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2

-: HAND WRITTEN NOTES:-
OF
CIVIL ENGINEERING

①

-: SUBJECT:-
STRUCTURAL ANALYSIS

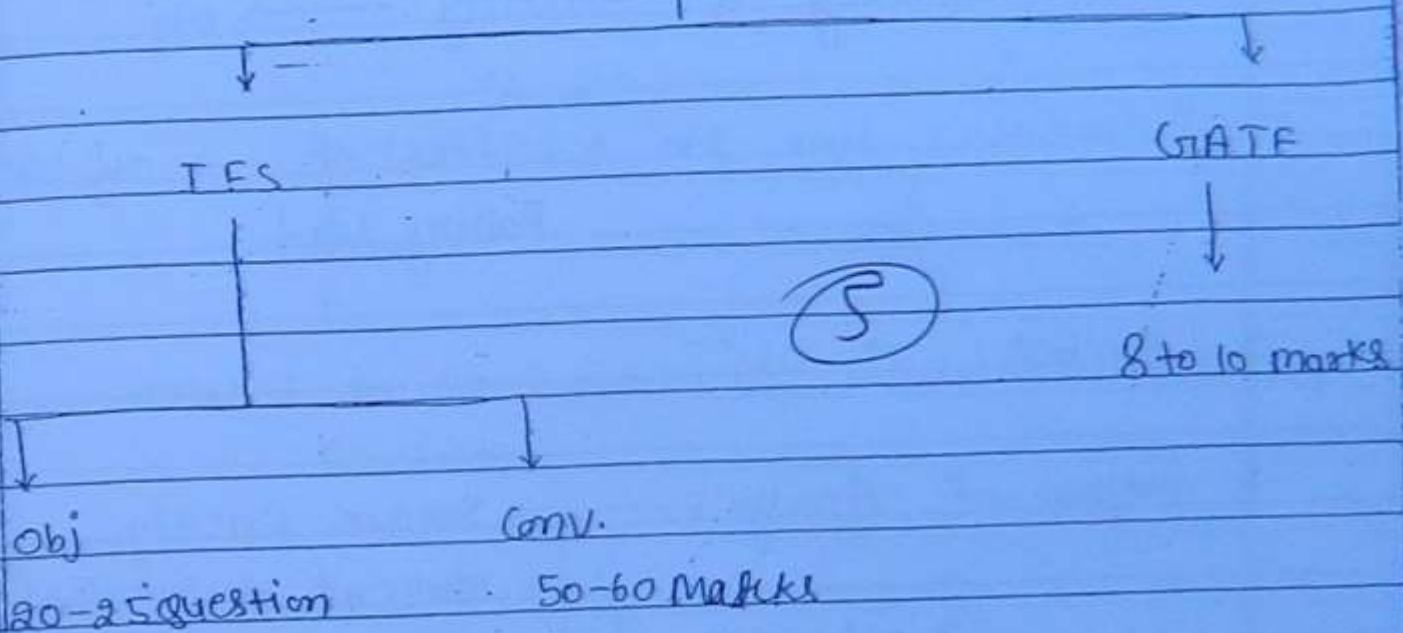
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School: _____

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		Theory of STRUCTURE		
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2	27-9-11	Influence line diag.	41-71	
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③

Structure Analysis



Nature of qns.

Easy to score

Type of subject

Conceptual

History of Teaching

50

②

Syllabus

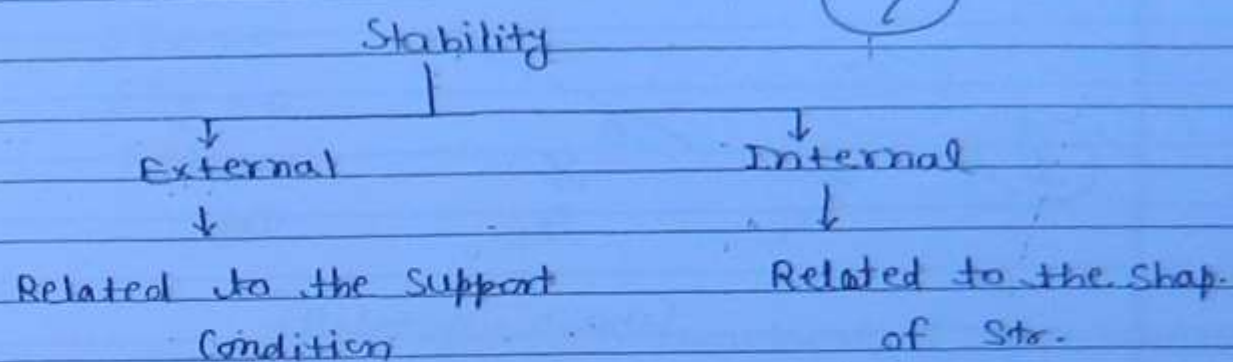
WWW.GATENOTES.IN

- Indeterminacy & Stability → obj
- Influence line Dia & Effect of Rolling load → obj + conv
- Arches → obj
- Method of Analysis
 - obj + conv.
 - Strain Energy
 - Moment dist method
 - Slope deflection Method
 - Matrix Method
- Trusses
 - Determinate
 - Indeterminateobj + conv.

Ref. Book

- ✓ ① Theory of Str. — by S. Ramamurtham
 - Strain Energy
 - Moment dist
 - Trusses
- ② Str. Analysis (L.S. Negi & Jandig)
 - Slope deflⁿ
 - Strain Energy
 - ILD
- ③ Str. Analysis (Gupta & Pandey)
 - Indeterminacy & stability
 - Matrix method
- ④ Structure Analysis → R.C. Hibbler → Basic concept 2- Truss

Indeterminacy & Stability of Structure



External Stability :-

→ Large displacement of the entire structure at the supports and joint is not permitted. However small elastic deformation may occur at free ends. To prevent rigid body displacement there should be enough reaction at the supports. At these arrangement should be adequate.

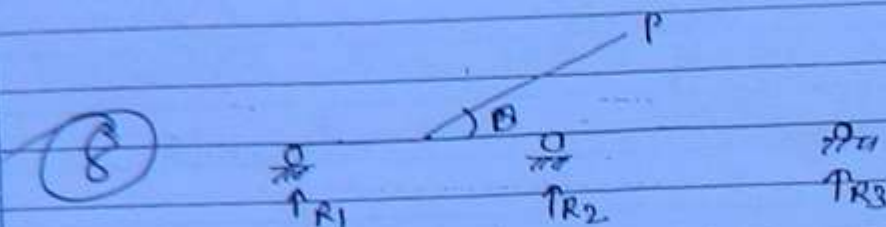
for plane structure (2-D), there should be min. of 3 independent reactions at the support which should be

- ① Non Parallel
- ② Non-Concurrent
- ③ Non Trivial

displacement if reaction is small which may permit large $\uparrow$ that it is called trivial

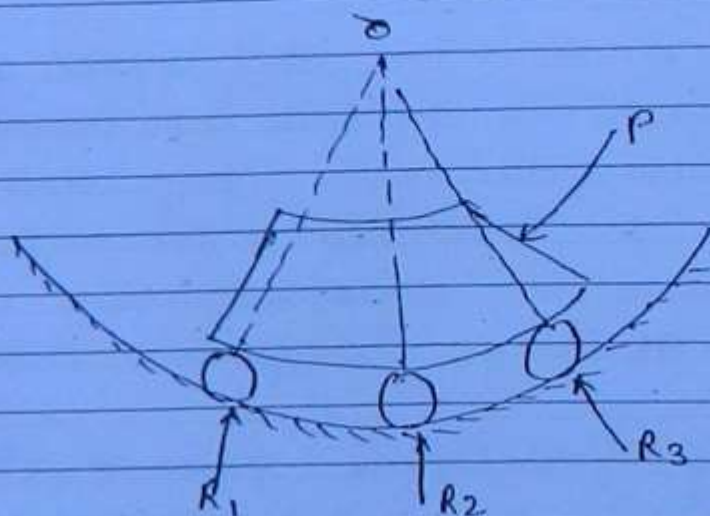
eg. A spring support with small stiffness

Note → All the support reaction are (1) than Linear instability will occur Reaction are Parallel.



Linear instability

→ If all reaction are concurrent. then angular instability will occur.



Angular / Rotational instability

→ For stability of 2D body (Plane is x-y is loading plane) there should be loading plane in

(1) x-direction ($\Delta x = 0$)

(2) y-direction ($\Delta y = 0$)

(3) about z-axis ($\Delta z = 0$)

To prevent above displacement following 3 condition of stable equilibrium should be satisfied.

① $\sum F_x = 0$

② $\sum F_y = 0$

③ $\sum M_z = 0$

⑨

For 3D structure (Space structure)

There should be no displacement. There should be no rigid displacement in following dirⁿ.

$\Delta x = 0$ $\Delta y = 0$ $\Delta z = 0$ $\Delta x = 0$ $\Delta y = 0$ $\Delta z = 0$

In order to prevent rigid body displacement for static equilibrium of space structure following 6-condition should be satisfied

$$\sum F_x = 0$$

$$\sum M_x = 0$$

$$\sum F_y = 0$$

$$\sum M_y = 0$$

$$\sum F_z = 0$$

$$\sum M_z = 0$$

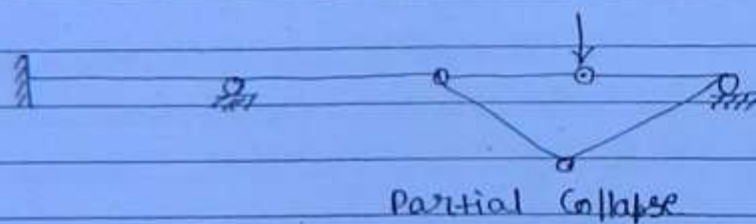
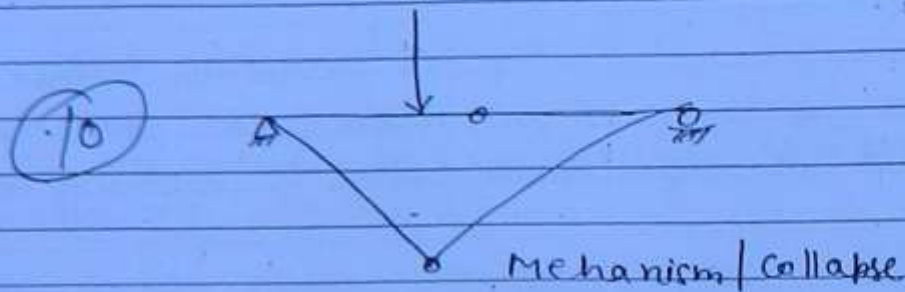
Internal Stability :-

No-part of the str. should move relative to the other part show as do ~~pressure~~ ^{other} shape of "Geometric" of structure, however, small elastic deformation may occur to preserve geometric enough no. of member & appropriate

arrangement is required. There should be no formation of condition of mechanism.

Mechanism means Rotation Collapse which may be complete str. or partial str.

Mechanism may occur 3- Coliner In a row hence Rotation is Permitted.

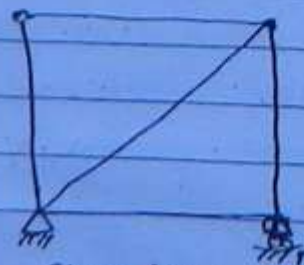


Internal Stability also called Geometric Stability.

In Truss Geometric Unstability occurs due to ~~defence~~ deficiency ~~may~~ off member and inadequate arrangement.



Unstable (Geometric)



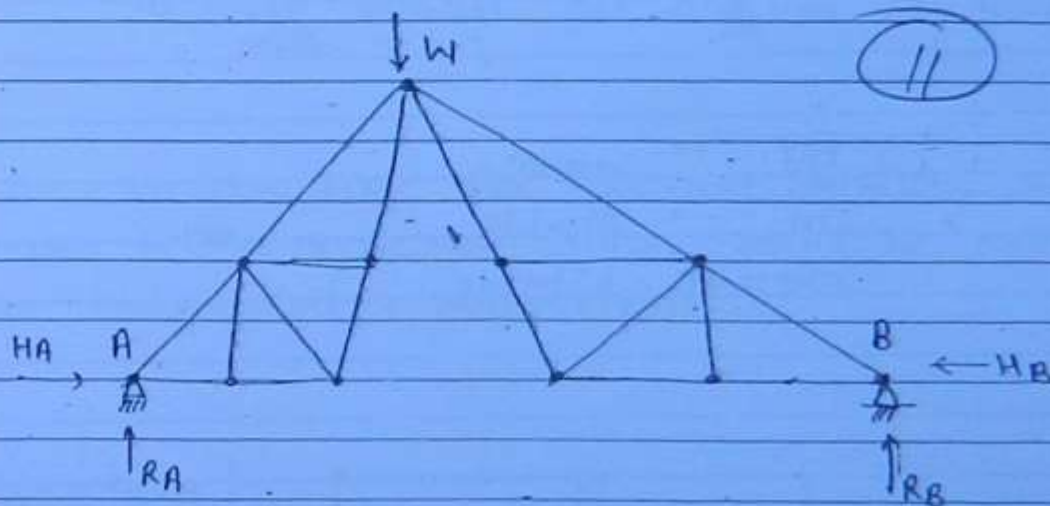
Stable (Internal)

over-all stability:-

For overall stability external

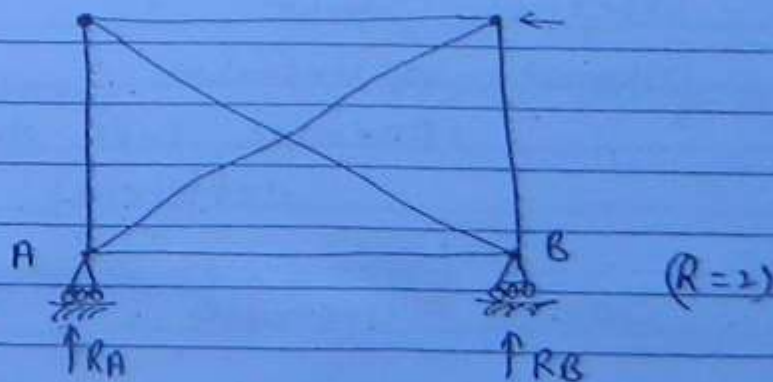
Stability is complex. In some cases overall stability may occur when structure is externally is over stiff and internally is unstable

that is extra external reaction may prevent internal deformation.



No. of External Reactions = 4

internally it is deficient to 1st order } but overall
& externally it is over stiff to 1st order } it stable

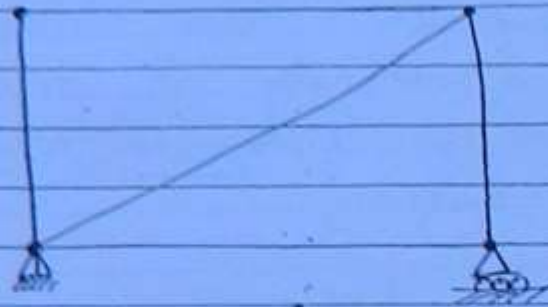


Externally deficient to 1st order but internally over stiff to 1st order

but over all is unstable

→ It is desirable to have internal and external stability both for overall stability.

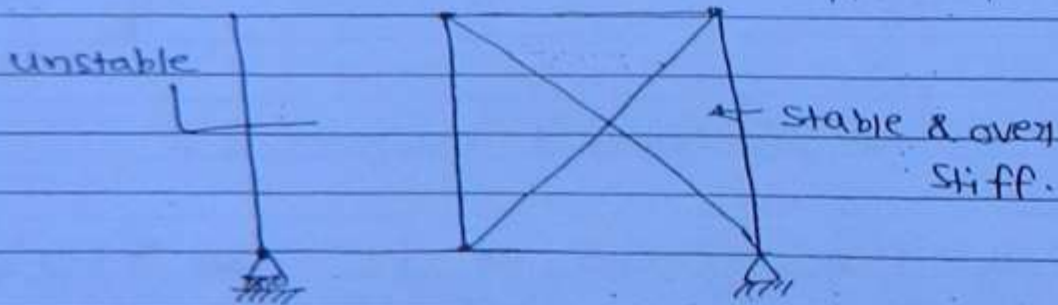
(12)



Ext. → stable

Int. → stable

overall → stable



unstable

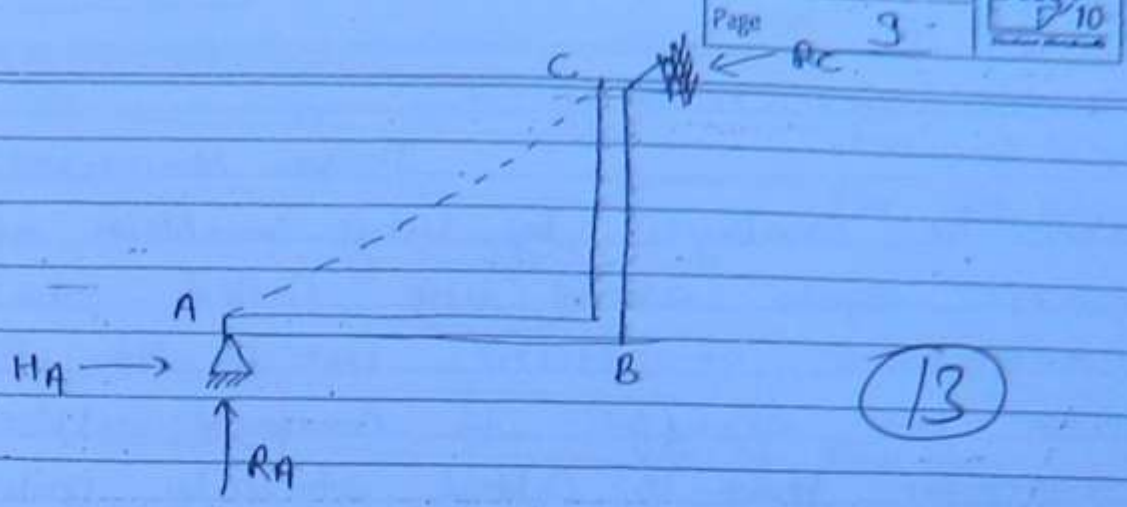
stable & over stiff.

External - Stable

Internal - is unstable

(Because Left Panel is deficient)

Overall - unstable



unstable because all reaction all concurrent

ie →

In - 3D structure for external stability there should be ^{min} 6-reaction which should be Non-Coplanar, Non-Concurrent, Non-trivial.

Indeterminacy :-

Static (Ds)

Related kinematic (Dk)
(free of to the degree of freedom and joints)

External (Dse)

(Related to Support Condition)

Internal (Dsi)

(Shape and Member arrangement)

$$[D_s = D_{se} + D_{si}]$$

Static indeterminacy :-

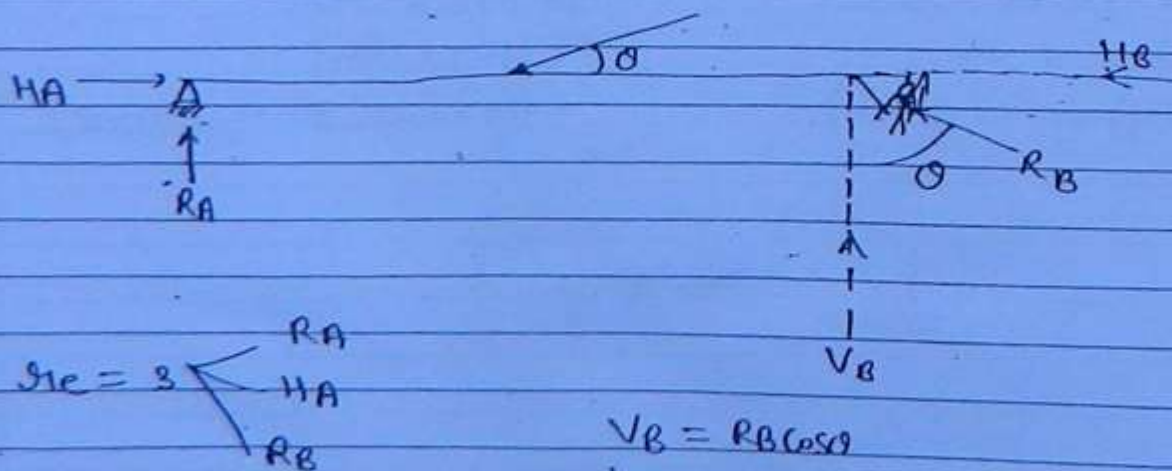
Those structures which can be analysed by using condition of static equb. alone are called statically determinate structure. but if No. of equation are not enough to compute all the Reaction that is called statically indeterminate / hyperstatic / Redundancy structure.

(14)

① External Static indeterminacy -

It is related to the support condition. If all the support reaction can be compute by using condition of static equilibrium alone than structure is statically determinate externally.

Let $\sum R_e$ is total No. of external support Reaction of all joint (No. of independent Reaction)



$\sum R_e = 3$

$V_B = R_B \cos \theta$
 $H_B = R_B \sin \theta$

$\left(\frac{H_B}{V_B} = \tan \theta \right) \rightarrow \text{depend.}$

$D_{se} = r_e - 3$ ————— For 2D Structure
 $= r_e - 6$ ————— For 3D Structure

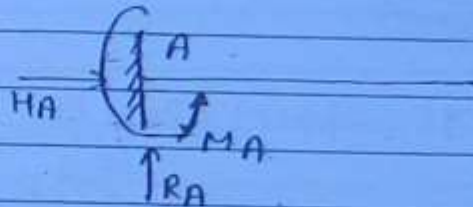
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Type of support

No. of Reaction

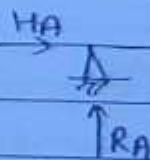
2D - Plane

① Fix support



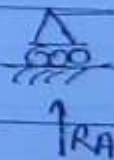
$r_e = 3$

② Hinge support



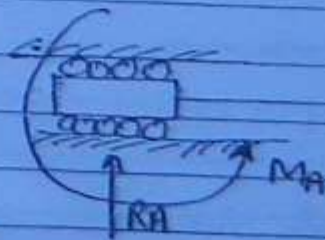
$r_e = 2$

③ Roller support

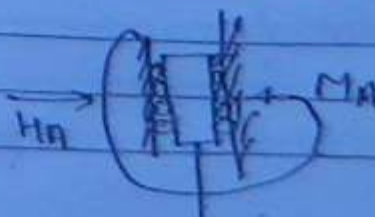


$r_e = 1$

④ Guided Roller support



$r_e = 2$



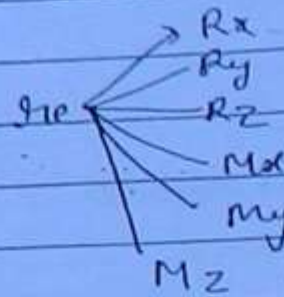
$r_e = 3$

Types of Support

No. of Reaction

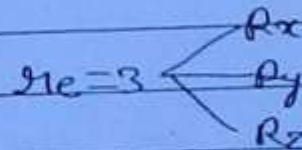
3D case

① Fix Support / Built-in Support



①/6

② Hinge support / Ball & Socket joint



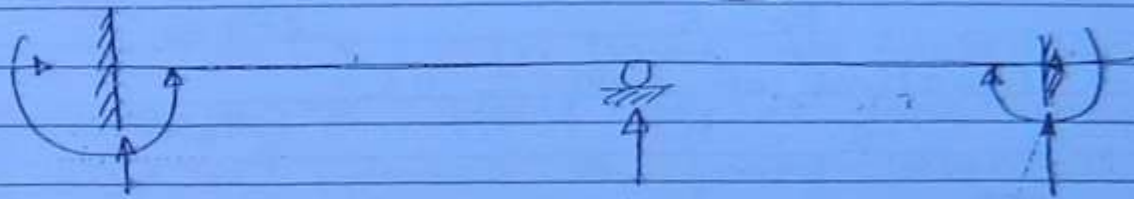
③ Roller Support

$R_e = 1$ (only Normal to Plane)

Ex → Find external static indeterminacy for the structure shown in figure.

(17)

Q



$$\text{Total } r_e = 3 + 1 + 3 = 7$$

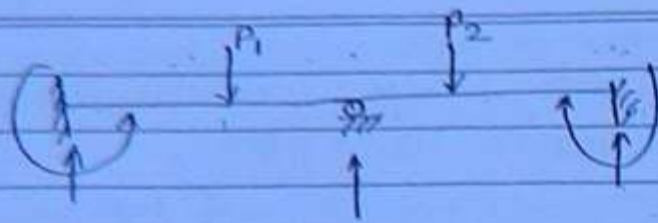
$$\text{Total eqm condition} = 3$$

$$Dse = r_e - 3 = 7 - 3 = 4$$

When loading is not mentioned then general loading is considered which may have a horizontal and vertical component both. Therefore at the support horizontal & vertical & moment reactions will be available.

If only vertical load is present then in frame, arches and trusses a vertical component of reaction will be present. Also moment depending upon support condition.

But in beams due to vertical loading alone horizontal reactions will not be present. Therefore to determine Dse horizontal reaction may be ignored and equation for horizontal forces may be ignored.

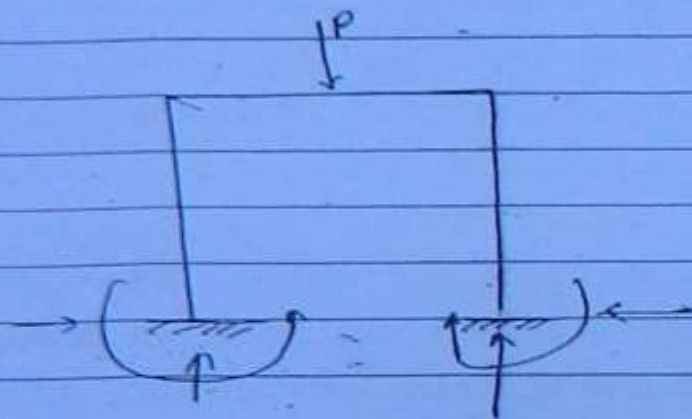


$r_c = 5$
 Equations of Eqm = 2

$$\begin{cases} \sum F_y = 0 \\ \sum M_2 = 0 \end{cases}$$

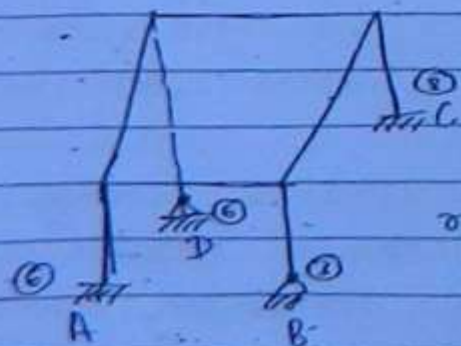
18

~~rc~~ Dse = $r_c - 2$
 $\Rightarrow 5 - 2 = 3$



$r_c = 3 + 3 = 6$
 $Dse = 6 - 3 = 3$

eg →



$r_c = 6 + 3 + 6 + 3 = 18$
 $Dse = r_c - 6$
 $\Rightarrow 18 - 6 = 12$

Internal static indeterminacy:- (Dsi)

Case-A 2D Truss/Plane Truss

(19)

In Truss all joints are Hinge, loading is applied only at joints and self weight of member is ignored. Hence all member carries ^{only} axial forces there fore each member has only one internal reaction that is axial force.

If there are m members than total no. of internal Reactions. let there are J no. of joints. at each joint there are 2 equation are equilibrium. $\begin{pmatrix} \sum F_x = 0 \\ \sum F_y = 0 \end{pmatrix}$

Total no. of equilibrium equation at all joint $\Rightarrow 2J$

out of which 3 equation are utilised to determine external support reaction Hence Net available equation two find Internal Forces $= (2J - 3)$

$Dsi = \text{Total No. of Internal Reaction} - \text{Total available Equation for int. reaction.}$

$$Dsi = m - (2J - 3)$$

if $D_{si} = 0$ Truss is $\left\{ \begin{array}{l} \text{internally determinate} \end{array} \right\}$

$D_{si} > 0$ Truss is indeterminate [Redundant / over stiff]

$D_{si} < 0$ internally unstable / deficient

(20)

Case-2 3-D Truss / Space Truss $\rightarrow$

In each member of Truss there is only one Internal force (axial force) but at each joint there are 3 equations ($\sum F_x = 0$, $\sum F_y = 0$ and $\sum F_z = 0$) Hence total eqn^s of all joint $\frac{3J}{(3J)}$

out of which 6 equation are use for external support reaction hence Net available equation To find internal reaction = $(3J - 6)$

$$D_{si} = m - (3J - 6)$$

If $D_{si} = 0$ - Truss is Internally determinate

Case-2. Rigid Frame (2-D, 3-D) $\rightarrow$

In Rigid frames internally internal indeterminacy will not exist. If structure has open configuration like a tree, beams are always internally determinate. To check internal indeterminacy following thumb rule can be applied.

(2)

Rule-1 If structure is internally determinate then it is not possible to provide a cut any where on the structure without splitting the structure in a two part.

eg.



Tree / open ~~tree~~ structure

If structure is statically determinate it is impossible to return back at any point without — the path it mean in internally determinate structure there will be no closed loops if

internal hinges, links, shear joint

etc.

2-D Rigid frame $\rightarrow$

in each member 2-D rigid member there will be 3 internal reaction (r_x , r_y and r_z) or axial force, shear force and Bending moment.

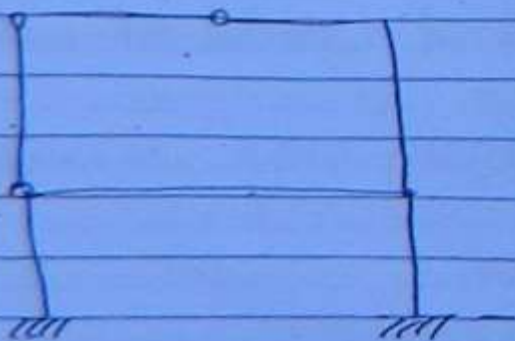
(22) if there is no close loop all the above three internal reaction can be found by providing it cut any section at suitable free body equilibrium for either left or right portion.

$$\sum F_x = 0, \sum F_y = 0, \sum M_z = 0$$

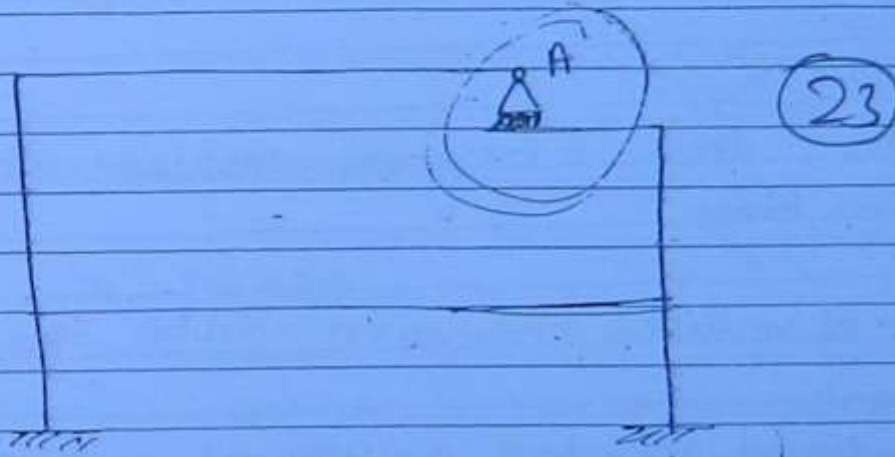
but if there are close loop then each close loop will have three internal indeterminacy. Hence if No. of close loops are c then total degree of internal indeterminacy $D_{si} = 3c$.

$$D_{si} = m - (3J - 6)$$

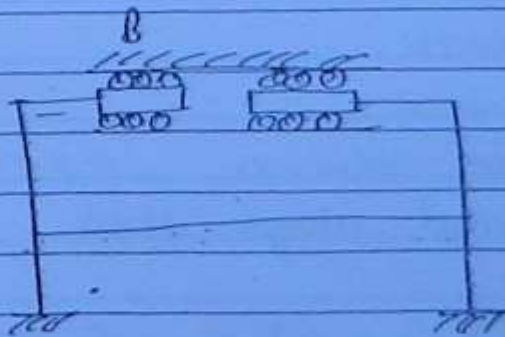
(c No. of close loops / No. of cuts required to convert close structure into an open structure)



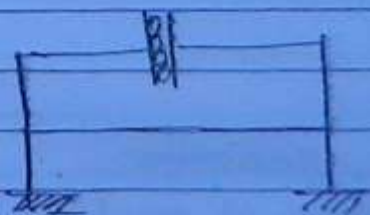
Note $\rightarrow$ If all joints are not rigid that is sum of the joint are hybrid due to presents of internal hinge, internal roller, shear roller then some inter of the internal reaction are related. No. of internal reaction released upon. Type of joint and No. of member meeting at the joint.



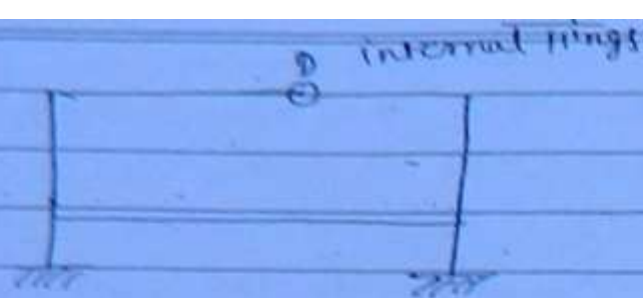
No. of int. Reaction released at A $\Rightarrow$
 $(r_{int}) = 2 \begin{cases} \rightarrow A.F. = 0 \\ \rightarrow B.M. = 0 \end{cases}$



r_{int} at B = 1
 i.e. A.F. at B = 0



r_{int} at C = 1
 (S.F. at C = 0)

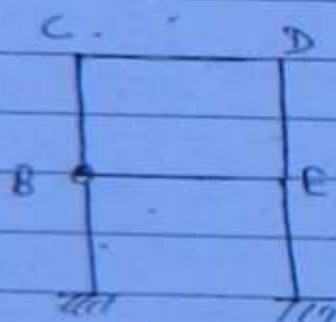


(24) at D. $B.M = 0$
 $\sum M = 1$

If more than 2 member meeting at internal hinge.

No. of released reaction will be $\Rightarrow r_r = (m' - 1)$

~~Ans~~ $m' =$ No. of member meeting at in Hinge.



$r_r = (m' - 1)$
 $\sum M_B = (3 - 1) = 2$

If 3D structure is internal Hinge then
 $\sum M = 3$

$\sum M = 3(m' - 1)$

main formula $\Rightarrow D.C.I = 3C - \sum M$

$\sum M =$ Total No. of Internal Reaction released at Hybrid joint.

3D - Rigid Frame

At each close loop there are 6 internal if No. of close loop that total No. of internal indeterminacy $DS_i = 6C$ and if sum of the joints are Hybrid $[DS_i = 6C - \sum r]$. for 2D Internal Hinge. $\circ \text{Hinge} = 3(m-1)$
 $m = \text{No. of members meeting at Hybrid joints.}$

(25)

overall static indeterminacy :-

1st method $DS = DS_e + DS_i$

$$DS = 0$$

Structure is determinate provided are Non Parallel, Non concurrent, Non trivial and There no condition of mechanism

$$DS > 0$$

stable & indeterminate (overstiff & Desirable

indeterminate str. are more economical than determinate because thinner section is required.

$$DS < 0$$

unstable structure.

Method 2 (2-D Buss):-

$$\textcircled{1} DS = \text{Total reaction (Internal reaction + external reaction)}$$

= Total Eqⁿ equation.

$$DS = m + r_e - 2j$$

② 3D Truss :-

$$D_s = m + r_e - 3J$$

②

③ 2D rigid frame/Beams :-

$$D_s = \text{Total reaction} - \text{Total equation at} \\ (\text{internal+external}) \quad \text{All joints}$$

$$D_s = (3m + r_e - 3J)$$

when all joints are rigid.

$$D_s = (3m + r_e - 3J) - r_{r1}$$

when of the joints are Hybrid

④ 3D Rigid frames/Beams →

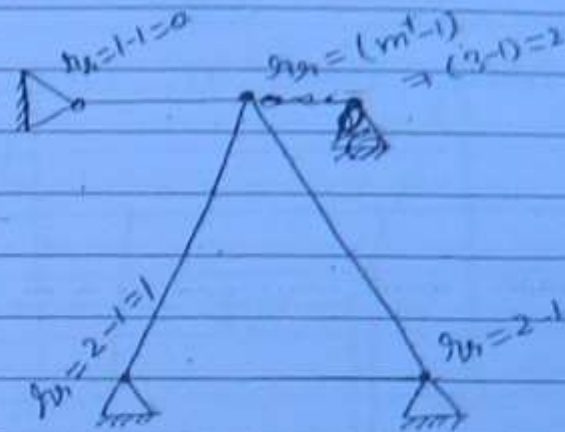
$$D_s = (6m + r_e - 6J) \quad \text{--- when all joint are rigid}$$

$$D_s = (6m + r_e - 6J) - r_{r1}$$

when some joint are Hybrid.

r_{r1} → Total No. of Released reaction.

Q. determine Ds for plane truss shown in figure.



→ 1st method →

$$D_{se} = r_{pe} - 3$$

$$\Rightarrow (2 + 2 + 2) - 3$$

$$D_{se} \Rightarrow 6 - 3 = 3$$

$$D_{si} = m - (2j - 3)$$

$$\Rightarrow 4 - (2 \times 4 - 3)$$

$$\Rightarrow 4 - 5 \Rightarrow -1$$

$$D_s = D_{se} + D_{si}$$

$$\Rightarrow 3 - 1 = 2$$

2nd method :-

$$D_s = m + r_{pe} - 2j$$

$$\Rightarrow 4 + 6 - 2 \times 4$$

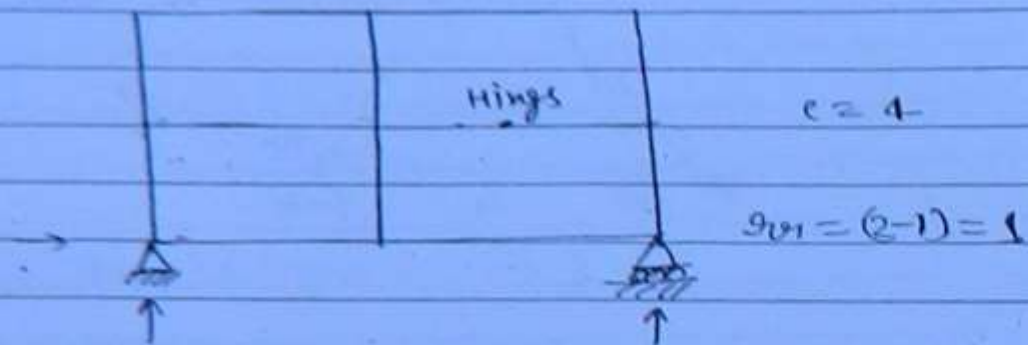
$$\Rightarrow 10 - 8 = 2$$

2nd method

Ques

(28)

Ques for 2D frame shown in fig. find Ds



1st method :-

$$J_e = 2 + 1 = 3$$

$$D_{se} = J_e - 3$$

$$\Rightarrow 3 - 3 = 0$$

$$D_{si} = 3c - J_h \Rightarrow 3 \times 4 - 1 \Rightarrow 12 - 1 = 11$$

$$D_s = D_{se} + D_{si}$$

$$\Rightarrow 0 + 11 \Rightarrow 11$$

2nd Method

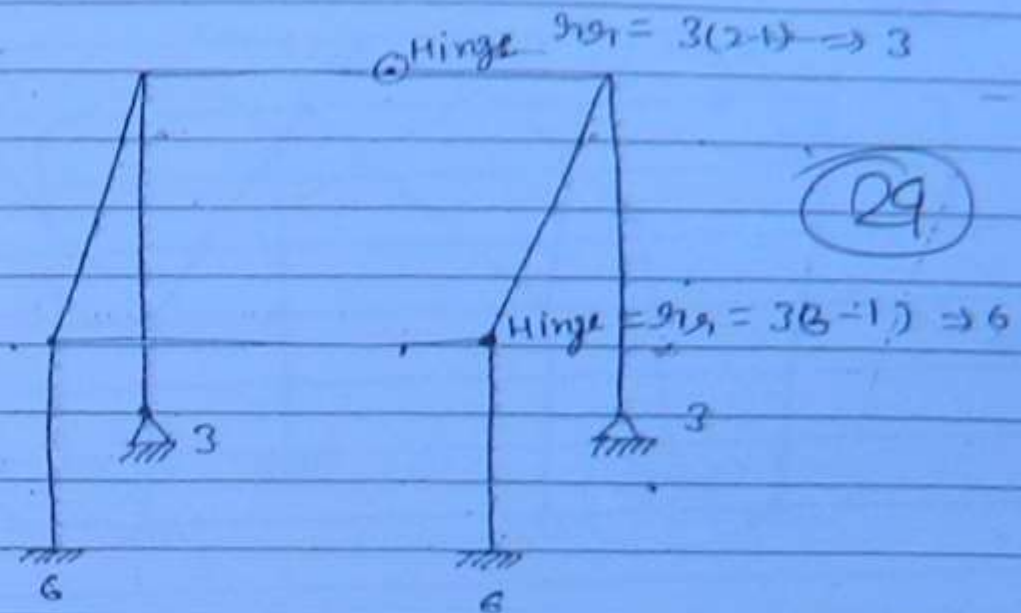
$$D_s = 3m + J_e - 3J - J_h$$

$$\Rightarrow 3 \times 3 + 3 - 3 \times 10 - 1$$

$$\Rightarrow 39 + 3 - 30 - 1$$

$$\Rightarrow 11$$

Q. For 3D rigid frame shown in fig. find D_s



Method -1

$$D_{se} = 8e - 6$$

$$= 18 - 6 = 12$$

$$D_{si} = 6c - 9r$$

$$\Rightarrow 6 \times 1 - (3 + 6)$$

$$\Rightarrow 6 - 9 = -3$$

$$D_s = D_{se} + D_{si} \Rightarrow 12 - 3 \Rightarrow 9$$

2nd Method

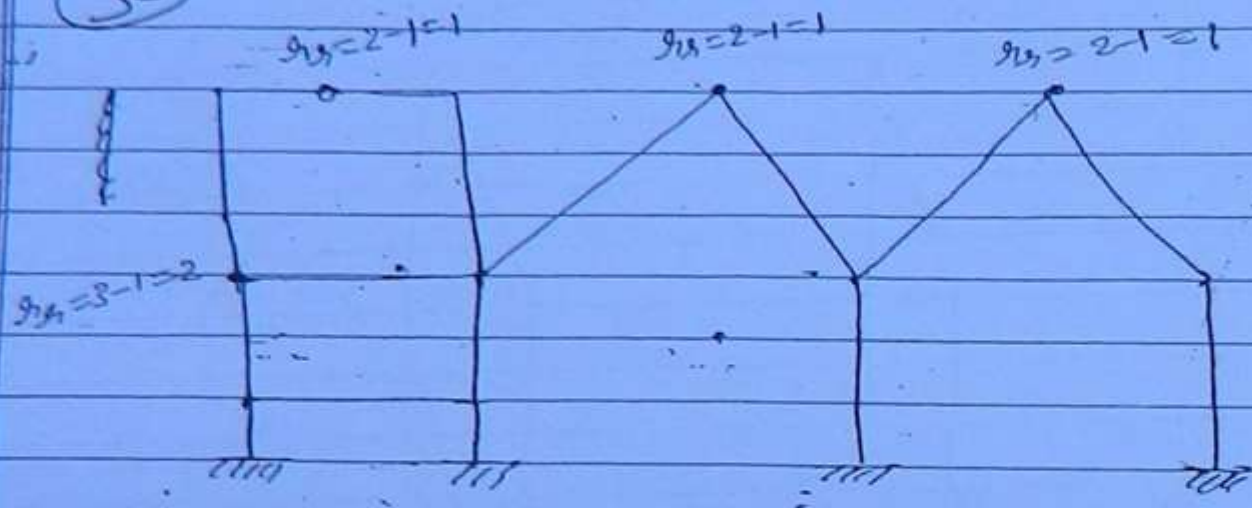
$$D_s = 6m + 9e - 6J - 9r$$

$$\Rightarrow 6 \times 9 + 18 - 6 \times 9 - 9$$

$$\Rightarrow 9$$

Q. for 2D rigid frame find D_s ?

(30)



1st method :-

$$J_e = 3 + 3 + 3 + 3 = 12$$

$$D_{se} = J_e - 3$$

$$\Rightarrow 12 - 3 = 9$$

$$D_{si} = 3C - J_{ij}$$

$$\Rightarrow 3 \times 2 - 5 \Rightarrow 1$$

$$D_s \Rightarrow D_{se} + D_{si} \Rightarrow 9 + 1 \Rightarrow 10$$

2nd method $\Rightarrow$

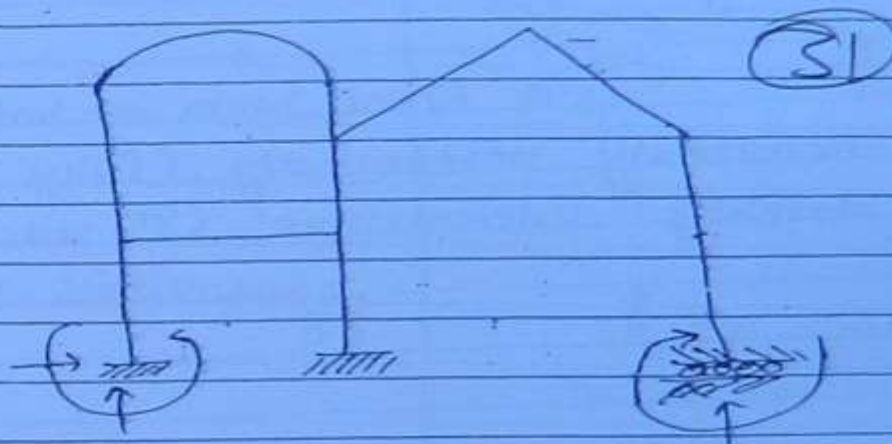
$$D_s = 3m + J_e - 3J - J_{ij}$$

$$\Rightarrow 3 \times 16 + 12 - 3 \times 15 - 5$$

$$\Rightarrow 48 + 12 - 45 - 5$$

$$\Rightarrow 10$$

Q. Find D_s Shown in Figure.



1st method

$$n_e = 3 + 1 + 2 = 6$$

$$D_{se} = n_e - 3 \Rightarrow 6 - 3 = 3$$

$$D_{si} = 3G - n_r \\ \Rightarrow 3 \times 1 - 0 \Rightarrow 3$$

$$D_s = D_{se} + D_{si} \Rightarrow 3 + 3 \Rightarrow 6$$

2nd Method

$$D_s = 3m + n_e - 3J \\ \Rightarrow 3 \times 8 + 6 - 3 \times 8 \\ \Rightarrow 18 + 6 - 24 \\ \Rightarrow 6$$

Kinematic indeterminacy :- "Kinematic Indeterminacy (D_K) - is refers to the total No. of available degree of freedom at all joint"

or

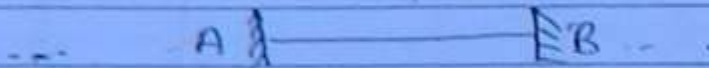
Kinematic indeterminacy (D_K)

D_K total No. of unrestrained displacement component at all joints.

A fully locked structure all joint kinematically determinate because it has no degree of freedom at any joints.

(32)

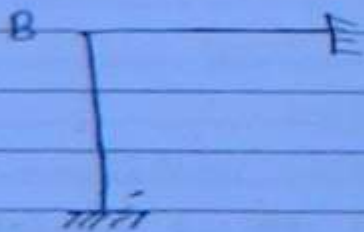
A fixed beam at both end is kinematically determinate (fully locked) but statically indeterminate (3rd order)



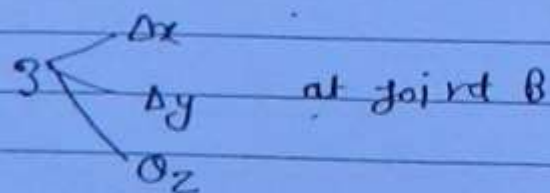
Notes The displacement at joints are elastic displacement. if a member is in ~~extension~~ inextension. then in any direction, then displacement in that dirⁿ will not be available at the joint.

Strained Joint) Types of joint

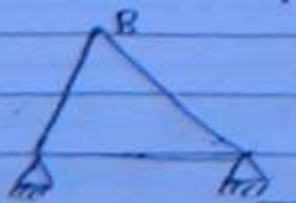
(1) 2D rigid joint



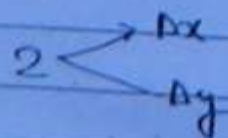
No. of displacement



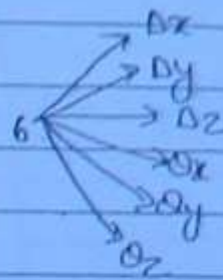
(2) 2D True joint / Pin joint



at joint B

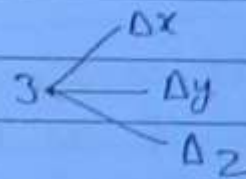


③ 3D Rigid joint



(33)

④ 3D Truss joint/Pin joint



Kinematic degree of determinacy :-

① For 2D plane Truss :-

$$D_k = 2J - r_e$$

At each ~~for~~ joints there may be two displacement $\Delta x, \Delta y$ out of which some of the joints are supported in the direction of support reaction displacement are not permitted. Hence net available displace. $\pm$ available degree of freedom

$$2J - r_e$$

For 3D TRUSS:-

$$DK = 3J - 9r$$

(34)

2D Rigid Frame:-

$$DK = 3J - 9r$$

Note → If some of the joint are hybrid.

Some of the internal reaction are released degree of freedom will increased
Hence DK will be

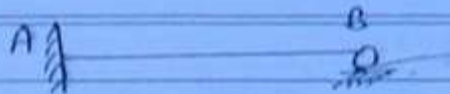
$$DK = 3J - r_r + r_r$$

r_r = released r_r at hybrid joint

if ^{some of} members of the members are axial rigid than due to axially rigidity ~~linear~~ axial displacement in the direction of member at some of the joint will not be available, Hence degree of freedom will get reduced

Let m → no. of axial displacement are not available at joint due to axial rigidity

$$DK = 3J - r_r + r_r - m$$



$$D_K = 3J - r_e$$

$$\Rightarrow 3 \times 2 - 4 = 2$$

 ΔB
 ΔB_x

If AB is axially rigid

$$\Delta B_x = 0$$

$$D_K = 3J - r_e - m$$

$$\Rightarrow 3 \times 2 - 4 - 1$$

$$\Rightarrow 1 \rightarrow \Delta B$$



$$D_K = 3J - r_e$$

$$\Rightarrow 3 \times 2 - 5 \Rightarrow 1 \rightarrow \Delta D$$

If CD is Axially rigid

then also

$$D_K = 1 \rightarrow (\Delta D)$$

Because in this case Δ_x is not available. Hence D_K will remain 1. In spite of angle member is axially rigid.

3D Rigid frame

$$D_K = 6J - r_e \text{ --- when all joints are rigid}$$

$$D_K = 6J - r_e + r_r \text{ --- Some of the joints are hinged.}$$

$$D_K = 6J - r_e + r_r - m \text{ if some of the members are axially rigid}$$

m = No. of axial displacement prevented due to axial rigidity.

degree of freedom at hybrid joint:-

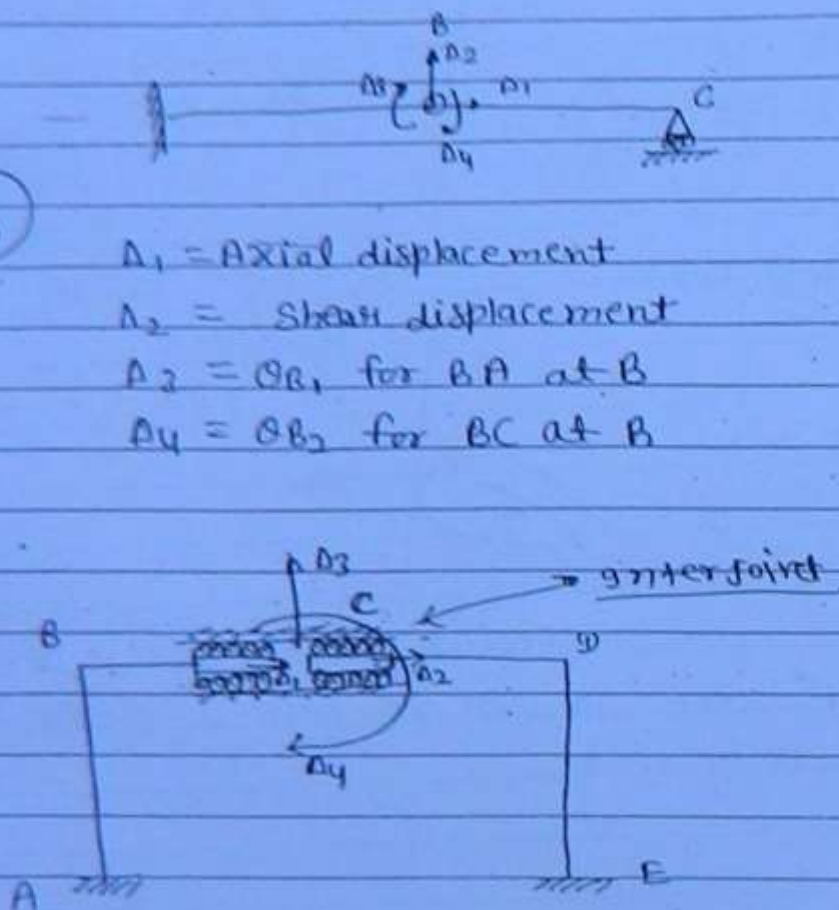
(36)

Δ_1 = Axial displacement

Δ_2 = Shear displacement

$\Delta_3 = \theta_B$ for BA at B

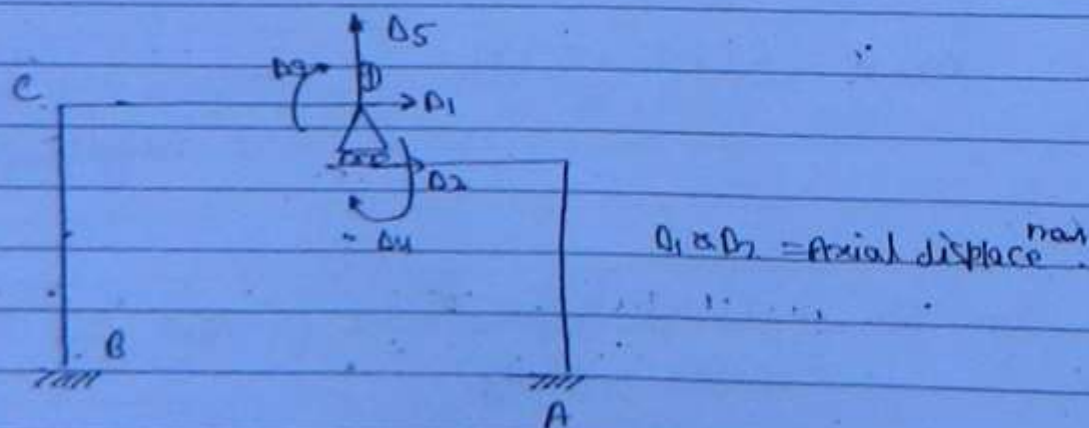
$\Delta_4 = \theta_B$ for BC at B



Δ_1 & Δ_2 are Axial displacement

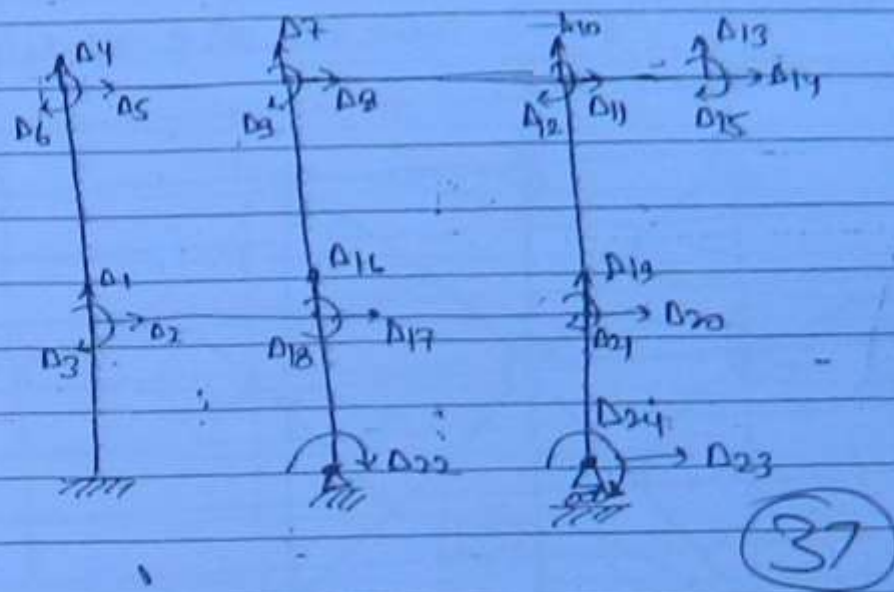
Δ_3 = Shear displacement

Δ_4 = θ Angle displacement



Δ_1 & Δ_2 = Axial displacement

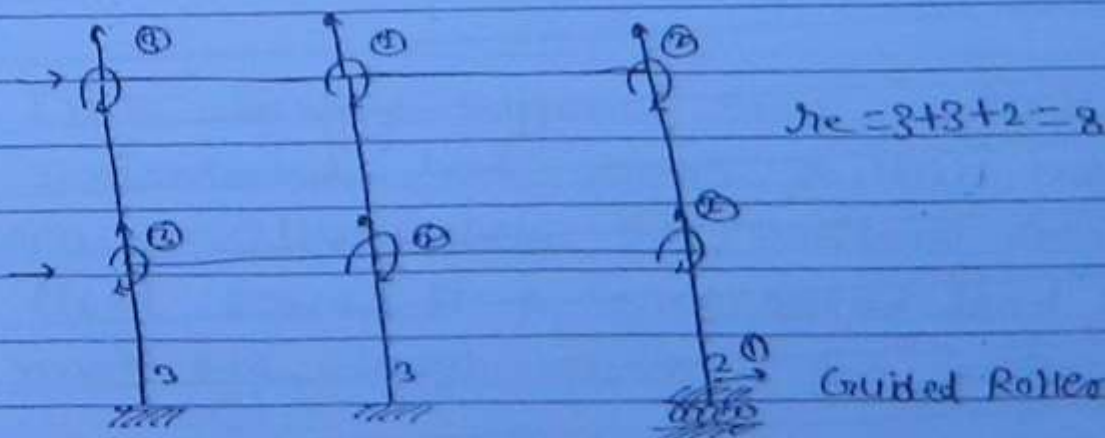
Q. Find degree of kinematic indeterminacy & represent all displacement at joints.



$$D_k \Rightarrow 3 \times J - r_e$$

$$3 \times 10 - 6 = 24$$

Q. for the rigid frame shown in figure find D_k of indeterminacy assuming beams as axial rigid.



$$r_e = 3 + 3 + 2 = 8$$

Guided Roller

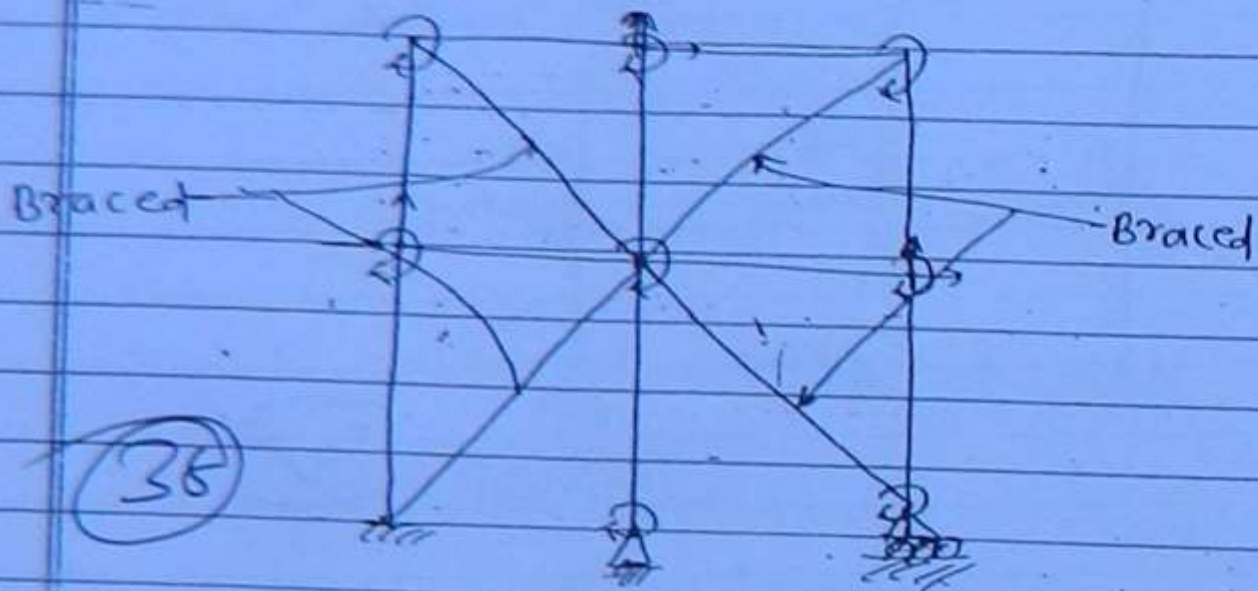
$$D_k = 3J - r_e + \frac{1}{2} \times 2 - 1$$

$$\Rightarrow 3 \times 9 - 8 - 1$$

$$D_k \Rightarrow 24 - 12 = 12$$

only beam are axial rigid.

Note In a elastic frame is connected by rigid
bracing than at all the braced joint
linear displacement will not be permitted
($\Delta x = 0$, $\Delta y = 0$) member are elastic there
are rigidly.

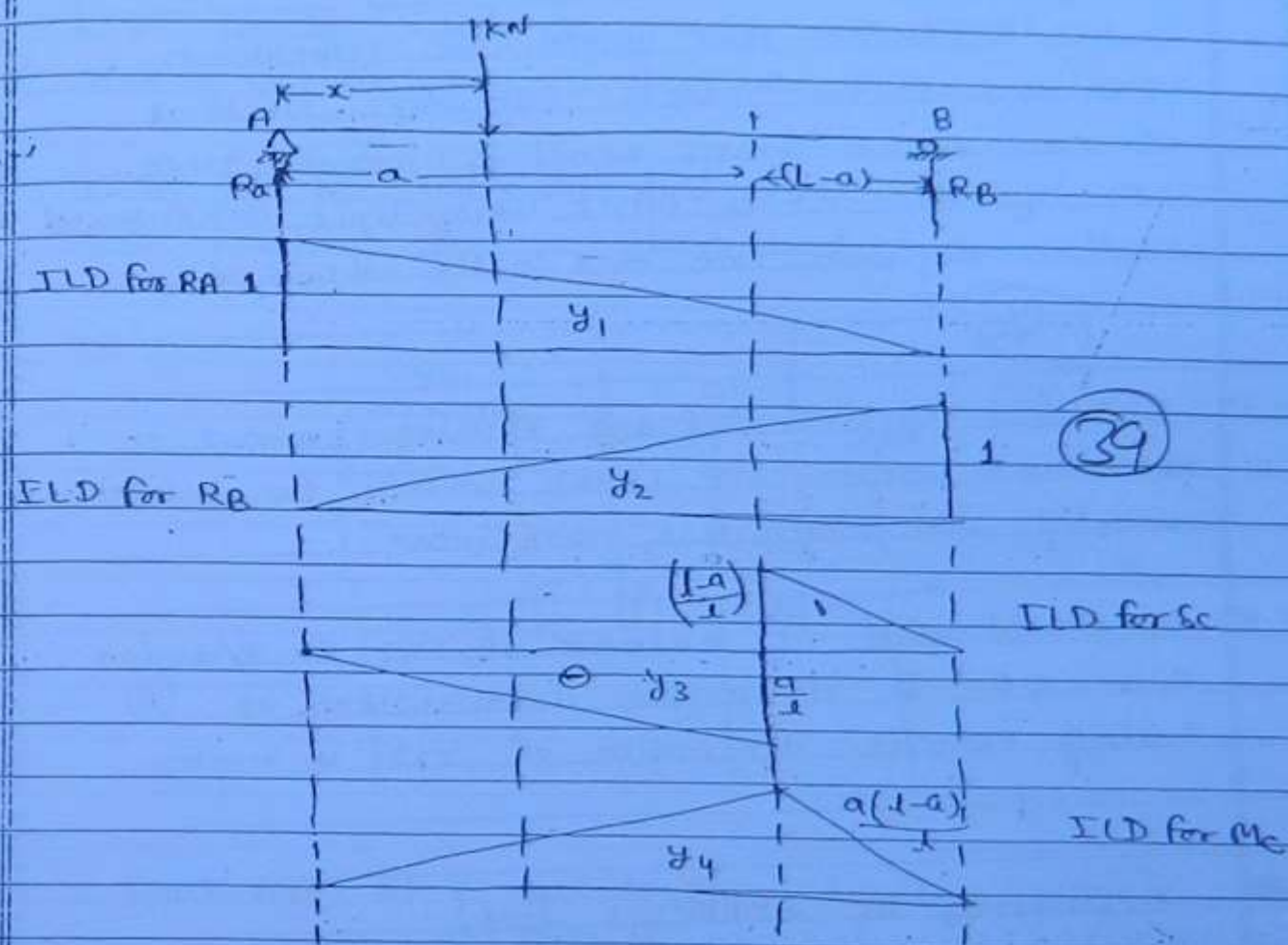


Influence Line Diagramme

Page

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10/10
10/10



Note: - (1) In a simply supported Beam or girder if a concentrated load moves then max^m B.M. occurs always below the position of load. And In a fixed Beam if a concentrated load moves the max^m B.M. always occurs at the support.

(2) The distance b/w two adjacent point of contraflexure is called focal length.

(3) The portion of a Beam or girder in which shear force is constant is known as shear span.

"Influence line diagramme represent variations of the stress function such as reaction, shear force, bending moment, slope or deflection at a point when unit concentrated load moves from one end to the other end"

(40)

Note ① ILD for reaction, S.F and Bending moment in determinate beam is linear whereas for indeterminate beam it is non linear.

② The dimension of ordinate of ILD for Reaction and S.F. is dimensionless whereas Bending moment dimension of ILD is meter.

③ Application of Influence line:-

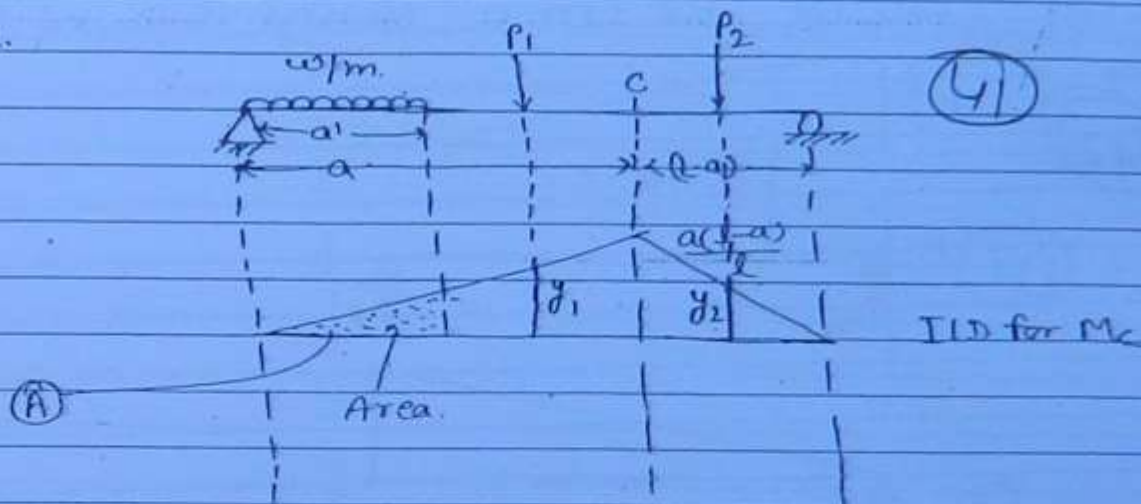
① ILD can be use to find position of live load which will produced max. value of a stress function. (load should place placed at that point where ordinate of ILD is max)

Max. (-ve) SF at C will occur when load is placed just left of C.

② ILD can be used to study the effect of moving load on the C structure.

③ ILD can be use to find total value of a stress function for a given load system in.

Ex.



Total B.M. at C due to given loading

$$\Rightarrow (w \times \text{Area of ILD below UDL}) + P_1 y_1 + P_2 y_2$$

$$M_c \Rightarrow (w \times A + P_1 y_1 + P_2 y_2)$$

Muller Breslau Principle

(Application for determinate & indeterminate ^{both})

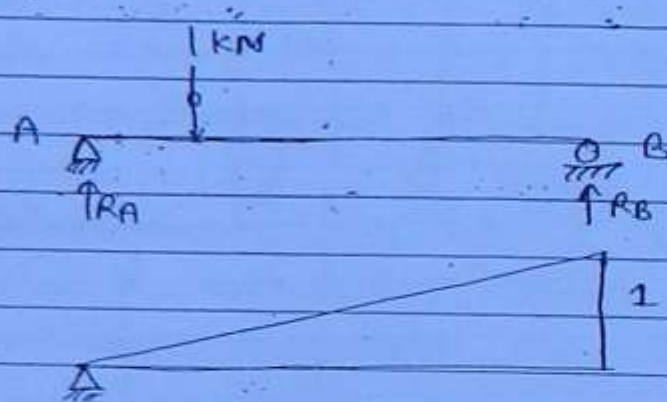
It gives qualitative influence line dia. for a stress function (reactions, sf & B.M.)

"The ILD for any stress function in a structure is represented by its deflected shape obtained by removing the restraint offered by that stress function & introducing a directly related generalised displacement in the dirⁿ that stress function"

"Influence line dia. any stress function can be obtained by given unit displacement in dirⁿ of that stress function and removing that stress function the resultant shape of beam ILD"

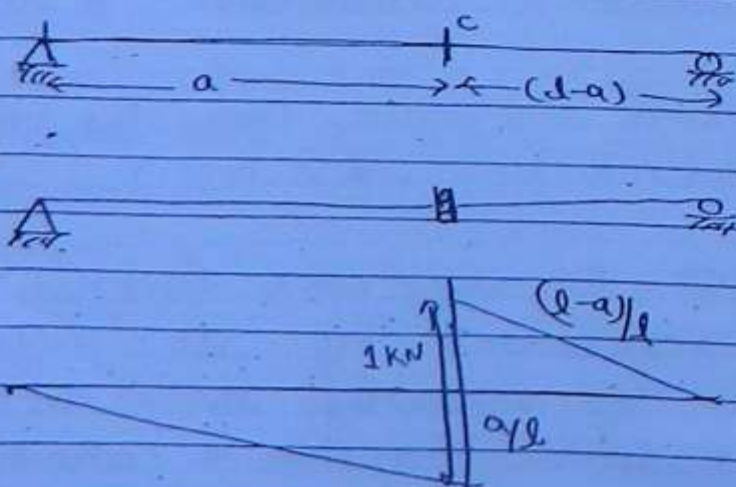
eg. ILD for R_B in a simply supported beam

(42)

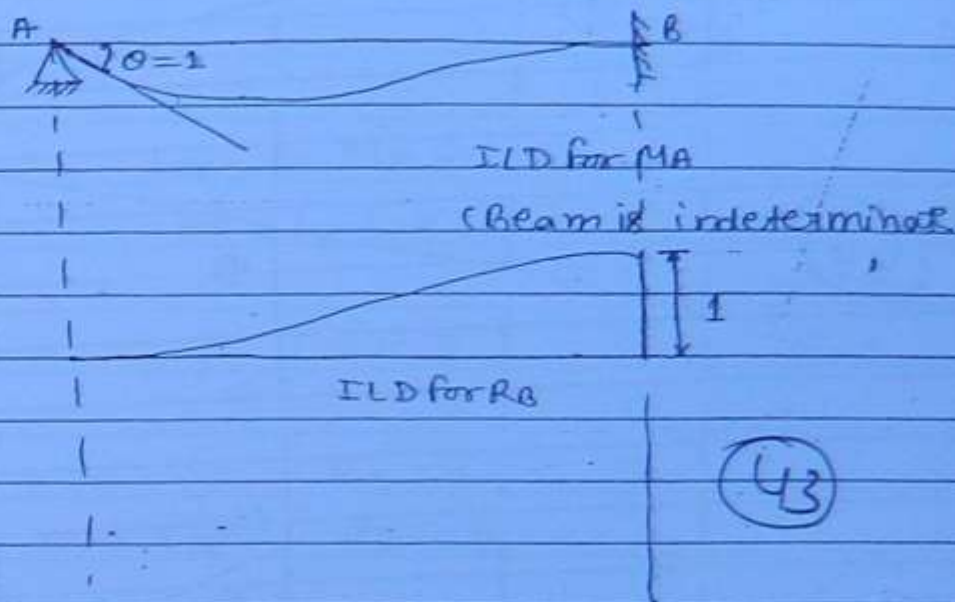
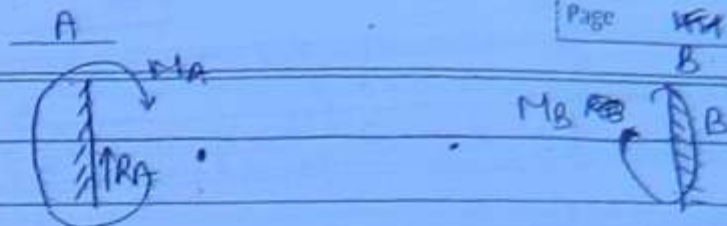


ILD for R_B

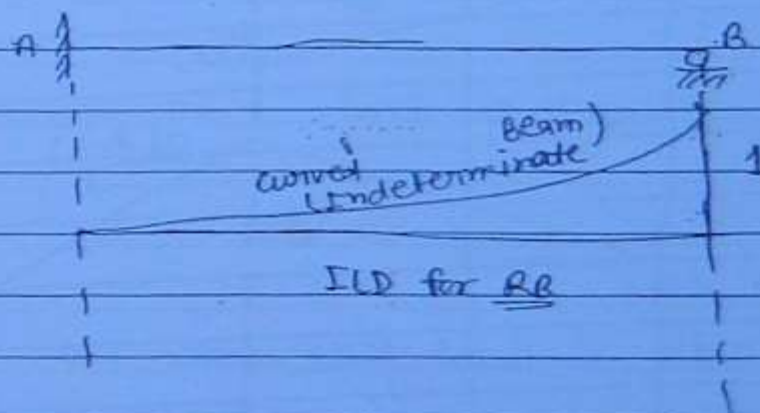
ILD for R_B can be obtained by removing R_B and giving ^{unit} displacement of B in the direction of R_B . The resultant deflection shape of beam will be ILD.



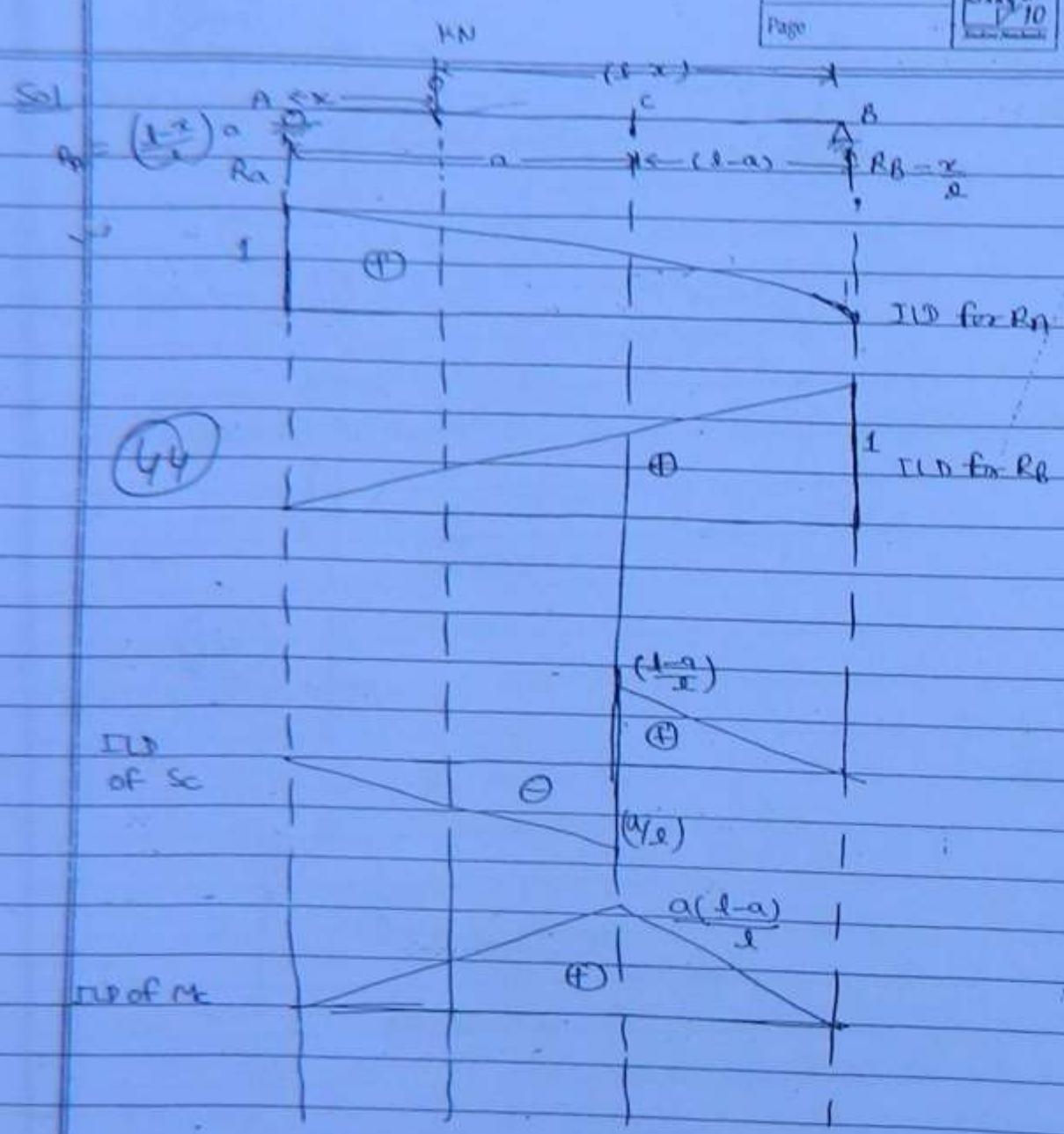
eg. 1



eg. 2



Q. using 1st Principal draw Influence line diagram for R_A , R_B , S_c & M_c for simply supported beam span L is shown in figure.



ILD for R_A

To Find R_A : Take $\sum M_B = 0$

$$R_A \cdot l - 1(l-x) = 0$$

$$R_A = \frac{(l-x)}{l}$$

when 1 kN load is at A ($x=0$)

$$R_A = 1$$

when 1 kN load is at RB ($x=2$)

$$R_A = 0$$

(45)

ILD for R_B :-

$$\text{Take } \sum M_A = 0$$

$$R_B \times 2 - 1 \cdot x = 0$$

$$R_B = \frac{x}{2}$$

$$\text{when } x=0, \quad R_B = 0$$

$$\text{when } x=2, \quad R_B = 1$$

* ILD for S_c

Case-I when unit load is the left of c

$$S_c = -R_B = -\frac{x}{2}$$

$$0 \leq x \leq a$$

$$S_c = 0 \quad \text{when 1 kN load is at a}$$

$$S_c = -\frac{a}{2} \quad \text{when 1 kN load is just to left of c}$$

Case-II when unit load is in CB

$$S_c = +R_{BA} \Rightarrow \frac{2-x}{2} \quad a \leq x \leq 2$$

$$\text{at } x=a \quad S_c = \frac{1-a}{2}$$

$$\text{at } x=2 \quad S_c = 0$$

x ILD for M_c :-

Case - I when unit load is in AC

$$M_c = R_A x (l - a)$$

(46)

$$M_c = \frac{x(l-x)}{l} \quad 0 \leq x \leq a$$

if $x=0$ (at A) 1 KN is at A)

$$M_c = 0$$

if $x=a$, $M_c = \frac{a(l-a)}{l}$

ILD for M_c :-

when unit load is in CB

$$M_c = R_A \cdot a$$

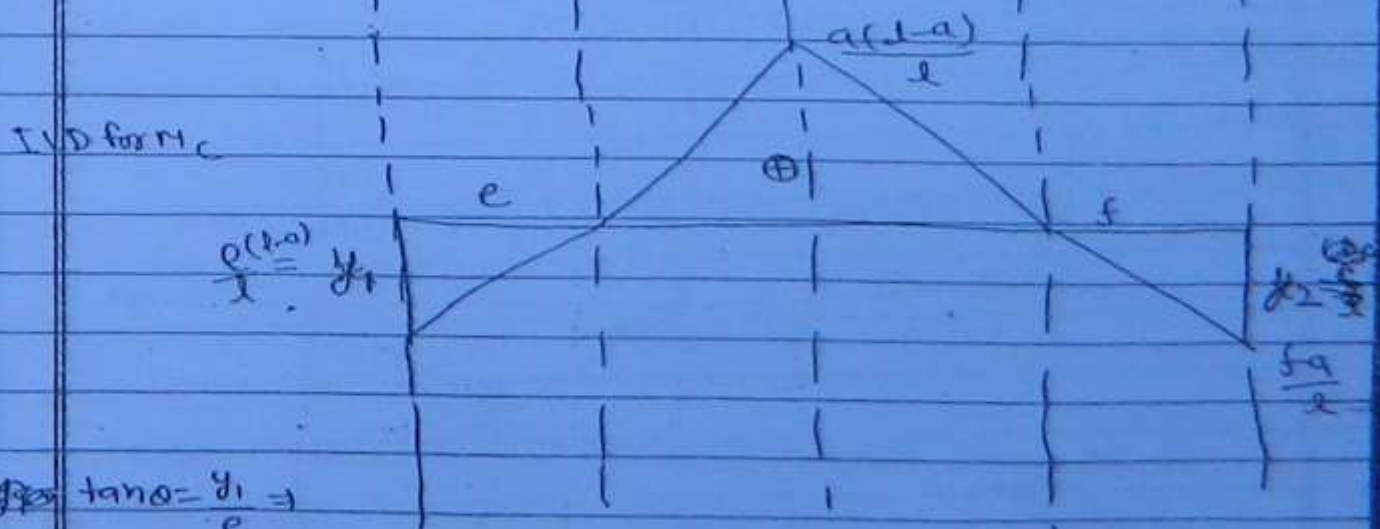
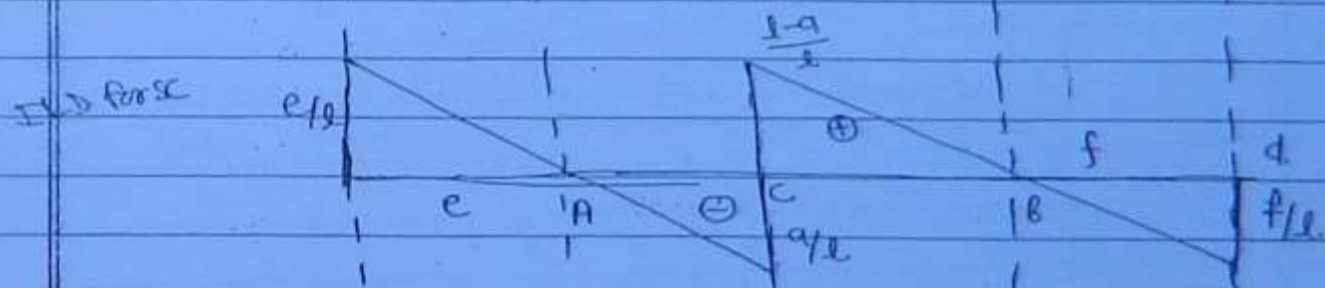
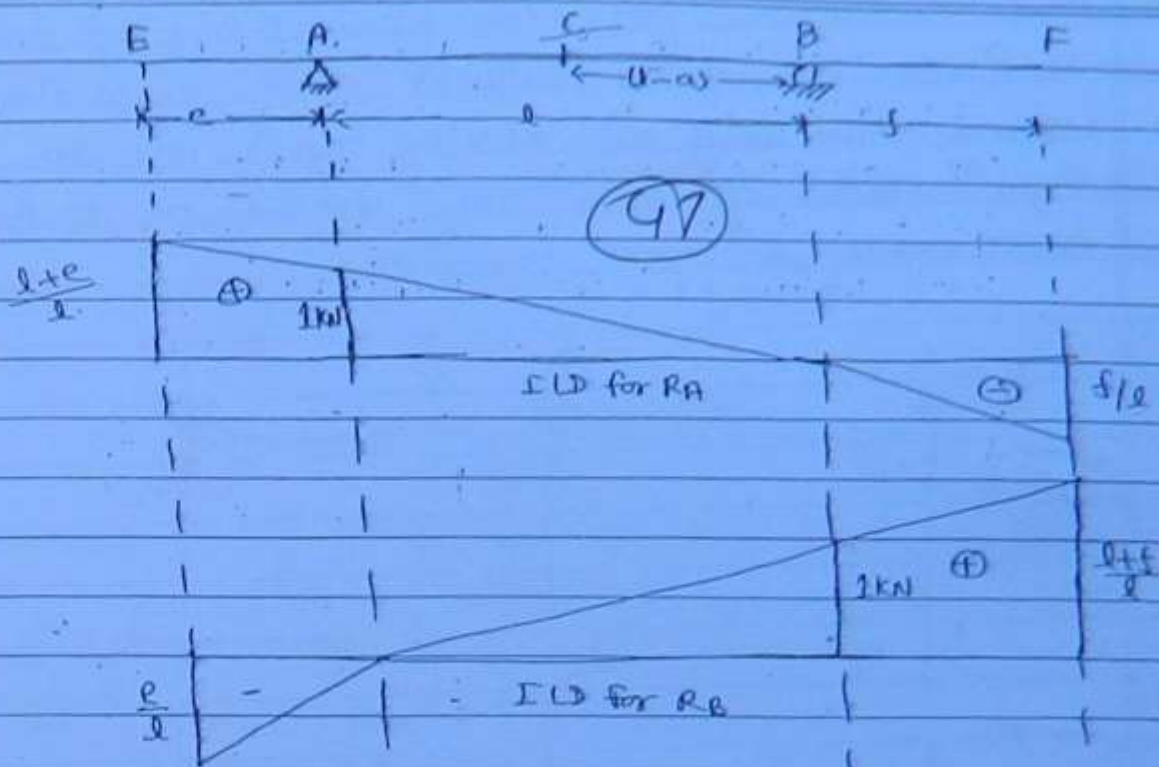
$$\Rightarrow \left(\frac{l-x}{l} \right) a \quad a \leq x \leq l$$

if $x=a$

$$M_c = \frac{(l-a) \cdot a}{l}$$

if $x=l$, $M_c = 0$

Note:- ILD for a overhanging beam can be obtained by linear extension of ILD for simply supported beam.



$$\tan \theta = \frac{y_2 - y_1}{x_2 - x_1}$$

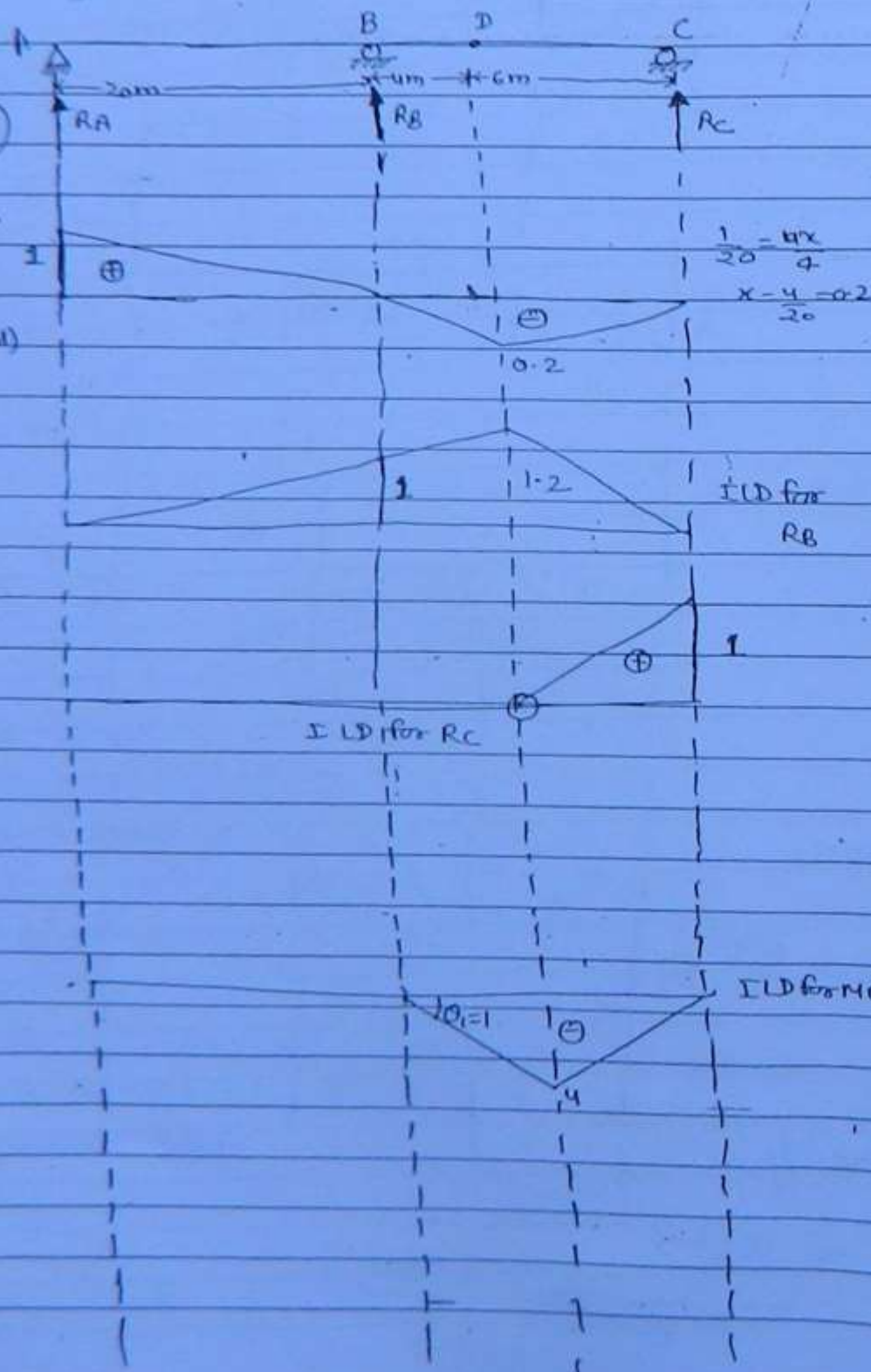
$$\Rightarrow \frac{y_2 - y_1}{x_2 - x_1} = \frac{e(1-a)}{l}$$

Q Draw Influence Dia. for R_A , R_B and R_C and M_B and also find max^m value of above stress function when a live load of 10 kN/m moves left to right which may have any length so as to produce max^m value of stress function.

