FLUID MECHANICS (8 marks)

Matter

- Solid
- Liquid
- Gas.

Liquid + Gas = fluid.

Fluid Mechanics.

- Fluid Statistics
- Fluid Dynamics.

A matter can exist either in Solid, Liquid or in Gaseous state. The liquid and the gaseous state, together it is called Fluid State. The solid state & the fluid state together exhibit different characteristics because of differences in the arrangement of molecular structure.

In a given volume, a solid comprises large number of molecules and the attraction b/w the molecules (cohesive forces) is very large ∴ a solid posses a rigid and compact shape. and it has a definite volume.

In the same given volume, a liquid contains relatively less no. of molecules and the cohesive forces b/w the molecules is small and therefore in the liquid mass, the molecules will be in motion. and a liquid occupies the shape of the container in which it is contained.

For the same given volume, a gas contains much less no. of molecules and the attractive forces b/w the molecules is very mall. and ∴ the gas molecules are in continuous random tion and it occupies the total vol. of the container in which it is pla

$$\rho = \frac{m}{V} \quad (kg/m^3)$$

Dimensional Formula $= ML^{-3}$

ρ_{H_2O} at $4^\circ c$ $= 1000\ kg/m^3$

For the gas,

$$P = \rho RT \qquad PV = mRT$$

$P\uparrow \quad \rho\uparrow$ $\qquad P = \left(\frac{m}{V}\right)RT = \rho RT$

$T\uparrow \quad \rho\downarrow$

For the liquids, no change of density wrt pressure and temp.

Any gas moving with mach number, $M < 0.3$, it is considered as an incompressible gas. Because for the incompressible, $\delta\rho = 0$

2. Specific Weight (Weight Density), w

$$w = \frac{\text{weight}}{\text{volume}} = \frac{W}{V} \quad (N/m^3)$$

$$w = \frac{W}{V} = \frac{mg}{V} = \left(\frac{m}{V}\right)g$$

$$\Rightarrow \boxed{w = \rho g}$$

Dimensional Formula $= ML^{-2}T^{-2}$.

Since weight density depends on 'g', hence it is not called an absolute quantity.

3. Specific Volume (v)

$$v = \frac{\text{Volume}}{\text{Mass}} = \frac{V}{m} \quad (m^3/kg).$$

Specific volume term is always given to the gases.

Since for a given mass of the gas, it may occupy different volumes.

4. Specific Gravity, S

$$S_{fluid} = \frac{\text{Density of fluid}}{\text{Density of std. fluid.}} \times \frac{g}{g.}$$

$$= \frac{\text{wt. density of fluid}}{\text{wt. density of std. fluid.}}$$

The standard fluid:- for liquids $\rightarrow$ water

for gases $\rightarrow$ air.

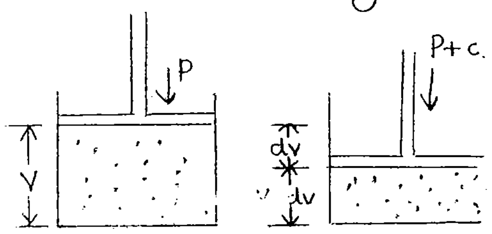
$S_{H_2O} = 1$; $S_{Hg} = 13.6$.

NOTE:

The term Relative Density is also sometimes used when the comparison is not made wrt to the standard fluid.

5. Bulk modulus of Elasticity (k) and Compressibility (β)

$$k = \frac{dP}{\left(\frac{-dv}{V}\right)} = \frac{P_2 - P_1}{-\left(\frac{V_2 - V_1}{V_1}\right)}$$



$V_1 = V$ $\quad$ $V_2 = V - dv$

$P_1 = P$ $\quad$ $P_2 = P + dP$

It is defined as the amount of incremental pressure (dP) required for the volume of gas, V decreases by an amount dv.

$$k = \frac{\text{compressive stress}}{\text{volumetric strain.}} = \frac{dP}{\left(\frac{-dv}{dV}\right)}$$

The $-$ve sign appears in the above eqn as with increase of pressure, volume decreases

* Compressibility (β)

It is defined as the property by virtue of which flui[ds] undergo a change of volume under the action of external pressure forces.

$$\therefore \beta = \frac{-\left(\frac{dv}{dv}\right)}{dP} = \frac{1}{k}.$$

$$\rho = \frac{m}{v}.$$

$$\rho \times v = m.$$

Differentiating on both sides,

$$\rho dv + v d\rho = 0. \qquad \text{(mass, m is constant).}$$

$$-\frac{dv}{v} = \frac{\partial \rho}{\rho}.$$

$$\therefore k = -\frac{dP}{\left(\frac{dv}{v}\right)} = \frac{dP}{\frac{d\rho}{\rho}}$$

$$\Rightarrow \boxed{k = \frac{\rho dP}{d\rho}}$$

For constant pressure process, $dp = 0$

$$\therefore k = 0.$$

For constant volume process, $dv = 0$

$$\therefore k = \infty$$

Isothermal Bulk modulus of Elasticity,

$$\frac{PV}{T} = c \qquad \text{(Characteristic Gas Equation)}$$

For isothermal process, T = constant.

$$\therefore PV = C.$$

$\therefore$ differentiating on both sides,

$$Pdv + vdp = 0.$$

$$\frac{-dv}{v} = \frac{dP}{P}.$$

$$k = \frac{dP}{-\left(\frac{dv}{v}\right)} = P \Rightarrow \boxed{k_{iso} = P}$$

Adiabatic bulk modulus of Elasiticity.

$$\frac{PV}{T} = C.$$

$$PV^{\gamma} = C \quad \text{(adiabatic process)}.$$

$$P\gamma V^{\gamma-1} dv + V^{\gamma} dp = 0.$$

$\gamma \rightarrow$ adiabatic index.

$$\gamma = \frac{C_P}{C_V}.$$

$C_P \rightarrow$ spec. heat at const. pr

$C_V \rightarrow$ sp. heat at const. volum

$$\frac{\gamma P V^{\gamma}}{V} dv = -V^{\gamma} dp.$$

$$\gamma P \frac{dv}{V} = -dp.$$

$$\therefore \quad -\frac{dv}{V} = \frac{dp}{P} \times \frac{1}{\gamma}.$$

$$k = \frac{dP}{\frac{dP}{P} \times \frac{1}{\gamma}} = \gamma p.$$

$$k_{adi} = \gamma p. = \gamma \times k_{iso}$$

Units $= N/m^2$.

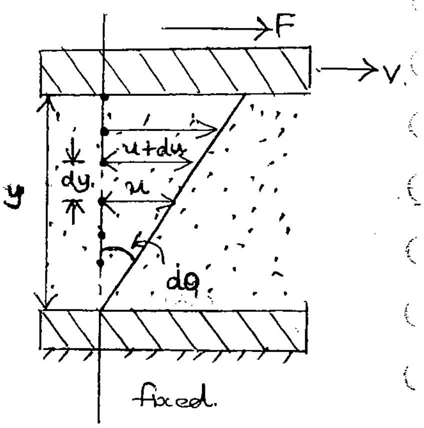
7th Aug, THURSDAY

6. Viscosity.

It is the internal resistance offered by the one fluid layer for the movement of adjacent fluid layer. It is due to the inter molecular ~~movementum~~ of exchange and the cohesive forces b/w the molecu

$$V = \frac{d}{t}$$

$$d = V \times t.$$



For the fluid particle which is on the top plate, travelled a distance $= V \times dt$.

$$\tan d\theta = \frac{V \times dt}{y}.$$

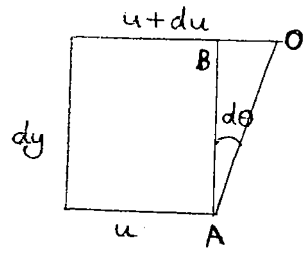
For small angles, $\tan d\theta \approx d\theta$.

$$d\theta = \frac{V \times dt}{y} \Rightarrow \boxed{\frac{d\theta}{dt} = \frac{V}{y}} \longrightarrow (1)$$

The term $\frac{d\theta}{dt}$ is called 'Rate of Angular deformation'

From ΔBAO,

$$\tan d\theta = \frac{du \cdot dt}{dy}$$



For small angles, $\tan d\theta \approx d\theta$.

$$\therefore d\theta = \frac{du \cdot dt}{dy} \Rightarrow \frac{d\theta}{dt} = \frac{du}{dy}. \longrightarrow (2)$$

$\frac{d\theta}{dt} \rightarrow$ rate of shear strain (or) rate of angular deformation

As shear force, $F \uparrow$, rate of angular deformation, $\frac{d\theta}{dt} \uparrow$

$$F \propto \frac{d\theta}{dt}$$

$$\tau = \frac{F}{A} \Rightarrow \tau \propto F \quad (A \rightarrow \text{area of plates is const.}$$

$$\therefore \tau \propto \frac{d\theta}{dt} \Rightarrow \boxed{\tau = \mu \frac{d\theta}{dt}} \longrightarrow (3)$$

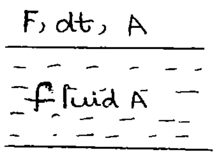
where μ is the proportionality constant and is known as 'Coeffecient of Dynamic Viscosity'.

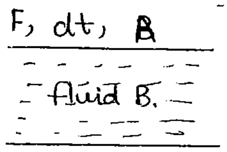
$$\mu = \frac{\tau}{\frac{d\theta}{dt}}.$$

If $\frac{d\theta}{dt} = 1$, then $\mu = \tau$.

$\therefore$ coeffecient of dynamic viscosity is defined as the amount of shear stress required to produce unit rate of angular deformation or rate of shear strain.

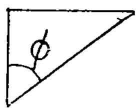
Q





Which fluid has more viscosity?

$$\frac{d\theta}{dt} < \frac{d\phi}{dt}.$$



$$\therefore \mu_B < \mu_A$$

$$\left(\text{Since } \mu \propto \frac{1}{d\theta/dt}\right)$$

From eqns (1), (2) & (3),

$$\tau = \mu \frac{V}{y} = \mu \frac{du}{dy}$$

$$\boxed{\tau = \mu \frac{du}{dy}} \Rightarrow \boxed{\text{Newton's Law of Viscosity}}$$

The term $\frac{du}{dy}$ has units of $\frac{m/s}{m} = s^{-1}$

The term $\frac{du}{dy}$ is called 'Velocity Gradient'

* If $\mu = 0$, then $\tau = 0$, the fluid is called an 'Ideal Fl...'

The fluid which satisfies Newton's Law of Viscosity is called 'Newtonian Fluid'.

$\therefore$ for Newtonian Fluid, $\tau = \mu\left(\frac{du}{dy}\right)$

Eg: Petrol, diesel, water etc.

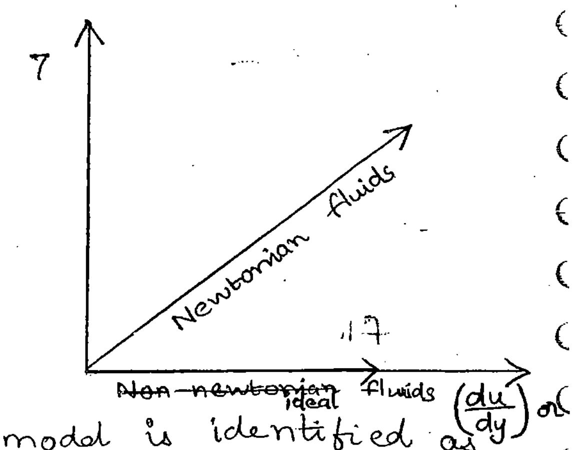
The fluids which does not obey Newton's Law of viscosity is called Non-newtonian Fluid.

The study of Non-Newtonian fluids is called "Rheology".

Eg: Blood, milk, clay etc.

The relationship between shear stress and velocity gradient for Non-Newtonian fluids is:

$$\tau = A\left(\frac{\partial u}{dy}\right)^n + B.$$



where A → flow consistency index,
B → flow behaviour index,
n → index.

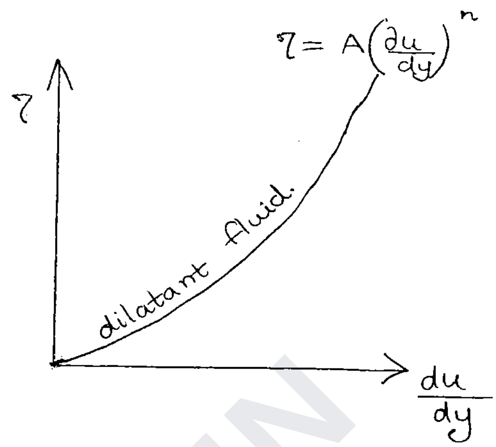
If $n = 1$, $A = \mu$, $B = 0$ ⇒ then the model is identified as Newtonian model.

Classification of Non-Newtonian Fluids :

1. Dilatant Fluid. $(n>1 \,,\, B=0)$
(Shear thickening fluids)

$$\tau = A\left(\frac{du}{dy}\right)^n$$

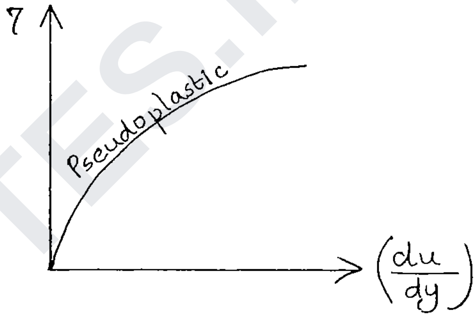
$$= \underbrace{A\left(\frac{du}{dy}\right)^{n-1}}_{\text{Apparent Viscosity } (\mu_{app})}\left(\frac{du}{dy}\right)$$



Eg: Milk

2. Pseudo plastic Fluid. $(n<1 \,,\, B=0)$
(Shear thinning fluids)

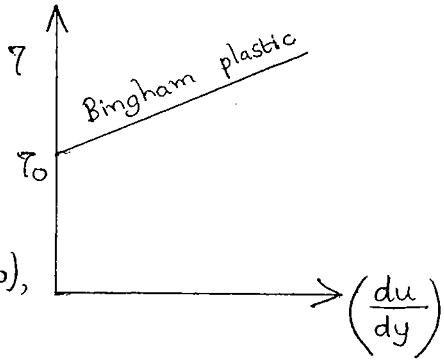
Eg: Blood.



3. Bingham Plastic Fluid $(n=1,\, B\neq 0)$

$$\tau = A\left(\frac{du}{dy}\right) + B$$

Eg: Toothpaste.



For these fluids, certain min. yield stress (τ_0), is required for the deformation.

4. Thixotropic Fluids.

For certain non-Newtonian fluids, the viscosity depends on time. They are called 'Thixotropic' & 'Rheopectic' fluids.

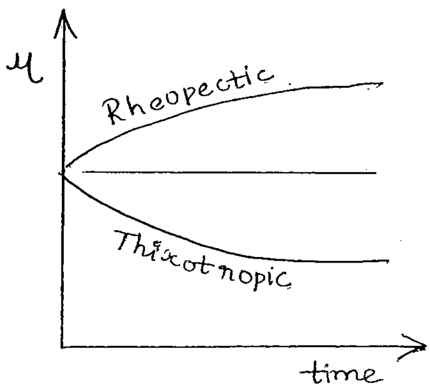
For thixotropic fluids, μ decreases with time.

Eg: Lipstick.

5. Rheopectic Fluids

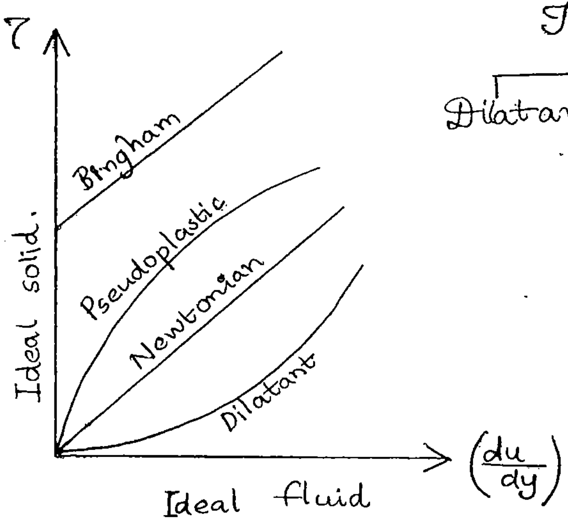
μ increases with time.

Eg: Gypsum solution in water



Fluids.

- Newtonian
- Non Newtonian.
  - Time independent
    - Dilatant
    - Pseudo
    - Bingham
  - Time dependent.
    - Thixotropic
    - Rheopectic.



Units of μ:

$$\tau = \mu\left(\frac{du}{dy}\right)$$

$$\mu = \frac{\tau}{\frac{du}{dy}} = \frac{N/m^2}{s^{-1}} = \frac{Ns}{m^2} = Pa-s.$$

$$1\ Pa-s = \frac{N}{m^2}\times s = \frac{kg\ m s^{-2}\times s}{m^2} = \frac{kg}{m\times s}\ (SI).$$

$$= \frac{g}{cm\times s}\ (c.g.s) \rightarrow \text{poise}$$

$$1\ \frac{kg}{ms} = \frac{1000\,g}{100\ cm\times s}$$

$$= \frac{10\ g}{cm\times s}$$

$$\therefore \boxed{1\ \frac{kg}{m.s} = 10\ \text{poise}}$$

$$1\ Pas = 1\ \frac{kg}{m\times s} = 10\ \text{poise..}$$

$$\boxed{1\ \text{poise} = 10^{-1}\ Pa-s}$$

* Kinematic Viscosity (ν)

$$\boxed{\nu = \frac{\mu}{\rho}}$$

Units :— $\dfrac{\frac{kg}{m \times s}}{kg/m^3} = m^2/s$ (SI) $= cm^2/s$ (c.g.s) → stoke

$\therefore$ 1 $m^2/s = 10^4$ stoke.

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Q.1. $V = 5.66\ m^3$, $m = 4765$ kg.

$$\rho = \frac{m}{V} = \frac{4765}{5.66} = 841.87\ kg/m^3$$

$$w = \rho g = 841.87 \times 9.81 = 8.26\ kN/m^3$$

$$G = \frac{\rho_f}{\rho_{H_2O}} = \frac{841.87}{1000} = \underline{\underline{0.842}}$$

Q.2 $\mu = 0.012$ poise.

R.D $= 0.79$

$$S_f = \frac{\rho_f}{\rho_{H_2O}}$$

$$\rho_f = 0.79 \times 1 = 0.79\ g/cm^3$$

$$\nu = \frac{\mu}{\rho_f} = \frac{0.012}{0.79} = \underline{\underline{0.0152}}\ \text{stokes}$$

Q.3. $u = 5 \sin(5\pi y)$

$\mu = 5$ poise $= 5 \times 10^{-1}$ Pa–s

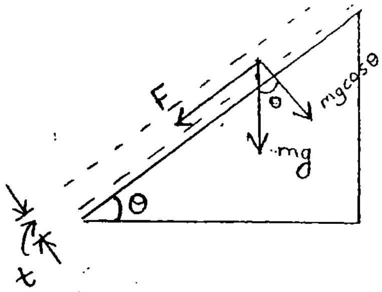
$\tau = \mu \dfrac{du}{dy}$ (valid for laminar flow).

$= 0.5 \times 5 \times 5\pi \cos(5\pi \times 0.05)$

$= \underline{\underline{27.76}}\ N/m^2$

Q.4.

$$\tau = \mu \frac{du}{dy}.$$



$$F = mg\sin\theta.$$

$$\tau = \frac{F}{A} = \frac{mg\sin\theta}{A}.$$

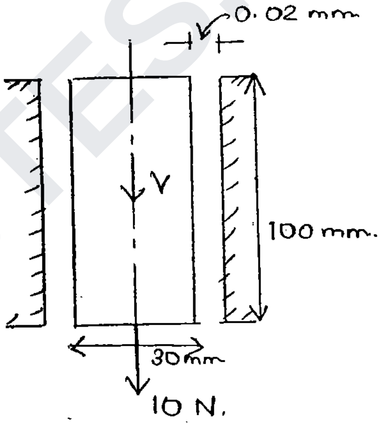
$$= \frac{\rho \times V \times g\sin\theta}{A}$$

$$\left\{ \frac{V}{A} = t \right\}$$

$$\tau = \underline{\underline{\rho g t \sin\theta}}$$

Q.5

$$\mu = 6 \text{ poise} = 6\times10^{-1} \text{ Pa-s}.$$



$$\tau = \mu \times \frac{du}{dy}.$$

$$= \mu \times \frac{V}{y}.$$

$$\tau = \frac{F}{A} = \frac{10\text{ N}}{\pi d l} = \frac{10}{\pi\times30\times10^{-3}\times100\times10^{-3}}$$

$$= \frac{6\times10^{-1}\times V}{0.02\times10^{-3}}$$

$$\therefore V = \underline{\underline{35.33}} \text{ mm/s}$$

Q.6 $\quad d = 350$ mm, $N = 200$ rpm, $\mu = 8$ poise $= 8\times10^{-1}$ Pa s.

$$P = \frac{2\pi NT}{60}$$

$$T = F\times r$$

$$F = \tau = \mu \frac{du}{dy}$$

$$\tau = 8\times10^{-1}\times\frac{\pi d N}{60\times0.002} = \frac{0.8\times\pi\times0.350\times200}{60\times0.002} = 1466 \text{ N/m}^2$$

$$\tau = \frac{F}{A} \Rightarrow F = 1466.076\times\pi\times0.35\times1$$

$$= 1612.03 \text{ N}$$

Torque, $T = F \times r$

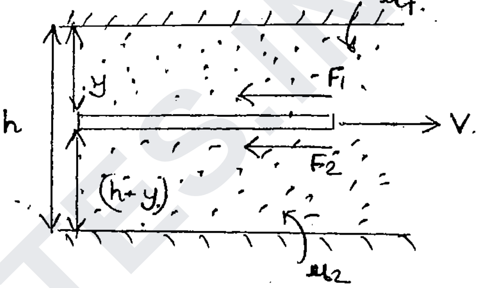
$$= 1612.035 \times \frac{0.35}{2} = 282.106$$

$$P = \frac{2\pi N T}{60} = \frac{2\pi \times 200 \times 282.106}{60} = 5908.4 \text{ W}$$

$$= \underline{\underline{5.908}} \text{ kW}$$

(or $P = F \times V$ where $V = \frac{\pi d N}{60}$

07 Let F be the total shear force acting on the thin plate moving with velocity, 'V'



$$F = F_1 + F_2 \quad \& \quad \frac{\partial F}{dy} = 0.$$

$$\tau_1 = \mu_1 \times \frac{du}{dy} = \frac{F_1}{A}.$$

$$F_1 = \frac{A \times \mu_1 \times V}{y}.$$

Similarly,

$$\tau_2 = \mu_2 \frac{du}{dy} = \frac{F_2}{A}.$$

$$F_2 = \frac{A \times \mu_2 \times V}{(h-y)}.$$

$$F = F_1 + F_2 = AV\left(\frac{\mu_1}{y} + \frac{\mu_2}{h-y}\right)$$

For minimum F,

$$\frac{dF}{dy} = 0.$$

$$-\frac{\mu_1}{y^2} + \frac{\mu_2}{(h-y)^2} = 0.$$

$$\underline{\underline{\frac{h-y}{y} = \sqrt{\frac{\mu_2}{\mu_1}}}} \Rightarrow \underline{\underline{\frac{y}{h-y} = \sqrt{\frac{\mu_1}{\mu_2}}}}$$

08. $V_1 = 1000\ cm^3$ $\quad V_2 = 995\ cm^3$

$P_1 = 1\ MN/m^2$ $\quad P_2 = 2\ MN/m^2$

$$k = \frac{P_2 - P_1}{-\left(\frac{V_2 - V_1}{V_1}\right)} = \frac{2-1}{\left(\frac{1000-995}{1000}\right)} = \underline{\underline{200\ MPa}}$$

7. Vapour Pressure :

Consider a closed container in which a liquid is filled by leaving an empty space above the liquid surface. If the temperature is increased, the vapour bubbles will starts forming and leaves from the liquid surface and get collected in empty space. The stage'll reach where the rate of formation of the new vapour bubbles will be equal to the rate of collapsing of vapour bubbles into the liquid surface. Under this condition, the vapour phase is said to be in phase eqbm with the liquid.

The pressure exerted by the vapour upon the liquid surface is called 'Vapour Pressure'.

The partial pressure of water vapour is less than the vapour pressure, then no water is present in the lake.

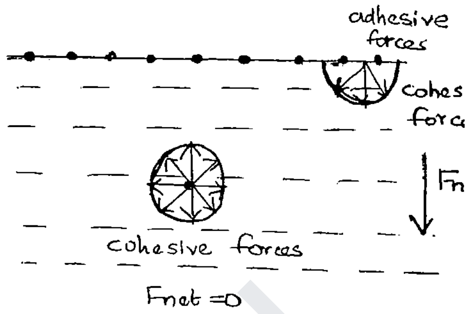
If the partial pressure of water vapour is equal to the vapour pressure then water is present in the lake.

In any fluid flow problems, at any section, the pressure should not fall less than or equal to vapour pressure, to avoid the cavitation.

If the pressure falls below the vapour pressure, then vapour bubbles will form and those will be carried along with flowing fluid into the high pr. regions where they will burst and it may damage the surfaces and the action is called the 'Pitting Action'

8. Surface Tension

Tension excerted by the fluid at the surface is called Surface Tension.



Consider a molecule which is completely inside the liquid and it is surrounded by equal no. of molecules. ∴ the net force acting upon such molecule is zero.

If we consider another molecule which is at the surface, it is under the influence of attractive force by its own molecule and the gas molecules' attractive force.

The cohesive forces are greater, than the adhesive force and ∴ it is acted by a net downward vertical force. As well as, the surface molecules are also subj. to cohesive forces by the molecules which lies to the left and right of the molecule. ∴ the surface of the liquid is under tension. This tension per unit length is called 'Surface Tension'

Units: N/m (or) J/m^2.

Eg:- a) A small needle can float
b) Collection of dust particles on the liquid surface.
c). Small birds can sit in the ponds for drinking water.
d) Formation of liquid droplets.

NOTE: $\sigma_{water-air} = 0.0736$ N/m at $20^\circ c$.

Case 1: Liquid Droplet

Let d = diameter of droplet.
σ = surface tension
p = pressure in excess of atmosphere (gauge pressure)

Under equilibrium,

Pressure force = surface tension

$$p \times \frac{\pi}{4} d^2 = \sigma \times \pi \times d.$$

$$\boxed{p = \frac{4\sigma}{d}}$$

Case 2: Soap Bubble.

$$p \times \frac{\pi}{4} d^2 = \sigma (2 \times \pi \times d)$$

$$\boxed{p = \frac{8\sigma}{d}}$$

Case 3: Liquid Jet.

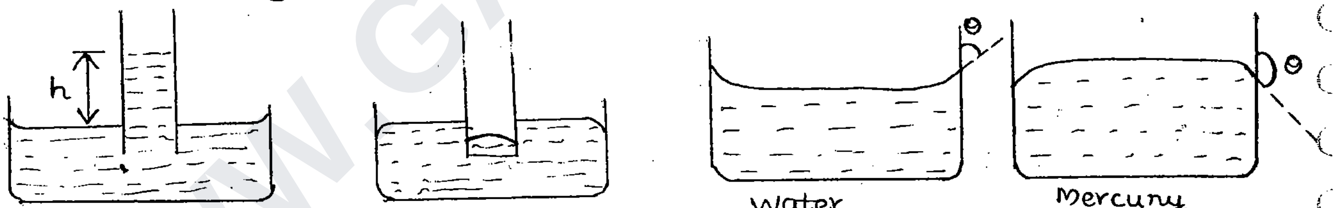
$$p \times l \times d = \sigma \times 2l.$$

$$\boxed{p = \frac{2\sigma}{d}}$$

Surface tension is due to cohesive forces. Surface tension decreases with increase in temperature. ∴ warm water is preferred for good washing of the clothes, thereby forming the liquid droplets into spherical form get minimised.

8th Aug, FRIDAY

9. Capillarity.



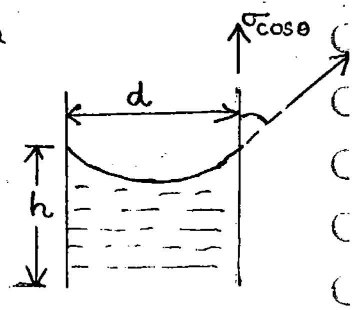
→ capillarity rise
→ adhesive > cohesive

→ capillarity depression
→ cohesive > adhesive

Water
→ concave up
→ $\theta < 90°$
→ wetted case
→ adhesive > cohesive

Mercury
concave down
$\theta > 90°$
not wetted case
cohesive > adhesive

The rise or fall of the liquid level in a narrow vertical tube when it's inserted in a liquid from the level of the liquid in a container.



Under equilibrium,
Surface tension force = weight of liquid in the tube.

$$\pi d \times \sigma \cos\theta = w \times \frac{\pi}{4} d^2 \times h.$$

$$h = \frac{4\sigma \cos\theta \times \pi d}{w \times \pi d^2}$$

$$\boxed{h = \frac{4\sigma \cos\theta}{w \times d} \quad \text{or} \quad \frac{4\sigma \cos\theta}{\rho g d}}$$

If $\theta > 90^\circ$, $\cos\theta = -ve$, $\therefore$ h becomes $-ve$ (capillarity fall)

As $d\uparrow$, $h\downarrow$

$\therefore$ for the tubes of manometer, to avoid the correctio for capillarity, the $d > 6\,mm$ is preferred.

For water, $\theta = 0^\circ$ For mercury, $\theta = 129^\circ$

$$h = \frac{4\sigma}{\rho g d}.$$

NOTE: Capillarity is due to both cohesion & adhesion.

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09. $d = 10^{-3}\,m$, $\sigma = 0.07\,N/m$.

$$p = \frac{4\sigma}{d} = \frac{4 \times 0.07}{10^{-3}} = 280\,Pa = \underline{\underline{0.28\,kPa}}$$

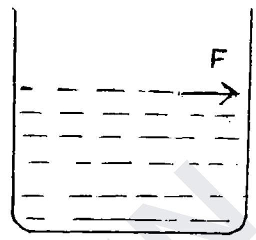
10. $d = 10 \times 10^{-3} = 10^{-2}\,m$, $\sigma = 0.04\,N/m$.

$$p = \frac{8\sigma}{d} = \frac{8 \times 0.04}{10^{-2}} = \underline{\underline{32\,N/m^2}}$$

Pressure Measurement

Intensity of Pressure:

The pressure is defined as the normal force excerted by static fluid per. unit area.

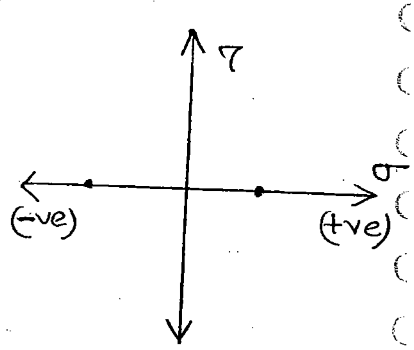


Static
($\tau=0$)

Intensity of pressure, $p = \dfrac{\text{normal force}}{\text{area.}}$

Pressure is compressive in nature. (fluid is subjected to compressive forces by walls of the container)

The intensity of pressure if it is located on Mohr's circle, it has zero radius ($\because \tau=0$)



Units: (i) $\dfrac{N}{m^2}$ (Pa)

(ii) kPa, MPa, GPa

(iii) mm of column of liquid.

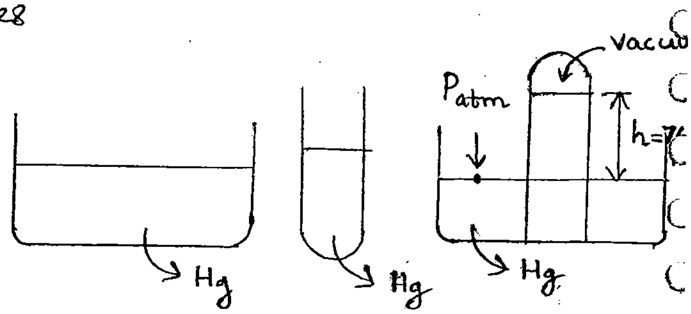
(iv) 1 bar = 10^5 Pa.

Types of Pressures:

1. Atmospheric Pressure (P_{atm})

It is defined as pressure excerted by the atmospheric air upon the surfaces

$P_{atm} = 760$ mm of Hg
$= 10.33$ m of H_2O



$$P_{atm} = \rho_{Hg} \times g \times h + P_v$$
$$= \rho_{Hg} \times g \times h.$$
$$= 13.6 \times 1000 \times 9.81 \times 760 \times 10^{-3} \ Pa$$
$$= 101.396 \ kPa.$$
$$= 1.01396 \ bar.$$
$$\approx 1.01325 \ bar$$
$$\approx 1 \ bar.$$

conducted by TORRICELL

(OR)

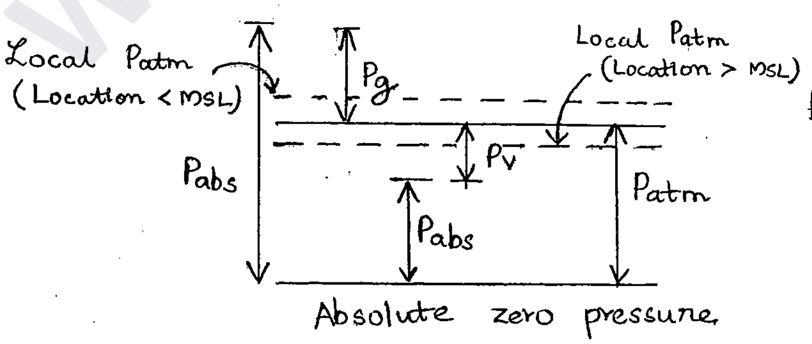
$$P_{atm} = \rho_{H_2O} \times g \times h_{H_2O}$$
$$= 1000 \times 9.81 \times 10.33.$$
$$= 1.01337 \ bar \ (at\ MSL)$$

2. Gauge Pressure

It is defined as the pressure measured by the gauge (instrument) in which atmospheric pressure is taken as datum. If the measured pressure is lower than atmospheric pressure, then it is called 'negative gauge pressure or vacuum pressure'.

3. Absolute zero pressure.

It is the pressure exerted by the absolute vacuum

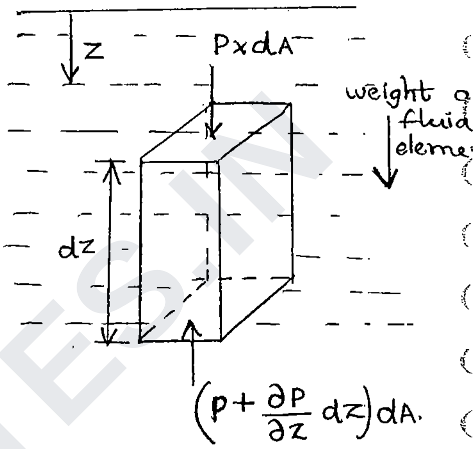


$$P_{abs} = P_{atm} + P_g$$
$$P_{abs} = P_{atm} - P_v$$

Hydrostatic Law:

It states that rate of Increase of pressure in the vertically downward direction in a static fluid is equal to the specific wt. of the fluid.

$$\boxed{\frac{\partial P}{\partial z} = w}$$



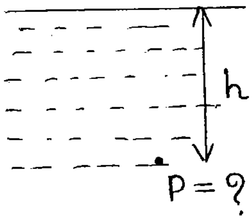
Under equilibrium,

$$\Sigma F_y = 0$$

$$-p \times dA + \left(p+\frac{\partial P}{\partial z}dz\right)dA - w = 0.$$

$$dz.dA\left(\frac{\partial P}{\partial z}\right) = \omega.dA.dz$$

$$\Rightarrow \boxed{\frac{\partial P}{\partial z} = \omega = \rho g}$$



$$\partial P = \rho g\, dz.$$

On integration,

$$p = \rho g z + c.$$

At $z=0$, $p = P_{atm}$.

$$\therefore \boxed{p_1 = \rho g z + P_{atm}.}$$

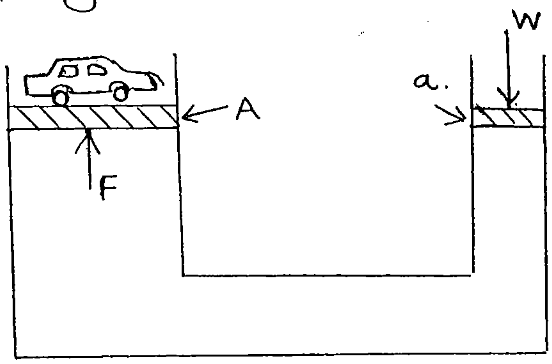
$$p - P_{atm} = \rho g z.$$

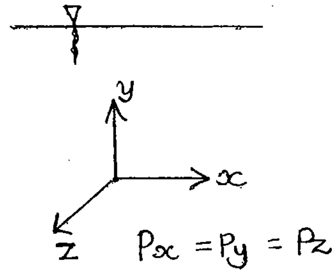
$$\boxed{\text{Gauge Pressure, } P_g = \rho g z}$$

At distance 'h' from free surface, $p = \rho g h$

Pascal's Law:

It states that in a static fluid, the pressure acts equally in all the directions.



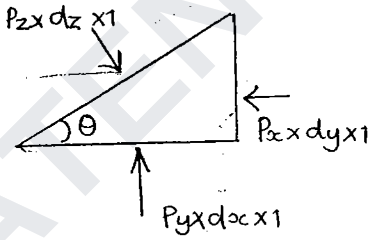


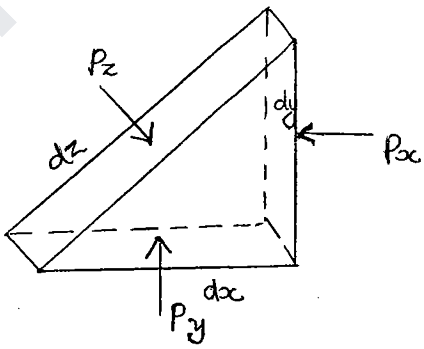
$\dfrac{W}{a} = \dfrac{F}{A}$; application of Pascal's Law (Hydraulic Lift)

$\Rightarrow F = \left(\dfrac{A}{a}\right) W$

Consider a wedge shaped fluid element,

Consider unit element,





Resolve the forces along x-direction,

$P_x \times dy \times 1 = P_z \times dz \times 1 \times \cancel{\cos\theta}\ \sin\theta \Rightarrow P_x\, dy = P_z\, dz\, \cancel{\cos}\sin\theta \rightarrow (1)$

Resolve forces along y-direction,

$P_y\, dx = P_z\, dz\, \cancel{\sin\theta}\cos\theta \rightarrow (2)$

$\sin\theta = \dfrac{dy}{dz} \quad \therefore dy = dz\sin\theta$

(1) becomes,

$P_x\, dy = P_z\, dy$

$\Rightarrow P_x = P_z$.

Similarly, (2) becomes,

$P_y = P_z$.

$\therefore \underline{P_x = P_y = P_z}$

Pressure Measurement Devices :

- Manometers.
  - Simple
  - Differential.
- Mechanical Gauges.

a) Piezometer

b) U- tube manometer

c) Single column manometer

(i) Vertical column.

(ii) Inclined column.

→ **Manometers:**

These are the devices which are used to measure the pressure at any point in the pipe by balancing the column of the fluid by the different kind of the fluid.

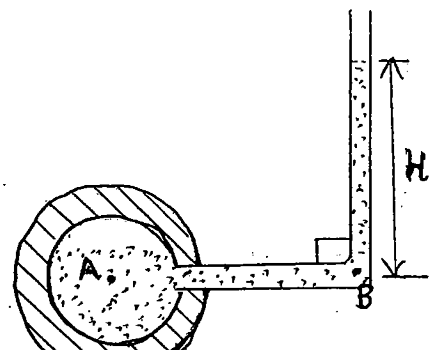
* **Simple Manometers**

These are the devices which contains a narrow tub whose one end is connected to the point where the pressure is to be measured and the other end is open to the atmosphere.

a) Piezometer.

$$\boxed{P_A = P_B = \rho g H + P_{atm}}$$

$$\therefore P_A = \rho g H \quad \text{(gauge pressure)}$$

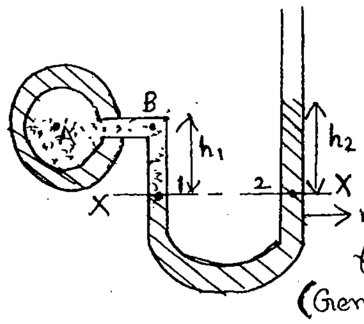


Piezometer is used for measurement of moderate pressures

Piezometer is not useful for measurement of gas pressures since gases never form a free surface with the atmosphere.

b) U-tube Manometer

At datum line X-X,



$$P_1 = P_2$$

$$P_B + \rho_1 g h_1 = \rho_2 g h_2 + P_{atm}.$$

$$P_A + \rho_1 g h_1 = \rho_2 g h_2 + P_{atm}.$$

$$P_A = \rho_2 g h_2 - \rho_1 g h_1 + P_{atm}.$$

$$\therefore \boxed{P_A = \rho_2 g h_2 - \rho_1 g h_1} \quad ; \text{gauge pressure}$$

$$P_A = \rho_1 g h.$$

If pipe contains oil, (ρ_0)

$$P_A = \rho_0 \times g \times h_0.$$

$$h_0 = \frac{P_A}{\rho_0 \times g} \text{ m of oil.}$$

If the pressure of oil given in terms of water column,

$$P_A = \rho_{H_2O} \times g \times h_{H_2O}.$$

$$h_{H_2O} = \frac{P_A}{\rho_{H_2O} \times g} = \frac{\rho_2 g h_2 - \rho_1 g h_1}{\rho_{H_2O} \times g} = \frac{\rho_2}{\rho_{H_2O}} \times h_2 - \frac{\rho_1}{\rho_{H_2O}} \times h_1$$

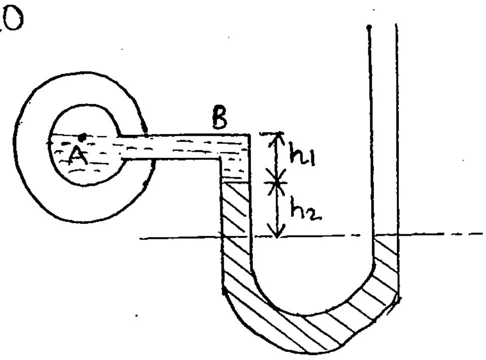
$$\boxed{h_{H_2O} = S_2 h_2 - S_1 h_1}$$

$$h_B + S_1 h_1 = S_2 h_2 + 0.$$

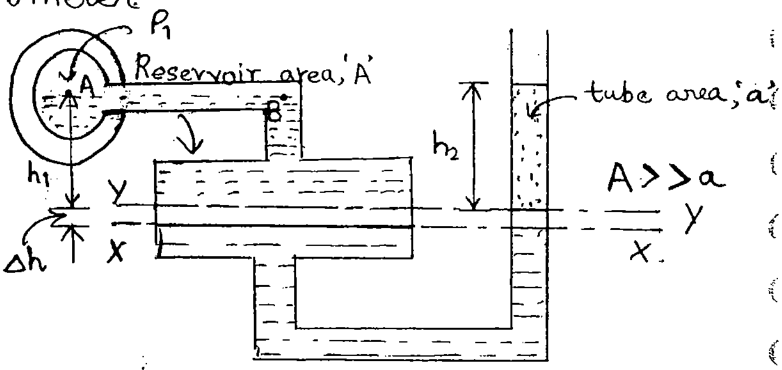
$$h_A + S_1 h_1 = S_2 h_2$$

$$\therefore \boxed{h_A = S_2 h_2 - S_1 h_1} \text{ m of } H_2O$$

Only disadvantage is the measurement of h_1 & h_2. (two values)



c) Single Column Manometer



At datum line,

$$\boxed{P_B + \rho_1 g (h_1 + \Delta h) = P_{atm} + \rho_2 g (h_2 + \Delta h)}$$

$$\boxed{P_A = (\rho_2 g h_2 - \rho_1 g h_1) + (\rho_2 - \rho_1) g \, \Delta h}$$

For volumes equal,

$$A \times \Delta h = a \times h_2$$

$$\Delta h = \frac{a h_2}{A}$$

Since $a << A$, then Δh can be neglected.

$$\boxed{P_A = \rho_2 g h_2 - \rho_1 g h_1} \quad \text{(gauge)}$$

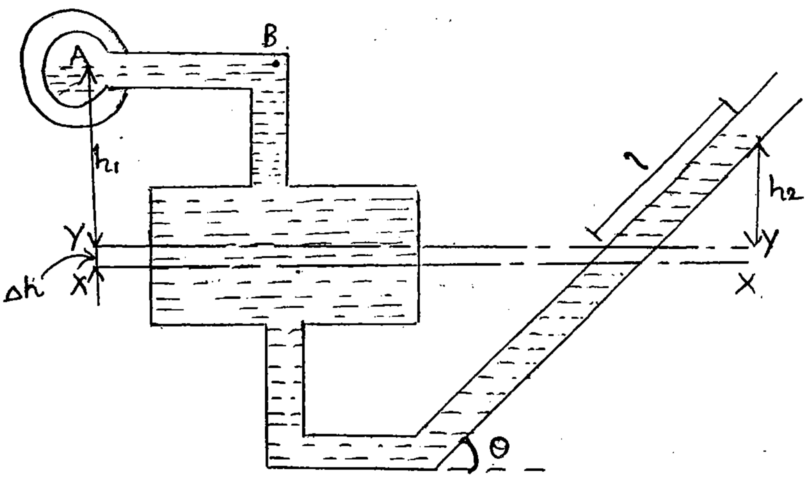
d) Single Column Inclined Manometer

$P = h \rho g$ ← vertical height

$h_2 = l \sin\theta$.

As 'θ' increases, sensitivity decreases.

$$\therefore \boxed{\text{Sensitivity} = \frac{1}{\sin\theta}}$$



θ	Sensitivity
0°	$\frac{1}{0} = \infty$
30°	$\frac{1}{1/2} = 2$
60°	$2/\sqrt{3} = 1.15$
90°	1

Q.2 $S_0 = 0.8$

$P_{H_2O} = \rho_{H_2O} \times g \times h_{H_2O}$

$= 1000 \times 9.81 \times 80 \text{ Pa}.$

$P_{oil} = \rho_{oil} \times g \times h_o$

$= 1000 \times 0.8 \times 9.81 \times h_o.$

$\because P_{H_2O} = P_{oil}$

$\Rightarrow h_o = \underline{\underline{100 \text{ m}}}$

Q.3. $\rho = 1000 + 0.008\, y^{3/2}.$

$\frac{dP}{dz} = \rho g.$

$\int dP = \int \rho g \, dz.$ $\qquad P = \rho g h \rightarrow$ incompressible flui (ρ const.)

$P = g\left(1000 + 0.008\, y^{3/2}\right).$

$900 \text{ kPa} = g\left(1000\, y + 0.008 \frac{y^{5/2}}{5/2}\right).$

$y = \underline{\underline{91.5 \text{ m}}}$

Q.4. Pressure at the bottom $= 0.9 \times 10 \times 2 + 10 \times 4$

$= \underline{\underline{58 \text{ kN/m}^2}}$

$\gamma_w = 10$

$\gamma_{oil} = 10 \times 0.9$

$p = \gamma H$

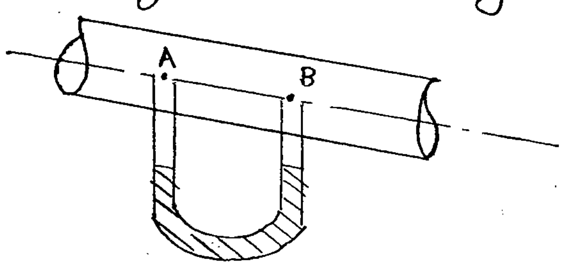
th Aug,
ONDAY

Q.5

Differential Manometer:

It is a device used to measure the difference of pressures b/w any two points that may lie in the single pipe or in different pipe lines

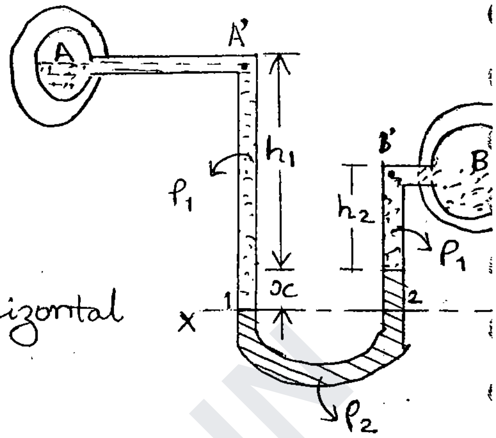
$P_A - P_B = ?$



At datum line $x-x$,

$$P_1 = P_2$$

$$P_A + \rho_1 g(h_1 + x) = P_B + \rho_1 g h_2 + \rho_2 g x.$$



$$\boxed{P_A - P_B = \rho_1 g h_2 - \rho_1 g h_1 + \rho_2 g x - \rho_1 g x}$$

If the points A & B lie in the same horizontal line, then $h_1 = h_2$,

$$P_A - P_B = (\rho_2 - \rho_1) g x.$$

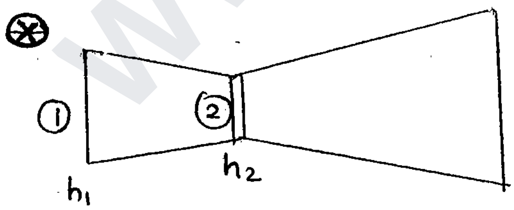
$$\therefore \quad h_A - h_B = \frac{(\rho_2 - \rho_1) g x}{\rho_1 g}.$$

$$= \left(\frac{\rho_2}{\rho_1} - 1\right) x.$$

$$= \left[\frac{\frac{\rho_2}{\rho_{H_2O}}}{\frac{\rho_1}{\rho_{H_2O}}} - 1\right] x$$

$$\Rightarrow \quad h_A - h_B = \left(\frac{S_2 - S_1}{S_1}\right) x$$

$$\boxed{\therefore \quad h_A - h_B = \left(\frac{S_m}{S_L} - 1\right) x}$$



Discharge of a venturimeter,

$$Q = \frac{a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

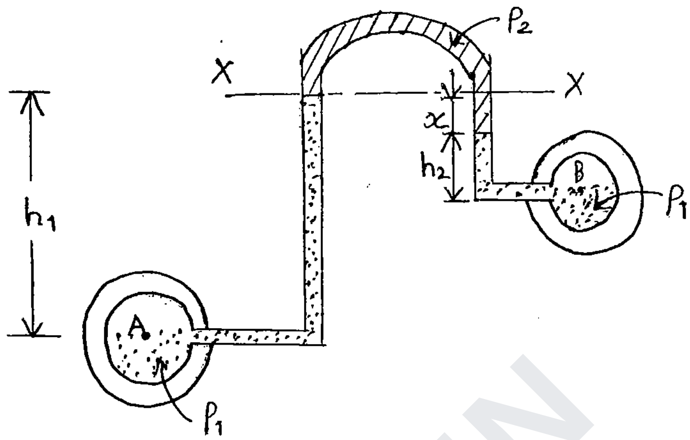
$$h = h_1 - h_2 = h_A - h_B.$$

$$= \left(\frac{\overset{S_{\text{manometric fluid}}}{13.6}}{\underset{S_{H_2O}}{1}} - x\right) = \underline{\underline{12.6x}} \quad \Rightarrow \quad \boxed{h = 12.6x}$$

Inverted U-tube Differential Manometer

At the datum line,

$$\boxed{P_A - \rho_1 g h_1 = P_B - \rho_1 g h_2 - \rho_2 g x}$$



Q.05.

$$P_A + 0.8\times10^3 \times 9.81 \times 0.31 = P_B + 0.8\times10^3\times9.81\times0.13 + 13.6\times10^3\times9.81\times0.08$$

$$P_A - P_B = \underline{\underline{9.26}}\ kPa$$

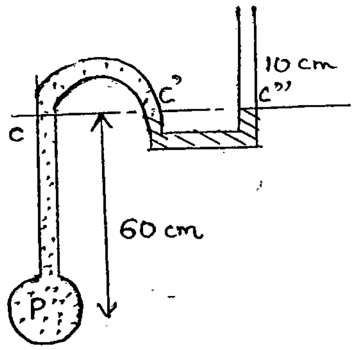
Q.06. At the datum line,

$$P_M - 1000\times9.81\times0.09 = P_N - 1000\times9.81\times(0.12+0.06) - 0.83\times1000\times9.81\times0.03$$

$$P_M - P_N = -1127.169\ Pa.$$

$$h_M - h_N = \frac{P_M - P_N}{0.8\times10^3\times9.81} = \underline{\underline{-13.84}}\ cm$$

Q.07. $P_{c''} = 13.6\times10^3\times9.81\times0.1 + \cancel{P_{atm}}^{\,0}$



$$P_c = P_{c'} = P_{c''} = 13.6\times10^3\times9.81\times0.1$$

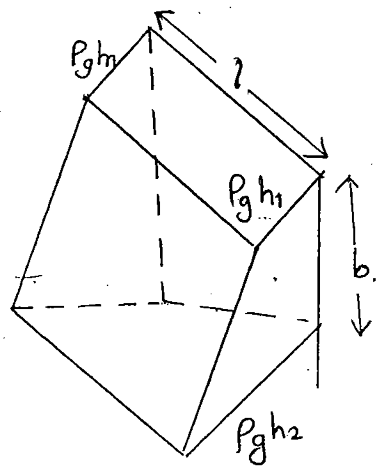
$$P_p = P_c + 0.85\times10^3\times9.81\times0.6.$$

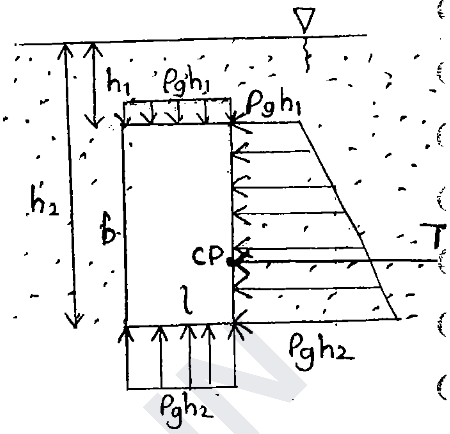
$$= 18.344\ kPa.$$

$$h_p = \frac{18.344\times10^3}{0.85\times10^3\times9.81} = \underline{\underline{2.2m}}$$

→ Hydrostatic Forces on Submerged Surfaces.

Pressure Prism:





Total Pressure (TP):

Whenever a surface either ~~or~~ a plane or curved is completely submerged in the static fluid, the pressure force variations will take place across the surface. The resultant of all the pressure force variations is called 'Total Pressure'. It has units of 'force' (N).

Centre of Pressure (CP):

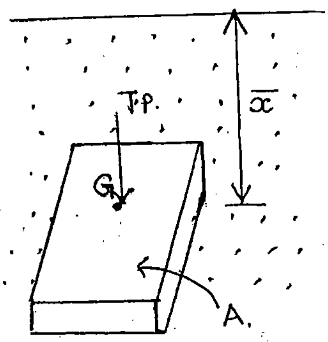
It is the point at which total pressure will act.

a) Horizontal Surface.

Let A → area of surface.

w → sp. wt. of liquid.

$\bar{x}$ → distance of C.G from free surface.

