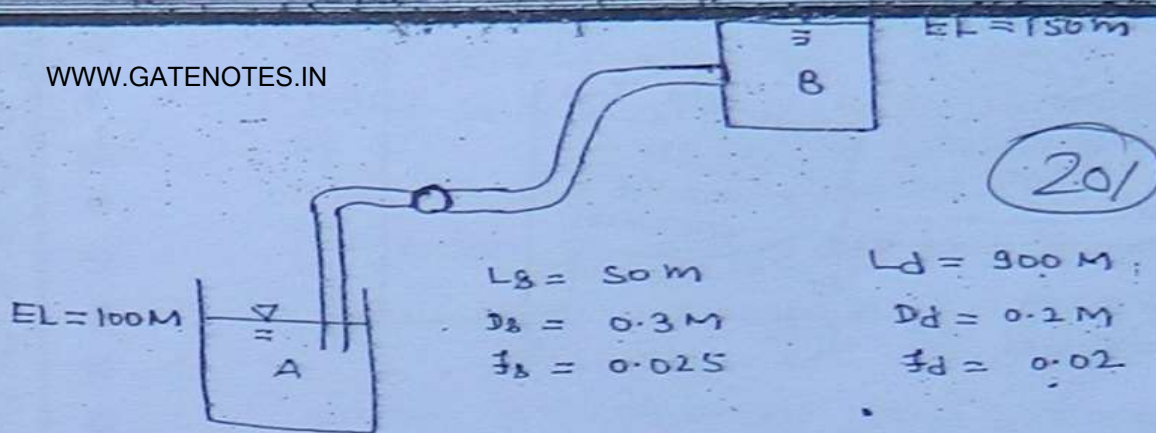




Head added by the pump.

$$H_p = \text{static head} + h_{fs} + h_{fd}$$

$$= (150 - 100) + \frac{f_s \cdot L_s \cdot Q^2}{12.1 D_s^5} + \frac{f_d \cdot L_d \cdot Q^2}{12.1 D_d^5}$$

$$\therefore h_{fs} = \frac{0.025 \times 50 \times Q^2}{12.1 \times (0.3)^5} = 42.5 Q^2$$

$$h_{fd} = \frac{0.02 \times 900 \times Q^2}{12.1 \times (0.2)^5} = 4648.76 Q^2$$

$$\therefore H_p = 50 + 42.5 Q^2 + 4648.76 Q^2$$

$$= 50 + 4691.26 Q^2 = 80 - 7000 Q^2$$

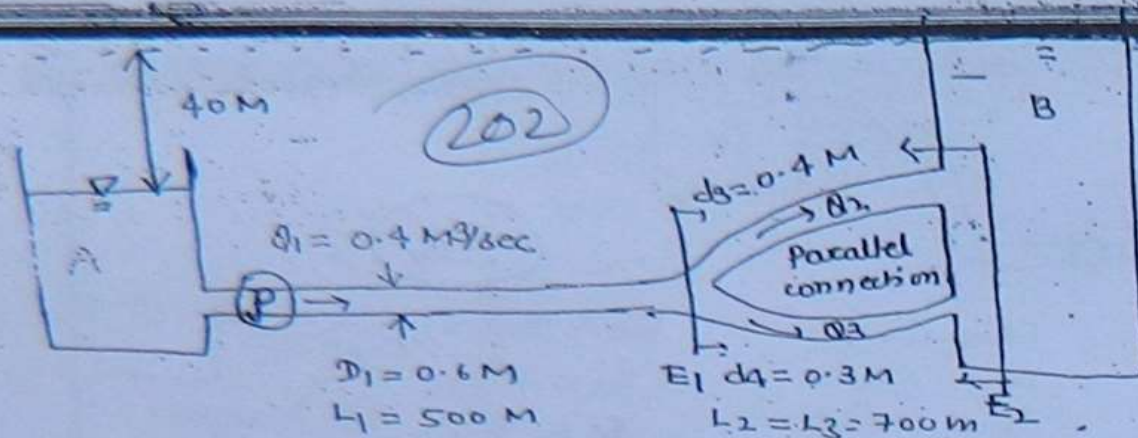
$$\therefore Q = 0.05 \text{ m}^3/\text{sec}$$

$$\therefore H_p = 80 - 7000 \times (0.05)^2$$

$$= 62.5 \text{ m}$$

$$\therefore \text{Power} = \omega Q H_p$$

Two reservoirs A & B are connected by a pipe system, consisting of ^{one} 60 cm pipe having ^{500m} ~~50m~~ length which branches thereafter into two pipes of 40 cm dia. & 30 cm dia. each 700 m long. A pump situated near reservoir A discharges 0.4 m³/sec through the pipe system. The difference in the reservoir level is such that reservoir ^{level} B is 40 m above than reservoir A. Assuming $f = 0.02$ determine the power required for the pump. ^{Assuming pump efficiency is 50%}



$$h_{f2} = h_{f3}$$

$$\therefore \frac{f L Q_2^2}{12.1 D_2^5} = \frac{f L Q_3^2}{12.1 D_3^5} \Rightarrow \frac{Q_2^2}{Q_3^2} = \left(\frac{D_2}{D_3}\right)^5 = \left(\frac{4}{3}\right)^5$$

$$\therefore Q_2/Q_3 = \sqrt[5]{4 \cdot 21} \quad \text{--- i)}$$

$$\left. \begin{aligned} Q_2 &= 0.268 \\ Q_3 &= 0.13 \end{aligned} \right\}$$

$$Q_2 + Q_3 = 0.4 \quad \text{--- ii)}$$

Total head added by the pump $[P = 309 \text{ kW}]$
 $= 40 + h_L$

NOTE: Head added by the pump is $\frac{h_s}{(40)} + \frac{h_{f1} + (h_{f2} + h_{f3})}{(3.97)}$
 However $h_{f2} = h_{f3}$

A town of 2 Lakh population is to be supplied water from a source, 2500 M away. The Lowest water level in the source is 15M below the water works of the town. The demand of the water is estimated as 150 Lit/capital/day. A pump of 300 H.P. is operated for 15 Hrs. & if the max^m demand is 150% of avg. demand and velocity of flow through pipe is 1.3M/sec and Efficiency of the pump is 70%. Determine the H.G. and friction factor. [Determine $\frac{P}{W+Z}$ the total water required]

$$P = \frac{W \cdot Q \cdot H_{\text{added by the pump}}}{\eta}$$

$\eta \rightarrow$ Efficiency of pump

Water flows in a 80 mm pipe at Reynolds No. 80,000. The pipe is estimated to have an equivalent sand grain roughness of size 0.16 mm. Determine the head loss expected in 500 m length of pipe. If pipe were to act as smooth pipe, how much head loss may be expected. ν of water = $\frac{106 \text{ m}^2}{\text{sec}}$

The following explicit eqn or f may be used

$$\frac{1}{f} = 1.14 - 2 \log_{10} \left[\left(\frac{k}{d} \right) + \frac{21.25}{Re^{0.9}} \right]$$

Where k = equivalent sand grain roughness = 0.16 mm

d = dia. of pipe = 80 mm

Re = Reynolds No. = 80,000

L = 500 m

203

$$Re = \frac{Vd}{\nu} \Rightarrow V = \frac{80,000 \times 10^{-6}}{0.08} = 1 \text{ m/sec.}$$

For Rough pipe $\downarrow$

$$\frac{1}{f} = 1.14 - 2 \log_{10} \left[\left(\frac{0.16}{80} \right) + \frac{21.25}{(80,000)^{0.9}} \right]$$

$$\Rightarrow f = 0.0257$$

$$\therefore \text{Head Loss} = \frac{fLV^2}{2gD} = 8.15 \text{ m}$$

For smooth pipe $\downarrow$

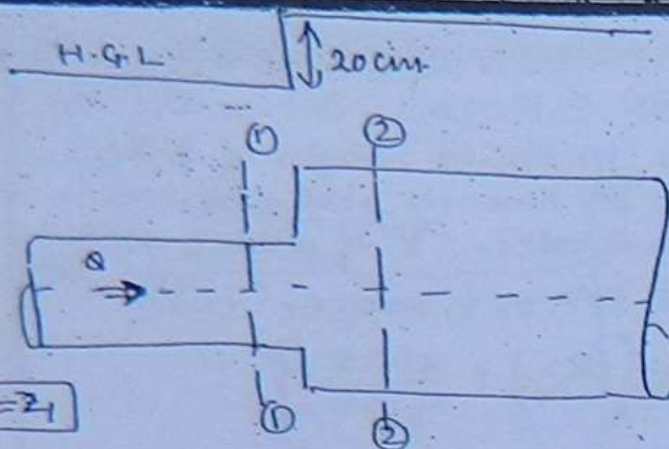
$$K=0$$

$$\frac{1}{f} = 1.14 - 2 \log_{10} \left[\frac{21.25}{(80,000)^{0.9}} \right]$$

$$\therefore f = 0.0187$$

$$\therefore \text{Head Loss} = \frac{8.15}{0.0257} \times 0.0187 = 5.96 \text{ m.}$$

A horizontal pipe having diameter of 0.5 m expands at a junction to one meter pipe having straight length of hydraulic gradient at junction rises by 20 cm. Find the flow rate in the pipe.



204

$$D_1 = 0.5 \text{ m} \quad D_2 = 1.0 \text{ m}$$

$$\text{H.G.L.} \Rightarrow \left(\frac{p_2}{\rho} + Z_2 \right) - \left(\frac{p_1}{\rho} + Z_1 \right) = 0.2$$

$$\therefore \frac{p_2}{\rho} - \frac{p_1}{\rho} = 0.2 \text{ m} \quad \text{--- (1)}$$

Head loss due to sudden expansion

Apply B. Eqn b/w ①-(1) & ②-(2)

$$Z_1 + \frac{p_1}{\rho} + \frac{V_1^2}{2g} = Z_2 + \frac{p_2}{\rho} + \frac{V_2^2}{2g} + \frac{(V_1 - V_2)^2}{2g}$$

$$\therefore \frac{p_2}{\rho} - \frac{p_1}{\rho} = \frac{V_1^2}{2g} - \frac{(V_1 - V_2)^2}{2g} - \frac{V_2^2}{2g}$$

$$0.2 = \frac{V_1^2}{2g} - \frac{V_1^2}{2g} - \frac{V_2^2}{2g} + \frac{2V_1V_2}{2g} - \frac{V_2^2}{2g}$$

$$\Rightarrow 0.2 = \frac{2V_1V_2 - 2V_2^2}{2g} \Rightarrow 2V_1V_2 - 2V_2^2 = 0.4 \times 9.81$$

$$\Rightarrow V_1V_2 - V_2^2 = 0.2 \times 9.81$$

$$V_1 = Q/A_1 = \frac{Q}{\pi/4 D_1^2}$$

$$V_2 = Q/A_2 = \frac{Q}{\pi/4 D_2^2}$$

$$Q = 0.635 \text{ m}^3/\text{sec}$$

→ To know direction of flow, know TE_1 & TE_2

If $TE_1 > TE_2$ then flow from ①-(1) to ②-(2)

205

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channel with hump: J

206

prob 1 (a) A Rect. channel has a width of 2.0 m and carries a discharge of $4.80 \text{ m}^3/\text{sec}$ with a depth of 1.6 m. At a certain section a small, smooth hump proposed to be built. calculate the Likely change in the water surface. Neglect the Energy Loss.

step 1

Determine the U/s Flow conditions i.e. whether Flow is subcritical or supercritical.

$$q = \frac{4.80}{2} = \frac{4.80}{2.0} = 2.40 \text{ m}^3/\text{sec/m}$$

$$V_1 = \frac{2.40}{1.6} = 1.50 \text{ m/sec}$$

$$F_1 = \frac{V_1}{\sqrt{g y_1}} = \frac{1.5}{\sqrt{9.81 \times 1.6}} = 0.378 < 1$$

[$y_1 = y_1$ since channel is rectangular]

U/s flow is subcritical so the hump will cause a drop in the water surface elevation.

step 2

$$E_1 = y_1 + \frac{V_1^2}{2g}$$

$$= 1.6 + \frac{(1.5)^2}{2 \times 9.81}$$

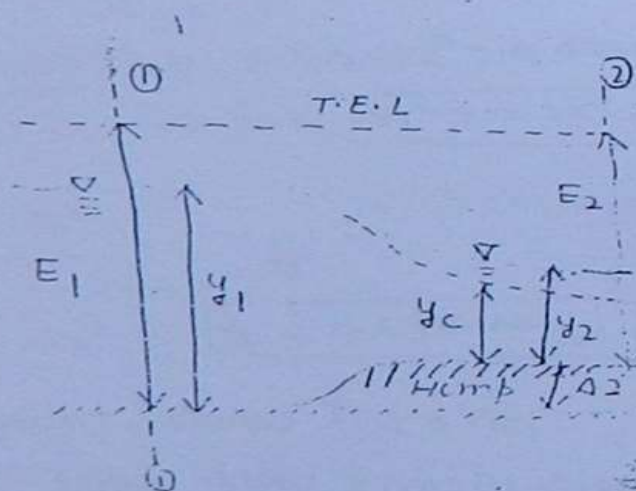
$$= 1.715 \text{ m}$$

similarly at section (2)

$$E_2 = E_1 - \Delta Z$$

$$= 1.715 - 0.10$$

$$= 1.615 \text{ m}$$



step 3

check if the flow condition at Section (2) is critical.

$$y_c = \left[\frac{q^3}{g} \right]^{1/3} = \left[\frac{(2.4)^3}{9.81} \right]^{1/3} = 0.837 \text{ m}$$

$$E_c = 1.5 y_c = 1.256 \text{ m}$$

Since $E_{c2} < E_2$ hence $y_2 > y_c$ and depth y_1 will be remain unchanged.

step 4) calculation of depth y_2

(207)

$$E_2 = y_2 + \frac{V_2^2}{2g} \quad \left[V_2 = \frac{Q}{y_2} \right]$$

$$E_2 = y_2 + \frac{Q^2}{2gy_2^2}$$

By trial & error method

$$1.615 = y_2 + \frac{(2.4)^2}{2 \times 9.81 \times y_2^2} \Rightarrow \boxed{y_2 = 1.481 \text{ m}}$$

(b) At the height of the hump is 0.5 m. Estimate the water surface elevation on the hump and at a section of the hump.

$$V_1 = 1.50 \text{ m/sec}$$

$$F_1 = 0.378 < 1$$

$$E_1 = 1.715 \text{ m}$$

$$y_2 = y_c = 0.837 \text{ m}$$

Available sp. energy at section (2)

$$\Rightarrow E_2 = E_1 - \Delta Z$$

$$E_2 = 1.715 - 0.5 = 1.215 \text{ m}$$

$$E_{c2} = 1.5 y_{c2} = 1.256 \text{ m}$$

→ Since sp. energy at section (2) is greater than E_2 , the available sp. energy at that section. Hence the depth at section (2) will be at the critical depth.

$$\text{Hence } y_2 = y_{c2} = 1.256 \text{ m}$$

The U/s depth y_1 will increase to a depth y_1' such that new sp. energy at the U/s section 1 is

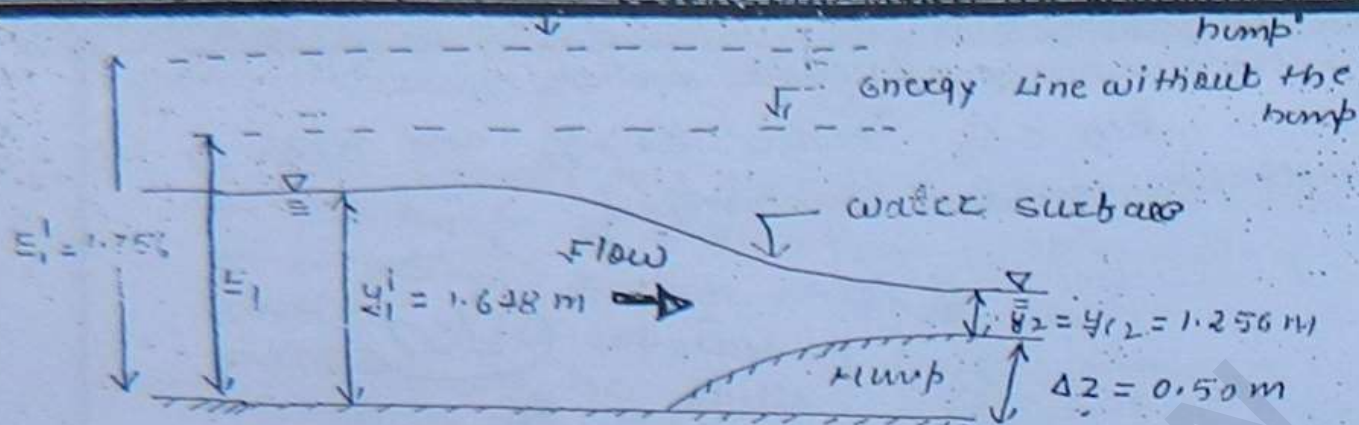
$$E_1' = E_{c2} + \Delta Z$$

$$E_1' = y_1' + \frac{V_1'^2}{2g} = E_{c2} + \Delta Z$$

$$\Rightarrow y_1' + \frac{Q^2}{2gy_1'^2} = 1.256 + 0.5$$

$$\Rightarrow y_1' + \frac{(2.4)^2}{2 \times 9.81 \times y_1'^2} = 1.756$$

$$\Rightarrow \boxed{y_1' = 1.648} \Rightarrow \boxed{y_1' > y_2}$$



Prob 2

A Rect. channel 2.5 m wide carries $6.0 \text{ m}^3/\text{sec}$ of flow at a depth of 0.50 m. Calculate the height of a flat topped hump required to be placed at a section to cause critical flow. The energy loss due to the obstruction by the hump can be taken as 0.1 times the U/s velocity head.

step 1)

$$q = 6.0 / 2.5 = 2.4 \text{ m}^2/\text{sec}/\text{m}$$

$$V_1 = 2.4 / 0.5 = 4.8 \text{ m/sec}$$

$$Fr_1 = \frac{4.8}{\sqrt{9.81 \times 0.5}} = 2.167 > 1 \Rightarrow \text{U/s Flow is supercritical.}$$

step 2)

$$E_1 = 0.50 + \frac{(4.8)^2}{2 \times 9.81} = 1.674 \text{ m}$$

step 3)

Since Flow at section (2) is critical

$$\text{So } \frac{y_c}{3} = \left(\frac{q^2}{g} \right)^{1/3} = \left[\frac{(2.4)^2}{9.81} \right]^{1/3} = 0.837 = y_2$$

$$\text{Now } \frac{V_c^2}{2g} = \frac{y_c}{3} = 0.419 = \frac{V_2^2}{2g}$$

∴ Applying Energy Equation b/w section (1) and (2)

$$E_1 - E_L = y_2 + \left(\frac{V_2^2}{2g} \right) + \Delta z$$

↳ height of hump

$$E_L = 0.1 \frac{V_1^2}{2g} = 0.117 \text{ m}$$

$$\Rightarrow 1.674 - 0.117 = 0.837 + 0.419 + \Delta z$$

$$\Rightarrow \boxed{\Delta z = 0.501 \text{ m}}$$

of $15.0 \text{ m}^3/\text{sec}$ at a depth of 2.0 m . It is proposed to reduce the width of the channel at a hydraulic structure. Assuming the transition to be horizontal and flow to be frictionless determine the water surface elevation y_1 and y_2 of the constriction when the constricted width is 2.50 m and 2.20 m .

Step 1 check the flow (subcritical or supercritical)

$$F_1 = \frac{V_1}{\sqrt{2g y_1}} \quad \left[V_1 = \frac{Q}{B_1 y_1} = \frac{15.0}{3.5 \times 2} = 2.143 \text{ m/sec} \right]$$

$$F_1 = \frac{2.143}{\sqrt{2 \times 9.81 \times 2.0}} = 0.484 < 1$$

The y_1 flow is subcritical and the transition will cause a drop in the water surface.

Step 2 $y_1 = y_1 + \frac{V_1^2}{2g} = 2.0 + \frac{(2.143)^2}{2 \times 9.81} = 2.234 \text{ m}$

Let B_{2m} = minimum width at section (2) which does not cause choking

then $E_{2m} = E_1 = 2.234 \text{ m}$

$$y_{2m} = \frac{2}{3} E_{2m} = \frac{2}{3} \times 2.234 = 1.489 \text{ m}^{**}$$

But $y_{2m}^3 = \left[\frac{Q^2}{g B_{2m}^3} \right]$

$$\Rightarrow B_{2m} = \left[\frac{Q^2}{g y_{2m}^3} \right]^{1/3} = 2.636 \text{ m}$$

Step 3 since $B_2 = 2.50 < B_{2m}$

Hence choking conditions would prevail

The depth at section (2) $y_{2c} = y_2$

$\therefore$ y_1 depth y_1 will increase to y_1'

$$y_2 = \frac{15.0}{B_2} = \frac{15.0}{2.5} = 6.0 \text{ m/sec}$$

$$y_{c2} = \left[\frac{q^2}{g} \right]^{1/3} = \left[\frac{(6)^2}{9.81} \right]^{1/3} = 1.542 \text{ m} \times \times$$

$$E_{c2} = 1.5 y_{c2} = 1.5 \times 1.542 = 2.3136 \text{ m}$$

$$E_1 = E_{c2} = 2.3136 \text{ with new u/s depth of } y_1$$

$$\text{such that } q_1 = y_1 V_1 = 15/3.5$$

$$= 4.2857 \text{ m}^3/\text{s}$$

$$y_1 + \frac{V_1^2}{2g} = 2.3136$$

$$y_1 + \frac{(4.2857)^2}{2 \times 9.81} = 2.3136$$

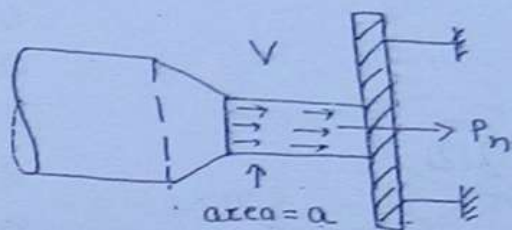
$$y_1 = 2.122 \text{ m}$$

NOTE:

For the same discharge when $B_2 < B_{2m}$ (under choking conditions) the depth at critical section will be different from y_{cm} and depends upon value of B_2 .

1. > Impact of JETS: ↓

Case Ist: when Jet of water strikes normally to a stationary Flat plate



(211)

If Loss of energy due to impact is negligible & surface is smooth so that friction loss is negligible, Force exerted by the jet on the plate is

$$P_n = \rho Q [V - 0]$$

$$P_n = \rho a V^2$$

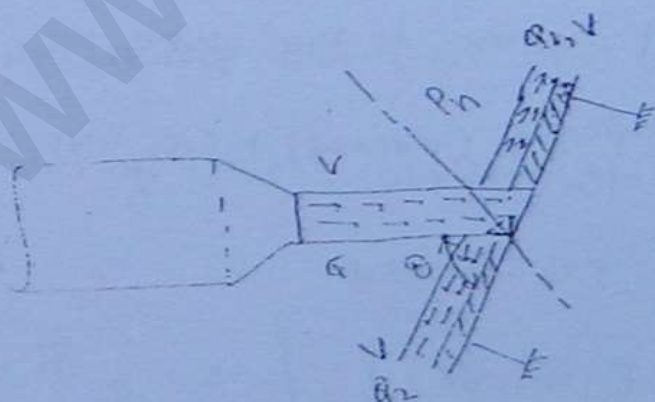
Jet on

work-done by the plate = zero.

[because plate remains stationary]

$a \rightarrow$ area of Jet
 $V \rightarrow$ initial velocity in the direction of flow

Case IInd: Jet strikes on an Inclined Flat stationary plate



P_n = Force exerted by Jet normal to plate

$$= \rho a [V \sin \theta - 0]$$

$$P_n = \rho a V^2 \sin \theta, \text{ [work-done} = 0]$$

Force exerted along the plate = 0

$$F = \rho Q V \cos \theta - \rho Q_1 V - (\rho Q_2 (-V)) = 0$$

$$\Rightarrow Q \cos \theta - Q_1 + Q_2 = 0 \quad \text{--- (i)}$$

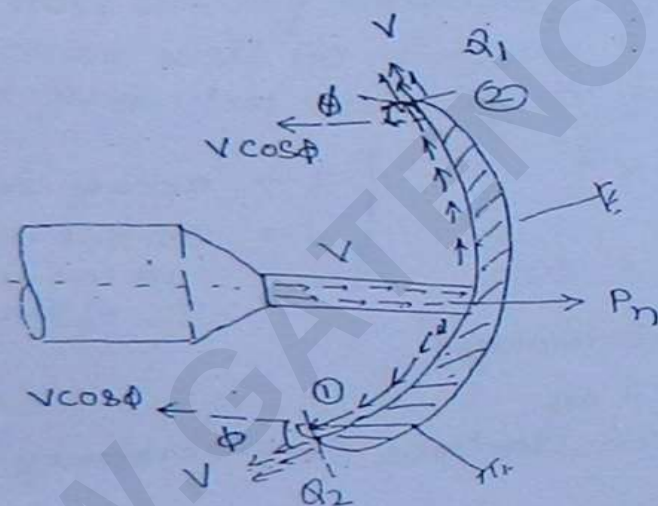
By eqn (i) & (ii)

$$\begin{aligned} Q_1 &= Q/2 [1 + \cos \theta] \\ Q_2 &= Q/2 (1 - \cos \theta) \end{aligned}$$

$$\frac{Q_1}{Q_2} = \frac{1 + \cos \theta}{1 - \cos \theta}$$

212

case IIIrd: Force exerted by Jet on a stationary curved plate



Force exerted by Jet normal to the plate

$$P_n = \rho Q V - [\rho Q_1 (-V \cos \phi) + \rho Q_2 (-V \cos \phi)]$$

$$= \rho Q V + \rho Q \cdot V \cos \phi$$

$$P_n = \rho Q V (1 + \cos \phi)$$

$$P_n = \rho a V^2 (1 + \cos \phi)$$

$$\text{work done} = 0$$

$$= \rho Q$$

$$= \rho \cdot a [v-u]$$

Force exerted by the jet on the plate

$$P_n = \rho \cdot a (v-u) [v-u]$$

$$P_n = \rho a (v-u)^2$$

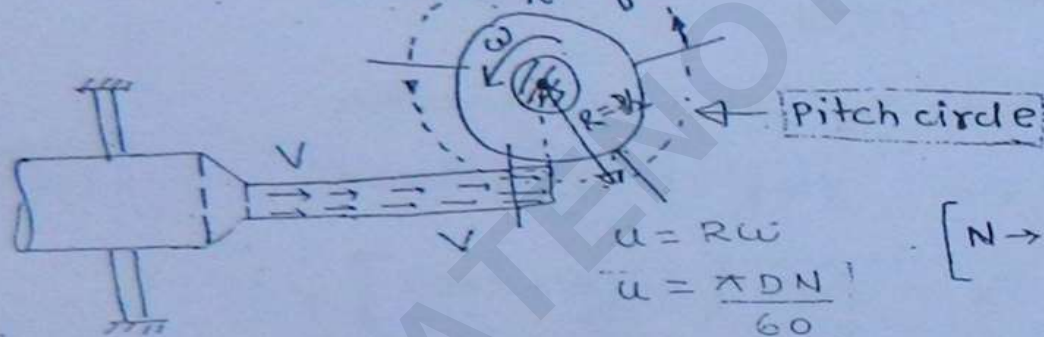
∴ Work-done per sec. by the Jet

$$W = \rho a (v-u)^2 \times u$$

(214)

asc VIth:

Jet strikes on a series of ~~fixed~~ Vanes mounted on a periphery of a wheel



Vane

Mass of water strikes the ~~per sec~~ per sec.

$$= \rho a v = \rho Q$$

$$[Q = av]$$

Force exerted by Jet on Vane will be

$$= \rho a v [v-u]$$

$$= \rho a [v^2 - vu]$$

Work-done by Jet per sec. = $\rho a [v^2 - vu] \times u$

$$= \rho a [v^2 u - u^2 v]$$

$$W = \rho Q [v-u] \times u$$

$$K.E / \text{sec of Jet} = \frac{1}{2} \times \text{Mass Flowing / sec} \times v^2$$

$$K.E. = \frac{1}{2} \rho a v^3$$

Efficiency of Jet

$$\eta = \frac{\text{work-done/sec}}{\text{K.E./sec}}$$

$$\therefore \eta = \frac{\rho a v [v-u] \cdot u}{\frac{1}{2} \rho a v^3} = \frac{2 [v u - u^2]}{v^2}$$

For Maxm. Efficiency of Jet, $\frac{d\eta}{du} = 0$ (215)

$$\therefore \frac{2}{v^2} [v - 2u] = 0$$

$$\Rightarrow \boxed{u = \frac{v}{2}}$$

tangential = $\frac{1}{2}$ Jet velocity

$$\therefore \eta_{\text{Max}} = \frac{2 \left[v \times \frac{v}{2} - \left(\frac{v}{2} \right)^2 \right]}{v^2} = 0.5$$

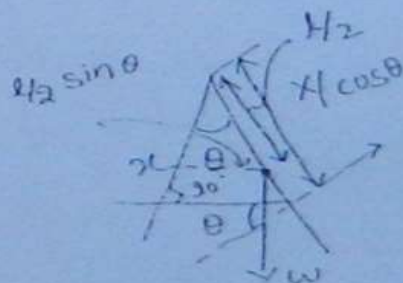
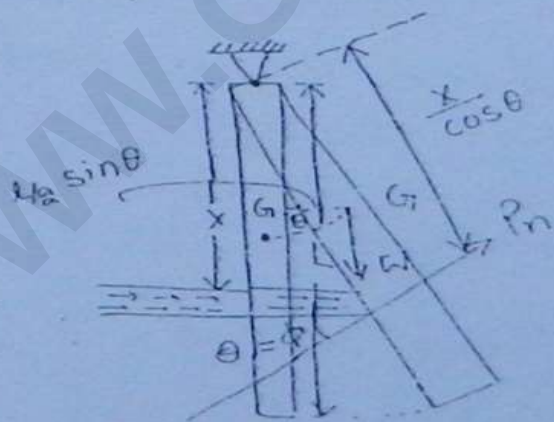
= 50% ans

NOTE:

In above analysis vanes are flat & if vanes are made curved then efficiency may be further increased as in case of Pelton wheel.

Case VIIIth:

Jet of water strikes normal to a hanging vertical plate



$$P_n = \rho a [v \cos \theta - 0]$$

$$P_n = \rho a v^2 \cos \theta$$

a $\rightarrow$ area of jet

$$\cos \theta = \frac{1}{2}$$

→ wt. of plate

$$\therefore (\rho a v^2 \cos \theta) \left(\frac{x}{\cos \theta} \right) = w \left(\frac{L}{2} \sin \theta \right)$$

$$\therefore \sin \theta = \frac{2 \rho a v^2 \cdot x}{w L}$$

special case:

When $x = L/2$ [Jet strikes on the c.g.]

$$\therefore \sin \theta = \frac{\rho a v^2}{w}$$

216

Prob. 1

A Jet of water of dia. 25 mm dia strikes to a 20 cm x 20 cm square plate of uniform thickness with a vel. of 10 m/sec at the centre of the plate which is suspended vertically by a hinge on its top edge. The weight of the plate is 98.1 N/m. The Jet strikes normal to the plate. What force must be applied at lower edge of plate so that plate is kept vertical. If the plate is allowed to deflect ^{freely} what will be the angle of def'n with vertical due to the force exerted by the Jet of water.

Ans: $\theta = 30^\circ$ $F = 28.5 \text{ N}$

Turbines

* used to convert hydraulic energy into mechanical energy which is further converted into electric energy by means of generators.

* It extracts energy

Pumps

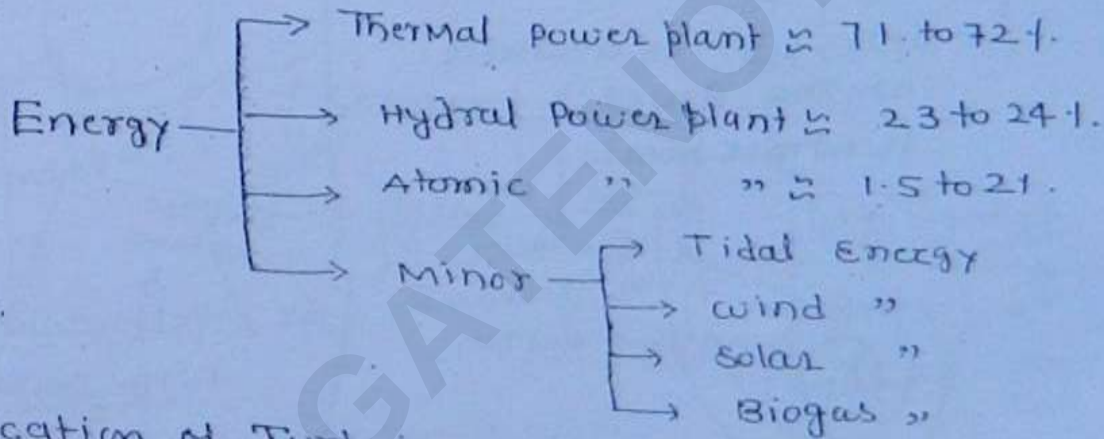
/ Fuel energy

* First electric energy is converted into mechanical energy by motor & mech. energy is converted into pressure energy or hydraulic energy by pumps.

* It adds energy.

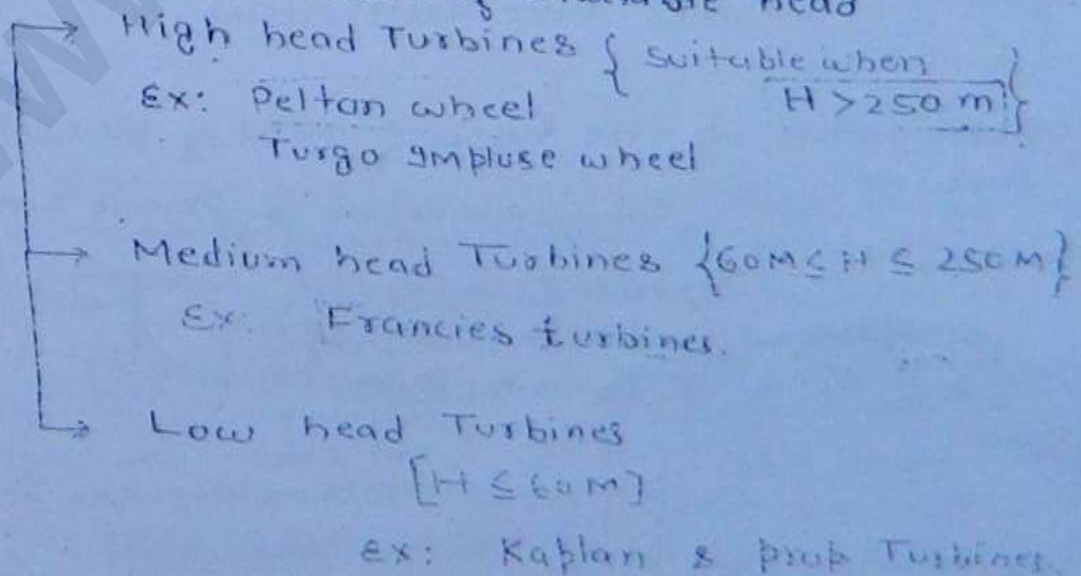
Turbines: ↓

(217)



classification of Turbines: ↓

a) classification on the basis of available head



- High [$V_1 = 300 \text{ to } 1000$]
ex: Kaplan & propeller
- Medium sp-speed [$V_2 = 60 \text{ to } 300$]
- Low sp-speed [$V_3 < 60 \text{ m/sec}$]
ex: Pelton wheel

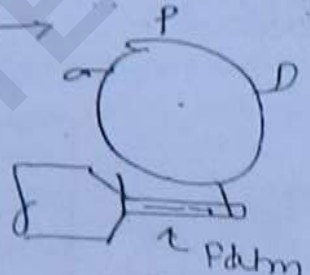
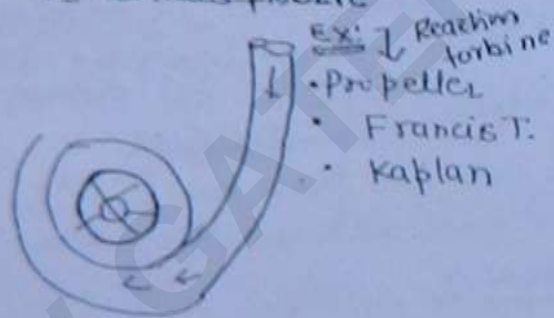
218

c) classification on the basis of available energy at the inlet of turbines

Reaction Turbine (Pressure Turbine) * at the inlet of turbine energy available is pressure energy & K.E.

Impulse turbine (velocity turbine)

only K.E. is available at the inlet & pressure is atmospheric



Impulse turbine

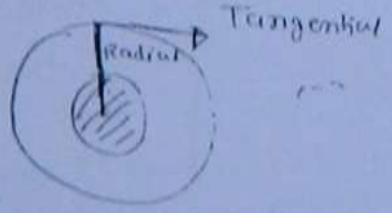
Ex: Pelton wheel
Turgo impulse wheel

• Reaction turbines have closed casing

with

d) classification on the basis of type of flow in the turbine

Radial Flow → Inward Radial flow → Francis Turbines
outward Radial flow → centrifugal pumps commonly but some turbines may also be designed



Tangential Flow (Parallel Flow)

Axial Flow

Mixed Flow
Ex: Francis T. (water may enter in radial dir. & may leave in axial direction)

Ex: (a) Pelton wheel

(b) Turgo impulse wheel

Ex: Kaplan, propeller

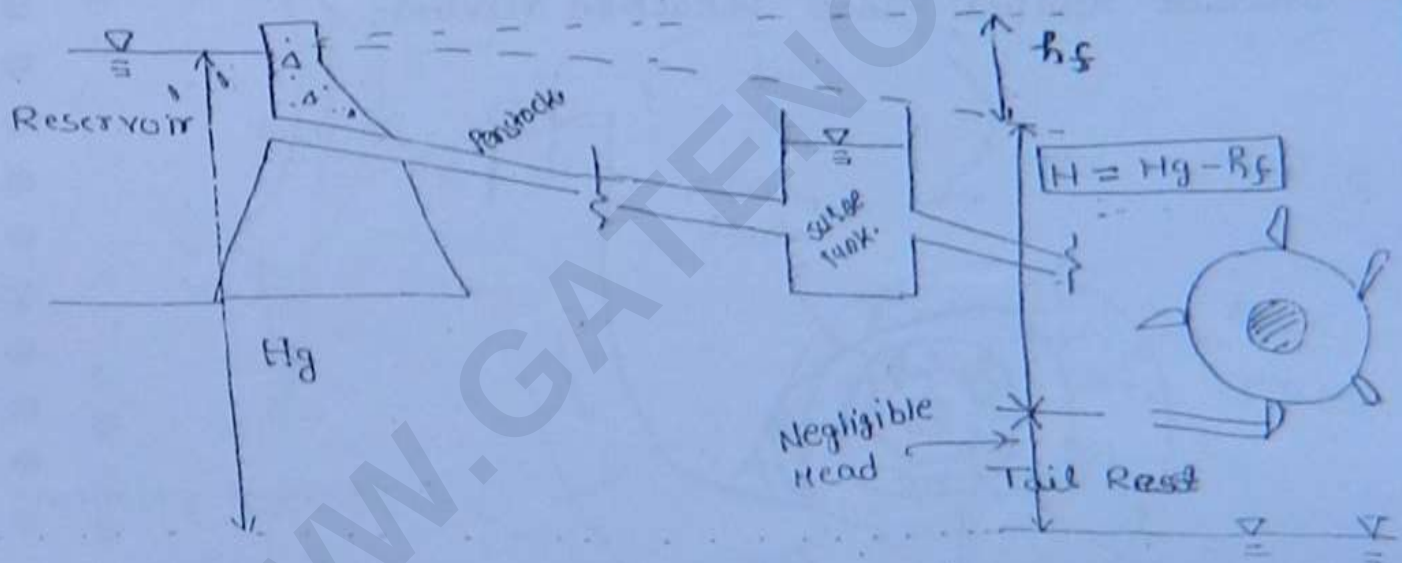
classification on the basis of which is in practical use.

is combination of last two classification.