Solⁿ:
Ans:-

(48)

ILD for R_A
(using Muller
Breslau's principle)



R_A will be maximum when only span AB is loaded.

$$R_{Amax} = \text{Loading Rate} \times \text{Area of ILD in AB}$$

(Q9)

$$\Rightarrow 10 \times \frac{1}{2} \times 20 \times 1$$

$$\Rightarrow 100 \text{ kN}$$

loading in BC produces $\oplus$ -ve Reaction at A.

if entire span is ~~ABC~~ ABC is loaded, then net reaction of ~~ABC~~ A will be:

$$R_{A\text{net}} = 10 \times [\text{Net Area of ILD}]$$

$$\Rightarrow 10 \times \left[+\frac{1}{2} \times 20 \times 1 - \frac{1}{2} \times 10 \times 0.2 \right]$$

$$\Rightarrow 10 \times [10 - 1] \Rightarrow 90 \text{ kN}$$

Maxim Reaction at B ^{will} ^{when} ~~load~~ ^{load} is present entire span ABC.

$$R_{Bmax} \Rightarrow 10 \times \frac{1}{2} \times 30 \times 1.2$$

$$\Rightarrow 180 \text{ kN}$$

R_{Cmax} will occur when at least span BC is loaded.

$$R_{Cmax} \Rightarrow 10 \times \frac{1}{2} \times 1 \times 6.5$$

$$\Rightarrow 30 \text{ kN}$$

max^m B.M. at B will be (-ve) i.e. hogging when at least span BC is loaded.

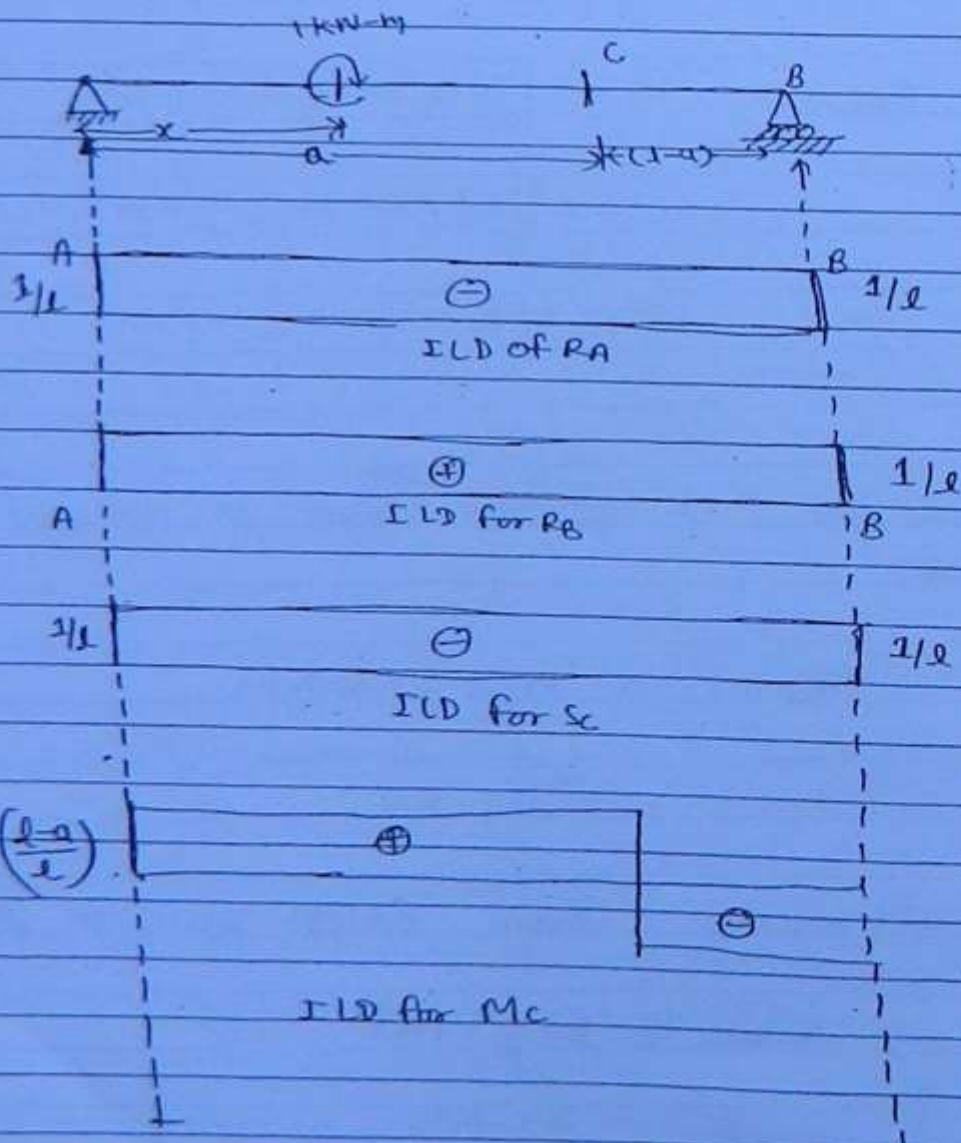
$$\therefore M_{\max} \Rightarrow 10 \times -\frac{1}{2} \times 4 \times 10$$

(56)

$$\Rightarrow -200 \text{ KN-m}$$

Q Draw ILD for simply supported girder of length l for R_A , R_B , S_C , M_C when a unit concentrated moment moves from A to B. as shown in fig.

Sol



$$\sum M_B = 0$$

$$R_A \cdot l - 1 \cdot l = 0$$

$$R_A = -\frac{1}{l}$$

$$\sum F_y = 0$$

$$R_A + R_B = 0$$

$$R_B = +\frac{1}{l}$$

(57)

ILD for S_c :-

$$S_c = -\frac{1}{l}$$

ILD for M_c :-

Case - I When 1 kN-m to the left of C

$$M_c = R_B \cdot (l - a)$$

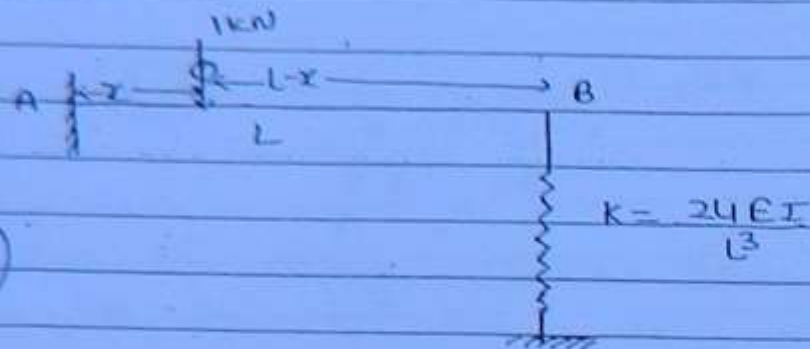
$$M_c = \frac{1}{l} (l - a) \Rightarrow \left(\frac{l - a}{l} \right)$$

Case - II When 1 kN-m is to the right of C

$$M_c = R_A \cdot a$$

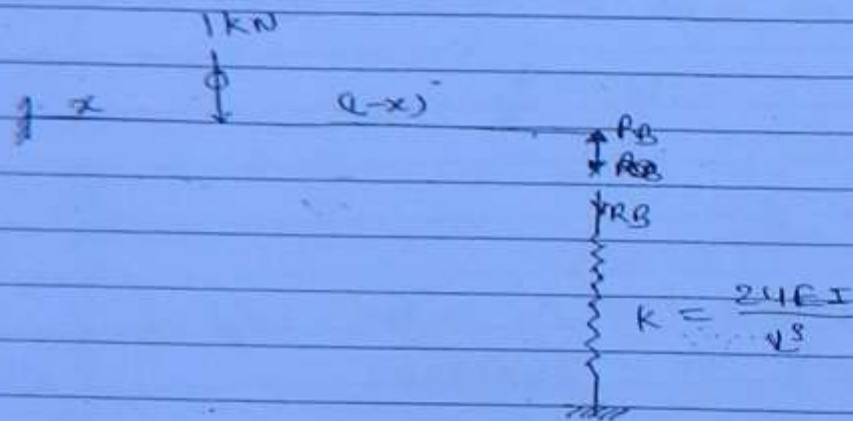
$$\Rightarrow -\frac{1 \cdot a}{l} = -\frac{a}{l}$$

- Q. A cantilever beam AB is supported over a spring as shown in fig. Draw influence line dia. for R_B and M_A when unit concentrated load moves from A to B.



(52)

Sol:-



Let internal Reaction at B is R_B

Downward deflection at B for AB = defⁿ of spring

$$\frac{1 \cdot x^3}{3EI} + \frac{1 \cdot x^2 (L-x)}{2EI} - \frac{R_B L^3}{3EI} \Rightarrow \frac{R_B}{K}$$

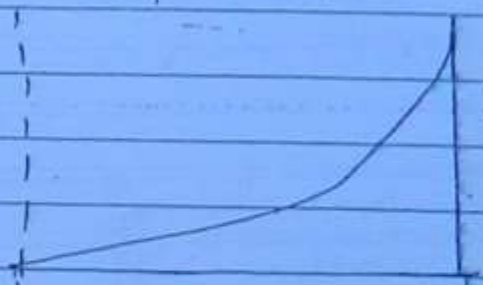
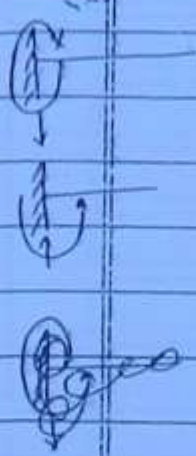
$$\frac{x^3}{3EI} + \frac{x^2 L}{2EI} - \frac{x^3}{2EI} - \frac{R_B L^3}{3EI} = \frac{R_B}{K}$$

$$R_B = \frac{4x^3 (3L-x)}{9L^3}$$

when $x=0$, $R_B=0$

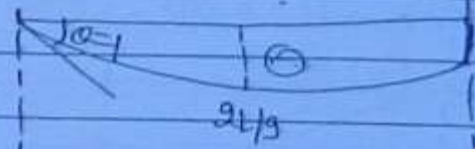
$x=L$, $R_B = \frac{8}{9}$

(53)



ILD for R_B

ILD for M_A



$$M_A = R_B \cdot L - 1 \cdot x$$

$$\Rightarrow \frac{4x^3}{9L^3} (3Lx)L - x$$

$$\Rightarrow \frac{44x^3}{9L^2} (3L-x) - x$$

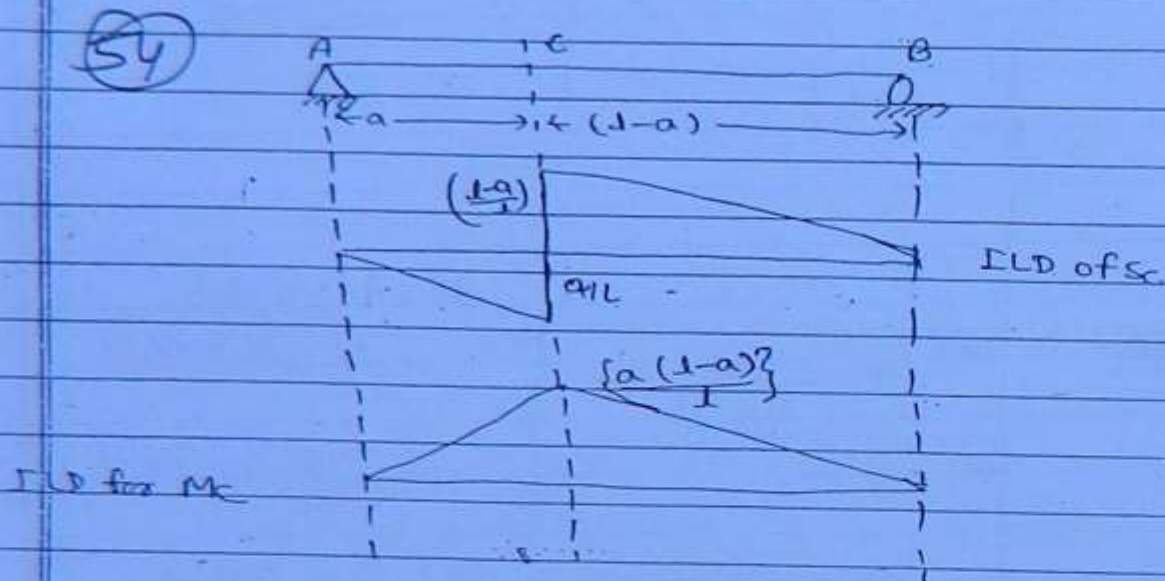
$$\text{At } x=0 \quad M_A=0$$

$$x=L \quad M_A = -4/9$$

$$\text{At } x=L/2 \quad M_A = -\frac{2L}{9}$$

Effect of Rolling load :-

Case-1 Find maximum shear force and B.M. at a section C. when a UDL passes over the girder having its length less than length of girder

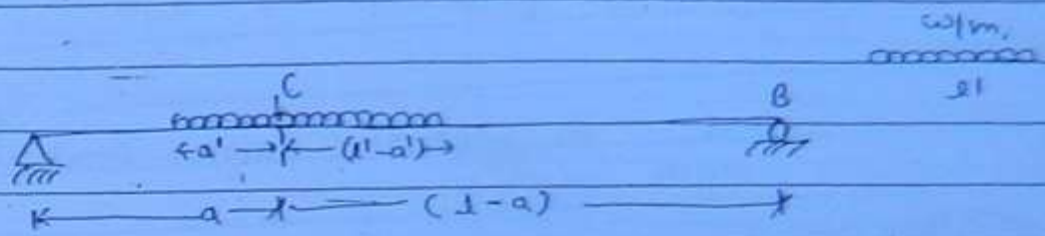


Max^m (-ive) Shear force at C will occur when head of the load is just to C. Load moving left to right.

Max^m (+ive) S.F. at C will occur when tail of the load is just to C. Load moving Right to left.

Max^m B.M. at C will occur average loading to the left of C is equal to average load. Right of C. It means section C divide the load in this same ratio as it divide to the span.

Let length of load is l'



$$\frac{a'}{a} = \frac{l'-a'}{l-a}$$

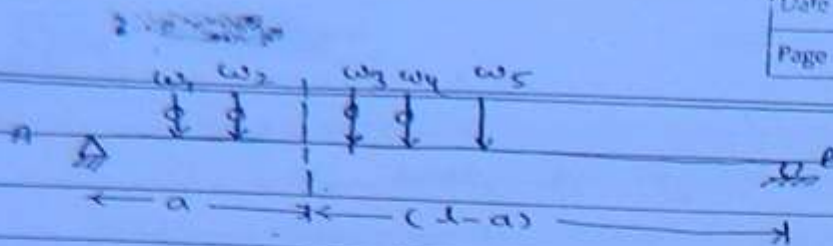
Ans ① The absolute max^m -ive. S.F. will occur at Support B when load is just to the left of B.

H

max^m (+ive) S.F. occur at A when load is just to be right of A.

② due to moving UDL absolute max^m B.M. will occur at the centre of span when avg. loading equal on both side of center means load is symmetrically placed at the centre.

Case 2 when a series of concentrated wheel load crosses a simply supported girder then ~~absolute max^m bending moment occur at the centre of span~~ max^m bending moment any section C occur when loading is placed such that average loading to the left of section C is equal to average loading to the right of section C.



load in left and right part is equal
 length

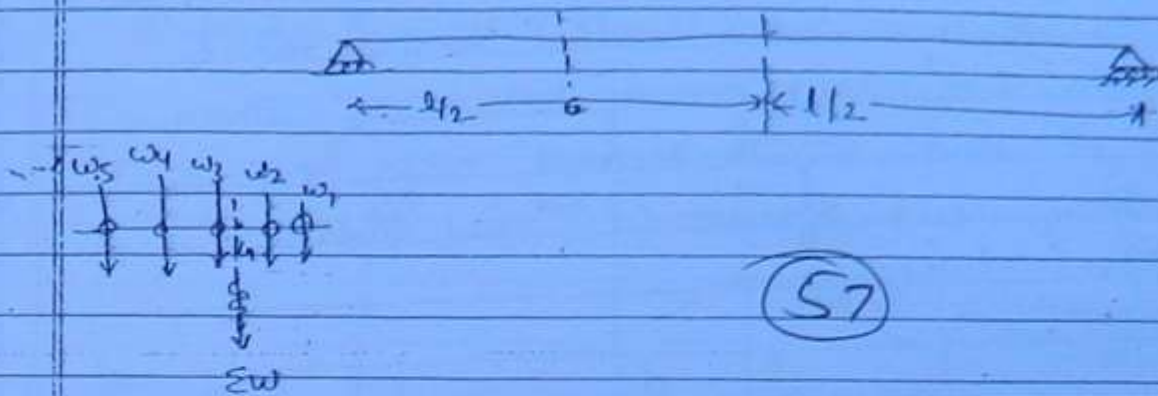
(56)



due to series of wheel load absolute max. bending moment occurs below any one of the wheel load and never between two wheel load when the position of load such that centre of the span is located mid-way b/w C.G. of load system and load. Consideration - load under consideration is either that load which is nearest to C.G. or that w_3 or that load which is next nearest to C.G. (w_2)

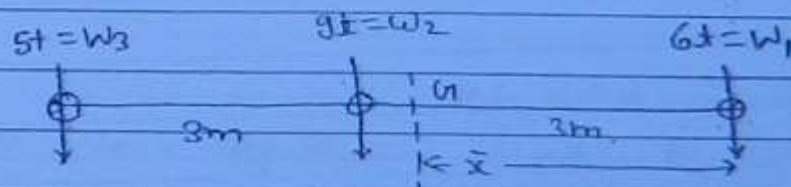
If $w_3 > w_2$ where w_3 is nearest load to C.G. then it is certain that max. bending moment will occur below w_3 but if $w_2 > w_3$ and w_2 is ^{next} nearest to load to C.G. then max. BM may occur either below w_2 or below w_3 , which ever is greater.

If C.G. of load system (G) coincides with any of the wheel load say w_3 then absolute max. bending moment will occur w_3 when w_3 is located just over centre.



Q. A series of 3 wheel load 5 tonne, 9 and 16 tonne spaced 3m center to center cross over a simply supported girder with a span of 10 m. when load move left to right with 6 tonne load leading then find position and magnitude of max. B.M. which may occur anywhere on the girder (that is mean absolute max. value.)

Sol



Distance of C.G. of loading system $\bar{x}$.

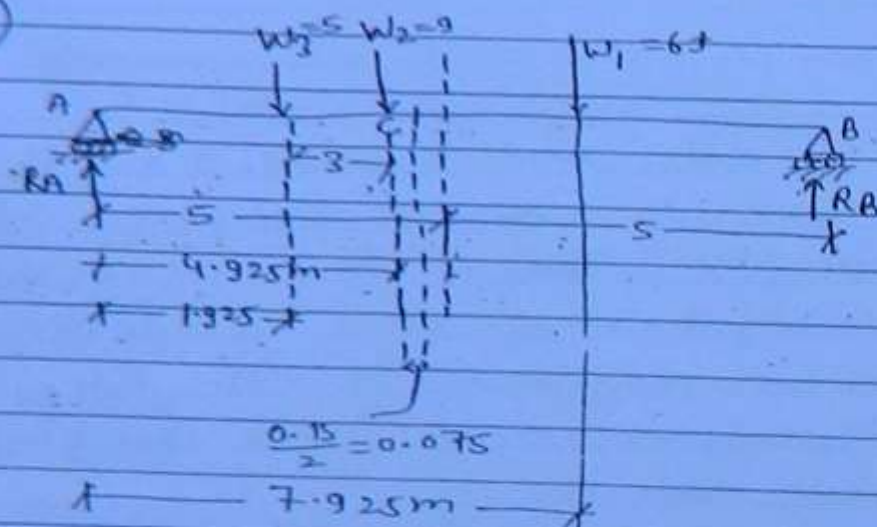
$$\bar{x} = \frac{w_1 \bar{x}_1 + w_2 \bar{x}_2 + w_3 \bar{x}_3}{w_1 + w_2 + w_3}$$

$$\bar{x} = \frac{6 \times 0 + 9 \times 3 + 5 \times 6}{5 + 9 + 6}$$

$$\bar{x} = 2.85 \text{ m}$$

The load nearest to C.G. is w_3 which is heavier than w_1 which is next nearest to C.G. there for maxm B.M. will occur below w_2 when centre of span C. is located mid-way b/w C.G. of load system in and load under consideration (w_3)

(SP)



$$R_A + R_B = 6 + 5 + 9$$

$$R_A + R_B = 20 \quad \text{--- (1)}$$

$$\sum M_A = 0$$

$$R_B \times 10 - 6 \times 7.925 - 9 \times 4.925 - 5 \times 1.925 = 0$$

$$R_B = 10.15 \text{ t}$$

$$R_A = 9.85 \text{ t}$$

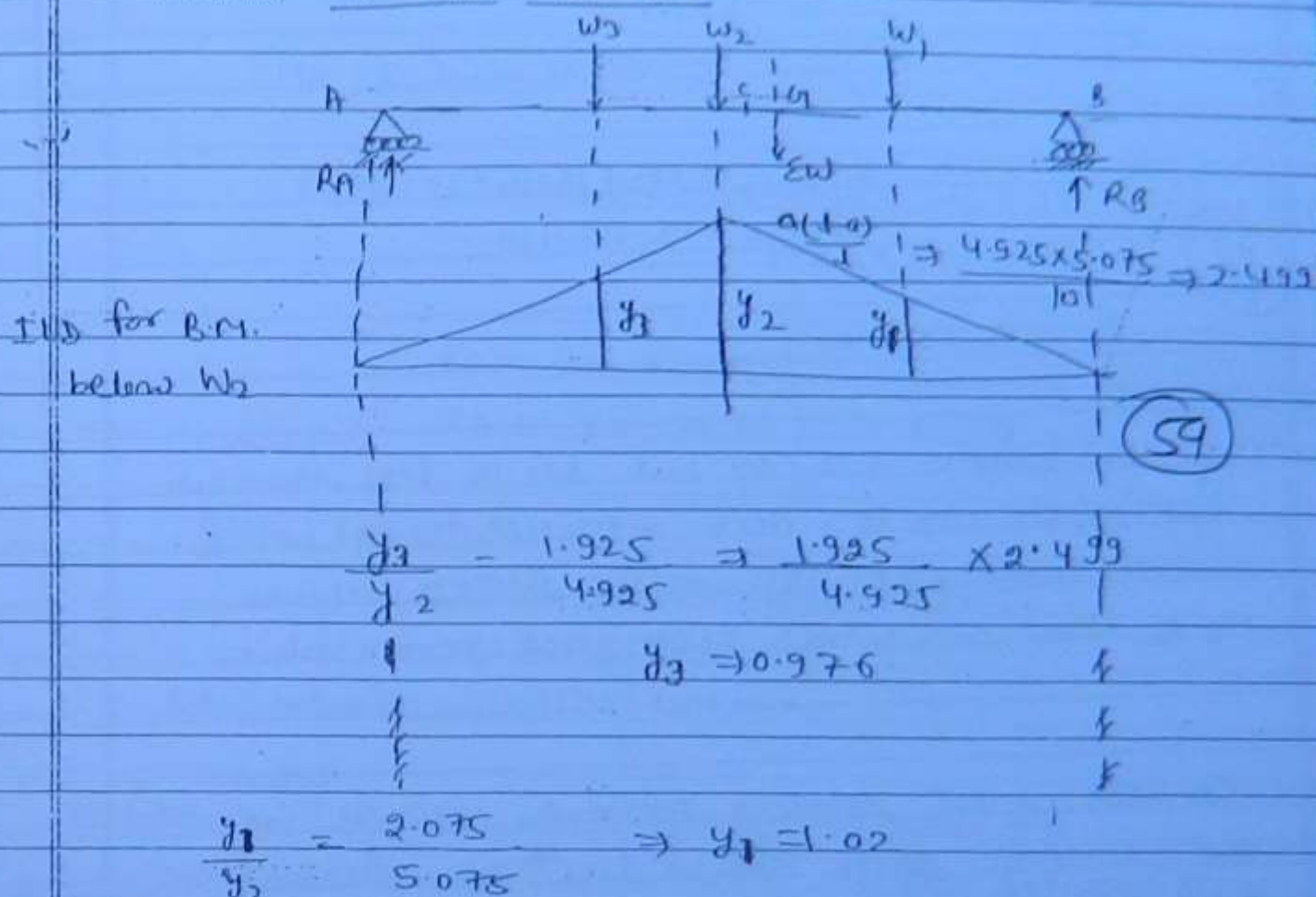
B.M. below w_2

$$R_A \times 4.925 - 5 \times 3 = 0$$

$$R_A = 9.85 \times 4.925 - 15 = 0$$

$$M_{\text{max}} = 33.5 \text{ t-m} \quad \text{occurs } 4.925 \text{ m from A}$$

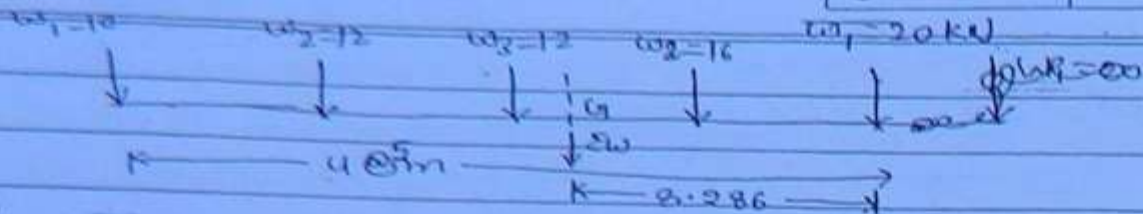
Method 2 Use of ILD



Total B.M. below $w_2 \Rightarrow$

$$\begin{aligned}
 & w_1 y_1 + w_2 y_2 + w_3 y_3 \\
 & \Rightarrow 6 \times 1.02 + 9 \times 2.499 + 5 \times 0.976 \\
 & \Rightarrow 33.5 \text{ k-m}
 \end{aligned}$$

- Q. 5 Point loads of 10kN, 12kN, 16kN and 20kN space at about 5m centre to centre over a simply supported girder of 80m. The load move left to right with 20kN load leading. then calculate position and magnitude of maxm B.M. which may occur any where on the girder.



$$\bar{x} = \frac{20 \times 0 + 16 \times 5 + 12 \times 10 + 12 \times 15 + 10 \times 20}{20 + 16 + 12 + 12 + 10}$$

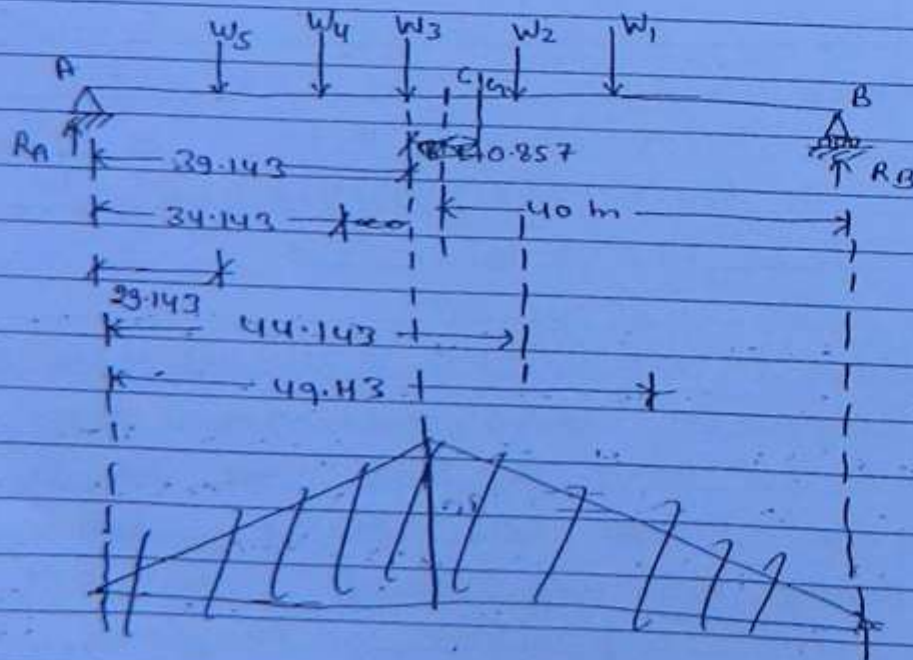
60

$$\bar{x} = 8.286 \text{ m}$$

$\therefore W_3$ is nearest to C.G. but W_2 is less than W_3 but where W_2 is next nearest to C.G. hence absolute maxm. BM will occur either below W_3 or below W_2 which ever gives greater value.

Case-1

Consider maxm. Bending moment occurs ^{below} W_3 . The centre span should lie ^{by} C.G. of load system. Load Under consideration (W_3)



$$R_A + R_B = 20 + 16 + 12 + 12 + 10 = 70$$

$$\Sigma M_A = 0$$

(6/)

$$R_B \times 80 - 20 \times 49.143 - 16 \times 44.143 - 12 \times 39.143 - 12 \times 34.143 - 10 \times 29.143 = 0$$

$$R_B = 35.753 \text{ kN}$$

$$R_A = 70 - 35.753 = 34.25 \text{ kN}$$

B.M. below w_2

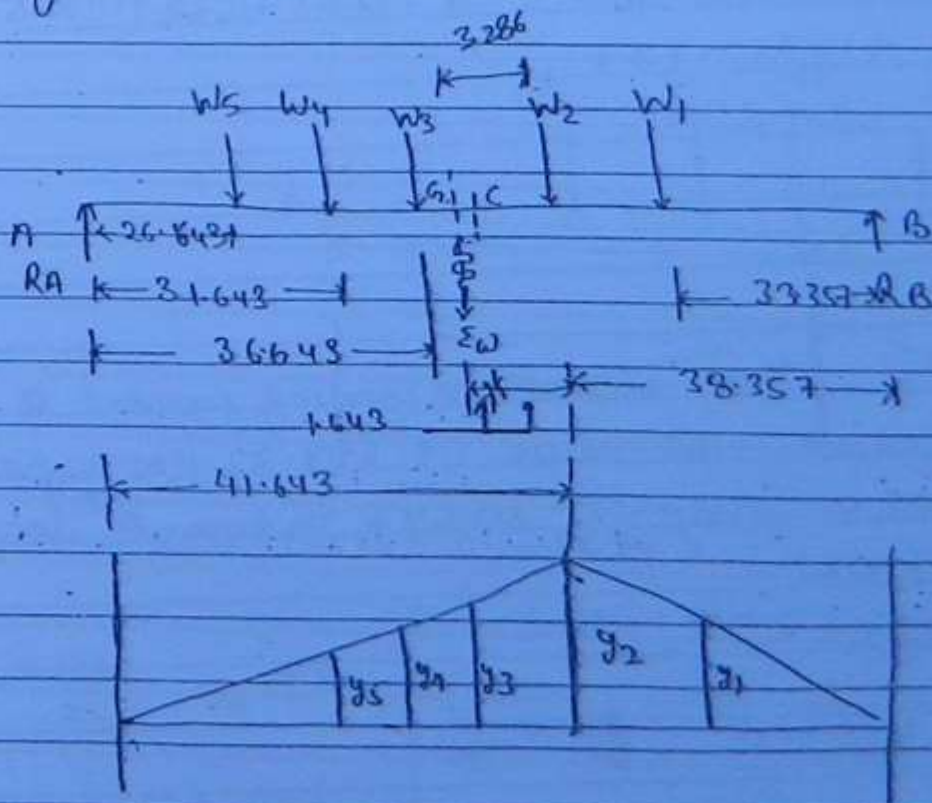
$$\rightarrow R_A \times 39.143 - 10 \times 10 - 12 \times 5$$

$$\Rightarrow 34.25 \times 39.143 - 100 - 60$$

$$\text{B.M. below } w_3 \Rightarrow 1180.65 \text{ kN-m.}$$

~~Method 2~~ Consider max^m B.M. occur below w_2

Hence, centre of span to be located between c.g. of local system and w_2 .



$$y_2 = \frac{a(1-a)}{1} \Rightarrow \frac{41.643 \times (80 - 41.643)}{80}$$

$$y_2 \Rightarrow \frac{41.643 \times 38.357}{80}$$

62

$$y_2 = 19.966$$

$$\frac{y_1}{y_2} = \frac{38.357}{38.357}$$

$$\Rightarrow y_1 = 17.367$$

$$\frac{y_3}{y_2} = \frac{36.643}{41.643}$$

$$\Rightarrow y_3 = 17.568$$

$$y_4 = 15.1715$$

$$y_5 = 12.774$$

Total BM below w_2

$$\Rightarrow w_1 y_1 + w_2 y_2 + w_3 y_3 + w_4 y_4$$

$$\Rightarrow 20 \times 17.363 + 16 \times 19.966 + 12 \times 17.568 + 12 \times 15.1755 + 10 \times 12.774$$

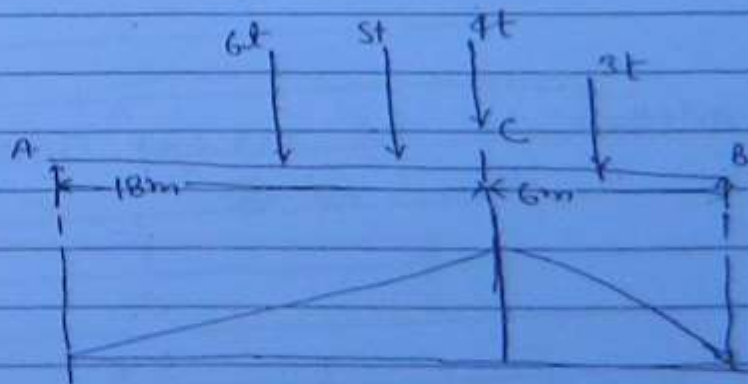
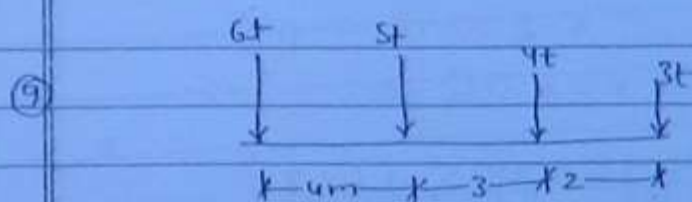
$$\Rightarrow 1187.33 \text{ kN}$$

The absolute bending moment is greater of the above so value i.e. 1187.33 kN-m which occurs at a distance 41.643 m from A ^{below} load 16 kN (w_2)

- | | | | |
|-----|-----|-----|------------------|
| ① b | ⑦ a | ⑬ b | ⑲ d |
| ② b | ⑧ d | ⑭ c | ⑳ only c correct |
| ③ b | ⑨ b | ⑮ b | ㉑ c |
| ④ b | ⑩ a | ⑯ d | ㉒ d |
| ⑤ a | ⑪ c | ⑰ d | ㉓ c |
| ⑥ c | ⑫ d | ⑱ b | ㉔ a |
| ㉕ d | ㉖ b | ㉗ d | ㉘ c |
| ㉙ a | | | |

63

② $\text{Load} \times \frac{1}{2} \times 16 \times 3$
 $\Rightarrow 2 \times 24$
 $\Rightarrow 48 \text{ m Ae}$



$$M_{C1} = 3 \times y_1 + 4 \times y_2 + 5 \times y_3 + 6 \times y_4$$

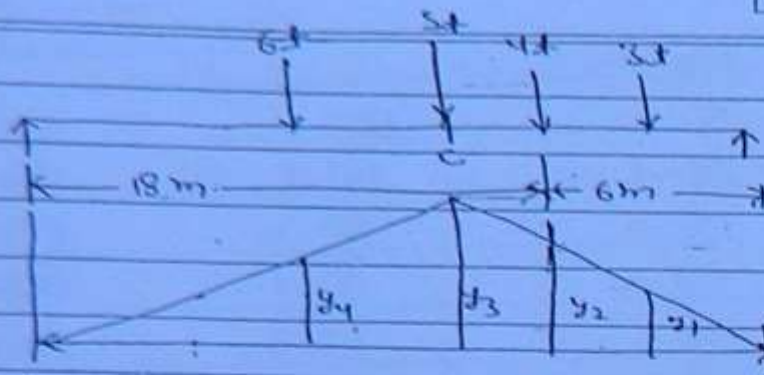
$$M_{C1} \Rightarrow 3 \times 3 + 4 \times 4.5 + 5 \times 3.75 + 6 \times 2.75 \Rightarrow 62.25$$

$$y_2 = \frac{18 \times 6}{24} \Rightarrow 4.5$$

$$\frac{y_3}{y_2} = \frac{18}{12} \quad y_3 = \frac{18}{12} \times 4.5 = 3.75$$

$$y_1 = \frac{y_1}{y_2} \Rightarrow \frac{4}{6} = 3$$

$$y_4 = \frac{11}{8} \times 3.75 \Rightarrow 2.75$$



64

$$y_3 = 4.5$$

$$\frac{y_1}{y_3} = \frac{1}{6} \Rightarrow \frac{4.5}{6} \Rightarrow 0.75$$

$$\frac{y_2}{y_3} = \frac{3}{6} \Rightarrow y_2 = 2.25$$

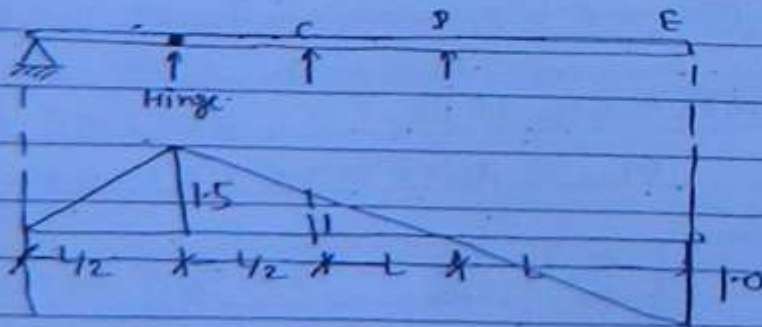
$$y_4 = \frac{14}{18} \times 4.5 = 3.5$$

$$M = 3 \times 0.75 + 2.25 \times 4 + 5 \times 4.5 + 6 \times 3.5$$

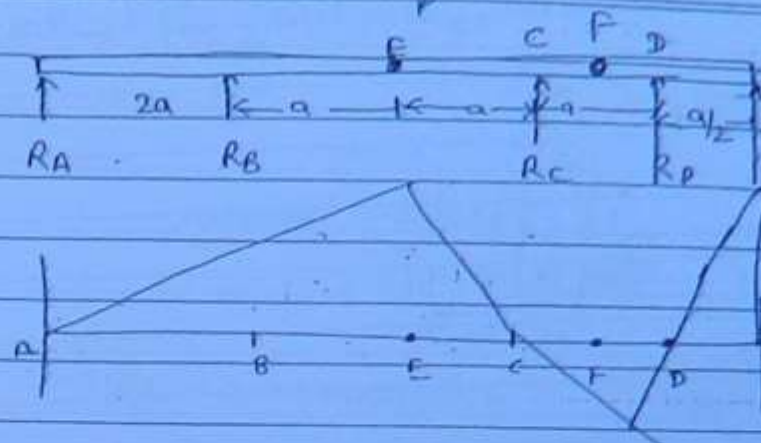
$$M = 54.75$$

- (10) indeterminate the I/D Not (linear)
than Ans - (A)

(15)

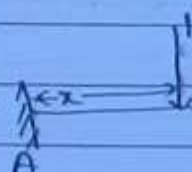


(17)



(65)

(19)


 $\uparrow B$
 R

$$R = f(x)$$

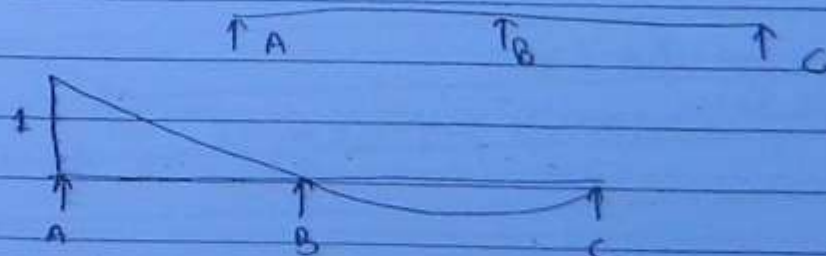
$$M_A = R \cdot l - 1 \cdot x$$

$$\Rightarrow f(x) l - x$$

$$\Rightarrow = [x - f_n(x) l]$$

(20)

(11)



(ii)



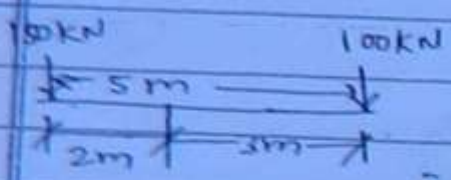
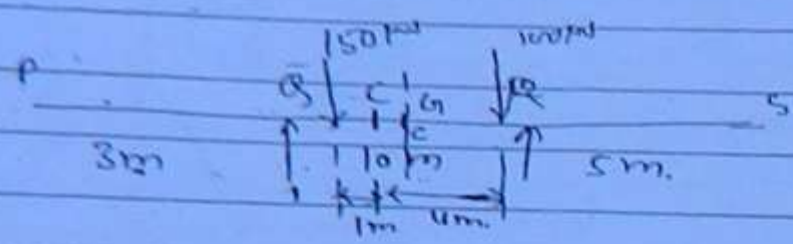
Only C statement is correct and all
 statements wrong.

21

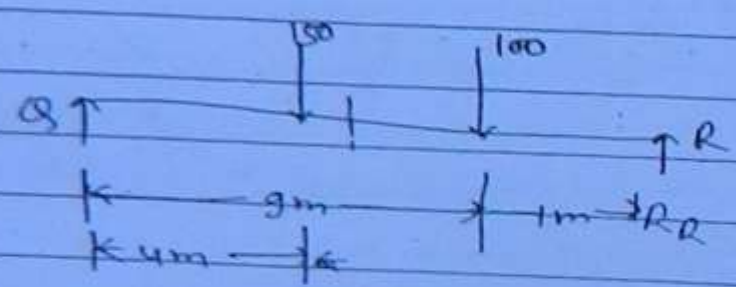
then determinate structure and find C

93

66



$$\bar{x} = \frac{150 \times 5 + 100 \times 3}{150 + 100} = \frac{1250}{250} = 5$$



$$R_B \times 10 - 100 \times 9 - 150 \times 4 = 0$$

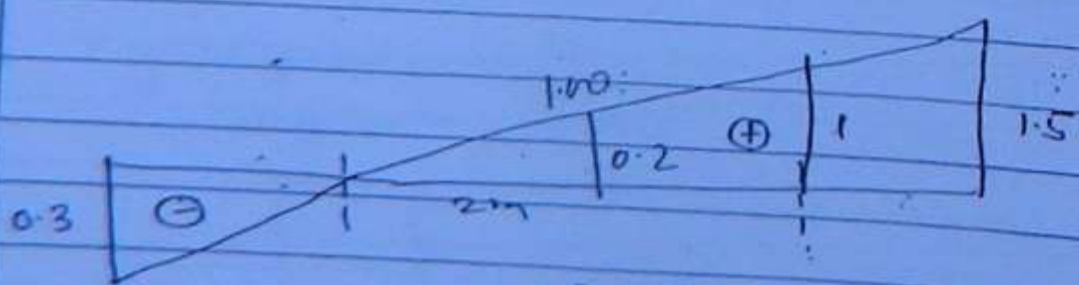
$$R_B = 150$$

$$R_A = 100$$

$$M_{max} = R_A \times 4$$

$$= 100 \times 4 = 400 \text{ kN-m}$$

24



ILD for Reaction of R

$$\text{Case-I} \Rightarrow 100 \times 1.5 + 150 \times 1$$

$$\Rightarrow 300$$

(max)

Ans

Case-II

$$150 \times 1.5$$

$$\Rightarrow 225$$

Case-III

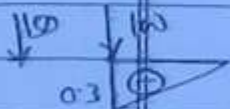
$$-150 \times 0.3 + 100 \times 0.2$$

$$\Rightarrow -25$$

Ans

than. 150 kN out of C.B.M. than.

$$-100 \times 0.3 \Rightarrow -30$$

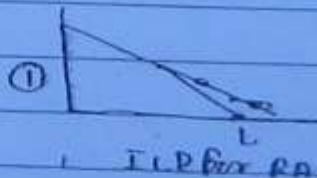


(25)

$$150 \times 5 = 750$$



(26)



not and

(27) not and

$$\textcircled{b} S_B = R_A - 1$$

$$R_A \times L - 1(L-x) = 0$$

$$R_A = \frac{(L-x)}{L}$$

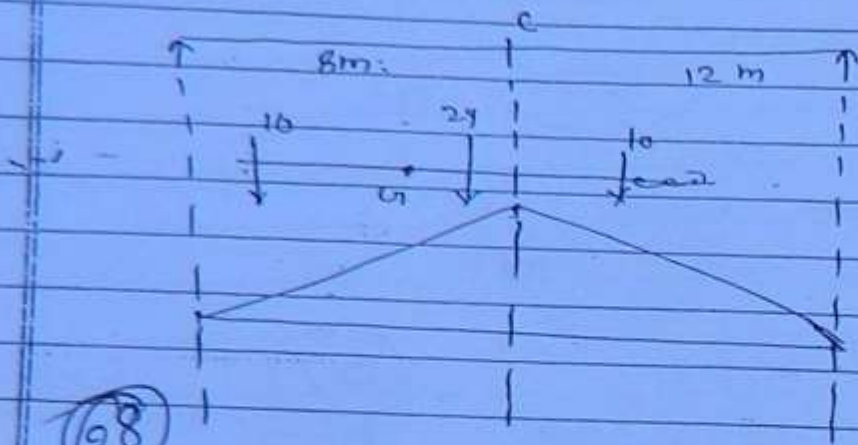
$$S_B = R_A - 1$$

$$= \frac{L-x}{L} - 1 = \frac{L-x-L}{L} = \frac{-x}{L}$$

$$S_B = R_A - 1$$

$$R_A + S_B = 1$$

27

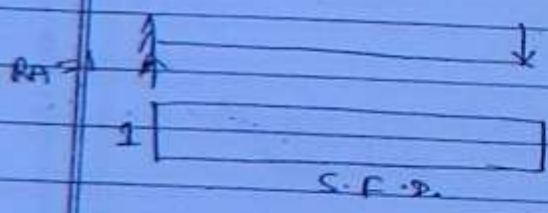
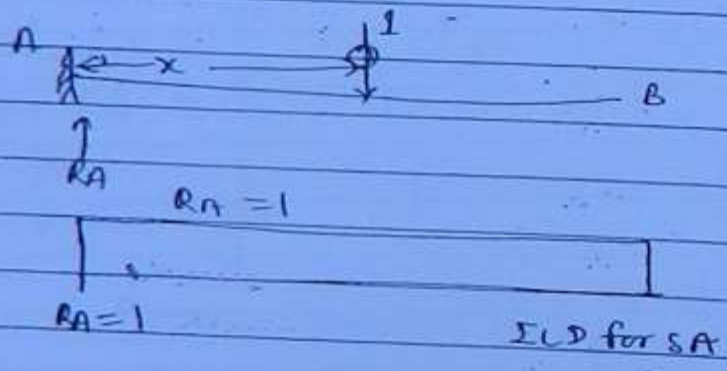


68

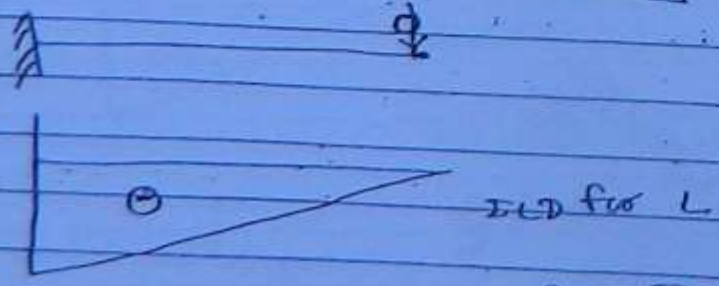
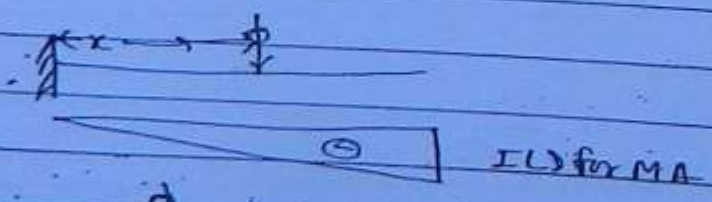
Ans - d

28 Statement

(A)



Statement B

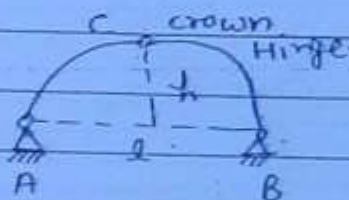


Ans - (C)

Arches

(69)

3-Hinged



2 Hinged



Fixed



A & B support Hinges

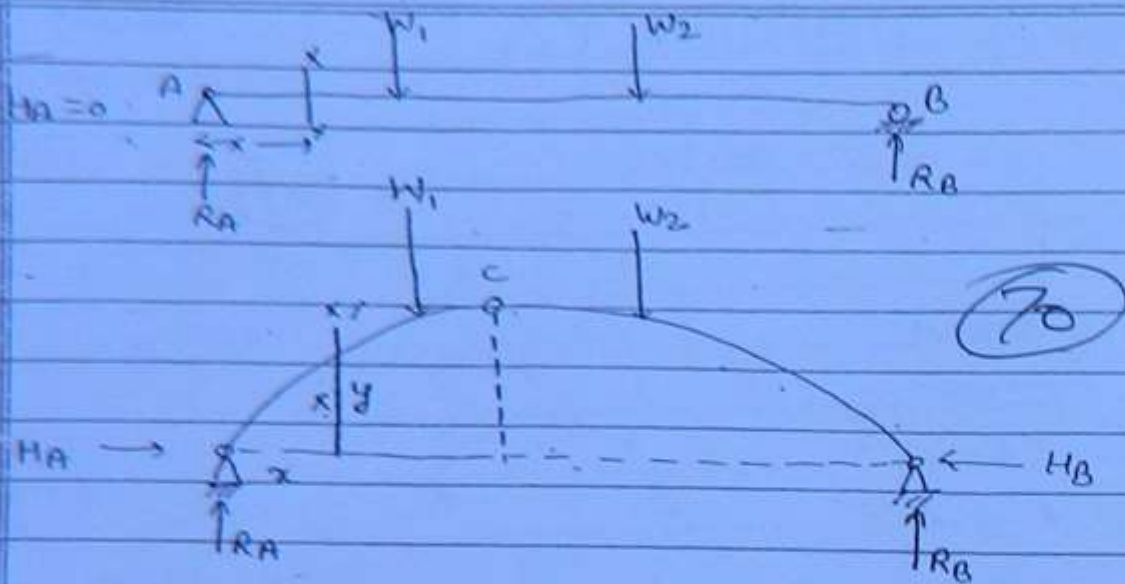
Indeterminate to
1st order

Indeterminate
to 3rd order

Determiner or
Abutment Hinges
or
Springings

Arches structure able to resist S.F., B.M. & Axial Thrust. All As compare to simply supported beam of equal span ~~is~~ under equal load free Hinged arch requires smaller section because in Arch.

Horizontal reactions are developed which reduced the B.M. Because B.M. produce by vertical reaction is sagging and B.M. produce by Horizontal reaction is hogging.



B.M. at x-x in Beam $= R_A \cdot x$

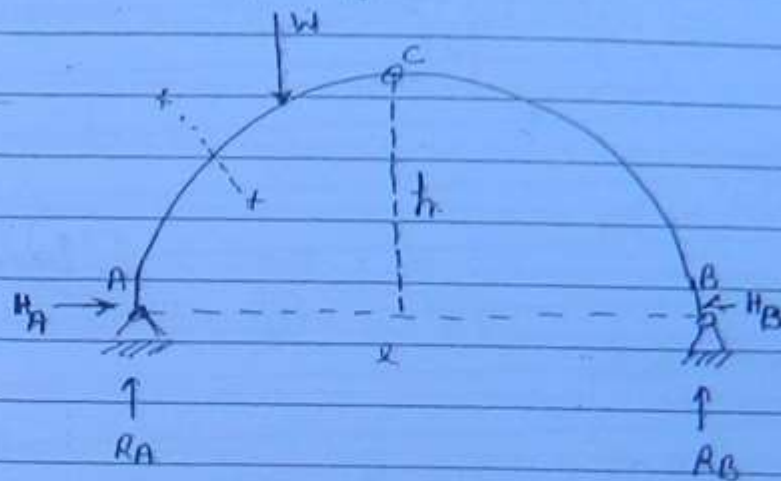
B.M. at x-x in Arch $= R_A \cdot x - H_A \cdot y$

Arch Moment $\Rightarrow$ Beam Moment - H moment

Generally for long span Arch structure are economical because Net B.M. Moment developed in Arches is less than net B.M. developed in Beam hence thinner section is needed however for small span beams are preferred.

In case of multi story structure and from aesthetic consideration arches are less perfect.

Analysis of three-hinged arch:-



Total Reaction for total equation

$$\text{Total Reaction} = 4$$

$$\text{Total Equation} = 3 + 1$$

$$\begin{array}{cccc} \downarrow & \downarrow & \downarrow & \downarrow \\ \Sigma F_x = 0 & \Sigma F_y = 0 & \Sigma M_z = 0 & M_c = 0 \end{array}$$

$$DS = 0 \quad \text{Stable \& Determinate.}$$

If there is no horizontal load

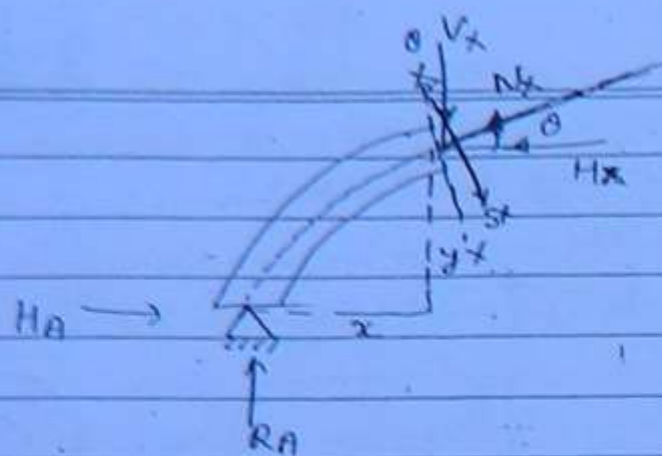
$$\text{then } \Sigma F_x = 0$$

$$H_A - H_B = 0$$

$$H_A = H_B = H \text{ (say)}$$

Following 3 types of forces are developed.





(72)

① B.M. at X-X

$$M_x = R_A \cdot x - H_A \cdot y$$

② S.F. at X-X / Radial Shear

$$S_x = V_x \cdot \cos \theta - H_x \cdot \sin \theta$$

③ Normal Thrust / Axial thrust (N_x) X-X

$$N_x = V_x \sin \theta + H_x \cos \theta$$

$$\tan \theta = \frac{dy}{dx}$$

For Parabolic Arch

$$y = \frac{4h}{l^2} (1-x)x$$

*** $\frac{dy}{dx} = \tan \theta \Rightarrow \frac{4h}{l^2} (1-2x)$

To compute V_x & H_x at $X-X$ consider free body equilibrium on ~~both~~ left of $X-X$

$$\Sigma F_x = 0$$

(73)

$$H_A - H_x = 0$$

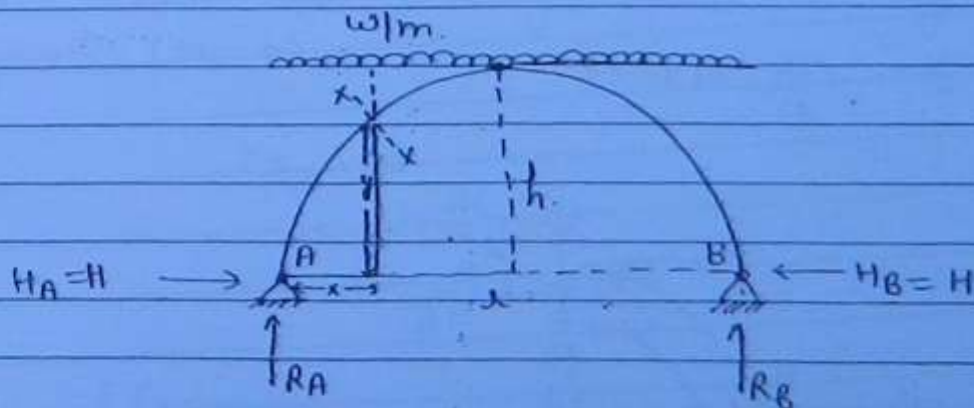
$$\boxed{H_A = H_x}$$

$$\Sigma F_y = 0$$

$$R_A - V_x = 0$$

$$\boxed{R_A = V_x}$$

Case-2 Three Hinge Parabolic Arch subjected ^{to} UDL over entire span.



$$\Sigma F_x = 0 \quad \text{--- (i)}$$

$$H_A - H_B = 0$$

$$H_A = H_B = H$$

$$\Sigma F_y = 0$$

$$R_A + R_B - wl = 0 \quad \text{--- (ii)}$$

$$\sum MB = 0$$

$$R_A \times 1 - W \times L \times \frac{L}{2} = 0$$

$$R_A = \frac{WL}{2}$$

$$R_B = \frac{WL}{2}$$

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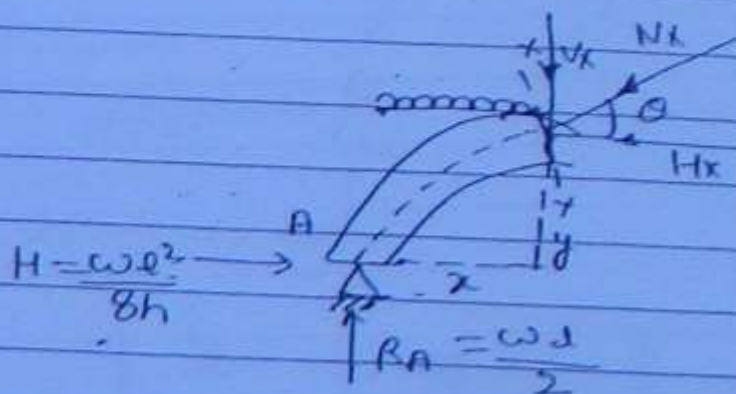
taking moment about C

$$R_A \cdot \frac{l}{2} - H \cdot h - W \cdot \frac{l}{2} \cdot \frac{l}{4} = 0$$

$$\frac{Wl^2}{4} - Hh - \frac{Wl^2}{8} = 0$$

* Imp. $H = \frac{Wl^2}{8h}$ Imp. *

$$y = \frac{4h}{l^2} x(1-x)$$



B.M. at $x-x$

$$M_x = R_A \cdot x - H \cdot y - \omega x \cdot \frac{x}{2}$$

put the value

$$M_x = \frac{\omega l}{2} \cdot x - \frac{\omega l^2}{8h} \cdot \frac{4h}{l^2} x(l-x) - \frac{\omega x^2}{2} \quad (75)$$

$$M_x = \frac{\omega l}{2} x - \frac{\omega l}{2} x + \frac{\omega x^2}{2} - \frac{\omega x^2}{2}$$

$$M_x = 0$$

Imp

"The 3-Hinge Parabolic Arch Subjected to u.d.l over entire span Bending moment at any section is zero" ($M_x = 0$)

$$\sum F_x = 0$$

$$H - H_x = 0$$

$$H = H_x = \frac{\omega l^2}{8h}$$

$$\sum F_y = 0$$

$$R_A - \omega \cdot x - V_x = 0$$

$$V_x = R - \omega \cdot x = \frac{\omega l}{2} - \omega x$$

Radial Shear/SF

$$S_x = V_x \cos \theta - H_x \sin \theta$$

$$= \left(\frac{\omega l}{2} - \omega \cdot x \right) \cos \theta - \frac{\omega l^2}{8h} \cdot \sin \theta$$

$$S_x = \cos \theta \cdot w \left[\frac{l}{2} - x - \frac{l^2}{8h} \tan \theta \right]$$

$$y = \frac{4h}{l^2} x(l-x)$$

$$\frac{dy}{dx} = \tan \theta = \frac{4h}{l^2} (l-2x)$$

(76)

$$S_x = \cos \theta \cdot w \left[\frac{l}{2} - x - \frac{l^2}{8h} \cdot \frac{4h}{l^2} (l-2x) \right]$$

$$S_x = \cos \theta \cdot w \left[\frac{l}{2} - x - \frac{l}{2} + x \right]$$

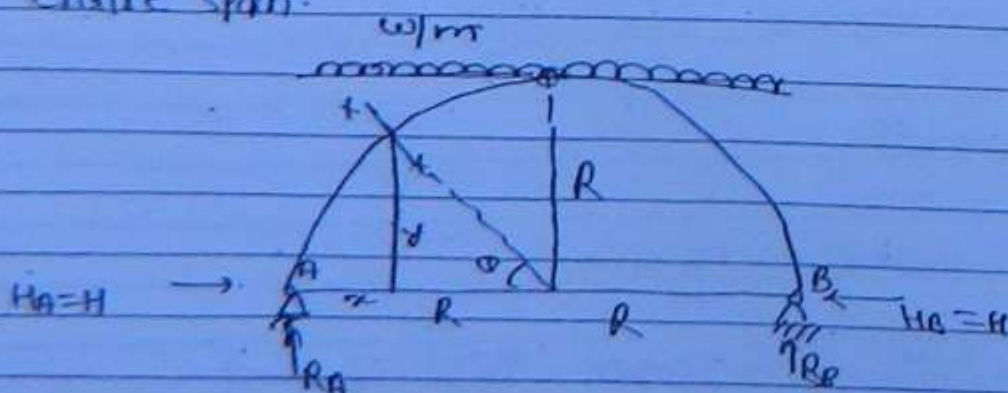
$$S_x = 0$$

"it means 3-Hinge Parabolic arch with udl on with entire span is free from S.F. and B.M."

$$S_x \text{ and } M_x = 0$$

However, Normal Thrust will exist

Case-2 3-Hinge semi-circle Arch subject to udl over entire span.



$$\sum F_y$$

$$R_A + R_B - w \cdot 2R = 0$$

$$R_A + R_B = 2wR$$

(77)

$$\sum M_A = 0$$

$$R_B \times 2R - w \times 2R \cdot R = 0$$

$$R_B = wR$$

$$R_A = wR$$

Moment about A $M_c = 0$

$$R_A \times R - H \cdot R - \frac{wR \cdot R}{2} = 0$$

$$wR \cdot R - HR - \frac{wR^2}{2} = 0$$

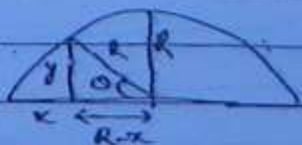
$$HR = \frac{wR^2}{2} - \frac{wR^2}{2} \Rightarrow \frac{wR^2}{2}$$

$$\boxed{H = \frac{wR}{2}}$$

Horizontal Reaction is $\frac{1}{2}$ of vertical Reaction.

B.M. at X-X.

$$M_x = R_A \cdot x - H \cdot y - \frac{w \cdot x \cdot x}{2}$$



$$\frac{y}{R} = \sin \theta$$

$$y = R \sin \theta$$

$$\frac{R-x}{R} = \cos \theta$$

$$R - x = R \cos \theta$$

$$x = R(1 - \cos \theta)$$

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$$M_x = R_H A \cdot x - H_y \cdot \omega \cdot x \cdot \frac{x}{2}$$

$$\Rightarrow \omega R \cdot R(1 - \cos \theta) - \frac{\omega R}{2} (R \sin \theta) - \frac{\omega}{2} R^2 (1 - \cos \theta)^2$$

$$M_x = \omega R^2 \left[\frac{1 - \cos \theta - \sin \theta}{2} - \frac{1}{2} (1 - \cos \theta)^2 \right]$$

$$\Rightarrow \omega R^2 \left[\frac{1 - \cancel{\cos \theta} - \sin \theta}{2} - \frac{1}{2} - \frac{\cos^2 \theta}{2} + \cos \theta \right]$$

$$M_x = \omega R^2 \left[\frac{1}{2} - \frac{\sin \theta}{2} - \frac{\cos^2 \theta}{2} \right]$$

$$M_x = \frac{\omega R^2}{2} [\cancel{\sin \theta} 1 - \sin \theta - \cos^2 \theta]$$

$$M_x = \frac{\omega R^2}{2} [\sin^2 \theta - \sin \theta]$$

$$M_x = -\frac{\omega R^2}{2} [\sin \theta - \sin^2 \theta]$$

$$\text{for } M_{\max} \quad \frac{dM}{d\theta} = 0$$

$$\Rightarrow [\cos \theta - 2 \sin \theta \cdot \cos \theta] = 0$$

$$\cos \theta (1 - 2 \sin \theta) = 0$$

$$\sin \theta = \frac{1}{2}$$

$$\text{or } \cos \theta = 0$$

$$\theta = 30^\circ$$

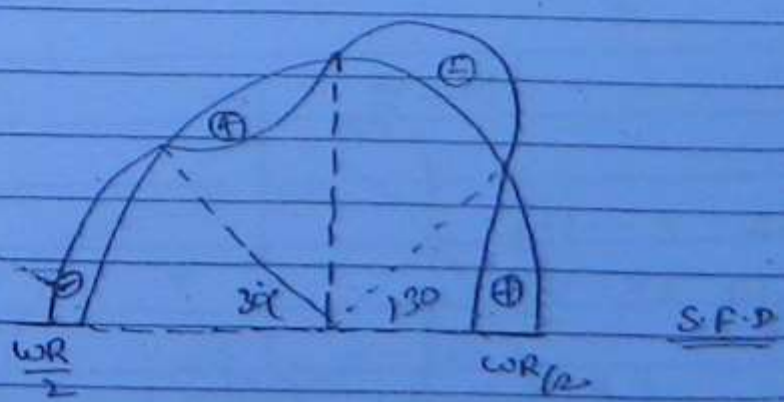
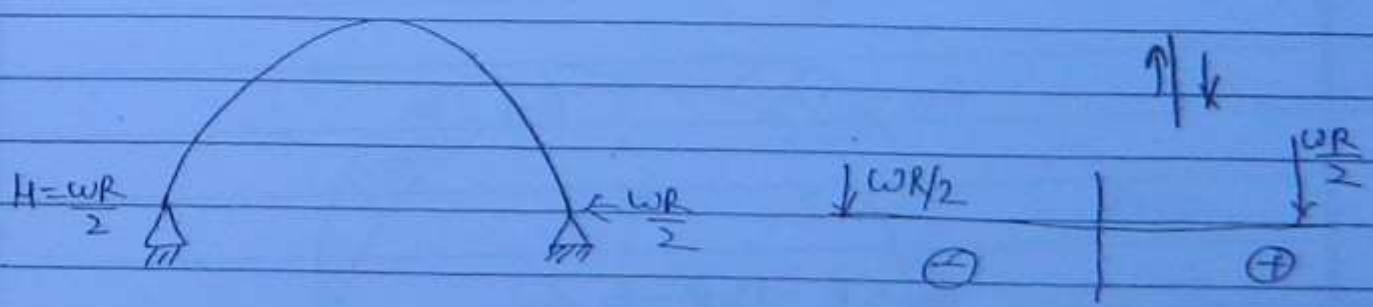
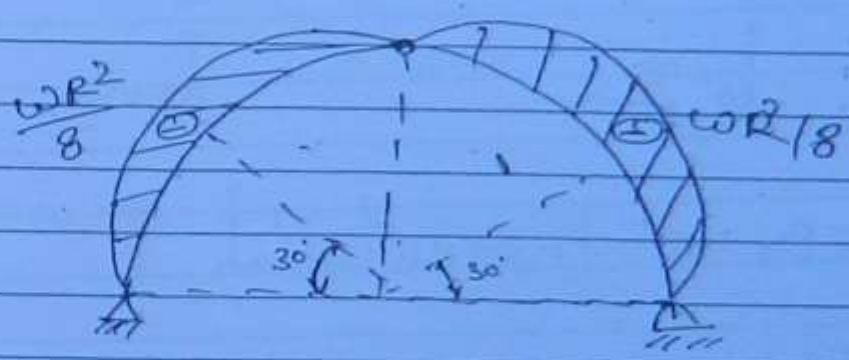
$$\cos \theta = 0 \Rightarrow \theta = 90^\circ$$

max^m Horizontal P.M. occurs at $\theta = 30^\circ$

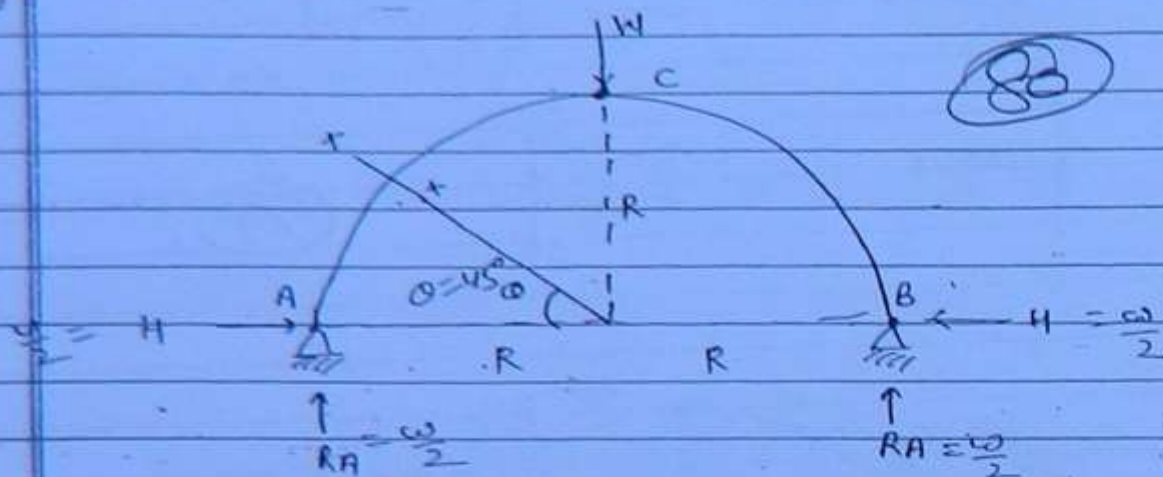
$$M_{max} = -\frac{wR^2}{2} \left[\frac{1}{2} - \frac{1}{4} \right]$$

$$M_{max} = -\frac{wR^2}{8}$$

79



Q. for 3 Hinged semi-circular Arch subject to concentrated load w at the crown find horizontal thrust & S.F. and B.M.



$$\sum F_y = 0 \quad R_A + R_B - w = 0$$

$$\sum M_A = 0$$

$$R_B \times 2R - w \times R = 0$$

$$R_B = \frac{w}{2}$$

$$R_A = \frac{w}{2}$$

$$M_C = 0$$

$$R_A \times R - H \cdot R = 0$$

$$\frac{w}{2} \times R - H \cdot R = 0$$

$$H = \frac{w}{2}$$

$$M_x = R_A \cdot x - H \cdot y$$

$$= \frac{w}{2} (x - y)$$

$$M_x = \frac{w}{2} [R(1 - \cos \theta) - R \sin \theta]$$

$$M_x = \frac{wR}{2} [1 - \cos\theta - \sin\theta]$$

$$\text{at } \theta = 0$$

$$M_x = 0$$

$$\text{at } \theta = \pi/2$$

$$M_x = 0$$

(8)

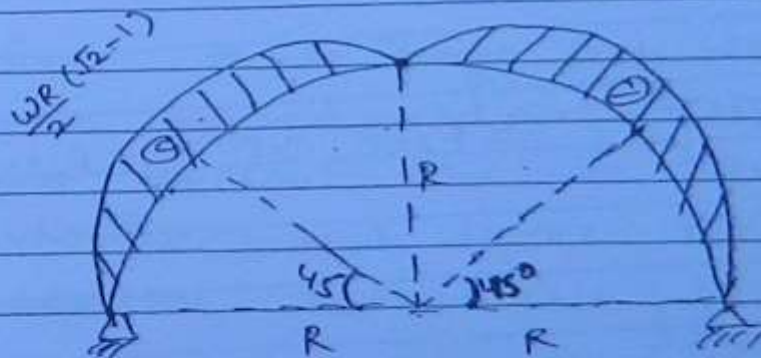
$$\frac{dM_x}{d\theta} = \frac{wR}{2} [0 + \sin\theta - \cos\theta] = 0$$

$$\Rightarrow \sin\theta - \cos\theta = 0$$

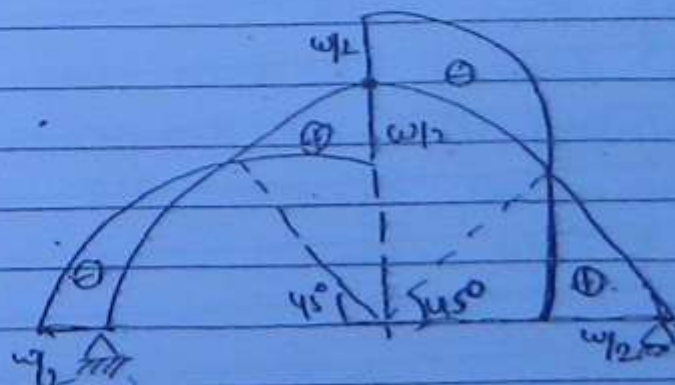
$$\theta = 45^\circ$$

$$M_{\max} = \frac{wR}{2} \left[1 - \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \right]$$

$$\Rightarrow \frac{wR}{2} (\sqrt{2} - 1)$$



BMD



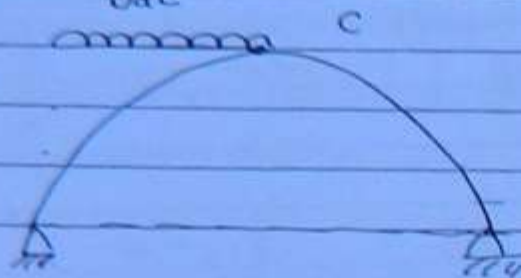
SFD

UdL for Half span.

H.W

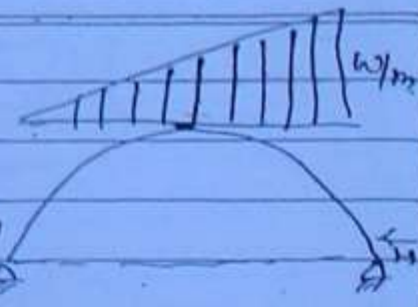
1/2

H



← H

H

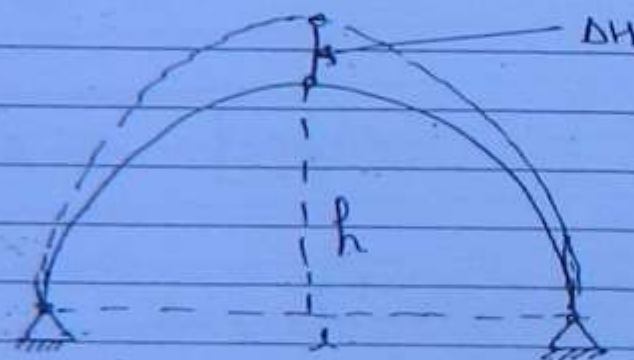


$$H = \frac{w r^2}{16h}$$

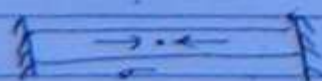
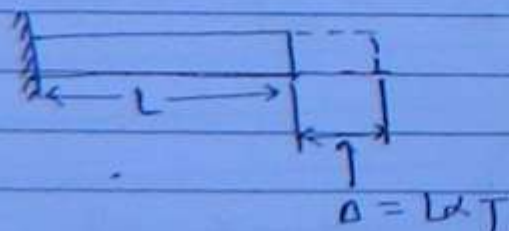
(82)

$$H = \frac{w r^2}{16h}$$

Temperature effect on 3-hinged Arches:-

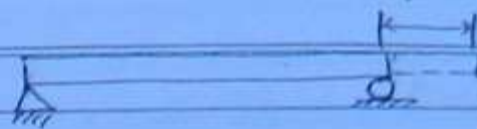


$$\Delta h = \left(\frac{l^2 + 4h^2}{4h} \right) \alpha T = \text{Free Exp}^n \text{ in Rise of Arch}$$



$$\Delta = 0$$

$$\sigma = E \alpha T$$



(83)

A 3-hinged Arch is a determinate structure and due to Rise in temp. Vertical Rise (h) will increase by Δh which is called free expansion.

A There will be two cases.

Case-1 If Arch is initially unloaded but when horizontal and vertical Reaction are not present then due to rise in temp. Vertical height will increase by Δh but horizontal and vertical reaction at the supports will remain zero.

Case-2 If before temp. change Arch is loaded due to which horizontal and vertical Reactions are present due to rise in temp. vertical height increase and hence Horizontal thrust is ~~reduced~~ reduced. ($H \propto \frac{1}{h}$)

There will be no change in vertical reaction let arch is loaded with udl due to load Horizontal thrust ($H = \frac{wl^2}{8h}$). if due to temp. rise by $T^\circ C$ 3-hinge Horizontal thrust is ΔH

$$\Delta H = \frac{wl^2}{8} \left(-\frac{1}{h^2} \Delta h \right)$$

$$\left[\frac{\Delta H}{H} = -\frac{\Delta h}{h} \right]$$

It means % decrease in Horizontal thrust is equal to % increase in vertical Rise.

Note

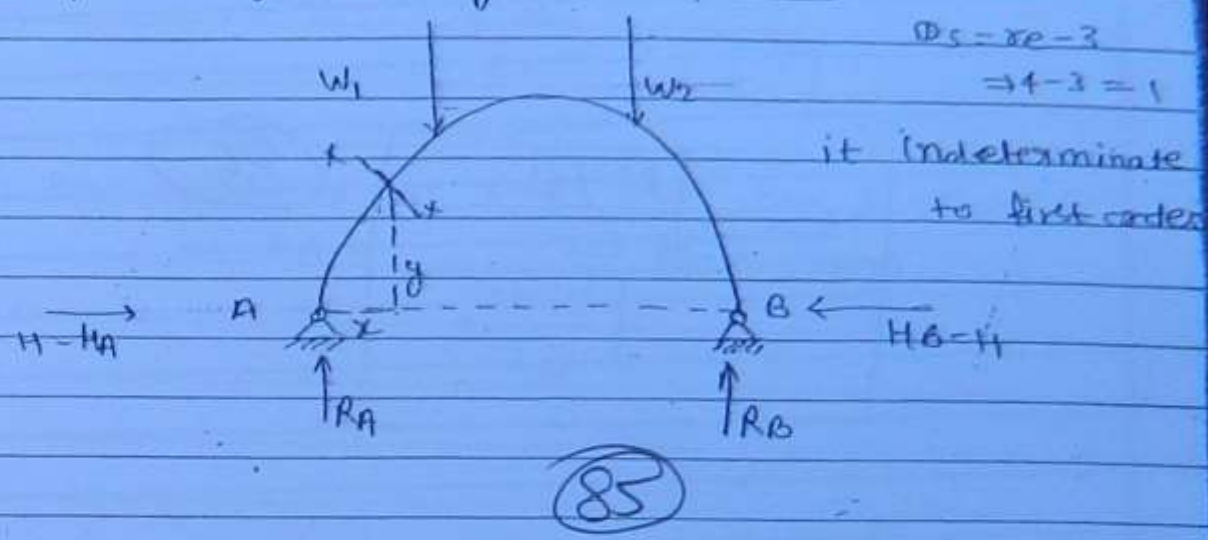
(84)

Generally due to temp. effect determinate structure undergoes free expn. Hence no reactions are produced due to temp. change, provided structure is unloaded but in indeterminate structure due to temp. change reactions are produced.

To this as

2-Hinge arch is an indeterminate structure hence due to temp. change Horizontal thrust is induced at the supports even when arches unloaded. And Horizontal thrust will be increase if Arch was initially loaded.

(H) Analysis of 2-Hinged Arches:-



2-Hinge arch indeterminate structure. Hence additional compatibility eqn is required. Since support at A and B are not moveable. Hence $\left(\frac{\partial U}{\partial H}\right) = 0$. U is total strain energy. and Since effect of bending moment is much greater, ~~the~~ greater than normal thrust and shear force, then U is bending strain energy.

$$U = \int \frac{M_x^2}{2EI} ds$$

$$M_x = R_A \cdot x - H \cdot y$$

$$\Rightarrow M - H \cdot y$$

M = B.M. due to vertical force only this is called = Beam moment

$$U = \int \frac{(M - Hy)^2}{2EI} ds$$

$$\frac{\partial U}{\partial H} = \int \frac{\partial}{\partial H} \left[\frac{(M - Hy)^2}{2EI} \right] ds = 0$$

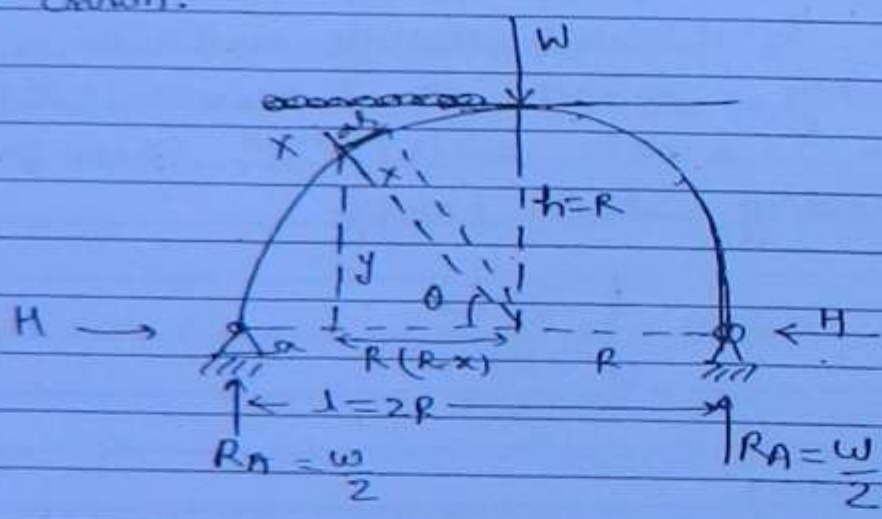
$$\frac{\partial v}{\partial H} = - \int \frac{M \cdot y \, ds}{EI} + H \int \frac{y^2 \, ds}{EI} = 0$$

$$H = \frac{\int M y \, ds}{\int \frac{y^2 \, ds}{EI}}$$

(86)

Ex-1

Find Horizontal Thrust 2-Hinge Semi-circular Arch. Subjected to a vertical load w at the crown.



$$\frac{y}{R} = \sin \theta$$

$$y = R \sin \theta$$

$$\frac{R-x}{R} = \cos \theta$$

$$R-x = R \cos \theta$$

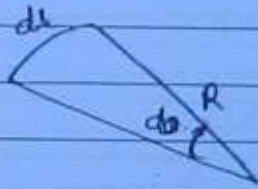
$$x = R(1 - \cos \theta)$$

$$H = \frac{\int \frac{M \cdot y \, ds}{EI}}$$

$$\int \frac{y^2 \, ds}{EI}$$

(87)

$M =$ Beam Moment at $x-x$ [B.M. due to vertical force only]
 $\Rightarrow RA \cdot x$
 $\Rightarrow \frac{w \cdot x}{2}$



$$ds = R \cdot d\theta$$

$$H = \frac{\int_0^{\pi/2} \frac{w}{2} R (1 - \cos\theta) R \cdot d\theta}{EI}$$

$$2 \int_0^{\pi/2} \frac{(R \sin\theta)^2 R \, d\theta}{EI}$$

$$H = \frac{\frac{w}{2} R^2 \int_0^{\pi/2} (1 - \cos\theta) \, d\theta}{R^3 \int_0^{\pi/2} \sin^2\theta \, d\theta}$$

$$H = \frac{\frac{w}{2} R^2 [0 + \sin\theta]_0^{\pi/2}}{R^3 \int_0^{\pi/2} \sin^2\theta \, d\theta}$$

$$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$$

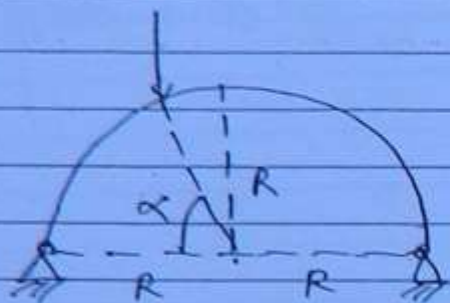
$$\frac{R^3}{2} [1 - \cos 2\theta]_0^{\pi/2}$$

* $H = \frac{w}{\pi}$ *

Special case-I

If concentrated load α radius vector α as shown in fig. then Horizontal thrust H is

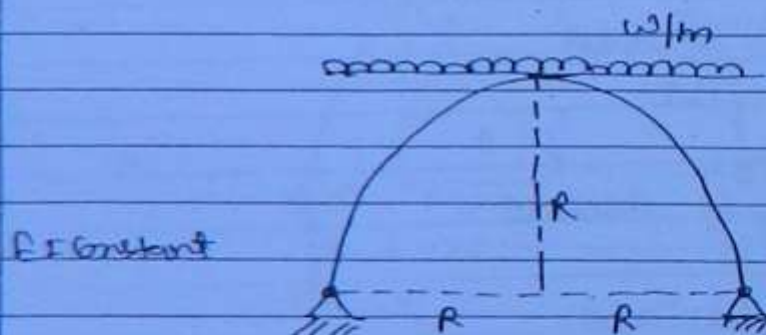
$$H = \frac{W}{\pi} \sin^2 \alpha$$



(88)

Special case-II

2 Hinge semi-circular Arch subjected to UDL over entire span



Eigenschaft

$$H_A = H_B = H$$

$$\rightarrow \frac{4}{3} \frac{wR}{\pi}$$

(III)

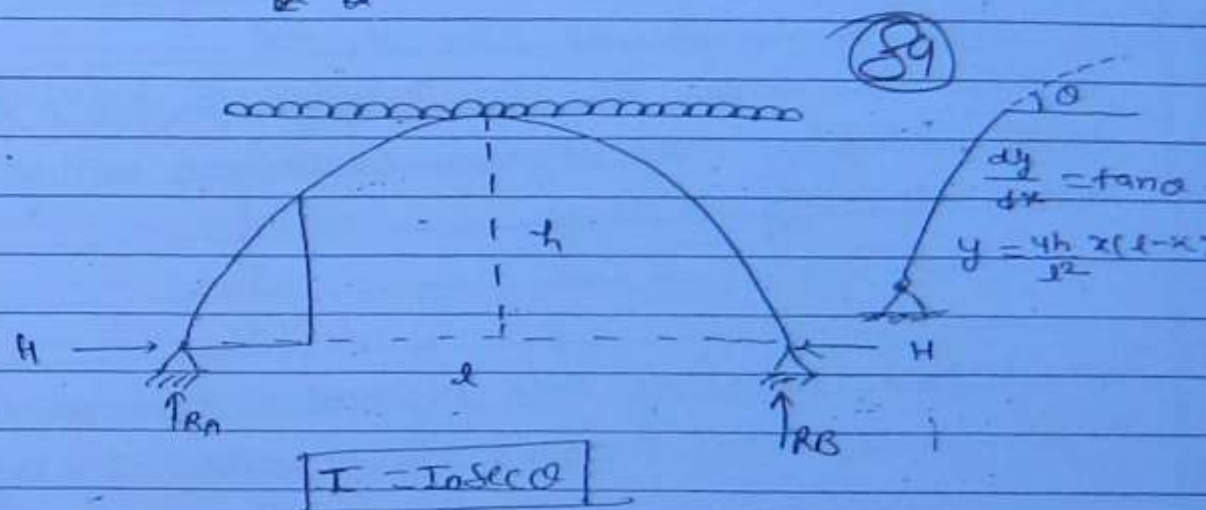
If UDL acts over half span, or ~~UDL~~ UDL acts over entire span then Horizontal thrust

$$H = \frac{2}{3} \frac{wB}{\pi}$$

Ques-2 2-Hinged Parabolic Arch Subjected to udl over entire Span.

