

(176)



-: HAND WRITTEN NOTES:-

OF

CIVIL ENGINEERING

(1)

-: SUBJECT:-

SURVEYING



Introduction: →

Earth: → Earth is an oblate spheroid.
(Flattened on poles)

(3)

Diameter - polar axis = 12714.80 km.

Equatorial axis = 12756.75 km.

difference = 42.95 km (0.34% less)

Types: →

① plane Surveying: → at least curvature is not considered
(Suitable for small area)

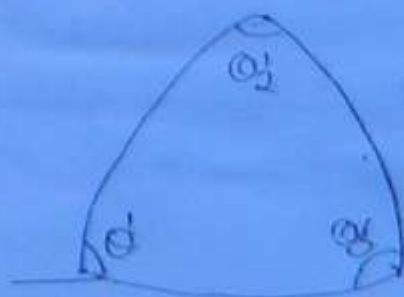
② Geodetic Surveying: → If earth curvature is considered
(Suitable for large area)

(a)



difference for 12 km length = 1 cm

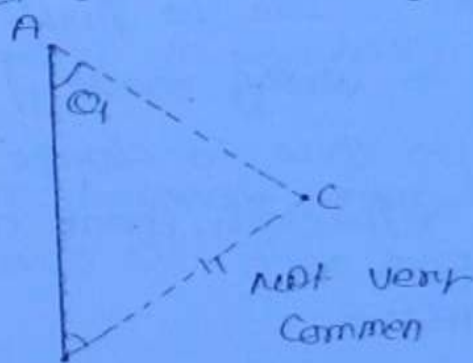
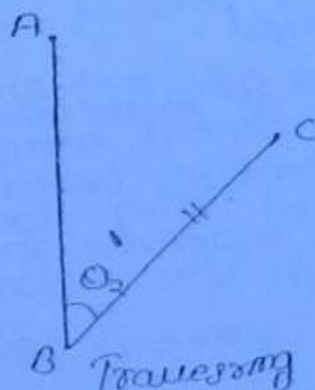
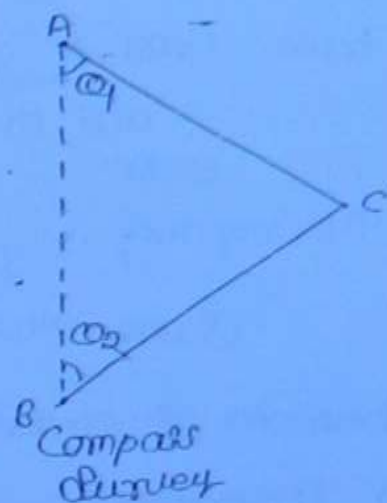
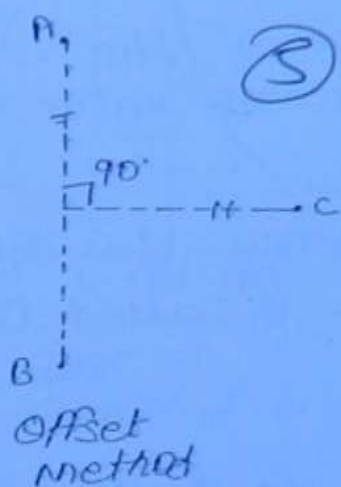
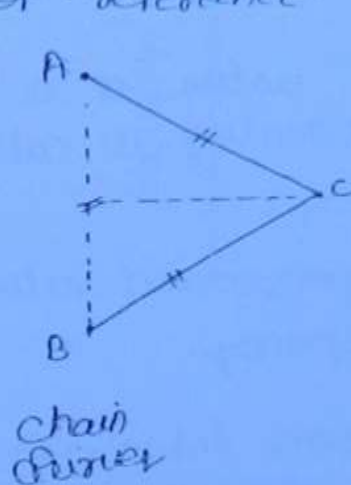
(b)



Difference of total angle
 $(\alpha_1' + \alpha_2' + \alpha_3') - (\alpha_1 + \alpha_2 + \alpha_3) = 1 \text{ Second} = 0^{\circ} 0' 1''$

(3) Methods of Surveying: $\rightarrow$

(a) Location of a point by measurement from two points of reference.



(b) Working from whole to part: $\rightarrow$ first major control points and - setting whole are fixed and distance are measured with higher accuracy then minor details can be taken even with less precision. Error involved in ~~can be taken~~ measurement will not be accumulated.

* Accuracy and Errors: $\rightarrow$

(1) Definitions

(1) Accuracy: $\rightarrow$ Degree of precision obtained in measurement of a quantity is called accuracy. By using proper instrument correct measurement and correct manner of taking measurement.

⑤ Precision:→

⑥ Degree of perfection used in the measurement is called precision.

⑦ True Error:→ Difference b/w true value of a quantity and measurement value of a quantity is called true error.

⑧ Discrepancy:→ Difference b/w two measured value of same quantity is called Discrepancy.

A Source of Errors:→

① Instrument :- Due to faulty instrument.

② Personal → Wrong reading/writing of measurement.

③ Nature → Due to change in temperature, humidity, refraction, local attraction magnetic declination.

A Kind of Errors:→

① Mistakes:→ Human error's due to less knowledge, carelessness, confusion in experience.

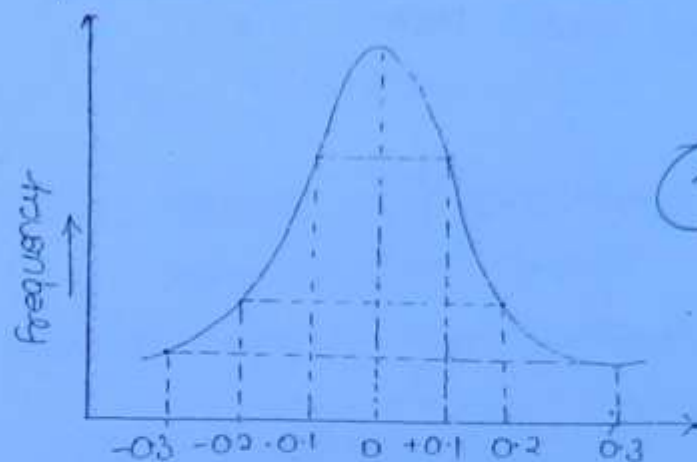
② Systematic Error:→ (Cumulative error)

Always have same size and directions under same condition of measurement may be either (+)ive or (-)ive

③ Accidental Error's → (Compensating error):→

These error's occur's same times in one direction and some times in opposite time and (-)ive error's compensate each others.

Heavy of probability



→ Accidental error's followed a definite rule method law of probability.

As per this law according to possibility distribution curve of error.

① Small error have higher frequency than large errors.

② positive and negative errors of same magnitude have same frequency.

* True value: → Exact value of a quantity.
(Almost impossible to measure)

* Most probable value: → The value of measurement which has chances of being the correct of a quantity than other measurement is called most probable value.

* Principal of least square: →

Most probable value of quantity is : for which sum of a square of a residual error is min.

Ex: → when all measurement have equal weight $x_1, x_2, x_3, \dots, x_n$ (equal weight = 1.0)

Residual error:-

If most probable value = x .

$$x = x_1$$

$$x = x_2$$

Square $(x-x_1)^2$

$(x-x_2)^2$

(8)

As per principle of least square

$$y = (x-x_1)^2 + (x-x_2)^2 + \dots = \min^m$$

$$\Rightarrow \frac{dy}{dx} = 2(x-x_1) + 2(x-x_2) + \dots = 0$$

$$nx = (x_1 + x_2 + \dots + x_n) = 0$$

$$n \cdot x = (x_1 + x_2 + \dots + x_n) = 0$$

$$x = \left(\frac{x_1 + x_2 + \dots + x_n}{n} \right) = \text{mean of all}$$

Case (iii) Unequal weights

Squares	measurement	weight	m.p.v	Residual error
$(x-x_1)^2 \times w_1$	x_1	w_1	$\uparrow$	$x-x_1$
$(x-x_2)^2 \times w_2$	x_2	w_2	$\uparrow$	$x-x_2$
-----	-----	-----	x	-----
$(x-x_n)^2 \times w_n$	x_n	w_n	$\downarrow$	$(x-x_n)$

Sum of square of residual error.

$$y = w_1(x-x_1)^2 + w_2(x-x_2)^2 + \dots = \min^m$$

$$\frac{dy}{dx} = 2w_1(x-x_1) + 2(w_2)(x-x_2) + \dots = 0$$

$$m.p.v \Rightarrow x = \frac{w_1x_1 + w_2x_2 + \dots}{w_1 + w_2 + \dots} = \frac{\sum w_i x_i}{\sum w_i}$$

★ The probable error of a single observation.

$$E_s = \pm 0.6745 \sqrt{\frac{\sum V^2}{(n-1)}} \quad (9)$$

V = difference b/w any single observation and mean of the series.

★ The probable error of the mean:-

$$E_m = \pm 0.6745 \sqrt{\frac{\sum V^2}{n(n-1)}}$$

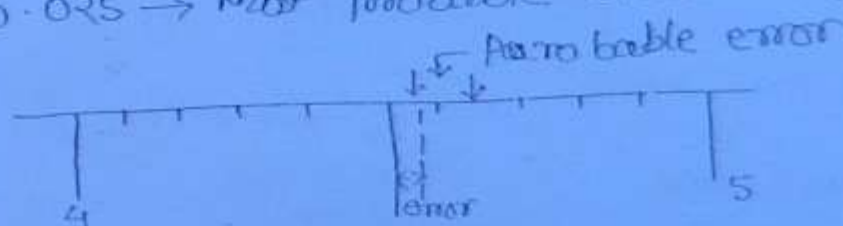
$$E_m = \frac{E_s}{\sqrt{n}}$$

★ Significant figures: $\rightarrow$ /max. error/ Most probable error: $\rightarrow$

4.6

error 0.05 $\leftarrow$ max. error

0.025 $\rightarrow$ Most probable error



$\rightarrow$ No. of Significant figures shows the accuracy of measurement

$\rightarrow$ If there are n Significant figure in a measurement (n-1) figure are called - certain figure

least figure is called - Uncertain figure.

$\rightarrow$ There are two types of error

Example Max. error Probable error

4.6

0.05

0.025

5.86

0.005

0.0025

① For max. error:→

Total error = Algebraic Sum of all errors.

$$e_T = \pm e_1 \pm e_2 \pm e_3$$

② For probable error:→

Sum = Root mean Square value

$$e_T = \pm \sqrt{e_1^2 + e_2^2 + e_3^2 + \dots}$$

→ Error's in computed Result:→

① Addition:→

If x and y are two measured value

$$S = x + y$$

$$\boxed{ds = dx + dy} \quad \text{--- ①}$$

If max. error's are $\pm \delta x$ and $\pm \delta y$

max. error

$$S \pm \delta S = (x \pm \delta x) + (y \pm \delta y)$$

$$\Rightarrow S \pm \delta S = (x + y) \pm (\delta x + \delta y)$$

$$\delta S = (\delta x + \delta y) \text{ max. error in } S$$

Probable error:-

are e_x and e_y sum of probable error's

$$e_S = \sqrt{e_x^2 + e_y^2}$$

$$\text{Range of } S = (S + e_S) \text{ to } (S - e_S)$$

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(ii) Subtraction: $\rightarrow$

$$S = x - y$$

$$dS = dx - dy$$

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Max. error if $\pm \delta x$ and $\pm \delta y$ are max. error

$$\text{for } (+) \Rightarrow \delta S = (+\delta x) - (-\delta y)$$

$$\delta S = +(\delta x + \delta y)$$

$$\text{or } = (-\delta x) - (+\delta y)$$

$$= -(\delta x + \delta y)$$

$$\delta S = \pm (\delta x + \delta y)$$

Range of S

$$S = (S + \delta S) \text{ to } (S - \delta S)$$

Probable error

$$e_s = \pm \sqrt{(e_x)^2 + (e_y)^2}$$

$$\text{Range} = (S + e_s) \text{ to } (S - e_s)$$

(iii) Multiplication: $\rightarrow$

$$S = x \cdot y$$

$$dS = x dy + y dx$$

Max. error

Respective error of x and $y \rightarrow \delta x$ and δy

$$\text{for } S \text{ in } x \rightarrow y \cdot \delta x$$

$$\text{in } y \rightarrow x \cdot \delta y$$

$$S = x \cdot \delta y + y \cdot \delta x \quad \text{--- (1)}$$

$$\text{Range } (S + \delta S) \text{ to } (S - \delta S)$$

Probable error: $\rightarrow$

Respective error of x and $y \rightarrow e_x$ and e_y

$$\text{for } S, \text{ error in } x = y \cdot e_x$$

$$y = x \cdot e_y$$

$$\Delta m = e_s = \sqrt{(y e_x)^2 + (x e_y)^2} \quad (12)$$

$$\Rightarrow \frac{e_s}{s} = \frac{1}{xy} \cdot \sqrt{(y e_x)^2 + (x e_y)^2}$$

$$\Rightarrow \frac{e_s}{s} = \sqrt{\left(\frac{e_x}{x}\right)^2 + \left(\frac{e_y}{y}\right)^2}$$

$$\Rightarrow e_s = s \times \sqrt{\left(\frac{e_x}{x}\right)^2 + \left(\frac{e_y}{y}\right)^2}$$

(q) Dividing: $\rightarrow s = x/y$

$$ds = \frac{y dx - x dy}{y^2}$$

$$ds = \left(\frac{dx}{y}\right) - \left(\frac{x}{y^2}\right) dy$$

Max error: - Respective error of x and $y \rightarrow \delta x$ and δy

For s in $x \rightarrow \frac{1}{y} \delta x$

in $y \rightarrow \frac{x}{y^2} \delta y$

$$\boxed{\Delta s = \frac{\delta x}{y} + \frac{x}{y^2} \delta y}$$

Range $(s + \Delta s)$ to $(s - \Delta s)$

Probable error: -

Respective error of x and $y \rightarrow e_x$ and e_y

For s error in $x = \frac{1}{y} e_x$

$y = \frac{x}{y^2} e_y$

$$\Delta m \quad e_s = \sqrt{\left(\frac{e_x}{y}\right)^2 + \left(x \cdot \frac{e_y}{y^2}\right)^2}$$

$$\Rightarrow \frac{e_s}{s} = \frac{y}{x} \sqrt{\left(\frac{e_x}{y}\right)^2 + \left(\frac{x}{y^2} e_y\right)^2}$$

$$\Rightarrow \left| \frac{e_s}{s} = \sqrt{\left(\frac{e_x}{x}\right)^2 + \left(\frac{e_y}{y}\right)^2} \right| \quad \text{--- 0}$$

Ques 1) A quantity S is equal to sum of two measured quantities x and y .

$$S = 5.68 + 3.648$$

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Find out probable error, max. error, most probable limits, max. limits of S .

$x = 5.68$	$0.005 \delta x$	$0.0025 \delta x$
$y = 3.648$	$0.0005 \delta y$	$0.00025 \delta y$
	max. error	probable error.

Solution: $\rightarrow$ (1) max. error

$$(\text{for } S) \delta S = \delta x + \delta y$$

$$\delta S = 0.005 + 0.0005$$

$$\delta S = 0.0055$$

$$\text{Range max. range} = S + \delta S \text{ to } (S - \delta S)$$

$$= 9.328 + 0.0055 \text{ to } 9.328 - 0.0055$$

$$9.3225$$

(2) most probable error: -

$$e_s = \sqrt{e_x^2 + e_y^2}$$

$$e_s = \sqrt{(0.0025)^2 + (0.00025)^2}$$

$$e_s = 0.002512$$

probable range of S

$$= S \pm e_s \text{ to } S - e_s$$

$$= 9.328 \pm 0.002512$$

$$= 9.330512 \text{ to } 9.325488$$

$$= 9.328 \pm 0.002512$$

$$= 9.3255 \text{ Ans}$$

Ques 3) Calculate the max. & probable error & range of a computed quantity.

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$$S = \frac{9.58}{4.6}$$

Solution:→

		max ^m	probable
x	9.58	$\delta x = 0.005$	$e_x = 0.0025$
y	4.60	$\delta y = 0.05$	$e_y = 0.025$

$$S = 2.0826$$

$$ds = \frac{y dx - x dy}{y^2}$$

$$ds = \frac{dx}{y} - \frac{x}{y^2} dy$$

max^m error

$$\delta_s = \frac{\delta x}{y} + \frac{\delta y}{y^2} x$$

$$\delta_s = \frac{0.005}{4.6} + \frac{9.58 \times 0.05}{4.6^2}$$

$$\delta_s = 0.0237$$

Range of S

2.0826	+0	2.0826
+0.0237		-0.0237
2.1063		2.0589

probable error

$$\Rightarrow \frac{e_s}{S} = \sqrt{\left(\frac{e_x}{x}\right)^2 + \left(\frac{e_y}{y}\right)^2}$$

$$\Rightarrow e_s = 2.0826 \sqrt{\left(\frac{0.0025}{9.58}\right)^2 + \left(\frac{0.025}{4.6}\right)^2}$$

$$\Rightarrow e_s = +0.0113$$

Range of S

2.0826	2.0826
+0.0113	-0.0113
2.0939	2.0713

Ans

(15)

* Fundamental definition: →

(Linear measurement)

Scale: → Scale is ratio of distance plotted on drawing to distance on the ground.

EX = $1 \text{ cm} = 1 \text{ km}$

$1 \text{ cm} = 1000 \text{ m}$

$1 \text{ cm} = 1000 \times 100 \text{ cm}$

$$R.F. = \frac{1}{100000} = \text{Representative fraction}$$

Types: →

① plane scale.

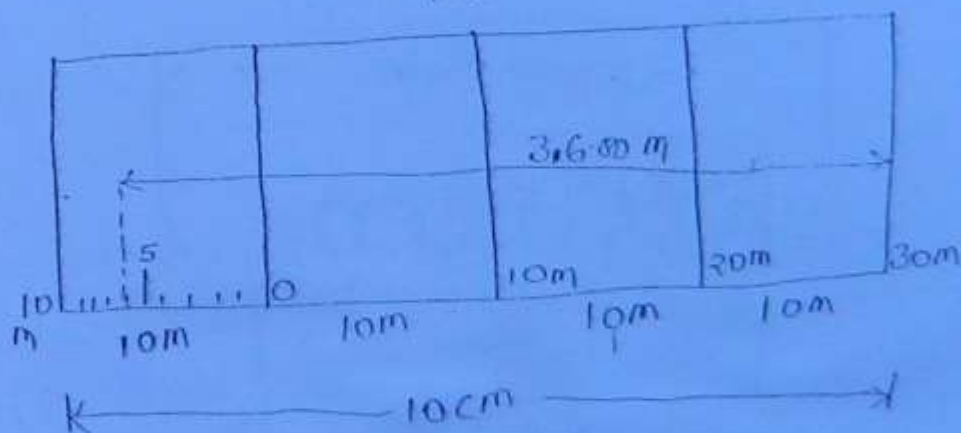
② Diagonal scale.

③ Vernier scale.

④ Chord scale.

① plane scale: → It measures upto two dimension onlyA scale $1 \text{ cm} = 4 \text{ m}$ to be prepared.

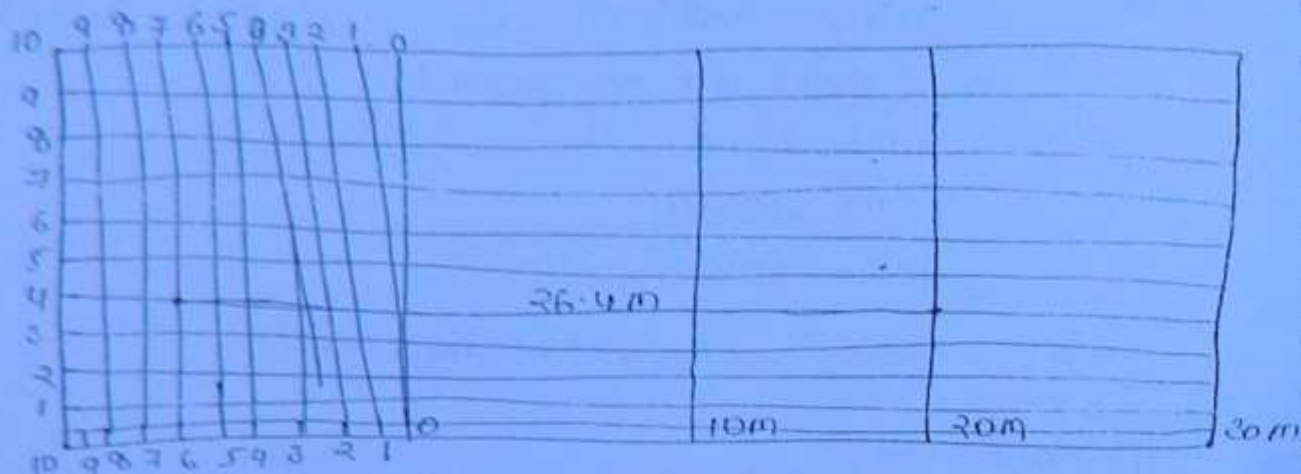
$$R.F. = \frac{1}{400}$$



→ In this scale, 10m and meter are the two dimension that can be measured.

② Diagonal Scale $\rightarrow$ It can measure upto 3 dimension.
Similar triangle theory is used in diagonal scale.

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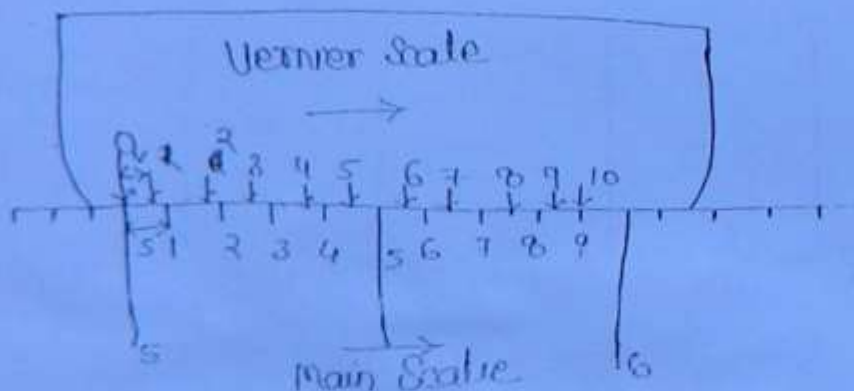


In three dimension in above case

- ① 10m
- ② meter
- ③ decimeter

③ Vernier Scale $\rightarrow$

(a) Direct Vernier Scale $\rightarrow$ In this case also upto 2 dimension can be read.



In case of direct Vernier scale

- ① Vernier scale moves in same direction as of main scale.
- ② $(n-1)$ parts of main scale is equal to n divisions of vernier scale.

$$n \cdot V = (n-1)S$$

$S =$ main scale
 $V =$ vernier scale

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$$V = \frac{(n-1) \cdot S}{n}$$

Least count $\rightarrow$ minimum value that can be used using a scale is called least count

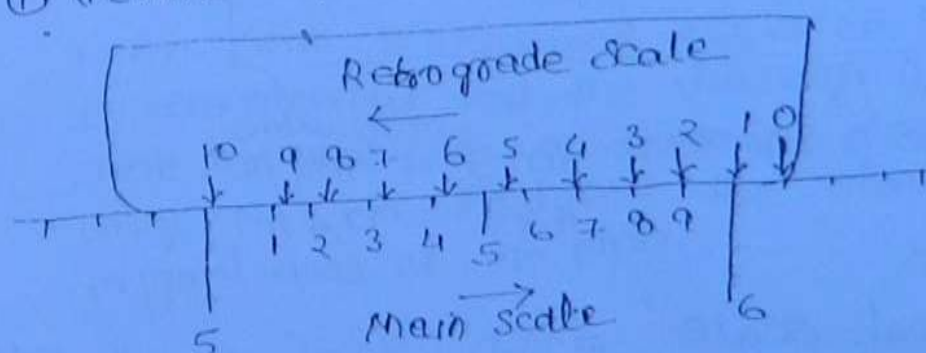
$$L.C. = S - V$$

$$L.C. = S - \frac{n-1}{n} S$$

$$L.C. = \frac{ns - ns + S}{n}$$

$$L.C. = \frac{S}{n} \quad \text{--- least count of this vernier scale.}$$

(6) Retrograde Scale $\rightarrow$ In case of Retrograde scale
(1) Vernier scale moves in opposite direction as the main scale.



(3) $(n+1)$ parts of main scale is equal n division (part) of Vernier scale.

$$n \cdot V = (n+1)S$$

$$V = \frac{(n+1) \cdot S}{n}$$

Least count $\rightarrow$ minimum value that can be read using a scale is called least count

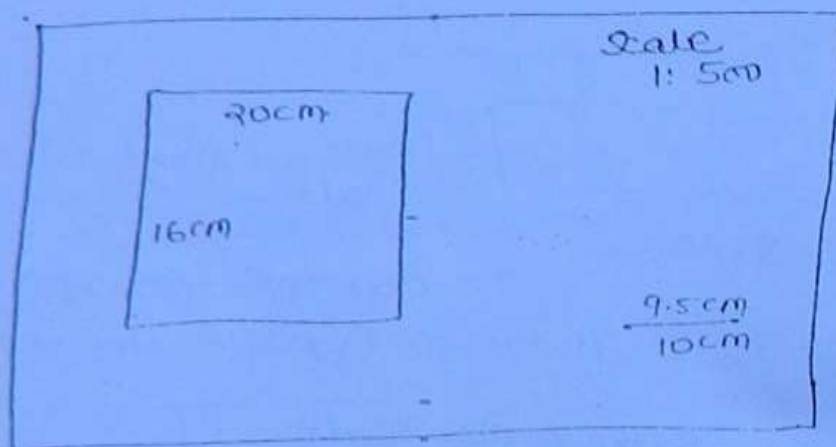
$$L.C. = \frac{n+1}{n} \times S - S$$

$$L.C. = \frac{(n+1) \times S - S}{n}$$

$$L.C. = \frac{ns + S - ns}{n}$$

$$L.C. = \frac{S}{n}$$

* Shrink Scale :
(and Shrinkage factor)



$$1: 500$$

$$1 \text{ cm} = 500 \text{ cm}$$

$$1 \text{ cm} = 5 \text{ m}$$

This line is represent - $20 \times 5 = 100 \text{ m}$ (Scale value)
 $16 \times 5 = 80 \text{ m}$

original scale

$$1: 500$$

$$10 \text{ cm} = 5000 \text{ cm}$$

Now

present - Scale

$$9.5 \text{ cm} = 5000 \text{ cm}$$

$$1 \text{ cm} = \frac{5000}{9.5} \text{ cm}$$

$$1 \text{ cm} = 526.316 \text{ cm}$$

$$1: 526.316$$

→ Shrinkage factor :- It is the ratio of Shrink length to original length of the line.

$$SF = \frac{\text{Shrink length}}{\text{original length}} \quad (19)$$

→ Shrink Scale = Shrinkage factor $\times$ Original Scale.

Ex. for given example.

$$SF = \frac{9.5}{10}$$

$$SF = 0.95$$

Shrink Scale = Shrinkage factor $\times$ Original Scale

$$= 0.95 \times \frac{1}{500}$$

$$= \frac{15}{526.316}$$

problem: → Area of a plan in a drawing plotted to a scale 1 cm = 50 m is measured 250 sq cm by planimeter. It was observed that the drawing has shrunk and line originally 10 cm drawn on drawing measured only 9.20 cm. Find out the shrink scale and original area of the plan.

Solution: → Shrink length = 9.2 cm

original length = 10 cm

$$\text{Shrinkage factor} = \frac{\text{Shrink length}}{\text{original length}}$$

$$= 9.2 / 10$$

$$= 0.92$$

original scale = 1 cm = 50 m

$$1 \text{ cm} = 5000 \text{ cm}$$

$$= 1/5000$$

$$\text{Shaunk scale} = S.F. \times 0.5$$

$$= 0.92 \times \frac{1}{5000}$$

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$$= \frac{1}{5434.78}$$

$$\text{Scale } 1\text{cm} = 54.35\text{m}$$

$$\text{original area} = 250\text{ cm}^2 \times (54.35)^2$$

$$= 738480.6\text{ m}^2$$

Ans:-

★ Error due to wrong length of chain / Tape :->

L = length (Designated) length of Tape / chain

L' = wrong length of Tape / chain [actual length]

J' = ^{wrong} Measured (written) length of a line

J = True of line measured

$$\boxed{\begin{array}{l} \text{True} \times \text{True} = \text{wrong} \times \text{wrong} \\ J \times L = L' \times J' \end{array}}$$

$$\text{True length of line } J = \left(\frac{L'}{L}\right) \times J'$$

Ex:-> If a 30m chain is actually 30.20 m long, what will be the actual length of a line which is measured 3052m using above tape.

Solution:->

$$L = 30\text{m}$$

$$L' = 30.20\text{m}$$

$$J' = 3052\text{m}$$

$$J = ?$$

$$L \times J = L' \times J'$$

$$J = \left(\frac{L'}{L}\right) \times J'$$

$$J = \frac{30.20}{30} \times 3052 = \boxed{3072.33\text{m}}$$

Formula

① $L = \left(\frac{L'}{L}\right) \times L'$ for length

② For Area

$$A = \left(\frac{L'}{L}\right)^2 \times A'$$

②

③ For Volume

$$V = \left(\frac{L'}{L}\right)^3 \times V'$$

★ Tape Correction : →

① Correction due to standardization. [Due to wear length of chain / Tape].

Total Correction required

$$Ca = \frac{\text{Total length of line (L')}}{L} \times c$$

$$Ca = \frac{L'}{L} \times c$$

* Ca = Total Correction

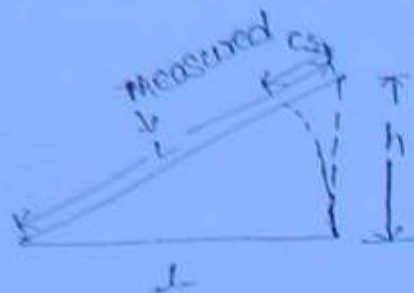
c = Correction required per chain length

L = Designated length of tape / chain

② Measured Length	③ Unaffected Length	④ Error	⑤ Correction	⑥ Case
more 30.50	less 30.0	(-)ve -0.50m	(+)ve +0.50m	Tape is long
less 29.50m	more 30.00m	(+)ve +0.50m	(-)ve -0.50m	Tape is short

(B) Correction due to slope (C_s)

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In case of chaining along a sloping area we need to measure the horizontal distance.

$$\text{Slope correction } (C_s) = L - L$$

$$C_s = L - (\sqrt{L^2 - h^2})$$

$$C_s = L - L [1 - h^2/L^2]^{1/2}$$

$$C_s = L - L + \frac{h^2}{2L} + \dots$$

$$C_s = \frac{h^2}{2L}$$

Error always positive (+ve) Correction is always (-)ve.

(C) Correction due to alignment $\Rightarrow (C_a) \Rightarrow$



If h is error in alignment.

L = length of line measured

Correction due to alignment

$$C_a = \frac{h^2}{2L} \leftarrow \text{Same as slope correction}$$

[Error always (+)ve] [Correction always (-)ve]

(d) Correction due to Temperature: $\rightarrow$ The correction required C_T

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$$C_T = L (T_m - T_0) \alpha$$

here L = length of line measured

T_m = Temperature at the time of measurement

T_0 = Temperature at the time of Standardization

Case (1) - IF T_m is more than T_0

$$(T_m > T_0)$$

Tape length is increase.

$$\text{Error} = L \alpha \text{ve}$$

$$\text{Correction} = (-) \text{ve}$$

Case (2) = IF $(T_m < T_0)$

Tape length is decrease

$$\text{Error} = (-) \text{ve}$$

$$\text{Correction} = (+) \text{ve}$$

(e) Full Correction: $\rightarrow$

If L = length of line measured.

P_m = Pull applied at the time of measurement.

P_0 = Pull applied at the time of Standardization

Pull Correction

$$C_P = \frac{(P_m - P_0) \cdot L}{AE}$$

A = c/s area of Tape/chain

E = Young's modulus of elasticity of Tape/chain

Case ① $P_m > P_0$

length increases

Error = (+) ve

Correction = (-) ve

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Case ② $P_m < P_0$

length is less

Error = (+) ve

Correction = (-) ve

⑦ Sag correction: →



Sag correction $C_{sag} = \frac{W^2 L}{24 P_m^2} = \frac{(wL)^2 L}{24 P_m^2}$

* W = Total weight of chain

$= wL$

w = weight per meter

L = length of chain/Tape

P_m = Pull applied at the time of measured.

Error-always = (+) ve

Correction-always = (-) ve

Ⓐ Normal Tension: →

If $(P_m > P_0)$ Pull correction is (+)ve
Sag correction is (-)ve

Pull correction and Sag correction neutralize each other

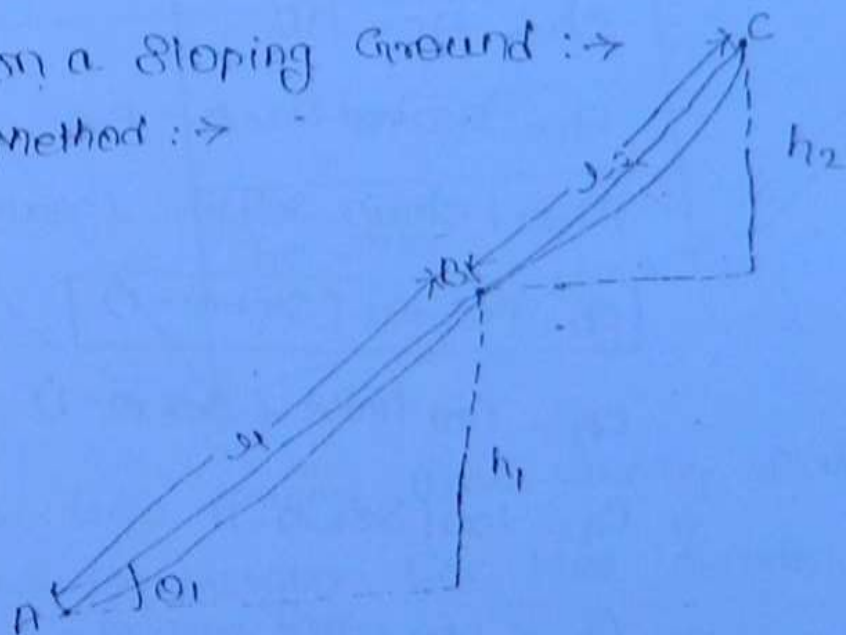
→ the value of pull (P_m) for which pull correction is equal to Sag correction is called Normal tension

$$\frac{(P_m - P_0)L}{AE} = \frac{w^2 L}{24 P_m^2}$$

This eq. can be solved for P_m to find total and error for (P_m)

Ⓑ Chaining on a Sloping Ground: →

① Indirect Method: →



Horizontal distance A to C

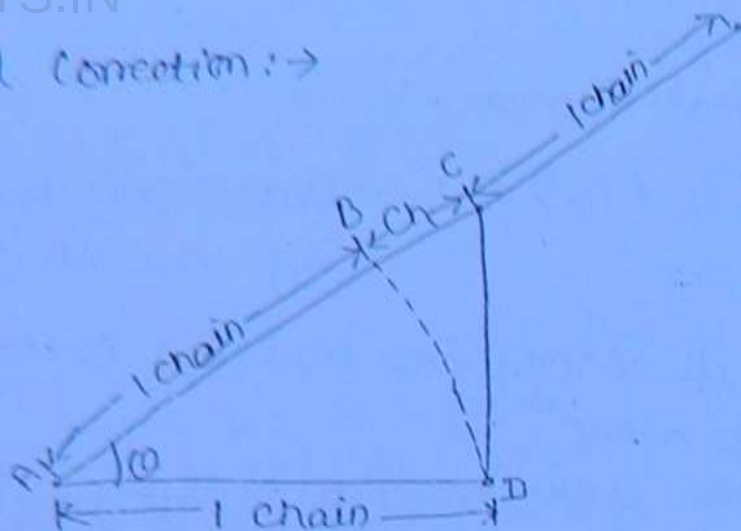
$$= L_1 \cos \theta_1 + L_2 \cos \theta_2 + \dots$$

② Measuring difference of level: →

$$\text{Horizontal distance} = \sqrt{L_1^2 - h_1^2} + \sqrt{L_2^2 - h_2^2} + \dots$$

③ Hypotenusal Correction: →

(26)



Hypotenusal correction $Ch = BC$

$$\frac{AD}{AC} = \cos \theta$$

$$Ch = AC - AB$$

$$Ch = AD \sec \theta - AB$$

$$Ch = 1 \text{ chain } \sec \theta - 1 \text{ chain}$$

$$Ch = 1 \text{ chain } (\sec \theta - 1)$$

$$Ch = 100 \text{ links } (\sec \theta - 1)$$

$$\Rightarrow Ch = 100 (\sec \theta - 1)$$

$$\Rightarrow Ch = 100 \times \frac{\theta^2}{2}$$

$$\Rightarrow Ch = 50 \theta^2 \text{ links}$$

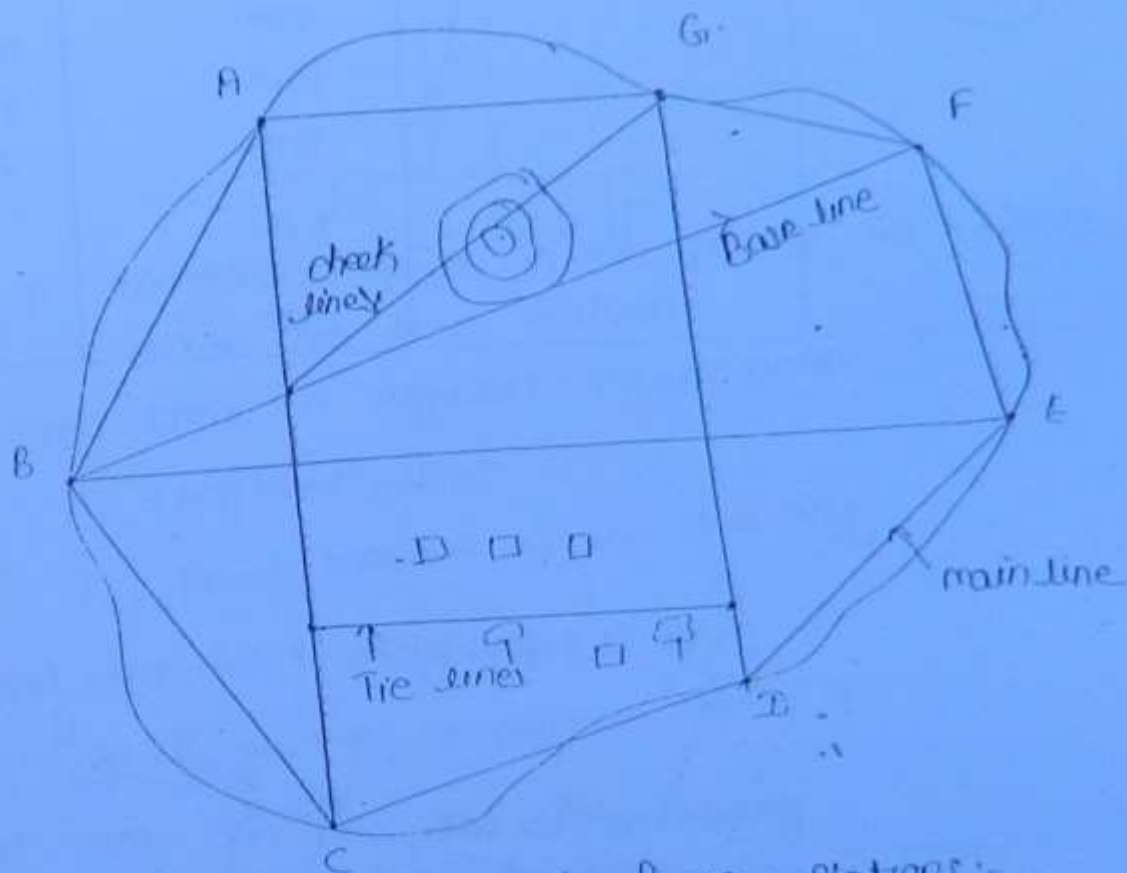
This correction is applied after each chain, and next chaining is started after correction.

