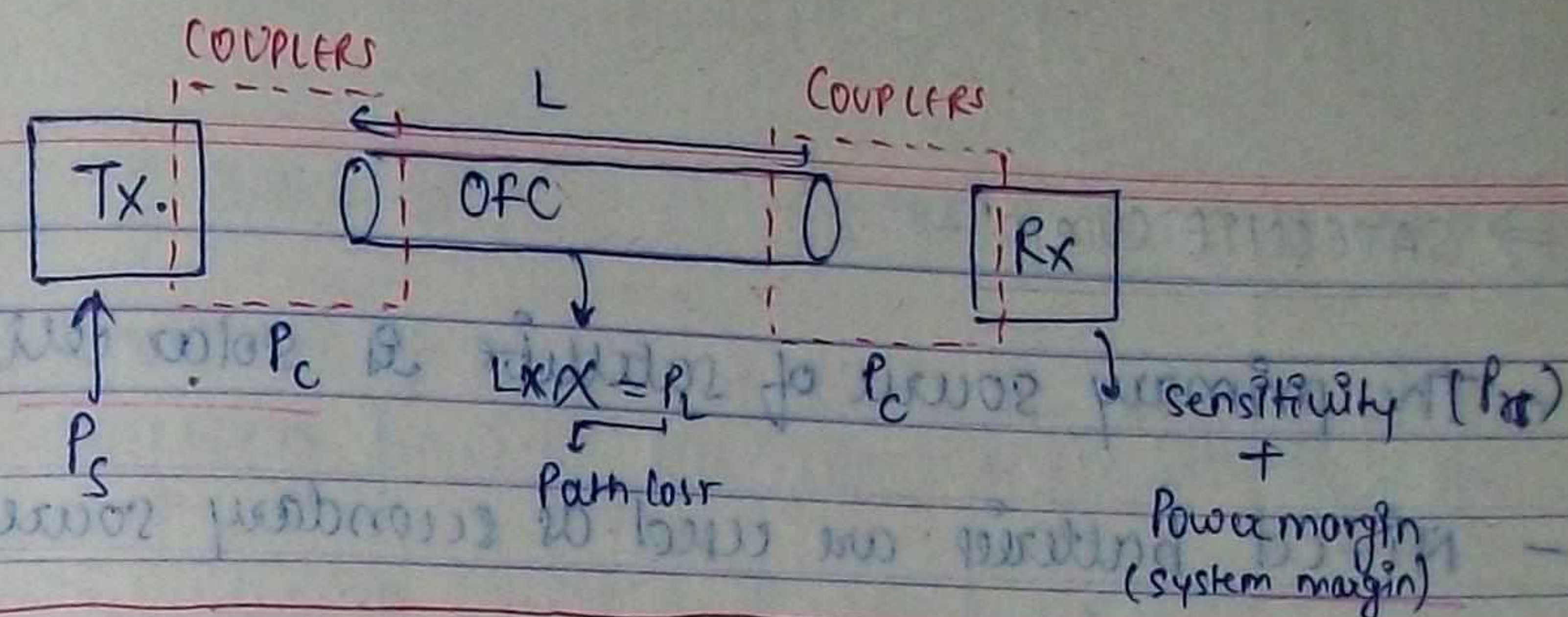
$$\text{Responsivity} = \frac{I}{P_i} = \frac{N_e \cdot e}{N_p \cdot E_p}$$

$$R = \left(\frac{N_e}{N_p} \right) \times \frac{e}{E_p}$$

$$\theta_E = \text{quantum efficiency} = \frac{N_e}{N_p} \quad (60\% - 80\%)$$

$$R = \theta_E \times \frac{e}{E_p}$$

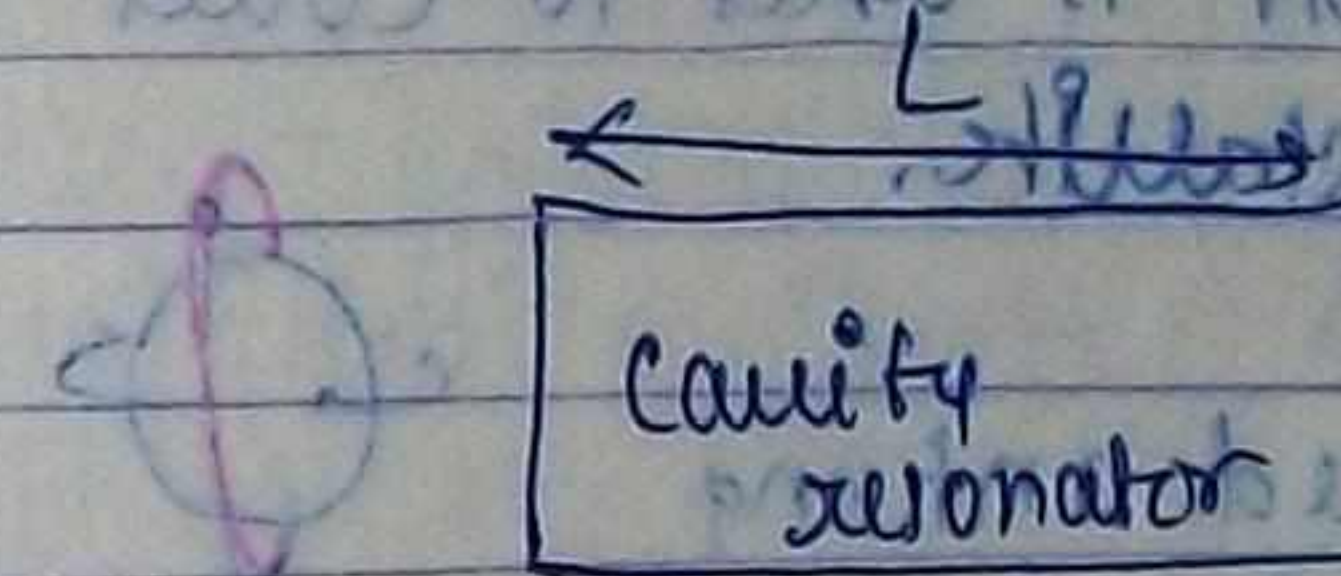
⇒ LINK BUDGET CALCULATION



$$P_s = 2P_c + \alpha L + P_r (\text{Receiver sensitivity}) + \text{Power margin (system margin)}$$

