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# -: HAND WRITTEN NOTES:-

## OF

# CIVIL ENGINEERING

1

## -: SUBJECT:-

# RCC & PRESTRESSED CONCRETE



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| S. No. | Date | Title | Page No. | Remarks |
|---|---|---|---|---|
| 1 | | Working Stress Method | | |
| | | (i) Singly reinforced | | |
| | | (ii) Doubly reinforced | | |
| | | (iii) T- beam | | |
| 2. | | Limit State Method | | |
| | | (i) Singly reinforced | | |
| | | (ii) Doubly | | |
| | | (iii) T- beam | | |

# WORKING STRESS METHOD

[redacted]

Introduction

R.C.C ~~is made up of~~ has two main Comp. Concrete and Steel reinforcement

properties of Concrete

① young's modulus of elasticity $E_c = 5000\sqrt{f_{ck}}$

$f_{ck}$ = characteristic strength of Concrete

→ This is the value of strength below which not more than 5% of test results are expected to fall.

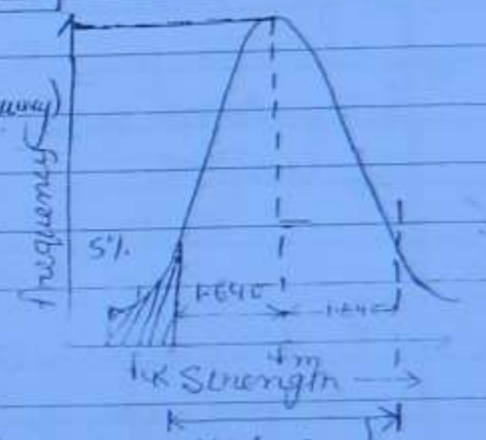
eg:- If we take 100 no. of Cubes to be tested then

17, 18.0 18.5 19.0 19.5, 20, 21, 22, ......

5% are fail — characteristic comp strength — 95%



M-20 Grade Concrete $\boxed{f_{ck} = 20 N/mm^2}$




$f_m$ — mean strength value (having max. frequency)

It is the average strength

$\boxed{f_m = f_{ck} + 1.64\sigma}$

σ = Standard deviation

for M20 & M25

M30 & above = 5N/mm²

→ Design mix is done for target mean strength value.

Confidence limit

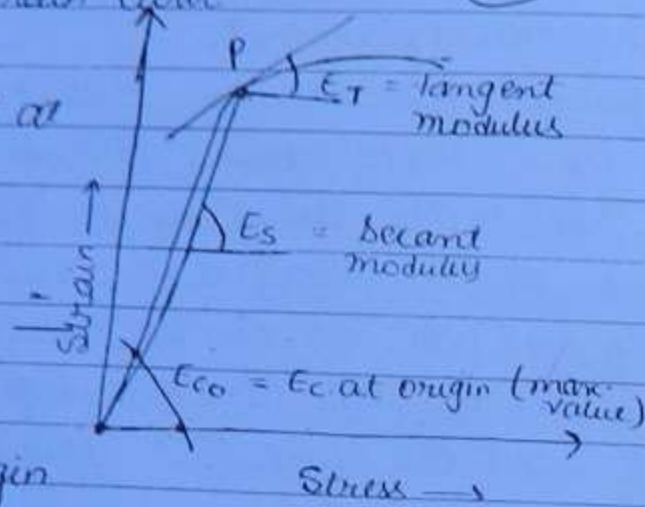
Probability of a single test result is more of falling betn the limit $(f_m - 1.64\sigma)$ to $(f_m + 1.64\sigma)$. This limit is called Confidence limit.

→ Probability % of lying a test result betn confidence limit is called Confidence level.

→ If % of test result falling below a value is 50% the strength value $f_m$

Young's modulus of elasticity ($E_c$)

$E_c$ is the slope of stress-strain curve




es :- ① Young's modulus of elasticity at origin: $E_c = 5000\sqrt{f_{ck}}$
(Considered by IS:456)

② Secant modulus - Slope of line joining any point on stress-strain curve with origin

③ Tangent modulus - Slope of tangent at any pt. on the curve

Long term modulus of elasticity (considering the effect of creep)

$$E_{ce} = \frac{E_c}{1+\theta} = \frac{5000\sqrt{f_{ck}}}{1+\theta}$$

$\theta$ creep coefficient



Here:-

| As per age of testing | $\theta$ Creep Coefficient |
|---|---|
| 7 days | 2.2 |
| 28 days | 1.6 |
| 1 year | 1.1 |

$$\text{Creep Coefficient} = \frac{\text{Creep Strain}}{\text{Elastic Strain}}$$

→ The value of young's modulus of elasticity reduces 2 to 3 times due to the effect of creep.

Strength of Concrete

(a) Compressive Strength

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## Permissible Stress in W.S.M

| Grades | Direct Compression $\sigma_{cc}$ | Bending Compression $\sigma_{cbc}$ |
|---|---|---|
| M15 | 4.0 | 5.0 |
| M20 | 5.0 | 7.0 |
| M25 | 6.0 | 8.5 |
| M30 | 8.0 | 10.0 |
| M35 | 9.0 | 11.5 |
| M40 | 10.0 | 13.0 |

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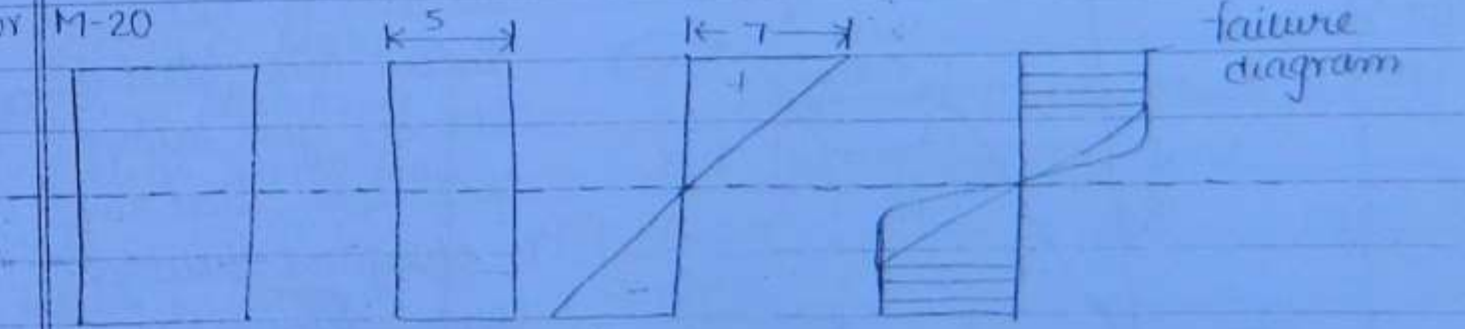
(b) Tensile Stresses

flexural tensile strength (bending) of Concrete

$f_{cr} = 0.7\sqrt{f_{ck}}$

| Grades | Direct tensile strength | Bending tensile strength |
|---|---|---|
| M15 | 2.0 | 2.71 |
| M20 | 2.8 | 3.13 |
| M25 | 3.2 | 3.5 |
| M30 | 3.6 | 3.83 |
| M35 | 4.0 | |
| M40 | 4.4 | |

for M-20




failure diagram

Direct Stress $P/A$ (All fibres are stressed to same value so chances of failure is more)

Bending Stress (Only top & bottom fibre are at max. stress so permissible stress is more than that of direct stress value)

Grade Of Concrete (As per IS: 456) (Table-2/ Pg-No-16)

| | |
|---|---|
| Ordinary Concrete | M10 to M20 |
| Standard Concrete | M25 to M55 |
| High Strength Concrete | M60 & above |

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mix (M)(25) → fck value at 28 days strength

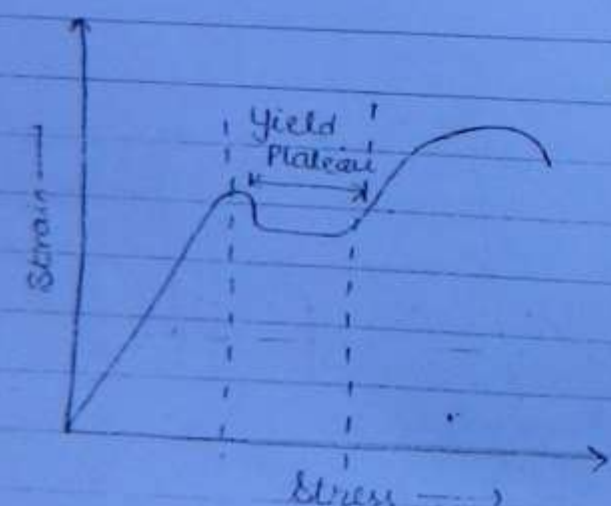
→ Min Recommended grade of Concrete for is M 20 (pg No-1) IS:456

Steel :- Types:-

1. Mild Steel — Ordinary / Hot Rolled
2. Medium tensile Steel — Ordinary / Hot Rolled
3. CTD bars (Cold twisted defined bars) or HYSD bars (High yield strength deformed bars)
4. TMT bars

Properties Of Hot Rolled mild steel bars --

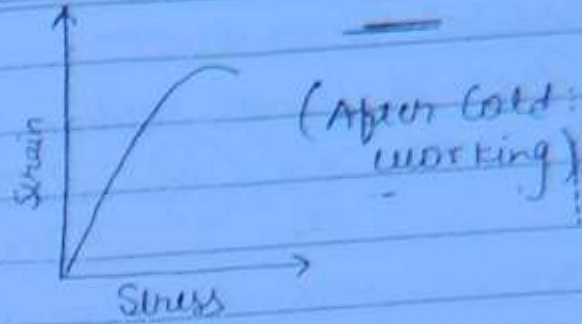
Hot Rolled mild Steel




(In case Of hot Rolled mild Steel if Stresses goes beyond yield limit, Steel reinf is elongated resulting large deformation and Cracking (i.e Undesirable).)

Remedy - (Cold working) → The Steel reinforcement is Stressed beyond yield limit either by Stretching or Twisting & then by unloading it.

Cold twisted bars are made using Cold working process

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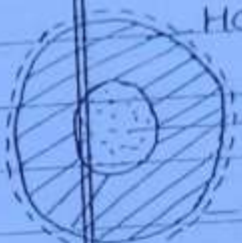
TMT bars -




Also called as thermo mechanically treated bars

Permissible stresses in steel (table no- 22 / P.no-82)

W.S.M

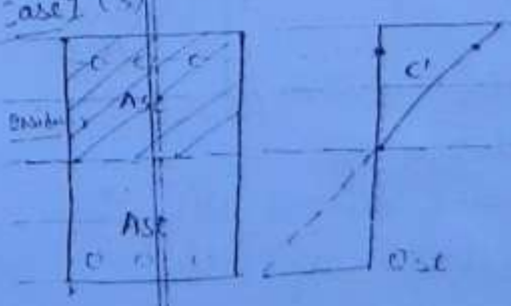
| | Mild steel fe 250 | Medium tensile Steel | CTD bars or HYSD bars fe 415 |
|---|---|---|---|
| ① Tension ($\sigma_{st}$ or $\sigma_{sc}$) | | | |
| upto 20 mm$\phi$ | 140 | 190 | 230 |
| > 20 mm$\phi$ | 130 | = | |
| ② Compression ($\sigma_{sc}$) All steel | 130 | | |
| ③ Compression in bar in case of beam or slab | | | |

Case I
When Comp resistance of Concrete is taking into account = 1.5 m·C'
C' = Stress in Concrete surrounding steel

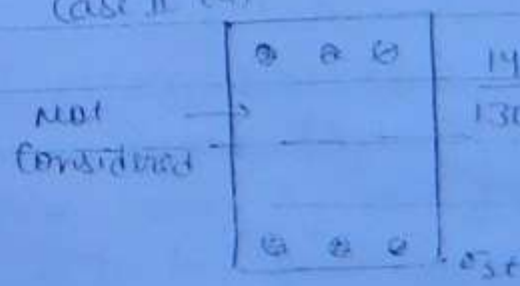
Case II
When Comp resistance is not taking into account

| | | | |
|---|---|---|---|
| upto 20mm | 140 | 190 | 190 |
| > 20mm | 130 | | |

Case I (3)




Case II (a)




$\frac{140}{130}$ | 190 | 190

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# SINGLY REINFORCED BEAM (W.S.M)

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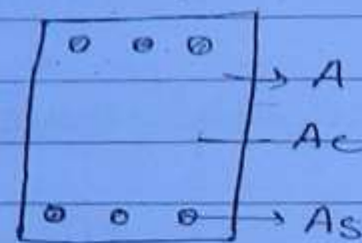
1 Modular Ratio (m)

Ratio of young's modulus of elasticity of steel to concrete

$$m = \frac{E_s}{E_c} = \frac{280}{3\sigma_{cbc}} \rightarrow \text{As per IS 456.}$$

| | M15 | M20 | M25 | M30 | M35 | M40 |
|---|---|---|---|---|---|---|
| $\sigma_{cbc}$ | 5 | 7 | 8.5 | 10 | 11.5 | 13.0 |
| m | 19 | 13 | 11 | 9 | 8 | 7 |

Consider a Column having total Area A

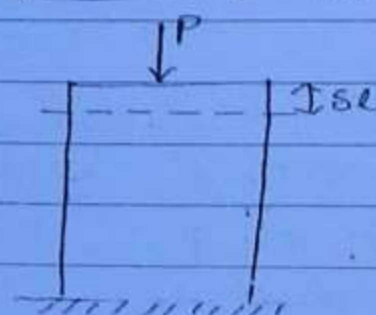



$A = A_c + A_s$

If total load P

$P = P_c + P_s$

$\boxed{P = A_s \cdot p_s + A_c \cdot p_c}$ —— (i)




Deformation δl shall be same in Concrete & Steel

$$\delta l = \frac{P_s \cdot l}{A_s \cdot E_s} = \frac{P_c \cdot l}{A_c \cdot E_c}$$

$$\frac{P_s}{A_s \cdot E_s} = \frac{P_c}{A_c \cdot E_c} \Rightarrow \frac{p_s}{E_s} = \frac{p_c}{E_c}$$

$$\Rightarrow \frac{p_s}{p_c} = \frac{E_s}{E_c} = m$$

If strain value is same $p_s = m \cdot p_c$

$t = m \cdot c$ $\qquad \sigma_{st} = m\,\sigma_{cbc}$

$\boxed{c = t/m}$

Value of ps in (i) we get

$P = A_s \cdot m\, p_c + A_c \cdot p_c$

$P = p_c (m A_s + A_c)$

Total Equivalent area in terms of Concrete $= A_c + m A_s$

for As area of steel, equivalent area in terms of Concrete $= m A_s$

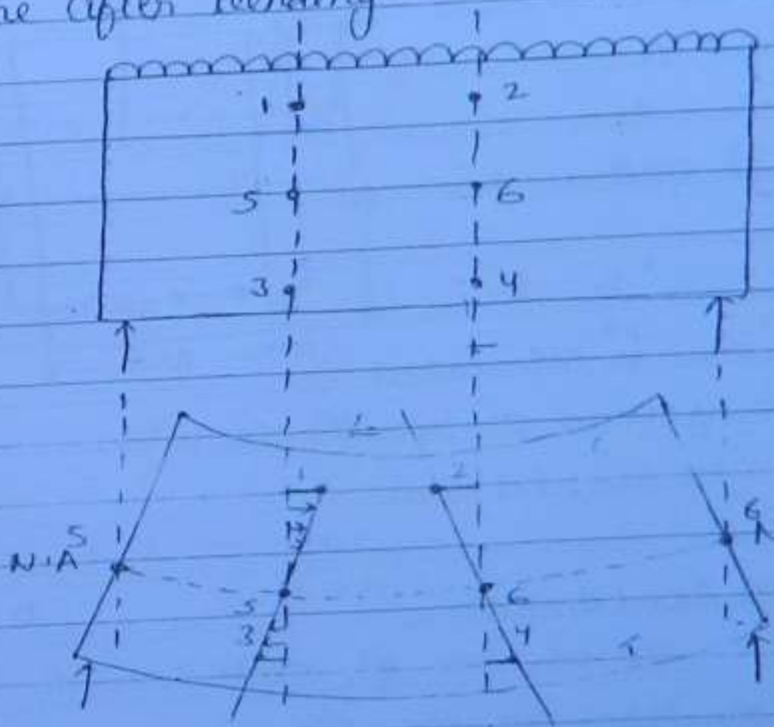
| Steel | Equivalent to Concrete |
| --- | --- |
| Ast | m·Ast |
| $\sigma_{st}$ | $\sigma_{st}/m$ |

(11)

→ modular ratio value is used in working stress method & W.S.M method is also called modular ratio method.

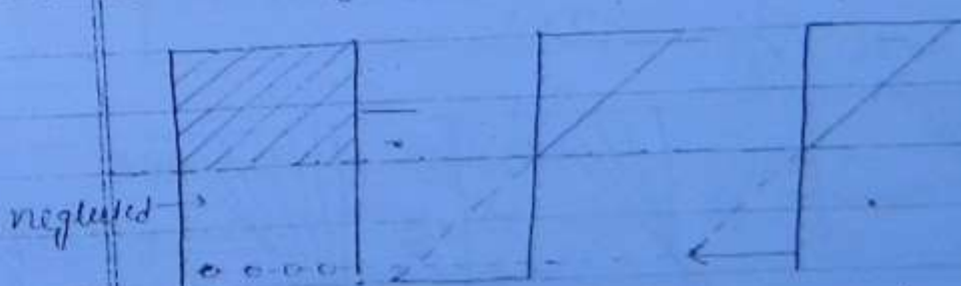
Assumptions :- (ANNEX – B) (~~B~~-1.3 p. no. 80)

1 At any cross-section plane section before bending remains plane after bending




1 & 2 → Compression
3 & 4 – tension
5 & 6 – Neutral axis

Result:- Strain diagram is linear




Strain dia    Stress diagram

2. All tensile stresses are taken by steel only. Tensile strength of concrete is to be ignored.
Concrete area in tension zone is completely neglected.

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(12)

3. $E_c$ value is constant
4. Stress-strain relationship under working load is a straight line. (Stress diagram is linear)
5. Modular ratio $m = \frac{280}{3\sigma_{cbc}}$

The exp given for m partially takes into account the effect of creep.

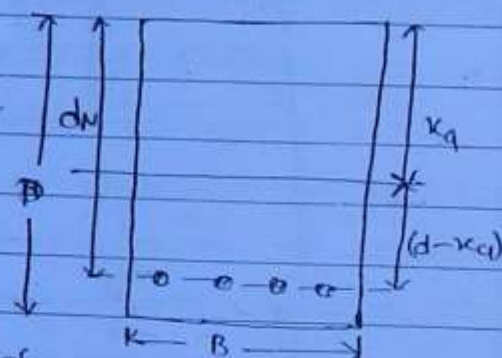
Actual depth of neutral axis

width B

Effective depth = d

Area of steel = Ast

If depth of actual neutral axis $= x_a$



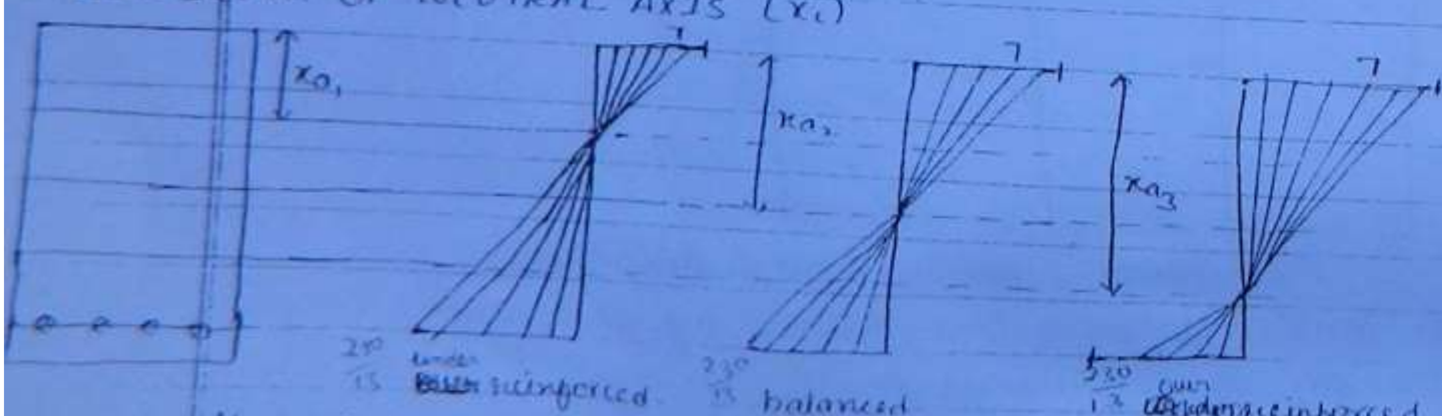

Depth of neutral axis is calculated equating moment of area on both the side of N.A

$$B \cdot x_a \cdot \frac{x_a}{2} = (m \cdot Ast) \cdot (d - x_a)$$

$$B \cdot \frac{x_a^2}{2} = m \cdot Ast (d - x_a)$$

For all purpose, calculation of MR, calculation of actual stress developed etc. $x_a$ value has to be used

ITICAL DEPTH OF NEUTRAL AXIS ($x_c$)




Here critical depth of N.A $= x_{a_2} = x_c$

Critical depth Of N·A is ~~such that~~ depth of N·A, so that stresses in Concrete & steel are attained to its max. permissible values at the same time at $x_a = x_c$ (Balanced Section)

Stress in Concrete = $\sigma_{cbc}$

Stress in steel = $\sigma_{st}$

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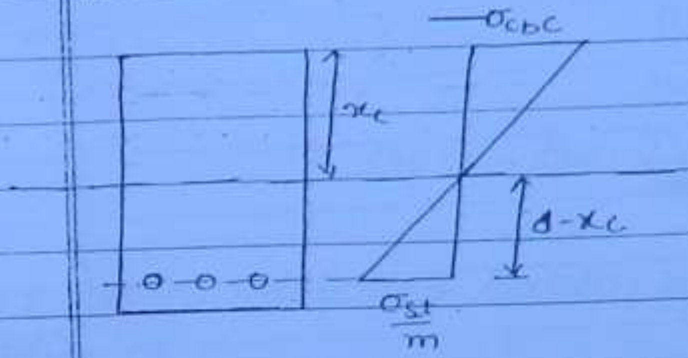
For balanced Section (for $x_c$)




Using similar $\Delta^{le}$

$$\frac{\sigma_{cbc}}{x_c} = \frac{\sigma_{st}/m}{d-x_c}$$

$$\frac{c}{x_c} = \frac{t/m}{d-x_c}$$

$$\frac{d-x_c}{x_c} = \frac{t}{mc}$$

$$\Rightarrow \frac{d}{x_c} = \frac{t}{m_c} + 1 \Rightarrow \frac{d}{x_c} = \frac{t+mc}{mc}$$

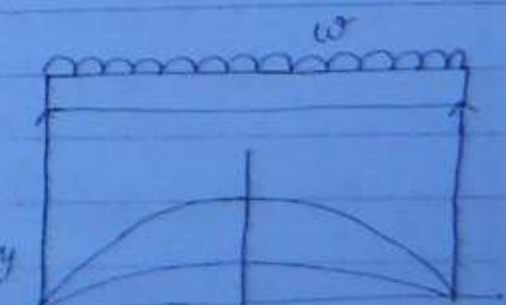
$$\frac{x_c}{d} = \frac{mc}{t+mc}$$

$$\frac{x_c}{d} = K = \frac{mc}{t+mc}$$

K = Critical depth Of N·A factor $= \dfrac{mc}{t+mc}$

$$\boxed{x_c = Kd = \frac{mc}{t+mc} \times d}$$

Moment Of Resistance.




Bending moment is the value Of moment developed due to load.

Moment of Resistance - It is the max. Capacity Of beam c/s to sustain moment

For moment of Resistance any one material (concrete or steel) should be at max. permissible value.

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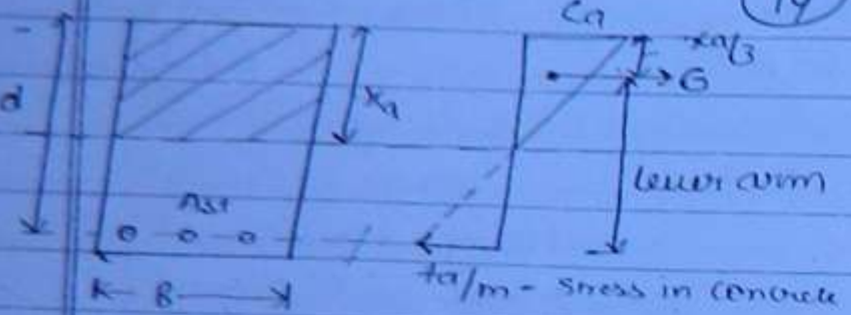


Total Compressive force

$C = $ Area $\times$ Avg. Stress

$B \cdot x_a \cdot \frac{C_a}{2}$

Total tensile force

$T = $ Area $\times$ Avg. Stress

$T = A_{st} \times t_a \quad \left(m \cdot A_{st} \times \frac{t_a}{m}\right)$

Lever arm $= d - \frac{x_a}{3}$

Moment of Resistance

$M_R = C \times$ lever arm

$B \cdot x_a \cdot \frac{C_a}{2}\left(d - \frac{x_a}{3}\right)$ —— (i) (from compression)

$M_R = T \times$ lever arm

$A_{st} \times t_a \left(d - \frac{x_a}{3}\right)$ —— (ii) (from tension)

Any one stress value should be at max. permissible stress value.

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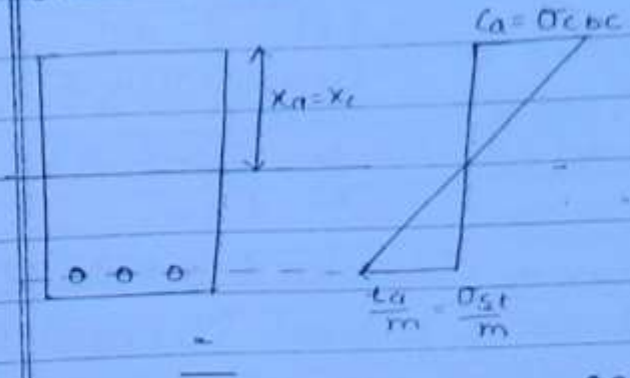
**CASE I** Balanced section :- In this case :- $x_a = x_c$

$$\frac{t_a}{m} = \frac{\sigma_{st}}{m}$$

$$C_a = \sigma_{cbc}$$



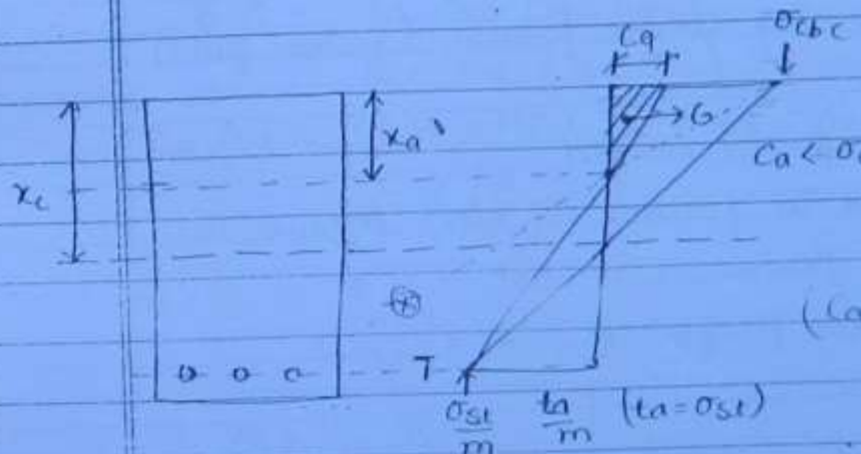

Now $M_R = B \cdot x_a \cdot \frac{C_a}{2}\left(d - \frac{x_a}{3}\right)$

$$\therefore M_R = B \cdot x_c \cdot \frac{\sigma_{cbc}}{2}\left(d - \frac{x_c}{3}\right) \text{ --- (I)}$$

again $M_R = A_{st} \cdot t_a \left(d - \frac{x_a}{3}\right)$

$$\Rightarrow M_R = A_{st} \cdot \sigma_{st}\left(d - \frac{x_c}{3}\right) \text{ --- (II)}$$

**CASE II** Under reinforced section :- $x_a < x_c$ (Area of steel is less than that req. for balanced sect.)




$C_a < \sigma_{cbc}$

$t_a = \sigma_{st}$

Moment of Resistance

$$M_R = B \cdot x_a \frac{C_a}{2}\left(d - \frac{x_a}{3}\right) \text{ (1)}$$

($C_a$ has to calculated) by similar $\Delta$

$$M_R = t_a \cdot A_{st}\left(d - \frac{x_a}{3}\right)$$

$$M_R = \sigma_{st} \cdot A_{st}\left(d - \frac{x_a}{3}\right) \text{ (2)}$$

direct formula

For (1) actual stress in concrete $C_a$ is to be calculated

$$\frac{C_a}{x_a} = \frac{\sigma_{st}/m}{d - x_a} \Rightarrow C_a = \left(\frac{x_a}{d - x_a}\right)\frac{\sigma_{st}}{m}$$

**CASE III** Over reinforced section :- $x_a > x_c$ ; $C_a = \sigma_{cbc}$ ; $t_a < \sigma_{st}$

$$M_R = B \cdot x_a \cdot \frac{\sigma_{cbc}}{2}\left(d - \frac{x_a}{3}\right) \text{ --- (1)}$$

$$M_R = t_a \cdot A_{st}\left(d - \frac{x_a}{3}\right) \text{ --- (2)}$$

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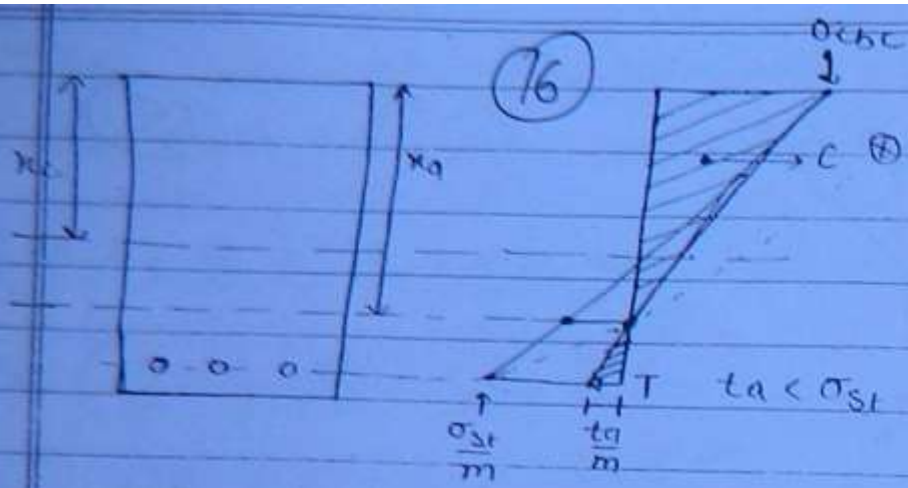


$\sigma_{cbc} = C_a$

for actual stress in steel -

$$\frac{C_a}{x_a} = \frac{t_a/m}{d - x_a}$$

$$\Rightarrow t_a = \frac{d - x_a}{x_a} m \cdot \sigma_{cbc}$$

# In Case Of Over reinforced Section
1. Concrete is at its max- permissible stress value so Concrete fails first.
2. Concrete failure is a brittle failure
3. It does not gives warning before failure (Sudden failure) It is to be avoided

# In case Of under reinforced Section
1. Failure is ductile due to failure Of steel because in this steel is at max. permissible steel.
2. This section should be preffered

Design formula

For a balanced section :- From Compression side

$$M_R = B \cdot x_a \cdot \frac{C_a}{2}\left(d - \frac{x_a}{3}\right)$$

$$M_R = B \cdot x_c \frac{\sigma_{cbc}}{2}\left(d - \frac{x_c}{3}\right)$$

$x_c = K d$

$$K = \frac{m\sigma_{cbc}}{\sigma_{st} + m \cdot \sigma_{cbc}}$$

put $x_c = Kd \; ; \; \sigma_{cbc} = C$

$$M_R = B \cdot K \cdot d \, \frac{C}{2}\left(d - \frac{K \cdot d}{3}\right)$$

$$= B \cdot d^2 \cdot K \cdot \frac{C}{2}\left(1 - \frac{K}{3}\right)$$

put $1 - \frac{K}{3} \rightarrow j$ lever arm depth factor

$M_R = \frac{1}{2} c \cdot j \cdot k \, Bd^2$

$M_R = Q\,Bd^2$

[ $Q = \frac{1}{2} c \, j \cdot k$ = Moment Coefficient

Where $k = \dfrac{mc}{t + mc}$

$j = 1 - k/3$

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For design

STEP1 Calculate load = $\omega$

STEP2 Calculate bending moment

STEP3 B.M = Moment of Resistance MR

$B \cdot M = Q \cdot Bd^2$

depth $d = \sqrt{\dfrac{B \cdot M}{Q \cdot B}} = \sqrt{\dfrac{MR}{Q \cdot B}}$

Area of steel

$B \cdot M = t_a \cdot A_{st} \left(d - \dfrac{x_a}{3}\right)$

$\sigma_{st} \cdot A_{st} \left(d - \dfrac{x_c}{3}\right)$

$\Rightarrow A_{st} = \dfrac{(B \cdot M = MR)}{\sigma_{st}\left(d - \dfrac{x_c}{3}\right)} \Rightarrow A_{st} = \dfrac{BM}{\sigma_{st}\left(d - Kd/3\right)}$

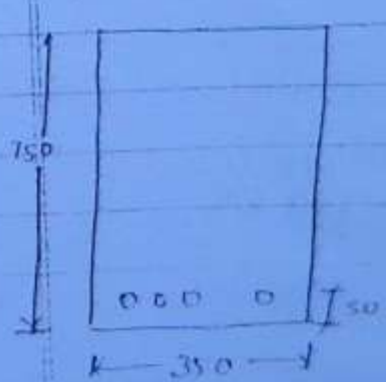
$= \dfrac{B \cdot M}{\sigma_{st}\left(1 - \dfrac{K}{3}\right) d}$

$$\boxed{A_{st} = \dfrac{B \cdot M}{\sigma_{st} \cdot j \cdot d}}$$

Ques:-1 Calculate Moment Of Resistance Of a RCC Rectangular beam Of size 350X750. fe 415 & M-20 is used

(1) $A_{st}$ = 4 Nos 16 mm $\phi$ bars

(2) $A_{st}$ = 6 Nos 25 mm $\phi$ bars




Calculate MR Of balanced Section & area Of steel required for balanced Section.

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Sol:- (I) Critical depth of N.A $x_c$

$$x_c = \frac{mc}{t+mc} \times d$$

IS

$$= \frac{13 \times 7}{230 + 13 \times 7} \times 700 = 198.44\ mm$$

(1) Where area of steel = 4nos - 16mm ϕ bar

$$A_{st} = 4 \times \frac{\pi}{4} \times 16^2 = 804.25\ mm^2$$

Actual depth of N.A

Equating moment of area on both side of N.A

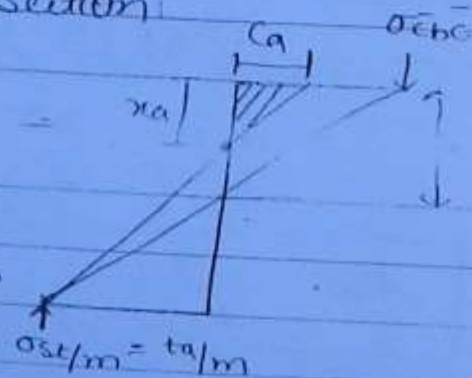
$$B \cdot \frac{x_a^2}{2} = m \cdot A_{st}(d - x_a)$$

$$350 \cdot \frac{x_a^2}{2} = 13 \times 804.25 (700 - x_a)$$

$$x_a = 176.80\ mm$$

$x_a < x_c \rightarrow$ under reinforced section

$C_a < \sigma_{cbc}$ & $t_a = \sigma_{st}$




rm tension side

mula - $M_R = \sigma_{st} A_{st} \left(d - \frac{x_a}{3}\right)$

$$= 230 \times 804.25 \left(700 - \frac{176.80}{3}\right) \times \frac{1}{10^6}$$

$M_R = 118.58\ KN\text{-}m$

pression side

$$C_a = \frac{x_a}{d - x_a} \times \frac{\sigma_{st}}{m} = \frac{176.80}{700 - 176.80} \times \frac{230}{13}$$

$C_a = 5.978\ N/mm^2$

Now $M_R = B \cdot x_a \cdot \frac{C_a}{2}\left(d - \frac{x_a}{3}\right)$

$$= 350 \times 176.80 \times \frac{5.978}{2}\left(700 - \frac{176.80}{3}\right) \times \frac{1}{10^6}$$

$= 118.57\ KN\text{-}m$

(2) When area of steel = 6nos - 25mm ϕ bars

$$6 \times \frac{\pi}{4} \times (25)^2 = 2945.24\ mm^2$$

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Actual depth of N·A → By equating moment on each side of N·A

$$\frac{B \cdot x_a^2}{2} = m \cdot A_{st}(d - x_a)$$

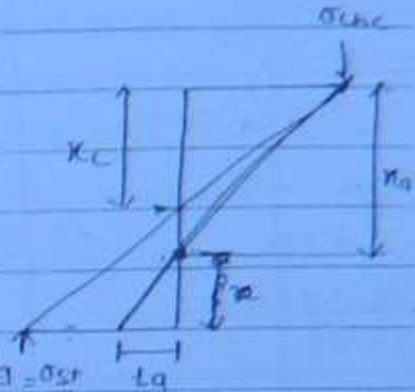
$$\frac{350 \cdot x_a^2}{2} = 13 \times 2945.24\,(700 - x_a)$$

$$x_a = 296.95$$

(19)

$x_a > x_c$ → Over reinforced section

$t_a < \sigma_{st}$ & $C_a = \sigma_{cbc}$



from direct formula of compression side

$$M_R = \frac{B \cdot x_a \cdot \sigma_{cbc}}{2}\left(d - \frac{x_a}{3}\right)$$

$$= 350 \times 296.95 \times \frac{7}{2}\left(700 - \frac{296.95}{3}\right)\frac{1}{10^6}$$

$$\frac{t_a}{m} = \frac{\sigma_{st}}{m}$$

$$= 218.6 \text{ KN-m}$$

tension side $\quad M_R = A_{st} \cdot t_a \left(d - \frac{x_a}{3}\right)$

$$\left\{ t_a = \frac{d - x_a}{x_a}\,\sigma_{cbc} \cdot m = \frac{700 - 296.95}{296.95} \times 7 \times 13 \right.$$

$$M_R = 2945.24 \times 123.51\left(700 - \frac{296.95}{3}\right) \times \frac{1}{10^6} \qquad t_a = 123.51$$

$$M_R = 218.63 \text{ KN-m}$$

Balanced section $\quad x_a = x_c = 198.44$ mm

$C_a = \sigma_{cbc} = 7 \quad ; \quad t_a = \sigma_{st} = 230$

$$M_R = B \cdot x_a \cdot \frac{C_a}{2}\left(d - \frac{x_a}{3}\right) \Rightarrow 350 \times 198.44 \times \frac{7}{2}\left(700 - \frac{198.44}{3}\right)\frac{1}{10^6}$$

$$M_R = 154.08 \text{ KN-m}$$

tension side $\quad M_R = \sigma_{st} \cdot A_{st}\left(d - \frac{x_a}{3}\right)$

$$154.08 = 230 \cdot A_{st}\left(700 - \frac{198.44}{3}\right)$$

$$A_{st} = \frac{154.08 \times 10^6}{230\,(700 - 198.44/3)}$$

$$A_{st} = 1056.9 \text{ mm}^2$$

provide

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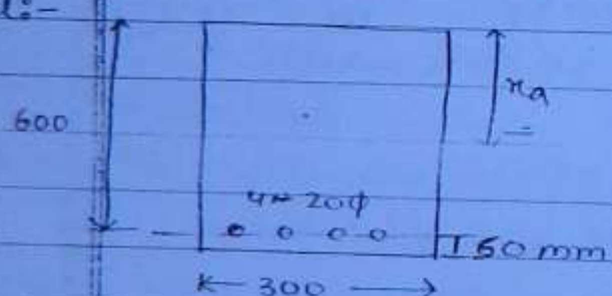
Comparison (20)

| Under Reinforced | Balanced | Overreinforced |
|---|---|---|
| B - 350 | same | same |
| d = 700 | | |
| $A_{st}$ = 4-16 | $A_{st}$ = 1056.9 $mm^2$ | $A_{st}$ = 2945.24 $mm^2$ |
| 804.25 $mm^2$ | | |
| x = 51.978 | $c_a = \sigma_{cbc} = 7$ | $c_a = \sigma_{cbc} = 7$ |
| $t_a$ = 230 | $t_a = \sigma_{st} = 230$ | $t_a$ = 123.5 |
| = 118.57 kN-m | $M_R$ = 154.08 kN-m | $M_R$ = 218.63 kN-m |
| = 176.08 | $x_a$ = 198.44 | $x_a$ = 296.95 |

Ques-21 A rectangular beam of size 300 x 650 mm is subjected to a U.D.L of 15 KN/m over a S.S. span of 5.6 m. (Load in excluding self wt.). Calculate stress developed in Concrete & Steel. $A_{st}$ = 4 nos 20 mm $\phi$ bars. m = 15mm cover = 50mm

Sol:-




① Actual depth of N.A

$$B \cdot \frac{x_a^2}{2} = m \cdot A_{st}(d - x_a)$$

$$300 \cdot \frac{x_a^2}{2} = 15 \times 4 \times \frac{\pi}{4} \times 20^2 (600 - x_a)$$

$$x_a = 218.85 \text{ mm}$$

② Load = 15 kN/m

Self wt = .3 x .65 x 1 x 25 = 4.875 kN/m

Total load w = 19.875 kN/m

$$B.M = \frac{wl^2}{8} \quad \frac{19.875 \times 5.6^2}{8} = 77.91 \text{ kN-m}$$

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Now $B.M = \sigma_{st} \cdot A_{st} \left(d - \frac{x_a}{3}\right)$

$$77.91 \times 10^6 = \sigma_{st} \cdot 4 \times \frac{\pi}{4} \times 20^2 \left(600 - \frac{218.85}{3}\right)$$

(2)

$$\sigma_{st} = 117.60 \ N/mm^2$$

[Also calculated from similar $\Delta^{le}$ ~~per~~]

$$B.M = B \cdot x_a \cdot \frac{c_a}{2} \left(d - \frac{x_a}{3}\right)$$

$$77.91 \times 10^6 = 300 \times 218.85 \times \frac{\sigma_{cbc}}{2} \left(600 - \frac{218.85}{3}\right)$$

$$\sigma_{cbc} = 4.503 N/mm^2$$

A reinforced concrete beam of size 350×700 mm is to be designed for a B.M of (i) 100 kN-m (ii) 200 kN-m. What will be steel reinforcement required for above B.M. M20, Fe415. What will be steel required for balanced section.

Effective cover = 50mm

Solⁿ MR of balanced section

$$MR = Q B d^2$$

$$Q = \frac{1}{2} \times 7 \times 0.905 \times 0.2835$$
$$= 0.898$$

$$MR = 0.898 \times 350 \times (650)^2 / 10^6$$
$$= 132.79 \ kN\text{-}m$$

$Q = \frac{1}{2} c \cdot j \cdot k$

$k = \frac{mc}{t + mc} = \frac{13 \times 7}{230 + 13 \times 7}$

$k = 0.2835$

$j = \left(1 - \frac{k}{3}\right) = 0.905$

(i) $B.M = 100 \ kN\text{-}m$

BM < MR singly reinforced balanced section so we need under reinforced section.

$$B.M = B \cdot x_a \cdot \frac{c_a}{2} \left[d - \frac{x_a}{3}\right] \quad \text{— (i)}$$

$$100 \times 10^6 = B \, x_a \, c_a/2 \left[d - x_a/3\right]$$

$$B.M = t_a \cdot A_{st} \left[d - \frac{x_a}{3}\right]$$

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for an under reinforced section —

$C_a < \sigma_{cbc}$

$\varepsilon_{s} = \sigma$

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So from eq$^n$ (2)

$$100 \times 10^6 = 230 \cdot A_{st} \left(650 - \frac{x_a}{3}\right) \quad \text{---(3)}$$

equating moment of area

$$\frac{B \cdot x_a^2}{2} = m \, A_{st} \, (d - x_a)$$

$$\frac{350 \times x_a^2}{2} = 13 \times A_{st} \, (650 - x_a)$$

$$A_{st} = \frac{350 \, x_a^2}{2 \times 13 \, (650 - x_a)}$$

$$A_{st} = \frac{13.46 \, x_a^2}{650 - x_a}$$

Putting the values of $A_{st}$ in eq (3)

$$100 \times 10^6 = 230 \times \frac{13.46 \, x_a^2}{(650 - x_a)} \times \left(650 - \frac{x_a}{3}\right)$$

$$100 \times 10^6 = \frac{3095.8 \, x_a^2 \, (1950 - x_a)}{3(650 - x_a)}$$

$$100 \times 10^6 = \frac{1031.93 \, (1950 - x_a) \, x_a^2}{(650 - x_a)}$$

$$\frac{(1950 - x_a) \, x_a^2}{(650 - x_a)} = 96905.46$$

$$1950 \, x_a^3 - x_a^3 = 62988549 - 96905.46 \, x_a$$

$$x_a^3 - 1950 \, x_a^2 - 96905.46 \, x_a + 62988.5499 = 0$$

$$x_a = 162.5 \text{mm}$$

$$A_{st} = \frac{13.46 \, x_a^2}{650 - x_a}$$

$$A_{st} = 729.08 \text{ mm}^2$$

Shortcut :- Only under-reinforced in beam

Considering a balanced section (23)

$x_a = x_c = 0.2835d = 0.2835 \times 650$
$= 184.275$

$A_{st} = \dfrac{BM}{\sigma_{st} \cdot j \cdot d} = \dfrac{100 \times 10^6}{230 \times 0.905 \times 650} = 739.9 \text{ mm}^2$

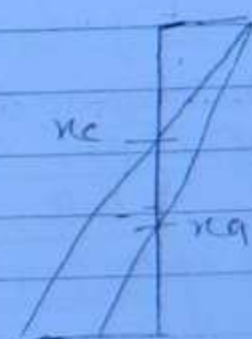
above formula can be used for slabs but not for beam.

(ii) $B.M = 200$ kN-m

$B.M > MR$ (over reinforced singly or doubly

$200 \times 10^6 = B \cdot x_a \cdot \dfrac{c_a}{2}\left[d - \dfrac{x_a}{3}\right]$

$200 \times 10^6 = 350 \times x_a \cdot \dfrac{c_a}{2}\left[650 - \dfrac{x_a}{3}\right]$




$163265.31 = x_a\left[650 - \dfrac{x_a}{3}\right]$

$489795.9 = 1950\, x_a - x_a^2$

$x_a = 296.14$ mm

From similar $\Delta^{le}$

$\dfrac{c_a}{x_a} = \dfrac{t_a/m}{d - x_a} \Rightarrow t_a = \dfrac{(d - x_a)\, m \cdot c_a}{x_a}$

$t_a = \dfrac{(650 - 296.14) \times 13 \times 7}{296.14}$

$t_a = 108.74 \text{ N/mm}^2$

$B.M = t_a \cdot A_{st}\left[d - \dfrac{x_a}{3}\right]$

$200 \times 10^6 = 108.74 \cdot A_{st}\left[650 - \dfrac{296.14}{3}\right]$

$A_{st} = 3336.3 \text{ mm}^2$

Area of steel required for bal. sec.

$A_{st} = \dfrac{MR}{\sigma_{st}\, j \cdot d} = \dfrac{132.79 \times 10^6}{230 \times .904 \times 650}$ $982.5 \text{ mm}^2$

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Ques. Design a rectangular R.C.C beam for a live load of 45 KN over a span of 7.50 m. use. M-20 & Fe 415. Load is excluding self wt. of beam. Consider width = 400mm

Soln  B = 400 mm   Span = 7.50m (effective)   24

1. Load / B.M → L.L = 45 KN/m

Self wt = $.4 \times .75 \times 1 \times 25$ = 7.50 kN/m

assume depth = 750 mm = Span/10

total $w$ = 45+7.50 = 52.50 KN/m

$B.M = \frac{wl^2}{8} = \frac{52.50 \times (7.5)^2}{8} = 369.14$ KN-m

Let us design for balanced section

$x_a = x_c$ ; $C_a = \sigma_{cbc}$ ; $t_a = \sigma_{st}$

(i) depth required –

$d = \sqrt{\frac{BM}{Q \cdot B}}$

$d = \sqrt{\frac{369.14}{1.11 \times 400}}$

d = 911.8

$Q = \frac{1}{2} c \cdot j \cdot K$

$K = \frac{mc}{t + mc} = \frac{11 \times 8.5}{230 + 11 \times 8.5}$

K = 0.289

$j = 1 - K/3 = .904$

Q = 1.11

eff cover = 50

d = 961.80 > 750 mm

Let us consider d = 1000 mm

Self wt = $.4 \times 1.0 \times 1 \times 25$ = 10 KN/m

total $w$ = 45 + 10 = 55 KN/m

BM = 386.76 KN-m

again $d = \sqrt{\frac{386.76}{1.11 \times 400}}$ = 933 mm

eff cover 50 ⇒ d = 983 mm < 1000 mm Ok

$A_{st} = \frac{BM}{\sigma_{st} \cdot j \cdot d} = \frac{386.72 \times 10^6}{230 \times .904 \times 950}$

$A_{st}$ = 1958 mm²

Calculate Moment Of Resistance Of section shown in fig.
use M-20 & fe 415

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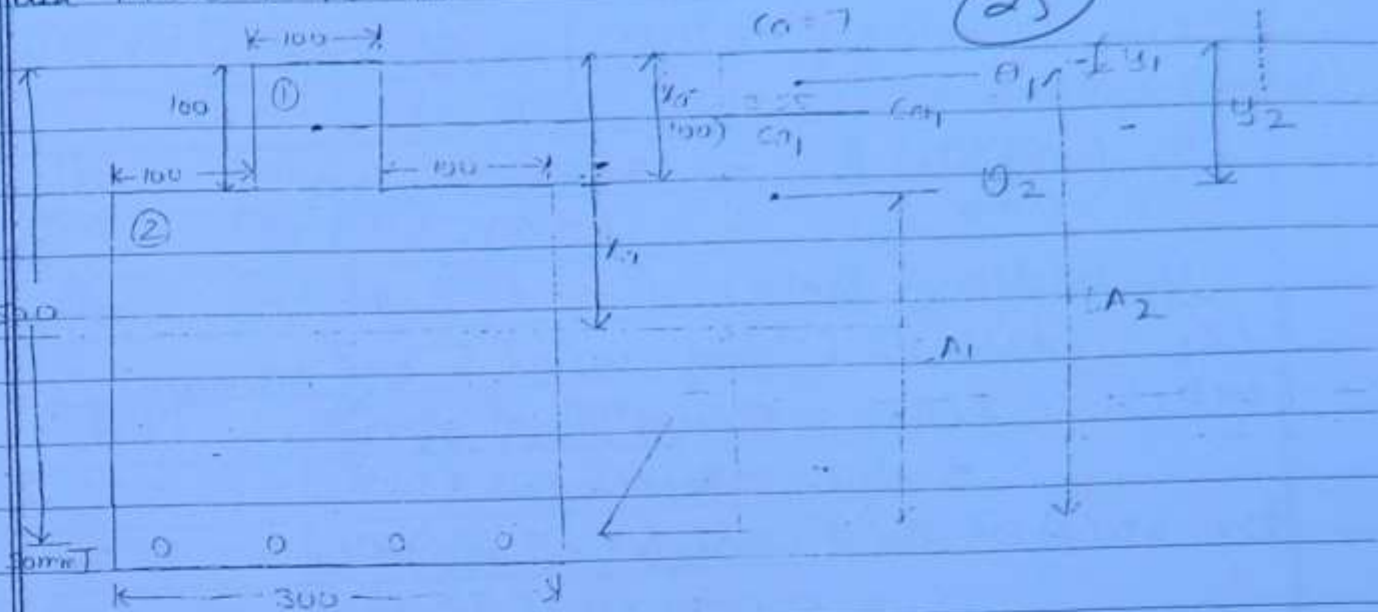



Critical depth Of N.A $\quad x_c = k.d$

$$x_c = \frac{mc}{t+mc} \times d = \frac{13\times7}{230+13\times7} \times 500 \Rightarrow 141.75\ mm$$

Actual depth Of N.A — Equating moment area On both side

assume $x_a > 100\ mm$

$100 \times 100\ (x_a - 50) + 300\ (x_a - 100) \frac{(x_a - 100)}{2} = m A_{st} [d - x_a]$

$66.67 \times (x_a - 50) + (x_a - 100)^2 = 69.70\ [500 - x_a]$

$x_a^2 - 200x_a + 10^4 + 66.67x_a - 3333.5 = 0$

$x_a = 202.68 \quad > 100\ mm$ (assumption is correct)

$x_a > x_c \quad ; \quad C_a = \sigma_{cbc} = 7 N/mm^2$

$\cdot\ t_a < \sigma_{st}$

Stress at junction

$$\frac{C_{a1}}{x_a - 100} = \frac{C_a}{x_a}$$

$$C_{a1} = (x_a - 100) \times \frac{7}{x_a}$$

$C_{a1} = 3.55\ N/mm^2$

Moment Of Resistance



for bar ① $C_1$ = area × avg. Stress

$$C_1 = 100\times100\times\left[\frac{7+3.55}{2}\right] = 52750N$$

$$\bar{y}_1 \left(\frac{a+2b}{a+b}\right)\frac{h}{3} = \frac{7+2\times3.55}{7+3.55}\times\frac{100}{3}$$

$$y_1 = 44.55mm$$

$$LA_1 = (d-y_1) = 500-44.55 = 455.45\ mm$$

$$MR_1 = C_1\times LA_1 = 52750\times455.45\times10^{-6}$$

$$= 24.025\ kN\text{-}m$$

for bar ② $C_2 = 300\times(x_a-100)\times\frac{ca_1}{2}$

$$= 54677.1\ N$$

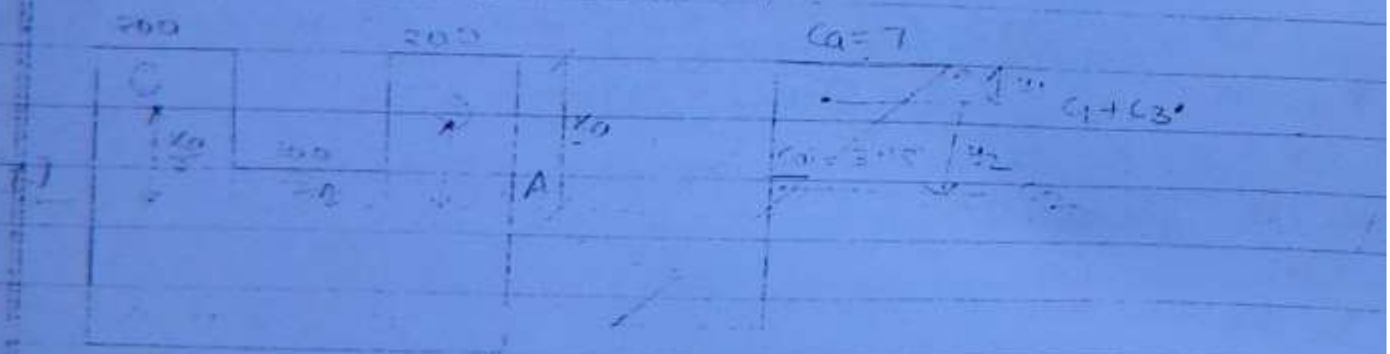
$$y_2 = 100+\left(\frac{x_a-100}{3}\right) = 133.89\ mm$$

$$LA_2 = d-y_2 = 366.11\ mm$$

$$(MR)_2 = C_2\times LA_2 = 20.02\ kN\text{-}m$$

Total M.R $= 24.025 + 20.02 = 44.04$ kN-m

A reinforced Concrete beam has shown in fig there are 4 - 20$\phi$ bar. Calculate N.A & Stress developed due to a B.M 85000 N-m




depth of N.A. equating moment of area

$$2\times200\times x_a\times\frac{x_a}{2} + 100\times(-100+x_a)\left(\frac{x_a-100}{2}\right)$$
$$= m\cdot A_{st}(d-x_a)$$

$$200x_a^2 + 50(x_a-100)^2 = 15\times\frac{\pi}{4}\times(20)^2(d-x_a)$$

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$200x_a^2 + 50(x_a^2 + 100^2 - 2\times100 x_a) = 4712.38(550 - x_a)$

$250x_a^2 + 500000 - 10000x_a = 2591809 - 4712.38x_a$

$250x_a^2 - 14712.38x_a + 2091809 = 0$

$x_a = 181.75$ mm $> 100$ (Assumption is correct)

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$\frac{C_a}{x_a} = \frac{C_{a1}}{(x_a - 100)}$

$C_{a1} = \frac{C_a(x_a - 100)}{x_a} = \frac{C_a(81.75)}{181.75} = 0.45C_a$

$B.M = 85\times10^6$ N-mm

$C_1 + C_3 = 2$(area x avg. stress)

$= 2(200\times x_a \times \frac{C_a}{2}) = 36350C_a$

$y_1 = \frac{x_a}{3} = 60.58$ mm

$LA_1 = 550 - y_1 = 489.416$

$(MR_1 + MR_3) = (C_1 + C_3)\times LA_1$

$= 36350C_a \times 489.416 = 17.79C_a$ KN-m

$C_2 = 100\times(x_a - 100)\times\frac{C_a}{2}$

$= 18.39.37C_a$

$y_2 = 100 + (x_a - 100)$

$y_2 = 127.25$

$LA_2 = 550 - 127.25 = 422.75$ mm

$MR_2 = 0.77$ KN-m

Total $= MR_1 + MR_2 + MR_3 = 18.56C_a$

$18.56C_a = 85 \Rightarrow C_a = 4.58$ N/mm²

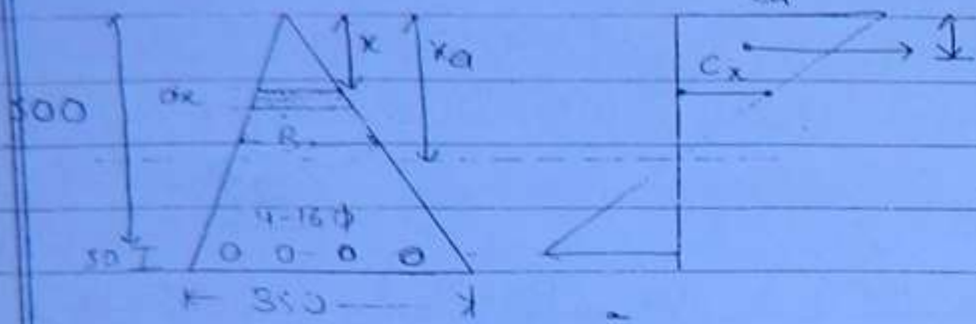
$C_{a1} = 0.45\times4.58 = 2.06$ N/mm²

$\frac{C_a}{x_a} = \frac{t_a/m}{d - x_a} \Rightarrow t_a = \frac{(d - x_a)(m\cdot C_a)}{x_a}$

$= \frac{(550 - 181.75)(15\times4.58)}{181.75}$

$t_a = 139.19$ N/mm²

Ques. Find Moment Of Resistance Of following Section

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Critical depth Of N.A $\bar{x}_c = \frac{mc}{t+mc} \times d = \frac{13\times7}{230+13\times7} \times 500$

$x_c = 141.75$ mm

$\frac{B_1}{x_a} = \frac{350}{550}$

Actual depth

$B_1 = \frac{7}{11} x_a$

$\frac{1}{2} \times B_1 \times x_a \cdot \frac{x_a}{3} = m \cdot A_{st} (d - x_a)$

$\frac{1}{2} \times \frac{7}{11} \times \frac{1}{3} \cdot x_a^3 = 13 \times 4 \times \pi/4 \times 16^2 (500 - x_a)$

$0.106 x_a^3 = 5227610.176 - 10455.22 x_a$

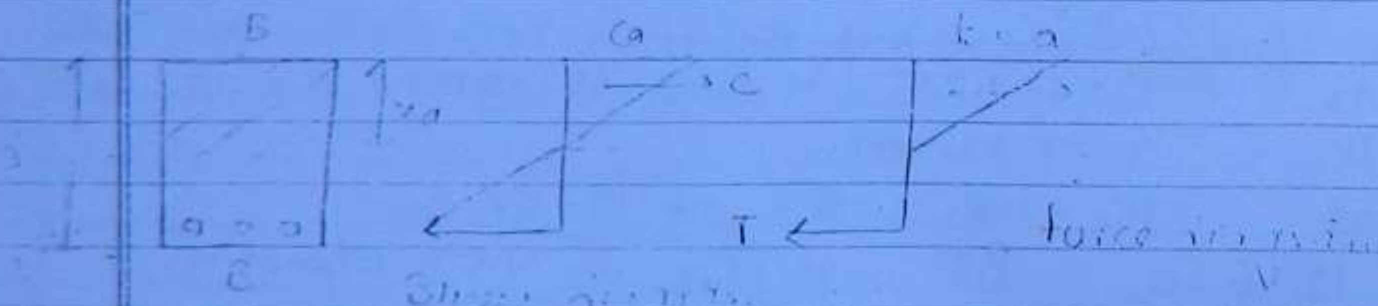
$.106 x_a^3 + 10455.22 x_a - 5227610.176 = 0$

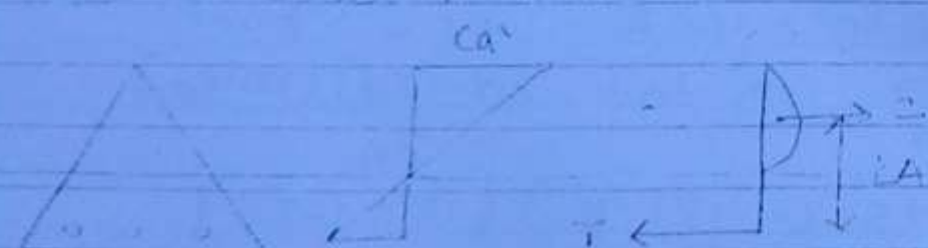
by trial & error $x_a = 279.2$ mm $> x_c$

(over reinforced)

$C_a = \sigma_{cbc} = T$

$t_a < \sigma_{st}$



Consider an elemental strip of $dx$ thickness at distance $x$ from top.

If width of secⁿ at N.A = $B_1$

$$\frac{B_1}{x_a} = \frac{350}{550} = \frac{7}{11}$$

$$B_1 = \frac{7}{11} x_a$$

$$B_x = \frac{7}{11} x \quad -(\text{doubt})$$

$$\frac{C_a}{x_a} = \frac{C_x}{(x_a - x)} \Rightarrow C_x = \frac{x_a - x}{x_a} C_a$$

$$\Rightarrow C_x \; \frac{279.2 - x}{279.2} \times 7$$

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$$C_x = \frac{279.2 - x}{39.88}$$

M.R of elemental strip $C_1 \times L.A$
= area × avg. stress × L.A

$$MR = (B_x \cdot dx)\,(C_x)\,(d - x)$$

$$\frac{7}{11} x \cdot dx \left(\frac{279.2 - x}{39.88}\right)(500 - x)$$

total M.R. →

$$\int_0^{x_a} MR = \int_0^{x_a} \frac{7}{11} x \left(\frac{279.2 - x}{39.88}\right)(500 - x)\, dx$$

$$\frac{7}{11} \int \left(\frac{279.2 - x - x^2}{39.88}\right) \cdot (500 - x)\, dx$$

$$\frac{7}{11} \int \frac{279.2x}{39.88} \times 500 - \frac{279.2x^2}{39.88} - \frac{500x^2}{39.88} + \frac{x^3}{39.88}\, dx$$

$$\frac{7}{11} \int_0^{x_a} 3500.50x - 7x^2 - 12.53x^2 + 0.025x^3\, dx$$

$$\frac{7}{11} \int_0^{x_a} 3500.50x - 19.55x^2 + 0.025x^3\, dx$$

$$\frac{7}{11} \left[\frac{0.025x^4}{4} - 19.55\frac{x^3}{3} + \frac{3500.50x^2}{2}\right]_0^{x_a}$$

$$M.R = \frac{7}{11}\left[\frac{0.025x_a^4}{4} - \frac{19.55x_a^3}{3} + 1750.25x_a^2\right]$$

$$MR = \frac{7}{11}\left(0.00625x_a^4 - 6.52x_a^3 + 1750.25x_a^2\right)$$

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# DOUBLY REINFORCED SECTION

\# Doubly reinforced section is needed if

1. Size of beam (width & depth) both are fixed
2. Beam has to sustain a higher B.M > M.R of singly reinforced balanced section

if size of beam = B x d

M.R of singly reinforced balanced section

$$M.R = Q.Bd^2$$

$$Q = \frac{1}{2}\, c.j.K$$

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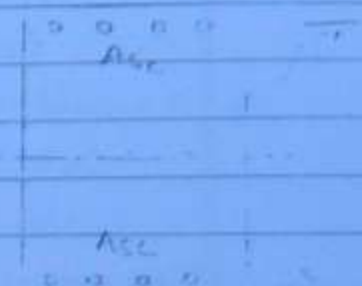


if B.M > MR

→ There are two option

(i) Over-reinforced section – avoided

(ii) Doubly-reinforced section – preffered

Permissible stress in compression steel

$= 1.5\, m.c'$

$c'$ = stress in concrete at the level of compression steel

Equivalent area for Asc area of steel in terms of concrete

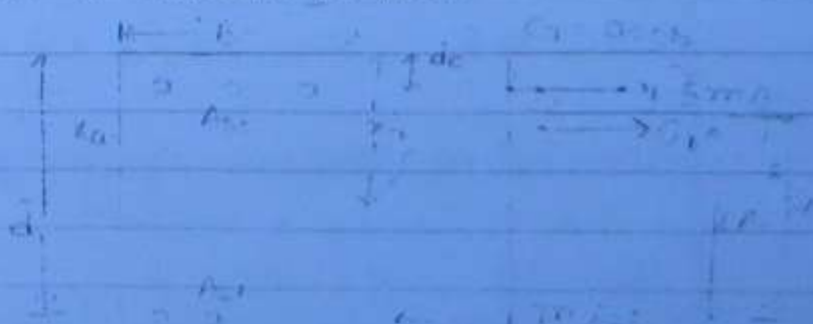
$= 1.5\, m.\,Asc$

1.5 m value is used for compression steel, is due to continued compressive strain for a long time (Creep.) Due to creep effect actual stress in steel is much higher than stress developed due to elastic strain.

Analysis

1. Critical depth of N.A

$$x_c = \frac{mc}{t+mc} \times d$$



2. Actual depth of N.A

Equating moment of area on both side of N.A

$$B.x_a\frac{x_a}{2} + 1.5m\,Asc(x_a - d_c) - Asc(x_a - d_c) = m.Ast(d - x_a)$$

concrete area not available

$$B.\frac{x_a^2}{2} + (1.5m - 1)A_{sc}(x_a - d_c) = m\,Ast(d - x_a)$$

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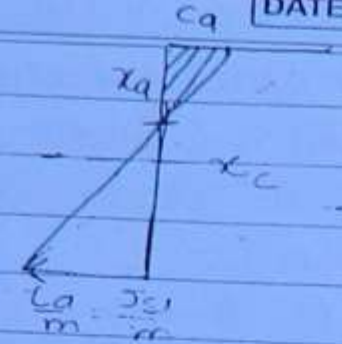
Compare —




if $x_a < x_c \rightarrow$ under reinforced

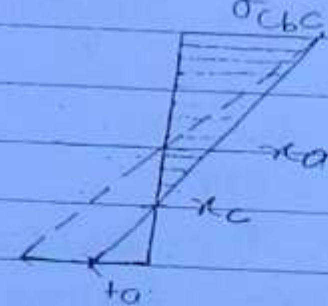
$c_a < \sigma_{cbc}$

$t_a = \sigma_{st}$ 32

if $x_a = x_c \rightarrow$ balanced section

$c_a = \sigma_{cbc}$

$t_a = \sigma_{st}$




if $x_a > x_c \rightarrow$ Over reinforced

$c_a = \sigma_{cbc}$ $t_a < \sigma_{st}$

Moment of Resistance fig(i) (Compression side)

for Concrete $C_1 = B \cdot x_a \frac{c_a}{2}$

$LA_1 = \left(d - \frac{x_a}{3}\right)$

$MR_1 = B \cdot x_a \cdot \frac{c_a}{2}\left(d - \frac{x_a}{3}\right)$

Compression Steel $C_2 = 1.5m \cdot A_{sc} \cdot c' - A_{sc} \cdot c'$

$(1.5m - 1) A_{sc} \cdot c'$

lever arm $LA_2 = d - d_c$

$MR_2 = (1.5m - 1) A_{sc}\, c'\, (d - d_c)$

Total $M.R = B \cdot x_a \cdot \frac{c_a}{2}\left(d - \frac{x_a}{3}\right) + (1.5m - 1) A_{sc}\, c'(d - d_c)$

Moment of Resistance from tension side.

$C_R = C_1 + C_2$

$y_1 = \frac{x_a}{3}$ ; $y_2 = d_c$ & $\bar{y} = \frac{C_1 y_1 + C_2 y_2}{C_1 + C_2}$

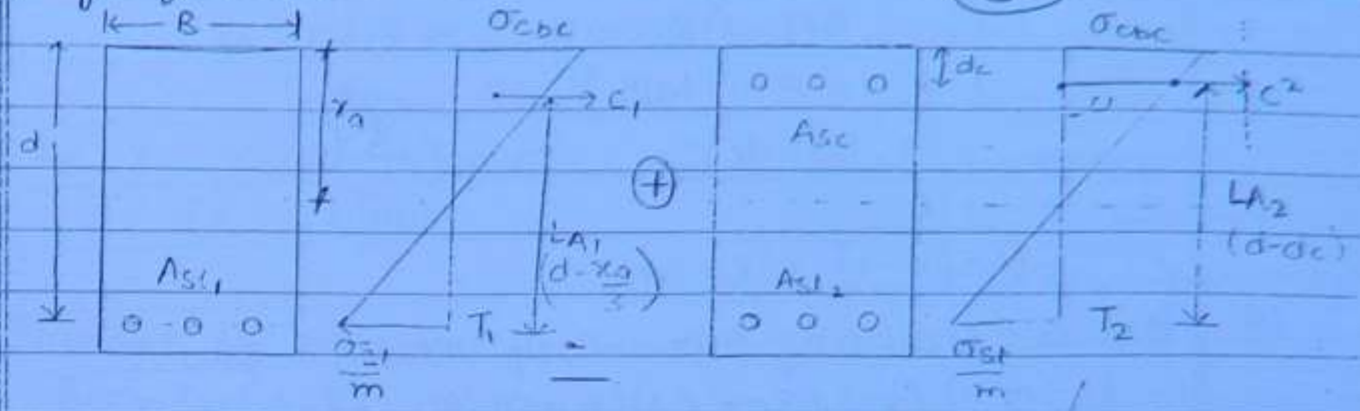
Net lever arm $= (d - \bar{y})$

$M.R = C_R (d - \bar{y})$

$= T \cdot (d - \bar{y})$

$M.R = A_{st} \cdot t_a (d - \bar{y})$

Design formula




$$C_1 = B \cdot x_a \cdot \frac{C_a}{2}$$
$$LA_1 = \left(d - \frac{x_a}{3}\right)$$
$$T_1 = t_a \cdot Ast_1$$

$$C_2 = (1.5m - 1) Asc \cdot c'$$
$$LA_2 = (d - d_c)$$
$$T_2 = t_a \, Ast_2$$

$$M.R = C_1 \cdot LA_1 + C_2 \cdot LA_2 \quad \text{(Compression)}$$
$$= B \cdot x_a \cdot \frac{C_a}{2}\left(d - \frac{x_a}{3}\right) + (1.5m-1) Asc \cdot c' (d - d_c)$$

for balanced section $M.R = (M.R_1)$ singly reinforced section

$$= B \cdot x_c \cdot \frac{\sigma_{cbc}}{2}\left(d - \frac{x_c}{3}\right) + (1.5m-1)\, Asc \cdot c' (d - d_c)$$

$$\overset{MR_1}{Q b d^2} + \overset{MR_2}{(1.5m-1) Asc \cdot c' (d - d_c)}$$

$$M.R = T_1 \cdot LA_1 + T_2 \, LA_2 \quad \text{(tension)}$$
$$= \overset{MR_1}{t_a \cdot Ast_1 \left(d - \frac{x_a}{3}\right)} + \overset{MR_2}{t_a \cdot Ast_2 (d - d_c)}$$

Design Steps :-

1. If beam has to be designed for a B.M = B.M
   $B.M > (MR_1 = Qbd^2)$
2. Check $MR_1 = Qbd^2$
   if $B.M > MR_1$ Then doubly reinforced section is required.
3. Calculate $Ast_1$ equating $MR_1$ on both tension & compression side.

$$Ast_1 = \frac{MR_1 = Qbd^2}{t_a \cdot (d - x_a/3)} = \frac{MR_1}{\sigma_{st}\,(d - x_c/3)}$$

DATE:

4. ~~Remaining~~ Remaining B.M

$$MR_2 = BM - MR_1$$

$$Ast_2 = \frac{(MR_2 = B.M - MR_1)}{ta \cdot (d - dc)}$$

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5 $$Asc = \frac{MR_2}{(1.5m-1) \cdot c'(d-dc)}$$

Or for Asc equating moment of area for part 2

$$(1.5m-1)\, Asc \cdot (xa - dc) = m \cdot Ast_2 (d - xa)$$

$$Asc = \frac{m \cdot Ast_2 (d - xa)}{(1.5m-1)(xa - dc)}$$ (Commonly used)

Generally M.R is calculated from compression side.

Ques:- A rectangular beam of 230×600 mm $\bar{c}$. 50 mm eff. cover is reinforced 5-25φ - on tension & 3-16 φ - Compression. It is carrying an imposed load of 50 kN/m over an effective span of 7m. Check adequacy of section (M-20) fe 415.

Sol:- Imposed load = 50 kN/m

Load I Self wt = .23 × .6 × 1 × 25 = 3.45 kN/m

total = 53.45 kN/m

$$B.M = \frac{wl^2}{8} = \frac{53.45 \times (7)^2}{8}$$

$$B.M = 321.38 \text{ kN-m}$$

Moment of Resistance

Critical depth $x_c = \frac{mc}{t + mc} \times d = \frac{13 \times 7}{230 + 13 \times 7} \times 550$

$= 155.92$ mm

Actual depth

$$B \cdot \frac{xa^2}{2} + (1.5m-1)\, Asc\, (xa - dc) = m \cdot Ast (d - xa)$$

$$230 \cdot \frac{xa^2}{2} + (1.5 \times 13 - 1) \cdot 603.20 (xa - 50) = 13 \times 2454.4 (550 - xa)$$

$x_a^2 + 97.04x_a - 4851.83 = 152600 - 277.45x_a$

$x_a^2 + 374.49 - 157451.83 = 0$

$x_a = 251.3 \text{ mm}$

$x_a > x_c \rightarrow$ over reinforced $C_a = \sigma_{cbc}$ $t_a < \sigma_{st}$

So $C_a = 7 N/mm^2$

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Moment of Resistance

$B \cdot x_a \cdot \frac{C_a}{2}\left(d - \frac{x_a}{3}\right) + (1.5m-1) A_{sc} \cdot C' (d - d_c)$

$\frac{C'}{x_a - d_c} = \frac{C_a}{x_a} \Rightarrow C' = \frac{7(251.3-50)}{251.3} = 5.607$

$MR = 230 \times 251.3 \times \frac{7}{2}\left(550 - \frac{251.3}{3}\right) + (1.5 \times 13 - 1) 603.20 \times 5.607 (550-50)$

$M.R = 125.60 \text{ KN-m}$

$\therefore B.M > M.R$ so it is not safe

Method II - Calculate the stresses developed in the beam

$\rightarrow$ due to B.M 327.38 KN-m

1. Calculate $x_a$ $x_a = 251.31 \text{ mm}$

If the stresses develop are $C_a$ = in concrete & $t_a$ = in steel

$\frac{C'}{x_a - d_c} = \frac{C_a}{x_a} \Rightarrow C' = 0.8 C_a$

Equating B.M & moment of resistance we get

$327.38 = B \cdot x_a \cdot \frac{C_a}{2}\left(d - \frac{x_a}{3}\right) + (1.5m-1) A_{sc} \cdot C' (d - d_c)$

$327.38 \times 10^6 = 230 \times 251.31 \times \frac{C_a}{2}\left(550 - \frac{251.3}{3}\right) + (1.5 \times 13 - 1) 603.2 \times .8C_a (550-50)$

$C_a = 18.25 N/mm^2$

$C_a > 7 N/mm^2$ failed.

$C' = .8 \times 18.25 = 14.6 N/mm^2$

DATE:

Stress in compression steel

$= 1.5\, m \times c' = 284.7\ N/mm^2$

Stress in tension steel

Using similar $\Delta^{le}$ $\frac{c_a}{x_a} = \frac{t_a/m}{d - x_a}$

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$$t_a = \frac{(d - x_a)m \cdot c_a}{x_a} = \frac{(550 - 251.3)\,13 \times 18.25}{251.31}$$

$$t_a = 281.9\ N/mm^2 > 230\ N/mm^2$$

(failed)

Ques:- Design a rectangular beam of size. 300X750 mm. overall depth for a live load of 45 kN/m over a span of 7.5 m Use M-25 & Fe 415

Sol. ① Live load = 45 kN/m

Self wt = $.3 \times .75 \times 1 \times 25 = 5.625$ kN/m

$$B.M = \frac{wl^2}{8} = \frac{50.625 \times (7.5)^2}{8} = 355.95\ kN\text{-}m.$$

② Calculate M.R of singly reinforced balanced section

$MR_1 = QBd^2$

$Q = \frac{1}{2} c \cdot j \cdot k = 1.11$

| | |
|---|---|
| $K = \frac{mc}{t + mc} = \frac{11 \times 8.5}{230 + 11 \times 8.5} = .289$<br>$j = 1 - \frac{k}{3} = .904$<br>$Q = 1.11$ | $MR_1 = 1.11 \times 230 \times 700^2$<br>$MR_1 = 163.17$ kN-m<br>$B.M > MR_1$<br>(doubly reinforced section) |

③ $A_{st_1} = \frac{MR_1}{\sigma_{st}(d - \frac{x_c}{3})} = \frac{MR_1}{\sigma_{st}\, j \cdot d}$

$= \frac{163.17 \times 10^6}{230 \times .904 \times 700}$ $1121.1\ mm^2$

DATE:

(4) $MR_2 = \overline{B.M} - MR_1$

$= 355.96 - 163.17 = 192.79$ kN-m

(5) $A_{st_2} = \dfrac{192.79}{230(700-50)} = 1289.56\ mm^2$ 37

(6) $A_{sc} = \dfrac{m \cdot A_{st_2}(d - x_a)}{(1.5m-1)(x_a - d_c)} = \dfrac{11 \times 1289.56(700 - 0.289 \times 700)}{(1.5 \times 11 - 1)(\cdot 289 \times 700 - 50)}$

$A_{sc} = 2990.6\ mm^2$

or $A_{sc} = \dfrac{MR_2}{(1.5m-1)c'(d-d_c)} = \dfrac{192.79 \times 10^6}{(1.5 \times 11 - 1) \times 6.4(700-50)}$

$C_a = \sigma_{cbc} = 8.5\ N/mm^2$

$c' = \dfrac{x_a - d_c}{x_a} \times C_a = \dfrac{0.289 \times 700 - 50}{\cdot 289 \times 700} \times 8.5$

$c' = 6.4\ N/mm^2$

$\therefore A_{sc} = 2989.9\ mm^2$

Total $A_{st} = A_{st_1} + A_{st_2} = 1121.1 + 1289.56$

$= 2410.66\ mm^2$

# T-BEAM

→ By working stress Method.