① Chain Surveying: →

② Limiting length of offset: →

(21)

Important terms: →



① Main line: → Lines joining main survey stations:-

② Base line: → The longest line that divide the whole area in two part.

③ Tie line: → A line drawn to collect the information or detail of objects nearby.

④ check line: → To check the accuracy of survey work

⑤ Weak condition triangle: → A triangle having no angle $< 30^\circ$

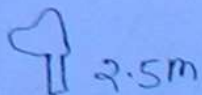
⑥ Off Sets and field book: →

Types { → Right angle off sets (Square offset) (at 90°)

→ Oblique. off sets: → at an angle with survey line

Hand Book: →

28



20m

16m

12m

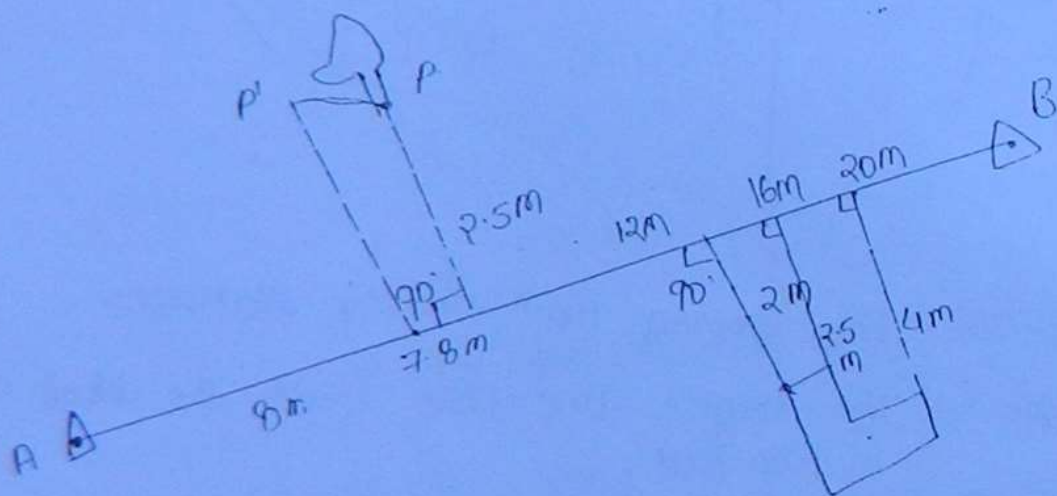
8m

ΔA

4

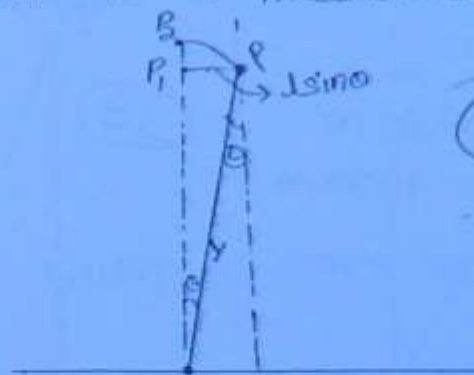
2.5

2m



Limiting length of offset $\rightarrow$

Case ② when error is in measurement of direction only



P = location of point on the ground for which offset has been taken.

θ = Error in angular measurement.

L = Length of offset.

P₂ = Point marked on the drawing.

Total error due to wrong angle is PP₂

$$\rightarrow PP_2 \approx PP_1 = L \sin \theta$$

If 1cm = S m is a scale of a drawing

$$\text{length of error on drawing} = \frac{L \sin \theta}{S} \text{ cm}$$

If error on the drawing is not more than 0.25 mm
(error shall not be reflected on the drawing)

$$\frac{L \sin \theta}{S} \text{ cm} = 0.025 \text{ cm}$$

$$L = \frac{0.025 \times S}{\sin \theta}$$

$$L = 0.025 \times S \times \frac{1}{\sin \theta} \rightarrow \text{Limiting length of offset}$$

Ques ① what is the limiting length of offset for a angular error in offset measurement of 3° and scale of drawing

① $1\text{cm} = 50\text{m}$

② $1\text{cm} = 1000\text{m}$

Solution: $\Rightarrow$ length of error on ground $= L \sin \theta$ m

length of error on drawing $= \frac{L \sin \theta}{S} \geq 0.025\text{cm}$

$L = 0.025 \times S \times \sin \theta$

① for Scale $1\text{cm} = 50\text{m}$

$L = 0.025 \times 50 \times \csc 3^\circ$

$L = 23.80\text{m}$

② for Scale $1\text{cm} = 1000\text{m}$

$L = 0.025 \times 1000 \times \csc 3^\circ$

$L = 477.7\text{m}$

Note: $\Rightarrow$ for a 23.80m offset on ground

length of error on ground

$= L \sin \theta$

$= 23.80 \times \sin 3^\circ$

$= 1.245\text{m}$

length of error on drawing

Scale $= 1\text{cm} = 50\text{m}$

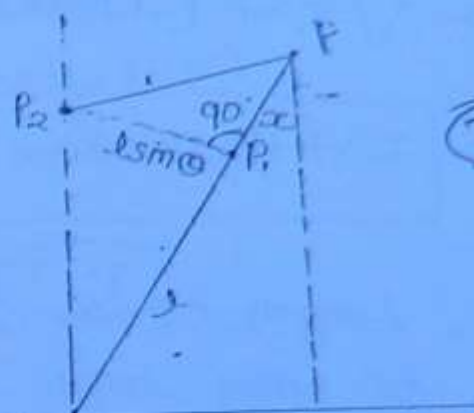
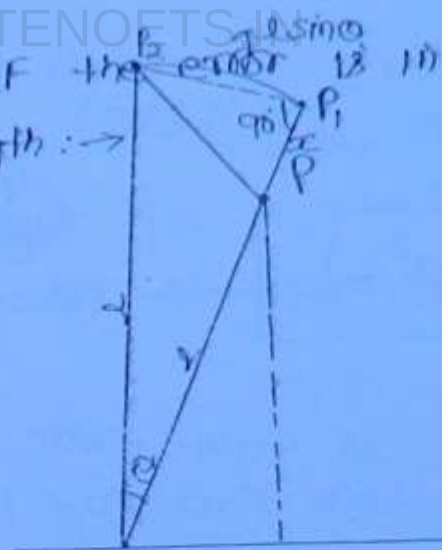
$= \frac{1.245}{50} = 0.0249\text{cm}$

R.F. $= 1\text{cm} = 5000\text{cm}$

$= 1:5000$

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Case ③ IF the error is in angle as well as measurement of length $\rightarrow$



③

P = Point on the ground for which offset is being taken
 l = Length of offset with error.
 PP_1 = Error in length measurement.
 $= x$

θ = Error in angular measurement.
 P_2 = Point marked on the drawing
 $P_1 P_2$ = Error due to angular measurement.
 $= l \sin \theta$

Assumption: $\rightarrow$ in both case
 $\angle PP_1 P_2 = 90^\circ$

So total error (length on ground)

$$PP_2 = \sqrt{PP_1^2 + P_1 P_2^2}$$

$$\Rightarrow PP_2 = \sqrt{x^2 + (l \sin \theta)^2}$$

If scale is $1 \text{ cm} = 5 \text{ m}$

length of error on the drawing

$$PP_2 = \frac{\sqrt{x^2 + (l \sin \theta)^2}}{5} > 0.025 \text{ cm}$$

$$\Rightarrow \sqrt{x^2 + (L \sin \theta)^2} = (0.025S)$$

$$\Rightarrow \boxed{x^2 + (L \sin \theta)^2 = \left(\frac{S}{40}\right)^2} \quad \text{--- (A)}$$

(32)

Que: $\Rightarrow$ Length of an offset is 20m. Error in measurement of offset length is 0.20m. Find max. permissible error in laying the direction of offset IF scale of the drawing is 1cm = 40m

Solution: $\Rightarrow L = 20 \text{ m}$

$$x = 0.20 \text{ m}$$

$$1 \text{ cm} = 40 \text{ m scale}$$

$$\therefore S = 40 \text{ m}$$

$$\theta = ?$$

$$\Rightarrow x^2 + (L \sin \theta)^2 = \left(\frac{S}{40}\right)^2$$

$$\Rightarrow (0.20)^2 + (20 \sin \theta)^2 = \left(\frac{40}{40}\right)^2$$

$$\Rightarrow 0.04 + 400 \sin^2 \theta = 1$$

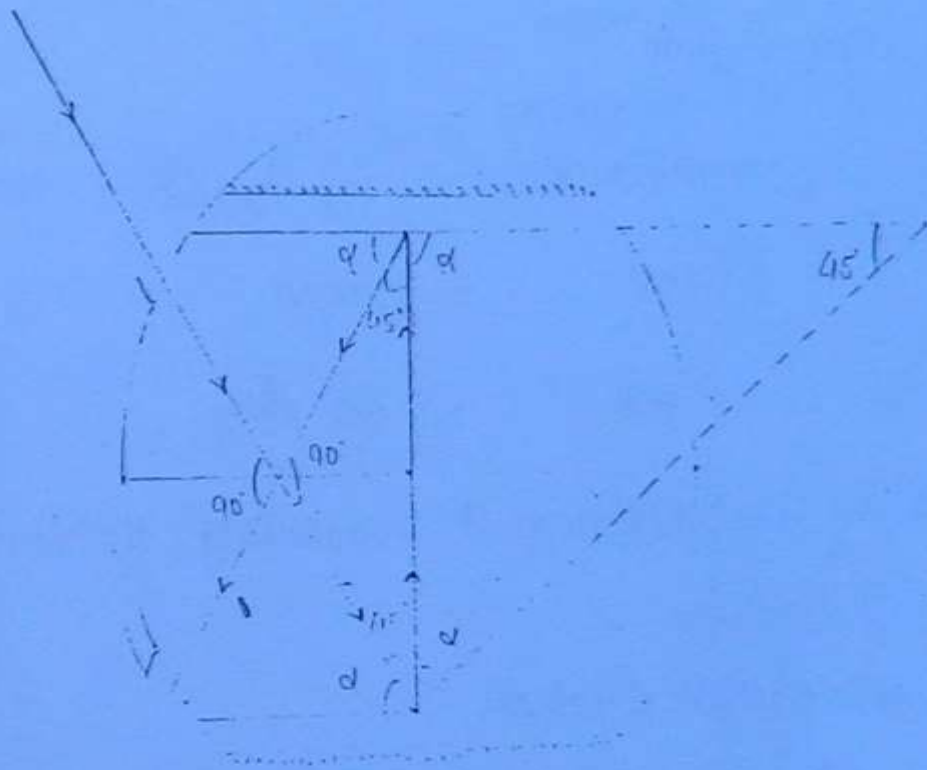
$$\Rightarrow \boxed{\theta = 2^\circ 48' 29''} \quad \text{Ans}$$

Optical Square: $\rightarrow$ Instrument used to set perpendicular on a line

$\rightarrow$ Angle b/w reflected rays (90°)

$\rightarrow$ Twice of angle b/w mirrors (45°)

(33)



Compass Survey →

② System of angle measurement: →

① Most widely used system

(34)

1 circumference = 360° degree

1 degree = 60 min

1 minute = 60 sec

② Hour System

1 circumference = 24 hours

1 hour = 60 min

1 min = 60 sec

Note → 1 circumference is according to the earth rotation

③ Centesimal System

1 circumference = 400 grade

1 grade = 100 centigrade

1 centigrade = 100 centi-centigrade

Important terms →

① Bearing → The direction of a line w.r. to a given meridian.
It is the angle b/w given meridian and the line.

② True meridian → Line joining true North and true South pole
the earth is called True meridian.

True north or south point are the point about which
earth is rotating.

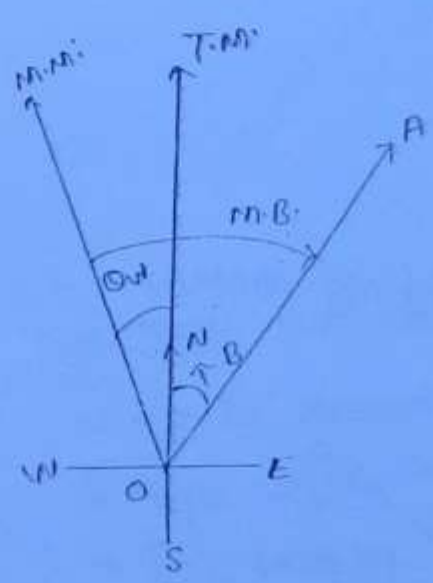
L

③ True bearing: → Bearing measured from true meridian.

④ Magnetic Meridian: → Line joining magnetic North and South poles. - (35)

Magnetic bearing: → Bearing measured from magnetic meridian.

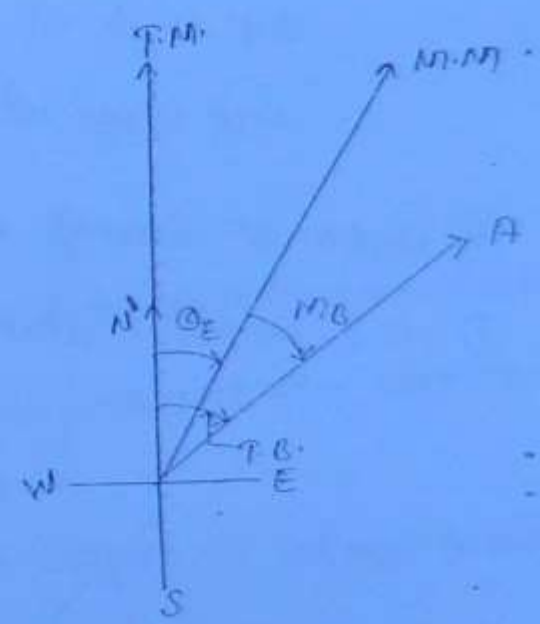
⑤ Magnetic declination: → Horizontal angle b/w true meridian and magnetic meridian at a particular place is called magnetic declination.



(Western Declination)

$$M.B. = T.B. + \theta_w$$

$$T.B. = M.B. - \theta_w$$



(Eastern declination)

$$M.B. = T.B. - \theta_E$$

$$T.B. = M.B. + \theta_E$$

⑦ Variation in Declination: →

Types: →

① Diurnal variation (Daily)

② Annual variation (Annually)

③ Secular variation (Due to the moment of magnet)

④ Irregular variation.

★ ⑥ Angle of Dip: $\rightarrow$ If a magnet is hanged freely from it's C.G. It aligns itself in the direction of magnetic flux in that area, angle made with horizontal direction of magnet is called dip angle.

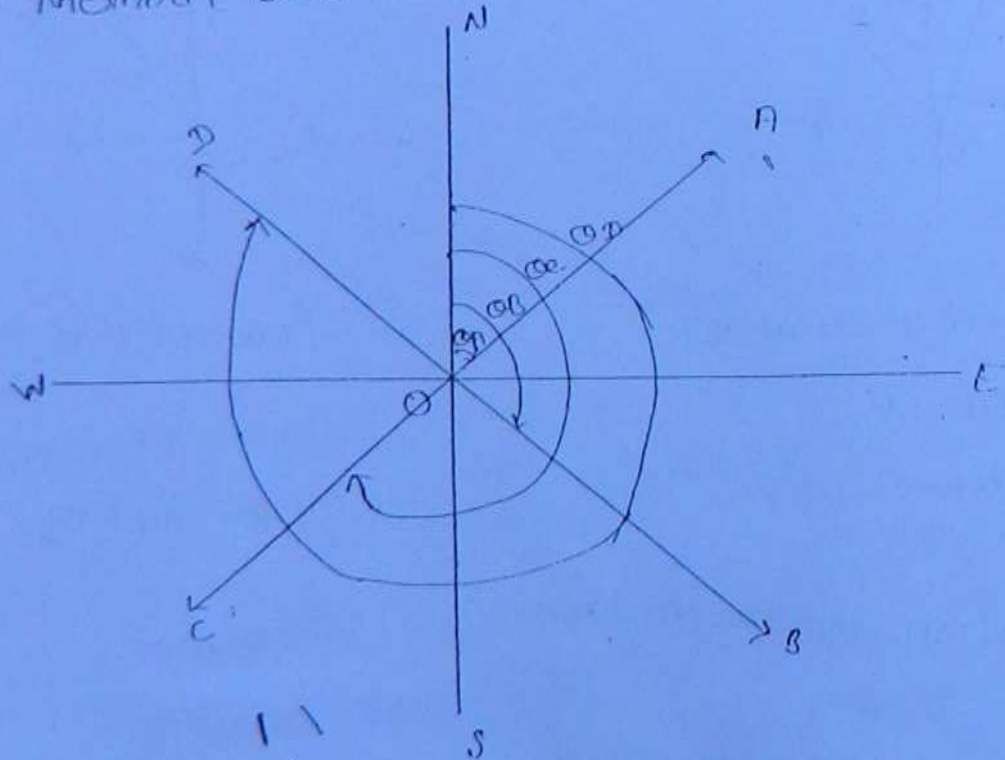
Dip angle at any point on earth is the angle b/w horizontal and direction of magnetic flux.

Dip angle at equator = 0°

Dip angle at poles = 90°

⑦ System of Bearing Measurement: $\rightarrow$

① WCB method (whole circle bearing method) $\rightarrow$

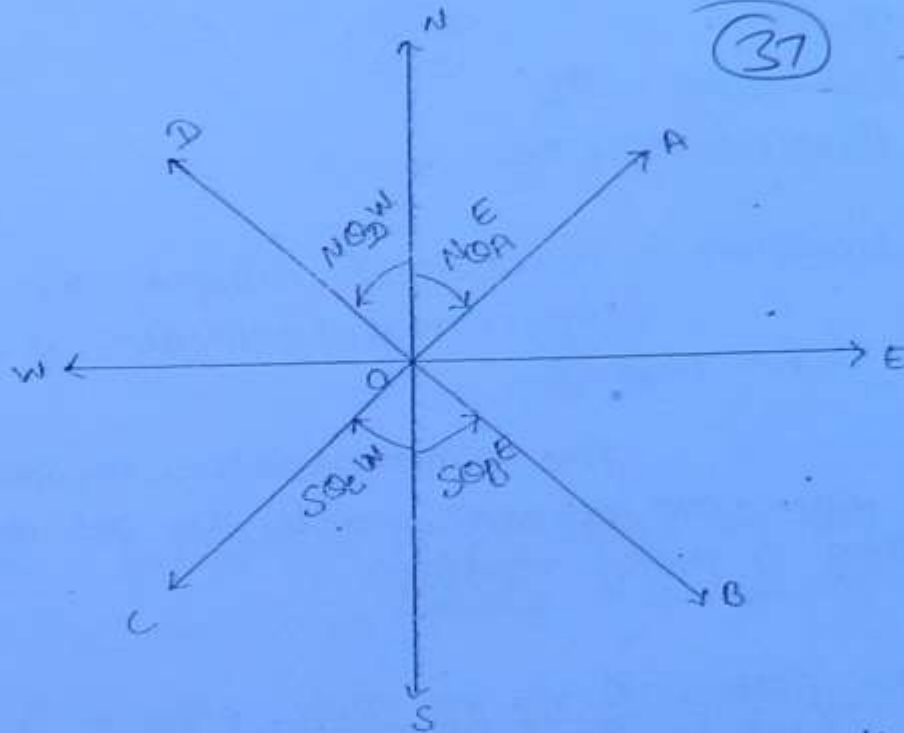


$\rightarrow$ All angle are measured from north.

$\rightarrow$ always in clockwise direction.

② C.S.B. method:→

(Quadrantal System of bearing)



- Angle are measured either from North or from South.
- Can be measured in clockwise / anticlockwise direction.
- Angle with direction are shown.
- This bearing system is also known as reduced bearing.

③ fore bearing and back bearing:→



→ for line AB

Angle at 1st point A = O_A = Fore bearing.

angle at 2nd point B = $\angle B$ = Back bearing

→ for line BA

Fore bearing = $\angle B$

Back bearing = $\angle A$

(38)

* Local Attraction: → Bearings of different line are measured using a magnetic needle. (in case of magnetic bearing)

Due to the presence of some iron objects near instrument, magnetic needle may get deflected, resulting in wrong readings.

ES-2002

Problem: → The following reading were noted in a closed traverse

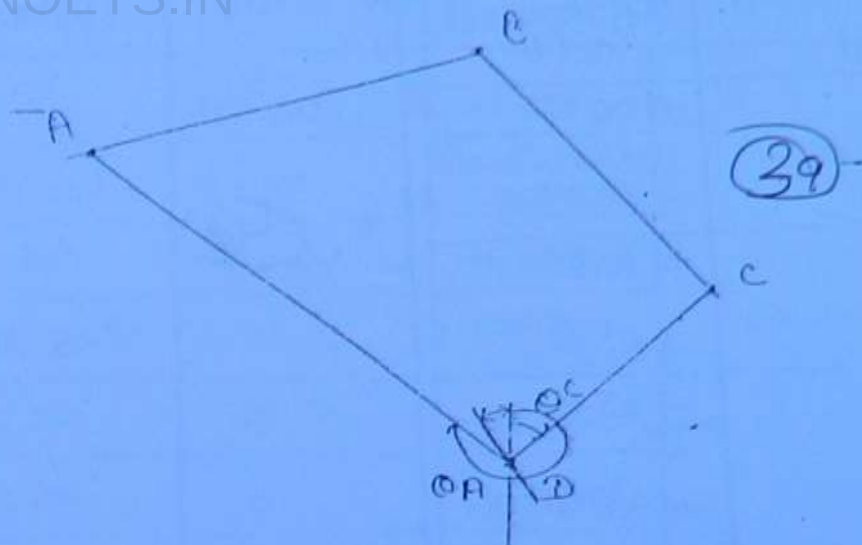
	Fore bearing	Back bearing
AB	32°	212° → 170°
BC	77°	262° → 185°
CD	112°	287° → 175°
DE	122°	302° → 180°
EA	265°	85° → 170°

This station is correct from local attraction

at what station do you suspect local attraction? find correct bearing of the line. what will be the true bearings (as reduced bearing is magnetic declination was 12'W)

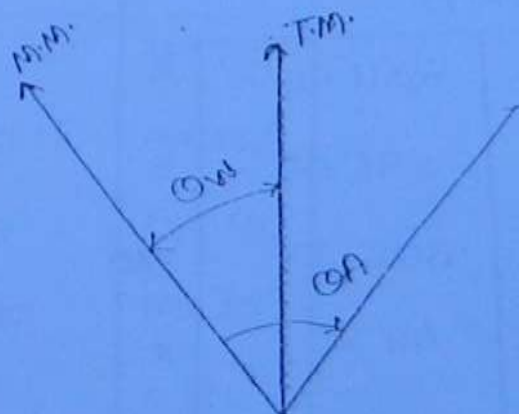
Solution: →

Line	Bearing	Correction	Corrected bearing
AB	32	✓ 0	32°
BA	212	✓ 0	212°
BC	77	✓ 0	77°
CB	262	✗ -5°	77 + 180 = 257
CD	112	✗ -5°	112 - 5 = 107
DC	287	✓ 0	107 + 180 = 287
DE	122	✓ 0	122°
ED	302	✓ 0	302°



→ Here all differences of FB/BB are 100, except for line BC and CD so station 'C' is affected from local attraction.

EA	265✓	0	265°
AE	85✓	0	85°



$$T.B = M.B - O.W.$$

True bearing	True bearing
Method	Q.C.B
20	N 20 E
200	S 20 W
65	N 65 E
245	S 65 W
95	S 95 E
275	N 85 W
110	S 70 E
290	N 70 W
253	S 73 W
73	N 73 E

(40)

Problem → following bearing were taken using a compass find out the correct bearing

AB	75° 5'	250° 20'	x
BC	115° 20'	296° 35'	x
CD	165° 35'	345° 35'	180°
DE	220° 50'	40° 5'	x
ED	300°	125° 5'	x

Solution: when station C and D are free from local attraction

Lines	Bearings	Correction	Corrected bearing
AB	75° 5'	+0° 30'	75° 35'
BA	250° 20'	+1° 15'	255° 35'
BC	115° 20'	+1° 15'	116° 35'
CB	296° 35'	0	296° 35'
CD	165° 35'	0	165° 35'
DC	345° 35'	0	345° 35'
DE	224° 50'	0	224° 50'
ED	44° 5'	+0° 45'	44° 50'
EA	304° 50'	+0° 45'	305° 35'
AE	125° 5'	+0° 30'	125° 35'

(41)

Problem:-> Following are the bearing of a closed traverse.

	FB	BB
AB	142° 30'	322° 30'
BC	223° 15'	40° 15'
CD	287°	107° 45'
DE	12° 45'	193° 15'
EA	60°	239°

Considered AB value as correct and call the correct bearing of all other lines.

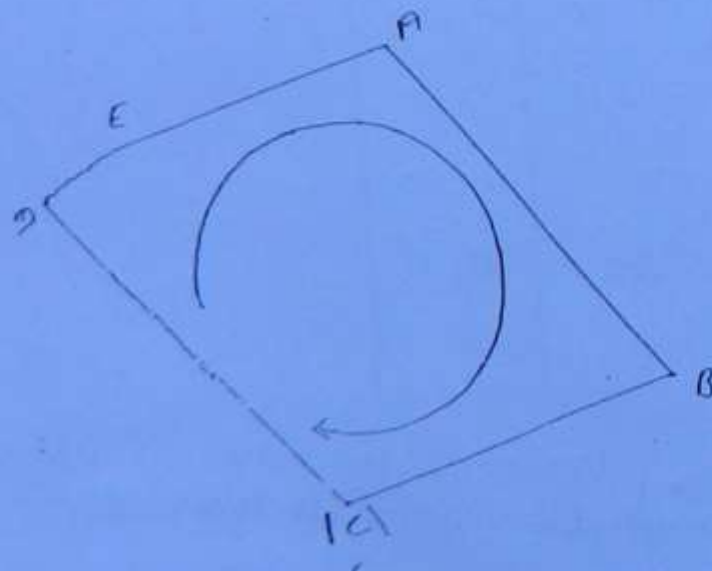
Solution:->

Lines	Bearing	Correction	Correct Bearing
AB	142° 30'	0	142° 30'
BA	322° 30'	0	322° 30'
BC	223° 15'	0	223° 15'
CB	44° 15'	-1° 0'	$\downarrow (223° 15' - 180°)$ 43° 15'
CD	287°	-1° 0'	$\downarrow$ 286°
DC	107° 45'	-1° 45'	$\downarrow$ 106°
DE	12° 45'	-1° 45'	$\downarrow$ 11°
ED	193° 15'	-2° 15'	$\downarrow$ 191° 15'
EA	60°	-2° 15'	$\downarrow$ 57° 45'
AE	239°	Not equal	$\downarrow$ 237° 05'

(42)

Method of Internal angle $\rightarrow$

1 Draw the traverse.



② Draw a clock wise circular direction.

③ Internal angle

(43)

$$A = A^{\text{before}}_E - A^{\text{after}}_B = 237 - 142'30'' = 96'30''$$

$$B = B_A - B_C = 322'30'' - 223'15'' = 99'15''$$

$$C = C_B - C_D = 44'15'' - 287'36'' = 117'015''$$

$$D = D_C - D_E = 107'45'' - 12'45'' = 95'$$

$$E = E_D - E_A = 193'15'' - 60'' = 133'15''$$

$$541'15''$$

④ If any angle is 360° deduct 360°

⑤ If any angle is negative add 360°

$$\begin{aligned} \text{Sum of all internal angles} &= (2n-4) \times 90^\circ \\ &= (2 \times 5 - 4) \times 90^\circ \\ &= 540^\circ \end{aligned}$$

$$\text{Difference in sum of all internal angle} = 1'15''$$

$$\text{Error in each angle} = \frac{1'15''}{5} = 0'15''$$

than correction in each angle = $(-) 0'15''$
corrected internal angle.

$$96'15''$$

$$99'$$

$$117'$$

$$194'45''$$

$$133'$$

$$\hline 540'$$

$$A = AE - AB = 96^{\circ} 15' = \angle A$$

$$B = BA - BC = 99^{\circ} = \angle B$$

(14)

$$C = CB - CD + 360^{\circ} = 117^{\circ}$$

$$D = DC - DE = 94^{\circ} 45'$$

$$E = ED - EA = 138^{\circ} = \angle E$$

Corrected bearing of line $\rightarrow$

$$\begin{array}{r} AB = 142^{\circ} 30' \\ + \angle A = + 96^{\circ} 15' \\ \hline \end{array}$$

$$\begin{array}{r} AE = 238^{\circ} 45' \\ - 180^{\circ} \\ \hline \end{array}$$

$$\begin{array}{r} EA = 58^{\circ} 45' \\ + \angle E = + 138^{\circ} \\ \hline \end{array}$$

$$\begin{array}{r} ED = 191^{\circ} 45' \\ - 180^{\circ} \\ \hline \end{array}$$

$$\begin{array}{r} DE = 11^{\circ} 45' \\ + \angle D = + 94^{\circ} 45' \\ \hline \end{array}$$

$$\begin{array}{r} DC = 106^{\circ} 30' \\ + 180^{\circ} \\ \hline \end{array}$$

$$\begin{array}{r} CD = 286^{\circ} 30' \\ + \angle C = + 117^{\circ} \\ \hline \end{array}$$

$$\begin{array}{r} - 360^{\circ} \\ \hline CB = 43^{\circ} 30' \end{array}$$

$$\begin{array}{r} CB = 43^{\circ} 30' \\ - 180^{\circ} \\ \hline \end{array}$$

$$\begin{array}{r} BC = 223^{\circ} 30' \\ + \angle B = + 99^{\circ} \\ \hline \end{array}$$

$$\begin{array}{r} BA = 322^{\circ} 30' \\ - 180^{\circ} \\ \hline \end{array}$$

$$AB = 142^{\circ} 30'$$

(14)

$$\begin{aligned}
 A &= AB & AE &= 191'30'' - 53'' = 138'30'' \\
 B &= BC & BA &= 69'30'' - 13'' = 56'30'' \\
 C &= CD & CB &= 32'15'' - 246'30'' = 115'45'' \\
 & & & \quad + 360'' \\
 D &= DE & DC &= 262'45'' - 210'30'' = 52'15'' \\
 E &= EA & ED &= 230'15'' - 80'45'' = 149'30'' \\
 & & & \quad \underline{542'30''}
 \end{aligned}$$

Total error = $2'30''$ (46)

Error in each angle = $\frac{2'30''}{5}$
 $= 0'30''$

Correction in each angle = $(-)$ $0'30''$

Corrected angle

138°

56°

$115^\circ 15'$

$52^\circ 15'$

149°

AB has min error

Corrected bearing AB = $191'30'' - 0'45'' = 190'45''$

" " BA = $13'' + 0'45'' =$

(this value is
 wrong than

corrected bearing AB = $191'30'' + 0'45'' = 192'15''$

" " BA = $13'' - 0'45'' = 12'15''$

L

$$AB = 192' 15''$$

$$\angle A = -130'$$

$$AE = 54' 15''$$

$$+ 180'$$

$$EA = 234' 15''$$

$$\angle E = -149'$$

$$ED = 85' 15''$$

$$+ 180'$$

$$DE = 265' 15''$$

$$-\angle D = -151' 45''$$

$$DC = 213' 30''$$

$$- 180'$$

$$CD = 33' 30''$$

$$\angle C = -145' 15''$$

$$+ 360$$

Because the value is negative than add.
360.

$$CB = 248' 15''$$

$$- 180'$$

$$BC = 68' 15''$$

$$-\angle B = -56'$$

$$BA = 12' 15''$$

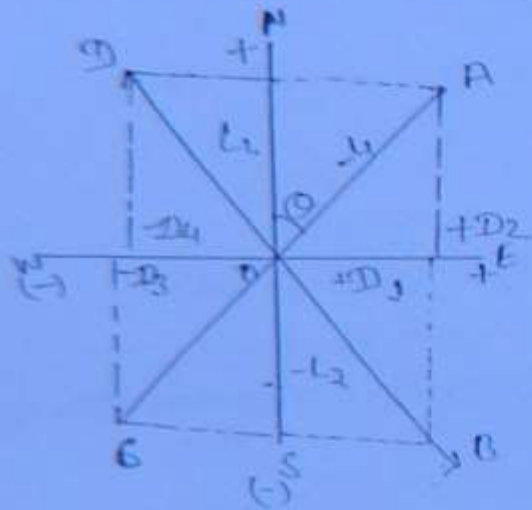
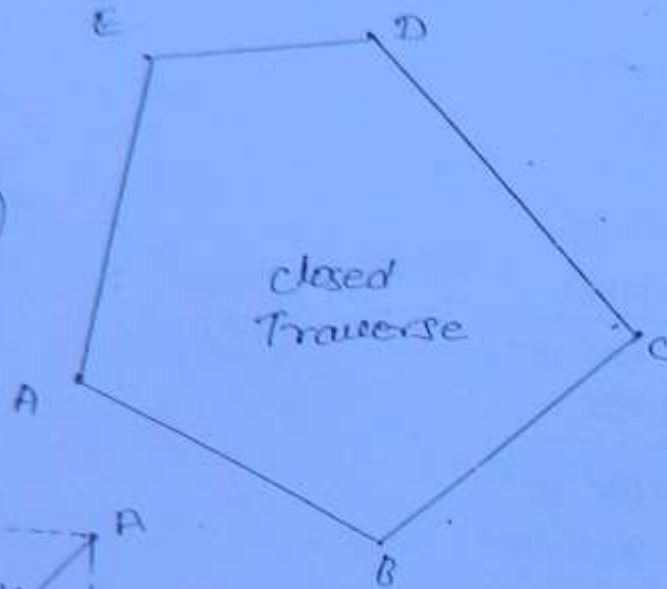
$$+ 180'$$

$$AB = 192' 15''$$

(47)

Traverse → Latitude and departure of different lines of a traverse.

(48)



→ Latitude is the projection of a line on N-S direction

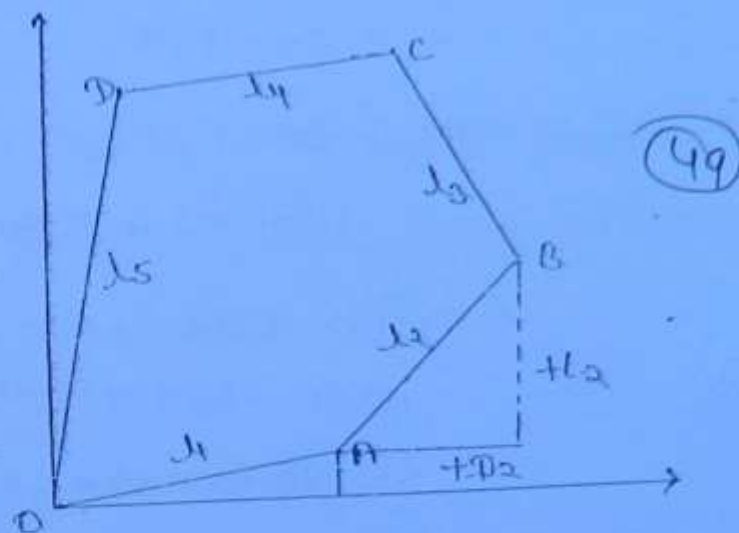
$$L = l \cos \theta$$

→ Departure is the projection of line on E-W direction

$$D = l \sin \theta$$

line	WCB	Latitude	Departure
l_1	$0 < \theta < 90^\circ$	N θ_1 E $+l_1 \cos \theta_1$	$+l_1 \sin \theta_1$
l_2		S θ_2 E $-l_2 \cos \theta_2$	$+l_2 \sin \theta_2$
l_3		S θ_3 W $-l_3 \cos \theta_3$	$-l_3 \sin \theta_3$
l_4		N θ_4 W $+l_4 \cos \theta_4$	$-l_4 \sin \theta_4$

(ii) Independent Co-ordinate $\rightarrow$



If co-ordinate of different points are measured w.r. to a fixed origin is called independent co-ordinate.

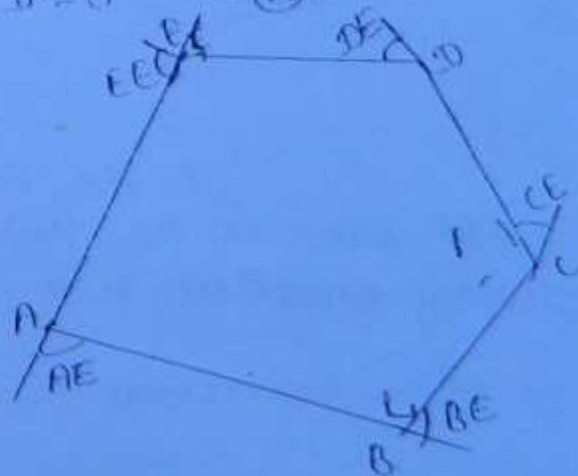
✓ Properties of a closed Traverse $\rightarrow$

① Sum of all latitude and departure should be zero
 $l_1 \cos \theta_1 + l_2 \cos \theta_2 + \dots$

$$\Sigma L = 0 \quad \text{--- (1)}$$

$$l_1 \cos \theta_1 + l_2 \cos \theta_2 + \dots$$

$$\Sigma D = 0 \quad \text{--- (2)}$$



② Sum of all internal angle of a closed traverse
 $= (2n-4) 90^\circ$

③ Sum of External angle of a closed traverse

$$AE + BE + \dots = (180 - A) + (180 - B) + \dots$$

$$= n \times 180 - (A + B + \dots)$$

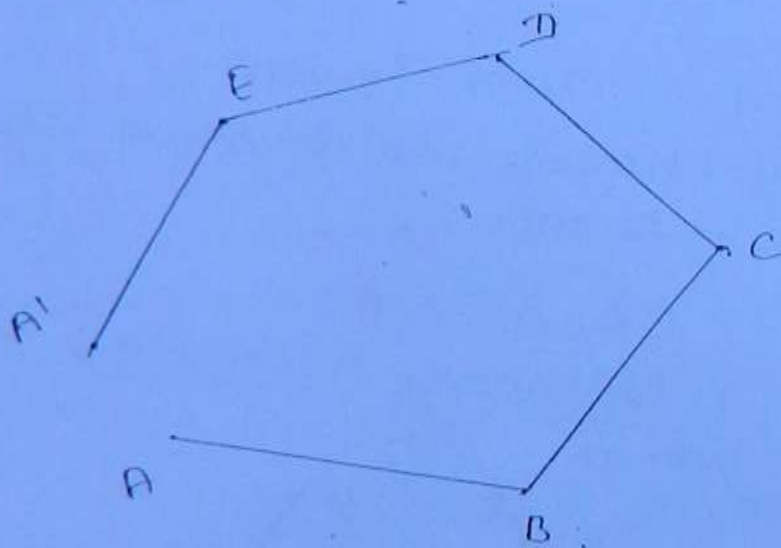
$$= 2n \times 90 - (2n-4) \times 90$$

$$= 2n \times 90 - 2n \times 90 + 4 \times 90$$

$$= 360^\circ$$

(50)

Closing Error $\rightarrow$ In case of a closed traverse



If the first point of the closed traverse is not same as the that point. the error is called closing error. AA' is the error

$\therefore A'A = \text{Correction}$

* Adjustment of closing error: $\rightarrow$

① $\Sigma L = 0$

$\Sigma D = 0$

② Sum of all internal angle = $(2n-4) \times 90^\circ$

method for balancing a closed traverse.