Find Horizontal Thrust at the supports if moment of inertia varies as $I = I_0 \sec \theta$ when θ is angle of tangent with the Horizontal

I_0 = moment of inertia at the crown
 ~~I_0~~



$$R_A = R_B = \frac{wl}{2}$$

$$H = \frac{\int \frac{M^2 y ds}{EI}}{\int \frac{y^2 ds}{EI}}$$

$\frac{dx}{ds} = \cos \theta$
 $ds = dx \cdot \sec \theta$

$$H = \frac{2 \int_0^{l/2} \left(\frac{wl}{2} \cdot x - \frac{wx^2}{2} \right) \cdot \frac{4h}{l^2} x(l-x) \cdot dx \sec \theta}{2 \int_0^{l/2} \left[\frac{4h}{l^2} x(l-x) \right]^2 dx \sec \theta}$$

$E I_0 \sec \theta$

$$H = \frac{w l^2}{8h} \int_0^{l/2} x^2 (l-x)^2 dx$$

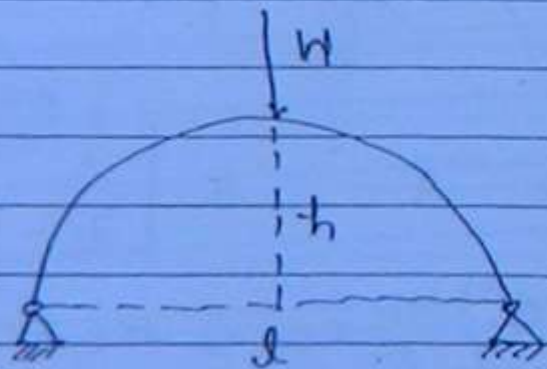
$$H = \frac{w l^2}{8h}$$

90

which is same as in 3-Hinge Parabolic Arch. Their Bending moment and shear force will be zero at any section.

Two-Hinged Parabolic Arch subjected to concentrated load w at the crown moment of Inertia $I = I_0 \sec \theta$

$$H = \frac{25}{128} \frac{w l^2}{h}$$



* Temperature Effect in 2-Hinged Arch: *

(91)



Due to rise in temp. the horizontal thrust induced given by

$$H = \frac{L \alpha T}{\int \frac{y^2 \cdot ds}{EI}}$$

Special Case-I

2-Hinged Semi-circular Arch Sub. to Rise in temp. by $T^\circ\text{C}$

$$H = \frac{4EI \alpha T}{\pi R^2}$$

for Semi-circular Arch

EI is constant.

Special Case-2 for 2-Hinged Parabolic Arch

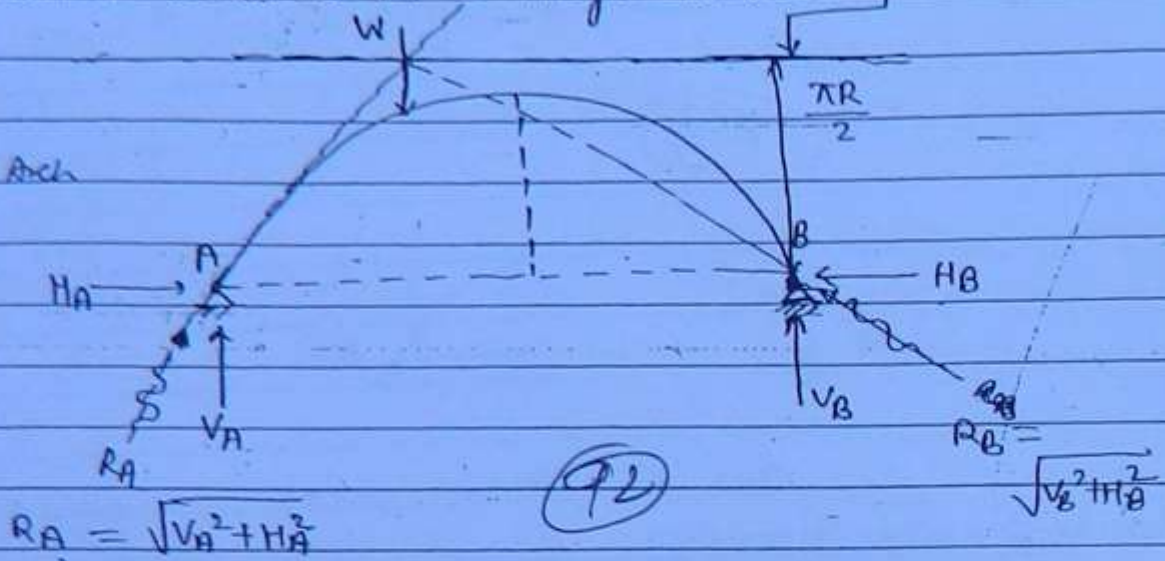
$$H = \frac{15EI \alpha T}{8h^2}$$



h = moment of inertia at the crown

* Reaction locus in 2-Hinge Arch:

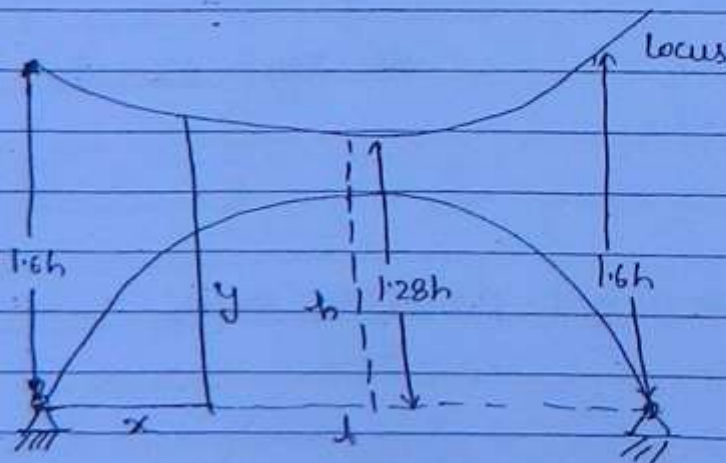
2 Hinge
Semi circular Arch



Case-I 2 Hinge semi circular Arch.

"Reaction locus is the locus of point of the intersection of 2-Resultant Reactions at the support when a point load moves from one end to the other end. for a 2-hinge Semi-circular Arch reaction locus is a straight line, whereas for a 2-hinged parabolic Arch reaction locus is a curve.

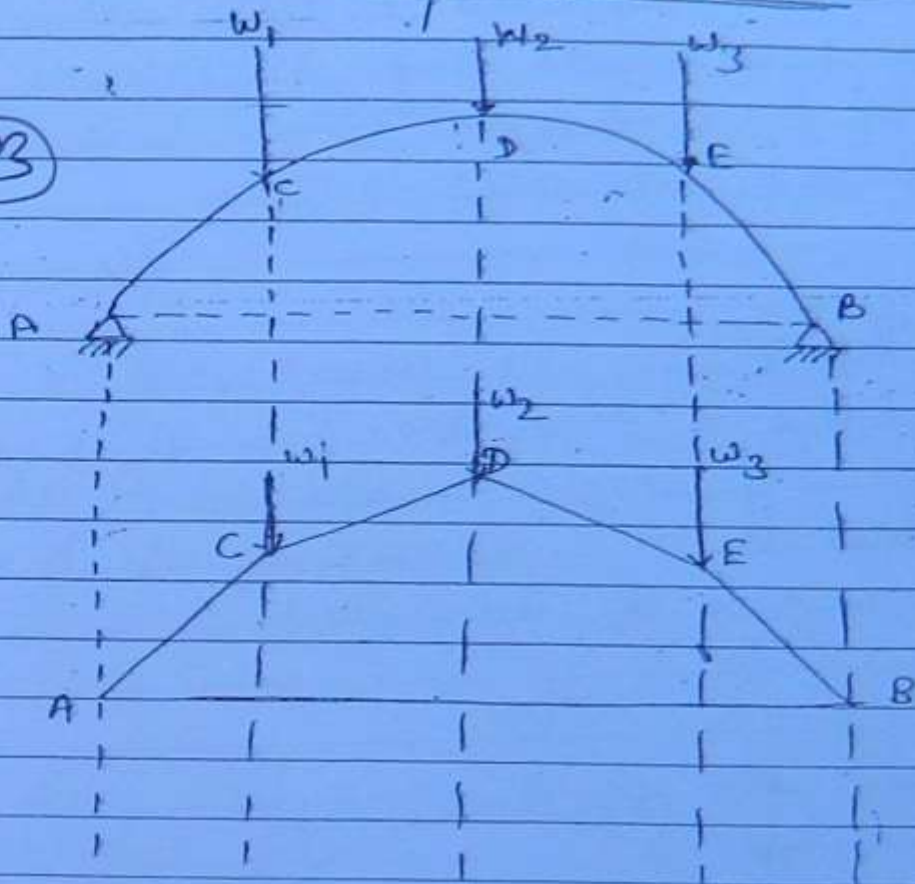
Parabolic Arch



$$y = \frac{1.6h^2}{12 + 12x - x^2}$$

Linear Arch / Theoretical Arch.

(Q3)



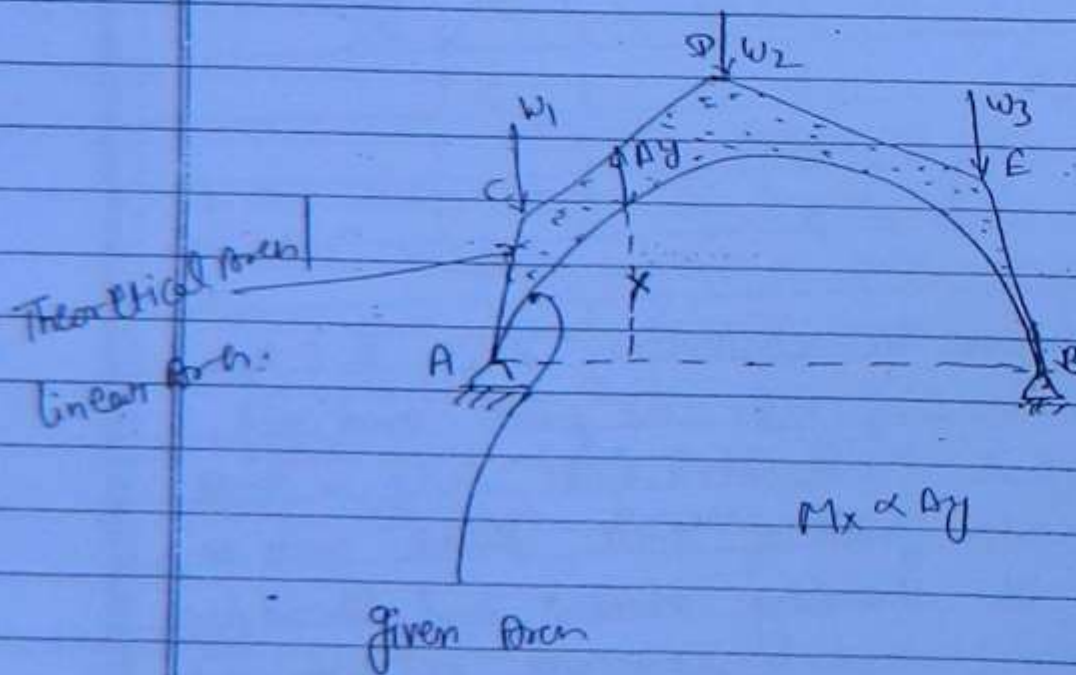
Imagine a structure A, C, D, E & B subjected to loads W_1, W_2, W_3 at joints and all joints A, C, D, E & B to be pin connected like a truss. If loads act on the joints of truss there will be only axial force. No bending moment and shear force. The shape of the above structure is similar to ~~the~~ Funicular Polygon.

A Linear Arch / theoretical Arch is a Imaginary structure which has pin joint member having shape of Funicular Polygon. The member of linear Arch carry only axial compression force & No S.F. & B.M.

Since Funicular polygon depends on loading & with the shape in position of load, Shape of funicular polygon ^{changes}. Hence it is difficult to Constructed a funicular polygon.

(94)

Eddy's Theorem :- If linear Arch is superimposed over the given Arch, then B.M. at the any section on the given arch is proportional to the ordinate of the intersect of the given Arch & linear Arch.

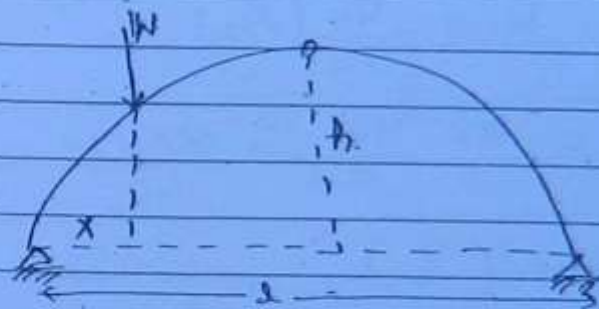


- ① B.C.
② a
③ b
④ c
⑤ a
⑥ c

- ⑦ a
⑧
⑨ b
⑩ a
⑪ b
⑫ a

- ⑬ a
⑭ a.c
⑮ b
⑯ a
⑰ only ① is right
⑱ 8

②



45

max B.M. will occur below crown, load is at a distance x .

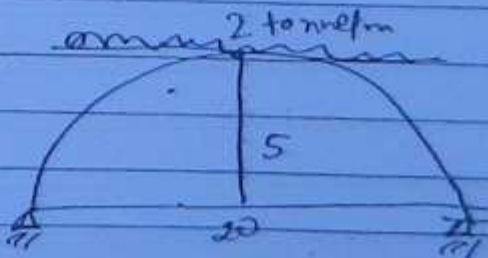
$$M_x = R_A \cdot x - H_A \cdot y$$

$$y = \frac{4h}{L^2} x(L-x)$$

for M_{max} $\frac{dM}{dx} = 0$

$M_{max} = R_A$ $\boxed{x = 0.211L}$

⑥

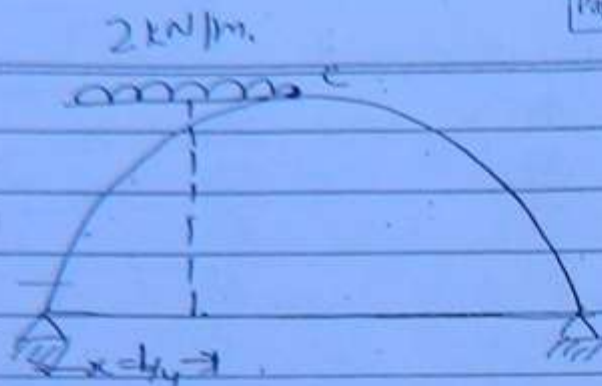


$$H = \frac{wL^2}{8H}$$

$$H = \frac{2 \times 20 \times 20}{8 \times 5}$$

$$H = 20 \text{ kN}$$

(12)



$$\theta = \tan^{-1} \left(\frac{1}{2} \right)$$

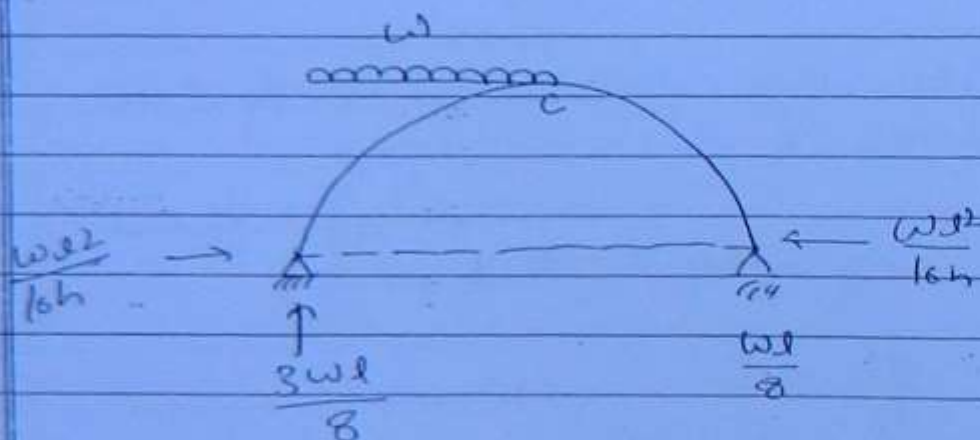
(96)

$$\tan \theta = \frac{4h}{x/2}$$

$$\tan \theta = \frac{4h}{x/2} = \frac{8 \times 10}{40} = \frac{1}{2}$$

$$\theta = \tan^{-1} \left(\frac{1}{2} \right)$$

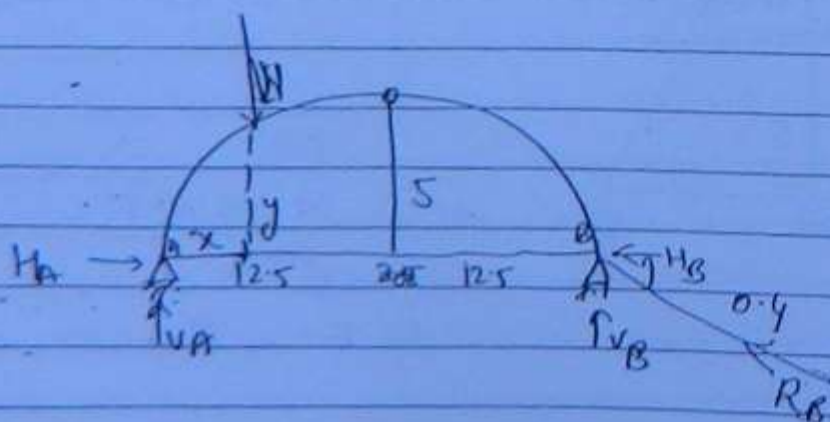
(18)



$$M_c = \frac{wl}{8} \times \frac{1}{2} - \frac{wl^2}{16h} \times h$$

$$M_c = 0$$

(19)



100%

$$H_B = R_B \cos \theta$$

$$V_B = R_B \sin \theta$$

$$\tan \theta = \frac{V_B}{H_B}$$

(97)

$$M_C = 0$$

$$V_B \times 12.5 - H_B \times 5 = 0$$

$$\frac{V_B}{H_B} = \frac{5}{12.5} = 0.4$$

Methods of Analysis

Methods of Analysis

Exact Methods

Approximate method

98

force method / method of consistent deformation

Displacement mtd / stiffness mtd
/ Equilibrium mtd

(Flexibility mtd) / Compatibility mtd

Ex. • Strain Energy / minⁿ potential Energy
/ Castigliano's Theorem.

Ex. • Cantilever mtd

• Factor method

• portal mtd.

→ Unit load method

→ Clapperton's three moment theorem.

→ virtual work method

→ column analogy method

→ Flexibility method

Ex.

→ Moment distribution method /

Hard Cross mtd / mtd of successive approximation

→ Slope deflⁿ method (G.A. maney mtd)

→ Kani's method / method of iteration

→ stiffness matrix method.

difference b/w force method & stiffness method.

FFC (force / Flexibility / Compatibility)

D/SF (displacement / stiffness / Equilibrium)

→ Basic unknowns are taken redundant forces which may support reaction and member forces.

→ Basic unknowns are taken joint displacement (A & D)

(99)

→ The no. of Redundant forces $\neq$ No. of Degree of Static indeterminacy (D_s)

→ the no. of displacement at joint / load is equal to degree of kinematic indeterminacy (D_k).
In Rigid frame & Beam, effect of Axial force is neglected.

→ To determine Redundant to Compatibility eqn. are written. no. of Compatibility eqn. is equal to degree of Static indeterminacy (D_s)

→ To Find Joint Displacement Joint eqn. conditions and shear eqn. are written. No. of equation equal to degree of kinematic indeterminacy

→ force method is suitable when $[D_s < D_k]$

Displacement method is suitable when $[D_k < D_s]$

Note-

- For Indeterminate Structure to be analysis by exact methods rigidity, stiffness of member should be known. If Indeterminate structure are to be analysis for which stiffness is not known then approximate method can be used.

- ② If D_s and D_r both are high such as case of multi story building & multispanne then neither force method nor displacement method is appropriate, under such condition approximate analysis is used.

Too

- ③ methods of Analysis of Indeterminate Structure may be applied for determinate structure also. But simple problem may give lengthy computation & solution.

Principle of Superposition:-

Assumption:-

- material is Isotropic, Homogeneous, Linear elastic in which hook's law is followed.
- Temp. is constant.
- Supports are unyielding.

Principle:- In a Beam, Truss or frame which may be determinate or indeterminate the resultant stress function due to multiple loading is equal to sum of effect of individual loading.

Note It is valid for those stress function which have linear relationship with loading

Castigliano Theorem:-

Theorem-1:-

(101)

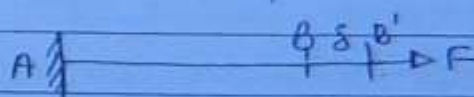
In any structure the material of which is linear elastic, temp. is constant and supports are unyielding then first partial derivate of total strain energy ~~to w~~ with respect to any displacement component is equal to force apply in the direction of that displacement.



$$\frac{\partial U}{\partial \Delta} = P$$

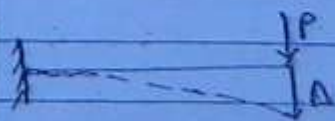


$$\frac{\partial U}{\partial \theta} = M$$



$$\frac{\partial U}{\partial \delta} = F$$

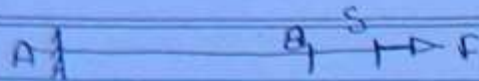
Theorem-2:- In a beam, truss or frame (any structure) the material of which is 'elastic, temp. is constant, supports are unyielding the first partial derivate of total strain energy with respect to any force is equal to displacement in the dirⁿ of the force.



$$\frac{\partial U}{\partial P} = \Delta$$



$$\frac{\partial U}{\partial M} = \theta$$



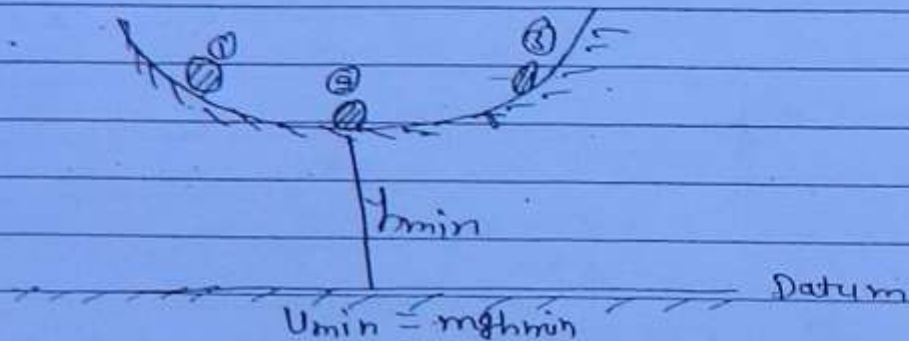
$$\frac{\partial U}{\partial F} = \delta$$



$$\frac{\partial U}{\partial P} = \delta$$

(102)

Principle of minimum potential energy:-



Statements "of all the geometrically compatible state of structure which satisfied deflection boundary conditions force eqn requirement will have final stable condition when total potential energy in the system is min."

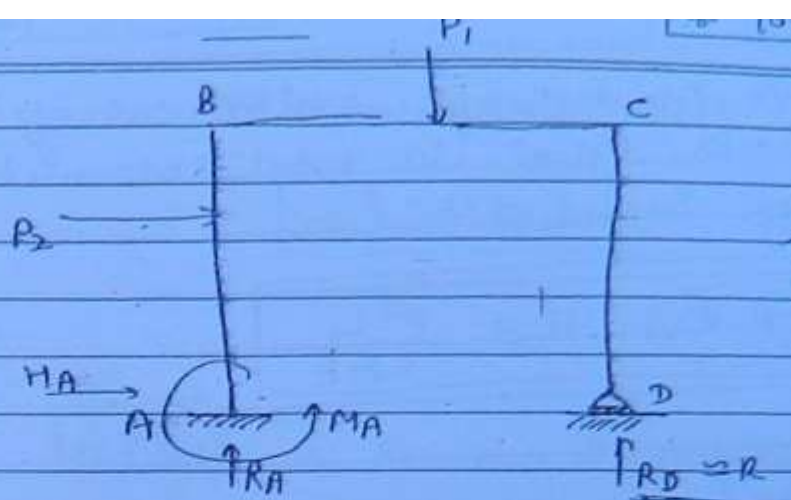
or

otherwise:-

If a structure is loaded and there are redundant reaction for which infinite solution may be possible then true solution will be that for which total minimum ~~state~~ potential energy min.

let in above frame Redundant is $R_2 = R$. True solution of R is such that U is min

$$\Rightarrow \left[\frac{\partial U}{\partial R} = 0 \right] \rightarrow \text{same as Castiglino theorem}$$



(103)

$$D_s = 4 - 3 = 1$$

No. of Redundants = $D_s = 1$

1. Betti's Law

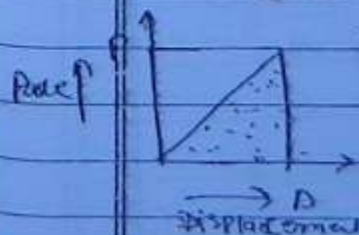
Betti's Law / Rayleigh Theorem:-

"In any structure the material of which is elastic and follows Hooke's law in which supports are unyielding, temp. is constant, external virtual work done by P system of forces during the displacement caused by Q system of forces is equal to external virtual work done by Q system of forces when deformation is caused by P system of forces."



Δ is displacement given by in its direction

Then work done by P is real work done

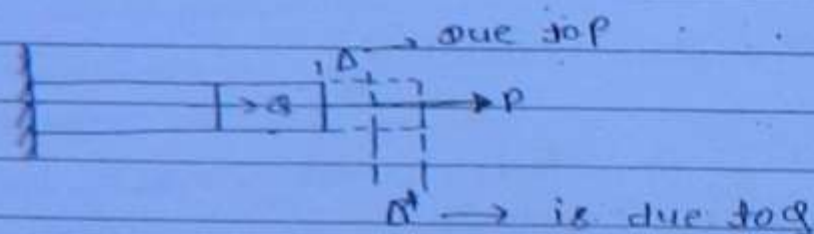
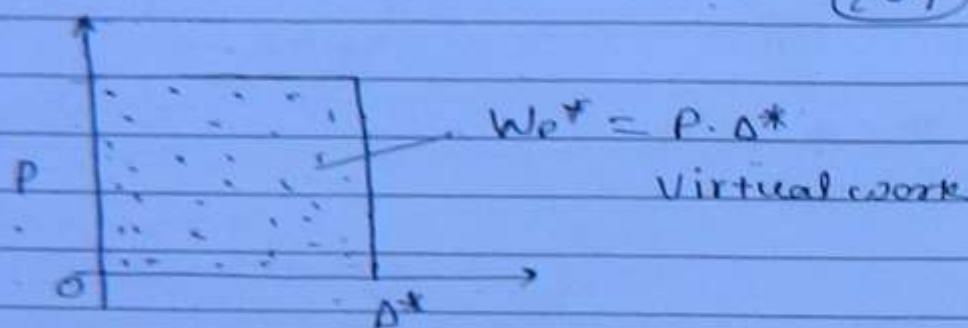


$$W_e = \frac{1}{2} \cdot P \cdot \Delta$$

If there is no loss of work in heat of or transmission it is equal to total Internal Strain energy stored. (U_i)

$$W_e = \frac{1}{2} \cdot P \cdot \Delta = U_i = \frac{P^2 L}{2AE}$$

(104)



Let Δ_1^* is displacement given in the direction of force P by any other force Q than external. work done by P is (W_{e1}^*)

$$W_{e1}^* = P \cdot \Delta_1^*$$

Δ_2^* is displacement given by P in the direction force Q than external virtual work done by Q is

$$W_{e2}^* = Q \cdot \Delta_2^*$$

According to Reldigh theorem / Betti's law

$$W_{e1}^* = W_{e2}^*$$

Special Cases:-

(105)

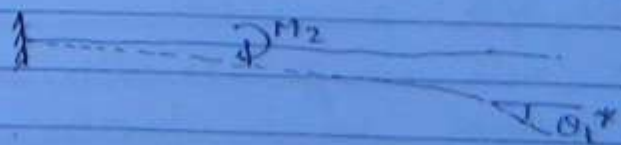
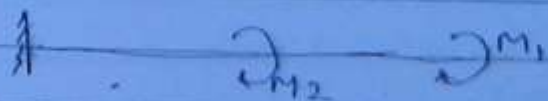
Let θ_1^*

Let θ_1^* is Rotational / Angular displacement in direction of M_1 (moment) produced by moment M_2 acting at some other point. And let θ_2^* is Rotational in direction of M_2 caused by moment M_1 , then external virtual work done by M_1 is equal to external virtual work done by M_2 .

Ex. virtual work done by $M_1 =$ Ex. virtual work done by M_2

$$M_1 \theta_1^* = M_2 \theta_2^*$$

if displacement is in direction of force work done will be +ve. If displacement is in opposite direction of force then work done will be (-ve)



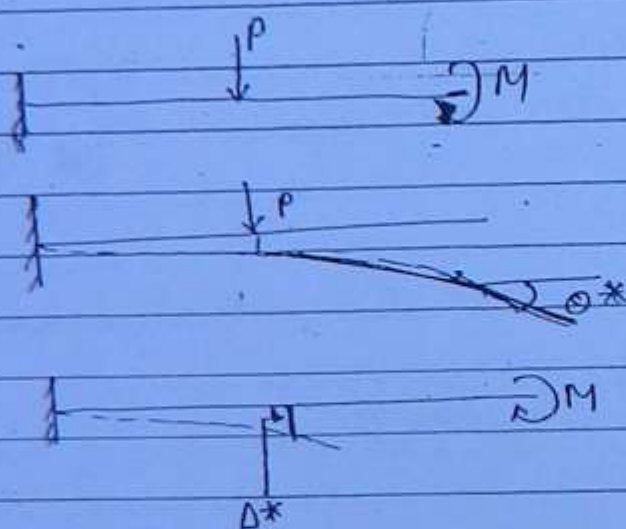
$$W_{e1}^* = M_1 \theta_1^*$$

$$W_{e2}^* = M_2 \theta_2^*$$



Special Case-II - If θ^* is angular displacement in the direction of moment in due to a linear force P acting at any point and let Δ^* is linear displacement in the direction of force P due to moment in acting at some other point. then virtual workdone by M is equal to virtual workdone by P

$$M\theta^* = P\Delta^*$$



(106)

Ex. for the semi-circular two-hinge Arch as shown in fig. a moment of 50 tonne-cm applied at B which produces a displacement of 0.5 cm at A. if a concentrated load of 10 tonne is applied at A ^{then} rotation at B in the arch will be.

(A) 0.01

(B) 0.1

(C) 0.05

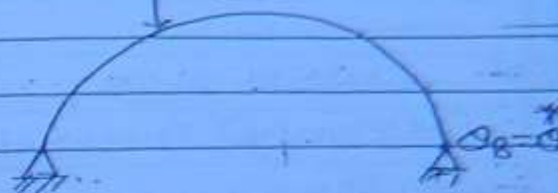
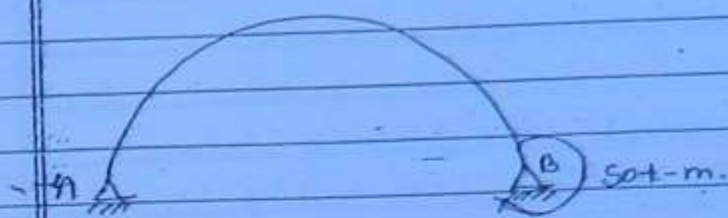
(D) 0.5

$$S_n = 0.5m = 8^*$$

Page 107

bill 10

P = 10 t



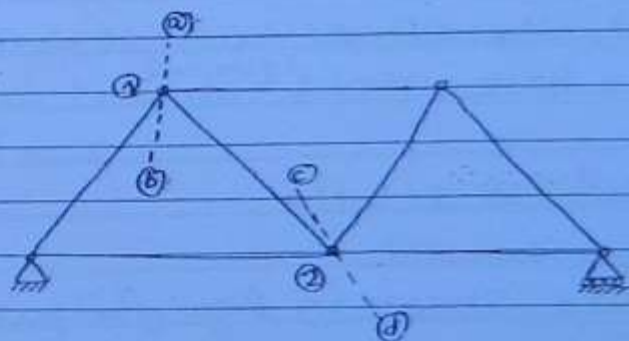
$$MO^* = P \cdot \Delta^*$$

$$50 \times 8^* = 10 \times 0.5$$

$$8^* = 0.1$$

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Maxwell Reciprocal theorem:-



→ It is an special case of Betti's Law in which $P=Q$
let S_{12} is displacement at point 1 in the dirⁿ of
ab due to point P placed at point ② in the direction
of cd.

let S_{21} is displacement at point ② in
the direction of cd due to point P and in the
direction of ab at point 1. then,

$$S_{12} = S_{21}$$

Point where disp. is measured Point where force is applied.

Principal of virtual work:-

Case-1 Elastic bodies:- The total virtual work done (External + Internal) for deformable bodies is equal to zero.

$$[W_e^* + W_i^* = 0]$$

Internal virtual work done by internal forces in ~~the~~ (generally) ~~five~~ because internal deformation is in opposite dirn to the internal forces.

$$|W_e^*| = |W_i^*| = U_i^*$$

(108)

Case-2 Rigid bodies:- Rigid bodies do not have internal deformation, therefore internal virtual work done is zero, hence total ^{External} virtual work done is zero.

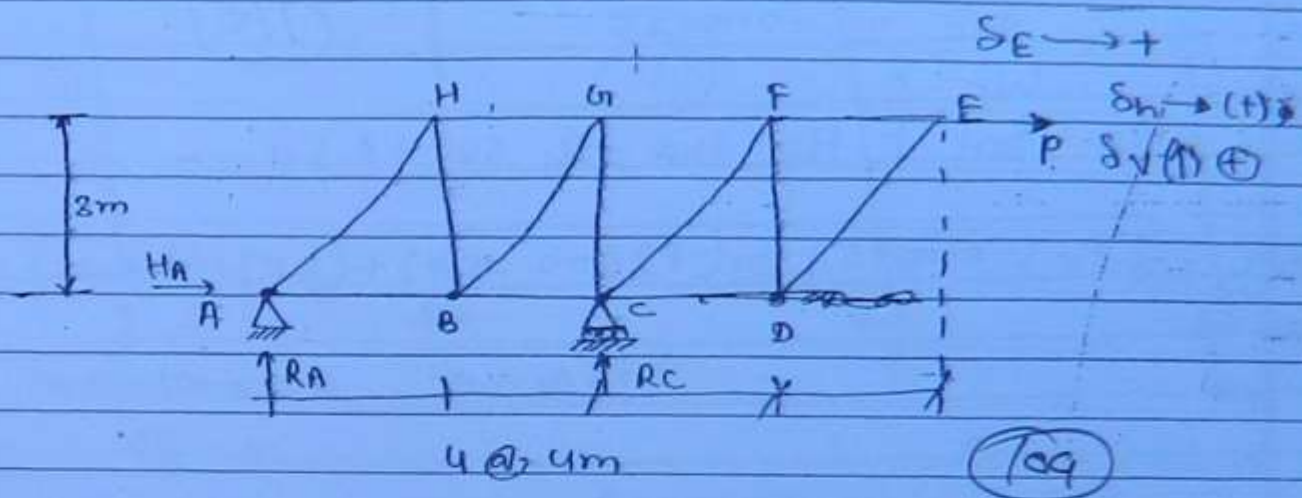
$$W_e^* = 0$$

for the truss shown in fig. compute Horizontal Component of displacement at joint E when following movement of support occur is ① support A moves horizontally by 0.0050 from Right to Left And 0.0075 Vertically down

② support C move vertically down by 0.0025m.

Note that there is no change in the length of any member, because all member are considered axially rigid.

Joint displacement is due to movement of support only.



To find horizontal displacement at E, say δE due to support settlement. Consider pseudo force P in horizontal direction at E.

$$\sum F_x = 0$$

$$H_A + P = 0$$

$$H_A = P$$

$$\sum F_y = 0$$

$$R_A + R_C = 0$$

$$\sum M_A = 0$$

$$R_C \times 8 - P \times 3 = 0$$

$$R_C = \frac{3}{8} P$$

If joint displacement are given in the direction of reaction forces then external virtual work will be done. For rigid body total external virtual work done is zero.

$$\delta_{HA} = -0.0050$$

$$\delta_{KA} = -0.0075$$

$$\delta_{VC} = -0.0025$$

1/0

$$W_E = 0$$

$$R_A \cdot \delta_{VA} + H_A \cdot \delta_{HA} + R_C \cdot \delta_{VC} + P \cdot \delta_E = 0$$

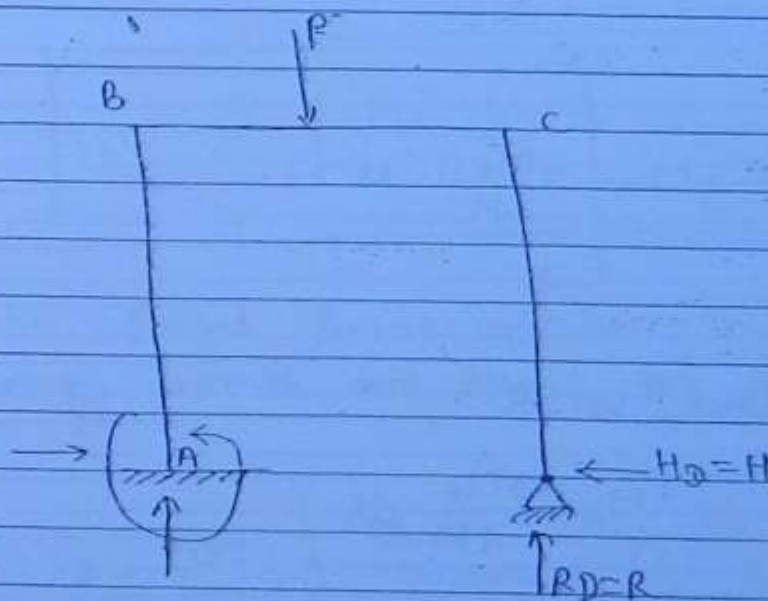
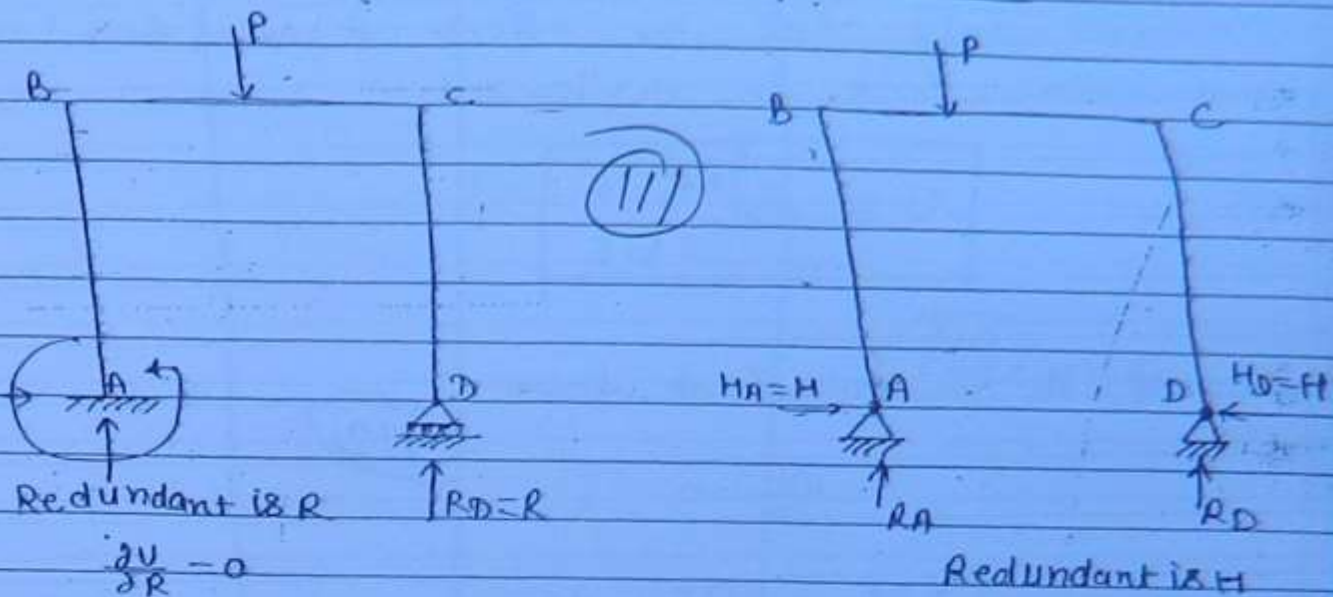
$$-\frac{3}{8}P \cdot (-0.0075) + (-P) \cdot (-0.0050) + \left(\frac{3}{8}P\right) \cdot (-0.0025)$$

$$+ P \delta_E = 0$$

$$\Rightarrow P \delta_E = -0.006785P$$

$$\Rightarrow \boxed{\delta_E = -0.006785 \text{ m.}} \quad (\leftarrow)$$

STRAIN - ENERGY METHOD



Redundants are H & R

$$\frac{\partial U}{\partial H} = 0$$

$$\frac{\partial U}{\partial R} = 0$$

note:- ① If a structure has Redundant reaction then for stable equilibrium its total strain energy w.r. to redundant forces should be minimum.

$$\frac{\partial U}{\partial R} = 0$$

$$\frac{\partial U}{\partial H} = 0$$

- ② The Strain Energy Considered is due to bending-moment only and the effect of axial force and shear force is neglected.

$$U = \frac{\sum \int M_x^2 ds}{2EI}$$

if H is redundant then.

(112)

$$\frac{\partial U}{\partial H} = 0$$

$$\Rightarrow \frac{\sum \int M_x \cdot \frac{\partial M_x}{\partial H} \cdot ds}{2EI}$$

$$\Rightarrow \frac{\sum \int M_x \frac{\partial M_x}{\partial H} \cdot ds}{EI}$$

if Redundant is R

$$\frac{\sum \int M_x \frac{\partial M_x}{\partial R} \cdot ds}{EI} = 0$$

if Redundant is MA

$$\frac{\sum \int M_x \frac{\partial M_x}{\partial MA} \cdot ds}{EI} = 0$$

$$\sum M_D = 0$$

$$R_A \times L - P \times \frac{L}{2} = 0$$

$$R_A = P/2$$

$$R_D = P/2$$

(114)

$$\sum F_x = 0$$

$$H_A - H_D = 0$$

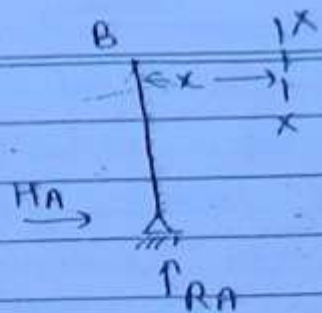
$$H_A = H_D = H \text{ (say)}$$

$$\frac{\partial U}{\partial H} = 0$$

$$\Rightarrow \frac{\sum \int M_x \cdot \frac{\partial M_x}{\partial H} ds}{EI} = 0$$

member	M_x	$\frac{\partial M_x}{\partial H}$	limit
AB	$-Hy$	$-y$	0 to L
BC	$\frac{P}{2}x - HL$	$-L$	0 to L/2

$$M_x = -Hy$$



$$M_x = R_A x - Hx$$

$$\Rightarrow \frac{P}{2}x - HL$$

$$\delta_{AB} = \sum \int \frac{M_x \cdot \frac{\partial M_x}{\partial H} \cdot ds}{EI} \neq 0$$

(1/5)

$$\Rightarrow \frac{2}{EI} \int_0^L \frac{(-Hy)(-y) \cdot dy}{EI} + \frac{2}{EI} \int_0^{L/2} \frac{(\frac{P}{2}x - HL)(-L) dx}{EI}$$

$$\Rightarrow \frac{2}{EI} \left[\frac{Hy^3}{3} \right]_0^L + \frac{2}{EI} \left[\frac{PL^2x}{2} - HL^2x \right]_0^{L/2}$$

$$\Rightarrow \frac{2}{EI} \left[\frac{HL^3}{3} + \frac{PL^3}{4} - HL^3 \right]$$

$$\Rightarrow \frac{2}{EI} \left[\frac{HL^3}{3} + \frac{PL^3}{4} - HL^3 \right]$$

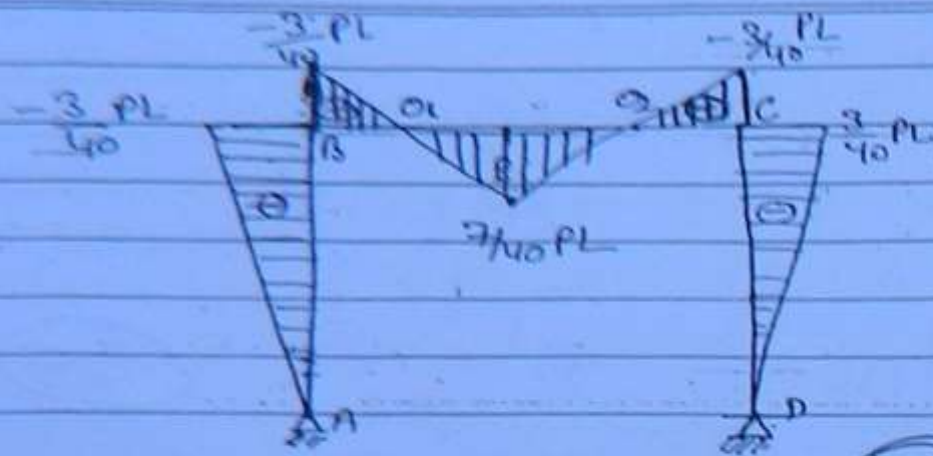
$$H \Rightarrow \frac{3P}{40}$$

$$\text{B.M. in AB } M_x = -Hy$$

$$\Rightarrow -\frac{3}{40} P \cdot y$$

$$\checkmark M_A = 0$$

$$\checkmark M_B = -\frac{3}{40} P L \Rightarrow -\frac{3}{40} PL$$



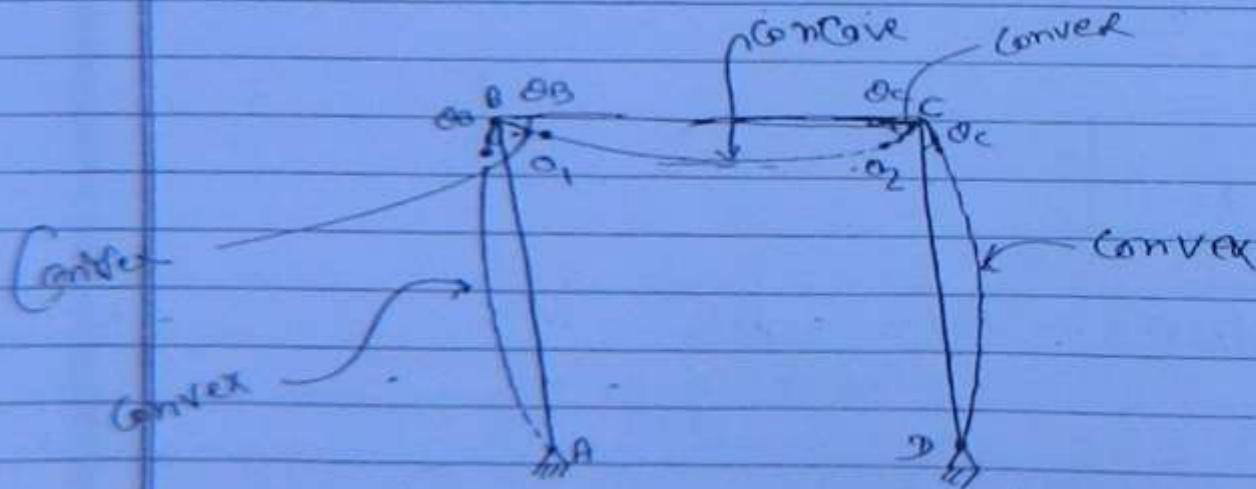
BM PE

$$M_x = \frac{P}{2} \cdot x - HL$$

$$M_B (x=0) = 0 - HL = -\frac{3}{40} PL$$

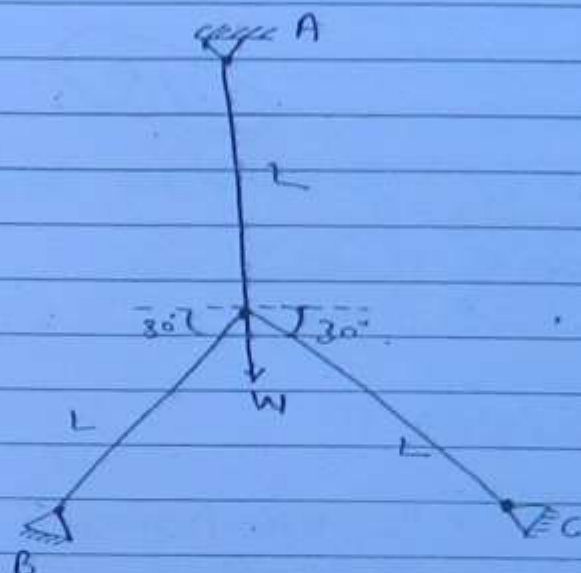
$$M_E (x=L/2) = \frac{P}{2} \times \frac{L}{2} - \frac{3}{40} PL$$

$$= +\frac{7}{40} PL$$



- Q. 3- Identically Rods OA, OB and OC are in same vertical plane and support load as shown in fig. force in all members and calculate vertical moment of joint. Area of each member is a & all joints is pin connected.

AF = constant



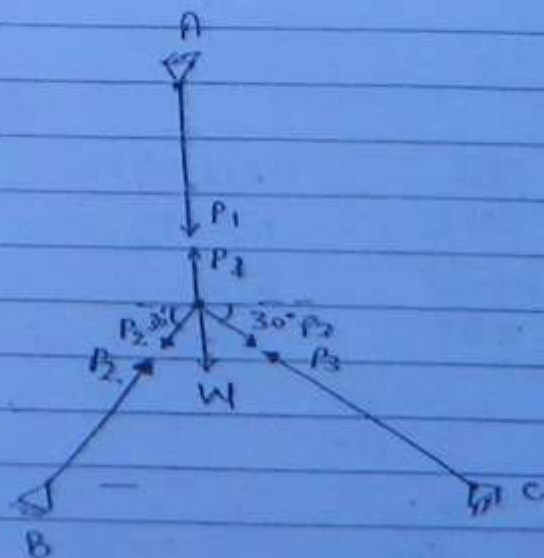
2D Truss

$$D = m + r - 2j$$

$$= 3 + 6 - 2 \times 4$$

$$= 1$$

Let force in members are P_1, P_2, P_3 in OA, OB, OC respectively

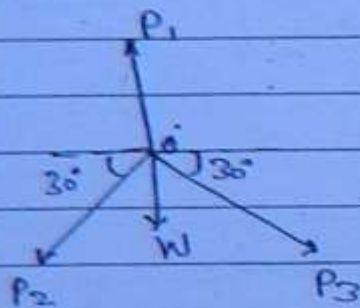


let P_1 is Redundent force:-

$$\frac{\partial U}{\partial P_1} = 0 \quad \text{--- (1)}$$

$$U = U_{AB} + U_{BC} + U_{AC}$$

$$U \Rightarrow \frac{P_1^2 L}{2AE} + \frac{P_2^2 L}{2AE} + \frac{P_3^2 L}{2AE}$$



$$\sum F_x = 0$$

$$P_3 \cos 30^\circ - P_2 \cos 30^\circ = 0$$

$$P_2 = P_3$$

$$\sum F_y = 0$$

$$P_1 - W - P_2 \sin 30^\circ - P_3 \sin 30^\circ = 0$$

$$P_1 - W - \frac{P_2}{2} - \frac{P_3}{2} = 0$$

$$\boxed{P_1 - W = P_2 = P_3}$$

$$U = \frac{P_1^2 L}{2AE} + \frac{2(P_1 - W)^2 L}{2AE}$$

$$\frac{\partial U}{\partial P_1} = 0 \Rightarrow \frac{\partial U}{\partial P_1} = \frac{2P_1 L}{2AE} + \frac{4(P_1 - W)L}{2AE}$$

$$\Rightarrow \frac{2P_1 L}{2AE} + \frac{2P_1 L}{2AE} - \frac{2WL}{2AE}$$

$$\Rightarrow \frac{3P_1 L}{AE} - \frac{2WL}{AE}$$

$$\frac{3P_1 L}{AE} = \frac{2WL}{AE}$$

$$P_1 = \frac{2W}{3}$$

(T)

(+ive) = Tension

$$P_2 = P_3 = P_1 - W$$

$$\Rightarrow \frac{2W}{3} = W$$

$$P_2 = P_3 = -\frac{W}{3}$$

(C)

Compression (-ive)

Method - 1

(119)

Vertical deflection of OA

$\Rightarrow$ Elongation of OA

$$\Rightarrow \frac{P_1 L}{AE}$$

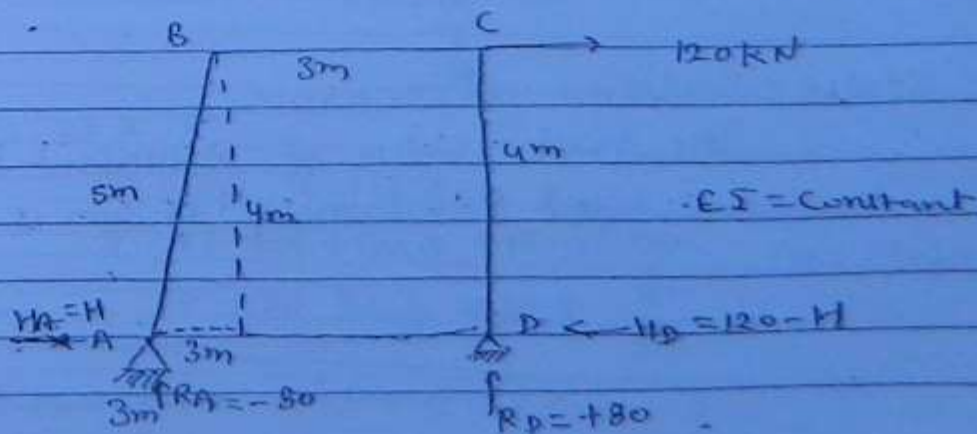
$$\Rightarrow \frac{\frac{2}{3} P \cdot L}{AE} = \frac{2PL}{3AE}$$

2nd Method

U is Total Strain Energy Stored.

$$\frac{dU}{dW} = \Delta_0$$

Q. Analyse a portal frame shown in fig. using strain energy method. Draw bending moment diagram.



$$(n_e)_{\text{reaction}} = 4$$

$$D_{sc} = n_e - 3 = 4 - 3 = 1$$

$$D_{si} = 0$$

$$\sum F_x = 0$$

$$H_A + H_D = 120$$

$$\{H_A = H\}$$

$$H + H_D = 120$$

$$H_D = 120 - H$$

$$\sum F_y = 0$$

$$H_A + H_D = 120$$

$$H + H_A = 120 \Rightarrow H.$$

$$120$$

$$\sum F_y = 0$$

$$R_A + R_D = 0 \quad \text{--- (ii)}$$

$$\sum M_A = 0$$

$$R_D \times 6 - 120 \times 4 = 0$$

$$R_D = \frac{120 \times 4}{6}$$

$$R_D = 80 \text{ KN}$$

$$R_A = -80 \text{ KN}$$

Say H is Redundant Reaction

for stable condition of structure

The True value of Redundent H is when total Pot. Energy of sym is $\min = 0$

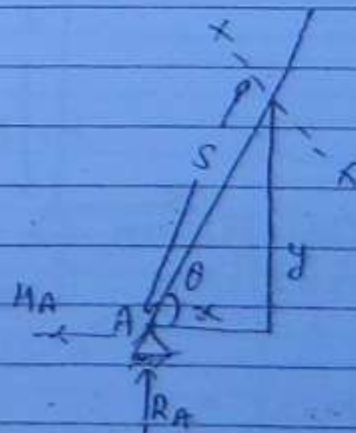
$$\frac{\partial U}{\partial H} = 0$$

$$U = \sum \int \frac{M_x^2 ds}{2EI}$$

$$\frac{\partial U}{\partial H} = \sum \int \frac{M_x \frac{\partial M_x}{\partial H} ds}{2EI} \quad (12)$$

member	M_x	$\frac{\partial M_x}{\partial H}$	limit of integration
AB	$-48s + 0.8Hs$	$-0.8s$	0 to 5
BC	$-80(3+x) + 4H$	4	0 to 3
DC	$-(120-H)y$	y	0 to 4

(1) For member AB



B.M. at $x \Rightarrow$

$$M_x = R_A \cdot x + H \cdot y$$

$$\Rightarrow (-80) s \cos \theta + H \cdot s \sin \theta$$

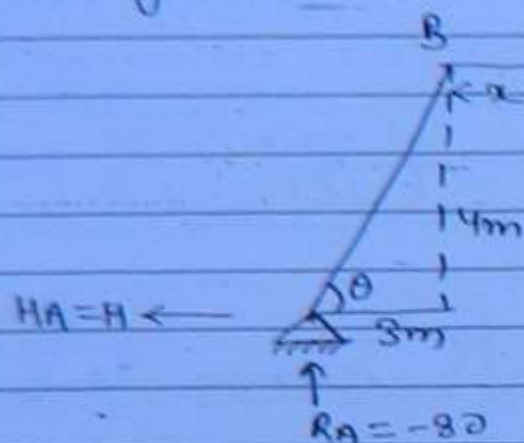
$$\Rightarrow -80 s \cdot \frac{3}{5} + H \cdot s \cdot \frac{4}{5}$$

$$B.M. at x \Rightarrow -48s + 0.8H \cdot s$$

②

At member BC

Free body diagram →



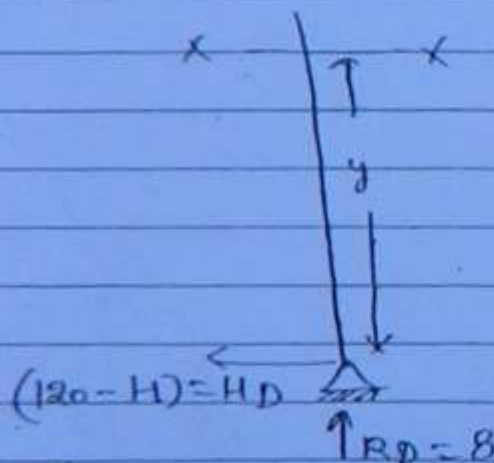
122

$$M_x = R_A (3+x) + H y$$

$$M_x \Rightarrow R - 80 (3+x) + 4H$$

③

At the member CD



$$M_x = -(120-H) y$$

Equation:-

$$\sum \int_{EI} M_x \frac{\partial M_x}{\partial H} ds = 0$$

Page 123

$$\Rightarrow \int_0^5 \frac{(-485 + 0.8HS)}{EI} \cdot 0.85 \, ds + \int_0^3 \frac{[-80(3+x) + 4H]}{EI} \cdot 4 \, dx + \int_0^3 \frac{-(120-H)2.5}{EI} \cdot 2.5 \, dx$$

$$H = 88.33 \text{ kN}$$

B.M. in AB

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$$M_x = -485 + 0.8HS$$

$$M_A = 0$$

$$M_B = -485 + 0.8 \times 88.33 \times 5$$

$$\Rightarrow +113.33$$

B.M. in BC

$$M_x = -80(3+x) + 4H$$

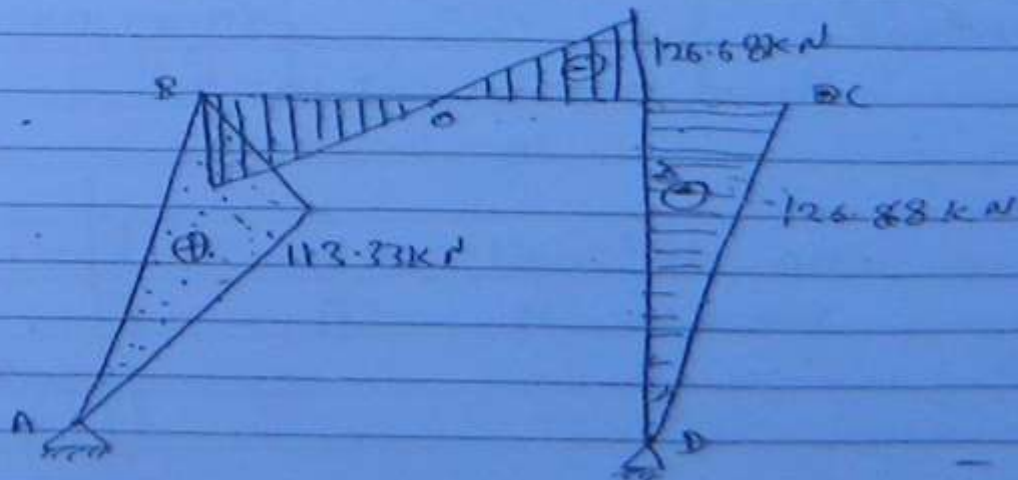
$$M_B = -80(3+0) + 4 \times 88.33 = +113.33$$

$$M_C = -80(3+3) + 88.33 \times 4 = -126.68$$

in BM in BC

$$M_D = 0$$

$$M_C = -126.68$$



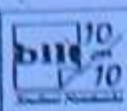
0 → contra flexure.

(hogging $\rightarrow$ concave
sagging $\rightarrow$ convex)

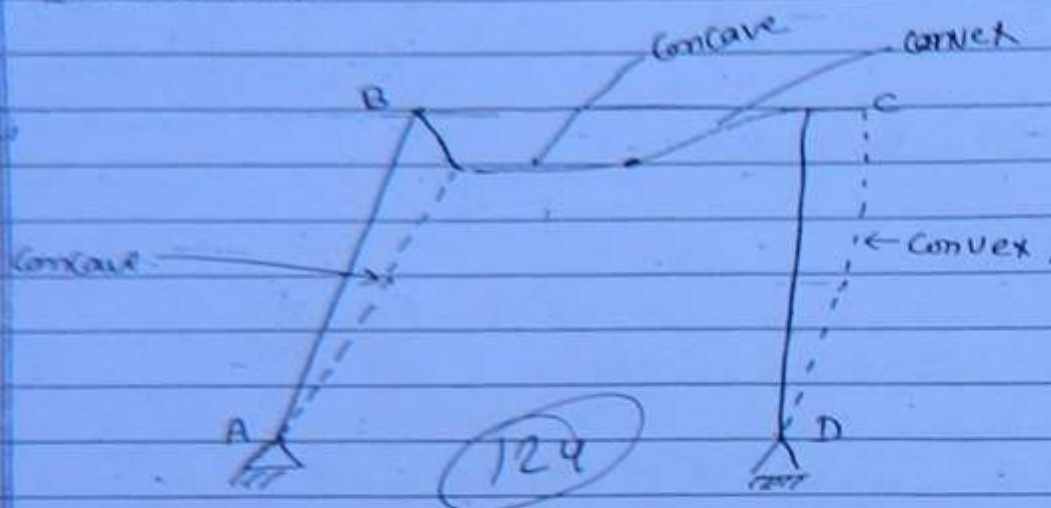
Elastic curve

Date / /

Page

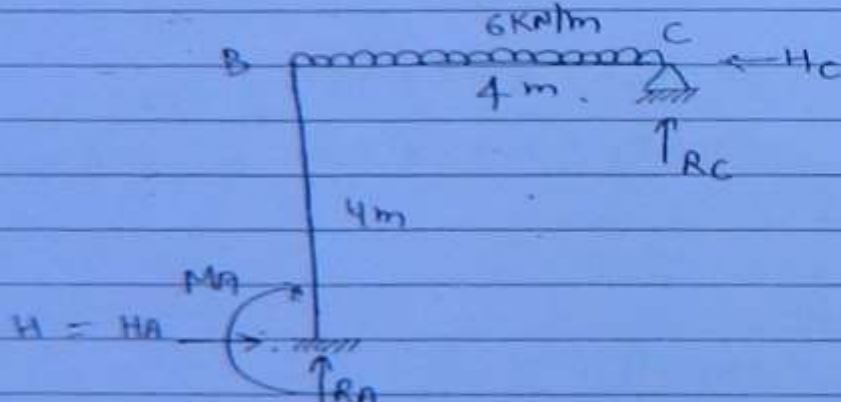


Elastic curve.



Q.

Analysing a Portal frame shown in fig using strain Energy method. $\{EI = \text{constant}\}$



$EI = \text{constant}$

$$\text{No. of reaction} = 3 + 2 = 5$$

$$D_s = 5 - 3 = 2$$

$$\sum F_x = 0 \quad H_A - H_C = 0$$

$$H_A = H_C = H \quad \text{--- (1)}$$

Consider Redundants are H & M_A

$$R_A + R_C = 6 \times 4 = 0$$

$$R_A + R_C = 24 \text{ KN} \quad \text{--- (1)}$$

$$E M_A = 0$$

$$M_A + R_A \times 4 - H \times 4 - 6 \times 4 \times 2 = 0$$

$$M_A + R_A \times 4 - H \times 4 - 48 = 0$$

$$4 R_A = 48 - M_A + 4 H$$

$$R_A = 12 - 0.25 M_A + H$$

(125)

from eqn (2)

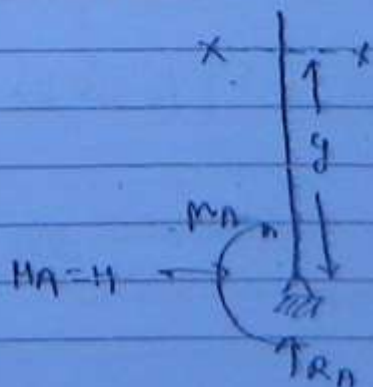
$$R_C = 24 - R_A$$

$$R_C = 24 - 12 + 0.25 M_A - H$$

$$R_C = 12 + 0.25 M_A - H$$

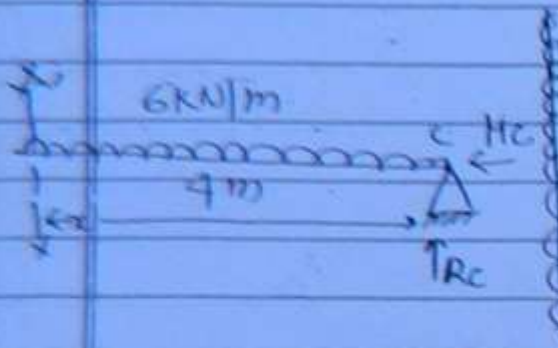
Member	M_x	$\partial M_x / \partial H$	$\partial M_x / \partial M_A$ Unit	Limit
AB	$M_A - H \cdot y$	$-y$	1	0 to 4
BC	$(12 + 0.25 M_A - H)x$ $- 3x^2$	$-x$	$-0.25x$	0 to 4

for member AB :-



$$M_x = M_A - H \cdot y$$

in form RCB



$$M_x = R_C x - w \cdot x \cdot \frac{x}{2}$$

$$R_C \cdot x - w \cdot x \cdot \frac{x}{2}$$

$$M_x = (12 + 0.25 M_A - H)x - \frac{6x^2}{2}$$

$$M_x = (12 + 0.25 M_A - H)x - 3x^2$$

Equation:-

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$$\frac{\partial U}{\partial H} = 0$$

$$\sum \int \frac{M_x}{EI} \cdot \frac{\partial M_x}{\partial H} \cdot ds = 0 \quad \text{--- (1)}$$

$$\frac{\partial U}{\partial M_A} = 0$$

$$\sum \int \frac{M_x}{EI} \cdot \frac{\partial M_x}{\partial M_A} \cdot ds = 0 \quad \text{--- (2)}$$

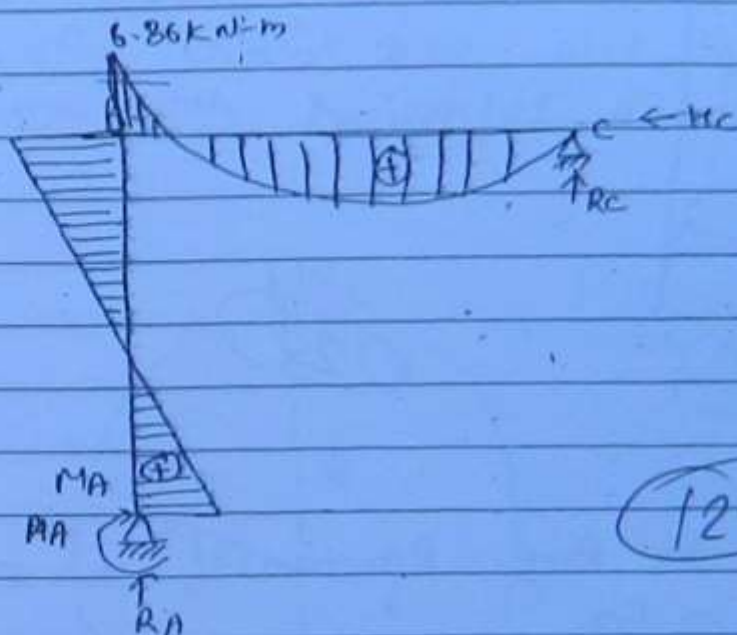
$$\Rightarrow \int_0^4 \frac{(M_A - Hy)(-y)}{EI} dy + \int_0^4 \frac{[(12 + 0.25 M_A - H)x - 3x^2](-x)}{EI} dx = 0 \quad \text{--- (1)}$$

$$\Rightarrow \int_0^4 \frac{(M_A - Hy) \cdot 1 \cdot dy}{EI} + \int_0^4 \frac{[(12 + 0.25 M_A - H)x - 3x^2] \cdot [0.25x]}{EI} dx = 0 \quad \text{--- (2)}$$

Solving (1) & (3)

$$M_A = +3.43 \text{ kN-m}$$

$$H = 2.57 \text{ kN}$$



At BMat AB

$$M_x = M_A - Hy$$

$$\Rightarrow 3.43 - 2.57y$$

$$M_A \Rightarrow +3.43$$

$$M_B \Rightarrow +3.43 - 2.57 \times 4$$

$$\Rightarrow -6.86$$

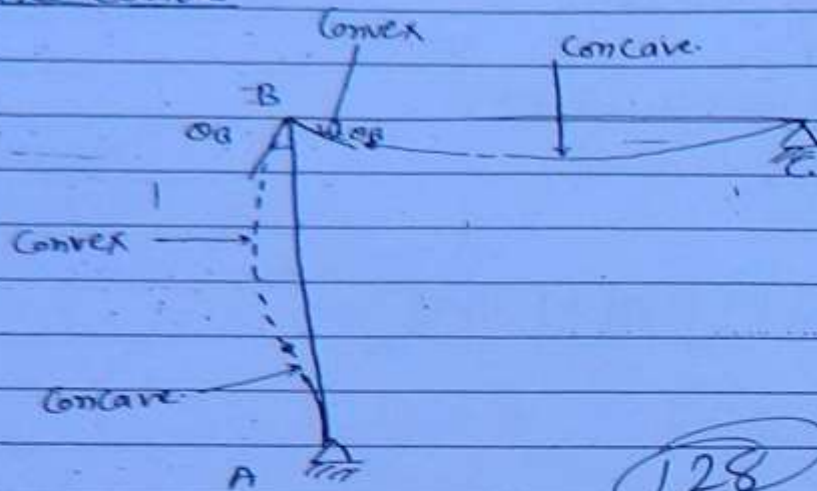
BM in CB

$$M_x = (12 + 0.25M_A - H)x \cdot 3x^2$$

$$M_C = 0$$

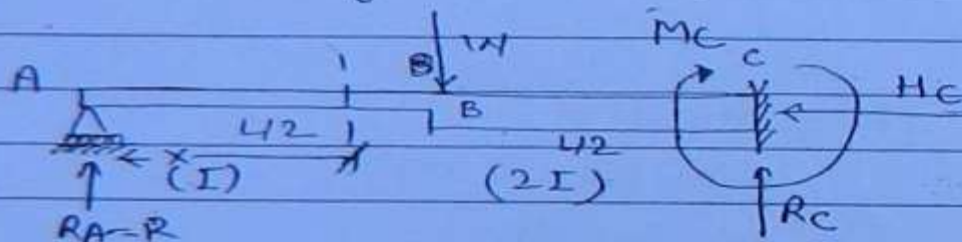
$$M_B = -6.86$$

Elastic curve



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Q. Determine the Prop- Reaction of the cantilever beam shown in fig.



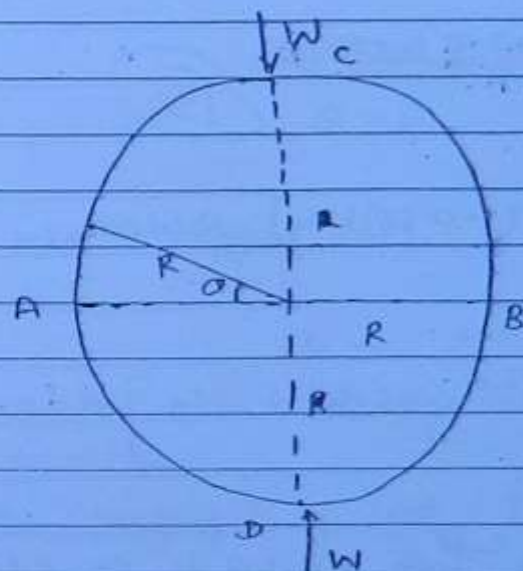
$$\frac{\partial U}{\partial R} = 0$$

$$U_{AB} = \int_0^{L/2} \frac{(M_x)^2 dx}{2EI} \rightarrow \int_0^{L/2} \frac{(R \cdot x)^2 dx}{2EI}$$

$$U_{BC} = \int_{L/2}^L \frac{[R_A x - w(L - L/2)]^2 dx}{2E(2I)}$$

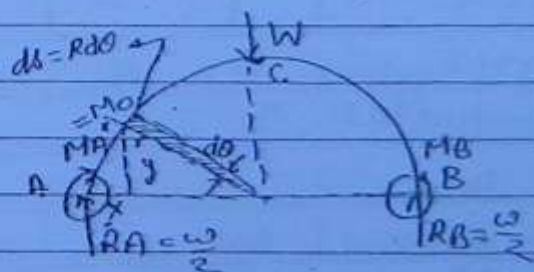
$$R_{prop} = \frac{5w}{8}$$

Q. A Ring of Radius R having Uniform cross-section ($EI = \text{constant}$) is subjected to 2 equal & opposite load w as shown in fig. determine max. Sagging & Hogging B.M. and also find decrease in length of vertical dia. and position of contraflexure with horizontal diameter.



(129)

for the analysis let us cut a Ring in two equal part. At the level of AB.



$$H_A = H_B = 0$$

For True value of Redundant $M_0 =$

$$\frac{\partial U}{\partial M_0} = 0$$

U for HALF RING

$$U = \frac{1}{2} \int_0^{\pi/2} \frac{M_x^2 ds}{EI}$$

BMat-xx

$$M_x \Rightarrow RA \cdot x + M_0$$

$$M_x \Rightarrow \frac{\omega}{2} R(1 - \cos \theta) + M_0$$

$$U = \frac{1}{2} \int_0^{\pi/2} \left[\frac{\omega}{2} R(1 - \cos \theta) + M_0 \right]^2 R d\theta$$



$$\frac{\omega}{2} R - \frac{\omega}{2} R \cos \theta + M_0$$

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$$\frac{\partial U}{\partial M} = \frac{1}{EI} \int_0^{\pi/2} 2 \left[\frac{\omega}{2} R(1 - \cos \theta) + M_0 \right] \cdot R \cdot d\theta$$

$$\frac{\partial U}{\partial M} \rightarrow \frac{2}{EI} \int_0^{\pi/2} \left[\frac{\omega}{2} R - \frac{\omega}{2} R \cos \theta + M_0 \right] R d\theta$$

$$\frac{\partial U}{\partial M} = \frac{2}{EI} \int_0^{\pi/2} \frac{\omega}{2} R^2 - \frac{\omega}{2} R \cos \theta + M_0 R d\theta$$

$$\Rightarrow \left[\frac{\omega}{2} R^2 \frac{\pi}{2} \right] - \left[-\frac{\omega}{2} R^2 1 \right] + \left[M_0 R \frac{\pi}{2} \right]$$

$$0 \Rightarrow \frac{\omega}{2} R^2 \frac{\pi}{2} + \frac{\omega}{2} R^2 + M_0 R \frac{\pi}{2}$$

$$M_0 R \frac{\pi}{2} \rightarrow -\frac{\omega R^2}{2} + \frac{\omega R^2 \pi}{2}$$

$$M_0 R \frac{\pi}{2} \rightarrow -\frac{\omega R^2}{2} (\pi - 2)$$

$$M_0 \rightarrow -\frac{\omega R}{2\pi} (\pi - 2)$$

$$M_x = \frac{wR}{2} (1 - \cos\theta + M_0) \quad \text{--- (1)}$$

for M_x to be max/min^m.

$$\frac{dM_x}{d\theta} = 0$$

$$\frac{wR}{2} (0 + \sin\theta) + 0 = 0$$

$$\sin\theta = 0$$

$$\theta = 0$$

(13)

Hence, max^m Hogging B.M. is at A & B.

$$M_0 = -\frac{wR}{2\pi} (\pi - 2)$$

$$\cos\theta = 0$$

from eq. (1) Max (Sagging) $\rightarrow \frac{wR}{2} [1 - 0] + \frac{-wR}{2\pi} (\pi - 2)$

$$\Rightarrow \frac{wR}{2} - \frac{wR}{2} + \frac{wR}{\pi}$$

$$\Rightarrow \frac{wR}{\pi}$$

for Point of contra flexure

$$M_x = 0$$

$$\Rightarrow \frac{wR}{2} (1 - \cos\theta) + M_0 = 0$$

$$\Rightarrow \frac{wR}{2} (1 - \cos\theta) - \frac{wR}{2\pi} (\pi - 2) = 0$$

$$\boxed{\theta = \cos^{-1}(2/\pi)}$$

Vertical deflection of CD

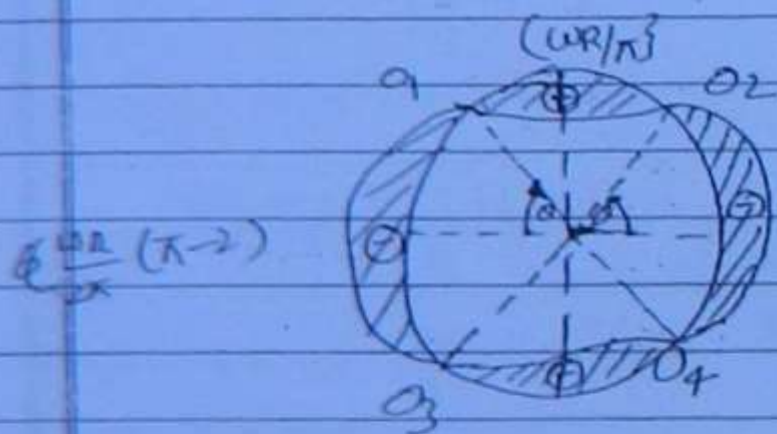
$$\Delta_{CD} = \frac{\partial U}{\partial W}$$

$$U = 4 \int_0^{\pi/2} \frac{M_x^2 ds}{2EI}$$

$$U = 4 \int_0^{\pi/2} \frac{\left[\frac{WR}{2} (1 - \cos \theta) + M_0 \right]^2 R \cdot d\theta}{2EI}$$

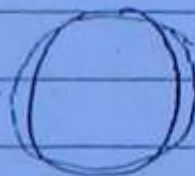
(1.32)

$$\Delta_{CD} = \frac{WR^3}{4\pi EI} (\pi^2 - 8)$$



$$\theta = \cos^{-1} \left(\frac{2}{\pi} \right)$$

$$\theta = 50.45^\circ$$



Elastic curve

Note:- In a closed str. Resultant ~~moment~~ member is internal and in open str. Resultant member is external.

MOMENT DISTRIBUTION METHOD

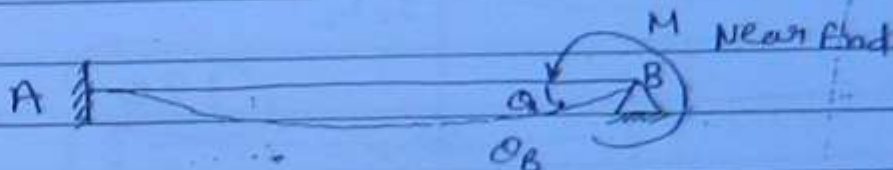
is also called →

- Hard - cross method
- Mtd. of successive approximation
- Base of stiffness concept.

(133)

Stiffness:-

Case-1



$$\theta_B = \frac{ML}{4EI} \quad \text{--- when far end is fixed}$$

$$M = \frac{4EI}{L} \cdot \theta_B \quad \text{if } \theta_B = 1, \text{ then } M = K \text{ (Stiffness)}$$

Stiffness of $BA = \frac{4EI}{L}$, when far end A is fixed.

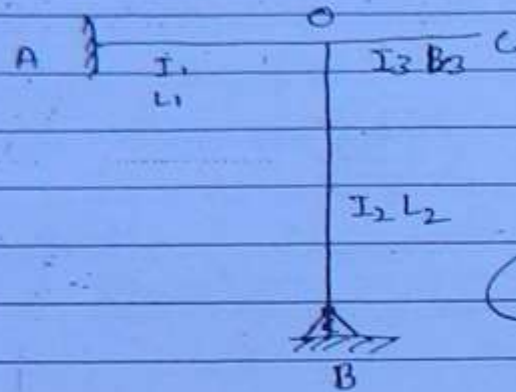


$$\theta_B = \frac{2M}{3EI} \quad \theta_A = \frac{ML}{6EI}$$

$$M = \frac{3EI}{L} \cdot \theta_B$$

if $\theta_B = 1$, $M = K$
 stiffness of $BA = \frac{3EI}{L}$ --- when far end A is hinged.

→ If far end is free, then stiffness of the member will be zero.



k_1 Stiffness of OA = $\frac{4E_1 I_1}{L_1}$ --- for end (A) is fixed.

k_2 Stiffness of OB = $\frac{3E_2 I_2}{L_2}$ --- for (B) is fixed

k_3 Stiffness of OC = 0 --- for C is free.

Total stiffness of all member = ΣK

$$\Sigma K = k_1 + k_2 + k_3$$

$$\Sigma K = \frac{4E_1 I_1}{L_1} + \frac{3E_2 I_2}{L_2} + 0$$

$$\Sigma K = K = \frac{4E_1 I_1}{L_1} + \frac{3E_2 I_2}{L_2}$$

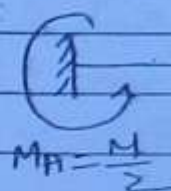
Relative Stiffness:-

$$R.B \Rightarrow \frac{\text{stiffness}}{4E}$$

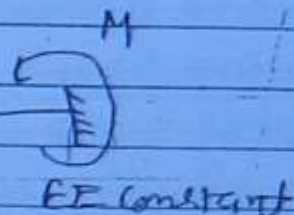
R.S. for OA = $\frac{I_1}{L_1} \left(\frac{I}{L} \right)$ ----- when far end is fix

R.S. for OB = $\frac{3}{4} \frac{I_2}{L_2}$ or $\frac{3I}{4L}$ ----- " " Hinge

R.S. for OC = 0



135



$$\frac{M_A}{M} = \frac{1}{2} = \text{Carry over factor}$$

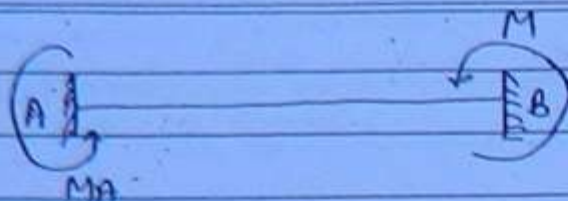
M_A = Carry over moment

→ it is always in the direction of applied moment.

→ If a moment m is applied at B (near end) then carry over moment at far end which is fixed will be in the direction of applied moment. The ratio of carry over moment to the applied moment is called Carry over factor.

→ If far end A is Hinged or free, then carry over moment is zero, Hence carry over factor is zero.