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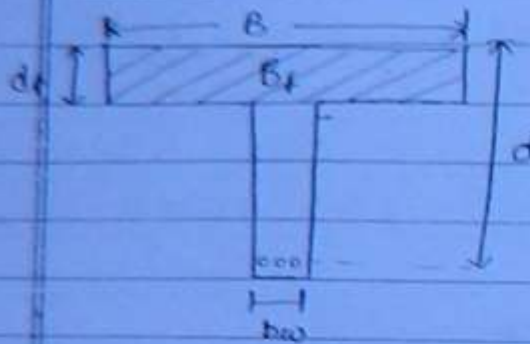
- Isolated beam



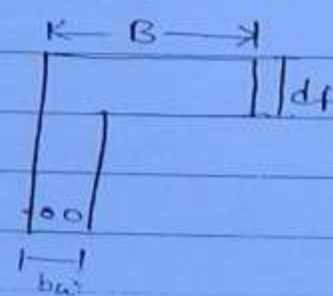

When width of flange is defined = B

Effective width of flange

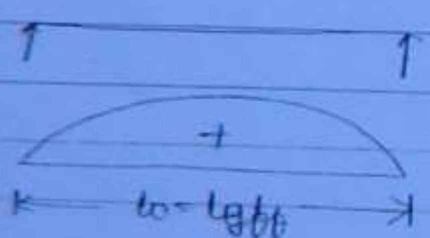
$$B_f = \frac{l_0}{\left(\frac{l_0}{B}\right) + 4} + b_w \quad \text{for T-beam}$$

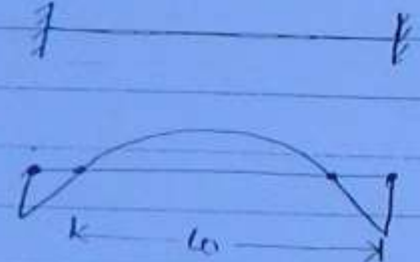
for L-beam

$$B_f = \frac{0.5\, l_0}{\left(\frac{l_0}{B}\right) + 4} + b_w$$




$l_0$ = disn betn point of zero moment of the beam




As per IS-456

for fixed or continous beam

$l_0 = 0.7\, l_{eff}$

$B_0$ = width of web flange

$b_{eff}$ = width of web

slab + beam type (continous type)

Effective width of flange

for T-beam $B_f = \frac{l_0}{6} + b_w + 6d_f$

For L-beams

$$B_f = \frac{l_0}{12} + b_w + 3d_f$$

OR for T-beam $B_f = b_w + \frac{l_1}{2} + \frac{l_2}{2}$

for L-beam $B_f = b_w + \frac{l_1}{2}$

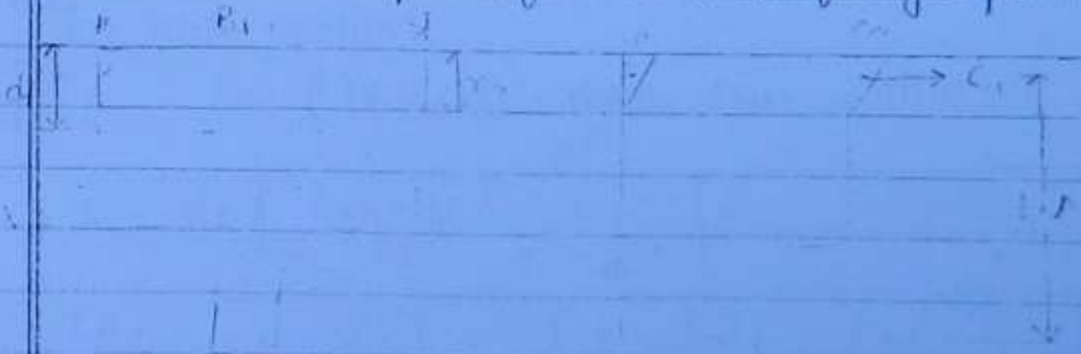
Effective width of flange is min of above two

Analysis of T-beam

(1) Critical depth of N.A

$$x_c = \frac{mc}{t + mc} \times d$$

Case I when actual depth of N.A is in flange portion $(x_a < d_f)$



Equating moment of area on both side of N.A

$$B_f \cdot x_a \cdot \frac{x_a}{2} = m \cdot A_{st} (d - x_a)$$

$$B_f \cdot \frac{x_a^2}{2} = m \cdot A_{st} (d - x_a)$$

$$x_a = ?$$

Here $\boxed{x_a < d_f}$

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(a) $x_a < x_c$ — under Reinforced Section

$c_a < \sigma_{cbc}$

$t_a = \sigma_{st}$

(b) if $x_a = x_c \longrightarrow$ Balanced Section

$c_a = \sigma_{cbc}$

$t_a = \sigma_{st}$

(c) if $x_a > x_c \longrightarrow$ over reinforced Section

$c_a = \sigma_{cbc}$

$t_a < \sigma_{st}$

Moment of Resistance

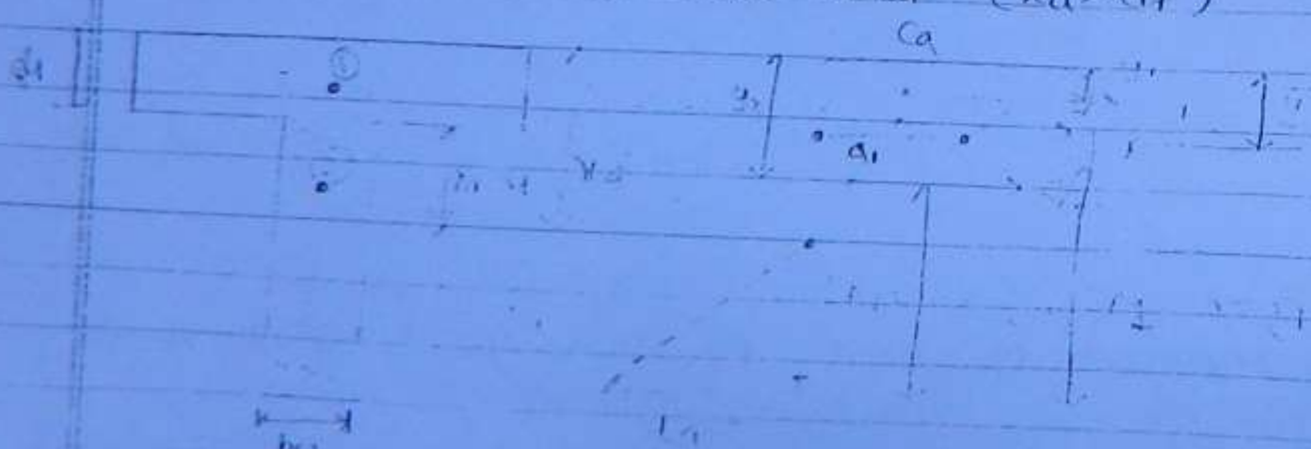
$MR = C_1 \cdot LA$

$$= B_f \cdot x_a \cdot \frac{\sigma_a}{2}\left(d - \frac{x_a}{3}\right) \text{ —— (1)}$$

$$M_R = t_a \times A_{st}\left(d - \frac{x_a}{3}\right) \text{ —— (2)}$$

NOTE - Case I is same as that of rectangular beam of size $(B_f \times d)$

CASE II When N·A is within web area $(x_a > d_f)$




(1) Actual depth of N A

Equating moment of area on both side of N·A

$$B_f \cdot d_f \cdot (x_a - d_f/2) + b_w (x_a - d_f)\left(\frac{x_a - d_f}{2}\right) = m A_{st}(d - x_a)$$

$$B_f \cdot d_f \left(x_a - \frac{d_f}{2}\right) + b_w \frac{(x_a - d_f)^2}{2} = m A_{st}(d - x_a)$$

2. Moment of Resistance

For flange portion - If stress @ top fibre = $c_a$

$$\frac{c_1}{x_a - d_f} = \frac{c_a}{x_a}$$

Stress at junction $c_1 = \frac{x_a - d_f}{x_a} \times c_a$

Flange portion $c_1 = B_f \cdot D_f \left(\frac{c_a + c_1}{2}\right)$

$$y_1 = \left(\frac{a+2b}{a+b}\right)\frac{h}{3} \Rightarrow \left(\frac{c_a + 2c_1}{c_a + c_1}\right)\frac{d_f}{3}$$

$LA_1 = d - y_1$

$$LA_1 = d - \left(\frac{c_a + 2c_1}{c_a + c_1}\right)\frac{d_f}{3}$$

$MR_1$ (for flange) $= c_1 \times LA_1$

$$= B_f \times d_f \cdot \left(\frac{c_a + c_1}{2}\right) \times \left[d - \left(\frac{c_a + 2c_1}{c_a + c_1}\right)\frac{d_f}{3}\right]$$

For web portion

$$C_2 = b_w (x_a - d_f)\frac{c_1}{2}$$

$$y_2 = d_f + \frac{(x_a - d_f)}{3}$$

$$LA_2 = d - y_2 = d - \left[d_f + \frac{(x_a - d_f)}{3}\right]$$

$MR_2 = C_2 \cdot LA_2$

$$= b_w (x_a - d_f)\frac{c_1}{2}\left[d - d_f - \left(\frac{x_a - d_f}{3}\right)\right]$$

Total Moment of Resistance $MR = MR_1 + MR_2$

$$= B_f \cdot d_f \cdot \left(\frac{c_a + c_1}{2}\right)\left[d - \left(\frac{c_a + 2c_1}{c_a + c_1}\right)\frac{d_f}{3}\right] + b_w (x_a - d_f)\frac{c_1}{2}$$

$$\left[d - d_f - \left(\frac{x_a - d_f}{3}\right)\right]$$

(42)

Moment of Resistance from Tension Side

Total Resultant Comp. force $C_R = C_1 + C_2$

$$\bar{y} = \frac{C_1 Y_1 + C_2 Y_2}{C_1 + C_2}$$

Net lever arm $= (d - \bar{y})$

$$MR = C_R (d - \bar{y}) = T (d - \bar{y}) = t_a A_{st} (d - \bar{y})$$

Design formula

It is based on following assumption

① Web area is neglected

② Lever arm $= 0.9d$

$$MR = B_f \cdot d_f \cdot \left(\frac{C_a + C_1}{2}\right) \times 0.9d$$

$$= B_f \cdot d_f \left[\frac{C_a + \left(\frac{x_a - d_f}{x_a}\right) C_a}{2}\right] \times 0.9d$$

$$= B_f \, d_f \, C_a \left[\frac{2x_a - d_f}{x_a}\right] \times 0.45d$$

$$MR = B_f \cdot d_f \cdot \sigma_{cbc} \left[\frac{2kd - d_f}{kd}\right] \times 0.45d$$

$$MR = 0.45 \cdot B_f \cdot d_f \, \sigma_{cbc} \left[\frac{2kd - d_f}{k}\right]$$

Equating $BM = MR$ we get value of d required

$$BM = t_a \cdot A_{st} \cdot 0.9d$$

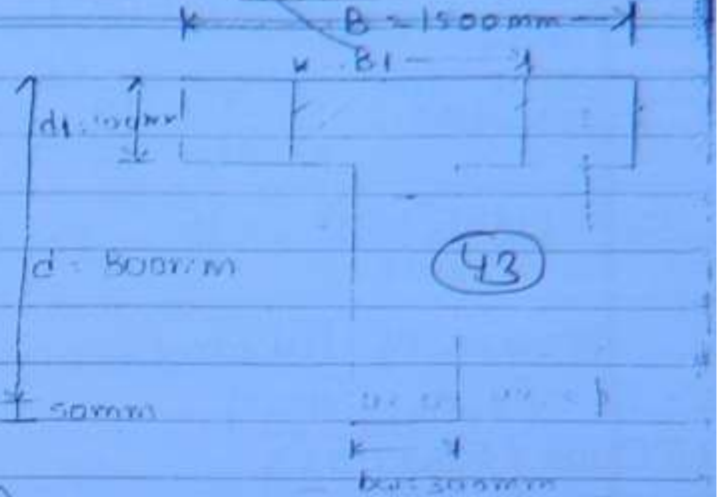
$$A_{st} = \frac{B.M}{t_a \times 0.9d}$$

ues

$Q^n$ Calculate MR of an Isolated T beam. Supported over a simply supported span $= 8m = l_o$

① Eff width of flange $(B_f)$

$$B_f = \frac{l_o}{(l_o/B) + 4} + b_w$$




$$B_f = \frac{8000}{\frac{8000}{1500}+4} + 300 = 1157.14$$

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Assuming N.A in flange area

$$B_f \cdot \frac{x_a^2}{2} = m \cdot A_{st}\,(d - x_a)$$

$$1157.14 \cdot \frac{x_a^2}{2} = 13 \times 4 \times \frac{\pi}{4}(25)^2 (800 - x_a)$$

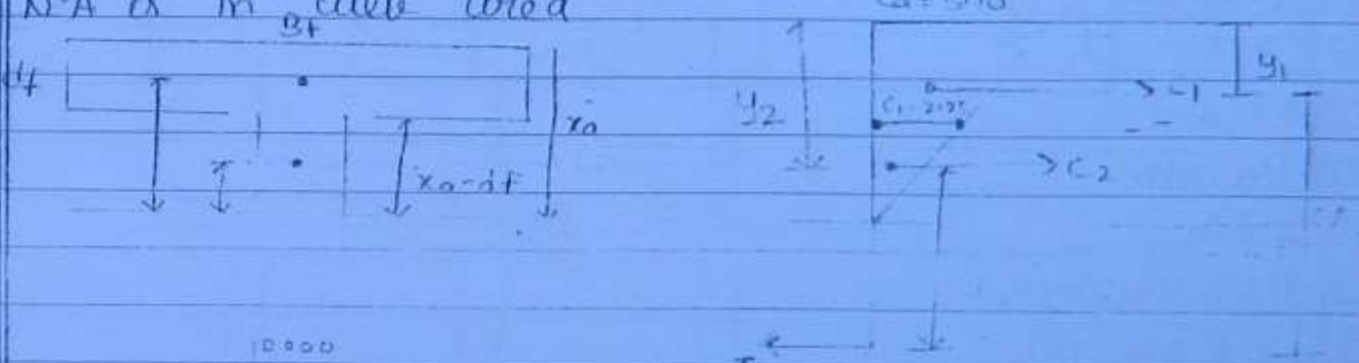
$$x_a^2 = 14768290.47\,(800 - x_a)$$

$$x_a^2 + 14768290.47\,x_a = 1.1814 \times 10^{10}$$

$$x_a = 167.09\text{ mm} \quad > 100\text{ mm}$$

$\therefore$ assumption is not correct

$\rightarrow$ N.A is in web area




$$B_f \cdot d_f\left(x_a - \frac{d_f}{2}\right) + b_w \frac{(x_a - d_f)^2}{2} = m \cdot A_{st} \cdot (d - x_a)$$

$$1157.14 \times 100 \times \left(x_a - \frac{100}{2}\right) + 300\left(\frac{x_a - 100}{2}\right)^2 = 13 \times 4 \times \frac{\pi}{4}(25)^2 (800 - x_a)$$

$$771.43\,x_a - 38571.67 + x_a^2 - 200\,x_a + 10^4 = 136135.68 - 170.17\,x_a$$

$$x_a^2 + 741.6\,x_a - 164707.35 = 0$$

$$x_a = 178.93\text{ mm}$$

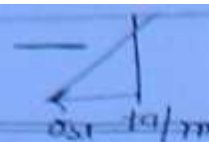
(2) Critical depth of N.A $-\; x_c = \frac{mc}{t_{\sigma st} + mc} \times d = \frac{13 \times 7}{230 + 13 \times 7} \times 800$

$= 226.8$ mm

$x_a < x_c$, It is under reinforced case

Cal $\sigma_{cbc}$ & $t_c$ : $\sigma_{st} = 230\,N/mm^2$

xa   d-xa   d-xa



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$= \frac{178.93}{800-178.93} \times \frac{230}{13} = 5.10\, N/mm^2$

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... at junction $\frac{C_1}{x_a - d_f} = \frac{C_a}{x_a} \Rightarrow C_1 = \frac{x_a - d_f}{x_a} \times C_a$

$C_1 = \frac{178.93-100}{178.93} \times 5.10 = 2.25\, N/mm^2$

3) Moment of Resistance

ge- $C_1 = B_f \cdot d_f \cdot \left(\frac{C_a + C_1}{2}\right)$

$1157 \times 100 \left(\frac{5.10 + 2.25}{2}\right)$

425248.95 N

$\left(\frac{C_a + 2C_1}{C_a + C_1}\right) \frac{d_f}{3}$

$\left[\frac{5.10 + (2.25 \times 2)}{5.10 + 2.25}\right] \times \frac{100}{3}$

= 43.537

= 756.46 mm

$= C_1 \times LA_1$

= 756.46 × 425248.95

= 321.68 kN-m

→ Web — $C_2 = b_w (x_a - d_f) \times \frac{C_1}{2}$

$= 300 \times (178.93 - 100) \times \frac{2.25}{3}$

$C_2 = 26638.875\, N$

$y_2 = d_f + \frac{x_a - d_f}{3} = 100 + \left(\frac{178.93 - 100}{3}\right)$

= 126.31

$LA_1 = (d - y_2) = 800 - 126.31 = 673.69\, mm$

$MR_2 = C_2 \times LA_2 = 26638.875 \times \frac{673.69}{10^6}$

→ $MR_2 = 17.94\, kN\text{-}m$

Total MR = $MR_1 + MR_2$

= 321.685 + 17.94

$\boxed{MR = 339.625\, kN\, m}$

Q Design a T-beam for a Commercial Complex with reference to the data as stated below

Clear span of T-beam = 10 m

Spacing of T-beam = 2.5 m

Live load = 4 kN/m²

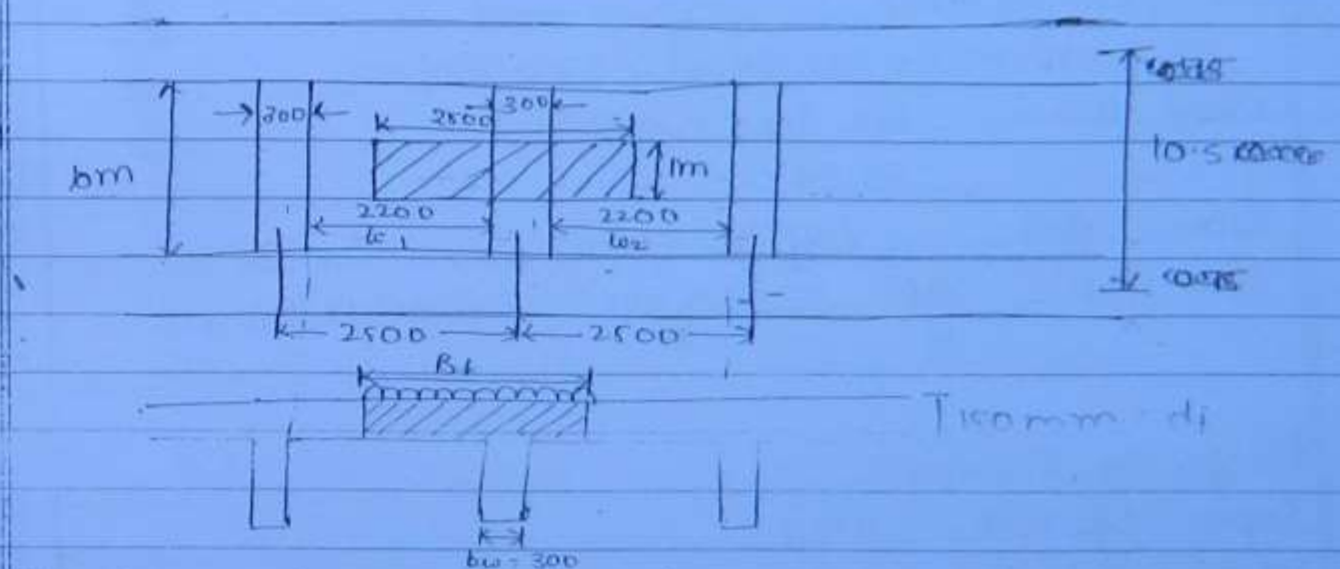
Thickness of slab 15 cm = .15 m

M 20 / Fe 415

Effective span of beam = 10.5 m

$b_w$ = 300 mm

Sol:-




① Effective width of flange

for Cont Slab + beam type

$$B_f = \frac{l_0}{6} + b_w + 6d_f$$

$$= \frac{10500}{6} + 300 + 6 \times 150 = 2950 \text{ mm}$$

② Max $B_f$ = c/c distance beam adjacent slab

$$= b_w + \frac{l_{01}}{2} + \frac{l_{02}}{2} = 300 + \frac{2200}{2} + \frac{2200}{2}$$

$B_f$ = 2500 mm

③ Load Calculation

over 1 beam for 1m length of beam

ⓐ dead

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Live load = 1 × 2.5 × 4 = 10 kN/m

Dead load of slab

= 0.15 × 1 × 2.5 × 25 = 9.37

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Self wt. of web portion of slab

Assume depth of T-beam

$= \frac{Span}{20} = \frac{10000}{20} = 500\ mm$

Slab thickness = 150 mm

Web depth = 350 mm

Dead load of web

= 0.30 × 0.35 × 1.0 × 25 = 2.625 kN/m

Total load = 10 + 9.37 + 2.625

= 21.995

say 22 kN/m

Max B.M = $\frac{wl^2}{8} = \frac{22 \times 10.5^2}{8} = 303.2$ kN/m

Design of T beam

Depth of N.A = 0.9d

Neglecting web portion

B.M = MR = $0.45 \cdot B_f\ d_f\ \sigma_{cbc} \left(\frac{2kd - d_f}{k}\right)$

$303.2 \times 10^6 = 0.45 \times 2500 \times 7 \times 150 \left(\frac{2 \times .2835 \times d - 150}{0.2835}\right)$

d = 392.89

say d = 400 mm

D = 400 + 50 = 450 mm

Area of steel $A_{st} = \frac{BM}{\sigma_{st}\ j \cdot d}$

$= \frac{303.2 \times 10^6}{230 \times 0.90 \times 400} = 3661.8\ mm^2$

No of 25 mm ϕ bars = $\frac{3661.8}{\pi/4\ (25)^2}$ = 7.45 (say 8 nos.)

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2nd Of option

If deflection criteria is considered, Min effective depth required

$= \frac{Span}{20} = \frac{10500}{20} = 525\ mm$

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Say 550 mm

+ 50 mm

Total = 600 mm

$A_{st} = \frac{303.2 \times 10^6}{230 \times 0.90 \times 550} = 2663 \ mm^2$

No. Of 25 mm $\phi$ bars = $\frac{2663}{\pi/4 \times 25^2} = 5.42$ (say 6 Nos)

# LIMIT STATE METHOD

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① Material Strength

(a) Characteristic Strength – The value of strength below which not more than 5% of test result are expected to fall

for Concrete → M25 $f_{ck} = 25\ N/mm^2$

for Steel – Fe 415 $f_y = 415\ N/mm^2$

(b) Design Strength

$$= \frac{\text{Characteristic Strength}}{\text{Partial factor of Safety}}$$

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① for Concrete $\gamma_{mc} = 1.50$

Permissible Stress in Concrete $= \frac{0.67 f_{ck}}{1.5}$ (As Per IS-456)

$$= 0.45 f_{ck}$$

② for Steel. $\gamma_{ms} = 1.15$

Permissible Stress $= \frac{f_y}{1.15} = .87 f_y$

③ load

Characteristic load – The load value that has 95% probability of not being exceeded during its life time of Structure. This value is called Characteristic load & used for design purpose.

Design load (In L.S.M)

$F_d = \gamma_f \times F$ F = Characteristic load

Value of $\gamma_f$

Ques

Partial Safety factor for loads (Table 18) P-68

| Load Combination | Limit State of Collapse D.L | L.L | W.L | Limit State of Serviceability D.L | L.L | W.L |
|---|---|---|---|---|---|---|
| D.L + L.L | 1.5 | 1.5 | – | 1.0 | 1.0 | – |
| D.L + (W.L or E.Q.L) | 1.5 / 0.90 | – | 1.5 | 1.0 | – | 1.0 |
| D.L + L.L + (W.L or E.Q.L) | 1.2 | 1.2 | 1.2 | 1.0 | 0.8 | 0.8 |

Example - If D.L.M = 50kN-m
LLM = 40kN-m & Q (G.L.M - 20kN-m)

① $1.5 \times 50 + 1.5 \times 40$ = 135 kN-m — not Critical
② $1.5 \times 50 + 1.5 \times 20$ = 105 kN-m
③ $1.2 \times 50 + 1.2 \times 10 + 1.2 \times 20$ = 132 kN-m

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Note ① 0.90 value is Consider when Stability against overturning is Critical
② WL & E.Q.L are not Considered Simultaneously.

| Limit State Of Collapse | Limit State Of Serviceability |
|---|---|
| ① Flexure | Deflection [DCVC] |
| ② Shear | Cracking |
| ③ Compression | Vibration |
| ④ torsion -- | Corrosion. |

→ Comparison

| Working Stress Method | Limit State Method |
|---|---|
| 1. Concrete | 1. Concrete |
| factor of safety for stress | Stress factor of safety |
| ① M15 = $\frac{Charac.}{Permissible} = \frac{15}{5} = 3$ | = $\frac{Charac.\ Strength}{Permissible\ stress} = \frac{f_{ck}}{0.45 f_{ck}} = 2.22$ |
| M20 = $\frac{20}{7} = 2.86$ | Load factor of safety = 1.50 |
| M25 = $25/8.5 = 2.94$ | Net F.O.S = $2.22 \times 1.50 = 3.33$ |
| average = 3.0 times | which is higher than that of W.S.M |
| factor of safety for load = 1.0 | |
| Net factor of safety = $3.0 \times 1.0$ = 3.0 | |

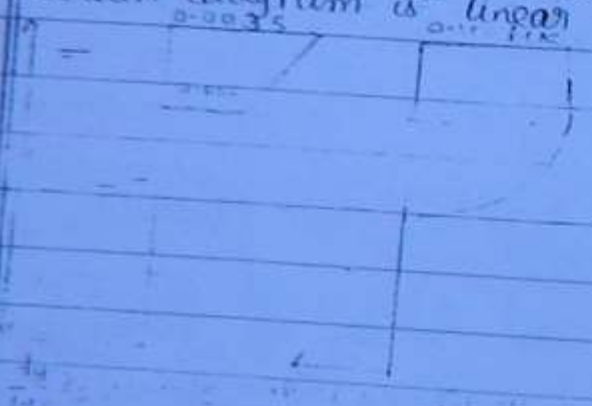
| W.S.M | L.S.M |
| --- | --- |
| 2. Steel | Steel |
| Stress F.O.S | Stress F.O.S = $\frac{f_y}{0.87 f_y} = 1.15$ |
| Mild Steel = $\frac{f_y}{\text{Permissible}}$ | Load F.O.S = 1.50 |
| 250 = 250/140 = 1.78 | |
| 415 = 415/230 = 1.8 | Net F.O.S = $1.15 \times 1.50$ |
| Stress F.O.S = 1.8 | = 1.725 |
| Load F.O.S = 1.00 | less than that of W.S.M |
| Net F.O.S = $1.80 \times 1 = 1.80$ | |

Limit State Method

# SINGLY REINFORCED BEAM

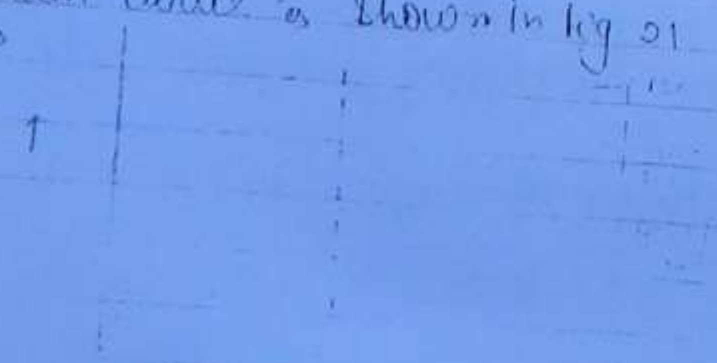
Assumptions (38.1 / Pg-69)

① Plane section before bending remains plane after bending
→ Strain diagram is linear



② The max. strain in concrete at outermost compression fibre is 0.0035

③ An acceptable stress-strain curve is shown in fig 21 for concrete →



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→ For design purpose the Comp. strength of Concrete shall be assumed to be 0.67 times characteristic strength. The partial F.O.S shall be applied in addition to it

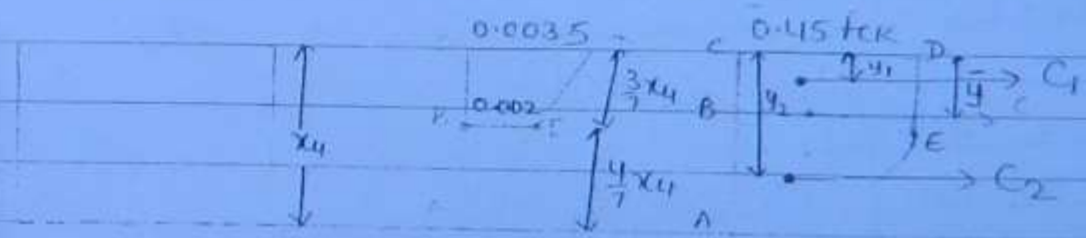
(4) Tensile strength of Concrete is ignored.

(5) The stress in reinforcement are derived from stress-strain curve for the type of steel used. Curves are given in fig 23. For design purpose partial safety factor of 1.15 shall be applied

Permissible stress in steel $= \frac{f_y}{1.15} = 0.87 f_y$

(6) The max. strain in tension reinforcement in the section at the time of failure shall not be less than $\frac{f_y}{1.15 E_s} + 0.002$

Analysis of stress block parameter




$T = 0.87 f_y A_{st}$

K— B —→|

If $x_u$ = Depth of neutral axis (Actual depth of N.A)

$\frac{AB}{AC} = \frac{0.002}{0.0035} = \frac{4}{7}$

$AB = \frac{4}{7} AC = \frac{4}{7} x_u$

$BC = \frac{3}{7} x_u$ (Depth of rectangular portion of stress diagram)

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Compressive force

$C_1$ = (width of beam × Area of stress diagram)

$= B \times 0.45 f_{ck} \times \frac{3}{7} x_u$

$= 0.1928 f_{ck} \cdot x_u \cdot B$

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$y_1 = \frac{1}{2} \times \frac{3}{7} x_u = \frac{3}{14} x_u$

$C_2 = B \times \frac{2}{3} \times 0.45 f_{ck} \times \frac{4}{7} x_u$

$= 0.1714 f_{ck} \cdot x_u \cdot B$

$y_2 = \frac{3}{7} x_u + \frac{3}{8} \times \frac{4}{7} x_u = 0.6429 x_u$

Q $C = C_1 + C_2$

$= .1928 f_{ck} \times x_u \cdot B + 0.1714 f_{ck} \cdot x_u \cdot B$

$= .3642 f_{ck} x_u \cdot B$

$\boxed{\text{As per IS} = .36 f_{ck} x_u \cdot B}$

$\bar{y} = \frac{C_1 \bar{y}_1 + C_2 y_2}{C_1 + C_2} = \frac{.1928 \times \frac{3}{14} + .1714 \times .6429}{.3642} \times x_u$

$= .416 x_u$

$\boxed{\bar{y} = .42 x_u}$ As per IS 456

Total tensile force $= T = 0.87 f_y A_{st}$

lever arm

= distance betn C & T

$(d - 0.42 x_u)$

(f) Actual depth of Neutral axis ($x_u$)

Total Compressive force = Total tensile force

C = T

$0.36 f_{ck} x_u \cdot B = .87 f_y \cdot A_{st}$

$x_u = \frac{.87 f_y A_{st}}{.36 f_{ck} B}$ imp.

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② Moment Of Resistance.

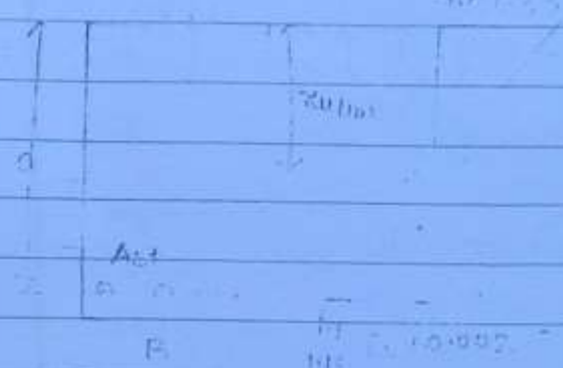
M.R = Total force Of Comp. x lever arm

$= .36 f_{ck}\, x_u \cdot B\,(d - 0.42\, x_u)$ — imp

M.R = Total force Of Tension x lever arm (53)

$= .87 f_y A_{st}\,(d - 0.42\, x_u)$ — imp.

③ limiting depth Of Neutral axis (based on strain diagram)



At $x_{u\,lim}$ - value Of strain should be at max limiting value

from similiar $\Delta^{le}$

$$\frac{x_{u,lim}}{d - x_{u\,lim}} = \frac{0.0035}{\frac{f_y}{1.15 E_s} + 0.002}$$

$$\Rightarrow \frac{d - x_{u\,lim}}{x_{u\,lim}} = \frac{\frac{f_y}{1.15 E_s} + 0.002}{0.0035}$$

$$\frac{d}{x_{u.\,lim}} = \frac{.87 f_y + 0.002 E_s}{0.0035 E_s} + 1$$

$$\frac{d}{x_{u.\,lim}} = \frac{.87 f_y + 0.0055\, E_s}{0.0035\, E_s}$$

$$\frac{d}{x_{u.\,lim}} = \frac{0.87 f_y + 0.0055 \times 2 \times 10^5}{0.0035 \times 2 \times 10^5}$$

$$\frac{d}{x_{u.\,lim}} = \frac{0.87 f_y \times 1100}{700}$$

$$\frac{x_{u\,lim}}{d} = \frac{700}{.87 f_y + 1100} \quad K$$

| $x_{u\,lim}/d$ | $f_y$ | $x_{u,lim}$ |
|---|---|---|
| .53 | fe 250 | .53d |
| .48 | fe 415 | .48d |
| .46 | fe 500 | .46d |

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$x_u \leq x_{u\,lim}$ —

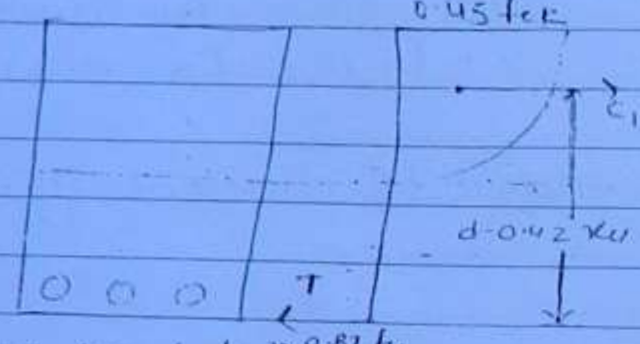
$x_u \not> x_{u\,lim}$ —

(54)

## Moment of Resistance

1. Under reinforced section

$$x_u = \frac{.87 f_y A_{st}}{.36 f_{ck} B}$$




$$x_{u\,lim} = \left.\begin{array}{l} 0.53d \\ 0.48d \\ 0.46d \end{array}\right\}$$

If $x_u < x_{u\,lim}$ - It is underreinforced secn

use $x_u$ for M.R

$M.R = 0.36\, f_{ck}\, B.\, x_u\,(d - 0.42\, x_u)$ —— ①

$\quad = 0.87\, f_y\, A_{st}\,(d - 0.42\, x_u)$ —— ②

Limiting section

If $x_u = x_{u\,lim}$

use $x_{u\,lim}$ for M.R

$M.R = 0.36\, f_{ck}\, B.\, x_{u\,lim}\,(d - 0.42\, x_{u\,lim})$ —— ①

$\quad = 0.87\, f_y\, A_{st}\,(d - 0.42\, x_{u\,lim})$ —— ②

Over reinforced section

If $\left( x_u = \dfrac{0.87 f_y A_{st}}{0.36 f_{ck} B} > x_{u\,lim} \right)$ It is an over reinforced secn

In this case $x_u$ is limited to $x_{u\,lim}$

$M.R = 0.36\, f_{ck} \cdot B \cdot x_{u\,lim}\,(d - 0.42\, x_{u\,lim})$ —— ①

Design formula

for a limiting secn

$B.M = M.R = 0.36\, f_{ck} \cdot B \cdot x_{u\,lim}\,(d - 0.42\, x_{u\,lim})$

$\quad x_{u\,lim} = K.d$

$B.M = 0.36\, f_{ck}\, B \cdot K.d\,(d - 0.42\, K.d)$

$B.M = [0.36\ f_{ck} \cdot \cancel{B} \cdot k \cdot (1 - 0.42k)]\, d^2 B$

$B.M = QBd^2$

$d = \sqrt{\dfrac{BM}{Q.B}}$

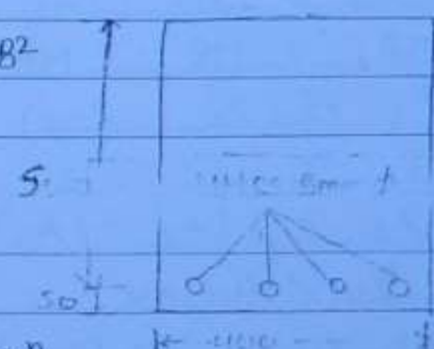
for Fe 415 $\quad Q = 0.36 \times f_{ck}\, (1 - 0.48 \times 0.42) \cdot .48$

$\quad = .138\ f_{ck}$

$M20 \quad Q = 2.75$

$M25 \quad Q = .138 \times 25 = 3.45$

Ques- Calculate M.R of a beam size 400 X 600 mm (overall depth) Eff. Cover = 50mm, M20, Fe 415 is used. Area of steel is
① 4 Nos 18 mm $\phi$ bars ② 8 Nos 25 mm $\phi$ bars
Calculate M.R of limiting section also & area of steel req. for this section.



Solⁿ (i) $x_u = \dfrac{.87 f_y A_{st}}{.36 f_{ck} B} = \dfrac{.87 \times 415 \times 4 \times \pi/4 \times 18^2}{.36 \times 20 \times 400}$

$x_u = 127.61$ mm

$x_{u\,lim} = .48 \times d = .48 \times 550$

$= 264$ mm

$x_u < x_{u\,lim}$ — under reinforced secⁿ

$\therefore x_u$ is used for the M.R

$MR = .36\, f_{ck} \cdot B \cdot x_u\, (d - 0.42\, x_u)$

$= .36 \times 20 \times 400 \times 127.61 (550 - .42 \times 127.61)$

$= 182.44$ kN.m

(ii) Area of steel = 8 Nos 25 mm $\phi$ bars

$A_{st} = 8 \times \dfrac{\pi}{4} \times 25^2 = 3926.99$

$x_u = \dfrac{.87 \times 415 \times 3926.99}{.36 \times 20 \times 400} = 492.30$

$x_{u\,lim} = .48 \times d = 264$ mm

$x_u > x_{u\,lim}$ It is over reinforced secⁿ

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$x_u = x_{u\,lim}$

$M.R = .36\, f_{ck}\, B.\, x_{ulim}\,(d - 0.42\, x_{u\,lim})$ 264

$= .36 \times 20 \times 400 \times 264\,(550 - 0.42 \times 264) \times \frac{1}{10^6}$

$= 333.87\ kN\text{-}m$

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M.R for limiting secⁿ

$= 333.87\ kN\text{-}m$

$333.87 \times 10^6 = .87 \times 415 \times A_{st}\,(550 - 0.42 \times 264)$

$A_{st} = \frac{333.87 \times 10^6}{158544.276}$

$A_{st} = 2105.85\ mm^2$

W.S.M
$x_{a_1} < x_{a_2} < x_{a_3}$

$C_1 < C_2 < C_3$

$LA_1 > LA_2 > LA_3$

$MR_1 < MR_2 < MR_3$

L.S.M
$C_1 < (C_2 = C_3)$

$T_1 < (T_2 = T_3)$

$LA_1 > (LA_2 = LA_3)$

$x_u > x_{u\,max}$

but $x_u$ lim is considered

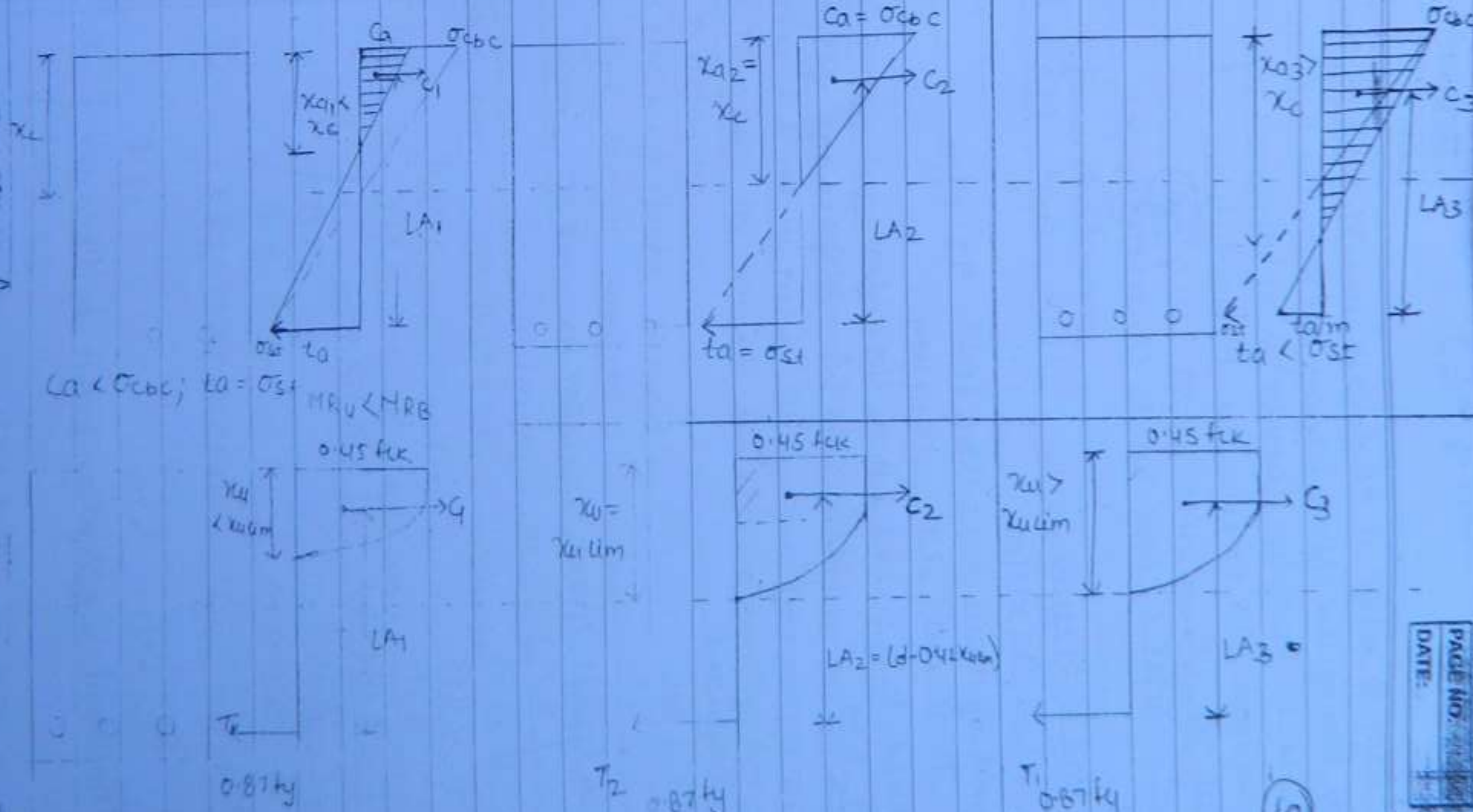
$MR_1 < (MR_2 = MR_3)$

Actual $(T_3 > T_2)$ but not considered

UNDER REINFORCED SECTION | BALANCED SECTION | OVER REINFORCED SECTION

Working Stress Method

Limit State Method




$C_a < \sigma_{cbc}$; $t_a = \sigma_{st}$

$MR_u < MR_B$

$C_1 < C_2 = C_3$

$T_1 < T_2 = T_3$

$LA_1 > LA_2 = LA_3$

$MR_1 < MR_2 = MR_3$

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span $\left[\frac{L}{8} \text{ to } \frac{L}{10}\right]$

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Ques- Use L.S.M to design a reinforced concrete rectangular beam having an eff S.S. span of 6m. The beam is required to have live (service) load = 14 kN/m & superimposed load 9.5 kN/m use Fe 415 & M-20

Adopt $d/b = 2$

$P_{t\,lim} = 0.955\%$, unit wt. of concrete 25 kN/m³

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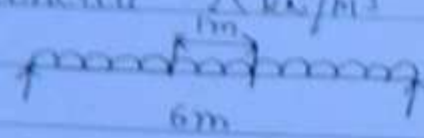
Solⁿ 1. Load




Assume $= \frac{6000}{10}$ $= 600$ mm, $= 300$ mm

Live load = 14 kN/m

S.I D.L = 9.5 kN/m

Self wt of beam $= .6 \times .3 \times 1 \times 25 = 4.5$ kN/m

Total load $w = 28$ kN/m

factored load $= 1.5 \times 28 = 42$ kN/m

Max. B.M $= \frac{wl^2}{8} = \frac{42 \times 6^2}{8} = 189$ kN-m

② Depth required

$$d = \sqrt{\frac{BM}{R \cdot b}}$$

$$R = .36 f_{ck} K(1 - 0.42K) = .138 \times f_{ck} = 2.75$$

$$d = \sqrt{\frac{B.M \times 2}{R \cdot d}}$$

$$d^2 = \frac{2BM}{R \cdot d}$$

$$d^3 = \frac{2 \times 189 \times 10^6}{2.75} = \boxed{516.08 \text{ mm} = d}$$

Say $d = 520$ mm

$$b = \frac{d}{2} = \frac{520}{2} = 260$$

$$D = 520 + 50 = 570$$

Load considered is slightly higher so ok.

Area of Steel Required

$A_{st} = \dfrac{B.M}{.87\, f_y\, (d - 0.42\, x_{ulim})}$ (59)

$= \dfrac{189 \times 10^6}{.87 \times 415\, (520 - 0.42 \times 0.48 \times 520)}$

$= 1261\ mm^2$

Check Limiting $A_{st} = \dfrac{.955}{100} \times 260 \times 520$

$= 1291\ mm^2$

Ques A simply supported RCC rectangular beam is supported over a eff. span of 8.0 m. live load over beam = 30 kN/m M25 / Fe 415 is used. Design the beam by WSM & LSM & Compare the results. Consider width of beam 350 mm in both case.

Sol^n (i) Working Stress Method

live load = 30 kN/m

Span = 8m ; Depth = 8000/10 = 800 mm

width = 350 mm

Self wt = $.35 \times .8 \times 1 \times 25 = 7$ kN/m

Total = 37 kN/m

$B.M = \dfrac{37 \times 8^2}{8} = 296$ kN·m

$d = \sqrt{\dfrac{B.M}{Q \cdot B}}$

$Q = \frac{1}{2} \times 8.5 \times .289 \times .904$

$= 1.11$

$d = \sqrt{\dfrac{296 \times 10^6}{1.11 \times 350}} = 872.87 > 0.8$

$+ 50$

$922\ mm$

$Q = \frac{1}{2}\, c\, j\, k$

$k = \dfrac{mc}{mc + t}$

$t = 230$, $m = 11$

$c = 8.5$

$k = \dfrac{11 \times 8.5}{11 \times 8.5 + 230}$

$= .289$

$j = .904$

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(60)

Consider $D = 950mm$

$d = 900mm$

Self wt $= .35 \times .95 \times 1 \times 25 = 8.3125\ kN/m$

total $w = 38.3125\ kN/m$

$B.M = \frac{wl^2}{8} = 306.5\ kN\text{-}m$

$d = \sqrt{\frac{306.5 \times 10^6}{1.11 \times 350}} = 888\ mm < 900\ mm$

OK.

Area of steel $= \frac{B.M}{\sigma_{st} \cdot j \cdot d}$

$= \frac{306.5 \times 10^6}{230 \times .904 \times 900}$

$= 1638\ mm^2$

Limit state method

live load $30\ kN/m$

Self wt $- .35 \times .8 \times 1 \times 25 = 7\ kN/m$

$w = 37\ kN/m$

factored $= 1.5 \times 37 = 55.5\ kN/m$

$B.M = \frac{wl^2}{8} = \frac{55.5 \times 8^2}{8} = 444\ kN\text{-}m$

$d = \sqrt{\frac{B.M}{Q.B}}$

$Q = .138 f_{ck} = .138 \times 25 = 3.45$

$d = \sqrt{\frac{444 \times 10^6}{3.45 \times 350}} = 606.3\ mm < 750mm.$

So consider $d = 600$

$D = 650mm$

Self wt $= .35 \times .65 \times 1 \times 25 = 5.6875$

$w = 1.5 \times 35.6875 = 53.53\ kN/m$

$$B.M = \frac{wl^2}{8} = 428.25$$

$$d = \sqrt{\frac{428.25\times10^6}{3.45\times350}} = 596 < 600\ mm\ ok$$

(61)

$$A_{st} = \frac{B.M}{.87\times f_y\times(d-.42\,x_{u\,lim})}$$

$$= \frac{428.25\times10^6}{.87\times415\times(600-.42\times.48\times600)}$$

$$= 2476\ mm^2$$

Results :-

| | W.S.M | L.S.M (Preferred less depth is less) |
|---|---|---|
| ① Depth D | 950 mm | 650 mm |
| ② Eff depth d | 900 mm | 600 mm |
| ③ Area of steel | 1638 mm² | 2476 mm² |

Area of Steel
Reinforced section (limiting section or under reinforced section) -

$$A_{st} = \frac{0.5 f_{ck}}{f_y}\left[1-\sqrt{1-\frac{4.6 M_u}{f_{ck}\,Bd^2}}\right]B\cdot d$$

eg. Size of beam = 400 x 700 mm & designed for 20 kN/m over a span of 7.5 m eff using 25 / fe 415. Calculate area of steel

$L.L = 20\ kN/m$

Self wt $= .4\times.7\times1\times25 = 7\ kN/m$

$w = 1.5\times27 = 40.5\ kN/m$

$$B.M = \frac{40.5\times7.5^2}{8} = 284.77\ kN\text{-}m$$

Moment of Resistance of singly Reinforced balanced secn

$M_{u\,lim} = Q\,Bd^2$

$= .138\times25\times400\times650^2/10^6$

$= 583.05\ kN\text{-}m$

B.M < Mulim under reinforced section

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$B\cdot M = M\cdot R$ (62)

$284.76\times10^6 = .36 f_{ck}\, B\cdot x_u\,(d - 0.42\,x_u)$

$= .36\times25\times400\, x_u\,(650 - 0.42\,x_u)$

$0.42\,x_u^2 - 650\,x_u + 79100 = 0$

$x_u = 133.147\ mm$

$$A_{st} = \frac{B\cdot M}{.87 f_y\,(d - 0.42\,x_u)}$$

$$= \frac{284.76\times10^6}{.87\times415\,(650 - 0.42\times133.147)}$$

$= 1327.60\ mm^2$

2nd Method Using direct formula we get

$$A_{st} = \frac{0.5 f_{ck}}{f_y}\left[1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck} B d^2}}\right] Bd$$

$$= \frac{0.5\times25}{415}\left[1 - \sqrt{1 - \frac{4.6\times284.76\times10^6}{25\times400\times650^2}}\right] 400\times650^2$$

$= 132.630 mm^2$

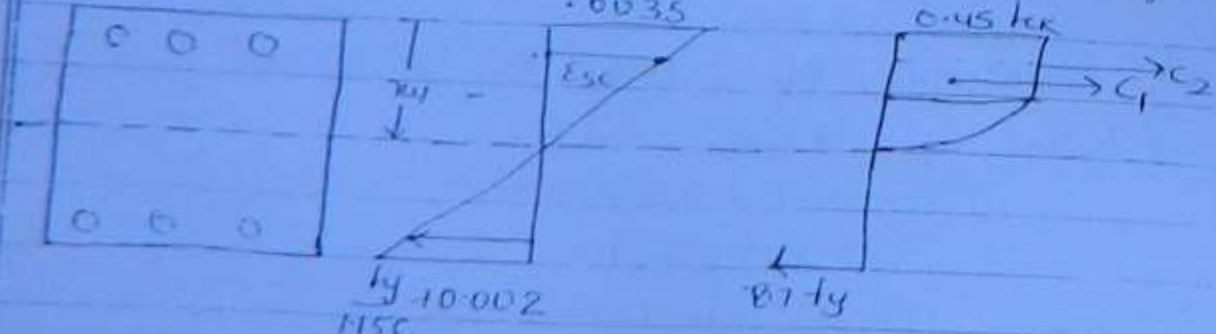
# DOUBLY REINFORCED SECTION

If width & depth both are restricted and beam has to sustain B.M higher than moment of resistance of singly reinforced section. Doubly reinforced beam is required.

B and D/d are defined

MR = QBd² — for singly reinforced section

B.M > MR Doubly reinforced section is req.



Max. Permissible Stress

for Concrete - $0.45 f_{ck}$

for tensile steel - $0.87 f_y$



# for Comp. Steel

As per assumption IS 456 - 2000

Stress in reinforcement is derived from Stress Strain Curve

If strain at Comp. Steel = $\varepsilon_{sc}$

$$\frac{\varepsilon_{sc}}{x_u - d_c} = \frac{0.0035}{x_u}$$

$$\varepsilon_{sc} = \left(\frac{x_u - d_c}{x_u}\right) \times 0.0035$$

As per value of $\varepsilon_{sc}$ Stress in Comp. Steel is read from Stress-strain Curve as per table.

Stress-Strain table

| fe 415 | | fe 500 | |
|---|---|---|---|
| Strain ($\varepsilon_{sc}$) | Stress ($f_{sc}$) | Strain | Stress |
| 0.00144 | 288.7 ($0.8 \times 0.87 f_y$) | 0.00174 | 347.8 |
| 0.00163 | 306.7 | 0.00195 | 369.6 |
| 0.00192 | 324.8 | 0.00226 | 391.3 |
| 0.00241 | 342.8 | 0.00277 | 413.0 |
| 0.00276 | 351.8 | 0.00312 | 423.9 |
| 0.00380 | 360.9 | 0.00417 | 434.8 |

Analysis of doubly reinforced section

(1) Limiting depth of N.A

$x_{u\,lim} = 0.53 d$ — fe 250

$0.48 d$ — fe 415

$0.46 d$ — fe 500

(2) Actual depth of N.A

Equating total force of comp to total force of tension

$$C = T$$

$$C_1 + C_2 = T$$

Area of concrete not present DATE:

$0.36 f_{ck} B \cdot x_u + A_{sc} \times f_{sc} - A_{sc} \times 0.45 f_{ck} = 0.87 f_y A_{st}$

64 Stress in conc

$0.36 f_{ck} B x_u + A_{sc} (f_{sc} - 0.45 f_{ck}) = 0.87 f_y A_{st}$

$x_u < x_{ulim}$ It is a under reinforced Section

Calculate moment of resistance using $x_u$

Moment of resistance

$= C_1 \times LA_1 + C_2 \times LA_2$

$M_u = 0.36 f_{ck} B \cdot x_u (d - 0.42 x_u) + (f_{sc} - 0.45 f_{ck}) A_{sc} (d - d_c)$

(ii) If $x_u = x_{ulim}$ ← limiting Case

use $x_{ulim}$ in place of $x_u$

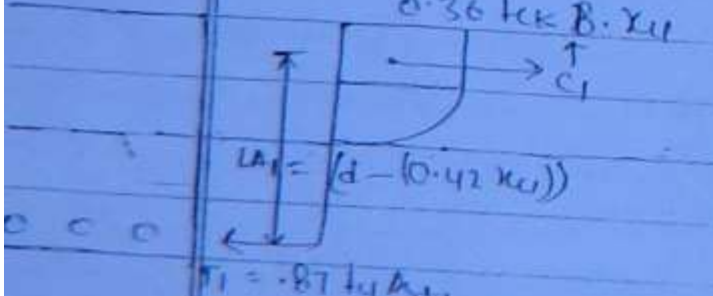
$M_u = 0.36 f_{ck} B \cdot x_{ulim} (d - 0.42 x_{ulim}) + (f_{sc} - 0.45 f_{ck}) A_{sc} (d - d_c)$

$= Q B D^2 + (f_{sc} - 0.45 f_{ck}) A_{sc} (d - d_c)$

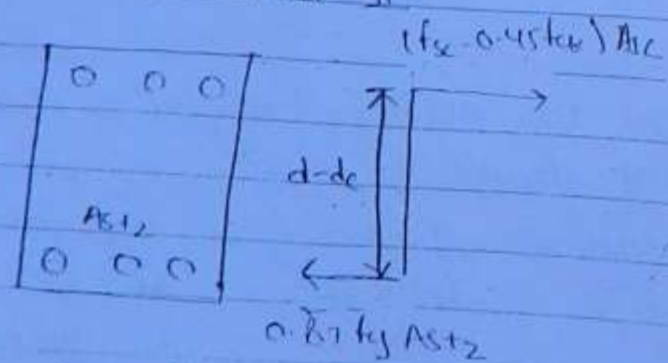
$= MR_1 + MR_2$

(iii) If $x_u > x_{ulim}$ use $x_u = x_{ulim}$ and moment of resistance shall be limited to $M_{u1}$

Part I




Part II




$C_1 = 0.36 f_{ck} B \cdot x_u$

$T_1 = 0.87 f_y A_{st1}$

$LA_1 = (d - 0.42 x_u)$

$C_2 = (f_{sc} - 0.45 f_{ck}) \cdot A_{sc}$

$T_2 = 0.87 f_y A_{st2}$

$LA_2 = (d - d_c)$

$M.R = 0.36 f_{ck} B \cdot x_u (d - 0.42 x_u) + (f_{sc} - 0.45 f_{ck}) A_{sc} (d - d_c)$

$= 0.87 f_y A_{st_1} (d - 0.42 x_u) + 0.7 f_y A_{st_2} (d - d_c)$

$MR_1 = (QBd^2) + (MR_2 = BM - MR_1)$

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Design formula

Step ① The beam has to be designed for B.M. Calculate M.R of singly reinforced limiting section

$MR_1 = QBd^2 = 0.36 f_{ck} B \cdot x_{u\,lim} (d - 0.42 x_{u\,lim})$

If $B.M > MR_1$ – Then a doubly reinf. Secⁿ is required

Step ② $A_{st1}$ for $MR_1$

$$A_{st1} = \frac{MR_1}{0.87 f_y (d - 0.42 x_{u\,lim})}$$

Step ③ Remaining B.M

$MR_2 = B.M - MR_1$

Step ④ Calculate $A_{st2}$

$$A_{st2} = \frac{(MR_2 = BM - MR_1)}{0.87 f_y \cdot (d - d_c)}$$

Step ⑤ Find Out $\varepsilon_{sc}$

$$\varepsilon_{sc} = \left(\frac{x_{u\,lim} - d_c}{x_{u\,lim}}\right) \times 0.0035$$

Step ⑥ $A_{sc}$ for $MR_1$

$$A_{sc} = \frac{MR_1}{(f_{sc} - 0.45 f_{ck})(d - d_c)}$$

Ques Determine ultimate moment capacity of a doubly reinf. Secⁿ

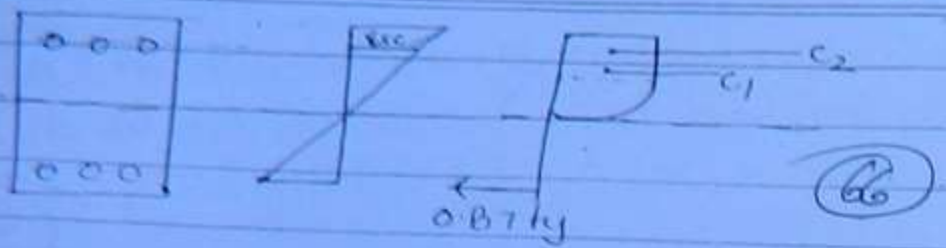
$B = 300$ mm ; $D = 600$ mm

$A_{st} = 2060$ mm² Eff. Cover = 50 mm

$A_{sc} = 804$ mm² M20 / Fe - 415

Sol. $M_u = MR_1 + MR_2$

DATE:

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limiting depth of N.A

$x_{u\,lim} = 0.48 \times d = 0.48 \times 550 = 264\,mm$

Actual depth of N.A

$C_1 + C_2 = T$

$0.36 f_{ck} B \cdot x_u + (f_{sc} - 0.45 f_{ck}) A_{sc} = 0.87 f_y A_{st}$

$0.36 \times 20 \times 300 \times x_u + (f_{sc} - 0.45 \times 20)\, 804 = 0.87 \times 415 \times 2060$

$2160\, x_u + 804 f_{sc} = 750999$

$2.68\, x_u + f_{sc} = 934.08$ —— (A)

rial (1) use $f_{sc} = 350\ N/mm^2$

$x_u = 217.94$

$\varepsilon_{sc} = \left(\frac{x_u - d_c}{x_u}\right) \times 0.0035$

$= \left(\frac{217.94 - 50}{217.94}\right) \times 0.0035 = 0.00269$

$f_{sc} = 342 + \frac{351 - 342}{(.00276 - 0.00241)} \times (0.00269 - 0.00241)$

$= 349.46$

$x_u = 217.94$

$x_u < x_{u\,lim}$

So It is under reinforced secn

$M.R = 0.36 f_{ck} B \cdot x_u (d - 0.42 x_u) + (f_{sc} - 0.45 f_{ck}) A_{sc} (d - d_c)$

$= 0.36 \times 20 \times 300 \times 217.4 (550 - 0.42 \times 217.4) + (349.46 - 0.45 \times 20) \times 804 \times (550 - 50)]$

$M.R = 191.91 \times 10^6 + 136.86 \times 10^6 = 328.77 \times 10^6$

$M.R = 352.4\ kN\text{-}m$ Ans

The size of a R.C beam is restricted to 250 x 500 mm. It carries a S.I load of 25 kN/m over a span of 6m. Determine the reinf. by L.S.D method M20/fe-415. Eff. cover = 40mm.

(67)

| | | | | | | |
|---|---|---|---|---|---|---|
| Stress | 0.8 fyd | .85 fyd | .90 fyd | 0.95 fyd | 0.975 fyd | 1.0 fyd |
| Strain | 0.00144 | 0.00163 | 0.00192 | 0.00241 | 0.00276 | 0.00380 |
| (fsc) | 288.84 | 306.89 | 324.95 | 343 | 352.02 | 361.05 |

$f_{yd} = .87 f_y$

$= 0.87 \times 415 = 361.05$

Load and B.M

Super imposed load = 25 kN/m

Self wt = $.25 \times .50 \times 1 \times 25 = 3.152$ kN/m

$w = 28.152$ kN/m

factored load = $1.5 \times 28.152 = 42.2$ kN/m

Max. B.M = $\frac{wl^2}{8} = \frac{42.2 \times 6^2}{8}$

M.R of singly reinforced section

$MR_1 = QBd^2$

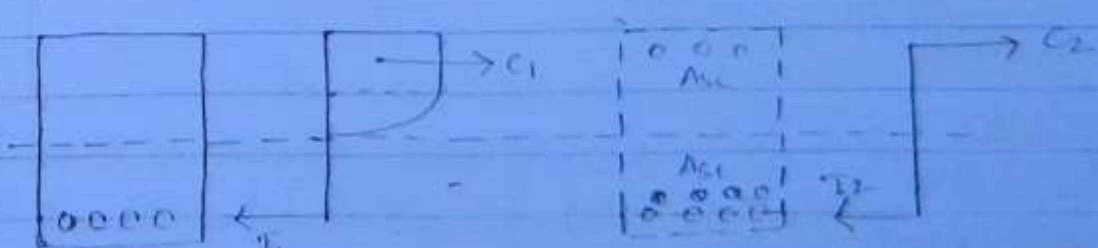
$= 0.36 f_{ck} \cdot B \cdot x_{u\,lim} (d - 0.42\, x_{u\,lim})$

$= 0.138 f_{ck} Bd^2$

$= 0.138 \times 20 \times 250 \times 450^2 \times \frac{1}{10^6}$

$MR_1 = 145.96$ kN-m

Req. MR > $MR_1$ so we have to use doubly reinf. section.




$MR_1 = 145.96$ + $MR_2 = 43.94$

$BM > MR_1$

$MR_2 = B.M - MR_1 = 189.9 - 145.96 = 43.94 \; kN\text{-}m$

Area Of Steel $A_{st1}$ for part 1

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$$A_{st1} = \frac{MR_1}{0.87 f_y (d - 0.42 x_{ulim})}$$

$$= \frac{145.96 \times 10^6}{.87 \times 415 (460 - 0.42 \times \cancel{0.48} \times 460)}$$

$$A_{st1} = 1100.75 \; mm^2$$

Area Of Steel for part ② $A_{st2}$

$$A_{st2} = \frac{MR_2}{0.87 \times 415 \times (460 - 40)}$$

$$= \frac{43.94 \times 10^6}{0.87 \times 415 \times (460 - 40)} = 289.76 \; mm^2$$

Total area of Steel $A_{st} = A_{st1} + A_{st2}$

$$= 1100.75 + 289.76$$

$$1390 = mm^2$$

Value of $\varepsilon_{sc}$

$$\varepsilon_{sc} = \frac{x_{ulim} - d_c}{x_{ulim}} \times 0.0035$$

$$= \frac{0.48 \times 460 - 40}{.48 \times 460} \times 0.0035$$

$$\varepsilon_{sc} = 0.00286$$

$$f_{sc} = 352 + \frac{361 - 352}{(.00380 - .00276)} (.00286 - .00276)$$

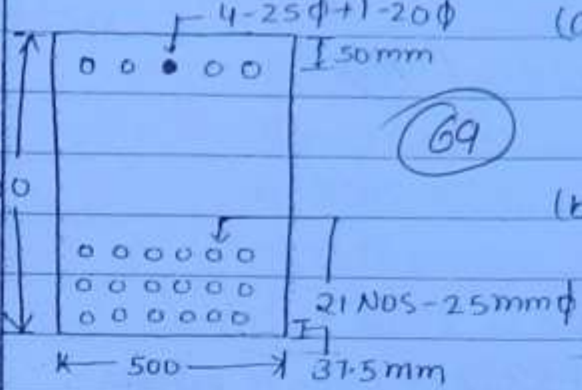
$$f_{sc} = 352.8$$

$$A_{sc} = \frac{MR_2}{(f_{sc} - 0.45 f_{ck})(d - d_c)}$$

$$= \frac{43.94 \times 10^6}{(352.8 - 0.45 \times 20) \times (460 - 40)}$$

$$A_{sc} = 304.94 \; mm^2$$

Ques A simply supported 18m effective span R.C.C rectangular beam 500x1500mm. M25 / Fe-415

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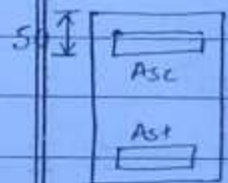
(a) Max. B.M at limit state of collapse
Stress in comp. rein − 0.9566 fyd
(b) What can be the super imposed load that can be applied over beam at working conditions.

Soln $M_u = MR_1 + MR_2$




$A_{sc} = 4 \times \frac{\pi}{4} \times 25^2 + \frac{\pi}{4} \times 1 \times 20^2 = 2277 \, mm^2$

$A_{st} = 21 \times \frac{\pi}{4} \times 25^2 = 10308.35 \, mm^2$

Eff Cover on tension side

$= 37.5 + 25 + 25 + \frac{25}{2} = 100 \, mm$

$B = 500 \, mm$

$d = 1400 \, mm$

$A_{sc} = 2277 \, mm^2$

$A_{st} = 10308 \, mm^2$

① Limiting depth of $x_u = 0.48 \times 1400 = 672 \, mm$

② Actual depth of N.A $C_1 + C_2 = T$

$0.36 f_{ck} B \cdot x_u + (f_{sc} - 0.45 f_{ck}) A_{sc} = 0.87 f_y A_{st}$

$0.36 \times 25 \times 500 \times x_u + (345.20 - 0.45 \times 25) \times 2277 = 0.87 \times 415 \times 10308$

$x_u = 658.06 \, mm$

Beam is under reinforced ($x_u < x_{ulim}$)

$MR = 0.36 f_{ck} B \cdot x_u (d - 0.42 x_u) + (f_{sc} - 0.45 f_{ck}) A_{sc} (d - d')$

$= 0.36 \times 25 \times 500 \times 658.16 (1400 - 0.42 \times 658.16) + (345.20 - 0.45 \times 25) \times 2277 \times (1400 - 50)$

$$M_u = 4354.25 \text{ KN-m}$$

(4) $B.M = \frac{W_u l^2}{8}$ (78)

$= \frac{8 \times 4354.25}{18^2} = W_u$

$W_u = 107.50 \text{ KN/m}$

Service load / unfactored load $= 71.66$ KN/m

deduct Self wt $= 0.50 \times 1.5 \times 1 \times 25 = 18.75$ KN/m

day
/2011
-2006

Service load that can be applied over beam $= 52.91$ KN/m.

Ques- A S.S.B of 5m eff. span is subjected to 24 KN/m live load $f_{ck} = 20 \text{ N/mm}^2$, $f_y = 415 \text{ N/mm}^2$. The overall depth of beam is 400 & width is 250 mm. Design the reinf^n of the beam $K = 0.138$; $j = 0.80$.

Sol^n. By limit state method we can done this Question

$$j = \left(1 - 0.42 \frac{x_{u\,max}}{d}\right) = (1 - 0.42 \times 0.48)$$
$$= .7984 = \text{say } .80$$

$$Q = K . f_{ck}$$
$$Q = .138 \times 20 = 2.76$$

1. Load & B.M

Live load $= 24$ KN/m

Self wt $= .4 \times .25 \times 1 \times 25 = 2.5$ KN/m

∴ Total load $W = 26.5$ KN/m

$$B.M = \frac{Wl^2}{8} = \frac{26 \times 1.5 \times 5 \times 5}{8}$$

$$= 124.22 \text{ KN-m}$$

MR of singly reinforced limiting section

$$MR_1 = Q . B . d^2$$

$$= .138 \times 20 \times 250 \times 350 \times 350 = 84.525 \text{ KN-m}$$

B.M > $MR_1$ So we have a doubly reinforced sections

3. Area Of Steel ($A_{st}$)

(a) $A_{st_1} = \dfrac{MR_1}{.87 f_y (d - 0.42 x_{u\,lim})}$ (71)

$= \dfrac{84.525 \times 10^6}{.87 \times 415 \, (350 - 0.42 \times 0.48 \times 350)} = \dfrac{84.525 \times 10^6}{100891.812}$

$= 837.78 \, mm^2$

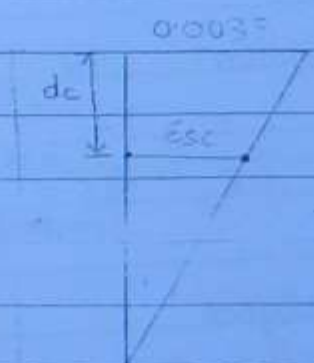
(b) $A_{st_2}$

$A_{st_2} = \dfrac{BM - MR_1}{.87 f_y \, (d - d_c)}$

$= \dfrac{39.695 \times 10^6}{.87 \times 415 \, (350 - 50)} = 366.48 \, mm^2$

Total $A_{st} = A_{st_1} + A_{st_2} = 837.78 + 366.48 = 1204.26 \, mm^2$

(c) $A_{sc} = \dfrac{B.M - MR_1}{(f_{sc} - 0.45 f_{ck}) \times (d - d_c)}$




value Of $f_{sc}$ depends upon $\varepsilon_{sc}$

$\varepsilon_{sc} = \dfrac{x_{u\,lim} - d_c}{x_{u\,lim}} \times 0.0035$

$= \dfrac{0.48 \times 350 - 50}{.48 \times 350} \times 0.0035 = 0.00246$

$f_{sc}$ Should be read for above values

Table is not given, So assuming $f_{sc} = 350 \, N/mm^2$

$A_{sc} = \dfrac{39.695 \times 10^6}{(350 - 0.45 \times 20)\,(350 - 50)}$

$= 388.02 \, mm^2$

# T-BEAM | L-BEAM

DATE: 3/10/2011

Effective width of flange

1. Isolated T-beam / L-beam

T-beam $B_f = \dfrac{l_0}{\left(\frac{l_0}{B}\right)+4} + b_w$ (72)

L-beam $B_f = \dfrac{0.5 l_0}{\left(\frac{l_0}{B}\right)+4} + b_w$

2. Slab + Beam type

1. T-beam $B_f = \dfrac{l_0}{6} + b_w + 6d_f$

$B_f$ = centre to centre betn adjacent slab $= b_w + \dfrac{l_1}{2} + \dfrac{l_2}{2}$

} Whichever is smaller

L-beam $B_f = \dfrac{l_0}{12} + b_w + 3d_f$

or $B_f = b_w + \dfrac{l_1}{2}$

} Whichever is smaller

Analysis

1 Limiting depth of N.A

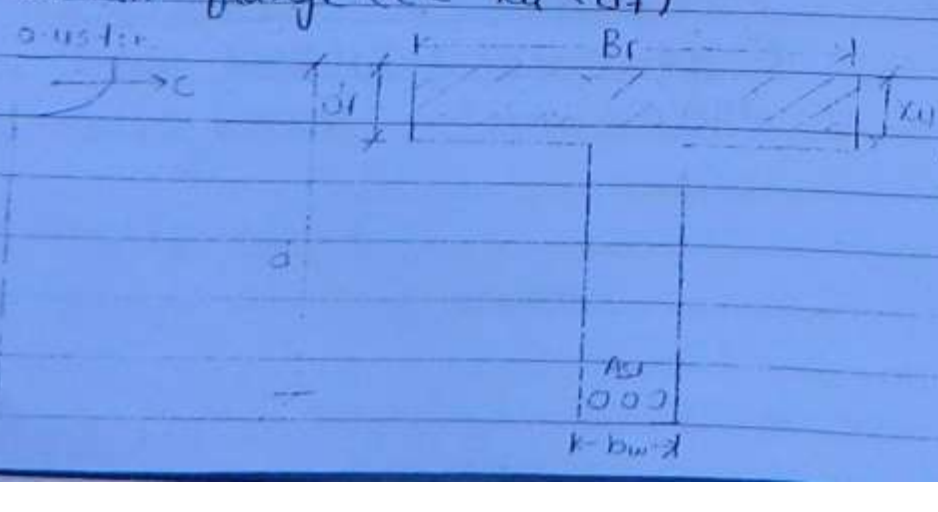
$x_{u\,lim} = 0.53d$ — fe 250

$0.48d$ — fe 415

$0.46d$ — fe 500

2. Actual depth of N.A & Moment of Resistance

CASE I When N.A lie within flange (i.e $x_u < d_f$)



$F_{cs} = 0.87 f_y A_{st}$ $\quad$ $F_{cc} < F_{cs} \rightarrow$ will PAGE NO:

$F_{cc} = F_{cs} =$ N.A lies at the junction of $d_f$ DATE:

It is similar case as of a Rectangular Section of $(B_f \times d)$

$$C = 0.36 f_{ck} B_f \cdot x_u$$

$$T = 0.87 f_y A_{st}$$

$$L.A = (d - 0.42 x_u)$$

(73)

Actual depth of N.A

$$C = T$$

$$0.36 f_{ck} B_f \cdot x_u = 0.87 f_y A_{st}$$

$$x_u = \frac{0.87 \cdot f_y \cdot A_{st}}{0.36 \cdot f_{ck} \cdot B_f}$$

Moment of Resistance

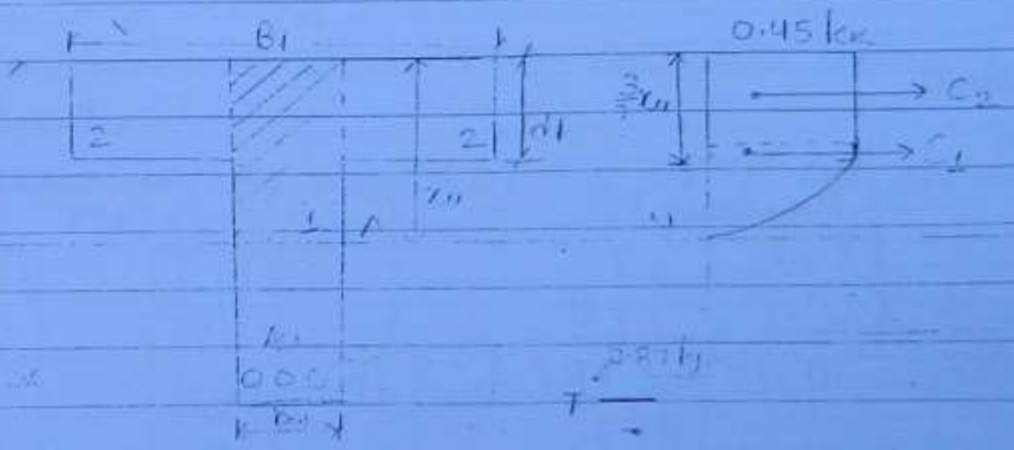
$$MR = C \cdot L.A \quad \text{or} \quad T \cdot LA$$

$$MR = 0.36 f_{ck} B_f \cdot x_u (d - 0.42 x_u) \quad \text{or} \quad 0.87 f_y A_{st} (d - 0.42 x_u)$$

CASE II When N.A lie within web area

SUB CASE I : $x_u > d_f$ (N.A is within web area)

& $d_f < \frac{3}{7} x_u$ (Depth of flange is within rectangular portion of stress diagram)




Actual depth of N.A

$$C_1 + C_2 = T$$

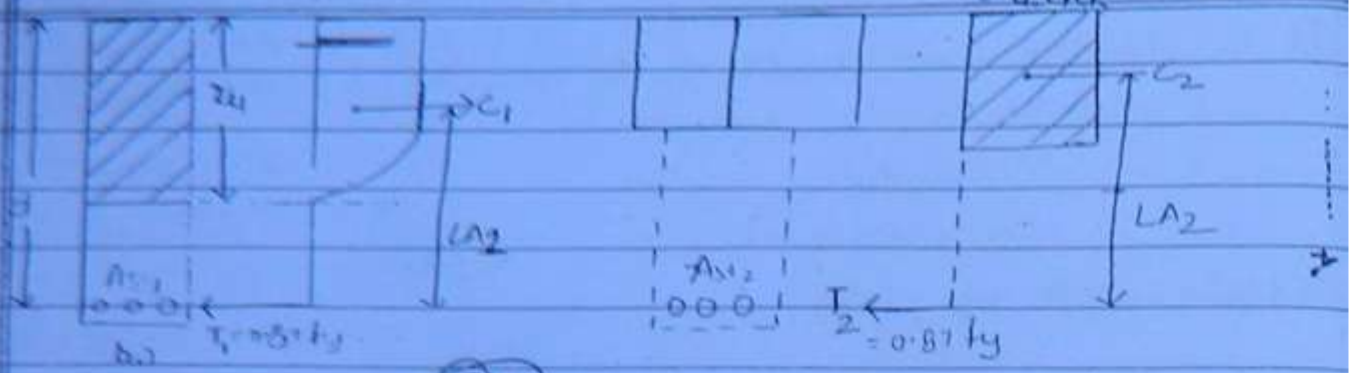
$$0.36 f_{ck} b_w \cdot x_u + (B_f - b_w) d_f \, 0.45 f_{ck} = 0.87 f_y \cdot A_{st}$$

$$0.36 f_{ck} b_w x_u + 0.45 f_{ck} (B_f - b_w) d_f = 0.87 f_y A_{st} \quad \text{— (A)}$$

Find $x_u = ?$

PAGE NO.
DATE:

PART I PART II

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$C_1 = 0.36 f_{ck} \cdot b_w \cdot x_u$ $\qquad$ $C_1 = 0.45 f_{ck} (B_f - b_w) d_f$

$T_1 = 0.87 f_y \cdot Ast_1$ $\qquad$ $T_2 = 0.87 f_y Ast_2$

$LA_1 = (d - 0.42 x_u)$ $\qquad$ $LA_2 = \left(d - \frac{df}{2}\right)$

Actual depth of N.A

$$C_1 + C_2 = T_1 + T_2$$

Moment of Resistance.

on Compression side $MR = C_1 \cdot LA_1 + C_2 \cdot LA_2$

$$MR = 0.36 f_{ck} \cdot b_w \cdot x_u (d - 0.42 x_u) + 0.45 f_{ck} (B_f - b_w) d_f \left(d - \frac{d_f}{2}\right)$$

on tension side $MR = T_1 \cdot LA_1 + T_2 \cdot LA_2$

$$MR = .87 f_y \cdot Ast_1 (d - 0.42 x_u) + 0.87 f_y Ast_2 \left(d - \frac{d_f}{2}\right)$$

Design formula

$$C_1 = T_1$$

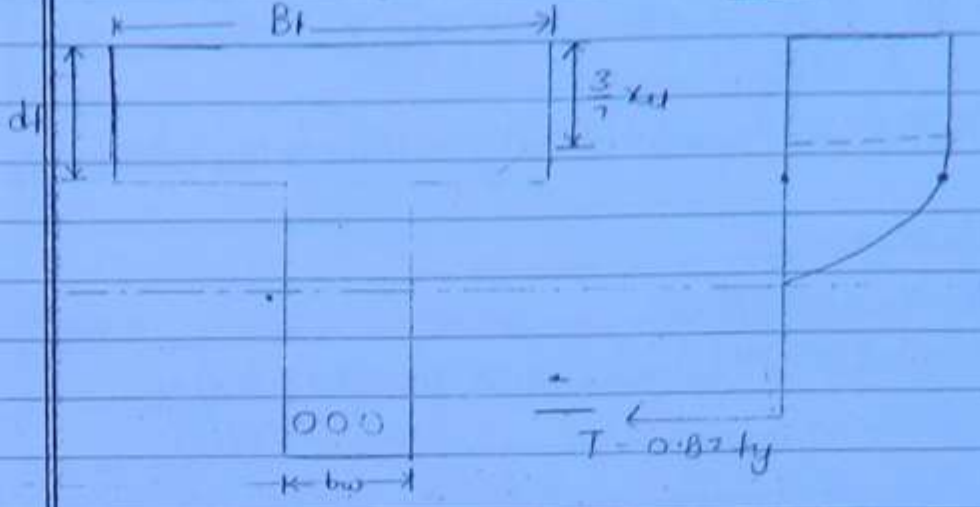
$$Ast_1 = \frac{0.36 f_{ck} \cdot b_w \cdot x_u}{.87 f_y} \quad — 1$$

again $C_2 = T_2$

we get $Ast_2 = \dfrac{0.45 f_{ck} (B_f - b_w) d_f}{.87 f_y} \quad — 2$

CASE II : If $x_u > D_f$ (N.A is within web area)
but $d_f > \frac{3}{7} x_u$ (Depth of flange is more than the rectangular portion of stress diagram)

As per
IS: 456:2000
ANNEX G
P.g No-97

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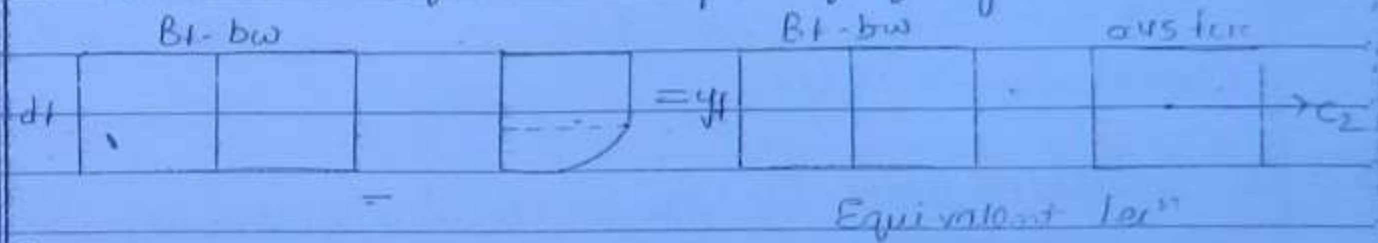
For flange area

Stress diagram is partly rectangular & partially parabolic.
In this case an equivalent depth of flange is considered

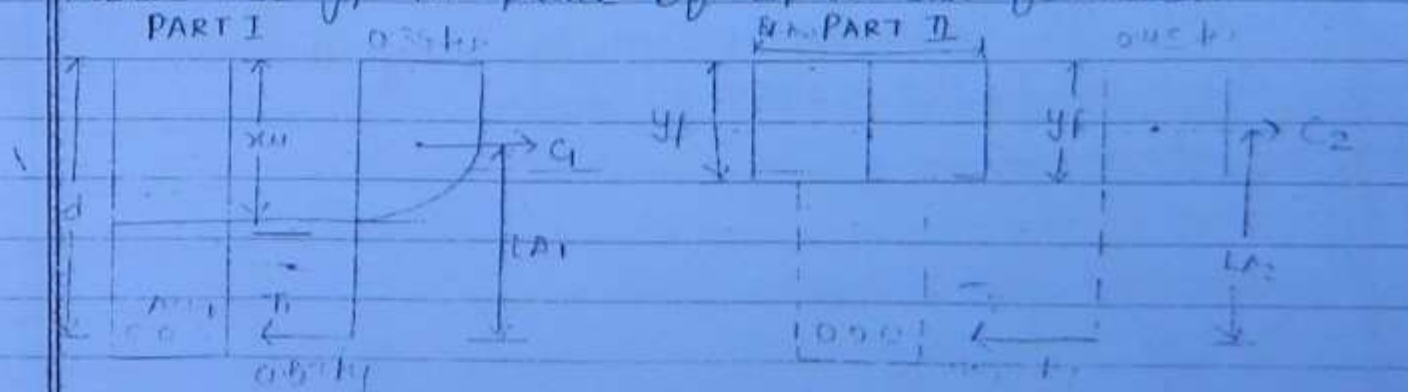



Depth of equivalent section

$y_f = 0.15 x_u + 0.65 d_f$ — A

Stress in overall depth of equivalent section is considered
0.45 fck

Now use $y_f$ in place of $d_f$ in all formula.