① Bowditch method.

② Transit method.

③ Graphical method

④ Area method.

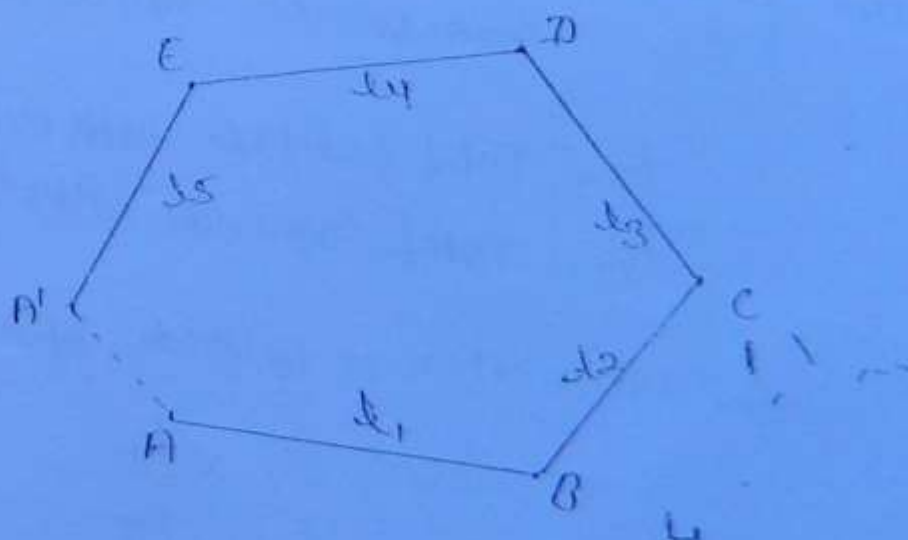
⑤

① Bowditch method: $\rightarrow$ This method is suitable when linear and angular measurements having been measured with equal precision. changes of error are same in linear/ angular measurements.

As per bowditch

① Error in linear measurement $\propto \sqrt{L}$

② Error in angular measurement $\propto \sqrt{L}$



ΣL = Total error in latitude.
(Sum of all latitude with sign)

ΣD = Total error in Departure
(Sum of all departure with sign)

ΣL = Sum of all length of different line

Correction in latitude of a particular line

$$(52) \quad C_L = \Sigma L \times \frac{l_i}{\Sigma L} \quad \text{--- (1)}$$

Correction in Departure

$$C_D = \Sigma D \times \frac{l_i}{\Sigma L} \quad \text{--- (2)}$$

⑤ Transit method $\rightarrow$ This method is suitable when angular measurement are more precise than linear measurement.

Error $\left\{ \begin{array}{l} \Sigma L = \text{Sum of all latitudes (with sign)} \\ \Sigma D = \text{Sum of all departure (with sign)} \end{array} \right.$

L_T = Total latitude with out sign

D_T = Total Departure with out sign

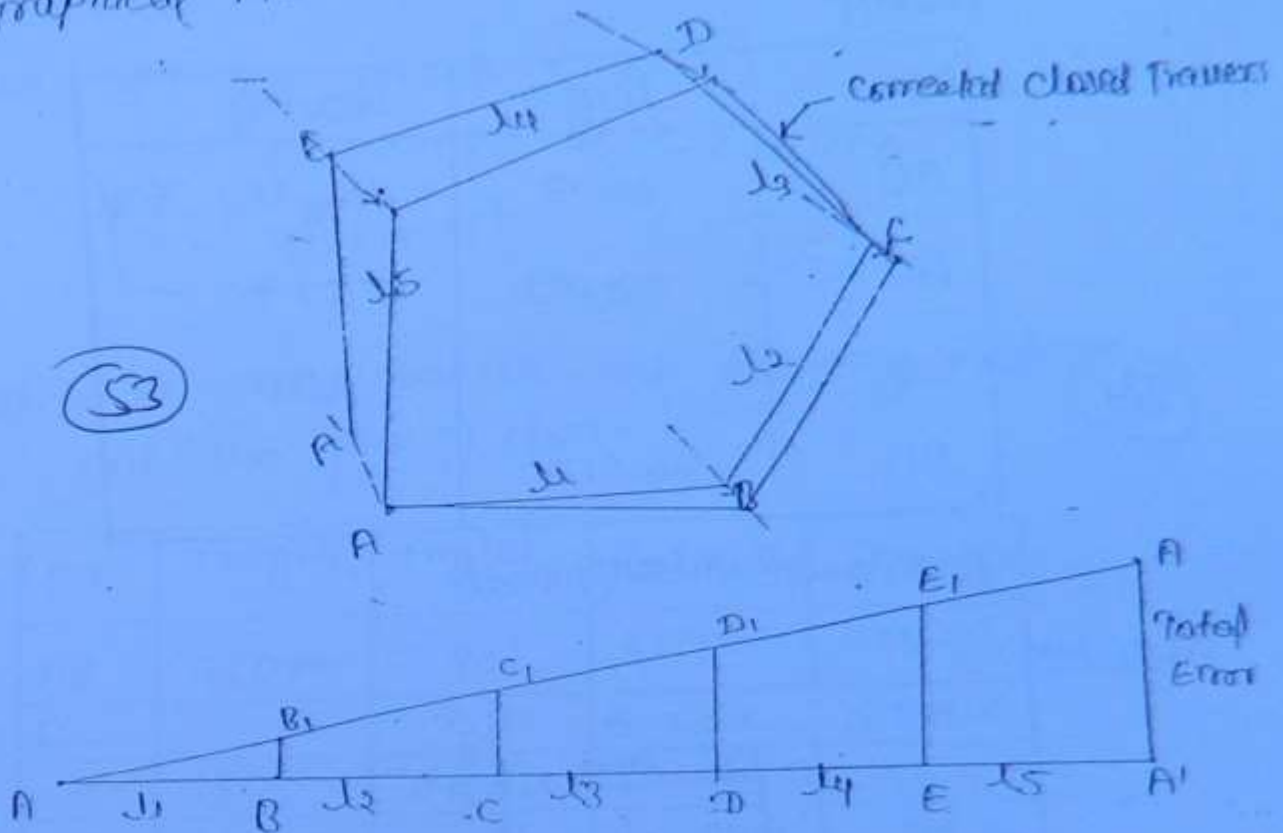
than correction in latitude of a part line

$$C_L = \Sigma L \times \frac{L_i}{L_T}$$

$$\text{--- } C_D = \Sigma D \times \frac{D_i}{D_T}$$

Graphical method: →

(S3)



all this connection is made and the mark point is same direction.

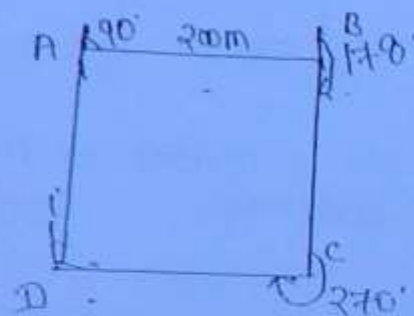
Ques. A closed traverse has the following length and bearing

Line	Length	Bearing
AB	200 m	Rightly East
BC	98 m	178°
CD	—	270°
DA	84.6 m	?

(34)

find out missing data

Solution:->



Bearing	Latitude	Departure
Rightly	$200 \cos 0$	$200 \sin 0$
178°	$= -97.94$	$= 3.42$
270°	$x \cos 270 = 0$	$x \sin 270 = -x$
84.6	86.39	$= 1.500$

$$\Sigma L = 0$$

$$\Sigma D = 0$$

From $\Sigma L = 0$

$$200 \cos \theta = 11.55$$

$$\cos \theta = 0.05775 \Rightarrow \theta = 86^\circ 41' 22'' \text{ from}$$

$$\Sigma D = 0$$

$$200 \sin 86^\circ + 3.42 + 1.508 - x = 0$$

$$x = 200 \sin 86^\circ + 4.928$$

$$x = 204.60 \text{ m}$$

Ans

(55)

Problem: $\rightarrow$ A closed traverse has following readings. Find out the missing data.

Line	Length	Angles in CB	Latitude	Departure
AB	250 m	85°	21.79	249.05
BC	$-x$	40°	$0.766x$	$0.643x$
CD	150	320°	114.907	-96.418
DE	220	295°	92.976	-199.39
EF	140	0°	$140 \cos 0$	$140 \sin 0$
FA	200	160°	-187.94	68.404

Solution: $\rightarrow$

$$\Sigma L = 0$$

$$\Rightarrow 21.79 + 0.766x + 114.907 + 92.976 - 187.94 = 0 \quad \text{--- (1)}$$

$$\Sigma D = 0$$

$$0.643x + 140 \sin 0 + 21.646 = 0 \quad \text{--- (2)}$$

from eq. (1)

$$\Rightarrow 140 \cos 0 = -0.766x - 41.733 \quad \text{--- (3)}$$

from eq. (2)

$$\Rightarrow 140 \sin 0 = -0.643x - 21.646 \quad \text{--- (4)}$$

$$\Rightarrow 140^2 (\cos^2 \theta + \sin^2 \theta) = (0.768 + 41.733)^2 + (0.6432 + 21.646)^2$$

$$\Rightarrow 19600 = x^2 + 91.77x + 2210.192 = 0$$

$$\Rightarrow x^2 + 91.77x - 17389.8 = 0$$

$$\boxed{x = 93.74} \text{ m}$$

Ans

(56)

from eq. (3)

$$\cos \theta = - \frac{173.54}{140} = -0.81098$$

from eq. (4)

$$\sin \theta = - \frac{81.92}{140} = -0.58515$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = 0.7215$$

$$\theta = 35' 45'' 52''$$

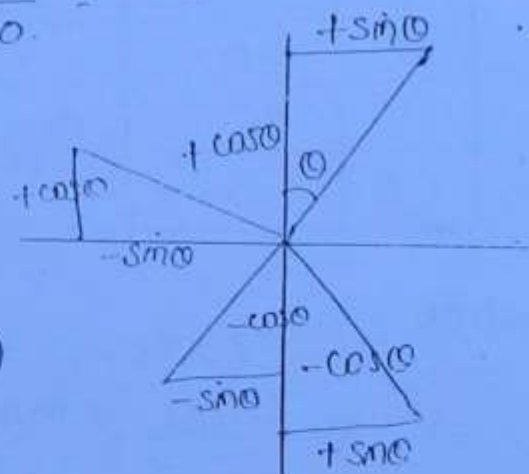
Bearing of Line C is in 3rd Q)

$$= 180^\circ + \theta$$

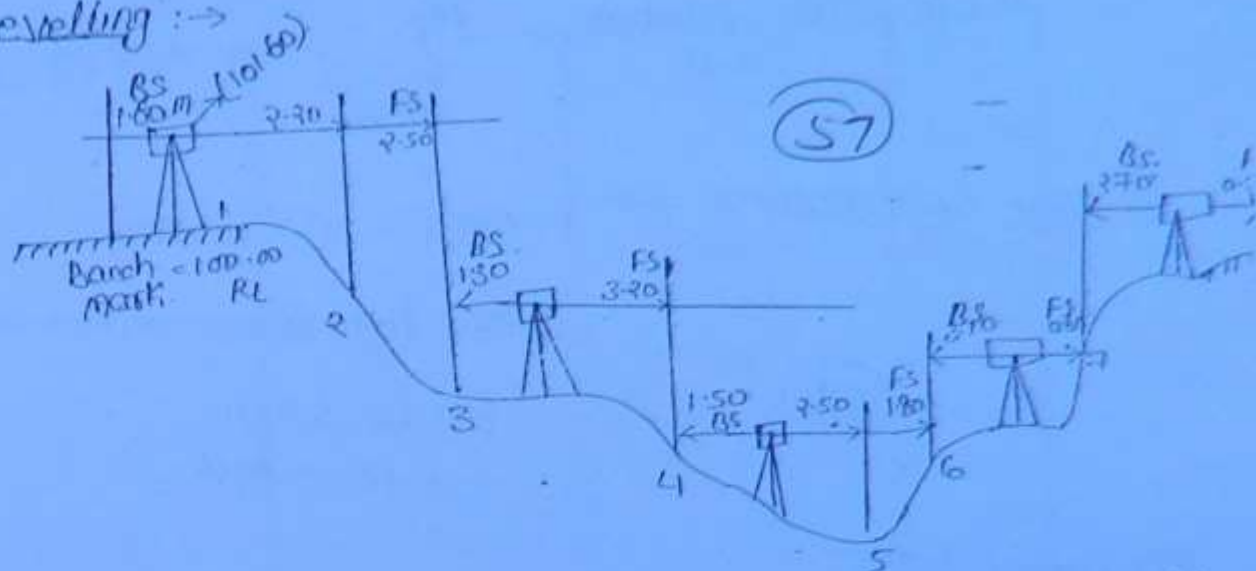
$$= 180^\circ + 35' 48'' 41.52''$$

$$\boxed{= 215^\circ 48' 41.52''}$$

Ans



★ Leveling :->



Terms :->

- ① Reduced Level :-> Reduced level of point on earth surface is the elevation of that point wrt to a fixed location (mean sea level) or (D.M.) length of certain bench marks.
- ② Back Sight (Reading) :-> First reading taken after setting up the instrument (leveling instr.) at a particular location.
- ③ Fore Sight Reading :-> Last reading taken from an instrument location after which ^{elevation} of instrument is being changed is called fore sight reading.
Last reading after which survey work is closed is also a fore sight.
- ④ Intermediate Sight :-> All other reading than BS and FS are intermediate sight.
- ⑤ Height of Instrument :-> The elevation of line of sight at particular instrument location.
$$H.I \text{ of } \underline{\text{any location}} = RL \text{ of BM} + BS$$

$$\text{H.I. of next station} = \underset{\substack{\uparrow \\ \text{Height}}}{\text{HI}} - \text{FS}_{(1)} + \text{BS}_{(2)}$$

⑥ Rise and fall $\rightarrow$ 1st reading - 2nd reading

(taken from same instrument station)

(58)

(+) ve $\rightarrow$ Rise

(-) ve $\rightarrow$ Fall

⑦ Level Book $\rightarrow$

Point	Back Side (BS)	Intermediate Side (IS)	Fore Side (FS)	Height of Instrument (HI)	R.L
1 BM	1.60			101.60	100.00 (Given)
2		2.20			99.40 = 101.60 - 2.20
3	1.30		2.50	99.40 + 1.30 = 100.70	99.10 = 101.60 - 2.50
4	1.50		3.20	97.20 + 1.50 = 98.70	97.20
5		2.50	4.80		96.20 = 98.70 - 2.50
6	3.10		4.80		
7	2.70		10.60		
8			10.70		

Rise and Fall Method :-

Level Book	Rise	Fall	R.L
	-	-	100.00
		0.60	99.40
	(59)	0.30	99.10
			10

(7) Check :- add All Back side reading =
 add All Fore side reading =
 the calculated difference b/w two reading
 then add All rise =
 add All Fall =
 difference b/w rise and Fall

$$\begin{aligned}
 \text{Note } ERS - EF_s &= \text{Last RL} - 1^{\text{st}} \text{ R.L.} \\
 &= E\text{Rise} - E\text{Fall}
 \end{aligned}$$

Problem:- In running fly levels from a BM of RL 120.750 the following readings were obtained

BS = 0.85 0.1885 1.182 0.965 0.49

FS = 0.555 1.150 1.945 1.755 -

60 From the last position of the instrument seven ^{page} at 10m interval are to be set out on a uniform descending gradient of 1 in 50. The first page is at RL 120.00m. Work out the staff readings for setting the top of pages. Fall the level book and carry out arithmetic checks

Solution:-

Points	BS	IS	FS	Rise	Fall	RL
1	0.85		-			120.750
2	1.285		0.555	0.295		121.045
3	1.182		1.150	0.135		121.180
4	0.965		1.945		0.763	120.417
5	0.49		1.755		0.790	119.627
6		0.117		0.373		120.00
7		0.317			0.20	119.80
8		0.517			0.20	119.60
9		0.717			0.20	119.40
10		0.917			0.20	119.20
11		1.117			0.20	119.00
12					0.20	118.80
check	Σ BS = 4.792	Σ IS = 6.737	Σ FS = 6.737			

Seven pages are to set
each at distance = 10m
at gradient b/w two pages

(61)

$$= 10 \times \frac{1}{150} = (-) 0.20 \text{ m}$$

for 1st page A.L = ~~120~~ 120.0m

Check ① $IS = EFS - EBS = 6.722 - 4.772$
 $= 1.95 \text{ m}$

② $RL = N^{\text{th}} AL - \text{Last } RL$
 $= 120.750 - 119.20$
 $= 1.95$

③ $E_{\text{rise}} + E_{\text{fall}} =$
 $-0.803 + 2.75 = 1.95 \text{ m}$

than OK Ans

— 0 —

problem → fill the missing data of a level book

point	BS	IS	FS	Rise	Fall	R.L.
1	3.125			X	X	123.60 (J)
2	(A) 2.265		(B) 1.80	1.325		125.005
3		2.32			0.055	124.95 (K)
4		(C) 1.92		0.40 (E)		125.350
5	(D) 1.04		2.655	(G) 0.735		124.615 (L)
6	1.620		3.205		2.165	122.45 (M)
7		3.625		(H) 2.005		120.445 (N)
8			(I) 1.40	2.145 (I)		122.390
EB = 8.05		EF = 7.14		ER = 3.87	EF = 4.96	

Solution

$$J = 125.005 - \text{Rise } (1.325)$$

$$J = 123.60$$

$$B = 3.125 - 1.325 = 1.80$$

$$A = 2.32 - 0.055 = 2.265$$

$$K = 125.005 - 0.055 = 124.95$$

$$F = 125.350 - 124.95 = 0.40$$

Rise

$$C = 2.32 - 0.40 = 1.92$$

$$G = 1.92 - 2.655 = -0.735 \text{ (Fall)}$$

$$L = 125.35 - 0.735 = 124.615$$

$$D = 3.205 - 2.165 = 1.04$$

$$M = 124.615 - 2.165$$

$$M = 122.45$$

$$H = 1.620 - 3.625 = -2.005 \text{ (Fall)}$$

$$N = 122.45 - 2.005 = 120.445$$

$$I = 122.59 - 120.445 = 2.145$$

(Rise)

$$E = 3.625 - 2.145 = 1.48$$

check =

$$\textcircled{1} EF_s - EB_s \Rightarrow 9.14 - 8.05 = 1.09$$

$$\textcircled{2} EF - ER \Rightarrow 4.96 - 3.87 = 1.09$$

$$\textcircled{3} 1^{st} PL - Last PL = 123.68 - 122.590 = 1.09$$

than of any

Answer -> following consecutive readings were taken from dumpy level.

BS $\boxed{6.25}$, 4.92, 6.52, $\boxed{8.35}$ FS

BS $\boxed{4.62}$, $\boxed{3.50}$, $\boxed{1.33}$, 3.97

IS $\boxed{6.8}$, $\boxed{4.83}$ BS 3.52, 2.30

IS $\boxed{1.50}$, $\boxed{2.40}$ BS 4.25, $\boxed{3.50}$ FS

(64)

level was shifted after 4th, 6th, 9th and 13th reading. IF R.L. of 1st point - BM = 100.00
Fill the level book and find out R.L. of different points.

Solution:->

Point	BS	IS	FS	Rise	Fall	R.L.
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						
11						
12						
13						
14						
15						

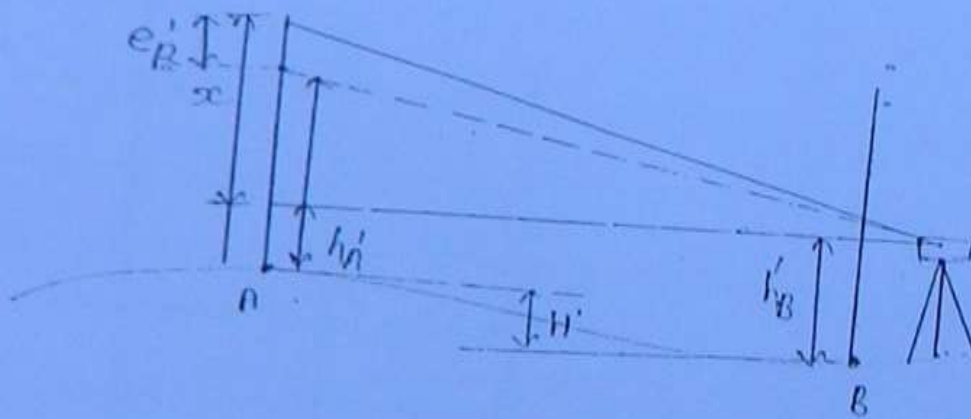
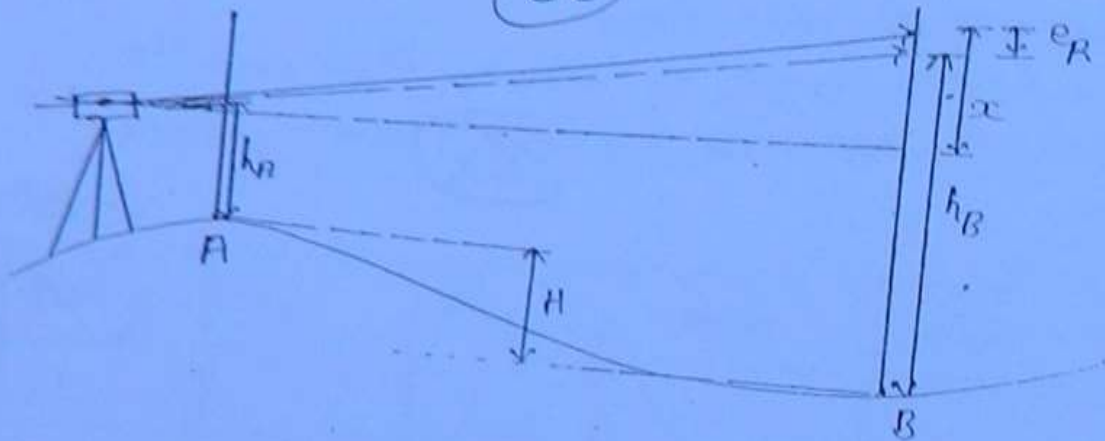
63

* Reciprocal leveling:-> Reciprocal leveling is used.

① Find out any error in the leveling instrument.
(Line of sight may not be in horizontal direction, even with bubble in the centre.)

② To eliminate the effect of earth curvature and refraction.

(66)



In case of reciprocal leveling, two points A and B at a distance about 100-200m. is selected. Staff readings are noted keeping instrument first first near A and then near B.

When instrument is at A
Reading at A = h_A

Reading at B = h_B

When Instrument is at B

reading at A = h_A'

Reading at B = h_B'

If Instrument is Faulty

$$h_B - h_A \neq h_B' - h_A' \quad (6)$$

When Instrument is at A

h_A = Correct reading

Correct reading at B should be = $h_B + e_R - x$

Correct difference of level b/w A and B

$$H = (h_B + e_R - x) - h_A \quad (1)$$

When Instrument is at B

h_B' = Correct reading

Correct reading at A should be = $h_A' + e_R' - x$

Correct difference of level b/w B and A

$$H' = h_B' - (h_A' + e_R' - x) \quad (2)$$

$$e_R' = e_R \text{ or } H' = H$$

Add eq. (1) and (2)

$$H + H' = h_B + e_R - x - h_A + h_B' - h_A' - e_R' + x$$

$$\Rightarrow 2H = (h_B - h_A) + (h_B' - h_A')$$

Correct difference of level b/w A and B

$$\Rightarrow H = \frac{(h_B - h_A) + (h'_B - h'_A)}{2}$$

(68)

Problem: $\rightarrow$ for a reciprocal leveling following reading was taken

Instrument	Reading		difference	Correct diff.
	A	B		
A	1.95 h_A	2.82 h_B	0.91	$\rightarrow 0.97$
B	0.925 h'_A	1.955 h'_B	1.03	

Distance b/w point 1 and 2 is 250 m. IF R.L. OF A is 120.50 m. Find out correct R.L. OF B. Find out the error in line of sight of instrument neglected error due to curvature and refraction.

Solution: $\Rightarrow$ Difference of reading when Instrument is at A

$$\Rightarrow h_B - h_A = 2.82 - 1.95 = 0.91$$

$\Rightarrow$ Difference of reading when Instrument is at B

$$h'_B - h'_A = 1.955 - 0.925 = 1.03$$

$\Rightarrow$ Correct difference of level b/w A and B

$$H = \frac{(h_B - h_A) + (h'_B - h'_A)}{2} = \frac{0.91 + 1.03}{2}$$

$$H = 0.97 \text{ m}$$

⇒ Correct Reading at A = 1.85

when Instrument is at A

(69)

A+B should be = 1.85 + 0.97 = 2.82 m

where reading taken = 2.76 < 2.82

So line of sight is inclined downward

R.L. of A = 120.50 m

R.L. of B = 120.50 m

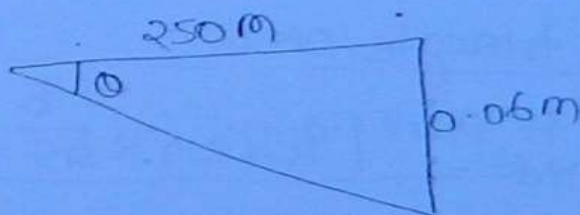
$$\begin{array}{r} - 0.97 \text{ m} \\ \hline 119.53 \text{ m} \end{array}$$

(B is at lower elevation than A)

Error in line of collimation

$$= 2.82 - 2.76$$

$$= 0.06$$



$$\tan \theta = \frac{0.06}{250} = 2.4 \times 10^{-4}$$

$$\theta = \frac{1}{4166.67}$$

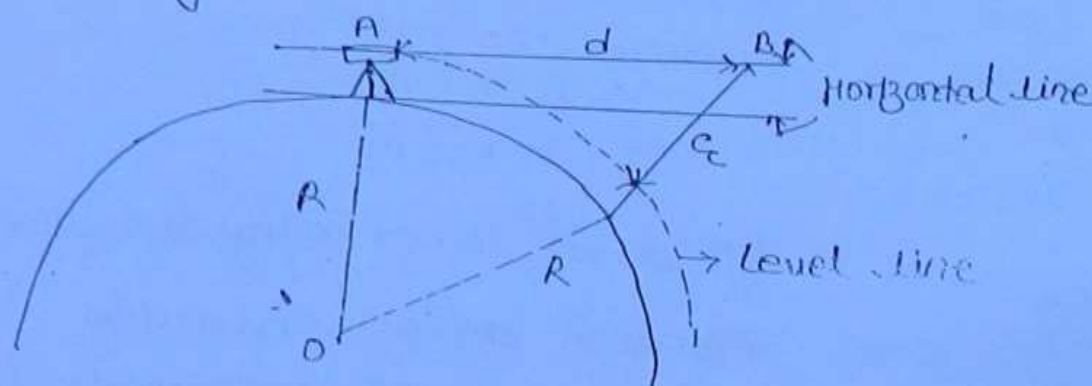
④ Correction due to curvature and refraction:→

① Correction due to curvature (Earth curvature) :→

→ Level line ⇒ A line parallel to earth surface (curved line)

70

→ Horizontal line ⇒ Tangent line at earth surface at any point.



In triangle OAB

$$\Rightarrow R^2 + d^2 = (R + C_c)^2$$

$$\Rightarrow R^2 + d^2 = R^2 + C_c^2 + 2RC_c$$

$$\Rightarrow d^2 = C_c (2R + C_c)$$

$$\therefore 2R + C_c \approx 2R$$

⇒ Correction due to curvature

$$C_c = \frac{d^2}{2R}$$

∴ R = Radius of Earth or
Radius of curvature
(6370 km)

Correction due to earth curvature

$$C_c = \frac{d^2}{2 \times 6370} \times 1000 = \text{meter}$$

$$\Rightarrow \boxed{C_c = 0.07849 d^2 \text{ m}} \quad (7)$$

here d = distance in km.

(#) for staff reading

Error = (+) ve

Correction = (-) ve

(#) For reduced levels calculated using wrong staff read.

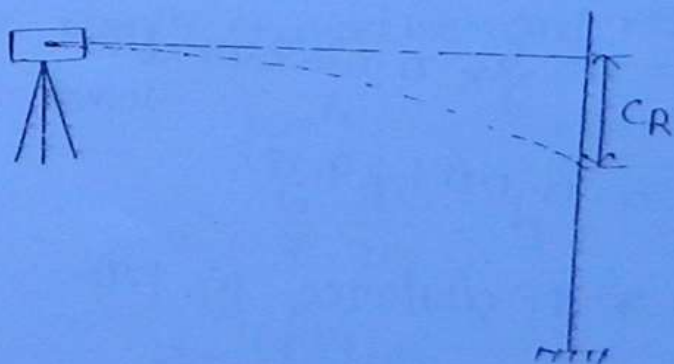
Error = (-) ve

Correction = (+) ve

Generally

$$\boxed{C_c = (-) \frac{d^2}{2R} = 0.07849 d^2}$$

(2) Correction due to refraction $\rightarrow$



This correction is required due to refraction.
The value is $(\frac{1}{7})$ of curvature

$$C_R = \frac{1}{7} \times \text{Correction due to curvature}$$

$$\Rightarrow C_R = \frac{1}{7} \times \frac{d^2}{2R} \quad (72)$$

$$\Rightarrow C_R = \frac{d^2}{14R}$$

$$\Rightarrow C_R = \frac{1}{7} \times 0.07849 d^2$$

$$\Rightarrow \boxed{C_R = 0.01121 d^2}$$

C_R is always (+) ve

Combined Correction $\Rightarrow$

Due to curvature and refraction

$$= - \left(\frac{d^2}{2R} \right) + \frac{1}{7} \left(\frac{d^2}{2R} \right)$$

$$= - \frac{6}{7} \times \frac{d^2}{2R}$$

$$= - \frac{6}{7} \times 0.07849 d^2$$

$$= 0.0672 d^2$$

* d = distance in km.

Note

$$C_c = (-) \frac{d^2}{2R} = -0.07849 d^2$$

$$C_R = (+) \frac{1}{7} \times \frac{d^2}{2R} = +0.01121 d^2$$

$$C = (-) \frac{6}{7} \times \frac{d^2}{2R} = (-) 0.06728 d^2$$

$$R = 6370$$

(73)

④ Distance of visible horizon →

A person at h height from sea level can see the point at sea surface upto a distance d .

Combined formula for correction due earth curvature and refraction is used

hence $C = h$

$$C = \frac{6}{7} \frac{d^2}{2R} = h$$

$$d = \sqrt{\frac{14}{6} Rh}$$

$$C = 0.06728 d^2$$

$$d = \sqrt{\frac{h}{0.06728}}$$

$$d = 3.855\sqrt{h}$$

* h = in meter

d = in km

(74)

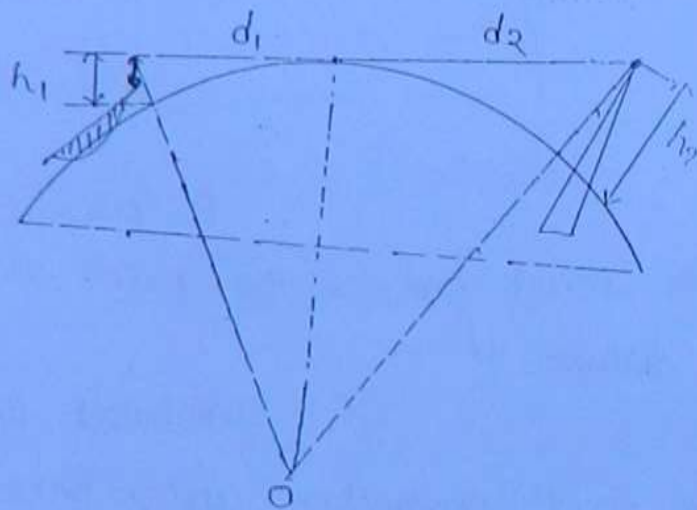
ES-1998

Problem $\Rightarrow$ D6

An observer standing on the deck of a ship just see a light house. The top of light house is 49m above sea level and the height of observer eye is 9m above sea level.

Find the distance of observer from light house.

Solution $\Rightarrow$



Distance of observation to light house = $d_1 + d_2$

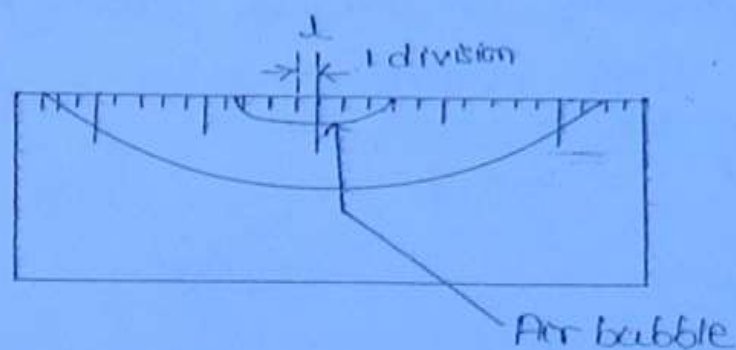
$$= 3.855\sqrt{h_1} + 3.855\sqrt{h_2}$$

$$= 3.855(\sqrt{9} + \sqrt{49})$$

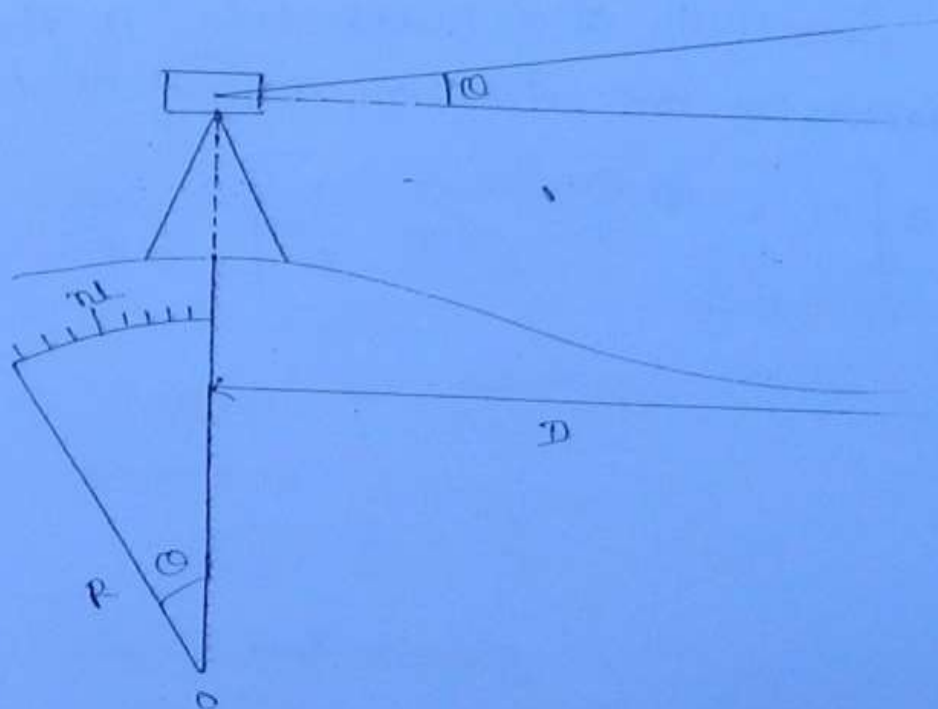
$$= 3.855 \times (3+7)$$

$$= 38.55 \text{ km Ans}$$

* Sensitiveness of a bubble Tube: $\rightarrow$



(75)



Experiment: $\rightarrow$

① fix the instrument at a location take

bubble in the centre at on a staff keep

② Now rotate the telescope such that bubble is at 1 division.

if l = length of one division.

Total moment of bubble = nl

(3) Radius of curvature of bubble tube = R

(4) Staff reading after rotation = S_2

(5) Staff intercept $S = S_2 - S_1$

total angle

(76)

$$\theta = \frac{S}{D} = \frac{nl}{R}$$

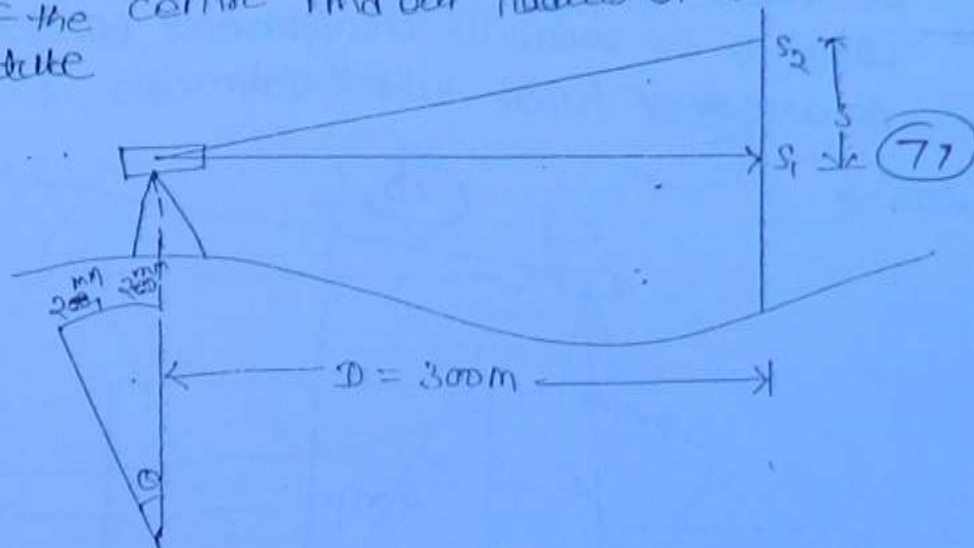
— (A)

Sensitivity of a bubble tube is the angle of rotation for one division movement of bubble

$$\Rightarrow \alpha = \frac{\theta}{n} = \frac{S}{nD} = \frac{1}{R} \quad \text{— (B)}$$

Problem $\rightarrow$ If a bubble tube has sensitivity of $\frac{1101}{25}$ Sec
length of one division of bubble tube is 2 mm. Find out the error in staff reading on a staff kept at 300 m distance. caused due to bubble 2 division out of the centre. Find out Radius of curvature of bubble of the centre.