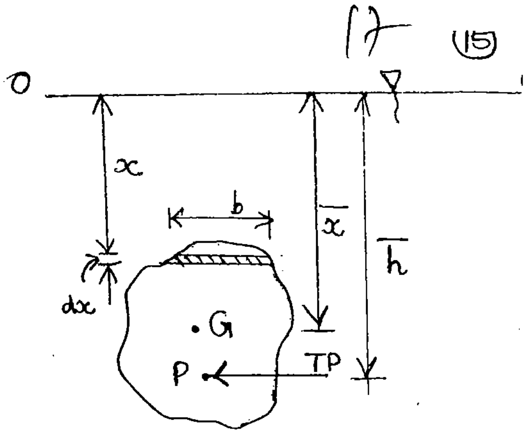


$$T.P = \text{pressure} \times \text{area}.$$

$$= \rho g \bar{x} \times A = \omega A \bar{x}$$

$$\boxed{TP = \omega A \bar{x} \quad \& \quad CP = \bar{x}}$$

b) Vertical Surface



Pressure force acting on elemental area $= \rho g x.(b\,dx)$.

Sum of the pressure forces

$$= \int \rho g (b\,dx) \times x.$$

$$= \rho g \int (b\,dx) x. \qquad \{(b\,dx)x = \text{first area moment}\}$$

$$TP = \rho g A\bar{x} = \omega A\bar{x}$$

"Sum of the moments of individual forces is equal to the moment caused by the resultant force" – Varignance Theore

Moment caused by pressure force acting on elemental area about free surface $=$ Force $\times x$

$$= (P \times A) \times x$$

$$= \rho g x.\ b\,dx.\ x.$$

$$= \rho g (b\,dx) x^2$$

Sum of moments $= \int \rho g (b\,dx) x^2$

$$= \rho g \int (b\,dx) x^2 \qquad \left[(b\,dx)x^2 = \text{second area mome} = I\right]$$

$$= \rho g I_0$$

Parallel Axis Theorem:

$$\rho g I_0 = \rho g (I_G + A\bar{x}^2) \longrightarrow (1)$$

Moment caused by TP about free surface $= TP \times \bar{h}$

$$= \omega A\bar{x} \times \bar{h} \longrightarrow (2)$$

From (1) & (2),

$$\rho g A\bar{x} \times \bar{h} = \rho g (I_G + A\bar{x}^2).$$

$$\therefore \bar{h} = \frac{I_G}{A\bar{x}} + \frac{A\bar{x}^2}{A\bar{x}}$$

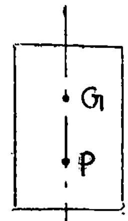
$$\left\{ \rho g A\bar{x} \times \bar{h} = \rho g I_0 \Rightarrow \bar{h} = \frac{I_0}{A\bar{x}} \right.$$

$$\bar{h} = \bar{x} + \frac{I_G}{A\bar{x}}$$; $I_G \rightarrow$ MI of the surface about an axis passing through C.G.

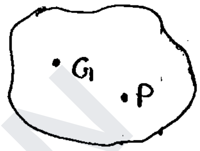
$\therefore \bar{h} > \bar{x} \Rightarrow$ C.P always lies below C.G

$$\bar{h} - \bar{x} = \frac{I_G}{A\bar{x}}$$

* For symmetrical surfaces, C.G & C.P lie along the same line.



Symmetrical



Unsymmetrical

*Volume of prism $= \frac{1}{2}(\rho g h_1 + \rho g h_2) \times b \times l$.

$$= \frac{1}{2}\rho g (h_1 + h_2) \times b \times l.$$

$$\bar{x} = h_1 + \frac{b}{2}$$

$$= h_1 + \frac{h_2 - h_1}{2} \Rightarrow \bar{x} = \frac{h_1 + h_2}{2}.$$

$\therefore$ Volume of pressure prism $= \frac{1}{2}(\rho g h_1 + \rho g h_2) \times b \times l$.

$$= \rho g \bar{x} \times A.$$

$$= wA\bar{x} = TP.$$

$\therefore$ $\boxed{\text{Volume of pressure prism} = TP}$

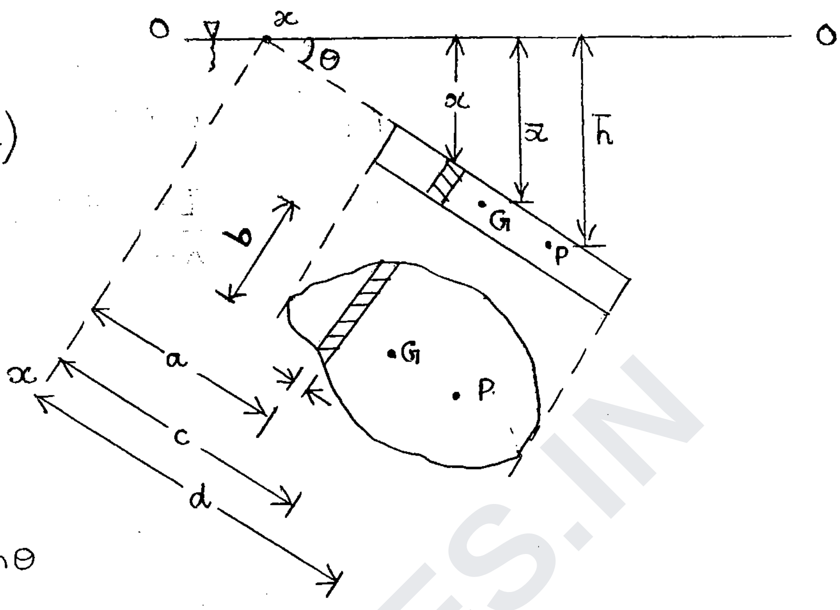
NOTE :

① Volume of pressure prism represents the '<u>total pressure</u>'

② The centroid of the volume of the prism will be equal to the '<u>Centre of Pressure</u>'.

$$\boxed{\begin{aligned} TP &= wA\bar{x} \\ CP &= \bar{x} + \frac{I_G}{A\bar{x}} \end{aligned}}$$

c) Inclined Surface.



Pressure force acting on elemental area $= \rho g x.(bdx)$

$$\sin\theta = \frac{x}{a} = \frac{\bar{x}}{c} = \frac{\bar{h}}{d}.$$

$$= \rho g (bdx)\, a\sin\theta.$$

Sum of pressure forces

$$= \int \rho g (bdx)\, a\sin\theta$$

$$= \rho g (\sin\theta) \int (bdx) a.$$

$$= \rho g \sin\theta \times A \times c$$

$$= \rho g \sin\theta \times A \times \frac{\bar{x}}{\sin\theta} = \rho g A\bar{x}$$

$$TP = wA\bar{x}$$

Moment of pressure force acting on elemental area

$$= \rho g x.(bdx) \cdot a.$$

$$= \rho g (a\sin\theta)(bdx)\, a$$

Sum of the moments $= \int \rho g (bdx) a^2 \sin\theta.$

$$= \rho g \sin\theta \int (bdx).a^2$$

$$= \rho g \sin\theta \cdot I_x.$$

$$= \rho g \sin\theta \left(I_G + Ac^2\right) \qquad c = \frac{\bar{x}}{\sin\theta}$$

$$= \rho g \sin\theta \left[I_G + A\left(\frac{\bar{x}}{\sin\theta}\right)^2\right] \longrightarrow (1)$$

Moment caused by total pressure about $x-x$, $= TP \times d.$

$$= wA\bar{x} \times d \longrightarrow (2)$$

From (1) & (2),

$$wA\bar{x}\, d = wA\bar{x}\frac{\bar{h}}{\sin\theta} = \rho g \sin\theta \left(I_G + A\frac{\bar{x}^2}{\sin^2\theta}\right)$$

$$\bar{h} = \frac{I_G \sin^2\theta}{A\bar{x}} + \bar{x}$$

$$\boxed{\begin{aligned} TP &= \omega A \bar{x} \\ CP &= \frac{I_G \sin^2\theta}{A\bar{x}} + \bar{x} \end{aligned}}$$

- For horizontal surface, $\theta = 0^\circ$,

$$\bar{h} = \bar{x}$$

- For vertical surface, $\theta = 90^\circ$,

$$\bar{h} = \bar{x} + \frac{I_G}{A\bar{x}}$$

Q.08. $TP = \omega A \bar{x}$

$= 1000 \times 9.81 \times \frac{\pi}{4} \times 4 \times 1 = \underline{\underline{30.76}}$ kN

Q.09. $TP = \omega A \bar{x}$

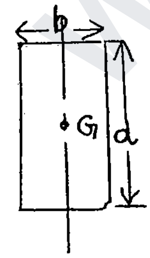
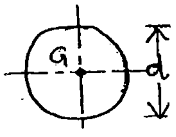
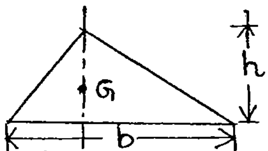
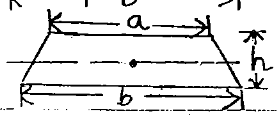
$= 1000 \times 10 \times 9 \times 1.5 = 135$ kN.

Moment of hydrostatic thrust about top edge $= TP \times \bar{h}$

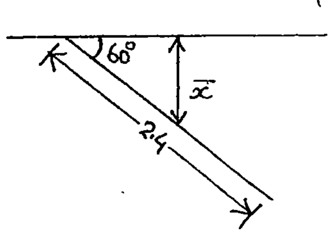
$= TP\left(\frac{I_G}{A\bar{x}} + \bar{x}\right)$

$= 135\left(\frac{3 \times 3^3/12}{9 \times 1.5} + 1.5\right)$

$= 135 \times 2 = \underline{\underline{270 \text{ kN}}}$

Shape	C.G from base	Area	I_G parallel to base
	$d/2$	$b \times d$	$bd^3/12$
	$d/2$	$\frac{\pi}{4} d^2$	$\frac{\pi}{64} d^4$
	$h/3$	$\frac{1}{2} bh$	$\frac{bh^3}{36}$
	$\left(\frac{2a+b}{a+b}\right)\frac{h}{3}$	$\frac{1}{2}(a+b)h$	$\frac{a^2+4ab+b^2}{36(a+b)} \times h^3$

10. $\bar{x} = 1.2 \sin 60$



$T.P = \omega A \bar{x}$

$= 850 \times 9.81 \times 0.75 \times 2.4 \times 1.2 \sin 60$

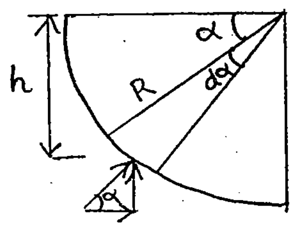
$= \underline{\underline{15.598}}$ kN

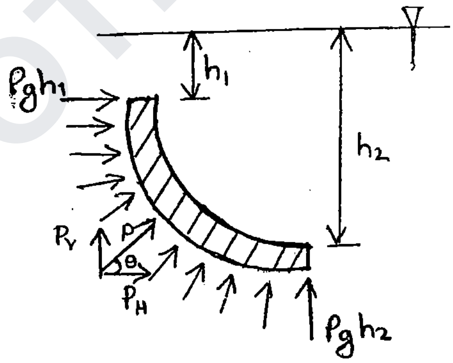
$$CP = \bar{x} + \frac{I_G \sin^2\theta}{A\bar{x}}$$

$$= 1.2 \sin 60 + \frac{0.75 \times 2.4^3/12 \times \sin^2 60}{0.75 \times 2.4 \times 1.2 \sin 60}$$

$= \underline{\underline{1.385}}$ m

11. d) Curved Surface

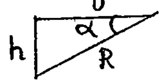




$dA = R\,d\alpha \times 1$

Pressure force acting on $dA = \rho g h \times dA$

$= \rho g h \times R\,d\alpha$



$\sin\alpha = \frac{h}{R}$

$h = R\sin\alpha$

$\therefore$ TP acting on $dA = \rho g (R\sin\alpha) R\,d\alpha = \rho g R^2 \sin\alpha\,d\alpha$

Horizontal component of TP $= \rho g R^2 \sin\alpha\,d\alpha \times \cos\alpha$

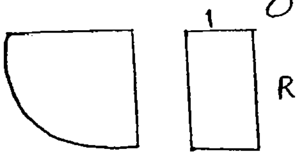
Sum of horizontal components $= \int_{\alpha=0}^{\pi/2} \rho g R^2 \sin\alpha \cos\alpha\,d\alpha$

$= \rho g R^2 \int \sin\alpha \cos\alpha\,d\alpha$

$= \rho g R^2 \left(\frac{-\cos 2\alpha}{2}\right)_0^{\pi/2}$

$= \frac{\rho g R^2}{2}$

$$= \rho g (R \times 1) \frac{R}{2} = \omega A \bar{x}$$



$\therefore$ horizontal component of TP acting on the curved surface will be equal to the total pressure acting on the projected area of curved surface onto the vertical plane.

Vertical component of TP acting on $dA = \rho g R^2 \sin^2\alpha \, d\alpha$.

Sum of vertical component for entire surface

$$= \int \rho g R^2 \sin^2\alpha \, d\alpha .$$

$$= \rho g R^2 \int_0^{\pi/2} \sin^2\alpha \, d\alpha . \qquad \int_0^{\pi/2} \sin^2\theta = \frac{\pi}{4}$$

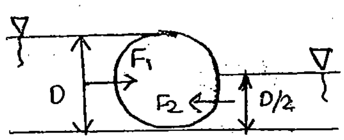
$$= \rho g . \left(\frac{\pi R^2}{4} \times 1 \right)$$

$= \omega \times$ volume of liquid supported by curved surface.

$=$ weight of liquid supported by curved su[rface] upto free surface.

$$\boxed{\begin{aligned} P_H &= \omega A \bar{x} \\ P_V &= W \text{ of liquid supported by curved surface.} \end{aligned}}$$

11. Net horizontal force $= F_1 - F_2$



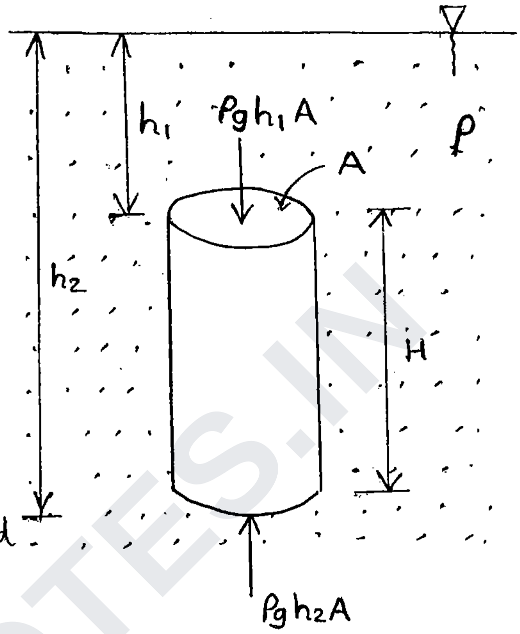
$$F_1 = \omega A \bar{x}$$

$$= \gamma \times D \times \frac{D}{2} = \frac{\gamma D^2}{2}$$

$$F_2 = \gamma \times \left(\frac{D}{2} \times 1 \right) \times \frac{D}{4} = \frac{\gamma D^2}{8}$$

$$F_1 - F_2 = \frac{3 \gamma D^2}{8}$$

3. BUOYANCY

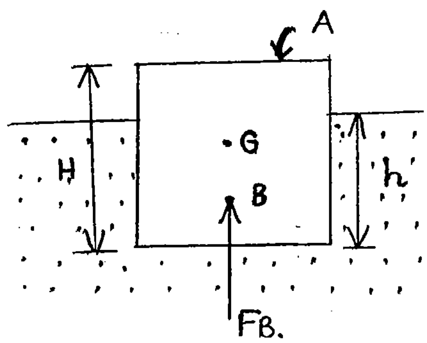


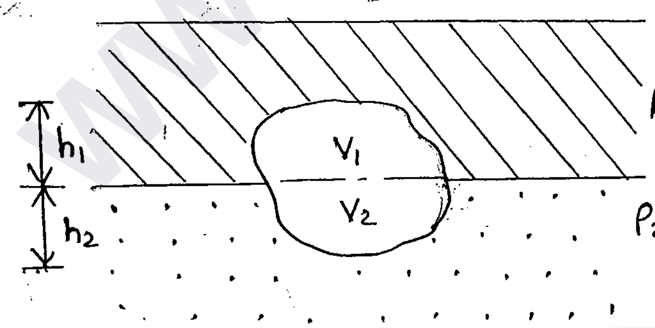
$$\text{Buoyancy Force} = \rho g h_2 A - \rho g h_1 A$$
$$= \rho g A (h_2 - h_1)$$
$$= \rho g A H = \rho g V_{body}$$
$$= \rho g V_{\text{fluid displaced}}$$
$$= \text{Weight of fluid displaced}$$

Whenever an object submerged partially or completely, it wil be subjected to the net resultant vertical upward force and is known as 'Buoyancy force' or 'Force of Buoyancy'

The point at which the force of buoyancy acts is known a 'Centre of Buoyancy' (B). The Centre of Buoyancy is the centroic of the volume of fluid displaced (volume of body immersed)

Volume of body immersed $= A \times h$





$$\boxed{\text{Buoyancy force} = \rho_1 g V_1 + \rho_2 g V_2}$$

At $p = 1$ bar, $T = 20 + 273 = 293$ K.

$$p = \rho R T$$

$$\therefore \rho = \frac{p}{RT} = \frac{100}{0.287 \times 293} = \underline{1.18\ kg/m^3}$$

F_B = weight of liquid displaced + weight of ~~to~~ air displaced.

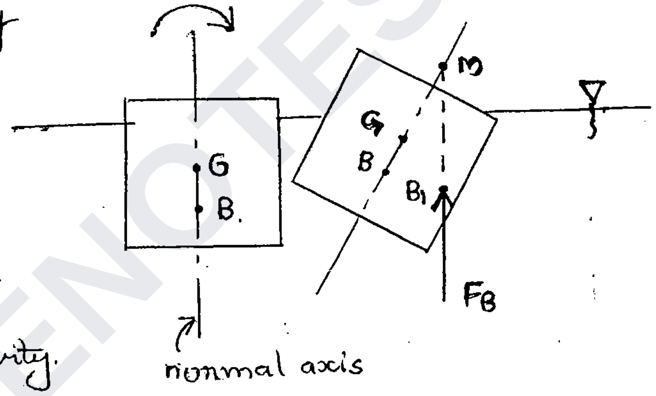
Since ρ of air is very small in magnitude, weight of air displaced will be neglected.

∴ For floating bodies,

$$\boxed{F_B = \text{weight of liquid displaced}}$$

→ Metacentre. (M)

It is defined as the point about which the body will try to oscillate.

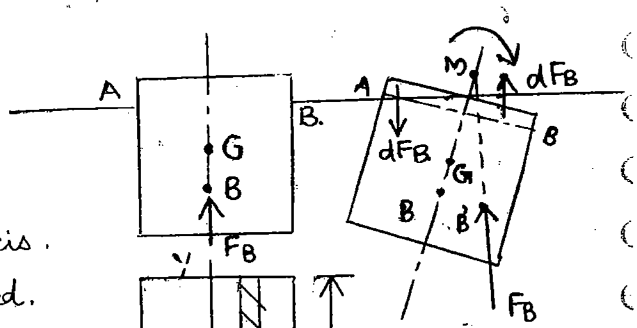


* Metacentric Height (GM)

It is the distance from metacentre to centre of gravity.

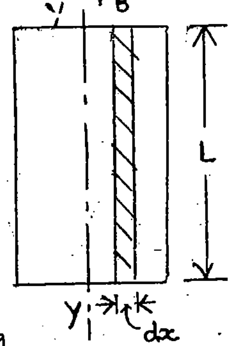
① Analytical Method.

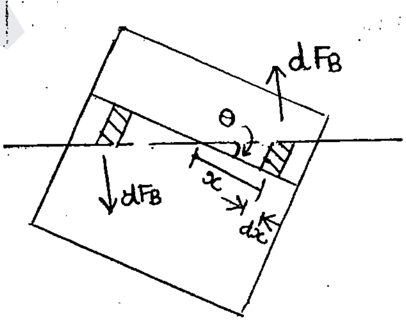
$$\boxed{GM = \frac{I_{Y-Y}}{\forall} - BG}$$

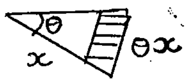


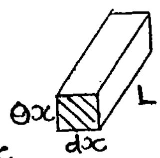
I_{YY} → moment of inertia about Y-Y axis.

$\forall$ → volume of liqui body immersed.









Moment due to $dF_B = dF_B \times 2x$

$$= \rho g \cdot \theta x \cdot dx \times L \times 2x$$

$$= \int \rho g \cdot \theta x \cdot dx \cdot L \times 2x$$

$$= \rho g \int 2x^2 \theta L\, dx$$

$$= \rho g \theta \int 2x^2 (L dx)$$

$$= \rho g \theta\, I_{yy}$$

$L dx \rightarrow$ top surface. area of δb

$\int y^2 dA = I$

$I_{yy} \rightarrow$ moment of inertia @ top liquid level.

Moment caused by change of centre of buoyancy $= F_B \times BB_1$

$$= F_B \times BM \times \theta$$

$\tan\theta = \frac{BB_1}{BM}$

$\tan\theta \approx \theta = \frac{B}{}$

$$\Rightarrow \rho g \theta\, I_{yy} = F_B \times BM \times \theta \quad (?)$$

$$BM = \frac{\rho g}{F_B} \times I_{yy}$$

$$= \frac{I_{yy}}{\forall}$$

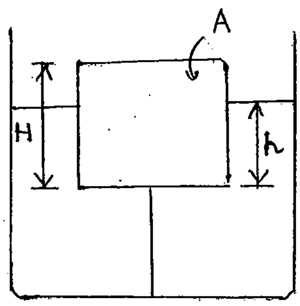
$F_B = \rho g \forall$

$\forall \rightarrow$ volume of immersed bo

$$GM = BM - BG = \frac{I_{yy}}{\forall} - BG$$

?> Due to change of centre of buoyancy, F_B will induce a mom. This moment will be exactly equal in magnitude to the moment caused by the forces, dF_B.

Q A cylindrical body of c/s area, A, height H, density ρ_s is immersed to a depth h in a liquid of density, ρ and tied to the bottom with a string. Tension in the string is



$$W + T = F_B$$

$$T = F_B - W$$

$$= \rho g h A - \rho_s g H A$$

$$\therefore T = (\rho h - \rho_s H) g A$$

When $F_B = W$, the object is freely floating.

$W > F_B$, the object will sink.

$F_B > W$, the body will move in upward direction to the point where $F_B = W$

For floating bodies, $T = 0$

$$\Rightarrow \rho h - \rho_s H = 0$$

$$\boxed{h = \frac{\rho_s}{\rho} H}$$; $h \rightarrow$ depth of body immersed in water

or $h = \dfrac{\text{R.D of solid}}{\text{RD of fluid}} \times$ total height.

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01. $h = \dfrac{0.5}{1} \times 2 = 1$ m.

Volume of cube submerged under water $= 1 \times (2 \times 2) = \underline{\underline{4 m^3}}$

02. $h = \dfrac{0.64}{1.025} \times 2 = \underline{\underline{1.248 \text{ m}}}$

03 $h = \dfrac{3.4}{13.6} \times H$

$\dfrac{h}{H} = 0.25$

∴ Fraction of volume under mercury. $= \underline{\underline{0.25}}$

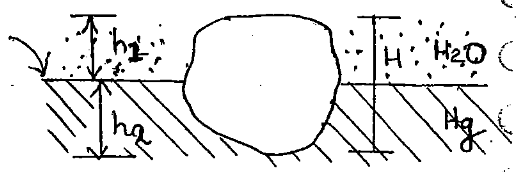
04. ~~Depth immersed in mercury~~ $= \cancel{\dfrac{7.6}{13.6}} \times \cancel{10} = \cancel{5.588 \text{ cm}}$

~~Height~~ of $S_s = \cancel{7.6}$, $S_{f1} = 13.6$, $S_{f2} = 1$

Let V be the volume of the solid, A be the c/s area of solid.

$H = 10$ cm

Liquid Interface.



$F_B = (F_B)_{H_2O} + (F_B)_{Hg}$

$(F_B)_{H_2O}$ = weight of water displaced

$= \rho g \times V_{H_2O}$

$= 1000 \times g \times A \times h_1 = 10 g h_1$

$(F_B)_{Hg}$ = weight of Hg displaced $= 13600 g \times A \times h_2 = 136 g h_2$

Total buoyancy force $= 1000\, g h_1 + 136\, g h_2$

At eqbm,

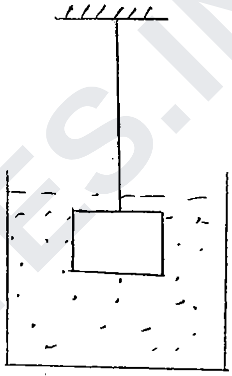
$$W = F_B.$$

$$\rho_s \times 0.1 \times 0.1 \times 0.1\, g = 10 g h_1 + 136 g (H - h_1).$$

$$7.6 = 10 h_1 + 136 (0.1 - h_1).$$

$$h_1 = \underline{\underline{4.76\ cm}}$$

5. $m = 12\ kg$, $S_F = 0.8$, $S_s = 2.4$, $g = 10\ m/s$



$W \downarrow$, $F_B \uparrow$, $T \uparrow$

$$W = F_B + T \Rightarrow T = W - F_B.$$

$W = mg = 12 \times 10 = \underline{120\ N}$

F_B = wt. of fluid displaced.

$= 0.8 \times 1000 \times 10 \times V_{body}$

$= 0.8 \times 1000 \times 10 \times \dfrac{12}{2.4 \times 1000}$ $\qquad V = \dfrac{m}{\rho}.$

$= 40\ N.$

$$T = 120 - 40 = \underline{\underline{80\ N}}$$

6. $\rho_{CH_4} = 0.75\ kg/m^3$

$\rho_{air} = 1.25\ kg/m^3$

Let V be the volume of the balloon.

$(F_B)_{air}$ = weight of balloon with CH_4 + weight of man.

$$\rho_{air} g \cdot V_b = \rho_{CH_4} \times g \times V_b + 750\ N.$$

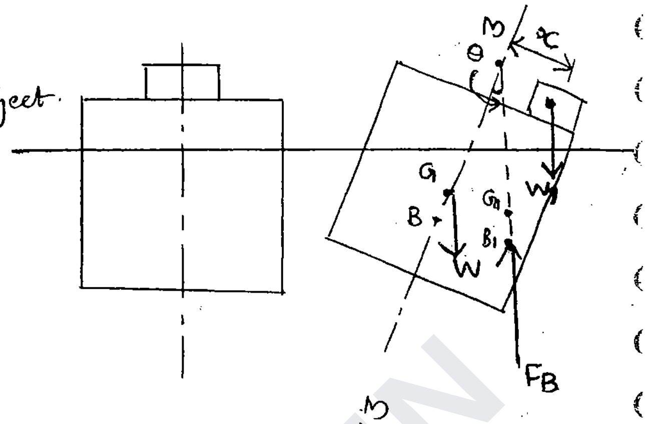
$$1.25 \times 9.81\ V_b = 0.75 \times 9.81 \times V_b + 750\ N.$$

$$V_b = \frac{750}{12.5 - 7.5} = \underline{\underline{150\ m^3}}$$

② Experimental Method.

Let W be the weight of object

and W_1 be the weight of small object.

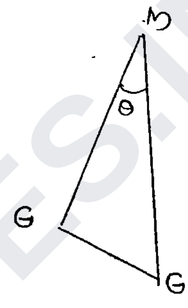


$$F_B \times GG_1 = W_1 \times x$$

$$W \times GG_1 = W_1 \times x.$$

$$W \times GM \times \tan\theta = W_1 \times x.$$

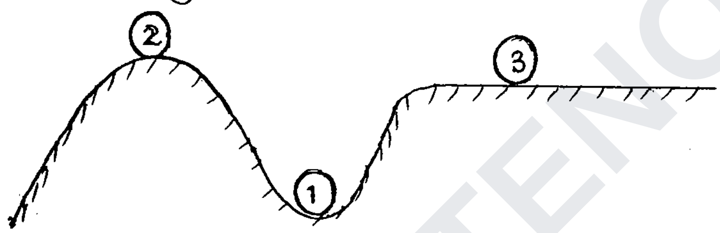
$$\therefore \boxed{GM = \frac{W_1 \times x}{W \tan\theta}}$$



$$\tan\theta = \frac{GG_1}{GM}$$

→ Conditions of Equilibrium

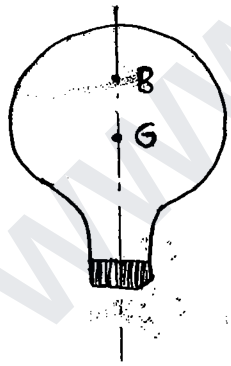
a) Submerged Bodies.

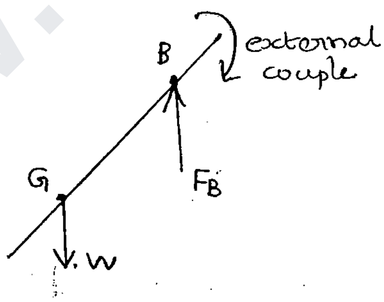


① → Stable Equilibrium

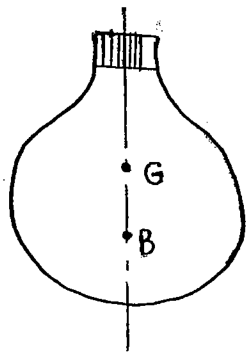
② → Unstable Equilibrium.

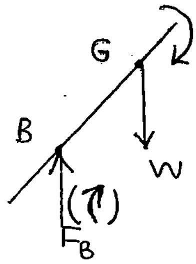
③ → Neutral Equilibrium.



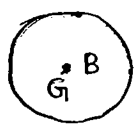


Stable Equilibrium.
(B is above G).



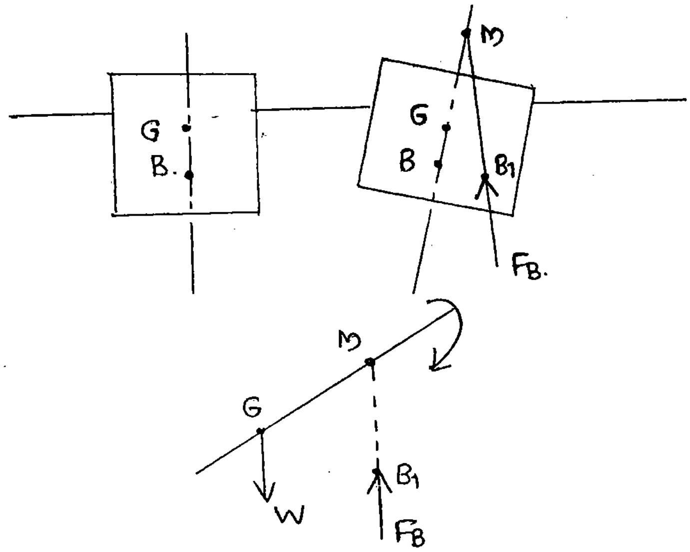


Unstable Equilibrium
(B is below G)

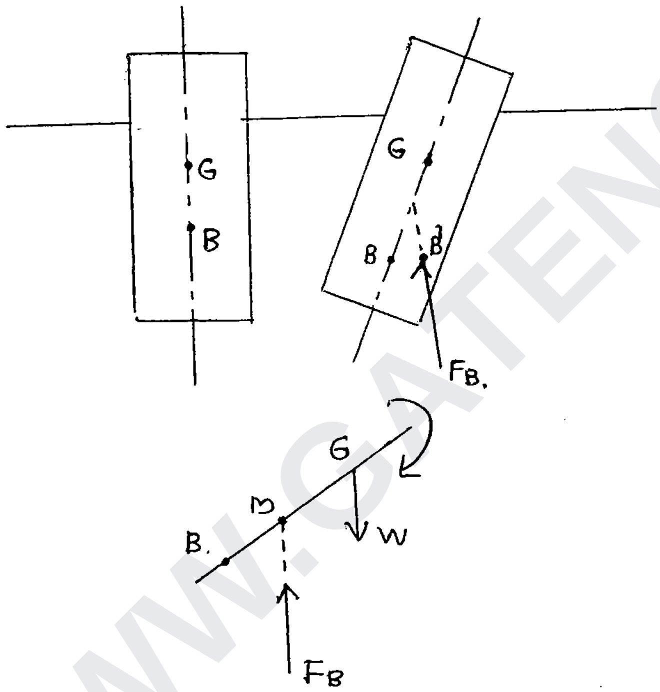


Neutral Equilibrium
(B coincides with G)

b) Floating Bodies.



Stable Equilibrium
(M is above G)



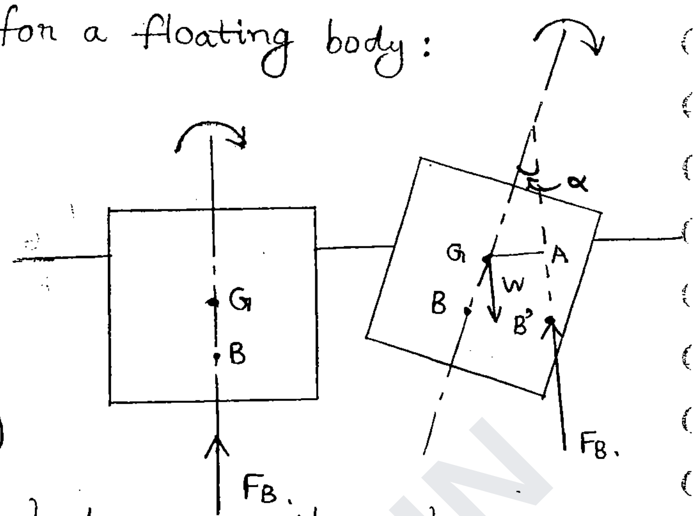
Unstable Equilibrium
(M is below G)

Neutral Equilibrium → M coincides with G

Time period of oscillation, $T = 2\pi \sqrt{\frac{K^2}{GM \times g}}$; GM → metacentric height.

When ship is loaded vertically as a stack, CG shifts upward and chances of M coming below G are more. So for the safer side, GM is increased, but it cannot be increased randomly considering the comfort factors. $\left(T \propto \frac{1}{\sqrt{GM}}\right)$

→ Time period of Oscillation for a floating body :



Restoring couple $= F_B \times GA$

$= W \times GA$

$= W \times GM \times \sin\theta$

$= W \times GM \times \theta$

$= W \times GM \times (-\theta)$

$-ve$ sign indicates, restoring couple decreases the angle

External couple $= I \times \alpha$

$= m \times k^2 \times \alpha$

$= \frac{W}{g} \times k^2 \times \frac{d^2\theta}{dt^2}$

Restoring couple = External couple.

$$-W \times GM \times \theta = \frac{W}{g} \times k^2 \times \frac{d^2\theta}{dt^2}$$

$$\frac{d^2\theta}{dt^2} = \frac{-GM \times g \times \theta}{k^2}.$$

$$\Rightarrow \boxed{\frac{d^2\theta}{dt^2} + \frac{GM\, g}{k^2}\,\theta = 0}$$

Soln. will be of form $\alpha \pm i\beta$.

$\alpha = 0$

$\beta = \sqrt{\frac{GM g}{k^2}}$

Solution, $\theta = \left[C_1 \cos \sqrt{\frac{GM \times g}{k^2}}\, t + C_2 \sin \sqrt{\frac{GM . g}{k^2}}\, t \right]$

Boundary Conditions :

(i) At $t = 0$; $\theta = 0$

(ii) At $t = \frac{T}{2}$; $\theta = 0$.

(i) $\Rightarrow$ $0 = C_1 + 0$ $\therefore C_1 = 0.$

(ii) $\Rightarrow$ $0 = C_2 \sin\left(\sqrt{\frac{GM . g}{k^2}} \times \frac{T}{2} \right)$

$$\sin\sqrt{\frac{GM.g}{k^2}}\cdot\frac{T}{2} = 0 = \sin n\pi \quad ; \; n = 1, 2, 3 \ldots$$

$$\sqrt{\frac{GM.g}{k^2}}\;\frac{T}{2} = \pi$$

$$\therefore \quad \boxed{T = 2\pi\sqrt{\frac{k^2}{GM.g}}}$$

; k → radius of gyration
GM → metacentric ht.

13th Aug,
WEDNESDAY

4. FLUID KINEMATICS

In the kinematics of fluid flow, the fluid characteristics like velocity, acceleration etc are studied without taking into consideration of the forces causing the motion.

The two methods by which the fluid flow problems can be analysed are:

(i) Lagrangian Method.

In this method, only a single fluid particle is considered for carrying out the study.

(ii) Eulerian Method

In this method, a point is considered in the fluid flow and study is carried out for those particles which are passing through given point. Only the Eulerian method is followed in all the fluid flow problems.

→ Types of Flow.

1. Steady & Unsteady flow

$$\left(\frac{\partial P}{\partial t}\right)_{(x,y,z)} = 0 \quad , \quad \left(\frac{\partial V}{\partial t}\right)_{(x,y,z)} = 0 \longrightarrow \text{Steady Flow}$$

$$\left(\frac{\partial P}{\partial t}\right)_{(x,y,z)} \neq 0 \quad , \quad \left(\frac{\partial V}{\partial t}\right)_{(x,y,z)} \neq 0 \longrightarrow \text{Unsteady Flow}$$

2. Uniform & Non uniform flow.

$$\frac{\partial V}{\partial s} = 0 \longrightarrow \text{Uniform Flow}$$

$$\frac{\partial V}{\partial s} \neq 0 \longrightarrow \text{Non uniform Flow.}$$

3. Laminar & Turbulent flow.

Laminar

Turbulent.

Reynold's number, $R_e = \frac{\text{Inertia force, } F_i}{\text{Viscous force, } F_v}$

$F_i = m \times a$

$= m \times \frac{\text{velocity}}{\text{time}} = \frac{\rho \times \text{volume}}{t} \times \text{velocity}$

$= \rho \times Q \times \text{velocity} = \underline{\underline{\rho A V^2}}$

$F_v = \tau . A$

$= \mu \frac{du}{dy} \times A = \frac{\mu V A}{L}$

$\therefore R_e = \frac{\rho A V^2}{\frac{\mu V A}{L}} = \frac{\rho V L}{\mu} \longrightarrow \text{plate} = \frac{VL}{\gamma} \quad \left(\gamma = \frac{\mu}{\rho}\right)$

$= \frac{\rho V D}{\mu} \longrightarrow \text{pipe} = \frac{VD}{\gamma}$

For a plate: (fluid is always a gas)

$Re < 5\times10^5 \longrightarrow$ Laminar

$5\times10^5 < Re < 10^7 \longrightarrow$ Transition

$Re > 10^7 \longrightarrow$ Turbulent

For the pipe:

$Re < 2000 \longrightarrow$ Laminar

$2000 < Re < 4000 \longrightarrow$ Transition

$Re > 4000 \longrightarrow$ Turbulent.

3. Compressible & Incompressible Flow

$\partial \rho = 0 \longrightarrow$ Incompressible

$\partial \rho \neq 0 \longrightarrow$ Compressible.

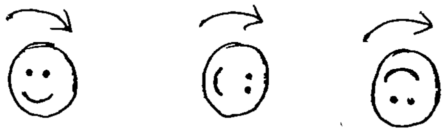
4. 1D, 2D & 3D flows.

$u = f(x), \; v = 0, \; w = 0 \longrightarrow 1D$

$u = f(x,y), \; v = f(x,y), \; w = 0 \longrightarrow 2D$

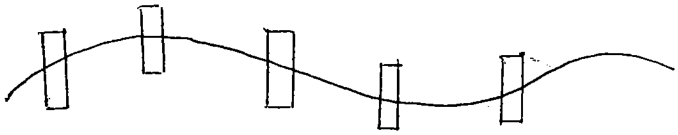
$u = f(x,y,z), \; v = f(x,y,z), \; w = f(x,y,z) \longrightarrow 3D$

5. Rotational & Irrotational Flows.

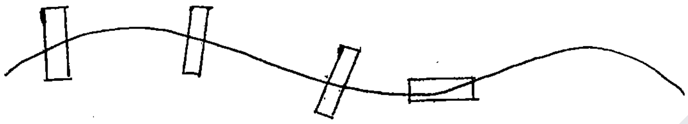


$\longrightarrow$ Rotational.

Fluid particles rotate about their mass centre.



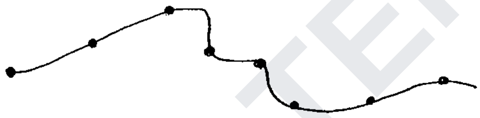
$\longrightarrow$ Irrotational



$\longrightarrow$ Rotational.

$\rightarrow$ Types of Lines

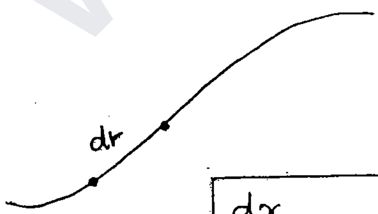
(i) Path Line. $\rightarrow$ path traced out by a fluid particle



(ii) Stream line $\rightarrow$ Imaginary line drawn in the flow field such that the tangent drawn at any point represents direction of velocity vector at that instant of time

NOTE:

- No flow takes place across the streamlines.
- No two streamlines cross each other.



$$\vec{dr} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$$

$$\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$$

$$\boxed{\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}} \rightarrow \underline{\text{Equation of Stream line}}$$

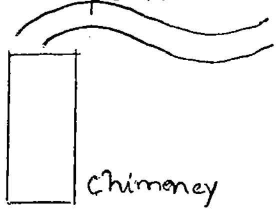
where u, v, w are velocity components along x, y & z directions

where V is the resultant velocity

(iii) Streakline

It is the locus or the path traced out by the fluid particles which have passed through the chosen point.

Eg: The path traced out by the chimney gases



NOTE:

- For steady flow, the pathline, streamline and streak line are identical.

→ Velocity & Acceleration :-

Velocity Vector, $\vec{V} = \vec{u}i + \vec{v}j + \vec{w}k$ where u, v, w are the velocity components along x, y & z direc

∴ Resultant velocity magnitude, $|V| = \sqrt{u^2 + v^2 + w^2}$

$u = f(x, y, z, t)$; $v = f(x, y, z, t)$, $w = f(x, y, z, t)$ ←

Acceleration along x-direction, $a_x = \frac{du}{dt}$

$$= \frac{\partial u}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial u}{\partial y} \frac{\partial y}{\partial t} + \frac{\partial u}{\partial z} \cdot \frac{\partial z}{\partial t} + \frac{\partial u}{\partial t} \cdot \frac{\partial t}{\partial t}.$$

$$\therefore \boxed{a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}}$$

Similarly, $a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$ ← local or temporal acceler

$$a_z = \underbrace{u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}}_{} + \frac{\partial w}{\partial t}.$$ ← convective acceleratio

∴ Acceleration vector, $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

Magnitude of resultant acceleration, $|a| = \sqrt{a_x^2 + a_y^2 + a_z^2}$

∴ Total Acceleration = convective + local.

Convective accl. is the acceleration produced because of change of velocity wrt change of space. $\left(\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y} \ldots\right)$

Local accl. is the acceleration produced because change of velocity wrt change of time $\left(\frac{\partial u}{\partial t}, \frac{\partial v}{\partial t} \;\&\; \frac{\partial w}{\partial t}\right)$

Type of Flow	Convective	Local.
Steady & uniform	×	×
Steady & nonuniform	✓	×
Unsteady & uniform	×	✓
Unsteady & non-uniform	✓	✓

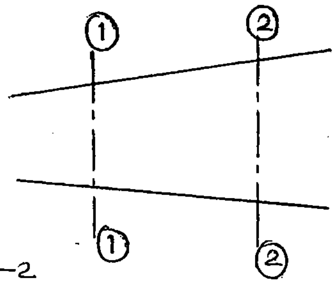
→ Discharge (Q)

$$Q = \frac{\text{volume of fluid flowing}}{\text{time}}$$

Unit: m^3/s

$$Q = \frac{A \times l}{t} = A \times \text{velocity}$$

* Continuity Equation in 1-D:



It is based on law of conservation of mass.

$$(\text{Mass flow rate})_{1-1} = (\text{mass flow rate})_{2-2}$$

$$\left(\frac{\text{mass}}{\text{time}}\right)_{1-1} = \left(\frac{\text{mass}}{\text{time}}\right)_{2-2}$$

$$\left(\frac{\text{Density} \times \text{volume}}{\text{time}}\right)_{1-1} = \left(\frac{\text{density} \times \text{volume}}{\text{time}}\right)_{2-2}$$

$$\rho_1 \times Q_1 = \rho_2 \times Q_2$$

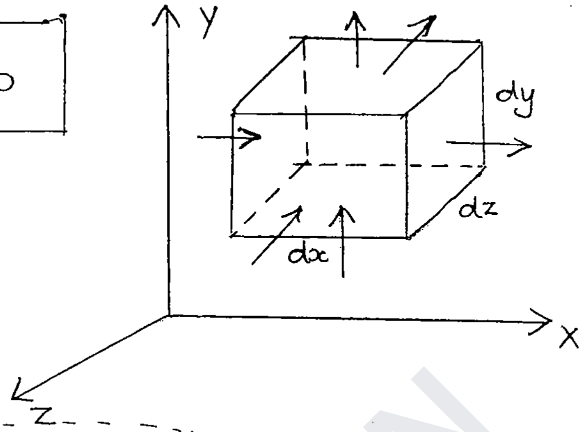
$$\boxed{\rho_1 A_1 V_1 = \rho_2 A_2 V_2}$$

For incompressible fluids, $\rho_1 = \rho_2$

$$\boxed{A_1 V_1 = A_2 V_2}$$

* Continuity Equation in Cartesian Co-ordinate System

$$\boxed{\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} = 0}$$



Mass flow rate entering along x-direction $= \rho(dy\,dz)u.$

Mass flow rate leaving along x-direction $= \rho(dy\,dz)u + \frac{\partial}{\partial x}(\rho u\, dy\, dz)\,dx$ ← change of mass flow rate for a length of 'dx'

Mass flow rate stored $= \rho u\, dy dz - \left(\rho u\, dy dz + \frac{\partial}{\partial x}(\rho u\, dy\, dz\, dx)\right)$

$$= -\frac{\partial}{\partial x}(\rho u\, dy\, dz)\,dx. \longrightarrow (1)$$

Mass flow rate stored because of flow taking place along y direction $= -\frac{\partial}{\partial y}(\rho v\, dx\, dz)\,dy. \longrightarrow (2)$

Mass flow rate stored because of flow taking place along z direction $= -\frac{\partial}{\partial z}(\rho z\, dx\, dy)\,dz. \longrightarrow (3)$

Total mass flow rate stored $= -\left(\frac{\partial}{\partial x}(\rho u\, \underline{dy dz})\,dx + \frac{\partial}{\partial y}(\rho v\, \underline{dx dz})\,dy + \frac{\partial}{\partial z}(\rho z\, \underline{dx dy})\,dz\right) \longrightarrow (4)$

Change in mass of element per unit time $= \frac{\partial m}{\partial t}$

$$= \frac{\partial}{\partial t}(\rho\, dx.dy.dz). \hookrightarrow (5)$$

Equating (4) & (5),

$$-\left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right] dx.dy.dz = \frac{\partial \rho}{\partial t}\, dx.dy.dz$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

For steady flow,

$$\frac{\partial \rho}{\partial t} = 0,$$

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

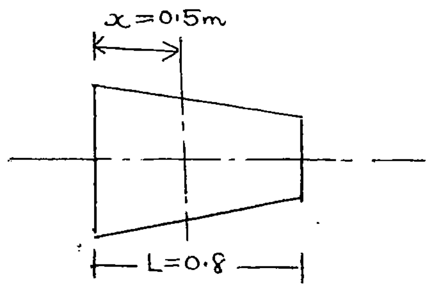
For steady, incompressible flow,

$$\rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0$$

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0}$$

P-61

2. $V = 2t\left(1 - \frac{x}{2L}\right)^2$



Local acceleration $= \frac{\partial V}{\partial t}$

$$= \frac{\partial}{\partial t}\left(2t\left(1 - \frac{x}{2L}\right)^2\right)$$

$$= 2\left(1 - \frac{x}{2L}\right)^2 = 2\left(1 - \frac{0.5}{2\times 0.8}\right)^2 = \underline{\underline{0.945}} \text{ m/s}$$

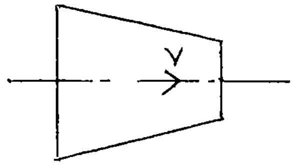
3. Convective acceleration $= a_x = u\frac{du}{dx} + v\cancel{\frac{du}{dy}}^{0} + w\cancel{\frac{\partial u}{\partial z}}^{0}$ (flow in x direction)

$$= 2t\left(1 - \frac{x}{2L}\right)^2 2t \times 2\left(1 - \frac{x}{2L}\right) \times \frac{-1}{2L}$$

$$= 2\times 3\left(1 - \frac{0.5}{1.6}\right)^2 \times 2\times 3\times 2\left(1 - \frac{0.5}{1.6}\right) \times \frac{-1}{2\times 0.8}$$

$$= \underline{\underline{-14.6}} \text{ m/s}^2$$

07. $A = 0.4 - 0.1x$



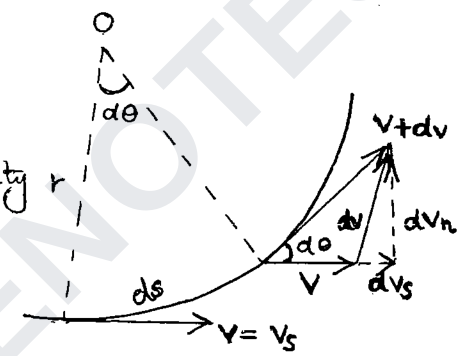
$$\frac{dQ}{dt} = \frac{0.12\ m^3/s}{s}$$

Velocity, $V = \frac{Q}{A}$.

$\therefore$ Local acceleration $= \frac{dV}{dt} = \frac{d}{dt}\left(\frac{Q}{A}\right)$

$$= \frac{1}{A}\frac{dQ}{dt}$$

$$= \frac{1}{0.4} \times 0.12 = \underline{\underline{0.3\ m/s^2}}$$



Let 'V_s' be the tangential velocity

'V_n' be radial or normal velocity

$V_s = f(s, n, t)$

$V_n = f(s, n, t)$

Tangential acceleration, $a_s = \frac{dV_s}{dt}$

$$= \frac{\partial V_s}{\partial s}\cdot\frac{\partial s}{\partial t} + \frac{\partial V_s}{\partial n}\cdot\frac{\partial n}{\partial t} + \frac{\partial V_s}{\partial t}\cdot\frac{\partial t}{\partial t}$$

$$\boxed{a_s = V_s \cdot \frac{\partial V_s}{\partial s} + V_n \frac{\partial V_s}{\partial n} + \frac{\partial V_s}{\partial t}}$$

For steady flow $\frac{\partial V_s}{\partial t} = 0$

Normal or radial acceleration, $a_n = \frac{dV_n}{dt}$

$$= \frac{\partial V_n}{\partial s}\cdot\frac{\partial s}{\partial t} + \frac{\partial V_n}{\partial n}\frac{\partial n}{\partial t} + \frac{\partial V_n}{\partial t}\cdot\frac{\partial t}{\partial t}$$

$$\boxed{a_n = V_s \frac{\partial V_n}{\partial s} + V_n \frac{\partial V_n}{\partial n} + \frac{\partial V_n}{\partial t}}$$

For steady flow $\frac{\partial V_n}{\partial t} = 0$

For steady flow,

$$a_s = V_s \frac{\partial V_s}{\partial s} + V_n \frac{\partial V_s}{\partial n} \quad \& \quad a_n = V_s \frac{\partial V_n}{\partial s} + V_n \frac{\partial V_n}{\partial n}.$$

Since there is no flow across streamline, then $V_n = 0$

$$a_s = V_s \frac{\partial V_s}{\partial s} \quad \& \quad a_n = V_s \frac{\partial V_n}{\partial s}$$

$$ds = r\,d\theta \longrightarrow ①$$

$$\tan d\theta = \frac{\partial V_n}{V} = d\theta \quad (\text{for small angles}) \longrightarrow ②$$

From ① & ②,

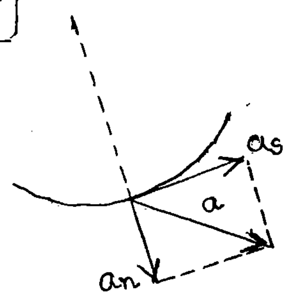
$$\frac{ds}{r} = \frac{dV_n}{V} = \frac{\partial V_n}{V_s} \qquad (\because V = V_s)$$

$$\frac{V_s}{r} = \frac{\partial V_n}{\partial s}$$

$$\therefore a_n = V_s \cdot \frac{\partial V_n}{\partial s}$$

$$= V_s \cdot \frac{V_s}{r}$$

$$\boxed{\text{Normal acceleration, } a_n = \frac{V_s^2}{r} = \frac{V^2}{r}}$$



$$a = \sqrt{a_s^2 + a_n^2}$$

NOTE:

⊙ Tangential acceleration is because of change in the magnitude of the tangential velocity

⊙ Normal or radial acceleration is because of the change in the direction of the tangential velocity.