$C_1 = 0.36\, f_{ck}\, b_w \cdot x_u$

$T_1 = 0.87\, f_y\, A_{st1}$

$LA_1 = (d - 0.42\, x_u)$

$C_2 = 0.45\, f_{ck}\, (B_f - b_w) y_f$

$T_2 = 0.87\, f_y\, A_{st2}$

$LA_2 = (d - y_f/2)$

Actual depth of N.A

$$C_1 + C_2 = T_1 + T_2$$

$$0.36 f_{ck} \cdot b_w \cdot x_u + 0.45 f_{ck} (B_f - b_w) y_f = 0.87 f_y A_{st_1} + 0.87 f_y A_{st_2}$$

Moment of Resistance

$$MR = C_1 \times LA_1 + C_2 \times LA_2$$

$$= 0.36 f_{ck} x_u \cdot b_w (d - 0.42 x_u) + 0.45 f_{ck} (B_f - b_w) y_f (d - y_f/2)$$

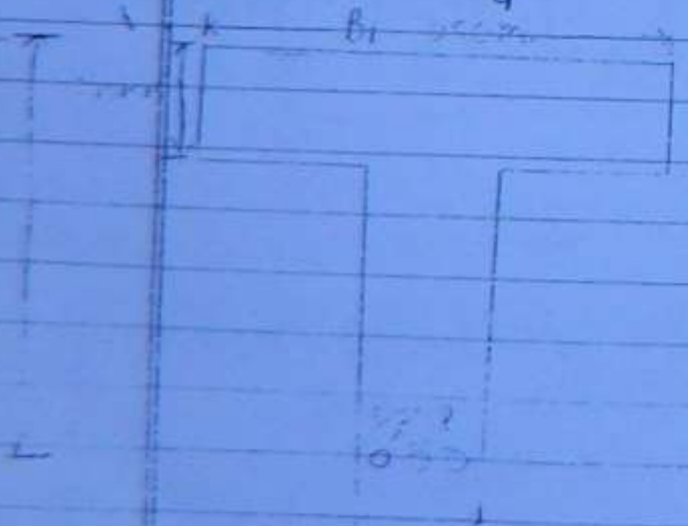
Design formula

$$A_{st_1} = \frac{0.36 f_{ck} \cdot b_w \cdot x_u}{0.87 f_y}$$

$$A_{st_2} = \frac{0.45 f_{ck} (B_f - b_w) y_f}{0.87 f_y}$$

Ques- An Isolated T-beam of span 6.0 m simply supported
Width of ~~beam~~ flange = B ~~Bf~~ = 1500 mm
Overall depth D = 800 mm
Effective cover = 50 mm — Use M-25, Fe415
Area of steel = 5-25 $\phi$
Width of web = 300 mm
Calculate MR of section if depth of flange is 100 mm

Soln $A_{st} = 5 \times \frac{\pi}{4} \times 25^2 = 2454.37 \text{ mm}^2$



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1 Eff. width of flange $B_f = \dfrac{l_o}{\left(\frac{l_o}{B}\right)+4} + b_w$

$$B_f = \frac{6000}{\left(\frac{6000}{1500}\right)+4} + 300 = 1050\,mm \quad (77)$$

2 Given $b_w = 300mm$ ; $D = 800mm$ ; $d = 750mm$

$d_f = 100\,mm$

3 Limiting depth of N.A

$x_{ulim} = 0.48 \times d = 0.48 \times 750 = 360mm$

4 Actual depth of Neutral axis

CASE I - Assume that N.A lie within flange ($x_u < d_f$)

$0.36 f_{ck} B_f \cdot x_u = 0.87 f_y A_{st}$

$$x_u = \frac{0.87 \times 415 \times 2454.37}{0.36 \times 25 \times 1050} = 93.77\,mm$$

So assumption is correct

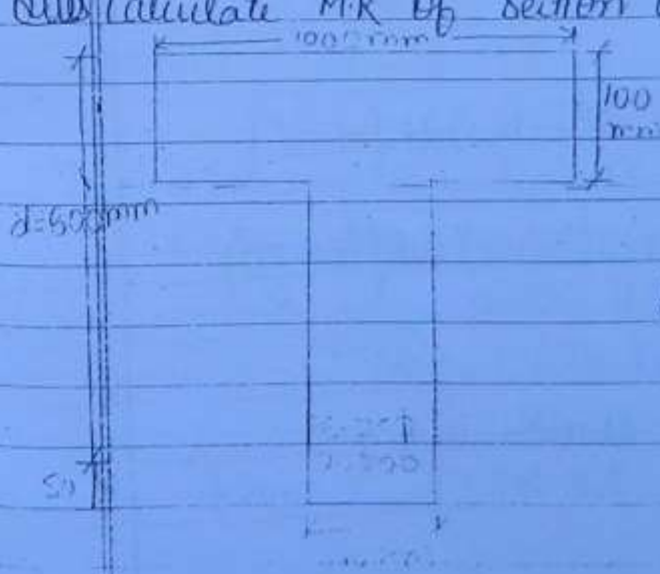
{ $x_u < x_{ulim}$ under reinforced Section

$MR = 0.36 f_{ck}\, x_u \cdot B_f \times (d - 0.42\, x_u)$

$= 0.36 \times 25 \times 93.77 \times 1050\,(750 - 0.42 \times 93.77)$

$= 629.70\ kN\text{-}m$

Ques Calculate M.R of Section as shown in fig.




Span of beam = 5.0 m

Disn. betn point of zero moment in the beam = 4.0 m

Soln Eff. width of flange

$$B_f = \frac{l_o}{\frac{l_o}{B}+4} + b_w$$

$$= \frac{5000}{\left(\frac{5000}{1000}\right)+4} + 300$$

$= 800mm$

Limiting depth of N.A

$x_{u\,lim} = 0.42 \times d = 0.42 \times 800 = 384\,mm$

Actual depth of N.A

1 Assuming $x_u < d_f$

$= 0.36\, f_{ck} \cdot x_u \cdot B_f = 0.87\, f_y \cdot A_{st}$

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$$x_u = \frac{0.87 \times 415 \times 6 \times \pi/4 \times 25^2}{0.36 \times 20 \times 800}$$

$= 184.61\, mm$

this assumtion is not Correct

2. Assuming $x_u > d_f$ but $d_f < \frac{3}{7} x_u$

using $d_f$ value we get

$= 0.36\, f_{ck} \cdot b_w \cdot x_u + 0.45\, f_{ck} (B_f - b_w) d_f = 0.87\, f_y A_{st}$

$= 0.36 \times 20 \times 300 \cdot x_u + 0.45 \times 20 (800 - 300) 100 = 0.87 \times 415 \times \frac{\pi}{4} \times 25^2 \times 6$

$$x_u = \frac{613380.0259}{.36 \times 20 \times 300}$$

$x_u = 283.97\, mm$

$d_f < \frac{3}{7} \times 283.97$

$d_f < 121.70$ Its OK

($x_u < x_{u\,lim}$ $x_u$ is used under reinforced Section)

So assumption is Correct

$MR = 0.36\, f_{ck} \cdot b_w \cdot x_u\, (d - 0.42\, x_u) + 0.45\, f_{ck} (B_f - b_w) d_f \left(d - \frac{d_f}{2}\right)$

680.73 $\times 0.36 \times 20 \times 300 \times 283.97 + 0.45 \times 20 (500) 100 \left(800 - \frac{100}{2}\right)$

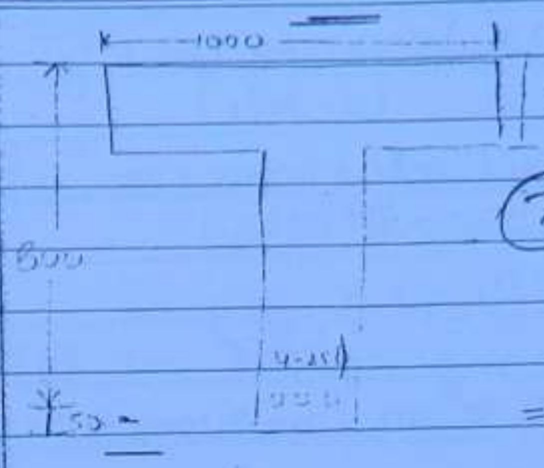
MR $= 755.04\, kN\text{-}m$

Ques- Calculate the MR of the section shown in fig.

use M 20/ fe 415

Span = 5m

$l_o = 4m$



$$B_f = \frac{4000}{\frac{4000}{1000}+4} + 300 = 800\,mm$$

(79) $x_{u\,lim} = 0.42 \times d = 384\,mm$

Actual depth of N.A

1. Assuming $x_u < d_f$

$= 0.36 f_{ck} \cdot x_u \cdot B = 0.87 f_y \cdot A_{st}$

$$x_u = \frac{0.87 \times 415 \times 4 \times \frac{\pi}{4} \times 25^2}{0.36 \times 20 \times 800}$$

$x_u = 123.08$ c is > than 100

So assumption is incorrect.

2. Assuming $x_u > d_f$ but $d_f < \frac{3}{7} x_u$

$= 0.36 f_{ck}\, b_w \cdot x_u + 0.45 f_{ck} (B_f - b_w) d_f = 0.87 f_y A_{st}$

$x_u = 119.87\,mm \rightarrow$ this is Correct

$\frac{3}{7} x_u = 51.37\,mm$

$d_f > 51.37$ So assumption is incorrect

Now we know that $d_f > \frac{3}{7} x_u$ & also $x_u > d_f$

use $y_f = 0.15 x_u + 0.65 d_f = 0.15 x_u + 65$

~~$y_f = 0.15 \times 119.87 + 65 = 82.98$~~

Actual depth of N.A

$C_1 + C_2 = T_1 + T_2$

$0.36 f_{ck}\, b_w \cdot x_u + 0.45 f_{ck} (B_f - b_w) y_f = 0.87 f_y A_{st}$

$0.36 \times 20 \times 300 \times x_u + 0.45 \times 20 (800 - 300)(0.15 x_u + 65) =$

$0.87 \times 415 \times 4 \times \frac{\pi}{4} \times 25^2$

$2160 x_u + 675 x_u + 292500 = 708920.0172$

$x_u = 146.88\,mm > d_f$ OK

$\frac{3}{7} x_u = \frac{3}{7} \times 146.88 = 62.95 < d_f\ (100\,mm)$ OK

$y_f = 0.15 \times 146.88 + 65 = 87.032\,mm$

DATE:

$MR = 0.36 f_{ck} x_u b_w (d - 0.42 x_u) + 0.45 f_{ck} (B_f - b_w) y_f (d - y_f/2)$

$= 0.36 \times 20 \times 146.88 \times 300 (800 - 0.42 \times 146.88) + 0.45 \times 20 (800 - 300) 87.032 (800 - 87.032/2)$

$=$ ~~[illegible]~~ $530.51$ KN-m

80

**Ques** In design section must be given or we can assume the section as balanced section.

| | |
|---|---|
| ethod- Step① If $x_u = d_f$, Cal$^c$ tR of the section | Given - Size of beam |
| $R = 0.36 f_{ck} \cdot B_f \cdot x_u (d - 0.42 x_u)$ | $B, B_f, b_w, d_f, d, D$ |
| $_u = 0.36 f_{ck} B_f \cdot x_u (d - 0.42 d_f)$ | Load or B.M |
| , or B.M < $M_{u1}$ ; the value of $x_u$ is $0 < x_u < d_f$ | design B.M = B.M |
| | Grade of Concrete & Steel |
| uating B.M = MR | find out area of steel for req. B.M. |

$= 0.36 f_{ck} \cdot B_f x_u (d - 0.42 x_u)$

find out $x_u = ?$

alculate area of steel $A_{st} = \dfrac{BM}{0.87 f_y (d - 0.42 x_u)}$

ep② If B.M > $M_{u1}$ then $x_u = \frac{7}{3} d_f$ & Calculate MR for this is equal to $M_{u2}$

It is case 2nd

So $d_f$ is use

$d_f = \frac{3}{7} x_u$

$M.R = M_{u_2} = 0.36 f_{ck} b_w \cdot x_u (d - 0.42 x_u) + 0.45 f_{ck} (B_f - b_w) d_f \left(d - \frac{d_f}{2}\right)$

Calculate $M_{ulim}$ $\boxed{\text{assuming } x_{ulim} > \frac{7}{3} d_f}$

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So $d_f < \frac{3}{7} x_{ulim}$ use $d_f$ formula

Calculate $M_{ulim}$ for $x_{ulim}$

$M_{ulim} = 0.36 f_{ck} b_w \cdot x_{ulim} (d - 0.42 x_{ulim}) + 0.45 f_{ck} (B_f - b_w) d_f (d - d_f/2)$

If B.M is less case 1 $M_{u_1}$ & case 2 $M_{u_2}$ then $x_u$ is less $x_u$ & $\frac{7}{3} d_f$ so $x_u < \frac{7}{3} d_f$

i.e $d_f > \frac{3}{7} x_u$ So use $y_f$ formula.

If B.M is less $M_{u2}$ & $M_{ulim}$ then $x_u$ is less $\frac{7}{3} d_f$ & $x_{ulim}$ $x_u > \frac{7}{3} d_f$ $\boxed{\text{i.e } d_f < \frac{3}{7} x_u}$

So $d_f$ will be consider.

p4 If B. $M_{u_1} < BM < M_{u_2}$ $\longrightarrow$ $x_u < 7/3 d_f$

So $d_f > \frac{3}{7} x_u$ So it is a $y_f$ formula.

Equating B.M = MR

$B.M = 0.36 f_{ck} b_w \cdot x_u (d - 0.42 x_u) + 0.45 f_{ck} (B_f - b_w) y_f (d_0 - y_f/2)$

$y_f = 0.15 x_u + 0.65 d_f$

Get $x_u$ = ?

Now find out area of steel

$A_{st_1} = \frac{0.36 f_{ck} \cdot b_w x_u}{0.87 f_y}$ ; $A_{st_2} = \frac{0.45 f_{ck} (B_f - b_w) y_f}{0.87 f_y}$

If $M_{u_2} < B.M < M_{ulim}$ $\longrightarrow$ $x_u > 7/3 d_f$ $\boxed{\text{i.e } d_f < \frac{3}{7} x_u}$

So $D_f$ formula is used

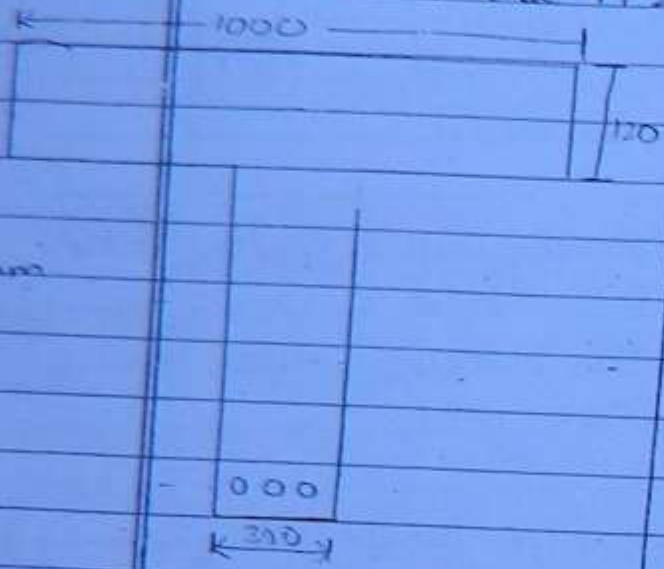
Equating $BM = MR = 0.36 f_{ck} b_w x_u (d - 0.42 x_u) + 0.45 f_{ck} (B_f - b_w) d_f (d - d_f/2)$

Get $x_u$ = ?

& also find area of steel $A_{st_1}$ & $A_{st_2}$

Ques- Design a T-beam of following cross-section. For bending moment due to service load ① 320 KN-m ② 430 KN-m. use M-20 / fe-415

(82)




Given $B = 1000\ mm$

Span = 7.5 m

$l_0 = 0.8$ times Span $= 0.8 \times 7.5 = 6m$

$b_w = 300mm$

$d_f = 120mm$

$d = 600mm$

Effective width $B_f = \dfrac{6000}{\frac{6000}{1000} + 4} + 300 = 900mm$

$x_{u\,lim} = .48 \times 600 = 288mm$

Design B.M

① $BM_1 = 1.5 \times 320 = 480\ KN\text{-}m$

$BM_2 = 1.5 \times 430 = 695\ KN\text{-}m$

Step I When $x_u = d_f$ $\quad x_u = 120mm$

$MR = M_u = .36\, f_{ck}\, B_f \cdot x_u\,(d - 0.42 x_u)$

$M_{u_1} = .36 \times 20 \times 900 \times 120\,(600 - 0.42 \times 120)$

$M_{u_1} = 427.36$

$M_{u_1} < BM_1$ So this is incorrect.

II When $x_u = \frac{7}{3} d_f = 280\ mm$

$d_f < 3/7\, x_u = d_f = 120 \ \angle \ \frac{3}{7} x_u = 120mm$

So use $d_f$ formula

$M_{u_2} = 0.36\, f_{ck}\, b_w \cdot x_u\,(d - 0.42 x_u) + 0.45 f_{ck}\,(B_f - b_w)\, d_f\,(d - d_f/2)$

$Mu_2 = 0.36 \times 20 \times 300 \times 280 (600 - 0.42 \times 280) + 0.45 \times 20 (900-300)$
$120 (600 - 120/2)$

$= 641.67$ KN-m

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$Mu_2 < BM_1$

$Mu_2 < BM_2$

III when $x_u = x_{ulim} = 288mm$

$\frac{3}{7} x_u = \frac{3}{7} \times 288 = 123.43mm$

$df < \frac{3}{7} x_u$ df formula is use

$M_{ulim} = 0.36 \times 20 \times 300 \times 288 (600 - 0.42 \times 288) + 0.45 \times 20$
$(900 - 300) \times 120 (600 - 120/2) \times 1/10^6$
$= 647.9$ KN-m

(1) for $BM_1$ it is b/w $Mu_1$ & $Mu_2$ So Calculate $x_{u1}$
So $x_u < \frac{7}{3} df$ So $df > \frac{3}{7} x_u$ ($y_f$ formula)

(2) for $BM_2$ it is b/w $Mu_2$ & $M_{ulim}$ & Calculate $x_{u2}$
$x_u > \frac{7}{3} df$ so $df < \frac{3}{7} x_u$ (df formula)

for (1) $B.M_1 = MR$
$BM_1 = 0.36 f_{ck} b_w \cdot x_u (d - 0.42 x_u) + 0.45 f_{ck} (B_f - b_w) y_f$
$(d - y_f/2)$

$y_f = 0.15 x_u + 0.65 df$

$480 \times 10^6 = 0.36 \times 20 \times 300 \times x_u (600 - 0.42 x_u) + 0.45 \times 20 (900 - 300)$
$(0.15 x_u + 0.65 \times 120) (600 - .075 x_u - 39)$

$480 \times 10^6 = 1296000 x_u - 907.2 x_u^2 + 5400 (.15 x_u + 78)(561 - 0.075 x_u)$

$480 \times 10^6 = 1296000 x_u - 907.2 x_u^2 + 5400 (.15 \times 561 x_u - .15 \times .075 x_u^2 + 43758 -$
$5.85 x_u)$

$129600 x_u - 907.2 x_u^2 + 454410 x_u - 60.75 x_u^2$
$+ 236293200 - 31590 x_u - 480 \times 10^6 = 0$

$-967.95 x_u^2 + 552420 x_u - 243706800 = 0$

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$x_u = 155.38$

$y_f = 0.15 \times 155.38 + 78$ (84)

$= 101.307\ mm$

$A_{st1} = \frac{0.36 f_{ck}\, b_w \cdot x_u}{0.87 f_y} = \frac{0.36 \times 20 \times 300 \times 155.38}{0.87 \times 415}$

$A_{st1} = 929.57\ mm^2$

$A_{st2} = \frac{0.45 f_{ck}(B_f - b_w) y_f}{0.87 f_y} = \frac{0.45 \times 20 (900-300) 101.307}{0.87 \times 415}$

$A_{st2} = 1515.185\ mm^2$

For $BM_2$ (df formula)

$BM_2 = MR$

$= 0.36 f_{ck}\, b_w \cdot x_u (d - 0.42 x_u) + 0.45 f_{ck}(B_f - b_w) d_f \left(d - \frac{d_f}{2}\right)$

$= 0.36 \times 20 \times 300\, x_u \times 600 - 0.36 \times 20 \times 300 \times 0.42 x_u^2 + 0.45 \times 20 (900-300) 120 \left(600 - \frac{120}{2}\right)$

$645 \times 10^6 = 1296000 x_u - 907.2 x_u^2 + 349920000 = 0$

$-907.2 x_u^2 + 1296000 x_u + 349920000 = 0$

$\cancel{x_u = 232.24}$

$\cancel{A_{st1} = \frac{0.36 \times 20 \times 300 \times 232.24}{0.87 \times 415}}$

$\cancel{A_{st2} = 0.45 \times 20 (900-300)}$

$-907.2 x_u^2 + 1296000 x_u - 29508 \times 10^4 = 0$

$x_u = \frac{-1296000 \pm \sqrt{(1296000)^2 - 4(907.2)(29508 \times 10^4)}}{-2 \times 907.2}$

$= \frac{-1296000 \pm 780275.3975}{-1814.4}$

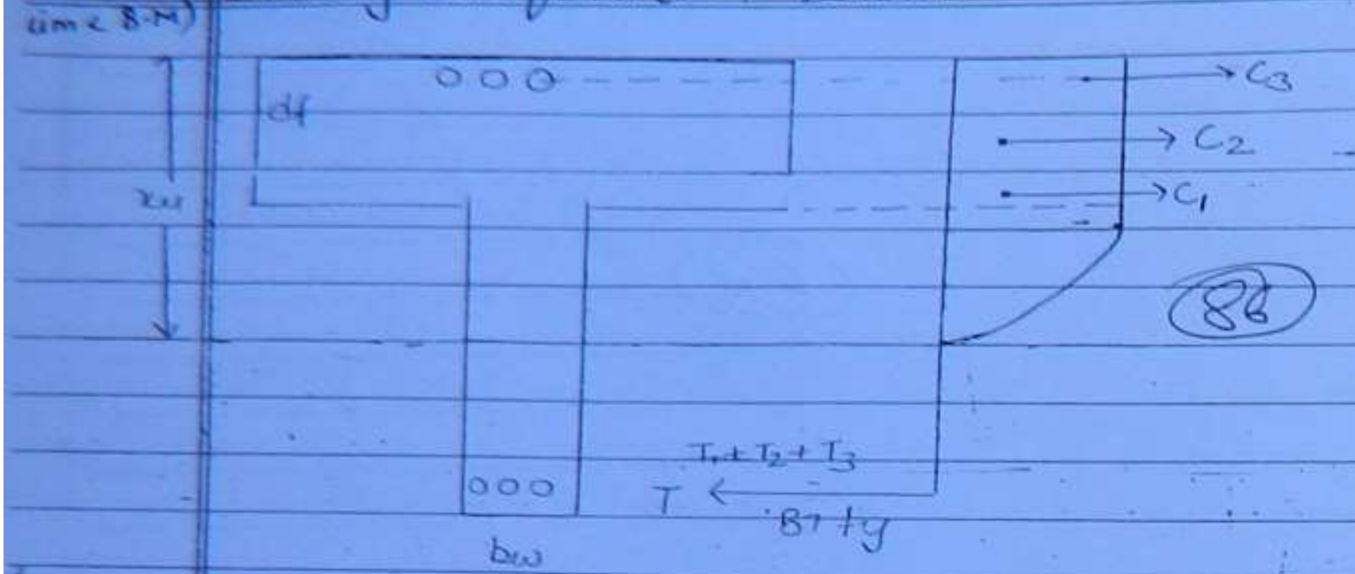
$= 284.24\ mm$

$A_{st1} = \frac{0.36 \times 20 \times 300 \times 284.24}{0.87 \times 415}$

$= 1700.48\ mm^2$

$A_{st2} = \frac{0.45 \times 20 (900-300) \times 120}{0.87 \times 415} = 1794.77\ mm^2$

# Doubly reinforced T-beam




When $x_u > df$

① $df < \frac{3}{7} x_u$ – df formula

② $df > \frac{3}{7} x_u$ – $y_f$ formula

$$y_f = 0.15 x_u + 0.65 df$$

For actual depth of N.A

$$C_1 + C_2 + C_3 = T_1 + T_2 + T_3$$

$$0.36 f_{ck} \cdot b_w \cdot x_u + 0.45 f_{ck} (B_f - b_w)(d_f \text{ or } y_f) + (f_{sc} - 0.45 f_{ck}) A_{sc} = 0.87 f_y A_{st_1} + 0.87 f_y A_{st_2} + 0.87 f_y A_{st_2}$$

Moment of Resistance

$$MR = C_1 \times LA_1 + C_2 \times LA_2 + C_3 \times LA_3$$

$$= 0.36 f_{ck} b_w \cdot x_u (d - 0.42 x_u) + 0.45 f_{ck} (B_f - b_w)(d_f / y_f)\left(d - \frac{y_f \text{ or } d_f}{2}\right) + (f_{sc} - 0.45 f_{ck}) A_{sc} (d - d_c)$$

Area of Steel –

$$A_{st_1} = \frac{0.36 f_{ck} \cdot b_w \cdot x_u}{.87 f_y}$$

$$A_{st_3} = \frac{(f_{sc} - 0.45 f_{ck}) \cdot A_{sc}}{0.87 f_y}$$

$$A_{st_2} = \frac{0.45 f_{ck} (B_f - b_w)(y_f \text{ or } d_f)}{0.87 f_y}$$

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# Doubly reinforced T-beam by working stress Method

Ques: Find the M.R of the hollow box section

Use M-20 | fe-415

$\sigma_{cbc} = 7$

$\sigma_{st} = 230$

$m = 13.33$ in tension

$= 20$ in Compression

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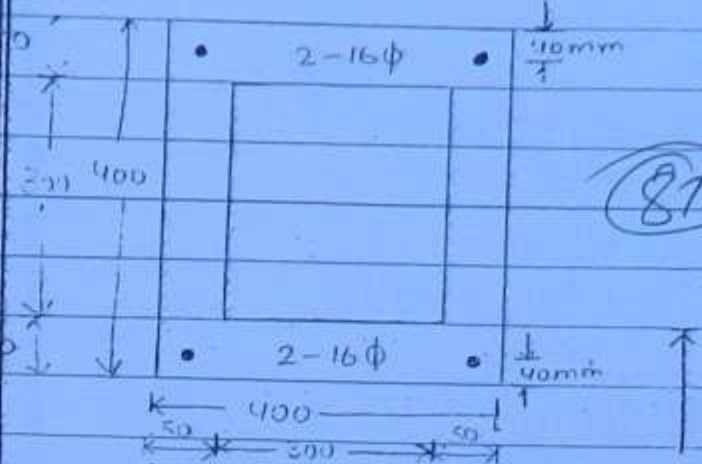


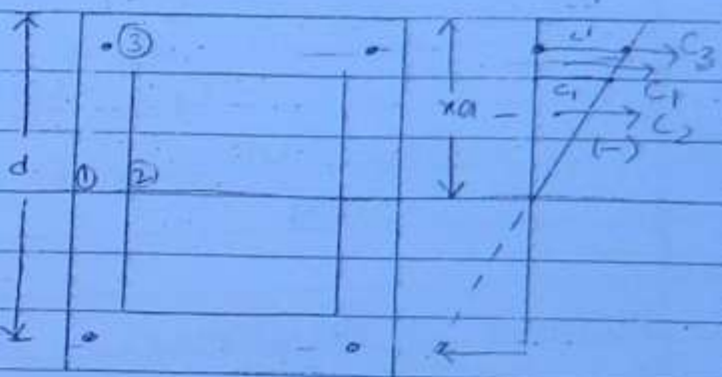


Sol$^n$

$d = 400 - 40 = 360\,mm$

$A_{st} = A_{sc} = 2 \times \frac{\pi}{4} \times 16^2 = 402.12\,mm^2$

$x_c = \frac{mc}{t + mc} \times d = \frac{13.33 \times 7}{230 + 13.33 \times 7} \times 360$

$x_c = 103.9\,mm$

Actual depth of N.A - Assuming $x_a > d_f = 50\,mm$

Equating moment of area on both side of N.A

$B x_a \cdot \frac{x_a}{2}$

$400 \times x_a \cdot \frac{x_a}{2} - 300 \times \frac{(x_a - 50)^2}{2} + (1.5m - 1) \times A_{sc}(x_a - 40) = m A_{st}(d - x_a)$

$200 x_a^2 - 150(x_a^2 - 100x_a + 2500) + (1.5 \times 13.33 - 1) \times 402.12 (x_a - 40) = 13.33 \times 402.12 (360 - x_a)$

$1.333 x_a^2 - x_a^2 + 100 x_a - 2500 + 50.92 - 2036.87 + 35.73 x_a - 12864.62 = 0$

$x_a = 81.39\,mm > 50\,mm$

assumption is correct

Moment of Resistance $= C_1 LA_1 + C_2 LA_2 + C_3 LA_3$

$MR_1 = C_1 \times LA_1 = 400 x_a$

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Stress — $x_a < x_c \rightarrow$ It is under-reinforced

$C_a < \sigma_{cbc}$

$t = \sigma_{st} = 230 \, N/mm^2$ (88)

$$\frac{C_a}{x_a} = \frac{t_a/m}{d - x_a} = \frac{C_a}{81.39} = \frac{230/13.33}{360-81.39}$$

$$C_a = 5.04 \, N/mm^2$$

hollow $\quad \frac{C_1}{x_a - 50} = \frac{C_a}{x_a} \Rightarrow C_1 = \frac{5.04\,(81.39-50)}{81.39}$

$$C_1 = 1.94 \, N/mm^2$$

Steel $\quad c' = \frac{81.39 - 40}{81.39} \times 5.04 = 2.56 \, N/mm^2$

Moment Of Resistance

$C_1 = 400 \times x_a \times \frac{C_a}{2} = 400 \times 81.39 \times \frac{5.04}{2} = 82041.32 \, N$

$y_1 = \frac{x_a}{3} = \frac{81.39}{3} = 27.13$

$LA_1 = d - y_1 = 360 - 27.13 = 332.87$

$MR_1 = 82041.32 \times 332.87 \times \frac{1}{10^6}$

$= (+)\, 27.31 \, kN\text{-}m$

(-ve) $\quad C_2 = 300 \times \overset{(x_a-50)}{\cancel{x_a}} \times \frac{C_1}{2} = 300 \times (81.39-50) \times \frac{1.94}{2} = 9134.49$

$y_2 = \frac{x_a - 50}{3} + 50 = 10.46 + 5 = 60.46$

$LA_2 = d - y_2 = 360 - 60.46 = 299.54$

$MR_2 = (-)\, 9134.49 \times 299.54 \times \frac{1}{10^6} = (-)\, 2.74 \, kN\text{-}m$

$C_3 = (1.5m - 1)\, A_{sc} \times C_s$

$(1.5 \times 13.33 - 1)\,(452.12) \times 2.56$

$= 19553.97 \, N$

$LA_3 = y_3 = 40$

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$LA_3 = 360 - 40 = 320\ mm$

$MR_3 = 19553.97 \times 320 \times \frac{1}{10^6} = 6.26\ kN\text{-}m$

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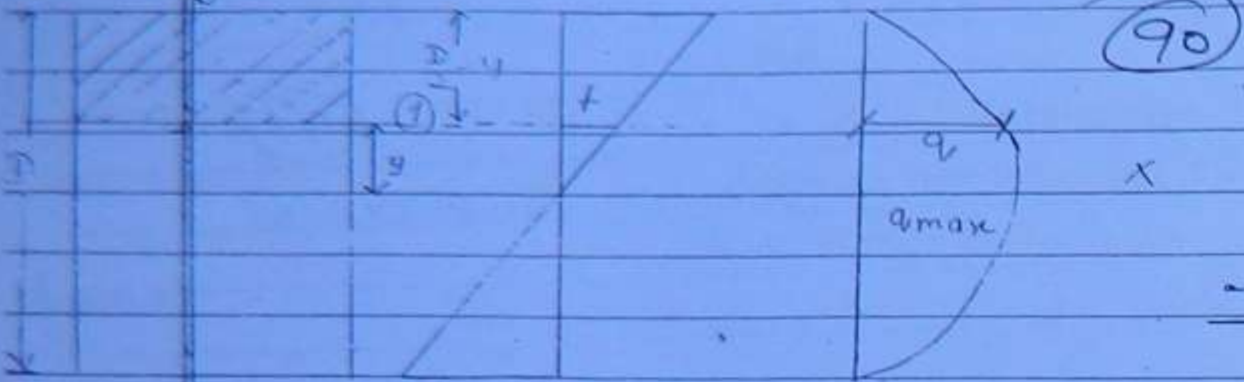
$\text{Total } MR = MR_1 + MR_2 + MR_3$

$= 27.31 - 2.74 + 6.26 = 30.83\ kN\text{-}m$

# SHEAR

Shear Stress in a homogenous Section




Bending Section

$$\frac{M}{I} = \frac{f}{y} = \frac{E}{R} \rightarrow \text{Bending}$$

bending stress $= f = \frac{M}{I}\bar{y}$

Shear stress at secⁿ 1-1

$$q = \frac{V}{IB}(A\bar{y})$$

V = Shear force
I = M.I of sec. abt X-X
B = Width of Section at 1-1
$A\bar{y}$ = Moment of area above 1-1 abt N.A

$$A\bar{y} = B\left(\frac{D}{2} - y\right)\left[\left\{y + \frac{1}{2}\left(\frac{D}{2} - y\right)\right\}\right]$$

$$= B\left(\frac{D}{2} - y\right) \times \left(y + \frac{D}{4} - \frac{y}{2}\right)$$

$$= B\left(\frac{D}{2} - y\right)\left(\frac{D}{4} + \frac{y}{2}\right)$$

$$= \frac{1}{2}B\left(\frac{D}{2} - y\right)\left(\frac{D}{2} + y\right)$$

$$= \frac{B}{2}\left(\frac{D^2}{4} - y^2\right)$$

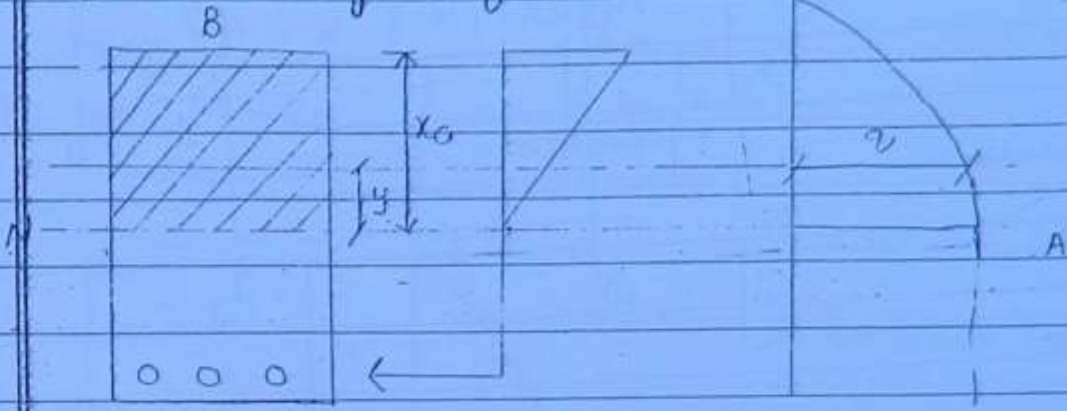
$$q = \frac{V}{IB}\,\frac{B}{2}\left(\frac{D^2}{4} - y^2\right)$$

$$q = \frac{V}{2I}\left(\frac{D^2}{4} - y^2\right)$$

Max. q at $y = 0$ at N.A

$$q_{max} = \frac{V}{2I}\left(\frac{D^2}{4}\right)$$

Shear Stress diagram for a R.C.C Beam. (91)




It is possible with zero shear stress at top & max. at N.A

Shear Stress at any section

$$q = \frac{V}{I_{eq} B}(A\bar{y})$$

Where $A\bar{y} = \frac{B}{2}(x_a^2 - y^2)$

$$q = \frac{V}{I_{eq}\cancel{B}} \frac{\cancel{B}}{2}(x_a^2 - y^2) \Rightarrow \boxed{q = \frac{V(x_a^2 - y^2)}{2I_{eq}}}$$

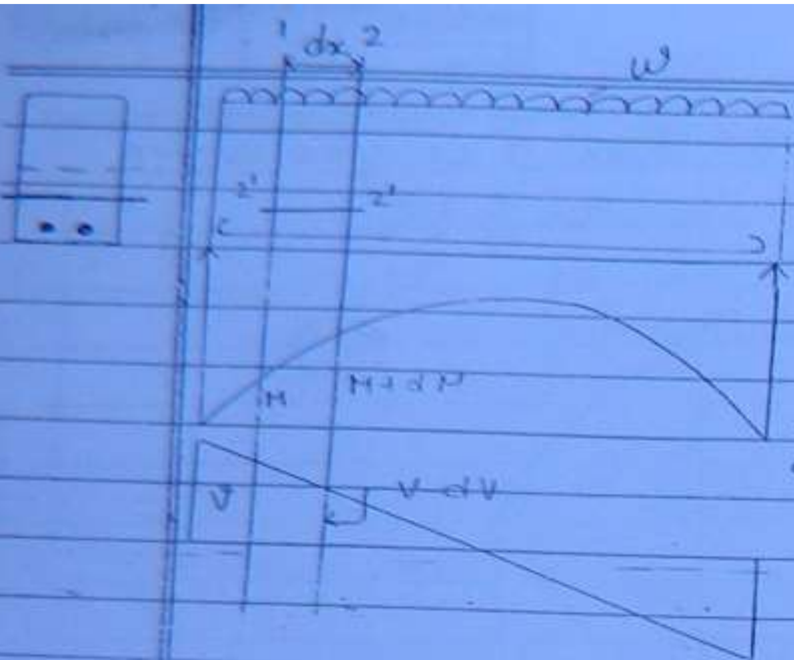
Shear Stress above N.A

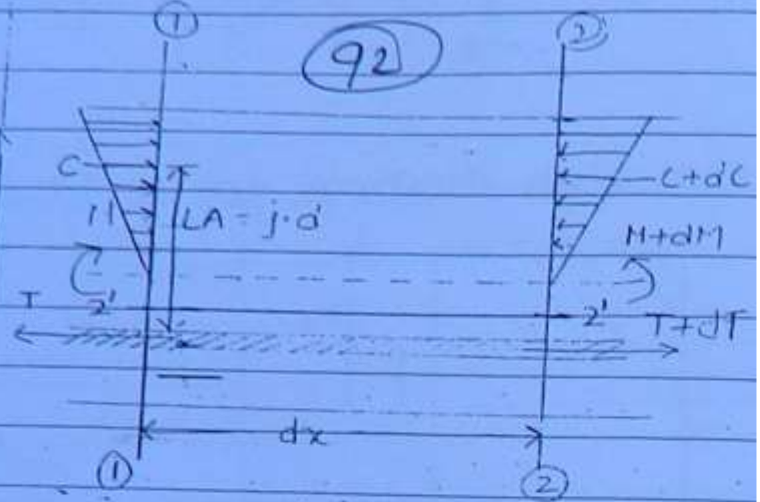
$$I_{eq} = \frac{B \cdot x_a^3}{3} + m A_{st}(d - x_a)^2$$

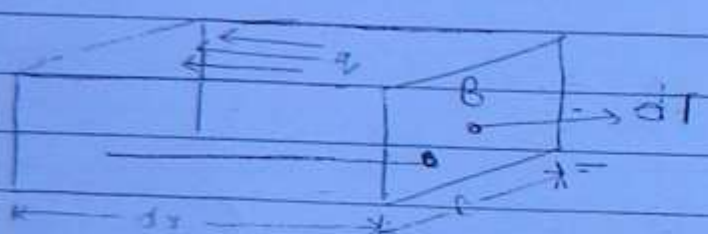
$$I_{base} = \frac{B x_a^3}{12} + B x_a\left(\frac{x_a}{2}\right)^2$$

$$= \frac{B x_a^3}{12} + \frac{B x_a^3}{4} = \frac{1+3}{12} B x_a^3$$

$$= \frac{B x_a^3}{3}$$




Consider two section 1-1 & 2-2 in the beam where moments are M & M+dM. At Section 1-1

Moment $M = C \cdot j \cdot d = T \cdot j \cdot d$ — 1

At section 2-2

Moment $M + dM = (C+dC)\, j \cdot d = (T+dT)\, j \cdot d$ — 2

(2) − (1)

$dM = (T+dT)\, j \cdot d - T \cdot j \cdot d$

$dM = dT\, j \cdot d$

$dM = dT = \dfrac{dM}{jd}$ This unbalanced dT forces in the reinforcement causes shear stress in concrete beam N.A & reinforcement

If q = Shear stress developed

Resulting force = $q \cdot B \cdot dx$ — (4)

Equating eq$^n$ (3) & (4)

$$q \cdot B \cdot dx = dT = \frac{dM}{j \cdot d}$$

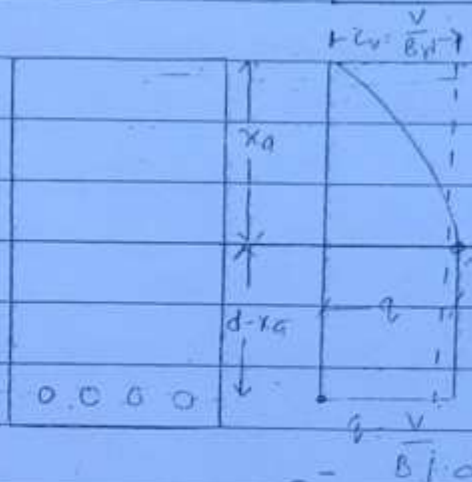
Shear Stress developed

$$q = \frac{dM}{B \cdot dx \cdot j \cdot d} = \frac{V}{B \cdot j \cdot d}$$

$$\boxed{q = \frac{V}{B \cdot j \cdot d}}$$ — L.S.M.



As per IS 456 $\boxed{q = \frac{V}{B d}}$ - L.S.M




This shear stress $q = \frac{V}{B \cdot j \cdot d}$ is constant below N.A

Shear Stress diagram is as shown in fig for a shear force = V

Shear force $V$ = (width of section) x (Area of shear stress dia)

$$= B\left(\frac{2}{3} \times q \cdot x_a + q(d - x_a)\right)$$

$$= B \cdot q \left(\frac{2}{3} x_a + d - x_a\right)$$

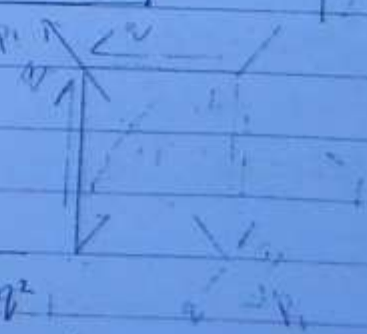
$$= B \cdot q \left(d - \frac{x_a}{3}\right) = B \cdot q \cdot j \cdot d$$

Shear value as per IS Code $\boxed{\tau_v = \frac{V}{B \cdot d}}$

Effect of Shear

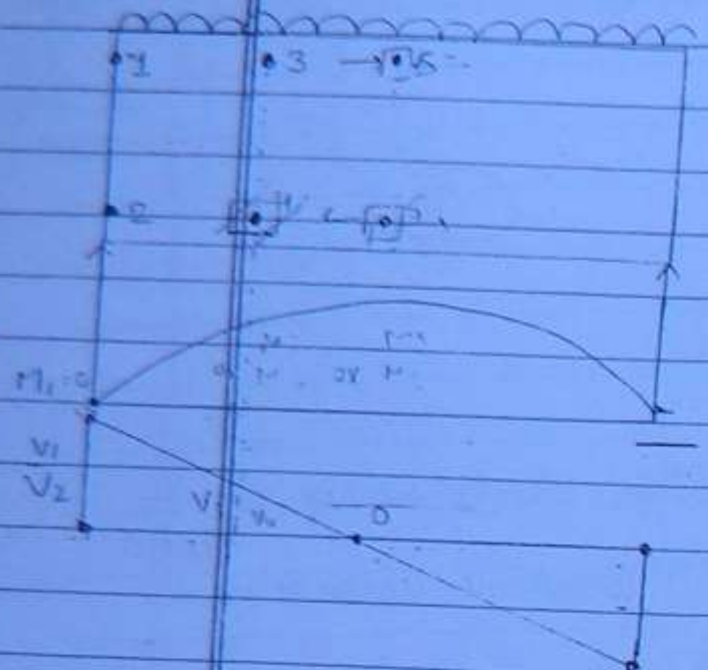
Point No. 4

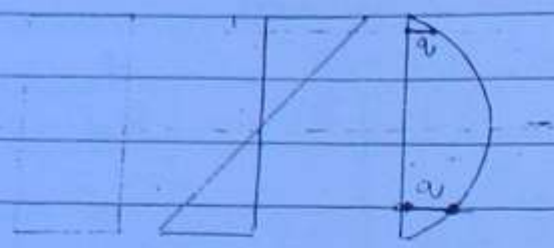
$f = \frac{M_4}{I} y \quad || \quad q = \frac{V_4}{I \cdot B}(A\bar{y})$




Principal Stresses $p_1 = \frac{f}{2} + \sqrt{\left(\frac{f}{2}\right)^2 + q^2}$ (tensile +ve)

$p_2 = \frac{f}{2} - \sqrt{\left(\frac{f}{2}\right)^2 + q^2}$ (Comp (-ve))





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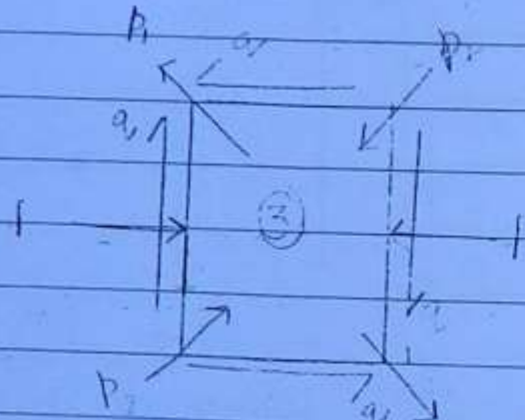
...l No. 3 $p_1 = -\frac{f}{2} + \sqrt{\left(\frac{f}{2}\right)^2 + q^2}$

$p_2 = -\frac{f}{2} - \sqrt{\left(\frac{f}{2}\right)^2 + q^2}$

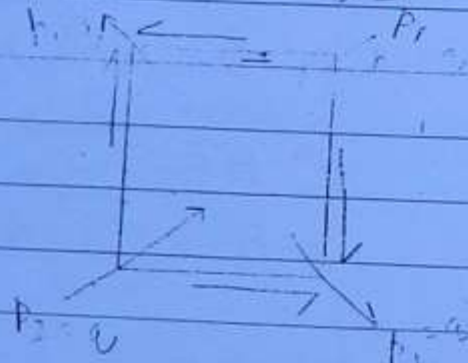


...l No. 2/1 $p_1 = q$

$p_2 = -q$

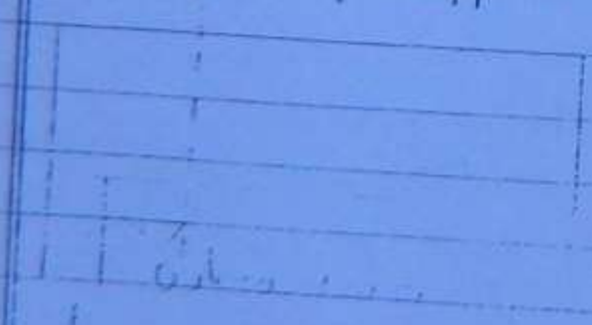
$\theta = 45^0$

...is Called diagonal tension



Design Of beam for Shear (IS:456)

1. Shear force value is Calculated
Design S.F Should be Calculated at d distance from face of Support



We can also design for max. S.F

DATE:

2 Nominal Shear Stress - It is the shear stress value developed in the section ($\tau_v$)

$$\tau_v = \frac{V}{B \cdot d} \quad \text{— } 1$$

This $\boxed{\tau_v \not> \tau_{max.}}$ in any case

95

3. Design Shear Strength of Concrete

(a) Without Shear reinforcement ($\tau_c$)

→ Shear Strength of Concrete without Shear reinforcement ($\tau_c$) depends upon

1 Grade of Concrete    2. % tension reinforcement

(TABLE - 23 - IS CODE - 456)

(b) With Shear reinforcement ($\tau_{c\,max}$)

(TABLE-20, 24)

| | M15 | M20 | M25 | M30 | M35 | M40 |
|---|---|---|---|---|---|---|
| WSM | 1.6 | 1.8 | 1.9 | 2.2 | 2.3 | 2.5 |
| LSM | 2.5 | 2.8 | 3.1 | 3.5 | 3.7 | 4.0 |

In any case $\tau_v < \tau_{max}$

If $\tau_v > \tau_{c\,max}$ → there is no other option then to revise the section, change the size.

Min Shear reinforcement

$$\frac{A_{sv}}{B \cdot S_v} \geq \frac{0.4}{.87\, f_y}$$ (26.5-1.6    Page No-48)

Design Step

1. Calculate $\tau_v = \frac{V}{b \cdot d}$

2. Calculate % of Steel $p = \frac{A_{st}}{B \cdot d} \times 100$

Find out $\tau_c$ from table

DATE:

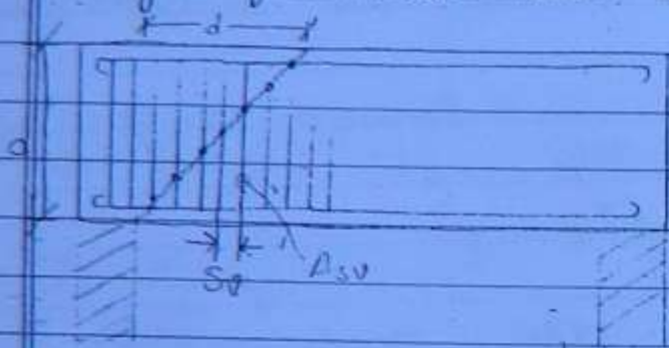
If $\tau_v < \tau_c$ → even ~~if~~ in this case - min shear reinforcement shall be provided (96)

5. If $\tau_v > \tau_c$

Net shear force $V_s$ for design of shear reinforcement

$$V_s = V - V_c$$
$$= V - \tau_c . B . d$$
$$= \tau_v B d - \tau_c . B . d$$
$$V_s = (\tau_v - \tau_c) B . d$$

6 Design of vertical shear reinforcement



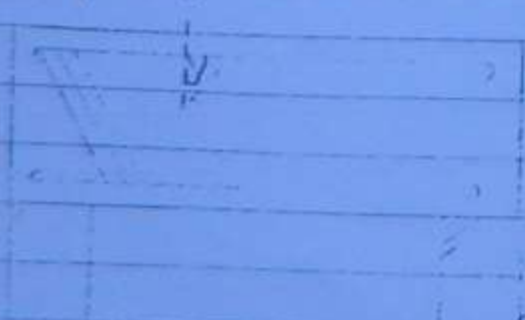

Shear strength of reinforcement

$$V_s = n \times A_{sv} \times \sigma_{sv}$$
$$= \frac{d}{S_v} \times A_{sv} \times \sigma_{sv}$$

Spacing of V.S.R $\quad \boxed{S_v = \frac{A_{sv} . \sigma_{sv} . d}{V_s}}$ Working Stress Method

Spacing V.S.R. $\quad \boxed{S_v = \frac{A_{sv} (0.87 f_y) d}{V_s}}$ Limit State Method

7 Design of Inclined shear reinforcement



W.S.M

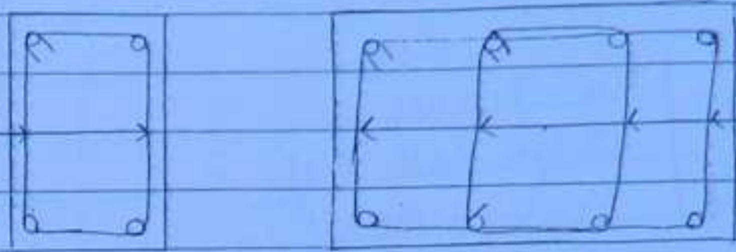
$$\boxed{S_v = \frac{A_{sv} . \sigma_{sv} . d}{V} (\sin\alpha + \cos\alpha)}$$

$$\boxed{S_v = \frac{A_{sv} (0.87 f_y) d}{V} (\sin\alpha + \cos\alpha)}$$

L.S.M

Area of Steel ($A_{sv}$) at one section

two legged Shear Stirrup



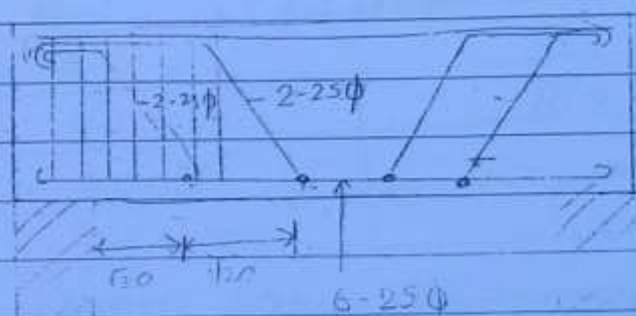
4-legged Shear Stirrups

97

$$A_{sv} = n \times \frac{\pi}{4} \times \phi^2$$

n = no. of legs

B Use of BENT UP BARS
(along $\bar{c}$ vertical shear reinforcement)




a = lever arm = j·d

1st At location of Bent up bars = $\sqrt{2}a$
$= \sqrt{2} \cdot j \cdot d$ distance from Support

2nd location at $\sqrt{2}\, j \cdot d$ distance from 1st Location

Generally bars should not be bend beyound $\frac{Span}{4}$ from support

Strength of bent up bars

$V_{sb} = A_{sb} \times \sigma_{sv} \sin\alpha \rightarrow$ WSM

$V_{sb} = A_{sb} \times (0.87 f_y) \sin\alpha$ — LSM

Contribution of bent up bars should be kept limited to $\frac{V_s}{2}$

Vertical shear reinf. should be designed for atleast

① $V_s/2$ ② $(V_s - V_{sb})$ } Whichever is more

DATE:

(98)

Ex. $\tau_v = ?$

$V = \cancel{400}\ KN$

$V_c = 100\ KN$

$V_s = 300\ KN$

| | |
|---|---|
| $V_{sb} = 100 KN$ | $V_{sb} = 200 KN$ |
| Design vertical shear reinf. for $(V_s - V_{sb}) = 200 KN$ | Design V.S.R $V_s/2$ |
| | $V_s - V_{sb} = 100 KN$ |
| | $V_s/2 = 150 KN$ ✓ |

Maximum spacing of shear reinforcement

(i) 0.75 d – for vertical stirrups

(ii) d – for inclined stirrups at 45°

(iii) 300 mm – in any case

E-2004

Design shear reinforcement at support if $\tau_c$

| p | 1.25 | 1.50 | 1.75 |
|---|---|---|---|
| $\tau_c$ | 0.70 | 0.74 | 0.78 |

Sol.

$B = 500 mm$

$D = 1500 mm$

$\bar{d} = 1400 mm$

$A_{st} = 21 \times \frac{\pi}{4} \times 25^2 = 10308\ mm^2$

$A_{sc} = 4 \times \frac{\pi}{4} \times 25^2 = 2277\ mm^2$

Service U.D.L = 52.92 KN/m

Span = 18m

Soln

Dead load = $25 \times .5 \times 1.5 = 18.75$ KN/m

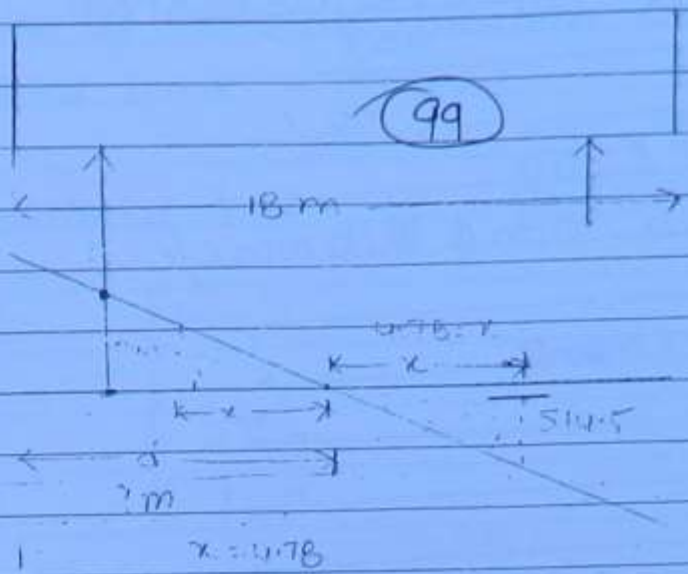
Total load = $52.92 + 18.75 = 71.67$ KN/m

Factored load = $1.5 \times 71.67 = 107.505$ KN/m

$V = \frac{wL}{2} = \frac{107.505 \times 18}{2} = 967.545 KN$

S.F.D

$\frac{x}{9} = \frac{514.5}{967.5}$ $x = 4.78 m$




In dotted area there is no requirement to provide Shear Reinforcement

→ Hence provide minimum Shear reinforcement.

Nominal Shear Stress $\tau_v = \frac{V}{B \cdot d} = \frac{967.50}{500 \times 1400} = 1.382 \ N/mm^2$

% of Steel

$p = \frac{A_{st}}{Bd} \times 100$

$= \frac{10308}{500 \times 1400} \times 100 = 1.47$

$\tau_c$ (from table by interpolation)

$\tau_c = 0.7 + \left(\frac{0.74 - 0.7}{1.5 - 1.25}\right)(1.47 - 1.25)$

$= 0.7352 \ N/mm^2$

Design Shear Strength of Concrete

$V_c = \tau_c \cdot B \cdot d = 0.735 \times 500 \times 1400$

$= 514.5 \ kN$

Shear reinforcement is required for

$V_s = V - V_c$

$= 967.5 - 514.5$

$= 453 \ kN$

Let us provide 2 legged 8mm $\phi$ Stirrups

$A_{sv} = 2 \times \frac{\pi}{4} \times 8^2 = 100.53$

DATE:

$$S_v = \frac{A_{sv} \times 0.87 f_y \times d}{V_s} = \frac{100.53 \times 0.87 \times 415 \times 1400}{453}$$

$= 112.17\ mm$ (100)

This value is betw 100 & 200

⇒ Hence provide 2-legged 8mm ϕ @ 110 mm c/c

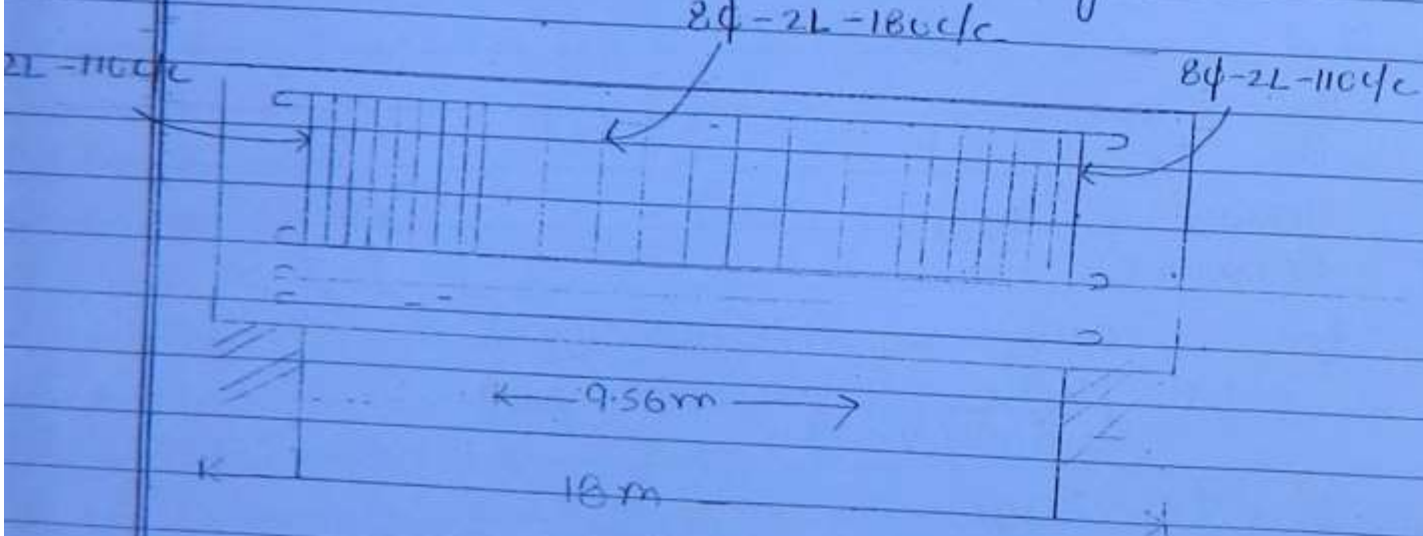
## Check

Min. Shear reinforcement

$$\frac{A_{sv}}{B \cdot S_v} \geq \frac{0.4}{.87 f_y}$$

$S_v \leq 181.4 mm$

Provide 2 legged 8mm ϕ @ 180mm c/c in central $(4.78 \times 2) = 9.56$ m length.




Maximum Spacing

1. For vertical stirrups – $0.75 \times 1400 = 1050 mm$
2. $= 300 mm$ ⇒ safe

A simply reinforced beam of size 300 x 750mm eff. depth is supported over a clear span of 6m & subjected to a U.D.L of 65KN/m excluding self wt. If 8 Nos of 25 mm ϕ has been provided as tension reinforcement Design shear reinforcement using bent up bars.

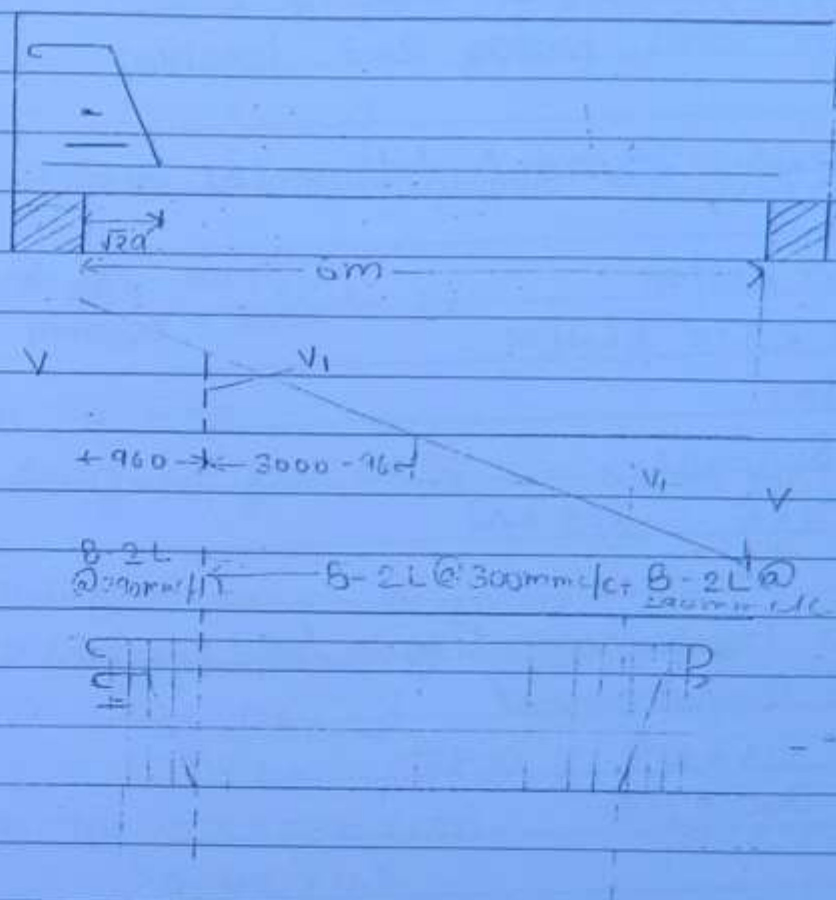
DATE:

Also at suitable location M-20/Fe-415 is used.

Total load $= L.L + D.L$

$= 65 + 5.625 = 70.625$ kN/m — (101)

factored load $= 1.5 \times 70.625 = 105.94$ kN/m —




Location of bent up bars

1st location of bent up bars

$= \sqrt{2}a = \sqrt{2} \cdot j \cdot d$

$j = 1 - \frac{K}{3}$

$K = \frac{mc}{t + mc} = \frac{13 \times 7}{230 + 13 \times 7}$

$= 0.2835$

$j = 1 - \frac{K}{3} = 0.905$

$= \sqrt{2} \times 905 \times 700$

$=$ 960 mm from Supports

IInd location of bent up bars

$2\sqrt{2}\,a = 1920$ mm from Support

$\frac{Span}{4} = \frac{6000}{4} = 1500mm$ 102

We Cannot use the 2nd bent up bars

⇒ Bars Cannot be bent from 2nd location

Let us bent 2 nos. - 25 mm ϕ from 1st location

2. Load / SF

Live load = 65 KN/M

750mm - eff depth

Total = 800mm

$D.L = .8 \times .3 \times 25 = 6 KN/M$

Total = 71 kN/m

Max. S.F at Support

$V = \frac{wL}{2} = \frac{71 \times 6}{2} = 213\ KN$

Design of Shear reinforcement with bent up bars

Nominal Shear Reinforcement

$\tau_v = \frac{V}{B \cdot d} = \frac{213 \times 10^3}{300 \times 750} = 0.95$

% Of Steel

$p = \frac{6 \times \pi/4 \times 25^2}{300 \times 750} \times 100 = 1.31\%$

$\tau_c = 0.42 + \frac{0.45 - 0.42}{.25} \times [1.31 - 1.25]$

$\tau_c = 0.427$

$V_c = 0.427 \times 300 \times 750 = 96.12\ KN$

Shear reinforcement designed for

$V_s = V - V_c$

$= 213 - 96.12$

$= 116.88 = 116.9\ KN$

Strength of bent up bars

$V_{sb} = A_{sb} \times \sigma_{sb} \times \sin\alpha$

$= \left(2 \times 25^2 \times \frac{\pi}{4}\right)(230) \sin 45^0$

(T93)

$= \frac{.7070 \times .785 \times 2 \times 625 \times 230}{10^3}$

$= 159.56 \text{ kN}$

In this case vertical shear reinforcement be designed for

$\frac{V_s}{2} = \frac{116.9}{2} = 58.45 \text{ kN}$

$V_s - V_{sb} = -ve$

It is designed for $V_s/2 = 58.45 \text{ kN}$

Shear reinforcement required.

use 2-legged 8mm $\phi$ vertical stirrups spacing

$S_v = \frac{A_{sv} \cdot \sigma_{sv} \cdot d}{V_s/2}$

$= \frac{100.53 \times 230 \times 150}{58.45 \times 10^3} = 296.69 \text{ mm}$

$\Rightarrow$ Provide 2-legged 8-mm dia @ 290 mm c/c

Shear force beyond bent up bars

$\frac{V_1}{V} = \frac{3000 - 960}{300} \Rightarrow V_1 = 144.84 \text{ kN}$

Design for this shear force

$\tau_v = \frac{V_1}{B \cdot d} = \frac{144.84 \times 10^3}{300 \times 750}$

$= 0.64$

% of steel

$p = \frac{8 \times \pi/4 \times 25^2}{300 \times 750} \times 100 = 0.0174 \times 100 = 1.74\%$

$\tau_c$ for 1.74% = 0.47

$V_c = \tau_c \cdot B \cdot d = 0.47 \times 300 \times 750$ (104)

$= 105.75$ kN

$V_s = V - V_c = 144.84 - 105.75 = 39.09$ kN

Spacing of 2-legged 8mm dia

$$S_v = \frac{A_{sv} \cdot \sigma_{sv} \cdot d}{V_s}$$

$$= \frac{100.53 \times 230 \times 750}{39.09 \times 10^3}$$

$= 443.63$ mm

Min Shear reinforcement

$$\frac{A_{sv}}{B \cdot S_v} = \frac{0.4}{0.87 f_y}$$

$$S_v = \frac{100.53 \times 0.87 \times 415}{0.4 \times 300}$$

$= 302.47$ mm $> 300$ mm

Max. Spacing provided = 300 mm

$S_v = 300$ mm.

In Rectangular R.C Beam grade M-15 / Fe-415 size 250x500mm effective. $V = 60$ kN. Under service load condition. Calculate spacing of 8mm dia 2-legged stirrups. Steel 3-16φ

| | | | | |
|---|---|---|---|---|
| P | 0.25 | 0.5 | 0.75 | 1.00 |
| $\tau_c$ | 0.35 | 0.45 | 0.54 | 0.64 |

Soln

Load - 60 kN

$D.L = 25 \times .25 \times .55 = 3.4375$ kN

Total load = 63.4375 kN

$$\tau_v = \frac{V}{B \cdot d} = \frac{60 \times 10^3 \times 1.5}{250 \times 500} = 0.72$$

$$p = \frac{A_{st}}{250 \times 500} = \frac{3 \times \pi/4 \times 16^2}{250 \times 500} \times 100 = 0.48$$

$$\tau_c = 0.35 + \frac{0.46 - 0.35}{0.50 - 0.25} \times (0.48 - 0.25) = 0.4512$$

DATE:

$V_c = \tau_c \times B \cdot d$

$= 0.4512 \times 250 \times 500$

$= 56.4 \text{ kN}$

(105)

$V_s = V - V_c = 90 - 56.4 = 33.6 \text{ kN}$

Spacing of 2 legged 8mm $\phi$ stirrups

$A_{sv} = 2 \times \frac{\pi}{4} \times 8^2$

$= 100.53$

$S_v = \frac{100.53 \times 0.87 \times 415 \times 500}{33600}$

$= 540.12$

Minimum shear reinforcement

$\frac{A_{sv}}{B \cdot S_v} \geq \frac{0.4}{0.87 f_y}$

$S_v = \frac{0.87 \times 415 \times 100.53}{0.4 \times 250}$

$= 362.96 \text{ mm}$

Max. Spacing

$0.75 \times d = 0.75 \times 500 = 375 \text{ mm}$

300 mm

provide 2-legged 8$\phi$ @ 300 mm c/c

# BOND / DEVELOPMENT LENGTH

Bond :-

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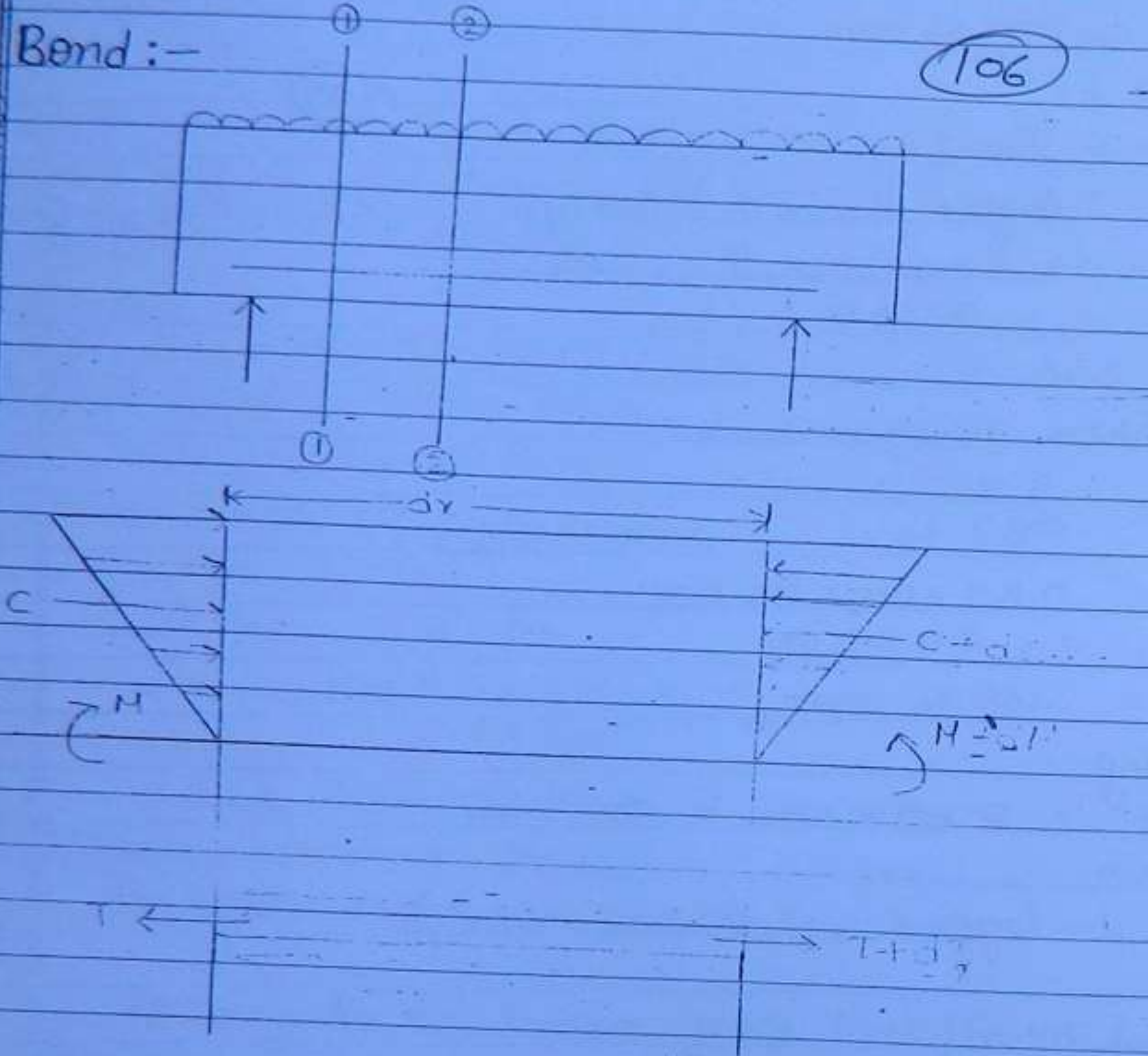



\# In a simply supported beam / any other kind of beam subjected to B.M & S.F

Bond stress (longitudinal shear stress developed b/w steel and concrete) is developed b/w steel & concrete in longitudinal dir$^n$ due to unbalanced dT force as shown in fig.

Unbalanced force dT = Bond force

$\Sigma O$ = Sum of all perimeter

$d \times \Sigma O \times \tau_{bd}$

$$\tau_{bd} = \frac{dT}{dx \times \Sigma o} \quad \text{— (1)}$$

(107)

At section (1)–(1)

$$M = C \cdot j \cdot d = T \cdot j \cdot d \quad \text{— (2)}$$

At section 2-2

$$M + dM = (C + dC) \cdot j \cdot d = (T + dT) \cdot j \cdot d$$

3 - 2

$$\Rightarrow dM = dT \cdot j \cdot d$$

$$dT = \frac{dM}{j \cdot d} \quad \text{—— (4)}$$

Bond stress developed

$$\tau_{bd} = \frac{dM}{dx \times \Sigma o \cdot j \cdot d} = \frac{V}{\Sigma o \cdot j \cdot d}$$

Here

$$\Sigma o = n \times \pi \times \phi$$

↑ Developed value of bond stress due to shear force

CHECK

Permissible values of Bond Stress (Table-21 Page-80

| For mild steel Reinf$^n$ in tension | LISM | LSM → Page No-43 |
|---|---|---|
| M-15 | 0.6 | x |
| M 20 | 0.8 | 1.2 |
| M 25 | 0.9 | 1.4 |
| M 30 | 1.0 | 1.5 |
| M 35 | 1.1 | 1.7 |
| M40 | 1.2 | 1.9 |
| M45 | 1.3 | 1.9 |
| M50 | 1.4 | 1.9 |

PAGE NO.
DATE:

# For HYSD bars ; values at given in table are increased by ⇒ 60% for both WSM & LSM
# For bars in Compression values in table are to be increased by 25% — WSM & LSM

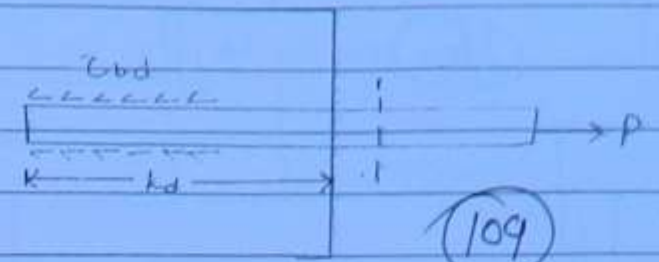
108

$\tau_{bd}$ Calculated $\ngtr$ $\tau_{bd}$ permissible.

Ex — $\tau_{bd}$ for M-30 Concrete for L.S.M for Fe-415 bars in Compression

$$\tau_{bd} = 1.5 \times 1.6 \times 1.25 = 3$$

Developement length. — Min length of steel requirement required to be embedded in concrete so that strength of bond is equal to strength of reinforcement in tension




(109)

$P = A_{st} \cdot \sigma_{st}$ = Bond strength

$A_{st} \cdot \sigma_{st} = \tau_{bd} \times \Sigma O \times L_d$

$\frac{\pi}{4} \times \phi^2 \times \sigma_{st} = \tau_{bd}\, \Sigma \times \pi \times \phi \times L_d$

$$\boxed{L_d = \frac{\phi\, \sigma_{st}}{4 \tau_{bd}}}$$ Working Stress method.

$$\boxed{L_d = \frac{\phi (0.87 f_y)}{4 \tau_{bd}}}$$ Limit State method.

Calulate developement length req. for a steel reinforcement fe-415 for tension and compression. Use M-25 & use LSM.

Soln For tension

$\tau_{bd} = 1.4 \times 1.60 = 2.24$

$L_d = \frac{\phi (0.87 f_y)}{4 \tau_{bd}} = \frac{\phi (0.87 \times 415)}{4 \times 2.24} = 4.295\, \phi$

In compression

$\tau_{bd} = 1.4 \times 1.6 \times 1.25 = 2.8\ N/mm^2$

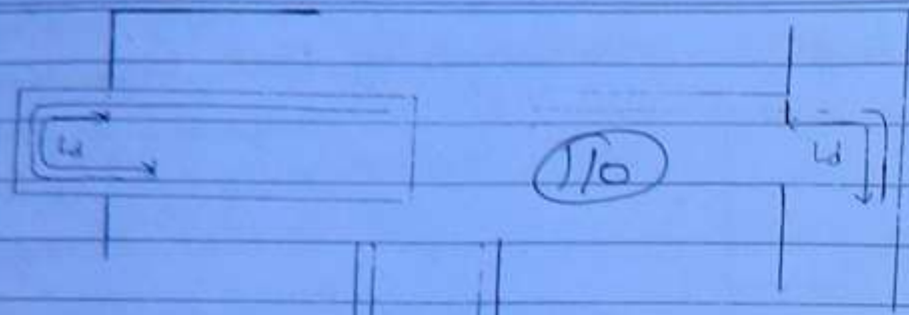
$L_d = \frac{\phi (.87 \times 415)}{4 \times 2.8} = 32.24\, \phi$

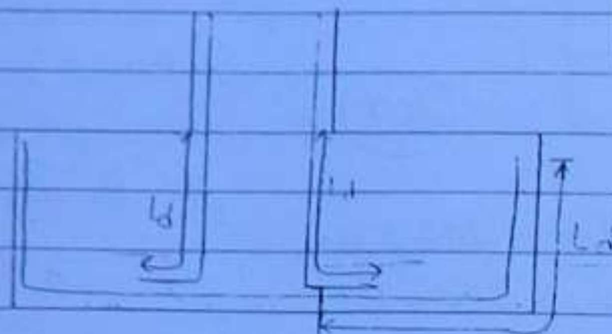
Use of developement length

1. For lap length of steel in slab / Beam / Column / foundation

   Lap length = $L_d$

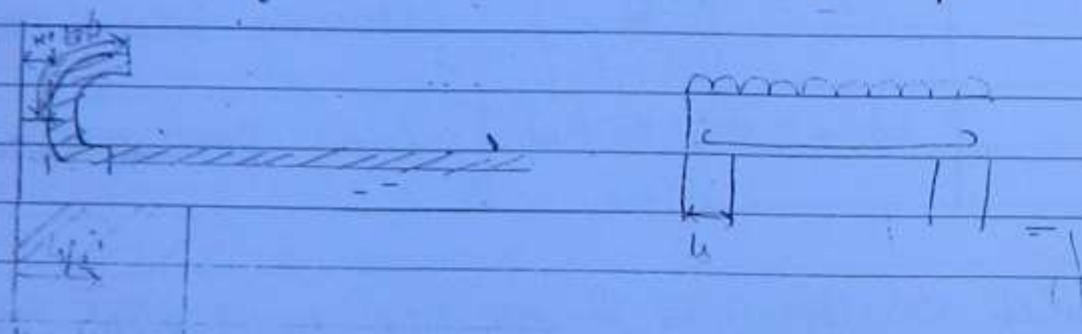
2. At corner in beam.





n Code
(26.2.3.3)
(c)

Development length requirement At Simple Support



positive moment reinforcement Shall be limited to a dia Such that

$$L_d \leq \frac{M_1}{V} + L_0$$

$M_1$ = Moment of resistance of Section assuming all reinforcement Stressed upto $\sigma_{st}$ or $0.87 f_y$

$A_{st} \cdot \sigma_{st} \cdot (jd)$ —— WSM

$A_{st} \cdot (.87 f_y)\, j \cdot d$ —— LSM

$V$ = Shear force at Support

$L_0$ = length of reinf. beyond Centre of Support

$L_0 = \frac{l_s}{2} - x' + 8\phi$ → (for mild Steel)

$L_0 = \frac{l_s}{2} - x'$ —— for HYSD Bars

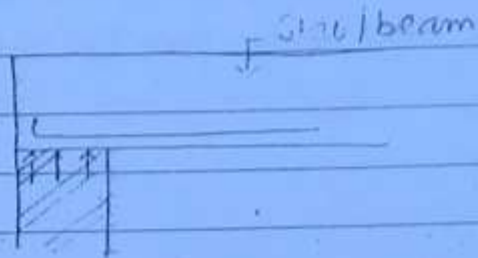
DATE:

If the end of the reinforcement are confined by a compressive reaction.

$$L_d \leq 1.3\frac{M_1}{V} + L_o \qquad \text{(III)}$$

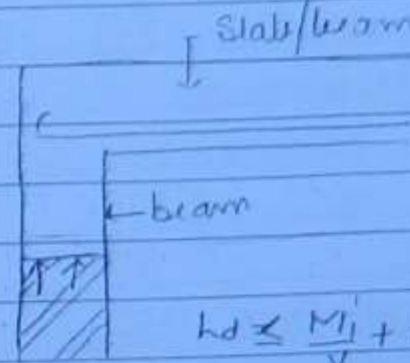
Example:-

(i)



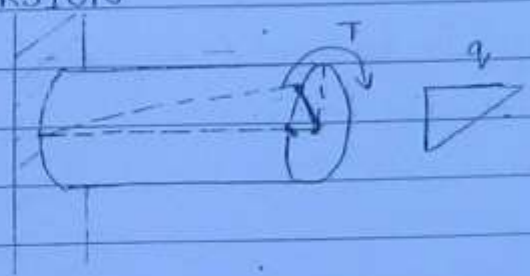

$$L_d \leq 1.3\frac{M_1}{V} + L_o$$

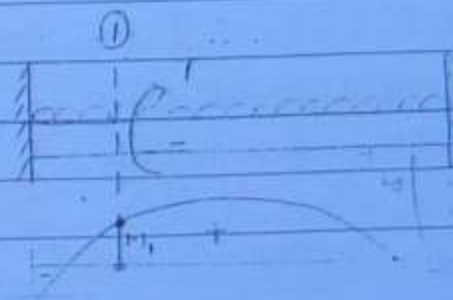
(ii)



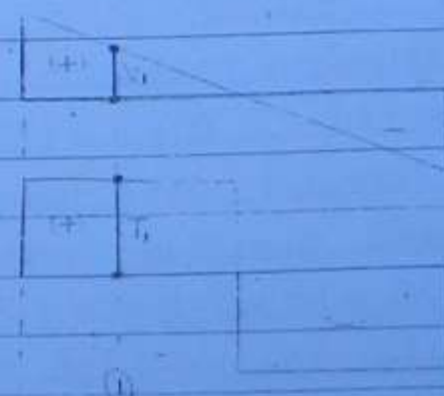

$$L_d \leq \frac{M_1}{V} + L_o$$

TORSION




If a beam is subjected to BM-M, S.F = V & torsional moment T at a particular cross-section



Design
(By working stress method)

1. Equivalent Shear

$$V_{eq} = V + \frac{1.6\,T}{B} \qquad -(1)$$

Nominal shear stress

$$\tau_{veq} = \frac{V_{eq}}{B \cdot d} \ngtr \tau_{c\,max}$$

If $\tau_{veq} > \tau_{c\,max}$ → change the size of beam

2. Equivalent Moment

$$M_{eq} = M + \frac{T}{1.7}\left(1 + \frac{D}{B}\right)$$ (112)

$$M_{eq} = M + M_T$$

CASE I Generally ($M_T < M$) In this case design only tension reinforcement

depth required

$$d = \sqrt{\frac{M_{eq}}{Q \cdot B}}$$

Area of steel (tension reinf.)

$$A_{st} = \frac{M_{eq}}{\sigma_{st} \cdot j \cdot d} = \frac{M_{eq}}{\sigma_{st}(d - x_a/3)}$$

CASE II If $M_T > M$ – In this case design in tension & compression reinforcement both.