Solution: tube



Sensitivity of bubble tube

$$\phi = 25 \text{ Sec} = \frac{25}{60 \times 60} \times \frac{\pi}{180} = \frac{25}{206265} \text{ rad.}$$

$$n = 2$$

length of one division $l = 2 \text{ mm}$

Sensitivity

$$\phi = \frac{\theta}{n} = \frac{l}{nD} = \frac{l}{R}$$

Error in staff reading

$$S = n \phi D = 2 \times \left(\frac{25}{206265} \right) \times 300$$

$$S = 0.0727 \text{ m}$$

$$S = 7.27 \text{ mm} \quad \text{Ans}$$

$$\Rightarrow \phi = \frac{l}{R} \quad R = \frac{l}{\phi} = \frac{2 \text{ mm}}{25/206265}$$

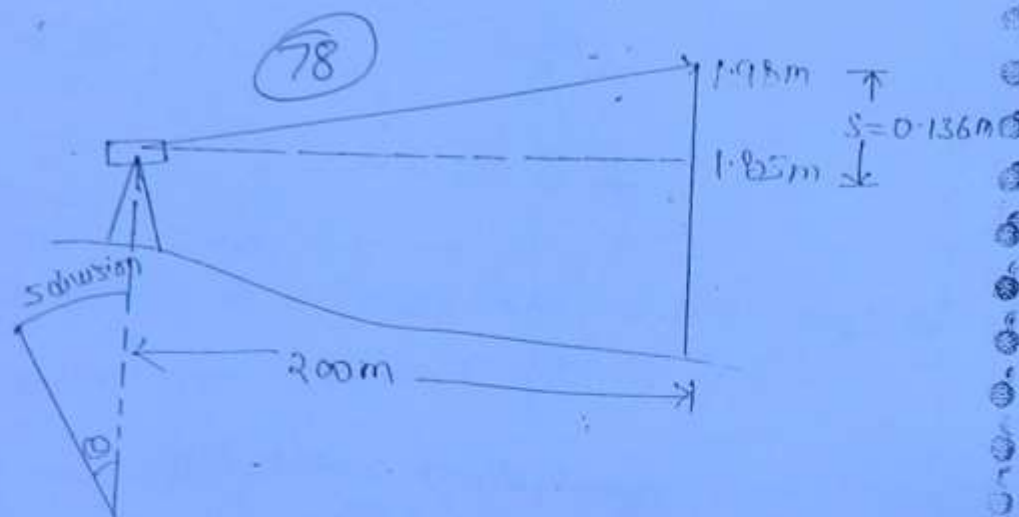
Radius of curvature of bubble of the centre

$$R = 16.502 \text{ m}$$

mm

Problem → The reading taken on a staff kept at 200m from an instrument with bubble at centre was 1.85m. Bubble is then moved by 5 division out of the centre and a staff reading observed was 1.98m. Find the sensitivity of bubble tube. What is the radius of curvature of bubble tube. Length of one division of bubble tube is 3mm

Solution →



$$S_1 = 1.85m$$

$$S_2 = 1.98m \text{ (after 5 division movement)}$$

$$S = S_2 - S_1$$

$$S = 1.98 - 1.85$$

$$S = 0.136m$$

$$n = 5$$

$$l = 3mm$$

$$\text{Sensitivity of bubble tube } \gamma = \frac{\theta}{n} = \frac{S}{nD} = \frac{l}{R}$$

$$\Rightarrow \gamma = \frac{S}{nD} = \frac{0.136}{5 \times 200} = 0.136 \times 10^{-4} = 1.36 \times 10^{-4} \times 60 \times 60 \times 180 = 28.05 \text{ arc sec}$$

Radius of curvature

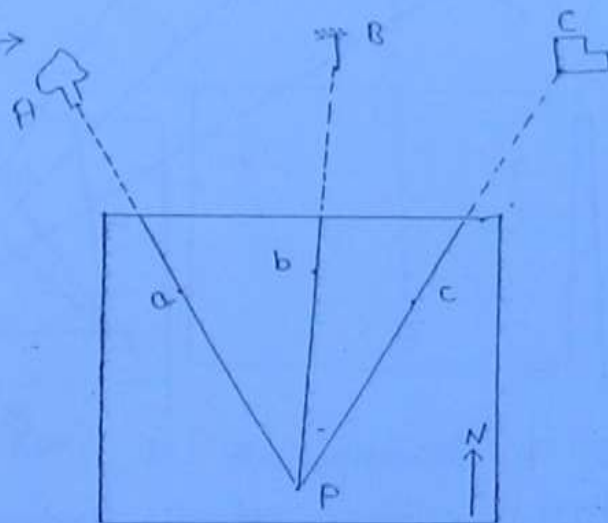
$$R = \frac{l}{\gamma} = \frac{3}{28.05 \times 1.72 \times 10^{-4}} = 22058.5 \text{ mm}$$

* Plane Table Surveying: →

There are four method.

- ① Radiation Method.
 - ② Intersection method.
 - ③ Traversing.
 - ④ Resection
- for collecting the objects details.
for locating the position of instrument station over drawing.

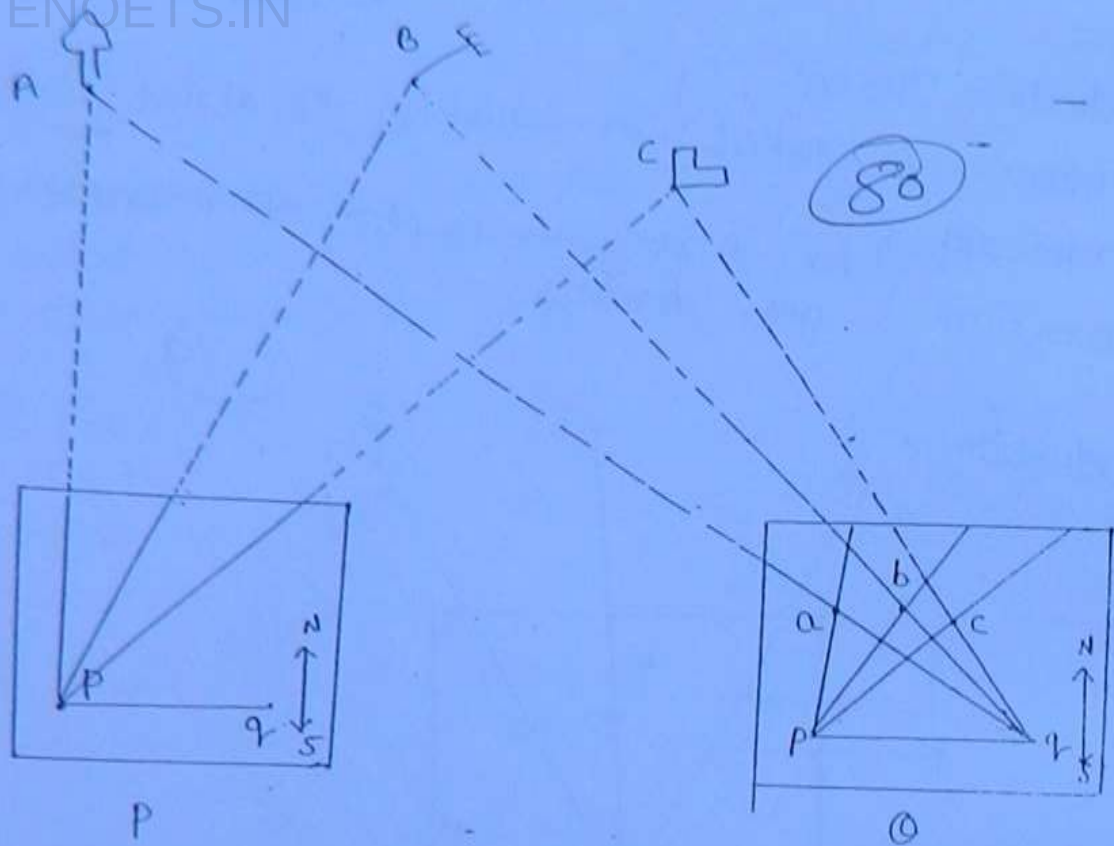
① Radiation: →



→ Orient the table in correct direction (N-S) using a line drawn on the drawing

- Position of instrument station P is marked over drawing
- For location object position, distances are measured by using any other instrument
- Lines are drawn towards different objects A, B and C and position of objects are marked by using a suitable scale

② Intersection: → first the instrument is set up at a station P and after orienting the table. Lines are drawn towards different objects, i.e. A, B and C.



→ Then another station Q is marked by measuring PQ distance and by drawing a line PQ distance and by drawing a line from P towards Q.

→ The instrument is changed to Q and after orienting the table by either compass or back orientation line are again drawn toward A, B and C.

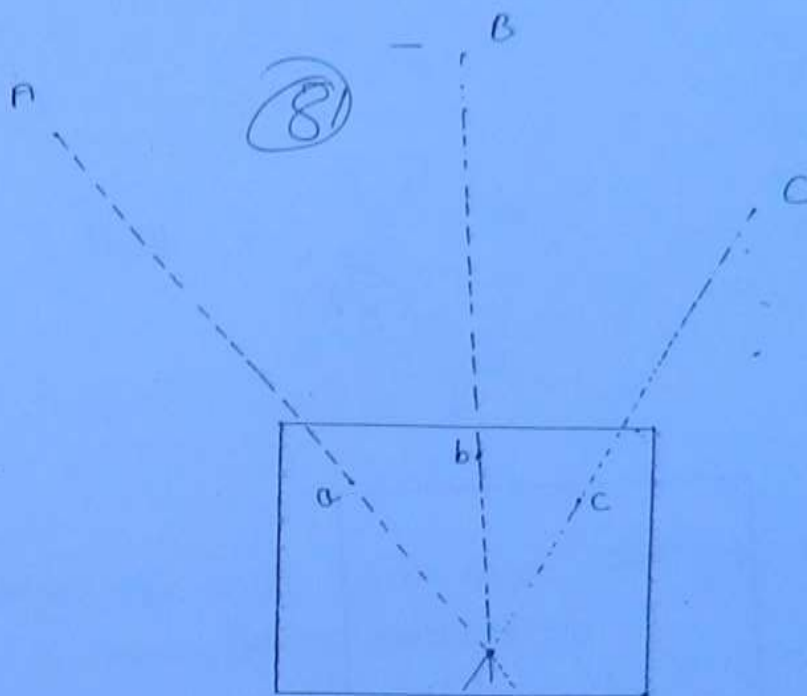
→ Intersection points are the location of objects.

③ Traversing: →

→ Different station of a closed traverse are marked by distance and orientation of table by back orientation.

→ Other details are collected either by radiation / Intersection L

④ Resection: →



→ Resection is the process of determining position of instrument station after orienting the table in correct direction.

- Methods
- ① Orientation by compass. (↑_S^N)
 - ② Orientation by back sighting.
 - ③ Three point problem.
 - ④ Two point problem.

⇒ Three point problem: → (using the known location of three points on the drawing)

- ① By Tracing paper method.
- ② By Bessel's graphical method.
- ③ By Lehman's rule method.

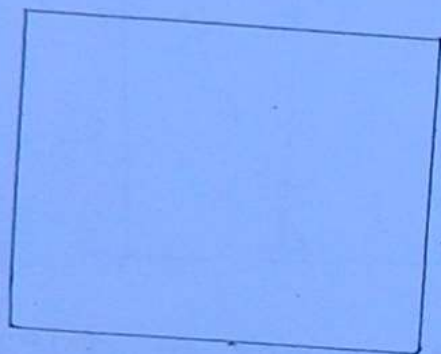
① Tracing paper method:->

A

B

C

(82)

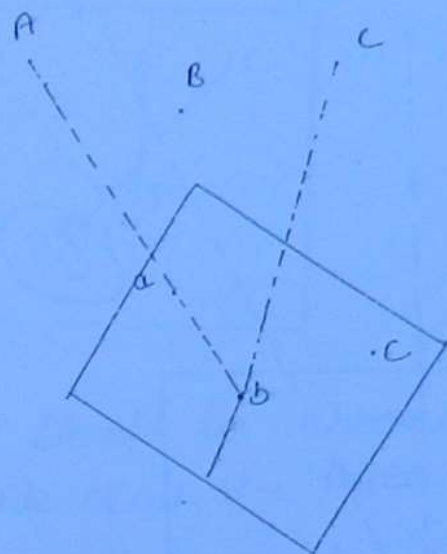


- fixed a Tracing paper on the drawing. Select any point on the tracing paper and draw three lines towards A, B and C.
- Now rotated the tracing paper over drawing such that the marked position a, b and c comes directly over the three lines drawn in tracing paper.
- Transfer the location of intersection point on the tracing drawing. This is the correct position of Instrument Station. Orient any one line towards the corresponding Object. This is the correct orientation.

③ Bessel's Graphical Method: →

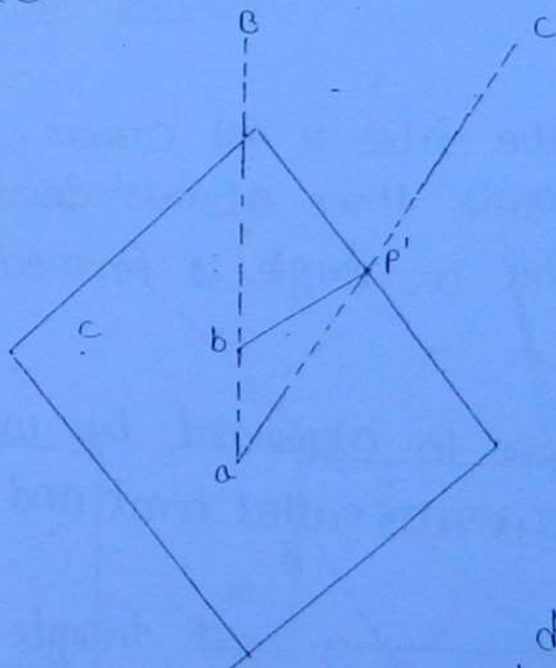
Step
①

(Q3)



→ After at setting the table at station P, rotating the table to make line $\overrightarrow{ba}$ from b toward C. Rotate

② Rotating the table such that line $\overrightarrow{ab}$ is now towards C, draw a line from a toward C.

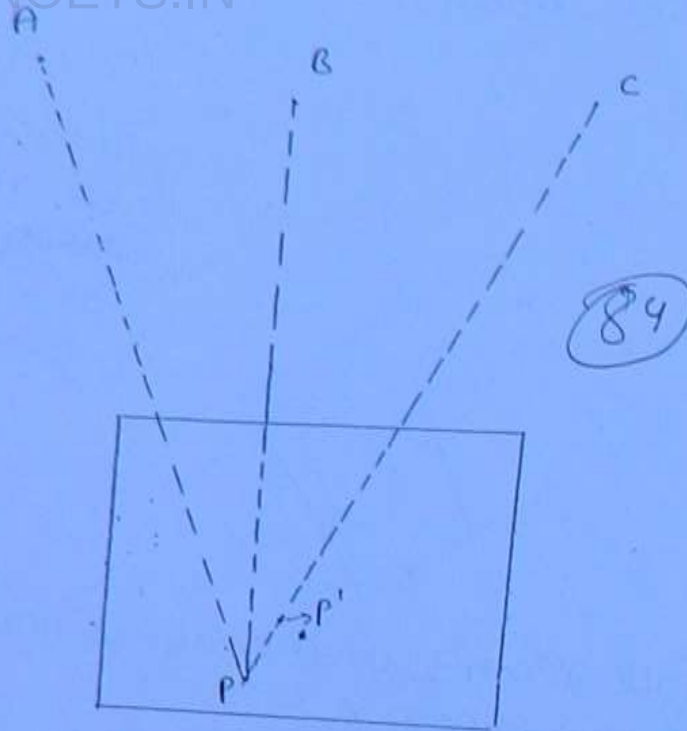


③ Intersection point of two lines drawn is P'

④ Now rotating the table and make line P'C to towards C. This is the correct orientation of the table, draw line ^{from} A, B and C that intersect line P'C at a point P that is the location of Instrument Station.

L

—



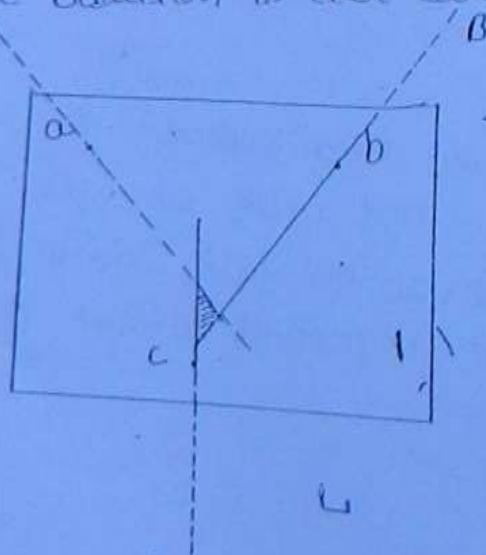
③ Lehman's method:→

→ If orientation of the table is not correct, the three lines drawn towards three objects does not intersect at a single point. This a triangle is formed called triangle of error.

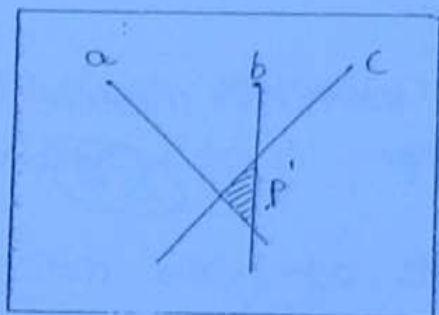
This triangle of error to be eliminated by using Lehman's Rules. The solution is also called trial and error solution.

Rules→

①



→ If triangle of error is within the map triangle possible location of point is within the triangle.

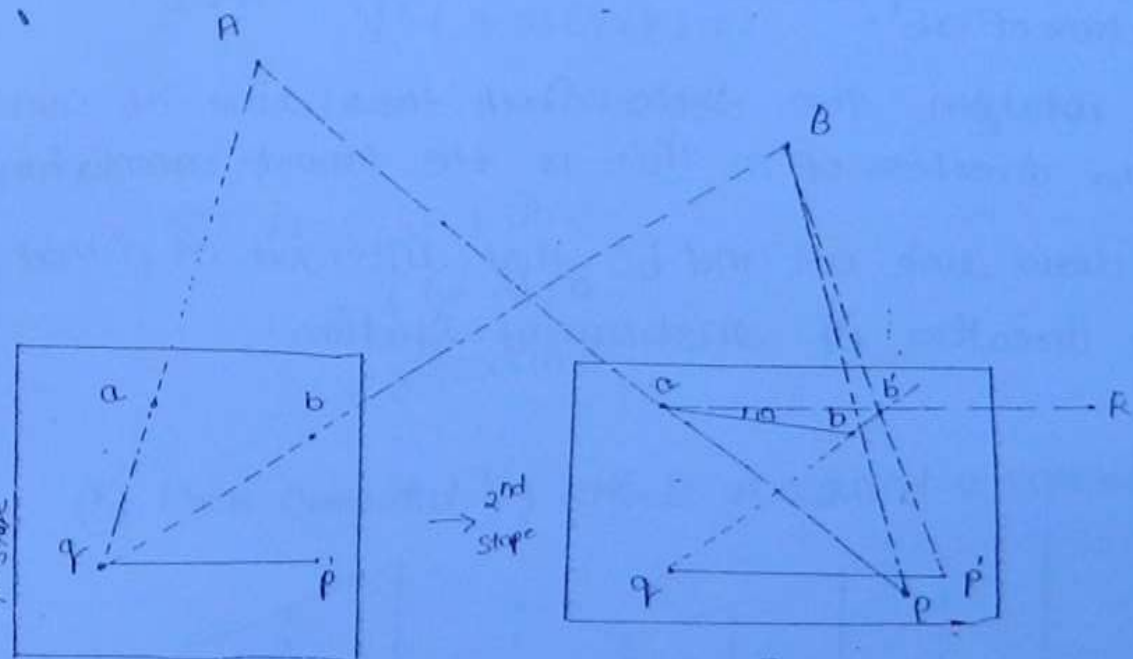


→ The point P' should be chosen such that, its distance from a, b and c are proportional to $P'A, P'B$ and $P'C$
(85)

→ The point P' should be chosen such that, it is to the same side of all the three rays Pa, Pb and Pc .

→ The P' should be opposite the triangle.

(4) Two point problem: →



Q

P

Procedure: →

(1) A and B are two points with which location are marked as a and b . L

(2) fixed the table at a station @ near P . Overlaping the tab

approximately draw line aA and bB that intersect at q .

(3) From q draw line toward P measure distance qP and plot the position of P' . (86)

(4) Now bring the table at P and orient the table by back orientation toward O . In this position orientation of table is same as it was at O .

(5) Now draw a line from P toward B that cut line qB at b' . $\angle Oab'$ is the error in orientation.

(6) To correct the error fix a flag at R near P in the direction of ab' .

(7) Now rotate the table such that line ab comes in the direction of R . This is the correct orientation.

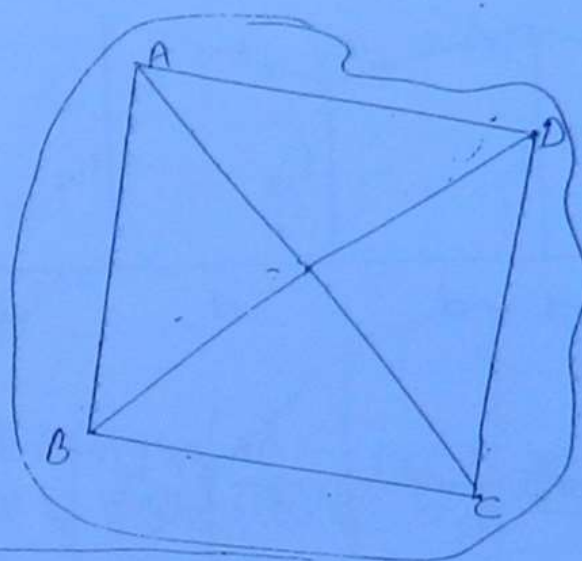
(8) Now draw line aA and bB that intersect at P' that is the direction of instrument station.

① Area :-

Different methods are

(a) Dividing into a number of Triangles.

(87)



$$(1) A = \sqrt{s(s-a)(s-b)(s-c)}$$

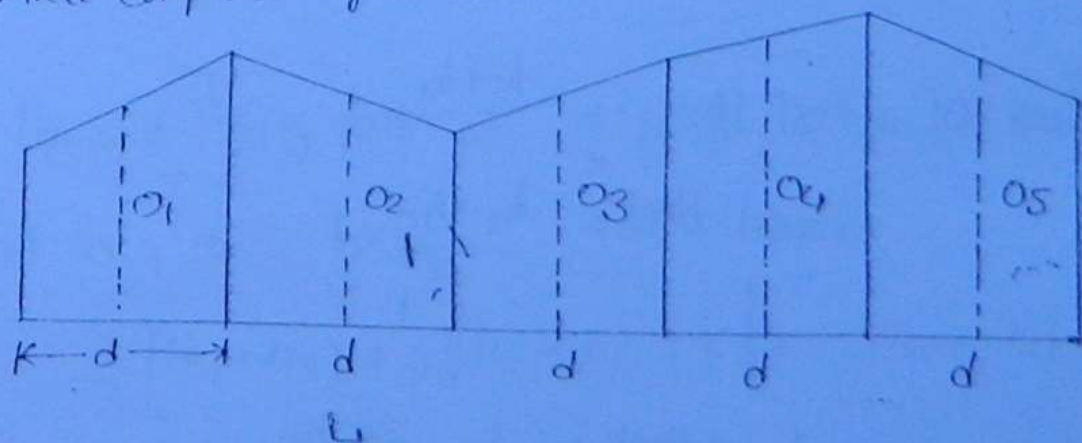
$$\text{where } s = \frac{a+b+c}{2}$$

$$(2) A = \frac{1}{2} a \cdot b \sin C$$

$$= \frac{1}{2} bc \sin A$$

$$= \frac{1}{2} ac \sin B$$

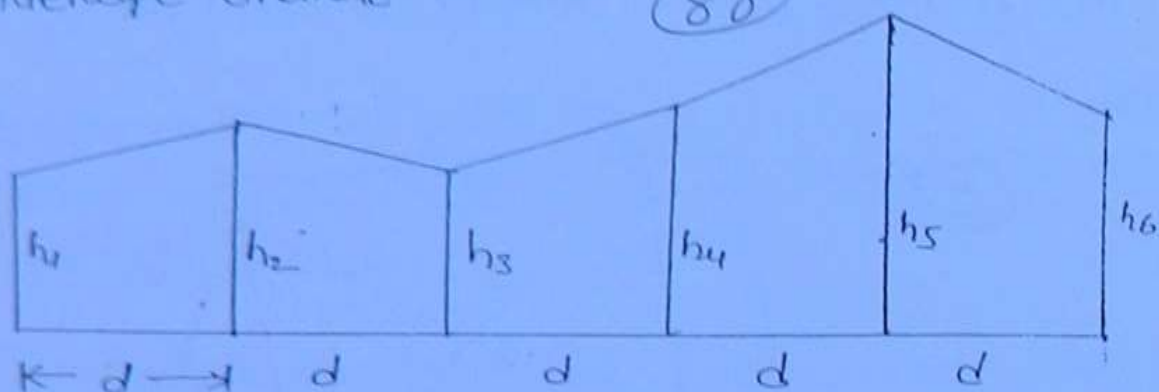
(b) Area computed by offsets measured at equal intervals.



① Mid ordinate method:-

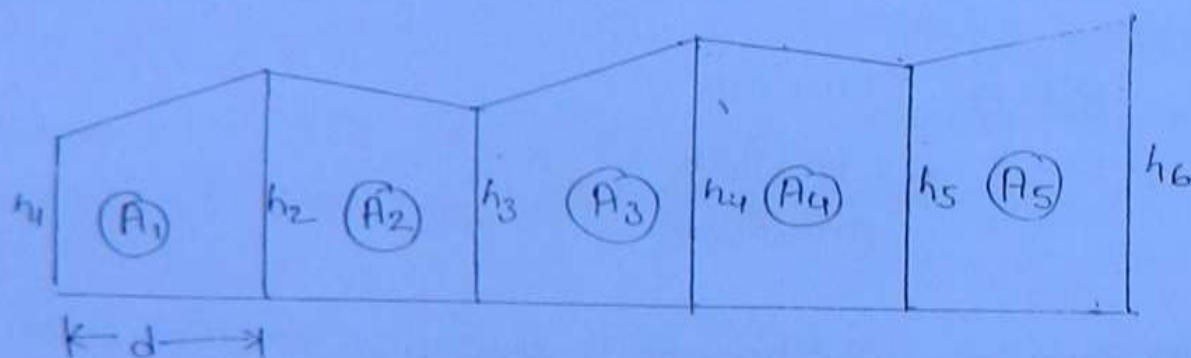
$$\text{Area} = d(O_1 + O_2 + O_3 + O_4 + \dots + O_n)$$

② Average ordinate rules:- $\rightarrow$ SS



$$A = \left(\frac{h_1 + h_2 + h_3 + \dots + h_n}{n} \right) \times (n-1) d$$

③ Trapezoidal Rule:- $\rightarrow$



Area of first block = $\frac{h_1 + h_2}{2} \times d$

Second block = $\frac{h_2 + h_3}{2} \times d$

Least block = $\frac{h_{n-1} + h_n}{2} \times d$

Total Area

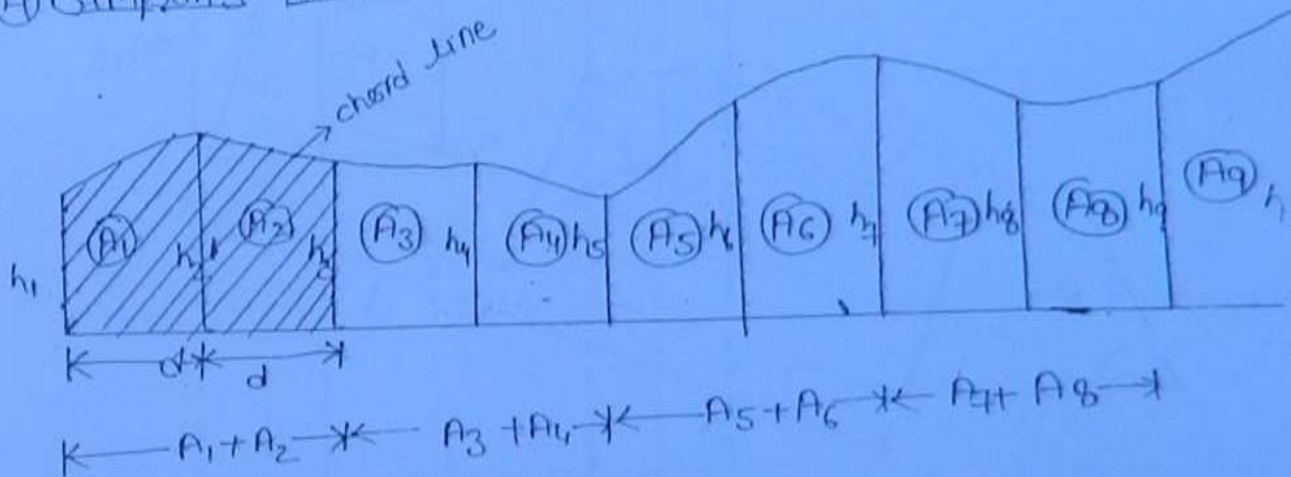
$$A = \frac{d}{2} \left[(h_1 + h_n) + 2(h_2 + h_3 + \dots + h_{n-1}) \right]$$

(89)

Trapezoidal Formula

$$A = d \left[\frac{h_1 + h_n}{2} + (h_2 + h_3 + h_4 + \dots + h_{n-1}) \right]$$

(A) Simpson's Rule: $\rightarrow$



Area of one pair $(A_1 + A_2) = \frac{d}{3} (h_1 + 4h_2 + h_3)$

Second pair $(A_3 + A_4) = \frac{d}{3} (h_3 + 4h_4 + h_5)$

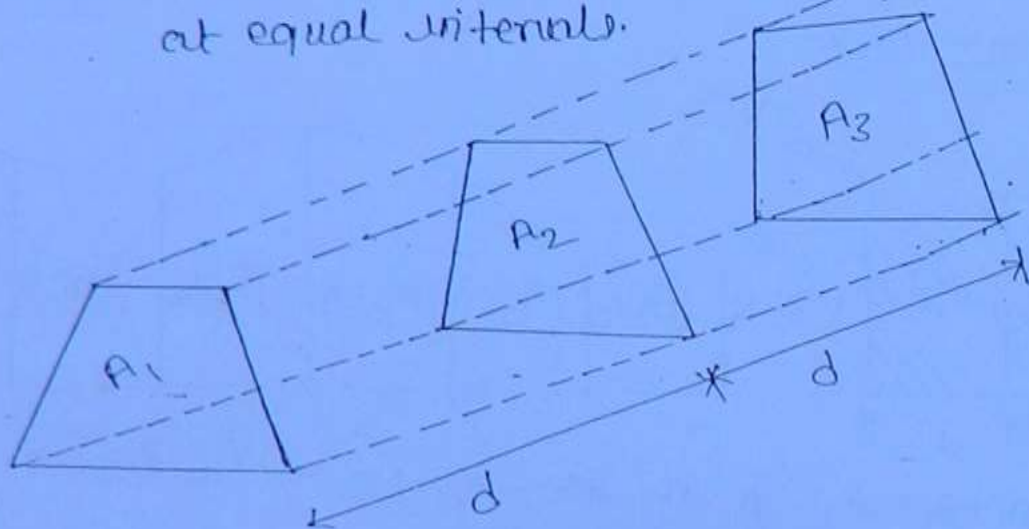
$A_{(n-2)} + A_{(n-1)}$ pair $= \frac{d}{3} (h_{n-2} + 4h_{n-1} + h_n)$

Total area: $\rightarrow$

$$A = \frac{d}{3} \left[(h_1 + h_n) + 4(h_2 + h_4 + h_6 + \dots) + 3(h_3 + h_5 + \dots) \right]$$

Note: $\rightarrow$ Above formula can be used for odd number of offsets only. If there are even No. of offsets either first block/last block area is calculated using trapezoidal rule and simply added. (90)

★ Volume: $\rightarrow$ If there are different parallel areas available at equal intervals.



(1) Trapezoidal formula: $\rightarrow$ (Method of End Area)

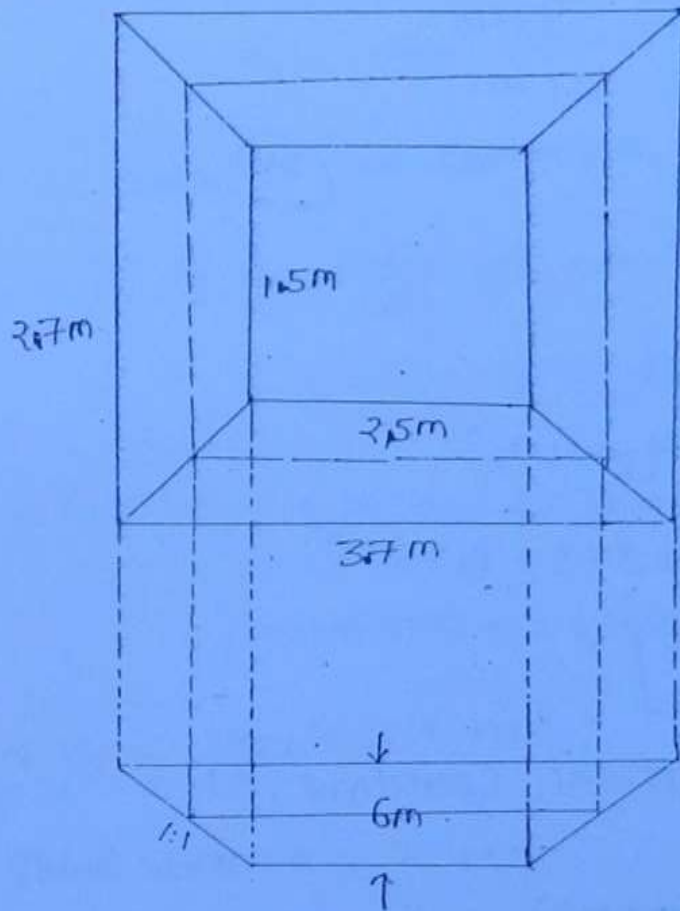
$$V = d \left[\frac{A_1 + A_n}{2} + (A_2 + A_3 + \dots + A_{n-1}) \right]$$

(2) Simpson Rule: $\rightarrow$

$$V = \frac{d}{3} \left[(A_1 + A_n) + 4(A_2 + A_4 + \dots) + 2(A_3 + A_5 + \dots) \right]$$

Note $\rightarrow$ The line joint in curved line then Simpson Rule is best and the line is joint in a straight line then Trapezoidal formula is best. L

Problem: → An excavation has shape as shown in Fig. Calculate volume of the excavation earth by
 (1) Trapezoidal Rule (2) Simpson's Rule.



(9)

Solution: - Trapezoidal Rule

$$\frac{37 \times 27}{6} \times \frac{25 \times 15}{6}$$

$$A_1 = 37 \times 27 = 999 \text{ m}^2$$

$$A_2 = 25 \times 15 = 375 \text{ m}^2$$

$$d = 6 \text{ m}$$

Volume

$$V = \frac{A_1 + A_2}{2} \times d = \frac{999 + 375}{2} \times 6$$

$$V = 4122 \text{ m}^3$$

③ Simpson's Rule: $\rightarrow$

Considered another area at centre. -

$$A_1 = 37 \times 27 = 999 \text{ m}^2$$

$$A_3 = 31 \times 21 = 651 \text{ m}^2$$

$$A_2 = 25 \times 15 = 375 \text{ m}^2$$

(92)

$$d = 3 \text{ m}$$

Volume

$$V = \frac{d}{3} (A_1 + A_3 + 4A_2)$$

$$V = \frac{3}{3} (999 + 375 + (4 \times 651))$$

$$V = 3978 \text{ m}^3$$

Problem: $\rightarrow$ Area of different counters of a reservoir area :-

Counters	Area (m ²)
V_1 104	$\rightarrow 4000 (A_0)$
105	$\rightarrow 15500 (A_1)$
106	$\rightarrow 90000 (A_2)$
107	$\rightarrow 160000 (A_3)$
108	$\rightarrow 850,000 (A_4)$
V_2 109	$\rightarrow 13,50,000 (A_5)$
110	$\rightarrow 16,40,000 (A_6)$
111	$\rightarrow 20,30,000 (A_7)$

Calculate volume of reservoir by Simpson's formula.
Volume below TBM may be neglected.