4. $\frac{\partial V_s}{\partial s} = \frac{1}{3}$; $r = 9\,m$; $V_s = V = 3\ m/s$

$$a_n = \frac{V^2}{r} = \frac{3^2}{9} = 1\ m/s^2.$$

$$a_s = V_s \cdot \frac{\partial V_s}{\partial s} = 3 \times \frac{1}{3} = 1\ m/s^2.$$

$$a = \sqrt{a_n^2 + a_s^2} = \sqrt{1+1} = \underline{\underline{\sqrt{2}\ m/s^2}}$$

06. $r = 3m$; $V = 3\sin\theta$

$$a_s = V_s \frac{\partial V_s}{\partial s}$$

But $\partial s = r\partial\theta$,

$$\therefore a_s = V_s \cdot \frac{\partial V_s}{r\partial\theta} = \frac{V_s}{r}\frac{\partial}{\partial\theta}(3\sin\theta)$$

$$= \frac{3\sin 45}{3} \times 3\cos 45$$

$$= \frac{3}{2} = \underline{\underline{1.5\ m/s^2}}$$

→ Velocity Potential Function (ϕ)

$$\phi = f(x,y,z,t) \Rightarrow u = -\frac{\partial\phi}{\partial x}\ ; \quad v = -\frac{\partial\phi}{\partial y}\ ; \quad w = -\frac{\partial\phi}{\partial z}$$

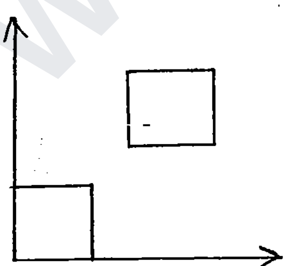
For steady incompressible flow,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad \text{(continuity equation)}$$

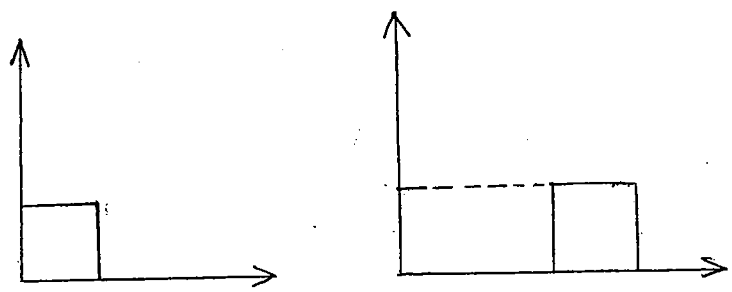
$$\frac{\partial}{\partial x}\left(-\frac{\partial\phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(-\frac{\partial\phi}{\partial y}\right) + \frac{\partial}{\partial z}\left(-\frac{\partial\phi}{\partial z}\right) = 0.$$

$$\boxed{\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0} \rightarrow \text{Laplace Equation}$$

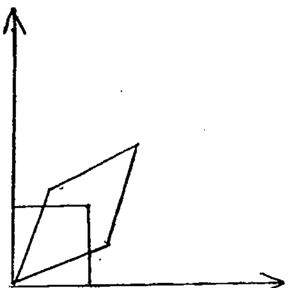
If ϕ exists for the flow, then it satisfies Laplace Equation



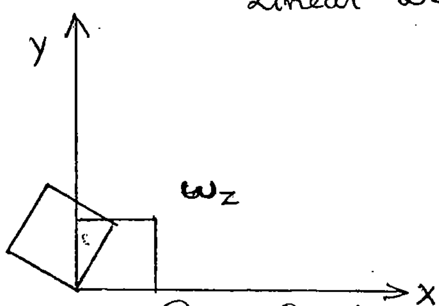
Linear Translation



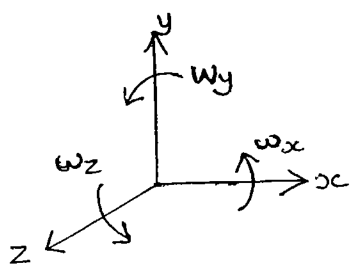
Linear Deformation



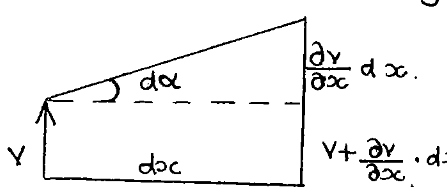
Angular Deformation

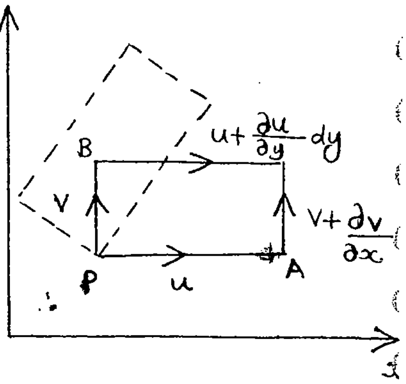


Pure Rotation



$$\tan d\alpha = \frac{\frac{\partial v}{\partial x} \cdot dx \cdot dt}{dx}$$





For small angles,

$\tan d\alpha = d\alpha$.

$$\frac{d\alpha}{dt} = \frac{\partial v}{\partial x}$$

Similarly,

$$\frac{d\beta}{dt} = \frac{-\partial u}{\partial y}$$ (-ve sign represents the element PB rotates in clockwise direction).

$$\omega_{PA} = \frac{d\alpha}{dt} = \frac{\partial v}{\partial x} \;; \qquad \omega_{PB} = \frac{d\beta}{dt} = \frac{-\partial u}{\partial y}$$

Rotation about z-axis,

$$w_z = \frac{1}{2}(\omega_{PA} + \omega_{PB}).$$

$$= \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right).$$

$$w_z = \frac{1}{2}\left(\frac{\partial}{\partial x}\left(\frac{-\partial \phi}{\partial y}\right) - \frac{\partial}{\partial y}\left(\frac{-\partial \phi}{\partial x}\right)\right)$$

$$= \frac{1}{2}\left(\frac{-\partial^2 \phi}{\partial x \partial y} + \frac{\partial^2 \phi}{\partial y \partial x}\right).$$

$$= 0 \qquad \Rightarrow \qquad \omega_x = \omega_y = \omega_z = 0$$

- If the velocity potential fn exists, then the flow is irrotational

$$\omega = \frac{1}{2} \begin{array}{ccc} + & - & + \\ \end{array}$$
$$\omega = \frac{1}{2}\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

$$w_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$w_y = -\frac{1}{2}\left(\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}\right) = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)$$

$$w_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)$$

→ Stream Function (Ψ)

$$\psi = f(x, y, t).$$

$$v = \frac{\partial \psi}{\partial x} \quad ; \quad u = -\frac{\partial \psi}{\partial y}$$

⊙ Stream function exists only for 2-D flow.

For steady incompressible flow,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial}{\partial x}\left(\frac{-\partial \psi}{\partial y}\right) + \frac{\partial}{\partial y}\left(\frac{\partial \psi}{\partial x}\right) = 0.$$

$$-\frac{\partial^2 \psi}{\partial x . \partial y} + \frac{\partial^2 \psi}{\partial y . \partial x} = 0 \Rightarrow \psi \text{ satisfies } \underline{\text{continuity equa}}$$

$$w_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}\left(\frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial x}\right) - \frac{\partial}{\partial y}\left(\frac{-\partial \psi}{\partial y}\right)\right)$$

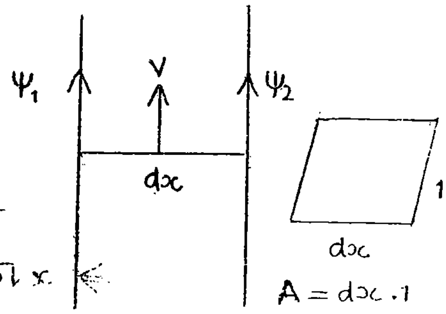
$$= \frac{1}{2}\left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}\right)$$

If ψ satisfies Laplace equation, then $\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$.

$\therefore$ $w_z = 0$. ⇒ flow is irrotational (not rotational).

Function	Continuity Equation	Laplace Equation	Rotational	Irrotatio
φ	✓	✓		✓
ψ	✓	✓		✓
		✗	✓	

$$Q = A \times V$$
$$= (dx \times 1) \times V$$
$$Q = V . dx \longrightarrow ①$$



$$\frac{\partial \psi}{\partial x} = \frac{\partial \psi}{\partial x} \cdot \frac{\partial x}{\partial x} + \frac{\partial \psi}{\partial y} \cdot \frac{\partial y}{\partial x}$$

$$= \frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial y} \cdot \frac{\partial y}{\partial x}$$

$$\partial \psi = \frac{\partial \psi}{\partial x} \cdot \partial x + \frac{\partial \psi}{\partial y} \cdot \partial y$$

$$\Rightarrow \partial \psi = V\,dx - u\,dy.$$

Since flow taking place along y-direction,

$$\therefore u = 0$$

$$d\psi = V\,dx \longrightarrow ②$$

From ① & ②,

$$dQ = d\psi$$

or $\boxed{dQ = \psi_2 - \psi_1}$

NOTE:

- The difference of magnitudes of the two stream fns will be equal to discharge per unit width.

Q1. $u = -x$, $V = 2y$.

$$u = -\frac{\partial \psi}{\partial y} \qquad V = \frac{\partial \psi}{\partial x}.$$

$$\psi = +\int x\,dy.$$

$$= xy + c.$$

$\frac{dx}{u} = \frac{dy}{V}$ $\Rightarrow$ Equation stream line.

$$\frac{dx}{-x} = \frac{dy}{2y}$$

Integrate on both sides,

$$-\int \frac{dx}{x} = \int \frac{dy}{2y}$$

$$-\log x = \frac{1}{2}\log y + \log c \qquad ①$$

$\Rightarrow x\sqrt{y} = c$ $(1,1) \Rightarrow 1\sqrt{1} = c$ $\Rightarrow x\sqrt{y} = 1$

08. $\frac{\pi}{4} \times (5\times10^{-3})^2 \times 7.5\times10^{-3} = \frac{\pi}{4} \times (0.5\times10^{-3})^2 \times V_{blood} + \underbrace{20\times10^{-9}}$ 3| $(A_1V_1 = A_2V_2)$

$V_{blood} = \underline{\underline{0.65\ m/s}}$

09. $\psi = x^2 - y^2.$

$V = \sqrt{u^2+v^2}$

$a = \sqrt{a_x^2 + a_y^2}$

$a_x = u.\frac{du}{dx} + v\frac{\partial u}{\partial y} + \cancel{w\frac{\partial u}{\partial z}}^{0} + \cancel{\frac{\partial u}{\partial t}}^{0}$

$a_y = u.\frac{dv}{\partial x} + v\frac{\partial v}{\partial y}.$

$u = \frac{-\partial\psi}{\partial y} = 2y.$

$v = \frac{\partial \psi}{\partial x} = 2x.$

$\Rightarrow a_x = 2y.0 + 2x.2$

$= 4x.$

$a_y = 2y.2 + 2x.0$

$= 4y.$

$a = a_x\hat{i} + a_y\hat{j} = \underline{\underline{4x\hat{i} + 4y\hat{j}}}$

10. $\phi = x^2+y^2$

$\frac{\partial\phi}{\partial x} = 2x$; $\frac{\partial^2\phi}{\partial x^2} = 2$

$\frac{\partial\phi}{\partial y} = 2y$; $\frac{\partial^2\phi}{\partial y^2} = 2$

$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 4 \neq 0$

$\phi = x^2 - y^2$

$\frac{\partial^2\phi}{\partial x^2} = 2$

$\frac{\partial^2\phi}{\partial y^2} = -2$

$\underline{\underline{\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0}}$

11. $\psi = \frac{3}{2}\left(y^2 - x^2\right)$.

$Q = \left|\psi_2 - \psi_1\right|$.

$\psi_1 = \frac{3}{2}\left(3^2 - 1^2\right) = 12$.

$\psi_2 = \frac{3}{2}\left(3^2 - 3^2\right) = 0$

$\therefore\ Q = |0 - 12| = \underline{\underline{12 \text{ units}}}$

12. $V = u\hat{i} + v\hat{j} = 3x\hat{i} + 4xy\hat{j}$

$w_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$.

$= \frac{1}{2}\left(4y - 0\right) = \frac{1}{2} \times 4 \times 2 = \underline{\underline{4}} \text{ rad/s}$.

13. $\psi = x^2 - y^2$

$\frac{\partial \phi}{\partial x} = -u \cancel{=} -\partial$ $\qquad u = -\frac{\partial \psi}{\partial y} = +2y$

$\frac{\partial \phi}{\partial y} = -v.$ $\qquad v = \frac{\partial \psi}{\partial x} = 2x.$

$\Rightarrow \frac{\partial \phi}{\partial x} = -2y.$

$\phi = -2yx + f(y)$ $\qquad \frac{\partial \phi}{\partial y} = -2x + f'(y).$

~~At origin,~~ $\cancel{\phi = 0} \cancel{\Rightarrow f(y)} = 0.$ $\qquad -v = -2x + f'(y).$

$\phi = \cancel{-2yx}$

$-\frac{\partial \psi}{\partial x} = -2x + f'(y)$

$-2x = -2x + f'(y)$

$\therefore f'(y) = 0 \quad \therefore f(y) = c.$

$\Rightarrow \phi = -2yx + c.$

At $(0,0)$; $\phi = 0$,

$0 = 0 + c$

$\therefore\ \phi = \underline{\underline{-2xy}}$

Lines of Constant velocity potential function:

$$\phi = c$$

$$d\phi = 0$$

But $\phi = f(x,y)$

$$d\phi = \frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} \cdot dy$$

$$0 = -u dx - v dy$$

$$\frac{dy}{dx} = \frac{-u}{v}$$

$\therefore$ Slope of constant velocity potential line is $\frac{-u}{v}$

Lines of Constant Stream Function:

$$\psi = c.$$

$$d\psi = 0$$

But $\psi = f(x,y)$

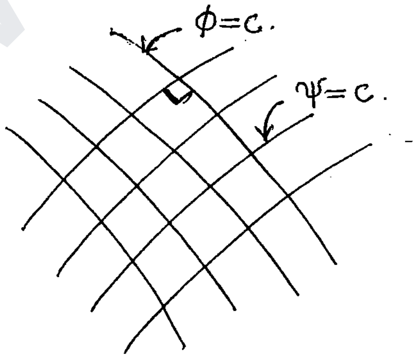
$$d\psi = \frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy$$

$$0 = v dx - u dy$$

$$\Rightarrow \frac{dy}{dx} = \frac{v}{u}$$

$\therefore$ slope of constant stream function line is $\frac{v}{u}$

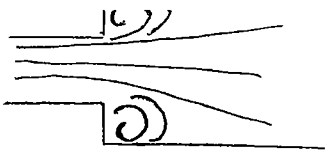
Product of slopes $= \frac{-u}{v} \times \frac{v}{u} = \underline{\underline{-1}}$



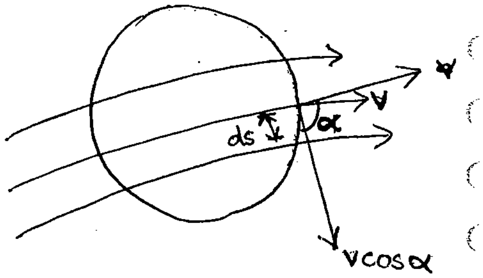
Flow net.

The line of constant velocity potential fn (ϕ) and line of constant stream function are orthogonal to each other.

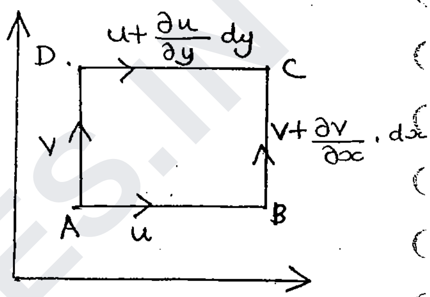
Circulation & Vorticity :



Circulation is defined as the flow of the fluid along a closed curve.



Mathematically, it is defined as the line integral of the tangential component of the velocity vector taken around a closed curve.



$$\text{Circulation, } \Gamma = \int_C V\cos\alpha \, . \, ds$$

Eg:-

$$\Gamma_{ABCD} = u\,dx + \left(v + \frac{\partial v}{\partial x}dx\right)dy - \left(u + \frac{\partial u}{\partial y}dy\right)dx - v\,dy.$$

$$= u\,dx + v\,dy + \frac{\partial v}{\partial x}\,.\,dx.dy - u\,dx - \frac{\partial u}{\partial y}dx.dy - v\,dy.$$

$$\Gamma_{ABCD} = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)dx.dy$$

$$* \ \text{Vorticity} = \frac{\Gamma}{A} = \frac{\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)dx.dy}{dx.dy} = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}.$$

$$= 2\,\omega_z$$

∴ $\boxed{\text{Vorticity is twice the rotation}}$

Vortex Flow :

The flow of the fluid mass along a curved path is known as Vortex Flow.

If torque is required to rotate the fluid mass it is called Forced Vortex Flow. If no torque is required, then it is known as Free vortex Flow.

Rotating mass of fluid in a vertical cylinder (forced vortex (31) flo..

In a free vortex flow, torque $=0$

Torque $=$ rate of change of angular momentum

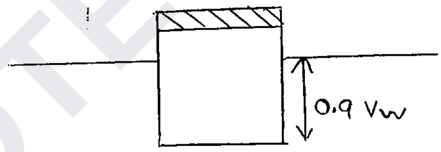
$$= \frac{\partial}{\partial t}(m \times v \times r)$$

$$\frac{\partial (mvr)}{\partial t} = 0.$$

$$mvr = C \Rightarrow \underline{\underline{vr = C}}$$

P-44

Q.18. F_B = weight of fluid displaced.



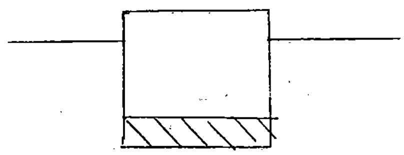
$$w = \rho_F \times g \times V_f.$$

$$mg = \rho_F \times g \times 0.9\ V_w$$

$$(m_w + m_l)g = 1000 \times 9.81 \times 0.9 \times \frac{m_w}{\rho_w}$$

$$(3.67 + m_l) = 1000 \times 0.9 \times \frac{3.67}{600}$$

$$\Rightarrow m_l = 1.835\ kg$$



Q.19. $F_B = (F_B)_w + (F_B)_l.$

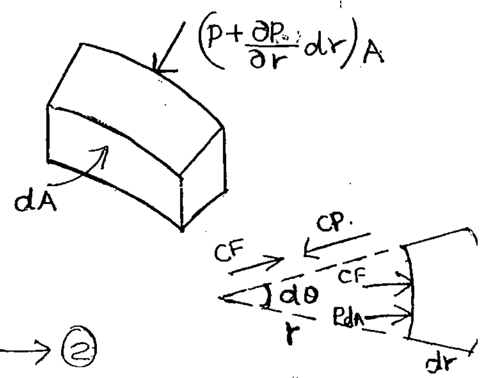
$$(m_w + m_l)g = (\rho_F \times g \times V)_w + (\rho_F \times g \times V)_l.$$

$$(3.67 + m_l)g = 1000 \times 9.81 \times 0.9\ \underbrace{V_w}_{\frac{3.67}{600}} + 1000 \times 9.81 \times \frac{m_l}{\rho_l}.$$

$$\underline{\underline{m_l = 2}}\ kg$$

* Equation of motion for Vortex Flow :

Under equilibrium,



$$p\,dA - \left(p + \frac{\partial p}{\partial r}dr\right)dA + CF = 0$$

$$\therefore \quad CF = \frac{\partial p}{\partial r}dr\,dA \longrightarrow (1)$$

$$\text{But} \quad CF = \frac{mv^2}{r} = \frac{\rho\,dA.dr \times v^2}{r} \longrightarrow (2)$$

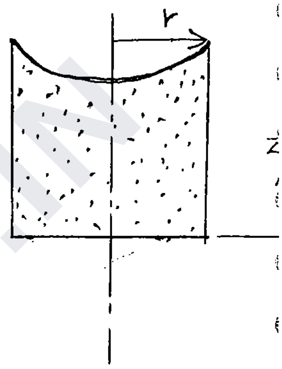
18th Aug, MONDAY

From (1) & (2),

$$\boxed{\frac{\partial P}{\partial r} = \frac{\rho v^2}{r}}$$

This equation represents the pressure gradient wrt. radius.

Also $\frac{\partial P}{\partial z} = -\rho g$.



$\therefore$ $p = f(r,z)$

$$dp = \frac{\partial p}{\partial r} dr + \frac{\partial p}{\partial z} . dz.$$

$$\boxed{dp = \frac{\rho v^2}{r} dr - \rho g \, dz}$$

* Equation of motion for Forced Vortex Flow

We have $V = r\omega$

$$\therefore \; dp = \frac{\rho \times r^2 \times \omega^2}{r} dr - \rho g \, dz.$$

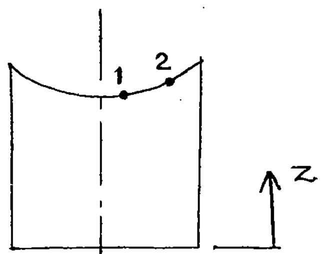
$$\int_1^2 dp = \int_1^2 \rho \omega^2 r dr - \int_1^2 \rho g \, dz$$

$$P_2 - P_1 = \left(\rho \omega^2 \frac{r^2}{2}\right)_1^2 - \rho g (z_2 - z_1)$$

$$= \frac{\rho \omega^2}{2} (r_2^2 - r_1^2) - \rho g (z_2 - z_1)$$

$$\Rightarrow \boxed{P_2 - P_1 = \frac{\rho}{2} (V_2^2 - V_1^2) - \rho g (z_2 - z_1)} \quad (\because V = r\omega)$$

⊙ If we consider two particles which are on the surface,



$$P_1 = P_2 = P_{atm}.$$

$$0 = \frac{\rho}{2} (V_2^2 - V_1^2) - \rho g (z_2 - z_1).$$

$$\Rightarrow \frac{V_2^2 - V_1^2}{2g} = z_2 - z_1$$

If point 1 is on the axis,

$$r_1 = 0 \Rightarrow r_1\omega = V_1 = \underline{\underline{0}}$$

$$\frac{V_2^2}{2g} = z_2 - z_1 \Rightarrow \frac{r_2^2\omega^2}{2g} = z_2 - z_1$$

Since z varies with square of radius, free surface of liquid in the forced vortex flow is parabolic

18th Aug,
MONDAY

DYNAMICS OF FLUID FLOW

Net force acting upon the fluid mass in the x direction is given by:

$$F_x = m \times a_x$$

where $m \rightarrow$ mass of the fluid.
$a_x \rightarrow$ acceleration in x direction.

The net force it comprises the following forces,

$$F_x = (F_g)_x + (F_p)_x + (F_t)_x + (F_v)_x + (F_c)_x$$

where $F_g \rightarrow$ force due to gravity.
$F_p \rightarrow$ force due to pressure.
$F_t \rightarrow$ force due to turbulence.
$F_v \rightarrow$ force due to viscosity.
$F_c \rightarrow$ force due to compressibility.

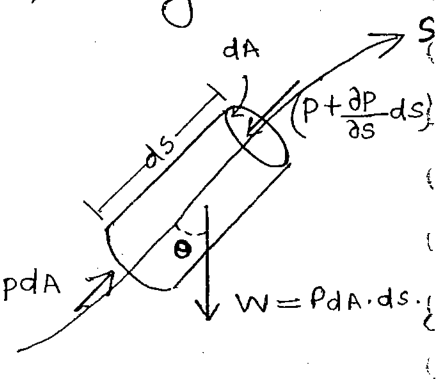
If the forces due to <u>gravity and pressure</u> are taken into consideration in the fluid motion by neglecting all other forces, then the resulting equation of motion is called "Euler's Equation of Motion."

Assumptions:

1. Fluid is inviscous $(F_v = 0)$ $\Rightarrow$ ideal fluid
2. Fluid is incompressible $(F_c = 0)$.
3. Fluid is irrotational $(F_t = 0)$ $\Rightarrow$ steady & laminar.

- Consider a cylindrical fluid element in motion,

Net force acting on fluid mass $= m \cdot a_s$ $(F_x = ma_x)$



$$p\,dA - \left(p + \frac{\partial p}{\partial s} ds\right) dA - w\cos\theta = m \times a_s$$

$$-\frac{\partial p}{\partial s} ds \cdot dA - \rho dA \cdot ds \cdot g\cos\theta = \rho dA \cdot ds \cdot a_s$$

$a_s = \frac{dv}{dt}$; $v = f(s,t)$.

$$\frac{dv}{dt} = \frac{\partial v}{\partial s} \cdot \frac{ds}{dt} + \frac{\partial v}{\partial t} \cdot \frac{dt}{dt}$$

$$a_s = v \cdot \frac{\partial v}{\partial s} + \frac{\partial v}{\partial t}$$

If flow is steady, then $\frac{\partial v}{\partial t} = 0$.

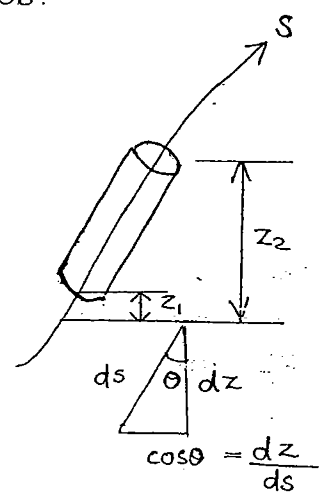
$$\therefore a_s = v \frac{\partial v}{\partial s}$$

$$\therefore -\frac{\partial p}{\partial s} \cdot ds \cdot dA - \rho dA \cdot ds \cdot g\cos\theta = \rho dA \cdot ds \cdot v\frac{\partial v}{\partial s}$$

$$-\frac{\partial p}{\partial s} - \rho g\cos\theta = \rho v\frac{\partial v}{\partial s}$$

$$-\frac{\partial p}{\partial s} - \rho g\frac{\partial z}{\partial s} = \rho v\frac{\partial v}{\partial s}$$

$$\partial p + \rho v \partial v + \rho g dz = 0$$



EULER'S EQUATION OF MOTION $\boxed{\frac{\partial p}{\rho} + v\partial v + g dz = 0}$

* Bernoulli's Equation of Motion

Bernoulli's eqn of motion is obtained by integrati Euler's eqn of motion.

$$\int \frac{\partial p}{\rho} + \int v dv + \int g dz = 0$$

$$\frac{p}{\rho} + \frac{v^2}{2} + gz = C$$

or $$\boxed{\frac{p}{\rho g} + \frac{v^2}{2g} + z = C}$$

For any two points ① & ②,

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

Assumptions:
(i) Fluid is ideal.
(ii) Fluid is irrotational.
(iii) Fluid is incompressible.
(iv) Flow is steady.

$\frac{P}{\rho g} \rightarrow$ pressure head.

$\frac{V^2}{2g} \longrightarrow$ kinetic (or) velocity head.

$z \longrightarrow$ datum (or) elevation head

Each term in the Bernoulli's equation represent energy per unit weight of the fluid.

$PE = mgz$

$PE/\text{unit wt} = \frac{mgz}{mg} = \underline{\underline{z}}$

$KE = \frac{1}{2} mV^2$

$KE/\text{unit wt} = \frac{\frac{1}{2} mV^2}{mg} = \underline{\underline{\frac{V^2}{2g}}}$

Units: $\frac{\text{Energy}}{\text{weight}} = \frac{Nm}{N} = \underline{\underline{m}}$

$$\boxed{P + \frac{\rho V^2}{2} + \rho g z = C}$$

Each term represents 'energy per unit volume' of fluid.

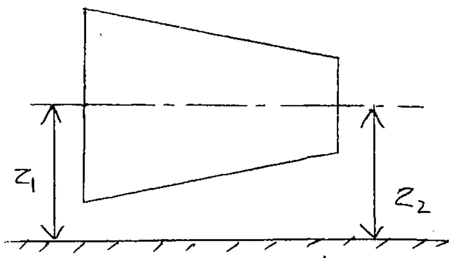
The term $\frac{\rho V^2}{2}$ is known as 'Dynamic Pressure'

- For a real fluid,

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_L$$

where h_L = head loss.

1. $\frac{P_1}{\rho g} = 10\,m$; $\frac{V_1^2}{2g} = 1\,m$; $d_1 = 100$ mm.



$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$10 + 1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}$$

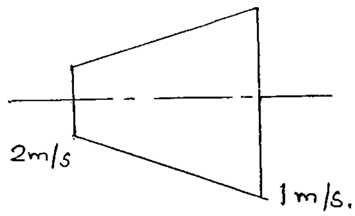
$$A_1 V_1 = A_2 V_2$$

$$V_2 = \frac{A_1 V_1}{A_2} = \frac{d_1^2 \times V_1}{d_2^2} = \frac{0.1^2 \times V_1}{0.05^2} = 4V_1$$

$$11 = \frac{P_2}{\rho g} + \frac{16\,V_1^2}{2g}$$

$$\frac{P_2}{\rho g} = 11 - 16 \times 1 = \underline{\underline{-5\,m}}$$

02. $z_1 = z_2$ (pipe is horizontal).



$$\frac{P_1}{\rho g}$$

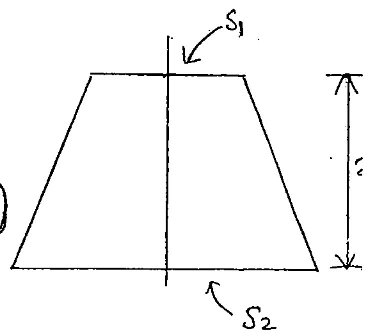
$$\frac{P}{\rho g} + \frac{V^2}{2g} = C \quad \boxed{V\uparrow, P\downarrow}$$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}$$

$$\frac{P_1 - P_2}{\rho g} = \frac{1^2}{2g} - \frac{2^2}{2g} = \frac{-3}{2g} = \underline{\underline{\frac{-1.5}{g}}}$$

Difference in pressure heads $= \underline{\underline{\frac{1.5}{g}}}$

03. $$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_L$$



$$2.5 + 1.27 + 2 = 5.407 + 0.203 + 0 + h_L \quad (\because z_1 = z_2 + 2)$$

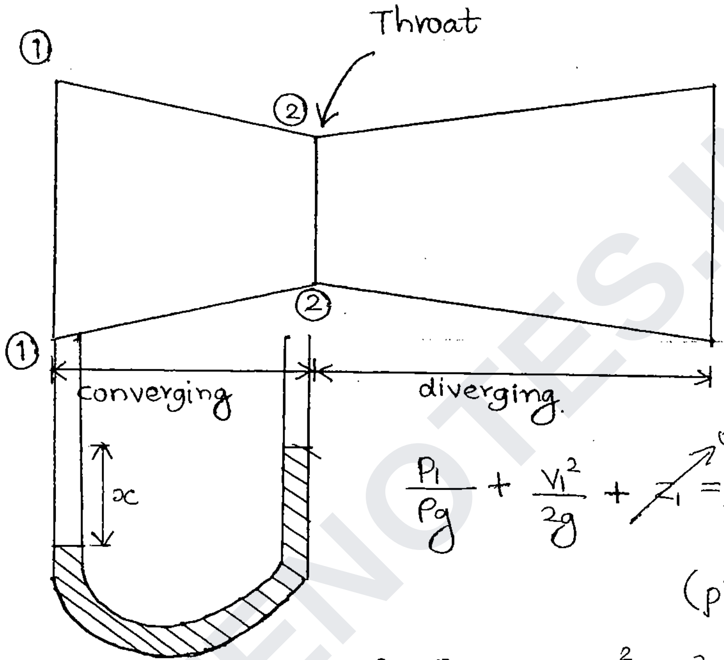
$$h_L = \underline{\underline{0.16\,m}}$$

Flow is from a section at higher energy to that at lower energy. ie from $\underline{\underline{S_1 \text{ to } S_2}}$

→ Applications of Bernoulli's Equation:

1. Venturimeter

It is used to measure the volume flow rate of fluid flowing in a pipe.



$A_2 < A_1$

$V_2 > V_1$

$P_2 < P_1$

$P_2 \nless P_v$ to avoid cavitation

Divergence angle should be very small and is about $5^\circ - 7^\circ$ to avoid flow seperation.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{z_1}^{\,0} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + \cancel{z}$$

(pipe is horizor

$$\frac{P_1 - P_2}{\rho g} = \frac{V_2^2 - V_1^2}{2g}$$

$$h = \frac{V_2^2 - V_1^2}{2g} \quad ; \; h \to \text{difference of pressure head.}$$

$$V_2^2 - V_1^2 = 2gh. \longrightarrow ①$$

$$a_1 V_1 = a_2 V_2$$

$$\therefore \; V_1 = \frac{a_2 V_2}{a_1} \longrightarrow ②$$

From ① & ②,

$$V_2^2 - \frac{a_2^2 V_2^2}{a_1^2} = 2gh$$

$$V_2^2 \left(\frac{a_1^2 - a_2^2}{a_1^2} \right) = 2gh$$

$$V_2^2 = \frac{2gh \cdot a_1^2}{a_1^2 - a_2^2}$$

$$\Rightarrow \; V_2 = \frac{a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

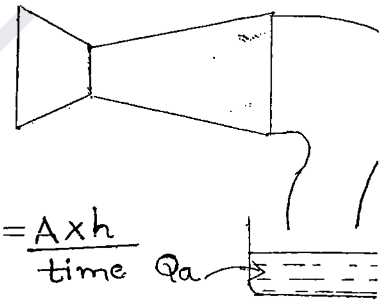
$$\boxed{Q_{th} = \frac{a_1 \times a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}}$$

$h = \left(\frac{S_m}{S_L} - 1\right)x$; when manometric fluid is heavier than liquid flowing.

$= \left(1 - \frac{S_m}{S_L}\right)x$; when manometric fluid is lighter than liquid flowing.

$$Q_a < Q_{th}.$$

Coeffecient of Discharge, $C_d = \frac{Q_a}{Q_{th}}$



$\therefore \quad Q_a = C_d \cdot Q_{th}$

$$\Rightarrow \quad Q_a = \frac{C_d \cdot a_1 \cdot a_2 \cdot \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

$Q_a = \frac{A \times h}{time}$

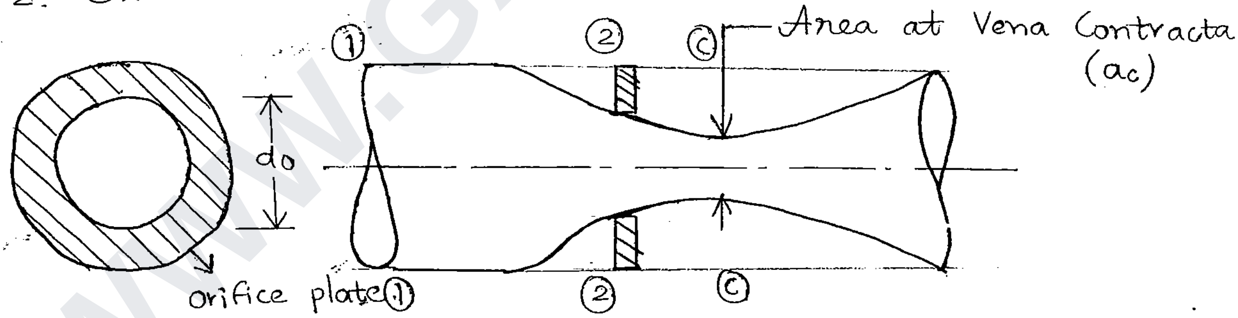
$\boxed{C_d = 0.98}$

time → time taken to rise 'h'

A → tank area

h → rise in tank.

2. Orificemeter



$d_o \longrightarrow$ orifice diameter

$a_o \longrightarrow$ orifice area.

Orifice meter is used to measure the volume flow rate.

$\boxed{C_d = 0.62}$

$$\boxed{Q_a = \frac{C_d \cdot a_1 \cdot a_o \cdot \sqrt{2gh}}{\sqrt{a_1^2 - a_o^2}}}$$

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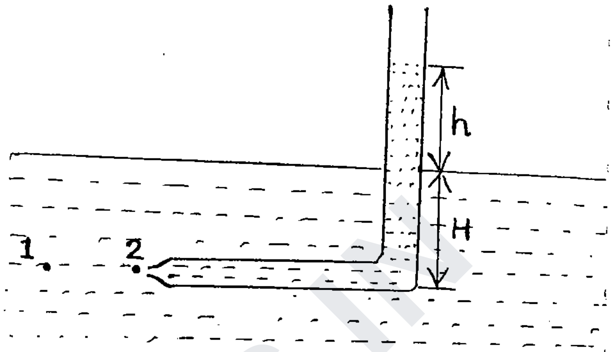
WEDNESDAY

3. Pitot Tube

It is a device used to measure the velocity of flow.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{z_1}^{\,0} = \frac{P_2}{\rho g} + \cancel{\frac{V_2^2}{2g}}^{\,0} + \cancel{z_2}^{\,0}$$



$$\frac{P_1}{\rho g} = H \quad \& \quad \frac{P_2}{\rho g} = H+h.$$

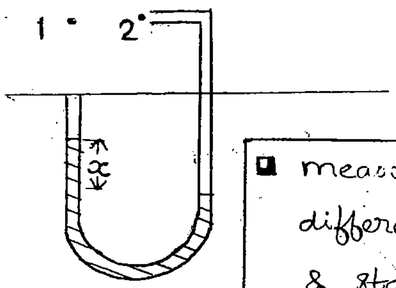
2 – stagnation point ($V_2=0$)

■ measures the stagnation pressure directly.

$$H + \frac{V_1^2}{2g} = H+h+0$$

$$\boxed{V_1 = \sqrt{2gh}}$$

4. Pitot Static Tube.



$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

■ measures the difference of static & stagnation pressure

$$\frac{V_1^2}{2g} = \frac{P_2 - P_1}{\rho g} = x\left(\frac{S_m}{S_L} - 1\right)$$

$$V_1^2 = 2gx\left(\frac{S_m}{S_L} - 1\right).$$

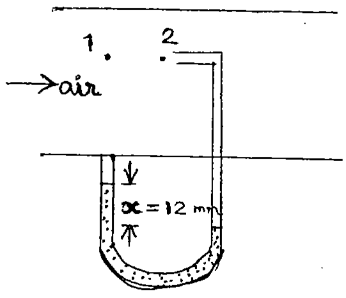
$$\therefore \boxed{V_1 = \sqrt{2gx\left(\frac{S_m}{S_L} - 1\right)}}$$

$$\boxed{\begin{aligned}(V_1)_a &= C_v\sqrt{2gh} \\ (V_1)_a &= C_v\sqrt{2gx\left(\frac{S_m}{S_L}-1\right)}\end{aligned}}$$

where $C_v \rightarrow$ coefficient of Pitot tube $(V_a < V_{th})$

10. $C_v = 1$; $\rho_{air} = 1.2 \ kg/m^3$

$$\frac{P_2 - P_1}{\rho g} = 12 \text{ mm of } H_2O = 12\left(\frac{S_{H_2O}}{S_{air}} - 1\right) \times 10^{-3}.$$



$$V_{air} = \sqrt{2gx}\sqrt{\frac{S_m}{S_L} - 1}$$

$$= \sqrt{2g \times 12 \times 10^{-3}\left(\frac{1}{1.2/1000} - 1\right)} = \underline{\underline{13.99}} \ m/s$$

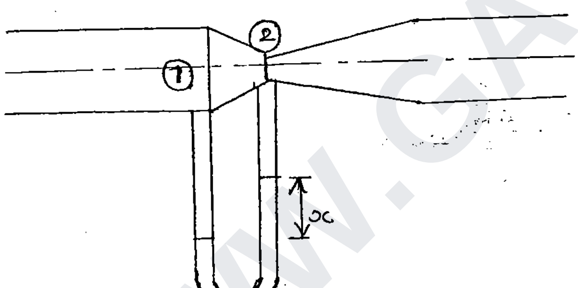
$$P_1 + \rho_{air} \times g \times y + \rho_{water} \times g \times x = P_2 + \rho_{air}(x+y)g.$$

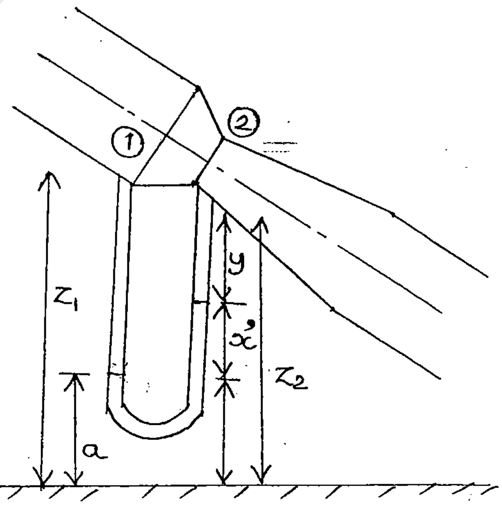
$$P_2 - P_1 = (\rho_{H_2O} - \rho_{air})g.x.$$

$$\frac{P_2 - P_1}{\rho_{air}\, g} = \frac{(\rho_{H_2O} - \rho_{air})gx}{\rho_{air} \times g.}$$

$$\frac{P_2 - P_1}{\rho_{air} \times g.} = \left(\frac{\rho_{H_2O}}{\rho_{air}} - 1\right)x.$$

9.





$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$\frac{P_1 - P_2}{\rho g} = \frac{V_2^2 - V_1^2}{2g.}$$

$$h = \frac{P_1 - P_2}{\rho g} = x\left(\frac{S_m}{S_L} - 1\right)$$

Also $\frac{V_2^2 - V_1^2}{2g} = x\left(\frac{S_m}{S_L} - 1\right) \rightarrow (1)$

$$P_1 + \rho_l g(z_1 - a) = P_2 + \rho_l g(z_2 - (x' + a)) + \rho_m g x'.$$

$$P_1 - P_2 = \rho_l g z_2 - \rho_l g x' + \rho_m g x' - \rho_l g z_1$$

$$\left(\frac{P_1}{\rho_l g} + z_1\right) - \left(\frac{P_2}{\rho_l g} + z_2\right) = \left(\frac{\rho_m}{\rho_l} - 1\right)x'$$

$$= \left(\frac{S_m}{S_l} - 1\right)x'$$

$$\Rightarrow \frac{V_2^2 - V_1^2}{2g} = \left(\frac{S_m}{S_l} - 1\right)x' \rightarrow (2)$$

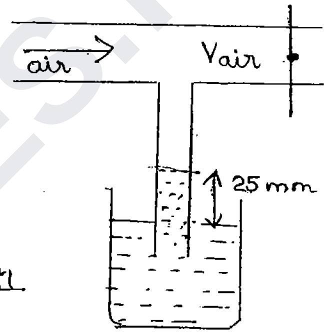
From ① & ②, $x = x'$.

$\therefore$ Manometer readings are independent of inclination of pipe. It depends only on rise of manometric fluid $(= x)$.

$\therefore h_1 = h_2 = h_3$

4. $\rho_{air} = 1.2\ kg/m^3$.

$$\frac{P_1}{\rho g} + \cancel{\frac{V_1^2}{2g}}^{\,0} + \cancel{z_1}^{\,0} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + \cancel{z_2}^{\,0\ (\text{not given})}$$



$$\frac{\rho_{air} \cdot V_{air}^2}{2} = \rho_w g h_w$$

$$V_{air}^2 = \frac{2 \rho_w g . h_w}{\rho_{air}} = \frac{2 \times 1000 \times 25 \times 10^{-3} \times 9.81}{1.2}$$

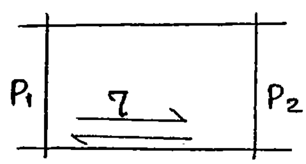
$$V_{air} = \underline{\underline{20.22}}$$

8. Laminar (Viscous) Flow:

$$Re = \frac{\rho V D}{\mu} < 2000 \quad (\text{for pipe flow}).$$

when $\mu \uparrow$, $Re \downarrow$
$V \downarrow$, $Re \downarrow$ $\Rightarrow$ when a fluid has high viscosity and flowing at low velocities, then flow becomes laminar.

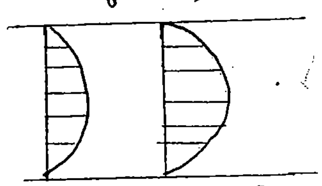
$P_2 < P_1$



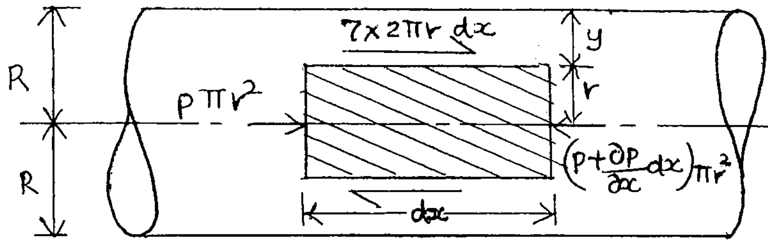
For a fully developed laminar flow,

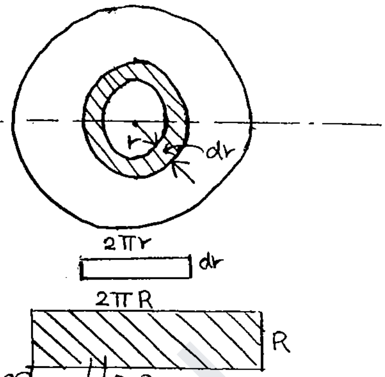
$$u = f(r)$$

$$\frac{\partial p}{\partial x} = \text{constant}$$



$u = f(r)$.





Forces acting are:

1. $p.\pi r^2$
2. $\left(p+\frac{\partial p}{\partial x}dx\right)\pi r^2$
3. $\tau.\, 2\pi r.\, dx$

Assumptions:

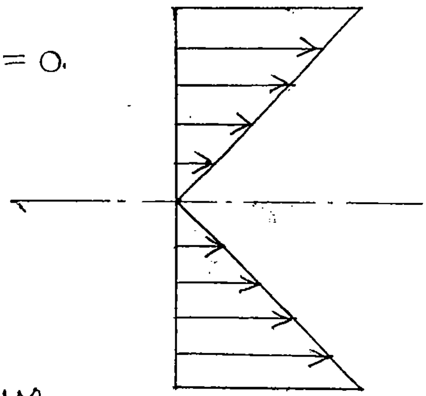
1. Fully developed flow.
2. Steady and uniform $(a=0)$.

$$F_{net} = m.\,a. = 0.$$

$$p\times\pi r^2 - \left(p+\frac{\partial p}{\partial x}dx\right)\pi r^2 - \tau\times 2\pi r\times dx = 0.$$

$$\frac{\partial p}{\partial x} r = -2\tau$$

$$\boxed{\tau = -\frac{\partial p}{\partial x}\frac{r}{2}.} \longrightarrow (1)$$



Shear Stress Distribution.

$\therefore$ Shear stress varies linearly with radius.

$\therefore$ At $r=0$, $\tau=0$.

$r=R$, $\tau = \tau_{max} = \tau_w$ (wall shear stress).

$$y = R-r$$
$$dy = -dr$$

$$\tau = \mu\frac{du}{dy} = -\mu\frac{du}{dr} \longrightarrow (2)$$

From (1) & (2),

$$-\mu\frac{du}{dr} = -\frac{\partial p}{\partial x}.\frac{r}{2}$$

$$du = \frac{1}{2\mu}\frac{\partial p}{\partial x} r.\, dr$$

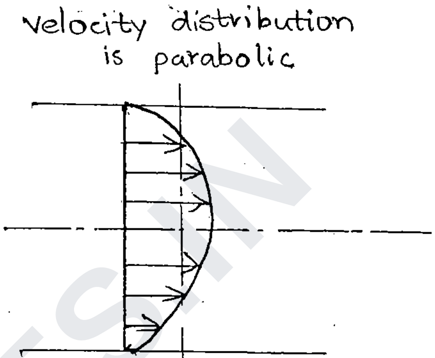
Integrating on both sides,

$$\boxed{u = \frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right) r^2 + C.}$$

At $r = R$, $u = 0$,

$$0 = \frac{1}{4\mu}\frac{\partial p}{\partial x}(R^2) + C.$$

or $$C = \frac{-1}{4\mu}\frac{\partial p}{\partial x}R^2.$$

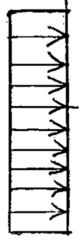


$$\therefore \quad \boxed{u = \frac{-1}{4\mu}\frac{\partial p}{\partial x}(R^2 - r^2).}$$

Since velocity u varies with square of radius 'r', therefore velocity distribution is parabolic.

$$Q = A\bar{u}$$

$$\bar{u} = \frac{Q}{\pi R^2}$$



$\bar{u}$, avg. velocity

$$Q = \int_0^R dQ.$$

$$dQ = (2\pi r . dr.) \times u$$

$$= 2\pi r . dr \left(\frac{-1}{4\mu}\frac{\partial p}{\partial x}(R^2 - r^2)\right).$$

$$\therefore Q = -\int_0^R \frac{2\pi}{4\mu}\left(\frac{\partial p}{\partial x}\right).(R^2 r - r^3)\,dr$$

$$= \frac{-\pi}{2\mu}\frac{\partial p}{\partial x}\left(R^2.\frac{r^2}{2} - \frac{r^4}{4}\right)_0^R$$

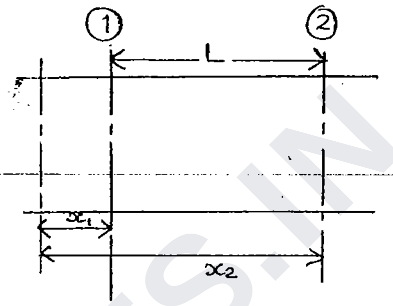
$$\boxed{Q = -\frac{\pi}{8\mu}\left(\frac{\partial p}{\partial x}\right). R^4}$$

$$\therefore \quad \boxed{\bar{u} = \frac{Q}{\pi R^2} = -\frac{R^2}{8\mu}\left(\frac{\partial p}{\partial x}\right)}$$

Max. velocity (at $r=0$), $u_{max} = -\frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right)R^2$

$$\Rightarrow \boxed{\frac{u_{max}}{\bar{u}} = 2.}$$

Max. velocity which occurs at the centre is called as 'Centre Line Velocity' (u_{max})



$$\bar{u} = -\frac{1}{8\mu}\left(\frac{\partial p}{\partial x}\right)R^2$$

$$\int_1^2 \partial p = \int_1^2 -\frac{8\mu\bar{u}}{R^2}\,.\,dx.$$

$$p_2 - p_1 = -\frac{8\mu\bar{u}}{R^2}(x_2 - x_1).$$

$$\boxed{p_1 - p_2 = \frac{8\mu\bar{u}L}{R^2} = \frac{32\mu\bar{u}L}{D^2}}$$

Head loss, $h_L = \frac{p_1 - p_2}{\rho g}$.

$$\boxed{h_L = \frac{32\mu\bar{u}L}{\rho g D^2}} \rightarrow \text{Halgen Poiseuille Equation}$$

2.127

2.01 $Q = ?$ $u_{max} = 1.5$ m/s ; $d = 40$ mm.

$$Q = A \times \bar{u}$$

$$= \frac{\pi}{4}d^2 \times \frac{1.5}{2} = \frac{3\pi}{10000} \text{ m}^3\text{/s}$$

02. $d = 15$ cm $= 0.15$m ; $p_1 - p_2 = 100$ kPa ; $L = 10$m.

$$\tau_w = -\frac{\partial p}{\partial x}\cdot\frac{r}{2} = \frac{100\times10^3}{10}\times\frac{0.15}{4} = 375 \text{ Pa}$$

3. $d = 0.1\ m,\ \tau = 250\ N/m^2,\ L = 10\ m.$

$$\tau = \frac{-\partial P}{\partial x}\frac{r}{2}$$

$$250 = \frac{\partial P}{10} \times \frac{0.05}{2}$$

$$\partial P = \underline{\underline{1 \times 10^5}}\ N/m^2$$

---

04. $$\tau = \frac{\partial p}{\partial x} \cdot \frac{r}{2}$$

$$= 2 \times 10^3 \times \frac{0.05}{4} = \underline{\underline{25\ Pa}}$$

05. $Q = 800\ mm^3/s = 800 \times 10^{-9}\ m^3/s.$

$L = 2m,\ D = 0.5 \times 10^{-3}\ mt,\ \frac{P_1 - P_2}{2} = 2 \times 10^6\ Pa.$

$$\bar{u} = \frac{Q}{\pi R^2} = \frac{800 \times 10^{-9}}{\pi \times \frac{(0.5 \times 10^{-3})^2}{4}} = 4.07\ m/s.$$

$$\bar{u} = \frac{-R^2}{8\mu}\left(\frac{\partial P}{\partial x}\right).$$

$$\mu = \frac{(-0.5 \times 10^{-3})^2/4}{8 \times 4.07}\left(\frac{2 \times 10^6}{2}\right) = 1.919 \times 10^{-3}\ NS/m^2.$$

---

06. $D = 0.2\ m,\ R = 0.1\ m.$

$$u_{max} = \frac{-1}{4\mu}\left(\frac{\partial P}{\partial x}\right) 0.1^2.$$

$$\frac{u_{max}}{u} = \frac{R^2}{R^2 - r^2}$$

$$1 = \frac{-1}{4\mu}\frac{\partial P}{\partial x} \times 0.1^2$$

$$u = \frac{-1}{4\mu}\frac{\partial P}{\partial x}(R^2 - r^2).$$

$$= \frac{1}{0.1^2}(0.1^2 - 0.05^2) = \underline{\underline{0.75\ m/s}}$$

07 $d = 0.025\ m$, $\frac{\partial P}{\partial x} = 0.12\ kgf/cm^2 = \frac{0.12 \times 9.81}{10^{-4}} = 11772\ N/m^2$ (39)

$u = 2.34\ m/s.$

09 $\mu = 0.01$ poise $= 0.01 \times 10^{-1}$ Pas.

$d = 0.01m$, $\bar{u} = 10\ mm/s = 0.01\ m/s.$

$$h_L = \frac{32 \mu \bar{u} L}{\rho g D^2}$$

$$h_L / 1000 = \frac{32 \mu \bar{u}}{\rho g D^2}$$

$$\therefore h_L = \frac{32 \times 0.001 \times 0.01}{1000 \times 9.81 \times 0.01^2} = \underline{\underline{0.32\ m}}$$

10. $h_L \propto \frac{1}{D^4}$

$$\frac{(h_L)_1}{(h_L)_2} = \frac{D_2^2}{D_1^2} = \frac{D_1^2 / 16}{D_1^2}$$

$$(h_L)_2 = \underline{\underline{16\ (h_L)_1}}$$

$d_2 = \frac{d_1}{2}$; $Q_2 = Q_1$; $L_2 = L_1$

$$h_L = \frac{32 \mu \times \frac{Q}{A} \times L}{\rho g d^2}$$

$$\boxed{h_L = \frac{128 \mu Q L}{\rho g \times \pi \times d^4} \Rightarrow h_L \propto \frac{1}{d^4}}$$

Re = :

12. $h_L = 2m$, $L = 10m$, $Re = 100$ (<2000), $Q_2 = 2Q_1$

$Q = A \times \bar{u}$ (d is constant, $Q \uparrow$ as $\bar{u} \uparrow$).

$$h_L = \frac{128 \mu Q L}{\rho g \pi d^4}$$ ρ, μ

$$h_L \propto Q.$$

$$\frac{h_{L1}}{Q_1} = \frac{h_{L2}}{Q_2}$$

$$\frac{2}{Q_1} = \frac{h_{L2}}{2Q_1}$$

$$\underline{\underline{h_{L2} = 4m}}$$

13. $d = 0.02\,m$, $Re = 1600$ (laminar), $h_L = 0.2$, $Q = ?$, $\gamma = ?$

$$h_L = \frac{32 \mu \bar{u} L}{\rho g D^2}$$

$$Re = \frac{\rho v d}{\mu}$$

$$Re = \frac{v d}{\gamma}$$

$$\frac{h_L}{L} = \frac{32 \mu \bar{u}}{\rho g D^2}$$

$$\Rightarrow \frac{\mu}{\rho} = \frac{\bar{u} d}{Re}$$

$$= 32 \left(\frac{\mu}{\rho}\right) \frac{\bar{u}}{g d^2}.$$

$$0.2 = 32 \times \left(\frac{\bar{u} \times d}{Re}\right)\left(\frac{\bar{u}}{g d^2}\right).$$

$$0.2 = \frac{32 \times \bar{u}^2}{1600 \times 9.81 \times 0.02}$$

$$\bar{u} = \cancel{0.0175}\ \underline{\underline{1.4}}\ m/s$$

$$Q = \frac{\pi}{4} D^2 \times \bar{u} = \frac{\pi}{4} \times 0.02^2 \times 1.4 = \underline{\underline{4.4 \times 10^{-4}}} \text{ cumecs.}$$

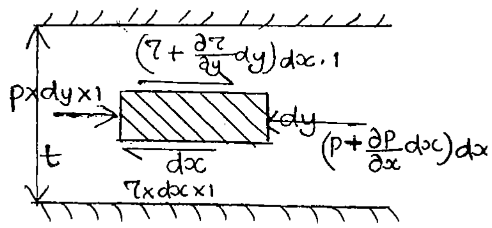
$$\gamma = \frac{\bar{u} d}{Re} = \frac{1.4 \times 0.02}{1600} = 1.75 \times 10^{-5}\ m^2/s = 1.75 \times 10^{-1} \text{ stokes}$$

$$= \underline{\underline{17.5}} \text{ centi stokes}$$

$$\boxed{1 \text{ stoke} = 10^{-4}\ m^2/s}$$

→ Flow between Two Parallel Plates.

a) Both Plates are fixed



Consider unit width of plates,

Forces acting are :

1. $p \times dy \times 1$
2. $\left(p + \frac{\partial p}{\partial x} dx\right) dy \times 1$
3. $\tau \times dx \times 1$
4. $\left(\tau + \frac{\partial \tau}{\partial y} dy\right) dx . 1.$

Flow is steady and uniform,

$$p \times dy \times 1 - \left(p + \frac{\partial p}{\partial x} dx\right) dy \times 1 - \tau \times dx \times 1 + \left(\tau + \frac{\partial \tau}{\partial y} . dy\right) dx . 1 = 0.$$

$$\frac{\partial \tau}{\partial y} = \frac{\partial p}{\partial x}.$$

For a laminar, $\tau = \mu \frac{du}{dy}.$

$$\frac{\partial}{\partial y}\left(\mu \frac{du}{dy}\right) = \frac{\partial p}{\partial x}.$$

$$\mu \frac{d^2u}{dy^2} = \frac{dp}{dx}.$$

$$\Rightarrow \frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu} \frac{\partial p}{\partial x}$$

Integrating on both sides,

$$\frac{\partial u}{\partial y} = \frac{1}{\mu} \frac{\partial p}{\partial x} . y + c_1$$

Once again integrate,

$$u = \frac{1}{2\mu} \frac{\partial p}{\partial x} y^2 + c_1 y + c_2.$$

Applying boundary conditions,

(i) at $y = 0$; $u = 0$ &

(ii) at $y = t$; $u = 0.$

$$C_2 = 0 \quad ; \quad C_1 = \frac{-1}{2\mu}\frac{\partial P}{\partial x} \cdot t$$

$$\therefore \quad u = \frac{1}{2\mu}\frac{\partial P}{\partial x} y^2 + \frac{-1}{2\mu}\frac{\partial P}{\partial x} ty$$

$$\boxed{u = \frac{-1}{2\mu}\left(\frac{\partial P}{\partial x}\right)(ty - y^2)}$$

Since u varies with square of y, therefore velocity distributi[on] is parabolic.

$$Q = \int_0^t dQ.$$

$$dQ = (dy \times 1)u. = -\frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right)(ty - y^2).\,dy.$$

$$Q = \int dQ = \frac{-1}{2\mu}\left(\frac{\partial P}{\partial x}\right)\left(\frac{ty^2}{2} - \frac{y^3}{3}\right)_0^t$$

$$\boxed{Q = \frac{-1}{12\mu}\left(\frac{\partial P}{\partial x}\right)t^3}$$

~~At~~ ~~y~~ = 8.