- i) Francis Turbines $\rightarrow$ Inward Radial flow Reaction turbine.
- ii) Pelton wheel / Turgo impulse wheel $\rightarrow$ Tangential flow impulse turbine.
- iii) Kaplan / Propeller $\rightarrow$ Axial flow Reaction turbine.
- iv) Modern Francis $\rightarrow$ Mixed Flow Reaction turbine.

Important units of hydropower plant:

(219)



a) Surge Tank:

Between the reservoir & turbine house, surge tank is provided in order to minimize water hammer pressure problem in penstock. Surge tank also helps in maintaining constant head at turbine.

b) Penstock:

It is the pipe through which water is brought from reservoir or from surge tank to the turbine chamber. There always takes head loss at the turbine.

energy is used to convert into work. Pressure at exit of turbine may fall below atmospheric therefore disposal of exit water directly into atmosphere is not safe hence a tube of gradually diverging ^{exit section} is used to carry water of turbine to tail rest & this is always submerged at some depth below tail rest level.

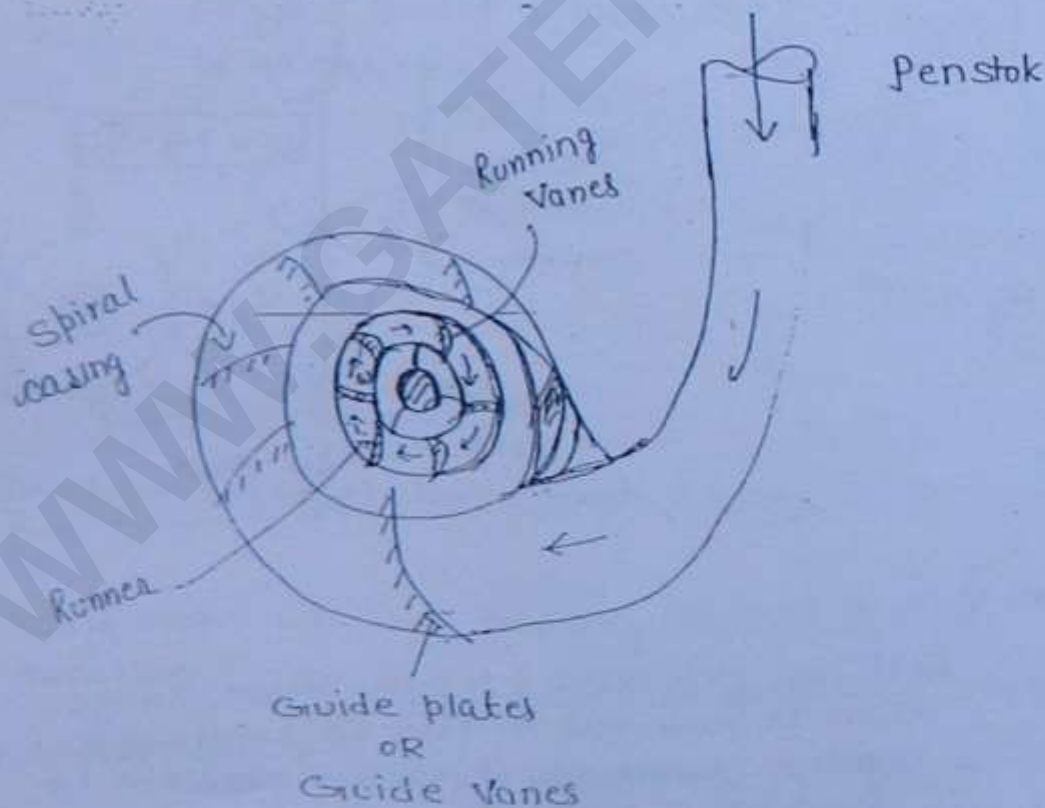
2. In case of impulse turbines, pressure remains const. hence draft tubes are not essential.

d) Turbine units: ↓

1) Francis Turbine: ↓

220

* Inward Radial Flow Reaction turbine



- Casing is a spiral chamber which is of gradually decreasing area in order to keep constant velocity at inlet of vanes. [As r decreases, so A is decreased] somewhat entered into runner.
- Guide plates are permanently attached to casing which guide the water to enter into runner.
- Runner is rotating unit on which curved vanes are

Katakna Runner tranter Energy to shaft & finally
 Shaft may be connected to generator.

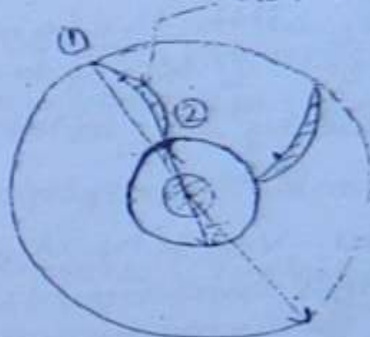
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> FRANCIS TURBINE: ↓

Runner ↓

(24)

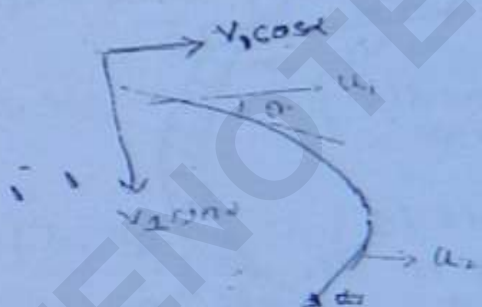
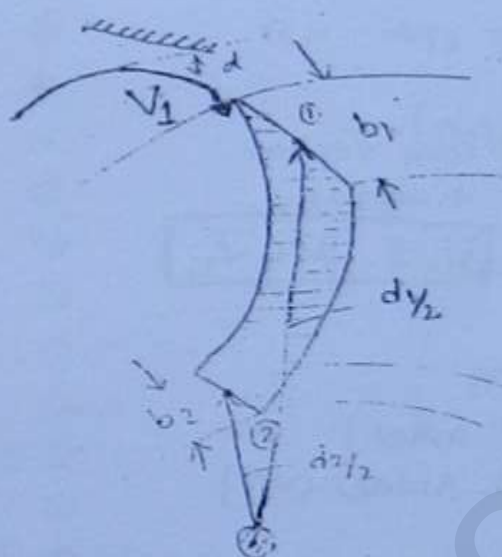
Runner Vane



① → Entry point
 ② → Exit point

Inner dia. = Dia. at exit
 = d_2

Outer diameter
 = Dia. at inlet
 = d_1



Velocity triangle ↓

Guide Blade (Fixed in the casing)

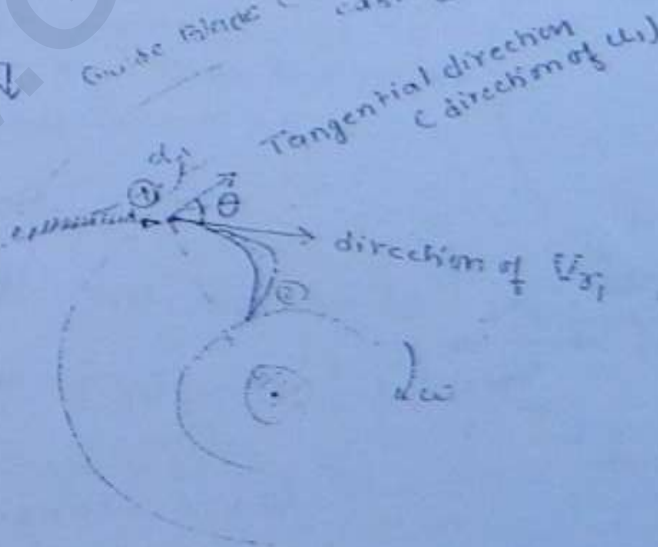
u = Tangential Velocity /
 Peripheral velocity
 of Runner at
 inlet

$$= r_1 \omega$$

$$= d_1 \omega$$

$$= d_1 \left(\frac{2\pi N}{60} \right)$$

$$u_1 = \frac{\pi d_1 N}{60} \text{ m/sec.}$$



Guide Blade Angle at Inlet

Runner

V_{a1} = Relative velocity at inlet (Rel. vel. of fluid with respect to Blade)

$$\theta > 90^\circ, = 90^\circ, < 90^\circ$$

If $\theta = 90^\circ$, Vanes are said to be radial at inlet

α = Guide blade angle or angle b/w u_1 & V_1

α = Angle of V_1 with the tangent of wheel / Runner at inlet [b/w 10° to 30°]

V_1 = Abs. velocity of fluid at inlet

$V_1 \cos \alpha$ = Tangential component of abs. velocity [wheel velocity at inlet u_1]

$V_1 \cos \alpha$ = This component is responsible for producing torque

$$V_1 \sin \alpha = V_{f1}$$

From vector diagram

$$\vec{V}_1 = \vec{u}_1 + \vec{V}_{r1}$$

$V_1 \sin \alpha$ = Radial component of V_1
(Radial velocity at inlet)
(Velocity of flow at inlet) [V_{f1}]



$$\vec{V}_{r2} + \vec{u}_2 = \vec{V}_2$$

V_{r2} = Relative velocity at exit which is in the direction of vane. In Rtn. turbine V_{r1} need not necessarily to be equal V_{r2}

u_2 = Tangential velocity of the Runner at exit

V_2 = Abs. velocity at exit

$\rightarrow$ β may be $> 90^\circ, = 90^\circ, < 90^\circ$

~~$V_2 \cos \beta = V_{f2}$ = Radial velocity at exit~~

$V_2 \sin \beta = V_{f2}$ = Radial (Flow) velocity at exit

NOTE: If β is 90° then Vanes are said to be radial at exit or if turbine discharges radially outward than $\beta = 90^\circ$

- In case of Francis Turbine in order to increase the efficiency β is purposely made 90°
- Following points may be noted w.r.t. Francis Turbines

(i) Discharge through Runner (Turbine)

(223)

$$Q = A_{f1} \times V_{f1}$$

$$= (\pi d_1 b_1) \times V_{f1} \quad \left\{ \begin{array}{l} \text{When thickness of} \\ \text{vane is negligible} \end{array} \right.$$

$$= [\pi d_1 - n t] b_1 V_{f1} \quad \left\{ \begin{array}{l} \text{If there are} \\ n \text{ vanes having} \\ t, \text{ thickness} \\ \text{each} \end{array} \right.$$

$$= K \cdot \pi d_1 b_1 V_{f1} \quad \left\{ \begin{array}{l} K \text{ is coeff. which} \\ \text{is account for} \\ \text{reduced area,} \\ \text{occupied vanes} \\ \text{thickness} \end{array} \right.$$

For eg: 5% area at circumference is occupied by Vane thickness than $K = 0.95$

At exit $Q = V_{f2} \times A_{f2}$

$$= (\pi d_2 b_2) \times V_{f2}$$

$$= [\pi d_2 - n t] \times b_2 V_{f2}$$

$$= K \pi d_2 b_2 V_{f2}$$

$$Q = \pi d_1 b_1 V_{f1} = \pi d_2 b_2 V_{f2}$$

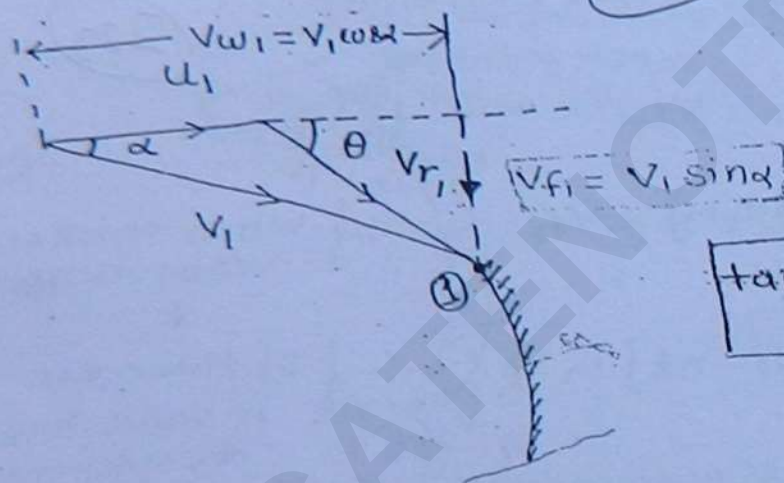
$$u_1 = \frac{\pi d_1 N}{60} = r_1 \omega$$

$$u_2 = \frac{\pi d_2 N}{60} = r_2 \omega$$

$$\therefore u_1 u_2 = d_1 d_2$$

B) velocity triangle at inlet.

case A) when $\theta < 90^\circ$



$$\tan \theta = \frac{V_{f1}}{V_{w1} - u_1}$$

NOTE:

if θ is not given then in order to obtain velocity triangle compare magnitudes of u_1 & V_{w1} . if

$$u_1 < V_{w1} \Rightarrow \theta < 90^\circ$$

$$\text{if } u_1 = V_{w1} \Rightarrow \theta = 90^\circ$$

$$\text{if } u_1 > V_{w1} \Rightarrow \theta > 90^\circ$$

case B)

When $\theta = 90^\circ$ means vanes are set radially at inlet

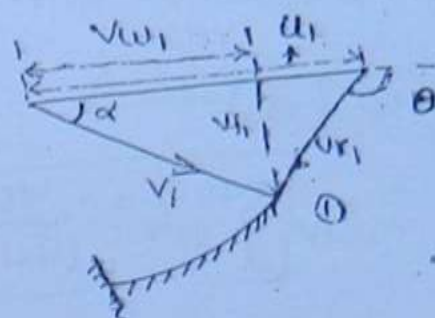
$$u_1 = V_{w1}$$

$$V_{r1} > 0$$

$$V_{f1} > 0$$

$$V_{r1} = V_{f1}$$

case 3) when $\theta > 90^\circ$

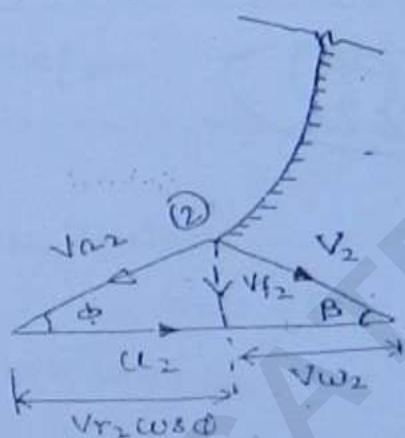


$$\tan(180 - \theta) = \frac{V_{f1}}{u_1 - V_{w1}}$$

(225)

4) velocity triangle at exit

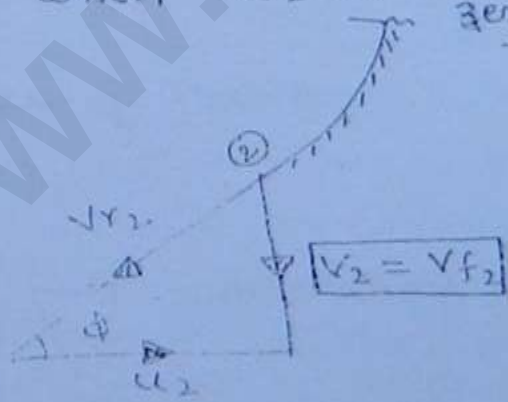
case 4) when V_{w2} is in direction of V_{w1}



$$|u_2| = |V_{r2} \cos \phi| + |V_{w2}|$$

VIMP case B)

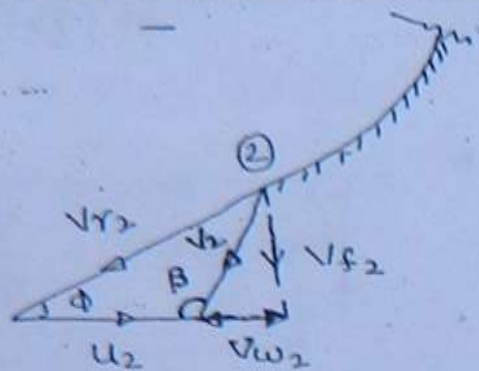
when $V_{w2} = 0$, whirl component at outlet is zero or turbine discharges radially outward.



$$V_2 = V_{f2}$$

- $\beta = 90^\circ$, $V_{w2} = 0$
- $|u_2| = |V_{r2} \cos \phi|$
- $\tan \phi = \frac{V_{f2}}{u_2}$

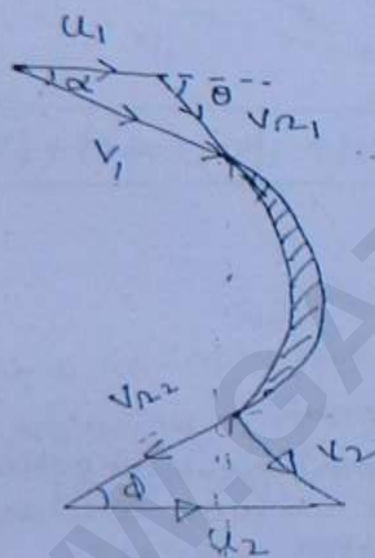
• common case of Francis Turbine.



$$\tan \phi = \frac{V_{f2}}{|u_2| + |V_{w2}|}$$

$$|V_{r2} \cos \phi| = |u_2| + |V_{w2}|$$

5> Work-done per sec by the water on Runner
(Runner Power)



226

Rate of change of Angular momentum = T

$$\text{Work-done/sec.} = T \times \omega$$

$T = \text{Angular momentum/sec. at inlet} - \text{Angular momentum/sec. at outlet}$

Ang. Momentum = mvr

$$\therefore \text{Ang. Momentum/sec} = \left(\frac{\text{mass}}{\text{sec}} \right) \cdot V \cdot r$$

$$\therefore T = \left(\text{mass/sec} \right) V \omega_1 r_1 - \left(\frac{\text{mass}}{\text{sec}} \right) V \omega_2 r_2$$

$$\therefore T = \rho Q [V \omega_1 r_1 - V \omega_2 r_2] \quad (22)$$

$$\begin{aligned} \therefore \text{work-done/sec.} &= T \times \omega \\ &= \rho Q [V \omega_1 r_1 \omega - V \omega_2 r_2 \omega] \\ &= \rho Q [V \omega_1 u_1 - V \omega_2 u_2] \end{aligned}$$

$$\therefore \boxed{\text{work-done/sec.} = \left(\frac{\omega}{g} \right) Q [V \omega_1 u_1 - V \omega_2 u_2]}$$

$[\omega \rightarrow \text{unit wt. of fluid}]$

For Francis Turbine

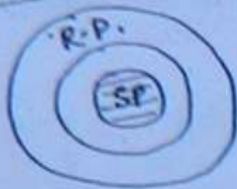
$$V \omega_2 = 0$$

$$\begin{aligned} \therefore \text{work-done/sec.} &= \text{R.P. (Runner Power)} \\ &= \rho Q [V \omega_1 u_1] \\ &= \frac{\omega Q}{g} [V \omega_1 u_1] \end{aligned}$$

6) Power of Turbine



$$H = \text{Net Head at inlet of turbine (Runner)} = H_g - H_t$$



228

a) Hydraulic Power (water power): It is the power available at the inlet of turbine

$$H.P. (W.P.) = \omega \cdot Q \cdot H \rightarrow \text{Kw} \quad \left[\omega = 9.81 \text{ KN/m}^3 \right]$$

$$= \frac{\rho \cdot Q \cdot H}{75} \rightarrow \text{H.P.} \{ \text{Horse Power} \}$$

b) Runner Power = work done by water on Runner

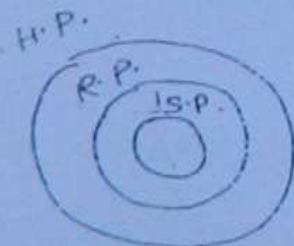
$$= \frac{\omega Q}{g} [V_{w1} u_1 - V_{w2} u_2]$$

c) Shaft-Power (Break Power): This is final power available at the shaft of turbine.

$$\therefore S.P. = R.P. - \text{Transmission Losses} \{ \text{Mechanical Losses} \}$$

7) Efficiency of Turbines: ↓

(a)



$$\text{Hydraulic efficiency} = \eta_h = \frac{R.P.}{\text{water Power}}$$

$$\eta_h = \frac{\left(\frac{\omega Q}{g} \right) [V_{w1} u_1 - V_{w2} u_2]}{\omega Q H}$$

$$\therefore \eta_h = \frac{V_{w1} u_1 - V_{w2} u_2}{g H}$$

$$\eta_B = \frac{V_{w1} u_1}{gH}$$

2) Mechanical efficiency

229

$$\eta_M = \frac{S.P.}{R.P.}$$

3) overall efficiency

$$\eta_o = \frac{S.P.}{H.P.} = \frac{S.P.}{R.P.} \times \frac{R.P.}{H.P.} = \eta_B \times \eta_M$$

$$\therefore \eta_o = \eta_B \times \eta_M$$

NOTE:

If Leakage in the turbine chamber are also considered than Volumetric efficiency may be considered as

$$\eta_v = \frac{Q'}{Q}$$

$Q' \rightarrow$ Discharge at exit

$Q \rightarrow$ Discharge at inlet

However this is more imp. for centrifugal pump & Negligible for turbines.

If it is also accounted than overall efficiency

$$\eta_o = \eta_B \times \eta_M \times \eta_v$$

NOTE:

(a) If it is given that velocity through the runner is constant then $V_{f1} = V_{f2}$ and hence $A_{f1} = A_{f2}$

(b) In case of some data is missing than Bernoulli's eqn may be applied b/w inlet & outlet of the runner by assuming No-Losses of head through the runner.

(c) Total Head at inlet = $\frac{\text{work done/sec/unit wt of water}}{\text{Energy head at outlet} + \text{Losses}}$

$$= \frac{\left(\frac{\omega Q}{g}\right) [V_{w1} u_1 - V_{w2} u_2]}{\omega Q} + \frac{V_2^2}{2g} + h_L$$

If $V_{w2} = 0$ & Losses are Negligible
than

[pressure
energy is
Negligible.

$$H = \frac{V_2^2}{2g} + \frac{V_{w1} u_1}{g}$$

230

Master Formula.

(d) If speed ratio is given than it means

$$\phi = \frac{u_1}{\sqrt{2gH}}$$

[it is only significant
for inlet point
 ϕ lies between 0.75 to
0.85

(e) If flow ratio is given than

$$\psi = \frac{V_{f1}}{\sqrt{2gH}}$$

[Also valid at inlet
 ψ lies b/w 0.15 to 0.35

(f) If width ratio is given than

$$= b/d \quad \left\{ b_1/d_1 \right\}$$

have an external diameter of 700 mm & a width of 180 mm. If the guide vanes are at 20° to the wheel tangent and the abs. velocity of water at inlet is 25 m/sec. Then find

- Discharge through the turbine
- Runner vane angle at inlet

$$N = 500 \text{ r.p.m.}$$

$$d_1 = 700 \text{ mm} \\ = 0.7 \text{ m}$$

$$b_1 = 180 \text{ mm} \\ = 0.18 \text{ m}$$

$$\alpha = 20^\circ$$

$$V_1 = 25 \text{ m/sec.}$$

$$\theta = ?$$

$$Q = ?$$

$$u_1 = \frac{\pi d_1 N}{60}$$

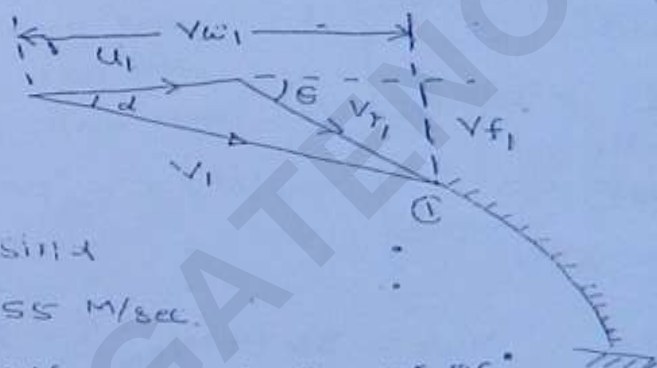
$$u_1 = 18.326 \text{ m/sec}$$

$$V_{w1} = V_1 \cos \alpha$$

$$= 25 \cos 20^\circ$$

$$= 23.49 \text{ m/sec}$$

$$\text{Since } u_1 < V_{w1} \Rightarrow \theta < 90^\circ$$



$$V_{f1} = V_1 \sin \alpha$$

$$= 8.55 \text{ m/sec.}$$

$$\tan \theta = \frac{V_{f1}}{V_{w1} - u_1} \Rightarrow \theta = 58.86^\circ$$

$$Q = A_{f1} V_{f1}$$

$$= (\pi d_1 b_1) V_{f1}$$

$$= \pi \times 0.7 \times 0.18 \times 8.55$$

$$= 3.354 \text{ m}^3/\text{sec}$$

Prob. 2

A reaction turbine works at 450 r.p.m. under a head of 120 M. If the dia. at inlet is 1.2 m & flow area at inlet is 0.4 m^2 . The angle made by absolute and relative velocity at the inlet with the tangent at wheel is 60° & 20° respectively. Then determine

- Flow rate
- Runner power developed
- Hydraulic efficiency

→ $N = 450$ R.P.M.

$H = 120$ M

$d_1 = 1.2$ M

$A_{f1} = 0.4$ M²

$\alpha = 20^\circ$

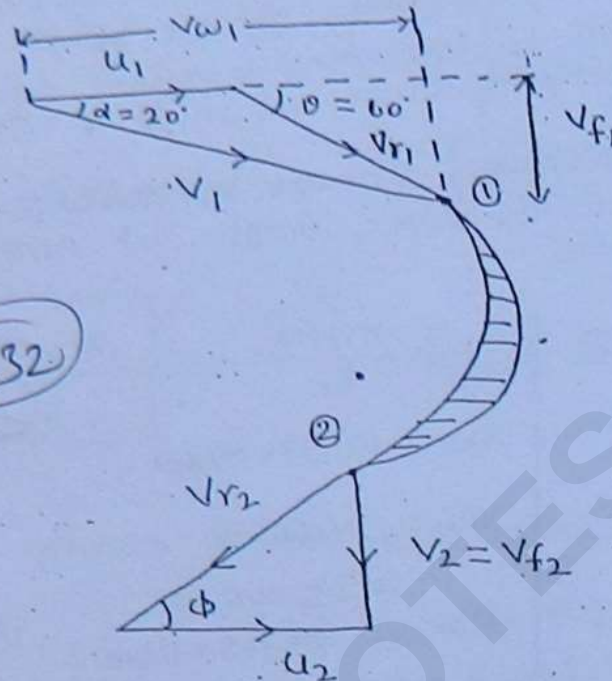
$\theta = 60^\circ$

$V_{w2} = 0$

$Q = ?$

R.P. = ?

$\eta_B = ?$



$$u_1 = \frac{\pi d_1 N}{60} = 28.27 \text{ M/sec.}$$

$$\tan \theta = \frac{V_{f1}}{V_{w1} - u_1} \quad \text{--- (1)}$$

$$\tan \alpha = \frac{V_{f1}}{V_{w1}} \quad \text{--- (2)}$$

By eqn (1) & (2)

$$V_{w1} = 35.78 \text{ M/sec}$$

$$V_{f1} = 13.03 \text{ M/sec.}$$

$$Q = A_{f1} \cdot V_{f1}$$

$$= 0.4 \times 13.03$$

$$Q = 5.21 \text{ m}^3/\text{sec.}$$

$$\eta_B = \frac{V_{w1} u_1 - V_{w2} u_2}{gH} = 85.96\%$$

$$\text{Runner power} = \frac{\rho g Q}{g} [V_{w1} u_1 - V_{w2} u_2] = 5270 \text{ kW}$$

An inward reaction flow turbine runs at 192 r.p.m. The dia & width at inlet are 600 mm & 150 mm while the outlet dia is 300 mm. The velocity of flow through the runner is constant at 1.5 m/sec. If the guide blades are 10° to the wheel tangent. Draw the inlet & outlet velocity diagram, if velocity of whirl at outlet is zero. Determine

- Runner Blade Angle
- Abs. Velocity of water leaving the guide blade
- Rel. velocity of water at inlet
- Width of the wheel at outlet
- Discharge through turbine
- Head supplied
- R.P. supplied (developed)
- Hydraulic Efficiency.

Given

$$N = 192 \text{ r.p.m.}$$

$$d_1 = 600 \text{ mm}$$

$$b_1 = 150 \text{ mm}$$

$$d_2 = 300 \text{ mm}$$

$$V_{f1} = V_{f2} = 1.5 \text{ m/sec}$$

$$\alpha = 10^\circ$$

$$V_{w2} = 0$$

$$\phi \text{ \& } \psi = ?$$

$$u_1 = \frac{\pi d_1 N}{60} = 6.03 \text{ m/sec}$$

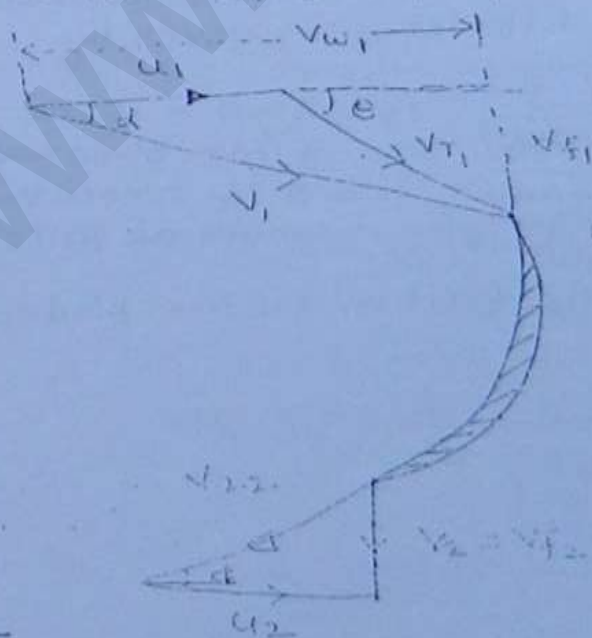
$$\tan \psi = \frac{V_{f1}}{V_{w1}}$$

$$\Rightarrow V_{w1} = 8.507 \text{ m/sec}$$

$$V_1 = \sqrt{V_{f1}^2 + V_{w1}^2}$$

$$= 8.63 \text{ m/sec}$$

$$\text{Since } u_1 < V_{w1} \Rightarrow \psi < 90^\circ$$



$$\tan \psi = \frac{V_{f1}}{V_{w1} - u_1}$$

$$\Rightarrow \psi = 31.19^\circ$$

$$\frac{u_1}{u_2} = \frac{d_1}{d_2}$$

$$\Rightarrow u_2 = u_1 \times \frac{d_2}{d_1} = 3.015 \text{ m/sec}$$

$$\tan \phi = \frac{V_{f2}}{u_2}$$

$$\Rightarrow \phi = 26.45^\circ$$

$$Q = \pi d_1 b_1 \cdot V_{f1}$$

$$= \pi \times 0.6 \times 0.15 \times 1.5$$

$$= 0.424 \text{ m}^3/\text{sec}$$

$$= \pi d_2 b_2 \cdot V_{f2}$$

$$\Rightarrow b_2 = 0.3 \text{ m} = 300 \text{ mm}$$

$$\sin \theta = \frac{V_{f1}}{V_{r1}} \Rightarrow V_{r1} = \frac{V_{f1}}{\sin \theta} = 2.89 \text{ m/sec}$$

$$R.P. = \frac{\omega G}{g} [V_{w1} u_1 - V_{w2} u_2]$$

$$= 21.74 \text{ kW}$$

• Apply Master Formula { Apply B. eqn b/w ① & ② }

$$H = \frac{V_2^2}{2g} + \frac{V_{w1} u_1}{g} \left[\begin{array}{l} \text{Assuming} \\ \text{Minor losses} \\ \text{to be neglected} \end{array} \right]$$

$$= 5.34 \text{ m}$$

$$\eta_H = \frac{V_{w1} u_1}{g H}$$

$$= 95.1\%$$

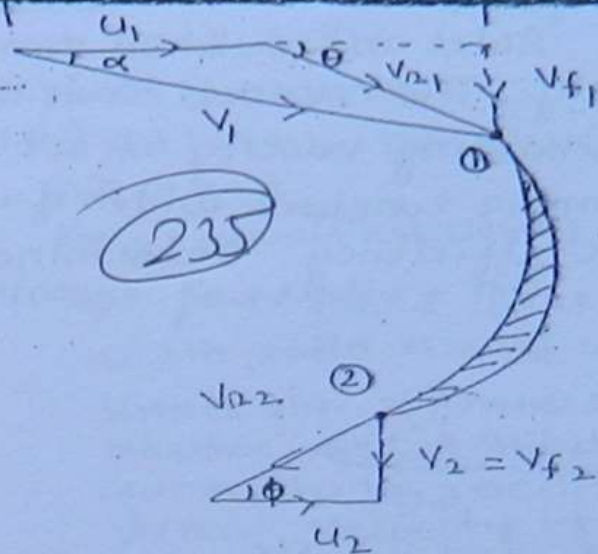
Prob. 4
CS/2003

An inward flow reaction turbine discharges radially the velocity of flow through the runner is const. so that hydraulic efficiency is given by

$$\eta_H = \frac{1}{1 + \frac{V_2 \tan^2 \alpha}{1 - \left(\frac{\tan \alpha}{\tan \theta} \right)}}$$

where α is the guide blade angle
& θ is runner vane angle at inlet.

Assume there is no friction on the blades.



$$\eta_B = \frac{V_{w1} u_1}{gH} \quad \text{--- (iii)}$$

Apply B. eqn V_{w1} (1) & (2)

$$H = \frac{V_2^2}{2g} + \frac{V_{w1} u_1}{g}$$

$$= \frac{V_2^2}{2g} + \frac{V_{w1} u_1}{g}$$

$$\frac{V_{w1} u_1}{g \eta_B} = \frac{V_2^2}{2g} + \frac{V_{w1} u_1}{g}$$

$$\Rightarrow \frac{1}{\eta_B} = \frac{V_2^2}{2u_1 V_{w1}} + 1$$

$$\eta_B = \frac{1}{1 + \frac{V_2^2}{2u_1 V_{w1}}}$$

$$= \frac{1}{1 + \frac{V_{w1}^2 \tan^2 \alpha / 2}{V_{w1} \times V_{w1} \left[1 - \frac{\tan \alpha}{\tan \theta} \right]}}$$

$$\therefore \eta_B = \frac{1}{1 + \left[\frac{\tan^2 \alpha / 2}{1 - \frac{\tan \alpha}{\tan \theta}} \right]}$$

$$\tan \theta = \frac{V_{f1}}{V_{w1}}$$

$$\Rightarrow V_{f1} = V_{w1} \tan \theta$$

$$V_2 = V_{f2} = V_{f1} = V_{w1} \tan \theta \quad \text{--- (1)}$$

$$\tan \alpha = \frac{V_{f1}}{V_{w1} - u_1}$$

$$\therefore \tan \theta = \frac{V_{w1} \tan \alpha}{V_{w1} - u_1}$$

$$\therefore V_{w1} - u_1 = V_{w1} \frac{\tan \alpha}{\tan \theta}$$

$$\therefore u_1 = V_{w1} \left[1 - \frac{\tan \alpha}{\tan \theta} \right] \quad \text{--- (2)}$$

9/2001

at an avg. head of 160 m with a discharge of 80 m³/sec. The inlet and outlet dia. are 4 m & 2 m respectively. The runner blade angle is 120°. Radial discharging velocity at outlet is 15 m/sec. Assuming constant width of wheel and 90% hydraulic efficiency. Determine H.P. produced, in MW. & R.P.M. of machine.

Given

$$H = 160 \text{ m}$$

$$Q = 80 \text{ m}^3/\text{sec}$$

$$d_1 = 4 \text{ m}$$

$$d_2 = 2 \text{ m}$$

$$\theta = 120^\circ$$

$$V_{f2} = V_2 = 15 \text{ m/sec.}$$

$$V_{w2} = 0$$

$$b_1 = b_2 = \text{const.}$$

$$\eta_h = 0.9$$

$$\text{H.P.} = ?$$

$$N = ?$$

236

$$u_1 = \frac{\pi d_1 N}{60}$$

$$Q = \pi d_2 b_2 V_{f2}$$

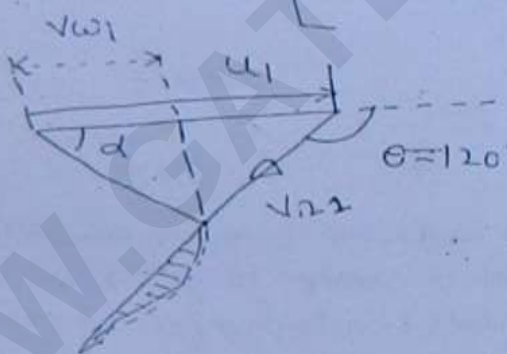
$$80 = \pi \times 2 \times b_2 \times 15$$

$$\Rightarrow b_2 = 0.84 \text{ m}$$

$$b_1 = b_2 = 0.84 \text{ m}$$

$$Q = V_{f1} \cdot \pi d_1 b_1$$

$$\Rightarrow V_{f1} = 7.5 \text{ m/sec.}$$



$$\tan(180^\circ - \theta) = \frac{V_{f1}}{u_1 - V_{w1}} \quad \text{--- (1)}$$

$$\eta_h = \frac{V_{w1} u_1}{g H}$$

$$\Rightarrow V_{w1} \cdot u_1 = 0.9 \times 9.81 \times 160 \quad \text{--- (2)}$$

By solving eqn (1) & (2)

$$u_1 V_{w1} = 39.81 \text{ m/sec} = \frac{\pi d_1 N}{60}$$

$$u_1 = 35.96 \text{ m/sec} \Rightarrow N = 190 \text{ R.P.M.}$$

$$\therefore \text{Hydraulic Power} = \rho Q g H$$

$$= 9.81 \times 80 \times 160 \text{ kW}$$

$$\therefore \text{H.P.} = 125.5 \text{ MW.}$$

Design a Francis turbine runner with the following

data:

Net head 68 m

speed $N = 750 \text{ r.p.m}$

output power = 330 kW (S.P.)

$\eta_B = 94\%$

η_{cy} (overall efficiency) = 85% = η_o

ψ = Flow ratio = 0.15 $\Rightarrow \psi = \frac{V_{f1}}{\sqrt{2gH}} = 0.15$

width ratio $\eta = b/d = 0.1$

Inner dia. of runner is $\frac{1}{2}$ of outer dia. Also assume 6% of the circumferential area of the runner to be occupied by the thickness of vanes. Velocity of flow remains constant & flow is radial at exit.