Solution:-> Volume b/w Contare 104 and 105 using Trapezoidal formula.

$$V_1 = \frac{4000 + 15500}{2} \times 1$$

(93)

$$V_1 = 950 \text{ } 9750 \text{ m}^3$$

Volume b/w Counters 105 to 111

$$\Rightarrow V_2 = \frac{d}{3} [(A_1 + A_7) + 4(A_2 + A_4 + A_6) + 2(A_3 + A_5)]$$

$$\Rightarrow V_2 = \frac{1}{3} [(15500 + 20,30,000) + 4(9,8000 + 9050,000 + 1840000) + 2(160,000 + 1350000)]$$

$$\Rightarrow V_2 = 5405033.3 \text{ m}^3$$

$$\text{Total Volume} = V_1 + V_2$$

$$V = 950 + 5405033.3 \text{ m}^3$$

$$V = 5415583.33 \text{ m}^3$$

Ans

ES-2006-600

Problem: → In a proposed reservoir the area containing within the contours are

Contours	Area (ha)	Using the method of end areas, calculate
100 →	32	① Capacity of reservoir when it is full at 100 m level. ② Elevation of water when it is 60% full. Ignore the volume below 65 m R.L. Given your remarks about method of end area.
95 →	26	
90 →	24	
85 →	18	
80 →	15	
75 →	13	
70 →	7	
65 →	2	

Solution: →

Contours	Area (ha)	Avg. Area	Volume (ha-m)	Cumulative Volume
100	32	29	145	600
95	26	25	125	455
90	24	21	105	330
85	18	16.5	82.5	225
80	15	14.0	70.0	142.5
75	13	10.0	50.0	72.5
70	7	4.5	22.5	22.5
65	2	0	0	0

① Capacity of reservoir when it is full at 100m level -

$$V = d \left[\frac{A_1 + A_n}{2} + A_2 + A_3 + \dots \right] \quad \text{--- (95)}$$

$$V = 5 \left[\frac{32+2}{2} + (7+13+15+18+24+26) \right]$$

$$\boxed{V = 600 \text{ ha-m}}$$

② Volume when reservoir is 60% Full

$$V = \frac{60}{100} \times 600 = 360 \text{ ha-m}$$

water level should be b/w 90 to 95 m

$$90 \text{ m} \rightarrow 330 \text{ ha-m}$$

$$95 \text{ m} \rightarrow 455 \text{ ha-m}$$

by interpolation

water level for 360 ha-m volume

$$= 90 + \frac{(95-90)}{(455-330)} \times (360-330)$$

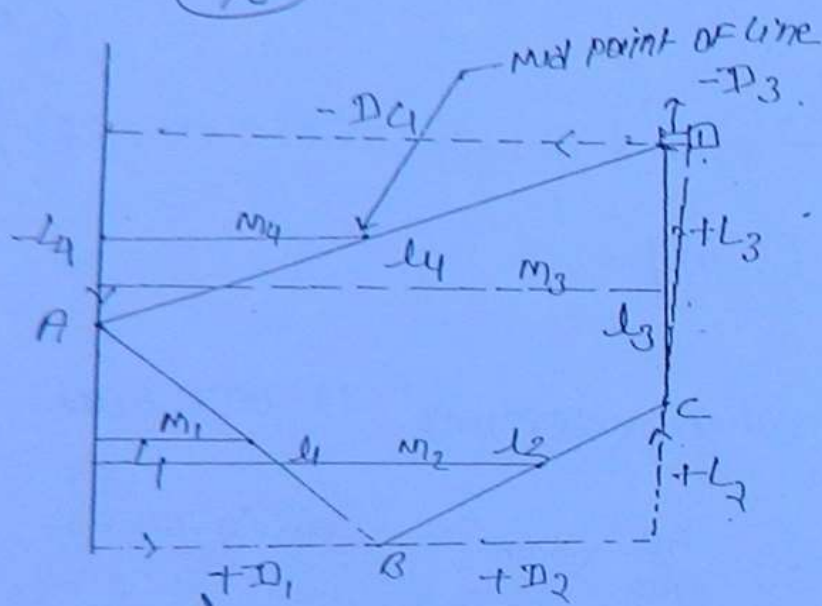
$$= \boxed{91.20 \text{ m}} \quad \text{Ans}$$

Remarks: $\rightarrow$ End area method is ^{used} trapezoidal formula for calculation of volume b/w two different area at equal interval and parallel to each others.

* Area enclosed within a closed Traverse:-

① M.D. Method:- (Meridian Distance Method):-

(96)



→ Meridian distance method is the distance of mid point of a line from a given meridian.

Meridian distance of

$$\text{Line AB} = m_1 = + \frac{D_1}{2}$$

$$\text{Line BC} = m_2 = + D_1 + \frac{D_3}{2}$$

$$= \frac{D_1}{2} + \frac{D_1}{2} + \frac{D_3}{2}$$

$$= m_1 + \left(\frac{D_1}{2} + \frac{D_3}{2} \right)$$

$$\text{Line CD} = D_1 + D_2 - \frac{D_3}{2}$$

$$= D_1 + D_2 + \left(- \frac{D_3}{2} \right)$$

$$= \left(\frac{D_1}{2} + \frac{D_1}{2} + \frac{D_2}{2} \right) + \frac{D_3}{2} + \left(-\frac{D_3}{2} \right)$$

$$= m_2 + \left(\frac{D_2}{2} + -\frac{D_3}{2} \right)$$

$$\text{M.D. of a line} = \left(\text{M.D. of previous line} \right) + \left(\frac{1}{2} \text{ Departure of previous line} \right) + \left(\frac{1}{2} \text{ Dep. of this line} \right)$$

$$\text{M.D. of line AD} = m_3 + \left[\left(-\frac{D_3}{2} \right) + \left(-\frac{D_4}{2} \right) \right]$$

Area within closed traverse.

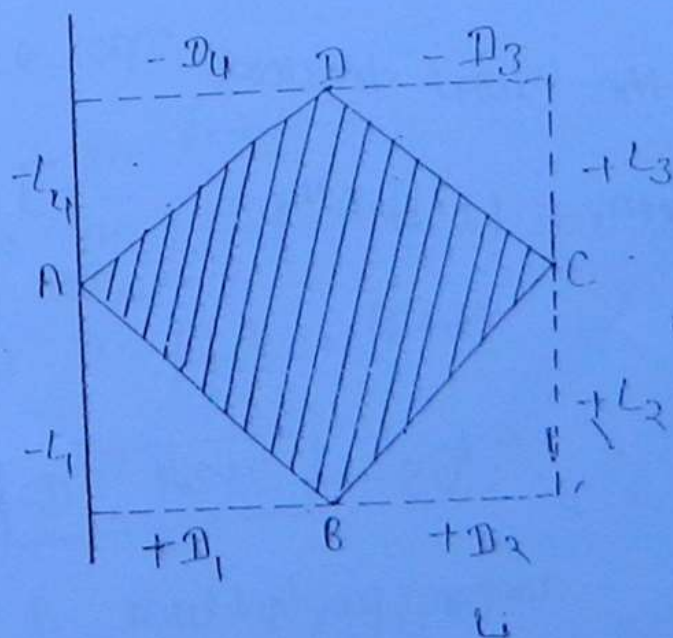
(97)

$$A = (\pm L_1) \times m_1 + (\pm L_2) m_2 + \dots$$

$$A = \sum (L \times m)$$

★ DMD Method :→

(Double Meridian Distance method)



Double meridian distance is the sum of the meridian distance of two extreme points of a line.

DMD of line

$$AB = M_1 = 0 + D_1$$

$$M_1 = D_1$$

$$\text{for } BC = M_2 = D_1 + (D_1 + D_2)$$

$$M_2 = M_1 + (D_1 + D_2)$$

$$\text{DMD of a line} = \left(\begin{array}{c} \text{DMD of} \\ \text{previous} \\ \text{line} \end{array} \right) + \left(\begin{array}{c} \text{Departure} \\ \text{of previous} \\ \text{line} \end{array} + \begin{array}{c} \text{Dep. of} \\ \text{this} \\ \text{line} \end{array} \right)$$

$$\text{for } CD = M_2 + [(+D_2) + (-D_3)]$$

$$DA = M_3 + [(-D_3) + (-D_4)]$$

Area within the closed traverse

$$A = \frac{1}{2} [(\pm L_1) \times M_1 + (\pm L_2) \times M_2 + \dots]$$

$$A = \frac{1}{2} \sum LM$$

(98)

$$\Rightarrow \frac{1}{F} = \frac{1}{u} + \frac{1}{v} \quad \text{--- (1)}$$

$$\frac{1}{u} = \frac{1}{F} - \frac{1}{v}$$

$$\frac{s}{u} = \frac{x}{v}$$

$$\boxed{v = \frac{ux}{s}} \quad \text{--- (2)}$$

(99)

$$\Rightarrow \frac{1}{u} = \frac{1}{F} - \frac{s}{ux}$$

$$\Rightarrow \frac{1}{u} \left(1 + \frac{s}{x}\right) = \frac{1}{F}$$

$$\Rightarrow F \left(1 + \frac{s}{x}\right) = u$$

$$\Rightarrow \boxed{u = F + \frac{F}{x} s} \quad \text{--- (3)}$$

The distance of object from axis of telescope

$$D = u + d$$

$$\Rightarrow D = F + \frac{F}{x} s + d$$

$$\Rightarrow D = \left(\frac{F}{x}\right) s + (F + d)$$

$$\boxed{D = ks + c} \quad \text{--- (A) } ^{1,1}$$

∴ k = multiplying constant

$$k = \frac{F}{x} \approx 100 \text{ (generally)}$$

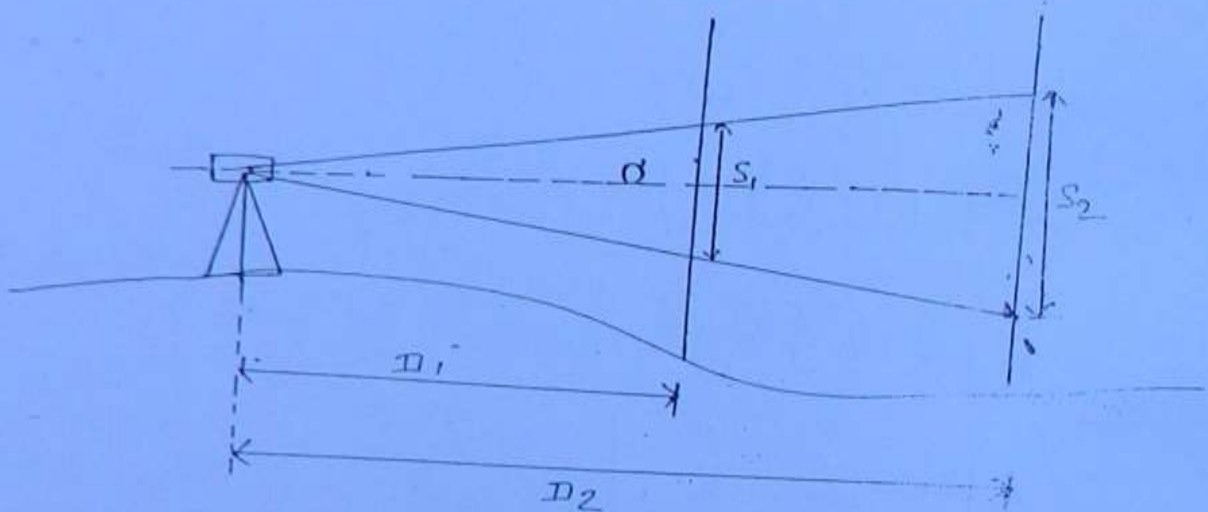
$C = \text{Additive Constant} = f(d) \approx 0$ (generally)

→ A telescope which have $k=100$, and $C=0$ is called anallatic telescope.

$S = \text{Staff intercept}$

100

⑧ Determination of k and C for a telescope :→



Instrument is fixed at a station and staff readings are taken at two location, at distance D_1 and D_2 , where staff intercept are S_1 and S_2 .

$$D_1 = kS_1 + C \quad \text{--- (1)}$$

$$D_2 = kS_2 + C \quad \text{--- (2)}$$

} Solve this two eq

$$\Rightarrow (D_1 - D_2) = k(S_1 - S_2)$$

$$k = \frac{(D_1 - D_2)}{(S_1 - S_2)} \quad \text{--- (A)}$$

$$\Rightarrow C = D_1 - kS_1$$

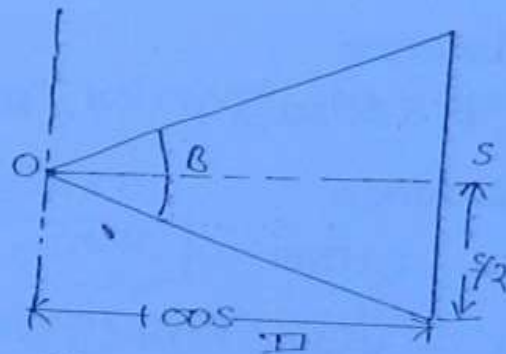
$$C = D_1 - \frac{(D_1 - D_2)}{(S_1 - S_2)} \cdot S_1$$

$$\Rightarrow C = \frac{D_1 S_1 - D_1 S_2 - D_2 S_1 + D_2 S_2}{(S_1 - S_2)}$$

$$C = \frac{D_2 S_1 - D_1 S_2}{S_1 - S_2} \quad \text{--- (B)}$$

(101)

$\Rightarrow$ for Anallatical Telescope.

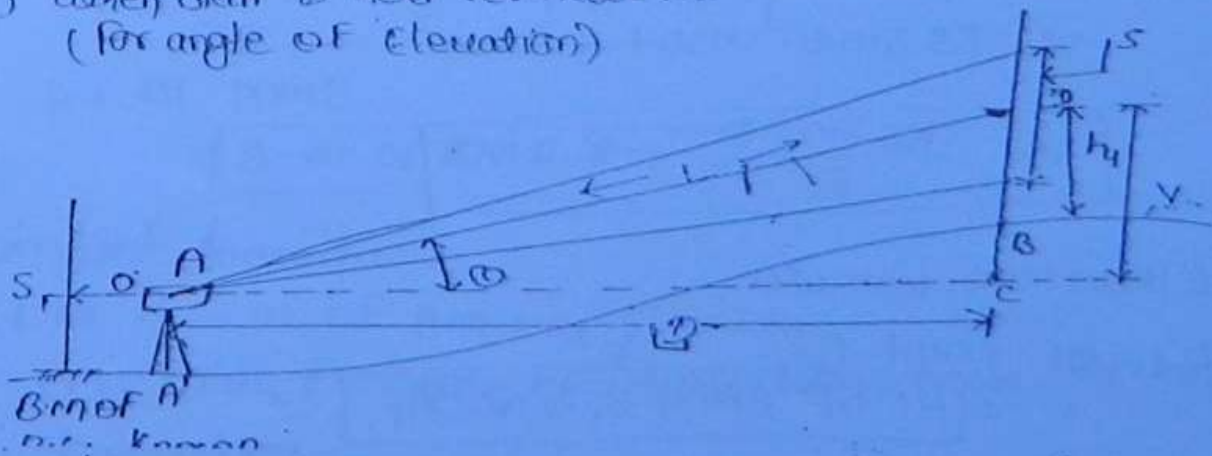


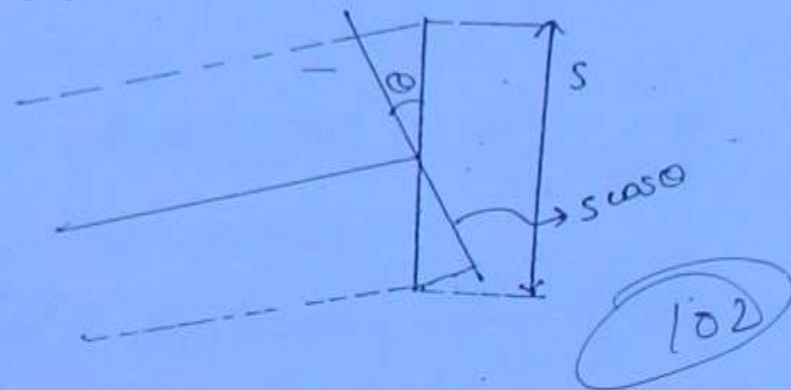
$$\tan \frac{B}{2} = \frac{S_2}{1008} = \frac{1}{200}$$

$$\therefore B = 0^\circ 30' 22.63''$$

$\star$ Distance and Elevation formula: $\rightarrow$

Case (1) when staff is held vertical: $\rightarrow$
(for angle of elevation)





★ when staff act vertical:→

Staff intercept for distance = $S \cos \theta$

Inclined distance

$$\Rightarrow L = k \cdot S \cos \theta + c$$

Horizontal distance

$$AC = D = L \cos \theta$$

$$D = (k S \cos \theta + c) \cos \theta$$

① Distance formula $\boxed{D = k S \cos^2 \theta + c \cos \theta}$ — A

Vertical distance

$$V = L \sin \theta$$

$$V = (k S \cos \theta + c) \sin \theta$$

$$V = k S \sin \theta \cdot \cos \theta + c \sin \theta$$

$$\boxed{V = \frac{k S \sin 2\theta}{2} + c \sin \theta}$$
 — B

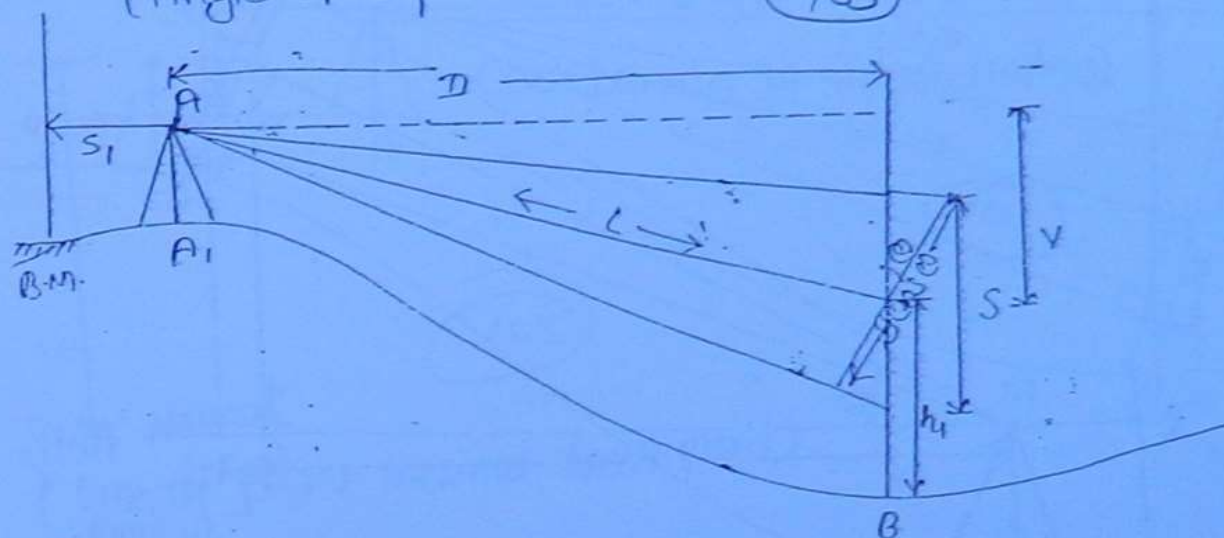
Elevation formula

R.L. of point B

$$= \boxed{RL \text{ of BM} + S_i + V - h_i}$$
 L

Case (2) Staff held vertical
(Angle of depression)

(103)



Staff intercept for distance = $S \cos \theta$

Inclined distance

$$L = k s \cos \theta + c$$

Horizontal distance formula

$$D = L \cos \theta$$

$$D = k s \cos^2 \theta + c \cos \theta$$

Elevation formula

$$V = L \sin \theta$$

$$V = k \cdot s \cdot \frac{\sin 2\theta}{2} + c \sin \theta$$

R.L. of point B

$$= \text{R.L. of BM} + S - \frac{V - h_1}{f \cdot \lambda}$$

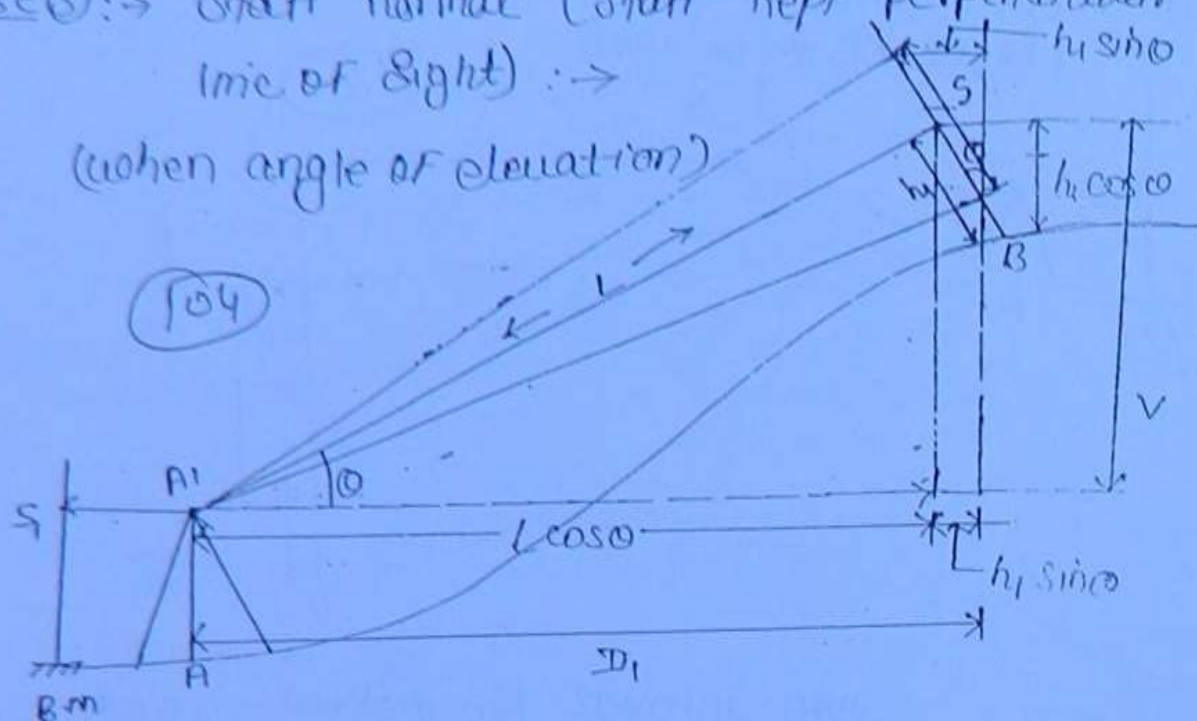
Combine formula.

$$\text{R.L. of B} = \text{R.L. of BM} + S_1 \pm V - h_1$$

Use +ve for angle of elevation, -ve for angle of depression.

Case ③: $\rightarrow$ Staff normal (staff kept perpendicular to line of sight) $\rightarrow$

(when angle of elevation)



Staff intercept for distance $L = S$

Inclined length

$$[L = ks + c]$$

Horizontal Distance

$$D = L \cos \theta + h_1 \sin \theta$$

$$[D = (ks + c) \cos \theta + h_1 \sin \theta] \quad \text{--- (A)}$$

Elevation formula.

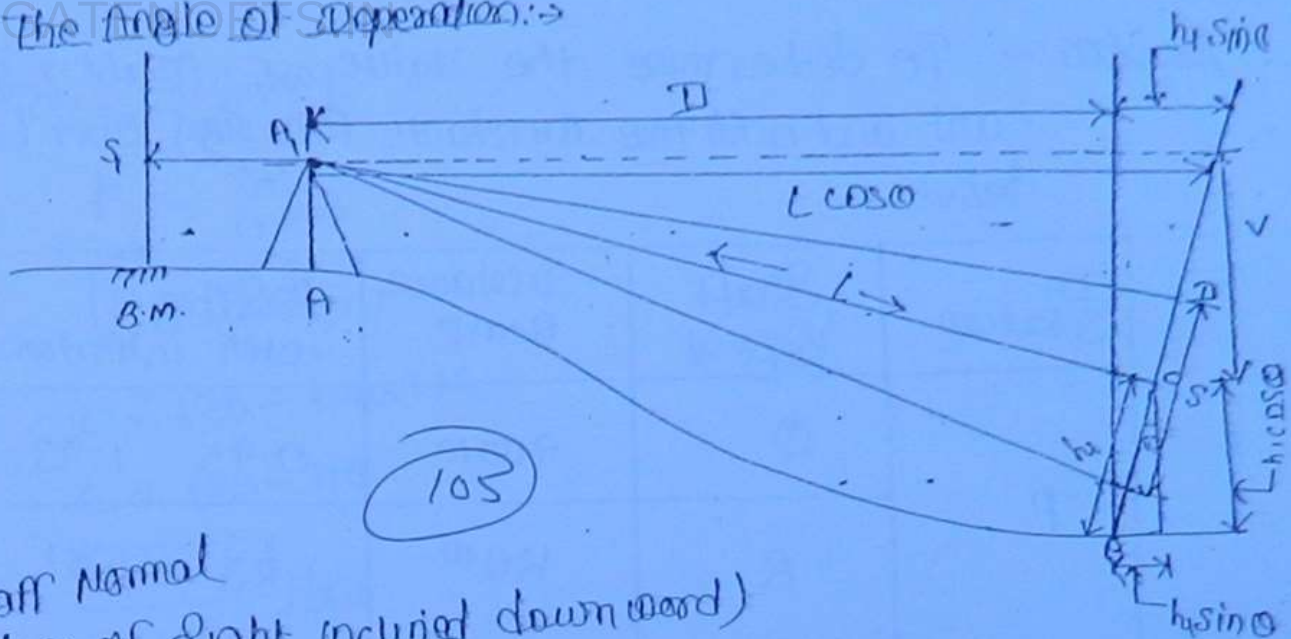
$$V = L \sin \theta$$

$$[V = (ks + c) \sin \theta]$$

R.L OF point B

$$= \text{R.L OF BM} + S_1 + V - h_1 \cos \theta$$

Case (ii) The Angle of Depression: $\Rightarrow$



Staff Normal
(Line of sight inclined downward)

Staff intercept = S
Inclined length $L = Ks + C$

Horizontal Distance formula

$$D = L \cos \theta - h_i \sin \theta$$

$$D = (Ks + C) \cos \theta - h_i \sin \theta$$

Elevation formula

$$V = L \sin \theta = (Ks + C) \sin \theta$$

R.L of B

$$RL \text{ of } BM + S_i - V - h_i \cos \theta$$

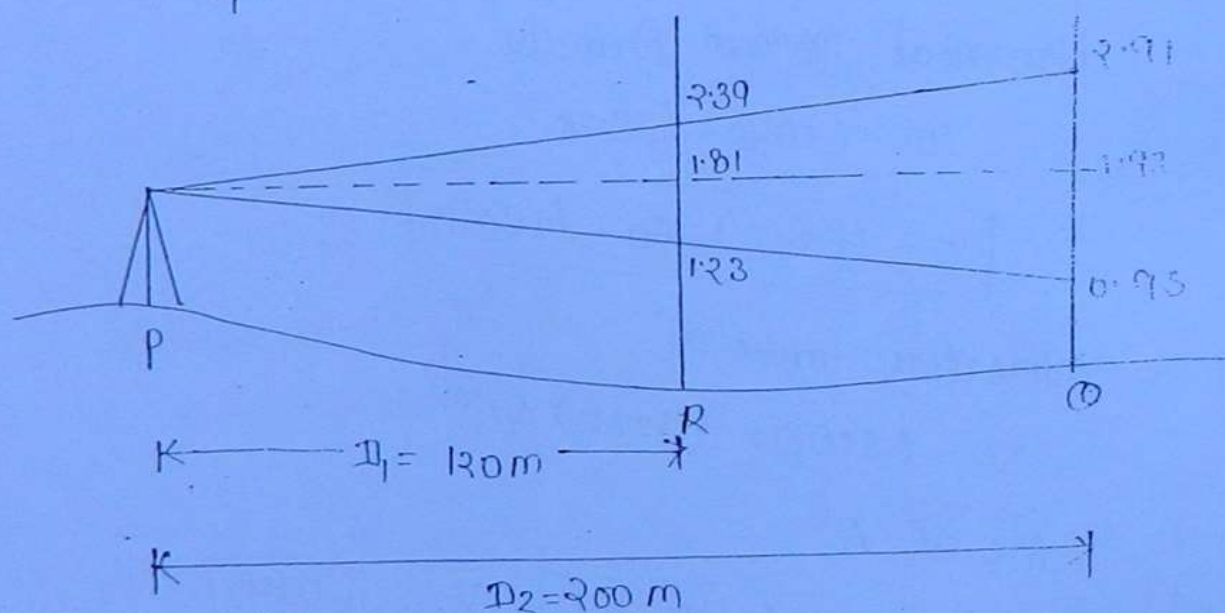
Problem → To determine the value of multiplying Constant and additive constant. following observation were taken.

In Station	Staff kept at	Distance from P	Reading with inclination of 0°
P	Q	200 m	0.95 1.93 2.91
	R	120 m	1.23 1.81 2.39

Solution →

$$S_1 = 2.39 - 1.23 \quad \left| \quad S_2 = 2.91 - 0.95 \right.$$

$$S_1 = 1.16 \quad \left| \quad S_2 = 1.96 \right. \quad (106)$$



$$D_1 = K S_1 + C$$

$$\Rightarrow 120 = K \times 1.16 + C \quad \text{--- (1)}$$

$$\Rightarrow D_2 = K \cdot S_2 + C$$

$$\Rightarrow 200 = K \times 1.96 + C \quad \text{--- (2)}$$

$$\text{④ ② ①}$$

$$\Rightarrow 90 = 0.80 \times k$$

$$k = \frac{90}{0.80}$$

$$\boxed{k = 100} \text{ Ans}$$

(107)

$$C = 120 - 100 \times 1.16$$

$$C = 120 - 116$$

$$\boxed{C = 4} \text{ Ans}$$

Problem $\Rightarrow$ From a theodolite set up at station A, readings were taken on a staff kept at station B and benchmark M.

Instrument Station	Staff location	Angle	Staff position	Readings
A	B	$+15^{\circ}30'$ (upward)	Kept vertical	2.35, 2.60, 2.85
A	M B.M RL 200.52 m	-15° (downward)	Kept normal	1.32, 1.55, 1.78

$$\text{If } k = 100, C = 0$$

height of instrument at A = 1.52 m, find out R.L. of A at B. Also find out distance of B and M from A.

Solution $\Rightarrow$ A to B

$$S = 2.85 - 2.35 = 0.50$$

Staff is vertical

$$\begin{aligned} \text{Staff intercept} &= S \cos \theta \\ &= 0.50 \cos 15^{\circ}30' \end{aligned}$$

108

$$D_1 = 100 \times 0.50 \times \cos^2 15.30^\circ$$

$$D_1 = 46.43 \text{ m}$$

$$V_1 = k_1 \frac{\sin 2\theta}{2} + \ell \sin \theta$$

$$V_1 = 100 \times 0.50 \times \frac{\sin(31)}{2}$$

$$V_1 = 13.876 \text{ m}$$

from A to ∞ wards

$$S = 1.79 - 1.32$$

$$S = 0.46$$

Distance formula →

$$D_2 = \frac{Ks}{K+1} \sin \theta$$

$$(Ks+c) \cos \theta - h_2 \sin \theta$$

$$D_2 = 100 \times 0.46 \times \cos 15^\circ - 1.55 \sin 15^\circ$$

$$D_2 = 44.03 \text{ m}$$

(109)

Elevation formula →

$$V_2 = L_2 \sin \theta$$

$$V_2 = (Ks+c) \sin \theta$$

$$V_2 = 0.46 \times 100 \times \sin 15^\circ$$

$$V_2 = 11.91 \text{ m}$$

R.L. OF A

$$= \text{R.L. of BM} + h_2 \cos \theta_2 + V_2 - 1.52$$

$$= 200.52 + 1.55 \times \cos 15^\circ + 11.91 - 1.52$$

$$= 212.41 \text{ m}$$

R.L. OF B

$$= \text{R.L. OF A} + 1.52 + V_1 - h_1$$

$$= 212.41 + 1.52 + 13.876 - 2.60$$

$$= 224.20 \text{ m}$$

Problem: → To determine the gradient b/w two points A and B a theodolite was setup at another station C, and the following observation were made.

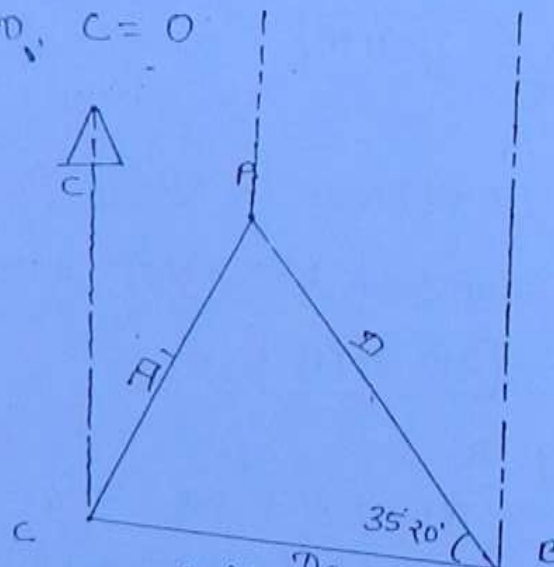
Station	Staff	Vertical angle	Staff reading
C	A	+04°20'0"	1.30, 1.61, 1.92
	B	+01°10'40"	1.10, 1.41, 1.72

(110)

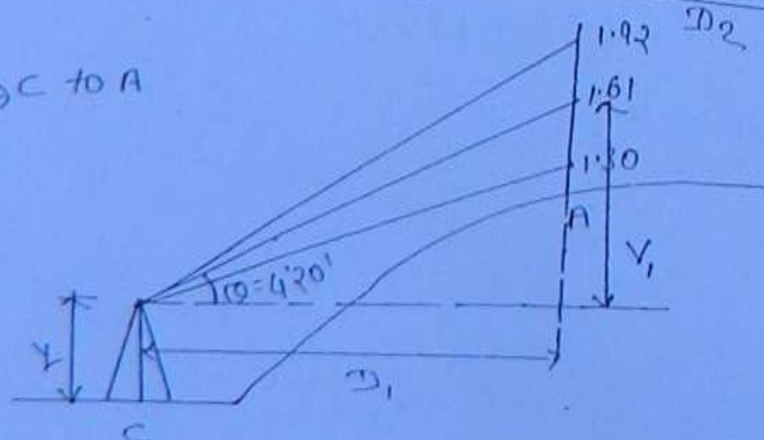
IF horizontal angle ABC = 35°20'
Determine average gradient b/w A and B.

$$K=100, C=0$$

Solution: →



→ C to A



R.L of C = x

$$S = 1.92 - 1.30 = 0.62$$

$$D_1 = K S \sin^2 \theta + \frac{K}{2} \cos^2 \theta$$

$$D_1 = 100 \times 0.62 \times \sin^2 42^\circ 20'$$

$$D_1 = 61.646m$$

$$V_1 = Ks \frac{\sin 2\theta}{2} + c \sin \theta$$

$$\Rightarrow V_1 = 100 \times 0.62 \times \frac{\sin 2 \times 0'20''}{2}$$

$$\Rightarrow \boxed{V_1 = 4.67 \text{ m}}$$

(iii)

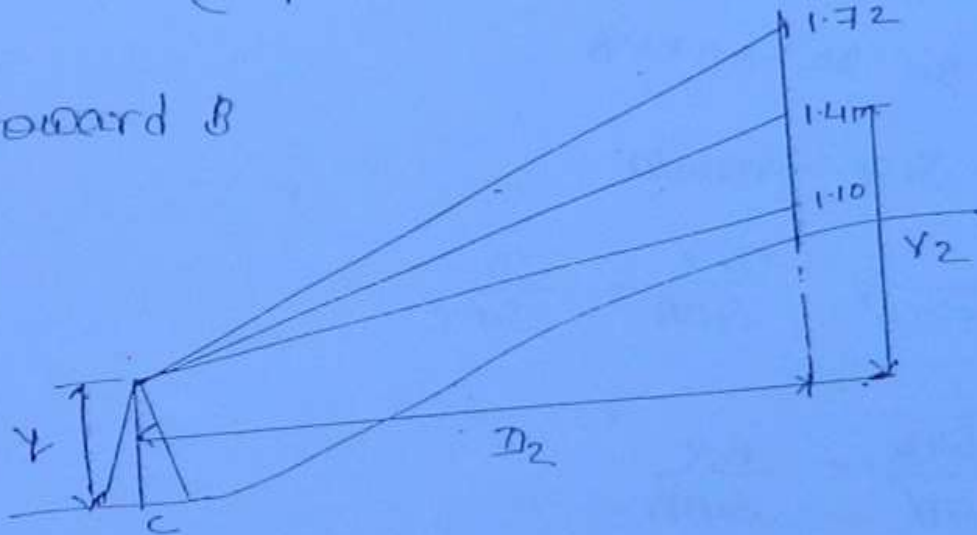
R.L. OF A

$$= x + y + V_1 - h_1$$

$$= x + y + 4.67 - 1.61$$

$$= (x + y + 3.06) \text{ m}$$

$\Rightarrow$ C toward B



$$S = 1.72 - 1.10$$

$$S = 0.62$$

$$\text{Distance } D_2 = Ks \cos^2 \theta + \frac{\gamma}{2} \cos \theta$$

$$D_2 = 100 \times 0.62 \cos^2 0'10'40''$$

$$\boxed{D_2 = 62 \text{ m}}$$

$$V_2 = Ks \frac{\sin 2\theta}{2} = 100 \times 0.62 \times \frac{\sin 2 \times 0'10'40''}{2}$$

$$\boxed{V_2 = 0.192 \text{ m}}$$

$$\begin{aligned} R.L. \text{ OF } B &= R.L. \text{ OF } C + 4 + V_2 - h_2 \\ &= (x+4) + 0.192 - 1.41 \\ &= (x+4) - 1.218 \end{aligned}$$

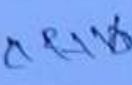
Difference of R.L. b/w A and B

(112)

A is higher

$$\begin{aligned} &= R.L. \text{ OF } A - R.L. \text{ OF } B \\ &= x + 4 + 3.06 - (x+4) - 1.218 \\ &= \boxed{4.278 \text{ m}} \end{aligned}$$

Distance from A to B

cos B = ~~cos A~~ 

Apply Sine-formula

$$\Rightarrow \frac{D_1}{\sin 35^\circ 20'} = \frac{D_2}{\sin A} = \frac{D}{\sin C}$$

$$\Rightarrow \frac{61.646}{\sin 35^\circ 20'} = \frac{6.2}{\sin A}$$

$$\Rightarrow \sin A = 35^\circ 30' 0.9''$$

$$\text{Angle } C = 180 - (35^\circ 30' 0.9'' + 35^\circ 30' 20'')$$

$$C = 109^\circ 5' 59.10''$$

$$D = \frac{D_1}{\sin 35^\circ 20'} \times \sin 109^\circ 5' 59.10''$$

$$\boxed{D = 100.72 \text{ m}}$$

$$\text{Gradient} = \frac{4.270}{100.72}$$

$$= \frac{1}{23541}$$

Ans

(1/3)

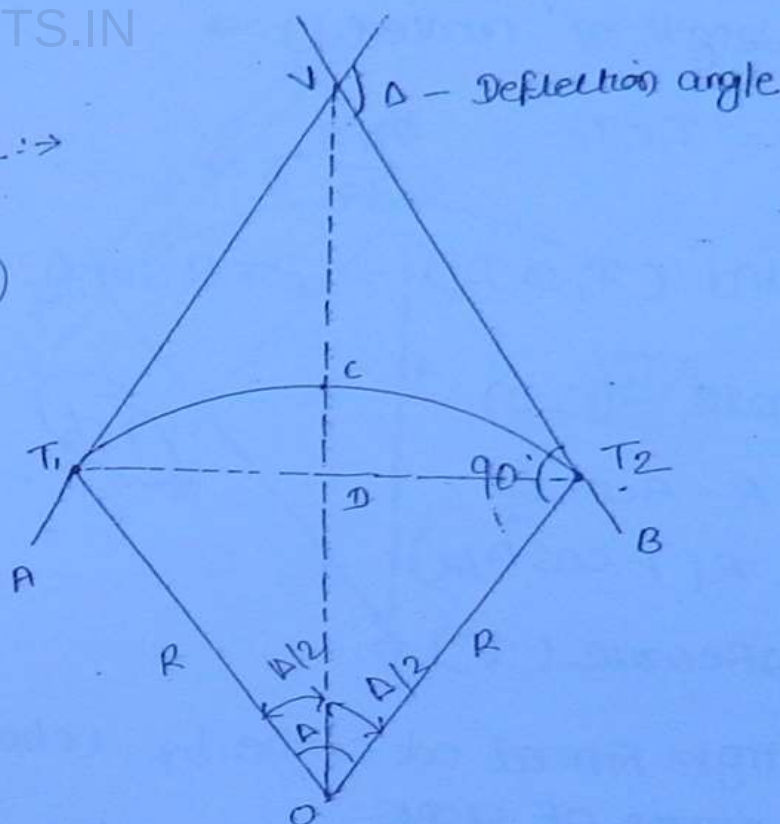
Es-1989

Problem 30

114

① Simple^I Curve $\rightarrow$

115



Important Terms: →

- (1) Back tangent $\rightarrow AT_1$
- (2) Forward tangent $\rightarrow BT_2$
- (3) Point of curve $\rightarrow$ Point T_1
- (4) Point of tangency $\rightarrow$ Point T_2
- (5) Deflection and angle (or intersection angle) Δ
- (6) Radius $= R$
- (7) of curve
centre is at O
- (8) Tangent distances $= VT_1 = VT_2 = R \cdot \tan \frac{\Delta}{2}$
- (9) External distances $= (VC) \rightarrow$ (Apex distance) $\dots$

$$V_C = R \sec \theta \frac{A}{2} - R$$

$$V_c = R \left(\sec \frac{A}{2} - 1 \right)$$

~~10.11.2020~~ Total length of curve (L) $\Rightarrow$

$$L = T_1 + T_2 = \frac{2\pi R}{360} \Delta$$

① Long Cord (T_1 & T_2) $= 2R \sin \frac{\Delta}{2}$

② Midordinate $:- (CD)$

$$CD = R - R \cos \frac{\Delta}{2}$$

$$CD = R(1 - \cos \Delta/2)$$

(116)

③ Degree of curve (D°) $\Rightarrow$

Angle formed at centre by 1 chain length is called degree of curve.