Average velocity, $\bar{u} = \frac{Q}{A} = \frac{\frac{-1}{12\mu}\left(\frac{\partial P}{\partial x}\right)t^3}{t \times 1} = \frac{-1}{12\mu}\left(\frac{\partial P}{\partial x}\right)t^2$

$$\boxed{\bar{u} = \frac{-1}{12\mu}\left(\frac{\partial P}{\partial x}\right)t^2}$$

At $y = t/2$, max. velocity occurs,

$$\boxed{u_{max} = \frac{-1}{8\mu}\left(\frac{\partial P}{\partial x}\right)t^2}$$

$$\boxed{\frac{u_{max}}{\bar{u}} = \frac{3}{2} = 1.5}$$

$u = \frac{-1}{2\mu}\left(\frac{\partial P}{\partial x}\right)(ty - y^2)$

At max. velocity,

$\frac{du}{dy} = 0$

$\Rightarrow \; t - 2y = 0$ or $y =$

$$\bar{u} = \frac{-1}{12\mu}\left(\frac{\partial P}{\partial x}\right) t^2$$

$$dp = \frac{-12\mu\bar{u}}{t^2} . \partial x$$

$$\int_1^2 dp = -\int_1^2 \frac{12\mu\bar{u}}{t^2} . \partial x$$

$$P_2 - P_1 = \frac{-12\mu\bar{u}}{t^2}(x_2 - x_1)$$

$$\boxed{P_1 - P_2 = \frac{12\mu\bar{u}L}{t^2}}$$

$$\boxed{\text{Head loss}, \ h_L = \frac{P_1 - P_2}{\rho g} = \frac{12\mu\bar{u}L}{\rho g t^2}}$$

11. $\rho = 880 \ kg/m^3$; $\nu = 7.4 \times 10^{-7} \ m^2/s$.

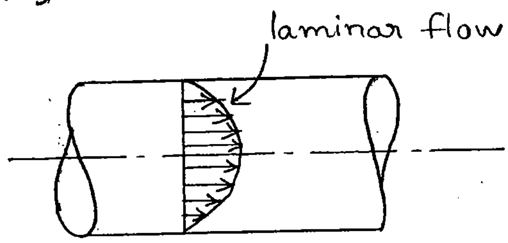
$\mu = \nu\rho = 6.512 \times 10^{-4}$ Pa.s.

$\tau = \mu \frac{dy}{dy} = 6.512 \times 10^{-4} \times 1000$

$= \underline{\underline{0.65 \ Pa}}$

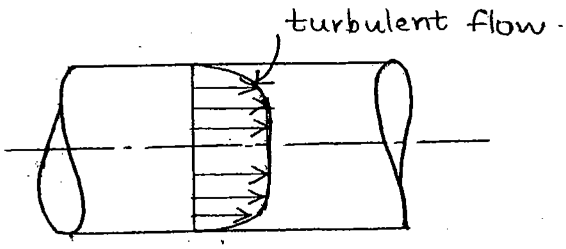
21 Aug, THURSDAY

→ Kinetic Energy Correction Factor (α)



$\alpha = \dfrac{\frac{KE}{S} \text{ of fluid particles based on actual velocity}}{\frac{KE}{S} \text{ of fluid particles based on average velocity.}}$

$\boxed{\alpha > 1}$



$u \approx \bar{u} \Rightarrow \alpha \approx 1$

$$\frac{KE}{S} = \frac{1}{2}\left(\frac{m}{t}\right) V^2 = \frac{1}{2} \dot{m} V^2$$

KE/s based on actual velocity $= \int_0^R \frac{1}{2} \rho . dQ \times u^2$

$= \int_0^R \frac{1}{2}\rho\,(2\pi r . dr \times u)\, u^2$

$= \pi\rho\left(\frac{-1}{64\mu^3}\right)\left(\frac{\partial P}{\partial x}\right)^3 \int r(R^2 - r^2)^3 d$

KE/s of fluid particles based on average velocity.

$$= \frac{1}{2}\rho Q \cdot \bar{u}^2$$

$$= \frac{1}{2}\rho \cdot A \cdot \bar{u} \cdot \bar{u}^2 = \frac{1}{2}\rho \pi R^2 \bar{u}^3$$

$$\alpha = \frac{\pi\rho \int_0^R r u^3 dr}{\frac{1}{2}\rho \pi R^2 \bar{u}^3} = \underline{\underline{2}}$$ (for laminar flow)

→ Momentum Correction Factor (α)

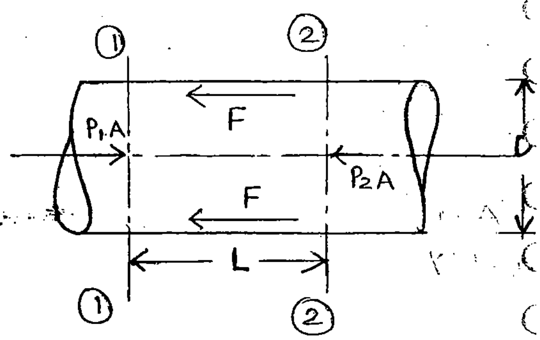
$$\beta = \frac{\text{momentum/s of fluid particles based on actual velocity}}{\text{momentum/s of fluid particles based on average velocity}}$$

$$= \frac{4}{3} = 1.33$$

$$\boxed{\begin{aligned}\alpha &= 2\\ \beta &= 1.33\end{aligned}}$$

→ Frictional Head Loss in a Pipe

$$\frac{P_1}{\rho g} + \cancel{\frac{V_1^2}{2g}}^{(V_1=V_2)} + \cancel{Z_1} = \frac{P_2}{\rho g} + \cancel{\frac{V_2^2}{2g}}^{(V_1=V_2)} + \cancel{Z_2} + h_f$$



$$h_f = \frac{P_1 - P_2}{\rho g} \longrightarrow (1)$$

($V_1 = V_2$ as A is constant)

Let F be the frictional force,

$$P_1 A_c - P_2 A_c - F = 0 \quad ; \quad A_c \rightarrow \text{c/s area.}$$

$$P_1 - P_2 = \frac{F}{A_c}$$

From experiments, William Froude derived the relationship,

$$F = f' A V^n \quad ; \quad n = 1.75 \text{ to } 2$$

$$A = \text{wetted area} = \pi D L$$

f' = frictional resistance per unit wetted area per unit velocity

$$\therefore F = f'(\pi D L) V^2$$

$$= f'(P.L) V^2 \quad ; \quad P \rightarrow \text{perimeter}$$

$$P_1 - P_2 = \frac{F}{A_c} = \frac{f'(\pi D L)\, v^2}{\pi/4 \, D^2}$$

$$= \frac{4 f' L V^2}{D} \longrightarrow (2)$$

From (1) & (2),

$$h_f = \frac{P_1 - P_2}{\rho g} = \frac{4 f' L V^2}{\rho g D}$$

Let $\frac{f'}{\rho} = \frac{f}{2}$ ← coeffecient of friction.

$$\therefore \boxed{h_f = \frac{4 f L V^2}{2 g D}}$$

$4f = f \longrightarrow$ Friction factor.

$$\boxed{h_f = \frac{f L V^2}{2 g D}}$$ DARCY'S WEISHBACH EQUATION.

Head loss in pipe flow, $h_L = \frac{32 \mu V L}{\rho g D^2}$; V = avg. velocity.

$$h_f = h_L$$

$$\frac{f L V^2}{2 g D} = \frac{32 \mu V L}{\rho g D^2}$$

$$f = \frac{64}{\frac{\rho V D}{\mu}}$$

$$\therefore f = \frac{64}{Re}$$

Coefficient of friction, $f = \frac{f}{4} = \frac{16}{Re}$

Friction Factor $= \frac{64}{Re}$
Coeffecient of friction $= \frac{16}{Re}$

$$\tau = \frac{-\partial P}{\partial x} \cdot \frac{r}{2}$$

$$\tau_w = -\frac{\partial P}{\partial x} \cdot \frac{R}{2} = -\frac{(P_2 - P_1)}{x_2 - x_1} \cdot \frac{R}{2}$$

$$= \left(\frac{P_1 - P_2}{L}\right) \cdot \frac{R}{2}$$

Wall shear stress, $\tau_w = \frac{(P_1 - P_2) D}{4L} \longrightarrow (1)$

$$h_f = \frac{P_1 - P_2}{\rho g}.$$

$$\therefore P_1 - P_2 = \rho g . h_f = \rho g . \frac{f L V^2}{2 g D}.$$

$$= \frac{\rho f L V^2}{2D} \longrightarrow (2)$$

From (1) & (2),

$$\tau_w = \left(\frac{\rho f L V^2}{2D}\right) \cdot \frac{D}{4L}$$

$$= \frac{f V^2 . \rho}{8}$$

$$\sqrt{\frac{\tau_w}{\rho}} = \sqrt{\frac{f V^2}{8}}.$$

Units of $\sqrt{\frac{\tau_w}{\rho}} = \sqrt{\frac{N/m^2}{kg/m^3}} = \sqrt{\frac{\frac{kg\,m}{s^2} \times \frac{1}{m^2}}{kg/m^3}} = \sqrt{\frac{m^2}{s^2}}$

$$= \frac{m}{s}$$

$\therefore \sqrt{\frac{\tau_w}{\rho}} = V^*$ (Shear velocity)

$$\boxed{V^* = \sqrt{\frac{f}{8}}\, V}$$

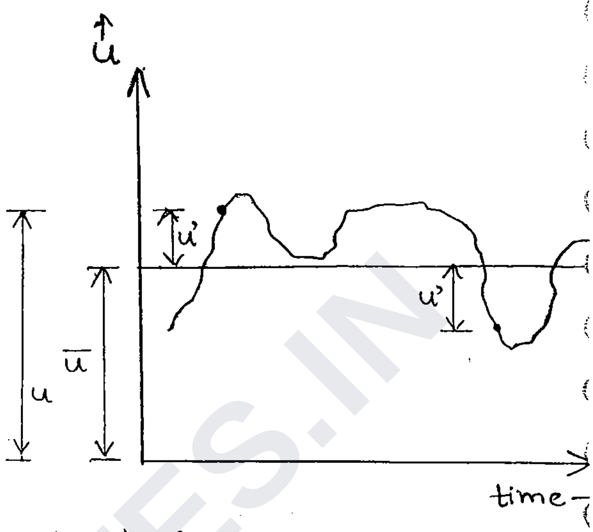
8. $S.G = 0.8$; $\mu = 1$ poise $= 10^{-1}$ PaS; $D = 0.05$ m ; $V = 2$ m/s.

$$\text{Friction factor} = \frac{64}{Re} = \frac{64}{\frac{\rho V D}{\mu}} = \frac{64}{\frac{0.8 \times 1000 \times 2 \times 0.05}{10^{-1}}}$$

$$= \underline{\underline{0.08}}$$

21st Aug,
THURSDAY

9. TURBULENT FLOW



$\bar{u} \rightarrow$ average velocity.

$u' \rightarrow$ fluctuation component of velocity.

$$u = \bar{u} \pm u'$$

Similarly,

$v' \rightarrow$ fluctuation component of velocity in y-direction.

Total shear stress in turbulent flow $= \mu \frac{du}{dy} + \eta \frac{du}{dy}$

$$= \tau_v + \tau_t.$$

where $\eta \rightarrow$ eddy viscosity; it is very difficult to measure

Prandtle suggested, $\tau = \rho u' v' \longrightarrow$ Reynolds.

$$= \rho \overline{u'}\, \overline{v'}$$

$l \rightarrow$ Prandtle mixing length is the distance measured in the normal direction b/w the two fluid layers such that one lump of fluid particles can jump into the another fluid layer and there is no change of momentum of fluid particles along flow direction.

$l = ky$; $k \rightarrow$ von Korman constant $(= 0.4)$

$$u' = v' = l\left(\frac{du}{dy}\right)$$

$$\tau_t = \tau = \rho u' v' = \rho l^2 \left(\frac{du}{dy}\right)^2$$

$$\boxed{\tau = \rho \times (0.4y)^2 \left(\frac{du}{dy}\right)^2}$$

$$\frac{\tau}{\rho} = (0.4y)^2 \left(\frac{du}{dy}\right)^2$$

$$\sqrt{\frac{\tau}{\rho}} = 0.4y \cdot \frac{du}{dy}$$

$$V_* = 0.4y \frac{du}{dy}$$

$$\Rightarrow \quad du = \frac{V_*}{0.4} \frac{dy}{y}$$

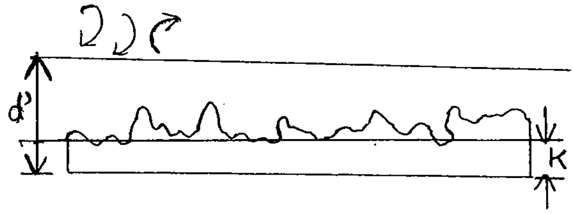
Integrating on both sides,

$$\boxed{u = 2.5 V_* \ln y + C}$$

$\therefore$ Velocity distribution in the case of turbulent flow is <u>logarithmic.</u>

→ Hydrodynamic Smooth & Rough Surfaces

Flow over a rough surface with protrusions (or irregularities)

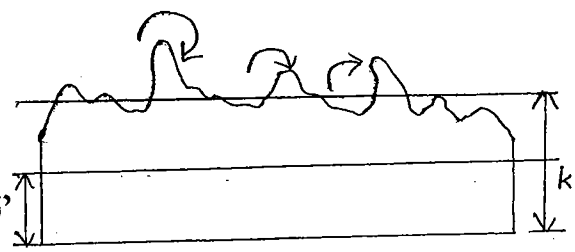


$k \rightarrow$ avg. height of protrusions.

$\delta' \rightarrow$ laminar sublayer thickness.

$\delta' > k \longrightarrow$ smooth

Laminar sublayer completely submerges the irregularities. All the turbulence will occur above this sublayer and turbulence will not touch the irregularities. So, a hydraulic smooth surface is formed.



$\delta' < k \longrightarrow$ rough surface.

$\frac{k}{\delta'} < 0.25$; $\longrightarrow$ Smooth

$\frac{k}{\delta'} > 6$; $\longrightarrow$ Rough

$0.25 < \frac{k}{\delta'} < 6$; $\longrightarrow$ Transition

$\frac{V_* k}{\nu} \rightarrow$ Reynold's Roughness number.

$\frac{V_* k}{\nu} < 3 \longrightarrow$ Smooth

$3 < \frac{V_* k}{\nu} < 70 \longrightarrow$ Transition

$\frac{V_* k}{\nu} > 70 \longrightarrow$ Rough

$$\boxed{\delta' = \frac{11.6\,\nu}{V_*}}$$

From Nikuradses Experiment.

$$u = 2.5\,V_* \ln y + c$$

At $y = R$, $u = u_{max}$ (pipe) ; y is measured from wall.

$$u_{max} = 2.5\,V_* \ln R + c.$$

$$c = u_{max} - 2.5\,V_* \ln R.$$

$$\therefore \quad u = 2.5\,V_* \ln y + u_{max} - 2.5\,V_* \ln R$$

$$u - u_{max} = 2.5\,V_* \ln\left(\frac{y}{R}\right)$$

$$\boxed{\frac{u - u_{max}}{V_*} = 2.5 \ln\left(\frac{y}{R}\right)}$$

$$\frac{u_{max} - u}{V_*} = \frac{2.5 \log_{10}\left(\frac{R}{y}\right)}{\log_{10} e.} \quad ; \quad \log_b a = \frac{\log_x a}{\log_x b}$$

'PRANDTLE VELOCITY DEFECT EQUATION'

$$\boxed{\frac{u_{max} - u}{V_*} = 5.75 \log_{10}\left(\frac{R}{y}\right)}$$

At the pipe boundary, ie, at $y = 0$

$$u = 2.5\,V_* \ln 0 + c.$$

ie at $y = 0$; $u = -\infty$

if $a^x = N$

$x = \log_a N$

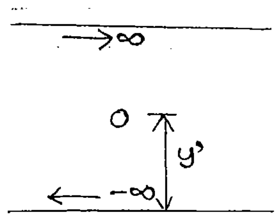
$\therefore \ln_e 0 = x$

$0 = e^x$

$\Rightarrow x = -\infty$

At $y=0$, $u=-\infty$

ie at $y=y'$, u becomes 0.



$$0 = 2.5\, V_* \ln y' + C.$$

$$C = -2.5\, V_* \ln y'$$

$$u = 2.5\, V_* \ln y - 2.5\, V_* \ln y'$$

$$\frac{u}{V^*} = 2.5 \ln\left(\frac{y}{y'}\right)$$

$$\frac{u}{V^*} = 5.75 \log_{10}\left(\frac{y}{y'}\right)$$ → valid for both smooth & rough pipes.

$y' = \dfrac{\delta'}{107}$ ⟶ smooth pipe.

$$\frac{u}{V^*} = 5.75 \log_{10}\left(\frac{y}{\delta'/107}\right)$$

$$= 5.75 \log_{10}\left(\frac{107 \times y}{\delta'}\right)$$

$$= 5.75 \log_{10}\left(\frac{107}{11.6} \times \frac{V_* y}{\gamma}\right)$$

$$= 5.75 \left(\log_{10}\left(\frac{107}{11.6}\right) + \log_{10}\left(\frac{V_* y}{\gamma}\right)\right)$$

$$\boxed{\frac{u}{V^*} = 5.55 + 5.75 \log_{10} \frac{V_* y}{\gamma}}$$ → smooth pipes

Similarly for rough pipes, $y' = \dfrac{k}{30}$

$$\boxed{\frac{u}{V^*} = 5.75 \log_{10}\left(\frac{y}{k}\right) + 8.5}$$ → rough pipes

Above equations given the value of local velocity, u.

For mean velocity, $\bar{u}$

$$\boxed{\frac{\bar{u}}{V^*} = 1.75 + 5.75 \log_{10} \frac{V_* R}{\gamma}}$$

smooth pipe

$$\boxed{\frac{\bar{u}}{V^*} = 4.75 + 5.75 \log_{10}\left(\frac{R}{K}\right)}$$

rough. pipe

$$\boxed{\frac{u - \bar{u}}{V_*} = 5.75 \log_{10}\left(\frac{y}{R}\right) + 3.75}$$ valid for Smooth & Rough pipes.

Q.1 Show that for a turbulent flow in a pipe, the ratio of u_{max} to $\bar{u}$ is $1.33\sqrt{f} + 1$.

$$\frac{u - \bar{u}}{V_*} = 5.75 \log_{10}\left(\frac{y}{R}\right) + 3.75$$

$\bar{u} = V =$ avg. velocity

At $y = R$, $u = u_{max}$,

$$\frac{u_{max} - V}{V^*} = \cancel{5.75 \log_{10}\left(\frac{R}{R}\right)}^{0} + 3.75$$

$$\sqrt{\frac{\tau_w}{\rho}} = \sqrt{\frac{f}{8}}\, V$$

$$\tau_w = \frac{\rho f V^2}{8}$$

$$\therefore \tau_0 = \frac{\rho f V^2}{8}$$

Friction factor, $f = \frac{8\tau_0}{\rho V^2}$

Coeffecient of friction, $f = \frac{2\tau_0}{\rho V^2}$

$$\frac{u_{max} - V}{V^*} = 3.75$$

$$V_* = \sqrt{\frac{f}{8}}\, V. \quad \text{(laminar flow).}$$

$$\therefore \quad u_{max} - V = \left(3.75 \sqrt{\frac{f}{8}} \times V\right).$$

$$\frac{u_{max}}{V} - 1 = \frac{3.75\sqrt{f}}{\sqrt{8}}$$

$$\Rightarrow \frac{u_{max}}{V} = 1.33\sqrt{f} + 1 \quad ; \; f \rightarrow \text{friction factor}$$

Q.02. $V = 2$ m/s, $f = 0.02$

u_{max} = centre line velocity

$$\frac{u_{max}}{V} = \left(1.33\sqrt{f} + 1\right) = \left(1.33 \times \sqrt{0.02} + 1\right).$$

$$\underline{\underline{u_{max} = 2.376 \text{ m/s}}}$$

Q.03 diameter, $d = 0.3$ m, $Re = 10^6$, $f = 0.025$.

$$\delta = \frac{11.6\,\nu}{V_*}$$

$$V_* = \sqrt{\frac{f}{8}}\, V.$$

$$= \sqrt{\frac{0.025}{8}}\, V. = 0.056\, V.$$

$$Re = \frac{\rho V D}{\mu} = \frac{VD}{\gamma}.$$

$$\frac{V}{\gamma} = \frac{10^6}{0.3}.$$

$$\delta = \frac{11.6\,\gamma}{0.056 V} = \frac{11.6 \times 0.3}{0.056 \times 10^6} = 6.21 \times 10^{-5}\ m.$$

$$= \underline{\underline{0.062}}\ mm$$

4. $L = 100m,\ d = 0.1\ m,\ f = 0.02,\ h_L = 10\ m.$

$$\tau_\omega = \frac{\partial p}{\partial x}\frac{r}{2}$$

$$= \frac{\rho g\, h_L}{L} \times \frac{D}{4} = \frac{1000 \times 9.81 \times 10 \times 0.1}{100 \times 4.}$$

$$= \underline{\underline{0.0245}}\ kPa$$

OR

$$\tau_\omega = \frac{f V^2 \rho}{8} \quad ; \quad h_L = \frac{f L V^2}{2gd.}$$

$$10 = \frac{0.02 \times 100 \times V^2}{2g \times 0.1}$$

$$\Rightarrow V^2 = 9.80665$$

$$\tau_\omega = \frac{0.02 \times 9.806 \times 1000}{8} = 24.516\ Pa = \underline{\underline{0.024}}\ kPa$$

5. $k = 0.15\,mm,\quad \tau = 4.9\ N/m^2,\quad \nu = 0.01$ stoke

$$= 0.01 \times 10^{-4}\ m^2/s.$$

$$V_* = \sqrt{\frac{\tau_\omega}{\rho}} = \sqrt{\frac{4.9}{1000}} = 0.07\ m/s$$

$$\frac{V_* k}{\gamma} = \frac{0.07 \times 0.15 \times 10^{-3}}{0.01 \times 10^{-4}} = 10.5 \qquad 3 < \frac{V_* k}{\gamma} < 70$$

$\Rightarrow$ transition bound

(OR)

$$d' = \frac{11.6\,\nu}{V_*} = \frac{11.6\times10^{-6}}{0.07} = 0.166 \text{ mm.}$$

$$\frac{k}{\delta'} = \frac{0.15}{0.166} = 0.9.$$

$$0.25 < \frac{k}{\delta'} < 6 \Rightarrow \text{transition boundary}$$

6. $d = 0.05$ m. $\quad \frac{m}{t} = \pi$ kg/s

$\mu = 0.001$ Pas.

$\rho = 1000.$

$$\rho Q = \pi$$

$$Q = \frac{\pi}{\rho} \text{ m}^3/\text{s}$$

$$V = \frac{Q}{A} = \frac{\pi/\rho}{\pi R^2} = \frac{4}{1000\times0.05^2} = 1.6 \text{ m/s.}$$

$$Re = \frac{\rho V D}{\mu} = \frac{1000\times1.6\times0.05}{0.001} = 80000 > 2000 \Rightarrow \text{turbulent}$$

$$f_D = \frac{64}{\cancel{Re}} = \frac{64}{\cancel{80000}} = \cancel{8\times10^{-4}}\ \frac{0.316}{(80000)^{1/4}} = 0.0187$$

$$h_f = \frac{fLV^2}{2gD} = \frac{\overset{0.0188}{\cancel{8\times10^{-4}}}\times1\times1.6^2}{2\times9.81\times0.05} = 0.049 \text{ m}$$

Pressure drop per unit length $= \rho g h_f$

$= 0.049\times1000\times10$

$= \underline{\underline{480}}$ Pa/m

The friction factor in the case of laminar flow depends on Reynold's number. $\left(f = \frac{64}{Re}\right)$

whereas in the case of turbulence flow, friction factor depends on Reynold's number and avg. height of irregularities.

For smooth pipe :

$$f = \frac{0.316}{(Re)^{1/4}} \quad (Re < 10^5) \qquad f = 0.032 + \frac{0.221}{(Re)^{0.23}} \quad (10^5 < Re < 10^7)$$

FLOW THROUGH PIPES

→ Head Loss

(i) Major loss

occurs because of friction

$$h_f = \frac{fLV^2}{2gd.}$$ (Darcy's Weisbach Equation)

$$f = \frac{64}{Re}$$ (Laminar Flow)

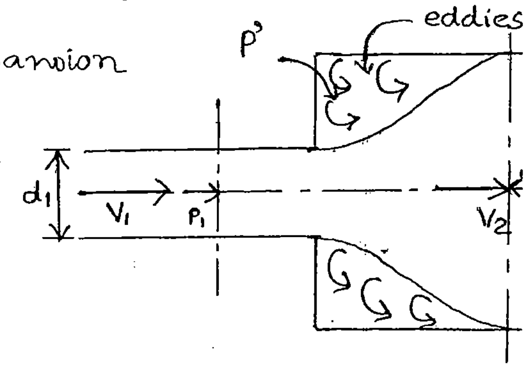
$$f = \frac{0.316}{(Re)^{1/4}}$$ (Turbulent flow in smooth pipe)

(ii) Minor Losses.

a) Sudden expansion of pipe.
b) Sudden contraction of pipe
c) Exit of the fluid from a pipe.
d) Inlet of the fluid to a pipe.
e) Pipe bend.
f) Obstruction in the pipeline.

→ Head loss due to sudden expansion.

Head loss due to sudden expansion is because of formation of eddies.



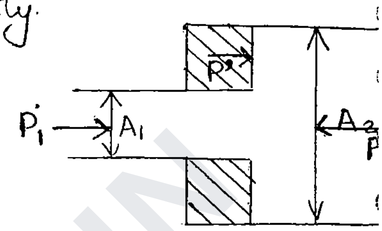
$$\boxed{h_e = \frac{(V_1 - V_2)^2}{2g}}$$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{Z_1}^{0} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + \cancel{Z_2}^{0} + h_e$$

$$\boxed{h_e = \frac{P_1 - P_2}{\rho g} + \frac{V_1^2 - V_2^2}{2g}}$$

$A_1 < A_2 \Rightarrow V_1 > V_2$, then there is a acceleration.

$$\therefore F_{net} = m \times a$$
$$= m \times \frac{dv}{dt}$$
$$= \frac{\text{mass}}{\text{time}} \times \text{change of velocity}$$
$$= \rho Q (V_2 - V_1) \longrightarrow (1)$$



But $F_{net} = P_1 A_1 - P_2 A_2 + P'(A_2 - A_1)$

From experiments, $P' = P_1$ (initial pressure)

$$\therefore F_{net} = P_1 A_1 - P_2 A_2 + P_1 A_2 - P_1 A_1$$
$$= (P_1 - P_2) A_2 \longrightarrow (2)$$

Equating (1) & (2),

$$(P_1 - P_2) A_2 = \rho Q (V_2 - V_1)$$

$$P_1 - P_2 = \rho V_2 (V_2 - V_1)$$

$$\frac{P_1 - P_2}{\rho g} = -\frac{V_1 V_2}{g} + \frac{V_2^2}{g}$$

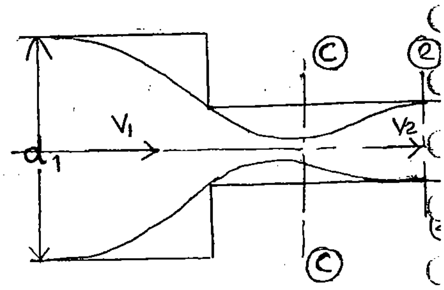
$$\therefore h_e = \frac{-V_1 V_2}{g} + \frac{V_2^2}{g} + \frac{V_1^2 - V_2^2}{2g}$$

$$= \frac{V_2}{g}(V_2 - V_1) + \frac{(V_1 - V_2)(V_1 + V_2)}{2g}$$

$$= \frac{V_1 - V_2}{2g}\left(V_1 + V_2 - 2V_2\right) = \underline{\underline{\frac{(V_1 - V_2)^2}{2g}}}$$

→ Head loss due to sudden Contraction.

(C)−(C) : section at vena contraction



Head loss due to sudden contraction is because of expansion of fluid from area at vena contracta to small diameter pipe.

Let $V_c \rightarrow$ velocity of fluid at vena contracta.

$$\therefore \boxed{h_c = \frac{(V_c - V_2)^2}{2g}}$$

Coeffecient of contraction, $C_c = \frac{V_2}{V_c}$.

$$a_c V_c = a_2 V_2$$

Coeffecient of contraction, $C_c = \frac{a_c}{a_2} = \frac{V_2}{V_c}$.

$$h_c = \frac{V_2^2}{2g}\left(\frac{V_c}{V_2} - 1\right)^2$$

$$\boxed{h_c = \frac{V_2^2}{2g}\left(\frac{1}{C_c} - 1\right)^2}$$

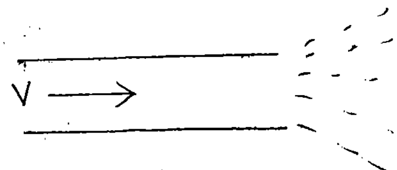
If C_c is not given, then,

$$\boxed{h_c = \frac{V_2^2}{2g}}$$

If C_c is given,

$$\boxed{h_c = \frac{0.5\, V_2^2}{2g}}$$

→ Head loss at the exit of Pipe.

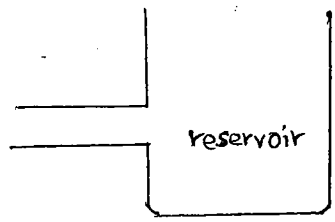


just like expansion.

$$A_1 V_1 = A_2 V_2$$

$$A_2 = \infty \quad ; \quad V_2 = 0.$$

$$h_o = \frac{(V_1 - V_2)^2}{2g}$$

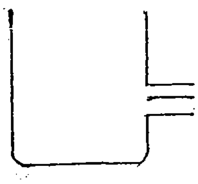


As $V_2 = 0$,

$$\boxed{h_o = \frac{V^2}{2g}}$$

→ Head loss at the inlet of the pipe

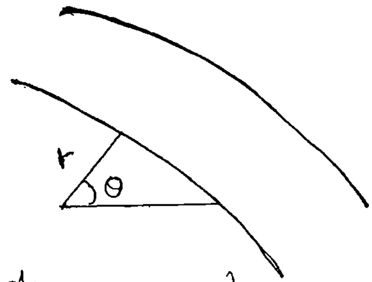
If C_c is being taken into consideration, then head loss due to sudden contraction, $h_c = 0.5 \frac{V_2^2}{2g}$.



$$\boxed{\text{Head loss at inlet of pipe, } h_i = 0.5 \frac{V_2^2}{2g}}$$

→ Head loss due to Pipe Bend.

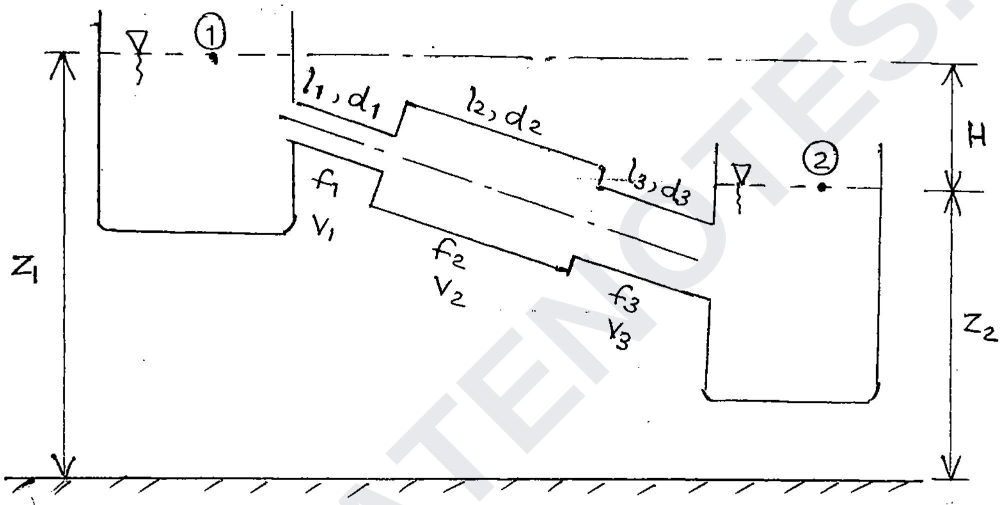
$$\boxed{h_b = k\frac{V_b^2}{2g}}$$



$k \rightarrow$ pipe bend coeffecient (depends on angle, radius).

→ Compound Pipe

(i) Pipes in Series



$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + H_L$$

For large reservoirs, the velocity of fluid becomes zero.
$(\therefore V_1 = V_2 = 0)$.

$P_1 = P_2 = P_{atm}$

$$\therefore h_L = z_1 - z_2$$

$$\boxed{h_L = H}$$

But h_L = major losses + minor losses.

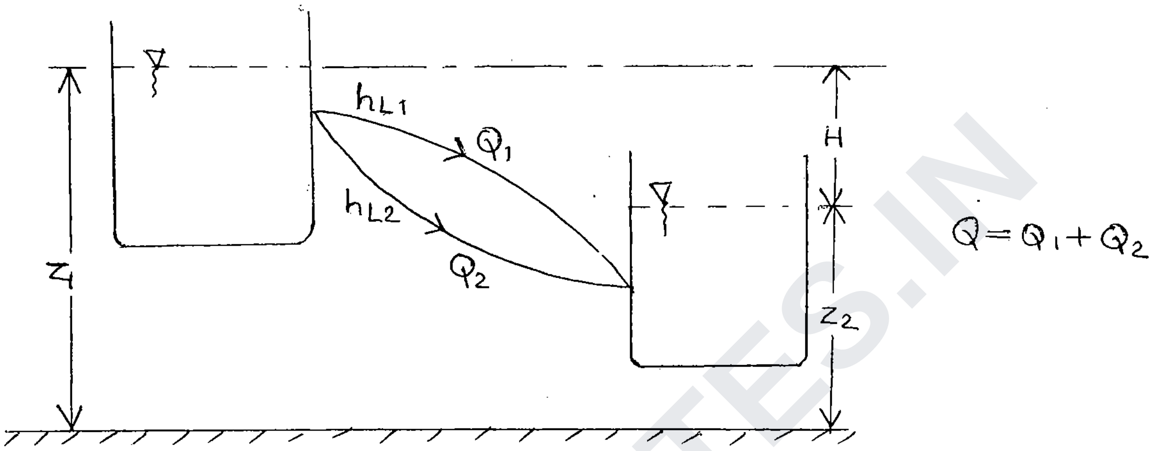
$$\therefore h_L = \underset{(h_i)}{\frac{0.5V_1^2}{2g}} + \underset{(h_f)}{\frac{f_1 l_1 V_1^2}{2g d_1}} + \underset{(h_e)}{\frac{(V_1 - V_2)^2}{2g}} + \underset{(h_f)}{\frac{f_2 L_2 V_2^2}{2g d_2}}$$

$$+ \underset{(h_c)}{\frac{0.5V_3^2}{2g}} + \underset{(h_f)}{\frac{f_3 l_3 V_3^2}{2g d_3}} + \underset{(h_o)}{\frac{V_3^2}{2g}} = H$$

Neglect all minor losses,

$$H = \frac{f_1 l_1 V_1^2}{2g d_1} + \frac{f_2 l_2 V_2^2}{2g d_2} + \frac{f_3 l_3 V_3^2}{2g d_3}$$

(ii) Pipes in Parallel



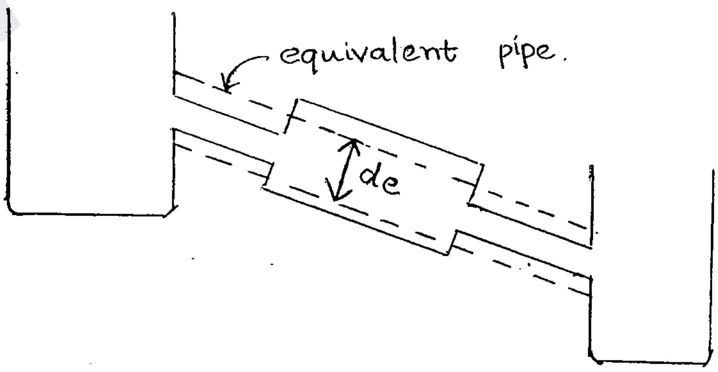
$$h_{L1} = h_{L2}$$

If we consider major loss,

$$\frac{f_1 l_1 V_1^2}{2g d_1} = \frac{f_2 l_2 V_2^2}{2g d_2}$$

→ Equivalent Pipe

It is the pipe of uniform diameter that replaces a compound pipe such that it allows the same discharge and it maintains the same head loss.



Compound pipe requires couplings; which may result in leakages and demands regular maintenances. To avoid that equivalent pipes are used.

* Head loss in compound pipe

Assume $f_1 = f_2 = f_3 = f$.

Assume that equivalent pipe also have, f.

Head loss in the compound pipe,

$$h_L = \frac{f l_1 V_1^2}{2g d_1} + \frac{f l_2 V_2^2}{2g d_2} + \frac{f l_3 V_3^2}{2g d_3}$$

$$Q = A_1 V_1 = A_2 V_2 = A_3 V_3.$$

$$V_1 = \frac{Q}{A_1} = \frac{Q}{\frac{\pi}{4} d_1^2} = \frac{4Q}{\pi d_1^2}$$

$$\therefore V_1^2 = \frac{16 Q^2}{\pi^2 d_1^4}$$

$$h_L = \frac{f l_1 \times 16 Q^2}{2g \times \pi^2 \times d_1^5} + \frac{f l_2 \times 16 Q^2}{2g \times \pi^2 \times d_2^5} + \frac{f l_3 \times 16 Q^2}{2g \pi^2 \times d_3^5} \longrightarrow (1)$$

Let $l_e \rightarrow$ length of equivalent pipe.

$$l_e = l_1 + l_2 + l_3.$$

$d_e \rightarrow$ diameter of equivalent pipe.

Head loss in equivalent pipe, $h_L = \dfrac{f l_e 16 Q^2}{2g \pi^2 \times d_e^5} \longrightarrow (2)$

$$(h_L)_{eq} = (h_L)_{comp.}$$

$$\frac{f l_e 16 Q^2}{2g \pi^2 d^5} = \frac{16 f Q^2}{2g \pi^2}\left(\frac{l_1}{d_1^5} + \frac{l_2}{d_2^5} + \frac{l_3}{d_3^5}\right)$$

DUPUIT'S EQUATION.

$$\boxed{\frac{l_e}{d_e^5} = \frac{l_1}{d_1^5} + \frac{l_2}{d_2^5} + \frac{l_3}{d_3^5}}$$

Dupuit's equation can be used to determine the equivalent diameter, d_e.

Q.01 $\gamma = 0.4$ stoke, $d = 8$ cm.

$Re = 2000$ (laminar)

$$2000 = \frac{\rho \times \bar{u}_{max} \times d}{\mu}$$

$$2000 = \frac{V \times 8 \times 10^{-2}}{0.4 \times 10^{-4}}$$

$$\bar{u} = V = 1 \text{ m/s}$$

$$u_{max} = 2\bar{u} = \underline{\underline{2 \text{ m/s}}}$$

Q.02. $\gamma = 8$ cm²/s, $d = 8$ cm, $Q = 3200\pi$ cm³/s.

$$V = \frac{Q}{A} = \frac{3200\,\pi}{\frac{\pi}{4} \times 8^2} = 200 \text{ cm/s.}$$

$$Re = \frac{\rho V d}{\mu} = \frac{200 \times 10^{-2} \times 8 \times 10^{-2}}{8 \times 10^{-4}} = 200 < 2000 \Rightarrow \underline{\underline{\text{laminar}}}$$

Q.03 $$h_L = \frac{fLV^2}{2gd} \Rightarrow h_L \propto \cancel{d}\,\frac{1}{d^5}.$$

$$\frac{(h_L)_A}{(h_L)_B} = \frac{(d_B)^5}{(1.2\,d_B)^5} = \underline{\underline{0.402}}$$

Q.04 $$\frac{2l}{d_e^5} = \frac{l}{(0.1)^5} + \frac{l}{(0.2)^5}$$ $\qquad l_e = l_1 + l_2 = l + l = \underline{\underline{2l}}$

$$d_e = \underline{\underline{11.42}} \text{ cm}$$

Q.05. For a parallel flow,

$$(h_L)_1 = (h_L)_2$$

$$\frac{f_1 L_1 V_1^2}{2g d_1} = \frac{f_2 L_2 V_2^2}{2g d_2}$$

$$\frac{f_1 L_1 Q_1^2}{2g d_1^5} = \frac{f_2 L_2 Q_2^2}{2g d_2^2}$$

Given $L_1 = L_2$; $d_1 = d_2$; $f_1 = 4 f_2$

$$\frac{4 f_2 \times L_1 \times Q_1^2}{2g\, d_1^5} = \frac{f_2 \times L_1 \times Q_2^2}{2g\, d_2^5}$$

$$\left(\frac{Q_1}{Q_2}\right)^2 = \frac{1}{4}.$$

$$Q_1 : Q_2 = \underline{\underline{1 : 2}}$$

6. $$h_L = \frac{f L Q^2}{2g\, d^5}.$$

$$Q^2 \propto d^5.$$

$$\left(\frac{Q_1}{Q_2}\right)^2 = \left(\frac{2d}{d}\right)^5 = 32$$

$$\frac{Q_1}{Q_2} = \underline{\underline{4\sqrt{2}}}$$

11. $$d_2 = 2 d_1.$$

$$h_e = \frac{(V_1 - V_2)^2}{2g}.$$

$$A_1 V_1 = A_2 V_2.$$

$$d_1^2 V_1 = d_2^2 V_2$$

$$V_2 = \left(\frac{d_1}{d_2}\right)^2 V_1.$$

$$= \frac{V_1}{4}.$$

$$h_e = \frac{\left(V_1 - \frac{V_1}{4}\right)^2}{2g} = \frac{V_1^2}{2g}\left(1 - \frac{1}{4}\right)^2.$$

$$\frac{h_e}{\frac{V_1^2}{2g}} = \left(1 - \frac{1}{4}\right)^2 = \underline{\underline{\frac{9}{16}}}$$

→ Equivalent Pipe for Pipes in Parallel.

$$Q = Q_1 + Q_2$$

If the two pipes have same diameter,

$$\Rightarrow \quad Q_1 = Q/2 \quad \& \quad Q_2 = Q/2$$

$$(h_L)_{eq} = (h_L)_{parallel.}$$

$$\frac{f l \times 16 Q^2}{2g\, \pi^2 \times d_e^5} = \frac{f l \times 16 \times (Q/2)^2}{2g \times \pi^2 \times d^5}$$

$$\frac{Q^2}{d_e^5} = \frac{Q^2}{4} \times \frac{1}{d^5}$$

$$d_e = (4)^{1/5} \times d.$$

NOTE : If n pipes of equal diameter are connected in parall

$$\boxed{d_e = n^{2/5} d}$$

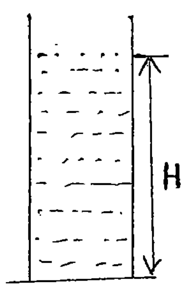
07. $d_e = 30\,cm$; $n = 2$.

$$d = \frac{d_e}{n^{2/5}} = \frac{30}{2^{2/5}} = \underline{\underline{22.74}}\ cm$$

$d \approx 25\ cm$

23th Aug
MONDAY

→ Power Transmission through Pipes



$$Power = \frac{Work\ done}{Time.}$$

$$= \frac{Force \times displacement}{Time.}$$

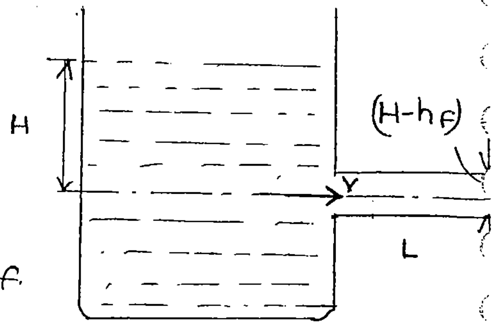
$$= \frac{weight \times H}{Time} = w \times \frac{volume}{time} \times H.$$

$$\therefore \quad \boxed{P = \rho g Q H}$$

Power at inlet of pipe $= \rho g Q H$

Let h_f be the frictional head loss in pipe.

Head available at outlet of pipe $= H - h_f$



Power at outlet of pipe $= \rho g Q (H - h_f)$.

$$\text{Efficiency of power transmission} = \frac{\text{Power at outlet}}{\text{Power at inlet}}$$

$$= \frac{\rho g Q (H - h_f)}{\rho g Q H}$$

$$\boxed{\eta = 1 - \frac{h_f}{H}}$$

Condition for Maximum Power Transmission

Power at outlet of pipe $= \rho g Q (H - h_f)$.

$$= \rho g \, \frac{\pi}{4} D^2 \times V \left(H - \frac{fLV^2}{2gD}\right)$$

$$= \rho g \frac{\pi}{4} D^2 \left(HV - \frac{fLV^3}{2gD}\right).$$

For maximum power,

$$\frac{dP}{dV} = 0.$$

$$H - \frac{3fLV^2}{2gD} = 0.$$

$$H - 3h_f = 0.$$

$$\therefore \quad H = 3h_f.$$

or $\boxed{h_f = \frac{H}{3}}$

$$\text{Efficiency of max power transmission} = 1 - \frac{h_f}{H} = 1 - \frac{1}{3}$$

$$= \underline{\underline{66.66\%}}$$

08. $H = 99$ m.

$h_f = \frac{H}{3} = 33$ m. (for max. power).

Max power transmitted $= \rho g Q (H - h_f)$.

$= 10^4 \times 1 (99 - 33)$.

$= 660 \times 10^3$ W

$= \underline{\underline{660 \text{ kW}}}$

09. $Q = 100$ m³/s, $H = 75$ m.

Power transmitted $= \rho g Q H$

$= 10^4 \times 100 \times 75$. $\quad$ 1 hp = 746 W

$= 75 \times 10^3$ kW.

$= 75 \times 10^3 \times 1.34$ hp

$= \underline{100576}$ hp

10. $Q = 0.1$ m³/s, $H = 10$ m, $h_f = 5$ m.

$P_{(in)} = \rho g Q H$

$= 10^4 \times 0.1 \times 10 = 10$ kW. (power required to pump to the height).

$P_{(out)} = \rho g Q (H - h_f)$.

$= 10^4 \times 0.1 \times 5 = 5$ kW. (power required to overcome friction loss).

Total power required by pump $= \rho g Q (h_L + H)$.

$= 10^4 \times 0.1 (5 + 10)$

$= \underline{\underline{15 \text{ kW}}}$

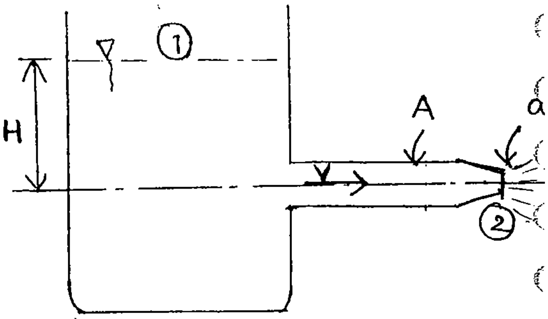
12. $h_L \propto \frac{1}{d^5}$.

$\frac{h_1}{h_2} = \frac{d_2^5}{d_1^5} = \left(\frac{d_1/2}{d_1}\right)^5 = \frac{1}{32}$.

$\underline{\underline{h_2 = 32 h_1}}$

→ Power Transmission Through Nozzle

$$\cancel{\frac{P_1}{\rho g}}^{P_{atm}} + \cancel{\frac{V_1^2}{2g}}^{0} + z_1 = \cancel{\frac{P_2}{\rho g}}^{P_{atm}} + \cancel{\frac{V_2^2}{2g}}^{\frac{v^2}{2g}} + z_2 + h_f.$$



$$\boxed{H = \frac{v^2}{2g} + h_f.}$$

$$h_f = \frac{fLV^2}{2g\,D.}$$

$$H = \frac{v^2}{2g} + h_f.\ \frac{fLV^2}{2g\,D.}$$

$$AV = av.$$

$$V = \frac{av}{A.}$$

$$H = \frac{v^2}{2g}\left(1 + \frac{fL\,a^2}{DA^2}\right).$$

$$v^2 = \frac{2g\,H}{1 + \frac{fLa^2}{A^2D.}}$$

$$\therefore \boxed{v = \sqrt{\frac{2g\,H}{1 + \frac{fLa^2}{A^2D.}}}}$$

Power at outlet of nozzle $= \dfrac{KE \text{ of jet}}{\text{time}}$

$$= \frac{1}{2}\dot{m}V^2$$

$$= \frac{1}{2}(\rho a.v)v^2$$

$$= \frac{1}{2}\rho a v^3$$

Power at inlet of nozzle $= \rho g Q (H - h_f)$.

where $Q = AV = av$

$$= \rho g\, av \frac{v^2}{2g.}$$

Power at inlet of pipe $= \rho g Q H$

$= \rho g\, a v \cdot H$

$$\eta = \frac{\text{power at outlet of nozzle}}{\text{power at inlet of pipe}}$$

$$= \frac{\frac{1}{2}\rho a v^3}{\rho g\, a v \times H} = \frac{v^2}{2gH}$$

$$\boxed{\eta = \frac{v^2}{2gH} = \frac{1}{1 + \frac{fLa^2}{DA^2}}}$$

Power at outlet of nozzle $= \frac{1}{2}\dot{m} v^2$

$$H - h_f = \frac{V^2}{2g}$$

$$av = AV$$

$$= \frac{1}{2}\rho a v\, v^2$$

$$= \frac{1}{2}\rho a v\, 2g\,(H - h_f)$$

$$= \frac{1}{2}\rho a v\, 2g\left(H - \frac{fLV^2}{2gD}\right)$$

$$= \frac{1}{2}\rho a \cdot 2g\left(Hv - \frac{fLa^2}{2gDA^2}v^3\right)$$

For max. power,

$$\frac{dP}{dV} = 0$$

$$H - \frac{fLa^2v^2}{2gDA^2}\cdot 3 = 0$$

$$H - \frac{fLV^2}{2gD}\cdot 3 = 0$$

$$\therefore \quad H = 3h_f$$

$$h_f = \frac{H}{3}$$

$$\frac{V^2}{2g} + h_f = H = 3h_f$$

$$\therefore \quad \frac{V^2}{2g} = 2\left(\frac{fLV^2}{2gD}\right)$$

$$\frac{V^2}{2g} = \frac{fL}{gD}\,\frac{a^2V^2}{A^2}$$

$$\frac{A^2}{a^2} = \frac{2fL}{D}$$

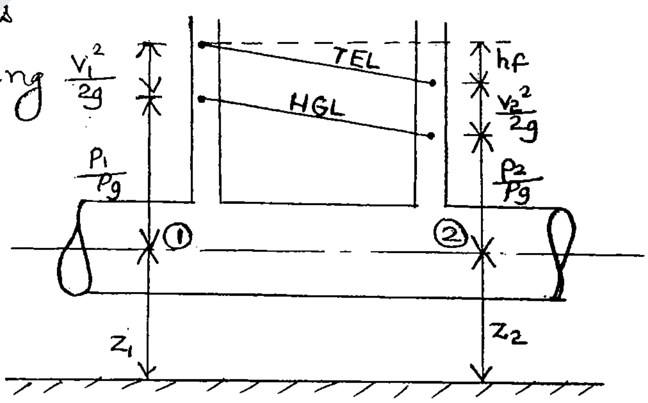
$$\Rightarrow \boxed{\frac{A}{a} = \sqrt{\frac{2fL}{D}}}$$

$$\frac{D^4}{d^4} = \frac{2fL}{D}$$

$$D^5 = 2fLd^4$$

$$\boxed{d = \left(\frac{D^5}{2fL}\right)^{1/4}}$$

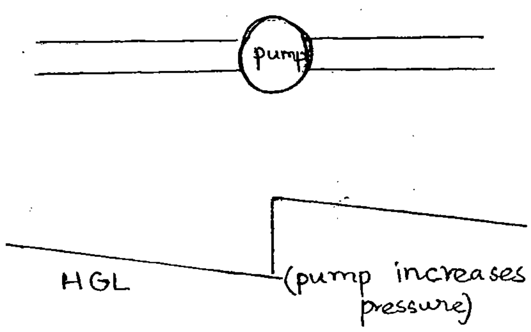
→ Total Energy Line (TEL) & Hydraulic Gradient Line (HGL)

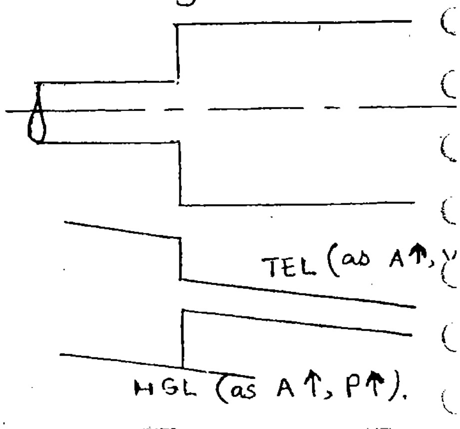


- HGL is the locus of all points which will be obtained by taking the piezometric heads $\left(\frac{P}{\rho g}+z\right)$ along the length of the pipe in the direction of flow.