Given

$K = 0.94$, $d_2 = d_1/2$, $b_1/d_1 = 0.1$, $V_{f1} = V_{f2}$

$$\eta_o = \frac{S.P.}{H.P.} = \frac{330}{H.P.} \Rightarrow H.P. = 388.235 \text{ kW}$$

$$\Rightarrow Q = 0.58 \text{ m}^3/\text{sec} \quad \left[\begin{array}{l} \omega = 10\pi \text{ rad/s} \\ = 9.816 \text{ rad/s} \end{array} \right]$$

$$\frac{V_{f1}}{\sqrt{2gH}} \Rightarrow V_{f1} = 5.48 \text{ m/sec}$$

$$Q = K \cdot (\pi d_1 b_1) V_{f1} = 0.94 \times \pi \times d_1 b_1 \times 5.48 = 0.58$$

$$b_1 d_1 = 0.036 \text{ m}^2 \quad \text{--- (11)}$$

$$b_1/d_1 = 0.1 \quad \text{--- (1)}$$

$$b_1 = 0.1 d_1$$

$$\Rightarrow d_1 \times 0.1 d_1 = 0.036$$

$$\Rightarrow 0.1 d_1^2 = 0.036$$

$$\Rightarrow d_1 = 0.6 \text{ m} = 600 \text{ mm}$$

$$b_1 = 0.06 \text{ m} = 60 \text{ mm}$$

$$Q = 0.94 \pi d_1 b_1 V_{f1} = 0.94 \pi d_2 b_2 V_{f2}$$

$$\Rightarrow b_2 = \frac{d_1 b_1 V_{f1}}{d_2 V_{f2}}$$

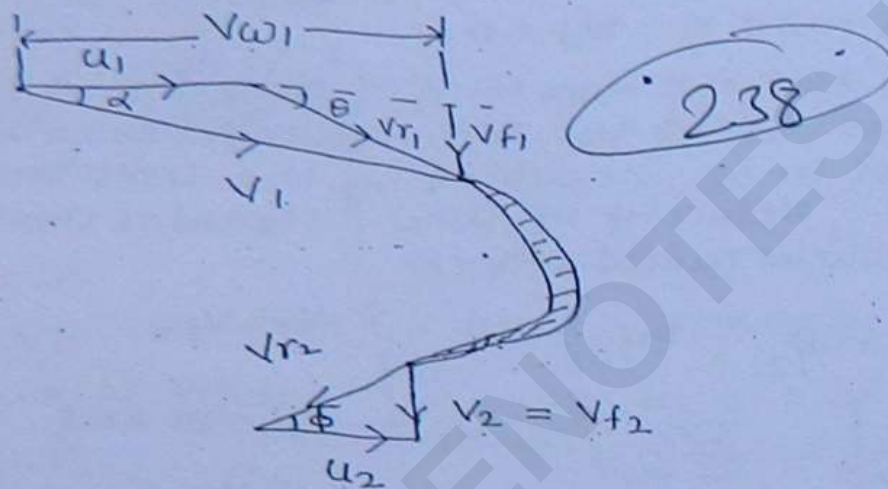
$$\Rightarrow b_2 = 120 \text{ mm}$$

$$\frac{1.5 \times 10^6}{60} = \frac{1.5 \times 0.6 \times 150}{60} = 23.56 \text{ m/sec.}$$

$$\eta_B = \frac{v_{w1} u_1}{gH} \Rightarrow v_{w1} = \frac{0.94 \times 9.81 \times 68}{23.56}$$

$$= 26.61 \text{ m/sec.}$$

$$\text{since } u_1 < v_{w1} \Rightarrow \theta < 90^\circ$$

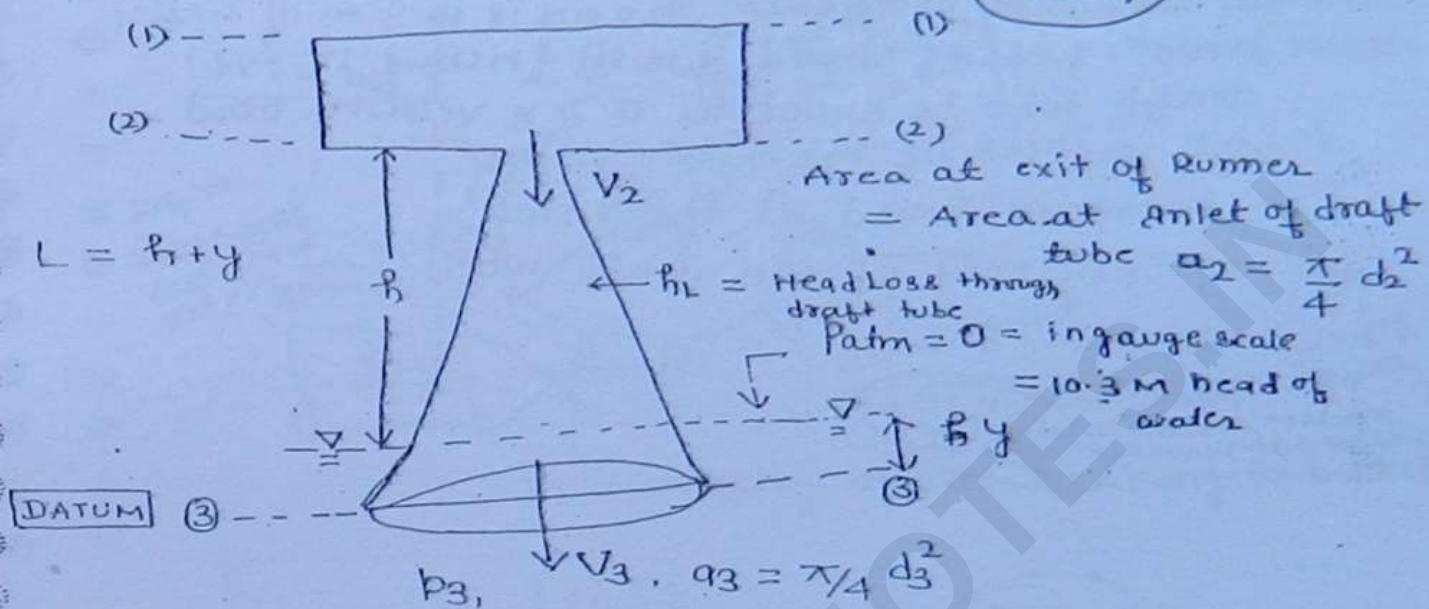


$$\tan \theta = \frac{v_{f1}}{v_{w1} - u_1} \Rightarrow \theta = 60.8^\circ$$

$$\tan \phi = \frac{v_{f2}}{u_2} = \frac{v_{f1}}{u_1/2} \Rightarrow \phi = 24.9^\circ$$

$$\tan \alpha = \frac{v_{f1}}{v_{w1}} \Rightarrow \boxed{\alpha = 11.64^\circ}$$

239



Apply B. Eqn b/w (2) & (3)

$$\begin{aligned}
 (h+y) + \frac{V_2^2}{2g} + \frac{p_2}{\rho g} &= 0 + \frac{V_3^2}{2g} + \frac{p_3}{\rho g} + h_L \\
 &= \frac{V_3^2}{2g} + \frac{p_{atm}}{\rho g} + y + h_L
 \end{aligned}$$

NOTE: Since p_2 is below atmosphere than it should be such that it should not fall below vapour pressure.

Efficiency of draft tube:

$$\eta = \frac{\text{Actual converging of K.H. into pressure head}}{\text{original K.H.}}$$

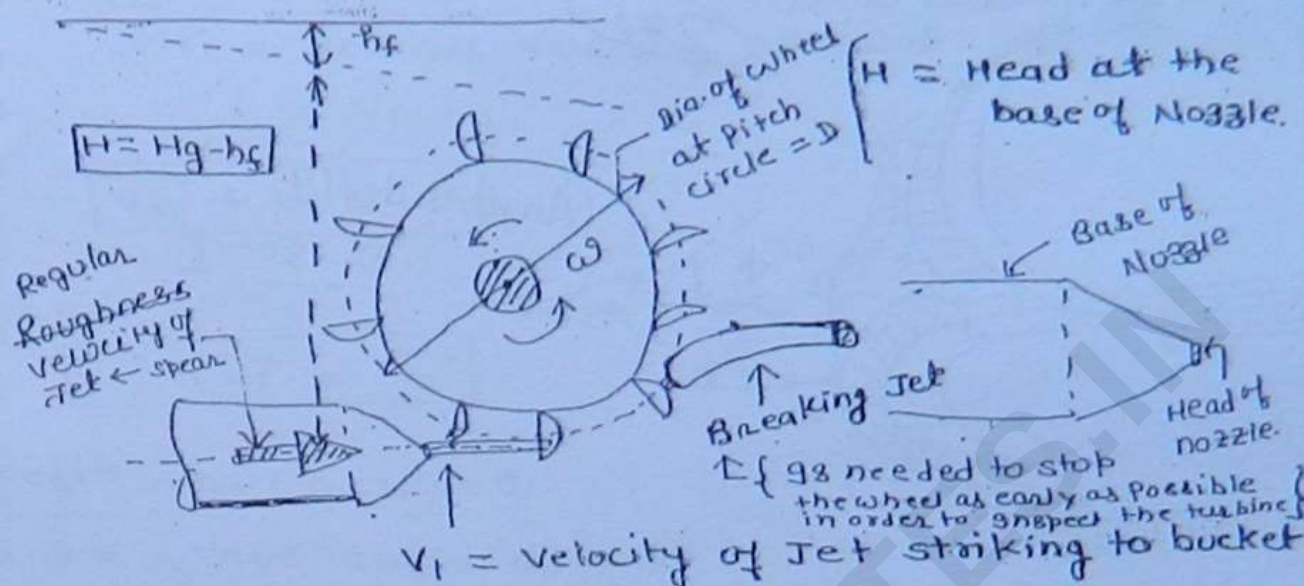
$$\eta = \frac{\frac{V_2^2}{2g} - \frac{V_3^2}{2g} - h_L}{\frac{V_2^2}{2g}}$$

1. A vertical draft tube having a diameter of 1 m & 1.5 m discharges water at outlet with velocity of 2.5 m/sec. The total length of the draft tube is 6 m & 120 m. Length of the draft tube is immersed in water. If the atmospheric pressure head is 10.3 m of water and loss of head due to friction in the draft tube is equal to $0.2 \times$ velocity head at outlet of the tube. Find

- (i) Pressure head at the inlet.
- (ii) Efficiency of the draft tube.

240

Tangential flow impulse Turbine



$$V_1 = C_v \sqrt{2gH}$$

$$C_v \approx 0.97 \text{ to } 0.98$$

(241)

→ When wheel is vertical and shaft is horizontal than it is called 'horizontal alignment', whereas 'vertical alignment' is that wheel is horizontal & shaft is vertical.

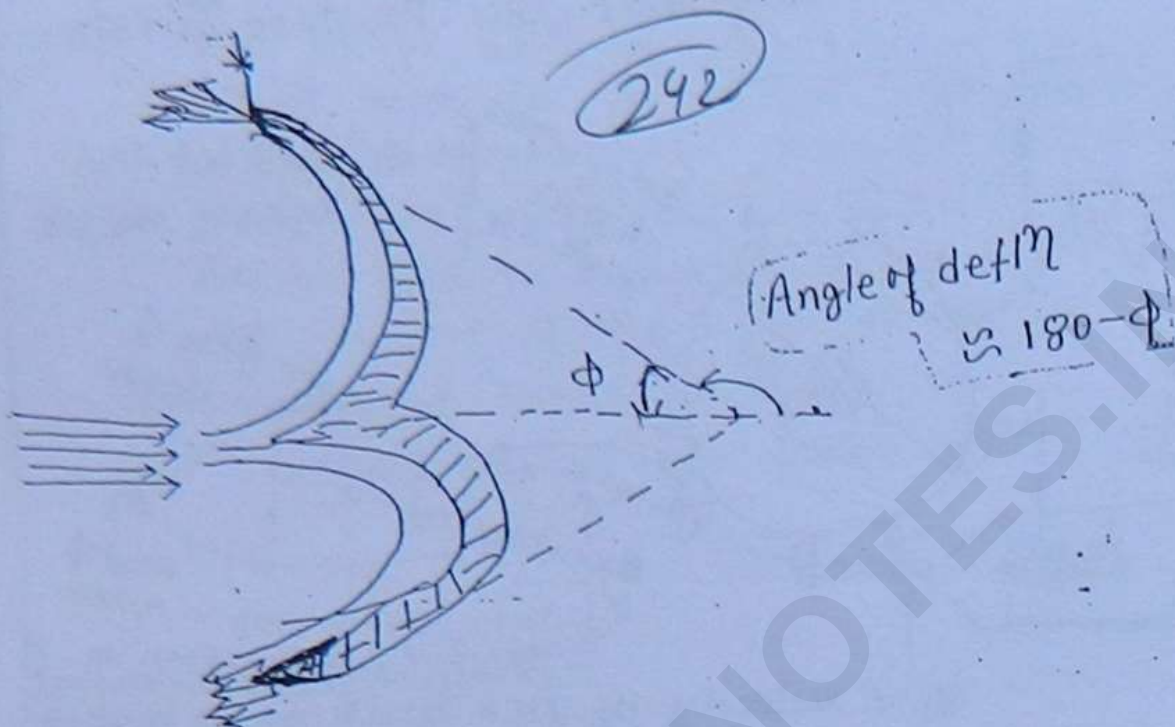
→ In the Pelton wheel hemispherical bucket are mounted on the pitch circle which may be 15 to 25 in number.

→ As far as possible less No. of vanes should be provided so as to minimize friction losses but specified minimum no. of vanes should be provided so as to prevent the loss of discharge without hitting the vanes. No. of vanes required depends upon Jet ratio which is defined as dia. of pitch circle to dia. of Jet.

$$m = \text{Jet Ratio} = \frac{\text{dia. of Pitch circle}}{\text{dia. of Jet}} = \frac{D}{d}$$

$$\text{No. of Vanes} = 15 + 0.5m$$

where m is generally b/w 10 to 15 ≈ 12



- Theoretically deflⁿ angle should be 180° in order to produce maxm. work-done but practically it causes retardation of coming vanes by hitting on the back of vanes hence appropriate angle of deflⁿ (160° to 165°) is desirable. In order to minimize friction losses vanes are polished on inside.
- Two hemispherical buckets are provided because it will cancel the y-component of force and alignment of bucket can be preserved.

$$u_1 = u_2 = \frac{\pi D N}{60}$$

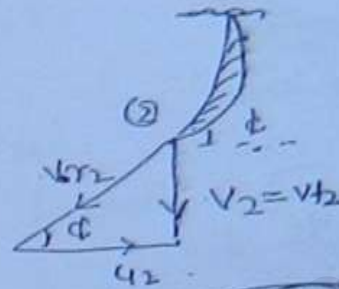
$d = \text{dia of jet}$
 $D = \text{dia of bucket (mean dia)}$

$\vec{V}_r = \text{Relative velocity at inlet}$
 $= V_1 - u_1$

$$\vec{V}_r + \vec{u}_1 = \vec{V}_1$$

At exit

$$\vec{V}_r + \vec{u}_2 = \vec{V}_2$$



$$\text{Def'n Angle} = 180 - \phi$$

$\phi = \text{Vane angle at exit}$

$\theta = \text{Angle b/w } u_1 \text{ \& } V_r$

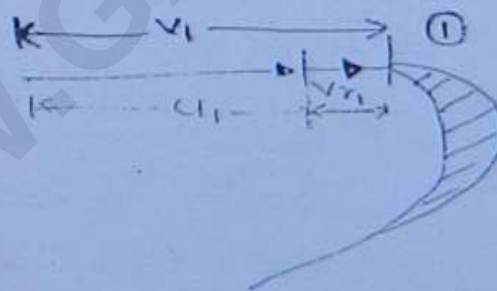
$\downarrow$
 Vane angle at inlet

243

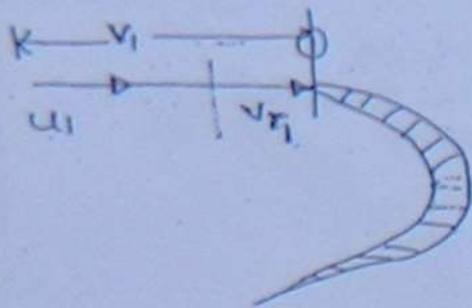
Velocity triangle at inlet: $\downarrow$

Since u_1, V_r, V_1 are in same direction

$\therefore$ Inlet vel. triangle reduces into a straight line.



Exit velocity diagram:



$$V_{r1} = V_1 - U_1$$

$$\begin{aligned} \theta &= 0^\circ \\ \alpha &= 0^\circ \end{aligned}$$

if $\theta = 0^\circ$

$$V_{w1} = V_1 \cos \alpha = V_1$$

$V_{f1} = 0$ { radial velocity at inlet }

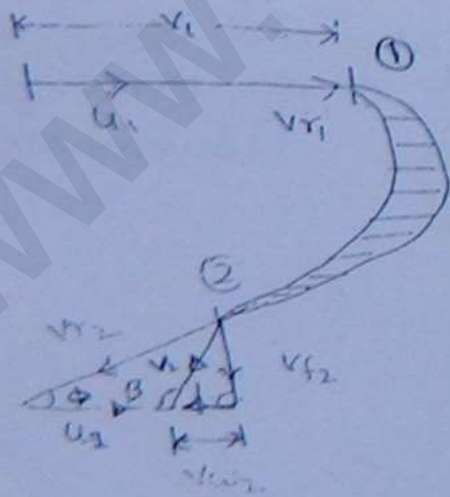
(244)

If Loss of friction on the surface of vane is negligible then $V_{r1} = V_{r2}$

$V_{r2} = V_{r1}$ ----- when friction loss is negligible

$V_{r2} = K V_{r1}$ ----- If friction losses are accounted. ($K < 1$)

Exit velocity diagram: ↓



$$\overline{V_{r2}} + \overline{U_2} = \overline{V_2}$$

$$U_2 = U_1 = \frac{\pi D N}{60}$$

$$\tan \phi = \frac{V_{f2}}{U_2 - V_{w2}}$$

where $V_{w2} = -ve$

Work done by water on the runner per second

$$= \frac{\omega Q}{g} \left[v_{w1} u_1 - [-v_{w2}] u_2 \right]$$

$$= \frac{\omega Q}{g} \left[v_{w1} u_1 + v_{w2} u_2 \right]$$

Discharge through turbine: ↓

(245)

$$= n \times \left(\frac{\pi d^2}{4} \right) \times v_1$$

where $n \rightarrow$ No. of vanes
 $d \rightarrow$ dia. of Jet
 $v_1 \rightarrow$ velocity of Jet
 $= C_v \sqrt{2gH}$
 $H \rightarrow$ Net head at the

Power available at the base of Nozzle: ↓ base of nozzle.

(1) $\frac{W.P.}{(H.P.)} = \omega Q H$ [Power available at the base of Nozzle]

(3) $R.P. = \frac{\omega Q}{g} \left[v_{w1} u_1 + v_{w2} u_2 \right] \left[u_1 = u_2 \right]$

245 Power available at the inlet of Jet/vane

30th

= K.E. of Jet per second

$$= \frac{1}{2} \times \text{mass flowing per second} \times v_1^2$$

$$= \frac{1}{2} \times \rho Q \times v_1^2$$

$$= \frac{1}{2} \left(\frac{\omega}{g} \right) (a v_1) \times v_1^2 \quad \left[a \rightarrow \text{area of Jet} \right]$$

$$= \frac{\omega a v_1^3}{2g}$$

H. = $\frac{R.P.}{P.E.H.}$ Jet velocity

(4) Power available at the shaft

S.P. or Break Power

$$= R.P. - \text{Losses}$$

a) Efficiency of Nozzle: ↓

$$\eta_N = \frac{K.E./\text{sec}}{\text{Power at the base of Nozzle}} = \frac{\frac{1}{2} \frac{wQ}{g} \times V_1^2}{wQH}$$

246

$$= \frac{V_1^2}{2gH} = \frac{C_v^2 (2gH)}{(2gH)} = C_v^2$$

∴ ~~η_N = C_v²~~

$$\eta_N = C_v^2$$

b) Hydraulic efficiency: ↓

$$\Rightarrow \eta_H = \frac{R.P.}{K.E./\text{sec of Jet}}$$

$$= \frac{\frac{wQ}{g} [V_{w1} u_1 + V_{w2} u_2]}{\frac{wQ}{2g} \times V_1^2}$$

NOTE! -

If hydraulic efficiency is calculated on the basis of power available at the base of nozzle then

$$\eta_H = \frac{R.P.}{\text{Power available at the base of Nozzle}}$$

$$= \frac{\frac{wQ}{g} [V_{w1} u_1 + V_{w2} u_2]}{wQH}$$

$$\eta_H = \frac{V_{w1} u_1 + V_{w2} u_2}{gH}$$

It means energy loss at the base of nozzle is neglected

c) Mechanical efficiency ↓

$$\eta_M = S.P./R.P.$$

d) Overall efficiency

Accounting the Nozzle Losses

$$\eta_O = \eta_N \times \eta_H \times \eta_M$$

If entire volume of Jet is not striking to the vane than volumetric efficiency

$$\eta_v = \frac{Q'}{Q}$$

Q' = total volume of water striking the Jet per sec.

$Q =$

247

Specification for design of Pelton wheel: ↓

(a) speed ratio $\phi = \frac{u_1}{\sqrt{2gH}}$

[ϕ is also called side clearance angle]

≈ 0.45 to 0.47

(b) Angle of deflection of Jet through the bucket is $\approx 165^\circ$ [if not given]

(c) Jet ratio = $\frac{\text{Dia. of pitch circle}}{\text{Dia. of Jet}} = D/d = m$

≈ 11 to 15 & commonly 9 is adopted 12.

(d) No. of buckets on the runner should be as less as possible in order to minimize friction losses. But in order to maximize volumetric efficiency optimum no. of buckets are given by

$$Z = 15 + \frac{D}{2d} = 15 + \frac{0.5}{0.1} m$$

[$m=12$]

≈ 21

[Range 18 to 25]

(e) No. of Jets (n) = $\frac{\text{Total discharge}}{\text{Discharge through one jet}}$

→ 6

(f) width of bucket may be taken as height times dia. of Jet & vertical depth of bucket may be taken as 15 times dia. of Jet

Pelton wheel

- i) Head at the base of Nozzle = 32 m (H)
- ii) discharge of the Nozzle = $0.18 \text{ m}^3/\text{sec}$ (Q)
- iii) area of Jet = 7500 mm^2 (a)
- iv) Power available at the shaft = 44 kW (S.P.)
- v) Mech. efficiency = 94% ($\eta_m = 0.94$)

calculate the power loss

- a) In the Nozzle
- b) In the Runner
- c) In the Mech. friction

$$\eta_m = \frac{\text{S.P.}}{\text{R.P.}}$$

(248)

$$\text{R.P.} = \frac{\text{S.P.}}{\eta_m} = \frac{44}{0.94} = 46.8 \text{ kW}$$

$$\text{velocity of Jet} = \frac{Q}{a} = \frac{0.18}{7500 \times 10^{-6}} = 24.0 \text{ m/sec}$$

$$\text{K.E. per second of the Jet} = \frac{1}{2} \times \left(\frac{wQ}{g} \right) \times V_1^2$$

$$= \frac{1}{2} \times \frac{9.81 \times 0.18}{9.81} \times (24)^2 = 51.84 \text{ kW}$$

Power available at the base of Nozzle

$$= wQH$$

$$= 9.81 \times 0.18 \times 32 = 56.5 \text{ kW}$$

Loss at Nozzle

$$= 56.5 - 51.84 = 4.66 \text{ kW}$$

Loss in Runner

$$= 51.84 - 46.8 = 5.04 \text{ kW}$$

$$\text{Mechanical Losses} = 46.8 - 44 = 2.8 \text{ kW}$$

$$= \text{S.P.} - \text{Losses}$$

$$= \text{S.P.} - \text{shaft power}$$

A Pelton wheel has a mean bucket speed of 12 m/sec. It is supplied with water at a rate of 750 lit/sec under a head of 35 m. If the buckets deflect the jet by an angle of 160° . Find the horse power & efficiency of the bucket taking $C_v = 0.98$ & Neglecting the friction in the losses.

$$u_1 = u_2 = 12 \text{ m/sec}$$

$$Q = 750 \text{ l/sec}$$

$$= 0.75 \text{ m}^3/\text{sec}$$

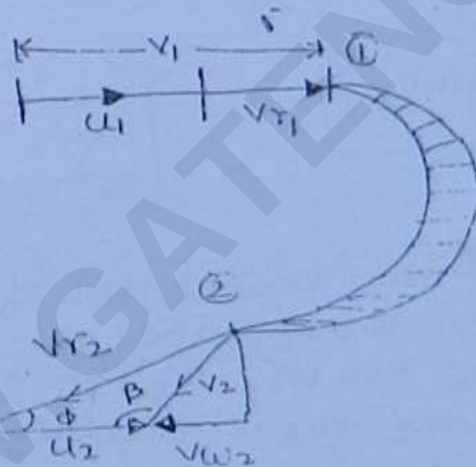
$$H = 35 \text{ m}$$

$$\text{Deflection angle} = 180^\circ - \phi$$

$$= 160^\circ$$

$\therefore$ side clearance angle or runner vane angle $(\phi) = 20^\circ$

$$C_v = 0.98$$



$$V_{r1} = V_1 - u_1$$

$$= 25.68 - 12$$

$$= 13.68 \text{ m/sec}$$

$$V_1 = C_v \sqrt{2gH}$$

$$= 0.98 \sqrt{2 \times 9.81 \times 35}$$

$$= 25.68 \text{ m/sec}$$

$$V_{r1} = V_{r2} = 13.68 \text{ m/sec}$$

$$\frac{20^\circ}{180^\circ} \left[V_{r2} \cos \phi \right] = |u_2| + |V_{w2}|$$

$$\Rightarrow 13.68 \cos 20^\circ = 12 + V_{w2}$$

$$\Rightarrow V_{w2} = 0.855 \text{ m/sec}$$

$$= \frac{\omega Q}{g} [V\omega_1 u_1 + V\omega_2 u_2]$$

$$= \frac{9.81 \times 0.75}{9.81} [25.68 + 0.855] \times 12$$

250

Efficiency: ↓

$$\text{Hydraulic efficiency } \eta_H = \frac{R.P.}{K.E./\text{second}}$$

$$K.E./\text{second} = \frac{1}{2} \times \left(\frac{\omega \Delta}{g} \right) \times V_1^2$$

$$= \frac{1}{2} \times \frac{9.81 \times 0.75}{9.81} \times 25.68^2$$

$$= 247.29 \text{ kW}$$

$$\therefore \eta_H = \frac{238.8}{247.29} \times 100 \%$$

$$= 96.56\%$$

rob3
sp004

A pelton wheel has mean bucket dia. of 1m and is running at 1000 R.P.M. The net head on the pelton wheel is 700m. If the side clearance angle is 15° and discharge through the nozzle $0.1 \text{ m}^3/\text{sec}$. Find

- Power available at the nozzle
- Hydraulic efficiency of turbine, take $C_v = 1$

$$u_1 = u_2 = \frac{\pi \cdot DN}{60} = \frac{\pi \times 1 \times 1000}{60} = 52.35 \text{ m/sec.}$$

Net head on the pelton wheel = 700m

$$\phi = 15^\circ$$

$$Q = 0.1 \text{ m}^3/\text{sec}, C_v = 1.0$$

$$V = C_v \sqrt{2gH}$$

$$= 1.0 \sqrt{2 \times 9.81 \times 700}$$

$$= 117.12 \text{ m/sec}$$

Can't be applied in calculator

$$\text{Power at Nozzle} = 9.81 \times 481 \times 700 \times 0.1 = 686.7 \text{ kW}$$

257

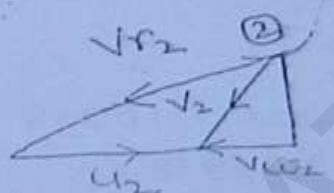
$$\eta_B = \frac{V_{w1} u_1 + V_{w2} u_2}{gH} \quad \left\{ \begin{array}{l} \text{Losses at} \\ \text{Nozzle are} \\ \text{negligible} \end{array} \right.$$

$$V_{w1} = V_1 = 117.19$$

$$V_{r1} = V_1 - u_1 = 117.19 - 52.35 = 64.83$$

Incoming bucket to be frictionless

$$V_{r2} = V_{r1} = 64.83 \text{ m/sec.}$$



$$|u_2| + |V_{w2}| = |V_{r2} \cos \alpha|$$

$$= 64.83 \cos 15^\circ$$

$$\therefore V_{w2} = 10.261 \text{ m/sec.}$$

$$\therefore \eta_B = 97.18\%$$

Prob 4

A pipeline 1200 m long supply water to three single pelton wheel. The head over the nozzle is 360 M. $CV = 0.98$, friction factor for the pipe $= 0.02$. The turbine efficiency based on the head of the nozzle is 0.85. If specific speed based on the head of nozzle is 15.3 [Ns. is such that $n = \text{r.p.m}$, $P = \text{kW}$, $H = \text{meter}$]. If head loss due to friction in the pipeline is 12 m and operating speed of each turbine is 300 r.p.m. determine

- Total power developed
- The dia. of pipe line
- The dia. of each nozzle.
- Discharge

$$N_s = \frac{N \sqrt{P}}{H^{5/4}}$$

$P \rightarrow H \cdot P$ Power/s
 $H \rightarrow$ Head

$$L = 1200 \text{ m}$$

no. of turbine = 3

$$H = 360 \text{ m}$$

$$f = 0.02$$

252

NOTE:

For multiple jet
 pelton wheel the
 sp. speed is based
 on Break power
 per jet.

$$\eta = 0.85 = \frac{\text{S.P.}}{\text{water available at the base of nozzle}}$$

$$0.85 = \frac{55.06}{WQH}$$

$$\Rightarrow Q = 1.834 \text{ m}^3/\text{sec.}$$

Total discharge supplied by pipe = 18.34

Discharge through each nozzle

$$= \frac{1.834}{3} = 0.611 \text{ m}^3/\text{sec}$$

$$V_1 = C_v \sqrt{2gH} = 0.98 \sqrt{2 \times 9.81 \times 360}$$

$$V_1 = 82.36 \text{ m/sec}$$

discharge dia. of each nozzle is

$$\frac{\pi d^2}{4} = \frac{0.611}{82.36} \Rightarrow d = 0.097 \text{ m} = 97 \text{ mm}$$

$$P_f = \frac{f L Q^2}{12.1 D^5} = \frac{0.02 \times 1200 \times 18.34^2}{12.1 D^5}$$

$$= 12$$

$$\Rightarrow D = 0.88 \text{ m}$$

Specific speed for various turbines:

[M.K.S. Unit]

a) Pelton wheel

P → H.P.

→ Single Jet $N_s = 10 \text{ to } 30$
→ Multi Jet $= 30 \text{ to } 60$

(253)

b) Francis turbine

$N_s \Rightarrow 60 \text{ to } 300$

c) Kaplan turbine

$N_s \Rightarrow 300 \text{ to } 1000$

Unit quantities: ↓

A turbine operates most efficiently at its design points at a particular combination of H , Q & N . But in practice these variables do not remain constant therefore concept of unit quantity is important to

(i) Predict the behaviour of turbine working at different condition

(ii) To make the comparison of performance of turbine of the same type but diff-size

(iii) It may be used to compare the performance of turbine of different type.

(iv) To correlate the use of experimental data.

a) unit speed: It is the theoretical speed at which a given turbine would operate under a head of 1M.

$$N_u = \frac{N}{\sqrt{H}}$$

for a turbine to run under a head of 1m

$$Q_u = \frac{Q}{\sqrt{H}}$$

(254)

> Unit power: It is the theoretical power which a turbine would produce under a head of 1m.

$$P_u = \frac{P}{H^{3/2}}$$

Model quantities: ↓

When the results obtain from the experiment conducted on the model are applied to the prototype in the field following similarity exists:

$$a) \frac{H}{N^2 D^2} = \text{constant}$$

$$i.e. \frac{H_M}{N_M^2 D_M^2} = \frac{H_P}{N_P^2 D_P^2}$$

$$b) \frac{P}{N^3 D^5} = \text{constant}$$

$$\Rightarrow \frac{P_M}{N_M^3 D_M^5} = \frac{P_P}{N_P^3 D_P^5}$$

$$c) \frac{Q}{N D^3} = \text{constant}$$

$$\Rightarrow \frac{Q_M}{N_M D_M^3} = \frac{Q_P}{N_P D_P^3}$$

3) Kaplan & Propeller Turbines

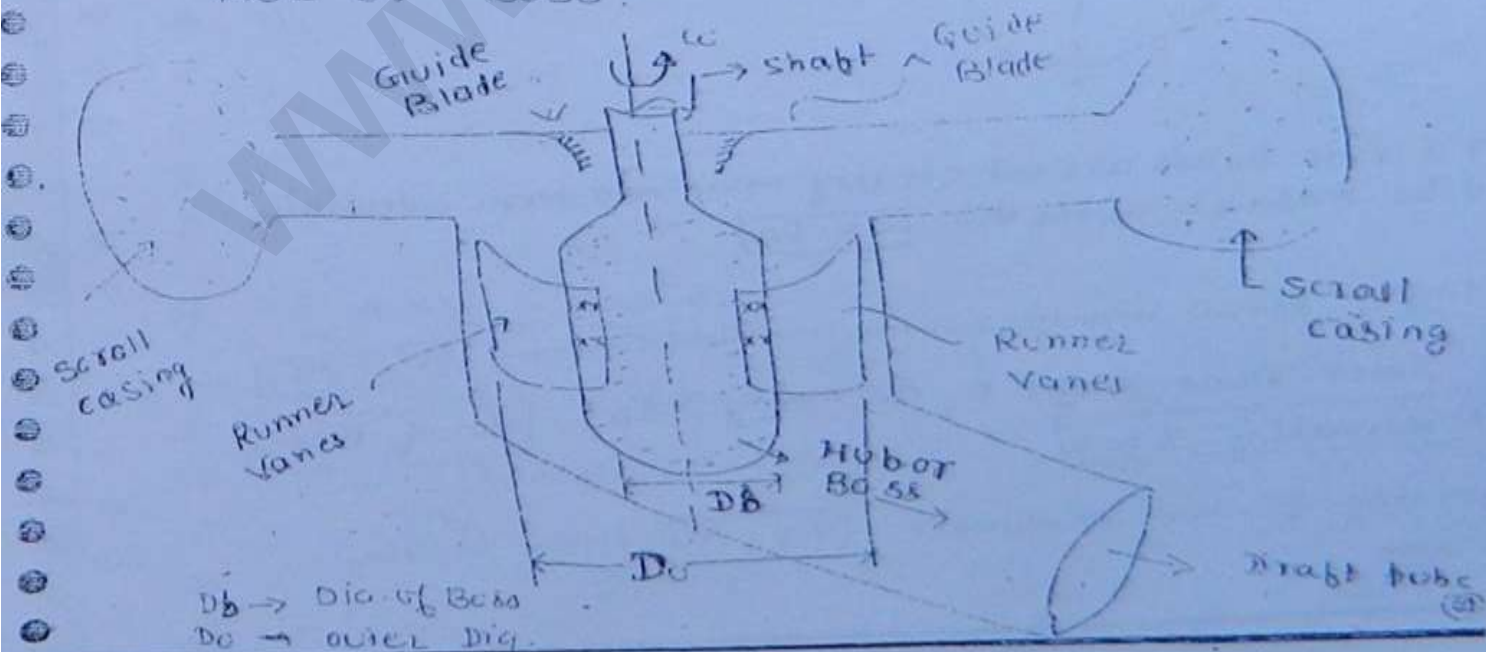
(255)

- Axial flow Rxn turbines
- Propeller turbines & Kaplan turbines are similar in principle except that Kaplan had improvement over propeller w.r.t. direction of the blades are flexible i.e. Propeller turbines has fixed vanes connected through rivets permanently where Kaplan has provided bolts & an additional bolt hole.



Important unit of Propeller turbines: ↓

- a) For the axial flow reaction turbine the shaft of the turbine is vertical, the lower end of the shaft is made larger which is known as Hub or Boss.



$$A_1 V_1 = A_2 V_2 = \frac{\pi D^2}{4} V$$

$$\begin{aligned} \text{ii)} \quad Q &= A_1 \cdot V_1 \\ &= A_2 \cdot V_2 \end{aligned}$$

$$\Rightarrow V_1 = V_2$$

256

iii) If friction losses are negligible then

$$V_{r1} = V_{r2}$$

iv) Vane angle from D_o to D_b may change

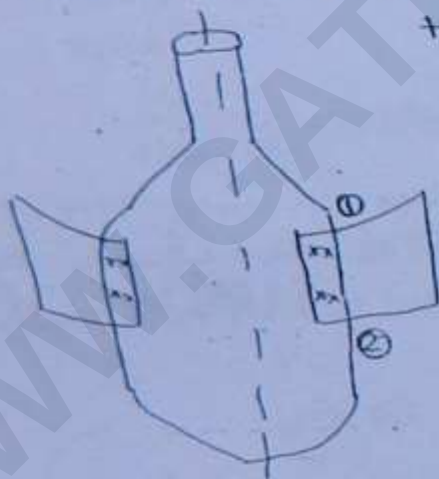
$$u_1 = u_2 = \frac{\pi D N}{60}$$

u_1 at $D = D_o$

u_2 at $D = D_b$

$$\left\{ D = D_{avg} = \frac{D_o + D_b}{2} \right\}$$

* Power & Efficiency calculation are similar to Francis turbine & velocity diagram also similar to Francis turbine.



$$H = \frac{V_2^2}{2g} + \frac{V_{w1} u_1}{g}$$

NOTE:

1) If entry & exit points are not clearly maintain then calculations should be made at outer dia. $d = D_o$

2) Velocity of flow through runner remains constant. $A_{f1} = A_{f2}$

3) The Inlet Vane Angle θ , normally $> 90^\circ$. No. of Blades is normally 3 to 6.

4) Ignoring friction resistance, $V_{r1} = V_{r2}$ may be taken.

5) Under assumptions B-E, may be applied at inlet and exit point

when $v_{r2} = 0$, friction on the blades is negligible

Prob 1
100/11

A propeller runner turbine runner has outer dia 4.5m & dia of hub is 2m. It is required to develop power 20600 kW when running at 150 r.p.m. under a head of 21 M. Assuming hydraulic efficiency 94% & overall efficiency of 88%. Determine the runner vane angle at inlet & outlet at the mean exit of vane. Assume velocity of whirl at outlet is zero. Also determine the vane angle at inlet & outlet at outer dia.