①

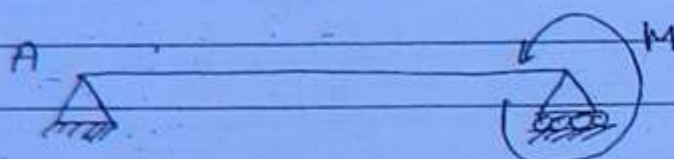


$$\frac{M_A}{M} = \frac{1}{2}$$

carry over factor

136

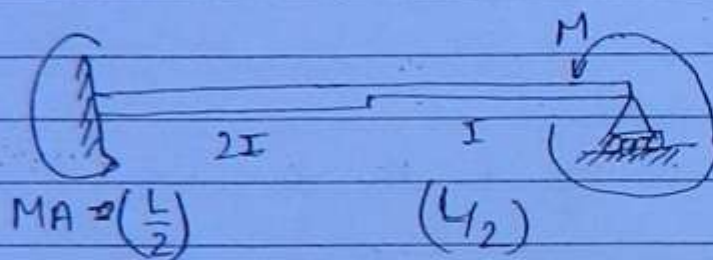
②



$$M_A = 0$$

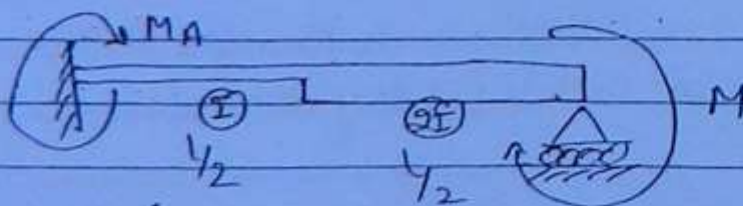
$$C.O.F = 0$$

③



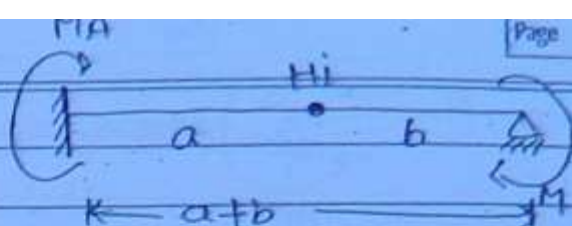
$$\frac{M_A}{M} = C.O.F > \frac{1}{2}$$

④



$$\frac{M_A}{M} = C.O.F < \frac{1}{2}$$

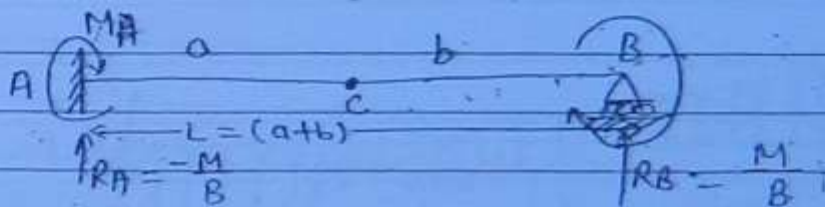
(c)

 $EI = \text{constant}$

$$\frac{M_A}{M} = \text{C.O.F.} = \frac{a}{b}$$

137

- Q Find stiffness of a member at joint B and carry over factor if far end A is fixed and beam contains internal hinge at C. Near end A is roller support as shown in fig.



$$M_C = 0$$

$$R_B \times b - M = 0$$

$$R_B = \frac{M}{b}$$

$$\text{C.O.F.} = \frac{M_A}{M}$$

$$\sum M_A = 0$$

$$R_B \times (a+b) - M - M_A = 0$$

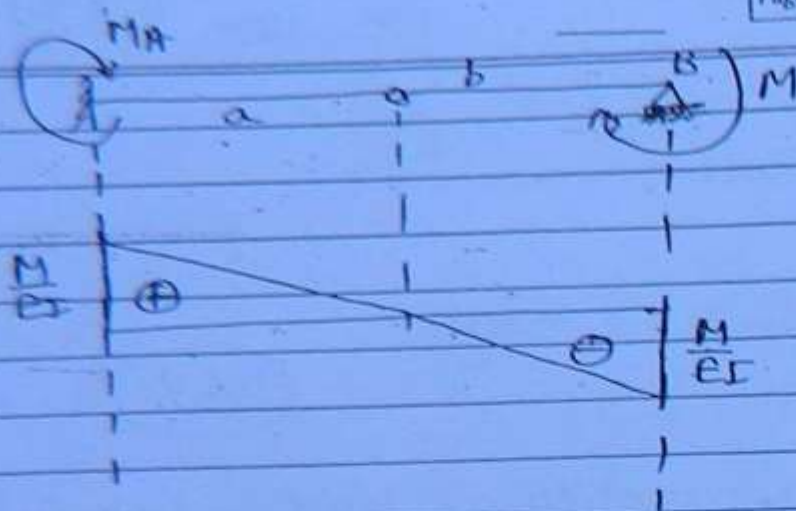
$$M_A = R_B(a+b) - M$$

$$M_A = \frac{M}{b}(a+b) - M$$

$$M_A = \frac{M}{b} \cdot a + \cancel{M} - \cancel{M}$$

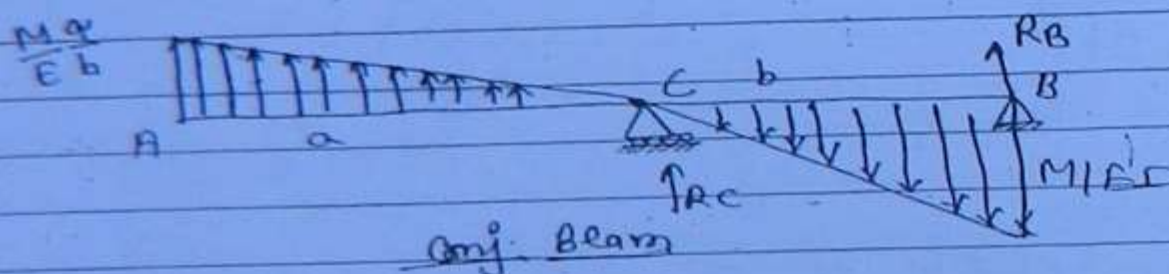
$$M_A = M \frac{a}{b}$$

$$\text{C.O.F.} = \frac{M_A}{M} = \frac{a}{b}$$



$\frac{M}{EI}$ or curvature diagram, for given beam.

(138)



Slope at B in real beam = S.F. at B in conj. beam = R_B

To find R_B take $\sum M_C = 0$

$$R_B \times b - \frac{M}{EI} \times \frac{1}{2} \times b \times \frac{2}{3}b - \frac{1}{2} \times a \times \frac{M}{EI} \times \frac{a}{3} \times \frac{2}{3}a = 0$$

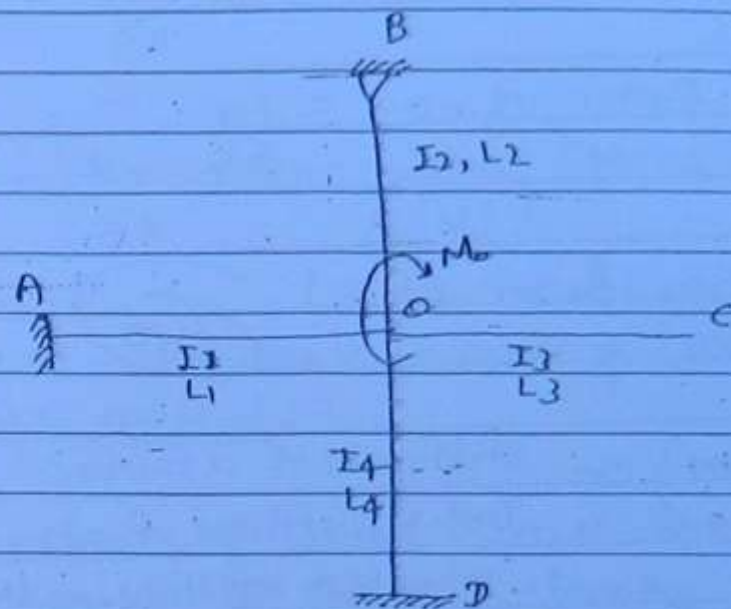
$$R_B = \frac{M \cdot b}{3EI} + \frac{M}{3EI} = \frac{a^3}{b^2} \approx 0$$

$$\Rightarrow \frac{M}{3EI} \left[\frac{b^3 + a^3}{b^2} \right]$$

$AB = 1$ $M = 1$

$$k = \left(\frac{3EI \cdot b^2}{a^3 + b^3} \right) \text{ or } k$$

Distribution theorem :-



(139)

O = Rigid joint

$$K_1 = \text{Stiffness of } OA = \frac{4EI_1}{L_1}$$

$$K_2 = \text{Stiffness of } OB = \frac{3EI_2}{L_2}$$

$$K_3 = \text{Stiffness of } OC = 0$$

$$K_4 = \text{Stiffness of } OD = \frac{4EI_4}{L_4}$$

Total stiffness of all member at O

$$\Sigma K = K_1 + K_2 + K_3 + K_4$$

At joint O

$$\text{Distribution factor of } OA = \frac{K_1}{\Sigma K}$$

$$\text{Distribution factor of } OB = \frac{K_2}{\Sigma K}$$

Page _____

$$\text{Distribution factor of oc} = \frac{K_3}{\Sigma K}$$

$$\text{Distribution factor of od} = \frac{K_4}{\Sigma K}$$

$$\text{Distribution factor } \Sigma K = 1 \quad (140)$$

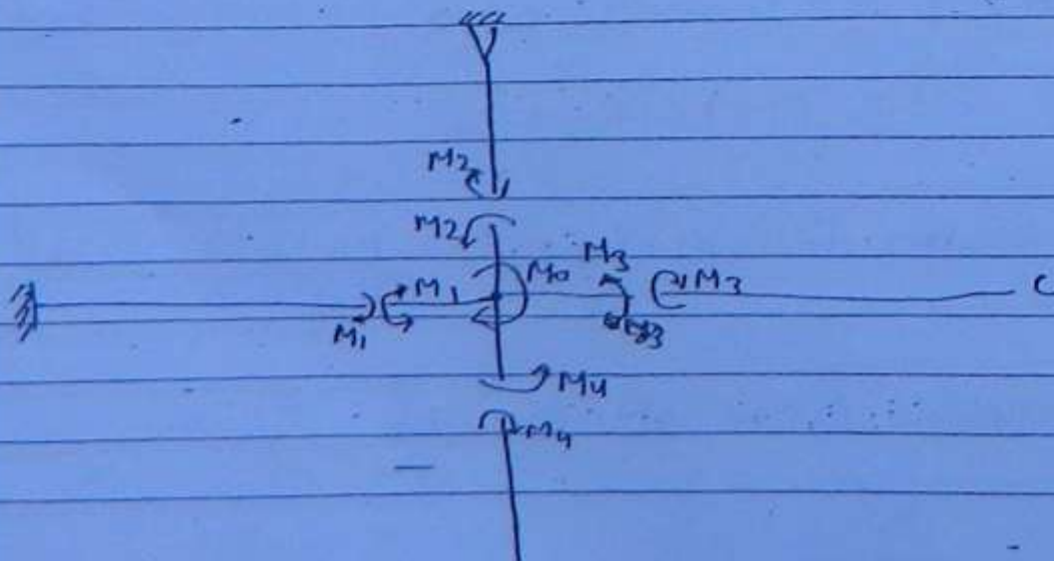
D.F. (Distribution factor) :- $\frac{\text{Stiffness of a factor}}{\text{Total stiffness of all member meeting at that joint}}$

$$\text{D.F.} \Rightarrow \frac{\text{R.S. of a member}}{\text{T.R.S.}}$$

Moment Distribution Theorem :-

Theorem ->

If At A rigid joint where more than 1 member are meeting ~~at~~ and external moment is applied M_0 , then it gets distribution in all member in the direction of applied moment in proportion of stiffness of each member.



$$M_1 + M_2 + M_3 + M_4 = 0$$

$$M_1 = \frac{K_1}{\sum K} \times M_0$$

$$M_2 = \frac{K_2}{\sum K} \times M_0$$

$$M_3 = \frac{K_3}{\sum K} \times M_0$$

$$M_4 = \frac{K_4}{\sum K} \times M_0$$

(141)

→ procedure to draw Bending moment dia in stiffness method:-

→ If the no. of spans/stories are large then force method is complex & lengthy such cases stiffness cases use.

In A stiffness method each span is considered ~~to~~ separately to draw BMD. At the both ends final moment are found each moment which are called end moments. ~~the~~ which can be found using any of the stiffness method.

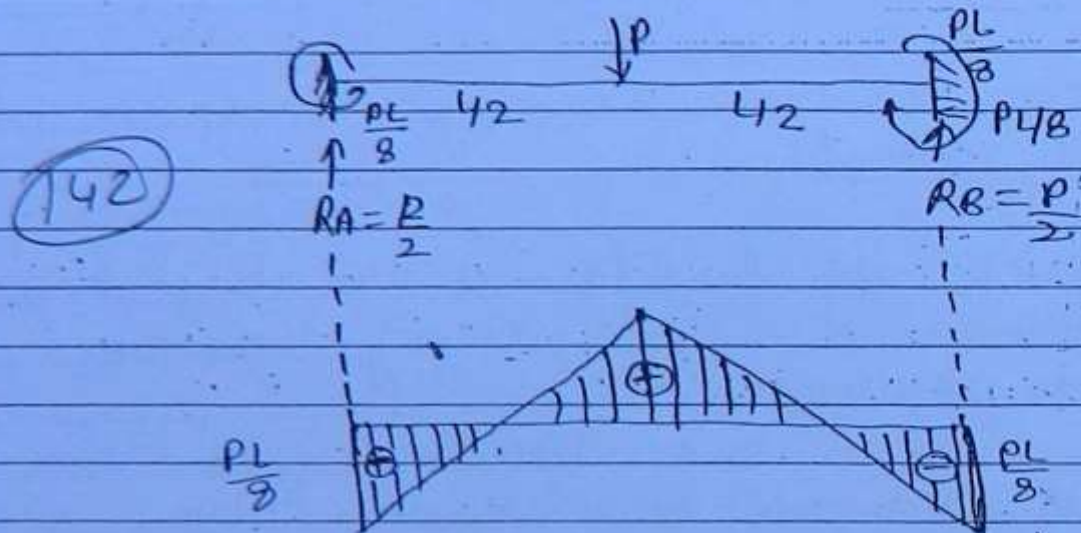
for every member end moment dia. is drawn in which sagging moment (-ive) are plotted below the beam/ Inside the frame.

And Hogging moment are plotted above the beam/ outside the frame.

for each member considering free simply supported & with given loading free BMD/ simply supported dia. is drawn separately. In which sagging above the beam is positive (+ive) Hogging is taken below the beam (-ive) inside the frame.

To obtain final diagram. End moment dia. And free BMD are superimposed.

The common are is cancelled.
At net diagram is resultant magnitude.

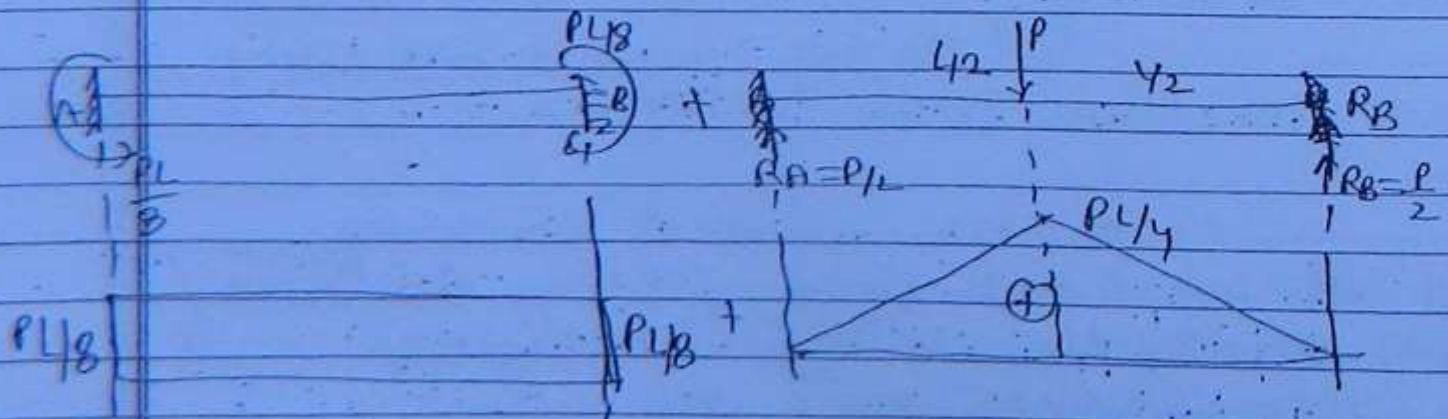


in B.M. in AC

$$M_x = R_A \cdot x - \frac{P \cdot x}{2}$$

$$M_A = -\frac{P \cdot L}{8}$$

$$M_C = \frac{P}{2} \cdot \frac{L}{2} - \frac{P \cdot L}{8} \Rightarrow \frac{+PL}{8}$$

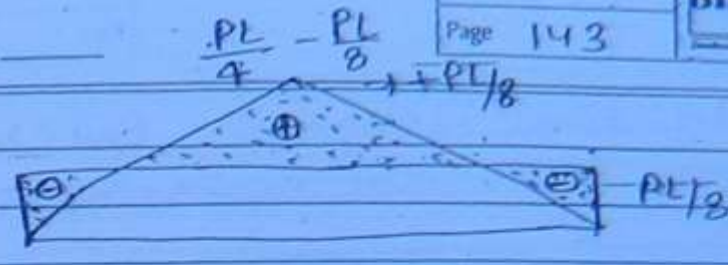


End Moment Diagram.

S.S. BMD. for BMD face.

Total Aq. $\Rightarrow$

$$\frac{PL}{8}$$



final B.M.D

(143)

Sign. Convention. for computation of end moment.

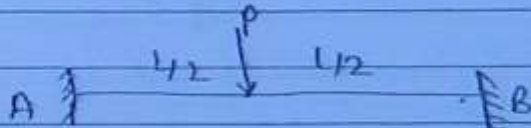
All clock-wise moment will be taken ~~the~~ (ive) and anti-clock wise moment will be (ive)

(+)

(-ve)

Fixed end moment ^{for} Standard loading condition:-

(1)

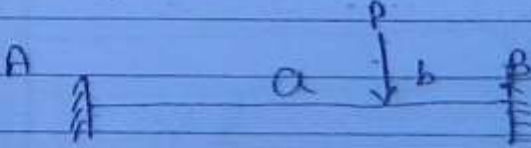


$\bar{M}_{AB}$ = Fixed End Moment at A for AB.

$$\Rightarrow -\frac{PL}{8}$$

$$\bar{M}_{BA} = +\frac{PL}{8}$$

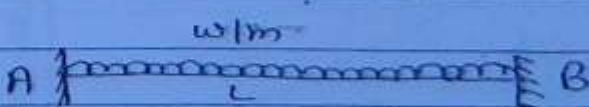
(2)



$$\bar{M}_{AB} = -\frac{Pa^2b^2}{L^2}$$

$$\bar{M}_{BA} = +\frac{Pa^2b^2}{L^2}$$

(3)

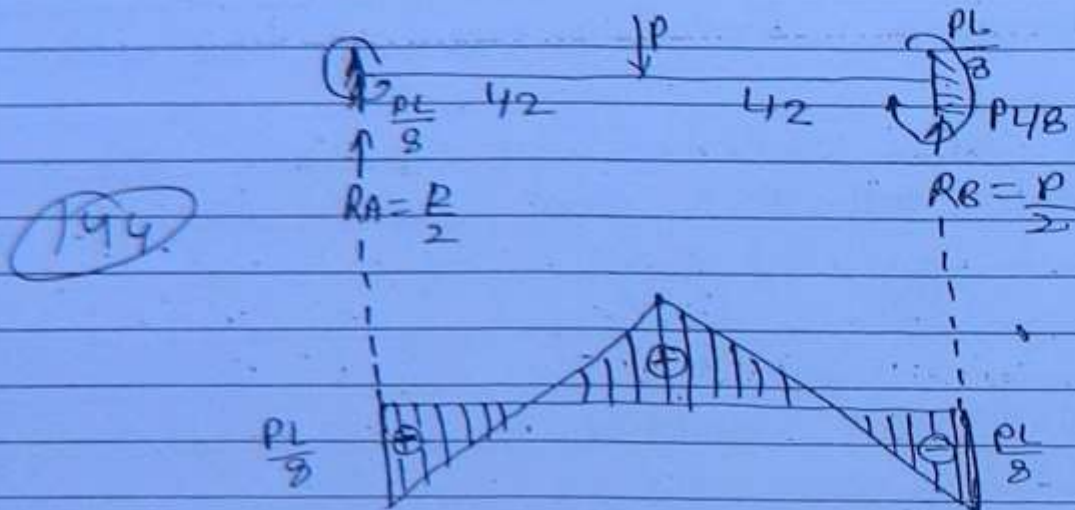


$$\bar{M}_{AB} = -\frac{wL^2}{8}$$

$$\bar{M}_{BA} = +\frac{wL^2}{8}$$

To obtain final diagram. End. moment dia.
and free BMD are superimposed.

The common one is cancelled.
At net diagram is resultant magnitude.

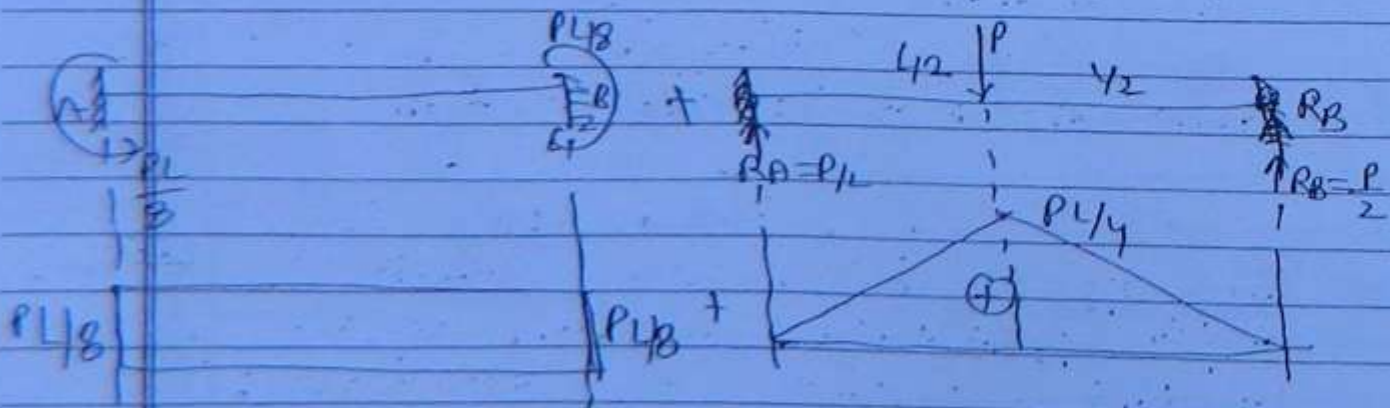


in B.M. in AC

$$M_x = R_A \cdot x - \frac{P \cdot x}{2}$$

$$M_A = -\frac{P \cdot L}{8}$$

$$M_C = \frac{P}{2} \cdot \frac{L}{2} - \frac{P \cdot L}{8} \Rightarrow \frac{+PL}{8}$$

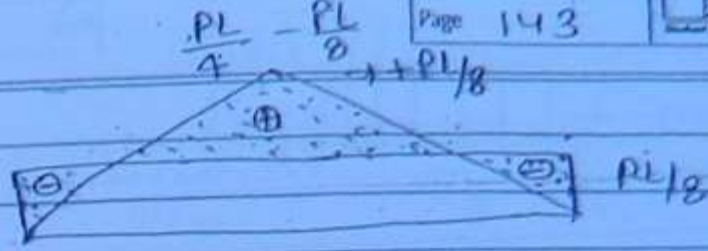


End Moment Diagram.

S.S. BMD. / For BMD
face.

Total Aq. $\Rightarrow$

$\frac{PL}{8}$



final B.M.D

(145)

Sign. Convention. for Computation of end moment.

All Clock-wise moment will be taken ~~the~~ (ive) and anti-clock wise moment will be (ive)

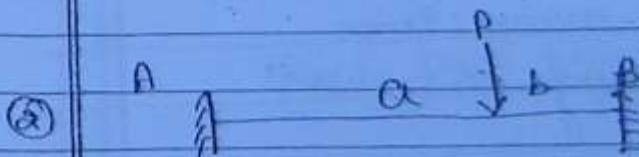
⊕

⊖

Fixed end moment ^{for} Standard loading condition:- $\bar{M}_{AB}$ = Fixed End Moment at A for AB.

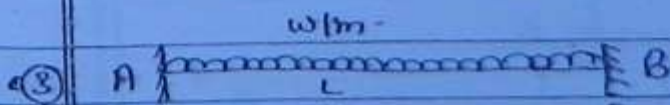
$\Rightarrow -\frac{PL}{8}$

$\bar{M}_{BA} = +\frac{PL}{8}$



$\bar{M}_{AB} = -\frac{Pab^2}{L^2}$

$\bar{M}_{BA} = +\frac{Pa^2b}{L^2}$



$\bar{M}_{AB} = -\frac{wL^2}{8}$

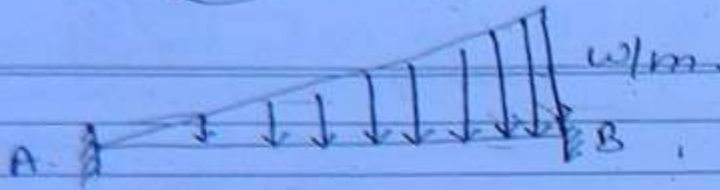
$\bar{M}_{BA} = +\frac{wL^2}{8}$

(-ive) (+ive)

Date / /
Page

10/5/20

④

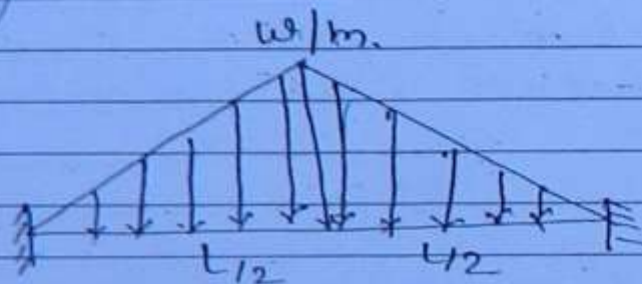


$$M_{AB} = -\frac{wL^2}{30}$$

$$M_{BA} = +\frac{wL^2}{30}$$

⑤

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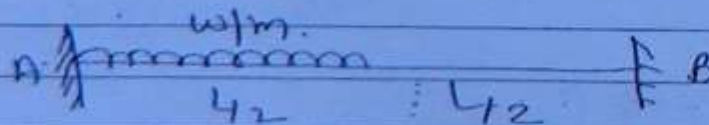


$$M_{AB} = -\frac{5wL^2}{96}$$

$$M_{BA} = +\frac{5wL^2}{96}$$

→ if arrow of loading points on the Reference face then loading is taken the (+ive) and if arrow of loading point away from Reference face then loading is (-ive)

⑥

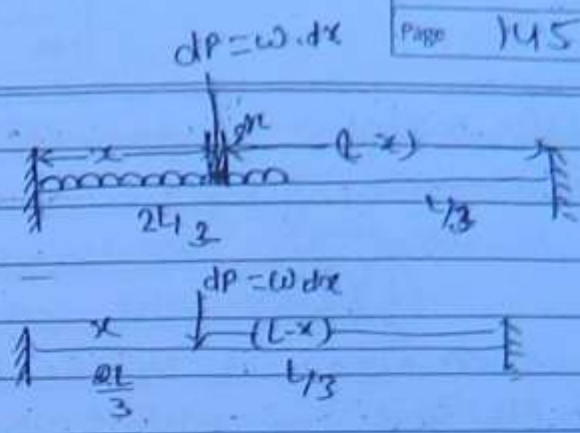


$$M_{AB} = -\frac{11wL^2}{192}$$

$$M_{BA} = +\frac{5wL^2}{192}$$

Note :-

(7)



147

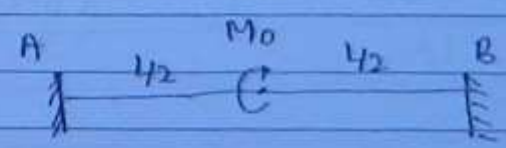
$$d\bar{M}_{AB} = - \frac{dP \cdot x \cdot (L-x)^2}{L^2} \quad \left\{ \frac{w \cdot dx \cdot x \cdot (L-x)^2}{L^2} \right\}$$

$$\Rightarrow - \frac{(w \cdot dx) \cdot x \cdot (L-x)^2}{L^2}$$

$$\bar{M}_{AB} = \int d\bar{M}_{AB} = - \int_0^{2L/3} \frac{w \cdot x \cdot (L-x)^2}{L^2} \cdot dx$$

$$\bar{M}_{BA} = \int d\bar{M}_{BA} = \int_0^{2L/3} \frac{w \cdot x^2 \cdot (L-x) \cdot dx}{L^2}$$

(8)



(only direction, not sagging & hogging than the value)

$$\bar{M}_{AB} = + \frac{M_o}{4}$$

$M_o \rightarrow$ +ive if (C)

$M_o \rightarrow$ -ive if (J)

$$\bar{M}_{BA} = + \frac{M_o}{4}$$

(9)



$$\bar{M}_{AB} = \frac{M_o b (3a-L)}{L^2}$$

$$\bar{M}_{BA} = \frac{M_o \cdot a \cdot (3b-L)}{L^2}$$

→ Note that if $a > \frac{L}{3}$, then $M_{AB} > 0$, M_{AB} is in the direction of M_0 .

(148)

→ if $b > \frac{L}{3}$, then M_{BA} is in the direction of M_0 . Hence, if M_0 acts in middle third strip, then fixed end moment at A & B. Both will be in dirⁿ of M_0 .

⑥ Effect of settlement of support



$$M_{AB} = -\frac{6EI\Delta}{L^2}$$

$$M_{BA} = -\frac{6EI\Delta}{L^2}$$

$$\sum F_y = 0$$

$$R_A + R_B = 0$$

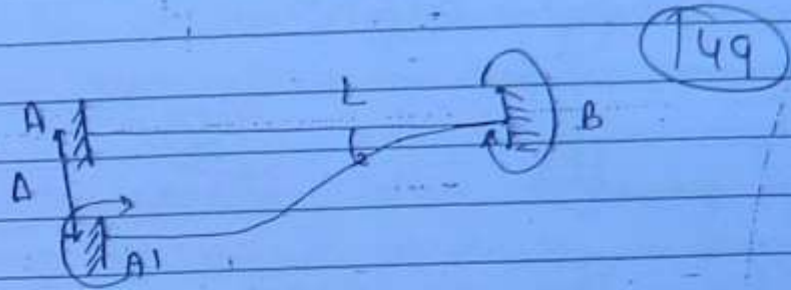
$$\sum M_B = 0$$

$$R_A \cdot L - \frac{6EI\Delta}{L^2} - \frac{6EI\Delta}{L^2} = 0$$

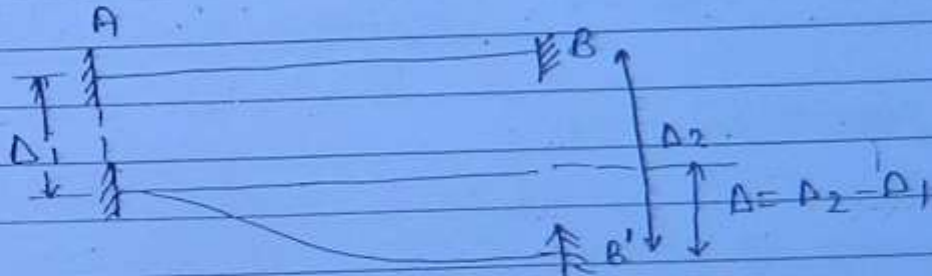
$$R_A = \frac{12EI\Delta}{L^3}$$

$$\text{s.f. in AB} = +R_A = \frac{12EI\Delta}{L^3}$$

Note:- If settlement of support cause clockwise rotation to the member then fixing moment developed at both ends will be anticlockwise and vice-versa.



$$\bar{M}_{AB} = \bar{M}_{BA} = +\frac{6EI\Delta}{L^2}$$



(11)



$$M_{AB} = -\frac{3EI\Delta}{L^2}$$

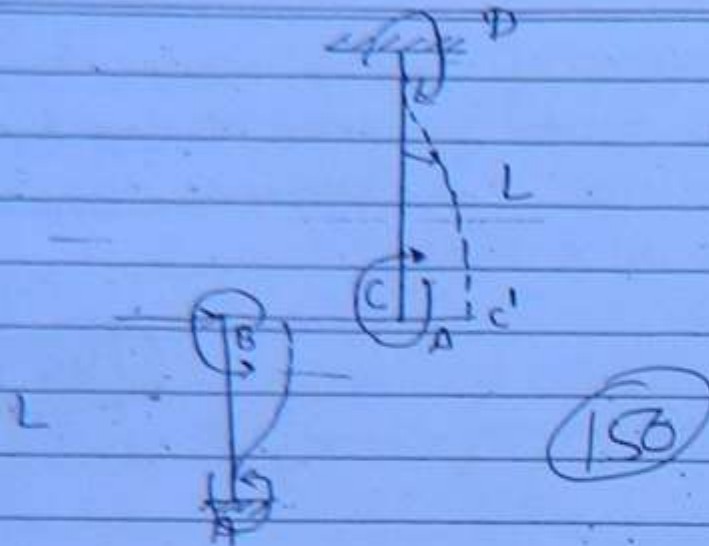
$$M_{BA} = 0$$

(12)



$$\bar{M}_{AB} = \bar{M}_{BA} = 0$$

(13)



(150)

$$\overline{M}_{AB} = \overline{M}_{BA} = \frac{-6EID}{L^2}$$

$$\overline{M}_{CD} = \overline{M}_{DC} = \frac{+6EID}{L^2}$$

$$\overline{M}_{BC} = \overline{M}_{CB} = 0$$

(14)

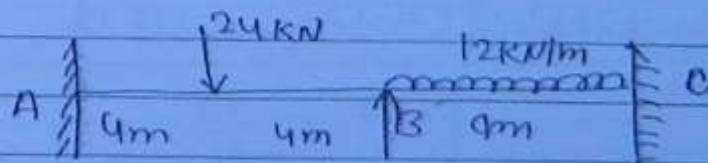
Procedure:-

- (1) Find Fixed end moment for each member for both ends. Considering clockwise moment +ve & Anti-clockwise moment (-ve)
- (2) Find distribution factor for each member at those joint where more than one member meet & joints are rigid.
- (3) Distribute the moment after balancing at each joint according to these distribution factor & then transfer carry over moment to the far ends if far ends are fixed.
- (4) find final end moment at the ends of tabular calculation +ve, -ve means Anticlockwise

(157)

- (5) Draw End Moment diagram and Super imposed S. S. dia. to obtained resultant moment diagram in B.M.D +ve Sagging -ve Hogging

Q For the beam shown in Fig. draw B.M.D using Hardy's cross method.



$EI = \text{constant}$

→ Fixed end moment

$$\bar{M}_{AB} = -\frac{PL}{8} = -\frac{24 \times 8}{8} = -24$$

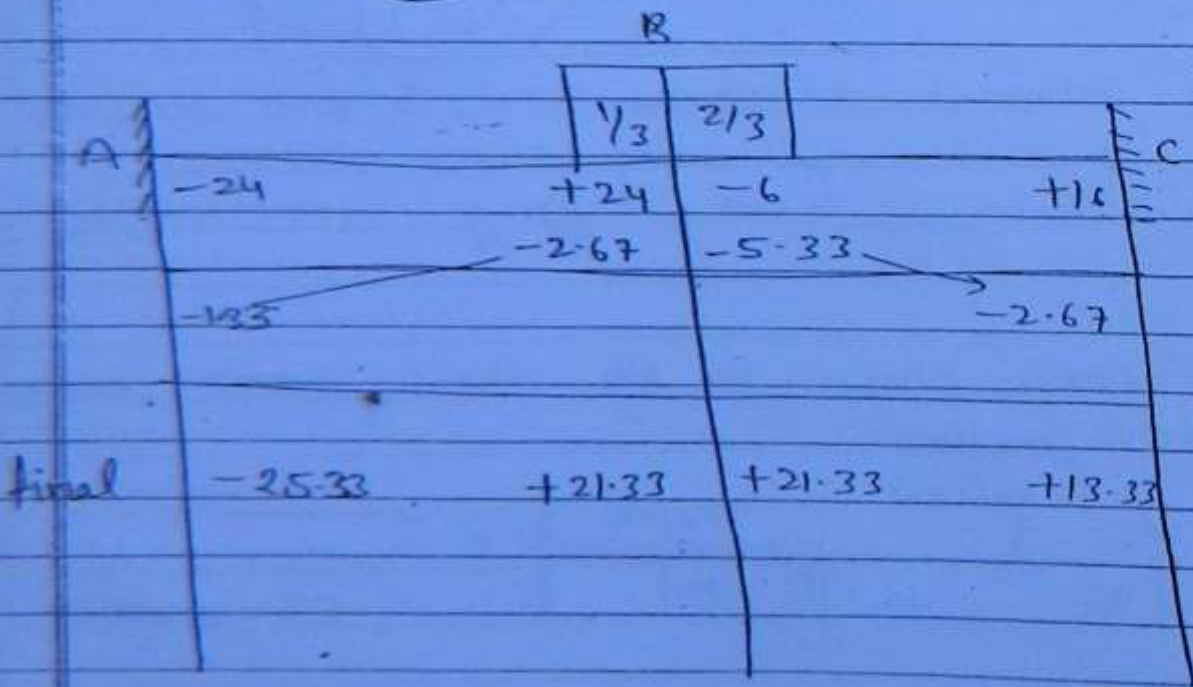
$$\bar{M}_{BA} = +\frac{PL}{8} = +24$$

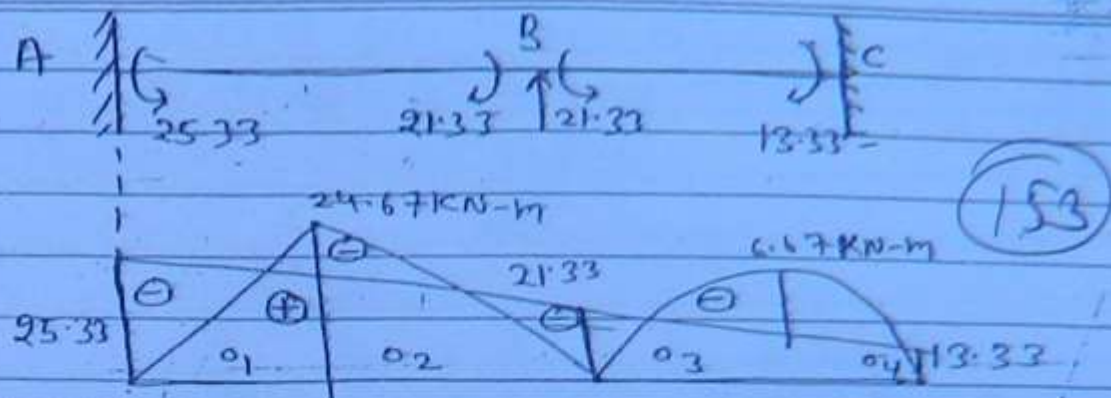
$$\bar{M}_{ac} = -\frac{wl^2}{12} \Rightarrow -\frac{12 \times 4^2}{12} \Rightarrow -16$$

$$\bar{M}_{cb} = +\frac{wl^2}{12} = +16$$

Joint	member	Stiffness	Total Stiffness	D.F. = $\frac{K}{\sum K}$
B	BA	$4EI/8$	$\frac{3}{2}EI$	$\frac{1}{3}$
	BC	$4EI/4$		

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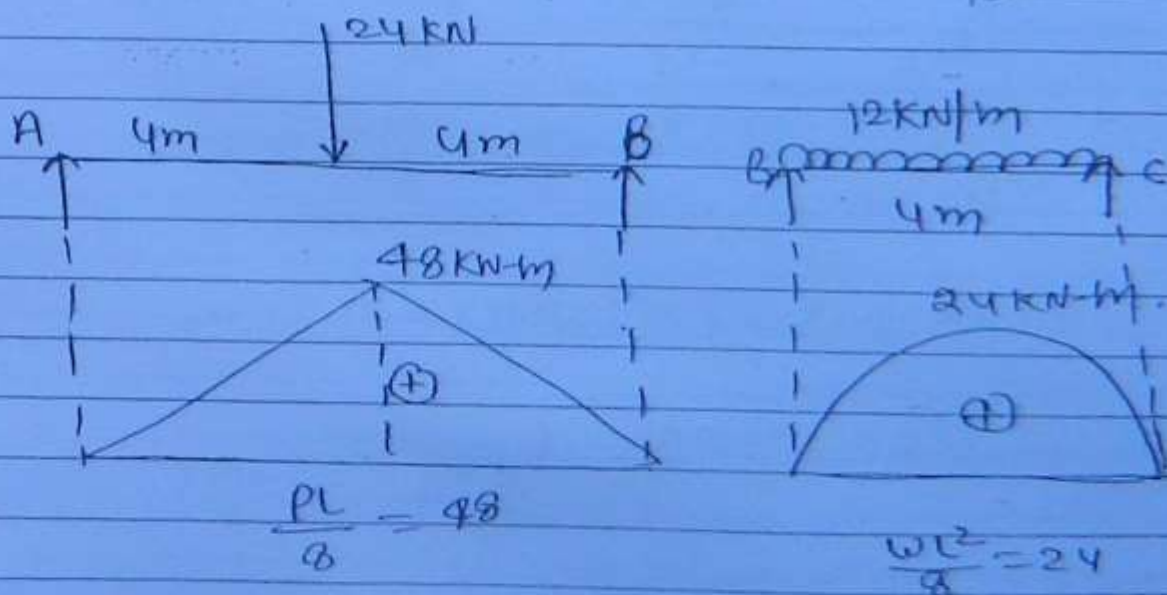


End moment dia

Resultant B.M. at centre of AB = $48 - \left(\frac{25.33 + 21.33}{2} \right)$

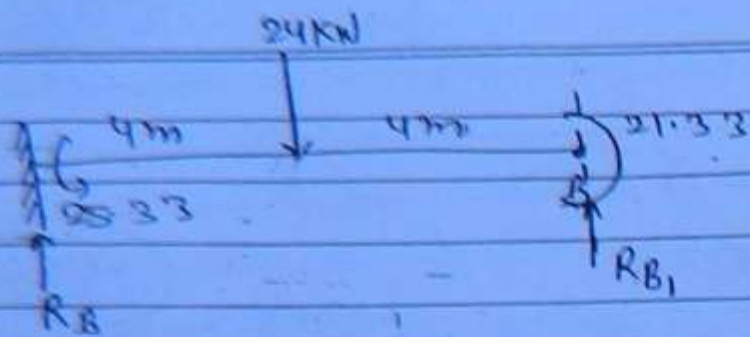
$\Rightarrow +24.67$

at BC = $24 - \left(\frac{21.33 + 13.33}{2} \right) \Rightarrow +6.67$



Computation of Reactions:

$\rightarrow$ To Find R_A consider free body equilibrium just to left of B.



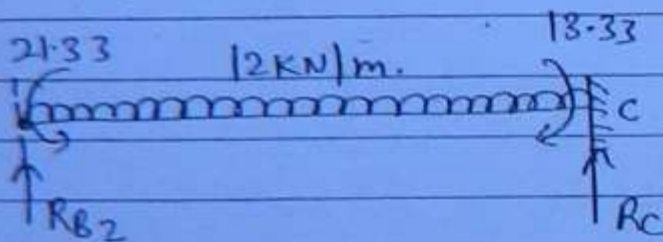
154

$$\sum M_B = 0$$

$$R_A \times 8 - 24 \times 4 + 21.33 = 0$$

$$R_A = 12.5 \text{ kN}$$

To find R_c consider free body equilibrium of span BC.



$$\sum M_B = 0$$

$$-R_c \times 4 + 13.33 + 12 \times 4 \times 2 - 21.33 = 0$$

$$R_c = 22 \text{ kN}$$

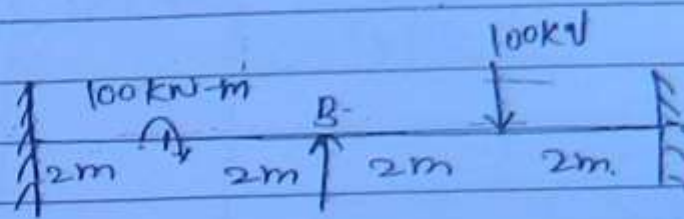
→ find $R_B = 0$

$$\sum F_y = 0$$

$$R_A + R_B + R_c - 24 - 12 \times 4 = 0$$

$$R_B = 37.5 \text{ kN}$$

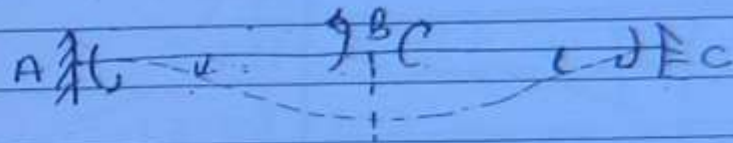
Prob 22 Draw B.M.D for beam ABC sway in Fig
Support B settles down by 1m $EI = \text{constant}$
i.e. Equal to 48000 kN/m²



155

$$\Delta B = 1 \text{ m} \downarrow$$

$$EI = 48000 \text{ kN-m}^2$$



$$\bar{M}_{AB} = +\frac{M_0}{4} - \frac{6EI\Delta}{L^2}$$

$$\Rightarrow \frac{+100}{4} - \frac{6 \times 48000 \times 1 \times 10^{-3}}{2}$$

$$\Rightarrow 25 - 18 \Rightarrow +7$$

$$\bar{M}_{BA} = +\frac{M_0}{4} - \frac{6EI\Delta}{L^2} \Rightarrow +7$$

$$\bar{M}_{BC} = -\frac{PL}{8} + \frac{6EI\Delta}{L^2} \Rightarrow \frac{-100 \times 4}{8} + 18$$

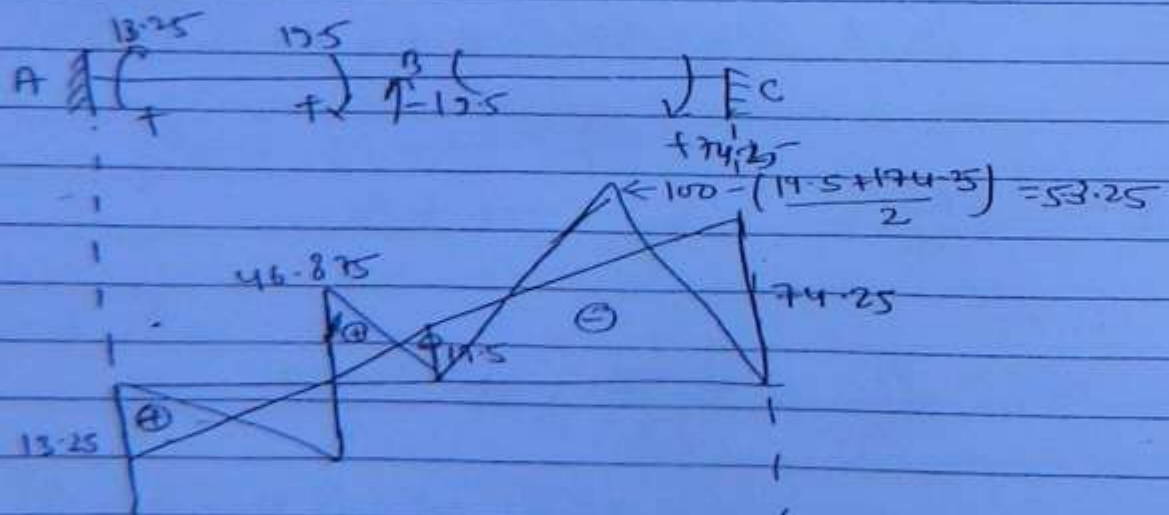
$$\Rightarrow -50 + 18$$

$$\bar{M}_{CB} = +\frac{PL}{8} + \frac{6EI\Delta}{L^2} = +50 + 18 = +68$$

Joint	member	R. stiff	T.R.S	D.F. = $\frac{R.S}{T.R.S}$
B	BA	I/4	2I/4	1/2
	BC	I/4		1/2

(156)

A	B		C	F.F.M Moment B.m.
	1/2	1/2		
+7	+7	-32	+68	
	+12.5	+12.5		
	+6.25		+6.25	
Total	+13.25	+19.5	-19.5	+74.25



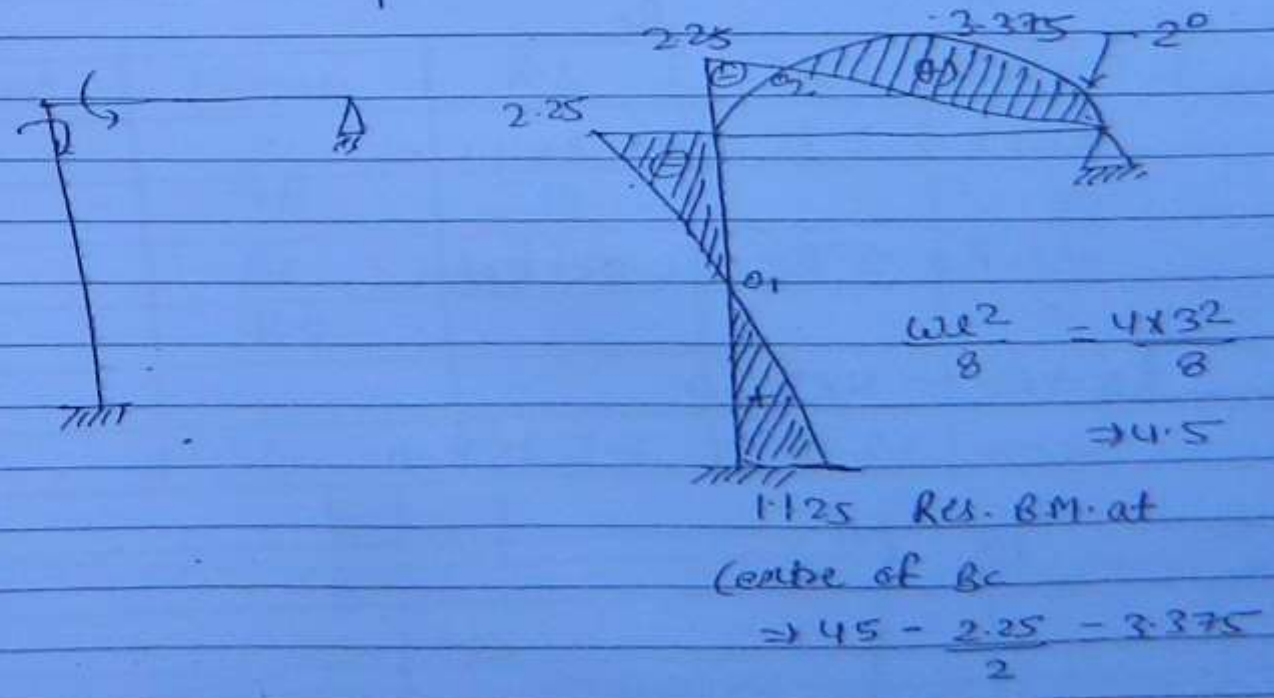
Net B.M at Centre of AB

$$7.50 - 3.125 = 53.125$$

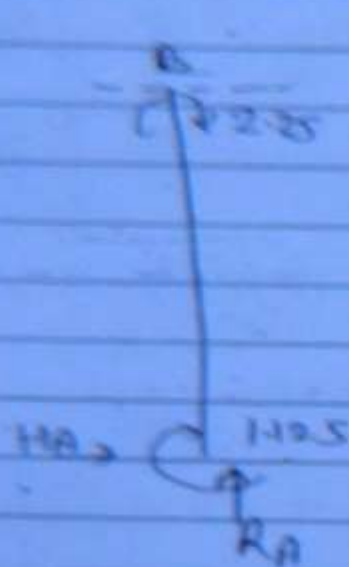
Joint	member	R.S.	T.R.C.	D.F.
B	BA	$I/4$	$\frac{25}{4}$	$\frac{1}{2}$
	BC	$\frac{3}{4} \times \frac{I}{3}$		$\frac{1}{2}$

(157)

	A	B	C	
	0	$\frac{1}{2}$ $\frac{1}{2}$	0	
	0	0	-3	+3
			+1.5	-3
				End correction
	0	0	-4.5	0
			+2.25	+2.25
		1.125		
final		1.125	+2.25	-2.25
				0



To find the consider force body equilibrium of AB



(158)

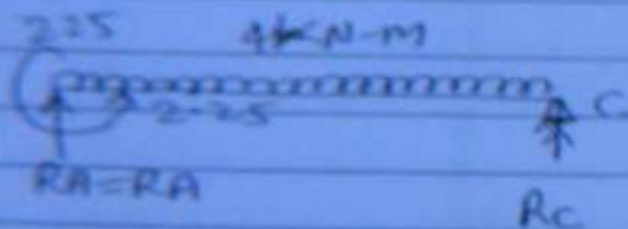
$$\sum M_B = 0$$

$$-H_A \times 4 + 1.125 + 2.25 = 0$$

$$H_A = \frac{1.125 + 2.25}{4}$$

$$= 0.84 \text{ kN}$$

To find RA Consider F.B. equi of BC



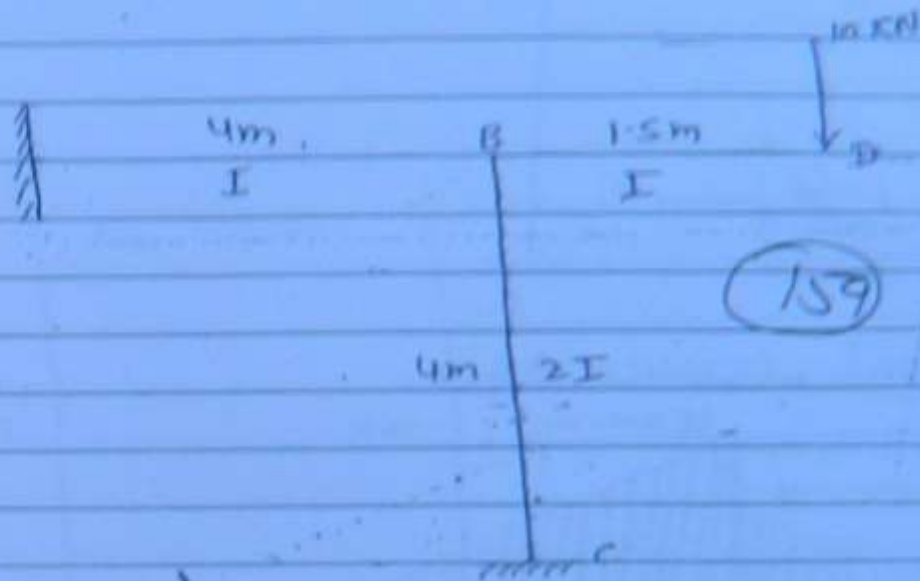
$$R_A \times 3 - 2.25 - 4 \times 3 \times 1.5 = 0$$

$$R_A = R_A = 6.75 \text{ kN}$$

$$R_A + R_C - 4 \times 3 = 0$$

$$R_C = 12 - 6.75 = 5.25 \text{ kN}$$

Q1. Analysis frame shown in fig. using M.D.M.
Draw B.M.D and elastic curve.



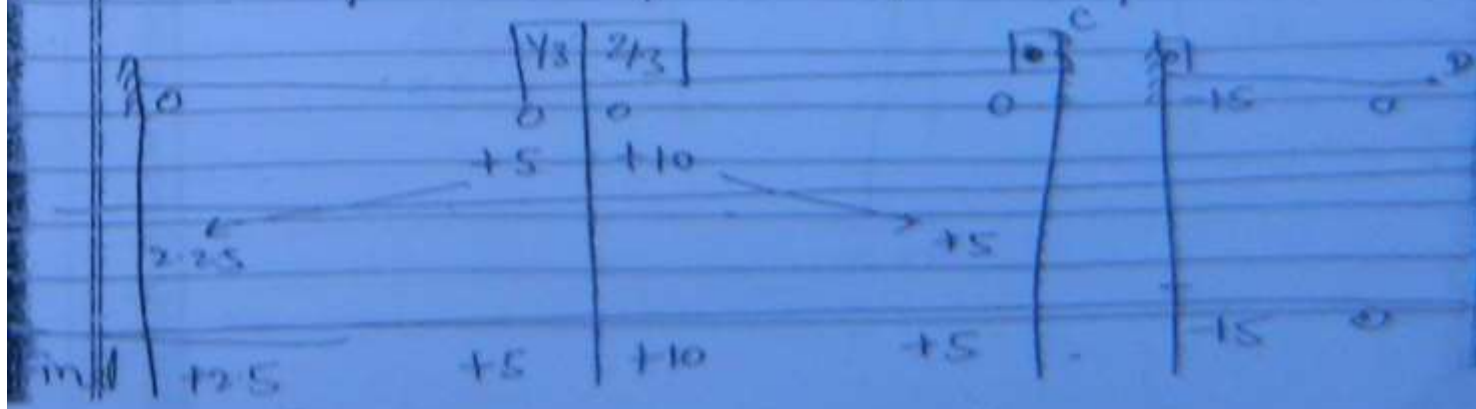
$$\bar{M}_{AB} = 0 = \bar{M}_{BA}$$

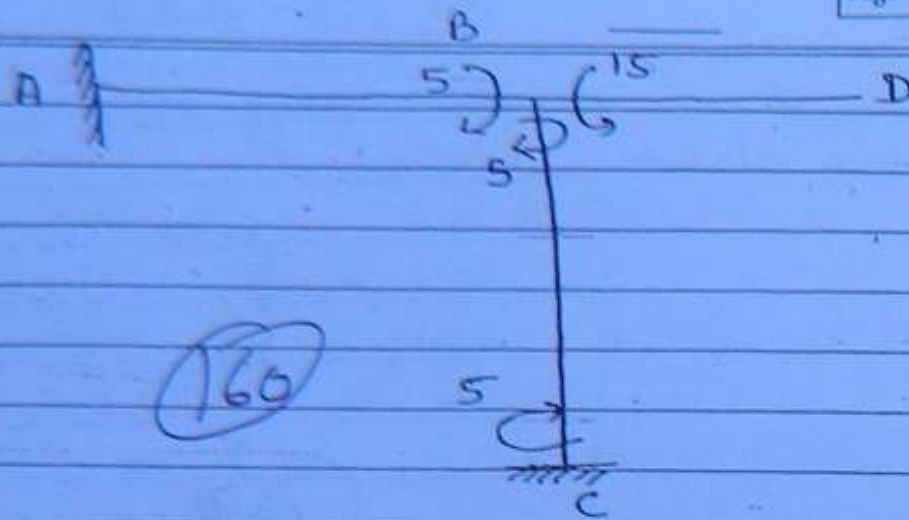
$$\bar{M}_{BC} = 0 = \bar{M}_{CB}$$

$$\bar{M}_{BD} = -15$$

$$\bar{M}_{DB} = 0$$

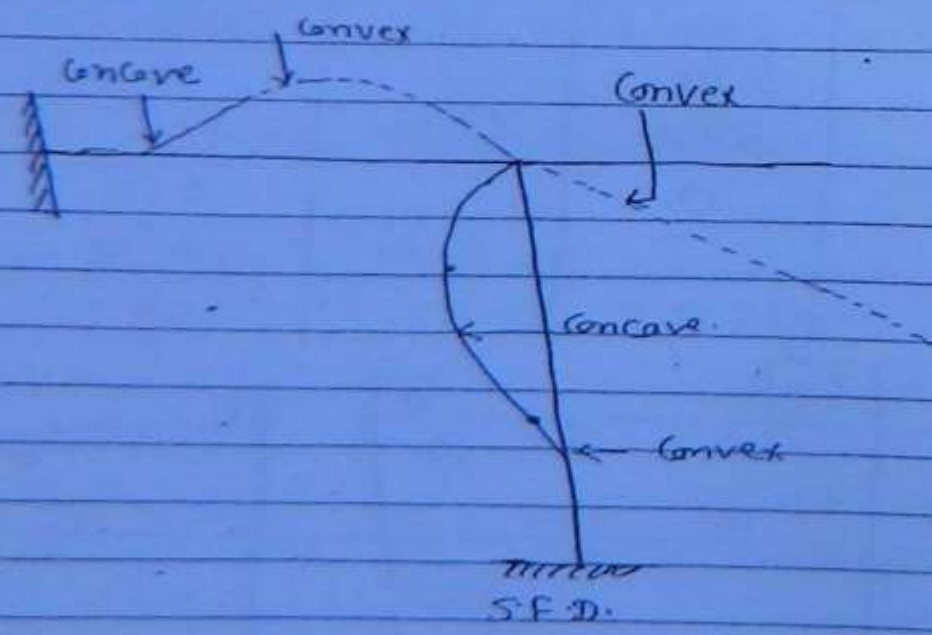
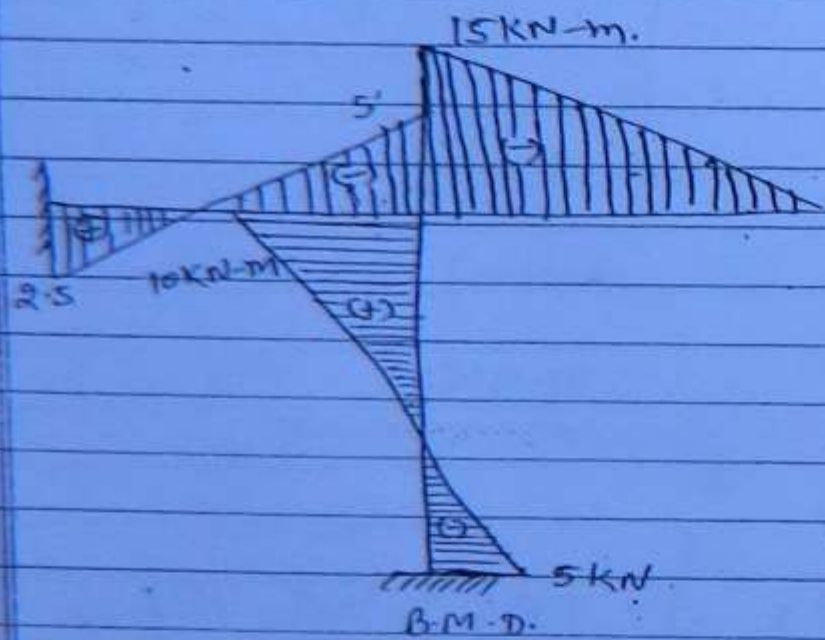
Joint	Member	R.S.	T.R.S.	$D.F. = \frac{R.S.}{T.R.S.}$
B	BA	$\frac{I}{4}$		$\frac{1}{2}$
	BC	$\frac{2I}{4}$	$\frac{3I}{4}$	$\frac{2}{3}$
	BD	0		0





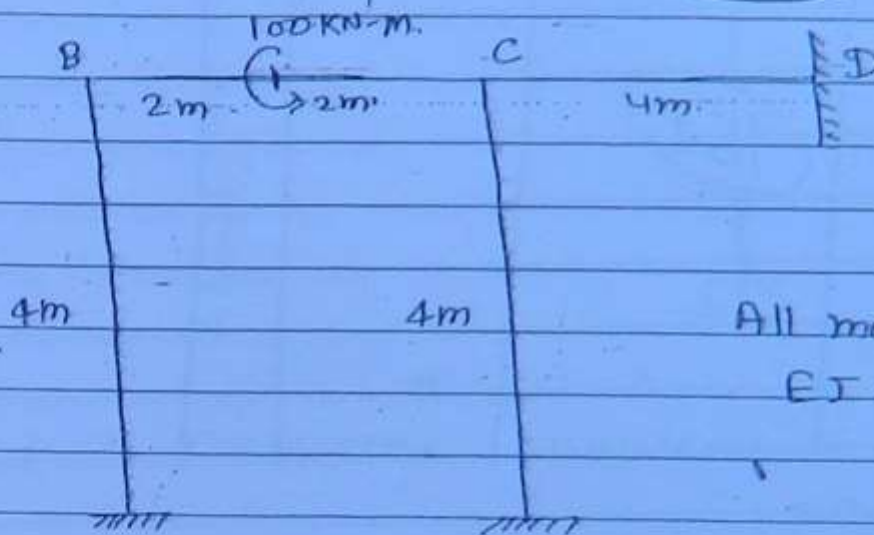
160

→ Horizontal Member AB →



Q.5 Analysis given Frame shown in Fig. using moment distribution Method and draw Bending moment Diagram.

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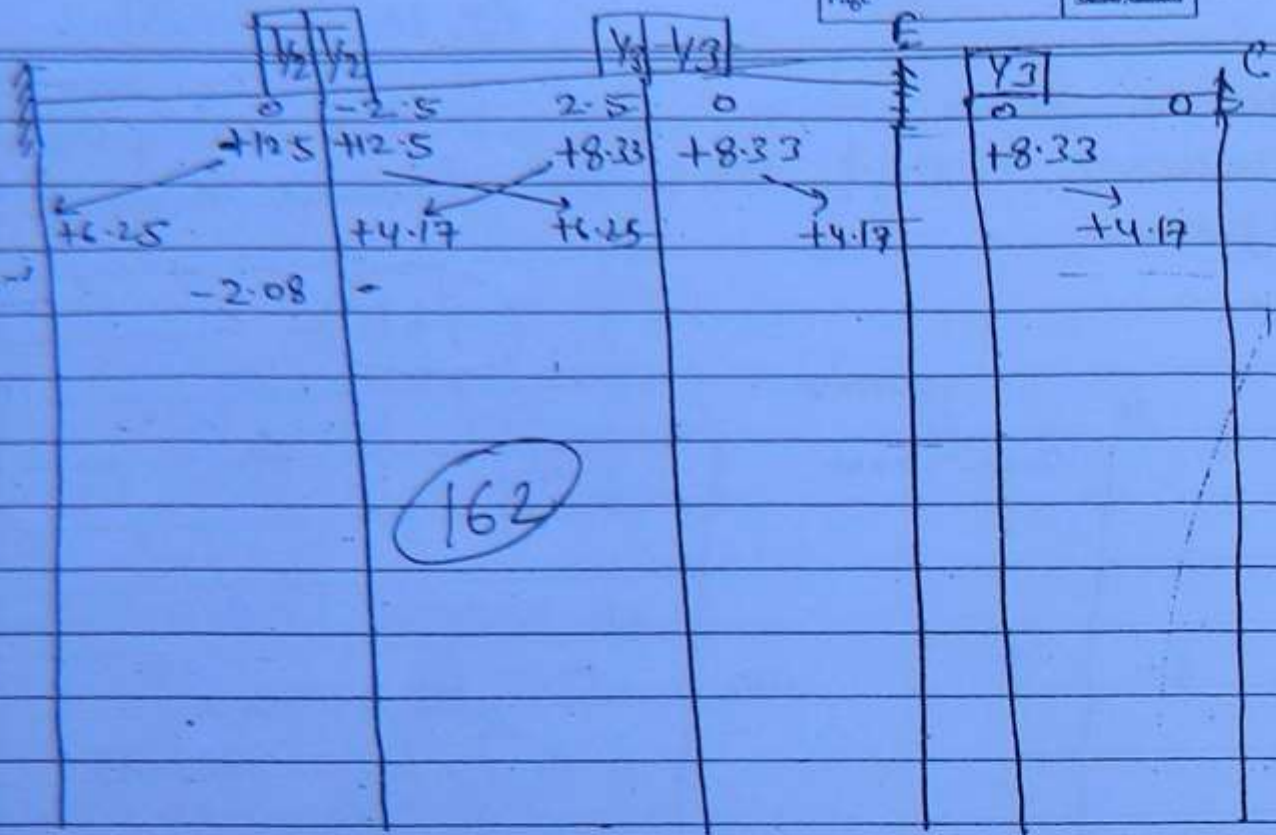
All member
 $EI = \text{constant}$

$$\bar{M}_{BC} = \frac{M_0}{4} = \left(-\frac{100}{4} \right) = -25$$

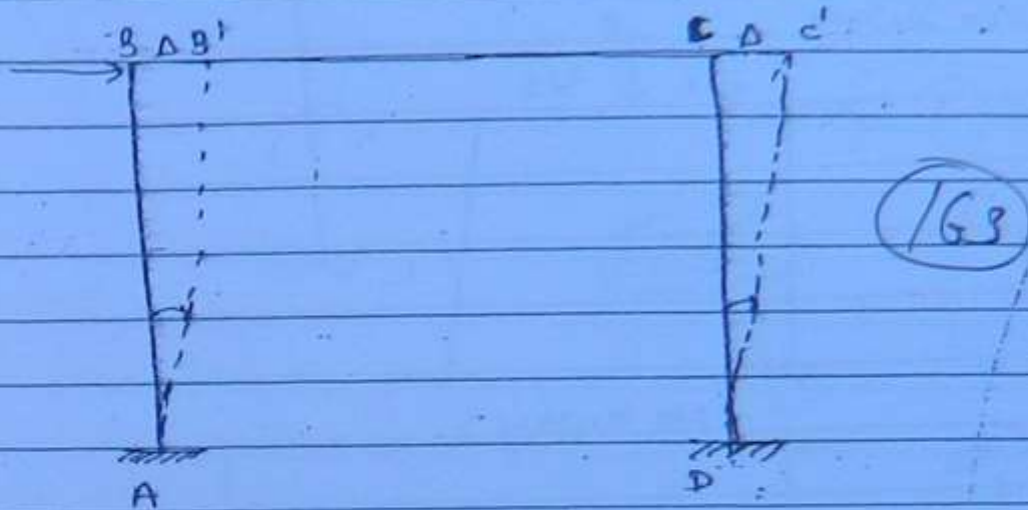
$$\bar{M}_{CB} = -25$$

Other member are zero

Joint	Member	R.S.	T.R.S.	D.F.
B	BA	$I/4$		$1/2$
	BC	$I/4$		$1/2$
C	CB			$1/3$
	CD			$1/3$
	CE			$1/3$



SWAY Analysis (Joint Sway)



$$BB' = CC' = \Delta$$

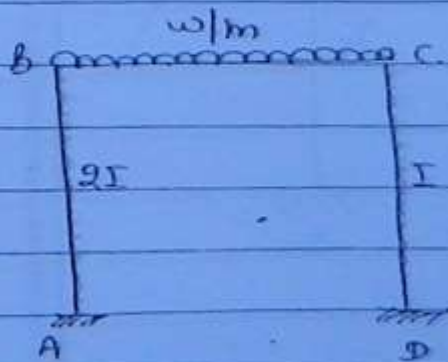
$\Delta \rightarrow$ Transverse displacement of member or shear displacement

$$\bar{M}_{AB} = \bar{M}_{BA} = -\frac{6EI\Delta}{L^2}$$

$$\bar{M}_{CD} = \bar{M}_{DC} = -\frac{6EI\Delta}{L^2}$$

$$\bar{M}_{BC} = \bar{M}_{CB} = 0$$

Thumb rule to find direction of sway.

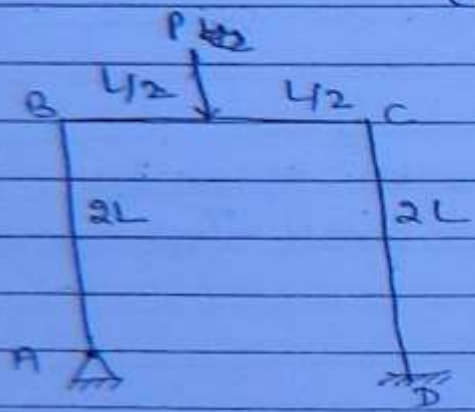
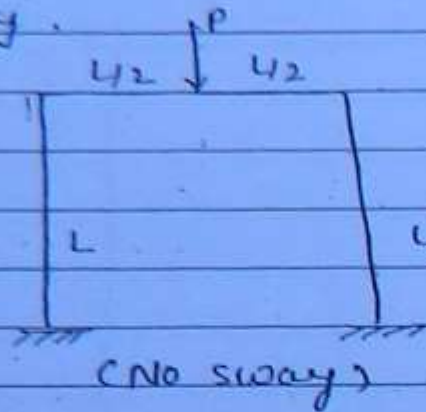


$\rightarrow$ sway Towards right
Because stiffness of CD
is less than stiffness of AB.

loading is symm. but stiffness is not symm.

If loading, stiffness and support condition are symm. about the centre of frame, there will be no sway.

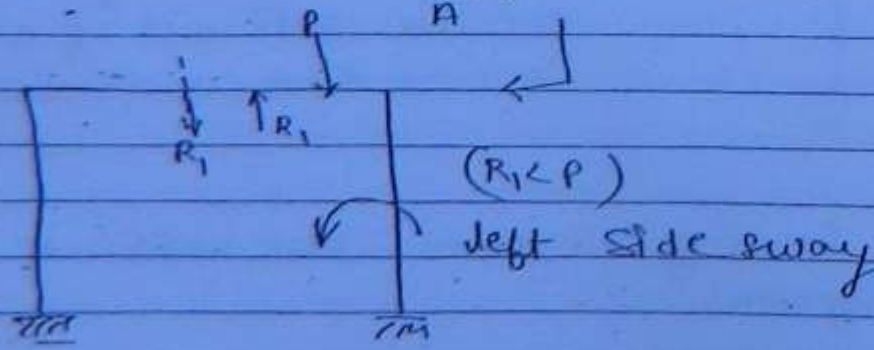
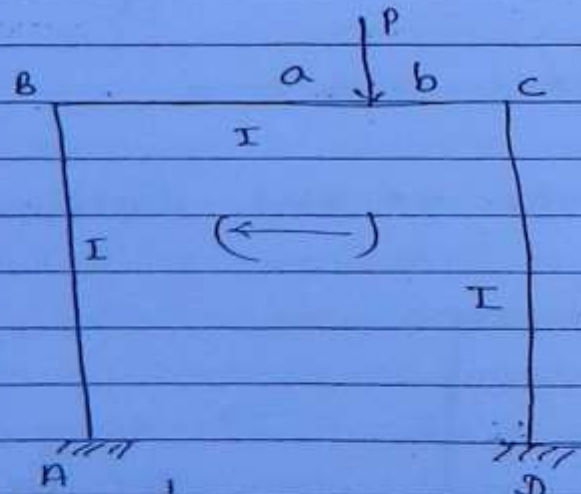
164

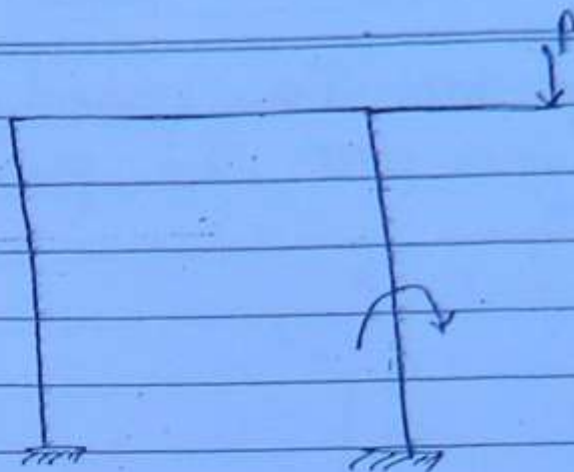


Sway towards left
because stiffness of AB
is less than stiffness of
CD.

→ Sway towards left if $a > b$

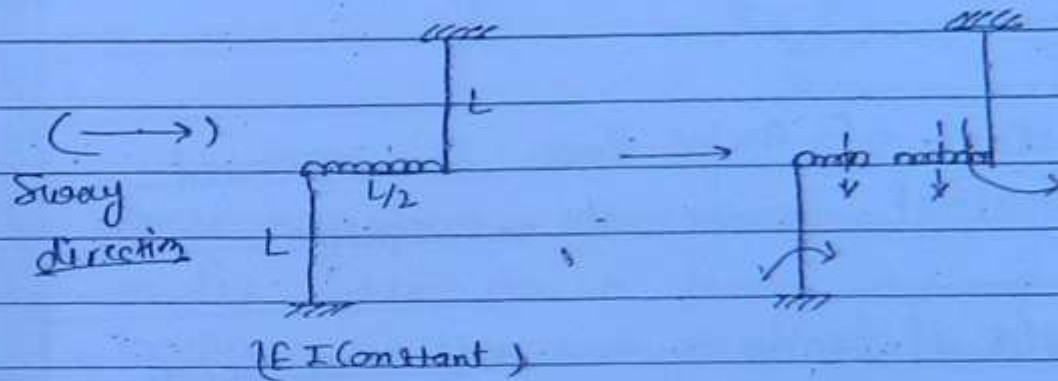
→ Sway towards Right if $a < b$



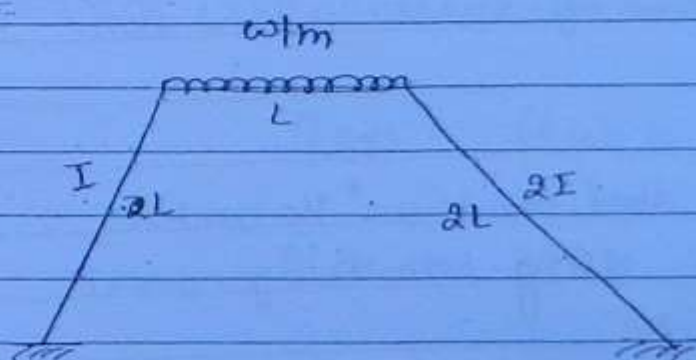


Toward Right.

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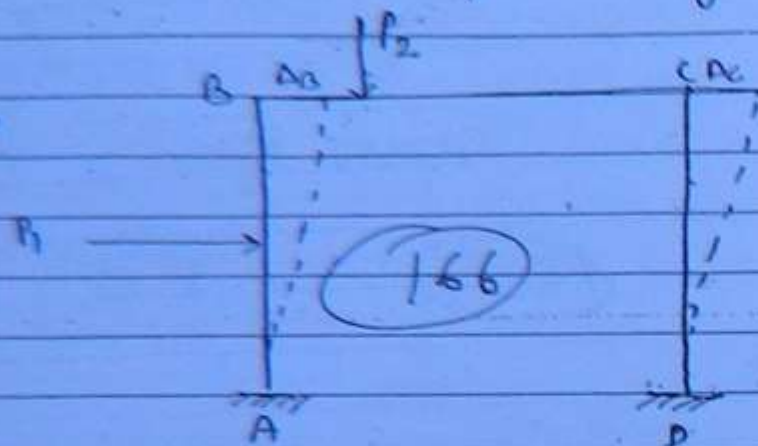


Resultant Sway Towards Right. ($\rightarrow$)



Sway may occur, which can be found by Computation.

Procedure to Analysis Sway cases when Δ is not known



$$\Delta_B = \Delta_C = \Delta$$

that is effect of Axial force are neglected
i.e. length of members do not change.

- ① initially analysis of frame is done neglected sway. It's called non sway analysis. In non sway analysis find fixed end moment due to loading, distribution factor at joints and compute the final ends moment at the end of table. called non. Sway moments.

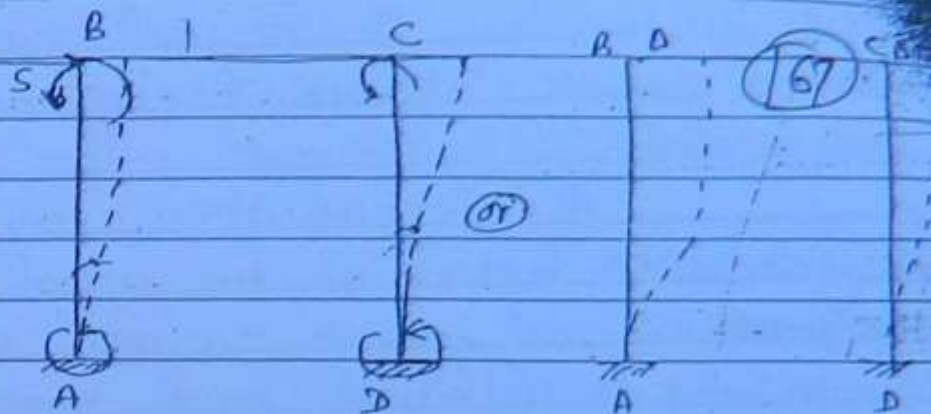
- ② if frame is vertical ~~that~~ ^{find} then ^{Horizontal} Reaction at the support using Non-sway moment & given loading and check

$$\sum F_x = H_A + H_D + P_1$$

if $\sum F_x = 0$ there will be no sway but
 $\sum F_x \neq 0$ then Sway force will be present.
and sway force is equal to $\sum F_x$.

$$S = \sum F_x$$

- ③ To find the effect of sway force remove the loading and apply sway force at the Panel



Under sway force Str. will go sway. A deform. in fig. due to a fixing moment will be produce. in those member for which transverse displacement is produced.

If a cause clockwise rotational to the member then fixing moment produced will Anti-clockwise and vice-versa.

$$\bar{M}_{AB} = \bar{M}_{BA} = \frac{6EI\Delta}{l^2}$$

$$\bar{M}_{BC} = \bar{M}_{CB} = 0$$

$$\bar{M}_{CD} = \bar{M}_{DC} = -\frac{6EI\Delta}{6l^2}$$

Since Δ is not known, hence following procedure is adopted. find Ratio of fixing moments produce it's ratio.

$$\bar{M}_{AB} : \bar{M}_{BA} : \bar{M}_{BC} : \bar{M}_{CB} : \bar{M}_{CD} : \bar{M}_{DC}$$

$$= \frac{-6EI\Delta}{l^2} : \frac{-6EI\Delta}{l^2} : 0 : 0 : \frac{-6EI\Delta}{6l^2} : \frac{-6EI\Delta}{6l^2}$$

$$\Rightarrow -1 : 0 : 0 : 0 : -1 : -1$$

Adopted any fixing moments in above ratio
say $-10 : -10 : 0 : 0 : -10 : -10$

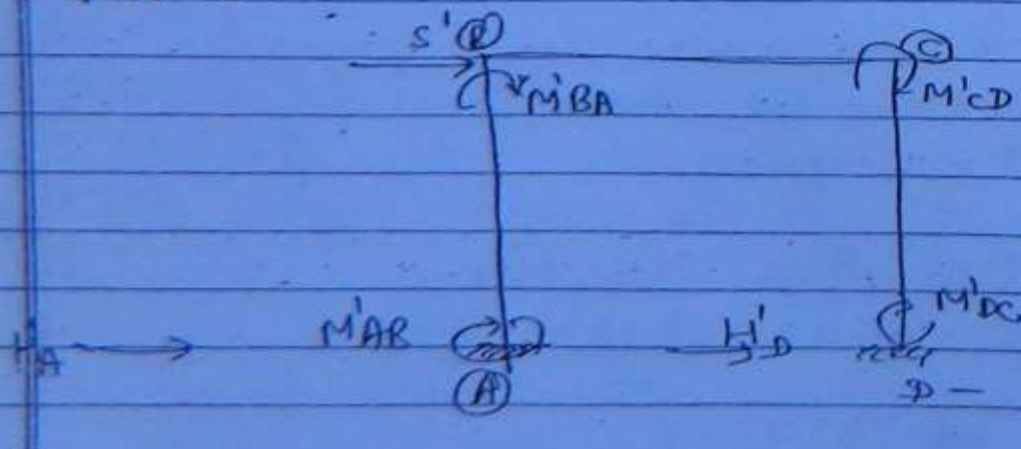
④ Enter above assume fixing moment
in this table.

& distribute them according to
the distribution factor, at the end of table.
There are

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A	B		C		D	
-10	-10	0	0	-10	-10	
Column (A)	M'_{AB}	M'_{BA}	M'_{CD}	M'_{CB}	M'_{DC}	M'_{BC}

→ The moment given in column (A) are not actual
sway moment, which are due to same sway
force S' .



$$\sum F_x = 0$$

$$S' + H_A + H_D = 0$$

①

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S' is known from eqn ① H_A and H_D can be computed by considering free body equilibrium of AB & CD with moments given in column A.

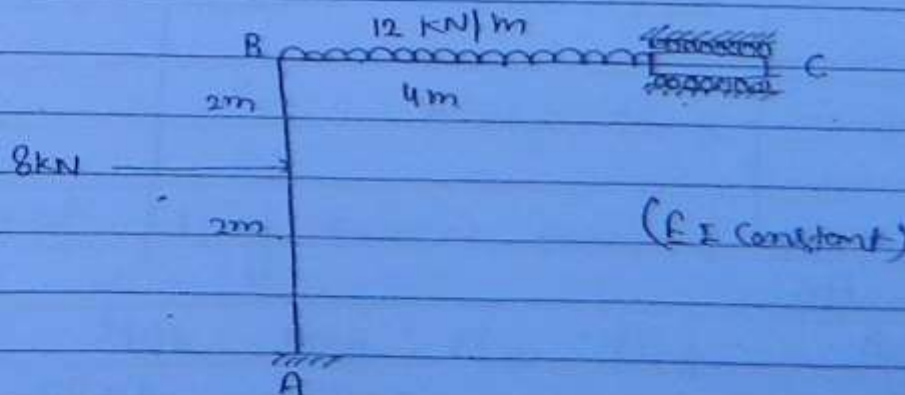
- ⑤ The moment obtain in column (a), are due to same sway forces (S') as found above. Hence actual sway moment actual sway force (S)

will be

$$\frac{\text{Column (a)} \times S}{S'}$$

- ⑥ final end moment will be algebraic sum of Non sway moment (Step-1) & actual sway moment obtain frame previous (Step 5)

Q. Analysis the frame shown in fig. the use Hardy cross method of moment distribution.



Since support C can carry moment Hence stiffness of BC is equal to $\frac{4EI}{L}$

Joint	Member	Stiffness Relative	total Stiffness	Distribution Factor
B	BA	$I/4$	$\frac{2I}{4}$	$1/2$
	BC	$I/4$		$1/2$

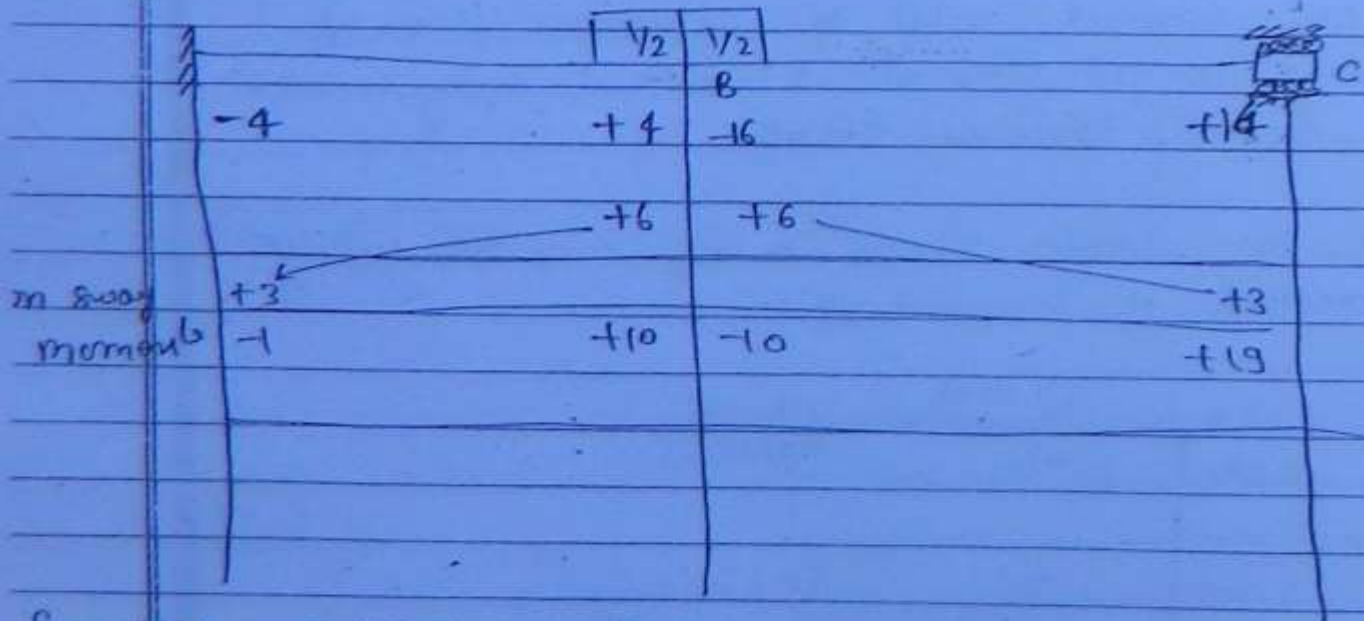
(170)

$$\bar{M}_{AB} = \frac{-PL}{8} \Rightarrow \frac{-8 \times 4}{8} \Rightarrow -4$$

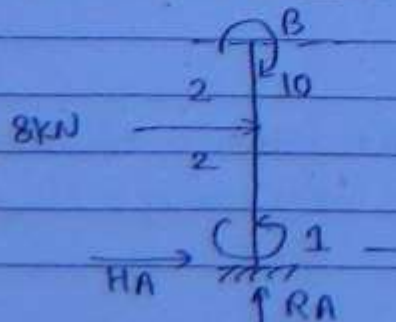
$$\bar{M}_{BA} = +4 \quad \bar{M}_{BC} = \frac{-wL^2}{12} \Rightarrow \frac{-12 \times 4 \times 4}{12} \Rightarrow -16$$

$$\bar{M}_{BC} = -16$$

$$\bar{M}_{CB} = +16$$



free body equilibrium AB



$$\sum M_B = 0$$

$$-H_A \times 4 - 1 - 8 \times 2 + 10 = 0$$

$$H_A = -1.75$$

$$H_C = 0$$

(171)

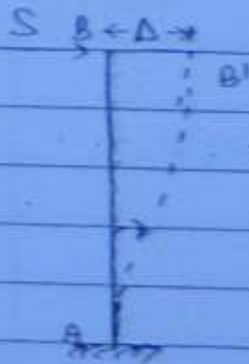
$$\sum F_x = 0$$

$$8 + H_A = 8 - 1.75 \Rightarrow 6.25 \text{ (} \rightarrow \text{)}$$

→ since $\sum F_x$ is not equal to zero Hence Sway will occur sway force

$$S = \sum F_x = 6.25 \text{ kN (} \rightarrow \text{)}$$

Part of question - SWAY ANALYSIS



$$\bar{M}_{AB} = \bar{M}_{BA} = -\frac{6EI\Delta}{L^2}$$

$$\bar{M}_{BC} = \bar{M}_{CB} = 0$$

Ratio of Fixing moment produced due to Δ.

$$\bar{M}_{AB} : \bar{M}_{BA} : \bar{M}_{BC} : \bar{M}_{CB}$$

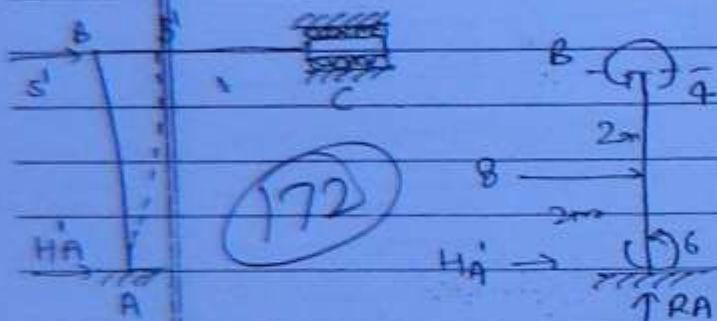
$$-\frac{6EI\Delta}{L^2} : -\frac{6EI\Delta}{L^2} : 0 : 0$$

$$-1 : -1 : 0 : 0$$

$$-6 : -6 : 0 : 0$$

	$\frac{1}{2}$	$\frac{1}{2}$	
	-8	-8	0
		+4	+4
	+2		+2
Column (a)	-6	-4	+4
USM	-15	-10	+10
USM	-1	+10	-10
total	16	0	0
			+24

The moment in given column (A) are due to some sway force S' such that $S' \neq H_A = 0$



$$M_B = 0$$

$$H_A \times 4 - 8 \times 6 \times 2 = 0$$

$$-H_A \times 4 - 6 \times 4 = 0$$

$$H_A = 2.5$$

from (1)

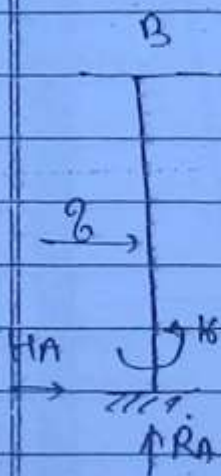
$$S' - 2.5 = 0 \quad [S' = 2.5]$$

the moments given in column A are due to some sway force S' Hence actual sway moment $\rightarrow$

	Column (a) $\times S'$	USM	Net Sway moment
$M_{AB} \Rightarrow$	$\frac{-8 \times 6 \times 2.5}{2.5} \Rightarrow -15$	-1	-16
$M_{BA} \Rightarrow$	$\frac{-4 \times 6 \times 2.5}{2.5} = -10$	+10	0

$$M_{CD} = \frac{+4 \times 6.25}{2.5} = +10 \quad -10 \quad 0$$

$$M_{DC} = \frac{+2 \times 6.25}{2.5} = +5 \quad +19 \quad +24$$



P1

$$\sum M_B = 0$$

$$-H_A \times 4 - 16 - 8 \times 2 = 0$$

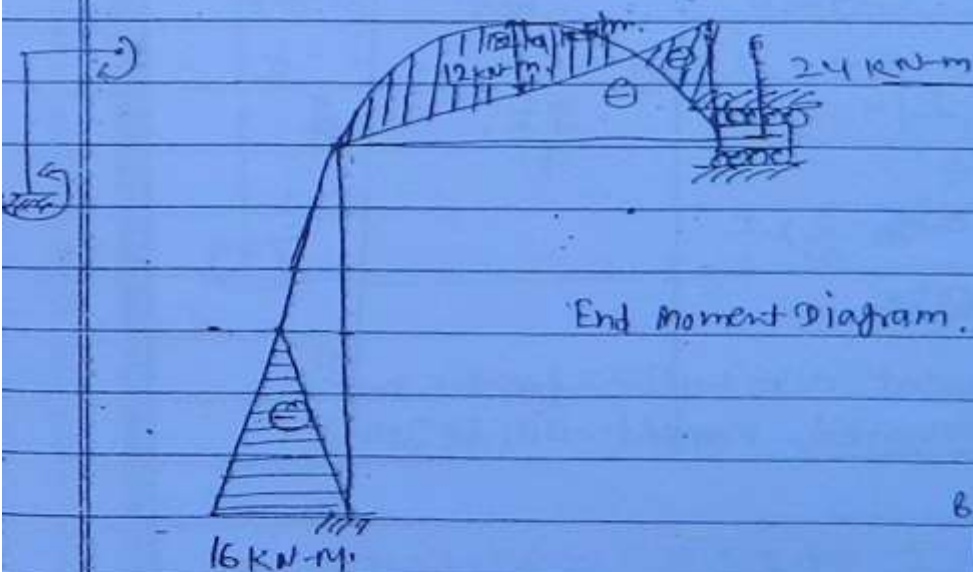
$$H_A = 8$$

$$H_C = 0$$

$$\sum F_x = 0$$

$$8 + H_A + H_C = 0$$

$$8 + 8 = 0 \quad (OK)$$

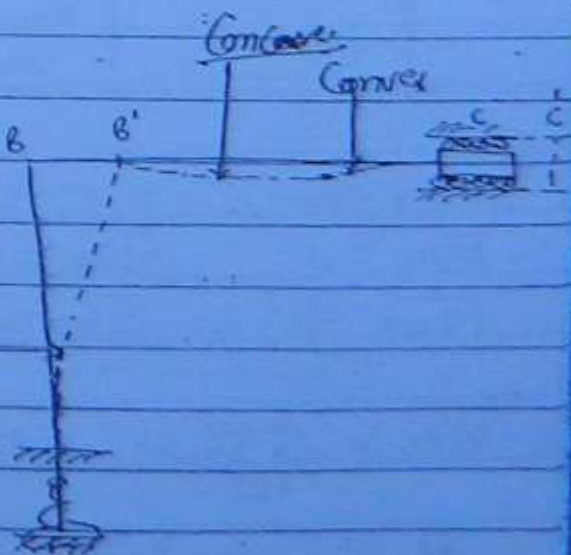


End Moment Diagram.

$$\frac{M}{I} = \frac{E}{R}$$

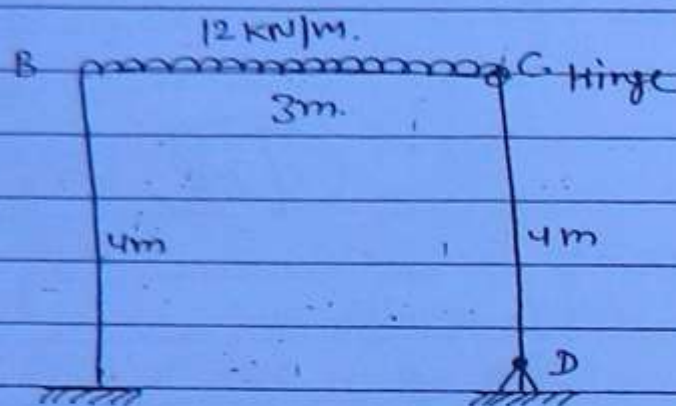
$$BB' = CC'$$

convex



Elastic curve.