① chain = 30 m

$$\therefore D = \frac{1720}{R} \quad \Bigg| \quad R = \frac{1720}{D}$$

② chain = 20 m

$$D = \frac{1146}{R}$$

$$R = \frac{1146}{D}$$

④ Setting out a Simple curve on Ground $\Rightarrow$

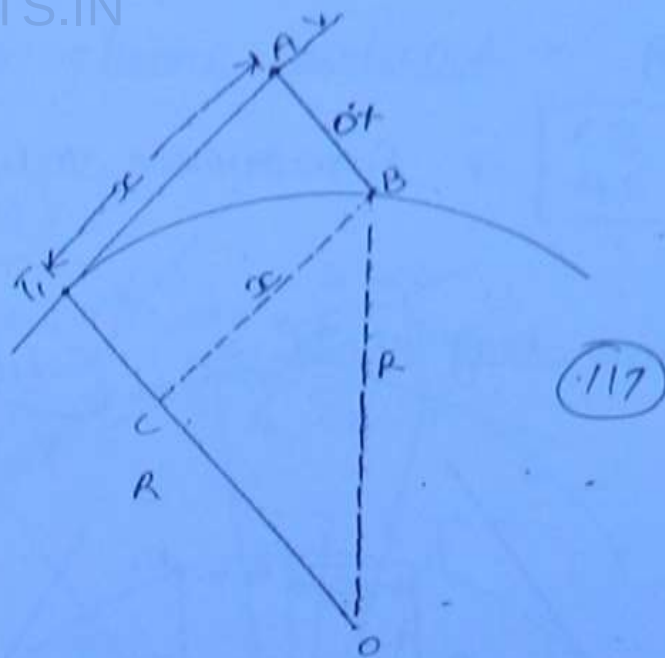
① Offset method $\Rightarrow$

① Perpendicular offsets $\Rightarrow$

$$OX = AB = T_1 C = OT_1 - OC$$

$$OX = R - \sqrt{R^2 - x^2} \quad \text{--- (A)}$$

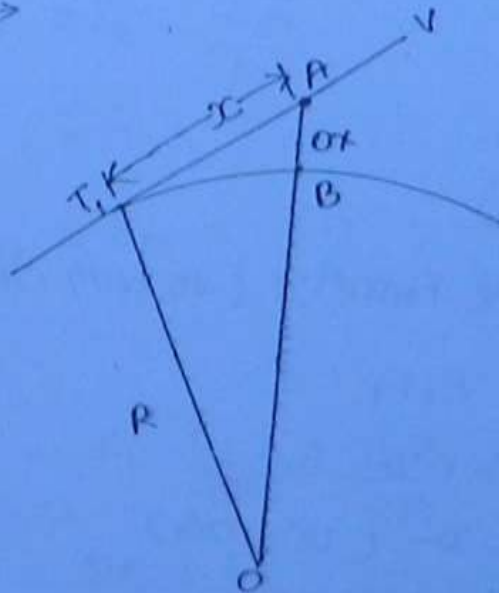
Exact formula



Expanding

$$\boxed{OX = \frac{x^2}{2R}} \rightarrow \text{approximate value}$$

⑤ Radial off set $\rightarrow$



$$OX = AB = OA - OB$$

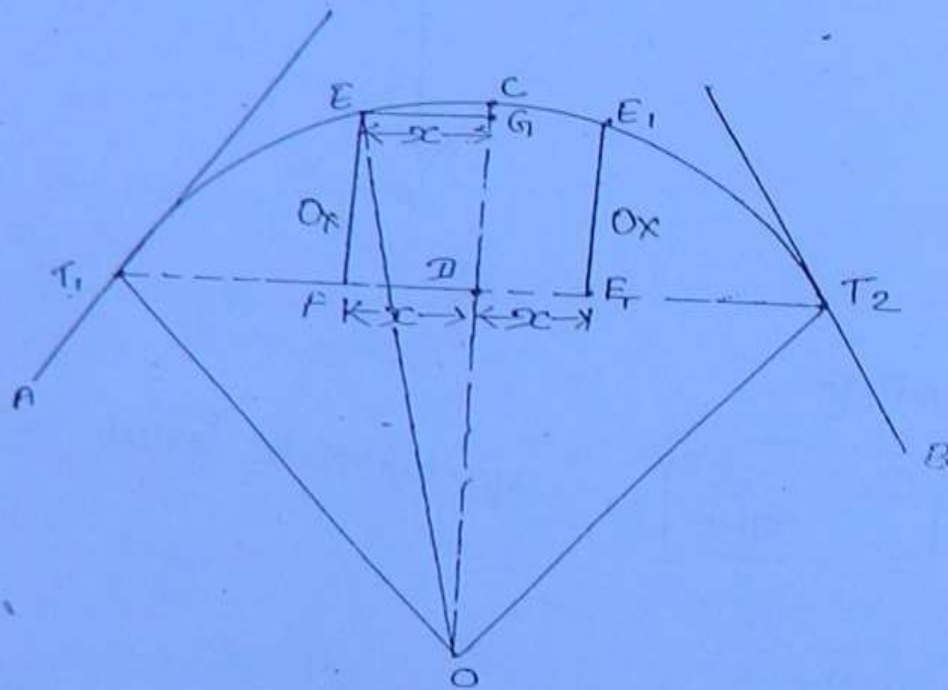
$$\boxed{OX = \sqrt{R^2 + x^2} - R} \rightarrow \text{exact formula}$$

Expanding

$$\boxed{O_x = \frac{x^2}{2R}} \rightarrow \text{Approximate value}$$

② offset from long chord: $\rightarrow$

(118)



$\Rightarrow CD = \text{Mid ordinate}$

$$CD = R(1 - \cos A/2)$$

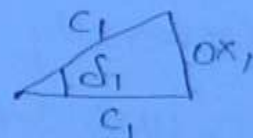
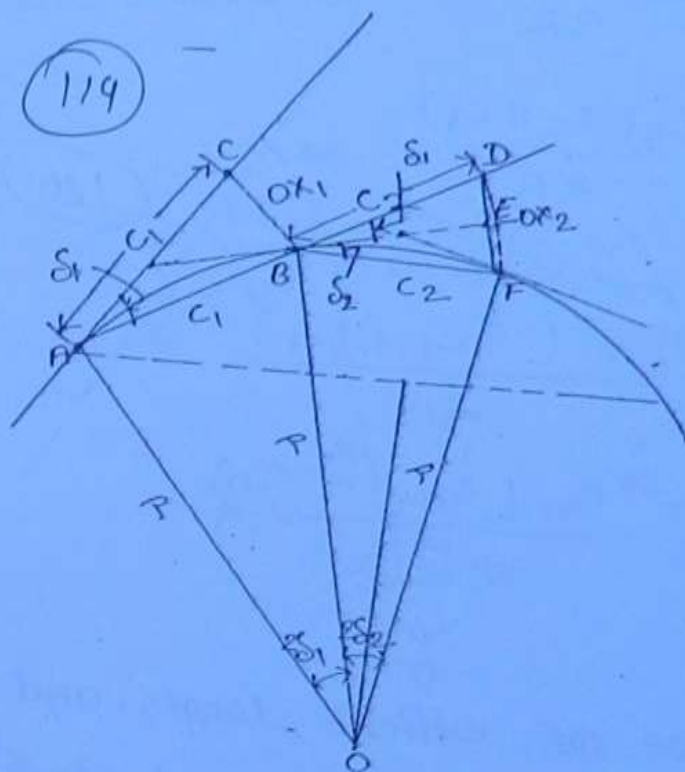
offset at x distance from D (on both side)

$$O_x = EF = EG$$

$$O_x = GD = CD - GC$$

$$O_x = CD - (OC - OG)$$

$$\boxed{O_x = R(1 - \cos A/2) - (R - \sqrt{R^2 - x^2})}$$



$$S_1 = \frac{OX_1}{C_1}$$

$$Ox_i = \delta_i c_i$$

$$\Rightarrow \phi_1 = \frac{C_1}{2R} \Rightarrow 2\phi_1 = \frac{C_1}{R}$$

$$\Rightarrow S_2 = \frac{C_2}{2R} \Rightarrow 2S_2 = \frac{C_1}{R}$$

$$\Rightarrow S_n = \frac{C_n}{2R} \Rightarrow 2S_n = \frac{C_n}{R}$$

Offset

$$Ox_1 = C_1 \cdot S_1 = C_1 \cdot \frac{C_1}{2R} = \frac{C_1^2}{2R}$$

$$OX_2 = DF = DE + EF$$

$$0x_2 = c_2\delta_1 + c_2\delta_2$$

$$Ox_2 = C_2 \frac{C_1}{2R} + C_2 \frac{C_2}{2R}$$

$$\Rightarrow OX_2 = \frac{C_2 (C_1 + C_2)}{2R}$$

$$\Rightarrow OX_3 = \frac{C_3 (C_2 + C_3)}{2R}$$

$$\Rightarrow OX_r = \frac{C_r (C_{r-1} + C_r)}{2R}$$

$$\Rightarrow OX_n = \frac{C_n (C_{n-1} + C_n)}{2R}$$

(120)

Generally

C_1 and C_n are of different length, and all other chords $C_2 = C_3 = C_4 = \dots = C_{n-1} = 1$ chain length

$$OX_1 = \frac{C_1^2}{2R}$$

$$OX_2 = \frac{C(C+C)}{2R}$$

$$OX_3 = \frac{C(C+C)}{2R} = \frac{C^2}{R}$$

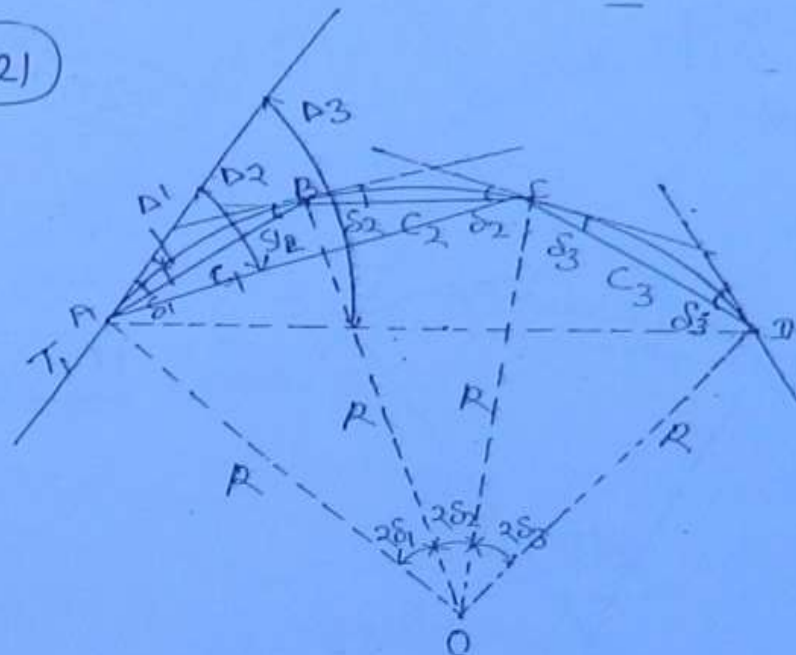
$$OX_3 = OX_4 = \dots = OX_{n-1}$$

$$OX_n = \frac{C_n(C+C_n)}{2R}$$

★ Rankine method :->

① One theodolite method :->

(12)



$$\frac{C_1}{2\pi R} = \frac{2\delta_1}{360^\circ}$$

$$\delta_1 = \frac{360 C_1}{4\pi R} \text{ degree}$$

$$\delta_1 = \frac{360 C_1 \times 60}{4\pi R} \text{ minute}$$

$$\delta_1 = 1718.9 \frac{C_1}{R} \text{ minute}$$

$$\delta_2 = 1718.9 \frac{C_2}{R} \text{ minute}$$

$$\delta_3 = 1718.9 \frac{C_3}{R} \text{ minute}$$

$$\delta_n = 1718.9 \frac{C_n}{R} \text{ minute}$$

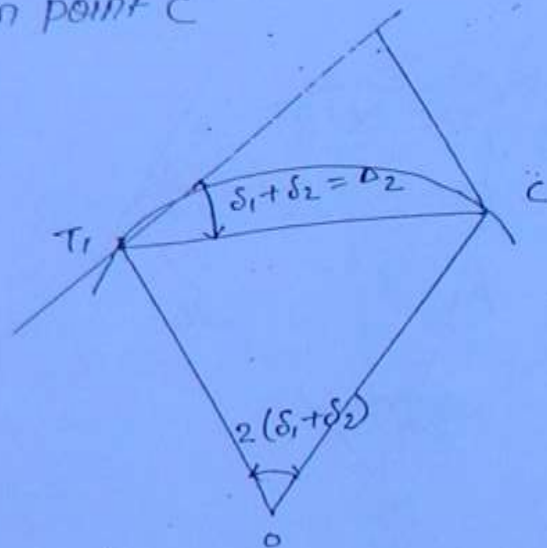
Total deflection angle: $\rightarrow$

for point B

$$\Delta_1 = \delta_1$$

from point C

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$$\Delta_2 = \delta_1 + \delta_2$$

$$\Delta_2 = \Delta_1 + \delta_2$$

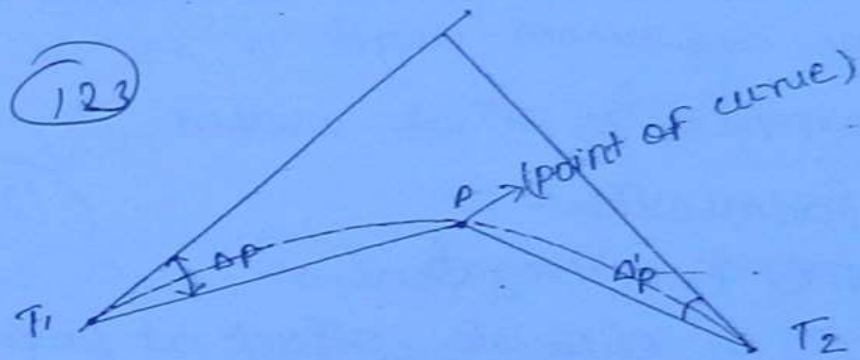
from point D

$$\Delta_3 = \delta_1 + \delta_2 + \delta_3$$

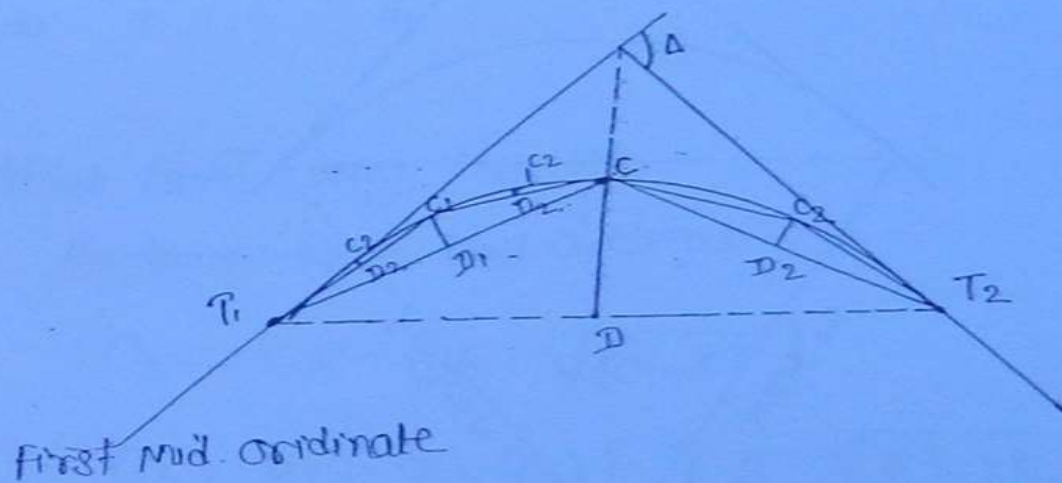
$$\Delta_3 = \Delta_2 + \delta_3$$

$$\Delta_n = \Delta_{n-1} + \delta_n$$

(b) Two Theodolite method: $\rightarrow$



★ By Bisection of chord :→



First mid. ordinate

$$CD = R(1 - \cos \Delta/2)$$

$$C_1D_1 = R(1 - \cos \Delta/4)$$

$$C_2D_2 = R(1 - \cos \Delta/8)$$

Problem:- For a circular curve of Radius 300m, calculate data required to set out a simple curve, if total deflection angle is 75° .

Use:- ① Perpendicular offset method.

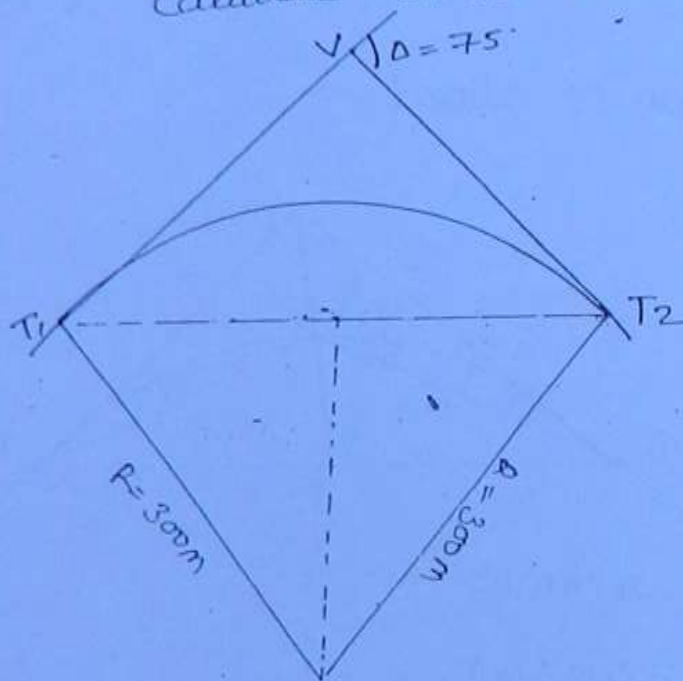
② Radial offset

③ Offset from long chord.

Calculate offset at every 20m distance

(124)

Solution:-



$$\begin{aligned} \Rightarrow VT_1 = VT_2 &= R \tan \frac{\Delta}{2} \\ &= 300 \times \tan \frac{75^\circ}{2} \\ &= 230.19 \text{ m} \end{aligned}$$

① Perpendicular offset

$$O_x = R - \sqrt{R^2 - x^2}$$

$$O_x = 300 - \sqrt{300^2 - x^2}$$

x	0	20	40	60	80m
Ox	0	0.67	2.68	6.06	

② Radial offset

$$O_x = \sqrt{R^2 - x^2} - R$$

$$O_x = \sqrt{300^2 - x^2} - 300$$

(125)

X	0	20	40	60	80 m
O _x	0	0.67 m	2.65 m	5.94 m	10.48 m
Using approximate formula	0	0.667 m	2.67	6.00	10.67

③ offset from long chord: →

Distance from mid ordinate = x

$$O_x = CD - (R - \sqrt{R^2 - x^2})$$

$$O_x = R(1 - \cos \theta_2) - (R - \sqrt{R^2 - x^2})$$

$$O_x = 61.99 - (300 - \sqrt{300^2 - x^2})$$

X	0	20	40	60	80	(upto 100)
O _x	61.99	61.32	59.01			

$$O_x = \sqrt{300^2 - x^2} - 238.01$$

Problem → ES - 1998 (55)

Two tangent intersect at chainage 126
50+60 (50 chain and 60 links)

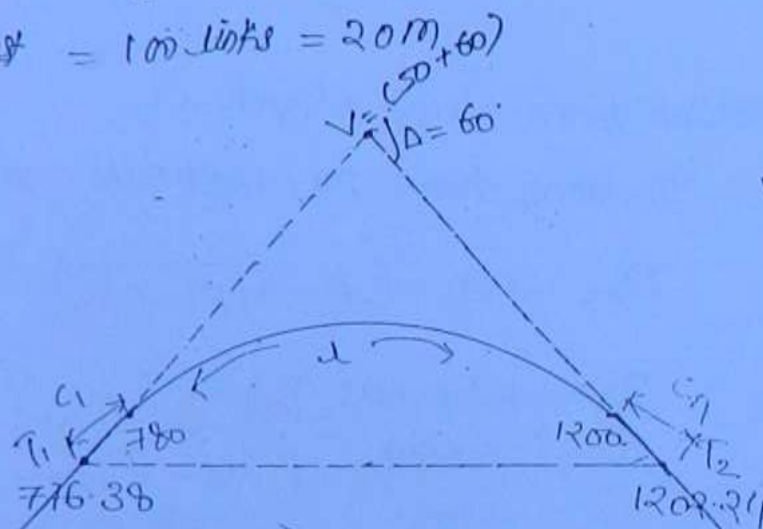
deflection angle being 61° , calculate the necessary data to set out a circular curve of 20 chain radius to connect the two tangents point by method of offset from chords. Take peg interval equal to 100 links with length of chain being 20m (100 links)

Solution → C = All chord length except

1st and last = 100 links = 20m

$$C_1 = ?$$

$$C_n = ?$$



Chainage of point of intersection

$$V = 50 + 60$$

$$V = 50 \text{ chains} + 60 \text{ chains}$$

$$V = 50 \times 20 + 60 \times 20$$

$$V = 1012 \text{ m} \quad R = 20 \text{ chains}$$

$$R = 20 \times 20 = 400 \text{ m}$$

Tangent length

$$VT_1 = VT_2 = R \tan \frac{\Delta}{2}$$

$$VT_1 = 400 \tan \frac{61^\circ}{2}$$

$$\Rightarrow VT_1 = 235.62 \text{ m}$$

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Length of curve

$$L = \frac{2\pi R}{360} \times \Delta$$

$$L = \frac{2\pi \times 400}{360} \times 61$$

$$L = 425.86 \text{ m}$$

change of T_1 = chainage of V - VT_1

$$T_1 = 1012 - 235.62$$

$$T_1 = 776.38 \text{ m}$$

chainage of T_2 :

$$T_2 = \text{chainage of } T_1 + L$$

$$T_2 = 776.38 + 425.86$$

$$T_2 = 1202.24 \text{ m}$$

Exact multiple of 20 m

after $T_1 = 780 \text{ m}$

$$\text{So first chord } C_1 = 780 - 776.38$$

$$C_1 = 3.62 \text{ m}$$

Exact multiple of 20 m

before $T_2 = 1200 \text{ m}$

$$\text{So last chord } C_n = 1202.24 - 1200 = 2.24 \text{ m}$$

All other chord length

$$C = 20 \text{ m}$$

offsets from chord:

$$Ox_1 = \frac{C_1^2}{2R} = \frac{3.62^2}{2 \times 400}$$

128

$$Ox_1 = 0.016 \text{ m}$$

$$Ox_2 = \frac{C(C_1 + C)}{2R} = \frac{20(3.62 + 20)}{2 \times 400}$$

$$Ox_2 = 0.59 \text{ m}$$

$$Ox_3 = Ox_4 = \dots = Ox_{n-1}$$

$$= \frac{C^2}{R} = \frac{20^2}{400} = 1 \text{ m}$$

$$Ox_n = \frac{C_n(C_n + C_{n-1})}{2R}$$

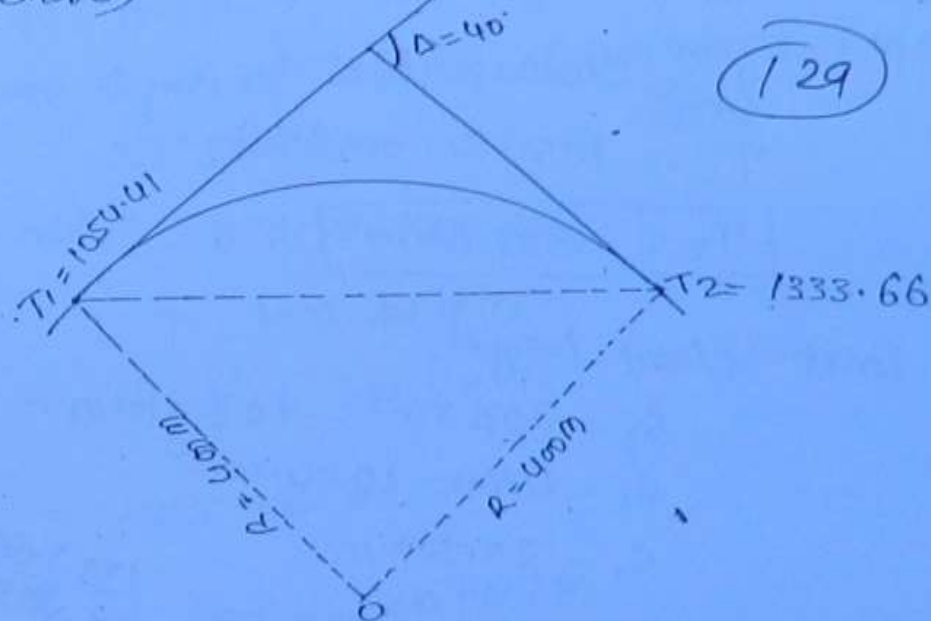
$$Ox_n = \frac{2.24(20 + 2.24)}{2 \times 400}$$

$$Ox_n = 0.062 \text{ m}$$

Problem: $\rightarrow$ 1990 6(b)

Two Tangents intersect at chainage 1200 m, deflection angle $= 40^\circ$, compute the data for setting out a 400 m radius curve by deflection angle and offset (chord). Take 30 m chord length in general case.

Solution: $\rightarrow$



Other chord length

$$C = 30 \text{ m}$$

$$C_1 = ? , C_n = ?$$

① Radius $R = 400 \text{ m}$

$$\Delta = 40^\circ$$

Tangent length

$$VT_1 = VT_2 = R \tan \Delta/2$$

$$= 400 \times \tan 40/2 = 145.59 \text{ m}$$

Length of curve

$$L = \frac{2\pi R}{360} \times \Delta = \frac{2 \times \pi \times 400}{360} \times 40$$

$$L = 279.25 \text{ m}$$

chainage of tangent point T_1

$$T_1 = \text{chainage } V - V T_1$$

$$T_1 = 1200 - 145.59$$

$$T_1 = 1054.41 \text{ m}$$

130

chainage of tangent point T_2

$$T_2 = \text{chainage } T_1 + L$$

$$T_2 = 1054.41 + 279.25$$

$$T_2 = 1333.66 \text{ m}$$

first chord length

$$C_1 = 30 \times 36 = 1080.00 \text{ m}$$

$$C_1 = 1080 - 1054.41$$

$$C_1 = 25.59 \text{ m}$$

$$C_n = 1333.66 - 1320 \rightarrow \frac{1333.66}{30} = 44.455$$

$$C_n = 13.66 \text{ m}$$

All other chord length $c = 30 \text{ m}$

$$S_1 = 1718.9 \frac{C_1}{R} = 1718.9 \times \frac{25.59}{400} = 1'49'58''$$

$$S_2 = S_3 = \dots = S_{n-1} = 1718.9 \times \frac{C}{R} = 1718.9 \times \frac{30}{400} = 2'08'55''$$

$$S_n = 1718.9 \frac{C_n}{R} = 1718.9 \times \frac{13.66}{400} = 0'58'42''$$

Points

S

A

T_1	1'49'58"	1'49'58"
1	2'08'55"	→ 3'58'53"
2	2'08'55"	
3	2'08'55"	6'7'48"
4	2'08'55"	
5	2'08'55"	

last 0'58'42" 26

RCCProblem $\Rightarrow$ solve last oneSolution $\Rightarrow$ loss of prestress = 20%

(13)

$$k = 1 - \frac{20}{100}$$

$$k = 0.80$$

Assume depth of slab

(considered 1 m width of slab)

$$D = 500 \text{ mm}$$

$$\Rightarrow \text{Self wt} = 0.50 \times 1 \times 1 \times 25$$

$$= 12.5 \text{ kN/m}$$

$$M_d = \frac{w_d l^2}{8} = \frac{12.5 \times 12^2}{8}$$

$$M_d = 225 \text{ kN-m}$$

$$\Rightarrow \text{Live load } w_l = 20 \text{ kN/m}$$

$$M_l = \frac{20 \times 12^2}{8}$$

$$M_l = 360 \text{ kN-m}$$

① Section modulus req.

$$Z = \frac{(1-k) M_d + M_l}{F_c}$$

$$Z = \frac{(1-0.80) \times 225 \times 10^6 + 360 \times 10^6}{19}$$

$$Z = 20928571.4 \text{ mm}^3$$

Depth required

$$D = \sqrt{\frac{6 \times 20928571.4}{B}} = 416.62$$

Provide 420 mm

$$\text{Self wt} = 0.43 \times 1 \times 1 \times 25$$

$$= 10.5 \text{ kN/m}$$

$$M_d = \frac{10.5 \times 12^2}{8}$$

$$M_d = 189 \text{ kN-m}$$

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② Prestressing force

$$P = \frac{A \cdot f_c}{2k} = \frac{1000 \times 420 \times 14}{2 \times 0.80}$$

$$P = 3675 \text{ kN}$$

$$\text{No. of Cable} = \frac{3675}{225} = 16.33 \approx 17 \text{ nos}$$

③ Eccentricity

$$e = \frac{(1+k) M_d + M_l}{2Pk}$$

$$e = \frac{(1+0.80) 189 \times 10^6 + 360 \times 10^6}{2 \times 0.80 \times 3675 \times 10^3}$$

$$e = 119 \text{ mm}$$

Problem: → A post tensimed prestressed concrete beam of rectangular section 240 mm wide, is to be designed for 25 kN/m live load u.d.l over an eff. span of 12 m. The stress in concrete is

17 MPa in compression

1.4 MPa in Tension

(133)

Considered loss of prestress = 15%.

(1) Calculated minimum possible depth of beam.

(2) Calculated p and e .

Solution: → Assume depth of beam.

$$D = 300 \text{ mm}$$

Self wgt.

$$w_d = 0.8 \times 0.24 \times 1 \times 25$$

$$w_d = 4.8 \text{ kN/m}$$

$$M_d = \frac{w_d l^2}{8}$$

$$M_d = \frac{4.8 \times 12^2}{8} = 86.4 \text{ kN-m}$$

$$M_l = \frac{w_l l^2}{8} = \frac{25 \times 12^2}{8}$$

$$M_l = 450 \text{ kN-m}$$

Section modulus required

$$Z = \frac{(1-k) M_d + M_l}{(k f_c - f_t)}$$

$$Z = \frac{(1-0.05) 86.4 + 450 \times 10^6}{k \cdot 0.85 \times 17 - (-1.4)}$$

$$Z = 29208052.8$$

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Depth required

$$D = \sqrt{\frac{62}{8}}$$

$$D = \sqrt{\frac{6 \times 292080032 \cdot 8}{8}}$$

134

$$D = 859.50 \text{ mm}$$

Considered $D = 900 \text{ mm}$

$$w_d = 0.24 \times 0.90 \times 25$$

$$w_d = 5.4$$

$$m_d = 5.4 \times \frac{12^2}{8}$$

$$m_d = 97.2$$

$$Z = \frac{0.15 \times 97.2 \times 10^6 + 450 \times 10^6}{(0.85 \times 17 + 1.4)}$$

$$Z = 29311041 \text{ mm}^3$$

Depth required

$$D = \sqrt{\frac{62}{8}}$$

$$D = 856.00 \text{ mm}$$

Take minimum possible depth = 860 mm

$$w_d = 0.24 \times 0.86 \times 25$$

$$w_d = 5.16$$

$$m_d = 5.16 \times \frac{12^2}{8}$$

$$m_d = 92.88 \text{ kN-m}$$

② Prestressing Force P/e

$$f_{sup} = \left(f_t - \frac{md}{z} \right)$$

$$f_{sup} = \left(-1.4 - \frac{92.80 \times 10^6 \times 6}{240 \times 860^2} \right)$$

$$f_{sup} = -4.56 \text{ N/mm}^2$$

(135)

$$f_{inf} = \frac{f_t}{k} + \frac{md + Md}{kz}$$

$$f_{inf} = \frac{-1.4}{0.85} + \frac{(92.80 + 450) \times 10^6 \times 6}{0.85 \times 240 \times 860^2}$$

$$f_{inf} = 19.94 \text{ N/mm}^2$$

prestressing force

$$P = \frac{A (f_{inf} + f_{sup})}{2}$$

$$P = \frac{240 \times 860 (-4.56 + 19.94)}{2}$$

$$P = 1587 \text{ kN}$$

eccentricity $\rightarrow$

$$e = \frac{z (f_{inf} - f_{sup})}{A (f_{sup} + f_{inf})}$$

$$e = \frac{240 \times 860^2}{6 \times 240 \times 860} \left(\frac{19.94 - (-4.56)}{19.94 + (-4.56)} \right)$$

$$e = 220 \text{ mm}$$

Ques 1) - b

Ques 2) a

Ques 3) a

Ques 4) d

Ques 5) c

Ques 6) c

Ques 7) b

Ques 8) c

Ques 9) b

Ques 10) a

Ques 11) c -

Ques 12) b d

Ques 13) c

Ques 14) d

Ques 15) a

Ques 16) b

Ques 17) b

Ques 18) a

Ques 19) b

Ques 20) c

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Ques 21) b

Ques 22) c

Ques 23) c

Ques 24)

Ques 25)

Ques 26)

Ques 27)

Ques 28)

Ques 29)

Ques 30)

Ques 31)

Ques 32)

Ques 33)

Ques 34)

Ques 35)

Ques 36)

Ques 37)

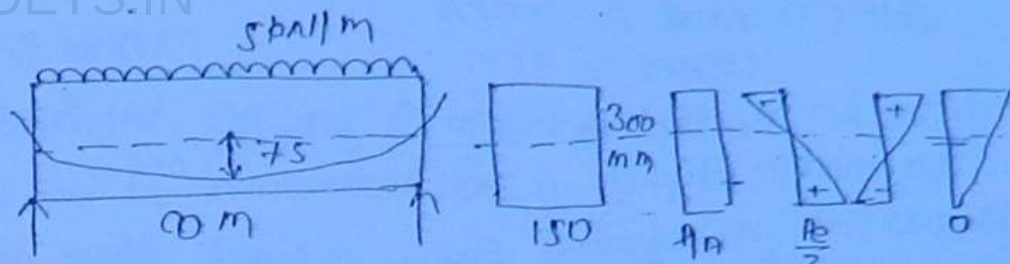
Ques 38)

Ques 39)

Ques 40)

Ques (4)

137



Dead load = $0.15 \times 0.3 \times 22$

Line load = $\frac{1.125 \text{ kN}}{5 \text{ kN}} = 6.125 \text{ kN}$

$M = \frac{wl^2}{8} = \frac{6.125 \times 8^2}{8} = 49 \text{ kN}$

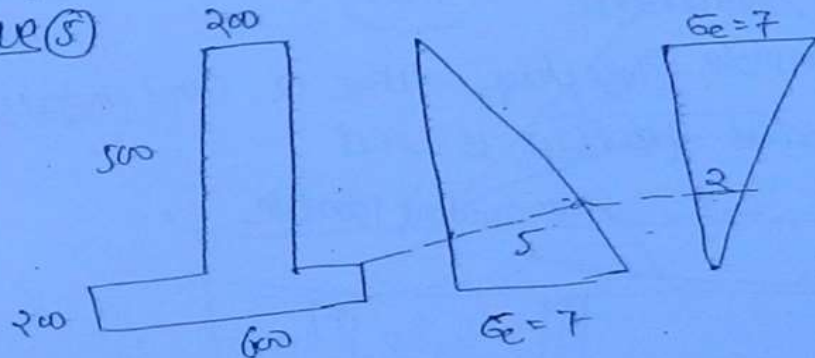
$\frac{P}{A} + \frac{Pe}{Z} - \frac{M}{Z} = 0$

$P \left(\frac{1}{A} + \frac{e}{Z} \right) = \frac{M}{Z} = P \left(\frac{Z+e}{AZ} \right) = \frac{M}{Z}$

$P = \frac{AM}{Z+ Ae} = \frac{150 \times 300 \times 49 \times 10^6}{150 \times 300^2 + 150 \times 300 \times 75}$

$P = 392 \text{ kN}$

Ques (5)



$P = 200 \times 500 \times \frac{f_t}{2} + 200 \times 600 \times \left(\frac{f_t}{2} \right)$

$P = 970 \text{ kN}$

$P = 200 \times 500 \times \left(\frac{f_t}{2} \right) + 200 \times 600 \times \frac{f_t}{2}$

$P = 570 \text{ kN}$

Ques (6)

40 mm

5 mm plus max size of aggregate

Ques (7)

$V_D = 0.67 BD \times \sqrt{f_{ct}^2 + 0.8 f_{cp} f_{ct}} + V_p$

$f_{ct} = 0.24 \sqrt{f_{ck}} = 0.24 \sqrt{45} = 1.61$

$f_{cp} = \frac{P}{BD} = \frac{200 \times 10^3}{150 \times 300} = 4.44$

$V_p = 200 \text{ mm} = 200 \times \frac{78 \times 7}{1000} = 78 \text{ kN}$

$V_D = 0.67 \times 150 \times 300 \sqrt{(1.61)^2 + 0.8 \times 4.44 \times 1.61} + 7.5$

$V_D = 94.40 \text{ kN}$

Ques 12) final $P = 500 \text{ kN}$

Losses = 15%

$$0.85 P_0 = P$$

$$P_0 = P / 0.85$$

$$\text{Initial } P_0 = \frac{500}{0.85} = 588.24 \text{ kN}$$

$$e = 75$$

$$\Rightarrow \frac{P}{A} + \frac{P e}{Z} \Rightarrow \frac{588.24 \times 10^3}{250 \times 300} + \frac{588.24 \times 10^3 \times 75}{250 \times 300^3}$$

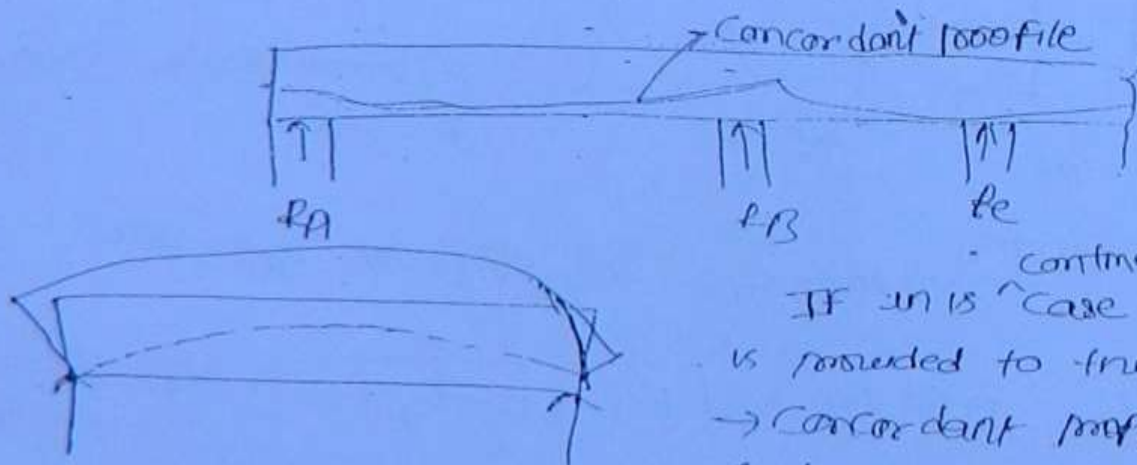
$$\Rightarrow 7.84 + 11.76$$

$$\sigma_{\text{top}} = -3.92$$

$$\sigma_{\text{bottom}} = 19.60 \text{ N/mm}^2$$

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Ques 13) (C) load balancing concept is applied only for determinate structure.
for indeterminate structure like a continuous beam, concordant profile is used



continuous beam

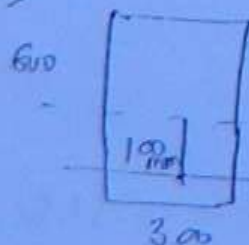
If in this case prestress cable is provided to this B.M.D.