Calculation at mean diameter: ↓

257

$$D_m = \frac{D_o + D_b}{2}$$

$$= 3.25 \text{ m}$$

$$\begin{aligned} u_1 = u_2 &= \frac{\pi D_m N}{60} \\ &= \frac{\pi \times 3.25 \times 150}{60} \\ &= 25.525 \text{ m/sec.} \end{aligned}$$

$$\text{Shaft Power} = 20600 \text{ kW}$$

$$\eta_o = 0.88$$

$$\eta_o = \frac{S.P.}{H.P.}$$

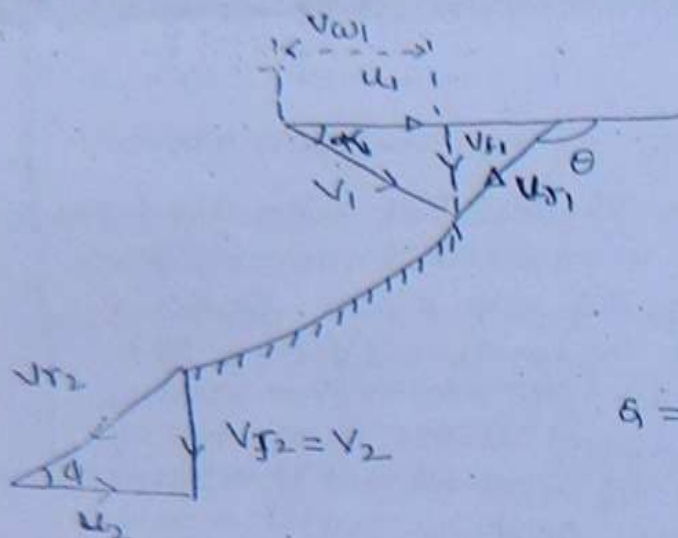
$$WQH = \frac{20600}{0.88}$$

$$\Rightarrow 4.81 \times 21 = \frac{20600}{0.88}$$

$$\Rightarrow Q = 113.63 \text{ m}^3/\text{sec.}$$

$$\text{Hydraulic efficiency} = \frac{v_{w1} u_1}{gH} = 0.94$$

$$\Rightarrow v_{w1} = 7.58 \text{ m/sec.}$$



$$Q = \frac{\pi}{4} (D_1^2 - D_2^2) \times V_{f1}$$

$$\Rightarrow V_{f1} = 8.90 \text{ m/sec}$$

$$V_h = V_{f2} = V_2 = 8.90 \text{ m/sec.}$$

$$\tan(180 - \theta) = \left(\frac{V_{f1}}{u_1 - V_{w1}} \right) = \frac{8.90}{(25.525 - 7.58)}$$

$$\tan(180 - \theta) =$$

$$\Rightarrow \theta = 153.6^\circ$$

From exit velocity diagram

$$\tan \phi = \frac{V_{f2}}{u_2} = \frac{8.95}{25.525}$$

$$\Rightarrow \phi = 19.22^\circ$$

Case IInd.

Repeat the calculation for $d = D_0$

NOTE

When it is given that runner dia is d m then it is the ^{outer} dia. of the runner & in case of the Pelton wheel pitch circle dia. should be taken

Runner

Prob 2

A Kaplan turbine has a dia. of 4 m. And hub dia. ^{9.2} meter. The discharge through turbine is 70 m³/sec. The H_B & H_m can be taken as 0.9 & 0.83 respectively. Assuming absence of visit at outlet & discharge is free from friction estimate the Net head available in the turbine & the power developed. Speed Ratio is 2.0. Also estimate specific speed.

$$V_{f1} = \frac{Q}{\frac{\pi}{4} (D_o^2 - D_b^2) (\frac{\pi}{2} \sin \theta)} = 6.12 \text{ m/sec}$$

$$= V_{f2} = V_2$$

$$H = \frac{V_2^2}{2g} + \frac{V_{w1} u_1}{g}$$

259

$$\Rightarrow H = \frac{6.12^2}{2 \times 9.81} + 0.9H$$

$$\Rightarrow H = 19.1 \text{ m}$$

$$\text{S.P.} = \frac{H \cdot P \cdot \eta_o}{1000} = \omega \rho H \times \eta_a \times \eta_m$$

$$\text{S.P.} = 10.972 \text{ kW}$$

$$\phi = \frac{u_1}{\sqrt{2gh}} \Rightarrow u_1 = 2.0 \times \sqrt{2 \times 9.81 \times 19.1}$$

$$u_1 = 38.71 \text{ m/sec}$$

$$u_1 = \frac{\pi D_o N}{60} \Rightarrow N = 184.76 \text{ rpm}$$

$$N_s = \frac{N \sqrt{P}}{H^{5/4}} = \frac{184.76 \sqrt{10.972}}{(19.1)^{5/4}} = 454.31$$

$$N_s (\text{specific speed}) = 454.31 \text{ (SI unit)}$$

The hub dia. of a Kaplan turbine working under a head of 12 m is 0.35 times dia. of runner, the turbine is running at 100 r.p.m. If the vane angle of extreme edge at outlet is 15° and

Flow ratio is 0.26.

a) Find dia. of runner $\rightarrow D_o = 6.55 \text{ m}$

b) Find dia. of boss $\rightarrow$

c) discharge through runner $\rightarrow 271.77 \text{ m}^3/\text{sec}$

d) Assume velocity of vort at outlet is zero

Prob 3

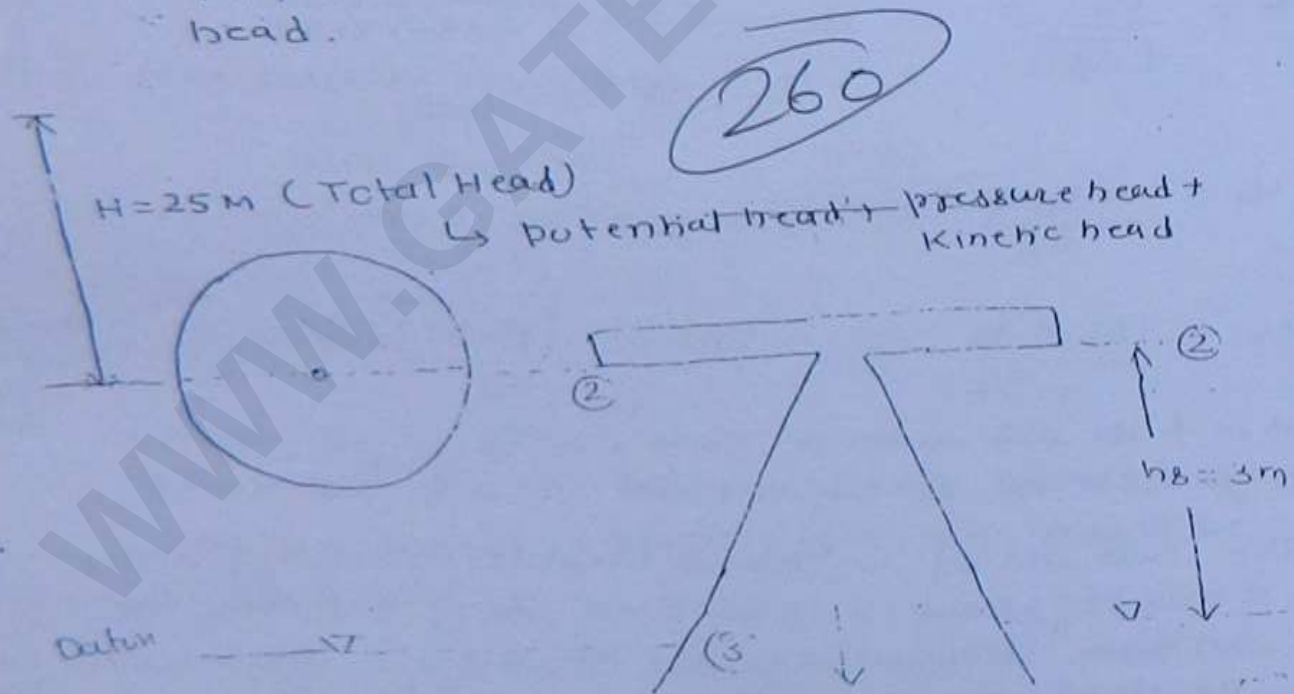
total head of 25m. The centre line of the machine is 3m above the water level, on the tail race. The abs. velocity of flow leaving the vanes on the runner wheel is in the radial direction. the outlet dia = $0.45 \times \text{inlet dia}$. The tangential velocity at exit the rim is 10.8 m/sec & velocity of runner

at entry is 3.3 m/sec. And velocity at the exit of the draft tube is 2.2 m/sec. Assuming that flow enters without shock. (the runner wheel)

determine

- 1) outlet angle of the guide blades (α)
- 2) inlet & outlet angle (θ & ϕ)
- 3) The pressure head at inlet & outlet of the runner

Assume that loss due to friction in the guide blade, runner blade plate & the draft tube is 4%, 6% & 5% respectively of the available head.



$$H = 25\text{m}$$

$$H_2 = 0$$

$$2 = 1.4 \times 10^4$$

$$V_1 = 10.8\text{ m/sec}$$

$$V_2 = 3.3\text{ m/sec}$$

$$\left[\begin{array}{l} 2 = 1 \\ 3 = 2 \\ p_1/\rho + V_1^2/2 = ? \end{array} \right]$$

$$V_3 = 2.2\text{ m/sec}$$

$$= 0.04 \left(\frac{V_1^2}{2g} \right) = 0.04 H$$

(26)

Loss in Runner Vanes

$$= 0.06 \left(\frac{V_2^2}{2g} \right) = 0.06 [0.96 H]$$

Loss in Draft tube

$$= 0.05 \left(\frac{V_3^2}{2g} \right)$$

Apply B.E. n B.W (2) & (3)

$$\left(\frac{V_2^2}{2g} + \frac{P_2}{\omega} + 3 \right) = \frac{P_{atm}}{\omega} + \frac{V_3^2}{2g} + 0 + h_L$$

$$=$$

$$\Rightarrow 0.95x = \frac{P_{atm}}{\omega} + \frac{V_3^2}{2g}$$

$$= 10.3 + \frac{2.2^2}{2 \times 9.81}$$

$$\Rightarrow x = \frac{10.5}{0.95} = 11.10$$

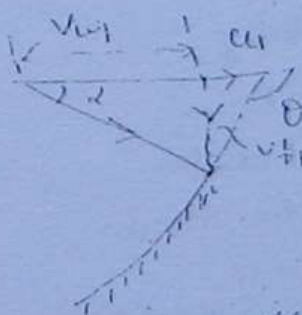
∴ Total head at the exit diameter = 11.10

B.E. n B.W Inlet & outlet of Runner

Head at Inlet = Head at outlet + work done on Runner / by unit of water

$$Y = 11.10 + \frac{\omega \cdot Q \cdot [V_{w1} \cdot U_1]}{\omega \cdot g} + 0.06y$$

$$0.94y = 11.10 + \frac{V_{w1} U_1}{g}$$



$$\Rightarrow \frac{V_{w1} U_1}{g} = 24 - 11.10$$

$$\Rightarrow V_{w1} = 10.409 \text{ m/sec}$$

Since $V_{w1} < U_1 \Rightarrow \theta > 90^\circ$

$$\tan \theta = \frac{V_{f1}}{V_{w1}} = \frac{3.3}{10.409}$$

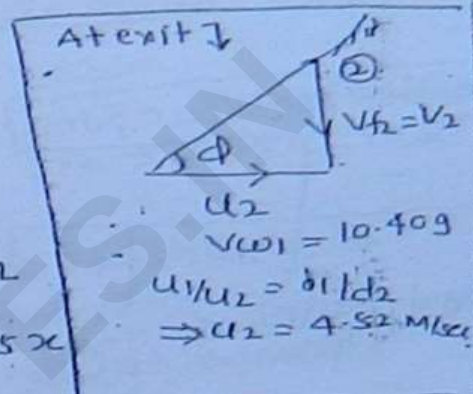
$$\tan (180^\circ - \theta) = \frac{V_{f1}}{U_1 - V_{w1}}$$

$$\Rightarrow \theta = 17.59^\circ$$

$$V_1 = \sqrt{V_{f1}^2 + V_{w1}^2}$$

$$= 10.92 \text{ m/sec}$$

$$\Rightarrow \theta = 96.76^\circ$$



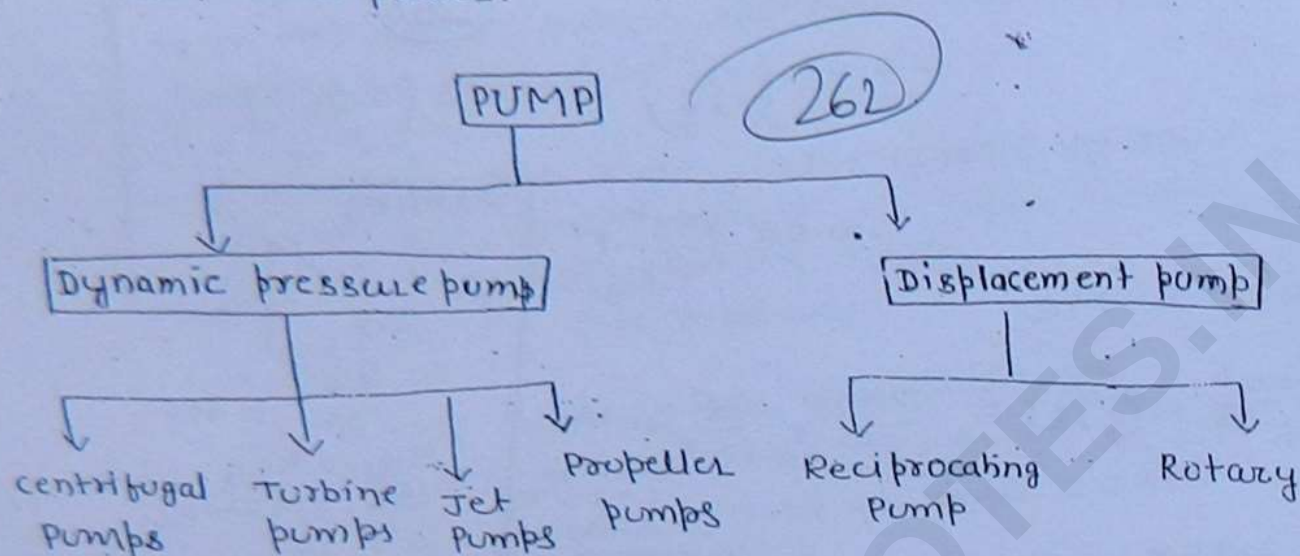
Total Head at Inlet

$$24 = 2 + \frac{P_1}{\omega} + \frac{V_1^2}{2g}$$

$$= 3 + \frac{P_{atm}}{\omega} + \frac{10.92^2}{2 \times 9.81}$$

$$\therefore P_1/\omega = 14.92 \text{ m}$$

- * These utilize mechanical power supplied by the shaft or utilize Man-power to convert it into hydraulic power or water power.



centrifugal pump → These work on the principle of forced Vortex - Motion

Reciprocating pump → works on the principle of suction pressure

* Centrifugal pump: ↓

1) These have high output & high efficiency & are used for Low head & high discharge.

a) When Head is Less than 15m these are called Low Head pump

b) $15\text{m} < H < 45\text{m}$ → medium head pump

c) $H > 45\text{m}$ → High head pump

→ Head here means suction head primarily

NOTE: when $\text{head is } > 40\text{m}$, single stage centrifugal pump is not desirable therefore Multistage pump in series should be used. It is practically observed that when head is $\text{btw } 12 \text{ to } 8\text{ m}$, centrifugal pump are most efficient.

* The Nos of vanes on Rotating unit are 6 to 12.

* centrifugal pumps are exactly inverse of Francis turbine. At Means these are outward radial flow pumps.

(263)

Principle: ↓

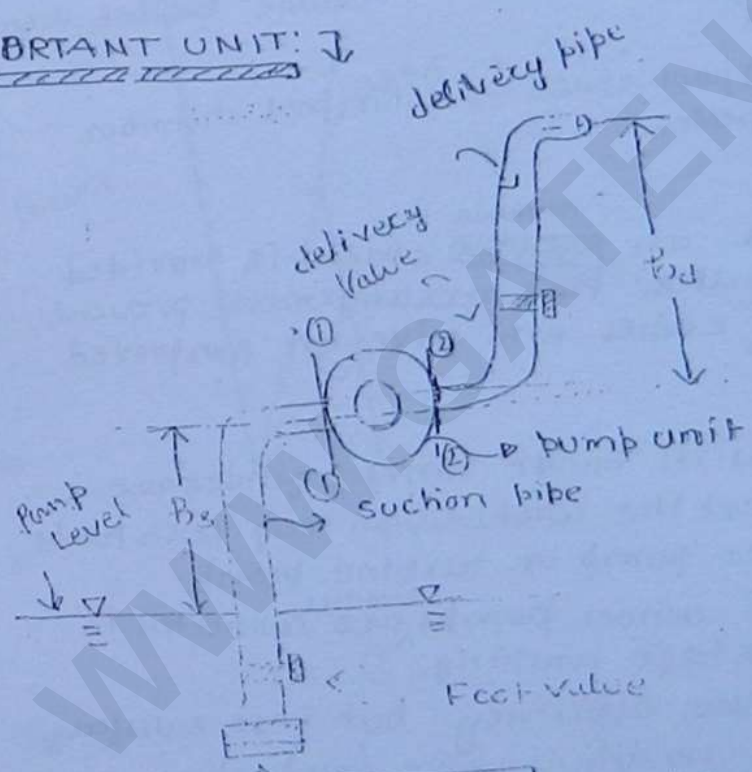
By rotating Impeller, pressure head difference is created.

$$= \frac{\omega^2 r_2^2}{2g} - \frac{\omega^2 r_1^2}{2g}$$

$r_1 \rightarrow$ inner radii
 $r_2 \rightarrow$ outer radii

→ This pressure head is created is utilized to Lift the water against manometric head (g + the head against which pump has to work)

IMPORTANT UNIT: ↓



strainer → used to prevent entry of blockage material

$$H_s = B_s + B_d$$

Static head = suction head + delivery head

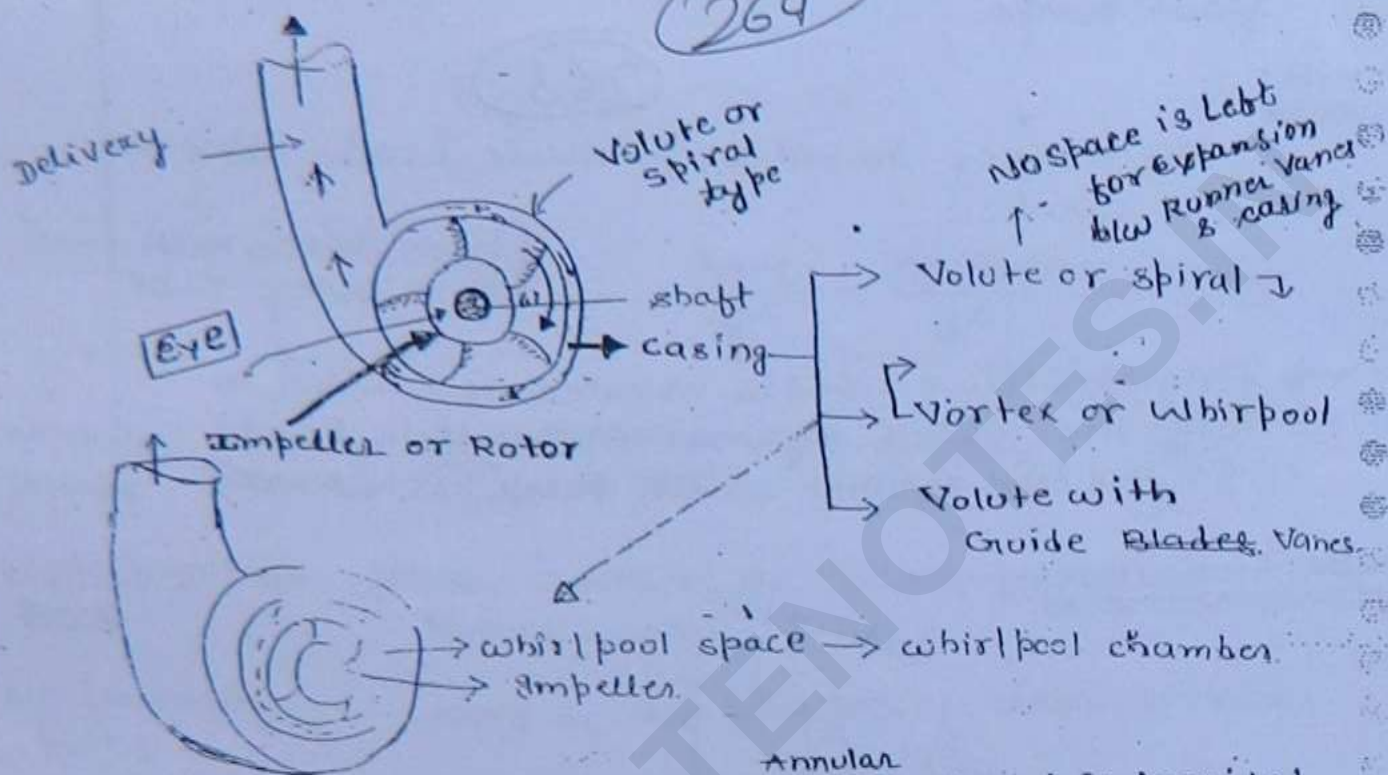
Pressure in suction pipe is always below atmospheric

- minimum pressure exist at (1-1) i.e. entry of pump
- This p_{min} should not fall vapour pressure of water

hammer problem. ($p_{min} = p_s$)

Imp parts of pump: ↓

264



— In whirlpool chamber an ~~Annular~~ spaced is provided b/w casing & impeller, this arrangement prevent the formation of eddies and gives an improved performance.

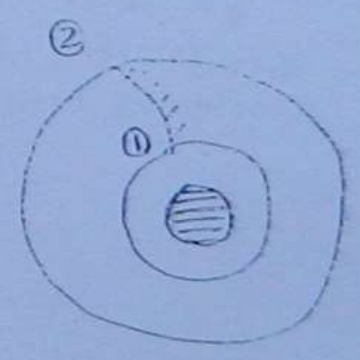
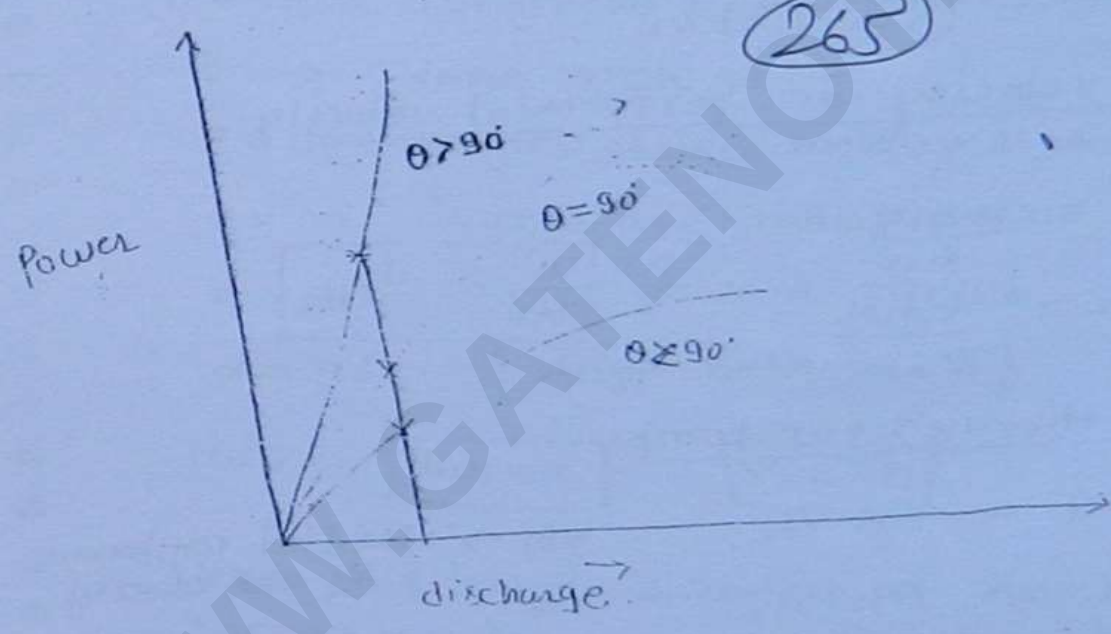
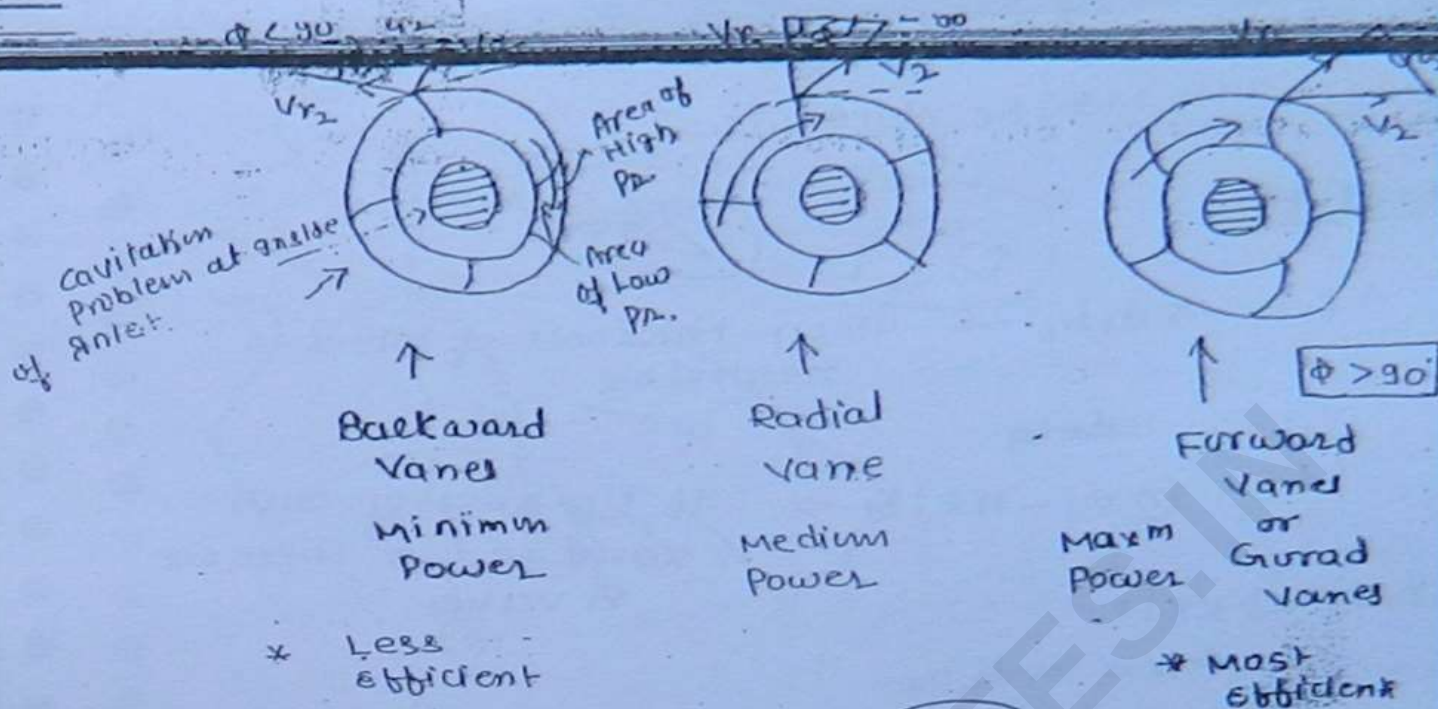
* In case of Volute with Guide Vanes, guides are provided to divert the water, properly such pumps are called diffuser pump or turbine pump. These are adopted when pumps ^{impeller} are connected in series for multistage pumping.

→ These have maximum efficiency but less satisfactory when operating conditions are fluctuating (Power or head)

* Impeller: ↓

It is the rotating unit of a pump similar to runner unit of turbine

→ Impeller has 6 to 12 curved Vanes.



d_1 → inner dia. (inlet dia)
 d_2 → exit dia. (outer dia)

Generally

$$d_1 \leq \frac{1}{2} d_2$$

* size of impeller means outer dia. d_2

Let b_2 be the outlet width
Following points may be noted: ↓

(1) Area of flow:

266

$A_{f1} = \pi d_1 b_1 \rightarrow$ when thickness of vanes is negligible

$$= \pi d_1 b_1$$

$= (\pi d_1 - n\ell) b_1 \rightarrow$ if thickness of each vane is ℓ & there are n vanes

similarly at exit

$$A_{f2} = \pi d_2 b_2$$

$$= (\pi d_2 - n\ell) b_2$$

(2) Tangential velocity or peripheral velocity:

$$u_1 = \frac{\pi d_1 N}{60}$$

$$u_2 = \frac{\pi d_2 N}{60}$$

$\Rightarrow$

$$\frac{u_1}{u_2} = \frac{d_1}{d_2}$$

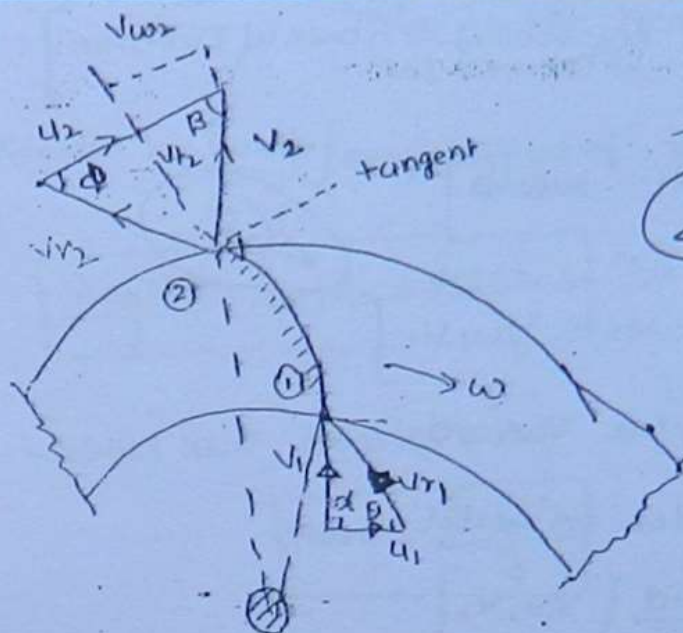
(3) Discharge through the pump:

$$Q = A_{f1} \cdot V_{f1}$$

$$= A_{f2} \cdot V_{f2}$$

$V_{f1} \rightarrow$ Radial component of abs. velocity
 V_{f2}

4) Velocity diagram :



267

u_1 & $u_2 \rightarrow$ Tangential or peripheral velocities

$\phi \rightarrow$ Vane angle at exit

$\phi < 90^\circ$: vanes called backward curved

$\phi = 90^\circ$: vanes are radial at exit

$\phi > 90^\circ$: vanes forward curved

$\alpha = 90^\circ$ { Angle betn u_1 & V_{f1} }

Then

$$V_{w1} = 0$$

$$V_1 = V_{f1}$$

$\rightarrow$ Hence discharge is radial at outlet, it means water enters at inlet without whirl and if there is no impact loss then discharge is said to be without whirl & shock

$$= - \left[\text{work done per second in case of Turbine} \right]$$

$$= - \frac{\omega Q}{g} \left[v_{w1} u_1 - v_{w2} u_2 \right] \quad (268)$$

$$\therefore \text{work/sec.} = \frac{\omega Q}{g} \left[v_{w2} u_2 - v_{w1} u_1 \right]$$

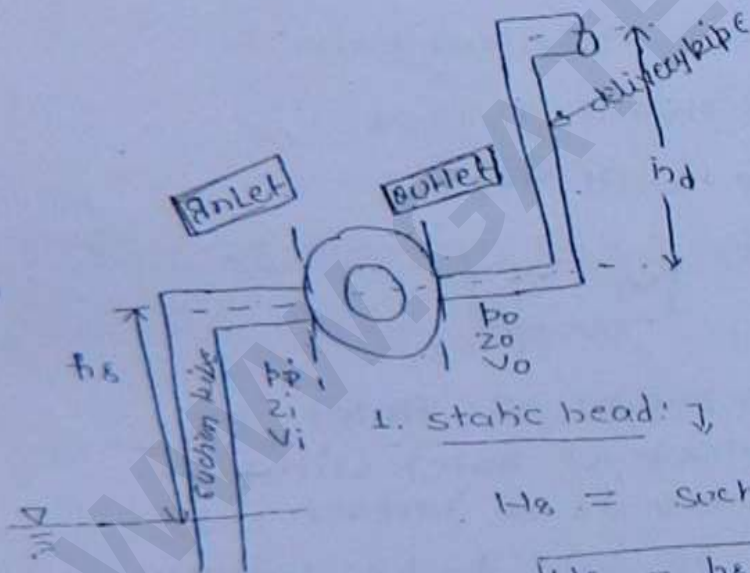
For maxm power $v_{w1} = 0$

work-done by impeller on water/sec

$$= \frac{\omega Q}{g} \left[v_{w2} u_2 \right]$$

$$= \text{Impeller power} / \text{Rotor power}$$

Important definitions: ↓



$$H_s = \text{suction head} + \text{delivery head}$$

$$H_s = h_s + h_d$$

2. Friction head: ↓
(h_f)

$$\text{Total friction head} = h_{fs} + h_{fd}$$

↓
friction head in
suction pipe

Manometric head: H_m

This is the head against which pump has to work.

Work done by pump / sec / unit wt of water = Manometric head + Losses.

$$\Rightarrow A) \quad \boxed{\frac{V_{w2} U_2}{g} = H_m + \text{Losses}} \quad \text{gcp} \quad [\text{Assuming } V_{w1} = 0]$$

— If Losses are negligible

$$\boxed{H_m = \frac{V_{w2} U_2}{g}}$$

269

B) $H_m = \text{Total energy head at outlet} - \text{Total energy head at inlet}$

$$= \left[\frac{p_o}{\rho} + z_o + \frac{v_o^2}{2g} \right] - \left[\frac{p_i}{\rho} + z_i + \frac{v_i^2}{2g} \right]$$

$v_o \rightarrow v_d$ (velocity in delivery pipe)

$v_i \rightarrow v_s$ (velocity in suction pipe)

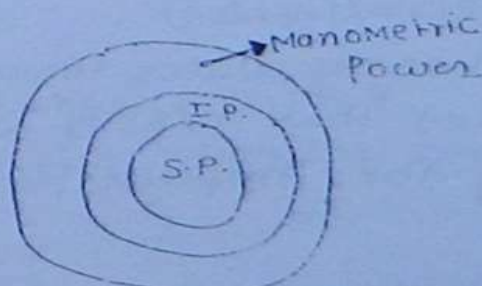
$$\boxed{z_o = z_i} \quad \boxed{p_o = p_d} \quad \boxed{p_i = p_s}$$

$$C) \quad H_m = (p_s + p_d) + (h_{fs} + h_{fd}) + \left(\frac{v_d^2}{2g} \right) \quad \downarrow \text{Negligible}$$

$$\boxed{H_m = (p_s + p_d) + (h_{fs} + h_{fd})} \quad \text{gcp}$$

$\uparrow$ Static head (H_s) $\uparrow$ Friction head

6) Efficiency & Powers of Pump: ↓



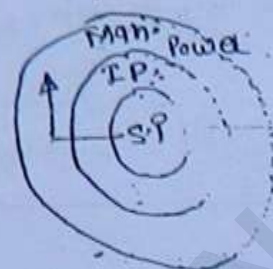
$$\boxed{S.P. > I.P. > \text{Manometric power (water)}} \quad \text{gcp}$$

→ Manometric power = $\omega Q H_m$

1> Mechanical ^{Efficiency} ~~Efficiency~~ (η_{mech}): ↓

$$\eta_{mech} = \frac{I.P.}{S.P.}$$

220



[Reverse calculation]

2> Manometric efficiency: ↓

$$\eta_{mano} = \frac{\text{Mano. power}}{\text{Imp. power}}$$

$$= \frac{\omega Q H_m}{\frac{\omega Q}{g} [V_{w2} u_2]}$$

$$= \frac{g H_m}{V_{w2} u_2}$$

define

$\eta_o \rightarrow \text{man } \eta_{mano}$
 $\eta_m \rightarrow$

3> overall efficiency: ↓

$$\eta_o = \frac{\text{Man. power}}{S.P.}$$

$$= \eta_{mech} \times \eta_{mano}$$

NOTE:

If Leakage losses are also accounted than volumetric efficiency

$$\eta_v = \frac{\text{Discharge at delivery point}}{\text{Discharge at Inlet}}$$

$$= Q / (Q + \Delta Q)$$

ΔQ = Loss of water through casing

It may be noted that Loss of water takes place after leaving the Impeller plates

overall efficiency

$$\eta_o = \eta_{mech} \times \eta_{mano} \times \eta_{volumetric}$$

6) Mean speed required to start the pumping of water

Minimum speed should be such that head developed should be greater than H_m .