Q. Analyse the frame shown in fig. and draw the BMD use Moment distribution method.



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Joint	Member	Relative Stiffness	total stiffness	D.F.
B	BA	$I/4$	$\frac{2I}{4}$	$\frac{1}{2}$
	0			
	BC	$\frac{3I}{4}$ $\frac{3}{4} \times \frac{1}{3}$		
C	CB	$\frac{3I}{4}$		
	CD	moment distribution factor not required. moment will be zero		

$$\bar{M}_{AB} = 0 \quad \bar{M}_{BA} = 0$$

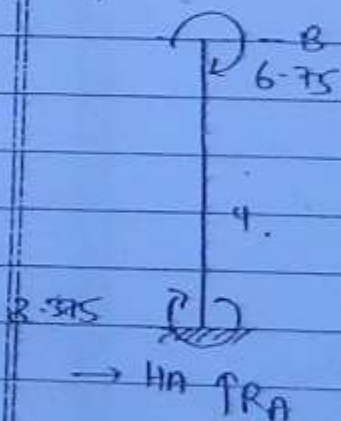
$$\bar{M}_{CD} = 0 \quad \bar{M}_{DC} = 0$$

$$\bar{M}_{BC} = -\frac{wL^2}{12} \Rightarrow -\frac{12 \times 3 \times 3}{12} = -9 \text{ kN/m}$$

$$\bar{M}_{CB} = \frac{wL^2}{12} = \frac{12 \times 3 \times 3}{12} = 9 \text{ kN/m}$$

	A	B	C	D
		$\frac{1}{2} \quad \frac{1}{2}$		
	0	0	-9	+9
End Correct			-4.5	-9
Corrected Δ	0	0	-13.5	0
	0	6.75	6.75	0
Non Sway Moment		+3.375	-6.75	0

Free body dia AB



$$-H_A \times 4 + 6.75 = 0$$

$$H_A = 6.75$$

$$= 2.53$$

$$\sum M_C = 0$$

$$H_D = 0$$

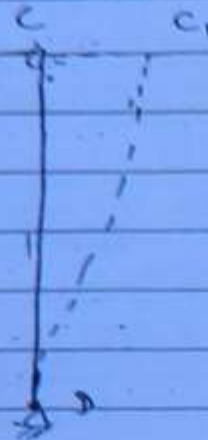
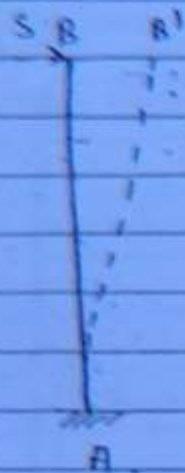
$$\sum F_x = 0$$

$$H_A + H_D = 2.53 + 0 = 2.53 \neq 0 \text{ (sway)}$$

$$S = \sum F_x = 2.53 \text{ kN} \cdot (-\rightarrow)$$

Remove the loading and applied sway force.
Right = 2.25 kN

Say



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Fixing moment due to say

$$\bar{M}_{AB} = \bar{M}_{BA} = -\frac{6EI\Delta}{L^2}$$

$$\bar{M}_{BC} = \bar{M}_{CB} = 0$$

$$\bar{M}_{CD} = \bar{M}_{DC} = 0 \quad \left\{ \begin{array}{l} \text{Both ends are} \\ \text{Hinged.} \end{array} \right.$$

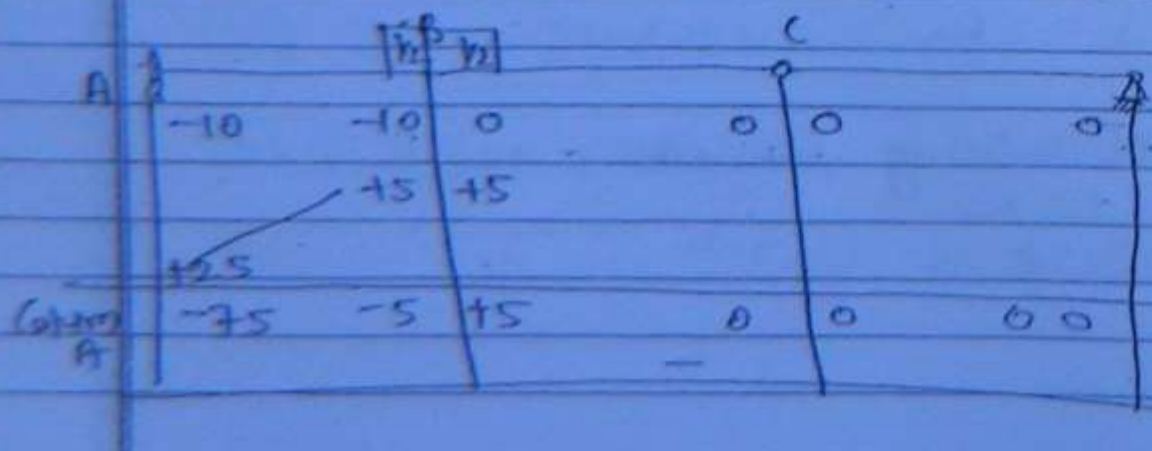
Fixing moment are

$$-\bar{M}_{AB} : \bar{M}_{BA} : \bar{M}_{BC} : \bar{M}_{CB} : \bar{M}_{CD} : \bar{M}_{DC}$$

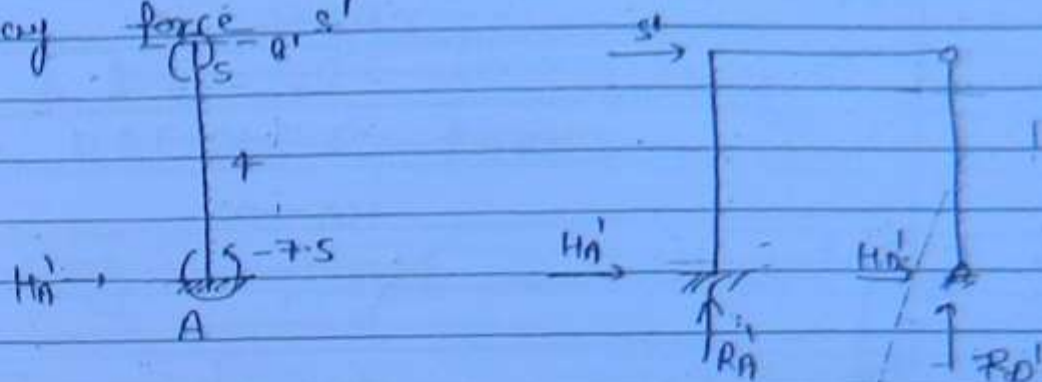
$\Rightarrow$ 6000

$$\Rightarrow -1 : -1 : 0 : 0 : 0 : 0$$

$$\Rightarrow -10 : -10 : 0 : 0 : 0 : 0$$



Moment obtain in Column A are, due to
Some sway force $(P_s - a's')$



for AB

$$H_A' + H_B' + S' = 0 \quad \text{--- (1)}$$

Taking moment About B

$$\sum M_B = 0$$

$$-H_A' \times 4 - 7.5 \times 5 = 0$$

$$H_A' = -3.125$$

$$H_B' = 0$$

from eqn (1)

$$H_A' + H_B' + S' = 0$$

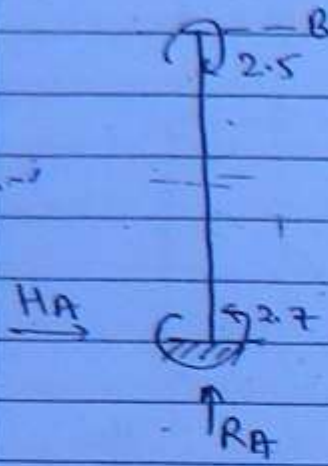
$$-3.125 + 0 + S' = 0$$

$$S' = 3.125 \text{ kN}$$

Actual sway moment

Column (a) X S

Column (a) X S	NSM table 1	Final moment
$M_{AB} = \frac{-7.5 \times 2.53}{3.125} = -6.07$	+3.375	-2.75
$M_{BA} = \frac{-5 \times 2.53}{3.125} = -4.05$	+6.75	+2.75
$M_{BC} = \frac{5 \times 2.53}{3.125} = +4.05$	-6.75	-2.75
$M_{CB} = 0$	0	0
$M_{CD} = 0$	0	0
$M_{DC} = 0$	0	0



$$\sum M_B = 0$$

$$-H_A \times 4 - 2.7 + 2.7 = 0$$

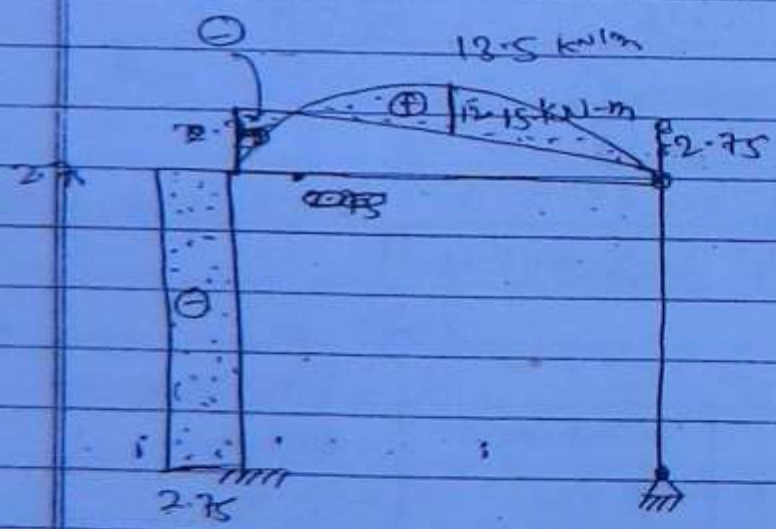
$$H_A = 0$$

$$\sum M_C = 0$$

$$H_D = 0$$

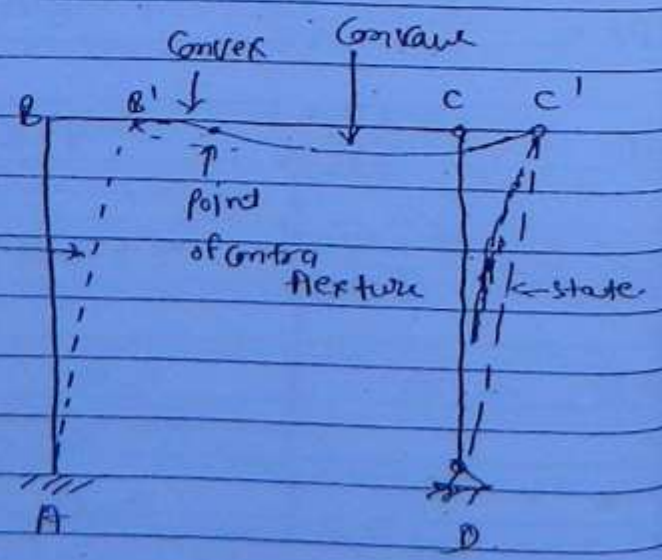
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$$\sum F_x = H_A + H_D = 0$$

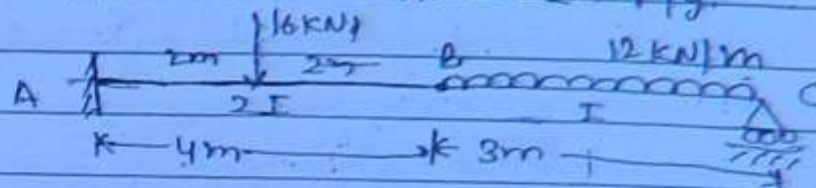


$$\frac{13.7 - 2.75}{2} = 12.15$$

Convex



Q. Analysis the beam shown in fig.



$$\bar{M}_{AB} = -\frac{Wab}{L} \Rightarrow -\frac{16 \times 2 \times 2}{4} = -16 \text{ kNm}$$

$$\bar{M}_{BA} = +\frac{Wab}{L} \Rightarrow +\frac{16 \times 2 \times 2}{4} = +16 \text{ kNm}$$

$$\bar{M}_{AB} = -\frac{PL}{8} \Rightarrow -\frac{16 \times 4}{8} = -8$$

$$\bar{M}_{BA} = +8$$

(179)

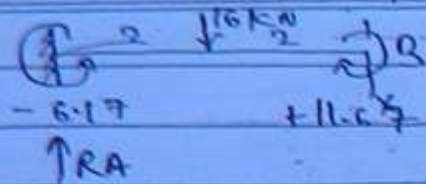
$$\bar{M}_{BC} = -\frac{WL^2}{12} \Rightarrow -\frac{12 \times 3 \times 3}{12} = -9$$

$$\bar{M}_{CB} = +9$$

member	join	R.S	T.R.S	D.P.
B	BA	$2I/4$	$\frac{3I}{4}$	$2/3$
	BC	$\frac{3}{4} \times \frac{I}{3}$		$\frac{1}{3}$

[2/3 B]				C	
-8	+8	-9		+9	
		-4.5	←	-9	End correction
-8	+8	-13.5		0	Corrected value
+11.83	+3.66	+11.83			
-6.17	-11.57	-11.69		0	
Nbr sway moment					

Consider free body equm AB

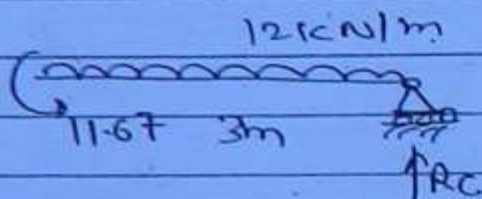


$$\sum M_B = 0$$

$$R_A \times 4 - 6.17 - 16 \times 2 + 11.67 = 0$$

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$$R_A = 6.625 \text{ (}\uparrow\text{)}$$



$$\sum M_B = 0$$

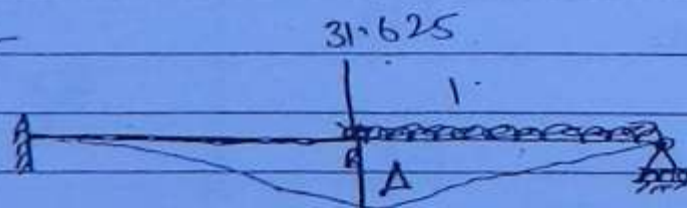
$$R_C \times 3 - 11.67 + 12 \times 3 \times 1.5 = 0$$

$$R_C = 14.11 \text{ (}\uparrow\text{)}$$

$$\sum F_y = 16 + 12 \times 3 - 6.625 - 14.11$$

$$S = \sum F_y \Rightarrow 31.625 \text{ (}\downarrow\text{)}$$

Sway Analysis :-



Due to D fixing moment

$$\bar{M}_{AB} = \frac{-6EID\Delta}{4^2} \Rightarrow \bar{M}_{BA} = \frac{3EID\Delta}{2 \times 4}$$

$$\bar{M}_{BC} = \frac{+63EID}{3^2} \Rightarrow \Delta$$

$$\bar{M}_{CB} = 0$$

$$\bar{M}_{AB} : \bar{M}_{BA} : \bar{M}_{BC} : \bar{M}_{CB}$$

$$= \frac{-3E\Delta}{4} : \frac{3E\Delta}{4} : \frac{8E\Delta}{3} : 0$$

$$\frac{3}{4} : \frac{3}{4} : 4 : 0$$

$$-4 : -4$$

Ratio of fixing moment

(18/)

$$\bar{M}_{AB} : \bar{M}_{BA} : \bar{M}_{BC} : \bar{M}_{CB}$$

$$\Rightarrow \frac{-12E\Delta}{16} : \frac{-12E\Delta}{16} : \frac{+3E\Delta}{9} : 0$$

$$\Rightarrow \frac{-3}{4} : \frac{-3}{4} : \frac{1}{3} : 0$$

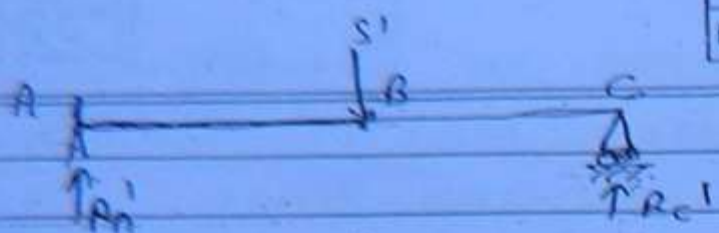
$$\Rightarrow -9 : -9 : +4 : 0$$

$$\Rightarrow -18 : -18 : +8 : 0$$

		24.3	8
	-18	-18	+8
		+16.7	+3.33
	+3.33		
Column ②	-14.67	-11.33	+11.33

Obtain

moments given a column after due to some sway force s'

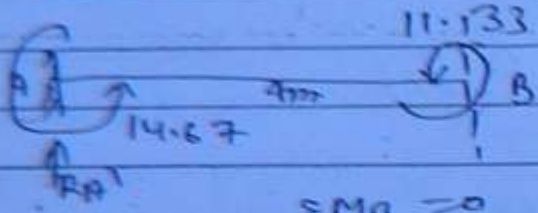


$$\sum F_y = 0$$

$$S' - R_A' - R_C' = 0$$

182

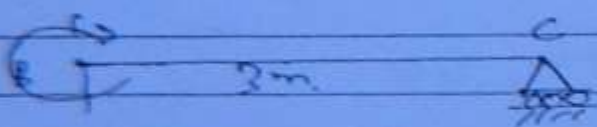
$$S' = R_A' + R_C'$$



$$\sum M_B = 0$$

$$R_A' \times 4 - 14.67 - 11.33 = 0$$

$$R_A' = +6.5 \text{ kN}$$

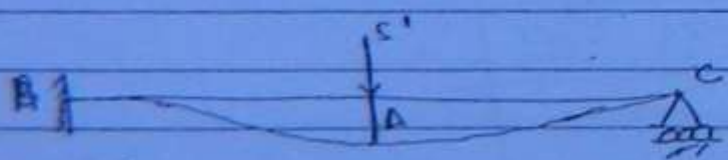


$$-R_C' + 3 + 11.33 = 0$$

$$R_C' = 3.77$$

$$S' = 6.5 + 3.77$$

$$= 10.27 \text{ kN (U)}$$



$$B = 11.67$$

Actual sway moment

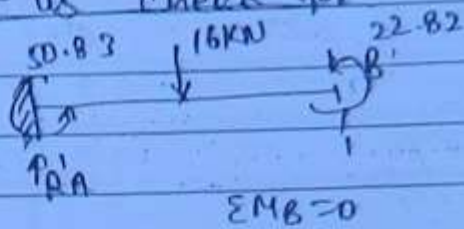
$$M_{AB} = \frac{-14.67 \times 31.625}{10.27} = -44.65$$

$$M_{BA} = \frac{11.33 \times 31.625}{10.27} = 34.73$$

NSM	Final
-6.17	50.82
+11.67	22.82

$$M_{BL} = \frac{113.3 \times 31.625}{10.27} \rightarrow 34.45 \quad | \quad -11.67 \quad | \quad +22.82$$

Let us check for

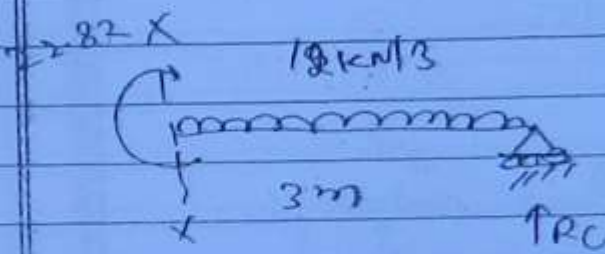


(183)

$$\sum M_B = 0$$

$$R_A \times 4 - 16 \times 2 - 22.82 - 50.83 = 0$$

$$R_A = 26.41 \text{ kN}$$



$$\sum M_B = 0$$

$$- R'_C \times 3 + 22.82 + 12 \times 3 \times 1.5 + 22.82 = 0$$

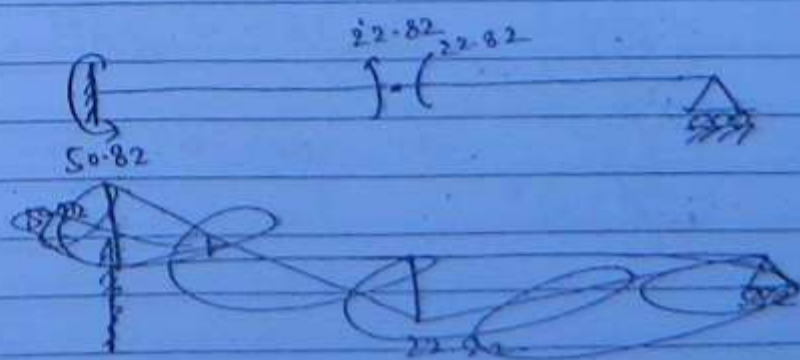
$$R_C = + 25.61$$

$$\sum F_y \Rightarrow R_A + R_C = 16 + 12 \times 3$$

$$\Rightarrow 16 + 39$$

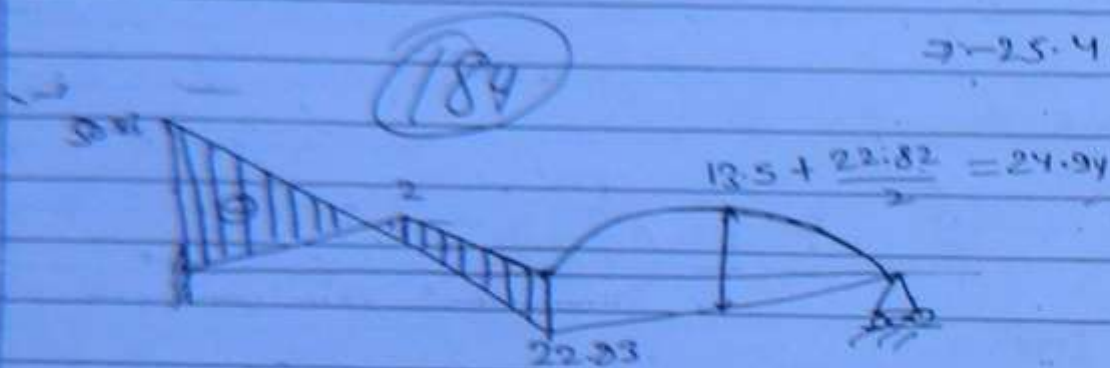
$$26.4 + 25.6 \Rightarrow 52$$

$\frac{52}{2} \rightarrow \frac{52}{2} = 26$ check save.



Centre ordinate and moment = $-\frac{50.83 + 22.82}{2}$

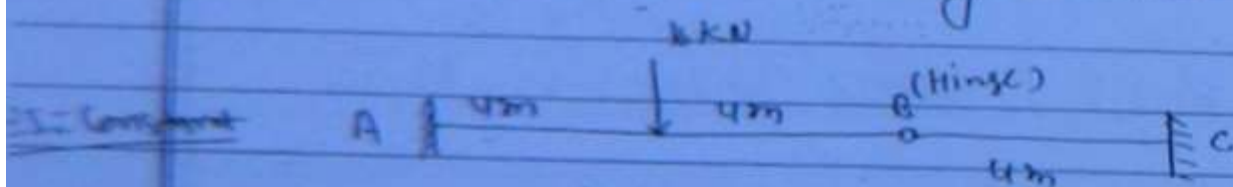
$$\Rightarrow -25.41$$



$$M_{AB} = \frac{PL}{8} = \frac{16 \times 4}{8} = 8$$

$$M_{BC} = \frac{WL^2}{8} = \frac{12 \times 3^2}{8} = 13.5$$

Q3:- Using moment distribution method analysis the beam and draw the bending moment diagram.



$$M_{AB} = -\frac{PL}{8} = -\frac{16 \times 4}{8} = -8 \text{ kNm}$$

$$M_{BA} = -16 \text{ kNm}$$

$$M_{BC} = M_{CB} = 0$$

Distribution factor

Member

Joint

R.I.S

TR.S

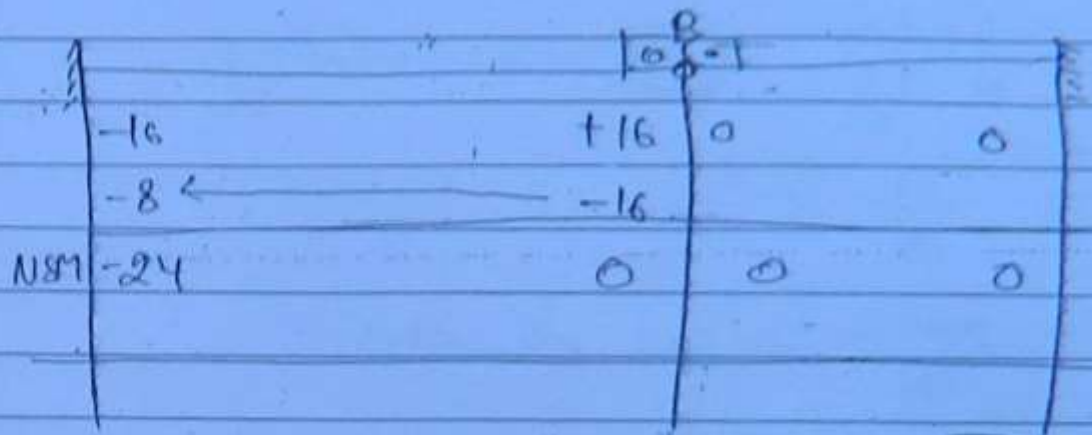
D.F.

B /

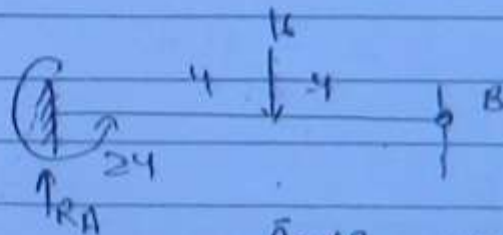
BA

$$\frac{\frac{3}{4} \times \frac{I}{8}}{\frac{3}{4} \times \frac{I}{8} + \frac{3}{4} \times \frac{I}{12}} = \frac{3}{12}$$

$$\frac{\frac{3}{4} \times \frac{I}{8}}{\frac{3}{4} \times \frac{I}{8} + \frac{3}{4} \times \frac{I}{12}} = \frac{3}{8}$$



185



$$\epsilon M_B = 0$$

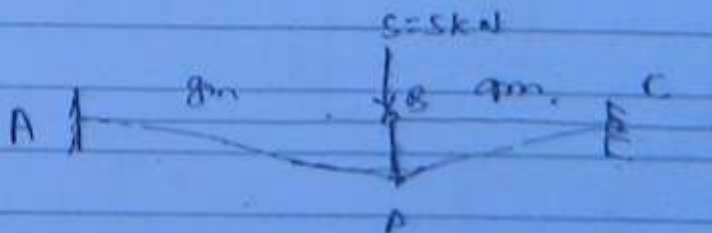
$$\bar{R}_A \times 8 - 24 + 16 \times 4 = 0$$

$$R_A = \frac{64 + 24}{8} = 11 \text{ kN}$$

Consider BC $\Rightarrow$

$$R_c = 0$$

$$\varepsilon f_y = 16 - 11 - 9 = 0$$



$$MAB = -3EIA \quad \text{a Men}$$

$$\overline{M}_{BC} = 0 = \overline{M}_{BD} \quad \overline{M}_{CB} = 36 \text{ kN}$$

Fixed moment

$$\overline{M}_{AB} : \overline{M}_{BA} : \overline{M}_{BC} : \overline{M}_{CB}$$

$$\Rightarrow -\frac{3EI\Delta}{8^2} : \frac{3EI\Delta}{8^2} : 0 : +\frac{3EI\Delta}{4^2}$$

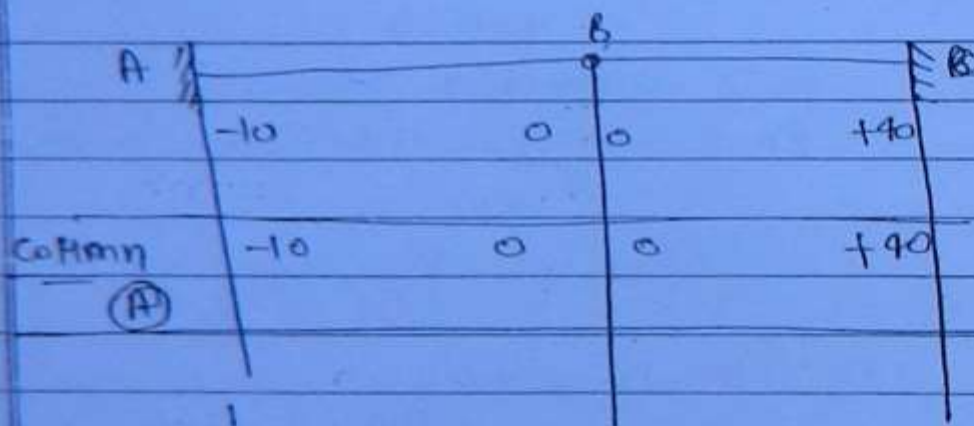
$$\Rightarrow -\frac{1}{64} : 0 : 0 : \frac{1}{16}$$

186

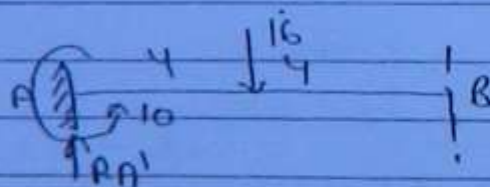
$$\Rightarrow -1 : 0 : 0 : +4$$

Say

$$\Rightarrow -10 : 0 : 0 : +40$$



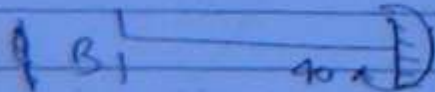
Consider AB



$$\sum M_B = 0$$

$$R_A' \times 8 - 16 \times 4 = 0$$

$$R_A' = 11.25$$



$$\sum M_B = 0$$

$$-R_C' \times 4 + 40 \times 4 = 0$$

$$R_C' = 10$$

$$S' = RA' + RC'$$

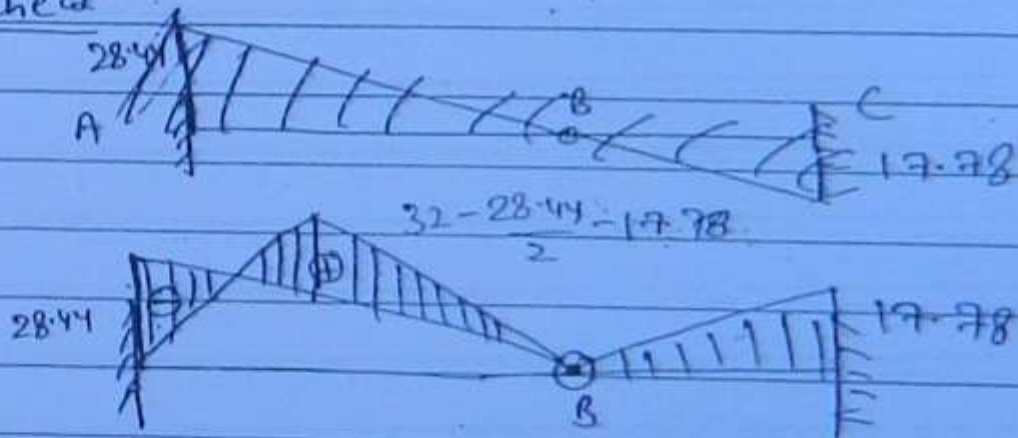
$$= 1.25 + 10$$

$$S' = 11.25$$

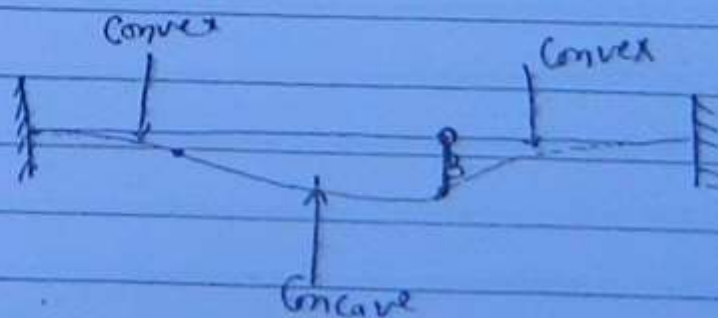
187

actual sway moment	NSM	Net moment
$M_{AB} = -10 \times 5 = -4.44$	-24	28.44
$M_{BA} = 11.25 = 0$	0	0
$M_{BC} = 0$	0	0
$M_{CB} = \frac{40 \times 5}{11.25} = 17.78$	0	17.78

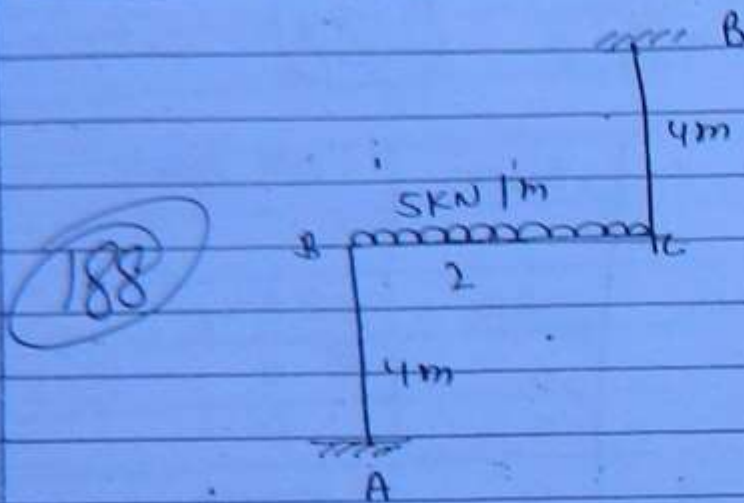
Let us check



$$M_{max} = \frac{PL}{4} = \frac{16 \times 8}{4} = 32$$



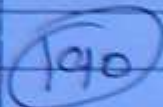
Q Analyse the frame shown in fig. using moment-distribution method.



$EI = \text{constant}$.

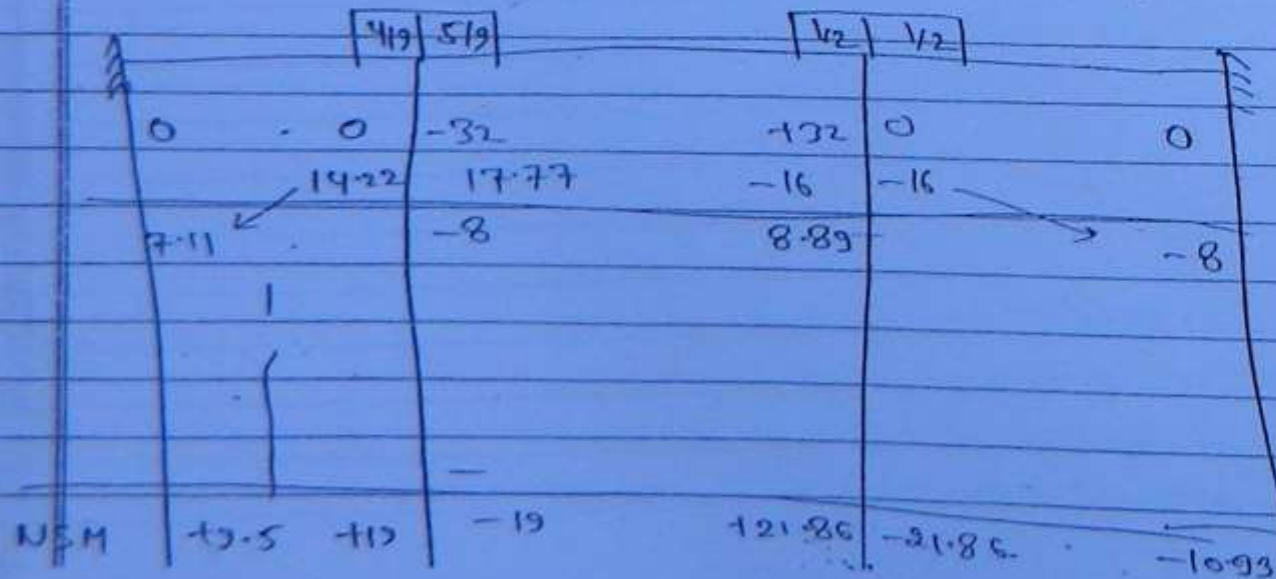
189

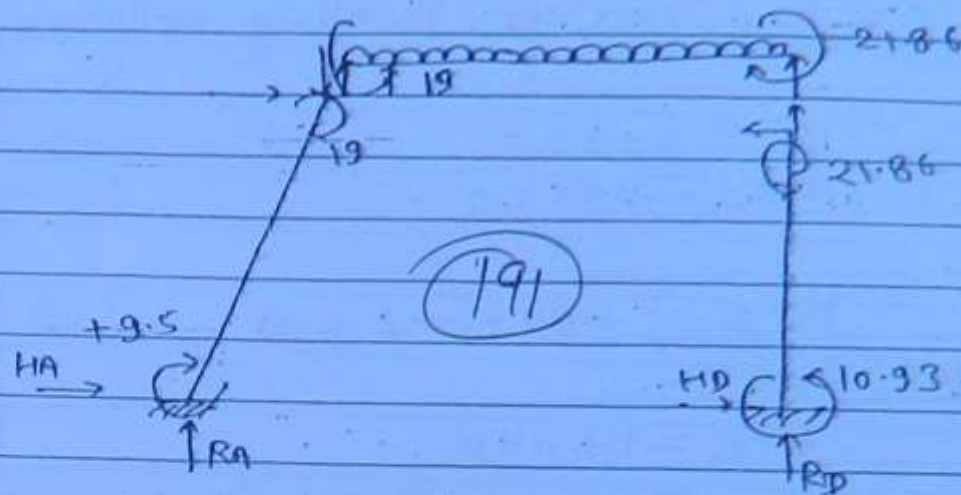
1910



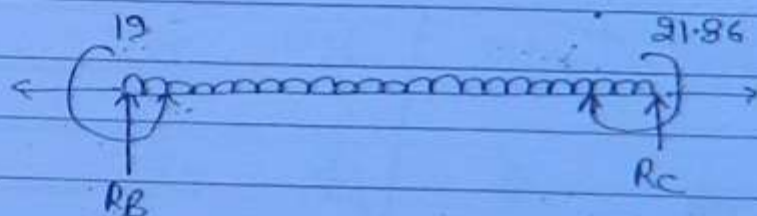
$$\bar{M}_{CD} = \bar{M}_{DC} = 0$$

Joint	member	Relative stiffness	Total Relative stiffness	D.F
B	BA	$I/5$	$\frac{9I}{20}$	$4/9$
	BC	$I/4$		$5/9$
C	CB	$I/4$	I	$1/2$
	CD	$I/4$	2	$1/2$





→ To find H_A in inclined member AB R_A is required since there is no vertical loading in AB. Hence $R_A = R_B$, hence R_B can be completed by taking free body equilibrium of BC.



$$\sum M_C = 0$$

$$R_B \times 24 - 19 - 24 \times 2 \times 12 + 21.86 = 0$$

$$R_A = R_B = 47.285 \text{ kN}$$

To find H_A , consider free body equilibrium of AB

$$\sum M_B = 0$$

$$-H_A \times 4 + R_A \times 3 + 9.5 + 19 = 0$$

$$H_A = \frac{3 \times 47.289 + 9.5 + 19}{4}$$

$$H_A = 42.58 \text{ (} \rightarrow \text{)}$$

To find H_D , consider freebody equilibrium of DC

$$\sum M_C = 0$$

$$-H_D \times 4 - 10.93 - 21.86 = 0$$

$$H_D = -81.97 (\rightarrow)$$

Net Horizontal force :-

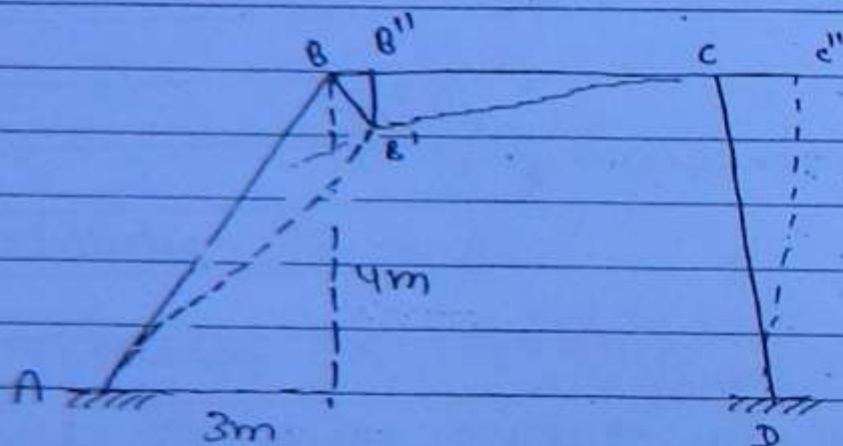
$$\sum F_x = H_A + H_D$$

$$\Rightarrow 42.598 - 81.97$$

$$\Rightarrow 34.398 \text{ kN } (\rightarrow)$$

(192)

Sway analysis :-



$$\text{let } BB' = \Delta$$

$$BB'' = \Delta \cos \theta = \frac{4}{5} \Delta$$

$$B'B'' = \Delta \sin \theta = \frac{3}{5} \Delta$$

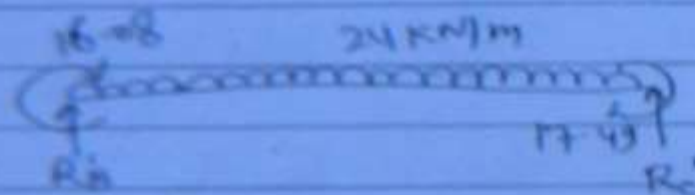
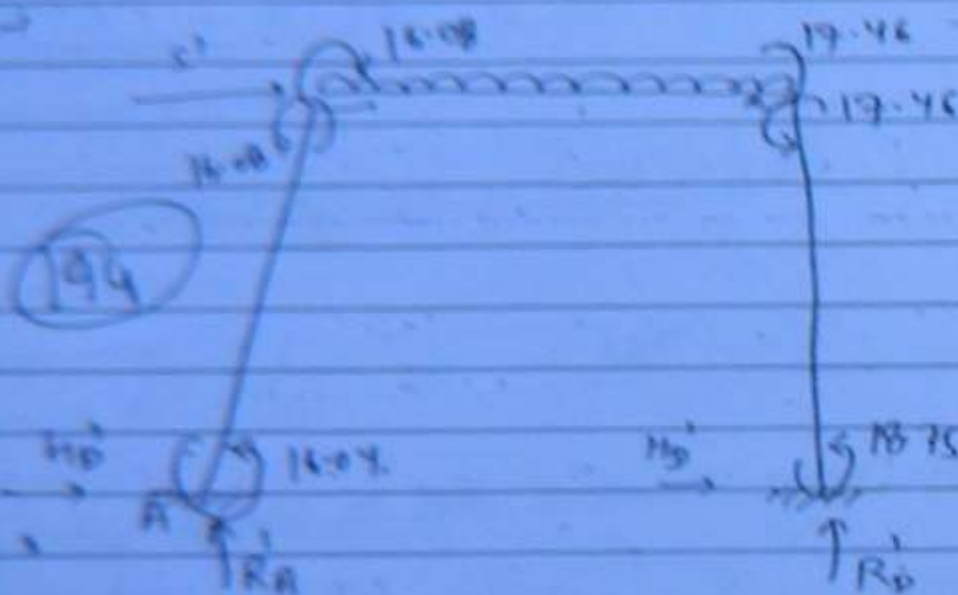
Horizontal comp. of sway at B & C are equal

$$BB' = CC'$$

$$CC' = \frac{4}{5} \Delta$$

~~How~~

Let us mem. given in each some way
force both.



$$R_A = R_B$$

To find R_B consider span BC

$$R_B \times 4 + 16.08 + 17.49 = 0 \quad \therefore$$

$$R_B = R_A = -8.785 \text{ kN}$$

To find H_A consider span AB $\rightarrow \sum M_B = 0$

$$R_A \times 3 - H_A \times 4 - 16.04 - 16.08 = 0$$

$$H_A = -14.92$$

To find H_b consider span CD

$$\sum M_c = 0$$

$$-H_b \times 4 - 18.75 - 19.46 = 0$$

$$S' + \frac{1}{2} H A' + H_b = 0$$

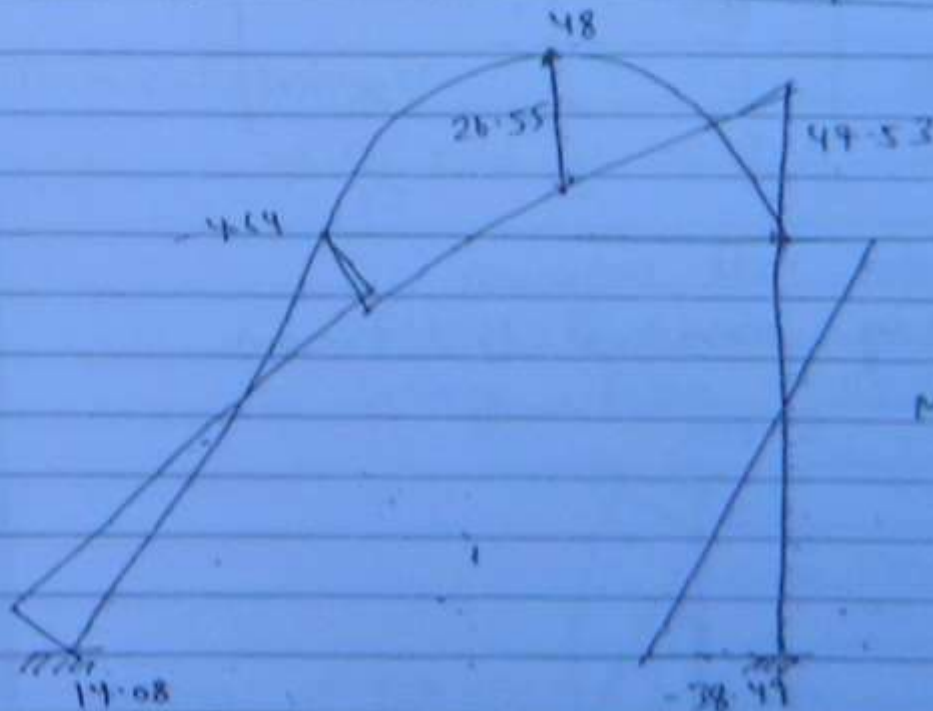
$$S' - 14.32 - 9.65 = 0$$

$$S' = 23.97 \text{ kN}$$

195

Moment given in girder over due to S' Hence actual sway moment

Equal sway moment						
Column @ X5 S'	-23.58	-23.64	+23.64	+23.58	-23.67	-23.67
NSM	+9.5	+19	+19	+21.86	-21.86	-10.93
Final moment	+14.08	-4.64	+4.67	+47.53	-47.53	-38.49



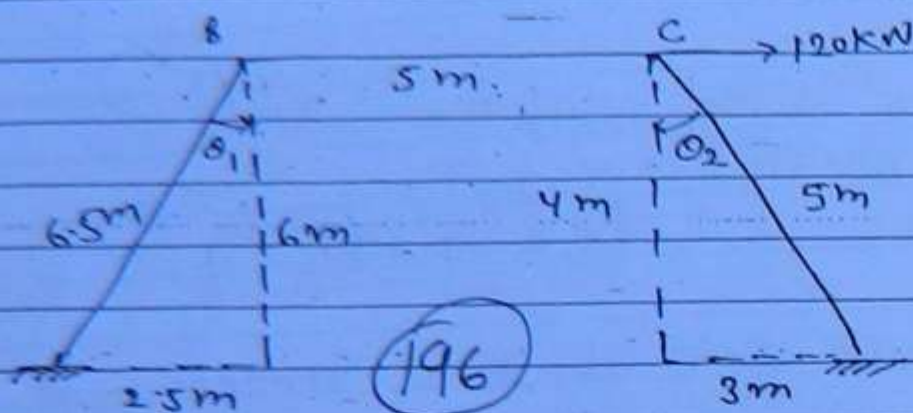
$$M_{BC} = \frac{wL^2}{8}$$

$$= \frac{24 \times 4^2 \times 4}{8}$$

$$= 48$$

$$\text{At centre} = 48 - \left(\frac{-4.64 + 47.53}{2} \right) = 26.55$$

Analysis the frame shown in fig. using mom.

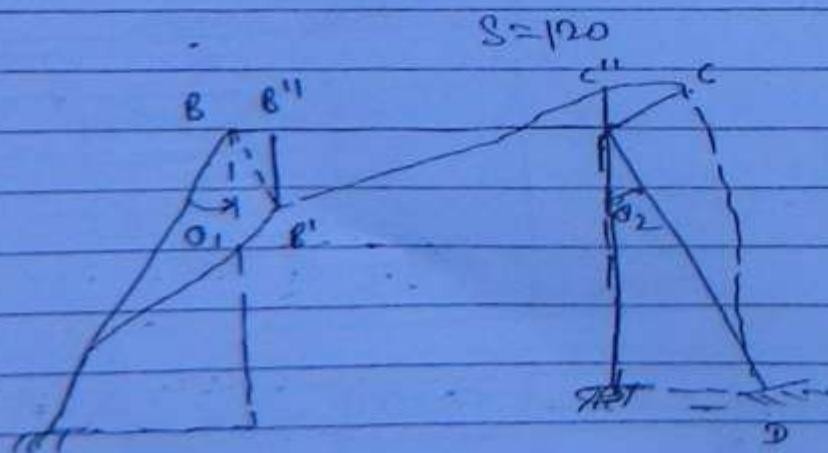


Distribution factor.

Joint	Member	R.S.	T.R.S.	D.F.
B	BA	$I/6.5$	$\frac{23}{6.5} I$	$10/23$
	BC	$I/5$		$13/23$
C	CB	$I/5$	$\frac{2}{5} I$	$1/2$
	CD	$I/5$		$1/2$

Fix. End moment all member.

Hence No sway moment not present



$$\text{let } BB' = A$$

$$BB' = A \cos \theta_1 = \frac{60}{65} \Rightarrow \frac{12 \cos \theta}{13}$$

197

$$B'B'' = A \sin \theta_1 \Rightarrow \frac{2.5 A}{6.5} = \frac{5 A}{13}$$

Horizontal displacement B = Horizontal displacement C

$$BB'' = C'C'$$

$$C'C' = \frac{12 A}{13}$$

$$\frac{C'C''}{C'C'} = \cos \theta_2$$

$$C'C' = \frac{C'C''}{\cos \theta_2} = \frac{12/13 A}{4/5} = \frac{15 A}{13}$$

$$C'C'' = C'C' \sin \theta_2$$

$$\Rightarrow \frac{15 A}{13} \times \frac{3}{5} = \frac{9 A}{13}$$

$$\text{Sway of AB} = BB' = A$$

$$\text{Sway of BC} = BB'' + C'C''$$

$$= \frac{5 A}{13} + \frac{9 A}{13}$$

$$\Rightarrow \frac{14 A}{13}$$

$$\text{Sway of CD} = C'C' = \frac{15 A}{13}$$

fixing moment due to sway:-

$$\bar{M}_{AB} = \bar{M}_{BA} = \frac{-6EI\Delta}{6 \cdot 5^2}$$

(198)

$$\bar{M}_{BC} = \bar{M}_{CB} = \frac{6EI\Delta}{5^2} = \frac{6EI(14/13)\Delta}{5^2} \rightarrow \frac{84EI\Delta}{325}$$

$$\bar{M}_{CD} = \bar{M}_{DC} = \frac{-6EI(15/13)\Delta}{5^2} \Rightarrow \frac{-90}{325}\Delta$$

Ratio of fixing moment:-

$$\Rightarrow \frac{-1}{6 \cdot 5^2} : \frac{-1}{6 \cdot 5^2} : \frac{14}{13 \times 5^2} : \frac{14}{13 \times 5^2} : \frac{-15}{13 \times 5^2} : \frac{15}{13 \times 5^2}$$

$$\Rightarrow \frac{3}{169} : \frac{-3}{169} : \frac{14}{325} : \frac{14}{325} : \frac{15}{325} : \frac{15}{325}$$

$$\Rightarrow \frac{-100}{13} : \frac{-100}{13} : +14 : +14 : -15 : -15$$

$$\Rightarrow -7.69 : -7.68 : 14 : 14 : -15 : -15$$

$$\Rightarrow -15.38 : -15.38 : 28 : 28 : -30 : 30$$

Distribute above movement according to a.f. and obtain Col @ let us movement given in Col @ are due to some sway force s' , such that

$$s' + H_D + H_A = 0$$

(199)

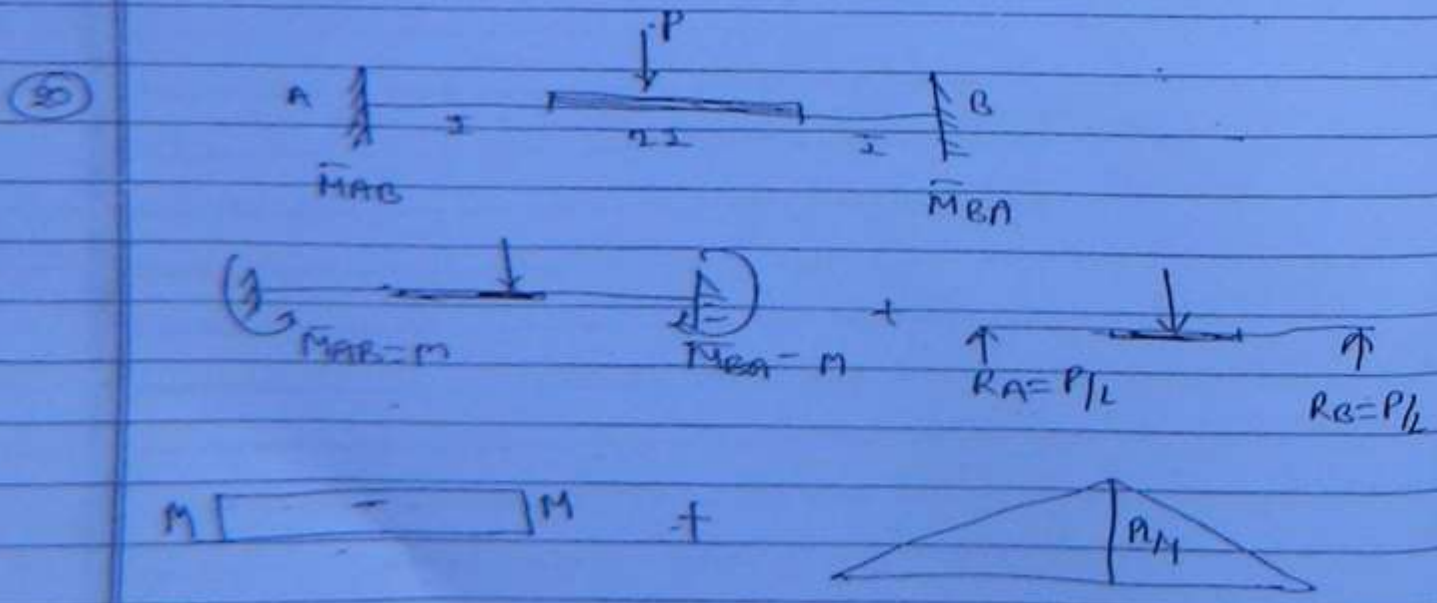
Find H_A and H_D , R_A and R_D are required R_A , R_D can be found by using free body eqn^m at span Bc with the given in Column (ii) hence s' as known.

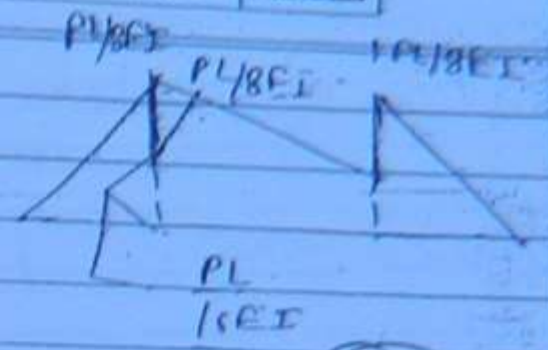
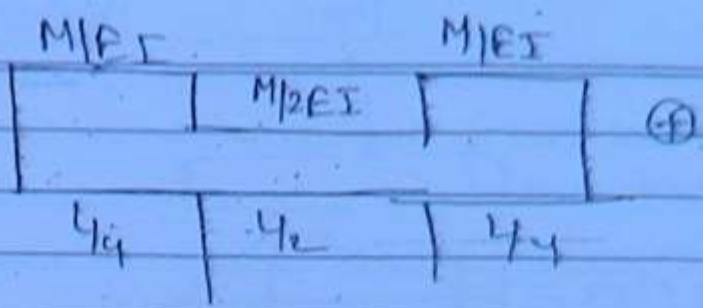
$$\text{final sway moment} = \frac{\text{col @} \times s}{s'}$$

∵ Moment sway moment are not given then final sway moment are final moment.

And moment dia will be final B.M.D. be there is no simply support dia.

- | | | | |
|--------|--------|--------|----------|
| (1) c | (19) b | (27) c | (35) 40 |
| (2) c | (18) a | (28) b | (36) 50 |
| (3) c | (17) B | (29) a | (37) 55 |
| (4) a | (16) C | (30) c | (38) 60 |
| (5) a | (15) b | (31) | (39) 65 |
| (6) d | (14) a | (32) | (40) 70 |
| (7) d | (13) b | (33) | (41) 75 |
| (8) a | (12) c | (34) | (42) 80 |
| (9) | (11) b | (35) | (43) 85 |
| (10) | (10) d | (36) | (44) 90 |
| (11) | (9) b | (37) | (45) 95 |
| (12) c | (8) b | (38) | (46) 100 |
| (13) b | (7) a | (39) | (47) 105 |
| (14) | (6) a | (40) | (48) 110 |
| (15) | (5) a | (41) | (49) 115 |
| (16) | (4) a | (42) | (50) 120 |
| (17) | (3) a | (43) | (51) 125 |
| (18) | (2) a | (44) | (52) 130 |
| (19) | (1) a | (45) | (53) 135 |
| (20) | | (46) | (54) 140 |
| (21) | | (47) | (55) 145 |
| (22) | | (48) | (56) 150 |
| (23) | | (49) | (57) 155 |
| (24) | | (50) | (58) 160 |
| (25) | | (51) | (59) 165 |
| (26) | | (52) | (60) 170 |
| (27) | | (53) | (61) 175 |
| (28) | | (54) | (62) 180 |
| (29) | | (55) | (63) 185 |
| (30) | | (56) | (64) 190 |
| (31) | | (57) | (65) 195 |
| (32) | | (58) | (66) 200 |
| (33) | | (59) | (67) 205 |
| (34) | | (60) | (68) 210 |
| (35) | | (61) | (69) 215 |
| (36) | | (62) | (70) 220 |
| (37) | | (63) | (71) 225 |
| (38) | | (64) | (72) 230 |
| (39) | | (65) | (73) 235 |
| (40) | | (66) | (74) 240 |
| (41) | | (67) | (75) 245 |
| (42) | | (68) | (76) 250 |
| (43) | | (69) | (77) 255 |
| (44) | | (70) | (78) 260 |
| (45) | | (71) | (79) 265 |
| (46) | | (72) | (80) 270 |





(20)

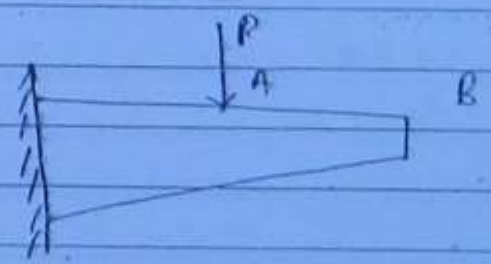
Area moment method.

Change in slope from A to B = 0
 $\theta_{AB} = 0$

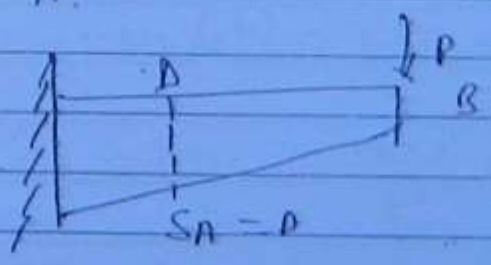
$$\rightarrow - \left[\frac{M}{EI} \times \frac{L}{4} + \frac{M}{2EI} \times \frac{L}{2} + \frac{M}{EI} \times \frac{L}{4} \right] +$$

$$\left[\left(\frac{1}{2} \times \frac{L}{4} \times \frac{PL}{8EI} \right) \times 2 + \frac{1}{2} \times \frac{PL}{16EI} \times \frac{L}{2} \times \frac{L}{16EI} \times \frac{L}{2} \right]$$

(27)

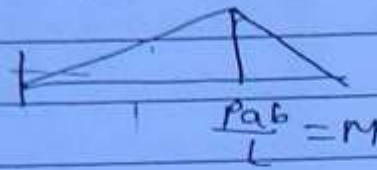
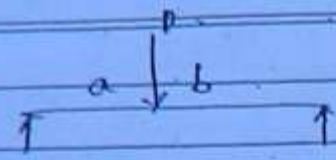
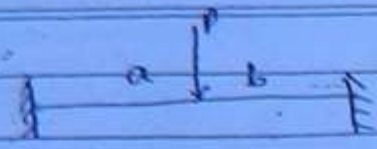


$\theta_B = 0$



Ans $\Rightarrow A$

31 (a)



$$M_A = -\frac{Pab^2}{L^2}$$

$$M_B = \frac{Pa^2b}{L^2}$$

$$\frac{Pab^2}{L^2} + \frac{Pa^2b}{L^2} \Rightarrow \frac{Pab(a+b)}{L^2} \Rightarrow \frac{Pa}{b} = M$$

32

48

$$OA + OB + OC + OD$$

$$\frac{4EI}{L} + \frac{4EI}{L} + \frac{3EI}{L} + 0$$

$$\Rightarrow \frac{11EI}{L}$$

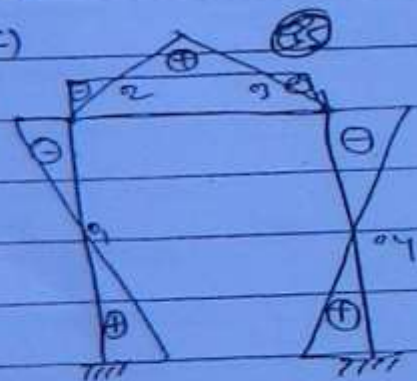
33

(a)

34

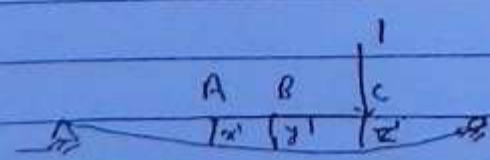
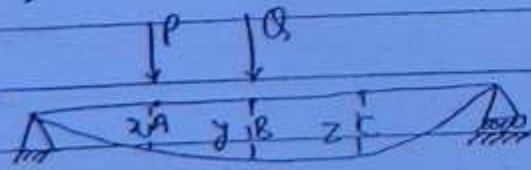
(c)

35



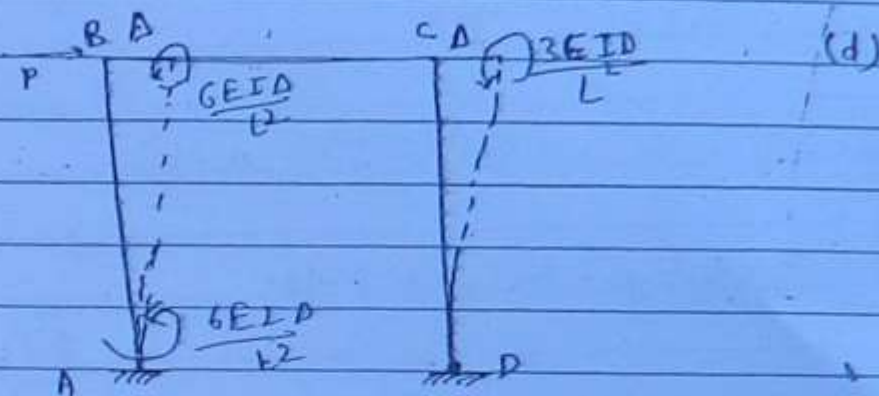
(d)

36



So virtual work done by unit load = virtual work done by P, Q
 1. $z = Px' + Qy'$

$$z = Px' + Qy' \quad (b)$$



$\Sigma M_B = 0$ in FBD of AB

$$Lx - H_A + \frac{6EID}{L^2} + \frac{6EID}{L^2} = 0$$

$$H_A = -\frac{12EID}{L^2}$$

$\Sigma M_C = 0$ in CD

$$-H_D L - \frac{3EID}{L^2} = 0$$

$$H_D = -\frac{3EID}{L^3}$$

$$P + H_D + H_A = 0$$

$$P - \frac{12EID}{L^3} - \frac{3EID}{L^3} = 0$$

$$P = \frac{15EID}{L^3}$$

(40)

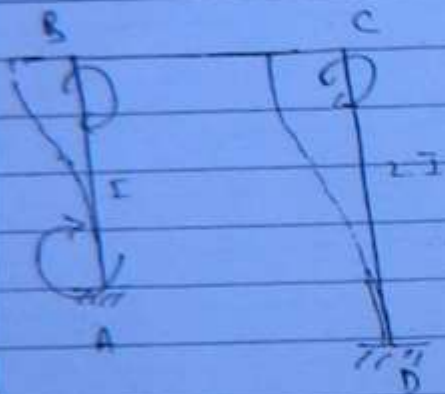


(202)

Stiffness of BA $\Rightarrow \frac{4EI}{2}$

Stiffness of CD $= \frac{4E(2I)}{3} = \frac{8EI}{3}$

Frame will sway toward left.



$$\bar{M}_{AB} = \bar{M}_{BA} = \frac{+6EI\Delta}{L^2(2^2)}$$

$$\bar{M}_{CD} = \bar{M}_{DC} = \frac{+6E(2I)\Delta}{3^2}$$

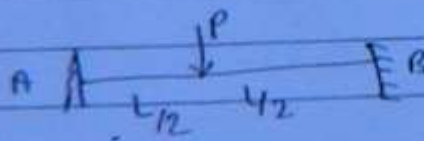
$$\frac{\bar{M}_{AB}}{\bar{M}_{CD}} = \frac{\frac{1}{2}}{\frac{2}{9}} = \frac{9}{8}$$

(41)

(6)

(42)

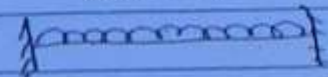
(c)



$$\frac{PL}{8} = 860$$

$$PL = 480$$

$$w \cdot L = P = w \cdot \frac{L}{2}$$



$$\bar{M} = \frac{wL^2}{12}$$

$$\Rightarrow \frac{P}{L} \times \frac{L^2}{12} = \frac{PL}{12}$$

$$\Rightarrow \frac{480}{12} = 40 \text{ kNm}$$

(43)

(2)

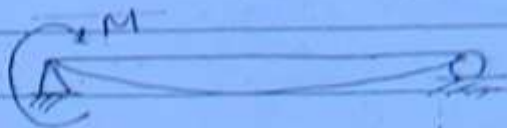
44 (c)

45 (c)

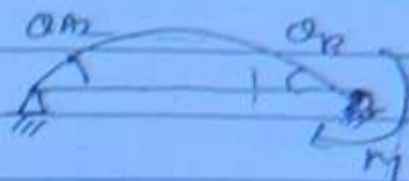
46

203

46



$$\theta_{A1} = \frac{ML}{3EI}$$



$$\theta_{A2} = \frac{ML}{6EI}$$

$$\text{Net } \theta_A = \theta_{A1} - \theta_{A2} \Rightarrow \frac{ML}{3EI} - \frac{ML}{6EI} = \frac{ML}{6EI}$$

(d)

47 (a)

48 (c)

49 (c)

50 (c)

51 (b)

53 (c)

54 d

55 c

56 (d)

stiffness of BC = $4(EI/L)$

$$B_n = 38(EI/L)$$

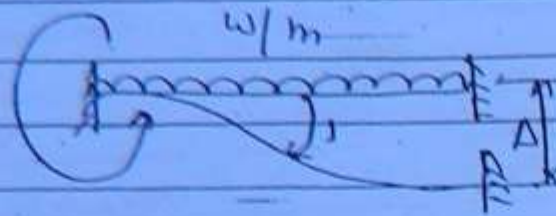
total 7

D.F. $\Rightarrow$

4/7

3/7

57 (H)

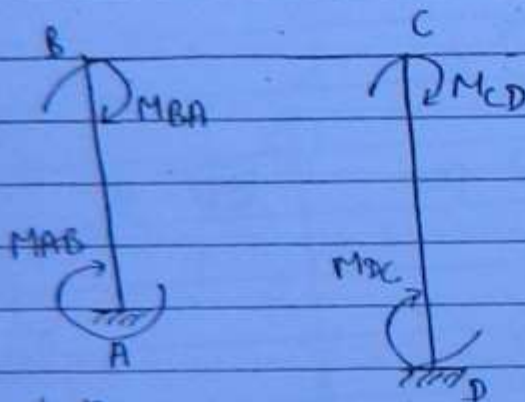


$$\bar{M}_{AB} = + \frac{wL^2}{12EI} - \frac{6EI\Delta}{L^2}$$

$$\Rightarrow \frac{wL^2}{12EI} - \frac{6EI}{L^2} \cdot \frac{wLY}{72EI}$$

$$\Rightarrow 0$$

(d)



Shear Eqn.

$$\Sigma F_x = 0$$

$$H_A + H_D + P = 0$$

$$\Sigma M_B = 0 \text{ for AB}$$

$$-H_A L_1 + M_{AB} + M_{BA} = 0$$

$$H_A = \frac{M_{AB} + M_{BA}}{L_1}$$

for DC

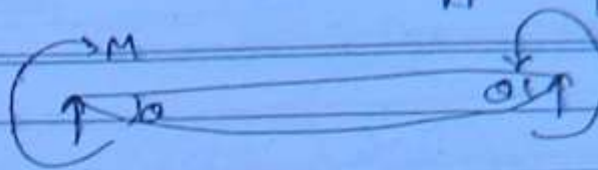
$$\Sigma M_C = 0$$

$$H_D = \frac{M_{CD} + M_{DC}}{L_2}$$

$$\frac{M_{AB} + M_{BA}}{L_1} + \frac{M_{CD} + M_{DC}}{L_2} + P = 0$$

(e)

(5)



(205)

$$\theta = \frac{M}{2EI}$$

$$M = \frac{2EI\theta}{L}$$

if $\theta = 1$

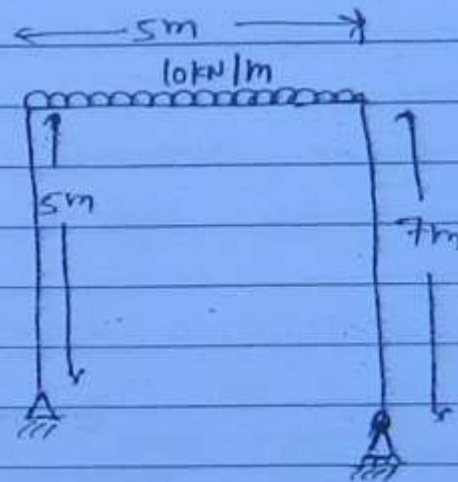
$$M = K$$

$$K = \frac{2EI}{L}$$

(6)

(62)

$$EI = 81380 \text{ kN-m}^2$$



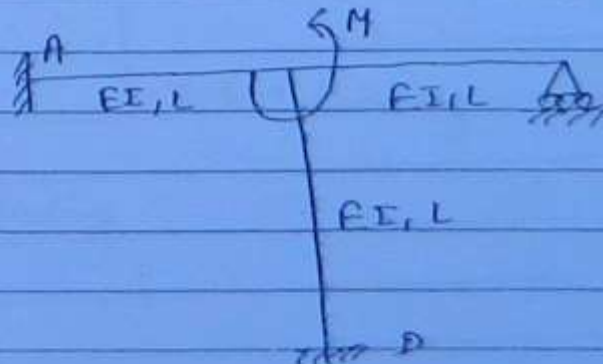
(a)

(63)

d

(64)

(b)



Stiffness of All member B = $\frac{4EI}{L} + \frac{4EI}{L} + \frac{3EI}{L}$

$$K = \frac{11EI}{L}$$

$$M = K \cdot \theta$$

$$M = \frac{11EI}{L} \cdot \theta$$

$$\theta = \frac{ML}{11EI}$$

(8)

(a)

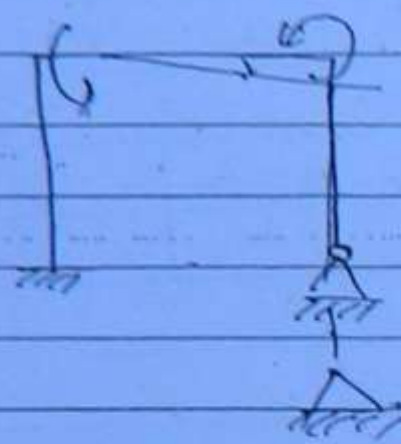
(6)

(b)

(206)

(5)

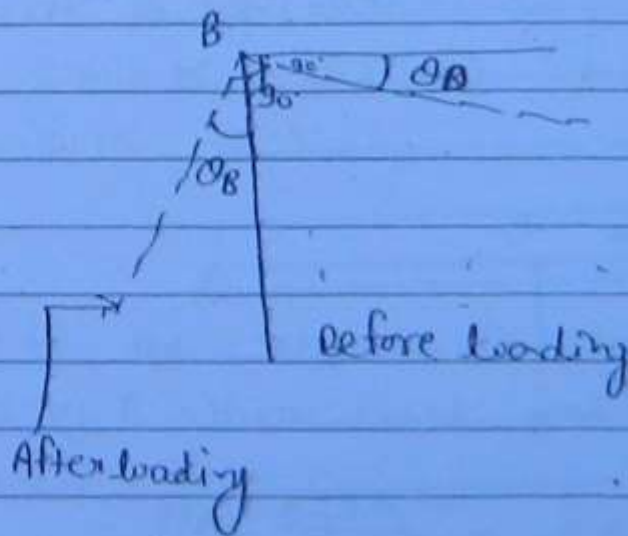
d



- Based on stiffness concept / Equilibrium concept / Displacement concept

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- Given by G.A. Maney
- Basic ~~unit~~ unknown are joint Displacement (O.D.)
- Joint Displacement are found using joint equilibrium condition & shear equation
- Joint Displacement are relate to the member force i.e. moments, such equation are called Slope-Deflection equation.
- Joint are considered rigid i.e. angle b/w member meeting at rigid joint do not change after loading.
- Joint rotates a whole



Derivation of Slope reflection Eqn:-

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① Sign convention:-

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(i) End moments:-

(+) clockwise

(-) Anticlockwise

(ii) Slope (Rotation of joints)

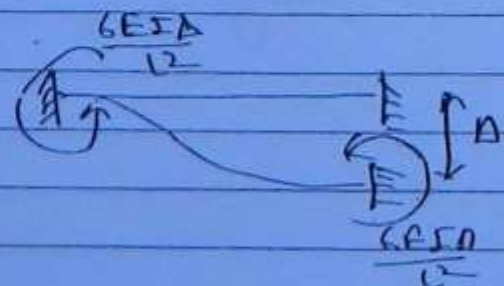
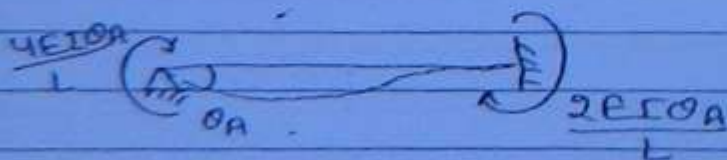
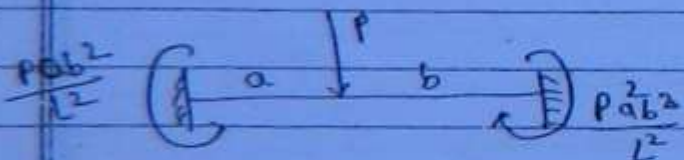
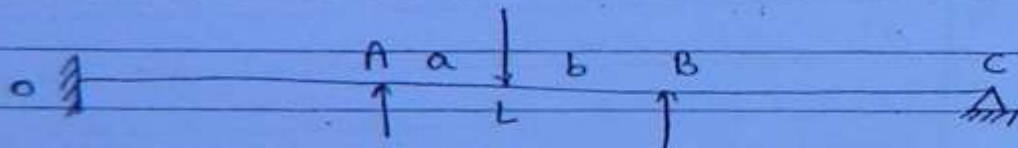
(+) Clockwise

(-) Anticlockwise

(iii) Linear displacement:-

Those Δ s which will produce Clockwise rotation to the member $\xrightarrow{\text{will be treated as}} +ve$

Those Δ s which produce anticlockwise rotation to the member $\xrightarrow{\text{will be treated as}} -ve$

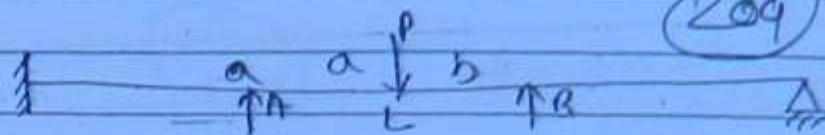


Derivation of slope deflection eqn.

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(209)

Step-I Consider AB to be fixed support then due to given loading fixed end moment will be M_{AB} and M_{BA}

$$M_{AB} = -\frac{Pab^2}{L^2}$$

Step-II If joint A rotates by θ_A then moment produced at A will be $= \frac{4EI}{L} \times \theta_A$

Carryover moment of $\theta_B = \frac{2EI}{L} \times \theta_B$

3) If supports B rotates by θ_B then moment produce B will be $\frac{4EI}{L} \theta_B$ then carry over moment of A will be $\frac{2EI}{L} \times \theta_B$

④ if one of the support settles (say) support B settles by Δ then fixed end ^{moment} produced to settlement

$$M_{AB} = \frac{6EI\Delta}{L^2} \quad M_{BA} = -\frac{6EI\Delta}{L^2}$$

where Δ is taken (+ve) if it produces clockwise rotation to the member.

* Final Effect of all the above 4 Cases.

Final Moment in AB at A (2/0)

$$M_{AB} = \bar{M}_{AB} + \frac{4EI}{L} \theta_A + \frac{2EI}{L} \theta_B - \frac{6EI\Delta}{L^2}$$

$$M_{AB} = \bar{M}_{AB} + \frac{2EI}{L} \left[2\theta_A + \theta_B - \frac{3\Delta}{L} \right] \quad \text{--- force-disp relation}$$

Final moment at B in AB

$$M_{BA} = \bar{M}_{BA} + \frac{2EI}{L} \left[2\theta_B + \theta_A - \frac{3\Delta}{L} \right]$$

$M \Rightarrow$ +ive $\curvearrowright$

$\theta \Rightarrow$ +ive $\curvearrowright$

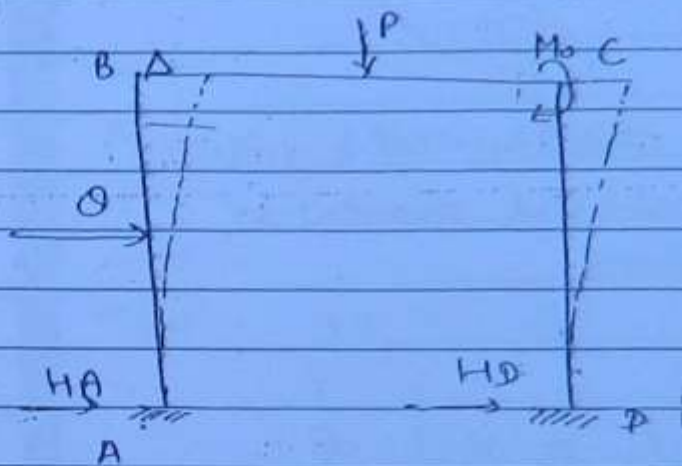
$\Delta \Rightarrow$ +ive if Produce $\curvearrowright$ rotⁿ to the member

The joints Displacements (θ, Δ) are basic unknown which are to be determine using joint equilibrium conditions and shear eqn. the no. of joint equilibrium condition will be equal to no. of rotational displacement(θ)

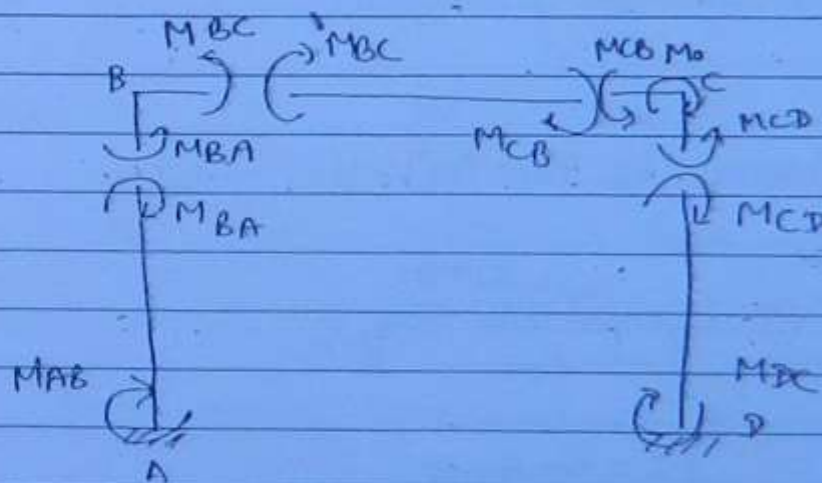
no. of Shear equation will be equal to No. of shear displacement(Δ).

→ In this analysis members are considered ~~inextensible~~ inextensible (axially rigid)

(211)



$$\theta_B, \theta_C, \Delta \quad (\Delta_B = \Delta_C = \Delta)$$



Eq^m of joint B (joint equi^m condition)

$$\sum M_B = 0$$

$$-M_{BA} - M_{BC} = 0$$

$$\boxed{M_{BA} + M_{BC} = 0} \quad \text{--- --- --- (I)}$$

Joint eq^m at C, $\sum M_C = 0$

$$-M_{CB} - M_{CD} + M_o = 0$$

$$\boxed{M_{CB} + M_{CD} = M_o} \quad \text{--- --- --- (II)}$$

Shear Equation $\sum F_x = 0$

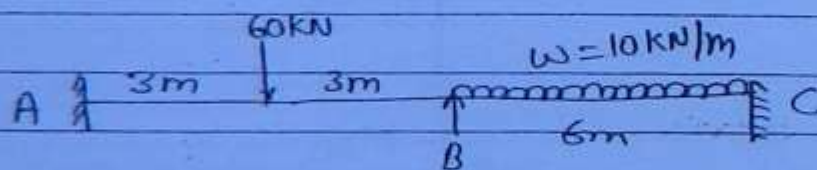
$$H_A + H_D + 0 = 0 \quad \text{--- (iii)}$$

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- Solving eqn (i) & (ii) θ_A, θ_B , and Δ will be known. and Hence final end moment in each member ~~can~~ can be found.