→ Concordant profile is a particular shape of cable used

for an indeterminate structure like continuous beam such that

that support reaction does not change. This will reduce the calculation of BM at different stage

Ques 14) :-



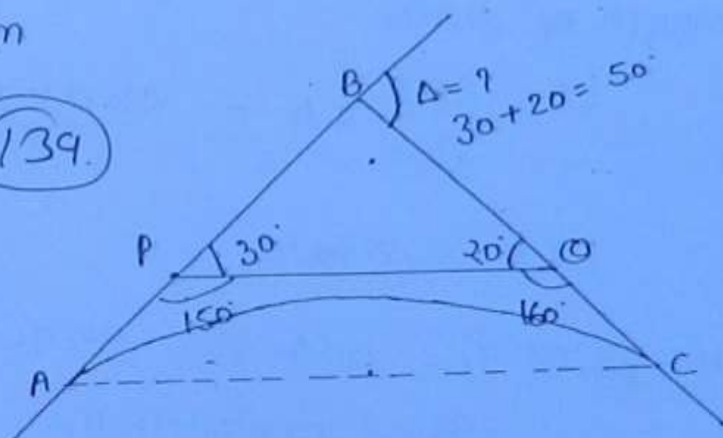
$$P = 1000 \text{ kN}$$

Ques: $\rightarrow$ 56- ES- 1991

Problem: $\rightarrow$ Two straight AB and BC meet in an inaccessible point B and to be connected by a simple curve of $R=600m$. Two points P and Q were selected on AB and BC such that $\angle APQ = 150^\circ$, $\angle CQP = 160^\circ$ then $PQ = 150m$. Make necessary calculations to set out curve by deflection angle method. Chainage of P = 1600m. Take chord length = 30m

Solution: $\rightarrow$

(139)



In Triangle BPQ

Apply Sin formula

$$\Rightarrow \frac{150}{\sin 130^\circ} = \frac{BP}{\sin 20^\circ}$$

$$\Rightarrow BP = \frac{\sin 20^\circ}{\sin 30^\circ} \times 150$$

$$\Rightarrow BP = 66.97m$$

$$\begin{aligned} \text{Chainage of B} &= 1600 + 66.97 \\ &= 1666.97m \end{aligned}$$

$$R = 600 \text{ m}$$

$$\Delta = 50^\circ$$

140

① Tangent length

$$VT_1 = R \tan \Delta/2 = 600 \times \tan \frac{50}{2}$$

$$VT_1 = 279.78 \text{ m}$$

length of curve

$$L = \frac{2\pi R}{360} \times \Delta = \frac{2 \times \pi \times 600}{360} \times 50$$

$$L = 523.60 \text{ m}$$

$$\text{chainage of } T_1 = 1666.97 - 279.78$$

$$T_1 = 1387.19 \text{ m}$$

$$\text{chainage of } T_2 = 1387.19 + 523.60$$

$$T_2 = 1910.79 \text{ m}$$

First chord length

$$C_1 = 1410 - 1387.19 \text{ m}$$

$$C_1 = 22.81 \text{ m}$$

$$C_n = 1910.79 - 1890$$

$$C_n = 20.79 \text{ m}$$

$1718.418 \frac{5}{2}$

Points	C	(141)	S ⁷	A
T ₁	0		0	0
1 →	22.81		1° 5' 21"	1° 5' 21"
2	30		1° 25' 57"	2° 31' 18"
3	30		1° 25' 257"	
4	30			
5	30			
6				
7				
8				
9				
~	20.79		0° 59' 34"	2° 5' 0"

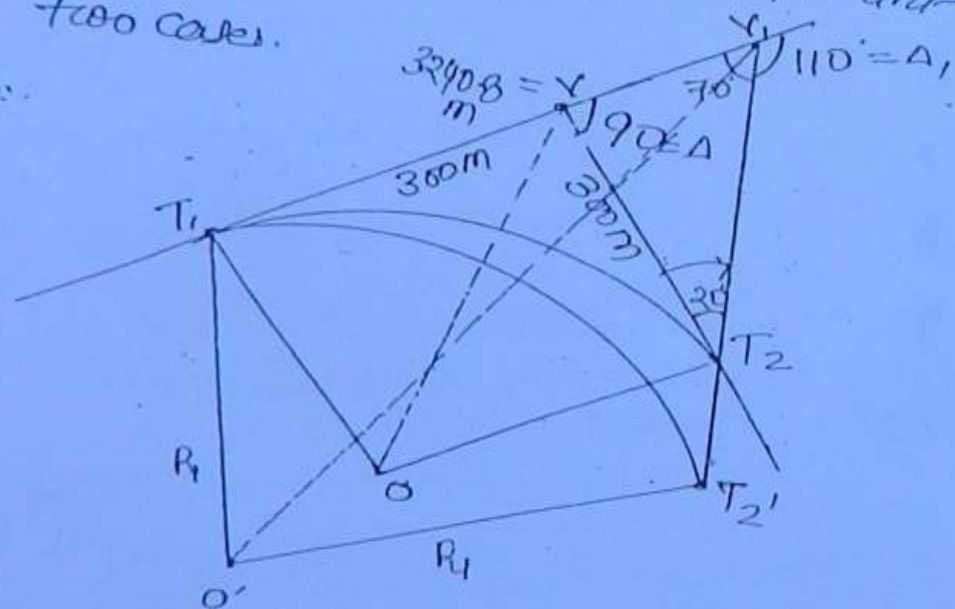
Problem:- Two tangents of a circular curve of 300m have a deflection angle 90° . If change the position of forward tangent it through 20° , such that deflection Δ is 110° (forward tangent it is rotated of tangency). Calculate the radius of

① Point of curve is not changed.

② Point of tangency is not changed.

If change of original P calculate change of important points two cases.

Solution:



$$R = 300m$$

$$\Delta = 90^\circ$$

Tangent length $VT_1 = VT_2 = 300 \tan \frac{90}{2}$
 $= 300m$

1 \

Case ① If point of curve is not changed.
 The new point of intersection is

In Triangle VV_1T_2

Apply sin

$$\Rightarrow \frac{VV_1}{VT_2} = \tan 20^\circ$$

(143)

$$\Rightarrow VV_1 = VT_2 \tan 20^\circ$$

$$\Rightarrow VV_1 = 300 \tan 20^\circ$$

$$\Rightarrow VV_1 = 109.19 \text{ m}$$

$$\frac{VT_2}{VT_2} = \frac{\cos}{\cos 20^\circ}$$

$$\Rightarrow V_1 T_2 = \frac{VT_2}{\cos 20^\circ}$$

$$\Rightarrow V_1 T_2 = \frac{300}{\cos 20^\circ}$$

$$\Rightarrow V_1 T_2 = 319.25 \text{ m}$$

New Tangent length

$$V_1 T_1 = VT_1 + VV_1 = 300 + 109.19$$

$$V_1 T_1 = 409.19 \text{ m}$$

$$V_1 T_1 = V_1 T_2$$

$$R_1 \tan \frac{\Delta_1}{2} = 409.19$$

$$\Rightarrow R_1 = \frac{409.19}{\tan \frac{110}{2}}$$

$$\Rightarrow \boxed{R_1 = 286.51 \text{ m}}$$

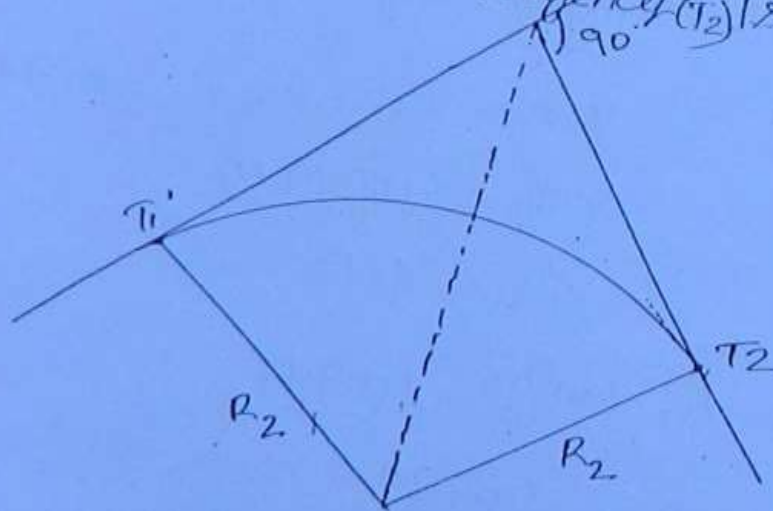
chainage of $T_1 = 3240.8 - 300$
 $= 2940.8 \text{ m}$

chainage of $V_1 = 3240.8 + 109.19$
 $= 3349.99 \text{ m}$

length of curve $\frac{2\pi R_1}{360} \times \Delta_1$
 $= \frac{2 \times \pi \times 286.51}{360} \times 110$
 $= 550.06 \text{ m}$

chainage of $T_2 = 2940.80 + 550.06 \text{ m}$
 $= 3490.86 \text{ m}$

Case (2) IF point of T_2 is not changed



V_1 = New point of intersection
 So new Tangent-length = $V_1 T_2$

$V_1 T_2 = 319.25$

$V_1 T_2 = V_1 T_1$

$$R_2 \tan \frac{\Delta_1}{2} = 319.25$$

$$R_2 = \frac{319.25}{\tan 110/2} \quad (145)$$

$$R_2 = 223.54 \text{ m}$$

$$\text{change of } T_1 = \text{change of } V_1 \theta - V_1 T_1$$

$$= 3349.99 - 319.25$$

$$= 3030.74 \text{ m}$$

$$\text{length of curve } L = \frac{2\pi R_2}{360} \times \Delta_1$$

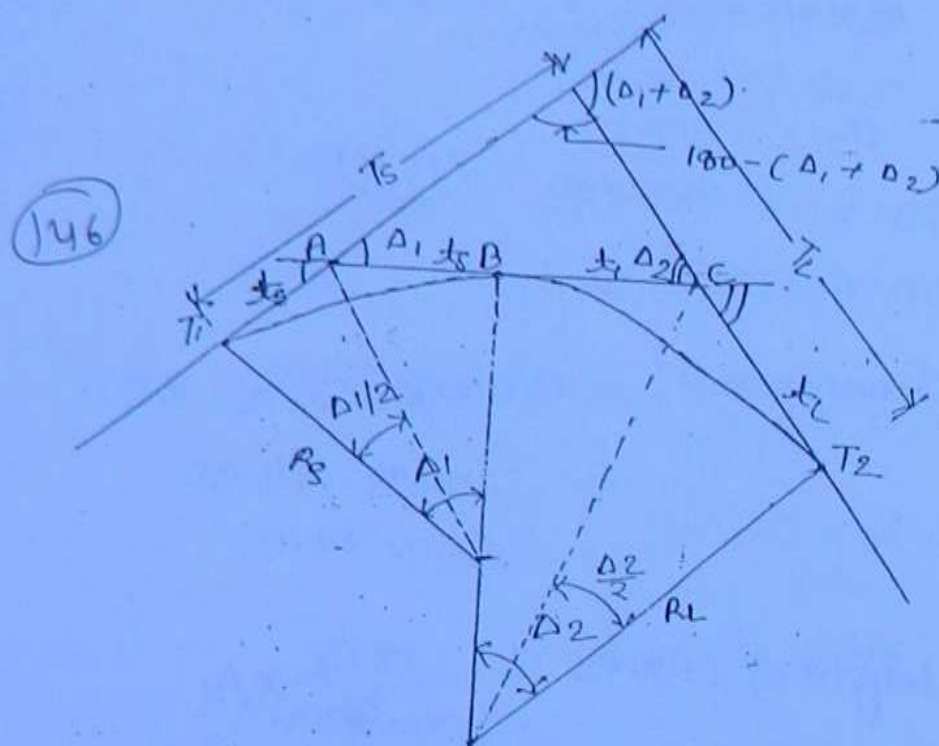
$$= \frac{2 \times \pi \times 223.54}{360} \times 110$$

$$= 429.16 \text{ m}$$

$$\text{change of } T_2 = 3030.74 + 429.16$$

$$= 3459.90 \text{ m}$$

* Compound Curve: $\rightarrow$



Components

① R_1 ② R_2 ③ t_s ④ t_c ⑤ T_1 ⑥ T_2 ⑦ Δ_1 ⑧ Δ_2 ⑨ $\Delta = \Delta_1 + \Delta_2$

Case ① Given values are

① Δ_1 ② Δ_2 ③ R_1 ④ R_2

Find out ① t_s ② t_c ③ t_s ④ T_1

Solution: $\rightarrow t_s = R_1 \tan \frac{\Delta_1}{2}$

$t_c = R_2 \tan \frac{\Delta_2}{2}$

Value of R_c = Common tangent = $t_s + t_c$

$= R_1 \tan \frac{\Delta_1}{2} + R_2 \tan \frac{\Delta_2}{2}$

In triangle VAC , Apply sine formula

$$\Rightarrow \frac{VA}{\sin \Delta_2} = \frac{VC}{\sin \Delta_1} = \frac{AC}{\sin (180 - \Delta)} = \frac{(t_s + t_L)}{\sin \Delta}$$

$$\Rightarrow VA = \frac{\sin \Delta_2}{\sin \Delta} (t_s + t_L) \quad (147)$$

$$\Rightarrow VC = \frac{\sin \Delta_1}{\sin \Delta} (t_s + t_L)$$

Total tangent length

$$\left. \begin{aligned} P_s &= t_s + (t_s + t_L) \frac{\sin \Delta_2}{\sin \Delta} \\ T_L &= t_L + (t_s + t_L) \frac{\sin \Delta_1}{\sin \Delta} \end{aligned} \right\} - (A)$$

Problem $\Rightarrow$ The following data refer to a compound curve

Total deflection angle = 93°

degree of first curve = 4°

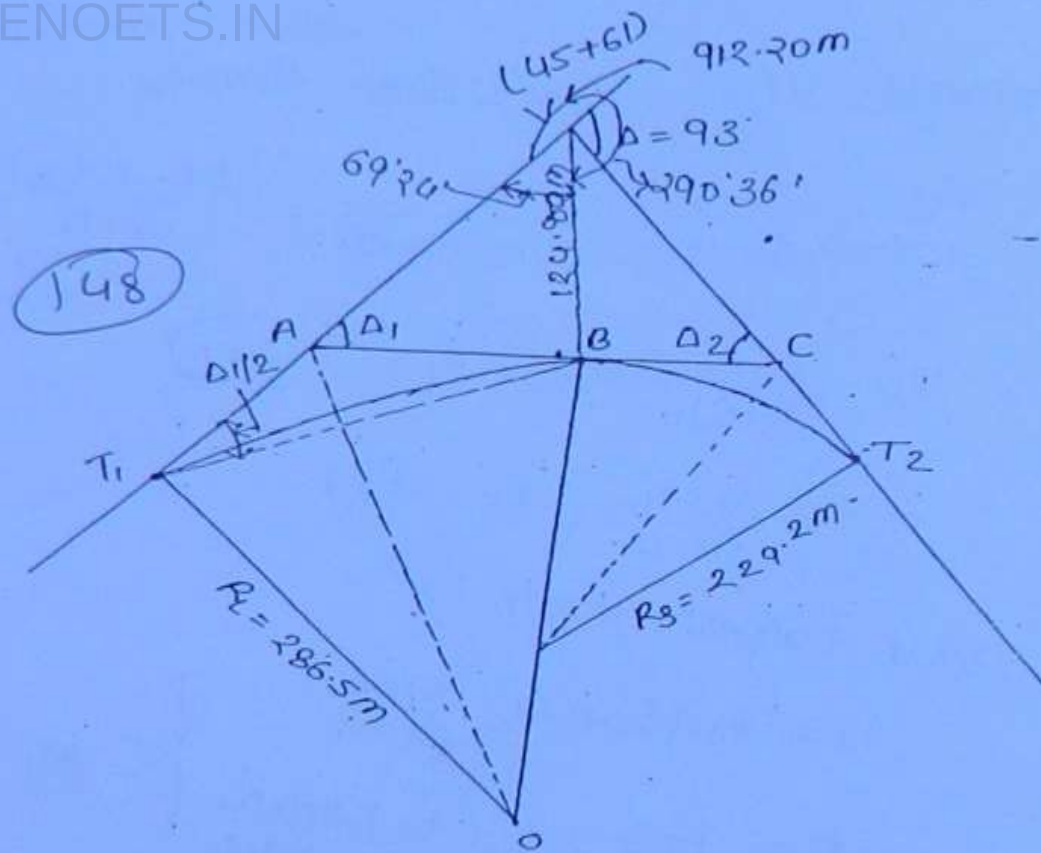
degree of second curve = 5°

point of intersection = $(45 + 61)$
(20 m chain used)

Determine the running distance to tangent point and point of compound curve, given that the latter point is $(6 + 24)$ from point of intersection at back angle of 90.36° from first tangent.

Solution $\Rightarrow R_1 = \frac{1146}{40} = 286.5 \text{ m} = R_1$

$R_2 = \frac{1146}{50} = 229.2 \text{ m} = R_2$



$$\begin{aligned} \text{chainage of P.L.} &= 45 \times 20 + 61 \times 0.20 \\ &= 912.20 \text{ m} \end{aligned}$$

$$\begin{aligned} V_B &= (6+24) \\ &= 6 \times 20 + 24 \times 0.20 \\ &= 124.80 \text{ m} \end{aligned}$$

$$\angle AVB = 360 - 290^\circ 36'$$

$$\angle AVB = 69^\circ 24'$$

$$AB = 286.50 \tan \frac{\Delta_1}{2}$$

Apply Sin Formula

$$\Rightarrow \frac{AB}{\sin 69^\circ 24'} = \frac{124.80}{\sin \Delta_1}$$

$$\Rightarrow \frac{286.50 \sin \Delta_1/2}{\sin 69^\circ 24' \cos \Delta_1/2} = \frac{124.80}{\sin \Delta_1}$$

(149)

$$\Rightarrow \frac{286.50 \sin \Delta_1/2}{\sin 69^\circ 24' \cos \Delta_1/2} = \frac{124.80}{2 \sin \frac{\Delta_1}{2} \cos \Delta_1/2}$$

$$\Rightarrow \sin^2 \Delta_1/2 = \frac{124.80}{2} \times \frac{\sin 69^\circ 24'}{286.50}$$

$$\Delta_1 = 53^\circ 46' 59.25''$$

$$\Delta_2 = 93^\circ - \Delta_1$$

$$\Delta_2 = 93^\circ - 53^\circ 41'$$

$$\Delta_2 = 39^\circ 19'$$

$$T_s = t_s + (t_s + t_L)$$

$$t_L = R_L \tan \frac{\Delta_1}{2}$$

$$\Rightarrow t_L = 286.50 \times \tan \frac{53^\circ 41'}{2}$$

$$\Rightarrow \boxed{t_L = 144.98 \text{ m}}$$

$$t_s = 229.20 \times \tan \frac{39^\circ 19'}{2}$$

$$\boxed{t_s = 81.88 \text{ m}}$$

$$\Rightarrow t_L + t_s = 226.86 \text{ m}$$

$$T_L = t_L + (t_s + t_L) \frac{\sin \Delta_2}{\sin \Delta}$$

$$T_L = 144.98 + 226.86 \times \frac{\sin 39^\circ 19'}{\sin 93^\circ}$$

$$T_L = 288.92 \text{ m}$$

$$T_S = 281.88 + 226.86 \times \frac{\sin 53'41''}{\sin 93'}$$

$$T_S = 264.92 \text{ m}$$

(150)

length of curve

$$\Delta \Rightarrow L_1 = \frac{2\pi R_L}{360} \times \Delta_1 = \frac{2\pi \times 286.50}{360} \times 53'41''$$

$$L_1 = 268.44 \text{ m}$$

$$\Rightarrow L_2 = \frac{2\pi R_S}{360} \times \Delta_2 = \frac{2\pi \times 229.20}{360} \times 39'19''$$

$$\Rightarrow L_2 = 157.28 \text{ m}$$

$$\text{chainage V} = 912.20$$

$$\Rightarrow -(T_L) = (-) 288.92$$

$$\text{chainage } T_1 = 623.28$$

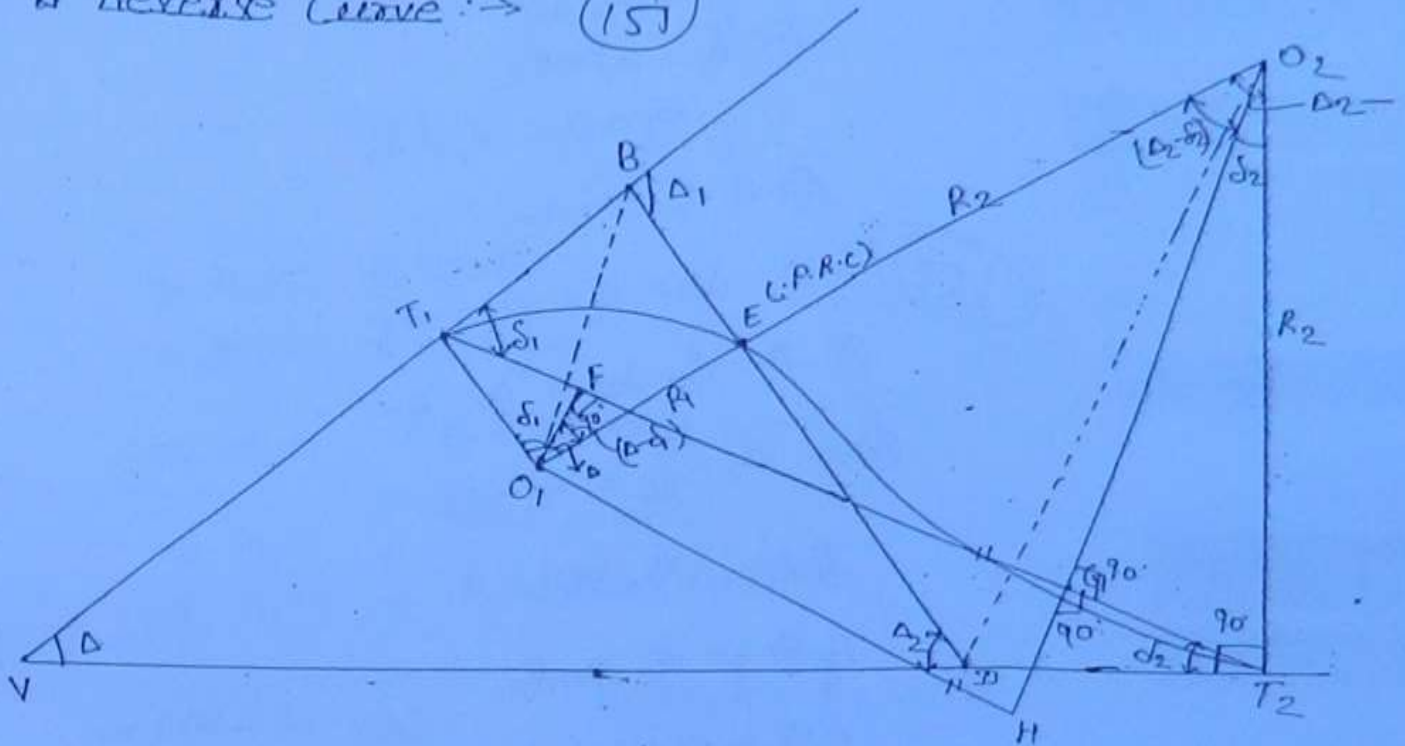
$$+ L_1 = + 268.44$$

$$\text{chainage (B)} \quad 891.72$$

$$+ L_2 = 157.28$$

$$\text{chainage (C)} \quad 1049 \text{ m}$$

A Reverse Curve $\rightarrow$ (15)



R_1 = first radius

R_2 = Second radius

E = point of Reverse curve

V = Point of Intersection.

Line O_1H is parallel to T_1T_2

$$O_1F \perp T_1T_2$$

$$O_2H \perp T_1T_2$$

$$\angle T_1O_1F = \delta_1$$

$$\angle T_2O_2G = \delta_2$$

$$\angle EOF = \Delta_1 - \delta_1$$

$$\angle EO_2F = \Delta_2 - \delta_2$$

($FG \parallel O_1H$)

Relation: $\rightarrow$

$$\textcircled{1} \delta_1 = \Delta + \delta_2$$

$$\text{so } \Delta = \delta_1 - \delta_2$$

$$\textcircled{2} \Delta_1 = \Delta + \Delta_2$$

$$\text{so } \Delta = \Delta_1 - \Delta_2$$

$$\textcircled{3} \Delta = \delta_1 - \delta_2 = \Delta_1 - \Delta_2$$

$$\textcircled{4} \Rightarrow \Delta_2 - \delta_2 = \Delta_1 - \delta_1$$

$$T_1 F = R_1 \sin \delta_1$$

$$T_2 G = R_2 \sin \delta_2$$

$$FG = O_1 H = O_1 O_2 \sin (\Delta_2 - \delta_2)$$

$$FG = (R_1 + R_2) \sin (\Delta_2 - \delta_2)$$

Total length

$$\Rightarrow T_1 T_2 = L = T_1 F + FG + T_2 G$$

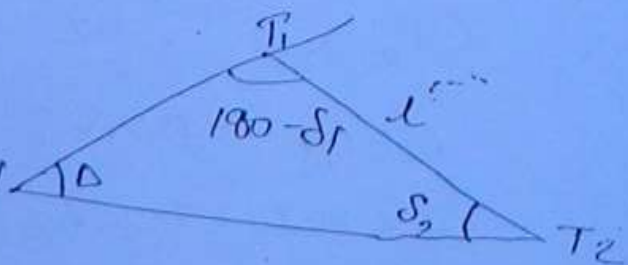
$$\Rightarrow T_1 T_2 = R_1 \sin \delta_1 + R_2 \sin \delta_2 + (R_1 + R_2) \sin (\Delta_1 - \delta_1)$$

$$\Rightarrow L = R_1 \sin \delta_1 + R_2 \sin \delta_2 + (R_1 + R_2) \sin (\Delta_1 - \delta_1)$$

Tangent length: $\rightarrow$

Apply sine formula

$$\Rightarrow \frac{VT_1}{\sin \delta_2} = \frac{VT_2}{\sin (180 - \delta_1)} = \frac{T_1 T_2}{\sin \Delta}$$



$$VT_1 = \frac{\sin \delta_2}{\sin A} \perp$$

$$VT_2 = \frac{\sin \delta_1}{\sin A} \perp$$

$$\Rightarrow O_1 F = R_1 \cos \delta_1$$

(153)

$$\Rightarrow O_2 G = R_2 \cos \delta_2$$

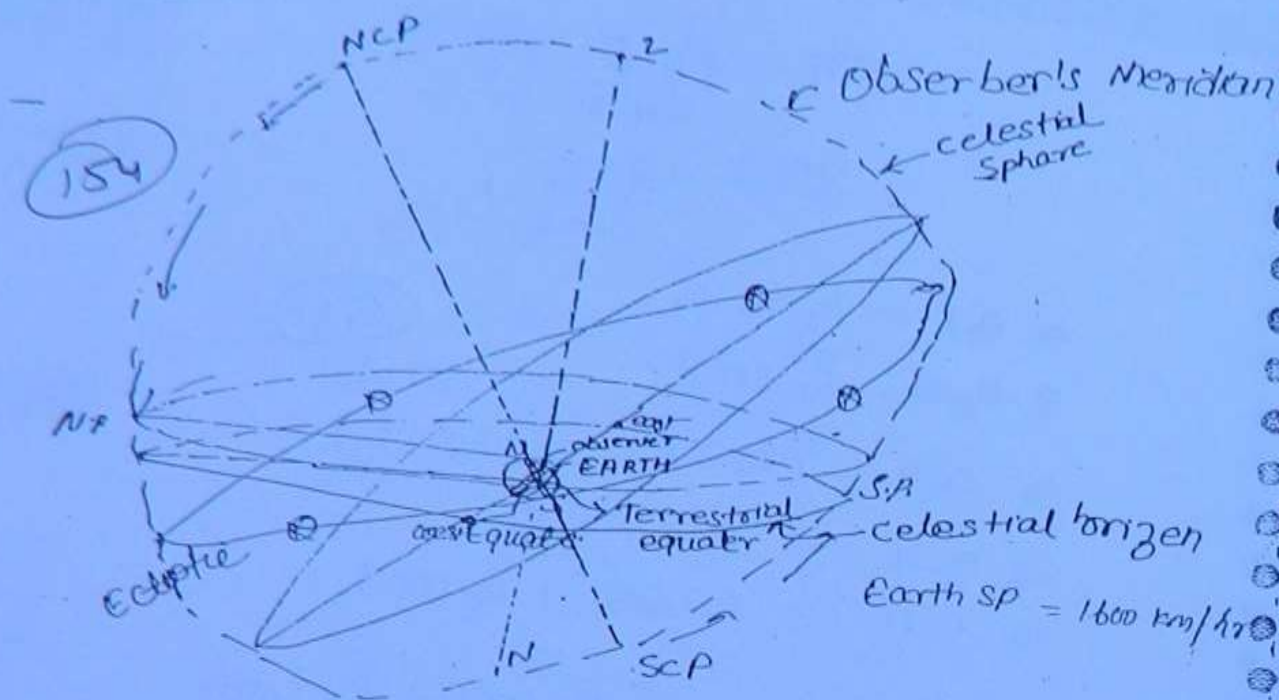
$$\Rightarrow O_2 H = (R_1 + R_2) \cos(\Delta_2 - \delta_2)$$

$$\Rightarrow O_2 H = O_1 F + O_2 G$$

$$\Rightarrow O_2 H = R_1 \cos \delta_1 + R_2 \cos \delta_2$$

$$\Rightarrow \cos(\Delta_2 - \delta_2) = \frac{R_1 \cos \delta_1 + R_2 \cos \delta_2}{(R_1 + R_2)}$$

$$\Rightarrow \boxed{\cos(\Delta_2 - \delta_2) = \cos(\Delta_1 - \delta_1)}$$



① The celestial sphere :-> The imaginary sphere on which all heavenly bodies like stars, sun, moon, planets etc. appear to us is called celestial sphere.

② Zenith :-> The point on celestial sphere exactly above the observer (opposite to centre of earth) is called zenith point.

③ Nadir point :-> Point on celestial sphere exactly below the observer position is called Nadir point.

④ zenith, nadir line :-> Line joining zenith and Nadir point.

⑤ Poles (on celestial sphere) :- If NP/SP of earth is extended to celestial sphere, we get two point called poles

① NCP - North Celestial pole.

② SCP - South Celestial Pole.

- (6) Celestial horizon: $\rightarrow$ The great circle (that is passing through centre as centre of earth) perpendicular to zenith Nadir line is called celestial horizon.
- (155)
- (7) Sensible horizon: $\rightarrow$ It is a circle having centre at observer's position. This circle is parallel perpendicular to zenith Nadir line. This circle is parallel to celestial horizon at R distance ($R = \text{radius of earth}$).
- (8) Terrestrial Equator: $\rightarrow$ Equator on earth surface.
- (9) Celestial Equator: $\rightarrow$ Equator extended on celestial sphere (great circle) is called Celestial Equator. It is perpendicular to North and South pole. It is not change.
- (10) Vertical circle: $\rightarrow$ All those great circles passing through zenith and nadir point are called vertical circle.
- (11) Observer's Meridian: $\rightarrow$ The great (vertical) circle, that is passing through North/South pole is called observer's meridian.
- (12) Prime Vertical: $\rightarrow$ The great circle (vertical) passing through zenith/nadir points and perpendicular to observer's meridian is called prime vertical.
- (13) North and South points: $\rightarrow$ Junction of observer's meridian with celestial horizon are North/South point.

(14) East / West point: → Junction of prime vertical with celestial horizon are east / west points.

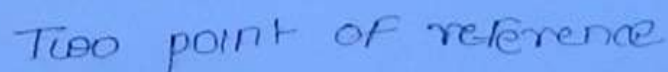
(15) Ecliptic: → The great circle of celestial sphere, on which the sun appears to lie in its movement in one year is called ecliptic w.r. to earth as centre.

(156)

(16) First point of Aries / First point of Libra: →

(Vernal Equinox)

(Autumnal Equinox)



- ① Pole.
- ② Equator.

Angle are measure with respect to

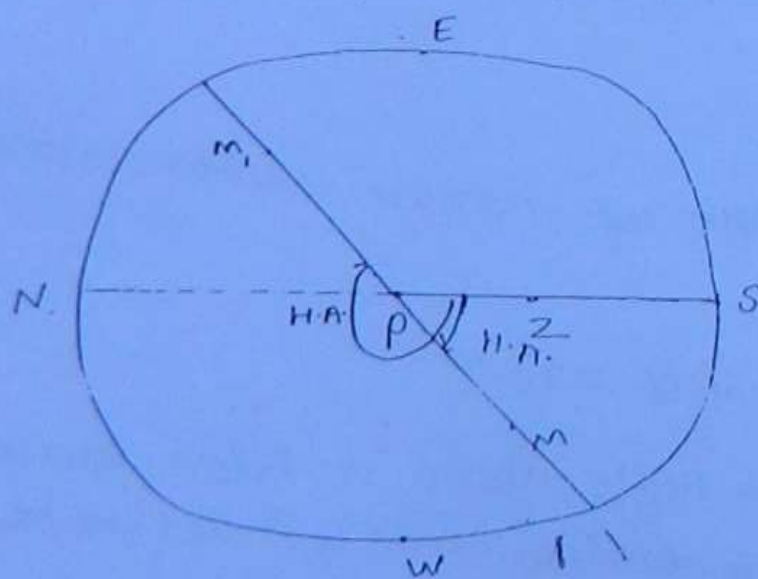
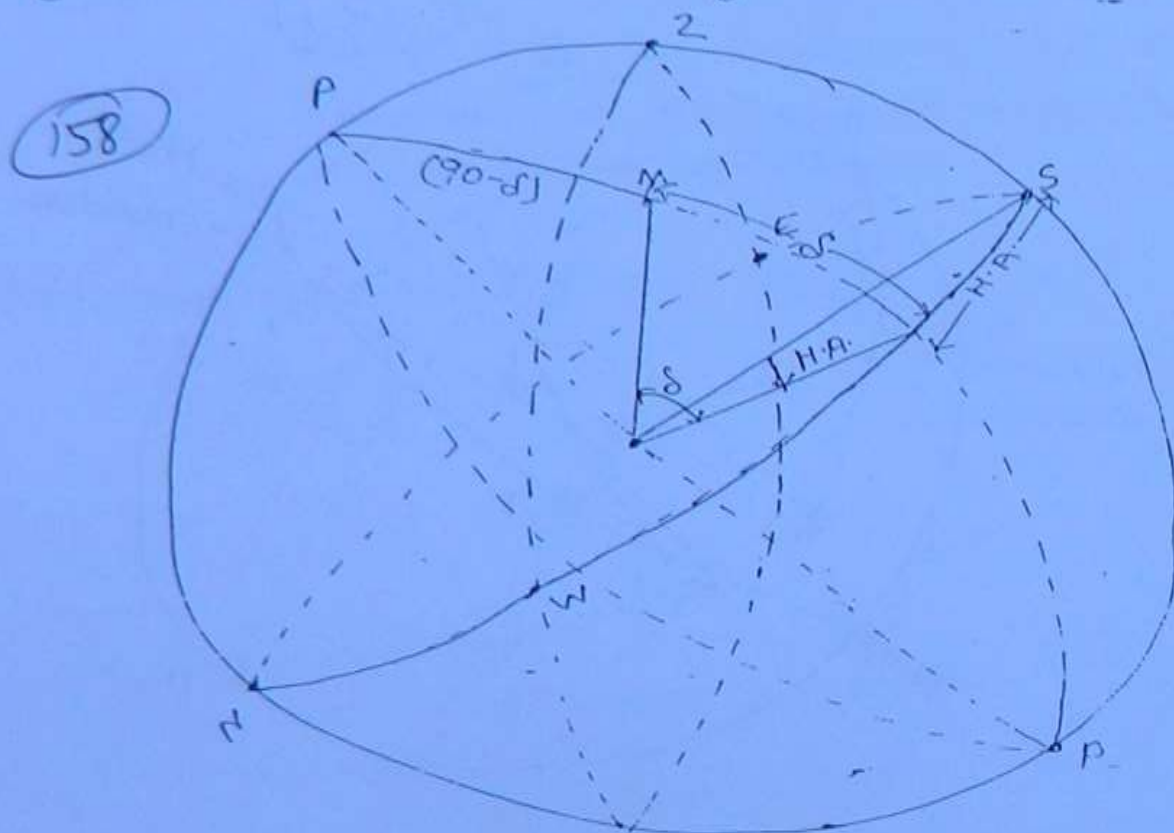
- ① Equator.
- ② First point of aries

① Declination: $\rightarrow$ Angle above or below equator, along declination circle is called declination.