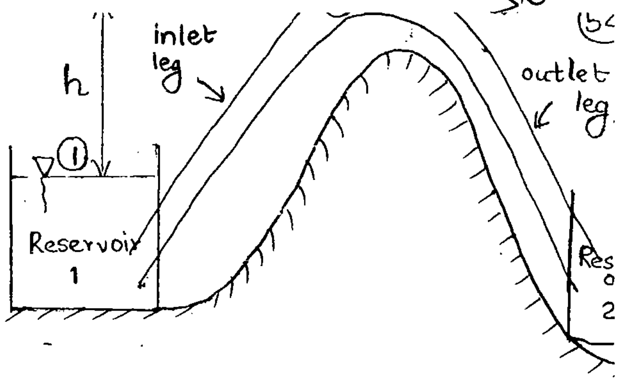
- TEL is the locus of all points which will be obtained by taking the total energy $\left(\frac{P}{\rho g}+\frac{V^2}{2g}+z\right)$ along the length of pipe in the direction of flow.

- TEL & HGL are seperated by kinetic head $\left(\frac{V^2}{2g}\right)$





→ Syphon



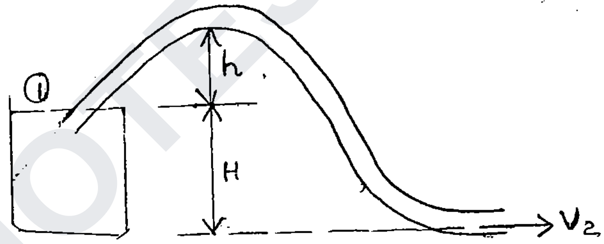
$$\frac{\cancel{P_1}^{P_{atm}}}{\rho g} + \frac{\cancel{V_1^2}^{\,0}}{2g} + \cancel{z_1}^{\,0} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + V_2$$

$$\frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h = 0$$

$$\frac{P_2}{\rho g} = -\left(h + \frac{V_2^2}{2g}\right).$$

$$P_2 = -\rho g\left(h + \frac{V_2^2}{2g}\right) \quad \text{[vacuum pressure]}$$

The pressure at the summit point is less than atmospheric pr. and ∴ the liquid will be transferred from the reservoir to the summit point because of the pressure difference. The pressure at the summit point is reduced lower than 2.5 m of absolute pressure for water. Then the dissolved air in the water will get collected ~~at~~ the summit point and it leads to the air lock. and it will obstruct the continuivity of flow.



$$\frac{\cancel{P_1}}{\rho g} + \frac{\cancel{V_1^2}}{2g} + z_1 = \frac{\cancel{P_2}}{\rho g} + \frac{V_2^2}{2g} + z_2$$

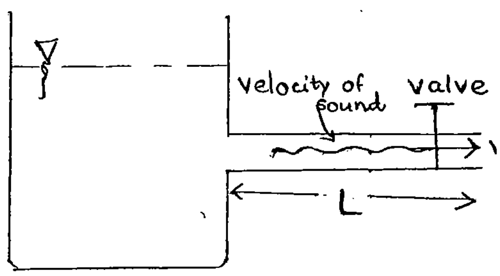
$$\frac{V_2^2}{2g} = H.$$

$$\boxed{V_2 = \sqrt{2gH}}$$

The pressure at the summit pt. cannot be reduced to vapour pressure of water to prevent cavitation.

→ Water Hammering

Rotarding force = pressure force.



$$m \times \left(\frac{V-0}{T}\right) = pA.$$

$$\frac{\rho \times \text{volume} \times V}{T} = pA.$$

$$\frac{\rho \times A \times L \times V}{T} = pA$$

$$\boxed{p = \frac{\rho L V}{T}}$$

> Incompressible flow
> Rigid pipe

$T \rightarrow$ time taken to reduce velocity.

- When the pipe is rigid and fluid is compressible,

Loss in KE of water = strain energy stored.

$$\frac{1}{2} m (V-0)^2 = \frac{p^2}{2K} \times A \times L$$

{Strain energy $= \frac{\sigma^2}{2E} \times$ volume

$$\frac{1}{2} \rho A L V^2 = \frac{p^2}{2K} AL$$

$$p^2 = \rho K V^2$$

Intensity of pressure, $p = V\sqrt{\rho K}$; Pipe rigid fluid compressible.

→ Velocity of Sound, $c = \sqrt{\frac{K}{\rho}}$

$$c^2 = \frac{K}{\rho}$$

$$p = V\sqrt{\frac{K}{c^2} \times K} \qquad \left(\because \rho = \frac{K}{c^2}\right)$$

$$\boxed{p = \frac{VK}{c}}$$

$$= V\sqrt{\rho \times \rho c^2}$$

$$\boxed{p = \rho V c.}$$

6. MOMENTUM EQUATION

→ Impact of Jets.

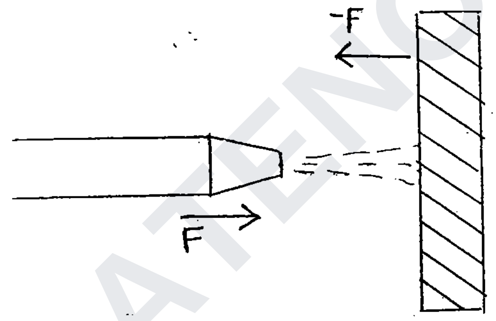
$$F_x = m \times a_x.$$
$$= m \times \frac{dv}{dt}.$$

$$F_x = \frac{d}{dt}(mv) \rightarrow \text{MOMENTUM EQUATION}$$

$\therefore$ Net force = Rate of change of momentum.

$$F_x \cdot dt = d(mv) \rightarrow \text{Impulse Momentum Equati}$$

impulse.

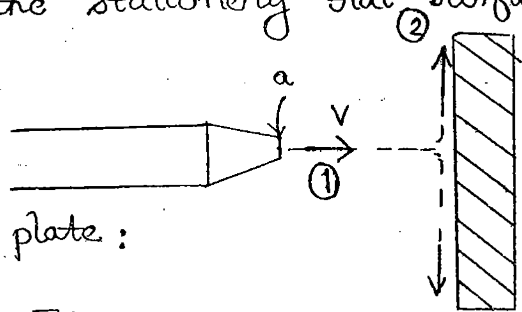


$$-F = \frac{\text{mass}}{\text{time}}(\text{Final velocity} - \text{Initial velocity}).$$

$$\boxed{F = \frac{\text{mass}}{\text{time}}(\text{Initial velocity} - \text{Final velocity})}$$

used to find the force excer by the jet on the obstructio

① Force exerted by the jet on the Stationery Flat Surfac



Let V be velocity of jet.
a be area of nozzle.

Force exerted by jet normal to the plate:

$$\cancel{\frac{P_1}{\rho g}} + \frac{V_1^2}{2g} + \cancel{Z_1} = \cancel{\frac{P_2}{\rho g}} + \frac{V_2^2}{2g} + \cancel{Z_2}$$ (assume there are no losses over the plate)

$$V_1 = V_2 = V.$$

$$F = \frac{m}{time}(\text{Initial velocity} - \text{final velocity}).$$

$$= \rho a v (v - 0)$$

(Final velocity component normal to the surface is zero)

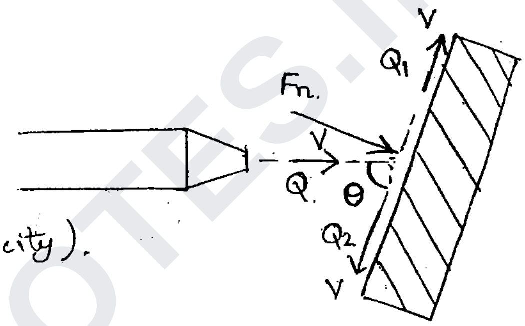
$$\boxed{F = \rho a v^2}$$

$F_x = F$

$F_y = 0$

$\because$ the plate is stationery, work done by the jet is zero.

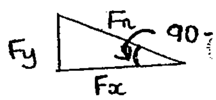
② Force excerted by the jet on a Flat stationery Inclined plate.



Force excerted by jet normal to the plate, $F_n = \frac{mass}{time}$ (ini. velo − final velocity).

$$F_n = \rho a v (v\sin\theta - 0)$$

$$\boxed{F_n = \rho a v^2 \sin\theta}$$



$F_x = F_n\cos\theta$

$= F_n \sin\theta$

$F_y = F_n\cos\theta.$

$F_x = F_n \sin\theta = \rho a v^2 \sin^2\theta$
$F_y = F_n \cos\theta = \rho a v^2 \sin\theta \cos\theta$

The net force acting parallel to the plate is zero,

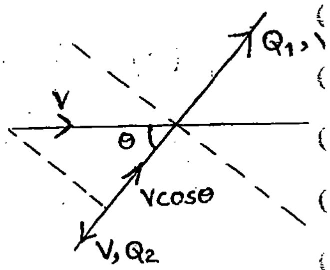
$$F_{net} = m \times a$$

$$0 = \frac{m}{time}(\text{initial velocity} - \text{final velocity})$$

$$= \rho Q v\cos\theta - (\rho Q_1 v - \rho Q_2 v)$$

$$Q\cos\theta = Q_1 - Q_2 \quad \rightarrow ①$$

$$Q = Q_1 + Q_2 \quad \rightarrow ②$$



$Q_1 = \frac{Q}{2}(1 + \cos\theta)$
$Q_2 = \frac{Q}{2}(1 - \cos\theta)$

Q.01 $F = \rho A V^2$

$= 1000 \times 10^{-4} \times 10^2$

$= \underline{\underline{10 N}}$

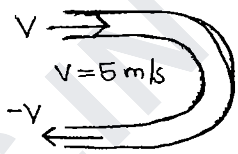
Q.02 $V = 5 m/s$, $A = 10^{-4} m^2$

$F = \frac{mass}{time}$ (initial velocity − final velocity).

$= \rho Q (v - (-v))$.

$= 2\rho a v^2 = 2 \times 1000 \times 10^{-4} \times 25$

$= \underline{\underline{5 N.}}$



Q.03 $V = 10 m/s$, $a = 5 cm^2 = 5 \times 10^{-4} m^2$

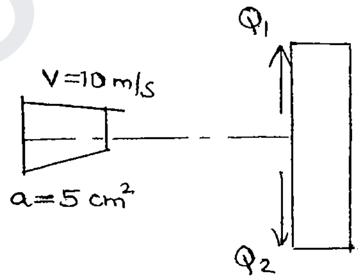
$Q = Q_1 + Q_2$

$= 2Q_1$

$aV = 2 a_1 V_1$

$5 \times 10^{-4} \times 10 = 2 \times 2.5 \times 10^{-4} \times V_1$

$V_1 = \underline{\underline{10 m/s}}$



Q.04. Magnitude of force, $F = \rho a V^2$

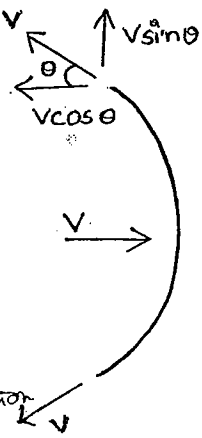
$= 1000 \times 5 \times 10^{-4} \times 10^2$

$= \underline{\underline{50 N}}$

?th Aug,
WEDNESDAY

③ Force excerted by the jet on the curved vane

(i). Jet striking the vane at the centre



$F_x = \rho a V (V - (-V\cos\theta))$

$F_x = \rho a V^2 (1 + \cos\theta)$

$F_y = \rho a V (0 - V\sin\theta) = -\rho a V^2 \sin\theta$

−ve sign indicates force excerted in vertically d/w direction

$$\frac{WD}{s} = \text{Force} \times \frac{\text{displacement}}{s}$$

$$= \text{Force} \times \text{velocity}.$$

For the fixed vane, velocity of vane is zero.

$\therefore$ WD is zero.

(ii) Jet strikes tangentially.

$$F_x = \rho Q\,(v\cos\theta - (-v\cos\theta))$$

$$= \rho Q\,(v\cos\theta + v\cos\theta)$$

$$= 2\,\rho a v^2 \cos\theta.$$

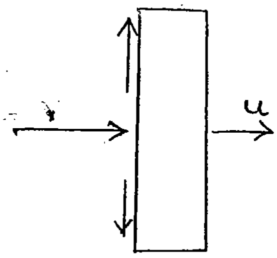
$$F_y = \rho a v\,(v\sin\theta - v\sin\theta) = 0$$

* MOVING VANES.

① Flat Vertical Plate

Let u be the velocity of the vane

In this case, the jet strikes the plate with relative velocity of $(v-u)$.



$$F_x = \rho a\,(v-u)\,[(v-u)-0]$$

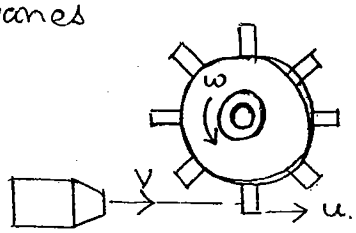
$$= \rho a\,(v-u)^2$$

$$F_x = \rho a\,(v-u)^2$$

$$\frac{W.D}{s} = F_x \cdot u = \rho a\,(v-u)^2 \times u.$$

⊙ If the jet is striking series of vanes

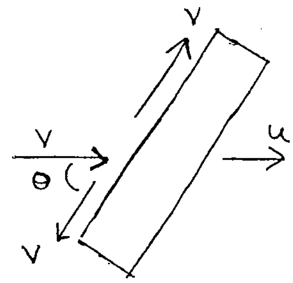
Mass of fluid striking the vane/s $= \rho a V$



$$F_x = \rho a V[(v-u) - 0] = \rho a V(v-u)$$

$$u = r\omega = \frac{\pi d N}{60}$$

2. Inclined Plate.

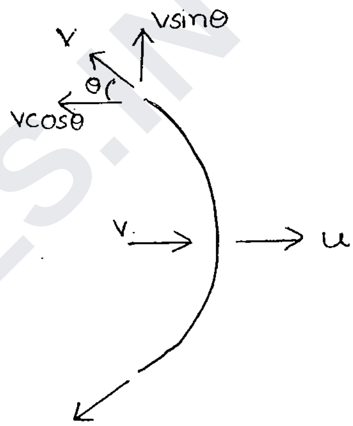


$$F_n = \rho a (v-u)\left[(v-u)\sin\theta - 0\right]$$

$$F_n = \rho a (v-u)^2 \sin\theta.$$

$F_x = F_n \sin\theta.$	$=$	$\rho a (v-u)^2 \sin^2\theta.$
$F_y = F_n \cos\theta$	$=$	$\rho a (v-u)^2 \sin\theta \cos\theta.$

3. Curved Plate



$$F_x = \rho a (v-u)\left[(v-u) - (-(v-u)\cos\theta)\right]$$

$$\boxed{F_x = \rho a (v-u)^2 (1+\cos\theta)}$$

$$\frac{WD}{s} = F_x . u$$

$$= \rho a (v-u)^2 (1+\cos\theta)\, u.$$

$$\eta = \frac{WD/s}{KE/s}$$

$$KE/s \text{ of jet} = \frac{1}{2}\left(\frac{m}{s}\right) v^2 = \frac{1}{2}\rho a v^3$$

$$\therefore \eta = \frac{\rho a (v-u)^2 (1+\cos\theta) u}{\frac{1}{2}\rho a v^3}$$

$$= \frac{2(v-u)^2(1+\cos\theta)u}{v^3}.$$

For the max. efficiency, $\frac{d\eta}{du} = 0.$

$$\frac{d}{du}\left(\frac{2(1+\cos\theta)}{v^3}(v-u)^2 u\right) = 0$$

$$(v-u)^2 + 2(v-u)u \times -1 = 0.$$

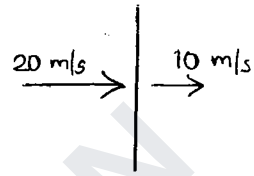
$$v-u-2u = 0.$$

$$\Rightarrow \boxed{u = \frac{v}{3}}$$

For the η_{max}, the blade velocity will be one third of jet velocity.

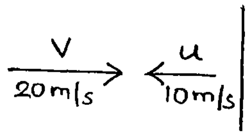
05. $V = 20\ m/s,\ u = 10\ m/s,\ a = 0.01\ m^2,\ \rho = 1000\ kg/m^3$

$$F = \rho a (V-u)^2$$
$$= 1000 \times 0.01\ (20-10)^2 = \underline{\underline{1000\ N}}$$



06. $\frac{WD}{S} = \text{Power} = Fu = 1000 \times 10 = \underline{\underline{10\ kW}}$

07. $$F = \rho a (v+u)^2$$
$$= 1000 \times 0.01 \times (20+10)^2 = \underline{\underline{9000}}\ N$$



08. $A = 0.2 \times 10^{-4}\ m^2 \qquad V = 1 \times 10^{-2}\ m/s$
$a = 0.07 \times 10^{-6}\ m^2$

$$AV = aV$$
$$0.2 \times 10^{-4} \times 10^{-2} = 0.07 \times 10^{-6} \times V$$
$$V = 2.857\ m/s$$

$$F = \rho a V^2 = 1000 \times 0.07 \times 10^{-6} \times (2.857)^2$$
$$= 5.7142 \times 10^{-4}\ N$$
$$= 5.\underline{\underline{827}} \times 10^{-5}\ kgf$$

1 kgf = 9.81

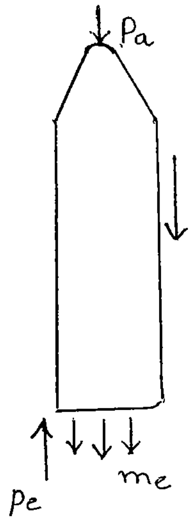
09. $V = 20\ m/s,\ u = 5\ m/s$

$F_1 = \rho a (v-u)^2 \qquad F_2 = \rho a V (v-u)$

$P_1 = \rho a (v-u)^2 \times u. \qquad P_2 = \rho a V (v-u) u$

$$\frac{P_1}{P_2} = \frac{\rho a (v-u)^2 \times u}{\rho a V (v-u) u} = \frac{v-u}{V} = \frac{20-5}{20} = \frac{15}{20} = \underline{\underline{0.75}}$$

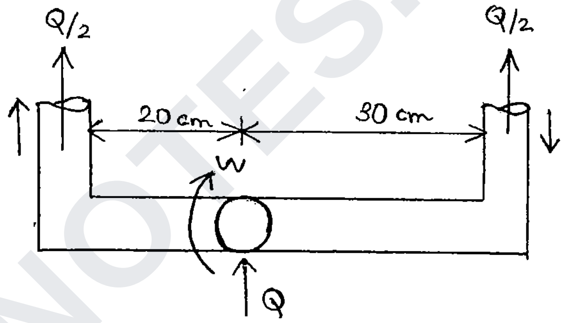
10. $G = 1.2$; Rocket Propulsion



M = fuel + oxidant.

⊙ Data insufficient

11. Velocity of jet from the nozzle $= \dfrac{\frac{Q}{2}}{A}$



$$= \frac{\left(\frac{144\times10^{-3}}{60}\right)/2}{1.2\times10^{-4}} = 10 \text{ m/s.}$$

Tangential velocity of arm end A $= r\omega$

$= 0.2\times8 = 1.6$ m/s.

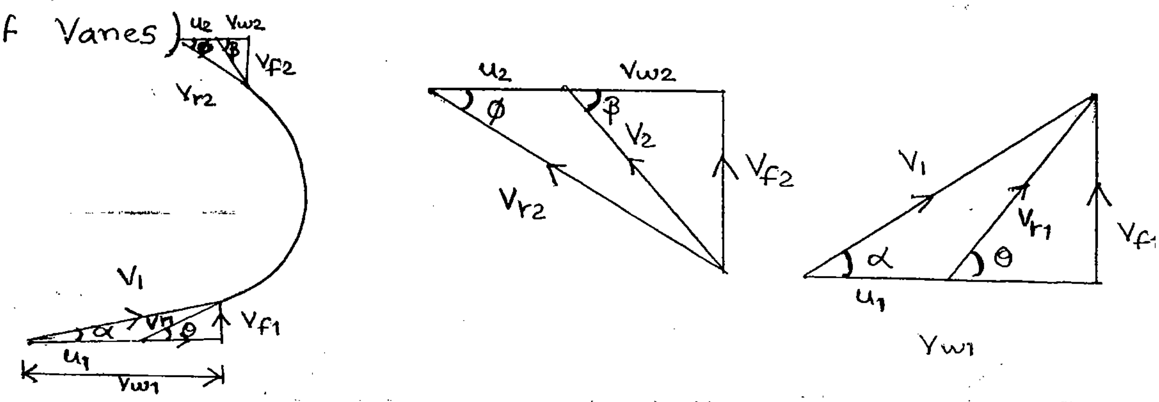
Tangential velocity of arm end $B = 0.3\times8 = 2.4$ m/s.

Absolute velocity of ~~a~~water at end A $= 10+1.6 = 11.6$ m/s.

Absolute velocity at end $B = 10-2.4 = 7.6$ m/s.

■ Unsymmetrical Curved Vane and the jet striking tangentially at one end and leaves tangentially.

(Series of Vanes)



$V_1 \rightarrow$ absolute velocity of jet

$V_{f1} \rightarrow$ velocity of flow

$V_{w1} \rightarrow$ velocity of whirl

$\alpha \rightarrow$ nozzle angle

$\theta \rightarrow$ inlet blade angle.

$\phi \rightarrow$ outlet blade angle

$u_1 \rightarrow$ blade velocity at inlet $= r_1 \omega$

$u_2 \rightarrow$ blade velocity at outlet $= r_2 \omega$

$$\frac{WD}{S} = \text{power} = \frac{2\pi N T}{60} = T \times \omega$$

$$T = \frac{\partial}{\partial t}(m V r) = \frac{\text{mass}}{\text{second}} \times \text{velocity} \times \text{radius}$$

Torque = rate of change of moment of momentum

Moment of momentum/s at inlet $= \rho a V_1 V_{w1} \times r_1$

Moment of momentum/s at outlet $= -\rho a V_1 V_{w2} \times r_2$.

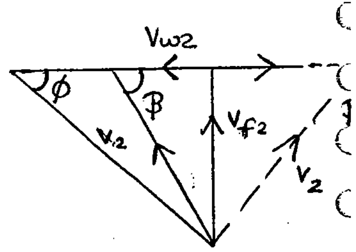
Torque $= \rho a V_1 V_{w1} r_1 - (-\rho a V_1 V_{w2} r_2)$ (same mass will be flowing out $= \rho a V_1$)

$$\frac{WD}{S} = T \times \omega$$

$$= \rho a V_1 V_{w1} \times r_1 \times \omega + \rho a V_1 V_{w2} \times r_2 \times \omega$$

$$= \rho a V_1 V_{w1} \times u_1 + \rho a V_1 V_{w2} \times u_2$$

$$\boxed{\frac{WD}{S} = \rho a V_1 (V_{w1} u_1 + V_{w2} u_2)}$$



- If $u_1 = u_2 = u$.

$$\frac{WD}{S} = \rho a V_1 (V_{w1} + V_{w2}) u.$$

- If $\beta = 90^\circ$, then $V_{w2} = 0$

$$\frac{WD}{S} = \rho a V_1 V_{w1} \times u.$$

- If $\beta > 90^\circ$,

$$\frac{WD}{S} = \rho a V_1 (V_{w1} - V_{w2}) u.$$

$$\boxed{\frac{WD}{S} = \rho a V_1 (V_{w1} u_1 \pm V_{w2} u_2)}$$

$$\frac{WD/s}{\frac{\text{wt. of fluid}}{S}} = \frac{\rho a V_1 (V_{w1} u_1 \pm V_{w2} u_2)}{\rho a V_1 \times g}$$

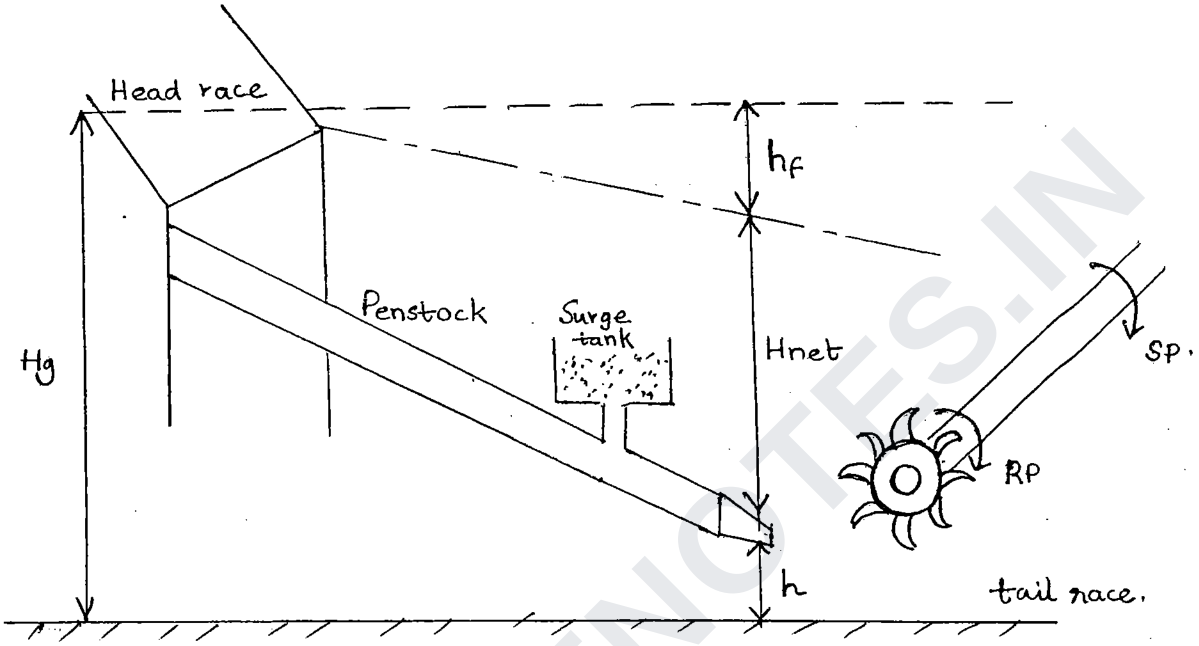
$$= \frac{V_{w1} u_1 \pm V_{w2} u_2}{g}$$

- When $\beta = 90^\circ$, $V_{w2} = 0$

$$= \frac{V_{w1} u_1}{g} \quad \text{(Francis Turbine)}$$

11. TURBOMACHINERY

Turbine : hydraulic energy into mechanical energy



$H_g \rightarrow$ gross head.

$H_{net} \rightarrow$ net head $\Rightarrow H_{net} = H_g - h_f$ ($\because$ h is small).

⊙ Hydraulic efficiency, $\eta_h = \frac{R.P}{W.P}$; R.P → runner power
W.P → water power.

$$RP = \rho a V_1 (V_{w1} u_1 \pm V_{w2} u_2) \ ; \ WP = \rho g Q H.$$

⊙ Mechanical efficiency ; $\eta_m = \frac{SP}{RP}$; SP → shaft power

⊙ Overall efficiency, $\eta_o = \frac{SP}{WP}$.

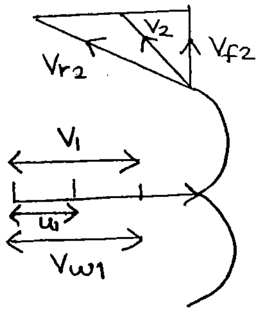
$$\boxed{\eta_o = \eta_h \times \eta_m}$$

$$= \frac{RP}{WP} \times \frac{SP}{RP} = \frac{SP}{WP}.$$

→ Pelton Wheel.

Turbine
- Energy — Impulse (KE only) / Reaction (KE & pressure)
- Speed
- Head.

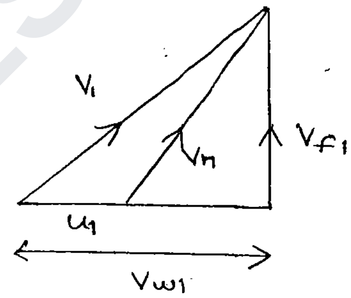
Pelton wheel is the low speed, high head tangential impulse turbine.

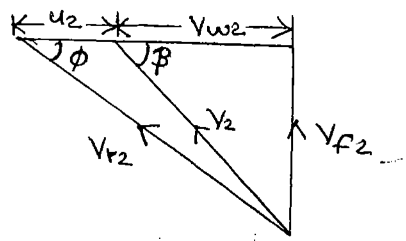


$\alpha = 0$

$u_1 = u_2 = u$

(Hemispherical bucket)





$$\frac{WD}{S} = \rho a V_1 (V_{w1} u_1 + V_{w2} u_2).$$

$$u_1 = u_2 = u.$$

$$= \rho a V_1 (V_{w1} + V_{w2}) u.$$

$$V_{w1} = V_1$$

$$V_{w2} = V_{r2} \cos\phi - u_2$$

If blade is smooth, then,

$$V_{r2} = V_{r1} = V_1 - u.$$

$$V_{w2} = (V_1 - u)\cos\phi - u.$$

$$\frac{WD}{S} = \rho a V_1 (V_1 + (V_1 - u)\cos\phi - u).u$$

$$= \rho a V_1 [(1+\cos\phi) V_1 - u(1+\cos\phi)] u.$$

$$\boxed{\frac{WD}{S} = \rho a V_1 (V_1 - u)(1+\cos\phi) u}$$

If blade is having friction,

$$V_{r2} = k \times V_{r1} \quad ; \quad k \rightarrow \text{blade friction coeffecient}$$

$$\frac{WD}{S} = \rho a V_1 (V_1 - u)(1 + k\cos\phi)\, u.$$

$$\eta = \frac{WD/s}{KE/s} = \frac{\rho a V_1 (V_1 - u)(1+\cos\phi)\, u}{\frac{1}{2}\rho a V_1^3.}$$

For max η, $\frac{\partial \eta}{du} = 0$;

$$(V_1 - u)1 + u(-1) = 0.$$

$$V_1 = 2u$$

$$\therefore \boxed{u = \frac{V_1}{2}}$$

For maximum efficiency, blade speed is equal to half of jet velocity.

1. $V_1 = C_v \sqrt{2gH}$

2. Blade speed ratio, $\phi = \frac{u}{\sqrt{2gH}}.$

3. Jet ratio, $m = D/d$; $D \rightarrow$ diameter of wheel
   $d \rightarrow$ diameter of jet.

4. No. of buckets, $z = 15 + \frac{D}{2d}.$

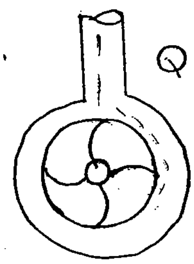
$$= 15 + 0.5\, m.$$

→ Radial Flow Turbines

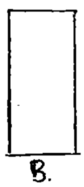
a) Inward Flow ($\beta = 90°$)

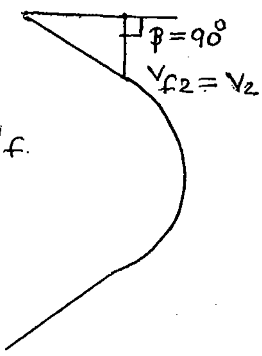
b) Outward Flow.

Radially enters
&
Axially leaves.



$Q = \pi D \times B \times V_f.$

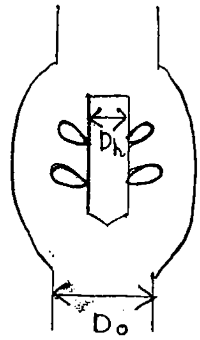




→ Axial Flow Turbines

a) Propeller — blades fixed to the hub.

b) Kaplan — adjustable vanes to the hub.



$$Q = \frac{\pi}{4}\left(D_o^2 - D_h^2\right) V_f.$$

P 267.

1. $T = 15915\ N\text{-}m$

$N = 600$ rpm

$$P = T \times \omega = \frac{15915 \times 2\pi \times 600}{60} = \underline{\underline{1000}}\ kW$$

2. $\eta_o = 80\%$; $Q = 50\ m^3/s$; $H = 7.5\ m.$

$WP = \rho g Q H.$

$= 1000 \times 10 \times 50 \times 7.5.$

$SP = \eta_o \times WP.$

$= 0.8 \times 1000 \times 10 \times 50 \times 7.5 = 3000\ kW$

$= 4000\ HP.$

5. $$\phi = \frac{u}{\sqrt{2gH}}.$$

$V_1 = C_v \sqrt{2gH}.$

If $C_v = 1$, $V_1 = \sqrt{2gH}$; $V_1 \rightarrow$ actual velocity.

$$\phi = \frac{\text{bucket speed (or) blade speed}}{\text{theoretical velocity of jet}}$$

6. $$\phi = \frac{u}{\sqrt{2gH}}.$$

$u = 0.46\sqrt{2g \times 400} = 40.74\ m/s$

$$r\omega = 40.744.$$

$$r \times \frac{2\pi N}{60} = 40.744$$

$$N = \frac{40.744 \times 60}{2.25 \times \pi} = \underline{\underline{345.84}} \text{ rpm}$$

→ Specific Speed (N_s)

It is the speed of the turbine which is identical in shape, blade angles, geometrical dimensions, gate openings etc with actual turbine but it will work under unit head and develops unit power

$$N_s = \frac{N\sqrt{P}}{H^{5/4}} \quad ; \; P \text{ in kW}$$

$$P = \frac{\rho g Q H}{1000}$$

$$P \propto QH$$

$$Q \propto D^2 \times V.$$

$$V \propto \sqrt{H}$$

$$u \propto V \propto \sqrt{H}$$

$$u \propto DN \quad (\because u = \pi DN/60)$$

$$D \propto \frac{u}{N} \propto \frac{\sqrt{H}}{N}$$

$$D^2 \propto \frac{H}{N^2}$$

$$Q \propto \frac{H}{N^2} \cdot \sqrt{H} \propto \frac{H^{3/2}}{N^2}$$

$$P \propto QH \propto \frac{H^{3/2}}{N^2} \times H.$$

$$\boxed{P \propto \frac{H^{5/2}}{N^2}}$$

$$P = \frac{K\,H^{5/2}}{N^2}$$

If $P=1$ & $H=1$, then $N=N_s$

$$1 = \frac{K\times 1}{N_s^2}$$

$$\Rightarrow N_s^2 = K$$

$$\therefore P = \frac{N_s^2 \times H^{5/2}}{N^2}$$

$$\therefore N_s = \frac{N\sqrt{P}}{H^{5/4}}.$$

N_s	Type of Turbine
8 to 30	Pelton wheel with single jet
30 to 55	Pelton wheel with two or more jets.
55 to 250	Francis Turbine.
250 to 880	Propeller or Kaplan.

3. $P = 8.1$ MW $= 8.1\times10^3$.

$H = 81$ m, $N = 540$.

$$N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{540\sqrt{8100}}{81^{5/4}} = \frac{540\times90}{243} = \underline{\underline{200}}$$

$55 < N_s < 250 \Rightarrow$ Francis Turbine.

4 Scale ratio $= \frac{D_m}{D_p} = \frac{1}{2} = \frac{D_2}{D_1}$; $D_m \rightarrow$ diameter of model, $D_p \rightarrow$ diameter of prototype

$H_1 = 20$ m, $N_1 = 500$ rpm. $\frac{DN}{\sqrt{H}} = C$.

$H_2 = 20$ m, $N_2 =$

$$\frac{D_1N_1}{\sqrt{H_1}} = \frac{D_2N_2}{\sqrt{H_2}} \Rightarrow \frac{500}{N_2} = \frac{D_2}{D_1} = \frac{1}{2} \Rightarrow \underline{\underline{N_2 = 1000}}$$

→ Unit Quantities.

To compare the performance of turbines, the head'll be reduced to unit quantity and the corresponding speed, discharge, power are called unit quantities for the same size of turbine.

① Unit Speed, N_u.

$$u \propto N \propto \sqrt{H} \qquad \left(u = \frac{\pi D N}{60}\right).$$

$$N \propto \sqrt{H}.$$

$$N = k\sqrt{H}.$$

when $H = 1$, $N = N_u$.

$$N_u = k.$$

$$\boxed{N_u = \frac{N}{\sqrt{H}}}$$

② Unit Discharge, Q_u.

$$Q \propto D^2 \times V$$

$$Q \propto V \propto \sqrt{H}.$$

$$Q = k\sqrt{H}.$$

If $H = 1\,m$, $Q = Q_u$

$$\boxed{Q_u = \frac{Q}{\sqrt{H}}}$$

③ Unit Power (P_u)

$$P = \rho g Q H.$$

$$P \propto QH$$

$$P \propto \sqrt{H}.\,H \propto H^{3/2}$$

$$P = k\,H^{3/2}$$

If $H = 1\,m$, $P = P_u$

$$\boxed{P_u = \frac{P}{H^{3/2}}}$$

10. BOUNDARY LAYER THEORY

Whenever a real fluid flows past the solid object, some of the fluid molecules will adhore to the surface of the solid and they will possess the velocity as that of the solid. This is known as 'no-slip condition'. If the solid object is stationery, then velocity of fluid particles which are on th surface have zero velocity. These molecules will retard the fluid molecules which are on the another layer. and therefor the velocity gradients will takes place in a narrow thin layer in the transverse direction and is known as the 'Boundary Layer.' In the boundary layer, the velocity will vary from zero to the free stream velocity.

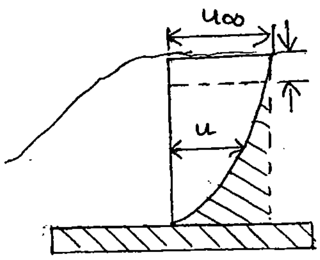
1. Boundary Layer Thickness (δ)

It is the thickness measured in the transverse direction from the surface in which the velocity will vary from zero to the 0.99 times of free stream velocity.

2. Displacement Thickness (δ^*)

It is the distance by which the solid object has to move in the transverse direction so that there is a compensation for loss of discharge due to the formation of boundary layer.

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{u_\infty}\right) dy$$

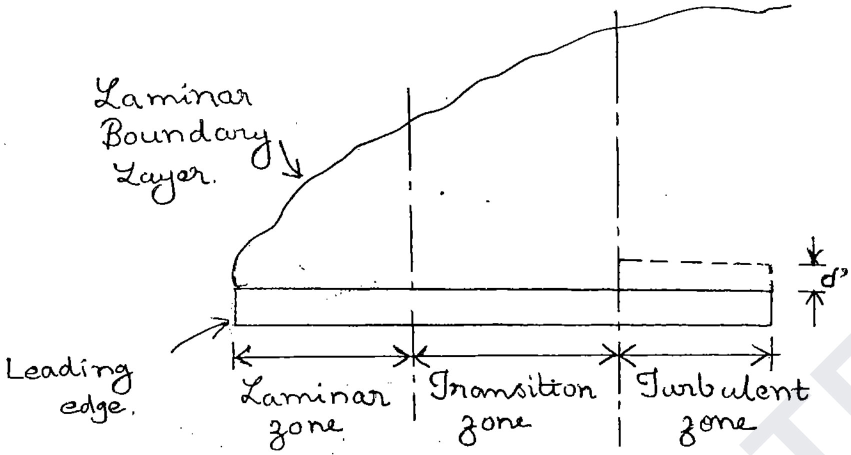


3. Momentum Thickness. (θ)

$$\theta = \int_0^\delta \frac{u}{u_\infty}\left(1 - \frac{u}{u_\infty}\right) dy.$$

4. Energy Thickness (δ^{**})

$$\delta^{**} = \int_0^{\delta}\left(1 - \frac{u^2}{u_\infty^2}\right) dy.$$



$\delta' \rightarrow$ laminar sublayer.

For a plate, $(Re)_{cr} = 5\times10^5$

$$Re = \frac{\rho u_\infty x}{\mu}$$; $x \rightarrow$ distance from leading edge upto which laminar zone exists.

Q For a laminar velocity profile, $\frac{u}{u_\infty} = \frac{3y}{2\delta} - \frac{y^3}{2\delta^3}$, find momentum thickness (θ).

$$\theta = \int_0^{\delta} \frac{u}{u_\infty}\left(1 - \frac{u}{u_\infty}\right) dy.$$

$$= \int_0^{\delta}\left(\frac{3y}{2\delta} - \frac{y^3}{2\delta^3}\right)\left(1 - \frac{3y}{2\delta} + \frac{y^3}{2\delta^3}\right) dy$$

$$= \int_0^{\delta}\left(\frac{3y}{2\delta} - \frac{9y^2}{4\delta^2} + \frac{3}{2}\frac{y^4}{\delta^4} - \frac{y^3}{2\delta^3} - \frac{y^6}{4\delta^6}\right) dy$$

$$= \left(\frac{3}{4}\delta - \frac{9}{12}\delta + \frac{3}{10}\delta - \frac{\delta}{8} - \frac{\delta}{28}\right)$$

$$= \frac{39}{280}\delta$$

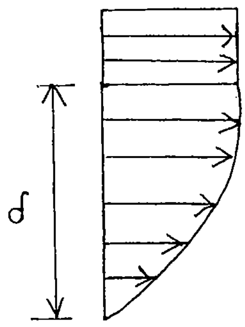
Boundary Conditions:

(i) At $x = 0$, $\delta = 0$.

(ii) At $y = 0$, $u = 0$

(iii) At $y = \delta$, $u = u_\infty$

(iv) At $y = \delta$, $\frac{du}{dy} = 0$



$\frac{du}{dy} = 0$ at $y \geq \delta$

Q. For a laminar velocity profile $\frac{u}{u_\infty} = \frac{3y}{2\delta} - \frac{y^3}{2\delta^3}$, find
(i) Boundary layer thickness.
(ii) shear stress on the surface of plate.

$$\frac{\tau_0}{\rho u_\infty^2} = \frac{\partial \theta}{\partial x}$$

$$\tau_0 = \rho u_\infty^2 \cdot \frac{\partial \theta}{\partial x}.$$

$$= \rho u_\infty^2 \frac{\partial}{\partial x}\left(\frac{39}{280}\delta\right).$$

$$\tau_0 = \rho u_\infty^2 \frac{39}{280} \frac{\partial \delta}{\partial x}. \longrightarrow (1)$$

$$\tau_0 = \mu \frac{du}{dy}.$$

$$= \mu \frac{d}{dy}\left(u_\infty\left(\frac{3y}{2\delta} - \frac{y^3}{2\delta^3}\right)\right)_{y=0}.$$

$$= \mu u_\infty \left(\frac{3}{2\delta} - \frac{3y^2}{2\delta^3}\right)_{y=0}$$

$$\tau_0 = \frac{3\mu u_\infty}{2\delta} \longrightarrow (2)$$

From (1) & (2),

$$\frac{3\mu u_\infty}{2\delta} = \rho u_\infty^2 \frac{\partial}{\partial x}\left(\frac{39}{280}\delta\right).$$

$$\frac{140\,\mu}{13\,\rho u_\infty}\,\partial x = \partial\delta \cdot \delta \Rightarrow \frac{140}{13} \times \frac{\mu}{\rho u_\infty} x = \frac{\delta^2}{2} + c.$$

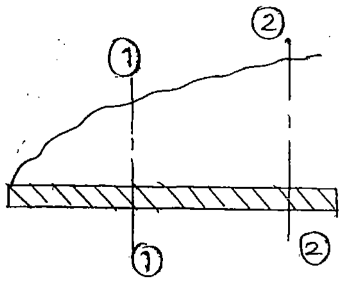
At $x = 0$, $\delta = 0$;

$$\Rightarrow c = 0.$$

$$\delta^2 = \frac{280}{13}\,\frac{\mu x}{\rho u_\infty}$$

$$\delta = 4.64\sqrt{\frac{\mu x}{\rho u_\infty}} = 4.64\sqrt{\frac{x^2}{\frac{\rho u_\infty x}{\mu}}} \Rightarrow \delta = \frac{4.64\,x}{\sqrt{Re_x}}.$$

$$\Rightarrow \boxed{\delta \propto \sqrt{x}.}$$



As distance from leading edge increases ($x\uparrow$), velocity gradient decreases ($\frac{du}{dy}\downarrow$)

Shear stress, $\tau = \mu\left(\frac{du}{dy}\right)$ As $x\uparrow$:

$\therefore\ \tau \propto \frac{1}{\sqrt{x}.}$ & $\delta \propto \sqrt{x}.$

(i) $\tau\downarrow$

(ii) $\delta\uparrow$

Von Korman Momentum equation:

$$\boxed{\frac{\tau_0}{\rho u_\infty^2} = \frac{\partial\theta}{\partial x}.}$$

$\delta \propto \sqrt{x} \Rightarrow \frac{\delta_1}{\sqrt{x_1}} = \frac{\delta_2}{\sqrt{x_2}}$
$\tau_0 \propto \frac{1}{\sqrt{x}.}$

Blasius equation: $\boxed{\delta_{lam} = \frac{5x}{\sqrt{Re_x}}}$

01. $u_\infty = 120$ m/s , $x_{cr} = ?$

$u_\infty = 60$ m/s , $x_{cr} = 125$ mm.

$$(Re)_{cr} = 5\times10^5 = \frac{\rho u_\infty x}{\mu.}$$

u_∞ $\frac{120}{60} = \frac{125}{x_1}$

$$\Rightarrow u_\infty \propto x.$$

$$\frac{(u_\infty)_1}{(u_\infty)_2} = \frac{x_2}{x_1} \Rightarrow \frac{60}{120} = \frac{x_2}{125} \Rightarrow x_2 = \frac{125}{2} = \underline{\underline{62.5\text{ mm}}}$$

2. $\delta = \frac{5x}{\sqrt{Re_x}} \Rightarrow \delta \propto \frac{1}{\sqrt{Re_x}}$

$$\frac{d_1}{d_2} = \frac{\sqrt{Re_2}}{\sqrt{Re_1}}$$

$$= \sqrt{\frac{256}{100}} = \underline{\underline{1.6}}$$

3. $\frac{\delta_1}{\delta_2} = \frac{\sqrt{x_1}}{\sqrt{x_2}}$ $(\because \delta \propto \sqrt{x}$ for laminar boundary lay

$$\frac{2}{3} = \frac{\sqrt{x}}{\sqrt{x+1}}$$

$$\frac{x}{x+1} = \frac{4}{9}.$$

$$x = \underline{\underline{0.8}}\,m$$

5. $\tau_0 \propto \frac{1}{\sqrt{x}}.$

$$\frac{(\tau_0)_1}{(\tau_0)_2} = \frac{\sqrt{x_2}}{\sqrt{x_1}} = \sqrt{\frac{4x_1}{x_1}} = 2$$

6. $\frac{u}{u_m} = 1.5\frac{y}{\delta}$; $\tau = \frac{K\mu u_m}{\delta}$

$$\theta = \int_0^{\delta} \frac{u}{u_m}\left(1 - \frac{u}{u_m}\right) dy$$

$$= \int_0^{\delta} \frac{1.5y}{\delta}\left(1 - \frac{1.5y}{\delta}\right) dy$$

$$= \int_0^{\delta}\left(\frac{1.5y}{\delta} - \frac{2.25y^2}{\delta^2}\right) dy.$$

$$= \frac{1.5}{2}\delta - \frac{2.25}{3}\delta = \frac{-\delta}{12}\ \underline{\underline{0}}$$ { question wrong }

$$\frac{\tau_0}{\rho u_\infty^2} = \frac{\partial\theta}{\partial x}$$

$$\tau_0 = \mu \frac{du}{dy}.$$

$$= \mu \left(\frac{1.5\, y \times u_\infty}{\delta} \right). \qquad \frac{u}{u_\infty} = \frac{1.5\, y}{\delta}.$$

$$\tau_0 = \frac{1.5\, \mu\, u_\infty \cdot 1}{\delta} = \frac{k\, \mu\, u_\infty}{\delta}$$

$$\therefore\ k = \frac{3}{2}$$

07 $$\frac{u}{u_\infty} = \frac{y}{\delta}$$

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{u_\infty}\right) dy.$$

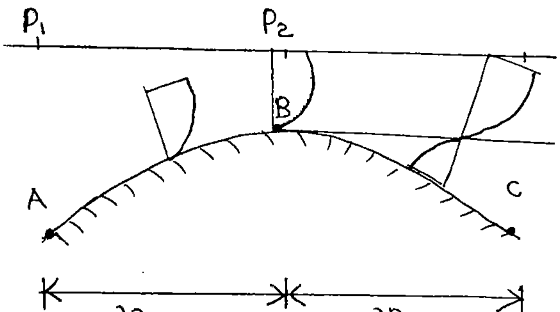
$$= \int_0^\delta \left(1 - \frac{y}{\delta}\right) . dy = \left(y - \frac{y^2}{2\delta}\right)_0^\delta$$

$$= \delta - \frac{\delta}{2} = \frac{\delta}{2}$$

$$\frac{\delta^*}{\delta} = \frac{\delta/2}{\delta} = \frac{1}{2}$$

→ Boundary Layer Seperation.

$P_2 < P_1$



A to B → no seperation.

B to C ⟶ seperation takes place.

B ⟶ point of seperation.

$\frac{\partial p}{\partial x} < 0$ $\quad$ $\frac{\partial p}{\partial x} > 0$ (seperati

$\frac{\partial u}{\partial x} > 0$ $\quad$ $\frac{\partial u}{\partial x} < 0$

09. $$(Re)_L = \frac{\rho u_\infty L}{\mu} = \frac{6\times 1}{0.15\times 10^{-4}} = 400000$$

$$\delta = \frac{4.64\,L}{\sqrt{(Re)_L}} = \underline{\underline{7.33}}\ \text{mm}$$

10. Shear stress at middle of plate:

Velocity profile is linear $\Rightarrow \frac{u}{u_\infty} = k\frac{y}{\delta}$.

$$\tau \propto \frac{1}{\delta}.$$

$$\frac{\tau_1}{\tau_2} = \frac{\delta_2}{\delta_1} = \sqrt{\frac{x_2}{x_1}}$$

$$\frac{\delta_{L/2}}{\delta_L} = \sqrt{\frac{L/2}{L}} = \frac{1}{\sqrt{2}}$$

$$\delta_{L/2} = \frac{7.33}{\sqrt{2}} = 5.183\ \text{mm}.$$

$\frac{\mu}{\rho} = \nu$

$$\tau_L = \mu\frac{du}{dy} = 0.15\times 10^{-4}\times 1.226\times\frac{6}{1} = 1.1034\times 10^{-4}.$$

$$\frac{\tau_{L/2}}{\tau_L} = \frac{\partial_L}{\partial_{L/2}}$$

$$\tau_{L/2} = 1.034\times 10^{-4}\times\frac{7.33}{5.183} = 1.56\times 10^{-4}$$

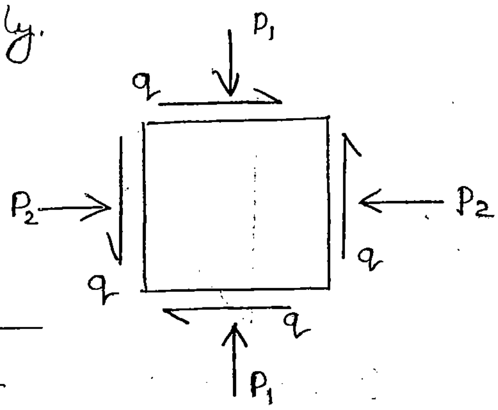
OPEN CHANNEL FLOW

→ Open Channel Flow vs. Pipe Flow.

Open Channel Flow	Pipe Flow
1. Open to atmosphere (pressure is zero)	1. Pressures other than zero (may be +ve or −ve)
2. Flow of open channel is gravitational., ie, from high level to low level	2. Pipe flow occurs from high total head to low total head. It may be from low level to high level or high level to low level
3. Water surface is the HGL. TEL of open channel is above HGL, with an ordinate of corresponding velocity head.	3. In case of pipes, HGL may be above the pipe line or below the pipe line. Eg: syphon.
4. Velocity distribution is logarithmic.	4. Velocity distribution may be parabolic (laminar) or almost rectangular (turbulent)
5. Shapes can be semicircular, trapezoidal, rectangular	5. Usually circular c/s

⊙ Atmospheric pressure is uniform on the free surface. According to Pascal's Law, pressure is uniform in all directions at a point in absence of shear stress only.

∴ if a stress element is as shown,



for uniform pressure,

$$P_1 = P_2 = P$$

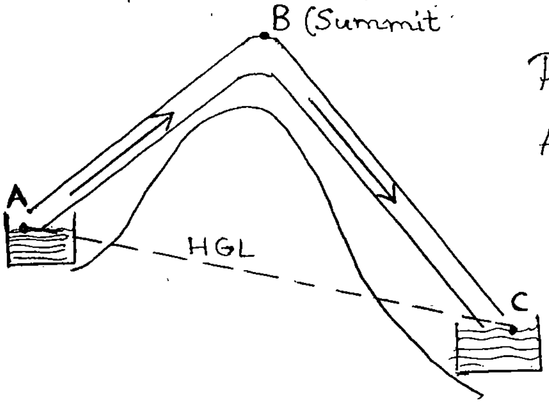
$$q = 0.$$

Radius of Mohr circle, $r = \sqrt{\left(\frac{P_1 - P_2}{2}\right)^2 + q^2}$

$$= 0.$$

∴ the Mohr Circle for uniform pressure at a point is a point on the normal stress axis.

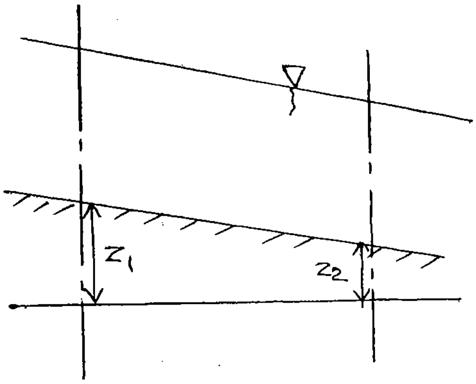
A free surface is subjected to uniform atmospheric pressure.
Uniform pressure is possible only if shear stress is zero.



Pressure inside the syphon from $A \rightarrow B$ is negative.

and from $B \rightarrow A$, pressure is positi

$\Rightarrow$ For a pipe flow, pressures other than zero.

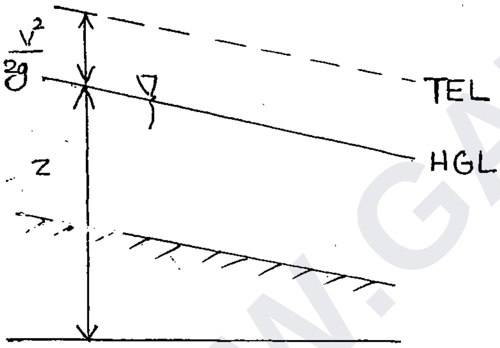


$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h$$

For constant depth of flow, $V_1 = V_2$.

$$P_1 = P_2 = P_{atm}$$

$\therefore$ $\underline{\underline{z_1 > z_2}}$ (gravitational flow)

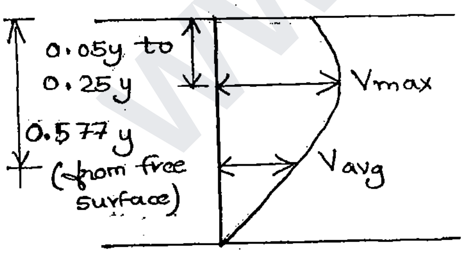


HGL = piezometric line.

$$= \left(\frac{P}{\rho g} + z\right) \text{ line } = z \text{ line.}$$

TEL of open channel flow

$$= \left(\frac{V^2}{2g} + z\right)$$

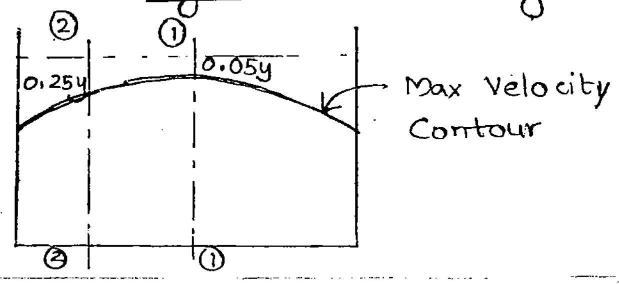


$Q = A \cdot V_{avg}$

$$V_{avg} = \frac{V_{0.2y} + V_{0.8y}}{2}$$ (2 pt. measurement velocities)

Current meter is used to measure velocities in Open channel fl.