- Bending moment dia is similar to that of moment distribution method.

Q. for analysis the beam by slope deflection method



$EI = \text{constant}$

Fixed End moment

$$\bar{M}_{AB} = -\frac{PL}{8} \Rightarrow \frac{60 \times 6}{8} \Rightarrow -45 \text{ kN-m}$$

$$\bar{M}_{BA} = +\frac{PL}{8} = +45 \text{ kN-m}$$

$$\bar{M}_{BC} = -\frac{wL^2}{12} \Rightarrow \frac{10 \times 6^2}{12} \Rightarrow -30 \text{ kN-m}$$

$$\bar{M}_{CB} = +30 \text{ kN-m}$$

StoP deflection equation.

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$$M_{AB} = \bar{M}_{AB} + \frac{2EI}{L} (2\theta_A + \theta_B - \frac{3\Delta}{L})$$

$$M_{AB} = -45 + \frac{2EI}{6} (\theta_B) \quad \text{--- (i)}$$

$$M_{BA} = 45 + \frac{2EI}{6} (2\theta_B) \quad \text{--- (ii)}$$

$$\bar{M}_{BC} = -30 + \frac{2EI}{6} (2\theta_B + \theta_C - \frac{3\Delta}{L})$$

$$\bar{M}_{BC} = -30 + \frac{2EI}{6} (2\theta_B) \quad \text{--- (iii)}$$

$$\bar{M}_{CB} = 30 + \frac{2EI}{6} (\theta_B) \quad \text{--- (iv)}$$

Joint Equm equation at B

$$M_{BA} + M_{BC} = 0$$

$$45 + \frac{2EI}{6} (\theta_B) + (-30) + \frac{2EI}{6} (2\theta_B) = 0$$

$$\frac{4EI\theta_B}{3} = 30 - 45$$

$$\therefore EI\theta_B = -15$$

$$\theta_B = \frac{-15 \times 3}{4EI} = \frac{-45}{4EI}$$

$$\boxed{\theta_B = \frac{-45}{4EI}}$$

substituting OR in eqn (i) (ii) (iii) (iv)

$$M_{AB} = -45 + \frac{2EI}{\beta_3} \times -\frac{45}{4EI}$$

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$$M_{AB} \Rightarrow -45 + 0.33 \Rightarrow -48.75$$

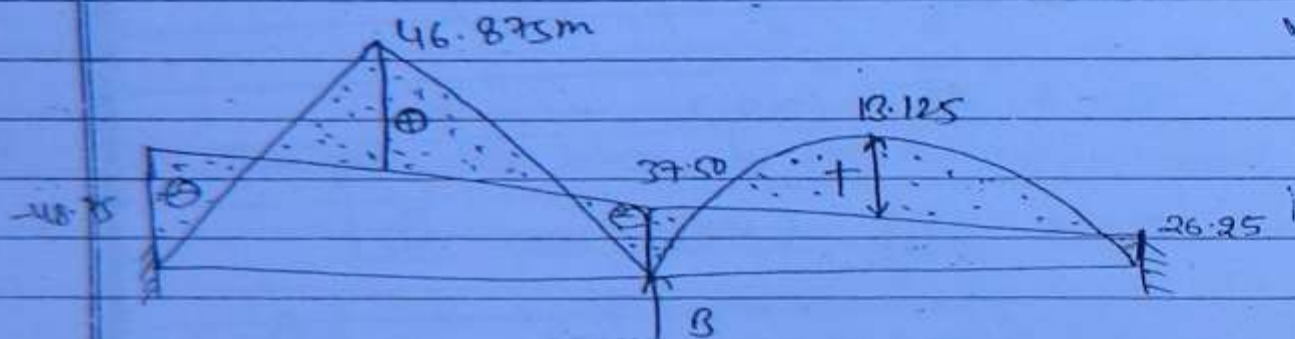
$$M_{BA} \Rightarrow$$

$$M_{AB} = -48.75$$

$$M_{BA} = +37.50$$

$$M_{BC} = -27.50$$

$$M_{CB} = +26.25$$



Simply support at Centre of AB = $\frac{PL}{4} = \frac{60 \times 6}{4} = 90$

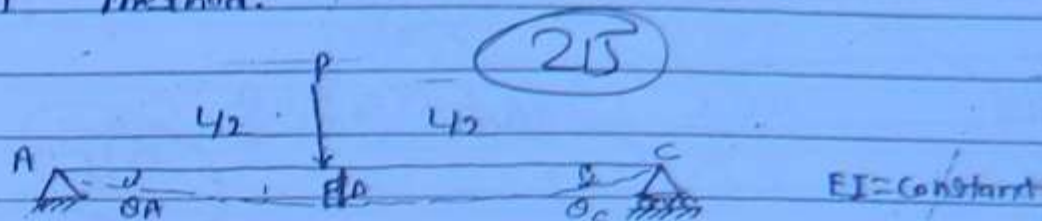
Net B.M. at Centre AB $\Rightarrow$

$$90 - \frac{(48.75 + 37.5)}{2} = 46.875$$

Simply supported in centre BC = $\frac{wL^2}{8} = \frac{10 \times 6^2}{8} = 45 \text{ KN-m}$

Net B.M. at Centre BC $\Rightarrow 45 - \frac{(37.5 + 26.25)}{2} = 13.125$

Q. Analyse the beam shown in fig. using Slope deflection method.



Note:- Above simply supported beam is determinate which can be analysed using equilibrium conditions alone. However, if method of indeterminate analysis are required to use then any of the stiffness method may be applied but computations will be lengthy.

Though slope deflection method considers above structure is made of two members AB and BC hence B is a assumed rigid ~~point~~ joint which is unsupported.

there will be 4 unknown

$$\theta_A, \theta_B, \theta_C, \Delta_B$$

Sol Fixed End moment

$$M_{AB} = M_{BA} = M_{BC} = M_{CB} = 0$$

Slope deflection equation.

$$M_{AB} = \overset{0}{\bar{M}_{AB}} + \frac{2EI}{L} \left(2\theta_A + \theta_B - \frac{3\Delta}{L} \right)$$

$$\Rightarrow 0 + \frac{2EI}{L/2} \left(2\theta_A + \theta_B - \frac{3\Delta}{L/2} \right) \quad \text{--- (1)}$$

$$M_{AB} = \frac{4EI}{L} \theta_A - \frac{6EI}{L^2} \Delta$$

$$\text{or } \frac{4EI}{L} (2\theta_A + \theta_B - \frac{3\Delta}{L}) \quad \text{--- (2)}$$

$$M_{BA} = 0 + \frac{4EI}{L} (2\theta_B + \theta_A - \frac{3\Delta}{L}) \quad \text{--- (i)}$$

$$M_{BA} = \frac{4EI}{L} (2\theta_B + \theta_A - \frac{6\Delta}{L}) \quad \text{--- (ii)}$$

$$M_{BC} = 0 + \frac{4EI}{L} (2\theta_C + \theta_B + \frac{6\Delta}{L}) \quad \text{--- (iii)}$$

$$M_{CB} = 0 + \frac{4EI}{L} (2\theta_C + \theta_B + \frac{6\Delta}{L}) \quad \text{--- (iv)}$$

Joint Eqm Equation.

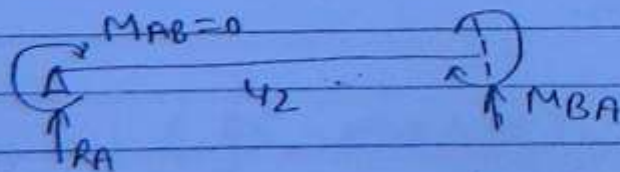
$$M_{AB} = 0 \quad \text{--- (A)}$$

$$M_{BA} + M_{BC} = 0 \quad \text{--- (B)}$$

$$M_C = 0 \quad \text{--- (C)}$$

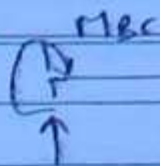
$$\text{Shear eqy. } R_A + R_C - P = 0 \quad \text{--- (D)}$$

for R_A & R_B



$$R_A \times \frac{L}{2} + M_{AB} + M_{BA} = 0$$

$$\boxed{R_A = \frac{M_{BA}}{2}} \quad \text{---}$$



(217)

$$-R_C \times \frac{L}{2} + M_{BC} + M_{CB} = 0$$

$$R_C = \frac{M_{BC}}{L/2}$$

putting

$$-\frac{M_{BA}}{L/2} + \frac{M_{BC}}{L/2} - P = 0$$

$$-M_{BA} + M_{BC} = \frac{PL}{2} \quad \text{--- (D)}$$

Solving eqn (A) (B) (C) & (D)

$$\theta_A = \frac{PLa^2}{16EI}$$

$$\theta_B = 0$$

$$\Delta = \frac{PL^3}{48EI}$$

$$\theta_C = -\frac{PL^2}{16EI}$$

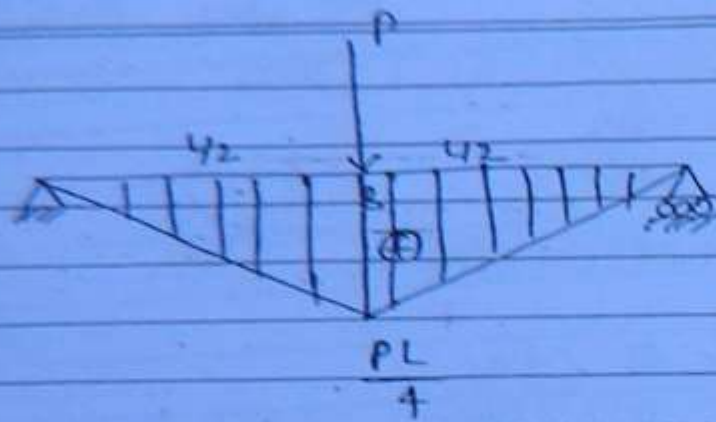
on substituting value in eqn (D) (iii) (iv) (v)

$$M_{AB} = 0$$

$$M_{BA} = -\frac{PL}{4}$$

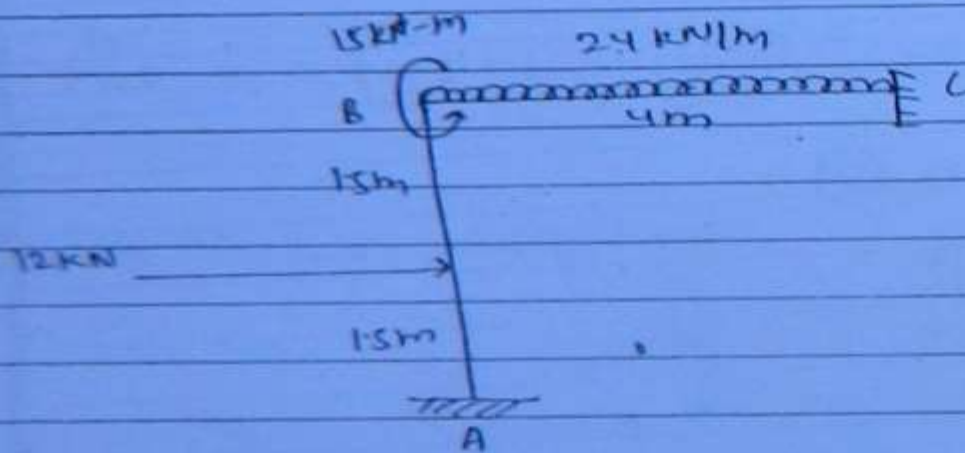
$$M_{BC} = +\frac{PL}{4}$$

$$M_{CB} = 0$$



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Q. Analyse the frame shown in fig. using Slope deflection method



Sol Fixed End moment

$$M_{AB} = -\frac{ML}{8} = -\frac{12 \times 3}{8} = -4.5 \text{ kN-m}$$

$$M_{BA} = +4.5 \text{ kN-m}$$

$$M_{BC} = -\frac{wL^2}{12} = -\frac{24 \times 4^2}{12} = -32 \text{ kN-m}$$

$$M_{CB} = +32 \text{ kN-m}$$

20p deflection eqn^m

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$$M_{AB} = \bar{M}_{AB} + \frac{2EI}{L} (2\theta_A + \theta_B - \frac{3\Delta}{L})$$

$$M_{AB} = -4.5 + \frac{2EI}{3} (\theta_B) \quad \text{--- (1)}$$

$$M_{BA} = 4.5 + \frac{2EI}{3} (2\theta_B) \quad \text{--- (2)}$$

$$M_{BC} = \bar{M}_{BC} + \frac{2EI}{L} (2\theta_B + \theta_C - \frac{3\Delta}{L})$$

$$\Rightarrow -32 + \frac{2EI}{2} (\theta_B)$$

$$M_{BC} = -32 + EI(\theta_B) \quad \text{--- (3)}$$

$$M_{CB} = +32 + \frac{2EI}{2} (\theta_B) \quad \text{--- (4)}$$

Joint Equ^m Equations

$$M_{BA} + M_{BC} = 0 - 15$$

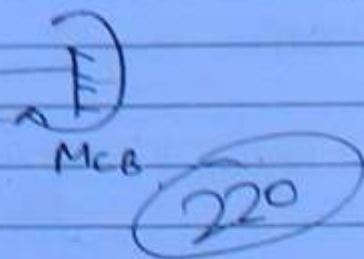
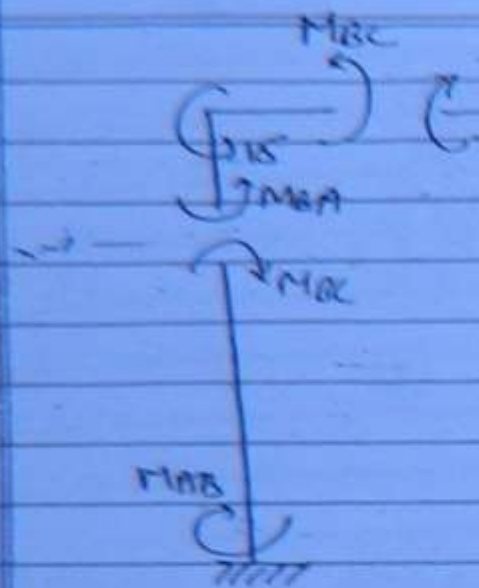
$$4.5 + \frac{2EI}{3} (2\theta_B) - 32 + EI\theta_B = -15$$

$$4.5 + 1.33EI\theta_B - 32 + EI\theta_B = -15$$

$$2.33EI\theta_B = -15 + 32 - 4.5$$

$$EI\theta_B \Rightarrow 5.125$$

$$\theta_B = \frac{5.125}{EI}$$



$$\sum M_B = 0$$

$$-M_{BA} - M_{BC} - 15 = 0$$

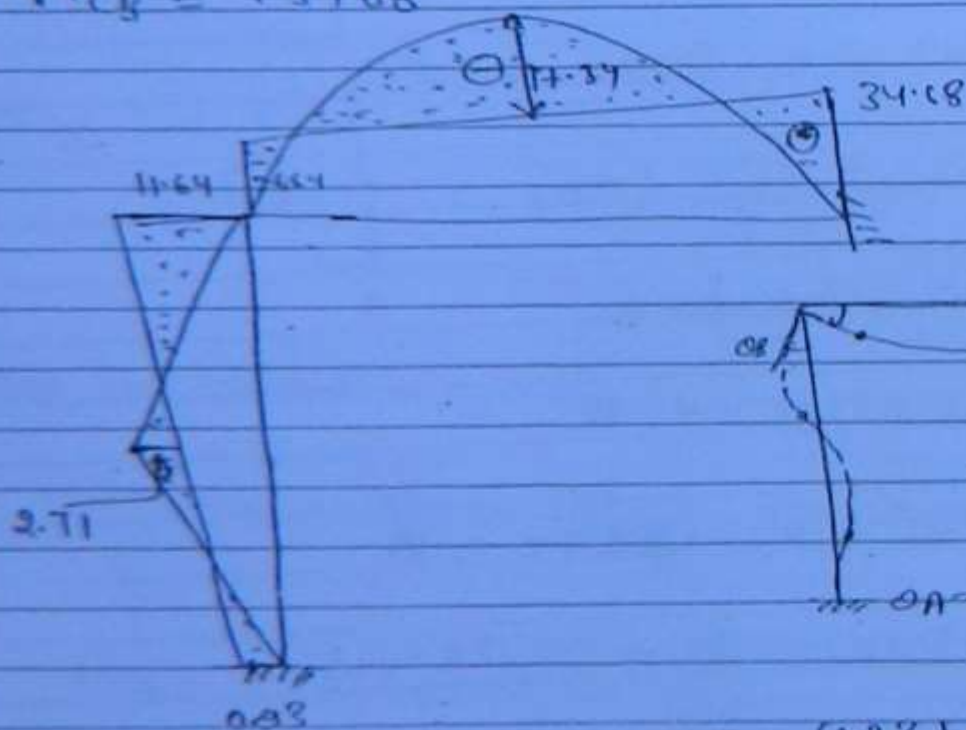
$$M_{BA} + M_{BC} = 15$$

$$M_{AB} = -0.93$$

$$M_{BA} = +11.64$$

$$M_{BC} = -26.64$$

$$M_{CB} = +34.68$$

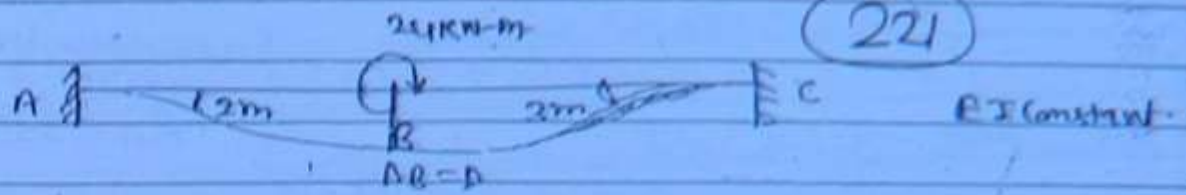


$$9 - \frac{(0.93 + 11.64)}{2} = 2.71$$

$$M_{AB} = \frac{PL}{4} \Rightarrow \frac{12 \times 3}{4} \Rightarrow 9$$

$$BC = \frac{wL^2}{8} = \frac{24 \times 2^2}{8} = 12 \rightarrow 12 - \frac{(26.64 + 34.68)}{2} = 17.34$$

Q. Analyse the Beam ABC using Slope deflection method.



Consider B is a unsupported joint. unknown θ_B, Δ_B

θ_B & ($\Delta_B = \Delta$) are unknown

Fixed End moments:-

$$\bar{M}_{AB} = \bar{M}_{BA} = \bar{M}_{BC} = \bar{M}_{CB} = 0$$

~~moment~~ Slope deflection eqn

$$M_{AB} = \bar{M}_{AB} + \frac{2EI}{L} (2\theta_A + \theta_B - \frac{3\Delta}{L}) = 0$$

$$\Rightarrow 0 + \frac{2EI}{2} (\theta_B - \frac{3\Delta}{2})$$

$$M_{AB} \Rightarrow EI (\theta_B - \frac{3\Delta}{2}) \quad \text{--- (1)}$$

$$M_{BA} = 0 + \frac{2EI}{2} (2\theta_B - \frac{3\Delta}{2}) \quad \text{--- (2)}$$

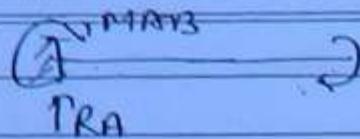
$$M_{BA} = EI (2\theta_B - \frac{3\Delta}{2}) \quad \text{--- (2)}$$

$$M_{BC} = \bar{M}_{BC} + \frac{2EI}{L} (2\theta_B + \theta_C + \frac{3\Delta}{L})$$

$$\bar{M}_{BC} = 0 + \frac{2EI}{2} (2\theta_B + \Delta) \quad \text{--- (3)}$$

$$M_{CB} = 0 + \frac{2EI}{2} (\theta_B + \Delta) \quad \text{--- (4)}$$

Joint equ.



$$M_{BA} + M_{AB} = 24$$

$$R_A \times 2 + M_{AB} + M_{BA} = 0$$

$$EI(2\theta_B - 1.5\Delta) +$$

$$R_A = \frac{(M_{AB} + M_{BA})}{2}$$

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$$M_{BA} + M_{BC} = 24$$

$$EI(2\theta_B - 1.5\Delta) + \frac{2}{3}EI(2\theta_B + \frac{\Delta}{8}) = 24$$

$$2\theta_B EI - 1.5\Delta EI + 1.32EI\theta_B + 0.66\Delta EI = 24$$

$$3.32\theta_B EI - 0.84\Delta = 24$$

$$3.32\theta_B - 0.84\Delta = \frac{24}{EI}$$

22

Shear equation $\sum F_y = 0$

$$R_C = \frac{M_{BC} + M_{CB}}{3} \Rightarrow$$

$$\Rightarrow \frac{0.66EI(2\theta_B + \Delta) + 0.66EI(\theta_B + \Delta)}{3}$$

$$\Rightarrow \frac{1.32\theta_B EI + 0.66EI\Delta + 0.66EI\theta_B + 0.66EI\Delta}{3}$$

$$\Rightarrow 0.44\theta_B EI + 0.22EI\Delta + 0.22EI\theta_B + 0.22EI\Delta$$

$$\Rightarrow 0.66EI\theta_B + 0.44EI\Delta$$

$$B = - \left[\frac{M_{AB} + M_{BC}}{2} \right]$$

$$\Rightarrow -\frac{1}{2} \left[EI(\theta_B - 1.5\Delta) + EI(2\theta_B - 1.5\Delta) \right]$$

$$\Rightarrow -\frac{1}{2} \left[EI\theta_B - 1.5\Delta EI + 2EI\theta_B - 1.5EI\Delta \right]$$

$$\Rightarrow -1.5EI\theta_B + 1.5EI\Delta = 0 \quad (223)$$

$$0.66EI\theta_B + 0.44EI\Delta = -1.5EI\theta_B + 1.5EI\Delta$$

$$2.16EI\theta_B + 1.06EI\Delta$$

$$\Delta = \frac{1.5EI\theta_B}{2.16EI} = 0.350\theta_B$$

Put (2)

$$3.32\theta_B - \frac{0.35}{EI} \theta_B = \frac{24}{EI}$$

$$2.97\theta_B = \frac{24}{EI}$$

$$\theta_B = \frac{24 \times 10^6}{2.97 \times EI} = \frac{8.06}{EI}$$

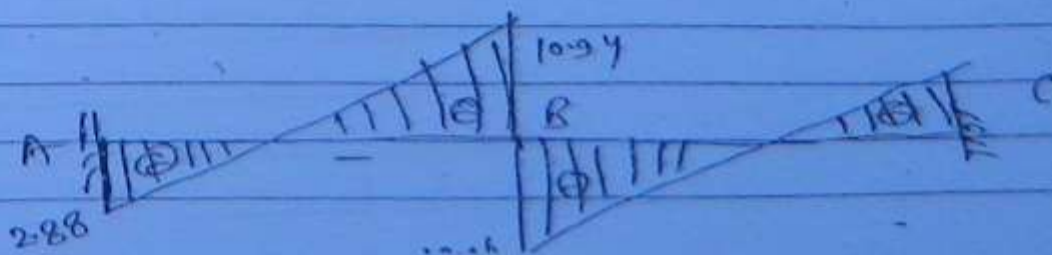
$$\Delta = \frac{3.456}{EI}$$

$$M_{AB} = \frac{M_0}{L^2} b(3a-L)$$

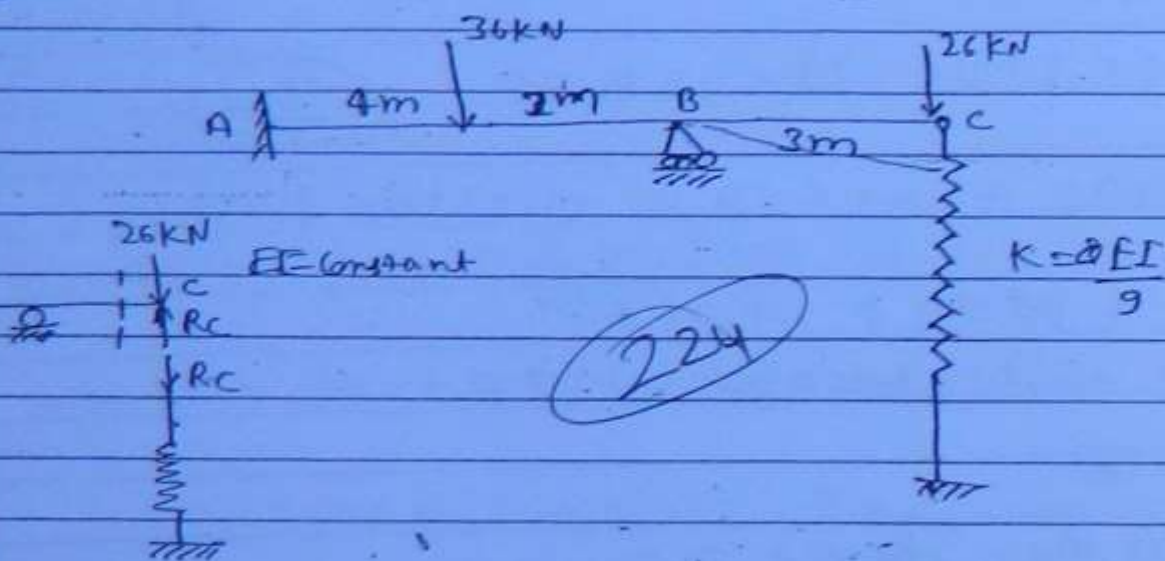
$$\Rightarrow \frac{24 \times 3}{3^2} (3 \times 2 - 5)$$

$$\Rightarrow \frac{24 \times 3 \times 1}{2.5} \Rightarrow +2.88$$

$$M_{AB} = 2.88, \quad M_{BA} = 10.94, \quad M_{BC} = 13.06, \quad M_{CB} = 7.14$$



Q. Analysis the Beam shown in fig. using slope deflection method. Joint C is supported over a spring with stiffness $K = \frac{EI}{9}$



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Fixed end moment

$$\bar{M}_{AB} = \frac{wab}{I} \quad \text{Unknowns are } \theta_B, \theta_C, \Delta_C$$

$$R_c = K \cdot \Delta_{\text{spring}}$$

$$\Delta_{\text{spring}} = \Delta_C$$

$$R_c = K \cdot \Delta_C \Rightarrow \frac{EI}{9} \Delta_C$$

Fixed end moment

$$M_{AB} = \frac{-wab^2}{L^2} \Rightarrow \frac{-36 \times 4 \times 2 \times 2}{6 \times 6} \Rightarrow -16 \text{ kN-m}$$

$$M_{BA} = \frac{wab^2}{L^2} \Rightarrow \frac{36 \times 4 \times 4 \times 2}{36 \times 6} \Rightarrow +32 \text{ kN-m}$$

$$\bar{M}_{BC} = 0$$

$$\bar{M}_{CB} = 0$$

Slope Deflection eqn

(225)

$$M_{AB} = -16 + \frac{2EI}{6} (2\theta_A + \theta_B - \frac{7\Delta}{6})$$

$$\Rightarrow -16 + \frac{EI}{3} (\theta_B - \frac{7\Delta}{6})$$

$$M_{AB} = -16 + \frac{EI\theta_B}{3} \quad \text{--- (i)}$$

$$M_{BA} = +32 + \frac{EI}{3} 2\theta_B \quad \text{--- (ii)}$$

$$M_{BC} = 0 + \frac{2EI}{12} (2\theta_B + \theta_C - 3\Delta_C)$$

$$\Rightarrow 0 + \frac{2EI}{3} (2\theta_B + \theta_C - \frac{\Delta_C}{3})$$

$$M_{BC} = \frac{2EI}{3} (2\theta_B + \theta_C - \Delta_C) \quad \text{--- (iii)}$$

$$M_{CB} = \frac{2EI}{3} (2\theta_C + \theta_B - \Delta_C) \quad \text{--- (iv)}$$

Joint equation at B

$$M_{BA} + M_{BC} = 0 \quad \text{--- (v)}$$

$$32 + EI(0.66)\theta_B + 0.66EI[2\theta_B + \theta_C - \Delta_C] = 0$$

$$32 + 0.66EI\theta_B + 1.32EI\theta_B + 0.66EI\theta_C - 0.66\Delta_C = 0$$

$$32 + 1.98EI\theta_B + 0.66EI\theta_C - 0.66\Delta_C = 0$$

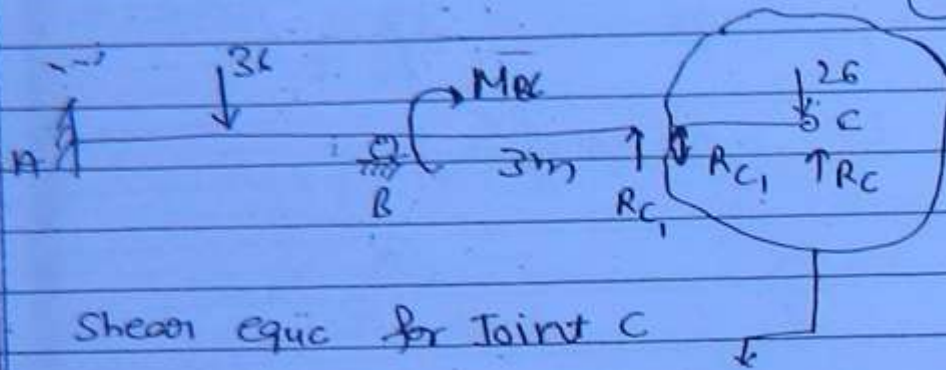
$$1.98EI\theta_B + 0.66EI\theta_C - 0.66\Delta_C = -32$$

$$\text{--- (vi)}$$

Joint eqn A+C

$$M_{CB} = 0$$

(B) 226



Shear eqn for Joint C

$$R_C - 26 - R_{C1} = 0$$

$$\frac{EI \Delta_C}{9} - 26 - R_{C1} = 0$$

$$\Sigma M_B = 0$$

$$-R_{C1} \times 3 + M_{BC} = 0$$

$$R_{C1} = \frac{M_{BC}}{3}$$

Shear eqn will be

$$\frac{EI \Delta_C}{9} - 26 - \frac{M_{BC}}{3} = 0 \quad \text{--- (C)}$$

$$M_{CB} = 0$$

$$0.66EI(2\theta_C + \theta_B - \Delta_C) = 0 \quad 0.66EI$$

$$1.32EI\theta_C + 0.66EI\theta_B - \Delta_C = 0$$

or

Solving eqn (A) & (C)

$$\theta_B = \frac{6}{EI}$$

$$\theta_C = \frac{60}{EI}$$

$$\Delta_C = \frac{126}{EI}$$

$$M_{AB} = -16 + \frac{EI}{8} \times \frac{8^2}{EI} \Rightarrow -14 \text{ kN-m}$$

$$M_{BA} = 32 + \cancel{EI} \frac{2}{8} \times \frac{8^2}{EI} \quad (227)$$

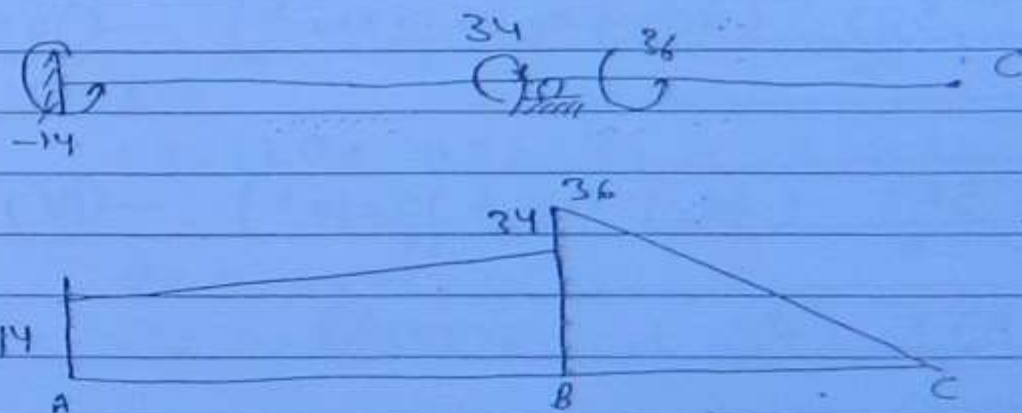
$$\Rightarrow 34 \text{ kN-m}$$

$$M_{BC} \Rightarrow \frac{2EI}{3} \left(2 \times \frac{6}{EI} + \frac{60}{EI} - \frac{126}{EI} \right)$$

$$\Rightarrow \frac{2}{3} \cancel{EI} \times \frac{-54}{EI} \times 18$$

$$M_{BC} \Rightarrow -36$$

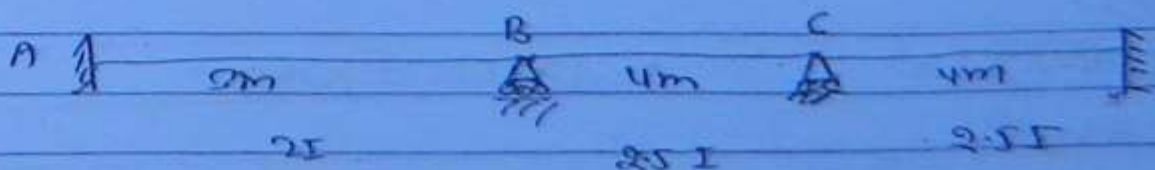
$$M_{CB} = 0$$



Q. 20M

Analysis the continuous beam shown in fig using slope deflection method. support B settled down 5mm.

$$E = 2 \times 10^5 \text{ N/mm}^2 \quad I = 36 \times 10^6 \text{ mm}^4$$



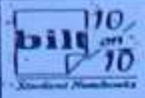
$$x = 2 \times 10^{-3} \text{ m}$$

$$I = 36 \times 10^6 \text{ mm}^4$$

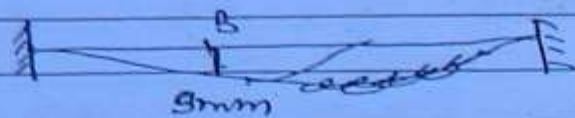
$$\Delta_B = 5 \text{ mm}$$

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$$M_{AB} = \frac{2EI}{3} (2\theta_A + \theta_B - \frac{3 \times 5 \times 10^{-3}}{3})$$

$$M_{AB} = \frac{4EI}{3} (\theta_B - 5 \times 10^{-3}) \quad \text{--- (i)}$$

$$M_{BA} = \frac{4EI}{3} (2\theta_B - 5 \times 10^{-3}) \quad \text{--- (ii)}$$

$$M_{BC} = \frac{2EI}{4} (2\theta_B + \theta_C + \frac{3 \times 5 \times 10^{-3}}{4})$$

$$M_{BC} = \frac{5EI}{4} (2\theta_B + \theta_C + \frac{15 \times 10^{-3}}{4}) \quad \text{--- (iii)}$$

$$M_{CB} = \frac{5EI}{4} (2\theta_C + \theta_B + \frac{15 \times 10^{-3}}{4}) \quad \text{--- (iv)}$$

$$M_{CD} = \frac{5EI}{4} (2\theta_C) \quad \text{--- (v)}$$

$$M_{DC} = \frac{5EI}{4} (\theta_C) \quad \text{--- (vi)}$$

Joint Equ At B

$$M_{BA} + M_{BC} = 0$$

$$1.33EI (2\theta_B - 5 \times 10^{-3}) + 1.25EI (2\theta_B + \theta_C + 3.75 \times 10^{-3}) = 0$$

1.33

$$1.33 \times 2 \times 10^5 \times 36 \times 10^6 \quad \therefore \text{or } 1.25 \times 2 \times 10^5 \times 36 \times 10^6$$

$$9.576 \times 10^{11}$$

$$1.9152 \times 10^{11}$$

$$2.66EI\theta_B - 6.66 \times 10^{-3} EI + 2.5EI\theta_B + 1.25EI\theta_C + 4.69 \times 10^{-3} EI = 0$$

$$5.16EI\theta_B + 1.25EI\theta_C - 1.97 \times 10^{-3} EI = 0$$

$$5.16\theta_B EI + 1.25EI\theta_C = 1.97 \times 10^{-3} EI$$

At joint C

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$$M_{CB} + M_{CD} = 0$$

$$1.25EI(2\theta_C + \theta_B + 3.75 \times 10^{-3}) + 1.25EI(\theta_C/2) = 0$$

$$2.50EI\theta_C + 1.25EI\theta_B + 4.68 \times 10^{-3} EI + 2.5EI\theta_C = 0$$

$$5EI\theta_C + 1.25EI\theta_B + 4.68 \times 10^{-3} EI = 0$$

5EI\theta_C

$$1.25EI\theta_B + 5EI\theta_C = -4.68 \times 10^{-3} EI$$

$$\theta_B = +6.49 \times 10^{-4}$$

$$\theta_C = -1.099 \times 10^{-3}$$

$$M_{AB} = -41.77$$

$$M_{CB} = +19.78$$

$$M_{BA} = -35.53$$

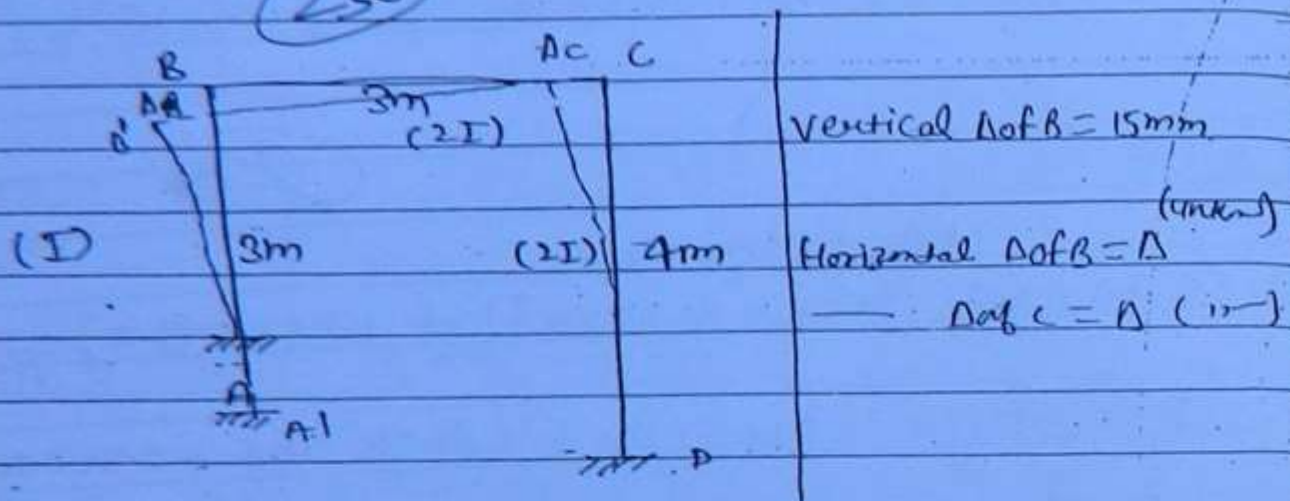
$$M_{CD} = -19.78$$

$$M_{BC} = +35.33$$

$$M_{DC} = -9.89$$

Q For the frame shown in fig. support A settles by 15 mm vertically down. Analyse the frame by slope-deflection method. And draw the BMD. Given that $EI = 48000 \text{ kN-m}$.

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FFM =

A MAB

$$M_{AB} = 0 + \frac{2EI}{L} \left(2\theta_A + \theta_B + \frac{3\Delta}{L} \right) \quad (-80/L)$$

$$M_{AB} = \frac{2EI}{3} (\theta_B + \Delta) \quad \text{--- (i)}$$

$$M_{BA} = \frac{2EI}{3} (2\theta_B + \Delta) \quad \text{--- (ii)}$$

$$M_{BC} = 0 + \frac{2E(2I)}{3} \left(2\theta_B + \theta_C + \frac{3 \times 15 \times 10^{-3}}{3} \right)$$

$$M_{BC} = \frac{4EI}{3} (2\theta_B + \theta_C + 15 \times 10^{-3}) \quad \text{--- (iii)}$$

$$M_{CB} = \frac{4EI}{3} (2\theta_C + \theta_B + 15 \times 10^{-3}) \quad \text{--- (iv)}$$

$$M_{CD} = 0 + \frac{2EI(3I)}{4} \left[2\theta_C + \theta_D + \frac{3\Delta}{4} \right]$$

$$M_{CD} = EI \left[2\theta_C + \frac{3\Delta}{4} \right] \quad \text{--- (V)}$$

$$M_{DC} = EI \left[\theta_C + \frac{3\Delta}{4} \right] \text{ or } 75\Delta \quad \text{--- (VI)}$$

Joint eqn At B

(23)

$$M_{BA} + M_{BC} = 0$$

$$0.67EI(2\theta_B + \Delta) + 1.33EI(2\theta_B + \theta_C + 15 \times 10^{-3}) = 0$$

$$\Rightarrow 1.34EI\theta_B + 0.67EID + 2.66EI\theta_B + 1.33EI\theta_C + 19.95 \times 10^{-3} = 0$$

$$\Rightarrow 4EI\theta_B + 2EID + 19.95 \times 10^{-3} = 0 \quad \text{--- (A)}$$

At joint C

$$M_{CB} + M_{CD} = 0$$

$$1.33EI(2\theta_C + \theta_B + 15 \times 10^{-3}) + EI(2\theta_C + 0.75\Delta) = 0$$

$$2.66EI\theta_C + 1.33\theta_B + 19.95 \times 10^{-3} + 2EI\theta_C + 0.75\Delta EI = 0$$

$$4.66EI\theta_C + 1.33\theta_B + 0.75\Delta + 19.95 \times 10^{-3} = 0 \quad \text{--- (B)}$$

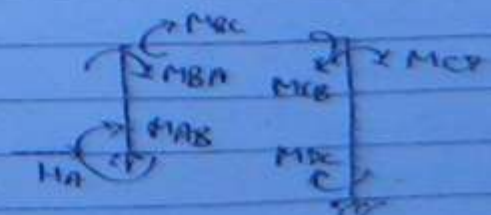
Shear eqn $H_A + H_D = 0$

To find H_A , consider FBD of AB

$$\sum M_B = 0$$

$$-H_A \times 3 + M_{AB} + M_{BA} = 0$$

$$H_A = \frac{M_{AB} + M_{BA}}{3}$$



To find H_D , consider FBD of CD

$$\sum M_C = 0$$

$$\rightarrow H_D \times 4 + M_{CD} + M_{DC} = 0$$

$$H_D = \frac{M_{CD} + M_{DC}}{4}$$

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$$H_A = \frac{M_{AB} + M_{BA}}{3}$$

$$\Rightarrow \frac{1}{3} [0.67 EI \theta_B + 0.67 \Delta + 1.34 EI \theta_B + 0.67 EI \Delta] = 0$$

$$\Rightarrow \frac{1}{3} [2 EI \theta_B + 1.34 \Delta] = 0$$

$$\Rightarrow 0.67 EI \theta_B + 0.45 \Delta = 0 \quad \text{--- (7)}$$

$$H_D = \frac{M_{CD} + M_{DC}}{4}$$

$$\Rightarrow \frac{1}{4} [2 EI \theta_C + 0.75 \Delta EI + EI \theta_C + 0.75 \Delta]$$

$$\Rightarrow 0.75 EI \theta_C + 1.5 \Delta = 0 \quad \text{--- (8)}$$

~~Equation~~

$$4 EI \theta_B + 2 EI \Delta + 19.95 \times 10^{-3} = 0$$

~~Equation~~

$$M_{AB} = +91.63 \text{ kN-m}$$

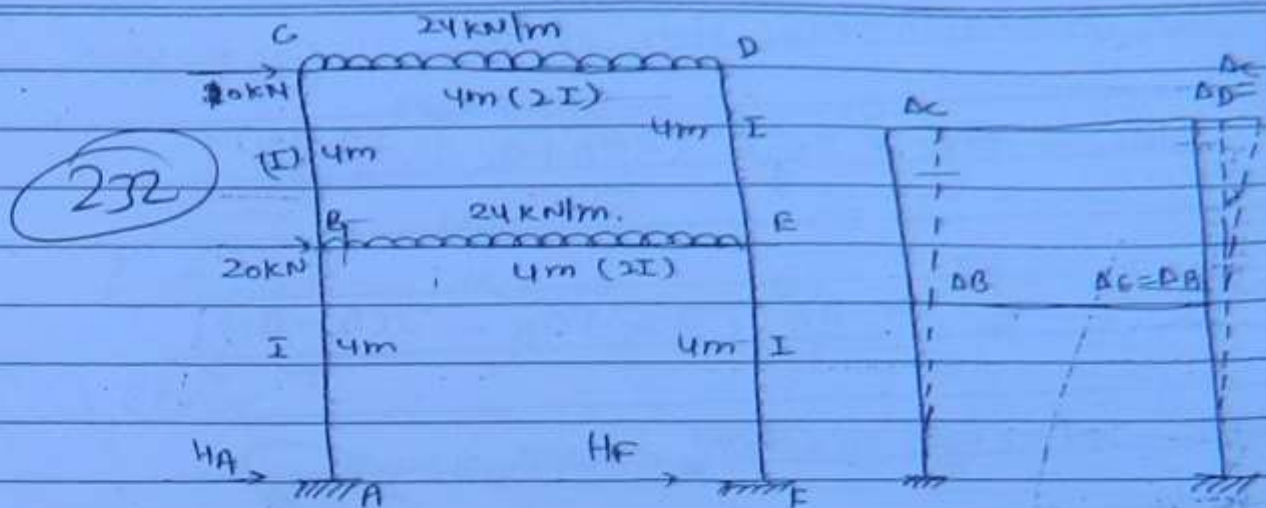
$$M_{BA} = -65.82 \text{ "}$$

$$M_{BC} = +65.82$$

$$M_{CB} = +116.35$$

$$M_{CD} = -116.35$$

$$M_{DC} = +81.93$$



Unknown $\theta_B, \theta_C, \theta_D, \theta_E, \Delta_B, \Delta_C$

Joint equation. -

At joint B:-

$$M_{BA} + M_{BC} + M_{BE} = 0 \quad \text{--- (i)}$$

At joint C

$$M_{CB} + M_{CD} = 0 \quad \text{--- (ii)}$$

At joint D

$$M_{DE} + M_{DC} = 0 \quad \text{--- (iii)}$$

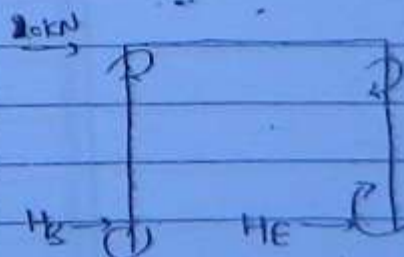
At joint F

$$M_{FE} + M_{FD} + M_{FB} = 0 \quad \text{--- (iv)}$$

Shear equation:-

$$\sum F_x = 0$$

$$H_A + H_F + 10 + 20 = 0 \quad \text{--- (5)}$$



$$\sum F_x = 0$$

$$H_B + H_E + 10 = 0 \quad \text{--- (6)}$$

→ Slope Deflection equation of BC. $\left(\frac{3\Delta}{4}\right)$

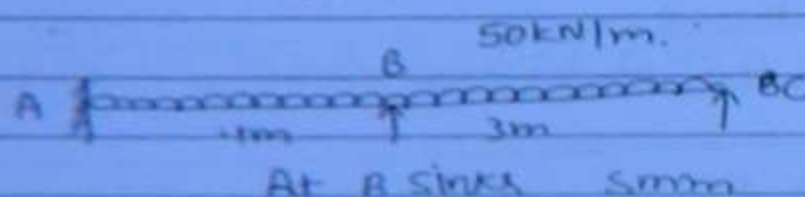
$$M_{BC} = \frac{M_B}{4} + \frac{2EI}{4} \left(2\theta_B + \theta_C + \frac{3(\Delta_2 - \Delta_1)}{4} \right)$$

$$M_{CB} = \frac{M_B}{4} + \frac{2EI}{4} \left(2\theta_C + \theta_B - \frac{3(\Delta_2 - \Delta_1)}{4} \right)$$

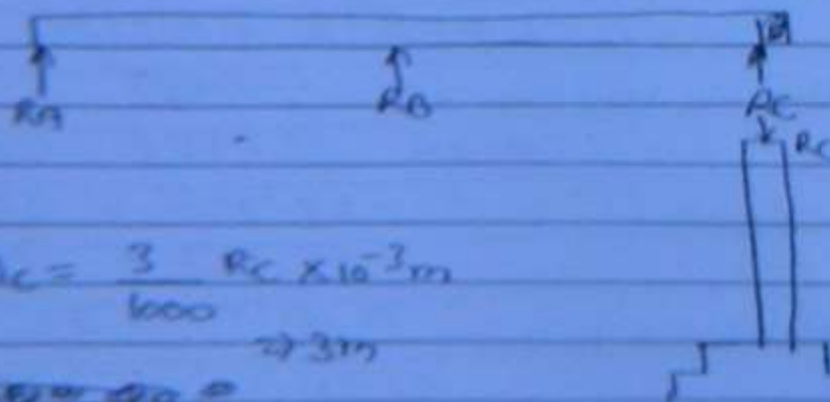
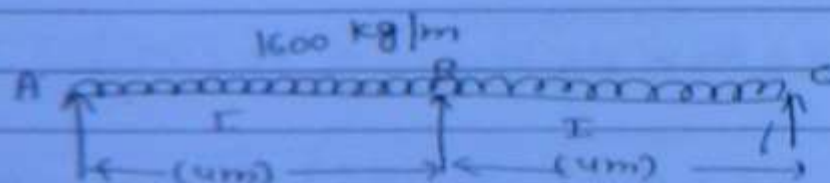
Slope Deflection equation for AB

$$M_{AB} = 0 + \frac{2EI}{4} \left(2\theta_A + \theta_B - \frac{3\Delta}{4} \right)$$

Q.16



Q.14(b)



$$\Delta_C = \frac{3}{600} R_C \times 10^{-3} m$$

$\approx 3m$

Red note

06.00.10 A1 B C unknown

$$\sum M_B = 0$$

$$M_{BA} + M_{BC} = 0 \quad \text{--- (1)}$$

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$$M_{AB} = 0 \quad \text{--- (2)}$$

$$M_{CB} = 0 \quad \text{--- (3)}$$

$$\sum F_y = 0$$

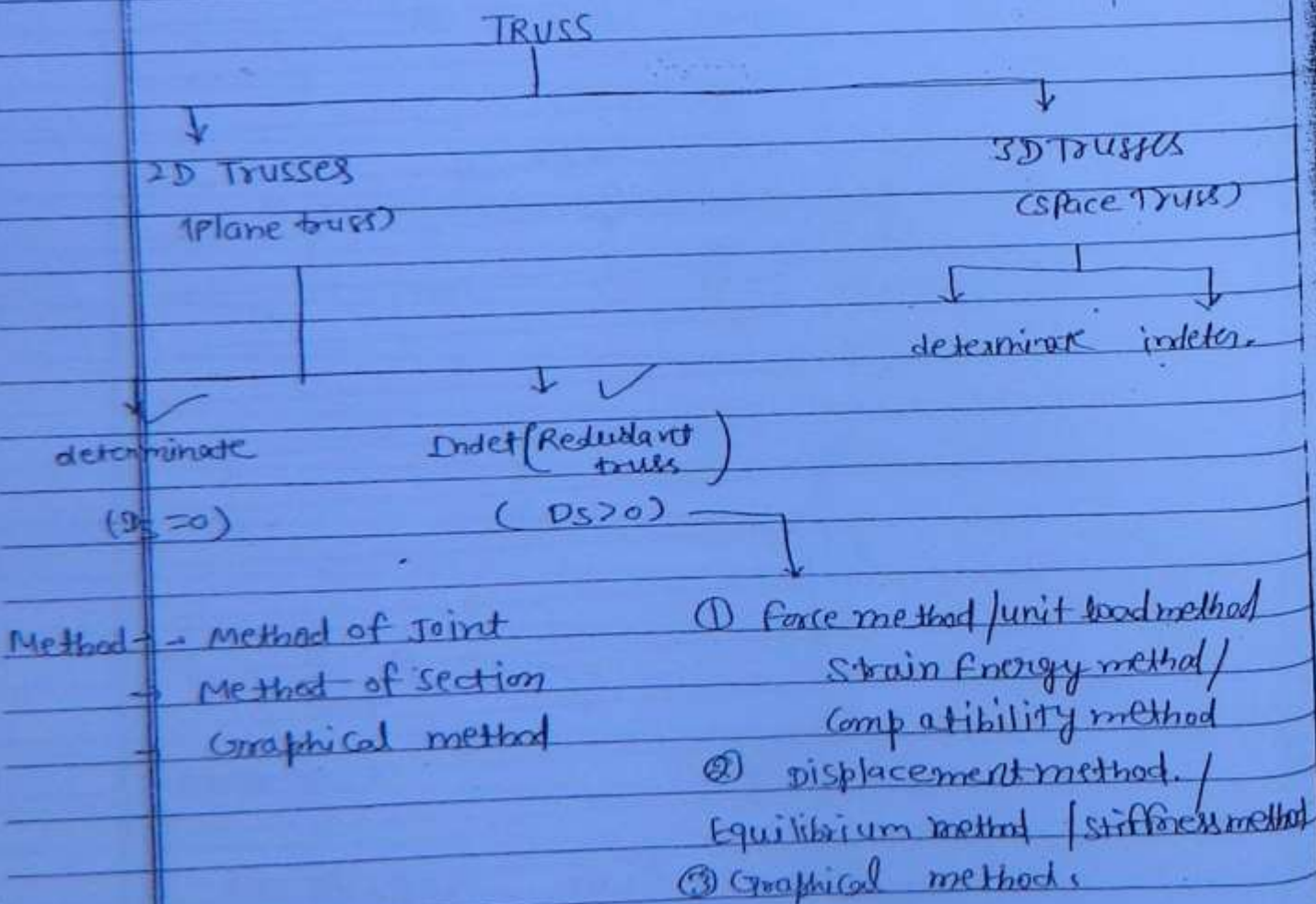
$$R_A + R_B + R_C = \overset{1600 \times 8}{\bullet} \quad \text{--- (4)}$$

TRUSSES:-

Assumptions

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- Pin joint frames
- All joints are connected by frictionless hinges
- All members are straight.
- loading is applied only at joint.
- self weight of member is neglected
- Members of truss carry only Axial forces (Compⁿ/Tension) No SF & BM.
- Hooke's Law is valid, The material is isotropic, Homogeneous and Linear Elastic.



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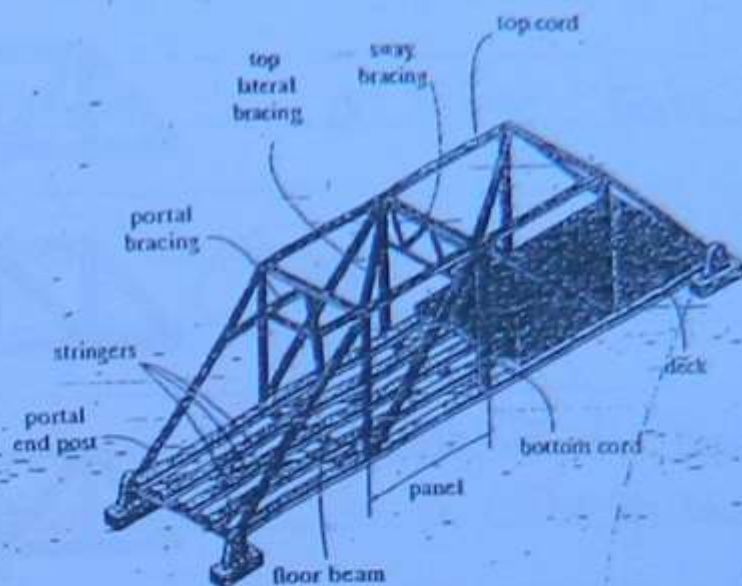
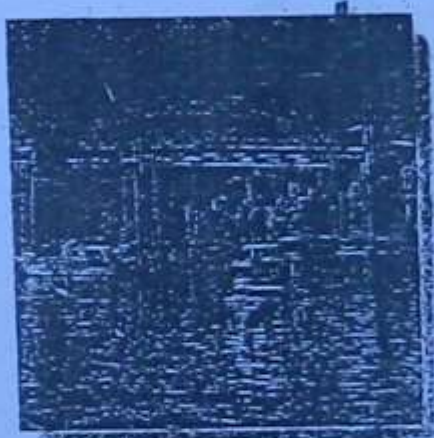


Fig. 3-4



Parker trusses are used to form this bridge.

Bridge Trusses. The main structural elements of a typical bridge truss are shown in Fig. 3-4. Here it is seen that a load on the *deck* is first transmitted to *stringers*, then to *floor beams*, and finally to the joints of the two supporting *side trusses*. The top and bottom cords of these side trusses are connected by top and bottom *lateral bracing*, which serves to resist the lateral forces caused by wind and the sideways caused by moving vehicles on the bridge. Additional stability is provided by the *portal* and *sway bracing*. As in the case of many long-span trusses, a roller is provided at one end of a bridge truss to allow for thermal expansion.

A few of the typical forms of bridge trusses currently used for single spans are shown in Fig. 3-5. In particular, the Pratt, Howe, and Warren trusses are normally used for spans up to 200 ft (61 m) in length. The most common form is the Warren truss with verticals, Fig. 3-5c. For larger spans, a truss with a polygonal upper cord, such as the Parker truss, Fig. 3-5d, is used for some savings in material. The Warren truss with verticals can also be fabricated in this manner for spans up to 300 ft (91 m). The greatest economy of material is obtained if the diagonals have a slope between 45° and 60° with the horizontal. If this rule is maintained, then for spans greater than 300 ft (91 m), the depth of the truss must increase and consequently the panels will get longer. This results in a heavy deck system and, to keep the weight of the deck within tolerable limits, *subdivided* trusses have been developed. Typical examples include the Baltimore and subdivided Warren trusses, Figs. 3-5e and 3-5f. Finally, the K-truss shown in Fig. 3-5g can also be used in place of a subdivided truss, since it accomplishes the same purpose.

BRIDGE TRUSSEL

SECTION 3-1 Common types of



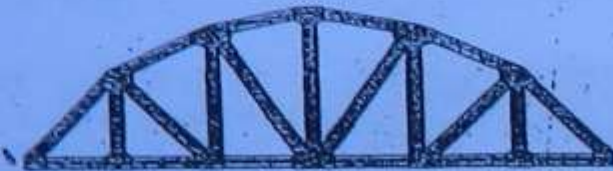
Pratt
(a)



Howe
(b)



Warren (with verticals)
(c)



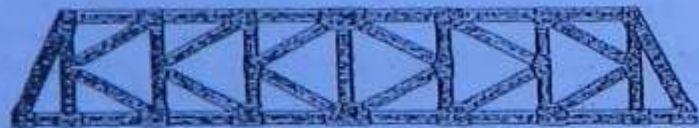
Parker
(d)



Baltimore
(e)



subdivided Warren
(f)



K-truss
(g)

Fig. 3-5

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Equilibrium method / stiffness method
③ Graphical method

Uses of Truss $\rightarrow$

(1) Roof Truss

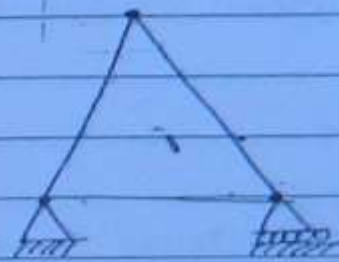
(2) Bridge Truss

Type of Truss $\rightarrow$

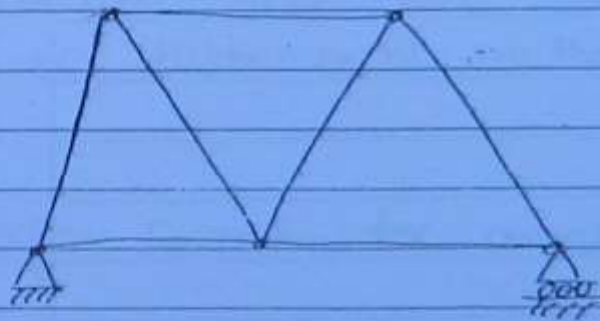
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(1) Simple Truss:-

The simple truss consists of triangular block. The simplest frame work is a triangular with three members.



Simple Truss



Compound Truss:- It is formed by 2 or more simple truss. It is often used for longer spans and is proved cheaper than a simple single truss for the same span.

Principal Simple Truss



2nd secondary Simple Truss

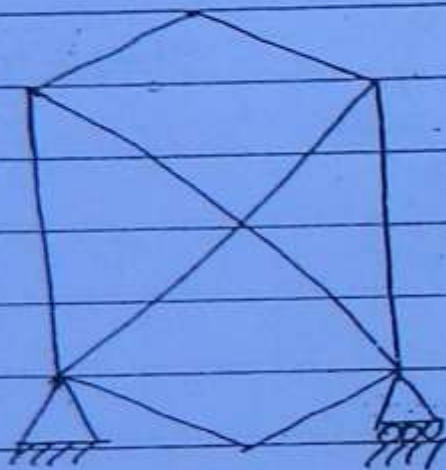
Simple Truss.

Simple Truss

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Compound

Complex Truss :- If Truss is neither simple nor compound then it will be complex truss. Generally a complex Truss has polygonal structure.



complex truss

Analysis of Trusses :-

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Step I

check the degree of static indeterminacy

$$D_s = (m + r_e) - 2J$$

→ if $D_s = 0$ then Truss is called determinate
Perfect frame.

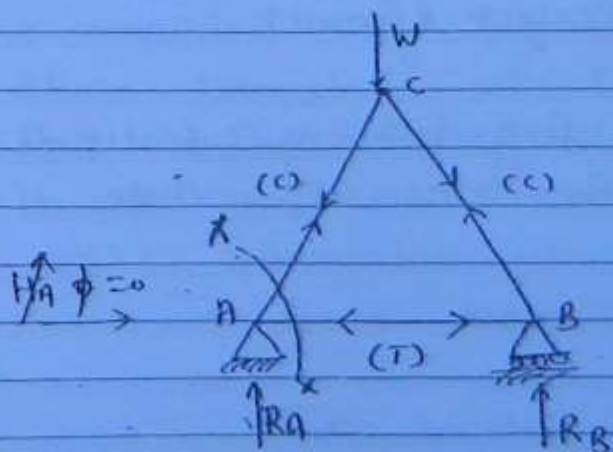
for such Trusses equilibrium equation are use
for analysis. And method of joint and method of
Section can be use.

Step 2:-

if $D_s > 0$ then Truss is called indeterminate
Redundants

→ Redundant may be internal and external. To
analysis such Truss ~~method~~ of force method
or displacement method can be use.

Sign Convention for member force:-



+ → Tension

- → Compression.

AC → ← Compression

AB → ← Tension

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Joint Free Body Diagram.

Member Free Body

Free Body dia of C.

Date / /

Page

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10
10

- At joints Tension force is represented by arrows moving away from joint.
- Compression force points towards the joint.
- In 2 dimension case (Plane Truss) for equilibrium of each joint following condition should be satisfied

① $\sum F_x = 0$

② $\sum F_y = 0$

Analysis of determinate ~~for~~ plane Trusses ($D=0$)
(Perfect frame)

rep-1 > Find support reactions using following 3 conditions of static equilibrium for truss as a whole.

i) $\sum F_x = 0$

ii) $\sum F_y = 0$

iii) $\sum M_z = 0$

Step-2 > To determine member forces use either method of joint or method of section.

(243)

(i) method of joint →

This method is suitable when force in all member of trusses are required. the free body diagram of each joint is considered and following 2 condition of static equilibrium are use to find member forces

$$\textcircled{1} \sum F_x = 0 \quad \textcircled{2} \sum F_y = 0$$

This method is applicable only when no. of unknown force at a joint is use than or equal to two. In this method the judicious selection of joint is done in such a way no. of unknown at joint are ≤ 2

(ii) method of section →

It is suitable when forces are required only in two member i.e. 1, 2, 3. After computation of support reaction two find member forces an imaginary section is cut such that not more than 3 member will be cut across the section. i.e. unknown are not more than 3.

The section assume must cut those member in which forces are required. the section can be vertical, inclined, horizontal

of 21g-2ag. free body equilibrium either from left portion or for right portion to the section is considered. And only following 3 condition are applied.

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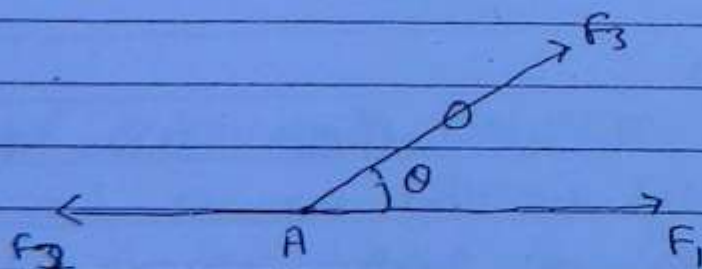
① $\sum F_x = 0$ ② $\sum F_y = 0$ ③ $\sum M_z = 0$

It means at a time more than 3 unknown force can be found.

for
diag
gmg

Thumb rule to find members which carry Zero forces

- 1) If at a joint only three members meet Two of them are co-linear there is no external force and Reaction at that joint then 3rd member (no-co-linear member) will carry zero force.



$$\theta \neq 0$$

$$\theta \neq 180$$

$$\boxed{F_3 = 0} \therefore \sum F_y = 0$$

$$F_3 \sin \theta = 0$$

$$\boxed{F_3 = 0}$$

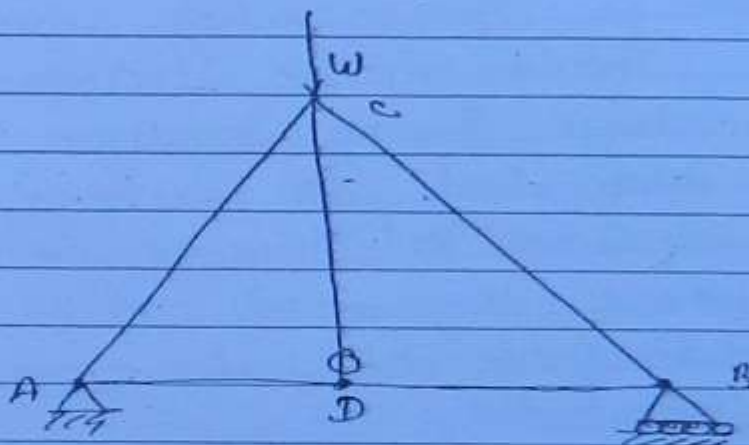
further notice that under such case both collinear member will have equal and alike forces.

$$F_1 = F_2$$

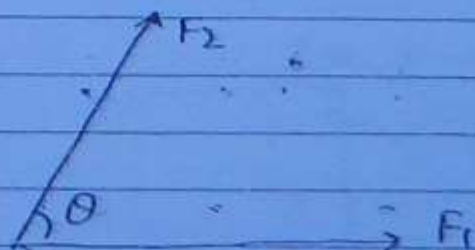
245

(Either both will be tensile or compression or Both will be zero)

eg.



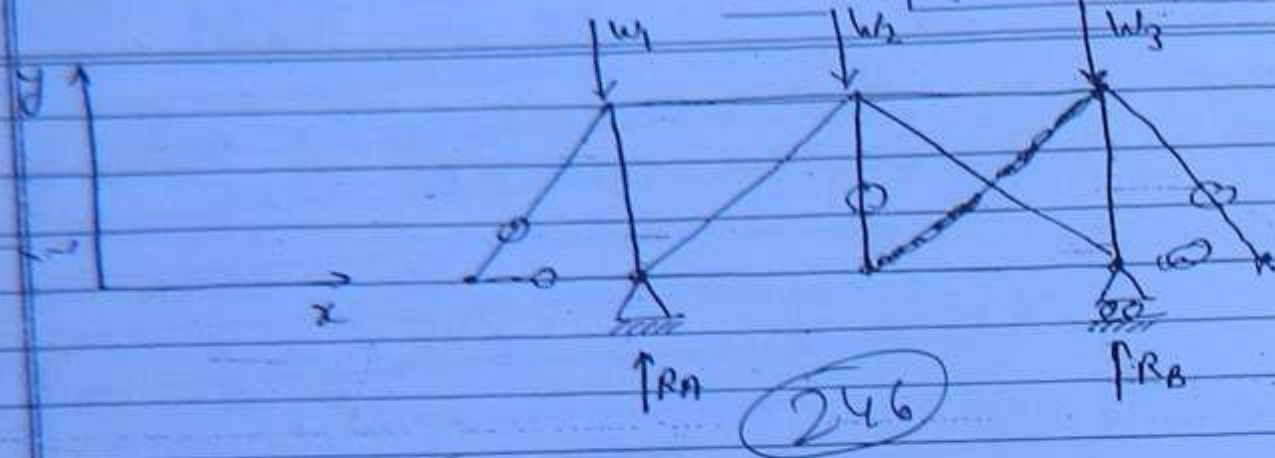
② If At joint only two member are present which are not co-linear and there is no external load or reaction at the joint then both members will carry zero force.



$$\theta \neq 0$$
$$\theta \neq 180$$

$$\sum F_y = 0$$
$$+ F_2 \sin \theta = 0$$
$$F_2 = 0$$

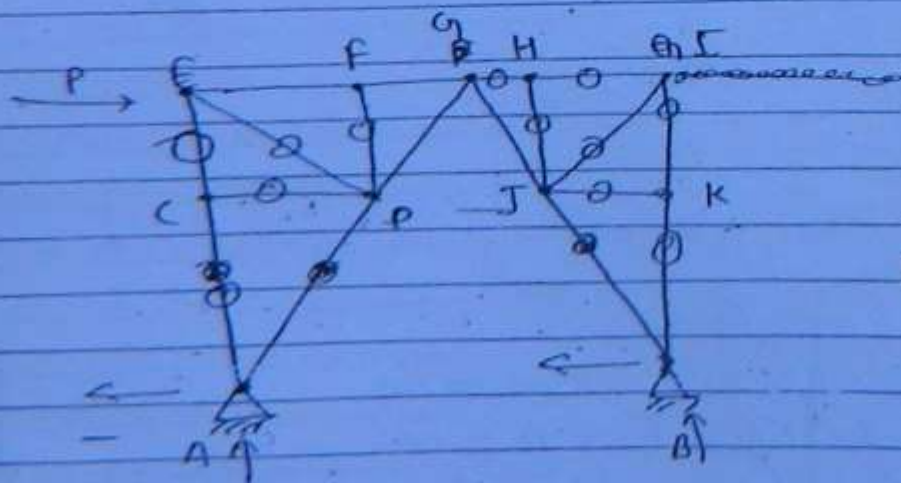
$$\sum F_x = 0$$
$$F_1 + F_2 \cos \theta = 0$$
$$F_1 + 0 = 0$$
$$F_1 = 0$$



Note - If sum of the members in a Truss carry zero force at a given loading than such member should not be removed. Because under change loading condition such members may carry load. ~~for~~ moreover a members are remove ~~and~~ may become (< 0) less than 0. Hence ~~the~~ geometry of the truss will not be preserved under general loading condition.

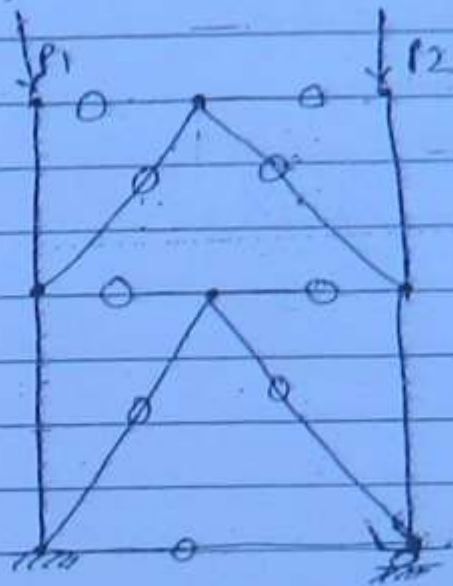
Q. In this truss shown in fig. Identify the no. of member which carry zero forces.

- (a) 8 (b) 10 (c) 12 (d) 14



Ans = 12

Q Identify the zero force member in the frame. Shown in fig.



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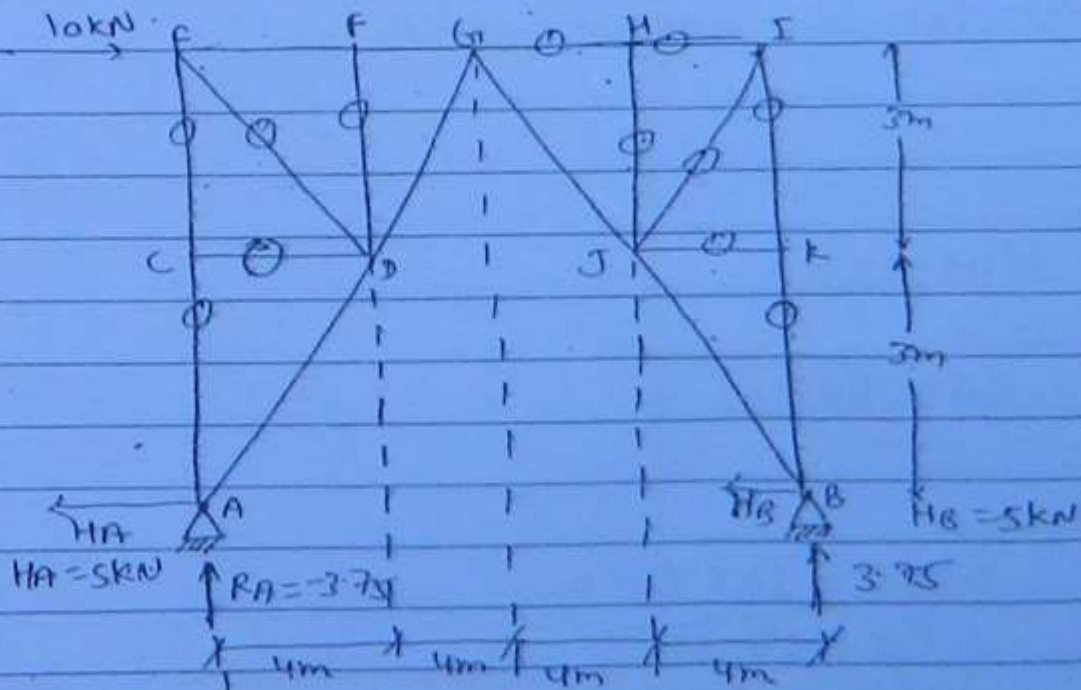
(A) 2

(B) 2.8

(C) 9

(d) None of these

Q Analyze the truss given below using method of joint and find forces in all member.



$$r_c = 4$$

$$D_{sc} = r_c - 3 \Rightarrow 4 - 3 = 1$$

$$D_{si} = m - (2j - 3)$$

$$\Rightarrow 18 - (2 \times 11 - 3)$$

$$\Rightarrow 18 - 19$$

$$\Rightarrow -1$$

248

$$D_s = D_{sc} + D_{si}$$

$$\Rightarrow 1 - 1 \Rightarrow 0$$

$$\sum F_x = 0$$

$$\sum F_y = 0, \sum MA = 0$$

$$\boxed{\sum M_{O(1)} = 0}$$

→ in this case all member connected at (1)
Hence, frame can be cut in 2 part and
each part can be rotated about (1).

$$\sum F_x = 0$$

$$H_A + H_B = 0$$

$$\sum F_y = 0$$

$$R_A + R_B = 0$$

$$\sum MA = 0$$

$$R_B \times 16 - 10 \times 8 = 0$$

$$\boxed{R_B = +3.75}$$

$$R_A = -3.75$$

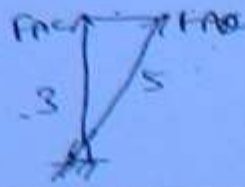
$$\sum M_G = 0$$

$$R_B \times 8 - H_B \times 6 = 0$$

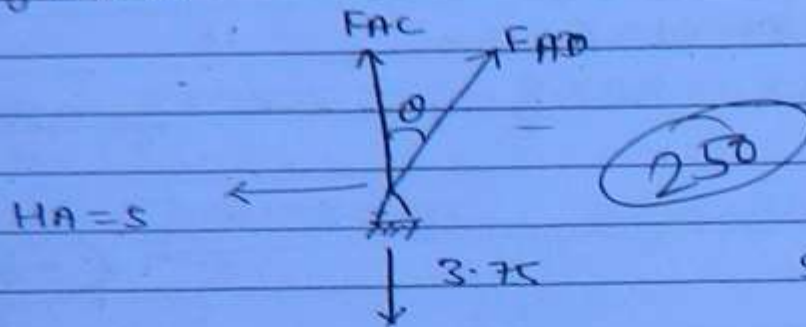
$$H_B = \frac{R_B \times 8}{6} \Rightarrow \frac{3.75 \times 8}{6} = 5 \text{ kN}$$

$$H_A = 5 \text{ kN}, H_B = 5 \text{ kN} \quad (249)$$

member	Force	+ (T) - (C)
AC	0	
AD	6.25	
CD	0	
CE	0	
DE	0	
DF	0	
DG	6.25	
EF	-10	
FG	-10	
GH	0	
HI	0	
GJ	-6.25	
HJ	0	
IJ	0	
JK	0	
IK	0	
JB	-6.25	
KB	0	



Consider joint A



$$\sin \theta = \frac{L}{K} = \frac{4}{5}$$

$$\sum F_x = 0$$

$$F_{AD} \sin \theta - 5 = 0$$

$$F_{AD} = \frac{5}{\sin \theta} = \frac{5}{4/5} = \frac{25}{4}$$

$$F_{AD} = 6.25 (+)$$

$$\sum F_y = 0$$

$$F_{AC} + F_{AD} \cos \theta - 3.75 = 0$$

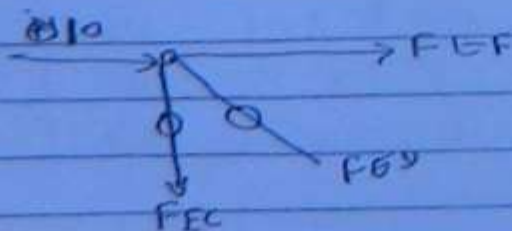
$$F_{AC} = 3.75 - F_{AD} \cos \theta$$

$$= 3.75 - 6.25 \cdot \frac{3}{5}$$

$$F_{AC} = 3.75 - 3.75 = 0$$

Consider joint D At D member AD & DB are co-linear and other member share zero forces. Hence both co-linear force. AD & DB will have equal forces

Joint E



$$\sum F_x = 0$$

$$F_{EF} + 10 + F_{CD} \cos 60 = 0$$

$$F_{EF} + 10 + 0 = 0$$

$$F_{EF} = -10$$

25/1

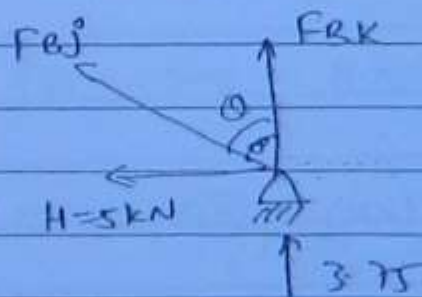
$$\sum F_y = 0$$

$$F_{EC}$$

At joint F

$$F_{EF} = F_{FE}$$

Consider joint B



$$\sum F_x = 0$$

$$F_{BK} + F_{BJ} \cos 60 + 3.75 = 0$$

$$-5 - F_{BJ} \sin 60 = 0$$

$$F_{BJ} = \frac{-5}{\sin 60} = -6.25$$

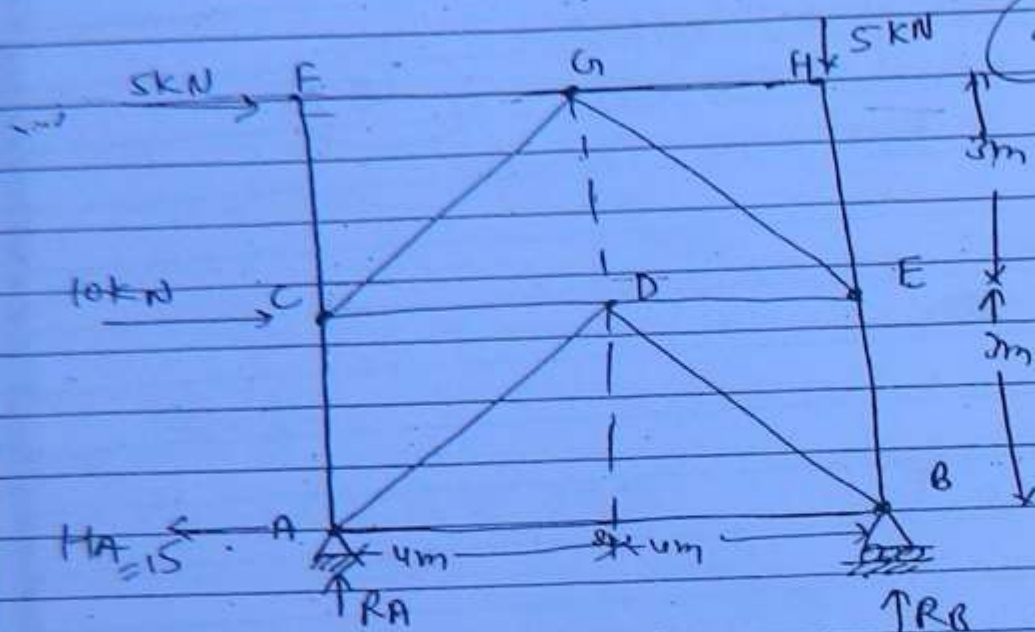
$$\sum F_y = 0$$

$$3.75 + F_{BK} + F_{BJ} \cos 60 = 0$$

$$3.75 + F_{BK} + (-6.25) \times \frac{1}{2} = 0$$

$$F_{BK} = 0$$

Q → Find force in all members of truss



$$m = 13 \quad r_e = 3$$

$$DS = m + r_e - 2j$$

$$\Rightarrow 13 + 3 - 2 \times 8$$

$$\Rightarrow 0$$

Serial No	Member	Force (kN) (+T -C)
1	AB	7.5
2	AC	+1.875
3	AD	+9.375
4	CD	-12.5
5	CG	+3.125
6	CF	0
7	DE	2.5
8	BD	-9.375
9	BE	-6.875
10	FG	-5
11	GH	0
12	EG	-3.125
13	HE	-5

$$\sum F_x = 0$$

$$5 + 10 - H_A = 0$$

$$H_A = 15$$

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$$\sum F_y = 0$$

$$R_A + R_B = 5$$

~~R_A~~

$$\sum M_A = 0$$

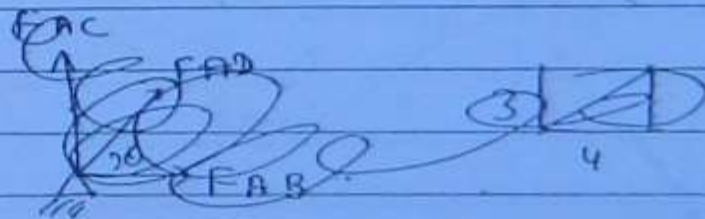
$$- R_B \times 8 + 5 \times 8 + 10 \times 3 + 5 \times 6 = 0$$

$$R_B = \frac{-40 + 30 + 30}{8}$$

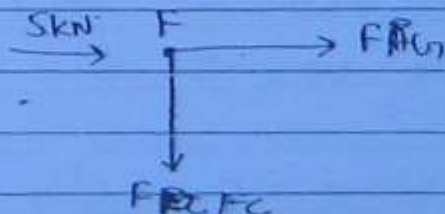
$$R_B = \frac{+20}{8} = +2.5$$

$$R_A = -7.5$$

At the consider member (A)



Consider member F



$$\sum F_x = 0$$

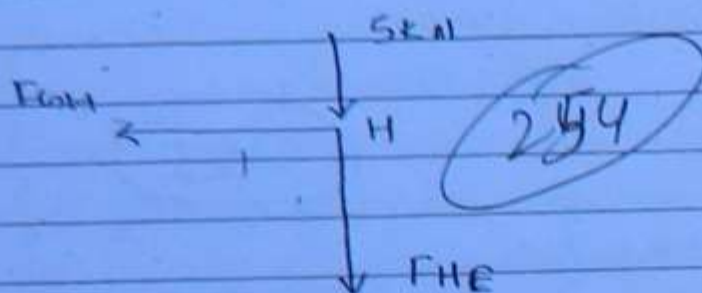
$$F_{FB} + 5 = 0$$

$$F_{FB} = -5$$

$$\sum F_y$$

$$F_{FC} = 0$$

③ Consider Joint H



$$\sum F_x = 0$$

$$F_{AH} = 0$$

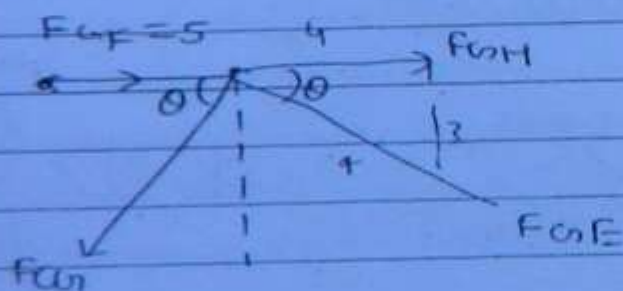
0

$$\sum F_y = 0$$

$$-F_{AH} - 5 = 0$$

$$F_{HE} = -5$$

Consider Joint G



$$\sum F_x = 0$$

$$F_{GN} \cos \theta + 5 - F_{GE} \cos \theta = 0 \quad \text{--- (i)}$$

$$F_{GN} \cos \theta - F_{GE} \cos \theta = -5$$

$$\sum F_y = 0$$

$$-F_{GN} \sin \theta - F_{GE} \sin \theta = 0 \quad \text{--- (ii)}$$

$$F_{GN} = -F_{GE}$$

-2

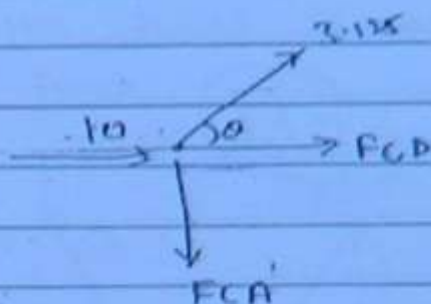
$$F_{GN} \cos \theta + 5 - F_{GN} \cos \theta = 0$$

$$-F_{GN} \cos \theta + 5 - F_{GN} \cos \theta = 0$$

$$-2F_{GN} \cos \theta = -5$$

$$+2F_{GN} \times \frac{4}{5} = 5$$

$$F_{GN} = \frac{5 \times 5}{8} = \frac{25}{8} = 3.125$$

Joint C

253

$$\sum F_y = 0$$

$$3.125 \sin 50^\circ - F_{CA} = 0$$

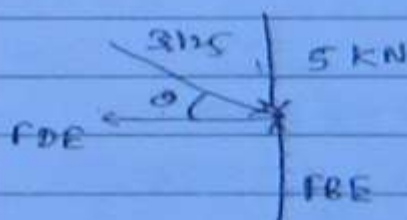
$$F_{CA} = 3.125 \times \frac{3}{5}$$

$$F_{CA} = +1.875$$

$$\sum F_x = 0$$

$$F_{CD} + 3.125 \cos 50^\circ + 10 = 0$$

$$F_{CD} = -12.5$$

Consider joint E

$$\sum F_y = 0$$

$$F_{DE} - 3.125 \cos 50^\circ = 0$$

$$F_{DE} = 3.125 \times \frac{4}{5}$$

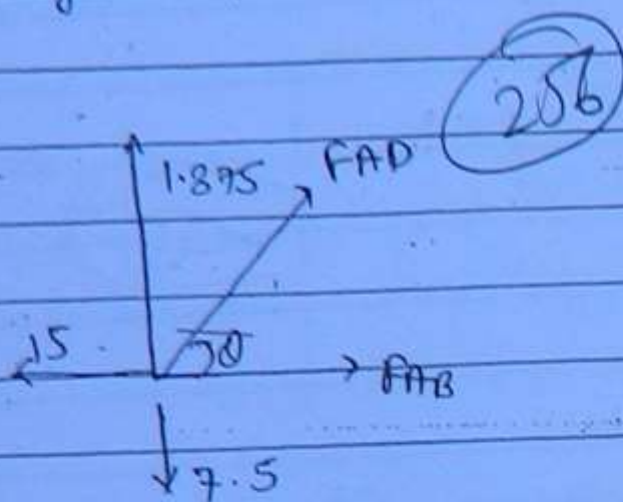
$$F_{DE} = 2.5$$

$$\sum F_y = 0 \quad - F_{BE} - 3.125 \sin 50^\circ = 0$$

$$F_{BE} = -6.875$$

⑤

Consider joint A



$$\sum F_y = 0$$

16

$$+ 1.875 + F_{AD} \sin 50^\circ - 7.5 = 0$$

$$1.875 + F_{AD} \sin 50^\circ - 7.5 = 0$$

$$1.875 + F_{AD} \times \frac{3}{5} - 7.5 = 0$$

$$F_{AD} \Rightarrow +9.375$$

$$\sum F_x = 0$$

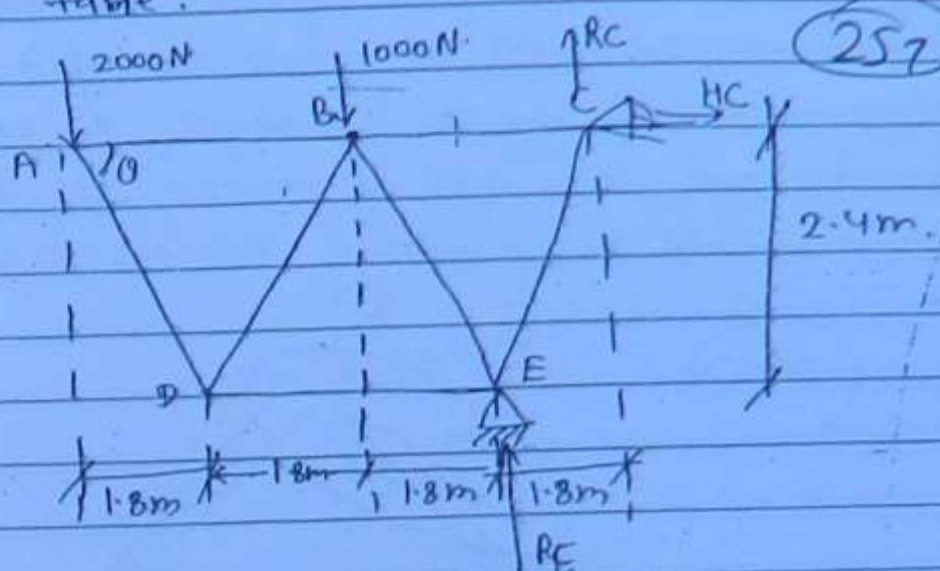
 $\cos 50^\circ$
~~sin 50~~

$$-15 + F_{AB} + F_{AD} \cos 50^\circ = 0$$

$$F_{AB} \Rightarrow 15 - 9.375 \times \frac{4}{5}$$

$$F_{AB} \Rightarrow +7.5$$

Q Analyse the truss shown in Fig. & list forces in the table.



$$\sum F_x = 0$$

$$H_c = 0$$

$$D_s = 0 \text{ (perfect frame)}$$

$$\sum F_y = 0$$

$$R_E + R_C = 3000 \text{ N}$$

$$\sum M_c = 0$$

$$R_E \times 7.2 - 2000 \times 7.2 - 1000 \times 3.6 = 0$$

$$R_E = \frac{14400 + 3600}{4}$$

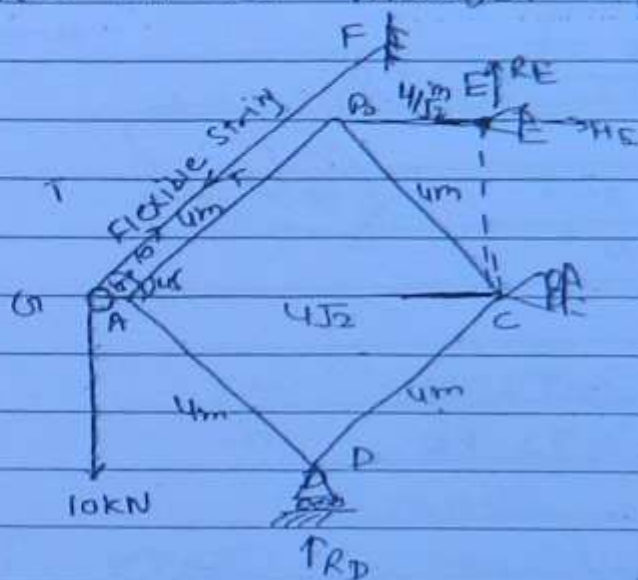
$$R_E = 4000 \text{ N}$$

$$R_C = -7000 \text{ N}$$

258

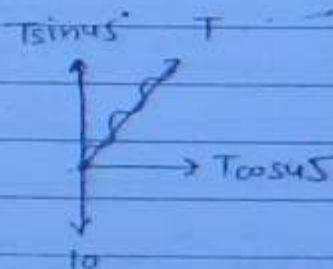
Q-

A Square Truss ABCD Shown in fig. Carry load of 10kN Apply through a spring GF which is passing over a frictionless pulley as shown in fig. GF & AB are parallel. The truss is supported on a roller at C & D. A link BE is shown in fig. is part of truss. find forces in all member of the truss.