Check $d = \sqrt{\frac{M_{eq}}{Q \cdot B}}$

$$A_{st} = \frac{M_{eq}}{\sigma_{st} \cdot j \cdot d}$$

Compression reinforcement is also provided for a moment $M_{e_2} = (M_T - M)$

The moment $M_{e_2}$ being taken as acting in the opposite sense of moment M.

If M = (+)ve B.M

∴ Consider $M_{e_2}$ as (–)ve BM

Compression steel

$$A_{sc} = A_{st_2} = \frac{M_{e_2}}{\sigma_{st} \cdot j \cdot d}$$

3. Design of shear reinforcement

Spacing of shear reinforcement

$$A_{sv} = \frac{T_u S_v}{b_1 d_1 (0.87 f_y) \sigma_{sv}} + \frac{V_u S_v}{2.5 d_1 (0.87 f_y) \sigma_{sv}}$$

$$\boxed{S_v = \frac{A_{sv}\cdot \overset{(0.87 f_y)}{\sigma_{sv}} \cdot d_1}{\left[\frac{T}{b_1} + \frac{V}{2.5}\right]}} \quad \text{T13}$$

Spacing of shear reinforcement

Max. Spacing of shear reinf$^n$ (page No-48)

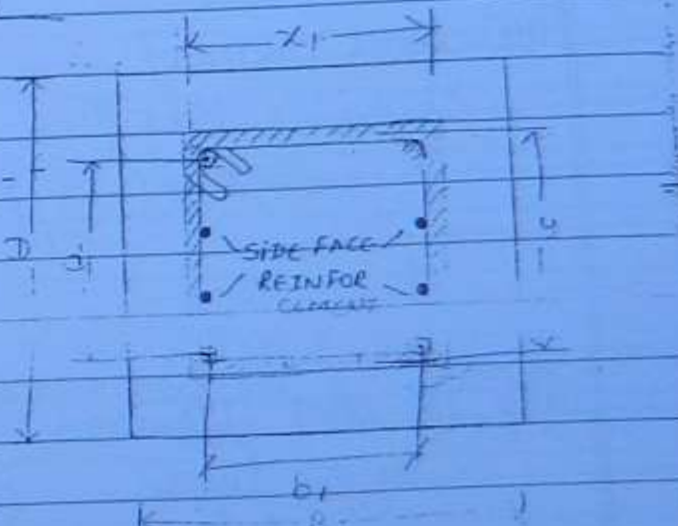
1. $x_1$
2. $\frac{x_1+y_1}{4}$ $\quad x_1, y_1 \rightarrow$ short & long dim. of stirrups
3. 300mm

Min. Shear reinforcement in case of torsion -(Page No-47)

Total transverse reinforcement $A_{sv}$ shall not be less than

$$A_{sv} \nless \frac{(\tau_{ve} - \tau_c)\, B\cdot S_v}{\sigma_{sv}}$$

$$\frac{A_{sv}}{B\cdot S_v} \nless \frac{(\tau_{ve} - \tau_c)}{\sigma_{sv}}$$




Side face Reinforcement

When depth of beam is more than

1. 750 mm – when beam is not subjected to torsion
2. 450mm – when subjected to torsion

Side face reinf$^n$ shall be provided equal to 0.1% of web area & distributed on both face

$$A_{sf} = \frac{0.1}{100} \times B\cdot D$$

Max. Spacing b/w side face reinf$^n$ = 300mm or web thickness whichever is less

DATE:

Limit State method

Factored values

$$B \cdot M = M_u = 1.5M$$

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$$T_u = 1.5T$$

$$V_u = 1.5V$$

Equivalent Shear

$$V_{ueq} = V_u + \frac{1.6\, T_u}{B}$$

Nominal Shear Stress $\tau_{veq} = \frac{V_{ueq}}{B \cdot d} \ngtr \tau_{c\,max}$

If $\tau_{veq} > \tau_{c\,max}$ revise the section

Equivalent moment

$$M_{ueq} = M_u + M_{Tu}$$

$$= M_u + \frac{T_u}{1.7}\left(1 + \frac{D}{B}\right)$$

$$d = \sqrt{\frac{M_{ueq}}{Q \cdot B}}$$

Area Of Steel (tension)

$$A_{st} = \frac{M_{ueq}}{0.87\, f_y \cdot j \cdot d}$$

$$= \frac{0.5 f_{ck}}{f_y}\left[1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck} B d^2}}\right] Bd$$

If $M_{Tu} > M_u \rightarrow$ provide Comp reinf$^n$ for $M_{ue_2}$ =

$$M_{ue_2} = M_{Tu} - M_u$$

$$A_{sc} = A_{st_2} = \frac{M_{ue_2}}{0.87\, f_y\, j\, d}$$

Shear reinf$^n$

$$S_v = \frac{A_{sv}(0.87 f_y) d_1}{\left[\frac{T_u}{b_1} + \frac{V_u}{2.5}\right]}$$

Min Shear reinf$^n$

$$\frac{A_{sv}}{B \cdot S_v} \geq \frac{\tau_{ve} - \tau_c}{\cdot 87 f_y}$$

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Max. Spacing

① $x_1$

② $\frac{x_1 + y_1}{4}$

③ 300mm

Side face reinf$^n$ is Same as L.S.M

Ques Design a beam subjected to a following at a particular section (use L.S.M) —

1. B.M = 300 kN-m
2. S.F = 200 kN-m
3. T.M = 80 kN-m

Use width of beam = 400mm

Use M-25 / Fe-415

Sol. Factored values

$M_u = 450$ kN-m

$T_u = 120$ kN-m

$V_u = 300$ kN-m

Equivalent Shear

$$V_{ueq} = V_u + \frac{1.6\, T_u}{B} = 300 + \frac{1.6 \times 120}{\cdot 400} = \cancel{300+48}\ 780\ \text{kN}$$

Assume depth = 600 mm (D)   d = 550

Nominal Shear Stress = $\tau_{veq} = \frac{V_{ueq}}{B \cdot d}$

$$\Rightarrow \tau_{veq} = \frac{780 \times 10^3}{400 \times 550} = 3.54$$

$\tau_{cmax}$ M-25 = 3.10

$\tau_{veq} > \tau_{cmax}$ so revise the section

assume depth = 900mm

$$\tau_{veq} = \frac{780 \times 10^3}{400 \times 850} = 2.29$$

Equivalent Moment $M_{ueq} = M_u + M_{tu} = \frac{T_u}{1.7}\left(1 + \frac{D}{B}\right)$

$$= 450 + \frac{120}{1.7}\left(1 + \frac{900}{400}\right) = 450 + \cancel{22}\ 229.41$$

$\cancel{228.429}$

DATE:

$M_{ueq} = 679.41$ kN-m

$d = \sqrt{\dfrac{M_{ueq}}{Q \cdot B}}$

$d = \sqrt{\dfrac{679.41 \times 10^6}{.138 \times 25 \times 400}}$ (116)

$d = 701.66$ mm

Say $= D = 750$ mm

~~$D = 800$ mm~~

$\tau_{veq} = \dfrac{V_{ueq}}{B \cdot D} = \dfrac{780 \times 10^3}{400 \times 700} = 2.8$

Safe

$M_{ueq} = 450 + \dfrac{120}{1.7}\left(1 + \dfrac{750}{400}\right)$

$= 450 + 202.94$

$M_{ueq}$ $652.9$ kN-m

~~Area of~~ $d = \sqrt{\dfrac{652.9 \times 10^6}{.138 \times 25 \times 400}} = 688$ mm $< 700$ mm provid..

Size $B = 400$ mm

$D = 750$ mm

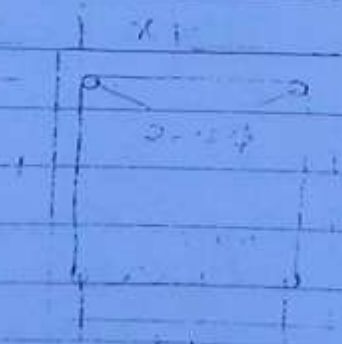
$d = 700$ mm

$A_{st} = \dfrac{M_{ueq}}{.87 f_y \cdot j \cdot d} = \dfrac{652.9 \times 10^6}{.87 \times 415 \times .80 \times 700}$

$= 3229.4$ mm$^2$ – 25mm$\phi \simeq$ 7 Nos.

Shear reinforcement

Spacing



$b_1 = 400 - 100 = 300$

$d_1 = 750 - 100 = 650$

$x_1 = 300 + 35 =$ ~~685~~ $335$

$650 + 35 = 685$

$S_v = \dfrac{A_{sv}(0.87 \times f_y)\, d_1}{\left(\dfrac{V_u}{2.5} + \dfrac{T_u}{b_1}\right)}$

DATE:

$$= \frac{2 \times \frac{\pi}{4} \times 10^2 \times 0.87 \times 415 \times 650}{\left( \frac{300 \times 10^3}{2.5} + \frac{120 \times 10^6}{300} \right)} = 70.9 \text{ mm}$$

117

Using 2 legged 12φ bars

$$= 70.89 \times \frac{12^2}{10^2} = 102.08 \text{ mm}$$

Use 2 legged @ 100 mm c/c

Min shear reinforcement

$$\frac{A_{sv}}{B \cdot S_v} \geq \frac{\tau_{uve} - \tau_c}{.87 f_y}$$

$$\frac{2 \times \pi/4 \times 12^2}{400 \times 100} \geq \frac{2.8 - 3.1}{.87 \times 415}$$

In a Rectangular R.C Beam grade M-15 / fe=415 size - 250 X 500 mm effective V = 60 KN. under service load condition. Calculate spacing of 8mm dia 2-legged stirrups. Steel - 3-16φ

| | | | | |
|---|---|---|---|---|
| p | 0.25 | 0.5 | 0.75 | 1.00 |
| $\tau_c$ | 0.35 | 0.45 | 0.54 | 0.64 |

Sol^n Load - 60 KN

D.L = 25 X .25 X .550 = 3.4375 KN

Total load = 63.4375 KN

$$\tau_v = \frac{V}{B \cdot d} = \frac{60 \times 10^3 \times 1.5}{250 \times 500} = \cancel{0}.72$$

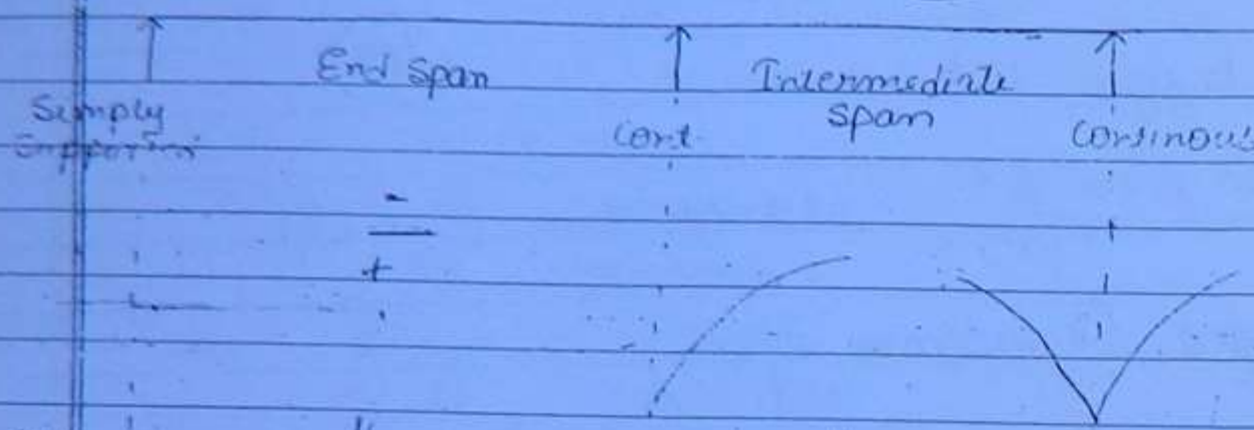
$$p = \frac{A_{st}}{250 \times 500} = \frac{3 \times \pi/4 \times 16^2}{250 \times 500} \times 100 = 0.48$$

$$\tau_c = 0.35 + \frac{0.46 - 0.35}{0.50 - 0.25} \times (0.48 - 0.25)$$

$$= 0.4512$$

# Bending moment and Shear force Coefficient for a Continous beam

1. For bending moment (118)

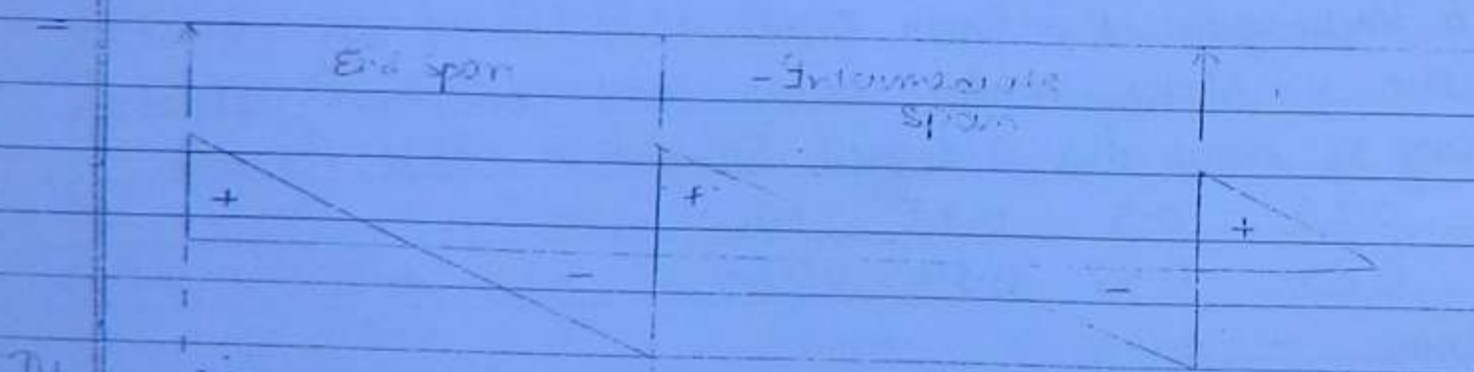



| | | | | |
|---|---|---|---|---|
| DL | 1/12 | −1/10 | 1/16 | −1/12 |
| LL | 1/10 | −1/9 | 1/12 | 1/9 |

B.M at any location $= K_d w_d l^2 + K_L w_L l^2$

At Support $= -\frac{1}{10} w_d l^2 - \frac{1}{9} w_L l^2$

For Shear force (TABLE-13 IS: 456)




| | | | | | |
|---|---|---|---|---|---|
| DL | 0.4 | 0.6 | 0.55 | 0.5 | $0.5 \times W_d \times l$ |
| LL | 0.45 | 0.6 | 0.6 | 0.5 | $0.5 \times W_L \times l$ |

Just to the left of Support (Outer Side) | Just to the Right of Support (Inner Side)

ES-1992

(a) 550x750 mm overall depth

$M_u = BM = 150 \text{ kN-m}$

$T_u = 50 \text{ kN-m}$

Use M-15 | Fe-415

Determine longitudinal reinforcement.

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| $p_t$ (%) | $\tau_c$ |
|---|---|
| 0.25 | 0.35 |
| 0.50 | 0.46 |

(b) If $V_u = 130\text{kN}$

For the beam in Ques above Determine the transverse bar reinforcement

Sol^n Equivalent Shear

$V_u = 0$ ; $T_u = 50\text{kN-m}$

$$V_{ue} = V_u + \frac{1.6 T_u}{B} = 0 + \frac{1.6 \times 50}{0.55} = 145.45 \text{ kN}$$

$$\tau_{ve} = \frac{V_{ue}}{Bd} = \frac{145.45 \times 10^3}{550 \times 700} = 0.38 < (\tau_{c\,max})$$

Safe

Equivalent moment

$$M_{ue} = M_u + M_{Tu} = M_u + \frac{T_u}{1.7}\left(1 + \frac{D}{B}\right)$$

$$= 150 + \frac{50}{1.7}\left(1 + \frac{750}{550}\right)$$

$$150 + 69.52 = 219.52 \text{ KN-m}$$

$\boxed{M_{Tu} < M_u}$ — Compression Steel not required

$$\text{Depth } d = \sqrt{\frac{M_{ue}}{Q \cdot B}} = \sqrt{\frac{219.52 \times 10^6}{.138 \times 15 \times 550}}$$

$$= 439.12 \text{ mm}$$

Available depth is = 700 mm OK

So We need under reinforced section because depth available is more than required

(120)

Area of Steel $A_{st} = \frac{0.5 f_{ck}}{f_y}\left[1-\sqrt{1-\frac{4.6 M_{ue}}{f_{ck} B d^2}}\right] Bd$

$A_{st} = \frac{0.5 \times 15}{415}\left[1-\sqrt{\frac{4.6 \times 219.52 \times 10^6}{15 \times 550 \times 700^2}}\right] 550 \times 700$

$A_{st} = 931.3 \ mm^2$

No. of 16mm dia bar $= \frac{931.3}{\frac{\pi}{4} \times 16^2} = 4.63$

Say 5 No.

(ii) Soln If Shear force $V_u = 130 \ kN$

$T_u = 50 \ kN\text{-}m$

Equivalent Shear

$V_{ue} = 130 + \frac{1.6 \times 50}{.550} = 275.45$

$\tau_{vue} = \frac{V_{ue}}{bd} = \frac{275.45 \times 10^3}{550 \times 700} = .715 \ N/mm^2$

$< \tau_{cmax}$

(2.5) - M-15

OK

(Safe)

Design of Shear reinforcement

$S_v = \frac{A_{sv}(0.87 f_y) \cdot d_1}{\left(\frac{V_u}{2.5} + \frac{T_u}{b_1}\right)}$

Use 2-legged 8mm $\phi$ shear reinforcement

$$A_{sv} = 2 \times \frac{\pi}{4} \times 8^2 = 100.53 \text{ mm}^2$$

(121)

$$S_v = \frac{100.53 \times .87 \times 415 \times 450 \times 650}{\left(\frac{130 \times 10^3}{25} + \frac{50 \times 10^6}{450}\right)}$$

$$S_v = 144.64$$

provide 2-legged 8mm $\phi$ @ 140mm c/c

Check – Min shear reinforcement

$$\frac{A_{sv}}{B \cdot S_v} \geq \frac{\tau_{vue} - \tau_c}{.87 f_y}$$

% of steel $p = \frac{A_{st}}{B \cdot d} \times 100 = \frac{5 \times \pi/4 \times 16^2}{5500 \times 700} \times 100$

$$= .26\%$$

$$\tau_c = .35 + \frac{0.46 - 0.35}{0.25} \times (0.26 - 0.25)$$

$$= 0.3544$$

$$\frac{100.53}{550 \times S_v} \geq \frac{(.715 - .3544)}{.87 \times 415}$$

$$S_v \leq \frac{.87 \times 415 \times 100.53}{\cancel{139.3678}\ (550\ (.715 - .3544))} = 182 \text{ mm}$$

139.3678

- Max. Spacing $x_1 = 450 + 16 + 8 = 474$

$y_1 = 650 + 16 + 8 = 674$

(1) $x_1 = 474$ mm

(2) $\frac{x_1 + y_1}{4} = \frac{474 + 674}{4} = 287$

(3) 300 mm.

Provide 2 legged - 8mm $\phi$ @ 140 mm c/c

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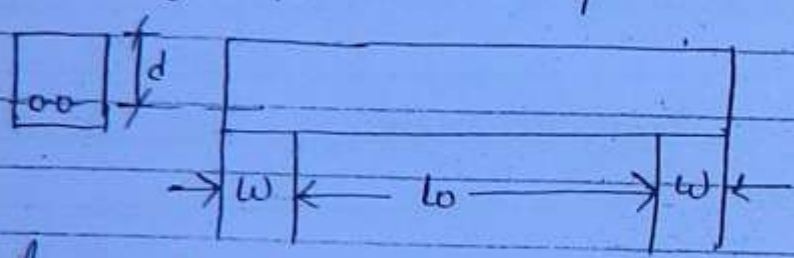
# DESIGN OF BEAM AND SLAB (One way Slab)

Important I.S Provision :—

① Effective span

(a) Simply supported beam/slab (22.2 – Page No-34/35)

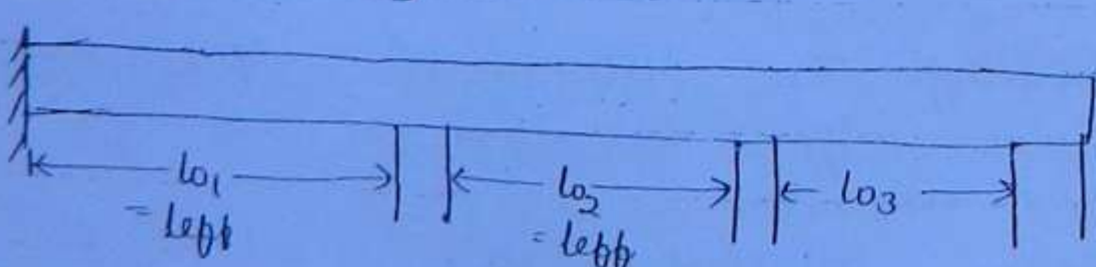


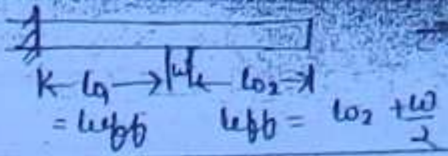

$l_{eff} = l_0 + d$ } whichever is less
$= l_0 + w$

(b) For a Continous beam or slab

1. If width of support is less than $\frac{1}{12}$ of clear span $\left(w < \frac{l_0}{12}\right)$ then eff span is same as that of simply supported

2. If $\left(w > \frac{l_0}{12}\right)$ or $w = 600\,mm$ } then $\boxed{Min}$






① For one end fixed & other Continous or Both end fixed Continous (intermediate span)

$l_{eff}$ = Clear span    (123)

2. With one end Continous & other free (Simply Supported)

$l_{eff}$ = Clear span + $\frac{W}{2}$ = $l_o + \frac{W}{2}$ } whichever is small

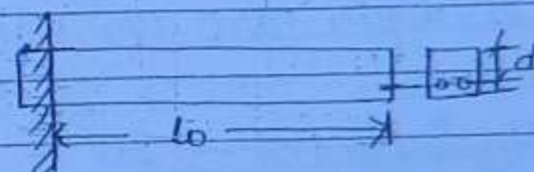
$l_{eff}$ = Clear Span + $d/2$ = $l_o + d/2$

(C)

Cantilever

(i) With fixed end

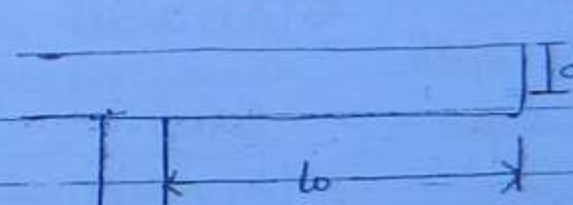
$l_{eff} = l_o + \frac{d}{2}$




d = eff depth

(ii) with Continous end

$l_{eff} = l_o + \frac{W}{2}$




(d)

Frames – c/c distance between beam/Column.

Control of deflection

(Important Criteria of limit state of serviceability)

(Page NO-37)

The deflection shall generally be limited to following-

(a) The final defl$^n$ of all load including the effect of temp, Creep, & Shrinkage & measured from the as-cast level of the support of floor, roof & all horizontal member should not normally exceed Span/250

(intermediate defl$^n$) (124)

(b) The deflection including the effect of Temp, Creep & Shrinkage occuring after erection of partition & the application of finishes should not normally exceed span/350 or 20mm whichever is less

Check for deflection

→ Vertical defl$^n$ limit may be generally be assumed satisfied if –

Span to depth ratio are not greater than

| | | |
|---|---|---|
| Cantilever | – 7 | |
| Simply Supported | – 20 | (A) values (upto 10m span) |
| Continous | – 26 | |

$$\frac{\text{Span}}{\text{Eff depth}} \leq \text{(A) values}$$

$$\text{Eff depth} \geq \frac{\text{Span}}{\text{(A) values}}$$

Except for cantilever if span is more than 10m the A value should be multiplied by $\frac{10}{\text{Span in metre}}$

$$\text{Eff depth} = \frac{\text{Span}}{\text{A value} \times \frac{10}{\text{Span (m)}}}$$

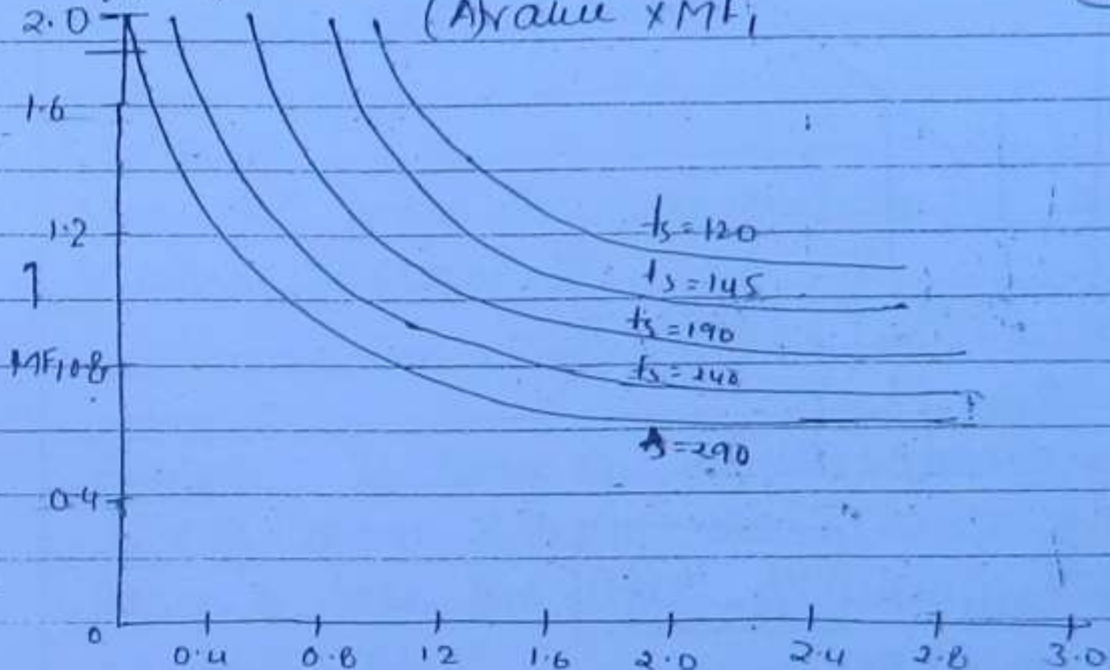
eg – For a 20m span S.S.B eff. depth req will

$$\text{Eff} = \frac{20 \times 10^3}{20 \times \frac{10}{20}} = 2\text{m or } 2000\text{mm}$$

Modification factor ($MF_1$) based on tension reinf$^n$
A value can be multiplied by a Modification factor $MF_1$ based on tension reinf provided.

$$\text{Eff depth} = \frac{\text{Span}}{(A)\text{ value} \times MF_1}$$

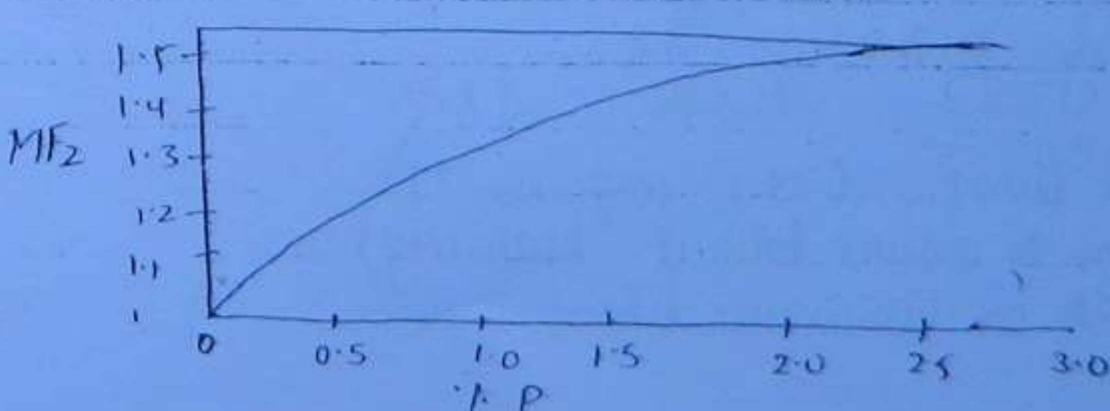
125




$$f_s = 0.58 f_y \frac{\text{Ast req}}{\text{Ast provided}}$$

Modification factor ($MF_2$) based on Comp. reinforcement

$$\text{Eff. depth} = \frac{\text{Span}}{(A)\text{ value} \times MF_1 \times MF_2}$$



For flanged section (T-beam / L-beam)
Reduction factor is used

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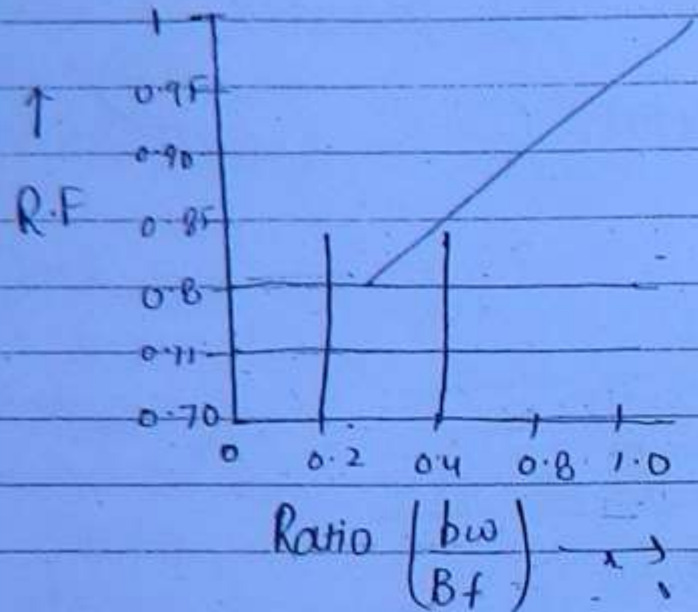


Two way slab $\boxed{l_y/l_x < 2}$

Check for deflection
Only for shorter span < 3.5 m (upto (load ≤ 3 kN/m²)
Span to overall depth ratio
Simply Supported = 35 } for Mild steel
Continous — 40

For HYSD Bars of grade Fe-415 the values shall be Multiplied by 0.8

S.S.B = 0.8 × 35 = 28

Continous - 0.8 × 40 = 32

Slenderness limit (23.3 - page No-39)
(For beams to ensure lateral stability) -
① For S.S.B or Continous beam

Clear dis$^n$ bet$^n$ lateral restraint $\not> 60b$ or
$\not> \dfrac{250b^2}{d}$ (small)

b → width
(127)
d → eff. depth of beam

Cantilever.
Clear dis$^n$ from the free end of the Cantilever
to lateral restraint $\not> 25b$
$\not> \dfrac{100b^2}{d}$ less

Example - If span of Cantilever beam is 5000 mm
Size of beam 250X450 mm Check whether
beam is safe for defl$^n$ & lateral stability
→ two way
Defl$^n$ →

$$\text{Eff depth} = \frac{\text{Span}}{7} = \frac{5000}{7} = 714.29 \text{ mm}$$

$$> 450 \text{ mm}$$

$$\text{lateral stability} = \frac{100\,b^2}{d}$$

$$= \frac{100 \times 250^2}{400} = 15625 \text{ mm}$$

(Safe

$$25 \times b = 25 \times 250 = 6250 \text{ mm}$$

$$> 5000 \text{ mm}$$

OK

## Steel Reinforcement (Beams)

Min tension reinforcement

$$\frac{A_{st\,min}}{bd} = \frac{0.85}{f_y}$$ (128)

Max tension reifn

= 4% of total Cross Section area

= $0.04\,bD$

Compression reifn

Max Comp reinfn $\not> 0.04\,bD$

## Slab

Min reinforcement

Mild Steel → 0.15% of total Cross section

$\frac{0.15}{100} \times BD$

HYSD Bars → $\frac{0.12}{100} \times B \cdot D$

Max spacing of bars

(i) For main reinfn

= 3d or 300 mm

(ii) For distribution bars

= 5d or 450 mm

~~Max dia~~ Max ~~Min~~ dia of bars for slab

$\not> \frac{\text{Depth}}{8} = \frac{1}{8} \times$ total thickness of slab.

If a slab is 75 mm thick

Max dia $= \frac{75}{8} = 9.375$

Max 8mm $\phi$ bars is used.

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DATE:

# Design of One way slab ($l_y/l_x > 2$)

A slab is said one way only if :-

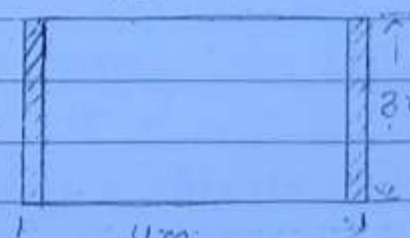
(i) If the slab is supported only on two support (opposite) (Span ratio is not to be seen) (fig i)

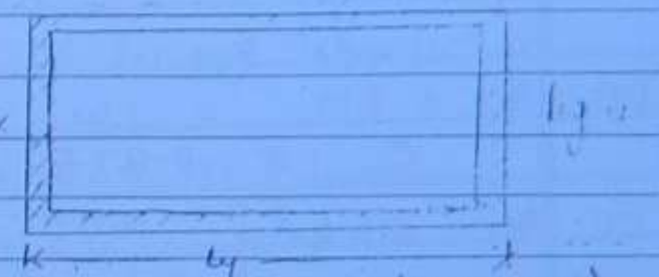
(ii) If the slab is supported on all four sides and span ratio $\frac{l_y}{l_x} > 2 \longrightarrow$ One way slab

(3)

$\frac{l_y}{l_x} \leq 2 \longrightarrow$ two way slab

fig (i)



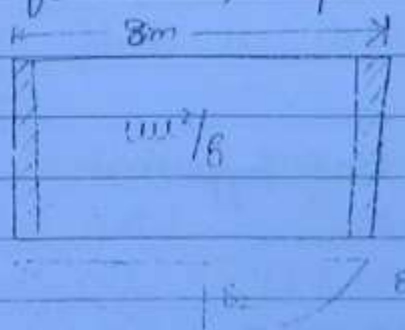

(One way) ($l_y/l_x > 2$)

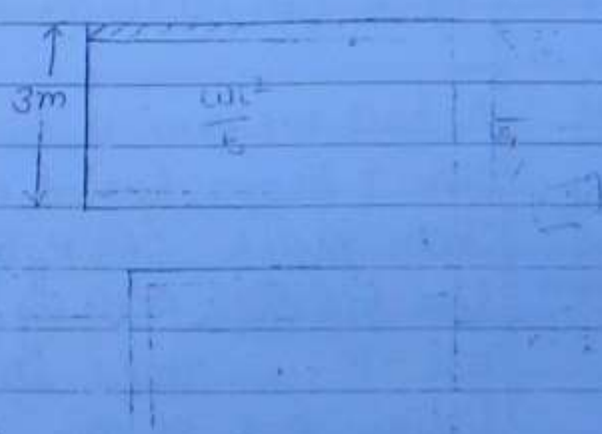
In fig (ii)

$l_y$ = longer span & $l_x$ = Shorter span

→ In this case slab is designed for shorter span only

→ In longer span dir$^n$ only distribution bars (min Steel reinforcement) is provided




Only shorter span is effective in design & design will be done for shorter span only due to above defl$^n$ calculated. As we know moment is proportional to defl$^n$ only so we in one way we design only for shorter span.

Design Steps

(1) Load over 1m x 1m of slab is calculated

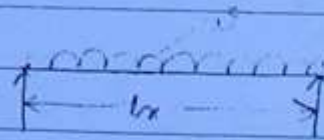
Dead load = t x 1 x 1 x 25 = $w_d$

Live load = $w_e$ x 1 x 1 = $w_e$

Floor finish = $t_f$ x 1 x 1 x 24 = $w_f$

Total load = $w$

(2) Bending Moment = $B.M = \dfrac{W \cdot l_x^2}{8}$




(3) Depth d = $\sqrt{\dfrac{B.M}{Q.B}}$ B = 1000 mm

(4) $A_{st} = \dfrac{BM}{\sigma_{st} \cdot j \cdot d} = \dfrac{M_u}{0.87 \cdot f_y \cdot j \cdot d}$ 132

or $A_{st} = \dfrac{0.5 f_{ck}}{f_y}\left[1 - \sqrt{1 - \dfrac{4.6 M_u}{f_{ck} B \cdot d^2}}\right] \times B \cdot d$

(Useful for under reinforced section & also for limiting section)

(5) Check for shear

(6) Check for Bond

(7) Check for developement length

:-1984

**Ques** A hall 10 x 40m is covered by R.C.C slab supported over T-beam spaced at 4m interval

Floor finish = 60 kg/m²

Live load = 200 kg/m²

Use M-15 mild steel use

Design one of the intermediate slab

(width of beam = 250mm)

Soln For one slab

Clear span $l_x$ = 3.75 m

$l_y$ = 10 m

Assume thickness of slab using defln criteria

Eff. depth = $\dfrac{Span}{26} = \dfrac{3750}{26}$ = 144mm

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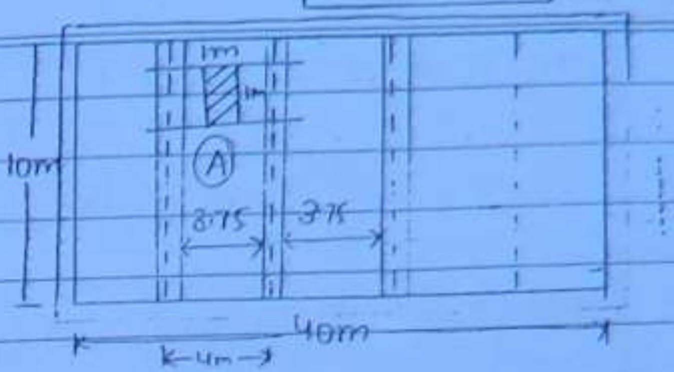
Eff. Cover = 25 mm

= 169 mm

say $D = 170$ mm

$d = 145$ mm




Effective span

Width = $W = 250$ mm

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$\frac{Span}{12} = \frac{3750}{12} = 312.5$ mm

$\left(W < \frac{Span}{12}\right) \Rightarrow l_{eff}$ is same as simply supported

$l_{eff} = l_o + d = 3.75 + 0.145 = 3.895$ m $\}$ small

or $l_o + w = 3.75 + .25 = 4.0$ m

$l_x = 3.895$ m

Similarly $l_y = 10.145$ m

Span ratio $\frac{l_y}{l_x} = \frac{10.145}{3.895} = 2.6 > 2$

Hence it is one way slab

Slab shall be designed for $l_x = 3.895$ m

Load

Consider 1m width of slab

Dead load

Self wt $= .17 \times 1 \times 1 \times 25 = 4.25$ kN/m²

floor finish : $60 \times 10 \times 10^{-3} \times 1 \times 1 = 0.60$ kN/m²

Live ~~load~~

$W_L = 2.0 \times 1 \times 1 = 2.0$ kN/m²

Bending moment (Form from Coefficient table)

$B.M = W_d \times \frac{1}{16} l_x^2 + W_L \times \frac{1}{12} \times l_x^2$

$M(+ve) = \frac{1}{16} \times 4.85 \times 3.895^2 + \frac{1}{12} \times 2 \times 3.895^2$

$M(+ve) = 7.13$ kN

(-ve) B.M $= -\frac{1}{10} W_d \, l_x^2 - \frac{1}{9} w_l \, l_x^2$

$= -\frac{1}{10} \times 4.85 \times 3.895^2 - \frac{1}{9} \times 2 \times 3.895^2$

(-ve) B.M = 10.73 KN

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Depth $d = \sqrt{\frac{B.M}{Q.B}}$

$d = \sqrt{\frac{10.73 \times 10^6}{0.87 \times 1000}}$

$d = 111$ mm $< 145$ mm

(Safe)

$k = \frac{mc}{t + mc}$

$= \frac{19 \times 5}{140 + 19 \times 5}$

$= 0.404$

$j = 1 - \frac{k}{3} = 0.865$

$Q = \frac{1}{2} c \cdot j \cdot k$

$= \frac{1}{2} \times 5 \times .404 \times .865$

$= .87$

$Ast = \frac{BM}{\sigma_{st} \cdot j \cdot d}$

(+ve) B.M $= \frac{7.13 \times 10^6}{140 \times 0.865 \times 145} = 406$ mm$^2$

$\frac{a_{st}}{Ast} = S$

Use 10mm $\phi$ bars spacing

$S = \frac{1000}{406} \times \frac{\pi}{4} \times (10)^2 = 193$ mm

provide 10mm $\phi$ bars @ 190 mm c/c

tip

(-ve) B.M $= \frac{10.73 \times 10^6}{140 \times 0.865 \times 145} = 611$ mm$^2$

Spacing of 10 mm $\phi$ mo bars

$S = \frac{1000}{611} \times \frac{\pi}{4} \times 10^2 = 128$ mm

provide 10mm $\phi$ bars @ 125 mm c/c

Check for Shear

$S.F = 0.55 \, w_d \cdot l_x + 0.60 \, w_l \cdot l_x$

$= 0.55 \times 4.85 \times 3.895 + 0.60 \times 2 \times 3.895$

$V = 15.06\ kN$

$\tau_v = \dfrac{V}{b \cdot d} = \dfrac{15.06 \times 10^3}{1000 \times 145} = 0.104$

$\tau_{c\,min} = 0.18$  $(\tau_v < \tau_c)$ safe

No shear reinforcement is required. (135)

Check for bond.

For (+ve) reinforcement 10 mm $\phi$ bars @ 190 c/c

S.F at support : $V = 15.06\ kN$

Bond stress developed

$\tau_{bd\,(developed)} = \dfrac{V}{\Sigma o \cdot j \cdot d} = \dfrac{15.06 \times 10^3}{\underbrace{\left(\frac{1000}{190}\right)}_{n} \times \pi \times 10 \times 0.865 \times 145}$

$\Sigma o = n \times \pi \phi$

$\tau_{bd} = 0.726\ N/mm^2$

$\tau_{bd\,(permissible)} = 0.6$

$\tau_{bd} > \tau_{bd\,(permi)}$ → failed

Change dia of bars provided

Use 8 mm dia bars for +ve reinforcement

Spacing $= 190 \times \dfrac{8^2}{10^2} = 121.16$

Provide 8 mm $\phi$ bars @ 120 mm c/c

$\tau_{bd}$ developed $= \dfrac{15.06 \times 10^3}{\left(\frac{1000}{120}\right) \times \pi \times 8 \times 0.865 \times 145}$

$= 0.57 < 0.6$ (Safe)

Distribution bars

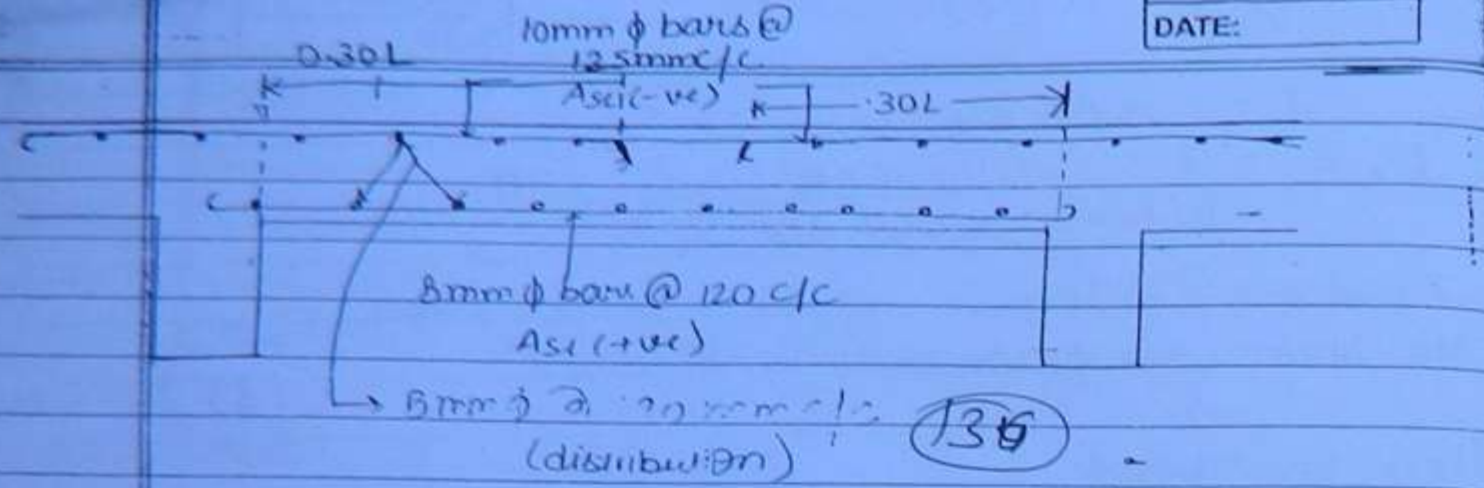
= 0.15% of total cross section area

$= \dfrac{0.15}{100} \times B \times D$

$= \dfrac{0.15}{100} \times 1000 \times 170 = 255\ mm^2$

Spacing of 8 mm $\phi$ bars is $\dfrac{1000}{255} \times \dfrac{\pi}{4} \times 8^2 = 197.12\ mm$

Provide 8 mm $\phi$ bars @ 190 mm c/c

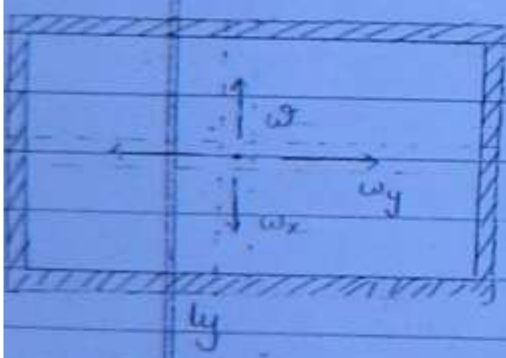



## Design of two way slab $\left(\frac{l_y}{l_x} \le 2\right)$

* A slab is said to be two way only if -
1. Slab should be supported on all four sides
2. Span Ratio $\le 2$

## Design

1. Rankine Grashoff Method
(Simply supported slab)
With corner not held down position




(Corners are free to lift in upward dirn)

No negative B.M will generate (pure simply supported case)

If $w$ = total load

$w = w_x + w_y$ — 1

defln $\delta_x = \frac{5}{384}\frac{w_x l_x^4}{EI}$

$\delta_y = \frac{5}{384}\frac{w_y l_y^4}{EI}$

In every case $\delta_x = \delta_y$ $\quad \frac{5}{384}\frac{w_x l_x^4}{EI} = \frac{5}{384}\frac{w_y l_y^4}{EI}$

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$$w_x l_x^4 = w_y l_y^4$$

$$\frac{w_x}{w_y} = \left(\frac{l_y}{l_x}\right)^4 = r^4$$

$$\boxed{w_x = r^4 w_y} \quad \text{—— (2)}$$

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Total load $w = r^4 . w_y + w_y$

$$(1+r^4) w_y$$

$$w_y = \left(\frac{1}{1+r^4}\right) w \quad \text{—— (3)}$$

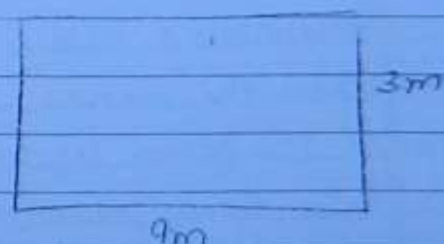
$$w_x = r^4 w_y = \left(\frac{r^4}{1+r^4}\right) w \quad \text{—— (4)}$$

BM in x-dir$^n$ $\Rightarrow$ $M_x = w_x \cdot \frac{l_x^2}{8} = \left(\frac{r^4}{1+r^4}\right) w \cdot \frac{l_x^2}{8}$

$$M_y = w_y \cdot \frac{l_y^2}{8} = \left(\frac{1}{1+r^4}\right) w \cdot \frac{l_y^2}{8}$$

This is as per rankine grashoff theory

Eg. $w_x = \left(\frac{r^4}{1+r^4}\right) w$




$r = 3 = l_y/l_x$ (one way slab)

$$w_x = \left(\frac{3^4}{1+3^4}\right) w = 0.99w$$

$$w_y = \left(\frac{1}{1+r^4}\right) w = 0.01 w$$

~~Me~~ a

2 Simply supported slab (When corners held down position)
(Negative B.M will develop at support and corner)
Method
(i) Pigeaud's method
(ii) Marcus Method
(iii) IS Code Method

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1. Pigeaud's Method

$$M_x = \alpha_x' \cdot \frac{W_x \cdot l_x^2}{8}$$

$$M_y = \alpha_y' \frac{W_y \cdot l_y^2}{8}$$ 138

here $\alpha_x'$ & $\alpha_y'$ are Coefficients values are read from table.

2. Marcus Method (Modification over Rankine grashoff Method)

Marcus Correction

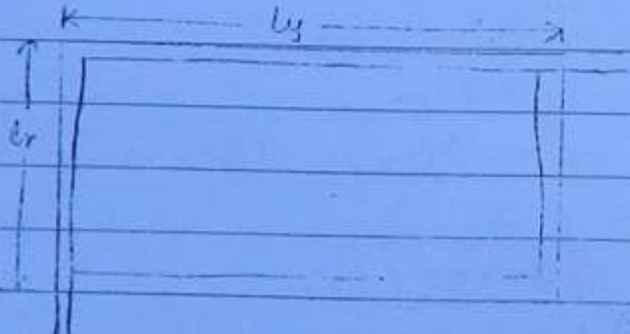
$$C = \left(1 - \frac{5}{6}\frac{\alpha^2}{1+\alpha^4}\right)$$ is applied

$$M_x = C \cdot w_x \cdot \frac{l_x^2}{8}$$

$$M_y = C \cdot w_y \cdot \frac{l_y^2}{8}$$

3. I.S Code Method (ANNEX D)

Restrained Slabs




→ When the Corners of
slab are prevented from lifting
(the corners held down position)
αx- B.M per unit width

$M_x = \alpha_x\, w l_x^2$

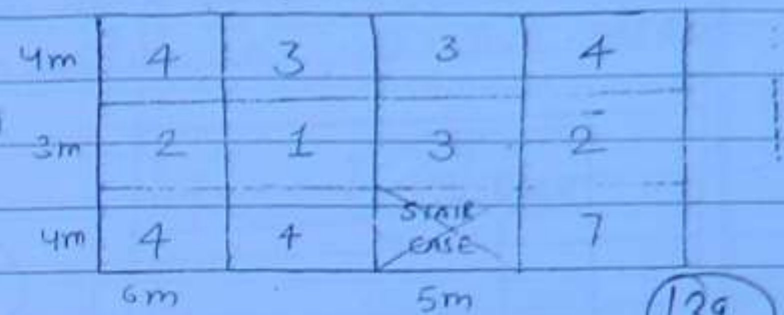
$M_y = \alpha_y\, w l_x^2$

& $\alpha_y$ are Coefficient based on Support Condition. (TABLE-26)

$l_x$ = Shorter span

$w$ = total load on slab per $m^2$ area

Four B.M are found
$M_x(+ve)$, $M_x(-ve)$, $M_y(+ve)$
$M_y(-ve)$

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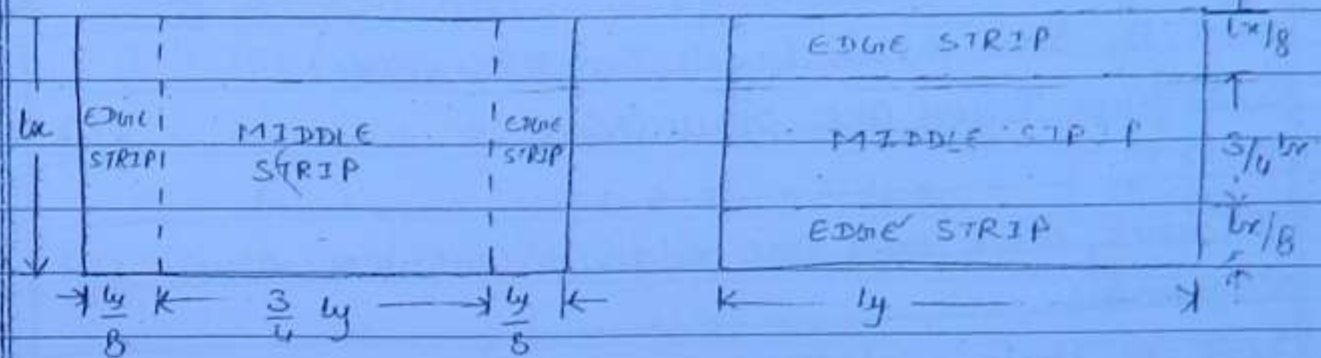
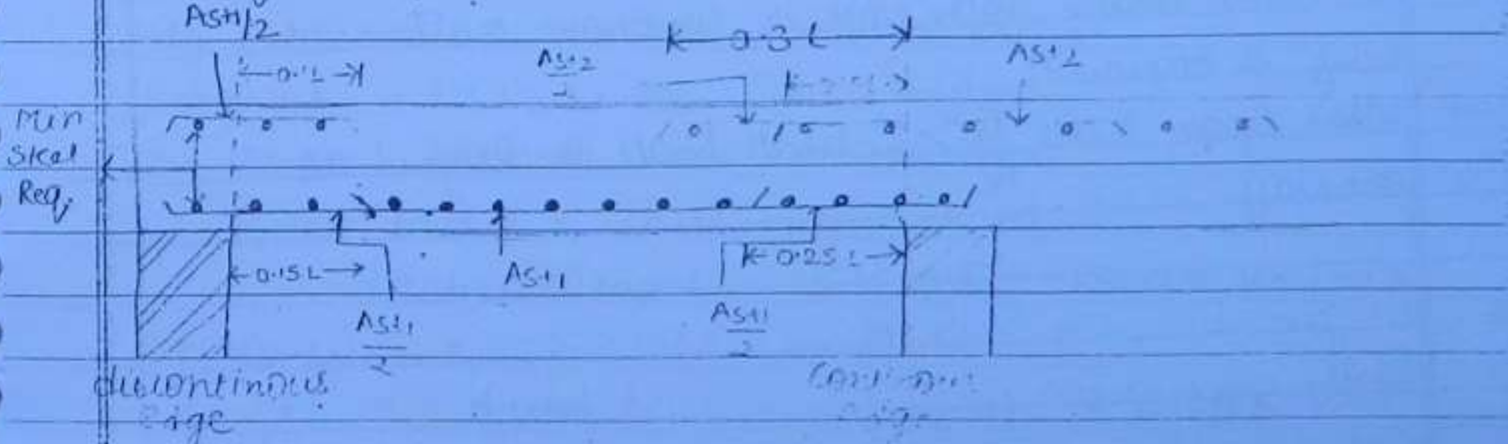
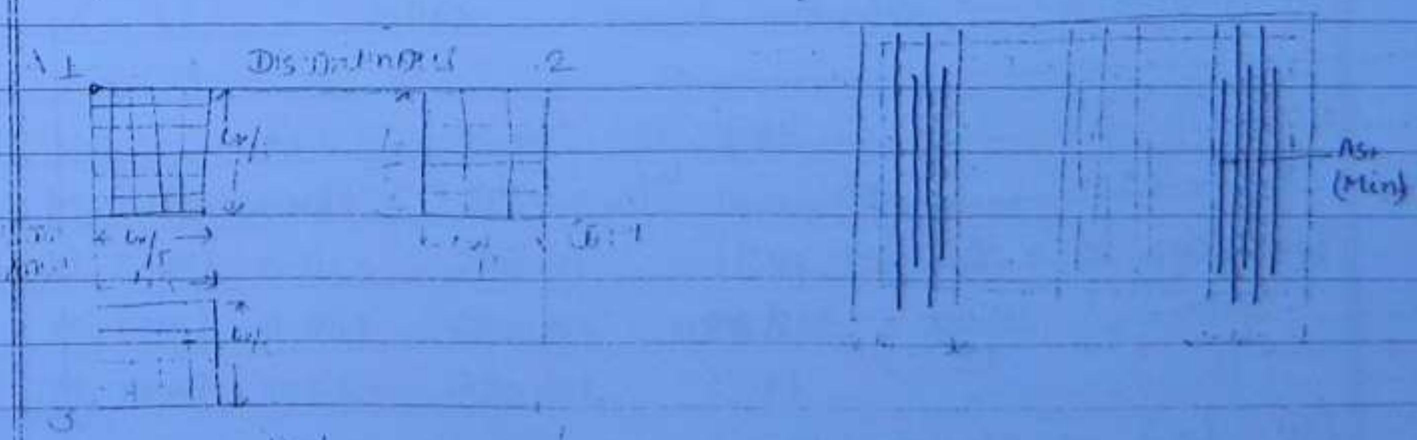
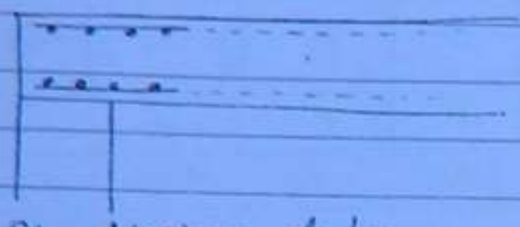



Moments $M_x$ & $M_y$ are applicable for middle strip only.
For edge strip min steel reinf^n required shall be provided.



→ Torsion reinforcement Shall be provided at Corner Where two edges are discontinous (Simply Supported).



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Size of Mesh = $l_x/5$

Area of Steel in each layer = $\frac{3}{4} \times A_{stx}(+ve)$

→ Such four layers are provided

→ At Corner When only one edge is Continous other discontinous

Size of Mesh = $l_x/5$

Area of Steel = $\frac{3}{8} \times A_{stx}\ (+ve)$

→ At Corner where both edge is Continous no Tension reinf^n is required.

→ When span ratio $\frac{l_y}{l_x} > 2$ Shall Shall be designed as one way.

Ques Design [illegible] of [illegible] 4.5m [illegible]
Slab [illegible]
[illegible]

Sol:- $l_x = 4.5m$ ; $l_y = 6$ $l_y/l_x = 1.3 < 2$ (two way)

Eff Span $t_1 = 300$ mm

$\frac{Span}{12} = \frac{4500}{12} = 375$ mm

$\left(w < \frac{l_x}{12}\right)$ → Effective span as per S.S. case

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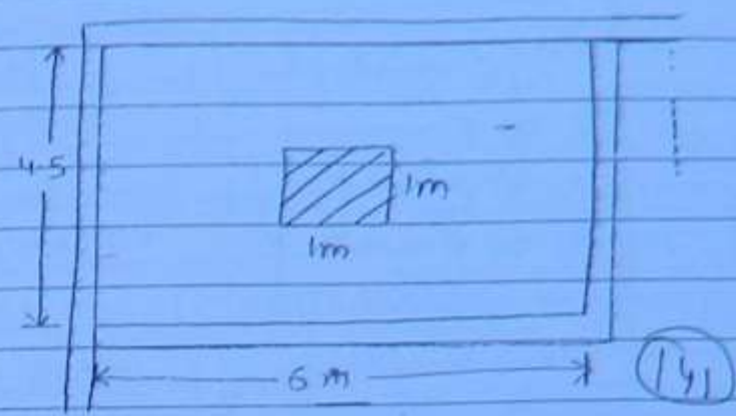
Assume thickness of slab as per deflection criteria

Continous: $0.8 \times 40 = 32$

Overall depth $= \frac{l_x}{32} = \frac{4500}{32}$




$D = 140.625$

Say $D = 150$ mm

$d = 150 - 30 = 120$ mm.

$l_{eff} = l_{x_0} + d = 4.50 + 0.12 = 4.62$ m
$\phantom{l_{eff} =} l_{x_0} + w = 4.50 + .30 = 4.8$

$l_{eff}$ 4.62 m is consider

$l_{eff} = l_{y_0} + d = 6.0 + 0.12 = 6.12$ m $= l_y$

$l_x = 4.62$ m

Span ratio $\frac{l_y}{l_x} = \frac{6.12}{4.62} = 1.32 < 2$ (two way slab)

Load - Calculation

Self wt $= .15 \times 1 \times 1 \times 25 = 3.75$ kN/m²

Floor finish $= 1 \times 1 \times 1 \times 1 = 1$ kN/m²

Live load $= 1 \times 1 \times 4 = 4$ kN/m²

Total load $= 8.75$ kN/m² $= w$

factored load $= 8.75 \times 1.5 = 13.125$ kN/m²

B.M Coefficient

$\alpha_x(+) = 0.050$ $\qquad \alpha_y(+) = \cancel{0.035}$ 0.035

$\alpha_x(-) = 0.066$ $\qquad \alpha_y(-) = \cancel{0.047}$ 0.047

~~Mx (+)~~

| | $\alpha$ | $w \cdot l_x^2$ | M | d |
|---|---|---|---|---|
| $M_x$ (+ve) | 0.05 | 280.145 | 14.0 | Req. 81.85 |
| $M_x$ (-ve) | 0.066 | 280.145 | 18.49 | Provide 120mm |
| $M_y$ (-ve) | 0.047 | 280.145 | 09.81 | |
| $M_y$ (+ve) | 0.035 | 280.145 | 13.17 | |

Depth $d = \sqrt{\frac{M_u}{Q \cdot B}} = \sqrt{\frac{18.49 \times 10^6}{1.38 \times 20 \times 1000}} = 81.85 < 120$ mm (Safe)

→ Ast

$$A_{st_1} = \frac{0.5 f_{ck}}{f_y}\left[1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck} B d^2}}\right] \times B d$$

$$= \frac{0.5 \times 20}{415}\left[1 - \sqrt{1 - \frac{4.6 \times 14.01 \times 10^6}{20 \times 1000 \times 120^2}}\right] 1000 \times 120$$

$$= 344.0 \ mm^2$$

$$A_{st_2} = \frac{0.5 \times 20}{415}\left[1 - \sqrt{1 - \frac{4.6 \times 18.4 \times 10^6}{20 \times 1000 \times 120^2}}\right] 1000 \times 120$$

$A_{st_2}$ = 464 mm

$A_{st_3}$ = 236 mm

$A_{st_4}$ = 322 mm

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Spacing of 10 mm $\phi$ bars

$$\frac{1000}{A_{st}} \times \frac{\pi}{4} \times 10^2 = \frac{78539.82}{A_{st_1}}$$

x (+) $S_1$ → 228 mm → 10 mm $\phi$ @ 220 mm c/c

x (−) $S_2$ → 169 mm → 10 mm $\phi$ @ 160 mm c/c

y (+) $S_3$ → 10 mm$\phi$ − 332 ; 8 mm$\phi$ − 213 → 8 mm $\phi$ @ 210 mm c/c

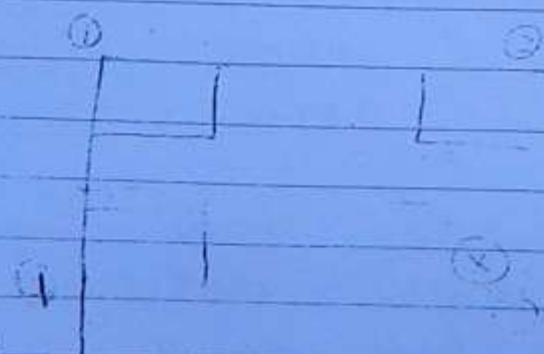
y (−) $S_4$ → 10 mm$\phi$ − 244 ; 8 mm$\phi$ − 156 → 8 mm $\phi$ @ 150 mm c/c

Torsion reinf$^n$

At Corner ①

Size of Mesh $= \frac{l_x}{5} = \frac{4620}{5}$

= 924 mm




Area of Steel $= \frac{3}{4} \times A_{st_x}(+ve)$

$$= \frac{3}{4} \times 344 = 258 \ mm^2$$

Spacing (10 mm $\phi$)

$$S = \frac{1000}{258} \times \frac{\pi}{4} \times 10^2 = 300 mm$$

8mm — Spacing = 194mm

Torsion reinf^n at Corner 1

$M_x (+)$ — Provide 10mm @ 220 c/c

$M_x (-)$ — 10mm $\phi$ @ 160 c/c

$M_y (+)$ — 8mm $\phi$ @ 105 c/c

$M_y (-)$ — 8mm $\phi$ @ 150 mm c/c

Corner 2/3

10 mm $\phi$ @ 320 mm c/c

10 mm $\phi$ @ 320 mm c/c

8 mm $\phi$ @ 210 mm c/c

8 mm $\phi$ @ 300 mm c/c

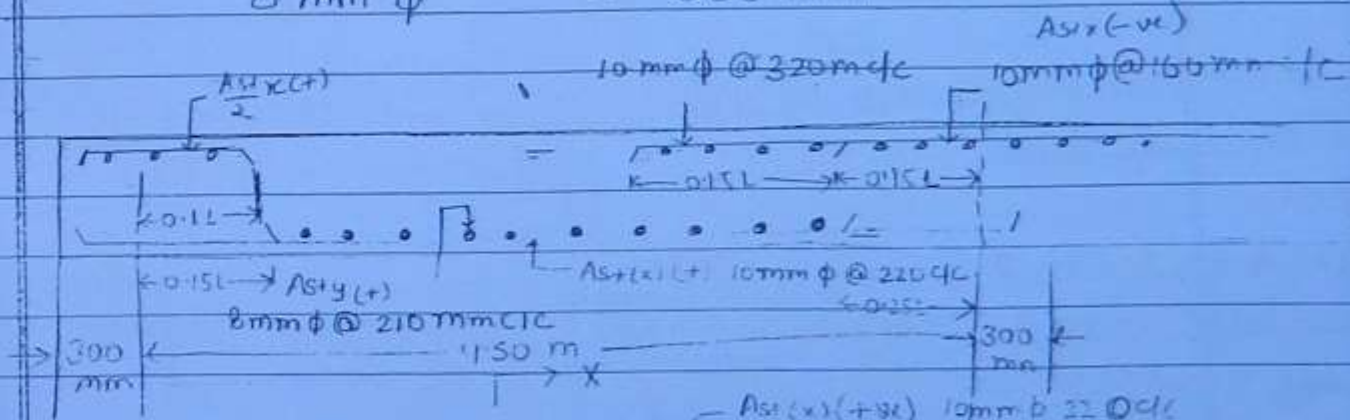
At Corner ②/③ 143

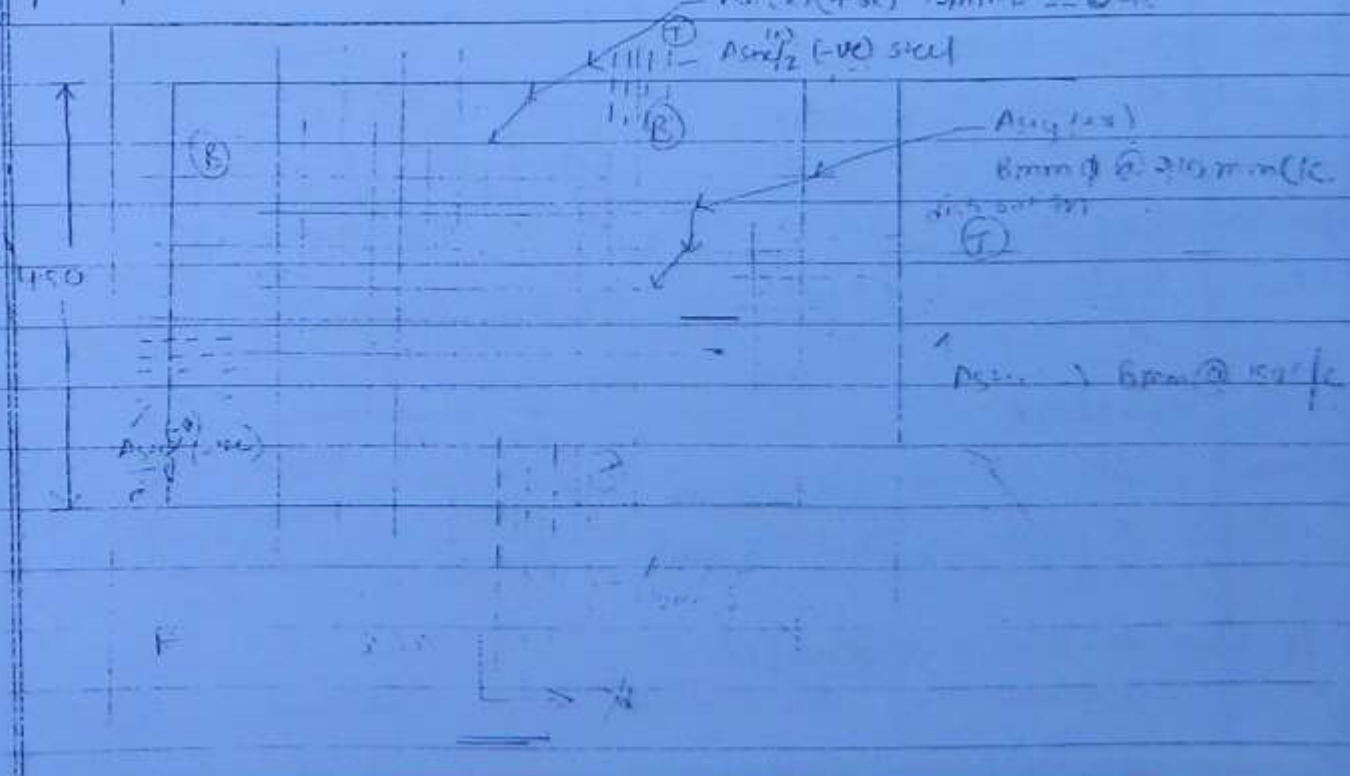
Size of Mesh $= l_x/5 = 924$ mm

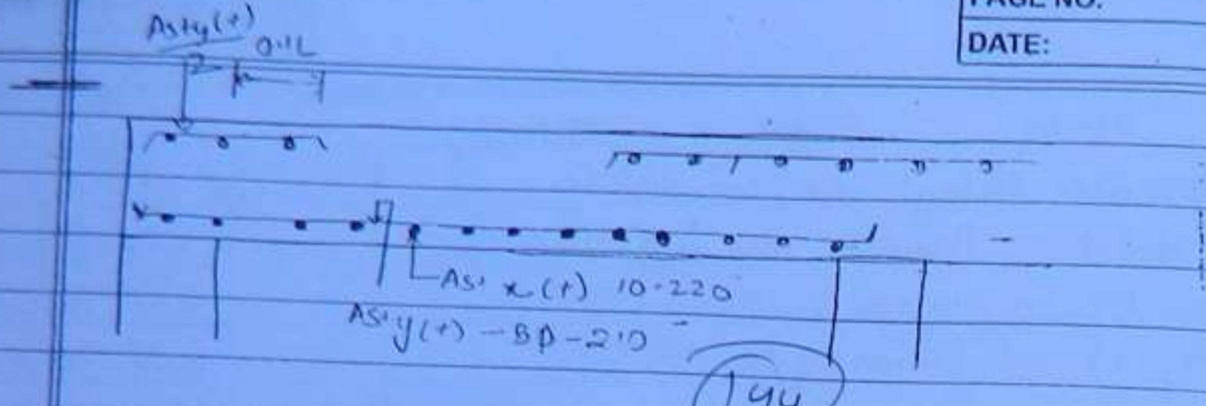
Area of Steel $= \frac{3}{8} \times A_{st\,x}(+ve) = \frac{3}{8} \times 344 = 129\ mm^2$

Spacing 10 mm $\phi$ = 600 mm

8 mm $\phi$ = 388 mm



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A rectangular [illegible] 2m x 3m is simply supported along shorter edge such that clear dist betn supporting wall is 2·7m. The slab is 15 cm thick. Steel 16mm φ @ [illegible] c/c at eff [illegible] 25mm along along larger edge 10mm φ @ 25 mm c/c along [illegible] edge. use M-15 / [illegible] is used. Permissible [illegible] Bending [illegible], Shear [illegible] kg/cm² [illegible] kg/cm² [illegible] calculate max safe [illegible] load the slab can [illegible].

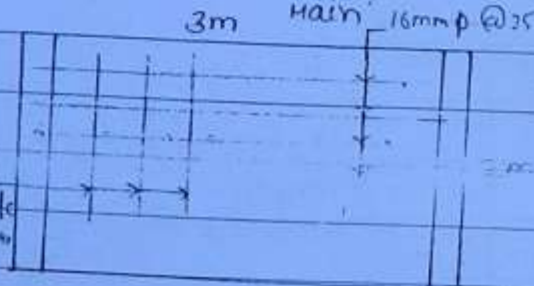
Sol<sup></sup>n – Let us consider 1m width of slab
It is one way slab

Eff. span

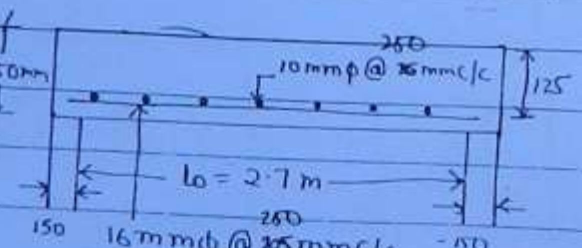
$l_{eff} = l_0 + d = 2.7 + .125 = 2.825\,m$

$l_0 + w = 2.7 + .15 = 2.85\,m$

$l_{eff} = 2.825$




Consider w kN/m is total load i.e. self wt the beam can carry.



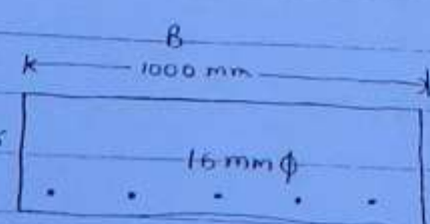

Check for bending :-

$$B.M = \frac{wl^2}{8} = \frac{w \times 2.825^2}{8} = .9976\,w\ \ kN\text{-}m$$

Moment of Resistance

$b = 1000$ mm , $d = 125$ mm ; $D = 150$ mm

$$A_{st} = \frac{1000}{250}\,\frac{\pi}{4}(16)^2 = 804.25\,mm^2$$




4 Nos of bars of 16mm φ is provided.

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$$x_c = \frac{mc}{t+mc} \times d = \frac{19\times5}{140+19\times5} = 50.53\text{mm}$$

$$\frac{B \cdot x_a^2}{2} = m \cdot A_{st}(d - x_a) \Rightarrow \frac{1000}{2} \cdot \cancel{50} x_a^2 = 19 \times 804.25(125 - x_a)$$

$$500x_a^2 = 1910093.75 - 1910093.75\, x_a$$

$$500x_a^2 + 1910093.75 - 1910093.75 = 0$$

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$$x_a = 48.38\text{ mm}$$

$x_a < x_c$ So it is under reinforced sec$^n$

$c_a < \sigma_{cbc}$ & $\sigma_{st} = t_a$

$$M.R = t_a \cdot A_{st} \left(d - \frac{x_a}{3}\right) = 140 \times 804.25\left(125 - \frac{48.38}{3}\right) \times \frac{1}{10^6}$$

$$= 12.26 \text{ kN-m}$$

$$\frac{c_a}{x_a} = \frac{t_a/m}{d - x_a} \Rightarrow c_a = \frac{x_a \cdot t_a}{(d-x_a)m} = \frac{48.38\times140}{(125-48.38)19} = 4.653 \text{ N/mm}^2$$

$$M.R = B \cdot x_a \cdot \frac{c_a}{2}\left(d - \frac{x_a}{3}\right) = 1000 \times 48.38 \times \frac{4.653}{2}\left(125 - \frac{48.38}{3}\right)\frac{1}{10^6}$$

$$= 12.26$$

Equating B.M = MR

$$0.9976\,w = \cancel{M.R}\ 12.26$$

$$w = 12.29 \text{ kN/m}$$

Check for Shear

$$V = \frac{wl}{2} = \frac{w \times 2.825}{2} = 1.4125\,w$$

$$\tau_v = \frac{V}{Bd} = \frac{1.4125\times10^3\,w}{1000\times125} = 0.0113\,w$$

$$\tau_v = \tau_c \Rightarrow 0.0113\,w = 0.30$$

$$w = 26.55 \text{ kN/m}$$

Check for Bond

$$\tau_{bd}\text{ (developed)} = \frac{V}{\Sigma o\, j d} = \frac{1.4125\,w \times 10^3}{4\times16\times\pi\times.865\times125}$$

$$= 0.06497\, w$$

$\tau_{bd}$ permissible $= 0.6$ N/mm$^2$ $\Rightarrow$ $\tau_{bd}$ developed

$\tau_{bd}$ (developed) = $\tau_{bd}$ (permissible)

$0.06497\, w = 0.6$

$w = 9.235$ kN/m

Self wt = $.15 \times 1 \times 1 \times 25 = 3.75$ kN/m

Live load that Can be provided = 5.485 kN/M over the slab.