$$\text{Bit rate} = \frac{P_r (\text{Power received})}{\text{Sensitivity}}$$

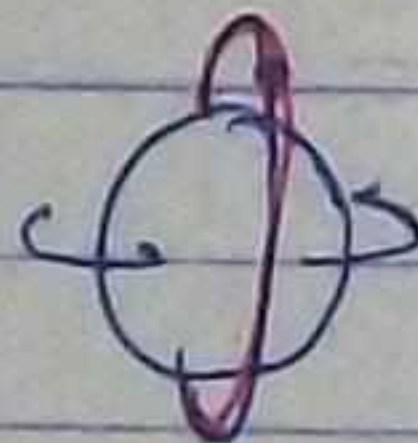
⇒ LASER



$$L = M_n \times \frac{\lambda}{2}$$

$$\Delta f = \frac{c}{2nL}$$

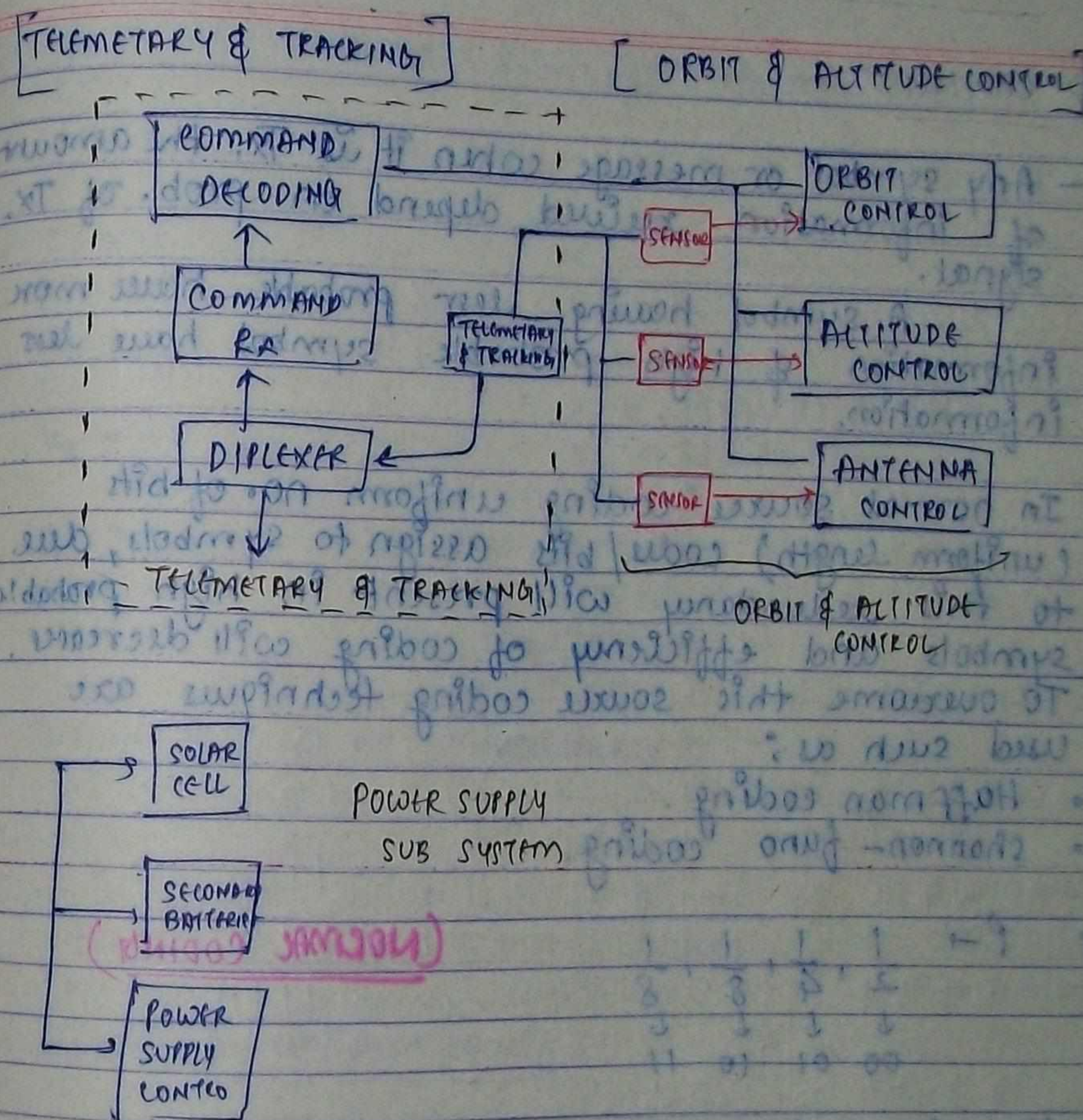
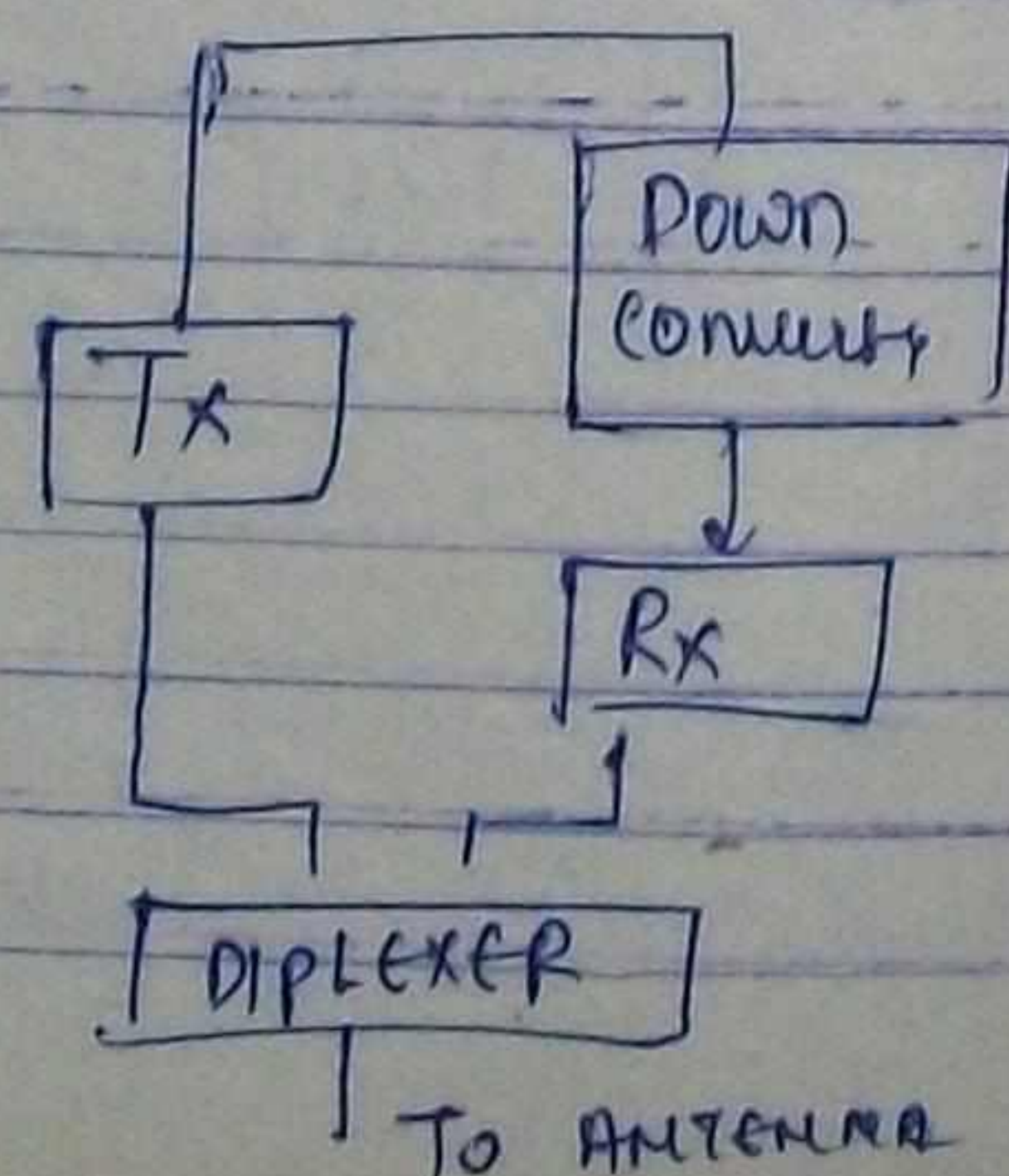
⇒ SATELLITE COMMⁿ-