(27)

$$\frac{\omega^2 r_2^2}{2g} - \frac{\omega^2 r_1^2}{2g} \geq H_m$$

gelp

$$\omega \rightarrow \text{Rad. Sec} = \frac{2\pi N}{60}$$

$$r_1 \rightarrow d_1/2$$

$$r_2 \rightarrow d_2/2$$

especial:

For a given value of ω & H_m it is possible to work out minimum dia. of impeller (outer dia.) which will be required for pumping of water

$$\Rightarrow \frac{\omega^2}{2g} \left[\frac{d_2^2}{4} - \frac{d_1^2}{4} \right] \geq H_m$$

$$\left[\begin{array}{l} \text{Take } d_2 = 2d_1 \\ \Rightarrow d_1 = d_2/2 \end{array} \right]$$

$$\therefore \frac{d_2^2}{4} [1 - 1/4] \geq \frac{2gH_m}{\omega^2}$$

$$\Rightarrow d_2 = \frac{10.23}{\omega} \sqrt{H_m}$$

gelp

7) Multi-stage centrifugal pump:

(a) To produce high head, impeller should be connected in series. If n no. of impellers are connected in series & each lifts manometric heads equal to (H_m) then

Series connection
 $Q = \text{const.}$

$$\text{Total head Lifted} = n H_m$$

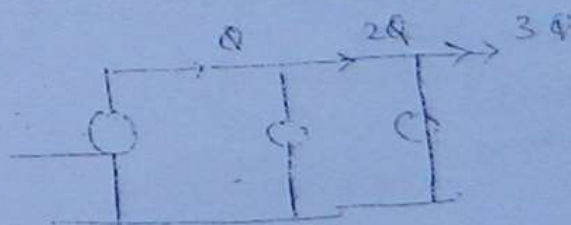
Here total discharge remains constant

(b) Parallel connection: - Impellers or pumps are mounted parallel to increase the discharge

$$\uparrow \text{Head} = \text{const.}$$

$$\therefore \text{Total discharge} = nQ$$

Total head remains constant



$$(1) \left(\frac{N \sqrt{Q}}{H_m^{3/4}} \right)_m = \left[\frac{N \sqrt{Q}}{(H_m)^{3/4}} \right]_p$$

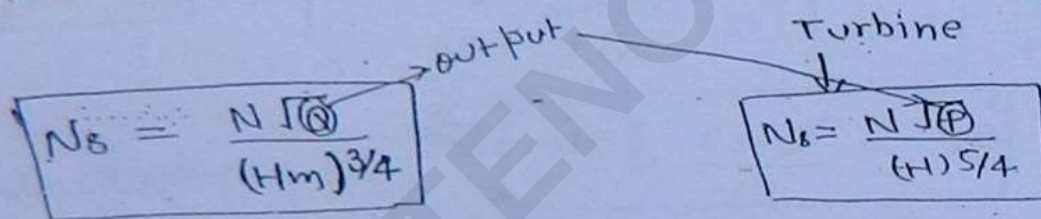
$$(2) \left(\frac{H_m}{D^2 N^2} \right)_m = \left(\frac{H_m}{D^2 N^2} \right)_p$$

272

$$(3) \left(\frac{Q}{N D^3} \right)_m = \left(\frac{Q}{N D^3} \right)_p$$

$$(4) \left(\frac{P}{N^3 D^5} \right)_m = \left(\frac{P}{N^3 D^5} \right)_p$$

9> specific speed of pumps: ↓



10> cavitation & ~~Thoma~~ Number: ↓

In centrifugal pump the pressure is minimum on the under side of vane at entry where vapour pressure may be formed, these vapour pressure carried to a region of high pr. near to exit where bubble collapse causing pitting & severe damage to metal surface. ~~apparently~~ entry vane tips at exit are the most susceptible for water hammer attack. Therefore harmful effect of cavitation are:

1> pitting & erosion of surface due to continuous hammering

2> sudden drop in head & decrease in the efficiency in the pump

3> Noise & vibration

4> corrosion problem

A centrifugal pump having outer dia equal to two times the inner dia & rotating at 1000 rpm works against a total head of 40m. The velocity of flow through impeller is constant & at 2.5 m/sec. The vanes are set back at an angle of 40° at outlet. At the outer dia of impeller is 150 cm & width of outlet is 5 cm

$N = 1000 \text{ rpm}$
 $H_m = 40 \text{ m}$

$V_{f1} = V_{f2} = 2.5 \text{ m/sec}$

$\phi = 40^\circ$

$d_2 = 0.5 \text{ m}$

$d_1 = 0.25 \text{ m}$

$b_2 = 0.05 \text{ m}$

than determine

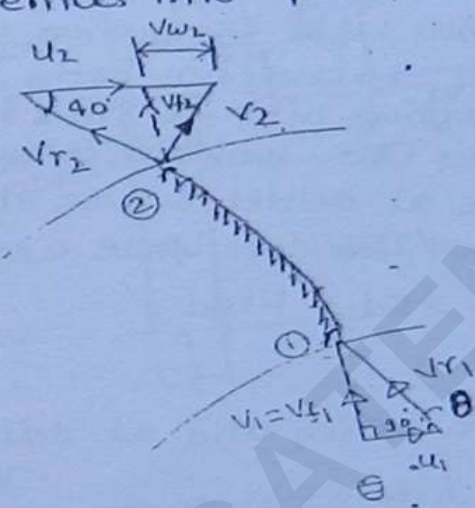
a) Vane Angle at inlet

b) work done by impeller/sec on water

c) Manometric efficiency

(273)

→ water enters into pump without whirl & shock. $[v_{w1} = 0]$



$\tan 40^\circ = \frac{V_{f2}}{u_2 - V_{w2}}$

$\Rightarrow V_{w2} = 23.19 \text{ m/sec}$

B) $\tan \theta = \frac{V_{f1}}{u_1}$

$\Rightarrow \theta = 10.81^\circ$

A) $u_2 = \frac{\pi d_2 N}{60}$
 $= \frac{\pi \times 0.5 \times 1000}{60}$
 $= 26.17 \text{ m/sec}$

C) Manometric efficiency
 $= \frac{g H_m}{V_{w2} u_2}$
 $= \frac{\text{Manometric Efficiency Power}}{\text{IP}}$

$Q = A_{f2} \times V_{f2} = (\pi d_2 b_2) \times V_{f2}$
 $= \pi \times 0.5 \times 0.05 \times 2.5$
 $= 0.1966 \text{ m}^3/\text{sec}$

work done by imp/sec. (IP)

$= \frac{w Q}{g} [V_{w2} u_1] = \frac{9.81 \times 0.1966 \times 23.19 \times 26.17}{9.81}$
 $= 119.9 \text{ kW}$

when running at 600 r.p.m. discharge at the rate of 8000 Lit/min against a head of 8.5 m. The water enters the impeller without whirl & shock. The inner dia. is 0.25 m & the vanes are set back at outlet at an angle of 45° & area of flow which is constant from inlet to outlet of the impeller of 0.06 m^2 . determine

- Manometric efficiency of pump
- The Vane Angle at inlet $\rightarrow 39^\circ$
- Minimum speed at which the pump commences to work.

274

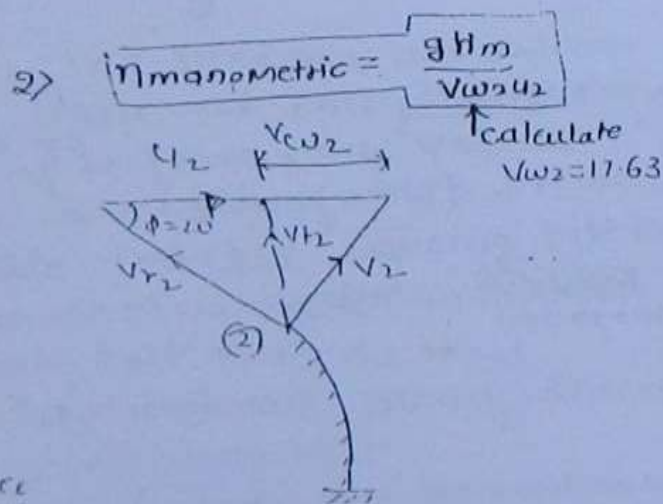
A.C.P. lifts water under a static lift of 40 m of which 3 m is suction lift. The suction & delivery pipes are of 30 cm dia. both. The friction loss in suction pipe is 2 m & in delivery pipe is 6 m. The impeller is 0.5 m dia. & 3 cm wide at outlet & runs at a speed of 1200 r.p.m. The exit blade angle is 20° and $\eta_{\text{mano}} = 85\%$. Find

- The discharge
- Pressure at the suction and at delivery point.

$$1) \quad h_m = h_s + h_f + h_d + h_{fd} \\ = 40 + 2 + 6 \\ = 48 \text{ m}$$

using exit velocity diagram

$$3) \quad \tan 20^\circ = \frac{V_{f2}}{u_2 - V_{w2}} \\ \Rightarrow V_{f2} = 5.02 \text{ m/sec}$$



$$Q = A_{f2} \times V_{f2} = (\pi d_2 b_2) \times V_{f2} = 0.237 \text{ m}^3/\text{sec}$$

$$\text{Absolute velocity at exit } V_2 = \sqrt{V_{f2}^2 + V_{w2}^2}$$

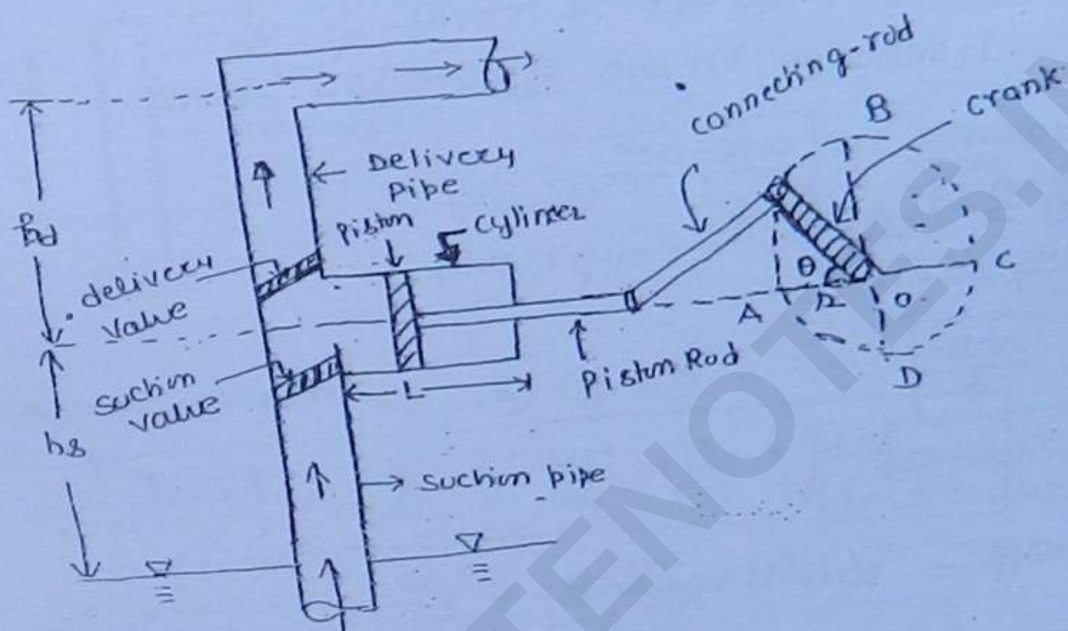
$$= \sqrt{(5.02)^2 + (17.63)^2} \\ = 18.33 \text{ m/sec}$$

27 Reciprocating pump.

These work on the principle of creating suction head.

Main parts: ↓

(27)



Working: ↓

The crank is rotated by an external source of power. When the crank is at A, suction stroke started and when $\theta = 0^\circ - 180^\circ$ suction stroke completes. The piston is moving in the cylinder forward & backward.

$A \rightarrow$ Area of cylinder = Area of piston

L = Length of cylinder chamber

$$L = 2r$$

when $\theta = 0^\circ$ to 90° , acceln takes place and

when $\theta = 90^\circ$ to 180° , deacceln. & at $\theta = 90^\circ$

the velocity of piston is maximum.

For $\theta = 180^\circ$ to $270^\circ \Rightarrow$ Acceln of delivery stroke
 For $\theta = 270^\circ$ to $360^\circ \Rightarrow$ Deacceln of delivery stroke
 at $\theta = 270^\circ \Rightarrow$ velocity is Max

It may be noted that while suction stroke only suction valve is open & delivery valve is closed & vice-versa.

a) Discharge through pump: ↓

Let N be the r.p.m. of crank
1 ~~pipe~~ ^{one} revolvⁿ of crank

$$\text{Vol. of water discharged} = AL$$

276

If in crank, there are N revⁿ.

than discharge in one minute is = ALN

∴ discharge in one sec.

$$\text{Sol} \quad Q = \frac{ALN}{60} \quad N \rightarrow \text{R.P.M}$$

$$A = \frac{\pi D^2}{4}$$

b) Power Required: ↓

$$P = \omega Q H_{\text{required}}$$

$$H_{\text{required}} = (h_s + h_d) + \underbrace{h_{fs} + h_{fd}}_{\substack{\rightarrow \text{if Neglected} \\ \text{then } H = h_s + h_d}}$$

$$\therefore P = \omega Q (h_s + h_d)$$

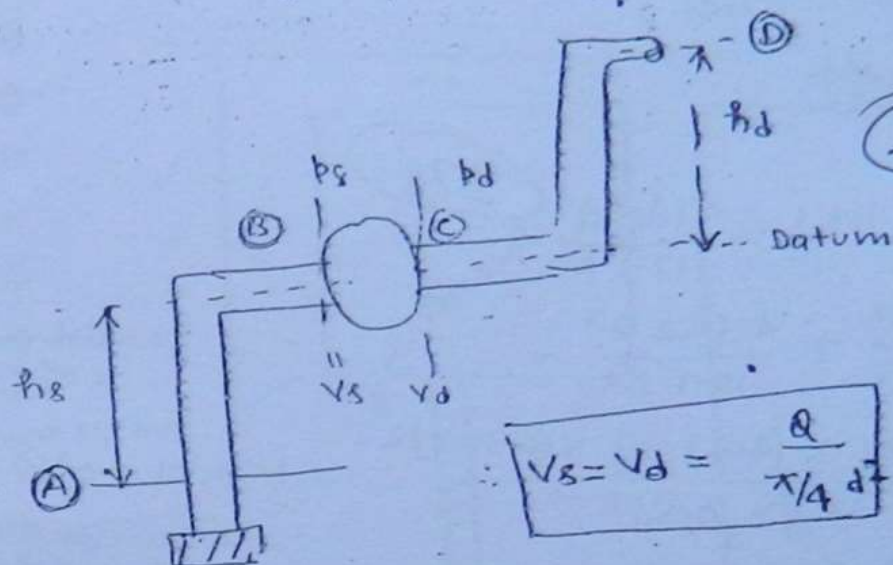
COMMENTS: ↓

1) The discharge through the pump is not continuous in nature therefore power required is fluctuating.

2) Reciprocating pumps are suitable for high suction head & low delivery head & low discharge. ^{more}

3) In order to make the continuous supply of water double acting pump may be used.

hence $V_H = V_1 = Q/A_1$



$$V_s = V_d = \frac{Q}{\pi/4 d^2} = \frac{0.237}{\pi/4 (0.33)^2} = 2.46 \text{ m/sec.}$$

Apply B. Eqn b/w (A) and (B)

$$\left(\frac{p_{atm}}{\rho} \right) = \frac{p_s}{\rho} + \frac{V_s^2}{2g} + [h_s + h_{fs}]$$

Loss in suction pipe

9m head of water $\rightarrow 10.3 = \frac{p_s}{\rho} + \frac{2.46^2}{2 \times 9.81} + 3 + 2$

$$\Rightarrow \frac{p_s}{\rho} = 5$$

$\geq 2.5 \text{ m of vapour pressure at } 20^\circ\text{C}$

Apply B. Eqn b/w (C) and (D)

$$\frac{p_d}{\rho} + \frac{V_d^2}{2g} + 0 = \frac{p_{atm}}{\rho} + \frac{V_d^2}{2g} + (h_d + h_{fd})$$

Loss in delivery pipe

$$p_d/\rho = 10.3 + 3.7 + 6 = 20 \text{ m}$$

Prob 4

In a pumping station 18000 m³ water is to be lifted per day from a intake well to a sedimentation tank under a static head of 21m. Length of suction & delivery pipes are 40m & 150m respectively. dia. of pipes is constant = 0.3m. There are two shifts of working each of 8hrs. If the efficiency of pump & water motor combined is 80% and friction coeff is 0.01. Recommend the unit of pumps each having B.H.P. of (30) per pump.

$$t = 16 \text{ hr}$$

$$\therefore Q = \frac{18000}{16 \times 60 \times 60} = 0.3125 \text{ m}^3/\text{sec}$$

$$H_s = 21 \text{ m}$$

$$h_{fs} = h_{fs} + h_{fd}$$

$$h_{fs} = \frac{4fL_s Q^2}{12.1 D^5}, \quad h_{fd} = \frac{4fL_d Q^2}{12.1 D^5}$$

$$\therefore h_f = \frac{4fL_s Q^2}{12.1 D^5} + \frac{4fL_d Q^2}{12.1 D^5}$$

$$= \frac{4 \times 0.01 \times (407.150) \times (0.3125)^2}{12.1 \times (0.5)^2} = 1.96 \text{ m}$$

$$\therefore H_m = H_s + h_f$$

$$= 21 + 1.96 = 22.96 \text{ m}$$

$$\therefore \text{Manometric power} = \frac{\rho Q H_m}{75} \rightarrow \text{KW}$$

$$= \frac{\rho Q H_m}{75} \text{ (H.P.)}$$

$$= \frac{1000 \times 0.3125 \times 22.96}{75}$$

$$= 95.67 \text{ H.P.}$$

$$\eta_o = \frac{\text{Mano. Power (H.P.)}}{\text{S.H.P.}}$$

$$\text{S.H.P.} = \frac{95.67}{0.80} = 119.58 \text{ (B.H.P.)}$$

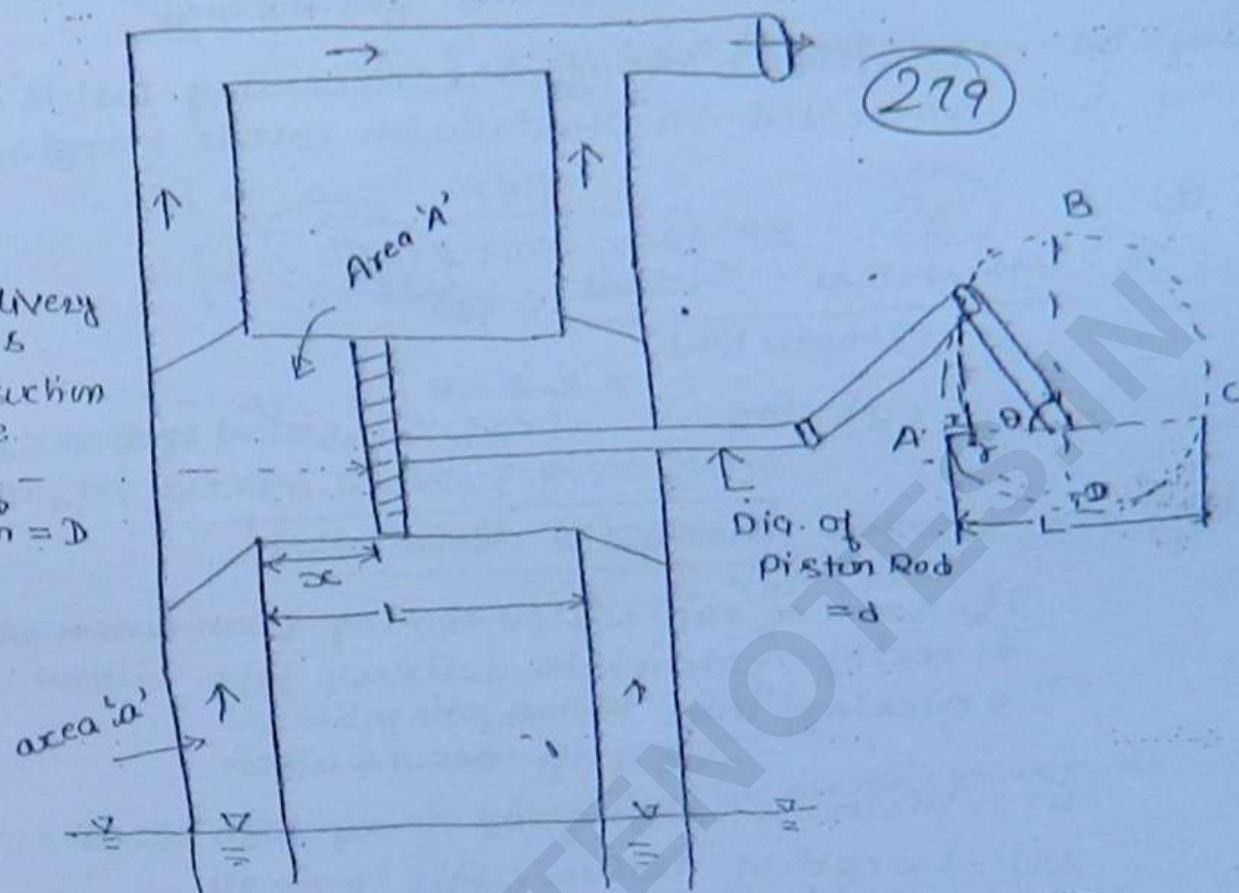
$$\text{No. of pump required} =$$

$$\frac{\text{Total B.H.P.}}{\text{B.H.P. of one pump}}$$

$$= \frac{119.58}{30} \approx 4$$

(279)

Two delivery pipes
two suction pipe.
Dia of piston = D



Area on the right of the piston = $\pi/4 D^2$
Area " " Left " = $\pi/4 (D^2 - d^2)$

Length $L = 2r$

So, Total volume of water delivered in one revolution of crank

$$V = \left[\pi/4 D^2 + \pi/4 (D^2 - d^2) \right] \times L$$

Volume of water delivered in one minute
= $\left[\pi/4 D^2 + \pi/4 (D^2 - d^2) \right] L \times N$

$$\text{Discharge/sec} = \frac{\pi/4 \left[D^2 + (D^2 - d^2) \right] L N}{60}$$

∴ If area, $d \ll D$ then

$$Q = \frac{2 \times \pi/4 D^2 \times L N}{60} = \frac{2 \pi D^2 L N}{60}$$

∴ discharge get doubled

Power Required:

$$P \propto Q$$

∴ Power required get doubled

It may be noted that though the operating cost is doubled but increased in installation cost is marginal

Slip: → (%)

$$= \frac{Q_{\text{theoretical}} - Q_{\text{actual}}}{Q_{\text{theoretical}}} \times 100$$

2.80

$$= (1 - C_d) \times 100$$

[C_d : coefficient of discharge]

-ve slip: → when $Q_{\text{actual}} > Q_{\text{theoretical}}$

If some of the water coming from suction pipe directly enters into delivery pipe without entering into piston chamber -
-ve slip occurs when

- (i) piston is moving at very high speed
- (ii) Length of delivery pipe is small

Effect of Accn of piston on the velocity of suction and delivery pipe: ↓

$$x = r(1 - \cos \theta)$$

velocity of piston

$$V = \frac{dx}{dt} = r \sin \theta \left(\frac{d\theta}{dt} \right) = r \omega \sin \theta$$

$$\boxed{\frac{dx}{dt} = r \omega \sin \theta} \quad \text{gib}$$

V_{max} occurs at $\theta = \pi/2$

Let v be the velocity of water in suction and delivery
a be the area of that pipe

$$\boxed{a v = A V} \quad \text{gib}$$

18 (A/a) $r \omega \sin \theta$ → velocity of water

$$\frac{dv}{dt} = \text{acceleration of water in pipe} \\ = (A/a) \times r \omega \cos \theta (d\theta/dt)$$

∴ $a = \text{accl}^n$ in pipe

$$a = (A/a) r \omega^2 \cos \theta \quad \text{sub}$$

(28)

a_{max} at $\theta = 0$ & π
required

• Force in suction water / pipe water

$$= \text{Mass} \times \text{accl}^n$$

$$= (\rho a l) \times (A/a) r \omega^2 \cos \theta$$

$$F = \rho l A r \omega^2 \cos \theta$$

• Pressure head in pipe due to piston movement

$$= F/a$$

$$= \rho l (A/a) r \omega^2 \cos \theta$$

∴ pressure head = $p/\rho g$

$$h = (l/g) \times (A/a) \times r \omega^2 \cos \theta \quad \text{sub}$$

(29)

• Friction head on suction / delivery pipe

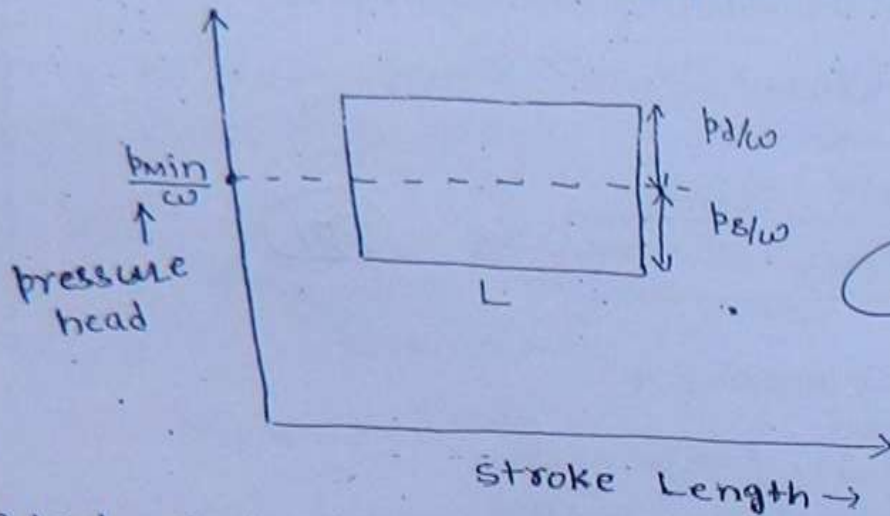
$$h_{fs} = \frac{f l_s v^2}{2 g d_s}$$

suction
[d → dia of pipe]

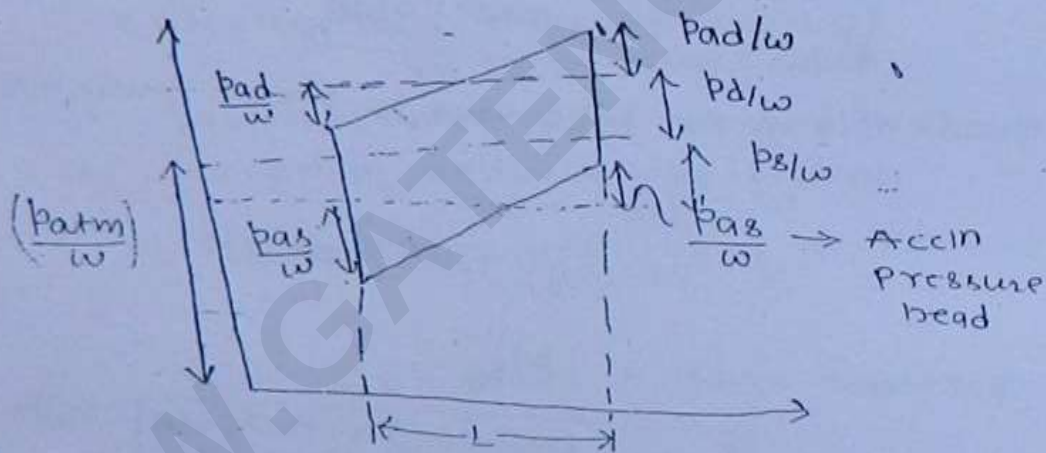
$$= \frac{f l_s \left[\left(\frac{A}{a} \right) r \omega \sin \theta \right]^2}{2 g d_s}$$

$$(h_f)_{\text{max}} = \frac{f l_s \left[(A/a) r \omega \right]^2}{2 g d_s}$$

distance travelled by the piston
in complete revolution of crank.



Ideal Indicator diagram when R_f is neglected & effect of accn. also neglected.



Significance: $\rightarrow$

1) Area of Indicator diagram is directly proportion to discharge and $Q \propto$ Power. Hence if a Indicator diagram of a pump is given then discharge output and power consumed can be compared of two pumps.

CS/2007
ES/2002

Prob 2

A. C. P. has an impeller of 0.5 m of outer dia. when running at 500 r.p.m. discharges 8000 L/min against a head of 8.5 m.

283

The cylinder bore dia. of a single acting reciprocating pump is 150 mm and stroke is 300 mm. The pump runs at 50 r.p.m. and water is lifted to a height of 25 m. The length of delivery pipe is $L_d = 22$ m and $d_d = 100$ mm.

Find the theoretical discharge & power required to running the pump, if actual discharge = 4.2 l/s. Find the slip. Also determine accn head at the beginning and middle of stroke.

$$A = \pi/4 D^2$$

$$D = 150 \text{ mm}$$

$$L = 2r = 300 \text{ mm}$$

$$N = 50$$

$$h_s = 25$$

$$L_d = 22$$

$$Q_{th} = \frac{A L N}{60} = \frac{\pi/4 (0.15)^2 \times 0.3 \times 50}{60} \text{ (m}^3/\text{sec)}$$

$$= 4.42 \times 10^{-3} \text{ m}^3/\text{sec}$$

$$= 4.42 \text{ l/sec}$$

$$Q_a = 4.2 \text{ l/sec}$$

$$\therefore C_d = 4.2 / 4.42 = 0.95$$

$$\therefore \% \text{ slip} = \frac{4.42 - 4.2}{4.42} \times 100 = 4.51\%$$

Accn head = pressure head due to accn

Accn head in suction pipe $\downarrow$

$$h_{as} = \left(\frac{L_s}{g} \right) \times \frac{A}{a_s} \times r \omega^2 \cos \theta$$

$$\left[\begin{aligned} \omega &= \frac{2\pi N}{60} \\ \Rightarrow r &= \frac{L}{2} \\ &= 150 \text{ mm} \end{aligned} \right]$$

Accn head in delivery pipe $\downarrow$

$$h_{ad} = \frac{L_d}{g} \times \frac{A}{a_d} \times r \omega^2 \cos \theta \rightarrow [C = 180^\circ \& 270^\circ]$$

velocity of flow from inlet to exit remain constant
 if the turbine discharges radially so that the degree
 of reaction (R) can be expressed as

$$R = \frac{1}{2} \left[1 - \frac{\cot \theta}{\cot \alpha - \cot \theta} \right] \quad (284)$$

where θ is the runner vane angle at inlet

α is the guide blade angle &

$R \rightarrow$ degree of rxn. defined as ratio of
 pressure head dropped to the

hydraulic work done in the runner.

Assume that losses in the runner are negligible.

$$R = \frac{\text{Pressure head drop b/w inlet \& outlet of Runner}}{\text{work done by the water on the runner / sec. / unit wt. of water}}$$

$$= \frac{P_1/\omega - P_2/\omega}{\frac{(V_{w1}u_1 - V_{w2}u_2)}{g}}$$

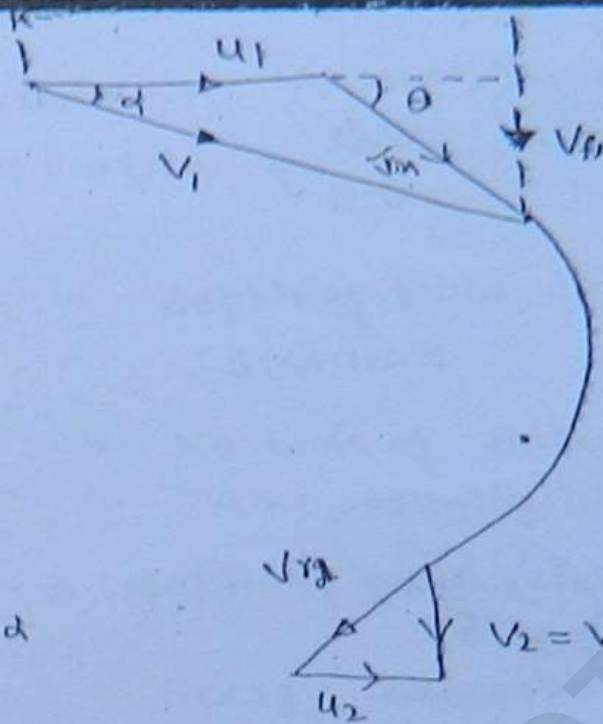
Apply the B. eqn b/w inlet & exit of runner

$$P_1/\omega + z_1 + \frac{V_1^2}{2g} = P_2/\omega + \frac{V_2^2}{2g} + z_2 + \frac{V_{w1}u_1}{g}$$

$$(P_1/\omega - P_2/\omega) = \left(\frac{V_2^2}{2g} - \frac{V_1^2}{2g} \right) + \frac{V_{w1}u_1}{g} \quad \left[\begin{array}{l} \text{No losses} \\ \text{are} \\ \text{considered} \end{array} \right]$$

$$R = \frac{\left[\frac{V_2^2}{2g} - \frac{V_1^2}{2g} \right] + \frac{V_{w1}u_1}{g}}{\frac{(V_{w1}u_1)}{g}}$$

$$R = 1 + \left(\frac{V_2^2 - V_1^2}{2 V_{w1}u_1} \right)$$



285

$$V_{f1} = V_{f2} = V_f$$

$$V_{w2} = 0$$

$$\frac{V_{w1}}{V_{f1}} = \cot \alpha$$

$$V_{w1} = V_{f1} \cot \alpha$$

$$V_2 = V_{f1}$$

$$V_1^2 = V_{f1}^2 + V_{w1}^2$$

$$V_1 = \sqrt{1 + \cot^2 \alpha} \cdot V_{f1}$$

$$V_1 = V_{f1} \sqrt{1 + \cot^2 \alpha} = V_{f1} \operatorname{cosec} \alpha \quad \text{--- (i)}$$

$$P = 1 - \frac{V_{f1}^2 - V_{f1}^2 \operatorname{cosec}^2 \alpha}{2 V_{f1} \cot \alpha \times V_{f1} (\cot \alpha - \cot \theta)}$$

$$P = 1 + \frac{V_{f1}^2 (1 - \operatorname{cosec}^2 \alpha)}{2 V_{f1}^2 \cot \alpha (\cot \alpha - \cot \theta)}$$

$$= 1 - \frac{\cot^2 \alpha}{2 \cot \alpha (\cot \alpha - \cot \theta)}$$

$$= 1 - \frac{\cot \alpha}{2 (\cot \alpha - \cot \theta)}$$

$$P = \frac{1}{2} \left[1 - \frac{\cot \theta}{(\cot \alpha - \cot \theta)} \right]$$

286

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O.C.F. $\left\{ \begin{array}{l} \text{Uniform} \rightarrow V = \text{const}, \bar{d} = \text{const}, A = \text{const} \left[\begin{array}{l} \text{channel} \\ \text{is} \\ \text{prismatic} \end{array} \right] \\ \text{Non-uniform} \left\{ \begin{array}{l} \text{GVF} \\ \text{RVF} \end{array} \right. \end{array} \right. \quad Q = \text{const}$

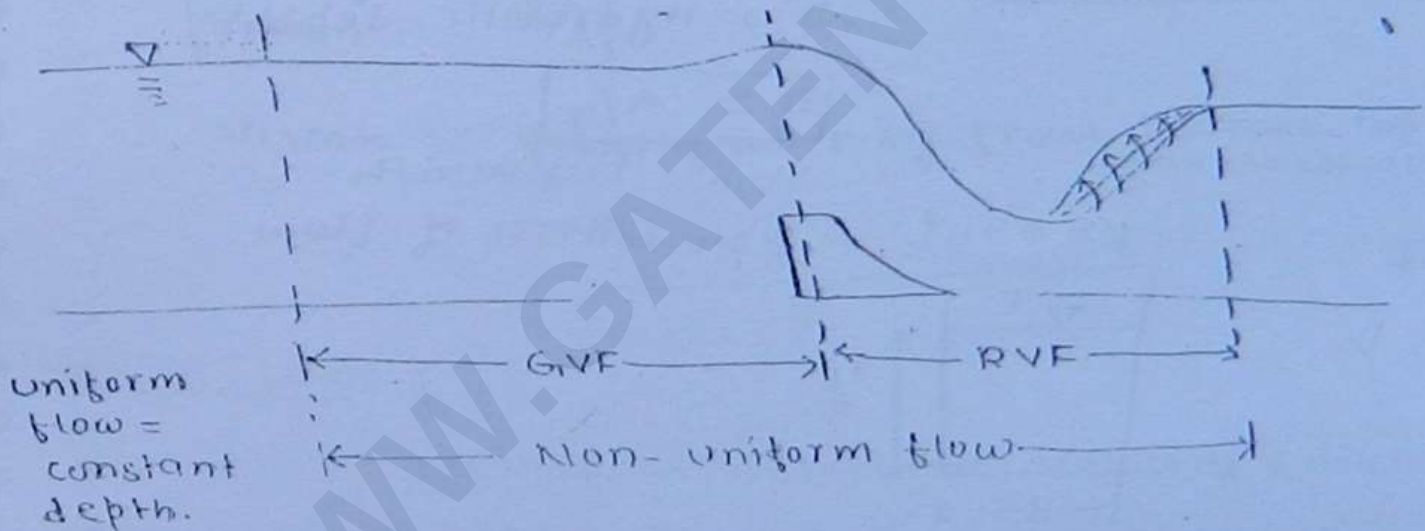
(287)

GVF $\rightarrow$ depth of flow changes over a long distance

$\rightarrow$ No loss of energy if friction losses are negligible.

RVF $\rightarrow$ depth of flow changes suddenly & dissipation

Energy dissipation takes place at the point of jump formation.