(146)

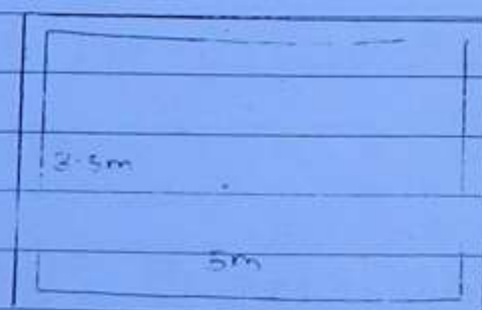
Ques Design a slab simply supported over a span of 3.5 m × 5 m Clear span the slab is loaded $\bar{c}$ live load of 4.00 kN/m² & floor finishes 50mm thick C-C flooring slab is prevented from lifting at corner & edges. use any method. use M-25 / Fe 415 W.S.M

width of support = 250mm

Use Marcus Method

Soln – $l_x = l_{xo} + d = 3.5 + .20 = 3.6$

$l_{xo} + W = 3.5 + .25 = 3.75$




Assume depth

Overall depth = $\frac{\text{Span}}{0.8 \times 35}$

$= \frac{3500}{0.8 \times 35} = 125$ mm

$D = 125$ mm

$d = 100$ mm $= 0.10$ m

Assume $l_x = 3.6$ m

$l_y = 5 + .1 = 5.1$ m

$\frac{l_y}{l_x} = 1.416 < 2$ two way

Load

Self wt = $.125 \times 1 \times 1 \times 25 = 3.125$

Live load = $4 \times 1 \times 1 = 4.00$ kN/m

Floor finish = $.05 \times 1 \times 1 \times 24 = 1.2$

8.325 kN/m

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B.M =

$$M_x = \left(1 - \frac{5}{6}\frac{r^2}{1+r^4}\right)\left(w \cdot \frac{l_x^2}{8}\right)\left(\frac{r^4}{1+r^4}\right)$$

$$= \left(1 - \frac{5}{6}\frac{1.42^2}{1+1.42^4}\right)\left(\frac{1.42^4}{1+1.42^4}\right)\left(8.325 \times \frac{3.6^2}{8}\right)$$

$$= 7.23 \text{ kN-m}$$

$$M_y = \left(1 - \frac{5}{6}\frac{r^2}{1+r^4}\right)\left(\frac{1}{1+r^4}\right)\left(w \cdot \frac{l_y^2}{8}\right)$$

$$= \left(1 - \frac{5}{6}\frac{1.42^2}{1+1.42^4}\right)\left(\frac{1}{1+1.42^4}\right)\left(8.325 \times \frac{5.1^2}{8}\right)$$

$$= 3.57 \text{ kN-m}$$

Depth of slab $d = \sqrt{\frac{M}{Q \cdot B}}$

$$d = \sqrt{\frac{7.23 \times 10^6}{1.11 \times 1000}} = 80.70 \text{ mm}$$

$< 100$ mm (safe) OK

$$K = \frac{mc}{t+mc} = \frac{11 \times 8.5}{230 + 11 \times 8.5} = 0.289$$

$$j = 0.904$$

$$Q = \frac{1}{2} c \cdot j \cdot k = 1.11$$

Let us use $d = 100$ mm

$D = 125$ mm

Area of steel $A_{st(x)} = \frac{M}{\sigma_{st} \, j \cdot d} = \frac{7.23 \times 10^6}{230 \times .904 \times 100} = 348 \text{ mm}^2$

$$A_{st(y)} = \frac{3.57 \times 10^6}{230 \times .904 \times 100} = 172 \text{ mm}$$

Use 10 mm $\phi$ bars

Spacing (x) $= \frac{1000}{348} \times \frac{\pi}{4} \times 10^2 = 225$ mm

8mm$\phi$ Spacing $= \frac{1000}{348} \times \frac{\pi}{4} \times 8^2 = 144$ mm

Use 8mm $\phi$ @ 140 mm c/c

Spacing (y) $= \frac{1000}{172} \times \frac{\pi}{4} \times 8^2 = 292$ mm

·12 % of BD

Min Steel $= \frac{\cdot 12}{100} \times 1000 \times 125$

$= 150\ mm^2$ (Ok

Max Spacing = 300mm

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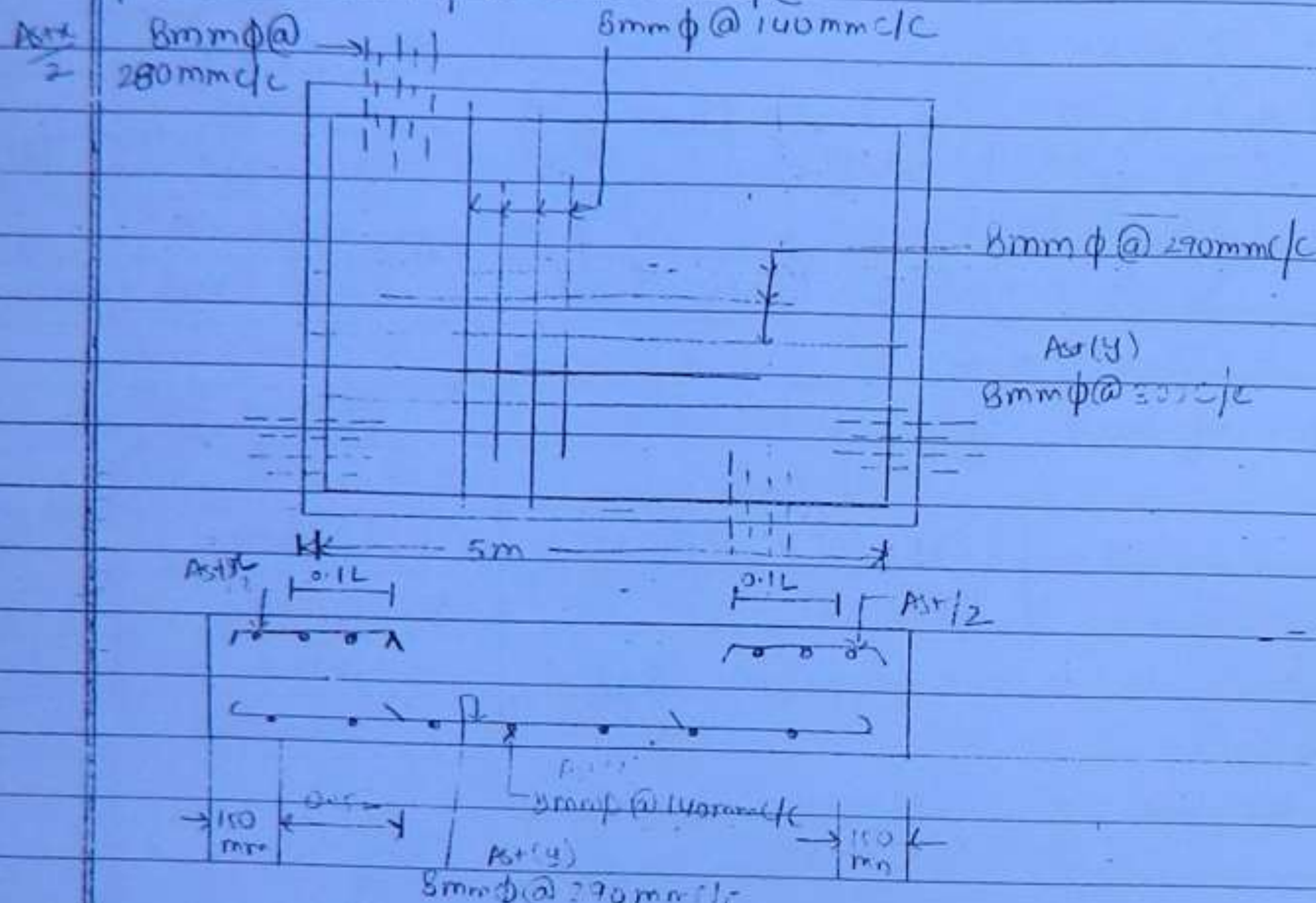
provide 8mm ϕ @ 290mm c/c



# COLUMNS

## Working Stress Method.

→ Long / Short Column

Slenderness Ratio = $\frac{\text{Eff. length}}{\text{least lateral dimension}}$

$\frac{l_{eff}}{B} < 12$ (Short Column)

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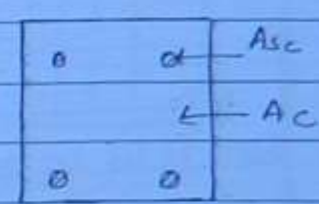
$\frac{l_{eff}}{B} > 12$ (Long Column)

— Load bearing Capacity of a Column

$P = A_{sc} \cdot \sigma_{sc} + A_c \cdot \sigma_{cc}$ — Short Column

$A_c = (A - A_{sc})$




For long Column

Load Carrying Capacity

$P = C_r \cdot [A_{sc} \cdot \sigma_{sc} + A_c \cdot \sigma_{cc}]$

Where

$C_r = \left(1.25 - \frac{l_{eff}}{48B}\right)$ Reduction Coefficient

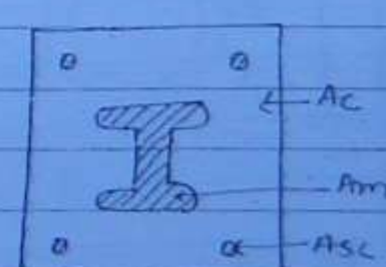
B = least lateral dimension

$C_r = \left(1.25 - \frac{l_{eff}}{160\, i_{min}}\right)$

$i_{min}$ = least radius of gyration $= \sqrt{\frac{I}{A}}$

For a Composite Column

$A_c = (A - A_{sc} - A_m)$




Load Carrying Capacity

$P = C_r\, [A_c \cdot \sigma_{cc} + A_{sc} \cdot \sigma_{sc} + A_m \cdot \sigma_{mc}]$

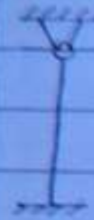
Effective length (TABLE 28)

Fixed – effectively held in position & restrained against Rotation

Theoritical – $0.50L$ – $L/2$

Recommended – $0.65L$

2 Effectively held in position but not restrained against Rotation (One end hinge & other end is fixed)



Theoritical - 0.70L $L/\sqrt{2}$

Recommeded - 0.80L

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3 Held in position but not restrained against Rotation (Both ends are hinge).

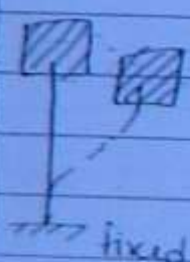
theoritical - 1.0 L

Recommeded = 1.0 L

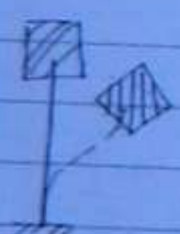
4 Not held in position but restrained against Rotation

Theoritical 1.0 L

Recommeded 1.2 L




5 Not held in position & partially restrained against Rotation



Theoritical - —

Recommended - 1.50 L

6


Theoritical - 2.0 L.

Recommended - 2.0 L

7


Theoritical 2·0L
Recommended 2·0L

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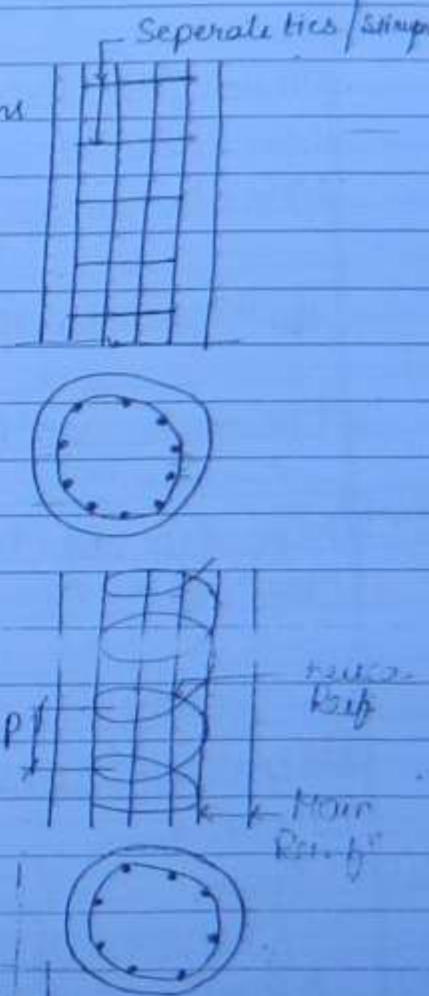
## Design Of Circular Column

(a) (with Seperate Stirupps)

Design same as for Sample R.C.C columns

$P = G_1 (A_c \cdot \sigma_{cc} + A_{sc} \cdot \sigma_{sc})$




(b) With helical Reinforcement (Tie)

Load carrying capacity

$P = 1.05\ G_1 (A_c \cdot \sigma_{cc} + A_{sc} \cdot \sigma_{sc})$

For helical Reinforcement following Criteria should be satisfied.

page No 71, $0.36 \frac{f_{ck}}{f_y} \left[ \frac{A_g}{A_c} - 1 \right] \leq \frac{V_h}{V_c}$ — A

Where

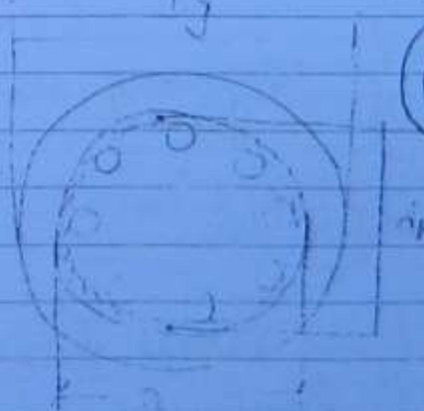
$A_g$ = gross area $= \frac{\pi}{4} (D_g)^2$

$D_g$ = gross diameter

$A_c$ = Core area

$D_c = D_g - 2 \times$ clear cover

$A_c = \frac{\pi}{4} D_c^2$



$V_c$ = Volume of core area for 1m length Of column

$\frac{\pi}{4} (D_c)^2 \times 1000$

$V_h$ – Volume of helical reinf^n in 1 unit length of column

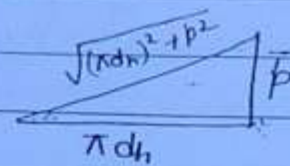
– (No. Of turns) x length in 1 turn X c/s area of helix

$$V_h = \left(\frac{1000}{p}\right) \times (\pi d_h) \times \left(\frac{\pi}{4} \times \phi_h^2\right)$$

Where $d_h = d_c - \phi_h$

length in one turn

$= \sqrt{(\pi d_h)^2 + p^2}$




$\phi_h$ = diameter of helix.

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Other I.S Recommendation

Min percentage of steel = 0.8 % of total area

Max percentage of steel = 6 % of total C/s area
(If bars are not lapped)

If bars are lapped = 4.0 % of total area

Minimum no. of bar — 4 – Rectangular
6 – Circular

Main Reinf. Min. dia of bars – 12 mm dia of bars

Max$^n$ dis$^n$ betn main bars (i.e longitudinal)
= 300 mm

Design of transverse reinforcement (Tie)

Diameter

(i) $\frac{\phi_{main}}{4}$ } whichever is greater

(ii) 6 mm

$\phi_{main}$ for larger dia

Spacing – ① Least lateral dimension
② 16 $\phi_{main}$ (min dia)
③ 300 mm

Pitch – Pitch of helical reinforcement

(1) ≯ 75 mm

(2) ≯ $d_c/6$

(3) ≮ 3$\phi$

DATE:

Design a Column to Carry a load of 1000 KN Column is 12m in length and restrained at both end use - M20 & Fe-415 Design a Square Column.

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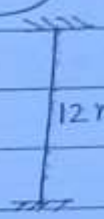
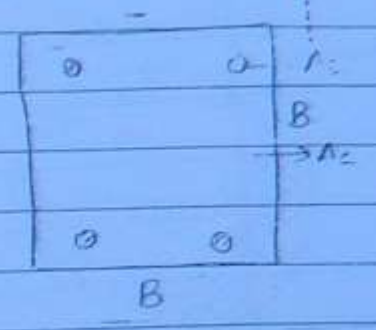
$l = 12m$




Effective length of Column

$l_{eff} = 0.65 l$

$= 0.65 \times 12 = 7.80 m$

Assume Steel 1% of Total c/s area

$A_{sc} = 0.01 A$

$A_c = A - A_{sc} = B^2 - 0.01 B^2 = \underline{0.99 B^2}$

Design of Column

Assuming Short Column

$P = A_{sc} \cdot \sigma_{sc} + A_c \cdot \sigma_{cc}$

$1000 \times 10^3 = 190 \cdot 0.01 B^2 + 0.99 B^2 \times 5$

$B = 382 mm$

| Grade | $\sigma_{cbc}$ | $\sigma_{cc}$ |
|---|---|---|
| M-15 | 5 | 4 |
| M-20 | 7 | 5 |
| M-25 | 8.5 | 6 |
| M-30 | 10 | 8 |
| M-35 | 11.5 | 9 |
| M-40 | 13 | 10 |
| Steel | $\sigma_{sc}$ | $\sigma_{st}$ |
| Fe 250 | 130 | 140 |
| Fe 415 | 190 | 230 |

Slenderness Ratio

$\frac{l_{eff}}{B} = \frac{7800}{382} = 21.4 \ngtr 12$

It is a long Column

For long Column

$P = C_r (A_{sc} \cdot \sigma_{sc} + A_c \cdot \sigma_{cc})$

$1000 \times 10^3 = \left(1.25 - \frac{l_{eff}}{48B}\right)\left(0.01 B^2 \times 190 + 0.99 B^2 \times 5\right)$

$1000 \times 10^3 = \left(1.25 - \frac{7800}{18336}\right)\left(0.01 B^2 \times 190 + 0.99 B^2 \times 5\right)$

$= 6.85 (1.25 B^2 - 162.5 B)$

$1.25 B^2 - 162.5 B - 145985.4 = 0$

$B = 412 mm$

Use $B = 420 mm$

$412^2$    1697.44

$A_{st} = 0.01 \times B^2 \Rightarrow 0.01 \times 420^2 = 1764 mm^2$

No. of 16 mm dia bars = 8.7

Use $4 - 20 mm \phi = 1256.64$

$4-12\phi = 452.39$

total $= 1709.03\ mm^2$

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Stirrups

① Diameter

$\frac{\phi_{main}}{4} = \frac{20}{4} = 5mm$ } use 6mm

② 6mm

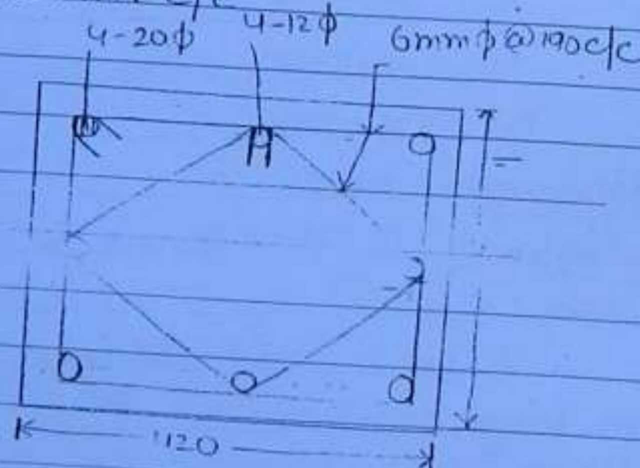
Spacing

① least lateral dimension = 420mm

② $16\times\phi$ main ~~(min dia)~~ $= 16\times12 = 192mm$

③ 300mm

provide 6mm φ bars @ 190 mm c/c




Design a Short Circular Column of 600mm diameter for a load of ~~2000~~ 2500 KN using helical reinforcement use M-25 & Fe-415. Calculate main bars, size & pitch of helical ties.

① Design a Short Column — D = 600mm

$P = (A_{sc}\cdot\sigma_{sc} + A_c\cdot\sigma_{cc})\,1.05$

$2500\times10^3 = 1.05\left(A_{sc}\cdot190 + \left(\frac{\pi}{4}\times600^2 - A_{sc}\right)6\right]$

$2380952.381 = 190A_{sc} + 1696460.033 - 6A_{sc}$

$684492.348 = 184A_{sc}$

$A_{sc} = 3720\ mm^2$

DATE:

No. of 25 mm $\phi$ bars required

$$n = \frac{3720}{\frac{\pi}{4} \times 25^2} = 7.57 \quad \text{say } 8 \text{ Nos.}$$

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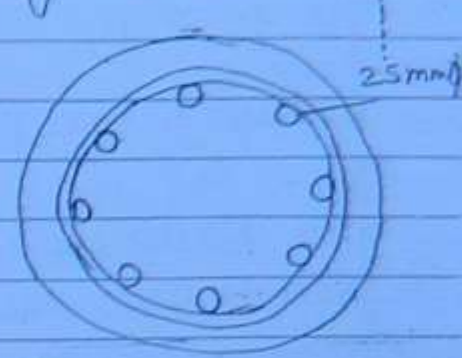



## Design of helical reinforcement

$$0.36 \frac{f_{ck}}{f_y} \left( \frac{A_g}{A_c} - 1 \right) \leq \frac{V_h}{V_c}$$

Where $D_g = 600\,mm$

$$A_g = \frac{\pi}{4} (600)^2 = 282743.33\,mm^2$$

Core diameter $D_c = D_g - 2 \times$ Clear Cover

$$600 - 2 \times 40 = 520\,mm$$

$$A_c = \frac{\pi}{4} D_c^2 = \frac{\pi}{4} \times 520^2 = 212371.66\,mm^2$$

$V_c$ for 1 m length

$$= A_c \times 1000$$

$$= 212371663.4\,mm^3$$

$V_h$ = Volume of helical reinf for 1m length of Column

Assuming dia of helical reinf$^n$

$$d_h = d_c - \phi_h = 520 - 8 = 512\,mm$$

$$V_h = \left( \frac{1000}{P} \right) \left( \pi \times \overset{d_h}{512} \right) \left( \frac{\pi}{4} \times \overset{\phi_h}{8^2} \right)$$

$$V_h = \frac{80851799.25}{P}$$

$$0.36 \times \frac{25}{415} (0.331) = \frac{80851799.25}{P \times 212371663.4}$$

$$7.186 \times 10^{-3} = \frac{0.3807}{P}$$

$$P = \underline{52.98\,mm}$$

DATE:

$p \ngtr 75$ mm OK

$p \ngtr \frac{d_e}{6} = \frac{520}{6} = 86.67$ mm

$p \nless 3 \times \phi_h = 3 \times 8 = 24$ mm

provide 8mm $\phi$ @ 50 mm c/c 156

IS Recommendation

① Slenderness ratio (Page NO - 42)

(a) unsupported length leem end restraint $\ngtr$ 60 times of least lateral dimension

Max. Slenderness ratio limit = 60

(b) If one end of column is unrestrained its unsupported length $l \ngtr \frac{100B^2}{D}$

② Minimum eccentricity

All columns shall be designed for a minimum eccentricity of = $\frac{\text{unsupported length of column}}{500} + \frac{\text{lateral dimens.}}{30}$

Subjected to minimum of 20 mm

$e_{min} = \frac{l}{500} + \frac{B \text{ or } D}{30}$ or 20 mm

Whichever is greater.

→ Combined bending & direct stress

P = Axial compressive load

$e_x$ = eccentricity abt Y-Y

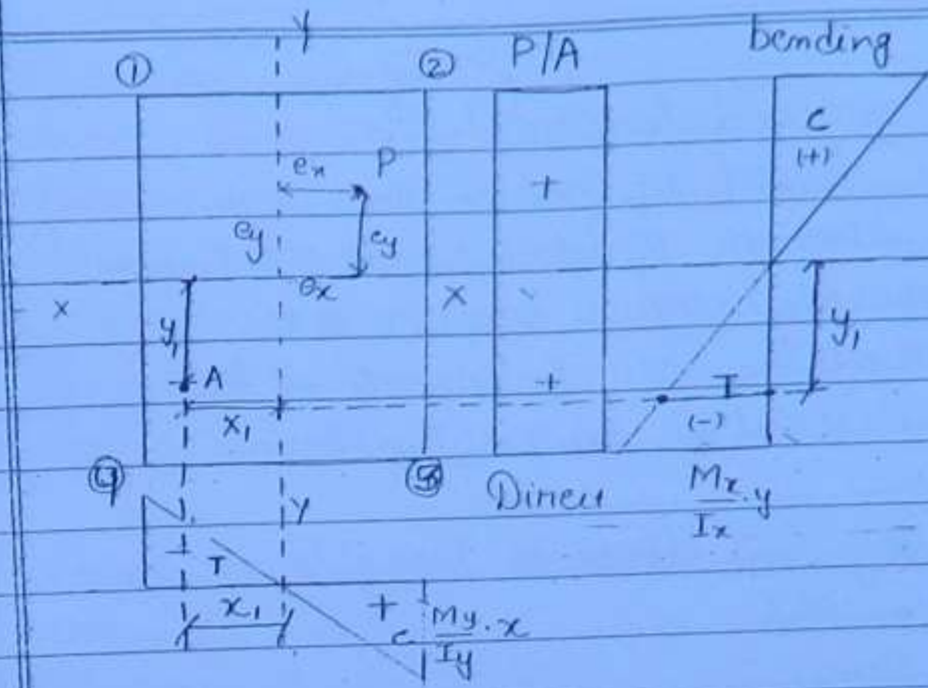
$e_y$ = " " X-X

$M_x$ = Moment abt X-X → $M_x = P.e_y$

$M_y$ → $P.e_x$

Stresses

① Direct stress = $f_0 = +\frac{P}{A}$

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(ii) Bending stress due to $M_x$

$$p_{bx} = \pm \frac{M_x}{I_x} y$$

(iii) Bending stress due to $M_y$ -

$$p_{by} = \pm \frac{M_y}{I_y} x$$

Final stresses at any point on the column section

$$p_0 + p_{bx} + p_{by}$$

$$= + \frac{P}{A} \pm \frac{M_x}{I_x} \cdot y \pm \frac{M_y}{I_y} \cdot x$$

At A point in fig. $(x_1, y_1)$

$$\text{final stress} = + \frac{P}{A} - \frac{M_x}{I_x} \cdot y_1 - \frac{M_y}{I_y} \cdot x_1$$

Stress at four corners of column

$$\text{Corner } 1 = \frac{P}{A} + \frac{M_x}{I_x} \cdot \left(\frac{D}{2}\right) - \frac{M_y}{I_y} \cdot \left(\frac{B}{2}\right)$$

$$2 = \frac{P}{A} + \frac{M_x}{I_x} \left(\frac{D}{2}\right) + \frac{M_y}{I_y} \left(\frac{B}{2}\right)$$

$$3 = \frac{P}{A} - \frac{M_x}{I_x} \left(\frac{D}{2}\right) + \frac{M_y}{I_y} \cdot \left(\frac{B}{2}\right)$$

$$4 = \frac{P}{A} - \frac{M_x}{I_x} \left(\frac{D}{2}\right) - \frac{M_y}{I_y} \left(\frac{B}{2}\right)$$

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## Design Criteria

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1. Design based on uncracked Section

→ If tensile Stresses develop in concrete are within permissible tensile Strength of Concrete, Area in tension need not to be ignored. Design is based on total c/s area, without ignoring the area of Concrete in tension. This design principle is called design - based on uncracked Section.

In this case following eq$^n$ should be satisfied.

$$\frac{\sigma_{cc\,cal}}{\sigma_{cc}} + \frac{\sigma_{cbc\,cal}}{\sigma_{cbc}} \leq 1$$

if satisfied, column is safe as uncracked section.

here $\sigma_{cc\,cal}$ = Direct Stress developed = $p_o = \frac{P}{A}$

$\sigma_{cbc\,cal}$ = Max Stress developed in Concrete in bending Compression at point no 2.

$$= \frac{M_x}{I_x}\left(\frac{D}{2}\right) + \frac{M_y}{I_y}\left(\frac{B}{2}\right)$$

6,8,9,10 $\sigma_{cc}$ – Max. permissible stress in direct Compression

8.5,10,11.5,13 $\sigma_{cbc}$ – Max. permissible Stress in bending Compression

Design based on Cracked Section

If the Criteria $\frac{\sigma_{cc\,cal}}{\sigma_{cc}} + \frac{\sigma_{cbc\,cal}}{\sigma_{cbc}} \leq 1$ is not Satisfied

the Column Section Should be Checked as Cracked Sec$^n$

① Ignoring the area of Concrete in tension

② Tensile Stresses are taken by Steel alone.

Actual Stresses developed in the Section are calculated. If the Stresses are less than permissible limits Column is Considered safe.

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Different Cases :-

① Small eccentricity case $(e < D/4)$ 159

Column is mostly designed as uncracked secⁿ

② Medium eccentricity case $(e > D/4)$ but $(e < 1.5B)$

(Complicated case.) but design acc. to cracked secⁿ

③ Long eccentricity case $(e > 1.5D)$

Column is design as cracked secⁿ due to high moment

→ Small eccentricity (Design based on uncracked secⁿ)

→ Uniaxial bending

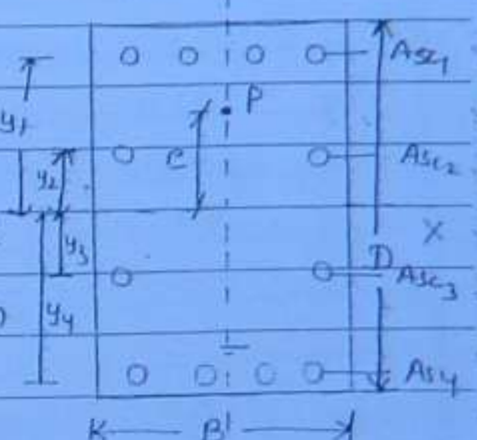
(A symmetrical secⁿ)

Load = P

eccentricity abt X-X = e

Moment abt $X\text{-}X = M_x = P.e$

If Column is safe as uncracked secⁿ




$$\frac{\sigma_{cc\,cal}}{\sigma_{cc}} + \frac{\sigma_{cbc\,cal}}{\sigma_{cbc}} \le 1$$

$$\sigma_{cc\,cal} = \frac{P}{A_{eq}} =$$

Where $A_{eq}$ = Equivalent area of Column secⁿ

$$= A_c + (1.5m - 1)\,A_{sc}$$

$$A_{eq} = BD + (1.5m - 1)(A_{sc_1} + A_{sc_2} + A_{sc_3} + A_{sc_4})$$

$$\sigma_{cbc\,cal} = \frac{M_x}{I_{eq}} \cdot \left(\frac{D}{2}\right) \text{ (at top stress)}$$

Where $I_{eq} = \underbrace{\frac{BD^3}{12}}_{\text{(Concrete)}} + (1.5m - 1)\left[A_{sc_1} \times y_1^2 + A_{sc_2} \times y_2^2 + A_{sc_3} \times y_3^2 + A_{sc_4} \times y_4^2\right]$

$$I_{eq} = \frac{BD^2}{12} + (1.5m - 1)\left[\Sigma A_{sc} \cdot y^2\right]$$

put these values in eqⁿ. if eqⁿ get satisfied then the column section is safe otherwise failed.

PAGE NO.
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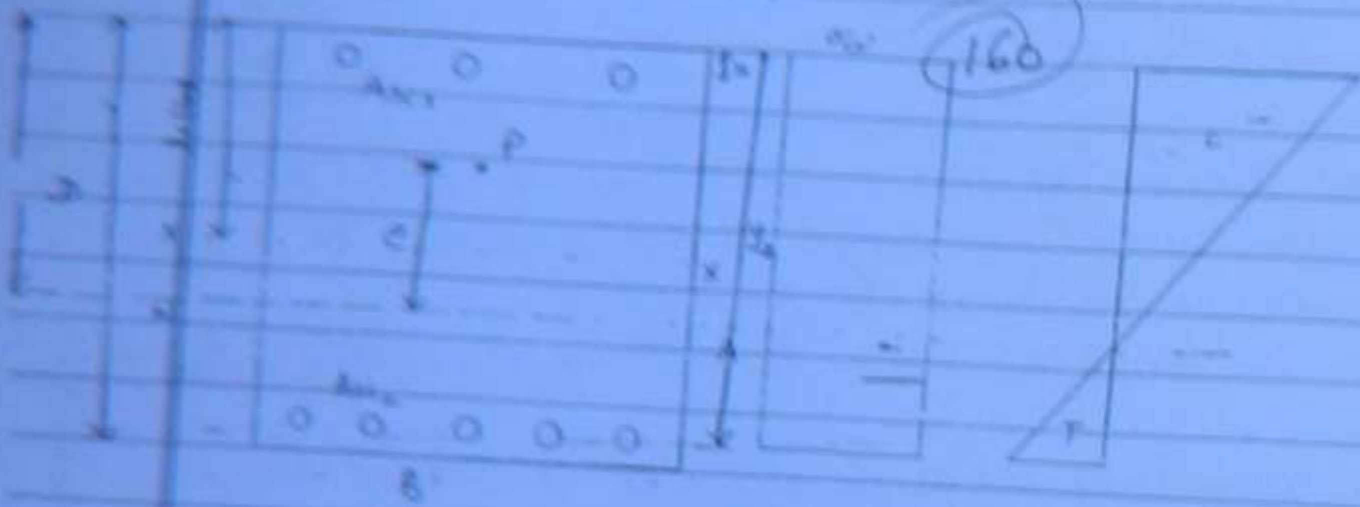
For unsymmetrical Section



If a load P is acting at an eccentricity e from x-x axis

$A_{eq} = B \times D + (1.5m - 1)(A_{sc_1} + A_{sc_2})$

C.G. of Section (N.A) from Top

$$\bar{y} = \frac{B \cdot D \cdot \frac{D}{2} + (1.5m-1)(A_{sc_1} \cdot y_1 + A_{sc_2} \cdot y_2)}{A_{eq}}$$

$$= \frac{\frac{BD^2}{2} + (1.5m-1)(A_{sc_1} y_1 + A_{sc_2} y_2)}{B \cdot D + (1.5m-1)(A_{sc_1} + A_{sc_2})}$$

Net eccentricity $e_{net} = e + \left(\bar{y} - \frac{D}{2}\right)$

Moment $= P e_{net} = M$

Equivalent moment of Inertia

$$I_{eq} = \frac{BD^3}{12} + BD\left(\bar{y} - \frac{D}{2}\right)^2 + (1.5m-1)\left[A_{sc_1}(\bar{y} - y_1)^2 + A_{sc_2}(\bar{y} + y_2)^2\right]$$

$$\sigma_{cc,cal} = \frac{P}{A_{eq}}$$

$$\sigma_{cbc,cal} = \frac{M}{I_{eq}}\bar{y} = \frac{P e_{net}}{I_{eq}} \times \bar{y}$$

Check $\frac{\sigma_{cc,cal}}{\sigma_{cc}} + \frac{\sigma_{cbc,cal}}{\sigma_{cbc}} \le 1$

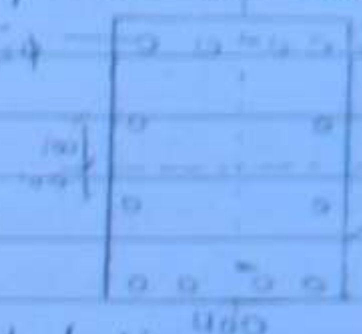
Hence safe

DATE:

A Column secⁿ of size 400 × 600 mm is reinforced with 12 bars of 25 mm φ as shown in fig. use M-25 / Fe-415. Find out

(i) max. eccentricity about X-X s.t. a load can be applied so that their is no tension in the section

(ii) Max. value of load that can be applied at eccentricity calculated above.



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Column should be safe as uncracked section.

Solⁿ Uniaxial bending case

$$A_{eq} = BD + (1.5m-1)[\Sigma A_{sc}]$$

$$= 400\times600 + (1.5\times11-1)\left(12\times\frac{\pi}{4}\times25^2\right)$$

$$= 331302.5 \text{ mm}^2$$

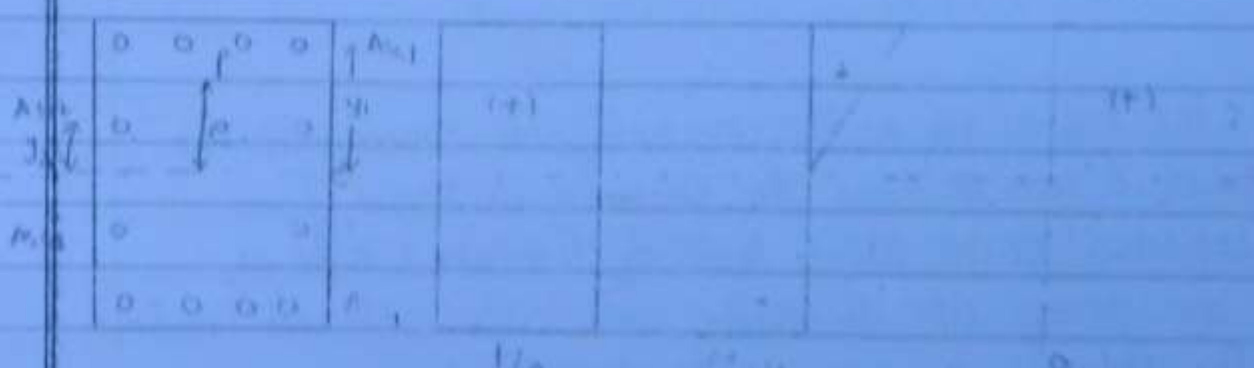
N.A is at XX

Moment of Inertia

$$I_{eq} = \frac{BD^3}{12} + (1.5m-1)\left[A_{sc1}\cdot y_1^2 + A_{sc2}\cdot y_2^2\right]$$

$$= \frac{400\times600^3}{12}(1.5m-1)\left[2\times4\times\frac{\pi}{4}\times25^2\times250^2 + 2\times2\times\frac{\pi}{4}\times25^2\times100^2\right]$$

$$= 1.13686\times10^{10}$$



If there is no tension, stress at bottom = 0

$$\frac{P}{A_{eq}} - \frac{M}{I_{eq}}\cdot y = 0$$

$$\frac{P}{A_{eq}} - \frac{P.e}{I_{eq}}\cdot y = 0 \Rightarrow \frac{P}{A_{eq}} = \frac{P.e}{I_{eq}}\cdot y$$

DATE:

Max. eccentricity

$$e_{max} = \frac{I_{eq}}{A_{eq} \cdot y} = \frac{1.13086 \times 10^{10}}{331302.5 \times 300}$$

$$= 113.78 \text{ mm}$$

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Value of load.

Criteria req. for uncracked Secn

$$\frac{\sigma_{cc\,cal}}{\sigma_{cc}} + \frac{\sigma_{cbc\,cal}}{\sigma_{cbc}} \leq 1$$

$$\sigma_{cc\,cal} = \frac{P}{A_{eq}} \quad \Big\| \quad \sigma_{cbc\,cal} = \frac{M}{I_{eq}} \bar{y} = \frac{P}{A_{eq}}$$

$\sigma_{cc} = 6$ $\qquad$ $\sigma_{cbc} = 8.5$

$$\frac{P/A_{eq}}{6} + \frac{M/I_{eq} \cdot y}{8.5} = 1$$

$$\frac{P/A_{eq}}{6} + \frac{P/A_{eq}}{8.5} = 1$$

$$\frac{P}{A_{eq}}\left(\frac{1}{6} + \frac{1}{8.5}\right) = 1$$

$$P = \frac{A_{eq}}{\left(\frac{1}{6} + \frac{1}{8.5}\right)} = \frac{331302.5}{\frac{1}{6} + \frac{1}{8.5}}$$

$$= 1165.271 \text{ KN}$$

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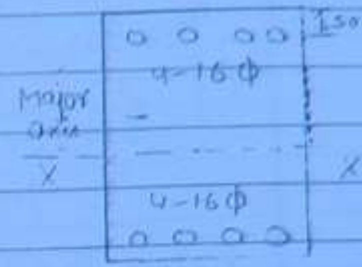
Ques Check the safety of a RCC rectangular column 250x350 mm reinf $\bar{c}$ 4-16 $\phi$ bars on each of its short face and subjected to an axial load of 450 KN & B.M of 15 kN-m acting abt its major axis
Can the column any more load if so how much?
M20 Conc, M=19 . As per code the section is safe

if $\dfrac{\sigma_{cc\,cal}}{\sigma_{cc}} + \dfrac{\sigma_{cbc,\,cal}}{\sigma_{cbc}} \leq 1$

DATE:

Sol$^n$ $P = 450$ kN & $M = 15$ kN-m




$A_{eq} = BD + (1.5m - 1) \Sigma A_{sc}$

$= 250 \times 350 + (1.5 \times 19 - 1) \times 4 \times \frac{\pi}{4} \times 16^2$

$= 131733.6 \text{ mm}^2$

$I_{eq} = \frac{BD^3}{12} + (1.5m - 1) \left\{ 2 \times \frac{\pi}{4} \times 4 \times 16^2 \times 125^2 \right\}$

$= \frac{250 \times 350^3}{12} + (1.5 \times 19 - 1) \left\{ 2 \times \frac{\pi}{4} \times 4 \times 16^2 \times 125^2 \right\}$

$= 1584.38 \times 10^6 \text{ mm}^4$

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$\sigma_{cc}\,cal = \frac{P}{A_{eq}} = \frac{450 \times 10^3}{131733.6} = 3.416 \text{ N/mm}^2$

$\sigma_{cbc}\,cal = \frac{M}{I} \cdot y = \frac{15 \times 10^6}{1584.38 \times 10^6} \times \left(\frac{350}{2}\right)$

$= 1.657 \text{ N/mm}^2$

$\sigma_{cc} = 5$ // $\sigma_{cbc} = 7$

Check $\frac{\sigma_{cc}\,cal}{\sigma_{cc}} + \frac{\sigma_{cbc},cal}{\sigma_{cbc}} = \frac{3.416}{5} + \frac{1.657}{7}$

$0.92 < 1$

Column is safe

If moment is kept same

$\frac{\sigma_{cc},cal}{\sigma_{cc}} + \frac{\sigma_{cbc},cal}{\sigma_{cbc}} = 1$

$\frac{P/131733.6}{5} + \frac{1.657}{7} = 1$

$P = 502752 \text{ N} = 502.75 \text{ kN}$

Extra load $= 502.752 - 450$

$= 52.75$ kN

If eccentricity value is kept same $e_1 = \frac{M}{P} = \frac{15 \times 10^6}{450 \times 10^3}$

$= 33.33$ mm

$\frac{\sigma_{cc},cal}{\sigma_{cc}} + \frac{\sigma_{cbc},cal}{\sigma_{cbc}} = 1$

DATE:

$$\frac{P}{A_{eq}\cdot\sigma_{cc}} + \frac{P\cdot e\cdot y}{I_{eq}\cdot\sigma_{cbc}} = 1$$

$$P\left[\frac{1}{131733.6\times 5} + \frac{33.33\times 350/2}{1584.38\times 10^{6}\times 7}\right] = 1$$

$$P = 489.21\,kN$$

Extra load $= 39.21\,kN$

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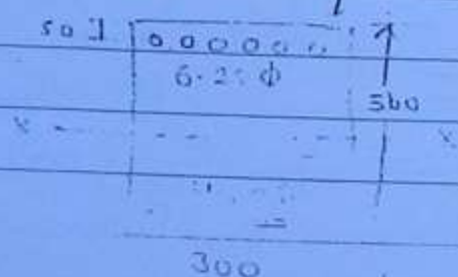
For a Column Section Shown in fig.

(i) Find out max. eccentricity value abt X-X so that No tension has developed in the Section

(ii) Find out max. load value that Can be applied On Column at eccentricity as Calculated above. Column should be safe as uncracked secn. use M-20 / Fe-415




Secn properties

$$= BD + (1.5m-1)\Sigma A_{sc}$$

$$= 300\times 500 + (1.5\times 13-1)\times\left[6\times\frac{\pi}{4}\times 25^2 + 4\times\frac{\pi}{4}\times 20^2\right]$$

$$= 227734.8\ mm^2$$

$$\frac{30\cdot D/2 + (1.5m-1)\left[A_{sc1}\,y_1 + A_{sc2}\cdot y_2\right]}{A_{eq}}$$

$$= \frac{300\times 500\times 500/2 + (1.5+13-1)\left[2945.2\times 50 + 1256.64\times 450\right]}{227734.8}$$

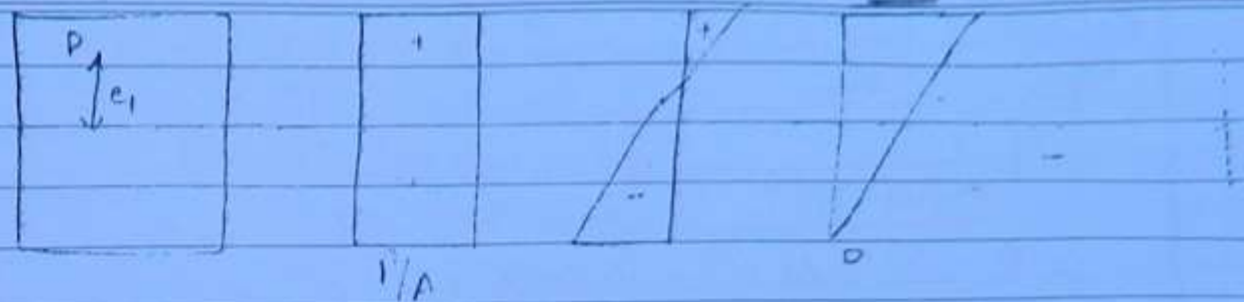
$$= 222.56\ mm$$

$$I_{eq} = \frac{BD^3}{12} + BD\left(\bar{y}-\frac{D}{2}\right)^2 + (1.5m-1)\left[A_{sc1}(\bar{y}-y_1)^2 + A_{sc2}(\bar{y}-y_2)^2\right]$$

$$= \frac{300\times 500^3}{12} + 300\times 500\,(222.56-250)^2 + (1.5\times 13-1)\left[2945.2\,(222.56-50)^2 + 1256.64\,(222.56-450)^2\right]$$

$$= 6062.9\times 10^{6}\ mm^4$$

CASE I




If e is above N.A stress at bottom = 0

$$\frac{P}{A_{eq}} - \frac{M}{I_{eq}} y_B = 0$$

$$\frac{P}{A_{eq}} = \frac{P.e_1}{I_{eq}} y_B \Rightarrow e_1 = \frac{I_{eq}}{A_{eq} \cdot y_B}$$

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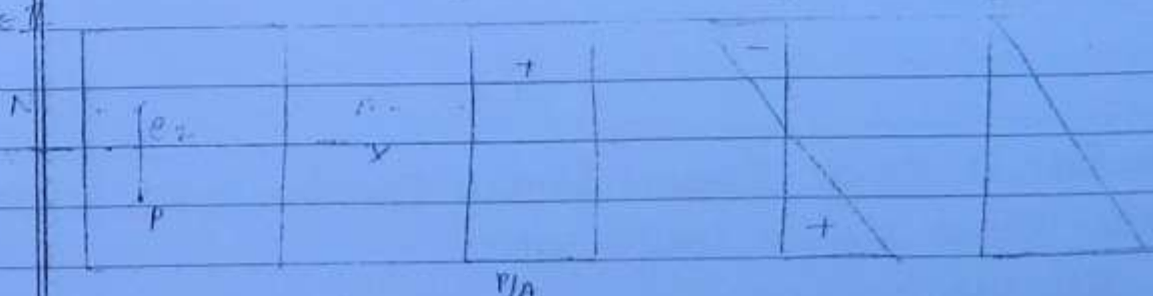
$$e_1 = \frac{6062.9 \times 10^6}{227734.78 \times (500 - 222.56)}$$

$$= 95.96 \text{ mm}$$

Eccentricity about x-x $= e_1 + \left(\frac{D}{2} - \bar{y}\right)$

$$= 95.96 + \left(\frac{500}{2} - 222.56\right) = 123.4 \text{ mm}$$

CASE II




If e is below N.A stress at top = 0

$$\frac{P}{A_{eq}} - \frac{M}{I_{eq}} y_t = 0$$

$$\frac{P}{A_{eq}} = \frac{P.e_2}{I_{eq}} \cdot y_t \rightarrow e_2 = \frac{I_{eq}}{A_{eq} \cdot y_t}$$

$$e_2 = \frac{6062.9 \times 10^6}{227734.78 \times 222.56} = 119.62 \text{ mm}$$

eccentricity abt x-x = $119.62 - (250 - 222.56) = 92.18$ mm

In first case eccentricity about x-x is max.

$e_{xx} = 123.4$ mm ; $e_{net} = 95.56$ mm

DATE:

$$\frac{\sigma_{cc\,cal}}{\sigma_{cc}} + \frac{\sigma_{cbc\,cal}}{\sigma_{cbc}} = 1 \Rightarrow \frac{P}{A_{eq}\cdot\sigma_{cc}} + \frac{P\cdot e_{net}\cdot y_t}{I_{eq}\cdot\sigma_{cbc}} = 1$$

$$P\left[\frac{1}{227734.8\times 5} + \frac{95.56\times 222.5.6}{6062.9\times 10^{6}\times 7}\right] = 1$$

$$P = 724985\,N = 724.985\,kN$$

- LARGE ECCENTRICITY CASE (Design based on Cracked Section) $e > 1.5D$

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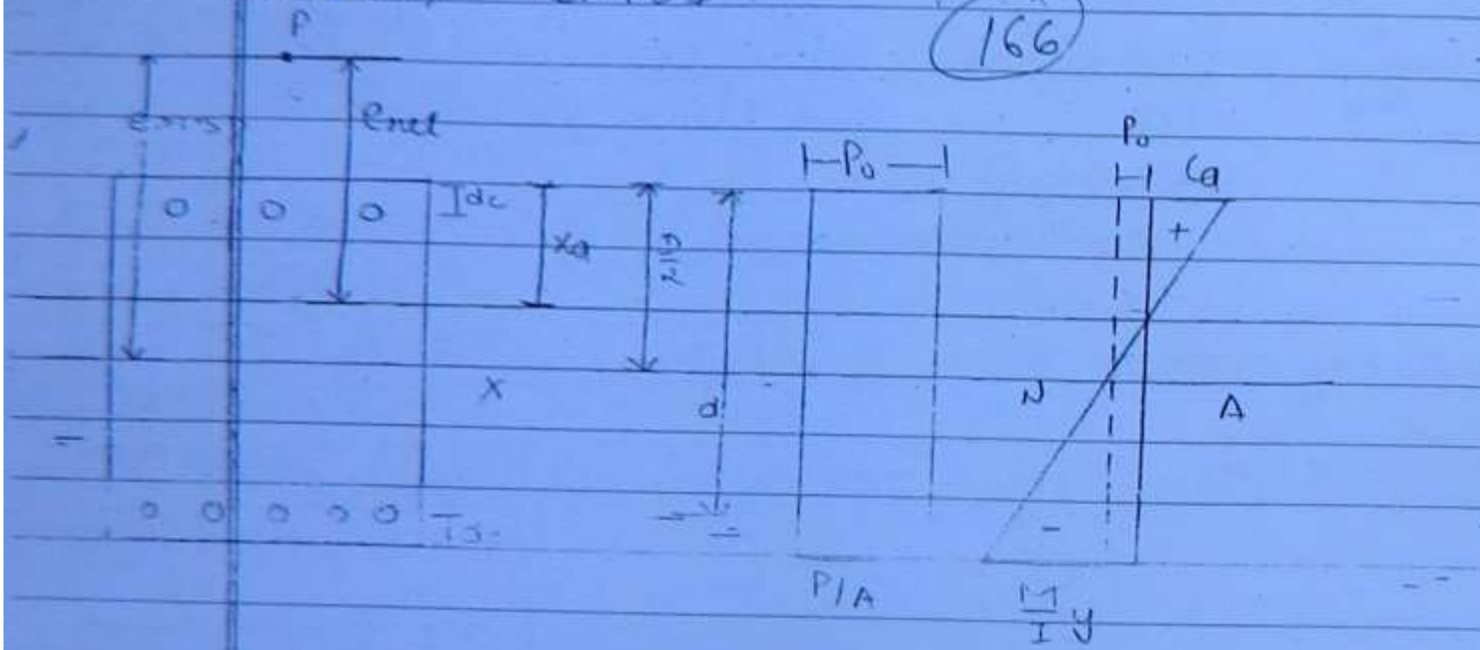


In this case a large tensile stresses are developed so area below N·A is neglected.

1. Depth of N·A

Equating moment of area on both side of N·A

$$B\cdot\frac{x_a^2}{2} + (1.5m-1)A_{sc}(x_a - d_c) = mA_{st}(d - x_a) \quad \text{---} ①$$

2. Net eccentricity

$$e_{net} = e - \left(\frac{D}{2} - x_a\right)$$

The Column is subjected to a load P and Moment $P\cdot e_{net}$

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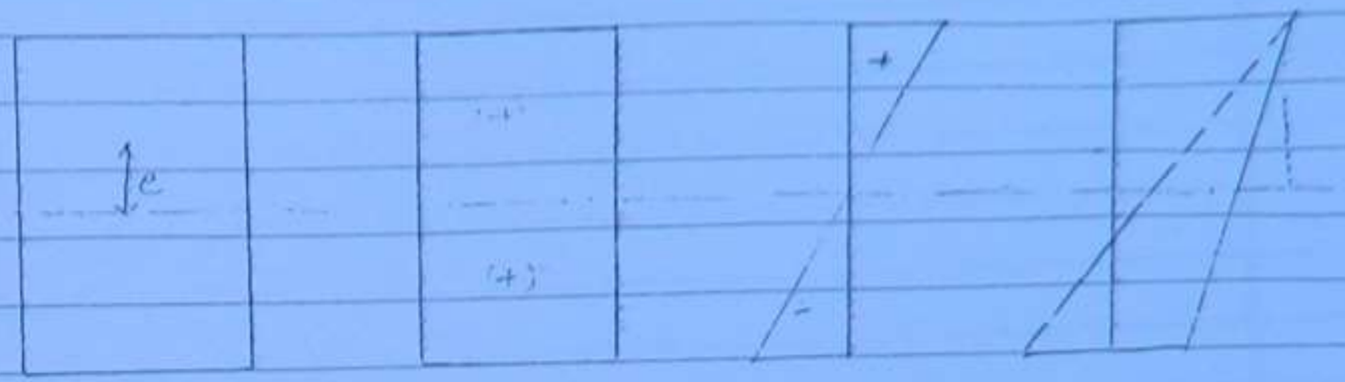

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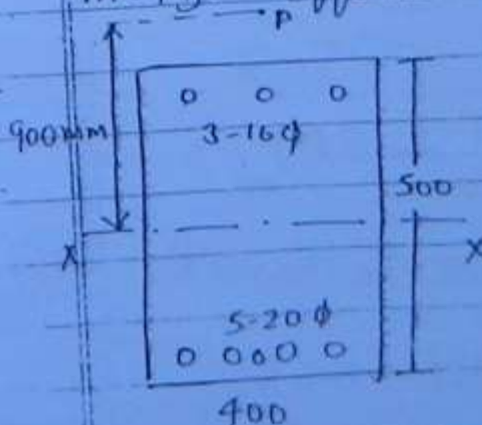
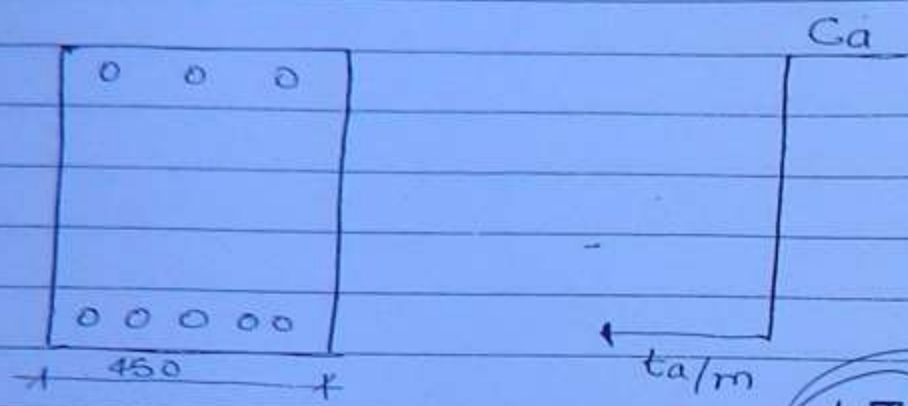
Ques A Column Sec$^n$ 400x500mm size is Subjected to a load of 80kN at an eccentricity of 900mm. use M-20/Fe-415 m = 13. Effective Cover = 30mm

Find not out actual Stress developed in the Section. Check the Safety of Section. 169



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$e = 900\,mm$

$1.5D = 1.5 \times 500 = 750\,mm$

$e > 1.5D$

This is a large eccentricity. Let us check the section as cracked section.

1. Depth of N.A.

$$B \cdot \frac{x_a^2}{2} + (1.5m - 1) A_{sc} (x_a - d_c) = m A_{st} \cdot (d - x_a)$$

$$400 \cdot \frac{x_a^2}{2} + (1.5 \times 13 - 1) \times$$

--

(2) Net eccentricity

$$e_{net} = e - \left(\frac{D}{2} - x_a\right)$$

$$= 900 - (250 - 155.5) = 805.5\,mm$$

$(> 1.5D)_{OK}$

(3). Moment

$$B.M. = P\,e_{net} = 80 \times \frac{805.5}{1000} = 64.44\ kN \cdot m$$

(4) If ~~actual~~ stress developed in concrete due to moment $P.e_{net} = C_a$

Stress in con in comp$^n$ steel $C'$

$$C' = \frac{x_a - d_c}{x_a} \times C_a = \frac{155.5 - 50}{155.5} \times C_a$$

$$= 0.678\, C_a$$

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(5) Equating B.M. = M.R

$$64.44 \times 10^{6} = B.x_a \frac{C_a}{2}\left(d - \frac{x_a}{3}\right) + (1.5m - 1) A_{sc}.c' \, (d - d_c)$$

$$= 400 \times 155.55 \times \frac{C_a}{2}\left(450 - \frac{155.5}{3}\right) + (1.5 \times 13 - 1)$$

$$\times 603.2 \times 0.678 C_a \times (450 - 50)$$

$$C_a = 4.18 \, N/mm^2$$

$$c' = 0.678 \times C_a = 2.84 \, N/mm^2$$

$$\frac{C_a}{x_a} = \frac{t_a/m}{d - x_a} \quad \Big\| \quad t_a = \frac{(d - x_a)\, m\, C_a}{x_a}$$

$$= \frac{(450 - 155.5) \times 13 \times 4.18}{155.55}$$

$$t_a = 102.91 \, N/mm^2$$

(6) Direct stresses :-

$$p_o = \frac{P}{A_{eqv.}}$$

$$= \frac{80 \times 10^3}{B \cdot x_a + (1.5m - 1) A_{sc} + m A_{st}}$$

$$= \frac{80 \times 10^3}{400 \times 155.5 + (1.5 \times 13 - 1) 603.2 + 13 \times 1570.8}$$

$$= 0.85 \text{ N/mm}^2$$

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Final stresses

(i) In concrete $= p_o + c_a = 0.85 + 4.18$
$= 5.03 \text{ N/mm}^2$

$5.03 < 7 \text{ N/mm}^2$ (OK safe)

In steel (comp.)

$= 1.5m(p_o + c') = 1.5 \times 13 \times (0.85 + 2.84) = 71.955$

$71.955 < \cancel{230}$ 190

In tensile steel

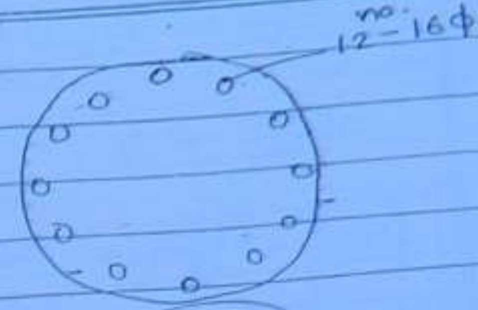
$t_a - m p_o = 102.91 - 13 \times 0.85 = 91.86 < 230$ OK

Question :- A circular column section of dia 600 mm, has 12 nos. 16 mm $\phi$ bars. It is subjected to a load of 1200 KN and B.M. of 150 KN-m M25/ Fe 415.
Check wheather column section is safe as uncracked section.

Sol$^n$ :- For the section to be safe as uncracked section

DATE:

$$\frac{\sigma_{cc}'}{\sigma_{cc}} + \frac{\sigma_{cbc}'}{\sigma_{cbc}} \leq 1$$

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(1) $\sigma_{cc}'$ for $P = 1200\,kN$

$$\sigma_{cc}' = \frac{P}{A_{eqv.}}$$

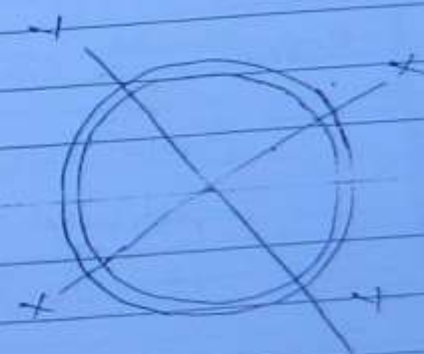
$$A_{eqv.} = \frac{\pi}{4}(D)^2 + (1.5m - 1)\Sigma A_{sc}.$$

(11).

$$= \frac{\pi}{4}(600)^2 + (1.5m - 1) \times 12 \times \frac{\pi}{4} \times (16)^2$$

$$= 320140.85$$

$$\sigma_{cc}' = \frac{1200 \times 10^{03}}{320140.85} = 3.75\ N/mm^2$$




(2) $$\sigma_{cbc}' = \frac{M}{I_{eqv.}} \times y$$

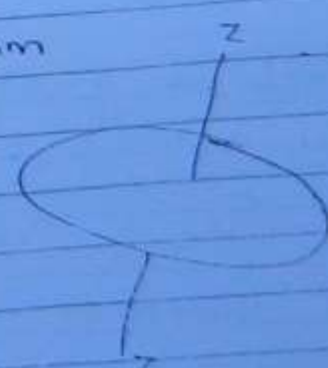
For $I_{eqv.}$

For steel

Consider steel equally distributed over a circular ring.

$$R = 300 - 50 = 250\,mm$$

M.I. about Z Z axes




$$I_{zz} = AR^2$$

$$= 18 \times \frac{\pi}{4} \times (16)^2 \times (250)^2$$

$$15079644.7 \cdot 4$$

Applying perpendicular axis theorem

$I_{zz} = I_{xx} + I_{yy}$

$I_{zz} = 2 I_D$

$I_D = \frac{I_{zz}}{2} = \frac{150796447.4}{2} = 75398223.69 \, mm^4$

Total M.I. of column about Diameter

$= \frac{\pi}{64}(D^4) + (1.5m - 1) \times I_{steel}$ 174

$= \frac{\pi}{64}(600)^4 + (1.5 \times 11 - 1) \times 75398223.69$

$= 7530.4 \times 10^{06} \, mm^4$

$\sigma_{cbc}' = \frac{M}{I_{eq}} \times y$

$= \frac{150 \times 10^{06}}{7530.4 \times 10^{06}} \times 300$

$\sigma_{cbc}' = \underline{5.97 \, N/mm^2}$

Check

$\frac{\sigma_{cc}'}{\sigma_{cc}} + \frac{\sigma_{cbc}'}{\sigma_{cbc}} = \frac{3.75}{6} + \frac{5.97}{8.5}$

$= 1.33$ failed.

Question A column of size $300 \times 400\ mm^2$ is subjected to (1) Load $P = 400\ KN$

(2) $BM_{xx} = 30\ KN\text{-}m$

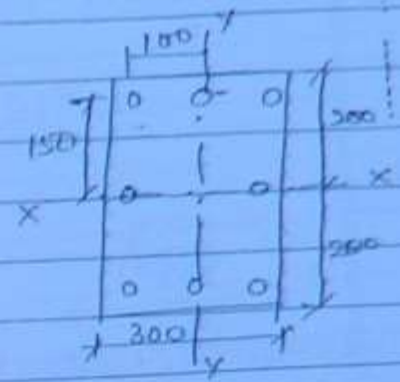
(3) $BM_{yy} = 40\ KN\text{-}m$

8 Nos. 20mm $\phi$ Bars provided

Check wheather column is

Safe as uncracked section

M-20, Fe-415.

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Solu Symmetrical section

(1) $A_{eqv.} = 300 \times 400 + (1.5 \times m - 1)\ \Sigma A_{sc}$

$= 300 \times 400 + (1.5 \times 13 - 1) \times 8 \times \frac{\pi}{4} \times (20mm)^2$

$A_{eqv} = 166495.57\ mm^2$

(2) $I_{eqv_{xx}} = \frac{BD^3}{12} + (1.5m - 1) \times \left(6 \times \frac{\pi}{4} \times (20)^2 \times 150^2\right)$

$= \frac{300 \times 400^3}{12} + 784612765.2$

$= 2384.61 \times 10^{06}\ mm^4$

(3) $I_{eqv_{yy}} = \frac{400 \times 300^3}{12} + (1.5 \times 13 - 1) \times \left(6 \times \frac{\pi}{4} \times 20^2 \times 100^2\right)$

$= 1248.7 \times 10^{06}$

(1) $p_0 = \frac{P}{A_{eqv}} = \frac{400 \times 10^{3}}{166495.6} = 2.40\ N/mm^2$

(2) $\sigma'_{cbc} = \frac{M_x}{I_{eqv_{xx}}} \times y + \frac{M_y}{I_{eqv_{yy}}} \times x$

DATE:

$$= \frac{30\times10^{06}}{2384.6\times10^{06}}\times200 + \frac{40\times10^{06}}{1248.7\times10^{6}}\times120$$

$$= 7.32\ N/mm^2$$

check.

$$\frac{\sigma_{cc}'}{\sigma_{cc}} + \frac{\sigma_{cbc}'}{\sigma_{cbc}} \le 1$$

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$$\frac{2.40}{5} + \frac{7.32}{7.0} = 1.5 > 1 \quad \text{failed}$$

## Column Design By LSM

(11) All column should be designed for min eccentricity of

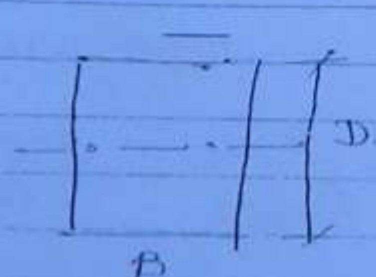
$$\left(\frac{l}{500} + \frac{D}{30}\right) \text{ or } 20\,mm \text{ whichever is less.}$$

$l$ = Unsupported length of column

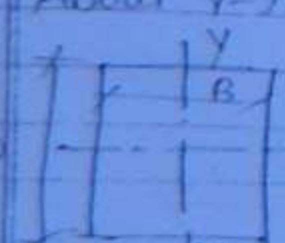
$D$ = lateral dimension.

About X-X

D = Lateral Dimension




About Y-Y




(2) B = Lateral Dimension

(II) Assumption.

All assumption for beam/ slab are allo valed for Column.

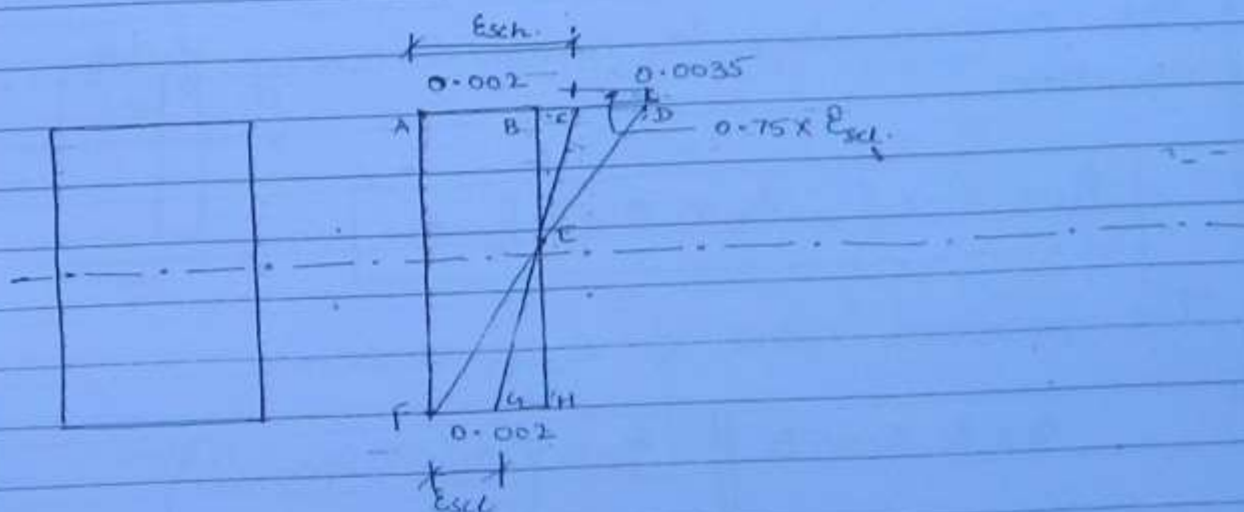
(1) Max$^m$ Comp. strain in concrete

In axial comp. = 0.002

(In bending comp. 0.0035) (177)

(2) The max$^m$ comp. strain at highly compressed fibre in concrete, subjected to axial comp. and when there is no tension in the section

= 0.0035 − 0.75 × strain at least comp fibre.





Ratio $\dfrac{BE}{EH} = \dfrac{BD}{FH} = \dfrac{0.0015}{0.002} = 0.75$

$$\frac{BE}{EH} = \frac{EC}{EG} = \frac{CD}{FG} = 0.75$$

$CD = 0.75\, FG$ या $0.75\, \varepsilon_{scl}$

So strain at highly comp. fibre

$$AC = 0.0035 - CD$$

$$\boxed{\varepsilon_{sch} = 0.0035 - 0.75 \times \varepsilon_{sce}}$$ 178

## (11) Design of Short Axially Loaded Column

→ All columns should be designed for min. eccentricity value.

except if $[e_{min} \leq 0.05D]$.

D = Lateral Dimension.

→ The column may be designed using following equation

$$\boxed{P_U = 0.4 f_{ck} \cdot A_c + 0.67 f_y \cdot A_{sc}}$$

The minimum value of $e_{min} = 20\,mm$.

$$20 \leq 0.05D \quad \Big\| \quad D \geq \left(\frac{20}{0.05} = 450\,mm\right).$$

For Circular column with Helical reinf

$$P_U = 1.05\left(0.4 f_{ck} \cdot A_c + 0.67 f_y A_{sc}\right)$$

and for helical Reinf

$$0.36 \frac{f_{ck}}{f_y}\left(\frac{A_g}{A_c} - 1\right) \leq \frac{V_h}{V_c}$$

$$f_y \ngtr 415\ N/mm^2$$

Question
ES 2002

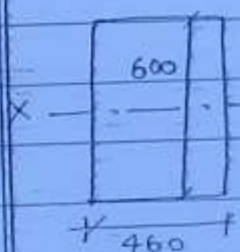
~~Design the~~
A reinforced concrete column of size 460×600 mm having effective length of 3.6m is to be designed using limit state method to support an axial service load of 2500 kN. Use M20/ Fe 415 steel.

Soln $l_{eff} = 3.60m.$ (179)

① Slenderness ratio $= \frac{l_{eff}}{B} = \frac{3600}{460} = 7.8 < 12.$

It is a short column

② Min. eccentricity about XX




Lateral dimension

$$e_{min} = \frac{l}{500} + \frac{D}{30}$$

$$= \frac{3600}{500} + \frac{600}{30} = 27$$

$> 20$ mm.

$0.05D = 0.05 \times 600 = 30$ mm.

$e_{min} < 0.05D.$

③ About Y-Y


$$e_{min} = \frac{3600}{500} + \frac{460}{30}$$

$= 22.53$ mm

$> 20$ mm

$0.05B = 0.05 \times 460 = 23$ mm

So we have $e_{min} \not> 0.05D.$

Now we can use the equation

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y \cdot A_{sc}$$

$$1.5\times 2500\times 10^3 = 0.4\times 20\times (A-A_{sc}) + 0.65\times 415\times A_{sc}$$

$$= 2208000 - 8A_{sc} + 269.75A_s$$

$A_{sc} = 5710\ mm^2$ 180

No. of 25mm $\phi$ Bar $= \dfrac{5710}{\frac{\pi}{4}\times(25)^2} = 12$ Nos

<u>Stirups</u>

① Dia $= \dfrac{\phi max}{4} = \dfrac{25}{4} = 6.25mm$

use 8mm $\phi$ Bars

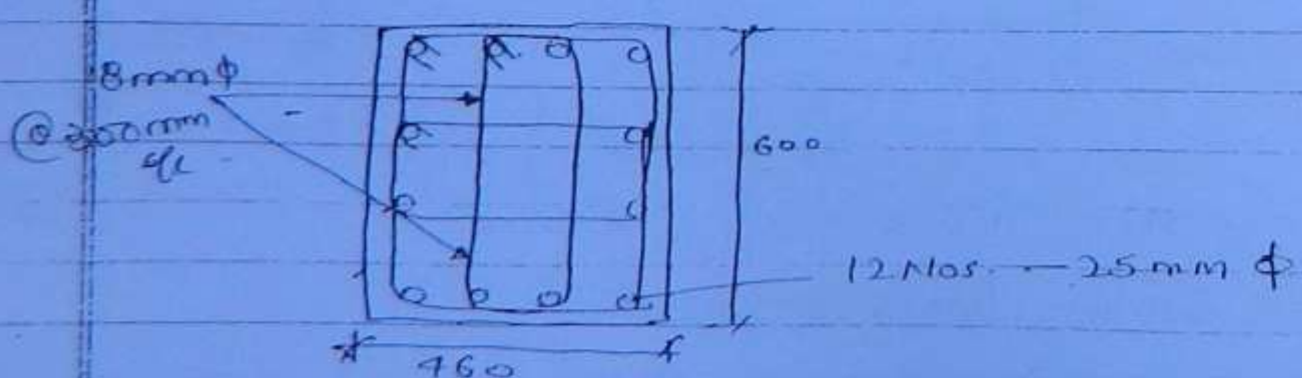
② Spacing.

1 LLD = 460mm

2 $16\phi_{main} = 16\times 25 = 400$

3. 300 mm

Provide 8mm $\phi$ @ 300mm c/c



Design Of column Subjected to axial load + Bending (uniaxial bending)
using interaction Chart

Load = $P_u$

Moment = $M_u$

Size Of Column = $B \times D$

Find Out area Of Steel

$A_{sc}$ = ?


Steps

1. Find Out $\frac{P_u}{f_{ck}BD}$ = y-value

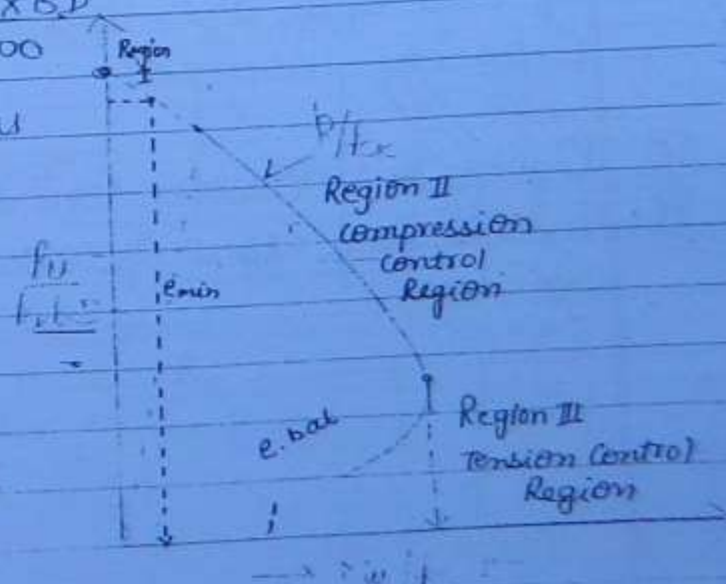
2. Find Out $\frac{M_u}{f_{ck}BD^2}$ = x-Value

Based on x & y value read $\frac{P}{f_{ck}}$ from interaction Chart

$\frac{P}{f_{ck}} = K$

$p = K \cdot f_{ck} = \frac{A_{sc}}{BD} \times 100$

Area Of Steel $A_{sc} = \frac{p \times BD}{100}$




Stirrups — Same as previous

Dia — $\frac{\Phi}{4}$ or 6mm

spacing - LLD, 16$\phi$, 300mm

For a column of size B×D

& fixed area Of Steel ($A_{sc}$)

$P/f_{ck}$ = One value, we get

One line On interaction chart

1. $P_{uz}$ = Max. load Carrying capacity of column

$P_{uz} = 0.45 f_{ck} \cdot A_c + 0.75 f_y \cdot A_{sc}$

also Called as ultimate load Carrying Capacity.

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② Region I – Min. eccentricity value or region column is subjected to less than min eccentricity value.
Effect of $P_u$ is more as compared to effect of $M_u$.

→ Column is subjected to compressive stress only.

③ Region II –

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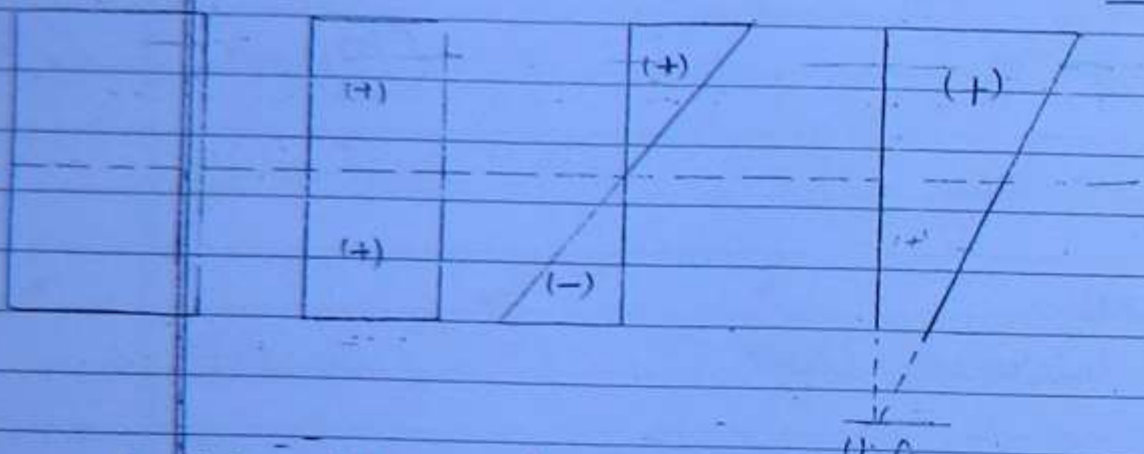


1. Entire Column Section is in Compression
2. N.A lie outside the Column Section
3. $\varepsilon_{sc} = 0.0035 - 0.75\,\varepsilon_{sc}$
4. Effect of load & moment are equally high
5. If load value are ~~considered~~ reduced moment carrying capacity increased

④ Region III – Tension Control Region

(i) In this case effect of moment is more as load values are less

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2. Tensile stresses are developed in the section
3. With reduction of load even moment carrying capacity also reduced due to this reason curve takes a invalid direction
4. Maximum strain = 0.0035
5. N.A lies within the section

(183)

Design of column subjected to combined axial load & Biaxial bending.

If column = $B \times D$ is to be designed for

1. Load $\rightarrow P_u$
2. Moment $\rightarrow M_{ux}$ abt X-X
3. Moment $\rightarrow M_{uy}$ abt Y-Y
4. $f_{ck}$ / $f_y$ are used
5. Using interaction chart - only for uniaxial bending.

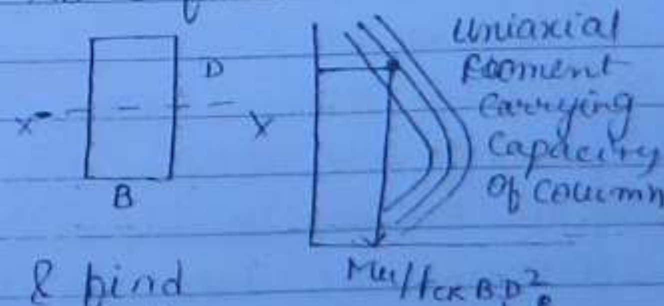
Steps (i) - Consider a percentage of steel = p%.

(ii) Find out $\frac{p}{f_{ck}}$ & $\frac{P_u}{f_{ck} B \cdot D}$ & $\frac{d'}{D}$

(About x-x)

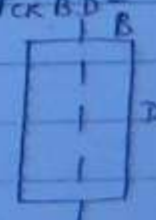
Read the appropriate chart No. & find out

$$\frac{M_u}{f_{ck} B D^2} = K_1$$




(i) $M_{ux_1} = K_1 \cdot f_{ck} B D^2$

About Y-Y - Check for $\frac{d'}{D}$ & find out value from chart $\frac{p}{f_{ck}}$ & $\frac{P_u}{f_{ck} B \cdot D}$




& read values of $\frac{M_u}{f_{ck} \cdot D B^2} = K_2$

$M_{uy_1} = K_2 f_{ck} D B^2$ - Uniaxial moment carrying capacity of column abt Y-axis

The Column Shall be Safe if

$$\left[\frac{M_{ux}}{M_{ux_1}}\right]^{\alpha_n} + \left[\frac{M_{uy}}{M_{uy_1}}\right]^{\alpha_n} \le 1.0$$

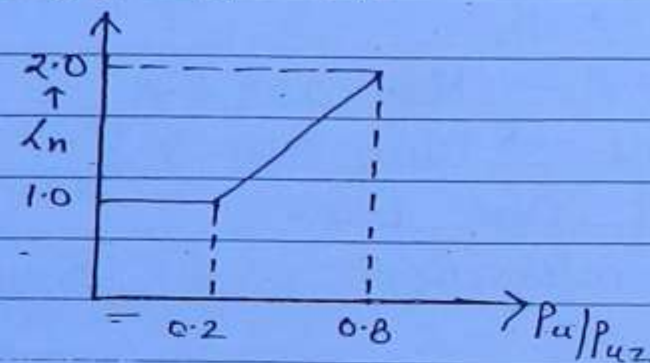
Where $\alpha_n$ = an exponent & value depends upon $P_u / P_{uz}$

$P_{uz} = 0.45 f_{ck} \cdot A_c + 0.75 f_y \cdot A_{sc}$

$\frac{P_u}{P_{uz}}$ 0.2 to 0.8 the value of $\alpha_n$ = 1.0 to 2.0

for value less than 0.2 $\alpha_n = 1.0$

for value greater than 0.8 $\alpha_n = 2.0$

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Design of biaxial long Columns

If a Column is subjected to

(i) Load $P_u$

(ii) Moment $M_{ux}$

(iii) Moment $M_{uy}$

(iv) Size of Column $B \times D$

(v) $f_{ck}$/ $f_y$ are used

(vi) To consider slenderness effect, some additional moment are added to above design moment.

$$M_{ax} = \frac{P_u D}{2000} \left\{\frac{l_{eff\,x}}{D}\right\}^2$$

$$M_{ay} = \frac{P_u b}{2000} \left\{\frac{l_{eff\,y}}{b}\right\}^2$$

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Now design the Column for

(i) $P_u$

(ii) $(M_{ux} + M_{ax})$

(iii) $(M_{uy} + M_{ay})$

Ques: Design a Column of size 400x600 for following data

Concrete - M15 & Steel - fe415

Load $P_u = 1600$ KN

$M_{ux} = 120$ KN-m ; $M_{uy} = 90$ KN-m

Reinf$^n$ are distributed equally on four sides.

Sol$^n$: Moment due to min eccentricity

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$e_{min} = 20mm$

$M_{umin} = P_u \cdot e_{min} = \frac{1600 \times 20}{1000} = 32$ KN

$M_{ux} > M_{umin}$ & $M_{uy} > M_{umin}$ Ok.