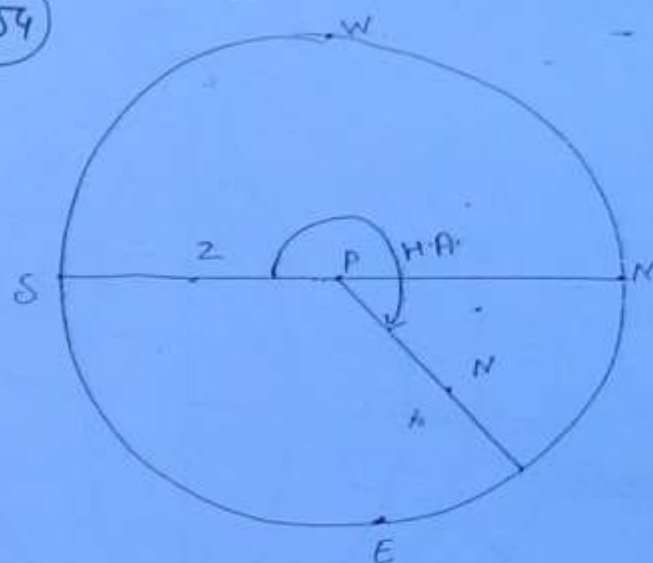
② Right Ascension: $\rightarrow$ Angle measured along equator, from the point of Aries towards east.

③ Co-declination Angle from pole to star = $90 - \delta$

④ Dependent equatorial system $\rightarrow$
 (Declination and hour angle)



(154)



Two point of reference:-

- (1) pole
- (2) South point

Two planes:-

- (1) Equator
- (2) Observer's meridian (point for South)

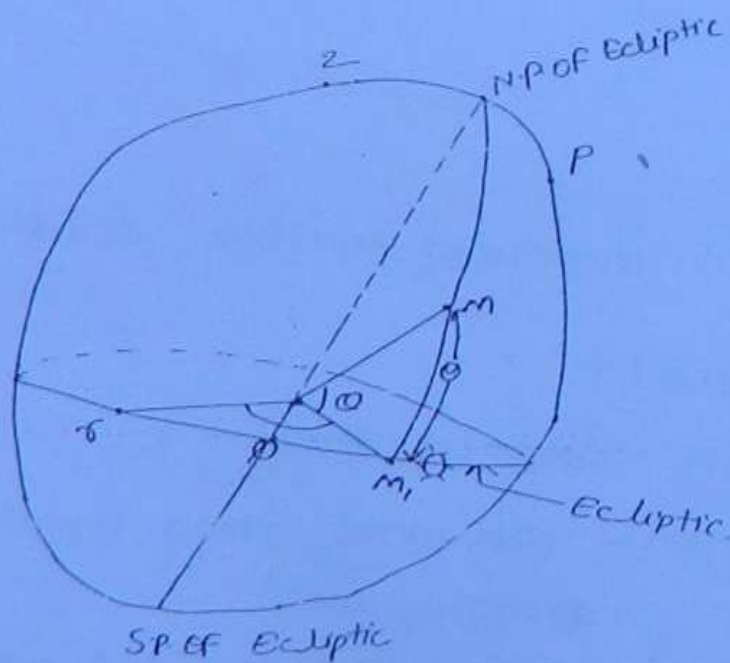
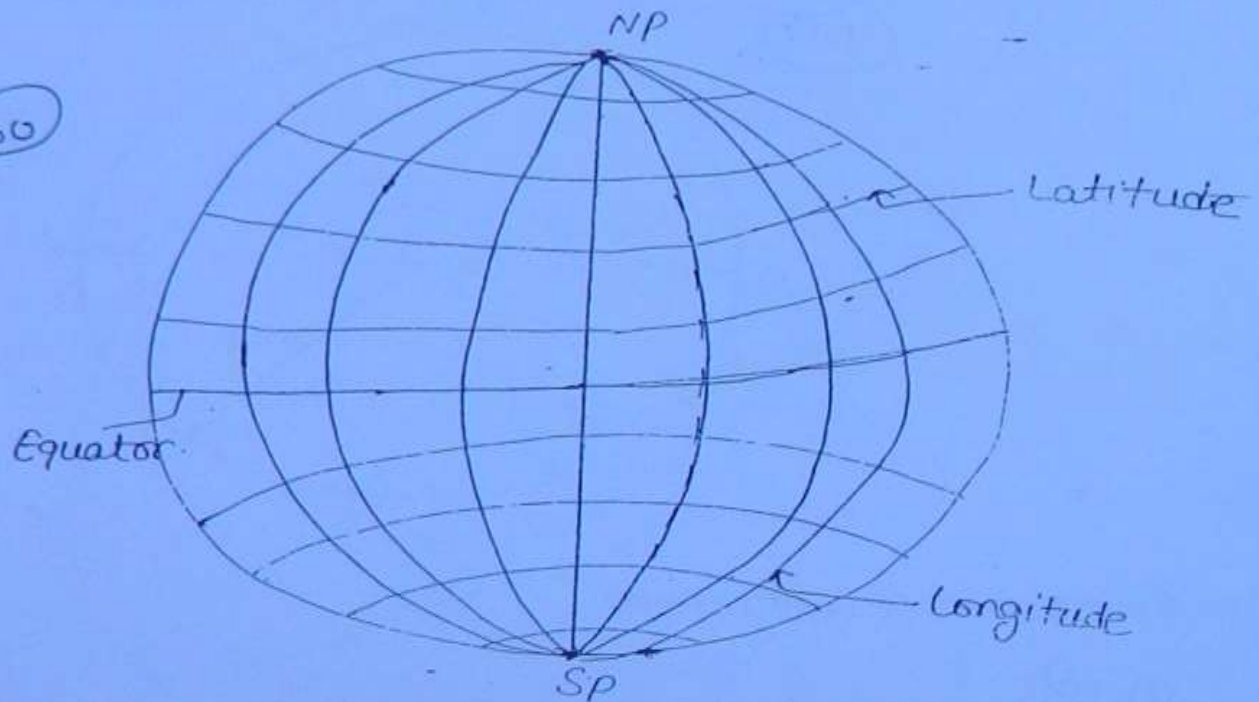
Angle measured:-

- (1) declination - same as above.
- (2) Hour Angle:- $\rightarrow$ Measured along equator from South going forward west.
- (3) Co-declination:- $\rightarrow (90^\circ - \delta)$ same as above

④ Celestial Latitude and Longitude System:

Terrestrial Latitude and Longitude

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Two point of reference.

(1) N.P. of ecliptic

(2) First point of Aries

Two planes:-

(61)

① Ecliptic

② plane, passing through first point of Aries.

Two angles:-

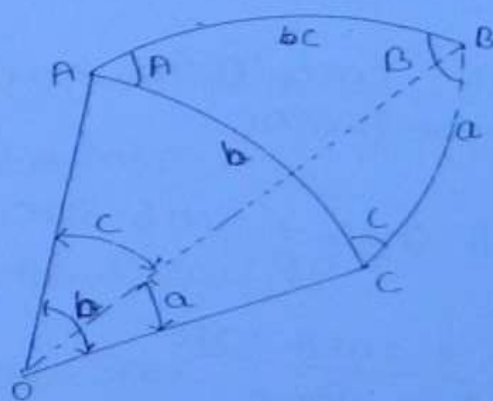
① Celestial Latitude:- Angle above or below Ecliptic

② Celestial Longitude $\rightarrow$ Angle measured along ecliptic

Starts from first point of Aries.

As per this system latitude of sun is always zero.

* Spherical Triangle $\rightarrow$



Angles $\rightarrow$ Angle b/w two planes

Sides $\rightarrow$ Angle b/w any two edge formed at centre of sphere.

Properties $\rightarrow$

① Any angle is less than $2 \times 90^\circ (\pi)$

$< 180^\circ$

② $180^\circ \leq A+B+C \leq 540^\circ$

③ Sum of any two side > third side

$$a + b > c$$

$$a + c > b$$

$$b + c > a$$

(162)

④ If Sum of any two side = 180° IF $a + b = 180°$
Sum of opposite two angle = 180°
 $A + B = 180°$

⑤ The smaller angle is opposite the smaller side

Formula :->

$$① \frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

$$② \cos A = \frac{b \cos a - \cos b \cdot \cos c}{\sin b \cdot \sin c}$$

$$\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$$

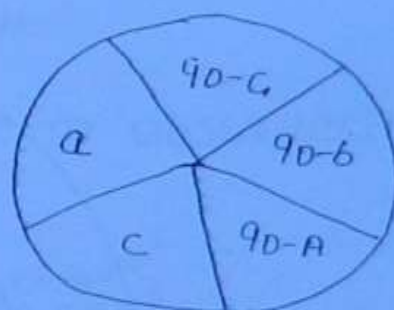
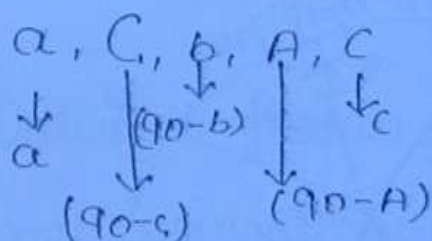
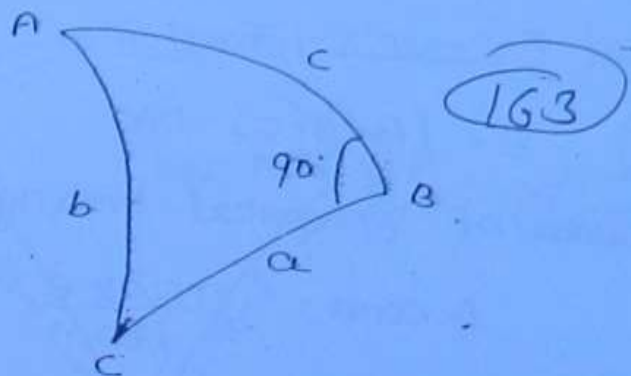
$$③ \cos a = \frac{\cos A - \cos B \cdot \cos C}{\sin B \cdot \sin C}$$

$$\cos A = -\cos B \cdot \cos C + \sin B \cdot \sin C \cdot \cos a$$

* Napier's Rule :-> This is for a right angle spherical triangle.

IF any one angle = 90° (A, B or C)
(not side)

L



$$\begin{aligned} \textcircled{1} \sin c &= \tan(90-A) \tan a \\ &= \cot A \tan a \end{aligned}$$

$$\begin{aligned} \textcircled{2} \sin c &= \cos(90-b) \cos(90-c) \\ &= \sec b \cdot \sec c \end{aligned}$$

$$\sin(\text{middle}) = \tan(\text{adjacent}_1) \times \tan(\text{adjacent}_2)$$

$$\sin(\text{middle}) = \cos(\text{opp}_1) \times \cos(\text{opp}_2)$$

$$\begin{aligned} \textcircled{3} \sin(90-b) &= \tan(90-c) \tan(90-A) \\ \cos b &= \cot c \cdot \cot A \end{aligned}$$

$$\begin{aligned} \textcircled{4} \sin(90-b) &= \cos a \cdot \cos c \\ \cos b &= \cos a \cdot \cos c \end{aligned}$$

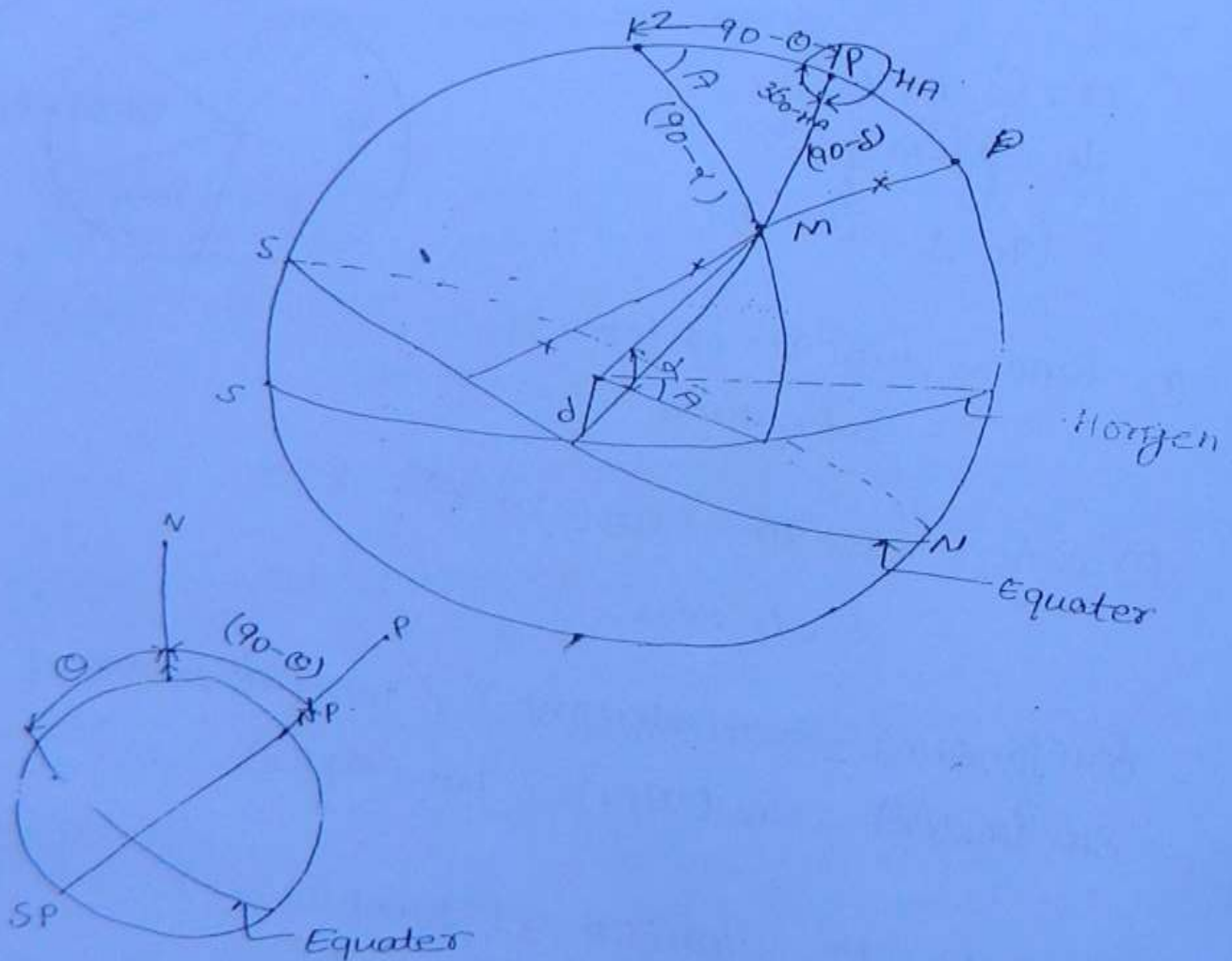
Total 10 such equation can be formed.

Spherical Excesses formed

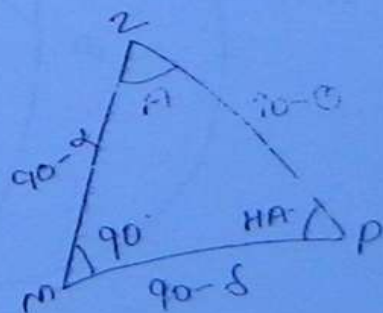
(164) $E = (A+B+C) - 180$

area of Spherical triangle

A area = $\frac{\pi R^2 \times E}{180}$

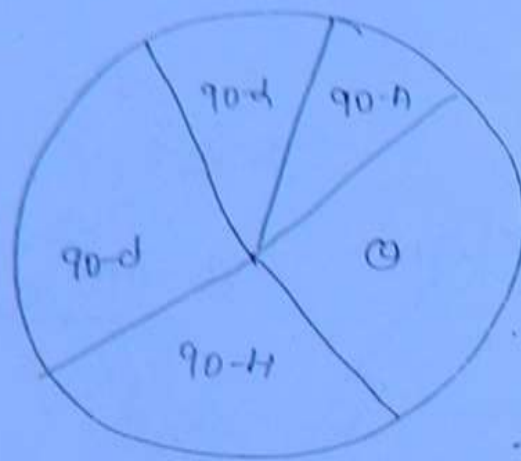


① Star at Elongation: $\rightarrow$



Angle - $(90-\alpha), A, (90-\beta), H, (90-\gamma)$
 $(90-\alpha) \quad (90-A) \quad C \quad (90-M) \quad (90-\delta)$

(166)



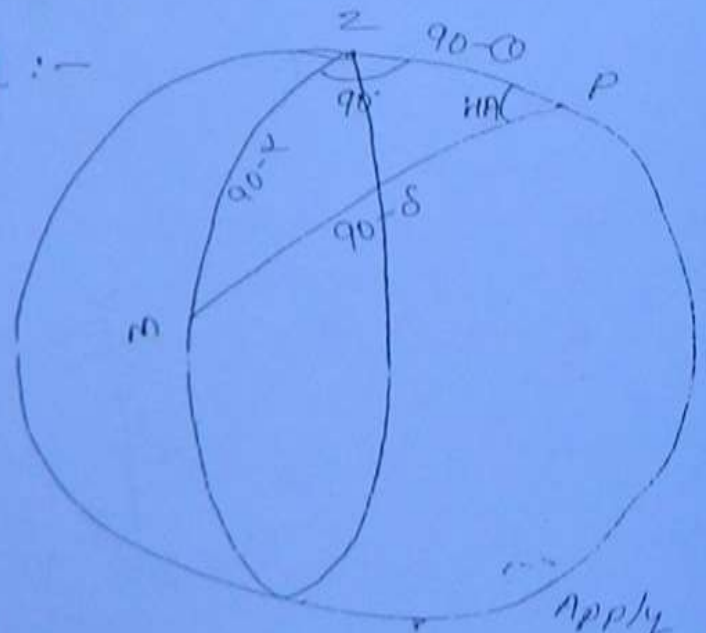
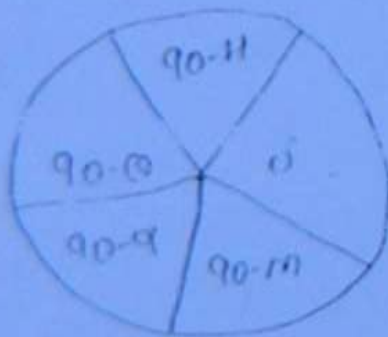
$$\sin(\text{middle}) = \tan(\delta_1) \times \tan(\alpha \delta_2)$$

$$\sin(\text{middle}) = \cos(\text{upp}_1) \times \cos(\text{upp}_2)$$

(2) Star of culmination:

m_1 and m_2
 $\downarrow$
 upper
 culmination
 $\downarrow$
 lower
 culmination

(3) Star at Prime Vertical:-



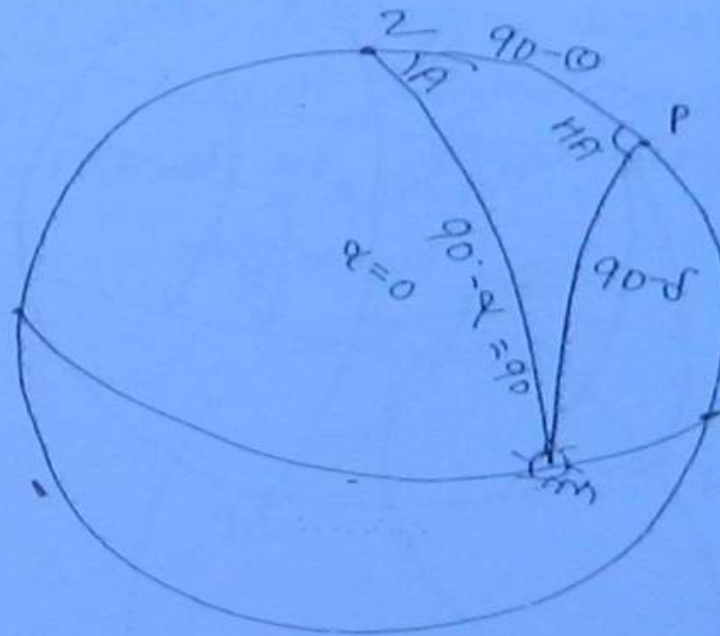
$90-\theta, \theta, 90-\phi, m, 90-\gamma$
 $\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$
 $(90-\theta) \quad (90-H) \quad \delta \quad (90-m) \quad (90-\gamma)$

Apply
Napier
rule

$$\sin(\text{middle}) = \frac{\tan(\alpha_1) \times \tan(\alpha_2)}{\cos(\phi_1) \times \cos(\phi_2)}$$

(167)

(4) Star at Horizon: →

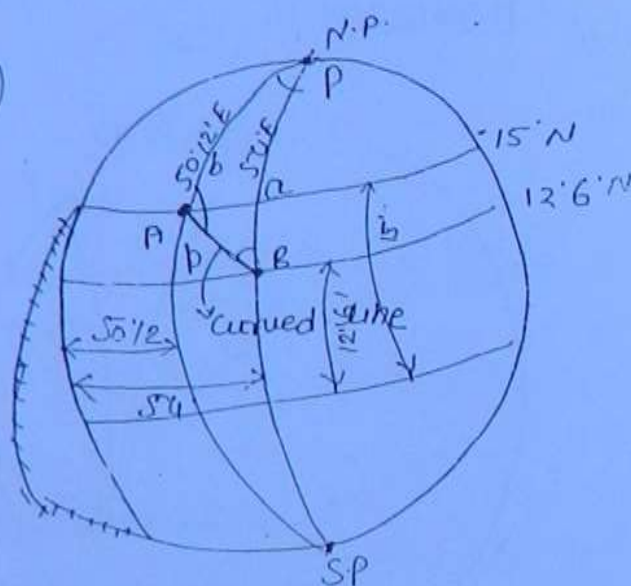


(5) Circumpolar Star :- Book -

① Find the shortest distance b/w two points A and B on earth's surface, given that latitude of A and B are $15^{\circ}N$ and $12^{\circ}6'N$, and their longitudes are $50^{\circ}12'E$ and $54^{\circ}E$. $R = 6370$ km

Solution: $\rightarrow$

(168)



$$a = 90^{\circ} - 12^{\circ}6' = 77^{\circ}54'$$

$$b = 90^{\circ} - 15^{\circ} = 75^{\circ}$$

$$P = 54^{\circ} - 50^{\circ}12'$$

$$P = \underline{3^{\circ}48'}$$

$$P = ?$$

$$\cos P = \frac{\cos a \cos b}{\sin a \sin b}$$

$$\cos P = \cos a \cos b + \sin a \sin b \cdot \cos P$$

$$\cos P = \cos 77^{\circ}54' \cos 75^{\circ} + \sin 77^{\circ}54' \sin 75^{\circ} \cos 3^{\circ}48'$$

$$\Rightarrow \cos P = 4^{\circ}41'46''$$

$$\text{Distance AB} = \frac{2\pi R}{360} \times p$$

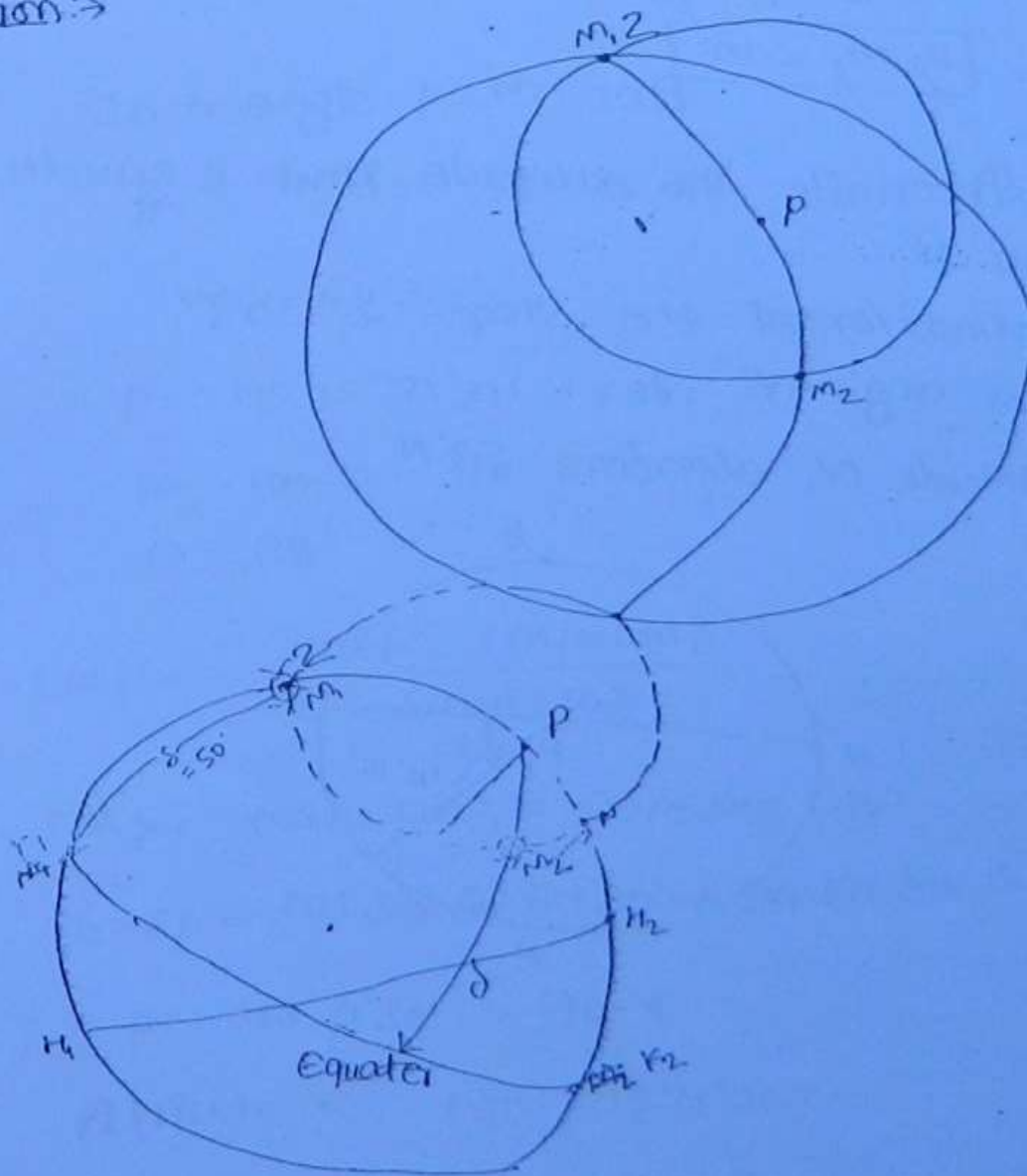
(169)

$$= \frac{2 \times \pi \times 6370}{360} \times 4^{\circ} 41' 46''$$

$$= \boxed{522.1 \text{ km}} \text{ Ans}$$

Problem $\rightarrow$ A star having a declination of $50^{\circ}N$ has its upper transit in the zenith of the place. Find the altitude of star at its lower transit.

Solution $\rightarrow$



Declination at upper transit

$$M_1 = 50 = 2H_1 = M_1$$

$$\text{So } 2p = 90 - 50 = 40'$$

at lower transit

$$PM_2 = 40'$$

(170)

Altitude of star at lower transit = $H_2 M_2 =$

$$2H_2 - 2p - PM_2$$

$$H_2 M_2 = 90 - 40 - 40$$

$$\boxed{H_2 M_2 = 10'} \quad \text{Ans}$$

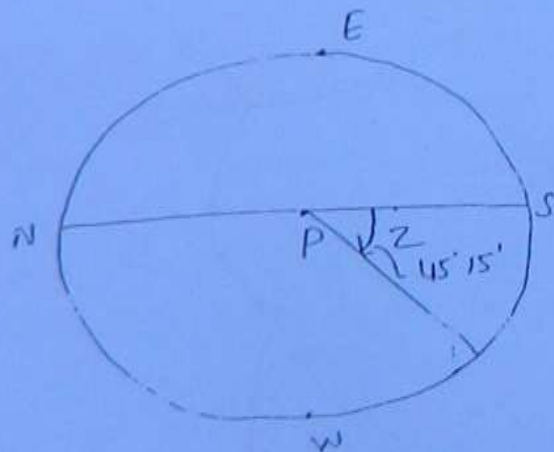
Problem:- Determine the altitude and azimuth of a star if

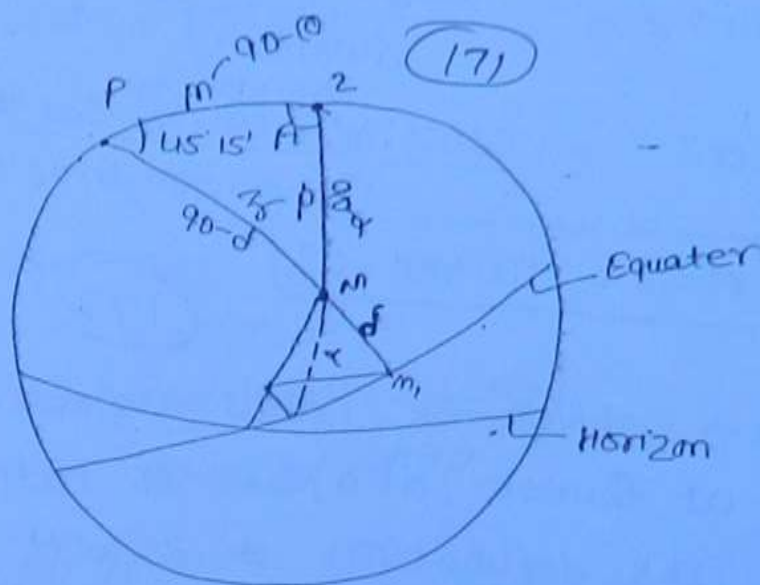
① Declination of the star = $25^\circ 30' N$

② Hour angle of star = $45^\circ 15'$

③ Latitude of observer = $52^\circ N$

Solution:-





In triangle PZM

$$Z = 90 - d = 90 - 35'30''$$

$$Z = 64'30''$$

$$p = 45'15''$$

$$m = 90 - 0 = 90 - 52'$$

$$m = 38'$$

$$\cos p = \frac{\cos p - \cos m \cos Z}{\sin m \sin Z}$$

$$\cos p = \cos m \cos Z + \sin m \sin Z \cos p$$

$$\cos p = \cos 38' \cos 64'30'' + \sin 38' \sin 64'30'' \cos 45'15''$$

$$p = 43'4'30'' = 90 - \alpha$$

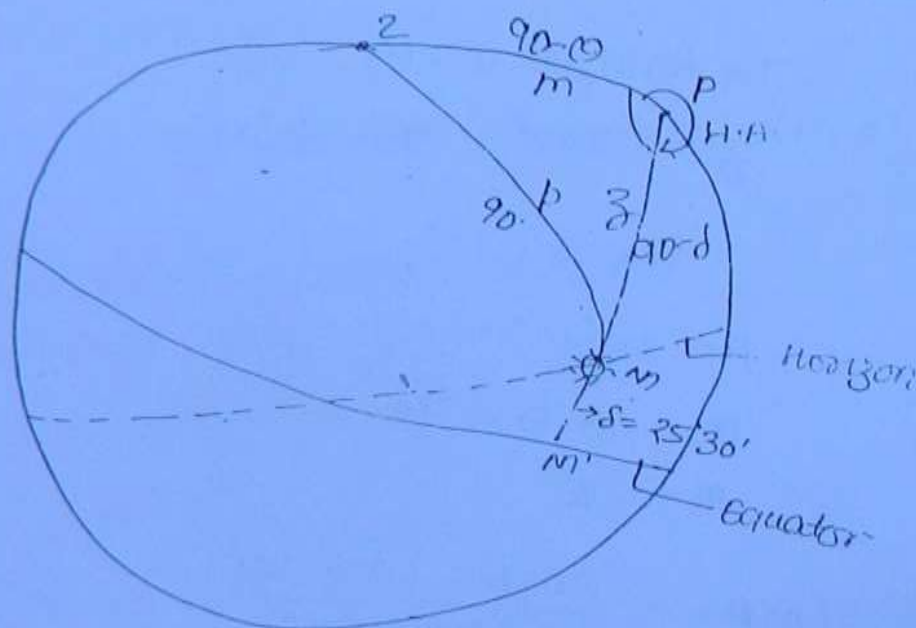
$$\text{Altitude } \alpha = 90 - 43'4'30''$$

$$\alpha = 46'55'30''$$

$$\cos 2 = \frac{\cos 64^{\circ} 30' - \cos 38^{\circ} \cos 43^{\circ} 4' 30''}{\sin 38^{\circ} \sin 43^{\circ} 4' 30''}$$

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Solution: $\rightarrow$



At Sunset

• $\lim_{n \rightarrow \infty} \delta u_n = 0$

$$\angle M = 90 - d = 90 - 0 = 90^\circ$$

Defination $M \sim N = 1.25 \times 30^\circ N$

$$PM = 90^\circ - \delta = 90^\circ - 35^\circ 30'$$

$$L = 64'30'' = d$$

WWW.GATEWAYSIN
① Latitude = $42^{\circ}30'$

$$Z_p = 90 - \phi = 90 - 42^{\circ}30'$$
$$= 47^{\circ}30' = m$$

$$\cos Z = \cos A = \frac{\cos \delta - \cos m \cdot \cos p}{\sin m \cdot \sin p}$$

$$A = \frac{\cos 64^{\circ}30' - \cos 47^{\circ}30' \cos 90}{\sin 47^{\circ}30' \sin 90}$$

$$A = 54^{\circ}16'24''$$

(173)

$$\cos p = \frac{\cos \delta - \cos m \cdot \cos Z}{\sin m \cdot \sin Z}$$

$$\cos p = \frac{\cos 90 - \cos 47^{\circ}30' \cos 64^{\circ}30'}{\sin 47^{\circ}30' \sin 64^{\circ}30'}$$

$$p = 115^{\circ}55'1''$$

$$H.A. = 360 - 115^{\circ}55'1''$$

$$\boxed{H.A. = 244^{\circ}4'59''} \text{ Ans}$$

Time

① Express following angle into time

$$95^{\circ} 30' 45'' \quad (174)$$

$$95^{\circ} = \frac{95}{15} = 5 \text{ hours } 40 \text{ min } 0 \text{ sec}$$

$$30' = \frac{30}{15} = 0 \text{ hour } 2 \text{ min } 0 \text{ sec}$$

$$45'' = \frac{45}{15} = 0 \text{ hr } 0 \text{ min } 3 \text{ sec}$$

$$5 \text{ hours } 42 \text{ min } 3 \text{ sec}$$

② 18 hr. 26 min 46 sec Convert into angle

$$18 \text{ hr} = 18 \times 15 = 270^{\circ} 0' 0''$$

$$26 \text{ min} = \frac{26 \times 15}{60} = 6^{\circ} 30' 0''$$

$$46 \text{ sec} = \frac{46 \times 15}{60} = 0^{\circ} 11' 30''$$

$$276^{\circ} 41' 30''$$

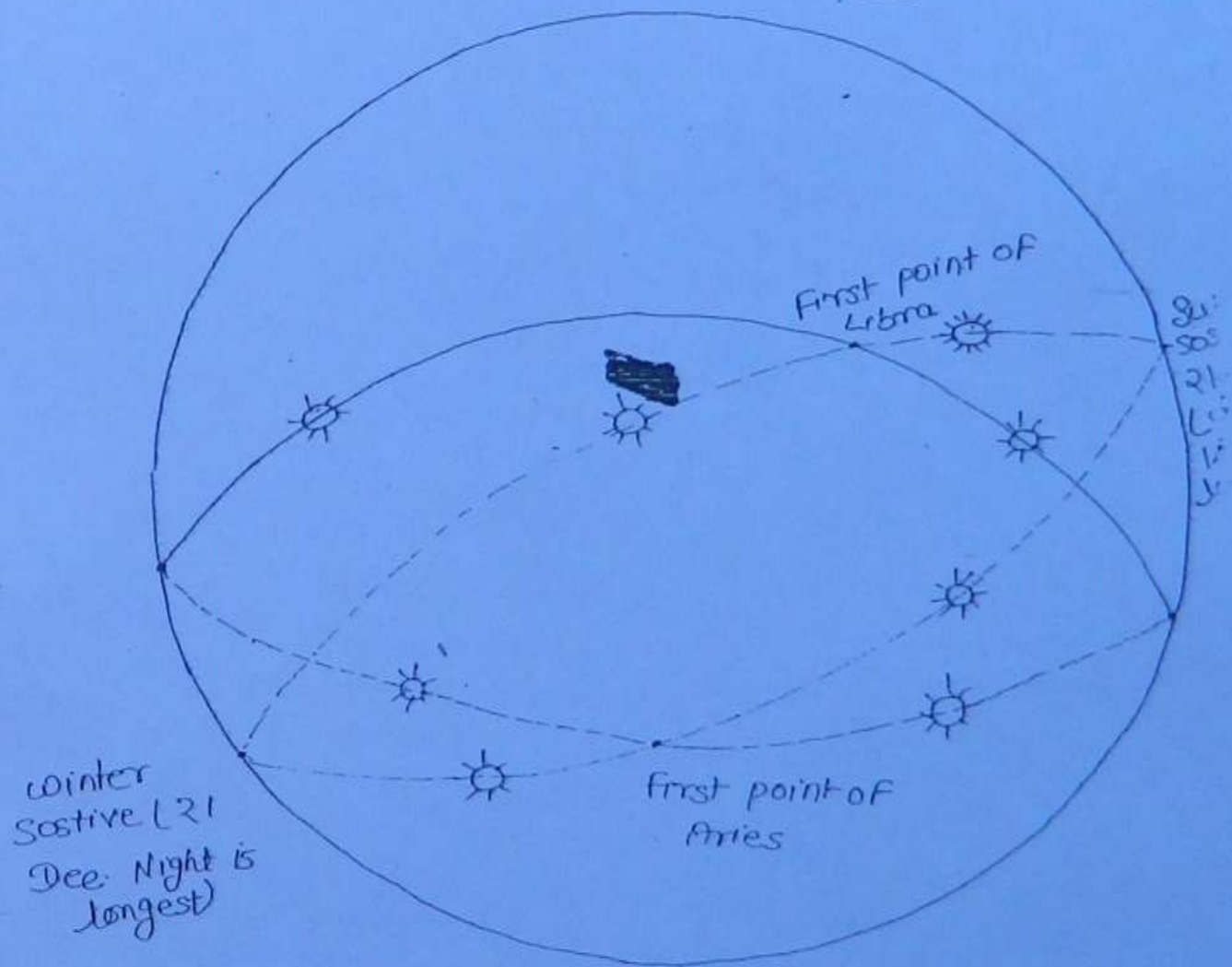
Day = Night = 21 September

Day = Night = 21 March

North east direction - Summer

South east direction - Rainy, Winter

(175)



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The end



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