* For Laminar Flow in open channel

$$Re \leq 500 \rightarrow \text{Laminar}$$

$$Re > 2000 \rightarrow \text{Turbulent}$$

$$Re = \frac{RVR}{\mu} = \frac{VR}{\nu}$$

$R \rightarrow$ Hydraulic Radius $= A/p \rightarrow$ wetted perimeter

$\rightarrow$ Hydraulic Mean depth

if $FR < 1$

Subcritical / Tranquil / Streaming / stable flow
(velocity Low, depth of flow high)

critical Flow: ↓

if $FR = 1 \Rightarrow$ critical Flow

supercritical Flow: ↓

288

if $FR > 1 \Rightarrow$ supercritical Flow / shooting
Rapid / unstable flow.
($V \uparrow$, depth of flow Low)

$$FR = \frac{V}{\sqrt{gD}}$$

$V \rightarrow$ Mean velocity = Q/A

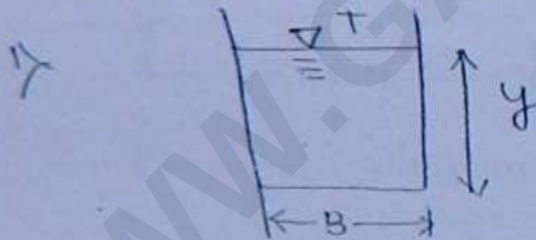
$D \rightarrow$ Hydraulic depth

$$D = A/T$$

$T =$ Top width

$A =$ Area of flow

For eg: ↓



$\therefore$ Hyd. depth = A/T

= $By/B = y$

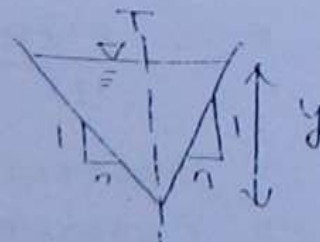
= depth of flow { in case of Rect. channel }

2) $T = 2ny$

$A = \frac{1}{2} \times y \times 2ny = ny^2$

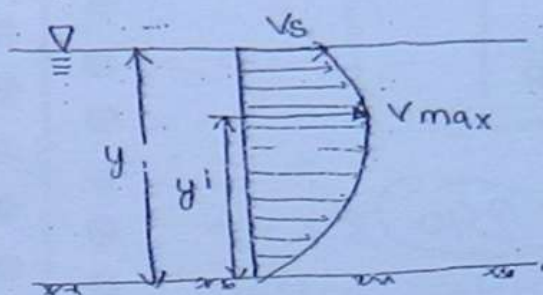
$\therefore$ Hydraulic depth = A/T

= $y/2$



[Side slope = $nH:1V$]

Velocity distribution in open channel



(289)

y_1 = distance of V_{max} $\approx 0.8y$ to $0.95y$ ^{gib}
from bottom

$V_s \approx 0.91 V_{max}$ ^{gib} $\rightarrow$ (May be b/w 0.85 to $0.95 V_{max}$)
 $\rightarrow$ velocity at surface

$$V_{mean} = Q/A$$

$V_{mean} \approx$ velocity at $0.6y$ from bottom top (freesurface)

$$\approx \frac{V_{at 0.2y} + V_{at 0.8y}}{2}$$

UNIFORM FLOW:

Methods to determine velocity & discharge

(a) chezy's eqn: $\downarrow$

$$V = C \sqrt{RS}$$

$$Q = A C \sqrt{RS}$$

chezy's constant can be calculated by the Kutter's or Bazin's eqn and C depends upon the surface roughness

S = slope of the channel bottom
 A = Area of flow
 R = Hydraulic Radius [Effective-length parameter]

C = chezy's constant
 $C = 148.7 \sqrt{R/n}$ ^{gib}

$$C = \frac{23 + \frac{0.00155}{S} + V_n}{\left[1 + \left(\frac{23 + \frac{0.00155}{S}}{S} \right) \left(\frac{n}{\sqrt{R}} \right) \right]}$$

n = Kutter's roughness coeff.
= Roughness of channel surface

$$C = 23 + \frac{0.00155}{S} + Y_N$$

$$\left[1 + \left(23 + \frac{0.00155}{S} \right) \right] \frac{n}{\sqrt{R}}$$

Bazin's eqn: ↓

240

$$C = \frac{157.6}{1.81 + K/\sqrt{R}}$$

• K = Bazin's coeff.

$$C = \frac{m/sec}{\sqrt{m}} = L^{1/2} T^{-1/2}$$

$$C = \frac{187}{1 + M/R}$$

Average or Mean velocity is find out

Manning's Equation: ↓

$$V = Y_N R^{2/3} S^{1/2}$$

N = Manning's
Rugosity constant

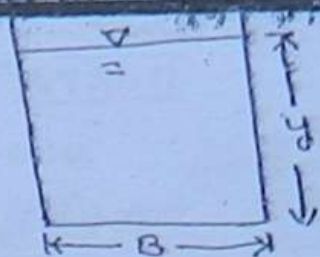
$$N = L^{1/3} T^{-1}$$

$$C = Y_N R^{1/6}$$

v. Imp.

$$C = \sqrt{8g/f} = Y_N R^{1/6}$$

A) Rectangular Section:



29/

$$A = By$$

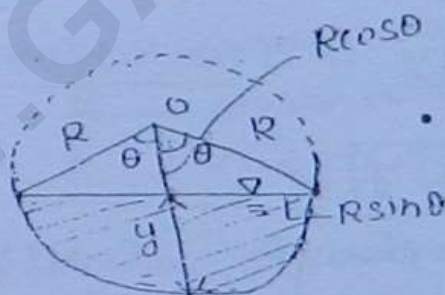
$$\text{Wetted Perimeter} = B + 2y$$

$$R = A/P = \frac{By}{B + 2y}$$

$$D = Hy \cdot \text{depth} = A/T = y$$

$$\begin{aligned} \text{Section Factor} &= \sqrt{\frac{A^3}{T}} \\ &= \left[\frac{(By)^3}{B} \right]^{1/2} = B \cdot y^{3/2} \end{aligned}$$

B) Circular Section:



• Area of Flow

$$= \theta R^2 - 2 \left[R^2(R-y)^2 \times \frac{1}{2} \times (R-y) \right]$$

[θ → Radian]

$$= R^2 \left[\theta - \frac{1}{2} \sin 2\theta \right]$$

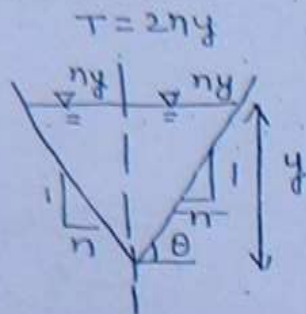
$$\boxed{A = R^2 \left[\theta - \frac{\sin 2\theta}{2} \right]}$$

- Top width $T = 2R \sin \theta$ ✓
- Wetted Perimeter $P = 2\theta R$

$$E_c = 5/4 y_c$$

$$f_c = \left(\frac{1.49^2}{g m^2} \right)^{1/5}$$

$$F = \sqrt{2} \frac{V}{\sqrt{g y}}$$



sideslopes

$$\boxed{nH:1V}$$

$$\tan \theta = y_n$$

$$A = ny^2$$

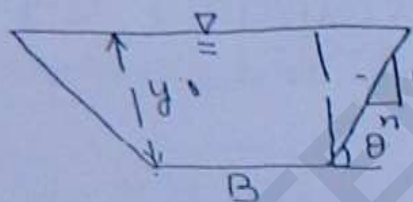
$$T = 2ny$$

$$R = A/P = \frac{ny}{2\sqrt{1+n^2}}$$

$$P = 2y\sqrt{1+n^2}$$

292

d) Trapezoidal section: ↓



$$T = B + 2ny$$

$$A = \left(\frac{B+T}{2} \right) y = \left(\frac{B+B+2ny}{2} \right) y$$

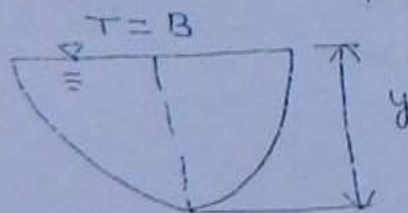
$$\boxed{A = (B+ny)y} \checkmark$$

wetted perimeter $\boxed{P = B + 2y\sqrt{1+n^2}}$

e) Parabolic channel: ↓

Hydraulic depth

$$\boxed{D = A/T = 2/3 y}$$



• Section Factor

$$Z = 2/9 \sqrt{6} B y^{3/2}$$

$$A = 2/3 T y = 2/3 B y$$

wetted perimeter $P = B + 8/3 \frac{y^2}{B}$
 $= \frac{3B^2 + 8y^2}{3B}$

Prob. Find the discharge for the channel section shown in figure whose bed slope is 0.001 and Manning's $N = 0.018$

$$S = 0.001$$

$$N = 0.018$$

0.012 to 0.025

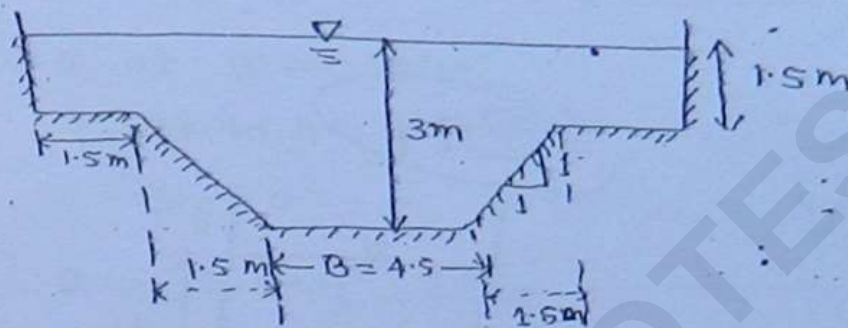
For smooth

Lined channels.

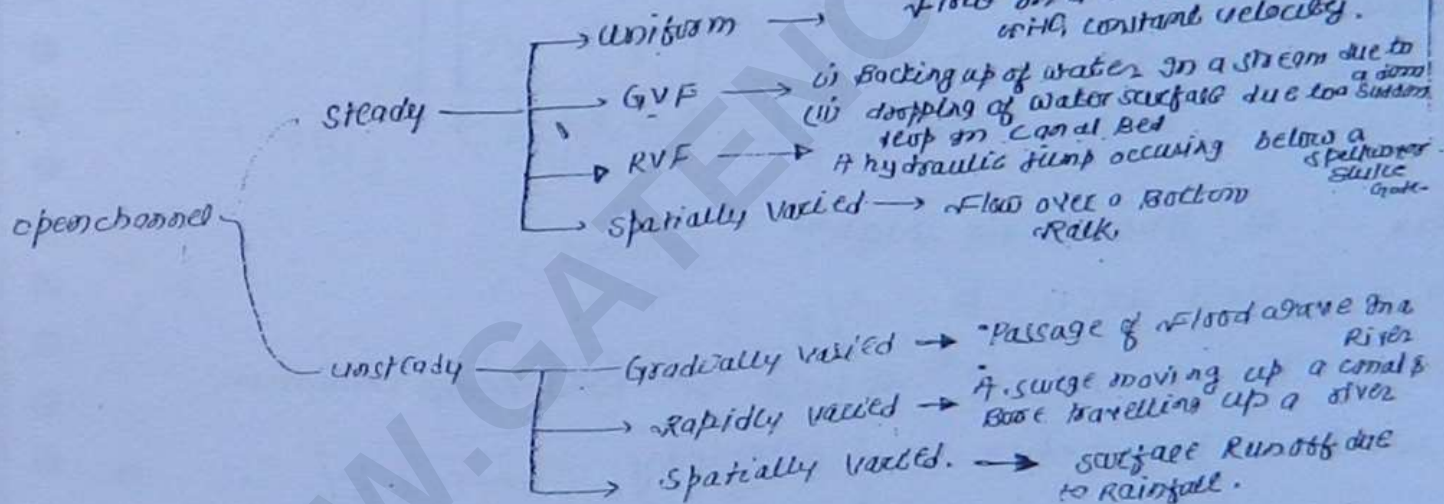
For Very Rough

Earthen channel

(293)



Flow in a prismatic channel with constant velocity.



* In G.V.F., frictional resistance plays an important role.

* In G.V.F., R.V.F. No flow is externally added or taken out of the system.

→ Spatially varied flow - either some flow is added or subtracted from the system.

→ Specific Force is sum of the Pressure Force + Momentum Flux per unit wt. of the fluid at a section.

Sp. Force is constant in a horizontal, frictionless channel.

critical flow condition is governed by the channel geometry and discharge. other channel properties such as the bed slope and roughness do not influence the critical flow condition for any given discharge.

A section of a channel is said to be economical when its cost of construction is least or for a given discharge and given area.

For a given sectional Area, dimension of section design in such a way that discharge carrying capacity is maximum.

rectangular section: ↓

$$A = By = \text{constant}$$

For Maxm Q

we know that

$$Q \propto V \propto R^{2/3} \rightarrow \text{Chezy's}$$

$$\propto R^{2/3} \rightarrow \text{Manning}$$

$Q_{\text{max}} \rightarrow R$ should be Maxm

For a given area $R = A/P$

For $R_{\text{max}} \rightarrow P$ should be ~~Maximum~~ minimum

$$\text{wetted perimeter } P = B + 2y$$

$$P = A/y + 2y$$

$$\text{For } P_{\text{min}} \quad dP/dy = 0$$

$$-A/y^2 + 2 = 0$$

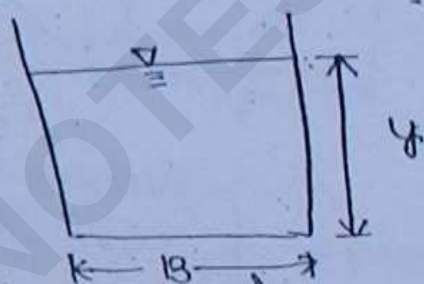
$$\Rightarrow \boxed{y = B/2} \quad \text{gap}$$

$$\boxed{\text{depth of flow} = \frac{1}{2} \text{ width}} \quad \text{gap}$$

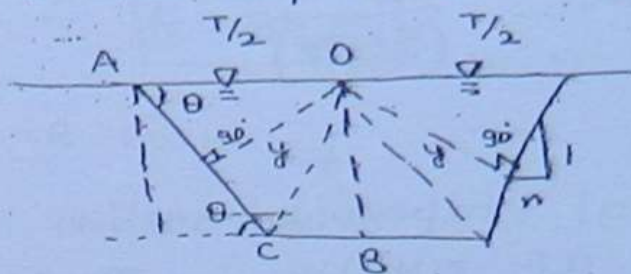
$$\text{Hydraulic Radius } R = A/P = \frac{By}{B+2y} = \frac{B \cdot B/2}{B+2 \cdot B/2}$$

$$\boxed{R = B/4 = y/2} \quad \text{gap}$$

294



case I: $\rightarrow$ side slopes are constant



$$A = (B + ny)y$$

$$P = B + 2y\sqrt{1+n^2}$$

$$T = B + 2ny$$

For Maxm Q at given area

P should be Minimum

$$\therefore \frac{dP}{dy} = 0$$

$$P = B + 2y\sqrt{n^2+1}$$

$$= A/y - ny + 2y\sqrt{n^2+1}$$

$$\therefore \frac{dP}{dy} = -A/y^2 - n + 2\sqrt{n^2+1}$$

$$= -\left(\frac{(B+ny)y}{y^2}\right) - n + 2\sqrt{n^2+1} = 0$$

$$= -(B+ny) - ny + 2y\sqrt{n^2+1} = 0$$

$$y\sqrt{n^2+1} = \frac{B+2ny}{2}$$

$$T/2 = \frac{B+2ny}{2}$$

$$\text{Side} = y\sqrt{n^2+1}$$

For Most Economical channel

$$\frac{1}{2} \text{ top width} = \text{one of sloping side length}$$

$$R = A/P = \frac{(B+ny)y}{B+2y\sqrt{n^2+1}}$$

$$= \frac{(B+ny)y}{B+2(B+2ny)}$$

$$= y/2$$

$$\Rightarrow R = y/2$$

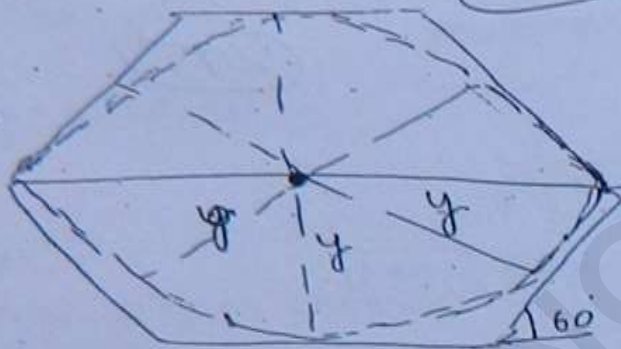
$$= y/AC = \frac{y}{y\sqrt{n^2+1}} = \frac{OB}{\left(\frac{B+2ny/2}{2}\right)}$$

$$\therefore \boxed{OB = y}$$

condition:

For an economical trapezoidal section side slope will be at $nH:1V$

296



$$\boxed{\theta = 60^\circ}$$

→ channel is most economical

$$\boxed{1H:3V}$$

Trapezoidal section is a part of Hexagon whose centre is at middle of top width.

Triangular section: ↓

$$A = ny^2$$

$$P = 2y\sqrt{1+n^2}$$

$$R = A/P$$

∴ For max B. P_{min}

$$\therefore \frac{dP}{dn} = 0$$

$$P = 2 \times \frac{\sqrt{A}}{\sqrt{n}} \sqrt{n^2+1}$$

$$P = 2\sqrt{A}\sqrt{1+n^2} \Rightarrow P^2 = 4A(n^2+1)$$

$$2P \left(\frac{dP}{dn} \right) = 4A(2n)$$

$$\boxed{\frac{dP}{dn} = 2\sqrt{A}}$$

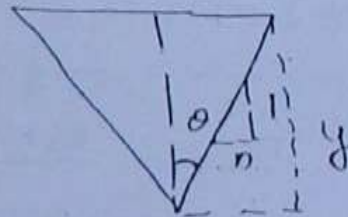
$$\frac{dP}{dn} = 0, n=1 \Rightarrow \boxed{\theta = 45^\circ}$$

i. Triangular section to be most economical

$$\theta = 45^\circ$$

and

$$\boxed{R = \frac{y}{2\sqrt{2}}}$$



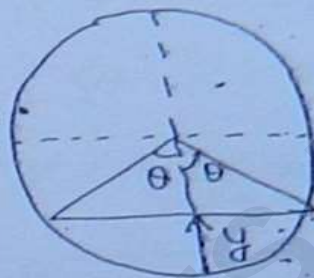
$$R = 8/2\sqrt{2}$$

CIRCULAR SECTION: $\downarrow$

(297)

1) For Maxm. velocity condition

2) For Maxm. discharge condition



$$V \propto (R = A/P)^{1/2} \quad \left| \quad A \propto R^2 \right.$$

$$\propto R^{2/3}$$

$\rightarrow$ Manning's eqn

$\propto A \cdot R^{2/3}$ Chezy's eqn.

For V_{max} - R should be Maxm.

$$A = R^2 \left[\theta - \frac{\sin 2\theta}{2} \right]$$

$$P = 2\theta R$$

$$\left[\begin{aligned} \frac{dA}{d\theta} &= R^2 [1 - \cos 2\theta] \\ \frac{dP}{d\theta} &= 2R \end{aligned} \right.$$

$$\therefore \frac{dR}{d\theta} = 0 \Rightarrow \frac{d}{d\theta} (A/P) = 0$$

$$\downarrow$$

$$= \frac{P \left(\frac{dA}{d\theta} \right) - A \cdot \frac{dP}{d\theta}}{P^2} = 0$$

$$\Rightarrow \frac{P R^2 [1 - \cos 2\theta] - A (2R)}{P^2} = 0$$

$$\Rightarrow 2R\theta [1 - \cos 2\theta] R^2 - R^2 \left[\theta - \frac{\sin 2\theta}{2} \right] \times 2R = 0$$

$$\Rightarrow \boxed{2\theta = \tan 2\theta}$$

$$\tan d = d \rightarrow \text{Trial and Error}$$

$$d = 4.5 \text{ rad}$$

$$\boxed{2\theta = 4.5 \text{ rad} = 257^\circ 30'}$$

mp

$$R_{max} = 0.60R$$

$$= 0.80D$$

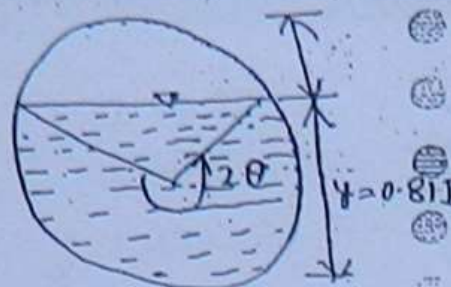
298

For V_{max} :

$$2\theta = 257^\circ 30'$$

$$y = 0.81D$$

$$R_{max} = 0.3D$$



Condition For Max^m discharge:

A) using chezy's eqn:

$$Q = CA R Y_2 S Y_2$$

$$\text{For } Q_{max} \Rightarrow (AR)^{1/2}_{max}$$

$$\therefore \frac{d}{d\theta} [AR Y_2] = 0$$

$$\Rightarrow \frac{d}{d\theta} \left[\frac{A^{3/2}}{P Y_2} \right] = 0$$

$$\Rightarrow \begin{cases} 2\theta = 308^\circ \\ y = 0.95D \\ R = 0.29D \end{cases}$$

B) using Manning eqn:

$$Q = Y_N \cdot A R^{2/3} S Y_2$$

$$Q_{max} = (AR^{2/3})_{max}$$

$$2\theta = 302^\circ 20'$$

$$y = 0.938D$$

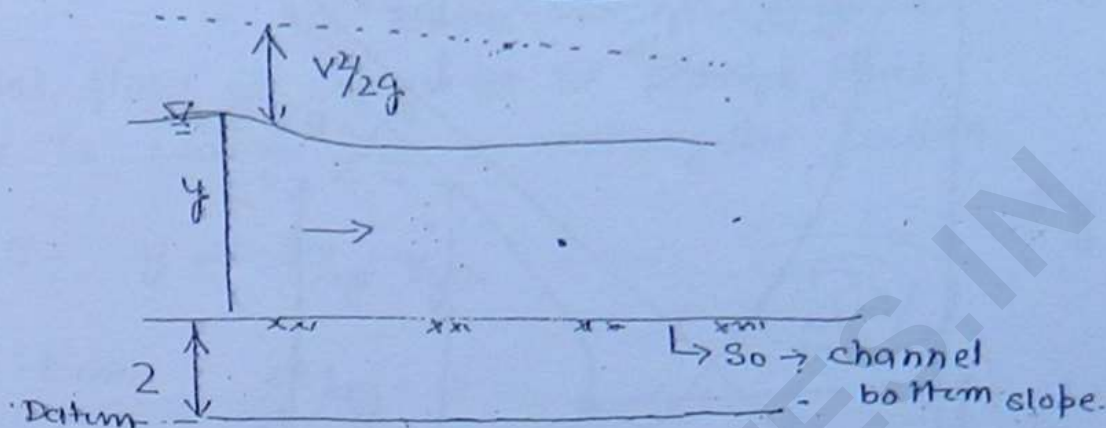
$$R = 0.29D$$

COMMENT: →

From technical consideration Mannings result are more realistic because Mannings N based on surface great roughness which can be computing directly whereas chezy's c is given arbitrary.

Gr.V.F. ↓

299



$$\text{Total energy head} = E = Z + y + \frac{v^2}{2g}$$

 Assuming uniform velocity of section $d=1$

Specific Energy: If the ~~channel~~ channel bottom is taken as datum then total energy per unit wt. is called specific energy.

For Gr.V.F. specific energy is constant

$$E = y + \frac{v^2}{2g}$$

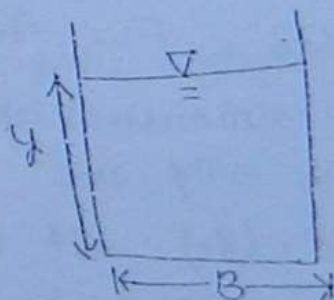
Case: 1st: For Rect. angular section

Specific Energy

$$E = E_p + E_k$$

$$E = y + \frac{v^2}{2g}$$

$$= y + \frac{Q^2}{2gA^3}$$

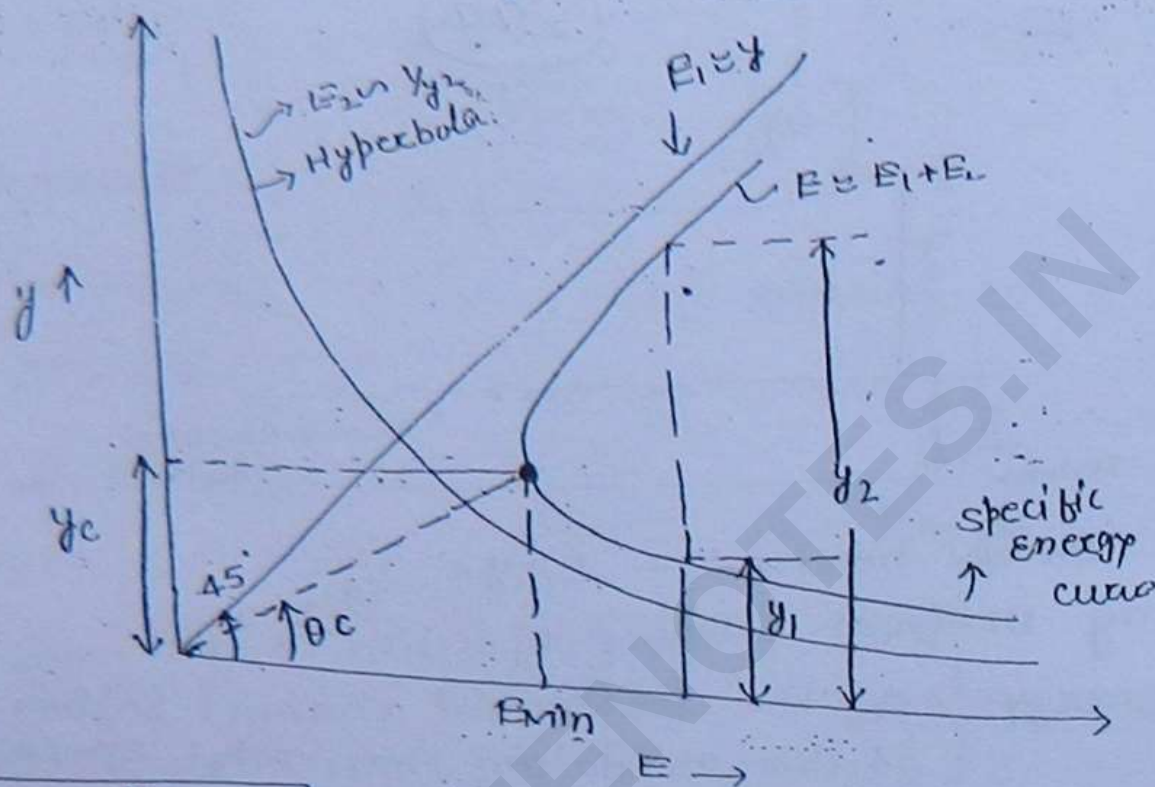


Let discharge per unit width $q = Q/B$

$$E = y + \frac{q^2}{2gy^3} = y + \left(\frac{q^2}{2g}\right) \times \frac{1}{y^3} = E_1 + E_2$$

$$E_2 = E_K = \frac{v^2}{2g} + y_2$$

300



$$\tan \theta = \frac{y_c}{E_{min}}$$

$$= \frac{2/3 E_{min}}{E_{min}}$$

$$\Rightarrow \theta = 33.7^\circ \quad \text{For Rectangular}$$

$$= 38.65^\circ \quad \text{For Triangular}$$

$$= 36.8^\circ \quad \text{For Parabolic}$$

For G.V.F. Minimum specific energy occurs at which there is only one depth of flow called critical depth of flow (y_c). It means at critical flow specific energy is minimum.

For other sp. energy there will be two depth of flow y_1 & y_2 known as alternate depths. For Rectangular section y_c is critical depth $2/3$ of E_{min} .

$$y_c = \frac{4}{5} E_{min} \quad \text{For Triangular}$$

$$= \frac{3}{4} E_{min} \quad \text{For Parabolic}$$

$$= \frac{4}{5} E_{min} \quad \text{For Triangular}$$

→ At critical flow it may be proved that discharge is maxm. For a rectangular section.

$$E = y + \frac{q^2}{2g} \cdot \frac{1}{y^3}$$

(30)

$$\therefore \text{For } E_{min}, \quad dE/dy = 0$$

$$\Rightarrow 1 + \frac{q^2}{2g} \left(-\frac{3}{y^4} \right) = 0$$

$$\Rightarrow y^3 = \left(\frac{q^2}{g} \right)$$

$$\therefore \text{At } E_{min}, \quad y = y_c$$

$$\therefore \boxed{y_c^3 = \left(\frac{q^2}{g} \right)} \rightarrow \text{Valid For Rectangular section.}$$

$$\text{If } y = y_c, \quad E = E_{min}$$

$$\therefore \text{at } y = y_c, \quad E_{min} = y_c + \frac{y_c^3}{2} \times \frac{1}{y_c^3}$$

$$\therefore \boxed{E_{min} = \frac{3}{2} y_c}$$

At critical flow, for rectangular section kinetic head is half of potential head.

* For parabolic section ↓

$$\boxed{E_{min} = \frac{4}{3} y_c} \\ = y_c + \frac{y_c}{3} \rightarrow \text{kinetic head}$$

↓ Triangular section ↓

$$\boxed{E_{min} = \frac{5}{4} y_c = \left(\frac{5}{4} y_c \right) \rightarrow \text{kinetic head.}}$$

$$\frac{V}{\sqrt{gD}} = 1$$

∴ For Rect section $D = A/T = \frac{By}{B} = y$

$$V_c = \sqrt{gy_c}$$

For Triangular section

$$D = A/T = y/2$$

$$V_c = \sqrt{gy_c/2}$$

For Parabolic section

$$D = A/T = 2/3 y$$

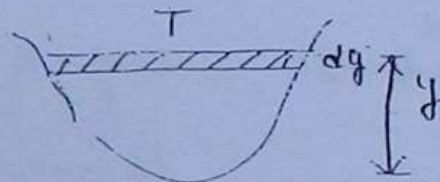
$$V_c = \sqrt{\frac{2gy_c}{3}}$$

critical depth for Non-uniform channel: ↓

$$dA = T dy$$

$$\therefore dA/dy = T$$

$$E = y + \frac{Q^2}{2gA^2}$$



For ~~critical~~ critical Flow $dE/dy = 0 = 1 + \frac{Q^2}{2g} \left(-\frac{2}{A^3} \right) \frac{dA}{dy}$

$$\therefore \frac{Q^2}{g} = \frac{A^3}{(dA/dy)} = A^3/T$$

$$\frac{Q^2}{g} = A^3/T$$

→ For critical Flow
Applicable for Rectangular or
Non-uniform section.

Dynamic Equations Gradually Varied Flow:

Assumptions:

- 1) Chezy's Formula & Manning Formula is used (with S_o as energy slope)
- 2) Bottom slope of the channel is very small
- 3) channel is prismatic
- 4) Energy correction factor is 1
- 5) Pressure distribution is only hydrostatic
- 6) Discharge is constant, flow is steady.
- 7) Roughness coeff. of channel is independent of the depth of flow and taken constant through the length of channel.



slope is falling in flow direction.

channel slope $\boxed{dz/dx = -S_o}$

If total energy is E

$$dE/dx = \text{Energy slope} - S_f$$

$$E = z + y + \frac{V^2}{2g}$$

$$dE/dx = dz/dx + dy/dx + d/dx \left[\frac{V^2}{2g} \right]$$

$$\therefore d/dx \left(\frac{V^2}{2g} \right) = d/dx \left[\frac{Q^2}{2gA^3} \right]$$

$$d/dx \left[\frac{Q^2}{2gB^2y^2} \right] = \frac{Q^2}{2gB^2} \times -2/y^3 dy/dx$$

$$[A = By]$$

$$= \frac{Q^2}{gB^2y^2} \times y dy/dx$$

304

$$= -\frac{V^2}{gY} \cdot dy/dx$$

$$\therefore dE/dx = dz/dx + dy/dx - \frac{V^2}{gY} \cdot dy/dx$$

$$dy/dx \left[1 - \frac{V^2}{gY} \right] = dE/dx - dz/dx$$

$$\therefore dy/dx = \frac{-S_f - (-S_0)}{1 - V^2/gY}$$

$$\boxed{dy/dx = \frac{S_0 - S_f}{1 - V^2/gY}} \rightarrow \text{dynamic eqn for G.V.F Rectangular section.}$$

$$\boxed{dy/dx = \frac{S_0 - S_f}{1 - F^2}} \rightarrow \text{For Rectangular channel}$$

For All sections:

$$\boxed{dy/dx = \frac{S_0 - S_f}{1 - \frac{Q^2 T}{g A^3}}}$$

$$[F^2 = \frac{V}{\sqrt{gY}}]$$

Energy slope

$$\boxed{S_f = h_t/L}$$

$$\boxed{S_0 - S_f = \frac{dE}{dx}}$$

↑ differential energy eqn of G.V.F

According to chezy's eqn

$$\boxed{dy/dx = S_0 \left[\frac{1 - (y_n/y)^3}{1 - (y_c/y)^3} \right]}$$

V.Imp
(obj)

$y_c \rightarrow$ critical depth of flow

$y_n \rightarrow$ Normal " "

According to Manning's Eqn: 7

$$\frac{dy}{dx} = S_0 \left[\frac{1 - (y_n/y)^{10/3}}{1 - (y_c/y)^{10/3}} \right]$$

(303)

$y \rightarrow$ Actual depth of flow

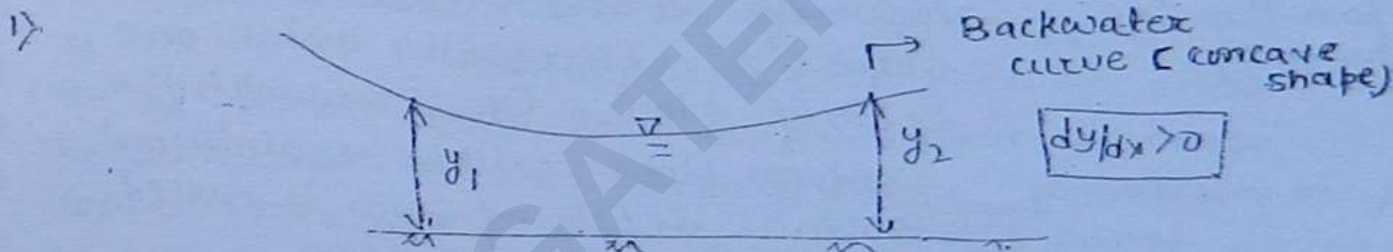
$y_n \rightarrow$ It is that depth of flow at which flow is uniform

$dy/dx \rightarrow$ Rate of change of depth of flow w.r.t. channel bottom

If $\frac{dy}{dx} > 0 \rightarrow$ depth of flow is increasing in direction of flow
 $\rightarrow$ Backwater flow curve

$dy/dx = 0 \rightarrow$ depth of flow is constant

$dy/dx < 0 \rightarrow$ depth of flow is decreasing
 $\rightarrow$ drawdown curve.



$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - \frac{V^2}{gy}}$$

$\frac{dy}{dx} > 0$ If

$\rightarrow$ channel slope $>$ energy slope

$\rightarrow$ Flow is subcritical.

$$\left[\begin{array}{l} S_0 > S_f \\ i > \frac{V^2}{gy} \end{array} \right]$$

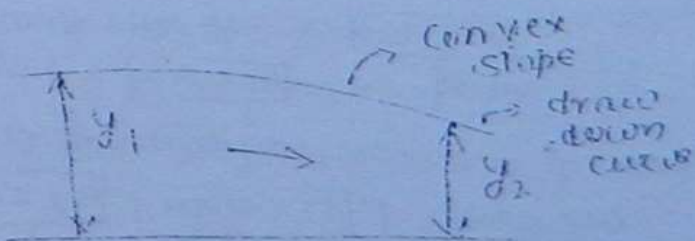
2)

$$\left\{ \begin{array}{l} S_0 < S_f \\ 1 < \frac{V^2}{gy} \\ \frac{dy}{dx} < 0 \end{array} \right.$$

$\rightarrow$ supercritical flow

drawdown curve

$\rightarrow$ convex shape.



$$\frac{V^2}{gy} \neq 1$$

306

iv) If $\frac{V^2}{gy} = 1$, $Fr = 1$

$$\frac{dV}{dx} = \infty \rightarrow$$

Since $m.g.v.F [dy/dx]$ is small depth of flow change over a larger length hence this condition beyond the assumption of G.V.F.

For a given discharge normal depth of flow can be calculated as follows: ↓

Normal depth of flow: ↓

↓
[depth of flow at which a given discharge flows as uniform flow in a given channel.]

For given values of Mannings N and chezy C and for given value of Q and channel bottom slope S_0 there will exist one depth of flow (y_n) at which the uniform flow will be maintained such a depth of flow is called Normal depth of flow.

- choking: - (i) u/s, water surface elevation is not affected by the conditions at section (2) till a critical stage is not achieved.

(2) in case of hump, for all $\Delta z \leq \Delta z_{max}$ — u/s water depth is constant
For all $\Delta z > \Delta z_{max}$ → y_1 increases in subcritical flow
→ y_2 decreases in super critical flow.

(3) in case of width contraction: -

$[B_2 > B_{2m}] \rightarrow$ u/s depth y_1 is constant under a
while for $[B_2 < B_{2m}] \rightarrow$ u/s depth y_1 will change

→ onset of critical condition at (2) is prerequisite to choking.

→ All cases $[\Delta z > \Delta z_{max}]$, $[B_2 < B_{2m}]$ → known as choked condition.

→ In subcritical flow, water surface will drop due to decrease in sp. energy
or supercritical flow, depth of flow increases due to reduction in sp. energy.

CRITICAL DEPTH

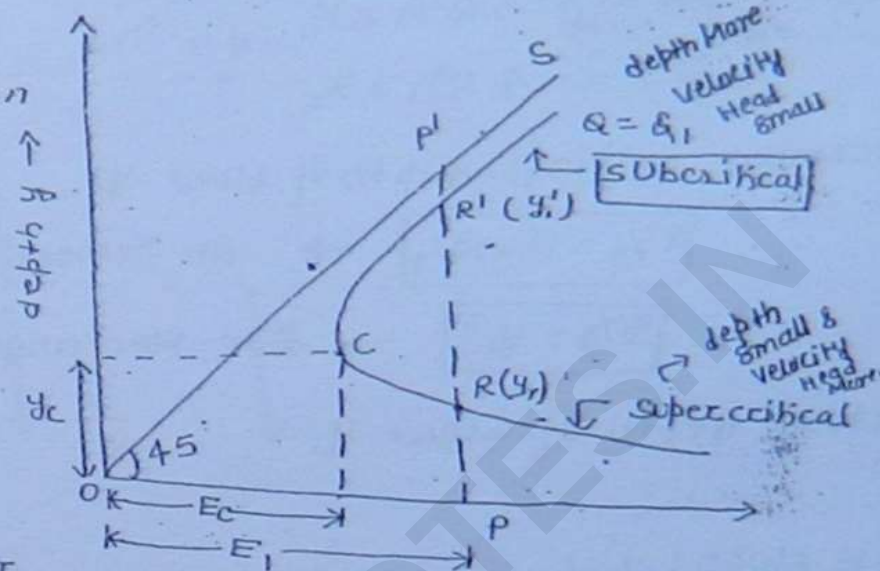
Constant discharge situation: $\downarrow$

$$E = y + \frac{Q^2}{2gA^2}$$

* For a channel of known geometry

$$E = f(y, Q)$$

Keeping $Q = \text{constant}$
 $Q = Q_1$ the variation of E with y is represented by a cubic parabola.



That at any particular discharge Q_1 can be passed in a given channel at two depths & still maintain the same sp. energy E .

The depth of flow can be either $PR = y_1$ or $PR' = y_1'$. These two have same sp. energy. The intercept PR' or PR represents the velocity head depth ($PR = y_1$) is smaller and has a large velocity head while other ($PR' = y_1'$) has a larger depth and consequently a smaller velocity head.

→ For a given Q , as the sp. energy is increased the difference b/w the two alternate depth increases.
 → If E is decreased, the difference $(y_1' - y_1)$ will decrease and at a certain value $E = E_c$.

→ At the lower limb CR of the sp. energy curve the depth $y_1 < y_c$ As such $V_1 > V_c$ and $F_1 > 1.0$
 → supercritical flow Region

→ In the upper limb CR' , $y_1' > y_c$ as such $V_1' < V_c$ and $F_1' < 1.0$ ⇒ subcritical flow Region.

1. determine normal depth of flow by y_n .

(308)

OR $Q = y_n \cdot A R^{2/3} S^{1/2}$

$Q = C \cdot A \cdot R^{1/2} S^{1/2}$

2. determine critical depth of flow y_c .