(1) Let us assume a percent of steel

$p = 1.4\%$

$\Rightarrow \frac{p}{f_{ck}} = \frac{1.4}{15} = 0.093$

$\frac{P_u}{f_{ck} BD} = \frac{1600 \times 10^3}{15 \times 400 \times 600} = 0.444$

About X-X — $\frac{d'}{D} = \frac{50}{600} = 0.083$

use 0.10 & Chart no 44. SP-16

$\frac{M_{ux}}{f_{ck} BD^2} = \frac{120 \times 10^6}{15 \times 400 \times 600^2} = 0.1 = K_1$

$M_{ux_1} = 0.1 \, f_{ck} \cdot B D^2$

$0.1 \times 15 \times 400 \times 600^2 = 216$ KN-m

About Y-Y — $\frac{d'}{B} = \frac{50}{400} =$

Chart no- 45 for value of $\frac{d'}{B} = 0.15$

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$$\frac{M_{uy}}{f_{ck} D B^2} = 0.10 = K_2$$

$M_{uxy_1} = 0.10 \times 15 \times 400^2 \times 600$

144 KN-m

$P_{u2} = 0.45\, f_{ck} A_c + 0.75\, f_{ck} A_{sc}$

$= 0.45 \times 15 \times (400 \times 600 - \frac{1.4}{100} \times 400 \times 600) + 0.75 \times 15 \times \frac{1.4}{100} \times 400 \times 600$

$P_{u2} = 2643.12$ kN

$$\frac{P_u}{P_{u2}} = \frac{1600}{2643.12} = 0.605$$

| $P_u/P_{u2}$ | $\alpha_n$ |
|---|---|
| 0.2 | 1.0 |
| 0.8 | 2.0 |

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for .605

$$\alpha_n = 1.0 + \frac{2-1}{0.8-0.2}(.605-0.2)$$

$= 1.575$

Check $\left(\frac{M_{ux}}{M_{ux_1}}\right)^{\alpha_n} + \left(\frac{M_{uy}}{M_{uy_1}}\right)^{\alpha_n} \leq 1.0$

$$\left(\frac{120}{216}\right)^{1.675} + \left(\frac{90}{144}\right)^{1.675}$$

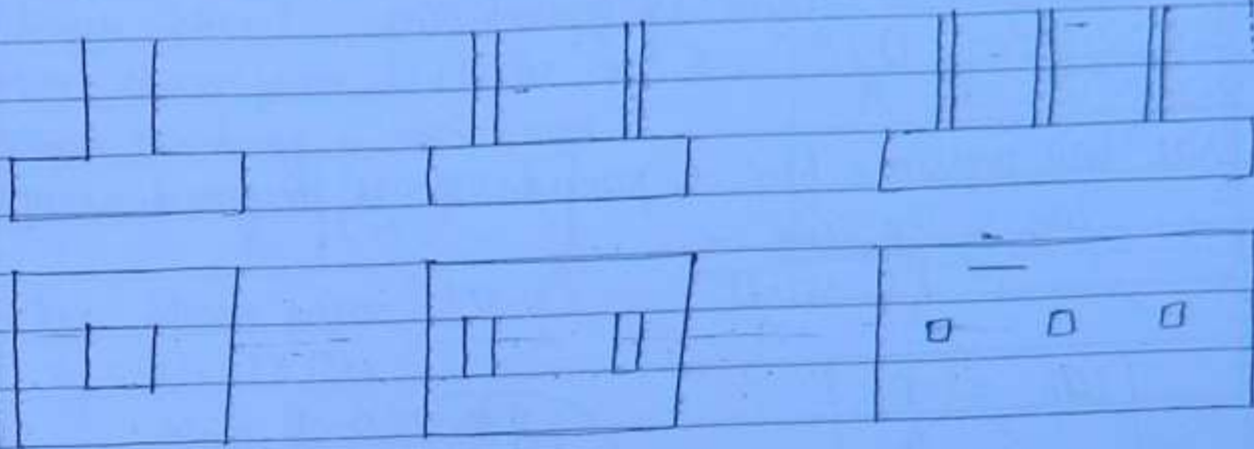
$.3764 + 0.455$

$0.83 \leq 1.0$ Safe

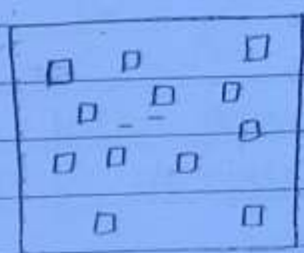
# Foundation

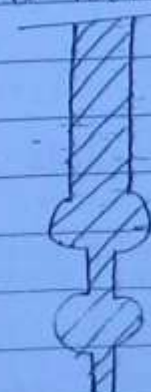
→ Types

(a) Isolated footing (imp)   (b) Combined footing   (c) Strip footing



(d) Raft   (e) Pile foundation   187





under reamed

## Isolated Footing Foundation

→ For a Rectangular footing of uniform thickness

Load from Column = $P$   $f_{ck}$ & $f_y$ are given

Size of Column = $a \times b$

Bearing Capacity of soil = $q_0$

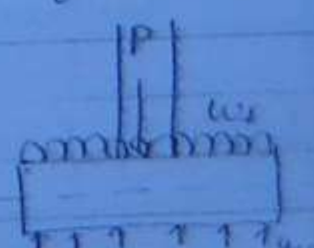
Step (1) Size of foundation

Load from Column = $P$

Weight of foundation = $0.10P$ (10% of load) (actual 15-20%)

Total load $P_T = 1.10P$

Area of foundation → $A = \frac{P_T}{q_0}$



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Find out length $L \times B$

$$A = L \times B$$

Pressure due to weight of foundation

$$w_f = \frac{0.1P}{A}$$

Net soil pressure for c foundation is designed

$$w_0 = q_0 - w_f$$

$$= \frac{P_T}{A} - \frac{0.1P}{A} \qquad \frac{P_T - 0.1P}{A}$$

$$\boxed{w_0 = \frac{P}{A}}$$

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For L.S.M it will be the factored.

$$\boxed{w_{u0} = \frac{1.5P}{A}}$$

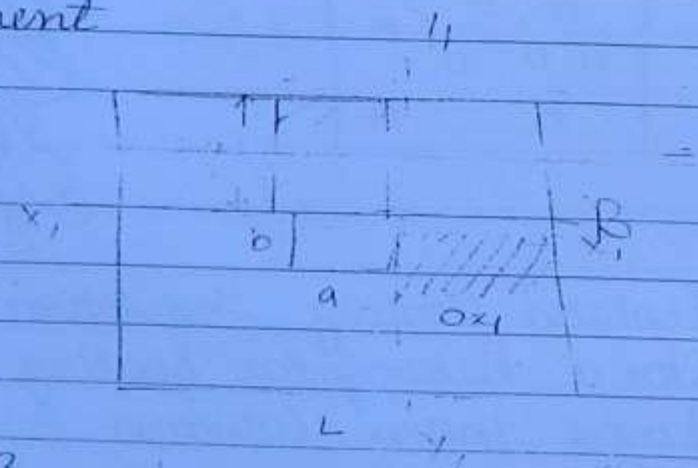
Step ② Check for Bending moment

Critical secn for B.M

→ At the face of Column

abt $X_1 - X_1$

Consider 1m width of foundation



overhang $OY_1 = \left(\frac{B-b}{2}\right)$

$$B.M \text{ at } X_1X_1 = w_0 \times 1 \times \frac{OY_1^2}{2}$$

$$= \frac{w_0}{2}\left(\frac{B-b}{2}\right)^2 = \boxed{\frac{w_0}{8}(B-b)^2}$$

$$B.M \text{ at } Y_1Y_1 = w_0 \times 1 \times \frac{OX_1^2}{2}$$

$$= \frac{w_0}{2}\left(\frac{L-a}{2}\right)^2 \qquad \boxed{\frac{w_0}{8}(L-a)^2}$$

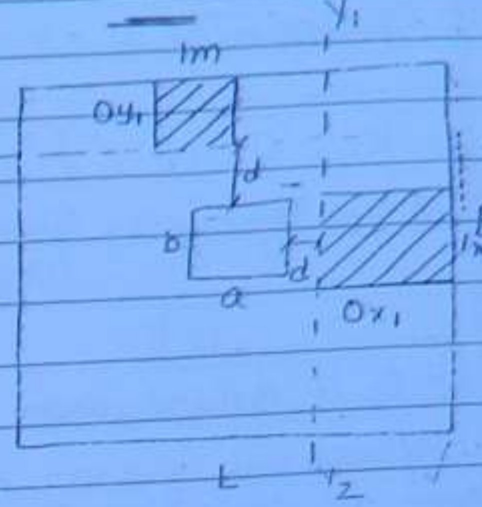
Depth of foundation $d = \sqrt{\frac{(M_x \text{ or } M_y)\text{ greater}}{Q \cdot B}}$

Step 3 Check for one way Shear

Critical Secⁿ for one way Shear (Single Shear)

Max. Shear force shall be at $Y_2$-$Y_2$ where overhang is maximum.

Overhang $Ox_1 = \left[\left(\frac{L-a}{2}\right) - d\right]$




Max. Shear force

$V_y = \omega_0 \times 1 \times Ox_1$

$\omega_0 \times \left[\left(\frac{L-a}{2}\right) - d\right]$

Nominal Shear Stress

$$\tau_v = \frac{V_y}{Bd} = \frac{V_y}{1000d} \le \tau_{c\,min}$$

$$\le K.\tau_c$$

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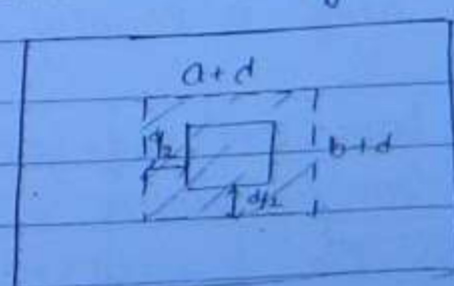
K = factor depending on Slab thickness

B.5.2.1.1 page No - 84

| overall depth of slab | 300 or more | 275 | 250 | 225 | 200 | 175 | 150 or less |
|---|---|---|---|---|---|---|---|
| K | 1.0 | 1.05 | 1.10 | 1.15 | 1.20 | 1.25 | 1.30 |

Step-4 Check for punching Shear failure or two way Shear




→ Net punching force.

$= P - \omega_0 \times (a+d) \times (b+d)$

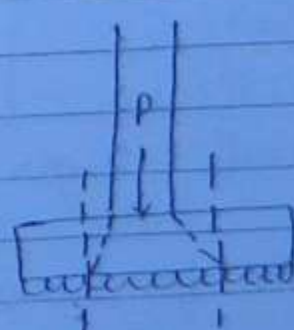
Net punching Shear Stress developed

$$= \frac{\text{Net punching force}}{\text{Resisting area}}$$

$$= \frac{P_0 - \omega_0 (a+b)(b+d)}{\text{Perimeter} \times \text{depth}}$$

$$= \frac{P - \omega_0 (a+d)(b+d)}{2\left[(a+d)+(b+d)\right] \times d}$$

$\le \tau_{vp}$ (permissible)



·12 % HYSD Bar

Cover = 50mm

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pg NO. 59 $\tau_{vp}$ (permissible) $= 0.25\sqrt{f_{ck}} \times k_s \rightarrow$ L.S.M

$= 0.16\sqrt{f_{ck}} \times k_s$ W.S.M

$k_s = 0.5 + \beta_c \not> 1$

$\beta_c = \frac{b}{a} = \frac{\text{Short side}}{\text{long side}}$

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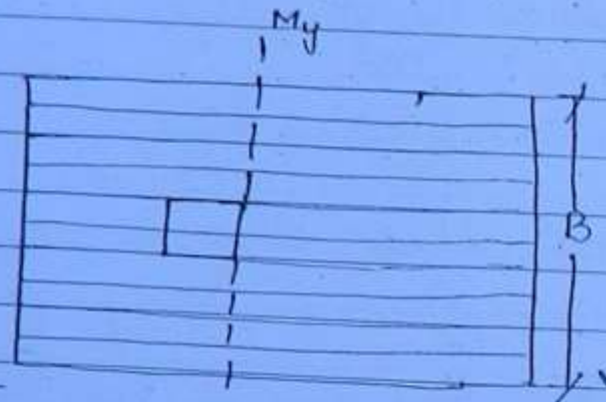
Step 5 Depth is decided, max. value required from above three criteria.

# Area of Steel

(a) Longer bars are req. for My. For 1m width




$A_{st} = \frac{M_y}{\sigma_{st} \cdot j \cdot d}$ — W.S.M

$= \frac{M_y}{0.87 f_y \cdot j \cdot d}$ — L.S.M

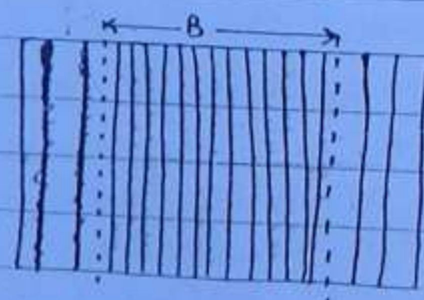
$= \frac{0.5 f_{ck}}{f_y}\left[1 - \sqrt{1 - \frac{4.6 M_{uy}}{f_{ck} \cdot B d^2}}\right] Bd$ — L.S.M

So for total B width area of steel = B·Ast

or No. of bars = B × (No. of bars req. for Ast)

This reinf$^n$ is equally distributed over 'B' width.

(b) Short bars are req. for Mx. For 1m width




$A_{st} = \frac{M_x}{\sigma_{st} \cdot j \cdot d}$ — W.S.M

$\sigma_{st} = 0.87 f_y$ — L.S.M

So for total L length area of steel = L·Ast

No. for bars for 1m width $n_1 = \frac{A_{st}}{\frac{\pi}{4}\phi^2}$

No. for bars for L width $n_T = L \times n_1$

NO. Of bars provided for in Central Band of width B is

$$n_c = n_T\left(\frac{2}{1+L/B}\right)$$

Remaining bars are are distributed equally on two side bands.

(191)

ES-1993

**Ques-** A rectangular RC column of size 60X40 cm carries a load of 80 tonnes. Design a rectangular footing use M-15 / fc-250

$q_0 = 20\,t/m^2$.

$a = 600mm$

$b = 400mm$

$80T = \frac{80\times1000\times10}{1000} = 800\,KN$

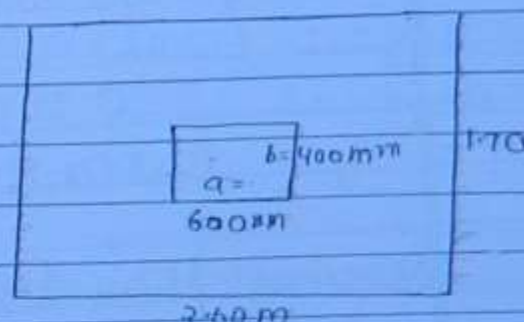
$P = 800\,KN$

$q_0 = 200\,KN/m^2$ $(1.10\times P)$

Total load $P_T = 880\,KN$

Area of foundation $= \frac{880\times1.5}{200\;1.5}$

$= 4.4\,m^2$




$\frac{L}{B} = \frac{a}{b} = 1.5$

Size of foundation $(L\times B)$ $= 1.5B^2 = 4.4$

$B = 1.71\,m$ & $L = 2.57\,m$

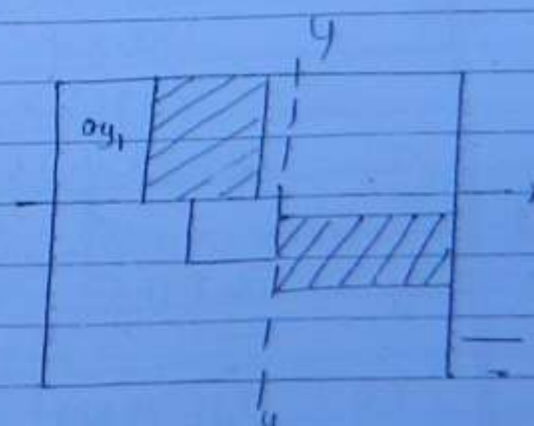
$B = 1.70\,m$

$L = 2.60$

$A = 4.42\,m^2$

$W_0 = \frac{1.5P}{A}$ (Net soil pressure)

$= \frac{1.5\times800}{4.42} = 271.5\,KN/m^2$




Check for B.M

At the face of Column abt $X_1$-$X_1$

$Oy_1 = \frac{1.70-.4}{2} = 0.65$

B.M at x-x $= 271.5\times\frac{0.65^2}{2}$

$57.35\,KN\cdot m$

B.M at y-y $= \frac{271.5}{2}\times\left(\frac{2.60-.6}{2}\right)^2$

$= 135.75\,KN\text{-}m$

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Depth of foundation $d = \sqrt{\dfrac{M_y}{Q \cdot B}}$

$$d = \sqrt{\frac{135.75 \times 10^6}{2.22 \times 1000}} = 247.29 \text{ mm}$$

say = 250 mm

$$Q = 0.36 f_{ck} \frac{x_{u\,lim}}{d}\left(1 - 0.42 \frac{x_{u\,lim}}{d}\right)$$

$$= 0.36 \times 15 \times 0.53 \left(1 - 0.42 \times 0.53\right) = 2.22$$

Check for One way Shear (192)

at Critical Secn Max. S.F shall be at Y-Y where overhang is max

$$Ox_1 = \left[\left(\frac{2.60 - 0.6}{2}\right) - .250\right]$$

$$= 0.75 \text{ m}$$

$$V_y = w_0 \times Ox_1 \times 1 = 271.5 \times 0.75$$

$$= 203.625 \text{ KN}$$

Nominal Shear Stress

$$\tau_v = \frac{V_y}{Bd} = \frac{203.625 \times 10^3}{1000 \times 250} = 0.8145 > \tau_{min}$$

$\boxed{\text{So fail}}$

Depth reqn

$$0.28 = \frac{203.625 \times 10^3}{1000 \times d}$$

$$= 727.23 \quad \text{say} =$$

$$d = \frac{727.23 + 250}{2} = 488 = \text{say } 500$$

$$V_y = w_0 \times 0.5 = 271.5 \times 0.75$$

$$= 135.75 \text{ KN}$$

$$\tau_v = \frac{135.75 \times 10^3}{1000 \times 500} = 0.2715 < 0.28 \ (\tau_{c\,min})$$

Hence safe

Check for two way shear or punching

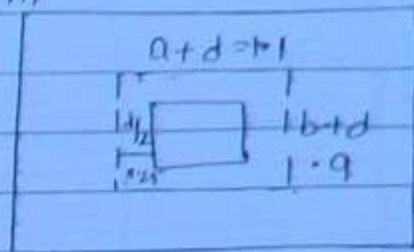
Net punching force $= P - w_0 (a+d)(b+d)$

$$1.5\times10^3$$

$$= 800 - 271.5(.6+.5)(.4+.5)\times10^3$$

$$= 931215\ N/m^2$$

Resisting area




$$= 2[(a+d)+(b+d)]\times d$$

$$= 2[1.1 + .9]\times .5$$

$$= \cancel{0.99\ m^2} = 2\times10^6$$

$$= 990\times10^6\ mm^2$$

$\tau_{vp}$ (developed) $= 0.465\ N/mm^2$

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$\tau_{vp}$(permissible) $= 0.25\sqrt{f_{ck}}\times K_s$

$$K_s = \left(0.5+\frac{b}{a}\right) = \left(0.5+\frac{400}{600}\right) = 1.67 > 1.0$$

So take 1.0

$\tau_{vp}$ (permissible) $= 0.25\sqrt{15}\times1.0$

$= 0.968\ N/mm^2$

$> \tau_{vp}$ (developed) – Safe

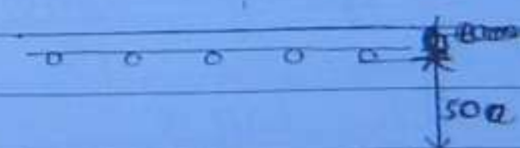
Depth required = 500 mm

Eff cover + 80 mm

Total depth = 580 mm

Eff cover = 50+16+8

= 74mm




Area of Steel

longer bar ($M_y$)

$$A_{st} = \frac{0.5\times15}{250}\left[1-\sqrt{1-\frac{4.6\times135.75\times10^6}{15\times1000\times500^2}}\right]1000\times500$$

$= 1305.75\ mm^2$

Use 16mm$\phi$ bars. No. of bars

$$= \frac{1305}{\pi/4\ 16^2} = 6.49\ \text{No}$$

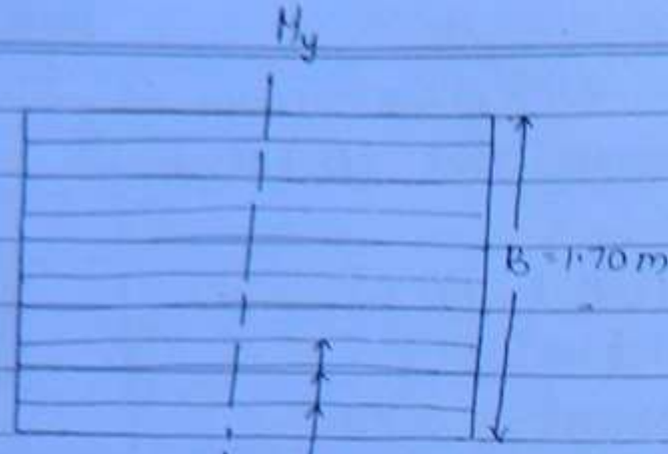
Total bars req. for B width $n_1 = 6.49\times1.7$

$= 11.03$

Say 12 Nos. of 16mm $\phi$ bars

Spacing = $\frac{1700}{12}$

= 141 mm




194

Shorter Side

$A_{st} = \frac{0.5\times15}{250}\left[1-\sqrt{1-\frac{4.6\times57.35\times10^6}{15\times1000\times500}}\right]1000\times500$

$537.24\ mm^2$

No. Of bars 10mmφ = $\frac{537.24}{\frac{\pi}{4}\times10^2} = 6.84$

Total NO = $2.6\times6.84 = 17.784$

say 18 Nos Of 10mmφ bars

Spacing = $\frac{2600}{18}$ = 144.44 mm

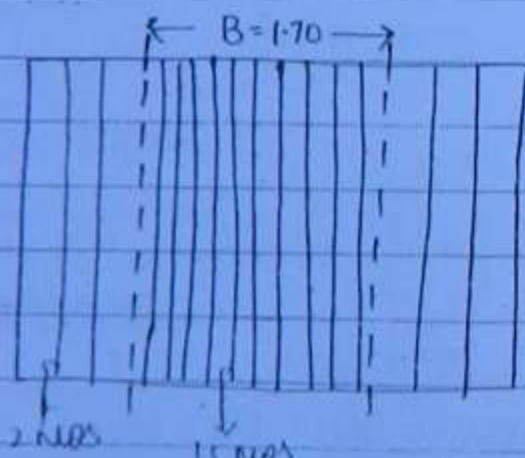
central band width

$n_c = 14.22$ say 15

Spacing = $\frac{1700}{15} = 113$

No. Of bars in side band

= $\frac{18-15}{2} = 1.5 = 2$ Nos.



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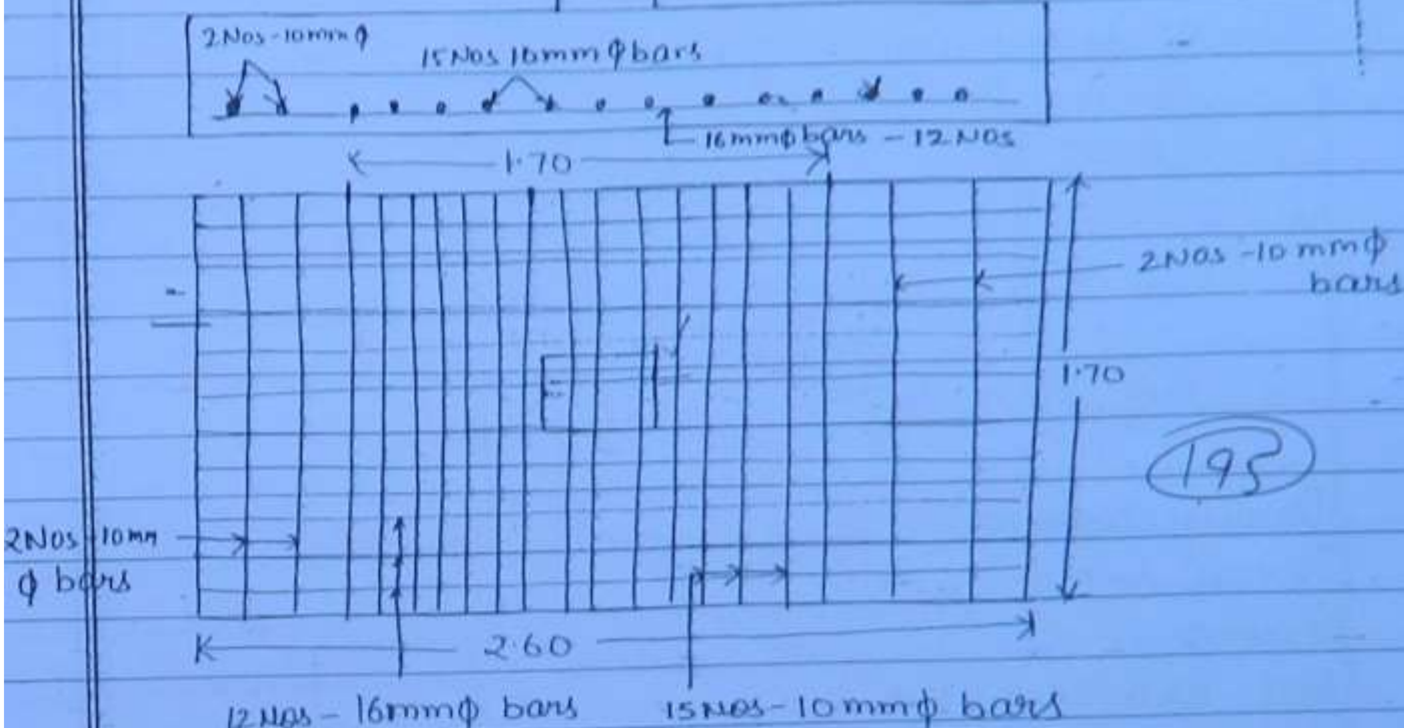


2 Nos - 10mm ϕ
15 Nos 16mm ϕ bars
16mm ϕ bars - 12 Nos
1·70
2 Nos - 10 mm ϕ bars
1·70
2 Nos 10mm ϕ bars
2·60
12 Nos - 16mm ϕ bars
15 Nos - 10 mm ϕ bars

DATE:

# Sloped Isolated footing

1. Size of foundation - Same

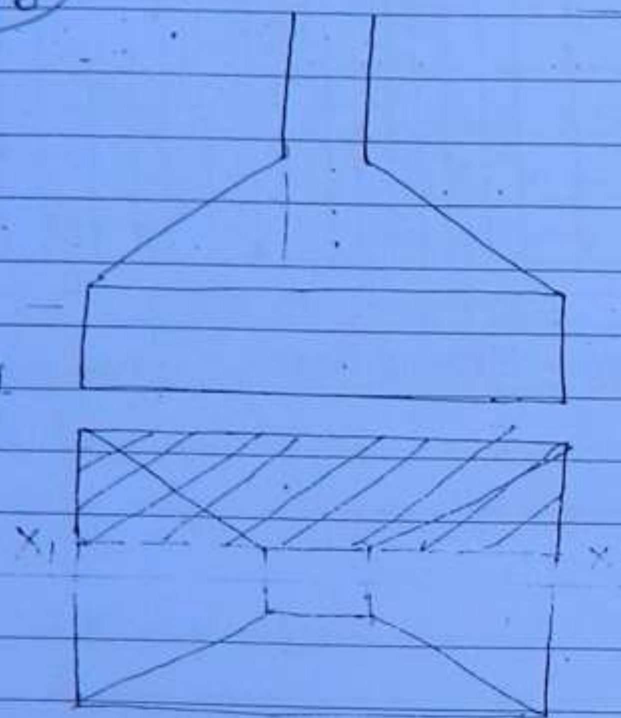
$$P + 0.10P = P_T$$

$$A = \frac{P_T}{q_0}$$

$$w_0 = \frac{P}{A}$$

196

$$B = \sqrt{A}$$

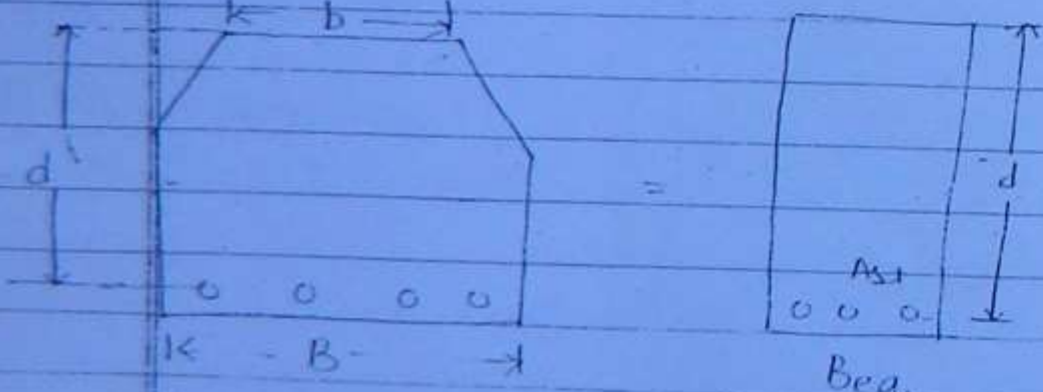



Bending moment

Critical secⁿ at face of Column

$$B.M = w_0 B \times \left(\frac{B-b}{2}\right)\left(\frac{B-b}{4}\right)$$

$$= w_0 B \times \frac{(B-b)^2}{8}$$




Width of equivalent secⁿ

$$B_{eq} = b + \frac{1}{8}(B-b)$$

Depth of secⁿ req.

$$d = \sqrt{\frac{BM}{Q \cdot B_{eq}}}$$

Check for one way shear

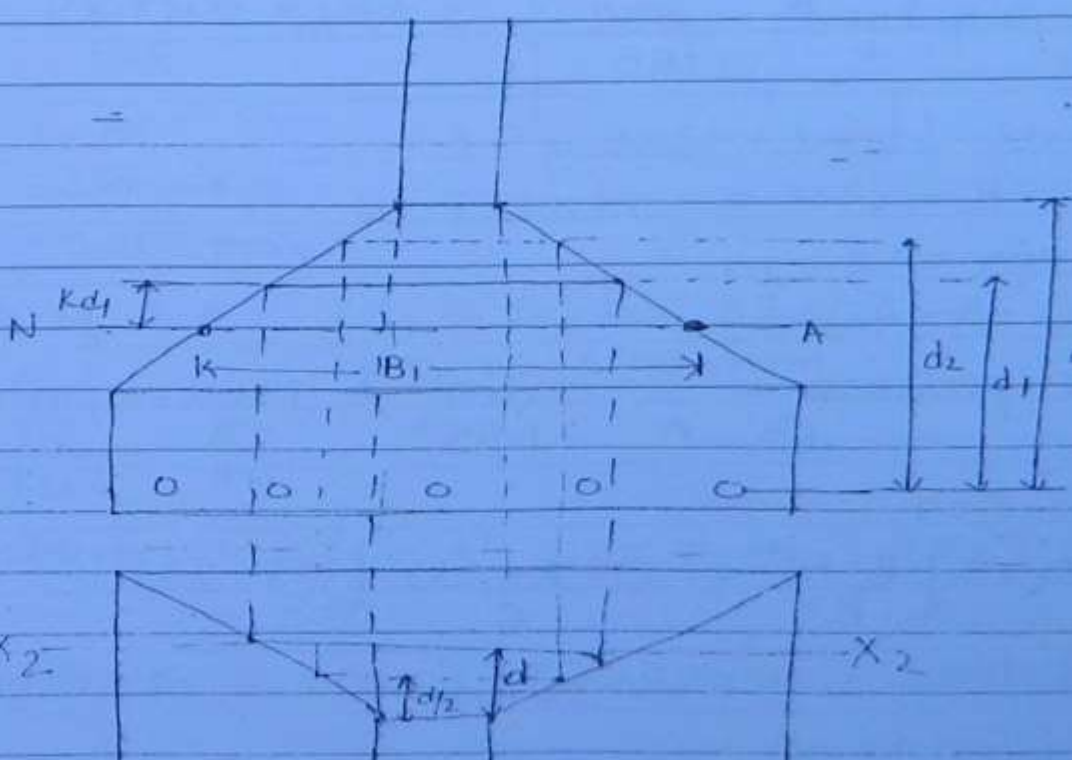
$$V = w_0 \times B \times \left[ \frac{B-b}{2} - d \right]$$

Nominal shear stress $\tau_v = \frac{V}{B_1 d_1}$ (197)

here

$B_1$ = width of footing at critical secⁿ at N.A

$d_1$ = depth of footing at critical secⁿ.



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Punching Shear

Punching Shear force $= P - \omega_0 (a+d)(b+d)$

Punching Shear Stress $= \dfrac{P - \omega (a+d)(b+d)}{2[(a+d)+(b+d)] \times d_2} \leq \bar{\tau}_{cp\,permissible}$

Area of Steel – Same.

2001

Ques Design a Sloped Square footing for RC Column, 300x300 mm Size, Carrying an axial load of 320 kN. The Safe bearing Capacity of Soil is 150 kN/sqm. Use M-20 / Fe-415 Use W.S.M

198

1. Size of foundation

Load from Column = 320 kN

foundation weight = 32 kN

352 kN

Area required $= \dfrac{352}{150} = 2.35\ m^2$

Size of Square footing

$B = \sqrt{2.35} = 1.55 \times 1.55$

Area = 2.4 m

B = 1550 mm

Net Soil pressure $\omega_0 = \dfrac{P}{A} = \dfrac{320}{1.55^2} = 133.2\ kN/m^2$

Check for B.M

$B.M = \omega_0 \times B \dfrac{(B-b)^2}{8} = \dfrac{133.2 \times 1.55}{8} (1.55 - 0.3)^2$

$= 40.32$ kN-m

$B_{eq} = b + \dfrac{1}{8}(B-b)$

$= 300 + \dfrac{1}{8}(1550 - 300) = 456.25$ mm

Depth required $d = \sqrt{\dfrac{B.M}{Q \cdot B_{eq}}}$

$$d = \sqrt{\frac{40.32 \times 10^6}{.898 \times 456.25}}$$

$$K = \frac{mc}{t+mc} = \frac{13 \times 7}{230+13\times7} = 0.2835$$

$d = 313.7$ mm

$j = 1 - K/3 = 0.9055$

Say $d = 350$ mm

$Q = \frac{1}{2} c \cdot j \cdot k = \frac{1}{2} \times 7 \times .9055 \times .2835$

$Q = .898$

Check for one way shear

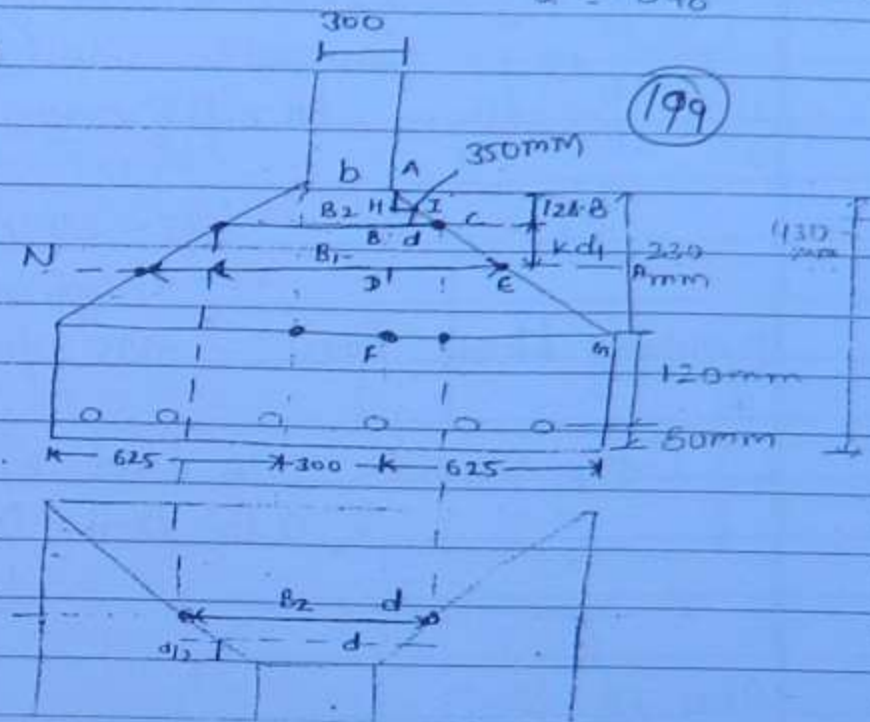
Total shear force

$= w_0 \cdot B \cdot \left[\frac{B-b}{2} - d\right]$

$= 133.2 \times 1.55 \left[\frac{1.55 - .3}{2} - .35\right]$

$= 56.78$ kN

(199)




$\frac{AB}{350} = \frac{AF = 230}{625}$

$AB = \frac{230 \times 350}{625} = 128.8$

$d_1 = d - AB$

$= 350 - 128.8$

$= 221.2$ mm

Depth of N.A $\quad Kd_1 = 0.2835 \times 221.2$

$BD = 62.71$ mm

$\frac{DE}{FG} = \frac{AD}{AF}$

$DE = \frac{128.8 + 62.71}{230} \times 625 = 520.4$ mm

Width $B_1 = 30 + 2 \times 520.4$ mm $= 1340.8$ mm

Check for single shear

$\tau_v = \frac{V}{B_1 d_1} = \frac{56.78 \times 10^3}{1340.8 \times 221.2 \text{mm}} = 0.19 \leq \tau_{c\,min} = 0.19$

(safe)

DATE:

Check for two way Shear (Punching Shear)

$$\text{Punching Shear Stress} = \frac{[320 - 133.2\,(.3+.35)(.3+.35)] \times 10^3}{10^6 \times 2\,[(.3+.35)+(.3+.35)]\ .2856]}$$

$HI = \frac{d}{2} = \frac{350}{2} = 175mm$

(200)

$\frac{AH}{AF} = \frac{HI}{FG} = \quad AH = \frac{175}{625} \times 230 = 64.4\,mm$

$d_2 = d - AH = 350 - 64.4mm$
$= 285.6mm$

Punching Shear Stress $= .355\ N/mm^2 < \tau_{vp}$ permiss

$\tau_{vp}$ permissible $= 0.16\sqrt{f_{ck}} \cdot k_1$
$= 0.16\sqrt{20} = 0.7155$

Safe

Area Of Steel

$A_{st} = \frac{40.32 \times 10^6}{230 \times .9055 \times 350} = \frac{B.M.}{\sigma_{st} \cdot j \cdot d}$

$= 553.14\ mm^2$

No. Of 10mm $\phi$ bars $= \frac{553.14}{\pi/4 \times 10^2} = 7.04$

Provide 8 Nos 10mm $\phi$ bars.

Circular footing

Ques. Design a circular Column Of 500mm dia having a load Of 1200 KN. Bearing Capacity Of Soil is 150 $KN/m^2$ Use M-25/ Fe 415 Steel. Use I.S.M

Sol$^n$ Size Of foundation

Load from Column = 1200 KN

Weight Of foundation 120 KN

$P_T$ = 1320 KN

Area required $= \dfrac{1320}{150} = 8.8\ m^2$

Dia of footing $D = \sqrt{\dfrac{4A}{\pi}} = \sqrt{\dfrac{4 \times 8.8}{\pi}} = 3.35\ m$

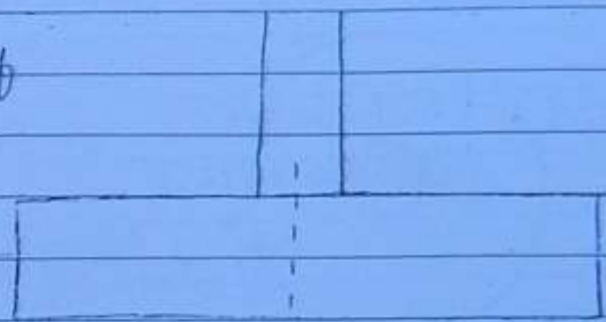
Provide $D = 3400\ mm$ 201

Radius $= 1700\ mm = 1.70$

Net Soil pressure $= \underset{W_u}{} \dfrac{1200 \times 1.5}{\pi \times 1.7^2} =$ ~~139~~ $198.26\ KN/m^2$

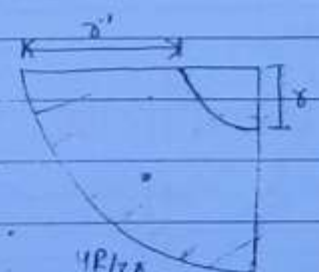
Check for B.M

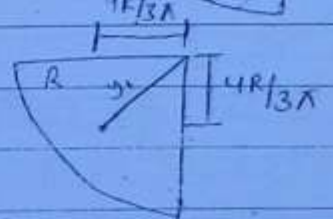
Critical Section is at face of Column



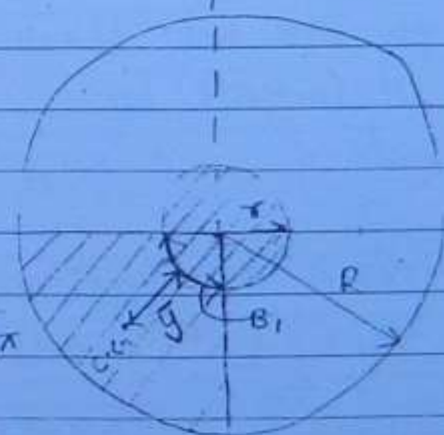
$B.M = W_u \times \left( \dfrac{\pi R^2 - \pi r^2}{4} \right) \bar{y}$

$\bar{y}$ value








Distance of C.G from Centre

$$y_1 = \sqrt{2} \times \frac{4R}{3\pi} = 0.6R$$

Distance of C.G of Shaded portion from Centre

$$= \frac{A_1 y_1 - A_2 y_2}{A_1 - A_2} = \frac{\frac{\pi R^2}{4} \times 0.6R - \frac{\pi r^2}{4} \times 0.6r}{\frac{\pi}{4}R^2 - \frac{\pi}{4}r^2}$$

$$= \frac{0.6(R^3 - r^3)}{(R^2 - r^2)}$$

Distance of C.G from face of column

$$\bar{y} = \frac{0.6(R^3 - r^3)}{R^2 - r^2} - r$$

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$$\boxed{B.M = w_u\left(\frac{\pi R^2 - \pi r^2}{4}\right)\left[\frac{0.6(R^3 - r^3)}{R^2 - r^2} - r\right]}$$

$$= 198.26 \times \frac{\pi}{4}(1.70^2 - 0.25^2) \times \left[\frac{0.6(1.70^3 - 0.25^3)}{1.70^2 - 0.25^2} - 0.25\right]$$

$$= 347.46 \text{ kN-m}$$

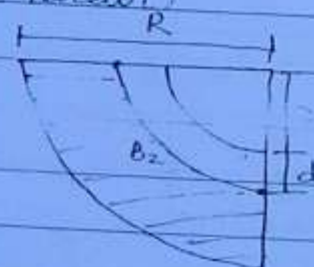
Depth req. $= \sqrt{\dfrac{BM}{Q \cdot B_1}}$ 202

Width $B_1 = \dfrac{2\pi r}{4} = \dfrac{2 \times \pi \times 0.25}{4} = 392.7\text{ mm}$

$$d = \sqrt{\frac{347.46 \times 10^6}{.138 \times 25 \times 392.7}} = 506.42\text{ mm}$$

Say $= 510\text{ mm} = d$

Check for one way Shear - (Single Shear)
Width at Critical Secn




$B_2 = \dfrac{2\pi(r+d)}{4} = \dfrac{\pi(\ldots)}{2}$

$= 1193.8$ mm ~~596.9 mm~~

$$V_u = W_u \times \frac{\pi}{4}\left[R^2 - (r+d)^2\right] \times B_2$$

$$= 198.25 \times \frac{\pi}{4}\left[1.70^2 - (.25 + 0.51)^2\right]$$

$= 360.25$ kN

$\tau_{vu} = \dfrac{V_u}{B_1 \cdot d} = \dfrac{360.25 \times 10^3}{1193.8 \times 510}$

$0.59 > 0.29$ fail

Depth req.

$d = \dfrac{V_u}{\tau_{c\,min} B_1} = \dfrac{360.25 \times 10^3}{0.29 \times 1193.8} = 1040.6$ mm

Sa d $= 1040.6 + \ldots$ $= 1115 \ldots$ Say 1100 mm

$V_u = 198.25 \times \frac{\pi}{4}\left[1.70^2 - (.25 + .8)^2\right]$

$= 278.32$ kN

$B_2 = \frac{\pi(250 + 800)}{2} = 1649.34$ mm

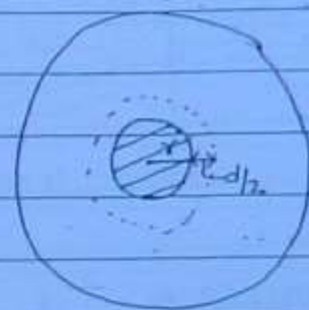
$\tau_{vu} = \frac{278.32 \times 10^3}{1649.34 \times 800} = .21 < 0.29$ (safe)

203

Check for two way Shear (Punching Shear)

Net punching Shear Stress

$\frac{P_u - W_u \pi\left(r + \frac{d}{2}\right)^2}{2\pi\left(r + \frac{d}{2}\right) \times d}$




$\frac{1.5 \times 1200 \times 10^3 - 198.25 \times \pi\left(.25 + \frac{.8}{2}\right)^2 \times 10^3}{2 \times \pi\left(250 + \frac{800}{2}\right) 800}$

$= 0.47$ N/mm²

permissible $\tau_{vp} = 0.25\sqrt{f_{ck}} = 0.25\sqrt{25}$

$= 1.25$ N/mm²

Safe

Area Of Steel

$A_{st} = \frac{0.5 \times 25}{415}\left[1 - \sqrt{1 - \frac{4.6 \times 347.46 \times 10^6}{25 \times 392.7 \times 800^2}}\right] 392.7 \times 800$

$B_1$

$= 1291.7$ mm²

No. of 12 mm $\phi$ bars $= \frac{1291.7 \times 4}{\pi \times 10^2} = 11.42$

Say 12 Nos.

provide 12 Nos of 12mm $\phi$ bars.

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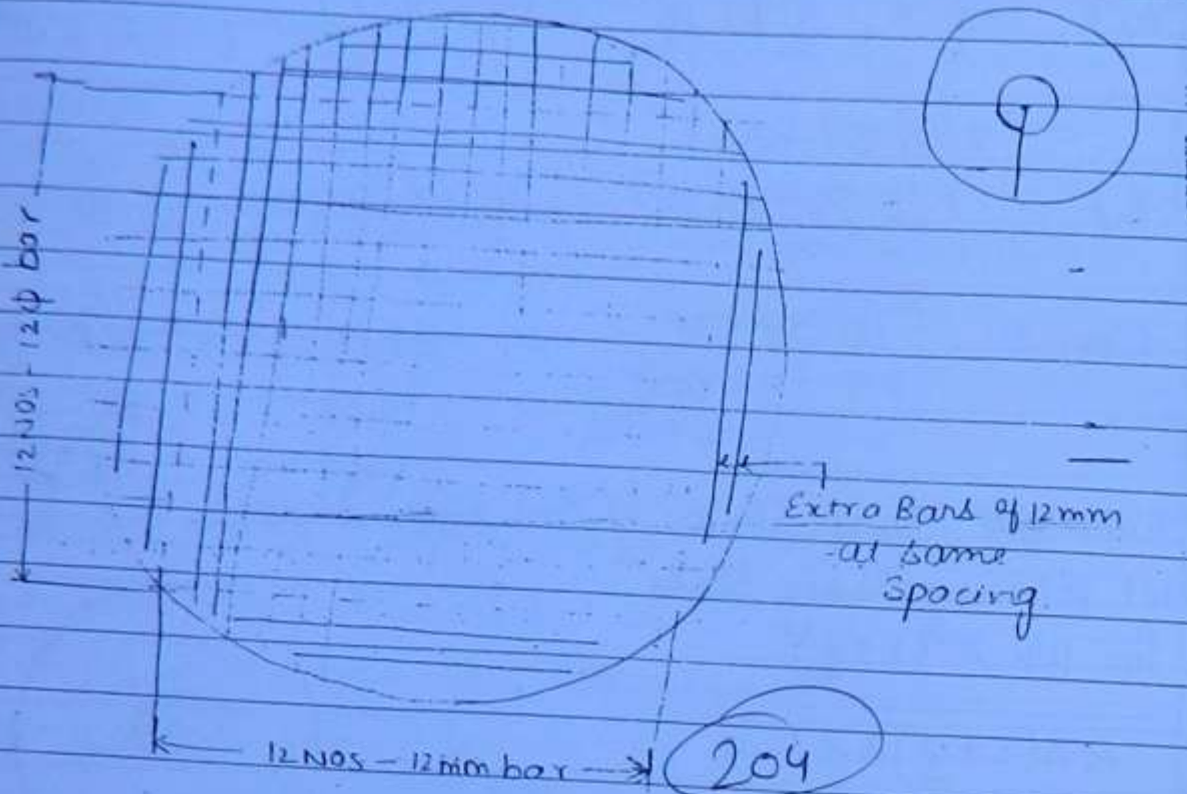


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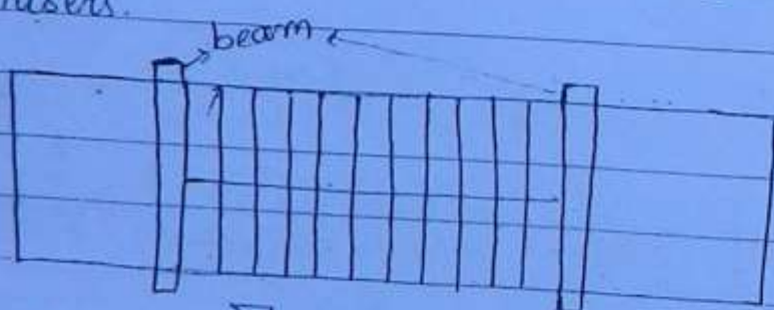
31|10|2011

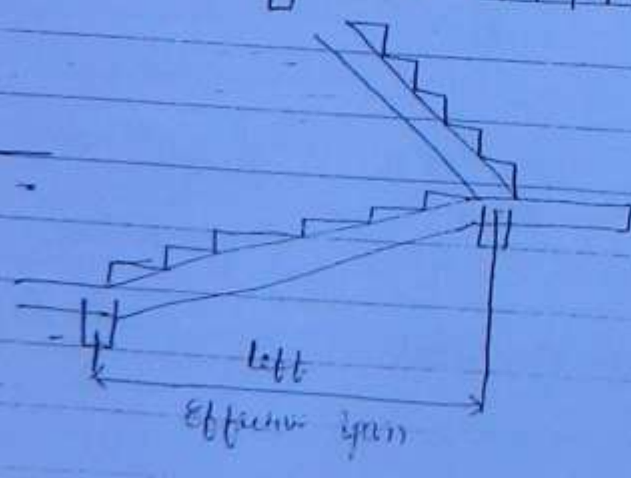
# Staircase :-

IS Code provision :- (As per 33·1 | P-63)

Effective Span of Staircase

(a) When Supported at top and bottom risers spanning parallel with risers.



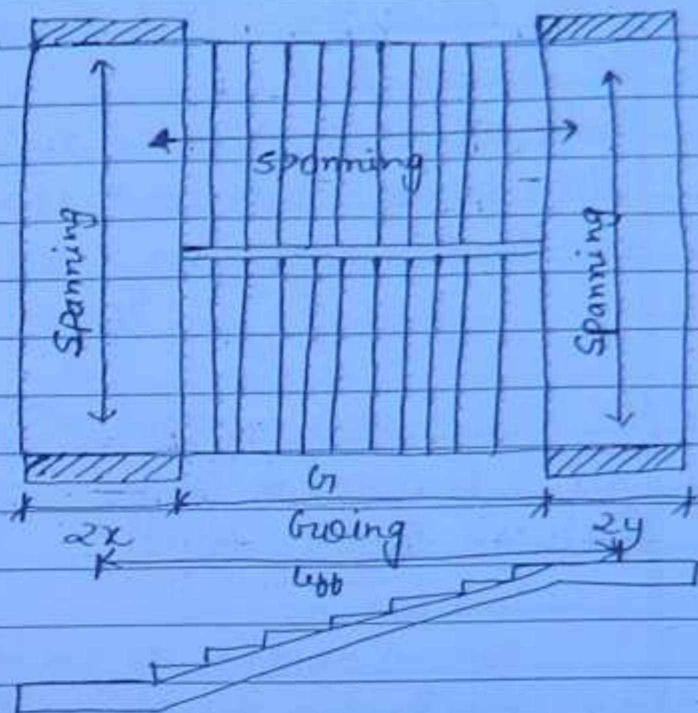

Effective Span = c/c distance betn beams

DATE:

(b) Landings are supported on side ~~bar~~ walls and waist slab are supported on landings.

Eff. Span = $l_{eff}$ = 205
$G + x + y$

Where value of $x$ or $y \not> 1m$



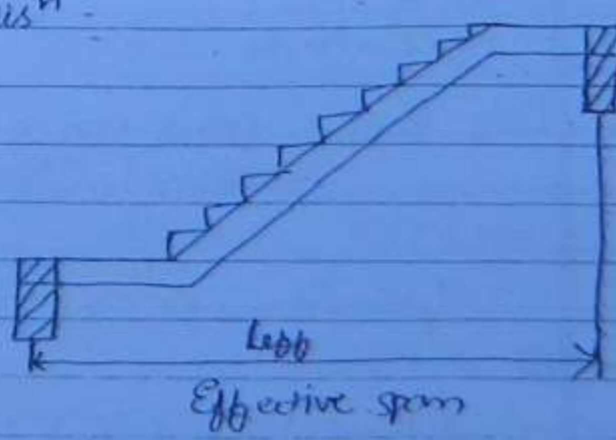

From Code.

| $x$ | $y$ | Eff. Span |
| --- | --- | --- |
| $<1m$ | $<1m$ | $G+x+y$ |
| $\geq 1m$ | $<1m$ | $G+1+y$ |
| $<1m$ | $\geq 1m$ | $G+x+1$ |
| $\geq 1m$ | $\geq 1m$ | $G+1+1$ |

G = going (horizontal length of waist slab)

(c) When supported on two beams provided at end. Landing slab is also spanning in same direction as of waist slab.

Eff span = Centre to centre dis$^n$ b/w beams.



Load Calculation:- For. 1m width of Stair

1. At landing Slab

Self wt = $1m \times 1m \times t \times 25$ = ?

Floor finish = $1m \times 1m \times t_f \times 24$ = ?

Live load = $1m \times 1m \times w_l$ = ?

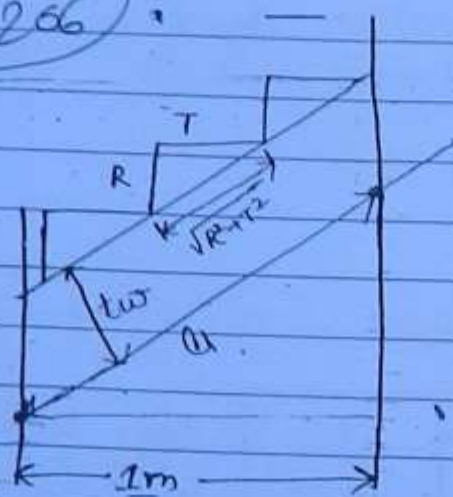
Total load = $w_1$

2. At waist slab portion (266)

For T horizontal length

Inclined length is

$$\sqrt{R^2+T^2}$$




For 1m horizontal length inclined length is

$$\frac{\sqrt{R^2+T^2}}{T} \times 1m = l_1$$

wt - waist slab

$$= l_1 \times 1m \times t_w \times 25$$

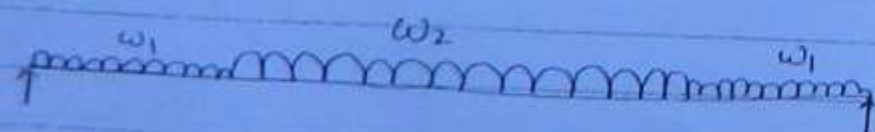
Self wt $= \frac{\sqrt{R^2+T^2}}{T} \times 1m \times 1m \times t_w \times 25$

Self wt. Steps $= \left(\frac{1}{T}\right) \times \frac{1}{2} \times R \times T \times 1m \times 25$

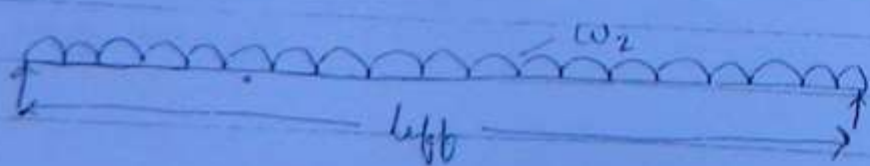
No. of steps if T is metre

Live load = $1m \times 1m \times w_L$

Total load $= w_2$




OR



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$$B.M = \frac{w_2 \times l_{eff}^2}{8}$$

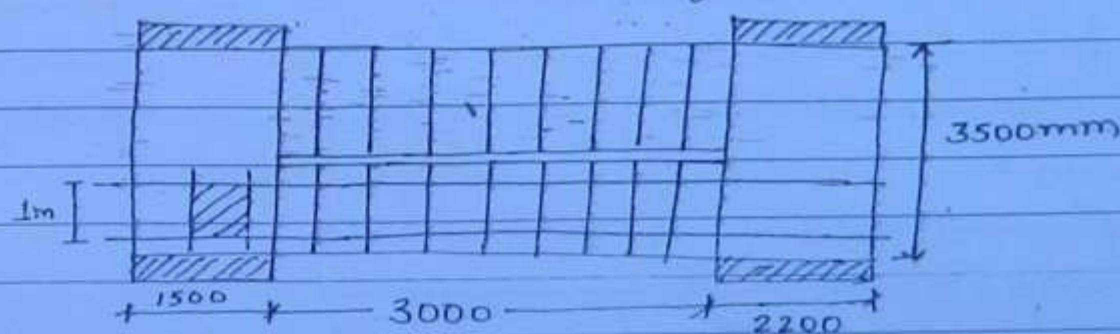
$$d = \sqrt{\frac{B.M}{Q.B}}$$

207

$$A_{st} = \frac{B.M}{.87 f_y \cdot j \cdot d} = \frac{BM}{\sigma_{st} \cdot j \cdot d}$$

$$\text{or } A_{st} = \frac{0.5 f_{ck}}{f_y}\left[1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck} B d^2}}\right] B d$$

Ques Design a dog-legged staircase as shown in fig.
Live load = 5 kN/m² & Floor finish = 1 kN/m²




Riser = 160mm
Treads = 300mm

Use - M-20 / Fe-415
Use L.S.M.

Soln Effective Span :- $l_{eff} = \underset{x}{\frac{1500}{2}} + \underset{G}{3000} + \underset{y}{\frac{2200}{2}} = 4850$

$$0.75 + 3.0 + 1 = 4.75 m$$

Load :—

Thickness of Slab (using defl$^n$ Criteria)

$$\text{thickness (Eff depth)} = \frac{\text{Span}}{20} = \frac{4750}{20} = 237.5 mm$$

$$+ 30 mm$$

$$267.5 mm$$

Use - 270 mm as thickness

DATE:

landing Slab

Self wt - $1m \times 1m \times .27 \times 25 = 6.75$

Floor finish : $1 \times 1m \times 1. \quad = 1.0$

live load = $1m \times 1m \times 5 \quad = 5$

Total - $12.75 \ KN/m^2$

208

Waist Slab

Self wt = $\dfrac{\sqrt{R^2+T^2}}{T} \times 1 \times 0.27 \times 25 =$

$1.13 \times 1 \times 0.27 \times 25 = 7.61 \ KN/m^2$

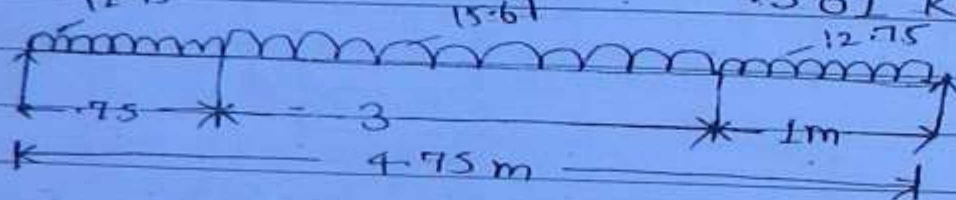
Self wt of Steps = $\left(\frac{1}{T}\right) \times \frac{1}{2} \times R \times T \times 1m \times 25$

$= \left(\dfrac{1000}{300}\right) \times \frac{1}{2} \times .16 \times .30 \times 1m \times 25 = 2.0$

Floor finish = $1 \times 1 \times 1 \quad = 1.0$

Live load = $1 \times 1 \times 5 \quad = 5.0$

$15.61 \ KN/m^2$




Max. B.M (Considering same load on entire span)

$B.M = \dfrac{15.61 \times 4.75^2}{8} = 44.02 \ KN-m$

Factored $= 1.5 \times B.M = 66.038 \ KN-m$

$d = \sqrt{\dfrac{B.M}{Q.B}} = \sqrt{\dfrac{66.04 \times 10^6}{.138 \times 20 \times 1000}}$

$d = 154.68 \ mm \not< 240 \ mm$

Safe

Use D = 270 mm

d = 240 mm

DATE:

$$A_{st} = \frac{0.5\times 20}{415}\left[1-\sqrt{1-\frac{4.6\times 66.04\times 10^6}{1000\times 240^2\times 20}}\right]1000\times 240^2$$

$= 820.75\ mm^2$

209

Use 16mm dia bars

Spacing $= \frac{1000}{820.75}\times\frac{\pi}{4}\times 16^2 = 244.9$ mm

Use 12mm

Spacing $= \frac{1000}{820.75}\times\frac{\pi}{4}\times 12^2 = 137$ mm

Use 12mm ϕ bars @ 130 mm c/c

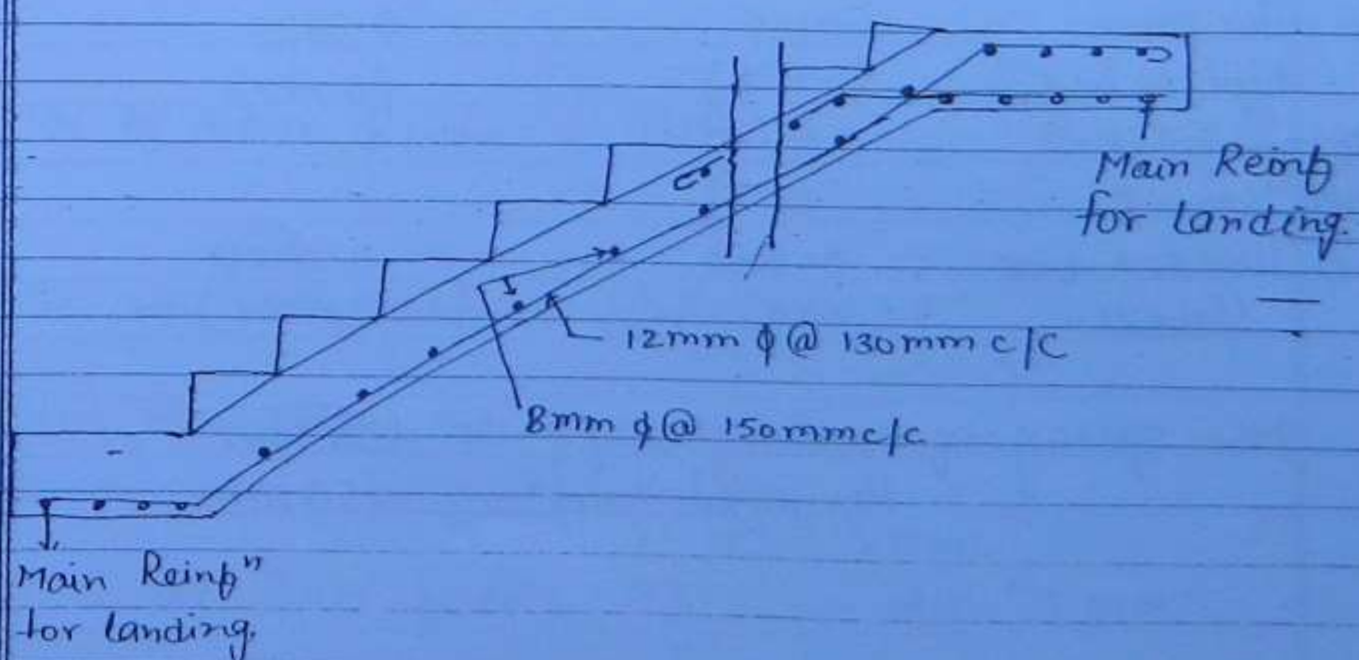
Distribution bars

$= \frac{0.12}{100}\times 1000\times 270 = 324\ mm^2$

Spacing of 8 mm ϕ bars $= \frac{1000}{324}\times\frac{\pi}{4}\times 8^2$

$= 155$ mm

Use 8mm ϕ bars @ 150 mm c/c



DATE:

→ Design of wall (As per $\overline{32}$ / P-61)
designed Same as Column

# 32.3.3 → Max. eff. height to thickness ratio

$$\frac{H_{eff}}{t} \not> 30$$

210

Effective height

(a) Restrained against rotation at both end
(i) By floor — 0.75 $H_w$
(ii) By intersecting walls = 0.75 $L_1$

(b) When not Restrained against rotation
(i) By floor = 1.0 $H_w$
By intersecting Walls = 1.0 $L_1$

Clause
32.5 — Min Reinf$^n$ required —
Horizontal & vertical both reinf$^n$ are provided.

| | | Vertical Reinf$^n$ | Horizontal Reinf$^n$ |
|---|---|---|---|
| (i) | Deformed bars $\not>$ 16mm | 0.0012 (0.12%) | 0.0020 (0.20%) |
| (ii) | For other type of bars (Mild Steel & greater than 16mm ϕ) | 0.0015 (0.15%) | 0.0025 (0.25%) |
| (iii) | Welded wire Fabrics | 0.0012 (0.12%) | 0.0020 (0.20%) |

Max. Spacing of vertical reinf$^n$ = 450 mm
Horizontal reinf$^n$ = 450 mm

DATE:

ES-1995

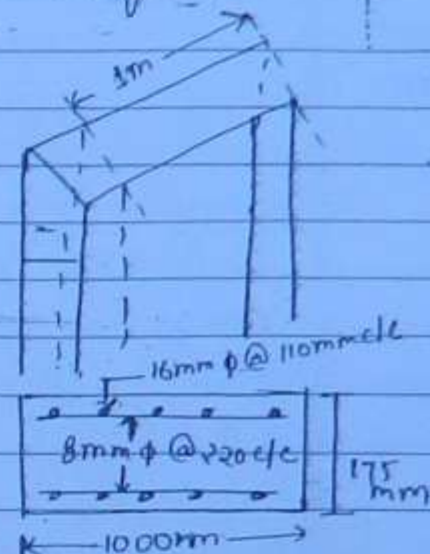
Ques A reinf Concrete wall 175 mm thickness & 3.2 ~~m~~ effective height is needed for a compressive load of 1000 kN/m. Design the wall. use - M20 / Mild steel reinf.

$P = 1000$ kN for 1m length of wall

Area $= 1000 \times 175$ mm

$A_{sc} =$

(211)




$$\frac{H_{eff}}{t} = \frac{3200}{175} = 18.29$$
$$> 12$$

Design as long Column.
Using W.S.M

$$P = C_r\,(A_c \cdot \sigma_{cc} + A_{sc} \cdot \sigma_{sc})$$

$$\left(1.25 - \frac{l_{eff}}{48B}\right)\left[(1000\times175 - A_{sc}) \times 4 + A_{sc} \times 130\right]$$

$$= 1000\times10^3 = \left(1.25 - \frac{3200}{48\times 175}\right)\left[1000\times175\times4 - 4A_{sc} + 130A_{sc}\right]$$

On each face. $A_{sc} = \dfrac{3576.86}{2} = 1788.43$

No. of 16mm φ bars $= \dfrac{1788.43}{\frac{\pi}{4}\times16^2} = 8.89$ say 9 Nos.

Min vertical reinf $= \frac{.15}{100} \times 1000 \times 175$
$= 262.5$ mm² OK

Min Horizontal reinf $= \dfrac{0.25}{100} \times 1000 \times 175$
$= 437.5$ mm²

On each face $= \dfrac{437.5}{2} = 218.75$ mm²

Spacing of 8mm $\phi$ bars

$$= \frac{1000}{218.75} \times \frac{\pi}{4} \times 8^2 = 229mm$$

Tuesday
|11/2011

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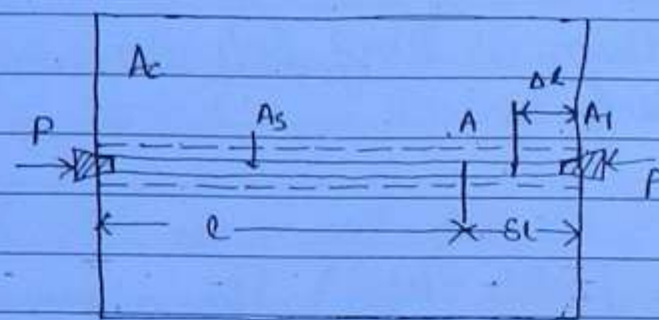
# PRESTRESSED CONCRETE

Concept :- If a steel reinforcement provided in a concrete is applied a force and is anchored after getting an elongation of $\delta l$. Now force is removed

Strain in steel

$$\Rightarrow e = \frac{\delta l}{l}$$




$= E$
$= eE$

Stress developed in steel

$$\Rightarrow p_s = e . E_s$$

Then $p_s = \frac{\delta l}{l} E_s$

If cross-section area of steel $= A_s$

Force developed due to $\delta l$ deformation

$$P_s = p_s \times A_s = \frac{\delta l}{l} E_s \cdot A_s$$

This force $P_s = P$ = Force applied on concrete

It is called prestressing force

$P_s$ = Tensile in steel

$P$ = Compressive in concrete

Stress developed in concrete $= p_c = \frac{P}{A_c}$

If due to any reason elongation provided is reduced, say by $\Delta l$. It will cause loss of prestress

If loss of elongation $= \Delta l$

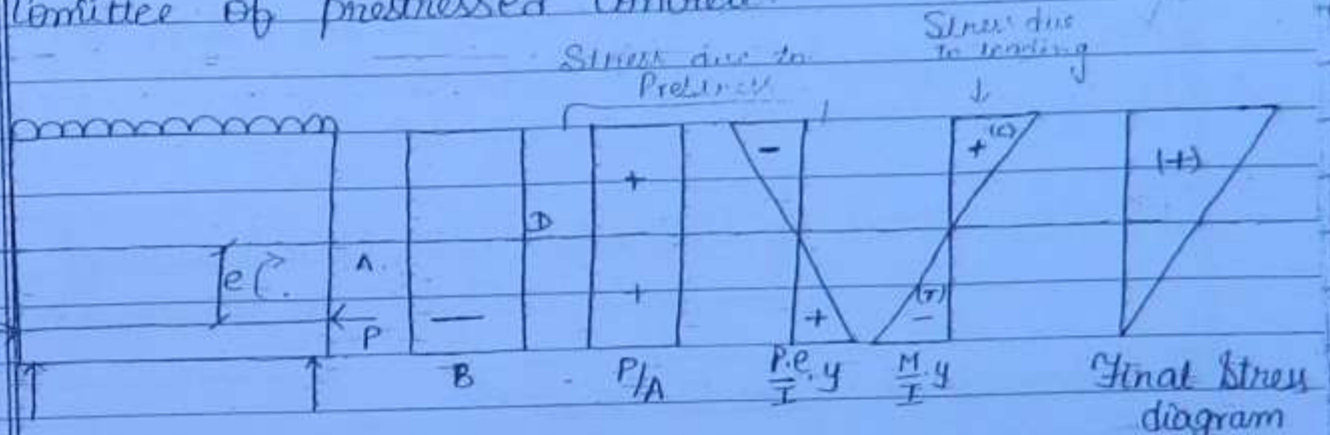
loss of strain $= \frac{\Delta l}{l}$

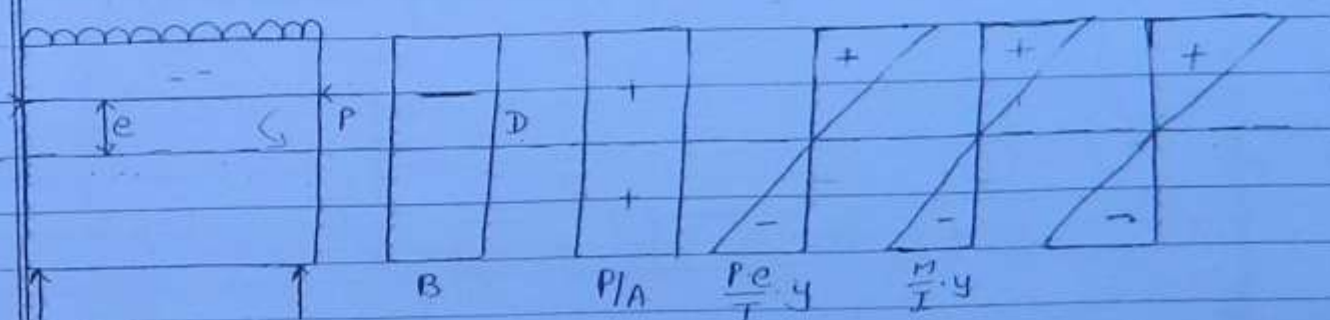
Loss of stress or prestress $= \frac{\Delta l}{l} E_s$

Net (Final) Stress available = $\frac{\delta l - \cancel{At}}{l} E_s$

(213)

Definition – "Prestress Concrete is one in which there have been introduced internal stresses of such magnitude and distribution, that stresses caused due to external loading are counterbalance upto a desired degree" As per ACI Committee of prestressed Concrete.




Necessity of high Strength Steel and Concrete

# Steel – In case of steel, losses are very high. Only due to Creep & Shrinkage loss of strain = 0.0008.

loss of stress due to Creep and Shrinkage = $0.0008 \times E_s$

$= 0.0008 \times 2 \times 10^5$

$= 160\ N/mm^2$

Total losses may be in the range of 200 to 300 $N/mm^2$

→ The losses should not be more that 10 to 20% of initial stresses provided in steel.

→ Initial ~~Stress~~ required —

1000 $N/mm^2$ to 2000 $N/mm^2$

Due to above reason we need a very high strength steel for PSC.

(214)

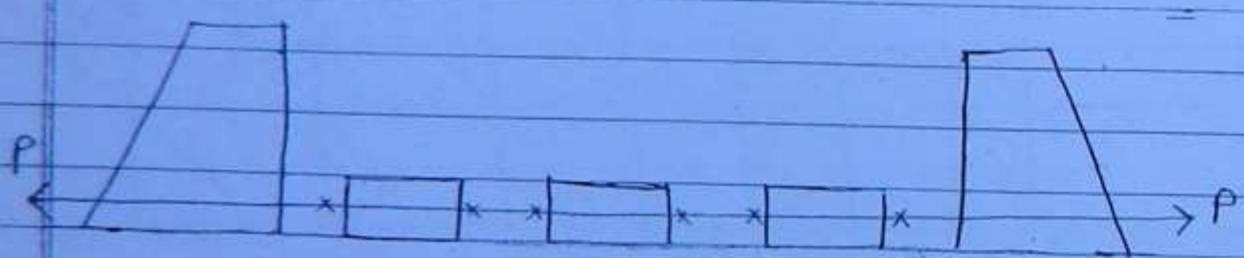
(II) Concrete -

1. To bear high bearing stresses developed at ends.
2. In case of pre tensioned members to bear high bond stress developed betn steel and concrete.
3. To reduce creep and shrinkage losses.

→ Minimum grade of concrete - M-30 (Post tensioned)
- M-40 (Pre tensioned)

Pretensioned and post tensioned

⇒ Pretensioned - (Hoyer Method or long line Method)

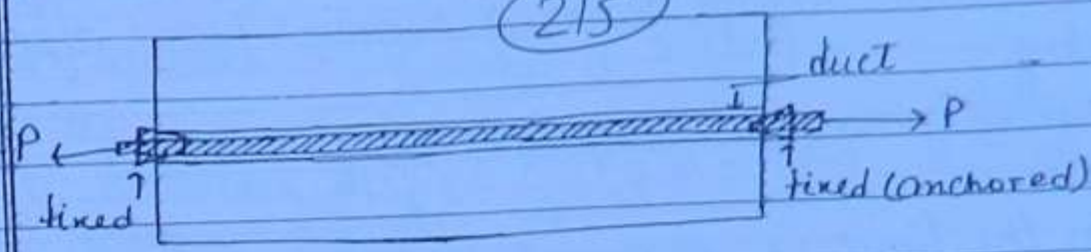



Properties

(i) Steel is in direct contact with concrete
(ii) Steel is tensioned before casting of concrete
(iii) After setting of concrete steel reinf are cut
(iv) Prestressing force is applied due to bond action betn steel and concrete.

→ Post tensioned.






· Properties

(i) Concrete is casted before prestressing of steel.

(ii) A duct is left in concrete during casting

(iii) After setting of concrete, steel reinf. (tendons) are inserted in duct and tensioned either from one end or both end, and anchored

(iv) Steel and concrete are not in direct contact.

→ Advantage of using prestressed concrete

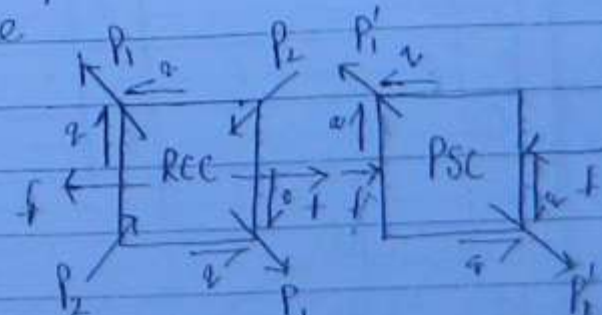
(i) Cross-section of concrete is utilised more effectively & entire cross-section is in compression (minor tensile stresses can be ignored)

(ii) In case of PSC, lighter c/s are required. Saving may be in supporting structure and foundation due to lighter weight of ~~suppo~~ superstructure.

(iii) Improve shearing resistance

$$p_1 = \frac{f}{2} \pm \sqrt{\left(\frac{f}{2}\right)^2 + q^2}$$

$$p_1' = -\frac{f}{2} + \sqrt{\left(\frac{f}{2}\right)^2 + q^2}$$



Assumption

1 Plane Section before bending remains plane after bending.

→ Strain diagram is uniform (216)

2. Within the limit of deformation hooke's law is valid. applicable to Concrete & Steel

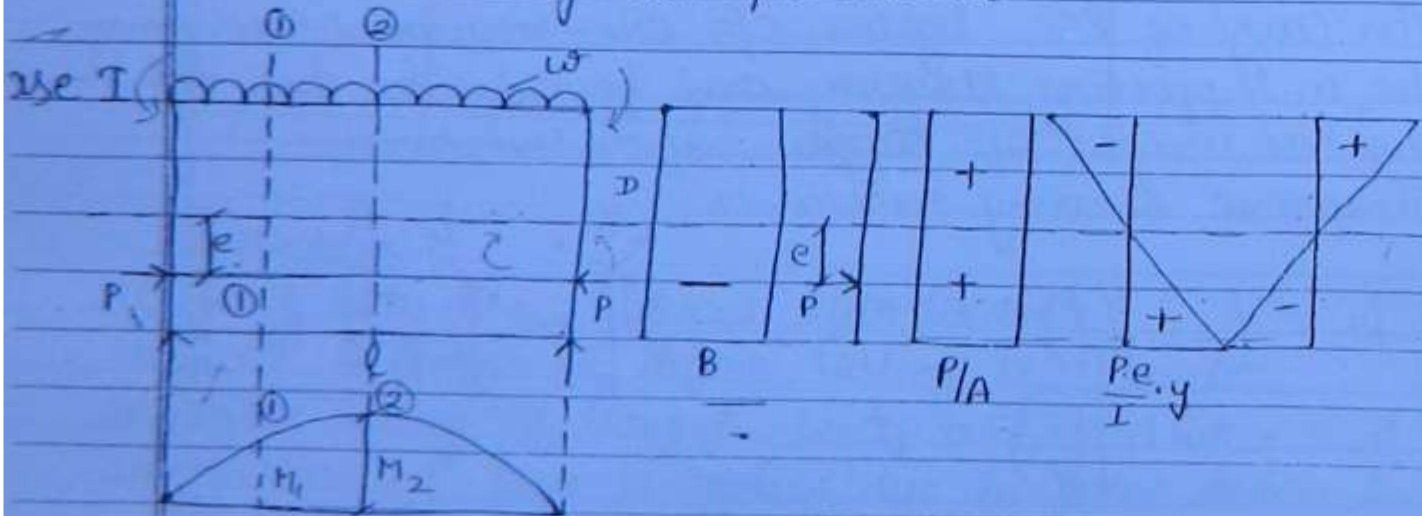
3. The Stresses in reinforcement doesn't changes along the length of reinforcement, Stress changes takes place only for concrete.

Variation of Stress in reinf$^n$ due to change in external loading is ignored.

Basic Concepts

1 Stress Concept method
2. Strength Concept method
3. Load balancing Concept method

Case I




Stresses developed

① Direct stress due to $P = + P/A = p_0$

2. Bending Stresses due to $P.e$ = $p_{b_1} = \mp \frac{P.e}{I} . y$

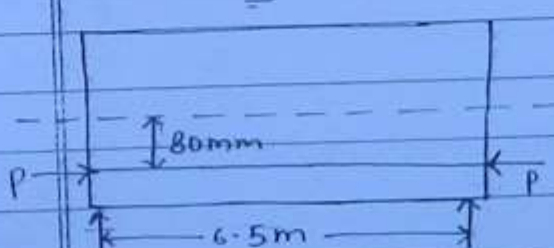
Moment due to load at sec$^n$ ①-① $= \pm \frac{M_1}{I} \cdot y$

→ Final stresses at top $= \frac{P}{A} - \frac{P.e}{I} \cdot y + \frac{M}{I} \cdot y$

Final stresses at bottom $= \frac{P}{A} + \frac{P.e}{I} \cdot y - \frac{M}{I} y$ 217

$$\boxed{\text{Final Stresses} = \frac{P}{A} \mp \frac{P.e}{I} \cdot y \pm \frac{M}{I} \cdot y}$$

A prestress Concrete beam 250mm x 300mm is prestress by 14 wires of 5 mmϕ (Straight Cable) provided at an eccentricity of 80mm Initial prestress = 1500 N/m². If span is 6.5m & beam is subjected to live load of 8kN/m. Calculate max. stress developed in beam at mid span at end. losses are Considered.