- The primary source of satellite is solar cell.
 - Ni-cd batteries are used as secondary source.
 - Horn antenna is used in satellite to send back signals.
 - Relative angular velocity of Geostationary satellite w.r.t ground is zero.
 - If ^{Polar} geosynchronous orbit is used to cover entire earth with single satellite.
 - Global coverage required antenna beam width of $17-3^\circ$
- 
- The diagram shows a circle representing Earth with a vertical line passing through its center, representing the polar axis. A curved arrow around the axis indicates rotation. A satellite is shown at a point on the axis, with a curved arrow indicating its orbital path around the Earth.

SATELLITE ON BOARDS -

satellite contain main part:

1. power supply sub system
2. orbit & altitude control
3. telemetry & tracking
4. commⁿ sub system.



INFORMATION THEORY

9/04/12

⇒ CODING

- Any symbol or message when it is Tx, the amount of information received depend on prob. of Tx signal.

A symbol having less probable have more information & high probable symbol have less information.

- In normal source coding uniform no. of bits (uniform length) code/bits assign to symbols, due to this redundancy will present in high probable symbols and efficiency of coding will decrease. To overcome this source coding techniques are used such as:

1. Huffman coding.
2. Shannon-fano coding.

Ex $P \rightarrow \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8}$
 $\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$
 00 01 10 11

(NORMAL CODING)

$$H = \frac{1}{2} \times 1 + \frac{1}{4} \times 2 + \frac{1}{8} \times 3 + \frac{1}{8} \times 3 = \frac{7}{4}$$