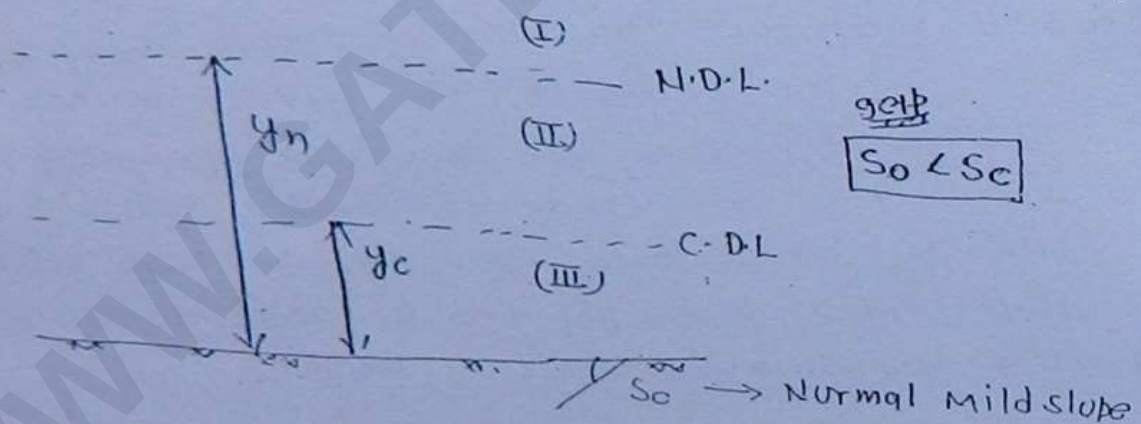
$\boxed{Q^2/g = A^3/T} \rightarrow \text{An General}$

$\boxed{Q^2/g = y_c^3} \rightarrow \text{For Rectangular channel}$

3. Actual depth of flow y .

(a) mild slope: $\downarrow$

when $\boxed{y_n > y_c}$



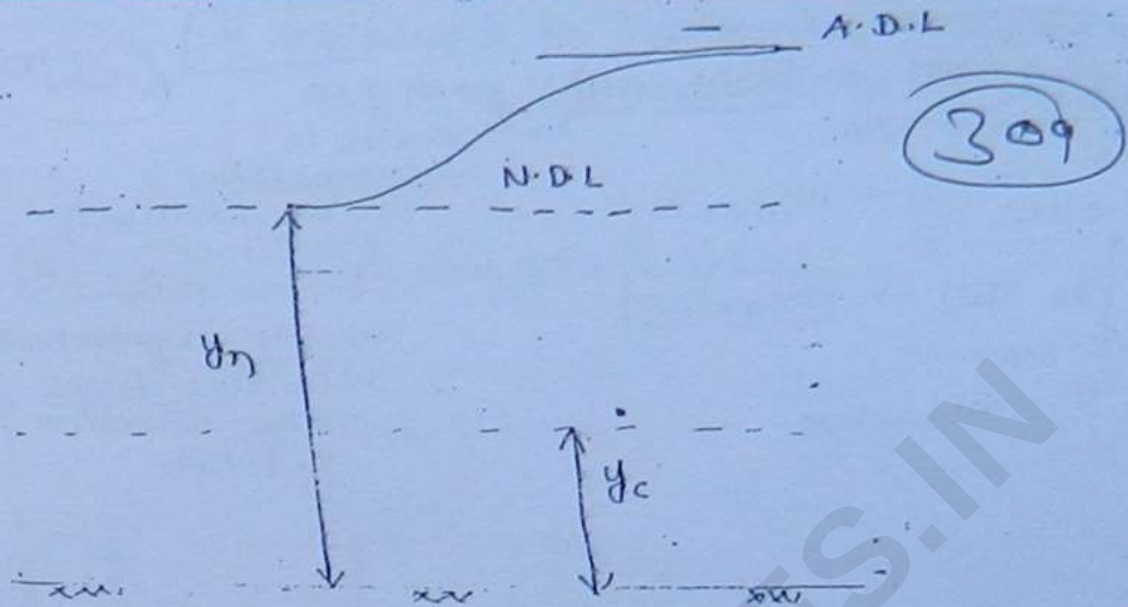
If actual depth of flow y is such that

$\boxed{y_c < y < y_n} \quad \boxed{y_n > y_c}$

$\rightarrow$ Then surface profile is M_1 type

M_1 profile means normal depth line is asymptotically and tend to become horizontal in d/s towards actual depth line. The curve water is, backwater and rising hence

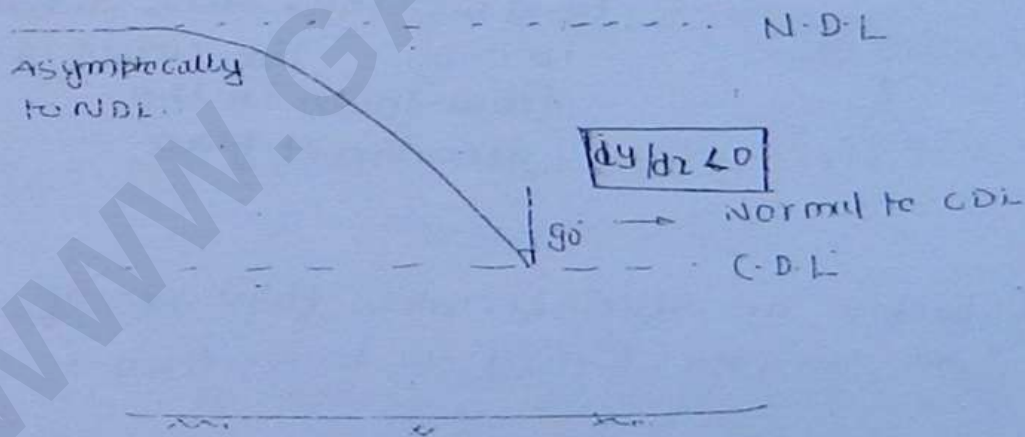
$\boxed{dy/dx > 0}$



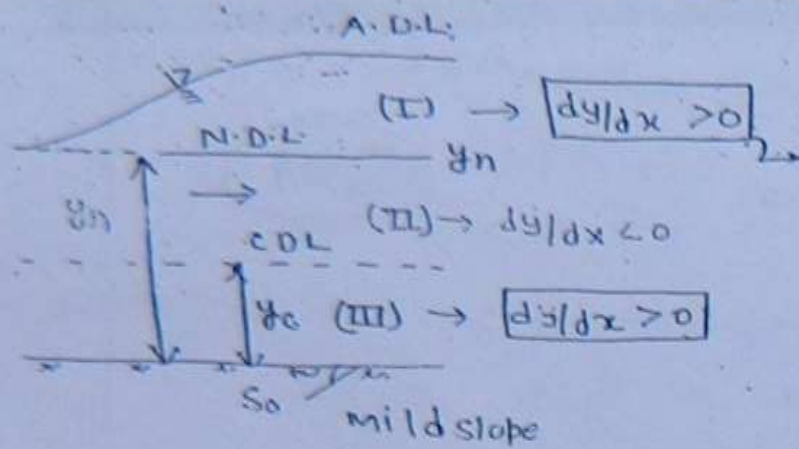
9f.

$y_c \leq y \leq y_n \rightarrow M_2$ profile will be formed

$\rightarrow$ Means asymptotically to normal depth line and Normally to critical depth in the direction of flow. The curve is drawdown.



M_1 & M_2 curve profile are formed when $Fr < 1$



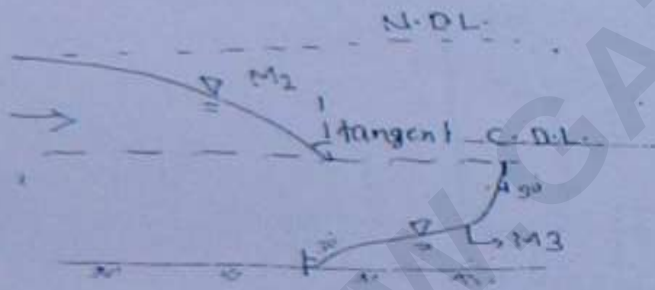
In mild slope

$$S_0 < S_c$$

3/0

curve is
Backwater
in the direction of
flow.
i.e. M1 profile
meets asymptotically to
N.D.L. and rises
in the direction
of flow.

When $y_c < y < y_n$, then M2 profile is formed



meets
M2 profile asymptotically
to N.D.L. and meets normally
to C.D.L. The curve is
draw-down in the
direction of flow.

NOTE: M1 & M2 profile are formed when flow are of
stable type having Froude No. Less than 1

When $0 < y < y_c \rightarrow$ M3 profile is formed

$\rightarrow$ M3 profile is backwater curve
which is normal to channel bottom
slope S_0 & C.D.L.

$\rightarrow$ M3 curve is formed when flow is
supercritical

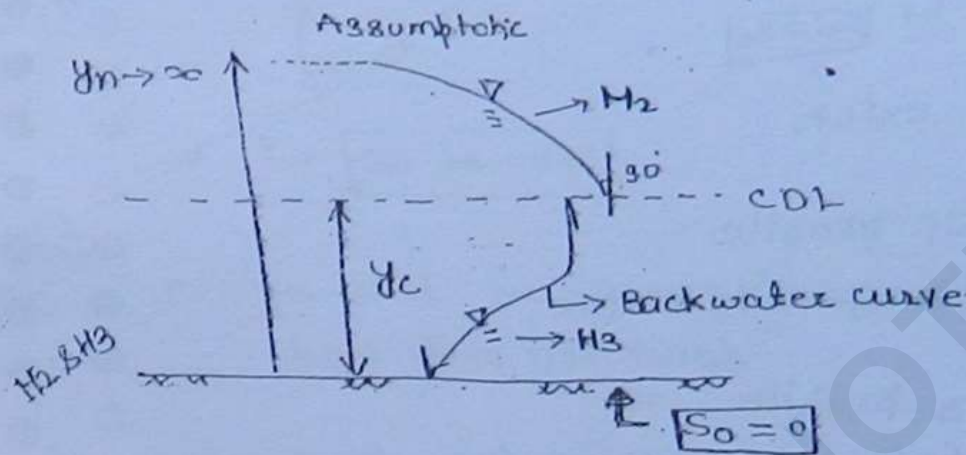
2) Horizontal slope: $\downarrow$

If slope is horizontal than normal depth of flow tends to infinity.

$$d. d. y_N \propto R^{2/3} S^{1/2} \rightarrow 0$$

(311)

$$\Rightarrow y_N \rightarrow \infty \quad (\text{For horizontal slope})$$



H_1 profiles are not formed and H_1 zone does not exist

If $y_c \leq y \leq y_N \rightarrow$ than H_2 profile is formed
 $\rightarrow$ drawdown curve.

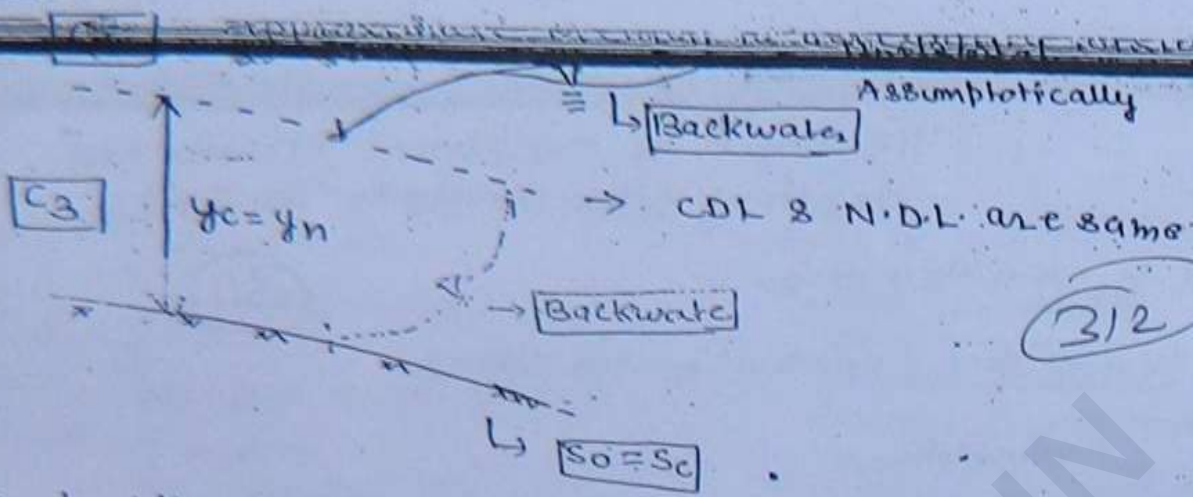
If $y < y_c \rightarrow$ than H_3 profile is formed.
 $\rightarrow$ Backwater curve.

3) critical slope: $\downarrow$

1) when channel bottom slope $S_0 = S_c$ at this stage flow is critical

2) y_N will be equal to y_c . $y_N = y_c$

(1903)



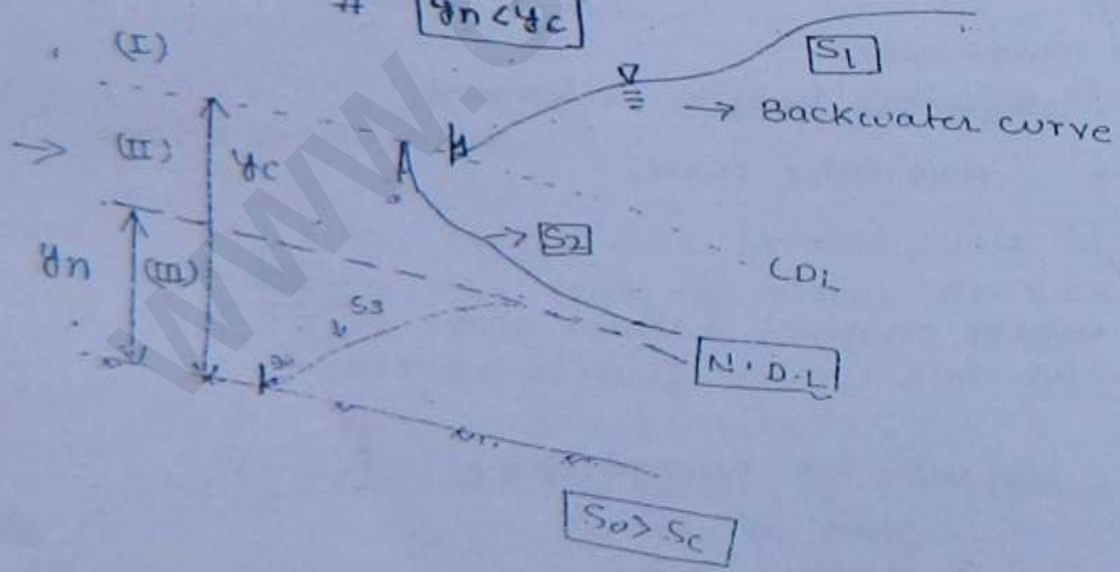
C_2 profile does not exist.

If $y > y_n \Rightarrow C_1$ profile
 If $y < y_n \Rightarrow (= y_c) \Rightarrow$ Backwater curve

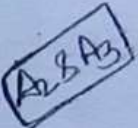
If $y < y_n \Rightarrow C_3$ profile
 If $y > y_n \Rightarrow (= y_c) \Rightarrow$ drawdown curve.
 Backwater

steep slope: $\downarrow$

$S_o > S_c$ imp.
 # $y_n < y_c$



S_1 profile $\Rightarrow$ when $y > y_c$
 S_2 profile $\Rightarrow$ when $y_n \leq y \leq y_c$
 S_3 profile $\Rightarrow$ when $0 < y < y_n$



$\neq$ A_1 zone will not exist and only A_2 & A_3 will exist.

Find Normal depth of flow

$$15 = 10 \text{ m}$$

$$f = 1.6 \text{ m}$$
$$y = 2.127$$

(314)

$$\therefore A = 10 y_n$$

$$Q = V_N A R^{2/3} S_0^{1/2}$$

$$30 = \frac{1}{0.015} (10 y_n) \left[\frac{10 y_n}{10 + 2 y_n} \right]^{2/3} (0.0001)^{1/2}$$

$$\therefore y_n^{5/2} = 1.209 (y_n + 5)$$

$$\Rightarrow y_n = 2.97 \text{ m} \quad (\text{By trial \& bit method})$$

2) Find critical depth

$$q = Q/B = 30/10 = 3 \text{ m}^3/\text{sec/m}$$

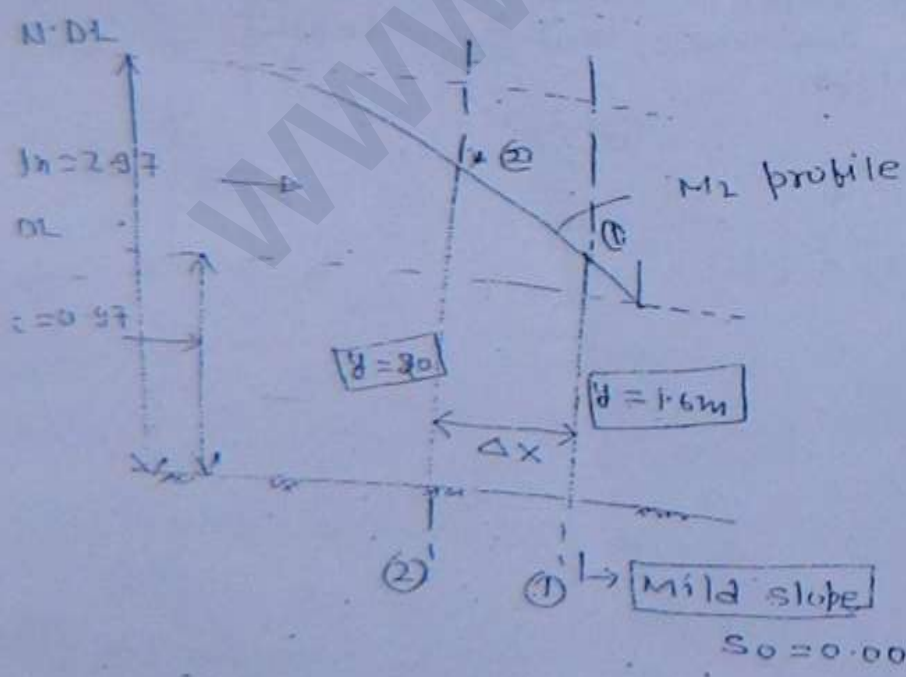
$$y_c^3 = (q^2/g) \rightarrow \text{For Rectangular}$$

$$\therefore y_c = \left(\frac{3^2}{9.81} \right)^{1/3} = 0.97 \text{ m}$$

since $y_n > y_c$ the given slope is mild slope and

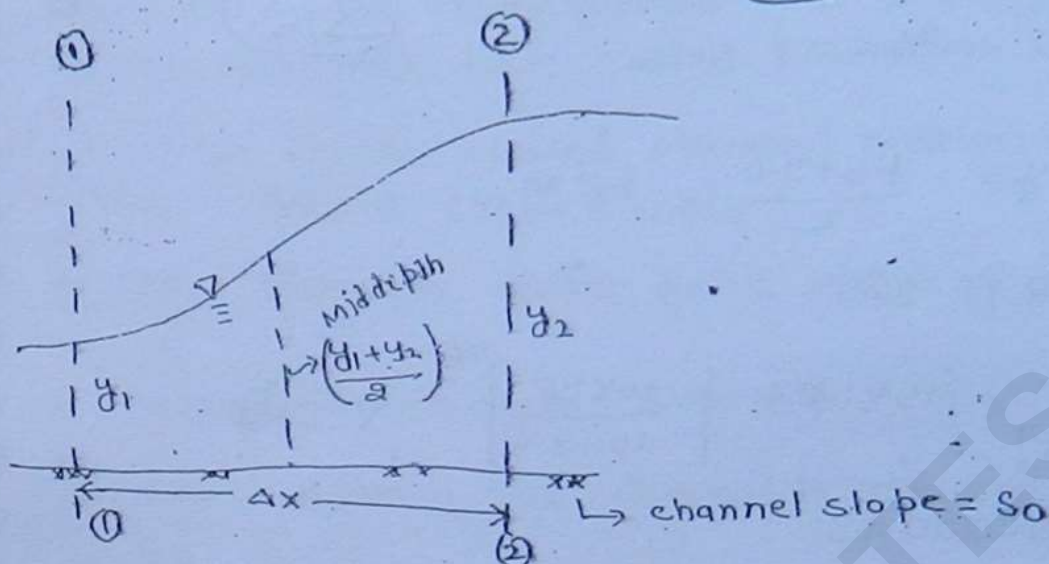
since actual depth of flow are b/w y_c & y_n

therefor surface profile will be M_2 .



Approximate Method to determine distance b/w two depths in G.V.F. ↓

(315)



$$E_1 = y_1 + \frac{V_1^2}{2g}$$

$$E_2 = \text{sp. energy at section 2} \\ = y_2 + \frac{V_2^2}{2g}$$

$$\Delta x = \frac{E_2 - E_1}{S_o - S_e}$$

slope of Energy line at mid section

$$\text{At mid section } y = \frac{y_1 + y_2}{2}$$

S_e is given by

$$R = \frac{V}{N} \quad A R^{2/3} S_e^{1/2}$$

$$A = B \cdot y \rightarrow \text{depth at mid section}$$

$$P = B + 2y, \quad R = A/P.$$

Ref. ex:

$$E_2 = y_2 + \frac{V_2^2}{2g} \\ = 2.0 + \frac{1.5^2}{2 \times 9.81}$$

$$= 2.11 \text{ m}$$

$$\left[V_2 = \frac{Q}{A_2} \right. \\ = \frac{30}{10 \times 2.0} = 1.5 \text{ m/s}$$

$$= 1.6 + \frac{1.875^2}{2 \times 9.81}$$

$$= 1.779$$

$$V = \frac{1.6 \times 10}{1.6 \times 10} = 1.0$$

(316)

At mid depth

$$y = \frac{1.6 + 2.0}{2} = 1.8 \text{ m}$$

$$Q = Y_N \cdot A R^{2/3} S_e^{1/2}$$

$$30 = \frac{1}{[0.015]} \times [10 \times 1.8] \left[\frac{10 \times 1.8}{10 + 3.6} \right]^{2/3} \times S_e^{1/2}$$

$$\Rightarrow S_e = 0.00043$$

$$\Delta X = \frac{E_2 - E_1}{S_0 - S_e} = \frac{2.11 - 1.779}{0.0001 - 0.00043} = (-) 1003$$

$$\therefore \Delta X = 1.003 \text{ km} \rightarrow \text{u/s to section (1)}$$

choking $\downarrow$

In the case of a channel with a hump and also in the case of a width contraction, it is observed that the u/s water-surface elevation is not affected by the conditions at section (2) till a critical stage is first achieved.

So in the case of a hump for all $\Delta Z \leq \Delta Z_{max}$, the u/s water depth is constant and for all $\Delta Z > \Delta Z_{max}$ the u/s depth is different from y_1 . Similarly in the case of the width contraction for $B_2 > B_{2min}$ the u/s depth y_1 is constant while for all $B_2 < B_{2min}$, the u/s depth undergoes a change.

This critical condition at section (2) is characterised by choking.

Thus for every case with $\Delta Z > \Delta Z_m$ or $B_2 < B_{2m}$ one known as choked conditions.

- 1) Analysis of flow through the channels when one section is transformed into another section such channels are called transition channel.
- 2) a) Flow over raised channel bottom slope.
Ex: Broad crested weir
- 3) b) Flow through sluice gate opening.

(317)

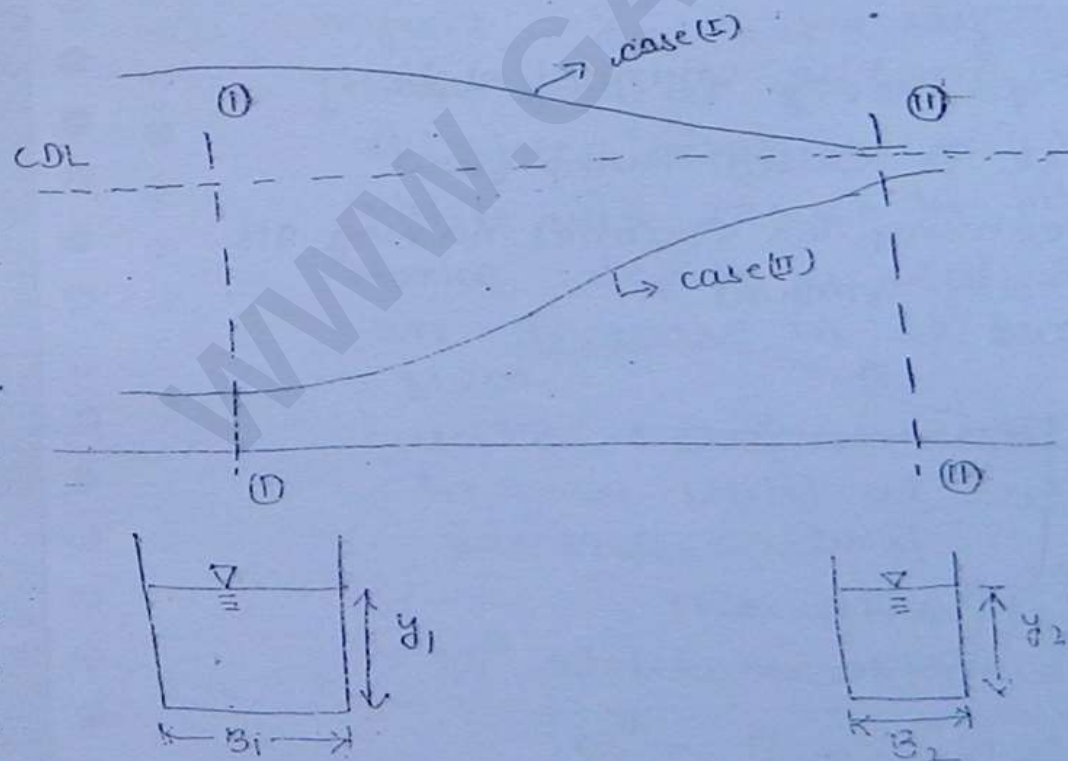
Flow through Rectangular channel Transition : ↓

when the width of the channel is reduced, it can be done by

- a) Sudden contraction
- b) Gradual contraction

In sudden contraction Loss of Energy is much more than Gradual contraction therefore all practical purpose

Gradual contraction is preferred.



Case I: - when at section (1)-(1) depth is y_1 & V_1

flow is subcritical ($Fr < 1$)

(3/8)

If flow is transform such that V_2 decreases than Flow will tend to become critical and depth will tend to become critical hence Lowering of water surface and type-2 curve will be formed

Case II: -

If flow is supercritical at section (1)-(1) than at section (2), flow tend to become critical hence flow will rise will be observed. In actual curve

If flow is through smooth transition and there is no change in bed level than $E_2 = E_1$

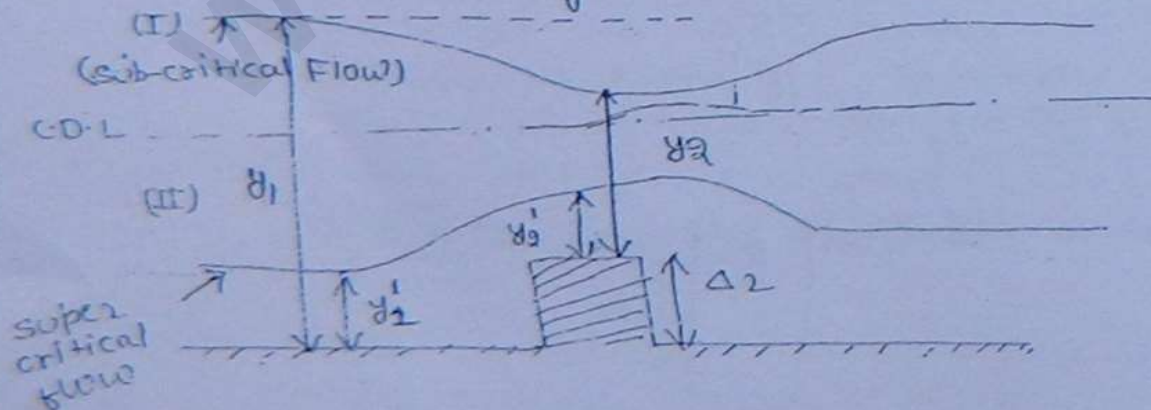
NOTE: - If B_2 is further contracted even after E_2 has reached E_{min} (corresponding to critical stage) than piling of water in U/s will occur and flow will not be possible at original depth y_2 .

Imp

Flow over Local rise in the channel slope: ↓

Flow over Local rise is called 'hump flow'.

Let's consider a rectangular channel having its bottom raised by an amount Δ .



(319)

$$(y_2 < y_1)$$

$$E_1 = \Delta Z + E_2$$

OR

$$E_1' = \Delta Z + E_2'$$

$$(y_2' > y_1')$$

→ If flow $Fr > 1 \rightarrow \{y_1' < y_c\}$

→ The Maxm height of hump ($\Delta Z = \text{Max}$) will be obtain when point (2) in the sp. energy curve or when depth of section (2) coincides with critical depth at that section. ($y_2 = y_c$)
Thus flow over raised section will be critical.

Then

$$E_1 = (\Delta Z)_{\text{Max}} + E_{\text{min}}$$

$$E_2 = E_{\text{min}}$$

$$(\Delta Z)_{\text{Max}} = E_1 - E_{\text{min}} \quad \text{setp}$$

NOTE:

if the height of hump is further ~~increased~~ beyond ΔZ , Let's say $\Delta Z' > (\Delta Z)_{\text{Max}}$ than flow at given sp. energy E_1 & at given depth y_1 will not be possible which will result in piling of water. Hence depth y_1 will increase, causing an increase in sp. energy of approaching flow.

water Level at section (1) will continue to rise until at section (2) flow becomes critical.

The increased sp. energy E_1'' at (1) is obtained as

$$E_1'' = E_{\text{min}} + \Delta Z'$$

$$= \frac{3}{2} y_c + \Delta Z'$$

→ For Rect channel.

A trapezoidal channel of base width 6m and side slope of 2H:1V carries a flow of 60 m³/sec at a depth of 2.5 m. There is a smooth transition to a rect. section 6m wide accompanied by a gradual lowering of channel bed by 0.6 m

- 1) Find the depth of water in the rect. section & change in water surface level.
- 2) In case the drop of water surface level is to be restricted to 0.30 m. what is the amount of bed must be lowered.

320

A lined channel ($N = 0.014$) is of trapezoidal section with one side vertical and other side on slope (1.5H:1V) If the channel is to be deliver 9 m³/sec when laid on a slope of 0.0002. calculate the dimensions of efficient section which requires minimum lining. Also calculate ^{corresponding} mean velocity.



$$\begin{cases} nH:1V \\ n = 1.5 \end{cases}$$

For minimum Lining, P should be minimum

$$\begin{aligned} P &= y + B + y \sqrt{n^2 + 1} \\ &= B + y [1 + \sqrt{3.25}] \end{aligned}$$

$$d =$$

$$A = \frac{B + (B + ny)}{2} \times y$$

$$A = \frac{(2.8 + 1.5y)}{2} \times y$$

$$A = (1.3 + 0.75y) \times y$$

$$\therefore B = A/y - 0.75y$$

$$\therefore P = A/y - 0.75y + 2.8y$$

$$\therefore dP/dy = -A/y^2 - 0.75 + 2.8 = 0$$

$$\Rightarrow y^2 = \frac{A}{2.05} \Rightarrow$$

$$A = 2.05y^2$$

$$B = 1.3y$$

(321)

For
Given Area
Perimeter
should be minimum.

By applying Manning's Equation

$$Q = V_N \cdot A \cdot R^{2/3} S^{1/2} \Rightarrow \text{Get } (y) = ?$$

channel with a hump:

a) subcritical Flow:

when $c < \Delta Z < \Delta Z_{max}$

the U/s water level remains stationary at y_1 while the depth of flow at section 2

decreases with ΔZ reaching at minimum

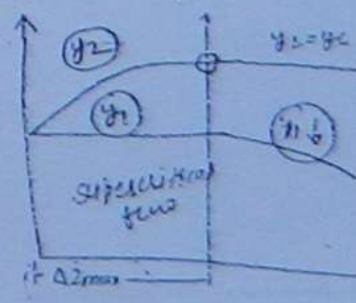
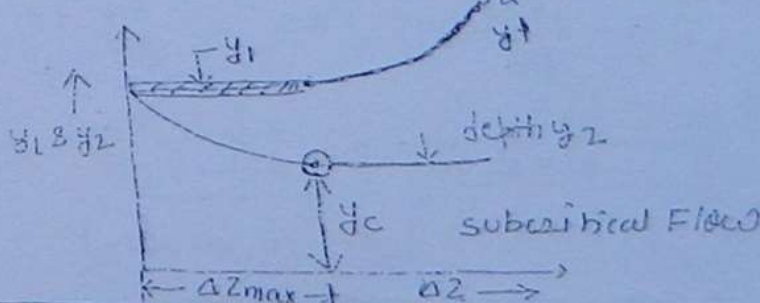
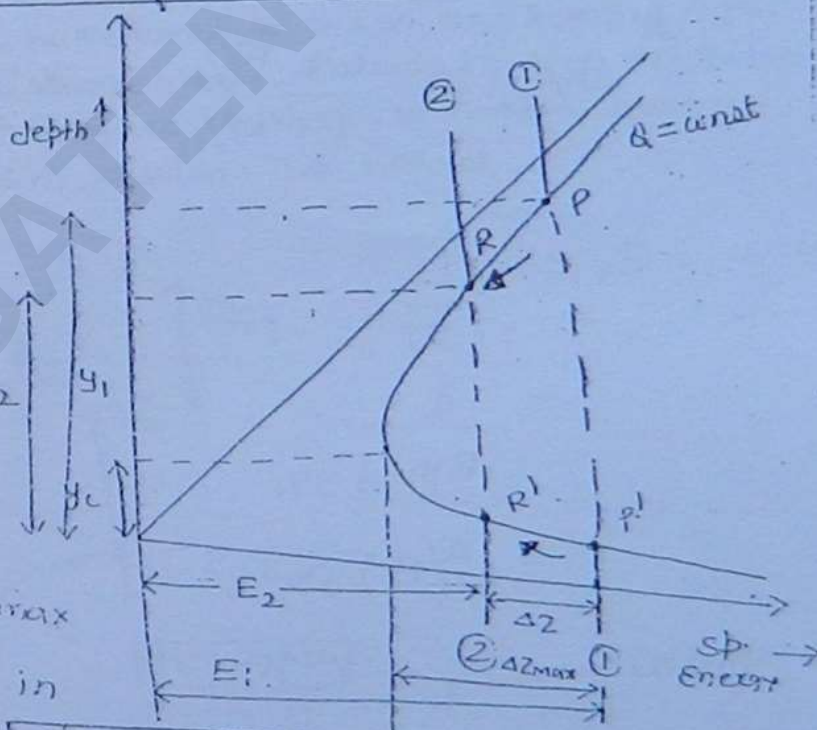
value of y_c at $\Delta Z = \Delta Z_{max}$

with further increase in

the value of ΔZ i.e.

$$\Delta Z > \Delta Z_{max}$$

y_1 will change to y'_1 while y_2 will continue to remain at y_c



8061
 In a flow through a rect. channel for a certain discharge the Froud No. corresponding to two alternate depths are F_1 & F_2 then prove that

$$(F_2/F_1)^{2/3} = \frac{2+F_2^2}{2+F_1^2}$$

322

$$F_2 = \frac{V}{\sqrt{gy}} = \frac{V}{\sqrt{gD}}$$

$$E_1 = E_2$$

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g}$$

$$\Rightarrow y_1 \left[1 + \frac{V_1^2}{2gy_1} \right] = y_2 \left[1 + \frac{V_2^2}{2gy_2} \right]$$

$$\therefore y_1/y_2 = \frac{1 + F_2^2/2}{1 + F_1^2/2} = \frac{2 + F_2^2}{2 + F_1^2}$$

$$\begin{aligned} \text{Now } E_1 &= \frac{V_1}{\sqrt{gy}} \\ &= \frac{Q}{By_1 \sqrt{gy_1}} \\ &= \frac{Q}{B \sqrt{gy_1^3}} \end{aligned}$$

$$\text{Similarly } F_2 = \frac{Q}{B \sqrt{gy_2^3}}$$

$$\therefore y_1^3 = \frac{Q^2}{3^2 g F_1^2}$$

$$y_2^3 = \frac{Q^2}{3^2 g F_2^2}$$

$$\left(\frac{F_2}{F_1}\right)^{1/3} = \frac{2 + F_2^2}{2 + F_1^2}$$

$$\left[F_1 = \frac{V_1}{\sqrt{g y_1}}, F_2 = \frac{V_2}{\sqrt{g y_2}} \right]$$

223

$$y_1^3 = \frac{Q^2}{B^2 g F_1^2}$$

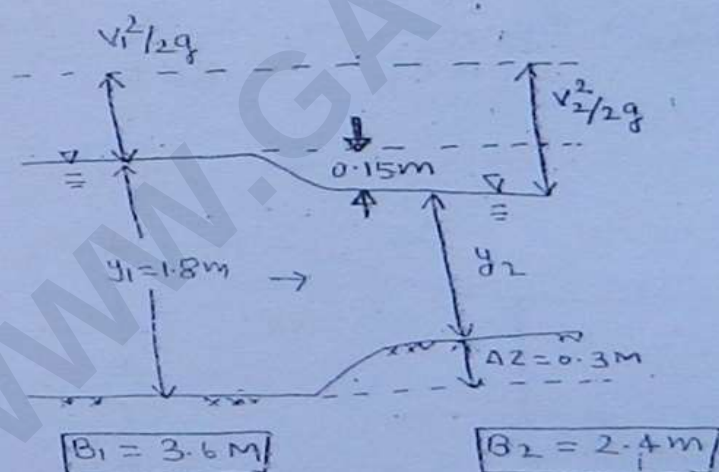
$$y_2^3 = \frac{Q^2}{B^2 g F_2^2}$$

$$y_1/y_2 = \left(\frac{F_2}{F_1}\right)^{2/3}$$

$$\left(\frac{F_2}{F_1}\right)^{2/3} = \frac{2 + F_2^2}{2 + F_1^2}$$

Prob 2

A 3.6 m wide rect. wide channel carries a water at a depth of 1.8 m. In order to measure the discharge the channel width is reduced to 2.4 m and hump of 0.3 m is provided in the bottom. calculate the discharge if the water surface in the contracted surface drop by 0.15 m assume no losses.



$$E_1 = \Delta Z + E_2$$

$$y_1 + \frac{V_1^2}{2g} = \Delta Z + y_2 + \frac{V_2^2}{2g} \quad \text{--- (1)}$$

$$\frac{V_2^2}{2g} - \frac{V_1^2}{2g} = 0.15 \text{ m} \quad \text{--- (2)}$$

From (2) in Eqn (1)

$$\Rightarrow y_2 = 1.8 - 0.3 - 0.15 = 1.35 \text{ m}$$

By using eqn (ii) -

$$\frac{Q^2}{2g A_2} - \frac{Q^2}{2g A_1} = 0.15$$

324

$$\Rightarrow \frac{Q^2}{2g} [y_{A_2} - y_{A_1}] = 0.15$$

$$\Rightarrow Q = 6.418 \text{ m}^3/\text{sec}$$

$$\begin{cases} A_1 = 3.6 \times 1.8 \\ A_2 = 2.4 \times 1.35 \end{cases}$$

2068
15/2005

A wide rect channel carries a flow of $2.76 \text{ m}^3/\text{sec}/\text{m}$ width.
The depth of flow is 1.524 m .

a) calculate the min. rise of flow at a section required to produce critical flow conditions

b) what is corresponding fall in the water level

Given

$$q = 2.76 \text{ m}^3/\text{sec}/\text{m}$$

$$y_1 = 1.524 \text{ m}$$

$$\Delta Z_{\max} = ? \quad (\text{For critical flow condition})$$

At critical Flow condition at ^{section} 2

$$E_1 = \Delta Z_{\max} + E_{\min}$$

$$\begin{aligned} v_1 &= q/y_1 \\ &= \frac{2.76}{1.524} \\ &= 1.81 \text{ m/sec} \end{aligned}$$

$$\begin{aligned} E_1 &= y_1 + \frac{v_1^2}{2g} \\ &= 1.524 + \frac{1.81^2}{2 \times 9.81} \\ &= 1.69 \text{ m} \end{aligned}$$

$$E_{\min} = \frac{3}{2} (y_c)$$

$$\begin{aligned} y_c &= (q^2/g)^{1/3} \\ &= 0.949 \text{ m} \end{aligned}$$

$$\begin{aligned} \Delta Z_{\max} &= E_1 - E_{\min} \\ &= 1.69 - 1.5 \times 0.949 \\ &= 0.342 \text{ m} \end{aligned}$$

$$\begin{aligned}
 b) \Delta y &= y_1 - y_2 = 1.524 - 0.914 \\
 &= 0.610 \text{ m} \\
 &= 0.294 \text{ m}
 \end{aligned}$$

(325)

Prob 4

A rect. channel 3.5 m wide laid at a slope of 0.0036, uniform flow occurs at a depth of 2 m. Find how high a hump can be raised on the channel bed without causing a change in U/s depth. If the U/s depth is to be raised to 2.4 m what should be the height of hump. Assume Manning's $N = 0.015$.

$$A_1 = 3.5 \times 2.0 = 7 \text{ m}^2$$

$$P_1 = 3.5 + 2 \times 2 = 7.5 \text{ m}$$

$$R_1 = A_1 / P_1 = 0.933$$

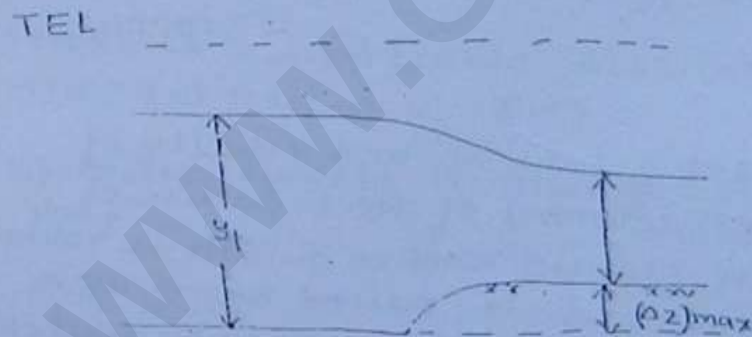
$$V_1 = \frac{1}{N} R_1^{2/3} S^{1/2}$$

$$= 3.82 \text{ m/sec}$$

check for Froude Number

$$F_1 = \frac{V_1}{\sqrt{g y_1}} = \frac{3.82}{\sqrt{9.81 \times 2}} = 0.86 < 1$$

↳ subcritical flow



9) If U/s depth is not to be changed than at equilibrium

$$E_1 = (y_2)_{\max} + E_{\min}$$

$$(y_2)_{\max} = E_1 - E_{\min}$$

$$Q = V_1 \times A_1 = 3.82 \times 7$$

$$Q/B_1 = \frac{1}{2} = 3.82 \times 7 / 9.81 = 7.64 \text{ m}^3/\text{sec}/\text{m}$$

$$\Delta Z_{\max} = E_1 - 3/2 \times 1.81$$

$$\begin{aligned} E_1 &= y_1 + \frac{v_1^2}{2g} \\ &= 2.0 + \frac{3.82^2}{2 \times 9.81} \\ &= 2.74 \text{ m} \end{aligned}$$

$$\begin{aligned} \Delta Z_{\max} &= 2.74 - 1.5 \times 1.81 \\ &= 0.026 \text{ m} \end{aligned}$$

b) If Now $y_1' = 2.4 \text{ m}$ (VL would change)
Discharge would remain same, y_1 would change.

$$y_1' + \frac{v_1'^2}{2g} = \Delta Z^1 + E_{\min} \quad [\Delta Z^1 > \Delta Z_{\max}]$$

$$2.4 + \frac{v_1'^2}{2g} = \Delta Z^1 + 3/2 \times 1.812$$

$$\Rightarrow 2.4 + \frac{(3.18)^2}{2 \times 9.81} = \Delta Z^1 + 1.812 \times 1.5$$

$$\Delta Z^1 = 0.198 \text{ m}$$

$$\begin{aligned} v_1' &= Q/A_1' \\ &= \frac{26.74}{3.5 \times 2.4} \\ &= 3.18 \text{ m/sec} \end{aligned}$$

Probs

Water flows at a depth of 1.6 m and velocity of 1.1 m/sec in a open channel of Rect. section of width 4m. At a certain section width is reduced to 3.5 m and the bed is raised by 0.35 m through a smooth flat hump. Calculate water surface elevation at the contracted section as well as the VLS section. Neglect losses.