$P = A_s \times P_s = 14 \times \frac{\pi}{4} \times 5^2 \times 1500 = 412.33$ KN

$e = 80$ mm

Loads – D.L $= 0.25 \times 0.30 \times 1 \times 25$

L.L $= 8$ KN/m

total $= 9.875$ KN/m

Max. B.M $= \frac{wl^2}{8} = m_2 = \frac{9.875 \times 6.5^2}{8} = 52.15$ KN-m

Area $= 250 \times 300 = 75 \times 10^3$ mm²

$I = \frac{250 \times 300^3}{12} = 5625 \times 10^5$ mm⁴

$y = \frac{300}{2} = 150$

$y_t = 150$ mm

$y_B = 150$ mm

DATE:

at ends

Stress at top $= \frac{P}{A} - \frac{P.e}{I} . y_t$

$\frac{412.33}{75\times10^3} - \frac{412.33\times80}{5625\times10^5}\times150$ 218

$= -3.296\ N/mm^2$

Stress at bottom $= \frac{P}{A} + \frac{P.e}{I} . y$

$\frac{412.33}{75\times10^3} + \frac{412.33\times80}{5625\times10^5}\times150 = 14.296 N/mm^2$

Stress at mid span at top

$= \frac{P}{A} - \frac{P.e}{I} . y + \frac{m_2}{I} . y$

$-3.296 + \frac{52.15\times10^6}{5625\times10^5}\times150 , = 10.61 N/mm^2$

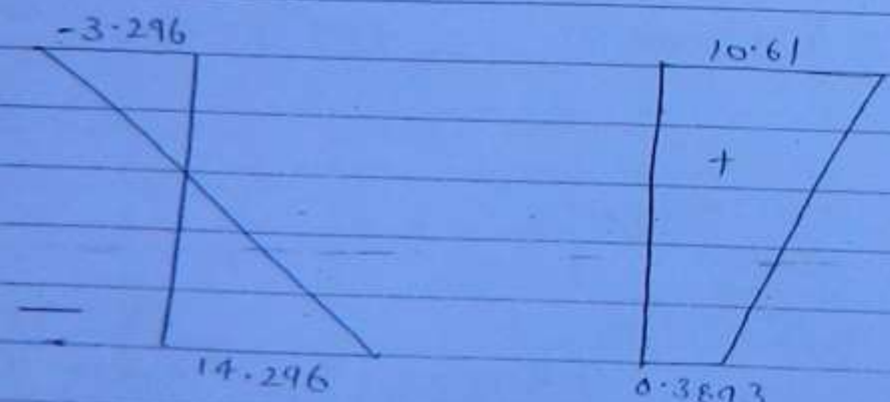
~~at bott~~

13.90

at bottom $= P/A + \frac{P.e}{I} . y - \frac{M_2}{I} . y$

$14.296 - 13.90 = .3893$

at end




# Effect of loading on stresses in tendons

Slope of beam at end due to U.DL - w

$\theta_1 = \frac{wl^3}{24EI}$

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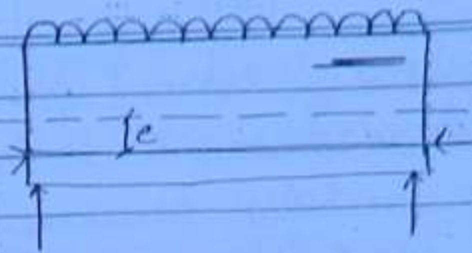
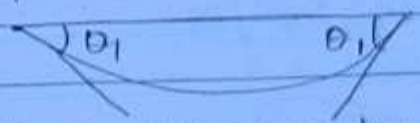
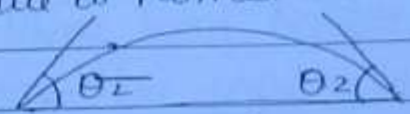
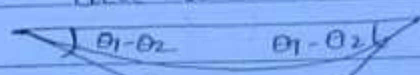



due to moment due to load




due to P·e




Net Slope due to combined effect

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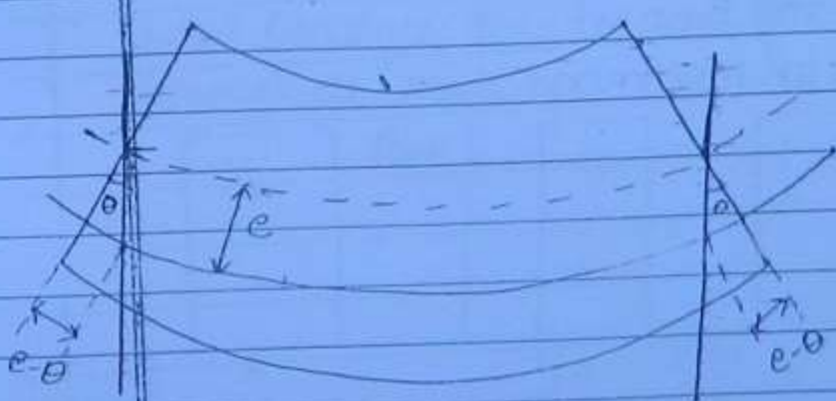
Slope of beam at ends due to moment P.e :

$$\theta_2 = \frac{P.e.l}{2EI}$$

(using mmt area method)

Net Slope $\theta = \theta_1 - \theta_2$

$$\theta = \frac{wl^3}{24E_cI} - \frac{P.e.l}{2E_cI}$$




total increase in length $= 2e\theta$

Total increase in length of steel r/f (tendons)

$$\Delta l = (+)\ 2\cdot e\cdot\theta$$

gain in stress due to $\Delta l$ increase in elongation

Stress increases $= \frac{\Delta l}{l} \times E_s$

IES 1990

DATE:

**Ques** A PSC beam of span 6m has a width of 150 mm & depth = 300mm. Initial stress = 1000 N/mm². Cable is at constant e = 50mm. $A_s = 100\ mm^2$. Find % increase in stress in wires if beam supports live load of 5kN/m.
Unit wt. of concrete = $24\ kN/m^3$
$E_c = 36\ kN/mm^2$
$E_s = 210\ kN/mm^2$

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**Sol^n** Load – $D.L = 0.15 \times 0.30 \times 1 \times 24 = 1.08$
$L.L = 5\ kN/m$
total = 6.08 kN/m

$P = A_s \times P_i = 100 \times 1000$
$e = 50$

Slope due to load = $\frac{wl^3}{24EI}$

$A = 150 \times 300 = 45 \times 10^3\ mm^2$

$I = \frac{150 \times 300^3}{12} =$

$\frac{wl^3}{24 E_c I} =$

$= 0.0045$

Slope due to moment $= \frac{Pe \cdot l}{2E_c I}$

$= 1.2345 \times 10$

Net slope (sagging) $\theta = \theta_1 - \theta_2$
$= 3.269 \times 10^{-3}$

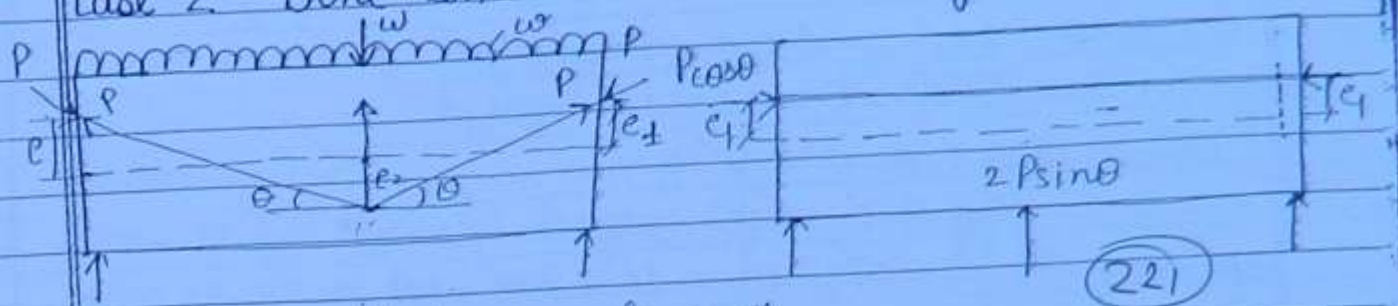
Gain in strength $= \frac{\Delta l}{l} \times E_s$

$= \frac{2 \cdot e \cdot \theta}{l} \times E_s$

$= 11.4415\ N/mm^2$

% increase $= \frac{11.44}{1000} \times 100 = 1.144$

Case-2. Bent tendons (Load balancing concept method)




Using load balancing concept.

1 Make the profile of cable straight from the point where it is at end

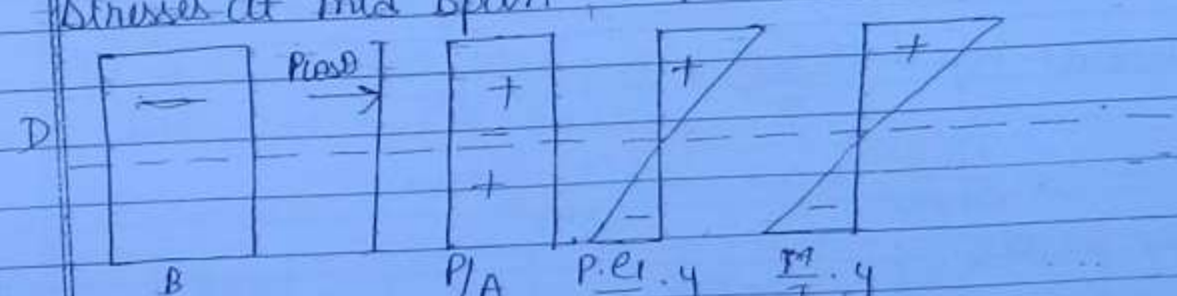
2 Upward balancing load due to profile of cable (Point load) $= 2P\sin\theta$ (↑)

downward load $= W$

Net point load $= (W - 2P\sin\theta)$

Now analyse the beam for equivalent beam shown

Stresses at mid span:



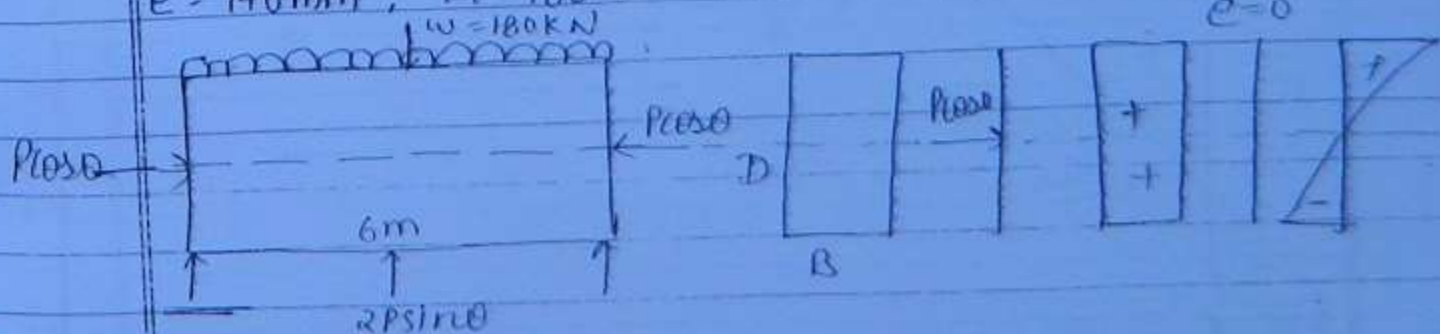

$P/A \qquad \frac{P.e_1}{I}.y \qquad \frac{M}{I}.y$

Where moment $M = \frac{(W - 2P\sin\theta).l}{4} + \frac{wl^2}{8}$

Final stresses $= \frac{P}{A} \pm \frac{P.e_1}{I}.y \pm \frac{M}{I}.y$

ES-2005 Ques A PSC beam 400x600 has span 6m. Calculate extreme fibre stress at mid span considering self wt also.

$e = 140mm$, $W = 180kN$ & $P = 1200kN$



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$P = 1200\,kN$

$\theta = \tan\theta = \frac{140}{3000} = 0.0467$ radian

$\theta = 2^\circ 40' 18.71''$

$P\cos\theta = 1200 \times \cos 2^\circ 40' 18.71''$

$= 1198.99 \cong 1200\,kN$

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upward force $= 2P\sin\theta = 111.878\,kN$

net point load $= W - 2P\sin\theta = 180 - 111.88$

$= 68.12\,kN$

Self wt $= 0.4 \times 0.6 \times 1 \times 25 = 6\,kN/m = w$

B.M at mid span $= = (W - 2P\sin\theta)\frac{l}{4} + \frac{wl^2}{8}$

$\frac{68.12}{4} \times 6 + \frac{6 \times 6^2}{8}$

$= 129.18\,kN\text{-}m$

$A = 400 \times 600 = 24 \times 10^4\,mm^2$

$I = \frac{400 \times 600^3}{12} = 72 \times 10^8\,mm^4$

Stresses at top $= \frac{P\cos\theta}{A} + 0 + \frac{M}{I} \cdot y$

$\frac{1200 \times 10^3}{24 \times 10^4} + 0 + \frac{129.18 \times 10^6}{72 \times 10^8} \times 300$

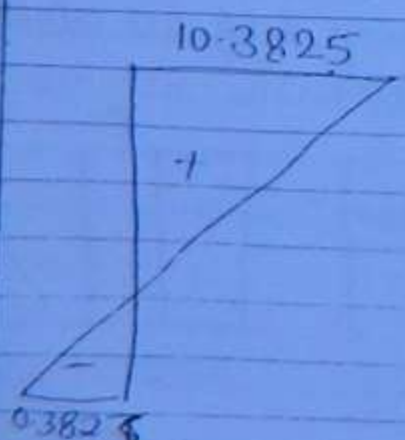
$5 + 5.3825$

$= 10.3825\,N/mm^2$

Stresses at bottom $= \frac{1200 \times 10^3}{24 \times 10^4} - \frac{M}{I} \cdot y$

$5 - 5.3825$

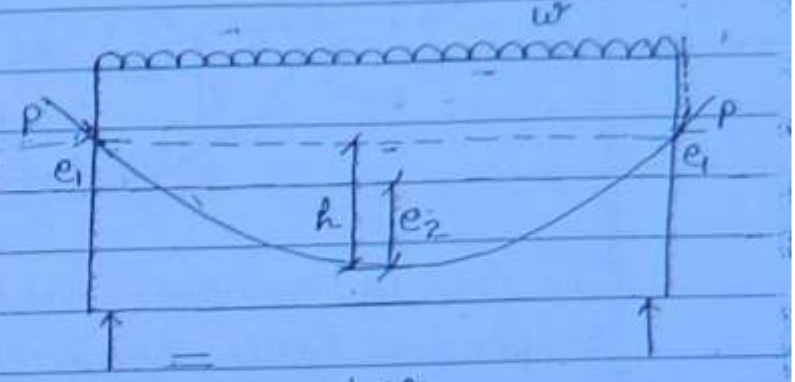
$= -0.3825\,N/mm^2$



Tendon of parabolic profile

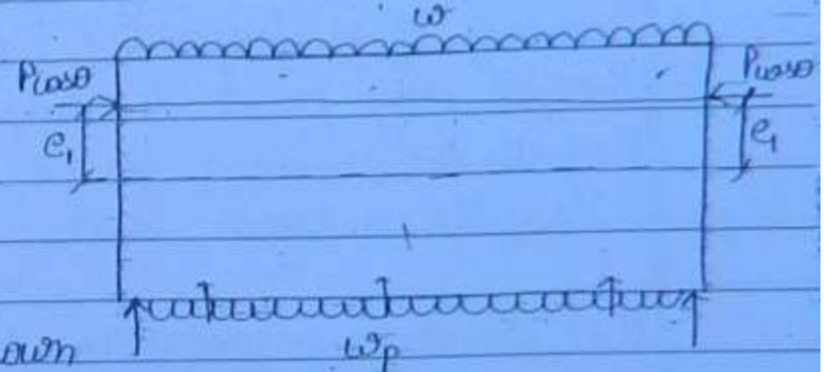
→ (load balancing concept)

In this case due to load balancing :-




1. Make profile of cable straight from pt. where the cable lie at ends

2. An upward balancing load (u.d.l) may be applied




$$\underset{u.d.l}{\text{upward}}\ w_p = \frac{8Ph}{l^2}$$

Now equivalent beam as shown in fig can be analysed.

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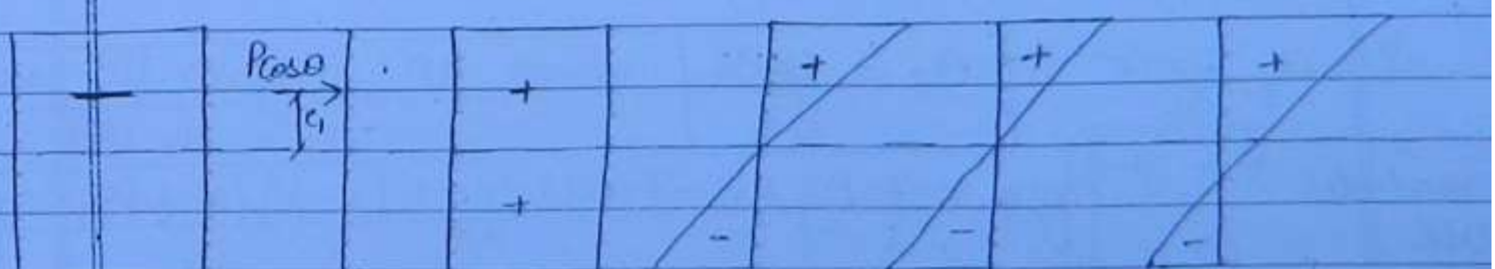
$$\text{Net u.d.l} = (w - w_p)$$

$$B.M = M = \frac{(w - w_p)l^2}{8}$$

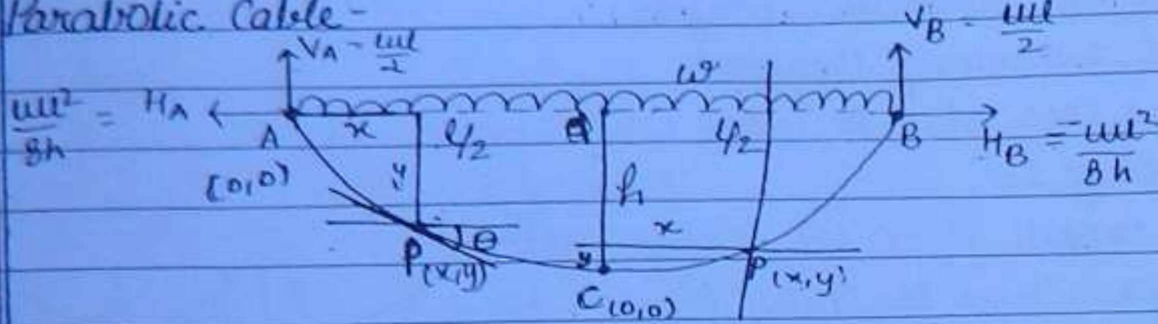



$\frac{P\cos\theta}{A}$ $\frac{P\cos\theta \cdot e_1}{I}\cdot y$ $\frac{M}{I}y$ Final stress

Final stresses

$$= \frac{P\cos\theta}{A} \pm \frac{P\cos\theta \cdot e_1}{I} y \pm \frac{M}{I} \cdot y$$

## Parabolic Cable -




The Case of parabolic cable is similar to the Case of a suspended Cable, subjected to U.d.l ($w$)

→ Property of a Suspended Cable.

:- B.M at any point in suspended Cable = 0.

B.M at C = 0

$V_A = V_B = \frac{wl}{2}$

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$$B.M \text{ at } C = \frac{wl^2}{8} + H_B \times h - V_B \times \frac{l}{2} = 0$$

$$\frac{wl^2}{8} + H_B \cdot h - \frac{wl^2}{4} = 0$$

$$H_B \cdot h = \frac{wl^2}{8}$$

$$\boxed{H_B = \frac{wl^2}{8h}}$$

For parabolic Cable $H_B = P\cos\theta \simeq P$

$w = w_p$

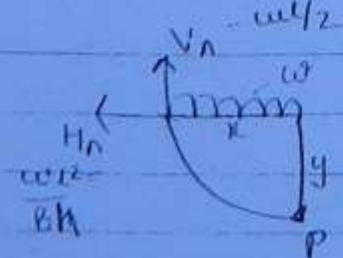
$P = \frac{w_p l^2}{8h}$

$$\boxed{w_p = \frac{8Ph}{l^2}}$$

Equation of parabolic tendon

Case 1 Origin is at ends.

B.M at P = 0



$$w\cdot x\cdot\frac{x}{2} + H_A\cdot y - V_A\cdot x = 0$$

$$\frac{wx^2}{2} + \frac{wl^2}{8h}y - \frac{wl}{2}x = 0$$

$$\frac{wl^2}{8h}y = \frac{wl}{2}x - \frac{wx^2}{2}$$

$$\frac{l^2}{4h}y = lx - x^2$$

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$$\frac{l^2}{4h}y = x(l-x)$$

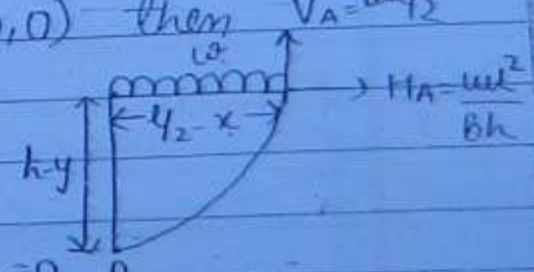
$$\boxed{y = \frac{4hx(l-x)}{l^2}}$$ Eq. of parabolic Cable (also called as deflection eq$^n$)

$$\tan\theta = \frac{dy}{dx} = \frac{4h}{l^2}(l-2x)$$ Slope equation

Slope at ends at $x=0$ or $x=l$

$$\frac{dy}{dx} = \tan\theta = \frac{4h}{l} = \theta$$ Slope at end.

If origin is at centre say C (0,0) then




B.M at P = 0

$$\frac{w}{2}\left(\frac{l}{2}-x\right)\left(\frac{l}{2}-x\right) + \frac{wl^2}{8h}(h-y) - \frac{wl}{2}\left(\frac{l}{2}-x\right) = 0$$

$$\frac{w}{2}\left(\frac{l^2}{4} - lx + x^2\right) + \frac{wl^2}{8} - \frac{wl^2}{8h}\cdot y - \frac{wl^2}{4} + \frac{wlx}{2} = 0$$

$$\cancel{\frac{wl^2}{8}} - \cancel{\frac{wlx}{2}} + \frac{wx^2}{2} + \cancel{\frac{wl^2}{8}} - \frac{wl^2}{8h}\cdot y - \cancel{\frac{wl^2}{4}} + \cancel{\frac{wlx}{2}} = 0$$

$$\frac{wx^2}{\cancel{2}} = \frac{wl^2}{4\,\cancel{8}h}\cdot y$$

$$\boxed{\frac{4x^2h}{l^2} \qquad y}$$ Eq. of Cable if origin is at Centre

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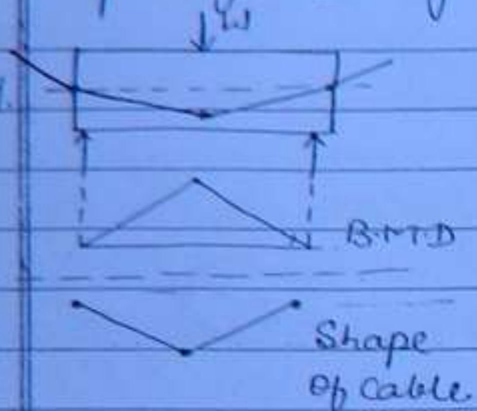
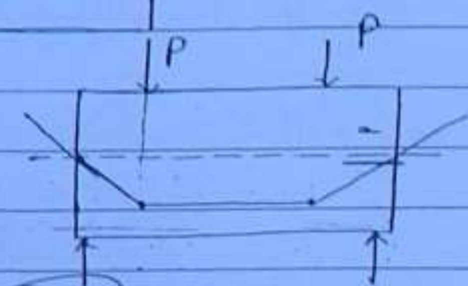
→ Shape of a load balancing Cable.

⇒ Shape of a load balancing Cable is similar to the Shape of bending moment diagram. (B.M.D)

Case 1.




$$\boxed{W = 2P\sin\theta}$$

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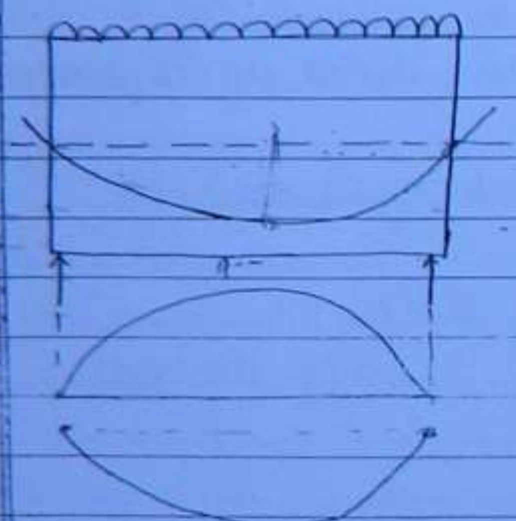
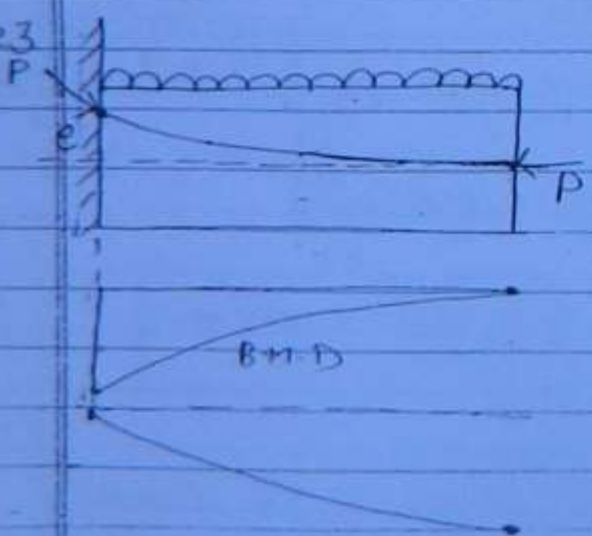
Case 2.

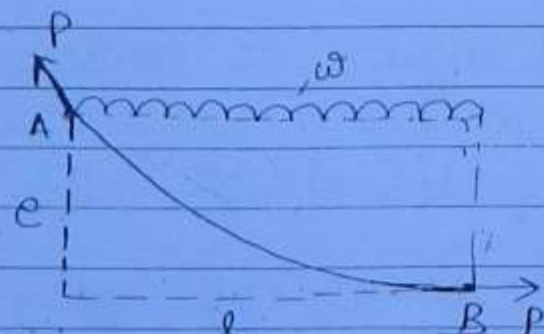


$$w_P = \frac{8Ph}{l^2}$$

$$\boxed{h = \frac{w_P l^2}{8P}}$$

Case 3




BM at A = 0

$$P \cdot e = wl \cdot \frac{l}{2}$$

$$\boxed{e = \frac{wl^2}{2P}}$$

DATE:

**Ques** Determine the profile of a load balancing Cable for a Cantilever beam Of the Span 4m Carrying an U.D.L Of 12 kN/m. Prestressing force is 1500 kN.

**Sol.** For a Cantilever Shape is shown On left hand Side page Shape Of Cable is mirror image of B.M.D as shown

eccentricity at Support = $e = \frac{wl^2}{2P}$

$$e = \frac{12 \times 4^2}{2 \times 1500} = 0.064\, m$$

$$\underline{e = 64 mm}$$

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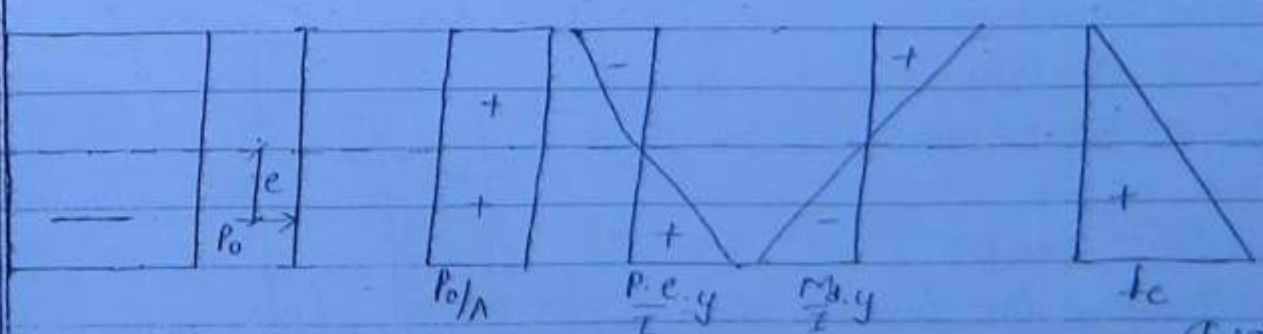
# Stresses in a beam ~~Of~~ at different Stage Of loading

1. At transfer
2. At final Stage Of loading

→ <u>At transfer</u>:- Just after transfer Of prestressing force in the beam.

1. Losses Of prestress are not Considered.
2. Live load is not Considered.
3. Dead load may be Considered (if not supported)
4. Dead load not to be Consider if beam is Still On its prestressing bed.




Total Stress diagram at transfer

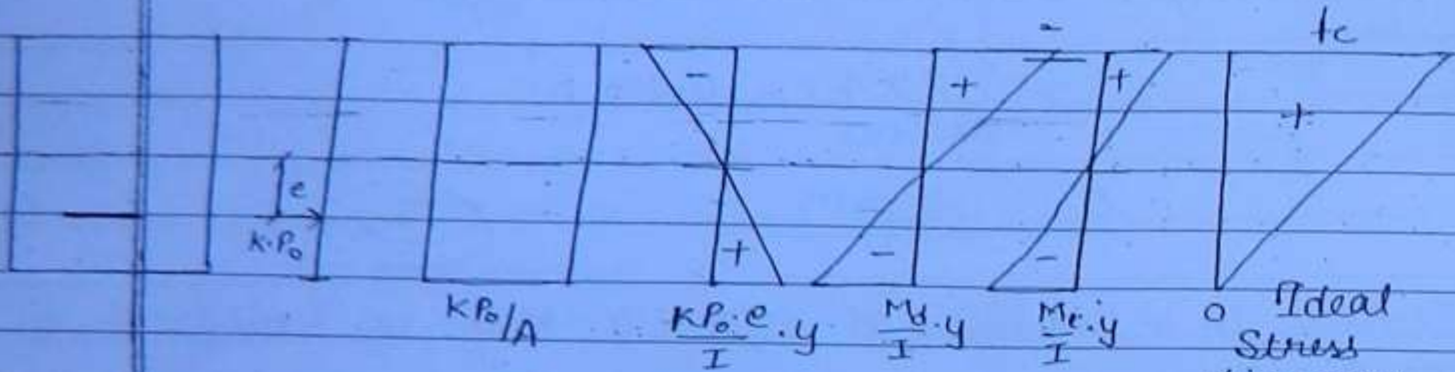
→ At final stage

1. Losses are to be Considered

Final $P = KP_0$ || if loss = 20%

$K$ = loss factor $(1 - \text{loss}\%)$ || $K = 0.80$

2. All loads are to be Considered.

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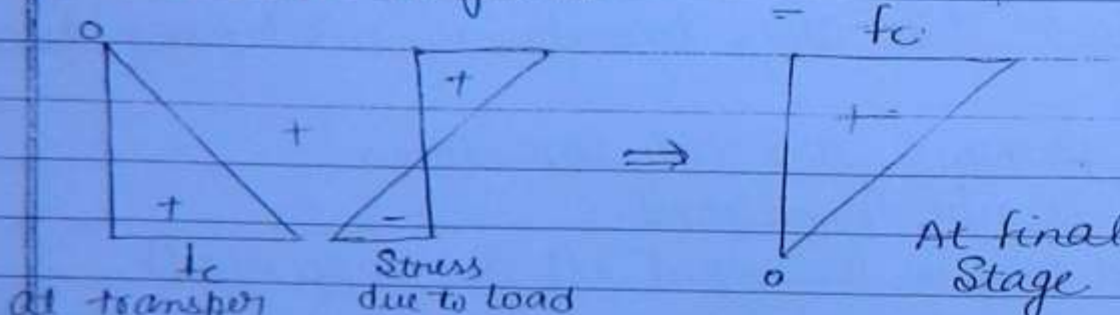
Ideal Stress diagram




**Ques** A post tensioned prestress Concrete is as shown in fig.

Span = 20m

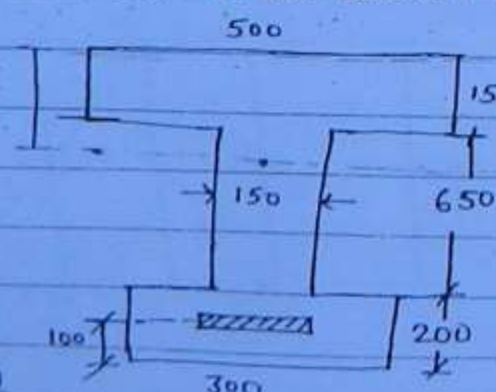
$p = 1100\ N/mm^2$

$A_s = 1385.44\ mm^2$

Loss = 15%

U.d.l = 12 KN/m

Determine the Stresses in beam at mid span at diff^n Stage of loading



Sol. $A = 500 \times 150 + 650 \times 150 + 200 \times 300 = 232500 \ mm^2$

$$\bar{y} = \frac{500 \times 150 \times 75 + (150 \times 650 \times 475) + (300 \times 200 \times 900)}{232500}$$

$\bar{y} = y_t = 455.65 \ mm$

$y_B = 1000 - 455.65 = 544.35 \ mm$

$$I = \left[ \frac{500 \times 150^3}{12} + 500 \times 150 \times (455.65 - 75) + \frac{150 \times 650^3}{12} + 150 \times 650 (455.65 - 475)^2 + \frac{300 \times 200^3}{12} + 300 \times 200 (455.65 - 900)^2 \right]$$

$I = 2.6524 \times 10^{10} \ mm^4$

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Dead load $= \frac{232500}{10^6} \cancel{\times F \times 25} = 5.8125 \ KN/m$

$M_d = \frac{W_d l^2}{8} = \frac{5.8125 \times 20^2}{8} = 290.625 \ KN\text{-}m$

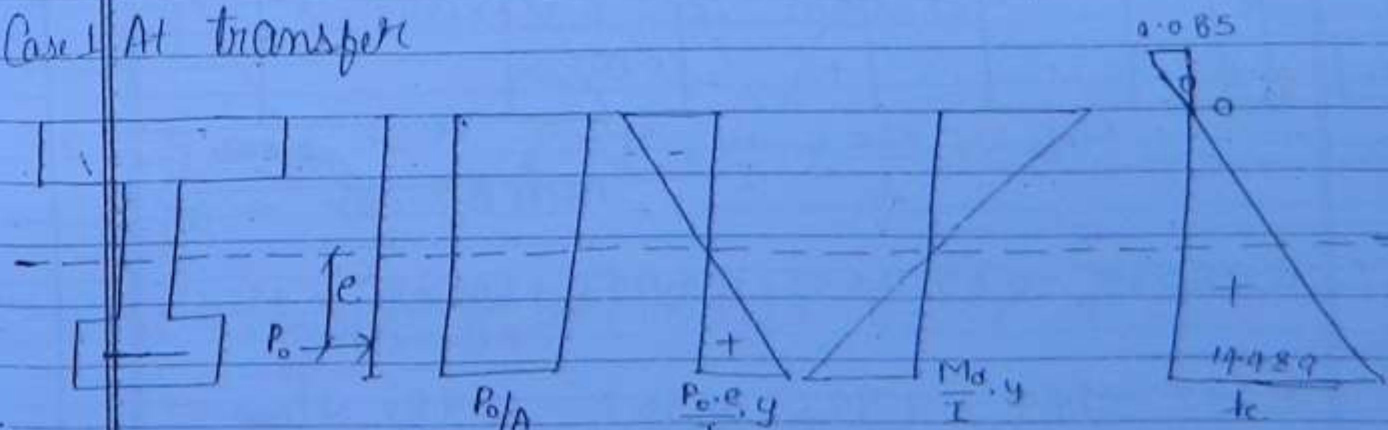
Live load $= 12 \ KN/m$

$M_l = \frac{12 \times 20^2}{8} = 600 \ KN\text{-}m$

$e = y_B - 100$

$= 444.35 \ mm$

Case 1 At transfer




Prestressing force $P_o = A_s \times p_o = 1385.44 \times 1100$

$= 1523984 \ N$

Stress at top

$$= \frac{P_0}{A} - \frac{P_0 \cdot e}{I} \cdot y_t + \frac{M_d}{I} \cdot y_t$$

$$= \frac{1523984}{232500} - \frac{1523984 \times 444.35}{2.6524 \times 10^{10}} \times 455.65 + \frac{290.625 \times 444.35}{2.6524 \times 10^{10}}$$

$$= 6.555 - 11.633 + 4.993 = -0.085 \text{ N/mm}^2$$

Stress at bottom

$\cancel{= 6.555 + 11.633 - 4.993 - 600}$

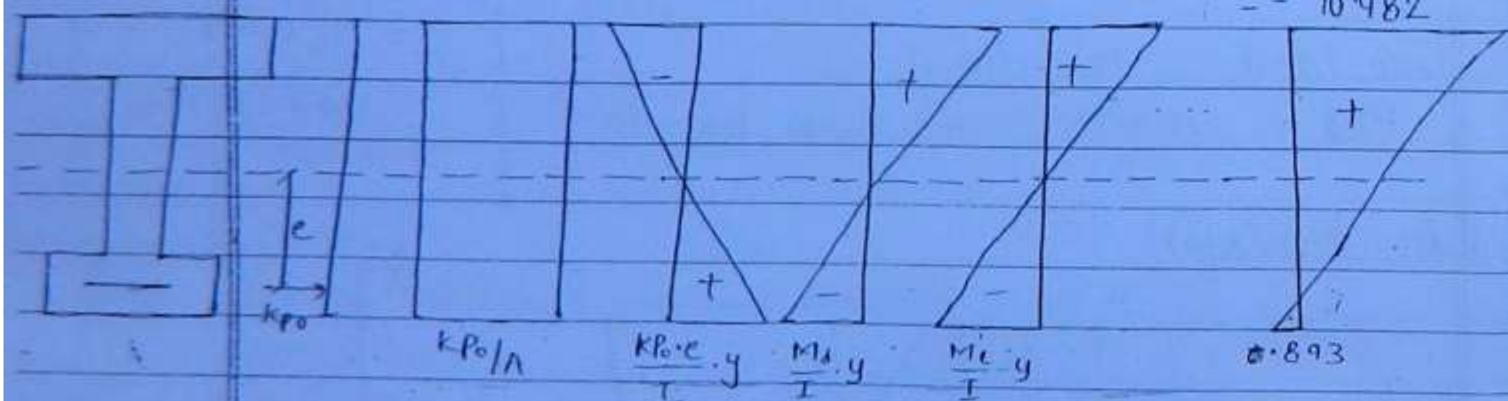
$$= \frac{P_0}{A} + \frac{P_0 \cdot e}{I} \cdot y_B - \frac{M_d}{I} \cdot y_B \qquad 230$$

$$= 6.555 + 11.633 \times \frac{544.35}{455.65} - 4.993 \times \frac{544.35}{455.65}$$

$$= 6.555 + 13.898 - 5.964 = 14.489 \text{ N/mm}^2$$

Final Stage

Final $P = KP_0 = .85 \times 1523984 = 1295386.4$




Stress at top $= .85 \times 6.555 - 0.85 \times 11.6333 + 4.993 + \frac{600 \times 10^6}{2.6524 \times 10^{10}} \times 455.65$

$$= 5.572 - 9.89 + 4.993 + 10.307 = 10.982 \text{ N/mm}^2$$

Stress at bottom $= 5.572 + 9.89 \times \frac{544.35}{455.65} - 4.993 \times \frac{544.35}{455.65} - 10.307 \times \frac{544.35}{455.65}$

$$= -0.893 \text{ N/mm}^2$$

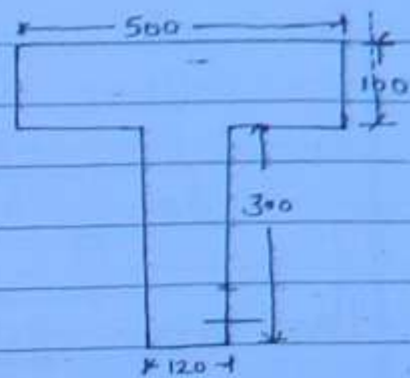
**Ques** A prestress concrete beam as shown in fig is to be designed for 11 kN/m live load, over a span of 6m. If stress in concrete should not exceed.

In Compression $= 15 N/mm^2$

In tension $= 0$

at any stage.

Find out prestressing force & eccentricity at which P should be applied.

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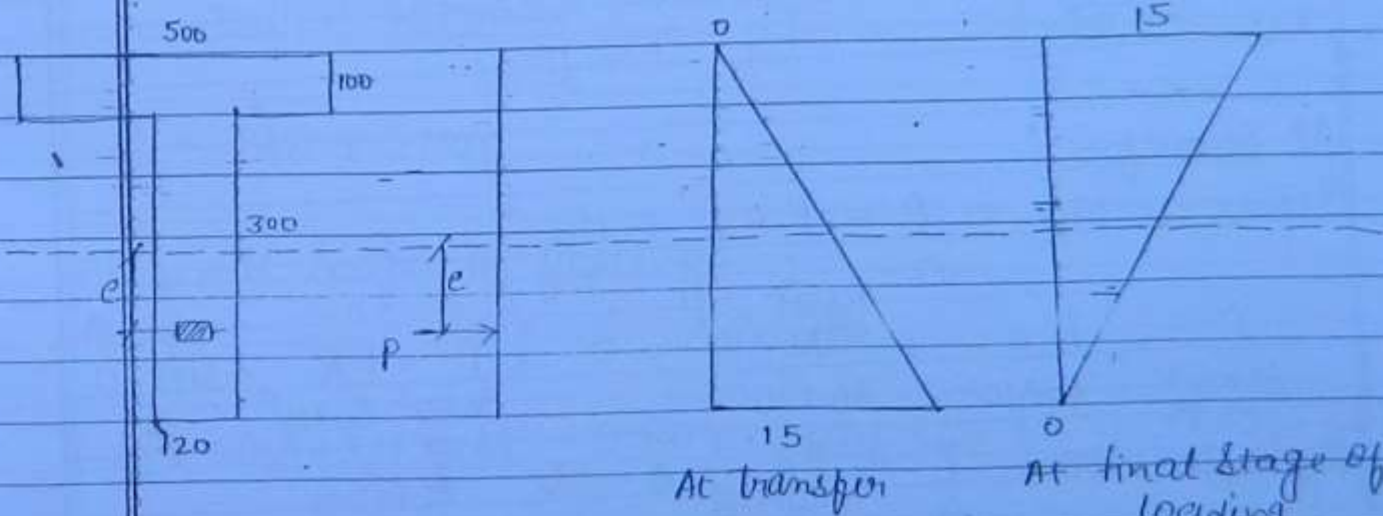
**Sol$^n$** Properties of Sec$^n$

$A = 500 \times 100 + 300 \times 120 = 86 \times 10^3 \, mm^2$




$$\bar{y} = \frac{500 \times 100 \times 50 + 300 \times 120 \times 250}{86 \times 10^3}$$

$$\bar{y} = 133.72 = y_t$$

$$y_B = 266.28 \, mm$$

$$I = \frac{500 \times 100^3}{12} + 500 \times 100 \, (133.72 - 50)^2 +$$

$$\frac{120 \times 300^3}{12} + 120 \times 300 \, (133.72 - 250)^2$$

$$= 1.15 \times 10^9 \, mm^4$$

DATE:

$$\frac{M}{I}y = \frac{M}{Z}$$

$$Z_t = \frac{I}{y_t} = \frac{1.15\times10^9}{133.72} = 86\times10^5$$

$$Z_B = \frac{I}{y_B} = \frac{1.15\times10^9}{266.28} = 43.19\times10^5$$

$$\text{Dead load} = \frac{86\times10^3}{10^6}\times1\times25 = 2.15\ kN/m$$

$$M_d = \frac{w_d l^2}{8} = \frac{2.15\times6^2}{8} = 9.675\ kN\cdot m$$

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$$\text{live load} = 11$$

$$M_L = \frac{11\times6^2}{8} = 49.5\ kN\cdot m$$

At transfer

$$\text{Stress at top} = \frac{P}{A} - \frac{P.e}{I}y_t + \frac{M_d}{Z_t} = 0$$

$$\frac{P}{86\times10^3} - \frac{P.e}{86\times10^5} + \frac{9.675\times10^6}{86\times10^5} = 0 \Rightarrow \frac{P}{86\times10^3} - \frac{P.e}{86\times10^5} + 1.125 = 0 \quad -(1)$$

$$\text{Stress at bottom}\ \frac{P}{A} + \frac{P.e}{I}.y - \frac{M_d}{Z} = 15$$

$$\frac{P}{86\times10^3} + \frac{P.e}{43.19\times10^5} - \frac{9.675\times10^6}{43.19\times10^5} = \frac{P}{86\times10^3} + \frac{P.e}{86\times10^5} - 1.125 = 15$$

$$\frac{P}{86\times10^3} + \frac{P.e}{86\times10^5} = 16.125 \quad -(2)$$

Multiply eqn (1) by $86\times10^5$

$$100P - P\,e + 9.675\times10^6 = 0$$

Multiply eqn (2)

$$50.22P + P.e - 9.675\times10^6 = 15\times43.19\times10^5$$

eq (1) $100P \quad P.e + 9.675\times10^6$

eq (2) $50.22P + P.e - 9.675\times10^6 = 15\times43.19\times10^5$

DATE:

eq ① → $\frac{P}{86\times10^3} - \frac{P.e}{86\times10^5} + 1.125 = 0$

eq ② → $\frac{P}{86\times10^3} + \frac{P.e}{86\times10^5} - 16.125 = 0$

Add both eqn

$\frac{P}{86\times10^3} + \frac{P}{86\times10^3} + 1.125 - 16.125 = 0$

$2.326\times10^{-5} P = 15$

$P = 431265\,N$

$e = \frac{100P + 9.675\times10^6}{P}$

$= \frac{100\times431265 + 9.675\times10^6}{431265}$

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$e = 122.43\,mm$

At final stage of loading

At top $= \frac{kP}{A} - \frac{kPe}{Z_t} + \frac{Md + Ml}{Z_t}$

$= \frac{0.80\times431265}{86\times10^3} - \frac{0.80\times431265\times122.43}{86\times10^5} + \frac{(9.675+49.5)}{86\times10^5}$

$= 4.01 - 4.911 + 6.88 = 5.979 < 15$ ok

At bottom $= \frac{kP}{A} + \frac{k.Pe}{Z_B} - \frac{Md+Ml}{Z_t}$

$4.01 + \frac{0.80\times431265\times122.43}{43.19\times10^5} - \frac{(9.675+49.5)}{43.19\times10^5}$

$4.01 + 9.78 - 13.70 = 0.09 \nless 0$

Hence safe

$P = 431265\,N$

$e$ 122.43 mm

DATE:

## Cracking moment

The value of moment at which visible tensile cracks are developed in the section is called cracking moment.

If tensile stress developed due to B.M, increases the tensile strength of concrete, the corresponding B.M is called cracking moment.

At bottom — If losses are not considered then

$$\frac{P}{A} + \frac{P.e}{I}y - \frac{M}{I}y > -f_t$$

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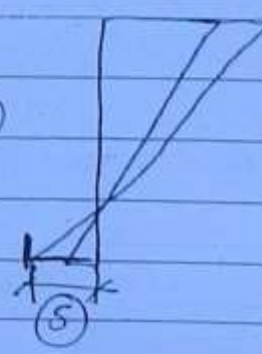



If losses are considered

$$\frac{KP}{A} + \frac{KP.e}{I}y - \frac{M}{I}y > (-f_t)$$

$f_t$ is tensile strength of concrete

ES-1990 Ques A rectangular concrete beam $150 \times 300\ mm^2$ is prestressed by cable carrying $P = 225$ kN & $e = 50$mm

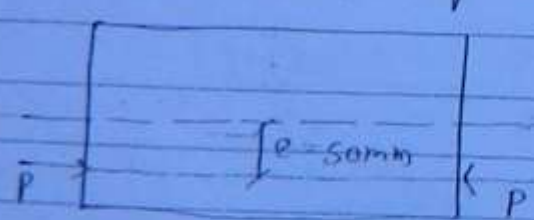
The beam supports a u.d.l 7.2 kN/m including (t) concentric self wt. of beam

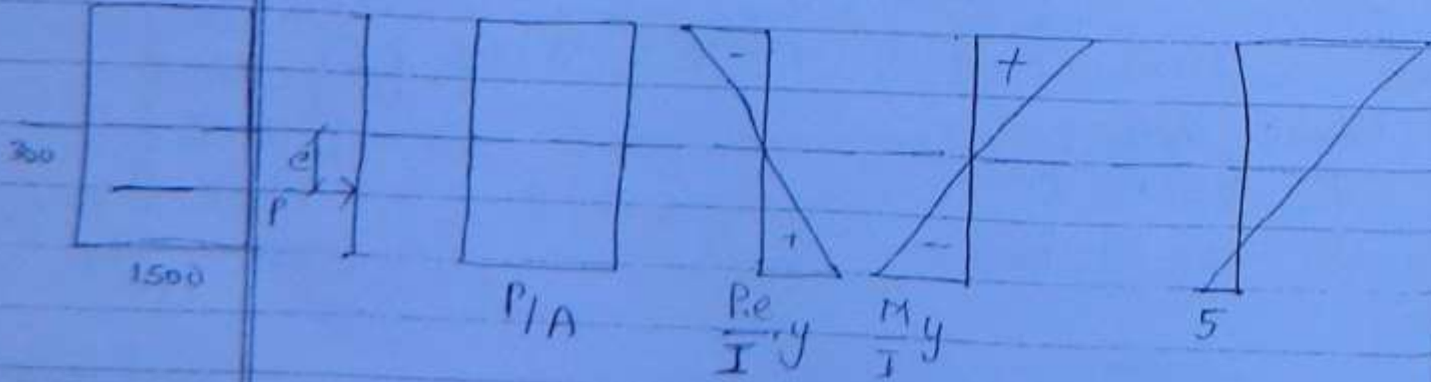
Span = 5m

If modulus of rupture = 5N/mm² (tensile stress)

Calculate load factor against cracking.

Soln



Stress at bottom of beam

$A = 45 \times 10^3$

$I = \frac{150 \times 300^3}{12} = 3375 \times 10^5$

$$\frac{P}{A} + \frac{P.e}{I} y - \frac{M}{I} y = -5$$

$$\frac{225 \times 10^3}{45 \times 10^3} + \frac{225 \times 10^3 \times 50}{3375 \times 10^5} \times 150 - \frac{M}{3375 \times 10^5} \times 150 = -5$$

$$5 + 5 + 5 = \frac{M}{3375 \times 10^5} \times 150$$

$M = 33.75 \times 10^6$ N-mm Cracking Moment

Cracking load $= \frac{w_c l^2}{8}$

$$33.75 = w_c \times \frac{5^2}{8}$$

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$w_c = 10.8$ kN/m

Service load or working load = 7.2 kN/m

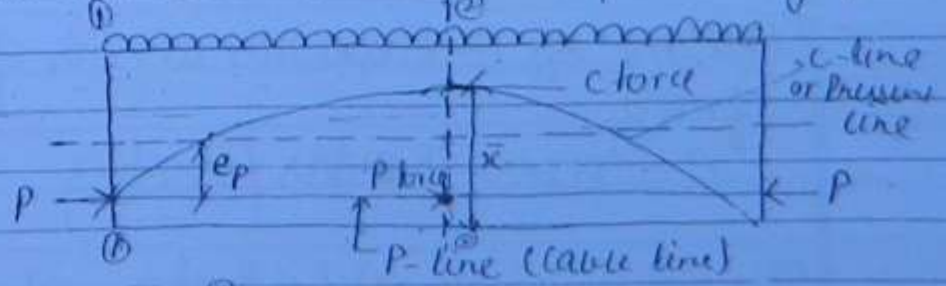
load factor = $\frac{\text{Cracking load}}{\text{working load}} = \frac{10.8}{7.2}$

$= 1.5$

imp

# P-Line | C-Line
(Cable line) (Pressure line)

P-line – is nothing but location of Cable itself. Prestressing force is acting at this location. Also called prestressing force line.



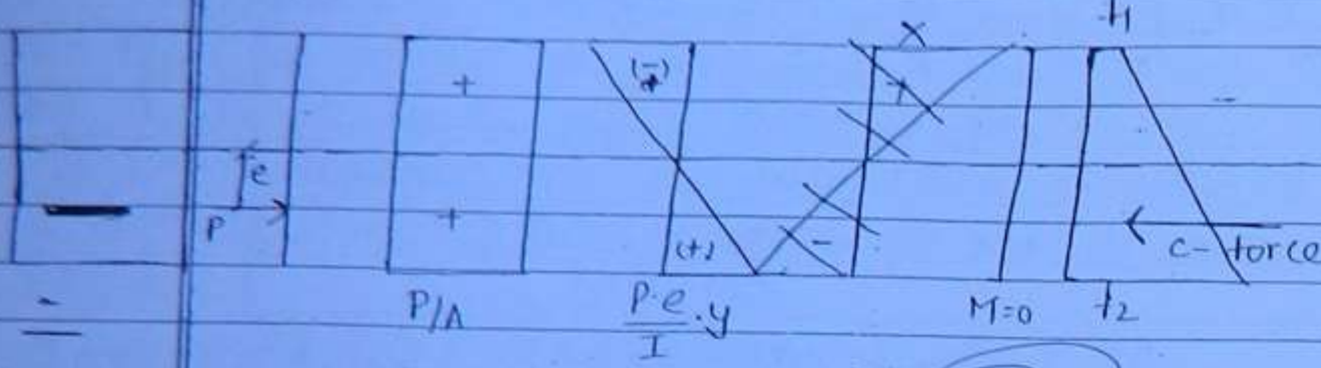

Prestressing force line

C-line – (Thrust line or Pressure line)

It is the location of Resultant thrust force due to final stress diagramme due to all loads. Pressure line is locus of Resultant force of stress diagramme in a beam.

DATE:

At Sec ①-① At end.

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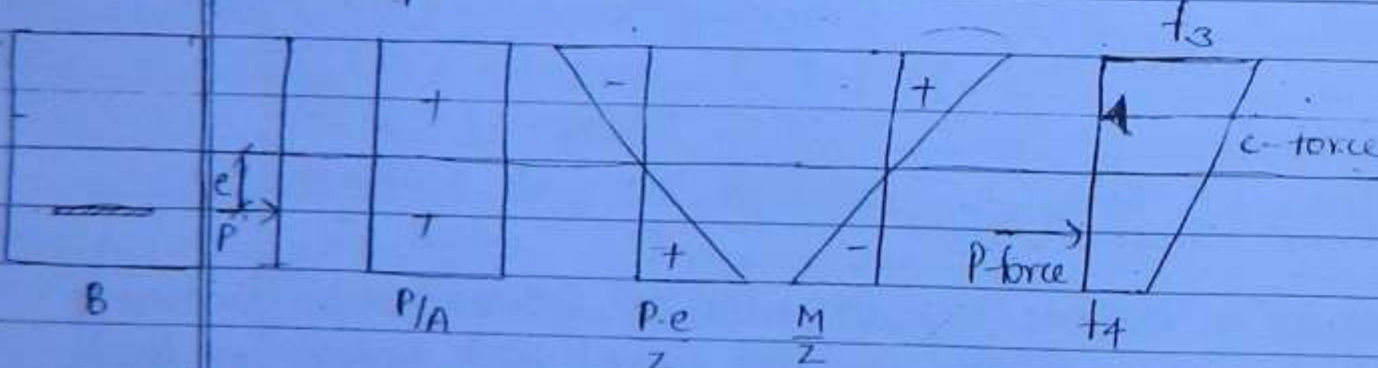
At end where $M = 0$

Resultant C-force always acts at same location of P-force

$$\text{C-force} = B \times D \times \left( \frac{f_1 + f_2}{2} \right) = \text{P-force}$$

At Mid span




Resultant of C-force act at CG of the final stress diagram.

$$\text{C-force} = B \times D \times \left( \frac{f_3 + f_4}{2} \right)$$

This value is equal to P-force always.

At all cross-section the value of c-force = P force

$\Rightarrow f_1 + f_2 \qquad f_3 + f_4$

it means Sum of Stresses at top & bottom is always constant at all section

Shift of c-line w.r.t P-line ($\bar{x}$)

It is always equal to $\bar{x} = \dfrac{M}{\text{P-force / c-force}}$

$$\boxed{\bar{x} = \frac{M}{P}}$$

Shape of c-line is same as the shape of B.M.D.

At each c/s.

Moment M = c-force $\times \bar{x}$

= P-force $\times \bar{x}$

c-force or Pressure line follow the path similar the B.M.D.

At any cross-section eccentricity of C line

$e_c = \bar{x} - e_p$ .

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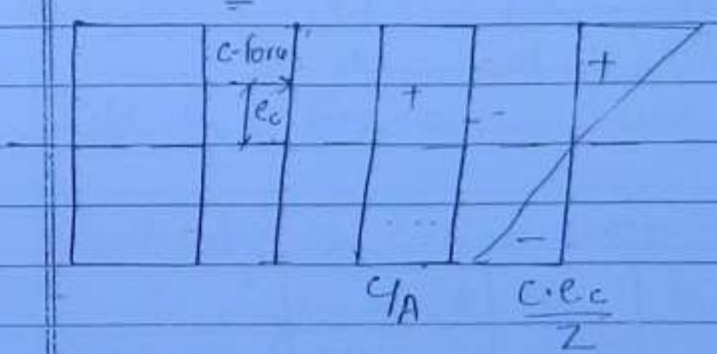
At Mid span




Final Stress $= \dfrac{C}{A} + \dfrac{C \cdot e_c}{Z}$ (top)

$= \dfrac{C}{A} - \dfrac{C \cdot e_c}{Z}$ Bottom

No moment is required to be considered.

(II) ~~Strength~~ Stress Concept Method. (P-force Method)

→ In this method stresses are found using actual location of prestressing force (P-force), without changing any value

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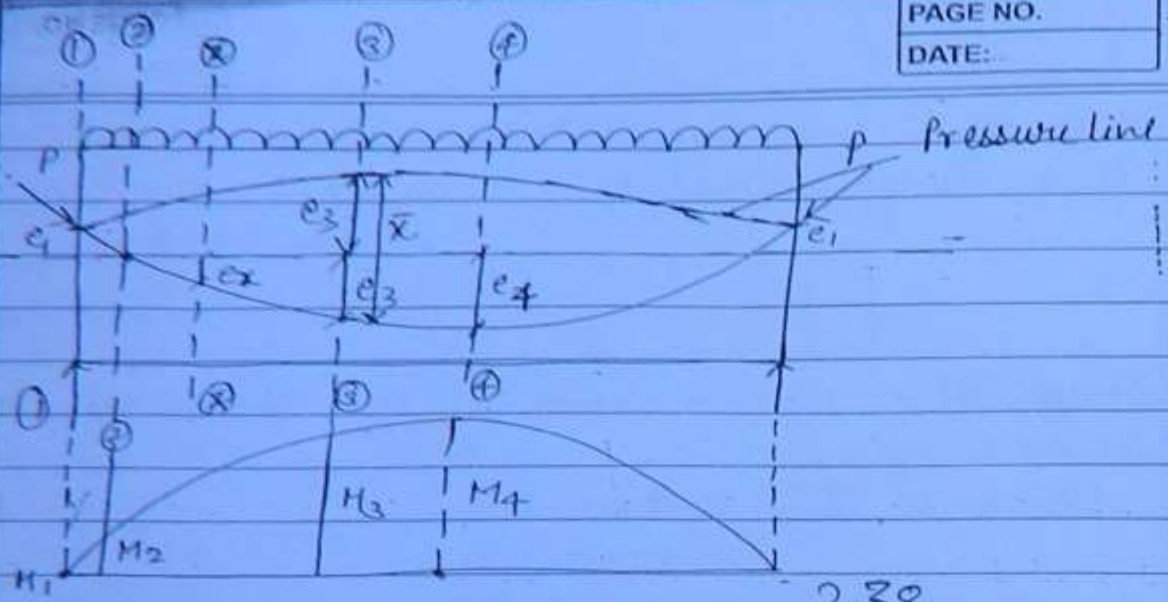


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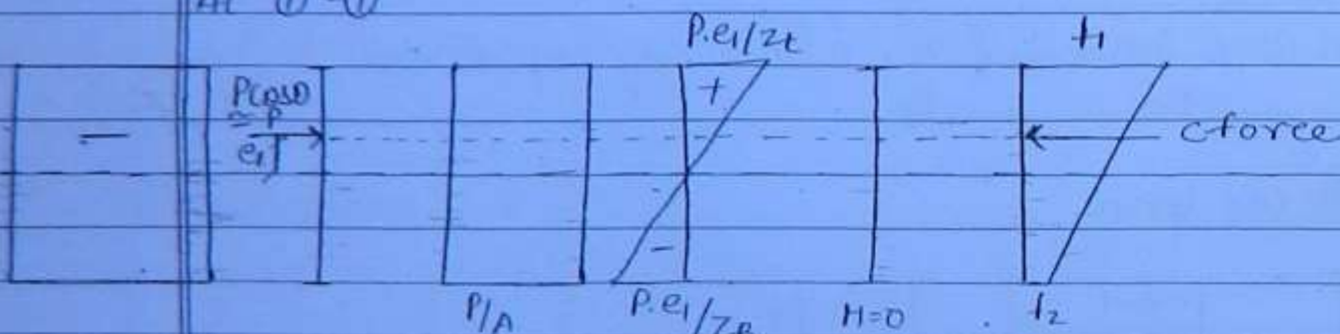
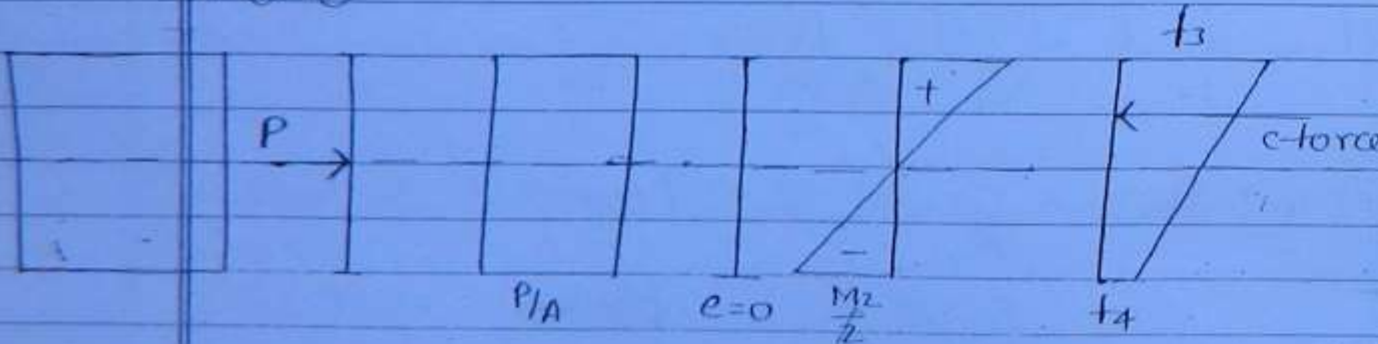
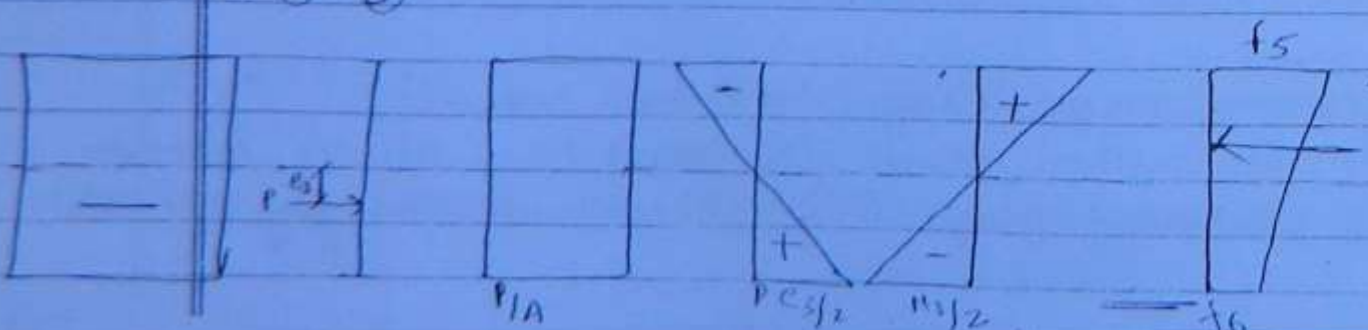
At ① - ①




A ② - ②




At ③ - ③



Final Stress — $f_5 | f_6$ $\quad \frac{P}{A} \mp \frac{P.e_3}{Z} \pm \frac{M_3}{Z}$

In this $\bar{x} = \frac{M_3}{P}$

$$\boxed{e_c = \bar{x} - e_3}$$

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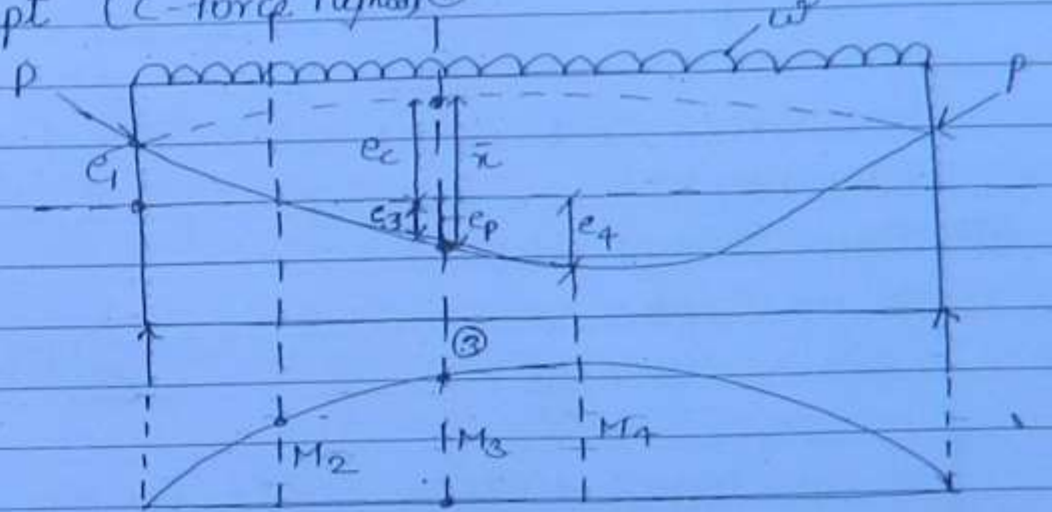
5-Nov-2011

Strength Concept (C-force Method) ③



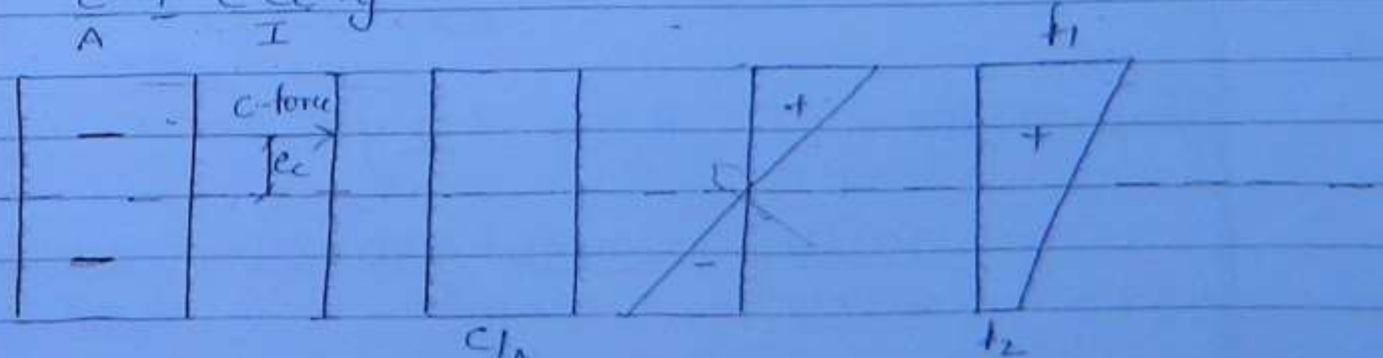

Shift of C line w.r.t P-line $\quad \bar{x} = \frac{M_3}{P}$

Eccentricity of C-line w.r.t N.A

$$e_c = \bar{x} - e_p = \bar{x} - e_3$$

Stress in section

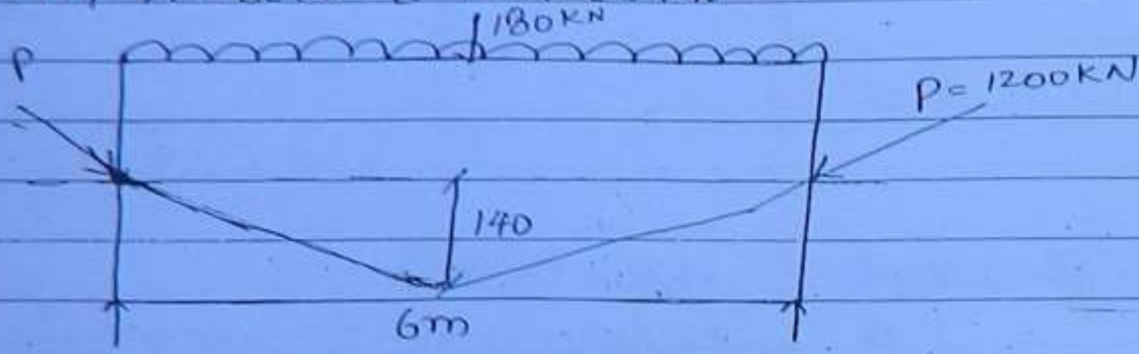
$$\frac{C}{A} \pm \frac{C.e_c}{I} \cdot y$$




In this Case, moment is not considered effect of moment is already considered in $e_c$

2005

**Ques** A PSC beam 400x600 has span 6m. Calculate extreme fibre stress at midspan considering self wt. also.
$e = 140$, $W = 180$ kN & $P = 1200$ kN




Dead load = $0.4 \times 0.6 \times 1 \times 25 = 6$ kN/m
$W = 180$ kN

(a) Stress Concept Method

At mid span

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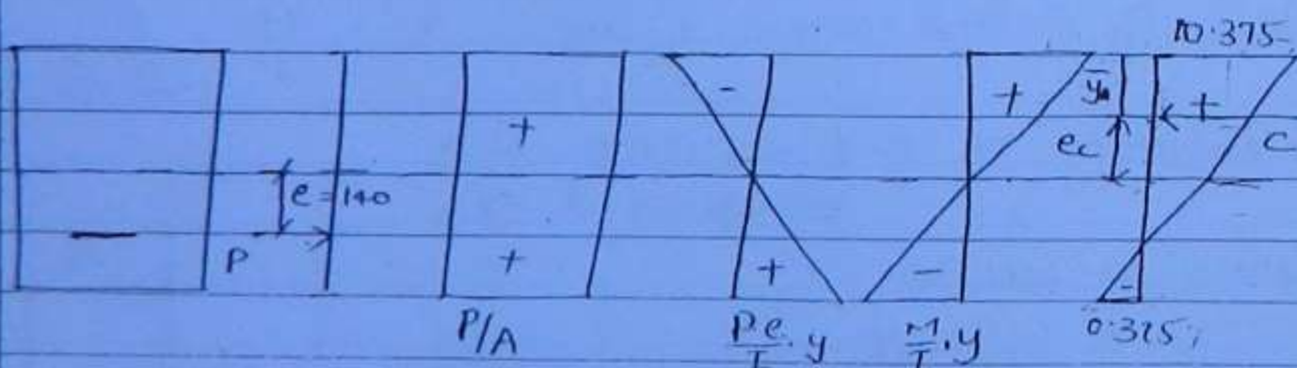
$$B.M = \frac{Wl}{4} + \frac{wl^2}{8}$$

$$= \frac{180 \times 6}{4} + \frac{6 \times 6^2}{8} = 297 \text{ kN-m}$$




$$\text{Stress at top} = \frac{P}{A} - \frac{P.e}{I}.y + \frac{M}{I}y$$

$$= \frac{1200 \times 10^3}{24 \times 10^4} - \frac{1200 \times 10^3 \times 140 \times 300}{72 \times 10^8} + \frac{297 \times 10^6 \times 300}{72 \times 10^8}$$

$$5 - 7 + 12.375$$

$$10.375$$

Stress at bottom $= 5 + 7 - 12.375$

$= -0.375$

DATE:

C-force

$$C = 400 \times 600 \left(\frac{10.375 - 0.375}{2}\right)$$

$$C = 1200{,}000\,N \Rightarrow 1200\,kN = P\ force$$

$$G = \bar{y} = \frac{a+2b}{a+b} \times \frac{h}{3}$$

$$= \frac{10.375 + 2(-0.375)}{10.375 + (-0.375)} \times \frac{600}{3}$$

$$= 192.5\,mm$$

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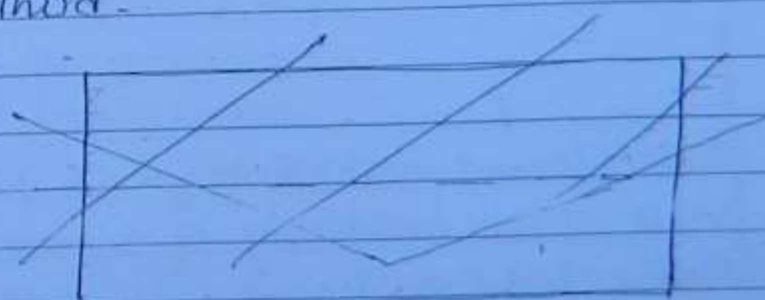
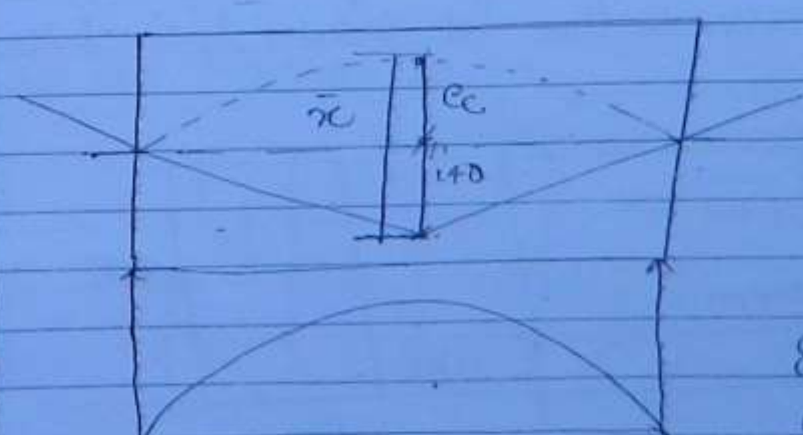
eccentricity of C-force.

$$e_c = 300 - 192.5 = 107.5$$

Strength Concept Method.






Shift of c-line w.r.t to P line

$$\bar{x} = \frac{M}{P} = \frac{297 \times 10^6}{1200 \times 10^3}$$

$$= 247.5$$

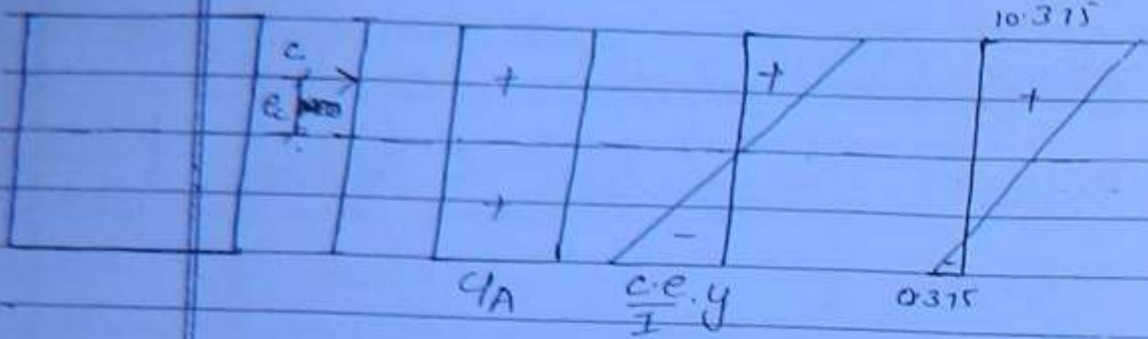
Eccentricity of C-line w.r.t to N.A

$$e_c = \bar{x} - e_p$$

$$247.5 - 140$$

$$= 107.5\,mm$$

(above N.A)




$$\text{Stress} = \frac{C}{A} + \frac{C.e}{I}.y \qquad 242$$

$$= \frac{1200\times10^3}{24\times10^4} + \frac{1200\times10^3\times107.5\times300}{72\times10^8}$$

$$5 \quad -5.375$$

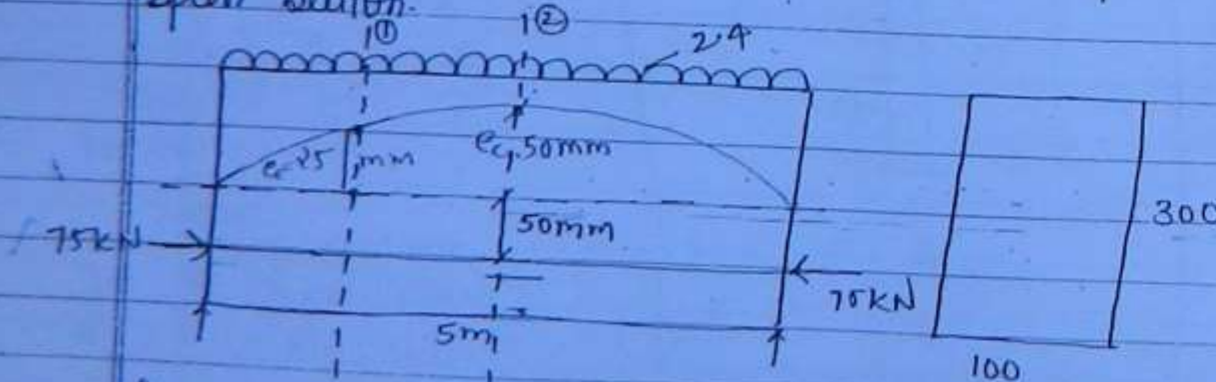
$$= 10.375\ N/mm^2$$

At bottom $= 5+5.375$

$$= -0.375\ N/mm^2$$

Es-1991 Ques

A PSC beam $100\times300\ mm^2$ is prestressed with a straight cable at an eccentricity of 50 mm, P = 75 KN, Span = 5m; Total load on beam including self wt = ~~24~~ 2.4 kN/m. Determine the pressure line at Quarter span & Mid span section.




Area = $3\times10^4\ mm^2$

$I = 225\times10^6\ mm^4$

$w = 2.4\ kN/m$

B.M at mid span $= \frac{2.4\times5^2}{8}$ 7.5 kN·m

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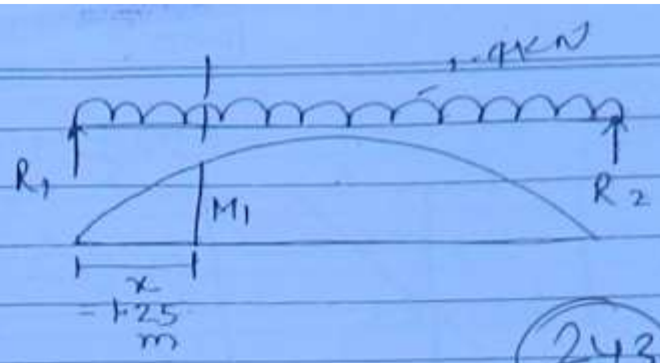


$R_1 = R_2 = \frac{2.4 \times 5}{2} = 6 kN$

$M_1 = R_1 \times 1.25 - \frac{w \times 1.25^2}{2}$

$= 6 \times 1.25 - \frac{2.4 \times 1.25^2}{2}$

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$= 5.625$ kN-m

Strength Concept Method

Shift of c-line w.r.t to P-force line

$\bar{x} = \frac{M \text{ at quarter span}}{P} = \frac{5.625 \times 10^6}{75 \times 10^3}$

$= 75$ mm

Eccentricity of c-line w.r.t to N.A

$e_c = \bar{x} - e_p$

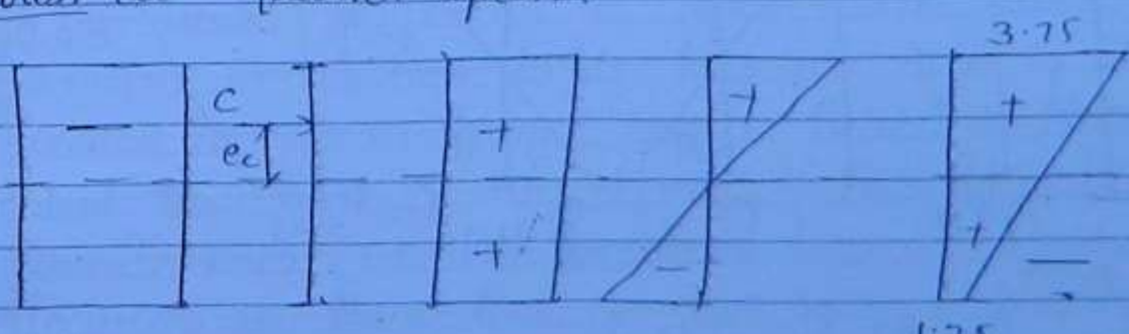
$= 75 - 50 = 25$ mm (above N.A)

Shift of c-line at mid span

$\bar{x} = \frac{M \text{ mid span}}{P} = \frac{7.5 \times 10^6}{75 \times 10^3} = 100$ mm

Eccentricity $= e_c = \bar{x} - e_p = 100 - 50 = 50$ mm

Stress at quarter span.