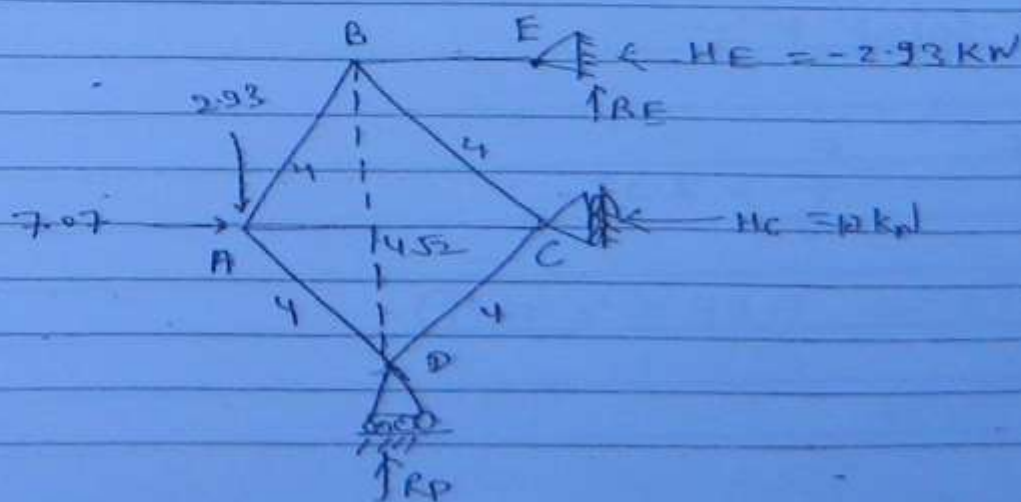


(259)

Net vertical force at G $\rightarrow 10 - \frac{10}{\sqrt{2}}$
 $\Rightarrow 2.93 \text{ kN}$

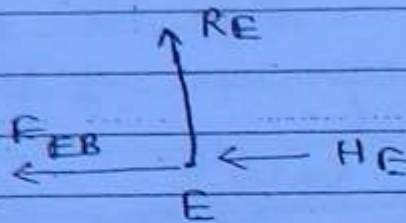


Net Horizontal at G $\rightarrow \frac{10}{\sqrt{2}} = 7.07$



$$\begin{aligned} D_s &= m + 9e - 25 \\ &\Rightarrow 6 \times 4 - 2 \times 5 \\ &\Rightarrow 0 \end{aligned}$$

Joint A+E



$$\sum F_y = 0$$

$$R_E = 0$$

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$$\sum F_D = 0$$

$$R_D + R_E - 2.93 = 0$$

$$R_D = 2.93 \text{ kN}$$

$$\sum F_x = 0$$

$$7.07 - H_C - H_E = 0$$

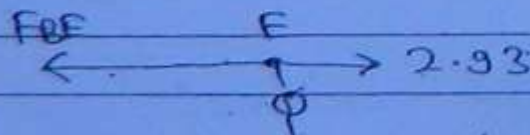
$$H_C + H_E = 7.07 \quad (1)$$

$$\sum M_C = 0$$

$$\frac{R_D \times 4}{\sqrt{2}} - 2.93 \times 4\sqrt{2} - H_E \times \frac{4}{\sqrt{2}} = 0$$

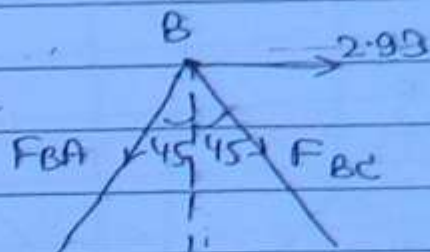
$$| H_E = -2.93 \text{ kN} |$$

Consider joint E's



$$\sum F_x = 0 \quad F_{BE} = 2.93 \text{ (T)}$$

Joint B



(261)

$$\sum F_x = 0$$

$$2.93 + F_{BC} \sin \theta - F_{BA} \sin \theta = 0$$

$$2.93 + \frac{F_{BC}}{\sqrt{2}} - \frac{F_{AB}}{\sqrt{2}} = 0 \quad \text{--- (i)}$$

$$\sum F_y = 0$$

$$-\frac{F_{AB}}{\sqrt{2}} - \frac{F_{BC}}{\sqrt{2}} = 0 \quad \text{--- (ii)}$$

from (i) & (ii)

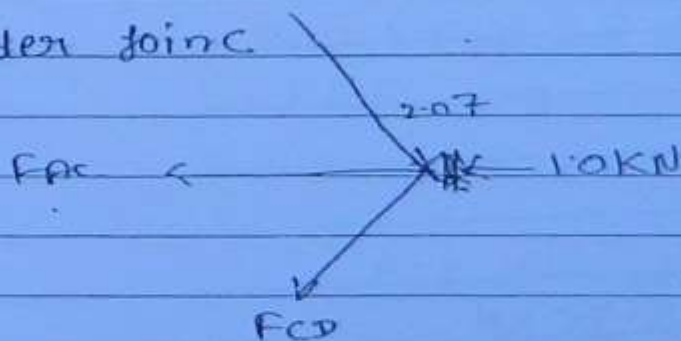
$$F_{AB} = -F_{BC}$$

 ~~$F_{AB} = +2.07$~~

$$F_{AB} = +2.07$$

$$F_{BC} = -2.07$$

③ Consider joint C

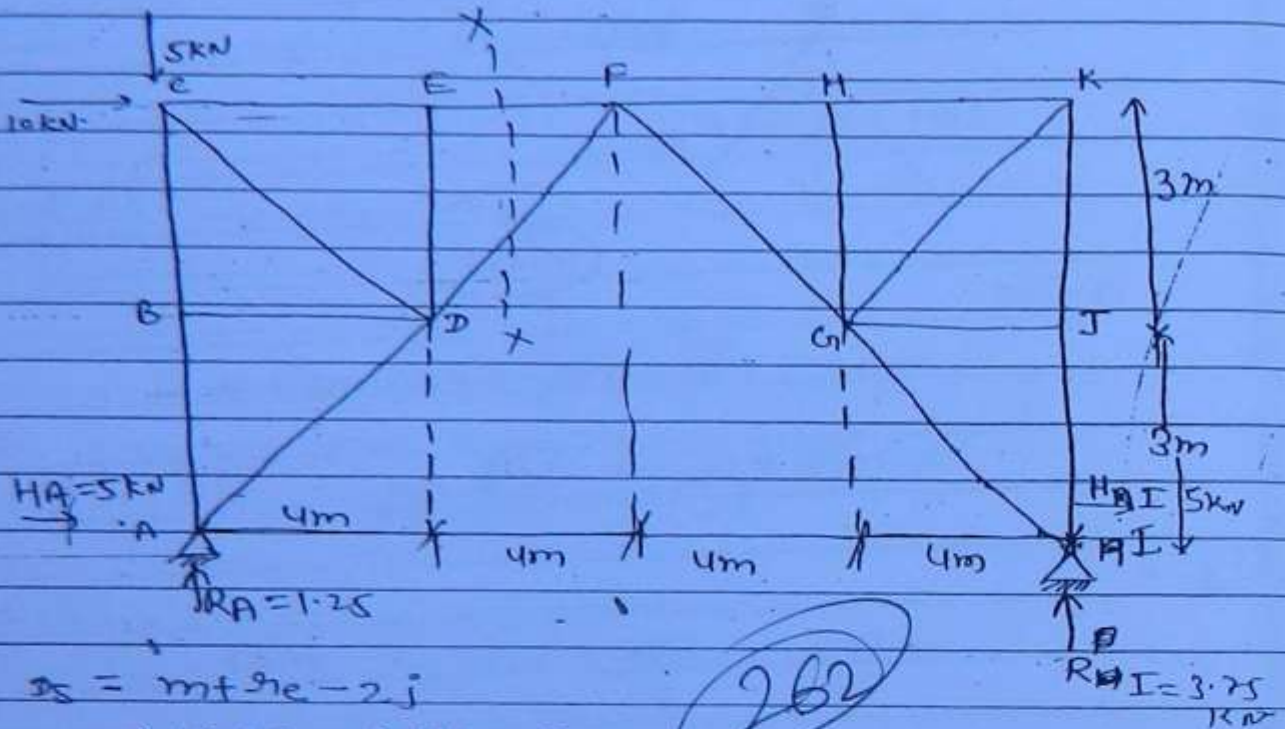


$$F_{CD} = -2.07$$

$$F_{AC} = -7.07$$

(Method of section)

3 Calculate forces in the members EF & DF.



$$\vec{r}_S = m + 9e - 2j$$

$$\Rightarrow 18 + 4 - 2 \times 11 = 0$$

$$\sum F_x = 0 \quad H_A + H_I = 10$$

$$\sum F_y = 0 \quad R_A + R_I = 5$$

$$\sum M_A = 0$$

$$-R_I \times 16 + 10 \times 6 = 0$$

$$R_I = \frac{60}{16} = 3.75 \text{ kN}$$

$$R_A = 1.25 \text{ kN}$$

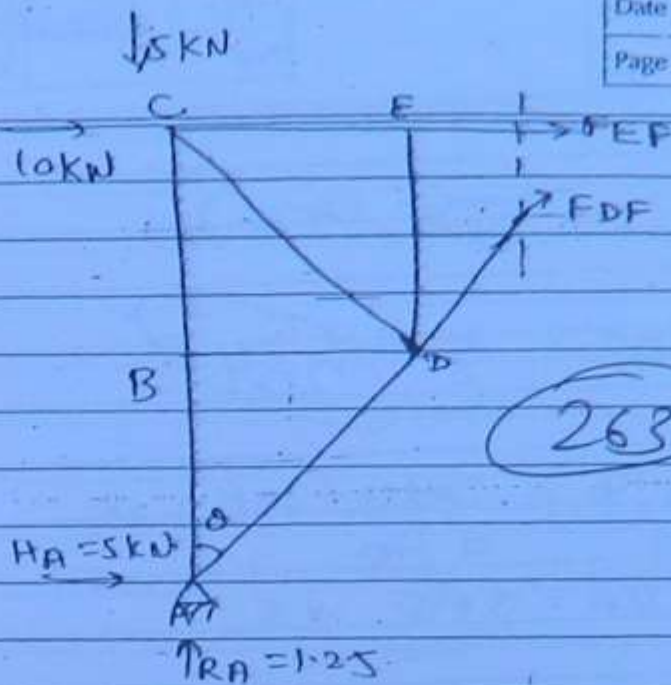
$$\left\{ \begin{array}{l} R_I = 3.75 \\ R_A = 1.25 \end{array} \right.$$

$$\sum M_F = 0$$

$$R_I \times 8 - H_I \times 6 = 0$$

$$H_I = \frac{3.75 \times 8}{6}$$

$$\left\{ \begin{array}{l} H_I = 5 \\ H_A = 5 \end{array} \right.$$



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$$\sum F_y = 0 \quad R_A - 5 + F_{DF} \cos 50 = 0$$

$$1.25 - 5 + F_{DF} \times \frac{3}{5} = 0$$

$$F_{DF} = +6.25 \text{ kN}$$

Ex 2.20

$$\sum F_x = 0$$

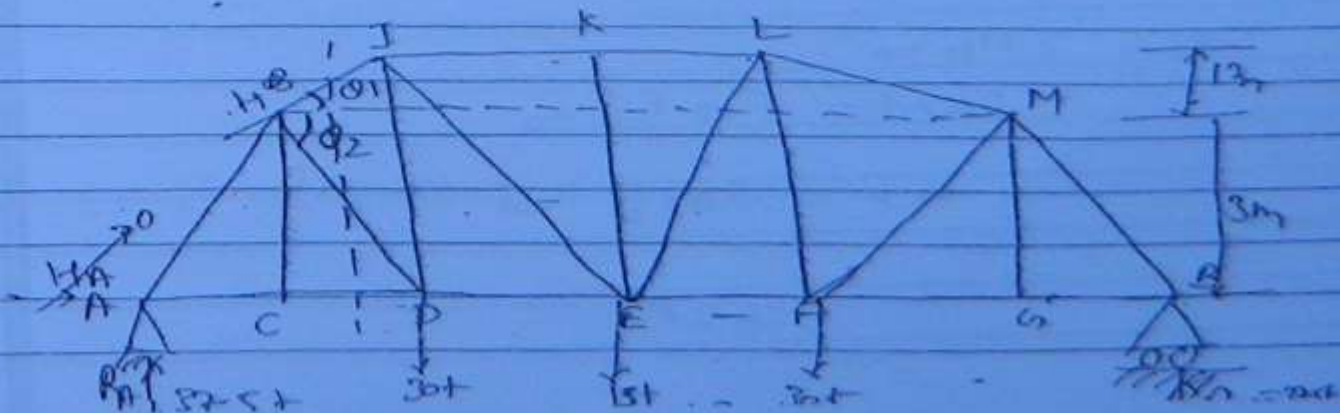
$$F_{EF} + F_{DF} \sin 50 + 10 - 5 = 0$$

$$F_{EF} = 5 - 10 - F_{DF} \cdot \sin 50$$

$$\Rightarrow -5 - 6.25 \times \frac{4}{5}$$

$$F_{EF} = -10$$

Find forces in member HD and HT.



$$DS = m + 9e - 2J$$

→ (All are triangular Block)

$$DS = 0$$

$$\sum F_x = 0$$

$$H_A = 0$$

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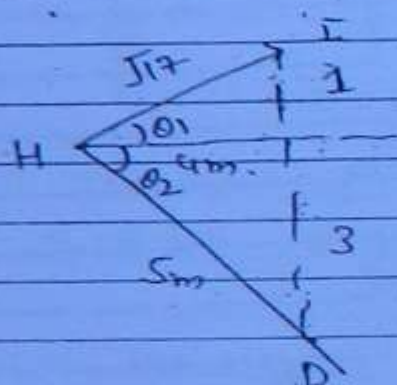
$$\sum F_y = 0$$

$$R_A + R_B = 30 + 10 + 30$$

$$R_A = R_B$$

$$2R_A = 70$$

$$R_A = 37.5 = R_B$$

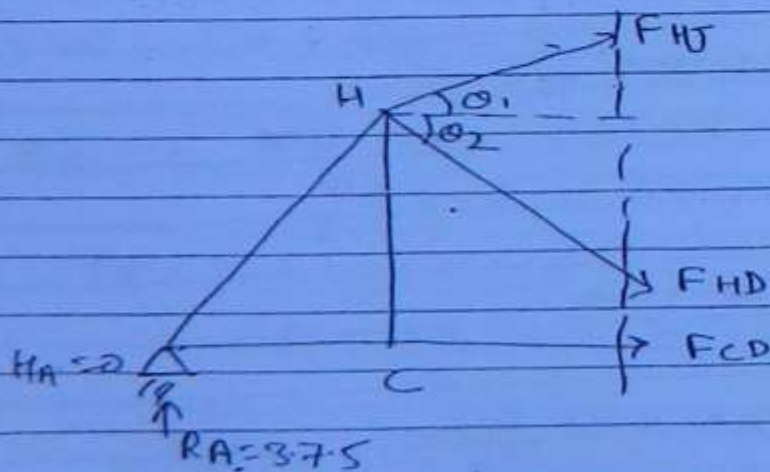


$$\sin \theta_1 = \frac{1}{5}$$

$$\cos \theta_1 = \frac{4}{5}$$

$$\sin \theta_2 = \frac{3}{5}$$

$$\cos \theta_2 = \frac{4}{5}$$



Taking $\sum M_H = 0$

$$R_A \times 4 - F_{CD} \times 3 = 0$$

$$F_{CD} = R_A \times \frac{4}{3} = 37.5 \times \frac{4}{3}$$

$$F_{CD} = 150$$

$$\sum F_x = 0$$

$$F_{CD} + F_{HD} \cos \theta_2 + F_{HJ} \cos \theta_1 = 0$$

$$50 + F_{HD} \frac{4}{5} + F_{HJ} \frac{4}{\sqrt{17}} = 0 \quad \text{--- (1)}$$

(0.8) (0.9701)

$$\sum F_y = 0$$

$$R_A + F_{HJ} \sin \theta_1 - F_{HD} \sin \theta_2 = 0$$

$$37.5 + F_{HJ} \times \frac{1}{\sqrt{17}} - F_{HD} \times \frac{3}{5} = 0 \quad \text{--- (2)}$$

(0.2427) 0.6

From eqn (1) & (2)

$$F_{HD} = 31.25 \text{ t}$$

$$F_{HJ} = -77.30 \text{ t}$$

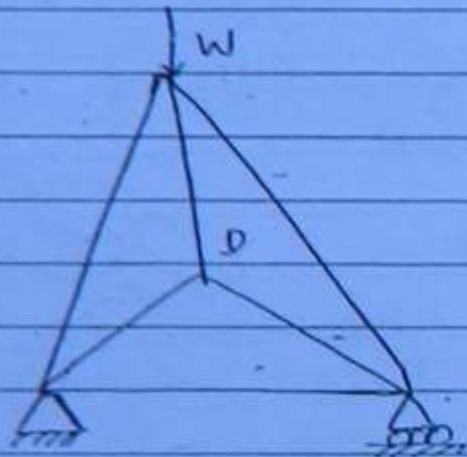
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Analysis of indeterminate truss

$$D_s > 0$$

= 1 or higher degree.

→ Force method / unit load method / strain energy method /
use of Castigliano's theorems.



$$D_s = m + r_e - 2j$$

$$\Rightarrow 6 + 3 - 2 \times 4 \Rightarrow 9 - 8 = 1$$

$$D_{se} = 0$$

$$D_{si} = m - (2j - 3)$$

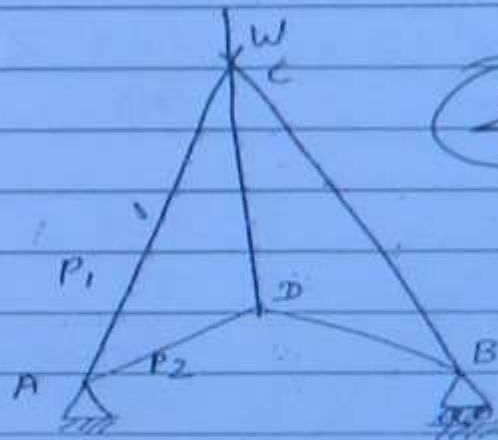
$$\Rightarrow 6 - (2 \times 4 - 3)$$

$$\Rightarrow 1$$

P-1 Find degree of static indeterminacy and check D_{se} & D_{si} , if degree of indet. is '1' it means there is one redundant reaction / force. identify the redundant is above. case redundant is internal force / force member / member force.

Let Redundant force is force in member AB.

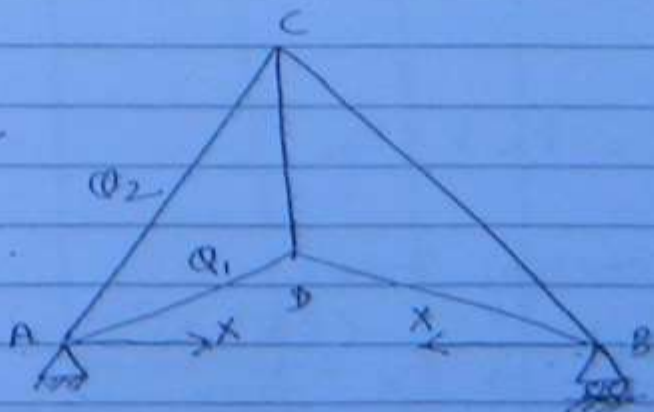
Step-2 due to given load force in truss member are $S_1, S_2, \dots, S_n$ which are required to compute. If redundant member is remove the rest of the truss or truss will be determinate & stable. Due to given loading in such determinate truss after removal of redundant, let force in member are $P_1, P_2, \dots, P_n$ which can be found by method of joint, method of section.



force in $AB = 0$

Step-3

Remove loading and assume the value of redundant force is 'X' since force in AB is 'X' hence apply 'X' force in A & B in the direction of AB .



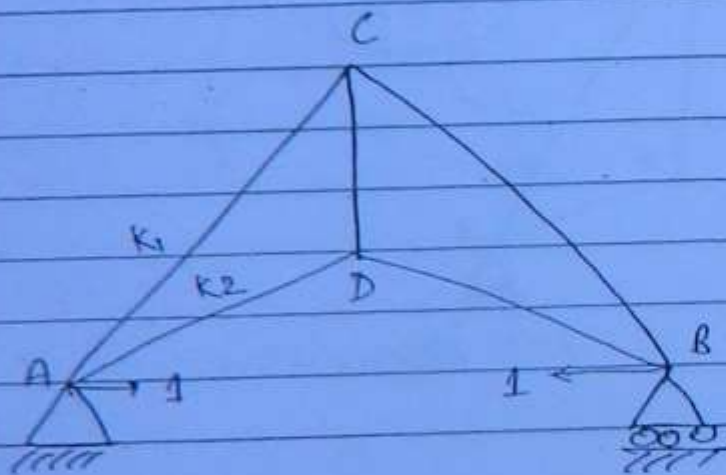
let due to X force in AB force developed in other member are $Q_1, Q_2, \dots, Q_n$

$$Q_{AB} = X$$

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Step-4

if ' X ' is taking common and unit load is applied, at $A-B$ in the direction of AB , then forces in member are $K_1, K_2, K_3, \dots, K_n$ then $Q = KX$



$$Q = KX$$

Step-5

The final force in each member $S_i = P_i + Q_i$

$$S_1 = P_1 + X \cdot K_1$$

$$S_2 = P_2 + X \cdot K_2$$

$$\vdots$$

$$S_n = P_n + X \cdot K_n$$

the total strain energy stored -

$$U = \sum \frac{S^2 L}{2AE} \Rightarrow \sum \frac{(P + K \cdot X)^2 \cdot L}{2AE}$$

- ① Check of Ds and identify Redundent
- ② Remove the Redundent find P force system due to given loading
- ③ Remove the loading and applied unit load in the direction of redundant And find K force system in all the member.
- ④ find value of Redundent force x

$$X = \frac{- \sum \frac{PKL}{AE}}{\sum \frac{K^2 L}{AE}}$$

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- ⑤ Find Final force in the member

$$[S = P + K \cdot X]$$

Special case →

→ if $D_s = 2$ means there are two redundant say Redundent force are $(X_1 \& X_2)$ Remove both the Redundants and find P force system in the determinate truss

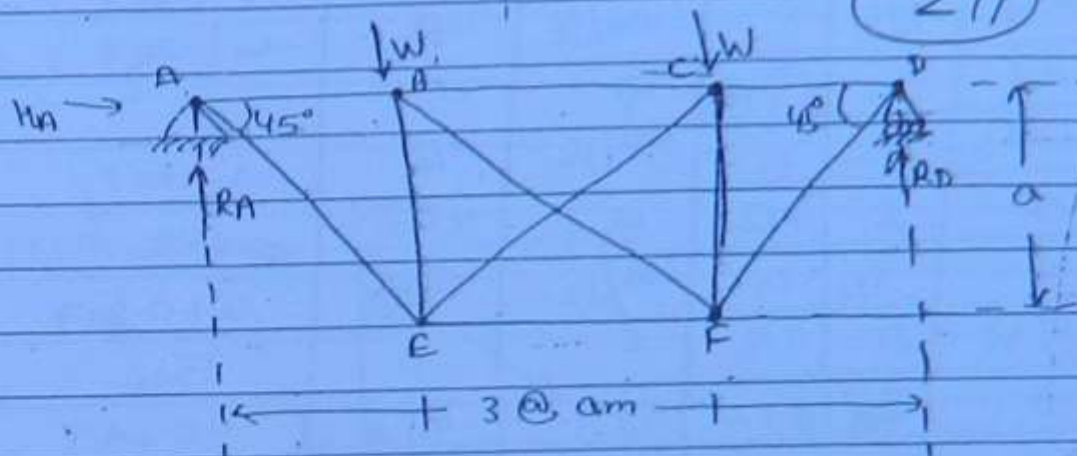
→ Apply Unit load in the direction of first Redundent and find K force system in all member.

→ Now Apply unit load in the direction of Second Redundent and find K' system of force in All members.

$\frac{\partial U}{\partial X_1} = 0$	$X_1 = \frac{- \sum \frac{PK_1 L}{AE}}{\sum \frac{K_1^2 L}{AE}}$	$X_2 = \frac{- \sum \frac{PK'_1 L}{AE}}{\sum \frac{K'^2_1 L}{AE}}$	$\frac{\partial U}{\partial X_2} = 0$
---------------------------------------	--	--	---------------------------------------

Q20

Analyse the truss shown in below and find force in all member if AE is axial rigidity. $\frac{AE}{L}$ = Axial stiffness.



$$D_s = m + r - 2j$$

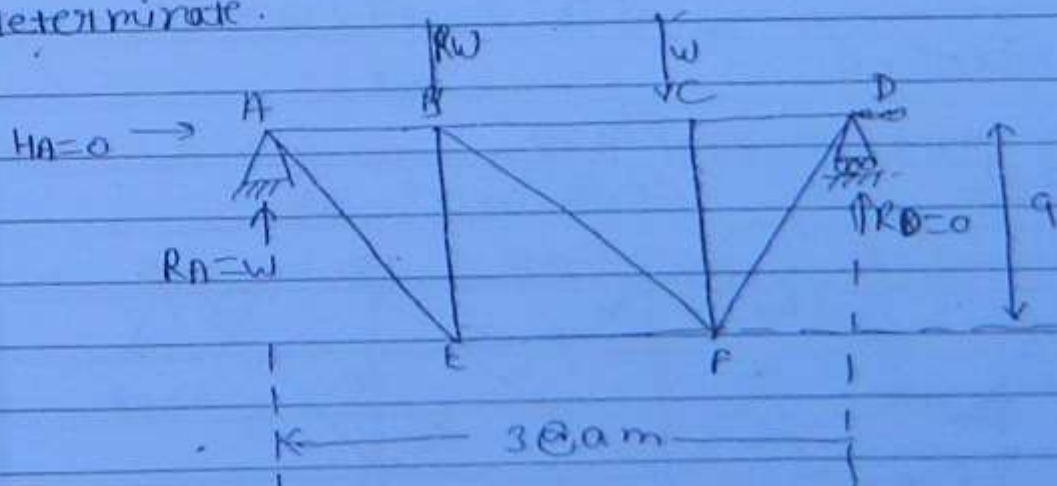
$$= 10 + 3 - 2 \times 6$$

$$D_s = 13 - 12 = 1$$

$$D_{sc} = 0$$

$$D_{si} = 1$$

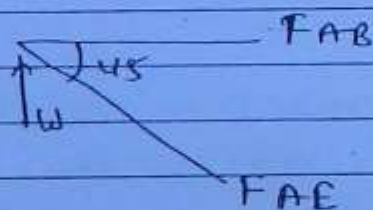
Let the Redundent member is ~~BE~~ EC. If Redundent is ~~Remove~~ remove, then rest of the truss will be determinate.



	member	P	K	L	$P \cdot K \cdot L$	$K^2 \cdot L$	$S = P + K \cdot X$
1	AB	-W	0	a	0	0	-W
2	AE	$\sqrt{2}W$	0	$a\sqrt{2}$	0	0	$2\sqrt{2}W$
3	BC	-W	$1/\sqrt{2}$	a	$W a / \sqrt{2}$	$a/\sqrt{2}$	$-W + 0.207$
4	BE	-W	$-1/\sqrt{2}$	a	$W a / \sqrt{2}$	$a/\sqrt{2}$	$-W + 0.207$
5	CD	-W	0	a	0	0	-W
6	CF	-W	$-1/\sqrt{2}$	a	$W a / \sqrt{2}$	$a/\sqrt{2}$	$-W + 0.207$
7	EC	0	1	$a\sqrt{2}$	0	$a/\sqrt{2}$	0.207
8	EF	W	$-1/\sqrt{2}$	a	$W a / \sqrt{2}$	$a/\sqrt{2}$	$W + 0.207$
9	FD	$\sqrt{2}W$	0	$a\sqrt{2}$	0	0	$\sqrt{2}W$
10	BF	0	$1/\sqrt{2}$	$a\sqrt{2}$	0	$a/\sqrt{2}$	-0.207
$\Sigma P \cdot K \cdot L$					$\sqrt{2}W a$	$2a(\sqrt{2}+1)$	
$\Sigma K^2 \cdot L$							

①

Consider joint A



$$\Sigma F_x = 0$$

$$F_{AB} = 0$$

$$F_{AB} + F_{AE} \cos 45^\circ = 0$$

$$F_{AB} = -\sqrt{2}W \frac{1}{\sqrt{2}}$$

$$F_{AB} = -W$$

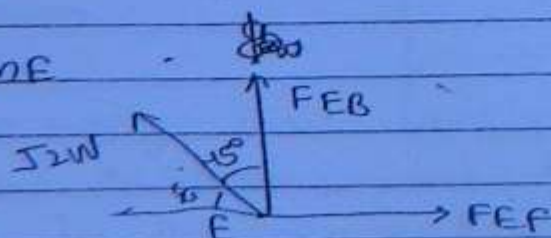
$$\Sigma F_y = 0$$

$$F_{AE} \sin 45^\circ = W$$

$$F_{AE} = \sqrt{2}W$$

②

Consider joint E



$$\Sigma F_y = 0$$

$$F_{EB} + \sqrt{2}W \cos 45^\circ = 0$$

$$F_{EB} = -\sqrt{2}W \times \frac{1}{\sqrt{2}} \Rightarrow -W$$

~~Ref~~ $\sum F_x = 0$

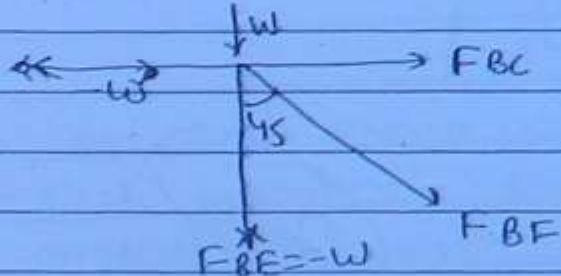
$$F_{EF} + \cancel{P} J_2 W \sin 45^\circ = 0$$

$$F_{EF} = W$$

②

Joint B

(223)



$$\sum F_y = 0$$

$$F_{BD} + \cancel{PBF} \cos 45^\circ$$

$$-W + W + PBF \cos 45^\circ = 0$$

$$PBF = 0$$

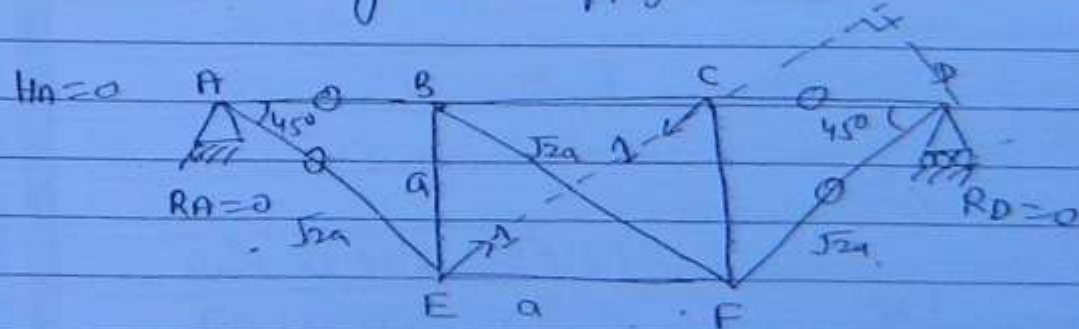
$$\sum F_x = 0$$

$$-W + F_{BC} + F_{BF} \cos 45^\circ = 0$$

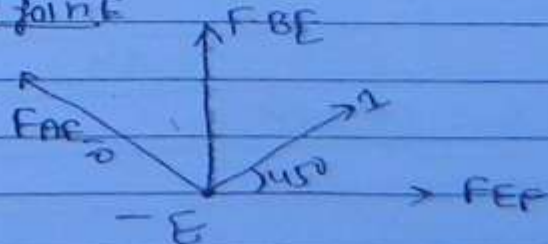
$$F_{BC} = -W$$

~~Ref~~

Remove loading and apply unit load at EDC



Consider joint E



$$\sum F_y = 0$$

$$F_{BE} + 1 \sin 45 = 0$$

$$\boxed{F_{BE} = -\frac{1}{\sqrt{2}}}$$

$$\sum F_x = 0$$

$$F_{EF} + 1 \sin 45 = 0$$

$$\boxed{F_{EF} = -\frac{1}{\sqrt{2}}}$$

Consider point C

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$$\sum F_y = 0$$

$$+F_{CF} - 1 \sin 45 = 0$$

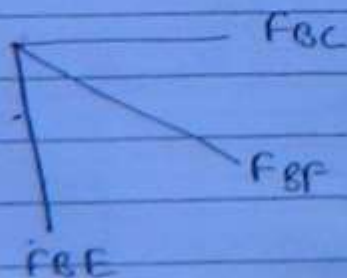
$$\boxed{F_{CF} = \frac{1}{\sqrt{2}}}$$

$$\sum F_x = 0$$

$$F_{BC} - 1 \cos 45 = 0$$

$$\boxed{F_{BC} = \frac{1}{\sqrt{2}}}$$

Consider point B



$$\sum F_x = 0$$

$$F_{BF} = 1$$

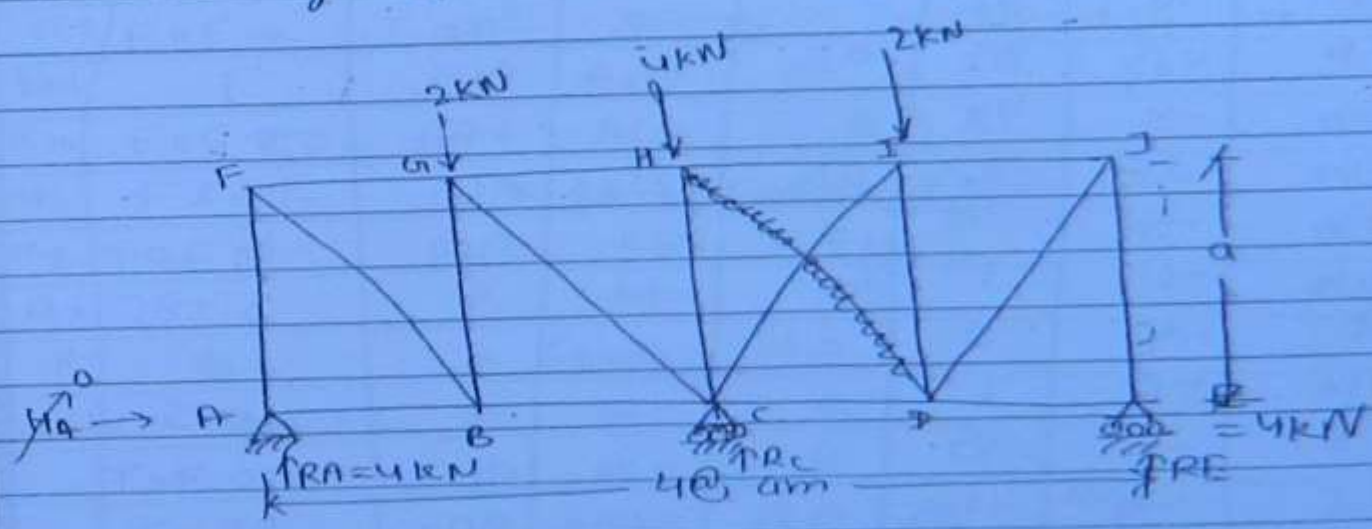
$$X = \frac{-\sum \frac{PKL}{AE}}{\sum \frac{k^2 L}{AE}}$$

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$$X = \frac{-\frac{5\sqrt{2}W}{AE}}{\frac{5\sqrt{2}A(\sqrt{2}+1)}{AE}} \Rightarrow \frac{-W\sqrt{2}}{\sqrt{2}(\sqrt{2}+1)}$$

$$X \Rightarrow 0.293 W$$

Q Analyze the truss shown in Fig and find the force in each member, the diagonal member makes angle of 45° with AE (constant).



$$m = 4$$

$$D_{se} = m - 3 \Rightarrow 1$$

$$D_{si} = m - (2J - 3) \\ \Rightarrow 4 - (2 \times 10 - 3) \\ = 0$$

$$\cancel{R_A + R_C + R_E = 9 + 4 + 2} \\ \cancel{R_A + R_C + R_E = 8}$$

$$\sum M_A = 0$$

$$R_E \times 4a - 2 \times 3a - 4 \times 2a - 2 \times a = 0$$

since R_E is External, Hence External.
Reaction is indeterminate

(R_E is indeterminate)

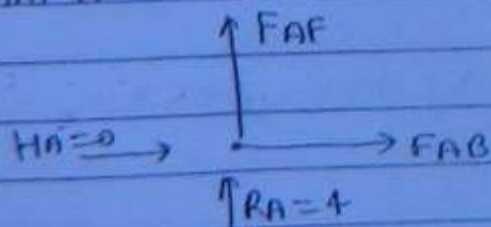
276

→ find a force system, remove Redundent (Say R_E)
 $R_A = R_E = 4$

S-NO	member	P	K	L	$P \times L$	$K^2 L$	$S = P + KX$	
1	AB	0	0	a	0	0	0	0
2	AF	-4	$1/2$	a	-2a	$1/4 a$	-0.707	0.72
3	BC	4	$-1/2$	a	-2a	$a/4$	0.707	0.72
4	BF	$4\sqrt{2}$	$1/\sqrt{2}$	$a\sqrt{2}$	$-4\sqrt{2}a$	$a/\sqrt{2}$	1	1.018
5	BG	-4	$1/2$	a	-2a	$+a/4$	-0.707	0.72
6	CG	$2\sqrt{2}$	$-1/\sqrt{2}$	$a\sqrt{2}$	$-2\sqrt{2}a$	$a/\sqrt{2}$	-1.827	-1.837
7	FG	-4	$1/2$	a	-2a	$a/4$	-0.707	0.72
8	GH	-6	1	a	-6a	a	0.585	0.51
9	HC	-4	0	a	0	0	-4	-4
10	IH	-6	1	a	-6a	a	0.585	
11	JI	-4	$1/2$	a	-2a	$a/4$	-0.707	
12	CI	$2\sqrt{2}$	$-1/\sqrt{2}$	$a\sqrt{2}$	$-2\sqrt{2}a$	$a/\sqrt{2}$	-1.827	
13	DI	-4	$1/2$	a	-2a	$+a/4$	-0.707	
14	DJ	$-4\sqrt{2}$	$1/\sqrt{2}$	$a\sqrt{2}$	$-4\sqrt{2}a$	$a/\sqrt{2}$	1	
15	DC	4	$-1/2$	a	-2a	$a/4$	0.707	
16	EJ	-4	$1/2$	a	-2a	$1/4 a$	-0.707	
17	ED	0	0	a	0	0	0	
EPIKL =					-44.97	6.828a		

$ERL =$

Consider joint A



$$\sum F_x = 0$$

$$F_{AB} = 0$$

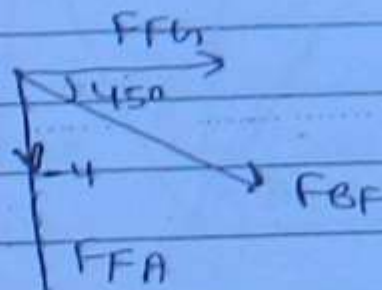
$$\sum F_y = 0$$

$$F_{AF} + 4 = 0$$

$$F_{AF} = -4$$

277

Consider Point F



$$\sum F_y = 0$$

$$F_{BF} \sin 45 = 4 \Rightarrow$$

$$F_{BF} = 4 \times \sqrt{2}$$

$$F_{BF} = 4\sqrt{2}$$

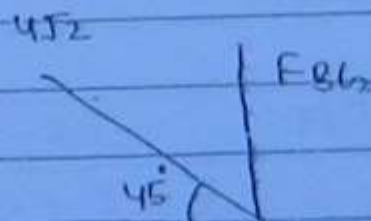
$$\sum F_x = 0$$

$$F_{GF} + F_{BF} \cos 45 = 0$$

$$F_{GF} + 4\sqrt{2} \times \frac{1}{\sqrt{2}} = 0$$

$$F_{GF} = -4$$

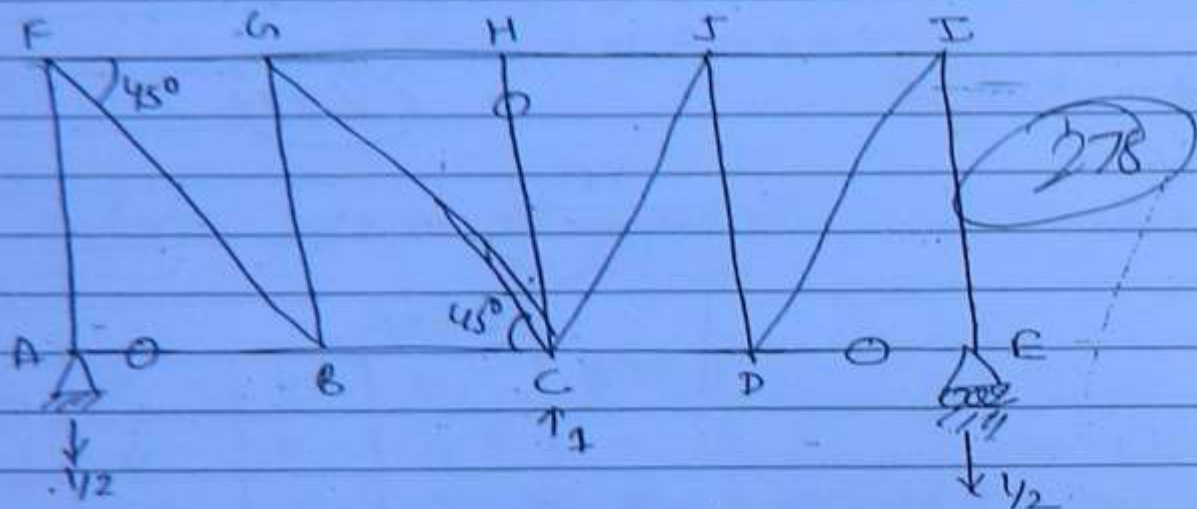
Consider point B



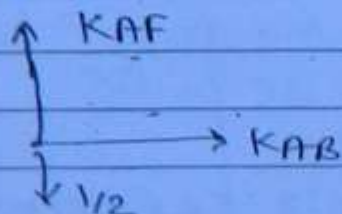
$$\sum F_y = 0 \quad F_{BG} + 4\sqrt{2} \sin 45 = 0$$

$$F_{BG} = -4$$

→ For K system of force apply unit load at C a unit load



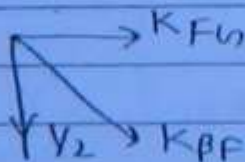
Consider joint A



$$K_{AB} = 0$$

$$K_{AF} = 1/2$$

Consider joint F



$$\sum F_y = 0$$

$$-K_{BF} \frac{1}{\sqrt{2}} - \frac{1}{2} = 0$$

$$K_{BF} = -\frac{1}{\sqrt{2}}$$

$$\sum F_x = 0$$

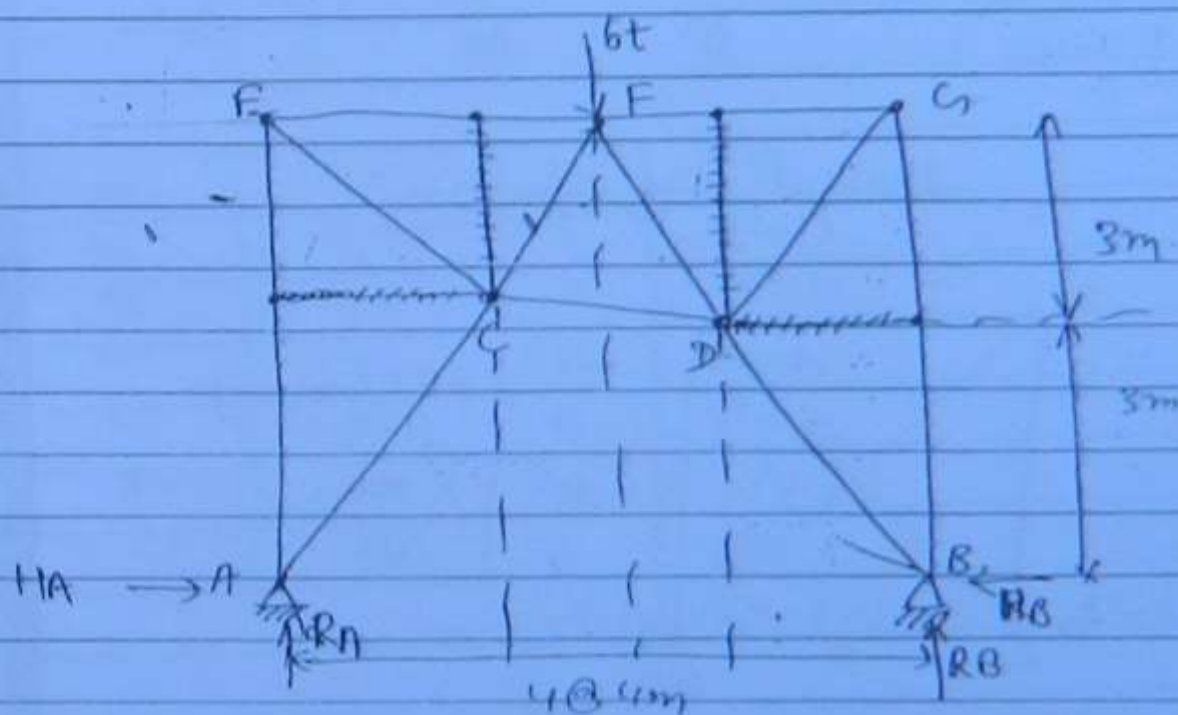
$$K_{BF} = 1/2$$

$$X = \frac{-\sum PKL/AE}{\sum K^2L/AE} \Rightarrow \left(\frac{-44.978}{6.8289} \right)$$

$$\Rightarrow +6.5852$$

279

Q. for the truss shown in fig all member have same axial Rigidity calculate Horizontal and vertical Reaction at supports.



$$D_S = mtrc - 2J$$

$$= 11 + 4 - 2 \times 7$$

$$D_S = 1$$

$$D_{se} = 8e - 3$$

$$= 4 - 3$$

$$= 1$$

$$\sum F_x = 0$$

$$H_A = H_B = H$$

$$\sum F_y = 0$$

$$R_A + R_B = 6$$

1

$$\sum M_A = 0$$

$$R_R \times 16 - 6 \times 8 = 0$$

$$R_R = \frac{36 \times 8}{16} = 3 \text{ kN}$$

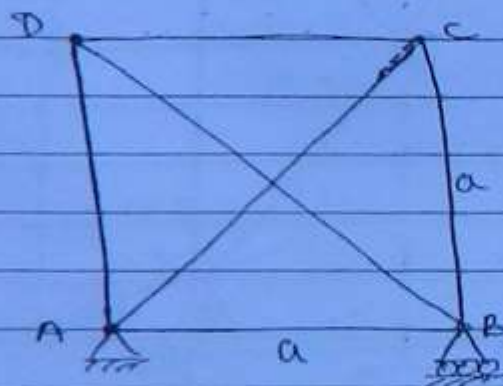
$$R_A = R_R = 3 \text{ kN} \uparrow$$

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→ Let the Redundent is H ($H_A = H_B = H$)

$$H = 3.27 \text{ t}$$

Lack of Fit Problems



$AE = \text{constant}$

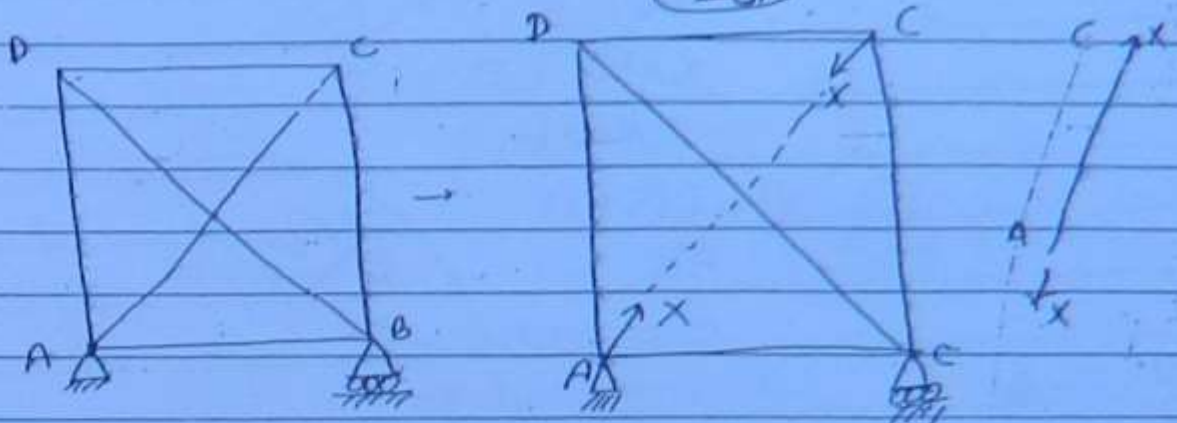
→ The Truss members are fabricated in the factory and assembled in the field. If any one member is fabricated either a too short or a too long. If defective member is force fully fitted, then all members may be sub. to forces.

→ Let a square truss ABCD is to be assembled but member AC is fabricated a too short.

for force full fitting of AC let us apply a force in AC.

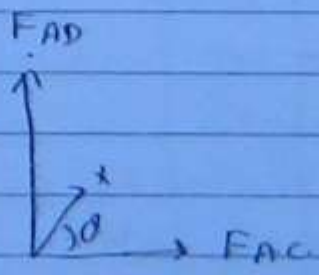
Hence joints A & C will be also under x force in direction AC. find forces in all members due to x force in AC.

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Member	force (Q)	L	$Q^2 L$
AB	$-x/\sqrt{2}$	a	$x^2 a/2$
AD	$-x/\sqrt{2}$	a	$x^2 a/2$
AC	x	$a\sqrt{2}$	$x^2 a\sqrt{2}$
BC	$-x/\sqrt{2}$	a	$x^2 a/2$
BD	$-x/\sqrt{2}$	$a\sqrt{2}$	$x^2 a\sqrt{2}$
CD	$-x/\sqrt{2}$	a	$x^2 a/2$
$\Sigma Q^2 L = 2x^2 a + 2\sqrt{2}x^2 a = 2x^2 a(\sqrt{2}+1)$			

Consider joint A



$R_A + R_B = 0$
 $H_A = 0$

$\Sigma F_x = 0$

@

$F_{AB} + x \cos 45^\circ = 0$

$F_{AB} = -\frac{x}{\sqrt{2}}$

$\Sigma F_y = 0$

$F_{AD} + x \sin 45^\circ = 0$

$F_{AD} = -\frac{x}{\sqrt{2}}$

Total Strain Energy in All member of Truss

$$U = \sum \frac{\alpha^2 L}{2AE}$$

$$\Rightarrow \frac{2x^2 a (\sqrt{2}+1)}{2AE}$$

$$U = \frac{x^2 a (\sqrt{2}+1)}{AE}$$

282

$$\frac{\partial U}{\partial x} = \Delta$$

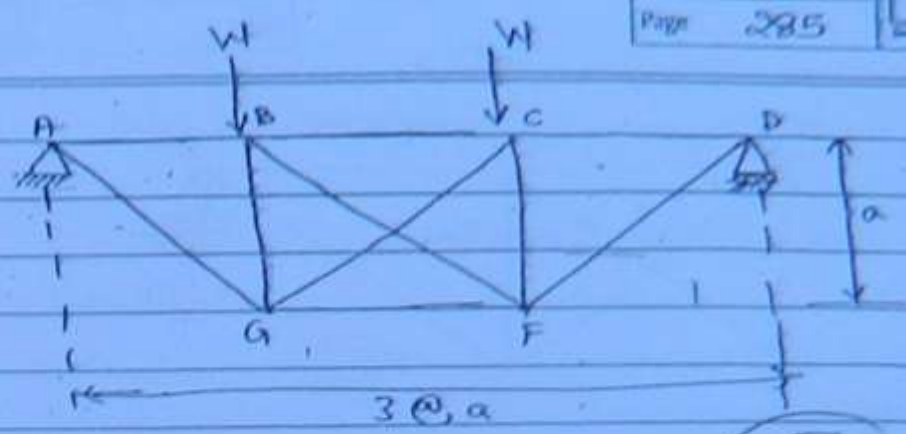
$$\frac{2x a (\sqrt{2}+1)}{AE} = \Delta$$

$$x = \frac{\Delta \cdot AE}{2a(\sqrt{2}+1)}$$

Note:- The Above forces shown in the table are due to leakage fit effect when there is no loading in the truss. If loading effect is also to be considered, then then separate analysis as per previous discussion will be required and combined effect algebraic sum of leakage fit effect and loading effect.

Q: in fig. Shows a Pin joint a truss A Having constant axial stiffness for each member. Calculate the axial force in member gf.

Q: Calculate the force in same member gf if it is fixed at a - ten short when structure carrying no loading.



283

So loading effect

$$D_s = m + r - 2j$$

$$\Rightarrow 10 + 3 - 2 \times 6$$

$$\Rightarrow 13 - 12 = 1$$

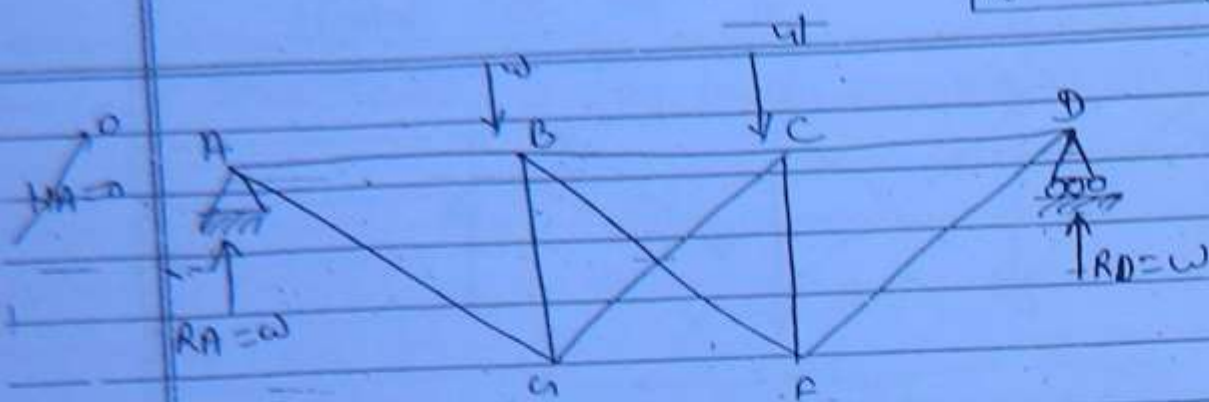
$$D_{se} = 0$$

$$D_s = 1$$

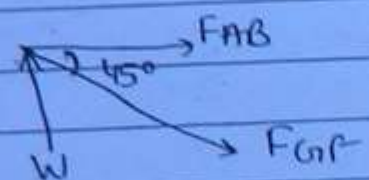
Hence redundant is internal force let member GF is redundant.

To find P force system remove the member and find forces in all member by method of joint.

S.No	P Member	member	K	P/K	K^2	$Q = KX^1$	Q^2
1	$-W$	AB	0	0	0	0	0
2	$W\sqrt{2}$	AG	0	0	0	0	0
3	$-2W$	BC	1	$-2W$	1	X^1	X^2
4	$-2W$	CG	1	$-2W$	1	X^1	X^2
5	$W\sqrt{2}$	BF	$-\sqrt{2}$	W	2	$-\sqrt{2}X^1$	$2X^1$
6	$-W$	CD	0	0	0	0	0
7	$-2W$	CE	1	$-2W$	1	X^1	X^2
8	$W\sqrt{2}$	EG	$-\sqrt{2}$	$-2W$	2	$-\sqrt{2}X^1$	$2X^2$
9	0	GF	1	0	1	X^1	X^2
10	$W\sqrt{2}$	DE	0	0	0	0	0



① consider joint a



284

$\sum F_x = 0$ $\sum F_y = 0$

$$w - F_{EF} \sin 45^\circ = 0$$

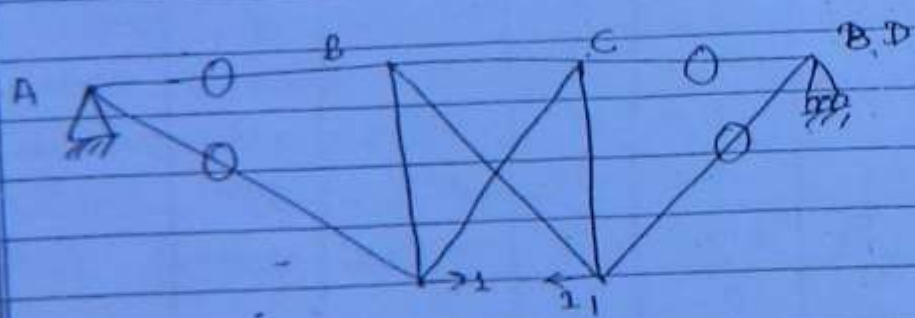
$$F_{EF} = w\sqrt{2}$$

$\sum F_x = 0$

$$\sum F_{AB} + w\sqrt{2} \cos 45^\circ = 0$$

$$F_{AB} = -\frac{w\sqrt{2} \times 1}{\sqrt{2}}$$

$$F_{AB} = -w$$



No reaction, because there are internal reaction

To find k force system remove given loading and apply unit load at C&F. At find k force method of joint.

Stiffness $\frac{AE}{L} = \text{constant}$

$X = \text{Redundant force in Cof will be}$

$$X \Rightarrow - \frac{\sum \frac{PKL}{AE}}{\sum \frac{K^2L}{AE}} \Rightarrow - \frac{\sum PK}{\sum K^2}$$

$$\Rightarrow - \frac{10w}{84} \Rightarrow -1.25w$$

(285)

Step 1

$$\Delta = 0.1 \times a$$

100

$$\Delta = \frac{a}{1000}$$

Let due to lack of fit force in member Cof is X'

$$\text{Hence } \frac{\partial U}{\partial X'} = \Delta = \frac{a}{1000}$$

$\frac{AE}{L} = \text{const. for all members}$

$$U = \sum \frac{Q^2L}{2AE} = \frac{L}{2AE} \sum Q^2$$

$Q_1, Q_2, \dots, Q_n$ are forces in Truss member due to X' force in Cof

$$U = \frac{L}{2AE} \sum Q^2 \Rightarrow$$

due to unit force in ~~each~~ of $K_1, K_2, K_3, \dots, K_n$ are the force in truss member, Hence due to x' forces in of forces in, thus member will be

$$Q_1 = K_1 x'$$

$$Q_2 = K_2 x'$$

$$\vdots$$

$$Q_n = K_n x'$$

286

Q

$$U = \frac{L}{2AE} \sum Q^2$$

$$\text{let } \frac{AE}{L} = C$$

$$U \Rightarrow \frac{1}{2C} \times 8x'^2$$

$$\frac{\partial U}{\partial x'} \Rightarrow \frac{2 \times 8x'}{2C} \Rightarrow \frac{8x'}{C} = \frac{a}{1000}$$

$$x' = \frac{aC}{8000}$$

$$\Rightarrow \boxed{\frac{a AE}{L 8000} = x'}$$

Deflection of Truss joints Methods

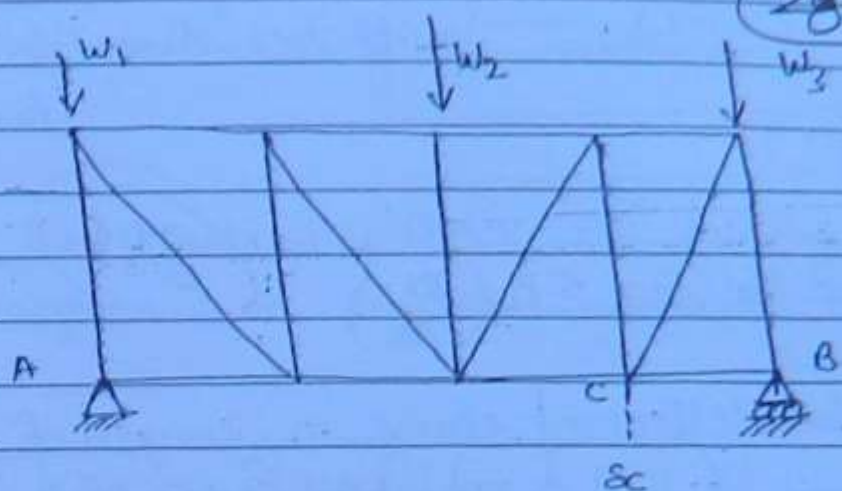
✓ ① force method / unit load method / Maxwell's mt / strain energy method

✓ ② Stiffness method / Equilibrium method

x ③ Graphical method.

④ Williot Mohr Diagram? applicable
⑤ Bar chain method } for truss

Unit load method / Maxwell's method :-



→ Let vertical Deflection - of joint C is required.

Procedure

Step-1 due to given load system find P-force system in all members unit method of joint say member forces are $P_1, P_2, \dots, P_n$.

Step-2 Remove given loading and apply unit load at C in the direction of desired deflection (vertical) and find K-force system say due to unit load at C member forces are $K_1, K_2, \dots, K_n$.

Step-3

the vertical deflection at C is given as

$$\delta_c = \sum \frac{P K L}{A E}$$

if S is (+ive) $\rightarrow$ ^{it is in the} direction of unit load.

if S is (-ive) $\rightarrow$ opposite direction of unit load.

Special Case $\rightarrow$

$$\delta_c = \sum P \left(\frac{PL}{AE} \right)$$

288

$\frac{PL}{AE}$ = Axial deflection / change in length of any member due to force P in that member

$P \rightarrow (+) T$

$P \rightarrow (-) C$

$$\delta_c = \sum_{i=1}^n K_i \Delta_i$$

if temp^s of some of the member is change Δ_i will be due to temp effect.

$$\Delta_i = (L \alpha T)_i$$

T = Temp. Change $^{\circ}C$

(+) if T is $\uparrow$

(-) if T is $\downarrow$

α = coeff. of Thermal change.

Δ_i in any member i may be due to defective manufacturing also if member is manufacture.

Δ too long $\rightarrow \Delta_i \rightarrow +ve$

$\Delta_i \rightarrow -ive$

289

$$\delta_c = \sum_{i=1}^n K_i \left(\frac{PL}{AE} + L \alpha T \right)$$
[illegible]
$$\Delta_{AB} = -3 \text{ m}^2$$
$$\Delta p_c = +2.17 \text{ m}$$
$$\Delta E_F = +6.0 \text{ eV}$$

vertical deflection of joint D

$$\Delta_D = \sum_{i=1}^n K_i \Delta_i$$

290

$$\Delta_D = K_{AB} \Delta_{AB} + K_{BC} \Delta_{BC} + K_{EF} \Delta_{EF} + K \Delta_{\text{other member}}$$

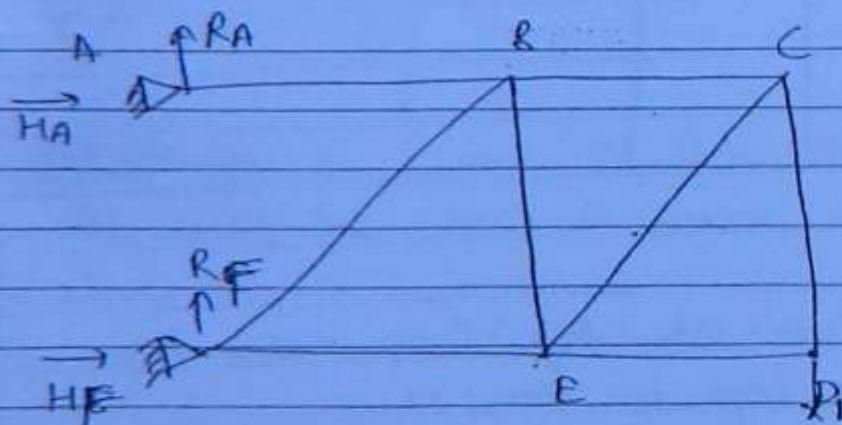
$$\Delta_D = K_{AB} \Delta_{AB} + K_{BC} \Delta_{BC} + K_{EF} \Delta_{EF}$$

$$\Delta_D = m + 9e - 2j$$

$$\Rightarrow 8 + 4 - 2 \times 6$$

$$\Rightarrow 0$$

Find K force is due to unit load at D in vertical direction.



$$M_B = 0$$

$$R_A \times 3 = 0$$

$$R_A = 0$$

or

At joint A

$$\sum F_y = 0$$

$$R_A = 0$$

$$\sum F_y = 0$$

$$R_A + R_F - 1 = 0$$

$$R_F = 1$$

$$\sum F_x = 0$$

$$H_A + H_F = 0$$

$$\sum M_F = 0$$

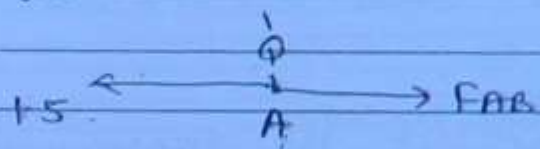
291

$$H_A \times 4 + 1 \times 6 = 0$$

$$H_A = -1.5$$

$$H_F = +1.5$$

Consider joint A

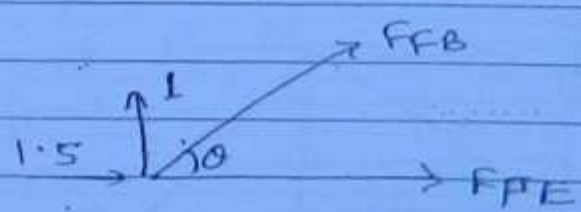


$$\sum F_x = 0$$

$$F_{AB} - 1.5 = 0$$

$$F_{AB} = 1.5$$

Consider joint F



$$\sum F_y = 0$$

$$K_{FB} \cdot \sin 30 + 1 = 0$$

$$K_{FB} = -\frac{1}{\frac{1}{2}} \times 5 = -1.25$$

$$\sum F_x = 0 \quad 1.5 + K_{FE} + K_{FB} \cdot \cos 30 = 0$$

$$K_{FE} = -1.5 + \frac{1.25 \times 3}{5}$$

$$= -0.75$$

Consider joint B



$$\sum F_x = 0$$

$$K_{BC} + 1.25 \cos 30 - 1.5 = 0$$

$$K_{BC} = 10.75$$

$$\Delta D = K_{AB} \cdot \Delta_{AB} + K_{AC} \cdot \Delta_{AC} + K_{EF} \cdot \Delta_{EF}$$

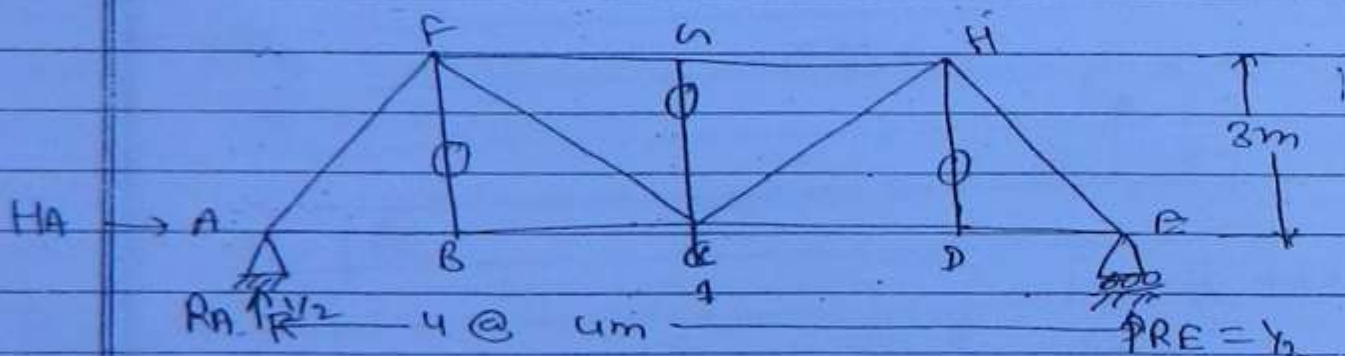
$$\Rightarrow (115) \times (-3) + (0.75)(2) + (0.75)(4)$$

$$\therefore \boxed{\Delta D = -6 \text{ mm}} \quad \uparrow$$

(2 of 2)

$\therefore \Delta D$ is (-) it means ΔD is in opposite direction of unit load.

Q. Compute vertical displacement of the displacement of joint C due to decrease in temp. by 50°C in bottom chord member only given that $\alpha = \frac{1}{150000} / ^\circ\text{C} \Rightarrow 6.67 \times 10^{-5}$



$$D_s = m + 9e - 2j$$

$$= 13 + 9 - 2 \times 8 = 0$$

Vertical deflection of joint C

$$\Delta_C = \sum_{i=1}^n K_i \Delta_i$$

Δ_i = change in length of member due to temp change.

Since Temp. change is only Bottom chord member Hence for other member $\Delta_i = 0$

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$$S_C = K_{AB} \Delta_{AB} + K_{BC} \Delta_{BC} + K_{CD} \Delta_{CD} + K_{DE} \Delta_{DE}$$

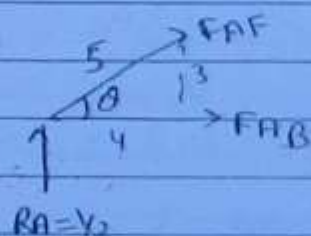
$$\Delta_{AB} = \Delta_{BC} = \Delta_{CD} = \Delta_{DE} = \Delta \times T$$

$$\Rightarrow 4 \times 6.67 \times 10^{-6} \times (-50) \times 10^3$$

$$\Rightarrow -1.33 \text{ mm}$$

$$\Rightarrow -1.33 \text{ mm}$$

Consider Joint A K force system.



$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$F_{AF} \sin \theta + \frac{1}{2} = 0$$

$$F_{AB} \cos \theta =$$

$$F_{AF} \times \frac{3}{5} = -\frac{1}{2} \Rightarrow F_{AF} =$$

$$\sum F_y = 0$$

$$F_{AF} \sin \theta + \frac{1}{2} = 0$$

$$F_{AF} = -\frac{1}{2} \times \frac{5}{3} = -\frac{5}{6} = -0.833$$

$$\sum F_x = 0$$

$$F_{AB} + F_{AF} \cos \theta$$

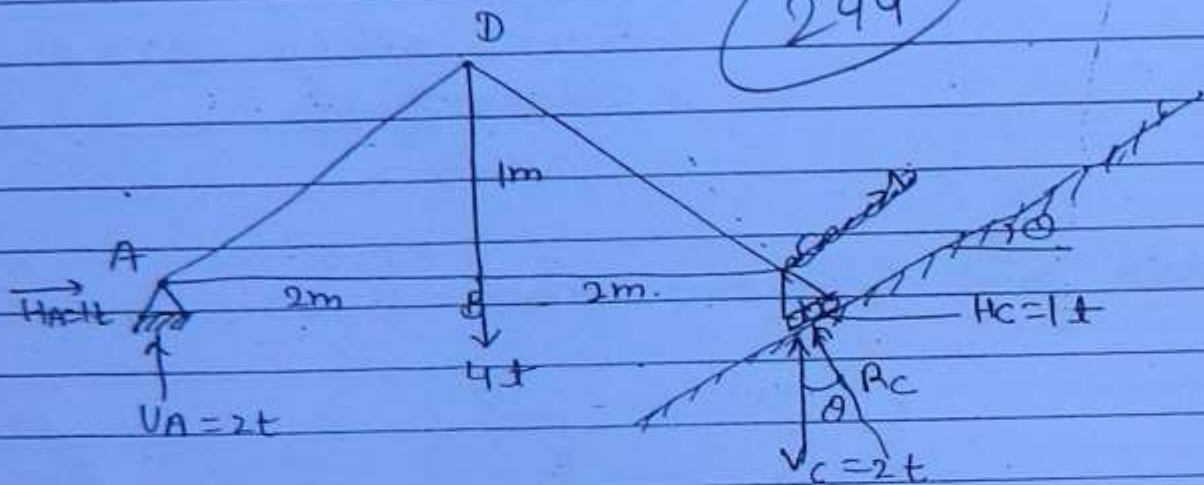
$$F_{AB} = -0.833 \times \frac{4}{5} = -\frac{5}{6} \times \frac{4}{5} = -\frac{2}{3}$$

$$F_{AB} = F_{CD} = F_{DE}$$

$$S_C = \sum K_i \Delta_i$$

$$= 4 \times \frac{2}{3} \times -1.33 = -3.55 \text{ mm}$$

Q A small truss as shown in fig. has all pin jointed members having area of cross section 20cm^2 each. The plane of roller is inclined and is parallel to AD. determine movement of joint C. When vertical load of 4t is applied at B given that $E = 2 \times 10^6 \text{ Kg/cm}^2$.



$$V_C = R_C \cos \theta$$

$$H_C = R_C \sin \theta$$

$$\frac{H_C}{V_C} = \tan \theta = \frac{1}{2}$$

$$\boxed{\frac{H_C = V_C}{2}}$$

$$\sum M_A = 0$$

$$V_C \times 4 - 4 \times 2 = 0$$

$$V_C = 2t$$

$$H_C = 1t$$

$$\sum F_x = 0$$

$$H_A - H_C = 0$$

$$H_A = H_C = 1t$$

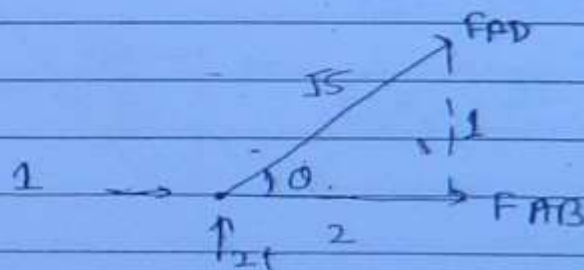
$$\sum F_y = 0 \quad V_A + V_C - 4 = 0$$

$$V_A = 2t$$

member	P E	K	L (m)	PKL
AB	+3	$I/12$	2	$6.708 \times 1000 \Rightarrow 6708$
AD	-4.47	0	5	$-4.472 \times 1000 \Rightarrow -4472$
BC	+3	$I/12$	2	$6.708 \Rightarrow 6708$
BD	+4	0	1	0 $\Rightarrow 0$
CD	-4.47	0	5	0 $\Rightarrow 0$
				$\Sigma PKL = 13146$

19-11

① Joint A



$$\Sigma F_x = 0$$

$$F_{AB} + F_{AD} \cos 10^\circ + 1 = 0$$

$$F_{AB} + F_{AD} \frac{2}{5} + 1 = 0$$

$$F_{AB} = -1 + 4.472 \times \frac{2}{5} \Rightarrow +3 \text{ kN}$$

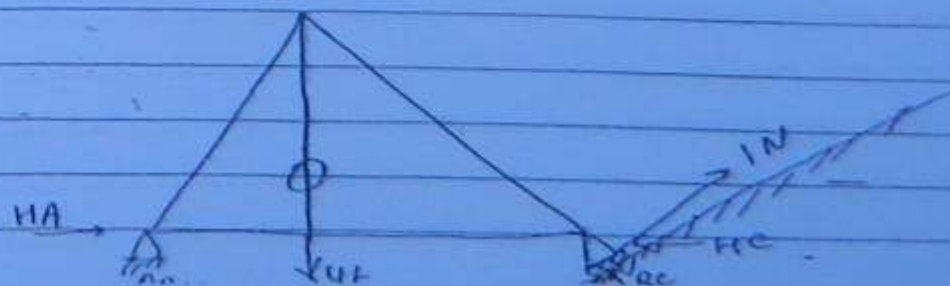
$$\Sigma F_y = 0$$

$$F_{AD} \sin 10^\circ + 2t = 0$$

$$F_{AD} = -2 \times 5 / 1$$

$$F_{AD} = -4.472$$

→ To find deflection in the direction of plane apply unit load $\perp$ to the plane. Unit load when θ with the horizontal.



$$\sum M_c = 0$$

$$V_A \times 4 = 0$$

$$V_A = 0$$

$$\sum F_x = 0$$

$$V_A + V_c + 1 \sin 45 = 0$$

$$V_c = -1 \sin 45$$

$$V_c = -1 \times \frac{1}{\sqrt{2}} = -\frac{1}{\sqrt{2}}$$

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$$H_c = \frac{V_c}{2} = -\frac{1}{2\sqrt{2}}$$

$$\sum F_x = 0$$

$$H_A + 1 \cdot \cos 45 + H_c = 0$$

$$H_A + \frac{1}{\sqrt{2}} - \frac{1}{2\sqrt{2}} = 0 \Rightarrow H_A = -\frac{1}{2\sqrt{2}}$$

$$A_c = \frac{\sum P K L}{A E}$$

$$A_c = \frac{13416}{20 \times 2 \times 10^6} \text{ m}$$

$$= 1.857 \times 10^{-3}$$

$$A_c = 0.32865$$

$$P \rightarrow K g$$

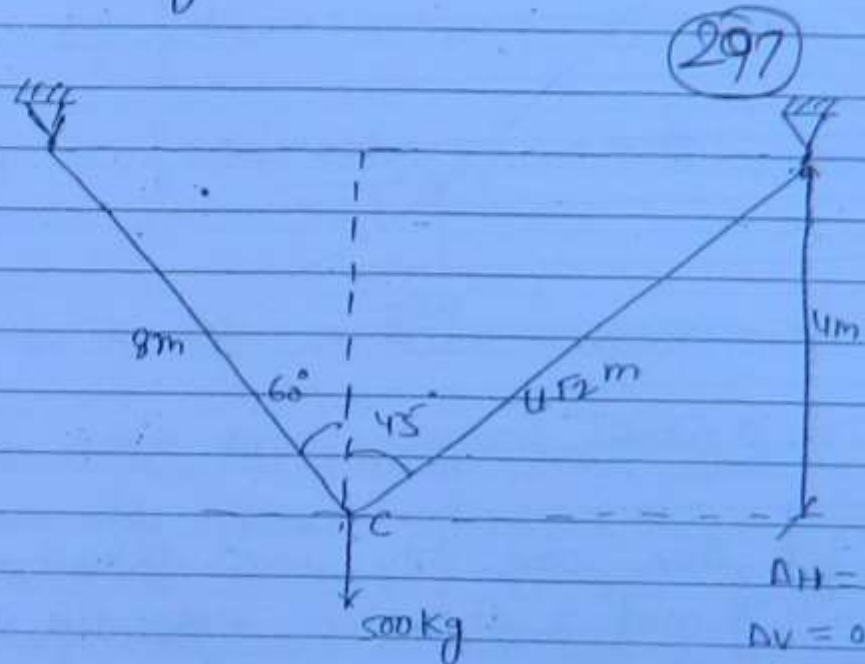
$$A = \text{cm}^2$$

$$E = \text{kg/cm}^2$$

$$L = \text{m}$$

A_c will be in

- Q 2 Two straight bars AC & BC 8mm in dia meet at joint C. Calculate horizontal and vertical component of deflection if all joints are hinge. An vertical load of 500 kg is apply at joint C. young modulus $2 \times 10^6 \text{ kg/cm}^2$.

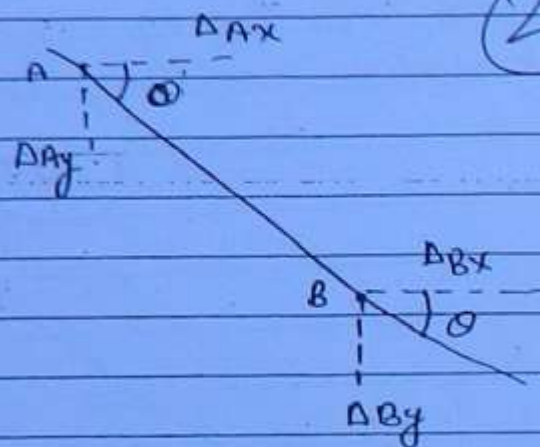


Ans:-

$$\Delta H = 0.828 \text{ mm} (\rightarrow)$$

$$\Delta V = 0.4329 \text{ mm} (\downarrow)$$

Force - Displacement Relation



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Consider AB is a truss member and A & B are truss joint.

Let ΔAx and ΔAy are displacement of joint A in x and y direction respectively. Similarly ΔBx and ΔBy are displacement of joint B.

The displacement of joint A in the direction of member AB:-

$$\Delta A = \Delta Ax \cos \theta + \Delta Ay \sin \theta$$

Displacement of joint B in the direction of member AB $\rightarrow$

$$\Delta B = \Delta Bx \cos \theta + \Delta By \sin \theta$$

θ is angle of member AB with Horizontal

Axial displacement of member AB $\Rightarrow$

$$\Delta A - \Delta B = \Delta Ax \cos \theta + \Delta Ay \sin \theta - \Delta Bx \cos \theta - \Delta By \sin \theta$$

$$\Delta_{AB} \Rightarrow (\Delta Ax - \Delta Bx) \cos \theta + (\Delta Ay - \Delta By) \sin \theta$$

$$\Delta_{AB} = \Delta_B - \Delta_A$$

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$$\Delta_{AB} = (\Delta_{Bx} - \Delta_{Ax}) \cos \theta + (\Delta_{By} - \Delta_{Ay}) \sin \theta$$

$$\Rightarrow \frac{P_{AB} \cdot L}{AE} = (\Delta_{Bx} - \Delta_{Ax}) \cos \theta + (\Delta_{By} - \Delta_{Ay}) \sin \theta$$

$$\Rightarrow P_{AB} = \frac{AE}{L} [(\Delta_{Bx} - \Delta_{Ax}) \cos \theta + (\Delta_{By} - \Delta_{Ay}) \sin \theta]$$

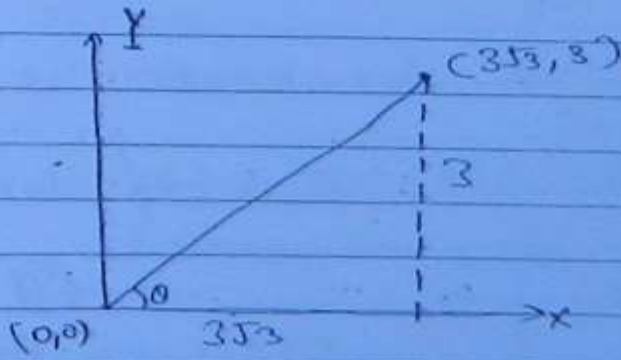
$$\Rightarrow K [(\Delta_{Bx} - \Delta_{Ax}) \cos \theta + (\Delta_{By} - \Delta_{Ay}) \sin \theta]$$

$$K = \frac{AE}{L} \Rightarrow \text{Axial stiffness}$$

Q. A BAR A, B of a Pin jointed Truss line in the x-y plane has the coordinates of A(0,0) and coordinates of B(3√3, 3) in m. joint A & B are displace and the displacement in x-y direction are ΔAx = 1 ΔAy = √3 ΔBx = 2 ΔBy = 2√3 mm. If a axial stiffness of BAR is 100 kN/m. the force induced in the bar above displacement is 100

- (A) 100 kN (B) 200 kN
(C) 173 kN (D) 273 kN

Sol



$$\tan \theta = \frac{3}{3\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\theta = 30^\circ$$

$$\cos \theta = \sin 30 = \frac{1}{2}$$

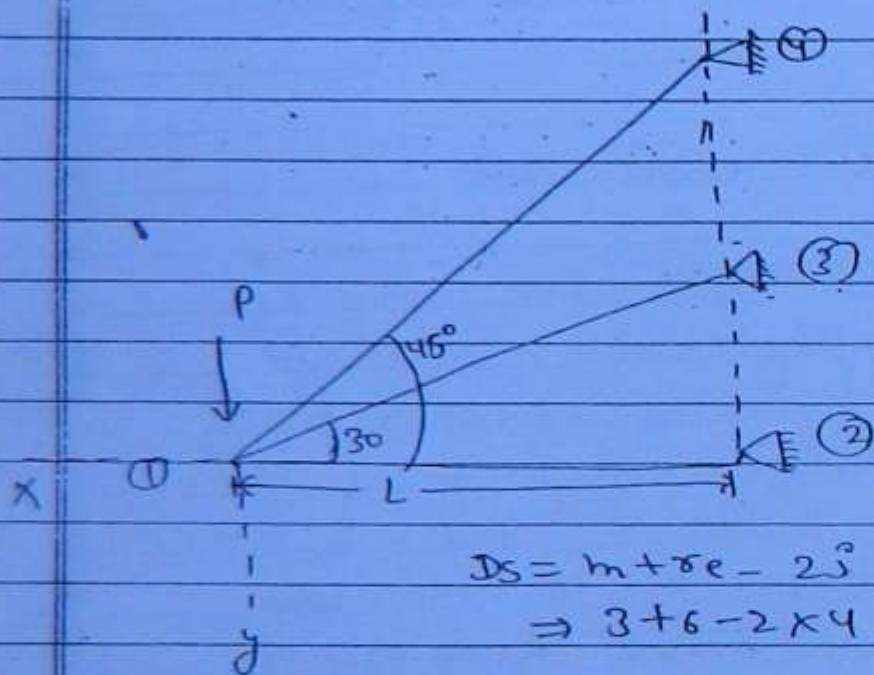
$$\sin \theta = \sin 30 = \frac{1}{2}$$

$$P_{AB} = 100 \left[\frac{(2-1) \times \sqrt{3}}{2} + \frac{(2\sqrt{3} - \sqrt{3}) \times 1}{2} \right]$$

$$\Rightarrow 100 \left[\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \right] = 100\sqrt{3} \Rightarrow 173.2 \text{ kN}$$

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Q. for three bar truss shown in fig. compute vertical displacement of node 1. using stiffness method.