At section (2) depth = 1.158 m
(y_2)

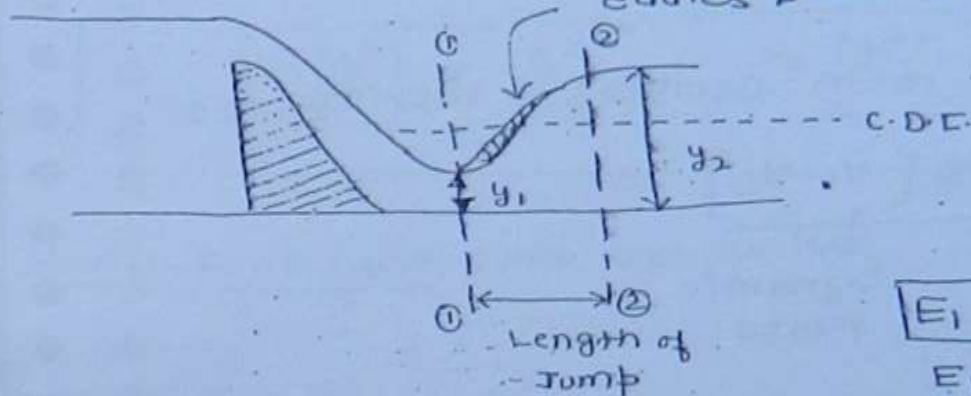
$$[y_2 > y_c =]$$

At section (1) elevation is =
raised

$$y_1 = 1.6 \text{ m}$$

(327)

Eddies & some energy is lost.



$$E_1 \neq E_2$$

$$E_1 > E_2$$

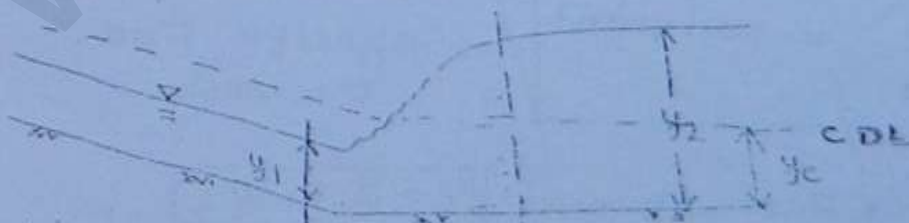
$$E_1 - E_2 = \Delta E \quad (\text{Loss of Energy})$$

An essential & necessary condition for hydraulic jump to occur Flow must change supercritical to subcritical & this change is over a small Length hence if flowing fluid having Froude No. greater than 1, jump may be created.

Hydraulic jump is defined as sudden & turbulent Passes of water supercritical to subcritical it is also called shooting, rapid, Tranquill, unstable type.

→ There is considerable dissipation of energy during the formation of Jmp.

sp. Force concept is always applied with hydraulic jump i.e. Flow changing from supercritical to subcritical

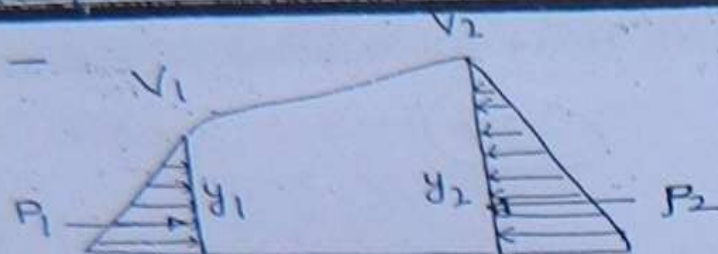


$$E_1 \neq E_2$$

$$y_1 < y_2 < y_c$$

- But $F_1 = F_2$ (sp. force)

y_1 & y_2 are subsequent depth or conjugate depth



328

If Resultant Force is in Equilibrium then $\Sigma F = 0$

$$\Rightarrow \underbrace{(P_1 - P_2)}_{\text{Static Force}} + \underbrace{PQ [V_1 - V_2]}_{\text{Dynamic Force}} = 0$$

$$P_1 = \omega A_1 \bar{x}_1 \quad \left[\bar{x}_1 = \text{Position of C.G. From Top surface} \right]$$

$$P_2 = \omega A_2 \bar{x}_2$$

$$\therefore \omega [A_1 \bar{x}_1 - A_2 \bar{x}_2] + PQ [V_1 - V_2] = 0$$

$$\Rightarrow \omega [A_1 \bar{x}_1 - A_2 \bar{x}_2] = PQ \left[\frac{Q}{A_2} - \frac{Q}{A_1} \right]$$

$$\Rightarrow \cancel{\omega} g [A_1 \bar{x}_1 - A_2 \bar{x}_2] = \cancel{Q} \left[\frac{Q}{A_2} - \frac{Q}{A_1} \right]$$

$$\Rightarrow g [A_1 \bar{x}_1 - A_2 \bar{x}_2] = \frac{Q^2}{A_2} - \frac{Q^2}{A_1}$$

$$\Rightarrow A_1 \bar{x}_1 + \frac{Q^2}{A_1 g} = A_2 \bar{x}_2 + \frac{Q^2}{A_2 g}$$

$$\Rightarrow \boxed{A\bar{x} + \frac{Q^2}{Ag} = \text{constant}} \rightarrow \text{specific Force } F = \text{const.}$$

Generally For Rect. channel $\bar{x} = \bar{z} = y/2$

[Either from top or bottom]

~~At minimum sp. force there is only one critical depth.~~

For other values of F there are two depths of flow y_1 & y_2 , called conjugate depth.

$$y_1 < y_c < y_2 \quad \text{self}$$

330

For critical Flow, sp. force is Minimum
for Rect. channel

$$F = \frac{Q^2 B}{g} \times y_y + \frac{B y^2}{2}$$

For $F(\min)$, $\frac{dF}{dy} = 0$

$$\Rightarrow \frac{Q^2 B}{g} (-y_y^2) + \frac{2 B y}{2} = 0$$

$$\Rightarrow y_c = \left(\frac{Q^2}{g} \right)^{1/3} \quad \text{--- (i)}$$

NOTE:

For Non-Rectangular channel

$$\left[\frac{Q^2}{g} = \frac{A^3}{T} \right] \quad \text{--- (ii) at critical Flow.}$$

$$F_1 = F_2$$

$$\Rightarrow \frac{Q^2}{A_1 g} + A_1 \bar{Z}_1 = \frac{Q^2}{A_2 g} + A_2 \bar{Z}_2$$

$$\left[Q = Q \cdot B \text{ For Rect. channel} \right]$$

$$\frac{Q^2 B^2}{B y_1 g} + B y_1 (y_1/2) = \frac{Q^2 B^2}{y_2 B g} + B y_2 (y_2/2)$$

$\downarrow$ on solving

$$2 \frac{Q^2}{g} = y_1 y_2 (y_1 + y_2) \quad \text{--- (iii)}$$

$$\Rightarrow y_c^3 = \frac{y_1 y_2 (y_1 + y_2)}{2} \quad \text{self --- (iv)}$$

For alternate depth $\rightarrow F_1 = F_2 \rightarrow y_c^3 = \frac{2 y_1^2 y_2^2}{(y_1 + y_2)} \quad \checkmark$

Using eqn (iii) we can find y_2 .

Let's find y_2

331

$$y_2^2 \cdot y_1 + y_1^2 \cdot y_2 - \frac{2q^2}{g} = 0$$

$$\Rightarrow x^2 a + b \cdot x + c = 0$$

$$\Rightarrow y_2 = \frac{-y_1^2 \pm \sqrt{y_1^4 + 4y_1 \left(\frac{2q^2}{g} \right)}}{2 \times y_1}$$

$$y_2 = +y_1/2 \left[-1 \pm \sqrt{1 + \frac{8q^2}{gy_1^3}} \right]$$

$$\frac{y_2}{y_1} = Y_2 \left[-1 \pm \sqrt{1 + \frac{8V_1^2}{gY_1}} \right]$$

$$\therefore \boxed{y_2/y_1 = Y_2 \left[-1 + \sqrt{1 + 8 Fr_1^2} \right]} \quad \text{self}$$

$$\text{OR} \quad \boxed{Fr_1 > 1}$$

> only For Rect. Section

$$\boxed{y/y_2 = Y_2 \left[-1 + \sqrt{1 + 8 Fr_2^2} \right]}$$

Loss of Energy in Jump: $\downarrow$

$$\Delta E = E_1 - E_2$$

$$= \left(y_1 + \frac{V_1^2}{2g} \right) - \left(y_2 + \frac{V_2^2}{2g} \right)$$

$$\boxed{\Delta E = \frac{[y_2 - y_1]^3}{4 y_1 y_2}} \quad \text{self}$$

OR

$$\boxed{\Delta E = \frac{[V_1 - V_2]^3}{2g (V_1 + V_2)}}$$

$$= \omega Q (\Delta E)$$

Height of Jump = $y_2 - y_1$

(330)

Length of Jump = 5 to 7 times $(y_2 - y_1)$

F_{r1}

a) 1-1.7

b) 1.7 to 2.5

c) 2.5 to 4.5 golf

d) 4.5 to 9.0 golf

e) > 9.0

Type of Jump

undular Jump

weak Jump

oscillating Jump

steady Jump

strong Jump

Prob 1

A Trapezoidal section having bottom width of 8 m & side slope is 1:1, carries a discharge of 30 m³/sec. Find the depth of conjugate to initial depth of 0.75 m before the jump. Also determine the loss of energy in the jump.

St Force

$$F_1 = F_2$$

$$\frac{Q^2}{A_1 g} + A_1 \bar{Z}_1 = \frac{Q^2}{A_2 g} + A_2 \bar{Z}_2$$

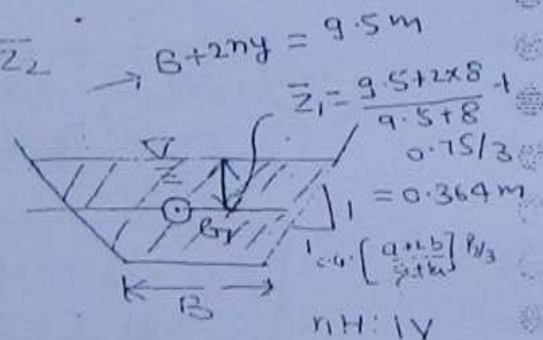
(1)

$$A_2 = (B + y_2) y_2$$

$$\bar{Z}_2 = \frac{(8 + 2y_2) + 2 \times 8}{8 + 2y_2 + 8} \times y_2 / 3$$

Solving for y_2 from eqn (1)

$$y_2 = 4.167 \text{ m}$$



$$A_1 = (B + ny_1) y_1 = (8 + 0.75) \times 0.75 = 6.56 \text{ m}$$

$$Z_1 = 0.364 \text{ m}$$

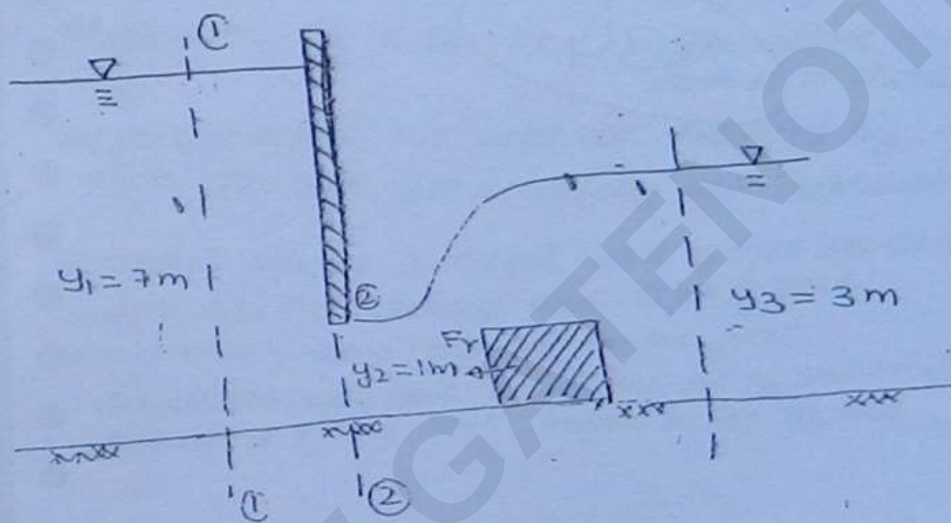
$$\text{Loss of Energy} = E_1 - E_2$$

333

$$\Delta E = \left(y_1 + \frac{V_1^2}{2g} \right) - \left(y_2 + \frac{V_2^2}{2g} \right)$$

Prob 2
ES/1992
A sluice across a channel is 6m wide, discharges a stream 1m deep. what is the flow rate when u/s of sluice is 7m. on the d/s side depth

a concrete block have been placed, to create the condition of hydraulic jump. Determine the force on the block if d/s depth is 3m.



$$Q = (y_1 \times 6) V_1 = (y_2 \times 6) V_2 = (y_3 \times 6) V_3$$

$$\Rightarrow 7V_1 = V_2 = 3V_3 \quad \text{--- (1)}$$

There is NO LOSS of Energy b/c (1)-(1) & (2)-(2)

$$\text{So } E_1 = E_2$$

$$\Rightarrow y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} \quad \text{--- (2)}$$

By Eqn (1) & (2)

$$\Rightarrow V_1 = 1.555 \text{ m/sec.}$$

$$V_2 = 10.77 \text{ m/sec.}$$

$$V_3 = 3.65 \text{ m/sec.}$$

$$F_r = (P_2 - P_3) + \rho g [V_2 - V_3]$$

$$= \rho g [A_2 \bar{x}_2 - A_3 \bar{x}_3] + \rho g [V_2 - V_3]$$

$$= 1000 \times 9.81 \left[(1 \times 6) \times y_2 - (3 \times 6) \times \frac{1.5}{3} \right] +$$

$$1000 \times 65.77 [10.95 - 3.65] \quad \text{N}$$

$$F_r = 243.83 \text{ kN}$$

334

(i) mild slope: - $y_0 > y_c \rightarrow$ Subcritical flow at normal depth.

(ii) steep slope: - $y_0 < y_c \rightarrow$ Supercritical flow at normal depth.

(13) (iii) critical slope: - $y_0 = y_c \rightarrow$ Critical flow at normal depth.

213 (iv) horizontal slope: - $S_0 = 0 \rightarrow$ constant uniform flow

213 (v) Adverse slope: - $S_0 < 0 \rightarrow$ " "

$\frac{dy}{dx} > 0$ if (a) $y > y_0$ and $y > y_c$ or
(b) $y < y_0$ and $y < y_c$

$\frac{dy}{dx} < 0$ if (i) $y_c > y > y_0$ or
(ii) $y_0 > y > y_c$

(i) As $y \rightarrow y_0 \Rightarrow \frac{dy}{dx} \rightarrow 0$ i.e. The water surface approaches the normal depth line asymptotically.

(2) As, $y \rightarrow y_c \Rightarrow \frac{dy}{dx} \rightarrow \infty$ i.e. water surface meets the critical depth line vertically.

(3) As, $y \rightarrow \infty, \frac{dy}{dx} \rightarrow S_0$ i.e. water surface meets a very large depth as a horizontal asymptote.

At critical depth the curves are indicated by dashed line to remind that the GVF eqn is strictly not applicable in that neighbourhood.

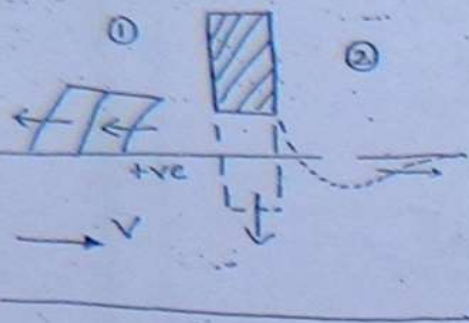
→ A control section is defined as a section in which a fixed relationship exists b/w the discharge and depth of flow.

- critical depth is also a critical control point.

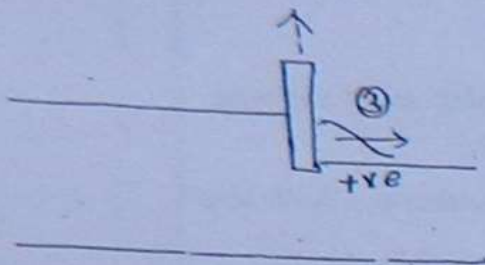
- Subcritical flows have controls at the D/S end while supercritical flows have controls at the U/S end.

335

336



→ If the flow in a channel is increased suddenly by means of opening gate a wave is formed which travels D/s.



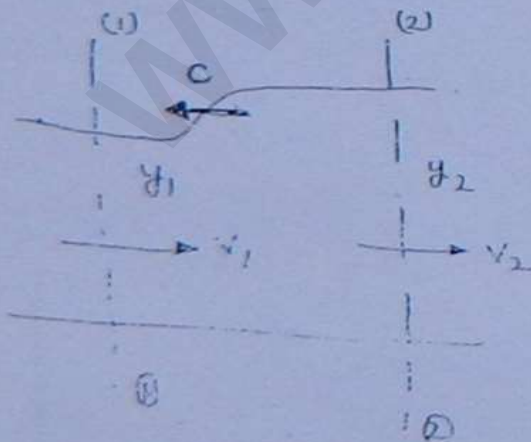
→ Similarly if a gate is suddenly closed and a flow is partially reduced, a wave formed travels such wave is called surge wave.

-ve surge wave:-

Also known as 'elevation surge wave' → If depth of water increases in the direction of motion of wave is called surge wave.

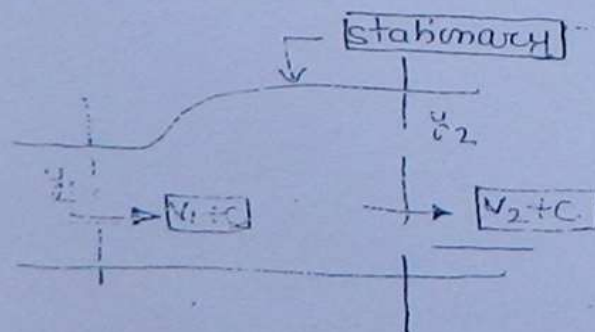
(1) & (3) are +ve surge wave & (2) is -ve surge wave.

Analysis ↓



Let vel. of wave is C .
{celerity of wave}

For Analysis ↓



For prismatic Rectangular channel:

$$Q = by_1(v_1 + c) = by_2(v_2 + c) \Rightarrow y_1(v_1 + c) = y_2(v_2 + c)$$

$$\therefore c = \frac{v_1 y_1 - v_2 y_2}{y_2 - y_1} \quad \text{--- (1)}$$

337

If there is no resistance force b/w (1) & (2) then

$$\text{Total Force} = 0$$

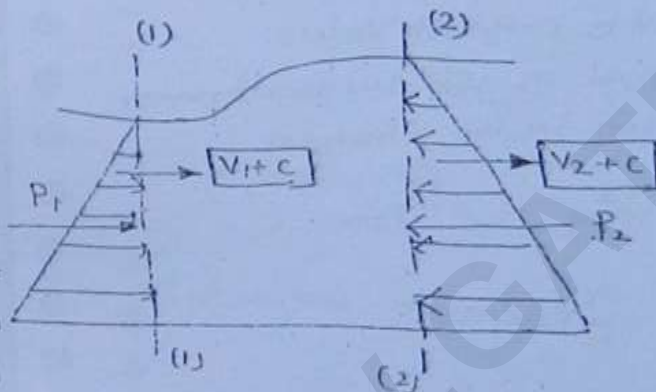
$$= \text{static force} + \text{dynamic force}$$

$$= (P_1 - P_2) + \rho A \{ [v_1 + c] - [v_2 + c] \} = 0$$

$$\Rightarrow P_1 - P_2 = \rho A [v_2 - v_1]$$

$$\therefore \omega b y_1 \times y_1/2 - \omega b y_2 \times y_2/2 = \rho A [v_2 - v_1]$$

$$\therefore \frac{\rho g b}{2} [y_1^2 - y_2^2] = \rho A [v_2 - v_1] \quad \text{--- (2)}$$



From eqn (1) & (2)

$$v_1 + c = \sqrt{\frac{g y_2}{2 y_1} (y_1 + y_2)}$$

$[v_1 + c] \rightarrow$ vel. of surge relative to water
{+ve surge}

$$\text{For -ve surge: } [c > v_1]$$

$$\therefore \text{Rel. vel.} = c - v_1 = \sqrt{\frac{g y_2}{2 y_1} (y_1 + y_2)}$$

Special case $\downarrow$

If surge is very small and depth of flow is large then $y_1 \approx y_2 \approx y$ then

$$v_1 + c = \sqrt{g y}$$

For Rectangular channel: $\downarrow$

$$\frac{v_1}{\sqrt{g y_1}} = F_1$$

$$\text{For critical Flow } v_1 = \sqrt{g y_1}$$

$$\text{For subcritical Flow } v_1 < \sqrt{g y_1}$$

$$\text{For supercritical Flow } v_1 > \sqrt{g y_1}$$

9m an open channel at any point external wave a small surge is created and at this surge ~~the~~ has zero celerity then $V_1 = \sqrt{gy} \Rightarrow V/\sqrt{gy} = Fr = 1$. Similarly if celerity is +ve means it causes upwards ^{flow} than $V_1 < \sqrt{gy}$. Therefore $Fr < 1$ Hence Flow is subcritical.

Similarly if celerity -ve wave wave travels d/s.

EX:1
A trapezoidal channel with base width of 6m & side slopes with 2H:1V conveys water at the rate of 17 m³/sec with a depth of flow of 1.5m. At this flow situation is subcritical or supercritical.

(338)

$$Fr = \frac{V}{\sqrt{gD}}, \quad D = A/T = \frac{(b+ny)y}{b+2ny} \quad [n=2]$$

EX:2
A rectangular horizontal channel of 3m width and 2m depth conveys water at 18 m³/sec. If the flow rate is suddenly reduced to 2/3 of its original value. Compute the magnitude ^{of salinity} and speed of U/s surge w.r.t water. Assume that there is no friction in the channel.

$$V_1 = Q/A_1 = 18/(b \cdot y_1) = \frac{18}{3 \times 2} = 3 \text{ m/sec}$$

$$V_2 = Q_2/A_2 = \frac{2/3 \times 18}{3 \times y_2} = 4/y_2$$

At stable condition

$$b y_1 (V_1 + C) = b y_2 (V_2 + C) \quad \text{--- (1)}$$

$$P_1 - P_2 = \rho g [V_2 - V_1] \quad \text{--- (2)}$$

$$C = \frac{V_1 y_1 - V_2 y_2}{y_2 - y_1} \quad \text{--- (3)}$$

$$= \frac{3 \times 2 - 4/y_2 \times y_2}{y_2 - 2}$$

$$C = \frac{2}{y_2 - 2}$$

$$V_1 + c = \frac{1.48 y_2 (4 + y_2)}{2 y_1}$$

$$3 + \frac{2}{y_2 - 2} = \frac{9.81 y_2 (2 + y_2)}{2 \times 2}$$

$$\Rightarrow y_2 = ?$$

→ clearly: - The velocity of the surge relative to the initial flow velocity in a canal is called celerity.

$$C_b = V_b - V_1 \quad \text{--- D/S}$$

$$C_s = V_w + V_1 \quad \text{--- U/S}$$

$$C_s = \sqrt{\frac{g y_1}{2} [y_1 + y_2]}$$

MODELS

339

Similarities: ↓

1) Geometric Similarities: - It includes physical parameters such as Length, width, height, volume, area, etc.

eg

$$\text{Area ratio} = \frac{\text{Area of prototype}}{\text{Area of Model}}$$

$$(A_r)$$

$$A_r = \frac{L_r^2}{L_r} = \left(\frac{L_p}{L_m} \right)^2 = \left(\frac{\text{Length of prototype}}{\text{Length of Model}} \right)^2$$

$$V_r = \frac{L_r^3}{L_r}$$

$$[L_r = 100:1 \text{ or } 1:100]$$

2) Kinematic similarity: Those parameters which involve effect of time such as vel. ~~area~~ accn, discharge etc.

$$a) V_r = \frac{V_p}{V_m} = \frac{L_r}{T_r}$$

$$b) \text{Accn. ratio} = a_r = \frac{V_r}{T_r} = \frac{L_r}{T_r^2}$$

$$c) \text{discharge ratio} = Q_r = \frac{L_r^3}{T_r}$$

3) Dynamic similarity: - It exists if the ratio of all the forces at homologous point model & prototype are similar.

a) Gravity Force = $F_g = m \cdot g$

$$= \rho L^3 \cdot (L/T^2 \rightarrow \text{same for Model \& prototype})$$

$$= \rho L^3 \cdot g$$

b) Inertial Force $\rightarrow$ Effect of mass and it always acts. $\Rightarrow$

$$F_i = M \times a$$

$$= \rho L^3 \times L/T^2$$

$$= \rho L^2 (L^2/T^2) = \rho L^2 v^2$$

c) viscosity Force:

$$F_v = \mu \times A$$

$$= \mu (V/L) \times L^2 = \mu V L$$

d) Pressure Force:

$$= p \times A$$

$$= p \times L^2$$

e) surface Tension Force:

$$F_s = \sigma \cdot L$$

f) Compressibility Force:

$\rightarrow$ Bulk modulus

$$F_c = K \times A = K \times L^2$$

Reynolds Number: $\downarrow$

For situation, viscosity forces are very predominant with Inertial forces but other forces are less significant than Reynolds number is defined as:

$$Re = \frac{F_i}{F_v}$$

$$= \frac{\rho L^2 v^2}{\mu V L}$$

$$Re = \frac{\rho V L}{\mu}$$

Eff.

$L \rightarrow$ Length parameter

$\rho \rightarrow$ mass density

$\mu \rightarrow$ dyn. coeff. of viscosity

Reynolds Model Law:

(341)

In the flow conditions where viscosity forces are very predominant than other forces than this Law is applied.

Example

- a) Flow in pipes under laminar conditions
- b) Flow of submarine & air plane but submarine are fully emerged.
- c) Flow around submerged structure.
- d) Flow through low speed turbo machines.

→ For such conditions Re for model will be equal to Re for prototype.

$$\left(\frac{\rho V L}{\mu}\right)_m = \left(\frac{\rho V L}{\mu}\right)_p$$

$$\Rightarrow \boxed{\frac{\rho_2 V_2 L_2}{\mu_2} = 1} \rightarrow \text{Reynolds Law}$$

$$\mu/\rho = \nu$$

$$\boxed{\frac{V_2 L_2}{\nu_2} = 1}$$

EX: 1)

$$T_2 = ?$$

$$\rho_2 \cdot \frac{L_2}{T_2} \times \frac{L_2}{\mu_2} = 1$$

$$\Rightarrow \boxed{T_2 = \frac{L_2^2}{\mu_2} \rho_2}$$

2)

Acc'n Ratio

$$a_2 = \frac{V_2}{T_2} = \frac{L_2}{T_2^2}$$

$$a_2 = \frac{L_2 \mu_2^2}{L_2^3 \rho_2^2}$$

$$\boxed{a_2 = \frac{\mu_2^2}{L_2^3 \rho_2^2}}$$

> Force Ratio:

$$F_r = M_r \cdot Q_r$$

$$= \rho_r \times L_r^3 \times \frac{u_r^2}{\rho_r^2 \times L_r^3}$$

$$F_r = \frac{u_r^2}{\rho_r}$$

342

> Power Ratio:

$$P_r = F_r \times V_r$$

$$P_r = \frac{u_r^2}{\rho_r} \times \frac{u_r}{L_r \cdot \rho_r}$$

$$P_r = \frac{u_r^3}{L_r \rho_r^2}$$

> Froude Number:

When gravity force is important apart from inertia force but other forces are less significant than Froude Number is defined as

$$F_r = \sqrt{F_i / F_g} = \sqrt{\frac{\rho L^2 v^2}{\rho L^3 g}}$$

$$F_r = \frac{v}{\sqrt{Lg}}$$

L: Length Parameter

: A/D [For open channel]

Froude Model Law: ↓

When gravity force is very important than this law is applicable.

Ex:

> Flow through open channels (waves & jumps)

Flow over spillway of a dam

- 3) Flow of Liquid jet of orifice
- 4) Flow over weir & Notches.
- 5) Motion of ship in rough and turbulent

According to this Law

$$(Fr)_p = (Fr)_m$$

343

$$\left(\frac{V}{\sqrt{gL}} \right)_p = \left(\frac{V}{\sqrt{gL}} \right)_m$$

$\Rightarrow$

$$\frac{V_p}{\sqrt{g_p L_p}} = 1$$

However $g_p = 1$

a) Time Ratio $T_p = ?$

$$L_p / T_p = V_p = \sqrt{g_p L_p}$$

$$T_p = \frac{L_p}{\sqrt{g_p L_p}}$$

$$T_p = \frac{L_p}{\sqrt{g_p L_p}}$$

$$T_p = \frac{L_p}{\sqrt{g_p L_p}} \quad \text{at } g_p = 1$$

b) Accn Ratio:

$$a_p = g_p = 1$$

c) Force Ratio:

$$F_p = \rho_p L_p^3 \times a_p$$

$$= \rho_p L_p^3$$

Disch

Discharge Ratio:

$$Q_p = \frac{L_p^3}{T_p} = \frac{L_p^3}{\frac{L_p}{\sqrt{g_p L_p}}}$$

$$Q_p = \frac{L_p^{5/2}}{\sqrt{g_p L_p}}$$

$$= \rho_2 L r^{3/2} g_2 \times L r^{1/2} g_2^{1/2}$$

$$= \rho_2 L r^{7/2} g_2^{3/2}$$

344

When compressibility force is significant apart from inertia force than mach number is important and it is defined as

$$M_a = \sqrt{F_i / F_c}$$

$$\therefore M_a = \sqrt{\frac{\rho L^2 v^2}{K \times L^2}}$$

$$M_a = v / c$$

$$= \frac{v}{\sqrt{K/\rho}}$$

$\sqrt{K/\rho} \rightarrow$ velocity of sound (c)

Mach number is significant when velocity is comparable with sound velocity

- at
- (i) $M_a > 1 \rightarrow$ Flow is supersonic
 - (ii) $M_a = 1 \rightarrow$ " " sonic
 - (iii) $M_a \gg 1 \rightarrow$ Flow is hypersonic
 - (iv) $M_a < 1 \rightarrow$ Flow is subsonic/ultrasonic
 - (v) $M_a < 0.3 \Rightarrow$ Effect of compressibility is neglected.
 $M_a < 0.4$

NOTE: Square of Mach number is called Cauchy's No.

$$M_a^2 = \text{Cauchy's No.}$$

$$\frac{v_a^2}{c^2} = "$$

$$= \frac{F_i}{F_c}$$

Mach Law: - mainly in compressible fluid flowing

with sound.

- ex:
- 1) flow of gases having high speed.
 - 2) water hammer problem
 - 3) aerodynamic testing
 - 4) Testing of torpedo's
 - 5) Motion of airplane with high speed
 - 6) Launching & Projectile of missile.

(345)

According to this Law

$$(Ma)_M = (Ma)_P$$

$$V_a/c_a = 1$$

4) Weber Model: ↓

When surface tension force apart from inertia force is important but other forces are less significant than Weber No. is defined as

$$We = \frac{F_i}{F_s}$$

$$We = \frac{\rho L^2 V^2}{\sigma L}$$

$$We = \frac{V}{\sqrt{\sigma/\rho L}}$$

Weber Model Law: ↓

Application

- a) capillary movement of water in soil
- b) flow of bloods in veins & arteries
- c) thin sheet flow
- d) Liquid atomization
- e) capillary tube flow.

According to this Law

$$(We)_P = (We)_M$$

$$\frac{V_P}{\sqrt{\sigma_P/\rho_P L_P}} = 1$$

346

When the pressure force is very important apart from another force as compared to other forces than Euler No. is defined as

$$Eu = \sqrt{F_i / F_p}$$

$$= \sqrt{\frac{\rho L^2 v^2}{p \times L^2}} = \frac{v}{\sqrt{p / \rho}}$$

$\Rightarrow$ $\sqrt{Eu}$ is called Newton Number.

$\Rightarrow$ Newton number square is called pressure coefficient.

$$\sqrt{Eu}^2 = \text{Newton Number} = \frac{F_p}{F_i} = \frac{p}{\rho v^2}$$

Euler Model Law: $\downarrow$

- ex:
- a) Flow through pipes under pressure
 - b) Flow over submerged bodies when pressure is important.
 - c) Pressure rise due to sudden closure of valve.
 - d) Discharge through weir & mouthpieces, under large head

According to this Law

$$(Eu)_m = (Eu)_p$$

NOTE

In some of the cases where viscosity force & gravity force both are important than Reynolds Law & Froude Law both should be applicable.

For ex: $\downarrow$

- a) Resistance to ship $\rightarrow$ gravity caused by viscosity & eddies formed by wave hence both law should be satisfied.

347

$$L_r^{2/3} = 14.49 / R_n$$

Above law satisfies both laws of velocity Franks Reynolds & Manning's Law
 An case of the rivers normally scale adopted for depth is different from scale for width & length therefore slope of bed is exaggerated and according to Manning's Law

$$V_n = \frac{R_n^{2/3} S_n}{n}$$

where

$$S_n =$$

$$\frac{D_n}{L_n}$$

Depth Ratio or

$$\frac{L_n}{L_n}$$

length Ratio /

$$A_n = L_n \times D_n$$

For wide Rivers (Perimeter)

$$P_n = L_n$$

$$R_n = \frac{A_n}{P_n} = \frac{L_n \times D_n}{L_n} = D_n$$

$$V_n = \frac{D_n^{2/3}}{n} \left(\frac{L_n^{1/2}}{L_n^{1/2}} \right)$$

$$V_n = \frac{D_n^{2/3}}{n} \frac{L_n^{1/2}}{L_n^{1/2}}$$

→ In case of the rivers in open channel, Fruid Model Law is applicable.

$$V_n = \frac{D_n^{2/3}}{n} \cdot \frac{g_n^{1/2}}{L_n^{1/2}} \quad \rightarrow 2$$

By eqn (1) & (2)

$$V_n = \frac{D_n^{2/3}}{n} \frac{L_n^{1/2}}{L_n^{1/2}}$$

$$V_n = \frac{D_n^{2/3}}{n} \frac{L_n^{1/2}}{L_n^{1/2}}$$

For dimensionless model

Put $g_n = 1$

348

$$\rightarrow \sin^2 \theta + \cos^2 \theta = 1$$

$$\rightarrow 1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$$

$$\rightarrow \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$\frac{\cos \theta}{\tan \theta}$	$\frac{\cos \theta}{\tan \theta}$
$\frac{\sin \theta}{\cos \theta}$	$\frac{\sin \theta}{\cos \theta}$
$\frac{\cos \theta}{\tan \theta}$	$\frac{\cos \theta}{\tan \theta}$

$$\rightarrow \sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2}$$

$$\rightarrow \sin C - \sin D = 2 \cos \frac{C+D}{2} \sin \frac{C-D}{2}$$

$$\rightarrow \cos C + \cos D = 2 \cos \frac{C+D}{2} \cos \frac{C-D}{2}$$

$$\rightarrow \cos C - \cos D = 2 \sin \frac{C+D}{2} \sin \frac{D-C}{2}$$

$$2 \sin A \sin B = \cos(A-B) - \cos(A+B)$$

$$2 \cos A \cos B = \cos(A+B) + \cos(A-B)$$

$$2 \sin A \cos B = \sin(A+B) + \sin(A-B)$$

$$2 \cos A \sin B = \sin(A+B) - \sin(A-B)$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

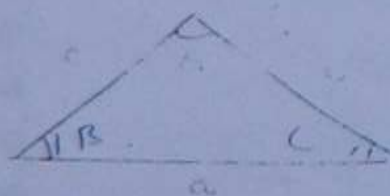
$$= 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

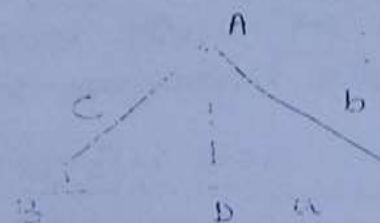
$$\sin 3A = 3 \sin A - 4 \sin^3 A$$

$$\cos 3A = 4 \cos^3 A - 3 \cos A$$

$$\tan 3A = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$$



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

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