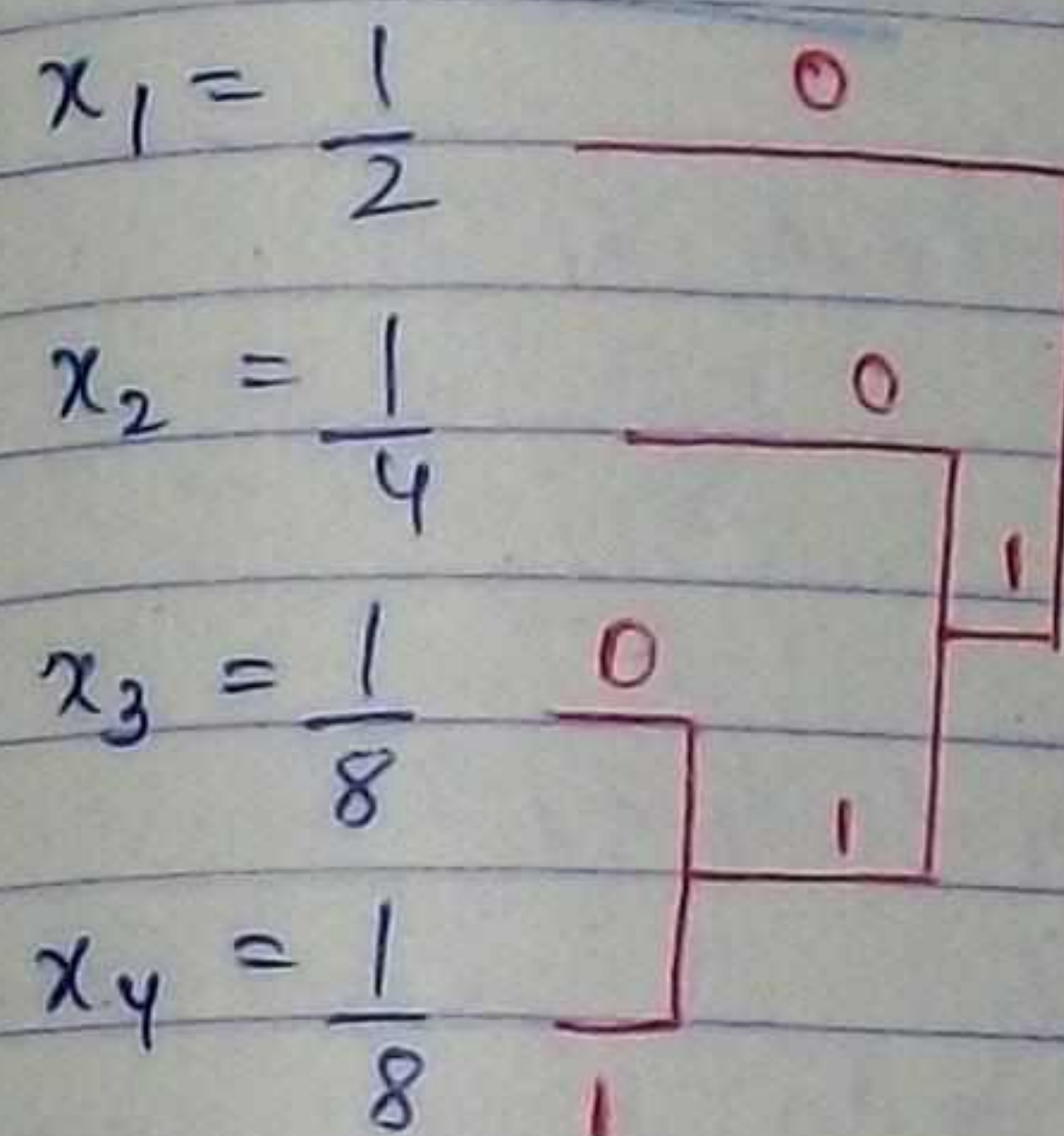
Average length of code

$$L = \frac{2 \times 1}{2} + \frac{2 \times 1}{4} + \frac{2 \times 1}{8} + \frac{2 \times 1}{8} = 2$$

$$\eta = \frac{H}{L} = \frac{1.75}{2} \times 100 = 87.5\%$$

⇒ HUFFMAN CODING - (Arrange in descending order)

$$x_1 = \frac{1}{2}, x_2 = \frac{1}{4}, x_3 = \frac{1}{8}, x_4 = \frac{1}{8}$$



$$\begin{aligned} x_1 &= 0 \\ x_2 &= 10 \\ x_3 &= 110 \\ x_4 &= 111 \end{aligned}$$

$$L = \frac{1 \times 1}{2} + \frac{2 \times 1}{4} + \frac{3 \times 1}{8} + \frac{3 \times 1}{8} = 1.75$$

$$\% \eta = \frac{H}{L} \times 100$$

$$\text{Redundancy} = 1 - \eta$$

Ques. 5 symbols have $P_e \rightarrow 0.4, 0.2, 0.2, 0.1, 0.1$ are transmitted in a commⁿ channel. Then determine η with uniform length coding & Huffman coding.

Solⁿ: (1) By uniform coding:

$$H = 0.4 \times \log_2 \frac{1}{0.4} + 0.2 \times \log_2 \frac{1}{0.2} + 0.2 \times \log_2 \frac{1}{0.2} + 0.1 \times \log_2 \frac{1}{0.1} + 0.1 \times \log_2 \frac{1}{0.1}$$

$$H = 0.4 \log_2 \frac{1}{0.4} + 0.2 \log_2 \frac{1}{0.2} + 0.2 \log_2 \frac{1}{0.2} + 0.1 \log_2 \frac{1}{0.1} + 0.1 \log_2 \frac{1}{0.1}$$

$$H = 2.12$$

$$\bar{L} = 3 \times 0.4 + 3 \times 0.2 + 3 \times 0.2 + 3 \times 0.1 + 3 \times 0.1 = 3$$

$$\eta = \frac{H}{L} \times 100 = \frac{2.12}{3} \times 100 = \underline{\underline{70.67\%}}$$

$$\text{Redundancy} = 1 - \eta = \underline{\underline{29.33\%}}$$

② Huffman coding:

0.4, 0.2, 0.2, 0.1, 0.1

$$\frac{1 \times 6}{8} + \frac{1 \times 6}{8} + \frac{1 \times 6}{4} + \frac{1 \times 6}{2} = 5$$