$AE = \text{const.}$

$$L_{12} = L$$

$$L_{13} = 2L\sqrt{3}$$

$$L_{23} = \sqrt{2}L$$

$$DS = m + re - 2j$$

$$\Rightarrow 3 + 6 - 2 \times 4 \Rightarrow 3 - 8 = -5$$

$$D_k = 2j - re$$

$$\Rightarrow 2 \times 4 - 6 \Rightarrow 2 \begin{matrix} \nearrow D_{1x} \\ \searrow D_{1y} \end{matrix}$$

for Displacement Relations for member 1,2

$$P_{12} = \frac{AE}{L} \left[(\Delta_{2x} - \Delta_{1x}) \cos 0^\circ + (\Delta_{2y} - \Delta_{1y}) \sin 0^\circ \right]$$

$$P_{12} = \frac{AE}{L} \left[(-\Delta_{1x}) \right] \Rightarrow \dots \Rightarrow \text{Answer}$$

$$P_{13} \Rightarrow \frac{AE}{24\sqrt{3}} \left[(\Delta_2 X - \Delta_1 X) \cos 30^\circ + (\Delta_2 Y - \Delta_1 Y) \sin 30^\circ \right]$$

$$\Rightarrow \frac{\sqrt{3}}{2} \frac{AE}{L} \left[-\frac{\Delta_1 X \cdot \sqrt{3}}{2} - \frac{\Delta_1 Y}{2} \right]$$

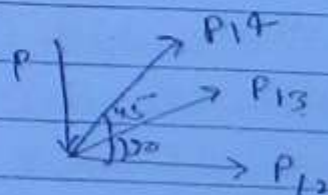
$$\Rightarrow -\frac{\sqrt{3}}{4} \frac{AE}{L} [\sqrt{3} \Delta_1 X + \Delta_1 Y] \quad \text{--- (3)}$$

for member P_{14}

$$P_{14} = \frac{AE}{J_2 L} \left[(\Delta_2 X - \Delta_1 X) \cos 45^\circ + (\Delta_2 Y - \Delta_1 Y) \sin 45^\circ \right]$$

$$P_{14} = -\frac{AE}{2L} [\Delta_1 X + \Delta_1 Y] \quad \text{--- (3)}$$

To find joint displacement nodal consider free body Equilibrium of node 1.



$$\sum F_x = 0$$

$$P_{12} + P_{13} \cos 30^\circ + P_{14} \cos 45^\circ = 0 \quad \text{--- (1)}$$

$$\sum F_y = 0$$

$$P_{13} \sin 45^\circ$$

$$P_{12} \sin 30^\circ + P_{14} \sin 45^\circ - P = 0 \quad \text{--- (2)}$$

Solving equ (1) & (2)

$$\Delta_1 Y = \frac{0.328 PL}{AE}$$

Any 7 3/28 PL
AEL

MATRIX METHOD

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Flexibility matrix method
(force mtd)

Stiffness method
(Displacement mtd)

Properties of matrix If a matrix is $[A]_{m \times n}$, then it means are m rows and n columns.

Flexibility matrix & Stiffness matrix will be always a square matrix.

- ② A unit matrix is that which has unit or element along the diagonal.
- ③ A diagonal matrix is that which has all member zero other than diagonal member.
- ④ The magnitude of a matrix represented by its determinate.
- ⑤ A determinate will have zero value, any two rows and any two column identical.
- ⑥ The sign of determinate will change if element of any two rows or any two column are interchange.

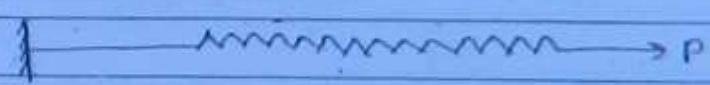
⑦ The inverse of a matrix A is

$$[A]^{-1} = \frac{\text{Adj.}[A]}{|A|}$$

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Adj[A] ~~of A~~ matrix obtain by transpose of cofactor of matrix A.

Flexibility and Stiffness :-



$$\Delta = \frac{P}{k}$$

$$\text{or } f = \frac{\Delta}{P}$$

$$k = \frac{P}{\Delta}$$

k $\rightarrow$ stiffness

f $\rightarrow$ flexibility

Stiffness is equal to force required to produce unit displacement ~~is~~ flexibility is displacement produce by unit force.

Properties of Flexibility and Stiffness matrix

$\rightarrow$ f & k matrix always square matrix $[n \times n]$

$$[A]_{n \times n} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}$$

→ The diagonal element will be always a non-zero and non-negative i.e. always positive.

→ The matrix will be always symmetrical about the diagonal. It means the transpose of matrix will be unchanged. It is because of applicability of Maxwell's Reciprocal Theorem.

$$f_{12} = f_{21}$$

$$f_{xy} = f_{yx}$$

$$\text{or } k_{12} = k_{21}$$

$$k_{xy} = k_{yx}$$

$$a_{xy} = a_{yx}$$

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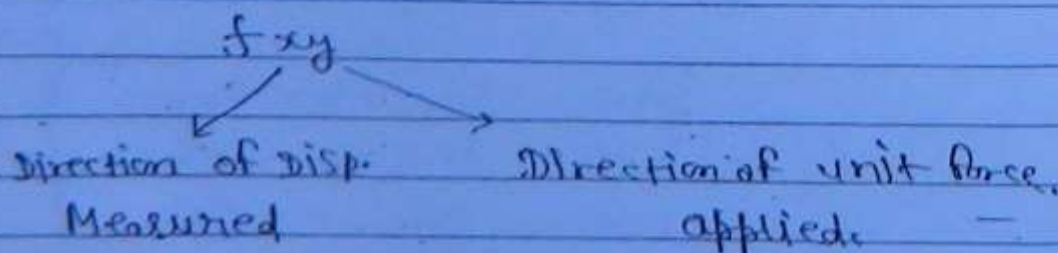
→ Procedure to develop flexibility matrix.

$$[f_n]_{n \times n} = \begin{bmatrix} f_{11} & f_{12} & \dots & f_{1n} \\ f_{21} & f_{22} & \dots & f_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ f_{m1} & f_{m2} & \dots & f_{mn} \end{bmatrix}$$

f_{11} = Displacement in the direction ① ^{due to} unit force in the direction of ①

f_{21} = Displacement in the direction ② ^{due to} unit force in the direction of ①

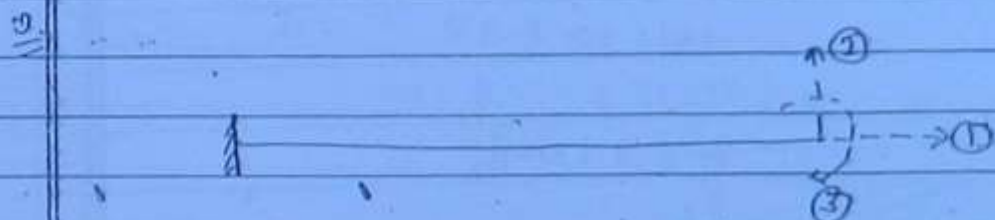
f_{xy} = Displacement in the direction ② ^{due to} unit force in the direction of ①



Step 1 To generate first column element of flexibility matrix give / apply unit force in dirⁿ ① only and measure the ~~direction in~~ displacement in direction ①, ② - - - ③

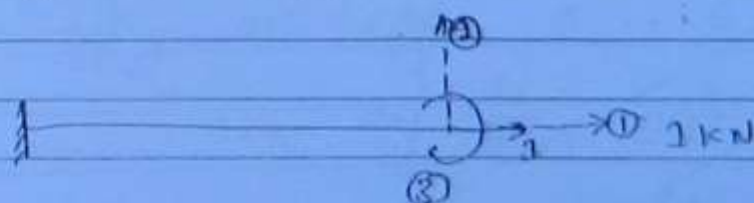
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Step 2 To obtain second column apply unit force in dirⁿ ② only & repeat the procedure.



L, A, E, I constant

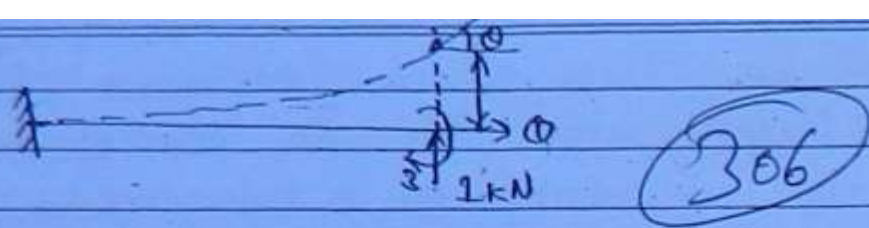
→ To develop first column of flexibility matrix apply unit force in direction ① only and measure the displacement in all the coordinate direction.



f_{11} = Displacement in dir ① due to unit force in the dir ①
 $\rightarrow \frac{L}{AE}$

f_{21} = Displ. in dir ② due to unit force in dir ①
 $= 0$
 $f_{31} = 0$

→ To generate second column of flexibility matrix apply unit force in dirⁿ of ② only and measured the displacement in ^{all} the coordinate direction.



$$f_{12} = 0$$

$$f_{22} = \frac{L^3}{3EI}$$

$$f_{32} = -\frac{L^2}{2EI}$$

To → To generate 3 column of flexibility apply unit force in the dirⁿ ③



$$f_{13} = 0$$

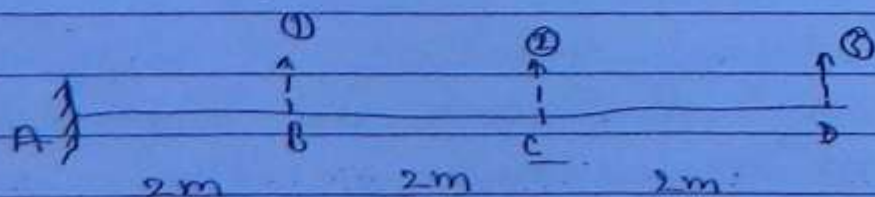
$$f_{23} = \frac{L^3}{2EI}$$

$$f_{33} = \frac{L}{EI}$$

$$[f] = \begin{bmatrix} L/AE & 0 & 0 \\ 0 & L^3/3EI & -L^2/2EI \\ 0 & -L^3/2EI & L/EI \end{bmatrix}$$

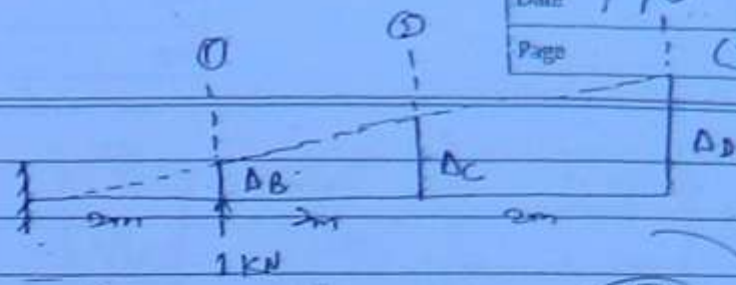
diagonal member are non zero and non-negative.

Q. Develop flexibility matrix for coordinate shown in fig.



$EI = \text{Constant}$

① First column.



$$f_{11} = \frac{1 \cdot (2)^3}{3EI} \rightarrow \frac{8}{3EI}$$

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$$f_{21} = \Delta_B + \theta_B \cdot L_{BC}$$

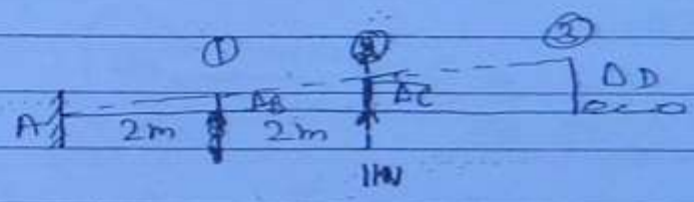
$$\Rightarrow \frac{8}{3EI} + \frac{1 \cdot (2)^2}{2EI} \times 2 \Rightarrow \frac{20}{3EI}$$

$$f_{31} = \Delta_D = \Delta_B + \theta_B \times L_{BD}$$

$$\Rightarrow \frac{8}{3EI} + \frac{1(2)^2}{2EI} \times 4 \Rightarrow \frac{32}{3EI}$$

Second column

②



$$f_{21} = \frac{1(4)^3}{3EI} \Rightarrow \frac{64}{3EI}$$

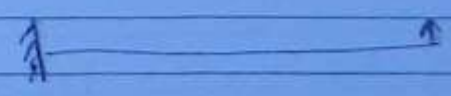
$$f_{22} = \frac{20}{3EI} = f_{21}$$

$$f_{22} = \frac{PL^3}{3EI} \Rightarrow \frac{1(4)^3}{3EI} \Rightarrow \frac{64}{3EI}$$

$$f_{22} \Rightarrow \frac{64}{3EI} + \frac{48}{3EI} \Rightarrow \frac{112}{3EI}$$

Third

③ Column



$$f_{13} = \frac{32}{3EI}$$

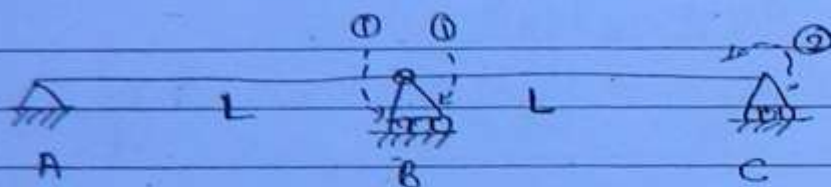
$$f_{23} = \frac{112}{3EI}$$

$$f_{33} = \frac{1(6)^3}{3EI} = \frac{36}{EI}$$

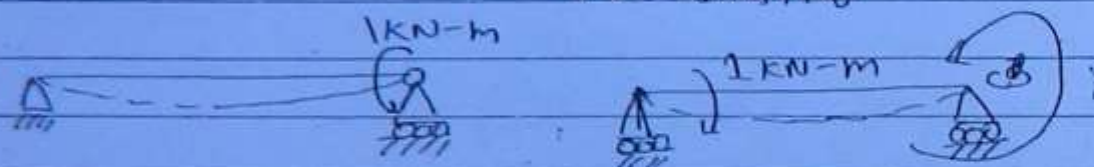
$$[f] = \frac{1}{3EI} \begin{bmatrix} 8 & 20 & 32 \\ 20 & -64 & 112 \\ 32 & 112 & 216 \end{bmatrix}$$

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Developed flexibility matrix for the beam coordinate shown in fig.



$EI = \text{constant}$



$$\theta_{B1} = \frac{L}{3EI}$$

$$\theta_{B2} = \frac{L}{3EI}$$

$$\theta_C = \frac{L}{6EI}$$

$$f_{11} = \frac{L}{3EI} + \frac{L}{3EI} = \frac{2L}{3EI}$$

$$f_{21} = \frac{6L}{6EI}$$



$$\theta = \frac{L}{3EI}$$

$$f_{12} = \frac{L}{6EI}$$

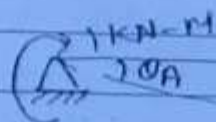
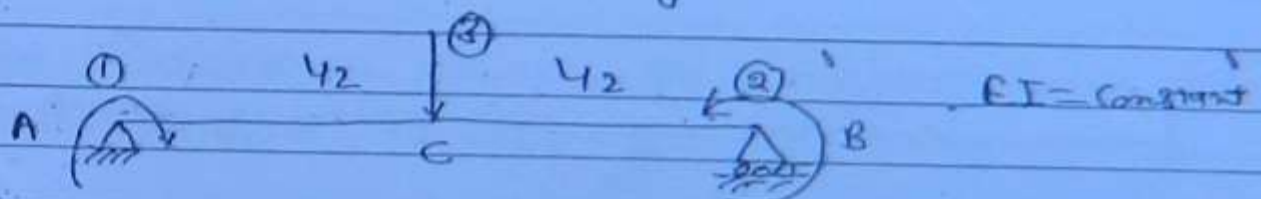
$$f_{22} = \frac{L}{3EI}$$

$$[f] = \begin{bmatrix} \frac{2L}{3EI} & \frac{L}{6EI} \\ \frac{L}{6EI} & \frac{L}{3EI} \end{bmatrix}$$

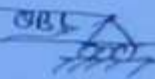
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$$[f] = \frac{L}{6EI} \begin{bmatrix} 4 & 1 \\ 1 & 2 \end{bmatrix}$$

Developed the flexibility matrix for the beam with coordinate shown in fig.



$$\theta_A = \frac{ML}{3EI}$$

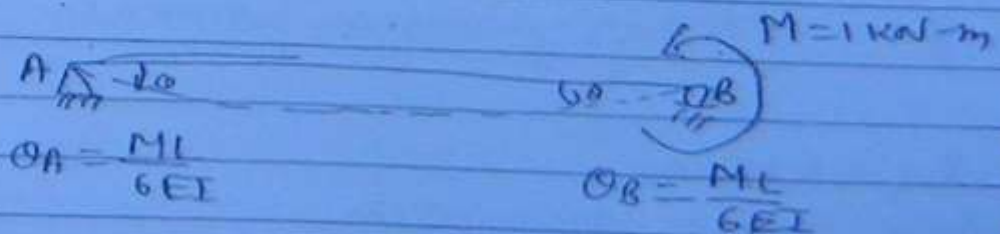


$$\theta_B = \frac{ML}{3EI}$$

$$f_{11} = \frac{L}{3EI}$$

$$f_{21} = \frac{L}{6EI}$$

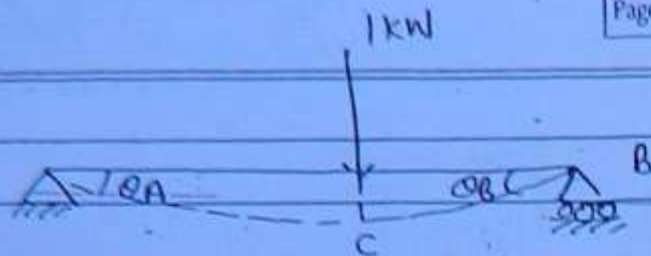
$$f_{31} = \frac{L^2}{16EI}$$



$$f_{12} = \frac{L}{6EI}$$

$$f_{22} = \frac{L}{3EI}$$

$$f_{32} = \frac{L^2}{16EI}$$



$$\theta_A = \frac{L^3}{16EI}$$

$$\theta_B = \frac{L^3}{16EI}$$

$$\delta_C = \frac{L^3}{48EI}$$

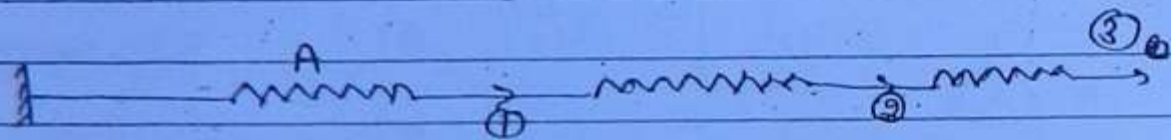
$$f_{12} = \frac{L^2}{16EI}$$

$$f_{23} = \frac{L^2}{16EI}$$

$$f_3 = \frac{L^3}{48EI}$$

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Q. develop the flexibility matrix method for the spring system shown in fig.



$$f_A = 0.05 \text{ cm/kN}$$

$$f_B = 0.10 \text{ "}$$

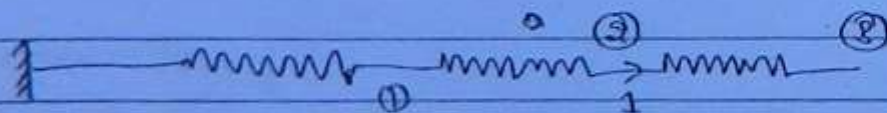
$$f_C = 0.20 \text{ "}$$

$$f = \frac{P}{k} \Rightarrow k = \frac{P}{f}$$

$$f_{11} = f_A \times 1 \Rightarrow 0.05 \text{ cm}$$

$$f_{21} = \delta_2 = \delta_1 = 0.05 \text{ cm}$$

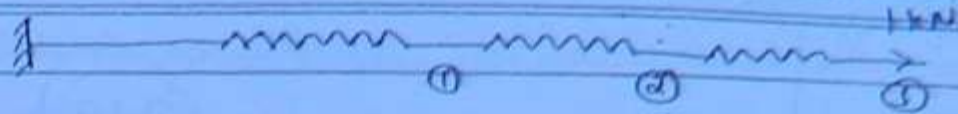
$$f_{31} = \delta_3 = \delta_1 = \delta_2 \Rightarrow 0.05 \text{ cm}$$



$$f_{12} = 0.05 \text{ cm}$$

$$\delta_{22} = 0.05 + 0.10 \Rightarrow 0.15 \text{ cm}$$

$$f_{22} = 0.15 \text{ cm}$$



$$f_{13} = 0.05 \text{ cm/kN}$$

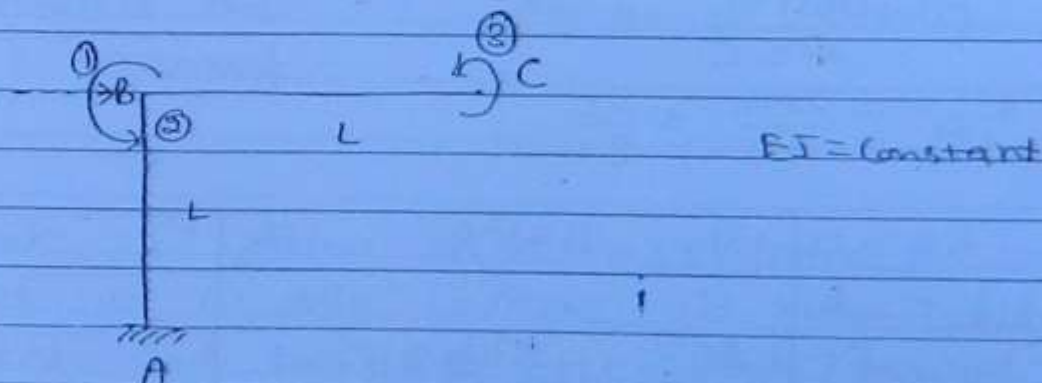
$$f_{23} = 0.05 + 0.10 = 0.15$$

$$f_{33} = 0.05 + 0.10 + 0.20 = 0.35$$

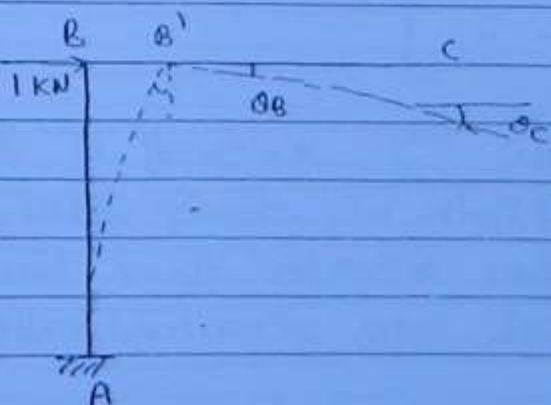
(311)

$$[f] = \begin{bmatrix} 0.05 & 0.05 & 0.05 \\ 0.05 & 0.15 & 0.15 \\ 0.05 & 0.15 & 0.35 \end{bmatrix}$$

Q. Develop flexibility matrix for cantilever beam with coordinate marked in fig.



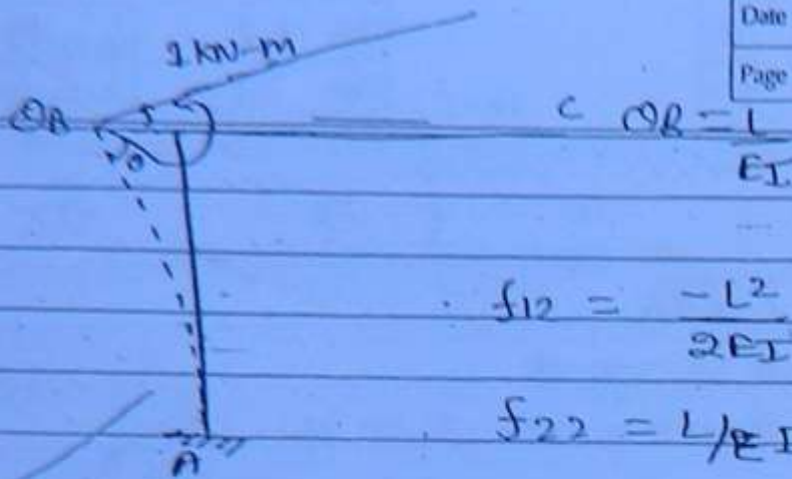
Sol.:-



$$f_{11} = \delta_{BB'} = \frac{L^3}{3EI}$$

$$f_{21} = -\frac{L^2}{2EI}$$

$$f_{31} = -\frac{L^2}{2EI}$$

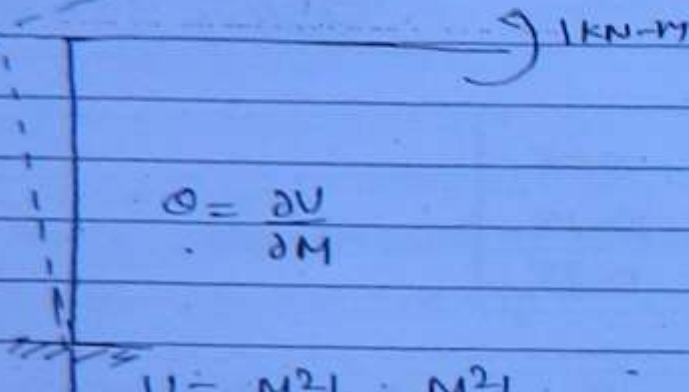


(3/2)

$$f_{12} = \frac{-L^2}{2EI}$$

$$f_{22} = L/EI$$

$$f_{32} = \frac{L}{EI}$$



$$\theta = \frac{\partial U}{\partial M}$$

$$F_{13} = \frac{-L^2}{2EI}$$

$$F_{23} = \frac{L}{EI}$$

$$U = \frac{M^2 L}{2EI} + \frac{M^2 L}{2EI}$$

$$\Rightarrow \frac{2M^2 L}{2EI}$$

$$F_{33} = \frac{2L}{EI}$$

$$\frac{\partial U}{\partial M} = \frac{2ML}{EI} \Rightarrow \frac{2L}{EI}$$

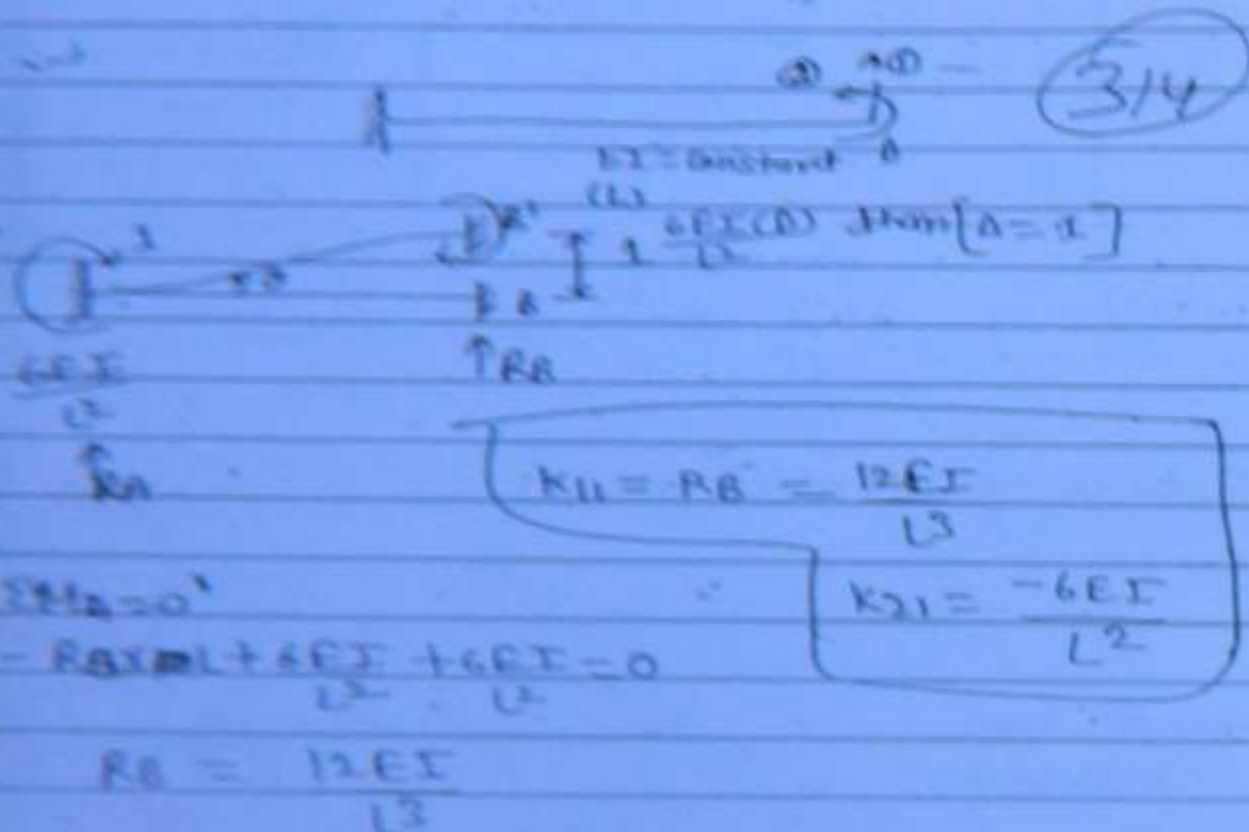
$$[f] = \frac{L}{EI} \begin{bmatrix} L^3/3 & -L^2/2 & -L^2/2 \\ -L^2/2 & L & L \\ -L^2/2 & L & 2L \end{bmatrix}$$



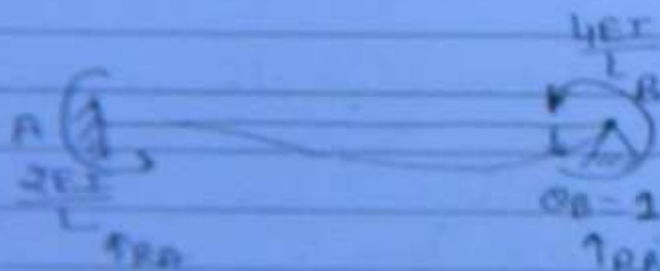




Q. develop stiffness matrix for beam AB with coordinate shown in fig.



2nd member stiffness



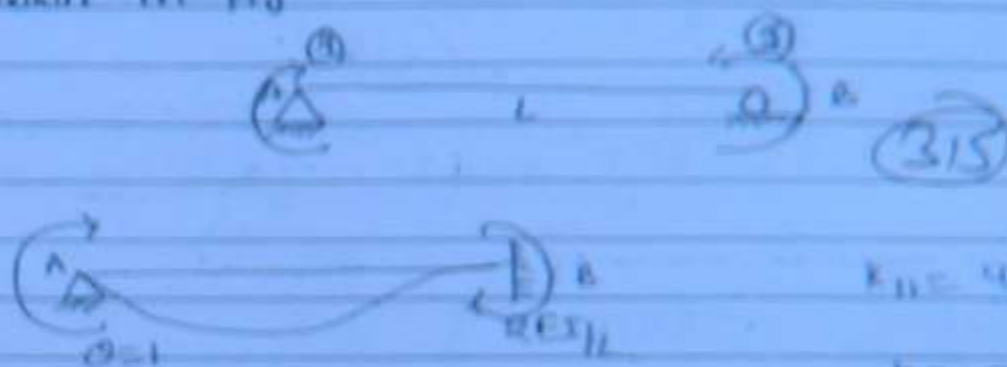
$$\begin{aligned} \sum M_A &= 0 \\ -R_B L + \frac{4EI}{L} - \frac{2EI}{L} &= 0 \\ R_B &= -\frac{6EI}{L^2} \end{aligned}$$

$$\begin{aligned} K_{12} &= R_B = -\frac{6EI}{L^2} \\ K_{22} &= \frac{4EI}{L} \end{aligned}$$

$$[K] = \frac{EI}{L^3} \begin{bmatrix} 12/L^2 & -6/L \\ -6/L & 4 \end{bmatrix}$$

Ans

Q. Generate stiffness matrix for the beam with coordinate shown in fig.



$$K_{11} = \frac{4EI}{L}$$

$$K_{21} = -\frac{2EI}{L}$$



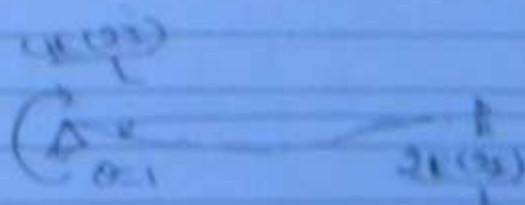
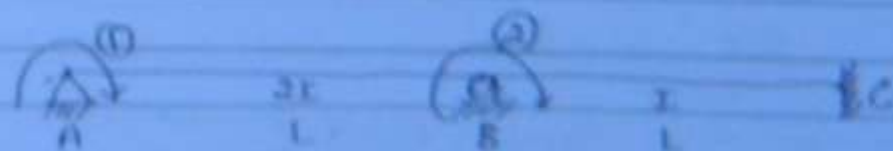
$$K_{12} = -\frac{2EI}{L}$$

$$K_{22} = \frac{4EI}{L}$$

$$[k] = \frac{EI}{L} \begin{bmatrix} 4 & -2 \\ -2 & 4 \end{bmatrix}$$

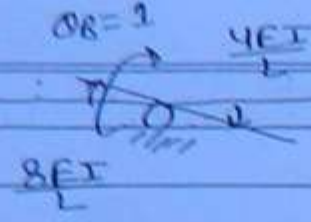
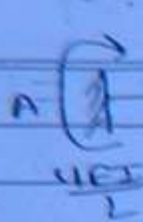
$$k = \frac{2EI}{L} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

Q. develop stiffness matrix for a beam shown in fig.



$$K_{11} = 3EI/L$$

$$K_{12} = 4EI/L$$



Page _____
 Student Name _____
 (3/10)

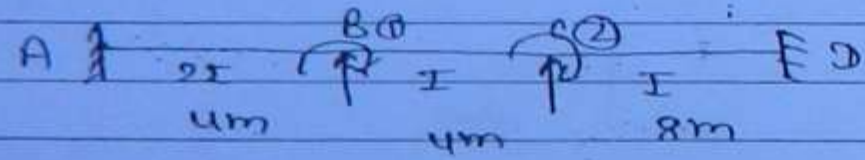
$$K_{12} = 4EI/L$$

$$K_{22} = \frac{8EI}{L} + \frac{4EI}{L} = \frac{12EI}{L}$$

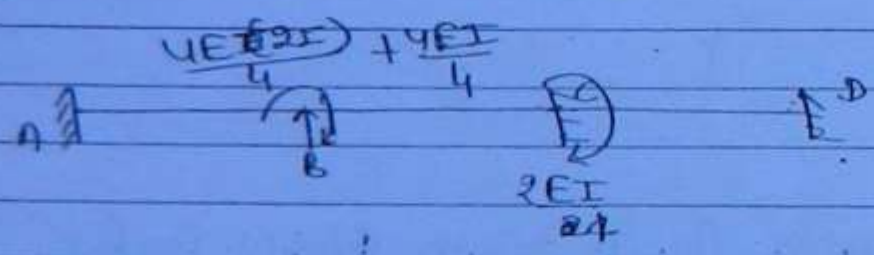
$$K = \begin{bmatrix} 8EI/L & 4EI/L \\ 4EI/L & 12EI/L \end{bmatrix}$$

$$K = \frac{4EI}{L} \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

Q. Develop stiffness matrix for beam shown in fig.

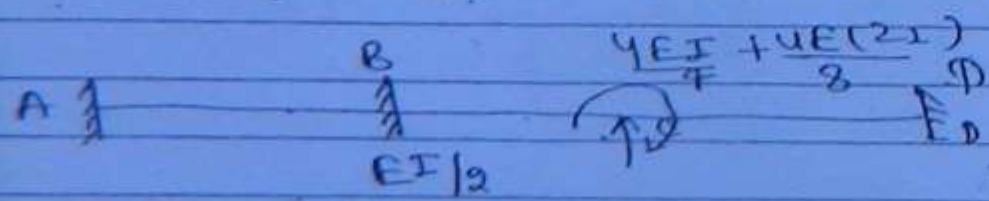


Sol



$$K_{11} = 2EI + EI = 3EI$$

$$K_{12} = \frac{EI}{2} = 0.5EI$$



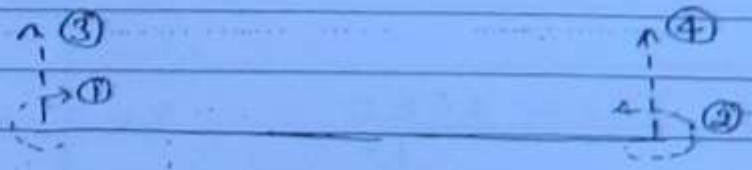
$$K_{12} = \frac{EI}{2} = 0.5EI$$

$$K_{22} = 2EI$$

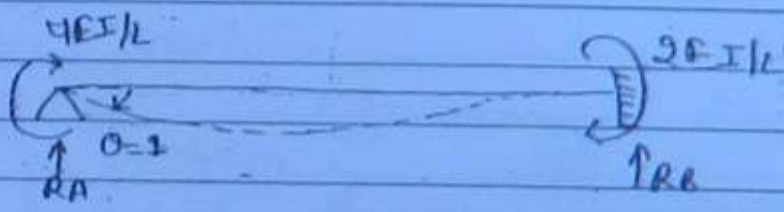
$$K = EI \begin{bmatrix} 3 & 0.5 \\ 0.5 & 2 \end{bmatrix}$$

Ans (317)

Q. Develop stiffness matrix for the beam with coordinate shown in fig.



So)



$\sum M_B = 0$

$$R_A \times L + \frac{4EI}{L} - \frac{2EI}{L} = 0$$

$$R_A = \frac{6EI}{L^2}$$

$$R_A + R_B = 0$$

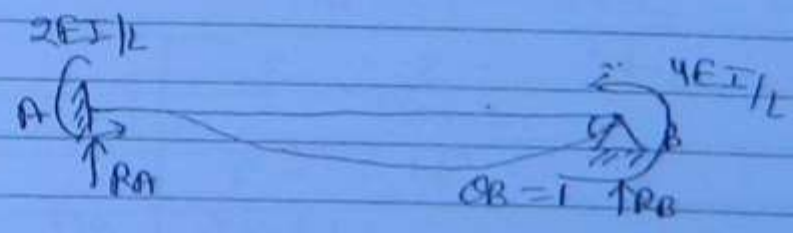
$$R_B = -\frac{6EI}{L^2}$$

$$K_{11} = \frac{4EI}{L}$$

$$K_{21} = -\frac{2EI}{L}$$

$$K_{31} = -\frac{6EI}{L^2}$$

$$K_{41} = \frac{6EI}{L^2}$$



$$K_{12} = -\frac{2EI}{L}$$

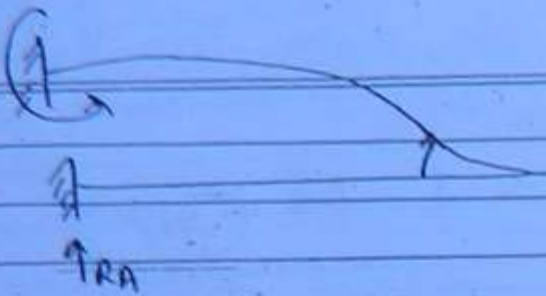
$$K_{22} = \frac{4EI}{L}$$

$$K_{32} = \frac{6EI}{L^2}$$

$$K_{42} = -\frac{6EI}{L^2}$$

$$[K] = \frac{EI}{L} \begin{bmatrix} 4 & -2 & -6 & 6 \\ -2 & 4 & 6 & -6 \end{bmatrix}$$

$$[R] = \frac{2EI}{L} \begin{bmatrix} 2 & -1 & -3 & 3 \\ -1 & 2 & 3 & -3 \end{bmatrix}$$



(3/8)

$$\sum M_B = 0$$

$$R_A = \frac{12EI}{L^3}$$

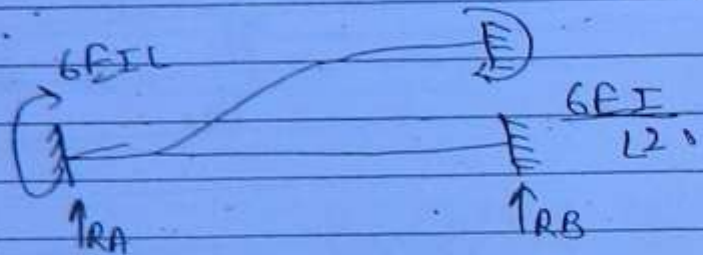
$$R_A = \frac{12EI}{L^3}$$

$$K_{12} = -\frac{6EI}{L^2}$$

$$K_{23} = +\frac{6EI}{L^2}$$

$$K_{33} = \frac{12EI}{L^3}$$

$$K_{34} = -\frac{12EI}{L^3}$$



$$R_A = -\frac{12EI}{L^3}$$

$$R_B = \frac{12EI}{L^3}$$

$$K_{14} = +\frac{6EI}{L^2}$$

$$K_{24} = -\frac{6EI}{L^2}$$

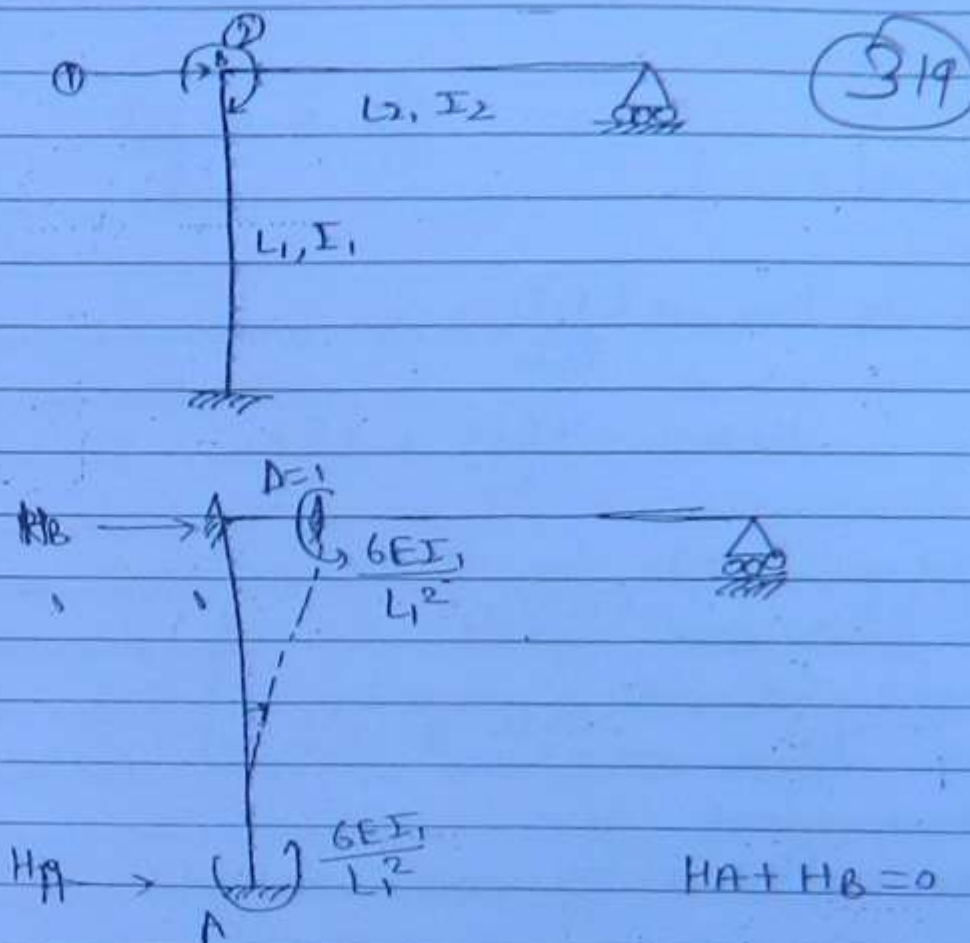
$$K_{34} = -\frac{12EI}{L^3}$$

$$K_{44} = +\frac{12EI}{L^3}$$

$$K = \frac{2EI}{L}$$

2	-1	-3	3
-1	2	3	-3
$-\frac{3}{L}$	$\frac{3}{L}$	$\frac{6}{L^2}$	$-\frac{6}{L^2}$
$\frac{3}{L}$	$-\frac{3}{L}$	$\frac{6}{L^2}$	$\frac{6}{L^2}$

Q. Develop stiffness matrix for the frame shown in fig. with coordinate indicated.



$$\sum M_B = 0$$

$$-H_A \times L_1 - \frac{6EI_1}{L_1^2} - \frac{6EI_1}{L_1^2}$$

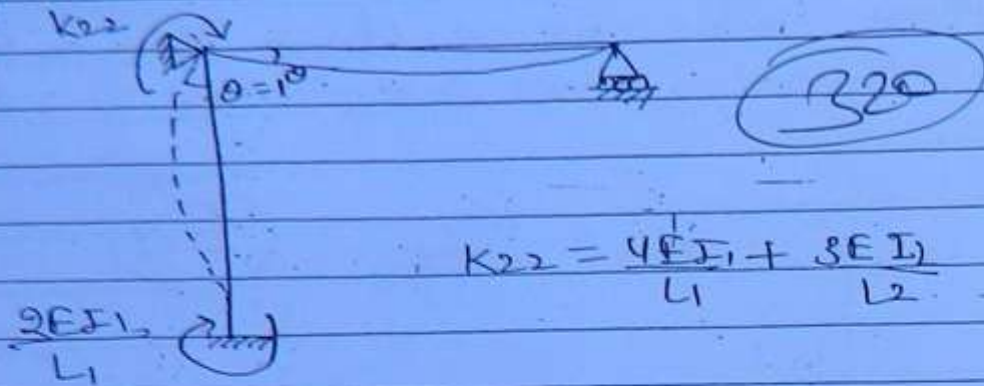
$$H_A = -\frac{12EI_1}{L_1^3}$$

$$H_B = +\frac{12EI_1}{L_1^3}$$

$$K_{11} = \frac{12EI_1}{L_1^3}$$

$$K_{22} = -\frac{6EI_1}{L_1^2}$$

to generate 2nd column:-



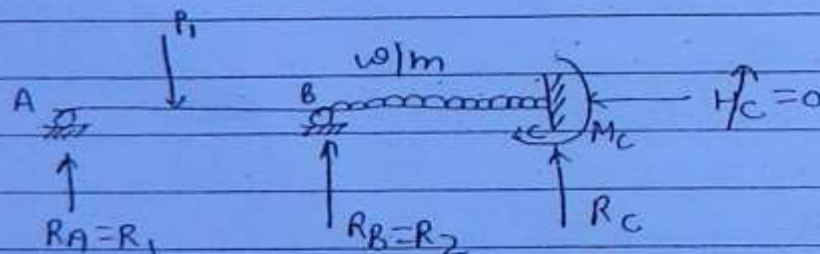
$$K_{22} = \frac{4EI_1}{L_1} + \frac{3EI_2}{L_2}$$

$$K_{12} = -\frac{6EI_1}{L_1^2}$$

$$K_{22} = \frac{4EI_1}{L_1} + \frac{3EI_2}{L_2}$$

$$K = \begin{bmatrix} \frac{12EI_1}{L_1^3} & -\frac{6EI_1}{L_1^2} \\ -\frac{6EI_1}{L_1^2} & \frac{4EI_1}{L_1} + \frac{3EI_2}{L_2} \end{bmatrix}$$

Flexibility matrix method.



Step 1 determine degree of static indeterminacy in beam neglected axial forces.

$$\begin{aligned} D_s &= D_e + D_f \\ &= 5 - 3 \\ &= 5 - 3 = 2 \end{aligned}$$

in above structure $D_s = 2$ Hence identify redundant
Say Redundant are R_A & R_B (Re).
(R1) —

Step 2 Remove the redundant and assign one coordinate in the direction of each redundant the No. of Co-ordinate will be equal to r , that is no. of Redundant.



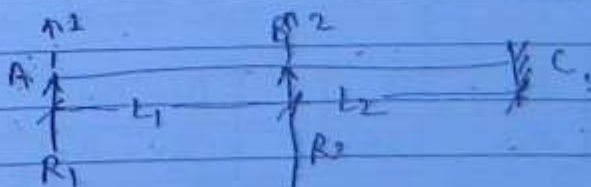
(32)

develop flexibility matrix for the above assign co-ordinates.

Step 3 Remove the Redundant and find the deflections in the direction of assign coordi coordinates due to given load system. When the Redundant are release, the str. are determinate which is called release structure basic determinate structure. Let due to given loading the deflections in the direction of co-ordinate are $\Delta_{1L}, \Delta_{2L} \dots$



Step 4 Remove the given loading and apply the Redundant Reaction in the direction of assigned Co-ordinate.



And find displacement in the direction of co-ordinate due to Redundent let Δ_{1R}, Δ_{2R} ---
 --- Δ_{1R}, Δ_{2R} the displacements cause by Redundent in the direction of co-ordinate.

$$\Delta_{1R} = f_{11}R_1 + f_{12}R_2$$

$$\Delta_{2R} = f_{21}R_1 + f_{22}R_2$$

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$$\begin{bmatrix} \Delta_{1R} \\ \Delta_{2R} \end{bmatrix} = \begin{bmatrix} f \\ f \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \end{bmatrix}$$

$$\begin{bmatrix} \Delta_{1R} \\ \Delta_{2R} \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \end{bmatrix}$$

cp-5 the final displacement in the direction of co-ordinate will be $\Delta_1 = \Delta_{1L} + \Delta_{1R}$
 $\Delta_2 = \Delta_{2L} + \Delta_{2R}$

but final displacement are

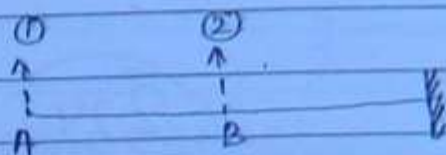
$$\Delta_{1R} = \overset{\Delta_1^0}{\Delta_1} - \Delta_{1L}$$

$$\Delta_{2R} = \overset{\Delta_2^0}{\Delta_2} - \Delta_{2L}$$

$$\begin{bmatrix} \Delta_{1R} \\ \Delta_{2R} \end{bmatrix} = \begin{bmatrix} \Delta_1 - \Delta_{1L} \\ \Delta_2 - \Delta_{2L} \end{bmatrix} \Rightarrow \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \end{bmatrix} = \begin{bmatrix} \Delta_1 - \Delta_{1L} \\ \Delta_2 - \Delta_{2L} \end{bmatrix}$$

$$\begin{bmatrix} R_1 \\ R_2 \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}^{-1} \begin{bmatrix} \Delta_1 - \Delta_{1L} \\ \Delta_2 - \Delta_{2L} \end{bmatrix}$$

$$\begin{bmatrix} R_1 \\ R_2 \end{bmatrix} = [f]^{-1} \begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix}$$



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f_{11} = displacement at ① due to unit force at ①

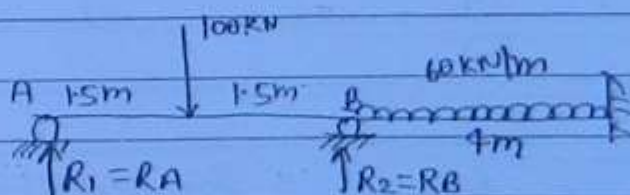
f_{12} = displacement at ① due to unit force at ②

$$\Delta_{1R} = f_{11} \cdot R_1 + f_{12} \cdot R_2$$

$$\Delta_{2R} = f_{21} \cdot R_1 + f_{22} \cdot R_2$$

Ex.

Analysis the beam shown in fig. find the reaction using flexibility matrix method.

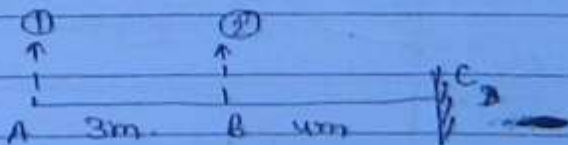


EI = constant

$$DS = 7 - 3 \Rightarrow 5 - 3 = 2$$

Let Redundent are R_A & R_B .

Assign one coordinate in dirn of each Redundent.

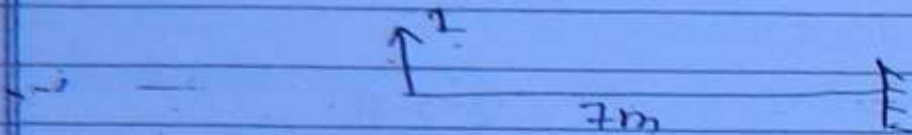


1st Column of Flexibility matrix

$$f_{11} = \frac{7^3}{3EI} \Rightarrow \frac{7^3}{3EI} = \frac{343}{3EI}$$

$$f_{21} = \frac{136}{3EI}$$

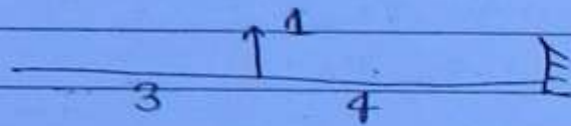
1st Column of flexibility matrix:



$$f_{11} = \frac{1 \cdot 7^3}{3EI} \rightarrow \frac{343}{3EI}$$

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2nd column of flexibility matrix



$$f_{21} = \frac{4 \cdot 4^3}{3EI} + \frac{1 \cdot 4^2}{2EI} \times 3$$

$$\Rightarrow \frac{64}{3EI} + \frac{24}{EI} \rightarrow$$

$$f_{22} = \frac{1 \cdot 4^3}{3EI} = \frac{64}{3EI}$$

$$[f] = \begin{bmatrix} \frac{343}{3EI} & \frac{136}{3EI} \\ \frac{136}{3EI} & \frac{64}{3EI} \end{bmatrix}$$

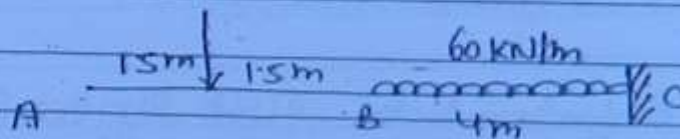
$$[f]^{-1} = \begin{bmatrix} \frac{64}{3EI} & -\frac{136}{3EI} \\ -\frac{136}{3EI} & \frac{343}{3EI} \end{bmatrix}$$

$$\Rightarrow \frac{343}{3EI} \times \frac{64}{3EI} - \left(\frac{136}{3EI} \right)^2$$

$$[f]^{-1} = EI \begin{bmatrix} 0.055 & -0.118 \\ -0.118 & 0.297 \end{bmatrix}$$

(325)

in Basic Release Structure find Δ_{11} and Δ_{21} due to given loading in the direction of axis Co-ordinate.



$$\Delta_{11} = - \left[\frac{100(5.5)^2}{3EI} + \frac{100(5.5)^2}{2EI} \times 1.5 + \frac{60 \times 4^3}{8EI} + \frac{60 \times 4^3}{6EI} \times 3 \right]$$

$$\Delta_{11} = \frac{-11654.58}{EI}$$

$$\Delta_{21} = \left[\frac{100 \times 4^3}{3EI} + \frac{100(4)^2}{2EI} \times 1.5 + \frac{60 \times 4^4}{80EI} \right]$$

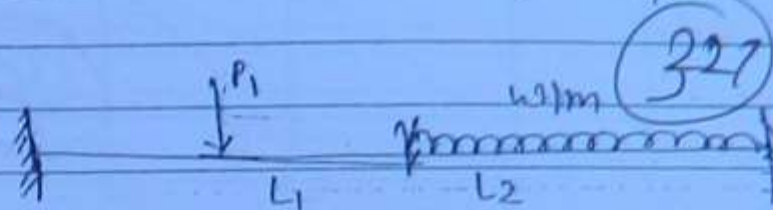
$$\Delta_{21} = \frac{-5253.33}{EI}$$

$$\begin{bmatrix} R_1 \\ R_2 \end{bmatrix} = [f]^{-1} \begin{bmatrix} \Delta^o - \Delta_{11} \\ \Delta^o - \Delta_{21} \end{bmatrix}$$

$$\begin{bmatrix} R_1 \\ R_2 \end{bmatrix} = \begin{bmatrix} 0.055EI & -0.118EI \\ -0.118EI & 0.297EI \end{bmatrix} \begin{bmatrix} 0 + 11654.58/EI \\ 0 + 5253.33/EI \end{bmatrix}$$

KB

Step 3 Let P_1' and P_2' are forces of the moment required/develop at coordinate ① & ② to lock the structure, due to given loading



$$\bar{M}_{AB} = P_1' = -\frac{PL_1}{8}$$

$$\bar{M}_{BA} + M_{Bx} = P_2' = +\frac{PL_1}{8} - \frac{wL_2^2}{12}$$

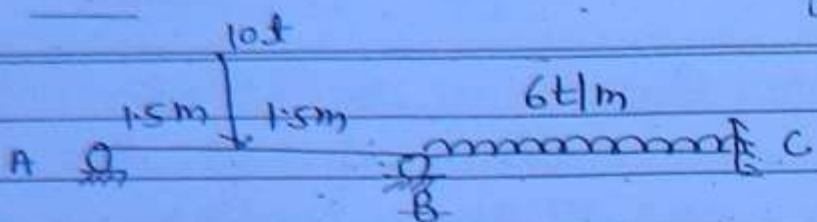
Let P_1, P_2 ----- are final forces/moment available in the direction of assign co-ordinate. if No external load or moment acts in direction of origin co-ordinate, then final value P_1 and $P_2 = 0$

$$\begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix} = \begin{bmatrix} K_1' \end{bmatrix}^{-1} \begin{bmatrix} P_1^0 - P_1' \\ P_2^0 - P_2' \end{bmatrix}$$

P_1' and P_2' Nothing that fixed end moment at co-ordinate ① and ② when structure is locked due to given loading.

If co-ordinates are linear then P_1' and P_2' will be reactions due to given loading in the direction of co-ordinate.

Q.

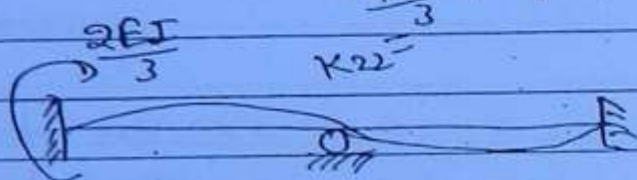


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$$K_{11} = \frac{4EI}{3}$$

$$K_{21} = \frac{2EI}{3}$$

$$\frac{4EI}{3} + \frac{4EI}{4} = \frac{7EI}{3}$$



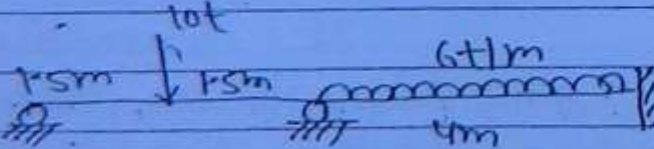
$$K_{21} = \frac{2EI}{3}$$

$$K_{22} = \frac{7EI}{3}$$

$$[K] = \begin{bmatrix} \frac{4EI}{3} & \frac{2EI}{3} \\ \frac{2EI}{3} & \frac{7EI}{3} \end{bmatrix}$$

$$K^{-1} = \begin{bmatrix} 7/8EI & -1/4EI \\ -1/4EI & 1/2EI \end{bmatrix}$$

then multiply



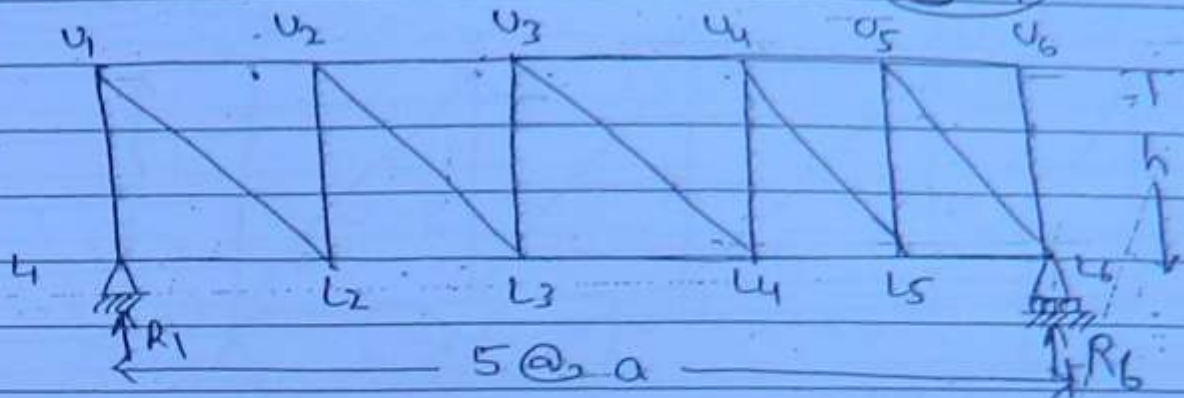
$$A'_1 = \bar{M}_{AB} = -\frac{10 \times 3}{8} \Rightarrow -3.75$$

$$B'_2 = \bar{M}_{BA} + \bar{M}_{BC} = +3.75 - \frac{6 \times 4^2}{12}$$

$$\Rightarrow 3.75 - 8 \Rightarrow -4.25$$

ILD for Truss

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→ In simply supported trusses when unit load moves from one end to the other end then top chord members are in comp. Bottom chord are in tension. And in inclined member force change ~~in~~ for comp to tension.

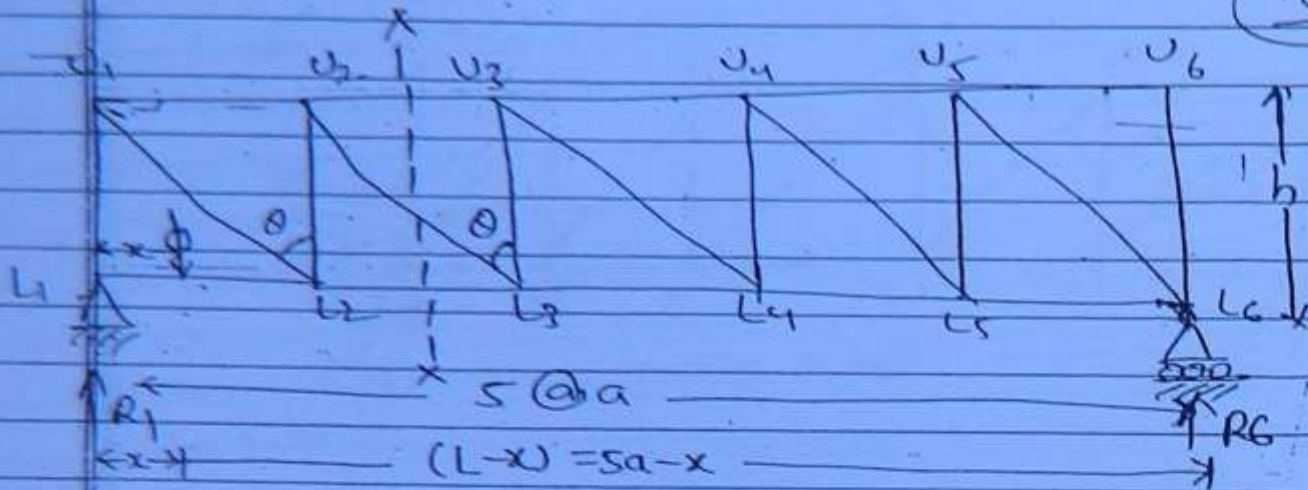
When unit load is left of member then force in inclined member is compression and when unit load is right of the member then force is tension. In vertical members force changes tension to comp.

If load moves over top chord then truss is called Deck type truss and if load moves over bottom chord then truss is called through type.

~~then~~

Q Draw ILD for $U_2 U_3$, $L_2 L_3$ and L_2 and L_3

330



Consider unit load placed at a distance x

$$L = 5a$$

$$\sum M_6 = 0$$

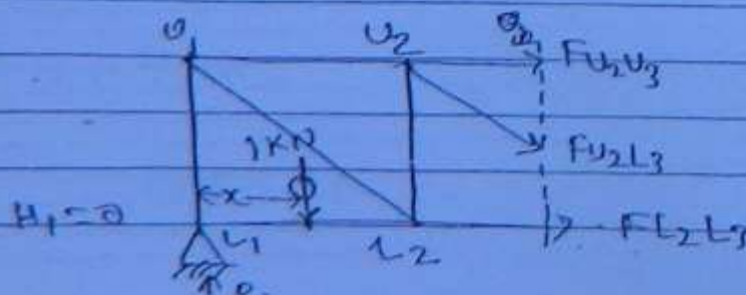
$$R_1 \times L - 1(L-x) = 0$$

$$R_1 = \frac{L-x}{L} \rightarrow \frac{5a-x}{5a}$$

$$R_6 = \frac{x}{L} = \frac{x}{5a}$$

ILD for $L_2 L_3$

Case-1 When unit load is to the left of L_2 / U_2
 $U_2 L_2$: Consider free body dia. to the left of $x-x$



To find $F_{L_2L_3}$ take moment about u_2

$$\sum M_{u_2} = 0$$

(33)

$$R_1 x a - 1x(a-x) - F_{L_2L_3} \cdot h = 0$$

$$F_{L_2L_3} = \frac{R_1 a - 1x(a-x)}{h}$$

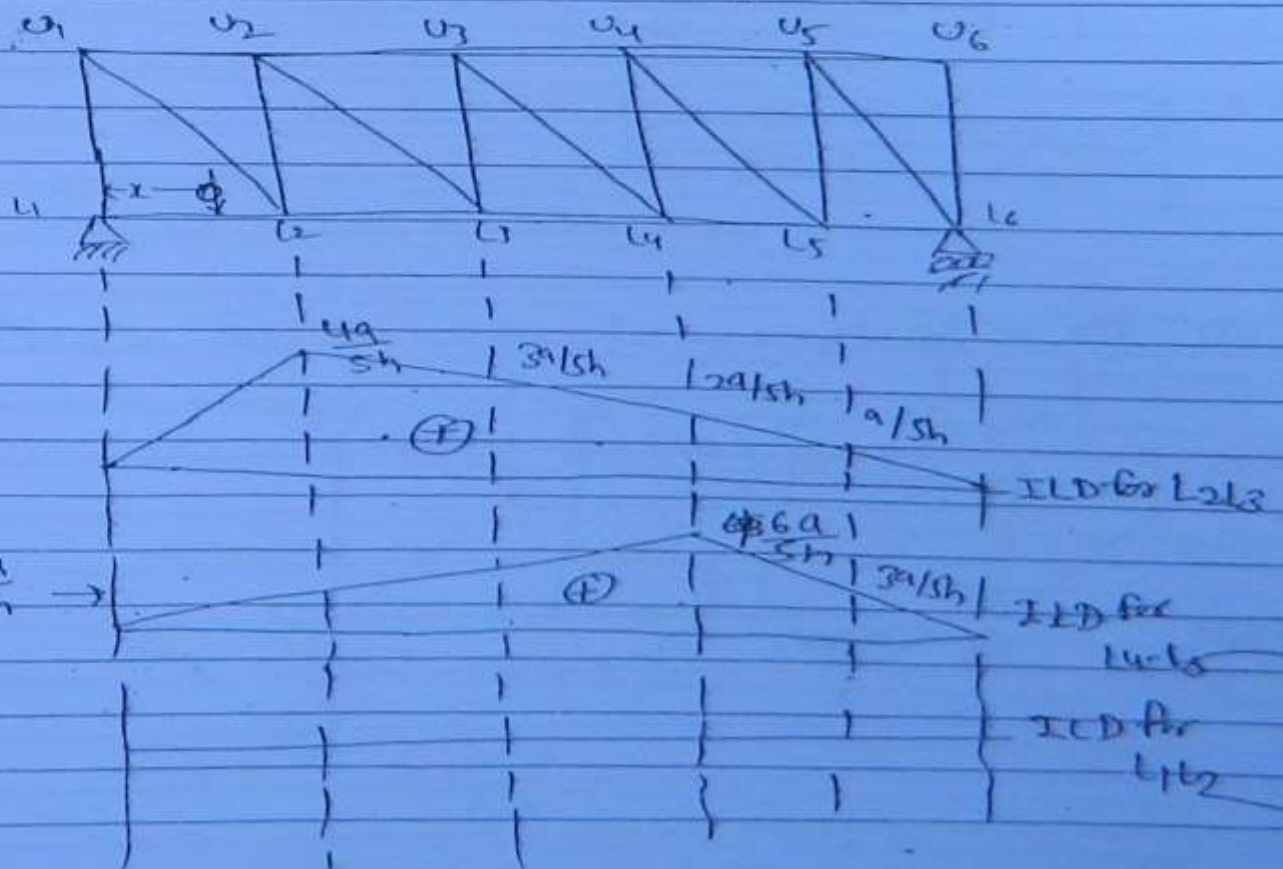
$$F_{L_2L_3} = \left[\frac{5a-x}{5a} \cdot a \right] - (a-x)$$

$$F_{L_2L_3} = \frac{5a-x-5a+5x}{5h}$$

$$F_{L_2L_3} = \frac{4x}{5h}$$

when $x=0$, $F_{L_2L_3} = 0$

when $x=a$, $F_{L_2L_3} = \frac{4a}{5h}$ (+) Tension.



Case-I when unit load is in $L_2 L_3$

$$\sum M_{u_2} = 0$$

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$$R_1 x a - F_{L_2 L_3} x h = 0$$

$$F_{L_2 L_3} = \frac{R_1 \cdot a}{h}$$

$$F_{L_2 L_3} = \left[\left(\frac{5a-x}{5a} \right) \right] \frac{1}{h}$$

$$F_{L_2 L_3} = \frac{(5a-x)}{5h}$$

if $x = a$

$$F_{L_2 L_3} = \frac{4a}{5h}$$

if $x = 2a$

$$F_{L_2 L_3} = \frac{3a}{5h}$$

if $x = 0$

$x = 5a$

$$F_{L_2 L_3} = 0$$

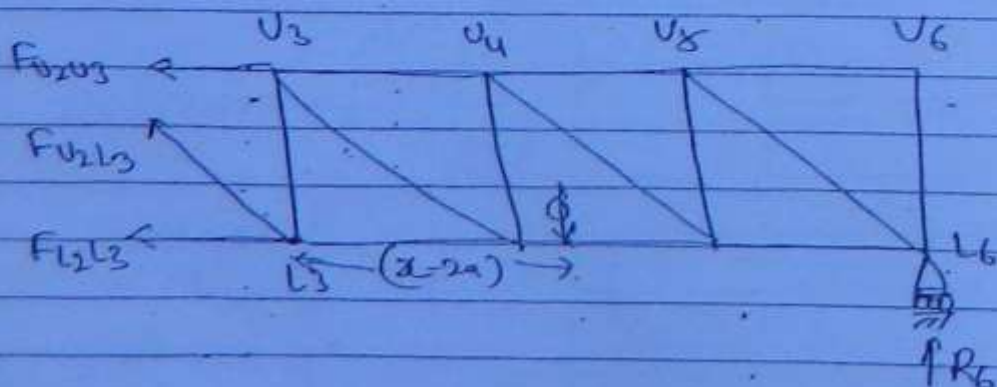
$$\left(\frac{a_1 \times a_2}{a_1 + a_2} \right) \times \frac{1}{h}$$

Revise formula

$$\frac{ax + ya}{a + y} \times \frac{1}{h}$$

$$\Rightarrow \frac{4a^2}{5ah} = \frac{4a}{5h}$$

ILD for U_2, U_3



$$\begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix} = \begin{bmatrix} K \\ \end{bmatrix} \begin{bmatrix} P_1 - P_1' \\ P_2 - P_2' \end{bmatrix}$$

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$$\begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix} = \begin{bmatrix} 7/8EI & -1/4EI \\ -1/4EI & 1/2EI \end{bmatrix} \begin{bmatrix} 0 + 3.75 \\ 0 + 4.25 \end{bmatrix}$$

$$\Delta_1 = \Delta_A = +2.22$$

EI

$$\Delta_2 = \Delta_B = +1.187$$

EI

To find final end moment write slope deflection equations.

$$M_{AB} = \bar{M}_{AB} + \frac{2EI}{L} \left[2\Delta_A + \Delta_B - \frac{3\Delta}{L} \right] \Rightarrow$$

$$\Rightarrow -3.75 + \frac{2EI}{3} \left[\frac{2 \times 2.22}{EI} + \frac{1.187}{EI} \right]$$

$$M_{AB} \Rightarrow -3.75 + 3.75 \Rightarrow 0$$

$$M_{BA} = \bar{M}_{BA} + \frac{2EI}{L} \left[2\Delta_B + \Delta_A - \frac{3\Delta}{L} \right]$$

$$\Rightarrow +3.75 + \frac{2EI}{3} \left[\frac{2 \times 1.187}{EI} + \frac{2.22}{EI} \right]$$

$$\Rightarrow +6.81 \text{ tm}$$

$$M_{BC} = \bar{M}_{BC} + \frac{2EI}{L} \left[2\Delta_B + \Delta_C - \frac{3\Delta}{L} \right]$$

$$\Rightarrow -8 + \frac{EI \times 1.187}{EI}$$

$$\Rightarrow -6.81 \text{ tm}$$

$$M_{CB} = +8.55$$

Chapter = 6

① c

② d

③ a

④ d

⑤ b

⑥ a

⑦ c

⑧ A

⑨ da

⑩ c

⑪ b

⑫ c

⑬ a d c

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Case 1 When unit load is in L_1, L_3

$$\sum M_{L_3} = 0$$

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$$F_{U_2} U_3 \times h + R_6 \times 3a = 0$$

$$F_{U_2} U_3 = -R_6 \times \frac{3a}{h} \Rightarrow -\frac{x}{5a} \times \frac{3a}{h}$$

$$\Rightarrow -\frac{3}{5} \frac{x}{h}$$

Case-2 When load is in L_3, L_6

$$\sum M_3 = 0$$

$$R_6 \times 3a + F_{U_2} U_3 \times h - 1(x - 2a) = 0$$

$$F_{U_2} U_3 = \frac{-R_6 \times 3a + (x - 2a)}{h}$$

$$F_{U_2} U_3 = \frac{-x/5a \times 3a + x - 2a}{h}$$

$$F_{U_2} U_3 \Rightarrow \frac{a}{5} \left(\frac{x - 5a}{h} \right)$$

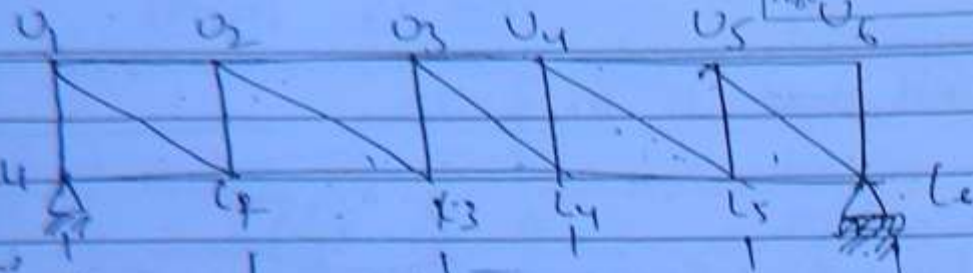
When $x = 2a$

$$F_{U_2} U_3 = -\frac{6}{5} \frac{a}{h}$$

When $x = 5a$

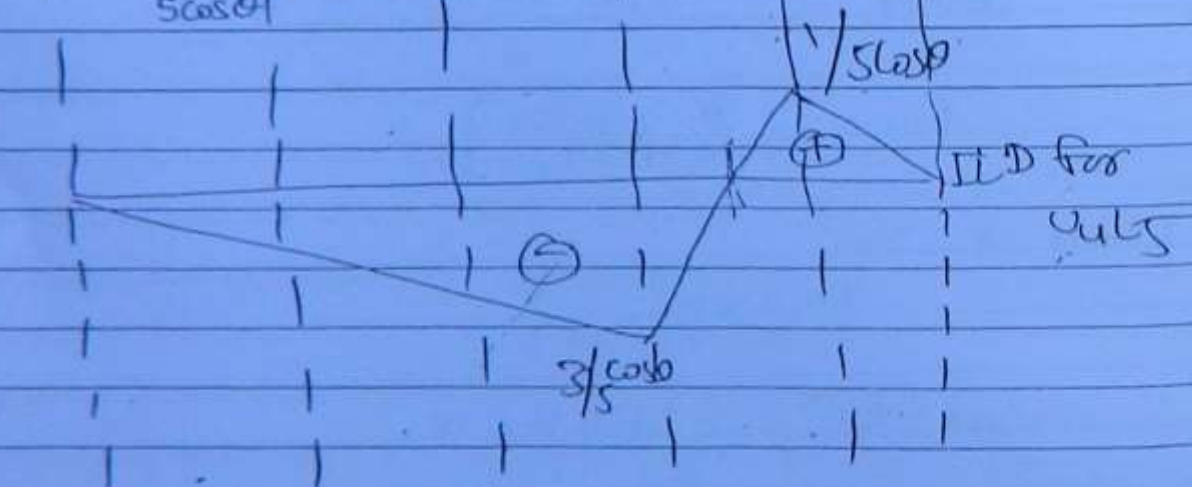
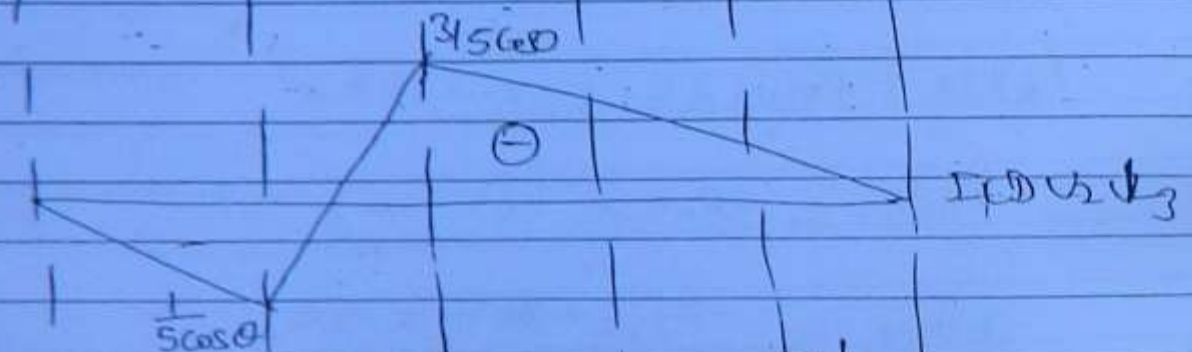
$$F_{U_2} U_3 = 0$$

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$$\frac{1}{5}$$

$$\frac{1 \times 10}{1.402}$$

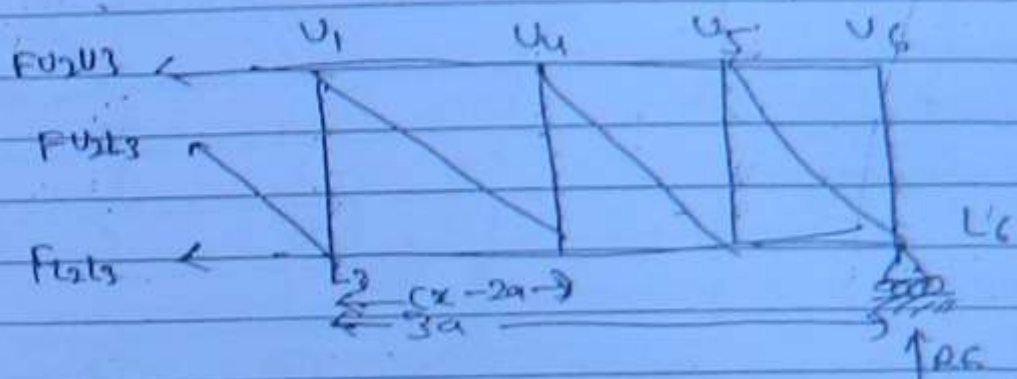


in
ILD $\uparrow$ $U_2 L_3$:-

Inclined member makes angle θ with the vertical

(337)

Case-1 when unit load is in $L_1 L_2$



$$\sum F_y = 0$$

$$F_{U2U3} \cos \theta + R_6 = 0$$

$$F_{U2U3} = \frac{-R_6}{\cos \theta} = \frac{-x}{5a} \times \frac{1}{\cos \theta}$$

Case-2 when 1KN is in $L_3 L_6$

$$F_{U2L3} \cos \theta + R_6 - 1 = 0$$

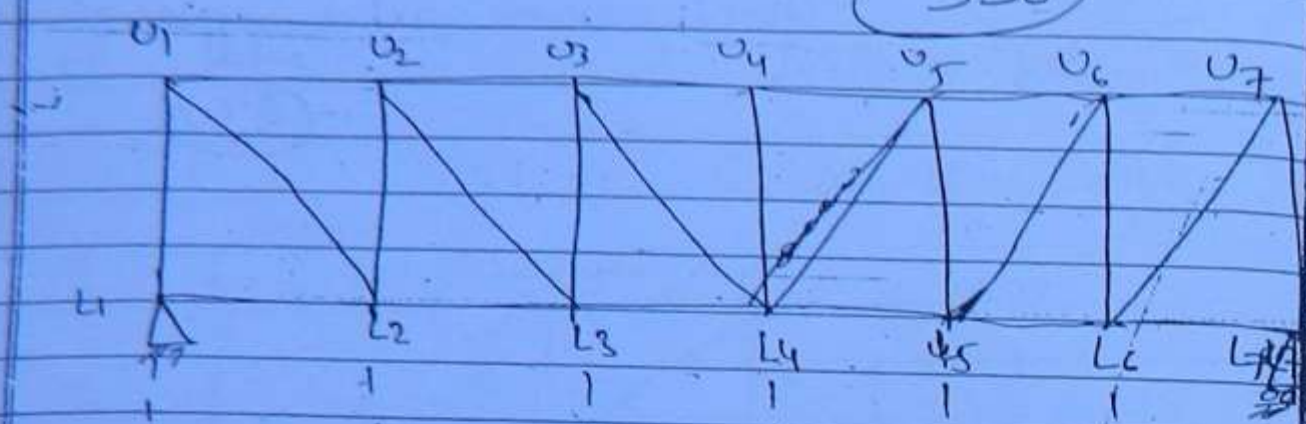
$$F_{U2L3} = \frac{1 - x/5a}{\cos \theta}$$

$$\text{if } x = 2a$$

$$F_{U2L3} = \frac{3}{5 \cos \theta}$$

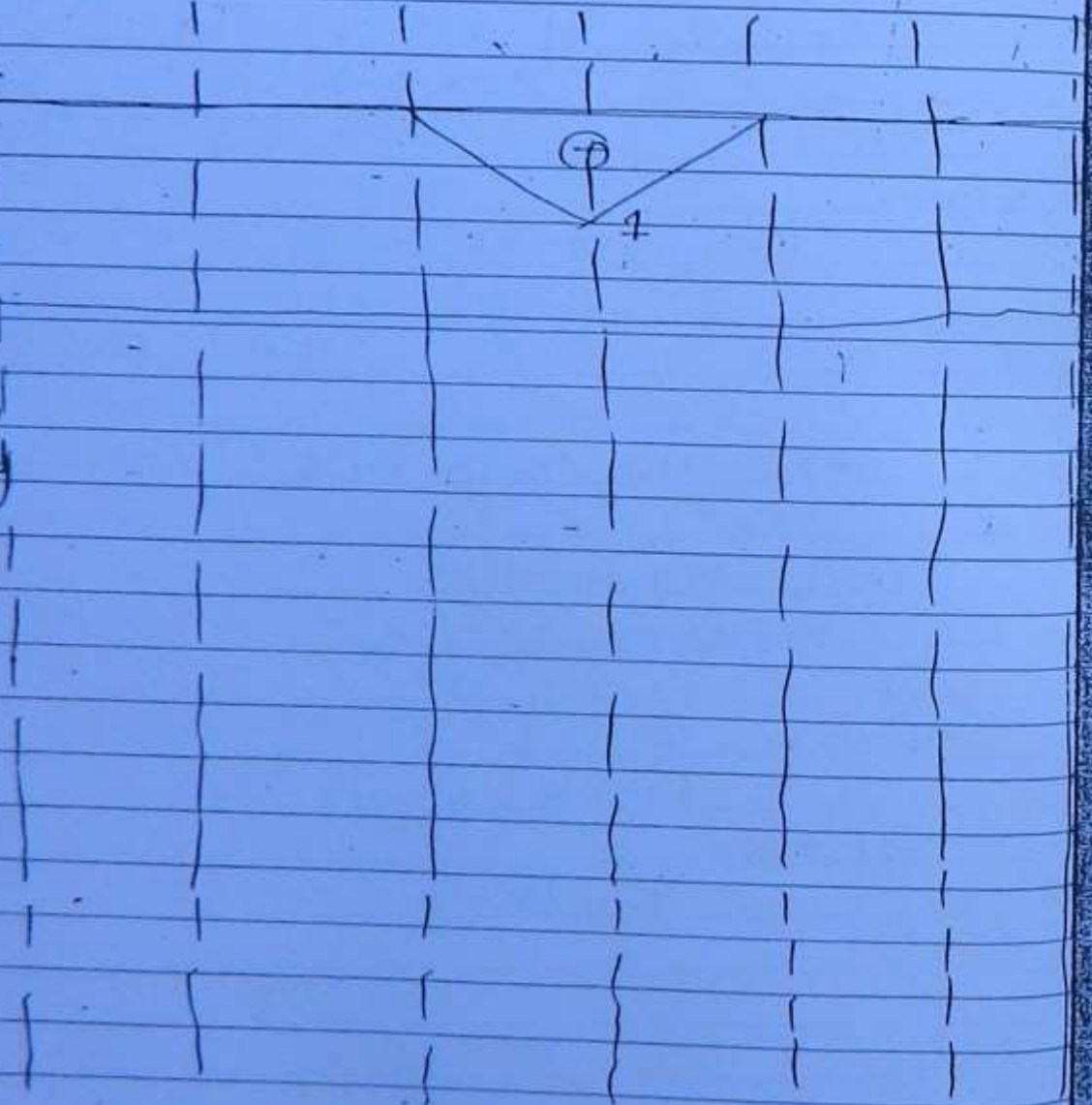
$$\text{if } x = 5a$$

$$F_{U2L3} = 0$$



ILD for U_{3L4}
 when chord is Deck
 type [Unit Load on
 Top chord]

ILD for U_{3L4} when
 chord is Trough
 type [Unit Load on
 Bottom chord]



Problem 5

- | | | |
|-------------------------------|---------------|----------|
| (1) b | (8) b | (13) c |
| (2) d | (9) a | (16) a b |
| (3) d | (10) b | (17) b |
| (4) b (by method of sections) | (11) None (e) | (18) b |
| (5) d | (12) b | (19) c |
| (6) d | (13) c | (20) a |
| (7) c | (14) d | (21) a |
| (8) d | (22) b | (24) . |
| | (23) b | |

(9) for max tension live load should be only in tension zone.

$$\begin{aligned}
 & \Rightarrow D.L \text{ (Net Area)} + L.L \times (\text{Area}) \\
 & \Rightarrow 20 \times [(-10 + 20) + [10 \times 20]] \\
 & \Rightarrow 400 \text{ kN}
 \end{aligned}$$

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