Velocity distribution of open channel flow is basically turbulent. All turbulent flows have logarithmic velocity distribution ($\propto y^{1/7}$)



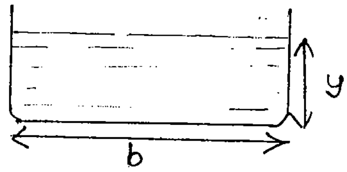
* Reasons for Velocity not Maximum at Free Surface.

(i) Water Currents. (wind forces)

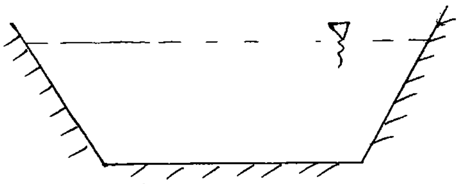
(ii) Surface Tension

(iii) Boundary Friction

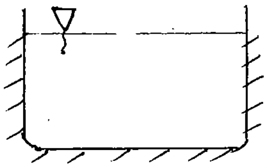
(iv) Aspect ratio. (y/b)



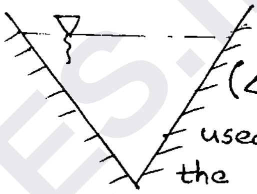
∴ In shallow channels, where aspect ratio is less, max. velocity point lies at less depth from free surface compared to that of deeper channel with more aspect ratio.



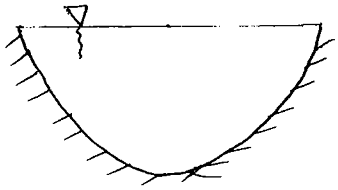
(soils)



(rock)



(Δ^r notch is used to measur the capacity of bore holes)



Semi circular section is most economical – Compared to other sections having sam area, semi-circle has least perimeter.

∴ cost of lining is the least for semi-circula channel sections.

→ Prismatic & Non Prismatic Channels.

If a channel has same slope, same size, same shape all along the length of channel, then its called Prismatic Channels. They are possible with artificial (lined) canals.

Shape, slope, size etc may change at diff. sections for a Non-prismatic channel. Eg:– Natural or unlined canals.

→ Classification of Open Channel Flow

(i) Steady Flow.

If the velocity of flow is constant at a given section at all times, it is called Steady flow.

∴ for steady flow, discharge ($= AV$) is also const. wrt time at a given section. ∴ $\frac{\partial Q}{\partial t} = 0$

In open channels, depth is important. ∴ for steady flow in open channels, $\frac{\partial y}{\partial t} = 0$.

* Unsteady Flow

$$\frac{\partial y}{\partial t} \neq 0.$$

It means the depth of flow, the velocity and discharge change from time to time at a given section.

(ii) Uniform Flow

If same velocity at all sections of an open chan[nel] at given time, it is called uniform flow.

$$\frac{\partial V}{\partial s} = 0$$

∴ depth of flow is constant at all sections at given time for uniform flow. ∴ $\frac{\partial y}{\partial x} = 0$

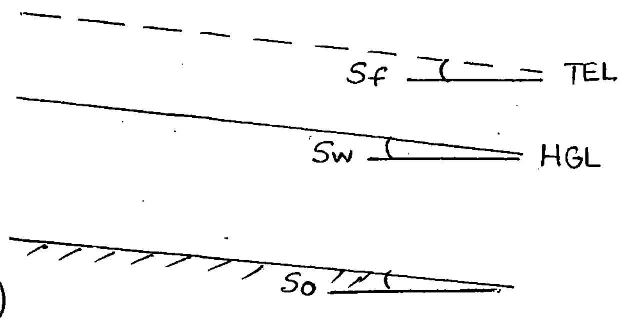
As depth is constant at all sections, water surface is parallel to channel bottom.

In uniform flow TEL, HGL and water surface are parallel to channel bottom.

$S_0 \rightarrow$ channel bed slope wrt horizontal.

$S_w \rightarrow$ water surface slope

$S_f \rightarrow$ Energy slope (water friction line slope)

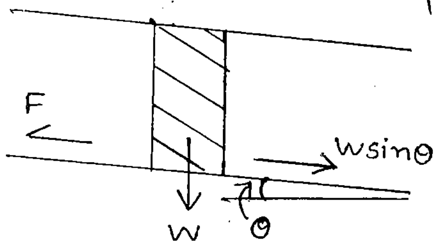


For uniform flow, $S_0 = S_w = S_f$

⊙ Uniform flow is possible in wide rectangular channels.

H ③

→ Condition for Uniform Flow.



For equilibrium,

d/w force along the slope.

= u/w force due to friction along the slope.

$$\boxed{F_S = W\sin\theta}$$

∴ Driving force = Resisting force.

Driving force, F_D = component of weight of fluid resolved in the direction of channel bottom.

Resisting force, F_R = due to boundary friction.

→ Terms related to Uniform flow:

y_n = normal depth.

Depth corresponding to uniform flow = y_n.

S_n = normal slope.

Slope corresponding to uniform flow, = S_n.

Q_n = normal discharge.

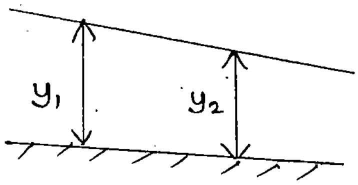
Discharge corresponding to uniform flow = Q_n.

For uniform flow, $S_0 = S_n$

$\therefore\ y_n = y_0$

→ * Non uniform Flow

Depth changes from section to section.



$$\frac{dy}{ds} \neq 0$$

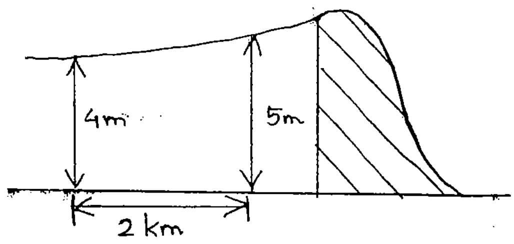
Non Uniform Flow
- Gradually Varied Flow (GVF)
- Rapid / Suddenly Varied Flow (RVF)

Backwater curve is a gradually varied flow.

Head due to velocity of approach – Afflux.

Hydraulic Jump is an example of ~~Spatially~~ rapidly varied flow.

Backwater Curve.



* Spatially Varied Flow (SVF) :

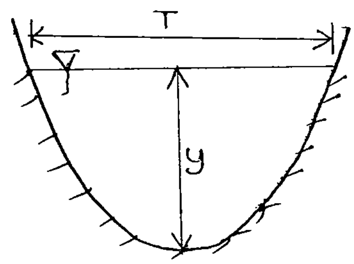
If the flow changes from point to point and time to time, it is called Spatially Varied Flow.

In spatially varied flow, the flow or discharge is either added or subtracted to the main canal system.

Eg:- Spreading of irrigation water in the field.

Taking out water from different outlets in canal system.

→ Channel Geometry :



y = depth of flow

= free surface to deepest point

T = Top width of flow

= width of free surface

P = wetted perimeter

= solid boundary in contact with fluid.

A = wetted area.

= c/s area of flow

R = hydraulic radius = hydraulic mean depth.

$$= \frac{\text{wetted area}}{\text{wetted perimeter}} = \frac{A}{P}$$

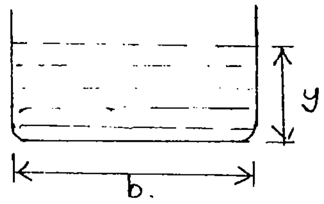
$$D = \text{hydraulic depth} = \frac{A}{T}$$

1. Rectangular Channels

$A = by$

$P = b + 2y$

$R = \frac{A}{P} = \frac{by}{b+2y}$



In case of wide rectangular channels, $y <<< b$. (Eg: rivers)

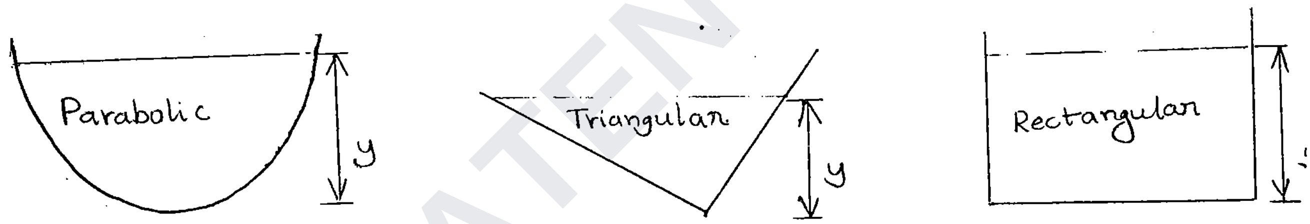
$R \approx \frac{by}{b} \approx y.$

$D = \frac{A}{T} = \frac{by}{b} = y.$

$$\boxed{\begin{aligned} T &= b \\ A &= by \\ R &\approx y \\ D &= y \end{aligned}}$$

⊙ Physical meaning of Hydraulic Depth = Average depth.

It cannot be applied in the case of closing shape of channels.

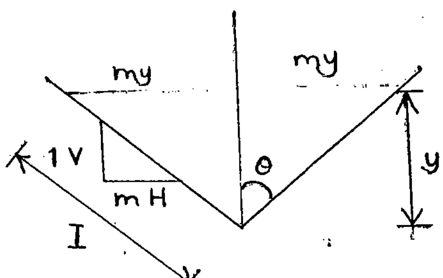


$D = \frac{2}{3} y$ $\qquad$ $D = \frac{y}{2}$ $\qquad$ $D = y.$

2. Triangular Channels

$\theta \rightarrow$ semi vertex angle.

$m \rightarrow$ side slope



$m = \tan\theta.$

Top width, $T = 2my.$

Area, $A = \frac{1}{2} \times 2my \times y$

$= my^2$

$\therefore \quad A = y^2 \tan\theta.$

Wetted perimeter, $P = 2I.$

$= (2\sqrt{1+m^2})y$

$= 2y\sqrt{1+m^2}$

$\therefore \quad P = 2y\sqrt{1+\tan^2\theta}$

$$R = \frac{A}{P} = \frac{my^2}{2y\sqrt{1+m^2}} = \frac{my}{2\sqrt{1+m^2}}$$

$$\text{Hydraulic depth} = \frac{A}{T} = \frac{my^2}{2my} = \frac{y}{2}$$

Hydraulic depth of triangle is independent of side slope.

If right angled triangle, $\theta = 45^0 \Rightarrow m = 1$

Area, $A = y^2$

$P = 2y\sqrt{2}$

$$R = \frac{y^2}{2y\sqrt{2}} = \frac{y}{2\sqrt{2}}$$

$$D = \frac{y}{2}$$

$T = 2my$

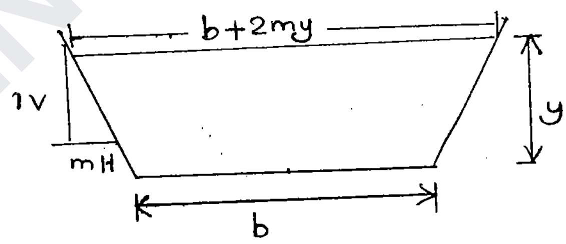
$A = my^2 = y^2\tan\theta$

$P = 2y\sqrt{1+m^2}$

$R = \dfrac{my}{2\sqrt{1+m^2}}$

$D = \dfrac{y}{2}$

3. Trapezoidal Channels.



$T = b + 2my$

$A = by + my^2$

$A = (b+my)y$

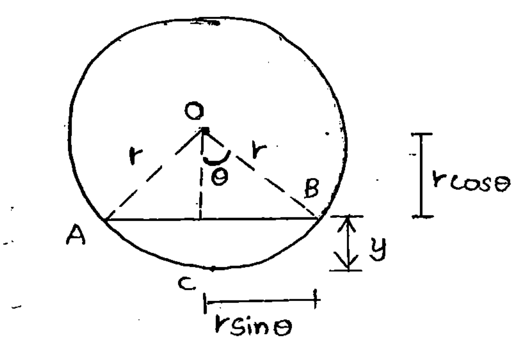
$P = b + 2y\sqrt{1+m^2}$

$$R = \frac{A}{P} = \frac{(b+my)y}{b+2y\sqrt{1+m^2}}$$

$$D = \frac{A}{T} = \frac{(b+my)y}{b+2my}$$

9th Sept,
TUESDAY

4. Circular Channels



$T = 2r\sin\theta$

A = sector area OACB − Area of Δ^{le} OAB

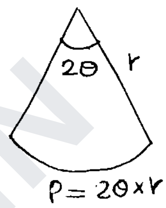
For $2\pi \rightarrow \pi r^2$

$2\theta \rightarrow r^2\theta$

$$\therefore \quad A = r^2\theta - \frac{1}{2} \times 2r\sin\theta \times r\cos\theta$$

$$= r^2\left(\theta - \frac{\sin 2\theta}{2}\right) = r^2\left(\frac{2\theta - \sin 2\theta}{2}\right)$$

$$= \frac{d^2}{8}\left(2\theta - \sin 2\theta\right)$$



$$P = 2r\theta = d\theta$$

$$R = \frac{A}{P} = \frac{d^2(2\theta - \sin 2\theta)}{8 \cdot d\theta}$$

$$R = \frac{d(2\theta - \sin 2\theta)}{8\theta}$$

$$D = \frac{A}{T} = \frac{d^2(2\theta - \sin 2\theta)}{8 \times 2r\sin\theta} = \frac{d(2\theta - \sin 2\theta)}{8 \cdot \sin\theta}$$

- If the circle is half full, $(2\theta = \pi)$.

Wetted area, $A = \frac{\pi r^2}{2}$

Wetted perimeter, $P = \pi r$

Hydraulic radius, $R = \frac{A}{P} = \frac{r}{2} = \frac{d}{4}$

$$\boxed{\begin{aligned} T &= 2r\sin\theta \\ A &= \frac{d^2}{8}(2\theta - \sin 2\theta) \\ P &= 2r\theta = d\theta \\ R &= \frac{d(2\theta - \sin 2\theta)}{8\theta} \\ D &= \frac{d(2\theta - \sin 2\theta)}{8\sin\theta} \end{aligned}}$$

⊙ Circular channel running full,

$$A = \pi r^2$$
$$P = 2\pi r$$
$$\Rightarrow R = \frac{r}{2} = \frac{d}{4}.$$

$$D = \frac{\pi r^2}{0} = \infty \text{ (not defined)}$$

NOTE :

$$\text{Froude's number} = \sqrt{\frac{\text{Inertia force}}{\text{Gravity force}}} = \frac{V}{\sqrt{gD}}$$

Infact Froude's number and hence hydraulic depth, D are defined for gravitational flow only. Circular channel running full is not gravitational flow, it is pressure flow. Hence D not defined. for circular channel running full.

▪ Perimeter of ellipse $= 2\pi\sqrt{\frac{a^2+b^2}{2}}$

a & b are semi-axis of ellipse.

Q. A circular channel of radius r was hit by a truck. Due to impact, c/s shape changed to an ellipse whose major axis is twice that of minor axis. Calculate the percentage change in hydraulic radius.

$$R_c = \frac{A}{P_c} \quad \& \quad R_e = \frac{A}{P_e}.$$

$$\frac{R_c}{R_e} = \frac{P_e}{P_c} = \frac{2\pi\sqrt{\frac{a^2+4a^2}{2}}}{2\pi r} = \frac{9.934\,a}{2\pi r.}$$ Say $b = 2a$.

Assuming area remains the same before and after impact,

$$\pi ab = \pi r^2$$

$$2\pi a^2 = \pi r^2$$

$$a^2 = \frac{r^2}{2}$$

$$\therefore \frac{R_c}{R_e} = \frac{9.934a}{2\pi \times \sqrt{2}a} = 1.118$$

$$R_e = 0.8944 R_c.$$

% change in hydraulic radius = $\underline{\underline{10.55}}$ % (↓)

* Froude's Number; (Fr)

$Fr < 1$; Sub critical Flow

$Fr = 1$; Critical Flow

$Fr > 1$; Super critical Flow

– For rectangular channel,

$$D = y$$

$$\therefore F_r = \frac{V}{\sqrt{gD}} = \frac{V}{\sqrt{gy}}.$$

⊙ For critical flow,

$$F_r = F_{rc} = 1$$

$$\therefore \frac{V_c}{\sqrt{g y_c}} = 1$$

⊙ For subcritical flow, $Fr < 1$

$$\frac{V}{\sqrt{gy}} < 1.$$

Assume V = constant,

$\therefore y > y_c$, then $Fr < 1$

Flows with <u>greater depth</u> are subcritical.

Assume $y =$ constant,

$\therefore \quad V < V_c \Rightarrow$ tranquil flow

Hence subcritical flows are '<u>Tranquil Flows</u>'.

⊙ For Supercritical flow,

$$Fr > 1$$

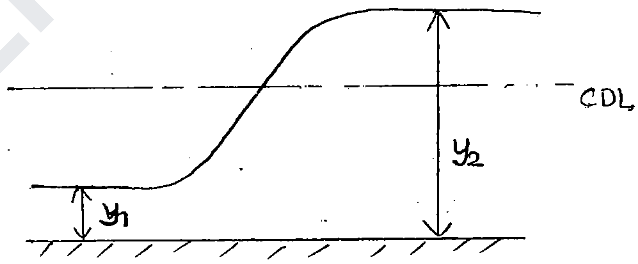
$$y < y_c$$

$$V > V_c$$

Hence supercritical flow occurs at shallow depths. If denominator $y =$ const; $V > V_c$. As $V > V_c$, supercritical flows are also called '<u>Torrential</u> flows'.

⊙ Hydraulic Jump occurs if supercritical flows meets subcritical flow.

Hydraulic ~~depth~~ jump. occurs if a flow whose depth less than critical depth meets another flow whose depth more than critical depth.



— For triangular channel

$$Fr = \frac{V}{\sqrt{gD}}$$

$$= \frac{V\sqrt{2}}{\sqrt{gy}} \quad ; \quad D = \frac{y}{2}$$

— For parabolic channel,

$$D = \frac{2y}{3}$$

$$Fr = \frac{V\sqrt{3}}{\sqrt{2gy}}$$

– For Circular Channel,

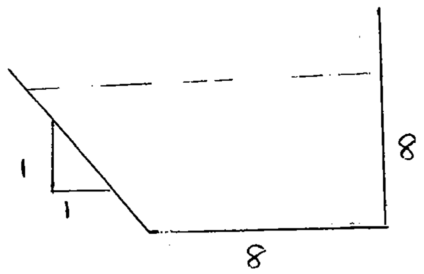
(i) Running half

$$Fr = \frac{V\sqrt{4}}{\sqrt{\pi r g}} \qquad D = \frac{\pi r}{4}$$

$$Fr = \frac{2V}{\sqrt{\pi g r}}$$

– For trapezoidal channel,

$$Fr = \frac{V}{\sqrt{gD}} \quad ; \quad D = \frac{A}{T}$$

Q. 

$Q = 10\ m^3/s$.

Find whether flow is critical, subcritical or supercritical.

$T = 16$.

$A = 8 \times 8 + \frac{1}{2} \times 8 \times \cancel{16}\,8$

$= \cancel{128}\ 96$

$D = \frac{96\ \cancel{128}}{16} = \underline{\underline{6}}$

$V = \frac{Q}{A} = \frac{10}{\cancel{128}\,96} = 0.1042\ m/s$

$Fr = \frac{V}{\sqrt{gD}} = \frac{0.1042}{\sqrt{6g}} = 0.0136 < 1$

∴ the flow is subcritical.

→ Reynold's Number (Re)

$$Re = \frac{\text{Inertia force}}{\text{Viscous force}}$$

$$Re = \frac{\rho V L}{\mu}$$

In case of open channels, L = hydraulic radius.

$$\therefore \quad Re = \frac{\rho V R}{\mu}.$$

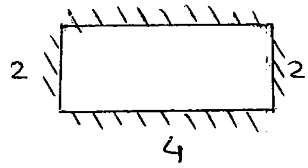
For circular pipe, $Re = \dfrac{\rho V d}{\mu}$ $\qquad (d = 4R)$

For non-circular pipe, $Re = \dfrac{\rho V 4R}{\mu}.$

Illy, head loss due to friction in turbulent flow of non-circular pipes, $h_f = \dfrac{fLV^2}{2gd} = \dfrac{fLV^2}{2g(4R)}$

Q. A gas of viscosity, μ, density ρ flows through a rectangular duct of 4m x 2m. Its Re is ...? Assume velocity

$$Re = \frac{\rho V d}{\mu} = \frac{\rho V (4R)}{\mu}.$$



$$R = \frac{A}{P} = \frac{4 \times 2}{2(4+2)} = \frac{2}{3}.$$

$$Re = \frac{\rho V \times \frac{4 \times 2}{3}}{\mu} = \frac{8 \rho V}{3 \mu}$$

→ Classification of Open Channels based on Re

$Re < 500$; Laminar

$500 < Re < 2000$; Transitional.

$Re > 2000$; Turbulent

→ Boundary Shear Stress of Open Channels.

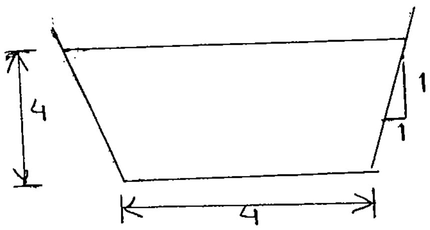
$$\tau_{avg} = \tau_{mean} = \gamma R S_0$$

where $\gamma \rightarrow$ wt. density

$R \rightarrow$ hydraulic radius

$S_0 \rightarrow$ channel bottom slope

Q.



$A = (b + my)y = (4 + 1 \times 4)4 = 32 \ m^2$

$P = b + 2y\sqrt{1+m^2} = 4 + 2 \times 4\sqrt{1+1}$

$= 15.31$

$$R = \frac{A}{P} = \frac{32}{15.31} = 2.08$$

$$\tau_{avg} = \gamma R S_0 = 9810 \times 2.08 \times \frac{1}{1000} = \underline{\underline{20.5}} \ N/m^2$$

'Preston Tube' is the device used for measuring boundary shear stress.

→ Uniform Flow Formulae :

(i) Chezy's formula

$$V = C\sqrt{RS}$$

$C_{lined} \approx 75$

$C_{unlined} \approx 40$

where $C \rightarrow$ Chezy's constant

'C' represents smoothness.

$V_{lined} > V_{unlined}$. Lined canals have less friction, more smooth compared to unlined canals. As $V \propto C$, C represents smoothness in fact.

Q. Arrange in increasing order of C
1. Sand 2. Boulder 3. Gravel 4. Silt

$2 < 3 < 1 < 4$

$C_{sea} < C_{river}$

* Dimensional formula for 'c':

$$C = \frac{V}{\sqrt{SR}} = \frac{L}{T\sqrt{L}} = \frac{\sqrt{L}}{T}$$

NOTE: Non-dimensional formula for $c = \frac{C}{\sqrt{g}}$

(ii) Manning's Formula.

$$V = \frac{1}{n} R^{2/3} S^{1/2};$$

$n \rightarrow$ Manning's rugosity coeffecient.

$$V \propto \frac{1}{n}$$

Manning's n represents roughness.

$$n_{lined} < n_{unlined}$$

$$n = \frac{R^{2/3} S^{1/2}}{V} = \frac{L^{2/3}}{L/T} = \frac{T}{L^{1/3}}$$

Dimensional formula : $[M^0 L^{-1/3} T]$

* Merits of Manning's Formula :

a) It is a simple formula.

b) As most of the channels are turbulent, it has experimental support compared to other formulae. $\therefore$ it is more accurate.

$$Q = AV$$
$$\propto y \cdot R^{2/3} \text{ (mannings formula).}$$
$$Q \propto y \cdot y^{2/3} \propto y^{5/3}$$
$$\Rightarrow \underline{\underline{Q \propto y^{5/3}}}$$
$$Q \propto y\sqrt{R} \text{ (Chezy's formula).}$$
$$\propto y\sqrt{y}$$
$$\underline{\underline{Q \propto y^{3/2}}}$$

Q. If the depth of a channel increases by 20%, % increase in discharge.

$$\frac{Q_1}{Q_2} \cancel{\propto} = \frac{y_1^{5/3}}{y_2^{5/3}}.$$

$$\frac{Q_1}{1.2\,\cancel{Q_1}} = \frac{\cancel{y_1}^{5/3}}{\cancel{y_2}^{5/3}} \qquad \frac{Q_2}{Q_1} = \left(\frac{y_2}{y_1}\right)^{5/3} = (1.2)^{5/3}$$

$$y_2^{5/3} = \cancel{0.833}\ \cancel{5}\, y_1^{5/3}. \qquad Q_2 = 1.35\,Q_1 \Rightarrow \underline{\underline{35\%}}$$

* Relation b/w Chezy's 'C' & Manning's 'n':

$$V = C\sqrt{RS} = \frac{1}{n} R^{2/3} S^{1/2}.$$

$$\boxed{C = \frac{1}{n} R^{1/6}}$$

(iii) Stickler's Formula.

$$\boxed{n = \frac{d_{50}^{1/6}}{21.1}}$$

where d_{50} → size below which 50% of particles are fines (m)

Q. In calculating value of n, instead of taking d_{50} in m, by mistake it is considered as mm. The error in the value of n is.

$$n = \frac{(d_{50})^{1/6}}{21.1}$$

Error $\propto (1000)^{1/6}$

$$\frac{n_1}{n_2} = \frac{1^{1/6}}{1000^{1/6}}$$

$$n_2 = 3.16\ n_1$$

$$\%\ \text{error} = \left(\frac{3.16 - 1}{1}\right) \times 100 = \underline{\underline{216\ \%}}$$

(iv) Meyerhoff's Modified Formula.

$$n = \frac{(D_{90})^{1/6}}{26}$$

$D_{90} \rightarrow$ size below which 90% particles are finer.

→ Conveyance of a Channel.

$$Q = K\sqrt{S}$$

Whatever may be the formula, whether Chezy's or Manning's, $Q = K\sqrt{S}$

where K is called Conveyence of channel.

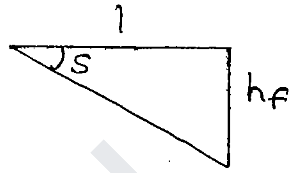
According to Manning's formula,

$$Q = A \cdot \frac{1}{n} R^{2/3} \sqrt{S}$$

$$K = \frac{AR^{2/3}}{n} = AC\sqrt{R}$$

* Relation b/w Chezy's c, Manning's n. & Darcy's f.

$$h_f = \frac{f l v^2}{2gd} = \frac{f l v^2}{2g.(4R)}$$



$$V = \sqrt{\frac{8g}{f}} \sqrt{R.\frac{h_f}{l}}$$

$$= \sqrt{\frac{8g}{f}} \sqrt{RS} = C\sqrt{RS}$$

$$\therefore \boxed{C = \sqrt{\frac{8g}{f}} = \frac{1}{n} R^{1/6}}$$

→ Hydraulically Efficient Channels

From a common man's perspective, a channel is said to be hydraulically efficient if it gives max. discharg

$$Q = AV$$

$$Q \propto V \quad ; \text{if } A \text{ is constant.}$$

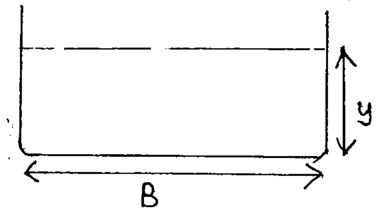
$$\propto C\sqrt{RS}$$

$$Q \propto \sqrt{R} \; ; \quad \text{if } c \text{ is constant \& } S \text{ is constar}$$

$$Q \propto \frac{A}{P} \propto \frac{1}{P}$$

If channel is said to be hydraulically efficient, if it has max. discharge and min. wetted perimeter. for a given area, nature of boundary and longitudinal slope.

(i) Rectangular Channel.



$A = By$

$P = B + 2y = \frac{A}{y} + 2y.$

For min. wetted perimeter,

$$\frac{dp}{dy} = 0$$

$$-\frac{A}{y^2} + 2 = 0.$$

$$A = 2y^2$$

ie $By = 2y^2 \Rightarrow \boxed{B = 2y}$

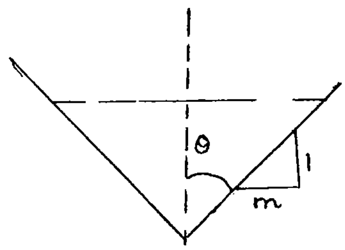
Q. Width of a rectangular channel 5m. Its hydraulic radius

$b = 5 = 2y_e.$

$y_e = 2.5$

$R = 0.5\, y_e = \underline{\underline{1.25\ m}}.$

(ii) Triangular Section

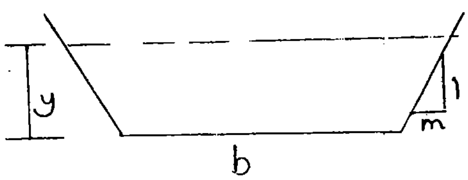


$\theta = 45^\circ$

Side slope, $m = 1$

∴ A right angle triangled is the efficient triangular section $\left(R = \frac{y}{2\sqrt{2}}\right)$

(iii) Trapezoidal Section



Condition 1: Assume m = const
y = variable

$A = (b + my)y$

$P = b + 2y\sqrt{1+m^2}$

$$\frac{dP}{dy} = 0.$$

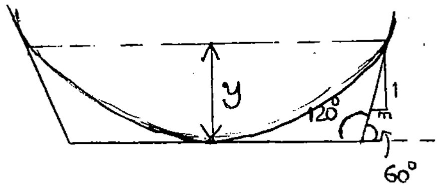
$$\frac{d}{dy}\left(\frac{A}{y} - my + 2y\sqrt{1+m^2}\right) = 0$$

$$\boxed{\begin{aligned} b + 2my &= 2y\sqrt{1+m^2} \\ \text{Top width} &= 2 \times \text{inclined side.} \\ R &= \frac{y}{2} \end{aligned}}$$

Condition 2: m (variable)
y (constant).

$$\frac{dP}{dm} = 0.$$

$$m = \frac{1}{\sqrt{3}}.$$



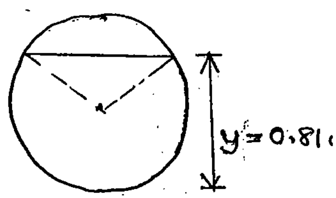
If centre of top width as origin, and depth of flow as radius, if a semi-circle is drawn, it will touch all the 3 sides tangentially; it means an efficient trapezoidal section is half of a regular hexagon.

(iv) Circular Channel Running Partially.

Condition 1: For max. velocity.

$y = 0.81\ d.$

$R = 0.3\ d.$



condition 2: For max Q,

$y = 0.93\ d$

$R = 0.29\ d.$

→ Hydraulic Efficiency vs. Economic Channels

Hydraulic efficiency is related to discharge carrying capacity (to the max) of a given shape of channel.

Different shapes of efficient channels may have same area of c/s. But considering the cost of construction econom shall be decided among various shapes. The perimeter of various shapes are as follows:-

Semi-circle < Trapezium < (Rectangle = Triangle)

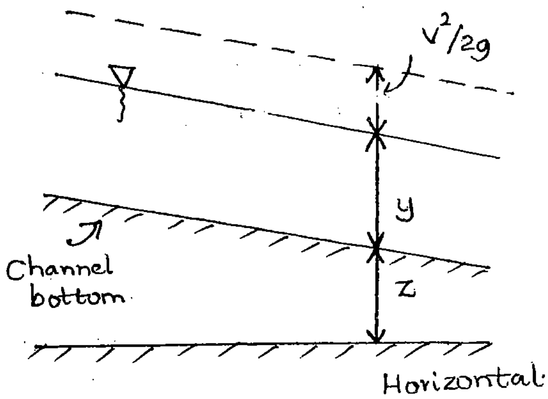
From the above, semi-circular shape is the most ~~efficient~~ economic shape. (satisfying hydraulic efficiency also)

3th Sept,
ATURDAY

→ Specific Energy:

The energy per unit ~~volume~~ weight of water wrt channel bottom as datum is called Specific Energy.

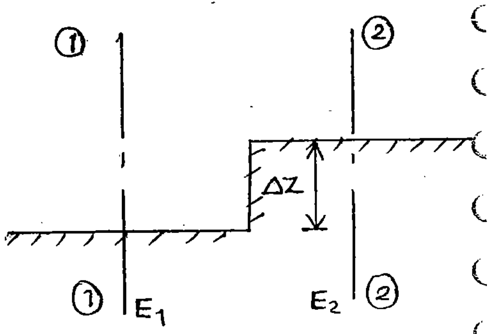
$$E = y + \frac{V^2}{2g}$$



Unit: m.

→ Specific Energy relations b/w Two sections of a Channel

Case 1: Channel bottom rise (or) hump (or) bump in the direction of flow.

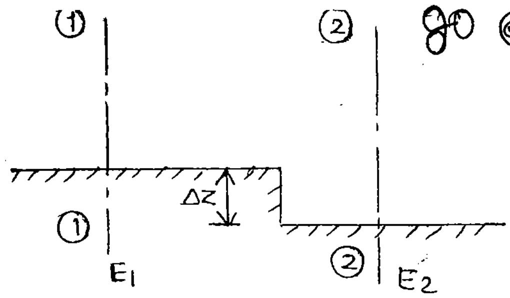


Deeper the channel bottom, more the specific energy.

Hence, $E_1 > E_2$

$$E_1 = E_2 + \Delta Z. \text{ (neglecting } h_f)$$

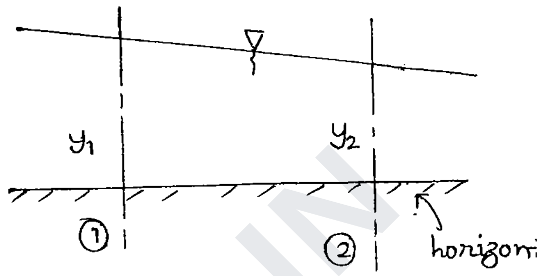
Case 2: Channel bottom drops



$$E_2 > E_1$$

$$E_1 = E_2 - \Delta Z.$$

Case 3: Channel bottom horizontal.



$$E_1 = E_2 \text{ (neglecting } h_f).$$

The two different depths possible for the same specific energy (E) are called '<u>Alternate Depths</u>'.

$$E_1 = E_2 \Rightarrow y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g}.$$

$$y_1\left(1 + \frac{V_1^2}{2gy_1}\right) = y_2\left(1 + \frac{V_2^2}{2gy_2}\right)$$

a) Rectangular Channel :-

$$Fr = \frac{V}{\sqrt{gy}} \; ; \; Fr^2 = \frac{V^2}{gy}.$$

$$\therefore \; y_1\left(1 + \frac{Fr_1^2}{2}\right) = y_2\left(1 + \frac{Fr_2^2}{2}\right)$$

NOTE :

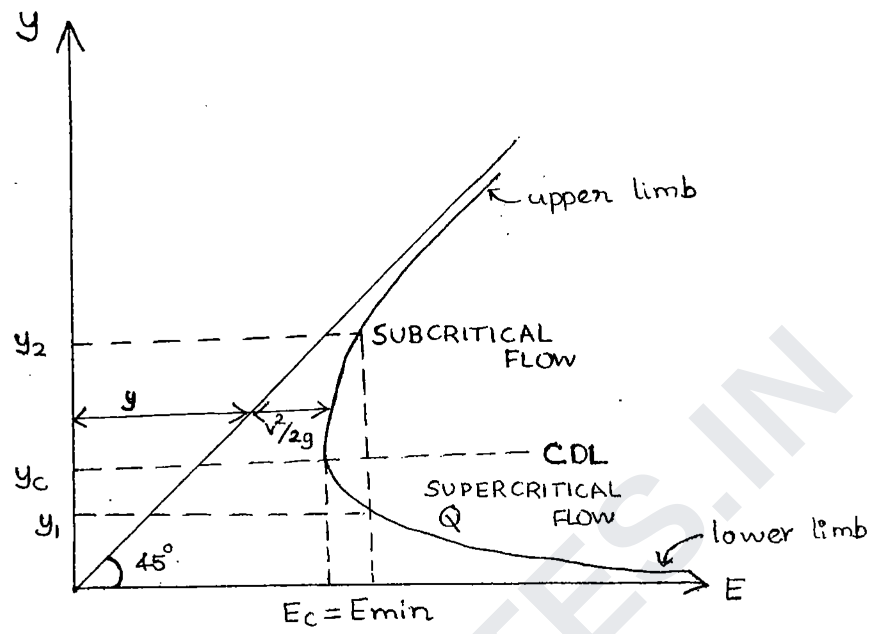
This equation shall be used if alternate depths are mentione

b) Triangular channel.

$$Fr = \frac{V\sqrt{2}}{\sqrt{gy}} \; ; \; Fr^2 = \frac{2V^2}{gy}.$$

$$\therefore \; y_1\left(1 + \frac{Fr_1^2}{4}\right) = y_2\left(1 + \frac{Fr_2^2}{4}\right)$$

→ Specific Energy Curve.



– Specific energy curve is drawn for a particular discharge. If discharge changes, specific energy curve changes.

A river may have number of specific energy curves depending upon the discharge which changes from season to season.

– Critical flow is the flow corresponding to which specific energy is minimum.

– b/w the CDL and the lower limb, $y < y_c$. Hence it is 'super critical region'. Similarly b/w CDL and upper limb, $y > y_c$. Hence it is 'subcritical region'

– For any other E more than Emin (min specific energy), two different depths are possible (y_1 & y_2).

y_1 corresponds to supercritical flow. y_2 corresponds to subcritical flow.

– The two possible depths (y_1 & y_2) corresponding same specific energy are called Alternate depths.

– Upper limb is asymptotical to ~~spec~~ '45° line' or 'potential energy line'

$$E = y + \frac{V^2}{2g}$$

$$= y\left(1 + \frac{V^2}{2gy}\right)$$

$$\frac{E}{y} = 1 + \frac{V^2}{2gy}$$

As $y \to \infty$, $\frac{E}{y} \to 1$

It means upper limb of curve is asymptotical to 45° line (potential energy line.

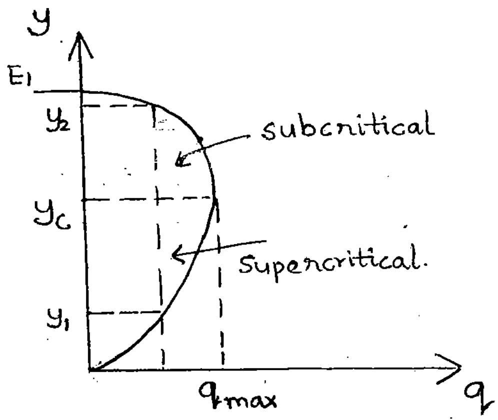
- Lower limb will be asymptotical to x-axis or specific energy axis.

As $y \to 0$, the specific energy curve will try to meet the x or specific energy axis asymptotically.

- From any point on the y-axis, the 'x offset' to the 45° line represents the corresponding potential energy line. Hence the 45° line is also called Potential energy line.

→ Discharge Diagram. (q vs y).

Discharge per 'm' width, $q = \frac{Q}{b}$



Discharge curve is drawn for a given u/s specific energy, E_1.

If u/s specific energy changes, discharge curve also changes.

Corresponding to critical flow, discharge is maximum.

For any other q less than q_{max}, two different depths are possible

→ Specific Force (F)

The force per unit weight of water is called Specific Force, F.

NOTE: Specific force is constant if the channel is horizontal at every section neglecting losses.

$$F = \frac{Q^2}{gA} + A\bar{z}$$

$\frac{Q^2}{gA} \rightarrow$ momentum of flow passing the channel section per unit time per unit weight of fluid

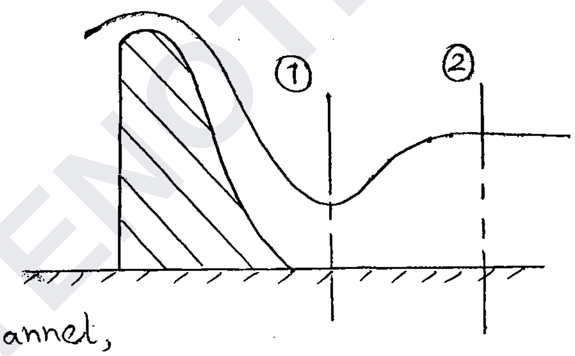
$A\bar{z} \rightarrow$ hydrostatic pressure force per unit weight of fluid.

Any parameter by unit time is called corresponding flux.

Hence $\frac{Q^2}{gA} \rightarrow$ momentum flux per unit weight of fluid.

$\bar{z} \rightarrow$ depth of CG of c/s from free surface.

Specific force concept is used in the analysis of hydraulic jump.



$$F_1 = F_2$$

$$\left(\frac{Q^2}{gA} + A\bar{z}\right)_1 = \left(\frac{Q^2}{gA} + A\bar{z}\right)_2$$

In this eqn, Q, width of channel, either y_1 or y_2 will be known. Hence only one unknown.

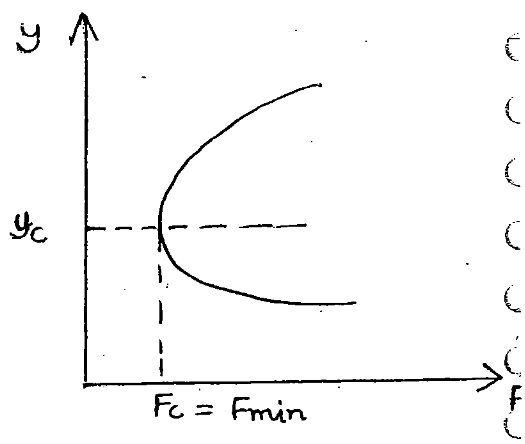
These two possible depths of same specific force are called Conjugate depths or Sequent depths.

– Corresponding to critical flow, specific force is minimum.

For any other specific force,

$F > F_{min}$,

two different depths (y_1 & y_2) are possible called Conjugate or Sequent depths



→ Critical Flow

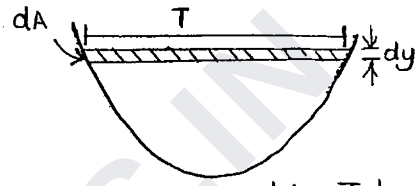
The flow corresponding to which specific energy is minimum is called Critical Flow

$$E = y + \frac{V^2}{2g}$$

$$E = y + \frac{Q^2}{2gA^2}$$

$$E_{min} \Rightarrow \frac{dE}{dy} = 0$$

$$1 + \frac{Q^2}{2g}\left(\frac{-2}{A^3}\right)\frac{dA}{dy} = 0.$$



$dA = T\,dy$.

CONDITION FOR CRITICAL FLOW $\Rightarrow$ $\boxed{\dfrac{Q^2}{g} = \dfrac{A^3}{T}}$

NOTE :

- This equation is valid for any shape of channel for critical flow.

- $$\frac{Q^2}{gA^2} = \frac{A}{T}$$

  $$\frac{V^2}{g} = D$$

  $$\frac{V}{\sqrt{gD}} = 1 = Fr$$

$\therefore$ $Fr = 1$ for critical flow.

- $$\frac{V^2}{g} = D.$$

  $$\frac{V^2}{2g} = \frac{D}{2}$$

For critical flow, velocity head $= \frac{1}{2}$ (hydraulic depth) for any shape of channel.

1. Rectangular Channel

$$\frac{Q^2}{g} = \frac{A^3}{T} = \frac{b^3 y_c^3}{b}$$

$$y_c^3 = \frac{Q^2}{g b^2}$$

$$\therefore \quad y_c = \left(\frac{Q^2}{g b^2}\right)^{1/3}.$$

$$q = \frac{Q}{b} \Rightarrow \boxed{y_c = \left(\frac{q^2}{g}\right)^{1/3}}$$

$$E_c = y_c + \frac{V_c^2}{2g} = y_c + \frac{D}{2}$$

$$= y_c + \frac{y_c}{2}$$

$$\boxed{E_c = \frac{3}{2} y_c}$$

2. Triangular Channel

$$\frac{Q^2}{g} = \frac{A^3}{T}$$

$$\frac{Q^2}{g} = \frac{(my^2)^3}{2my}$$

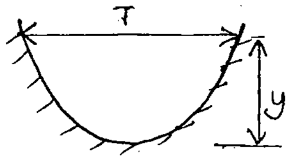
$$\frac{2Q^2}{gm^2} = y_c^5$$

$$\therefore \boxed{y_c = \left(\frac{2Q^2}{gm^2}\right)^{1/5}}$$

$$E_c = y_c + \frac{V^2}{2g} = y_c + \frac{D}{2} = y_c + \frac{y_c/2}{2}$$

$$\boxed{E_c = 1.25\, y_c}$$

3. Parabolic Channel.



$A = \frac{2}{3} T y$

$$\frac{Q^2}{g} = \frac{A^3}{T}$$

$$= \left(\frac{2}{3}\right)^3 \frac{(Ty)^3}{T}$$

$$y_c^{\,3} = \left(\frac{3}{2}\right)^3 \frac{Q^2}{gT^2}$$

$$\therefore \boxed{y_c = \frac{3}{2}\left(\frac{Q^2}{gT^2}\right)^{1/3}}$$

$$E_c = y_c + \frac{V_c^2}{2g} = y_c + \frac{D}{2}$$

$$= y_c + \frac{2}{3}\frac{y_c}{2}$$

$$\boxed{E_c = \frac{4}{3} y_c}$$

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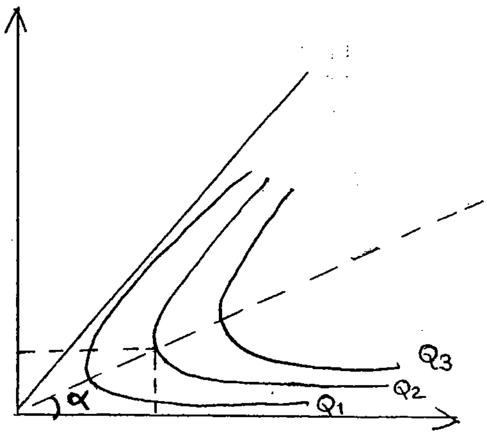
MONDAY

The line joining the origin with minimum specific energy points for different discharges for a given shape of channel is a straight line with an angle α with the horizontal.



$Q_3 > Q_2 > Q_1$.

$$\alpha = \tan^{-1}\left(\frac{y_c}{E_c}\right)$$

$= \tan^{-1}\left(\frac{2}{3}\right) = 33.69^\circ$; for rectangular channel

$= \tan^{-1}\left(\frac{3}{4}\right) = 36.86^\circ$; for parabolic channel

$= \tan^{-1}\left(\frac{4}{5}\right) = 38.65^\circ$; for triangular channel.

→ Sector Factor, K_s

$$K_s = A\sqrt{D}$$

$$\boxed{K_s = A\sqrt{\frac{A}{T}} = \frac{Q_c}{\sqrt{g}}}$$

* Physical importance of K_s.

— Based on the values of section factor, the critical discharges of different channels can be calculated straightaway.

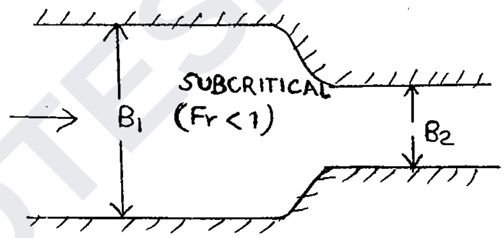
→ Channel Transitions

Case 1 : Channel width reduction.

(i) Approaching flow is sub-critical.

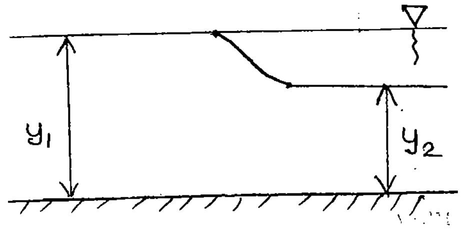
$B_2 < B_1$

$Q_1 = Q_2 = Q$



$q_1 = \dfrac{Q}{B_1}$ & $q_2 = \dfrac{Q}{B_2}$

$q_1 < q_2$



Assume channel bottom as horizontal,

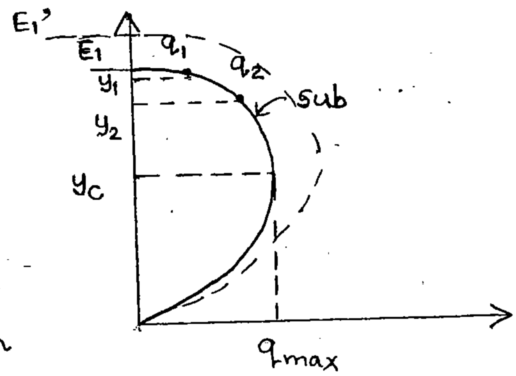
$\therefore \; E_2 = E_1$

NOTE :

As $E_2 = E_1$, specific energy curve (E vs. y) is not useful in understanding the value of y_2. As 'q' changes from section 1 to section 2, discharge diagram (q vs. y) can be used

Referring discharge curve in the sub-critical region, as $q_2 > q_1$

$\Rightarrow y_2 < y_1$

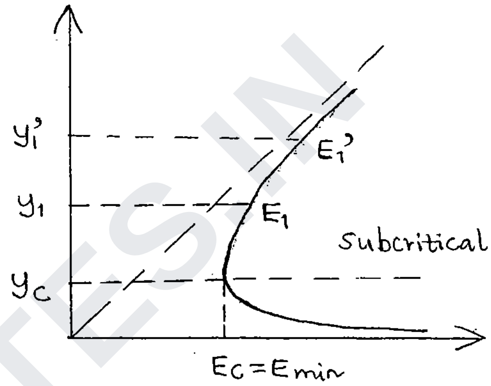


It means the level of water falls or decreases in the throat or constriction part.

Assume width of throat further decreases, then q_2 further increases and hence y_2 further decreases. If $B_2 = B_c$, ie width of throat reduces to a critical value, then $q_2 = q_{max}$ & $y_2 = y_c$.

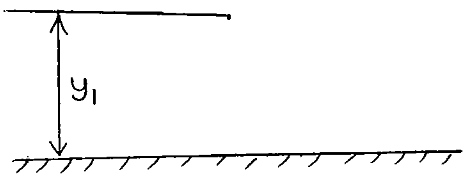
If $B_2 < B_c$, then $q_2 > q_{max}$ which is not possible for given u/s specific energy E_1. In order to allow $B_2 < B_c$, the u/s flow gets disturbed, with increase in specific energy from E_1 to E_1'. To know the wave pattern on the upstream once it gets disturbed, specific energy curve shall be used.

Referring specific energy curve in the sub-critical region, as E_1 increases to E_1', then y_1 increases to y_1', ie the u/s level rises from y_1 to y_1'



(ii) Approaching flow is Supercritical.

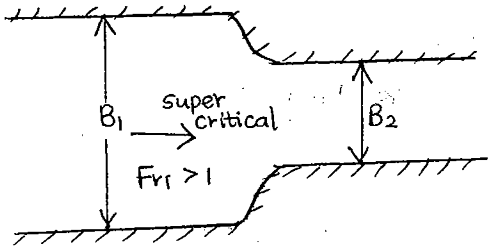
$B_2 < B_1 \Rightarrow q_2 > q_1$



Channel bottom horizontal, $E_2 = E_1$.

$q_2 > q_1$

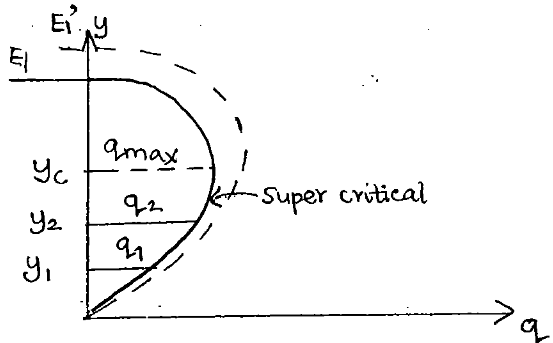
$\Rightarrow y_2 > y_1$



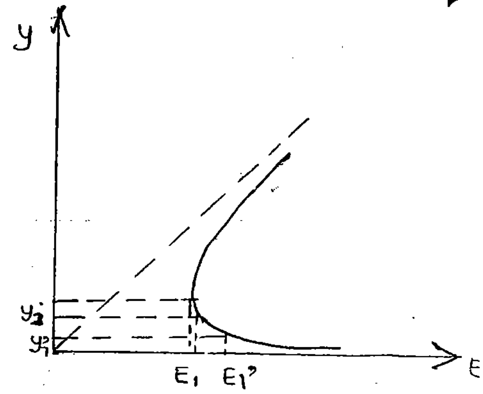
∴ depth of water rises in the contracted portion if approaching flow is supercritical.

If $B_2 = B_c$, critical flow exists in the throat portion, with.

$q_2 = q_{max}$ & $y_2 = y_c$



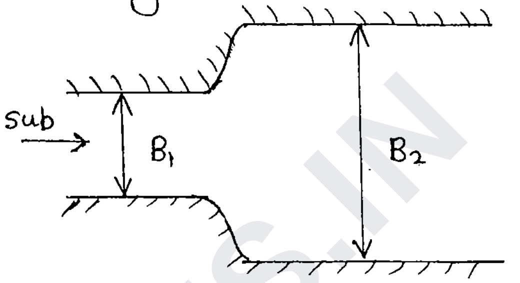
If $B_2 < B_c$, then $q_2 > q_{max}$ which is not possible for a given u/s spec. energy. In order to allow $B_2 < B_c$, the u/s flow gets disturbed with increase in spec. energy from E_1 to E_1'. It means u/s flow gets disturbed



Referring specific curve in super critical region, as E_1 increases to E_1', the u/s level decreases from y_1 to y_1'

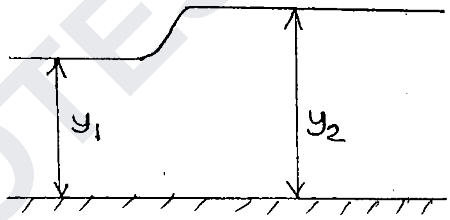
Case 2 : Channel width increasing.

(i) Approaching flow Subcritical.



$B_2 > B_1$

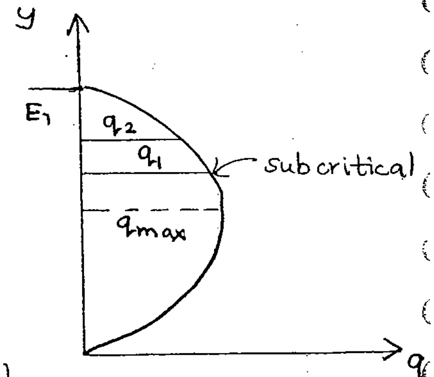
$\Rightarrow q_2 < q_1$



Channel bottom horizontal,

$\therefore E_2 = E_1$

Referring discharge curve in subcritical region,



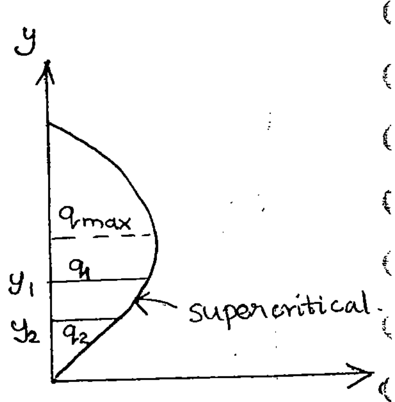
for $q_2 < q_1 \Rightarrow y_2 > y_1$

It means level of water raises in the expanded portion

NOTE :

If $B_2 \to \infty$, $q_2 \to 0$ and hence no flow.