$$x_1 = 0.4$$

$x_2 = 0.2$

$$x_3 = 0.2$$

$$x_y = 0.1$$

$$x_y = 0.1$$

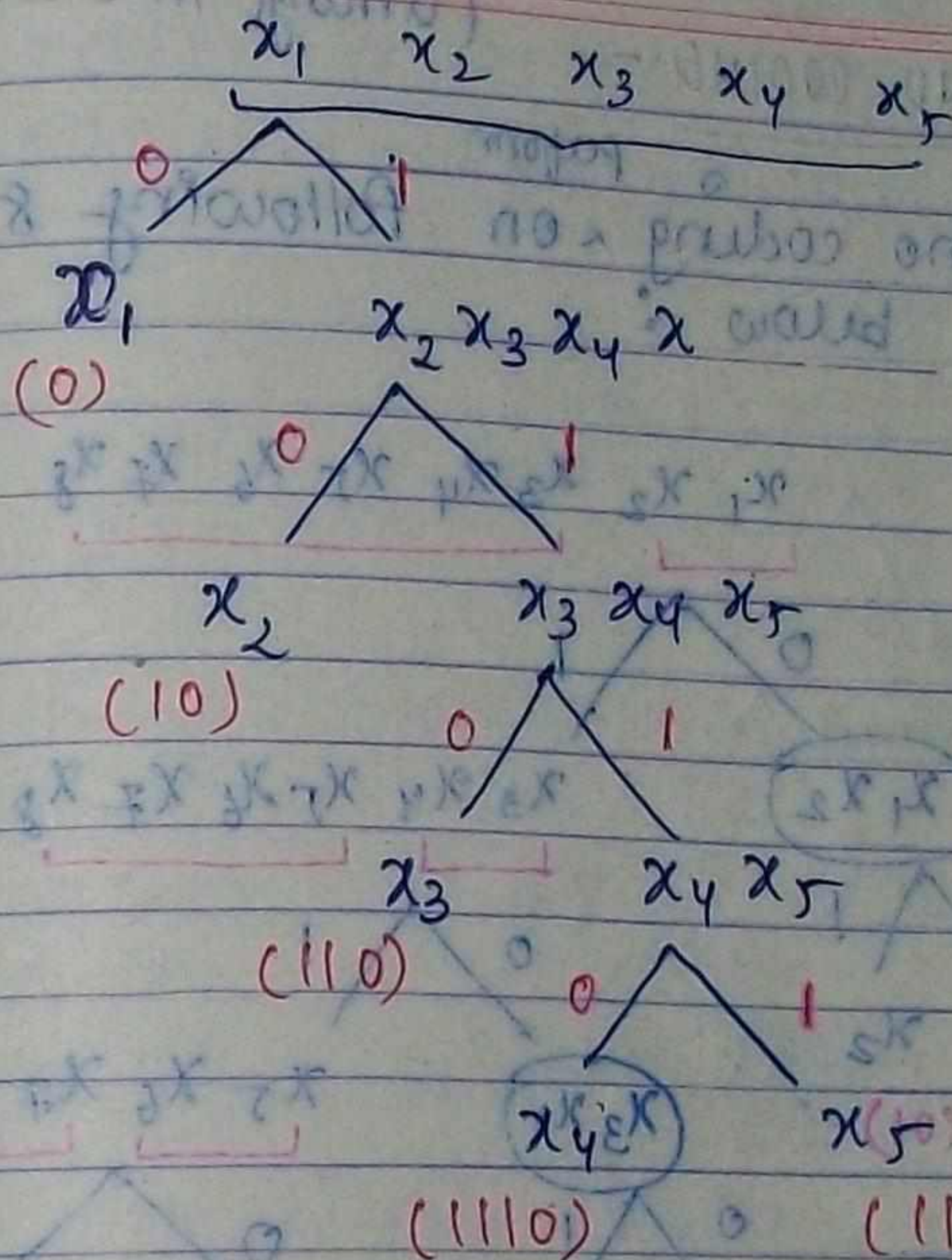
$$\bar{b} = 0.4 \times 1 + 0.2 \times 2 + 0.2 \times 3 + 4 \times 0.1 + 4 \times 0.1$$

$$\eta = \frac{H}{I} = \frac{2.12}{2.2}$$

$$= \underline{\underline{96\%}}$$

Redundancy = 4 %

#



Huffman coding

$$\# H(x) = \sum_{i=1}^L p_i \log_2 \frac{1}{p_i}$$

$$= \sum_{i=1}^L P(x_i) \log_2 \frac{1}{P(x_i)}$$

P-618
Conventional

⇒ SHANNON - FANO CODING - (arrange in decreasing order)

- In shannon-fano coding ^{perform} on following 8 symbol prob. are given below:

$$x_1 = 0.25$$

$$x_2 = 0.25$$

$$x_3 = 0.125$$

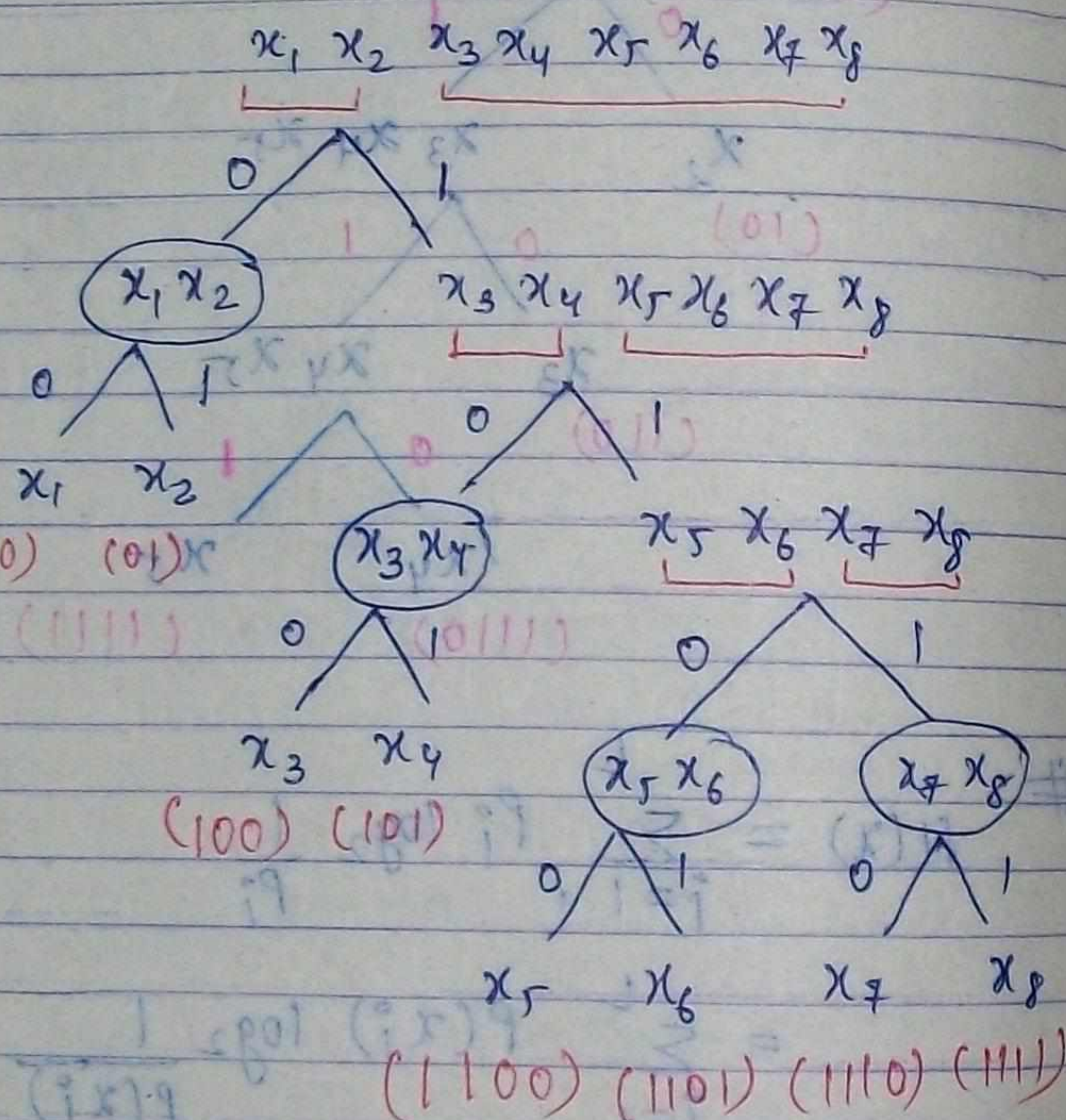
$$x_4 = 0.125$$

$$x_5 = 0.0625$$

$$x_6 = 0.0625$$

$$x_7 = 0.0625$$

$$x_8 = 0.0625$$



$$H = 0.25 \log_2 \frac{1}{0.25} + 0.25 \log_2 \frac{1}{0.25} + 0.125 \log_2 \frac{1}{0.125} + 0.125 \log_2 \frac{1}{0.125} + 4 \left(0.0625 \log_2 \frac{1}{0.0625} \right)$$

$$H = 2.44$$

$$\bar{L} = 2 \times 0.25 + 2 \times 0.25 + 3 \times 0.125 + 3 \times 0.125 + 4 \times 0.0625 + 4 \times 0.0625 + 4 \times 0.0625 + 4 \times 0.0625$$

$$\bar{L} = 2.75$$

$$\eta = \frac{2.44}{2.75} \times 100 =$$

Ques 1. P-34 workbook. (normal coding)

$$x_1 = 1/2 \rightarrow 000$$

$$x_2 = 1/4 \rightarrow 001$$

$$x_3 = 1/8 \rightarrow 010$$

$$x_4 = 1/16 \rightarrow 011$$

$$x_5 = 1/32 \rightarrow 100$$

$$x_6 = 1/64 \rightarrow 101$$

$$x_7 = 1/128 \rightarrow 110$$

$$x_8 = 1/256 \rightarrow 111$$

$$\textcircled{1} H = \sum_{i=1}^L p_i \log_2 \frac{1}{p_i}$$

$$= 1.96$$

$$\textcircled{2} P = \left[3 \times \frac{1}{2} + 2 \times \frac{1}{4} + 2 \times \frac{1}{8} + 1 \times \frac{1}{16} + 2 \times \frac{1}{32} + 1 \times \frac{1}{64} + 1 \times \frac{1}{128} + 0 \times \frac{1}{256} \right] \frac{1}{3}$$