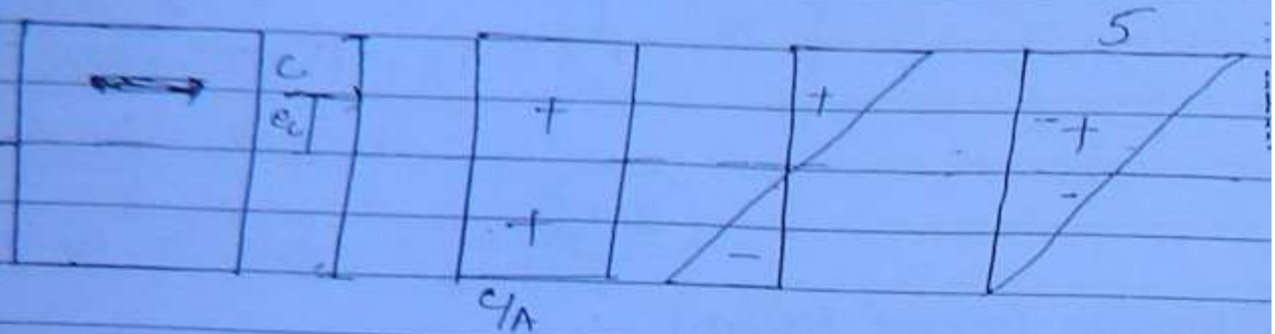

At top $= \frac{C}{A} + \frac{C e_c}{I} y = \frac{75 \times 10^3}{3 \times 10^4} + \frac{75 \times 10^3 \times 25}{225 \times 10^6} \times 150$

$= 2.5 + 1.25$

$= 3.75$

At bottom $= 2.5 - 1.25 = 1.25$

At mid span Stresses




At top $= \dfrac{C}{A} + \dfrac{C \cdot e_c}{I} \cdot y$

$= \dfrac{75\times10^3}{3\times10^4} + \dfrac{75\times10^3 \times 50}{225\times10^6} \times \cancel{300}\ 150$

$= 2.5 + 2.5$

$= 5$

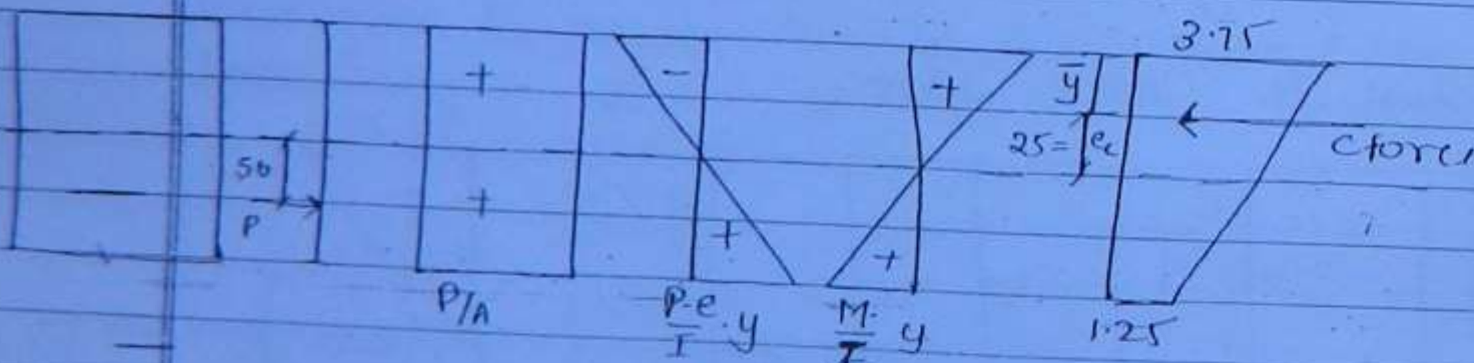
At bottom: $2.5 - 2.5$    2481

$= 0$

(b) Stress Concept Method

At Quarter Span

B.M = 5.625 KN-m




At top $= \dfrac{P}{A} - \dfrac{P e}{I} \cdot y + \dfrac{M}{I} y$

$= \dfrac{75\times10^3}{3\times10^4} - \dfrac{75\times10^3 \times 50}{225\times10^6} \times 150 + \dfrac{5.625\times10^6}{225\times10^6} \times 150$

$2.5 - 2.5 + 3.75$

$= 3.75\ N/mm^2$

DATE:

at bottom $\frac{P}{A} + \frac{P.e}{I}.y - \frac{M}{I} y$

$= 2.5 + 2.5 - 3.75$

$= 1.25 \ N/mm^2$

Shift $\bar{x} = \frac{M}{P} = \frac{5.625 \times 10^6}{75 \times 10^3} = 75mm$

$e_c = 75 - 50 = 25mm$ 245

$C.G = \frac{a+2b}{a+b} \times \frac{h}{3} = \frac{3.75 + 2(1.25)}{3.75 + 1.25} \times \frac{300}{3}$

$= 125 \ mm$

$C\text{-force} = B \times D \times \frac{(f_1 + f_2)}{2}$

$= 100 \times 300 \left(\frac{5}{2}\right)$

$= 75 \ KN = P$ force

$e_c = 150 - 125 = 25 \ mm$

2006-ES

**Ques** A PSC beam 300x600mm span = 12m Carries a U.D.L 12 KN/m in addition to its self wt. Area of steel is 2000 $mm^2$ located at 175mm from soffit. Cable is straight initially stress to a level of 800 $N/mm^2 = P_0$. It is bonned to concrete (i.e Pre. tensioned beam). Determine the location of thrust line at mid span & end span

$\frac{E_s}{E_c} = 6$

Sol. Prestress force $= P_0 \times A_s = 2000 \times 800 \times 10^{-3}$

$= 1600KN$

B.M at Mid Span $= \frac{12 \times 12^2}{8} = \frac{wl^2}{8}$

$= 216$ KN-m

$A = 300 \times 600 = 18 \times 10^4$ mm$^2$

$I = \frac{300 \times 600^3}{12} = 54 \times 10^8$

At end Moment $= 0$

Shift of C-line w.r.t to P-line

$\bar{x} = \frac{M}{P} = 0$

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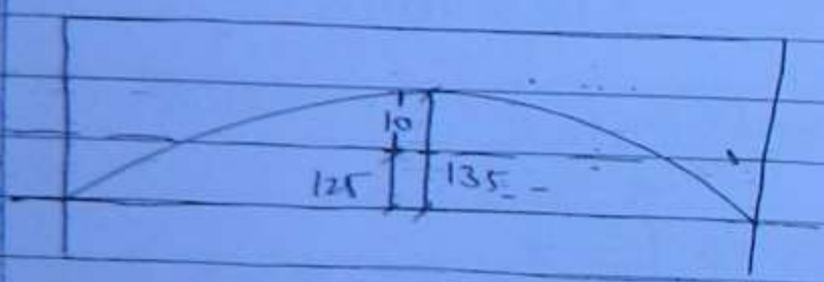
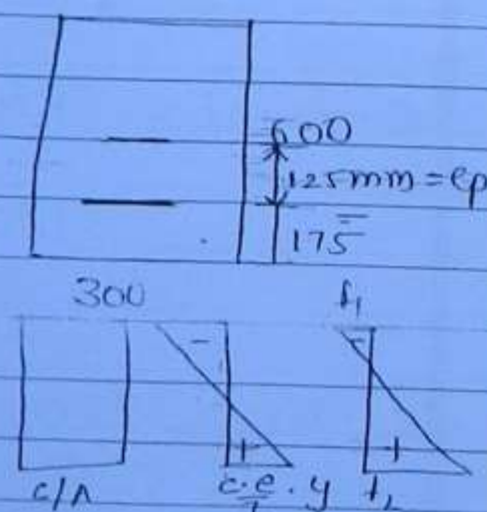
Eccentricity of C-line $= e_c = \bar{x} - e_p$




$e_c = 0 - 125$

$= -125$

125 mm below N.A

Stress $= \frac{C}{A} \mp \frac{C.e_c}{I}.y$

$= \frac{1600 \times 10^3}{18 \times 10^4} \mp \frac{1600 \times 10^3 \times 125}{54 \times 10^8} \times 300$

$8.889 \mp 11.11$

At Top $= -2.227$ N/mm$^2$

at bottom $= 19.999$ N/mm$^2$

$f_1 + f_2 = 17.78$ N/mm$^2$

b). At Mid Span

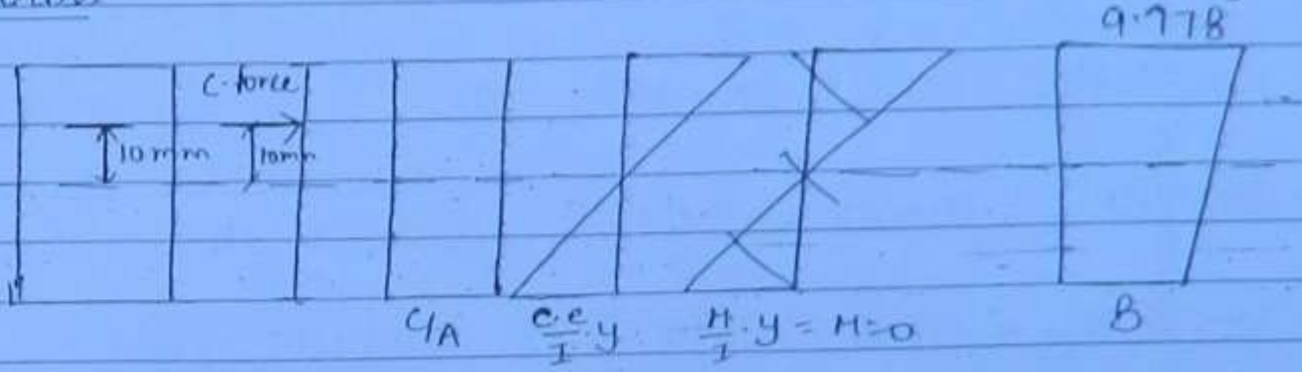
Shift $\bar{x} = \frac{M}{P} = 135$ mm.

Eccentricity $= e_c = \bar{x} - e_p$

$= 135 - 125$

$e_c = 10$

Stress




Stresses $= \frac{C}{A} + \frac{C.e}{I}y \pm \frac{M}{I}.y$ (247)

$= 8.889 \pm \frac{1600 \times 10^3 \times 10}{54 \times 10^8} \times 300 \pm \cancel{\frac{216 \times 10^6}{54 \times 10^8} \times 300}$

$8.889 + 0.889 \cancel{\pm 12}$

at top $= 9.778$

at bottom $= 8$

Ques A psc beam is as shown in fig.

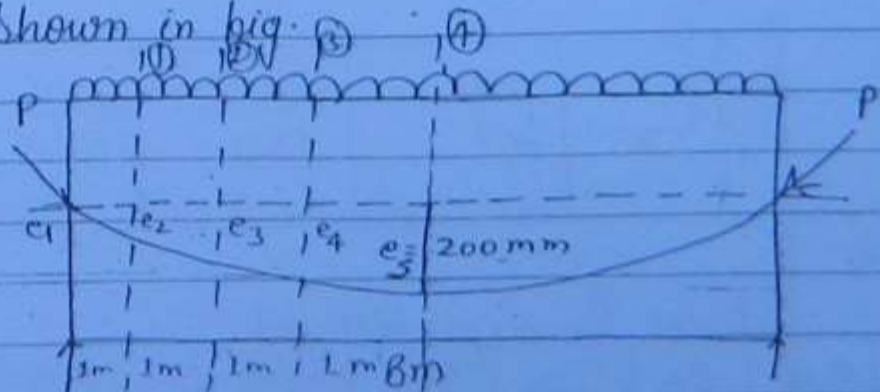
$w = 30$ kN/m

excluding self wt.

Size of beam $=$ $400 \times 750$ mm

$e = 200$ mm

$P = 1500$ kN




Show the position of thrust line at every 1m distance & find out stresses at top & bottom.

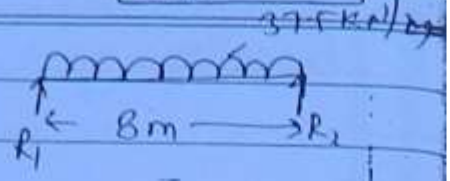
Soln $A = 3 \times 10^5$ mm$^2$

$I = 1.41 \times 10^{10}$ mm$^4$

Self wt $= .4 \times .75 \times 1 \times 25 = 7.5$ kN/m

total $w = 37.5$ kN/m.

At (DD)




$R_1 = R_2 = 150 = \frac{37.5 \times 8}{2}$

$B.M = 150 \times 1 - \frac{37.5 \times 1^2}{2}$

1m $B.M_1 = 131.25$ kN-m $= M_1$

2m $BM_2 = 150 \times 2 - \frac{37.5 \times 2^2}{2} = 225$ kN-m

3m $BM_3 = 150 \times 3 - \frac{37.5 \times 3^2}{2} = 281.25$ kN-m

4m $BM_4 = \frac{37.5 \times 8^2}{8} = 300$

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Eccentricity max eccentricity

$y = \frac{4h}{l^2} x(l-x)$

$= \frac{4 \times 200}{8000^2} x(8000-x) = \frac{x(8000-x)}{80000}$

y = 375

| x | 1m | 2m | 3m |
|---|---|---|---|
| | 87.5 | 150 | 187.5 |

Stress Concept Method

| Point | B.M | $e_P$ | P/A | $\frac{P.e}{I}.y$ | $\frac{M}{I}.y$ | Stress at top | Stress at Bottom | $\bar{y}$ | $\bar{y}$-375 $e_c$ |
|---|---|---|---|---|---|---|---|---|---|
| 0m | 0 | 0 | 5 | ∓ 0 | ± 0 | 5 | 5 | | |
| 1m | 131.25 | 87.5 | 5 | ∓ 3.49 | ± 3.49 | 5 | 5 | 562.5 | |
| 2m | 225 | 150 | 5 | ∓ 5.98 | ± 6 | 5 | 5 | 562.5 | |
| 3m | 281.25 | 187.5 | 5 | ∓ 7.5 | ± 7.5 | 5 | 5 | 562.5 | |
| 4m | 300 | 200 | 5 | ∓ 8.0 | ± 8.0 | 5 | 5 | 562.5 | |
| | | | | | | | | 562.5 | |

$P = 2400\,kN$

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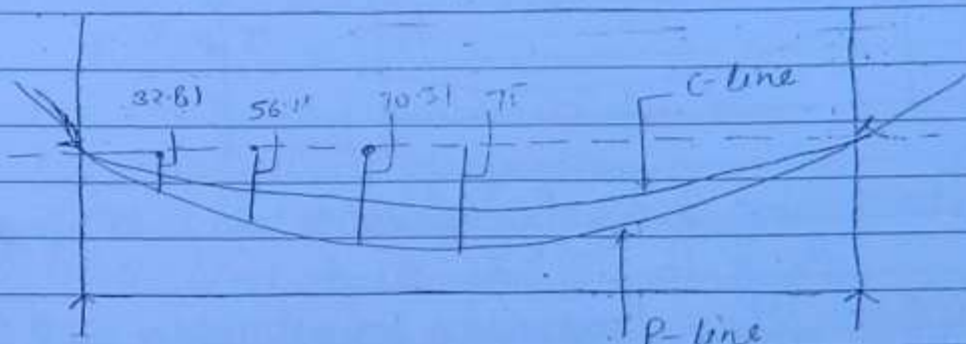
| Point | BM | $e_p$ | $P/A$ | $\frac{P.e}{I}.y$ | $\frac{M}{I}.y$ | Stress at top | Stress at Bottom | $\bar{y}$ | $e_c$ ($\bar{y}-375$) |
|---|---|---|---|---|---|---|---|---|---|
| 0 m | 0 | 0 | ± 8 | ∓ 0 | ± 0 | 8 | 8 | 375 | 0 |
| 1 m | 131.25 | 87.5 | ± 8 | ∓ 5.6 | ± 3.5 | 5.9 | 10.1 | 407.8 | 32.8 ~~320.3~~ |
| 2 m | 225 | 150 | ± 8 | ∓ 9.6 | ± 6 | 4.4 | 11.6 | 431.25 | 56.25 ~~281.25~~ |
| 3 m | 281.25 | 187.5 | ± 8 | ∓ 12.0 | ± 7.5 | 3.5 | 12.5 | 445.31 | 70.31 |
| 4 m | 300 | 200 | + 8 | ∓ 12.8 | ± 8 | 3.2 | 12.8 | 450 | 75 |

below N.A

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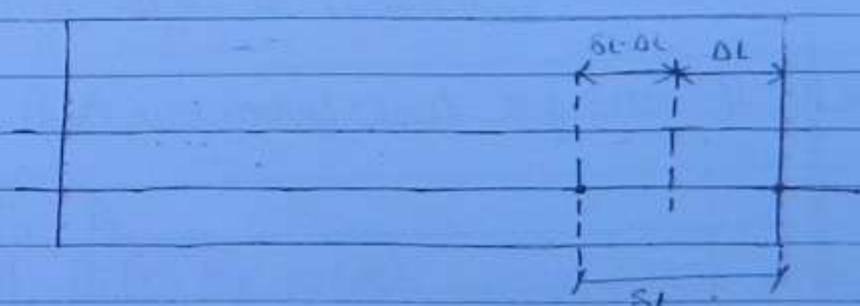
# Losses (Loss of Prestress)




If an initial elongation of $\delta l$ is provided

$$p_0 = \frac{\delta l}{l} \times E_s$$

If $\Delta l$ = loss of elongation provided (due to any reason)
Loss of Prestress

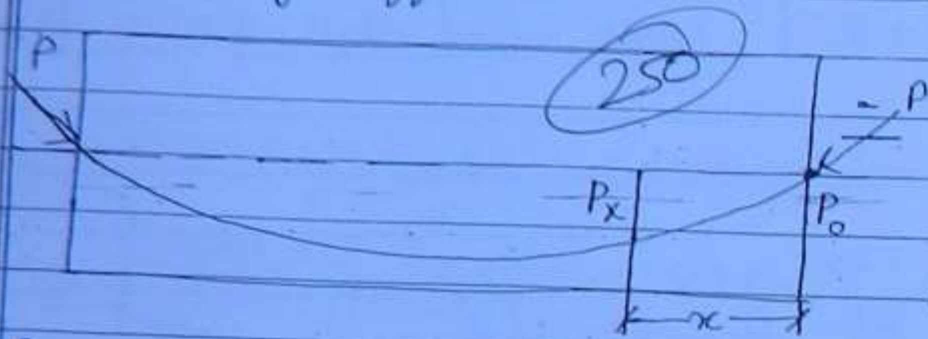
$$P_{loss} = \frac{\Delta l}{l} \times E_s$$

Final stress is due to $(\delta l - \Delta l)$ ~~if an initial elongation of $\delta l$ is provided~~ $p_{final} = \frac{\delta l - \Delta l}{l} \times E_s$

Type of losses

1\. Loss due to friction (During tensioning process)
(Due to Combined effect of length & Curvature).

→ Only for post tensioned beam or member
→ Due to length effect




Prestressing force $P_x$ available at x disⁿ from free end

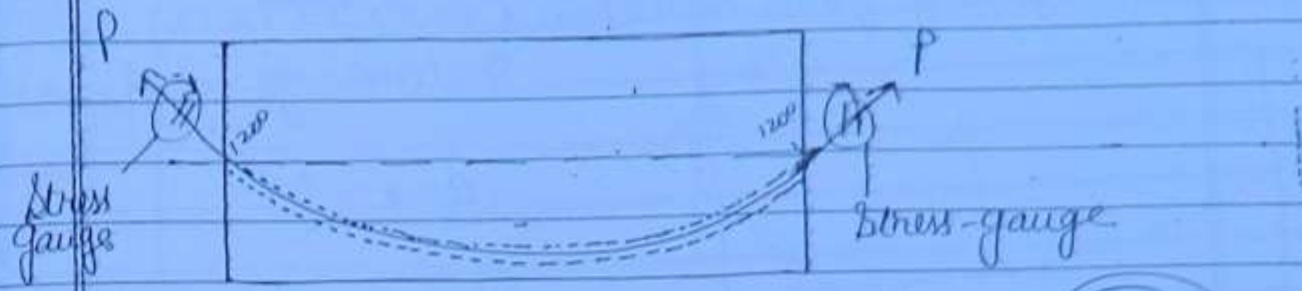
$$P_x = P_0 \cdot e^{-Kx}$$

Where $P_0$ = Prestressing force at free end.
$x$ = distance
$K$ = Wobble Correction factor a constant
$= (15 \times 10^{-4}$ to $50 \times 10^{-4})$ per meter

Case 1 - If jacking is done from one end.


For max loss ⇒ $x = l$ at other end.

Case 2 When jacking is at both ends
For max loss $x = \frac{l}{2}$

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→ Due to Curvature effect.

Prestressing force at x disn from free end.

$$P_x = P_0 e^{-\mu\alpha}$$

Where $P_0$ = Prestressing force at free end.

$\mu$ = Coefficient of friction

= 0.55 – Steel is moving over Concrete

⇒ 0.35 – Steel duct tendon is moving on steel duct.

⇒ 0.25 – Steel tendon are moving over lead.

$\alpha$ = Change in gradient betn two pt. of Consideration

→ Combined effect of length & Curvature

$$P_x = P_0 \cdot e^{-Kx - \mu\alpha}$$

$$P_x = P_0 (1 - (Kx + \mu\alpha))$$

Loss of prestress.

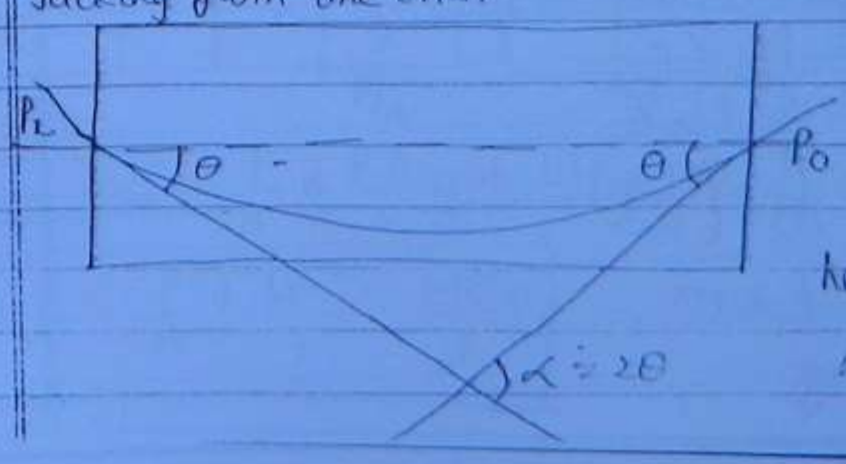
$$= P_0 - P_x$$

$$= P - P_0 (1 - Kx + \mu\alpha)$$

$$= P_0 (Kx + \mu\alpha)$$ – same as recommended by IS code.

Value of $\alpha$ – Eq. of Parabolic Cable $y = \frac{4h}{l^2} x(l - x)$

Jacking from one end.




Slope $= \frac{dy}{dx} = \frac{4h}{l}(l - 2x)$

at $x = 0$

$\theta = \tan\theta = \frac{4h}{l}$ radian

here Change in gradient

$\alpha = 2\theta = \frac{8h}{l}$ radian

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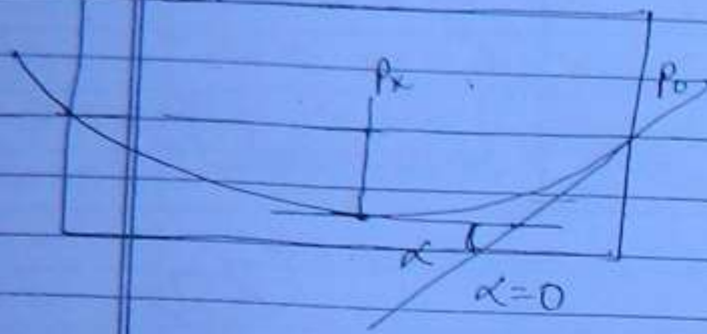
When jacking from both end.




$\theta = \tan\theta = \frac{dy}{dx} = \frac{4h}{l^2}(l-2x)$

at $x = \frac{l}{2}$

$$\boxed{\alpha = \theta = \frac{4h}{l} \text{ radian}}$$

2. Loss due to Anchorage slip, deformation etc. (Post tensioned) (during Anchoring process)

If $\Delta l$ = loss of elongation

loss $= \frac{\Delta l}{l} \times E_s$

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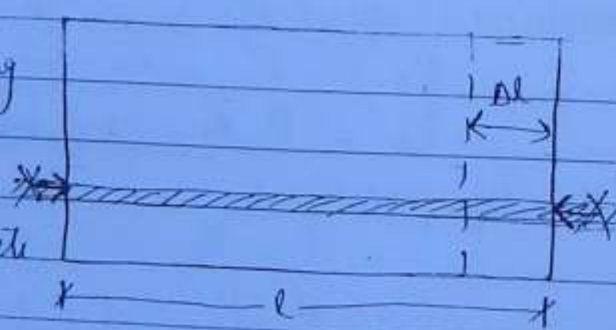
3. Loss occuring subsequently.

(a) Due to elastic shortening

→ For pretensioned Members.

(a) Just after Cutting the reinforcement prestressing force is applied on Concrete.




Elastic Shortening of Concrete due to P compressive force

$\Delta l = \frac{Pl}{A_c E_c}$

The same deformation will be in steel also

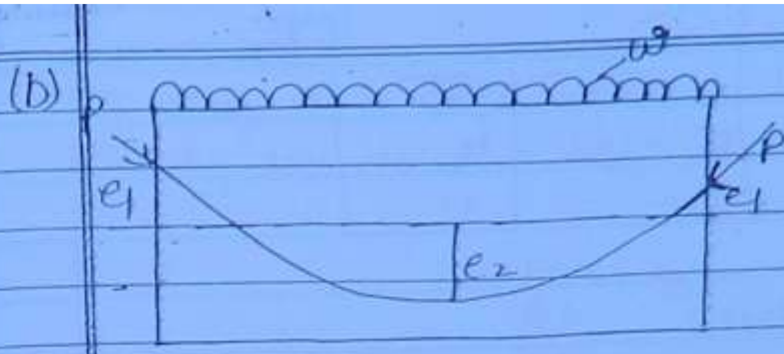
Loss of Prestress in steel $= \frac{\Delta l}{l} \times E_s$

$= \frac{Pl}{A_c E_c} \cdot \frac{E_s}{l}$

$= \frac{P}{A_c}\left(\frac{E_s}{E_c}\right)$

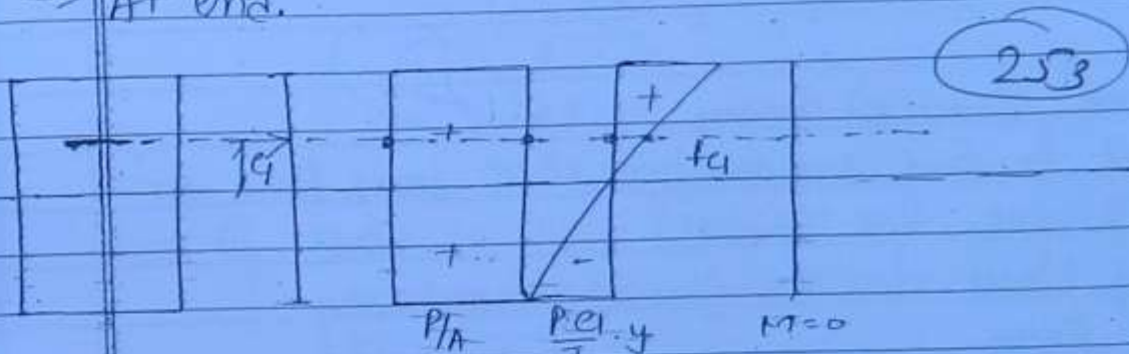
Loss of prestress due to elastic shortening $f_c \times m$

(b)




Stress in concrete at the level of steel is found at end & at mid span.

→ At end.

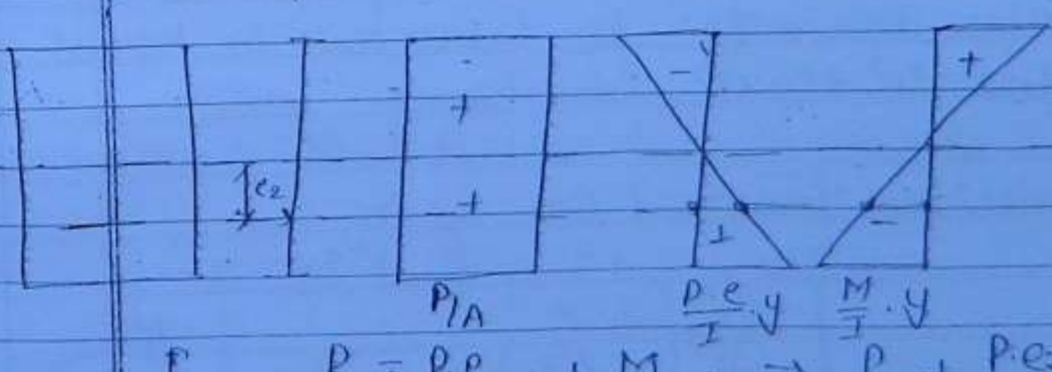



Stress in concrete at the level of steel

$$f_{c_1} = \frac{P}{A} + \frac{P.e_1}{I}.y = \frac{P}{A} + \frac{P.e_1}{I}e_1$$

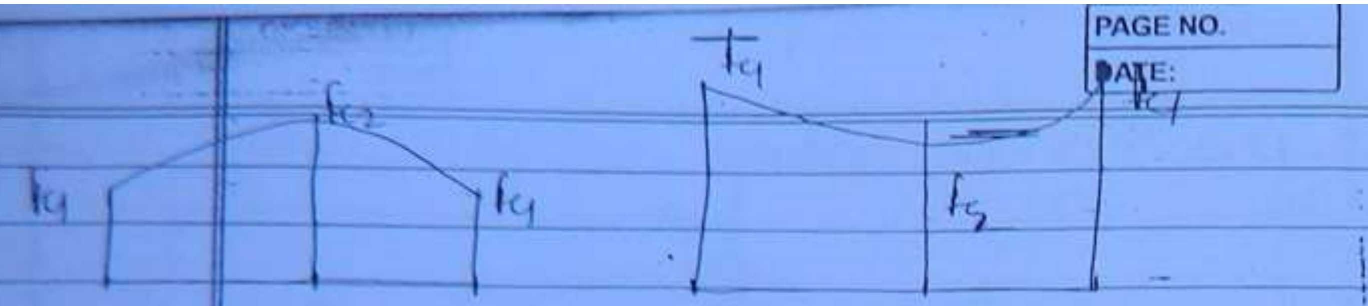
$$\boxed{f_{c_1} = \frac{P}{A} + \frac{P.e_1^2}{I}}$$

At Mid span




$$f_{c_1} = \frac{P}{A} \mp \frac{P.e}{I}y \pm \frac{M}{I}y \Rightarrow \frac{P}{A} + \frac{P.e_2}{I}y - \frac{M}{I}y$$

$$\boxed{f_{c_1} = \frac{P}{A} + \frac{Pe_2^2}{I} - \frac{Me_2}{I}}$$




→ Average $f_c$ is Calculated.

① $f_c = \frac{f_1 + f_2}{2}$ ← B.C Punmia

② $f_c = f_{c1} + \frac{2}{3}(f_{c2} - f_{c1})$ ← K. Raju

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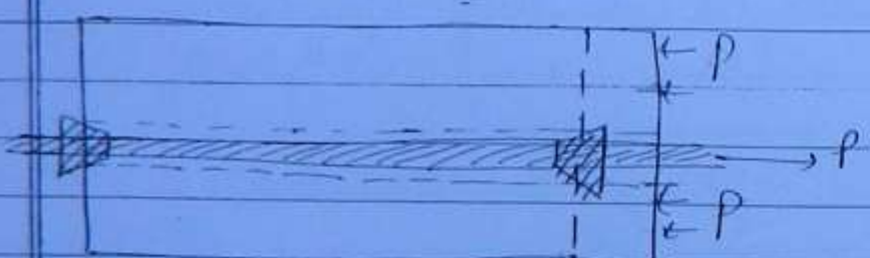
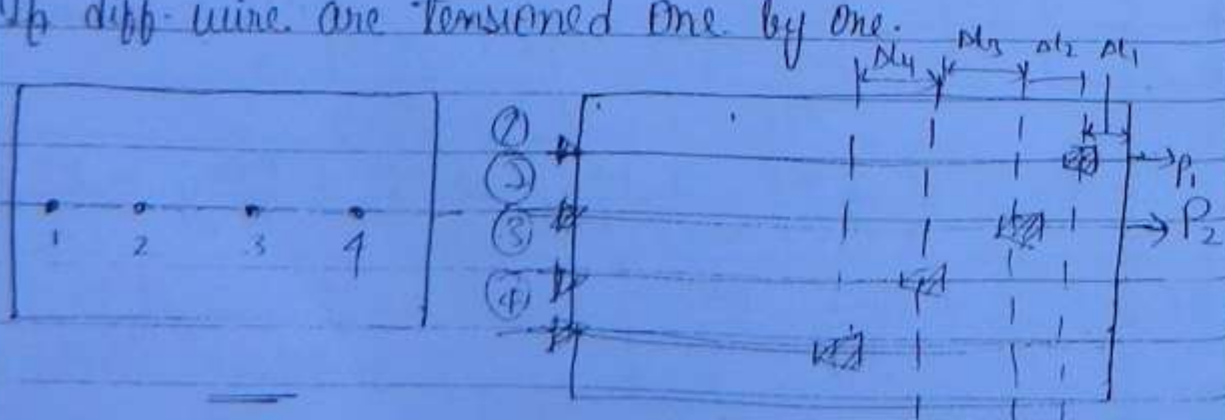
→ For post tensioned beam.

Case 1. If all tendons are tensioned at same time or There is only one tendon.
Loss of stress due to elastic shortening
= zero.




In this case elastic shortening take place before anchoring the reinforcement so there will be no further elastic shortening after anchoring. So there will be no loss in steel due to elastic shortening.

Case 2. If diff. wire are tensioned one by one.



If Reinf$^n$ 1 is anchored 1 & 4 th is anchored las

Finally loss in steel. NO - 4 $= 0$

" " " - 3 = Due to prestressing force $P_4$

$$= \frac{\Delta l_4}{l} \times E_s$$

$$= \frac{P_4 \cdot l}{A_c E_c \times l} \cdot E_s = f_{c4} \cdot m$$

Loss of stress in wire (2) - Due to prestressing force $P_3$ & $P_4$

$$= \frac{\Delta l_3 + \Delta l_4}{l} \times E_s$$

$$\frac{P_c + P_4}{A_c} \times m = (f_{c3} + f_{c4}) m$$



Loss of stress in wire (1)

$$\frac{\Delta l_2 + \Delta l_3 + \Delta l_4}{l} \times E_s$$

$$= (f_{c2} + f_{c3} + f_{c4}) m =$$

Loss due to Creep of Concrete

Loss of stress = Creep strain $\times E_s$

Creep Coefficient $= \dfrac{\text{Creep strain}}{\text{Elastic strain}}$

Creep strain = Creep Coefficient $\times$ Elastic strain

$$\phi \times \frac{\delta l}{l}$$

Loss of stress due to Creep $= \phi \times \dfrac{\delta l}{l} \times E_s$

$$= \phi \times \frac{P \cdot l}{A_c \cdot E_c \cdot l} \times E_s$$

$$= \phi \left(\frac{P}{A_c}\right)\left(\frac{E_s}{E_c}\right)$$

$$= \phi \cdot m \cdot f_c$$

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Creep Coefficient —

| | | | |
|---|---|---|---|
| $\phi$ = | 7 days | 28 days | 1 year |
| $\phi$ = | 2.2 | 1.6 | 1.1 |

Loss due to Creep of Steel (Relaxation of steel)

Loss = 1 to 5% of $P_o$

Generally 2 to 3% of $P_o$ is Considered.

Loss due to Shrinkage of Concrete.

Loss of Stress = Shrinkage Strain $\times E_s$

(i) Pretensioned PSC Member

$$= 3\times10^{-4}\times E_s$$

(ii) Post tensioned PSC Member

$$= \frac{2\times10^{-4}}{\log_{10}(T+2)}\times E_s$$

T = Age of loading in days.

256.

| TYPE OF LOSS | PRETENSIONED | POST-TENSIONED |
|---|---|---|
| → During tensioning (due to friction) Length & Curvature | — | $P_o(kx+\mu\alpha)$ |
| → During Anchoring due to anchorage slip) | — | $\frac{\Delta l}{l}\times E_s$ |
| Due to elastic Shortening | $(m\times f_c)$ | ① If one & all tendon tensioned at same time = 0<br>② Subsequent tensioning $=\Sigma\frac{\delta l}{l}\times E_s = \frac{\Sigma P}{A}\times m$ |
| Due to Creep of Concrete | $\phi\cdot m\cdot f_c$ | $\phi\cdot m\cdot f_c$ |
| Due to Creep of Steel | 1 to 5% | 1 to 5% |
| Due to Shrinkage of Concrete | $3\times10^{-4}\times E_s$ | $\frac{2\times10^{-4}}{\log_{10}(T+2)}\times E_s$ |

Total loss of Prestress

| | TYPE OF LOSS | PRETENSIONED | POST TENSIONED |
|---|---|---|---|
| 1. | Elastic Shortening | 3% | 1% |
| 2. | Creep of Concrete | 6% | 5% |
| 3. | Creep of Steel | 2% | 3% |
| 4. | Shrinkage of Concrete | 7% | 6% |
| | | 18% | 15% |

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ES-2001

Ques A simply supported Post tensioned Concrete beam span 15m has rectangular C/S 300 X 600 mm. The prestress force is 1150 KN $\bar{c}$ zero eccentricity at end & at mid span is 200 mm. Cable is parabolic. Friction Coeffⁿ = 0.15 per 100m (K & μ = 0.35. Determine the loss due to friction at centre of beam

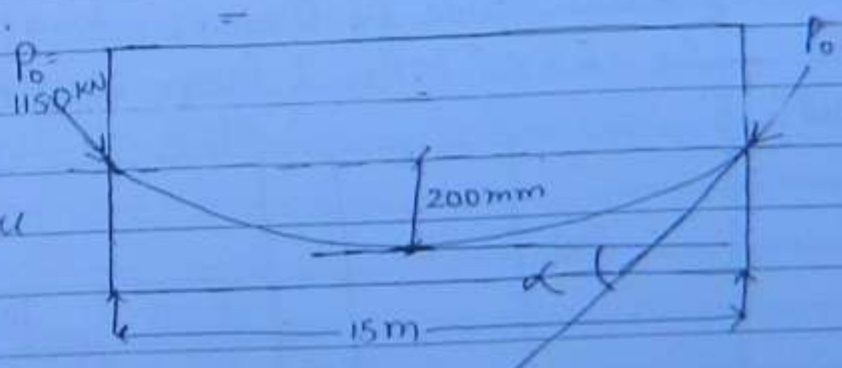
Solⁿ $P_0 = 1150\,KN$

$K = \frac{0.15}{100}$ per metre

$= 0.0015$ per metre

$\mu = 0.35$

$x = 7.5m$




Value of $\alpha$

$\alpha = \theta$

Eqⁿ of Cable. $y = \frac{4h}{l^2} x(l-x)$

$\frac{dy}{dx} = \frac{4h}{l^2}(l-2x)$

Slope at end at $x = 0$

$\theta = \tan\theta = \frac{dy}{dx} = \frac{4h}{l}$

$\Rightarrow \alpha = \theta = \frac{4 \times 200}{15000} = 0.5333$ radian.

$$\theta = 3^\circ 3^\circ \cancel{10.55''}$$

$$= \frac{3^\circ 3^\circ 10.38 \times \pi}{180} = 0.053328 \text{ radians}$$

Loss of prestress due to friction

$$= P_0 (Kx + \mu\alpha)$$

$$= 1150 (0.0015 \times 7.5 + 0.35 \times 0.0533)$$

$$= 34.4 \text{ KN}$$

$$\% \text{ loss} = \frac{34.4}{1150} \times 100 = 2.99\%$$

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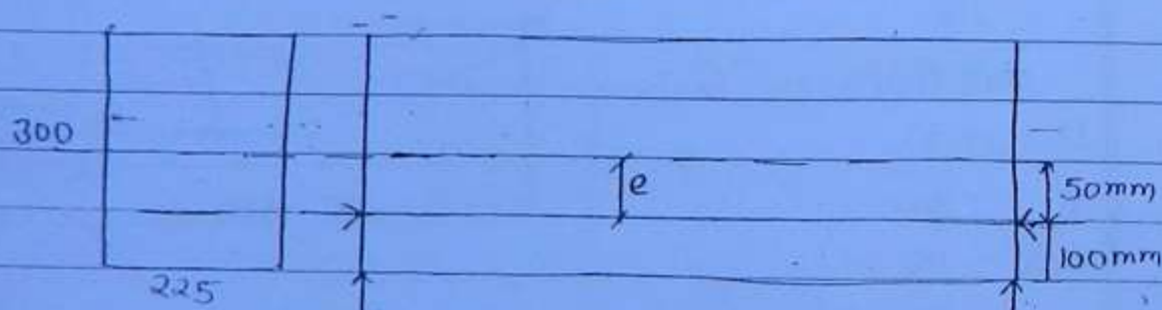
5-2003

**Ques** A Pretensioned beam of size 225 × 300 mm is prestressed by 12 wires of 5 mm ϕ initially prestressed at 1100 MPa. The centroid of prestressing wire is located at 100 mm from bottom. Estimate the loss of prestress due to all reason. Grade of Concrete – M-40, Relaxation of steel = 5%. $E_s = 2 \times 10^5$ MPa & Creep Coefficient = 1.6

Sol.



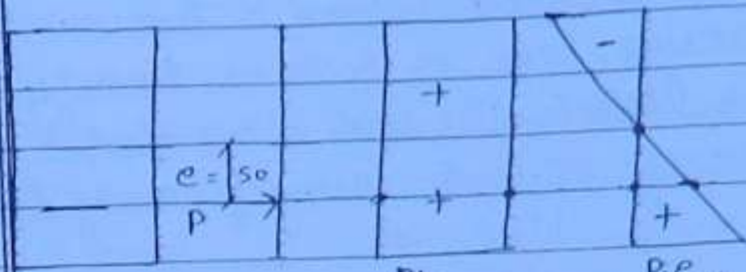

$$A_s = 12 \times \frac{\pi}{4} \times 5^2 = 235.62 \text{ mm}^2$$

$$P_0 = A_s \times p_0 \Rightarrow 235.62 \times 1100$$

$$= 259.18 \text{ KN}$$

$p_0 = 1100 \text{ N/mm}^2$ (initially Prestress)

1. Loss due to elastic shortening $= m \times f_c$



$P/A$ $\quad \frac{P.e}{I}.y$

Stress in Concrete At the level of Steel -

$$= \frac{P}{A} + \frac{P.e}{I}y = \frac{259.18\times10^3}{300\times225} + \frac{259.18\times10^3\times50^2}{50625\times10^4}$$

$f_c = 3.84 + 1.28$

$f_c = 5.12\ N/mm^2$

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loss $= m \times f_c$

$\frac{E_s}{E_c} \times f_c$

$\frac{2\times10^5}{5000\sqrt{40}} \times 5.12 =$

$= 32.38\ N/mm^2$

% loss $= \frac{32.38}{1100}\times100 = 2.94\%$.

Creep of Concrete $= \phi \cdot m \cdot f_c$

$= 1.6\times32.38$

loss $= 51.81\ N/mm^2$

Creep of Steel $= 5\%$ of $P_o$

$= \frac{5}{100}\times1100 = 55\ N/mm^2$

Shrinkage of Concrete $= 3\times10^{-4}\times E_s$

$= 3\times10^{-4}\times2\times10^5$

$= 60\ N/mm^2$

Total loss $= 32.38+51.81+55+60 = 199.19$

% loss $= \frac{199.19}{1100}\times100 = 18.11\%$.

ES 1993

**Ques** A Post tensioned, Prestressed Concrete beam is tensioned initially by a Prestressing force of 1458 kN transfered at 28 days Strength of Concrete. The Cable is parabolic with an eccentricity of 520 mm at Centre. using following data

$A_c = 2.42 \times 10^5\ mm^2$ , $A_s = 1385\ mm^2$

$I = 5.3 \times 10^{10}\ mm^4$ $f_s = 1059\ N/mm^2$ at transfer

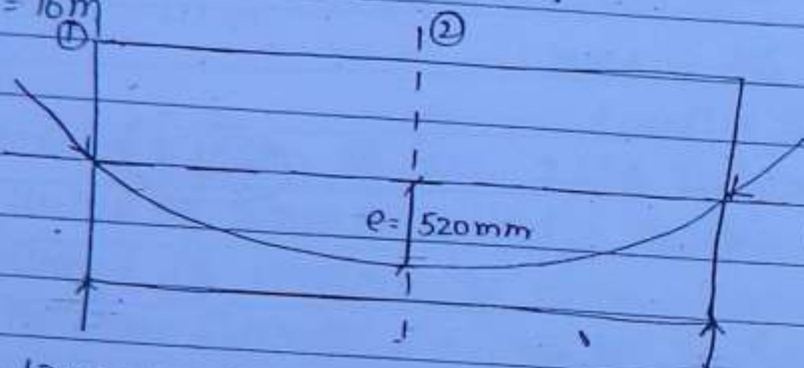
$E_s = 2.1 \times 10^5\ N/mm^2$

$E_c = 0.382 \times 10^5\ N/mm^2$ , Anchorage Slip $= 2.5mm$

$k = 0.0015$ /metre , $\mu = 0.25$ | Calculate all losses.

Span = 16m

Soln

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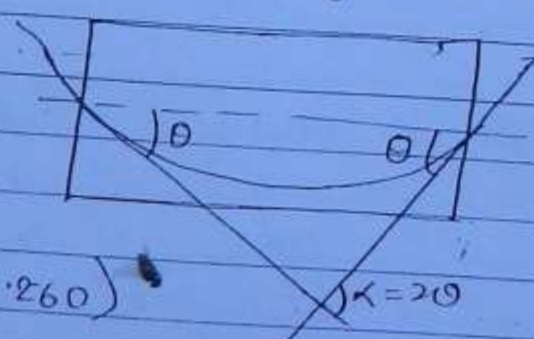
$P = 1385 \times 1059 = 1466.7\ kN.$

1. Loss due to friction $= P_0 (Kx + \mu\alpha)$

Assuming jacking is done from one end only. Max loss will be at other end

$\alpha = 2\theta = \frac{8h}{l}$

$= \frac{8 \times 520}{16 \times 10^3} = .260$ radian




loss ⇒ $1059\,(0.0015 \times 16 + 0.25 \times .260)$

$= 94.251\ N/mm^2$

2. Loss due to Anchorage slip $= \frac{\Delta l}{l} \times E_s$

$= \frac{2.5}{16 \times 10^3} \times 2.1 \times 10^5$

$= 32.81\ N/mm^2$

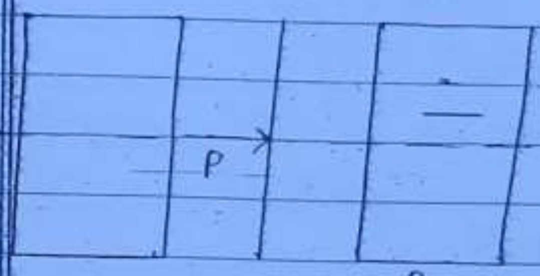
DATE:

3. Loss due to elastic Shortening of Concrete = 0

4. Loss due to Creep of Concrete = $\phi \cdot m \cdot f_c$

Stress in Concrete at the level of Steel.

At ends

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$P/A$

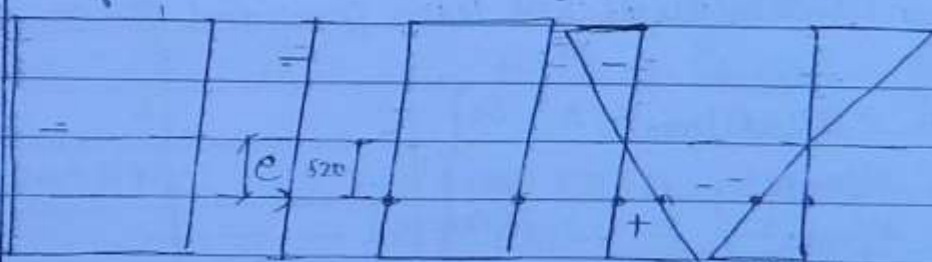
$$f_{c_1} = \frac{P}{A} = \frac{1466.7 \times 10^3}{2.42 \times 10^5} = 6.06 \text{ N/mm}^2$$

At Mid span

$$M_d = \frac{w_d \times l^2}{8}$$

$$w_d = \frac{2.42 \times 10^5}{10^6} \times 1 \times 25 = 6.05$$

$$M_d = 193.6 \text{ kN-m}$$



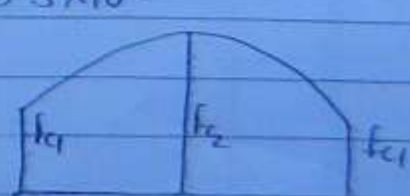

$P/A$ $\quad \frac{P.e}{I}.y$ $\quad \frac{M}{I}.y$

$$f_{c_2} = \frac{P}{A} + \frac{P.e^2}{I} - \frac{M.y}{I} = \frac{1466.7 \times 10^3}{2.42 \times 10^5} + \frac{1466.7 \times 10^3 \times 520^2}{5.3 \times 10^{10}} - \frac{193.6 \times 10^6 \times 520}{5.3 \times 10^{10}}$$

$$f_{c_2} = 6.06 + 7.48 - 1.899$$

$$= 11.641$$

$$f_c = \frac{f_{c1} + f_{c2}}{2} = 8.85 \text{ N/mm}^2$$




$$\frac{E_s}{E_c} = m = 5.497$$

$$\text{Loss} = \phi \times m \times f_c = 1.6 \times 5.497 \times 8.85$$

$\phi = 1.6$ at 28 days. $\quad = 77.84 \text{ N/mm}^2$

5. ~~Loss~~ Loss due to Creep of Steel $= 1 \text{ to } 5\%$ of $P_0$

$= 3\%$ of $P_0$

$= \frac{3}{100} \times 1059 = 31.77 \, N/mm^2$

6. Loss due to Shrinkage $= \frac{2 \times 10^{-4}}{\log_{10}(T+2)} E_s$

$= \frac{2 \times 10^{-4}}{\log_{10}(28+2)} \times 2.1 \times 10^5$

$= 28.43 \, N/mm^2$

Total loss $= 265.101 \, N/mm^2$

$\%$ loss $= \frac{265.101}{1059} \times 100$

$= 25\%$

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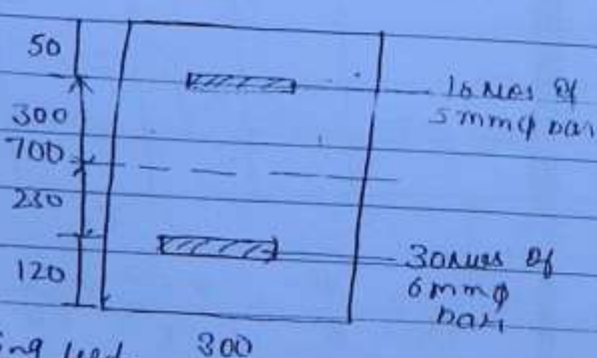
**Ques** A PSC beam (Pretensioned) has been provided tendons as shown in fig

Initial Prestress $= 1200 \, N/mm^2$

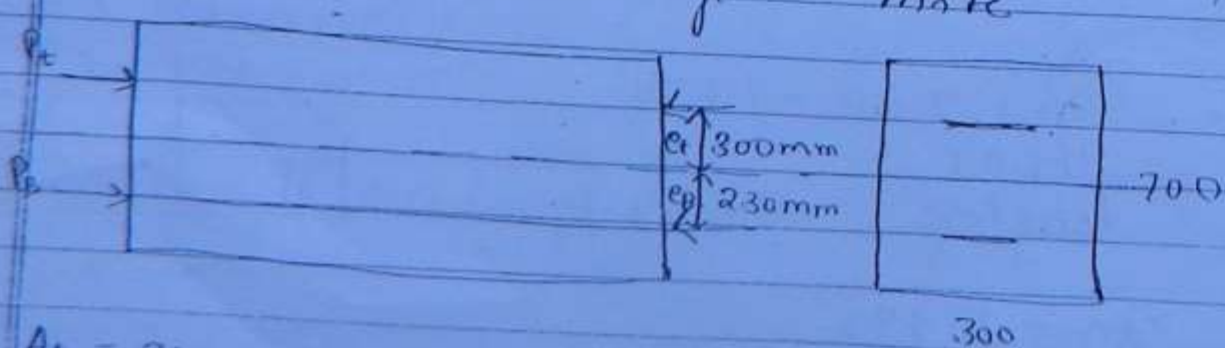
$E_s = 210 \times 10^3 \, N/mm^2$

$E_c = 36 \times 10^3 \, N/mm^2$

Calculate loss in wires due to elastic shortening. The Member is still supported on its prestressing bed.




**Sol** Loss due elastic Shorteninge $= m \times f_c$

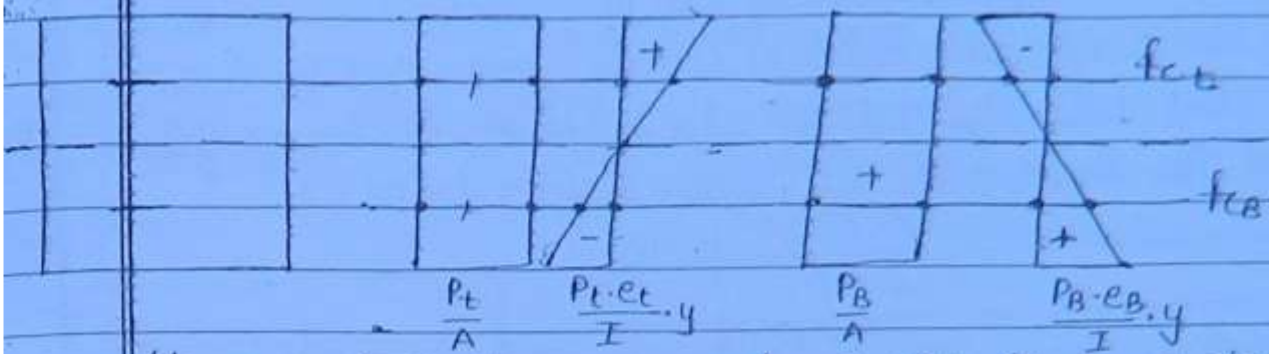



$A_t = 314.16$

$P_t = 1200 \times 314.16 = 377 \, KN$

$A_B = 30 \times \frac{\pi}{4} \times 6^2 = 848.23$

$P_B = 1017.9\ kN$ $\qquad I = 8575 \times 10^6\ mm^4$




Stress in concrete at the level of steel at top

$$f_{ct} = \frac{P_t}{A} + \frac{P_t \cdot e_t}{I} \cdot y_{(et)} + \frac{P_B}{A} - \frac{P_B \cdot e_B}{I} \cdot y_{(et)}$$

$$= \frac{377 \times 10^3}{21 \times 10^4} + \frac{377 \times 10^3 \times 300^2}{8575 \times 10^6} + \frac{1017.9 \times 10^3}{21 \times 10^4} - \frac{1017.9 \times 10^3 \times 230 \times 300}{8575 \times 10^6}$$

$$= 1.795 + 3.96 + 4.85 - 8.19$$

$$= 2.415\ N/mm^2$$

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$$f_{cB} = \frac{P_t}{A} - \frac{P_t e_t}{I} \cdot e_B + \frac{P_B}{A} + \frac{P_B \cdot e_B^2}{I}$$

$$= 1.795 - \frac{377 \times 10^3 \times 300 \times 230}{8575 \times 10^6} + 4.85 + \frac{1017.9 \times 10^3 \times 230^2}{8575 \times 10^6}$$

$$= 1.795 - 3.03 + 4.85 + 6.28$$

$$= 9.895\ N/mm^2.$$

Loss of stress in top steel $= m \cdot f_{ct}$

$$= \frac{E_s}{E_c} \times f_{ct}$$

$$= 5.83 \times 2.415 = 14.08\ N/mm^2$$

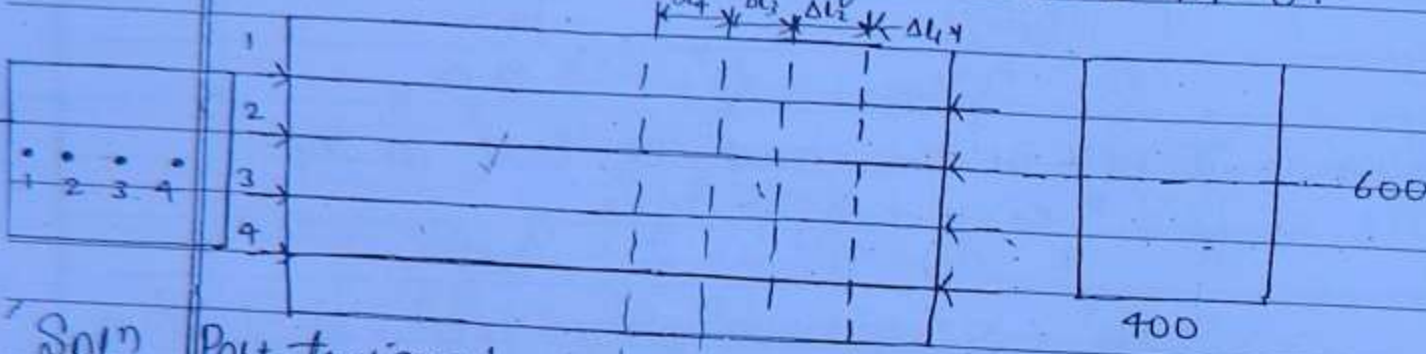
Loss of stress in bottom $= m \cdot f_{cB}$

$$= 5.83 \times 9.895$$

$$= 57.68\ N/mm^2$$

DATE:

Ques (a) A Post-tensioned Concrete beam 20 m long has c/s $400 \times 600 \ mm^2$ is prestressed by 4-Cable of $300 \ mm^2$ each provided without any eccentricity Each tendon is initially prestressed to $1800 \ N/mm^2$ Find out loss of prestress due to elastic shortening of concrete Wires are tensioned one after other & $m = 6$.




Soln Post-tensioned

$A_s = 300 \ m^2$ each $\qquad E_s = 2 \times 10^5 \ N/mm^2$

$P_s = 1800 \ N/mm^2 \qquad E_c =$

$P = 300 \times 1800 = 540 \ KN$

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$\Delta l_1 = \Delta l_2 = \Delta l_3 = \Delta l_4$

$$\Delta l = \frac{Pl}{A_c E_c} = \frac{540 \times 10^3 \times 20 \times 10^3 \times 6}{400 \times 600 \times 2 \times 10^5} = 1.35 mm$$

Loss of stress due to one wire $= \frac{\Delta l}{l} \times E_s = \frac{1.35}{20 \times 10^3} \times 2 \times 10^5 = 13.5 \ N/mm^2$

loss in steel due to wire no 4 $= 0$,

due to wire no. 3 $= \frac{\Delta l_4}{l} \times E_s$

$= 13.5 \ N/mm^2$

due to wire no. 2 $\quad \frac{\Delta l_3 + \Delta l_4}{l} \times E_s$

$= \cancel{13.5 \times 2} \ \frac{1.35 \times 2}{20 \times 10^3} \times 2 \times 10^5$

$= 27 \ N/mm^2$

due to wire no 1 $= \frac{\Delta l_2 + \Delta l_3 + \Delta l_4}{l} \times E_s$

$= \frac{3 \times 1.35}{20 \times 10^3} \times 2 \times 10^5 = 40.5 \ N/mm^2$

(b) If it is desired that a final stress of 1800 $N/mm^2$ is maintained after losses how much initial stress should be given to diffn wires.

Soln Wire no - 4 → loss of stress = 0

Initial stress = 1800 $N/mm^2$

" " 3 → loss = 13.5 $N/mm^2$

$$\text{or} = \frac{300 \times 1800}{400 \times 600} \times 6 = 13.5 \ N/mm^2$$

Initial stress = 1800 + 13.5 = 1813.5 $N/mm^2$

wire no. 2 → loss due to $P_3$ & $P_4$

$$\frac{P_3 + P_4}{A_c} \times m =$$ 265

$$= \frac{300 \times 1813.5 + 300 \times 1800}{400 \times 600} \times 6 = 27.10 \ N/mm^2$$

Initial stress = 1800 + 27.10 = 1827.10 $N/mm^2$

wire no. 1 — loss due to $P_2$ & $P_3$ & $P_4$

$$= \frac{P_2 + P_3 + P_4}{A_c} \times m$$

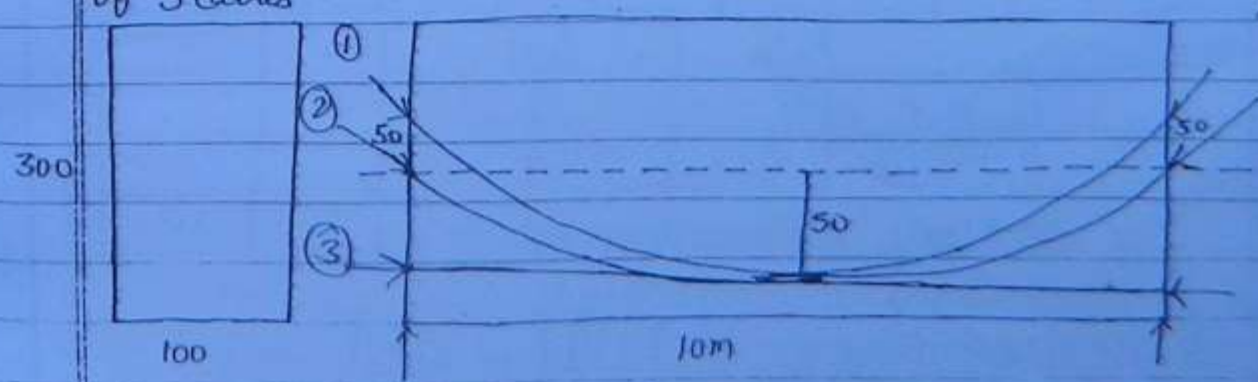
$$= \frac{(1827.10 + 1813.5 + 1800)\,300}{400 \times 600} \times 6$$

$= 40.8 \ N/mm^2$

Initial stress = 1840.8 $N/mm^2$

Ques A Post tensioned concrete beam is stressed by successive tensioning of 3 cables

$m = 6$




c/s of each cable = 200 $mm^2$, Initial stress in each cable = 1200 $N/mm^2$

DATE:

Estimate % loss of stress in each cable due to elastic shortening only (Neglect self wt of & live load)

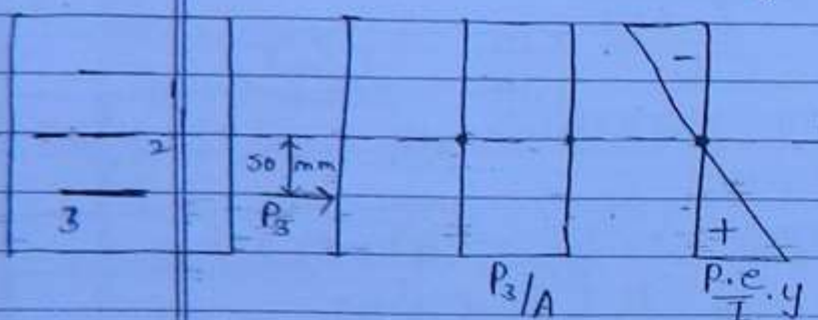
Sol$^n$ wire No. 3 – loss of stress = 0

wire No. 2 – due to wire No. 3 loss will occur.

$$= \frac{\Delta l_3}{l} \times E_s = \frac{P_3 \cdot l}{A_c \cdot E_c \cdot l} E_s$$

$$= \frac{P_3}{A_c} \cdot \frac{E_s}{E_c} = f_{c_3} \times m$$

At end – stress in concrete at the location of wire no. 2 due to prestressing of wire no. 3

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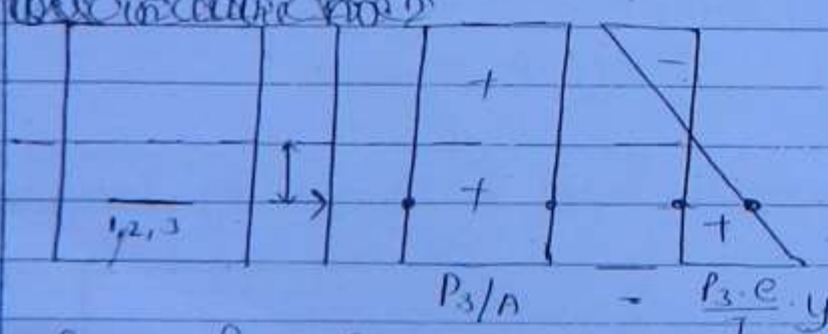
$$f_{c_3} = \frac{P_3}{A}$$

$$= \frac{200 \times 1200}{300 \times 100} = 8\ N/mm^2$$

Mid span




Stress in concrete at the level of wire no. ② due to prestressing force $P_3$

$$f_{c_2} = \frac{P_3}{A} + \frac{P_3 . e}{I} . y$$

$$= \frac{200 \times 1200}{300 \times 100} + \frac{200 \times 1200 \times 50^2}{225 \times 10^6}$$

$$= 8 + 2.67$$

$$= 10.67\ N/mm^2$$

DATE:

$$\text{loss} = \overline{\text{Average stress}} = \frac{8 + 10.67}{} \quad f_{c_1} + \frac{2}{3}(f_{c_2} - f_{c_1})$$

$$= 8 + \frac{2}{3}(10.67 - 8)$$

$$f_{c_3} = 9.78 \ N/mm^2$$

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$$\text{loss of prestress} = 9.78 \times 6$$

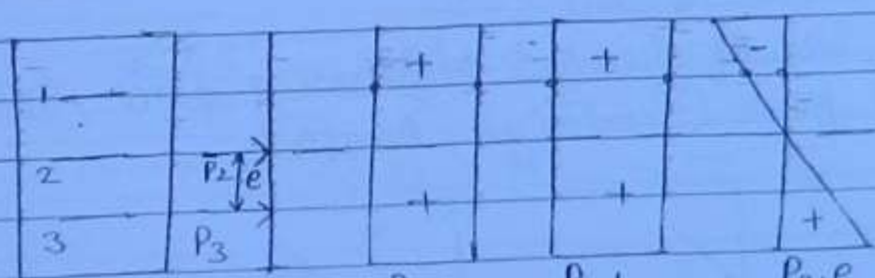
$$= 58.68 \ N/mm^2$$

Wire No. 1 – due to wire no 2 & 3 loss will occur

$$\text{loss} = (f_{c3} + f_{c2})\, m$$

→ Stress in Concrete at the location of wire no. 1 due to force in wire no. 3 & 2

At end

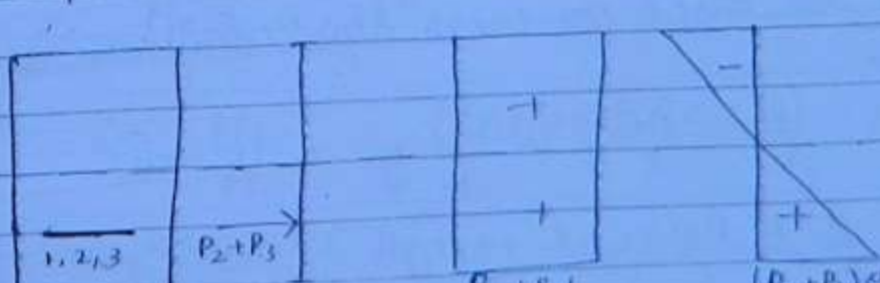



$$f_{c_1} = \frac{P_2}{A} + \frac{P_3}{A} - \frac{P_3 \cdot e}{I} \cdot y$$

$$= \frac{200 \times 1200}{100 \times 300} + \frac{200 \times 1200}{300 \times 100} - \frac{200 \times 1200 \times 50}{225 \times 10^6} \times 50$$

$$f_{c_1} \quad 8 + 8 - 2.67 = 13.33$$

At mid span




$$f_{c_2} = \frac{P_2 + P_3}{A} + \frac{(P_2 + P_3)e}{I} \cdot y \Rightarrow \frac{2(200 \times 1200)}{100 \times 300} + \frac{2(200 \times 1200)\,50^2}{225 \times 10^6}$$

$$= 8 \times 2 + 5.33 = 21.33 \ N/mm^2$$

$$\text{average} = 13.33 + \frac{2}{3}(21.33 - 13.33)$$

$$= 18.66\ N/mm^2$$

$$\text{loss} = 18.66 \times 6$$

$$= 111.98\ N/mm^2$$

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# Design of Prestressed beam

Design of Rectangular beam (Symmetrical Secⁿ)

TYPE 1- Rectangular secⁿ:- for a given value of B.M

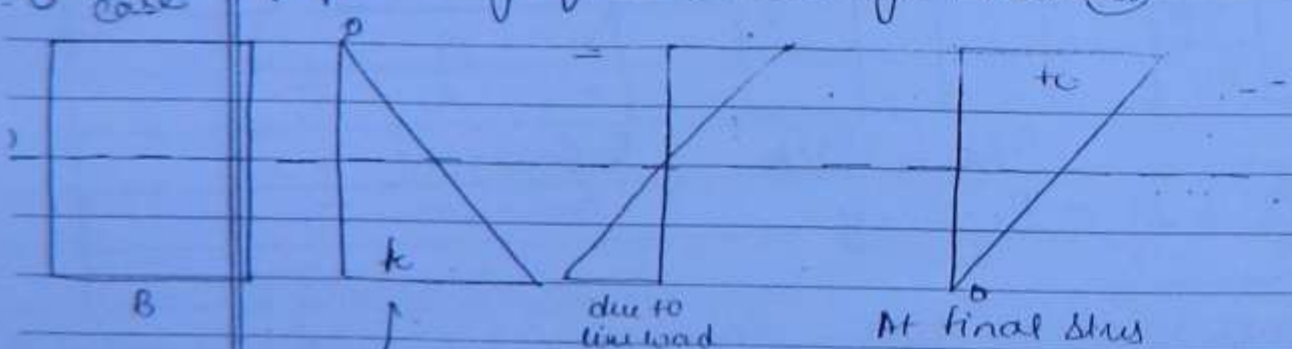
Given values -

(1) Permissible stresses in Concrete in Compression $f_c$
in tension = 0

2. Permissible stress of steel = $f_s$

To find out size of beam, prestressing force, location of prestressing force (eccentricity), Area ($A_s$)

stress of stress = 0 in this case




At transfer

Design formula are based on ideal stress diagram as shown in fig.

$$\text{At transfer (top)} = \frac{P}{A} - \frac{P.e}{I}y + \frac{M_d}{I}y = 0 \quad -(1)$$

$$\text{Bottom} = \frac{P}{A} + \frac{P.e}{I}.y - \frac{M_d}{I}y = f_c \quad -(2)$$

$$\text{At final stage (top)} = \frac{P}{A} - \frac{P.e}{Z} + \frac{(M_d + M_L)}{Z} = f_c \quad -(3)$$

$$\text{bottom} = \frac{P}{A} + \frac{P.e}{Z} - \left(\frac{M_d + M_L}{Z}\right) = 0 \quad \text{—} (4)$$

From (3) − (1)

$$\frac{M_L}{Z} = f_c \quad \text{— I}$$

(Section Modulus) $Z = M_L / f_c$

(264)

decide B & D so that $Z = \frac{BD^2}{6}$

$$A = B \times D$$

From (1) $\frac{Pe}{Z} = \frac{P}{A} + \frac{M_d}{Z}$

From (4) $\frac{P.e}{Z} = \frac{-P}{A} + \frac{M_d}{Z} + \frac{M_L}{Z}$

$$2\frac{P.e}{Z} = \frac{2M_d}{Z} + \frac{M_L}{Z}$$

$$e = \frac{2M_d + M_L}{2P} \quad \text{— II}$$

From (3) + (4)

$$\frac{2P}{A} = f_c \qquad \boxed{P = \frac{A.f_c}{2}} \quad \text{— III}$$

Design Step

1. Find out live load Moment

$$M_L = \frac{w_L \times l^2}{8}$$

2. $Z = \frac{M_L}{f_c}$ (Section Modulus)

3. Decide size of beam so that $Z = \frac{BD^2}{6}$

4. Calculate Area $B \times D$

5. Calculate $P = \frac{A \times f_c}{2}$

6. Calculate Area Of Steel $= A_s = \frac{P}{f_s}$

7. Calculate dead load

$\Rightarrow w_d = B \times d \times 1 \times 25 =$

$M_d = \frac{w_d l^2}{8}$

8. Eccentricity $e = \frac{2M_d + M_e}{2P}$ 270

5-1998

**Ques** Design a prestressed Concrete Slab Span in 12m, Carrying an imposed load Of $20 kN/m^2$, M-40 Concrete & Steel with ultimate Strength Of $1600 N/mm^2$ are available. Permissible Stresses in Concrete in Comp $= 14 N/mm^2$ tension $= 0$. Neglect the loss In prestress. Cable Of 12 wires Of 5mm ϕ are there Carrying an eff. prestress $= 225$ kN each

Sol:- 1 $w_l = 20 kN/m^2$

$M_l = \frac{w_l \cdot l^2}{8} = \frac{20 \times 12^2}{8} = 360$ kN-m

Section Modulus $Z = \frac{M_e}{f_c} = \frac{360 \times 10^6}{14}$

$Z = 25714285.71$

$\frac{BD^2}{6} = 25714285.71$

$BD^2 = 154285714.3$

Width Of Slab $= 1000 = B$

$D^2 = 154285.7143$

$D = 392.79$ mm $\simeq 392.8$ mm

Say $D = 400$ mm

DATE:

Area = $400 \times 1000 = 4 \times 10^5 \, mm^2$

Prestressing force = $\frac{4 \times 10^5 \times 14}{2} = 2800 \, KN$

~~Area of Steel $A_s = \frac{2800 \times 10^3}{1600} = 1750 \, mm^2$~~

No. of Cables req (of 12 wires of 5mm dia)

$= \frac{2800}{225} = 12.44 \rightarrow 13$ Nos. Cable

~~Total No. of wire = $13 \times 12 = 156$~~

~~Area of Steel $A_s = 13 \times \frac{\pi}{4}$~~

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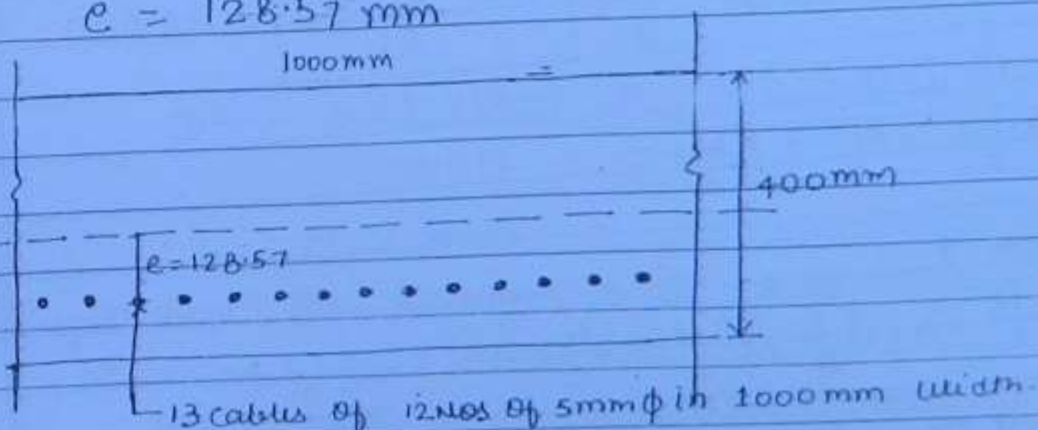
Dead load $= 1 \times \cdot 4 \times 1 \times 25 = 10 \, KN/m^2$

$M_d = \frac{10 \times 12^2}{8} = 180 \, KN\text{-}m$

$e = \frac{2 \times 180 \times 10^6 + 360 \times 10^6}{2 \times 2800 \times 10^3}$

$e = 128.57 \, mm$



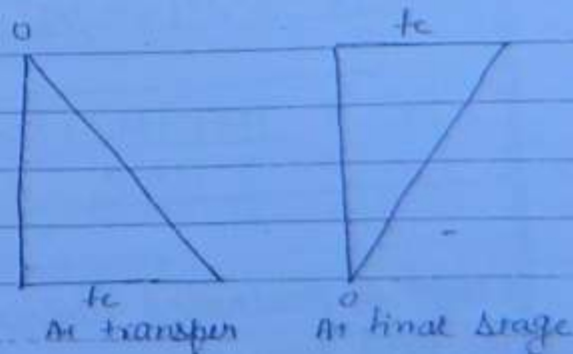

Case 2 If losses are Considered.

Permissible stress in Concrete

in Compression = $f_c$

in Tension = 0

loss factor = K




At transfer

At top = $\frac{P}{A} - \frac{Pe}{Z} + \frac{M_d}{Z} = 0$ — (1)

At bottom $= \frac{P}{A} + \frac{P.e}{Z} - \frac{M_d}{Z} = f_c$ —— (2)

At final stage of loading

At top $= \frac{KP}{A} - \frac{KPe}{Z} + \frac{M_d}{Z} + \frac{M_l}{Z} = f_c$ —— (3)

At bottom $\frac{KP}{A} + \frac{KP.e}{Z} - \frac{M_d}{Z} - \frac{M_l}{Z} = 0$ —— (4)

From (3) − (1)K

$$\frac{M_d}{Z} + \frac{M_l}{Z} - \frac{K.M_d}{Z} = f_c$$

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$$\frac{(1-K)M_d + M_l}{Z} = f_c$$

Section Modulus $\boxed{\frac{(1-K)M_d + M_l}{f_c} = Z}$ —— (A)

From (3) + 4

$$\frac{2KP}{A} = f_c \quad \boxed{P = \frac{A \cdot f_c}{2K}}$$ —— (B)

From (1) × K

$$\frac{K.Pe}{Z} = \frac{-KP}{A} + \frac{KM_d}{Z}$$

From 4

$$\frac{K.Pe}{Z} = -\frac{KP}{A} + \frac{M_d}{Z} + \frac{M_l}{Z}$$

Ading these two we get

$$\frac{2KPe}{Z} = \frac{(1+K)M_d + M_l}{Z}$$

$$\boxed{e = \frac{(1+K)M_d + M_l}{2KP}}$$ —— (C)

Ques Design a prestressed Concrete beam for a span of 20m, live load over beam is 35 KN/m. Permissible stress in con in compression = 18 N/mm² & in tension = 0

Permissible stress in steel = 1600 N/mm²

Loss of prestress = 15%.

Design a rectangular beam & show the prestressing force & its location.

Sol:- live load = 35 KN/m

$$M_L = \frac{35 \times 20^2}{8} = 1750 \text{ KN-m}$$

Let us assume c/s of rectangular beam

Size of beam 400×600 mm

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$$w_d = .4 \times .6 \times 1 \times 25 = 6 \text{ KN/m}$$

$$M_d = \frac{6 \times 20^2}{8} = 300 \text{ KN-m}$$

Now $$Z = \frac{(1-.85)300 \times 10^6 + 1750 \times 10^6}{18}$$

$$Z = 99722222.22$$

If B = 400

$$\frac{BD^2}{6} = Z$$

$$D^2 = \frac{6 \times 99722222.22}{400}$$

$$D = 1223 \text{ mm}$$ 1050

Keep a size B = 600 mm & D = 1050 mm

$$w_d = .6 \times 1.05 \times 1 \times 25 = 15.75 \text{ KN/m}$$

$$M_d = \frac{15.75 \times 20^2}{8} = 787.5 \text{ KN-m}$$

$$Z = \frac{(1-.85)\,787.5 \times 10^6 + 1750 \times 10^6}{18}$$

$$Z = 103784722.2$$

$$D = 1018 \text{ mm} \quad \& \quad B = 600 \text{ mm}$$

Size provided = 600×1050 mm

Area = $63 \times 10^4$ mm²

prestressing force $$P = \frac{A f_c}{2k} = \frac{63 \times 10^4 \times 18}{2 \times .85} = 6670.59 \text{ KN}$$

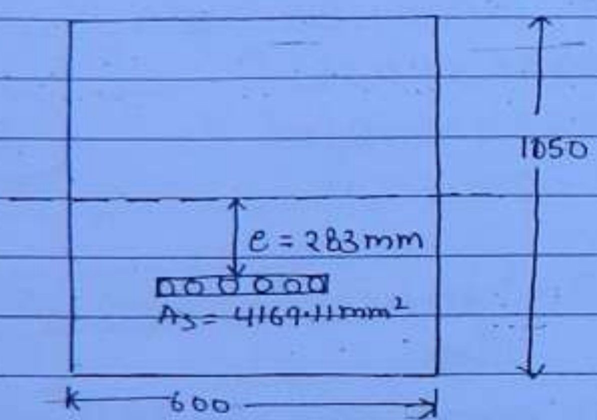
DATE:

Area Of Steel $= \dfrac{P}{f_s} = \dfrac{6670.6 \times 10^3}{1600} = 4169.11$

$$e = \frac{(1+k)M_d + M_l}{2kP}$$

$$= \frac{(1+.85)\,787.5\times10^6 + \cancel{800}^{1750}\times10^6}{2\times.85\times6670.6\times10^3}$$

$e = 283\,mm$

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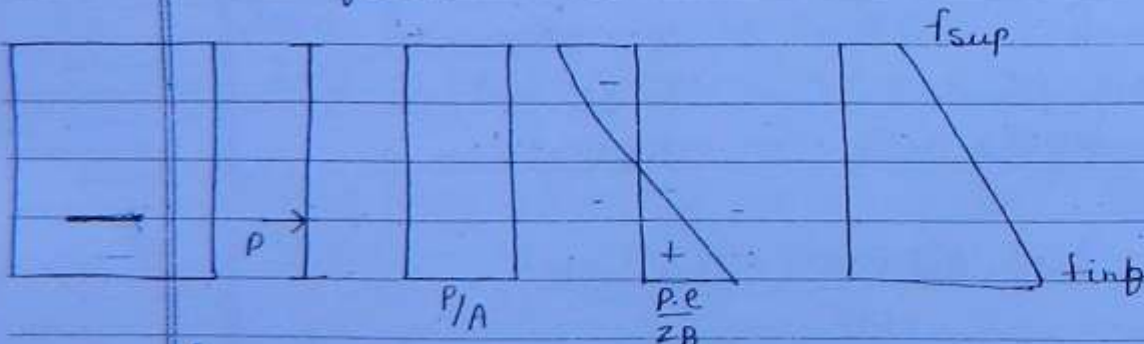
Case 3 If permissible stresses in Concrete
in Compression $= f_c$
in tension $= -f_t$
loss factor $= K$




$f_{sup} = \dfrac{P}{A} - \dfrac{P.e}{Z_t}$

$f_{inf} = \dfrac{P}{A} + \dfrac{P.e}{Z_B}$

Design Steps

1. Assume a section

Calculate $w_d$ & $M_d$

2. $M_e = \frac{w_e \cdot l^2}{8}$

3. Section modulus required

$$Z = \frac{(1-K)M_d + M_e}{(K \cdot f_c - f_t)}$$

4. $f_{sup} = \left(f_t - \frac{M_d}{Z_t}\right) = \left(f_t - \frac{M_d}{Z}\right)$

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5. $f_{inf} = \frac{f_c}{K} + \frac{M_d + M_e}{K Z_B}$

6. Prestressing force

$$P = \frac{A\,(f_{inf} \cdot Z_B + f_{sup} \cdot Z_t)}{Z_B + Z_t}$$

If $Z_B = Z_t = Z$

then $P = \frac{A\,(f_{inf} + f_{sup})}{2}$

7. Eccentricity $= e = \frac{Z_t \cdot Z_B\,(f_{inf} - f_{sup})}{A\,(f_{sup} Z_t + f_{inf} Z_B)}$

If $Z_t = Z_B = Z$

$$e = \frac{Z\,(f_{inf} - f_{sup})}{A\,(f_{sup} + f_{inf})}$$

ES-2007

Ques A Post-tensioned PSC Beam of rect. secⁿ 240 mm wi
is to be designed for a live load of 25 kN/m over an ef
span of 12m. Permissible stresses in concrete in compr
$= 17\ N/mm^2$ & tension $= 1.41\ N/mm^2$
Loss = 15%. Calculate min. possible depth of beam
P & e

DATE:

Sol Assume size of beam

$B = 240\,mm$

$D = 600\,mm$

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dead load $= .24 \times .6 \times 1 \times 25 = 3.6\ kN/m$

$$M_d = \frac{3.6 \times 12^2}{8} = 64.8\ kN\text{-}m$$

$$M_e = \frac{25 \times 12^2}{8} = 450\ kN\text{-}m$$

$$Z = \frac{(1-K)M_d + M_e}{(K f_c - f_t)}$$

$$= \frac{(1-.85)\,64.8 \times 10^6 + 450 \times 10^6}{(.85 \times 17 + 1.4)}$$

$$= 29004416.4$$

$\frac{BD^2}{6}$ $D = 851.53\ mm$

Assum $D = 860\ mm$

$W_d = .24 \times .86 \times 1 \times 25 = 5.16$

$M_d = 92.88\ kN\ m$

$$Z = \frac{(1-.85)\,92.88 \times 10^6 + 450 \times 10^6}{(.85 \times 17 + 1.4)}$$

$Z = 29270157.73$

$D = 855\ mm$ ~~= Min depth req~~ = 855

say $= D = 860\,mm$ = Min depth req.

Size of beam $= 240 \times 860\ mm^2$

$Z = 29584 \times 10^3$

4. $$f_{sup} = \left(f_t - \frac{M_d}{Z}\right)$$

$$= -1.4 - \frac{92.88 \times 10^6}{\cancel{29270157.73}\ \frac{240 \times 860^2}{6}}$$

$f_{sup} = -4.54\ N/mm^2$