(ii) Approaching flow Supercritical



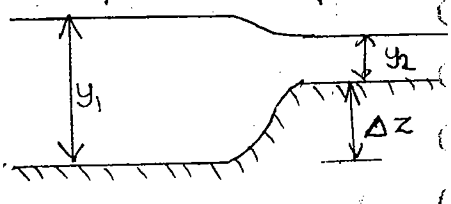
$B_2 > B_1 \Rightarrow q_2 < q_1$

Referring discharge curve in supercritical region, $q_2 < q_1 \Rightarrow y_2 < y_1$

It means level of water falls in the expanded portion

Case 3: Channel bottom rises (or) hump (or) bump.

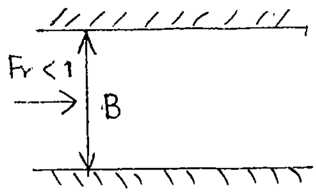
a) Approaching flow subcritical.

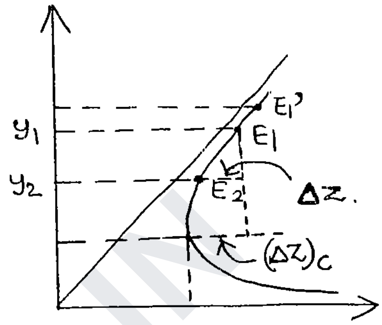


elevation

Assume width of channel is constant.

As width is assumed const, q is const. Hence discharge curve is not useful in analysing y_2. As channel bottom level rises, then $E_2 < E_1$





$$E_1 = E_2 + \Delta Z.$$

Referring specific energy curve in the sub-critical region,

for $E_2 < E_1$, then $y_2 < y_1$. It means the level of water decreases in the hump portion

NOTE: 'Hump' is the friend of 'contraction' in wave pattern

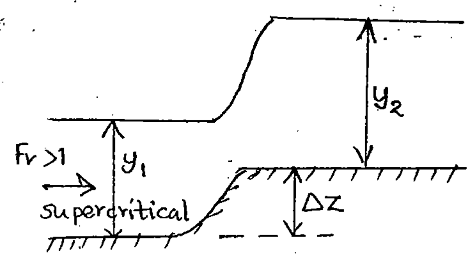
If the hump height further increases, E_2 decreases for a given u/s specific energy. E_1, y_2 further decreases.

If $\Delta Z = (\Delta Z)_{max} = (\Delta Z)_c$ (critical height of hump), then E_2 reduces to minimum or critical specific energy. It means the flow in the hump becomes critical.

$$E_1 = E_c + (\Delta Z)_c$$

If $\Delta Z > (\Delta Z)_{max}$, then E_2 shall be less than E_c which is not possible for a given u/s specific energy. In order to allow ΔZ more than $(\Delta Z)_{max}$, the u/s flow gets disturbed with increase in specific energy from E_1 to E_1'. As seen in the specific energy curve, as E_1 increases to E_1', y_1 increases to y'.

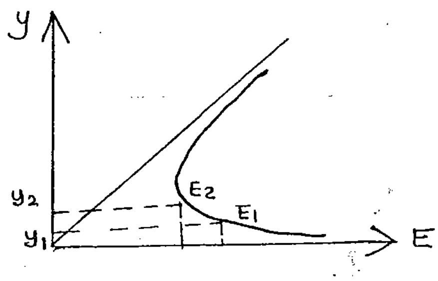
b) Approaching flow Supercritical.



$$E_2 < E_1$$

$$\Rightarrow y_2 > y_1 \text{ (y–E diagram)}$$

Level of water raises in the hump portion.

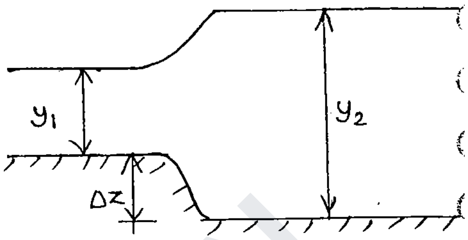


When $\Delta Z > \Delta Z_c$, E_2 shall be less than E_c which is not possible. Hence u/s flow gets disturbed with increase in sp. energy from E_1 to E_1'

Referring supercritical region of specific energy, as E_1 increases to E_1', y_1 decreases to y_1'

Case 4: Channel bottom drops

Channel width is assumed to be constant

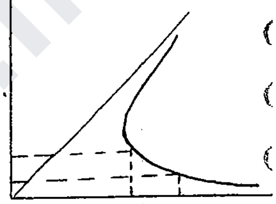


$E_2 > E_1$

(i) Approaching flow Subcritical.

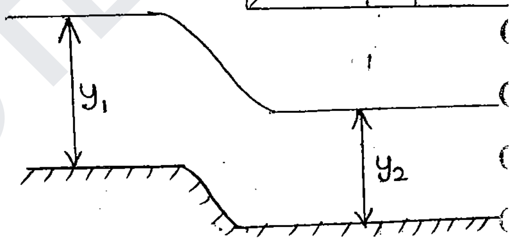
$E_2 > E_1 \Rightarrow y_2 > y_1$.

There is no limiting condition for depth of drop.



(ii) Approaching flow Supercritical.

$E_2 > E_1 \Rightarrow y_2 < y_1$



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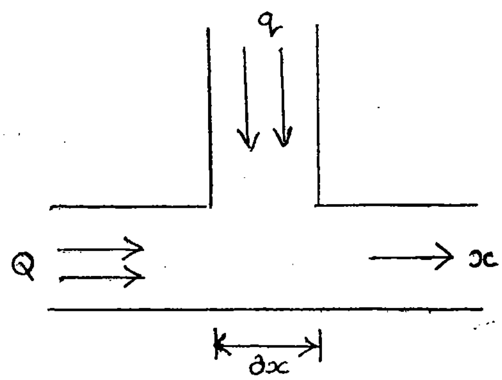
1. If there is no friction, it is called slip flow.

2. $\dfrac{\partial Q}{\partial x} = q$

$\Rightarrow \dfrac{\partial Q}{\partial x} - q = 0$



3. $Q \propto \sqrt{S}$

$$\frac{15}{\sqrt{1/1440}} = \frac{Q}{\sqrt{1/1000}}$$

$\Rightarrow Q = \underline{\underline{18\ m^3/s}}$

4. $Q = AV = by.\ \frac{1}{n}\ y^{2/3} S^{1/2}$ (wide rectangular channel, $R = y$)

$Q \propto y^{5/3}$

$$\frac{Q_2}{Q_1} = \left(\frac{1.2\ y_1}{y_1}\right)^{5/3} \Rightarrow Q = \underline{\underline{1.35}}\ Q_1 \qquad \underline{\underline{35\%}}$$

$$\frac{Q_{half}}{Q_{full}} = \left(\frac{y_{half}}{1.52}\right)^{5/3} = \frac{1}{2}$$

$$y_{half} = \underline{\underline{1}} \text{ m.}$$

06. $V = C\sqrt{RS} = C\sqrt{yS}$

$$Fr = \frac{V}{\sqrt{gy}} = \frac{C\sqrt{yS}}{\sqrt{gy}} = \underline{\underline{\frac{C\sqrt{S}}{\sqrt{g}}}}$$

07. $\tau = \gamma R S_0$

$$RS_0 = \frac{6}{9810} = 6.116 \times 10^{-4}$$

$$C = \frac{V}{\sqrt{RS}} = \frac{1.5}{\sqrt{6.116 \times 10^{-4}}} = \underline{\underline{60.6}}$$

08 $A = (4 + 1 \times 2)2 = 12 \text{ m}^2$

$$P = 2y\sqrt{1+m^2} = 4\sqrt{2}$$

$$R = \frac{A}{P} = \frac{12}{4\sqrt{2}} = 2.12 \text{ m.}$$

$$\tau_{avg} = \gamma R S_0 = 9810 \times 2.12 \times 0.002$$

$$= \underline{\underline{41.6}} \text{ N/m}^2$$

09. $A = my^2$

$$4 = 1 \times y^2 \Rightarrow y = 2$$

$$Fr = \frac{V\sqrt{2}}{\sqrt{gy}} = \frac{1.25\sqrt{2}}{\sqrt{2g}} = 0.399 \approx \underline{\underline{0.4}}$$

10 $P = 2y\sqrt{1+m^2} = 2y\sqrt{2}$

$$\frac{P}{y} = 2.828$$

$P = 3I$ (for efficient trapezoidal channel = half of hexagon)

$$= 3y\sqrt{1+\frac{1}{3}}$$

$$\frac{P}{y} = \underline{\underline{3.46}}$$

12. $y_1 = 0.4\ m$, $y_2 = 1.6\ m$ are alternate depths.

$$y_1 + \left(\frac{q}{y_1}\right)^2 \times \frac{1}{2g} = y_2 + \left(\frac{q}{y_2}\right)^2 \times \frac{1}{2g} \qquad V = \frac{q}{y}$$

$$0.4 + \frac{q^2}{0.4^2} \times \frac{1}{2g} = 1.6 + \frac{q^2}{1.6^2} \times \frac{1}{2g}.$$

$$q = 2\ m^2/s$$

$$\therefore V_1 = \frac{q}{y_1} = \frac{2}{0.4} = 5\ m/s.$$

$$E_1 = y_1 + \frac{V_1^2}{2g} = 0.4 + \frac{5^2}{2g} = \underline{\underline{1.6746}}\ m$$

13. $y = 1.6\ m$, $E = 2.8\ m$

$$E = y + \frac{V^2}{2g}.$$

$$2.8 = 1.6 + \frac{V^2}{2g} \Rightarrow V = 4.85\ m/s$$

$$Fr = \frac{V}{\sqrt{gy}} = \frac{4.85}{\sqrt{9\times 1.6}} = 1.225 > 1 \Rightarrow \underline{\underline{\text{supercritical}}}$$

14. $Fr = 0.8$, $y = 1.5\ m$.

$$Fr = \frac{V}{\sqrt{gy}} \Rightarrow 0.8 = \frac{V}{\sqrt{1.5g}} \qquad \therefore V = 3.068\ m/s$$

$q = Vy = 3.06 \times 1.5 = 4.59 \ m^2/s$

$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{4.59^2}{g}\right)^{1/3} = \underline{\underline{1.29}} \ m$

15. $(Fr)_n = 2.83$

$\frac{y_c}{y_n} = ?$

$Fr = \frac{V}{\sqrt{gy}} = \frac{q}{y\sqrt{gy}}$

$\Rightarrow Fr \propto \frac{1}{y^{3/2}}$; similar equation used in hydraulic jump.

$\frac{(Fr)_n}{(Fr)_c} = \left(\frac{y_c}{y_n}\right)^{3/2}$

$\frac{2.83}{1} = \left(\frac{y_c}{y_n}\right)^{3/2} \Rightarrow \frac{y_c}{y_n} = \underline{\underline{2}}$

NOTE : $y_1\left(1 + \frac{Fr_1^2}{2}\right) = y_2\left(1 + \frac{Fr_2^2}{2}\right)$ cannot be used here, as it corresponds to same specific energy at both sections. In given problem, the critical flow and uniform flow have different specific energies. Hence, a new relation is to be used.

16. $y_1\left(1 + \frac{0.2^2}{2}\right) = y_2\left(1 + \frac{4^2}{2}\right).$

$\frac{y_1}{y_2} = \frac{9}{1.02} = \underline{\underline{8.823}}$

17. $E_c = 2.5 \ m.$

$y_c = \frac{E_c}{1.25} = 2 \ m.$

$y_c = \left(\frac{2Q^2}{g m^2}\right)^{1/5}$

$2 = \left(\frac{2Q^2}{9.81 \times 1.5^2}\right)^{1/5} \Rightarrow Q = \underline{\underline{18.79}} \ m^3/s$

19. $b_1 = 2m, \quad y_1 = 1.2\ m \qquad q_1 = \frac{Q}{b_1} = \frac{3}{2} = 1.5$

$b_2 = 1.5m, \quad y_2 = ? \qquad q_2 = \frac{Q}{b_2} = \frac{3}{1.5} = 2$

$$y_1 + \left(\frac{q}{y_1}\right)^2 \times \frac{1}{2g} = y_2 + \left(\frac{q}{y_2}\right)^2 \times \frac{1}{2g}.$$ (Assume channel bottom as horizontal $\therefore E_1 = E_2$)

$$1.2 + \left(\frac{1.5}{1.2}\right)^2 \times \frac{1}{2g} = y_2 + \frac{1}{y_2^2} \times \frac{2^2}{2g}.$$

$$1.28 = y_2 + \frac{0.2039}{y_2^2}$$

$$\underline{\underline{y_2 = 1.12\ m}}$$

Solving the above cubic equation,

$$y_2 = 1.12, \ -0.35, \ 0.52\ m.$$

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{2^2}{g}\right)^{1/3} = 0.742\ m$$

$$V_1 = \frac{Q}{b_1 y_1} = 1.25$$

$$Fr_1 = \frac{1.25}{\sqrt{g \times 1.2}} = 0.36 < 1 \Rightarrow \underline{\underline{\text{subcritical}}}$$

We know that, flow is subcritical. $\therefore$ $y_2 > y_c$ at section 2. $\therefore$ $y_2 = 0.52$ & $y_2 = -0.35$ are neglected.

20. $y_1 = 1.2\ m, \ V_1 = 2.4\ m/s \Rightarrow q_1 = 2.88\ m^2/s$

$\Delta Z = 0.6\ m.$

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{2.88^2}{g}\right)^{1/3} = 0.945$$

$$y_1 > y_c \Rightarrow \text{subcritical.}$$

If the approaching flow is subcritical, the level of water will fall in hump portion. Hence option (b) seems to be correct. It is correct only if height of hump provided less than critical height of hump.

$$E_1 = E_2 + \Delta z$$

$$= E_c + (\Delta z)_c$$

$$y_1 + \frac{V_1^2}{2g} = \frac{3}{2} y_c + (\Delta z)_c$$

$$1.2 + \frac{2.4^2}{2g} = \frac{3}{2} \times 0.945 + (\Delta z)_c$$

$$\therefore (\Delta z)_c = 0.076 \text{ m}$$

Height of hump provided, 0.6 m is more than than that of $(\Delta z)_c$. Hence u/s flow gets disturbed. with increase in its level for subcritical flow.

$q = By.$

● Moving waves are called surges.

Value of increase in u/s level, $\Delta y_1 = (y_1' - y_1) = ?$

$Q = By.v =$

$$E_1' = y_1' + \frac{V_1'^2}{2g}$$

$$= y_1' + \frac{q^2}{2g\, y_1'^2}$$

$$E_1' = E_c + (\Delta z)\text{provided.}$$

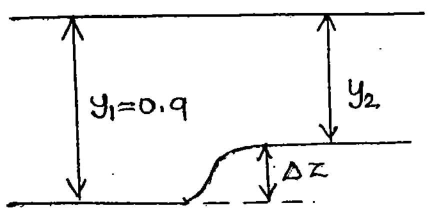
$$= 1.41 + 0.6 = 2.01 \text{ m}.$$

$$2.01 = y_1' + \frac{2.88^2}{2g \times y_1'^2}$$

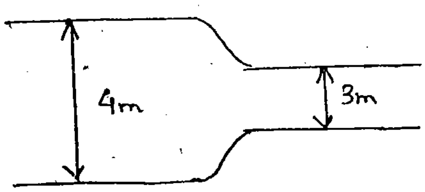
$$y_1' = 1.89, -0.42, 0.53$$

$$y_1' > y_1 \Rightarrow \underline{\underline{y_1' = 1.89}}$$

$$\Delta y_1 = 1.89 - 1.2 = \underline{\underline{0.69 \text{ m}}}$$



21. $B_1 = 4$m



As water surface remains horizontal,

$$y_2 = y_1 - \Delta z.$$

$$E_1 = E_2 + \Delta z$$

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + \Delta z$$

$$\Rightarrow V_1 = V_2 \quad \text{(flow is uniform).}$$

Water surface remains horizontal means, the flow is uniform for which $V_1 = V_2$.

$$Q = A_1 V_1 = A_2 V_2$$

$$A_1 = A_2$$

$$b_1 y_1 = b_2 y_2.$$

$$\frac{y_2}{y_1} = \frac{b_1}{b_2}$$

$$\Rightarrow y_2 = 0.9 \times \frac{4}{3} = 1.2 \text{ m}.$$

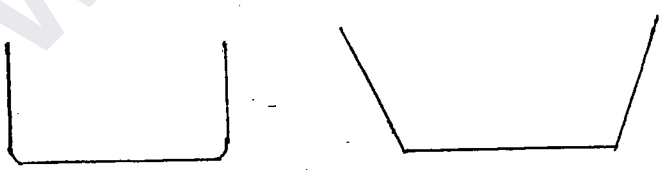
$$y_2 = y_1 - \Delta z$$

$$1.2 = 0.9 - \Delta z$$

$\therefore \Delta z = -0.3$ m (−ve indicates that assumption of hump is wrong)

Hence channel level drops by $\underline{\underline{0.3 \text{ m}}}$

22. $Q = 12 \text{ m}^3/\text{s}$



$y_1 = 2$ m $\qquad$ $b_2 = 1.8$ m

$b_1 = 2.4$ m $\qquad$ $m = 1$

$y_2 = 1.6$ m.

$$E_1 = E_2 + \Delta z$$

$$V_1 = \frac{Q}{A_1} = \frac{12}{2.4 \times 2} = 2.5 \text{ m/s}$$

$$E_1 = 2 + \frac{2.5^2}{2 \times 9.81} = 2.32 \text{ m.}$$

$$E_2 = y_2 + \frac{V_2^2}{2g}.$$

$$V_2 = \frac{Q}{A_2} = \frac{12}{(1.8 + 1 \times 1.6)1.6} = 2.21 \text{ m/s.}$$

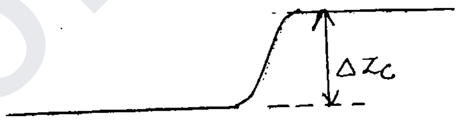
$$E_2 = 1.6 + \frac{2.21^2}{2g} = 1.85 \text{ m.}$$

$$\Delta Z = E_1 - E_2 = 2.32 - 1.85$$

$$= \underline{\underline{0.47}} \text{ m}$$

23. $b_1 = 4\text{m}$, $Q = 11.04 \text{ m}^3/\text{s}$, $y_1 = 1.5\text{m}$.

Assume width of channel as constant.



$$y_c = \left(\frac{Q^2}{gb^2}\right)^{1/3} = \left(\frac{(11.04)^2}{9.81 \times 4^2}\right)^{1/3} = 0.92 \text{ m.}$$

$$E_c = \frac{3}{2} y_c = 1.38 \text{ m.}$$

$$E_1 = y_1 + \frac{q^2}{y_1^2 \times 2g} = 1.67 \text{ m}$$

$$1.67 = 1.38 + (\Delta Z)_c$$

$$(\Delta Z)_c = \underline{\underline{0.29 \text{ m}}}$$

24. In this problem, channel bottom is assumed as horizontal.

$$\therefore E_1 = E_c$$

$$1.67 = \frac{3}{2} y_c.$$

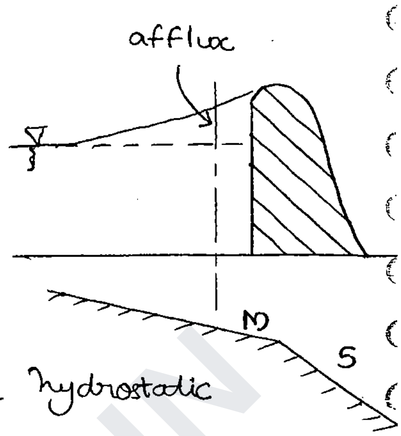
$$\Rightarrow y_c = 1.11 \text{ m.}$$

$$y_c = \left(\frac{Q^2}{g\, b_c^2}\right)^{1/3}$$

$$1.11 = \left(\frac{11.04^2}{9.81\ b_c^2}\right)^{1/3} \Rightarrow b_c = \underline{\underline{3}} \text{ m}$$

17th Sept, WEDNESDAY → Gradually Varied Flow

* Reasons



- Obstruction to flow, such as weir
- Change of c/s and shape.
- Change of bed slope

* Assumption

- Pressure distribution is assumed to be hydrostatic (streamlines are straight and parallel)
- The uniform flow formulae can be ~~cal~~ used to calculate velocity at any section.

$$V = \frac{1}{n} R^{2/3} S_0^{1/2} \quad ; \text{ uniform flow}$$

$$V = \frac{1}{n} R^{2/3} S_f^{1/2} \quad ; \text{ GVF}$$

S_0 : bed slope & S_f : energy slope

- Kinetic energy correction factor is assumed to be unity

$\frac{P}{\gamma} + \frac{V^2}{2g} + z =$ constant ; in the energy equation velocity distribution is assumed to be rectangular (uniform). But in open channel flow, it is logarithmic. So a correction factor is assumed as α, known as kinetic energy correction factor

→ Differential Equation of GVF :

$$\boxed{\frac{dy}{dx} = \frac{S_0 - S_f}{1 - \frac{Q^2 T}{g A^3}}}$$

$\frac{dy}{dx}$ → slope of water surface wrt channel bottom

$$\frac{Q^2 T}{g A^3} = \frac{Q^2}{A^2} \cdot \frac{1}{g} \cdot \frac{1}{A/T} = \frac{V^2}{g D}$$

$$\Rightarrow \quad \frac{Q^2 T}{g A^3} = Fr^2$$

$$\therefore \boxed{\frac{dy}{dx} = \frac{S_0 - S_f}{1 - F_r^2}}$$

Using Manning's formula, the equation of GVF is :-

$$\frac{dy}{dx} = S_0 \left[\frac{1 - \left(\frac{y_n}{y}\right)^{10/3}}{1 - \left(\frac{y_c}{y}\right)^3} \right]$$

Using Chezy's formula,

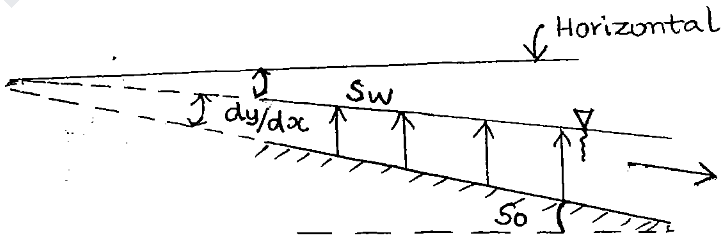
$$\frac{dy}{dx} = S_0 \left[\frac{1 - \left(\frac{y_n}{y}\right)^3}{1 - \left(\frac{y_c}{y}\right)^3} \right]$$

→ **Water Surface Slope wrt to Horizontal (S_w)**

Case (i) : Rising water surface. – depth is increasing in the direction of flow.

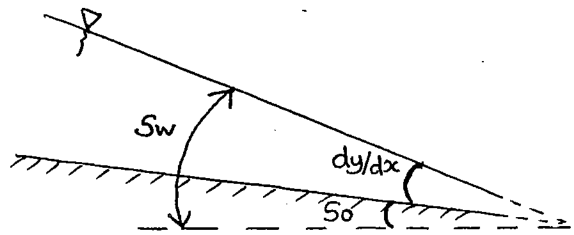
$$S_0 = S_w + \frac{dy}{dx}.$$

$$\therefore S_w = S_0 - \frac{dy}{dx}.$$



Case (ii): Water surface falling – depth of water decreasing

$$S_w = S_0 + \frac{dy}{dx}$$

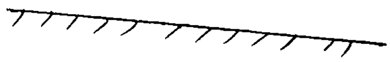


→ Differential Energy Equation of GVF

$$\frac{dy}{dx}(1 - F_r^2) = S_0 - S_f.$$

$$\boxed{\frac{\Delta E}{\Delta x} = S_0 - S_f}$$

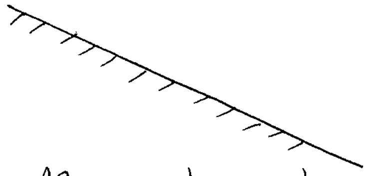
→ Classification of Slopes



Less Slope – More depth

$S_o < S_c$

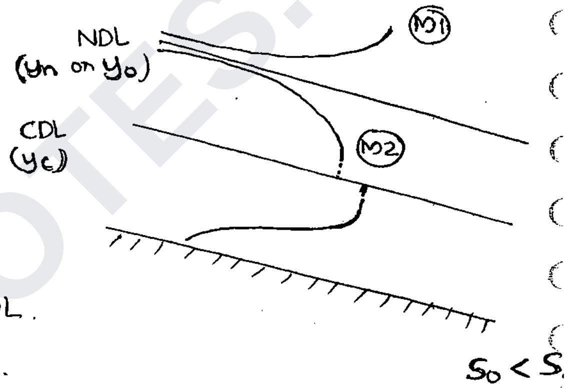
y_o (or y_n) $> y_c$



More slope – less depth

$S_o > S_c$

y_o (or y_n) $< y_c$



* Mild Slope :

First Profiles are above NDL & CDL.

Second Profiles are in b/w NDL & CDL.

Third profiles are below NDL & CDL.

First & Third profiles are rising profiles.

2nd profiles are falling or drawdown curves.

$$\frac{dy}{dx} = S_o \left[\frac{1 - \left(\frac{y_n}{y}\right)^3}{1 - \left(\frac{y_c}{y}\right)^3} \right]$$

For first profile, $y > y_n > y_c$

$$\frac{dy}{dx} = S_o \left(\frac{+ve}{+ve} \right) = S_o > 0 \Rightarrow \text{rising profile.}$$

For 3rd profile (M_3), $y_n > y_c > y$.

$$\frac{dy}{dx} = S_o \left(\frac{-ve}{-ve} \right) = +ve\, S_o > 0 \Rightarrow \text{Rising profile.}$$

For 2nd profile, ($M2$); $y_n > y > y_c$

$$\frac{dy}{dx} = S_o \left(\frac{-ve}{+ve} \right) = -ve \Rightarrow \text{drawn down or falling curve}$$

⊙ M_1 @ u/s : $y \longrightarrow y_n$

$$\frac{dy}{dx} = S_0 \left[\frac{1-\left(\frac{y_n}{y}\right)^3}{1-\left(\frac{y_c}{y}\right)^3} \right] = 0.$$

$\frac{dy}{dx}=0 \Rightarrow$ the water surface is parallel to channel bottom.

Further, we know that NDL is parallel to channel bottom.

As $y \to y_n$ (not exactly $y=y_n$), all water surface profiles meet the NDL ~~asymptotically~~. asymptotically.

⊙ M1 @ d/s : $y \longrightarrow \infty \implies \frac{dy}{dx} = S_0$

As $\frac{dy}{dx} \to S_0$, M1 profile tends to become horizontal at the d/s end.

⊙ M_2 @ d/s : $y \longrightarrow y_c \implies \frac{dy}{dx} \to \infty$

As $\frac{dy}{dx} \to \infty$, all water surface profiles will meet the CDL normally or perpendicularly.

⊙ M_3 @ d/s meets the CDL normally. At u/s near the channel boundary it is not properly defined.

$M_1 \rightarrow y > y_n > y_c$ (subcritical)

$M_2 \rightarrow y_n > y > y_c$ (subcritical)

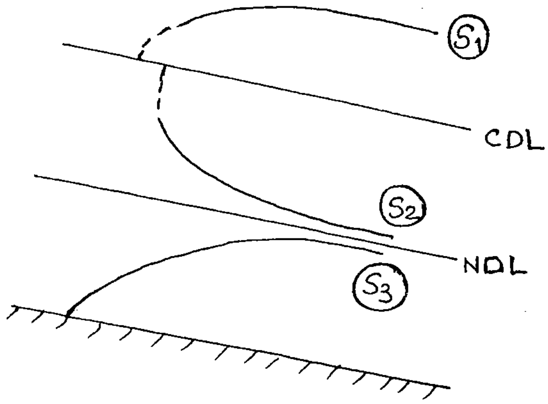
$M_3 \rightarrow y_n > y_c > y$ (Supercritical)

NOTE: All the third profiles are supercritical and hence hydraulic jump is possible in all the third profiles.

* Steep Slope (S)

$S_0 > S_c$

$y_n < y_c$



$S_1 \rightarrow y > y_c > y_n$ (Subcritical)

$S_2 \rightarrow y_c > y > y_n$ (Supercritical)

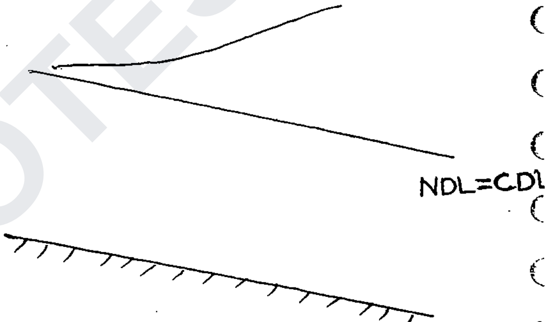
$S_3 \rightarrow y_c > y_n > y$ (Supercritical)

S_3 : Hydraulic Jump S_2 : Hydraulic drop (possible only in

* Critical Slope (C)

$S_0 = S_c$

$y_n = y_c$



NOTE :
C_2 will not exist as NDL & CDL coincides.

Transitional depth – Depth where NDL & CDL coincide is called transitional depth. At transitional depth, the water surface tends to become horizontal

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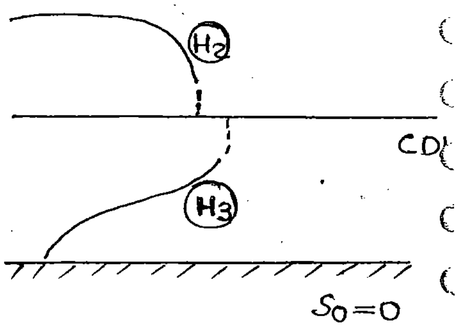
* Horizontal Profile. (H)

$S_0 = 0$

$y_n \rightarrow \infty$



∴ NDL do not exist, only CDL exists

$H_2 \rightarrow y > y_c$ (Subcritical)

$H_3 \rightarrow y < y_c$ (Supercritical)

H_3 : Hydraulic Jump.

* Adverse Profiles (A)

– channel bottom rising in the direction of flow

$S_0 = -ve$

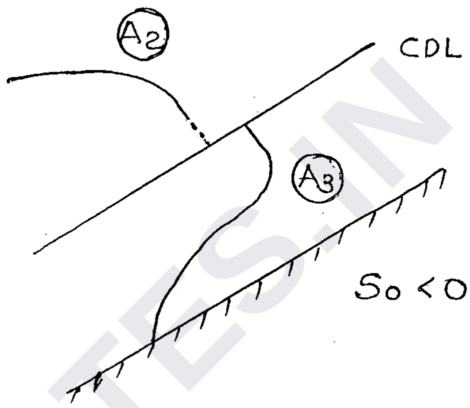
$y_n \rightarrow$ do not exist ; $\therefore A_1$ do not exist.

– unstable.

$A_2 \rightarrow y > y_c$; (Subcritical)

$A_3 \rightarrow y < y_c$; (Supercritical)

A_3 : Hydraulic Jump (unstable)



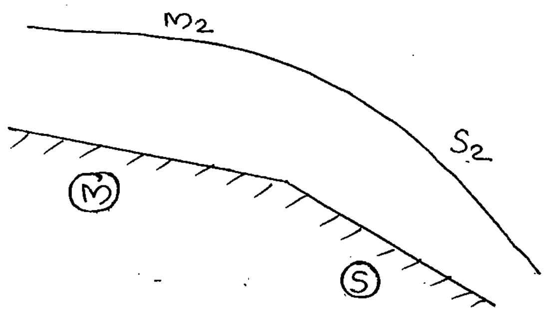
Q. Hydraulic jump is possible in which of the following profiles ...

a) M_2 b) H_2 c) M_3 d) A_3.

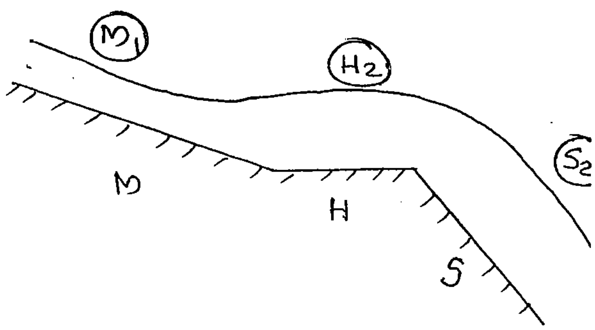
Though hydr. jump is possible in A_3, as it is unstable it is not the right answer.

→ Combinations of Water Surface Profiles :

If 2^{nd} slope is more than 1^{st} slope, the surface profiles are 'drawdowns'.

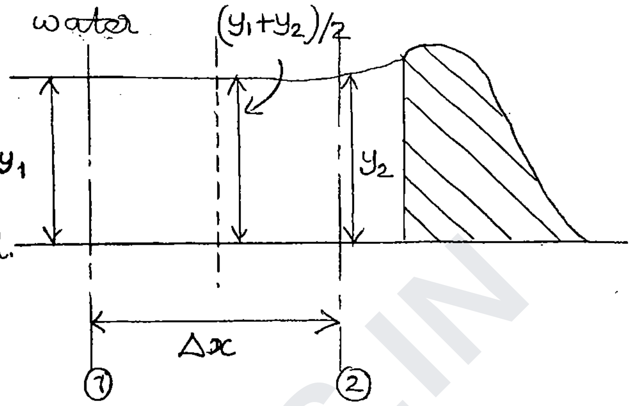


If more slope followed by less slope, profile is rising.



→ Length of Water Surface Profiles (Δx) of GVF :

Using the diff. energy equation, one can calculate the length of water surface profiles.



Method 1 : Single Step or Direct Step Method.

$$\Delta x = \frac{\Delta E}{S_0 - \overline{S_f}}$$

$$\Delta E = E_1 - E_2$$

$\overline{S_f}$ = average energy slope. $= \dfrac{S_{f1} + S_{f2}}{2}$

To save time, energy slope can be worked out for middle stream also.

Energy slope can be calculated by using either Chezy or Manning's formula.

* Demerits:
  - It is less accurate as the energy slope is not properly taken into account.
  - It is useful for prismatic (lined canals only) and not suitable for non-prismatic.
* Merits :
  - Quick solution.

Method II : Numerical or Graphical Integration Method.

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - F_r^2}$$

$$dx = \left(\frac{1 - F_r^2}{S_0 - S_f}\right) dy.$$

$$\therefore \quad x = \int_0^l \left(\frac{1 - F_r^2}{S_0 - S_f}\right) dy.$$

* Merits :

– It is more accurate compared to direct step method as water surface profile is more appropriately taken into account.

– It is useful for both prismatic & non-prismatic canals.

* Demerits :

– It is more time consuming.

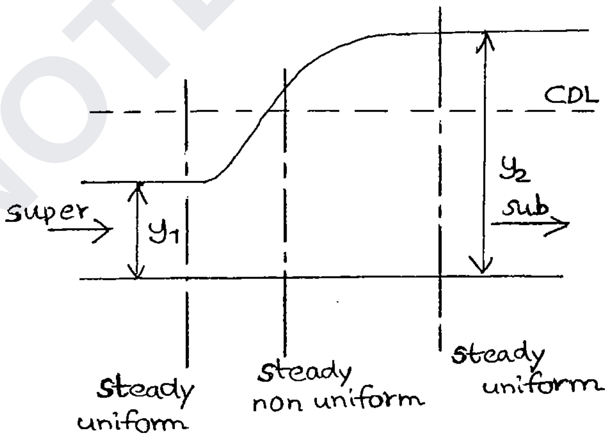
→ Hydraulic Jump

It is a steady rapidly varied flow.

y_1 : Initial / Prejump Heights

y_2 : Final / Post jump Heights

y_1 & y_2 together are called 'Conjugate or Sequent' depths.



- $\frac{y_2}{y_1}$ → Sequent depth ratio
- $(y_2 - y_1)$ → Jump height
- Length of jump (L_j) = (5 to 7) h_j

Supercritical flow meeting sub critical flow is the condition for hydraulic jump.

* Uses of Hydraulic Jump :

– energy dissipation at d/s of spillway, so that spillway is safe.

– effective mixing of chemicals in industries.

In the derivations of hydraulic jump, all the 3 fundamental equations of fluid mechanics, such as :-

(i) Energy equations

(ii) Continuity equations

(iii) Momentum equations.

are used.

NOTE:

- Wherever impact, change of momentum are involved with huge loss of energy, momentum equation must be used.
- In the gradually varied flow, change of momentum is negligible. Hence in its derivations only continuity and energy equations are used.

* Assumptions in the Analysis of Hydraulic Jump.

(i) Channel bottom is assumed to be horizontal. (to apply specific force concept)

(ii) Length of jump is small and hence friction is neglected

(iii) Before and after the jump, the flow is assumed to be uniform and pressure distribution is hydrostatic.

→ Relations for Hydraulic Jump of any shape of Channel.

1. Relation b/w y_1 & y_2

Equate $F_1 = F_2$

$$\left(\frac{Q^2}{gA} + A\bar{z}\right)_1 = \left(\frac{Q^2}{gA} + A\bar{z}\right)_2.$$

2. Energy loss in the hydraulic jump.

$$\Delta E = E_1 - E_2$$

3. Power loss (P)

$$P = \gamma Q(\Delta E)$$

* Standard Equations for Rectangular Channel

① $$\frac{2q^2}{g} = y_1 y_2 (y_1 + y_2)$$

② $$y_2 = \frac{y_1}{2}\left(-1 + \sqrt{1 + 8Fr_1^2}\right)$$

③ $$y_1 = \frac{y_2}{2}\left(-1 + \sqrt{1 + 8Fr_2^2}\right)$$

$$Fr_1^2 = \frac{q^2}{g y_1^3} \; ; \quad Fr_2^2 = \frac{q^2}{g y_2^3}$$

④ $$\Delta E = \frac{(y_2 - y_1)^3}{4 y_1 y_2} = \frac{(V_1 - V_2)^3}{2g(V_1 + V_2)}$$

⑤ $$\frac{y_1}{y_2} = \left(\frac{Fr_2}{Fr_1}\right)^{2/3}$$

* Classification of Hydraulic Jump.

It is based on Froude number of incoming flow.

→ Surges & Wavefronts

Hydraulic Jump	Surge.
(i) Occurs because of supercritical flow meeting subcritical flow.	(i) Occur due to lifting or drop the gates. across a channel.
(ii) It is stationery or fixed at a given location, hence also called stationery jump or standing wave front.	(ii) Surges are moving hydraulic jumps.
(iii) It is steady, rapidly varied flow.	(iii) Unsteady, rapidly varied flow

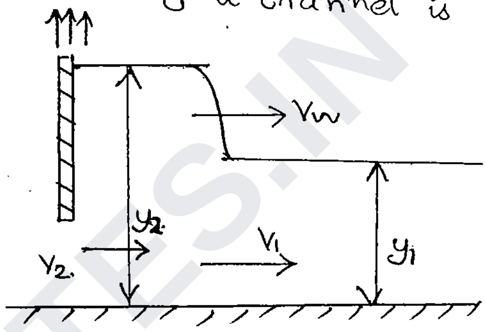
Surges:

(i) +ve surges:

The sudden rise of water by the operation of gates.

a) +ve surge moving D/S :-

– If a gate at the head or u/s of a channel is suddenly lifted.



$y_1 \rightarrow$ initial ht. of surge.

$y_2 \rightarrow$ final depth.

$y_2 - y_1 \rightarrow$ height of surge.

$V_1 \rightarrow$ initial velocity

$V_w \rightarrow$ Velocity of elementary (or) surge wave (or) wave

It occurs from u/s to d/s.

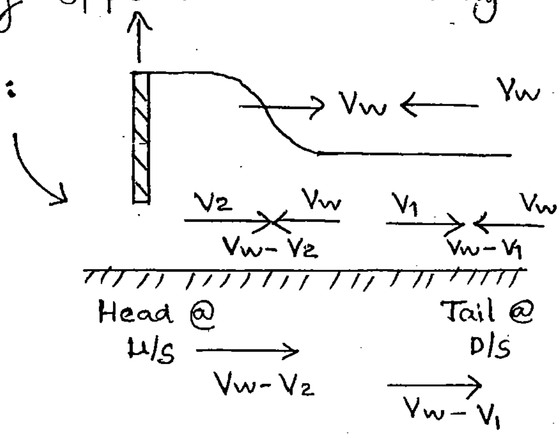
* Celerity of wave (c) : It is the relative velocity of V_w & V_1.

In this problem, $c = V_w - V_1 = \sqrt{\dfrac{g y_2 (y_1 + y_2)}{2 y_1}}$

* Important points in the derivation :

a) Surge is basically unsteady in nature. There is no analysis for unsteady flow directly. Unsteady flows are converted to equivalent steady state by applying opposite wave velocity.

b) Equivalent steady state is as flows:



c) Momentum equation is used to finally derive the relation.

The above equation is useful for ~~both~~ large waves. ~~and~~

Large waves $\Rightarrow (y_2 - y_1) >>> y_1$

Eg: Tidal bore, Tsunami.

Small waves ⇒ $y_2 - y_1 <<< y_1$ (elementary waves)

Eg: Stones dropped in water.

In case of small waves,

$$C \approx \sqrt{g D_1}$$

$$C \approx \sqrt{g y_1}$$; for rectangular waves.

NOTE: '+ve surge' occurs if more flow or more water or more discharge occurs in b/w head & tail (or b/w two sections,

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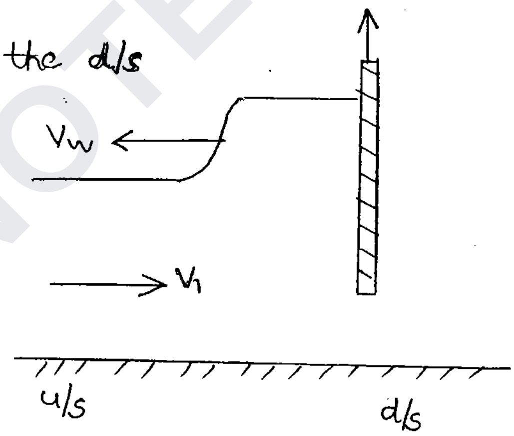
b) +ve surge moving u/s

By closing a gate at the d/s



$$C = V_w + V_1$$

$$= \sqrt{\frac{g y_2 (y_1 + y_2)}{2 y_1}}$$

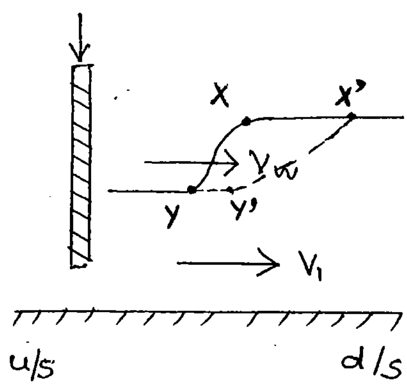
$$= \sqrt{g y_1}$$; for small wave.

* –ve surges

– Fall of level of water suddenly
– less flow of water b/w u/s & d/s

a) –ve surge moving d/s

By closing a gate at the u/s

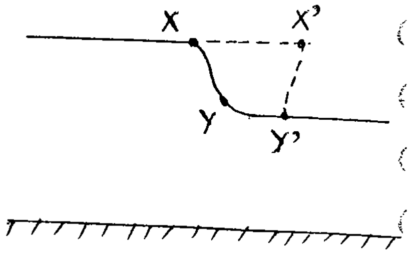


• –ve surge analysis is complicated as they are unstable.

In surges, top layers move faster than bottom layers. As shown in fig, point x of –ve surge moves by greater distance (xx') compared to smaller distance of bottom point. Y (YY'). Due to this, the surge will lose its shape. and ∴ unsta

NOTE :

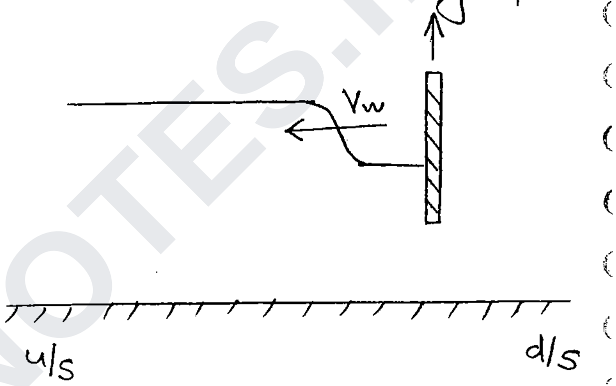
- +ve surges are stable.



In +ve surges or the surge can be maintained. ∴ +ve surges are stable.

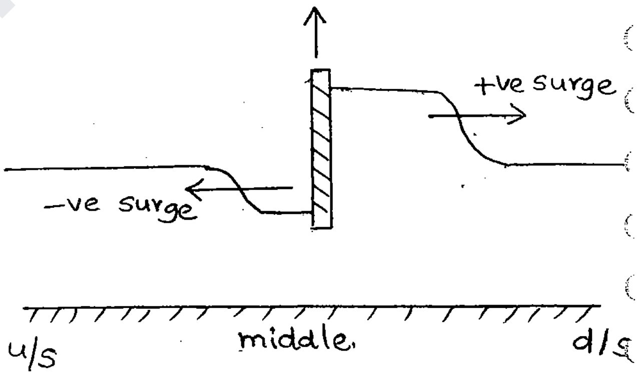
b) −ve surge moving u/s

The gate at the d/s of a channel suddenly opened

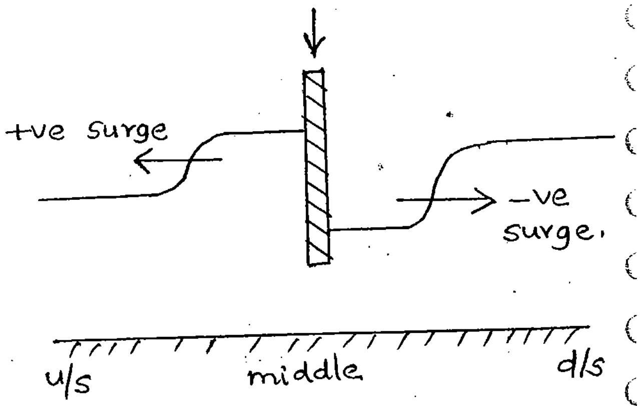


* Combination of Surges.

a) Gate at the middle of a channel opened.



b) Gate at the middle of a channel closed.



→ Control Section.

The sections where flow measurement is done is called Control section.

Theoretically, critical sections are better as control control section. In other flows, control sections are choosen away from disturbances.

* In supercritical flow, selection of control section :

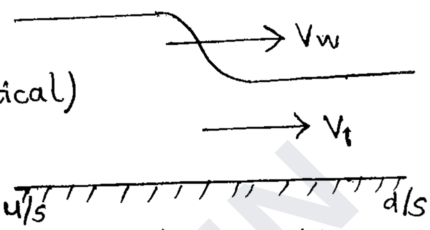
$$C = V_w - V_1 = \sqrt{g y_1}$$

For critical flow, $Fr = 1$

$$V_c = \sqrt{g y}.$$

(supercritical)

$$\therefore \quad V_w = V_1 + V_c \Rightarrow \underline{V_w > V_c}$$



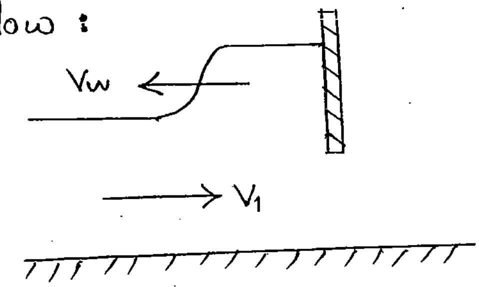
Wave travels towards d/s in case of supercritical flow.

If a stone is dropped in a river and if the waves travel towards d/s, it indicates supercritical flow.

In supercritical flow, control sections are selected on the u/s reaches of channel, away from disturbances

* Control sections in subcritical flow :

$$V_w + V_1 = \sqrt{g y_1}$$
$$= V_c.$$

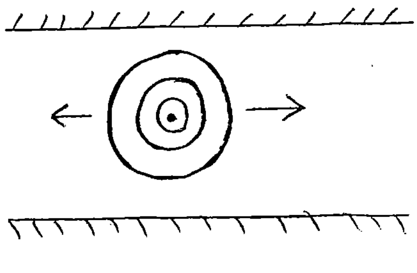


$$V_w = V_c - V_1$$
$$\Rightarrow V_w < V_c \quad \text{(Subcritical)}$$

In subcritical flow, the waves or disturbances travel towards u/s. Hence, in subcritical flow, control sections are selected on the d/s reaches of channel.

If a stone is dropped in a river and if the wave or disturbances travel towards u/s, it indicates subcritical flow.

- In critical flow, the waves will travel in both the directions symmetrically, if a stone is dropped.



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13. DIMENSIONAL ANALYSIS

→ Purpose of Dimensional Analysis

(i) Model study (or) experimental investigations.

(ii) Deriving π terms (non-dimensional terms)

(iii) Out of the various possible solution parameters, to find out the actual parameters.

→ Fourier's Theory of Dimensional Homogeneity.

– $F = ma$; rational formula valid throughout the world, hence dimensionally homogeneous.

– $F = 9.81\, m$; empirical formulae are useful in any one system only. Value of constant changes from system to system. Hence they are dimensionally non-homogeneous.

– Chezy's formula, Manning's formula – non homogeneous

→ Derivation of π terms :

$$\underset{\substack{\downarrow \\ \text{dependent variable.}}}{A} = f\underset{\substack{\downarrow \\ \text{independent variable.}}}{(\alpha, \beta, \gamma, \mu)}.$$

$$f(A, \alpha, \beta, \gamma, \mu) = 0.$$

Total number of variables, n = No: of independent variables + dependent variable.

So here, $n = 4 + 1 = \underline{\underline{5}}$

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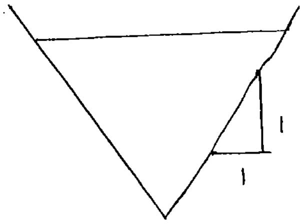

25. $S_0 = 0.001$, $Q = 0.2\ m^3/s$, $n = 0.015$

$$y_c = \left(\frac{2Q^2}{gm^2}\right)^{1/5} = \left(\frac{2\times0.2^2}{9.81\times1}\right)^{1/5}$$

$$= \underline{\underline{0.38\ m}}$$

$$V = \frac{Q}{A} = \frac{0.2}{my_n^2} = \frac{0.2}{1\times y_n^2} = \frac{1}{0.015}\left(\frac{y_n}{2\sqrt{2}}\right)^{2/3}(0.001)^{1/2}$$

$$y_n = 0.56\ m$$

$y_0 = y_n > y_c \Rightarrow S_0 < S_c$

$\therefore$ Mild Slope.

26 $y = 0.4\ m$.

$y_c < y < y_n \Rightarrow M_2$ profile.

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$b = 15\ m$; $y = 3m$; $S_0 = \frac{1}{5000}$; $Q = 29\ m^3/s$; $C = 65$.

Depth is increasing in the direction of flow.

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - \frac{Q^2T}{gA^3}}$$

Energy slope, S_f can be calculated using Chezy's Formula.

$$V = C\sqrt{RS_f}$$

$$\frac{29}{15\times3} = 65\sqrt{\frac{45}{21}S_f}$$

$$S_f = 4.587\times10^{-5}$$

$$\frac{dy}{dx} = \frac{\frac{1}{5000} - 4.587\times10^{-5}}{1 - \frac{29^2\times15}{9\times45^3}} = \underline{\underline{1.56\times10^{-4}}}$$

Water surface slope, $S_w = S_0 - \frac{dy}{dx}$ (depth ↑ direction of flow).

$$= \frac{1}{5000} - 1.56 \times 10^{-4}$$

$$= \underline{\underline{4.366 \times 10^{-5}}}$$

28. $q = 8\ m^2/s$,

$S_0 = 0.004$,

$n = 0.015$, $y_2 = 1$ m.

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{64}{g}\right)^{1/3} = \underline{\underline{1.86\ m}}$$

$$\frac{q}{y_n} = \frac{1}{n} y_n^{2/3} S_0^{1/2}.$$

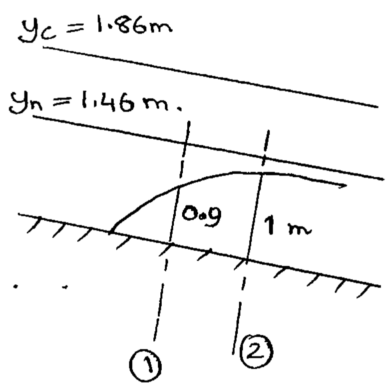
$$\frac{8}{y_n} = \frac{1}{0.015} y_n^{2/3}\, 0.004^{1/2}$$

$$\Rightarrow y_n = \underline{\underline{1.46}}\ m$$

$y_c > y_n \Rightarrow S_c < S_0 \Rightarrow$ Steep slope.

$y_c > y_n > y \Rightarrow S_3$ profile. (raising profile).

29. As S_3 is a raising profile, 0.9 m section will be on the u/s of 1 m section.



$$\Delta x = \frac{\Delta E}{S_0 - \overline{S_f}} \quad \text{(direct step method)}.$$

$$E_1 = y_1 + \frac{q^2}{2g y_1^2} = 0.9 + \frac{64}{2g \times 0.9^2}$$

$$= \underline{\underline{4.92\ m}}$$

$$E_2 = y_2 + \frac{q^2}{2g\ y_2^2} = 1 + \frac{64}{2g \times 1^2} = \underline{\underline{4.26\ m}}$$

$$\Delta E = 4.26 - 4.92 = \underline{\underline{-0.657}}\ m$$

Calculation $\overline{S_f}$ for middle section (stream). of height $\frac{0.9+1}{2} = 0.95$

$$V = \frac{q}{y_m} = \frac{8}{0.95} = \frac{1}{0.015} \times (0.95)^{2/3} \times (S_f)^{1/2}$$

$$\overline{S_f} = \underline{\underline{0.017}}$$

$\overline{S_f} > S_o \Rightarrow$ raising profile. (Energy slope > Bed slope)

$$\Delta x = \frac{-0.657}{(0.004-0.017)} = \underline{\underline{50.21\ m}}$$

NOTE :

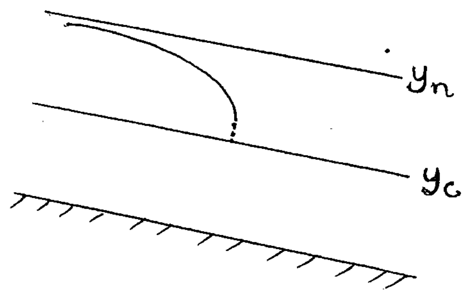
Energy slope > Bed slope $\Rightarrow$ Rising Profile

33 Free fall $\longrightarrow$ Draw down.

$n = 0.012$, $S_o = 0.0004$, $q = 2\ m^3/s/m$

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{4}{g}\right)^{1/3} = \underline{\underline{0.74\ m}}$$

mild slope:-



$$V = \frac{q}{y_n} = \frac{1}{n} y_n^{2/3} S_o^{1/2}.$$

$$\frac{2}{y_n} = \frac{1}{0.012} y_n^{2/3} (0.0004)^{1/2}$$

$$y_n = \underline{\underline{1.115\ m}}$$

$y_n > y_c \Rightarrow$ mild slope.

After several kilometres long, it will touch CDL.

As it is a falling profile, it is M_2. After several km long 2nd profile will meet CDL. Hence depth after a long distance is equal to y_c.