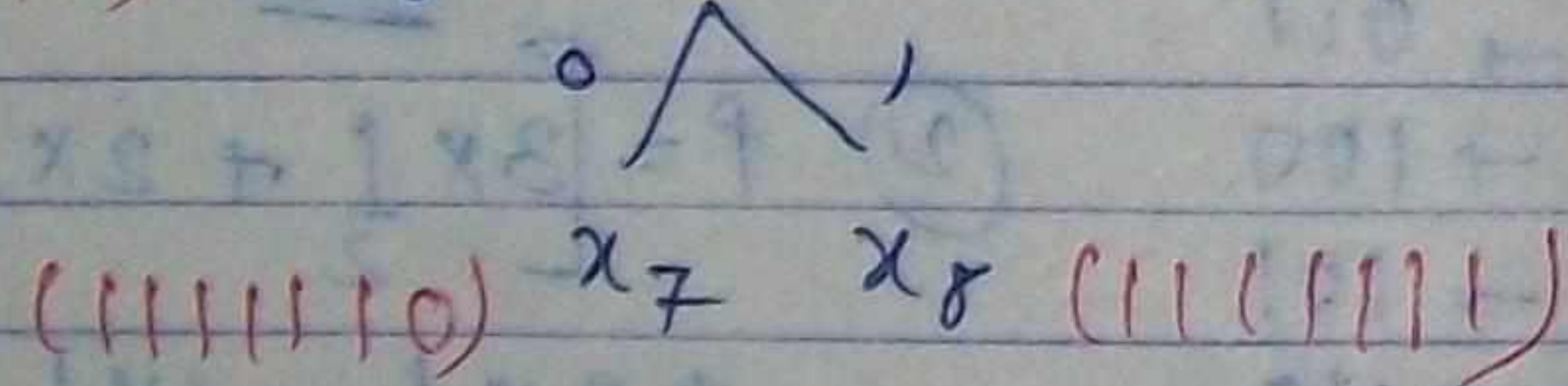
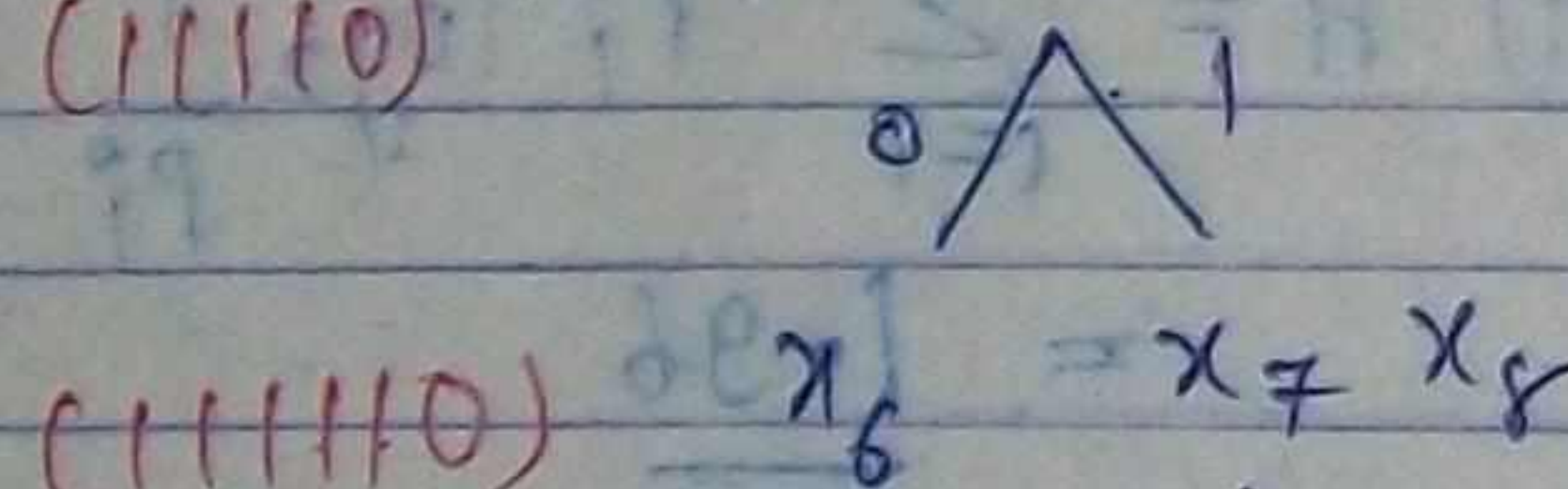
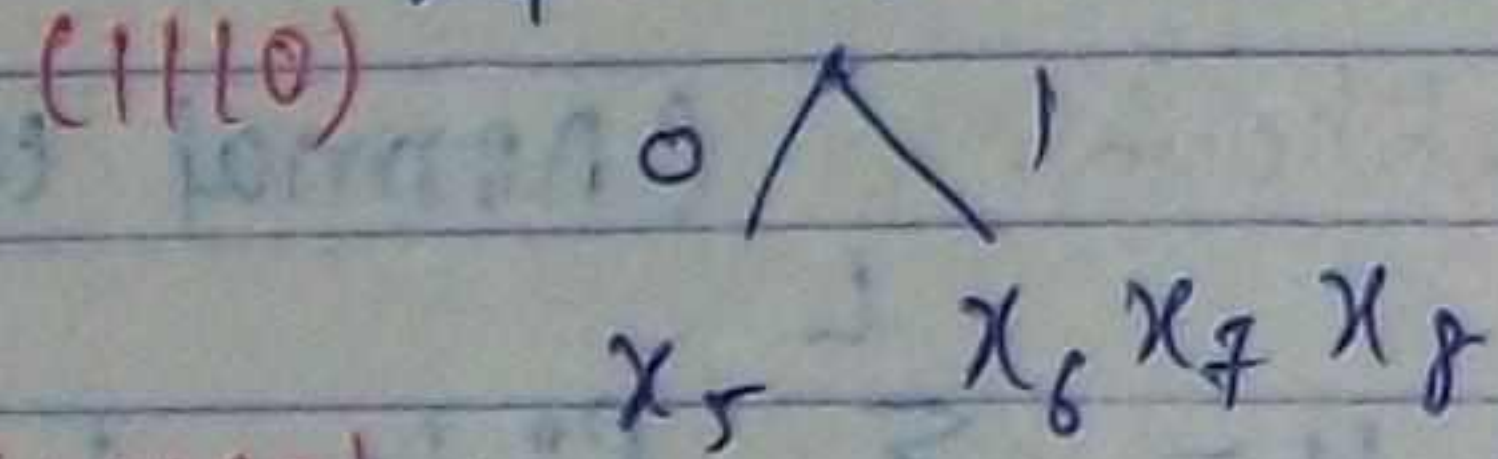
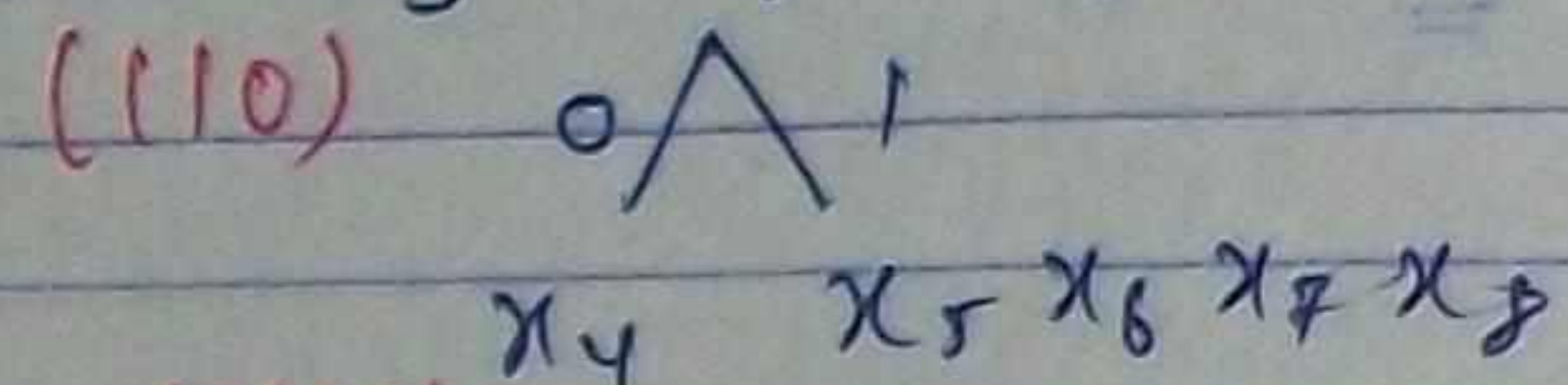
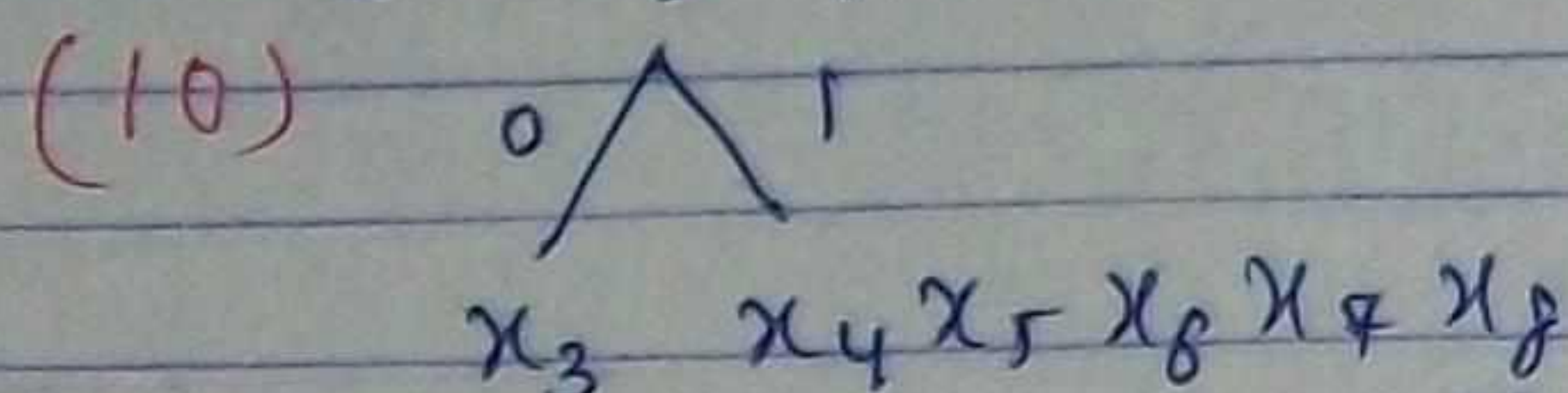
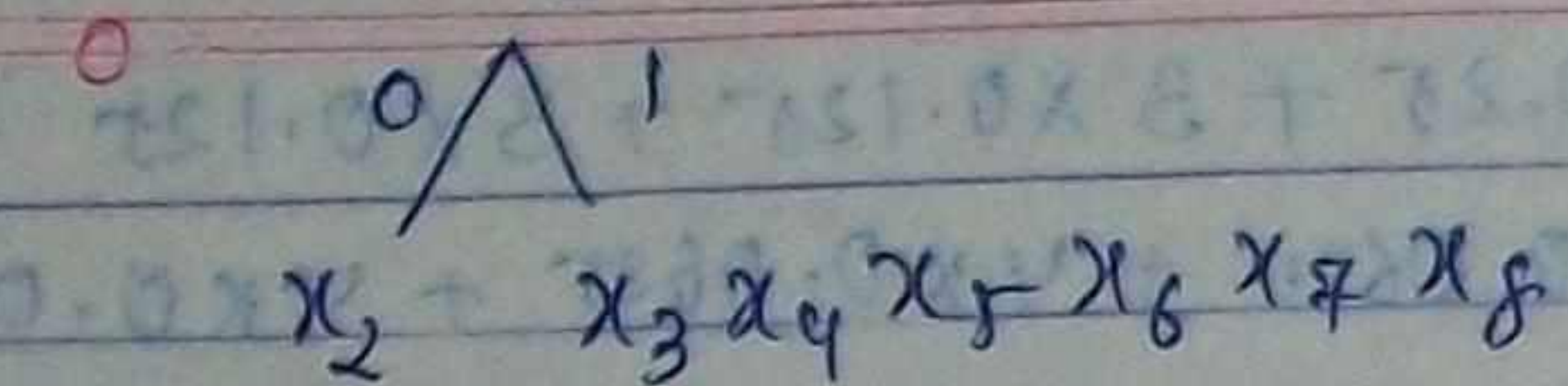
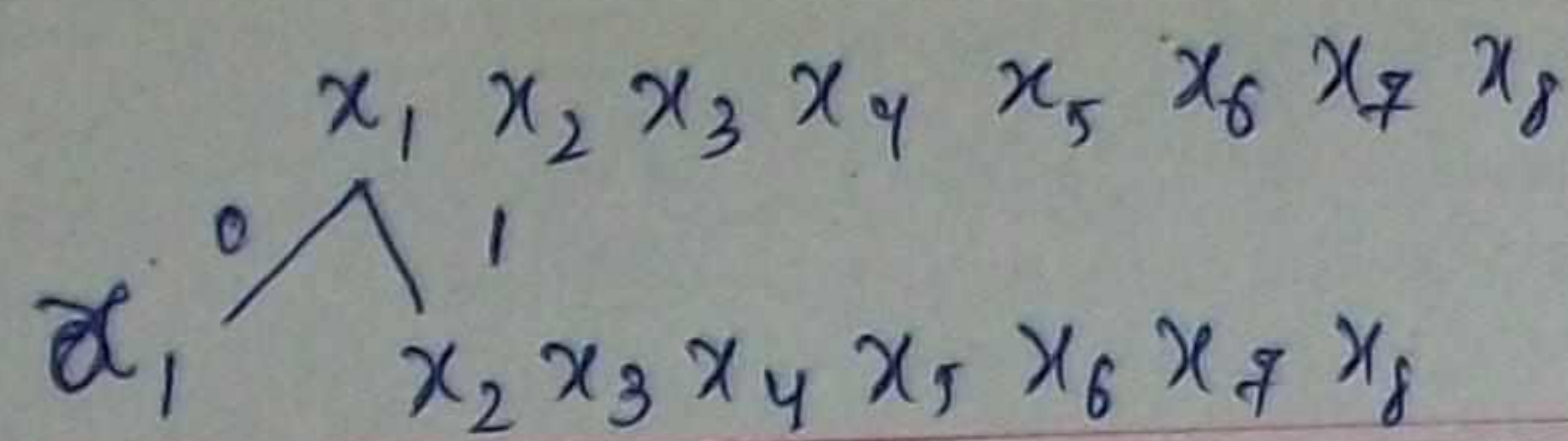
$$P(0) = 0.799 \approx 0.8$$

$$P(1) = 1 - P(0) = 0.2$$

$$\textcircled{3} \bar{L} = 3$$

$$\eta = \frac{1.96}{3} = 65.9\%$$

④ By shannon coding:



- Since all symbols are having different prob.
 ∴ Shannon coding becomes Huffman coding

- Bit
- $x_1 \rightarrow 1$
 - $x_2 \rightarrow 2$
 - $x_3 \rightarrow 3$
 - $x_4 \rightarrow 4$
 - $x_5 \rightarrow 5$
 - $x_6 \rightarrow 6$
 - $x_7 \rightarrow 7$
 - $x_8 \rightarrow 7$

(5) $L = \frac{1}{2} \times 1 + \frac{1}{4} \times 2 + \dots = 1.96$

$\eta = \frac{H}{L} = \frac{1.96}{1.96} = 100\%$

Second order extension code.

Ques 2. P-34 workbook -

Solⁿ: (b)

- $x_1 x_1 =$
- $x_1 x_2 =$
- $x_1 x_3 =$
- $x_2 x_1 =$
- $x_2 x_2 =$
- $x_2 x_3 =$
- $x_3 x_1 =$
- $x_3 x_2 =$
- $x_3 x_3 =$