34 Relative loss $= \frac{\Delta E}{E_1}$

35. $q = 10 \ m^2/s, \ V = 20 \ m/s$

$$y_1 = \frac{q}{V} = \frac{10}{20} = 0.5 \ m.$$

$$\frac{2q^2}{g} = y_1 y_2 (y_1 + y_2).$$

$$\frac{2 \times 10^2}{g} = 0.5 y_2 (0.5 + y_2).$$

Tail water depth, $y_2 = \underline{\underline{6.14}} \ m$

36. $Fr_1 = 2.5$

$$\frac{y_2}{y_1} = \frac{1}{2}\left(-1 + \sqrt{1 + 8 Fr_1^2}\right)$$

$$\frac{y_2}{y_1} = \frac{1}{2}\left(-1 + \sqrt{1 + 8 \times 2.5^2}\right) = \underline{\underline{3.071}}$$

$$\Delta E = \frac{(y_2 - y_1)^3}{4 y_1 y_2} = \frac{y_1^3 \left(\frac{y_2}{y_1} - 1\right)^3}{4 y_1 y_2}$$

$$\frac{\Delta E}{y_1} = \frac{\left(\frac{y_2}{y_1} - 1\right)^3}{4\left(\frac{y_2}{y_1}\right)} = \frac{(3.071 - 1)^3}{4 \times 3.071} = \underline{\underline{0.723}}$$

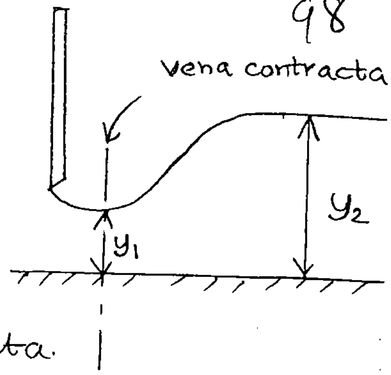
37. $Q = 1 \ m^3/s, \ y_1 = 0.5 \ m^3/s.$

$$\left(\frac{Q^2}{gA} + A\bar{z}\right)_1 = \left(\frac{Q^2}{gA} + A\bar{z}\right)_2.$$

$$\frac{1}{g \times 0.5^2} + 0.5^2 \times \frac{0.5}{3} = \frac{1}{g \times y_2^2} + y_2^2 \times \frac{y_2}{3}.$$

$$0.45 \quad 0.435 = \frac{1}{g \ y_2^2} + \frac{y_2^3}{3}.$$

$$\Rightarrow y_2 = \underline{\underline{1.02}} \ m$$



38. $y_2 = 3\,m$, $Fr_2 = \dfrac{1}{2\sqrt{2}}$; $C_c = 0.9$

Vena contracta is the section where c/s area of jet is least.

Say y_1 = initial depth = depth at vena contracta.

$$y_1 = \frac{y_2}{2}\left(-1 + \sqrt{1+8Fr_2^2}\right)$$

$$= \frac{3}{2}\left(-1 + \sqrt{1+8\times\frac{1}{8}}\right) = \underline{\underline{0.62\ m}}$$

Coeffecient of contraction, $C_c = \dfrac{\text{Area at vena contracta}}{\text{Area of gap or opening}}$

$$0.9 = \frac{y_1 \times 1}{y \times 1} \quad \text{(assumed metre width)}.$$

$$\Rightarrow \quad y = \underline{\underline{0.69}}\ m$$

39. $B = 4\,m$, $Q = 16\ m^3/s$

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{16}{g}\right)^{1/3} = \underline{\underline{1.177}}\ m$$

$$y = \frac{q}{V} = \frac{4}{3} = \underline{\underline{1.333\ m}}$$

$\Rightarrow y > y_c \Rightarrow$ subcritical.

$\therefore$ No $\underline{\underline{\text{jump}}}$ occurs

41. Supercritical flow $\rightarrow$ disturbances travelling d/s.

42. $y_1 = 1.2\,m$, $V_1 = 2\ m/s$.

$$V_w + V_1 = \sqrt{g y_1} \quad \text{(wave travelling u/s)}.$$

$$V_w = \sqrt{1.2g} - 2$$

$$= \underline{\underline{1.43\ m/s}}$$

In this problem, if the wave is travelling towards d/s

$$V_w = \sqrt{g y_1} + V_1 = \sqrt{1.2 g} + 2 = \underline{\underline{5.43 \text{ m/s}}}$$

43. $y_2 - y_1$ = amplitude = 2 cm.

$y_2 = (y_1 + 0.02)$ m.

$$V_w = 1.80 \text{ m/s.}$$

For a pond, $V_1 = 0$. (velocity of pond is zero).

$$V_w \pm V_1 = \sqrt{\frac{g y_2}{2 y_1}(y_1 + y_2)}$$

$$V_w = \sqrt{\frac{g(y_1+0.02)(y_1+y_1+0.02)}{2y_1}}$$

$$1.8 = \sqrt{\frac{g(y_1+0.02)^2(y_1^2+0.02y_1)}{2y_1}}$$

$$y_1 = \underline{\underline{0.3}} \text{ m}$$

⊙ If the season changes from summer to winter, part water on surface solidifies as ice in river. The level of surface of river increases. Volume of ice is more than that of equivalent wt. of water

→ Buckingham's π theorem (or) Method of Repeating Variables.

No: of π terms $= n - m$

where $n \rightarrow$ total no: of variables

$m \rightarrow$ no: of repeating variables.

- Repeating variables shall contain all the fundamental or basic or primary units (such as M, L, T).
- Dependent variable shall not be choosen as repeating variable.
- Repeating variables consist of :-
  a) a geometrical property (Area, volume, perimeter, diameter etc)
  b) a kinematic property (property which describes motion of fluid)
  Eg:- Velocity, acceleration, discharge.
  c) a fluid or dynamic property
  Eg:- mass density, viscosity.

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Q.01 $F = \phi(\overset{L}{D}, \overset{T}{V}, \omega, \mu, \overset{M}{\rho})$

$n = 6$

Mass density (ρ), velocity (V) and diameter (D) are choosen as repeating variables.

Term	Dimensions
F	MLT^{-2}
D	L
V	LT^{-1}
ω	T^{-1}
μ	$ML^{-1}T^{-1}$
ρ	ML^{-3}

No: of π terms $= n - m$
$= 6 - 3$
$= \underline{\underline{3}}$

π_1 term :

$\pi_1 = M^0 L^0 T^0$

$= \rho^{a_1} V^{b_1} D^{c_1} \cdot F$

$$\Pi_1 = M^0L^0T^0 = (ML^{-3})^{a_1} (LT^{-1})^{b_1} L^{c_1} . (MLT^{-2}).$$

$a_1 + 1 = 0. \Rightarrow a_1 = -1.$

$-3a_1 + b_1 + c_1 + 1 = 0. \Rightarrow 3 - 2 + c_1 + 1 = 0 \Rightarrow c_1 = -2.$

$-b_1 - 2 = 0. \Rightarrow b_1 = -2.$

$$\Pi_1 = M^0L^0T^0 = (ML^{-3})^{-1} (LT^{-1})^{-2} L^{-2} (MLT^{-2})$$

$$\Pi_1 = \rho^{-1} V^{-2} D^{-2} F. = \frac{F}{\rho V^2 D^2}$$

Π_2 term :

$$\Pi_2 = M^0L^0T^0 = \rho^{a_2} V^{b_2} . D^{c_2} . \omega$$

$$= (ML^{-3})^{a_2} (LT^{-1})^{b_2} . L^{c_2} . T^{-1}.$$

$a_2 = 0. ; b_2 = -1, c_2 = 1.$

$$\therefore \Pi_2 = \frac{\omega D}{V.}$$

Π_3 term :

$$\Pi_3 = M^0L^0T^0 = \rho^{a_3} . V^{b_3} . D^{c_3} . \mu$$

$$= (ML^{-3})^{a_3} (LT^{-1})^{b_3} . L^{c_3} . M^1L^{-1}T^{-1}$$

$a_3 + 1 = 0 \Rightarrow a_3 = -1.$

$-b_3 - 1 = 0 \Rightarrow b_3 = -1.$

$-3a_3 + b_3 + c_3 - 1 = 0 \Rightarrow c_3 = -1.$

$$\Pi_3 = \frac{\mu}{\rho V D.}$$

* Repeating variables are called so as they are repeatedly used in the derivation of various possible Π terms.

Q Time period of a pendulum is a fn of length of pendulum and g. No: of possible Π terms are :

$$T = fn(l, g)$$

$n = 3, \quad m = 2.$

$n - m = 1$ possible π terms.

→ Advantages of Buckingham's Theorem

(i) It is simple even for complex problems with more no: of variables.

(ii) No: of possible π terms can be worked out initially.

→ Rayleigh's Method.

It is impossible to estimate the no: of possible π terms in this method. Hence outdated.

→ Dimensional Matrix Method

No: of π terms $= n - r$

where $n \rightarrow$ total no. of variables.

$r \rightarrow$ rank of matrix.

This method avoids the confusion of adopting MLT, FLT system.

$$\begin{array}{c|c}
 & \begin{matrix} F & \rho & V & D & \omega & \mu \end{matrix} \\
\begin{matrix} M \\ L \\ T \end{matrix} & \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & -3 & 1 & 1 & 0 & -1 \\ -2 & 0 & -1 & 0 & -1 & -1 \end{bmatrix}
\end{array}$$

r, rank → No: of non zero row.

24th sept, WEDNESDAY

→ Model

Small scale replica of original structure.

* Scale ratio :-

$$L_r = \frac{L_m}{L_p} = \frac{B_m}{B_p} = \frac{y_m}{y_p}$$

Models

Undistorted models | Distorted Models

Undistorted models

(i) Geometrically similar.

Corresponding geometrical ratios are equal.

$$\frac{y_m}{y_p} = \frac{b_m}{b_p} = L_r$$

$$L_H = \frac{b_m}{b_p} = \frac{L_m}{L_p} \ ; \ L_V = \frac{y_m}{y_p}$$

$$L_H = L_V = L_r$$

(ii) Materials of model and prototype are same.

Distorted Models

(i)

$$L_H = \frac{6}{6000} = \frac{1}{1000} \ ; \ L_V = \frac{1}{6}$$

$$L_H \neq L_V$$

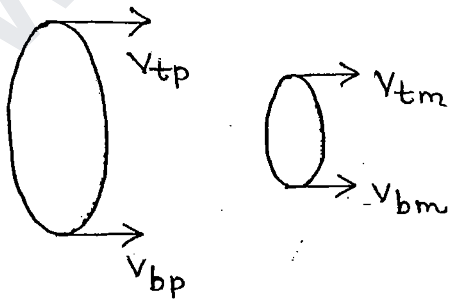
Eg: River, harbour, tidal models are distorted models.

* Similitude Criteria.

A model and prototype are completely similar, if they are

a) Geometrically Similar. – similarity of shapes

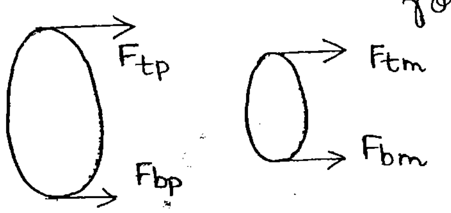
b) Kinematically Similar – similar kinematic parameters.



$$\frac{V_{tm}}{V_{tp}} = \frac{V_{bm}}{V_{bp}}.$$

ie, corresponding velocity ratios are equal.

c) Dynamic Similarity – corresponding points have similar force ratios.



$$\frac{F_{tm}}{F_{tp}} = \frac{F_{bm}}{F_{bp}}$$

If all the three similarities are satisfied, then, a model and prototype are said to be completely similar

- In hydraulic machine, a model and prototype are said to be completely similar if they are homologous.

Two fluid systems are said to be homologous if they have similar stream lines.

→ Froude's Law of Similarity

$$* \quad F_r = \sqrt{\frac{\text{inertia force}}{\text{gravity force}}} = \frac{V}{\sqrt{gD}}.$$

$$(F_r)_m = (F_r)_p$$

$$\left(\frac{V}{\sqrt{gD}}\right)_m = \left(\frac{V}{\sqrt{gD}}\right)_p.$$

* Velocity ratio, $V_r = \dfrac{V_m}{V_p} = \sqrt{\dfrac{g_m}{g_p}} \sqrt{\dfrac{D_m}{D_p}}$

$V_r = \sqrt{g_r} . \sqrt{L_V}$; distorted model

where $L_V \rightarrow$ vertical or depth scale ratio.

$V_r = \sqrt{g_r} . \sqrt{L_r}$; undistorted models.

If the model and prototype are at same location,

$$g_r = \frac{g_m}{g_p} = 1$$

$$\Rightarrow \quad V_r = \sqrt{L_r}$$

$$\boxed{\begin{array}{l} g_{equator} = 9.78 \ m/s^2 \\ g_{polar} = 9.83 \ m/s. \end{array}}$$

$g_{moon} = \left(\frac{1}{6}\right)^{th} g_{earth}.$

All the rocket launching stations are located close to equator as $g_{equator} < g_{polar}$.

* Use of Froude's Law:

Froude's Law is important wherever gravitational force is important. For eg; in the analysis of open channels, spillways, notches, weirs, tides, rivers, reservoirs etc.

* Area ratio, $A_r = by = L_H \cdot L_V$; distorted model

If do $A_r = L_r^2$; undistorted model

* Volume ratio, $(Vol)_r = Lby = (L_H)^2 L_V$; distorted model

$(Vol)_r \cancel{Vol} = (L_r)^3$; undistorted model

* Time ratio $(T_r) = \dfrac{L_r}{V_r}$ $\left\{ V = \dfrac{L}{T} \right\}$

$T_r = \dfrac{L_H}{\sqrt{L_V}}$; distorted model

$T_r = \dfrac{L_r}{\sqrt{L_r}} = \sqrt{L_r}$; undistorted model.

* Acceleration ratio $(a_r) = \dfrac{V}{T}$

$= \dfrac{\sqrt{L_V}}{L_H/\sqrt{L_V}}$

$\Rightarrow a_r = \dfrac{L_V}{L_H}$; distorted model.

$a_r = 1$; undistorted model.
$(L_V = L_H = L_r)$.

$\boxed{a_m = a_p}$; as per Froude's Law of Undistorted models.

* Discharge ratio, $Q_r = A_r \cdot V_r$

$= L_H \cdot L_V \cdot \sqrt{L_V}$

$\Rightarrow Q_r = L_H L_V^{3/2}$; distorted model

$Q_r = (L_r)^{5/2}$; undistorted model.

* Discharge per m width, $q_r = \frac{Q}{L_H} = \frac{L_H L_V^{3/2}}{L_H}$

$$q_r = L_V^{3/2} \text{ ; distorted model}$$

$$q_r = L_r^{3/2} \text{ ; undistorted model.}$$

* Mass ratio, $(\text{Mass})_r = \rho_r \times (\text{vol})_r$

$$= \rho_r \times (L_r)^3$$

* Momentum ratio $= M_r \cdot V_r$

$$= \rho_r \cdot L_r^3 \sqrt{L_r}$$

$$(\text{Momentum})_r = \rho_r (L_r)^{7/2}$$

In case the fluid of model and prototype are same, then $\rho_r = 1$.

$$\therefore (\text{momentum})_r = (L_r)^{7/2}$$

* $(\text{Work or Energy})_r = F_r \times L_r$

$$= (L_r)^4$$

* Force ratio, $(\text{force})_r = M_r \times a_r$

$$= (L_r)^3$$

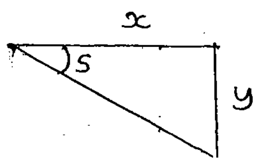
* Pressure ratio, $(\text{pressure})_r = \frac{F_r}{A_r} = \frac{L_r^3}{L_r^2} = \underline{\underline{L_r}}$

* Power ratio. $= F_r \times V_r$

$$= L_r^3 \cdot \sqrt{L_r} = (L_r)^{7/2}$$

$3 + \frac{1}{2}$

* Manning's n ratio :



$$V = \frac{1}{n} R^{2/3} S^{1/2}$$

$$n = \frac{R^{2/3} S^{1/2}}{V} = \frac{L_V^{2/3} \sqrt{\frac{L_V}{L_H}}}{\sqrt{L_V}}$$

$$(n)_r = \frac{L_V^{2/3}}{\sqrt{L_H}} \quad ; \text{ distorted model.}$$

$\frac{2}{3}-\frac{1}{2}$

$$(n)_r = (L_r)^{1/6} \quad ; \text{ undistorted model}$$

* Chezy's C ratio:

$$C = \frac{V}{\sqrt{RS.}} = \frac{\sqrt{L_V}\,\sqrt{L_H}.}{\sqrt{L_V.\,L_V.}}$$

$$C = \sqrt{\frac{L_H}{L_V}} \quad ; \text{ distorted model}$$

$$C = 1 \quad ; \text{ undistorted model.}$$

* Hydraulic Jump ratio

$$(h_j)_r = \frac{h_{jm}}{h_{jp}} = L_V \quad ; \text{ distorted model.}$$

$$(h_j)_r = L_r \quad ; \text{ undistorted model.}$$

* Specific Energy ratio

$$(E)_r = L_V$$

→ Reynold's Law of Similarity

$$Re = \frac{\text{Inertia force}}{\text{Viscous force}}$$

$$(Re)_m = (Re)_p$$

$$\left(\frac{\rho V L}{\mu}\right)_m = \left(\frac{\rho V L}{\mu}\right)_{p.}$$

* Velocity ratio, $V_r = \frac{V_m}{V_p} = \frac{\mu_m}{\mu_p} \cdot \frac{\rho_p}{\rho_m} \cdot \frac{L_p}{L_m}$

$$\boxed{V_r = \frac{\mu_r}{\rho_r} \cdot \frac{1}{L_r}} \qquad V_r = \frac{\gamma_r}{L_r}$$

If fluids of model & prototype are same,

$\mu_r = 1, \; \rho_r = 1, \; \gamma_r = 1$

$$\boxed{V_r = \frac{1}{L_r}}$$

* Use of Reynold's Law:

Wherever viscous forces are important such as in flow through pressure pipes, motion of fully submerged bodies like submarine & parachute etc.

* $(Area)_r = L_r^2$
* $(Volume)_r = L_r^3$
* $T_r = \frac{L_r}{V_r} = \frac{L_r . L_r}{\gamma_r} = \frac{L_r^2}{\gamma_r} = \frac{L_r^2 . \rho_r}{\mu_r}$

If fluids are same,

$$T_r = L_r^2$$

* Acceleration ratio, $a_r = \frac{1}{L_r} . \frac{1}{L_r^2} = \frac{1}{L_r^3}$

* Discharge ratio, $Q_r = A_r . V_r$

$$= L_r^2 \times \frac{1}{L_r} = L_r$$

* Mass ratio, $M_r = \rho_r L_r^3$

* Momentum ratio $= M_r . V_r$

$$= \rho_r L_r^3 . \frac{1}{L_r} = \rho_r . L_r^2$$

* Force ratio $= M_r . a_r$

$$= \rho_r L_r^3 . \frac{1}{L_r^3} = \rho_r$$

* Work ratio $= F_r . L_r = \rho_r L_r$

* Pressure ratio $= \frac{F_r}{A_r} = \frac{\rho_r}{L_r^2}$

* Power ratio $= F_r . V_r$

$$= \frac{P_r .}{L_r}$$

* Darcy's friction factor ratio.

$$h_f = \frac{4fLV^2}{2gD.}$$

$$f = \frac{h_f . 2g . D}{L V^2}$$

$$\cancel{(f)_r = L_r . \frac{1}{L_r^3} \times L_r^2 = 1}$$

$$f_r = (h_f)_r . \frac{g_r}{V_r^2} . \left(\frac{D}{L}\right)$$

For same location, $(g_r = 1)$.

$$f_r = \frac{L_r . 1}{1/L_r^2} = \underline{\underline{L_r^3}}$$

→ Euler's Law of Similarity

$$\text{Euler's no., } E_u = \sqrt{\frac{\text{Inertia force}}{\text{Pressure force}}}$$

$$\frac{P}{\rho g} = . \frac{V^2}{2g}.$$

$$\frac{V}{\sqrt{\frac{2P}{\rho}}} = \text{constant} = \text{Euler's no.}$$

$$(E_u)_m = (E_u)_p . \Rightarrow \left(\frac{V}{\sqrt{\left(\frac{2P}{\rho}\right)}}\right)_m = \left(\frac{V}{\sqrt{\frac{2P}{\rho}}}\right)_p .$$

$$V_r = \frac{V_m}{V_p} = \sqrt{\left(\frac{\Delta P}{\rho}\right)_r}$$

Eulers law is important where pressure force is important such as cavitation studies.

→ Mach Law of Similarity

$$Ma = \sqrt{\frac{\text{Inertia force}}{\text{Elastic force}}}$$

Mach law is important wherever elastic forces are important such as high speed bodies like rockets, in compressible flows.

$$Ma = \frac{V}{C} = \frac{\text{velocity of body or flow}}{\text{velocity of } \cancel{\text{flow}} \text{ sound}}$$

$$\text{Velocity of sound, } C = \sqrt{\frac{K}{\rho}}$$

where $K \rightarrow$ bulk modulus of elasticity of fluid system.

$\rho \rightarrow$ density of fluid

$$Ma = \frac{V}{C} = \frac{V}{\sqrt{K/\rho}}$$

$$(Ma)_m = (Ma)_p$$

$$\left(\frac{V}{\sqrt{K/\rho}}\right)_m = \left(\frac{V}{\sqrt{\frac{K}{\rho}}}\right)_p$$

$$\Rightarrow V_r = \sqrt{\left(\frac{K}{\rho}\right)_r}$$

⊙ If mach no., $m < 0.3$, fluids are treated as incompressible ie, compressibility is neglected.

$m_a < 1$; Subsonic flow

$m_a = 1$; Sonic flow

$1 < m_a < 3$; Supersonic

$m_a > 3$; Hypersonic flow

→ Weber's Law of Similarity

$$\text{Weber no., } W = \sqrt{\frac{\text{inertia force}}{\text{surface tension}}}$$

$$= \frac{V}{\sqrt{\frac{\sigma}{\rho l}}}$$

$$(W)_m = (W)_p$$

$$\left(\frac{V}{\sqrt{\frac{\sigma}{\rho l}}}\right)_m = \left(\frac{V}{\sqrt{\frac{\sigma}{\rho l}}}\right)_p$$

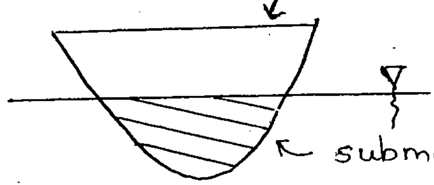
$$\Rightarrow V_r = \sqrt{\frac{\sigma_r}{\rho_r L_r}}$$

Weber's Law is important wherever surface tension is important such as in the analysis of capillarity studies and formation of spherical bubbles or droplets.

25th Sept, THURSDAY

→ Scale Effect

In certain fluid systems more than one force will be predominant. If only effect of one force is considered and others are neglected, the resulting error is called 'Scale Effect'.



For eg: in the case of a ship, the submerged part is influenced by viscous force. Hence Reynold's Law to be applied. The open part is influenced by gravitational force and hence Froude's Law to be applied. In case only one force is considered (gravitational force), the error is scale effect.

In case, both gravity and viscous force to be considered the relation is:

$$V_r = \sqrt{g_r}\sqrt{L_r} \rightarrow \text{Froude's Law.}$$

$$= \frac{\nu_r}{L_r} \rightarrow \text{Reynold's Law}$$

$$\frac{\nu_r}{L_r} = \sqrt{g_r}\sqrt{L_r}$$

Say $g_r = 1$; $\therefore \boxed{\nu_r = (L_r)^{3/2}}$

07. $V_r = \frac{1}{L_r}$

$$\frac{V_m}{V_p} = \frac{1}{1/16}$$

$$\frac{V_m}{8} = 16. \Rightarrow V_m = \underline{\underline{128\ m/s}}$$

08. $\nu_1 = 0.01$ stoke $\nu_2 = 0.03$ stoke

$V_1 = 2 m/s$ $V_2 = ?$

$d_1 = 15$ cm. $d_2 = 15$ cm.

Flow through pressure pipes is governed by Reynold's Law.

$$\left(\frac{Vd}{\nu}\right)_1 = \left(\frac{Vd}{\nu}\right)_2$$

$$\frac{V_1}{V_2} = \frac{\nu_1}{\nu_2}$$

$$\Rightarrow V_2 = \frac{0.03}{0.01} \times 2 = \underline{\underline{6\ m/s}}$$

09. $L_r = 1/30$.

Model	Prototype.
Tunnel	Ocean
$\nu_m = 1.51 \times 10^{-5}$	$V_p = 15$ km/hr.
	$\nu_p = 1.02 \times 10^{-6}$

$$\left(\frac{Vd}{\upsilon}\right)_m = \left(\frac{Vd}{\upsilon}\right)_p.$$

$$L_r = \frac{d_m}{d_p} = \frac{1}{30}$$

$$\frac{V_m}{V_p} = \frac{\upsilon_r}{L_r} = \frac{1.51\times10^{-5}/1.02\times10^{-6}}{1/30}$$

$$V_m = \frac{1.51\times10}{1.02}\times 30 \times 15 \times \frac{5}{18} = \underline{\underline{1850.5}}\ m/s$$

10 According to Froude's Law, $a_r = 1$.

$$\therefore\ a_m = a_p = 1.3\ m/s^2$$

11. $L_r = 1/16.$

$$\frac{Q_m}{Q_p.} = (L_r)^{5/2}.$$

$$Q_m = 1024 \times \left(\frac{1}{16}\right)^{5/2} = \underline{\underline{1\ m^3/s}}$$

12. $T_r = \sqrt{L_r}$

$$\frac{T_m}{T_p} = \sqrt{\frac{1}{25}}$$

$$\frac{10}{T_p} = \frac{1}{5} \Rightarrow T_p = \underline{\underline{50}}\ \text{minutes}$$

13. $L_r = \dfrac{1}{10}$

$F_m = 50\ N.$ $\qquad$ $F_p = ?$

$V_m = 5\ m/s$ $\qquad$ $V_p = 10\ m/s.$

According to Reynold's Law, $V_r = \dfrac{1}{L_r}$

According to Froude's Law, $V_r = \sqrt{L_r}$

* The given data is not satisfying any of these laws.

Hence we have to work out from fundamentals.

$$F = ma = \rho L^3 \frac{L}{T^2} = \rho L^2 \frac{L^2}{T^2} = \rho L^2 v^2$$

$$\left(\frac{F}{\rho L^2 V^2}\right)_m = \left(\frac{F}{\rho L^2 V^2}\right)_p$$

$$F_p = F_m \cdot \frac{V_p^2}{V_m^2} \times \left(\frac{L_p}{L_m}\right)^2$$

$$= 50 \times \left(\frac{10}{5}\right)^2 \times 10^2 = \underline{\underline{20000\ N}}$$

14. $L_r = \frac{1}{25}$

$V_p = 10\ m/s$

$$\frac{V_m}{V_p} = \sqrt{L_r}$$

$$V_m = 10\sqrt{\frac{1}{25}} = \underline{\underline{2\ m/s}}$$

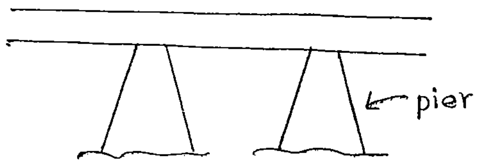
(Data is silent about viscosities. Hence ship analysis shall be done using Froude's Law)

15. $(h_j)_r = L_r$

$$\frac{(h_j)_m}{(h_j)_p} = \frac{1}{20}$$

$$\frac{0.2}{(h_j)_p} = \frac{1}{20} \Rightarrow (h_j)_p = \underline{\underline{4\ m}}$$

16. $F_r = L_r^3$ (Froude's Law).



$$\frac{F_m}{F_p} = \left(\frac{1}{25}\right)^3$$

$$F_p = 5 \times 25^3 = \underline{\underline{78.125\ kN}}$$

The wave force acting on bridge piers is subj. to gravitation force. Hence Froude's Law to be applied.

17. $n_r = (L_r)^{1/6}$

$$\frac{n_m}{n_p} = \left(\frac{1}{100}\right)^{1/6} \Rightarrow n_p = 0.013 \times 100^{1/6}$$

$$= \underline{\underline{0.028}}$$

18. $\dfrac{2q_p^2}{g} = y_1 y_2 (y_1 + y_2).$

$$q_p = \left(\frac{0.5 \times 1.5\,(0.5+1.5)}{2} \times g\right)^{1/2} = 2.712$$

$$\frac{q_m}{q_p} = L_r^{3/2}$$

$$q_m = 2.712 \times \left(\frac{1}{9}\right)^{3/2} = \underline{\underline{0.1\ m^3/s/m}}$$

21. $L_H = \dfrac{1}{40}\ ;\ L_V = 1/9.$

$$\frac{Q_m}{Q_p} = L_H \cdot (L_V)^{3/2}$$

$$\frac{Q_m}{Q_p} = \frac{1}{40} \cdot \left(\frac{1}{9}\right)^{3/2}$$

$$\Rightarrow Q_p = \underline{\underline{1080}}\ lps.$$

23. $L_H = \dfrac{1}{1000}\ ;\ L_V = \dfrac{1}{100}$

$$\frac{q_m}{q_p} = (L_V)^{3/2}.$$

$$\frac{0.1}{q_p} = \left(\frac{1}{100}\right)^{3/2} \Rightarrow q_p = \underline{\underline{100}}\ m^2/s$$

24. $L_H = \dfrac{1}{150}\ ,\ L_V = \dfrac{1}{60}$

The actual time interval b/w two successive high tides in a sea, T_p = 12 hours 24 minutes = 744 minutes.

For distorted models, according to Froude's law,

$$T_r = \frac{L_H}{\sqrt{L_V}}.$$

$$\frac{T_m}{744} = \frac{1/150}{1/\sqrt{60}} \Rightarrow T_m = \underline{\underline{38.42}}\ \text{minutes}$$

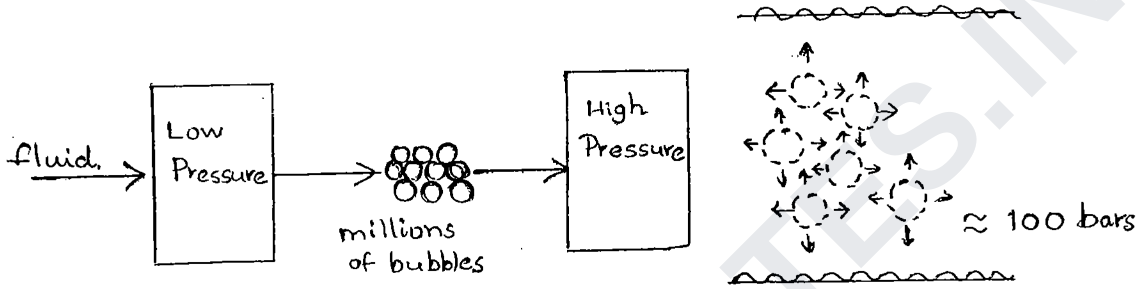
025 $V_r = 0.0894.$

$$V_r = (L_r)^{3/2}$$

$$L_r = (V_r)^{2/3} = (0.0894)^{2/3} = 0.199$$

$$\Rightarrow L_r = 1:5$$

→ Cavitation in Hydraulic Machines.



Cavitation is due to low pressure (less than vapour pressure of flowing fluid).

Vapour pressure of water at room temp. is 2.5 m of water (absolute) or −7.8 m of water (gauge). This is also called 'Separation Head of water'.

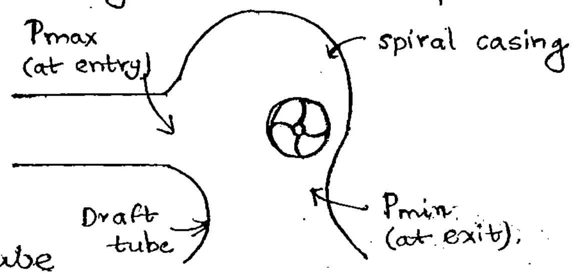
The formation of bubbles is called Cavitation. When these bubbles reach a high pr. region, they blast or blow or condens. Due to this, lot of energy is released, which acts on adjacent machine parts damaging them. This damage is called **Pitting**.

NOTE: Efficiency of machines will reduce sharply due to cavitation.

* Cavitation in Turbines

a) Pelton wheel : no cavitation

Because all its parts are subjected to atmospheric pressure.



b) Reaction Turbine

- cavitation possible at exit of reaction turbine or entry of draft tube as pressure is min. at this point.

- Outlet tip of blade of reaction turbine is prone for cavitation.
- Convex sides of blades are prone for cavitation

Cavitation studies were done in hydraulic machines by Thoma. Thoma has conducted no. of experiments and gave a coeffecient called "Critical Cavitation Parameter" (σ_c) in terms of specific speed.

For any reaction turbine, actual cavitation parameter, σ

$$\sigma = \frac{H_a - H_v - H_s}{H_{net}}$$

$H_a \rightarrow$ atmospheric pressure head (in m) (absolute).

$H_v \rightarrow$ vapour pressure head (in m) (absolute)

$H_s \rightarrow$ turbine setting (height at which turbine is installed above tail race)

$H_{net} \rightarrow$ net operating head of the turbine

For no cavitation, $\sigma \not< \sigma_c$

* Measures to control cavitation :

(i) If H_s increases, σ decreases and the turbine is prone for cavitation. Hence to avoid cavitation, if required, turbine may be submerged under water, which gives −ve value of H_s.

(ii) The last resort will be to reduce specific speed so that σ_c reduces

Q. Critical cavitation parameter, $\sigma_c = 0.1$. Net operating head, h_{net} = The vapour pressure head of water = −7.8 m. Atmospheric pr. head = 10.3 (absolute). The height at which turbine to be installed to avoid cavitation is

$H_v = 10.3 - 7.8 = \underline{\underline{2.5\ m}}$

$H_a = 10.3\ m$ (abs).

$H_{net} = 30\ m$.

$$\sigma_c = \frac{H_a - H_v - H_s}{H_{net}}$$

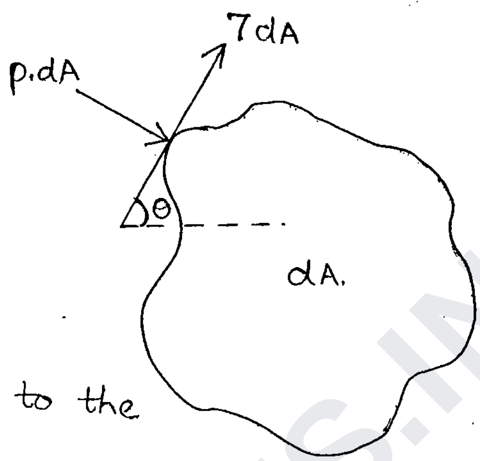
$$0.1 = \frac{10.3 - 2.5 - H_s}{30}$$

$$H_s = \underline{\underline{4.8\ m}}$$

4th Sept,
SATURDAY

11 FORCES ON SUBMERGED BODIES

Consider an infinitesmally small element of area dA. Tangent to which makes an angle of θ with horizontal.



$\tau dA \rightarrow$ shear force tangential to the element.

$p.dA \rightarrow$ normal force due to pressure.

* **Drag Force, F_D**

The component of pressure and shear forces resolved in the direction of flow.

$$F_D = \int(\underbrace{\tau dA \cos\theta}_{\text{shear drag (or) friction drag}} + \underbrace{p\, dA \sin\theta}_{\text{pressure drag (or) form drag}})$$

Pressure drag is due to differential pressures on either side of a body.

In case of ideal fluids, viscosity is zero and hence shear or skin friction drag is zero.

In case of symmetrical bodies, at high Reynold's numbers pressures will be same on both the sides. Hence they balance each other. ∴ pressure drag is zero.

∴ Total drag is zero in case of fluid flowing is ideal and the body is perfectly symmetrical.

* Lift Force, F_L

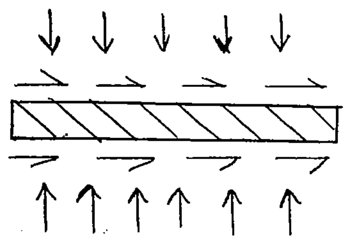
Component of pressure and shear forces in a direction ⊥r to direction of flow.

$$F_L = \int \tau dA \sin\theta - \int p dA \cos\theta.$$

NOTE :

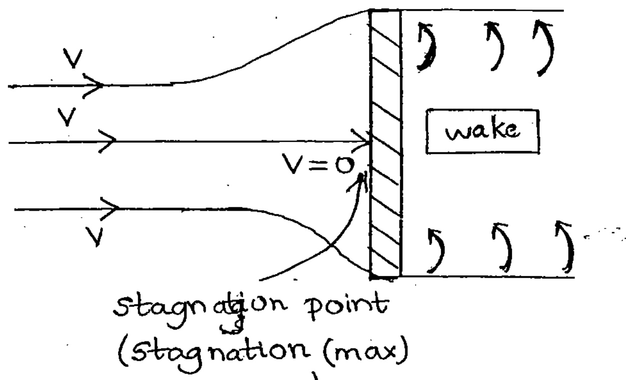
For Ideal fluids flowing over symmetrical bodies, $F_L = 0$.

Case 1 : Flow Over Horizontal Flat Plate.



In case of flow over flat plate, only shear friction f drag exists. Pressure or form drag is negligible because of perfect symmetry.

Case 2 : Flow over Vertical Disc of negligible thickness



At the u/s, velocity tends to zero, called 'Stagnation point' and the pressure is maximum on u/s. The disturbed region on the d/s of an obstruction is called wake.

In order to suppress the disturbances, in the wake region lot of energy is destroyed. and pressure tends to be less. Hence there will be differential pressure on u/s & d/s. This diff. pressure causes pressure or form drag. Hence in this case, total drag primarily consist of pressure drag as shear drag is negligible due to negligible plate thickness.

Case 3: Flow over a Sphere.

(i) Approaching Flow has Laminar Boundary Layer.

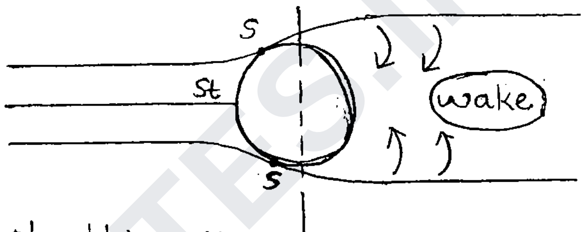
For flow over sphere,

$$Re < 2 \times 10^5$$

For flow over flat plate,

$$Re < 5 \times 10^5$$

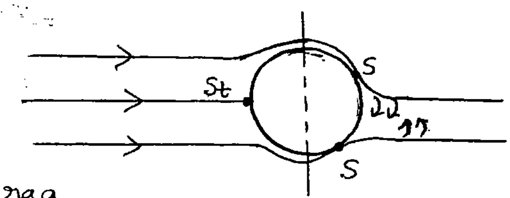
In this case, two seperations points will form towards u/s of sphere with an angle of approximately 60°.



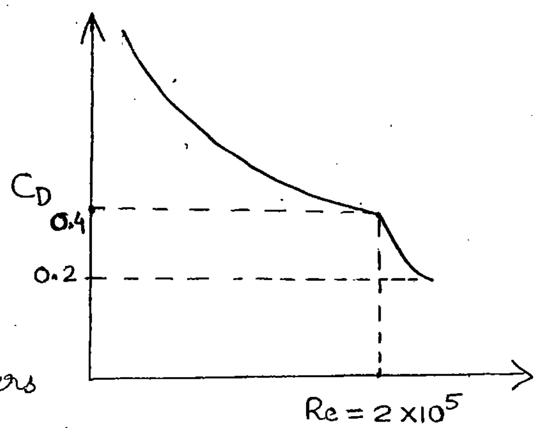
Because of this the size of wake is very large and pressure drag is more. Even shear drag is present, but the magnitude of it is less than that of pressure drag.

(ii) Approaching Flow has Turbulent Boundary Layer.

In this case seperation points will form on d/s. Hence the size of wake is small. Hence pressure drag is less compared to that of case (i) of LBL.

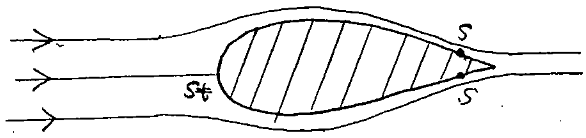


At around $Re = 2 \times 10^5$, the C_D decreases to half. It means drag resistance decreases. It has practical applications in cricket & golf. When a new ball is given, players rub it make it rough and reduce the drag resistance as the BL converts from LBL to TBL



Case 4: Aerofoil Shape

In an aerofoil, seperation points 'll form almost towards d/s.



Streamlined Body

Hence the pressure drag is negligible and shear drag is predominant one.

Streamlined body: The body which suppresses the wake to the max. extend.

Bluff body: The body which has max. size of wake
For eg: vertical disc, spherical ball, cylindrical bodies.

$$F_{D\,(\text{streamlined})} \approx \frac{1}{40} F_{D\,(\text{vertical disc})} \quad \{\text{shear drag} \lll \text{pressure drag}\}$$

$$F_{D\,(\text{sphere})} \approx \frac{1}{3} F_{D\,(\text{vertical disc})}$$

→ Formula for Drag Force.

$$F_D = \frac{C_D A \rho V^2}{2}$$

where $C_D \rightarrow$ coeffecient of drag.

$A \rightarrow$ characteristic area. (= exposed area in case of plates, spheres)

A = contact surface area in case of boats, ships.

$\rho \rightarrow$ mass density of fluid.

$V \rightarrow$ free stream velocity. $(= u_\infty)$

$$F_D \propto \frac{1}{Re} \quad \& \quad C_D \propto \frac{1}{Re}$$

$$C_D = \frac{24}{Re} \; ; \text{ for } Re < 0.1$$

$$F_D = \frac{C_D A \rho v^2}{2} = \frac{24}{Re} \frac{A \rho v^2}{2}$$

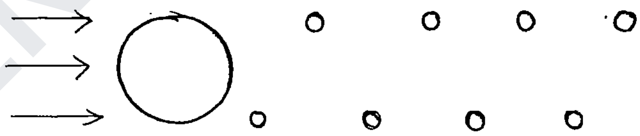
$$= \frac{24}{\frac{\rho v d}{\mu}} \cdot \frac{\rho v^2}{2} \cdot \frac{\pi d^2}{4}$$

$$\boxed{F_D = 3\pi \mu V d} \quad ; \text{ for sphere}$$

Ratio of pressure & shear drag $= 1:2$

Case 5: Flow over cylindrical body

If $Re > 30$, alternate released of vortices called Rankine Vortex street (or) Rankine Vortex trail which causes vibration of cylindrical bodies like chimneys, cables etc.



At around $Re > 250$, the frequency of vortex shedding is equal to natural frequency of the fluid system causing Resonance. It gives music or singing of cable.

$$S = \frac{nd}{V}$$

where $S \rightarrow$ Strouhal Number

$n \rightarrow$ frequency of vortex shedding

$d \rightarrow$ diameter of cylinder.

$V \rightarrow$ free stream velocity.

Strouhal no. depends upon Reynold's number.

For $Re > 1000$, $S = 0.21$

For $Re < 1000$, $S = 0.20\left(1 - \frac{20}{Re}\right)$

$$F_L = \frac{C_L A \rho v^2}{2}$$

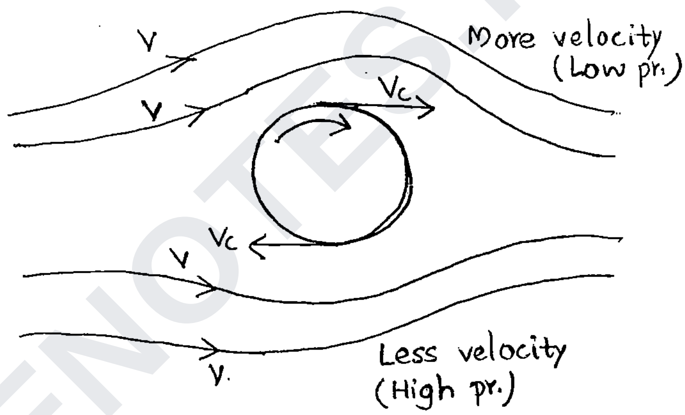
$C_L \rightarrow$ coeffecient of lift.

$A \rightarrow$ projected area (plan projection)

* Magnus Effect :

The lift force obtained by the product of spinning (circulation) with translation.

when the spin is given in downward direction, the body gets lifted in upward direction.



Q Of the following cases, which will have more skin friction drag ?

a) Tennis ball b) Parachute c) Arrow d). Cyclist.

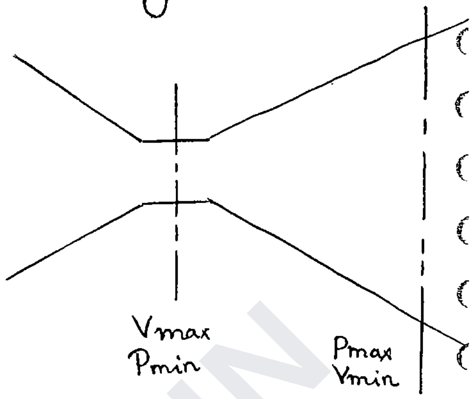
Parachute, arrow & cyclist will have more skin friction drag.

Q. Drag on cylinders and spheres reduces if the reynold's number is about 2×10^5 as flow seperation is delayed due to onset of turbulence.

Pressure drag is due to formation of disturbed region called 'wake'. Disturbance is caused due to formation of eddices and vortices. which takes place due to flow seperation.

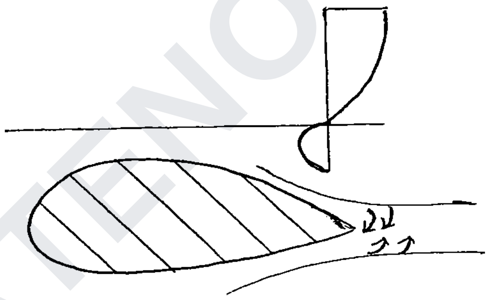
Flow seperation is because of :

(i) Adverse Pressure gradient (or) +ve pressure gradient.

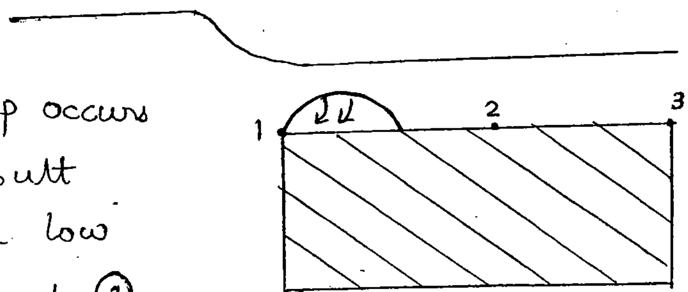


When a pipe diverges from small area, pressure increases in the direction of flow. (pressure hill) ; ie +ve pressure gradient. The and streamlines may not attach to the diverging boundary properly. Hence KE of flow cannot counter the +ve uphill. Hence, there will be a reverse flow in the form of eddies and vortices.

(ii) Velocity tends to zero (or) -ve velocity with an influx point.



(iii) Sharp corners over the bodies subjected to flow



When flow occurs over sharp corners, a jump occurs the sharp point. As a result eddies are formed and a low pressure region is created at 1.
So cavitation occurs there.
To minimise the effect, chamfered corners are provided.

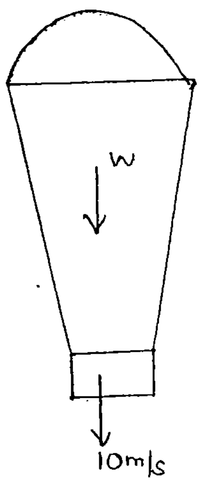
P-161

1. $P = C_D \rho A \frac{V^2}{2} \cdot V$

$\Rightarrow P \propto V^3$

$\frac{P_2}{P_1} = \left(\frac{V_2}{V_1}\right)^3 = \left(\frac{2V}{V}\right)^3 = \underline{\underline{8}}$

2.



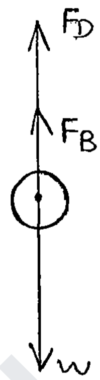
$w = 80\,kg$

$= 784.8\ N.$

$\rho_{air} = 1.2\ kg/m^3$

$C_D = 0.8$

Neglecting buoyancy forces,



$$W = F_D$$

$$784.8 = C_D\,\rho\,\frac{AV^2}{2}$$

$$= 0.8 \times 1.2 \times \frac{\pi D^2}{4} \times \frac{10^2}{2}$$

$$D = \underline{\underline{4.56\,m}}$$

3 $\rho_{air} = 1.2\ kg/m^3$, $\nu = 1.6$ stokes $= 1.6 \times 10^{-4}\ m^2/s.$

$V = 8\,m/s$, $D = 60\,mm = 0.06\ m.$

$$Re = \frac{\rho V D}{\mu} = \frac{8 \times 0.06}{1.6 \times 10^{-4}} = 3000$$

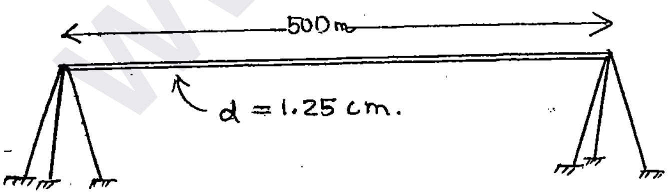
$C_D = 0.5$ for $100 < Re < 2 \times 10^5.$

Weight of ball, $w = F_D = \frac{C_D\,\rho A V^2}{2.}$ (neglecting F_B)

$$= \frac{0.5 \times 1.2 \times \frac{\pi}{4} \times 0.06^2 \times 8^2}{2}$$

$$= \underline{\underline{0.054}}\ N$$

4.



$V = 100$ kmph $= \underline{27.7}$ m/s

$\gamma_{air} = 13.4\ N/m^3.$

$\nu_{air} = 1.4 \times 10^{-5}\ m^2/s$

$\because Re > 10^4 \Rightarrow C_D = \underline{\underline{1.2}}$

V in transverse direction

$$Re = \frac{VD}{\nu} = \frac{27.7 \times 0.0125}{1.4 \times 10^{-5}}$$

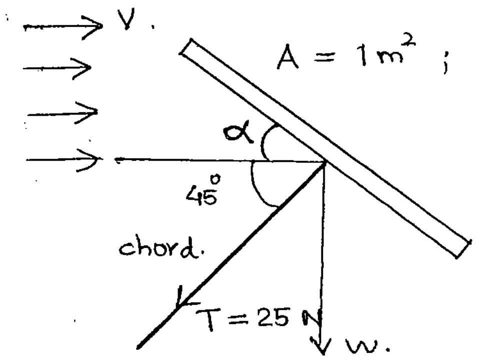
$$= \underline{\underline{24732.14}}$$

Projected area is a rectangle. ($L \times d$)

$$F_D = C_D \frac{\rho A V^2}{2} = 1.2 \times 500 \times 0.0125 \times \frac{13.4}{9.81} \times \frac{27.7^2}{2}$$

$$= \underline{\underline{3931.651 \text{ N}}}$$

5.



$A = 1 m^2$; $V = 54$ kmph $= 15$ m/s

$\gamma_{air} = 12.25$ N/m^3.

$\alpha \rightarrow$ angle of attack.

Drag Force, F_D = resultant component of various forces in the direction of free stream velocity.

$$F_D = C_D \frac{\rho A V^2}{2} = 25 \cos 45.$$

$$C_D \times 1 \times \frac{12.25}{9.81} \times \frac{15^2}{2} = 25 \cos 45.$$

$$C_D = \underline{\underline{0.126}}$$

Lift Force = Resultant of all forces in the direction normal to free stream velocity.

$$F_L = C_L \frac{\rho A V^2}{2} = 2.5 + 25 \sin 45.$$

$$C_L \times \frac{12.25}{9.81} \times 1 \times \frac{15^2}{2} = \underline{20.177}$$

$$\Rightarrow \quad C_L = \underline{\underline{0.1436}}$$

6. $C_{D1} = 1 \qquad C_{D2} = 0.75$

$P_1 = P_2$

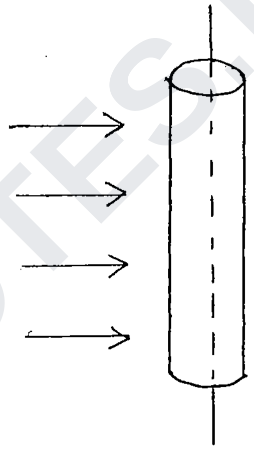
$C_{D1} V_1^3 = C_{D2} V_2^2$

$$\left(\frac{V_2}{V_1}\right)^3 = \frac{C_{D1}}{C_{D2}} = \frac{1}{0.75}$$

$$V_2 = 1.1 \; V_1$$

$\Rightarrow 10.06\ \%$ increase in velocity.

8. ~~Data is missing to~~



$$F_D = C_D \, \rho A \frac{V^2}{2}$$

$$30 = C_D \cdot S \cdot \rho_w \times d \times L \times \frac{V^2}{2}$$

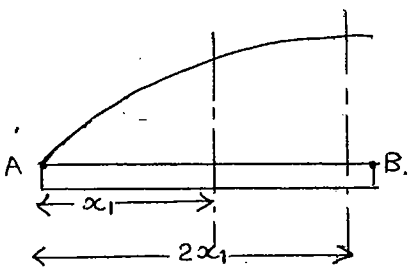
$\frac{\rho_s}{\rho_w} = S$

$$= C_D \cdot 0.8 \times 1000 \times 0.08 \times 0.2 \times \frac{0.5^2}{2}$$

$$\Rightarrow C_D = \underline{\underline{18.75}} \quad (?)$$

Boundary layer thickness increases in the direction of flow: a) at the leading edge (or) at the front part of plate velocity gradient is steep (or) high. For a given fluid, viscosity is constant. We know,

$$\text{Shear stress, } \tau = \mu \cdot \frac{du}{dy}$$



$\therefore \; \tau_{\text{leading edge}} > \tau_{\text{trailing edge}}$

Initial boundary layer formed will act as an obstruction for the advancing layer. and a sequence follows

$\delta \propto \sqrt{x}$	; for	LBL
$\delta \propto x^{4/5}$	; for	TBL

NOTE :

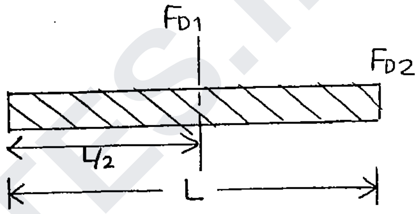
Growth of TBL is faster than growth of LBL, other conditions remaining same as $x^{4/5} > x^{1/2}$

In the given question,

$$\frac{\delta_2}{\delta_1} = \sqrt{\frac{x_2}{x_1}} = \sqrt{\frac{2x_1}{x_1}}$$

$$\Rightarrow \quad \delta_2 = \sqrt{2}\,\delta_1$$

Q. $\frac{F_{D1}}{F_{D2}} = ?$



Assume $C_D \propto (Re\,x)^{-1/2}$

ie $C_D \propto \frac{1}{\sqrt{Re\,.\,x}}$

Question is assumed with LBL over a flat plate for which the relation is :

$$\frac{\delta}{x} = \frac{K}{\sqrt{Re\,.\,x}}$$

$$\delta = \frac{kx}{\sqrt{\frac{V}{\nu}x}} = k\sqrt{\frac{\nu x}{V}}$$

$$\delta \propto \frac{1}{\sqrt{Re\,x}}$$

According to Blasius, $K = 5$.

$$F_D = C_D\,A\,\frac{\rho V^2}{2}$$

$$F_D \propto C_D\,.\,A$$

(L changing from L/2 to L ∴ A not constant)

$$\frac{F_{D1}}{F_{D2}} = \frac{C_{D1}\,.\,A_1}{C_{D2}\,.\,A_2} = \sqrt{\frac{Re_2}{Re_1}} \times \frac{L}{2L} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

But $Re = \frac{VL}{\nu} \Rightarrow Re \propto L \quad \therefore \frac{Re_2}{Re_1} = \frac{2L}{L} = 2$